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# MATHEMATICS

## Core & Extended for Cambridge IGCSE®

Ian Bettison
Mathew Taylor

Oxford excellence for Cambridge IGCSE®

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# MATHEMATICS

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# Contents

You will find the numerical answers to all questions with worked solutions and commentaries to all Raise your grade exam-style questions available at www.oxfordsecondary.com/esg-for-caie-igcse

Matched to the latest Cambridge assessment criteria, this in-depth Exam Success Guide brings clarity and focus to exam preparation with detailed and practical guidance on raising attainment in IGCSE® Mathematics.

This Exam Success Guide:

- is **fully matched** to the latest Cambridge IGCSE® syllabus
- includes a comprehensive list of **syllabus objectives** at the start of each chapter where you can build a record of your revision as well as **Key skills** features within the chapters to guide you through your revision and **Recap** features to review the key information
- provides exam-style questions at the end of each chapter to equip you to **Raise your grade**. These questions have fully worked solutions with commentaries as part of the online resources
- will guide you through answering exam questions with extensive use of **Worked examples** with **Exam tips**
- will help you to avoid common mistakes with **Watch out!** features.

This Exam Success Guide has been designed to maximise exam potential.

The key features which will help you include:

- **Your revision checklist** table with a list of the objectives covered in each section at the start of each unit to focus your learning, monitor progress and build a record of your revision. Objectives that are for the **extended** syllabus only are shown in **bold**.

| Your revision checklist | | | |
|---|---|---|---|
| Core/**Extended** curriculum | 1 | 2 | 3 |
| 1.1 | | | |
| 1.14 | | | |

- **Recap** features: Use these to review the key information

  ◀◀ **Recap**
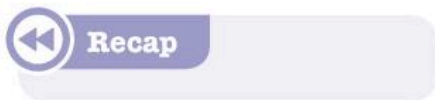
- **Apply:** short activities to help you remember some of the key facts and ideas

  ✏ **Apply**

- **Key skills:** each section contains several key skills that you must master in order to succeed

  🔑 **Key skills**

- **Worked examples:** every section is rich in Worked examples which guide you step-by-step through answering exam-style questions
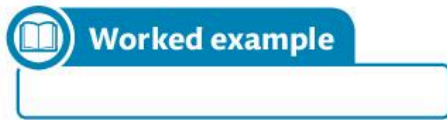
📖 **Worked example**

- **Exam tips:** alongside each Worked example are Exam tips which will help you to answer the question and set out your working correctly

**Exam tip**

- **Watch out!:** these boxes point out key misconceptions and how to avoid them

👁 **Watch out!**
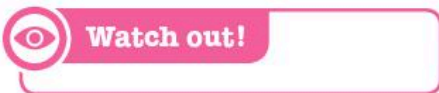
- **Questions:** at the end of each section there are questions to help you practise the key skills

❓ **Questions**

- **Raise your grade:** each chapter has a Raise your grade section at the end with sample exam-style questions for you to attempt.

🌐 You will find the numerical answers to all questions with worked solutions and commentaries to all Raise your grade exam-style questions at **www.oxfordsecondary.com/esg-for-caie-igcse**.
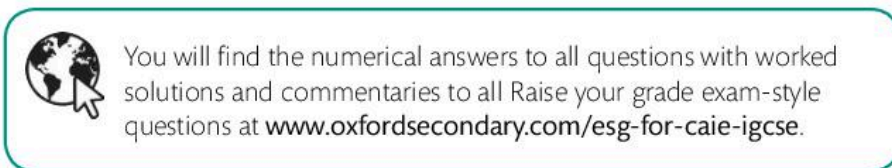
You could also create a revision planner like the one below to plan your revision timetable.

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|--------|---------|-----------|----------|--------|----------|--------|
|        |         |           |          |        |          |        |
|        |         |           |          |        |          |        |
|        |         |           |          |        |          |        |
|        |         |           |          |        |          |        |
|        |         |           |          |        |          |        |
|        |         |           |          |        |          |        |

## 'Extended only' content

This book is intended for use by candidates studying both the Core and Extended syllabuses.

- The syllabus objectives at the start of each chapter indicate **Extended only** content in **bold**.
- Within each chapter, where content is in the Extended syllabus but *not* in the Core syllabus, it is clearly shaded and indicated with a red 'Extended' bar or an 'E' icon.
- Raise Your grade questions for the Extended syllabus only are also clearly shaded and indicated with an 'E' icon.

**2.2**

**Extended**

(a) $x^2 + 7x + 6$    Find two numbers that multiply to give 6 and add up to 7: 6 and 1.

So $x^2 + 7x + 6 = (x + 6)(x + 1)$

(b) $x^2 + 3x - 28$    Find two numbers that multiply to give –28 and add up to 3: 7 and –4.

So $x^2 + 3x - 28 = (x + 7)(x - 4)$

(c) $x^2 - 7x + 12$    Find two numbers that multiply to give 12 and add up to –7: –3 and 4.

So $x^2 - 7x + 12 = (x - 3)(x - 4)$

(d) $x^2 - 2x - 15$    Find two numbers that multiply to give –15 and add up to –2: 3 and –5.

So $x^2 - 2x - 15 = (x + 3)(x - 5)$

**Exam tip**

When you have factorised an expression, expand the expression again if you have time to check that you got it right.

$= x^2 - 5x - 3x + 15 = x^2 - 8x + 15$

(d) $(2x + 3)(5x - 9)$   $= (2x) \times (5x) + (2x) \times (-9) + 3 \times (5x) + 3 \times (-9)$

$= 10x^2 - 18x + 15x - 27 = 10x^2 - 3x - 27$

**E**

**Key skills**

You need to be able to expand three brackets.

**Worked example**

Expand and simplify

$(x - 1)(x + 2)(x + 4)$            **[3 marks]**

$(x - 1)(x + 2)(x + 4)$

$= (x^2 - x + 2x - 2)(x + 4)$

$= (x^2 + x - 2)(x + 4)$

$= x^3 + 4x^2 + x^2 + 4x - 2x - 8$

$= x^3 + 5x^2 + 2x - 8$

Expand and simplify the first pair of brackets.

Multiply the quadratic expression by the third bracket.

**Key skills**

You must be able to factorise algebraic expressions.

**Worked example**

Factorise these expressions.

(a) $9ab + 3b^2$             **[1 mark]**

(b) $9x + 5x + 18y + 10y$      **[1 mark]**

## Aiming for success

This Exam Success Guide will help you to understand what is in the Cambridge IGCSE® Mathematics exam and to improve your key skills to achieve higher grades.

When you take your exam, you will sit two compulsory papers. Work through the chapters in order to help you become more familiar with the demands of each section of the syllabus and to achieve greater success in the exam. Use the full worked solutions and commentaries for each Raise your grade question. These can be found at www.oxfordsecondary.com/esg-for-caie-igcse.

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**Worked example**

7  If a train travels for 50 km at a speed of 40 km h$^{-1}$, then travels a further 60 km at a speed of 30 km h$^{-1}$, find:

(a) The total time taken **[3 marks]**

(b) the average speed at which the train travelled, accurate to 3 significant figures. **[3 marks]**

---

(a)  Speed = distance ÷ time

Time = distance ÷ speed ← Write down the formula for speed and rearrange it

$t_1 = 50 ÷ 40 = 1.25$

$t_2 = 60 ÷ 30 = 2$ ← Calculate the times for the two different parts of the journey

Total time taken = 1.25 + 2 = 3.25 hours ← Calculate the total time

(b)  Average speed = total distance ÷ total time ← Write down the formula for average speed

Total distance = 50 + 60 = 110 km ← Calculate the total distance

Average speed = 110 ÷ 3.25 = 33.8 kmh$^{-1}$

Divide your total distance by your answer to part **a** and round it to 3 significant figures

---

Each question in the Raise your grade section is mapped to the relevant section in the chapter, so that you can go back and revise the key ideas if you get stuck.

# Introduction

The Cambridge IGCSE® Mathematics qualification (syllabus 0580) is examined at either Core tier (expected IGCSE® grades D to G) or at *Extended* tier (expected IGCSE® grades A* to C).

The assessment for each tier is the same. Candidates sit one short-answer paper and one paper with structured questions. The following tables give the details of each paper.

| Core candidates take: | | Extended candidates take: | |
|---|---|---|---|
| **Paper 1 (Core)** | 1 hour 35% | **Paper 2 (Extended)** | 1 hour 30 minutes 35% |
| 56 marks<br>Short-answer questions<br>Questions will be based on the Core curriculum<br>Externally assessed | | 70 marks<br>Short-answer questions<br>Questions will be based on the Extended curriculum<br>Externally assessed | |
| **and:** | | **and:** | |
| **Paper 3 (Core)** | 2 hours 65% | **Paper 4 (Extended)** | 2 hours 30 minutes 65% |
| 104 marks<br>Structured questions<br>Questions will be based on the Core curriculum<br>Externally assessed | | 130 marks<br>Structured questions<br>Questions will be based on the Extended curriculum<br>Externally assessed | |

- Candidates should have a scientific calculator for all papers.
- Three significant figures will be required in answers (or one decimal place for answers in degrees) except where otherwise stated.
- Candidates should use the value of $\pi$ from their calculator or the value of 3.142.

Questions on the Core papers will be based only on the Core curriculum.

Questions on the Extended papers will be based on the Extended curriculum (which include all of the Core syllabus content).

## Advice and guidance

**Before the exams:**

✓ Check the dates of the exams

✓ Plan your revision carefully. Make a timetable that includes each section of the syllabus

✓ Focus on the areas that you have found difficult in the past

✓ Practise skills first, then applications to exam-style questions

✓ Create mind-maps, flash cards or revision posters if you think they will help

✓ Understand the command words in each question

| Command word | What it means |
| --- | --- |
| Calculate | work out from given facts, figures or information, generally using a calculator |
| Construct | make an accurate drawing |
| Describe | state the points of a topic/give characteristics and main features |
| Determine | establish with certainty |
| Explain | set out purposes or reasons/make the relationships between things evident/provide why and/or how and support with relevant evidence |
| Give | produce an answer from a given source or recall/memory |
| Plot | mark point(s) on a graph |
| Show (that) | provide structured evidence that leads to a given result |
| Sketch | make a simple freehand drawing showing the key features |
| Work out | calculate from given facts, figures or information with or without the use of a calculator |
| Write | give an answer in a specific form |
| Write down | give an answer without significant working |

✓ Use mark schemes and worked solutions to assess yourself

✓ Make sure that you have all of your equipment for the exam. You need a scientific calculator, pens, pencils, a ruler, a protractor and a pair of compasses

# How will you be assessed?

**During the exams:**

- ✓ Plan your time. Allow one minute per mark so you have some time at the end to check your answers
- ✓ If you can't do a question, move on quickly and come back to it
- ✓ Read the questions carefully and identify the command word(s)
- ✓ Set out your working clearly so it can be followed by the examiner
- ✓ Make sure that your final answer is clearly indicated
- ✓ Check that you have included the units, if appropriate
- ✓ Check that you have rounded your answer to a suitable or specified degree of accuracy
- ✓ Check that your answer is sensible
- ✓ Do not round intermediate values in working – this may lead to rounding errors in your final answer
- ✓ Make sure that you use correct mathematical terminology when giving reasons
- ✓ Go back and attempt questions you missed out at the end
- ✓ Leave time at the end for checking through all of your answers again

# 1 Number

## Your revision checklist

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 1.1 | Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square and cube numbers, common factors and common multiples, rational and irrational numbers (e.g. $\pi$, $\sqrt{2}$ ), real numbers, reciprocals. | | | |
| 1.2 | Understand notation of Venn diagrams. Definition of sets. **Use language, notation and Venn diagrams to describe sets and represent relationships between sets.** | | | |
| 1.3 | Calculate with squares, square roots, cubes and cube roots and other powers and roots of numbers. | | | |
| 1.4 | Use directed numbers in practical situations. | | | |
| 1.5 | Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. Recognise equivalence and convert between these forms (**including recurring decimals**). | | | |
| 1.6 | Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, >, <, ≥, ≤. | | | |
| 1.7 | Understand the meaning of indices (fractional, negative and zero) and use the rules of indices. Use the standard form $A \times 10^n$ where $n$ is a positive or negative integer, and $1 \leq A < 10$. | | | |
| 1.8 | Use the four rules for calculations with whole numbers, decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets. | | | |
| 1.9 | Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem. | | | |
| 1.10 | Give appropriate upper and lower bounds for data given to a specified accuracy. **Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy.** | | | |
| 1.11 | Demonstrate an understanding of ratio and proportion. **Increase and decrease a quantity by a given ratio.** Calculate average speed. Use common measures of rate. | | | |
| 1.12 | Calculate a given percentage of a quantity. Express one quantity as a percentage of another. Calculate percentage increase or decrease. **Carry out calculations involving reverse percentages.** | | | |
| 1.13 | Use a calculator efficiently. Apply appropriate checks of accuracy. | | | |
| 1.14 | Calculate times in terms of the 24-hour and 12-hour clock. Read clocks, dials and timetables. | | | |
| 1.15 | Calculate using money and convert from one currency to another. | | | |
| 1.16 | Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest. Extract data from tables and charts. | | | |
| 1.17 | **Use exponential growth and decay in relation to population and finance.** | | | |

**You need to:**

- Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square and cube numbers, common factors and common multiples, rational and irrational numbers (e.g. $\pi$, $\sqrt{2}$), real numbers, reciprocals.

**Natural numbers** are the numbers you use to count. So the natural numbers are 1, 2, 3, 4,…

**Integers** are 'whole numbers'. They can be positive or negative (with zero in between). So the integers are the numbers …−3, −2, −1, 0, 1, 2, 3,…
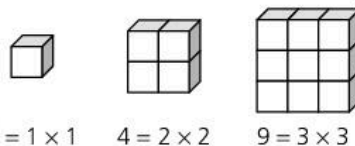
**Positive integers** are the numbers 1, 2, 3, 4,…

**Negative integers** are the numbers −1, −2, −3, −4,…

**Prime numbers** have only two (different) factors (i.e. 1 and itself). So 1 is not a prime number. The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19,… (The number of primes is infinite).

**Square numbers**

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, … So the numbers 1, 4, 9,… are square numbers.

$1 = 1 \times 1$     $4 = 2 \times 2$     $9 = 3 \times 3$

**Cube numbers**

$1^3 = 1$, $2^3 = 8$, $3^3 = 27$, … So the numbers 1, 8, 27,… are cube numbers.

**Apply**

Make a list of:
- the first 15 prime numbers
- the first 15 square numbers
- the first 8 cube numbers.

**Key skills**

You need to be able to write any number as the **product of prime factors**.

**Worked example**

Express 504 as the product of prime factors.          **[2 marks]**

Divide 504 by the smallest possible prime number, in this case 2. Continue until you have only prime numbers in your tree.

```
        504
       /   \
      2    252
          /   \
         2    126
             /   \
            2     63
                 /   \
                3     21
                     /   \
                    3     7
```

Hence $504 = 2^3 \times 3^2 \times 7$

**Exam tip**

Use a factor tree to help you. Give your answer using index notation.
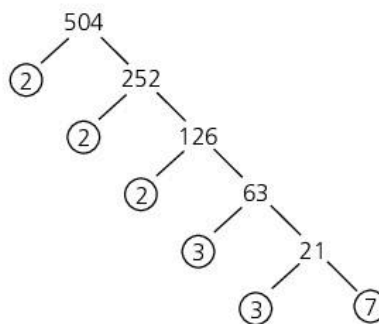
3 is a **common factor** of 9 and 12 since 3 is a factor of both 9 and 12.

30 is a **common multiple** of 6 and 15 since 30 is a multiple of both 6 and 15.

---

**Worked example**

Find the highest common factor and lowest common multiple of 60 and 504.                                    **[4 marks]**

---

$60 = 2^2 \times 3 \times 5$

$504 = 2^3 \times 3^2 \times 7$

The highest common factor is the product of the prime factors *common* to both numbers, in this case, $2^2$ and $3$.

$HCF = 2^2 \times 3 = 12$

The lowest common multiple is the product of the largest power of each prime that appears in either number, in this case, $2^3$, $3^2$, $5$ and $7$.

$LCM = 2^3 \times 3^2 \times 5 \times 7 = 2520$

---

**Exam tip**

Write both numbers as the product of prime factors first. $2^2$ is common to both numbers, but not $2^3$. Likewise, 3 is common to both numbers, but not $3^2$.

A **rational number** is a number which can be expressed in the form $\frac{p}{q}$ where $p$ and $q$ are whole numbers.

- All decimals which recur are rational. For example, $0.\dot{3} = 0.333333...$ is rational since $0.\dot{3} = \frac{1}{3}$
- All decimals which terminate (i.e. which end) are rational. For example, 0.625 is a rational number since 0.625 can be written as $\frac{625}{1000} = \frac{5}{8}$

An **irrational number** is a number which cannot be expressed in the form $\frac{p}{q}$ where $p$ and $q$ are integers. For example, $\pi$, $\sqrt{2}$ and $\sqrt[3]{7}$ are all irrational numbers.

A **real number** is any rational or irrational number that can be represented on a number line.
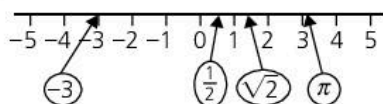


**👁 Watch out!**

$\sqrt{2\frac{1}{4}}$ doesn't look rational but it is since $\sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

**Worked example**

$\sqrt{4}$  $\sqrt{15}$  $\sqrt{25}$  $\sqrt{36}$  $\sqrt{144}$

From the list above, write down:

(a) an odd prime number
(b) two factors of 42
(c) an irrational number
(d) a multiple of 4. **[4 marks]**

(a) $\sqrt{25}$ = **5** which is an odd prime number
(b) $\sqrt{4}$ = **2** and $\sqrt{36}$ = **6**, and both 2 and 6 are factors of 42
(c) $\sqrt{15}$ is an irrational number
(d) $\sqrt{144}$ = **12** which is a multiple of 4

**Exam tip**

Look for square numbers in the list and work out the square roots of these first.

The **reciprocal** of a number is 'one over' the number. For example, the reciprocal of 4 is $\frac{1}{4}$ and the reciprocal of $n$ is $\frac{1}{n}$.

If you are asked to find the reciprocal of a fraction, you just 'flip it over'. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

## Questions

**1** Which of these numbers is **not** a rational number?
$\frac{3}{5}$  $\sqrt{7}$  0.6  $-1\frac{2}{3}$  $\sqrt{25}$

**2** An integer $n$ is such that $80 \leq n < 90$.
Write down a value of $n$ which is:
**a** a multiple of both 3 and 4
**b** a prime number
**c** a factor of 1700.

**3** Find:
**a** a prime number which is a factor of 49
**b** an even prime number.

**4** **a** Write down all the factors of 21.
**b** Write down all the factors of 28.
**c** Hence find the HCF of 21 and 28.

**5** $\sqrt{9}$  $\sqrt{15}$  $\sqrt{49}$  $\sqrt{16}$  $\sqrt{64}$  $\sqrt{121}$
From the list above, write down:
**a** a prime number less than 10
**b** a factor of 22

**c** a power of 2
**d** an irrational number.

**6** Write each number as the product of prime factors (e.g. $72 = 2^3 \times 3^2$).
**a** 30  **b** 24  **c** 18
**d** 28  **e** 105  **f** 64

**7** Use question **6** to find the highest common factor of each pair of numbers.
**a** 30 and 24
**b** 28 and 64
**c** 18 and 105

**8** Use question **6** to find the lowest common multiple of each pair of numbers.
**a** 30 and 64
**b** 24 and 105
**c** 28 and 18

**9** Write down the reciprocal of the following numbers:
**a** 5  **b** $\frac{1}{3}$  **c** $\frac{4}{5}$  **d** $\frac{17}{13}$

**You need to:**

- Understand notation of Venn diagrams.
- Definition of sets.

- Use language, notation and Venn diagrams to describe sets and represent relationships between sets. (Extended)
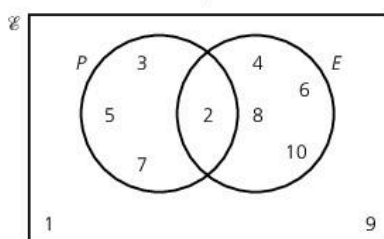
A **set** is a collection of items. These may be numbers, people, letters, etc.

You use 'curly' brackets to represent sets.

For example, the set $P$, of prime numbers less than or equal to 10, can be represented as $P = \{2, 3, 5, 7\}$.

The number of **elements** in set $P$ is denoted by $n(P)$, so in this case $n(P) = 4$.

A **Venn diagram** can be used to represent sets.



In the Venn diagram, $P$ is the set of prime numbers less than or equal to 10 and $E$ is the set of even numbers less than or equal to 10.

The symbol $\mathscr{E}$ is the symbol for the **universal set**. This is the set of all things that are being considered at the time, in this case, the set of integers from 1 to 10.

The element '2' lies inside both circles. This region of the Venn diagram is known as the **intersection** and contains all of the elements that are in **both** sets.

**◄◄ Recap**

The symbol for intersection is $\cap$.

In the Venn diagram above:

$P \cap E = \{2\}$

$n(P \cap E) = 1$

Elements which lie in either $P$ or $E$ or both lie in the **union** of $P$ and $E$.

**◄◄ Recap**

The symbol for union is $\cup$.

In the Venn diagram above:

$P \cup E = \{2, 3, 4, 5, 6, 7, 8, 10\}$

$n(P \cup E) = 8$

Elements that do not lie in $P$ are called **complementary**.

**◄◄ Recap**

The symbol for the complement of set $P$ is $P'$.

In the Venn diagram above:

$P' = \{1, 4, 6, 8, 9, 10\}$

$n(P') = 6$

## Worked example

In a class of 33 students, 20 like chess, 12 like draughts and 5 like neither.

(a) Represent this information in a Venn diagram.  **[3 marks]**
(b) How many students like only one of chess or draughts?

**[1 mark]**

---

**Exam tip**

Draw a Venn diagram with two circles overlapping and add in the 5 students who like neither game outside of both circles.
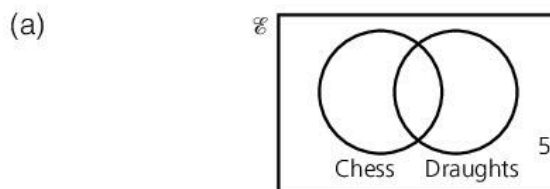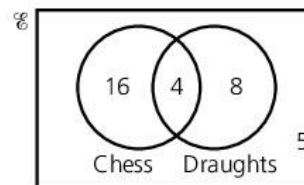
**Exam tip**

Add up the number of students who like chess and who like draughts then subtract the number of students left to find the number of students who like both.

Fill in the 4 in the intersection. $20 - 4 = 16$ students like chess only. $12 - 4 = 8$ students like draughts only.

(a)



There are 28 students left.

There are 20 who like chess and 12 who like draughts.

$20 + 12 - 28 = 4$



(b) The number of students who like only one of chess or draughts is $16 + 8 = 24$

---

## Extended

### Recap

You need to know several other symbols and notations for sets.

$\in$ means '… is an element of…'

$\notin$ means '… is not an element of…'

$\varnothing$ means the empty set or { }

$A \subseteq B$ means 'A is a subset of B' (and A *can* equal B)

$A \nsubseteq B$ means 'A is not a subset of B'

$A \subset B$ means 'A is a proper subset of B' (and A *cannot* equal B)

$A \not\subset B$ means 'A is not a proper subset of B'

### Apply

$A = \{$Integers from 1 to 10$\}$
$B = \{$Odd numbers between 0 and 20$\}$
$C = \{2, 4, 6, 8, 10\}$

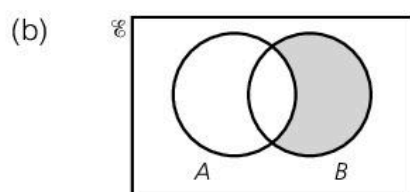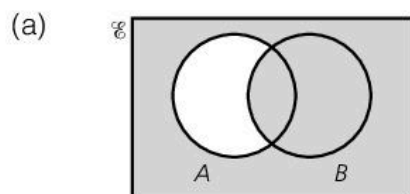Which of the following statements are true and which are false?

- $2 \in C$
- $4 \in B$
- $8 \notin A$
- $14 \notin B$
- $A \subseteq B$
- $C \subset A$
- $B \cap C = \varnothing$

**Worked example**

Draw Venn diagrams and shade the region(s) that represent:
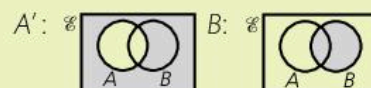
(a) $A' \cup B$

(b) $A' \cap B$ **[2 marks]**

(a) 

(b) 

🔑 **Key skills**

You need to be able to shade Venn diagrams to represent different situations.

**Exam tip**

Imagine shading the region $A'$ and the region $B$ separately.

$A'$:  $B$: 

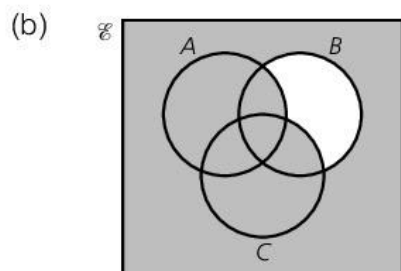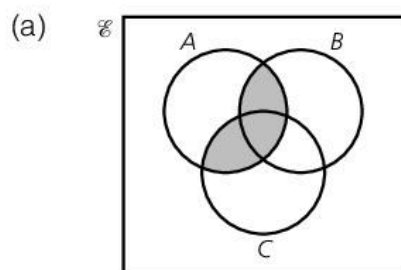The union is any region that is shaded in either diagram.

The intersection is any region that is shaded in *both* of the separate diagrams.

**Worked example**

Draw Venn diagrams and shade the region(s) that represent:

(a) $(B \cup C) \cap A$

(b) $(A \cup C) \cup B'$ **[2 marks]**

(a) 

(b) 

🔑 **Key skills**

You need to be able to work with Venn diagrams containing three circles.

**Exam tip**

Look for the regions that lie in $B$ or $C$ and **also** in $A$.

**Exam tip**

The only region **not** shaded is the one that lies in $B$, but not in $A$ or $C$.

**? Questions**

1  In a group of 100 students, 70 enjoy Maths, 50 enjoy French and 20 enjoy neither.
   a   Draw a Venn diagram showing this information.
   b   Use your diagram to find the number of students who enjoy both subjects.

2  On an athletics day 150 athletes take part. 60 are in the 100 metres race, 50 are in the 200 metres race and 80 are in neither.
   a   Draw a Venn diagram showing this information.
   b   Use the diagram to find the number of athletes who ran in only one race.

3  In a shop there were 120 customers on a certain day. Of these, 60 paid using notes, 30 paid using coins and 50 paid using cards. There were no customers who paid using both cards and cash.
   a   Draw a Venn diagram showing this information.
   b   Use your diagram to find the number of customers who used both notes and coins.

4  Describe the shaded regions:

   a

   b

   c

   d
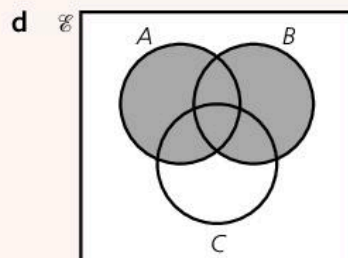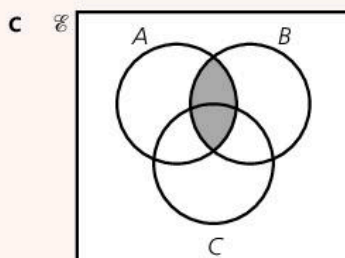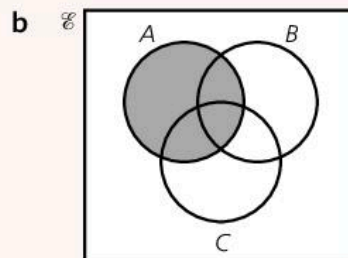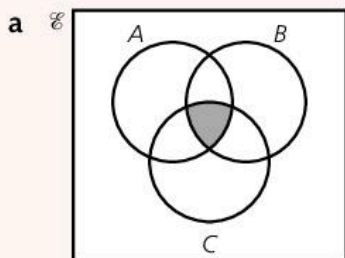
## You need to:

- Calculate with squares, square roots, cubes and cube roots and other powers and roots of numbers.

The **square** of a number $n$ is $n \times n = n^2$, so the square of 5 is $5^2 = 25$

[5] [$x^2$] [=] on some calculators

The **cube** of a number $n$ is $n \times n \times n = n^3$, so the cube of 2 is $2^3 = 8$

[2] [$x^3$] [=] on some calculators

$25 = 5^2$
5 is the **square root** of 25. The square root of $n$ is represented by $\sqrt{n}$. So $\sqrt{25} = 5$

[$\sqrt{x}$] [2] [5] [=] on some calculators

$8 = 2^3$
2 is the **cube root** of 8. The cube root of $n$ is represented by $\sqrt[3]{n}$. So $\sqrt[3]{8} = 2$

SHIFT [$\sqrt[3]{x}$] [8] [=] on some calculators

> **Apply**
>
> Investigate how to enter powers and roots on your scientific calculator.

> **Recap**
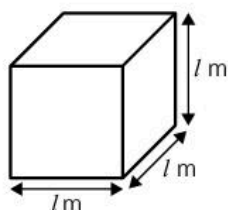>
> $2^4 = 16$ so $\sqrt[4]{16} = 2$
>
> $3^5 = 243$ so $\sqrt[5]{243} = 3$

> **Key skills**
>
> You need to be able to work with other powers and roots, for example powers of 4 and fourth roots.

---

**Worked example**

A cube of side $l$ metres has a volume of 30 cubic metres.



$l$ m
$l$ m
$l$ m

(a) Calculate the value of $l$.
(b) Calculate the area of one face of the cube. **[3 marks]**

---

(a) $l = \sqrt[3]{30} = 3.107...$
     $= 3.11$ metres (to 3 s.f.)

(b) $(3.107...)^2 = 9.654...$
     $= 9.65$ square metres (to 3 s.f.)

> **Exam tip**
>
> Use your calculator to find the cube root of 30 since $l^3 = 30$.
>
> In part (b), use the *exact* answer from part (a) rather than the rounded answer. This ensures no rounding errors are carried forwards.
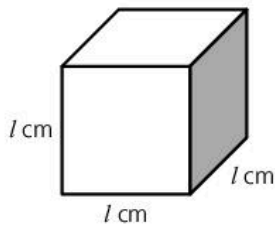
**Worked example**

Use your calculator to work out $6^4 \div \sqrt[6]{1000}$. Round your answer to two decimal places. **[2 marks]**

$6^4 \div \sqrt[6]{1000} = 409.831\ldots$

$= 409.83$ (to 2 d.p.)

**? Questions**

1 Find $l$ (to 3 s.f.) given that the volume of the cube is 50 cm³.



$l$ cm
$l$ cm
$l$ cm

2 Calculate the value of each expression (to 3 s.f. where necessary):

   **a** $8^2$  **b** $11^3$  **c** $7.1^2$

   **d** $\sqrt{17}$  **e** $\sqrt[3]{-100}$  **f** $\sqrt[3]{6859}$

3 Find the side length of a square whose area is 49 cm².

4 Find the side length of a cube whose volume is 216 cm³.

5 Write the following in ascending order (smallest first):

   **a** $0.6^3$  $\sqrt[3]{0.6}$  $\sqrt{0.6}$  $0.6$  $0.6^2$

   **b** $6^3$  $\sqrt[3]{6}$  $\sqrt{6}$  $6$  $6^2$

6 Calculate the value of each expression, rounding to 3 s.f. where appropriate.

   **a** $3^2 \times \sqrt[4]{16}$

   **b** $2^5 \div \sqrt[3]{8}$

   **c** $5^4 \times \sqrt{7}$

   **d** $4^6 \div \sqrt[5]{20}$

**You need to:**

• **Use directed numbers in practical situations.**

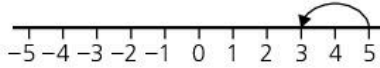To add a positive number, or subtract a negative number, move to the right on a number line.

$(-4) + 6 = 2$
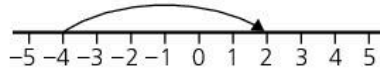
To subtract a positive number, or add a negative number, move to the left on a number line.

$5 + (-2) = 5 - 2 = 3$

Multiplying or dividing two negative numbers or two positive numbers gives a positive number.

Multiplying or dividing a negative number and a positive number gives a negative number.

$(-4) \times (-2) = 8$          $3 \times (-5) = -15$

$(-10) \div (-2) = 5$          $(-6) \div 3 = -2$

**Watch out!**

You can remember the rule 'two negatives make a positive' when multiplying or dividing.

When adding or subtracting, the same rule does not work so use the number line method.

**Worked example**

In June, the average temperature in Moscow was 14°C.

In January, the average temperature was 25°C lower than in June.

What was the average temperature in Moscow in January?

**[2 marks]**

$14 - 25 = -11°C$

**Key skills**

You need to be able to use **directed numbers** in practical situations.

**Exam tip**

Sketch a number line if necessary and remember to move 25 places to the left.

**Questions**

1   The average temperature each month in Montreal is shown in the table.

| Month | Jan | Feb | Mar | Apr | May | Jun |
|---|---|---|---|---|---|---|
| Avg Temp | −11 | −9 | −3 | 5 | 13 | 18 |

| | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|
| | 21 | 19 | 15 | 8 | 1 | −7 |

   a   Find the difference between the highest and lowest average monthly temperatures.
   b   The average minimum temperature for December is usually 5° lower than the average December temperature. Find the average minimum temperature for December.

2   Calculate the following:
   a   $17 + (-3)$     b   $(-4) \times (-2)$     c   $(-7) - (-1)$
   d   $15 - (-4)$     e   $(-18) \div (-3)$     f   $(-40) \div 10$

3   A man's bank balance is −£345.20. He then withdraws £50. What is his new balance?

4   The water on a gauge is at −20 cm, that is 20 cm below the flood level. If the water rises by 25 cm what level does the gauge now show?

## You need to:

- Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts.
- Recognise equivalence and convert between these forms (including recurring decimals). (Extended)

If a cake is split into five equal pieces and Joe eats one of the five pieces then the amount he eats can be expressed:

as a **fraction**, that is $\frac{1}{5}$ of the whole cake

as a **decimal**, that is 0.2 of the whole cake

as a **percentage**, that is 20% of the whole cake.

these three expressions are equivalent

---

### 🔑 Key skills

You need to be able to simplify fractions.

### Exam tip

Divide both the numerator and denominator by the highest common factor of 24 and 30, 6.

---

### Worked example

Write $\frac{24}{30}$ in simplest form. **[2 marks]**

---

$$\frac{24}{30} = \frac{24 \div 6}{30 \div 6}$$

$$= \frac{4}{5}$$

---

### 🔑 Key skills

You need to be able to convert between fractions, decimals and percentages.

### Exam tip

Multiplying by 100% does not change the value of the number since 100% = 1.

---

### Worked example

(a) Express 0.15 as:
   (i) a fraction
   (ii) a percentage. **[3 marks]**

(b) Express $\frac{2}{5}$ as a decimal. **[1 mark]**

---

(a) (i) $0.15 = \frac{15}{100} = \frac{3}{20}$

   (ii) $0.15 \times 100\% = 15\%$

(b) $\frac{2}{5} = \frac{4}{10} = 0.4$

### Recap

To convert a recurring decimal to a fraction, multiply the recurring decimal by the power of 10 equal to the number of places of recursion. For example, for 2 place recursion, multiply by 100.

### Worked example

Convert $0.\dot{1}\dot{2}$ into a fraction in its simplest form. **[3 marks]**

Let $x = 0.1212...$

$100x = 12.1212...$

$100x - x = 12.1212... - 0.1212...$

$99x = 12$

$x = \dfrac{12}{99} = \dfrac{4}{33}$

### Key skills

You need to be able to convert from a recurring decimal to a fraction in simplest form.

### Exam tip

Multiply $x$ by 100 since it is 2 place recursion.

Subtract $x$ from $100x$ and then simplify the resulting fraction if possible.

### Questions

1 Simplify the following fractions.
a $\dfrac{10}{15}$   b $\dfrac{18}{20}$
c $\dfrac{35}{40}$   d $\dfrac{120}{150}$

2 Express 0.375 as:
a a fraction
b a percentage.

3 Express $\dfrac{3}{25}$ as:
a a decimal
b a percentage.

4 Express 55% as:
a a decimal
b a fraction.

5 0.037   37%   0.307   $\dfrac{3}{10}$   $\dfrac{3}{100}$   3.7%
From the numbers listed above, write down:
a the smallest number
b the largest number
c the two numbers which are equal.

6 Convert the following recurring decimals to fractions. **E**
a $0.\dot{2}$
b $0\dot{6}\dot{1}$
c $0.\dot{1}3\dot{5}$
d $0.\dot{6}1\dot{6}$

## You need to:

- Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, >, <, ≥, ≤.

---

### Worked example

Place these numbers in ascending order:

$\dfrac{34}{50}$  $\dfrac{2}{3}$  0.66  67%  $\dfrac{13}{20}$ **[2 marks]**

---

$\dfrac{34}{50} = 0.68$  $\dfrac{2}{3} = 0.\dot{6}$  0.66  67% = 0.67  $\dfrac{13}{20} = 0.65$

So in ascending order they are:

$\dfrac{13}{20}$  0.66  $\dfrac{2}{3}$  67%  $\dfrac{34}{50}$

### Worked example

$a = 5$
$b = -4$

Choose one of the symbols =, < or > to complete each of these statements:

(a) $a + b \ldots b + a$
(b) $a^2 \ldots b^2$
(c) $a - b \ldots b - a$ **[3 marks]**

---

(a) $5 + (-4) = 1$ and $(-4) + 5 = 1$
   so $a + b = b + a$

(b) $5^2 = 25$ and $(-4)^2 = 16$
   so $a^2 > b^2$

(c) $5 - (-4) = 9$ and $(-4) - 5 = -9$
   so $a - b > b - a$

---

### ❓ Questions

1 Write these numbers in ascending order:
$\dfrac{41}{50}$  0.8  $\dfrac{21}{25}$  81%  0.85

2 If $x = 9$ and $y = 11$ write the correct sign, <, = or >, in these expressions:
   a $x \ldots y$
   b $y - x \ldots x - y$
   c $12x \ldots y^2$

**You need to:**

- Understand the meaning of indices (fractional, negative and zero) and use the rules of indices.
- Use the standard form $A \times 10^n$ where $n$ is a positive or negative integer, and $1 \le A < 10$.

When you write $3 \times 3 \times 3 \times 3$ as $3^4$ you are using **index notation**.

The number '3' is referred to as the **base** and the little number '4' is the **index**.

To multiply powers of the same base, **add** the indices.

$$9^3 \times 9^4 = 9^7$$

To divide powers of the same base, **subtract** the indices.

$$2^9 \div 2^4 = 2^5$$

To find a power of a power, **multiply** the indices.

$$(5^3)^4 = 5^{12}$$

### 🔑 Key skills

You need to be able to use the rules of indices to simplify expressions.

### ⏪ Recap

The three laws of indices can be written in a more general form.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$

### 👁 Watch out!

Remember to **subtract** the indices when dividing; a common mistake is to divide them as well.

### ✏️ Apply

Match the expressions that have the same value. Do not use a calculator.

$$5^6 \quad 5^2 \quad 5^4 \times 5^2 \quad 5^8 \quad 5 \times 5^2 \quad \frac{5^5}{5^3} \quad \frac{5 \times 5^8}{5^3} \quad 5^5 \times 5^3$$

$$(5^4)^2 \quad 25^3 \quad 5^{11} \div 5^8$$

### Worked example

Express these in the form $a^n$, where $a$ is an integer:

(a) $(11^3)^2 \times (11^2)^5$

(b) $\dfrac{(7^3)^4 \times 7^8}{(7^2)^5}$ **[4 marks]**

(a) $(11^3)^2 \times (11^2)^5 = 11^6 \times 11^{10}$

$$= 11^{16}$$

(b) $\dfrac{(7^3)^4 \times 7^8}{(7^2)^5} = \dfrac{7^{12} \times 7^8}{7^{10}}$

$$= \dfrac{7^{20}}{7^{10}}$$

$$= 7^{10}$$

### Exam tip

Make sure you set out your working clearly and apply each rule in turn.

🔑 **Key skills**

You need to understand and work with the zero index, negative indices and fractional indices.

- $a^0 = 1$
- $a^{-1} = \dfrac{1}{a}$
- In general, $a^{-n} = \dfrac{1}{a^n}$
- $a^{\frac{1}{2}} = \sqrt{a}$
- In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$
- In general, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

---

**Worked example**

Calculate:

(a)  $2^5$                                                                    **[1 mark]**

(b)  $3^{-2}$                                                                **[1 mark]**

(c)  $5^0$                                                                    **[1 mark]**

---

(a)  $2 \times 2 \times 2 \times 2 \times 2 = 32$

(b)  $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

(c)  $5^0 = 1$

---

**Exam tip**

Remember that $a^{-b} = \dfrac{1}{a^b}$, and that $a^0 = 1$, as long as $a \neq 0$.

---

**Worked example**

Evaluate these numbers:

(a) $8^{\frac{4}{3}}$

(b) $25^{-\frac{1}{2}}$

(c) $\left(\sqrt{\dfrac{3}{2}}\right)^4$                                          **[6 marks]**

---

(a) $8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4$
    $= 2^4$
    $= 16$

(b) $25^{-\frac{1}{2}} = \dfrac{1}{\sqrt{25}}$
    $= \dfrac{1}{5}$

(c) $\left(\sqrt{\dfrac{3}{2}}\right)^4 = \left(\left(\dfrac{3}{2}\right)^{\frac{1}{2}}\right)^4$
    $= \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

---

**Exam tip**

In part (a), apply the cube root first.

**Exam tip**

In part (b), remember the negative in the power is 'one over'.

**Exam tip**

In part (c), convert the square root to a power of a half and use the power of a power rule.

**Worked example**

If $\sqrt{128} = 2^k$, find the value of $k$.                           **[2 marks]**

---

$128 = 2^7$

$\sqrt{2^7} = \left(2^7\right)^{\frac{1}{2}} = 2^{\frac{7}{2}}$

Hence $k = \dfrac{7}{2}$

✏️ **Apply**

You should learn the first eight powers of 2; try listing them now.

**Exam tip**

Use the rules of indices and make sure you actually answer the question by stating the value of $k$.

The mass of the Earth is about 5 974 200 000 000 000 000 000 000 kg, a very large number. The time taken for light to travel 1 km is about 0.000 003 335 56 seconds, a very small number. Standard form is useful when writing very large and very small numbers.

To write a number in standard form, express it as a number between 1 and 10 multiplied by the appropriate power of 10.

🔑 **Key skills**

You need to be able to convert numbers into **standard form**.

$$A \times 10^n$$

$A$ is a number between 1 and 10 ($1 < A < 10$)

$n$ is a whole number, positive for large numbers, negative for small numbers.
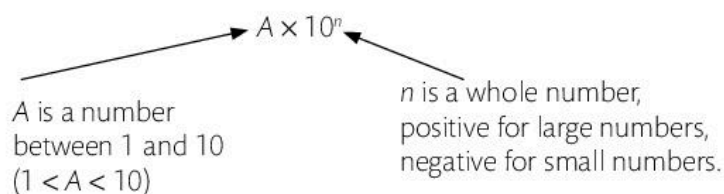
So the mass of the Earth $= 5.9742 \times 10^{24}$ kg

The time taken for light to travel 1 km $= 3.335\,56 \times 10^{-6}$ seconds

⏪ **Recap**

To convert large numbers into standard form, move the digits to the right through the decimal point until only one non-zero digit remains. Count the number of places the digits have moved; this is $n$.

To convert small numbers into standard form, move the digits to the left through the decimal point until only one non-zero digit remains. Count the number of places the digits have moved; this is $n$, but remember $n$ is *negative*.

**Worked example**

Write these numbers in standard form:

(a) 3 723 000

(b) 0.000000312                                     **[2 marks]**

---

(a) $3.723 \times 10^6$

(b) $3.12 \times 10^{-7}$

**Exam tip**

Move the digits six places to the right so the decimal point comes after the first 3.

**Exam tip**

Move the digits seven places to the left so the decimal point comes after the 3.

You need to be able to calculate with numbers in standard form.

## Worked example

The speed of light is $3 \times 10^9$ ms$^{-1}$.

The distance from Earth to the moon is $3.844 \times 10^5$ km.

Calculate, to three significant figures, the time it takes light from the moon to reach the Earth. **[3 marks]**

Speed = distance ÷ time, so time = distance ÷ speed

$$\frac{3.844 \times 10^8}{3 \times 10^9} = 0.128 \text{ seconds (to 3 s.f.)}$$

### Exam tip

Convert $3.844 \times 10^5$ km into metres first by multiplying it by 1000 or $10^3$.

It is usually fine to give your answer as a normal number or in standard form, but read the question carefully to check.

✏️ **Apply**

Investigate how you can enter numbers in standard form into your scientific calculator.

❓ **Questions**

1  Calculate the values of these expressions:

a  $2^6$        b  $3^4$        c  $5^3$

d  $11^2$       e  $2^{-3}$      f  $10^{-2}$

g  $19^0$       h  $13^2$       i  $4^{-3}$

j  $\left(\frac{1}{2}\right)^3$    k  $\left(\frac{2}{3}\right)^2$    l  $\left(\frac{5}{3}\right)^4$

m  $\left(\frac{1}{2}\right)^{-2}$   n  $\left(\frac{2}{5}\right)^{-3}$   o  $\left(\frac{2}{3}\right)^{-4}$

2  Find $x$ when:

a  $32^x = 2$      b  $81^x = 3$      c  $125^x = 5$

d  $49^x = 7$      e  $121^x = 11$    f  $27^x = 3$

g  $243^x = 3$     h  $256^x = 16$    i  $3^x = \frac{1}{3}$

j  $81^x = \frac{1}{3}$   k  $125^x = \frac{1}{5}$   l  $512^x = \frac{1}{2}$

3  Evaluate these expressions without using decimals. Show all working clearly.

a  $3^{-4}$       b  $5^{-3}$       c  $7^{-2}$

d  $8^{\frac{1}{3}}$       e  $16^{\frac{3}{4}}$       f  $25^{\frac{3}{2}}$

g  $\left(\frac{3}{4}\right)^2$    h  $\left(\frac{27}{64}\right)^{\frac{1}{3}}$    i  $\left(\frac{16}{625}\right)^{\frac{1}{4}}$

4  The distance from London to Beirut is approximately 3460 km. Express this number in standard form.

5  The distance from Auckland to Rio de Janeiro is approximately 12 260 km. Express this number in standard form.

6  The radius of the Earth is 6 378 100 m. Express this number in standard form.

7  The speed of light is given as $2.998 \times 10^8$ ms$^{-1}$. Express this as an ordinary number.

8  Express these numbers in decimal form (to 3 s.f.):

a  $0.2^6$      b  $0.3^{15}$

c  $0.15^7$     d  $0.22^8$

9  Give the values of these expressions in standard form (to 3 s.f.):

a  $2^{30}$      b  $3^{20}$      c  $\left(\frac{1}{2}\right)^6$

d  $\left(\frac{1}{3}\right)^3$    e  $\sqrt{0.005}$    f  $\left(\frac{1}{11}\right)^2$

g  $5^{13}$      h  $\sqrt{0.007}$    i  $\left(\frac{3}{4}\right)^{10}$

10  Give your answers to these calculations in standard form (to 3 s.f. where necessary):

a  $(3.4 \times 10^7) \times (4.2 \times 10^5)$

b  $(2.9 \times 10^{15}) \times (2.1 \times 10^7)$

c  $(7.2 \times 10^4) \times (1.3 \times 10^{-1})$

d  $(3.91 \times 10^{-5}) \div (2.35 \times 10^{-7})$

e  $(9.21 \times 10^7) \div (2.31 \times 10^{-5})$

f  $(1.21 \times 10^{-5}) \div (1.24 \times 10^9)$

**11** The population of the world at the end of 1995 was $5.2 \times 10^9$ people.

   **a** The population was projected to grow by 4% in 1996. Calculate the projected population at the end of 1996, giving your answer in standard form (to 2 s.f.).

   **b** In fact, the population at the end of 1996 was $5.5 \times 10^9$. What was the percentage increase (to 2 s.f.) in the population over 1996?

   **c** The projected population at the end of 2020 is $1.8 \times 10^{10}$. How many more people is this than at the end of 1995? (Give your answer in standard form to 2 s.f.)

**12** The density of water is $1 \times 10^3$ kg m$^{-3}$. Find the following (all in standard form):

   **a** the mass of water (in kg) in a cuboid measuring 2 m by 3 m by 5 m

   **b** the volume (in m$^3$) of water whose mass is $5 \times 10^8$ tonnes (one tonne is 1000 kg)

   **c** the volume (in cm$^3$) of 1 m$^3$ of water

   **d** the mass (in g) of 1 m$^3$ of water

   **e** the density of water in g cm$^{-3}$

   **f** the mass of water (in g) in a cuboid measuring 6 cm by 3 cm by 10 cm.

**13** The population of a certain country is $5.7 \times 10^8$ and its area is $7.21 \times 10^4$ km$^2$. Find the population density (people per m$^2$) of this country in standard form (to 3 s.f.).

**14** The diameter of the Earth is $1.3 \times 10^7$ m. Assuming that the Earth is a perfect sphere, find its circumference in km. Write the answer in standard form (to 2 s.f.).

**15** The adult population of a country is 60 million. The average annual income per adult is $43,000. Find in standard form the total annual income from the adult population.

**You need to:**
- Use the four rules for calculations with whole numbers, decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets.

⏪ **Recap**

When adding or subtracting whole numbers or decimals, make sure you line up the digits according to their **place value**.

$3647 + 451$:

$$\begin{array}{r} \overset{1}{3647} \\ +\ 451 \\ \hline 4098 \end{array}$$

$67.3 - 15.61$:

$$\begin{array}{r} \overset{6^1\ 2^1}{6\cancel{7}.\cancel{3}0} \\ -\ 15.61 \\ \hline 51.69 \end{array}$$

Check that you carry or borrow correctly.

⏪ **Recap**

When multiplying whole numbers, set out your working carefully as a column multiplication.

$241 \times 72$:

$$\begin{array}{r} 241 \\ \times\ 72 \\ \hline 482 \\ 16870 \\ \hline 17352 \end{array}$$

Make sure to add a '0' when you multiply by the 10s digit.

⏪ **Recap**

When multiplying decimals, ignore the decimal points in your long multiplication and then count the number of digits after the decimal points to give the correct final answer.

$13.2 \times 1.4$:

$$\begin{array}{r} 132 \\ \times\ 14 \\ \hline 528 \\ 1320 \\ \hline 1848 \end{array}$$

Hence $13.2 \times 1.4 = 18.48$ (two digits after the decimal point)

⏪ **Recap**

Division of whole numbers and decimals can be done using the 'bus stop' method.

$1446 \div 6$: $\quad 6\overline{)14^24^26}\ \ ^{241}$

$16.5 \div 11$: $\quad 11\overline{)16.^55}\ \ ^{1.5}$

**Worked example**

Calculate:

(a) $\dfrac{3}{4}+\dfrac{1}{5}$

(b) $2\dfrac{1}{2}-1\dfrac{2}{7}$

(c) $2\dfrac{2}{3}\times 4\dfrac{1}{2}$

(d) $\dfrac{6}{7}\div\dfrac{2}{5}$  **[4 marks]**

(a) $\dfrac{3}{4}+\dfrac{1}{5}=\dfrac{15}{20}+\dfrac{4}{20}=\dfrac{19}{20}$

(b) $2\dfrac{1}{2}-1\dfrac{2}{7}=\dfrac{5}{2}-\dfrac{9}{7}=\dfrac{35}{14}-\dfrac{18}{14}=\dfrac{17}{14}=1\dfrac{3}{14}$

(c) $2\dfrac{2}{3}\times 4\dfrac{1}{2}=\dfrac{8}{3}\times\dfrac{9}{2}=\dfrac{72}{6}=12$

(d) $\dfrac{6}{7}\div\dfrac{2}{5}=\dfrac{6}{7}\times\dfrac{5}{2}=\dfrac{30}{14}=\dfrac{15}{7}=2\dfrac{1}{7}$

**Recap**

Arithmetic with fractions:

- When adding and subtracting fractions, use a common denominator.
- When multiplying fractions, multiply the numerators and multiply the denominators.
- When dividing fractions, 'flip' the second one over and multiply.
- If you have a mixed number, write the fraction as an **improper fraction** first.

**Exam tip**

Remember to use the correct rules for arithmetic with fractions.

**B**rackets – for example $(3+4)$
**I**ndices – for example $2^3$ or $\sqrt{3}$
**D**ivision – for example $8\div 2$
**M**ultiplication – for example $3\times 4$
**A**ddition – for example $5+2$
**S**ubtraction – for example $7-2$

→ **BIDMAS**

**Key skills**

You need to be able to apply the correct order of operations when carrying out multi-step calculations.

**Worked example**

Calculate $(3+4)^2\times 3+(5+7)\div 6-1$  **[3 marks]**

Using the B of BIDMAS gives $(3+4)^2\times 3+(5+7)\div 6-1$
$=7^2\times 3+12\div 6-1$

Using the I of BIDMAS gives $7^2\times 3+12\div 6-1$
$=49\times 3+12\div 6-1$

Using the D of BIDMAS gives $49\times 3+12\div 6-1$
$=49\times 3+2-1$

Using the M of BIDMAS gives $49\times 3+2-1=147+2-1$

Using the A of BIDMAS gives $147+2-1=149-1$

Using the S of BIDMAS gives $149-1=148$

**Exam tip**

Show each step of working since there will be method marks available for certain key steps.

**?** **Questions**

**1** Calculate:

    **a** $3752 + 681$

    **b** $1563 - 789$

    **c** $461 \times 63$

    **d** $686 \div 7$

**2** Calculate:

    **a** $14.78 + 11.41$

    **b** $19.31 - 7.9$

    **c** $1.23 \times 7.81$

    **d** $97.2 \div 12$

**3** A dowelling rod has length 150 cm. It is cut into pieces of length $1\frac{1}{4}$ cm. How many such pieces can be cut from the original rod?

**4** A bottle of orange juice holds $2\frac{1}{2}$ litres of water. A glass holds $\frac{1}{8}$ litre. How many glasses can be filled from one bottle of orange juice?

**5** **a** Express 0.375 as a fraction.

    **b** Express $\frac{3}{25}$ as a decimal.

**6** Calculate the following:

    **a** $3\frac{1}{2} \times 5\frac{1}{3}$

    **b** $7\frac{3}{4} + 6\frac{2}{5}$

    **c** $8\frac{5}{6} - 3\frac{3}{8}$

    **d** $4\frac{1}{5} \div 2\frac{3}{7}$

**7** Evaluate the following expressions:

    **a** $(6 - 2) \times 7^2$

    **b** $8 \times 2 - (4 - 1)^2$

    **c** $(2^2 + 1) \div 5^2$

    **d** $(4 + 7) \times (6^2 - 7^2)^2$

**8** Add one set of brackets to each calculation to make it correct:

    **a** $4 + 15 \div 5 - 2 \times 5 = 29$

    **b** $3 + 2^2 \times 3 = 15$

    **c** $5 + 3 \times 2 + 7 = 32$

**You need to:**
- Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem.

## ⏪ Recap

When rounding to a given number of **decimal places (d.p.)**, the rounded number must have exactly that number of digits after the decimal point.

| 13.565132 | rounded to 1 d.p. is 13.6 |
| | rounded to 2 d.p. is 13.57 |

When rounding to a given number of **significant figures (s.f.)**, the first non-zero digit is the first significant figure.

| 13.565132 | rounded to 1 s.f. is 10 |
| | rounded to 5 s.f. is 13.565 |
| 0.004615 | rounded to 1 s.f. is 0.005 |
| | rounded to 3 s.f. is 0.00462 |

### 👁 Watch out!

Remember that 5 rounds *up*.

### 👁 Watch out!

Remember to include the units digit; 13 lies between 10 and 20 and closer to 10. Do not just write '1'.

### 🔑 Key skills

You need to be able to estimate the answer to a calculation by rounding.

### Worked example

Find an approximate answer to $19.79 - 2.31 \times 3.15$ by rounding each number to 1 significant figure. **[2 marks]**

19.79 rounds to 20 (to 1 s.f.)

2.31 rounds to 2 (to 1 s.f.)

3.15 rounds to 3 (to 1 s.f.)

Hence the calculation becomes $20 - 2 \times 3 = 14$

### Exam tip

Write each number as a 1 s.f. approximation first.

Don't forget the rules of BIDMAS when working out your final answer.

## ❓ Questions

**1 a** Estimate the values of these expressions by rounding each number to 1 s.f:

  i   $$\frac{4.1 + 3.9 \times 2.1}{3.2}$$

  ii  $$\frac{9.1 \times 8.1 + 3.8 \times 7.2}{10.1}$$

**b** Calculate the values of the expressions in part **a**, giving your answers to 3 s.f.

**2 a** Find an estimate of the area of a circle of radius 9.97 cm.

**b** Calculate (to 3 s.f.) the area of a circle of radius 9.97 cm.

**3 a** Without using your calculator, and showing all your working, estimate (to 1 s.f.) the answer to this calculation:

$$\frac{2104.3 - (9.81)^2}{0.096}$$

**b** Using your calculator, find the answer to the calculation in part **a** to 3 s.f.

**You need to:**

- Give appropriate upper and lower bounds for data given to a specified accuracy.

- Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy. **(Extended)**

---

🔑 **Key skills**

You need to be able to give upper and lower bounds for rounded numbers.

👁 **Watch out!**

5.35 rounds up to 5.4 but you state the upper bound is 5.35 and use the 'less than' inequality. Never give the upper bound as 5.349 or worse still as 5.34.

**Exam tip**

Give your answers using inequalities.

---

If the length of a rope is given as 5.3 m (to 2 s.f.) then you can calculate the lower and upper bounds for the length of the rope.

To calculate the lower and upper bounds, think of the two numbers (to 2 s.f.) immediately below and above 5.3. These are 5.2 and 5.4.



The lower bound is halfway between 5.2 and 5.3. So 5.25 is the lower bound. The upper bound is halfway between 5.3 and 5.4. So 5.35 is the upper bound.

Bounds can be given using inequalities:

5.25 m $\leq$ length of rope < 5.35 m

---

**Worked example**

Find lower and upper bounds for the following:

(a) The perimeter of a circle given as 15 cm (to 2 s.f.)

(b) The population of a city given as 340 000, correct to the nearest ten thousand **[2 marks]**

---

(a) 14.5 cm $\leq$ perimeter < 15.5 cm

(b) 335 000 $\leq$ population < 345 000

---

**Extended**

🔑 **Key skills**

You need to be able to calculate bounds for simple problems involving rounded values.

**Exam tip**

Write out the bounds of the numbers you are given. You will get a mark for this in the exam.

---

**Worked example**

The dimensions of a rectangle are 12 cm and 8 cm to the nearest cm. Calculate, to 3 s.f, the smallest possible area as a percentage of the largest possible area. **[4 marks]**

---

The lower and upper bounds for the 12 cm side are 11.5 cm and 12.5 cm respectively.

The lower and upper bounds for the 8 cm side are 7.5 cm and 8.5 cm respectively.

So the smallest possible area is $7.5 \times 11.5 = 86.25$ cm$^2$.

The largest possible area is $8.5 \times 12.5 = 106.25$ cm$^2$.

So the smallest possible area as a percentage of the largest possible area

$$= \frac{86.25}{106.25} \times 100\%$$

$$= 81.2\% \text{ (to 3 s.f.)}$$

### Recap

Use this table to ensure you use the correct bounds in calculations:

| Operation | Lower bound | Upper bound |
|---|---|---|
| Addition | LB + LB | UB + UB |
| Subtraction | LB − UB | UB − LB |
| Multiplication | LB × LB | UB × UB |
| Division | LB ÷ UB | UB ÷ LB |

### ? Questions

**1** Mount Kenya is 17 060 ft high, correct to the nearest twenty feet. Find the smallest possible height of Mount Kenya.

**2** Write down the upper and lower bounds for each of these numbers:

  **a** $w = 73.43$      (to 2 d.p.)

  **b** $x = 7320$       (to 3 s.f.)

  **c** $y = 7320$       (to 4 s.f.)

  **d** $z = 147.037$   (to 3 d.p.)

  **e** $a = 100$        (to 3 s.f.)

  **f** $b = 100$        (to 1 s.f.)

**3** The distance between Nairobi and Dar es Salaam is 671 km.

Find this distance:

  **a** to the nearest 10 km

  **b** to the nearest 20 km

  **c** to the nearest 50 km.

**4** The population of Nairobi in 2007 was estimated at 2 940 000 correct to the nearest ten thousand. Find the upper and lower bounds for the population.

**5** A man runs a 100 m race and his time is measured as 10.3 s. If the track is accurate to the nearest metre and his time is accurate to the nearest 0.1 s, find the lower and upper bounds (to 1 d.p.) for his average speed. **E**

**6** The area of a rugby field is 6950 m², correct to 3 s.f. The length of the field is 95 m, correct to 2 s.f.

  **a** Find the lower and upper bounds for the area of the field.

  **b** Find the lower and upper bounds for the length of the field.

  **c** Use the bounds from parts **a** and **b** to calculate the lower and upper bounds (to 3 s.f.) for the width of the field.

**7** The formula for the distance $s$ travelled by a body with initial speed $u$ and constant acceleration $a$ after a time $t$ is given by $s = ut + \frac{1}{2}at^2$. Find the least and greatest possible values (to 3 s.f.) of $s$ when $u = 6.1$, $a = 4.5$, $t = 13.6$, all correct to 1 d.p.

**8** Pythagoras' theorem states that $a^2 + b^2 = c^2$ where $a$, $b$ and $c$ are the three lengths of the sides of a right-angled triangle and $c$ is the hypotenuse. If $a = 4.3$ cm and $c = 12.1$ cm, both correct to 1 d.p, find the smallest and largest possible values for $b$ (to 1 d.p.).

**9** The formula $s = \dfrac{v^2 - u^2}{2a}$ is used to find the distance travelled by an object whose initial speed is $u$, whose final speed is $v$ and whose acceleration is $a$. Find an inequality for $s$ (to 2 s.f.) if $v = 15$, $u = 11$ and $a = 2.3$, all correct to 2 s.f.

To **Raise your grade** now try question 2, page 49

**You need to:**
- Demonstrate an understanding of ratio and proportion.
- **Increase and decrease a quantity by a given ratio. (Extended)**
- Calculate average speed.
- Use common measures of rate.

---

### 🔑 Key skills

You need to be able to simplify ratios and divide amounts into a given ratio.

### Exam tip

Divide both numbers by the highest common factor.

### Exam tip

Add up the different parts of the ratio and then divide the total to find the value of each part.

Multiply the value of each part by the number of parts.

---

**Worked example**

(a) Simplify the ratio 150 : 100
(b) Divide 64 into the ratio 5 : 4 : 7                    **[3 marks]**

---

(a) 150 : 100
   = 3 : 2

(b) 5 + 4 + 7 = 16
   64 ÷ 16 = 4
   5 × 4 = 20
   4 × 4 = 16
   7 × 4 = 28
   Hence 64 divided into the ratio 5 : 4 : 7 is 20 : 16 : 28

---

### 🔑 Key skills

You need to be able to use the **unitary method** to solve simple problems of proportion.

### Exam tip

Work out the cost of 1 kg of carrots.

Multiply the **unit cost** by 5 to find the cost of Lotte's carrots.

---

**Worked example**

Peter buys 3 kg of carrots from a market and pays $4.50.

Lotte buys 5 kg of carrots from the same market.

Calculate the cost of Lotte's carrots.          **[2 marks]**

---

3 kg costs $4.50
1 kg costs $1.50
5 kg costs $1.50 × 5 = $7.50

**Worked example**

(a) Increase \$30 in the ratio 5 : 4

(b) Decrease 54 kg in the ratio 5 : 9 **[2 marks]**

(a) $\dfrac{5}{4} = \dfrac{x}{30}$

$\Rightarrow x = 30 \times \dfrac{5}{4} = \$37.50$

(b) $\dfrac{5}{9} = \dfrac{x}{54}$ kg

$\Rightarrow x = 54 \times \dfrac{5}{9}$

$= 30$ kg

**Key skills**

You need to be able to increase and decrease a quantity by a given ratio.

**Exam tip**

Write the ratio as a fraction equal to the new amount divided by the original amount.

The approach works whether increasing or decreasing by a given ratio.

**Recap**

average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

or $S = \dfrac{D}{T}$

This can be rearranged to make either $D$ or $T$ the subject:

$D = S \times T$

$T = \dfrac{D}{S}$

**Watch out!**

Make sure you work in consistent units. For example, if speed is in m s$^{-1}$, make sure distance is in metres and time in seconds.

**Worked example**

A car travels 189 km at an average speed of 60 km h$^{-1}$.

How long does the journey take? **[2 marks]**

$T = \dfrac{D}{S} = \dfrac{189}{60}$

$\dfrac{189}{60} = 3.15$ hours

3.15 hours = 3 hours and 9 minutes

**Exam tip**

Use the formula for time in terms of $D$ and $S$.

Work your answer out as a decimal number of hours and then convert to hours and minutes.

## Key skills

You need to be able to work with other measures of rate.

## Worked example

A 90 litre tank is filled full of water in 2 minutes and 15 seconds.

What is the rate of flow (in litres per minute) of the water into the tank? **[2 marks]**

---

2 minutes and 15 seconds = 2.25 minutes

$\dfrac{90}{2.25} = 40$ litres per minute

## Exam tip

Look at the rate required in the question. In this case it is litres per minute, so divide 90 by 2.25.

## Questions

1　Anatole, Brij and Christophe receive $560 from their great aunt, to be divided in the ratio Anatole : Brij : Christophe = 3 : 5 : 6.
　　**a**　Calculate how much each receives.
　　**b**　Christophe puts all his share into a venture with Dimitri. If Dimitri adds to his share and puts in $300 altogether then find the ratio of Christophe's investment to Dimitri's investment.

2　When David's car was repaired, the charge for labour was $200. This was $\dfrac{4}{7}$ of the total bill. What was the total bill?

3　**a**　Increase $50 in the ratio 7 : 5
　　**b**　Decrease 45 grams in the ratio 8 : 9

4　A coach leaves London at 0655 and arrives in Glasgow at 1612, a distance of 667 km. Find the average speed in kilometres per hour.

5　A plane travels from Windhoek to Johannesburg in 1 h 45 min. If the distance is 1190 km, find the average speed.

6　A bucket is filled with water at a rate of 0.04 litres per second. If it takes 9 minutes to fill the bucket, what is the capacity of the bucket?

To **Raise your grade** now try questions 5, 6 and 7 page 49

## You need to:

- Calculate a given percentage of a quantity.
- Express one quantity as a percentage of another.
- Calculate percentage increase or decrease.
- **Carry out calculations involving reverse percentages. (Extended)**

**◀◀ Recap**

A percentage multiplier can be used to find a percentage of a given quantity.

For example, to find 67% of 200 kg, multiply 200 by 0.67

**✎ Apply**

Write down the multiplier you would use to find:

a  25% of 120

b  40% of 300

c  82% of 450

d  91% of 1250

e  125% of 560.

**Worked example**

A road is 1300 metres long and 12% of it requires resurfacing.

Calculate the length of road that needs resurfacing.  **[2 marks]**

---

$1300 \times 0.12$

$= 156$ metres

**Exam tip**

Make sure you write down the calculation you are going to do, even if you use a calculator.

**◀◀ Recap**

To find one quantity as a percentage of another, you divide the first quantity by the second and then multiply by 100%.

**Worked example**

A student scored 75 out of 120 in a test.

Express this as a percentage.  **[2 marks]**

---

Percentage $= \dfrac{75}{120} \times 100\%$

$= 62.5\%$

**Exam tip**

You are asked to find 75 as a percentage of 120 so divide 75 by 120 and then multiply by 100%.

**Worked example**

(a) One year, a football club's average crowd attendance was 41 200. The following year the attendance rose by 4%. What was its new average attendance?

(b) A car worth $5000 loses 15% of its value in a year.

What is it worth after one year? **[4 marks]**

(a) 41 200 × 1.04 = 42 848
(b) 5000 × 0.85 = $4250

**Exam tip**

A percentage **increase** of 4% converts to a multiplier of 1 + 0.04 = 1.04

A percentage **decrease** of 15% converts to a multiplier of 1 − 0.15 = 0.85

**Extended**

**Key skills**

You need to be able to find the original quantity when you are given the quantity after a percentage increase or decrease. This is known as **reverse percentage change**.

**Exam tip**

Set up and solve an equation with the original price, $x$, and the multiplier for a 10% decrease, 0.9.

**Exam tip**

You can use the equation method whether there has been a percentage increase or decrease.

**Worked example**

(a) The price of a skirt is reduced by 10% in a sale.

If it cost £31.50 in the sale, what was the original price of the skirt?

(b) The price of a camera including sales tax of 17.5% is $94.

What was the price before sales tax was added? **[4 marks]**

(a) Let the original cost be $x$.
$x \times 0.9 = 31.50$

$x = \dfrac{31.5}{0.9} = 35$, so the original price of the skirt was £35.

(b) Let the pre-tax cost of the camera be $y$.

$y \times 1.175 = 94$

$y = \dfrac{94}{1.175} = 80$, so the price of the camera before sales tax was added was $80.

### Questions

1  A man buys plane tickets for himself, his wife and his four children.
   The adult fare is $172 and the child fare is 67% of the adult fare.
   Find the total cost of the journey.

2  In April a lawnmower cost £265. In the September sale it was only
   £225.25. What was the percentage discount?

3  A bleach bottle is labelled '900 ml for the price of 750 ml: $x$% extra free'.
   The $x$ is smudged and illegible. What is $x$?

4  The sale price of a garden table is £48 and it has a sign saying
   'Reduction of 20%'. What was the price of the table before the sale?   **E**

5  A school claims that the students' average mark in an exam has increased
   by 15% over 5 years. Two boys are told that the average mark is now 85.1.
   George thinks that the average mark five years ago was 72.335 but James
   thinks it was 74. Who is correct, and how is the correct answer obtained?

6  In a sale, all items are reduced by 15%. A carpet now costs £15.30 per
   square metre. What was the price before the sale?

7  A document is photocopied so that the lengths of the copy are 70% of
   the original lengths. If the copy measures 12.6 cm by 17.5 cm, what are
   the dimensions of the original document?

8  The number of students at a school in 2006 was 85% of the number at
   the school in 2005. In 2006 the number of students was 1020.
   How many students were there in 2005?

9  Find the original price of a car which was sold for $1200 at a loss of 4%.

10 Find the original price of an antique which was sold at £545 at a profit of 9%.

11 The profit of a company in 2004 was £1 500 000. In 2005 the profit was
   25% higher than it was in 2004 but in 2006 the profit fell by 40%.
   a  Show that the profit in 2005 was £1 875 000.
   b  What was the profit in 2006?

**You need to:**
- Use a calculator efficiently.
- Apply appropriate checks of accuracy.

## 🔑 Key skills

You need to be able to use your calculator correctly and efficiently to carry out potentially complex calculations.

You also need to be able to check your answer to a calculation using a suitable estimate.

### Worked example

(a) Estimate the answer to $\sqrt{7.62 + 8.13}$ by first rounding each number to 1 s.f.

(b) Calculate the exact answer to $\sqrt{7.62 + 8.13}$, giving it as a decimal, and showing all of the digits on your calculator display. **[3 marks]**

---

(a) $7.62 = 8$ (to 1 s.f.) and $8.13 = 8$ (to 1 s.f.)

$\Rightarrow \sqrt{7.62 + 8.13} \approx \sqrt{8 + 8}$

$\qquad\qquad\qquad = \sqrt{16}$

$\qquad\qquad\qquad = 4$

(b) $\sqrt{7.62 + 8.13} = 3.968626967$

**Exam tip**

Write down your approximations and show all steps in the working.

## ❓ Questions

1. **a** Estimate the values of these expressions by rounding all the numbers to 1 s.f:

   **i** $\dfrac{14.1 + 6.9 \times 8.3}{8.9 - 2.1}$

   **ii** $\dfrac{10.1^2 + 2.1 \times 20.7}{23.1 - 3.2}$

   **b** Calculate the values of the expressions in part **a**, giving your answers to 3 s.f.

2. Calculate the value of each expression (to 3 s.f.):

   **a** $\sqrt{7.2 \times 3.5 + 2.1 \times 5.7}$

   **b** $\dfrac{6.1 - 3.5}{2.1 + 4.7 \times 1.8}$

   **c** $\dfrac{2.7^2 + 4.2^3}{1.9^4}$

   Check your answers using sensible estimates.

## 👁 Watch out!

There are many different calculators available for use in your exam. Make sure that you can use the one that you have rather than following a set series of instructions or key strokes from a textbook.

**You need to:**
- Calculate times in terms of the 24-hour and 12-hour clock.
- Read clocks, dials and timetables.

**Worked example**

(a) A flight departed at 1514 and arrived at 1842. How long was the flight?

(b) A girl arrived at a rehearsal at 1935 and left at 2129. For how long was she at the rehearsal? **[2 marks]**

(a) Three hours on from 1514 is 1814.
1842 is another 28 minutes on from that.
So the flight took 3 h 28 min.

(b) One hour on from 1935 is 2035.
2035 to 2100 is another 25 minutes. 2100 to 2129 is a further 29 minutes. 25 + 29 = 54
So she was at the rehearsal for 1 hour 54 minutes.

**Worked example**

Derek wanted to get from Twickenham to Wembley Stadium. A website gave him the information shown in the table.

(a) How long did the journey take in total?
(b) At which station did Derek wait for 8 minutes? **[2 marks]**

(a) He departed at 1158 and arrived at 1302. One hour would bring him to 1258 so there is another 4 minutes.
So the journey took 1 h 4 min.

(b) There is an 8 minute gap between 1203 and 1211.
So he waited for 8 minutes at Richmond Rail Station.

| Depart Twickenham Rail Station | 1158 |
|---|---|
| Arrive Richmond Rail Station | 1203 |
| Depart Richmond Rail Station | 1211 |
| Arrive Willesden Junction Underground Station | 1230 |
| Depart Willesden Junction Underground Station | 1235 |
| Arrive Wembley Central Station | 1242 |
| Depart Wembley Central Station | 1248 |
| Arrive Wembley Stadium | 1302 |

**Questions**

1 A plane leaves Nairobi airport at 2330 and arrives in London the next day at 0520. The time in Nairobi is 3 hours ahead of the time in London.

a How long does the flight take?

The return flight leaves London at 1005 and arrives in Nairobi at 2135.

b How long does the return flight take?

c Calculate how much longer the outward journey is than the return journey.

2 A flight from Singapore to London leaves at 0130 local time and arrives the same day at 0555 local time. The airline website says that journey takes 12 h 25 min.

a How many hours ahead of London is Singapore?

b A traveller arriving at Singapore rings home when the time in Singapore is 0700. What is the time in London when he rings?

## You need to:

- Calculate using money and convert from one currency to another.

🔑 **Key skills**

You need to be able to calculate currency conversions.

When you travel abroad, you often have to exchange your domestic currency for the currency of the country to which you are travelling.

You do this using an **exchange rate**.

Exchange rates change frequently and they may also depend on where you change your currency, for example at the airport or using a local bank.

✏️ **Apply**

At the time of writing this book, the exchange rate from GBP (£) to Dirhams (AED) was £1 to 4.82 AED.

Investigate the exchange rate of your local currency.

Find at least five different foreign currencies to compare.

**Exam tip**

You will be told the exchange rate to use in an exam question.

**Worked example**

Susan wanted to change some Canadian dollars into Kenyan shillings. The exchange rate was 0.01592 Canadian dollars to one Kenyan shilling.

(a) How many Kenyan shillings did Susan get if she exchanged 500 Canadian dollars?

(b) At the end of her trip, she exchanged 2000 Kenyan shillings back into Canadian dollars. How many Canadian dollars did she get? **[4 marks]**

---

(a) Susan got one Kenyan shilling for every 0.01592 Canadian dollars.
So she got $\dfrac{500}{0.01592} = 31407$ Kenyan shillings (to the nearest shilling)

(b) For each Kenyan shilling she got 0.01592 Canadian dollars.
So she got $2000 \times 0.01592 = 31.84$ Canadian dollars.

❓ **Questions**

1 A Bureau de Change offers $1.568 per £ sterling but charges a commission fee of £3. How many dollars (to the nearest cent) does Joe get for £75?

2 If £1 = €1.27 then find:

 a the cost (to the nearest £) of a holiday house which costs €450 per week to rent

 b how much (in €) a British holidaymaker would get for £300 at the foreign exchange.

3 Mr Smith wants to convert 600 euros into Chinese yuan. The bank offers him a rate of 1 euro for 8.37 yuan.

 a How many yuan does he get?

 b At the end of his holiday Mr Smith has 100 Chinese yuan. If he changes his yuan back into euros at the same rate of exchange, how many euros does he get?

## You need to:

- Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest.
- Extract data from tables and charts.

People who earn money often pay tax on their earnings. This is called **income tax**.

They usually receive a tax-free allowance to use first.

---

**Worked example**

Juan earns $45 000 per year. He gets a tax free allowance of $8000 and pays tax at a rate of 25% on the next $20 000.

He then pays tax at a rate of 30% on his remaining income. How much tax does Juan pay? **[3 marks]**

---

$45 000 − $8000 = $37 000

25% of $20 000 = 20 000 ÷ 4 = $5000

$37 000 − $20 000 = $17 000

30% of $17 000 = 17 000 × 0.3 = $5100

Total tax = $5000 + $5100 = $10 100

---

**Exam tip**

Set out your working clearly in a step-by-step format. You will get credit for clear correct working, even if you get the final answer wrong.

---

If you have a savings account or an investment, you usually get interest paid to you on these.

If you have a loan or a credit card, you usually pay interest on what you borrow.

The amount you invest or borrow is called the **principal**.

Interest falls into two categories:

**Simple interest** is paid or payable only on the original principal.

**Compound interest** is added to the principal at the end of each period of time (usually a year) and the next year's interest is paid on the new amount.

---

**Worked example**

Joseph invests $500 at 4% per annum simple interest.

How much will he have at the end of 5 years? **[2 marks]**

---

Interest = 4% of $500 = $\frac{4}{100} \times 500 = \$20$ per year

Total interest in 5 years = $20 × 5 = $100

Joseph will have $500 + $100 = $600

---

**Exam tip**

'Per annum' means each year.

Simple interest is paid only on the principal so it will be the same amount each year.

## Worked example

Ali invests $200 at 3% compound interest. What amount will he have after 3 years?   **[2 marks]**

---

3% of $200 = $6

$200 + $6 = $206 so Ali has $206 at the start of Year 2.

3% of $206 = $6.18

$206 + $6.18 = $212.18 so Ali has $212.18 at the start of Year 3.

3% of $212.18 = $6.37 to 2 d.p.

$212.18 + $6.37 = $218.55

Ali will have $218.55 after 3 years.

**Recap**

The formula for compound interest is:

Amount after $n$ years
= $P \times$ (percentage multiplier)$^n$

where $P$ is the principal and $n$ is the number of years.

## Worked example

Use the formula for compound interest to calculate:

(a) how much Rose owes on a bank loan of $800 after 2 years if the rate of interest she pays is 7%.

(b) how much an $18 000 car is worth after 3 years if its value depreciates by 30% per year.   **[4 marks]**

---

(a) $800 \times 1.07^2 = 915.92$
Rose owes $915.92 after two years.

(b) $18\,000 \times 0.7^3 = 6174$
The car is worth $6174 after 3 years.

## Questions

1  Julienne earns $73 000 per year. She gets a tax-free allowance of $7000 and then pays tax at a rate of 20% on the next $50 000. The tax rate on any additional income is then 40%. How much tax does Julienne pay overall?

2  A bank offers 5% compound interest on investments. A man invests £2000.
   a   What is his investment worth after 2 years?
   b   What is the total percentage increase?

3  An investment fund has increased in value by a total of 21% over the last two years.

   a   A man invested £1000 in the fund two years ago. What is it worth now?
   b   Calculate the yearly rate of interest assuming that it was:
      i  compound interest    ii  simple interest.

4  A bank offers 2% simple interest per year. A woman opens an account with a deposit of €750. She closes the account 11 months later. How much money does she withdraw?

5  Jack invests €80 in an account offering him 3.6% simple interest. He removes his money after 10 months. How much interest does he get?

## You need to:

- Use exponential growth and decay in relation to population and finance. (Extended)

The principle of compound interest can be extended to model growth and decay in many different situations. For example, the growth or decline of a population of organisms, the decay of a radioactive element and the value of a financial product such as shares.

### ⏪ Recap

The formula for **exponential growth** (or **decay**) is:

Amount after $n$ time intervals $= A \times$ (growth or decay factor)$^n$

where $A$ is the initial amount and the growth or decay factor is a percentage multiplier.

### 🔑 Key skills

You need to be able to calculate exponential growth and decay.

### Worked example

A colony of bacteria increases at a rate of 10% per hour.

Initially there are 120 bacteria.

Calculate the number of bacteria in the colony after 8 hours.

**[2 marks]**

Number of bacteria after 8 hours $= 120 \times 1.1^8$

$\qquad\qquad\qquad\qquad\qquad\quad = 257$

### Exam tip

Use the formula for exponential growth with a growth factor of $1 + 0.1 = 1.1$

Your answer must be a whole number since you are dealing with organisms.

### Worked example

A radioactive substance decays at a rate of 7% per day.

Initially there are 1000 atoms of the substance.

Calculate the number of atoms after 5 days. **[2 marks]**

Number of atoms after 5 days $= 1000 \times 0.93^5$

$\qquad\qquad\qquad\qquad\qquad\quad = 696$

### Exam tip

The multiplier represents a percentage decrease since you are modelling decay.

The answer should again be a whole number.

### Extended

**Exam tip**

You can combine the growth and the decay parts into a single calculation using the two different multipliers.

Make sure you show all of your working, even if you are using a calculator.

**Worked example**

The value of a financial bond increases at a rate of 4% for the first 6 months, and then decreases at a rate of 2% for the next 4 months.

The bond is initially worth $500.

Calculate the value of the bond after 10 months.     **[3 marks]**

---

Value of bond after 10 months = $500 \times 1.04^6 \times 0.98^4$
$$= \$583.54$$

---

**? Questions**

**1** The population of a colony of rabbits grows at a rate of 20% per month.
Initially there are 70 rabbits.
Calculate the number of rabbits after 7 months.

**2** A colony of bacteria is treated with an antibiotic that reduces the amount of bacteria by 30% every hour. Initially there are 3000 bacteria.
**a** Calculate the number of bacteria after 3 hours.
**b** After how many hours will the bacteria colony be less than 200?

**3** A radioactive substance decays at a rate of 14% per year. Initially there are 2000 atoms of the substance.
**a** Calculate the number of atoms after 2 years.

The 'half-life' of a radioactive substance is the time it takes for half of the atoms to decay.

**b** Calculate the half-life for this radioactive substance. Give your answer as a decimal number of years, correct to 1 d.p.

**4** A financial product decreases in value by 7% each month for the first 3 months, and then increases in value by 7% for the next 3 months.

Adil, a broker, says that the product is now worth the same as it was at the start of the 6-month period.

**a** Show that Adil is wrong.
**b** Find, correct to 2 d.p, the percentage change in the value of the product over the 6-month period.

1 Work out the value of $3 + \cfrac{1}{7 + \cfrac{1}{16}}$. Give your answer correct to 6 decimal places. **[2 marks]**

2 A man walked 1000 miles, correct to the nearest 10 miles. If he walked on average 10 miles per day, correct to the nearest mile, what are the lower and upper bounds, to the nearest day, for the number of days he spent walking? **[4 marks]**

3 How many prime numbers are there between 90 and 100? **[1 mark]**

4 In 2016, Pete bought a car for $20 000.

    **a** This was 25% more than he paid for his previous car.
       How much did he pay for his previous car? **[2 marks]**

    **b** One year later, Pete's new car was worth the same as he paid for his old car.
       By what percentage had his new car decreased in value? **[2 marks]**

5 Genoveva and Apolonia share a large container of cherries in the ratio 3 : 5. If Apolonia gives Genoveva 20 cherries from her share, their new shares are in the ratio 7 : 5. How many cherries are there altogether? **[3 marks]**

6 The Titanic ocean liner was $2.7 \times 10^2$ m long. If a domestic cat can run at a speed of 40 kmh$^{-1}$, how long would it have taken a cat to run from one end of the Titanic to the other? Give your answer to the nearest second. **[2 marks]**

7 If a train travels for 50 km at a speed of 40 kmh$^{-1}$, then travels a further 60 km at a speed of 30 kmh$^{-1}$, find:

    **a** the total time taken **[3 marks]**

    **b** the average speed at which the train travelled, accurate to 3 significant figures. **[3 marks]**

# 2 Algebra and graphs

Tick these boxes to build a record of your revision

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 2.1 | Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in **complicated** formulae. Rearrange simple formulae. Construct simple expressions and set up simple equations. | | | |
| 2.2 | Manipulate directed numbers. Use brackets and extract common factors. Expand products of algebraic expressions, **including three brackets**. **Factorise, where possible, expressions of the form $ax + bx + kay + kby$, $a^2x^2 - b^2y^2$, $a^2 + 2ab + b^2$, $ax^2 + bx + c$** | | | |
| 2.3 | **Manipulate algebraic fractions.** **Factorise and simplify rational expressions.** | | | |
| 2.4 | Use and interpret positive, negative and zero indices. **Use and interpret fractional indices.** Use the rules of indices. | | | |
| 2.5 | Derive and solve simple linear equations in one unknown. Derive and solve simultaneous linear equations in two unknowns. **Derive and solve simultaneous equations, involving one linear and one quadratic.** **Derive and solve quadratic equations by factorisation, completing the square and by use of the formula.** **Derive and solve linear inequalities.** | | | |
| 2.6 | **Represent inequalities graphically and use this representation to solve simple linear programming problems.** | | | |
| 2.7 | Continue a given number sequence. Recognise patterns in sequences including the term-to-term rule and relationships between different sequences. Find and use the $n$th term of a sequence, **including exponential sequences.** | | | |
| 2.8 | **Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities.** | | | |
| 2.9 | **Use function notation, e.g. $f(x) = 3x - 5$, $f: x \mapsto 3x - 5$, to describe simple functions.** **Find inverse functions $f^{-1}(x)$. Form composite functions as defined by $gf(x) = g(f(x))$.** | | | |
| 2.10 | Interpret and use graphs in practical situations including travel graphs and conversion graphs. Draw graphs from given data. **Apply the idea of rate of change to simple kinematics involving distance-time and speed-time graphs, acceleration and deceleration. Calculate distance travelled as area under a speed-time graph.** | | | |
| 2.11 | Construct tables of values for functions of the form $ax + b$, $\pm x^2 + ax + b$, $\dfrac{a}{x}(x \neq 0)$, where $a$ and $b$ are integers, **and functions of the form $ax^n$ (and simple sums of these) and functions of the form $ab^x + c$. Draw and interpret these graphs.** Solve linear and quadratic equations approximately, including finding and interpreting roots by graphical methods. **Draw and interpret graphs representing exponential growth and decay problems.** Recognise, sketch and interpret graphs of functions. | | | |
| 2.12 | **Estimate gradients of curves by drawing tangents.** | | | |
| 2.13 | **Understand the idea of a derived function.** **Use the derivatives of functions of the form $ax^n$, and simple sums of not more than three of these.** **Apply differentiation to gradients and turning points (stationary points).** **Discriminate between maxima and minima by any method.** | | | |

**You need to:**

- Use letters to express generalised numbers and express basic arithmetic processes algebraically.
- Substitute numbers for words and letters in **complicated** formulae.
- Rearrange simple formulae.
- Construct simple expressions and set up simple equations.

### ⏪ Recap

In algebra you use letters to represent numbers, either because you don't know their value or because you want to be able to change their value.

### 🔑 Key skills

You must be able to substitute numbers for words and letters in formulae.

---

**Worked example**

If $D = \dfrac{(a-b)^2}{c^3}$, find the value of $D$ when $a = 7$, $b = -2$ and $c = 3$. **[2 marks]**

---

$D = \dfrac{(7-(-2))^2}{3^3}$

$= \dfrac{9^2}{3^3}$

$= 3$

### 🟩 Exam tip

Remember to use the correct order of operations when solving questions like this one.

---

### 🟩 Exam tip

Here are some general guidelines for rearranging formulae.

**Step 1** Clear the fractions.

**Step 2** Multiply out any brackets involving $x$.

**Step 3** Put all $x$-terms together on one side of the equation.

**Step 4** Divide both sides by the coefficient of the $x$-term.

### 🔑 Key skills

You must be able to rearrange simple formulae.

---

**Worked example**

Make $x$ the subject of the formula $2x + 3b = c$. **[2 marks]**

---

| **Step 1** | Clear the fractions. | No fractions. |
| **Step 2** | Multiply out any brackets involving $x$. | No brackets to multiply out. |

**Step 3** Put $x$-terms together on one side.

$$2x = c - 3b$$

**Step 4** Divide by the coefficient of the $x$-term.

$$x = \frac{c - 3b}{2}$$

Coefficient of $x$-term is 2.

---

**Worked example**

Make $x$ the subject of the formula $\dfrac{ax + b}{x} = c$.  **[2 marks]**

---

**Step 1** Clear the fractions.

$$ax + b = c \times x = cx$$

**Step 2** Multiply out any brackets involving $x$.

No brackets to multiply out.

**Step 3** Put $x$-terms on one side and write as $x(\ ) = \ldots$

$$cx - ax = b$$

$$x(c - a) = b$$

**Step 4** Divide by the coefficient of the $x$-term.

$$x = \frac{b}{c - a}$$

> **Exam tip**
>
> Here $x$ appears twice, so factorise to give $x(c - a) = b$.

---

**Worked example**

Make $x$ the subject of the formula $\dfrac{a}{x} + b = c$.  **[2 marks]**

---

**Step 1** Clear the fractions.

$$a + bx = cx$$

**Step 2** Multiply out any brackets involving $x$.

No brackets.

**Step 3** Put $x$-terms on one side and write as $x(\ ) = \ldots$

$$a = cx - bx$$

$$a = x(c - b)$$

**Step 4** Divide by the coefficient of the $x$-term.

$$x = \frac{a}{c - b}$$

## Worked example

Make $x$ the subject of the formula $a = \dfrac{bx^2 + c}{d}$. **[2 marks]**

**Step 1**   Clear the fractions.
$$ad = bx^2 + c$$

**Step 2**   Multiply out any brackets involving $x^2$.    No brackets.

**Step 3**   Put $x^2$-term on one side.
$$ad - c = bx^2$$

**Step 4**   Divide by the coefficient of the $x^2$-term.
$$x^2 = \dfrac{ad - c}{b}$$

**Step 5**   Take square roots.
$$x = \pm\sqrt{\dfrac{ad - c}{b}}$$

### Exam tip

If the equation involves $x^2$ then, first of all, make $x^2$ the subject and then take the square root of both sides to find $x$. Include the $\pm$ sign when taking square roots.

## Worked example

Jane thinks of a number, multiplies it by 4, adds 7 and gets the answer 19.

Construct an equation to represent this situation and then solve it. **[3 marks]**

Let $x$ represent Jane's number.
$$4x + 7 = 19$$
Then    $4x = 19 - 7 = 12$
$$x = \dfrac{12}{4} = 3$$
Therefore Jane's number was 3.

### 🔑 Key skills

You must be able to construct simple expressions and set up simple equations.

### ❓ Questions

1. The acceleration of a body, moving with uniform acceleration, is given by the formula
$$a = \dfrac{v - u}{t}.$$
In this formula $a$ is the uniform acceleration, $u$ is the initial velocity and $v$ is the velocity at time $t$. Use the formula to find $a$ (in $\text{m s}^{-2}$) if:

   a   $v = 5 \, \text{m s}^{-1}$,     $u = 1 \, \text{m s}^{-1}$,     $t = 8 \, \text{s}$

   b   $v = 0.1 \, \text{m s}^{-1}$,     $u = 0.02 \, \text{m s}^{-1}$,     $t = 2 \, \text{s}$

2. Use the formula $s = vt - \dfrac{1}{2}at^2$ to calculate $s$ (to 2 s.f.) given that
$v = 27.27 \, \text{m s}^{-1}$, $a = 9.81 \, \text{m s}^{-2}$, $t = 1.73 \, \text{s}$.

**3** The formula for the volume, $V$, of a sphere of radius $r$ is $V = \dfrac{4}{3}\pi r^3$.

    **a** Use this formula to calculate the volume of a sphere (to 3 s.f.) of radius 2 m.

    **b** Make $r$ the subject of the formula.

    **c** Use part **b** to calculate the radius (to 3 s.f.) of a sphere of volume 200 mm³.

**4** Evaluate $a^2 b$ when $a = 5000$ and $b = 300$.

**5** Given that $s = \dfrac{v^2 - u^2}{2a}$, find $s$ when $u = 60$, $v = 80$ and $a = 400$.

**6** If $\sqrt{\dfrac{py + q}{r}} = s$, find $y$ (to 3 s.f.) when $p = 132$, $q = 251$, $r = 158$ and $s = 17$.    **E**

**7** Make $x$ the subject of these formulae:

    **a** $mx + n = p$         **b** $a(x + b) = c$

    **c** $\dfrac{x + p}{q} = r$         **d** $\dfrac{p(x + q)}{r} = t$

    **e** $\dfrac{h}{x} = u$         **f** $\dfrac{k}{x + b} = w$

    **g** $\dfrac{d}{ax + b} = c$         **h** $\dfrac{a}{x} + b = c$

**8** Make the given variable the subject of each formula:

    **a** $w$   $\dfrac{aw + b}{c} = d$     **b** $y$   $\dfrac{y + b}{x + t} = c$

    **c** $z$   $\dfrac{z}{a} = \dfrac{b}{c}$     **d** $h$   $\dfrac{a}{h} = b$

    **e** $o$   $\dfrac{o}{h} = \dfrac{1}{2}$     **f** $r$   $\dfrac{r}{t} = t$

    **g** $t$   $\dfrac{t + b}{a + c} = d$     **h** $g$   $\dfrac{a}{g} = \dfrac{p}{q}$

**9** Make $x$ the subject of these formulae:

    **a** $r + mx = nx$         **b** $ax + b = cx + d$

    **c** $x = \dfrac{d + bx}{a}$         **d** $n - x = \dfrac{m + qx}{p}$

    **e** $\dfrac{A}{x} = \dfrac{B}{x} + C$         **f** $\dfrac{ax}{x + b} = c$

**10** Make the given variable the subject of each formula:

    **a** $A$   $\dfrac{A + s}{A} = t$     **b** $R$   $\dfrac{aR}{R + 1} = b$

    **c** $e$   $\dfrac{ae + b}{ce + d} = 1$     **d** $p$   $\dfrac{ap}{bp + c} = d$

    **e** $Q$   $\dfrac{bQ}{Q + d} = c$     **f** $x$   $\dfrac{\sqrt{ax - b}}{c} = d$

    **g** $y$   $\sqrt{\dfrac{my + n}{p}} = q$     **h** $q$   $\dfrac{n - aq}{bq + m} = c$

    **i** $t$   $\sqrt{\dfrac{a - bt}{t}} = c$     **j** $k$   $\left(\dfrac{ak}{k + b}\right)^2 = p$

To **Raise your grade** now try question 6, page 92

**You need to:**

- Manipulate directed numbers.
- Use brackets and extract common factors.
- Expand products of algebraic expressions, **including three brackets. (Extended)**

- Factorise, where possible, expressions of the form $ax + bx + kay + kby$, $a^2x^2 - b^2y^2$, $a^2 + 2ab + b^2$, $ax^2 + bx + c$ (Extended)

---

**Worked example**

The temperature in Moscow was $-1°C$ at 23:00. By 04:00 the next day it had fallen by $3°C$. What was the temperature at 04:00? **[1 mark]**

---

The temperature was $(-1) - 3 = -4°C$



$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$
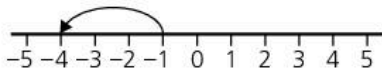
🔑 **Key skills**

You must be able to manipulate directed numbers.

**Exam tip**

You can use a number line to show directed numbers.

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$

---
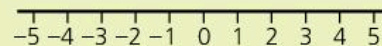
**Worked example**

Expand these expressions.

(a) $4a(2a + 3b)$ **[1 mark]**

(b) $5p(2p - 5q)$ **[1 mark]**

(c) $6s(2s + 7t) - 2s(5s - 2t)$ **[1 mark]**

---

(a) $4a(2a + 3b) = 8a^2 + 12ab$   | $8a^2$ is shorthand for $8 \times a \times a$.

(b) $5p(2p - 5q) = 10p^2 - 25pq$

(c) $6s(2s + 7t) - 2s(5s - 2t)$
$= 12s^2 + 42st - 10s^2 + 4st$
$= 2s^2 + 46st$

🔑 **Key skills**

You must be able to expand products of algebraic expressions and reverse the process by factorising.

---

**Worked example**

Expand these expressions.

(a) $(x + 2)(x + 5)$    (b) $(x + 7)(x - 4)$

(c) $(x - 3)(x - 5)$    (d) $(2x + 3)(5x - 9)$ **[5 marks]**

---

Use the FOIL method:

(a) $(x + 2)(x + 5)$   $= x \times x + x \times 5 + 2 \times x + 2 \times 5$
$= x^2 + 5x + 2x + 10 = x^2 + 7x + 10$

**Exam tip**

Remember the FOIL method means 'First – Outside – Inside – Last'.

(b) $(x+7)(x-4)$ $= x \times x + x \times (-4) + 7 \times x + 7 \times (-4)$

$= x^2 - 4x + 7x - 28 = x^2 + 3x - 28$

(c) $(x-3)(x-5)$ $= x \times x + x \times (-5) + (-3) \times x + (-3) \times (-5)$

$= x^2 - 5x - 3x + 15 = x^2 - 8x + 15$

(d) $(2x+3)(5x-9)$ $= (2x) \times (5x) + (2x) \times (-9) + 3 \times (5x) + 3 \times (-9)$

$= 10x^2 - 18x + 15x - 27 = 10x^2 - 3x - 27$

**E**

### 🔑 Key skills

You need to be able to expand three brackets.

**Worked example**

Expand and simplify

$(x-1)(x+2)(x+4)$ **[3 marks]**

$(x-1)(x+2)(x+4)$

$= (x^2 - x + 2x - 2)(x+4)$

$= (x^2 + x - 2)(x+4)$

$= x^3 + 4x^2 + x^2 + 4x - 2x - 8$

$= x^3 + 5x^2 + 2x - 8$

> Expand and simplify the first pair of brackets.

> Multiply the quadratic expression by the third bracket.

### 🔑 Key skills

You must be able to factorise algebraic expressions.

### Exam tip

Expanding expressions and factorising expressions are the opposites of each other.

**Worked example**

Factorise these expressions.

(a) $9ab + 3b^2$ **[1 mark]**

(b) $9x + 5x + 18y + 10y$ **[1 mark]**

(c) $9r + 4s + 5r + 3s$ **[1 mark]**

(a) $9ab$ and $3b^2$ have an HCF of $3b$.

$9ab + 3b^2 = 3b(3a + b)$.

(b) $9x + 5x + 18y + 10y = 14x + 28y$ and $14x$ and $28y$ have an HCF of 14.

$9x + 5x + 18y + 10y = 14x + 28y = 14(x + 2y)$

(c) $9r + 4s + 5r + 3s = 14r + 7s$ and $14r$ and $7s$ have an HCF of 7.

$9r + 4s + 5r + 3s = 14r + 7s = 7(2r + s)$

## Extended

### 🔑 Key skills

You must be able to factorise quadratic expressions of the forms $a^2 + 2ab + b^2$ and $ax^2 + bx + c$.

**Worked example**

Factorise (a) $x^2 + 7x + 6$ (b) $x^2 + 3x - 28$

(c) $x^2 - 7x + 12$ (d) $x^2 - 2x - 15$ **[4 marks]**

(a) $x^2 + 7x + 6$   Find two numbers that multiply to give 6 and add up to 7: 6 and 1.

So $x^2 + 7x + 6 = (x + 6)(x + 1)$

(b) $x^2 + 3x - 28$   Find two numbers that multiply to give −28 and add up to 3: 7 and −4.

So $x^2 + 3x - 28 = (x + 7)(x - 4)$

(c) $x^2 - 7x + 12$   Find two numbers that multiply to give 12 and add up to −7: −3 and −4.

So $x^2 - 7x + 12 = (x - 3)(x - 4)$

(d) $x^2 - 2x - 15$   Find two numbers that multiply to give −15 and add up to −2: 3 and −5.

So $x^2 - 2x - 15 = (x + 3)(x - 5)$

**Exam tip**

When you have factorised an expression, expand the expression again if you have time to check that you got it right.

**Worked example**

Factorise

(a) $x^2 - 16$ **[1 mark]**

(b) $x^2 + 7x$. **[1 mark]**

(a) This is the difference of two squares, so $a^2 - b^2 = (a + b)(a - b)$. In this case $a = x$ and $b = 4$, therefore $x^2 - 16 = (x + 4)(x - 4)$.

(b) $x^2 + 7x = x(x + 7)$

**Exam tip**

If there is no middle term, it is called 'the difference of two squares'.

**Worked example** **[2 marks]**

Factorise $2x^2 + 7x + 6$.

**Step 1** The brackets must be of the form $(2x +$   $)(x +$   $)$.

**Step 2** The missing numbers must multiply to give 6: 1 and 6 or 2 and 3.

**Step 3** Try the four possible combinations.

$(2x + 1)(x + 6)$ gives $x$-term $12x + x = 13x$

$(2x + 6)(x + 1)$ gives $x$-term $2x + 6x = 8x$

$(2x + 2)(x + 3)$ gives $x$-term $6x + 2x = 8x$

$(2x + 3)(x + 2)$ gives $x$-term $4x + 3x = 7x$

Only $(2x + 3)(x + 2)$ gives the $7x$ term in the middle, so

$2x^2 + 7x + 6 = (2x + 3)(x + 2)$

**Exam tip**

Factorising an equation in this way is sometimes called factorising 'by inspection'.

**? Questions**

**1** Expand these expressions.

   **a** $7(p + 3q)$

   **b** $6(5m - 7n)$

   **c** $3(5a + 2b) - 6(2a - 3b)$

**2** Multiply out the brackets in these expressions.

   **a** $(2x + 1)(3x + 2)$    **b** $(5x + 2)(3x + 4)$

   **c** $(6t - 1)(2t - 3)$    **d** $(2y - 9)(3y - 1)$

   **e** $(7z - 1)(2z + 3)$    **f** $(9r - 2)(3r + 2)$

   **g** $(7e - 11)(2e + 1)$    **h** $(8q + 1)(5q - 3)$

   **i** $(3p - 1)(3p + 1)$    **j** $(7y + 2)(7y - 2)$

   **k** $(2k + 1)(k + 3)$    **l** $(2v - 1)(5v + 1)$

**3** Multiply out the brackets and simplify these expressions.

   **a** $(x + 2)(x + 3)$    **b** $(x + 5)(x + 4)$

   **c** $(t + 1)(t + 2)$    **d** $(3q + 1)(2q - 1)$

   **e** $(5y + 2)(2y - 3)$    **f** $(5m - 1)(5m + 1)$

   **g** $(2y + 1)(2y - 1)$    **h** $(3p + 2)^2$

   **i** $(2q - 1)^2$    **j** $(5d + 2e)(2d - 3e)$

   **k** $(5p + 3q)(4p + q)$    **l** $(7s - 3t)(2s - t)$

**4** Expand and simplify these expressions.

   **a** $(x + 3)^2$    **b** $(y + 5)^2$

   **c** $(y - 4)^2$    **d** $(z - 6)^2$

   **e** $(2w - 3)^2$    **f** $(5t - 2)^2$

   **g** $(3a + b)(2a + b)$    **h** $(3m - 2n)(5m - n)$

   **i** $(5p + 2q)(3p - 4q)$    **j** $(2x - 3y)(5x - 2y)$

   **k** $(3c + 2d)^2$    **l** $(5p - 3q)^2$

**5** Factorise these expressions.

   **a** $2x - 4xy$    **b** $3xy + 4yz$

   **c** $12x^2 + 14xy$    **d** $16abc - 24a^2b$

   **e** $8de^3 - 8d^3e$    **f** $35a^2bc - 21ab + 7bc^2$

**6** Factorise these quadratic expressions. **E**

   **a** $x^2 + 9x + 18$    **b** $x^2 - x - 20$

   **c** $x^2 - 7x + 10$    **d** $x^2 + 3x - 40$

   **e** $x^2 - x - 42$    **f** $x^2 + 7x + 12$

   **g** $x^2 + 2x - 24$    **h** $x^2 - 16$

   **i** $x^2 + 3x$    **j** $x^2 - 25$

**7** Factorise these quadratic expressions. **E**

   **a** $x^2 - 5x - 6$    **b** $x^2 + 5x + 6$

   **c** $x^2 + 5x - 6$    **d** $x^2 - 5x + 6$

   **e** $x^2 - 4x - 60$    **f** $x^2 + 5x - 36$

   **g** $x^2 - 20x + 99$    **h** $x^2 - 1$

   **i** $x^2 + x - 132$    **j** $x^2 + 6x + 9$

   **k** $x^2 - 10x + 25$    **l** $x^2 - 100$

**8** Factorise these quadratic expressions.

   **a** $x^2 + 7x + 12$    **b** $x^2 + 7x + 10$

   **c** $x^2 - 5x - 6$    **d** $x^2 - 5x + 6$

   **e** $2x^2 + 5x - 12$    **f** $3x^2 + 11x + 6$

   **g** $4x^2 + 12x + 5$    **h** $5x^2 + 13x + 8$

**9** Factorise these quadratic expressions.

   **a** $a^2 + 5ab$    **b** $r^2 + 2r$

   **c** $t^2 - 36$    **d** $b^2 + 11b + 24$

   **e** $4p^2 + 20p + 9$    **f** $5q^2 - 8q - 4$

**10** Factorise these quadratic expressions.

   **a** $x^2 + 3x + 2$    **b** $y^2 - 9$

   **c** $z^2 + 2z$    **d** $n^2 - n - 6$

   **e** $4p^2 - 8p - 5$    **f** $3q^2 - 8q + 4$

**11** Factorise these quadratic expressions.

   **a** $x^2 - 9x + 20$    **b** $x^2 - 3x - 10$

   **c** $4x^2 - 11x + 6$    **d** $6x^2 - 13x - 5$

   **e** $8x^2 - 13x + 5$    **f** $6x^2 + 17x + 5$

**12** Expand and simplify:

   **a** $(x + 1)(x - 2)(x + 3)$

   **b** $(x - 3)(x + 5)(x - 6)$

   **c** $(2x + 1)(x - 5)(x + 4)$

   **d** $(3x - 1)(2x + 3)(4x - 5)$

# You need to:

- **Manipulate algebraic fractions. (Extended)**
- **Factorise and simplify rational expressions. (Extended)**

◀◀ **Recap**

An algebraic fraction is a fraction that contains algebraic expressions.

🔑 **Key skills**

You must be able to manipulate algebraic fractions.

**Worked example**

Simplify $\dfrac{x+1}{3} + \dfrac{x-3}{4}$. **[2 marks]**

LCM of 3 and 4 is 12 so express both terms as fractions with denominator 12.

$$\frac{x+1}{3} = \frac{4x+4}{12} \text{ and } \frac{x-3}{4} = \frac{3x-9}{12}$$

So

$$\frac{x+1}{3} + \frac{x-3}{4} = \frac{4x+4}{12} + \frac{3x-9}{12} = \frac{7x-5}{12}$$

**Exam tip**

The main thing to remember is that, with algebra, you are allowed to do only the things that you are allowed to do with actual numbers.

**Worked example**

Simplify $\dfrac{3(4x-1)}{2} - \dfrac{2(5x+3)}{3}$. **[2 marks]**

LCM of 2 and 3 is 6 so express both terms as fractions with denominator 6.

$$\frac{3(4x-1)}{2} = \frac{36x-9}{6} \text{ and } \frac{2(5x+3)}{3} = \frac{20x+12}{6}$$

So

$$\frac{3(4x-1)}{2} - \frac{2(5x+3)}{3} = \frac{36x-9}{6} - \frac{20x+12}{6} = \frac{16x-21}{6}$$

**Worked example**

Simplify $\dfrac{3}{1-x} - \dfrac{2}{1+x}$. **[2 marks]**

LCM of $1-x$ and $1+x$ is $(1-x)(1+x)$ so

$$\frac{3}{1-x} = \frac{3(1+x)}{(1-x)(1+x)} = \frac{3+3x}{(1-x)(1+x)} \text{ and}$$

$$\frac{2}{1+x} = \frac{2(1-x)}{(1+x)(1-x)} = \frac{2-2x}{(1+x)(1-x)}$$

**Extended**

So

$$\frac{3}{1-x} - \frac{2}{1+x}$$

$$= \frac{3+3x}{(1-x)(1+x)} - \frac{2-2x}{(1+x)(1-x)}$$

$$= \frac{1+5x}{(1+x)(1-x)}$$

🔑 **Key skills**

You must be able to factorise and simplify rational expressions.

**Exam tip**

When you see a question like this, you should expect the expressions on the top and bottom to have a bracket in common when factorised. This is often (although not always) the case, but when it is, it will help you to factorise the expression more quickly.

**Worked example**

Factorise and simplify

(a) $\dfrac{x^2-4x}{x^2-x-12}$  **[2 marks]**

(b) $\dfrac{x^2-7x+12}{2x^2-7x+3}$  **[2 marks]**

___

(a) $\dfrac{x^2-4x}{x^2-x-12}$

$x^2 - 4x = x(x-4)$

$x^2 - x - 12 = (x-4)(x+3)$

Hence

$$\frac{x^2-4x}{x^2-x-12} = \frac{x(x-4)}{(x-4)(x+3)}$$

$$= \frac{x}{(x+3)}$$

(b) $\dfrac{x^2-7x+12}{2x^2-7x+3}$

$x^2 - 7x + 12 = (x-3)(x-4)$

$2x^2 - 7x + 3 = (x-3)(2x-1)$

Hence

$$\frac{x^2-7x+12}{2x^2-7x+3} = \frac{(x-3)(x-4)}{(x-3)(2x-1)}$$

$$= \frac{(x-4)}{(2x-1)}$$

**? Questions**

**1** Simplify, leaving your answers as fractions in their lowest form:

a $\dfrac{2x+1}{3}+\dfrac{x+1}{5}$

b $\dfrac{2x-5}{2}+\dfrac{x+3}{3}$

c $\dfrac{2x+3}{4}+\dfrac{3x-2}{6}$

d $\dfrac{3x+1}{2}+\dfrac{x+2}{7}$

e $\dfrac{3x+5}{3}-\dfrac{x+2}{4}$

f $\dfrac{5x+1}{5}-\dfrac{3x-2}{6}$

g $\dfrac{x-1}{6}-\dfrac{5x+3}{12}$

h $\dfrac{6x-1}{4}-\dfrac{2x-5}{5}$

**2** Simplify, leaving your answers as fractions in their lowest form:

a $\dfrac{x}{2}+\dfrac{3x+1}{5}+\dfrac{2x-1}{10}$

b $\dfrac{x-1}{4}+\dfrac{4x+1}{5}+\dfrac{2x+3}{20}$

c $\dfrac{x-3}{2}+\dfrac{2x-1}{3}+\dfrac{x+1}{6}$

d $x+\dfrac{x}{2}\;\left(\text{write as }\dfrac{2x}{2}+\dfrac{x}{2}\right)$

e $x+\dfrac{2x+1}{2}+\dfrac{x}{3}$

f $x+1+\dfrac{3x+2}{3}+\dfrac{x-2}{4}$

**3** Simplify, leaving your answers as fractions in their lowest form:

a $\dfrac{2}{x+1}+\dfrac{3}{x+2}$

b $\dfrac{3}{x+2}+\dfrac{4}{x+3}$

c $\dfrac{5}{x+1}+\dfrac{7}{x-1}$

d $\dfrac{5}{x+3}-\dfrac{2}{x+2}$

e $\dfrac{3}{2x+3}+\dfrac{4}{3x-1}$

f $\dfrac{5}{3x+1}+\dfrac{4}{4x-3}$

**4** Simplify these fractions.

a $\dfrac{x^2+3x+2}{x+2}$

b $\dfrac{x^2+5x+6}{x+3}$

c $\dfrac{2x^2+3x+1}{x+1}$

d $\dfrac{x^2+x-12}{x+4}$

e $\dfrac{x^2-7x+10}{x-2}$

f $\dfrac{4x^2-8x+3}{2x-3}$

g $\dfrac{x^2-1}{x-1}$

h $\dfrac{25x^2-1}{5x+1}$

To **Raise your grade** now try question 14, page 93

**You need to:**

- Use and interpret positive, negative and zero indices.
- **Use and interpret fractional indices. (Extended)**
- Use the rules of indices.

$2^5$

5 is the index

2 is the base.
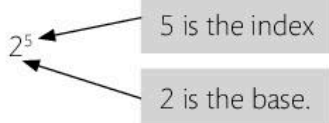
**Recap**

When you write $2 \times 2 \times 2 \times 2 \times 2$ as $2^5$ you are using index notation.

When the index is a positive whole number, such as 5, $2^5$ means 'five 2s multiplied together'.

**Key skills**

You must be able to use the rules of indices.

**Recap**

The three laws of indices are:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$

---

**Worked example**

Simplify these expressions.

(a)  $a^2 \times a^3$

(b)  $(p^3)^4$

(c)  $\dfrac{q^{11}}{q^2}$

(d)  $\dfrac{r^4}{r^7}$

**[4 marks]**

---

(a)  $a^2 \times a^3 = a^5$

(b)  $(p^3)^4 = p^{12}$

(c)  $q^{11} \div q^2 = q^9$

(d)  $r^4 \div r^7 = r^{4-7} = r^{-3}$

---

**Worked example**

Simplify $\dfrac{(3x^2y)^2 \times (2x^3y^2)^3}{(2x^5y^7)^2}$

**[3 marks]**

---

$$\frac{\left(3x^2y\right)^2 \times \left(2x^3y^2\right)^3}{\left(2x^5y^7\right)^2} = \frac{9x^4y^2 \times 8x^9y^6}{4x^{10}y^{14}}$$

$$= \frac{72x^{13}y^8}{4x^{10}y^{14}}$$

$$= 18x^3y^{-6}$$

$$= \frac{18x^3}{y^6}$$

**Exam tip**

Remember to apply the power of each bracket to the numbers as well as the letters.

Set out your working step by step and use only one rule of indices on each line.

You can use the rule for negative indices to write your final answer as a fraction.

**Worked example**

Simplify these expressions.

(a) $\left(\sqrt{a}\right)^{-4}$ **[1 mark]**

(b) $2a^{\frac{1}{2}}b^{\frac{2}{3}} \times 3a^{-\frac{3}{2}}b^{\frac{2}{3}}$ **[3 marks]**

(a) $\left(a^{\frac{1}{2}}\right)^{-4} = a^{-2}$

(b) $2a^{\frac{1}{2}}b^{\frac{2}{3}} \times 3a^{-\frac{3}{2}}b^{\frac{2}{3}} = 2 \times 3 \times a^{\frac{1}{2}} \times a^{-\frac{3}{2}} \times b^{\frac{2}{3}} \times b^{\frac{2}{3}}$
$= 6a^{-1}b^{\frac{4}{3}}$

🔑 **Key skills**

You must be able to use and interpret fractional indices.

**Exam tip**

Reorder the terms and then use the rules of indices.

**❓ Questions**

**1** Simplify these expressions.

a $a^2 \times a^3$  b $b^3 \times b^{14}$

c $c^6 \times c^5$  d $x^6 \times x^7$

e $y^9 \div y^2$  f $z^{11} \div z^4$

g $(q^3)^4$  h $(w^2)^6$

i $(r^2)^3 \div r^4$  j $(s^5)^3 \times s^6$

k $\dfrac{t^2 \times t^5}{t^3}$  l $\dfrac{(w^3)^4 \times w^6}{w^2}$

**2** Find $a$ when:

a $2^3 \times 2^a = 2^7$  b $x^5 \times x^a = x^6$

c $y^2 \times y^a = y^2$  d $r^a \div r^{11} = r^9$

e $s^a \div s^9 = s^7$  f $2^9 \div 2^a = 2^2$

g $(3^5)^a = 3^{20}$  h $(u^a)^a = u^9$

i $(x^a)^{a+1} = x^{42}$

**3** Simplify these fractions.

a $\dfrac{(2x^8) \times (6x^5)}{4x^9}$  b $\dfrac{(4y^6) \times (5y^7)}{10y^{11}}$

c $\dfrac{(8z^2) \times (3z^3)}{2z^4}$  d $\dfrac{(6a^3) \times (4a^9)}{(2a^2) \times (3a^8)}$

e $\dfrac{(12c^3) \times (4c^9)}{(2c^2) \times (3c^6)}$  f $\dfrac{(10h^2) \times (18h^5)}{(5h^4) \times (3h)}$

**4** Simplify these fractions.

a $\dfrac{(4a^3b^4) \times (10a^2b^5)}{5a^4b^2}$  b $\dfrac{(6mn^2) \times (10m^3n^4)}{4mn^2}$

c $\dfrac{(2r^2t^4) \times (10r^5t^3)}{5r^2t^5}$  d $\dfrac{(3g^2h^5) \times (2g^3h^6)}{6g^3h}$

**5** Simplify these expressions.

a $(2x^2y)^3$  b $(3x^3y^4)^2$

c $(8x^2y^3)^2$  d $(5x^2y^5)^3$

e $(9x^6y^7)^2$  f $(8x^7y^3)^2$

g $(2x^2y^3)^4 \times (3x^2y^3)^3$  h $(2x^3y^5)^3 \times (5x^3y^4)^2$

**6** Simplify these expressions. **E**

a $\left(\sqrt[3]{x}\right)^{-4}$

b $\sqrt{y^{-7}}$

c $\sqrt[5]{p^6 \times p^{-7}}$

**7** Simplify these expressions.

a $4x^{\frac{1}{2}} \times 3x^{-\frac{1}{3}}$

b $6x^{\frac{1}{3}}y^{\frac{1}{2}} \times 5x^{-1}y^{\frac{2}{5}}$

c $\dfrac{8x^{\frac{4}{5}}y^{\frac{1}{3}}}{4x^{-\frac{1}{5}}y^{-\frac{1}{3}}}$

## You need to:

- Derive and solve simple linear equations in one unknown.
- Derive and solve simultaneous linear equations in two unknowns.
- **Derive and solve simultaneous equations, involving one linear and one quadratic. (Extended)**
- **Derive and solve quadratic equations by factorisation, completing the square and by use of the formula. (Extended)**
- **Derive and solve linear inequalities. (Extended)**

### 🔑 Key skills

You must be able to derive and solve simple linear equations in one unknown.

### ◀◀ Recap

In a linear equation the highest power of $x$ is 1.

For example, $3x + 2 = 17$ is a linear equation.

When you solve the equation you find the value of $x$ which makes the left-hand side of the equation equal to the right-hand side.

The solution of $3x + 2 = 17$ is $x = 5$ since $3 \times 5 + 2 = 17$.

You solve an equation by doing the same operation to both sides.

### Exam tip

As a general rule, to solve a linear equation:

**Step 1** Clear the fractions.

**Step 2** Expand the brackets.

**Step 3** Put all the $x$-terms on one side of the equation.

**Step 4** Simplify the equation.

**Step 5** Check your solution.

---

**Worked example**

Solve the equation $3x - 5 = 16$. **[2 marks]**

---

| | |
|---|---|
| $3x - 5 = 16$ | Add 5 to both sides. |
| $3x = 21$ | Divide both sides by 3. |
| $x = 7$ | |

---

**Worked example**

Solve the equation $2(x + 4) = 14$. **[2 marks]**

---

| | |
|---|---|
| $2(x + 4) = 14$ | Expand the brackets. |
| $2x + 8 = 14$ | Subtract 8 from both sides. |
| $2x = 6$ | Divide both sides by 2. |
| $x = 3$ | |

**Worked example**

Solve the equation $\dfrac{3x}{4} = 12$. **[2 marks]**

---

$\dfrac{3x}{4} = 12$    Clear the fractions by multiplying both sides by 4.

$3x = 48$    Divide both sides by 3.

$x = 16$

---

**Worked example**

Apples cost $a$ cents and bananas cost $b$ cents.

Avni buys 3 apples and 1 banana and pays 90 cents.
Parmjit buys 1 apple and 2 bananas and pays 80 cents.

(a) Derive two simultaneous equations from this information.

**[2 marks]**

(b) Solve the simultaneous equations to find $a$ and $b$.

**[3 marks]**

---

(a) The cost of 3 apples = $3a$ cents and the cost of 1 banana = $b$ cents.

So, from Avni, $3a + b = 90$       (1)

The cost of 1 apple = $a$ cents and the cost of 2 bananas = $2b$ cents.

So, from Parmjit, $a + 2b = 80$       (2)

(b) You can solve these simultaneous equations by **elimination**.

Multiply equation (1) by 2 so that both equations have a $2b$ term. Give this new equation the label (3).

$6a + 2b = 180$       (3)

Subtract equation (2) from equation (3) to eliminate the $2b$ terms.

$(6a + 2b) - (a + 2b) = 180 - 80$

$5a = 100$, so $a = 20$

Use the value of $a$ in equation (1) to find $b$.

$3 \times 20 + b = 90$, so $b = 30$

Therefore, an apple costs 20 cents and a banana costs 30 cents.

🔑 **Key skills**

You must be able to derive and solve simultaneous linear equations in two unknowns.

**Exam tip**

They are called **simultaneous** equations because the solutions satisfy both of the equations at the same time.

**Exam tip**

It is a good idea to label your simultaneous equations with a number, so you can keep track of which one is which as you change and combine them.

**Exam tip**

The elimination method works by multiplying one or both of the equations by a number (different numbers if you are multiplying both equations), so that the size of the coefficient of one of the variables is the same in both equations.

When using the elimination method, you aim to get the equations into one of these forms:

**1 Same coefficients and same signs** or **2 Same coefficients but different signs**

$$3x + 2y = 19 \quad (1)$$
$$6x + 2y = 34 \quad (2)$$

$$2x + 6y = 26 \quad (1)$$
$$7x - 6y = 10 \quad (2)$$

**SAME SIGNS SUBTRACT**

In this case **subtract** the equations: (2) − (1)

$$6x - 3x + 2y - 2y = 34 - 19$$
$$3x = 15$$
$$x = 5$$

Put $x = 5$ in equation (1).

$$3 \times 5 + 2y = 19$$
$$15 + 2y = 19$$
$$2y = 4$$
$$y = 2$$

**DIFFERENT SIGNS ADD**

In this case **add** the equations: (1) + (2)

$$2x + 7x + 6y - 6y = 26 + 10$$
$$9x = 36$$
$$x = 4$$

Put $x = 4$ in equation (1).

$$2 \times 4 + 6y = 26$$
$$8 + 6y = 26$$
$$6y = 18$$
$$y = 3$$

**Exam tip**

The elimination method is the quickest method, when it works. Often though, you will need to use substitution.

---

**Worked example**

Solve the simultaneous equations $\quad 2x + 5y = 19$

and $\quad y = x + 1$ **[3 marks]**

---

First, number your two equations.

$$2x + 5y = 19 \quad \quad (1)$$
$$y = x + 1 \quad \quad (2)$$

Equation (2) gives $y$ in terms of $x$ so substitute $(x + 1)$ for $y$ in equation (1).

$$2x + 5(x + 1) = 19$$
$$7x + 5 = 19$$
$$7x = 14, \text{ so } x = 2$$

Put $x = 2$ in equation (2).

$$y = 2 + 1 = 3$$

The solution is $x = 2$, $y = 3$

### Recap

Quadratic equations are equations where the highest power of any variable is 2. There are three main ways to solve them.

**Worked example**

Solve the $x^2 + 5x - 24 = 0$ **[3 marks]**

This can be solved by factorising.

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

Either $x + 8 = 0$ giving $x = -8$

or $x - 3 = 0$ giving $x = 3$

**Worked example**

Solve $x^2 + 8x + 5 = 0$ by completing the square. **[3 marks]**

$x^2 + 8x + 5 = (x + 4)^2 - 11$

$8 \div 2 = 4$     $5 = 4^2 - 11$

So you need to solve $(x + 4)^2 - 11 = 0$

Hence $(x + 4)^2 = 11$

So $x + 4 = \pm\sqrt{11}$ and so $x = -4 \pm\sqrt{11}$.
$x = -0.683$ or $x = -7.32$ (to 3 s.f.)

### 🔑 Key skills

You must be able to derive and solve quadratic equations by factorisation, completing the square and by use of the formula.

### Exam tip

Factorising is always the quickest and easiest method when the equation is easy to factorise. Always start by seeing if this is possible.

### Exam tip

Completing the square is most useful when the coefficient of $x^2$ is 1 and the coefficient of $x$ is an even number. If this is not the case, it can become messy, and you might want to just use the quadratic formula.

### 👁 Watch out!

Don't forget the 'plus or minus' when you square root both sides.

### Recap

The quadratic formula states that the solutions to the equation

$ax^2 + bx + c = 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The two solutions are usually combined in the form $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $\pm$ means 'plus or minus'.

To use this method replace the letters $a$, $b$, and $c$ with the numbers that come from the particular equation you want to solve.

**Worked example**

Solve the equation $3x^2 - 2x - 7 = 0$. **[3 marks]**

The equation can't be factorised, and the coefficient of $x^2$ is not 1, so solve it using the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Write down the values of $a$, $b$ and $c$.

$a = 3$, $b = -2$, $c = -7$

Substitute these values in the formula.

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -7}}{6}$$

$$= \frac{2 \pm \sqrt{88}}{6}$$

$$= \frac{2 \pm 9.3808\ldots}{6}$$

$$= 1.90 \text{ or } -1.23 \text{ to 3 s.f.}$$

### Watch out!

It is probably not a good idea to type the whole formula into the calculator in one go. Instead, break the calculation up into smaller parts.

### Key skills

You must be able to derive and solve linear inequalities.

### Recap

You can solve inequalities in the same way as you solve equations. You can:

- add and subtract any number from both sides of an inequality
- multiply and divide both sides of an inequality by a positive number.

**If you multiply or divide by a negative number** you have to reverse the inequality sign e.g. $-3 < 1$ but $3 > -1$. Try to avoid doing this as it is easy to make a mistake.

**Worked example**

Solve $x + 2 > 7$. **[1 mark]**

$x + 2 > 7$     Subtract 2 from both sides.

$x > 5$

**Worked example**

Solve $-7x > 21$. **[2 marks]**

| | |
|---|---|
| $-7x > 21$ | Add $7x$ to both sides. |
| $0 > 21 + 7x$ | Subtract 21 from both sides. |
| $-21 > 7x$ | Divide both sides by 7. |
| $-3 > x$ | |
| $x < -3$ | |

or

| | |
|---|---|
| $-7x > 21$ | Divide by $-7$ and reverse inequality sign. |
| $x < -3$ | |

**👁 Watch out!**

This method is quicker but you can easily make a mistake.

You can express inequalities on a number line.

$x > -3$ can be expressed as



$x \leq 4$ can be expressed as



$1 < x \leq 7$ can be expressed as



How can you remember whether to fill in the circle or not?

The signs $\geq$ and $\leq$ use more ink than $>$ and $<$.

• uses more ink than ∘

'Less than or equal to' and 'greater than or equal to' use more ink than 'less than' or 'greater than'.

$\leq$ goes with 'less than or equal to' and •
$\geq$ goes with 'greater than or equal to' and •
$<$ goes with 'less than' and ∘
$>$ goes with 'greater than' and ∘

**Key skills**

You must be able to derive and solve simultaneous equations, involving one linear and one quadratic.

**Apply**

If one equation is quadratic and one is linear, what are the maximum and minimum number of solutions the simultaneous equations could have?

**Worked example**

Solve the simultaneous equations $y = x^2 + 3$ and $y = 10 - \dfrac{3}{2}x$

**[3 marks]**

Because both equations start with '$y=$', you can equate the right hand sides to give

$$x^2 + 3 = 10 - \frac{3}{2}x.$$

Multiplying both sides by 2: $\qquad 2x^2 + 6 = 20 - 3x$

Bringing everything to one side: $\qquad 2x^2 + 3x - 14 = 0$

Factorising: $\qquad (2x + 7)(x - 2) = 0$

Therefore: $x = -\dfrac{7}{2}$ or $x = 2$

But $y = x^2 + 3$, so when $x = 2$, $y = 2^2 + 3 = 7$

and when $x = -\dfrac{7}{2}$, $y = \left(-\dfrac{7}{2}\right)^2 + 3 = \dfrac{49}{4} + 3 = 15.25$

**Questions**

1 The three angles in a triangle are $a$, $a + 20$ and $a + 25$.
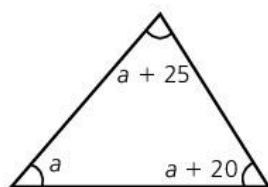


   a Write down an equation involving $a$.

   b Solve this equation to find $a$.

2 Three consecutive whole numbers add up to 144. If the lowest number is $n$ then:

   a write down expressions for the other two numbers in terms of $n$

   b write down an equation involving $n$

   c find $n$ and hence find the other two numbers.

3 Solve the following equations.

   a $3(x - 1) = 2(x + 1)$

   b $2(5x + 2) = 6(3x - 2)$

   c $5(7x - 3) = 4(9x - 4)$

4 Solve the following equations.

   a $\dfrac{15}{x} = 5$  b $\dfrac{6}{x} = 2$

   c $\dfrac{42}{x + 3} = 6$  d $\dfrac{35}{2x + 1} = 5$

5 Solve these simultaneous equations by elimination.

   a $2x + y = 11$
   $3x - y = 14$

   b $3u + 2v = 10$
   $7u - v = 29$

   c $11p + 3q = 71$
   $5p - q = 37$

   d $9a + 2b = 41$
   $5a - 4b = 33$

   e $7p - 3q = 15$
   $5p + 2q = 19$

   f $13b - 7c = 47$
   $7b - 9c = 41$

6 Solve these simultaneous equations (by the substitution method).

   a $5m + 3n = 27$
   $m = 7 - n$

   b $2p + 7q = 3$
   $p = 6 + q$

   In parts **c – f** first rearrange one of the equations.

   c $u + 2v = 7$
   $2u + 3v = 11$

   d $3p + 2q = 21$
   $p - 3q + 4 = 0$

   e $7r + 2s = 17$
   $r - 3s = 9$

   f $5x - 7y + 5 = 26$
   $x - y - 9 = 0$

7 A man buys 5 first class tickets and 2 second class tickets which cost him €246. Another man buys 2 first class and 3 second class tickets which cost him €149.

   Let the price of a first class ticket be €$x$ and the price of a second class ticket be €$y$.

   a Write down a pair of simultaneous equations involving $x$ and $y$.

   b Find $x$ and $y$.

**8** Solve these equations.

**a** $x^2 + 7x + 12 = 0$  **b** $y^2 + 13y + 22 = 0$

**c** $m^2 - 5m - 6 = 0$  **d** $a^2 - 5a + 6 = 0$

**e** $z^2 - 4z - 12 = 0$  **f** $z^2 + 2z + 1 = 0$

**g** $c^2 + 15c + 36 = 0$  **h** $t^2 - 18t + 81 = 0$

**i** $r^2 - 6r = 0$  **j** $t^2 + 11t = 0$

**k** $w^2 - 3w = 0$  **l** $k^2 + k = 0$

**9** Solve the following equations by completing the square (leaving square roots in your answers).

**a** $x^2 + 2x - 1 = 0$  **b** $x^2 - 4x - 3 = 0$

**c** $x^2 + 12x + 36 = 0$  **d** $x^2 + 20x + 5 = 0$

**e** $x^2 + 8x - 9 = 0$  **f** $x^2 - 2x - 7 = 0$

**10** Solve the following equations by completing the square. First write them in the form $x^2 + px + q = 0$. Leave square roots in your answers.

**a** $2x^2 + 4x - 6 = 0$  **b** $3x^2 + 15x - 12 = 0$

**c** $2x^2 + 10x + 1 = 0$  **d** $2x^2 + 8x - 12 = 0$

**11** **a** If $f(x) = x^2 + 4x + 5$ then show that the equation can be written as $f(x) = (x + 2)^2 + 1$.

**b** Use the result from part **a** to explain why $f(x)$ cannot take a value lower than 1.

**c** Use the result from part **a** to also explain why $f(x)$ takes the minimum value of 1 when $x = -2$.

**12** *Edible Reptiles* claims that its bags of snakes contain 5 more snakes than the bags from *Snakes Alive* and that they charge 1 cent less per snake than *Snakes Alive*. Anjana buys a bag from *Snakes Alive* for $5.00. Dhruv buys a bag of the same snakes for $5.70 from *Edible Reptiles*.

**a** If $n$ is the number of snakes in a bag from *Snakes Alive* then find, in terms of $n$:

**i** how many snakes are in a bag from *Edible Reptiles*

**ii** the cost per snake (in cents) at both shops.

**b** Set up an equation involving $n$ and solve it to find $n$.

**13** A rectangular box is 23 cm longer than it is wide. Its diagonal is 65 cm. If $x$ is the width of the box then:

**a** use Pythagoras' theorem to find an expression for the length of the box in terms of $x$

**b** show that $x^2 + 23x - 1848 = 0$

**c** solve this equation to find the exact value of $x$.

**14** Solve the following inequalities.

**a** $3x - 7 < 17$

**b** $10 - 2x \geq 4$

**c** $6x - 3 > 3(x + 2)$

**d** $7 - 4x \leq 9 - x$

**15** Solve the following pairs of simultaneous equations.

**a** $y = x^2 - 6x + 10$
    $y = x + 4$

**b** $y = 8x^2 - 22x + 6$
    $y = 8x - 7$

**c** $x^2 + y^2 = 25$
    $y = 3x - 5$

**d** $y = \dfrac{8}{x}$
    $y = 2.56x + 3.2$

**You need to:**
- **Represent inequalities graphically and use this representation to solve simple linear programming problems. (Extended)**

**Extended**

## Key skills

You must be able to represent inequalities graphically.

## Exam tip

Use solid lines to show that the points on the line are included in the region described by an inequality.

Use dotted lines to show that the points on the line are not included in the region described by an inequality.

This is true regardless of whether you are shading the region that is included or not included.

## Watch out!

Make sure you know if the question wants you to shade the area that *is* in the region, or the area that *isn't*.

## Key skills

You must be able to solve simple linear programming problems.

## Recap

Just as equations can be represented graphically by lines and curves, inequalities can be represented by regions.

### Worked example

By shading the unwanted region on the diagram, show the region defined by the inequalities $x \geq 2$, $y > 1$, $x + y < 6$

**[4 marks]**

First draw the three lines $x = 2$, $y = 1$, $x + y = 6$

Use dotted lines for $x + y = 6$, $y = 1$ and a solid line for $x = 2$.
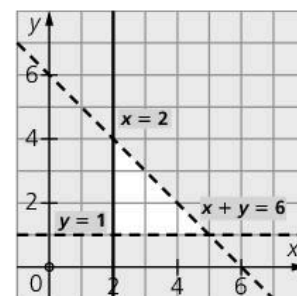


$x \geq 2$ describes the region to the right of $x = 2$. So shade to the left of $x = 2$.

$y > 1$ describes the region above the line $y = 1$. So shade below $y = 1$.

$x + y < 6$ describes the region below the line $x + y = 6$. So shade above $x + y = 6$.

The region $x \geq 2$, $y > 1$, $x + y < 6$ is shown unshaded.

## Recap

Shading inequalities is used mainly for solving linear programming problems.

Linear programming is a graphical method of finding the best solution to a problem that is defined by simultaneous inequalities.

### Worked example

(a) Leave unshaded the region defined by the inequalities $x \leq 4$, $y < 2x + 1$, $5x + 2y > 20$ **[3 marks]**

(b) Find the maximum value of $x + y$ for points that have integer coordinates in this region. **[2 marks]**

(a)  First draw the three lines $x = 4$, $y = 2x + 1$, $5x + 2y = 20$

Use dotted lines for $y = 2x + 1$ and $5x + 2y = 20$ (because points on these lines are not in the solution set) and a solid line for $x = 4$ (because points on this line are in the solution set).

Shade to the right of $x = 4$.

Shade above $y = 2x + 1$.

Shade below $5x + 2y = 20$



**Exam tip**

In linear programming questions, you always shade the area that is *not* in the region defined by the inequality.
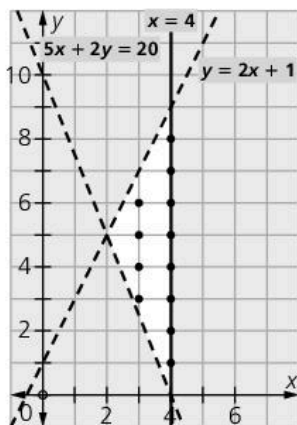
(b)  In the unshaded region (the solution set) mark the points which have integer coordinates.

The possible points are marked as black dots on the diagram (in the solution to part (a)).

The table shows the value of $x + y$ at each of the possible points.

| $x$ | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
|-----|---|---|---|---|---|---|---|---|---|----|----|----|
| $y$ | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $x + y$ | 6 | 7 | 8 | 9 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

The maximum value of $x + y = 12$.

---

**? Questions**

1  Ashley is baking cupcakes and brownies to sell in his cake shop.

Each cupcake costs 10 cents to make, and sells for 50 cents.
Each brownie costs 25 cents to make, and sells for 75 cents.

Ashley has $10 to spend on ingredients.

He must bake at least 20 cupcakes and at least 10 brownies for regular customers.

Let $x$ be the number of cupcakes he bakes.
Let $y$ be the number of brownies he bakes.

a  Write down three inequalities which must be satisfied.

b  Assuming he sells all the cupcakes and brownies he bakes, how many of each type should he bake in order to make the maximum profit?

**You need to:**

- Continue a given number sequence. Recognise patterns in sequences including the term-to-term rule and relationships between different sequences. Find and use the $n$th term of a sequence, **including exponential sequences. (Extended)**

---

🔑 **Key skills**

You must be able to continue a given number sequence.

⏪ **Recap**

A sequence is a set of numbers that follows a pattern.

🔑 **Key skills**

You must be able to recognise patterns in sequences, including the term-to-term rule and relationships between different sequences.

🔑 **Key skills**

You must be able to find and use the $n$th term of sequences.

---

**Worked example**

Write down the next two terms in each of these sequences:

(a)   5, 8, 11, 14, 17,…            **[1 mark]**

(b)   2, 6, 18, 54,…               **[1 mark]**

(c)   22, 15, 8, 1,…               **[1 mark]**

---

(a)   The term-to-term rule is to 'add 3' so the next two terms are 20 and 23.

(b)   The term-to-term rule is to 'multiply by 3' so the next two terms are 162 and 486.

(c)   The term-to-term rule is to 'subtract 7' so the next two terms are –6 and –13.

---

**Worked example**

Find the formula for the $n$th term of this sequence:

   4, 10, 16, 22, 28,…             **[2 marks]**

---

The gap between the terms is always 6, therefore it is a linear sequence.

This means the $n$th term formula is of the form:
$n$th term $= an + b$

$a$ represents the gap between consecutive terms, therefore the formula is:

   $n$th term $= 6n + b$

   When $n = 1$        $4 = 6 \times 1 + b$

   therefore        $b = -2$

so the formula is        $n$th term $= 6n - 2$

---

**Worked example**

(a)   What is the $n$th term of 19, 13, 7, 1,…?    **[2 marks]**

(b)   Which term of 19, 13, 7, 1,… is equal to –47?    **[2 marks]**

(a) The differences are all –6.

So the $n$th term is $-6n + 25$
which can be written as $25 - 6n$.

(b) From part (a) the $n$th term is $25 - 6n$

so $25 - 6n = -47$

$n = 12$

So the 12th term is equal to –47.

---

**Worked example**

Find the $n$th term formula for this sequence:

3, 5, 9, 17, 33,…                                    **[2 marks]**

---

The sequence is exponential

$2^n$ gives 2, 4, 8, 16, 32,…

So the $n$th term formula is $2^n + 1$

Compare the sequence to a known exponential sequence, in this case the sequence $2^n$.

🔑 **Key skills**

You must be able to find and use the $n$th term of an exponential sequence.

Ⓔ

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**? Questions**

**1** A football club has 35 000 supporters at its first home match. The attendance increases by 250 at each home game.

  **a** How many supporters will be at the $n$th home game?

  **b** If there are 37 750 at the last home game of the season then how many home games did the club play?

**2** Joshua decides to save money in the following way:

He saves £1 in the first week, £1.20 in the second week, £1.40 in the third week, and so on.

  **a** How much would he save in the $n$th week?

  **b** How much would he save in the 8th week?

  **c** In which week would he first save at least £5?

  **d** After 10 weeks, Joshua wants to buy a tennis racquet which costs £19.99. He realises that he hasn't saved quite enough, but by how much is he short?

**3** Find the first three terms in a sequence whose $n$th term is:

  **a** $5n - 1$          **b** $n^2 - 1$
  **c** $\frac{1}{2}n(n+1)$          **d** $\frac{n^3 + 1}{3n}$

**4** Find the next two terms in each sequence.

  **a** 5, 10, 20, 40, 80,…

  **b** 1, 8, 27, 64, 125,…

  **c** 2, 8, 18, 32, 50,…

  **d** $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36},…$

**5** Find the $n$th term formulae for these sequences:

  **a** 7, 9, 11, 13,…          **b** 16, 13, 10, 7,…

  **c** 2, 5, 10, 17, 26,…          **d** 2, 8, 18, 32, 50,…

  **e** 0, 7, 26, 63, 124,…

**6 a** Write down the next two terms in each of these sequences:   Ⓔ

  **i** 4, 6, 10, 18, 34,…          **ii** 3, 6, 12, 24, 48,…

  **b** Find the $n$th term formulae for these sequences:

  **i** 1, 3, 7, 15, 31,…          **ii** 6, 12, 24, 48, 96,…

**To Raise your grade now try questions 2, 10 and 19, pages 92–93**

**You need to:**

- Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities. (Extended)

### Extended

🔑 **Recap**

Two variables are directly proportional if there is always a constant ratio between them.

🔑 **Key skills**

You must be able to express direct proportion in algebraic terms and use this form of expression to find unknown quantities.

🔑 **Key skills**

You must be able to express inverse proportion in algebraic terms and use this form of expression to find unknown quantities.

$y \propto x$ means that $y$ is directly proportional to $x$.

$y \propto x$ can be rewritten as $y = kx$ where $k$ is a constant that you can find.

For example, the number of apples, $A$, on a tree is directly proportional to the number of branches, $b$. If $b$ doubles then $A$ doubles.

In a similar way, $y \propto (x + 1)^2$ means that $y$ is directly proportional to $(x + 1)^2$. This can be rewritten as $y = k(x + 1)^2$ where $k$ is a constant.

$y \propto \dfrac{1}{x}$ means that $y$ is inversely proportional to $x$. This can be rewritten as $y = \dfrac{k}{x}$ where $k$ is a constant.

For example, the average speed of a runner, $v$, in a race is inversely proportional to the time, $T$, he takes to run the race. If $T$ doubles then $v$ halves.

In a similar way, $y \propto \dfrac{1}{\sqrt{x+2}}$ means that $y$ is inversely proportional to $\sqrt{x+2}$.

This can be rewritten as $y = \dfrac{k}{\sqrt{x+2}}$ where $k$ is a constant.

### Solving problems involving proportion

In all cases the method is the same:

- Write the relation between $x$ and $y$ as an equation using $k$.
- Find $k$ using a pair of values given in the question and replace $k$ in the equation with this value.
- Use the equation to find other values of $x$ and $y$.

---

**Worked example**

$y$ is inversely proportional to $x$ and $y = 24$ when $x = 5$. Find the value of

(a) $y$ when $x = 2$

(b) $x$ when $y = 30$. **[2 marks]**

---

Write the relation between $x$ and $y$ as an equation using $k$.

$$y = \frac{k}{x}$$

Find $k$ using the pair of values given in the question.

When $x = 5$, $y = 24$ so $24 = \dfrac{k}{5}$ and so $k = 120$

Hence $y = \dfrac{120}{x}$

Use the equation to find values of $x$ and $y$.

(a) When $x = 2$, $y = \dfrac{120}{2} = 60$

(b) When $y = 30$, $30 = \dfrac{120}{x}$ and so $x = 4$

**Extended**

## Questions

1   Rewrite these statements using a constant $k$.

   **a**   $S \propto t$

   **b**   $F \propto \dfrac{1}{r^2}$

   **c**   $G$ varies as $r^2$.

   **d**   $E$ is proportional to $m$.

2   $P$ is proportional to $S$. If $P = 16$ when $S = 4$, calculate:

   **a**   the value of $P$ when $S = 7$

   **b**   the value of $S$ when $P = 80$.

3   Competitors in a 'World's Strongest Man' competition are required to carry large spherical stones called Atlas Stones, which range from 100 to 160 kilograms.

   If the mass of each stone is proportional to the cube of its radius, and a 100 kg stone has a radius of 21.9 cm, what will be the radius of a 160 kg stone? (Give your answer correct to 1 decimal place.)

4   The distance, $s$, travelled by an object is directly proportional to the square of the time, $t$, for which it has been travelling. When $t = 5$, $s = 75$.

   **a**   Evaluate $k$, the constant of proportionality and write down an equation for $s$ in terms of $t$.

   **b**   Find the value of $s$ when $t = 7$.

   **c**   Find $t$ when $s = 363$.

   **d**   Describe what happens when $t$ is doubled

5   The mass, $m$, of an object is directly proportional to the cube of its side length, $l$. The mass of a cube with side length 3 cm is found to be 216 g.

   **a**   Calculate the constant of proportionality and write down an equation for $m$ in terms of $l$.

   **b**   Find the mass of an object with side length 7 cm.

   **c**   Find the side length of an object which has mass 9261 g.

   **d**   Describe what happens to $l$ when $m$ increases by 33.1%.

6   The light intensity, $I$, is measured at a distance $d$ away from a lamp. It is found that $I \propto \dfrac{1}{d^2}$. It is observed that $I = 180$ when $d = 7$.

   **a**   Find the constant of proportionality and write down an equation involving $I$ and $d$.

   **b**   Find the value of $I$ when $d = 2$.

   **c**   Find the value of $d$ when $I = 45$.

   **d**   Describe what happens to $d$ when $I$ decreases by 75%.

7   The Eiffel Tower in Paris is repainted every 7 years. The job takes 25 painters working for 18 months to complete.

   Assuming they didn't get in each other's way, find how long the job would take:

   **a**   50 painters

   **b**   75 painters.

## You need to:

- Use function notation, e.g. $f(x) = 3x - 5$, $f: x \mapsto 3x - 5$, to describe simple functions. (Extended)
- Find inverse functions $f^{-1}(x)$. Form composite functions as defined by $gf(x) = g(f(x))$. (Extended)

## Extended

**Key skills**

You must be able to use function notation, e.g. $f(x) = 3x - 5$, $f: x \mapsto 3x - 5$, to describe simple functions.

**Recap**

A **function** is a mapping in which each element in the domain has one, and only one, image in the range.

Functions are usually denoted by the letters f, g etc, so you can write

$f: x \mapsto x + 1$   or   $f(x) = x + 1$   and   $g: x \mapsto x^2$   or   $g(x) = x^2$

**Exam tip**

The $f(x)$ notation and the arrow notation are used interchangeably. They mean exactly the same thing.

**Recap**

To find the inverse of $f(x)$:

- put $f(x) = y$ and make $x$ the subject of the formula in terms of $y$
- replace each $y$ with $x$ to find $f^{-1}(x)$.

**Key skills**

You must be able to find inverse functions $f^{-1}(x)$.

**Watch out!**

Only one-to-one functions have inverses.

### Worked example

Find the inverse of $f(x) = \dfrac{3x - 5}{2}$    **[3 marks]**

- Putting $f(x) = y$:    $y = \dfrac{3x - 5}{2}$

  Making $x$ the subject of the formula: $x = \dfrac{2y + 5}{3}$

- Replacing $y$ with $x$ gives $f^{-1}(x) = \dfrac{2x + 5}{3}$

**Key skills**

You must be able to form composite functions as defined by $gf(x) = g(f(x))$.

**Recap**

If $f(x) = x + 1$ and $g(x) = x^2$ then 'gf' means 'do f first, then g'.

So $gf(x)$ means $g(f(x))$.

### Worked example

If $f(x) = x + 3$ and $g(x) = x^2$ then find:

(a)  the value of $x$ for which $f(x) = 8$    **[2 marks]**

(b)  the values of $x$ for which $g(x) = 36$    **[2 marks]**

(c)  $fg(4)$    **[2 marks]**

(d)  $gf(x)$    **[2 marks]**

**Watch out!**

$fg(x)$ is not the same as $gf(x)$. Work from the inside outwards.

(a) Solving $f(x) = 8$ means solving $x + 3 = 8$. So $x = 5$.

(b) Solving $g(x) = 36$ means solving $x^2 = 36$. So $x = 6$ or $-6$.

(c) $g(4) = 4^2 = 16$ so $f(g(4)) = f(16) = 16 + 3 = 19$

(b) $f(x) = x + 3$ so $gf(x) = g(x + 3) = (x + 3)^2$

## Questions

**1** If $f(x) = x^2 + 3$, find:

  **a** $f(2)$

  **b** $f(-1)$

  **c** a value of $x$ such that $f(x) = 3$.

**2** Given the function $f : x \mapsto \dfrac{3x^2 + 1}{2}$, find:

  **a** $f(0)$

  **b** $f(2)$

  **c** $f(-3)$

**3** If $f(x) = \dfrac{x + 1}{x - 2}$ then find $f^{-1}(x)$.

**4** If $f(x) = x^2$ and $g(x) = x + 1$, find:

  **a** $fg(2)$

  **b** $fg(x)$

  **c** a value of $x$ such that $fg(x) = 16$

  **d** $gg^{-1}(-3)$

**5** The functions f and g are as follows:

$$f : x \mapsto 2x + 5$$

$$g : x \mapsto 2 + \sqrt{x}$$

  **a** Calculate $f(-3)$.

  **b** Given that $f(a) = 17$, find the value of $a$.

  **c** Find the inverse function of g.

**6** Find the inverses of the following functions, in the form '$x \mapsto \ldots$'

  **a** $f : x \mapsto 4x - 1$    **b** $f : x \mapsto \dfrac{3(x + 4)}{2}$

**You need to:**

- Interpret and use graphs in practical situations including travel graphs and conversion graphs.
- Draw graphs from given data.
- **Apply the idea of rate of change to simple kinematics involving** distance-time and speed-time graphs, acceleration and deceleration. Calculate distance travelled as area under a speed-time graph. (Extended)

◀◀ **Recap**

Graphs are often useful for practical things, such as performing unit conversions (for example, converting one currency into another) or analysing a journey.

🔑 **Key skills**

You must be able to interpret and use conversion graphs.

**Worked example**



The graph shows the amount that a shop charges for hiring a bike for up to 8 hours in a day.

There is an initial charge and then an hourly charge.

(a) What is the initial charge? **[1 mark]**

(b) What is the hourly charge? **[1 mark]**

(c) How much would it cost to hire a bike for 3 hours? **[1 mark]**

(d) How many hours' hire would cost $28? **[1 mark]**

........................................................
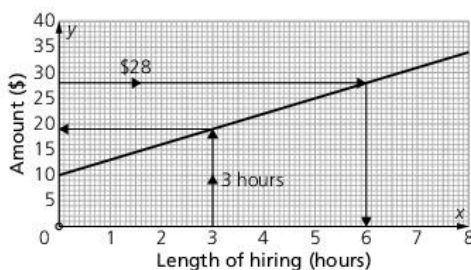


(a) The initial charge is the cost when the time is zero. This is $10.

(b) From the graph you can see that 1 hour costs $13. So the hourly charge is $3.

(c) Reading off from the graph gives $19.

(d) Reading off from the graph gives 6 hours.

**Extended**

🔑 **Key skills**

You must be able to interpret travel graphs.

🔑 **Key skills**

You must be able to apply the idea of rate of change to simple kinematics problems involving distance-time and speed-time graphs, acceleration and deceleration.

## Distance-time graphs

Gradient on a distance-time graph represents speed.

This graph shows the journey of a boy cycling. From the graph you can tell how the boy's speed changes.

**What is happening to the speed...**

Between 0 s and 2 s the boy is speeding up.

Between 2 s and 5 s the boy is travelling at a constant speed.

Between 5 s and 7 s the boy is slowing down.

Between 7 s and 10 s the boy is stationary.

Between 10 s and 12 s the boy is speeding up.

From 12 s to 15 s the boy is travelling at a constant speed.

From 15 s to 17 s the boy is slowing down.
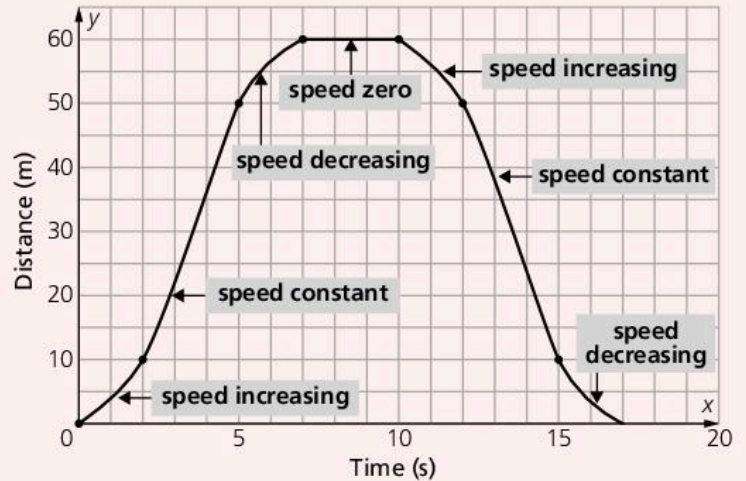
## Speed-time graphs

Gradient on a speed-time graph represents acceleration.

Area under a speed-time graph represents distance travelled.

This graph shows the speed of a toy car. From the graph you can tell how the car's speed and acceleration change.

**What is happening to the speed**

Between 0 s and 17 s the car is speeding up to reach a top speed of $12\,\mathrm{m\,s^{-1}}$.

Between 17 s and 25 s the car has a constant speed of $12\,\mathrm{m\,s^{-1}}$.

From 25 s onwards the car is slowing down (decelerating) to come to rest.

**What is happening to the acceleration**

Between 0 s and 2 s the acceleration is increasing.

Between 2 s and 12 s the acceleration is constant.

Between 12 s and 17 s the acceleration is decreasing.

Between 17 s and 25 s the acceleration is zero.

Between 25 s and 30 s the deceleration is increasing.

From 30 s to 35 s the deceleration is constant.

From 35 s to 40 s the deceleration is decreasing.
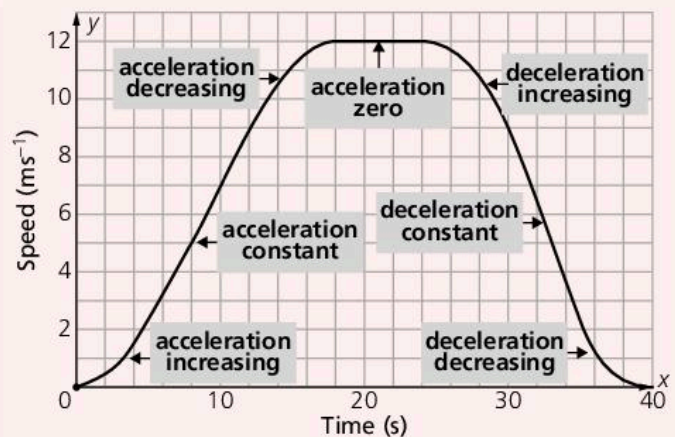
🔑 **Key skills**

You must be able to calculate distance travelled as area under a speed-time graph.

**Extended**

**Worked example**

A coach was hired to take a school team to a football game. The graph shows the distance travelled from the school.

(a) How far did the coach travel before stopping? **[1 mark]**

(b) For how long did the coach stop? **[1 mark]**

(c) How much further did the coach travel to the game? **[1 mark]**

(d) How long did the coach wait at the game? **[1 mark]**

(e) How long did the return journey take? **[1 mark]**

(f) What was the average speed of the coach on the return journey? **[3 marks]**
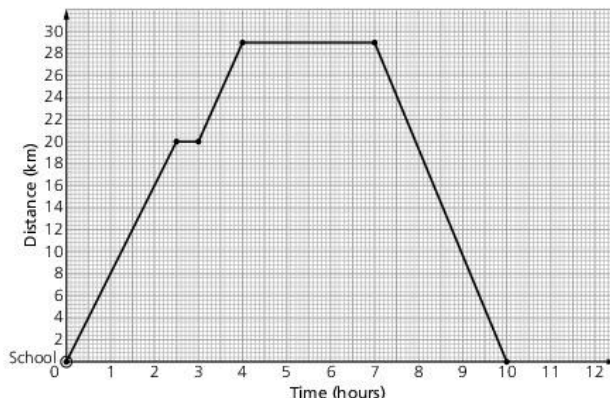
(a) The first line segment stops at a height of 20 km, so the answer is 20 km.

(b) The first flat line segment goes from 2.5 to 3 on the horizontal axis, so the answer is 30 min.

(c) The next time the coach stops is at 29 km. 29 − 20 = 9, so the answer is another 9 km.

(d) The second flat line segment goes from 4 to 7 on the horizontal axis, so the answer is 3 hours.

(e) The final section of the graph goes from 7 to 10 on the horizontal axis, so the answer is 3 hours.

(f) On the return journey, they travel 29 km in 3 hours, so the average speed is $29 \div 3 = 9.67 \text{ kmh}^{-1}$
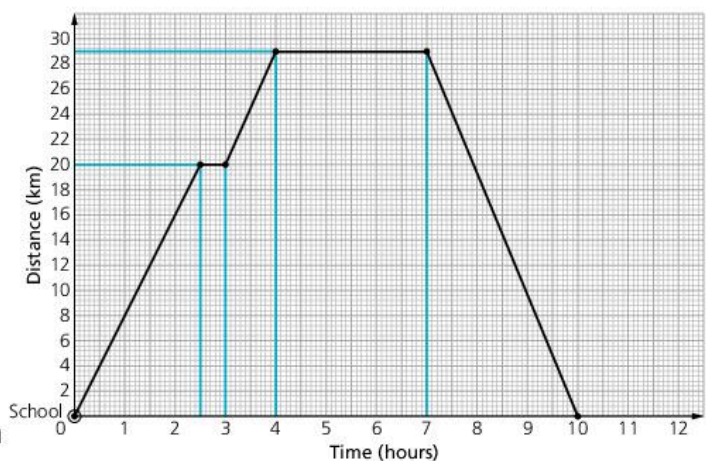
**Exam tip**

Begin by drawing horizontal and vertical lines that separate the graph at the important places, and show clearly where these places are, relative to the horizontal and vertical axes.

**Exam tip**

Remember: speed = distance ÷ time

In the above Worked example, distance is in kilometres and time is in hours, so speed is in kilometres per hour.

'kilometres' are a 'distance', 'per' is 'divided by' and 'hour' is a 'time'.

This idea can be used with different units of compound measure.

**? Questions**

1   The graph below shows the amount an electrician charges for up to five hours' work.
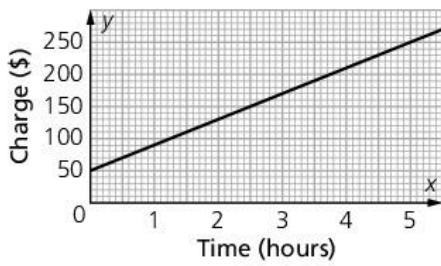


a   What does the electrician charge for being called out?

b   How much does he charge for 2 hours' work?

c   If he charged Mr Bali $210, then how many hours work did he do?

d   A second electrician has a call out fee of $60 and an hourly fee of $35. For how many hours work would the two electricians charge the same amount?

2   Hasnain drove to see a friend. The graph **E** shows his journey to and from his friend's house and the time he spent with his friend.
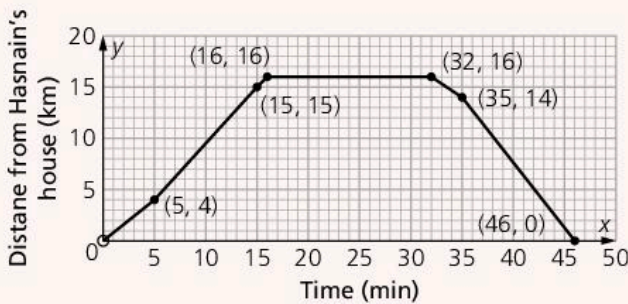


a   How long did it take for Hasnain to reach a constant speed?

b   What was happening to his speed in the first 5 minutes?

c   What was the constant speed in km h$^{-1}$?

d   How long did Hasnain spend at his friend's house?

e   What was his constant speed for the last 11 minutes of his journey?

3   Awais was on a fast road and the graph shows his journey.
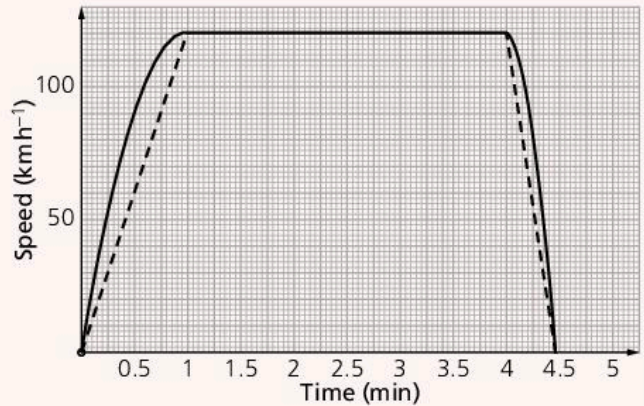


a   How long did it take Awais to reach a constant speed?

b   How far did he travel at this constant speed?

c   What was happening to his acceleration in the first minute?

d   By using the dotted lines for the first minute and the last 30 seconds, estimate how far Awais travelled.

e   Is your answer to part **d** an under-estimate or an over-estimate of the distance that Awais travelled? Give a reason for your answer.

4   The graph shows the journey made by Karen on her bicycle. She cycled from her home to a café 50 km away and then back.
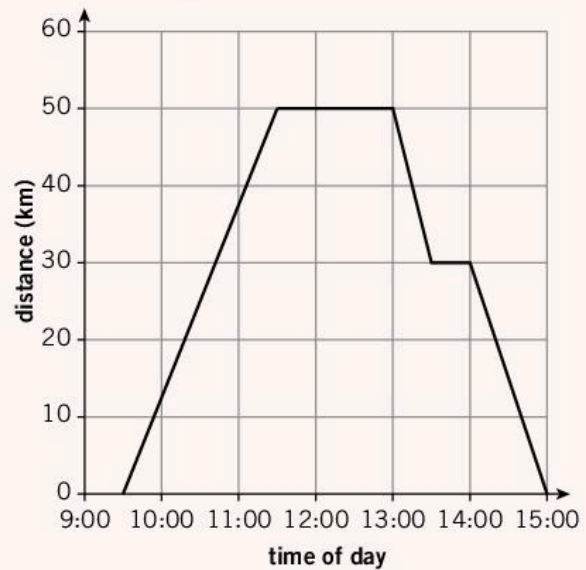


a   How long did she stay at the café?

b   What was her speed during the first part of her journey?

c   What was her average speed while cycling on the way back?

**You need to:**

- Construct tables of values for functions of the form $ax + b$, $\pm x^2 + ax + b$, $\frac{a}{x} (x \neq 0)$ where $a$ and $b$ are integers, and functions of the form $ax^n$ (and simple sums of these) and functions of the form $ab^x + c$. **(Extended)**
- Draw and interpret these graphs.
- Solve linear and quadratic equations approximately, including finding and interpreting roots by graphical methods.
- Draw and interpret graphs representing exponential growth and decay problems. **(Extended)**
- Recognise, sketch and interpret graphs of functions.

### 🔑 Key skills

You need to be familiar with several different types of graphs.

### E 🔑 Key skills

The graphs may also include functions of the form $ax^n$ (and simple sums of these) and functions of the form $ab^x + c$.

### 🔑 Key skills

You must be able to construct tables of values for functions of the form $ax + b$, $\pm x^2 + ax + b$, $\frac{a}{x} (x \neq 0)$, where $a$ and $b$ are integers, draw and interpret these graphs, and use them to find approximations to roots of equations.

### Exam tip

When plotting the line, you don't have to plot *all* the points, but you should plot at least 3 of them. Remember that the $x$-axis is horizontal and that the $y$-axis is vertical (alphabetical order in both): $x$, $y$; $h$, $v$.

---

**Worked example**

The table gives the values of $x$ and $y$ for the function $y = 3x + 2$.

| $x$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|----|----|----|----|----|----|----|
| $y$ | | 2 | 5 | 8 | 11 | | |

(a) Fill in the missing values of $y$. **[1 mark]**

(b) Sketch the graph of $y = 3x + 2$. **[2 marks]**

(c) Use your graph to solve the equation $3x + 2 = 10$. **[1 mark]**

---

(a) To find the $y$-value when $x = -1$ substitute $-1$ for $x$ into $3x + 2$.

$$3 \times -1 + 2 = -1$$

Repeating this for $x = 4$ and $x = 5$:

| $x$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|----|----|----|----|----|----|----|
| $y$ | −1 | 2 | 5 | 8 | 11 | 14 | 17 |

(b) Mark the points on the graph, that is (−1, −1), (0, 2), etc.    Join the points using a straight line.
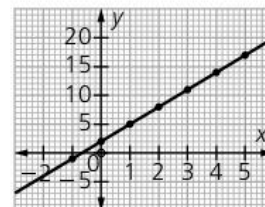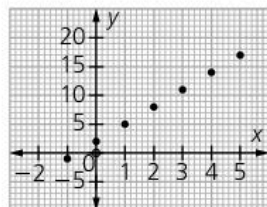


(c) The solution to $3x + 2 = 10$ is the $x$-coordinate of the point on the graph where $y = 3x + 2$ meets $y = 10$



$y = 3x + 2$

$y = 10$

$x$-coordinate is about 2.7.
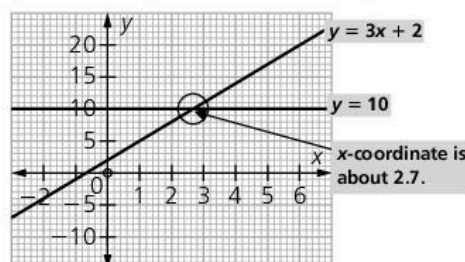
From the graph, the solution to $3x + 2 = 10$ is 2.7 (to 1 d.p.).

**Worked example**

The table gives the values of $x$ and $y$ for the function $y = x^2 - 4x + 4$.

| $x$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $y$ | | 4 | 1 | | 1 | | 9 |

(a) Fill in the missing values of $y$. **[1 mark]**

(b) Sketch the graph of $y = x^2 - 4x + 4$. **[2 marks]**

(c) Use the graph to solve $x^2 - 4x + 4 = 7$. **[2 marks]**

---

(a) To find the value of $y$ when $x = -1$ substitute $-1$ for $x$ into $x^2 - 4x + 4$.

Use brackets on the calculator, so type in $(-1)^2 - 4(-1) + 4$ to get 9. Repeating this for the values 2 and 4 gives:

| $x$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |

(b) Mark the points on the graph, that is $(-1, 9)$, $(0, 4)$, etc.    Join the points using a smooth curve.
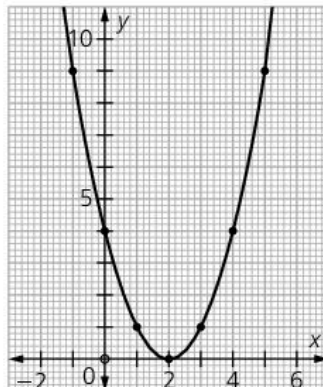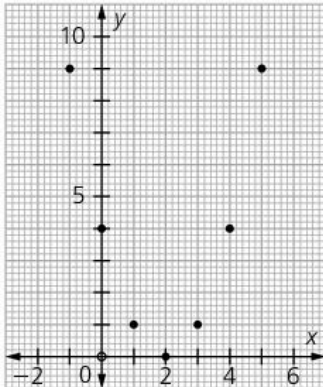


(c) The solutions to $x^2 - 4x + 4 = 7$ are the $x$-coordinates of the points on the graph where $y = x^2 - 4x + 4$ meets $y = 7$.



$y = x^2 - 4x + 4$

$y = 7$

*x-coordinates are about −0.6 and 4.6.*
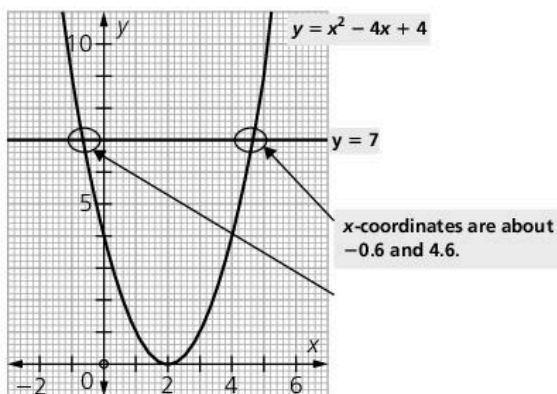
From the graph, the solutions to $x^2 - 4x + 4 = 7$ are −0.6 and 4.6 (to 1 d.p.).

**Exam tip**

The basic shape of the graph of $y = ax^2 + bx + c$ is shown in these diagrams.

(i) $a > 0$
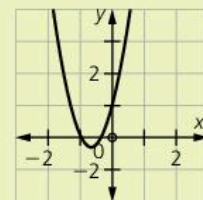


(ii) $a < 0$
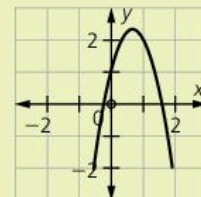


A positive value of $a$ gives the graph a smiley shape, because it is feeling positive.

A negative value of $a$ gives the graph a frowny shape, because it is feeling negative.

**Worked example**

The table gives the values of $x$ and $y$ for the function $y = \dfrac{24}{x}$.

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-6$ | $-8$ | | | $-$ | | 12 | 8 | |

(a) Fill in the missing values of $y$. **[1 mark]**

(b) Sketch the graph of $y = \dfrac{24}{x}$. **[2 marks]**

---

(a) To find the value of $y$ when $x = -2$ substitute $-2$ for $x$ into $\dfrac{24}{x}$ to give $y = -12$. Repeating this for $-1$, 2 and 4 gives:

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-6$ | $-8$ | $-12$ | $-24$ | $-$ | 24 | 12 | 8 | 6 |

(b) Mark the points on the graph, that is $(-4, -6)$, $(-3, -8)$ etc. Join up the points using a smooth curve.
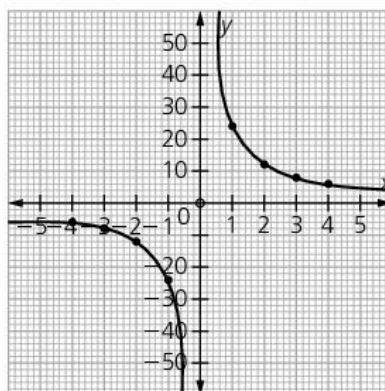


---

**Recap**

An exponential graph is a graph of a function where the variable (usually $x$) is the power, rather than the base.

For example:

$x^2$ is a quadratic function

$2^x$ is an exponential function

**Graph of $y = a^x$**

When $a > 0$ the graph of $y = a^x$ has this basic shape:

Crosses $y$-axis at $(0, 1)$.

Tails off towards the $x$-axis for 'large' negative values of $x$.

Gradient is always positive and always increasing.
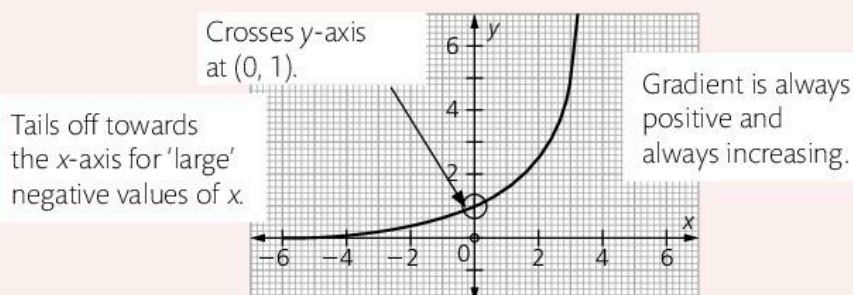
**1**  This table of values is for the graph $y = x^2 - 6x + 1$.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | | | −7 | | | | |

   **a**  Copy and complete the table.
   **b**  Draw a scale from 0 to 6 on the $x$-axis (2 cm per unit) and from −10 to 5 on the $y$-axis (1 cm per unit).
   **c**  Draw a sketch of the curve $y = x^2 - 6x + 1$.
   **d**  Use your graph to find the values of $x$ (to 1 d.p.) when $y = 0$ and $y = -5$.

**2**  This table of values is for $y = x^2 - 2x - 20$.

| $x$ | −5 | −4 | −3 | −2 | −1 | 0 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 15 | 4 | −5 | | −17 | | −20 | −17 | | −5 | 4 | |

   **a**  Copy and complete the table.

   **b**  Draw a scale from −5 to 7 on the $x$-axis (1 cm per unit) and from −30 to 40 on the $y$-axis (2 cm per 5 units). Draw a sketch of the curve $y = x^2 - 2x - 20$.

   **c**  Copy and complete the table of values for $y = \dfrac{24}{x}$.

| $x$ | −5 | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | −4.8 | −6.0 | | | −24.0 | − | 24.0 | 12.0 | 8.0 | | 4.8 | | 3.4 |

   **d**  On the same diagram as part **b** draw the graph of $y = \dfrac{24}{x}$ for $-5 \le x \le 7$.

   **e**  Write down the $x$-coordinate of the two intersection points of the two graphs.

      These two $x$-values are the solutions to an equation. Write down and simplify this equation.

**3**  The table of values is for the graph $y = x^2 - 2x - 4$.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | 4 | |

   **a**  Copy and complete the table.
   **b**  Draw a scale from −3 to 5 on the $x$-axis (2 cm per unit) and from −8 to 12 on the $y$-axis (1 cm per unit).
   **c**  Draw a sketch of the curve $y = x^2 - 2x - 4$.
   **d**  Use your graph to solve (to 1 d.p.) the equation $x^2 - 2x - 4 = 0$.
   **e**  Use your graph again to solve (to 1 d.p.) the equation $x^2 - 2x - 4 = 5$.
   **f**  What is the smallest value of $x^2 - 2x - 4$ and which value of $x$ achieves this smallest value?

**4**  Draw the graph of $y = 2^x$ using $x$ values from −1 to 5 (1 cm per unit) and a scale of 1 cm per 5 units on the $y$-axis. Find the approximate value of $x$ when $y = 10$. **E**

## You need to:

- **Estimate gradients of curves by drawing tangents. (Extended)**

**Extended**

Unlike straight lines, finding the gradient of a curve at a given point can be tricky.

One way to estimate it is to draw a tangent to the curve and find the gradient of the tangent.

**Key skills**

You must be able to estimate gradients of curves by drawing tangents.

**Exam tip**

Remember that the gradient of a curve at a point is the same as the gradient of the tangent to the curve at that point.

**Worked example**

Approximate the gradient of the curve $y = x^2 + 1$ at the point (2, 6) by drawing a tangent. **[3 marks]**

First plot the graph as accurately as you can, near to the point where you want to find the gradient.

Draw in a tangent and then, using the most convenient points available, make a right-angled triangle underneath it. From the sides of the triangle, you can work out the gradient of the line.
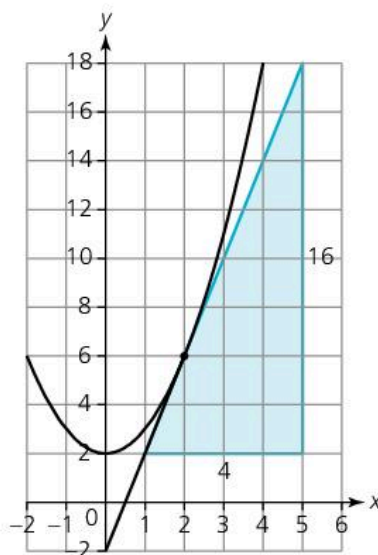


In this example, the estimate of the gradient is $16 \div 4 = 4$.

**Watch out!**

Be careful to check how the axes are calibrated. Don't just count the squares.

**Questions**

1. Draw a tangent to find an approximation of the gradient of the curve $y = (x - 2)^2 + 1$ when $x = 3$.
2. Draw a tangent to find an approximation of the gradient of the curve $y = x^2 - 3x + 5$ when $x = 2$.
3. Draw a tangent to find an approximation of the gradient of the curve $y = 5 - x^2$ at the point (2, 1).

**You need to:**

- Understand the idea of a derived function. (Extended)
- Use the derivatives of functions of the form $ax^n$, and simple sums of not more than three of these. (Extended)
- Apply differentiation to gradients and turning points (stationary points). (Extended)
- Discriminate between maxima and minima by any method. (Extended)

**Recap**

A derived function is obtained by differentiating another function.

Differentiation is part of an important area of mathematics called Calculus.

Given any function $y = f(x)$, the function $\frac{dy}{dx}$ is obtained by differentiating the original function.

Although differentiation has many applications, you should primarily think of $\frac{dy}{dx}$ as being the function that tells you the gradient of $y = f(x)$ for any given value of $x$.

**Recap**

The derivative of a function of the form $ax^n$ is $anx^{n-1}$. In other words, multiply by the power, then subtract 1 from the power.

$\frac{d}{dx}ax^n = anx^{n-1}$.

**Recap**

If the expression has several terms, you just differentiate each term, one at a time.

**Worked example**

Differentiate the following functions:

(a) $y = x^2$      [1 mark]
(b) $y = 3x^4$      [1 mark]
(c) $y = 5x^2 + 3x + 2$      [1 mark]

---

(a) $\frac{dy}{dx} = 2 \times x = 2x$

(b) $\frac{dy}{dx} = 4 \times 3x^{4-1} = 12x^3$

(c) $\frac{dy}{dx} = 2 \times 5x^{2-1} + 3 = 10x + 3$

**Extended**

**Key skills**

You must understand the idea of a derived function.

**Watch out!**

It is not a good idea, when you are new to calculus, to think of $\frac{dy}{dx}$ as being a fraction. It is better just to think of it as being the name of a function.

**Key skills**

You must be able to use the derivatives of functions of the form $ax^n$, and simple sums of not more than three of these.

**Exam tip**

The derived function is often called the derivative. They mean exactly the same thing.

**Exam tip**

$\frac{d}{dx}$ is a function that means 'the derivative of'.

**Exam tip**

Remember that when you differentiate a constant term, it becomes zero (i.e. it disappears).

**Extended**

A stationary point on a graph is any place where the gradient is zero. A turning point is a type of stationary point.

A turning point can be a 'local maximum' or a 'local minimum'.

Usually you just say 'maximum' or 'minimum', and if you are talking about more than one you call them 'maxima' or 'minima'.

🔑 **Key skills**

You must be able to apply differentiation to gradients and turning points (stationary points).

**Worked example**

Find the $x$-coordinates of any stationary points of the function $y = x^2 - 6x$. **[3 marks]**

---

If $y = x^2 - 6x$ then $\dfrac{dy}{dx} = 2x - 6$

A stationary point is where $\dfrac{dy}{dx} = 0$, so $2x - 6 = 0$

Therefore $x = 3$

**Exam tip**

Note that this turning point could probably have been found more quickly by completing the square.

✏️ **Apply**

Find the turning point of the quadratic curve in the above Worked example by completing the square.

Completing the square was recapped in section 2.5.

↩ **Recap**

You need to be familiar with two methods for discriminating between different types of turning points.

1. Using the second derivative.
2. Considering the gradients on both sides of the point.

🔑 **Key skills**

You must be able to discriminate between maxima and minima by any method.

**Exam tip**

To 'discriminate' means to recognise which kind of turning point it is.

**Exam tip**

To 'determine their nature' is another phrase commonly used to mean 'work out which kind of turning point they are'.

**Worked example**

Find the $x$-coordinates of the turning points of the function $y = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 - 2x - 1$, and determine the nature of the points. **[5 marks]**

---

If $y = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 - 2x - 1$ then $\dfrac{dy}{dx} = x^2 - x - 2$

A stationary point is where $\dfrac{dy}{dx} = 0$, so $x^2 - x - 2 = 0$

This factorises as $(x-2)(x+1)=0$

Therefore $x=-1$ or $x=2$

**Method 1 Using the second derivative**

$$\frac{d^2y}{dx^2} = 2x-1$$

When $x=-1$, $\frac{d^2y}{dx^2} = (2\times-1)-1=-3$, which is negative,

therefore $x=-1$ is a maximum.

When $x=2$, $\frac{d^2y}{dx^2} = (2\times2)-1=3$, which is positive,

therefore $x=2$ is a minimum.

**Method 2 Considering the gradients on both sides of the point**

When $x=-1$, you need to look at the gradients on each side.

| $x$ | −2 | −1 | 0 |
|---|---|---|---|
| $\frac{dy}{dx}$ | 4 | 0 | −2 |

Because the gradient is positive before the stationary point and negative directly after it, you can tell that $x=-1$ is a maximum.

When $x=2$, you need to look at the gradients on each side.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $\frac{dy}{dx}$ | −2 | 0 | 4 |

Because the gradient is negative before the stationary point and positive directly after it, you can tell that $x=-2$ is a minimum.

**Exam tip**

$\frac{d^2y}{dx^2}$ is called 'the second derivative'. To find $\frac{d^2y}{dx^2}$ you differentiate the expression for $\frac{dy}{dx}$.

**Exam tip**

Remember that if the second derivative is also zero, you must use Method 2.

**◉ Watch out!**

By taking a step of one whole unit either side of the stationary point, you must be careful that nothing else significant happens to the curve within that interval.

Exam questions, however, are usually designed so that nothing significant does happen.

You can, if you prefer, use points that are nearer to the stationary point.

**◉ Watch out!**

Sometimes it might seem obvious which point is a maximum and which is a minimum, because you know the shape of the graph, but you still need to prove it using algebra.

**? Questions**

1 Find the coordinates of the stationary point of the function $y = 2x^2 + 4x - 3$ and determine its nature.

2 Find the turning point of the function $y = x^4 + 1$ and determine its nature.

3 Find the x-coordinates of the stationary points of the function $y = x^3 - 3x^2 - 24x + 8$ and determine their nature.

**1** Simplify $\dfrac{6x^4y^2}{3x^2y}$ **[2 marks]**

**2** A sequence begins 1, 3, 9, 27, 81,…

Write down:

   **a** the $n$th term rule **[1 mark]**
   **b** the 15th term in the sequence. **[1 mark]**

**3** Find the point of intersection of these straight lines:
$$3x + 4y = 22$$
$$-5x + 2y = 24$$
**[4 marks]**

**4** Expand and simplify $(x+1)(2x+1)(x-3)$ **[2 marks]**

**5** In this diagram, the rectangle $ABCD$ is attached to the top of the trapezium $CDEF$. $AB = 2(x+1)$ cm, $BC = 2(x-1)$ cm, $EF = 4x$ cm, and the perpendicular height of the trapezium is $(2x-3)$ cm.

   **a** Write an expression for the area of rectangle $ABCD$. **[1 mark]**

   **b** Write an expression for the area of trapezium $CDEF$. **[1 mark]**

   **c** Write an expression for the total area of $ABCD$ and $CDEF$. **[1 mark]**

   **d** The total area of $ABCD$ and $CDEF$ is 91 cm². Calculate the value of $x$. **[4 marks]**
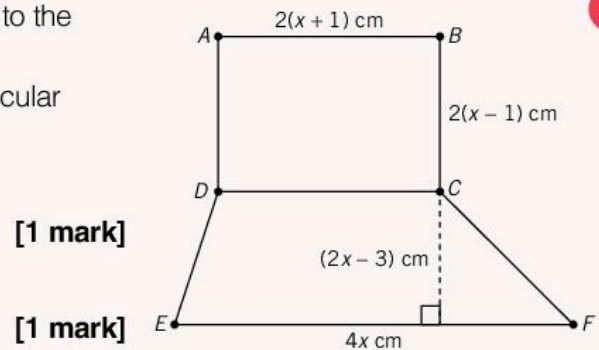
**6** The volume $V$ of a cone with base radius $r$ and height $h$ is given by the formula $V = \dfrac{1}{3}\pi r^2 h$.

   **a** Calculate the volume of a cone with base radius 6 cm and height 12 cm. **[2 marks]**

   **b** Rearrange the formula to make $h$ the subject. **[1 mark]**

   **c** A cone has a volume of 960 cm³ and a base radius of 12 cm. Calculate its height. **[2 marks]**

   **d** A cone has a volume of $72\pi$ cm³, and its radius and height are equal. Calculate its height. **[2 marks]**

**7** $x^2 - 6x + 14$ can be written in the form $(x + p)^2 + q$. Find the values of $p$ and $q$. **[2 marks]**

**8** Find the value of $x$ in this equation: $\dfrac{7}{5} + \dfrac{16}{10} = \dfrac{x}{4}$ **[3 marks]**

**9** Solve the inequality $\dfrac{x-3}{5} < \dfrac{2x-3}{4}$ **[3 marks]**

**10** A sequence begins 10, 7, 4, 1, –2,…

Write down:

   **a** the $n$th term rule **[1 mark]**

   **b** the smallest value of $n$ for which the $n$th term is less than –100. **[1 mark]**

**11** The diameter of Saturn is 10 times the diameter of Venus, and the diameter of Venus is 2.4 times the diameter of Neptune. If the diameter of Saturn is $x$ times the diameter of Neptune, what is $x$? **[5 marks]**

**12** The sum $S$ of the first $n$ positive integers can be found using the formula:

$$S = \frac{n(n+1)}{2}$$

If the sum of the first $n$ positive integers is 153:

    **a** show that $n^2 + n - 306 = 0$ **[2 marks]**

    **b** solve this quadratic equation to find the value of $n$. **[3 marks]**

**13** If $x = -\dfrac{1}{4}$, from the expressions $x^{-2}$, $x^{-1}$, $x^0$, $x$ and $x^2$, write down the one that has:

    **a** the largest value **[3 marks]**

    **b** the smallest value. **[1 mark]**

**14** Show that the expression $\dfrac{a}{b} + \dfrac{a-b}{ab} - \dfrac{b}{a}$ can be rearranged to $\dfrac{(a-b)(a+b+1)}{ab}$. **[4 marks]**

**15** Given that $y = 3x^3 - 4x^2 + 2x + 7$, find:

    **a** $\dfrac{dy}{dx}$ **[2 marks]**

    **b** $\dfrac{d^2y}{dx^2}$ **[2 marks]**

**16** Factorise $3x^2 + x - 2$ **[2 marks]**

**17** On the right is an accurate graph of $y = f(x)$.

    **a** Use the graph to solve:

        **i** $f(x) = 0$ **[1 mark]**

        **ii** $f(x) = -1$. **[1 mark]**

    **b** Write down the equation of the line that would need to be added to the graph to solve the equation $f(x) - 2x + 1 = 0$. **[1 mark]**

    **c** Estimate the gradient of the curve at the point where $x = 2$. **[1 mark]**

    **d** Estimate the value(s) of $x$ where the gradient $= 0$. **[2 marks]**
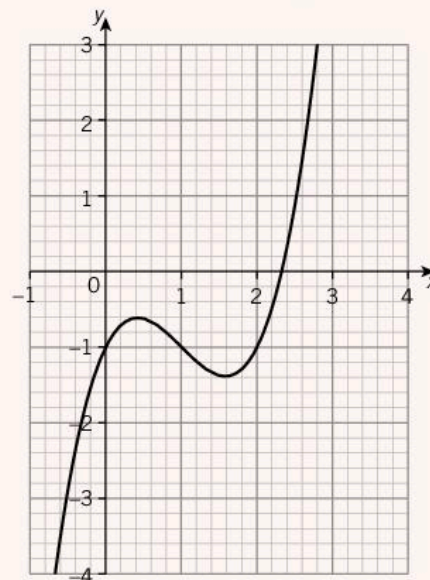
**18** Given that f: $x \mapsto x + 1$ and g: $x \mapsto 3x$, find the following composites in the form '$x \mapsto \ldots$':

    **a** fg **[2 marks]**

    **b** gf **[2 marks]**

**19** A sequence begins $\dfrac{1}{2}, \dfrac{3}{5}, \dfrac{5}{8}, \dfrac{7}{11}, \dfrac{9}{14}, \ldots$ Write down:

    **a** the $n$th term rule **[2 marks]**

    **b** the 20th term in the sequence. **[1 mark]**

**20** The graph below shows the train journey between Birmingham and Stratford-upon-Avon.

   **a** Find the acceleration of the train for the first two minutes of the journey. **[2 marks]**

   **b** Find the distance, in km, between Birmingham and Stratford-upon-Avon. **[2 marks]**

   **c** Find the average speed of the train journey. Give your answer in kmh⁻¹. **[2 marks]**
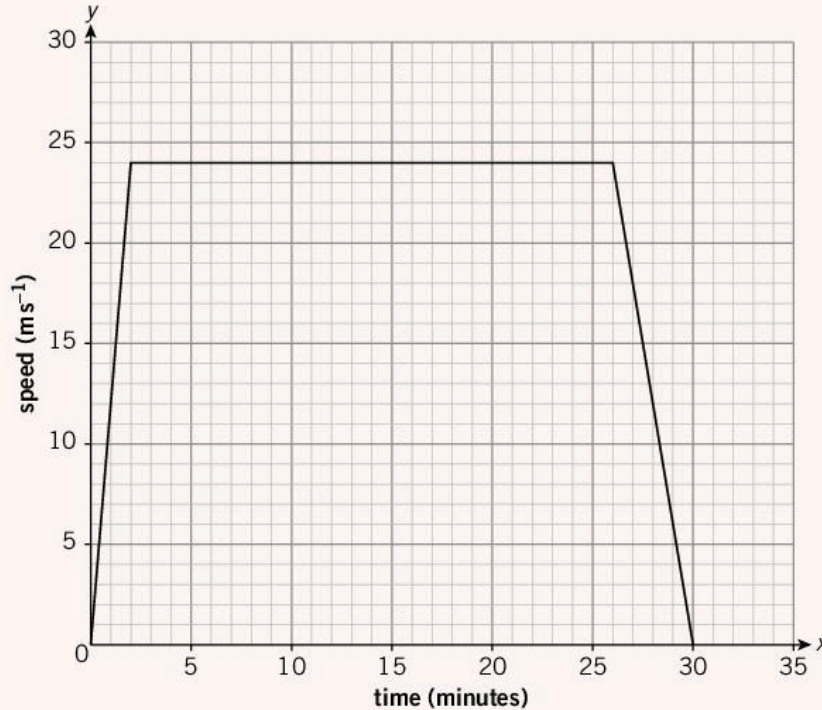


**21** $f(x) = x^2 + x - 2$ and $g(x) = \dfrac{x-1}{2}$

Find:

   **a** $g^{-1}(x)$ **[2 marks]**

   **b** $gg^{-1}(4)$ **[2 marks]**

   **c** $fg(x)$ in the form $\dfrac{x^2 - a}{b}$, where $a$ and $b$ are both integers. **[2 marks]**

**22** When a beach ball is inflated, its surface area $A$ is proportional to the square of its radius $r$.

   **a** What happens to $A$ when $r$ is doubled? **[2 marks]**

   **b** What happens to $A$ when $r$ is decreased by 20%? **[2 marks]**

**23** Find the inverses of the following functions, in the form '$x \mapsto \ldots$':

   **a** $f: x \mapsto \dfrac{1}{2}(x^2 + 6) + 5$ **[2 marks]**

   **b** $f: x \mapsto \dfrac{\sqrt{x^2 + 7}}{3}$ **[2 marks]**

**24** Find the turning points of the function

$$f(x) = 2x^3 + 3x^2 - 36x + 4$$

and determine their nature. **[10 marks]**

# 3 Coordinate geometry

## Your revision checklist

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 3.1 | Demonstrate familiarity with Cartesian coordinates in two dimensions. | | | |
| 3.2 | Find the gradient of a straight line. <br> Calculate the gradient of a straight line from the coordinates of two points on it. | | | |
| 3.3 | **Calculate the length and the coordinates of the midpoint of a straight line from the coordinates of its end points.** | | | |
| 3.4 | Interpret and obtain the equation of a straight line graph in the form $y = mx + c$. | | | |
| 3.5 | Determine the equation of a straight line parallel to a given line. | | | |
| 3.6 | **Find the gradient of parallel and perpendicular lines.** | | | |

## You need to:

• Demonstrate familiarity with Cartesian coordinates in two dimensions.

### Key skills

You must be able to use and interpret two-dimensional coordinates.

### Recap

Coordinates are pairs of numbers that determine a location on a grid, relative to a fixed point called the 'origin'. The origin has the coordinates (0, 0) and is the point where the two axes cross: the horizontal axis is the $x$-axis, and the vertical axis is the $y$-axis.

In a two-dimensional coordinate pair, the first number tells you the horizontal distance from the origin (the distance along the $x$-axis), and the second number tells you the vertical distance (the distance along the $y$-axis).

### Exam tip

Remembering that the letter $x$ looks like 'a cross' (across) is a common way of remembering which axis is which.

### Exam tip

Notice from the way the axes are labelled that positive coordinates are to the right and upwards, and negative coordinates are to the left and downwards.

### Exam tip

Coordinates are always written inside round brackets with a comma separating the two numbers.

### Worked example

What are the coordinates of the four points $A$, $B$, $C$ and $D$?
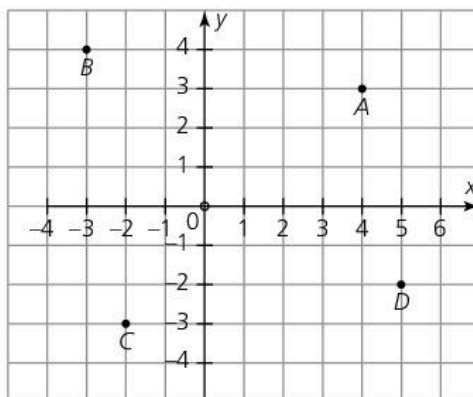


[4 marks]

To get to the point $A$ from the origin, you go across by 4 and up by 3. This means $A$ has the coordinates (4, 3). The coordinates of $B$, $C$ and $D$ are (–3, 4), (–2, –3) and (5, –2) respectively.

### Questions

1   What are the coordinates of the points $E$, $F$, $G$, $H$, $I$ and $J$?
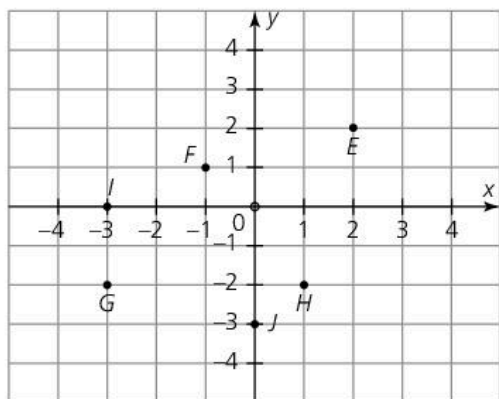
## You need to:

- Find the gradient of a straight line.
- Calculate the gradient of a straight line from the coordinates of two points on it.

To find the gradient of a straight line, choose two points $(x_1, y_1)$ and $(x_2, y_2)$ on the line and divide the difference between the two $y$-coordinates by the difference between the two $x$-coordinates:

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Key skills**

You must be able to find the gradient of a straight line, using two points on the line.

### Worked example

Find the gradient of the line that passes through the points (2, 1) and (4, 5).
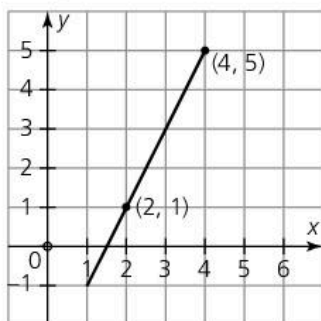


**[2 marks]**

Here $(x_1, y_1) = (2, 1)$, and $(x_2, y_2) = (4, 5)$

The gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 1}{4 - 2} = \dfrac{4}{2} = 2$

**Apply**

Note that in the above Worked example $(x_1, y_1)$ was chosen as (2, 1) and $(x_2, y_2)$ as (4, 5), but this is an arbitrary choice. It could have been the other way around. Recalculate the gradient with the coordinates the other way around and comment on your answer.

**Exam tip**

Usually you will choose which point to subtract from the other based on reducing the amount of negative numbers involved in the calculation.

**? Questions**

**1** For each pair of points, find the gradient of the straight line that joins them.

  **a** (1, 2) and (4, 6)           **b** (5, 7) and (3, 1)

  **c** (−1, 4) and (−3, 9)        **d** (−2, 7) and (−5, −1)

  **e** (−5, −6) and (−8, −11)   **f** (−10, −12) and (14, 0)

**2** Find the gradient of each straight line in the diagram.
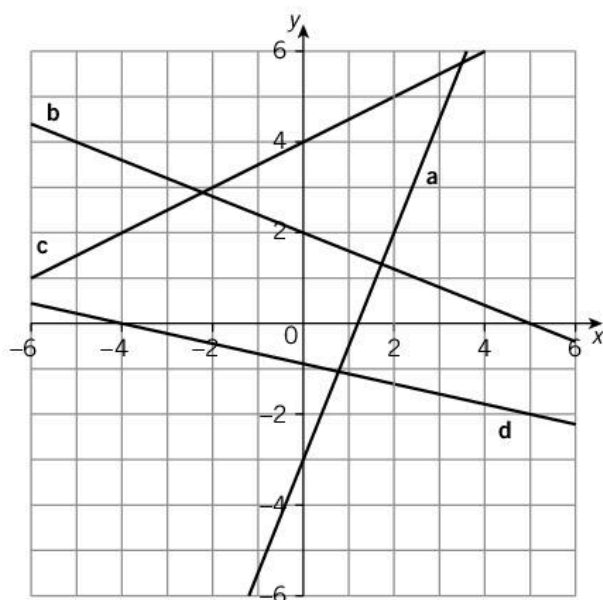


**3** Find the value of $a$, given that the line joining (5, $a$) and (3, 10) has a gradient of −3.

**4** Find the value of $b$, given that the line joining (1, 6) and ($b$, 2) has a gradient of $-\frac{1}{2}$.

**5** Find the value of $c$, given that the line joining (−7, −2) and (−6, $c$) has a gradient of −3.

**6** Find the value of $d$, given that the line joining ($d$, 6) and (3, $d$) has a gradient of 2.

**7** Find the gradient of the line passing through the points ($4e$, $12e$) and ($7e$, $9e$).

**8** The line through (5, $p$) and (3, 2) and the line through (2, −4) and (6, $p$) have the same gradient.

  **a** Find $p$.

  **b** Find the gradient of the lines.

**You need to:**

- Calculate the length and the coordinates of the midpoint of a straight line from the coordinates of its end points. (Extended)

**Extended**

⏪ **Recap**

To find the length of a line segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$, use the formula:

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

🔑 **Key skills**

You need to be able to calculate the length of a line segment joining two points, using the coordinates of the two points.

⏪ **Recap**

To find the midpoint of a line segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$, use the formula:

$$\text{midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

🔑 **Key skills**

You need to be able to find the midpoint of a line segment joining two points, using the coordinates of the two points.

**Worked example**

Point $A$ is (2, 1) and point $B$ is (8, 5).

(a) Find the midpoint $M$ of the line segment $AB$.  **[2 marks]**

(b) Find the length of $AB$.  **[2 marks]**
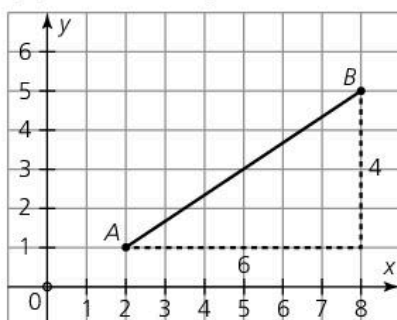


(a) The midpoint $M = \left( \dfrac{2+8}{2}, \dfrac{1+5}{2} \right) = (5, 3)$

(b) The length of $AB = \sqrt{(8-2)^2 + (5-1)^2}$
$= \sqrt{36 + 16} = \sqrt{52} = 7.21$ (to 2 d.p.)

**Exam tip**

This is the same as taking the mean of the two $x$-coordinates and the mean of the two $y$-coordinates.

**Exam tip**

This formula is just a re-arrangement of Pythagoras' theorem, where the line segment joining the two points is the hypotenuse of a triangle.

The triangle in the diagram shows this. See section 6.2 for revision of Pythagoras' theorem.

❓ **Questions**

1   Point $A$ has coordinates (4, 6), and point $B$ is at (−2, 3).

   **a**  Find the length of the line segment $AB$.

   **b**  The midpoint of $AB$ is point $M$. Find the coordinates of $M$.

2   $A$ has coordinates (5, 4). The midpoint of the line segment $AB$ is (1, 1).

   **a**  Find the coordinates of $B$.

   **b**  Find the length of the line segment $AB$.

## You need to:

- Interpret and obtain the equation of a straight line graph in the form $y = mx + c$.

### Key skills

You must be able to interpret the equation of a straight line graph in the form $y = mx + c$.

### Exam tip

The place where the line crosses the y-axis is usually called the y-intercept.

### Recap

**Equations of straight lines**

The equation of any non-vertical straight line can be written in the form

$$y = mx + c$$

where $m$ is the gradient and $c$ is the y-intercept.

The gradient, $m$, is a measure of how steep the line is.

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$m$ is positive if the line slopes upwards from left to right.

$m$ is negative if the line slopes downwards from left to right.

The y-intercept of a straight line is the y-coordinate of the point where the line crosses the y-axis.
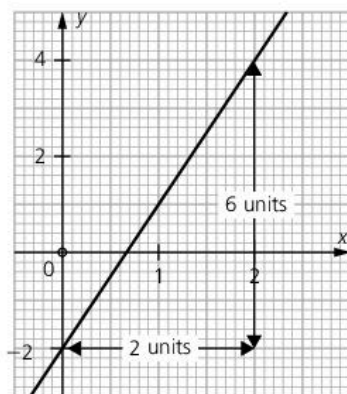


**The y-intercept is –2.**

**To find the gradient of a straight line take any two points on it.**

$$\frac{\text{change in } y}{\text{change in } x} = \frac{4 - (-2)}{2 - 0} = \frac{6}{2} = 3, \text{ so the gradient is 3.}$$

**So the equation of the line is $y = 3x - 2$.**

### Exam tip

Straight lines with positive gradients go uphill as they move from left to right, so they look like this :

Straight lines with **N**egative gradients go downhill as they move from left to right, so they look like this:

Negative
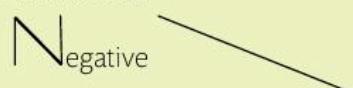
**Worked example**

A line has the equation $y = 3x + 1$.

(a) What is the gradient of the line? **[1 mark]**

(b) Where does this line cross the $y$-axis? **[1 mark]**

---

$y = 3x + 1$ is of the form $y = mx + c$, with $m = 3$ and $c = 1$.

(a) $m$ is the gradient of the line,
   so the gradient of $y = 3x + 1$ is 3.

(b) $c$ is the $y$-intercept,
   so the line crosses the $y$-axis at (0, 1).

🔑 **Key skills**

You must be able to obtain the equation of a straight line graph in the form $y = mx + c$ using two points on the line.

**Worked example**

What is the equation of the straight line that passes through the points (1, 3) and (4, 9)?

Give your answer in the form $y = mx + c$. **[3 marks]**

---

The gradient $m = \dfrac{9 - 3}{4 - 1} = \dfrac{6}{3} = 2$, so the equation is $y = 2x + c$.

To work out the $y$-intercept, substitute one of the points into this equation.

When $x = 1$, $y = 3$ 
$$y = 2x + c$$
$$3 = 2 \times 1 + c$$
$$3 = 2 + c$$
$$c = 1$$

so the equation is $y = 2x + 1$

**Exam tip**

It doesn't matter which of the two points you substitute in to find $c$.

✏️ **Apply**

In the above Worked example, try working out the value of $c$ by substituting $x = 4$ and $y = 9$ into $y = 2x + c$ instead of $x = 1$ and $y = 3$ so you can see that you get the same answer.

You must be able to draw a straight line graph from its equation.

**Exam tip**

Plot three points, not two, in case you make a mistake with one of them.

**Worked example**

Draw the line $y = 5 + 2x$.                                    **[3 marks]**

This line has a $y$-intercept of 5 and a gradient of 2.

Set up a table of three values:

| $x$ | 0 | 2 | 3 |
|-----|---|---|----|
| $y$ | 5 | 9 | 11 |
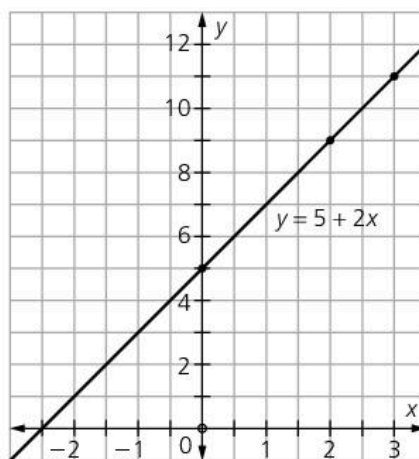


**Exam tip**

Sometimes you will need to rearrange the equation into the form $y = mx + c$ so you can read off the gradient and $y$-intercept. This will also help you when substituting in values of $x$ and $y$.

**Exam tip**

It is really important that you are confident working with negative numbers. See section 1.4 for revision of negative numbers.

❓ **Questions**

1   What is the equation of the straight line that passes through the points $(-1, -2)$ and $(1, 4)$?

Give your answer in the form $y = mx + c$.

2   Draw the lines:

**a**  $y = 8 - \dfrac{2}{3}x$

**b**  $3x - 5y = 15$

3   Find an equation of the line which:

**a**  passes through $(0, 6)$ and has a gradient of 2

**b**  passes through $(2, 3)$ and has a gradient of $-1$

**c**  passes through $(-2, -5)$ and has a gradient of $\dfrac{1}{2}$.

## You need to:

- Determine the equation of a straight line parallel to a given line.
- **Find the gradient of parallel and perpendicular lines. (Extended)**

### 🔑 Key skills

You must be able to find the equation of a line parallel to another line, given the equation of the other line and a point the line passes through.

### ⏪ Recap

Two lines that are parallel have the same gradient. So $y = 3x + 1$ and $y = 3x - 5$ are parallel lines since they both have gradient 3.

### Exam tip

Remember that when an equation is in the form $y = mx + c$, the gradient is $m$.

### Worked example

Find the line parallel to $y = 3x - 2$ which crosses the $y$-axis at the point (0, 5). **[3 marks]**

A line parallel to $y = 3x - 2$ has gradient 3.

A line passing through (0, 5) has $y$-intercept 5.

So the equation of the line is $y = 3x + 5$.

### Exam tip

If the point you are given is the $y$-intercept, this makes the question a lot easier.

### ⏪ Recap

If two lines are perpendicular, the product of their gradients is −1:

$$m_1 \times m_2 = -1$$

### Extended

### Worked example

Find the gradient of the lines perpendicular to these lines:

(a) $y = 4x - 3$
(b) $y = -0.5x + 1$
(c) $2x + 3y = 12$ **[4 marks]**

(a) Looking at $y = 4x - 3$, which is already in the form $y = mx + c$, the gradient is 4.

Therefore let $m_1 = 4$

$m_1 \times m_2 = -1$

so $m_2 = -1 \div m_1 = -1 \div 4 = -\dfrac{1}{4}$

### 🔑 Key skills

You must be able to find the gradient of a line that is perpendicular to a given line.

(b) Looking at $y = -0.5x + 1$, which is already in the form $y = mx + c$, the gradient is $-0.5$.

Therefore let $m_1 = -0.5$

$m_1 \times m_2 = -1$

so $m_2 = -1 \div m_1 = -1 \div (-0.5) = 2$

(c) First rearrange the equation $2x + 3y = 12$ into the form $y = mx + c$.

$3y = -2x + 12$

$y = -\dfrac{2}{3}x + 4$, so $m_1 = -\dfrac{2}{3}$

The negative reciprocal of $-\dfrac{2}{3}$ is $\dfrac{3}{2}$, so $m_2 = \dfrac{3}{2}$

**Exam tip**

It is often useful to rearrange an equation into the form $y = mx + c$ to find the gradient or $y$-intercept of the line.

**Exam tip**

A quick way to find the perpendicular gradient is to write the original gradient as a fraction, then just turn the fraction upside–down and change the sign. This is called finding the 'negative reciprocal'.

**Key skills**

You must be able to find the equation of a line perpendicular to another line, given the equation of the other line and a point your line passes through.

**Worked example**

Find the equation of the straight line through (3, 1) that is perpendicular to the line $y = 0.5x + 1$.          **[3 marks]**

Gradient of the perpendicular line: $-1 \div 0.5 = -2$

The equation of a straight line is $y = mx + c$ and $m = -2$, so $y = -2x + c$

The line passes through (3, 1), so

$1 = -2 \times 3 + c$

$c = 7$

Hence the equation is $y = -2x + 7$

**? Questions**

**1** Find the equation of the line parallel to $y = 5x + 10$ that passes through the point (0, 2).

**2** Find the equation of the line parallel to the line $y = \frac{1}{2}x - 6$ that passes through the point (–2, 2).

**3** Find the equation of the line that passes through (4, 1) and is parallel to the line $y = 6x - 2$.

**4** Find the equation of the line parallel to $3x - 4y = 24$ that passes through the point (–4, 1).

Give your answer in the form $ay - bx = c$, where $a$, $b$ and $c$ are all whole numbers.

**5** Find the gradient of lines perpendicular to the following lines: **E**

**a** $y = 3x - 1$　　　**b** $y = -2x + 3$
**c** $y = \frac{1}{4}x - 5$　　　**d** $2x - 3y = 7$

**6** Find the equation of the line which:

**a** passes through (2, 3) and is perpendicular to $y = x - 4$
**b** passes through (–3, 5) and is perpendicular to $y = -2x + 1$
**c** passes through (6, –1) and is perpendicular to $2y = 3x + 5$
**d** passes through (–1, –2) and is perpendicular to the line through points (2, 3) and (4, 6).

**7** Find the equation of the straight line through (4, 2) that is perpendicular to the line $y = -2x + 3$.

**8** **a** In the diagram below, find the equation of the line CD.
**b** The line AB has the equation $2y - x = -2$.
Show that the lines AB and CD are perpendicular.
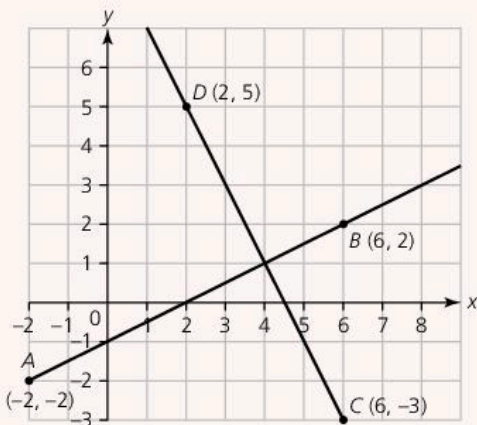


**9** Find the gradient of the line perpendicular to the line $2y - 5x = 11$.

**10** Two points A and B have coordinates (5, 7) and (–1, 3) respectively.

**a** Find the midpoint of the line segment AB.
**b** Find the gradient of AB.
**c** Find the equation of the perpendicular bisector of the line segment AB.

To **Raise your grade** now try questions 3, 4 and 5 page 106

1.   Find the value of $j$, given that the line joining points $(j, 2)$ and $(5, 7)$ has a gradient of $-1$.

     **[3 marks]**

2.   Find the value of $k$, given that the line joining points $(3, k)$ and $(k, 5)$ has a gradient of $2$.

     **[3 marks]**

3.   Find the equation of the line which:    **E**

   **a**   passes through $(6, -1)$ and is perpendicular to $2y = 3x + 5$    **[3 marks]**

   **b**   passes through $(-1, -2)$ and is perpendicular to the line through $(2, 3)$ and $(4, 6)$.    **[4 marks]**

4. **a**   Write down the gradient of the line joining the points $(m, 3n)$ and $(2, -6)$.    **[1 mark]**

   **b**   Find the value of $m$ if the line is parallel to the $y$-axis.    **[2 marks]**

   **c**   Find the value of $n$ if the line is parallel to the $x$-axis.    **[2 marks]**

5.   A straight line passes through two points with coordinates $A(4, 5)$ and $B(8, 11)$. Find the equation of the straight line which passes through the midpoint of $AB$ and is perpendicular to the line through $A$ and $B$.    **[7 marks]**

6.   Point $A$ has coordinates $(4, 6)$, and point $B$ is at $(-2, 3)$.

   **a**   Find the length of the line segment $AB$.    **[2 marks]**

   **b**   The midpoint of $AB$ is point $M$. Find the coordinates of $M$.    **[2 marks]**

   **c**   Find the coordinates of point $P$ which divides $AB$ in the ratio $2 : 1$.    **[2 marks]**

# 4 Geometry

Tick these boxes to build a record of your revision

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 4.1 | Use and interpret the geometric terms: point, line, parallel, bearing, right-angle, acute, obtuse and reflex angles, perpendicular, similarity and congruence. | | | |
| | Use and interpret the vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures including nets. | | | |
| 4.2 | Measure and draw lines and angles. | | | |
| | Construct a triangle given the three sides, using a ruler and pair of compasses only. | | | |
| 4.3 | Read and make scale drawings. | | | |
| 4.4 | Calculate lengths of similar figures. | | | |
| | **Use the relationship between area of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids.** | | | |
| 4.5 | Recognise congruent shapes. | | | |
| | **Use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS).** | | | |
| 4.6 | Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions. | | | |
| | **Recognise symmetry properties of a prism (including a cylinder) and a pyramid (including a cone).** | | | |
| | **Use the following symmetry properties of circles:** | | | |
| | • **equal chords are equidistant from the centre** | | | |
| | • **the perpendicular bisector of a chord passes through the centre** | | | |
| | • **tangents from an external point are equal in length.** | | | |
| 4.7 | Calculate unknown angles using the following geometric properties: | | | |
| | • angles at a point | | | |
| | • angles at a point on a straight line and intersecting straight lines | | | |
| | • angles formed within parallel lines | | | |
| | • angle properties of triangles and quadrilaterals | | | |
| | • angle properties of regular polygons | | | |
| | • angle in a semicircle | | | |
| | • angle between tangent and radius of a circle | | | |
| | • **angle properties of irregular polygons** | | | |
| | • **angle at the centre of a circle is twice the angle at the circumference** | | | |
| | • **angles in the same segment are equal** | | | |
| | • **angles in opposite segments are supplementary; cyclic quadrilaterals** | | | |
| | • **alternate segment theorem.** | | | |

**You need to:**

- Use and interpret the geometric terms: point, line, parallel, bearing, right-angle, acute, obtuse and reflex angles, perpendicular, similarity and congruence.

- Use and interpret the vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures including nets.

**Recap**

An **acute angle** is less than 90°.



An **obtuse angle** is between 90° and 180°.



A **reflex angle** is larger than 180° but less than 360°.



A **right-angle** is 90°.
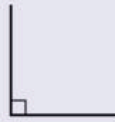


**Recap**

**Perpendicular lines** intersect at right angles.
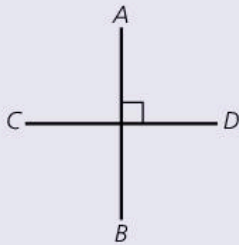


**Parallel lines** never meet. Arrows are used to indicate parallel lines.
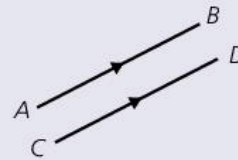


**Recap**

A **polygon** is a closed 2-dimensional shape with straight edges.
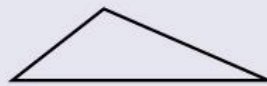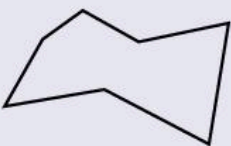


A **regular polygon** has all the sides equal and all the angles equal.



regular hexagon    square    equilateral triangle

**Recap**

An **isosceles** triangle has two equal sides and two equal angles.

An **equilateral** triangle has three equal sides and three equal angles.
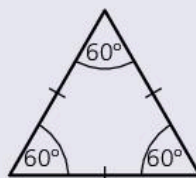
A **right-angled** triangle has one right-angle.



The marks indicate that the sides are equal.

A **quadrilateral** is a four-sided polygon.

**Square**
4 equal sides, 4 right-angles
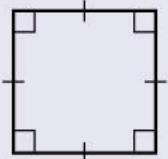
**Rectangle**
4 sides, 4 right-angles, opposite sides equal
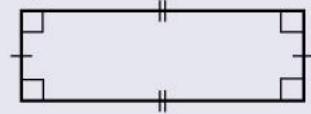
**Parallelogram**
4 sides, 2 sets of parallel sides, opposite sides equal, opposite angles equal
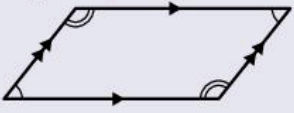
**Rhombus**
4 equal sides, opposite sides parallel, opposite angles equal
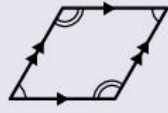
**Trapezium**
4 sides, one set of opposite sides parallel
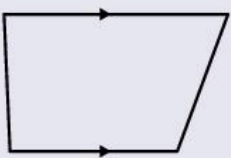
**Kite**
4 sides, 2 pairs of adjacent sides equal, 1 pair of opposite angles equal

**Key skills**

You need to be able to recognise and use the geometrical properties of quadrilaterals and other polygons.

**Apply**

Answer **yes** or **no** to these questions. Explain your answer.

- Is a square a rectangle?
- Is a parallelogram a rhombus?
- Is a rectangle a parallelogram?
- Is a rhombus a kite?

**Recap**

If you cut along the edges of a 3-dimensional shape and lay it out flat, you have a **net**.

3 cm

2 cm

4 cm

Back

Left  Base  Right

Front

Top

3

2

4
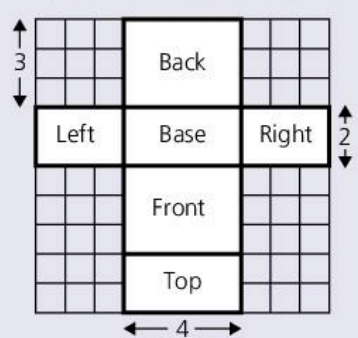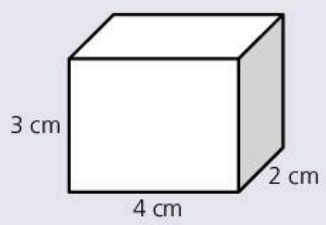
**?** **Questions**

**1** Name these polygons.

**a**

**b**

3 cm

3 cm

**c**

**d**

4 cm

4 cm

**e**

**f**

**g**

**h**

**i**

3 cm    3 cm
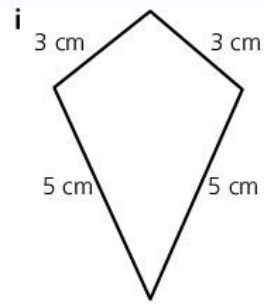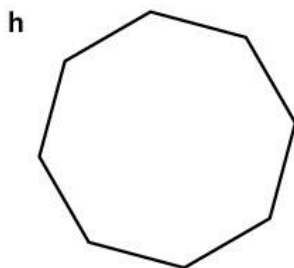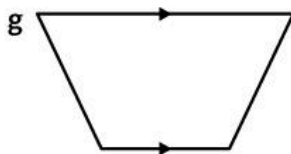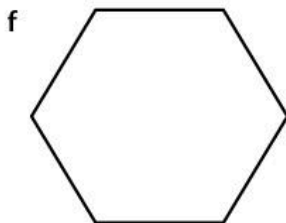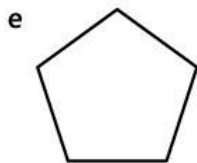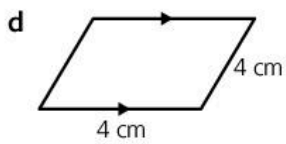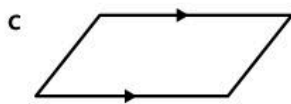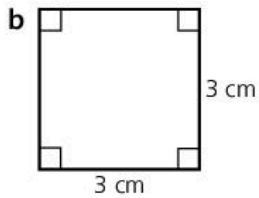
5 cm    5 cm

**2** Which of the nets below can be used to make a cube?

**a**

**b**

**c**

**c**
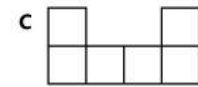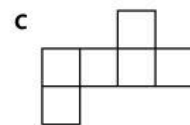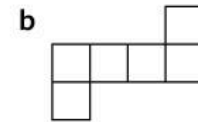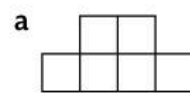
**3** Sketch the net of a cuboid with side lengths 3 cm, 5 cm and 7 cm.

**4** Using a scale of 1 cm to 2 m, draw a net of this isosceles triangular prism.

7 m

4 m

12 m
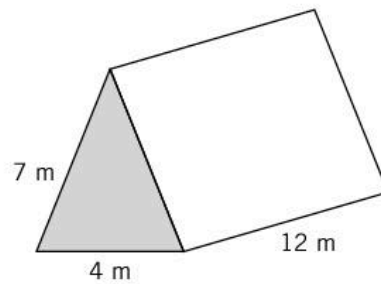
## You need to:

- Measure and draw lines and angles.
- Construct a triangle given the three sides, using a ruler and pair of compasses only.

**Worked example**

Construct a triangle with side lengths 5 cm, 6 cm and 8 cm.
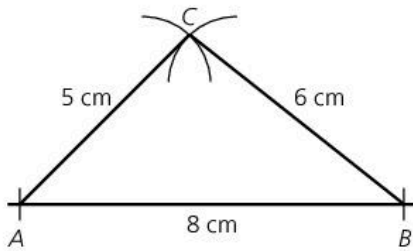
**[3 marks]**

**Exam tip**

Draw the longest side first using a ruler. Then draw little arcs of radius 5 cm and 6 cm above your line. Where the arcs cross will be the third corner of the triangle and you can draw the other sides in.

**? Questions**

1  Construct an equilateral triangle *ABC* where *AB* is

    **a** 5 cm          **b** 3.7 cm

2
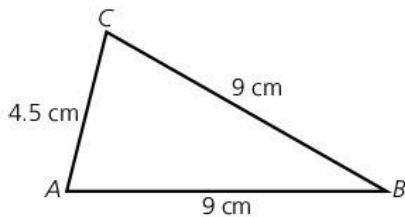


Make an accurate full-sized copy of this diagram.

3
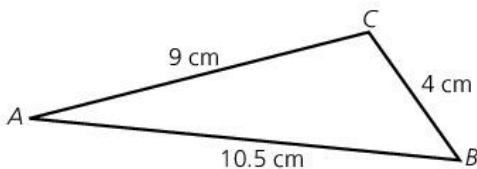


Make an accurate full-sized copy of this diagram.

4  Explain why it is not possible to construct a triangle *ABC* where $AB = 10$ cm, $BC = 5$ cm and $CA = 4$ cm.

5
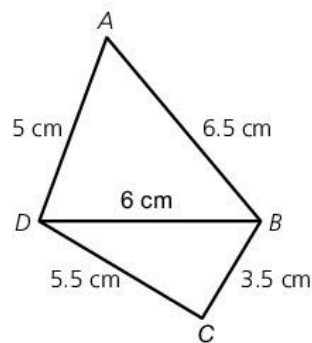


Make an accurate full-sized copy of this diagram.

## You need to:

• **Read and make scale drawings.**

🔑 **Key skills**

You need to be able to make an accurate scale drawing from information given in either a diagram or text.

**Exam tip**

Use a sensible scale, for example 1 cm = 10 m.

Measure the angle *BAC* using a protractor.

**Exam tip**

Measure the side length of your scale drawing and use your scale to write down the actual length of side *BC*.

**Worked example**

A triangular field *ABC* has sides *AB* = 120 m and *AC* = 80 m. Angle *BAC* = 40°.

By making an accurate scale drawing of the field, find the length of side *BC*. **[3 marks]**
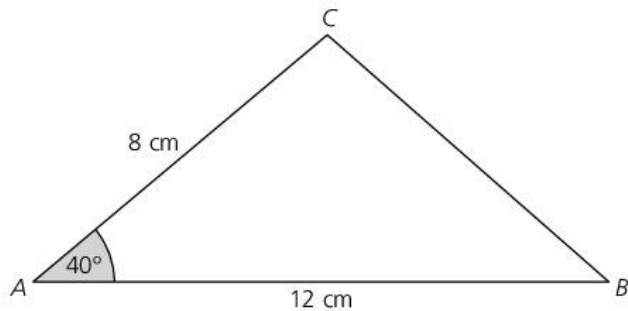


*BC* = 7.8 cm

So the length of side *BC* of the triangular field is 78 m.

❓ **Question**

1 A garden has four sides as shown in the diagram.
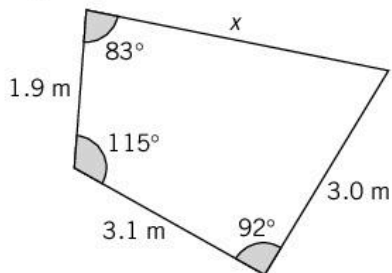


Use an accurate scale drawing to find the length marked *x*.

4.4

## You need to:

- Calculate lengths of similar figures.
- Use the relationship between area of similar triangles, with corresponding results for similar

figures and extension to volumes and surface areas of similar solids. (Extended)

Two shapes are **similar** if the ratio of every pair of corresponding sides is the same. You can think of one of the shapes being an **enlargement** of the other.



In the two shapes above, all of the corresponding side length ratios are equal to 0.75 so the shapes are similar.

### Worked example

(a) These two shapes are similar.
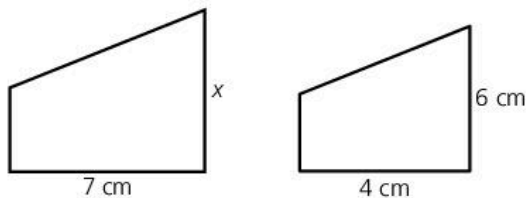


Find $x$. **[2 marks]**

(b) Triangles $ABC$ and $ADE$ are similar.
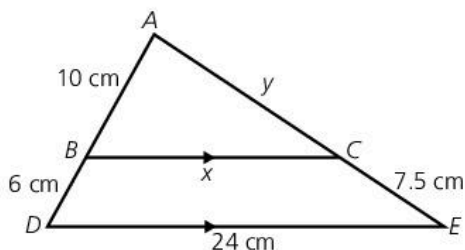


Find $x$ and $y$. **[4 marks]**

---

(a) $\dfrac{7}{4} = 1.75$

$x = 1.75 \times 6 = 10.5$ cm

(b)
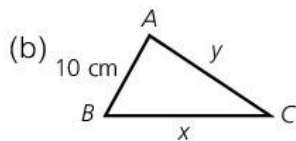


$\dfrac{16}{10} = 1.6$

$x = 24 \div 1.6 = 15$ cm

$\dfrac{y + 7.5}{y} = 1.6 \Rightarrow y + 7.5 = 1.6y$

$7.5 = 0.6y \Rightarrow y = 12.5$ cm
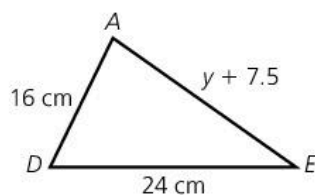
**🔑 Key skills**

You need to be able to find a missing side length when you are given two similar shapes.

**Exam tip**

Calculate the ratio of corresponding sides.

**Exam tip**

Draw the two similar triangles separately and mark on all of the lengths before calculating the ratio of corresponding sides.

**Extended**

**Exam tip**

Multiply the area of the smaller triangle by the *square* of the ratio of side lengths.

**Worked example**

These two triangles are similar.



5 cm        8 cm

The area of the smaller triangle is 24 cm². Find the area of the larger triangle. **[3 marks]**

$$A = 24 \times \left(\frac{8}{5}\right)^2$$
$$= 61.44 \text{ cm}^2$$

**Exam tip**

Multiply the volume of the smaller cylinder by the *cube* of the ratio of side lengths.

**Worked example**

Two similar cylinders have radius 3 cm and 5 cm respectively.

The volume of the smaller cylinder is 54 cm³.

Find the volume of the larger cylinder. **[3 marks]**

$$V = 54 \times \left(\frac{5}{3}\right)^3$$
$$= 250 \text{ cm}^3$$

🔑 **Key skills**

You need to be able to find a side length when given two areas or two volumes.

You also need to be able to find an area given two volumes and a volume given two areas.

**Exam tip**

If the ratio of volumes is the cube of the ratio of lengths, the ratio of lengths is the *cube root* of the ratio of volumes.

**Worked example**

Two bottles are similar.

Bottle *A* has volume 1200 cm³ and height 10 cm.

Bottle *B* has volume 1900 cm³.

Find the height of bottle *B*. **[3 marks]**

$$\frac{h}{10} = \sqrt[3]{\frac{1900}{1200}}$$

$$h = 10 \times \sqrt[3]{\frac{1900}{1200}} = 11.7 \text{ cm (to 3 s.f.)}$$

**Worked example**

Two containers are similar.

Container $D$ has a volume of 8000 cm³ and a surface area of 5000 cm².

Container $E$ has a volume of 4000 cm³.

Find the surface area of container $E$.          **[3 marks]**

Ratio of volumes $= \dfrac{4000}{8000} = 0.5$

Ratio of side lengths $= \sqrt[3]{0.5}$

Ratio of surface areas $= \left(\sqrt[3]{0.5}\right)^2$

So $\dfrac{A}{5000} = \left(\sqrt[3]{0.5}\right)^2 \Rightarrow A = 3150$ cm² (to 3 s.f.)

**Exam tip**

Use the ratio of volumes to work out the ratio of side lengths and then convert this to the ratio of areas.

**? Questions**

1  Ben and Sarah want to measure the height of a building. Ben is 1.8 m tall and Sarah suggests that he stands next to the building and compares the shadows. She measures his shadow to be 2.4 m long and the shadow of the building to be 16 m long. How tall is the building?

2  A photocopier is set to reduce the lengths of copies to $\dfrac{2}{3}$ of the original size. If the original document measures 12 cm by 15 cm, what will be the dimensions of the copy?

3  A photography shop produces enlargements of photos. A 15 cm × 10 cm photo was enlarged so that its longest side was 24 cm. What was the length of the shorter side?

4  A map is reduced to $\dfrac{3}{5}$ of its original size.
   A field on the original map measured 25 mm × 35 mm. What will be its dimensions on the image?

5  A map that measures 24 cm by 30 cm is reduced to $\dfrac{2}{3}$ of its original size. What are the dimensions of the reduced map?

6  In the triangle in the diagram, $BD = 8$ cm, $AB = 10$ cm, $AD = 6$ cm, $AC = x$ and $CD = y$.



   a  Draw the two triangles $ABC$ and $DBA$ in the same orientation and mark on all their angles.
   b  Hence explain why triangles $ABC$ and $DBA$ are similar.
   c  Write down an equation involving $x$.
   d  Solve the equation to find $x$.
   e  Calculate the value of $y$.

7  A rectangle $P$ is enlarged to a rectangle $Q$. The dimensions of $P$ are 5 m by 12 m. The shortest side of $Q$ is 6 m.

   a  What is the scale factor of enlargement?
   b  What is the length of the longer side of $Q$?
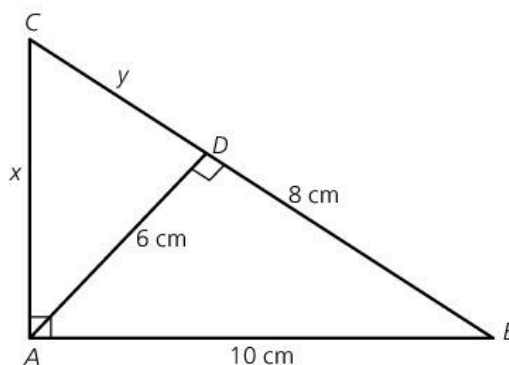
**8**  A right-angled triangle $P$ is enlarged to triangle $Q$. The hypotenuse of $P$ is 12 cm and the hypotenuse of $Q$ is 15 cm.

    **a**  What is the scale factor of enlargement?

    **b**  If the shortest side of $P$ is 8 cm, find the shortest side of $Q$.

**9**  A photo 8 cm high and 10 cm wide has a border 2 cm high along the bottom and the top of the photo and $w$ cm wide on each side. Find $w$ if the original photo is similar to the photo with its border.

**10**
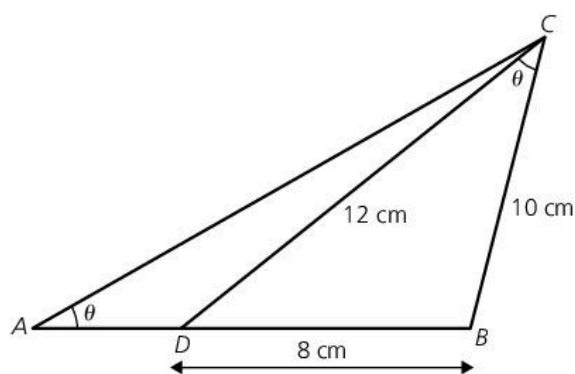


In the diagram $\angle DCB = \angle CAB = \theta$, $DB = 8$ cm, $DC = 12$ cm and $CB = 10$ cm.

    **a**  To which triangle is triangle $ABC$ similar?

    **b**  Draw triangle $ABC$ and the triangle from part **a** so that they have the same orientation and mark each side clearly.

    **c**  Find the length $AB$.

    **d**  Find the length $AC$.

**11**  A cone of radius 6 cm and height 15 cm has a cone of height 9 cm removed from its top. What is the radius of the removed cone?

**12**  The distance between Delhi and Calcutta is 1310 km. On a map they are 26.2 cm apart. Find the scale of the map in the form $1:n$.

**13**  The scale of a map is $1:20\,000\,000$. **E**
On the map the area of a state is 5 cm². Calculate the actual area of the state in km².

**14**  In the diagram, $AB = 5$ cm, $BC = 4$ cm and the area of the triangle $ABE$ is 23 cm². Given that $BE$ is parallel to $CD$ find (to 2 s.f.) the area of the triangle $ACD$.
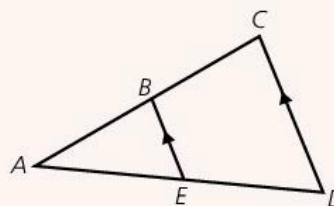


**15**  Two large water tanks are similar. One holds 5 m³ and the other holds 12 m³. If the height of the smaller one is 1.2 m, what is the height (to 3 s.f.) of the larger one?

**16**  Two pictures are similar. The area of one is 54 cm² and of the other is 216 cm². If the length of the larger one is 18 cm, find the length of the smaller one.

**17**  Three layers of a wedding cake are similar. The middle layer has a surface area of 3600 cm² and a mass of 5 kg.

    **a**  What is the mass (to 3 s.f.) of the bottom layer if its surface area is 8000 cm²?

    **b**  What is the surface area (to 3 s.f.) of the top layer if its mass is 3 kg?



**18**  Two cuboids are similar. One has volume 6 m³ and the other has volume 11 m³. If the surface area of the smaller one is 22 m², what is the surface area (to 3 s.f.) of the larger one?

To **Raise your grade** now try question 2, page 131

## You need to:

- Recognise congruent shapes.
- Use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS). **(Extended)**

**Recap**

Two shapes are **congruent** if the corresponding sides and the corresponding angles are equal.

**Recap**

To prove that two triangles are congruent, you need to demonstrate one of the following **congruence criteria:**

- All three pairs of corresponding sides are equal in length **(SSS)**
- Two pairs of corresponding angles and one pair of corresponding sides are equal **(ASA)**
- Two pairs of corresponding sides and the *included* angle are equal **(SAS)**
- The hypotenuse and one pair of corresponding sides are equal in a *right-angled* triangle **(RHS)**

**Extended**

**Apply**

To demonstrate that SSA is *not* sufficient to prove congruency. Sketch two different triangles that have sides 6 cm and 7 cm and an angle of 30° such that the triangles are *not* congruent.

**Key skills**

You need to be able to prove that two triangles are congruent.

**Worked example**
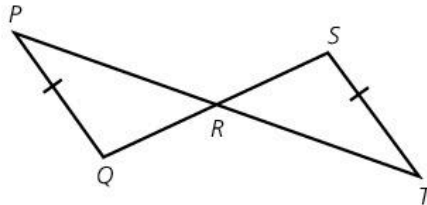


$PQ = ST$

$R$ is the midpoint of lines $PT$ and $QS$.

Prove that triangles $PQR$ and $RST$ are congruent. **[3 marks]**

---

$PQ = ST$ (information given in the question)

$QR = RS$ (since $R$ is the midpoint of $QS$)

$PR = RT$ (since $R$ is the midpoint of $PT$)

So by the congruence criteria SSS, triangles $PQR$ and $RST$ are congruent.

**Exam tip**

Give a reason for all of your deductions and make sure you write a full conclusion including the congruence criteria that you have used.

**Questions**

1  Identify pairs of congruent shapes:
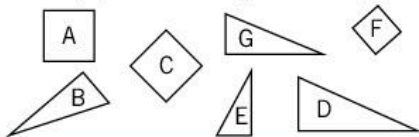


2  Triangle $ABC$ is isosceles with $AB = BC$. Points $P$ and $Q$ are on $AB$ and $BC$ respectively such that $AP = CQ$. Prove that triangles $ABQ$ and $CBP$ are congruent. (Hint: draw a sketch) **E**

3  In the diagram, congruent triangles $ABC$ and $BDE$ are right-angled (at $B$ and $D$ respectively). Prove that angle $AFE$ is a right-angle.
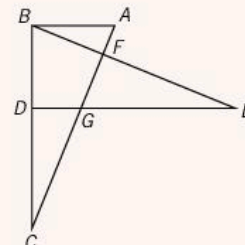
- Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions.
- Recognise symmetry properties of a prism (including a cylinder) and a pyramid (including a cone). **(Extended)**

- Use the following symmetry properties of circles:
  - equal chords are equidistant from the centre
  - the perpendicular bisector of a chord passes through the centre
  - tangents from an external point are equal in length. **(Extended)**

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**Recap**

An equilateral triangle has three lines of symmetry.

---
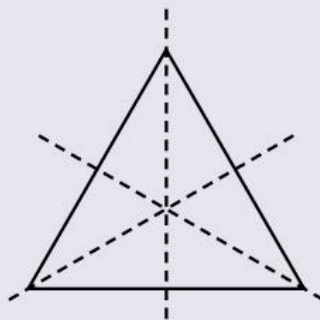
**Recap**

A square has four lines of symmetry.
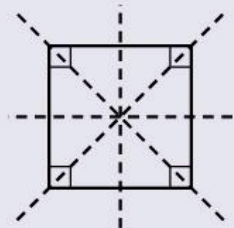
A rectangle has two lines of symmetry.
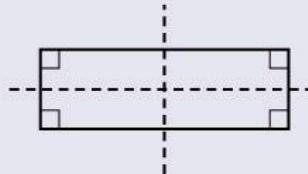
A parallelogram has no lines of symmetry.
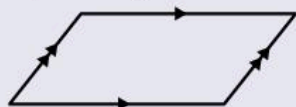
A rhombus has two lines of symmetry.
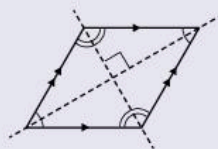
A trapezium has no lines of symmetry.

A kite has one line of symmetry.

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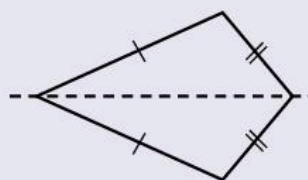
**Recap**

A regular $n$-sided polygon has $n$ lines of symmetry.
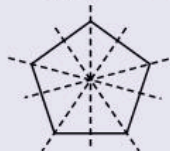
For example, a regular pentagon has five lines of symmetry.

## Recap

An equilateral triangle has rotational symmetry of order 3.
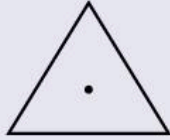
A square has rotational symmetry of order 4.
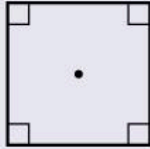
A rectangle has rotational symmetry of order 2.

A parallelogram has rotational symmetry of order 2.
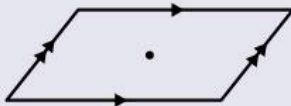
A rhombus has rotational symmetry of order 2.
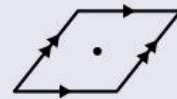
A trapezium has rotational symmetry of order 1.

A kite has rotational symmetry of order 1.
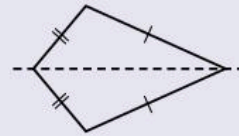
A regular $n$-sided polygon has rotational symmetry of order $n$.

**Worked example**

For these shapes:

(a) draw all the lines of symmetry

(b) write down the order of rotational symmetry. **[4 marks]**

(i)    (ii)

(a) (i)

(ii)
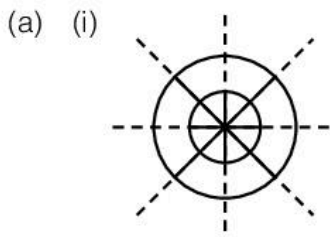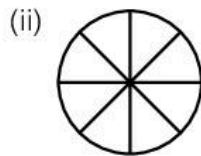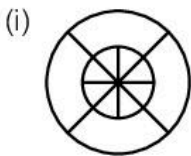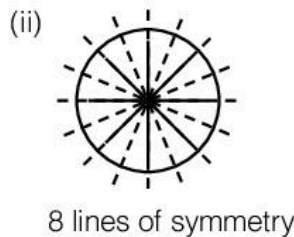
8 lines of symmetry

4 lines of symmetry

(b) (i) Order of rotational symmetry 4.

(ii) Order of rotational symmetry 8.

🔑 **Key skills**

You need to be able to draw lines of symmetry, state the number of lines of symmetry and find the order of rotational symmetry.

**Exam tip**

Imagine placing a mirror on the line of symmetry. The reflected part of the shape should appear identical.

**Exam tip**

For shape (i), rotating by 90°, 180°, 270° and 360° gives the same shape.

**⏪ Recap**

A cylinder and cone both have an **axis** of symmetry.
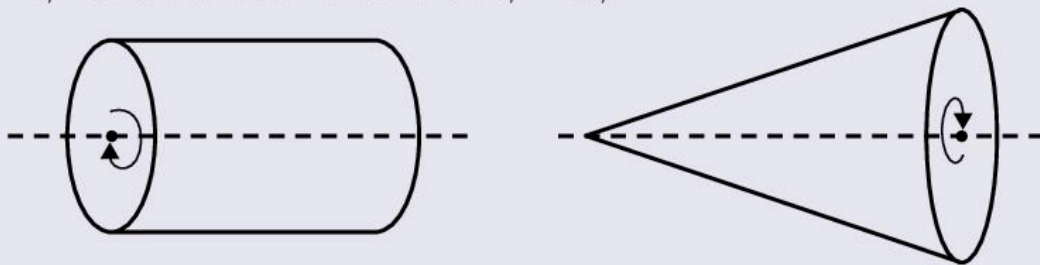


Prisms with a regular $n$-sided polygon cross-section have order of rotational symmetry $n$.
For example, a pentagonal prism has order of rotational symmetry 5.
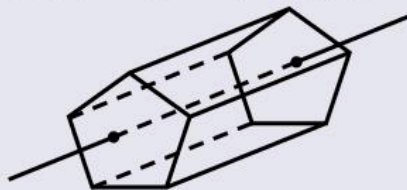


**⏪ Recap**

A cuboid has, in general, three **planes of symmetry**.
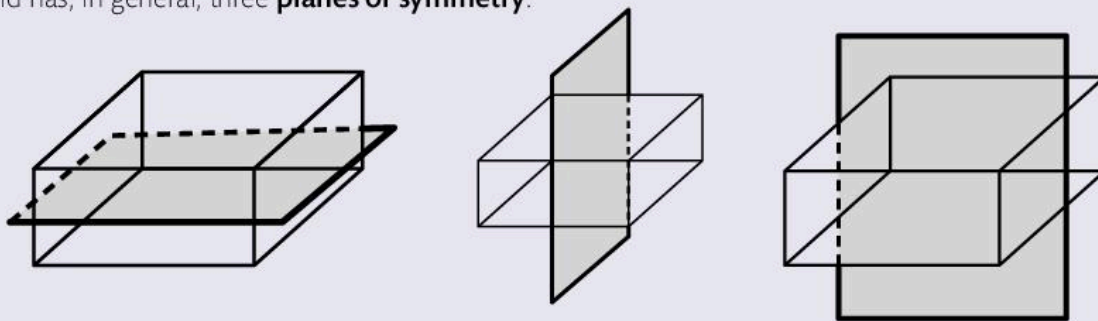


Prisms with a regular $n$-sided polygon cross-section have $n + 1$ planes of symmetry.
For example, a pentagonal prism has 6 planes of symmetry.
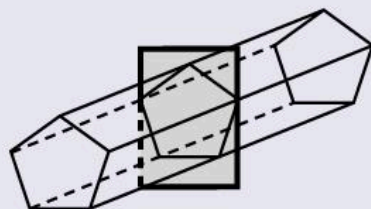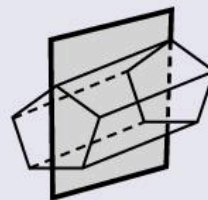


One plane like this       5 planes like this

**✏️ Apply**

If a cuboid has one pair of opposite faces square, it will have five planes of symmetry.

Sketch a cuboid with this feature and draw the planes of symmetry.

How many planes of symmetry does a cube have?

**Worked example**

(a) On a copy of the diagram, sketch one of the three planes of symmetry of the cuboid.
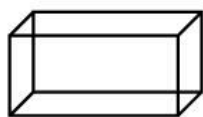


(b) Write down the order of rotational symmetry of an equilateral triangular prism. **[2 marks]**

(a)
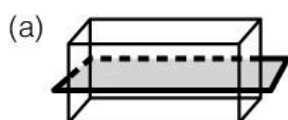


(b) The order of rotational symmetry is 3.

**Exam tip**
You could have sketched either of the other two planes of symmetry.

**Exam tip**
A prism with a regular *n*-sided polygon has order of rotational symmetry *n*.

**Recap**

**Equal chords**
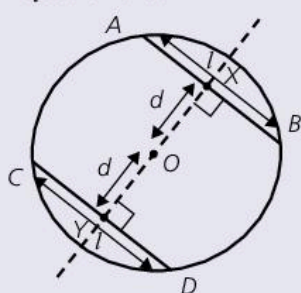


If two chords AB and CD have equal length then they are the same perpendicular distance from the centre of the circle.

OX = OY

**Bisector of chord**
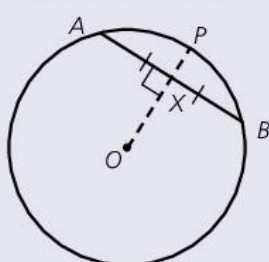


The perpendicular line from the centre of a circle to a chord bisects the chord.

AX = XB

**Recap**

**Tangents**

The tangents from a point P outside a circle to two points Q and R on the circle are equal in length.
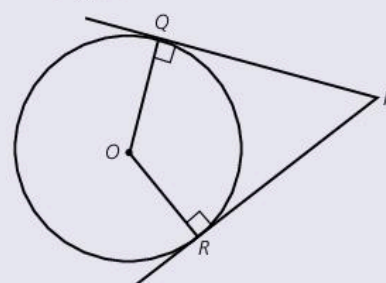


PQ = PR

**Watch out!**
These facts are usually used in the context of questions on circle theorems (section 4.7) or in questions on Pythagoras' theorem and trigonometry (chapter 6).

**? Questions**

**1** Copy the following shapes and
 **a** draw on all the lines of symmetry
 **b** state the order of rotational symmetry.

 **i** $   **ii** ∩   **iii** ⊘   **iv** ✝
 **v** 8   **vi** B   **vii** T   **viii** M

**2**
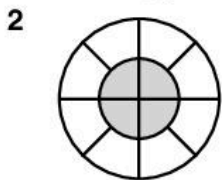


For this shape write down:

 **a** the order of rotational symmetry
 **b** the number of lines of symmetry.

**3** Draw axes with values of $x$ and $y$ from −10 to 10. Draw the shapes and fill in a copy of the table.

| | Shape | Vertices | Missing vertices | Lines of symmetry |
|---|---|---|---|---|
| **a** | Rectangle | (1, 1), (5, 1), (5, 3) | (__,__) | $x = 3$, $y = $__ |
| **b** | Rectangle | (6, 1), (6, 9) | (__,__) (__,__) | $x = 8$, $y = 5$ |
| **c** | Isosceles triangle | (3, 5), (5, 5), (4, 9) | | $x = $__ |
| **d** | Isosceles triangle | (0, 5), (3, 7) | (__,__) | $y = 7$ only |
| **e** | Rhombus | (−3, 0), (−1, 1), (0, 3) | (__,__) | $y = −x$, $y = $__ |
| **f** | Trapezium | (3, −4), (2, −7), (7, −7) | (__,__) | $x = 4.5$ only |
| **g** | Parallelogram | (−6, −6), (−5, −3), (−2, −3) | (__,__) | None |
| **h** | Square | (−7, 3), (−5, 3) | (__,__) (__,__) | $y = x + 8$  $x = $__ <br> $y = $__   $y = $__ |

**4** **a** Shade one square in each diagram so that there is: **E**

 **i** one line of symmetry
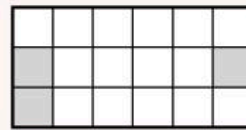


 **ii** rotational symmetry of order 2.
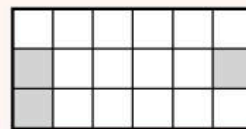


 **b** The cuboid shown below has no square faces. How many planes of symmetry does it have?
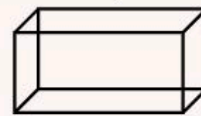


 **c** How many planes of symmetry has a regular hexagonal prism?
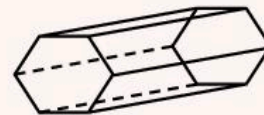


 **d** Write down the order of rotational symmetry of the regular hexagonal prism about the axis shown.
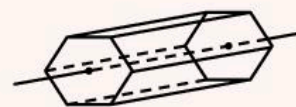


**5** How many planes of symmetry does a right square-based pyramid have?

## You need to:

- Calculate unknown angles using the following geometric properties:
  - angles at a point
  - angles at a point on a straight line and intersecting straight lines
  - angles formed within parallel lines
  - angle properties of triangles and quadrilaterals
  - angle properties of regular polygons
  - angle in a semicircle
  - angle between tangent and radius of a circle

- angle properties of irregular polygons (Extended)
- angle at the centre of a circle is twice the angle at the circumference (Extended)
- angles in the same segment are equal (Extended)
- angles in opposite segments are supplementary; cyclic quadrilaterals (Extended)
- alternate segment theorem. (Extended)

---

### ◀◀ Recap

**Angles at a point add up to 360°.**

$a + b + c + d = 360°$.

**Angles on a straight line add up to 180°.**

$a + c = b + d = 180°$.

**Vertically opposite angles are equal**.

$a = b$ and $c = d$.

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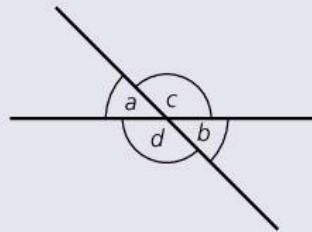
### ◀◀ Recap
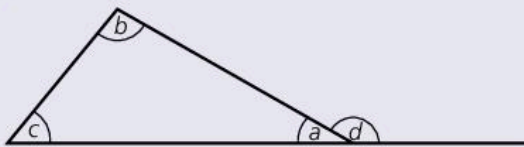
**The angle sum in a triangle is 180°.**

$a + b + c = 180°$.

**The exterior angle of a triangle is equal to the sum of the two interior opposite angles.**

$d = b + c$.

**The angle sum of a quadrilateral is 360°.**

### ✏ Apply

Construct a proof that the exterior angle of a triangle is equal to the sum of the two interior opposite angles.

---

### ◀◀ Recap
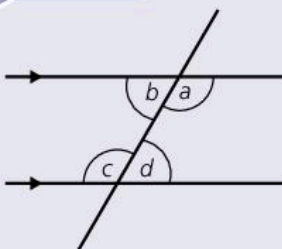
**Alternate angles are equal.**
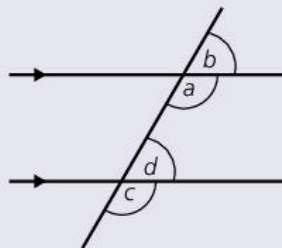
$a = c$ and $b = d$.

**Corresponding angles are equal.**

$a = c$ and $b = d$.

**Allied (co-interior) angles add up to 180°.**

$a + d = 180°$.

## Key skills

You need to be able to use angle rules to find missing angles.

### Worked example

Find the angles marked with letters in the diagram below.
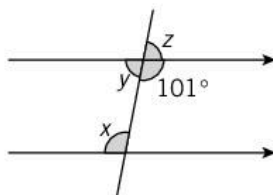


Give a reason for each answer.                    **[4 marks]**

---

$x = 101°$ (Alternate angles are equal)

$y = 180 - 101 = 79°$ (Angles on a straight line add up to 180°)

$z = 79°$ (Opposite angles are equal)

### Exam tip

State your reasons clearly using the correct terminology.
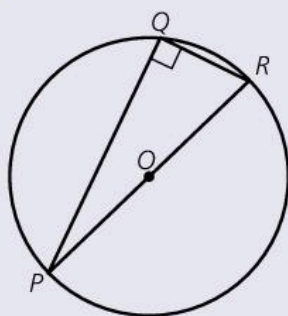
## Recap



$PR$ is a diameter of the circle and point $Q$ lies on the circumference.

**The angle in a semi-circle is 90°.**

Angle $RQP = 90°$.

## Recap
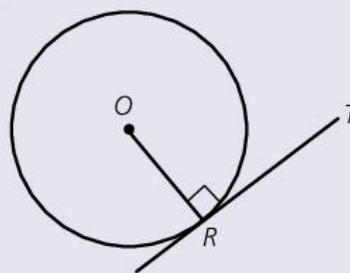


$OR$ is a radius of the circle and $RT$ is a tangent to the circle at $R$.

**The angle between a radius and a tangent is 90°.**

Angle $ORT = 90°$.

## Extended



## Recap

The sum of the **exterior angles** of a polygon is 360°.

If the polygon is regular, each exterior angle is equal to $\dfrac{360}{n}$.
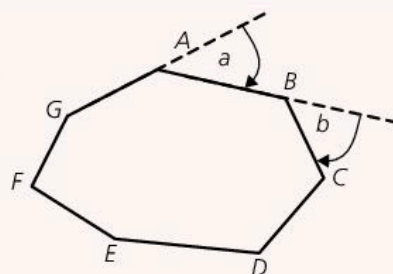
Imagine standing at $G$ and walking clockwise around the polygon. You would turn through angle $a$ at $A$, angle $b$ at $B$, etc.

By the time you have returned to $G$, you would have completed a full turn.

## Recap

In general, the sum of the interior angles in an $n$-sided polygon is $(n - 2) \times 180°$.
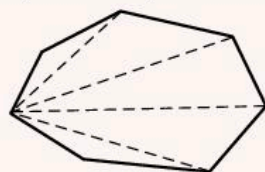


This 7-sided polygon can be split up into 5 triangles.

The sum of the **interior angles** of the polygon is $5 \times 180 = 900°$.

## Recap
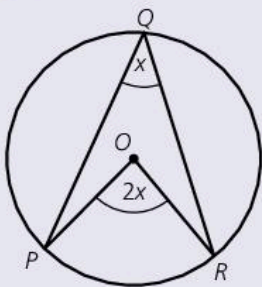


**The angle at the centre of a circle is twice the angle at the circumference.**

## Recap



**Angles in the same segment are equal.**

## Extended

**Watch out!**

This rule works even if the angles are not in the shape of an 'arrow-head'.
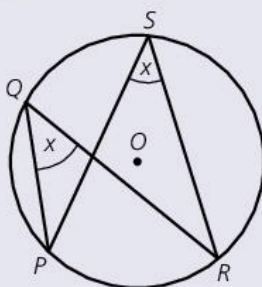


**Worked example**

Find the values of *x* and *y* in the following diagram.
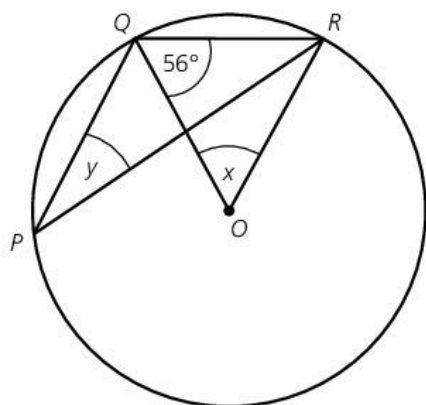


Give a reason for each stage of your working. **[4 marks]**

*OQ* and *OR* are radii of the circle so triangle *OQR* is isosceles.

Therefore, angle *QRO* = 56°.

$x = 180 - 2 \times 56 = 68°$ (Angles in a triangle)

$y = \dfrac{68}{2} = 34°$ (Angle at the centre is twice the angle at the circumference)
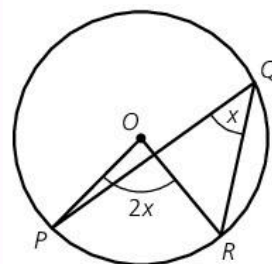
**Key skills**

You need to be able to use circle theorems to find missing angles.

**Exam tip**

There will be marks in the exam for clear explanations and reasons.

Make sure you use the correct terminology too.

**Extended**

A **cyclic quadrilateral** has all four vertices on the circumference of a circle.
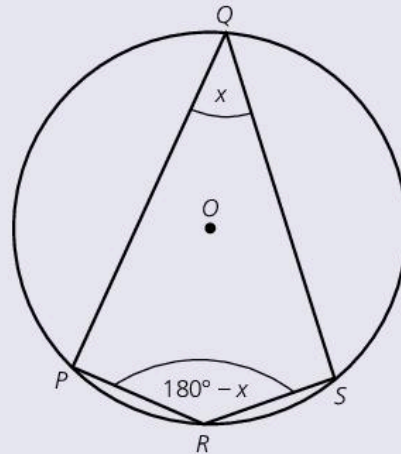


**Opposite angles in a cyclic quadrilateral add up to 180°.**

---

**Worked example**

Find the values of $x$, $y$ and $z$ in the following diagram.
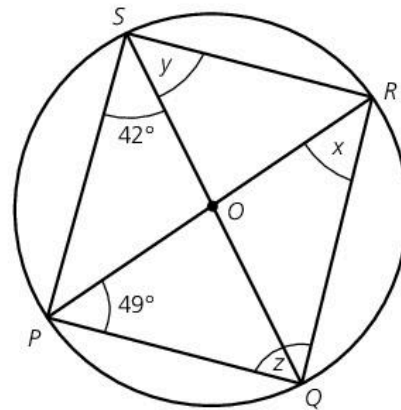


Give a reason for each stage of your working.   **[4 marks]**

---

$x = 42°$ (Angles in the same segment are equal)

$y = 49°$ (Angles in the same segment are equal)

$z = 180 - (42 + 49) = 89°$ (Opposite angles in a cyclic quadrilateral add up to 180°)

**Exam tip**

Make sure to show each stage of your working and give clear reasons.
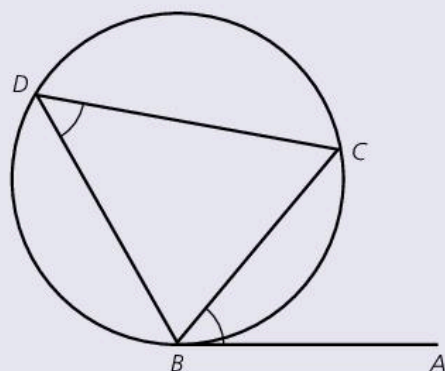
**The angle between a tangent and a chord is equal to the angle in the alternate segment.**

(This is known as the **Alternate Segment Theorem**).

---

**Worked example**

Find the angle marked $g$ in this diagram.
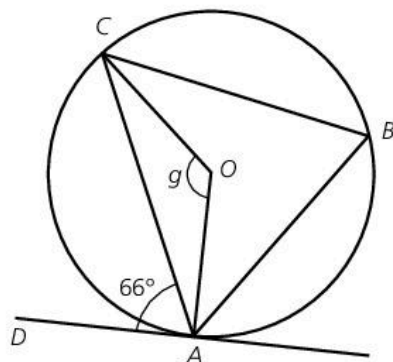


Give a reason for each stage of your working. **[3 marks]**

---

Angle $ABC = 66°$ (Alternate segment theorem)

$g = 66 \times 2 = 132°$ (Angle at the centre is twice the angle at the circumference)

**Exam tip**

You can write 'Alternate segment theorem' as a reason without explaining it any further.

**?** **Questions**

**1** Find the angles marked with letters:

**a**

65°

*a* 140°

**b**

*b*

110° 126°

**c**

91° 107°

84° *c*

**d**

42°

*g* 67°

**e**

4*h*

*h*
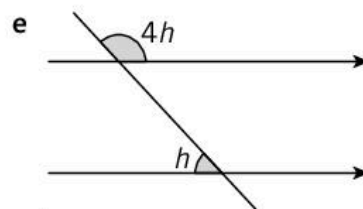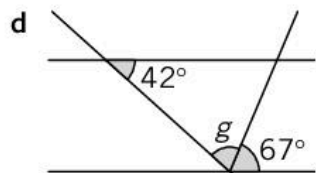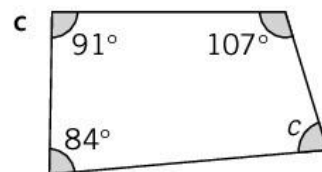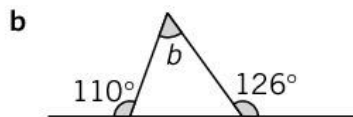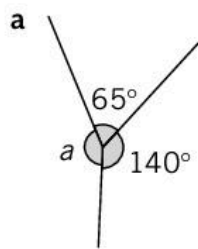
**2** Find the value of *x* in these quadrilaterals.

**a**

$x + 15°$

$x + 10°$

$x - 10°$

$x - 5°$

**b**

$3x - 40°$

$5x + 20°$

$2x + 10°$
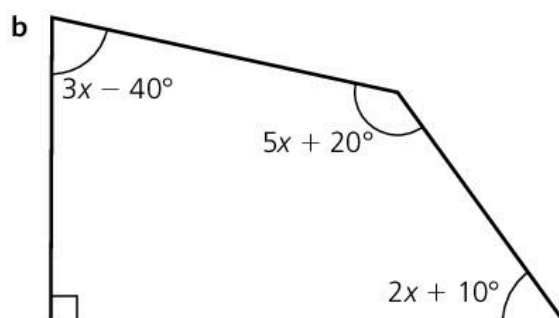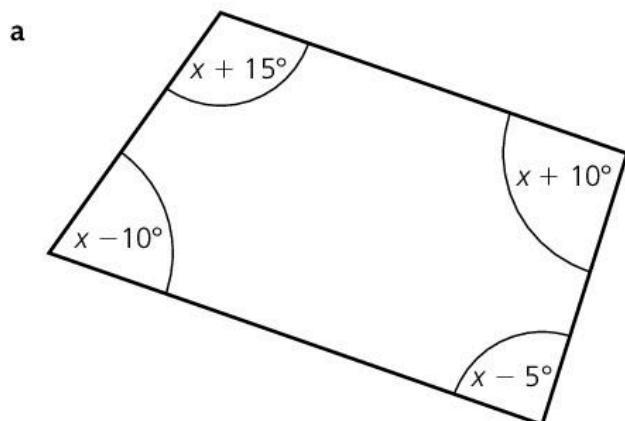
**3** Calculate the largest angle in a quadrilateral in which one angle is 5 times each of the other angles.

**4** In an isosceles trapezium, the smaller angle is equal to 53°. Find the larger angle.

**5** A triangle has angles in the ratio 2 : 3 : 4. Find the size of the largest angle.

**6** Name a polygon in which the sum of the interior angles is:

**a** 180° **b** 540° **c** 360° **d** 720°

**7** The sum of the interior angles of a regular polygon is 1440°. Find the number of sides.

**8** In a regular polygon, each exterior angle is twice the size of each interior angle. Find the number of sides.

**9** Find the angles marked with letters.

**a**

d
151°
88°
d
132° 125°

**b**

114°
98°
94°
e

**10** Find, giving reasons, the angles shown by letters.

**a**
Q
x
O
88°
P R

**b**
R
O
x
76°
Q
P

**c**
Q
64° O
x
R
P

**d**
R
32°
S
x
O
y
28°
P Q

**e**
R
S x
y
O
73°
Q
41°
P

**f**
Q
z
55°
O R
P
y
x
S

**g**
S
x
43°
R
O
29°
P
y
Q

**h**
R
y
S x
67°
O
z
Q
26°
P

**i**
Q
58°
R
y
x
P
O

**j**
O
Q
x
31°
P y R

**k**
Q
O 27°
x
R
y
z
P

**l**
R
S w
y
x
O
z
61°
Q
P
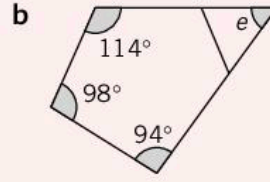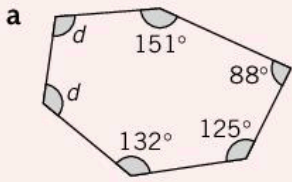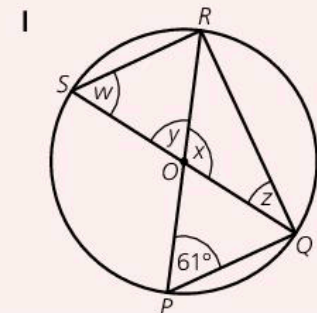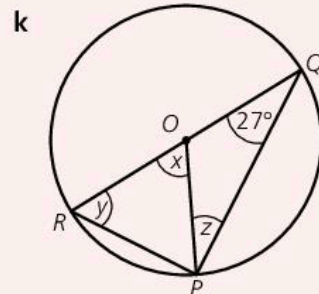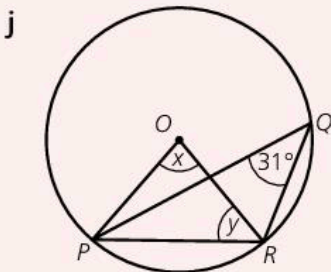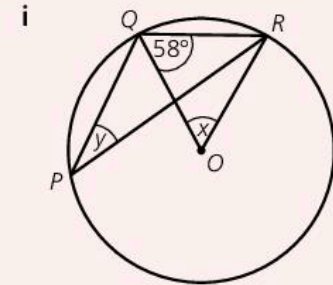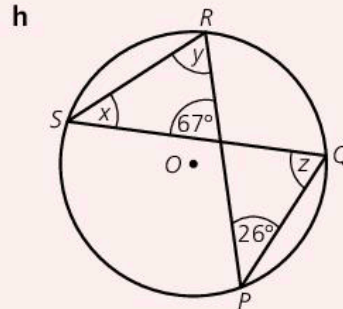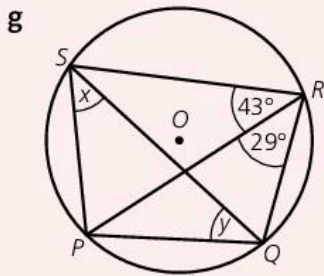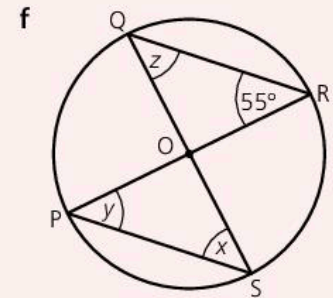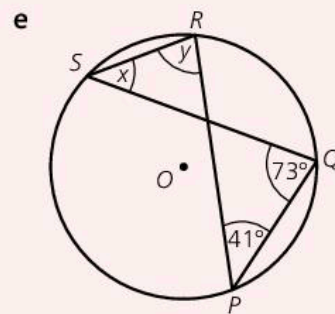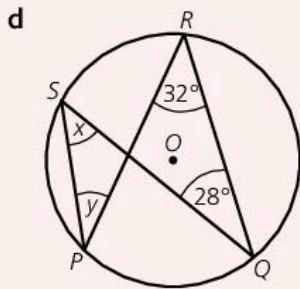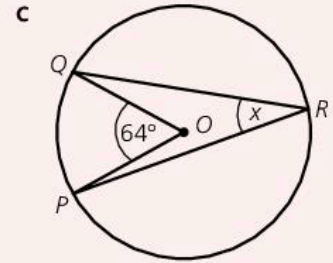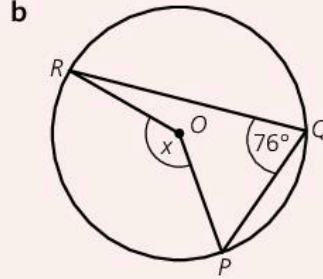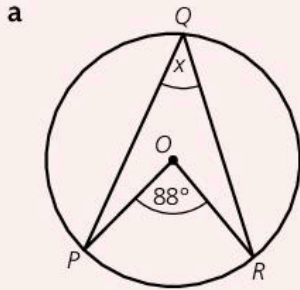
**11** Find, giving reasons, the angles shown by letters.

**a**



**b**



**c**



**d**



**e**



**f**



**g**



**h**



**i**
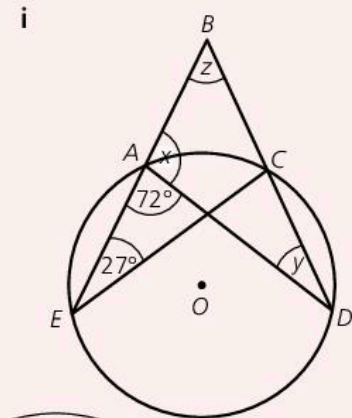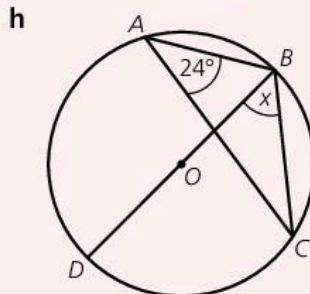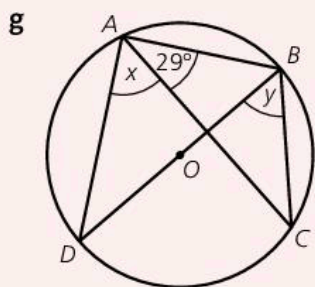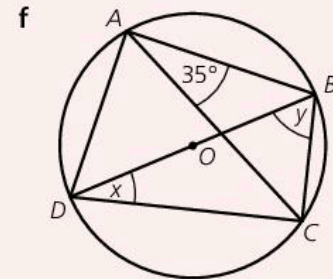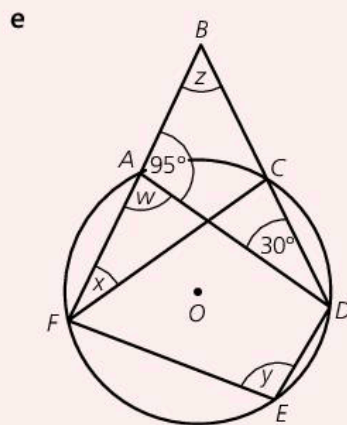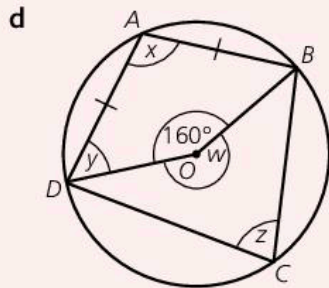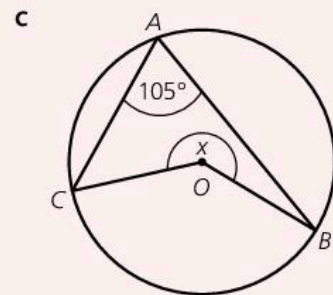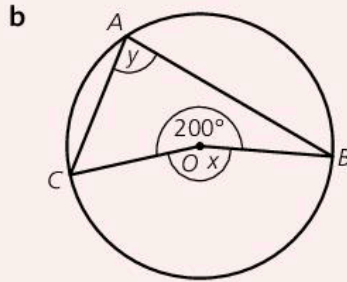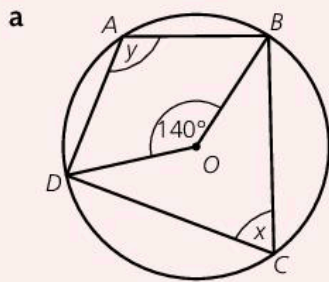


**12** Find, giving reasons, the angles shown by letters.

**a**



**b**



**c**



**d**
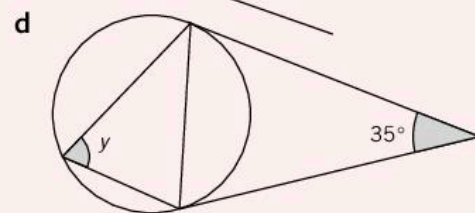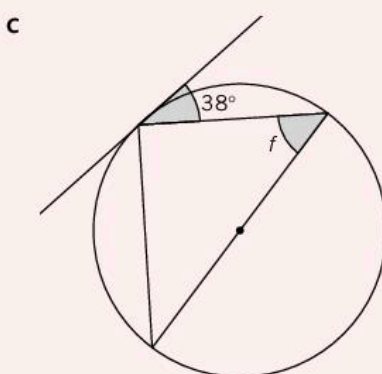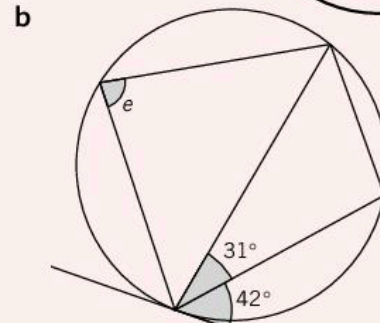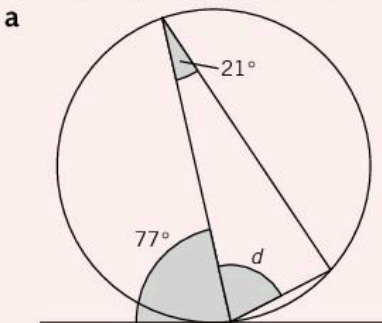


To **Raise your grade** now try questions 1, 3, 4 and 5, page 131

**1** A regular decagon rests on side *AB*. *ABC* is a straight line.
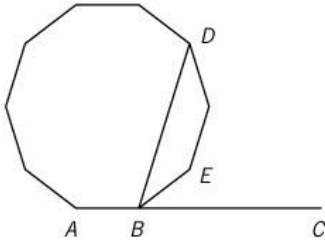


Calculate:

**a** angle *EBC* **[3 marks]**

**b** angle *ABD*. **[3 marks]**

**2** Two similar barrels have heights in the ratio 6 : 5.

**a** The surface area of the larger barrel is 2.4 m². Find the surface area of the smaller barrel. **[3 marks]**

**b** The volume of the smaller barrel is 1.2 m³. Find the volume of the larger barrel. **[3 marks]**

**3** *A*, *B* and *C* lie on the circumference of a circle with centre *O*.
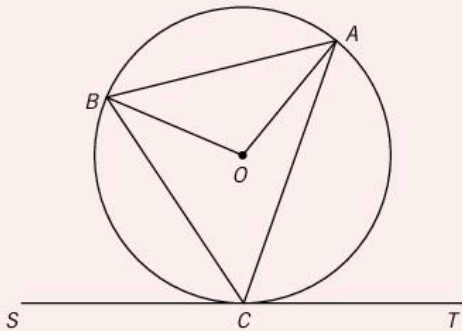


Given that the reflex angle *BOA* = 250° and angle *ABC* = 82°:

**a** find angle *BCA* **[3 marks]**

**b** find angle *BCS*. **[3 marks]**

**4** A triangle has angles in the ratio 2 : 3 : 4. Find the size of the largest angle. **[3 marks]**

**5** Find the size of angle *x* and the size of angle *y*: **[4 marks]**
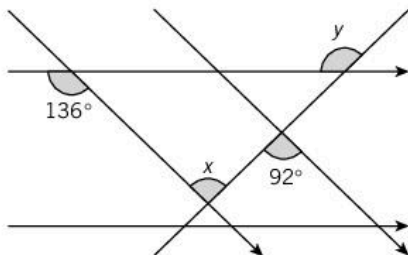
# 5 Mensuration

Tick these boxes to build a record of your revision

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 5.1 | Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units. | | | |
| 5.2 | Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium, and compound shapes derived from these. | | | |
| 5.3 | Carry out calculations involving the circumference and area of a circle.<br>Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle. | | | |
| 5.4 | Carry out calculations involving the surface area and volume of a cuboid, prism and cylinder.<br>Carry out calculations involving the surface area and volume of a sphere, pyramid and cone. | | | |
| 5.5 | Carry out calculations involving the areas and volumes of compound shapes. | | | |

## You need to:

- Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units.

### ⏪ Recap

**Units of length**

$10\,mm = 1\,cm$

$100\,cm = 1\,m$

$1000\,m = 1\,km$

**Units of mass**

$1000\,g = 1\,kg$

$1000\,kg = 1\,tonne$

**Units of capacity**

$1\,cm^3 = 1\,ml$

$1000\,ml = 1\,litre$

**Units of area**

$100\,mm^2 = 1\,cm^2$

$10\,000\,cm^2 = 1\,m^2$

$1\,000\,000\,m^2 = 1\,km^2$

**Units of volume**

$1000\,mm^3 = 1\,cm^3$

$1\,000\,000\,cm^3 = 1\,m^3$

### 👁 Watch out!

Units of area and units of volume have different conversions to the standard units of length. It is useful to **learn** these conversions and remember to use them in the exam.

### Worked example

Calculate the number of square centimetres in 5 square metres.

**[2 marks]**



5 m × 1 m
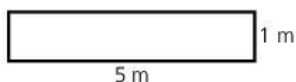
$1\,m \times 5\,m = 100\,cm \times 500\,cm$

$\qquad\qquad = 50\,000\,cm^2$

### Exam tip

Imagine a rectangle with dimensions such that the area is 5 square metres.

Alternatively, use your learnt conversion from $m^2$ to $cm^2$ so $5 \times 10\,000 = 50\,000$.

### ❓ Questions

1 Complete these unit conversions.

   **a** $1\,m^2 = \ldots\ldots\ldots\ldots\ldots cm^2$

   **b** $1\,km^2 = \ldots\ldots\ldots\ldots\ldots m^2$

   **c** $20\,000\,cm^2 = \ldots\ldots\ldots\ldots\ldots m^2$

   **d** $50\,000\,m^2 = \ldots\ldots\ldots\ldots\ldots km^2$

   **e** $14.8\,km^2 = \ldots\ldots\ldots\ldots\ldots m^2$

   **f** $1\,380\,000\,cm^2 = \ldots\ldots\ldots\ldots\ldots m^2$

   **g** $0.25\,m^2 = \ldots\ldots\ldots\ldots\ldots cm^2$

   **h** $0.00436\,km^2 = \ldots\ldots\ldots\ldots\ldots m^2$

   **i** $0.00000547\,km^2 = \ldots\ldots\ldots\ldots\ldots mm^2$

2 Complete these unit conversions.

   **a** $1\,cm^3 = \ldots\ldots\ldots\ldots\ldots mm^3$

   **b** $1\,m^3 = \ldots\ldots\ldots\ldots\ldots cm^3$

   **c** $200\,000\,cm^3 = \ldots\ldots\ldots\ldots\ldots m^3$

   **d** $6500\,mm^3 = \ldots\ldots\ldots\ldots\ldots cm^3$

   **e** $1000\,cm^3 = \ldots\ldots\ldots\ldots\ldots litres$

   **f** $1\,m^3 = \ldots\ldots\ldots\ldots\ldots litres$

   **g** $6.7\,litres = \ldots\ldots\ldots\ldots\ldots cm^3$

   **h** $1\,km^3 = \ldots\ldots\ldots\ldots\ldots m^3$

## You need to:

- Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium, and compound shapes derived from these.

### ⏪ Recap

**Rectangle**

Area = $l \times h$
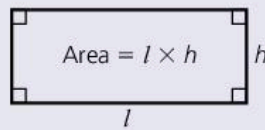
Perimeter = $2l + 2h$

**Triangle**

Area = $\dfrac{b \times h}{2}$
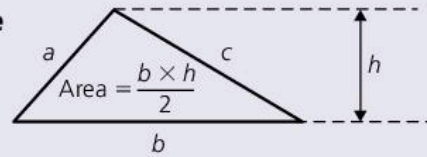
Perimeter = $a + b + c$

**Parallelogram**

The area of the parallelogram is the same as the area of the dotted rectangle.

So area of parallelogram = $b \times h$, that is base × height
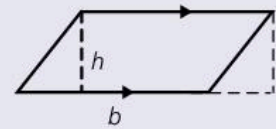
**Trapezium**

The area of a trapezium is $\dfrac{1}{2}(a+b) \times h$
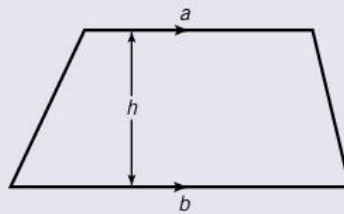
### 🔑 Key skills

You need to be able to use the above formulae to find the area and/or perimeter of shapes made up from the above shapes. These are called **compound** shapes.

### Exam tip

Split the calculation into two parts and make sure your working is set out clearly.

### Worked example
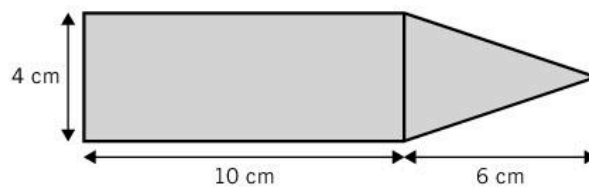
Calculate the area of this shape. **[3 marks]**

Area of rectangle = $4 \times 10 = 40$ cm²

Area of triangle = $\dfrac{4 \times 6}{2} = 12$ cm²

Total area = $40 + 12 = 52$ cm²

**Questions**

**1** Find the area and perimeter of these shapes.

**a**
8 cm
6 cm

**b**
5 cm
8 cm
11 cm
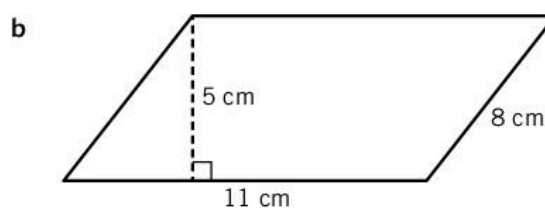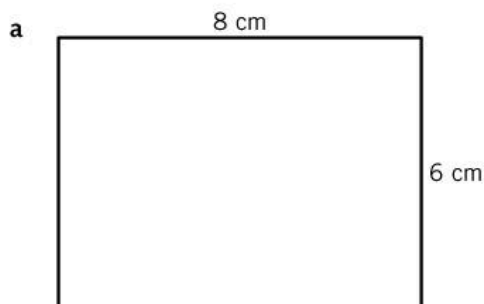
**2** Find the area of these shapes.

**a**
9 cm
12 cm

**b**
8 cm
6 cm
12 cm

**c**
10 cm
8 cm
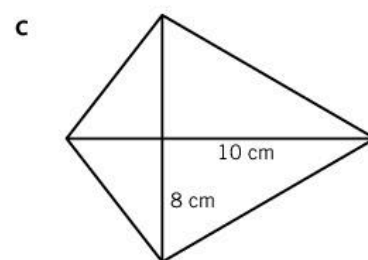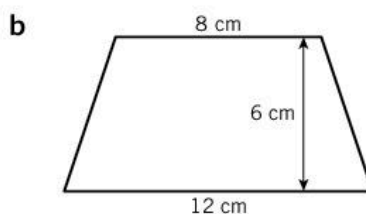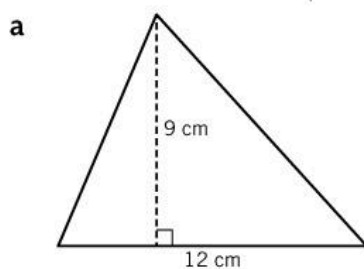
**3** Find the area of this compound shape.

4 m
8 m
3.5 m
6 m
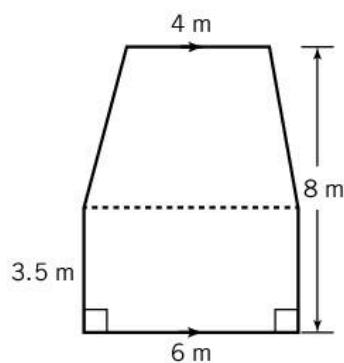
**4** A rectangle has area 126 cm² and width 4.5 cm. Calculate:

   **a** the length of the rectangle
   **b** the perimeter of the rectangle.

**5** A triangle has area 52 cm² and base 2.6 cm. Calculate the height of the triangle.

**6** A trapezium of area 210 cm² has parallel sides of length 10 cm and 18 cm.
   How far apart are the parallel sides?

**7** The area of this compound shape is 56 cm².
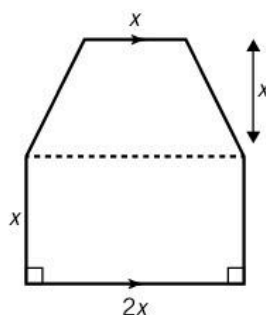
$x$
$x$
$x$
$2x$

Find the value of $x$.

**You need to:**

- Carry out calculations involving the circumference and area of a circle.
- Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle.

⏪ **Recap**

**Circle**
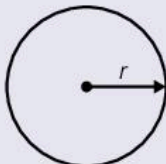
Area of circle $= \pi r^2$

Circumference of circle $= 2\pi r$

**Sector of circle**
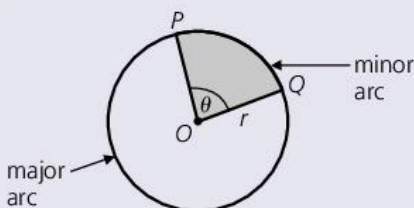
Length of minor arc $= \dfrac{\theta}{360} \times 2\pi r$
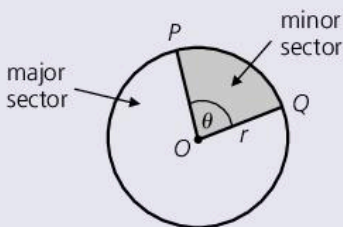
Area of minor sector $= \dfrac{\theta}{360} \times \pi r^2$

👁 **Watch out!**

You may be asked for the answer as a rounded decimal or as a multiple of $\pi$.

If it doesn't specify, it is best to give both.

🔑 **Key skills**

You need to be able to use the formulae for the length of an arc and the area of a sector.

**Worked example**

A large circular cheese has radius 12 cm.

A wedge is cut from the cheese in the shape of a sector of angle 40°.

Calculate:

(a)　the cross-sectional area of the wedge

(b)　the total perimeter of the wedge.　　　　　　**[5 marks]**

12 cm
40°
12 cm
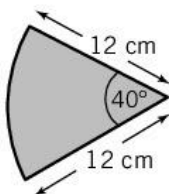
(a)　Area of sector $= \dfrac{40}{360} \times \pi \times 12^2 = 16\pi = 50.3$ cm² (to 3 s.f.)

(b)　Length of arc $= \dfrac{40}{360} \times 2 \times \pi \times 12 = \dfrac{8}{3}\pi = 8.38$ cm (to 3 s.f.)

　　　Perimeter $= \dfrac{8}{3}\pi + 2 \times 12 = 32.4$ cm (to 3 s.f.)

**Exam tip**

Don't forget to add on the two radii for the **total** perimeter.

Use the exact answer for the arc length to ensure no rounding errors are carried forwards.

**? Questions**

**1** Find, in terms of $\pi$, the area which is enclosed between two circles with the same centre, one with radius 7 cm and the other with radius 5 cm.
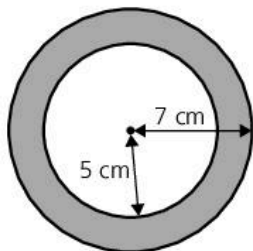


**2** Show that the area of a shape consisting of a square of side length 4 cm with a semicircle of radius 2 cm added to one side is $16 + 2\pi$.
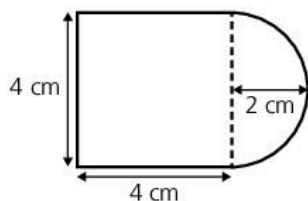


**3** Find the radius of the circle which has the same area as the combined area of a circle of radius 12 cm and a circle of radius 5 cm.

**4** A running track is 60 metres wide. The inside of the inner lane consists of two straight sections of length 80 m and two semicircles of radius 24 m. The outside of the outer lane consists of two straight sections of length 80 m and two semicircles of radius 30 m.
  **a** Find, in terms of $\pi$, the perimeter of the inside of the inner lane.
  **b** Find, in terms of $\pi$, the perimeter of the outside of the outer lane.
  **c** Find, in terms of $\pi$, the area of the track.

**5** Calculate the area of this shape. Each of the three smaller semicricles has diameter 4 cm. Leave your answer in terms of $\pi$.
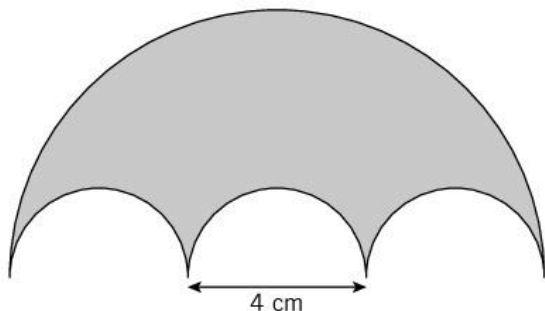


**6** A circle has an area of 100 cm². What is its radius? Give your answer correct to 3 significant figures.

**7** A circle has a circumference of 50 m. What is its area? Give your answer correct to 3 significant figures.

**8** Find the area and perimeter of these sectors, correct to 3 significant figures.

  **a**



  **b**
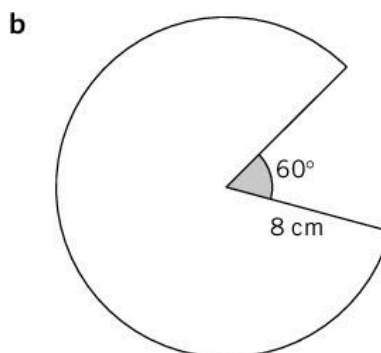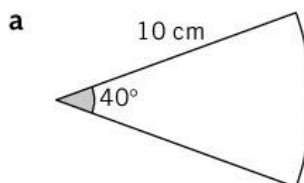


**9** The perimeter of this sector is 14.33 cm.
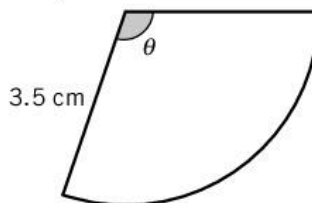


Find the value of $\theta$.

## You need to:

- Carry out calculations involving the surface area and volume of a cuboid, prism and cylinder.
- Carry out calculations involving the surface area and volume of a sphere, pyramid and cone.
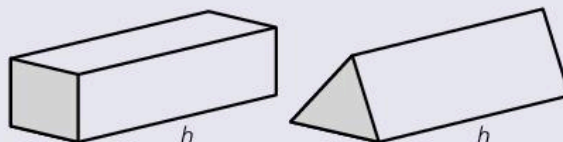- Carry out calculations involving the areas and volumes of compound shapes.

### ◀◀ Recap

**Cuboid**

Volume of cuboid $= b \times l \times h$

Surface area of cuboid $= 2hl + 2hb + 2bl = 2(hl + hb + bl)$

**Prism**

Volume of a prism = cross-sectional area × height

> **👁 Watch out!**
>
> There is no formula as such for the surface area of a prism. It depends on the shape and dimensions of the cross-section.

### ◀◀ Recap

**Cylinder**

A cylinder is a prism with a circular cross-section.
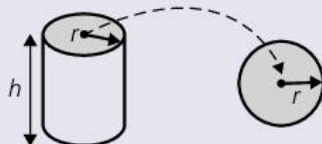
Cross-sectional area $= \pi r^2$
Volume of cylinder $= \pi r^2 h$

The **curved surface area** of a cylinder $= 2\pi rh$ (or $\pi dh$ where $d$ is the diameter)

The **total surface area** of a cylinder $= 2\pi rh + 2\pi r^2$

> **👁 Watch out!**
>
> Make sure you read the question carefully to ensure that you calculate the correct surface area.

> **🔑 Key skills**
>
> You need to be able to calculate the volume of a cylinder in context.

> **Exam tip**
>
> The formula for volume requires the *radius* of the cylinder.

**Worked example**

A cylindrical can of soup has a diameter of 7.5 cm and a height of 11 cm.

Calculate the volume of the can. **[2 marks]**

---

Radius = 3.75 cm
Volume $= \pi \times 3.75^2 \times 11 = 485.965\ldots$
$= 486 \text{ cm}^3$ (to 3 s.f.)

**Worked example**

A cylindrical can of soup has a diameter of 7.5 cm and a height of 11 cm. Calculate:

(a)   the curved surface area of the can

(b)   the total surface area of the can.   **[4 marks]**

---

(a)   Curved surface area $= \pi \times 7.5 \times 11$
$$= 259.181\ldots$$
$$= 259 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(b)   Total surface area $=$ curved surface area $+ 2\pi r^2$
$$= 259.181\ldots + 2 \times \pi \times 3.75^2$$
$$= 347.538\ldots$$
$$= 348 \text{ cm}^2 \text{ (to 3 s.f.)}$$

**Exam tip**

Use the given diameter to calculate the curved surface area.

**Exam tip**

Use the exact value of the curved surface area so as not to introduce rounding errors.

**Worked example**

The cross-section of a water pipe is a circle of radius 10 cm. Water flows along the pipe at a depth of 4 cm, as shown in the diagram.
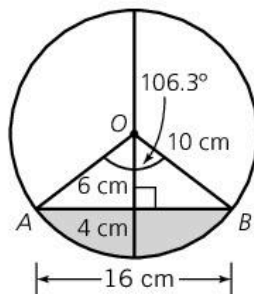


$OB = 10$ cm, $AB = 16$ cm and angle $AOB = 106.3°$

Given that the water is moving at 1.2 m per minute, calculate the volume of water which flows along the pipe in one hour.

Give your answer to 3 significant figures.   **[6 marks]**

---

Area of cross-section of water $=$ area of minor segment bounded by chord $AB$

Area of minor segment $=$ area of minor sector $-$ area of triangle $AOB$

Area of minor sector $= \dfrac{106.3}{360} \times \pi \times 10^2 = 92.729\ldots \text{ cm}^2$

Area of triangle $AOB = \dfrac{6 \times 16}{2} = 48 \text{ cm}^2$

Hence area of cross-section of water $= 92.729\ldots - 48 = 44.729\ldots \text{ cm}^3$

Rate of flow $= 120$ cm per minute

Volume of water in 1 minute $= 44.729\ldots \times 120 = 5371.71\ldots \text{ cm}^3$

So in 1 hour, volume of water $= 5371.71\ldots \times 60 = 322\,000 \text{ cm}^3$ (to 3 s.f.)

The region shaded grey in this diagram is a segment of the circle. A segment of a circle is the region bounded by a chord and an arc.

👁 **Watch out!**

6 marks indicate that this is going to be a multi-stage solution, so make sure you set your working out neatly and logically. You might want to work in 'rough' first.
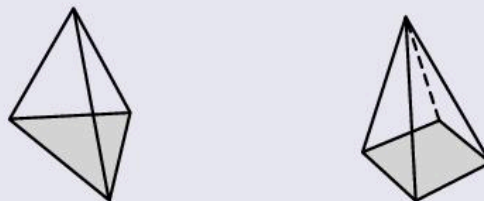
**Exam tip**

Write down what you intend to calculate and how you are going to get there, then work the calculation through.

**Exam tip**

Avoid using a rounded value for the cross-sectional area of the water. This ensures that your final answer will be correct when rounded.
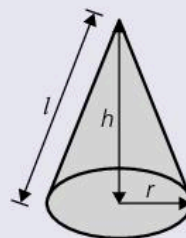
**Recap**

**Pyramid**
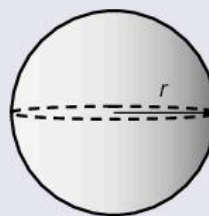
Triangular pyramid or tetrahedron    Square-based pyramid

Volume of pyramid $= \dfrac{1}{3} \times$ base area $\times$ height

**Cone**

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

The **curved surface area** of a cone $= \pi rl$ where $l$ is the 'slant height'.

**Sphere**

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

**Key skills**

You need to be able to work out the volume of compound solids. The formulae for the surface area and volume of a sphere, cone and pyramid will be given in the question when required. You do not need to learn them.

**Exam tip**

Work out the two volumes separately. Don't forget to halve the formula for the volume of a sphere.

**Worked example**

An ice cream is modelled as a cone with a hemisphere on top. The height of the cone is 10 cm and the radius of the cone is 3 cm. Find the total volume of the ice cream, giving your answer as an exact multiple of $\pi$.                                **[4 marks]**
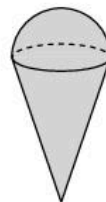
Volume of cone $= \dfrac{1}{3} \times \pi \times 3^2 \times 10 = 30\pi \, \text{cm}^3$

Volume of hemisphere $= \dfrac{\frac{4}{3} \times \pi \times 3^3}{2} = 18\pi \, \text{cm}^3$

Total volume $= 30\pi + 18\pi = 48\pi \, \text{cm}^3$

## ❓ Questions

**1** A cylindrical glass of radius 4 cm and height 9 cm is filled with water. The water is then poured into an upturned cone of base radius 5 cm and height 15 cm until the cone is full. How much water (to 2 s.f.) will be left in the glass?

**2** A company makes spherical and cubical ice holders. What is the radius (to 3 s.f.) of the spherical container if it has the same volume as the cubical container with a side length 13 mm?

**3** An ice cream scoop is designed to make spheres. Ice cream is taken from a container measuring 20 cm by 15 cm by 13 cm. If the scoop always picks up perfect spheres of radius 2.4 cm, how many scoops (to the nearest whole number) can be filled from the container (assuming no waste).

**4** A 3-litre pot of paint is used to cover the surface of a large sphere. The instructions say that one litre of paint will cover 5 $m^2$. What is the maximum radius of the sphere if it is to be completely covered?

**5** A cone of height 45 cm has to hold at least 3 litres of water. What is the least possible value (to 3 s.f.) of the base radius?

**6** A railway tunnel is constructed in the shape of a hollow cylinder. It is 1 km long and has a radius of 3 m. A gravel bed is laid in order to support the track. A cross-section of the tunnel is shown in the diagram, with the shaded area representing the gravel. AB represents the horizontal surface of the gravel.

X is the midpoint of AB and $\angle OAB = 60°$.
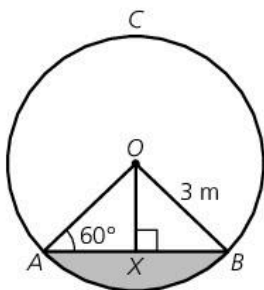


**a** Show that OX = 2.60 m (to 3 s.f.).
**b** Find the area (to 3 s.f.) of the triangle OAB.
**c** By considering the area of the sector OAB and your answer to part **b**, find (to 3 s.f.) the shaded area.
**d** Hence find (to 3 s.f.) the volume of gravel required for the tunnel.
**e** Find (to 3 s.f.) the length of the major arc ACB.
**f** The wall of the tunnel above the gravel level is to be painted. Find (to 3 s.f.) the surface area to be painted.

**7** A ditch is cut in the ground in such a way that its cross-section is a trapezium as shown in the diagram.
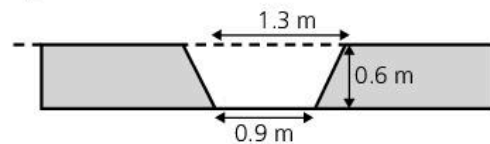


The ditch is 400 m long and it is filled with water.

**a** What is the area of the cross-section of the ditch?
**b** What is the volume of water which the ditch contains?
**c** The water flows at a rate of 1.2 m s⁻¹. What volume of water passes one point in a minute?

A pipe with a square cross-section is then placed in the ditch and the rest of the ditch is filled in with soil.

**d** If the pipe has the largest possible cross-sectional area, how much soil is put back into the ditch?

**8** The Earth is roughly a sphere with a radius of 6371 km. If about 30% of the Earth's surface is land, roughly what area is this in $km^2$? Give your answer correct to 3 significant figures.

**1** Sam has a bowl made in the shape of an upside down frustum. Its height is 4 cm. Inside the bowl, the base has a radius of 2 cm and the top has a radius of 5 cm.



Sam also has some metal spheres, each with a radius of 1 cm.

**a** What is the volume of the inside of the bowl?

[4 marks]

**b** What is the volume of each metal sphere?

[2 marks]

**c** If 100 cm³ of water is poured into the bowl first, how many spheres can then be placed into the bowl before the water starts to overflow? [3 marks]

**2** Anne has a rectangular piece of paper on her desk. The paper has a length of 29.7 cm and a width of 21 cm.

Because she is bored, Anne starts punching little circles out of the paper using a hole punch that makes circles with a diameter of 6 mm.

Assuming no holes overlap, how many holes must she punch out before she has less than half of the original piece of paper left?
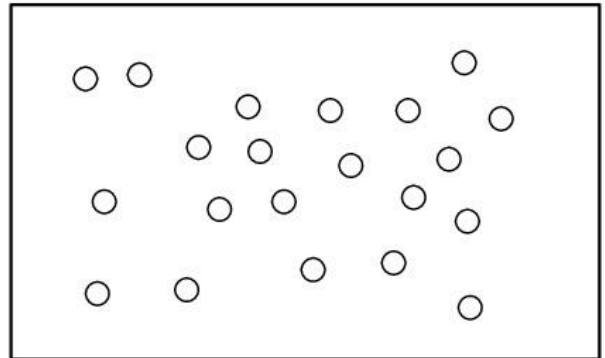


[5 marks]

**3** Part of a parade float includes a number of papier mâchè cones, each with radius 4 cm and height 29 cm. The cones do not have a base. If a small tin of paint can cover an area of 12 m², how many such cones will this paint cover completely? [5 marks]

**4** **a** A tin of soup has a diameter of 7.5 cm and a height of 11 cm.

Show that its volume is 486 cm³, correct to 3 significant figures. [2 marks]

**b** The soup is poured from the tin into a bowl, whose shape is an upside-down, hollow frustum. The diameter of the top of the bowl is 15 cm, the diameter of the base of the bowl is 8 cm, and the height of the bowl is also 8 cm.

If you assume that the tin is completely full, would its contents fit in the bowl?
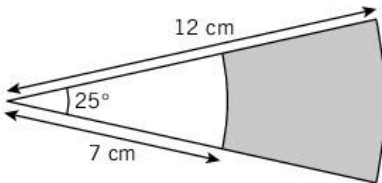
Explain your reasoning. [5 marks]

**5** In the diagram, the shape *AFBE* is called a lune. It has two curved sides, *AFB* and *AEB*. *AFBO* is a quarter circle of radius 10 cm, centred at *O*. *AEB* is a semi-circle, of radius *AD*, centred at *D*, where *D* is the midpoint of *AB*.
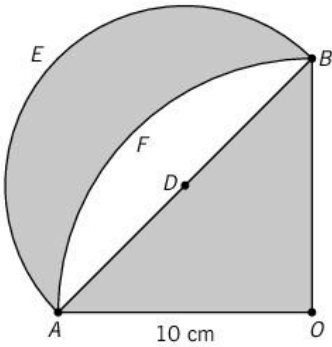


What is the area of the lune *AFBE*?

**[5 marks]**

**6**



**a** Calculate the area of the shaded region. **[3 marks]**

**b** Calculate the perimeter of the shaded region. **[3 marks]**

**7** The area of a right-angled triangle is 51.2 cm². If the two perpendicular sides are in the ratio 1 : 1.6, find the lengths of all three sides of the triangle. **[5 marks]**

# 6 Trigonometry

## Your revision checklist

Tick these boxes to build a record of your revision

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 6.1 | Interpret and use three-figure bearings. | | | |
| 6.2 | Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle. **Solve trigonometric problems in two dimensions involving angles of elevation and depression.** **Know that the perpendicular distance from a point to a line is the shortest distance to the line.** | | | |
| 6.3 | **Recognise, sketch and interpret graphs of simple trigonometric functions.** **Graph and know the properties of trigonometric functions.** **Solve simple trigonometric equations for angle values between 0° and 360°.** | | | |
| 6.4 | **Solve problems using the sine and cosine rules for any triangle and the formula:** **Area of triangle** $= \dfrac{1}{2} ab \sin C$. | | | |
| 6.5 | **Solve simple trigonometric problems in three dimensions including angle between a line and a plane.** | | | |

## You need to:

- Interpret and use three-figure bearings.

### ⏪ Recap

Bearings tell you a direction as an angle measured clockwise from the North line.
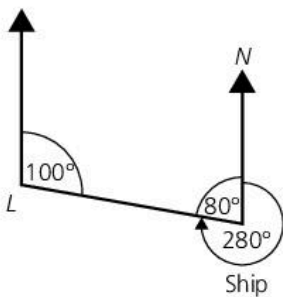


York is at the centre of the diagram.
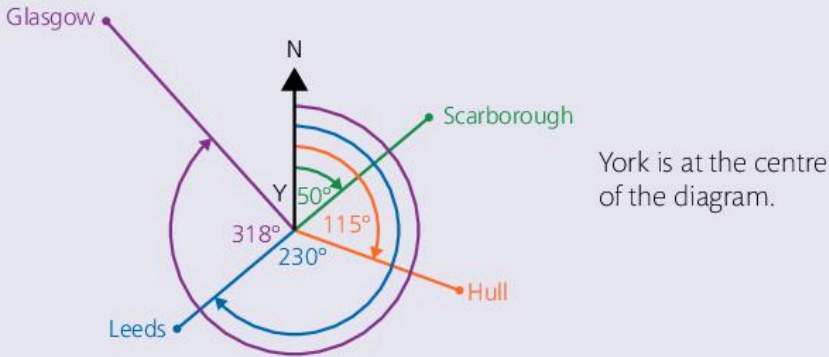
Scarborough is on a bearing of 050° from York.

Hull is on a bearing of 115° from York.

Leeds is on a bearing of 230° from York.

Glasgow is on a bearing of 318° from York.

### Worked example

The bearing of a ship from a lighthouse is 100°.

Find the bearing of the lighthouse from the ship. **[2 marks]**



Acute angle

$LSN = 180 - 100 = 80°$

Bearing $= 360 - 80 = 280°$

### 🔑 Key skills

If you are given the bearing of $B$ from $A$, you need to be able to find the bearing of $A$ from $B$. This is known as the **back bearing**.
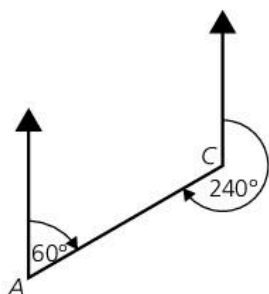
### Exam tip

Draw a little diagram and use angle rules.

**Worked example**

Clinton walks towards Abi's house on a bearing of 240°.

Write down the bearing Abi should take if she walks
to meet Clinton.  **[1 mark]**



$240 - 180 = 060°$

**Exam tip**

A quick way of working out
the back bearing is to add
180° if the initial bearing is less
than 180° or subtract 180° if it
is greater than 180°.

**? Questions**

1  The bearing of Cambridge from Oxford is 065°.

   Work out the bearing of Oxford from Cambridge.

2  A plane flies from an airport on a bearing of 320°.

   Work out the bearing on which it needs to fly to
   get back to the airport.

3  Use a scale drawing to work out the answers to
   each of these questions.

   a  A ship sails 300 km on a bearing of 080° and
      then a further 250 km on a bearing of 110°.
      Find how far the ship is from where it started,
      and the bearing it should sail on to get directly
      back to port.

   b  Two ports, A and B, are 60 km apart and B is
      due south of A. There is a yacht which is on a
      bearing of 095° from A, and on a bearing of
      036° from B. A sailboat is on a bearing of 240°
      from A, and on a bearing of 315° from B.
      How far apart are the yacht and sailboat?

   c  Two cats, Fluffy and Saff, are playing in a field.
      They start from the same place and Fluffy runs
      at a constant speed of 4 m s⁻¹ on a bearing of
      054°. Saff runs at a constant speed of 5 km h⁻¹
      on a bearing of 285°. How far apart are the
      cats after 10 seconds?

4  The diagram shows each of three landmarks in a
   village.

   The bearing of the clock tower from the village
   cross is 280°.

   The church is due West of the clock tower and
   the distance from the village cross to the clock
   tower is the same as the distance from the clock
   tower to the church.
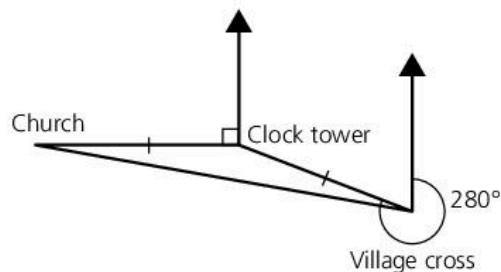


   Calculate the bearing of the village cross from
   the church.

**You need to:**

- Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle.
- Solve trigonometric problems in two dimensions involving angles of elevation and depression. (Extended)
- Know that the perpendicular distance from a point to a line is the shortest distance to the line. (Extended)
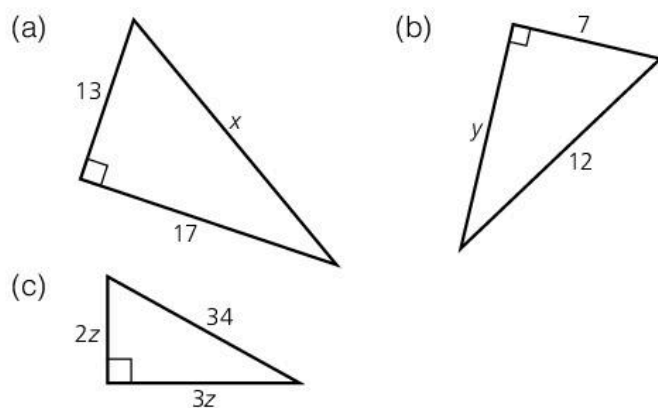
◀◀ **Recap**

Pythagoras' theorem states that:

'In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.'

$a^2 + b^2 = c^2$
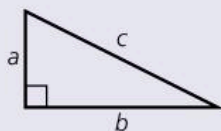


**Worked example**

Find $x$, $y$ and $z$ in these triangles.

(a)



(b)



(c)



**[7 marks]**

(a) $13^2 + 17^2 = x^2$

$x^2 = 169 + 289 = 458$

$x = \sqrt{458} = 21.4$ (to 3 s.f.)

(b) $y^2 + 7^2 = 12^2$

$y^2 = 144 - 49 = 95$

$y = \sqrt{95} = 9.75$ (to 3 s.f.)

(c) $(2z)^2 + (3z)^2 = 34^2$

$4z^2 + 9z^2 = 1156$

$13z^2 = 1156$

$z = \sqrt{\dfrac{1156}{13}} = 9.43$ (to 3 s.f.)

🔑 **Key skills**

You need to be able to use Pythagoras' theorem to find a missing side length of a right-angled triangle.

**Exam tip**

Show all of your working, even if you use a calculator.

**Exam tip**

Rearrange the formula to find the length of one of the shorter sides.

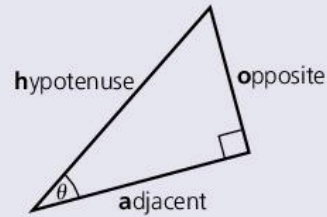👁 **Watch out!**

$(2z)^2 = 4z^2$, not $2z^2$.

## Key skills

You need to be able to use trigonometry to find a missing side length in a right-angled triangle.

## Worked example

Find *x* and *y* in these triangles.

(a)

24 cm

29°

*x*

(b)

*y*

18 mm

64°
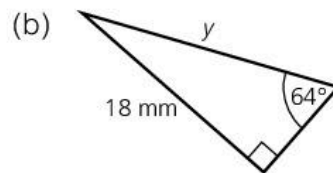
**[4 marks]**

(a) $\cos\theta = \dfrac{A}{H}$

$\cos 29° = \dfrac{x}{24}$

$\Rightarrow x = 24 \times \cos 29°$

$x = 21.0$ cm (to 3 s.f.)

(b) $\sin\theta = \dfrac{O}{H}$

$\sin 64° = \dfrac{18}{y}$

$\Rightarrow y = \dfrac{18}{\sin 64°}$
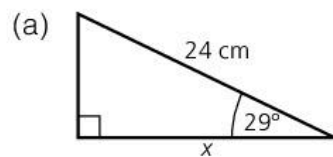
$y = 20.0$ mm (to 3 s.f.)

## Exam tip

Write down the ratio you are going to use before substituting in.

## Exam tip

Rearrange the equation carefully since your unknown is in the denominator of the fraction.

**Worked example**
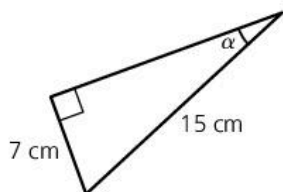
Find $\alpha$ in this triangle.



7 cm
15 cm
$\alpha$

**[2 marks]**

$\sin\alpha = \dfrac{O}{H}$

$\sin\alpha = \dfrac{7}{15}$

$\alpha = \sin^{-1}\left(\dfrac{7}{15}\right)$

$= 27.8°$ (to 1 d.p.)

**Exam tip**

Use the inverse trigonometric function on your calculator to find an angle.

Angles should be rounded to one decimal place unless otherwise stated.

◀◀ **Recap**

The **angle of elevation** is the angle you look up at to see something above you.

The **angle of depression** is the angle you look down at to see something below you.
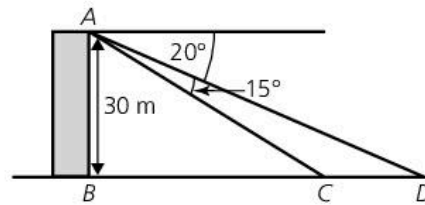


angle of depression → $\alpha$

angle of elevation

$\theta$

**Extended**
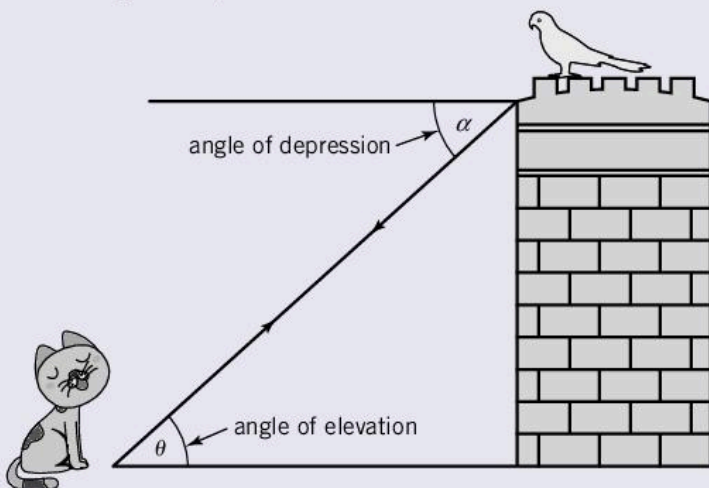
## Worked example

A surveyor standing on the top of a building 30 m tall sees two points $C$ and $D$ due north of her.

The angles of depression of $C$ and $D$ are 35° and 20° respectively.



Find the distance $CD$. **[4 marks]**



$$\tan 55° = \frac{x}{30} \Rightarrow x = 30 \times \tan 55°$$

$$\tan 70° = \frac{y}{30} \Rightarrow y = 30 \times \tan 70°$$

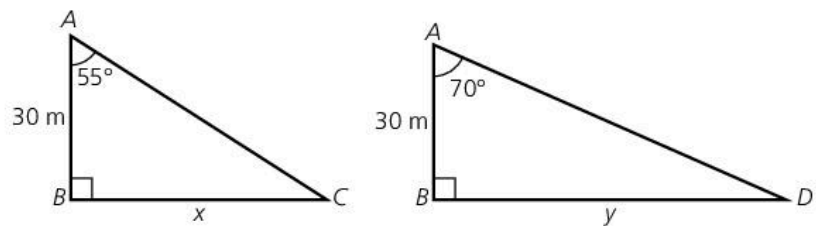$CD = 30 \tan 70° - 30 \tan 55° = 39.6$ m (to 3 s.f.)

### Exam tip

Use the information in the question to draw some clearly labelled diagrams to help.

The distance $CD$ will be $y - x$.

### Exam tip

Work exactly and do not evaluate anything until the end. This prevents rounding errors creeping in.

### 🔑 Key skills

You need to be able to find the shortest distance from a point to a line.

### ⏪ Recap

The shortest distance from a point to a line is the perpendicular distance.



Shortest distance
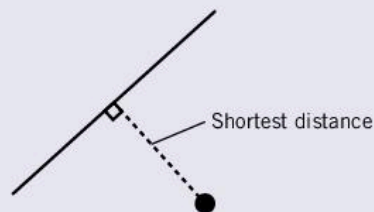
**Extended**

### Worked example

Find the shortest distance from the point (3, 0) to the line with equation $y = \frac{1}{2}x + 1$ **[5 marks]**

$\frac{1}{2} \times m = -1 \Rightarrow m = -2$
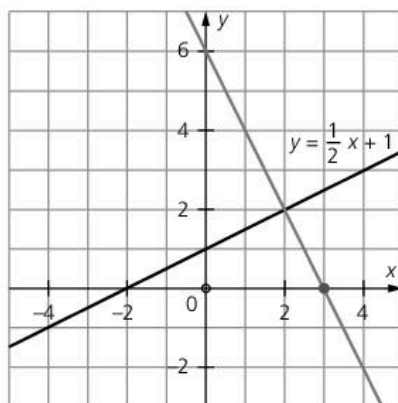
So $y = -2x + c$

Using point (3, 0):

$0 = -2 \times 3 + c \Rightarrow c = 6$

Hence $y = -2x + 6$

$\frac{1}{2}x + 1 = -2x + 6$

$\Rightarrow x = 2$

When $x = 2$, $y = 2$

$\sqrt{(2-0)^2 + (2-3)^2} = \sqrt{5}$ units

$y = \frac{1}{2}x + 1$

**Watch out!**

This problem is in the context of coordinate geometry so it is always best to draw a diagram.

**Exam tip**

Find the equation of the line perpendicular to the given line through the given point.

**Exam tip**

Find the point of intersection of the two lines.

**Exam tip**

Use Pythagoras' theorem to find the distance between the given point and the point of intersection.

Give your answer as an exact value unless otherwise stated.

### ? Questions

**1** A piece of A4 paper measures 298 mm by 210 mm. Find the length of the longest straight line which can be drawn on it.

**2** A square field has side length 45 m. Find the length of the diagonal (to 3 s.f.).

**3** A man walks 1.5 km north and then 1 km east. How far is he from his starting point (to the nearest m)?

**4** An equilateral triangle has side length 8 cm. Find the height of the triangle (to 3 s.f.).

8 cm    8 cm

8 cm
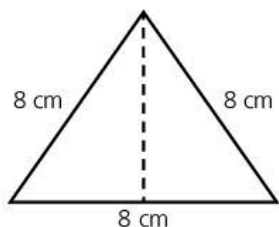
**5** Find the side lengths marked with x in these triangles.

**a**

3 cm    50°

x

**b**

62°

7.3 cm

x

**c**



13 mm
71°
x

**d**



55°
14.7 m
x
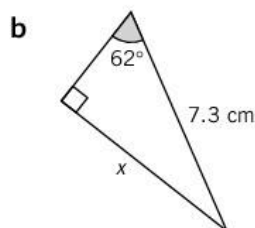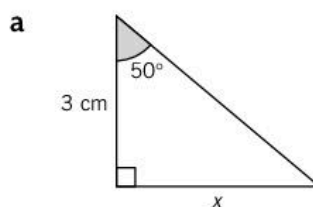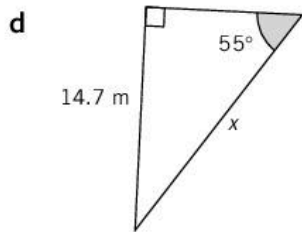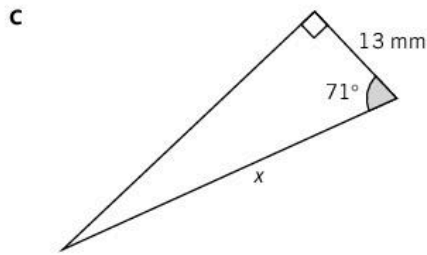
**6** Find the angles marked with x in these triangles.

**a**



x
14 cm
9 cm

**b**



2.1 cm
x
6.8 cm

**7** In the trapezium, AD = 32 cm, FE = 20 cm, FB = 12 cm and angle FAB = 60°.



F    20 cm    E
12 cm
60°
A    B        C    D
32 cm
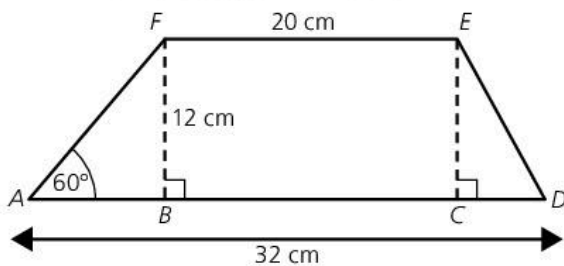
**a** Show that AB = 6.9 cm (to 1 d.p.).
**b** By first finding CD, calculate the angle EDC, giving your answer to one decimal place.
**c** Find the area of the trapezium.

**8** In the diagram:



D
A  26°      53°
B    C
42 m
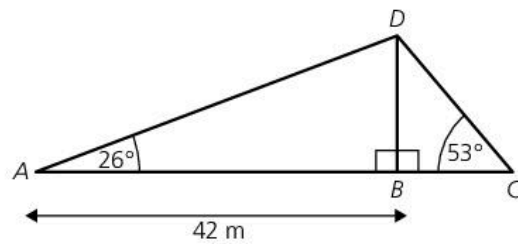
**a** show that the length BD is 20.5 m (to 3 s.f.).
**b** use this to calculate the length DC (to 3 s.f.).
**c** calculate the angle ADC.

**9** In the diagram, D is the midpoint of AB.
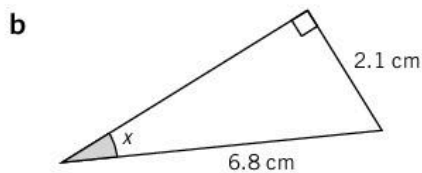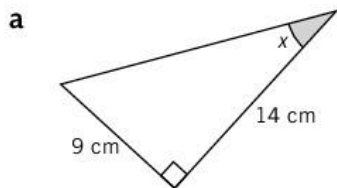


C
x        7 cm
θ        27°
A        B
y    D    y
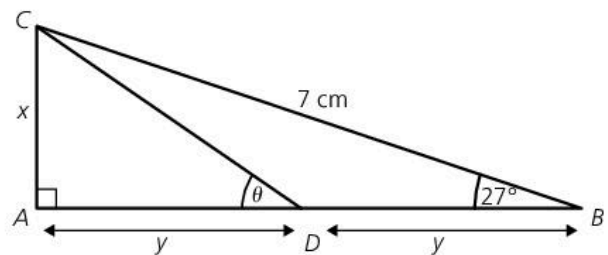
**a** Find x (to 3 s.f.).
**b** Find y (to 3 s.f.).
**c** Hence show that $\theta = 45.5°$ (to 1 d.p.).

**10** Find the length marked x in the diagram below:



76°
38°
x
20°
31 cm
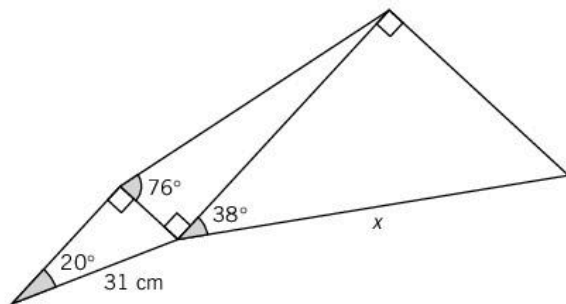
**11** The town of Hinsdale is 30 km due south of Princeton. The village of Charlbury is 25 km due east of Princeton. Find the bearing of Hinsdale from Charlbury.

**12** Arturo uses a clinometer to find the angle of elevation of a clock tower from the ground. **E**

His reading says it is 31°.
Arturo is 120 m from the base of the tower on horizontal ground.
Find the height of the tower.

**13** Find the shortest distance from the point (2, 6) to the line with equation $y = \frac{1}{3}x + 2$. Give your answer in surd form.

## You need to:

- **Recognise, sketch and interpret graphs of simple trigonometric functions. (Extended)**
- **Graph and know the properties of trigonometric functions. (Extended)**
- **Solve simple trigonometric equations for values between 0° and 360°. (Extended)**

### Recap

You can find the sine, cosine or tangent of any angle.

You need to be able to work with angles in the interval $0° \leq x \leq 360°$.

The graph of $y = \sin x$ looks like this:
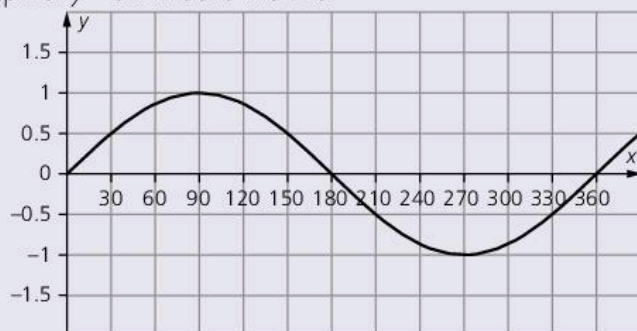


The graph of $y = \cos x$ looks like this:
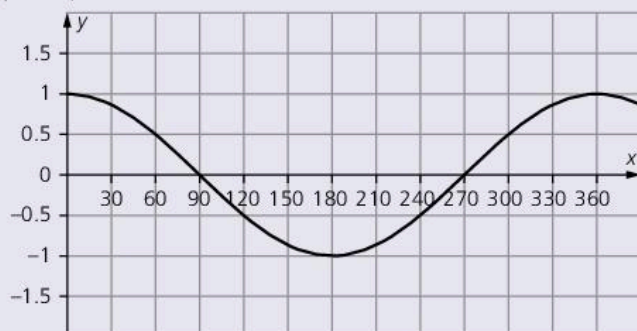


The graph of $y = \tan x$ looks like this:
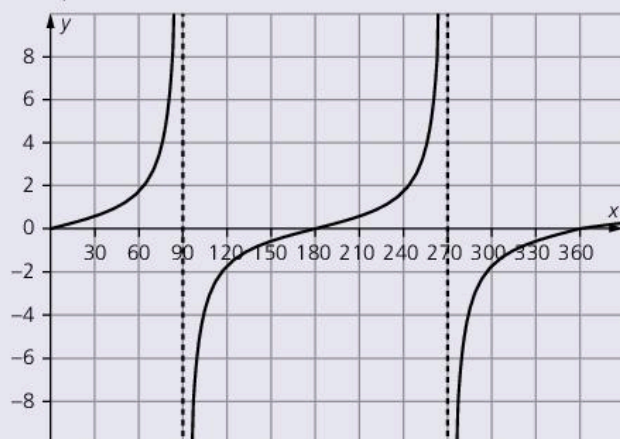


The dashed vertical lines drawn through $x = 90°$ and $x = 270°$ are called **asymptotes** and represent a discontinuity in the graph. This is because tan $x$ is **undefined** when $x = 90°$ or $x = 270°$.

### Key skills

You need to be able to use the graphs of the trigonometric functions to solve simple trigonometric equations in the interval $0° \leq x \leq 360°$.

**Extended**

### Exam tip
Use your calculator to find the inverse sine of 0.5. This is known as the **principal solution**.

### Exam tip
Draw a sketch graph of the sine function and add a line going across at $y = 0.5$. You can then read off other solutions to the equation and check them on your calculator.

### Exam tip
Rearrange the equation into the form $\tan x = \ldots$, then use the inverse tangent function to find the principal solution.

### Exam tip
Again, draw a sketch graph. Notice for the tan function, it repeats itself every 180° so you can find additional solutions accurately by adding/subtracting multiples of 180° to the principal solution.

---

**Worked example**

Solve, in the interval $0° \leq x \leq 360°$, the following equations.

(a) $\sin x = 0.5$

(b) $2 \tan x = 7$ **[4 marks]**

---

(a) $x = \sin^{-1} 0.5 = 30°$
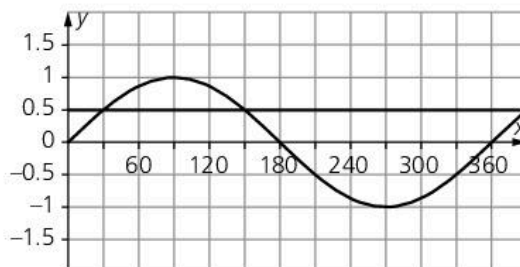


$\sin x = 0.5$ is also true when $x = 150°$

Hence $x = 30°$ or $150°$

(b) $2 \tan x = 7 \Rightarrow \tan x = 3.5$

$x = \tan^{-1} 3.5 = 74.054\ldots = 74.1°$ (to 1 d.p.)
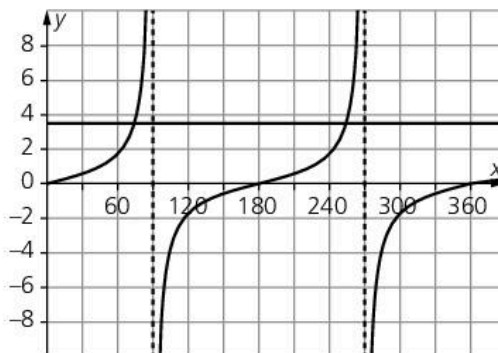


$\tan x = 3.5$ is also true when $x = 254.1°$

Hence $x = 74.1°$ or $254.1°$ (both to 1 d.p.)

---

### ? Questions

1 Solve, in the interval $0° \leq x \leq 360°$, the following equations.
   a  $\sin x = 0.4$
   b  $\cos x = 0.5$
   c  $\tan x = 2$
   d  $\sin x = -0.6$
   e  $\cos x = -0.8$
   f  $\tan x = -1$

2 Solve, in the interval $0° \leq x \leq 360°$, the following equations.
   a  $2 \sin x = 0.6$
   b  $3 \cos x = -1$
   c  $5 \tan x = 24$
   d  $1 + 3 \sin x = 2$
   e  $2 - 3 \cos x = 4$
   f  $6 - 5 \tan x = -2$

## You need to:

- **Solve problems using the sine and cosine rules for any triangle and the formula:**
  **Area of triangle $= \dfrac{1}{2} ab \sin C$. (Extended)**

◀◀ **Recap**

The **sine rule** states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When finding an angle rewrite the rule as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Use the sine rule if you know one side and its opposite angle and one other measurement.

The **cosine rule** states that

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use the cosine rule if you know

- two sides and the enclosed angle (the angle between the two sides)

or

- all three sides.
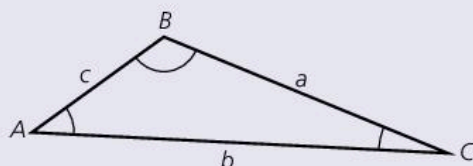
**Extended**

🔑 **Key skills**

You need to be able to use the sine and cosine rules to find missing side lengths or unknown angles in non-right-angled triangles.

👁 **Watch out!**

To find an angle using the cosine rule you need to carefully rearrange the formula.

---

**Worked example**

Find $x$ and $\theta$ in these triangles.

(a)

(b)

**[4 marks]**

---
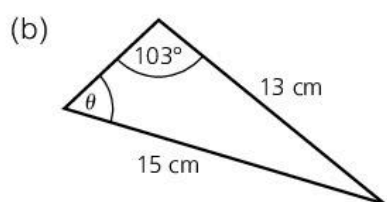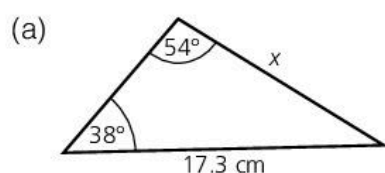
(a) $\dfrac{x}{\sin 38°} = \dfrac{17.3}{\sin 54°}$

$x = \dfrac{17.3}{\sin 54°} \times \sin 38°$

$= 13.165... = 13.2$ cm (to 3 s.f.)

(b) $\dfrac{\sin \theta}{13} = \dfrac{\sin 103°}{15}$

$\sin \theta = \dfrac{\sin 103°}{15} \times 13 = 0.8444...$

$\theta = \sin^{-1}(0.8444...) = 57.61... = 57.6°$ (to 1 d.p.)

**Exam tip**

Use the sine rule since you are working with two angles and two sides.

**Exam tip**

Use the alternative version of the sine rule since you are finding an angle.

**Extended**

## Worked example

Find $x$ and $\theta$ in these triangles.

(a)

142 cm
32°
154 cm
$x$

(b)

24 cm
19 cm
$\theta$
32 cm

**[4 marks]**

**Exam tip**

Use the cosine rule since you are working with all three sides and one angle.

**Exam tip**

Evaluate the numbers and then carefully rearrange the equation to make $\cos\theta$ the subject.
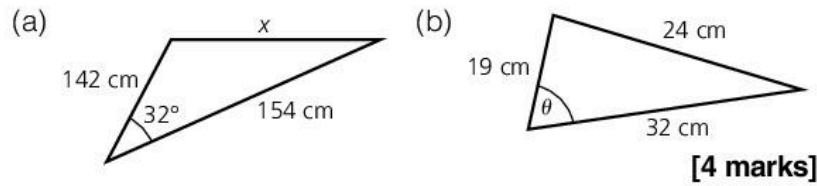
(a) $x^2 = 142^2 + 154^2 - 2 \times 142 \times 154 \times \cos 32°$

$x^2 = 6789.768\ldots$

Hence, $x = 82.4$ cm (to 3 s.f.)

(b) $24^2 = 32^2 + 19^2 - 2 \times 32 \times 19 \times \cos\theta$

$576 = 1385 - 1216 \cos\theta$

$576 + 1216 \cos\theta = 1385$

Hence $\cos\theta = \dfrac{1385 - 576}{1216} = 0.6652\ldots$

$\theta = \cos^{-1}(0.6652\ldots) = 48.3°$ (to 1 d.p.)

### Key skills

You need to be able to calculate the area of a triangle using the general formula.

### Recap

Suppose you know the lengths of $a$, $b$ and the angle $C$.

$\text{Area} = \dfrac{1}{2}ab \sin C$
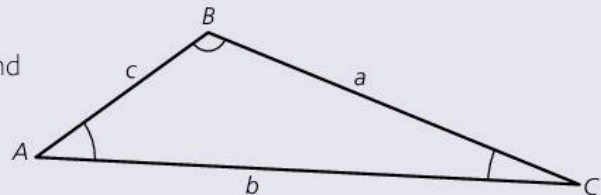


You can use this formula when you know the lengths of two sides in a triangle and the angle between them.

## Worked example


142 cm
32°
154 cm

Find the area of the triangle.

**[2 marks]**

**Exam tip**

Use the general formula for the area of a triangle since you have two sides and the *included* angle.
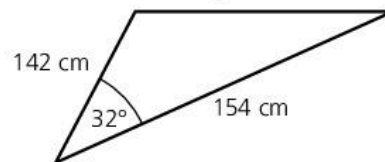
$\dfrac{1}{2} \times 142 \times 154 \times \sin 32° = 5794.13\ldots$

$= 5790$ cm$^2$ (to 3 s.f.)

### Worked example

The diagram shows four cities, $A$, $B$, $C$ and $D$. $AB = 700$ km and $AC = 800$ km. Angle $BAC = 13°$, angle $CAD = 36°$ and angle $ADC = 95°$. Calculate, giving your answers to three significant figures:
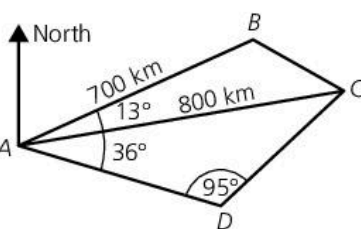
(a)  the distance $BC$

(b)  the distance $AD$

(c)  the area of land enclosed by the quadrilateral $ABCD$.

**[8 marks]**

(a)  $BC^2 = 700^2 + 800^2 - 2 \times 700 \times 800 \times \cos 13°$

  $BC^2 = 38705.52\ldots$

  Hence, $BC = 196.73\ldots = 197$ km (to 3 s.f.)

(b)  Angle $ACD = 180 - 95 - 36 = 49°$

  $\dfrac{AD}{\sin 49°} = \dfrac{800}{\sin 95°} \Rightarrow AD = \dfrac{800}{\sin 95°} \times \sin 49°$

  Hence, $AD = 606.07\ldots = 606$ km (to 3 s.f.)

(c)  Area of $ABCD$ = Area of triangle $ABC$ + Area of triangle $ACD$

  Area of $ABC = \dfrac{1}{2} \times 700 \times 800 \times \sin 13° = 62\,986.29\ldots$

  Area of $ACD = \dfrac{1}{2} \times 800 \times 606\ldots \times \sin 36° = 142\,479.14\ldots$

  Hence area of $ABCD = 205\,465.4\ldots = 205\,000$ km² (to 3 s.f.)

**Key skills**

You need to be able to solve practical problems using the sine and cosine rules and the general formula for the area of a triangle.

**Exam tip**

Use the cosine rule to find the third side of the triangle $ABC$.

**Exam tip**

To use the sine rule on triangle $ACD$, you first need to find the angle opposite side $AD$.

**Exam tip**

Work as accurately as possible before rounding your final answer.

### Questions

1  Find the length of each side marked $x$.

**a**

**b**

**c**
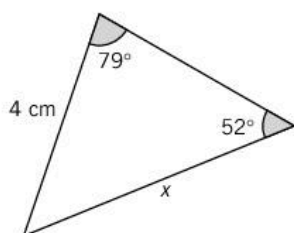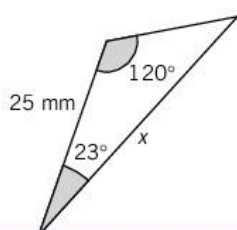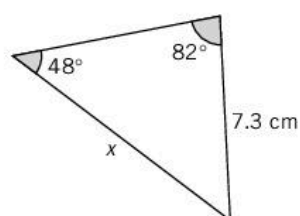
**d**
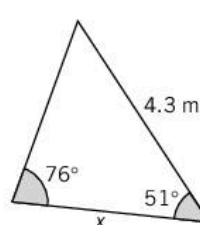
**Extended**

**2** Find each angle marked with $x$.

**a**



81°  12 cm
$x$
14 cm

**b**



33°
10.1 cm
$x$  6.8 cm
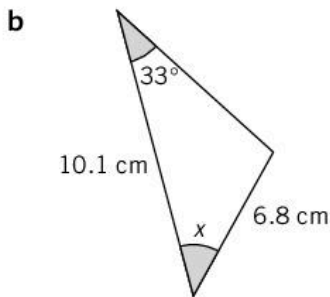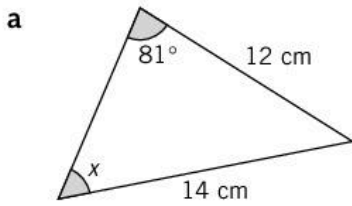
**3** A garden, $ABC$, is in the shape of a triangle. If $AB = 7.8$ m, angle $ACB = 65°$ and angle $BAC = 39°$, find the length of $BC$.

**4** An aircraft flies due north for 65 km and then turns onto a bearing of $x°$ for a further 80 km. If the bearing of the aircraft from its original starting point is 065°, find $x$.

**5** Find each side length marked with an $x$.

**a**



7.9 cm
85°
6.8 cm  $x$

**b**



28 cm
80°
$x$  37 cm

**6** Find each angle marked with an $x$.

**a**



57 cm
36 cm
$x$
62 cm

**b**



61 cm
41 cm
$x$  33 cm
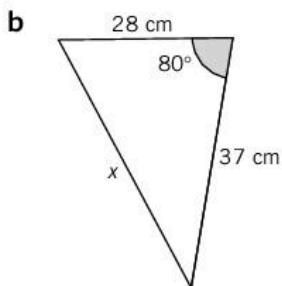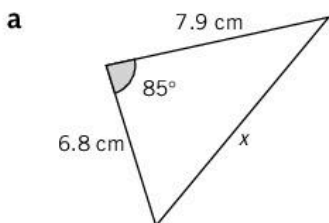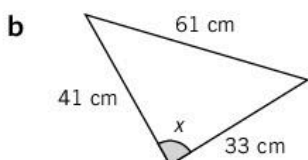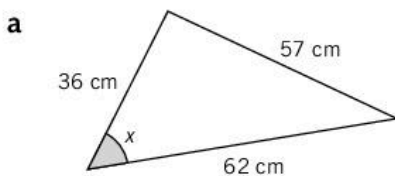
**7** Hilda the hedgehog walks due north for 75 m and then turns through a bearing of 073° and walks a further 64 m. How far is she from her starting point?

**8** The diagram shows the sail on a yacht. Find the length of $AB$ and the angle it makes with the horizontal boom. The mast $AC$ is vertical.



$C$
37°  6.3 m
9.7 m
$B$
$A$  Boom
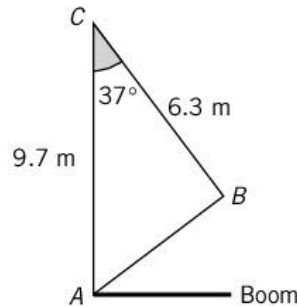
**9** Find the area of this triangle, correct to 2 decimal places.



$B$
6 cm
$A$  44°
10 cm  $C$
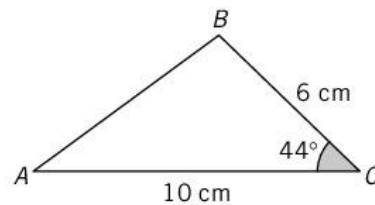
**10** A car travels 15 km from $A$ on a bearing of 100° to a point $B$. It then travels 12 km from $B$ on a bearing of 175° to a point $C$. The car then returns to $A$.

**a** Calculate the area of triangle $ABC$ (to 3 s.f.).
**b** Calculate the time (to the nearest minute) it takes to travel from $C$ to $A$ if the car maintains a steady speed of 80 km h$^{-1}$.

**11**



21 cm  $C$
$B$  11 cm
120°  73°  $D$
18 cm
$A$
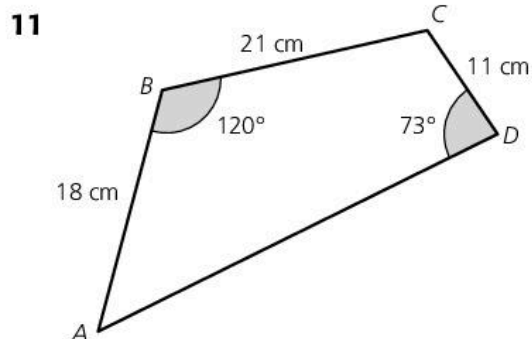
**a** Find the length of $AC$ (to 3 s.f.) in the quadrilateral.
**b** Find the area (to 3 s.f.) of the triangle $ABC$.
**c** Find the angle $CAD$ (to 1 d.p.) and so find the angle $ACD$ (to 1 d.p.).
**d** Hence find the area of the quadrilateral $ABCD$ (to 2 s.f.).

**To Raise your grade now try questions 1, 4, 6 and 8, pages 161–162**

## You need to:

- **Solve simple trigonometric problems in three dimensions including angle between a line and a plane. (Extended)**

### 🔑 Key skills

You need to be able to find missing side lengths and unknown angles in 3D shapes.

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**Worked example**
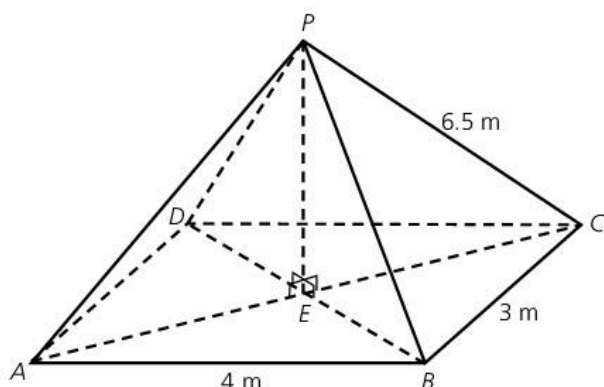


The diagram shows a model of an Egyptian pyramid.

$AB = 4$ m, $BC = 3$ m and $PC = 6.5$ m.

$ABCD$ is a horizontal rectangular base and point $E$ lies at the intersection of lines $AC$ and $BD$.

$P$ is vertically above $E$.

Calculate:

(a)   the height of the model

(b)   angle $PCA$

(c)   angle $PBC$                                                    **[9 marks]**

---

(a)   Pythagoras' theorem for triangle $ABC$ gives:

$x^2 = 3^2 + 4^2 = 25$

Hence, $x = 5$ m and $AE = 2.5$ m
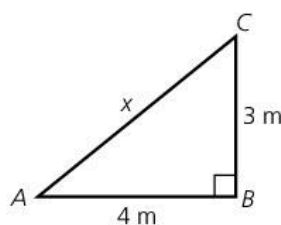


Pythagoras theorem for triangle $AEP$ gives:

$h^2 = 6.5^2 - 2.5^2 = 36$

Hence, $h = 6$ m
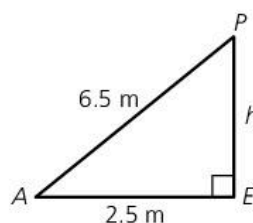


**Exam tip**

Break the problem down into stages and draw labelled diagrams of the triangles you are using.

### Extended

**Exam tip**

Look for right-angled triangles so you can use simple trigonometry.

**Exam tip**

Don't forget to round angles to 1 decimal place.

(b) Angle $PCA$ = angle $PCE$

Using trigonometry,

$\cos \theta = \dfrac{2.5}{6.5}$ hence $\theta = \cos^{-1}\left(\dfrac{2.5}{6.5}\right)$

Angle $PCA = 67.4°$ (to 1 d.p.)



(c) Angle $PBC$ = angle $PBM$ where $M$ is the midpoint of $BC$.

$\cos \beta = \dfrac{1.5}{6.5}$ hence $\beta = \cos^{-1}\left(\dfrac{1.5}{6.5}\right)$

Angle $PBC = 76.7°$ (to 1 d.p.)
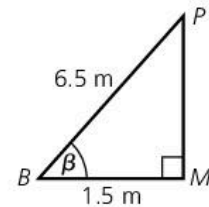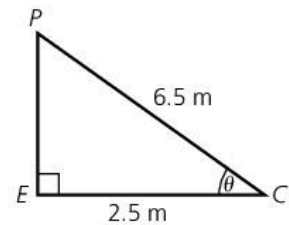


**? Questions**

**1** Find the length of the longest diagonals in this cuboid.
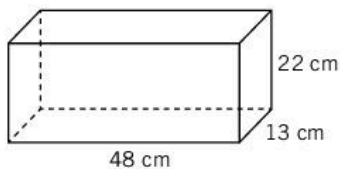


**2** A cuboid has a longest diagonal equal to 75 cm. If two of the sides are 16 cm and 40 cm, find the length of the third side.

**3** A right square-based pyramid has a perpendicular height of 14 m and a base length of 12 m. Find the angle between the base and one of the sloping sides.

**4** Find the height of this square-based pyramid.
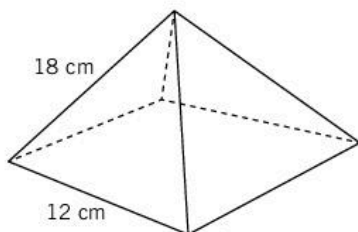


**5**



A mouse runs up this massive block of cheese, directly from point $A$ to point $B$. Find how far it runs and the angle of the slope it runs up.

**6** In the pyramid $ABCDE$, the square base $ABCD$ is horizontal and $EM$ is vertical. $M$ is the midpoint of $AC$. $AB = 15$ cm and $EM = 11$ cm.
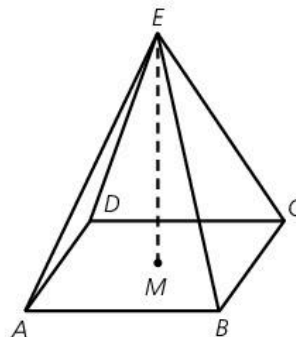


**a** Find $AM$ (to 3 s.f.).
**b** Show that $AE = 15.3$ cm (to 3 s.f.).
**c** Calculate the angle (to 1 d.p.) between the line $AE$ and the plane $ABCD$.

To **Raise your grade** now try questions **7** and **9**, page 162

**1** In triangle $ABC$, $AB = 14$ cm, $AC = 10$ cm and angle $CAB = 20°$.

Calculate:

**a** the length $BC$



10 cm

$A$ ◄ 20°

14 cm

$C$

$B$
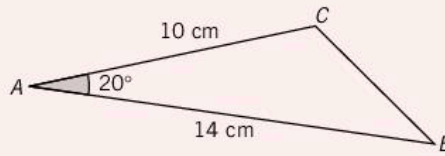
**[2 marks]**

**b** the area of the triangle. **[2 marks]**

**2** A circle is inscribed in a square as shown.

Given that the radius of the circle is 5 cm, find the length of the diagonal of the square.
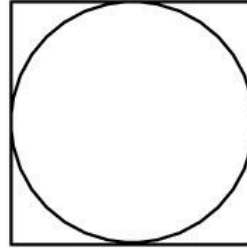
**[4 marks]**

**3** Given that $\sin x = \dfrac{\sqrt{2}}{2}$ and $0° \leq x \leq 180°$, find the two values of $x$ which satisfy the equation.

**[3 marks]**

**4** The triangle $ABC$ has area 120 cm² and angle $ACB = 50°$.

$C$

50°

$A$

$B$
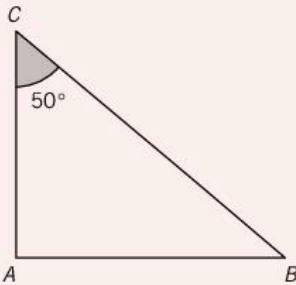
Given that $BC = 30$ cm, find, correct to three significant figures:

**a** $AC$ **[3 marks]**

**b** $AB$. **[3 marks]**

**5** Johnny is looking up at the top of a bell tower which is perpendicular to the ground. Given that the angle of elevation of the bell tower is 35°, as measured from Johnny's feet, and Johnny is standing 120 m from the tower, find the height of the bell tower. **[3 marks]**

**6** A boat sails from Aville to Beetown and then to Ceeford. Given that the bearing of Beetown from Aville is 031° and that the distances are as shown in the diagram, find the bearing of Aville from Ceeford.

Beetown

84 km

31°

102 km

Aville

92 km

**[6 marks]**

Ceeford
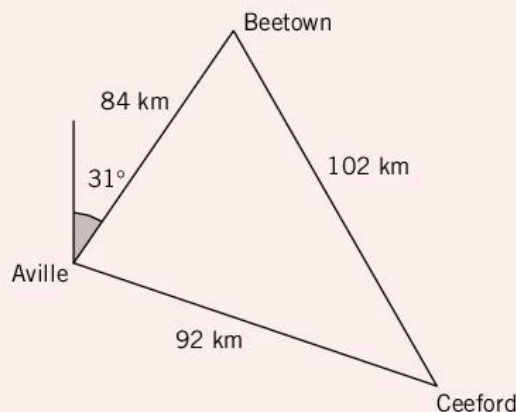
**7** A right square-based pyramid has base length 3.6 cm and perpendicular height 5.1 cm. Find the length of edge *AB*.

E

B

5.1 cm

A

3.6 cm
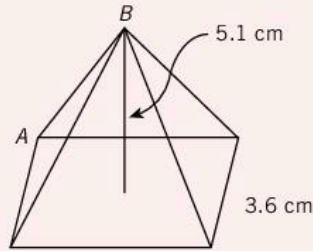
**[6 marks]**

**8** The area of this triangle is 45 cm². Find the length marked *x*.

14 cm

x

12 cm
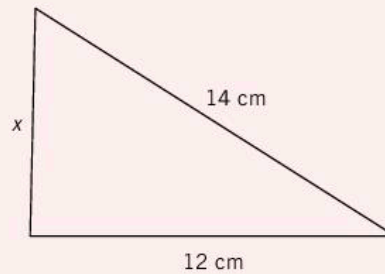
**[5 marks]**

**9** Two cuboids are stacked as shown, with the top cuboid exactly in the centre of the bottom one. Find the length marked *PQ*.

**[6 marks]**

12 m

Q

31 m

9 m

P

23 m

18 m

35 m
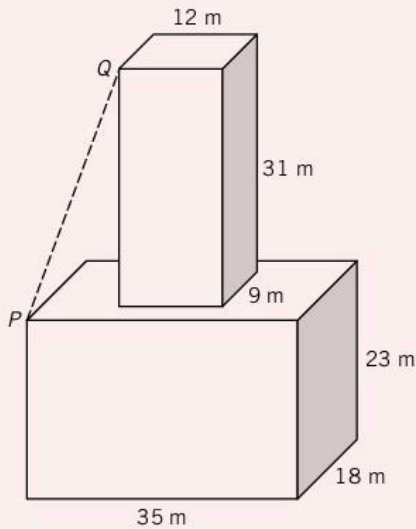
**10** In the diagram below, *AB* is a tangent to the circle. *AB* = 6 cm and *BD* = 5 cm.

C

D

5 cm

A

6 cm

B
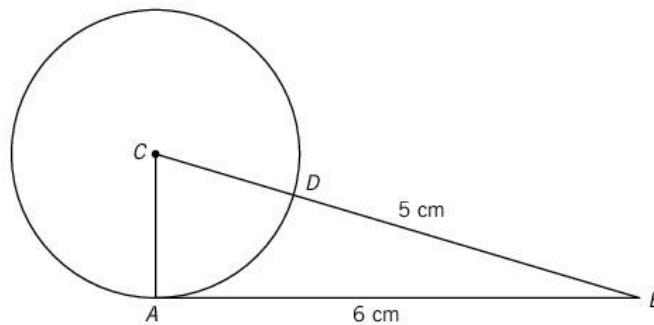
Find the radius of the circle.

**[5 marks]**

# 7 Vectors and transformations

## Your revision checklist

| Core/**Extended** curriculum | 1 | 2 | 3 |
|---|---|---|---|
| 7.1 | Describe a translation by using a vector represented by e.g. $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a**. <br><br> Add and subtract vectors. Multiply a vector by a scalar. | | | |
| 7.2 | Reflect simple plane figures in horizontal or vertical lines. Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°. Construct translations and enlargements of simple plane figures. Recognise and describe reflections, rotations, translations and enlargements. | | | |
| 7.3 | **Calculate the magnitude of a vector** $\begin{pmatrix} x \\ y \end{pmatrix}$ **as** $\sqrt{x^2 + y^2}$. <br><br> **Represent vectors by directed line segments. Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. Use position vectors.** | | | |

**You need to:**

- Describe a translation by using a vector represented by e.g. $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a**.

- Add and subtract vectors. Multiply a vector by a scalar.

### Exam tip

All that really defines a vector is its length (called its 'magnitude') and its direction. Its position is not usually considered. If two vectors have the same magnitude and direction then they are equal.

### 🔑 Key skills

You must be able to describe a translation using a vector in the three different formats.

### ⏪ Recap

If the point $A$ (−3, 2) is translated to the point $B$ (−1, 5) then $A$ moves 2 units to the right and 3 units up. The translation can be represented by the column vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

The notation $\overrightarrow{AB}$ is used to describe the translation from $A$ to $B$.

So in this example $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.



The same translation moves the point $C$ (1, 3) to $D$ (3, 6).

The notation $\overrightarrow{CD}$ is used to describe the translation from $C$ to $D$.

So $\overrightarrow{CD} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

So $\overrightarrow{AB} = \overrightarrow{CD}$.

The two vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ have the same direction and the same magnitude (length).

The vector $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ will translate $B$ back to $A$.

A vector can also be represented by a lowercase letter. In text books these are printed in bold, for example $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Such vectors should be underlined when written by hand.

---

### Worked example

The points $A$, $B$, $C$ and $D$ are (2, 1), (6, 3), (1, 3) and (5, 5) respectively.

(a) Describe the vector from $A$ to $B$ as a column vector.

**[1 mark]**

(b) What can you say about the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$?

**[1 mark]**

---

(a) To get from $A$ to $B$ you go across by 4 and up by 2.

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

(b) They are equal.
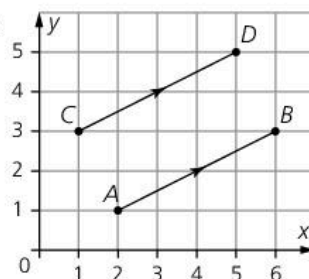
You must be able to add and subtract column vectors.

**Worked example**

If $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, write the following as column vectors:

(a)  $\mathbf{a} + \mathbf{b}$               **[1 mark]**
(b)  $\mathbf{a} - \mathbf{b}$               **[1 mark]**
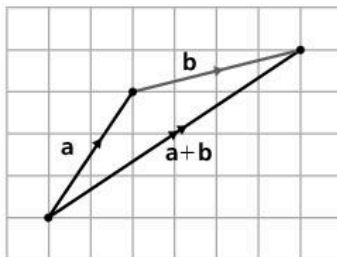
(a)  $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

             $= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

(b)  $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

             $= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$
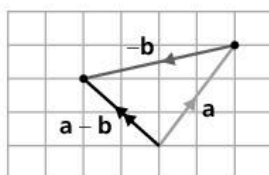
⏪ **Recap**

To add or subtract vectors you just add or subtract the corresponding horizontal and vertical components.

You must be able to multiply a vector by a scalar.

**Exam tip**

Remember that a scalar just means a number.

**Worked example**

If $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, write as column vectors:

(a)  $5\mathbf{a}$                                  **[1 mark]**
(b)  $3\mathbf{a} + 2\mathbf{b}$                       **[1 mark]**

(a)  Multiply each component by the scalar, 5.

     $5\mathbf{a} = \begin{pmatrix} 5 \times 2 \\ 5 \times 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

(b)  $3\mathbf{a} + 2\mathbf{b} = 3\begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

              $= \begin{pmatrix} 6 \\ 9 \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

              $= \begin{pmatrix} 14 \\ 11 \end{pmatrix}$
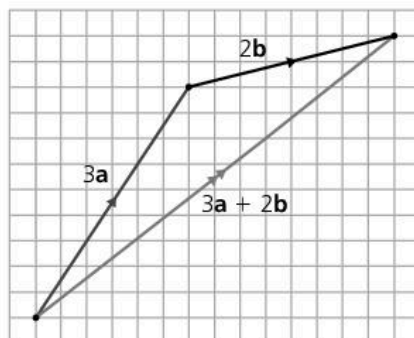
⏪ **Recap**

To multiply a vector by a scalar, multiply each component of the vector by the scalar.

**? Questions**

**1**   If $\mathbf{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$, write as a single column vector:

**a**  $\mathbf{a} + \mathbf{b}$

**b**  $3\mathbf{a}$

**c**  $-\mathbf{b}$

**d**  $2\mathbf{b} - \mathbf{a}$

**e**  $2\mathbf{a} - \mathbf{c}$

**f**  $-\mathbf{a} - \mathbf{c}$

**g**  $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$

**h**  $2(\mathbf{a} + \mathbf{b} - \mathbf{c})$

**i**  $-4\mathbf{c}$

**j**  $2\mathbf{c} - 3\mathbf{a}$

**k**  $3\mathbf{a} - 2\mathbf{b}$

**l**  $-2\mathbf{a} - \mathbf{b}$

**m** $-3\mathbf{c} + \mathbf{b}$

**n**  $\frac{1}{2}(\mathbf{b} - \mathbf{c})$

**o**  $\frac{1}{2}(\mathbf{c} - \mathbf{b})$

To **Raise your grade** now try question 1, page 176

# You need to:

• Reflect simple plane figures in horizontal or vertical lines. Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°. Construct translations and enlargements of simple plane figures. Recognise and describe reflections, rotations, translations and enlargements.

◀◀ **Recap**

You can transform a shape by **reflecting** (as in a mirror), **rotating** (turning), **translating** (moving) or **enlarging** it.

The shape you start with is called the **object** and the shape you end with is called the **image**.

◀◀ **Recap**

You describe a reflection by naming the mirror line.

Triangle $B$ is a reflection of triangle $A$ in the line $x = 1$.

Triangle $A$ is the object.

Triangle $B$ is the image.
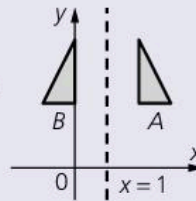


**Exam tip**

Notice that the distance from triangle $A$ to the mirror line is the same as the distance from triangle $B$ to the mirror line.

## Worked example

The triangle $ABC$ has vertices at $A$ (–1, 1), $B$ (0, 1) and $C$ (1, 3).

Draw the image of triangle $ABC$ when it is reflected in the line $y = x$, labelling the new vertices $A'$, $B'$ and $C'$.          **[3 marks]**

First draw the triangle $ABC$.

Connect each vertex to the mirror line with a line perpendicular to the mirror line.
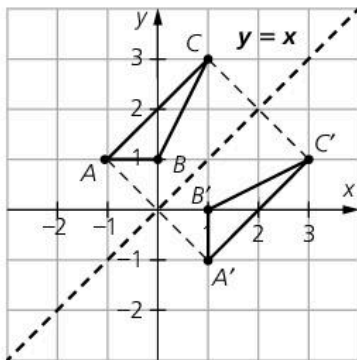


The distance from the line to the image is the same as the distance from the original point to the line.

🔑 **Key skills**

You must be able to reflect a simple plane figure and describe the transformation.

**Exam tip**

Notice that when $(p, q)$ is reflected in the line $y = x$ its image is $(q, p)$.

You describe a rotation by naming the **centre**, the **angle** and the **direction** (clockwise or anticlockwise) of the rotation.

The diagram shows an L-shape rotated through angles of 90°, 180° and 270° about a fixed point.

Original position | 90° anticlockwise | 180° | 270° anticlockwise
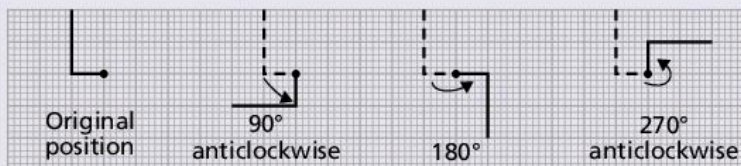
The fixed point is called the centre of rotation.

🔑 **Key skills**

You must be able to rotate a simple plane figure and describe the transformation.

**Worked example**

Find the image of the triangle with vertices $A$ (2, 2), $B$ (4, 5) and $C$ (6, 3) when it is rotated by 90° anticlockwise about the point (0, 4).
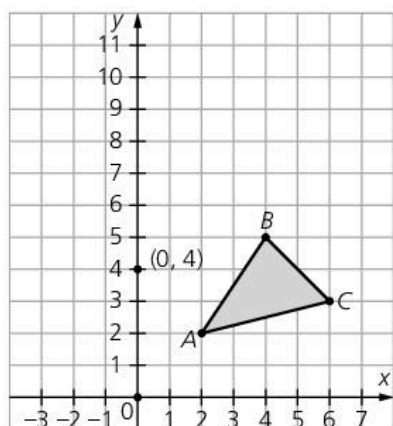
**[3 marks]**

Imagine L-shapes joining (0, 4) to the three vertices and then rotate these L-shapes as shown in the diagrams.

$A$ (2, 2) gets mapped to (2, 6)   $B$ (4, 5) gets mapped to (−1, 8)
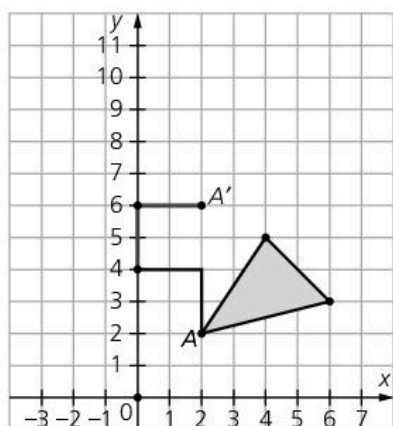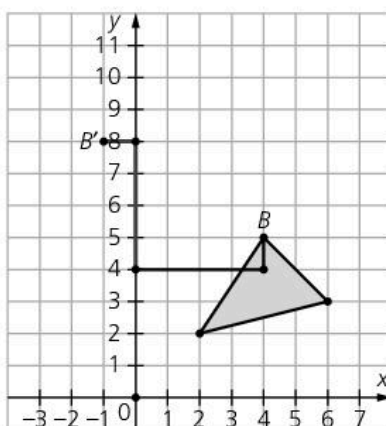
$C$ (6, 3) gets mapped to (1, 10).
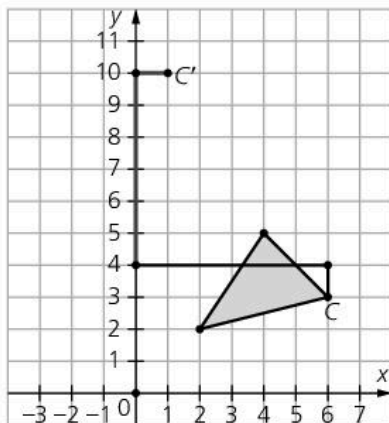


The diagram shows triangle $ABC$ and its image $A'B'C'$.
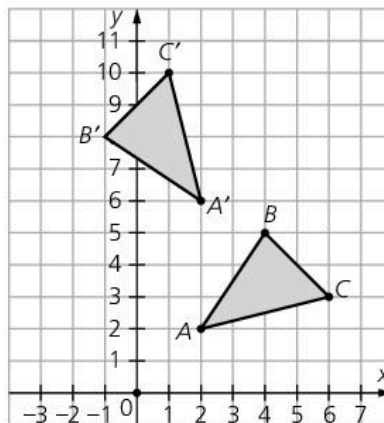
You describe a translation with a **vector**.

In the Worked example below, triangle $T$ is translated to triangle $S$ by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

- The top number in the vector tells you the horizontal movement (positive right, negative left).

- The bottom number in the vector tells you the vertical movement (positive up, negative down).

**Worked example**

Translate triangle $T$ by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.
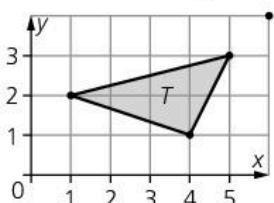


**[2 marks]**

Each corner of the triangle moves across by 2 and up by 3.
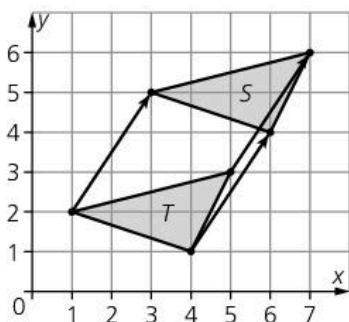


🔑 **Key skills**

You must be able to translate a simple plane figure and describe the transformation.

You describe an enlargement by naming the **centre of enlargement** and the **scale factor**. When a shape is enlarged, the image is mathematically similar to the object.

- If the scale factor is greater than 1, then the object gets bigger.

- If the scale factor is between 0 and 1, then the object gets smaller.

- If the scale factor is 1, then the object remains the same size.

If $X$ is the centre of the enlargement and $k$ is the scale factor, then $A$ gets mapped to $A'$ where $A'$ lies on the line $XA$ (extended if necessary) and $XA' = k \times XA$.

**Key skills**

You must be able to enlarge a simple plane figure and describe the transformation.

---

**Worked example**

Enlarge the triangle $ABC$ with vertices $A$ (2, 4), $B$ (4, 0) and $C$ (2, 2) by scale factor 3 with centre of enlargement $X$ (1, 2). **[3 marks]**

---

To find $A'$ draw the line $XA$ extended through $A$.

Mark the point $A'$ on this line such that $XA' = 3 \times XA$.

So $A'$ is the point (4, 8).

Repeat for $B'$ and $C'$.
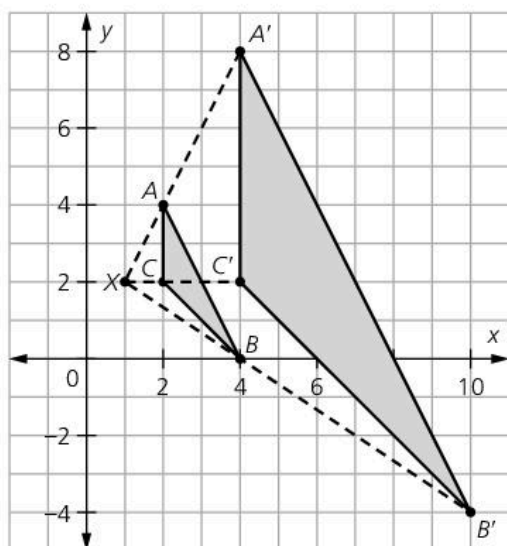


The triangle $A'B'C'$ is an enlargement of triangle $ABC$.

You use the same principle when $k$ is a fraction.

**Worked example**

Enlarge the quadrilateral ABCD with vertices A (−1, 1), B (1, 3), C (3, 3) and D (3, −1) by scale factor $\frac{1}{2}$ with centre of enlargement X (−3, −3).　　　　**[3 marks]**

To find A′ draw the line XA. Mark the point A′ on this line such that $XA' = \frac{1}{2} \times XA$.

Repeat for B′, C′ and D′.

Join A′B′C′D′.

The quadrilateral A′B′C′D′ with A′(−2, −1), B′(−1, 0), C′(0, 0) and D′(0, −2) is an enlargement of quadrilateral ABCD. Note that it is still referred to as an enlargement even when the image is smaller than the object.
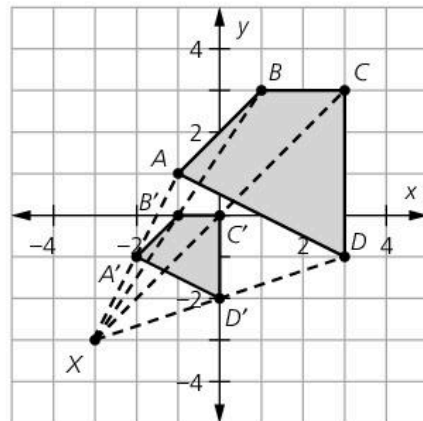


**Worked example**

Enlarge the quadrilateral ABCD with vertices A (2, 1), B (4, 2), C (4, 1) and D (3, −1) by scale factor −2 with centre of enlargement X (1, 0).　　　　**[3 marks]**

To find A′ draw the line AX and extend past X.

Mark the point A′ on this line such that XA′ = 2 × XA.

Repeat for B′, C′ and D′.

The quadrilateral A′B′C′D′ with A′(−1, −2), B′(−5, −4), C′(−5, −2), D′(−3, 2) is an enlargement of quadrilateral ABCD.
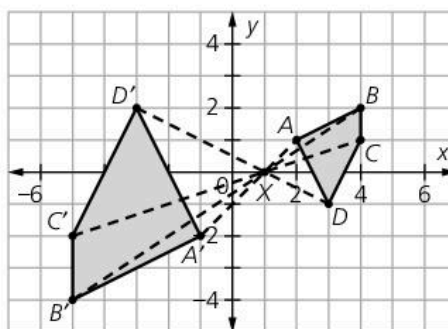


**Exam tip**

When the scale factor of enlargement is negative, the image is on the other side of the centre of enlargement from the object.

**?** **Questions**

**1**
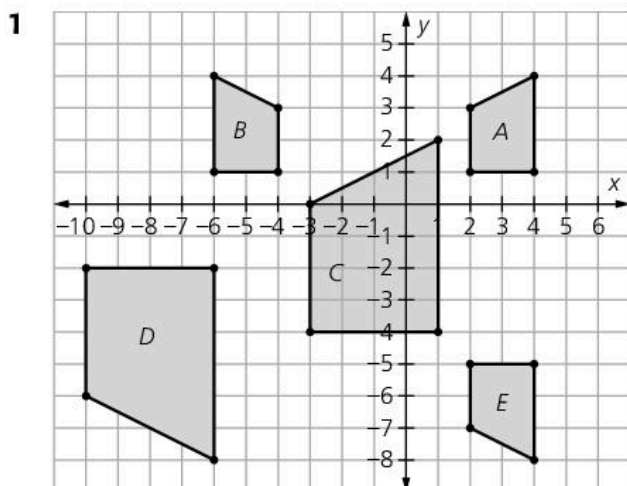


Describe the transformation which maps:
**a** *A* onto *B*  **b** *A* onto *C*
**c** *B* onto *D*  **d** *A* onto *E*.

**2** **a** Draw a pair of axes with *x* and *y* from −8 to 8 and 1 cm per unit on both.
Draw and label the triangle $T_1$ which has vertices (2, 2), (5, 2) and (5, 4).

**b i** Draw and label the triangle $T_2$ which is the image of $T_1$ under a rotation of 90° anticlockwise, centre (7, 2).
**ii** Draw and label the triangle $T_3$ which is the image of $T_2$ under a reflection in the line $y = -4$.

**c i** Draw and label the triangle $T_4$ which is the image of $T_1$ under a rotation of 90° clockwise, centre (−4, 2).
**ii** Draw and label the triangle $T_5$ which is the image of $T_4$ under a reflection in the *y*-axis.
**iii** Draw and label the triangle $T_6$ which is the image of $T_1$ under a rotation of 90° anticlockwise, centre (−2, 0).

**d** Show that the vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ translates $T_3$ onto $T_5$.

**e** Find the vector which translates $T_2$ onto $T_6$.

**3** **a** Draw a pair of axes with *x* and *y* from −8 to 8 and 1 cm per unit on both.
Draw and label the parallelogram $P_1$ which has vertices (2, 2), (5, 2), (3, 4) and (6, 4).

**b i** Draw and label the parallelogram $P_2$ which is the image of $P_1$ under a rotation of 180°, centre the origin.
**ii** Draw and label the parallelogram $P_3$ which is the image of $P_2$ under a reflection in the line $x = 1$.
**c** Draw and label the parallelogram $P_4$ which is the image of $P_1$ under a reflection in the *y*-axis.
**d** Find the vector which translates $P_3$ onto $P_4$.
**e** To what point is (2, 2) mapped under the transformation described in **b i** followed by the transformation described in **b ii** followed by the transformation described in **d**?
**f** To what point is (2, 2) mapped under the transformation described in **c**?

**4** **a** Draw a pair of axes with *x* and *y* from −8 to 8 with 1 cm per unit on both.
Draw and label the triangle with vertices *A*(1, 1), *B*(2, 5) and *C*(4, 3).
**b** Draw the rotation of triangle *ABC* by 90° anticlockwise centre (−1, 2). Label the image triangle $A_1B_1C_1$.
**c** Draw the reflection of triangle *ABC* in the line $y = -x$. Label the image triangle $A_2B_2C_2$.

To **Raise your grade** now try question 2, page 176

## You need to:

- Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$. (Extended)
- Represent vectors by directed line segments. Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. Use position vectors. (Extended)

**Extended**

### ◀◀ Recap

The magnitude of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is its length, and is calculated using the formula $\sqrt{x^2 + y^2}$.

The magnitude of a vector **a** is written $|\mathbf{a}|$.

If $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \mathbf{a}$ you can find $|\mathbf{a}|$ by using Pythagoras' theorem.

$$AB^2 = 2^2 + 3^2$$
$$= 4 + 9$$
$$= 13$$
$$AB = \sqrt{13} = 3.61 \text{ (to 3 s.f.)}$$
so $|\mathbf{a}| = 3.61$
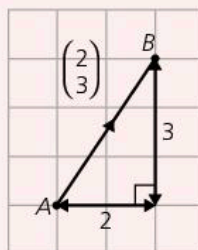
### 🔑 Key skills

You must be able to calculate the magnitude of a vector.

### 🔑 Key skills

You must be able to represent a vector using a directed line segment.

Look back at the examples in section 7.1. 'Directed line segments' is just the name given to the arrows used on diagrams to illustrate vectors.

### Worked example

Find the magnitude of the vector $\mathbf{a} + \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$. **[2 marks]**

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

$$|\mathbf{a} + \mathbf{b}| = \sqrt{10^2 + 7^2} = 149 = 12.2 \text{ (to 3 s.f.)}$$

### 🔑 Key skills

You must be able to use the sum or difference of two coplanar vectors to express another given vector.

### Exam tip

The word 'coplanar' just means 'in the same two-dimensional plane'. When you are working with vectors that only have two components, all vectors are coplanar so you don't have to worry about this.

**Extended**

---

**Worked example**

Express the vector **c** in terms of the two coplanar vectors **a** and **b**.
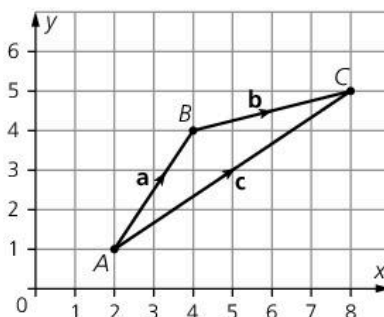


[1 mark]

The vector **c** takes you from point $A$ to point $C$.

Another way to get from point $A$ to point $C$ is to go via point $B$.

This is vector **a** followed by vector **b**.

Therefore $\mathbf{c} = \mathbf{a} + \mathbf{b}$.

---

**Apply**

Look again at the vectors in the above Worked example.

How would you express vector **a** in terms of the vectors **b** and **c**?

How would you express vector **b** in terms of the vectors **a** and **c**?

---

**Key skills**

You must be able to use position vectors.

**Recap**

A position vector is just a vector that begins at the origin.

Look at this diagram.
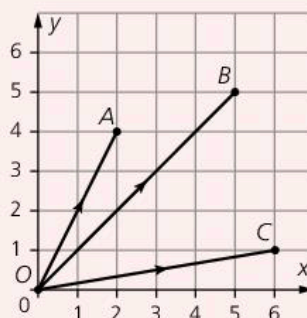


The vector $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. This is a position vector because it starts with $O$.

Similarly $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$.

An extremely useful formula is $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

**Extended**

### Exam tip
Although this formula is often used this way, using position vectors, there is nothing special about the O, which can be exchanged for any other more useful reference point on your diagram if necessary.

---

### Worked example

The position vector $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. The position vector $\overrightarrow{OB} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$.

Write the position vector $\overrightarrow{AB}$ as a column vector. **[1 mark]**

........................................................................................................

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

so $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

---

### ❓ Questions

**1** Calculate the magnitudes of the following vectors, giving your answers correct to 3 significant figures.

**a** $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$　　**b** $\begin{pmatrix} 1 \\ 14 \end{pmatrix}$　　**c** $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

**d** $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$　　**e** $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

**2** $ABCD$ is a parallelogram such that $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$. Find these vectors in terms of $\mathbf{p}$ and $\mathbf{q}$:

**a** $\overrightarrow{CD}$　　**b** $\overrightarrow{AD}$　　　**c** $\overrightarrow{AC}$
**d** $\overrightarrow{AM}$ where $M$ is the midpoint of $AB$
**e** $\overrightarrow{AN}$ where $N$ is the midpoint of $AC$
**f** $\overrightarrow{AP}$ where $P$ is the point along $AC$ which is twice as far from $A$ as from $C$ ($P$ is between $A$ and $C$).

**3** $ABCD$ is a parallelogram (labelled anticlockwise) with $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$. $M$ is the midpoint of $AD$, $N$ is the midpoint of $AB$ and $R$ is one-quarter of the way along $AC$ from $A$.
**a** Find the following in terms of $\mathbf{p}$ and $\mathbf{q}$ (in the form __$\mathbf{p}$ + __$\mathbf{q}$):
　　**i** $\overrightarrow{AM}$　　　　**ii** $\overrightarrow{AN}$
　　**iii** $\overrightarrow{AC}$　　　　**iv** $\overrightarrow{AR}$
　　**v** $\overrightarrow{MR}$　　　　**vi** $\overrightarrow{RN}$
**b** Hence show that $M$, $N$ and $R$ all lie on a straight line (by showing that $MR$ is parallel to $RN$).
**c** Find the ratio $MR : MN$.

**4** $ABC$ is a triangle (labelled anticlockwise) with $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$. $X$, $Y$ and $Z$ are the midpoints of $AB$, $BC$ and $CA$ respectively.
**a** Find the following in terms of $\mathbf{p}$ and $\mathbf{q}$ (in the form __$\mathbf{p}$ + __$\mathbf{q}$):
　　**i** $\overrightarrow{BC}$　　　　**ii** $\overrightarrow{BY}$
　　**iii** $\overrightarrow{XB}$　　　　**iv** $\overrightarrow{XY}$
　　**v** $\overrightarrow{YC}$　　　　**vi** $\overrightarrow{YZ}$
　　**vii** $\overrightarrow{AZ}$　　　　**viii** $\overrightarrow{XZ}$
**b** Hence show that $XZ$ is parallel to $BC$.
**c** Find the ratio $XZ : BC$.

**5** $ABCD$ is a parallelogram (labelled anticlockwise) with $\overrightarrow{AB} = \mathbf{r}$ and $\overrightarrow{AD} = \mathbf{s}$. $X$ is two-thirds of the way along $BD$ from $B$ and $Y$ is one-third of the way along $AD$ from $A$.
**a** Find the following in terms of $\mathbf{r}$ and $\mathbf{s}$ (in the form __$\mathbf{r}$ + __$\mathbf{s}$):
　　**i** $\overrightarrow{BD}$　　　　**ii** $\overrightarrow{AC}$
　　**iii** $\overrightarrow{BX}$　　　　**iv** $\overrightarrow{AX}$
　　**v** $\overrightarrow{AY}$　　　　**vi** $\overrightarrow{YX}$
**b** Hence show that $YX$ is parallel to $AC$.
**c** Find the ratio $YX : AC$.

**1** If $\mathbf{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$, write as a single column vector:

   **a**  $-7\mathbf{c}$      **[2 marks]**

   **b**  $\frac{1}{2}\mathbf{a}$      **[2 marks]**

   **c**  $-\frac{1}{2}(\mathbf{b} + \mathbf{c})$      **[2 marks]**

**2** Consider the triangle *ABC* shown below.

   **a**  Translate triangle *ABC* by the vector $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$ and label the image Δ2.

       **[2 marks]**

   **b**  Reflect Δ2 in the line $y = 1$ and label this image Δ3.

       **[2 marks]**

   **c**  Rotate Δ3 by 180° about (0, 0) and label this image Δ4.

       **[2 marks]**

   **d**  Describe fully the single transformation that maps *ABC* onto Δ4.
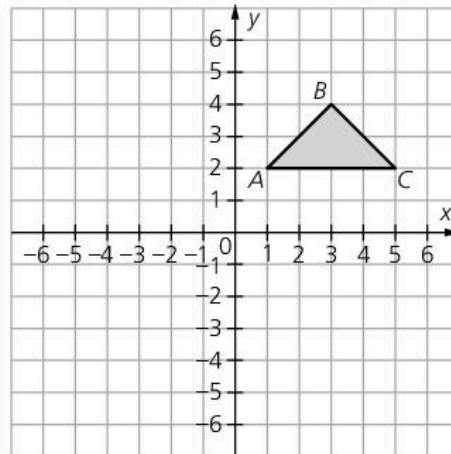
       **[2 marks]**

**3** Calculate the magnitudes of the following vectors, giving your answers correct to 3 significant figures.

   **a**  $\begin{pmatrix} -7 \\ 15 \end{pmatrix}$      **[2 marks]**

   **b**  $\begin{pmatrix} 11 \\ -4 \end{pmatrix}$      **[2 marks]**

   **c**  $\begin{pmatrix} 0 \\ 25 \end{pmatrix}$      **[1 mark]**

**4** In the diagram, *M* is the midpoint of *AC* and *N* is the midpoint of *BD*.

To prove that *M* and *N* are actually in the same place, and that the diagonals of the parallelogram bisect each other, you simply need to show that $\overline{AM} = \overline{AN}$.

Explain clearly each step in your working.

   **a**  Express the vector $\overline{AM}$ in terms of **a** and **b**.      **[2 marks]**

   **b**  Express the vector $\overline{AN}$ in terms of **a** and **b**.      **[3 marks]**
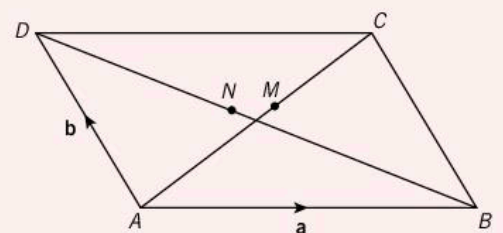
**5**   In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$M$ is the midpoint of the line segment $AB$, and $C$ is the midpoint of segment $OA$.

The point $N$ is such that $CN : NB = 1 : 2$.
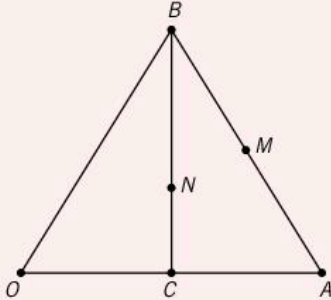


**a**   Find, in terms of $\mathbf{a}$ and $\mathbf{b}$:

   **i**   $\overrightarrow{AB}$ **[2 marks]**

   **ii**   $\overrightarrow{ON}$ . **[2 marks]**

**b**   If the line $ON$ is extended, so that it passes through the line segment $AB$, prove that this line passes through the point $M$. **[2 marks]**

**6**   In this diagram, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BM} = \mathbf{b}$, where $M$ is the midpoint of $\overrightarrow{BC}$, and $\overrightarrow{AN} : \overrightarrow{NC} = 2 : 1$.
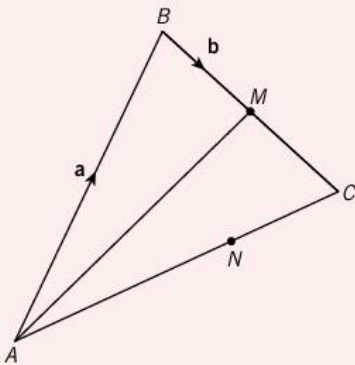


**a**   Express $\overrightarrow{AN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **[2 marks]**

**b**   Show that $\overrightarrow{NM} = \dfrac{1}{3}(\mathbf{a} - \mathbf{b})$. **[3 marks]**

**c**   Show that $\overrightarrow{BN} = \dfrac{1}{3}(4\mathbf{b} - \mathbf{a})$. **[3 marks]**

# 8 Probability

## Your revision checklist

| Core/**Extended** curriculum | | 1 | 2 | 3 |
|---|---|---|---|---|
| 8.1 | Calculate the probability of a single event as either a fraction, decimal or percentage. | | | |
| 8.2 | Understand and use the probability scale from 0 to 1. | | | |
| 8.3 | Understand that the probability of an event occurring = 1 − the probability of the event not occurring. | | | |
| 8.4 | Understand relative frequency as an estimate of probability. Predict the expected frequency of occurrences. | | | |
| 8.5 | Calculate the probability of simple combined events, using possibility diagrams, tree diagrams and Venn diagrams. | | | |
| 8.6 | **Calculate conditional probability using Venn diagrams, tree diagrams and tables.** | | | |

## You need to:

- Calculate the probability of a single event as either a fraction, decimal or percentage.

### ⏪ Recap

The probability of an event is a number between 0 and 1 that tells you how likely it is that the event will occur.

Examples of probability:

The probability of Tottenham Hotspur winning their next game is 0.75.

The probability of scoring a six when a dice is thrown is $\frac{1}{6}$.

The probability of a part made in a factory being faulty is 0.02.

The probability of rain tomorrow is 30%

**Worked example**

There are 7 red balls, 2 blue balls and 6 yellow balls in a bag. A ball is chosen at random. Find the probability that the ball is:

(a) blue            **[1 mark]**

(b) red             **[1 mark]**

(c) either red or blue.    **[1 mark]**

There are $7 + 2 + 6 = 15$ balls in the bag.

(a) 2 out of 15 of the balls are blue, so the probability of picking a blue ball is $\frac{2}{15}$.

(b) 7 out of 15 of the balls are red, so the probability of picking a red ball is $\frac{7}{15}$.

(c) The events of being red and blue are mutually exclusive, so:

probability of ball being red or blue

$=$ probability of being red $+$ probability of being blue

$= \frac{7}{15} + \frac{2}{15}$

$= \frac{9}{15}$

$= \frac{3}{5}$

**1** A fair six-sided dice is rolled. Find the probability that it shows:

**a** an even number

**b** a prime number

**c** a number less than 5

**d** a number greater than 7.

**2** A bag contains 7 blue marbles, 6 red marbles and 3 yellow marbles. One marble is picked at random from the bag. Find the probability that it is:

**a** yellow

**b** red

**c** blue or yellow

**d** green.

**3** The letters of the word RWANDA are written on cards and placed into a hat. One card is drawn at random. Find the probability that the letter shown is:

**a** the letter W

**b** the letter A

**c** a consonant

**d** in the first half of the alphabet.

**4** 16 tiles, numbered from 1 to 16, are placed in a bag. One tile is drawn at random and the number is recorded. Find the probability that it is:

**a** a factor of 16

**b** a multiple of 3

**c.** a prime number

**d** less than 10

**e** greater than 17

**f** either a factor of 12 or a factor of 15.

**5** Two identical spinners, like the one shown here, are spun. The area occupied by each number on the spinners is equal.

**a** How many possible outcomes are there?

**b** What is the probability of getting a 2 on both spinners?



**c** What is the probability of getting an odd number on *either* spinner?

**d** What is the probability of getting a total of 4?

**e** What is the probability of getting a total of 1?
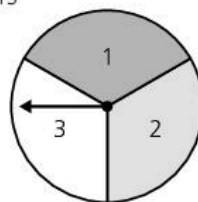
## You need to:

- Understand and use the probability scale from 0 to 1.
- Understand that the probability of an event occurring = 1 − the probability of the event not occurring.

### ⏪ Recap

All probabilities are numbers between 0 and 1.

**It is more likely that an event occurs as the probability moves along this line.**

P = 0 means impossible

0      0.5      1

P = 1 means certainty
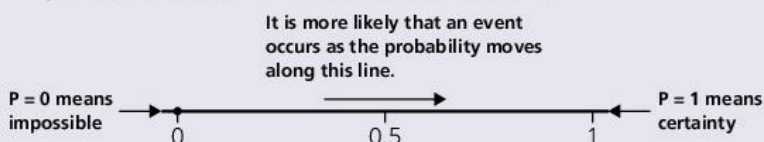
P(event not occurring) = 1 − P(event occurring)

### 🔑 Key skills

You must be aware that probabilities occur only between 0 and 1.

### 🔑 Key skills

You must be aware that the probability of an event occurring = 1 − the probability of the event not occurring.

### Exam tip

In real life, not many things actually have probabilities of 0 or 1, because not many things are totally impossible (0) or completely certain (1).

### Exam tip

This is because 'an event occurring' and 'an event not occurring' are one way of describing all the things that could happen regarding an event, therefore their probabilities add up to 1.

---

### Worked example

The probability of the traffic lights being red is 0.3.

The probability of them being green is 0.5.

(a) Is it more likely that the lights are red or green? **[1 mark]**

(b) Find the probability that the lights are not red. **[1 mark]**

---

(a)

0   0.1  0.2  (0.3) 0.4  (0.5) 0.6  0.7  0.8  0.9   1
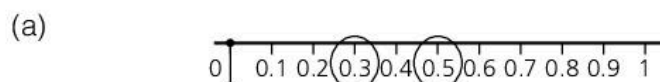
The probability of the lights being red is 0.3. P(red) = 0.3

The probability of the lights being green is $\frac{1}{2}$, or 0.5. P(green) = 0.5

It is more likely that the lights are green since 0.5 is greater than 0.3.

(b) P(not red) = 1 − P(red) = 1 − 0.3 = 0.7

### 👁 Watch out!

If you ever calculate a probability and it is less than 0 or more than 1, it always means you have made a mistake. There are no exceptions!

**? Questions**

1. An incomplete pack of playing cards contains only 45 cards. All the cards are either red or black. The probability of choosing a red card from the pack is 0.6.

   a What is the probability of choosing a black card?

   b Which colour card is more likely to be chosen?

   c How many red cards are there in the pack?

2. A standard pack of 52 playing cards contains four suits (hearts, clubs, spades and diamonds) and each suit has 13 cards. Each suit has numbered cards from 1 to 10, plus a jack, a queen and a king. One card is drawn at random. Work out the probability that it is:

   a a heart

   b a king

   c a number less than 6

   d a jack or queen

   e the seven of hearts

   f not a club.

3. A bag contains 3 red balls, 4 green balls and 5 yellow balls. One ball is chosen at random from the bag, its colour noted, and it is then replaced. A second ball is then chosen from the bag. Find the probability that:

   a both balls are red

   b both balls are green

   c one ball is red and the other is yellow

   d neither ball is yellow.

4. The letters from the word FANTASTIC are written onto individual pieces of paper and placed in a basket. One letter is chosen at random and then replaced. A second letter is then chosen. Find the probability that:

   a the letter A is obtained twice

   b the letter N is obtained twice

   c the letter F is selected first, followed by a vowel

   d a consonant and a vowel are selected.

**You need to:**
- Understand relative frequency as an estimate of probability.
- Predict the expected frequency of occurrences.

🔑 **Key skills**

You must be able to calculate relative frequency.

⏪ **Recap**

The **relative frequency** of an event occurring is defined as

$$\text{relative frequency} = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

**Exam tip**

Remember that 'relative frequency' is a probability that you obtain from data.

The more data you have, the more reliable the relative frequency is.

**Worked example**

Scarlett has a bag that she knows contains 100 marker pens. She also knows that the bag contains only red, orange and purple marker pens, but she has no idea how many of each colour there are. Scarlett takes 20 marker pens from the bag at random. Of these, 8 are red, 5 are orange, and 7 are purple. She then puts the pens back into the bag and takes out another pen at random. Find the probability that:

(a) the pen is red                                      **[1 mark]**
(b) the pen is not red.                                 **[1 mark]**

She puts that pen back, and takes out 30 pens at random. 18 of them are red.

(c) Explain how this would affect your answers to parts (a) and (b).                          **[2 marks]**

---

(a) Out of the 20 pens, 8 are red, so the relative frequency of red pens is $\frac{8}{20} = \frac{2}{5}$.

(b) $P(\text{not red}) = 1 - P(\text{red})$
$$= 1 - \frac{2}{5}$$
$$= \frac{3}{5}$$

(c) 30 is bigger than 20, so the new information gives a more reliable relative frequency.

The new relative frequency for red pens is $\frac{18}{30} = \frac{3}{5}$.

This new information would give the following answers:

(a) $\frac{3}{5}$

(b) $\frac{2}{5}$

**Exam tip**

Simplify your fractions if you want to, but it is not essential.

**Exam tip**

Use the P notation to mean 'the probability of whatever it says in the bracket'.

**? Questions**

**1** Jenny rolls a dice 18 times. She gets a 6 twice.

**a** If the dice is fair, how many times would she have expected to get a 6?

She rolls the dice another 32 times and gets a 6 three more times.

**b** Based on this data, what is the probability of rolling a 6 with this dice?

**2** Kieran takes 12 sweets at random from a bag. Four of them are red.

**a** Based on this data, what is the probability that a sweet taken from the bag is red?

**b** If there are 60 sweets in the bag, roughly how many would you expect to be red?

**3** Derren wanted to test whether a particular coin he owned was a fair coin.

**a** If the coin was fair, and he flipped the coin 40 times, roughly how many times should he expect it to land on heads?

**b** Derren flipped the coin 40 times and it landed on heads 17 times. He said, 'It isn't a fair coin, because 17 is less than 20.' How might you respond to this statement?

**4** Lucy wanted to know how many students in her school owned a pet. She asked 15 students at random if they owned a pet and 10 said yes.

**a** Based on this data, what is the probability that a randomly selected student owns a pet?

**b** If there are 900 students in her school, roughly how many of them should Lucy expect to own a pet?

Lucy asks a different 15 students if they own a pet and this time only 8 say yes.

**c** Based on all the data combined, how many students would she now expect to own a pet?

To **Raise your grade** now try questions 2, 6, 8 and 10, pages 193–194

**You need to:**
- Calculate the probability of simple combined events, using possibility diagrams, tree diagrams and Venn diagrams.

**Recap**

Possibility diagrams display all the possible outcomes of an event in a grid.

---

**Worked example**

A red dice and a blue dice are rolled at the same time.
The possibility diagram displays all the possible outcomes.
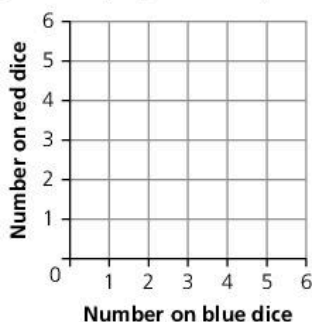


Find the probability of obtaining:

(a)    a total of 9                                    **[1 mark]**
(b)    a total of 3                                    **[1 mark]**
(c)    two sixes.                                      **[1 mark]**

---

There are 36 possible outcomes, shown where the lines cross.
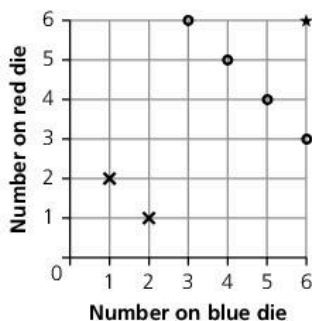


(a)    There are four ways of obtaining a total of 9, therefore

$$P(\text{a total of } 9) = \frac{4}{36} = \frac{1}{9}$$

(b)    There are two ways of obtaining a total of 3, therefore

$$P(\text{a total of } 3) = \frac{2}{36} = \frac{1}{18}$$

(c)    There is only one way of obtaining two sixes, therefore

$$P(\text{two sixes}) = \frac{1}{36}$$

⏪ **Recap**

Tree diagrams are a clear way of representing the possible outcomes of combined events.

On tree diagrams:

as you move across, multiply probabilities

as you move down, add probabilities.

**Worked example**

A bag contains 10 balls, seven of which are red and the rest green. A boy randomly takes out a ball from the bag, notes its colour and puts it back. He then repeats this process.

(a) Draw a tree diagram to represent this information.

**[2 marks]**

(b) Find the probability that he chooses two red balls.

**[2 marks]**

(c) Find the probability that he chooses two balls of different colours.

**[2 marks]**

(a)

1st ball    2nd ball



Notice that the probabilities on the branches that start from the same point always add up to 1.
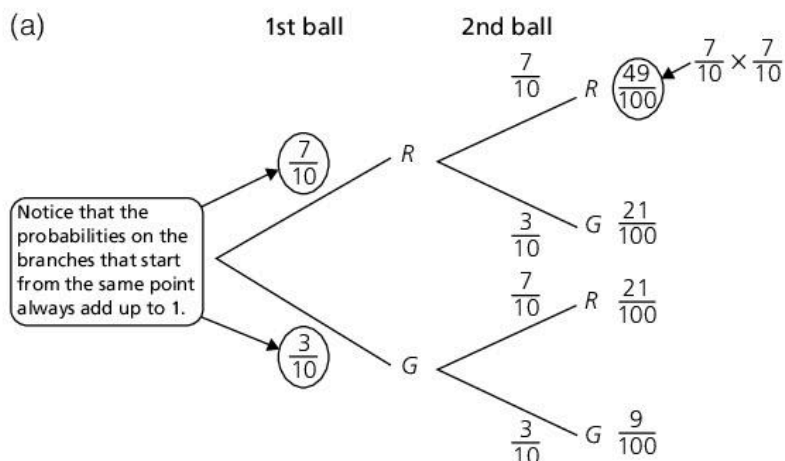
(b) Probability of two reds is $\dfrac{7}{10} \times \dfrac{7}{10} = \dfrac{49}{100}$

(c) $P(RG) = \dfrac{7}{10} \times \dfrac{3}{10} = \dfrac{21}{100}$ and $P(GR) = \dfrac{3}{10} \times \dfrac{7}{10} = \dfrac{21}{100}$

The probability of choosing two balls of different colours

$= \dfrac{21}{100} + \dfrac{21}{100} = \dfrac{21}{50}$

You must be able to calculate probabilities of combined events using a Venn diagram.

⏪ **Recap**

Venn diagrams are sometimes useful for solving probability problems.

The sets represent the outcomes of an event.

The diagram can contain either probabilities or frequencies.

---

**Worked example**

In a class of 33 students, 20 like chess, 12 like draughts and 5 like neither.

(a)  Represent this on a Venn diagram.  **[2 marks]**

(b)  Find the probability that a randomly selected student:

   (i)  likes chess but not draughts  **[1 mark]**

   (ii)  likes draughts but not chess.  **[1 mark]**

---

(a)  The two circles can overlap. Imagine that one circle has total 20 (students who like chess), one has total 12 (students who like draughts).

   The combined total has to be 28 (33 − 5).

   If the circles did not overlap their combined total would be 32 (20 + 12), so the overlap must have 4 (32 − 28).



There are 20 altogether who like chess so 16 must go in here.

There are 12 altogether who like draughts so 8 must go in here.
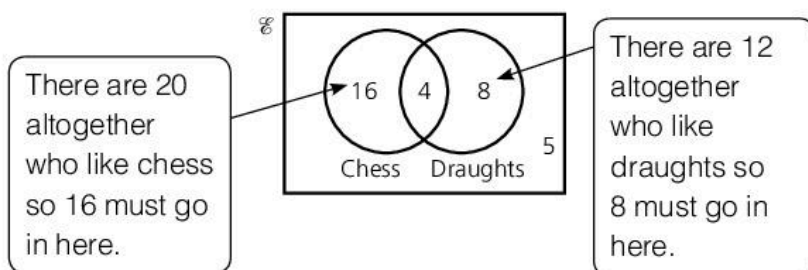
(b)  (i)  $P(\text{student likes chess but not draughts}) = \dfrac{16}{33}$

   (ii)  $P(\text{student likes draughts but not chess}) = \dfrac{8}{33}$

1 A bag contains 6 blue counters and 5 green counters. A counter is drawn from the bag and replaced. A second counter is then drawn.

   **a** Draw a tree diagram showing the possible outcomes.

   **b** What is the probability of drawing two blue counters?

   **c** What is the probability of drawing one counter of each colour?

2 A road has two sets of traffic lights. The probability of being stopped at each set is 0.3.

   **a** Draw a tree diagram showing the possible outcomes.

   **b** What is the probability of getting stopped at both sets?

   **c** What is the probability of getting stopped at neither set?

3 In a year of 100 students, 70 enjoy Maths, 50 enjoy French and 20 enjoy neither.

   **a** Draw a Venn diagram to display this information.

   **b** Use your diagram to work out the probability that a randomly selected student likes both Maths and French.

4 A bag contains 4 green balls, 3 red balls and 2 yellow balls. A ball is chosen at random from the bag and then replaced. A second ball is then chosen.

   **a** Draw a tree diagram showing the possible outcomes.

   **b** What is the probability that both balls are green?

   **c** What is the probability that the balls are the same colour?

5 The probability that Sheila is late to school is 0.2.

   **a** Draw a tree diagram showing all of the possibilities for Sheila's arrivals on three consecutive days.

   **b** What is the probability that Sheila is late on all three days?

   **c** What is the probability that she is late just once?

**You need to:**

- **Calculate conditional probability using Venn diagrams, tree diagrams and tables. (Extended)**

### Worked example

Out of 30 students in a class, 20 like pop music, 14 like rock music and 4 like neither.

(a)   Illustrate this information using a Venn diagram.

**[2 marks]**

(b)   Find the probability that a randomly selected student:

   (i)   likes pop music and rock music   **[2 marks]**

   (ii)   likes pop music, given that they don't like rock music   **[2 marks]**

   (iii)   likes rock music, given that they don't like pop music.   **[2 marks]**

---

(a)   
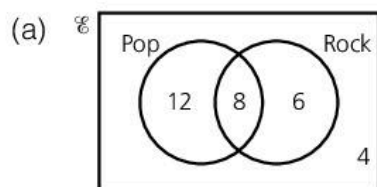


(b)   (i)   $P(\text{pop and rock}) = \dfrac{8}{30} = \dfrac{4}{15}$

   (ii)   There are 16 people who don't like rock music, and 12 of those like pop music, therefore the probability that a student likes pop music, given that they don't like rock music is:

$$P(\text{pop} \mid \text{rock'}) = \dfrac{12}{16} = \dfrac{3}{4}$$

   (iii)   There are 10 people who don't like pop music, and 6 of those like rock music, therefore the probability that a student likes rock music, given that they don't like pop music is:

$$P(\text{rock} \mid \text{pop'}) = \dfrac{6}{10} = \dfrac{3}{5}$$

### 🔑 Key skills

You must be able to calculate a conditional probability using a Venn diagram.

### Exam tip

Conditional probability questions often contain the word 'given'.

### Exam tip

Remember the notation for the probability of 'A given B' is $P(A \mid B)$

### Exam tip

In a conditional probability question, the denominator of the fraction is smaller.

🔑 **Key skills**

You must be able to calculate a conditional probability using a tree diagram.

**Worked example**

There are 3 red balls and 2 green balls in a bag. One ball is chosen at random, its colour is noted and *it is not replaced*. A second ball is then chosen and its colour is also noted. Find the probability that:

| | | |
|---|---|---|
| (a) | both balls are red | **[2 marks]** |
| (b) | both balls are the same colour | **[2 marks]** |
| (c) | at least one of the balls is green | **[2 marks]** |
| (d) | the first ball is red, given that the second ball is green. | **[2 marks]** |

**Exam tip**

The fact that the first ball is not replaced means that the probability of the second ball being a particular colour is conditional and depends on the colour of the first ball.

If a red ball is chosen first, then there are now 4 balls in the bag and only 2 of them are red. The probability of choosing a red is $\frac{2}{4}$.

When the 5 balls are in the bag, the probability of choosing a red is $\frac{3}{5}$, since 3 of the 5 balls are red.



1st ball — 2nd ball
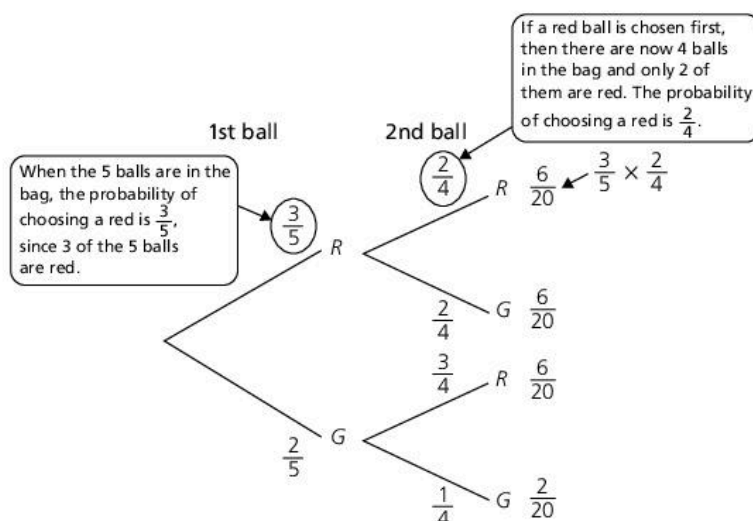
$$\frac{3}{5} \times \frac{2}{4}$$

(a) $\quad P(RR) = \dfrac{3}{5} \times \dfrac{2}{4} = \dfrac{6}{20} = \dfrac{3}{10}$

(b) $\quad P(\text{same colour}) = P(RR \text{ or } GG)$

$$= P(RR) + P(GG) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

(c) When two balls are chosen, either 'At least one of the balls is green' or both of them are red.

So the probabilities of these two events add up to 1.

$P(\text{at least one } G) + P(RR) = 1$

So $P(\text{at least one } G) = 1 - P(RR)$

From the tree diagram $P(RR) = \dfrac{3}{10}$

And so

$P(\text{at least one } G) = 1 - P(RR)$

$$= 1 - \frac{3}{10} = \frac{7}{10}$$

**Exam tip**

This is an example of an important short cut.

You **do not** need to think of all the cases when there is at least one green ball.

(d)   P(first ball red | second ball green)

To calculate this you divide the probability of 'the first ball is red, then the second ball is green' by the probability that the second ball is green.

$$\frac{\frac{6}{20}}{\frac{6}{20}+\frac{2}{20}} = \frac{6}{8} = \frac{3}{4}$$

**Exam tip**

From the second half of the tree diagram you can see that

$P(R\,|\,R) = \frac{2}{4}$ and $P(R\,|\,G) = \frac{3}{4}$.

**Apply**

Think about how the method of solving part (d) shown in the above Worked example relates to the Venn diagram method.

**Key skills**

You must be able to calculate a conditional probability using a table.

**Worked example**

Here is a table showing the meal choices made by 60 people at a wedding.

|  | Ice-cream | Cake | Brownie | **Total** |
|---|---|---|---|---|
| Chicken | 9 | 16 | 12 | 37 |
| Fish | 6 | 12 | 5 | 23 |
| **Total** | 15 | 28 | 17 | 60 |

Find the probability that a randomly selected person:

(a)   chose a brownie, given that they chose chicken

**[2 marks]**

(b)   chose chicken, given that they chose ice-cream.

**[2 marks]**

(a)   The number of people who chose chicken was 37, and 12 of those people chose a brownie,

so        $P(\text{brownie}\,|\,\text{chicken}) = \frac{12}{37}$

(b)   The number of people who chose ice-cream was 15, and 9 of those people chose chicken,

so        $P(\text{chicken}\,|\,\text{ice-cream}) = \frac{9}{15} = \frac{3}{5}$

**? Questions**

1 A boy is late 60% of the time when it is raining and 30% of the time when it is dry. It rains on 25% of days.

   **a** Draw a tree diagram to represent the above information.

   **b** Find the probability that:

      **i** it is raining and he is late
      **ii** he is late
      **iii** he is on time given that it is dry.

2 On an athletics day 150 athletes take part. 60 are in the 100 metres race, 50 are in the 200 metres race and 80 are in neither.

   **a** Draw a Venn diagram showing this information.

   **b** Find the probability that a randomly selected athlete will be:

      **i** taking part in both races
      **ii** taking part in the 100 m race, given that they are taking part in the 200 m race
      **iii** taking part in the 200 m race, given that they are taking part in the 100 m race.

3 At a café, customers can choose between tea and coffee. Both drinks come in three different sizes. The café sells 50 drinks as displayed in the following table.

|  | Small | Medium | Large | **Total** |
|---|---|---|---|---|
| Tea | 12 | 14 | 9 | 35 |
| Cofee | 5 | 3 | 7 | 15 |
| **Total** | 17 | 17 | 16 | 50 |

Find the probability that a randomly selected customer will have chosen:

   **a** a small drink, given that they chose tea

   **b** coffee, given that they chose a large drink

   **c** a medium drink, given that they did not choose coffee.

1    Dipika rolls two dice.

     What is the probability that both dice land on a six?    **[2 marks]**

2    The spinner shown is spun 400 times.
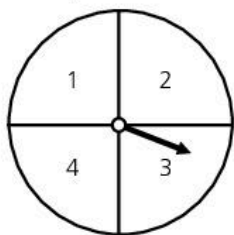


     How many times would you expect the spinner to land on:
     **a**    the sector numbered 1    **[1 mark]**
     **b**    a sector with an even number    **[1 mark]**
     **c**    a sector with a number of more than 3?    **[1 mark]**

3    At a sports club, 30 members are asked if they like hockey (H), rugby (R) or both.
     Some of the results are shown in the Venn diagram.
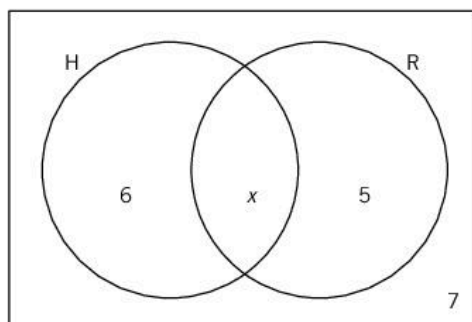


     **a**    Find the value of $x$.    **[2 marks]**
     **b**    Two members of the club are chosen at random. Find the probability that both
              members like either hockey or rugby or both.    **[2 marks]**

4    A spinner is equally likely to land on the numbers 1, 2, 3, 4, 5 or 6.
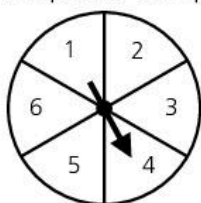


     Find the probability that the spinner lands on:
     **a**    an even number    **[1 mark]**
     **b**    a number greater than 4    **[1 mark]**
     **c**    an even number or a number greater than 4.    **[1 mark]**

5    The probability of a girl wearing glasses is 0.3 and the probability of a girl having
     blonde hair is 0.4. Joshua thinks that the probability of a girl wearing glasses or having
     blonde hair is 0.7. Is he correct?    **[2 marks]**

**6**  Mr Choudry drives to work each morning. The probability that he parks his car at the front of the building is 0.4. The probability that he parks his car at the side of the building is 0.15. The rest of the time he parks at the back of the building.

    **a**  What is the probability that Mr Choudry will park at the back of the building on any particular morning? **[1 mark]**

    **b**  What is the probability that he will park either at the back or at the side of the building on any particular morning? **[1 mark]**

    **c**  In the next year, he works for 220 days. On approximately how many days will Mr Choudry park either at the back or at the side of the building? **[2 marks]**

**7**  Alexander shoots three arrows at a target. With each arrow, the probability that he hits the target is 0.3. Whether or not he hits the target with any given arrow is independent of what happened to the previous arrows.
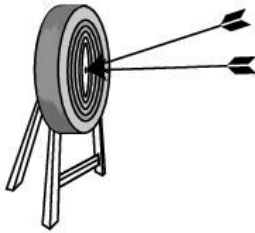
Find the probability that he hits the target with the first two arrows but misses with the third. **[2 marks]**

**8**  A boy picks out a marble from a bag of 20 coloured marbles. He records its colour and then puts it back. He does this fifty times. Ten of the marbles he takes out are red. How many of the 20 marbles in the bag do you estimate to be red? **[2 marks]**

**9**  A box contains 12 counters of which two are blue, three are red and the rest are purple. Two counters are chosen at random, the first being replaced before the second is chosen.

Find the probability that

    **a**  the first counter is purple and the second is blue **[1 mark]**

    **b**  both counters are red **[2 marks]**

    **c**  one of the counters is blue and the other is red. **[2 marks]**

**10**  A coin is biased so that heads appears twice as often as tails.

    **a**  If $p$ is the probability of getting tails, write down the probability of getting heads in terms of $p$. **[1 mark]**

    **b**  Hence show that $p = \dfrac{1}{3}$. **[2 marks]**

    **c**  The coin is tossed twice. Find the probability of getting:

      **i**  two heads

      **ii**  a head first and then a tail

      **iii**  two tails. **[4 marks]**

**11** A circular spinner has four sections, numbered 1, 2, 3 and 4. The area for 2 is twice the area for 1, the area for 3 is three times the area for 1 and the area for 4 is four times the area for 1.

If $p$ is the probability of getting a 1 then:

**a** write down, in terms of $p$, the probabilities of getting 2, 3 and 4 **[2 marks]**

**b** find $p$ **[2 marks]**

**c** find the probability of getting an even number. **[2 marks]**

**12** Mr Danyata travels to school by car. The probability that he is delayed in traffic on a particular day is 0.4. If he is delayed in traffic, the probability that he is late to school is 0.7. If he is not delayed in traffic, the probability that he is late is 0.05. **E**

Find the probability that Mr Danyata is late to school. **[3 marks]**

**13** Five yellow balls and three red balls are placed in a bag and two are removed, one at a time, without replacement.

**a** Draw a tree diagram to represent the above information. **[3 marks]**

**b** Find the probability that:

   **i** both balls are red

   **ii** both balls are the same colour

   **iii** the second ball is yellow given that the first is red. **[6 marks]**

**14** Fifty people attending a conference were offered a choice of tea, coffee or hot chocolate to drink, along with biscuits or a piece of cake. Their choices are displayed in this table.

| | Tea | Coffee | Hot chocolate | Total |
|---|---|---|---|---|
| **Biscuits** | 12 | 6 | 8 | 26 |
| **Cake** | 10 | 9 | 5 | 24 |
| **Total** | 22 | 15 | 13 | 50 |

Work out the probability that a randomly selected member of the group:

**a** chose cake **[1 mark]**

**b** chose coffee **[1 mark]**

**c** chose biscuits, given that they chose tea **[1 mark]**

**d** chose tea, given that they chose biscuits. **[1 mark]**

# 9 Statistics

## Your revision checklist

| Core/**Extended** curriculum | 1 | 2 | 3 |
|---|---|---|---|
| 9.1 | Collect, classify and tabulate statistical data. | | | |
| 9.2 | Read, interpret and draw inferences from tables and statistical diagrams.<br>Compare sets of data using tables, graphs and statistical measures.<br>Appreciate restrictions on drawing conclusions from given data. | | | |
| 9.3 | Construct and interpret bar charts, pie charts, pictograms, stem-and-leaf diagrams, simple frequency distributions, histograms with equal **and unequal** intervals and scatter diagrams. | | | |
| 9.4 | Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used. | | | |
| 9.5 | **Calculate an estimate of the mean for grouped and continuous data.**<br>**Identify the modal class from a grouped frequency distribution.** | | | |
| 9.6 | **Construct and use cumulative frequency diagrams.**<br>**Estimate and interpret the median, percentiles, quartiles and interquartile range.**<br>**Construct and interpret box-and-whisker plots.** | | | |
| 9.7 | Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram. | | | |
| 9.8 | Draw, interpret and use lines of best fit by eye. | | | |

**You need to:**

- Collect, classify and tabulate statistical data.
- Read, interpret and draw inferences from tables and statistical diagrams.
- Compare sets of data using tables, graphs and statistical measures.
- Appreciate restrictions on drawing conclusions from given data.

- Construct and interpret bar charts, pie charts, pictograms, stem-and-leaf diagrams, simple frequency distributions, histograms with equal **and unequal (Extended)** intervals and scatter diagrams.

🔑 **Key skills**

You need to be able to tabulate both discrete and continuous data, read and interpret various statistical diagrams, and compare sets of data, drawing conclusions while appreciating the restrictions involved.

🔑 **Key skills**

You must be able to construct and interpret bar charts.

⏪ **Recap**

In a bar chart, the frequency is represented by the height of the bar.

**Worked example**

The homework scores of 20 students were as follows:

20  20  15  20  17
16  18  18  20  15
20  18  19  17  18
20  19  18  20  18

Represent this information using a bar chart.　　**[3 marks]**

Start by putting the scores into a frequency table, and then from the table draw the bar chart.

| Score | Frequency |
|-------|-----------|
| 15 | 2 |
| 16 | 1 |
| 17 | 2 |
| 18 | 6 |
| 19 | 2 |
| 20 | 7 |



Homework scores
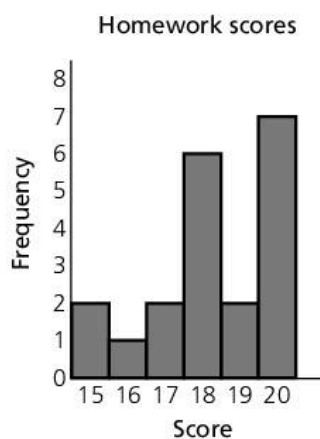
**Exam tip**
Frequency always goes on the vertical axis.

⏪ **Recap**

In a pie chart, you use sectors to represent the data.

An angle of $x$ in the pie represents $\dfrac{x}{360}$ of the total.



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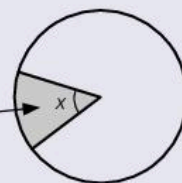
**Worked example**

The pie chart shows the answers 120 students gave to a question on a multiple choice paper. How many students put $A$ as the answer? **[2 marks]**
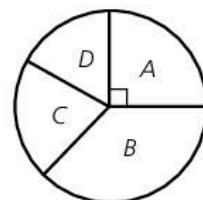


The right angle in the sector representing $A$ shows that $\dfrac{90}{360}$ or one-quarter of the students chose $A$ as their answer.

$\dfrac{1}{4}$ of $120 = \dfrac{1}{4} \times 120 = 30$

30 students gave the answer $A$.

---

**Worked example**

James asked 36 students in his class where they had been on holiday. He recorded the results in this table:

| Country | UK | France | Spain | Other |
|---|---|---|---|---|
| Number of students | 9 | 6 | 11 | 10 |

Represent this information on a pie chart. **[3 marks]**

At the centre of the pie there is an angle of 360°. There are 36 students so 10° represents one student.

| Country | UK | France | Spain | Other |
|---|---|---|---|---|
| Number of students | 9 | 6 | 11 | 10 |
| Angle of sector | 90° | 60° | 110° | 100° |
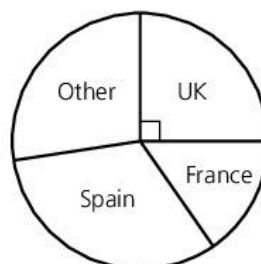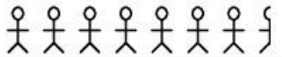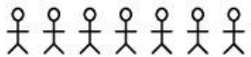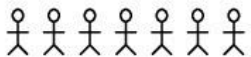
**Worked example**

The number of students in each year at a school are as follows.

| Year | Frequency |
|------|-----------|
| 7 | 80 |
| 8 | 75 |
| 9 | 70 |
| 10 | 70 |
| 11 | 100 |

Represent this information in a pictogram. **[4 marks]**

The symbol ☥ represents 10 students.

| Year | Frequency |
|------|-----------|
| 7 | ☥ ☥ ☥ ☥ ☥ ☥ ☥ ☥ |
| 8 | ☥ ☥ ☥ ☥ ☥ ☥ ☥ ⸲ |
| 9 | ☥ ☥ ☥ ☥ ☥ ☥ ☥ |
| 10 | ☥ ☥ ☥ ☥ ☥ ☥ ☥ |
| 11 | ☥ ☥ ☥ ☥ ☥ ☥ ☥ ☥ ☥ ☥ |

**Key skills**

You must be able to construct and interpret stem-and-leaf diagrams.

**Recap**

A stem-and-leaf diagram is a way of displaying data in groups.

**Worked example**

A class of 25 students all took a history test that was marked out of 50. Here are their marks.

| 22 | 18 | 8 | 42 | 25 | 35 | 47 | 33 | 28 | 9 |
| 25 | 13 | 46 | 50 | 31 | 44 | 26 | 10 | 49 | 17 |
| 39 | 21 | 30 | 27 | 34 | | | | | |

Display this data on a stem-and-leaf diagram. **[4 marks]**

Use the stem part of the diagram to represent the tens digit, and the leaf part to represent the units digit.

| Stem | Leaf | | | | | | Key 1|3 means 13 |
|------|------|---|---|---|---|---|---|
| 0 | 8 | 9 | | | | | |
| 1 | 0 | 3 | 7 | 8 | | | |
| 2 | 1 | 2 | 5 | 5 | 6 | 7 | 8 |
| 3 | 0 | 1 | 3 | 4 | 5 | 9 | |
| 4 | 2 | 4 | 6 | 7 | 9 | | |
| 5 | 0 | | | | | | |

**Exam tip**

This kind of stem-and-leaf diagram is called an 'ordered stem-and-leaf diagram' because the numbers in the leaf section are in ascending order. It is sometimes a good idea to make an unordered diagram first, particularly if the data set is large.

**Watch out!**

You must always remember to include a key with your diagram.

**Exam tip**

Notice how the key is reversed on the left hand side of the diagram.

**Worked example**

Two teams, of nine runners each, competed in a 1500 metre running race. Their times were as follows:

Team 1: 7 m 11 s, 7 m 15 s, 7 m 30 s, 7 m 43 s, 8 m 22 s, 8 m 37 s, 8 m 38 s, 9 m 1 s, 9 m 10 s

Team 2: 6 m 20 s, 6 m 48 s, 6 m 51 s, 7 m 9 s, 7 m 20 s, 8 m 12 s, 8 m 22 s, 8 m 35 s, 8 m 49 s

Display this data in a back-to-back stem-and-leaf diagram.

**[4 marks]**

Construct the diagram so that Team 1 goes to the left and Team 2 goes to the right.

|  | Team 1 |  |  |  |  | Team 2 |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|
| Key (Team 1) |  |  |  |  | 6 | 20 | 48 | 51 |  | Key (Team 2) |
| 43\|7 means | 43 | 30 | 15 | 11 | 7 | 09 | 20 |  |  | 7\|20 means |
| 7 min 43 sec |  | 38 | 37 | 22 | 8 | 12 | 22 | 35 | 49 | 7 min 20 sec |
|  |  |  | 10 | 01 | 9 |  |  |  |  |  |

**Recap**

A histogram is a kind of bar chart, with the following general features:

- They are often used for continuous data
- The bars often have different widths
- The vertical axis is often labelled 'frequency density'
- There are no gaps between the bars (unless there is a bar with zero height)
- The horizontal axis is labelled, not the bars
- The frequency is determined by the area of the bar

**Key skills**

You must be able to construct and interpret histograms with equal class widths.

**Exam tip**

A histogram with equal class intervals is not much different to a regular bar chart.

**Worked example**

Draw a histogram using the following table of data which shows the distribution of the ages of 100 people attending a music festival.

| Age | Frequency |
|---|---|
| $0 \leq x < 20$ | 22 |
| $20 \leq x < 40$ | 45 |
| $40 \leq x < 60$ | 19 |
| $60 \leq x < 80$ | 14 |

**[4 marks]**
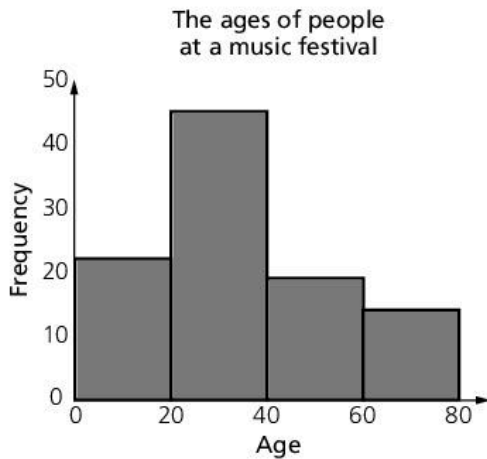
The ages of people at a music festival

## Extended

### Worked example

The lengths of time 120 cars were parked in a short stay car park are recorded in the following table.

| Time (min) | Frequency |
|------------|-----------|
| $0 \le x < 5$ | 10 |
| $5 \le x < 15$ | 20 |
| $15 \le x < 30$ | 45 |
| $30 \le x < 60$ | 30 |
| $60 \le x < 90$ | 15 |

Display this data in a histogram. **[6 marks]**

First, work out the frequency density for each class interval.

| Time (min) | Frequency | Frequency density |
|------------|-----------|-------------------|
| $0 \le x < 5$ | 10 | 2 |
| $5 \le x < 15$ | 20 | 2 |
| $15 \le x < 30$ | 45 | 3 |
| $30 \le x < 60$ | 30 | 1 |
| $60 \le x < 90$ | 15 | 0.5 |

**Exam tip**

The frequencies are represented by the area of each bar.

Minutes spent in a car park
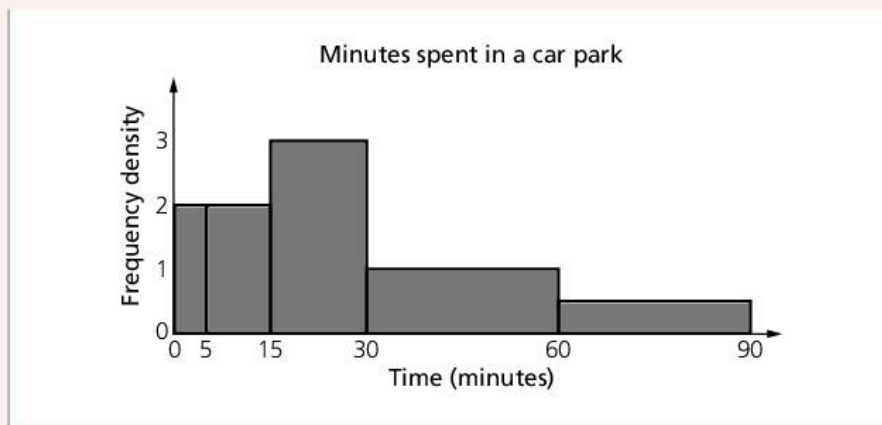


**Key skills**

You must be able to construct and interpret scatter diagrams.

**Recap**

You can use a scatter diagram to find out if there is any correlation (connection) between two sets of variables, such as shoe size and height, or sales of gloves and the outdoor temperature.

**Worked example**

During the first week of the summer holidays, Jessie wrote down the temperature in her garden at noon, and also the number of hours she spent watching TV that day, to the nearest hour.

| Temperature | 28 | 27 | 20 | 22 | 18 | 14 | 25 |
|---|---|---|---|---|---|---|---|
| Hours spent watching TV | 1 | 1.5 | 3 | 4 | 4.5 | 6 | 2 |

Draw a scatter diagram to illustrate this data. **[4 marks]**

Because you suspect that the number of hours spent watching TV may depend on the temperature, you put temperature on the horizontal axis.
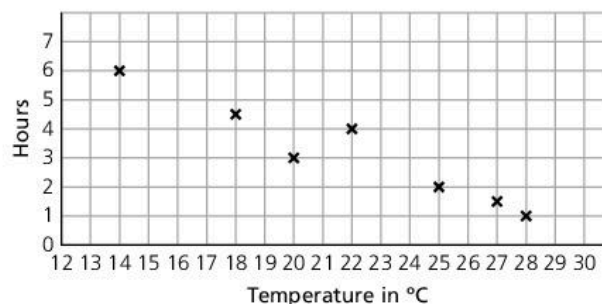


You will look at the interpretation of scatter diagrams in section 9.7/9.8.

**? Questions**

**1** Draw a bar chart for this data set:

| Colour of car | Frequency |
|---|---|
| Red | 1 |
| Blue | 12 |
| Silver | 8 |
| Black | 2 |

**2** The pie chart shows the nationalities of a group of 150 delegates at a conference. Given that 30 delegates came from Ethiopia, find:
  **a** the values of $p$ and $q$
  **b** how many delegates came from each of Kenya, Uganda and Sudan.

Sudan
$q°$
Kenya 144°
Ethiopia $p°$
Uganda 120°
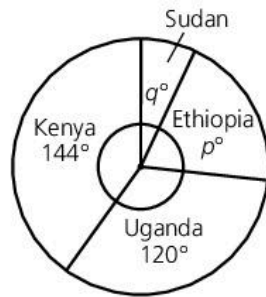
**3** 11 students are asked to close their eyes for what they think is 60 seconds. Their actual times in seconds are:

70, 64, 49, 52, 61, 53, 72, 57, 53, 67, 48

Make a stem-and-leaf diagram to display this data.

**4** Summarised below are the prices of the goods (to the nearest £) sold by an electrical shop on a certain day.

| Price of goods (£) | Frequency | Frequency density |
|---|---|---|
| $20 < x \le 40$ | 20 | 1 |
| $40 < x \le 50$ | 37 | |
| $50 < x \le 60$ | | 6.2 |
| $60 < x \le 70$ | 51 | |
| $70 < x \le 90$ | 30 | |
| $90 < x \le 130$ | | 0.2 |

  **a** Copy and complete the table.
  **b** Draw a histogram to illustrate these data using 1 cm per £10 on the horizontal axis (from 20 to 130) and 2 cm per unit on the vertical axis (from 0 to 7).

**5** A boy drew a histogram for the data shown in the table below. He added a column to the table to show the width of each bar.

| Mass (kg) | Frequency | Frequency density (height of bar) | Width of bar |
|---|---|---|---|
| 10- | 8 | | |
| 30- | 16 | | |
| 50- | 38 | | |
| 60- | 40 | 4 cm | 1 cm |
| 70- | 28 | | |
| 75- | 30 | | |
| 80- | 21 | | |
| 85- | 11 | | |
| 90-120 | 6 | | |

On the boy's histogram the '60-' bar was 1 cm wide and 4 cm high as shown in the table.
  **a** Copy and complete the table.
  **b** Hence draw the boy's histogram using the scale that he used.

To **Raise your grade** now try question 10, page 216

**You need to:**

- Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used.

---

### Key skills

You must be able to calculate the mean, median, mode and range for individual and discrete data.

### Exam tip

Mean, median and mode are all averages, and are a way of representing a set of numbers using only one number. The range tells you how spread out the data is.

---

### Recap

**Mean, median, mode and range**

The **mean** is $\dfrac{\text{sum of all the values}}{\text{number of values}}$, which can be written as $\bar{x} = \dfrac{\sum x}{n}$

The **median** is the 'middle' value when the data have been written in ascending or descending order. If there are $n$ values then the median is the $\dfrac{n+1}{2}$ th value.

When $n$ is odd...

For example with 3 values, the median is the $\dfrac{3+1}{2}$ = second value.

When $n$ is even...

For example with 4 values, the median is the $\dfrac{4+1}{2} = 2.5$ th value, that is halfway between the second and third values.

The **mode** is the value which appears most frequently.

The **range** is the difference between the largest and smallest values.

---

### Worked example

Here are some data: 14, 16, 10, 15, 13, 13, 19, 15, 13, 12, 14
Find:

(a)   the mean                                      **[2 marks]**

(b)   the median                                    **[1 mark]**

(c)   the mode                                      **[1 mark]**

(d)   the range.                                    **[1 mark]**

---

(a)   Mean $= \dfrac{14+16+10+15+13+13+19+15+13+12+14}{11}$

$= \dfrac{154}{11} = 14$

(b)   First write the data in ascending order...

10, 12, 13, 13, 13, ⑭ 14, 15, 15, 16, 19

Median $= 14$

(c)   Mode $= 13$

(d)   Range $= 19 - 10 = 9$

---

### Exam tip

There are 11 values and so the median is the $\dfrac{11+1}{2}$ = sixth value

**1** Here is a set of data:

7, 5, 2, 10, 10, 8, 11

Find:

**a** the mean

**b** the median

**c** the mode

**d** the range.

**2** Jane took five maths tests. Her scores were as follows:

12,  25,  14,  22,  17

Find:

**a** her mean score

**b** her median score

**c** the range of her scores.

Jane then took another test, and her mean score is now 19.

**d** What did she score in the sixth test?

**3** A dice was rolled many times and its score ($x$) is shown in the table.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|
| $f$ | 3 | 8 | 13 | 10 | 6 | 5 |

**a** Find the mean of the frequency distribution (to 3 s.f.).

**b** Find the median of the frequency distribution.

**c** Write down the mode and the range of the frequency distribution.

**4** The average height of 5 students in a class is 152 cm. If another student whose height is 146 cm joins the class, what is the new average height?

**5** The mean of 16 numbers is 17.5.
The mean of 12 of the numbers is 17.
What is the mean of the other 4 numbers?

**You need to:**

- Calculate an estimate of the mean for grouped and continuous data. (Extended)
- Identify the modal class from a grouped frequency distribution. (Extended)

**Extended**

🔑 **Key skills**

You must be able to calculate the mean of a set of grouped discrete data.

⏪ **Recap**

You will sometimes need to calculate the mean from data in a table.

👁 **Watch out!**

Note that the mean of a set of whole numbers can be a decimal number. In fact, when using real data, it would be a decimal most of the time.

👁 **Watch out!**

Make sure you use the highest and lowest values of the data to calculate the range, rather than the highest and lowest frequencies.

🔑 **Key skills**

You must be able to identify the modal class from a grouped frequency distribution.

**Worked example**

Here is a table showing the number of siblings that 30 students have.

| Number of siblings | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 5 | 8 | 10 | 4 | 2 | 1 |

Calculate:

(a)  the mean number of siblings                    **[2 marks]**

(b)  the range of the number of siblings.          **[1 mark]**

---

(a)  First calculate the sum of the number of siblings multiplied by the frequency.

$(0 \times 5) + (1 \times 8) + (2 \times 10) + (3 \times 4) + (4 \times 2) + (5 \times 1) = 53$

There are 30 students, so the mean $= 53 \div 30 = 1.77$

(b)  $5 - 0 = 5$

**Worked example**

This table shows the number of pages in a randomly selected set of 25 books.

| Number of pages | $0 \leq x < 100$ | $100 \leq x < 250$ | $250 \leq x < 500$ | $500 \leq x < 1000$ |
|---|---|---|---|---|
| Frequency | 5 | 10 | 8 | 2 |

(a)  Calculate an estimate of the mean number of pages in the books.                    **[2 marks]**

(b)  Why is your answer to part (a) only an estimate?    **[1 mark]**

(c)  Which is the modal class?                    **[1 mark]**

(a) Because the data has been given in classes, you need to use the mid-point of each class as an estimate.

Calculate the sum of the number of pages multiplied by the frequency.

$(50 \times 5) + (175 \times 10) + (375 \times 8) + (750 \times 2) = 6500$

There are 25 books,
so the mean number of pages = $6500 \div 25 = 260$

(b) Because you do not know how many pages any of the books have.

(c) The modal class is $100 \le x < 250$.

**Watch out!**

The modal class is not 10. That is the frequency.

**? Questions**

1  The scores of a class in a test are given in the table.

| Score ($x$) | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|
| Frequency ($f$) | 2 | 5 | 7 | 6 | 3 | 1 |

Find: **a** the mean
  **b** the median
  **c** the mode
  **d** the range.

2  The table shows the masses, in grams, of 100 oranges.

| Mass, $x$ (g) | 120- | 130- | 140- | 150- | 160- | 170–180 |
|---|---|---|---|---|---|---|
| Frequency | 25 | 19 | 23 | 16 | 12 | 5 |

Find: **a** an estimate of the mean
  **b** the interval which contains the median
  **c** the modal class.

3  The weights of elephants are recorded in the table below.

| Weight, $w$ (kg) | Frequency |
|---|---|
| $2000 \le w < 2500$ | 7 |
| $2500 \le w < 3000$ | 10 |
| $3000 \le w < 3500$ | 15 |
| $3500 \le w < 4000$ | 21 |
| $4000 \le w < 4500$ | 13 |
| $4500 \le w < 5000$ | 7 |
| $5000 \le w < 5500$ | 4 |
| $5500 \le w < 6000$ | 3 |

**a** Estimate the mean weight.
**b** Write down the median class.
**c** Calculate an estimate for the range.

**You need to:**

- Construct and use cumulative frequency diagrams. (Extended)
- Estimate and interpret the median, percentiles, quartiles and interquartile range. (Extended)
- Construct and interpret box-and-whisker plots. (Extended)

**Extended**

### Worked example

The table shows the masses, in grams, of 100 apples.

| Mass, $x$ (g) | 120– | 130– | 140– | 150– | 160– | 170–180 |
|---|---|---|---|---|---|---|
| Frequency | 25 | 19 | 23 | 16 | 12 | 5 |

(a) Construct a frequency table for this data, with a column for cumulative frequency. **[2 marks]**

(b) Draw a cumulative frequency diagram for this data. **[4 marks]**

(c) Estimate:
(i) the median **[1 mark]**
(ii) the 80th percentile **[1 mark]**
(iii) the interquartile range. **[2 marks]**

(a)

| Mass $x$ (g) | Frequency | Cumulative frequency |
|---|---|---|
| $120 \leq x < 130$ | 25 | 25 |
| $130 \leq x < 140$ | 19 | 44 |
| $140 \leq x < 150$ | 23 | 67 |
| $150 \leq x < 160$ | 16 | 83 |
| $160 \leq x < 170$ | 12 | 95 |
| $170 \leq x < 180$ | 5 | 100 |

←25 + 19
←44 + 23

(b) and (c)



The curve will generally be smooth and must always have a positive gradient.
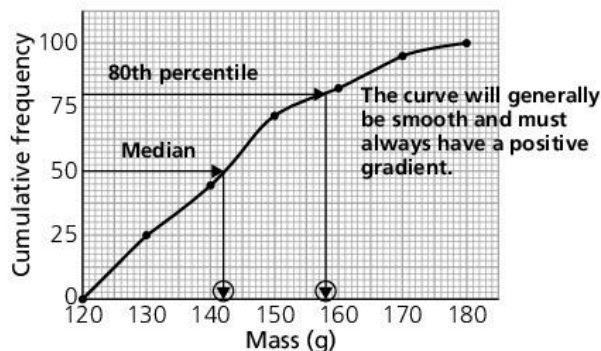
(i) From the diagram, the median ≈ 142 (to 3 s.f.)

(ii) From the diagram, the 80th percentile ≈ 158 (to 3 s.f.)
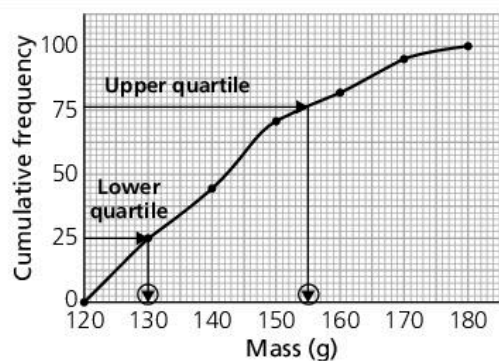
(iii) From the diagram, the upper quartile ≈ 155 (to 3 s.f.)
The lower quartile ≈ 130 (to 3 s.f.)
The interquartile range ≈ 155 − 130 = 25

## Recap

Box-and-whisker plots show the median and quartiles of a data set, along with the highest and lowest values.
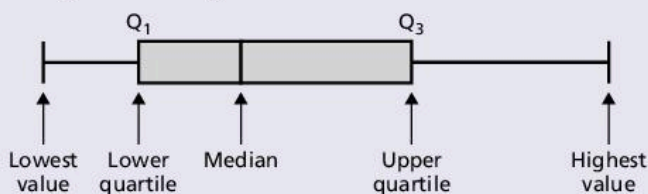


## Key skills

You must be able to construct and interpret box-and-whisker plots.

**Exam tip**

The lower and upper quartiles are sometimes called $Q_1$ and $Q_3$.

### Worked example

A data set, based on the individual mass of 100 apples, has the following features.

Lowest value = 120          Highest value = 180
Lower quartile = 130          Upper quartile = 155
Median = 142

Illustrate this data using a box-and-whisker plot.          **[3 marks]**
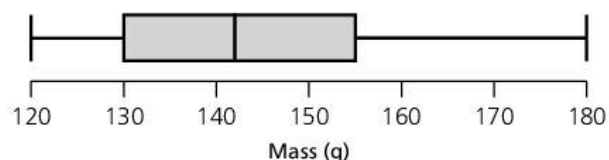


**Exam tip**

Box-and-whisker plots can be drawn directly from cumulative frequency diagrams.

**Exam tip**

Remember: a box-and-whisker plot can also be displayed vertically.

**Extended**

## Exam tip

One comment should compare the averages and another comment should compare the spread.

## Exam tip

The interquartile range is the width of the box part of the diagram.

### Worked example

Here is a box-and-whisker plot showing the test scores from two classes.
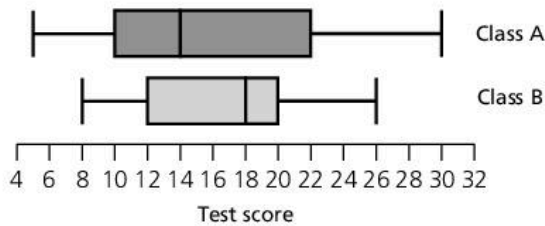


Compare the results from the two classes. **[2 marks]**

On average, class B scored higher, as this class had a higher median.

Class A had a wider range of scores, as this class had a higher range and interquartile range.

## Questions

1 The masses (in kg) of 100 animals are recorded in the table.

| Mass (kg) | 20– | 25– | 30– | 35– | 40– | 45–50 |
|---|---|---|---|---|---|---|
| Frequency | 8 | 21 | 28 | 19 | 15 | 9 |

Write down the coordinates of the points through which the cumulative frequency curve should pass.

2 The number of minutes Lucy and Zac spend reading each day are recorded for several weeks, and their results are displayed using a box-and-whisker plot.
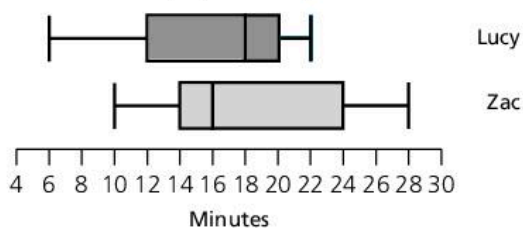


Compare the amounts of time they spend reading.

3 A biologist recorded the lengths of a group of 200 insects and displayed his results in the table.

| Length (mm) | 10– | 11– | 12– | 13– | 14– | 15– | 16–17 |
|---|---|---|---|---|---|---|---|
| Frequency | 17 | 38 | 53 | 49 | 21 | 12 | 10 |

a Draw a cumulative frequency graph for these data using a scale of 2 cm per 1 mm on the horizontal axis and 1 cm per 10 units on the vertical axis. (The horizontal axis should go from 10 mm to 17 mm).

b Use the graph to find an estimate of the median.

c Use the graph to find estimates of the lower and upper quartiles.

**You need to:**
- Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram.
- Draw, interpret and use lines of best fit by eye.

🔑 **Key skills**

You must be able to determine the nature of a correlation by examining a scatter diagram.

**Exam tip**

Looking for a correlation is the main reason to draw scatter diagrams.

⏪ **Recap**

On a scatter diagram, 'correlation' means the way in which the two variables are related.

These points all lie close to a straight line with positive gradient.

There is said to be 'strong (linear) positive correlation'.

These points all lie close to a straight line with negative gradient.

There is said to be 'strong (linear) negative correlation'.

These are called the 'lines of best fit'.



These points are spread out all over the graph.

There is said to be 'no (linear) correlation'.

**Exam tip**

Most people's lines of best fit will be slightly different. You should, however:

- make sure your line is a straight line (always use a ruler!)
- make sure there are no points you cannot get to from your line by moving vertically
- make sure your line is roughly in the direction of the points
- make sure there are roughly the same amount of points on both sides of the line.

**Worked example**

The scatter diagram shows the number of hours a child spent online and the temperature at noon, over a seven day period.
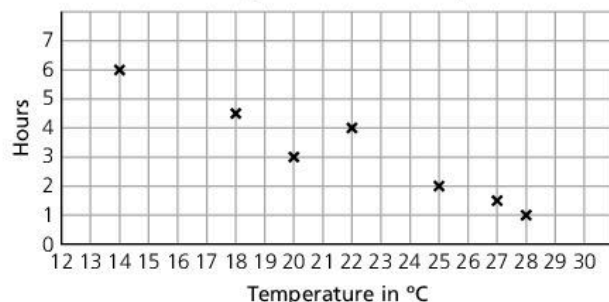


(a) Draw a line of best fit by eye. **[1 mark]**

(b) Describe the correlation. **[1 mark]**

(c) Give an interpretation of the correlation. **[1 mark]**

(d) If the temperature on one of the days had been 21°C, roughly how many hours would you expect the child to have spent online? **[1 mark]**

(a)
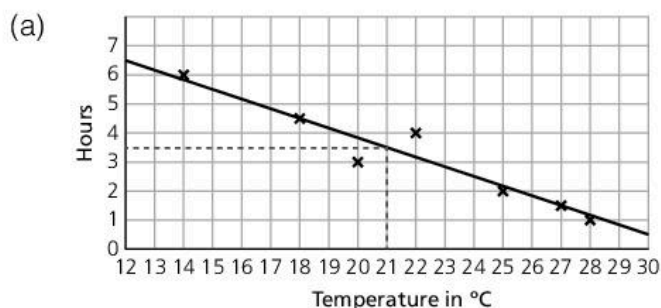


(b) This is a negative correlation.

(c) The higher the temperature, the less time spent online.

(d) 3.5 hours

**Exam tip**

Drawing the line 'by eye' just means 'without calculating the line's equation'. It means deciding on a sensible place for the line just by looking at the graph.

**Exam tip**

You can see the correlation easily by looking at the diagram. It does not need to be worked out.

**Watch out!**

Remember that a correlation between two variables does not necessarily mean that one causes the other.

In the above Worked example, it might not be warmer temperatures that caused the child to spend less time online.

**Watch out!**

Your line of best fit does not necessarily have to go through **any** of the points.

**Watch out!**

Your line of best fit does not have to go through the origin!

**Exam tip**

Remember that using the line of best fit to make predictions within the range of the given data is called 'interpolation' and is considered to be reliable, but making predictions outside the range of the given data is called 'extrapolation' and is considered to be unreliable.

### ? Questions

**1** Describe the correlation shown in the scatter diagram.
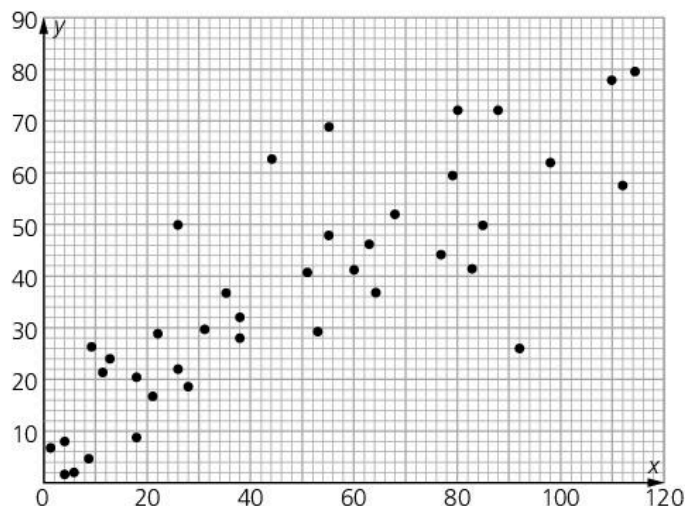


**2** The table shows the speed of a motorcycle in miles per hour and its petrol consumption in miles per gallon.

| Speed (mph) | 20 | 25 | 30 | 35 | 45 | 50 | 56 | 70 |
|---|---|---|---|---|---|---|---|---|
| Petrol consumption (mpg) | 80 | 76 | 70 | 64 | 54 | 50 | 47 | 40 |

**a** Draw a scatter diagram to show this information.
**b** Draw the line of best fit.
**c** Estimate the fuel consumption when the motorcycle is travelling at 40 mph.
**d** State, with a reason, whether or not it is sensible to predict the fuel consumption when the motorcycle is travelling at 120 mph around a racetrack.

**3** The table shows the marks scored by 10 students in practical and theoretical biology tests.

| Practical | 45 | 36 | 14 | 22 | 25 | 31 | 44 | 38 | 27 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|
| Theoretical | 39 | 33 | 18 | 21 | 27 | 29 | 48 | 43 | 27 | 32 |

**a** Draw a scatter diagram to show this information.
**b** Draw the line of best fit.
**c** Another student took the practical test but missed the theoretical test. If she scored 28 on the practical test, use your line of best fit to estimate her mark on the theoretical test.
**d** State, with a reason, whether or not it is sensible to predict the practical test score of a student who scored 65 marks on the theoretical test.

To **Raise your grade** now try question 5, page 215

**1**  Here is a set of data:      6, 8, 11, 14, 15, 15, 17
Find:

**a**   the mean                                                                                  **[1 mark]**
**b**   the median                                                                              **[1 mark]**
**c**   the mode                                                                                **[1 mark]**
**d**   the range.                                                                              **[1 mark]**

**2**  Twenty students were asked to estimate a time interval of 30 seconds.
The following stem-and-leaf diagram shows their actual times in seconds.

Time in seconds

```
1 | 9
2 | 1 3 3 4 6 7 7 9
3 | 0 1 1 2 2 2 4 5 6 6
4 | 0
```

Key 1 | 9 = 19 seconds

Write down:

**a**   the mode                                                                                **[1 mark]**
**b**   the median                                                                              **[1 mark]**
**c**   the range.                                                                              **[1 mark]**

**3**  Here is a set of data:      1.6, 1.2, 1.6, 1.3, 1.4, 1.5, 1.6, 1.7
Find:

**a**   the mean                                                                                  **[1 mark]**

**b**   the median                                                                              **[1 mark]**

**c**   the mode                                                                                **[1 mark]**

**d**   the range.                                                                              **[1 mark]**

**4**  The marks scored by 49 students in a test are recorded in the table below.    **E**

| Score, $x$ | Number of students, $f$ |
|---|---|
| 12 | 1 |
| 13 | 7 |
| 14 | 8 |
| 15 | 5 |
| 16 | 6 |
| 17 | 8 |
| 18 | 12 |
| 19 | 2 |

Find:

**a**   the mean score                                                                      **[2 marks]**

**b**   the median score                                                                    **[2 marks]**

**c**   the modal score.                                                                    **[2 marks]**

**5** The scatter diagram below shows the performance of a group of students in their autumn term and summer term examinations.
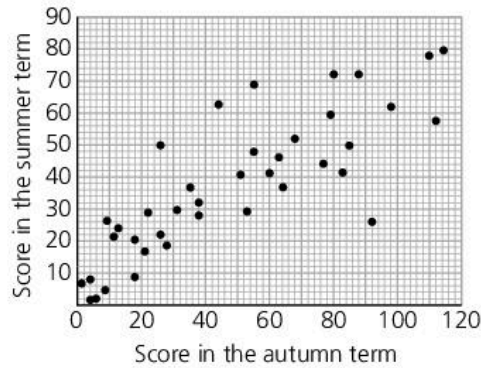


**a** Draw in a line of best fit by eye. **[1 mark]**

**b** What type of correlation does the diagram show? **[1 mark]**

**c** One student was absent for the summer term examination. If she scored 45 in the autumn term, what score would you predict she would get in the summer term?

**[2 marks]**

**6** The cumulative frequency curve shows the mass of African civets in a tropical rainforest.

**a** Estimate the interquartile range of the masses. **[3 marks]**

**b** Estimate the number of civets with a mass greater than 21 kg. **[2 marks]**
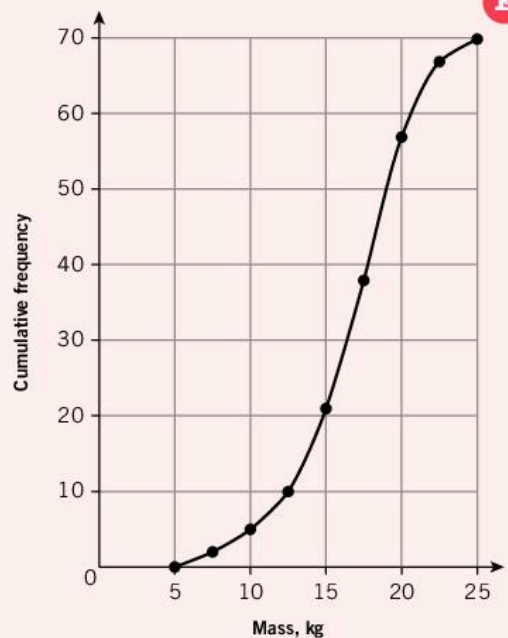


**7** The mass of 120 African bushpigs is shown in the table:

| Mass, kg | Frequency |
|---|---|
| $60 \leq m < 90$ | 5 |
| $90 \leq m < 110$ | 24 |
| $110 \leq m < 120$ | 51 |
| $120 \leq m < 130$ | 31 |
| $130 \leq m < 150$ | 9 |

Find an estimate for the mean mass of the bushpigs. **[8 marks]**

**8** Here is a box-and-whisker plot showing the amount of time, in minutes, that a group of people spent in two different shops.

Shop A

Shop B

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Time spent in each shop in minutes
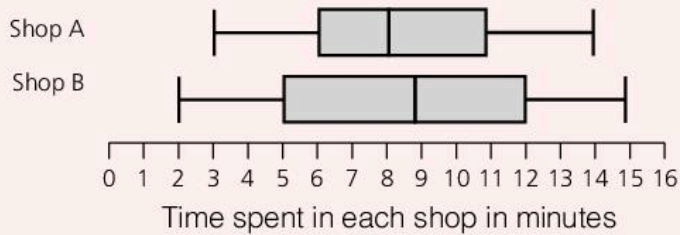
   **a** Find the interquartile range for Shop B.                          **[2 marks]**

   **b** Find the range for Shop A.                                        **[2 marks]**

   **c** Compare the two box-and-whisker plots.                            **[2 marks]**

**9** Giraffe heights at a Cotswold zoo are recorded in the table below.

| Height, $h$ (metres) | Frequency |
|---|---|
| $3.5 \leq h < 4.0$ | 12 |
| $4.0 \leq h < 4.5$ | 18 |
| $4.5 \leq h < 5.0$ | 15 |
| $5.0 \leq h < 5.5$ | 8 |

Find:

   **a** an estimate for the mean height                                   **[3 marks]**

   **b** the modal class.                                                  **[1 mark]**

**10** At a girls' school, a random sample of students was taken and each student recorded her intake of milk (in ml) during a given day.

Some of the results and part of the histogram are shown.

| Milk intake (ml) | 10– | 30– | 60– | 100– | 150– | 200– | 300–500 |
|---|---|---|---|---|---|---|---|
| No. of students | 4 | 9 | 32 | | 25 | | |
| Frequency density | | | | | | | |



Milk intake (ml)
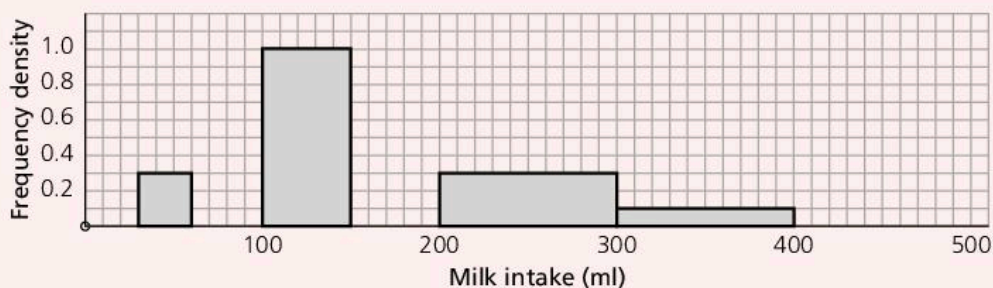
   **a** Calculate the frequency density for the '30– ' category.         **[2 marks]**

   **b** Copy the table, calculating the frequency density in all the cases where the number of students is stated.                                        **[2 marks]**

   **c** Copy and complete the histogram to illustrate these data using 1 cm per 50 ml on the horizontal axis and 1 cm per 0.1 on the vertical axis. Use the table to complete the histogram.                                            **[3 marks]**

   **d** Use the histogram to complete the table.                          **[3 marks]**

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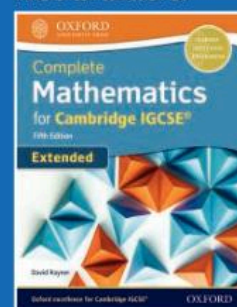
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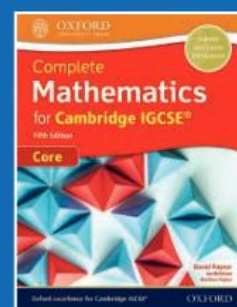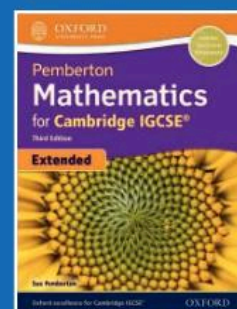- ✓ Progression to the next educational stage

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