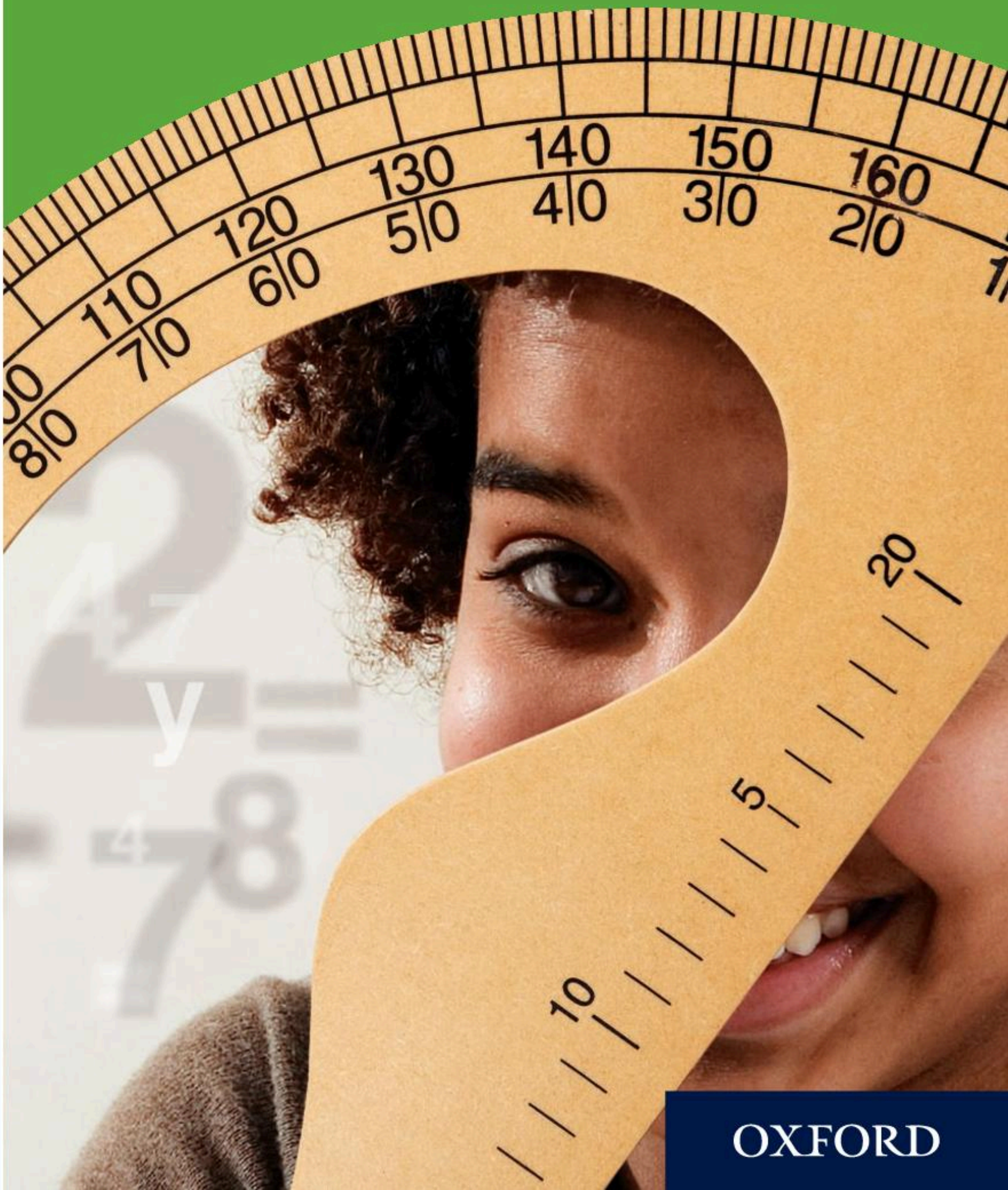
Oxford excellence for the Caribbean

Book 1

STP Mathematics for Jamaica

GRADE 7

SECOND EDITION



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Contents

Introduction

To the student

This new edition of *STP Mathematics for Jamaica Student Book 1* attempts to meet your needs as you begin your study of Mathematics at the secondary school level. Your learning experiences at this stage lay the foundation for future achievement in CSEC Mathematics and beyond. We are very conscious of your need for success and enjoyment in doing Mathematics, which comes from solving problems correctly. With this in mind, we have divided most of the exercises into three types of question:

Type 1 questions

These are identified by numbers written in bold print, e.g. **12**. They help you to see if you understand the topic being discussed and should be attempted in every chapter you study.

Type 2 questions

These are identified by a single underline under the bold print, e.g. **<u>12</u>**. They are extra questions for you to do and are not more difficult. They should be attempted if you need extra practice or want to do revision at a later time.

Type 3 questions

These are identified by a double underline under the bold print, e.g. **<u>12</u>**. They are for those of you who completed Type 1 questions fairly easily and want to attempt questions that are more challenging.

Multiple choice questions

Multiple choice questions are included in the book to help you become more familiar with the format of your assessments at CSEC.

Mixed exercises

Most chapters end with Mixed exercises to help you advance your critical thinking, problem-solving and computational skills. These exercises will also help you revise what you have done, either when you have finished the chapter or as you prepare for examinations.

Use of calculator

You should be able to use a calculator accurately before you leave school. We suggest that you use a calculator mainly to check your answers. Whether you use a calculator or do the computations yourself, always estimate your answer first and always ask the question, 'Does my answer make sense?'

Suggestions for use of student book

- Break up the material in a chapter into manageable parts.
- Have paper and a pencil with you always when you are studying mathematics.
- Write down and look up the meaning of all new vocabulary you encounter.
- Read all questions carefully and rephrase them in your own words.
- Remember that each question contains all the information you need to solve the problem. Do not look only at the numbers that are given.
- Practise your mathematics. This will ensure your success!

You are therefore advised to try to solve as many problems as you can.
Above all, don't be afraid to make mistakes as you are learning. The greatest mathematicians all made many mistakes as they tried to solve problems.

You are now on your way to success in mathematics – GOOD LUCK!

To the teacher

In writing this series, the authors attempted to present the topics in such a way that students will understand the connections among topics in mathematics, and be encouraged to see and use mathematics as a means to make sense of the real world. The exercises have been carefully graded to make the content more accessible to students.

This new edition is designed to:

1 Assist you in helping students to

- attain important mathematical skills
- connect mathematics to their everyday lives and understand its role in the development of our contemporary society
- see the importance of critical thinking skills in everyday problems
- discover the fun of doing mathematics both individually and collaboratively
- develop a positive attitude towards doing mathematics.

2 Encourage you to include historical information about mathematics in your teaching.

Topics from the history of mathematics have been incorporated to ensure that mathematics is not dissociated from its past. This should lead to an increase in the level of enthusiasm, interest and fascination among students, thus enriching the teaching and learning experiences in the mathematics lessons.

Investigations

'Investigation' is included in this revised STP Mathematics for Jamaica series. This is in keeping with the requirements of the latest CSEC syllabus.

Investigations are used to provide students with the opportunity to explore hands-on and minds-on mathematics. At the same time, teachers are presented with open-ended explorations to enhance their mathematical instruction.

It is expected that the tasks will

- encourage problem solving and reasoning
- develop communication skills and the ability to work collaboratively
- connect various mathematical concepts and theories.

Suggestions

1 At the start of each lesson, give a brief outline of the topic to be covered in the lesson. As examples are given, refer back to the outline to show how the example fits into it.

2 List terms that you consider new to the students and solicit additional words from them. Encourage students to read from the text and make their own vocabulary list. Remember that mathematics is a foreign language. The ability to communicate mathematically must involve the careful use of the correct terminology.

3 Have students construct different ways to phrase questions. This helps students to see mathematics as a language. Students, especially in the junior classes, tend to concentrate on the numerical or 'maths' part of the question and pay little attention to the information that is required to solve the problem.

4 When solving problems, have students identify their own problem-solving strategies and listen to the strategies of others. This practice should create an atmosphere of discussion in the class centred on different approaches to solving the same problem.

As the students try to solve problems on their own they will make mistakes. This is expected, as this was the experience of the inventors of mathematics: they tried, guessed, made many mistakes and worked for hours, days and sometimes years before reaching a solution.

There are enough problems in the exercises to allow the students to try and try again. The excitement, disappointment and struggle with a problem until a solution is found will create rewarding mathematical experiences.

1 Working with numbers

At the end of this chapter you should be able to...

1 approximate a given whole number to the nearest ten, hundred, thousand, …
2 estimate the product of two whole numbers to the nearest ten, hundred, thousand, …
3 use a calculator to work with whole numbers after finding approximate answers
4 solve problems involving whole numbers
5 perform operations involving a combination of +, −, × and ÷
6 solve problems using brackets
7 identify square, rectangular and triangular numbers
8 list the factors of a given number
9 write down multiples of a given number
10 use indices to write products of repeated numbers
11 classify numbers as prime or composite
12 express a given number as a product of primes
13 find the HCF or LCM of a group of numbers
14 solve problems requiring the use of the HCF or LCM of a set of numbers.

Activity

The Romans did not use symbols for numbers, but used letters of the alphabet. For example the Romans used X for ten, V for five; XV means 'ten and five', i.e. 15.

The Roman way of writing numbers is still used today. (When you write your CXC examinations, your grades are written using Roman numerals. If you study hard and do very well you will get Grade I.)

The numbers one to six are written I, II, III, IV, V, VI.

1 Why do you think I is written before V for the number 4, and I is written after V for the number 6?

2 Write the numbers that you think IX and XI mean.

3 The numbers 7 and 8 are written VII and VIII.

The letter L is used for 50, the letter C is used for 100 and the letter M is used for 1000.

Write the value of XIV, CLX and MLII.

4 Write the following numbers in Roman numerals: 25, 152, 1854, 2006.

5 Find LXII – XXIV and write your answer in Roman numerals.

6 In Roman numerals, the year 2019 is written MMXIX.

Which year, written in Roman numerals, uses the most letters?

Hint: it is in the 19th century.

You need to know…

- ✔ place values
- ✔ multiplication tables up to 12×12
- ✔ how to add, subtract, multiply and divide whole numbers
- ✔ how to carry out long multiplication and long division.

Key words

addition, approximation, associative, commutative, composite number, distributive, divisible, divisibility rules, division, even number, factor, highest common factor (HCF), identity element, index (plural indices), inverse element, lowest common multiple (LCM), multiple, multiplication, natural number, non-commutative, odd number, place value, prime number, product, rectangular number, square number, subtraction, triangular number, twin primes, whole number

Laws of numbers

From previous work you know that

$$8 + 17 = 17 + 8 \quad \text{and that} \quad 8 \times 17 = 17 \times 8$$

i.e. for *addition* and *multiplication* the order of the numbers does not matter. We say that numbers are *commutative* under addition and multiplication.

On the other hand 8 – 17 and 17 – 8 do not give the same answer; neither do $8 \div 17$ and $17 \div 8$.

In this case numbers are *non-commutative* under *subtraction* and *division*.

You have also seen that

$$5 + (6 + 7) = (5 + 6) + 7 = 5 + 6 + 7$$

and that $$4 \times (6 \times 8) = (4 \times 6) \times 8 = 4 \times 6 \times 8$$

i.e. the brackets can be removed without changing the answer, so there is no ambiguity in writing $5 + 6 + 7$ or $4 \times 6 \times 8$.
This illustrates the *associative* law for the addition and multiplication of numbers.

On the other hand, for $7 - (5 + 6)$ and $(6 + 9) \div 3$ the answers will be different if the brackets are removed. Subtraction and division of numbers **do not** satisfy the associative law.

In other calculations you know that $5\,(10 + 9) = 5 \times 10 + 5 \times 9$.

Here multiplication is distributing itself over addition. This is the *distributive* law and is the only law expressing a relation between two basic operations.

The identity element

In a set of numbers when 0 (zero) is added to any number it preserves the identity of that number, i.e. it does not change its value, e.g. $5 + 0 = 5$ and $0 + 7 = 7$ so 0 is the *identity element* for addition.

The identity element for multiplication is 1. Multiplying a number by 1 does not change its value, e.g. $6 \times 1 = 6$ and $1 \times 10 = 10$.

The inverse element

For every number, another number can be found so that the result of adding it to the original number is the identity element for addition, namely 0. The second number is the *inverse element* under addition. For example, -5 is the inverse of 5 under addition, as $5 + -5 = 0$.

Similarly for multiplication, for every number except 0, another number can be found so that when it multiplies the given number, the result is the identity element for multiplication, namely 1. For example, the inverse element under multiplication for the number 2 is $\frac{1}{2}$, as $2 \times \frac{1}{2} = 1$.

Exercise 1a

In questions **1** to **10**, which law, if any (associative, commutative or distributive), does each of the statements illustrate?

1 $3 \times (4 \times 3) = (3 \times 4) \times 3$

2 $10 + (5 + 7) = (10 + 5) + 7$

3 $9 + 12 = 12 + 9$

4 $7 \times (3 + 5) = 7 \times 3 + 7 \times 5$

5 $7 + 3 + 2 = 2 + 7 + 3$

6 $(4 \times 6) \times 2 = 4 \times (6 \times 2)$

7 $8 \times (4 - 1) = 8 \times 4 + 8 \times (-1)$

8 $3 \times (3 + 2) = 3 \times 3 + 3 \times 2$

9 $5 \times 4 = 4 \times 5$

10 $6 \times (6 - 2) = 6 \times 6 - 6 \times 2$

11 In the statement $10 + 0 = 0 + 10$ how would you describe the zero (0)?

12 In the statement $1 \times 8 = 8 \times 1$ how would you describe the 1?

13 In the statement $10 + (-10) = 0$ how would you describe the -10?

14 What is the inverse of 9 under addition?

15 What is the inverse of 9 under multiplication?

16 What is the inverse of -5 under addition?

In questions **17** to **19**, write down the letter that goes with the correct answer.

17 The inverse of 4 under addition is

A -4 **B** 0 **C** $\frac{1}{4}$ **D** 4

18 The inverse of 4 under multiplication is

A 4 **B** -4 **C** $\frac{1}{4}$ **D** 0

19 $6 + 0 = 6$ so 0 is

A not a member of the set of integers
B the identity element under addition
C the inverse under addition
D the inverse under multiplication.

Addition, subtraction, multiplication and division with whole numbers

The following exercises will help you revise your knowledge of place value and the basic operations of +, –, × and ÷ on whole numbers.

Exercise 1b

Without using a calculator, find:

1	390×400	5	$6000 \times 43\,656$
2	867×600	6	808×4000
3	732×3000	7	$10\,600 \times 3000$
4	$81\,823 \times 900$	8	$66\,500 \times 700\,000$

Remember numbers can be multiplied in any order, e.g. $100 \times 56 = 56 \times 100$

Exercise 1c

Do not use a calculator for this exercise.

1 Write the value of the 4 in these numbers.

a 5047 **b** 6403 **c** 3304 **d** 4056 **e** 48 976

2 Write these numbers in order with the smallest first.

a 8451, 8876, 534, 10 880

b 43 624, 734 921, 2 000 843, 933 402

3 Without using a calculator find the value of

a $\begin{array}{r} 1030 \\ +\,2057 \\ \hline \end{array}$ **b** $\begin{array}{r} 2501 \\ 282 \\ +\,7043 \\ \hline \end{array}$ **c** $\begin{array}{r} 90\,078 \\ 8\,203 \\ 32\,004 \\ +\,80\,720 \\ \hline \end{array}$

4 Our local council have allocated \$16 000 000 for education, \$13 500 000 for the care of the elderly, \$750 000 for keeping the area clean and tidy and \$2 350 000 for all their other expenses. How much do they need to cover all these expenses?

5 **a** 821 – 415 **c** 670 504 – 500 680 **e** 200 707 – 193 000

b 526 – 308 **d** 100 123 – 78 000 **f** 910 027 – 452 009

6 A book gives the populations of three countries as follows:

USA	287 365 732
Japan	152 845 756
Canada	34 815 377

How much larger is the population of the USA than the combined population of the other two countries?

7 Find the missing digit; it is marked with □:

a 27 + 28 = □5 **b** 128 + □59 = 1087 **c** 25 – 1□ = 6

In questions **8** to **10** find without using a calculator:

8 **a** 25 – 6 + 7 – 9

b 14 + 2 – 8 – 3

c 95 – 161 + 75 + 34

d 9500 – 1010 – 2050 + 4300

It is the sign *in front* of a number that tells you what to do with that number. For example 128 – 56 + 92 means '128 take away 56 and add on 92'. This can be done in any order with addition and subtraction so we could add on 92 and then take away 56, i.e.

$$128 - 56 + 92 = 220 - 56 = 164$$

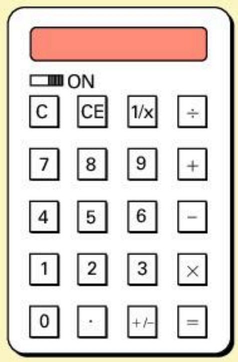
9 **a** 54×6 **d** 8082×4 **g** $70\,450\,000 \times 7$ **j** 54×30
b 204×6 **e** $36\,000 \times 5$ **h** $31\,500\,000 \times 3$ **k** 204×50
c 408×8 **f** $705\,500 \times 7$ **i** $5\,700\,500 \times 4$ **l** 448×80

10 **a** 390×90 **d** 409×206 **g** $40\,502 \times 5060$
b 556×70 **e** 632×107 **h** $73\,006 \times 3080$
c 81×3000 **f** 903×3060 **i** $83\,007 \times 15\,040$

11 Calculate the following and give the remainder when there is one.
a $78 \div 6$ **d** $673 \div 9$ **g** $5\,058\,000 \div 6$ **j** $48\,400\,560 \div 4$
b $85 \div 7$ **e** $800 \div 7$ **h** $6\,290\,805 \div 4$ **k** $8\,108\,100 \div 9$
c $54 \div 8$ **f** $706 \div 3$ **i** $8\,004\,602 \div 3$ **l** $1\,750\,005 \div 5$

12 Calculate the following and give the remainder when there is one.
a $7514 \div 34$ **d** $1608 \div 25$ **g** $4321 \div 56$ **j** $4001 \div 36$
b $5829 \div 43$ **e** $7092 \div 35$ **h** $7974 \div 17$ **k** $3900 \div 43$
c $6372 \div 27$ **f** $2694 \div 30$ **i** $103 \div 35$ **l** $2800 \div 14$

Approximation



Calculators are very useful and can save a lot of time. Calculators do not make mistakes but *we* sometimes do when we use them. So it is important to know roughly if the answer we get from a calculator is right. By simplifying the numbers involved we can get a rough answer in our heads.

One way to simplify numbers is to make them into the nearest number of hundreds. For example

1276 is roughly 13 hundreds or 1300

and

1234 is roughly 12 hundreds or 1200

We say that 1276 is rounded up to 1300 and 1234 is rounded down to 1200. In mathematics we say that 1276 is approximately equal to 13 hundreds.

We use the symbol $\approx$ to mean 'is approximately equal to'. We would write

$1276 \approx 13$ hundreds
$1234 \approx 12$ hundreds

When a number is halfway between hundreds we always round up. We say

$1250 \approx 13$ hundreds

Exercise 1d

Write each of the following numbers as an approximate number of tens:

1 84 **2** 151 **3** 632 **4** 228 **5** 155

Write each of the following numbers as an approximate number of hundreds:

6 830 **7** 256 **8** 1221 **9** 1350 **10** 3780

Write each number as an approximate number of thousands:

11 30 978 **12** 876 434 **13** 710 256 **14** 979 797 **15** 267 262

Write each number as an approximate number of millions:

16 45 672 189 **17** 6 665 555 **18** 35 454 430 **19** 94 325 432

By writing each number correct to the nearest number of tens find an approximate answer for:

20 $153 + 181$

21 $68 + 143 + 73$

22 $369 - 92 + 85$

23 $295 + 304 - 451$

24 $63 + 29 + 40 + 37 + 81$

25 $13 + 29 + 83 + 121 + 5$

26 $103 + 125 + 76 + 41 + 8$

27 $260 + 145 - 36 - 118$

28 $142 - 89 + 64 - 101$

Now use your calculator to find the exact answers to questions **20** to **28**. Remember to look at your rough answer to check that your calculator answer is probably correct.

Puzzle

Copy the following sets of numbers. Put +, −, ×, or ÷ in each box so that the calculations are correct.

1 $9 \square 4 = 5$ **2** $7 \square 3 = 21$ **3** $28 \square 4 = 7$ **4** $8 \square 2 = 4$

Mixed operations of +, −, ×, ÷

When a calculation involves a mixture of the operations +, −, ×, ÷ we always do

multiplication and division first

For example

$$2 \times 4 + 3 \times 6 \quad \text{(multiplication first)}$$
$$= 8 + 18 = 26$$

Exercise 1e

Find $5 - 10 \times 2 \div 5 + 3$

$$\begin{aligned} & 5 - 10 \times 2 \div 5 + 3 \\ &= 5 - 20 \div 5 + 3 \quad (\times \text{ done}) \\ &= 5 - 4 + 3 \quad (\div \text{ done}) \\ &= 1 + 3 \\ &= 4 \end{aligned}$$

Find:

1 $8 \div 2 + 6 \times 3$

2 $14 \times 2 \div 7 - 3 + 6$

3 $5 + 4 \times 3 + 8 \div 2$

4 $5 - 4 \div 2 + 7 \times 2$

5 $6 \times 3 - 8 \times 2$

6 $9 \div 3 + 12 \div 6$

7 $12 \div 3 - 15 \div 5$

<u>8</u> $9 + 3 - 6 \div 2 + 1$

<u>9</u> $6 - 3 \times 2 + 9 \div 3$

10 $5 \times 3 \times 2 - 2 \times 3 \times 4$

11 $10 \times 3 \div 15 + 6$

12 $8 + 7 \times 4 \div 2$

13 $3 \times 8 \div 4 + 7$

14 $9 \div 3 + 7 \times 2$

15 $4 - 8 \div 2 + 6$

<u>16</u> $5 \times 4 \div 10 + 6$

<u>17</u> $6 \times 3 \div 9 + 2 \times 4$

<u>18</u> $7 + 3 \times 2 \div 6$

<u>19</u> $8 \div 4 + 6 \div 2$

<u>20</u> $12 \div 4 + 3 \times 2$

<u>21</u> $19 + 3 \times 2 - 8 \div 2$

<u>22</u> $7 \times 2 - 3 + 6 \div 2$

<u>23</u> $8 + 3 \times 2 - 4 \div 2$

<u>24</u> $7 \times 2 - 4 \div 2 + 1$

<u>25</u> $6 + 8 \div 4 + 2 \times 3 \times 4$

<u>26</u> $5 \times 3 \times 4 \div 12 + 6 - 2$

<u>27</u> $5 + 6 \times 2 - 8 \div 2 + 9 \div 3$

<u>28</u> $7 - 9 \div 3 + 6 \times 2 - 4 \div 2$

<u>29</u> $9 \div 3 - 2 + 1 + 6 \times 2$

<u>30</u> $4 \times 2 - 6 \div 3 + 3 \times 2 \times 4$

Using brackets

If we need to do some addition and/or subtraction before multiplication and division we use brackets round the section that is to be done first. For example $2 \times (3 + 2)$ means work out $3 + 2$ first.

So
$$\begin{aligned} & 2 \times (3 + 2) \\ &= 2 \times 5 \\ &= 10 \end{aligned}$$

For a calculation with brackets and a mixture of $\times$, $\div$, $+$ and $-$ we first work out the inside of the Brackets, then the order is to do the Multiplication and Division, and lastly the Addition and Subtraction.

You can remember this order from the acronym BODMAS.

Another way of remembering is using the first letters of the words in the sentence

Bless My Dear Aunt Sally.

Exercise 1f

Find $2 \times (3 \times 6 - 4) + 7 - 12 \div 6$

$$\begin{aligned} 2 \times (3 \times 6 - 4) + 7 - 12 \div 6 &= 2 \times (18 - 4) + 7 - 12 \div 6 \\ &= 2 \times 14 + 7 - 12 \div 6 && \text{(inside bracket first)} \\ &= 28 + 7 - 2 && (\times \text{ and } \div \text{ next}) \\ &= 33 && (\text{lastly } + \text{ and } -) \end{aligned}$$

Find:

1 $12 \div (5 + 1)$

2 $8 \times (3 + 4)$

3 $(5 - 2) \times 3$

4 $(6 + 1) \times 2$

5 $(3 + 2) \times (4 - 1)$

6 $(3 - 2) \times (5 + 3)$

7 $7 \times (12 - 5)$

8 $(6 + 2) \div 4$

9 $(8 + 1) \times (2 + 3)$

10 $(9 - 1) \div (6 - 2)$

11 $2 + 3 \times (3 + 2)$

12 $7 - 2 \times (5 - 3)$

13 $8 - 5 + 2 \times (4 + 3)$

14 $2 \times (7 - 2) \div (16 - 11)$

15 $4 + 3 \times (2 - 1) + 8 \div (9 - 7)$

16 $6 \div (10 - 8) + 4$

17 $7 \times (12 - 6) - 12$

18 $12 - 8 - 3 \times (9 - 8)$

19 $4 \times (15 - 7) \div (17 - 9)$

20 $5 \times (8 - 2) + 3 \times (7 - 5)$

21 $6 \times 8 - 18 \div (2 + 4)$

22 $10 \div 5 + 20 \div (4 + 1)$

23 $5 + (2 \times 10 - 5) - 6$

24 $8 - (15 \div 3 + 4) + 1$

25 $(2 \times 3 - 4) + (33 \div 11 + 5)$

26 $(18 \div 3 + 3) \div (4 \times 4 - 7)$

27 $(50 \div 5 + 6) - (8 \times 2 - 4)$

28 $(10 \times 3 - 20) + 3 \times (9 \div 3 + 2)$

29 $(7 - 3 \times 2) \div (8 \div 4 - 1)$

30 $(5 + 3) \times 2 + 10 \div (8 - 3)$

Investigation

1 *Do not use a calculator for parts* **a** *to* **d**.

a Multiply 123 456 789 by 3 and then multiply the result by 9.
What do you notice?

b Repeat part **a** multiplying first by a different number less than 9.

c Repeat part **a** again using a third number less than 9.

d Is there a rule for predicting the answer when 123 456 789 is multiplied by one of the numbers 2, 3, 4, 5, 6, 7 or 8, and the result is multiplied by 9? If you find one, write it down and test it.

2 *Now try using your calculator.* What do you notice?

Types of number

Whole numbers and natural numbers

Natural numbers are the numbers 1, 2, 3, 4, 5, … .
Whole numbers are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, … So whole numbers are the natural numbers plus zero.

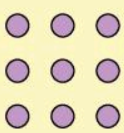
Even and odd numbers

Whole numbers that divide by 2 to give whole number answers are called *even numbers*. For example 10, 456 and 83 450 are even numbers. Even numbers end in 0, 2, 4, 6 or 8.

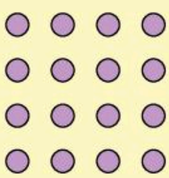
Whole numbers that, when divided by 2, do not give a whole number answer are called *odd numbers*. For example 9, 389 and 997 733 are odd numbers. Odd numbers end in 1, 3, 5, 7 or 9.

Square numbers

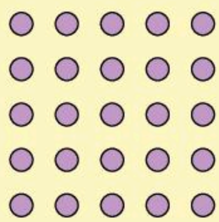
Square numbers can be represented by a number of dots arranged in a square formation. For example 9, 16 and 25 are square numbers because they can be arranged in square formations as shown below.



$9 = 3 \times 3$



$16 = 4 \times 4$

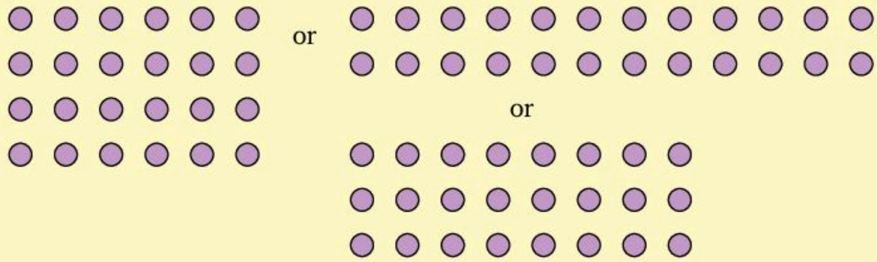


$25 = 5 \times 5$

The smallest square number is 1 because $1 = 1 \times 1$.

Rectangular numbers

Any number that can be shown as a rectangular pattern of dots is called a *rectangular number*. For example, 24 is a rectangular number because 24 dots can be arranged as

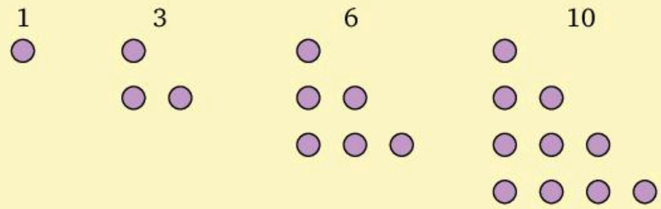


5 is NOT a rectangular number because a line of dots is not a rectangle.

Triangular numbers

A *triangular number* can be shown as dots arranged in rows so that each row is one dot longer than the row above.

These are the first four triangular numbers:



Exercise 1g

1 Consider the following pattern:

$$\begin{aligned} 1 &= 1 = 1 \times 1 \\ 1+3 &= 4 = 2 \times 2 \\ 1+3+5 &= 9 = 3 \times 3 \\ 1+3+5+7 &= 16 = 4 \times 4 \end{aligned}$$

Write the next three lines in this pattern.

Now try and write (without adding them up) the sum of

a the first eight odd numbers **b** the first twenty odd numbers.

2 Consider the following pattern:

$$2 = 2 = 1 \times 2$$
$$2 + 4 = 6 = 2 \times 3$$
$$2 + 4 + 6 = 12 = 3 \times 4$$
$$2 + 4 + 6 + 8 = 20 = 4 \times 5$$

Write the next three lines in this pattern.

How many consecutive even numbers, beginning with 2, have a sum of 156? ($156 = 12 \times 13$)

3 Which of the following numbers are square numbers?

4, 6, 8, 9, 12, 18, 30, 36, 40, 61, 140, 169

4 Which of the following numbers are rectangular numbers?

8, 6, 11, 14, 15, 72, 91, 323, 403

Give a reason for your answer.

5 Show that 12 is a rectangular number in two different ways.

6 Show that 18 is a rectangular number in two different ways.

7 Show that 36 is a rectangular number in three different ways.

8 Draw dot patterns for the next three triangular numbers after 10.

9 Without drawing dot patterns, write down the next three triangular numbers after 28.

10 Look at the pattern.

1

1 2 1

1 2 3 2 1

1 2 3 4 3 2 1

What total do you get for each line in this pattern?

Are all these totals rectangular numbers and/or square numbers?

11 What type of number do you get by adding the numbers in each row of this pattern?

1

$1 + 2$

$1 + 2 + 3$

$1 + 2 + 3 + 4$

$1 + 2 + 3 + 4 + 5$

12 Write down the numbers between 1 and 12 that are

- **a** square numbers
- **b** rectangular numbers
- **c** triangular numbers.

13 Which of the numbers between 24 and 40 are

- **a** square numbers
- **b** rectangular numbers
- **c** triangular numbers?

Factors

The number 2 is a *factor* of 12, since 2 will divide exactly into 12 six times. There is no remainder.

The number 12 may be expressed as the *product* of two factors in several different ways, namely:

1×12 $\quad 2 \times 6 \quad$ or $\quad 3 \times 4$

The numbers 1, 2, 3, 4, 6 and 12 will divide exactly into 12.

All the factors of 12 are 1, 2, 3, 4, 6, 12.

Exercise 1h

Express each of the following numbers as the product of two factors, giving all: possibilities:

1	18	**5**	30	**9**	48	**<u>13</u>**	80	**<u>17</u>**	120
2	20	**6**	36	**10**	60	**<u>14</u>**	96	**<u>18</u>**	135
3	24	**7**	40	**11**	64	**<u>15</u>**	100	**<u>19</u>**	144
4	27	**8**	45	**12**	72	**<u>16</u>**	108	**<u>20</u>**	160

Exercise 1i

List all the factors for each of the numbers in Exercise 1h.

Multiples

A *multiple* of a number is that number multiplied by a whole number.
12 is a multiple of 2 since $2 \times 6 = 12$.

The multiples of 2 are 2, 4, 6, 8, 10, 12, . . .

Similarly 15 is a multiple of 3 since $3 \times 5 = 15$. The multiples of 3 are 3, 6, 9, 15, 18, 21, …

Exercise 1j

1 Write down the multiples of 3 between 20 and 40.
2 Write down the multiples of 5 between 19 and 49.
3 Write down the multiples of 7 between 25 and 60.
4 Write down the multiples of 11 between 50 and 100.
5 Write down the multiples of 13 between 25 and 70.

Prime numbers

A *prime number* is a whole number whose only factors are 1 and itself. For example, the only factors of 3 are 1 and 3 and the only factors of 5 are 1 and 5. The numbers 3 and 5 are both prime numbers. Note that 1 is *not* a prime number.

A whole number, other than 1, which is not prime is a *composite number*, e.g. 6 ($6 = 2 \times 3$).

Investigation

Sieve of Eratosthenes

This is a way of finding prime numbers.

Start with an array of whole numbers:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1 is not a prime number – cross it out.

Cross out all numbers, apart from 2, that are *divisible* by 2 (some have been done).

Then cross out all numbers, apart from 3, that are divisible by 3 (some have been done).

4 has already been crossed out. The next number is 5. Cross out all numbers apart from 5 that are divisible by 5, and so on for 7, 11, etc.

The numbers that are left are the prime numbers less than 100.

Extend this idea to find the prime numbers between 100 and 200.

Exercise 1k

1 Which of the following numbers are prime numbers?

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

2 Write down the prime numbers between 20 and 30.

3 Write down the prime numbers between 30 and 50.

4 Which of these numbers are prime numbers?

5, 10, 19, 29, 39, 49, 61

5 Which of these numbers are prime numbers?

41, 57, 91, 101, 127

6 Are the following statements true or false?
  a All prime numbers are odd numbers.
  b All odd numbers are prime numbers.
  c All prime numbers between 10 and 100 are odd numbers.
  d The only even prime number is 2.
  e There are six prime numbers less than 10.

Investigation

The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Two prime numbers that differ by 2 are called *twin primes*, so 3 and 5 are twin primes, and so are 17 and 19.

1 What is the next pair of twin primes after 19?

2 71 is a prime number. What is the next prime number
  a smaller than 71   b larger than 71?

  Hence find out whether or not 71 is one of a pair of twin primes.

3 313 is one of a pair of twin primes. Find the other one.

Indices

The accepted shorthand way of writing $2 \times 2 \times 2 \times 2$ is 2^4.

We read this as '2 to the power of 4' or '2 to the four'.

The 4 is called the *index*. We say 2^4 is the index form of $2 \times 2 \times 2 \times 2$.

Hence $16 = 2 \times 2 \times 2 \times 2 = 2^4$ and similarly $3^3 = 3 \times 3 \times 3 = 27$.

Expressing a number using indices gives a convenient way for writing large numbers. For example, it has been discovered that

$$2^{216091} - 1$$

is a prime number. This is a very large number that would fill two newspaper pages if it were written in full, as it contains 65 050 digits. Using indices we are able to write it in the short form shown above.

Exercise 1I

Write the following products in index form:

1 $2 \times 2 \times 2$

2 $3 \times 3 \times 3 \times 3$

3 $5 \times 5 \times 5 \times 5$

4 $7 \times 7 \times 7 \times 7 \times 7$

5 $2 \times 2 \times 2 \times 2 \times 2$

6 $3 \times 3 \times 3 \times 3 \times 3 \times 3$

7 $13 \times 13 \times 13$

8 19×19

9 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

10 $6 \times 6 \times 6 \times 6$

Find the value of:

11 2^5

12 3^3

13 5^2

14 2^3

15 3^2

16 7^2

17 3^4

18 4^2

19 10^2

20 10^3

21 10^4

22 10^1

Express the following numbers in index form:

23 4

24 9

25 8

26 27

27 49

28 25

29 32

30 64

Place values and powers of ten

We saw earlier that we can write a number under *place value* headings, where the heading gives the value of a digit.

	thousands	hundreds	tens	units
For example, 1507 can be written	1	5	0	7

These place value headings can be written as powers of 10 because $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, $10^4 = 10\,000$, and so on.

You can see that the value of 10^3 is 1000 (1 followed by three zeros), and the value of 10^1 is 10 (1 followed by one zero), and so on.

So the number of zeros in the value of $10^{\text{any index}}$ is the value of the index.

Continuing this pattern, it follows that $10^0 = 1$.

	10^3	10^2	10^1	10^0
So 1507 can be written	1	5	0	7
and 210 can be written		2	1	0

The value of the digit 2 in the number 210 can be written as 2×10^2 and the value of the digit 1 in the number 1507 can be written as 1×10^3.

Expressing a number as a product in index form

We can now write any number as the product of prime numbers in index form.

Consider the number 108:

$$\begin{aligned} 108 &= 12 \times 9 \\ &= 4 \times 3 \times 9 \\ &= 2 \times 2 \times 3 \times 3 \times 3 \end{aligned}$$

i.e. $108 = 2^2 \times 3^3$

Therefore 108 expressed as the product of prime numbers or factors in index form is $2^2 \times 3^3$.

Similarly

$$\begin{aligned} 441 &= 9 \times 49 \\ &= 3 \times 3 \times 7 \times 7 \\ &= 3^2 \times 7^2 \end{aligned}$$

Exercise 1m

1 Write down the value of the digit 6 in each number. Give your answer in the form $6 \times 10^{\text{a power}}$.

a 60 **b** 600 **c** 6 **d** 6000

2 Write these numbers as ordinary numbers.

a 3×10^2 **b** 2×10^3 **c** $2 \times 10^2 + 5 \times 10^0$ **d** $3 \times 10^4 + 6 \times 10^1$

5×10^2 means $5 \times 100 = 500$ and $4 \times 10^3 + 3 \times 10^2$ means $4000 + 300 = 4300$.

Write the following products in index form:

3 $2 \times 2 \times 7 \times 7$

The number of times the same number is multiplied together gives the index for that number. There are two 2s multiplied together so they give 2^2 and two 7s multiplied together so they give 7^2. You can only use an index when the same number is multiplied together two or more times. 2×7 cannot be written as one number in index form.

4 $3 \times 3 \times 3 \times 5 \times 5$

5 $5 \times 5 \times 5 \times 13 \times 13$

6 $2 \times 3 \times 3 \times 5 \times 2 \times 5$

7 $2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 5$

8 $3 \times 11 \times 11 \times 2 \times 2$

9 $7 \times 7 \times 7 \times 3 \times 5 \times 7 \times 3$

10 $13 \times 5 \times 13 \times 5 \times 13$

11 $3 \times 5 \times 5 \times 3 \times 7 \times 3 \times 7$

12 $2 \times 3 \times 2 \times 5 \times 3 \times 5$

Find the value of:

13 $2^2 \times 3^3$ **16** $2^2 \times 3^2$

14 $3^2 \times 5^2$ **17** $2^2 \times 3^2 \times 5$

15 $2^4 \times 7$ **18** $2 \times 3^2 \times 7$

> The index shows how many times the same number is multiplied together, so here 2 is multiplied by 2 and 3 is multiplied by 3 and by 3 again.

Find the value of:

19 $1^3 + 5^3 + 3^3$ **20** $3^3 + 7^3 + 1^3$ **21** $3^3 + 7^3 + 0^3$

Can you find any other numbers like these three?

Finding prime factors

The following *divisibility rules* may help us to decide whether a given number has certain prime numbers as factors:

A number is divisible

- by 2 if the last digit is even
- by 3 if the sum of the digits is divisible by 3
- by 5 if the last digit is 0 or 5
- by 6 if it is an even number and divisible by 3
- by 9 if the sum of the digits is divisible by 9.

Exercise 1n

Is 1683 divisible by 3?

The sum of the digits is $1 + 6 + 8 + 3 = 18$, which is divisible by 3.
Therefore 1683 is divisible by 3.

1 Is 525 divisible by 3?

2 Is 747 divisible by 5?

> The number is divisible by 5 if it ends in 0 or 5.

3 Is 2931 divisible by 3?

4 Is 740 divisible by 5?

5 Is 543 divisible by 5?

6 Is 1424 divisible by 2?

> The number is divisible by 2 if the last digit is an even number.

7 Is 9471 divisible by 3?

8 Is 2731 divisible by 2?

Is 8820 divisible by 15 ?

15 is the product of the prime numbers 3 and 5, so you need to test to see if 8820 is divisible by both 3 and 5.

8820 is divisible by 5 since it ends in 0.

8820 is divisible by 3 since $8 + 8 + 2 = 18$ which is divisible by 3.

8820 is therefore divisible by both 5 and 3, i.e. it is divisible by 5×3 or 15.

9 Is 10 752 divisible by 6 ?

10 Is 21 168 divisible by 6 ?

11 Is 30 870 divisible by 15 ?

Puzzle

It is a curious fact that $12 \times 12 = 144$ and if the digits in all three numbers are reversed you have $21 \times 21 = 441$ which is also true.

Find other examples with this property.

Expressing a number as a product of prime factors

To express a number in prime factors start by trying to divide by 2 and keep on until you can no longer divide exactly by 2. Next try 3 in the same way, then 5 and so on for each prime number until you are left with a quotient of 1.

Exercise 1p

Express 720 as the product of prime factors in index form.

(Test for the prime factors in order, 2 first, then 3, and so on.)

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

Therefore $720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

i.e. $720 = 2^4 \times 3^2 \times 5$

Express each of the following numbers as the product of prime factors in index form:

1 24 **3** 63 **5** 136 **7** 216 **9** 405

2 28 **4** 72 **6** 84 **8** 528 **10** 784

11 List every even number from 20 to 30 as the sum of two primes.

Highest Common Factor (HCF)

The *highest common factor* of two or more numbers is the largest number that divides exactly into each of them.

For example 8 is the HCF of 16 and 24 and 15 is the HCF of 45, 60 and 120.

Exercise 1q

State the HCF of:

1 9, 12

2 8, 16

3 12, 24

4 14, 42

5 25, 50, 75

6 22, 33, 44

7 21, 42, 84

8 39, 13, 26

<u>9</u> 25, 35, 50, 60

<u>10</u> 36, 44, 52, 56

<u>11</u> 15, 30, 45, 60

<u>12</u> 10, 18, 20, 36

Investigation

Two or more counting numbers (1, 2, 3, ...) are called relatively prime if their highest common factor is 1. For example 2 and 3 are relatively prime, so are 3 and 8.

Find as many relatively prime numbers as you can less than 20.

Lowest Common Multiple (LCM)

The *lowest common multiple* of two or more numbers is the smallest number that divides exactly by each of the numbers.

For example the LCM of 8 and 12 is 24 since both 8 and 12 divide exactly into 24.

Similarly the LCM of 4, 6 and 9 is 36.

Exercise 1r

State the LCM of:

1	3, 5	**4**	9, 12	**7**	12, 16, 24	**<u>10</u>**	18, 27, 36
2	6, 8	**5**	3, 9, 12	**8**	4, 5, 6	**<u>11</u>**	9, 12, 36
3	5, 15	**6**	10, 15, 20	**<u>9</u>**	9, 12, 18	**<u>12</u>**	6, 7, 8

Using prime factors to find the HCF and LCM

When the HCF or the LCM of two or more numbers is not easy to spot, we can find them by using the prime factors of each number.

Exercise 1s

Find the HCF of 28 and 36.

$28 = 4 \times 7 = 2 \times 2 \times 7$

$36 = 4 \times 9 = 2 \times 2 \times 3 \times 3$

The HCF $= 2 \times 2 = 4$

First write each number as a product of prime factors. Then pick out the factors that are common to each number.

This shows an alternative method for finding the prime factors: start by writing the number as a product of any two factors, then express each of those which are not prime as the product of any two factors and continue this until all the factors are prime.

Find the HCF of the following sets of numbers.

1 **a** 45, 60 **b** 64, 72

2 **a** 32, 52, 56 **b** 18, 54, 72

3 **a** 351, 648 **b** 432, 768

4 **a** 105, 147, 189 **b** 273, 975, 1638

Find the LCM of 15 and 20.

$15 = 3 \times 5$

$20 = 4 \times 5 = 2 \times 2 \times 5$

The LCM $= 3 \times 5 \times 2 \times 2 = 60$

Find the LCM of

5 **a** 45, 60 **b** 64, 72

6 **a** 56, 84 **b** 104, 169

7 **a** 44, 121, 66 **b** 36, 48, 108

8 For these three numbers: 20, 30, 35, find

a the HCF **b** the LCM.

Problems involving HCFs and LCMs

Exercise 1t

Mrs Walcott buys a box of chocolates for her party. She is unsure whether there will be 8, 9 or 12 people altogether, but she is sure that whichever number it is everybody can have the same number of chocolates. What is the least number of chocolates that needs to be in the box?

You need to find the smallest number that 8, 9 and 12 will divide into exactly.

First express each number in prime factors.

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$12 = 2 \times 2 \times 3$

You must write three 2s so that 8 will divide into the number and you must write two 3s so that 9 will divide into the number. The two 2s and one 3 for 12 are already there so there is no need to write them again.

The smallest number that 8, 9 and 12 divide into exactly is
$2 \times 2 \times 2 \times 3 \times 3$
i.e. $8 \times 9 = 72$

1. Some edging tiles are sold in widths of 10 cm and 25 cm. What is the shortest length that can be made up using either an exact number of 10 cm tiles or an exact number of 25 cm tiles?
2. Find the least sum of money into which \$24, \$30 and \$54 will divide exactly.
3. Find the smallest length that can be divided exactly into equal sections of length 5 m or 8 m or 12 m.
4. A room measures 450 cm by 350 cm. Find the side of the largest square tile that can be used to tile the floor without any cutting.
5. Two toy cars travel around a racetrack, the one completing the circuit in 6 seconds and the other in $6\frac{1}{2}$ seconds. If they leave the starting line together how long will it be before they are again side by side?
6. If I go up a flight of stairs two at a time I get to the top without any being left over. If I then try three at a time and again five at a time, I still get to the top without any being left over. Find the shortest flight of stairs for which this is possible. How many would remain if I were able to go up seven at a time?

7. In the first year of a large comprehensive school it is possible to divide the pupils into equal sized classes of either 24 or 30 or 32 and have no pupils left over. Find the size of the smallest entry that makes this possible. How many classes will there be if each class is to have 24 pupils?
8. Find the largest number of children that can equally share 72 sweets and 54 chocolates.

Puzzle

1 Using the four digits 2, 3, 6 and 9 once only you can make several pairs of two-digit numbers, e.g. 26 and 93. Find 26×93.

Now pair the digits in a different way, e.g. 39 and 62 and find 39×62.

What do you notice?

Can you find another four digits with the same property?

2 The church at Arima has a peal of four bells. No. 1 bell rings every 5 seconds, No. 2 bell every 6 seconds, No. 3 bell every 7 seconds and No. 4 every 8 seconds. They are first tolled together. Investigate how long it will be before they all sound together again.

Mixed exercises

Exercise 1u

Find:

1 $392 + 6250 + 307$

2 $540\,078 - 74\,500$

3 912×6

4 $72 \div 8$

5 $(7 + 30) \times 6 - 137$

6 $4\,650\,000 - 177\,000 \div 3$

7 $668 - 242 + 127 \times 2$

8 $934 + 823 - 1277 - 404$

9 $9 - (15 \div 3 + 5) + 7$

10 How many times can 16 be taken away from 200?

11 The contents of a carton of sweets weigh 5 kilograms. The sweets are divided into packets each weighing 500 grams. How many packets of sweets can be made up? (1 kilogram = 1000 grams)

12 Ann and Ben were two candidates in an election. Ann got 441 more votes than Ben. Ben got 320 votes. There were 34 spoilt voting papers. 293 people could have voted but failed to do so.

a How many votes did Ann get? **b** How many people could have voted?

Exercise 1v

Select the letter that gives the correct answer.

1 The multiples of 6 between 15 and 32 are

A 12, 24, 30 **B** 18, 20, 30 **C** 18, 24, 28 **D** 18, 24, 30

2 Which of the numbers 10, 13, 17, 21, 26, 27, 29 are prime numbers?

A 10, 13, 26 **B** 13, 17, 21 **C** 13, 17, 27 **D** 13, 17, 29

3 Written in index form $2 \times 2 \times 5 \times 5 \times 2$ is

A $2^2 \times 5$ **B** $2^2 \times 5^2$ **C** $2^3 \times 5$ **D** $2^3 \times 5^2$

4 The LCM (least common multiple) of 4, 6 and 8 is

A 16 **B** 24 **C** 32 **D** 48

5 Expressed as the product of its prime factors 36 is

A $2^2 \times 3$ **B** 2×3^2 **C** $2^2 \times 3^2$ **D** $2^3 \times 3$

6 The HCF (highest common factor) of 22, 33 and 66 is

A 2 **B** 3 **C** 6 **D** 11

7 48 written as the product of its prime factors is

A $2^2 \times 3$ **B** $2^3 \times 3$ **C** $2^4 \times 3$ **D** $2^5 \times 3$

Investigation

The ancient Greeks discovered a set of numbers, each of which is equal to the sum of its factors, excluding itself. These are called perfect numbers.

For example, the factors of 6, excluding 6, are {1, 2, 3}.

$1 + 2 + 3 = 6$. Hence 6 is a PERFECT number.

Find some other perfect numbers. Consider all factors, not just prime factors.

Can a prime number be a perfect number? Explain your answer.

Investigate 496 to see if it is a perfect number.

Did you know?

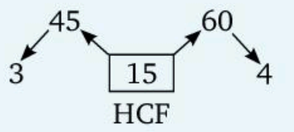
The following method may be used to find the LCM of two numbers.

e.g. Find the LCM of 45 and 60.

First find the HCF of 45 and 60. This is 15.

Divide each number by the HCF, 15. We get 3 and 4.

The LCM is the product of these answers and the HCF, i.e. $3 \times 4 \times 15 = 180$.



$$\text{LCM} = 3 \times 15 \times 4 = 180$$

In 1999 Nayan Hajratwala found the largest prime number known up to that time. It contains 2 098 960 digits. It comes from the calculation of $2^{6\,972\,593} - 1$.

A prime expressed in this way is called a Mersenne Prime.

This was the first-known million-digit prime.

In this chapter you have seen that…

- ✔ you can do a rough check on calculations by rounding the numbers to the nearest 10, 100, 1000, …
- ✔ to solve word problems, you need to read the question carefully to make sure that you understand what you have been asked to do
- ✔ the sign in front of a number tells you whether you add or subtract that number
- ✔ brackets are used to show what needs to be done first
- ✔ when there are no brackets do multiplication and division before addition and subtraction
- ✔ square numbers can be shown as a square pattern of dots
- ✔ rectangular numbers can be shown as a rectangular pattern of dots
- ✔ triangular numbers can be shown as a triangular pattern of dots
- ✔ a prime number is any number bigger than 1 whose only factors are 1 and itself

- ✔ you can use indices to shorten the way you write an expression that is the same number multiplied together several times
- ✔ numbers that are not prime can be expressed as the product of prime factors in index form
- ✔ you can use prime factors to find the largest number that will divide exactly into each number in a group by picking out the prime factors that are common to all the numbers and multiplying them together. It is called the Highest Common Factor (HCF) of the group
- ✔ you can use prime factors to find the lowest number that all the numbers of a group will divide into exactly by picking out all the prime factors that appear anywhere in the group, each one to its highest index, and multiplying them together. It is called the Lowest Common Multiple (LCM).

2 Directed numbers

At the end of this chapter you should be able to...

1 use positive or negative numbers to describe displacements on one side or the other of a given point on a line

2 apply positive and negative numbers, where appropriate, in a physical situation

3 perform operations of addition, subtraction, multiplication or division on positive and negative numbers.

Did you know?

Wherever you are a million is always a million (1 000 000).

However, a BILLION is not always a billion.

In the USA, 1 billion = 1000 × 1 000 000

But in some countries, 1 BILLION = 1 000 000 × 1 000 000.

You need to know...

✔ multiplication tables up to 12 × 12

✔ how to add, subtract, multiply and divide whole numbers.

Key words

directed numbers, integers, negative numbers, number line, positive numbers

Positive and negative numbers

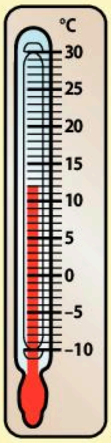
There are many quantities that can be measured above or below a natural zero.

For example, we are used to temperatures above and below 0 °C (Celsius) which is the freezing point of water.

A temperature of 5 °C below freezing point is writen as –5 °C.

Most people would call –5 °C 'minus 5 °C' but we will call it 'negative 5 °C' and there are good reasons for doing so because in mathematics 'minus' means 'take away'.

A temperature of 5 °C above freezing point is called 'positive 5 °C' and can be written as +5 °C.



Most people would just call it 5 °C and write it without the positive symbol.

A number without any symbol in front of it is a *positive number*,

i.e.	2 means +2
and	+3 can be written as 3

Positive and *negative numbers* are collectively known as *directed numbers*.

Directed numbers can be used to describe any quantity that can be measured above or below a natural zero. For example, a distance of 50 m above sea level and a distance of 50 m below sea level could be written as + 50 m and – 50 m respectively.

They can also be used to describe time before and after a particular event. For example, 5 seconds before the start of a race and 5 seconds after the start of a race could be written as –5 s and +5 s respectively.

Directed numbers can also be used to describe quantities that involve one of two possible directions. For example, if a car is travelling north at 70 km/h and another car is travelling south at 70 km/h they can be described as going at +70 km/h and –70 km/h respectively.

Exercise 2a

Draw a Celsius thermometer and mark a scale on it from –10° to +10°. Use your drawing to write the following temperatures as positive or negative numbers:

1 10° above freezing point
2 7° below freezing point
3 3° below zero
4 5° above zero
5 8° below zero
6 freezing point

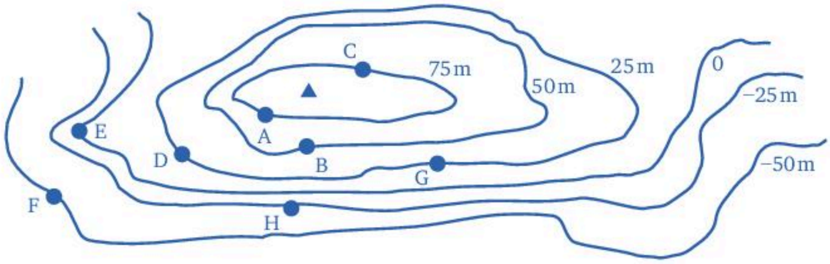
Write in words, the meaning of the following temperatures:

7 –2 °C
8 +3 °C
9 4 °C
10 –10 °C
11 +8 °C
12 0 °C

Which temperature is higher?

13 +8° or +10°
14 12° or 3°
15 –2° or +4°
16 –3° or –5°
17 –8° or 2°
18 –2° or –5°
19 1° or –1°
20 +3° or –5°
21 –7° or –10°
22 –2° or –9°

23 The contour lines on the map below show distances above sea level as positive numbers and distances below sea level as negative numbers.



Write down in words the position relative to sea level of the points A, B, C, D, E, F, G and H.

In questions **24** to **34** use positive or negative numbers to describe the quantities.

A ball thrown up a distance of 5 m.

Up is above the start point so a positive number describes this.

+5 m

24 5 seconds before blast off of a rocket.

25 5 seconds after blast off of a rocket.

26 \$500 in your purse.

27 \$500 owed.

28 1 minute before the train leaves the station.

29 A win of \$50 000 on a lottery.

30 A debt of \$5000.

31 Walking forwards five paces.

32 Walking backwards five paces.

33 The top of a hill which is 200 m above sea level.

34 A ball thrown down a distance of 5 m.

35 At midnight the temperature was –2 °C. One hour later it was 1 ° colder. What was the temperature then?

36 At midday the temperature was 18 °C. Two hours later it was 3 ° warmer. What was the temperature then?

37 A rock climber started at +200 m and came a distance of 50 m down the rock face. How far above sea level was he then?

38 At midnight the temperature was –5 °C. One hour later it was 2 ° warmer. What was the temperature then?

39 At the end of the week my financial state could be described as –\$25. I was later given \$50. How could I then describe my financial state?

40 Positive numbers are used to describe a number of paces forwards and negative numbers are used to describe a number of paces backwards. Describe where you are in relation to your starting point if you walk +10 paces followed by –4 paces.

The number line

If we draw a straight line and mark a point on it as zero, then we can describe the whole numbers as equally spaced points to the right of zero as 1, 2, 3, 4, 5, … These are called positive numbers.

Numbers to the left of zero are called negative numbers and can be described as equally spaced points to the left of zero.

This line is called a *number line*.

On this number line, 5 is to the *right* of 3

and we say that 5 is *greater* than 3

or $5 > 3$ (> means 'is greater than')

Also –2 is to the *right* of –4

and we say that –2 is *greater* than –4

or $-2 > -4$

So 'greater' means 'higher up the scale'.

(A temperature of –2 °C is higher than a temperature of –4 °C.)

Now 2 is to the *left* of 6

and we say that 2 is *less* than 6

or $2 < 6$ (< means 'is smaller than')

Also –3 is to the *left* of –1

and we say that –3 is *less* than –1

or $-3 < -1$

So 'less than' means 'lower down the scale'.

Note that the numbers … –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, … are called *integers*.

Exercise 2b

Draw a number line.

In questions **1** to **12** write either > or < between the two numbers:

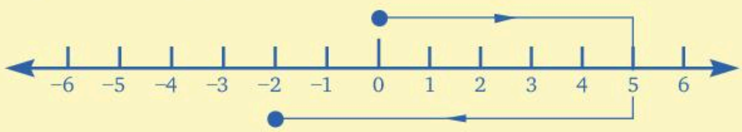
1 3 2	**4** −3 −1	**7** 3 −2	**10** −7 3
2 5 1	**5** 1 −2	**8** 5 −10	**11** −1 0
3 −1 −4	**6** −4 1	**9** −3 −9	**12** 1 −1

In questions **13** to **24** write down the next two numbers in the pattern

13 4, 6, 8	**16** −4, −2, 0	**19** 5, 1, −3	**22** −10, −8, −6
14 −4, −6, −8	**17** 9, 6, 3	**20** 2, 4, 8	**23** −1, −2, −4
15 4, 2, 0	**18** −4, −1, 2	**21** 36, 6, 1	**24** 1, 0, −1

Addition and subtraction of positive numbers

If you were asked to work out 5 – 7 you would probably say that it cannot be done. But if you were asked to work out where you would be if you walked 5 steps forwards and then 7 steps backwards, you would say that you were 2 steps behind your starting point.



On the number line, 5 – 7 means

start at 0 and go 5 places to the right

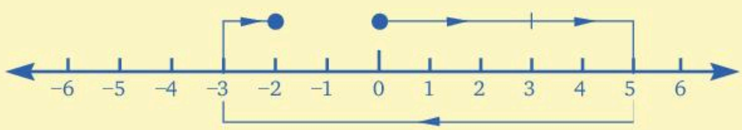
and then go 7 places to the left

So $5 - 7 = -2$

i.e. 'minus' a positive number means move to the left

and 'plus' a positive number means move to the right.

In this way $3 + 2 - 8 + 1$ can be shown on the number line as follows:



Therefore $3 + 2 - 8 + 1 = -2$.

Exercise 2c

Find, using a number line if it helps:

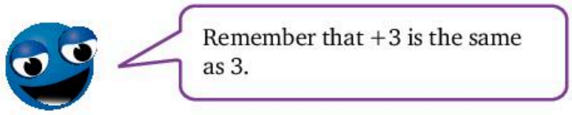
1 3 − 6	**3** 4 − 6	**5** 4 − 2	**7** −2 + 3	**9** −5 − 7
2 5 − 2	**4** 5 − 7	**6** 5 + 2	**8** −3 + 5	**10** −3 + 2

(+4) − (+3)

(+4) − (+3) = 4 − 3   (+4 = 4 and +3 = 3)

= 1

11 (+3) + (+2)	**21** −4 + 2 + 5
12 (+2) − (+4)	**22** −3 + 1 − 4
13 (+5) − (+7)	**23** 5 − 6 − 9
14 (−3) + (+2)	**24** −3 − 4 + 2
15 (−1) + (+5)	**25** − 2 − 3 + 9
16 5 − 2 + 3	**26** (+3) + (+4) − (+1)
17 7 − 9 + 4	**27** (+2) − (+5) + (+6)
18 5 − 11 + 3	**28** (+9) − (+7) − (+2)
19 10 − 4 − 9	**29** (−3) + (+5) − (+5)
20 3 + 6 − 10	**30** (−8) − (+4) + (+7)



Addition and subtraction of negative numbers

Most of you will have some money of your own, from pocket money and other sources. Many of you will have borrowed money at some time.

At any one time you have a balance of money, i.e. the total sum that you own or owe!

If you own \$3 and you borrow \$4, your balance is a debt of \$1. We can write this as

$$(+3) + (-4) = (-1)$$

or as $$3 + (-4) = -1$$

But $$3 - 4 = -1$$

∴ $+(-4)$ means -4

If you owe \$2 and then take away that debt, your balance is zero. We can write this as

$$(-2) - (-2) = 0$$

You can pay off the debt on your balance only if someone gives you \$2.
So subtracting a negative number is equivalent to adding a positive number, i.e. – (–2) is equivalent to +2.

$$-(-2) \quad \text{means} \quad +2$$

Exercise 2d

Find:

1 $3 + (-1)$

2 $5 + (-8)$

3 $4 - (-3)$

4 $-1 - (-4)$

5 $-2 + (-7)$

6 $-2 - (-5)$

7 $4 + (-7)$

8 $-3 - (-9)$

9 $-4 + (-10)$

10 $2 - (-8)$

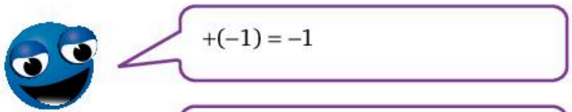
<u>**11**</u> $-7 + (-7)$

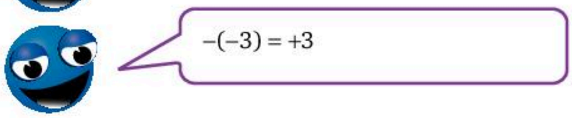
<u>**12**</u> $-3 - (-3)$

<u>**13**</u> $+4 + (-4)$

<u>**14**</u> $+2 - (-4)$

<u>**15**</u> $-3 + (-3)$





$2 + (-1) - (-4)$

$$+(-1) = -1 \text{ and } -(-4) = +4$$

$$2 + (-1) - (-4) = 2 - 1 + 4$$

$$= 5$$

16 $5 + (-1) - (-3)$

17 $(-1) + (-1) + (-1)$

18 $4 - (-2) + (-4)$

19 $-2 - (-2) + (-4)$

20 $6 - (-7) + (-8)$

<u>**21**</u> $9 + (-5) - (-9)$

<u>**22**</u> $8 - (-7) + (-2)$

<u>**23**</u> $10 + (-9) + (-7)$

<u>**24**</u> $12 + (-8) - (-4)$

<u>**25**</u> $9 + (-12) - (-4)$

Addition and subtraction of directed numbers

We can now use the following rules:

- plus a positive number and minus a negative number are both equivalent to a positive number
- plus a negative number and minus a positive number are both equivalent to a negative number

i.e.

$+(+a) = +a$ and $-(-a) = +a$

$+(-a) = -a$ and $-(+a) = -a$

Exercise 2e

Find:

1 3 + (–2)

2 –3 – (+2)

3 6 – (–3)

4 4 + (+4)

5 –5 – (–7)

6 9 – (+2)

7 7 + (–3)

8 8 + (+2)

9 10 – (–5)

10 –2 – (–4)

11 12 + (–7)

12 –4 – (+8)

13 3 – (–2)

14 –5 + (–4)

15 8 + (–7)

16 4 – (–5)

17 7 + (–3) – (+5)

18 2 – (–4) + (–6)

19 5 + (–2) – (+1)

20 8 – (–3) + (+5)

21 7 + (–4) – (–2)

22 3 – (+2) + (–5)

23 –9 + (–2) – (–3)

24 8 + (+9) – (–2)

25 7 + (–9) – (+2)

26 4 + (–1) – (+7)

27 –3 + (+5) – (–2)

28 –4 + (+8) + (–7)

29 –9 – (+4) – (–10)

30 –2 – (+8) + (–9)

–8 – (4 – 7)

$$\begin{aligned} -8-(4-7) &= -8-(-3) \quad \text{(brackets first)} \\ &= -8+3 \\ &= -5 \end{aligned}$$

31 3 – (4 – 3)

32 5 + (7 – 9)

33 4 + (8 – 12)

34 –3 – (7 – 10)

35 6 + (8 – 15)

36 (3 – 5) + 2

37 5 – (6 – 10)

38 (4 – 9) – 2

39 (7 + 4) – 15

40 $8+(3-8)$

41 $(3-8)-(9-4)$

42 $(3-1)+(5-10)$

43 $(7-12)-(6-9)$

44 $(4-8)-(10-15)$

45 Add (+7) to (–5).

46 Subtract 7 from –5.

47 Subtract (–2) from 1

48 Find the value of '8 take away –10'.

49 Add –5 to +3

50 Find the sum of –3 and +4.

51 Find the sum of –8 and +10.

52 Subtract positive 8 from negative 7.

53 Find the sum of –3 and –3 and –3.

54 Find the value of twice negative 3.

55 Find the value of four times –2.

Multiplication of directed numbers

From previous work we know that

a $(+3)\times(+2)=+6$

This is just the multiplication of positive numbers,

i.e. $(+3)\times(+2)=3\times 2=6$

b $(-3)\times(+2)=-6$

Here we could write $(-3)\times(+2)=-3(2)$.

This is equivalent to subtracting 3 twos, i.e. subtracting 6.

c $(+4)\times(-3)=-12$

This means four lots of –3,

i.e. $(-3)+(-3)+(-3)+(-3)=-12$

d $(-2)\times(-3)=+6$

This can be thought of as taking away two lots of –3,

i.e. $-2(-3)=-(-6)$

We have already seen that taking away a negative number is equivalent to adding a positive number, so $(-2)\times(-3)=+6$.

To summarise:

- when two positive numbers are multiplied, the answer is positive
- when two negative numbers are multiplied, the answer is positive
- when a positive number and a negative number are multiplied the answer is negative.

Exercise 2f

Calculate: **a** $(+2) \times (+4)$ **b** 2×4.

a $(+2) \times (+4) = 8$

b $2 \times 4 = 8$

This shows that $(+2) \times (+4)$ means the same as 2×4.

Calculate: **a** $(-3) \times (+4)$ **b** -3×4.

a $(-3) \times (+4) = -12$

b $-3 \times 4 = -12$

This shows that $(-3) \times (+4)$ means the same as -3×4.

Because order does not matter when two quantities are multiplied together, $(+4) \times (-3)$ gives the same answer of -12.

So -3 and $+4$ can be multiplied together in two different ways, but they mean the same thing.

Calculate:

1 $(-3) \times (+5)$

2 $(+4) \times (-2)$

3 $(-7) \times (-2)$

4 $(+4) \times (+1)$

5 $(+6) \times (-7)$

6 $(-4) \times (-3)$

7 $(-6) \times (+3)$

8 $(-8) \times (-2)$

<u>9</u> $(+5) \times (-1)$

<u>10</u> $(-6) \times (-3)$

<u>11</u> $(-3) \times (-9)$

<u>12</u> $(-2) \times (+8)$

13 $7 \times (-5)$

14 $-6(-4)$

15 -3×5

16 $5 \times (-9)$

17 $-6(4)$

18 $-2(-4)$

19 $-(-3)$

20 $4 \times (-2)$

<u>21</u> $3(-2)$

<u>22</u> 5×3

<u>23</u> $6 \times (-3)$

<u>24</u> $-5(-4)$

25 $6 \times (-4)$

26 $-3(+8)$

27 $(+5) \times (+9)$

28 -4×5

<u>29</u> $7(-4)$

<u>30</u> $(-4) \times (-9)$

Division of directed numbers

The rules for multiplying directed numbers also show us what happens when we divide with directed numbers. We also use the fact that because $2 \times 3 = 6$ it follows that $6 \div 3 = 2$.

a In the same way, $(-3) \times 4 = -12$, so $(-12) \div 4 = -3$.

Notice that the order *does* matter in division, e.g.

$$(-12) \div 4 = -3$$

but $4 \div (-12) = -\frac{4}{12}$ which simplifies to $-\frac{1}{3}$.

b Also $(-4) \times (-2) = 8$ so it follows that $8 \div (-2) = -4$

c Now $3 \times (-2) = -6$ so again it follows that $(-6) \div (-2) = 3$

a and **b** show that:

When a negative number is divided by a positive number and when a positive number is divided by a negative number the answer is negative.

c shows that:

When a negative number is divided by a negative number the answer is positive.

Exercise 2g

Calculate: **a** $-8 \div 4$ **b** $8 \div (-4)$

a $-8 \div 4 = -2$

b $8 \div (-4) = -2$

These examples show that the answer is negative when a division involves one positive number and one negative number.

Calculate: **a** $8 \div 4$ **b** $(-8) \div (-4)$

a $8 \div 4 = 2$

b $(-8) \div (-4) = 2$

These examples show that the answer is positive when a division involves two numbers with the same sign.

Calculate:

1 $-12 \div 6$

2 $25 \div 5$

3 $16 \div (-4)$

4 $(-24) \div (-12)$

5 $3 \div (-3)$

6 $(-28) \div (-7)$

7 $36 \div (-12)$

8 $(-2) \div (-2)$

9 $-18 \div 6$

10 $20 \div (-4)$

11 $15 \div (-12)$

12 $-5 \div 3$

13 $-36 \div (-10)$

14 $1 \div (-1)$

15 $44 \div (-10)$

16 $\frac{-8}{4}$

17 $\frac{12}{-6}$

18 $\frac{27}{-3}$

19 $\frac{-27}{-9}$

20 $\frac{45}{-40}$

Calculate: **a** $-2 + 6 \div (-3)$ **b** $-3 \div 6 \times (-9)$ **c** $5 \times (2 - 5) \div 3$

a $-2 + 6 \div (-3) = -2 + (-2)$

$= -4$

Remember that multiplication and division is done before addition and subtraction.

b $-3 \div 6 \times (-9) = (-3) \times (-9) \div 6$

$= 27 \div 6 = \frac{27}{6} = \frac{9}{2} = 4\frac{1}{2}$

Remember that multiplication and division can be done in any order.

c $5 \times (2 - 5) \div 3 = 5 \times (-3) \div 3$

$= 5 \times (-1) = -5$

Remember that numbers in brackets are calculated first.

Calculate:

21 $16 \div 4(3 - 5)$

22 $24 - 3 \times (-4)$

23 $3 \times (4 - 7)$

24 $5 \div (8 - 7)$

25 $6 + 8(-2)$

26 $2 + 6 \div (-3)$

27 $12 - 2(1 + 5)$

28 $7 - 3(2 - 4)$

29 $7 \times 2 - (6 + 4)$

30 $(-8) \div 2(12 + 4)$

31 $3(2 - 6) + 4\ (12 - 10)$

32 $2(6 - 3) \div 2(4 - 6)$

33 $3 - 7 \div 2(5 - 3)$

34 $(-3)(2 - 5) \times 4(7 - 6)$

35 $5(7 + 8) \div 6 \times (-2)$

Mixed exercises

Exercise 2h

1 Which is the higher temperature, $-5°$ or $-8°$?

2 Write $<$ or $>$ between **a** $-3, \quad 2$ **b** $-2, \quad -4$

Find:

3	$-4+6$	**7**	$-2+(-3)-(-5)$	**11**	$(-5)\times(-2)$
4	$3+2-10$	**8**	$4-(2-3)$	**12**	$-2(7)$
5	$2+(-4)$	**9**	$6\times(-4)$	**13**	$(+4)\times(-6)$
6	$3-(-1)$	**10**	$-36\div 3$	**14**	$-2(-3)+1$

Exercise 2i

Find:

1	$4\div(-2)$	**5**	$3-4\div(-2)$	**9**	$4(6-3)\div 2(4-6)$
2	$(-5)\times(-10)$	**6**	$(-2-7)\div 3\times 6$	**10**	$3(5+1)\times(-2)(3-2)$
3	$(-5)\div(-10)$	**7**	$2\times 4-3(6-4)$	**11**	$(-1)(5-6)\div 2(4-3)$
4	$(-6)\div 6$	**8**	$2\times 4\div 3(6-4)$	**12**	$3\times 2+4\times 6-2\div(-4)$

Did you know?

Did you know that Astragalia is the name given to the large number of bones claimed to have been discovered by archaeologists at prehistoric sites?

It is said that these bones were used as dice in ancient games.

In this chapter you have seen that...

- ✔ directed numbers is the collective name for positive and negative numbers
- ✔ directed numbers can be used to describe quantities that can be measured above or below a natural zero
- ✔ the rules for addition and subtraction are:

  $+(+a)$ and $-(-a)$ both give $+a$ and

  $+(-a)$ and $-(+a)$ both give $-a$

  You can remember these as

  SAME SIGNS GIVE POSITIVE, DIFFERENT SIGNS GIVE NEGATIVE
- ✔ $5 \times (-3)$, $(+5) \times (-3)$,

  $(-3) \times 5$, $(-3) \times (+5)$

  ALL MEAN THE SAME
- ✔ the rules for multiplication and division are:
  - when a negative number is divided by a positive number and when a positive number is divided by a negative number the answer is negative
  - when a negative number is divided by a negative number the answer is positive.

3 Fractions

At the end of this chapter you should be able to...

1 express one quantity as a fraction of another
2 write a fraction equivalent to a given fraction
3 order a set of fractions according to magnitude
4 add and subtract fractions
5 solve word problems involving fractional operations
6 perform operations involving multiplication and division of fractions including mixed numbers.

Did you know?

The system of writing one number above the other, as in $\frac{1}{2}$, is attributed to a Hindu mathematician, Brahmagupta. The bar between the two numbers as in $\frac{1}{2}$ was first used by the Arabs, about CE 1150.

You need to know...

- ✔ how to add and subtract whole numbers
- ✔ how to divide by a whole number
- ✔ what LCM means and how to find it.

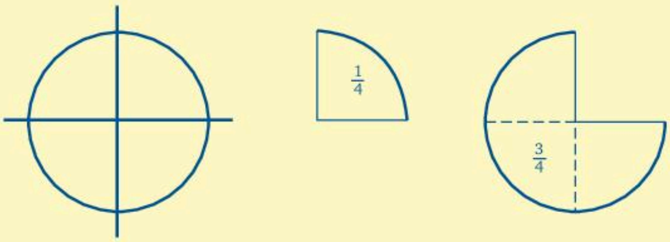
Key words

common denominator, common factor, denominator, equivalent fraction, expression, fraction, improper fraction, lowest common multiple (LCM), lowest terms, mixed number, mixed operations, numerator, proper fraction, rational number simplify a fraction

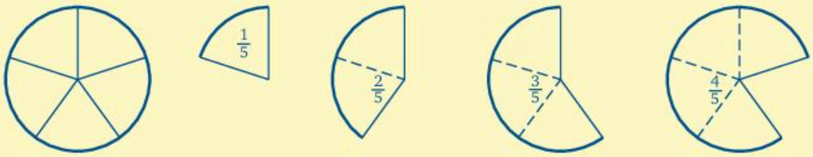
The meaning of fractions

This section is a revision of the meaning of fractions.

When we cut a cake into four equal pieces, each piece is one quarter, written $\frac{1}{4}$, of the cake. When one piece is taken away there are three pieces left, so the *fraction* that is left is three quarters, or $\frac{3}{4}$.



When the cake is divided into five equal slices, one slice is $\frac{1}{5}$, two slices is $\frac{2}{5}$, three slices is $\frac{3}{5}$ and four slices is $\frac{4}{5}$ of the cake.



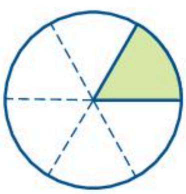
Notice that the top number in each fraction (called the *numerator*) tells you *how many* slices and the bottom number (called the *denominator*) tells you about the total number of slices to make a whole cake.

Fractions where the numerator and denominator are whole numbers are called *rational numbers*.

Exercise 3a

Write the fraction that is shaded in each of these sketches:

1



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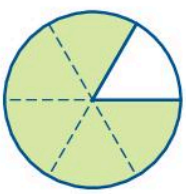

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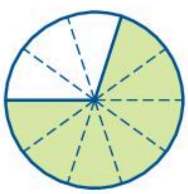
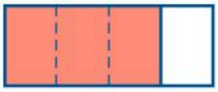


It is not only cakes that can be divided into fractions. Anything at all that can be split up into equal parts can be divided into fractions.

Write the fraction that is shaded in each of the following diagrams:

4 

5 

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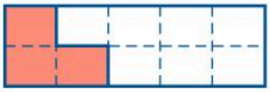
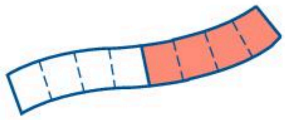
7 

8 

9 

10 

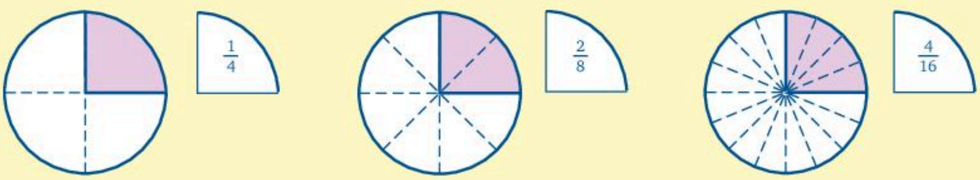
11 

Equivalent fractions

In the first sketch below, a cake is cut into four equal pieces.
One slice is $\frac{1}{4}$ of the cake.

In the second sketch the cake is cut into eight pieces.
Two slices is $\frac{2}{8}$ of the cake.

In the third sketch the cake is cut into sixteen equal slices.
Four slices is $\frac{4}{16}$ of the cake.



But the same amount of cake has been taken each time.

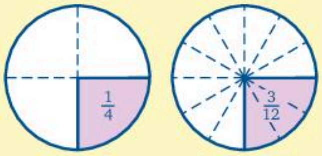
Therefore $\quad \frac{1}{4} = \frac{2}{8} = \frac{4}{16}$

and we say that $\frac{1}{4}, \frac{2}{8}$ and $\frac{4}{16}$ are *equivalent fractions*.

Now $\quad \frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8} \quad$ and $\quad \frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

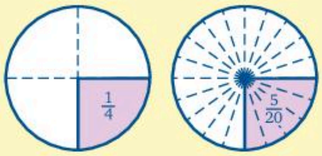
So all we have to do to find equivalent fractions is to multiply the numerator and the denominator by the same number. For instance

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$



and

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$



Any fraction can be treated in this way.

Exercise 3b

Fill in the missing numbers to make equivalent fractions:

a $\frac{1}{5} = \frac{3}{\quad}$ **b** $\frac{1}{5} = \frac{\quad}{20}$

a If $\frac{1}{5} = \frac{3}{\quad}$ the numerator has been multiplied by 3 so we need to multiply the denominator by 3.

$$\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

b If $\frac{1}{5} = \frac{\quad}{20}$ the denominator has been multiplied by 4 so you need to multiply the numerator by 4.

$$\frac{1}{5} = \frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

In questions **1** to **18** fill in the missing numbers to make equivalent fractions.

1 $\frac{9}{10} = \frac{\ }{40}$

2 $\frac{1}{6} = \frac{3}{\ }$

3 $\frac{1}{3} = \frac{\ }{12}$

4 $\frac{2}{5} = \frac{6}{\ }$

5 $\frac{3}{7} = \frac{\ }{28}$

6 $\frac{9}{10} = \frac{90}{\ }$

7 $\frac{2}{9} = \frac{4}{\ }$

8 $\frac{3}{8} = \frac{\ }{80}$

9 $\frac{5}{11} = \frac{\ }{22}$

10 $\frac{4}{5} = \frac{8}{\ }$

11 $\frac{1}{10} = \frac{10}{\ }$

12 $\frac{2}{9} = \frac{\ }{36}$

13 $\frac{1}{10} = \frac{100}{\ }$

14 $\frac{2}{9} = \frac{20}{\ }$

15 $\frac{3}{8} = \frac{3000}{\ }$

16 $\frac{5}{11} = \frac{\ }{121}$

17 $\frac{4}{5} = \frac{400}{\ }$

18 $\frac{1}{10} = \frac{1000}{\ }$

Write $\frac{2}{3}$ as an equivalent fraction with denominator 24.

You need to write $\frac{2}{3}$ as $\frac{?}{24}$; 3×8 is 24 so you need to multiply the top and bottom of $\frac{2}{3}$ by 8.

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$$

19 Write each of the following fractions as an equivalent fraction with denominator 24:

a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **d** $\frac{3}{4}$ **e** $\frac{5}{12}$ **f** $\frac{3}{8}$

20 Write each of the following fractions in equivalent form with denominator 45:

a $\frac{2}{15}$ **b** $\frac{4}{9}$ **c** $\frac{3}{5}$ **d** $\frac{1}{3}$ **e** $\frac{14}{15}$ **f** $\frac{1}{5}$

21 Change each of the following fractions into an equivalent fraction with numerator 12:

a $\frac{1}{6}$ **b** $\frac{3}{4}$ **c** $\frac{6}{7}$ **d** $\frac{4}{5}$ **e** $\frac{2}{3}$ **f** $\frac{1}{2}$

22 Some of the following equivalent fractions are correct but two of them are wrong. Find the wrong ones and correct them by altering the numerator:

a $\frac{2}{5} = \frac{6}{15}$ **b** $\frac{2}{3} = \frac{4}{9}$ **c** $\frac{3}{7} = \frac{6}{14}$ **d** $\frac{4}{9} = \frac{12}{27}$ **e** $\frac{7}{10} = \frac{77}{100}$

Investigation

Using the numbers 1, 2, 4 and 8 write down all the fractions you can think of that are equal to or smaller than 1. Use a single number for the numerator and a single number for the denominator. You can use a number more than once in the same fraction.

1 **a** How many fractions can you find?

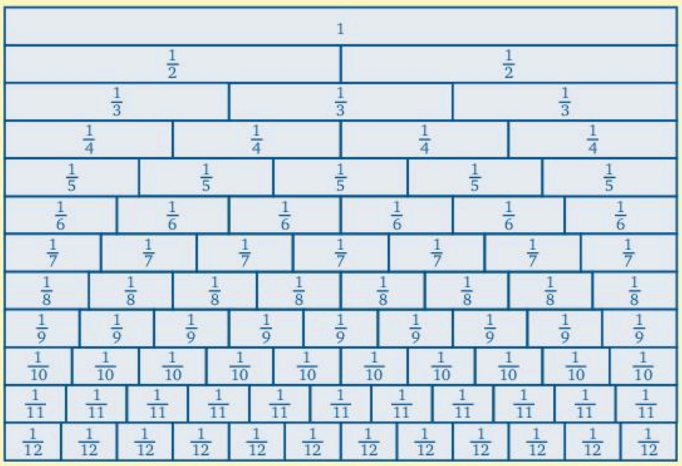
b Which fractions are equivalent fractions?

c What fraction does not have an equivalent fraction in the list you have written down?

2 Add 16 to the list of numbers 1, 2, 4, 8 and repeat part **1**.

3 Two-digit numbers, such as 14 and 82, can be made from the digits 1, 2, 4 and 8. Use such numbers to repeat part **1**.

This is a fraction wall. You can use it to check your answers for the questions in Exercise 3c.



Exercise 3c

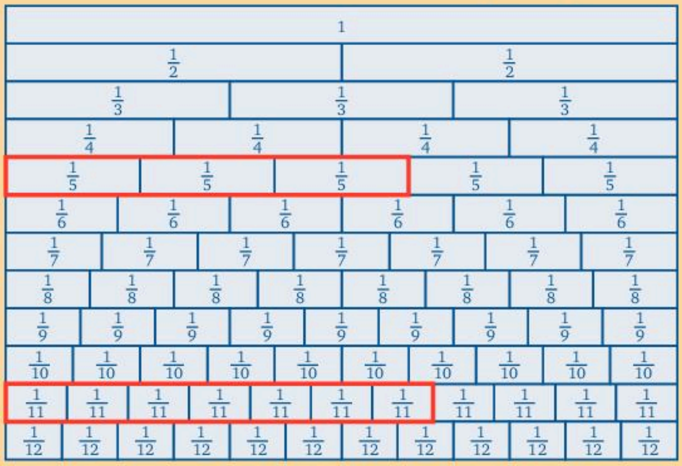
Which is the bigger fraction, $\frac{3}{5}$ or $\frac{7}{11}$?

You need to change $\frac{3}{5}$ and $\frac{7}{11}$ into equivalent fractions with the same denominator, so you need to find the *lowest common multiple* (*LCM*) of 5 and 11.

$\frac{3}{5} = \frac{33}{55}$ and $\frac{7}{11} = \frac{35}{55}$ (55 divides by 5 and by 11)

$\frac{35}{55} > \frac{33}{55}$ so $\frac{7}{11}$ is the bigger fraction.

A quick check on the fraction wall confirms that $\frac{7}{11}$ is bigger than $\frac{3}{5}$.



In questions **1** to **20** find which is the bigger fraction:

1 $\frac{1}{2}$ or $\frac{1}{3}$

2 $\frac{3}{4}$ or $\frac{5}{6}$

3 $\frac{2}{3}$ or $\frac{4}{5}$

4 $\frac{2}{9}$ or $\frac{1}{7}$

5 $\frac{2}{7}$ or $\frac{3}{8}$

6 $\frac{2}{5}$ or $\frac{3}{7}$

7 $\frac{5}{6}$ or $\frac{3}{5}$

8 $\frac{3}{8}$ or $\frac{1}{5}$

9 $\frac{4}{5}$ or $\frac{6}{7}$

10 $\frac{3}{5}$ or $\frac{4}{7}$

11 $\frac{1}{4}$ or $\frac{3}{11}$

12 $\frac{5}{7}$ or $\frac{3}{5}$

13 $\frac{3}{8}$ or $\frac{5}{11}$

14 $\frac{3}{10}$ or $\frac{4}{11}$

15 $\frac{1}{4}$ or $\frac{2}{7}$

16 $\frac{2}{9}$ or $\frac{3}{11}$

17 $\frac{5}{7}$ or $\frac{7}{9}$

18 $\frac{9}{11}$ or $\frac{7}{9}$

19 $\frac{2}{5}$ or $\frac{1}{3}$

20 $\frac{4}{7}$ or $\frac{3}{5}$

In questions **21** to **30**, put either > or < between the fractions:

21 $\frac{1}{4}$ $\frac{2}{7}$

22 $\frac{2}{3}$ $\frac{5}{8}$

23 $\frac{3}{7}$ $\frac{1}{2}$

24 $\frac{5}{8}$ $\frac{7}{10}$

25 $\frac{3}{10}$ $\frac{1}{4}$

26 $\frac{3}{5}$ $\frac{2}{3}$

27 $\frac{2}{9}$ $\frac{1}{5}$

28 $\frac{4}{9}$ $\frac{5}{11}$

29 $\frac{2}{11}$ $\frac{1}{7}$

30 $\frac{8}{11}$ $\frac{3}{4}$

You need to change $\frac{1}{4}$ and $\frac{2}{7}$ into equivalent fractions with the same denominator. Then you can see if $\frac{1}{4}$ is bigger or smaller than $\frac{2}{7}$.

Arrange these fractions in ascending order: $\frac{3}{4}, \frac{7}{10}, \frac{1}{2}, \frac{4}{5}$

$$\frac{3}{4} = \frac{15}{20}$$

$$\frac{7}{10} = \frac{14}{20}$$

$$\frac{1}{2} = \frac{10}{20} \quad \text{(20 divides by 4, 10, 2 and 5)}$$

$$\frac{4}{5} = \frac{16}{20}$$

So the ascending order is $\frac{1}{2}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}$.

Arrange the following fractions in ascending order:

31 $\frac{2}{3}, \frac{1}{2}, \frac{3}{5}, \frac{7}{30}$

32 $\frac{13}{20}, \frac{3}{4}, \frac{4}{10}, \frac{5}{8}$

33 $\frac{1}{3}, \frac{5}{6}, \frac{1}{2}, \frac{7}{12}$

34 $\frac{2}{5}, \frac{3}{8}, \frac{17}{20}, \frac{1}{2}, \frac{7}{10}$

35 $\frac{5}{7}, \frac{11}{14}, \frac{3}{4}, \frac{17}{28}, \frac{1}{2}$

36 $\frac{7}{10}, \frac{2}{5}, \frac{3}{5}, \frac{14}{25}, \frac{1}{2}$

Arrange the following fractions in descending order:

37 $\frac{5}{6}, \frac{1}{2}, \frac{7}{9}, \frac{11}{18}, \frac{2}{3}$

38 $\frac{13}{20}, \frac{3}{5}, \frac{1}{2}, \frac{3}{4}, \frac{7}{10}$

39 $\frac{7}{12}, \frac{1}{6}, \frac{2}{3}, \frac{17}{24}, \frac{3}{4}$

40 $\frac{7}{10}, \frac{11}{15}, \frac{2}{3}, \frac{23}{30}, \frac{4}{5}$

41 $\frac{7}{16}, \frac{1}{2}, \frac{5}{8}, \frac{19}{32}, \frac{3}{4}$

42 $\frac{4}{5}, \frac{7}{12}, \frac{5}{6}, \frac{1}{2}, \frac{3}{4}$

Simplifying fractions

Think of the way you find equivalent fractions. For example

$$\frac{2}{5} = \frac{2 \times 7}{5 \times 7} = \frac{14}{35}$$

Looking at this the other way round we see that

$$\frac{14}{35} = \frac{\cancel{7} \times 2}{\cancel{7} \times 5} = \frac{2}{5}$$

In the middle step, 7 is a factor of both the numerator and the denominator and it is called a *common factor*. To get the final value of $\frac{2}{5}$ we have 'crossed out' the common factor. What we have really done is to divide the top and the bottom by 7 and this *simplifies* the fraction.

When all the simplifying is finished we say that the fraction is in its *lowest terms*.

Exercise 3d

Simplify $\frac{66}{176}$ by dividing top and bottom by the common factors.

$$\frac{\overset{3}{\cancel{\overset{33}{\cancel{66}}}}}{\underset{8}{\cancel{\underset{88}{\cancel{176}}}}} = \frac{3}{8}$$

(We divided top and bottom by 2 and then by 11.)

Simplify the following fractions:

1 $\frac{2}{6}$

2 $\frac{30}{50}$

3 $\frac{3}{9}$

4 $\frac{6}{12}$

5 $\frac{12}{18}$

6 $\frac{10}{20}$

7 $\frac{8}{32}$

8 $\frac{8}{28}$

9 $\frac{16}{56}$

10 $\frac{10}{30}$

11 $\frac{36}{72}$

12 $\frac{15}{75}$

When you simplify, check that you have divided numerator and denominator by ALL possible common factors. For example, if you divide top and bottom of $\frac{6}{12}$ by 2, you are left with $\frac{3}{6}$; this has a common factor of 3 so will simplify further.

Adding and subtracting fractions

Suppose there is a bowl of oranges and apples. First you take three oranges and then two more oranges. You then have five oranges; we can add the 3 and the 2 together because they are the same kind of fruit. But three oranges and two apples cannot be added together because they are different kinds of fruit.

For fractions it is the denominator that tells us the kind of fraction, so we can add or subtract fractions if they have the same denominator but not while their denominators are different.

Exercise 3e

Find $\frac{2}{5} + \frac{1}{3}$

You need to change $\frac{2}{5}$ and $\frac{1}{3}$ into equivalent fractions with the same denominator.

5 and 3 both divide into 15 so choose 15 as the new *common denominator*.

$$\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

Find:

1 $\frac{2}{5}+\frac{1}{5}$ **3** $\frac{1}{5}+\frac{1}{6}$ **5** $\frac{3}{10}+\frac{2}{3}$ **7** $\frac{3}{7}+\frac{1}{6}$ **9** $\frac{1}{6}+\frac{2}{7}$ **11** $\frac{3}{11}+\frac{5}{9}$

2 $\frac{1}{7}+\frac{3}{7}$ **4** $\frac{2}{5}+\frac{3}{7}$ **6** $\frac{4}{7}+\frac{1}{8}$ **8** $\frac{2}{3}+\frac{2}{7}$ **10** $\frac{5}{6}+\frac{1}{7}$ **12** $\frac{2}{9}+\frac{3}{10}$

More than two fractions can be added in this way. The common denominator must be divisible by *all* of the original denominators.

Find $\frac{1}{8}+\frac{1}{2}+\frac{1}{3}$

(8, 2 and 3 all divide into 24)

$$\begin{aligned}\frac{1}{8}+\frac{1}{2}+\frac{1}{3} &= \frac{3}{24}+\frac{12}{24}+\frac{8}{24}\\ &= \frac{3+12+8}{24}\\ &= \frac{23}{24}\end{aligned}$$

Find:

13 $\frac{1}{5}+\frac{1}{4}+\frac{1}{2}$ 16 $\frac{5}{12}+\frac{1}{6}+\frac{1}{3}$ **19** $\frac{1}{2}+\frac{3}{8}+\frac{1}{10}$ **22** $\frac{2}{9}+\frac{2}{3}+\frac{1}{18}$

14 $\frac{1}{8}+\frac{1}{4}+\frac{1}{3}$ **17** $\frac{1}{7}+\frac{3}{14}+\frac{1}{2}$ **20** $\frac{1}{3}+\frac{2}{9}+\frac{1}{6}$ **23** $\frac{2}{15}+\frac{1}{10}+\frac{2}{5}$

15 $\frac{3}{10}+\frac{2}{5}+\frac{1}{4}$ **18** $\frac{1}{3}+\frac{1}{6}+\frac{1}{2}$ **21** $\frac{7}{20}+\frac{3}{10}+\frac{1}{5}$ **24** $\frac{1}{4}+\frac{1}{12}+\frac{1}{3}$

Find $\frac{7}{9}-\frac{1}{4}$

(The denominators are not the same so we use equivalent fractions with the LCM of the two denominators, 36.)

$$\begin{aligned}\frac{7}{9}-\frac{1}{4} &= \frac{28}{36}-\frac{9}{36}\\ &= \frac{28-9}{36}\\ &= \frac{19}{36} \quad \text{(This will not simplify.)}\end{aligned}$$

25 $\frac{7}{10}-\frac{2}{10}$ 27 $\frac{3}{4}-\frac{1}{5}$ **29** $\frac{5}{7}-\frac{2}{7}$ **31** $\frac{2}{3}-\frac{3}{7}$

26 $\frac{6}{17}-\frac{1}{17}$ **28** $\frac{9}{10}-\frac{1}{2}$ **30** $\frac{19}{20}-\frac{7}{20}$ **32** $\frac{4}{7}-\frac{1}{3}$

33 $\frac{11}{15}-\frac{4}{15}$	**36** $\frac{8}{13}-\frac{1}{2}$	**39** $\frac{5}{8}-\frac{2}{7}$	**42** $\frac{7}{12}-\frac{1}{3}$
34 $\frac{13}{18}-\frac{7}{18}$	**37** $\frac{11}{12}-\frac{5}{6}$	**40** $\frac{7}{15}-\frac{1}{5}$	**43** $\frac{13}{18}-\frac{5}{9}$
35 $\frac{7}{9}-\frac{2}{3}$	**38** $\frac{19}{100}-\frac{1}{10}$	**41** $\frac{3}{4}-\frac{5}{8}$	**44** $\frac{13}{15}-\frac{3}{5}$

Fractions can be added and subtracted in one problem in a similar way.

For example:

$$\frac{7}{9}+\frac{1}{18}-\frac{1}{6}=\frac{14}{18}+\frac{1}{18}-\frac{3}{18}$$

$$=\frac{14+1-3}{18}$$

$$=\frac{15-3}{18}$$

$$=\frac{\cancel{12}^{2}}{\cancel{18}_{3}}$$ This will simplify by dividing top and bottom by 6.

$$=\frac{2}{3}$$

It is not always possible to work from left to right in order because we have to subtract too much too soon. In this case we can do the adding first. Remember that it is the operation (i.e. add or subtract) *in front* of a number that tells you what to do with that number.

Exercise 3f

Find $\frac{1}{8}-\frac{3}{4}+\frac{11}{16}$

$$\frac{1}{8}-\frac{3}{4}+\frac{11}{16}=\frac{2}{16}-\frac{12}{16}+\frac{11}{16}=\frac{2}{16}+\frac{11}{16}-\frac{12}{16}$$

$$=\frac{2+11-12}{16}=\frac{13-12}{16}$$

$$=\frac{1}{16}$$

Find:

1 $\frac{3}{4}+\frac{1}{2}-\frac{7}{8}$

2 $\frac{6}{7}-\frac{9}{14}+\frac{1}{2}$

3 $\frac{3}{8}+\frac{7}{16}-\frac{3}{4}$

4 $\frac{11}{12}+\frac{1}{6}-\frac{2}{3}$

Remember that the operation (+ or –) in front of a number tells you what to do with that number only.

5 $\frac{3}{5}+\frac{3}{25}-\frac{27}{50}$

6 $\frac{2}{3}+\frac{1}{6}-\frac{5}{12}$

7 $\frac{4}{5}-\frac{7}{10}+\frac{1}{2}$

8 $\frac{7}{9}-\frac{2}{3}+\frac{5}{6}$

9 $\frac{7}{10}-\frac{41}{100}+\frac{1}{20}$

10 $\frac{5}{8}-\frac{21}{40}+\frac{2}{5}$

11 $\frac{7}{12}-\frac{1}{6}+\frac{1}{3}$

12 $\frac{2}{3}-\frac{7}{18}+\frac{2}{9}$

13 $\frac{2}{9}-\frac{1}{3}+\frac{1}{6}$

14 $\frac{1}{6}-\frac{2}{3}+\frac{7}{12}$

15 $\frac{2}{5}-\frac{1}{2}+\frac{3}{10}$

16 $\frac{1}{8}-\frac{13}{16}+\frac{3}{4}$

17 $\frac{1}{6}-\frac{5}{18}+\frac{1}{3}$

18 $\frac{1}{5}-\frac{7}{10}+\frac{17}{20}$

19 $\frac{1}{4}-\frac{5}{8}+\frac{1}{2}$

20 $\frac{2}{3}-\frac{5}{6}+\frac{1}{2}$

21 $\frac{3}{10}-\frac{61}{100}+\frac{1}{2}$

22 $\frac{1}{8}-\frac{7}{24}+\frac{5}{12}$

23 $\frac{1}{3}-\frac{5}{18}+\frac{2}{9}$

24 $\frac{3}{10}+\frac{2}{15}-\frac{2}{5}$

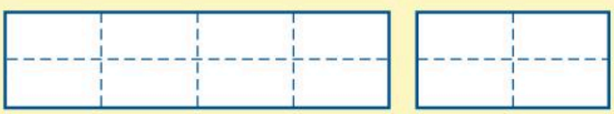
Remember that you can do the addition before you do the subtraction. For example, to find $\frac{4-6+3}{18}$, you can add 4 and 3 before taking 6 away.

Puzzle

An old lady went to market with a basket of eggs. To the first customer she sold half of what she had plus half an egg. The second customer bought half of what remained plus half an egg and the third customer bought half of what now remained and half an egg. That left the lady with thirty-six eggs. There were no broken eggs at any time. How many eggs did she have in her basket at the start?

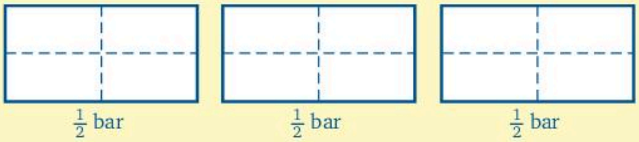
Mixed numbers and improper fractions

Most of the fractions we have met so far have been less than a whole unit. These are called *proper fractions*. But we often have more than a whole unit. Suppose, for instance, that we have one and a half bars of chocolate:



We have $1\frac{1}{2}$ bars, and $1\frac{1}{2}$ is called a *mixed number*.

Another way of describing the amount of chocolate is to say that we have three half-bars.



We have $\frac{3}{2}$ bars and $\frac{3}{2}$ is called an *improper fraction* because the numerator is bigger than the denominator.

But the amount of chocolate in the two examples is the same, so

$$\frac{3}{2} = 1\frac{1}{2}$$

Improper fractions can be changed into mixed numbers by finding out how many whole units there are. We know that three thirds $\left(\frac{3}{3}\right)$ is one whole, therefore

$$\frac{8}{3} = \frac{3}{3} + \frac{3}{3} + \frac{2}{3} \quad \text{(gives two wholes and two thirds)}$$
$$= 2\frac{2}{3}$$

Exercise 3g

In questions **1** to **8** change the improper fractions into mixed numbers:

1 $\frac{9}{4}$

2 $\frac{19}{4}$

3 $\frac{37}{6}$

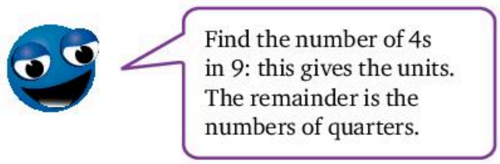
4 $\frac{53}{10}$

5 $\frac{41}{8}$

6 $\frac{127}{5}$

7 $\frac{114}{11}$

8 $\frac{109}{8}$



We can also change mixed numbers into improper fractions. For instance, in $2\frac{4}{5}$ we have two whole units and $\frac{4}{5}$. In each whole unit there are five fifths, so in $2\frac{4}{5}$ we have ten fifths and four fifths, i.e.

$$2\frac{4}{5} = \frac{10}{5} + \frac{4}{5} = \frac{14}{5}$$

Exercise 3h

Change $3\frac{1}{7}$ into an improper fraction.

$$\begin{aligned} 3\frac{1}{7} &= 3 + \frac{1}{7} \\ &= \frac{21}{7} + \frac{1}{7} \\ &= \frac{22}{7} \end{aligned}$$

In $3\frac{1}{7}$ we have 3 whole units and $\frac{1}{7}$. There are seven sevenths in 1 whole, 14 sevenths in 2 wholes and 21 sevenths in 3 wholes. So in $3\frac{1}{7}$ we have 21 sevenths plus one seventh. Altogether, we have 22 sevenths.

In questions **1** to **20** change the mixed numbers into improper fractions:

1 $4\frac{1}{3}$	**5** $8\frac{1}{7}$	**9** $3\frac{2}{3}$	**13** $3\frac{4}{5}$	**17** $1\frac{9}{10}$
2 $8\frac{1}{4}$	**6** $6\frac{3}{5}$	**10** $5\frac{1}{2}$	**14** $4\frac{7}{9}$	**18** $6\frac{2}{3}$
3 $1\frac{7}{10}$	**7** $2\frac{6}{7}$	**11** $7\frac{2}{5}$	**15** $8\frac{3}{4}$	**19** $7\frac{3}{8}$
4 $10\frac{8}{9}$	**8** $4\frac{1}{6}$	**12** $2\frac{4}{9}$	**16** $10\frac{3}{7}$	**20** $10\frac{1}{10}$

The meaning of 13 ÷ 5

$13 \div 5$ means 'how many fives are there in 13?'.

There are 2 fives in 13 with 3 left over, so $15 \div 5 = 2$, remainder 3.

Note that the remainder, 3, is $\frac{3}{5}$ of 5. Thus we can say that there are $2\frac{3}{5}$ fives in 13

i.e. $13 \div 5 = 2\frac{3}{5}$

But $\frac{13}{5} = 2\frac{3}{5}$ Therefore $13 \div 5$ and $\frac{13}{5}$ mean the same thing.

Exercise 3i

Find 27 ÷ 8

$$27 \div 8 = \frac{27}{8}$$

There are 3 eights in 27 with 3 left over: so there are 3 units and 3 eighths.

$$= 3\frac{3}{8}$$

Calculate the following divisions, giving your answers as mixed numbers:

1 $36 \div 7$

2 $59 \div 6$

3 $52 \div 11$

4 $20 \div 8$

5 $82 \div 5$

6 $29 \div 4$

7 $41 \div 3$

8 $64 \div 9$

9 $98 \div 12$

10 $107 \div 10$

11 $37 \div 5$

12 $52 \div 8$

Adding mixed numbers

If we want to find the value of $2\frac{1}{3}+3\frac{1}{4}$ we first convert the mixed numbers to improper fractions,

i.e. $2\frac{1}{3}+3\frac{1}{4}=\frac{7}{3}+\frac{13}{4}$

and then add the fractions.

This gives $\frac{28}{12}+\frac{39}{12}=\frac{67}{12}$.

Then convert the improper fraction to a mixed number: $\frac{67}{12}=5\frac{7}{12}$

Therefore $2\frac{1}{3}+3\frac{1}{4}=5\frac{7}{12}$

Exercise 3j

Find:

1 $2\frac{1}{4}+3\frac{1}{2}$

2 $1\frac{1}{2}+2\frac{1}{3}$

3 $4\frac{1}{5}+1\frac{3}{8}$

4 $5\frac{1}{9}+4\frac{1}{3}$

5 $3\frac{1}{4}+2\frac{5}{9}$

6 $1\frac{1}{3}+2\frac{5}{6}$

7 $3\frac{1}{4}+1\frac{1}{5}$

8 $2\frac{1}{7}+1\frac{1}{14}$

9 $6\frac{3}{10}+1\frac{2}{5}$

10 $8\frac{1}{7}+5\frac{2}{3}$

Convert $2\frac{1}{4}$ and $3\frac{1}{2}$ to improper fractions and then add them. If the answer is an improper fraction, convert it to a mixed number.

11 $7\frac{3}{8}+3\frac{7}{16}$

12 $1\frac{3}{4}+4\frac{7}{12}$

13 $3\frac{5}{7}+7\frac{1}{2}$

14 $6\frac{1}{2}+1\frac{9}{16}$

15 $8\frac{7}{8}+3\frac{3}{16}$

16 $9\frac{2}{3}+8\frac{5}{6}$

17 $2\frac{4}{5}+7\frac{3}{10}$

18 $6\frac{3}{10}+4\frac{4}{5}$

19 $1\frac{1}{4}+3\frac{2}{3}+6\frac{7}{12}$

20 $5\frac{1}{7}+4\frac{1}{2}+7\frac{11}{14}$

21 $4\frac{4}{5}+9\frac{4}{15}+1\frac{1}{3}$

22 $4\frac{3}{5}+8\frac{7}{10}+2\frac{1}{2}$

23 $3\frac{7}{10}+9\frac{21}{100}+1\frac{3}{5}$

24 $4\frac{1}{4}+7\frac{1}{8}+6\frac{1}{32}$

25 $1\frac{5}{7}+11\frac{1}{2}+9\frac{1}{14}$

Subtracting mixed numbers

If we want to find the value of $5\frac{3}{4}-2\frac{2}{5}$ we can use the same method as for adding:

$$\begin{aligned}5\frac{3}{4}-2\frac{2}{5} &= \frac{23}{4}-\frac{12}{5}\\ &= \frac{115-48}{20}\\ &= \frac{67}{20}\\ &= 3\frac{7}{20}\end{aligned}$$

Exercise 3k

Find:

1 $2\frac{3}{4}-1\frac{1}{8}$

2 $3\frac{2}{3}-1\frac{4}{5}$

3 $1\frac{5}{6}-\frac{2}{3}$

4 $3\frac{1}{4}-2\frac{1}{2}$

5 $7\frac{3}{4}-2\frac{1}{3}$

6 $3\frac{5}{6}-2\frac{1}{3}$

7 $2\frac{6}{7}-1\frac{1}{2}$

8 $4\frac{1}{2}-2\frac{1}{5}$

9 $5\frac{3}{4}-2\frac{1}{2}$

10 $8\frac{4}{5}-5\frac{1}{2}$

11 $5\frac{7}{9}-3\frac{5}{7}$

12 $4\frac{5}{8}-1\frac{1}{3}$

13 $6\frac{3}{4}-3\frac{6}{7}$

14 $7\frac{1}{2}-5\frac{3}{4}$

15 $4\frac{3}{5}-1\frac{1}{4}$

16 $7\frac{6}{7}-4\frac{3}{5}$

17 $2\frac{1}{2}-1\frac{3}{4}$

18 $5\frac{4}{7}-3\frac{4}{5}$

19 $3\frac{1}{4}-1\frac{7}{8}$

20 $5\frac{3}{5}-2\frac{9}{10}$

21 $4\frac{1}{6}-2\frac{2}{3}$

22 $6\frac{2}{3}-3\frac{5}{6}$

23 $7\frac{3}{4}-4\frac{7}{8}$

24 $9\frac{7}{10}-5\frac{4}{5}$

Puzzle

Fred is an old man. He lived one-eighth of his life as a boy, one-twelfth as a youth, one-half as a man and has spent 28 years in his old age. How old is Fred now?

Multiplying fractions

When fractions are multiplied the result is given by multiplying together the numbers in the numerator and also multiplying together the numbers in the denominator.

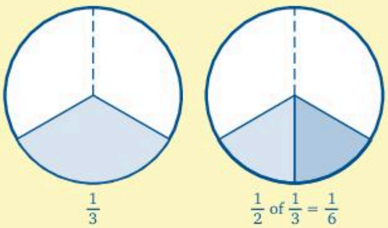
For example

$$\frac{1}{2}\times\frac{1}{3}=\frac{1\times 1}{2\times 3}$$
$$=\frac{1}{6}$$

If we look at a cake diagram we can see that $\frac{1}{2}$ of $\frac{1}{3}$ of the cake is $\frac{1}{6}$ of the cake.

So $\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$

and $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$



We see that 'of' means 'multiplied by'.

Simplifying

Sometimes we can simplify a product by dividing by the common factors.

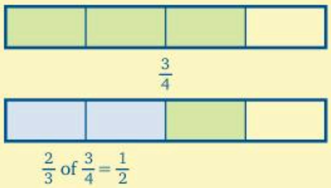
For example

$$\frac{2}{3}\times\frac{3}{4}=\frac{2\times 3}{3\times 4}=\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{3}^{1}}{\cancel{4}_{2}}=\frac{1\times 1}{1\times 2}$$
$$=\frac{1}{2}$$

You can see that 3 is a common factor of the numerator and denominator, so we can divide both by 3. Similarly 2 is a common factor of the numerator (2) and the denominator (4), so we can also divide both by 2.

The diagram shows that

$\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$



Exercise 3I

Find $\frac{4}{25}\times\frac{15}{16}$

$\frac{{}^{1}\cancel{4}}{{}_{5}\cancel{25}}\times\frac{\cancel{15}^{3}}{\cancel{16}_{4}}$ Divide numerator and denominator by 5, which is a common factor of 15 and 25, and by 4, which is a common factor of 4 and 16.

$=\frac{1\times 3}{5\times 4}=\frac{3}{20}$

Find:

1. $\frac{3}{4}\times\frac{1}{2}$
2. $\frac{2}{3}\times\frac{5}{7}$
3. $\frac{2}{5}\times\frac{1}{3}$
4. $\frac{1}{2}\times\frac{7}{8}$
5. $\frac{3}{4}\times\frac{4}{7}$
6. $\frac{4}{9}\times\frac{1}{7}$
7. $\frac{3}{7}\times\frac{2}{5}$
8. $\frac{2}{5}\times\frac{3}{5}$
9. $\frac{5}{6}\times\frac{1}{4}$
10. $\frac{2}{3}\times\frac{7}{9}$
11. $\frac{3}{4}\times\frac{1}{5}$
12. $\frac{1}{7}\times\frac{3}{5}$
13. $\frac{7}{8}\times\frac{4}{21}$
14. $\frac{3}{4}\times\frac{16}{21}$
15. $\frac{21}{22}\times\frac{11}{27}$
16. $\frac{8}{9}\times\frac{33}{44}$
17. $\frac{7}{9}\times\frac{3}{21}$
18. $\frac{3}{4}\times\frac{5}{7}$
19. $\frac{4}{5}\times\frac{15}{16}$
20. $\frac{10}{11}\times\frac{33}{35}$
21. $\frac{4}{15}\times\frac{25}{64}$
22. $\frac{2}{3}\times\frac{33}{40}$
23. $\frac{3}{7}\times\frac{28}{33}$
24. $\frac{48}{55}\times\frac{5}{12}$

Find:

25. $\frac{3}{7}\times\frac{5}{9}\times\frac{14}{15}$
26. $\frac{11}{21}\times\frac{30}{31}\times\frac{7}{55}$
27. $\frac{15}{16}\times\frac{8}{9}\times\frac{4}{5}$
28. $\frac{5}{7}\times\frac{3}{8}\times\frac{21}{30}$
29. $\frac{1}{2}\times\frac{7}{12}\times\frac{18}{35}$
30. $\frac{7}{11}\times\frac{8}{9}\times\frac{33}{28}$

Express as $\frac{3\times 5\times 14}{7\times 9\times 15}$ then look for common factors between the numerator and denominator.

Multiplying mixed numbers

Suppose that we want to find the value of $2\frac{1}{3}\times\frac{5}{21}\times 1\frac{1}{5}$.

We cannot multiply mixed numbers together unless we change them into improper fractions first. So we change $2\frac{1}{3}$ into $\frac{7}{3}$ and we change $1\frac{1}{5}$ into $\frac{6}{5}$.

Then we can use the same method as before, so

$$2\frac{1}{3}\times\frac{5}{21}\times 1\frac{1}{5}=\frac{7\times 5\times 6}{3\times 21\times 5}$$

Now look for common factors to simplify the fraction.

Exercise 3m

Find $2\frac{1}{3}\times\frac{5}{21}\times1\frac{1}{5}$

$$2\frac{1}{3}\times\frac{5}{21}\times1\frac{1}{5}=\frac{7}{3}\times\frac{5}{21}\times\frac{6}{5}$$

$$=\frac{\overset{1}{\cancel{7}}\times\overset{1}{\cancel{5}}\times\overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}}\times\underset{3}{\cancel{21}}\times\underset{1}{\cancel{5}}}$$

$$=\frac{2}{3}$$

Find:

1 $1\frac{1}{2}\times\frac{2}{5}$

2 $2\frac{1}{2}\times\frac{4}{5}$

3 $3\frac{1}{4}\times\frac{3}{13}$

4 $4\frac{2}{3}\times2\frac{2}{5}$

5 $2\frac{1}{5}\times\frac{5}{22}$

6 $1\frac{1}{4}\times\frac{2}{5}$

7 $3\frac{5}{7}\times1\frac{1}{13}$

8 $8\frac{1}{3}\times3\frac{3}{5}$

9 $2\frac{1}{10}\times7\frac{6}{7}$

10 $6\frac{3}{10}\times1\frac{4}{21}$

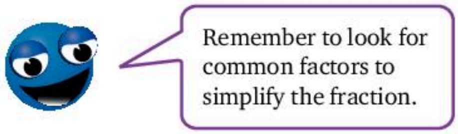
11 $4\frac{2}{7}\times2\frac{1}{10}$

12 $6\frac{1}{4}\times1\frac{3}{5}$

13 $3\frac{2}{3}\times1\frac{1}{5}\times1\frac{3}{22}$

14 $1\frac{1}{18}\times1\frac{4}{5}\times3\frac{1}{3}$

15 $4\frac{4}{5}\times1\frac{5}{8}\times3\frac{3}{4}$



A whole number can be written as a fraction with a denominator of 1. For instance $6=\frac{6}{1}$.

Doing this makes it easier to multiply a whole number by a fraction or a mixed number.

Find $6\times7\frac{1}{3}$

$$6\times7\frac{1}{3}=\frac{6}{1}\times\frac{22}{3}$$

$$=\frac{\overset{2}{\cancel{6}}\times22}{1\times\underset{1}{\cancel{3}}}\quad\text{(divide numerator and denominator by 3)}$$

$$=\frac{2\times22}{1\times1}=44$$

Find:

16 $5\times4\frac{3}{5}$

17 $2\frac{1}{7}\times14$

18 $3\frac{1}{8}\times4$

19 $4\frac{1}{6}\times9$

20 $18\times6\frac{1}{9}$

21 $4\times3\frac{3}{8}$

22 $3\frac{3}{5}\times10$

23 $2\frac{5}{6}\times3$

24 $5\frac{5}{7}\times21$

25 $3\times6\frac{1}{9}$

26 $1\frac{3}{4}\times8$

27 $28\times1\frac{4}{7}$

Fractions of quantities

Exercise 3n

Find three-fifths of 95 metres.

Remember that 'of' means 'multiply', so to find $\frac{3}{5}$ of 95 you have to multiply $\frac{3}{5}$ by 95.

This is the same as multiplying by $\frac{95}{1}$.

$$\frac{3}{5}\times\frac{95}{1}=\frac{3\times\overset{19}{\cancel{95}}}{\underset{1}{\cancel{5}}\times1}\quad\text{(dividing numerator and denominator by 5)}$$

$$=\frac{3\times19}{1\times1}$$

$$=57$$

$\frac{3}{5}$ of 95 metres is 57 metres.

Find:

1 $\frac{1}{3}$ of 18

2 $\frac{1}{5}$ of 30

3 $\frac{1}{7}$ of 21

4 $\frac{2}{3}$ of 24

5 $\frac{5}{7}$ of 14

<u>6</u> $\frac{1}{4}$ of 24

<u>7</u> $\frac{1}{6}$ of 30

<u>8</u> $\frac{1}{8}$ of 64

<u>9</u> $\frac{3}{5}$ of 20 metres

10 $\frac{5}{9}$ of 45 dollars

11 $\frac{9}{10}$ of 50 litres

12 $\frac{3}{8}$ of 88 miles

13 $\frac{7}{16}$ of 48 gallons

<u>14</u> $\frac{4}{9}$ of 18 metres

<u>15</u> $\frac{5}{8}$ of 16 dollars

<u>16</u> $\frac{4}{9}$ of 63 litres

17 $\frac{1}{4}$ of 2 m

18 $\frac{2}{9}$ of 36 cm

19 $\frac{3}{10}$ of \$100

20 $\frac{2}{7}$ of 42 cm

21 $\frac{4}{5}$ of 1 year (365 days)

<u>22</u> $\frac{3}{8}$ of 1 day (24 hours)

<u>23</u> $\frac{1}{7}$ of 1 week

<u>24</u> $\frac{1}{3}$ of \$9

Dividing by fractions

When we divide 6 by 3 we are finding how many threes there are in 6 and we say $6 \div 3 = 2$.

In the same way, when we divide 10 by $\frac{1}{2}$ we are finding how many halves there are in 10; we know that in 1 whole number there are 2 halves,

i.e. $\frac{1}{1} \div \frac{1}{2} = 2$ (1 divided by one-half equals 2)

But $\frac{1}{1} \times \frac{2}{1} = 2$ (1 times 2 equals 2)

So $\frac{1}{1} \div \frac{1}{2} = \frac{1}{1} \times \frac{2}{1}$ (therefore 1 divided by one-half gives the same result as 1 times 2)

Also, in 2 wholes there are 4 halves:

i.e. $\frac{2}{1} \div \frac{1}{2} = 4$ (2 divided by one-half equals 4)

and $\frac{2}{1} \times \frac{2}{1} = 4$ (2 times 2 equals 4)

So $\frac{2}{1} \div \frac{1}{2} = \frac{2}{1} \times \frac{2}{1} = 4$ (therefore 2 divided by one-half gives the same result as 2 times 2)

Continuing in this way, we see that in 10 wholes there are 20 halves,

i.e. $\frac{10}{1} \div \frac{1}{2} = 20$ (10 divided by one-half equals 20)

and $\frac{10}{1} \times \frac{2}{1} = 20$ (10 times 2 equals 20)

So $\frac{10}{1} \div \frac{1}{2} = \frac{10}{1} \times \frac{2}{1} = 20$ (therefore 10 divided by one-half gives the same result as 10 times 2)

From the above examples, we divided by a fraction by 'turning the fraction upside down' (inverting it) and then multiplying.

This rule holds for division by fractions.

To divide by a fraction we turn that fraction upside down and multiply.

Dividing by whole numbers and mixed numbers

If we want to divide 3 by 5 we can say

$$3 \div 5 = \frac{3}{1} \div \frac{5}{1} \qquad \text{(3 divided by 5)}$$

$$= \frac{3}{1} \times \frac{1}{5} \qquad \text{(3 times one-fifth)}$$

$$= \frac{3}{5} \qquad \text{(three-fifths)}$$

So $3 \div 5$ is the same as $\frac{3}{5}$. Similarly $7 \div 11$ is $\frac{7}{11}$.

Division with mixed numbers can be done as long as all the mixed numbers are first changed into improper fractions. For example if we want to divide $2\frac{1}{2}$ by $1\frac{1}{4}$ we first change $2\frac{1}{2}$ into $\frac{5}{2}$ and $1\frac{1}{4}$ into $\frac{5}{4}$. Then we can use the same method as before.

Exercise 3p

Find:

1 $8 \div \frac{4}{5}$

2 $18 \div \frac{6}{7}$

3 $40 \div \frac{8}{9}$

4 $72 \div \frac{8}{11}$

5 $28 \div \frac{14}{15}$

6 $44 \div \frac{4}{9}$

7 $27 \div \frac{9}{13}$

8 $36 \div \frac{4}{7}$

9 $34 \div \frac{17}{19}$

10 $\frac{21}{32} \div \frac{7}{8}$

11 $\frac{8}{75} \div \frac{4}{15}$

12 $\frac{15}{26} \div \frac{5}{13}$

Find the value of $2\frac{1}{2} \div 1\frac{1}{4}$.

$$2\frac{1}{2} \div 1\frac{1}{4} = \frac{5}{2} \div \frac{5}{4} \qquad \text{(Express each mixed number as an improper fraction.)}$$

$$= \frac{5}{2} \times \frac{4}{5} \qquad \text{(To divide by a fraction turn it upside down and multiply.)}$$

$$= \frac{20}{10} = 2$$

Give the answer to:

13 $5\frac{4}{9} \div \frac{14}{27}$

14 $3\frac{1}{8} \div 3\frac{3}{4}$

15 $7\frac{1}{5} \div 1\frac{7}{20}$

16 Divide $8\frac{1}{4}$ by $1\frac{3}{8}$

17 Divide $6\frac{2}{3}$ by $2\frac{4}{9}$

18 $4\frac{2}{7} \div \frac{9}{14}$

19 $6\frac{4}{9} \div 1\frac{1}{3}$

20 Divide $5\frac{1}{4}$ by $2\frac{11}{12}$

21 Divide $7\frac{1}{7}$ by $1\frac{11}{14}$

22 $10\frac{2}{3} \div 1\frac{7}{9}$

23 $8\frac{4}{5} \div 3\frac{3}{10}$

24 Divide $11\frac{1}{4}$ by $\frac{15}{16}$

25 $31\frac{1}{2} \div 5\frac{5}{8}$

26 $9\frac{3}{4} \div 1\frac{5}{8}$

27 $12\frac{1}{2} \div 8\frac{3}{4}$

28 Divide $10\frac{5}{6}$ by $3\frac{1}{4}$

29 Divide $8\frac{2}{3}$ by $5\frac{7}{9}$

30 $22\frac{2}{3} \div 1\frac{8}{9}$

Mixed multiplication and division

Suppose we want to find the value of an *expression* like $2\frac{1}{4} \times \frac{3}{14} \div 1\frac{2}{7}$.
Two things need to be done:

Step 1 If there are any mixed numbers, change them into improper fractions.

Step 2 Turn the fraction *after* the ÷ sign upside down and change ÷ into ×.

Then

$2\frac{1}{4} \times \frac{3}{14} \div 1\frac{2}{7} = \frac{9}{4} \times \frac{3}{14} \div \frac{9}{7}$ (mixed numbers changed to improper fractions)

$= \frac{\overset{1}{\cancel{9}}}{4} \times \frac{3}{\underset{2}{\cancel{14}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{9}}}$ ($\frac{9}{7}$ turned upside down and ÷ changed to ×)

$= \frac{3}{8}$

Mixed operations

When brackets are placed round a pair of fractions it means that we have to work out what is *inside* the brackets before doing anything else.

For example
$$\left(\frac{1}{2}+\frac{1}{4}\right) \times \frac{5}{7} = \left(\frac{2+1}{4}\right) \times \frac{5}{7}$$
$$= \frac{3}{4} \times \frac{5}{7}$$
$$= \frac{15}{28}$$

If we meet an expression in which +, −, × and ÷ occur, we need to know the order in which to do the calculations. We use the same rule for fractions as we used for *mixed operations* with whole numbers, that is

Brackets first, then Multiply and Divide, then Add and Subtract.

You may remember this order from the sentence

Bless My Dear Aunt Sally.

Exercise 3q

Find:

1 $\frac{5}{8}\times1\frac{1}{2}\div\frac{15}{16}$

2 $2\frac{3}{4}\times\frac{5}{6}\div\frac{11}{12}$

3 $\frac{2}{3}\times1\frac{1}{5}\div\frac{12}{25}$

4 $\frac{4}{7}\times\frac{8}{9}\div\frac{16}{21}$

5 $\frac{2}{5}\times\frac{9}{10}\div\frac{27}{40}$

6 $\frac{3}{4}\times2\frac{1}{3}\div\frac{21}{32}$

7 $3\frac{2}{5}\times\frac{4}{5}\div\frac{8}{15}$

8 $\frac{3}{7}\times2\frac{1}{2}\div\frac{10}{21}$

9 $\frac{3}{5}\times\frac{9}{11}\div\frac{18}{55}$

10 $\frac{1}{4}\times\frac{11}{12}\div\frac{22}{27}$

11 $\frac{3}{7}\times\frac{2}{5}\div\frac{8}{21}$

12 $\frac{14}{25}\times\frac{5}{9}\div\frac{7}{18}$

Find $\frac{2}{5}-\frac{1}{2}\times\frac{3}{5}+\frac{1}{10}$

$\frac{2}{5}-\frac{1}{2}\times\frac{3}{5}+\frac{1}{10}=\frac{2}{5}-\frac{3}{10}+\frac{1}{10}$ (do the multiplication first)

$=\frac{4-3+1}{10}$ (use the LCM to find the common denominator, then add and subtract)

$=\frac{2}{10}$ (simplify)

$=\frac{1}{5}$

Calculate:

13 $\frac{1}{2}+\frac{1}{4}\times\frac{2}{5}$

14 $\frac{2}{3}\times\frac{1}{2}+\frac{1}{4}$

15 $\frac{4}{5}-\frac{3}{10}\div\frac{1}{2}$

16 $\frac{2}{7}\div\frac{2}{3}-\frac{3}{14}$

17 $\frac{4}{5}+\frac{3}{10}\times\frac{2}{9}$

18 $\frac{1}{3}-\frac{1}{2}\times\frac{1}{4}$

19 $\frac{3}{4}\div\frac{1}{2}+\frac{1}{8}$

20 $\frac{1}{7}+\frac{5}{8}\div\frac{3}{4}$

21 $\frac{5}{6}\times\frac{3}{10}-\frac{3}{16}$

Work out the value inside the brackets first.

22 $\frac{3}{7}-\frac{1}{4}\times\frac{8}{21}$

23 $\left(\frac{4}{9}-\frac{1}{3}\right)\times\frac{6}{7}$

24 $\frac{3}{5}\times\left(\frac{2}{3}+\frac{1}{2}\right)$

25 $\frac{7}{8}\div\left(\frac{3}{4}+\frac{2}{3}\right)$

26 $\left(\frac{3}{10}+\frac{2}{5}\right)\div\frac{7}{15}$

27 $\left(\frac{5}{11}-\frac{1}{3}\right)\times\frac{3}{8}$

28 $\frac{3}{8}\div\left(\frac{2}{3}+\frac{1}{4}\right)$

29 $\left(\frac{4}{7}+\frac{1}{3}\right)\div3\frac{4}{5}$

30 $\frac{5}{9}\times\left(\frac{2}{3}-\frac{1}{6}\right)$

31 $\left(\frac{6}{11}-\frac{1}{2}\right)\div\frac{3}{4}$

32 $\frac{9}{10}\div\left(\frac{1}{6}+\frac{2}{3}\right)$

33 $\frac{1}{6}\times\left(\frac{2}{3}-\frac{1}{2}\right)\div\frac{7}{12}$

34 $\frac{7}{10}\div\left(\frac{2}{5}+\frac{4}{15}\times\frac{3}{5}\right)$

35 $\left(2\frac{1}{4}+\frac{3}{8}\right)\times\frac{2}{3}-1\frac{1}{2}$

36 $1\frac{3}{11}-\frac{6}{7}\times1\frac{5}{9}+\frac{13}{33}$

37 $\frac{5}{8}\times\left(\frac{4}{9}-\frac{1}{6}\right)\div1\frac{9}{16}$

38 $\frac{2}{9}+\left(\frac{6}{7}\div\frac{3}{4}\right)\times3\frac{1}{2}$

39 $1\frac{1}{10}\times\frac{23}{24}\div\left(\frac{3}{5}+\frac{1}{6}\right)$

40 $2\frac{2}{5}-\frac{7}{10}\times\left(\frac{4}{7}-\frac{1}{3}\right)$

41 $\frac{5}{9}\div\left(1\frac{1}{3}+\frac{4}{9}\right)+\frac{3}{8}$

42 $1\frac{2}{9}+\left(\frac{5}{6}-\frac{3}{4}\div4\frac{1}{2}\right)$

State whether each of the following statements is true or false:

43 $\frac{1}{2}\times\frac{2}{3}+\frac{1}{3}=\frac{1}{3}+\frac{1}{3}$

44 $\frac{1}{3}\times\frac{3}{4}+\frac{1}{4}=\frac{1}{3}\times1$

45 $\frac{1}{4}\div\frac{3}{4}+\frac{1}{2}=\frac{1}{3}+\frac{1}{2}$

46 $\frac{1}{3}+\frac{2}{3}\times\frac{1}{4}=\frac{1}{3}+\frac{1}{6}$

47 $\frac{1}{2}+\frac{1}{4}\div\frac{1}{2}=\frac{3}{4}\times\frac{2}{1}$

48 $\frac{3}{4}-\frac{1}{2}\times\frac{2}{3}=\frac{1}{4}\times\frac{2}{3}$

49 $\frac{2}{3}-\frac{1}{4}+\frac{1}{2}=\frac{2}{3}+\frac{1}{2}-\frac{1}{4}$

50 $\frac{3}{5}\times\frac{2}{3}+\frac{1}{2}=\frac{3}{5}+\frac{1}{2}\times\frac{2}{3}$

51 $\frac{4}{7}-\frac{1}{4}\div\frac{1}{3}=\left(\frac{4}{7}-\frac{1}{4}\right)\div\frac{1}{3}$

52 $\frac{3}{8}\div\frac{1}{4}-\frac{1}{4}=\frac{3}{8}\times\frac{4}{1}-\frac{1}{4}$

Problems

Exercise 3r

If Jane can iron a shirt in $4\frac{3}{4}$ minutes, how long will it take her to iron 10 shirts?

You know the time to iron 1 shirt. It will take 10 times as long to iron 10 shirts.

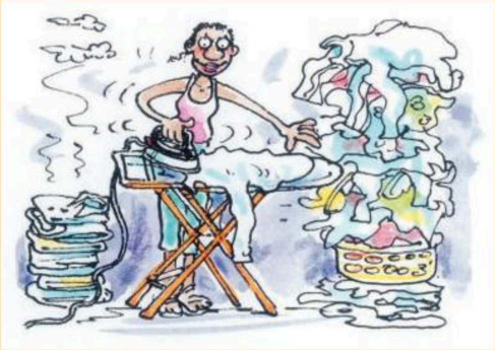
Time to iron 1 shirt $= 4\frac{3}{4}$ minutes

Time to iron 10 shirts $= 4\frac{3}{4} \times 10$ minutes

$= \frac{19}{4_2} \times \frac{10^5}{1}$ minutes

$= \frac{95}{2}$ minutes

$= 47\frac{1}{2}$ minutes



A piece of string of length $22\frac{1}{2}$ cm is to be cut into small pieces each $\frac{3}{4}$ cm long. How many pieces can be obtained?

Number of small pieces = length of string ÷ length of one short piece

You want to find the number of $\frac{3}{4}$ cm pieces in $22\frac{1}{2}$ cm so you need to find $22\frac{1}{2} \div \frac{3}{4}$.

Number of small pieces $= 22\frac{1}{2} \div \frac{3}{4}$

(Express $22\frac{1}{2}$ as an improper fraction and multiply by $\frac{3}{4}$ turned upside down.)

$= \frac{{}^{15}45 \times 4^{2}}{{}_{1}2 \times 3_{1}} = 30$

Thus 30 pieces can be obtained.

1 A bag of flour weighs $1\frac{1}{2}$ kilograms. What is the weight of 20 bags?

2 A cook adds $3\frac{1}{2}$ cups of water to a stew. If the cup holds $\frac{1}{10}$ of a litre how many litres of water were added?

3 My journey to school starts with a walk of $\frac{1}{2}$ km to the bus stop, then a bus ride of $2\frac{1}{5}$ km followed by a walk of $\frac{3}{10}$ km. How long is my journey to school?

4 It takes $3\frac{1}{4}$ minutes for a cub scout to clean a pair of shoes. If he cleans 18 pairs of shoes to raise money for charity, how long does he spend on the job?

5 A burger bar chef cooks some beefburgers and piles them one on top of the other. If each burger is $9\frac{1}{2}$ mm thick and the pile is 209 mm high, how many did he cook?

6 If you read 30 pages of a book in $\frac{3}{4}$ of an hour, how many minutes does it take to read each page?

Mixed exercises

Exercise 3s

1 Calculate:

a $\frac{2}{3}+\frac{4}{7}$

b $\frac{5}{6}-\frac{3}{8}$

c $\frac{3}{8}+\frac{1}{9}$

d $1\frac{1}{2}+\frac{2}{3}$

e $2\frac{1}{4}-1\frac{1}{3}$

2 Simplify:

a $\frac{54}{24}$

b $3\frac{15}{75}$

3 Write the first quantity as a fraction of the second quantity:

a 3 days; 1 week

b 17 children; 30 children

4 Write the following fractions in ascending size order:

a $\frac{1}{2}, \frac{7}{10}, \frac{3}{5}, \frac{13}{20}$

b $\frac{3}{4}, \frac{7}{12}, \frac{5}{6}, \frac{2}{3}$

c $\frac{3}{5}, \frac{7}{10}, \frac{17}{20}, \frac{71}{100}$

5 Write either > or < between the following pairs of fractions, to make true statements:

a $\frac{5}{12}\quad\frac{7}{16}$

b $\frac{3}{8}\quad\frac{7}{24}$

c $\frac{13}{22}\quad\frac{19}{33}$

6 A cricket club consists of 7 members who are good batsmen only, 5 who are good bowlers only, 4 all-rounders and some non-players. If there are 22 people in the club, what fraction of them are

a non-players

b good batsmen only?

7 Find how many times $2\frac{1}{4}$ goes into $13\frac{1}{2}$.

8 What is $\frac{7}{9}$ of $1\frac{1}{14}$?

9 Find $\frac{3}{5}+1\frac{1}{2}\times\frac{7}{10}$.

10 Arrange the following fractions in ascending order of size: $\frac{7}{10}, \frac{3}{5}, \frac{2}{3}$.

11 Find:

a $4\frac{1}{7} \times 4\frac{2}{3}$ **b** $3\frac{3}{8} \div 2\frac{1}{4}$.

12 What is $\frac{3}{4}$ of $\frac{8}{9}$ added to $1\frac{1}{2}$?

13 Find $\left(1\frac{7}{8} + 2\frac{1}{4}\right) \times 1\frac{5}{11}$.

14 What is $\frac{2}{7}$ of 1 hour 3 minutes (in minutes)?

15 Find $7\frac{1}{5} - 4\frac{1}{8} \div 1\frac{1}{4}$.

16 Fill in the missing numbers:

a $\frac{7}{9} = \frac{21}{}$ **b** $\frac{10}{11} = \frac{}{44}$

17 Express as mixed numbers:

a $\frac{13}{5}$ **b** $\frac{31}{8}$ **c** $\frac{27}{5}$

18 State whether the following statements are true or false:

a $\frac{4}{11} > \frac{3}{10}$ **b** $\frac{3}{7}$ of $5 = \frac{3}{7} \times \frac{5}{1}$ **c** $2\frac{1}{7} = \frac{7}{15}$

Exercise 3t

1 Calculate:

a $\frac{4}{5} - \frac{2}{3}$ **c** $\frac{7}{11} - \frac{1}{2}$ **e** $3\frac{1}{6} - 2\frac{2}{3}$

b $\frac{9}{10} + \frac{4}{5}$ **d** $2\frac{1}{3} + 4\frac{1}{4}$ **f** $5\frac{1}{4} - 2\frac{3}{5}$

2 Simplify:

a $\frac{84}{96}$ **b** $\frac{77}{42}$ **c** $\frac{84}{91}$

3 Write the first quantity as a fraction of the second quantity:

a 13c; \$1 **b** 233 days; 1 leap year

4 Write either $<$ or $>$ between the following pairs of fractions:

a $\frac{13}{20}\ \frac{7}{15}$ **b** $\frac{5}{9}\ \frac{11}{18}$ **c** $\frac{5}{6}\ \frac{7}{8}$

5 Write the following fractions in ascending size order:

a $\frac{7}{20}, \frac{3}{8}, \frac{2}{5}, \frac{3}{10}$ **b** $\frac{2}{5}, \frac{7}{15}, \frac{3}{10}, \frac{1}{2}$ **c** $\frac{9}{16}, \frac{3}{4}, \frac{5}{8}, \frac{17}{32}$

6 In a class of 28 children, 13 live in houses with gardens, 7 live in houses without gardens and the rest live in apartments. What fraction of the children

a do not live in houses with gardens **b** live in apartments?

Exercise 3u

1 Find:

a $4\frac{1}{2}\times 3\frac{1}{3}$ b $3\frac{2}{5}\div\frac{3}{10}$

2 Find:

a $\frac{8}{9}+\frac{21}{27}$ b $2\frac{1}{3}+\frac{4}{9}+1\frac{5}{6}$

3 Put > or < between the following pairs of numbers:

a $\frac{4}{7}\quad\frac{5}{8}$ b $\frac{11}{9}\quad 1\frac{3}{10}$

4 Calculate:

a $5\frac{1}{4}-1\frac{2}{3}\div\frac{2}{5}$ b $3\frac{3}{8}\times\left(8\frac{1}{2}-5\frac{5}{6}\right)$

5 Arrange in ascending order: $\frac{7}{15},\frac{1}{3},\frac{2}{5}$.

6 What is $1\frac{1}{2}$ subtracted from $\frac{2}{3}$ of $5\frac{1}{4}$?

7 Find:

a $4\frac{1}{2}\times 3\frac{2}{3}-10\frac{1}{4}$ b $3\frac{1}{2}\div\left(2\frac{1}{8}-\frac{3}{4}\right)$

8 What is $1\frac{2}{3}$ of 1 minute 15 seconds (in seconds)?

9 Fill in the missing numbers:

a $\frac{4}{5}=\frac{\quad}{30}$ b $\frac{2}{7}=\frac{6}{\quad}$

10 Express as mixed numbers:

a $\frac{25}{8}$ b $\frac{49}{9}$ c $\frac{37}{6}$

11 A car travels $5\frac{1}{4}$ km north, then $2\frac{1}{2}$ km west and finally $4\frac{3}{8}$ km north. What is the total distance travelled (in kilometres)? What fraction of the journey was travelled in a northerly direction?

12 A man can paint a door in 1 hour 15 minutes. How many similar doors can he paint in $7\frac{1}{2}$ hours?

Did you know?

Did you know that the almanac writer Benjamin Banneker (1731–1806) was the son of a freed slave? He taught himself mathematics and chemistry. It is said that he became a long time correspondent of Thomas Jefferson.

In this chapter you have seen that…

- ✔ you can find a fraction equivalent to a given fraction by multiplying the numerator and denominator by the same number
- ✔ you can simplify fractions by dividing the numerator and denominator by their common factors
- ✔ you can add and subtract fractions by changing them to equivalent fractions with the same denominator
- ✔ you can find one quantity as a fraction of another by first making sure that they are in the same units, then by placing the first quantity over the second
- ✔ 'of' means 'multiplied by'
- ✔ you can multiply and divide whole numbers by fractions, by writing the whole number over 1
- ✔ to multiply fractions by fractions you multiply the numerators together and you multiply the denominators together
- ✔ to divide by a fraction you turn the fraction upside down and multiply
- ✔ you can multiply and divide with mixed numbers by turning the mixed numbers into improper fractions.

4 Decimals

At the end of this chapter you should be able to...

1 write a given number in expanded form – under the headings hundreds, tens, units, etc.
2 write a decimal number as a fraction and vice versa
3 add and subtract decimal numbers
4 multiply and divide decimal numbers by 10, 100, 1000, . . .
5 multiply and divide decimal numbers by whole numbers
6 solve problems using operations on decimal numbers
7 multiply two decimal numbers
8 write a given fraction as a recurring decimal
9 write a number correct to a given number of decimal places
10 divide by a decimal number
11 change a fractional number to a decimal number
12 order a set of numbers by size.

Did you know?

Theano, wife of Pythagoras, and two of her daughters were members of Pythagoras' mathematical school that included women who supported each other.

She wrote a biography of her husband, and it is believed that she and her daughters were responsible for attaching his name to a theorem which was well known years before his time.

You need to know...

✔ how to add and subtract whole numbers
✔ how to do short and long division
✔ how to multiply whole numbers
✔ how to multiply fractions together
✔ the meaning of decimals.

Key words

decimal, denominator, equilateral triangle, fraction, numerator, product, quadrilateral, rectangle, recurring decimal, regular pentagon, triangle

Decimals

The next exercise revises the meaning of place values in *decimals*.

Exercise 4a

Write the following numbers in headed columns:

	tens	units		tenths	hundredths
34.62 =	3	4	.	6	2

	units		tenths	hundredths	thousandths	ten-thousandths
0.0207 =	0	.	0	2	0	7

1 2.6
2 32.1
3 6.03
4 0.09
5 101.3
6 0.000 07
7 1.046
8 12.001
9 6.34
10 0.604
11 15.045
12 0.0092

Changing decimals to fractions

Exercise 4b

Write the following decimals as *fractions* in their lowest terms (using mixed numbers where necessary):

	units		tenths	
0.6 =	0	.	6	$= \frac{6}{10}$ Now simplify.
				$= \frac{3}{5}$

	tens	units		tenths	hundredths	
12.04 =	1	2	.	0	4	$= 12\frac{4}{100}$ Now simplify the fraction.
						$= 12\frac{1}{25}$

1	0.2	**4**	0.0007	**7**	0.7	**10**	1.7
2	0.06	**5**	0.001	**8**	2.01	**11**	15.5
3	1.3	**6**	6.4	**9**	1.8	**12**	8.06

Write 0.302 as a fraction.

		units		tenths	hundredths	thousandths	
0.302	=	0	.	3	0	2	$= \frac{3}{10} + \frac{2}{1000}$

Now write these with a common denominator. $= \frac{300}{1000} + \frac{2}{1000}$

$= \frac{302}{1000}$ Now simplify.

$= \frac{151}{500}$

You can miss out the first two steps and go straight to one fraction.

Write as fractions:

13	0.73	**16**	0.0029	**19**	0.071	**22**	0.63
14	0.081	**17**	0.000 67	**20**	0.3001	**23**	0.031
15	0.207	**18**	0.17	**21**	0.0207	**24**	0.47

Write as fractions in their lowest terms:

25	0.25	**28**	0.0305	**31**	0.35	**34**	0.125
26	0.072	**29**	0.15	**32**	0.0016	**35**	0.48
27	0.38	**30**	0.025	**33**	0.044	**36**	0.625

Addition and subtraction of decimals

To add decimals you can write them in columns. It is important to keep the decimal points in line.

		tens	units		tenths	
$4.2 + 13.1 = 17.3$			4	.	2	2 tenths + 1 tenth = 3 tenths
	+	1	3	.	1	
		1	7	.	3	
$5.3 + 6.8 = 12.1$			5	.	3	3 tenths + 8 tenths = 11 tenths
	+		6	.	8	= 1 unit and 1 tenth
		1	2	.	1	
		1	1			

The headings above the digits need not be written as long as we know what they are and the decimal points are in line (including the invisible point after a whole number, e.g. $4 = 4.0$).

Exercise 4c

Find $3 + 1.6 + 0.032 + 2.0066$

Write the numbers in a column, keeping the decimal points in line.

$$\begin{array}{r} 3 \\ 1.6 \\ 0.032 \\ +\,2.0066 \\ \hline 6.6386 \\ \hline \end{array}$$

$3 + 1.6 + 0.032 + 2.0066 = 6.6386$

Find:

1 $7.2 + 3.6$

2 $6.21 + 1.34$

3 $0.013 + 0.026$

4 $3.87 + 0.11$

5 $4.6 + 1.23$

6 $13.14 + 0.9$

7 $4 + 3.6$

8 $9.24 + 3$

9 $3.6 + 0.08$

10 $7.2 + 0.32 + 1.6$

11 $0.0043 + 0.263$

12 $0.002 + 2.1$

13 $0.000\,52 + 0.001\,24$

14 $0.068 + 0.003 + 0.06$

15 $4.62 + 0.078$

16 $0.32 + 0.032 + 0.0032$

17 $4.6 + 0.0005$

18 $16.8 + 3.9$

19 $1.62 + 2.078 + 3.1$

20 $7.34 + 6 + 14.034$

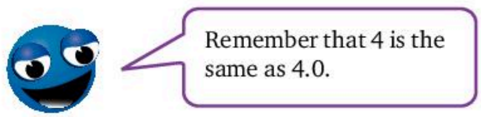
21 Add 0.68 to 1.7.

22 Find the sum of 3.28 and 14.021.

23 To 7.9 add 4 and 3.72.

24 Evaluate $7.9 + 0.62 + 5$.

25 Find the sum of 8.6, 5 and 3.21.



Subtraction also may be done by writing the numbers in columns, making sure that the decimal points are in line.

Find $24.2 - 13.7$

$$\begin{array}{r} 24.2 \\ -\,13.7 \\ \hline 10.5 \\ \hline \end{array}$$

$24.2 - 13.7 = 10.5$

Find:

26 $6.8 - 4.3$

27 $9.6 - 1.8$

28 $32.7 - 14.2$

29 $0.62 - 0.21$

30 $0.0342 - 0.0021$

31 $17.23 - 0.36$

32 $3.273 - 1.032$

33 $0.262 - 0.071$

34 $102.6 - 31.2$

35 $7.32 - 0.67$

36 $54.07 - 12.62$

37 $7.063 - 0.124$

Exercise 4d

It may be necessary to add zeros so that there is the same number of digits after the point in both cases.

Find 4.623 – 1.7 Fill 'empty' places with zeros.

$$\begin{array}{r} 4.623 \\ -\ 1.700 \\ \hline 2.923 \\ \hline \end{array}$$

4.623 – 1.7 = 2.923

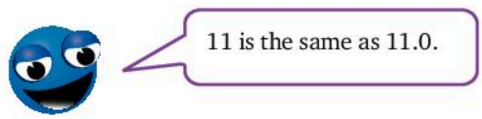
Find 4.63 – 1.0342 Fill 'empty' places with zeros.

$$\begin{array}{r} 4.6300 \\ -\ 1.0342 \\ \hline 3.5958 \\ \hline \end{array}$$

4.63 – 1.0342 = 3.5958

Find:

1 3.26 – 0.2
2 3.2 – 0.26
3 14.23 – 11.1
4 6.8 – 4.14
5 11 – 8.6
6 7.98 – 0.098
7 7.098 – 0.98
8 3.2 – 0.428
9 11.2 – 0.0026
10 0.000 32 – 0.000 123
11 0.0073 – 0.0006
12 0.0073 – 0.006
13 0.006 – 0.000 73
14 0.06 – 0.000 73
15 6 – 0.73
16 6 – 0.073
17 7.3 – 0.06
18 730 – 0.6
19 0.73 – 0.000 06
20 0.73 – 0.6
21 Take 19.2 from 76.8.
22 Subtract 1.9 from 10.2.
23 From 0.168 subtract 0.019.
24 Evaluate 7.62 – 0.81.



Find the value of:

25 8.62 + 1.7
26 8.62 – 1.7
27 3.8 – 0.82
28 0.08 + 0.32 + 6.2
29 100 + 0.28
30 100 – 0.28
31 0.26 + 0.026
32 0.26 – 0.026
33 38.2 + 1.68
34 38.2 – 1.68
35 0.84 + 2 + 200
36 16 + 1.6 + 0.16
37 0.02 – 0.013
38 0.062 + 0.32
39 6.83 – 0.19
40 17.2 + 20 + 1.62

Problems

Exercise 4e

The distance all round this *triangle* is 6.5 cm.

What is the length of the third side?

As the distance round the three sides is 6.5 cm, you can find the third side by adding the lengths of the two sides that you know, then take the result from 6.5 cm.

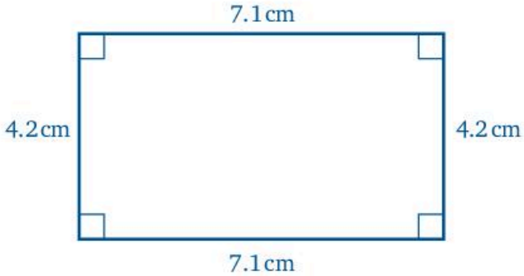
1.6 cm + 2.3 cm = 3.9 cm

$$\begin{array}{r} 1.6 \\ +\,2.3 \\ \hline 3.9 \end{array}$$

The length of the third side is 6.5 cm − 3.9 cm = 2.6 cm

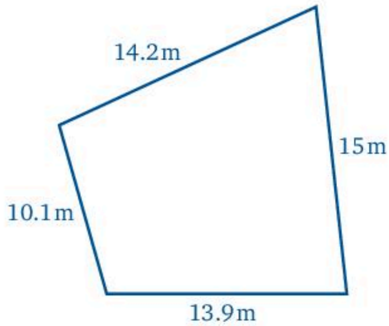
$$\begin{array}{r} 6.5 \\ -\,3.9 \\ \hline 2.6 \end{array}$$

1. A tape is to be placed round this *rectangle*. Find the length of tape required.



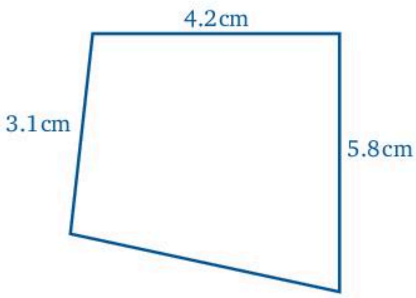
2. A piece of webbing is 7.6 m long. If 2.3 m is cut off, how much is left?
3. Find the total bill for three articles costing \$5, \$6.52 and \$13.25.
4. The bill for two books came to \$28.48. One book cost \$7.44. What was the cost of the other one?
5. Add 2.32 and 0.68 and subtract the result from 4.

6 The diagram shows the measurements of the sides of a field. Find the length of fencing required to enclose the field.



7 The bill for three meals was \$30. The first meal cost \$7.15 and the second \$13.60. What was the cost of the third?

8 The total distance round the sides of this *quadrilateral* is 19 cm. What is the length of the fourth side?



Multiplication and division by 10, 100, 1000, ...

Consider 0.2×10. When multiplied by 10, tenths become units $\left(\frac{1}{10} \times 10 = 1\right)$, so

units		tenths				units
0	.	2	$\times$	10	$=$	2

The digit has moved one place value to the left.

Multiplying by 100 means multiplying by 10 and then by 10 again, so the digits move 2 place values to the left.

	tens	units		tenths	hundredths	thousandths		
		0	.	4	2	6	$\times$	100
$=$	4	2	.	6				

Notice that the digits move to the left while the point stays put, but without headings it looks as though the digits stay put and the point moves to the right.

When necessary we fill in an empty space with a zero.

units		tenths				hundreds	tens	units
4	.	2	×	100	=	4	2	0

When we divide by 10, hundreds become tens and tens become units.

hundreds	tens	units		tens	units
6	4	0	÷ 10 =	6	4

The digits move one place value to the right and the number becomes smaller but it looks as though the decimal point moves to the left so

$2.72 \div 10 = 0.272$

To divide by 100 the point is moved two places to the left.
To divide by 1000 the point is moved three places to the left.

Exercise 4f

Find the value of:

a 368×100 $\qquad 368 \times 100 = 36\,800$

b 3.68×10 $\qquad 3.68 \times 10 = 36.8$

c 3.68×1000 $\qquad 3.68 \times 1000 = 3680$

Find the value of:

1 72×1000

2 8.24×10

3 0.0024×100

4 46×10

5 32.78×100

6 $0.043 \times 10\,000$

7 0.0602×100

8 3.206×10

9 72.81×1000

10 $0.000\,006\,3 \times 10$

11 $0.007\,03 \times 100$

12 $0.0374 \times 10\,000$

Find the value of:

a $3.2 \div 10$ $\qquad$ **b** $320 \div 10\,000$

a $3.2 \div 10 = 0.32$

b The units become ten-thousandths, the tens become thousandths, and so on.

$320 \div 10\,000 = 0.0320 = 0.032$

The final zero can be omitted because it doesn't affect the value of anything.

Find the value of:

13 $277.2 \div 100$
14 $76.26 \div 10$
15 $0.000\,24 \div 10$
16 $1.4 \div 100$
17 $27 \div 10$
18 $6.8 \div 100$
19 $0.26 \div 10$
20 $15.8 \div 1000$
21 $426 \div 10\,000$
22 $13.4 \div 10$
23 $3.74 \div 1000$
24 $0.92 \div 100$

Find:

25 $1.6 \div 10$
26 1.6×10
27 0.078×100
28 $0.078 \div 100$
29 14.2×100
30 0.068×100
31 $1.63 \div 100$
32 $2 \div 1000$
33 $140 \div 1000$
34 $7.8 \times 10\,000$
35 $24 \div 100$
36 0.063×1000
37 0.32×10
38 $7.9 \div 100$
39 $0.000\,78 \times 100$
40 $2.4 \div 10$
41 11.1×1000
42 $0.038 \div 100$
43 $0.38 \div 100$
44 $3.8 \times 100\,000$
45 $0.024 \div 100$
46 $0.3 \div 100\,000$
47 0.0041×1000
48 0.1004×100

49 Share 42 m of string equally amongst 10 people.

50 Find the total cost of 100 articles at $1.52 each.

51 Evaluate $13.8 \div 100$ and 13.8×100.

52 Multiply 1.6 by 100 and then divide the result by 1000.

53 Add 16.2 and 1.26 and divide the result by 100.

54 Take 9.6 from 13.4 and divide the result by 1000.

Division by whole numbers

We can see that

units		tenths			units		tenths
0	.	6	÷ 2	=	0	.	3

because 6 tenths ÷ 2 = 3 tenths. So we may divide by a whole number using the same layout as we do with whole numbers as long as we keep the digits in the correct columns and the points are in line.

Exercise 4g

Find the value of:

1 $0.4 \div 2$
2 $3.2 \div 2$
3 $0.63 \div 3$
4 $7.8 \div 3$
5 $0.9 \div 9$
6 $0.95 \div 5$
7 $0.672 \div 3$
8 $26.6 \div 7$
9 $42.6 \div 2$
10 $7.53 \div 3$
11 $6.56 \div 4$
12 $0.75 \div 5$

It may sometimes be necessary to fill spaces with zeros.

0.000 36 ÷ 3

3)0.000 36
0.000 12

0.000 36 ÷ 3 = 0.000 12

0.45 ÷ 5

5)0.45
0.09

0.45 ÷ 5 = 0.09

6.12 ÷ 3

3)6.12
2.04

6.12 ÷ 3 = 2.04

13	0.057 ÷ 3	**19**	1.62 ÷ 2	**25**	0.0076 ÷ 4	**31**	0.038 ÷ 2
14	0.000 65 ÷ 5	**20**	4.24 ÷ 4	**26**	0.81 ÷ 9	**32**	4.62 ÷ 6
15	0.008 72 ÷ 4	**21**	1.232 ÷ 4	**27**	0.5215 ÷ 5	**33**	14.749 ÷ 7
16	0.168 ÷ 4	**22**	0.6552 ÷ 6	**28**	0.000 075 ÷ 5	**34**	1.86 ÷ 3
17	0.012 ÷ 6	**23**	0.0285 ÷ 5	**29**	6.3 ÷ 7	**35**	0.222 ÷ 6
18	0.000 36 ÷ 6	**24**	0.1359 ÷ 3	**30**	0.0636 ÷ 6	**36**	6.24 ÷ 8

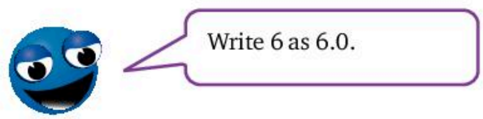
It may be necessary to add zeros at the end of a number in order to finish the division.

2.9 ÷ 8

5 2 4
8)2.9000
0.3625

2.9 ÷ 8 = 0.3625

Find the value of:



37	6 ÷ 5	**44**	7 ÷ 4				
38	7.4 ÷ 4	**45**	9.1 ÷ 2				
39	0.83 ÷ 2	**46**	0.000 31 ÷ 2	**51**	2.6 ÷ 5	**56**	3.014 ÷ 5
40	0.9 ÷ 6	**47**	9.4 ÷ 4	**52**	7.62 ÷ 4	**57**	6.83 ÷ 8
41	3.6 ÷ 5	**48**	0.062 ÷ 5	**53**	13 ÷ 5	**58**	14.7 ÷ 6
42	0.0002 ÷ 5	**49**	0.5 ÷ 4	**54**	0.3 ÷ 6	**59**	2.3 ÷ 4
43	7.1 ÷ 8	**50**	0.31 ÷ 8	**55**	0.01 ÷ 4	**60**	0.446 ÷ 8

If we divide 7.8 m of tape equally amongst 5 people, how long a piece will they each have?

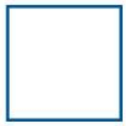
We need to divide 7.8 m into 5 equal lengths, so we need to find 7.8 ÷ 5.

Length of each piece = 7.8 ÷ 5 m

= 1.56 m

$$5\overline{)7.80}$$
$$1.56$$

61 The total distance round the sides of a square is 14.6 cm. What is the length of a side?



62 Divide 32.6 m into 8 equal parts.

63 Share 14.3 kg equally between 2 people.

64 The total distance round the sides of a *regular pentagon* (a five-sided figure with all the sides equal) is 16 cm. What is the length of one side?

65 Share $36 equally amongst 8 people.

We can also use long division. The decimal point is used only in the original number and the answer, not in the lines of working below this.

Exercise 4h

Find 2.56 ÷ 16

2.56 ÷ 16 = 0.16

$$\begin{array}{r} 0.16 \\ 16\overline{)2.56} \\ 1\,6 \\ \hline 96 \\ 96 \\ \hline \end{array}$$

4.2 ÷ 25

4.2 ÷ 25 = 0.168

$$\begin{array}{r} 0.168 \\ 25\overline{)4.200} \\ 2\,5 \\ \hline 1\,70 \\ 1\,50 \\ \hline 200 \\ 200 \\ \hline \end{array}$$

Find the value of:

1	$26.4 \div 24$	7	$0.0615 \div 15$	13	$35.52 \div 111$	19	$20.79 \div 99$
2	$2.1 \div 14$	8	$0.864 \div 24$	14	$7.28 \div 28$	20	$0.01426 \div 20$
3	$1.56 \div 13$	9	$8.48 \div 16$	15	$1.296 \div 54$	21	$23.4 \div 45$
4	$9.45 \div 21$	10	$5.2 \div 20$	16	$0.00805 \div 35$	22	$71.76 \div 23$
5	$11.22 \div 22$	11	$7.84 \div 14$	17	$54.4 \div 17$	23	$39.48 \div 47$
6	$80 \div 25$	12	$25.2 \div 36$	18	$21.93 \div 51$	24	$0.2556 \div 45$

Multiplication with decimals

We can calculate the *product* 0.2×0.3 by first changing the decimals to fractions.

Then $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$ and $\frac{6}{100} = 0.06$.

Therefore $0.2 \times 0.3 = 0.06$

Similarly $0.04 \times 0.6 = \frac{4}{100} \times \frac{6}{10} = \frac{24}{1000}$ and $\frac{24}{1000} = 0.024$

In the example above, if we add together the number of digits (including zeros) after the decimal points in the original two numbers, we get the number of digits after the point in the answer.

The number of digits after the point is called the number of decimal places.

In the first example, 0.2 has one decimal place and 0.3 has one decimal place. The answer, 0.06, has two decimal places, which is the sum of one and one.

In the second example, 0.04 has two decimal places and 0.6 has one decimal place. The answer, 0.024, has three decimal places, which is the sum of two and one.

We can use this fact to work out 0.3×0.02 without using fractions. Multiply 3 by 2 ignoring the decimal points; count up the number of decimal places after the decimal points and then put the point in the correct position in the answer, writing in zeros where necessary, i.e. $0.3 \times 0.02 = 0.006$.

Any zeros that come after the point must be included when counting the decimal places.

Exercise 4i

Find 0.08×0.4

(First ignore the decimal points; just multiply the numbers together, i.e. $8 \times 4 = 32$. Now count the number of decimal places in each of the two numbers you are multiplying together. Adding them gives the number of places in the answer, counting back from the right-hand digit. Sometimes you have to put a zero in too, because there aren't enough decimal places.)

0.08	×	0.4	=	0.032	$8 \times 4 = 32$
(2 places)		(1 place)		(3 places)	

Find 6×0.002

6	×	0.002	=	0.012	$6 \times 2 = 12$
(0 places)		(3 places)		(3 places)	

Calculate the following products:

1	0.6×0.3	**7**	0.5×0.07	**13**	0.07×12
2	0.04×0.06	**8**	8×0.6	**14**	4×0.009
3	0.009×2	**9**	0.08×0.08	**15**	0.9×9
4	0.07×0.008	**10**	3×0.0006	**16**	0.0008×11
5	0.12×0.09	**11**	0.7×0.06	**17**	7×0.011
6	0.07×0.0003	**12**	9×0.08	**18**	0.04×7

Zeros appearing in the multiplication in the middle or at the right-hand end must also be considered when counting the places.

0.252×0.4

0.252	×	0.4	=	0.1008
(3 places)		(1 place)		(4 places)

$$\begin{array}{r} 252 \\ \times \quad 4 \\ \hline 1008 \\ \hline \end{array}$$

2.5×6

2.5	×	6	=	15.0
(1 place)		(0 places)		(1 place)

$$\begin{array}{r} 25 \\ \times \quad 6 \\ \hline 150 \\ \hline \end{array}$$

300×0.2

300	$\times$	0.2	=	60.0	$300 \times 2 = 600$
(0 places)		(1 place)		(1 place)	

Calculate the following products:

19	0.751×0.2	**<u>27</u>**	320×0.07	**<u>35</u>**	4×1.6
20	3.2×0.5	**<u>28</u>**	0.4×0.0055	**<u>36</u>**	5×0.016
21	0.35×4	**<u>29</u>**	0.5×0.06	**<u>37</u>**	$0.000\,04 \times 0.000\,16$
22	1.52×0.0006	**<u>30</u>**	0.04×0.352	**<u>38</u>**	$16\,000 \times 0.05$
<u>23</u>	400×0.6	**31**	1.6×0.4	**<u>39</u>**	0.16×4
<u>24</u>	31.5×2	**32**	1.6×0.5	**<u>40</u>**	0.0016×5
<u>25</u>	5.6×0.02	**33**	160×0.004	**<u>41</u>**	0.072×0.6
<u>26</u>	0.008×256	**34**	0.16×0.005	**<u>42</u>**	310×0.04

Find 0.26×1.3

0.26	$\times$	1.3	=	0.338
(2 places)		(1 place)		(3 places)

$$\begin{array}{r} 26 \\ \times 13 \\ \hline 78 \\ 260 \\ \hline 338 \\ \hline \end{array}$$

Calculate the following products:

43	4.2×1.6	**<u>49</u>**	13.2×2.5	**55**	14.4×4.5	**<u>61</u>**	0.28×0.28
44	52×0.24	**<u>50</u>**	0.0082×0.034	**56**	1.36×0.082	**<u>62</u>**	0.34×0.31
45	0.68×0.14	**<u>51</u>**	17.8×420	**57**	0.081×0.032	**<u>63</u>**	14×0.123
46	48.2×26	**<u>52</u>**	3.2×37	**58**	1.6×1.6	**<u>64</u>**	1.9×9.1
<u>47</u>	310×1.4	**<u>53</u>**	39×0.23	**<u>59</u>**	0.16×16	**<u>65</u>**	8.2×2.8
<u>48</u>	1.68×0.27	**<u>54</u>**	0.264×750	**<u>60</u>**	0.0016×1600	**<u>66</u>**	0.047×0.66

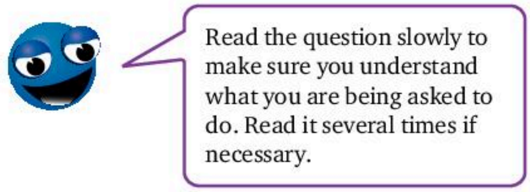
Find the cost of 6 books at \$2.35 each.
The total cost is equal to the cost of 1 book multiplied by the number of books.

$\therefore$ Cost = \$2.35 × 6
= \$14.10

$$\begin{array}{r} 235 \\ \times \quad 6 \\ \hline 1410 \\ \hline \end{array}$$

67 Find the cost of 10 articles at \$32.50 each.

68 The total distance round the sides of a square is 17.6 cm.
Find the length of one side of the square.



69 Divide 26.6 kg into 7 equal parts.

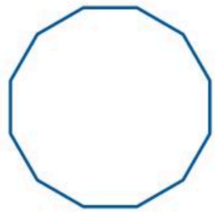
70 Find the total distance round the sides of a square of side 4.2 cm.

71 Find the cost of 62 notebooks at 68 c each, first in cents and then in dollars.

72 Multiply 3.2 by 0.6 and divide the result by 8.

73 If 68.25 m of ribbon is divided into 21 equal pieces, how long is each piece?

74 The length of a side of a regular twelve-sided polygon (a shape with 12 equal sides) is 4.2 m.
Find the total distance round the sides of the polygon.



Investigation

If we divide 2 by 5 we get an exact answer, namely 0.4.

However, if we divide 2 by 3 the answer is 0.666 666 6… going on forever, i.e. $2 \div 3$ does not give an exact decimal. We say that 6 recurs. Some of the numbers we get when we divide one whole number by another are interesting and are worth investigating.

(You should not use a calculator for this work before you have used pencil and paper methods.)

1 Find $0.4 \div 7$. Continue working until you have at least 12 non-zero digits in your answer. Use a calculator to check the first 8 non-zero digits of your answer. What do you notice about the pattern of digits in your answer?

2 Now try $0.2 \div 7$. Do you get the same digits in the same order?
How does this answer differ from the answer you got in part **1**?

3 Can you find any other decimals which, when divided by 7, give the same pattern of digits, but start in a different place in the pattern?

4 Now try any numbers larger than 1 that 7 does not divide into exactly.
For example $1.5 \div 7$ and $2.2 \div 7$.

5 Is it true to say that every decimal, when divided by 7, gives either an exact answer, or a recurring answer that involves the same cycle of digits?

6 Repeat parts **1** to **5** but divide by 9 instead of 7.

7 What happens if you divide by the other odd prime numbers less than 10?

8 What happens if you divide by the even numbers less than 10?

Changing fractions to decimals (exact values)

We may think of $\frac{3}{4}$ as $3 \div 4$ and hence write it as a decimal.

Exercise 4j

Express $\frac{3}{4}$ as a decimal.

$\frac{3}{4} = 3 \div 4 = 0.75$

$$4\overline{)3.00}$$
$$0.75$$

Express the following fractions as decimals:

1 $\frac{1}{4}$

2 $\frac{3}{8}$

3 $\frac{3}{5}$

4 $\frac{3}{25}$

5 $\frac{1}{25}$

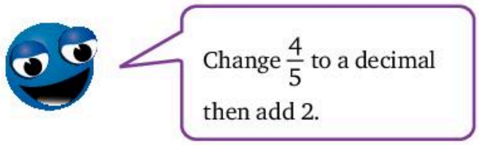
6 $2\frac{4}{5}$

7 $\frac{5}{8}$

8 $\frac{7}{16}$

9 $\frac{3}{25}$

10 $\frac{1}{32}$



Recurring decimals

Consider the calculation

$3 \div 4 = 0.75$ $\qquad 4\overline{)3.00}$ $\quad 0.75$

By adding two zeros after the point we are able to finish the division and give an exact answer. Now consider

$2 \div 3 = 0.666\ldots$ $\qquad 3\overline{)2.0000\ldots}$ $\quad 0.6666\ldots$

We can see that we will continue to obtain 6s for ever, also written as ..., and we say that the 6 *recurs*.

Consider

$31 \div 11 = 2.8181\ldots$ $\qquad 11\overline{)31.0000}$ $\quad 2.8181$

Here 81 recurs.

Sometimes it is one digit which is repeated and sometimes it is a group of digits. If one digit or a group continues to *recur* we have a *recurring decimal*.

Exercise 4k

Calculate $0.2 \div 7$

$0.2 \div 7 = 0.028\,571\,428\,571\,4\ldots$

$7\overline{)0.200\,000\,000\,000\,000\ldots}$
$0.028\,571\,428\,571\,428\ldots$

Calculate:

1. $1.4 \div 6$
2. $0.03 \div 11$
3. $4 \div 7$
4. $0.43 \div 3$
5. $0.03 \div 7$
6. $1.1 \div 9$

Express $\frac{4}{3}$ as a decimal.

$\frac{4}{3} = 4 \div 3 = 1.333\ldots$

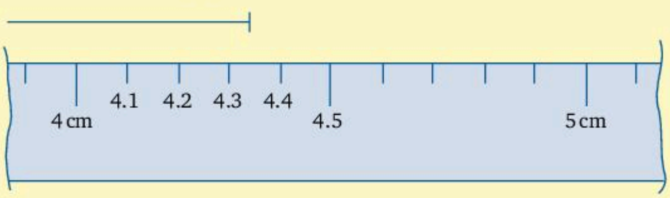
$3\overline{)4.00}$
1.33

Express the following fractions as decimals:

7. $\frac{4}{9}$
8. $\frac{2}{3}$
9. $\frac{2}{11}$
10. $\frac{5}{7}$
11. $\frac{7}{9}$
12. $\frac{8}{7}$

Correcting to a given number of decimal places

Often we need to know only the first few digits of a decimal. For instance, if we measure a length with an ordinary ruler we usually need an answer to the nearest $\frac{1}{10}$ cm and are not interested, or cannot see, how many $\frac{1}{100}$ cm are involved.



Look at this enlarged view of the end of a line which is being measured. We can see that with a more accurate measure we might be able to give the length as 4.34 cm. However on the given ruler we would probably measure it as 4.3 cm because we can see that the end of the line is nearer 4.3 than 4.4. We cannot give the exact length of the line but we can say that it is 4.3 cm long to the nearest $\frac{1}{10}$ cm. We write this as 4.3 cm correct to 1 decimal place.

Consider the numbers 0.62, 0.622, 0.625, 0.627 and 0.63. To compare them we write 0.62 as 0.620 and 0.63 as 0.630 so that each number has 3 digits after the point. When we write them in order in a column:

0.620

0.622

0.625

0.627

0.630

we can see that 0.622 is nearer to 0.620 than to 0.630 while 0.627 is nearer to 0.630 so we write

$0.62\vdots 2 = 0.62$ (correct to 2 decimal places)

$0.62\vdots 7 = 0.63$ (correct to 2 decimal places)

It is not so obvious what to do with 0.625 as it is halfway between 0.62 and 0.63. To save arguments, if the digit after the cut-off line is 5 or more we add 1 to the digit before the cut-off line, i.e. we round the number *up*, so we write

$0.62\vdots 5 = 0.63$ (correct to 2 decimal places)

Exercise 4I

Give 10.9315 correct to

a the nearest whole number **b** 1 decimal place **c** 3 decimal places.

a $10.\vdots 9315 = 11$ (correct to the nearest whole number)

b $10.9\vdots 315 = 10.9$ (correct to 1 decimal place)

c $10.931\vdots 5 = 10.932$ (correct to 3 decimal places)

Give 4.699 and 0.007 correct to 2 decimal places.

$4.69\vdots 9 = 4.70$ (correct to 2 decimal places)

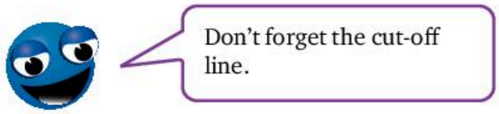
$0.00\vdots 7 = 0.01$ (correct to 2 decimal places)

Give the following numbers correct to the nearest whole number:

1	13.9	**3**	26.5	**5**	4.45	**7**	109.7	**9**	74.09
2	6.34	**4**	2.78	**6**	6.783	**8**	6.145	**10**	3.9999

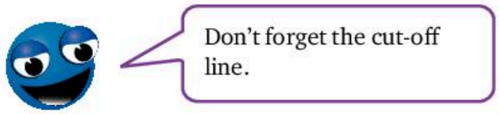
Give the following numbers correct to 3 decimal places:

11	0.3627	**16**	0.0843
12	0.026 234	**17**	0.084 47
13	0.007 14	**18**	0.3251
14	0.0695	**19**	0.032 51
15	0.000 98	**20**	3.9999



Give the following numbers correct to the number of decimal places indicated in the brackets:

21	1.784	(1)	**26**	1.639	(2)
22	42.64	(1)	**27**	1.639	(1)
23	1.0092	(2)	**28**	1.689	(nearest whole number)
24	0.009 42	(4)	**29**	3.4984	(2)
25	0.7345	(3)	**30**	3.4984	(1)



Find 4.28 ÷ 6 giving your answer correct to 2 decimal places.

$4.28 \div 6 = 0.71|3\ldots$

$= 0.71$ (correct to 2 decimal places)

```
6)4.280
  0.713...
```

Calculate 302 ÷ 14 correct to 1 decimal place.

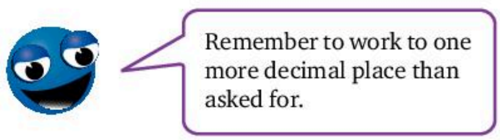
$302 \div 14 = 21.5|7\ldots$

$= 21.6$ (correct to 1 decimal place)

```
      21.57...
  14)302.00
     28
      22
      14
       8 0
       7 0
       1 00
         98
```

Calculate, giving your answers correct to 2 decimal places:

31	0.496 ÷ 3	**36**	2.35 ÷ 15		
32	6.49 ÷ 7	**37**	0.68 ÷ 16		
33	3.12 ÷ 9	**38**	0.99 ÷ 21		
34	12.2 ÷ 6	**39**	5.68 ÷ 24	**41**	1.73 ÷ 8
35	25.68 ÷ 9	**40**	3.85 ÷ 101	**42**	48.4 ÷ 51



Calculate, giving your answers correct to 1 decimal place:

43	$32.9 \div 8$	**46**	$9.76 \div 11$	**49**	$45 \div 21$	**52**	$8.4 \div 13$
44	$402 \div 7$	**47**	$124 \div 17$	**50**	$15.1 \div 16$	**53**	$26 \div 15$
45	$15.3 \div 6$	**48**	$16.2 \div 14$	**51**	$213 \div 22$	**54**	$519 \div 19$

Find, giving your answers correct to 3 decimal places:

55	$0.023 \div 4$	**58**	$0.23 \div 11$	**61**	$0.654 \div 23$	**64**	$0.321 \div 17$
56	$0.123 \div 7$	**59**	$0.45 \div 12$	**62**	$0.98 \div 32$	**65**	$1.26 \div 32$
57	$1.25 \div 3$	**60**	$0.012 \div 13$	**63**	$0.2584 \div 16$	**66**	$0.88 \div 24$

Give $\frac{4}{7}$ as a decimal correct to 3 decimal places.

$\frac{4}{7} = 4 \div 7 = 0.5714$

$= 0.571$ (correct to 3 decimal places)

$$7\overline{)4.0000}$$
$$0.5714$$

(This is an approximate answer.)

Give the following fractions as decimals correct to 3 decimal places:

67	$\frac{3}{7}$	**71**	$\frac{9}{11}$	**75**	$\frac{1}{3}$	**79**	$\frac{6}{13}$	**83**	$\frac{4}{15}$
68	$\frac{4}{9}$	**72**	$\frac{6}{7}$	**76**	$\frac{4}{11}$	**80**	$\frac{4}{21}$	**84**	$\frac{7}{18}$
69	$\frac{1}{6}$	**73**	$\frac{8}{7}$	**77**	$\frac{3}{14}$	**81**	$\frac{3}{19}$	**85**	$\frac{3}{22}$
70	$\frac{2}{3}$	**74**	$\frac{1}{9}$	**78**	$\frac{4}{17}$	**82**	$\frac{3}{17}$	**86**	$\frac{4}{33}$

Division by decimals

$0.012 \div 0.06$ can be written as $\frac{0.012}{0.06}$. We know how to divide by a whole number so we need to find an equivalent fraction with *denominator* 6 instead of 0.06. Now $0.06 \times 100 = 6$. Therefore we multiply the *numerator* and denominator by 100.

$$\frac{0.012}{0.06} = \frac{0.012 \times 100}{0.06 \times 100} = \frac{1.2}{6}$$
$$= 0.2$$

To divide by a decimal, the denominator must be made into a whole number but the numerator need not be. We can write, for short,

$$0.012 \div 0.06 = \frac{0.01\vdots 2}{0.06\vdots} = \frac{1.2}{6} \quad \text{(keeping the points in line)}$$

The dashed line indicates where we want the point to be so as to make the denominator a whole number.

Exercise 4m

Find $0.024 \div 0.6$

$$0.024 \div 0.6 = \frac{0.0\vdots 24}{0.6\vdots} = \frac{0.24}{6} = 0.04$$

(See that the decimal points are lined up one beneath the other. Draw a line through the fraction where you want the decimal point to be.)

$6\overline{)0.24}$
$\quad 0.04$

Find $64 \div 0.08$

$$64 \div 0.08 = \frac{64.00\vdots}{0.08\vdots} = \frac{6400}{8} = 800$$

(Multiplying the top and bottom by 100 makes the denominator a whole number.)

$8\overline{)6400}$
$\quad 800$

Find the exact answers to the following questions:

1 $0.04 \div 0.2$

2 $0.0006 \div 0.03$

3 $4 \div 0.5$

4 $0.8 \div 0.04$

5 $90 \div 0.02$

6 $0.48 \div 0.04$

7 $0.032 \div 0.2$

8 $3.6 \div 0.6$

9 $3.6 \div 0.06$

10 $3 \div 0.6$

11 $6.5 \div 0.5$

12 $8.4 \div 0.07$

13 $72 \div 0.09$

14 $1.08 \div 0.003$

15 $0.0012 \div 0.1$

16 $0.009 \div 0.9$

17 $0.9 \div 0.009$

18 $0.92 \div 0.4$

19 $16.8 \div 0.8$

20 $0.001\,32 \div 0.11$

21 $0.000\,068\,4 \div 0.04$

22 $20.8 \div 0.0004$

23 $0.0012 \div 0.3$

24 $4.8 \div 0.08$

25 $1.76 \div 2.2$

26 $144 \div 0.16$

27 $0.496 \div 1.6$

28 $0.0288 \div 0.18$

29 $34.3 \div 1.4$

30 $10.24 \div 3.2$

31 $0.0204 \div 0.017$

32 $102.5 \div 2.5$

33 $9.8 \div 1.4$

34 $0.168 \div 0.14$

35 $1.35 \div 0.15$

36 $0.192 \div 2.4$

Find the value of $16.9 \div 0.3$ giving your answer correct to 1 decimal place.

$$16.9 \div 0.3 = \frac{16.9}{0.3} = \frac{169}{3}$$

$$= 56.33\ldots$$

$$= 56.3 \quad \text{(correct to 1 decimal place)}$$

$$3\overline{)169.00}$$
$$56.33\ldots$$

Calculate, giving your answers correct to 2 decimal places:

Work to 3 decimal places, then correct to 2.

37 $3.8 \div 0.6$

38 $0.59 \div 0.07$

39 $15 \div 0.9$

40 $5.633 \div 0.2$

41 $0.796 \div 1.1$

42 $1.25 \div 0.03$

43 $0.0024 \div 0.09$

44 $0.65 \div 0.7$

45 $0.0072 \div 0.007$

46 $5 \div 7$

Calculate, giving your answers correct to the number of decimal places indicated in the brackets:

Remember to work to one more decimal place than you need in your answer. For an answer correct to 1 decimal place you must work to 2 decimal places.

47 $0.123 \div 6$ (2)

48 $2.3 \div 0.8$ (1)

49 $90 \div 11$ (1)

50 $0.0078 \div 0.09$ (3)

51 $12 \div 9$ (4)

52 $0.23 \div 0.007$ (1)

53 $16.2 \div 0.8$ (1)

54 $0.21 \div 6.5$ (3)

55 $85 \div 0.3$ (3)

56 $1.37 \div 0.8$ (1)

57 $56.9 \div 1.6$ (nearest whole number)

58 $0.89 \div 0.23$ (1)

59 $0.75 \div 4.5$ (3)

60 $0.023 \div 0.021$ (1)

61 $3.2 \div 1.4$ (1)

62 $0.045 \div 0.012$ (nearest whole number)

63 $12.3 \div 17$ (2)

64 $0.0054 \div 0.021$ (4)

65 $0.012 \div 0.021$ (2)

66 $0.52 \div 0.21$ (1)

Mixed multiplication and division

Exercise 4n

Calculate, giving your answers exactly:

1 0.48×0.3
2 $0.48 \div 0.3$
3 2.56×0.02
4 $2.56 \div 0.02$
5 3.6×0.8
6 9.6×0.6
7 0.0042×0.03
8 $0.0042 \div 0.03$
9 16.8×0.4
10 $1.68 \div 0.4$
11 20.4×0.6
12 $5.04 \div 0.06$

Find $\dfrac{0.12 \times 3}{0.006}$

$\dfrac{0.12 \times 3}{0.006} = \dfrac{0.36}{0.006}$ (Multiply 0.12 by 3 first.) $12 \times 3 = 36$

$= \dfrac{360}{6}$ (Multiply top and bottom by 1000.)

$= 60$

Find the value of:

13 $\dfrac{0.2 \times 0.6}{0.4}$
14 $\dfrac{1.2 \times 0.04}{0.3}$
15 $\dfrac{4.8 \times 0.2}{0.6 \times 0.4}$
16 $\dfrac{3.2}{4 \times 0.2}$
17 $\dfrac{3}{0.6 \times 0.5}$
18 $\dfrac{4.4 \times 0.3}{11}$
19 $\dfrac{2.5 \times 0.7}{3.5 \times 4}$
20 $\dfrac{5.6 \times 0.8}{6.4}$
21 $\dfrac{0.9 \times 4}{0.5 \times 0.6}$

Relative sizes

To compare the sizes of numbers they need to be in the same form, either as fractions with the same denominators, or as decimals.

Exercise 4p

Express 0.82, $\frac{4}{5}$, $\frac{9}{11}$ as decimals where necessary and write them in order of size with the smallest first.

$\frac{4}{5} = 0.8$

$\frac{9}{11} = 0.8181\ldots$ $\qquad 11\overline{)9.000}$ gives 0.8181

In order of size: $\frac{4}{5}, \frac{9}{11}, 0.82$

Express the following sets of numbers as decimals or as fractions and write them in order of size with the smallest first:

1 $\frac{1}{4}$, 0.2 **4** $\frac{1}{3}$, 0.3, $\frac{3}{11}$ **7** $\frac{3}{8}$, $\frac{9}{25}$, 0.35 **10** $0.\dot{7}$, $\frac{8}{11}$

2 $\frac{2}{5}$, $\frac{4}{9}$ **5** $\frac{8}{9}$, 0.9, $\frac{7}{8}$ **8** $\frac{3}{5}$, $\frac{4}{7}$, 0.59 **11** $0.\dot{3}$, $\frac{5}{12}$

3 $\frac{1}{2}$, $\frac{4}{9}$ **6** $\frac{3}{4}$, $\frac{17}{20}$ **9** $\frac{3}{7}$, $\frac{5}{11}$, $\frac{6}{13}$ **12** $\frac{1}{2}$, 0.45, $\frac{9}{19}$

Mixed exercises

Exercise 4q

Select the letter that gives the correct answer.

1 When 0.68 is multiplied by 1000 the result is

A 6.8 **B** 68 **C** 680 **D** 6800

2 $\frac{7}{8}$ expressed as a decimal is

A 0.0875 **B** 0.75 **C** 0.875 **D** 0.88

3 The decimal 2.999 correct to 2 decimal places is

A 2.99 **B** 3.00 **C** 3.01 **D** 3.10

4 $3.2 \times 1.4 =$

A 3.48 **B** 3.58 **C** 4.38 **D** 4.48

5 $16.1 - 4.28 =$

A 11.73 **B** 11.82 **C** 12.73 **D** 12.82

6 Which is the largest of the numbers 6.4, $6\frac{2}{3}$, 6.6, 6.5?

A $6\frac{2}{3}$ **B** 6.4 **C** 6.5 **D** 6.6

Exercise 4r

1 Express 0.06 as a fraction in its lowest terms.

2 Divide 6.24 by **a** 100 **b** 12.

3 Add 3.2 and 0.9 and subtract the result from 5.8.

4 The total distance round the sides of an *equilateral triangle* (a triangle with three equal sides) is 19.2 cm. Find the length of one side.

5 Divide 0.0432 by 0.9.

6 Express $\frac{6}{25}$ as a decimal.

7 Find the cost of 24 articles at \$2.32 each.

8 Give 7.7815 correct to

  a the nearest whole number **b** 1 decimal place **c** 3 decimal places.

Exercise 4s

1 Give $\frac{5}{7}$ as a recurring decimal.

2 Divide each number by 100: **a** 6.4 **b** 0.064.

3 Multiply 14.8 by 1.1.

4 Express 0.62 as a fraction in its lowest terms.

5 Add 6.7, 0.67, 0.067 and 0.0067 together.

6 Divide 16.4 by 8.

7 Which is bigger, 0.7 or $\frac{7}{9}$?

8 How many pieces of ribbon of length 0.3 m can be cut from a piece 7.5 m long?

Exercise 4t

1 Express $\frac{4}{25}$ as a decimal.

2 Find $6.43 \div 0.7$ correct to 3 decimal places.

3 Find 0.06×0.06.

4 Express 0.0095 as a fraction in its lowest terms.

5 Find $13.8 + 2.43 - 1.6$.

6 Find the cost of 3.5 m of ribbon at 58 c per metre.

7 Find $\frac{0.6 \times 0.3}{0.09}$.

<u>8</u> Write $0.\dot{6}$ in another way as a decimal. Why is it not easy to find $0.7 - 0.\dot{6}$?

Did you know?

Pythagoras was a member of a closely knit brotherhood. He was founder of the famous Pythagorean school, which was devoted to the study of philosophy, mathematics and natural science.

These Pythagoreans believed that natural numbers were the building blocks of everything, and attached special significance to certain natural numbers.

Some examples are:

One – the number of reason

Two – the first female number, represented diversity of opinion

Three – the first male number, represented harmony

Four – suggested the squaring of accounts

Five – the union of the first male and female numbers, represented marriage.

In this chapter you have seen that...

- ✔ the decimal point divides the units from the tenths
- ✔ you can add and subtract decimals by writing them in columns, making sure that the decimal points are in line
- ✔ you can multiply decimals by 10, 100, . . . by moving the digits the appropriate number of place values to the left
- ✔ you can divide decimals by 10, 100, . . . by moving the digits to the appropriate number of place values the right
- ✔ a decimal can be changed to a fraction by writing the numbers after the point as tenths, hundredths, . . . and simplifying.
- ✔ you must take great care with the position of the decimal point when you multiply decimals: the sum of the decimal places in the numbers that are multiplied together gives the number of decimal places in the answer

- ✔ you must also be careful with division: to divide by a decimal, multiply the top and the bottom by the same number so that the denominator becomes a whole number
- ✔ some answers are not exact so must be given correct to a given number of places. Remember to work to one more place than you need in the answer
- ✔ fractions can be changed into decimals by dividing the bottom number into the top number. When some fractions are changed into decimals, you get a repeating pattern of digits. These are called recurring decimals
- ✔ you should learn that $\frac{1}{2} = 0.5$, $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$, $\frac{1}{8} = 0.125$
- ✔ sizes of numbers can be compared by converting them into decimals.

5 Percentages

At the end of this chapter you should be able to...

1 express given percentages as fractions and vice versa
2 express percentages as decimals
3 solve problems involving percentages
4 express one quantity as a percentage of another
5 calculate a percentage of a given quantity.

Did you know?

It is thought that every prime number can be expressed as the sum of not more than four square numbers.

For example, $5 = 2^2 + 1^2$ and $19 = 4^2 + 1^2 + 1^2 + 1^2$

Try to express some prime numbers as the sum of four or fewer square numbers.

You need to know...

- ✔ how to simplify fractions
- ✔ how to change a mixed number to an improper fraction
- ✔ how to multiply by a fraction
- ✔ how to find one quantity as a fraction of another quantity
- ✔ the meaning of decimals
- ✔ how to multiply and divide fractions and decimals by 100
- ✔ the units of length, area, mass and capacity
- ✔ how to change units.

Key words

decimal, fraction, percentage

Expressing percentages as fractions

'Per cent' means per hundred, i.e. if 60 per cent of the workers in a factory are women it means that 60 out of every 100 workers are women. If there are 700 workers in the factory, $60 \times 7 = 420$ are women, while if there are 1200 workers, $60 \times 12 = 720$ are women.

In mathematics we are always looking for shorter ways of writing statements and especially for symbols to stand for words. The symbol that means 'per cent' is %, i.e. 60 per cent and 60% have exactly the same meaning.

60 per cent means 60 per hundred and this can be written as $60\% = 60 \times \frac{1}{100} = \frac{60}{100} = \frac{3}{5}$

i.e. 60% of a quantity is exactly the same as $\frac{60}{100}\left(\text{or } \frac{3}{5}\right)$ of that quantity.

If there are 800 cars in a car park and 60% of them are British, then $\frac{60}{100}$ of the cars are British.

i.e. the number of British cars is $\frac{60}{100} \times 800 = 480$

Exercise 5a

Express **a** 40% **b** $22\frac{1}{2}\%$ as fractions in their lowest terms.

a $40\% = \frac{40}{100} = \frac{2}{5}$

b $22\frac{1}{2}\% = \frac{45}{2}\% = \frac{45}{2 \times 100} = \frac{9}{40}$

Express as fractions in their lowest terms:

1 20%	**8** 50%	**<u>15</u>** 70%	**<u>22</u>** 95%
2 45%	**9** 65%	**<u>16</u>** 75%	**<u>23</u>** 15%
3 25%	**10** 56%	**<u>17</u>** 48%	**<u>24</u>** 8%
4 72%	**11** 37%	**<u>18</u>** 69%	**<u>25</u>** 82%
5 $33\frac{1}{3}\%$	**12** $66\frac{2}{3}\%$	**<u>19</u>** $37\frac{1}{2}\%$	**<u>26</u>** $87\frac{1}{2}\%$
6 $12\frac{1}{2}\%$	**13** $62\frac{1}{2}\%$	**<u>20</u>** $5\frac{1}{3}\%$	**<u>27</u>** $6\frac{1}{4}\%$
7 $2\frac{1}{2}\%$	**14** 125%	**<u>21</u>** $17\frac{1}{2}\%$	**<u>28</u>** 150%

Express **a** 54% **b** $6\frac{1}{2}\%$ **c** $27\frac{1}{3}\%$ as decimals.

a $54\% = \frac{54}{100} = 0.54$

b $6\frac{1}{2}\% = \frac{6.5}{100} = 0.065$

c $27\frac{1}{3}\% = \frac{82}{3}\% = \frac{82}{3 \times 100} = 0.273$ to 3 decimal places

Express the following percentages as decimals, giving your answers correct to 3 d.p. where necessary:

29	47%	**34**	58%	**<u>39</u>**	92%	**<u>44</u>**	8%
30	12%	**35**	30%	**<u>40</u>**	65%	**<u>45</u>**	3%
31	$5\frac{1}{2}\%$	**36**	$62\frac{1}{4}\%$	**<u>41</u>**	120%	**<u>46</u>**	180%
32	145%	**37**	350%	**<u>42</u>**	231%	**<u>47</u>**	$5\frac{1}{3}\%$
33	$58\frac{1}{3}\%$	**38**	$48\frac{2}{3}\%$	**<u>43</u>**	$85\frac{2}{3}\%$	**<u>48</u>**	$54\frac{1}{7}\%$

Expressing fractions and decimals as percentages

If $\frac{4}{5}$ of the pupils in a school have been away for a holiday, it means that 80 in every 100 have been on holiday,

i.e. $\frac{4}{5}$ is the same as 80%.

A *fraction* may be converted into a percentage by multiplying that fraction by 100%. This does not alter its value, since $100\% = \frac{100}{100} = 1$, which is the multiplicative identity.

A *decimal* may be converted into a percentage by multiplying it by 100%.

Exercise 5b

Express $\frac{7}{20}$ as a percentage.

$$\frac{7}{20} = \frac{7}{\cancel{20}_1} \times \cancel{100}^5\% = 35\%$$

Express the following fractions as percentages, giving your answers correct to 1 decimal place where necessary:

1	$\frac{1}{2}$	**5**	$\frac{21}{40}$	**9**	$\frac{3}{8}$	**<u>13</u>**	$\frac{7}{5}$	**<u>17</u>**	$\frac{7}{20}$
2	$\frac{7}{10}$	**6**	$\frac{1}{4}$	**10**	$\frac{23}{60}$	**<u>14</u>**	$\frac{5}{8}$	**<u>18</u>**	$\frac{31}{25}$
3	$\frac{13}{20}$	**7**	$\frac{3}{20}$	**<u>11</u>**	$\frac{3}{4}$	**<u>15</u>**	$\frac{8}{3}$	**<u>19</u>**	$\frac{7}{8}$
4	$\frac{1}{3}$	**8**	$\frac{4}{25}$	**<u>12</u>**	$\frac{9}{20}$	**<u>16</u>**	$\frac{3}{5}$	**<u>20</u>**	$\frac{8}{5}$

Express **a** 0.7 **b** 1.24 as percentages.

a $0.7 = 0.7 \times 100\% = 70\%$ **b** $1.24 = 1.24 \times 100\% = 124\%$

Express the following decimals as percentages:

21	0.5	**25**	0.625	**29**	2.64	**<u>33</u>**	1.25	**<u>37</u>**	0.16
22	0.22	**26**	0.9	**30**	0.845	**<u>34</u>**	3.41	**<u>38</u>**	1.39
23	0.83	**27**	0.04	**<u>31</u>**	0.25	**<u>35</u>**	0.075	**<u>39</u>**	6.35
24	1.72	**28**	0.55	**<u>32</u>**	0.74	**<u>36</u>**	0.36	**<u>40</u>**	0.1825

Exercise 5c

1 Express as fractions in their lowest terms:

a 30% **b** 85% **c** $42\frac{1}{2}\%$ **d** $5\frac{1}{4}\%$

2 Express as decimals:

a 44% **b** 68% **c** 170% **d** $16\frac{1}{2}\%$

3 Express as percentages:

a $\frac{2}{5}$ **b** $\frac{17}{20}$ **c** $\frac{1}{8}$ **d** $\frac{17}{15}$

4 Express as percentages:

a 0.2 **b** 0.62 **c** 0.845 **d** 1.78

Copy and complete the following table:

	Fraction	Percentage	Decimal
	$\frac{3}{4}$	75%	0.75
5	$\frac{4}{5}$		
6		60%	
7			0.7
8	$\frac{11}{20}$		
9		44%	
10			0.32

Problems

Suppose that in the town of Doxton 25 families in every 100 own a car. We can deduce from this that 75 in every 100 families do not. Since every family either owns a car or does not own a car, if we are given one percentage we can deduce the other.

Exercise 5d

If 14% of homes have a landline, what percentage do not?

All homes (i.e. 100% of homes) either have, or do not have, a landline.

If 14% have a landline, then (100 – 14)% do not,

i.e. 86% do not.

1 If 48% of the pupils in a school are girls, what percentage are boys?

2 If 87% of households have a computer, what percentage do not?

3 In the fourth year, 64% of the pupils do not study chemistry. What percentage study chemistry?

4 In a box of oranges, 8% are bad. What percentage are good?

5 Twelve per cent of the persons taking a driver's test fail to pass first time. What percentage pass first time?

6 A hockey team won 62% of their matches and drew 26% of them. What percentage did they lose?

7 A rugby team drew 12% of their matches and lost 45% of them. What percentage did they win?

8 Deductions from a youth's wage were: income tax 18%, other deductions 14%. What percentage did he keep?

9 In an election, 40% of the electorate voted for Mrs Long, 32% for Mr Singhe and the remainder voted for Miss Berry. What percentage voted for Miss Berry if there were only three candidates and 8% of the electorate failed to vote?

10 In a school, 36% of the pupils study French and 38% study German. If 12% study both languages, what percentage do not study either?

11 85% of the first year pupils in a school study craft and 72% study photography. If 60% study both subjects, what percentage study neither?

12 A concert is attended by 1200 people. If 42% are adult females and 37% are adult males, how many children attended?

13 The attendance at an athletics meeting is 14 000. If 68% are men and boys and 22% are women, how many are girls?

14 In a book, 98% of the pages contain text, diagrams or both. If 88% of the pages contain text and 32% contain diagrams, what percentage contain

a neither text nor diagrams
b only diagrams
c only text
d both text and diagrams?

Puzzle

Alice's Adventures in Wonderland was created by Lewis Carroll, whose real name was Charles Lutwidge Dodgson. Dodgson was a mathematician who thought up many puzzles. This is one, with the title of 'Casualties':

If 70% have lost an eye, 75% an ear, 80% an arm, 85% a leg, what percentage, at least, must have lost all four?

Expressing one quantity as a percentage of another

If we wish to find 4 as a percentage of 20, we know that 4 is $\frac{4}{20}$ of 20

and $\frac{4}{20} = \frac{4}{20} \times 100\%$

i.e. 4 as a percentage of 20 is

$$\frac{4}{20} \times 100\% = 20\%$$

To express one quantity as a percentage of another, we divide the first quantity by the second and multiply this fraction by 100%.

Exercise 5e

Express 20 cm as a percentage of 300 cm.

The first quantity as a percentage of the second quantity is

$$\frac{20}{300} \times 100\% = \frac{20}{3}\% = 6\frac{2}{3}\%$$

Express the first quantity as a percentage of the second:

1 3, 12

2 30 cm, 50 cm

3 3 m, 9 m

4 4 in, 12 in

5 15, 20

6 24 cm, 40 cm

7 60 cm, 4 m

8 10 ft, 40 ft

9 20 m^2, 80 m^2

10 75 cm^2, 200 cm^2

11 25 cm^2, 125 cm^2

12 4 litres, 10 litres

13 3 pints, 5 pints

14 5, 50

15 2 cm, 10 cm

16 600 m, 2 km

17 $3\frac{1}{2}$ yd, 7 yd

18 40, 20

19 35 m, 56 m

20 50 cm, 5 m

21 8 in, 12 in

22 200 mm^2, 800 mm^2

23 198 mm^2, 275 mm^2

24 50 m^2, 15 m^2

25 3.6 t, 5 t

26 33.6 g, 80 g

Make sure that both quantities are measured in the same unit.

Finding a percentage of a quantity

To find a percentage of a quantity, change the percentage to a fraction and multiply by the quantity.

Exercise 5f

Find the value of **a** 12% of 450 **b** $7\frac{1}{3}$% of 3.75 m

a $12\%\text{ of }450 = \frac{12}{100} \times 450 = 54$ (the term 'of' means to multiply by)

b $7\frac{1}{3}\%\text{ of }3.75\text{ m} = 7\frac{1}{3}\%\text{ of }375\text{ cm} = \frac{22}{3}\%\text{ of }375\text{ cm}$

$= \frac{22}{3 \times 100} \times 375\text{ cm}$

$= 27.5\text{ cm}$

Find the value of:

1 40% of 120

2 12% of 800 g

3 74% of 75 cm

4 44% of 650 km

5 8% of $2

6 77% of 4 kg

7 70% of 360

8 86% of 1150 g

9 55% of 8.6 m

10 96% of 215 cm^2

11 63% of 4 m

12 96% of 15 m^2

13 45% of 740

14 33% of 600 kg

15 6% of 24 m

16 15% of $10

17 17% of 2 km

18 32% of 5 litres

19 30% of $250

20 66% of 300 m

21 $33\frac{1}{3}$% of 270 g

22 $5\frac{1}{4}$% of 56 mm

23 $37\frac{1}{2}$% of 48 cm

24 $22\frac{1}{2}$ % of 40 m^2

25 $66\frac{2}{3}$% of 480 m^2

26 $32\frac{1}{7}$% of 140 km

27 $62\frac{1}{2}$% of 8 km

28 $74\frac{1}{2}$% of 200 cm^2

29 $33\frac{1}{3}$% of $42

30 $82\frac{1}{5}$% of $65

31 12% of $4

32 $7\frac{1}{2}$% of 80 g

33 $2\frac{1}{3}$% of 90 m

34 $16\frac{2}{3}$% of $60

35 $3\frac{1}{8}$% of 64 kg

36 $87\frac{1}{2}$% of 16 mm

Investigation

Banks offer many different savings accounts. Get an up-to-date leaflet from one bank that gives details of all its different accounts and the rate of interest offered on each.

Write a short report on which account you would use, and why, if

1 you are saving to buy a pair of trainers

2 a relative has given you \$100 000 and you want to keep it safe until you leave school.

Problems

Exercise 5g

In the second year, 287 of the 350 pupils study geography. What percentage study geography?

Express 287 as a fraction of 350, then multiply by 100%.

$$\text{Percentage studying geography} = \frac{287}{350} \times 100\%$$
$$= 82\%$$

1 There are 60 boys in the third year, 24 of whom study chemistry. What percentage of third year boys study chemistry?

Read the question carefully to make sure that you understand what you are being asked to find. Read it several times if necessary.

2 In a history test, Victoria scored 28 out of a possible 40. What was her percentage mark?

3 Out of 20 drivers tested in one day for a driver's licence, 4 of them failed. What percentage failed?

4 There are 60 photographs in a book, 12 of which are coloured. What is the percentage of coloured photographs?

5 Forty-two of the 60 choristers in a choir wear spectacles. What percentage do not?

6 Each week a boy saves \$400 of the \$1600 he earns. What percentage does he spend?

7 A secretary takes 56 letters to the post office for posting. 14 are registered and the remainder are ordinary mail. What percentage go by ordinary mail?

8 Monique obtained 80 marks out of a possible 120 in her end of term maths examination. What was her percentage mark?

9 Sherry earns $16 000 per week, but after paying taxes and other deductions, her 'take home' pay is only $10 400.

What percentage is this of her total earnings before deductions?

10 If 8% of a crowd of 24 500 at a football match were females, how many females attended?

If 54% of the 1800 pupils in a school are boys, how many girls are there in the school?

Method 1 First find the number of boys in the school: this is 54% of 1800.

$$\text{Number of boys} = \frac{54}{100} \times 1800$$
$$= 972$$

Now you can find the number of girls:

$$\text{Number of girls} = 1800 - 972$$
$$= 828$$

Method 2 As 54% of the pupils are boys, 100% – 54%, i.e. 46% of the pupils are girls.
So the number of girls is 46% of 1800 $= \frac{46}{100} \times 1800 = 828$

11 In a garage, 16 of the 30 cars which are for sale are second hand. What percentage of the cars are

a new **b** second hand?

12 There are 80 houses in my street and 65% of them have an internet connection. How many houses

a have the internet **b** do not have the internet?

13 In my class there are 30 pupils and 40% of them have a bicycle. How many pupils

a have a bicycle **b** do not have a bicycle?

14 Yesterday, of the 350 international flights leaving Miami International Airport, 2% were bound for Kingston. How many of these flights

a flew to Kingston **b** did not fly to Kingston?

15 In a particular year, 64% of the 16 000 Jewish immigrants into Israel came from Eastern Europe. How many of the immigrants did not come from Eastern Europe?

16 There are 120 shops in the High Street, 35% of which sell food. How many High Street shops do not sell food?

17 Last year the amount I paid in rates on my house was $64 000. This year my rates will increase by 12%. Find the increase.

18 A mathematics book has 320 pages, 40% of which are on algebra, 25% on geometry and the remainder on arithmetic. How many pages of arithmetic are there?

Mixed exercises

Exercise 5h

Select the letter that gives the correct answer.

1 Expressed as a percentage $2\frac{3}{4}$ is

A 27.5% **B** 75% **C** 95% **D** 275%

2 15 metres expressed as a percentage of 35 metres correct to 2 decimal places is

A 42.80% **B** 42.86% **C** 43.80% **D** 44.85%

3 Expressed as a percentage of $4, 25 cents is

A 0.625% **B** 6.25% **C** 6.30% **D** 62.5%

4 55% of 240 cm is

A 115 cm **B** 120 cm **C** 132 cm **D** 142 cm

5 48% expressed as a vulgar fraction in its lowest term is

A $\frac{5}{12}$ **B** $\frac{1}{2}$ **C** $\frac{12}{25}$ **D** $\frac{24}{50}$

6 $\frac{3}{8}$ as a percentage is

A 37% **B** 37.5% **C** 38% **D** 40%

Exercise 5i

1 Express 36%

a as a fraction in its lowest terms

b as a decimal.

2 Express as a percentage, giving your answer correct to 1 d.p. if necessary:

a $\frac{5}{8}$ **b** $1\frac{1}{3}$ **c** 2.5

3 Express 250 g as a percentage of 2000 kg.

4 Find 85% of 340 m^2.

5 The cost of insuring a car in Barbados is about 8% of its value.
Find the cost of insuring a car valued at $2400 000.

Exercise 5j

1 Find the first quantity as a percentage of the second quantity:

   a 10 m, 80 m **b** \$0.75, \$2 **c** 150 cm, 300 cm

2 Express as a percentage, giving your answer correct to 1 d.p. where necessary:

   a $\frac{2}{7}$ **b** 0.279 **c** $1\frac{2}{9}$

3 Express $12\frac{1}{2}\%$ as

   a a fraction in its lowest terms

   b a decimal.

4 Find 36% of \$2.50.

5 There are 450 children in a primary school, 12% of whom do not speak English at home. Find the number of children for whom English is not their home language.

Puzzle

If 5 September falls on a Friday, on which day of the week will Christmas Day fall?

Did you know?

You can use this calculator method to find the number of gifts received in the song *The Twelve Days of Christmas*.

1 The number of gifts received on the *n*th day is calculated by

   [AC] [1] [+] [2] [+] [3] [+] ··· [+] [n] [=]

2 The total number of gifts is found by

   [AC] [1] [M+] [+] [2] [M+] [+] [3] ··· [M+] [+] [1] [2] [M+] [MR]

In this chapter you have seen that...

- ✔ a percentage can be expressed as a fraction by putting it over 100 and simplifying, e.g. $70\% = \frac{70}{100} = \frac{7}{10}$
- ✔ a percentage can be expressed as a decimal by dividing it by 100, e.g. $65\% = 65 \div 100 = 0.65$
- ✔ a fraction, or a decimal, can be expressed as a percentage by multiplying it by 100, e.g. $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$ and $0.8 = 0.8 \times 100\% = 80\%$
- ✔ to express one quantity as a percentage of another, first express the quantity as a fraction of the second and multiply by 100, e.g. 4 as a percentage of 20 is $\frac{4}{20} \times 100\% = 20\%$
- ✔ to find a percentage of a quantity, multiply the percentage by the quantity and divide by 100, e.g. 20% of 80 cm $= \frac{20}{100} \times 80\,\text{cm} = 16\,\text{cm}$.

REVIEW TEST 1: CHAPTERS 1–5

In questions **1** to **12** choose the letter for the correct answer.

1 To the nearest 10, 187 is

A 100 B 180 C 190 D 200

2 Written as a fraction 1.4 is

A $1\frac{4}{7}$ B $1\frac{4}{9}$ C $1\frac{4}{10}$ D $1\frac{4}{11}$

3 The LCM of 2, 4, 5 is

A 20 B 30 C 40 D 50

4 Written as a decimal $\frac{9}{10}+\frac{7}{1000}$ is

A 0.097 B 0.907 C 0.97 D 9.07

5 $0.3 \times 0.02 =$

A 0.006 B 0.60 C 0.600 D 6.000

6 The HCF of 12, 15 and 30 is

A 3 B 6 C 60 D 180

7 All the factors of 12 are

A 3, 6, 9, 12 B 2, 3, 4, 6 C 2, 4, 5, 8 D 1, 2, 3, 4, 6, 12

8 $2 \times 1\frac{3}{4} =$

A $2\frac{3}{4}$ B $2\frac{3}{8}$ C $3\frac{1}{2}$ D $3\frac{3}{4}$

9 $0.8 \div 0.4 =$

A 0.02 B 0.2 C 0.32 D 2

10 The remainder when 70 is divided by 15 is

A 5 B 10 C 15 D 60

11 Tony cuts lengths of 16 m, 7.42 m and 12.83 m from a coil of rope 50 m long. How long is the remaining piece of rope?

A 9.25 m B 10.25 m C 12.75 m D 13.75 m

12 Which of the following fractions gives a recurring decimal?

A $\frac{3}{4}$ B $\frac{3}{5}$ C $\frac{3}{7}$ D $\frac{3}{10}$

13 Write the number(s) between 26 and 42 inclusive that are

a square numbers

b rectangular numbers

c triangular numbers.

14 **a** Simplify $\frac{7}{12} \div \frac{21}{4}$.

b Write the next two numbers in this pattern: 1, 3, 6,...

c Calculate $3 \times 10 + 7 \times 10 \div (2 \times 10) + 0 \times 1$

15 **a** Arrange the following fractions in ascending order:

$\frac{2}{3}, \frac{7}{9}, \frac{3}{4}, \frac{5}{12}$

b Find 8 months as a fraction of 2 years.

16 Put > or < between the following pairs of numbers:

a $\frac{3}{7} \quad \frac{3}{8}$

b $\frac{11}{7} \quad 1\frac{3}{10}$

17 Find

a $18 + 1.8 + 0.18$

b $5.349 - 1.652$

c 2.3×27

d $25.65 \div 9$

18 **a** Arrange the following fractions is descending order:

$\frac{7}{12}, \frac{3}{4}, \frac{7}{8}, \frac{17}{24}$

b Find 86% of 24 m.

19 Express

a 35% as a fraction in its lowest terms

b 0.24 as a percentage

c $\frac{19}{25}$ as a percentage.

20 Find

a $8 + (-3) + 5$

b $-8 - (+4) - (-9)$

c $(5 - 7) - (10 - 12)$

d $(-5) \times (-6)$

21 Find

a $\frac{7}{12} - \frac{1}{3} + \frac{5}{6}$ **b** $6\frac{7}{10} + 3\frac{4}{5}$ **c** $7\frac{3}{5} - 3\frac{3}{4}$ **d** $3\frac{3}{7} \div 21$

22 Find

a $\left(\frac{7}{11} - \frac{1}{3}\right) \times \frac{11}{20}$

b $\frac{3}{7} \times \frac{8}{9} \div \frac{12}{21}$

6 Measurement

At the end of this chapter you should be able to...

1 use suitable metric units of measure for length and mass
2 change from small to large metric units of measure and vice versa
3 add and subtract metric quantities
4 change from a.m. and p.m. time to 24-hour clock times and vice versa
6 express quantities in given imperial units of measure
7 make rough equivalence between imperial and metric units
8 convert between temperatures measured in degrees Celsius and degrees Fahrenheit
9 recognise volume as a measure of space
10 measure volume using standard units.

Did you know?

The story of zero – the number that we use so often

The word zero came from Italian. It was not always as important as it is today.

We were using numbers for thousands of years before zero (0) was introduced to us.

Zero is special:

- If we add or subtract 0 from a number, the result is the original number.
- If we multiply a number by 0, the result is zero.
- If we raise a number other than 0 to the power 0, the result is 1.
- If we divide 0 by a number other than 0, the result is 0.

We cannot define a number divided by 0.

You need to know...

✔ how to multiply and divide by 10, 100 and 1000
✔ how to multiply by any number
✔ the basic number facts including your tables
✔ how to deal with simple decimals.

Key words

approximation, capacity, centimetre, cubic unit, degrees Celsius, degrees Fahrenheit, foot, gram, hundredweight, inch, kilogram, kilometre, litre, mass, metre, mile, milligram, millilitre, millimetre, ounce, perimeter, pound, ton, tonne, volume, yard

The need for standard units

Tyrone needs a new shutter for his window. He describes the size by saying it is as wide as his table and as high as the length of his walking stick. These measurements are no use for ordering the shutter. Measurements have to be in units that everyone understands.

There are two standard sets of units. Metric units are used in most countries but imperial units are used in the USA and a few other countries.

Whenever we want to measure a length, or weigh an object, we find the length or mass in standard units. We might for instance give the length of a line in millimetres or the mass of a bag of apples in pounds. The millimetre belongs to a set of units called the metric system. The pound is one of the imperial units.

The metric system was developed in France in 1790 so that units in the system would be related to each other by a factor of ten.

The basic unit of capacity is the litre. The litre has the volume of a cube of side 10 centimetres.

Units of length

The basic unit of length is the *metre* (m). To get an idea of how long a metre is, remember that a standard bed is about 2 m long. However, a metre is not a useful unit for measuring either very large things or very small things so we need larger units and smaller units.

We get the larger unit by multiplying the metre by 1000. We get the smaller units by dividing the metre into 100 parts or 1000 parts.

1000 metres is called 1 *kilometre* (km)

(It takes about 15 minutes to walk a distance of 1 km.)

$\frac{1}{100}$ of a metre is called 1 *centimetre* (cm)

$\frac{1}{1000}$ of a metre is called 1 *millimetre* (mm)

(You can see centimetres and millimetres on your ruler.)

Many rulers show both inches and centimetres.

Most of us use rulers and tapes for everyday measurement. More specialised measuring instruments are Vernier callipers. These are used for measurements such as internal and external diameters of pipes, and for measurements that are required to 0.1mm.
The picture shows a digital Vernier calliper.

Also used are electronic instruments for measuring distances, for example lengths of rooms.
These typically use lasers or ultrasound and give the lengths digitally.

Some uses of metric units of length

Millimetres (mm) for lengths of nails and screws, widths of film, tapes and ribbons.

Centimetres (cm) for body sizes, i.e. height, chest, etc., widths of wallpaper, belts and ties.

Metres (m) for sizes of rooms, swimming pools, garden beds, hoses, ladders, etc.

Kilometres (km) for road signs, maps, distances between places.

Exercise 6a

1 Which metric unit would you use to measure

- **a** the length of your classroom
- **b** the length of your pencil
- **c** the length of a soccer pitch
- **d** the distance from Castries to Roseau
- **e** the length of a page in this book
- **f** the thickness of your exercise book?

2 Use your ruler to draw a line of length

a	10 cm	**d**	50 mm	**g**	15 mm	**j**	16 mm
b	3 cm	**e**	20 mm	**h**	12 cm	**k**	5 cm
c	15 cm	**f**	4 cm	**i**	25 mm	**l**	75 mm

3 Estimate the length, in centimetres, of the following lines:

a

b

c

d

e

Now use your ruler to measure each line.

4 Estimate the length, in millimetres, of the following lines:

a

b

c

d

e

Now use your ruler to measure each line.

5 Use a straight edge (not a ruler with a scale) to draw a line that is approximately

a 10 cm long **b** 5 cm long **c** 15 cm long **d** 20 mm long

Now measure each line to see how good your *approximation* was.

6 Estimate the width of your classroom in metres.

7 Estimate the length of your classroom in metres.

8 Measure the length and width of your exercise book in centimetres. Draw a rough sketch of your book with the measurements on it. Find the *perimeter* (the distance all round) of your book.

9 Each side of a square is 10 cm long. Draw a rough sketch of the square with the measurements on it. Calculate the perimeter of the square.

10 A sheet is 200 cm wide and 250 cm long. What is the perimeter of the sheet?

Activity

This shows a woman near a tree.

The woman is 170 cm tall.

1 Estimate the height of the tree.

2 Use a person or an object (e.g. a door) whose height you know to estimate the height and width of the main building in your school.

3 Explain how you could estimate the length and height of a bridge.

Changing from large units to smaller units

The metric units of length are the kilometre, the metre, the centimetre and the millimetre where

1 km = 1000 m   1 m = 100 cm

1 m = 1000 mm   1 cm = 10 mm

Exercise 6b

Express 3 km in metres.

1 km is 1000 m, so 3 km is 3 times 1000 m

$$3\text{ km} = 3 \times 1000\text{ m}$$
$$= 3000\text{ m}$$

Express 3.5 m in centimetres.

1 m is 100 cm, so 3.5 m is 3.5 times 100 cm

$$3.5\text{ m} = 3.5 \times 100\text{ cm}$$
$$= 350\text{ cm}$$

Express the given quantity in terms of the unit in brackets:

1	2 m	(cm)	**13**	1.5 m	(cm)
2	5 km	(m)	**14**	2.3 cm	(mm)
3	3 cm	(mm)	**15**	4.6 km	(m)
4	4 m	(cm)	**16**	3.7 m	(mm)
5	12 km	(m)	**17**	1.9 m	(mm)
6	15 cm	(mm)	**18**	3.5 km	(m)
7	6 m	(mm)	**19**	2.7 m	(cm)
8	1 km	(cm)	**20**	1.9 km	(cm)
9	3 m	(mm)	**21**	3.8 cm	(mm)
10	2 km	(mm)	**22**	9.2 m	(mm)
11	5 m	(cm)	**23**	2.3 km	(m)
12	7 m	(mm)	**24**	8.4 m	(cm)

Units of mass

The most familiar units used for weighing are the *kilogram* (kg) and the *gram* (g). We shall use the term '*mass*', not 'weight'.

Most groceries that are sold in tins or packets have masses given in grams. For example the mass of the most common packet of butter is 250 g. One eating apple weighs roughly 100 g, so the gram is a small unit of mass. Kilograms are used to give the mass of sugar or flour: the mass of the most common bag of sugar is l kg and the most common bag of flour weighs 1.5 kg.

For weighing large loads (timber or steel for example) a larger unit of mass is needed, and we use the *tonne* (t). For weighing very small quantities (for example the mass of a particular drug in one pill) we use the *milligram* (mg).

The relationships between these masses are

$$1\,\text{t} = 1000\,\text{kg}$$

$$1\,\text{kg} = 1000\,\text{g}$$

$$1\,\text{g} = 1000\,\text{mg}$$

Instruments for measuring mass are called scales or balances and they come in a variety of forms.

A spring balance is used for weighing heavy objects such as luggage and gas cylinders. The scale is marked in kilograms and pounds.

These kitchen scales are also calibrated in kilograms and pounds.

These are digital kitchen scales. Most digital scales give the option of weighing in grams or pounds and ounces.

Exercise 6c

Express 2 t in grams.

First change tonnes to kg, then change kg to grams.

1 t is 1000 kg and 1 kg is 1000 g

$$
\begin{aligned}
2\,\text{t} &= 2 \times 1000\,\text{kg} \\
&= 2000\,\text{kg} \\
&= 2000 \times 1000\,\text{g} \\
&= 2\,000\,000\,\text{g}
\end{aligned}
$$

Express each quantity in terms of the unit given in brackets:

1	12 t	(kg)	**4**	1 t	(g)	**7**	6 g	(mg)
2	3 kg	(g)	**5**	1 kg	(mg)	**8**	2 t	(g)
3	5 g	(mg)	**6**	13 kg	(g)	**9**	4 kg	(g)

10	2 kg	(mg)	**15**	1.8 g	(mg)	**20**	2.5 kg	(g)
11	3 t	(kg)	**16**	0.7 t	(kg)	**21**	7.3 g	(mg)
12	4 g	(mg)	**17**	5.2 kg	(mg)	**22**	0.3 kg	(mg)
13	1.5 kg	(g)	**18**	0.6 g	(mg)	**23**	0.5 t	(kg)
14	2.7 t	(kg)	**19**	11.3 t	(kg)	**24**	0.8 g	(mg)

Mixed units

When you use your ruler to measure a line, you will probably find that the line is not an exact number of centimetres. For example the width of this page is 19 cm and 5 mm. We can say that the width of this page is 19 cm 5 mm or we could give the width in millimetres alone.

Now $19\text{ cm} = 19 \times 10\text{ mm}$

$= 190\text{ mm}$

So $19\text{ cm } 5\text{ mm} = 195\text{ mm}$

Exercise 6d

Express each quantity in terms of the unit given in brackets:

4 kg 50 g (g)

Change 4 kg to grams, then add 50 g

$4\text{ kg} = 4 \times 1000\text{ g}$

$= 4000\text{ g}$

Therefore $4\text{ kg } 50\text{ g} = 4050\text{ g}$

1	1 m 36 cm	(cm)	**11**	3 kg 500 g	(g)
2	3 cm 5 mm	(mm)	**12**	2 kg 8 g	(g)
3	1 km 50 m	(m)	**13**	5 g 500 mg	(mg)
4	4 cm 8 mm	(mm)	**14**	2 t 800 kg	(kg)
5	2 m 7 cm	(cm)	**15**	3 t 250 kg	(kg)
6	3 km 20 m	(m)	**16**	1 kg 20 g	(g)
7	5 m 2 cm	(cm)	**17**	1 g 250 mg	(mg)
8	5 km 500 m	(m)	**18**	3 kg 550 g	(g)
9	20 cm 2 mm	(mm)	**19**	2 t 50 kg	(kg)
10	8 m 9 mm	(mm)	**20**	1 kg 10 g	(g)

Changing from small units to larger units

Exercise 6e

Express 400 cm in metres.

100 cm = 1 m, so 1 cm = 1 ÷ 100 m

So 400 cm = 400 ÷ 100 m

= 4 m

In questions **1** to **20**, express the given quantity in terms of the unit given in brackets:

1	300 mm	(cm)	**11**	1500 kg	(t)
2	6000 m	(km)	**12**	3680 g	(kg)
3	150 cm	(m)	**13**	1500 mg	(g)
4	250 mm	(cm)	**14**	5020 g	(kg)
5	1600 m	(km)	**15**	3800 kg	(t)
6	72 m	(km)	**16**	86 kg	(t)
7	12 cm	(m)	**17**	560 g	(kg)
8	88 mm	(cm)	**18**	28 mg	(g)
9	1250 mm	(m)	**19**	190 kg	(t)
10	2850 m	(km)	**20**	86 g	(kg)

Express 5 m 36 cm in metres.

First change 36 cm to metres, then add 5.

36 cm = 36 ÷ 100 m

= 0.36 m

So 5 m 36 cm = 5.36 m

In questions **21** to **40** express the given quantity in terms of the unit given in brackets:

21	3 m 45 cm	(m)	**28**	4 m 5 mm	(m)
22	8 cm 4 mm	(cm)	**29**	1 km 10 cm	(km)
23	11 km 2 m	(km)	**30**	8 cm 5 mm	(km)
24	2 km 42 m	(km)	**31**	5 kg 142 g	(kg)
25	4 cm 4 mm	(cm)	**32**	48 g 171 mg	(g)
26	5 m 3 cm	(m)	**33**	9 kg 8 g	(kg)
27	7 km 5 m	(km)	**34**	9 g 88 mg	(g)

35	12 kg 19 g	(kg)	38	5 g 3 mg	(g)
36	4g 111 mg	(g)	39	250 g 500 mg	(kg)
37	1 t 56 kg	(t)	40	850 kg 550 g	(t)

Exercise 6f

Find 1 kg + 158 g in **a** grams **b** kilograms.

a $1\text{ kg} = 1000\text{ g}$

$\therefore\ 1\text{ kg} + 158\text{ g} = 1158\text{ g}$ ($\therefore$ means 'therefore' or 'it follows that')

b
$$158\text{ g} = 158 \div 1000\text{ kg}$$
$$= 0.158\text{ kg}$$
$$\therefore\ 1\text{ kg} + 158\text{ g} = 1.158\text{ kg}$$

Find the sum of 5 m, 4 cm and 97 mm in **a** metres **b** centimetres.

a
$$4\text{ cm} = 4 \div 100\text{ m} = 0.04\text{ m}$$
$$97\text{ mm} = 97 \div 1000\text{ m} = 0.097\text{ m}$$
$$\therefore\ 5\text{ m} + 4\text{ cm} + 97\text{ mm} = (5 + 0.04 + 0.097)\text{ m}$$
$$= 5.137\text{m}$$

b
$$5\text{ m} = 5 \times 100\text{ cm} = 500\text{ cm}$$
$$97\text{ mm} = 97 \div 10\text{ cm} = 9.7\text{ cm}$$
$$\therefore\ 5\text{ m} + 4\text{ cm} + 97\text{ mm} = (500 + 4 + 9.7)\text{ cm}$$
$$= 513.7\text{ cm}$$

Alternatively, use your answer from **a**: $5.137\text{ m} = 5.137 \times 100\text{ cm}$
$$= 513.7\text{ cm}$$

Quantities must be expressed in the same units before they are added or subtracted.

Find, giving your answer in metres:

1 5 m + 86 cm

2 92 cm + 115 mm

3 3 km + 136 cm

4 51 m + 3 km

5 36 cm + 87 mm + 520 cm

6 120 mm + 53 cm + 4 m

If you are changing to a smaller unit, e.g. from metres to centimetres, *multiply*. If you are changing to a larger unit, e.g. from grams to kilograms, *divide*.

Find, giving your answer in millimetres:

7 36 cm + 80 mm

8 5 cm + 5 mm

9 1 m + 82 cm

10 2 m + 45 cm + 6 mm

11 3 cm + 5 m + 2.9 cm

12 34 cm + 18 mm + 1 m

Find, giving your answer in grams:

13 3 kg + 250 g

14 5 kg + 115 g

15 5.8 kg + 9.3 kg

16 1 kg + 0.8 kg + 750 g

17 116 g + 0.93 kg + 680 mg

18 248 g + 0.06 kg + 730 mg

Find, expressing your answer in kilograms:

19 2 t + 580 kg

20 1.8 t + 562 kg

21 390 g + 1.83 kg

22 1.6 t + 3.9 kg + 2500 g

23 1.03 t + 9.6 kg + 0.05 t

24 5.4 t + 272 kg + 0.3 t

Find, expressing your answer in the unit given in brackets:

25 8 m – 52 cm (cm)

26 52 mm + 87 cm (m)

27 1.3 kg – 150 g (g)

28 1.3 m – 564 mm (cm)

29 2.05 t + 592 kg (kg)

30 20 g – 150 mg (mg)

31 36 kg – 580 g (g)

32 1.5 t – 590 kg (kg)

33 3.9 m + 582 mm (cm)

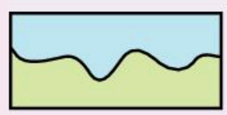
34 0.3 m – 29.5 cm (mm)

Investigation

1 This is a map of an island. Explain how you could estimate the length of its coastline.

2 This is the same island, drawn to a larger scale. Would you get the same answer for the length of its coastline from this drawing?

3 This shows the coastline of part of the island drawn to a much larger scale. If you used a map of the whole island drawn with this scale, how would your estimate of the length of the coastline compare with your first estimate?



4 Do you think it is possible to measure the length of the coastline exactly?
(Think of a bit of coastline you know and imagine measuring a short length of it.)

5 Now suppose that you want to measure the length of the table you are sitting at.
You could measure it with a ruler.
You could measure it with a tape measure marked in centimetres and millimetres.
You could measure it with a precision instrument that will read lengths to tenths of a millimetre, or even hundredths of a millimetre.
You could measure the length in several different places.
Write down, with reasons, whether it is possible to find the length exactly.
Do you think it is possible to give any measurement exactly?

Multiplying metric units

Exercise 6g

Calculate, expressing your answer in the unit given in brackets:

$3 \times 2\,\text{g}\ 741\,\text{mg}$ (g)

First express the mass in grams.

$$2\,\text{g}\ 741\,\text{mg} = 2.741\,\text{g}$$

$$\therefore \quad 3 \times 2\,\text{g}\ 741\,\text{mg} = 3 \times 2.741\,\text{g}$$

$$= 8.223\,\text{g}$$

$$\begin{array}{r} 2741 \\ \times \quad 3 \\ \hline 8223 \\ \hline \end{array}$$

1 $4 \times 3\,\text{kg}\ 385\,\text{g}$ (g)

2 $9 \times 5\,\text{m}\ 88\,\text{mm}$ (mm)

3 $3 \times 4\,\text{kg}\ 521\,\text{g}$ (kg)

4 $5 \times 2\,\text{m}\ 51\,\text{cm}$ (m)

5 $10 \times 3\,\text{t}\ 200\,\text{kg}$ (t)

6 $2 \times 5\,\text{cm}\ 3\,\text{mm}$ (cm)

7 $6 \times 2\,\text{g}\ 561\,\text{mg}$ (mg)

8 $8 \times 3\,\text{km}\ 56\,\text{m}$ (km)

9 $3 \times 7\,\text{t}\ 590\,\text{kg}$ (t)

10 $7 \times 2\,\text{km}\ 320\,\text{m}$ (m)

Puzzle

If a box of bananas weighs 7 kilograms and half of its own mass, how much does a box and a half of bananas weigh?

Problems

Exercise 6h

Find, in kilograms, the total mass of a bag of flour of mass 1.5 kg, a jar of jam of mass 450 g and a packet of rice of mass 500 g.

The total mass means the sum of the three masses.

First change each mass to kg, then add them.

$$\text{The mass of the jar of jam} = 450 \div 1000\,\text{kg}$$
$$= 0.45\,\text{kg}$$
$$\text{The mass of the packet of rice} = 500 \div 1000\,\text{kg}$$
$$= 0.5\,\text{kg}$$
$$\text{The total mass} = (1.5 + 0.45 + 0.5)\,\text{kg}$$
$$= 2.45\,\text{kg}$$

Read each question carefully to make sure that you understand what you are asked to find. Read it several times if necessary.

1 Find the sum, in metres, of 5 m, 52 cm, 420 cm.

2 Find the sum, in grams, of 1 kg, 260 g, 580 g.

3 Subtract 52 kg from 0.8 t, giving your answer in kilograms.

4 Find the difference, in grams, between 5 g and 890 mg.

5 Find the total length, in millimetres, of a piece of wood 82 cm long and another piece of wood 260 mm long.

6 Find the total mass, in kilograms, of 500 g of butter, 2 kg of potatoes, 1.5 kg of flour.

7 One can of baked beans has a mass of 220 g. What is the mass, in kilograms, of ten of these cans?

8 One fence post is 150 cm long. What length of wood, in metres, is needed to make ten such fence posts?

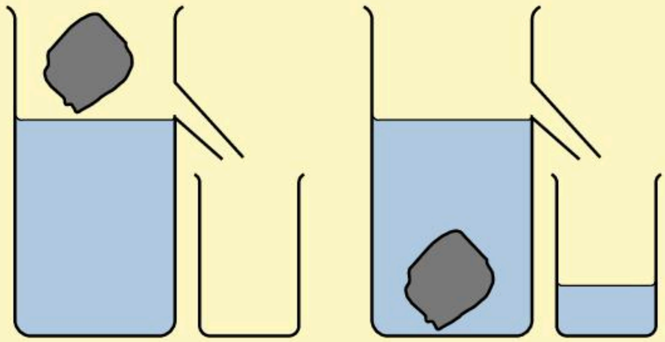
9 Find the perimeter of a square if each side is of length 8.3 cm. Give your answer in centimetres.

10 A wooden vegetable crate and its contents have a mass of 6.5 kg. If the crate has a mass of 1.2 kg what is the mass of its contents?

Metric units of volume and capacity

Volume measures the amount of space that an object occupies.

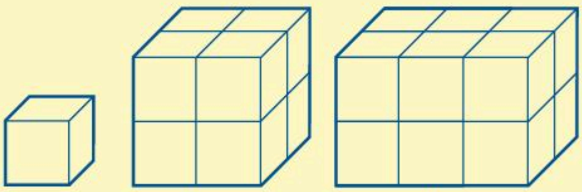
In the science laboratory you may well have seen a container with a spout similar to the one shown in the diagram (some people call this a Eureka can; do you know why?).



The container is filled with water to the level of the spout. Any solid which is put into the water will force a quantity of water into the measuring jug. The volume of this water will be equal to the volume of the solid. The volume of a solid is the amount of space it occupies.

Cubic units

As with area, we need a convenient unit for measuring volume. The most suitable unit is a cube. A cube has six faces. Each face is a square.



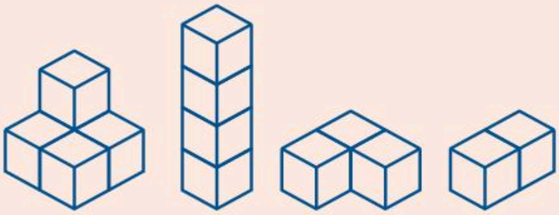
How many of the smallest cubes are needed to fill the same space as each of the solids A and B? Careful counting will show that 8 small cubes fill the same space as solid A and 12 small cubes fill the same space as solid B.

A cube with a side of 1 cm has a volume of one cubic centimetre which is written 1 cm^3.

Similarly a cube with a side of 1 mm has a volume of 1 mm^3 and a cube with a side of 1 m has a volume of 1 m^3.

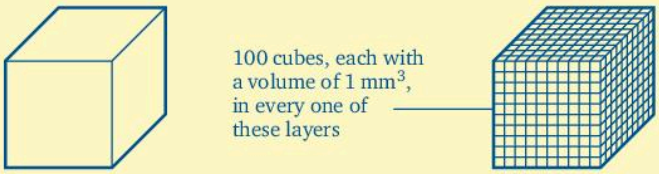
Puzzle

Which two of these shapes will fit together to form a cube?



Changing units of volume

Consider a cube of side 1 cm. If each edge is divided into 10 mm the cube can be divided into 10 layers each layer with 10 × 10 cubes of side 1 mm.



So $$1\,\text{cm}^3 = 10 \times 10 \times 10\,\text{mm}^3$$

i.e. $$1\,\text{cm}^3 = 1000\,\text{mm}^3$$

Similarly, since $1\,\text{m} = 100\,\text{cm}$

$$1 \text{ cubic metre} = 100 \times 100 \times 100\,\text{cm}^3$$

i.e. $$1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3$$

Exercise 6i

Express $2.4\,\text{m}^3$ in **a** cm^3 **b** mm^3.

a Since $1\,\text{m}^3 = 100 \times 100 \times 100\,\text{cm}^3$

$2.4\,\text{m}^3 = 2.4 \times 100 \times 100 \times 100\,\text{cm}^3$

$= 2\,400\,000\,\text{cm}^3$

b Since $1\,\text{m}^3 = 1000 \times 1000 \times 1000\,\text{mm}^3$

$2.4\,\text{m}^3 = 2.4 \times 1000 \times 1000 \times 1000\,\text{mm}^3$

$= 2\,400\,000\,000\,\text{mm}^3$

1 Which metric unit would you use to measure the volume of

a a room **b** a teaspoon **c** a can of cola?

Express in mm^3:

2 $8\,cm^3$ **3** $14\,cm^3$ **4** $6.2\,cm^3$ **5** $0.43\,cm^3$ **6** $0.092\,m^3$ **7** $0.04\,cm^3$

Express in cm^3:

8 $3\,m^3$ **10** $0.42\,m^3$ **12** $22\,mm^3$

9 $2.5\,m^3$ **11** $0.0063\,m^3$ **13** $731\,mm^3$

Capacity

When we buy a bottle of milk or a can of engine oil we are not usually interested in the external measurements or volume of the container. What really concerns us is the *capacity* of the container, i.e. how much milk can the bottle hold, or how much engine oil is inside the can.

The most common unit of capacity in the metric system is the *litre*. (A litre is usually the size of a large bottle of water.) A litre is much larger than a cubic centimetre but much smaller than a cubic metre. The relationship between these quantities is:

$$1000\,cm^3 = 1 \text{ litre}$$

i.e. a litre is the volume of a cube of side 10 cm

and $$1000 \text{ litres} = 1\,m^3$$

When the amount of liquid is small, such as dosages for medicines, the *millilitre* (ml) is used. A millilitre is a thousandth part of a litre, i.e.

$$1000\,ml = 1 \text{ litre} \quad \text{or} \quad 1\,ml = 1\,cm^3$$

Exercise 6j

1 Which unit would you use to give the volume of

a the book **d** a spoonful of water **g** a petrol can

b the room you are in **e** a lorry load of rubble **h** a \$20 coin

c one vitamin pill **f** a packet of cornflakes **i** a concrete building block?

Express 5.6 litres in cm^3.

$$1 \text{ litre} = 1000 \text{ cm}^3$$

so

$$5.6 \text{ litres} = 5.6 \times 1000 \text{ cm}^3$$

$$= 5600 \text{ cm}^3$$

Express in cm^3:

2 2.5 litres **4** 0.54 litres **6** 35 litres

3 1.76 litres **5** 0.0075 litres **7** 0.028 litres

Express in litres:

8 7000 cm^3 **9** 4000 cm^3 **10** 24 000 cm^3 **11** 600 cm^3

Express in litres:

12 5 m^3 **13** 12 m^3 **14** 4.6 m^3 **15** 0.067 m^3

Imperial units of length

Some imperial units are still used. For instance, in some places distances on road signs are still given in miles. One *mile* is roughly equivalent to $1\frac{1}{2}$ km. A better approximation is

5 miles is about 8 kilometres

Yards, *feet* and *inches* are other imperial units of length that are still used. In this system units are not always divided into ten parts to give smaller units so we have to learn 'tables'.

12 inches (in) = 1 foot (ft)

3 feet = 1 yard (yd)

1760 yards = 1 mile

Exercise 6k

Express 2 ft 5 in in inches.

First convert the number of feet into inches then add the number of inches.

$$
\begin{aligned}
2\text{ ft} &= 2 \times 12\text{ in} \\
&= 24\text{ in} \\
\therefore \quad 2\text{ ft } 5\text{ in} &= 24 + 5\text{ in} \\
&= 29\text{ in}
\end{aligned}
$$

Express the given quantity in the unit in brackets:

1	5 ft 8 in	(in)	**6**	2 miles 800 yd	(yd)
2	4 yd 2 ft	(ft)	**7**	5 yd 2 ft	(ft)
3	1 mile 49 yd	(yd)	**8**	10 ft 3 in	(in)
4	2 ft 11 in	(in)	**9**	9 yd 1 ft	(ft)
5	8 ft 4 in	(in)	**10**	9 ft 10 in	(in)

Express 52 inches in feet and inches.

There are 12 inches in 1 foot so we need to find how many complete 12's there are in 52.

A number of inches may be left over.

$$
\begin{aligned}
52\text{ in} &= 52 \div 12 \\
&= 4\text{ ft } 4\text{ in}
\end{aligned}
\qquad 12\overline{)52}\ \ 4\text{ r }4
$$

11	36 in	(ft)	**16**	2000 yd	(miles and yd)
12	29 in	(ft and in)	**17**	75 in	(ft and in)
13	86 in	(ft and in)	**18**	100 ft	(yd and ft)
14	9 ft	(yd)	**19**	120 in	(ft and in)
15	13 ft	(yd and ft)	**20**	30 000 yd	(miles and yd)

Imperial units of mass

The imperial units of mass that are still used are *pounds* and *ounces*. Other units of mass that you may still see are *hundredweights* and *tons* (not to be confused with tonnes).

16 ounces (oz) = 1 pound (lb)

112 pounds = 1 hundredweight (cwt)

20 hundredweight = 1 ton

Exercise 6I

Express the given quantity in terms of the units given in brackets:

1	2 lb 6 oz	(oz)	**6**	24 oz	(lb and oz)
2	1 lb 12 oz	(oz)	**7**	18 oz	(lb and oz)
3	4 lb 3 oz	(oz)	**8**	36 oz	(lb and oz)
4	3 tons 4 cwt	(cwt)	**9**	30 cwt	(tons and cwt)
5	1 cwt 50 lb	(lb)	**10**	120 lb	(cwt and lb)

Rough equivalence between metric and imperial units

If you shop in a supermarket you will find that nearly all goods are sold in grams or kilograms. However, some shops still sell goods in pounds and ounces. It is often useful to be able to convert, roughly, pounds into kilograms or grams into pounds. For a rough conversion it is good enough to say that

1 kg is about 2 lb

although one kilogram is slightly more than two pounds.

One metre is slightly longer than one yard but for a rough conversion it is good enough to say that

1 m is about 1 yd

Remember that the symbol ≈ means 'is approximately equal to' so

$$1\text{ kg} \approx 2\text{ lb}$$

$$1\text{ m} \approx 1\text{ yd or } 3\text{ ft}$$

Exercise 6m

Write 5 kg roughly in terms of the unit in brackets: 5 kg (lb)

$1\,\text{kg} \approx 2\,\text{lb}$, so 5 kg is approximately 5 times 2 lb

$$5\,\text{kg} \approx 5 \times 2\,\text{lb}$$

$$\therefore \quad 5\,\text{kg} \approx 10\,\text{lb}$$

Write 10 ft roughly in terms of the unit in brackets: 10 ft (m)

$3\,\text{ft} \approx 1\,\text{m}$, so you need to find the number of 3's in 10

$$10\,\text{ft} \approx 10 \div 3\,\text{m}$$

$$\therefore \quad 10\,\text{ft} \approx 3.3\,\text{m (to 1 d.p.)}$$

In questions **1** to **10**, write the first unit roughly in terms of the unit in brackets:

1 3 kg (lb)

2 2 m (ft)

3 4 lb (kg)

4 9 ft (m)

5 1.5 kg (lb)

6 5 m (ft)

7 3.5 kg (lb)

8 8 ft (m)

9 250 g (oz)

10 500 g (lb)

In questions **11** to **16** use the approximation 5 miles ≈ 8 km to convert the given number of miles into an approximate number of kilometres:

11 10 miles

12 20 miles

13 15 miles

14 100 miles

15 75 miles

16 40 miles

17 I buy a 5 lb bag of potatoes and two 1.5 kg bags of flour. What mass, roughly, in pounds do I have to carry?

18 A window is 6 ft high. Roughly, what is its height in metres?

19 I have a picture which measures 2 ft by 1 ft. Wood for framing it is sold by the metre. Roughly, what length of framing, in metres, should I buy?

20 In the supermarket I buy a 4 kg packet of sugar and a 5 lb bag of potatoes. Which is heavier?

21 In one catalogue a table cloth is described as measuring 4 ft by 8 ft. In another catalogue a different table cloth is described as measuring 1 m by 2 m. Which one is bigger?

22 The distance between Antigua and St Kitts is about 50 miles. The distance between Dominica and Martinique is about 140 kilometres. What is the difference, in miles, between the distances the two pairs of islands are apart?

23 A recipe requires 250 grams of flour. Roughly, how many ounces is this?

Converting from inches to centimetres and from centimetres to inches is often useful. For most purposes it is good enough to say that 1 inch $\approx 2\frac{1}{2}$ cm.

24 An instruction in an old knitting pattern says knit 6 inches. Maria has a tape measure marked only in centimetres. How many centimetres should she knit?

25 The instructions for repotting a plant say that it should go into a 10 cm pot. The flower pots that Tom has in his shed are marked 3 in, 4 in and 5 in. Which one should he use?

26 Mr Smith wishes to extend his gas pipe lines which were installed several years ago in 1 in and $\frac{1}{2}$ in diameter copper tubing. The only new piping he can buy has diameters of 10 mm, 15 mm, 20 mm or 25 mm. Use the approximation 1 in $\approx$ 2.5 cm to determine which piping he should buy that would be nearest to

a the 1 in pipes

b the $\frac{1}{2}$ in pipes.

27 A carpenter wishes to replace a 6 in floorboard. The only sizes available are metric and have widths of 12 cm, 15 cm, 18 cm and 20 cm. Use the approximation 1 in $\approx$ 2.5 cm to determine which one he should buy.

28 A shop sells material at \$210 per metre while the same material is sold in the local market at \$180 per yard. Using 4 in $\approx$ 10 cm find which is cheaper.

Investigation

1 Some imperial units have specialised uses, for example, furlongs are used to measure distances in horse racing courses and fathoms are used to measure the depth of water.

a Use reference materials to find out the relationships between these units and the more common imperial units of length.

b Find out as much as you can about other imperial units of distance and mass.

c Nautical miles are used to measure distances at sea. Find out what you can about nautical miles, including the rough equivalence of 1 nautical mile in miles and in kilometres.

2 A group of young secondary school pupils were asked to write down their heights and masses on sheets of paper which were gathered in.

This is a list of *exactly* what was written down.

a This group of children used a mixture of units. Some of the entries are unbelievable.

Which are they?

Give some of the reasons for these unbelievable entries.

Height	**Mass**
141 cm	35 kg
1.38 cm	4 stone
1.8 m	6.26 stone
4 feet 5 inches	4 kg
52 feet	6 stone
5 foot 4	8 stone
1 metre 53	$7\frac{1}{2}$ stone
1 metre 41 cm	28.0 kg
141 cm	5 stone 4 pounds
4 feet 7 inches	32 kg

b Find out how your group know their heights and masses; each of you write down your own height and mass on a piece of paper. Use whatever unit you know them in, and do not write your name on it.

Collect in the pieces of paper and write out a list like the one above.

c What official forms do you know about that ask for height?

What unit is required?

d Write down your own height and mass in both metric and imperial units.

Time

Time is measured in millennia (1 millennium = 1000 years), centuries, decades, years, months, weeks, days, hours, minutes and seconds.

There are 12 months in a year but the number of days in a month varies.

There are 365 days in a year, except for leap years when there are 366.

Remember:
Thirty days hath September, April, June and November. All the rest have thirty-one except for February clear, which has twenty-eight and twenty-nine in each leap year.

The relationships between weeks, days, hours, minutes and seconds are fixed:

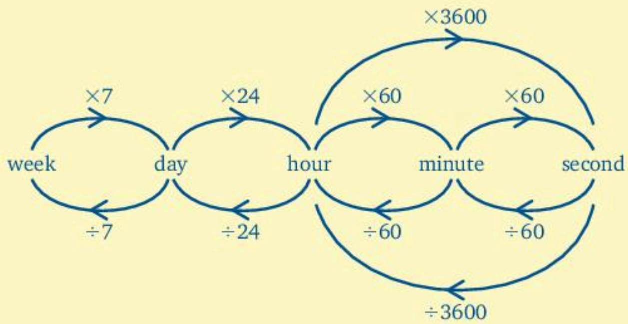
1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

When you change units of time, remember that you multiply when you change to a smaller unit, and you divide when you change to a larger unit.



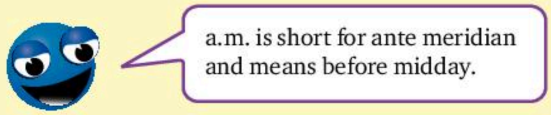
There are two ways of measuring the time of day: the 24-hour clock and the 12-hour clock.

The 24-hour clock uses the full 24 hours in a day, measuring from midnight through to the next midnight.

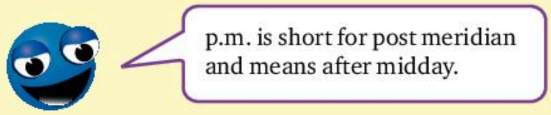
The time is given as a four-figure number, for example, 1346 hr and 0730 hr. The first two figures give the hours and the second two figures give the minutes. So 1346 hr means 13 hours and 46 minutes after midnight and 0730 hr means 7 hours and 30 minutes after midnight.

Sometimes there is a space or a colon between the hours and the minutes, for example 13 46 or 13:46.

The 12-hour clock uses the 12 hours from midnight to midday as a.m. times.



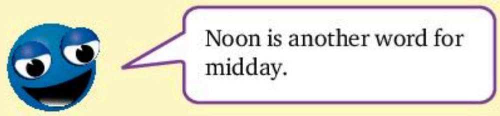
and the 12 hours from midday to midnight as p.m. times.



The time is written as a number of hours and a number of minutes followed by a.m. or p.m.
The hours and the minutes are usually separated by a stop, for example 6.30 a.m. means 6 hours and 30 minutes after midnight and 10.05 p.m. means 10 hours and 5 minutes after midday.

In the 24-hour clock, it is clear that 0000 hr means midnight and 1200 hr means midday.

But in the 12-hour clock, you need to write 'midnight' or 'midday' because 12.00 could mean either.



The time on this clock can be read as 2.56 p.m. or 1456 hr.

Exercise 6n

1 Look at this calendar.

Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

a Which month is this – August, September or October?

b Today is the 9th of the month. What day of the week is it?

c Today is the 24th of the month. What was the date a week ago today?

d The day after tomorrow is the third Wednesday of the month. What is the date today?

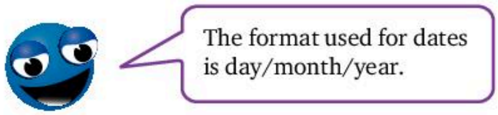
2 Elspeth goes on holiday on 8 June.
She returns on 21 June.
How many nights is she away?

3 David starts work on 1 September.
He gets paid on the twentieth of each month.
How many times does he get paid before Christmas?

4 The dates of birth of three people are:

Julie 14–3–93 Dennis 14–1–92 Johanne 14–8–93

a Who is the eldest?
b Who is the youngest?
c In which year will the youngest be 30?



5 The president of the local cricket club is elected every year at the Annual General Meeting.
This is a list of the presidents since the club was formed.

1928–42	S. Green
1942–48	P. Cave
1948–57	D. S. Short
1957–74	P. Baldrick
1974–82	H. Anthony
1982–96	D. S Short
1996–99	W. May
1999–07	C. D. Bowen
2007–	O. D. Williams

a For how many years was P. Baldrick president?
b Who was president for the greatest number of years without a break?
c Assuming that O. D. Williams continues as president, which year will he be elected to begin his 15th year?

6 Write

a 190 minutes in hours and minutes
b 450 hours in days and hours.

7 Write

a 5 minutes and 8 seconds in seconds.
b $3\frac{1}{2}$ hours in minutes.

8 Find

a 20 minutes as a fraction of an hour

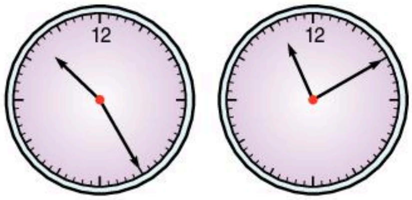
b 36 seconds as a fraction of an hour.

9 These clock faces show the time at the beginning and end of a history lesson.

a What time did the lesson start?

b What time did the lesson end?

c How long was the lesson?



Lesson begins Lesson ends

10 My ferry is due at 5.34 p.m.

a How long should it be before it arrives?

b Write 5.34 p.m. in 24-hour time.

11 Find the number of hours and minutes between

a 9.30 a.m. and 11.15 a.m. the same day

b 8.30 a.m. and 5.10 p.m. the same day

c 10.20 p.m. and 12.30 a.m. the next day.

12 The time needed to cook a chicken is 40 minutes per kilogram plus 20 minutes. How long should it take to cook a $3\frac{1}{2}$ kg chicken?

13 Susan's bus is due at 2005 hr.

a How many minutes should she have to wait?

b Write 2005 hr as an a.m. or p.m. time.

14 Find the period of time between

a 0320 hours and 0950 hours on the same day.

b 0535 hours and 1404 hours on the same day.

c 2100 hours and 0500 hours next day.

d 0000 hours and 0303 hours next day.

15 A plane leaves Kingston for Port of Spain.

The flight should take 2 hours 10 minutes.

The plane leaves Kingston on time at 1450 hours and is 15 minutes late arriving in Port of Spain. When does the plane arrive in Port of Spain?

16 The bus service from Westwick to Plimpton runs twice a day. This is the timetable.

Westwick		0945	1420
Red Farm Hill		1004	1439
Astleton	arr	1056	1531
	dep	1116	1545
Morgan's Hollow		1129	1559
Plimpton		1207	1637

a How long does each bus take to go from Westwick to Plimpton?

b Which two bus stops do you think are closest together?

Give a reason for your answer.

Temperature

There are two commonly used units for measuring temperature.
One is *degrees Celsius*.

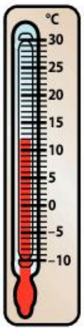
The freezing point of water is zero degrees Celsius. This is written 0 °C.
The boiling point of water is 100 degrees Celsius. This is written 100 °C.

The other unit is *degrees Fahrenheit*.
The freezing point of water is 32 degrees Fahrenheit. This is written 32 °F.
The boiling point of water is 212 °F.

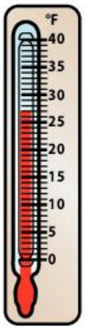
Some thermometers have both scales on them.

Exercise 6p

1 **a** What is the temperature shown on this thermometer?



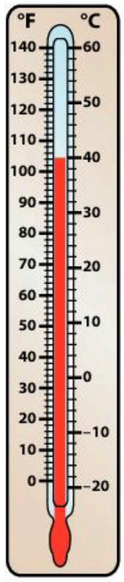
b What is the temperature shown on this thermometer?



c Which thermometer shows the higher temperature?

Give a reason for your answer.

2 This thermometer is marked in degrees Fahrenheit and in degrees Celsius.



a What Fahrenheit temperature does the thermometer show?

b What Celsius temperature does the thermometer show?

The temperature goes down by 20 °C.

c What is the new Celsius reading?

d What is the new Fahrenheit reading?

3 Use the thermometer in question **2** to convert

a 10 °C to degrees Fahrenheit

b 5 °C to degrees Fahrenheit

c 80 °F to degrees Celsius

d 35 °F to degrees Celsius

4 In August 2003, the temperature in London reached a record high of 101 °F.

Use these instructions to convert 101 °F to degrees Celsius.

1 Subtract 32°.

2 Divide your answer by 9.

3 Multiply your answer by 5.

Check your answer on the thermometer in question **2**.

5 The temperature in December in Scotland can go as low as −8 °C.

Use the thermometer in question **2** to convert −8 °C to degrees Fahrenheit.

6 Use these instructions to convert −8 °C to degrees Fahrenheit.

1 Multiply −8 by 9.

2 Divide your answer by 5.

3 Add 32 to your answer.

How does your result compare with your answer to question **5**?

Did you know?

There are two other temperature scales, Kelvin and Rankine. They both measure temperature from absolute zero. (Absolute zero is the lowest possible temperature, when nothing can move and there is no heat.)

Kelvin is based on degrees Celsius, so that an increment of 1 degree Kelvin (1 °K) = an increment of 1 °C, and Rankine is based on degrees Fahrenheit, so that an increment of 1 degree Rankine (1 °R) = an increment of 1 °F where

$$0\,°\text{K} = -273.15\,°\text{C}$$

$$0\,°\text{R} = -459.67\,°\text{F}$$

Time to do an internet search: find out what each of these scales is used for.

Mixed exercises

Exercise 6q

Express the given quantity in terms of the unit given in brackets:

1	4 km	(m)	**4**	250 g	(kg)	**7**	1 m 50 cm	(m)
2	30 g	(kg)	**5**	0.03 km	(cm)	**8**	2.8 cm	(mm)
3	3.5 m	(cm)	**6**	1250 m	(km)	**9**	65 g	(kg)

10 A can of meat has mass 429 g. What is the mass, in kilograms, of ten such cans?

11 Express 8 cm^3 in

a mm^3 **b** m^3.

12 Express 3500 cm^3 in litres.

Exercise 6r

Express the given quantity in terms of the unit in brackets:

1	236 cm	(m)	**5**	4 km 250 m	(km)
2	0.02 m	(mm)	**6**	3.6 t	(kg)
3	5 kg	(g)	**7**	2 kg 350 g	(kg)
4	500 mg	(g)	**8**	2 g	(mg)

9 Give 2 kg as an approximate number of pounds.

10 Use 1 inch ≈ 2.5 cm to give a rough conversion of 12 inches to cm.

Exercise 6s

Express the given quantity in terms of the unit in brackets:

1	5.78 t	(kg)	**5**	780 days	(years and days)
2	350 kg	(t)	**6**	$1\frac{1}{2}$ hours	(minutes)
3	0.155 mm	(cm)	**7**	2 km 50 m	(km)
4	1 t 560 kg	(t)			

8 A road sign in the UK says road works in 600 yards. Approximately how many metres is this?

9 Use 1 mile ≈ 1.6 km to convert 50 miles to an approximate number of kilometres.

10 Express 0.009 m^3 in

a cm^3 **b** mm^3

11 Express 0.44 litres in cm^3.

Did you know?

Why are there 112 pounds in a hundredweight?

Years ago, when a farmer had to pay tithes (a tithe is a tenth part), he had to pay the church one tenth of what he produced. One tenth of 112 is 11.2. Take this from 112 and you're left with 100.8. Rounded down to the nearest whole number this is 100. So, to have 100 pounds of wheat to sell a farmer needed to bring 112 pounds from the field. This is why there are 112 pounds in a hundredweight.

In this chapter you have seen that...

- ✔ the metric units of length in common use are the kilometre, the metre, the centimetre and the millimetre, where

$$1\,\text{cm} = 10\,\text{mm}$$
$$1\,\text{m} = 100\,\text{cm}$$
$$1\,\text{km} = 1000\,\text{m}$$

- ✔ the metric units of mass in common use are the tonne, the kilogram, the gram and the milligram, where

$$1\,\text{g} = 1000\,\text{mg}$$
$$1\,\text{kg} = 1000\,\text{g}$$
$$1\,\text{t} = 1000\,\text{kg}$$

- ✔ common imperial units of length are inches (in), feet (ft), yards (yd), and miles and the relationships between them are

$$12\,\text{in} = 1\,\text{ft}$$
$$3\,\text{ft} = 1\,\text{yd}$$
$$1760\,\text{yd} = 1\,\text{mile}$$

- ✔ common imperial units of mass are ounces (oz), pounds (lb), hundredweights (cwt) and tons and the relationships between them are

$$16\,\text{oz} = 1\,\text{lb}$$
$$112\,\text{lb} = 1\,\text{cwt}$$
$$20\,\text{cwt} = 1\,\text{ton}$$

- ✔ the time of day can be measured as a.m. or p.m. times or as 24-hour time
- ✔ to change to a smaller unit, e.g. km to m, multiply
- ✔ to change to a larger unit, e.g. mm to cm, divide
- ✔ you can roughly convert between metric and imperial units using

$$1\,\text{kg} \approx 2\,\text{lb}$$
$$1\,\text{m} \approx 1\,\text{yd}$$
$$5\,\text{miles} \approx 8\,\text{km}$$

- ✔ volume and capacity are measures of space
- ✔ degrees Celsius and degrees Fahrenheit are two different scales for measuring temperature. The freezing point of water is 0 °C and 32 °F. The boiling point of water is 100 °C and 212 °F.

7 Area and perimeter

At the end of this chapter you should be able to...

1 identify area as a measure of the amount of surface enclosed by the given shape
2 find an approximate area of a shape by counting the number of equal squares enclosed by the shape
3 identify a unit square as a square with side of unit length
4 use the unit square as the unit of measure for area
5 calculate the area of a square given the length of its side
6 calculate the area of a rectangle given its length and width
7 calculate the area of a compound rectilinear plane figure
8 identify the distance around a given shape as its perimeter
9 calculate the perimeter of a figure given the necessary measurements of its sides
10 calculate the number of squares, of given size, required to cover a given rectangle
11 convert from one metric unit of area to another.
12 calculate the length (width) of a rectangle, given its width (length) and area.

Did you know?

The original definition of a metre was the length of a platinum–iridium bar kept in controlled conditions in Paris, but in 1960 it was redefined as the length of the path travelled by light in a vacuum in an interval of $\frac{1}{299\,792\,458}$ of a second.

You need to know...

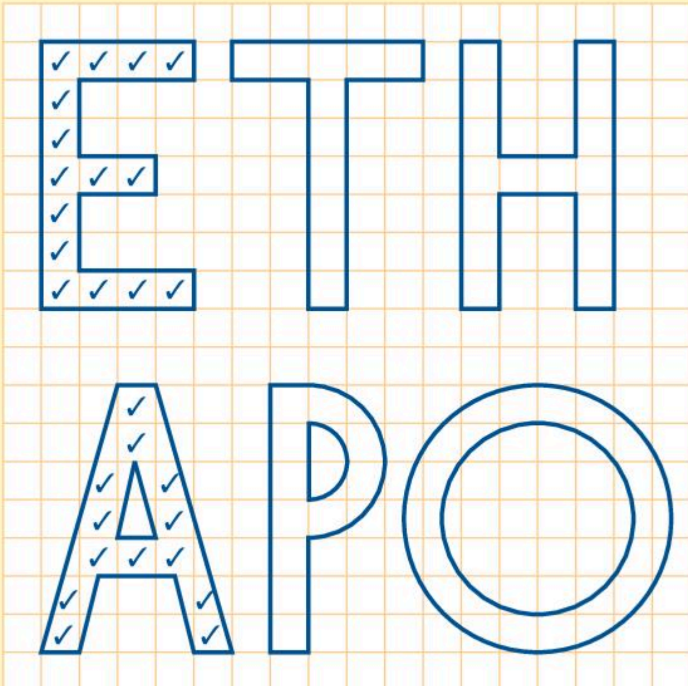
- ✔ how to multiply and divide by 10, 100, 1000, ...
- ✔ the multiplication tables up to 10×10
- ✔ how to multiply fractions.

Key words

area, perimeter, rectangle, square, square centimetre, square kilometre, square metre, square millimetre

Counting squares

The *area* of a shape or figure is the amount of surface enclosed within the lines which bound it. Below, six letters have been drawn on squared paper.



We can see by counting squares, that the area of the letter E is 15 squares.

Exercise 7a

What is the area of:

1 the letter T

2 the letter H?

What is the approximate area of the letter A?

Sometimes the squares do not fit exactly on the area we are finding. When this is so we count a square if more than half of it is within the area we are finding, but exclude it if more than half of it is outside.

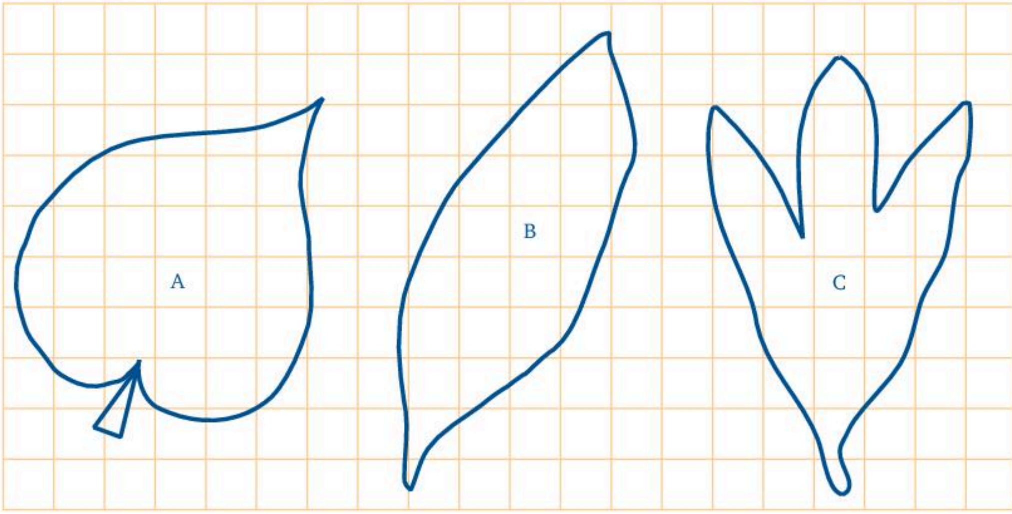
By counting squares in this way the approximate area of the letter A is 13 squares.

What is the approximate area of:

3 the letter P

4 the letter O?

The next set of diagrams shows the outlines of three leaves.



By counting squares find the approximate area of:

5 the leaf marked A

6 the leaf marked B

7 the leaf marked C.

8 Which leaf has

  a the largest area

  b the smallest area?

In each of the following questions find the area of the given figure by counting squares.

The following method may be used to find the area:

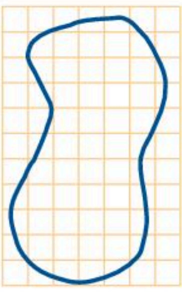
1 Count the number of complete squares.

2 Count the number of incomplete squares and divide this number by 2.

3 Add the results obtained in steps **1** and **2** above.

The answer is the required area.

9



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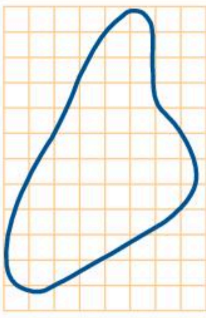

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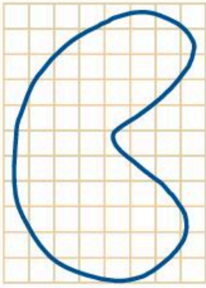

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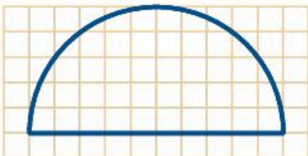

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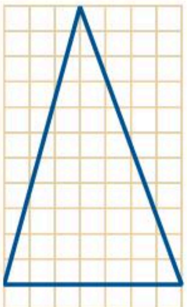

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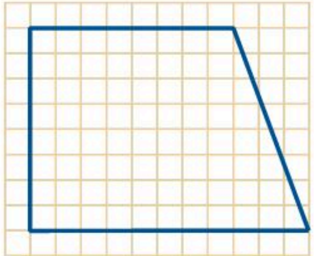

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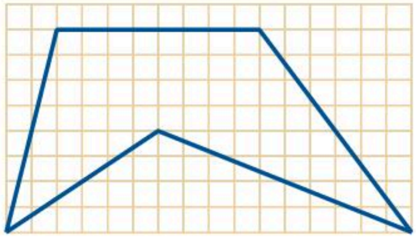


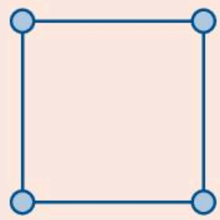
16

Puzzle

A man has a square swimming pool in his garden with a concrete post at the outside on each corner as shown.

He wants to double the area of his pool and still keep it square, so that none of the posts have to be moved and are still outside the pool.

How can he do this?



Units of area

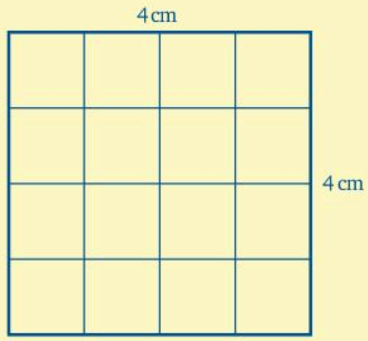
There is nothing special about the size of square we have used. If other people are going to understand what we are talking about when we say that the area of a certain shape is 12 squares, we must have a square or unit of area which everybody understands and which is always the same.

A metre is a standard length and a square with sides 1 m long is said to have an area of one *square metre*. We write one square metre as $1\,m^2$. Other agreed lengths such as millimetres, centimetres and kilometres, are also in use. The unit of area used depends on what we are measuring.

We could measure the area of a small coin in *square millimetres* (mm^2), the area of the page of a book in *square centimetres* (cm^2), the area of a roof in square metres (m^2) and the area of an island in *square kilometres* (km^2).

Area of a square

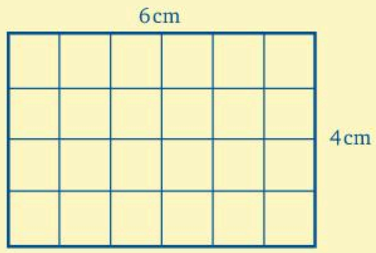
The *square* is the simplest figure of which to find the area. If we have a square whose side is 4 cm long it is easy to see that we must have 16 squares, each of side 1 cm, to cover the given square,



i.e. the area of a square of side 4 cm is $16\,cm^2$.

Area of a rectangle

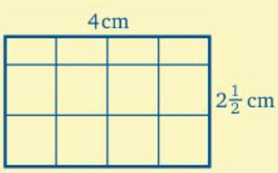
If we have a *rectangle* measuring 6 cm by 4 cm we require 4 rows each containing 6 squares of side 1 cm to cover this rectangle,



i.e. the area of the rectangle $= 6 \times 4\,cm^2$

$= 24\,cm^2$

A similar result can then be found for a rectangle of any size; for example a rectangle of length 4 cm and breadth $2\frac{1}{2}$ cm has an area of $4 \times 2\frac{1}{2}\,cm^2$.



In general, for any rectangle

area = length × breadth

Exercise 7b

Find the area of each of the following shapes, clearly stating the units involved:

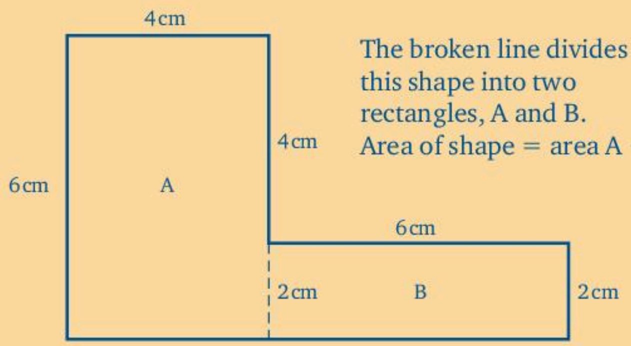
1 A square of side 2 cm
2 A square of side 8 cm
3 A square of side 10 cm
4 A square of side 5 cm
5 A square of side 1.5 cm
<u>6</u> A square of side 2.5 cm
<u>7</u> A square of side 0.7 m
<u>8</u> A square of side 1.2 cm
<u>9</u> A square of side $\frac{1}{2}$ km
<u>10</u> A square of side $\frac{3}{4}$ m
11 A rectangle measuring 5 cm by 6 cm
12 A rectangle measuring 6 cm by 8 cm
13 A rectangle measuring 3 m by 9 m
14 A rectangle measuring 14 cm by 20 cm
15 A rectangle measuring 1.8 mm by 2.2 mm
<u>16</u> A rectangle measuring 35 km by 42 km
<u>17</u> A rectangle measuring 1.5 m by 1.9 m
<u>18</u> A rectangle measuring 4.8 cm by 6.3 cm
<u>19</u> A rectangle measuring 95 cm by 240 cm
<u>20</u> A rectangle measuring 150 mm by 240 mm

Compound figures

Exercise 7c

You can often find the area of a figure by dividing it into two or more rectangles.

Find the area of the following figure.



The broken line divides this shape into two rectangles, A and B.
Area of shape = area A + area B

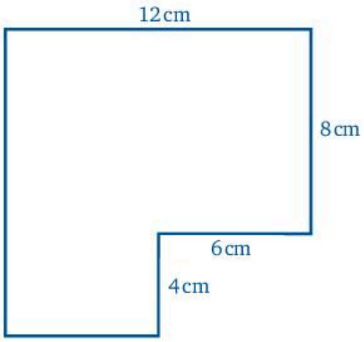
Area of A $= 6 \times 4\,\text{cm}^2 = 24\,\text{cm}^2$

Area of B $= 6 \times 2\,\text{cm}^2 = 12\,\text{cm}^2$

Therefore area of whole figure $= 24\,\text{cm}^2 + 12\,\text{cm}^2 = 36\,\text{cm}^2$.

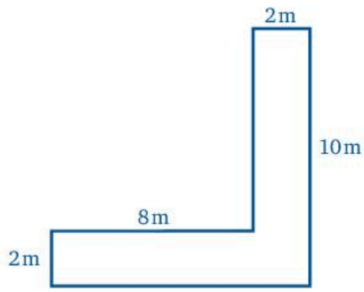
Find the areas of the following figures by dividing them into rectangles.

1



Sketch the diagram. Draw a line to divide it into two rectangles. There is often more than one way of doing this. Label the rectangles A and B. Work out and mark in any extra lengths you need to find the areas of the rectangles.

2



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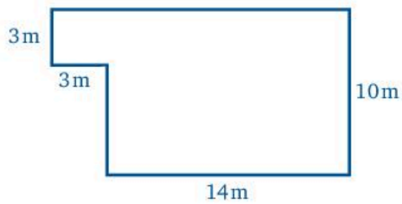

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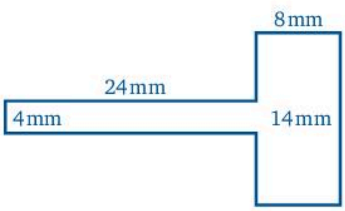

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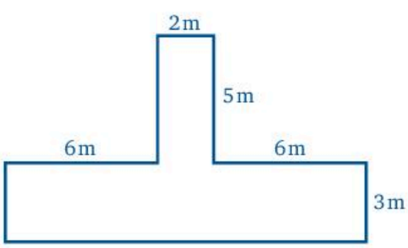

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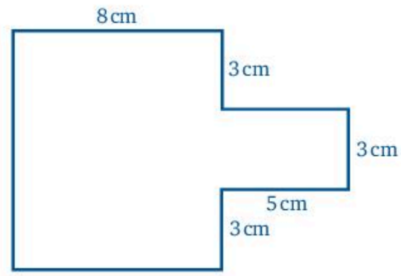

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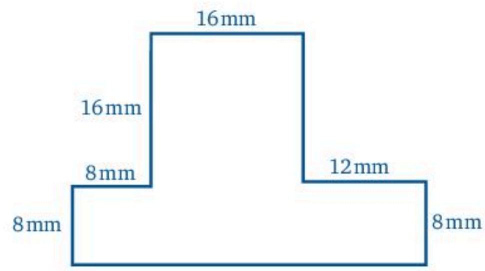

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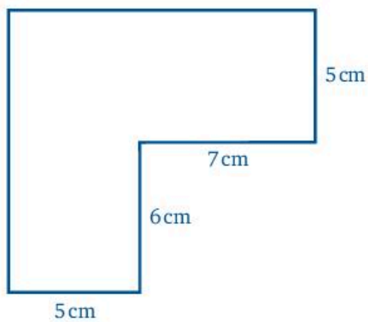

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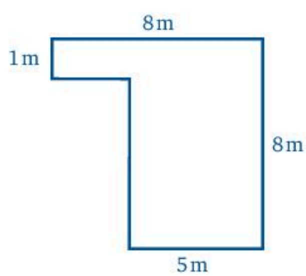

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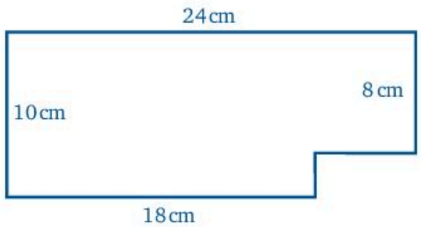
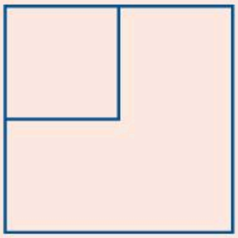


Puzzle



A farmer has a square field. He has already planted a quarter of the field with sugar cane as shown in the diagram. He now wants to divide the remainder of the field into four equal plots all the same size and shape. How will he do it?

Perimeter

The *perimeter* of a shape is the total length of all its sides.

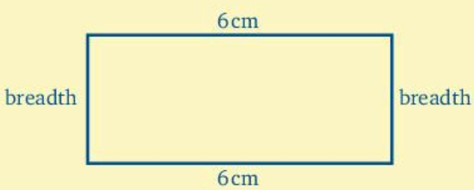
The perimeter of the square on page 150 is

$$4\,\text{cm} + 4\,\text{cm} + 4\,\text{cm} + 4\,\text{cm} = 16\,\text{cm}$$

Exercise 7d

Find the perimeter of each shape given in Exercise 7b, clearly indicating units.

If we are given a rectangle whose perimeter is 22 cm and told that the length of the rectangle is 6 cm it is possible to find its breadth and its area.



The two lengths add up to 12 cm so the two breadths add up to (22 – 12) cm = 10 cm.

Therefore the breadth is 5 cm.

$$\begin{aligned}\text{The area of this rectangle} &= 6 \times 5\,\text{cm}^2\\ &= 30\,\text{cm}^2\end{aligned}$$

Exercise 7e

The following table gives some of the measurements for various rectangles. Fill in the values that are missing:

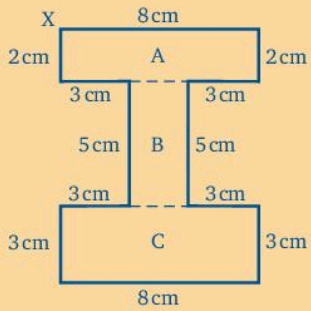
	Length	Breadth	Perimeter	Area
1	4 cm		12 cm	
2	5 cm		14 cm	
3		3 m	16 m	
4		6 mm	30 mm	
5	6 cm			30 cm^2
6	12 m			120 m^2
7		4 km		36 km^2
8		7 mm		63 mm^2
9		5 cm	60 cm	
10	21 cm			1680 cm^2

Problems

Exercise 7f

Find for the following figure

a the perimeter **b** the area.



a Starting at X, the distance all round the figure and back to X is

$8 + 2 + 3 + 5 + 3 + 3 + 8 + 3 + 3 + 5 + 3 + 2\text{ cm} = 48\text{ cm}$.

Therefore the perimeter is 48 cm.

b Divide the figure into three rectangles A, B and C.

Then the area of A $= 8 \times 2\,\text{cm}^2 = 16\,\text{cm}^2$

the area of B $= 5 \times 2\,\text{cm}^2 = 10\,\text{cm}^2$

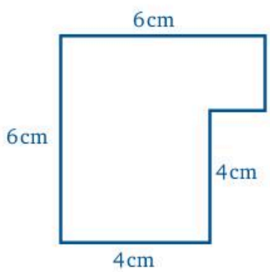
and the area of C $= 8 \times 3\,\text{cm}^2 = 24\,\text{cm}^2$

Therefore the total area $= (16 + 10 + 24)\,\text{cm}^2 = 50\,\text{cm}^2$.

Find for each of the following figures

a the perimeter **b** the area.

1



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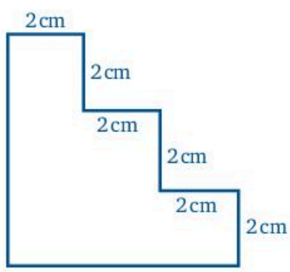

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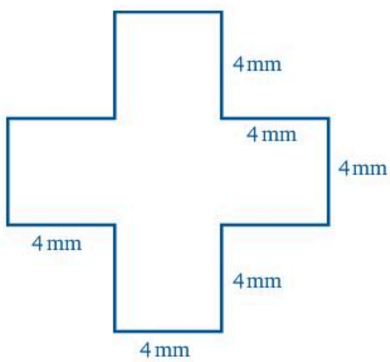

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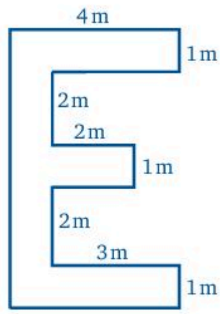

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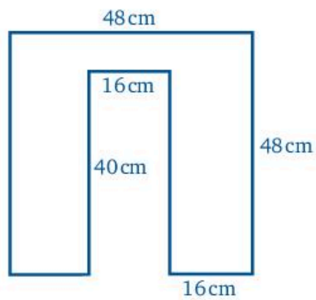
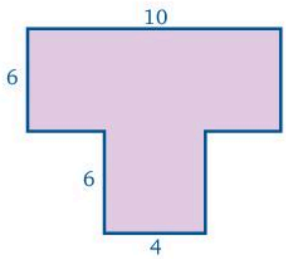


Find the areas of the following figures in square centimetres.
The measurements are all in centimetres.

6



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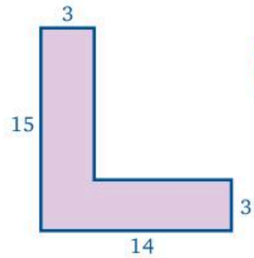

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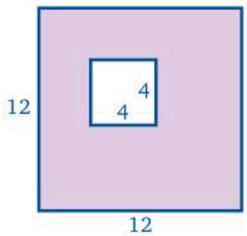

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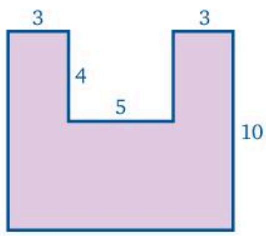
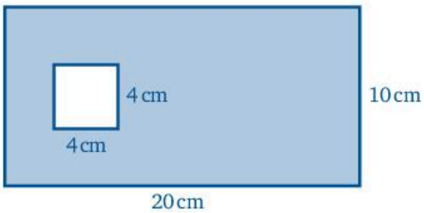
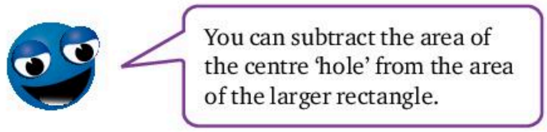


In each of the following figures find the area that is shaded:

10





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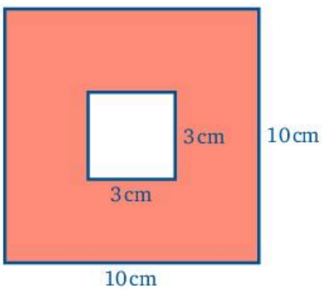

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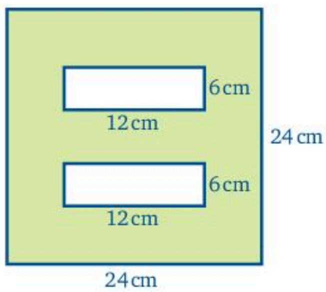

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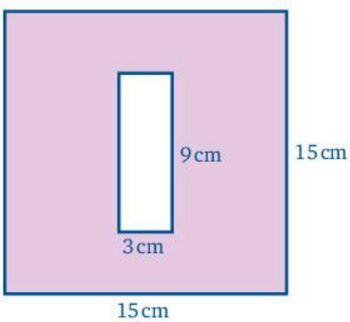

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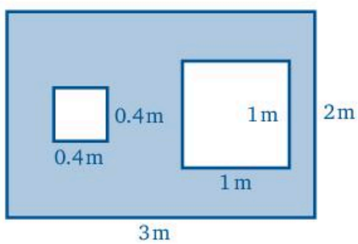
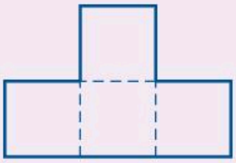


Investigation

This shape is made with 1 cm squares.



1 This shape has a perimeter of 10 cm. What is its area?

2 Find other shapes made from 1 cm squares that also have a perimeter of 10 cm.

Which one has the largest area?

3 Find other shapes that have the same area as the shape above.

Which shape has the shortest perimeter?

4 Investigate different shapes with a perimeter of 16 cm.

Find the shape with the largest possible area.

5 Investigate different shapes with an area of 6 cm^2.

Which shape has the shortest perimeter?

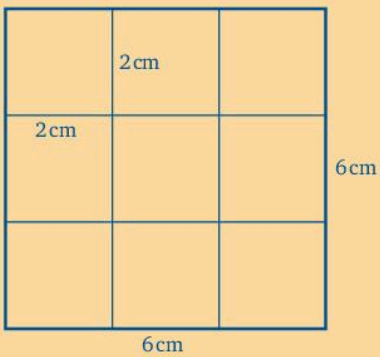
6 For a given area, what shape has the shortest perimeter?

7 A rectangle has the same number of square centimetres of area as it has centimetres of perimeter.

Find possible whole number values for the length and breadth of this rectangle. (There are two different rectangles with this property.)

Exercise 7g

Draw a square of side 6 cm. How many squares of side 2 cm are required to cover it?



Three 2 cm squares will fit along each edge.

We see that 9 squares of side 2 cm are required to cover the larger square whose side is 6 cm.

1 Draw a square of side 4 cm. How many squares of side 2 cm are required to cover it?

2 Draw a square of side 9 cm. How many squares of side 3 cm are required to cover it?

3 Draw a rectangle measuring 6 cm by 4 cm. How many squares of side 2 cm are required to cover it?

4 Draw a rectangle measuring 9 cm by 6 cm. How many squares of side 3 cm are required to cover it?

5 How many squares of side 5 cm are required to cover a rectangle measuring 45 cm by 25 cm?

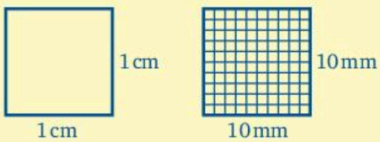
6 How many squares of side 4 cm are required to cover a rectangle measuring 1 m by 80 cm?

Changing units of area

A square of side 1 cm may be divided into 100 equal squares of side 1 mm,

$$1\text{ cm} \times 1\text{ cm} = 10\text{ mm} \times 10\text{ mm}$$

i.e. $1\text{ cm}^2 = 100\text{ mm}^2$



Similarly since $1\text{ m} = 100\text{ cm}$

$$1\text{ m} \times 1\text{ m} = 100\text{ cm} \times 100\text{ cm}$$

i.e. $1\text{ m}^2 = 10\,000\text{ cm}^2$

and as $1\text{ km} = 1000\text{ m}$

$$1\text{ km} \times 1\text{ km} = 1000\text{ m} \times 1000\text{ m}$$

i.e. $1\text{ km}^2 = 1\,000\,000\text{ m}^2$

When we convert from a unit of area which is large to a unit of area which is smaller we must remember that the number of units will be bigger,

e.g. $2\text{ km}^2 = 2 \times 1000\text{ m} \times 1000\text{ m} = 2 \times 1\,000\,000\text{ m}^2$

$= 2\,000\,000\text{ m}^2$

and $12\text{ m}^2 = 12 \times 10\,000\text{ cm}^2$

$= 120\,000\text{ cm}^2$

If we convert from a unit of area which is small into one which is larger the number of units will be smaller,

e.g. $500\,\text{mm}^2 = \frac{500}{100}\,\text{cm}^2 = 5\,\text{cm}^2$

Exercise 7h

Express $5\,\text{m}^2$ in **a** cm^2 **b** mm^2.

a Since $1\,\text{m}^2 = 100\,\text{cm} \times 100\,\text{cm}$

$5\,\text{m}^2 = 5 \times 100\,\text{cm} \times 100\,\text{cm} = 50\,000\,\text{cm}^2$

b Since $1\,\text{cm}^2 = 100\,\text{mm}^2$

$50\,000\,\text{cm}^2 = 50\,000 \times 100\,\text{mm}^2 = 5\,000\,000\,\text{mm}^2$

Therefore $5\,\text{m}^2 = 50\,000\,\text{cm}^2 = 5\,000\,000\,\text{mm}^2$.

1 Express in cm^2:

a $3\,\text{m}^2$ **b** $12\,\text{m}^2$ **c** $7.5\,\text{m}^2$ **d** $82\,\text{m}^2$ **e** $8\frac{1}{2}\,\text{m}^2$

2 Express in mm^2:

a $14\,\text{cm}^2$ **b** $3\,\text{cm}^2$ **c** $7.5\,\text{cm}^2$ **d** $26\,\text{cm}^2$ **e** $32\frac{1}{2}\,\text{cm}^2$

3 Express $0.056\,\text{m}^2$ in

a cm^2 **b** mm^2

Express $354\,000\,000\,\text{mm}^2$ in **a** cm^2 **b** m^2.

a Since $100\,\text{mm}^2 = 1\,\text{cm}^2$

$354\,000\,000\,\text{mm}^2 = \frac{354\,000\,000}{100}\,\text{cm}^2 = 3\,540\,000\,\text{cm}^2$

b Since $100 \times 100\,\text{cm}^2 = 1\,\text{m}^2$

$3\,540\,000\,\text{cm}^2 = \frac{3\,540\,000}{100 \times 100}\,\text{m}^2 = 354\,\text{m}^2$

Therefore $354\,000\,000\,\text{mm}^2 = 3\,540\,000\,\text{cm}^2 = 354\,\text{m}^2$.

4 Express in cm^2:

a $400\,\text{mm}^2$ **b** $2500\,\text{mm}^2$ **c** $50\,\text{mm}^2$ **d** $25\,\text{mm}^2$ **e** $734\,\text{mm}^2$

5 Express in m^2:

a $5500\,\text{cm}^2$ **b** $140\,000\,\text{cm}^2$ **c** $760\,\text{cm}^2$ **d** $18\,600\,\text{cm}^2$ **e** $29\,700\,000\,\text{cm}^2$

6 Express in km^2:

a $7\,500\,000\,\text{m}^2$ **b** $430\,000\,\text{m}^2$ **c** $50\,000\,\text{m}^2$ **d** $245\,000\,\text{m}^2$ **e** $176\,000\,000\,\text{m}^2$

Sometimes questions ask us to find the area of a rectangle in different square units from those in which the length and breadth are given. When this is so, we must change the units of the measurements we are given so that they 'match' the square units required in the answer.

Exercise 7i

Find the area of a rectangle measuring 50 cm by 35 cm.
Give your answer in m^2.

(Since the answer is to be given in m^2 we express both the length and breadth in m.)

Breadth of rectangle = 35 cm = 0.35 m

Length of rectangle = 50 cm = 0.5 m

Therefore area of rectangle = $0.35 \times 0.5 = 0.175\ m^2$

Find the area of each of the following rectangles, giving your answer in the unit in brackets:

	Length	Breadth	
1	10 m	0.5 m	(cm^2)
2	6 cm	3 cm	(mm^2)
3	50 m	0.35 m	(cm^2)
4	1.4 m	1 m	(cm^2)
5	400 cm	200 cm	(m^2)
6	3 m	$\frac{1}{2}$ m	(cm^2)
7	$2\frac{1}{2}$ m	$1\frac{1}{2}$ m	(cm^2)
8	1.5 cm	1.2 cm	(mm^2)
9	0.4 km	0.3 km	(m^2)
10	0.45 km	0.05 km	(m^2)

Finding a length when the area is given

Exercise 7j

Find the length of a rectangle of area 20 cm² and width 2.5 cm.

$$\text{Area} = \text{length} \times \text{width}, \quad \text{or} \quad A = l \times b,$$

then

$$\text{length} = \frac{\text{area}}{\text{width}} \quad \text{or} \quad l = \frac{A}{b}$$

$$\text{length} = \frac{20}{2.5}\ \text{cm}$$

$$= \frac{200}{25}\ \text{cm}$$

$$= 8\ \text{cm}$$

Find the missing measurements for the following rectangles:

	Area	Length	Width
1	2.4 cm²	6 cm	
2	20 cm²	4 cm	
3	36 m²		3.6 m
4	108 mm²	27 mm	
5	3 cm²		0.6 cm
6	6 m²	4 m	
7	20 cm²		16 cm
8	7.2 m²		2.4 m
9	4.2 m²		0.6 m
10	14.4 cm²	2.4 cm	

Mixed problems

Exercise 7k

In questions **1** to **4** find

a the area of the playing surface

b the perimeter of the playing surface.

1 A soccer field measuring 110 m by 75 m.

2 A rugby pitch measuring 100 m by 70 m.

3 A playing field measuring 120 m by 70 m.

4 A tennis court measuring 26 m by 12 m.

5 A roll of wallpaper is 10 m long and 50 cm wide. Find its area in square metres.

6 A school hall measuring 20 m by 15 m is to be covered with square floor tiles of side 50 cm.
How many tiles are required?

7 A rectangular carpet measures 4 m by 3 m. Find its area. How much would it cost to clean at 75 c per square metre?

8 The top of my desk is 150 cm long and 60 cm wide. Find its area.

9 How many square linen serviettes, of side 50 cm, may be cut from a roll of linen 25 m long and 1 m wide?

10 How many square concrete paving slabs, each of side $\frac{3}{4}$ m, are required to pave a rectangular yard measuring 9 m by 6 m?

Exercise 7l

Select the letter that gives the correct answer.

1 The area of a square of side 12 cm is

A 48 cm^2 **B** 72 cm^2 **C** 144 cm^2 **D** 288 cm^2

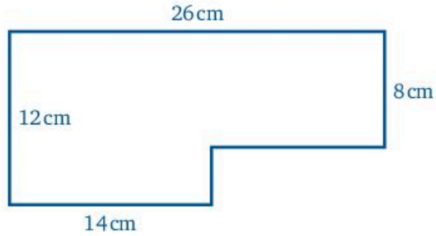
2 The area of a rectangle measuring 4 cm by 8 cm is

A 16 cm^2 **B** 32 cm^2 **C** 64 cm^2 **D** 84 cm^2

3 The area of a rectangle measuring 8 cm by 8 mm is

A 6.4 cm^2 **B** 32 cm^2 **C** 64 cm^2 **D** 128 cm^2

4 The area of this shape is



A $208\,cm^2$
B $256\,cm^2$
C $264\,cm^2$
D $312\,cm^2$

5 The perimeter of the shape given in question **4** is

A 64 cm B 68 cm C 74 cm D 76 cm

6 The area of a rectangle is $168\,cm^2$. If it is 14 cm long its breadth is

A 10 cm B 12 cm C 14 cm D 16 cm

Did you know?

Why study mathematics?

Mathematics is found in:

- Your home

  Mathematics is used in home building.

  Frames are strengthened by triangle-bracing.

  Geometric shapes are used to beautify houses.

- Your diet

  A good diet contains proper amounts of basic food nutrients. Grams and percentages of daily amounts of nutrients are written on the packaging of foods.

  Persons on special diets use metric scales to measure their intake.

- Your career

  The career you choose may require mathematics. Some are Accountants, Architects, Bank workers, Dieticians, Draftsmen, Engineers, Electricians, Carpenters, Surveyors and others.

- Your sports

  We use mathematics to calculate batting and other averages.

  To find the chance of a victory we use probability.

These are but a few uses of mathematics.

In this chapter you have seen that...

- ✔ you can find the area of an irregular shape by putting it on a grid and counting squares
- ✔ the unit of area used has to be a standard size square. Those in common use are
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)

  The relationships between them are
  - $1\,cm^2 = 100\,mm^2$
  - $1\,m^2 = 10\,000\,cm^2$
  - $1\,km^2 = 1\,000\,000\,m^2$
- ✔ the area of a square is found by multiplying the length of a side by itself
- ✔ the area of a rectangle is found by multiplying its length by its breadth
- ✔ compound shapes can often be divided into two or more rectangles
- ✔ the perimeter of a figure is the total length of all its sides
- ✔ when you convert to a smaller unit of area you multiply, and when you convert to a larger unit of area you divide.

8 Solids and nets

At the end of this chapter you should be able to...

1. use the correct geometrical terms
2. identify the different special quadrilaterals
3. draw tessellations
4. draw nets to make solids.

You need to know...

✔ how to use square grid paper to copy diagrams.

Key words

cube, cuboid, edge, face, hexagon, kite, line, line segment, net, parallel, parallelogram, pentagon, perpendicular, point, plane figure, polygon, prism, pyramid, quadrilateral, ray, rectangle, regular polygon, rhombus, solid, square, tessellation, trapezium, triangle, vertex (plural vertices), 2D (two-dimensional), 3D (three-dimensional)

Terms used in geometry

Geometry is about investigating shapes.

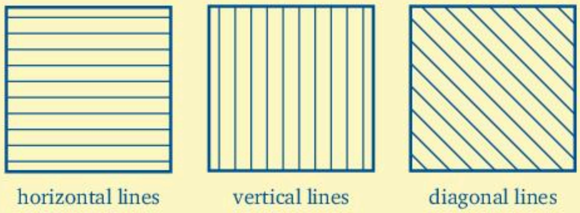
The terms we use in geometry are

- points: a *point* has no width, length or depth. We cannot draw a point, so we draw a small dot.
- lines: a *line* can be straight or curved. A line has no start and no end. A line has no thickness.

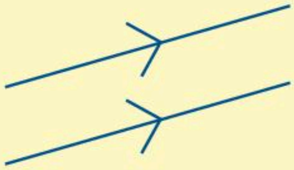
- line segments: a *line segment* is part of a line between two points.

- rays: a *ray* is a part of a line with only one end point.

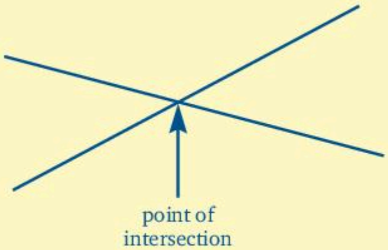
- horizontal, vertical and diagonal lines.



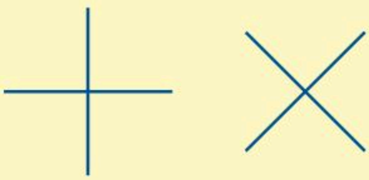
- parallel lines: *parallel* lines are always the same distance apart. Parallel lines are marked with arrows.



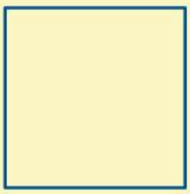
- intersecting lines: *intersecting* lines cross each other.



- perpendicular lines: a north–south line and an east–west line are *perpendicular*. The lines remain perpendicular when they are rotated.



- plane figures: these are two-dimensional (flat) shapes bounded by line segments, for example a square. A *plane figure* is a *2D* shape.

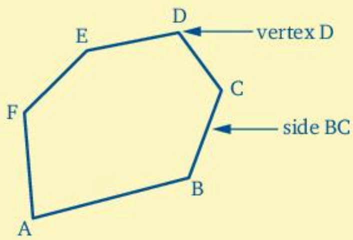


- solids: a *solid* occupies three-dimensional space, for example a cube. A solid is a *3D* shape.

Plane figures

A plane figure bounded by straight line segments is called a *polygon*. Polygons have sides and *vertices* (singular *vertex*). A vertex is where two sides meet.

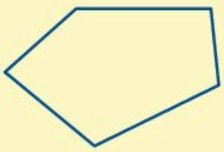
Vertices are labelled with capital letters and a side is named by the letters at each end. The figure is named by all the letters in order around its vertices, so the figure in the diagram is ABCDEF.



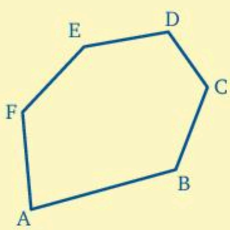
A plane figure bounded by three straight line segments is called a *triangle*.

A plane figure bounded by four straight line segments is called a *quadrilateral*.

A plane figure bounded by five straight line segments is called a *pentagon*.

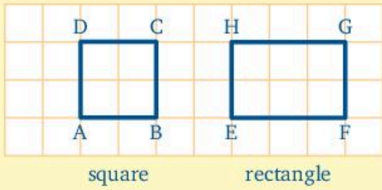


The figure ABCDEF shown here has six sides and is called a *hexagon*.



Special quadrilaterals

A *square* and a *rectangle* are both quadrilaterals. A square is a rectangle, but not all rectangles are squares.



In a square

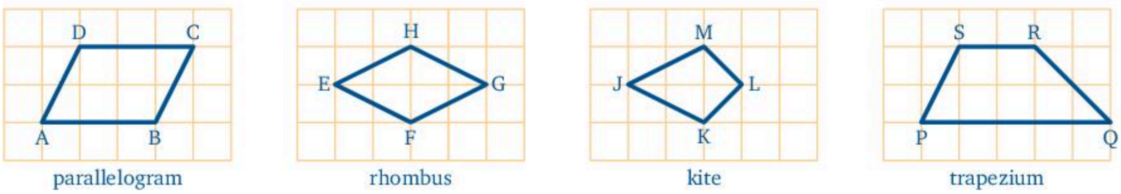
- all the sides are the same length (i.e. AB = BC = CD = DA)
- opposite sides are parallel (i.e. AB is parallel to DC; BC is parallel to AD)
- adjacent sides are perpendicular. (i.e. AB is perpendicular to BC; BC is perpendicular to CD; CD is perpendicular to AB)

In a rectangle

- the opposite sides are the same length (i.e. EF = GH and FG = HE)
- opposite sides are parallel (i.e. EF is parallel to HG and EH is parallel to FG)
- adjacent sides are perpendicular. (i.e. EF is perpendicular to FG; EG is perpendicular to GH; GH is perpendicular to HE and HE is perpendicular to EF)

The other special quadrilaterals are the *parallelogram*, the *rhombus*, the *kite* and the *trapezium*.

Exercise 8a



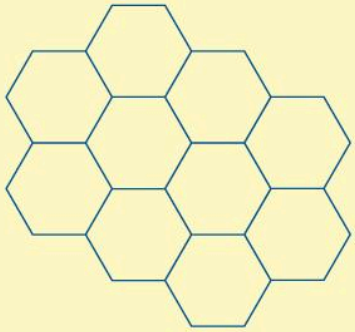
Each of the figures in the diagram is a quadrilateral. Copy them onto 1 cm squared paper.

For each one, name the sides, if any, which are

a the same length
b parallel
c perpendicular.

Tessellations

Shapes that fit together on a flat surface without leaving any gaps are said to tessellate. The diagram shows a *tessellation* using regular hexagons. (A *regular polygon* has all sides the same length, and all angles at the vertices are the same size.)



Exercise 8b

1 On 1 cm squared paper, draw two different tessellations using a rectangle.

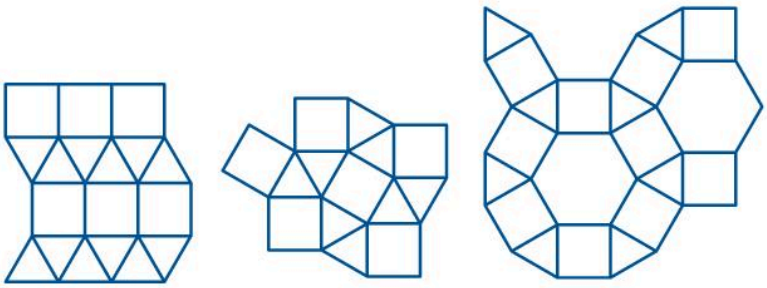
2 Repeat question 1 using the parallelogram in Exercise 8a.

3 Repeat question 1 using the rhombus in Exercise 8a.

4 Draw any triangle. Trace it several times and then cut out your triangles. Make some tessellations using your triangles.

5 Will all triangles tessellate? Try repeating question **4** with different triangles.

6 Regular hexagons, squares and regular triangles can be combined to make interesting patterns. Some examples are given below:



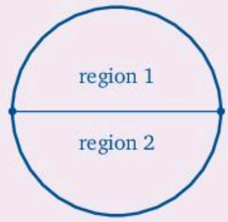
Copy these patterns and extend them. (If you make templates to help you, make each shape of side 2 cm.)

7 Make some patterns of your own using the shapes in question **6**.

Investigation

The diagram shows a circle with 2 evenly spaced dots and a line segment joining the dots.

The diagram has 2 regions.

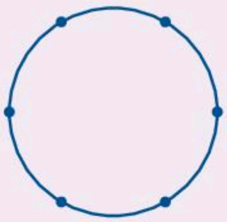


These diagrams show circles with 3 evenly spaced dots, 4 evenly spaced dots and 5 evenly spaced dots respectively. Lines are drawn between the dots.

1 Copy and complete this table.

Number of dots	2	3	4	5			
Number of regions	2	4					

2 Trace this circle and its dots. Draw lines joining all the dots and count the regions. Add these numbers to your table.

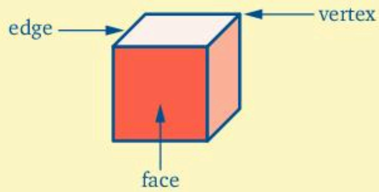


3 Can you see a pattern connecting the number of dots and the number of regions? You can try adding further circles with more dots, but make sure that the dots are evenly spaced.

4 Explain why it is important that the dots are evenly spaced.

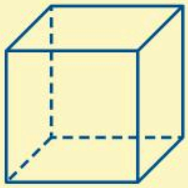
Solids

The *solids* we investigate in this chapter have plane *faces*, *edges* and vertices (singular vertex). The edge is where the faces meet and the vertex is where the edges meet.



This drawing of a *cube* shows the hidden edges as broken lines. It shows that a cube has

6 faces
12 edges
8 vertices.



Nets

Any solid with flat faces can be made from a flat sheet.

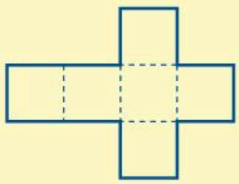
(We are using the word 'solid' for any object that takes up space, i.e. for any three-dimensional object, and such an object can be hollow.)

A cube can be made from six separate squares.

We can avoid a lot of unnecessary sticking if we join some squares together before cutting out.

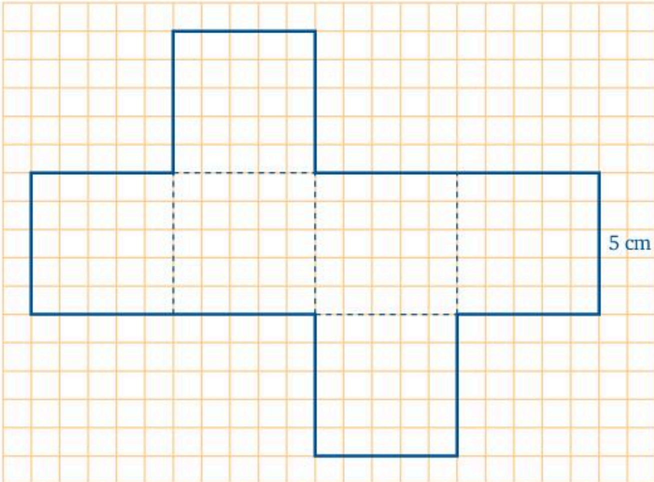
This is called a *net*.

There are other arrangements of six squares that can be folded up to make a cube. Not all arrangements of six squares will work however, as we will see in the next exercise.



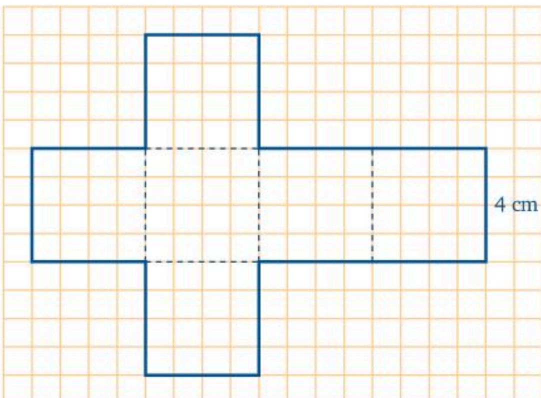
Exercise 8c

1 Below is the net of a cube of edge 5 cm.



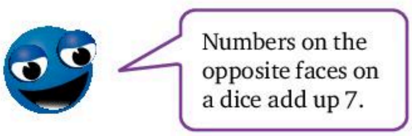
Draw the net on 1 cm squared paper and cut it out. Fold it along the broken lines. Fix it together with sticky tape.

2 Below is the net of a cube of edge 4 cm.

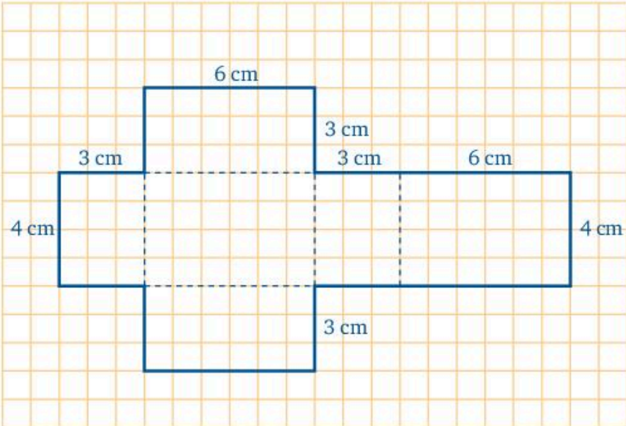


Draw the net on 1 cm squared paper and cut it out. Fold it along the broken lines. Fix it together with sticky tape.

If you mark the faces with the numbers 1 to 6, you can make a dice.



3 Draw this net full-size on 1 cm squared paper.



Cut the net out and fold it along the dotted lines. Stick the edges together.

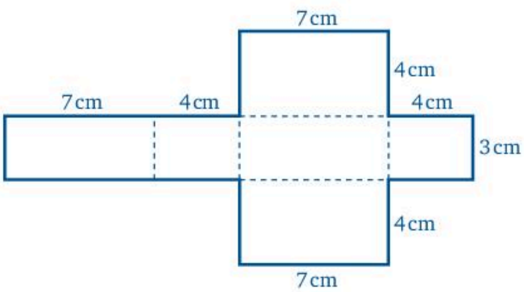
a **i** How many faces are rectangles measuring 6 cm by 3 cm?

ii How many faces are rectangles measuring 6 cm by 4 cm?

iii What are the measurements of the remaining faces?

b Draw another arrangement of the rectangles that will fold up to make the same cuboid.

4 Draw this net full-size on 1 cm squared paper.



Cut the net out and fold along the dotted lines. Stick the edges together. This solid is called a *cuboid*.

a **i** How many faces are rectangles measuring 7 cm by 4 cm?

ii How many faces are rectangles measuring 7 cm by 3 cm?

iii What are the measurements of the remaining faces?

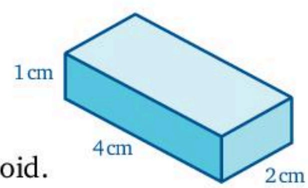
b Draw another arrangement of the rectangles to give a different net which will fold up to make the same cuboid.

5 This cuboid is 4 cm long, 2 cm wide and 1 cm high.

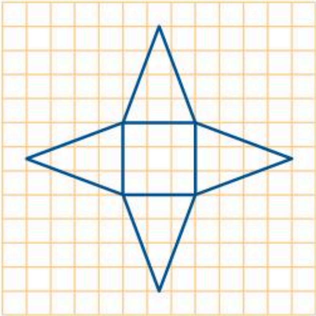
a How many faces does this cuboid have?

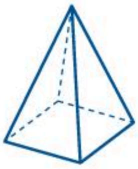
b Sketch the faces, showing their measurements.

c On 1 cm squared paper, draw a net that will make this cuboid.



6 This net makes a solid called a square-based *pyramid*.





Copy this net onto 2 cm squared paper.

Cut out your net and fold the triangles up.

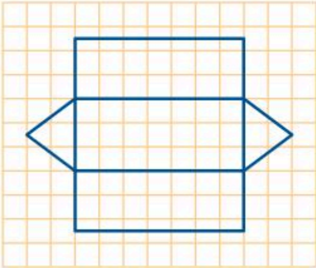
Stick the edges together with sticky tape.

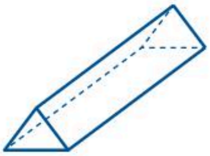
a How many faces does this pyramid have?

b How many edges does this pyramid have?

c How many vertices does this pyramid have?

7 This net makes a solid called a *prism* with a triangular cross-section.





Copy this net onto 2 cm squared paper.

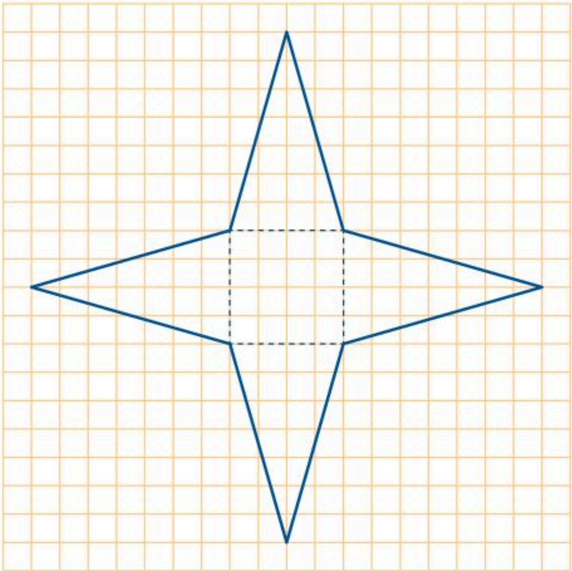
Cut your net out and fold it up to make the prism.

a How many faces does this prism have?

b How many edges does this prism have?

c How many vertices does this prism have?

8 Draw this net full-size on 1 cm squared paper. The square, which is shown with dotted lines has an edge of 4 cm.



Cut the net out and fold it along the dotted lines. Stick the edges together.

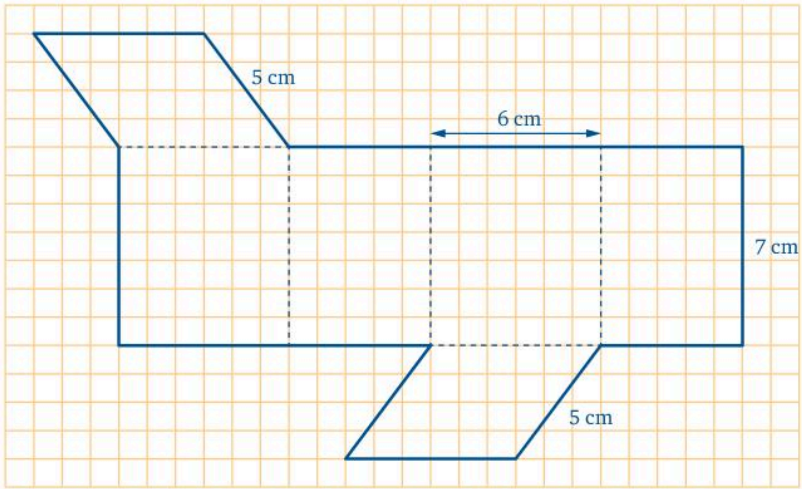
a What is the name of this solid?

b How many faces does this solid have?

c How many edges does this solid have?

d How many vertices does this solid have?

9 Draw this net full-size on 1 cm squared paper.



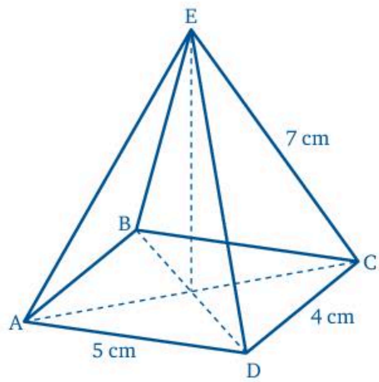
a Cut the net out and fold it along the dotted lines. Stick the edges together.

b What name do we give to this solid?

c What shape is the cross-section?

d Find the length of the perimeter of the shape at one end.

10 The diagram shows a pyramid with a square base.

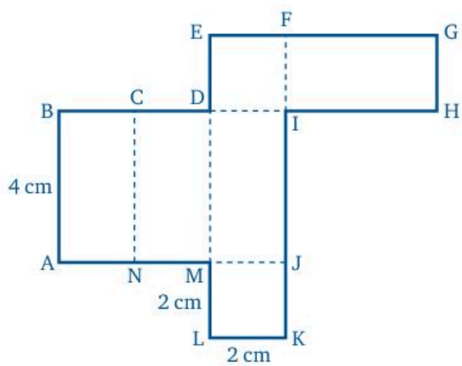


a Sketch a net for this solid. Label it and insert the measurements.

b How many triangles are exactly the same shape and size as triangle AED? Name them.

c How many triangles are exactly the same shape and size as triangle ECD? Name them.

11 This net will make a cuboid.

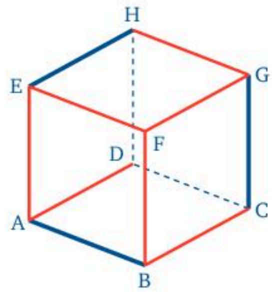


a Sketch the cuboid, and show its measurements.

b Which edge joins HI?

c Which corners meet at A?

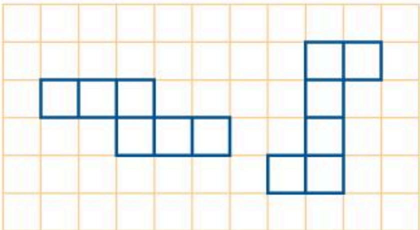
12 This cube is cut along the edges drawn with a coloured line and flattened out.
Draw the flattened shape.



13 Here are two arrangements of six squares.

a Draw as many other arrangements of six squares as you can find.

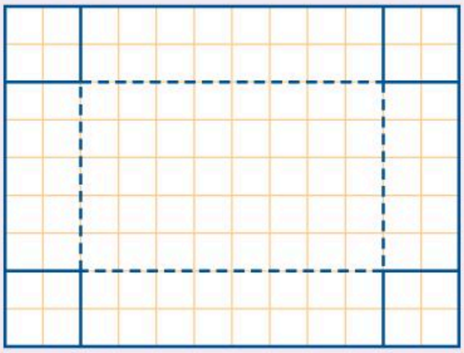
b Which of your arrangements, including the two given here, will fold up to make a cube? If you cannot tell by looking, cut them out and try to make a cube.



Investigation

A rectangular piece of card measuring 12 cm by 9 cm is to be used to make a small rectangular box.

The diagram shows one way of doing this.



1 How many different open boxes can be made from this piece of card if the length and breadth of the base is to be a whole number of centimetres?
Draw each possibility on squared paper.

2 For each different box find

a its dimensions **b** its total external surface area **c** its capacity.

3 Use the information you have found in part **2** to state whether each of these statements is true (T) or false (F).

A The box with the largest base has the greatest capacity.

B The box with the smallest total external surface area has the smallest capacity.

C The box with the greatest capacity is as deep as it is wide.

4 Find the dimensions of the largest cubical open box that can be made from a rectangular card measuring 12 cm by 9 cm. How much card is wasted?

5 Investigate other sizes of card to find the dimensions of the box with the largest volume that can be made from each size.

Did you know?

There is a formula, called the Euler formula, that gives the connection between the number of edges (E), number of faces (F) and number of vertices (V) of a solid made from plane faces. The formula works for any convex solid with no holes in it. (Convex means that all the faces face outwards.)

The formula is

$$V - E + F = 2$$

Try it on the solids you made from the nets in Exercise 8c.

In this chapter you have seen that...

- ✔ the special quadrilaterals each have a name and can be identified by the properties of their sides
- ✔ some polygons will tessellate
- ✔ solids with plane faces can be made from a net.

9 Geometry

At the end of this chapter you should be able to...

1. use geometrical instruments to draw lines and angles
2. use geometrical instruments to measure line segments and angles
3. bisect a line segment using a ruler and a pair of compasses
4. draw a perpendicular to a line using a ruler and a pair of compasses
5. draw a line through a given point parallel to a given line using a ruler and protractor.

Did you know?

The area between two concentric circles (circles having the same centre) is called an annulus.

You need to know...

✔ how to add and subtract whole numbers.

Key words

acute, alternate angles, base line, bisect, centre, circle, circumference, diameter, line segment, midpoint, obtuse, pair of compasses, parallel, perpendicular bisector, protractor, radius, rhombus, right angle

Constructions

When a new object, for example a new car, is designed there are many jobs that have to be done before it can be made. One of these jobs is to make accurate drawings of the parts.

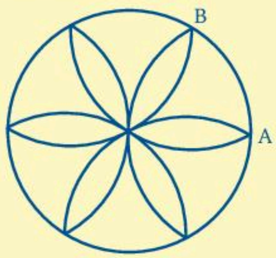
To draw accurately you need

a *sharp* pencil
a ruler
a pair of compasses
a protractor.

Using a pair of compasses

Using a *pair of compasses* is not easy: it needs practice. Draw several circles. Make some of them small and some large. You should not be able to see the place at which you start and finish. Holding the compasses at the tip only will give the most accurate results.

Now try drawing the daisy pattern below.



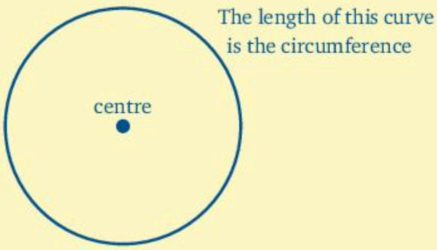
Draw a circle of radius 5 cm. Keeping the radius the same, put the point of the compasses at A and draw an arc to cut the circle in two places, one of which is B. Move the point to B and repeat. Carry on moving the point of your compasses round the circle until the pattern is complete.

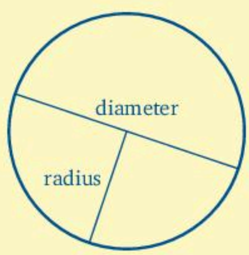
Repeat the daisy pattern but this time draw complete circles instead of arcs.

There are some more patterns using compasses in the Activity on pages 180 and 181.

Diameter, radius and circumference

When you use a pair of compasses to draw a *circle*, the place where you put the point is the *centre* of the circle. The length of the curve that the pencil draws is the *circumference* of the circle.





Any straight line joining the centre to a point on the circumference is a *radius*.

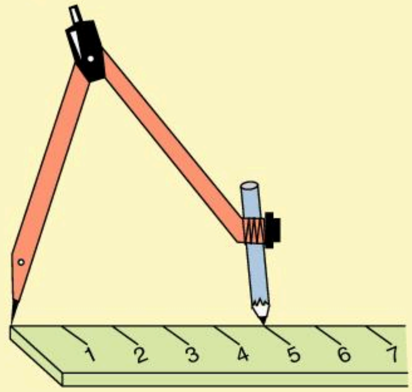
A straight line across the full width of a circle (i.e. going through the centre from one point on the circumference to the opposite point on the circumference) is a *diameter*.

Drawing straight line segments

To draw a straight *line segment* that is 5 cm long, start by using your ruler to draw a line that is *longer* than 5 cm.

Then mark a point on this line near one end as shown. Label it A.

Next use your compasses to measure 5 cm on your ruler.



Then put the point of the compasses on the line at A and draw an arc to cut the line as shown.

The length of line between A and B should be 5 cm. Measure it with your ruler.

Exercise 9a

Draw, as accurately as you can, straight line segments of the following lengths:

1 6 cm	**3** 12 cm	**5** 8.5 cm	**7** 4.5 cm
2 2 cm	**4** 9 cm	**6** 3.5 cm	**8** 6.8 cm

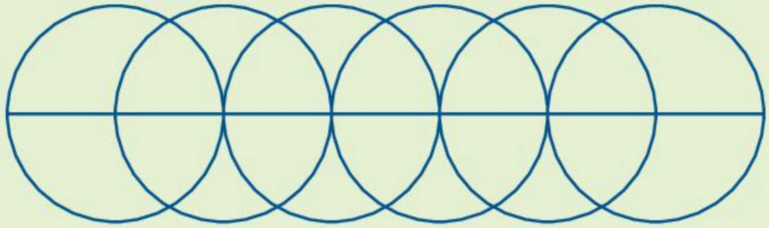
Draw circles with the following radii. If you draw accurately, you should not be able to see where you started and ended.

9 8 cm	**10** 6 cm	**11** 9 cm	**12** 5 cm

Activity

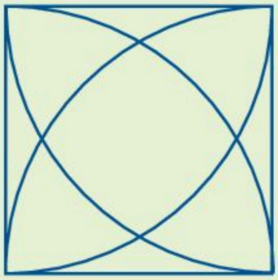
The patterns below are made using a pair of compasses. Try copying them. Some instructions are given which should help.

1

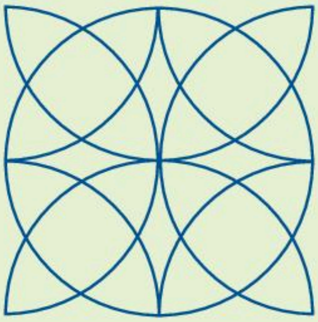


Draw a straight line. Open your compasses to a radius of 3 cm and draw a circle with its centre on the line. Move the point of the compasses 3 cm along the line and draw another circle. Repeat as often as you can.

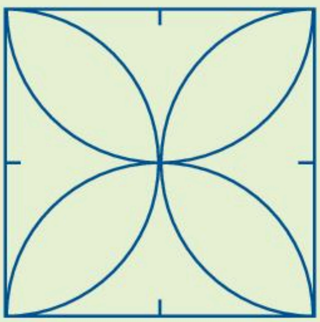
2 Draw a square of side 4 cm. Open your compasses to a radius of 4 cm and with the point on one corner of the square draw an arc across the square. Repeat on the other three corners.



Try the same pattern, but leave out the sides of the square; just mark the corners. A block of four of these looks good.

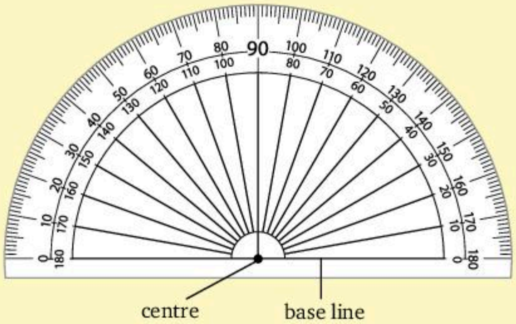


3 Draw a square of side 8 cm. Mark the midpoint of each side. Open your compasses to a radius of 4 cm, and with the point on the middle of one side of the square, draw an arc. Repeat at the other three midpoints.



Using a protractor

A *protractor* looks like this:

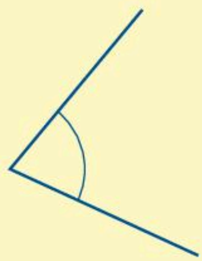


It has a straight line at or near the straight edge. This line is called the *base line*.

The *centre* of the base line is marked.

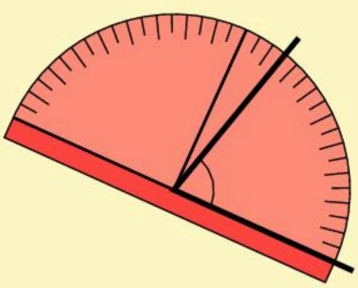
The protractor has two scales, an inside one and an outside one.

To measure the size of this angle, first decide whether it is *acute* or *obtuse*.



This is an acute angle because it is *less* than 90°.

Next place the protractor on the angle as shown.



One arm of the angle is on the base line.

The vertex (point) of the angle is at the centre of the base line.

Choose the scale that starts at 0° on the arm of the base line. Read off the number where the other arm cuts this scale.

Check with your estimate to make sure that you have chosen the right scale.

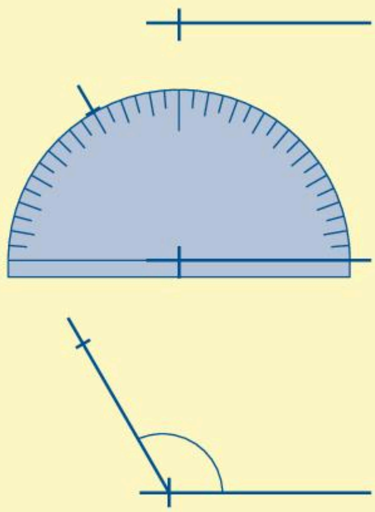
To draw an angle of 120° start by drawing one arm and mark the vertex.

Place your protractor as shown in the diagram. Make sure that the vertex is at the centre of the base line.

Choose the scale that starts at 0° on your drawn line and mark the paper next to the 120° mark on the scale.

Remove the protractor and join your mark to the vertex.

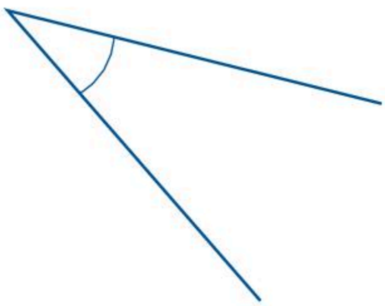
Now look at your angle: does it look the right size?



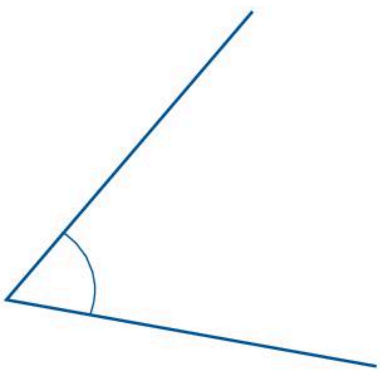
Exercise 9b

Measure the following angles (if necessary, turn the page to a convenient position):

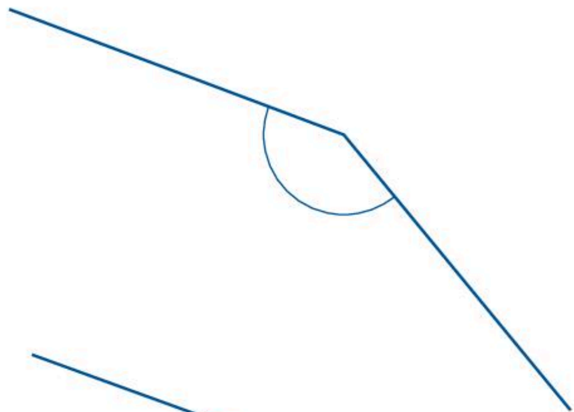
1



2



3



4

Use your protractor to draw the following angles accurately:

5	25°	**8**	160°	**11**	110°	**14**	125°
6	37°	**9**	83°	**12**	49°	**15**	175°
7	55°	**10**	15°	**13**	65°	**16**	72°

Construction to bisect a line

To *bisect* a line we have to find the *midpoint* of that line. To do this we construct a *rhombus* with the given line as one diagonal, but we do not join the sides of the rhombus.

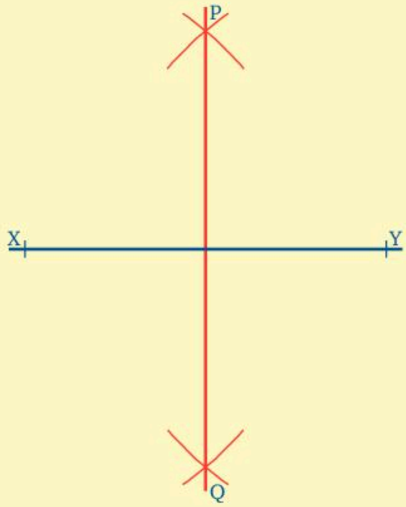
To bisect XY, open your compasses to a radius that is about $\frac{3}{4}$ of the length of XY.

With the point of the compasses on X, draw arcs above and below XY.

Move the point to Y (being careful not to change the radius) and draw arcs to cut the first pair at P and Q.

Join PQ.

The point where PQ cuts XY is the midpoint of XY.
PQ is called the *perpendicular bisector* of XY.
The angle between PQ and XY is a *right angle*.

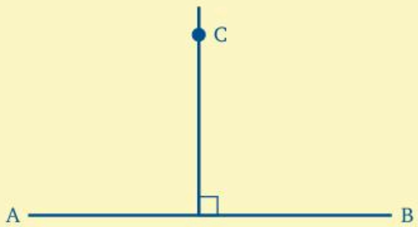


(XPYQ is a rhombus because the same radius is used to draw all the arcs, i.e. XP = YP = YQ = XQ. PQ and XY are the diagonals of the rhombus so PQ bisects XY.)

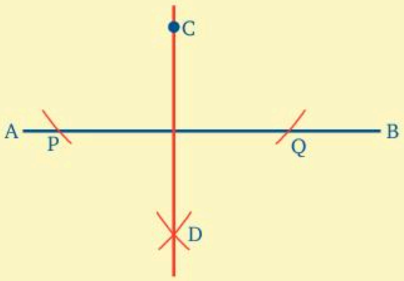
Note: When you are going to bisect a line, draw it so that there is plenty of space for the arcs above *and* below the line.

Dropping a perpendicular from a point to a line

If you are told to drop a perpendicular from a point, C, to a line, AB, this means that you have to draw a line through C which is at right angles to the line AB.



To drop a perpendicular from C to AB, open your compasses to a radius that is about $1\frac{1}{2}$ times the distance of C from AB.



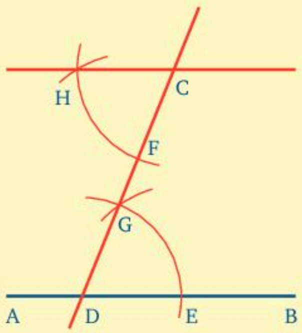
With the point of the compasses on C, draw arcs to cut the line AB at P and Q.

Move the point to P and draw an arc on the other side of AB. Move the point to Q and draw an arc to cut the last arc at D.

Join CD.

CD is then perpendicular to AB.

To construct a line parallel to a given line through a given point



C is a point above the line AB.

To draw a line through C parallel to AB proceed as follows:

- Draw a line through C to cut AB at D.
- With centre D and any suitable radius (it must not be too small) draw an arc to cut AB at E and CD at G.
- With centre C and the same radius draw an arc to cut CD at F.
- With centre E open out your compasses to a radius that passes through G.
- With this radius place the point of you compasses at F and draw the first arc at H.
- Join CH.

CH is *parallel* to AB as you have constructed angles EDG and FCH as equal angles. You have constructed *alternate angles*.

Exercise 9c

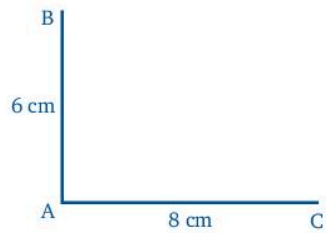
1 **a** Draw a line segment AB of length 8 cm.

b Construct the perpendicular bisector of AB.
Letter the perpendicular bisector PQ where P is above AB and Q is below AB but nearer to AB than P is.

c Join PA and PB. Measure the lengths of PA and PB. How do these lengths compare?

d Join QA and QB. Measure the lengths of QA and QB. How do these lengths compare?

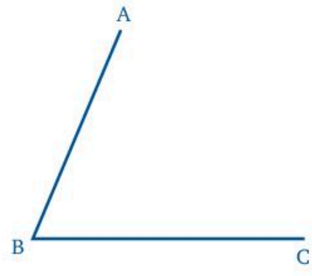
2 **a** Use the edges of your set-square to draw two lines as shown in the diagram.
Mark the corner A. Measure AB = 6 cm up the page and AC = 8 cm along the page.



b Construct the perpendicular bisectors of AB and AC.
Mark the point where they cross X.

c With the point of your compasses at X and radius the distance XA draw a circle.
Does this circle go through any particular points?

3 **a** Draw any angle ABC similar to the one shown in this diagram.



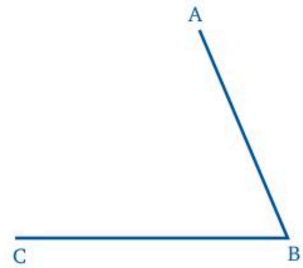
b Construct the line through A parallel to BC.

c Construct the line through C parallel to AB to cross the line through A at D.

d Measure the lengths of BC and AD.
How do these lengths compare?

e Measure the lengths of BA and CD.
How do these lengths compare?

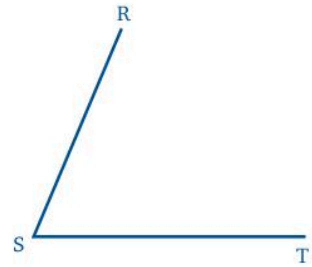
4 a Draw any angle ABC similar to the one shown in this diagram.



b Construct the perpendicular bisectors of AB and AC. Mark the point where these bisectors cross X.

c With centre X and radius XA draw a circle. Does this circle pass through the points B and C?

5 a Draw any angle RST similar to the one shown in this diagram.



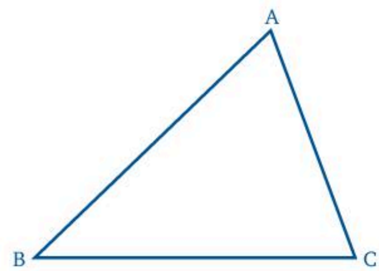
b Construct the line through R parallel to ST.

c Construct the line through T parallel to SR to cross the line through R at U.

d Join SU and RT. Mark the point where they cross X.

e Measure and record the lengths of XR, XS, XT and XU. How do these lengths compare?

6 a Draw any triangle ABC.



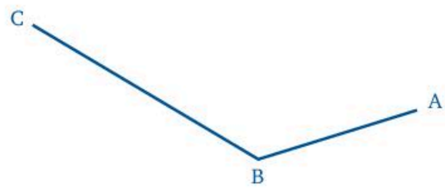
b Construct the perpendicular from A to BC to meet BC at D.

c Construct the perpendicular from B to AC to meet AC at E.

d Construct the perpendicular from C to AB to meet AB at F.

e What do you notice?

7 a Draw an angle ABC similar to the angle in the diagram

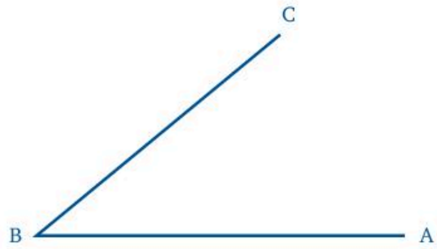


b Construct the perpendicular bisetors of AB and BC.

c Mark X, the point where these bisectors intersect.

d With centre X and radius XA draw a circle. What special points does this circle pass through?

8 Draw an angle ABC similar to the angle in the diagram.



a Draw a line through C parallel to BA.

b Draw a line through A parallel to BC.

c Mark the point where your two lines cross D.
Measure the angles ABC and CDA. They should be equal.

d Measure the lines BA and CD. They should be the same length.

Puzzle

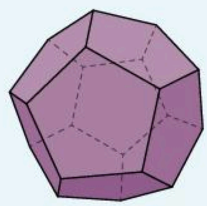
How many cubes can you see?

Did you know?

In a platonic solid, every face is the same size of regular polygon.

One such solid is a cube.

Another platonic solid is called a dodecahedron; all its faces are regular pentagons.



A solid you will probably recognise is made from 20 hexagons and 12 pentagons. What is it?

In this chapter you have seen that...

- ✔ you can use geometrical instruments to draw and measure line segments and angles
- ✔ a line segment can be bisected using a ruler and a pair of compasses
- ✔ a perpendicular to a line can be constructed using a ruler and a pair of compasses
- ✔ you can construct a line through a given point parallel to a given line using a ruler and a pair of compasses.

10 Transformations

At the end of this chapter you should be able to...

1 identify shapes that have lines of symmetry
2 complete drawings of shapes, given their lines of symmetry
3 draw in lines of symmetry for given shapes
4 find the mirror line given a reflection
5 identify shapes that have rotational symmetry
6 state the order of rotational symmetry of a given shape
7 differentiate between rotational and line symmetry in a shape
8 construct shapes having line or rotational symmetry.

Did you know?

Many well-known trademarks have line symmetry, for example logos on cars. Which trademarks can you find that have line symmetry?

You need to know...

✔ how to use a protractor to draw an angle

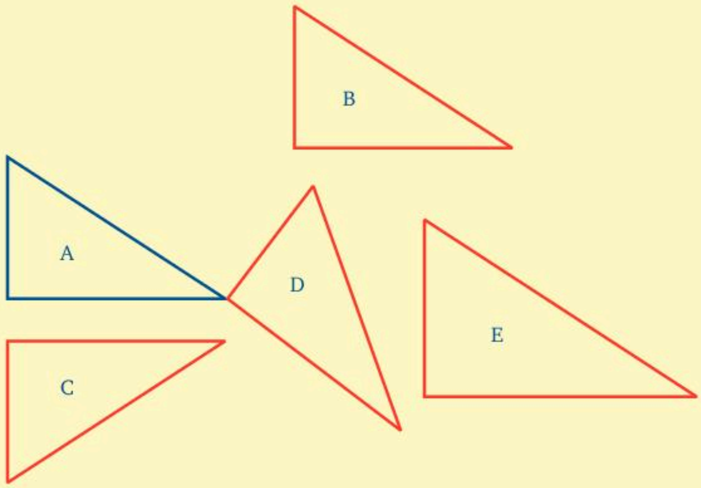
✔ how to construct the perpendicular bisector of a line segment.

Key words

bilateral symmetry, centre of rotation, corresponding sides, image, line symmetry, line of symmetry, mapping, midpoint, mirror line, object, order of rotational symmetry, reflection, rotation, rotational symmetry, symmetrical, translation

Transformations

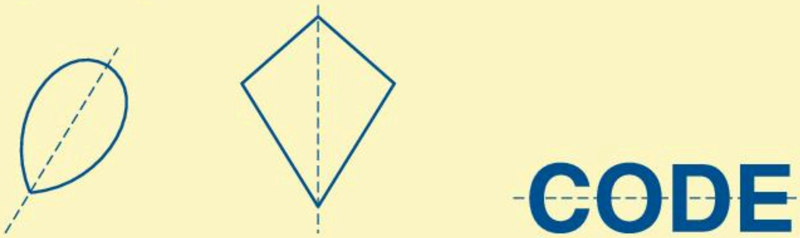
A transformation of a shape moves it or reflects it or rotates it or enlarges it. This diagram shows different transformations of a triangle marked A.



The *object* is triangle A. The *images* are transformations of A. Triangle B is a translation of A, triangle C is a reflection of A, triangle D is a rotation and triangle E is an enlargement.

In this chapter we concentrate on *reflections*, *translations* and *rotations*.

Line symmetry



Shapes like these are *symmetrical*. They have *line symmetry* (or *bilateral symmetry*); the dashed line is the *line of symmetry* because if the shape were folded along the dashed line, one half of the drawing would fit exactly over the other half.

Exercise 10a

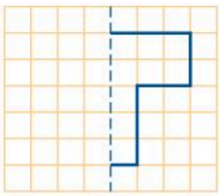
1 Which of the following shapes have a line of symmetry?

A


B

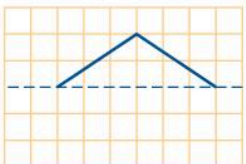

C

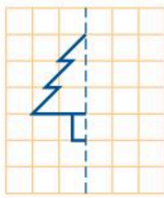

Copy the following drawings on square grid paper and complete them so that the dashed line is the line of symmetry.

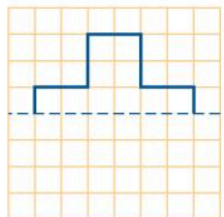
2


4

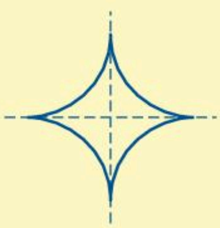
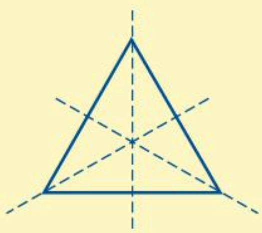
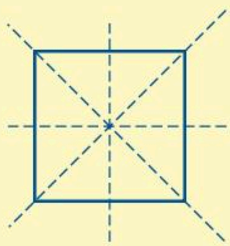

6


3


5


7


Two or more lines of symmetry

Shapes can have more than one line of symmetry. In the drawings above, the lines are shown by dashed lines and it is clear that the first shape has two lines of symmetry, the second has three and the third has four.

Exercise 10b

Sketch or trace the shapes in questions **1** to **12**. Mark in the lines of symmetry and say how many there are. (Some shapes may have no line of symmetry.)

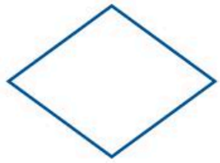
1


2


3

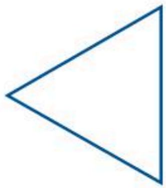

4


5


6


7


8

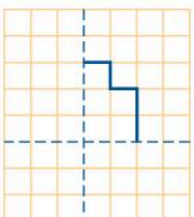

9


10

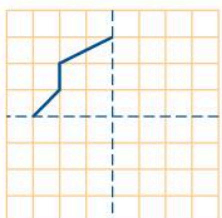

11

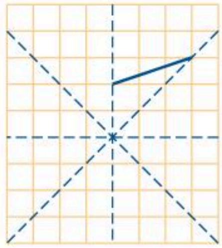

12

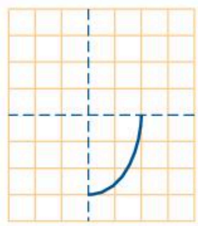

Copy and complete the following drawings on square grid paper. The dashed lines are the lines of symmetry.

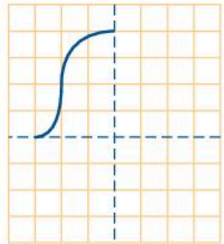
13


14


15


16


<u>**17**</u>


<u>**18**</u>


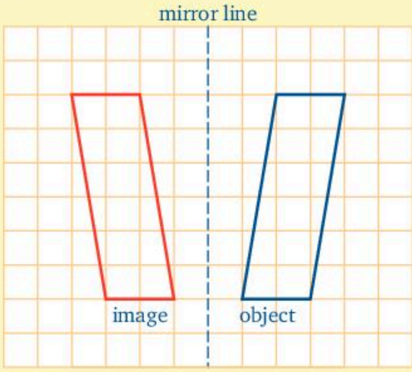
19 Draw, on square grid paper or on plain paper, shapes of your own with more than one line of symmetry.

Puzzle

Show how sixteen counters can be arranged in ten rows with exactly four counters in each row.

Reflections

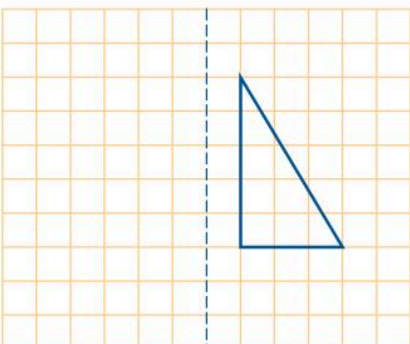
When we reflect an object in a line (called the *mirror line*), the object and its image together form a symmetrical shape.



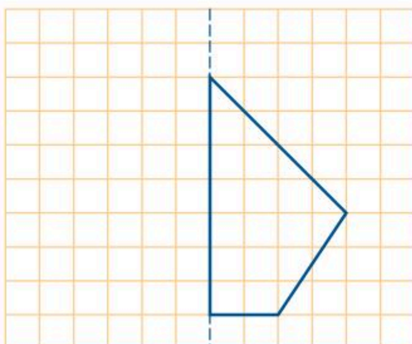
Exercise 10c

In each question copy the object and the mirror line on to square grid paper and draw the image of each object.

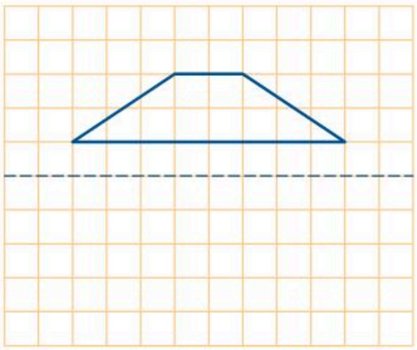
1



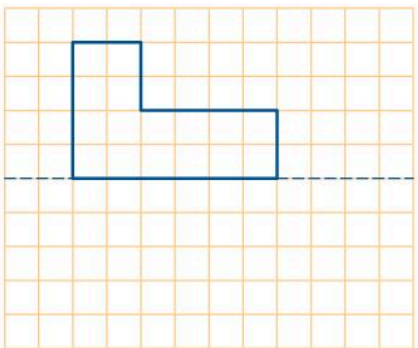
2



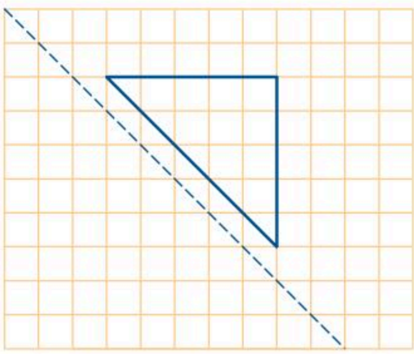
3



4



5



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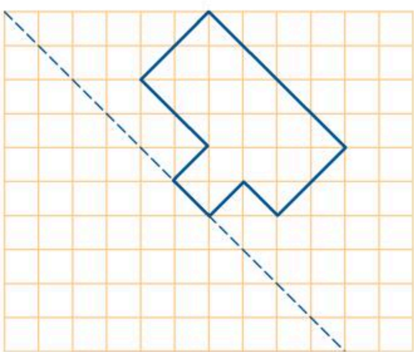

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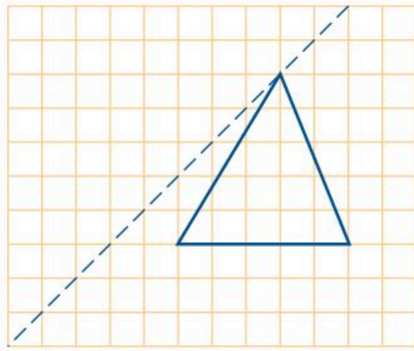
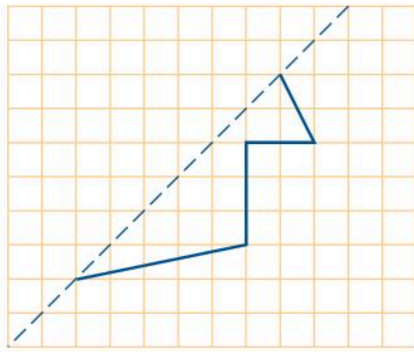


8



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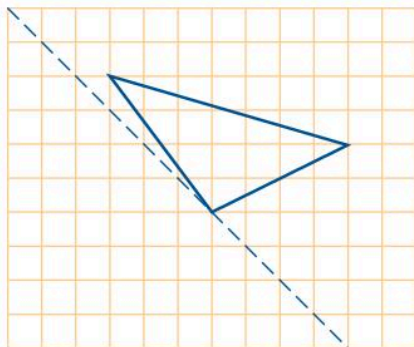
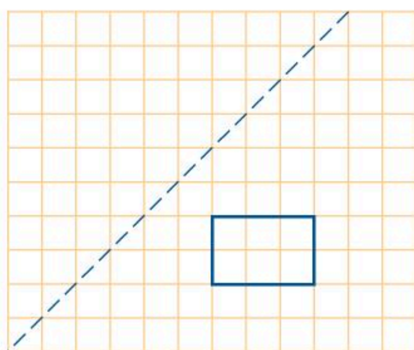
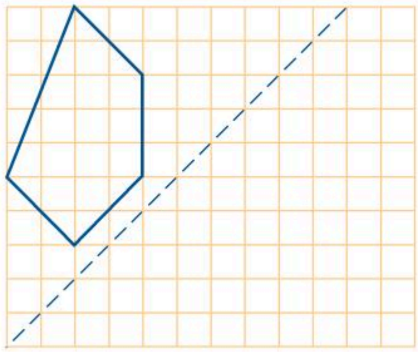


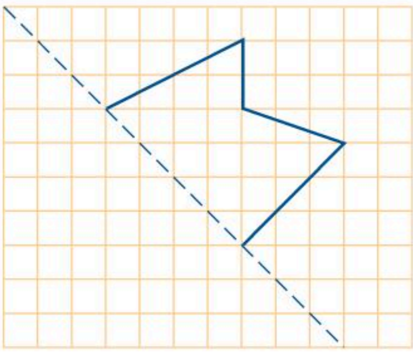
10



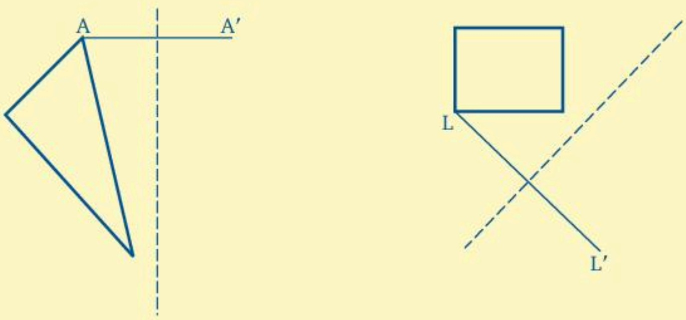
11



12



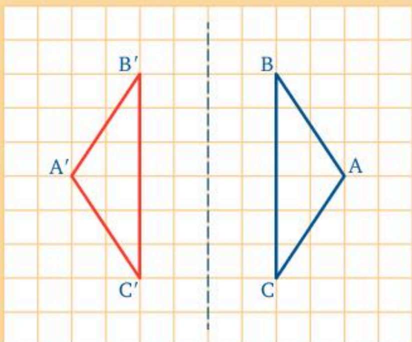
Finding the mirror line



We can see from these diagrams, and from the work in the previous exercise, that the object and image points are at equal distances from the mirror line, and the lines joining them (e.g. AA′ and LL′) are perpendicular (at right angles) to the mirror line.

Exercise 10d

Find the mirror line if △A′B′C′ is the image of △ABC.

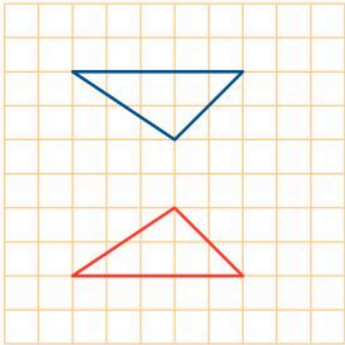


The mirror line is halfway between an object point and its image and perpendicular to the line through them.

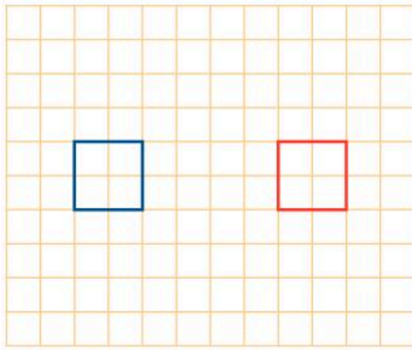
So the mirror line is halfway between B and B′ and perpendicular to the line BB′. Check that it also goes through the *midpoint* of CC′.

Copy the diagrams and draw the mirror lines.

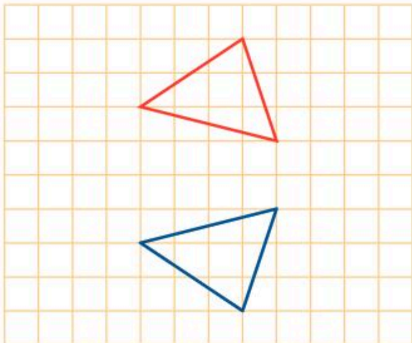
1



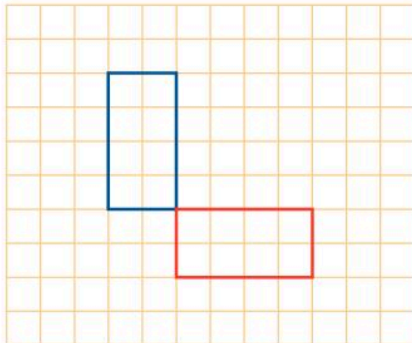
5



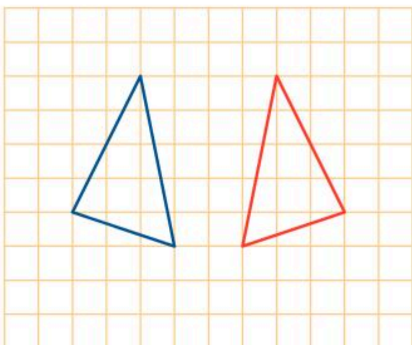
2



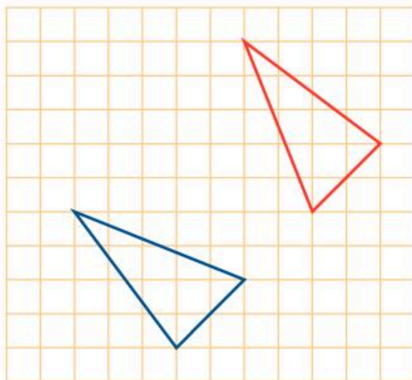
6



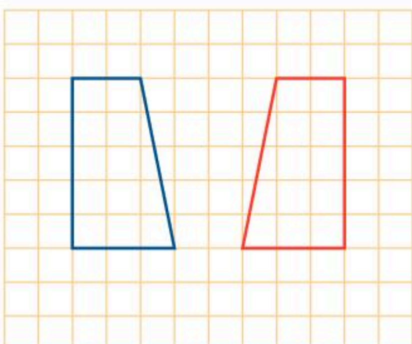
3



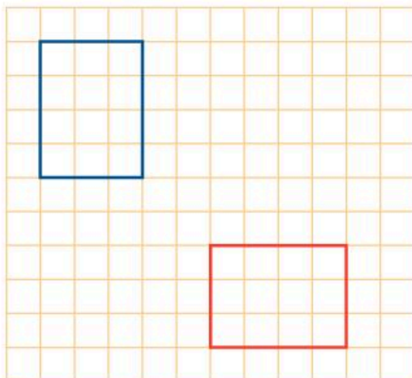
7



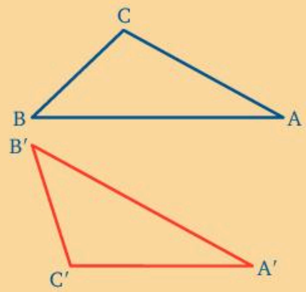
4



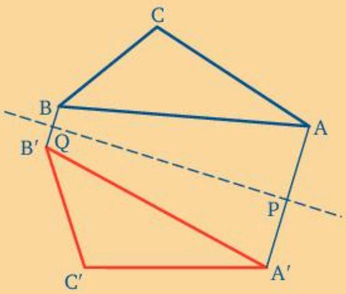
8



If A′B′C′ is the reflection of ABC, draw the mirror line.



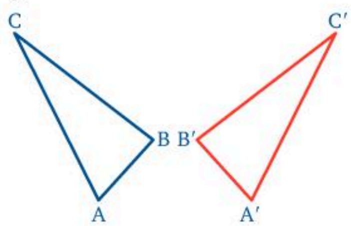
(Join AA′ and BB′ and find their midpoints, marking them P and Q. Then PQ is the mirror line.)



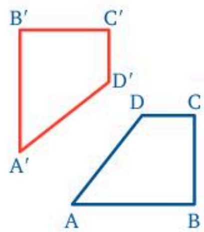
Whenever you attempt to draw a mirror line in this way, always check that the mirror line is at right angles to AA′ and BB′. If it is not, then A′B′C′ cannot be a reflection of ABC.

9 Trace the diagrams and draw the mirror lines.

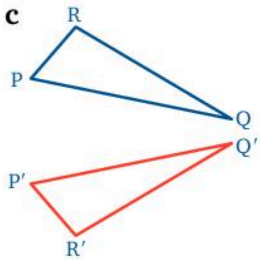
a



b

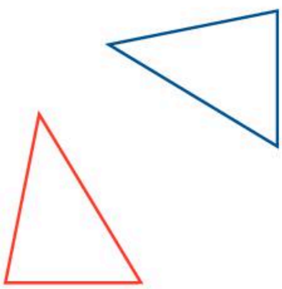


c

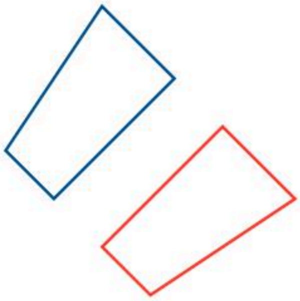


10

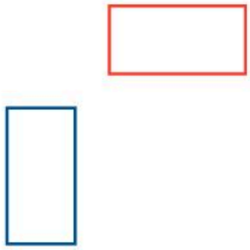
11



12

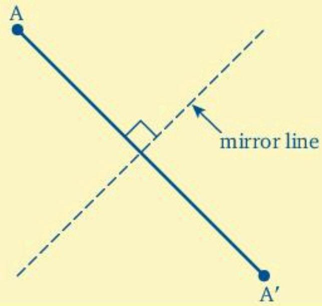


13



Construction of the mirror line

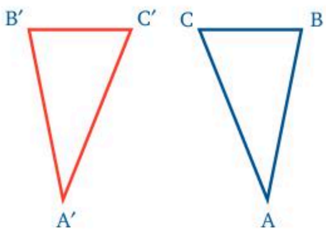
If we have only one point and its image, and we cannot use squares to guide us, we can use the fact that the mirror line goes through the midpoint of AA′ and is perpendicular to AA′. The mirror line is therefore the perpendicular bisector of AA′ and can be drawn.



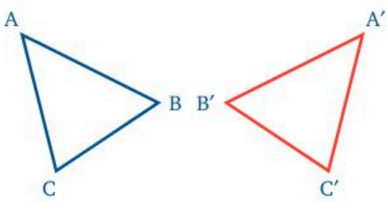
Exercise 10e

In this exercise copy each diagram on to plain paper. In questions **1** and **2**, triangle A′B′C′ is the image of triangle ABC under a reflection. Join A to A′ and construct the perpendicular bisector of AA′. Hence draw the mirror line.

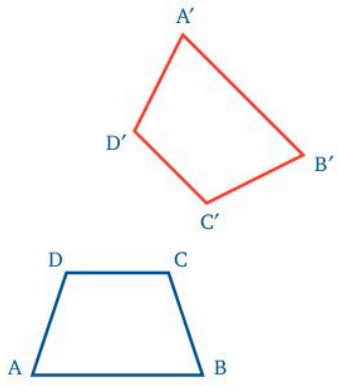
1



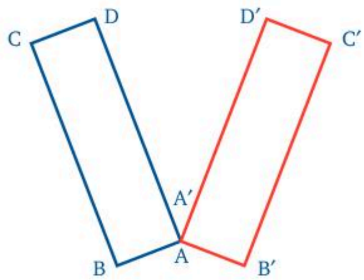
2



3 Trace the diagram onto plain paper. A′B′C′D′ is the image of ABCD under a reflection. Join D to D′ and construct the perpendicular bisector of DD′. Hence draw the mirror line.

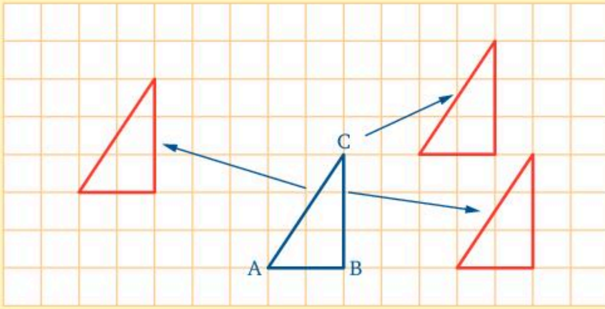


4 Trace the diagram onto plain paper. A′B′C′D′ is the image of ABCD under a reflection. Join D to D′ and construct the perpendicular bisector of DD′. Hence draw the mirror line.



Translations

Consider the movements in the diagram:

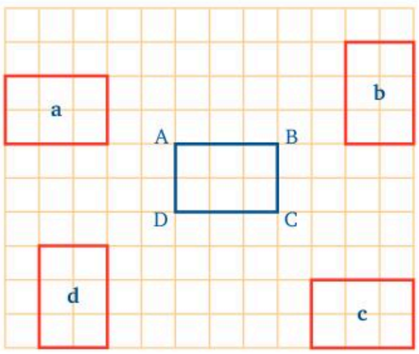


All these movements are of the same type. The side AB remains parallel to the horizontal grid lines and the triangle continues to face in the same direction. This type of movement is called a *translation*.

Although not a reflection we still use the words *object* and *image*.

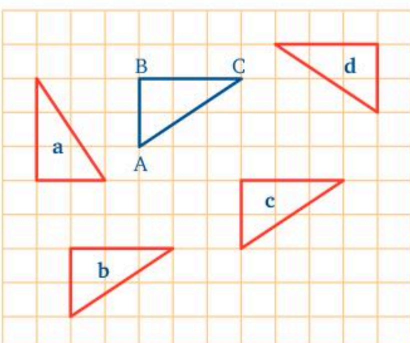
Exercise 10f

1 Which images of rectangle ABCD are given by a translation?

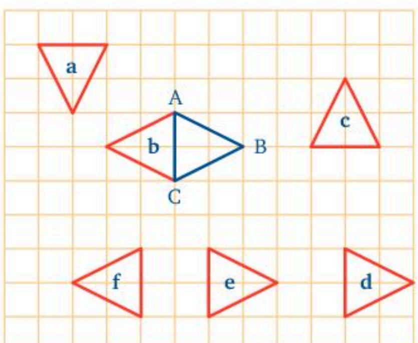


In questions **2** and **3**, which images are given by a translation?

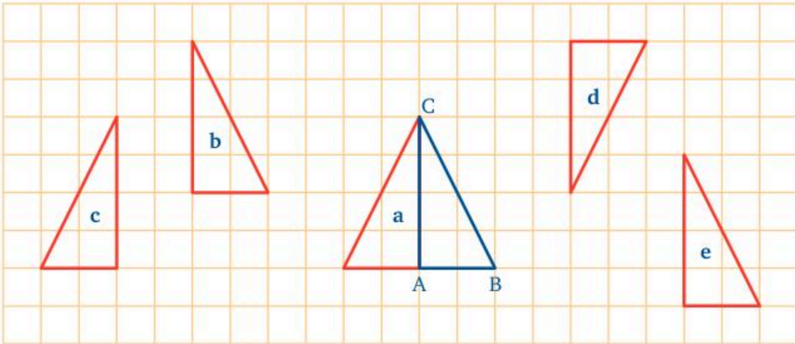
2



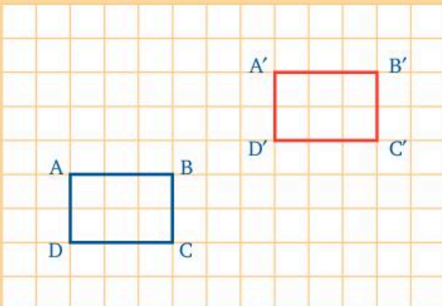
3



4 In the following diagram, which images of $\triangle ABC$ are given by a translation, which by a reflection and which by neither?



Describe the translation that translates rectangle ABCD to A′B′C′D′.



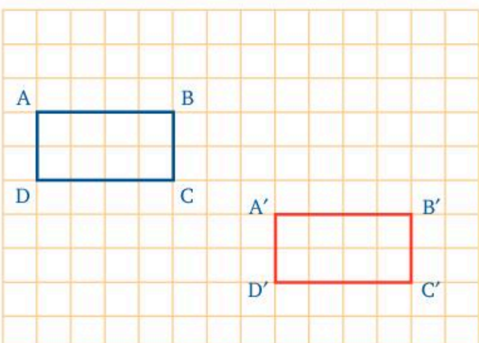
ABCD is moved 6 squares to the right (or in the positive direction) and 3 squares up.

(Moving to the right is positive, moving to the left is negative.

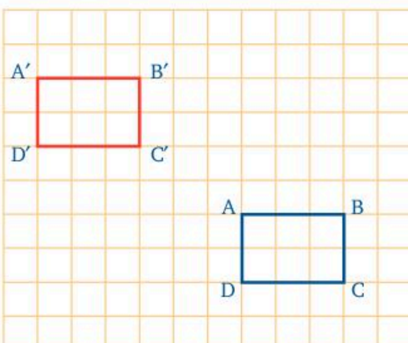
In a perpendicular direction we move up the page or down the page.)

In questions **5** and **6** describe the translation that translates rectangle ABCD to rectangle A′B′C′D′.

5

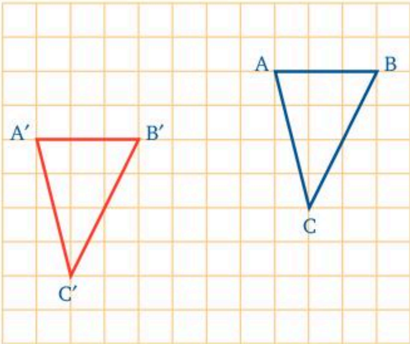


6

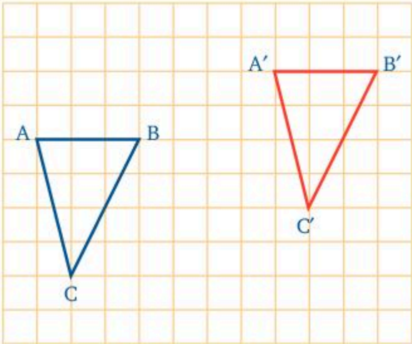


In questions **7** and 8 describe the translation that translates triangle ABC to triangle A′B′C′.

7

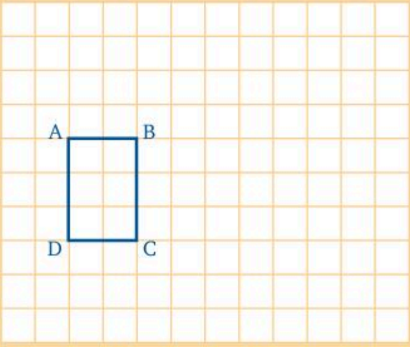


8

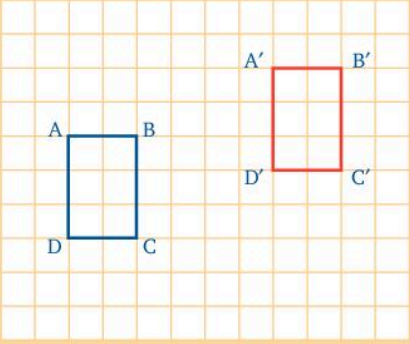


Sometimes a shape is given together with the instructions how to get the image.

Find the image of ABCD under a translation 6 squares to the right and 2 squares up. Mark the image A′B′C′D′.

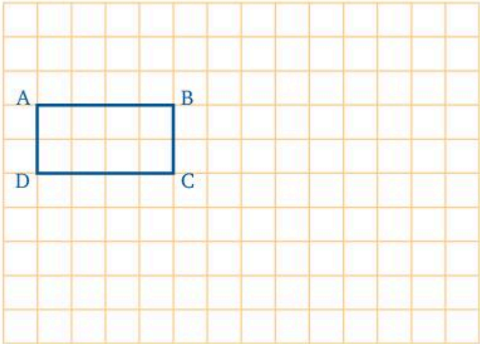


This diagram shows the result of translating the shape 6 squares to the right and 2 squares up.



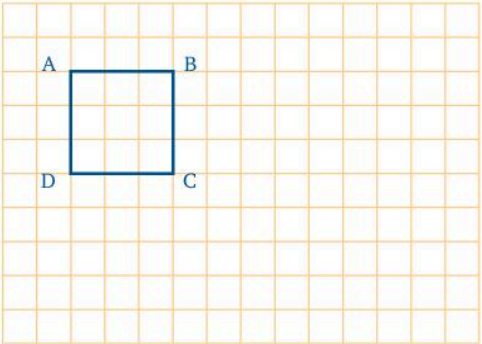
In questions **9** to **15** copy the diagram on to square grid paper.
Now translate the given shape according to the instruction.

9



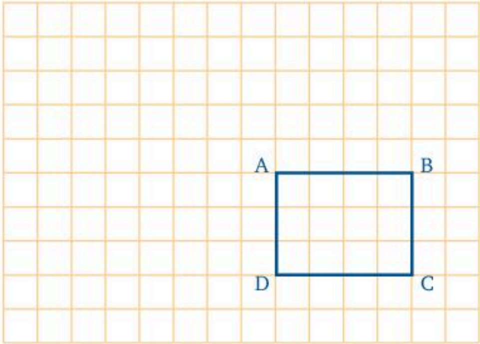
8 squares to the right and
2 squares down

12



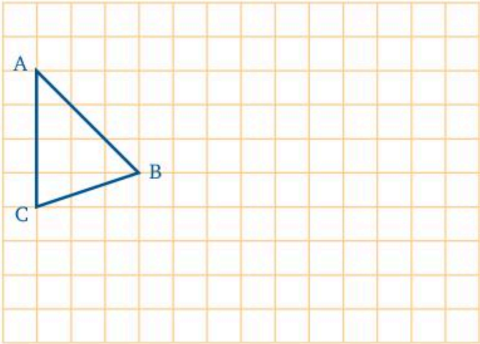
7 squares to the right and
3 squares down.

10



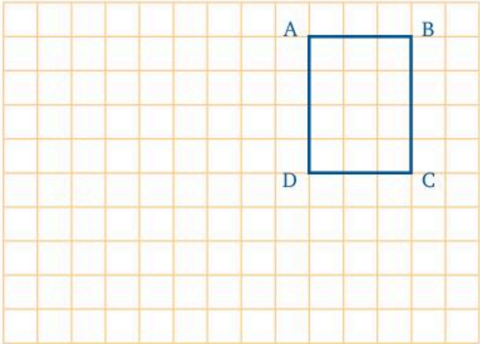
5 squares to the left and
4 squares up.

13



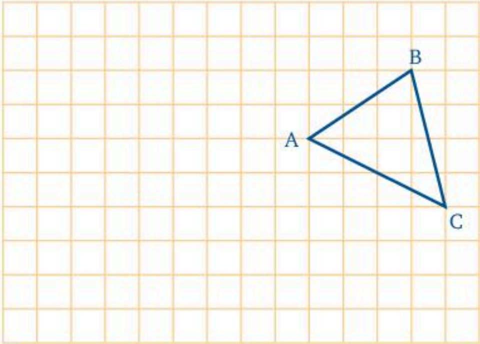
9 squares to the right and
2 squares down.

11



7 squares to the left and
3 squares down.

14

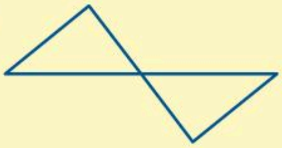


8 squares to the left and
1 squares down.

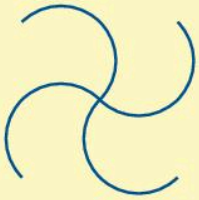
Rotational symmetry

Some shapes have a type of symmetry different from line symmetry.

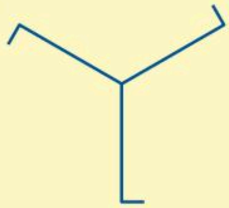
a



b



c



These shapes do not have an axis of symmetry but can be turned or rotated about a centre point and still look the same. Such shapes are said to have *rotational symmetry*. The centre point is called the *centre of rotation*.

Exercise 10g

1 **a**

b

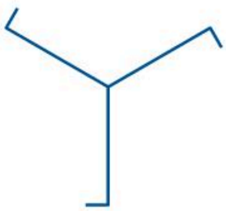
c

Trace each of the shapes above, then turn the tracing paper about the centre of rotation (put a compass point or a pencil point in the centre). Turn until the traced shape fits over the original shape again. In each case state through what fraction of a complete turn the shape has been rotated.

2 Which of the following shapes have rotational symmetry?

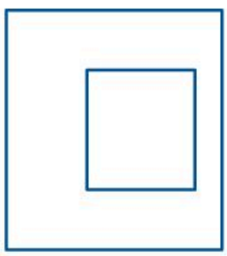
a

b



c

d



Order of rotational symmetry

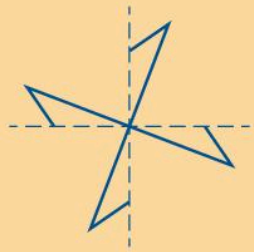
If the smallest angle that a shape needs to be turned through to fit is a third of a complete turn, then it will need two more such turns to return it to its original position. So, starting from its original position, it takes three turns, each one-third of a revolution, to return it to its starting position.

It has rotational symmetry of order 3.

Exercise 10h

Give the order of rotational symmetry of the following shape.

You need to decide whether turning this shape about its centre through some angle leaves it looking unchanged to the eye. If so, how big is this angle? How many times can you do this before getting back to the starting position?

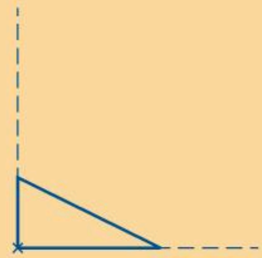


The smallest angle turned through is a right angle or one-quarter of a complete turn.

The shape has rotational symmetry of order **4**.

1–12 Give the order of rotational symmetry, if any, of the shapes in Exercise 10b, questions **1** to **12**.

Copy and complete the diagram, given that there is rotational symmetry of order 4.

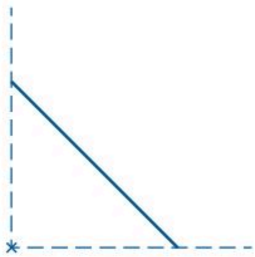


If the order of rotational symmetry is to be 4 you must turn the given shape about the cross (×) through one-quarter of a turn, i.e. through 90°. Repeat this until you get back to the starting position.

Each of the diagrams in questions **13** to **18** has rotational symmetry of the order given and × marks the centre of rotation.

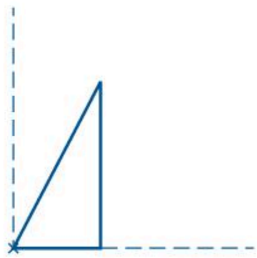
Copy and complete the diagrams. (Tracing paper may he helpful.)

13



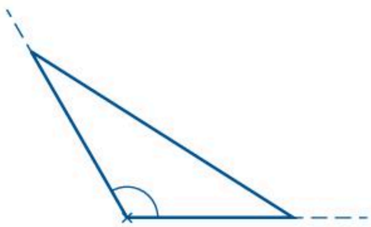
Rotational symmetry of order 4

16



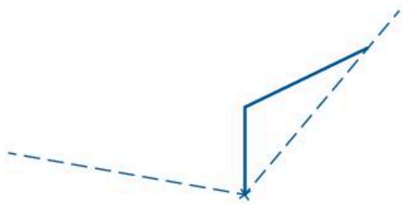
Rotational symmetry of order 4

14



Rotational symmetry of order 3

17



Rotational symmetry of order 3

15

Rotational symmetry of order 2

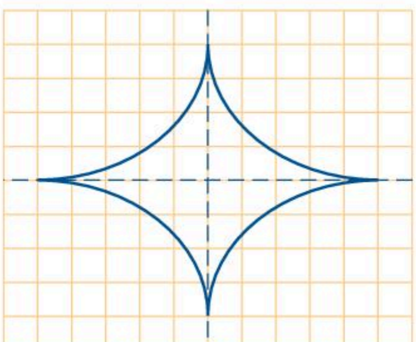
18

Rotational symmetry of order 2

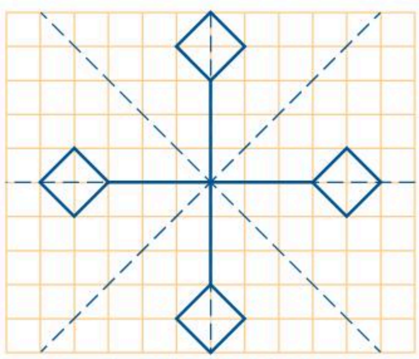
<u>**19**</u> In questions **13** to **18**, give the size of the angle, in degrees, through which each shape is turned.

Exercise 10i

Some shapes have both line symmetry and rotational symmetry:



Two axes of symmetry
Rotational symmetry of order 2



Four axes of symmetry
Rotational symmetry of order 4

Which of the following shapes have

a rotational symmetry only
b line symmetry only
c both?

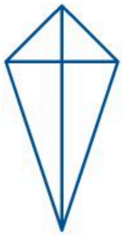
You can spot line symmetry by seeing if any line acts as a mirror; you can spot rotational symmetry if you can rotate the shape about a point that leaves it looking unchanged.

1

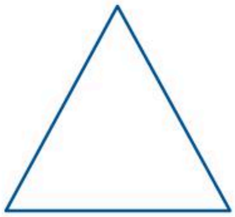
2

3

4



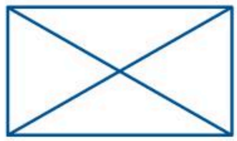
5



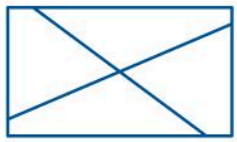
6

7

8



9



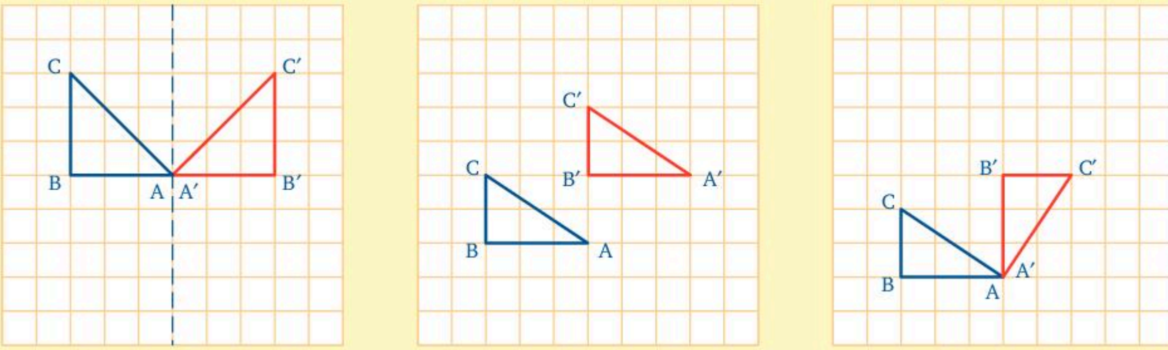
10 Make up three shapes that have rotational symmetry only. Give the order of symmetry and the angle of turn, in degrees.

11 Make up three shapes with line symmetry only. Give the number of axes of symmetry.

12 Make up three shapes that have both line symmetry and rotational symmetry.

13 The capital letter X has line symmetry (two axes) and rotational symmetry (of order 2). Investigate the other letters of the alphabet.

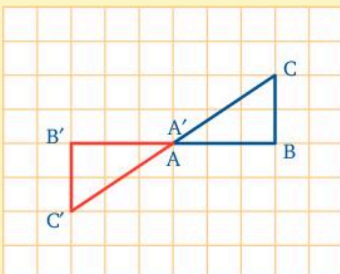
Transformations: rotations



So far, in transforming an object we have used reflection, as in **a**, and translation, as in **b**, but for **c** we need a rotation.

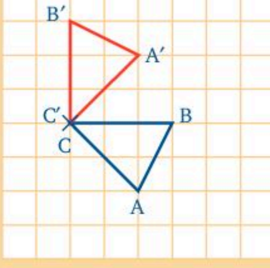
In this case we are rotating △ABC about A through 90° clockwise. We could also say △ABC was rotated through 270° anticlockwise. Clockwise rotation is sometimes referred to as negative rotation or rotating through a negative angle. Anticlockwise rotation is rotating through a positive angle.

For a rotation of 180° we do not need to say whether it is clockwise or anticlockwise rotation.



Exercise 10j

Give the angle of rotation when △ABC is *mapped* to △A′B′C′

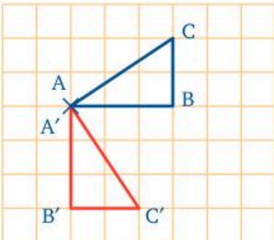


If you rotate △ABC anticlockwise about C it can be turned to position △A′B′C′. You must now measure the angle it has turned through. Compare the position of one side of the object triangle with the *corresponding* side of the image triangle.

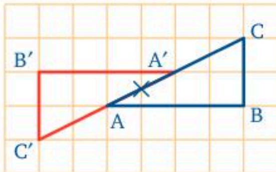
The angle of rotation is 90° anticlockwise.

In questions **1** to **4**, give the angle of rotation when △ABC is mapped to △A′B′C′.

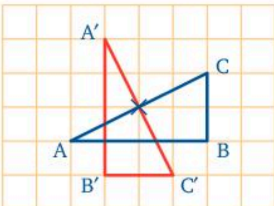
1



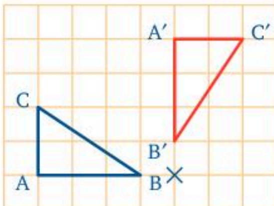
3



2

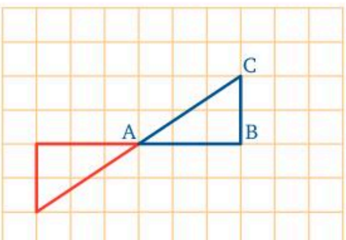


4

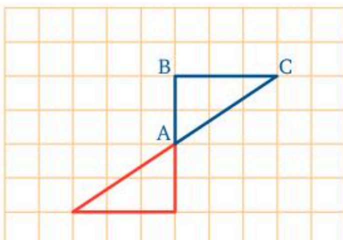


In questions **5** to **10**, copy the diagram, mark the centre of rotation with a cross and give the angle of rotation. Triangle ABC is the object in each case.

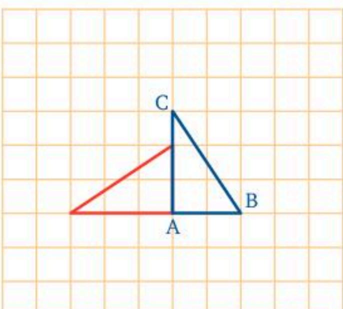
5



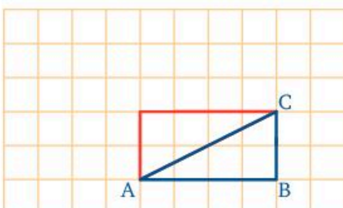
7



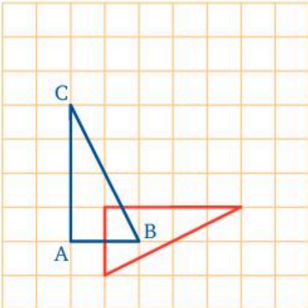
6



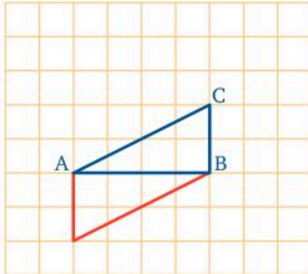
8



9

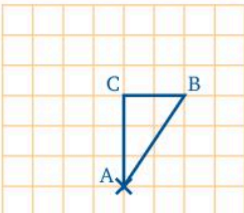


10



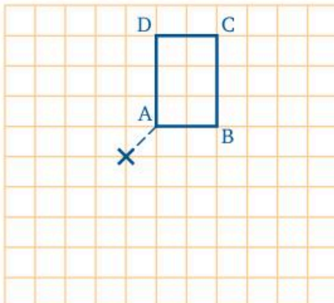
Copy the diagrams in questions **11** to **18**, using 1 cm to 1 unit. Find the images of the given objects under the rotations described. The centre of rotation is marked with a cross.

11



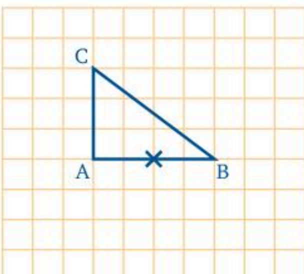
Angle of rotation 90° anticlockwise

14



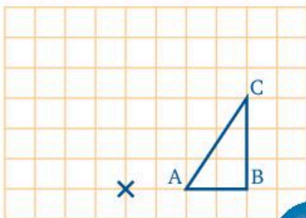
Angle of rotation 180° (As the centre of rotation is not a point on the object, join it to A first.)

12

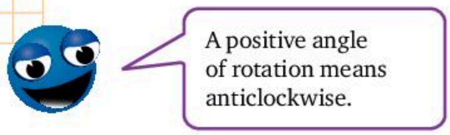


Angle of rotation 180°

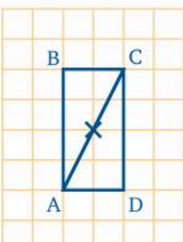
15



A positive angle of rotation of 90°

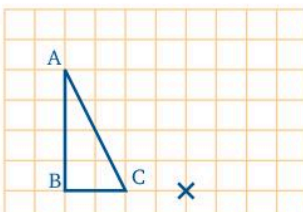


13



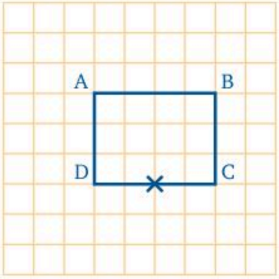
Angle of rotation 180°

16



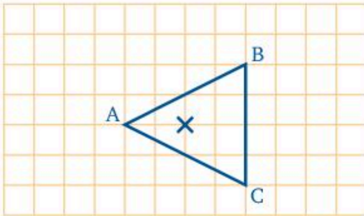
A negative angle of rotation of 90°

17



Angle of rotation of 90° anticlockwise

18



Angle of rotation of 180°

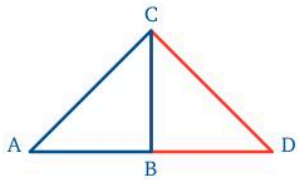
Mixed questions on reflections, translations and rotations

Exercise 10k

Sometimes we do not know which point is the image of a particular object point. In such cases there could be more than one possible transformation.

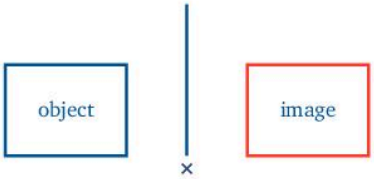
(Remember that a rotation of 90° anticlockwise is the same as a rotation of 270° clockwise. Do not give these as two independent transformations.)

1



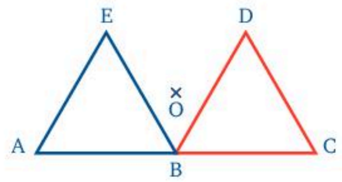
Name and describe two possible transformations that will map the object △ABC to the image △BCD.

2



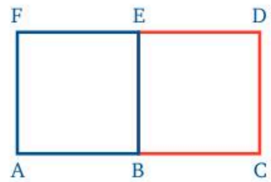
Name and describe three possible transformations that will map the object to the image.

3



Name and describe four possible transformations that will map the left-hand triangle to the right-hand triangle.

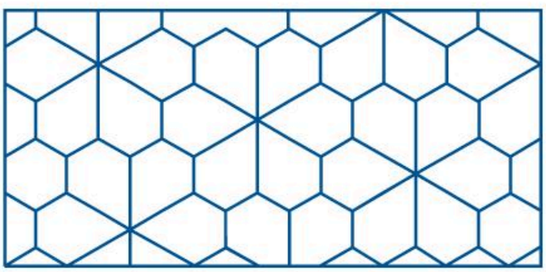
4



Name and describe five possible transformations that will map the left-hand square to the right-hand square.

5 A car is turning a corner and two of its positions are shown. Trace the drawing, allowing plenty of space above and below, and find the centre of the turning circle.

6 Look at the diagram below. Taking one of the shapes as the object, what types of transformations will map it to other shapes in the diagram?



Puzzle

A dried-out well is 12 metres deep. A snail starts from the bottom and tries to climb out. It climbs up 3 m every night and falls back 2 m every day. How long will it take the snail to climb out?

In this chapter you have seen that...

- ✔ when an object is reflected in a mirror line, the object and the image are symmetrical about the mirror line
- ✔ the mirror line is the perpendicular bisector of the line joining a point on the object to the corresponding point on the image
- ✔ a translation moves an object without turning it or reflecting it
- ✔ a shape has rotational symmetry if it can be rotated about a point to a different position but still look the same
- ✔ the order of rotational symmetry is the number of times the shape can be rotated and still look the same until it returns to its original position
- ✔ when rotational symmetry is described as turning about a point through a given angle you must say whether the turning is clockwise or anticlockwise.

REVIEW TEST 2: CHAPTERS 6–10

In questions **1** to **12** choose the letter for the correct answer.

1 The floor area of a rectangular room measuring 8 m by 4 m is

A $24\,m^2$ B $32\,m^2$ C $48\,m^2$ D $64\,m^2$

2 Expressed in cm^2, $0.004\,m^2$ is

A $40\,cm^2$ B $400\,cm^2$ C $4000\,cm^2$ D $40\,000\,cm^2$

3 Expressed in litres, $5500\,cm^3$ is

A $550\,l$ B $55\,l$ C $5.5\,l$ D $0.55\,l$

4 Which unit would you use to measure the volume of a cell phone?

A litres B m^3 C cm^3 D mm^3

5 A plane figure bounded by 5 line segments is called

A a triangle C a pentagon
B a quadrilateral D a hexagon

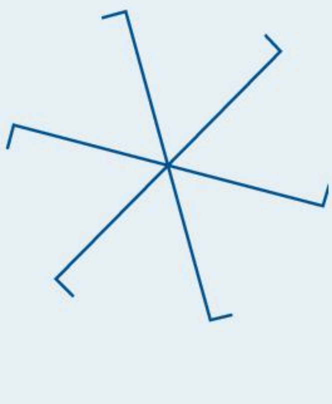
6 Which of the following statements are true?

i all the sides of a rectangle are the same length
ii the opposite sides of a square are equal
iii all the angles of a rectangle are equal

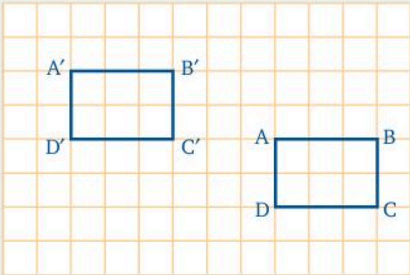
A none B **i** and **ii** C **i** and **iii** D **ii** and **iii**

7 The order of rotational symmetry of this figure is

A 2 B 4 C 6 D 8



8



The translation that translates ABCD to A′B′C′D′ is

A 6 squares to the right and 2 squares up

B 6 squares to the right and 2 squares down

C 3 squares to the left and 2 squares up

D 6 squares to the left and 2 squares up

9 Which of the following statements are true?

i a cube has 6 faces

ii a cuboid has 8 faces

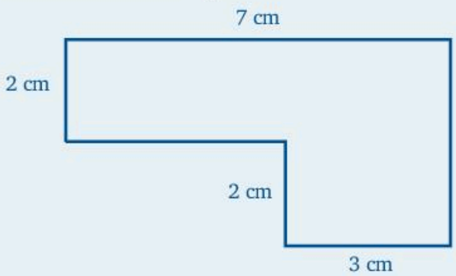
iii a square-based pyramid has 5 faces

A none B **i** and **ii** C **i** and **iii** D **ii** and **iii**

10 How many axes of symmetry does a square have?

A 1 B 2 C 3 D 4

Questions **11** and **12** refer to this diagram:



11 The perimeter of this shape is

A 14 cm B 18 cm C 20 cm D 22 cm

12 The area of this shape is

A 16 cm^2 B 18 cm^2 C 20 cm^2 D 22 cm^2

13 Express

a 0.04 m in cm

b 500 mm^2 in cm^2

c 10 cm^3 in mm^3

14 Express

a 3.49 t in kg

b $1\frac{3}{4}$ hours in minutes

c 0.4 mm in cm

15 Express

a 0.006 m^2 in cm^2

b 0.08 litres in cm^3

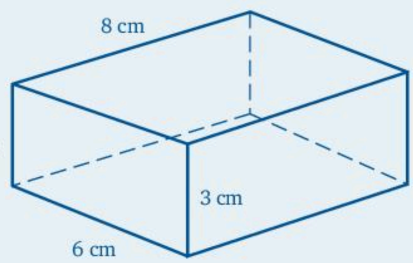
c 0.000 05 litres in mm^3

16 **a** How many vertices does this cuboid have?

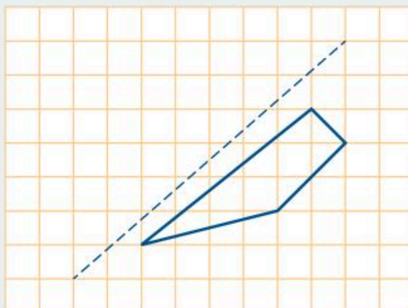
b How many faces measure 8 cm by 6 cm?

c How many faces measure 6 cm by 3 cm?

d What are the measurements of the remaining faces?

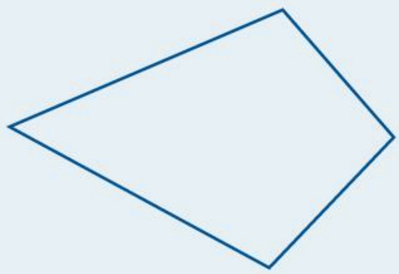


17 Copy the object and the mirror line on to square grid paper.
Draw the image of the object.



18 **a** What name do we give to this shape?

b How many sides (if any) are perpendicular?
Give a reason for your answer.



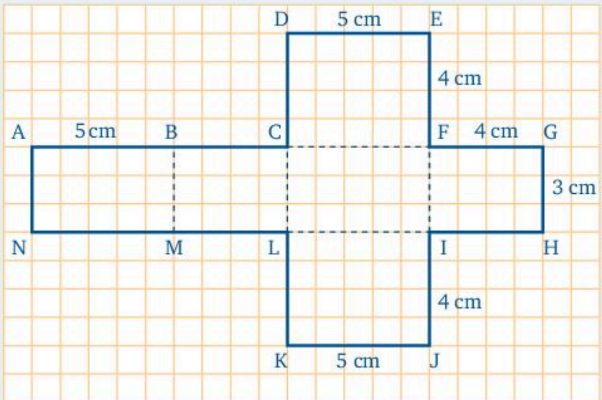
19 This shape has rotational symmetry of order 4 about ×.
Copy the diagram and complete it.

20 This net will make a cuboid.

a How many rectangular faces does it have?

b Calculate the external surface area of the cuboid.

c Name the vertices that meet at E.



21 Using only a ruler and a pair of compasses, construct triangle ABC where angle A = 90°, AB = 6 cm and AC = 8 cm.
Measure and record the length of BC.

11 Ratio and proportion

At the end of this chapter you should be able to...

1 express the ratio of one number to another
2 express the ratio of one unit of measurement to another
3 write a ratio equivalent to a given ratio
4 divide a quantity in a given ratio
5 use the map ratio to calculate the actual distance between two places given their distance apart on a map
6 solve problems on direct proportion using the unitary method.

You need to know...

✔ the meaning of equivalent fractions
✔ how to simplify fractions
✔ how to multiply a fraction by a whole number
✔ the meaning of lowest common multiple.

Key words

denominator, directly proportional, equivalent fractions, map ratio, numerator, ratio, relative size, representative fraction

Simplifying ratios

Suppose that Peter makes a model of his father's boat. If the model is 1 m long while the actual boat is 20 m long, we say that the *ratio* of the length of the model to the length of the actual boat is 1 m to 20 m or, more simply, 1 : 20.
We can also write the ratio as the fraction $\frac{1}{20}$.
If Peter built a larger model which was 2 m long then the ratio would be

length of model : length of actual boat = 2 : 20

As a fraction it would be

$$\frac{\text{length of model}}{\text{length of boat}} = \frac{2}{20} \text{ which simplifies to } \frac{1}{10}.$$

So 2 : 20 = 1 : 10
Ratios are therefore comparisons between related quantities.

Exercise 11a

Express the ratios in their simplest form: **a** 24 to 72 **b** 2 cm to 1 m

a $\frac{24}{72} = \frac{3}{9} = \frac{1}{3}$ (dividing both numbers by 8 and then by 3)

or $24:72 = 3:9 = 1:3$

so $24:72 = 1:3$

b (Before we can compare 2 cm and 1 m they must be expressed in the same unit.)

$\frac{2\text{ cm}}{1\text{ m}} = \frac{2\text{ cm}}{100\text{ cm}} = \frac{1}{50}$ or $2\text{ cm}:1\text{ m} = 2\text{ cm}:100\text{ cm} = 2:100 = 1:50$

so $2\text{ cm}:1\text{ m} = 1:50$

Express the following ratios in their simplest form:

1 $8:10$

2 $20:16$

3 $12:18$

4 $2\text{ cm}:8\text{ cm}$

5 $32\text{c}:96\text{c}$

6 $45\text{ g}:1\text{ kg}$

7 $\$4:75\text{c}$

8 $48\text{c}:\$2.88$

9 $288:306$

10 $10\text{ cm}^2:1\text{ m}^2$

Simplify the ratio $24:18:12$

(As there are three numbers involved, this ratio cannot be expressed as a single fraction.)

$24:18:12 = 4:3:2$ (dividing each number by 6)

Simplify the ratios:

11 $4:6:10$

12 $18:24:36$

13 $2:10:20$

14 $9:12:15$

15 $20:24:32$

<u>16</u> $7:56:49$

<u>17</u> $15:20:35$

<u>18</u> $16:128:64$

<u>19</u> $144:12:24$

<u>20</u> $98:63:14$

We know that we can produce *equivalent fractions* by multiplying or dividing both *numerator* and *denominator* by the same number,

so that $\frac{2}{3}=\frac{4}{6}$ or $\frac{12}{18}$ or $\frac{20}{30}$.

We can do the same with a ratio in the form 3 : 6.

$3:6=6:12$ (multiplying both numbers by 2)

and $2:\frac{1}{3}=6:1$ (multiplying both numbers by 3)

We can use this to simplify ratios containing fractions.

Exercise 11b

Express the following ratios in their simplest forms:

1 $5:\frac{1}{3}$

2 $2:\frac{1}{4}$

3 $\frac{1}{2}:\frac{1}{3}$

4 $\frac{3}{4}:\frac{1}{4}$

5 $\frac{1}{3}:\frac{3}{4}$

6 $\frac{7}{12}:\frac{5}{6}$

7 $\frac{5}{4}:\frac{6}{7}$

8 $3:\frac{4}{3}$

9 $2\frac{2}{3}:1\frac{1}{6}$

10 $\frac{2}{3}:\frac{7}{15}$

11 24 : 15 : 9

12 $\frac{4}{9}:\frac{2}{3}$

13 $4:\frac{9}{10}$

14 $\frac{4}{5}:6$

15 $7\frac{1}{2}:9\frac{1}{2}$

16 $\frac{1}{4}:\frac{1}{5}$

17 $1\frac{1}{2}:3:4\frac{1}{2}$

18 $6:4\frac{1}{2}$

19 $\frac{1}{6}:\frac{1}{8}:\frac{1}{12}$

20 6 : 8 : 12

Relative sizes

Exercise 11c

Which ratio is the larger, 6 : 5 or 7 : 6?

(Express each ratio as a fraction. We need to compare the sizes of $\frac{6}{5}$ and $\frac{7}{6}$ so we express both with the same denominator.)

$$\frac{6}{5}=\frac{36}{30} \quad \text{and} \quad \frac{7}{6}=\frac{35}{30}$$

so 6 : 5 is larger than 7 : 6

1 Which ratio is the larger, 5 : 7 or 2 : 3?

2 Which ratio is the smaller, 7 : 4 or 13 : 8?

3 Which ratio is the larger, $\frac{5}{8}$ or $\frac{7}{12}$?

4 Which ratio is the smaller, $\frac{3}{4}$ or $\frac{7}{10}$?

In the following sets of ratios some are equal to one another. In each question identify the equal ratios.

5 $6:8$, $24:32$, $\frac{3}{4}:1$

Simplify each ratio, then you can see which are equal.

6 $10:24$, $\frac{5}{9}:\frac{4}{5}$, $\frac{5}{9}:\frac{4}{3}$

7 $8:64$, $2:14$, $\frac{1}{16}:\frac{1}{2}$

8 $\frac{2}{3}:3$, $4:18$, $2:6$

Ratio problems

Exercise 11d

A family has 12 pets of which 6 are cats or kittens, 2 are dogs and the rest are birds. Find the ratio of the numbers of

a birds to dogs **b** birds to pets.

a There are 4 birds and 2 dogs

So number of birds : number of dogs $= 4:2$

$= 2:1$

b There are 4 birds and 12 pets

So number of birds : number of pets $= 4:12$

$= 1:3$

In each question give your answer in its simplest form.

1 A couple have 6 grandsons and 4 granddaughters. Find

a the ratio of the number of grandsons to that of granddaughters

b the ratio of the number of granddaughters to that of grandchildren.

2 Square A has side 6 cm and square B has side 8 cm. Find the ratio of

a the length of the side of square A to the length of the side of square B

b the area of square A to the area of square B.

3 Tom walks 2 km to school in 40 minutes and John cycles 5 km to school in 15 minutes. Find the ratio of

a Tom's distance to John's distance

b Tom's time to John's time.

4 Mary has 18 sweets and Jane has 12. As Mary has 6 sweets more than Jane she tries to even things out by giving Jane 6 sweets. What is the ratio of the number of sweets Mary has to the number Jane has

a to start with b to end with?

5 If $p:q = 2:3$, find the ratio $6p:2q$.

6 Rectangle A has length 12 cm and width 6 cm while rectangle B has length 8 cm and width 5 cm. Find the ratio of

a the length of A to the length of B

b the area of A to the area of B

c the perimeter of A to the perimeter of B

d the size of an angle of A to the size of an angle of B.

7 A triangle has sides of lengths 3.2 cm, 4.8 cm and 3.6 cm. Find the ratio of the lengths of the sides to one another.

8 Two angles of a triangle are 54° and 72°. Find the ratio of the size of the third angle to the sum of the first two.

9 For a school bazaar, Mrs Jones and Mrs Brown make marmalade in 1 lb jars. Mrs Jones makes 5 jars of lemon marmalade and 3 jars of orange. Mrs Brown makes 7 jars of lemon marmalade and 5 of grapefruit. Find the ratio of the numbers of jars of

a lemon to orange to grapefruit

b Mrs Jones' to Mrs Brown's marmalade

c Mrs Jones' lemon to orange.

Finding missing quantities

Some missing numbers are fairly obvious.

Exercise 11e

Find the missing numbers in the following ratios:

a $6:5 = \quad :10$ b $\frac{4}{3} = \frac{}{9} = \frac{24}{}$

a 10 is 5×2, so the missing number is 6×2. Therefore $6:5 = 12:10$

b Think of these as equivalent fractions. $\frac{4}{3} = \frac{12}{9} = \frac{24}{18}$

Find the missing numbers in the following ratios:

1 $2:5 = 4:\ $

2 $\ :6 = 12:18$

3 $24:14 = 12:\ $

4 $\frac{6}{\ } = \frac{9}{3}$

5 $3:\ \ = 12:32$

6 $\ :15 = 8:10$

7 $9:6 = \ :4$

8 $\frac{\ }{4} = \frac{15}{10}$

9 $\frac{6}{8} = \frac{\ }{12}$

10 $6:9 = 8:\ $

Further problems

Exercise 11f

Two speeds are in the ratio 12 : 5. If the first speed is 8 km/h, what is the second speed?

$8 = 12 \div 3 \times 2$

So the second speed is $5 \div 3 \times 2$ km/h

$= \frac{10}{3}$ km/h $= 3\frac{1}{3}$ km/h

1 The ratio of the amount of money in David's pocket to that in Indira's pocket is 9 : 10. Indira has $250. How much has David got?

2 Two lengths are in the ratio 3 : 7. The second length is 42 cm. Find the first length.

3 If the ratio in question **2** were 7 : 3, what would the first length be?

4 In a rectangle, the ratio of length to width is 9 : 4. The length is 24 cm. Find the width.

5 The ratio of the perimeter of a triangle to its shortest side is 10 : 3. The perimeter is 35 cm. What is the length of the shortest side?

6 A length, originally 6 cm, is increased so that the ratio of the new length to the old length is 9 : 2. What is the new length?

7 A class is making a model of the school building and the ratio of the lengths of the model to the lengths of the actual building is 1 : 20. The gym is 6 m high. How high, in centimetres, should the model of the gym be?

8 The ratio of lengths of a model boat to those of the actual boat is 3 : 50. Find the length of the actual boat if the model is 72 cm long.

Division in a given ratio

Exercise 11g

Share \$60 between Anne and John so that Anne's share and John's share are in the ratio 3 : 2.

Anne has 3 portions and John has 2 portions so they have 5 portions between them. So one portion is $\frac{1}{5}$ of \$60, i.e. $\$\frac{60}{5} = \$12$.

We know that Anne has 3 portions,

$$\therefore \quad \text{Anne's share} = 3 \times \$12 = \$36$$

$$\text{John's share} = 2 \times \$12 = \$24$$

Check: $\$36 + \$24 = \$60$

1 Divide \$80 into two parts in the ratio 3 : 2.

2 Divide 32 cm into two parts in the ratio 3 : 5.

3 Divide \$45 into two shares in the ratio 4 : 5.

4 Dick and Tom share the contents of a bag of peanuts between them in the ratio 3 : 5. If there are 40 peanuts, how many do they each get?

5 Maria is 10 years old and Eleanor is 15 years old. Divide \$750 between them in the ratio of their ages.

6 In a class of 30 pupils the ratio of the number of boys to the number of girls is 7 : 8. How many girls are there?

7 Divide \$2000 into two parts in the ratio 1 : 7.

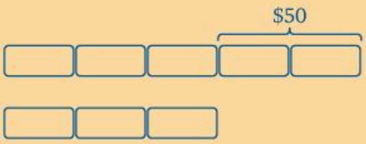
8 In a garden the ratio of the area of lawn to the area of flowerbed is 12 : 5. If the total area is 357 m², find the area of

a the lawn **b** the flowerbed.

9 In a bowl containing oranges and apples, the ratio of the numbers of oranges to apples is 4 : 3. If there are 28 fruit altogether, how many apples are there?

Gabriella and Ronnel get a share of an inheritance in the ratio of 5 : 3. Gabriella receives $50 more than Ronnel. Find the total inheritance.

You can visualise this problem by drawing a block for each part of the money that each receives.



Now you can see that the $50 is two of eight equal shares.

Therefore 1 share is $\$\frac{50}{2} = \$25$

Therefore the total inheritance is $8 \times \$25 = \$200$

10 A wooden plank was sawn into two pieces in the ratio 5 : 7. What remained was 44 cm longer than the length cut off.

a How long was the original plank?

b What length was cut off?

11 After Jo had peeled an orange she found the ratio of what she could eat to what was wasted was 3 : 2. If she could eat 42 g more than she wasted what was the mass of

a the original orange

b the part she could eat?

12 The Education and Social Services departments of a region divide the total grant they receive from the government between the two departments in the ratio 5 : 4. As a result the Education department receives $546 000 more than Social Services.

How much does each receive?

13 Uncle Rohan gives Misha and John a sum of money to share between them so that Misha gets $1000 more than John. He says, 'Divide it in the ratio 5 : 7.'

How much does each receive?

14 A plank of wood is cut into two pieces. The shorter piece is $\frac{3}{8}$ of the original plank.

What is the ratio of the lengths of the two pieces?

15 A sum of money is divided between Eleanor and Rohan in the ratio 8 : 5.

If Rohan receives $4000, how much does Eleanor receive?

16 Sarah and Amy share the chocolates in a large box between them in the ratio 4 : 7. If Sarah receives 16 chocolates, how many does Amy receive?

17 The money raised at a school bazaar is divided between the Art department and the Music department in the ratio 5 : 6. If the Art department receives $155 000, how much money was raised?

Divide 6 m into three parts in the ratio 3 : 7 : 2.

There are 3 + 7 + 2, i.e. 12 portions altogether, so one portion is $\frac{1}{12}$ of 600 cm,

$$\therefore \quad \text{first part} = \frac{3}{12} \times 600\,\text{cm} = 150\,\text{cm}$$

$$\text{Similarly second part} = \frac{7}{12} \times 600\,\text{cm} = 350\,\text{cm}$$

$$\text{and third part} = \frac{2}{12} \times 600\,\text{cm} = 100\,\text{cm}$$

Check: 150 + 350 + 100 = 600 cm = 6 m

18 Divide $2600 amongst three people so that their shares are in the ratio 4 : 5 : 4.

19 The perimeter of a triangle is 24 cm and the lengths of the sides are in the ratio 3 : 4 : 5. Find the lengths of the three sides.

20 In a garden, the ratio of the areas of lawn to beds to paths is $3 : 1 : \frac{1}{2}$. Find the three areas if the total area is 63 m².

Puzzle

A man left $799 500 to be divided between his widow, four sons and five daughters. He directed that every daughter should have three times as much as a son and that every son should have twice as much as their mother. What was the widow's share?

Map ratio (or representative fraction)

The *map ratio* of a map is the ratio of a length on the map to the length it represents on the ground. This ratio or fraction is given on most maps in addition to the scale. It is sometimes called the *representative fraction* of the map, or RF for short.

If two villages are 6 km apart and on the map this distance is represented by 6 cm, then the ratio is

$$\begin{aligned} 6\,\text{cm} : 6\,\text{km} &= 6\,\text{cm} : 600\,000\,\text{cm} \\ &= 1 : 100\,000 \end{aligned}$$

so the map ratio is $1 : 100\,000$ or $\frac{1}{100\,000}$

Any length on the ground is 100 000 times the corresponding length on the map.

Exercise 11h

Find the map ratio of a map if 12 km is represented by 1.2 cm on the map.

$$\begin{aligned} \text{RF} &= 1.2\,\text{cm} : 12\,\text{km} \\ &= 1.2\,\text{cm} : 1\,200\,000\,\text{cm} \quad \text{(changing both to the same unit)} \\ &= 12 : 12\,000\,000 \quad \text{(multiplying both numbers by 10)} \\ &= 1 : 1\,000\,000 \quad \text{(dividing both numbers by 12)} \end{aligned}$$

Find the map ratio of the maps in the following questions:

1 2 cm on the map represents 1 km.

2 The scale of the map is 1 cm to 5 km.

3 10 km is represented by 10 cm on the map.

4 3.2 cm on the map represents 16 km.

5 $\frac{1}{2}$ cm on the map represents 500 m.

6 100 km is represented by 5 cm on the map.

If the map ratio is 1 : 5000 and the distance between two points on the map is 12 cm, find the actual distance between the two points.

1 cm on the map represents 5000 cm on the ground.

12 cm on the map represents 12×5000 cm on the ground,

i.e. 60 000 cm = 600 m

7 The map ratio of a map is 1 : 50 000. The distance between A and B on the map is 6 cm. What is the true distance between A and B?

8 The map ratio of a map is 1 : 1000. A length on the map is 7 cm. What real length does this represent?

9 The map ratio of a map is 1 : 10 000. Find the actual length represented by 2 cm.

10 The map ratio of a map is 1 : 200 000. The distance between two towns is 20 km. What is this in centimetres? Find the distance on the map between the points representing the towns.

11 The map ratio of a map is 1 : 2 000 000. Find the distance on the map which represents an actual distance of 36 km.

Puzzle

If four hens lay four eggs in four days, how long will it take twelve hens to lay 36 eggs?

Proportion

When comparing quantities, words other than ratio are sometimes used. If two varying quantities are *directly proportional* they are always in the same ratio.

Sometimes it is obvious that two quantities are directly proportional, e.g. the cost of buying oranges is proportional to the number of oranges bought. In cases like this you would be expected to know that the quantities are in direct proportion.

Exercise 11i

A book of 250 pages is 1.5 cm thick (not counting the covers).

a How thick is a book of 400 pages?

b How many pages are there in a book 2.7 cm thick?

a If 250 pages are 15 mm thick then 1 page is $\frac{15}{250}$ mm thick

so 400 pages are $\frac{\cancel{15}^{3}}{\cancel{250}_{\cancel{50}_{1}}} \times \cancel{400}^{8}$ mm thick that is, 24 mm or 2.4 cm thick

b If 15 mm contains 250 pages then 1 mm contains $\frac{250}{15}$ pages

so 27 mm contains $\frac{\cancel{250}^{50}}{\cancel{15}_{\cancel{3}_{1}}} \times \cancel{27}^{9}$ pages that is, 450 pages

1 Sam covers 9 m when he walks 12 paces. How far does he travel when he walks 16 paces?

2 I can buy 24 bottles of a cold drink for $2500 when buying in bulk. How many bottles can I buy at the same rate for $7500?

3 If 64 seedlings are allowed 24 cm^2 of space, how much space should be allowed for 48 seedlings? How many seedlings can be planted in 27 cm^2?

4 A ream (500 sheets) of paper is 6 cm thick. How thick a pile would 300 sheets make?

5 At a school picnic 15 sandwiches are provided for every 8 children. How many sandwiches are needed for 56 children?

A family with two pets spends $750 a week on pet food. If the family gets a third pet, how much a week will be spent on pet food?

We are not told what sort of animals the pets are. Different animals eat different types and quantities of food so the amount spent is not in proportion to the number of pets.

Beware! Some of the quantities in the following questions are not in direct proportion. Some questions need a different method and some cannot be answered at all from the given information.

6 Two tea towels dry on a clothes line in 2 hours. How long would 5 tea towels take to dry?

7 Three bricklayers build a wall in 6 hours. How long would two bricklayers take to build the wall working at the same rate?

8 House contents insurance is charged at the rate of $450 per hundred thousand dollars worth of the contents. How much is the insurance if the contents are worth $1360 000?

9 If the insurance paid on the contents of a house is $9000, at the rate of $500 per hundred thousand dollars worth, what are the house contents worth?

10 It takes Margaret 45 minutes to walk 4 km. How long would it take her to walk 5 km at the same speed? How far would she go in 1 hour?

11 It takes a gardener 45 minutes to dig a flower bed of area 7.5 m^2. If he digs at the same rate, how long does he take to dig 9 m^2?

12 Fencing costs $12 000 per 1.8 m length. How much would 7.5 m cost?

13 Mrs Brown and Mrs Jones make 4 dozen sandwiches in half an hour in Mrs Jones' small kitchen. If they had 30 friends in to help, how many sandwiches could be made in the same time?

14 A recipe for 12 scones requires 2 teaspoons of baking powder and 240 g of flour. If a larger number of scones are made, using 540 g of flour, how much baking powder is needed?

Investigation

People come in all shapes and sizes but we expect the relative sizes of different parts of our bodies to be more or less the same.

For example, we do not expect a person's arms to be twice as long as their legs! We might expect the ratio of arm length to leg length to be about 2 : 3.

1 Gather some evidence and use it to find out if the last statement is roughly true.

2 Does the age of the person make any difference?

3 Investigate the ratio of shoe size to height.

Mixed exercises

Exercise 11j

1 Express the ratio $10\,\text{mm}^2 : 1\,\text{cm}^2$ in its simplest form.

2 Simplify the ratio $\frac{7}{8} : \frac{3}{4}$.

3 Adrian has $240 and Brian has $360. Give the ratio of the amount of Adrian's money to the total amount of money.

4 Which ratio is the larger, 16 : 13 or 9 : 7?

5 What is the map ratio of a map with a scale of 1 cm to 5 km?

6 Find the missing number in the ratio 7 : 12 =   : 9.

7 Share $2600 amongst three people in the ratio 6 : 3 : 4.

8 The ratio of boys to girls in a school is 10 : 9. There are 459 girls. How many boys are there?

Exercise 11k

1 Express the ratio 96 : 216 in its simplest form.

2 Simplify the ratio $\frac{1}{4}:\frac{2}{5}$.

3 Divide \$10 000 into three parts in the ratio 10 : 13 : 2.

4 Two cubes have edges of lengths 8 cm and 12 cm. Find the ratio of

  a the lengths of their edges

  b their volumes.

5 Find the missing number in the ratio $\square : 18 = 11 : 24$.

6 What does 1 cm represent on a map with map ratio 1 : 10 000?

7 If $x : y = 3 : 4$, find the ratio $4x : 3y$.

8 It costs \$2250 to feed a dog for 12 days. At the same rate, how much will have to be spent to feed it for 35 days?

Exercise 11l

1 Express the ratio 1028 : 576 in its simplest form.

2 Which ratio is the smaller, 32 : 24 or 30 : 22?

3 An alloy is made of copper and zinc in the ratio 11 : 2. How much zinc does 65 kg of alloy contain?

4 Increase a length of 24 m so that the ratio of the new length to the old length is 11 : 8.

5 Anne has twice as many crayons as Martin, who has three times as many as Susan. Give the ratio of the number of crayons owned by the three children.

6 The map ratio of a map is 1 : 50 000. Find the length on the ground represented by 6.4 cm on the map.

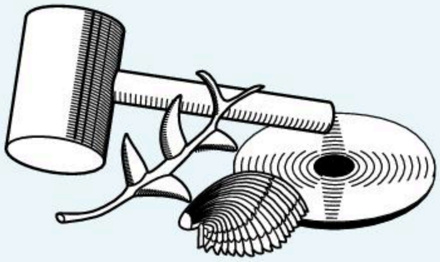
7 Simplify the ratio $\frac{13}{12}:\frac{5}{21}$.

8 Tiling to cover a floor of area 15 m^2 costs \$27 500. How much would you expect to pay for similar tiling to cover an area measuring 5 m by 4.2 m?

Did you know?

Bhaskara, who lived in India around CE 1150, did some very interesting work in arithmetic.

Bhaskara's work was full of puzzles and stories.
He had a problem about Hari, a god who had four hands.
He wanted to pick up a hammer, a shell, a flower and a discus. Hari wanted to know in how many ways he can pick up these four things. Can you figure it out?
A chart like the one below will help you.



	Hand 1	**Hand 2**	**Hand 3**	**Hand 4**
1st way	Hammer	Shell	Flower	Discus
2nd way	Hammer	Shell	Discus	Flower
3rd way				

Complete the table. How many ways are possible?

In this chapter you have seen that…

- ✔ a ratio can be simplified by dividing (or multiplying when fractions are involved) all parts in the ratio by the same number
- ✔ you can compare ratios by expressing them with the same denominator
- ✔ you can divide a quantity in the ratio $a:b$ by first dividing it into $(a+b)$ equal parts
- ✔ the scale of a map is sometimes given as a ratio, e.g. 1 : 25 000, and it is called the map ratio; it is sometimes given as a fraction, e.g. $\frac{1}{25\,000}$, when it is called the representative fraction
- ✔ two quantities that are directly proportional are always in the same ratio.

12 Statistics

At the end of this chapter you should be able to...

1 organise statistical data in a frequency table
2 calculate the arithmetic mean of a set of numbers
3 find the missing number from a set, given the mean and the remaining numbers
4 find the mode of a given set of numbers
5 find the median of a given set of numbers
6 find the range of a set of data.

Did you know?

Florence Nightingale gathered statistics during the Crimean War (1853–1856) and put them to good use. When she arrived at Scutari in Crimea she found that 42% of the soldiers admitted to the hospital died, mainly from disease, not war wounds. With a team of nurses she reduced this number to 2%. She went on to set up the first professional nursing service.

You need to know...

✔ how to work with whole numbers and with decimals and fractions.

Key words

arithmetic average, data, frequency, frequency table, mean, median, mode, range, tally

Frequency tables

The branch of mathematics called statistics is used for dealing with large collections of information in the form of numbers. This information is known as *data*. The number of items of data can run into thousands as, for instance, when the incomes of everyone in Trinidad and Tobago are being considered, but to learn the methods we start with smaller collections.

If we ask 72 people how many holidays outside Jamaica they took in the previous year, and record their answers, we are faced with a disorganised set of numbers:

1	4	1	3	4	5	4	2	6
7	3	5	2	2	3	1	0	1
0	1	2	0	1	1	2	1	2
1	0	1	3	2	1	3	2	2
1	2	2	5	6	4	0	2	1
3	1	1	2	2	0	1	3	5
6	1	2	2	1	0	1	1	4
1	1	0	2	2	1	3	4	3

To make sense of these numbers we must put them in order. One way of doing this is to form a *frequency table*. The first column lists the number of holidays taken abroad, from the least to the most. Work down the columns of data, making a *tally* mark, |, in the tally column opposite the appropriate number of holidays in the first column of the frequency table. (Do *not* go through the data looking for particular numbers.)

Count up the tally marks and write the total in the *frequency* column. Check by adding up the numbers in the frequency column. The total of the frequency should be the same as the total number of pieces of data recorded. (Arrange the tally marks in fives, either by leaving a gap between blocks, or by crossing four tally marks with the fifth, as is done in this table.)

Number of holidays abroad	Tally	Frequency
0	𝍸 |||	8
1	𝍸 𝍸 𝍸 𝍸 |||	23
2	𝍸 𝍸 𝍸 |||	18
3	𝍸 ||||	9
4	𝍸 |	6
5	||||	4
6	|||	3
7	|	1
	Total	72

We can see now that the greatest number of people took one holiday away from Jamaica in the previous year, and far fewer people took five or more holidays abroad than took one or two holidays abroad.

Exercise 12a

For each question draw up a frequency table like the one on the previous page.

1 The list shows the US shoe size of each of 72 women questioned in a survey.

4	11	10	6	7	10	7	8	6
7	8	9	7	9	8	9	9	8
6	9	10	9	8	9	8	10	10
8	10	9	8	7	8	8	7	7
8	7	7	12	6	9	10	9	9
11	9	9	8	9	10	11	8	5
5	8	7	7	8	10	6	6	7
8	8	10	8	7	7	6	7	11

What fraction of women take the most common shoe size?

2 This list gives the total number of goals scored in the 38 UK Premier League games played by Manchester United in the 2018/2019 season.

3	5	3	2	3	2
4	5	4	3	3	4
0	4	4	5	4	6
4	5	2	1	3	4
1	3	0	4	5	2
2	3	3	3	4	2
2	2				

In how many games were more than 4 goals scored in total?

3 The following list shows the number of bedrooms in the homes of 60 people surveyed.

3	2	4	3	3	3
2	5	6	2	3	3
3	2	7	2	2	3
3	4	4	4	2	3
3	3	1	1	5	2
2	3	3	3	4	2
2	2	3	1	2	2
1	1	3	5	4	3
3	2	1	5	6	1
2	3	3	3	3	4

Which numbers of bedrooms are equally common?

Averages

We are frequently looking for ways of representing a set of numbers in a simple form. Can we choose a single number that will adequately represent a set of numbers?

We try to do this by using averages.

Three different types of average are used, each with its own individual advantages and disadvantages. They are the *arithmetic average* or *mean*, the *mode* and the *median*.

The arithmetic average or mean

Consider a group of five children. When they are asked to produce the money they are carrying the amounts collected are \$550, \$1400, \$950, \$600 and \$750 respectively. If the total of this money (\$4250) is shared equally amongst the five children, each will receive \$850. This is called the arithmetic average or mean of the five amounts.

> The arithmetic average or mean of a set of numbers is the sum of the numbers divided by the number of numbers in the set.

For example, the average or mean of 12, 15, 25, 42 and 16 is

$$\frac{12+15+25+42+16}{5}=\frac{110}{5}=22$$

One commonplace use of the arithmetic average is to compare the marks of pupils in a group or form. The pupils are given positions according to their average mark over the full range of subjects they study. An advantage is that we can compare the results of pupils who study 7 subjects with those who study 11 subjects. A disadvantage is that one very poor mark may pull the mean down significantly.

The mean may also be rather artificial, for example, giving $5\frac{1}{3}$ dollars to each of a group of people, or having a mean shoe size of 5.1, or a mean family size of 2.24 children.

Exercise 12b

Find the arithmetic average or mean of the following sets of numbers:

1 3, 6, 9, 14

2 2, 4, 9, 13

3 12, 13, 14, 15, 16, 17, 18

4 23, 25, 27, 29, 31, 33, 35

5 19, 6, 13, 10, 32

6 34, 14, 39, 20, 16, 45

7 1.2, 2.4, 3.6, 4.8

8 18.2, 20.7, 32.5, 50, 78.6

9 6.3, 4.5, 6.8, 5.2, 7.3, 7.1

10 3.1, 0.4, 7.2, 0.7, 6.1

11 38.2, 17.6, 63.5, 80.7

12 0.76, 0.09, 0.35, 0.54, 1.36

Nikolai's examination percentages in 8 subjects were 83, 47, 62, 49, 55, 72, 58 and 62. What was his mean mark?

Mean mark for 8 subjects

$$= \frac{\text{sum of the marks in the 8 subjects}}{\text{number of subjects}}$$

$$= \frac{83+47+62+49+55+72+58+62}{8}$$

$$= \frac{488}{8}$$

$$= 61$$

13 In the Christmas terminal examinations Lisa scored a total of 504 in 8 subjects. Find her mean mark.

14 A darts player scored 2304 in 24 visits to the board. What was his average number of points per visit?

15 A bowler took 110 wickets for 1815 runs. Calculate his average number of runs per wicket.

16 Kemuel's examination percentages in 7 subjects were 64, 43, 86, 74, 55, 53 and 66. What was his mean mark?

17 In six consecutive English examinations, Joy's percentage marks were 83, 76, 85, 73, 64 and 63. Find her mean mark.

<u>18</u> A football team scored 54 goals in 40 league games. Find the average number of goals per game.

<u>19</u> The first Hockey XI scored 14 goals in their first 16 matches. What was the average number of goals per match?

<u>**20**</u> In a dancing competition the recorded scores for the winners were 5.8, 5.9, 6.0, 5.8, 5.8, 5.8, 5.6 and 5.7. Find their mean score.

<u>**21**</u> The recorded rainfall each day at a holiday resort during the first week of my holiday was 3 mm, 0, 4.5 mm, 0, 0, 5 mm and 1.5 mm. Find the mean daily rainfall for the week.

<u>**22**</u> The masses of the members of a rowing eight were 82 kg, 85 kg, 86 kg, 86 kg, 84 kg, 88 kg, 92 kg and 85 kg. Find the average mass of the 'eight'. If the cox weighed 41 kg, what was the average mass of the crew?

On average my car travels 28.5 miles on each gallon of petrol. How far will it travel on 30 gallons?

If the car travels 28.5 miles on 1 gallon of petrol it will travel 30×28.5 miles, i.e. 855 miles, on 30 gallons.

23 My father's car travels on average 33.4 miles on each gallon of petrol. How far will it travel on 55 gallons?

24 Olga's car travels on average 12.6 km on each litre of petrol. How far will it travel on 205 litres?

25 The average daily rainfall in Puddletown during April was 2.4 mm. How much rain fell during the month?

26 The daily average number of hours of sunshine during my 14-day holiday in Florida was 9.4. For how many hours did the sun shine while I was on holiday?

Elaine's average mark after 7 subjects is 56 and after 8 subjects it has risen to 58. How many does she score in her eighth subject?

We can find the total scored in 7 subjects and the total scored in 8 subjects. Then the score in the 8th subject is the difference between these two totals.

Total scored in 7 subjects is, $56 \times 7 = 392$

Total scored in 8 subjects is, $58 \times 8 = 464$

Score in her eighth subject

= total for 8 subjects – total for 7 subjects

$= 464 - 392$

$= 72$

Therefore Elaine scores 72 in her eighth subject.

27 Zachary's batting average after 11 completed innings was 62. After 12 completed innings it had increased to 68. How many runs did he score in his twelfth innings?

28 Richard was collecting money for charity. The average amount collected from the first 15 houses at which he called was $30, while the average amount collected after 16 houses was $35. How much did he collect from the sixteenth house?

29 After six examination results Tom's average mark was 57. His next result increased his average to 62. What was his seventh mark?

30 Anne's average mark after 8 results was 54. This dropped to 49 when she received her ninth result, which was for French. What was her French mark?

31 During a certain week the number of lunches served in a school canteen were: Monday 213, Tuesday 243, Wednesday 237 and Thursday 239. Find the average number of meals served daily over the four days. If the daily average for the week (Monday–Friday) was 225, how many meals were served on Friday?

32 A paperboy's sales during a certain week were: Monday 84, Tuesday 112, Wednesday 108, Thursday 95 and Friday 131. Find his average daily sales. When he included his sales on Saturday his daily average increased to 128. How many papers did he sell on Saturday?

33 The number of hours of sunshine in Barbados for successive days during a certain week were 11.1, 11.9, 11.2, 12.0, 11.7, 12.9 and 11.8. Find the daily average. The following week the daily average was 11 hours. How many more hours of sunshine were there the first week than the second?

34 Tissha's marks in the end of term examinations were 46, 80, 59, 83, 54, 67, 79, 82 and 62. Find her average mark. It was found that there had been an error in her mathematics mark. It should have been 74, not 83. What difference did this make to her average?

35 The heights of the 11 girls in a hockey team are 162 cm, 152 cm, 166 cm, 149 cm, 153 cm, 165 cm, 169 cm, 145 cm, 155 cm, 159 cm and 163 cm. Find the average height of the team. If the girl who was 145 cm tall were replaced by a girl 156 cm tall, what difference would this make to the average height of the team?

36 During the last five years the distances I travelled in my car, in miles, were 10 426, 12 634, 11 926, 14651 and 13 973. How many miles did I travel in the whole period? What was my yearly average? How many miles should I travel this year to reduce the average annual mileage over the six years to 11 984?

37 The average mass of the 18 boys in a class is 63.2 kg. When two new boys join the class the average mass increases to 63.7 kg. What is the combined mass of the two new boys?

38 The average height of the 12 boys in a class is 163 cm and the average height of the 18 girls is 159 cm. Find the average height of the class.

You can find the total height of the boys and the total height of the girls. From this you can find the total height of all 30 pupils. Then you can work out the average height.

39 The average mass of the 15 girls in a class is 54.4 kg while the average mass of the 10 boys is 57.4 kg. Find the average mass of the class.

40 In a school the average size of the 14 lower school forms is 30, the average size of the 16 middle school forms is 25 and the average size of the 20 upper school forms is 24. Find the average size of form for the whole school.

41 Northshire has an area of 400 000 hectares and last year the annual rainfall was 274 cm, while Southshire has an area of 150 000 hectares and last year the annual rainfall was 314 cm. What was the annual rainfall last year for the combined area of the two counties? Give your answer to the nearest cm.

42 After 10 three-day matches and 8 one-day matches, the average *daily* attendances for a cricket season were 2160 for three-day matches and 4497 for one-day matches. Calculate the average *daily* attendance for the 18 matches.

Puzzle

How is it possible for a student, whose average examination mark is 53, to increase that average by scoring 35 in the next examination?

Mode

The mode of a set of numbers is the number that occurs most frequently, e.g. the mode of the numbers 6, 4, 6, 8, 10, 6, 3, 8 and 4 is 6, since 6 is the only number occurring more than twice.

It would obviously be of use for a company with a chain of shoe shops to know that the mode or modal size for men's shoes in one part of the country is 8, whereas in another part of the country it is 7. Such information would influence the number of pairs of shoes of each size kept in stock.

If all the numbers in a set of numbers are different, there cannot be a mode, for no number occurs more frequently than all the others. On the other hand, if two numbers are equally the most popular, there will be two modes.

Exercise 12c

Find the mode of each of the following sets of numbers:

1 10, 8, 12, 14, 12, 10, 12, 8, 10, 12, 4

2 3, 9, 7, 9, 5, 4, 8, 2, 4, 3, 5, 9

3 1.2, 1.8, 1.9, 1.2, 1.8, 1.7, 1.4, 1.3, 1.8

4 58, 56, 59, 62, 56, 63, 54, 53

5 5.9, 5.6, 5.8, 5.7, 5.9, 5.9, 5.8, 5.7

6 26.2, 26.8, 26.4, 26.7, 26.5, 26.4, 26.6, 26.5, 26.4

7 The table shows the number of goals scored by a football club last season.

Numbers of goals	0	1	2	3	4	5	6
Frequency	12	16	7	4	2	0	1

Find the modal number of goals scored by this football club.

8 Given below are the marks out of 10 obtained by 30 girls in a history test.

8, 6, 5, 7, 8, 9, 10, 10, 3, 7, 3, 5, 4, 8, 7, 8, 10, 9, 8, 7, 10, 9, 9, 7, 5, 4, 8, 1, 9, 8

a Make a table similar to the table in question **7** to show the marks scored by the 30 girls in a history test.

b Find the mode for this data.

9 The heights of 10 girls, correct to the nearest centimetre, are:

155, 148, 153, 154, 155, 149, 162, 154, 156, 155

What is their modal height?

10 The number of letters in the words of a sentence were:

2, 4, 3, 5, 2, 3, 8, 2, 5, 7, 9, 3, 6, 3, 7, 3, 4, 9, 2, 3, 8, 3, 5, 2, 10, 3, 4, 6, 2, 3, 4

How many words were there in the sentence? What is the modal number of letters per word?

11 The shoe sizes of pupils in a class are:

4, 4, 7, 6, 5, 5, 6, 6, 6, 4, 5, 8, 6, 7, 4, 7, 9, 6,
5, 7, 6, 7, 8, 6, 4, 4, 4, 5, 5, 7, 7, 7, 5, 8, 6, 5

How many pupils are there in the class?

What is the modal shoe size?

Median

The median value of a set of numbers is the value of the middle number when they have been placed in ascending (or descending) order of magnitude.

Imagine nine children arranged in order of their height.

The height of the fifth or middle child is 154 cm,

i.e. the median height is 154 cm.

Similarly 24 is the median of 12, 18, 24, 37 and 46. Two numbers are smaller than 24 and two are larger.

To find the median of 16, 49, 53, 8, 32, 19 and 62, rearrange the numbers in ascending order:

8, 16, 19, 32, 49, 53, 62

then we can see that the middle number of these is 32,

i.e. the median is 32.

If there is an even number of numbers, the median is found by finding the average or mean of the two middle values after they have been placed in ascending or descending order.

Exercise 12d

Find the median of each of the following sets of numbers:

1 1, 2, 3, 5, 7, 11, 13

2 26, 33, 39, 42, 64, 87, 90

3 13, 24, 19, 13, 6, 36, 17

4 4, 18, 32, 16, 9, 7, 29

5 1.2, 3.4, 3.2, 6.5, 9.8, 0.4, 1.8

6 5, 7, 11, 13, 17, 19

7 34, 46, 88, 92, 104, 116, 118, 144

8 34, 42, 16, 83, 97, 24, 18, 38

9 1.92, 1.84, 1.89, 1.86, 1.96, 1.98, 1.73, 1.88

10 15.2, 6.3, 14.8, 9.5, 16.3, 24.9

Comparing the mean, median and mode

Each of these 'averages' has advantages and disadvantages compared with the other two.

The **mean** can be calculated exactly and uses all the data but can be misleading if there is one very high or very low value.

The **median** is easy to understand and is unaffected by a very high or a very low value.

The **mode** is simple to understand and is most useful for retailers ordering such things as shoes, clothes or hats.

Investigation

Investigate whether or not it is possible to write down a set of seven different whole numbers such that their mean, mode and median are all the same.

Range

The *range* of a set of data is the difference between the largest and the smallest values.

The answer is always a single number.

For example, the range of the masses 5 kg, 8 kg, 10 kg 12 kg is 12 kg − 5 kg = 7 kg

Exercise 12e

1 Six students got these marks in a test: 7, 9, 4, 6, 8, 6

What is the range of marks?

2 In four different shops, the cost of a particular soft drink was \$57, \$62, \$58 and \$60.

What was the range of the prices?

3 The ages, in years, of the girls in a hockey team were:

10, 9, 10, 11, 13, 10, 12, 14, 9, 10, 11

Write down the range of ages.

4 The ages of the members of a family are:

32, 24, 43, 7, 3, 49, 82, 16, 73

a How large is the family?

b What is the range of their ages?

5 The table shows the number of letters per word in a paragraph from a book.

Number of letters	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	4	10	18	15	14	8	9	7	6	8	3	2

What is the range in the number of letters per word in the words used in the paragraph?

6 The table shows the number of goals scored by the teams in a school football league.

Number of goals	0	1	2	3	4	5
Frequency	5	6	4	3	1	2

a How many teams are there in the league?

b Find the range of the number of goals scored.

7 The table shows the last round scores of the golfers taking part in a competition.

Score	66	68	69	70	71	72	73	74	75	76	77
Frequency	2	2	3	6	4	8	9	10	6	8	9

a How many scores are recorded?

b Find the range of scores.

8 The table shows the number of adults living in the houses in a street.

Number of adults	0	1	2	3	4	5	6	7
Frequency	2	2	3	5	8	7	4	1

a How many houses are there in the street?

b How many adults live in these houses?

c Find the range of the number of adults per house living in the street.

Mixed exercises

Exercise 12f

For each set of numbers in questions **1** to **5** find:

a the mean **b** the mode **c** the median **b** the range.

1 21, 16, 25, 21, 19, 32, 27

2 67, 71, 69, 82, 70, 66, 81, 66, 67

3 43, 46, 47, 45, 45, 42, 47, 49, 43, 43

4 84, 93, 13, 16, 28, 13, 32, 63, 45

5 30, 27, 32, 27, 28, 27, 26, 27

6 In seven rounds of golf a golfer returned these scores:

72, 87, 73, 72, 86, 72, 77

Find the mean, mode, median and range of these scores.

7 The heights (correct to the nearest centimetre) of a group of girls are: 159, 155, 153, 154, 157, 162, 152, 160, 161, 157. Find:

a their mean height
b their modal height
c their median height
d their range.

8 The marks, out of 100, in a geography test for the members of a class were: 64, 50, 35, 85, 52, 47, 72, 31, 74, 49, 36, 44, 54, 48, 32, 52, 53, 48, 71, 52, 56, 49, 81, 45, 52, 80, 46. Find:

a the mean mark
b the modal mark
c the median mark
d their range.

9 Find the mean, mode, median and range of the following golf scores:

85, 76, 91, 83, 88, 84, 84, 82, 77, 79, 80, 83, 86, 84

10 The table shows how many pupils in a form were absent for various numbers of sessions during a certain school week.

Number of sessions absent	0	1	2	3	4	5	6	7	8	9	10
Frequency	20	2	4	0	2	0	1	2	0	0	1

Find:

a the mode **b** the median **c** the mean **d** the range.

11 The table shows the number of children per family in the families of the pupils in a class.

Number of children	1	2	3	4	5	6	7
Frequency	1	3	9	5	5	2	1

Find:

a the mode **b** the median **c** the mean **d** the range.

Puzzle

Find the missing number in this set.

6	10	5	3
7	6	7	8
8	9	8	

Did you know?

You may become a statistic by census, sampling or record. These are only three of the ways.

Census

A census is a direct counting of a nation, state or school. From time to time the government of your island takes a census, recording the number and age of persons in the region and other information.

See if you can find the National Census Report for your island from the CARICOM 2000 Round of Population and Housing Census.

Sampling

A sample is a subset of a population. Conducting a census may be expensive or time-consuming if the population is large. In this case a sample will be used to get an indication of the situation for the population.

At election time predictions are made using samples.

Records

Brian Lara holds the record for the highest score in a test match. What is the score? Who was the last person to hold this record before Brian Lara?

Some records are obtained by chance while others are attained after long practice.

In this chapter you have seen that...

- ✔ large quantities of information can be made sense of by organising the data into a frequency table. The frequency is the number of times that a particular value occurs
- ✔ the arithmetic average or mean of a set of numbers is their sum divided by the number of them
- ✔ the mode is the number that occurs most often
- ✔ the median is the middle number when the values have been placed in ascending or descending order
- ✔ the range of a set of data is the difference between the largest and smallest values.

13 Algebra 1

At the end of this chapter you should be able to...

1 use letters to represent numbers in mathematical statements
2 use index notation to write down products
3 simplify simple expressions in one or more variables
4 substitute numbers into expressions.

Did you know?

Arabic numerals became known in the West through a book by the Arabian mathematician Muhammad ibn Musa al-Khwarizmi, written in the year 820 CE under the title al-Jabr wa'l-Muqabala. It is said that the word 'algebra' came from the title of this book.

You need to know...

✔ how to work with simple numbers
✔ how to express a number in index form
✔ how to work with brackets.

Key words

base, coefficient, expression, index (plural indices), like terms, unlike terms, variable

The idea of algebra

Algebra uses letters for numbers so that some facts can be discovered when the numbers involved are not known. For example think of a bag of sweets. Different people will think of varying numbers of sweets in the bag. If we use the letter n, we can say that there are n sweets in the bag. Letters used in this way are called *variables*.

Expressions

'I think of a number and take away 3.'

If we use the letter x for the number, we can write the sentence as $x - 3$.

$x - 3$ is called an *expression*.

Exercise 13a

Form an expression from the sentence 'I think of a number and add 4'.

Let the number be x. Then adding 4 to x can be written as

$$x + 4$$

Form expressions from the following sentences.

1. Think of a number and subtract 3.
2. Think of a number and add 1.
3. Think of a number and subtract 6.
4. The number 5 is subtracted from another number.

Think of a number and multiply it by 3.

Let the number be x.

The expression is $3 \times x$.

5. A number is doubled.
6. A number is multipled by 4.
7. 7 is multiplied by an unknown number.
8. 6 times an unknown number.

$4 \times n$ Any letter can be used to describe an unknown number.

$4 \times n$ means 4 times an unknown number.

Write sentences to show the meaning of the following expressions.

9 $3 \times n$ **11** $n - 5$ **13** $8 \times n$ **15** $12 + n$

10 $x + 9$ **12** $x + 8$ **14** $7 - x$ **16** $x \div 6$

Simplifying expressions

Expressions like $4 \times n$ are abbreviated to $4n$.

This means that, for example, $5x$ means $5 \times x$.

The known number multiplied by an unknown is called the *coefficient* of the unknown.

So 4 is the coefficient of n and 5 is the coefficient of x.

Like terms

Consider $3x + 5x - 4x + 2x$.

This is an expression and can be simplified to $6x$.

$3x$, $5x$, $4x$ and $2x$ are all *terms* in this expression. Each term contains x. They are of the same type and are called *like terms*.

Exercise 13b

Simplify $4h - 6h + 7h - h$

You can do the addition before the subtraction, i.e. $4h + 7h - 6h - h$

$$4h - 6h + 7h - h = 4h$$

Simplify:

1 $3x + x + 4x + 2x$

2 $3x - x + 4x - 2x$

3 $8x - 6x$

4 $6 - 1 + 4 - 7$

5 $9y - 3y + 2y$

6 $2 - 3 + 9 - 1$

7 $5 - 3 - 1$

8 $3x - 2x - x$

Remember that the sign in front of a number applies to that number only.

Unlike terms

$3x + 2x - 7$ can be simplified to $5x - 7$, and $5x - 2y + 4x + 3y$ can be simplified to $9x + y$.

Terms containing x are different from terms without an x. They are called *unlike terms* and cannot be collected. Similarly $9x$ and $5y$ are unlike terms; therefore $9x - 5y$ cannot be simplified.

Exercise 13c

Simplify $3x - 4 + 7 - 2x + 4x$

You can rearrange this to have the like terms together, i.e.

$3x - 2x + 4x + 7 - 4$

$$3x - 4 + 7 - 2x + 4x = 5x + 3$$

Simplify $2x + 4y - x + 5y$

$$2x + 4y - x + 5y = 2x - x + 4y + 5y = x + 9y$$

Simplify:

1 $2x + 4 + 3 + 5x$

2 $2x - 4 + 3x + 9$

3 $5x - 2 + 3 - x$

4 $4a + 5c + 6a$

5 $6x + 5y + 2x - 3y$

6 $6x + 5y + 2x + 3y$

7 $6x + 5y - 2x - 3y$

8 $6x + 5y - 2x + 3y$

9 $4x + 1 + 3x + 2 + x$

10 $6x + 9 + 2x - 1$

11 $7x - 3 + 9 - 4x$

12 $9x + 3y + 10x$

13 $6x + 5y + 2x + 3y + 2x$

14 $6x + 5y - 2x - 3y + 7x + y$

15 $30x + 2 - 15x - 6 + 4$

16 $2z + 3x + 4y + 6z + x - 3y$

17 $4x + 3y - 4 + 6x - 2y + 7 - x$

18 $7x + 3 - 9 - 9x + 2x + 6 + 11$

There are three sets of like terms here: x's, y's and z's.

Brackets

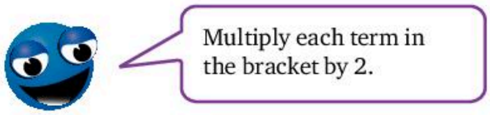
Sometimes brackets are used to hold two quantities together. For instance, if we wish to multiply the sum of x and 3 by 4 we write $4(x + 3)$. The multiplication sign is invisible just as it is in $5x$, which means $5 \times x$.

$4(x + 3)$ means 'four times everything in the brackets'

so we have $4 \times x$ and 4×3, and we write $4(x + 3) = 4x + 12$.

Exercise 13d

Multiply out the brackets:



1 $2(x+1)$
2 $3(3x+2)$
3 $5(x+6)$
4 $4(3x+3)$
5 $2(4+5x)$
6 $2(6+5a)$
7 $5(a+b)$
8 $4(4x+3)$
9 $3(6+4x)$
10 $5(x+1)$
11 $7(2+x)$
12 $8(3+2x)$

To simplify an expression containing brackets we first multiply out the brackets and then collect like terms.

Exercise 13e

Simplify $2+(3x+7)$

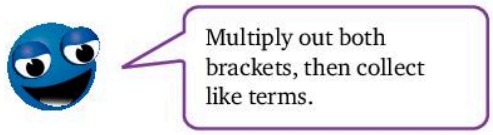
First deal with the brackets, then collect like terms.

$2+(3x+7)$ (This means $2+1(3x+7)$)

$=2+3x+7$

$=3x+9$

Simplify the following expressions:



1 $2x+4(x+1)$
2 $3+5(2x+3)$
3 $3(x+1)+4$
4 $7+2(2x+5)$
5 $2(x+4)+3(x+5)$
6 $3x+(2x+5)$
7 $4+(3x+1)$
8 $3x+2(3x+4)$

Investigation

Meg wanted to find out Malcolm's age without asking him directly what it was.

The following conversation took place.

Meg: Think of your age but don't tell me what it is

Malcolm: Right.

Meg: Multiply it by 5, add 4 and take away your age.

Malcolm: Yes.

Meg: Divide the result by 4 and tell me your answer.

Malcolm: 15

Meg: That means you are 14.

Malcolm: Correct. How do you know that?

However many times Meg tried this on her friends and relations she found their age by taking 1 away from the number they gave.

Does it always work?

Can you use simple algebra to prove that it always gives the correct answer?

Indices

We have already seen that the shorthand way of writing $2 \times 2 \times 2 \times 2$ is 2^4.
In the same way we can write $a \times a \times a \times a$ as a^4. The 4 is called the *index* and a is called the *base*.

Exercise 13f

Write the following expressions in index form:

1 $z \times z \times z$

2 $a \times a$

3 $b \times b \times b \times b \times b$

4 $y \times y \times y \times y \times y$

5 $s \times s \times s$

6 $z \times z \times z \times z \times z \times z$

Give the meanings of the following expressions:

7 a^3

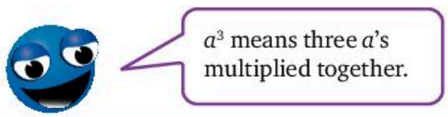
8 x^4

9 a^5

10 b^2

<u>**11**</u> x^6

<u>**12**</u> z^4



Simplify $2 \times x \times y \times x \times 3$

(Write the numbers first, then the letters in alphabetical order.)

$$2 \times x \times y \times x \times 3 = 2 \times 3 \times x \times x \times y \qquad (2 \times 3 \text{ is } 6 \text{ and } x \times x \text{ is } x^2)$$
$$= 6x^2y$$

Simplify the following expressions:

13 $2 \times a$

14 $4 \times x \times x$

15 $3 \times a \times 4$

16 $a \times a \times b$

17 $3 \times z \times x \times 5 \times z$

18 $5 \times a \times b \times b \times a$

Write each expression in full without using indices:

19 $3z^2$

20 $2abc$

21 $4zy^2$

22 $6a^2b$

<u>**23**</u> $2x^3$

<u>**24**</u> $3a^4b^2$

Simplify the following expressions:

25 $3x \times 2z$

26 $x \times 6x^2$

27 $4a^2 \times 3$

28 $3a \times 2a \times a$

29 $a \times b \times c \times 2a$

30 $4x \times 3y \times 2x$

31 $z \times z \times z \times z$

32 $2z \times 3z$

33 $4x^2 \times 6$

34 $2 \times 4 \times x \times 2$

35 $4s^2 \times s$

36 $x^2 \times x^4$

37 $y \times z \times y \times z$

38 $2x \times 5z \times y$

39 $a \times a \times a \times a \times a \times a \times a$

40 $4x^2 \times 2x^2$

41 $x \times y \times z \times a$

42 $s^4 \times s^3$

$3x \times 2z = 3 \times x \times 2 \times z$
You can change the order to $3 \times 2 \times x \times z$.
What is a simpler way of writing $x \times z$?

Substituting numbers into expressions

We can find a value for an expression when we substitute numbers for the variables in an expression. (Substituting numbers for variables means giving the variables numerical values.) For example, when $x = 6$, we replace x with 6

so the value of $2x - 4$ is $2 \times 6 - 4$

$= 12 - 4 = 8$

Exercise 13g

1 Find the value of $x - 7$ when **a** $x = 12$ **b** $x = 4$ **c** $x = -2$

2 Find the value of $16 - x$ when **a** $x = 9$ **b** $x = 16$ **c** $x = 20$

3 Find the value of $16 + x$ when **a** $x = 4$ **b** $x = -6$ **c** $x = -20$

4 Find the value of $5 - x$ when **a** $x = 4$ **b** $x = 6$ **c** $x = -2$

5 Find the value of $3x + 4$ when **a** $x = 2$ **b** $x = 3$ **c** $x = -1$

6 Find the value of $6 - 2x$ when **a** $x = 2$ **b** $x = 3$ **c** $x = -2$

7 Find the value of $6 + y$ when **a** $y = 4$ **b** $y = -5$ **c** $y = -10$

8 Find the value of $3n - 4$ when **a** $n = 3$ **b** $n = 1$ **c** $n = -1$

9 Find the value of $2n^2$ when **a** $n = 2$ **b** $n = 5$ **c** $n = -2$

10 Find the value of y^3 when **a** $y = 2$ **b** $y = 3$ **c** $y = -2$

$2n^2 = 2 \times n \times n$

Find the value of $\frac{x}{2} - \frac{3y}{4}$ when $x = 1$ and $y = 3$

Replacing x with 1 and y with 3 gives

$$\frac{1}{2} - \frac{3 \times 3}{4} = \frac{1}{2} - \frac{9}{4}$$
$$= \frac{2-9}{4}$$
$$= \frac{-7}{4} = -1\frac{3}{4}$$

11 Find the value of $\frac{3x}{2} - \frac{3}{4}$ when $x = 1$.

12 Find the value of $3xy$ when $x = 4$ and $y = 2$.

13 Find the value of $\frac{3x}{2} - \frac{y}{4}$ when $x = 3$ and $y = 5$.

14 Find the value of $n(2n-1)$ when $n = 10$.

15 Find the value of $\frac{n}{2}(n-1)$ when $n = 6$.

16 Find the value of $p(q-9)$ when $p = 10$ and $q = 12$.

17 Find the value of $\frac{n^2}{4}$ when $n = 8$.

18 Find the value of $\frac{5x}{4} - \frac{y}{3}$ when $x = 2$ and $y = 11$

19 Find the value of $\frac{PRT}{100}$ when $P = 50$, $R = 10$ and $T = 4$.

20 Find the value of $b^2 - 4ac$ when $a = 6$, $b = 5$ and $c = 2$.

Mixed exercises

Exercise 13h

Select the letter that gives the correct answer.

1 $3x - 2y + 7x + 3y$ simplifies to

A $4x + y$ **B** $8x + y$ **C** $10x - y$ **D** $10x + y$

2 $3(7x + 1)$ simplifies to

A $7x + 3$ **B** $14x + 3$ **C** $21x + 1$ **D** $21x + 3$

3 What does $4a^3$ mean?

A $4 \times a \times a$ **B** $4 \times a \times a \times a$ **C** $4 \times a \times a \times a \times a$ **D** $64 \times a \times a \times a$

4 Simplify $2x \times 4x \times 7x$

A $8x^3$ **B** $14x^3$ **C** $28x^3$ **D** $56x^3$

5 $3(2x+5)$ simplifies to

A $5x+5$ B $5x+8$ C $6x+5$ D $6x+15$

6 The value of $5xy+24$ when $x=3$ and $y=-2$ is

A -30 B -12 C -6 D 0

Exercise 13i

1 Simplify $4(x+3)+1$

2 Simplify $3a \times 5b \times 4c$

3 Simplify $4x+3(x+4)$

4 Find the value of $3xy-6$ when $x=3$ and $y=-2$.

5 Simplify $a^3 \times a^3$.

Exercise 13j

1 Simplify $3+2(x+2)$

2 Simplify $3(x-1)+4(2x+3)+5(x+1)$

3 What does x^5 mean?

4 Form an expression from the sentence 'I think of a number, multiply it by 3 and subtract 10.'

5 Find the value of $n^2(n-1)$ when $n=3$.

Exercise 13k

1 Peter had 14 marbles and lost x of them. John had 8 marbles and won y marbles. The two boys put all their marbles into one bag. Write an expression for the number of marbles in the bag.

2 Find the value of $\frac{1}{x}+\frac{1}{y}$ when $x=4$ and $y=3$.

3 Simplify $3a \times 3 \times 4a$

4 Find the value of $3(2-x)+4(3-y)$ when $x=1$ and $y=-2$.

5 Simplify $2x \times 4x \times 3y$.

Investigation

Each student must bring a calendar page for the month of his or her birth and choose any four by four grid of sixteen days, not including blank squares. Outline this grid and find

1 the sum of the four centre numbers

2 the sum of the four numbers on each diagonal.

What do you notice?

Compare your results with those of other members of the class.

Do calendars from the same month give the same or different sums when using different four by four squares?

What happens if other months are used that begin on a different day?

In this chapter you have seen that...

- ✔ an expression is a collection of terms without an 'is equal to' sign
- ✔ like terms are groups of numbers or groups of terms with the same letter
- ✔ you can add and subtract like terms to simplify them but you cannot add and subtract unlike terms
- ✔ you can simplify algebraic expressions involving brackets by multiplying out the brackets
- ✔ index form with letters means the same as with numbers
- ✔ you can substitute numbers for variables to find a value of an expression.

14 Algebra 2

At the end of this chapter you should be able to...

1 solve simple equations
2 construct and solve simple equations to solve problems
3 form and solve equations containing brackets.

Did you know?

In his treatises, René Descartes signed his name in Latin – Renatus Cartesius. It is from this that the name Cartesian originated.

When you plot points on a graph, you will use Cartesian coordinates.

You need to know...

- ✔ how to distinguish between like and unlike terms
- ✔ how to collect like terms
- ✔ how to work with directed numbers.

Key words

equation, expression, like terms, perimeter, solve, unlike terms

The idea of equations

'I think of a number, and take away 3; the result is 7.'

We can see the number must be 10.

Using a letter to stand for the unknown number we can write the first sentence as an *equation*:

$$x - 3 = 7$$

Then if $x = 10$

$$10 - 3 = 7$$

so $x = 10$ fits the equation.

Note that an *expression* does not include an 'is equal to' sign whereas an equation does. For example, $x - 3$ is an expression and $x - 3 = 7$ is an equation.

Exercise 14a

Form equations to illustrate the following statements and find the unknown numbers:

I think of a number, add 4 and the result is 10.

Let the number be x.

Then add 4 to x. This gives $x + 4$ which we know is 10, i.e. $x + 4$ and 10 are the same so they are equal. This gives the equation.

The equation is $x + 4 = 10$.

The number is 6.

1 I think of a number, subtract 3 and get 4.
2 I think of a number, add 1 and the result is 3.
3 If a number is added to 3 we get 9.
4 If 5 is subtracted from a number we get 2.

I think of a number, multiply it by 3 and the result is 12.

Let the number be x.

Multiplying by 3 gives $3 \times x$ which can be shortened to $3x$.

We know that $3x$ gives 12.

So the equation is $3x = 12$.

The number is 4.

5 I think of a number, double it and get 8.
6 If a number is multiplied by 7 the result is 14.
7 When we multiply a number by 3 we get 15.
8 Six times an unknown number gives 24.

$4x = 20$

$4x = 20$ means 4 times an unknown number gives 20, or, I think of a number, multiply it by 4 and the result is 20.

Write sentences to show the meaning of the following equations:

9 $3x = 18$
10 $x + 6 = 7$
11 $x - 2 = 9$
12 $5x = 20$
13 $5 + x = 7$
14 $x - 4 = 1$
15 $4x = 8$
16 $x + 1 = 4$

Puzzle

In three years' time I shall be three times as old as I was three years ago.

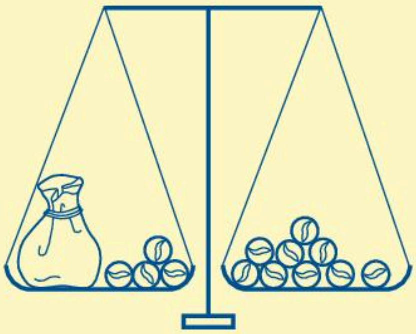
How old am I?

Solving equations

Some equations need an organised approach, not guesswork.

Imagine a balance:

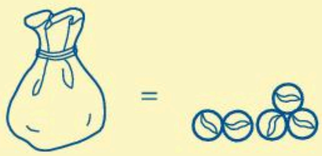
On this side there is a bag containing an unknown number of marbles, say x marbles, and 4 loose marbles.



On this side, there are 9 separate marbles, balancing the marbles on the other side.

$$x + 4 = 9$$

Take 4 loose marbles from each side, so that the two sides still balance.



$$x = 5$$

We write: $\quad x + 4 = 9$

Take 4 from both sides $\quad x + 4 - 4 = 9 - 4$

$$x = 5$$

When we have found the value of x we have *solved* the equation.

As a second example suppose that:

On this side there is a bag that originally held x marbles but now has 2 missing.

On this side, there are 5 loose marbles.

$$x - 2 = 5$$

We can make the bag complete by putting back 2 marbles but, to keep the balance, we must add 2 marbles to the right-hand side also.

So we write $x - 2 = 5$

Add 2 to both sides $x - 2 + 2 = 5 + 2$

$x = 7$

Whatever you do to one side of an equation you must also do to the other side.

Exercise 14b

Solve the following equations:

$y + 4 = 6$

$y + 4 = 6$

Take 4 from both sides $y = 2$

1 $x + 7 = 15$
2 $x + 9 = 18$
3 $10 + y = 12$
4 $2 + c = 9$
5 $a + 3 = 7$
6 $x + 4 = 9$
7 $a + 5 = 11$
8 $9 + a = 15$
9 $a + 1 = 6$
10 $a + 8 = 15$
11 $7 + c = 10$
12 $c + 2 = 3$

You can 'see' the solution of these equations without doing any working: use this to check your answers.

$x - 6 = 2$

$x - 6 = 2$

Add 6 to both sides $x = 8$

13 $x - 6 = 4$
14 $a - 2 = 1$
15 $y - 3 = 5$
16 $x - 4 = 6$
17 $c - 8 = 1$
18 $x - 5 = 7$
19 $s - 4 = 1$
20 $x - 9 = 3$
21 $a - 4 = 8$
22 $x - 3 = 0$
23 $c - 1 = 1$
24 $y - 7 = 2$

Exercise 14c

Sometimes the letter term is on the right-hand side instead of the left.

$3 = x - 4$

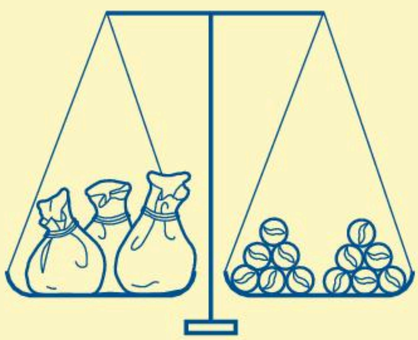
	$3 = x - 4$	
Add 4 to both sides	$7 = x$	($7 = x$ is the same as $x = 7$)
	$x = 7$	

Solve the following equations:

1 $4 = x + 2$
2 $6 = x - 3$
3 $7 = a + 4$
4 $6 = x - 7$
5 $1 = c - 2$
6 $5 = s + 2$
7 $x + 3 = 10$
8 $c + 4 = 4$
9 $3 = b + 2$
10 $6 + c = 10$
11 $7 = x + 3$
12 $x + 1 = 9$
13 $x + 3 = 15$
14 $y - 6 = 4$
15 $x - 7 = 4$
16 $6 = x - 4$
17 $x - 4 = 2$
18 $x - 9 = 2$
19 $x - 1 = 4$
20 $10 = a - 1$
21 $c - 7 = 9$
22 $x - 4 = 8$
23 $y - 1 = 9$
24 $x - 3 = 6$
25 $c - 7 = 10$
26 $4 = b - 1$
27 $x - 4 = 12$
28 $y - 9 = 14$
29 $2 = z - 2$
30 $x + 1 = 8$
31 $x - 1 = 8$
32 $x - 8 = 1$
33 $c + 5 = 9$
34 $d - 3 = 1$
35 $1 = c - 3$
36 $z + 3 = 5$

Multiples of x

Imagine that on this side of the scales there are 3 bags each containing an equal unknown number of marbles, say x in each.



On this side there are 12 loose marbles.

$$3 \times x = 12$$
$$3x = 12$$

We can keep the balance if we divide the contents of each scale pan by 3.

$$x = 4$$

Exercise 14d

Solve $6x = 12$

$$6x = 12$$

Divide both sides by 6 $\qquad 6x \div 6 = 12 \div 6$

$$x = 2$$

Solve $3x = 7$

$$3x = 7$$

Divide both sides by 3 $\qquad x = \frac{7}{3}$

$$x = 2\frac{1}{3}$$

Solve the following equations:

1	$5x = 10$	**7**	$3a = 1$	**13**	$6x = 36$	**19**	$3x = 27$
2	$3x = 9$	**8**	$6z = 18$	**14**	$6x = 6$	**20**	$8x = 16$
3	$2x = 5$	**9**	$5p = 7$	**15**	$6x = 1$	**21**	$4y = 3$
4	$7x = 21$	**10**	$2x = 40$	**16**	$5z = 10$	**22**	$5x = 6$
5	$4b = 16$	**11**	$7y = 14$	**17**	$5z = 9$	**23**	$2z = 10$
6	$4c = 9$	**12**	$6a = 3$	**18**	$2y = 7$	**24**	$7x = 1$

Mixed operations

Exercise 14e

Solve the following equations:

1	$x + 4 = 8$	**<u>10</u>**	$x - 2 = 11$	**<u>19</u>**	$x + 3 = 5$
2	$x - 4 = 8$	**<u>11</u>**	$12 = x + 4$	**<u>20</u>**	$3x = 5$
3	$4x = 8$	**<u>12</u>**	$x - 12 = 4$	**<u>21</u>**	$z - 5 = 6$
4	$5 + y = 6$	**13**	$8 = c + 2$	**22**	$c + 5 = 5$
5	$5y = 6$	**14**	$3x = 10$	**23**	$5a = 25$
6	$4x = 12$	**15**	$20 = 4x$	**24**	$a + 5 = 25$
7	$4 + x = 12$	**16**	$7y = 2$	**25**	$a - 5 = 25$
8	$x - 4 = 12$	**17**	$3x = 8$	**<u>26</u>**	$a - 25 = 5$
<u>9</u>	$2x = 11$	**<u>18</u>**	$3 = a - 4$	**<u>27</u>**	$25a = 5$

Two operations

Exercise 14f

Solve $7 = 3x - 5$

The aim is to get the letter term on its own $\quad 7 = 3x - 5$

Add 5 to both sides (to isolate the x term) $\quad 12 = 3x$

Divide both sides by 3 $\quad 4 = x$

i.e. $\quad x = 4$

Solve $2x + 3 = 5$

$$2x + 3 = 5$$

Take 3 from both sides (to get $2x$ on its own) $\quad 2x = 2$

Divide both sides by 2 $\quad x = 1$

(It is possible to check whether your answer is correct. We can put $x = 1$ in the left-hand side of the equation and see if we get the same value on the right-hand side.)

Check: If $x = 1$, left-hand side $= 2 \times 1 + 3 = 5$

Right-hand side $= 5$, so $x = 1$ fits the equation.

Solve the following equations:

1 $6f + 2 = 26$

2 $4x + 7 = 19$

3 $17 = 7x + 3$

4 $4d - 5 = 19$

5 $7x + 1 = 22$

6 $3a + 12 = 12$

7 $10 = 10x - 50$

8 $6 = 2h - 4$

9 $3p - 4 = 4$

10 $3x + 4 = 25$

11 $2x + 15 = 25$

12 $13 = 3e + 4$

13 $5z - 9 = 16$

14 $20 = 12x - 4$

15 $9g + 1 = 28$

16 $9 = 8x - 15$

17 $8 = 8 + 3z$

18 $5x - 4 = 5$

19 $15 = 1 + 7c$

20 $9x - 4 = 14$

21 $3x - 2 = 3$

22 $7 = 2z + 6$

23 $5 = 7x - 23$

24 $2x + 6 = 6$

25 $19x - 16 = 22$

26 $3x + 1 = 11$

27 $16 = 7x - 1$

28 $10x - 6 = 24$

29 $5x - 7 = 4$

30 $8 = 3x + 7$

31 $9 = 6a - 27$

32 $4z + 3 = 4$

33 $2x + 4 = 14$

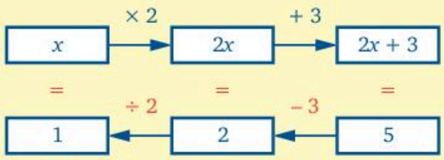
34 $3 = 7x - 3$

Using a flow chart to solve an equation

A flow chart solves an equation by working backwards from the answer.

To solve $2x + 3 = 5$, start by forming the left-hand side of the equation in the top flow chart. So starting with x, we multiply it by 2 to get $2x$, then the next step is to add 3.

Then starting with the answer, and working in the opposite direction in the lower flow chart, perform the opposite operations. So instead of adding 3, subtract 3, then instead of multiplying by 2, divide by 2.



Exercise 14g

Use a flow chart to solve the following equations.

1 $x + 5 = 9$

2 $3x + 5 = 8$

3 $2x - 7 = 3$

4 $5x - 8 = 7$

5 $3 - 2x = 7$

Equations with letter terms on both sides

Some equations have letter terms on both sides. Consider the equation

$$5x + 1 = 2x + 9$$

We want to have a letter term on one side only, so we need to subtract $2x$ from both sides. This gives

$$3x + 1 = 9$$

and we can go on to solve the equation as before.

Notice that we want the letter term on the side which has the greater numbers of x's to start with.

If we look at the equation

$$9 - 4x = 2x + 4$$

we can see that x's have been subtracted on the left-hand side, so there are more x's on the right-hand side.

Add $4x$ to both sides and then the equation becomes

$$9 = 6x + 4$$

and we can go on as before.

Exercise 14h

Deal with the letters first, then the numbers.

Solve $5x + 2 = 2x + 9$

Subtract $2x$ from both sides $\quad 3x + 2 = 9$

Subtract 2 from both sides $\quad 3x = 7$

Divide both sides by 3 $\quad x = \frac{7}{3} = 2\frac{1}{3}$

Deal with the x terms first.

Solve the equations.

1 $3x + 4 = 2x + 8$

2 $x + 7 = 4x + 4$

3 $2x + 5 = 5x - 4$

4 $3x - 1 = 5x - 11$

5 $7x + 3 = 3x + 31$

6 $6x + 4 = 2x - 4$

7 $7x - 25 = 3x - 1$

8 $11x - 6 = 8x + 9$

Solve $9 + x = 4 - 4x$

$$9 + x = 4 - 4x$$

Add $4x$ to both sides $\quad 9 + 5x = 4$

Subtract 9 from both sides $\quad 5x = -5$

Divide both sides by 5 $\quad x = -1$

Check: If $x = -1$, $\quad$ left-hand side $= 9 + (-1) = 8$

right-hand side $= 4 - (-4) = 8$

So $x = -1$ is the solution.

The left-hand side contains the greater number of x's, so we add $4x$ to both sides to remove the x's from the right-hand side.

Solve the equations.

9 $4x - 3 = 39 - 2x$

10 $5 + x = 17 - 5x$

11 $7 - 2x = 4 + x$

12 $24 - 2x = 5x + 3$

13 $5x - 6 = 3 - 4x$

14 $12 + 2x = 24 - 4x$

15 $32 - 6x = 8 + 2x$

16 $9 - 3x = -5 + 4x$

Problems

Exercise 14i

I think of a number, double it and add 3. The result is 15.

What is the number?

Let the number be x. Double it is $2x$, then add 3 gives $2x + 3$.

The result is 15 so the equation is $2x + 3 = 15$

Take 3 from both sides $2x = 12$

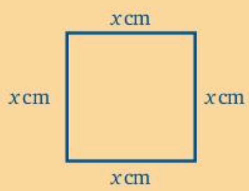
Divide both sides by 2 $x = 6$

The number is 6.

The side of a square is x cm. Its *perimeter* is 20 cm.

Find x.

Draw a diagram.



Now you can see that the perimeter is $(x + x + x + x)$ cm which is $4x$ cm.

You also know that the perimeter is 20 cm.

so the equation is $4x = 20$

Divide both sides by 4 $x = 5$

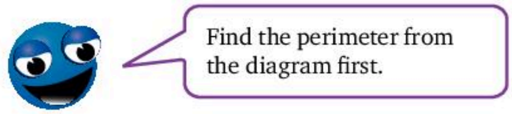
Check: $5 + 5 + 5 + 5 = 10 + 5 + 5 = 15 + 5 = 20$.

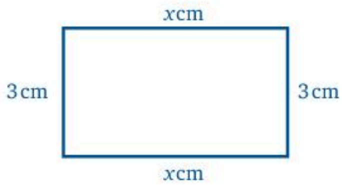
Form equations and solve the problems:

1. I think of a number, multiply it by 4 and subtract 8. The result is 20. What is the number?
2. I think of a number, multiply it by 6 and subtract 12. The result is 30. What is the number?
3. I think of a number, multiply it by 3 and add 6. The result is 21. What is the number?
4. When 8 is added to an unknown number the result is 10. What is the number?

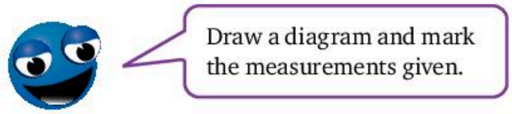
5 I think of a number, multiply it by 3 and add the result to 7. The total is 28. What is the number?

6 The sides of a rectangle are x cm and 3 cm. Its perimeter is 24 cm. Find x.





7 The lengths of the three sides of a triangle are x cm, x cm and 6 cm. Its perimeter is 20 cm. Find x.



8 Cassie and Amelia each have x sweets and Leela has 10 sweets. Amongst them they have 24 sweets. What is x?

9 Three boys had x sweets each. Amongst them they gave 9 sweets to a fourth boy and then found that they had 18 sweets left altogether. Find x.

10 I have two pieces of ribbon each x cm long and a third piece 9 cm long. Altogether there are 31 cm of ribbon. What is the length of each of the first two pieces?

Equations containing brackets

Remember that $3(x - 2)$ means 'three times everything in the bracket',

so $$\begin{aligned} 3(x-2) &= 3 \times x + 3 \times (-2) \\ &= 3x - 6 \end{aligned}$$

If we wish to solve equations containing brackets we first multiply out the brackets and then collect *like terms*. The number terms and x terms are *unlike terms*, so we need to collect the x terms on one side of the equals sign and the number terms on the other.

Exercise 14j

Solve $4 + 2(x + 1) = 22$.

Remember whatever you do to one side of an equation you must do to the other.

	$4 + 2(x + 1) = 22$	First multiply out the brackets
	$4 + 2x + 2 = 22$	
	$2x + 6 = 22$	You want the x term on its own.
Take 6 from both sides	$2x = 16$	
Divide both sides by 2	$x = 8$	

Check: If $x = 8$, left-hand side $= 4 + 2(8 + 1)$

$= 4 + 2 \times 9$

$= 22$

Right-hand side $= 22$, so $x = 8$ is the solution.

Solve the following equations:

1 $6 + 3(x + 4) = 24$

2 $3x + 2 = 2(2x + 1)$

3 $5x + 3(x + 1) = 14$

4 $5(x + 1) = 20$

5 $2(x + 5) = 6(x + 1)$

<u>6</u> $28 = 4(3x + 1)$

<u>7</u> $4 + 2(x + 1) = 12$

<u>8</u> $7x + (x + 2) = 22$

<u>9</u> $8x + 3(2x + 1) = 7$

10 $4x - 2 = 1 + (2x + 3)$

11 $7x + x = 4x + (x + 1)$

12 $3(x + 2) + 4(2x + 1) = 6x + 20$

13 $6x + 4 + 5(x + 6) = 56$

14 $2 + 3(x + 8) = 4(2x + 1)$

Solve the equation $4 - (2 - x) = 5$.

$4 - (2 - x) = 5$	Remember that $-(2 - x)$ means $(-1) \times (2 - x)$
$4 - 2 + x = 5$	$(-1) \times (2) = -2$ and $(-1) \times (-x) = +x$
$2 + x = 5$	
$x = 3$	

Solve the following equations:

15 $2-(x-4)=2$

16 $3-(3x+1)=1$

17 $5-(3-x)=10$

18 $6x-2(x-4)=1$

19 $x-4(3-x)=3$

20 $2(5-x)-3(4-2x)=6$

21 $3(2x-4)-2(x-5)=10$

22 $5(1-x)-3(1-3x)=10$

23 $2(x+7)-3(x-5)=21$

24 $2(x-3)+x-(1-x)=1$

Puzzle

Hindu problem solving

The ancient Hindus were fond of doing number puzzles. A mathematician named Aryabhata who lived in India during the sixth century CE liked this kind of puzzle:

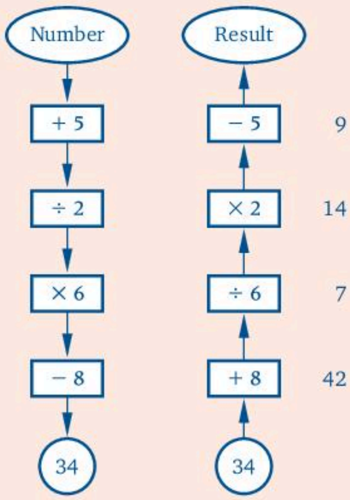
> If 5 is added to a certain number, the result divided by 2, that result multiplied by 6, and then 8 subtracted from that result, the answer is 34. Find the number.

Aryabhata solved this problem using the method of inversion, i.e. he worked backwards and did the inverse, or opposite steps as he went along. Adding and subtracting are inverse steps. Dividing and multiplying are inverse steps.

Let us set out Aryabhata's problem using a diagram.

We will set out the problem on the left starting at the top.

On the right we will start at the bottom and do the inverse steps:



Try this method yourself on questions **1**, **2** and **4** of the next exercise.

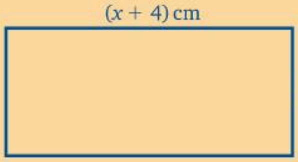
Problems to be solved by forming equations

Exercise 14k

The width of a rectangle is x cm. Its length is 4 cm more than its width. The perimeter is 48 cm. What is the width?

The width is x cm so the length is $(x + 4)$ cm.

The perimeter is the distance all around the rectangle; from the diagram this is $x + (x + 4) + x + (x + 4)$.



You also know this is 48 cm

so $\quad x + (x + 4) + x + (x + 4) = 48$

Removing the brackets and collecting like terms gives

$$4x + 8 = 48$$

Take 8 from each side $\quad 4x = 40$

Divide each side by 4 $\quad x = 10$

Therefore the width is 10 cm.

An ice cream bar costs x dollars and a cone costs \$200 more. One ice cream bar and two cones together cost \$1360.

How much is an ice cream bar?

An ice cream bar costs x dollars and a cone costs $(x + 200)$ dollars.

It follows that 2 cones cost $2 \times (x + 200)$ dollars

so the cost of an ice cream bar and two cones is $x + 2(x + 200)$ dollars

But you know that the total cost is \$1360

so $\quad x + 2(x + 200) = 1360 \quad$ Now multiply out the bracket

$x + 2x + 400 = 1360 \quad$ Collect like terms

$3x + 400 = 1360 \quad$ Take 400 from each side

$3x = 960 \quad$ Divide each side by 3

$x = 320$

Therefore a cone costs \$320.

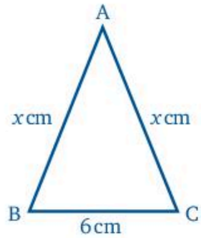
Solve the following problems by forming an equation in each case. Explain, either in words or on a diagram, what your letter stands for and always end by answering the question asked.

1. I think of a number, double it and add 14. The result is 36. What is the number?

> Start by letting the number be x. Next write down what double this is and then add 14. You can now form an equation in x.

2. I think of a number and add 6. The result is equal to twice the first number. What is the first number?

3. In triangle ABC, AB = AC. The perimeter is 24 cm. Find AB.



4. A bun costs x dollars and a cake costs $30 more than a bun. Four cakes and three buns together cost $1590. How much does one bun cost?

> Find the cost of 1 cake, then the cost of 4 cakes. Add on the cost of 3 buns. The total cost is given, so you can form an equation in x and solve it to find x.

5. A bus started from the terminus with x passengers. At the first stop another x passengers got on and 3 got off. At the next stop, 8 passengers got on. There were then 37 passengers. How many passengers were there on the bus to start with?

Mixed exercises

Exercise 14I

Select the letter that gives the correct answer.

1 The solution of the equation $2x - 5 = 7$ is $x =$

A 4 **B** 5 **C** 6 **D** 7

2 The solution of the equation $4x - 3 = 2x + 3$ is $x =$

A 1 **B** 2 **C** 3 **D** 4

3 I think of a number, double it and add 5. The result is 19. What number did I think of?

A 6 **B** 7 **C** 8 **D** 9

4 I think of a number, treble it and subtract 9. The answer is 12.
What number did I think of?

A 6 **B** 7 **C** 8 **D** 9

5 The solution of the equation $7 - 2x = 3$ is $x =$

A 2 **B** 3 **C** 4 **D** 5

6 The value of x that satisfies the equation $5(x - 1) - 3 = 3(x + 2)$ is

A 5 **B** 6 **C** 7 **D** 8

Exercise 14m

1 Solve the equation $4x - 5 = 3$.

2 Solve the equation $4(x + 3) = 16$.

3 Solve the equation $3x - 2 = 4 - x$.

4 When I think of a number, double it and add three, I get 11.
What number did I think of?

5 Solve the equation $3x - 4 - x + 6 = 8$.

6 Solve the equation $3 - 4(5 - x) = 3$.

Exercise 14n

1 Solve the equation $2x - 9 = 2$.

2 Solve the equation $2x + 8 + 3x - 6 = 4$.

3 Solve the equation $5 - 3x + 2 + 7x = 11$.

4 I think of a number and double it and subtract 3 and I get 7.
What number did I think of?

5 When shopping, Mrs Jones spent \$$x$ in the first shop, the same amount in the second shop, \$200 in the third and \$800 in the last. The total amount she spent was \$1800. Form an equation. How much did she spend in the first shop?

6 In a quadrilateral ABCD, angle A is $x°$, angle B is twice the size of angle A, angle C is 40° less than angle A and angle D is 25° more than angle A. Find the size of angle D.

Puzzle

1 May bought a certain number of \$50 stamps and a certain number of \$160 stamps. Altogether she paid \$930 for these stamps. How many of each did she buy?

2 Lance wrote down a lot of plus sevens and minus fours. Altogether he wrote 22 digits. When he came to add them all up the total was zero. How many plus sevens were there?

Investigation

P is any point on a diagonal of a 6 cm × 4 cm rectangle.

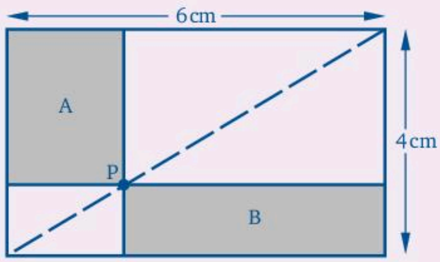
Investigate the relationship between the area of rectangle A and the area of rectangle B.

Which of these is true?

area A > area B

area A = area B

area A < area B



In this chapter you have seen that...

- ✔ an equation means the equality of two expressions
- ✔ you can do whatever you like with an equation as long as you do the same to both sides
- ✔ to solve an equation, aim to have the letter terms on one side of the equals sign and the number terms on the other side
- ✔ you can check your solution by replacing the letter by the number in each side independently: they will be equal if your answer is correct
- ✔ you can solve problems by forming an equation.

REVIEW TEST 3: CHAPTERS 11–14

In questions **1** to **12** choose the letter for the correct answer.

1 The expression $3(4-x)-(7-x)$ simplifies to

A $5-2x$ B $5-4x$ C $-2x$ D $5+2x$

2 The value of x that satisfies the equation $5-2x=15-7x$ is

A $\frac{20}{7}$ B $\frac{1}{2}$ C -2 D 2

3 The arithmetic mean of 2, 4, 6 and 8 is

A 3 B 4 C 5 D 6

4 The mode of the set of numbers 8, 5, 7, 8, 3, 5, 4, 2, 5, 6

A 5 B 6 C 7 D 8

5 The median of the set of numbers 8, 4, 9, 3, 6, 8, 7 is

A 5 B 6 C 7 D 8

6 The ratio of a map is 1 : 50 000. On the map the distance between A and B is 2.5 cm.

On the ground the distance the distance between A and B, in kilometres, is

A 1.25 B 12.5 C 125 D 1250

7 The range of the set of numbers 15, 28, 14, 15, 23, 15 is

A 12 B 14 C 15 D 16

8 $2x+3y-x+4y-3x$ simplifies to

A $4x-7y$ B $6x+7y$ C $7y-4x$ D $7y-2x$

9 $3x+5(2x-3y)$ simplifies to

A $13x-15y$ B $10x-15y$ C $13x-3y$ D $10x-15y$

10 $3x\times 2y\times 4x\times 3y$ simplifies to

A $36x^2y^2$ B $72x^2y$ C $72x^2y^2$ D $72xy^2$

11 The value of $2n(3n-4)$ when $n=10$ is

A 260 B 320 C 420 D 520

12 When $x=2$ the value of $5x-3+4-3x$ is

A 5 B 8 C 9 D 11

13 **a** Which ratio is the larger, 7 : 5 or 12 : 7?
b Which ratio is the smaller, $\frac{5}{8}$ or $\frac{7}{12}$?
c Divide \$480 between two people in the ratio 3 : 7.
d What is the map ratio if 1 cm on the map represents 500 m?

14 **a** Which ratio is the smaller, 3 : 5 or 5 : 8?
b Which ratio is the larger, $\frac{3}{8}$ or $\frac{5}{12}$?
c Divide \$330 between two sisters in the ratio 7 : 4.
How much more does one sister get than the other?
d Divide \$1800 between three brothers Adam, Ben and Carter in the ratio 2 : 3 : 4.
How much more does Carter get than Adam?

15 **a** Solve the equation $5x - 5 - 2x + 9 = 13$
b The lengths of the three sides of a triangle are $2x$ cm, $(x + 7)$ cm and $(x - 1)$ cm. If the perimeter of the triangle is 26 cm, find the length of the longest side.

16 The table shows the number of goals scored by the teams in a league.

Number of goals	0	1	2	3	4	5	6
Frequency	4	8	6	4	2	0	1

a How many teams are there in the league?
b What is the range in the number of goals scores?

17 Find the mean, mode, median and range of the following golf scores:
86, 77, 92, 84, 89, 85, 85, 83, 78, 80, 81, 84, 87, 85

18 The marks, out of 10, scored by 25 boys in a spelling test are given in the table:

Mark	5	6	7	8	9	10
Frequency	8	7	4	3	2	1

Find a the range of these marks
b the modal mark
c the median mark
d the mean mark

19 a Solve the equation $3x - 7 = 8$

b I think of a number, treble it, and subtract 5.
The answer is twice the number I first thought of.
What was the number I first thought of?

20 a Solve the equation $5x + 7 = 2(x + 8)$

b Solve the equation $5x - 4 = 8 - x$

15 Sets

At the end of this chapter you should be able to...

1 identify and give examples of well-defined sets
2 list the members of a given set
3 describe a given set in words
4 use correctly the symbols $\in$, $\notin$, $\subset$, U, $\cap$, $\cup$
5 write statements using set notations and vice versa
6 classify sets as finite or infinite
7 determine when two sets are equal or equivalent
8 identify empty sets and use the correct symbol for such a set $\varnothing$ or $\{ \}$
9 write down the subsets of a given set
10 give a suitable universal set for a given set
11 draw Venn diagrams to display given sets
12 obtain and interpret information from Venn diagrams
13 find the complement of a set.

Did you know?

Charles Dodgson (1832–1899), who is better known as Lewis Carroll, the author of *Alice in Wonderland*, was an Oxford mathematician who did a lot of work on sets.

You need to know...

- ✔ what is meant by a set
- ✔ how to identify the members of a given set
- ✔ how to find and list members in the intersection or union of two sets
- ✔ how to draw a Venn diagram to show the union or intersection of two sets.

Key words

complement of a set, disjoint sets, element, empty set, equal set, equivalent set, finite set, infinite set, intersection of sets, member, null set, proper subset, set, subset, union of sets, universal set, Venn diagram, the symbols $\in$, $\notin$, U, $\varnothing$, $\{ \}$, $\subset$, $\cup$, $\cap$

Set notation

The branch of mathematics known as Set Theory was founded by Georg Cantor.

A *set* is a clearly defined collection of things that have something in common. We talk about a set of drawing instruments, a set of cutlery and a set of books.

Things which belong to a set are called *members* or *elements*. When written down, these members or elements are usually separated by commas and enclosed by curly brackets or braces { }.

Instead of writing 'the set of musical instruments'

we write {musical instruments}

Exercise 15a

1 Use the correct set notation to write down

- **a** the set of rivers of the world
- **b** the set of pupils in my class
- **c** the set of subjects I study at school
- **d** the set of furniture in this room.

You can write down any two members you can think of. For example the set of rivers includes any river you can think of, such as the Mississippi, and so on.

2 Write down two members from each of the sets given in question **1**.

Describing members

We do not have to list all the members of a set; frequently we can use words to describe the members in a set.

For example instead of {Sunday, Monday, . . . , Saturday} we could say {the days of the week} and instead of {5, 6, 7, 8, 9} we could say {whole numbers from 5 to 9 inclusive}.

We could write {a, b, c, d, e} = {the first five letters of the alphabet}.

Exercise 15b

In questions **1** to **10** describe in words the given sets:

1 {w, x, y, z}

2 {January, June, July}

3 {June, July, August}

4 {Grenada, St Vincent, St Lucia, Dominica}

5 {Plymouth, St John's, Basseterre}

6 {2, 4, 6, 8, 10, 12}

7 {1, 2, 3, 4, 5, 6}

8 {2, 3, 5, 7, 11, 13}

9 {45, 46, 47, 48, 49, 50}

10 {15, 20, 25, 30, 35}

In questions **11** to **15** describe a set which includes the given members and state another member of it:

11 {Anil, Kyle, David, Richard}

12 {jacket, raincoat, sweater, anorak}

13 {rice puffs, corn flakes, bran flakes, granola}

14 {hibiscus, croton, bougainvillea}

15 {*Macbeth*, *Julius Caesar*, *King Lear*, *Romeo and Juliet*}

In the remaining questions list the members in the given sets:

{months of the year beginning with the letter M} = {March, May}

16 {whole numbers greater than 10 but less than 16}

17 {the first eight letters of the alphabet}

18 {the letters used in the word 'mathematics'}

19 {the four main islands forming the Windward group}

20 {the four main islands forming the Leeward group}

21 {subjects I study}

22 {oceans of the world}

23 {foods I ate for breakfast this morning}

24 {prime numbers less than 20}

25 {even numbers less than 20}

A prime number cannot be divided exactly by any number other than itself and one.

26 {odd numbers between 20 and 30}

27 {multiples of 3 between 10 and 31}

28 {multiples of 7 between 15 and 50}

29 {capital cities in the Windward Islands}

30 {Caricom states}

Puzzle

Write down a set using five odd digits whose sum is 14.

The symbol ∈

Instead of writing

'August is a member of the set of months of the year'

we write August ∈ {months of the year}

The symbol ∈ means 'is a member of' or 'is an element of'.

Exercise 15c

Write the following statements in set notation:

1 Apple is a member of the set of fruit.

2 Shirt is a member of the set of clothing.

3 Dog is a member of the set of domestic animals.

4 Geography is a member of the set of school subjects.

5 Carpet is a member of the set of floor coverings.

6 Hairdressing is a member of the set of occupations.

The symbol ∉

We are all aware that August is *not* a member of the set of days of the week.

Since we have chosen ∈ to mean 'is a member of' we use ∉ to mean 'is *not* a member of'. We can therefore write

'August is not a member of the set of days of the week'

as August ∉ {days of the week}

Exercise 15d

Write the following statements in set notation:

1 Orange is not a member of the set of animals.

2 Cat is not a member of the set of fruit.

3 Table is not a member of the set of trees.

4 Shirt is not a member of the set of subjects I study.

5 Anne is not a member of the set of boys' names.

6 Chisel is not a member of the set of buildings.

7 Cup is not a member of the set of bedroom furniture.

8 Cherry is not a member of the set of Japanese cars.

9 Aeroplane is not a member of the set of foreign countries.

10 Curry is not a member of the set of breeds of dogs.

Now write each of the following in set notation:

11 Porridge is a member of the set of breakfast cereals.

12 Electricity is not a member of the set of building materials.

13 Water is not a member of the set of metals.

14 Spider is a member of the set of living things.

15 Saturday is a member of the set of days of the week.

16 A salmon is a fish.

17 August is not the name of a day of the week.

18 Spain is a European country.

19 Brazil is not an Asian country.

Write down the meaning of:

20 Football $\in$ {team games}

21 Shoes $\notin$ {beverages}

22 Hockey $\notin$ {electrical appliances}

23 Needle $\in$ {metal objects}

24 Danielle $\notin$ {boys' names}

25 Using the correct notation write down

  a three members that belong to {dairy produce}

  b three members that do not belong to {dairy produce}.

26 Using the correct notation, write down

  a three members that belong to {clothes}

  b three members that do not belong to {clothes}.

Finite and infinite sets

We often need to refer to a set several times. When this is so we label the set with a capital letter. For example

$$A = \{\text{months of the year beginning with the letter J}\}$$

or $$A = \{\text{January, June, July}\}$$

In many cases it is not possible to list all the members of a set. When this is so we write down the first few members followed by dots.

For example if $N = \{\text{positive whole numbers}\}$

we could write $N = \{1, 2, 3, 4, \ldots\}$

Similarly if $X = \{\text{even numbers}\}$ and $Y = \{\text{odd numbers}\}$

we could write $X = \{2, 4, 6, 8, \ldots\}$ and $Y = \{1, 3, 5, 7, \ldots\}$

Sets like N, X and Y are called *infinite sets* because there is no limit to the number of members each contains. When we can write down, or count, all the members in a set, the set is called a *finite set*.

Equal sets

When two sets have exactly the same members they are said to be *equal sets*.

If $A = \{2, 4, 6, 8\}$ and $B = \{6, 4, 8, 2\}$

then $A = B$

Similarly, if $X = \{\text{prime numbers less than 8}\}$

$= \{2, 3, 5, 7\}$

and $Y = \{\text{whole numbers up to 7 inclusive except 1, 4 and 6}\}$

$= \{2, 3, 5, 7\}$

then $X = Y$

The order in which the members are listed does not matter; neither does the way in which the sets are described.

Equivalent sets

Sets are *equivalent* when they contain the same number of elements. The elements in equivalent sets are not usually the same (if they were, the sets would be equal sets).

For example $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ are equivalent sets.

Exercise 15e

Determine whether or not the following sets are equal:

1 $A = \{\text{chair, table, desk, blackboard}\}$
$B = \{\text{desk, blackboard, table, chair}\}$

2 $X = \{\text{d, i, k, f, w}\}$
$Y = \{\text{f, w, k, i}\}$

3 $V = \{4, 6, 8, 10, 12\}$
$W = \{\text{even numbers from 4 to 12 inclusive}\}$

4 $C = \{\text{i, e, a, u, o}\}$
$D = \{\text{vowels}\}$

5 $P = \{\text{capital cities of all the West Indian islands}\}$
$Q = \{\text{Roseau, Kingston, Plymouth, San Fernando}\}$

Determine whether or not the following sets are equivalent:

6 $A = \{0, 1, 2, 3, 4\}$
$B = \{\text{the first five letters of the alphabet}\}$

7 $C = \{\text{apple, orange, banana, grape}\}$
$D = \{\text{fruit sold in the local market}\}$

8 $P = \{\text{students in your school's baseball team}\}$
$Q = \{\text{students in your school's football team}\}$

Empty set

Have you ever seen a woman with three eyes or a man with four legs?
We hope not, for neither exists. There are no members in either of these sets.
Such a set is called an *empty set* or *null set* and is written { } or ∅.

Exercise 15f

1 Give some examples of empty sets.

2 Which of the following sets are empty?

a {dogs with wings}
b {men who have landed on the Moon}
c {children more than 5 m tall}
d {cars that can carry 100 people}
e {men more than 100 years old}
f {dogs without tails}

Subsets

If $A = \{\text{Andrew, Stefan, Matthew, Maria, Jay}\}$ and $B = \{\text{Maria, Jay}\}$ we see that all the members of B are also members of A.

We say that B is a *subset* of A and write this $B \subset A$.

If $X = \{a, b, c\}$ then $\{a, b, c\}$, $\{a, b\}$, $\{b, c\}$, $\{a, c\}$, $\{a\}$, $\{b\}$, $\{c\}$ and $\varnothing$ are all subsets of X.

Note that both X and $\varnothing$ are considered to be subsets of X. Subsets that do not contain all the members of X are called *proper subsets*. All the subsets given above except $\{a, b, c\}$ are therefore proper subsets of X.

Exercise 15g

1 If $A = \{w, x, y, z\}$ write down all the subsets of A that have two members.

2 If $B = \{\text{Amelia, Brendan, Kevin, Leela}\}$ write down all the subsets of B that have two female members.

3 If $N = \{1, 2, 3, \ldots, 10\}$ list the following subsets of N:

$A = \{\text{odd numbers}\}$ $\quad B = \{\text{even numbers}\}$ $\quad C = \{\text{prime numbers}\}$

4 Give a subset with at least three members for each of the following sets:

a {islands in the Caribbean}
b {rivers}
c {oceans}
d {basketball teams}

5 If $X = \{1, 2, 3, 5, 7, 11, 13\}$ which of the following sets are proper subsets of X?

a {positive odd numbers less than 6}
b {positive even numbers less than 4}
c {positive prime numbers less than 14}
d {positive odd numbers between 10 and 14}

The universal set

Consider the set {natural numbers less than 16}, i.e. the set $\{1, 2, 3, 4, \ldots, 15\}$.

Now consider the sets A, B and C whose members are such that

$$A = \{\text{prime numbers}\} = \{2, 3, 5, 7, 11, 13\}$$

$$B = \{\text{multiples of 3}\} = \{3, 6, 9, 12, 15\}$$

$$C = \{\text{multiples of 5}\} = \{5, 10, 15\}$$

The original set {1, 2, 3, 4, . . . , 15} is called the *universal set* for the sets A, B and C. It is a set that contains all the members that occur in the sets A, B and C as well as some other members that are not found in any of these three. The universal set is denoted by the symbol U.

For example, a universal set for {cup, plate, saucer} could be {crockery}.

We write U = crockery.

Exercise 15h

Suggest a suitable universal set for:

1 {8, 12, 16, 17, 20}

2 {vowels}

3 {rivers in Guyana}

4 {prefects}

5 {cats with three legs}

6 {sparrows}

There is no one correct answer for these. Another universal set for {cup, plate, saucer} could be U = {tableware}.

U = {boys' names}

Two subsets are {John, Peter, Paul} and {Dino, Fritz, Alec}.

Write down two subsets, each with at least two members, for each of the following universal sets:

7 U = {girls' names}

8 U = {European countries}

9 U = {African countries}

10 U = {Members of Parliament}

11 U = {school subjects}

12 U = {colours}

Suggest a universal set for:

13 {set squares, protractors, rulers}

14 {houses, apartments, bungalows}

15 {cars, vans, trucks, motorbikes}

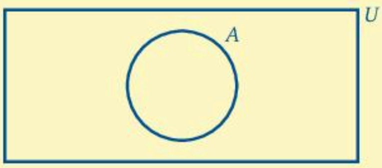
16 {trainers, shoes, sandals, boots}

17 {golfers, football players, netball players, sprinters}

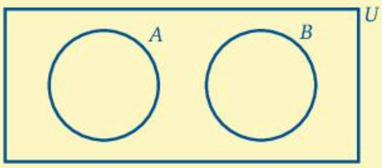
Venn diagrams

Many years ago a Cambridge mathematician named John Venn (1834–1923) studied the algebra of sets and introduced the diagrams which now bear his name. In a *Venn diagram* the universal set (U) is usually represented by a rectangle and subsets of the universal set are usually shown as circles inside the rectangle. There is nothing special about circles – any convenient enclosed shape would do.

If U = {school children} then A could be {pupils in my school}, i.e. A is a subset of U.

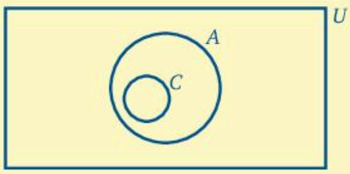


Similarly, if B = {pupils in the next school to my school} the diagram would be

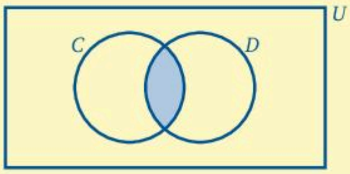


Two sets like these, which have no common members, are called *disjoint sets*.

If C = {pupils in my class} then, because all the members of C are also members of A, i.e. C is a proper subset of A, the Venn diagram is



If D = {my school friends} the corresponding Venn diagram could be



The shaded region shows the friends I have who are in my class. These friends belong to both sets. The unshaded region of D represents friends I have in school who are not in my class.

Union of two sets

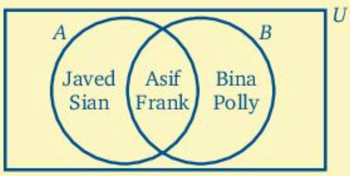
In my class

$$A = \{\text{pupils good at maths}\} = \{\text{Frank, Javed, Asif, Sian}\}$$

and

$$B = \{\text{pupils good at French}\} = \{\text{Bina, Asif, Polly, Frank}\}$$

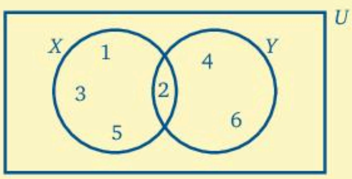
If the universal set is {all the pupils in my class} the names could be placed in a Venn diagram as follows:



If we write down the set of all the members of my class who are good at *either* maths *or* French we have the set {Javed, Sian, Asif, Frank, Bina, Polly}. This is called the *union* of the sets A and B and is denoted by

$$A \cup B$$

Similarly if $X = \{1, 2, 3, 5\}$ and $Y = \{2, 4, 6\}$ we can illustrate these sets in the following Venn diagram:



and write the union of the two sets X and Y

$$X \cup Y = \{1, 2, 3, 4, 5, 6\}$$

To find the union of two sets, write down all the members of the first set, then all the members of the second set which have not already been included.

Exercise 15i

Find the union of $A = \{3, 6, 9, 12\}$ and $B = \{4, 6, 8, 10, 12\}$.

6 and 12 are in both sets so they do not need to be included twice.

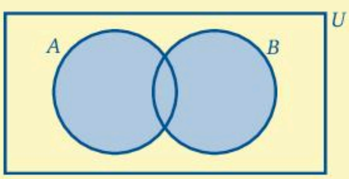
$A \cup B = \{3, 4, 6, 8, 9, 10, 12\}$

Find the union of the two given sets in each of the following:

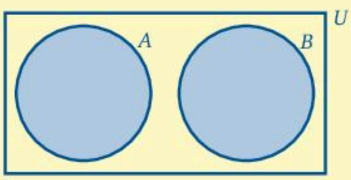
1. A = {Peter, James, John} B = {John, Andrew, Paul}
2. $X = \{3, 6, 9, 12\}$ $Y = \{4, 8, 12, 16\}$
3. $P = \{a, e, i, o, u\}$ $Q = \{a, b, c, d, e\}$
4. $A = \{a, b, c\}$ $B = \{x, y, z\}$
5. $A = \{p, q, r, s, t\}$ $B = \{p, r, t\}$
6. $X = \{2, 3, 5, 7\}$ $Y = \{1, 3, 5, 7\}$
7. $X = \{5, 7, 11, 13\}$ $Y = \{6, 8, 10, 12\}$
8. P = {whole numbers that divide exactly into 12}
   Q = {whole numbers that divide exactly into 10}
9. A = {letters in the word 'classroom'}
   B = {letters in the word 'school'}
10. P = {letters in the word 'arithmetic'}
    Q = {letters in the word 'algebra'}

To represent the union of two sets in a Venn diagram we shade the combined region representing the two sets. This shaded area may occur in three ways.

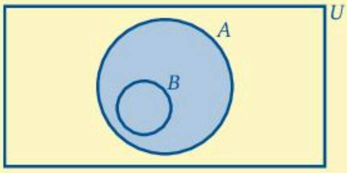
1 When sets A and B have some common members:



2 When A and B have no common member, i.e. when they are disjoint:



3 When B is a proper subset of A:



Exercise 15j

Show the union of $P = \{3, 6, 9, 12, 15\}$ and $Q = \{3, 5, 7, 9, 11, 15\}$ on a Venn diagram.

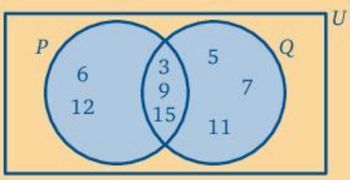
3, 9 and 15 are in both sets so these go in the overlapping part.

6 and 12 are only in P so they go in the left-hand part of the circle marked P.

5, 7, and 11 go in the right-hand part of the circle marked Q.

The union is the combination of both sets so both circles are shaded.

$$P \cup Q = \{3, 5, 6, 7, 9, 11, 12, 15\}$$



Note that we don't repeat elements in a set, so elements 3, 9 and 15 are only written once in the union of P and Q.

Draw suitable Venn diagrams to show the unions of the following sets:

1	$A = \{p, q, r, s\}$	$B = \{r, s, t, u\}$
2	$X = \{1, 3, 5, 7, 9\}$	$Y = \{2, 4, 6, 8, 10\}$
3	$P = \{a, b, c, d, e, f, g\}$	$Q = \{c, d, g\}$
4	E = {rectangles}	F = {squares}
5	G = {even numbers}	H = {odd numbers}
6	M = {triangles}	N = {squares}
7	$A = \{3, 6, 9, 12, 15\}$	$B = \{4, 6, 8, 10, 12, 14\}$
8	P = {letters in the word 'Donald'}	Q = {letters in the word 'London'}
9	X = {Marc, Leslie, Joe, Claude}	Y = {Leslie, Sita, Joe, Yvette}
10	A = {letters in the word 'metric'}	B = {letters in the word 'imperial'}

Intersection of sets

If we return to the set of pupils in my class

$$A = \{\text{pupils good at maths}\}$$
$$= \{\text{Frank, Javed, Asif, Sian}\}$$

and

$$B = \{\text{pupils good at French}\}$$
$$= \{\text{Bina, Asif, Polly, Frank}\}$$

then Frank and Asif form the set of pupils who are good at *both* maths and French. The members that are in both sets give what is called the *intersection* of the sets A and B.

The intersection of two sets A and B is written $A \cap B$,

i.e. for the given sets, $A \cap B = \{\text{Frank, Asif}\}$

Exercise 15k

Find the intersection of $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{1, 2, 3, 5, 7\}$.

The intersection contains the elements that are in both X and Y

$$X \cap Y = \{1, 2, 3, 5\}$$

Find the intersection of the following pairs of sets:

1 $A = \{3, 6, 9, 12\}$ $\quad B = \{5, 6, 7, 8, 9\}$

2 $X = \{4, 8, 12, 16, 20\}$ $\quad Y = \{4, 12, 20\}$

3 $P = \{\text{Zane, Gabriel, Colin, Alice}\}$ $\quad Q = \{\text{Alice, Tyrell, Hans, Zane}\}$

4 $C = \{\text{o, p, q, r, s, t}\}$ $\quad D = \{\text{a, e, i, o, u}\}$

5 $A = \{\text{tomato, cabbage, apple, pear}\}$ $\quad B = \{\text{cabbage, tomato}\}$

6 $M = \{\text{prime numbers less than 12}\}$ $\quad N = \{\text{odd numbers less than 12}\}$

7 $P = \{4, 8, 12, 16\}$ $\quad Q = \{8, 16, 24, 48\}$

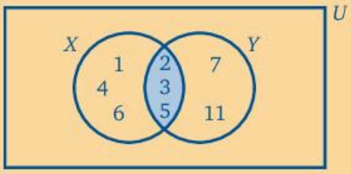
8 $A = \{\text{factors of 12}\}$ $\quad B = \{\text{factors of 10}\}$

9 $X = \{\text{letters in the word 'twice'}\}$ $\quad Y = \{\text{letters in the word 'sweat'}\}$

10 $P = \{\text{letters in the word 'metric'}\}$ $\quad Q = \{\text{letters in the word 'imperial'}\}$

Exercise 15I

Show the intersection of $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{2, 3, 5, 7, 11\}$ on a Venn diagram.



$X \cap Y = \{2, 3, 5\}$

The elements in $X \cap Y$ are in the overlap, so this is the part to shade.

Draw suitable Venn diagrams to show the intersections of the following sets:

1	$A = \{1, 3, 5, 7, 9, 11\}$	$B = \{2, 3, 4, 5, 6, 7\}$
2	P = {John, David, Dino, Kay}	Q = {Pete, Dino, Omar, John}
3	X = {a, e, i, o, u}	Y = {b, f, o, w, u}
4	A = {oak, ash, elm, pine}	B = {teak, oak, sapele, elm}
5	X = {poodle, greyhound, boxer}	Y = {pug, collie, boxer, cairn}
6	$P = \{4, 8, 12, 16\}$	$Q = \{8, 16, 24, 48\}$
7	A = {factors of 12}	B = {factors of 20}
8	X = {letters in the word 'think'}	Y = {letters in the word 'flint'}
9	A = {letters in the word 'arithmetic'}	B = {letters in the word 'geometry'}
10	P = {prime numbers less than 10}	Q = {odd numbers less than 15}

Puzzle

The number 30 can be written as the product $2 \times 3 \times 5$.

We may write these numbers as the set $\{2, 3, 5\}$.

Writing another number as a product gives this set $\{2, 5, 7, 11\}$.

1 What is the second number?

2 a Write down the intersection of the two sets.

  b What is the number that comes from multiplying the elements of the intersection?

  c What is the relationship of this number to the first two numbers?

3 a Write down the union of the sets $\{3, 5, 7\}$ and $\{2, 3, 7\}$.

  b What number comes from the product of the elements in the union?

  c Describe the relationship of this number to the two original numbers.

Complement of a set

If U = {pupils in my school}

and A = {pupils who represent the school at games}

then the *complement* of A is the set of all the members of U that are not members of A.

In this case, the complement of A is

{pupils in my school who do not represent my school at games}

The complement of A is denoted by A'.

Similarly if U = {the whole numbers from 1 to 10 inclusive}

and $A = \{1, 3, 5, 7, 9\}$

the complement of A, i.e. $A' = \{2, 4, 6, 8, 10\}$.

Exercise 15m

Give the complement of P where

P = {Thursday, Friday} if U = {days of the week}

P' = {Monday, Tuesday, Wednesday, Saturday, Sunday}

Give the complement of each of the following sets.

1 $A = \{5, 15, 25\}$ if $U = \{5, 10, 15, 20, 25\}$

2 $B = \{7, 8, 9, 10\}$ if $U = \{5, 6, 7, 8, 9, 10, 11\}$

3 V = {a, e, i, o, u} if U = {letters of the alphabet}

4 P = {consonants} if U = {letters of the alphabet}

5 A = {Monday. Wednesday, Friday} if U = {days of the week}

6 X = {children} if U = {human beings}

7 M = {British motor cars} if U = {motor cars}

8 S = {male tennis players} if U = {tennis players}

9 C = {Jamaican towns} if U = {Caribbean towns}

10 D = {squares} if U = {quadrilaterals}

11 E = {adults over 80 years old} if U = {adults}

12 F = {female doctors} if U = {doctors}

If $A = \{\text{men}\}$ and $A' = \{\text{women}\}$, what is U?

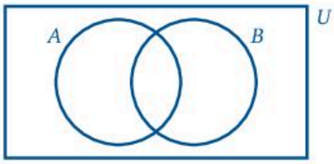
$U = A + A' = \{\text{adults}\}$

13 If $A = \{\text{homes with television sets}\}$ and $A' = \{\text{homes without television sets}\}$, what is U?

14 If $A = \{\text{vowels}\}$ and $A' = \{\text{consonants}\}$, what is U?

15 If $X = \{a, b, c, d, e\}$ and $X' = \{f, g, h, i, j\}$, what is U?

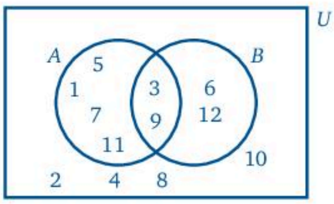
16



a Copy the Venn diagram and shade the region representing A'.

b Copy the Venn diagram and shade the region representing B'.

17



Use this Venn diagram to list the following sets:

a A'

b B'

c $A \cup B$

d the complement of $(A \cup B)$

18 $U = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ and $A = \{\text{prime numbers}\}$
List the sets

a A

b A'

19 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{\text{factors of } 12\}$ and $B = \{\text{odd numbers}\}$
Show U, A and B on a Venn diagram. Hence list the sets

a A' **b** B' **c** $A' \cap B'$ **d** $A' \cup B'$

20 $U = \{\text{different letters in the word 'complement'}\}$
$P = \{\text{different letters in the word 'temple'}\}$
$Q = \{\text{different letters in the word 'cement'}\}$
Show U, P and Q on a Venn diagram. Hence list the sets

a P' **b** Q' **c** $P' \cap Q'$ **d** $P' \cup Q'$

Mixed exercise

Exercise 15n

1 Write the following statements in set notation:

a Jupiter is a member of the set of planets

b saucer is not a member of the set of motorcars.

2 Determine whether or not sets P and Q are equal.

$P = \{\text{prime numbers less than 12}\}$

$Q = \{\text{factors of 12}\}$

3 Give a subset with at least five members for each of the following sets:

a {South American countries}

b {Canadian provinces}

4 Find the union of the sets $A = \{d, e, f, g\}$ and $B = \{f, g, h, i\}$.

5 Draw a Venn diagram to show the union of the sets

$E = \{\text{different letters in the word 'Jamaica'}\}$

and $F = \{\text{different letters in the word 'Barbados'}\}$.

6 For the sets $A = \{6, 8, 10, 12\}$ and $B = \{8, 12, 16, 24\}$, list the set that is

a the union of A and B

b the intersection of A and B.

7 $Y = \{\text{different letters in the word 'ratio'}\}$

$Z = \{\text{different letters in the word 'proportion'}\}$

$U = \{\text{letters of the alphabet}\}$

Show these sets on a Venn diagram and use this diagram to list the sets

a Y'

b Z'

c $Y \cup Z$

d $(Y \cup Z)'$

8 $U = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$A = \{\text{multiples of 4}\}$ and $B = \{\text{even numbers}\}$

Show U, A and B on a Venn diagram. Hence list the sets

a A'

b B'

c $A \cup B$

d $A' \cup B'$

e $(A \cup B)'$

Investigation

Consider the following sets X and Y.

$X = \{0, 2, 4, 6, \ldots\}$, $Y = \{1, 3, 5, 7, \ldots\}$.

1 Describe X and Y in words.

2 Give the next three numbers in X and in Y.

3 Choose two numbers from X and find their sum. Is this sum a member of X? Is this always true?

4 Choose two numbers from Y and find their sum. Which set contains this sum? Is this always true?

5 When will the sum of any two whole numbers always be in Y? Explain your answer with examples.

In this chapter you have seen that…

- ✔ a set is a collection of things that have something in common
- ✔ an infinite set has no limit on the number of members in it
- ✔ in a finite set, all the members can be counted or listed
- ✔ a proper subset of a set A contains some, but not all, of the members of A
- ✔ the union of two sets contains all the members of the first set together with the members of the second set that have not already been included
- ✔ the intersection of two sets contains the elements that are in both sets
- ✔ when two sets contain the same number of elements, they are said to be equivalent
- ✔ when two sets have exactly the same members, they are said to be equal
- ✔ a set which has no members is called an empty or null set and is written { } or $\varnothing$
- ✔ the complement of a set A is the set of all the members of the universal set U that are not members of A. The complement of A is denoted by A'.

16 Consumer arithmetic

At the end of this chapter you should be able to...

1 work out the total price given the quantity and the unit price
2 work out the unit price given the quantity and the total price
3 work out the quantity given the total price and the unit price
4 identify best buys and bargains by comparison of unit costs
5 calculate profit and loss in monetary and percentage terms
6 convert Jamaican dollars to other currencies and vice versa.

Did you know?

Marjorie Lee Browne (1914–79) was one of the first two black women to receive a doctorate in mathematics in the USA. Her father was considered to be a 'whiz' at mental arithmetic. He first stimulated her interest in maths as a child and later he learned maths from her.

You need to know...

✔ how to work with decimals and fractions
✔ how to work with percentages
✔ how to substitute numbers into expressions
✔ how to correct a number to a given place value.

Key words

cost price, currency, exchange rate, loss, profit, selling price

Bank notes

The *currency* used in many Caribbean countries is dollars.

These are the bank notes used in Jamaica.

Exercise 16a

1 What is the smallest number of bank notes needed to make $4350?

2 How many $50 bank notes are needed make $1000?

3 Give two possible mixtures of $50, $100 and $500 bank notes that can be used to make $2450.

4 Britney had two $1000 bank notes, four $500 bank notes and one $100 bank note. She needed to make up a total of $5000. When she went to the bank they offered her $50 bank notes and $100 bank notes. Give two possible combinations of bank notes that Britney could get to add to the bank notes she already had to make a total of $5000.

Multiple costs

We often buy more than one of a given article.

For example, if we buy 10 articles at $5000 each, the total cost is 10 × $5000.

Alternatively, if we buy a multipack costing $40 000 and the cost of one pack is $8000, we can find the number of packs by dividing 40 000 by 8000.

Also, if a multipack costs $90 000 and contains 6 packs then the cost of each pack is $(90 000 ÷ 6).

Exercise 16b

Mrs Gayle buys packets of candy to give to the children who are coming to her son's birthday party. The packets cost \$250. She spends \$3500 to buy one packet for each child. How many children will there be at the party?

Number of children = total amount spent ÷ cost of each packet
= \$3500 ÷ \$250
= 14

There will be 14 children at the party.

1 Bottles of apple soda cost \$70 each. How many bottles will I get for \$1680?

2 Mini chocolate eggs cost \$87.50 each. How many will be in bag costing \$1050?

3 Tablets for my washing machine cost \$25.20. A large box of these tablets costs \$3150. How many tablets will it contain?

4 A case of bathroom tissue costs \$1400. There are 24 rolls in a case. Find, to the nearest dollar, the cost of one roll.

5 A baker delivers a consignment of 800 g loaves. One loaf costs \$340 and the total cost of the delivery is \$32 300. How many loaves were delivered?

The remaining questions depend on the same relationship between the three quantities.

6 Find the total cost of 18 cakes at \$220 each.

7 How much will 36 ceramic tiles cost at \$282 each?

8 Bus rides to school cost Mellie \$410 each way. Find the cost of going back and forth to school for a week.

9 Packs of 24 bags of ginger tea cost \$276. What is the cost of 1 bag?

10 A can of tuna in oil contains 170 g of tuna and costs \$561.
What is the cost of **a** 1 g **b** 100 g?

Best buys

These two jars of coffee contain different amounts of coffee and cost different amounts of money.

\$4750 \$8500

You can find which of these two jars is the better value for money (or best buy). There are two ways you can do this.

The first way is to find the cost of the same amount of coffee for each jar.

The smaller jar holds 250 g and costs \$4750.

The larger jar holds 500 g and costs \$8500.

To compare the cost of the coffee in the larger jar with the cost of the coffee in the smaller jar, you can find the cost of 250 g of coffee in the larger jar.

$500\text{ g} = 2 \times 250\text{ g}$ so 250 g of coffee in the larger jar costs $\$8500 \div 2 = \$4250$.

This shows that the cost of 250 g of coffee in the larger costs less than the same amount in the smaller jar. The larger jar is therefore the better value for money.

The second method is to find the mass of coffee that the same amount of money will buy.

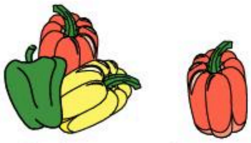
The smaller jar costs \$4750 for 250 g so \$100 will buy $\frac{250}{47.5}\text{ g} = 5.3\text{ g}$ to 1 d.p.

The larger jar costs \$8500 for 500 g so \$100 will buy $\frac{500}{85}\text{ g} = 5.9\text{ g}$ to 1 d.p.

Now you can see that \$100 will buy more in the larger jar than in the smaller jar.
(Do not assume that the larger of two jars with the same total contents as two smaller jars is always the better value.)

Exercise 16c

1



3 for $924 $340 each

Peppers are sold in packs of three or singly.
Which is the better value?
Give a reason for your answer.

You can compare these prices by either finding the cost of 3 of the single peppers or by finding the cost of one of the peppers in the pack. Whichever way you choose, remember to show your working.

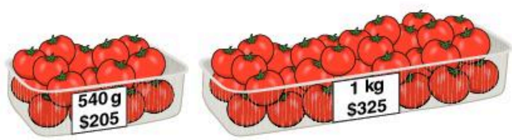
2 Cans of cola are sold in packs of 4 cans and in packs of 6 cans.

Which pack is better value for money?
Give a reason for your answer.

3 These are two different bags of paper clips.
Which bag is better value for money?
Explain your answer.

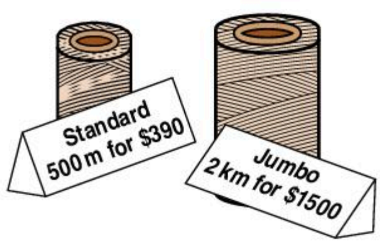
You can compare the number of clips per dollar or the cost of one clip.

4



Which pack of tomatoes is better value for money?
Give a reason for your answer.

5 String is sold in rolls of two sizes.
Jane said that the larger roll is better value for money.
Is Jane correct?
Give a reason for your answer.

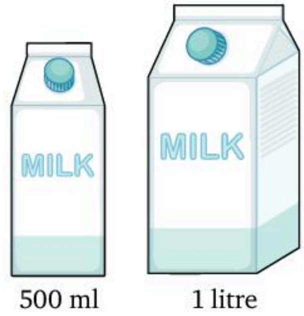


6 In a supermarket milk is sold in two sizes.

The litre container costs \$440 while the half-litre container costs \$230.

Which container is the better value for money?

Justify your answer.



7

Which jar is better value?

Profit and loss

When a store buys items the price it pays is called the *cost price* (CP). The store then sells the items at a price called the *selling price* (SP).

When the selling price is greater than the cost price, the store makes a *profit*.

When the selling price is less than the cost price, the store makes a *loss*.

For example, SportCom purchased some pairs of trainers for \$10 800 a pair. They sold them for \$18 000 a pair. Therefore the profit on each pair is \$18 000 − \$10 800 = \$7200.

Some pairs of trainers did not sell at the full price, so SportCom sold them for \$8640.

Therefore the store made a loss on those trainers of \$2160.

Profit and loss are often given as percentages. The percentage profit or loss is always found as a percentage of the cost price. Therefore the percentage profit on the trainers sold at the full price is \$7200 as a percentage of \$10 800, which is

$$\frac{7200}{10\,800} \times 100\% = 66.66\ldots\% = 66.7\% \text{ correct to 1 decimal place.}$$

The percentage loss on the trainers sold at \$8640 is \$2160 as a percentage of \$10 800, which is

$$\frac{2160}{10\,800} \times 100\% = 20\%$$

Exercise 16d

In questions **1** to **6** find the profit or loss.

1 A calculator costing \$2600 is sold for \$3120

2 A food mixer costing \$4800 is sold for \$7200

3 A car costing \$3360 000 is sold for \$2240 000

4 A concert ticket costing \$3840 is sold for \$2560

5 A chair costing \$9350 is sold for \$6510

6 A computer desk that cost \$12 000 is sold for \$7500

7 Monique bought a cell phone for \$15 000 and sold it two years later for \$8000.
What was Monique's loss?

8 Kyle bought an old chair for \$1700. He renovated the chair and sold it for \$8500.
What profit did Kyle make?

A second-hand car dealer bought a car for \$3500 000 and sold it for \$4340 000.
Find his percentage profit.

$$\text{Profit} = \text{SP} - \text{CP}$$
$$= \$4340\,000 - \$3500\,000 = \$840\,000$$
$$\%\ \text{profit} = \frac{\text{profit}}{\text{CP}} \times 100\%$$
$$= \frac{\$840\,000}{\$3500\,000} \times 100\% = 24\%$$

Therefore the percentage profit is 24%.

Find the percentage profit:

9 CP \$12 000, profit \$3000

10 CP \$28 000, profit \$8400

<u>11</u> CP \$16 000, profit \$4000

<u>12</u> CP \$55 000, profit \$5500

A retailer bought a leather chair for \$37 500 and sold it for \$28 500. Find his percentage loss.

$$\text{Loss} = \text{CP} - \text{SP}$$
$$= \$37\,500 - \$28\,500 = \$9000$$
$$\%\ \text{loss} = \frac{\text{loss}}{\text{CP}} \times 100\%$$
$$= \frac{\$9000}{\$37\,500} \times 100\% = 24\%$$

Therefore the percentage loss is 24%.

Find the percentage loss:

13 CP \$2000, loss \$400

14 CP \$12 500, loss \$2500

15 CP \$6400, loss \$960

16 CP \$16 000, loss \$3840

An article costing \$3000 is sold at a profit of 25%. Find the selling price.

Method 1 Find the profit then add it to the cost.

$$\text{Profit} = 25\%\ \text{of}\ \$3000 = \frac{25}{100} \times \$3000 = \$750$$
$$\text{SP} = \$3000 + \$750$$

Therefore the selling price is \$3750

Method 2 First find the SP as a percentage of the CP.

SP = 125% of CP.

$$\therefore \qquad \text{SP} = \frac{125}{100} \times \$3000$$

Therefore the selling price is \$3750

Find the selling price:

17 CP \$5000, profit 12%

18 CP \$6400, profit $12\frac{1}{2}\%$

19 CP \$2900, profit 110%

20 CP \$3600, loss 50%

21 CP \$7500, loss 64%

22 CP \$12 800, loss $37\frac{1}{2}\%$

23 An article bought for \$204 000 is sold for \$265 200. Find the percentage profit.

24 A pack of wallpaper bought for \$8750 retails at \$10 500. Calculate the percentage profit.

25 A jeweller bought a pendant for \$85 000 and sold it for \$76 500. Find his percentage loss.

26 A table bought for \$20 400 is sold for \$17 340. Find the percentage loss.

27 The cost price of a set of furniture is \$1050 000. The retailer sells it for \$1176 000. Find the percentage profit.

28 Phil bought an antique coin for \$37 375 but found after over a year that he was unable to sell it. Reluctantly he reduced the price and eventually sold it at a loss of 20%.

a Find the sale price.

b How much did he lose on the deal?

29 Jo bought two identical sets of china for \$10 000. She was able to sell one set at a profit of 30% but forced to sell the other set at a loss of 20%.

a Find **i** the profit she made on the first sale

ii the loss she made on the second sale.

b Did Jo make a profit or a loss overall? Justify your answer?

Exchange rates

When we shop abroad, prices given in the local currency often give us little idea of value so we tend to convert prices into the currency we are familiar with. To do this we need to know the *exchange rate*.

This tells us how many units of currency are equivalent to one unit of our own currency.

For example, using an exchange rate of 1 US dollar (US\$1) = J\$130 means that

100 US dollars = 100 × 130 Jamaican dollars = 13 000 Jamaican dollars

or 1 Jamaican dollars = $\frac{1}{130}$ US dollars = 0.007 69 US dollars.

Therefore 100 Jamaican dollars = 100 × $\frac{1}{130}$ US dollars = 0.769 US dollars

Exercise 16e

This table gives the equivalent of Jamaican dollars in various currencies.

Jamaican $	US$	UK£	Barbadian $	Trinidad $	Canadian $
100	0.77	0.58	1.58	5.14	1.01

Use this table to convert **a** 6345 Jamaican dollars into US$

b £100 into Jamaican dollars

a 100 Jamaican dollars is equivalent to US$ 0.77

Therefore

6345 Jamaican dollars is equivalent to US$ $\frac{0.77}{100} \times 6345 =$ US$48.86 (correct to the nearest cent)

b 100 Jamaican dollars is equivalent to 0.58 UK pounds

Therefore

£1 is equivalent to $\frac{100}{0.58}$ Jamaican dollars = 172.41 Jamaican dollars

so £100 = 100 × 172.41 Jamaican dollars

= 17 241 Jamaican dollars

Use the table above to convert

1 800 Jamaican dollars into US dollars

2 16 500 Jamaican dollars into US dollars

3 95 000 Jamaican dollars into UK pounds

4 63 450 Jamaican dollars into UK pounds

5 5500 Jamaican dollars into Barbadian dollars

6 77 600 Jamaican dollars into Barbadian dollars

7 334 400 Jamaican dollars into Trinidad and Tobago dollars

8 25 200 Jamaican dollars into Canadian dollars

9 US$55 into Jamaican dollars

10 US$500 into Jamaican dollars

11 US$4500 into Jamaican dollars

12 555 Jamaican dollars into US dollars

13 7500 Jamaican dollars into US dollars

14 84 840 Jamaican dollars into US dollars

15 £45 into Jamaican dollars

16 £350 into Jamaican dollars

17 £1200 into Jamaican dollars

18 3430 Jamaican dollars into UK pounds

19 12 500 Jamaican dollars into UK pounds

20 64 800 Jamaican dollars into UK pounds

Mixed exercises

Exercise 16f

1 George buys a jacket for $12 000 and sells it at a profit of 20%. Find the selling price.

2 Sadie pays $8500 for a concert ticket but is unable to attend. With some difficulty she finds she is able to sell it for $5100. Find her percentage loss.

3 A watch bought for $24 000 is sold at a profit of $17\frac{1}{2}$%. Find the selling price.

4 If 1 pound sterling is equivalent to 165 Jamaican dollars and to 1.72 Canadian dollars, find the equivalent of

a £550 in Jamaican dollars

b 7000 Jamaican dollars in Canadian dollars.

5 If 100 Jamaican dollars is equivalent to 1.58 Barbadian dollars, convert

a 7750 Jamaican dollars into Barbadian dollars

b 560 Barbadian dollars into Jamaican dollars.

Exercise 16g

1 **a** If 1 glue stick costs \$380, find the cost of 12.

b If 7 packs of adhesive labels cost \$14 280, find the cost of one pack.

c A pack of wire-bound notebooks costs \$9800. If each notebook costs \$1225, how many notebooks are there in a pack?

2 The cost price of a tie is \$3750 and the sale price is \$5000. Find the percentage profit.

3 A shirt is sold for \$2400 at a loss of \$1280.

Find **a** the cost price **b** the percentage loss.

4 A pack of 50 sandwich bags costs \$675. Find:

a the cost of one bag

b the number of bags I could buy for \$432 if they are priced at the same rate

c the number of bags in a pack if they cost \$15 each but the cost of the pack remained the same.

5 Convert

a 750 Barbadian dollars into UK pounds if each Barbadian dollar is equivalent to £0.35

b £550 into Barbadian dollars if £1 = 2.76 Barbadian dollars.

6 If 100 Jamaican dollars is equivalent to 5.16 Trinidad and Tobago dollars, convert

a 6750 Jamaican dollars into Trinidad and Tobago dollars

b 4460 Trinidad and Tobago dollars into Jamaican dollars.

In this chapter you have seen…

✔ how to use the fact that

$$\text{total cost} = \text{cost of one article} \times \text{number of articles}$$

to find the value of any one of these unknowns given the other two

✔ how to decide which of two offers is the better buy

✔ that profit and loss can be calculated as a percentage of the cost price

✔ that exchange rates are used to convert from one currency to another.

17 Relations

At the end of this chapter you should be able to...

1 recognise a relation
2 describe a relation in words
3 use an arrow diagram to show a relation
4 use an ordered pair to show a relation
5 find the domain and range of a relation
6 identify the different types of relation
7 use tables and equations to represent relations
8 substitute values into an equation.

You need to know...

✔ about sets
✔ what x^2 and x^3 mean.

Key words

domain, mapping, ordered pair, range, relation, table of values

Relations

Look at the pairs of numbers in the set {(1, 2), (2, 4), (6, 12)}.

There is the same relation between the numbers in each pair: the second number in each pair is twice the first number.

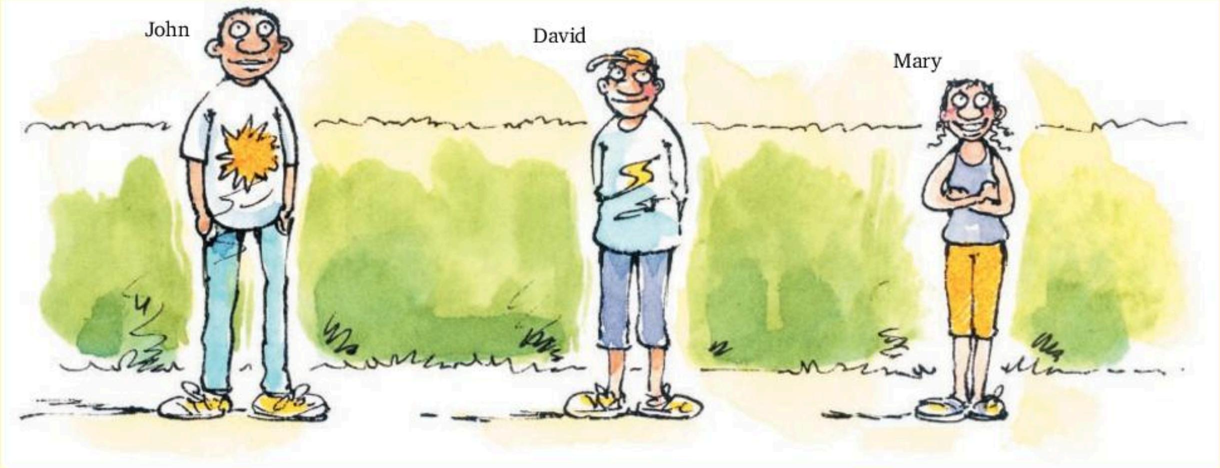
The pairs (1, 2), (2, 4) and (6, 12) are called *ordered pairs* because the order of the numbers in them is important. This is because if, for example, we change (1, 2) to (2, 1), it is no longer true that the second number is twice the first.

The set {(1, 2), (2, 4), (6, 12)} is an example of a relation.

A *relation* is a set of ordered pairs with a rule that connects the two objects in each pair.

The objects do not have to be numbers, and the relation does not have to be mathematical.

For example, John, David and Mary are friends.



John is taller than David and David is taller than Mary; each of these is a relation between two children.

We can write this information as a set of pairs: (David, John), (Mary, David).

John must also be taller than Mary, so we can add another pair with the same relation: (Mary, John).

Again, the order of the two names in each pair is important. For example, for the pair (John, David), the relation is not true because David is not taller than John.

We can describe the relation as 'the second child in each pair is taller than the first child'. We can write this relation as the set of ordered pairs

{(David, John), (Mary, David), (Mary, John)}

Exercise 17a

1 Describe the relation between the second and the first number in each pair in this set.

$$\{(1, 2), (2, 3), (5, 6), (10, 11)\}$$

2 Describe the relation between the second and the first number in each pair in this relation.

$$\{(1, 3), (2, 4), (6, 8), (10, 12)\}$$

3 Describe the relation between the second and the first number in each pair in this relation.

$$\{(2, 4), (3, 9), (4, 16), (5, 25)\}$$

4 This table shows the subject and number of pages in three school books.

Title	Number of pages
Maths	160
Spanish	210
Science	140

Write the set of ordered pairs in the relation described as 'The second book in each pair has more pages than the first book.'

5 This is a set of shapes {□, ⌂, ▲}.

Write the relation described as the set of ordered pairs where the first object is the number of sides and the second object is the shape.

Remember that a relation is a set of ordered pairs.

6 This table lists some countries and their populations.

Country	Population
Jamaica	2 500 000
Trinidad	1 300 000
Barbados	300 000
St Lucia	150 000

Give the relation described as 'the second country has a larger population than the first country'.

7 The second number in each pair in this relation is the square of the first number. Fill in the missing numbers.

$$\{(2, 4), (5, \quad), (\quad, 64)\}$$

8 The second number in each pair in this relation is the next prime number that is larger than the first number. Fill in the missing numbers.

{(8, 11), (6,   ), (1,   ), (14,   )}

Domain and range

The *domain* of a relation is the set of the first objects in the ordered pairs.

For example, the domain of the relation {(1, 2), (2, 4), (6, 12)} is the set {1, 2, 6} and the domain of the relation {(David, John), (Mary, David), (Mary, John)} is the set {David, Mary}.

Notice that we do not include Mary twice as it is the same person.

The *range* of a relation is the set of the second objects in each ordered pair.

For example, the range of the relation {(1, 2), (2, 4), (6, 12)} is the set {2, 4, 12} and the range of the relation {(David, John), (Mary, David), (Mary, John)} is the set {John, David}.

Exercise 17b

1 Write the domain and the range of the relation {(1, 2), (2, 3), (5, 6), (10, 11)}.

2 Write the domain and range of each relation.

a {(a, b), (a, c), (b, c)} **b** {(▭, □), (△, △), (▱, □)}

3 The set {2, 4, 6} is the domain of a relation. The second number in each ordered pair is the square of the first number. What is the range?

4 Fred, Dwayne and Scott are three boys. Fred is older than Dwayne and Dwayne is older than Scott.

a Write the relation described as 'the second boy in each pair is older than the first boy.'

b Give the domain and range.

5 Write the domain and range of each relation.

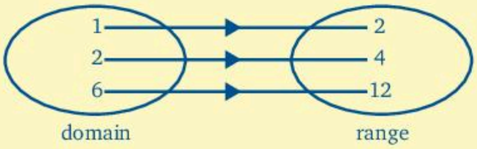
a {(10°, acute), (150°, obtuse), (45°, acute), (175°, obtuse)}

b {(t, t), (t, u), (s, t), (s, w)}

Mapping diagrams

We can represent a relation with a *mapping* diagram.

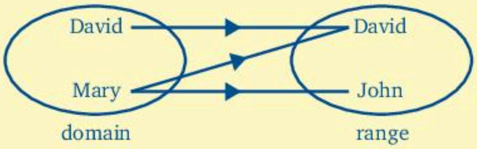
This mapping diagram represents the relation {(1, 2), (2, 4), (6, 12)}



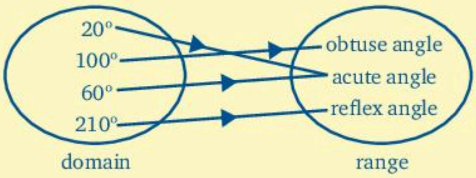
The members of the domain are placed in one oval and the members in the range are placed in a second oval. The arrows show the association between the members in the domain and the members in the range.

We say that 1 maps to 2, 2 maps to 4 and 6 maps to 12.

This mapping diagram represents the relation {(David, David), (Mary, David), (Mary, John)}.



This shows clearly that David maps to David and that Mary maps to John and David. The mapping diagram below represents another relation.



We can use this diagram to write down the relation as a set of ordered pairs:

{(20°, acute angle), (100°, obtuse angle), (60°, acute angle), (210°, reflex angle)}

Exercise 17c

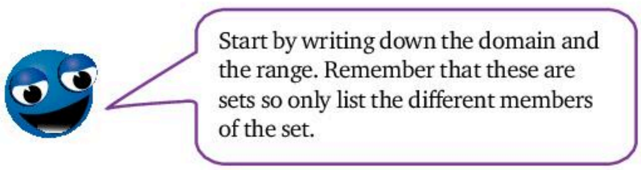
1 Draw a mapping diagram to represent these relations.

a {(1, 2), (2, 3), (5, 6), (10, 11)}

b {(a, b), (a, c), (b, c)}

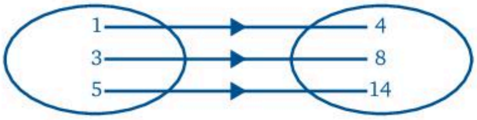
c {(2, 4), (3, 9), (4, 16), (5, 25)}

d {(a, $2a$), (b, $2b$), (c, $2c$)}



2 Each diagram represents a relation. Write the relation as a set of ordered pairs.

a



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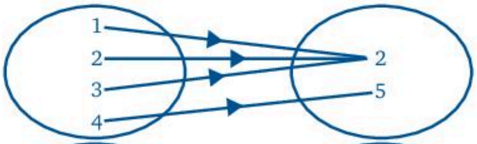
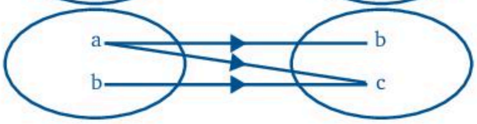
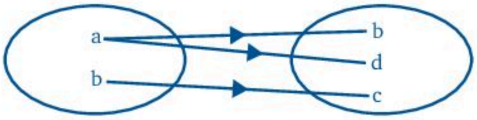


c



d



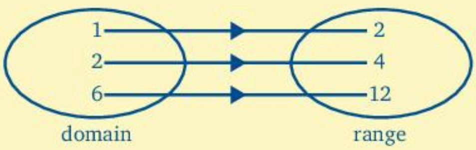
3 A relation is described as 0 maps to 0, 90 maps to 1 and 180 maps to 0. Draw a diagram to represent this relation.

Types of relation

Look again at the relation {(1, 2), (2, 4), (6, 12)}.

No two ordered pairs have the same first number, and no two ordered pairs have the same second number.

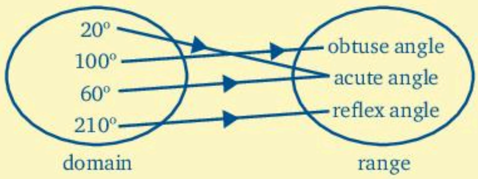
We can see this clearly from the mapping diagram:



There is only one arrow from every member of the domain. There is only one arrow to every member of the range. Every member of the domain maps to only one member of the range, and every member of the range comes from only one member of the domain.

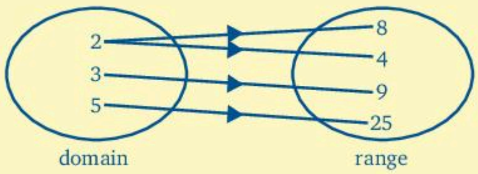
Any relation where this is true is called a 'one-to-one' relation.
This is written as 1 : 1 or 1–1.

There are other types of relation.
One type is where more than one member of the domain maps to one member of the range.
This is the case with this relation.



This type is called a 'many-to-one' relation, which we write as $n:1$.

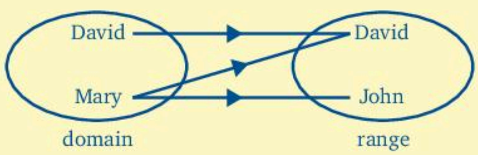
Another type is where a member of the domain maps to more than one member of the range.
This is the case with this relation.



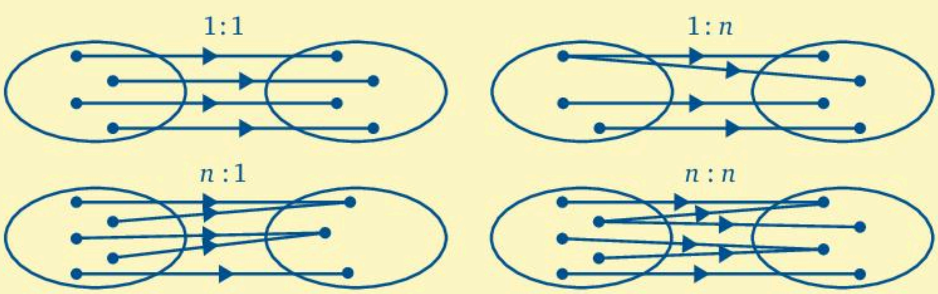
This is an example of a 'one-to-many' relation, written as $1:n$.

The last type of relation is where a member of the domain maps to more than one member of the range *and* more than one member of the domain maps to one member of the range.

This is the case with the relation {(David, John), (Mary, David), (Mary, John)}.



This type is called a 'many-to-many' relation. We write this as $n:n$.
This diagram summarises the different types of relation.



Exercise 17d

Describe the type of relation in each question in Exercise 17c.

Investigation

Kim, David, Jenny and Clare are Emma's family. They are Emma's mother, father, younger brother and younger sister.

1. David is older than Emma.
2. Clare is not Emma's younger brother.
3. Jenny is not Emma's father. She is also not Emma's younger sister.

Who is the mother, father, younger brother and younger sister?

Using tables

When the ordered pairs in a relation are numbers, such as {(1, 2), (2, 4), (6, 12)}, we can represent them in a *table of values*.

We use x to stand for the values of the first number in each pair and y to stand for the values of the second number in each pair.

So the relation {(1, 2), (2, 4), (6, 12)} can be represented by the table

x	1	2	6
y	2	4	12

The values of x give the members of the domain and the values of y give the members of the range.

Exercise 17e

1. Represent each relation as a table of values of x and y.
   - **a** {(1, 2), (2, 4), (3, 6), (4, 8)}
   - **b** {(2, 1), (4, 2), (6, 3), (9, 4.5)}
   - **c** {(10, 0), (7, 3), (5, 5), (0, 10)}
   - **d** {(0, 0), (1, 4), (2, 6), (3, 8), (4, 6)}
2. What type of relation is each one in question **1**?
3. The table represents a relation.

x	1	4	4	8	8
y	1	0	2	0	6

What type of relation is this? Give a reason for your answer.

4 Repeat question **3** for these tables.

a

x	1	1	4	4	9
y	–1	1	–2	2	–3

b

x	1	2	2	4	3
y	1	1	2	2	–3

c

x	1	2	3	4	5
y	1	2	3	4	5

Equations

We have already seen that we can describe the connection between the two numbers in each ordered pair in the relation {(1, 2), (2, 4), (6, 12)} as 'the second number in each pair is twice the first number'.

Using x to stand for the first number and y to stand for the second number, we can describe the connection more briefly as the equation $y = 2x$.

If we also give the values that x can have, we can use the equation to define the relation as

$$\{(x, y)\} \quad \text{where} \quad y = 2x \quad \text{for } x = 1, 2, 6$$

Now consider the relation $\{(x, y)\}$ where $y = 2x - 1$ for $x = 2, 4, 6, 8$.

This means that the value of x in the first ordered pair is 2 and we use the equation $y = 2x - 1$ to find the value of y by substituting 2 for x.

Remember that $2x$ means $2 \times x$, so when $x = 2$,

$$\begin{aligned} y &= 2 \times 2 - 1 \\ &= 4 - 1 \text{ (do multiplication before subtraction)} \\ &= 3 \end{aligned}$$

We can find the other ordered pairs in the same way and represent the relation in a table.

x	2	4	6	8
y	3	7	11	15

Exercise 17f

1 A relation is given by $\{(x, y)\}$ where $y = 3x$ for $x = 1, 2, 3$.

Copy and complete this table of values.

x	1	2	3
y		6	

Remember that $3x$ means $3 \times x$.
To find y when $x = 1$, use the equation $y = 3x$ and substitute 1 for x, i.e. when $x = 1$, $y = 3 \times 1$.

2 A relation is given by $\{(x, y)\}$ where $y = 4x - 1$ for $x = 1, 2, 3$.

Copy and complete this table of values.

x	1	2	3
y		7	

Remember that multiplication and division must be done before addition and subtraction.

3 A relation is given by $\{(x, y)\}$ where $y = 10 - x$ for $x = 2, 4, 6, 8$.

Copy and complete this table of values.

x	2	4	6	8
y		6		

4 A relation is given by $\{(x, y)\}$ where $y = 12 - 2x$ for $x = 1, 2, 5, 6$.

Copy and complete this table of values.

x	1	2	5	6
y			2	

5 A relation is given by $\{(x, y)\}$ where $y = x^2 + 1$ for $x = 1, 2, 3, 4$.

Copy and complete this table of values.

x	1	2	3	4
y			10	

Remember that x^2 means $x \times x$, so when $x = 2$, $x^2 + 1 = 2 \times 2 + 1$.

6 A relation is given by $\{(x, y)\}$ where $y = 2x^2 - 1$ for $x = 1, 2, 3, 4$.

Copy and complete this table of values.

x	1	2	3	4
y			17	

$2x^2$ means $2 \times x \times x$, so when $x = 4$, $2x^2 - 1 = 2 \times (4 \times 4) - 1$.

7 A relation is given by $\{(x, y)\}$ where $y = x^2 - 3x + 4$ for $x = 1, 2, 3, 4$.

Copy and complete this table of values.

x	1	2	3	4
y		2		

8 A relation is given by $\{(x, y)$ where $y = x^2 - x$ for $x = 1, 2, 3, 4$.

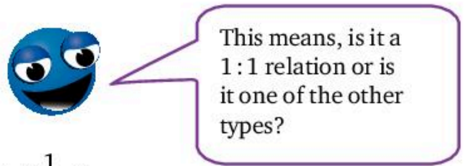
a Copy and complete this table of values.

x	1	2	3	4
y		2		

b Write the domain and range.

c Represent the relation with an arrow diagram.

d What type of relation is this?



9 A relation is given by $\{(x, y)\}$ where $y = 3x + \frac{1}{2}$ for $x = 1, 1\frac{1}{2}, 2$.

a Copy and complete this table of values.

x	1	$1\frac{1}{2}$	2
y	$3\frac{1}{2}$		

b Write the domain and range.

c Represent the relation with an arrow diagram.

d What type of relation is this?

10 A relation is given by $\{(x, y)\}$ where $y = x^2 - 5x + 6$ for $x = 1, 2, 3, 4$.

a Copy and complete this table of values.

x	1	2	3	4
y		0	0	

b Write the domain and range.

c Represent the relation with an arrow diagram.

d What type of relation is this?

11 A relation is given by $\{(x, y)\}$ where $y = x^3 - 8x^2 + 15x$ for $x = 0, 2, 3, 5$.

a Copy and complete this table of values.

x	0	2	3	5
y			0	

b Write the domain and range.

c Represent the relation with an arrow diagram.

d What type of relation is this?

Exercise 17g

Select the letter that gives the correct answer.

1 The relation between the first number and the second number in each of the pairs in the set {(2, 3), (3, 5), (4, 7), (5, 9), (6, 11)} is

A double the first number and subtract 1

B double the first number and add 1

C treble the first number

D treble the first number and subtract 3.

2 The second number in each pair in the following relation is double the first plus 3:

{(3, 9), (6,), (8, 19)}

The missing number is

A 12 **B** 14 **C** 15 **D** 18

3 The domain of the relation {(3, 5), (4, 7), (5, 9), (6, 11)} is the set

A {3, 4, 5} **B** {3, 4, 5, 6}

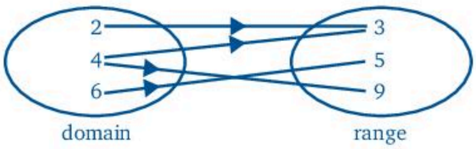
C {4, 5, 6} **D** {5, 7, 9, 11}

4 The range of the relation {(3, 9), (4, 12), (5, 15), (6, 18)} is the set

A {3, 4, 5, 6} **B** {4, 5, 6}

C {9, 12, 15} **D** {9, 12, 15, 18}

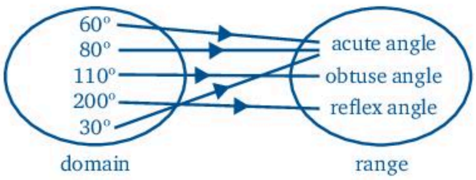
5



Which type of relation is this?

A 1 : 1 **B** 1 : n **C** n : 1 **D** n : n

6



Which type of relation is this?

A 1 : 1 **B** 1 : n **C** n : 1 **D** n : n

7 What type of relation is represented by the set {(a, b), (a, c), (b, c), (c, b)}?

A 1 : 1 **B** 1 : n **C** n : 1 **D** n : n

8 The table represents a relation.

x	2	5	4	5	7
y	1	0	3	2	5

What type of relation is this?

A 1 : 1 **B** 1 : n **C** n : 1 **D** n : n

9 A relation is given by {(x, y)} where $y = 8 - 2x$ for $x = 1, 2, 4, 6$

x	1	2	4	6
y	6	4		–4

The missing number from this table is

A –2 **B** 0 **C** 2 **D** 4

10 A relation is given by {(x, y)} where $y = x^2 - 3x + 4$ and x takes the values 1, 2, 3, 4.

x	1	2	3	4
y	2		4	8

The missing number from this table is

A 0 **B** 1 **C** 2 **D** 4

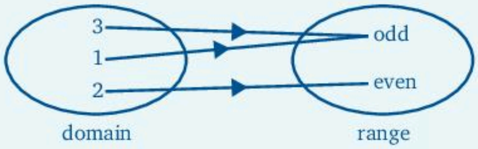
Did you know?

One of the most important relations in physics is the relation between the position of a subatomic particle and its momentum. It is called the 'uncertainty relation' and was stated by the German theoretical physicist Werner Heisenberg in 1927 as:

> 'The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.'

In this chapter you have seen that...

- ✔ a relation is a set of ordered pairs
- ✔ the set of the first objects in each pair is the domain and the set of the second objects in each pair is the range
- ✔ a relation can be represented by a mapping diagram, e.g.



- ✔ a relation where a number maps to a number can be represented by an equation, e.g. $y = 2x$, together with the values that x can take
- ✔ there are four types of relation: $1:1$, $n:1$, $1:n$ and $n:n$.

18 Coordinate geometry

At the end of this chapter you should be able to...

1 describe the position of a point with reference to a pair of perpendicular axes
2 plot points on a rectangular grid given the coordinates
3 write the coordinates of given points on a rectangular grid
4 know the meaning of the equation of a straight line and plot its graph.

Did you know?

The ideas used in this chapter are part of Descartes' *Geometry*. He thought of these ideas as he watched a fly crawling along a ceiling.

You will learn more about Descartes later.

You need to know...

✔ how to work with directed numbers

✔ what a relation is.

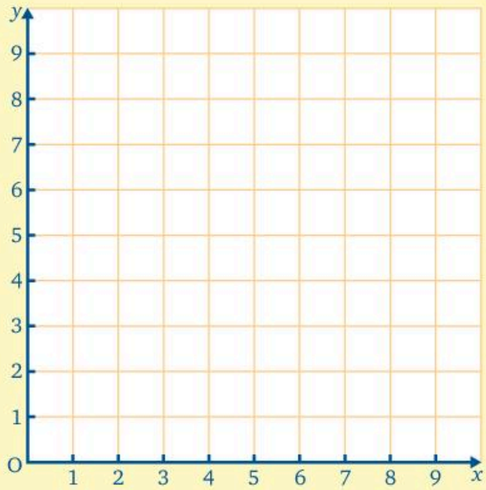
Key words

axis (plural axes), Cartesian coordinate system, Cartesian plane, coordinates, relation

Plotting points using positive coordinates

There are many occasions when you need to describe the position of an object. For example, telling a friend how to find your house, finding a square in the game of battleships or describing the position of an aeroplane showing up on a radar screen. In mathematics we need a quick way to describe the position of a point.

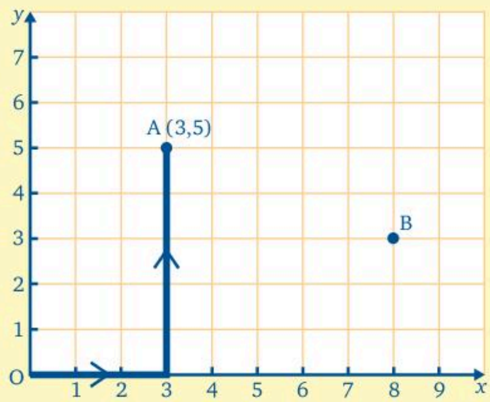
We do this by using squared paper and marking a point O at the corner of one square. We then draw a line through O across the page. This line is called O*x*. Next we draw a line through O up the page. This line is called O*y*. Starting from O we then mark numbered scales on each line, or *axis*.



O is called the origin
O*x* is called the *x*-axis
O*y* is called the *y*-axis

We can now describe the position of a point A as follows:

start from O and move 3 units along O*x*,
then move 5 units up from O*x* parallel to O*y*.



We always use the same method to describe the position of a point:

start from O, *first* move *along* and *then up*.

We can now shorten the description of the position of the point A to the number pair (3, 5).

The number pair (3, 5) is referred to as the *coordinates* of A.

The first number, 3, is called the *x*-coordinate of A.

The second number, 5, is called the *y*-coordinate of A.

Now consider another point B

whose *x*-coordinate is 8
and whose *y*-coordinate is 3.

If we simply refer to the point B(8, 3) this tells us all that we need to know about the position of B.

The origin is the point (0, 0).

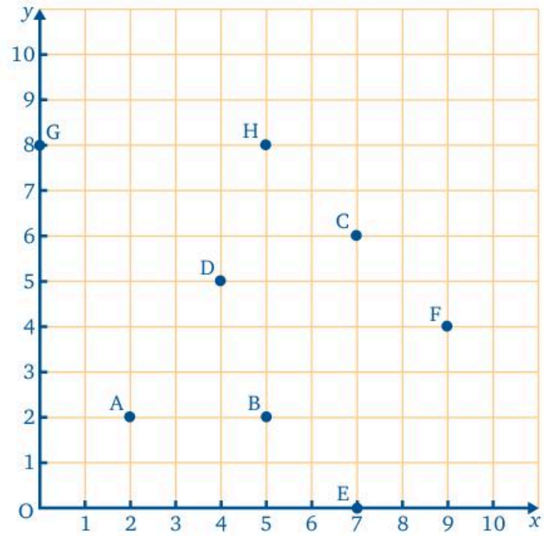
The coordinates of a point are another example of an ordered pair.

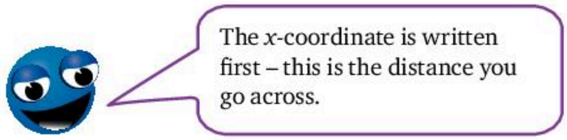
The *x*-coordinate always comes first and the *y*-coordinate is always second.

This way of specifying the position of a point is called the *Cartesian coordinate system*.

Exercise 18a

1 Write the coordinates of the points A, B, C, D, E, F, G and H.





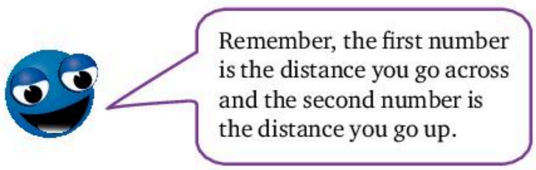
2 Draw a set of axes of your own. Along each axis mark points 0, 1, 2, …, 10 units from O. Mark the following points and label each point with its own letter:

A(2, 8) B(4, 9) C(7, 9) D(8, 7) E(8, 6) F(9, 4) G(8, 4)
H(7, 3) I(5, 3) J(7, 2) K(7, 1) L(4, 2) M(2, 0) N(0, 2)

Now join your points together in alphabetical order and join A to N.

3 Draw a set of axes and give them scales from 0 to 10.
Mark the following points:

A(2, 5) B(7, 5) C(7, 4) D(8, 4) E(8, 3)
F(9, 3) G(9, 2) H(6, 3) I(6, 1) J(7, 1)
K(7, 0) L(5, 0) M(5, 2) N(4, 2) P(4, 0)
Q(2, 0) R(2, 1) S(3, 1) T(3, 2) U(0, 2)
V(0, 3) W(1, 3) X(1, 4) Y(2, 4)



Now join your points together in alphabetical order and join A to Y.

4 Mark the following points on your own set of axes:

A(2, 7) B(8, 7) C(8, 1) D(2, 1)

Join A to B, B to C, C to D and D to A. What is the name of the figure ABCD?

5 Mark the following points on your own set of axes:

A(2, 2) B(8, 2) C(5, 5)

Join A to B, B to C and C to A. Describe fully the triangle ABC.

<u>6</u> Mark the following points on your own set of axes:

A(4, 0) B(6, 0) C(6, 4) D(4, 4)

Join A to B, B to C, C to D and D to A. Name the figure ABCD.

7 Mark the following points on your own set of axes:

A(5, 2) B(8, 5) C(5, 8) D(2, 5)

Join the points to make the figure ABCD. What is ABCD?

<u>8</u> On your own set of axes mark the points A(8, 4), B(8, 8) and C(14, 6). Join A to B, B to C and C to A. Describe fully the figure ABC.

For each of questions **9** to **14** you will need to draw your own set of axes.

9 The points A(2, 1), B(6, 1) and C(6, 5) are three corners of a square ABCD. Mark the points A, B and C. Find the point D and write the coordinates of D.

10 The points A(2, 1), B(2, 3) and C(7, 3) are three vertices of a rectangle ABCD. Mark the points and find the point D. Write the coordinates of D.

11 The points A(1, 4), B(4, 7) and C(7, 4) are three vertices of a square ABCD. Mark the points A, B and C and find D. Write the coordinates of D.

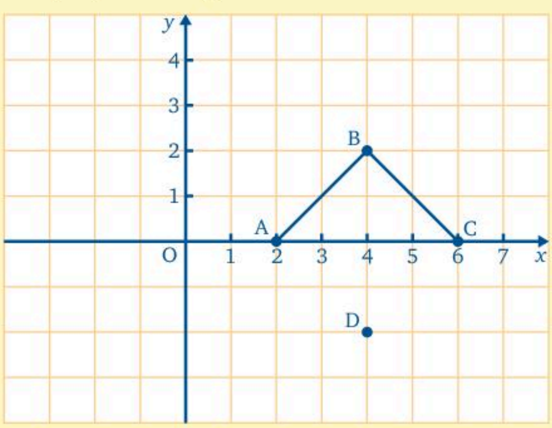
12 Mark the points A(2, 4) and B(8, 4). Join A to B and find the point C which is the midpoint (the exact middle) of the line AB. Write the coordinates of C.

13 Mark the points P(3, 5) and Q(3, 9). Join P and Q and mark the point R which is the midpoint of PQ. Write the coordinates of R.

14 Mark the points A(0, 5) and B(4, 1). Find the coordinates of the midpoint of AB.

Negative coordinates

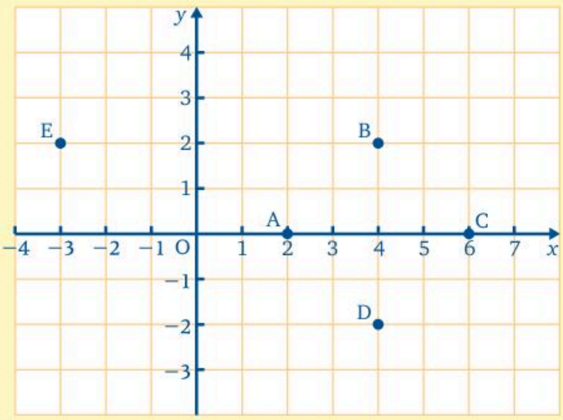
If A(2, 0), B(4, 2) and C(6, 0) are three corners of a square ABCD, we can see that the fourth corner, D, is two squares below the x-axis.



To describe the position of D we need to extend the scale on the *y*-axis below zero. To do this we use the negative numbers

$$-1, -2, -3, -4, \ldots$$

In the same way we can use the negative numbers –1, –2, –3, ... to extend the scale on the *x*-axis to the left of zero.



The set of the perpendicular *x* and *y* axes defines what is called the *Cartesian plane*.

The *y*-coordinate of the point D is written –2 and is called 'negative 2'.

The *x*-coordinate of the point E is written –3 and is called 'negative 3'.

The numbers 1, 2, 3, 4, ... are called positive numbers. They could be written as +1, +2, +3, +4, ... but we do not usually put the + sign in.

Now D is 4 squares to the right of O so its *x*-coordinate is 4

and 2 squares below the *x*-axis so its *y*-coordinate is –2,

D is the point (4, –2)

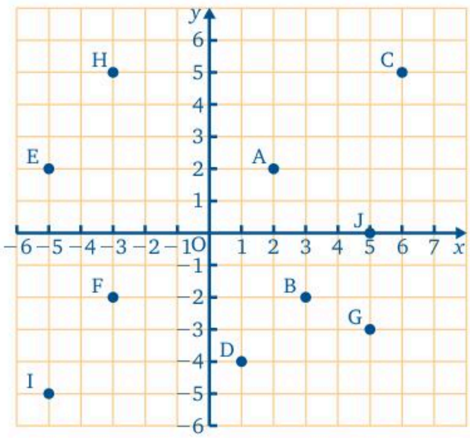
E is 3 squares to the left of O so its *x*-coordinate is –3

and 2 squares up from O so its *y*-coordinate is 2,

E is the point (–3, 2)

Exercise 18b

Use this diagram for questions **1** and **2**.



1 Write down the *x*-coordinate of each of the points A, B, C, D, E, F, G, H, I, J and O (the origin).

2 Write the *y*-coordinate of each of the points A, B, C, D, E, H, I and J.

In questions **3** to **8** draw your own set of axes and scale each one from –5 to 5.

3 Mark the points A(–3, 4), B(–1, 4), C(1, 3), D(1, 2), E(–1, 1), F(1, 0), G(1, –1), H(–1, –2), I(–3, –2).

Join the points in alphabetical order and join I to A.

4 Mark the points A(4, –1), B(4, 2), C(3, 3), D(2, 3), E(2, 4), F(1, 4), G(1, 3), H(–2, 3), I(–3, 2), J(–3, –1).

Join the points in alphabetical order and join J to A.

5 Mark the points A(2, 1), B(–1, 3), C(–3, 0), D(0, –2).

Join the points to make the figure ABCD. What is the name of the figure?

6 Mark the points A(1, 3), B(–1, –1), C(3, –1).

Join the points to make the figure ABC and describe ABC.

7 Mark the points A(–2, –1), B(5, –1), C(5, 2), D(–2, 2).

Join the points to make the figure ABCD and describe ABCD.

8 Mark the points A(–3, 0), B(1, 3), C(0, –4).

What kind of triangle is ABC?

In questions **9** to **18**, the points A, B and C are three corners of a square ABCD. Mark the points and find the point D. Give the coordinates of D.

9	A(1, 1)	B(1, –1)	C(–1, –1)	**14**	A(–3, –1)	B(–3, 2)	C(0, 2)
10	A(1, 3)	B(6, 3)	C(6, –2)	**15**	A(0, 4)	B(–2, 1)	C(1, –1)
11	A(3, 3)	B(3, –1)	C(–1, –1)	**16**	A(1, 0)	B(3, 2)	C(1, 4)
12	A(–2, –1)	B(–2, 3)	C(–6, 3)	**17**	A(–2, –1)	B(2, –2)	C(3, 2)
13	A(–5, –3)	B(–1, –3)	C(–1, 1)	**18**	A(–3, –2)	B(–5, 2)	C(–1, 4)

In questions **19** to **28**, mark the points A and B and the point C, the midpoint of the line AB. Give the coordinates of C.

19	A(2, 2)	B(6, 2)	**24**	A(2, 1)	B(6, 2)
20	A(2, 3)	B(2, –5)	**25**	A(2, 1)	B(–4, 5)
21	A(–1, 3)	B(–6, 3)	**26**	A(–7, –3)	B(5, 3)
22	A(–3, 5)	B(–3, –7)	**27**	A(–3, 3)	B(3, –3)
23	A(–1, –2)	B(–9, –2)	**28**	A(–7, –3)	B(5, 3)

Straight lines

When the ordered pairs in a *relation* are numbers, as in {(1, 2), (2, 4), (6, 12)}, we can think of them as sets of coordinates.

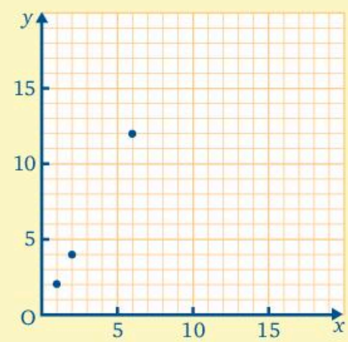
This means we can represent them in a table and plot them as points, (x, y), on a plane.

We use x to stand for the values of the first number in each pair and y to stand for the second number in each pair.

So the relation {(1, 2), (2, 4), (6, 12)} can be represented by the table

x	1	2	6
y	2	4	12

The values of x give the members of the domain and the values of y give the members of the range. We can then represent these ordered pairs as points on a graph.



Exercise 18c

1 Represent each relation as a table of values of x and y and illustrate them on a graph. In each case state the type of relation represented.

a {(1, 2), (2, 4), (3, 6), (4, 8)}

b {(2, 1), (4, 2), (6, 3), (9, 4.5)}

c {(10, 0), (7, 3), (5, 5), (0, 10)}

d {(0, 0), (1, 4), (2, 6), (3, 8), (4, 6)}

2 A relation is represented by this table.

x	2	6	10
y	1	3	5

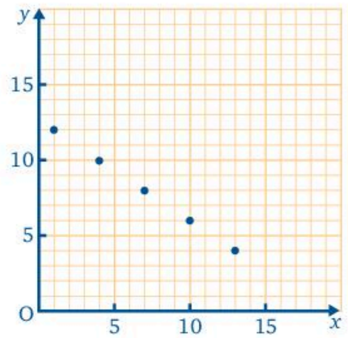
Illustrate the relation on a graph and state what type of relation this represents.

3 This graph illustrates a relation.

a Represent the relation as a table.

b Give the relation as a set of ordered pairs.

c What type of relation is this?



4 A relation is represented by this table.

x	1	1	4	4
y	–1	1	–2	2

a Illustrate this relation on a graph.

b Do these points lie on a straight line?

c What type of relation is this?

5 A relation is represented by this table.

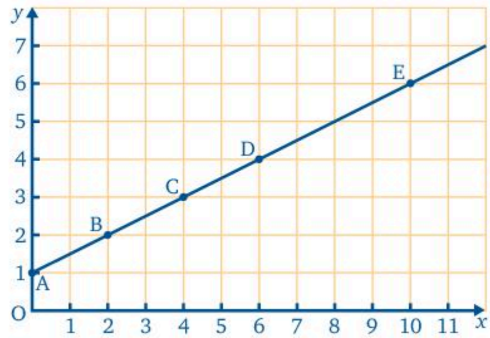
x	0	0	3	–3
y	–3	3	0	0

a Illustrate this relation on a graph.

b Do these points lie on a straight line?

c What type of relation is this?

6 The points A, B, C, D and E illustrate a relation.



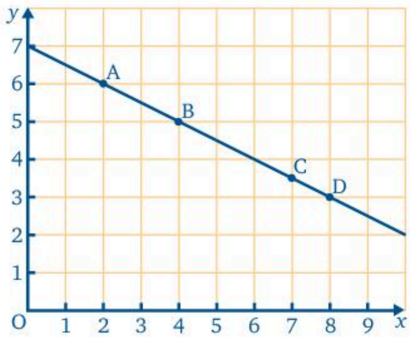
a Represent these points as a table.

b How is the y-coordinate of each point related to its x-coordinate?

c The points A, B, C, D and E all lie on the same straight line.
G is another point on this line. Its x-coordinate is 8; what is its y-coordinate?

d F, H and I are also points on this line. Find the missing coordinates.
(5,), (16,), (a,)

7 The points A, B, C and D illustrate a relation.



a Represent these points as a table.

b How is the y-coordinate of each point related to its x-coordinate?

c The points A, B, C, and D all lie on the same straight line.
E is another point on this line. Its x-coordinate is 5; what is its y-coordinate?

d F, H and I are also points on this line. Find the missing coordinates.
(1,), (12,), (a,)

In the following questions we are going to investigate the properties of the diagonals of the special quadrilaterals. You will need your own set of axes for each question. Mark a scale on each axis from –5 to +5. Mark the points A, B, C and D and join them to form the quadrilateral ABCD.

8 A(5, –2) B(2, 4) C(–3, 4) D(0, –2)

a What type of quadrilateral is ABCD?

b Join A to C and B to D. These are the diagonals of the quadrilateral. Mark with an E the point where the diagonals cross.

c Measure the diagonals. Are they the same length?

d Is E the midpoint of either, or both, of the diagonals?

e Measure the four angles at E. Do the diagonals cross at right angles?

For questions **9** to **12**, repeat question **8** for the following points.

9 A(2, –2) B(2, 4) C(–4, 4) D(–4, –2)

10 A(2, –2) B(5, 4) C(–3, 4) D(–1, –2)

11 A(2, 0) B(0, 4) C(–2, 0) D(0, –4)

12 A(1, –4) B(1, –1) C(–5, –1) D(–5, –4)

13 Name the quadrilaterals in which the two diagonals are of equal length.

14 Name the quadrilaterals in which the diagonals cut at right angles.

15 Name the quadrilaterals in which the diagonals cut each other in half.

Investigation

Draw your own set of x and y axes and scale each of them from –6 to +8.

Plot the points A(–1, 3), B(3, –1) and C(–1, –5).

1 Can you write

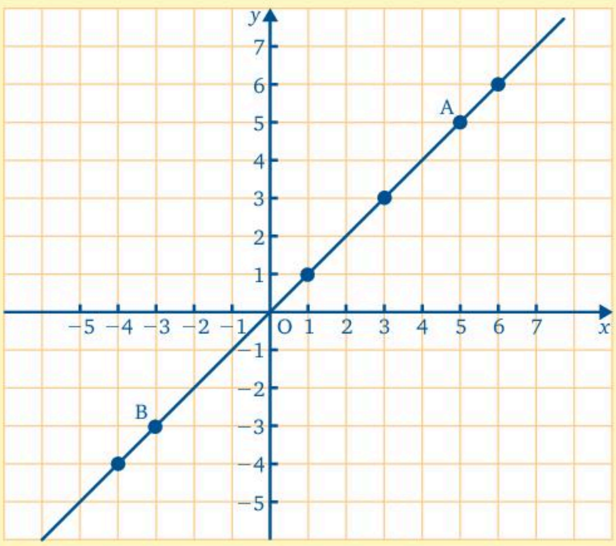
a the coordinates of a point D such that ABCD is a square

b the coordinates of a point E such that ACBE is a parallelogram

c the coordinates of a point F such that CDEF is a rectangle?

2 Can you give the name of the special quadrilateral EDBF?

The equation of a straight line



If we plot the points with coordinates (–4, –4), (1, 1), (3, 3) and (6, 6), we can see that a straight line can be drawn through these points that also passes through the origin.

For each point the y-coordinate is the same as the x-coordinate.

This is also true for any other point on this line,

e.g. the coordinates of A are (5, 5) and the coordinates of B are (–3, –3).

Hence y-coordinate = x-coordinate

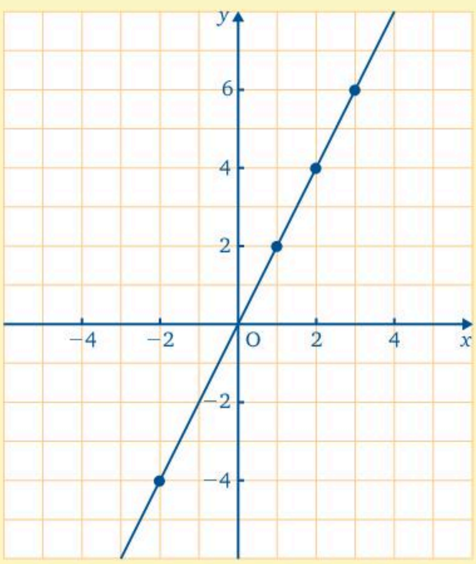
or simply $y = x$

This is called the equation of the line.

We can also think of a line as a set of points, i.e. this line is the set of points, or ordered number pairs, such that $\{(x, y)\}$ satisfies the relation $y = x$.

It follows that if another point on the line has an x-coordinate of –5, then its y-coordinate is –5 and if a further point has a y-coordinate of 4, its x-coordinate is 4.

In a similar way we can plot the points with coordinates (–2, –4), (1, 2), (2, 4) and (3, 6).



These points also lie on a straight line passing through the origin.

In each case the y-coordinate is twice the x-coordinate.

The equation of this line is therefore $y = 2x$ and we often refer to it simply as 'the line $y = 2x$'.

If another point on this line has an x-coordinate of 4,

its y-coordinate is 2×4, i.e. 8,

and if a further point has a y-coordinate of –5,

its x-coordinate must be $-2\frac{1}{2}$.

Exercise 18d

1 Find the y-coordinates of points on the line $y = x$ that have x-coordinates of

a 2 **b** 3 **c** 7 **d** 12.

2 Find the y-coordinates of points on the line $y = x$ that have x-coordinates of

a -1 **b** -6 **c** -8 **d** -20.

3 Find the y-coordinates of points on the line $y = -x$ that have x-coordinates of

a $3\frac{1}{2}$ **b** $-4\frac{1}{2}$ **c** 6.1 **d** -8.3

4 Find the x-coordinates of points on the line $y = -x$ that have y-coordinates of

a 7 **b** -2 **c** $5\frac{1}{2}$ **d** -4.2.

5 Find the y-coordinates of points on the line $y = 2x$ that have x-coordinates of

a 5 **b** -4 **c** $3\frac{1}{2}$ **d** -2.6.

6 Find the x-coordinates of points on the line $y = -3x$ that have y-coordinates of

a 3 **b** -9 **c** 6 **d** -4.

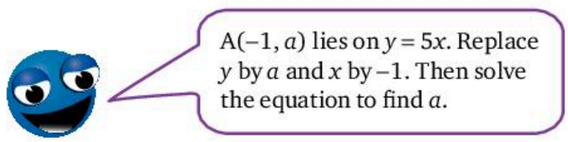
7 Find the x-coordinates of points on the line $y = \frac{1}{2}x$ that have y-coordinates of

a 6 **b** -12 **c** $\frac{1}{2}$ **d** -8.2.

8 Find the x-coordinates of points on the line $y = -4x$ that have y-coordinates of

a 8 **b** -16 **c** 6 **d** -3.

9 If the points $(-1, a)$, $(b, 15)$ and $(c, -20)$ lie on the straight line with equation $y = 5x$, find the values of a, b and c.



10 If the points $(3, a)$, $(-12, b)$ and $(c, -12)$ lie on the straight line with equation $y = -\frac{2}{3}x$, find the values of a, b and c.

11 Using 1 cm to 1 unit on each axis, plot the points $(-2, -6)$, $(1, 3)$, $(3, 9)$ and $(4, 12)$. What is the equation of the straight line that passes through these points?

12 Using 1 cm to 1 unit on each axis, plot the points (–3, 6) (–2, 4), (1, –2) and (3, –6). What is the equation of the straight line that passes through these points?

13 Using the same scale on each axis, plot the points (–6, 2), (0, 0), (3, –1) and (9, –3). What is the equation of the straight line that passes through these points?

14 Using the same scale on each axis, plot the points (–6, –4), (–3, –2), (6, 4) and (12, 8). What is the equation of the straight line that passes through these points?

15 Which of the points (–2, –4), (2.5, 4), (6, 12) and (7.5, 10) lie on the line $y = 2x$?

16 Which of the points (–5, –15), (–2, 6), (1, –3) and (8, –24) lie on the line $y = -3x$?

17 Consider these points:
(2, 2), (–2, 1), (3, 0), (–4.2, –2) and (–6.4, –3.2).
Which of the points lie

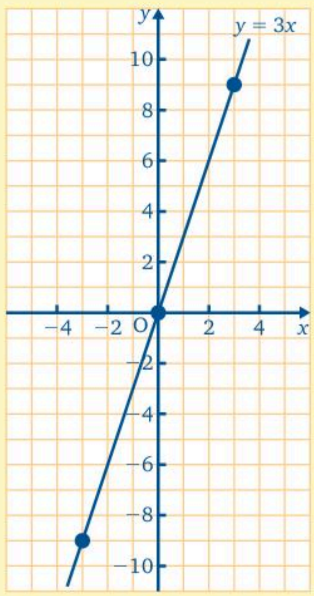
a above the line $y = \frac{1}{2}x$

b below the line $y = \frac{1}{2}x$?

Plotting the graph of a given equation

If we want to draw the graph of $y = 3x$ for values of x from –3 to +3, then we need to find the coordinates of some points on the line.

As we know that it is a straight line, two points are enough. However, it is sensible to find three points, the third point acting as a check on our working. It does not matter which three points we find, so we will choose easy values for x, one at each extreme and one near the middle.



If $\quad x = -3, \quad y = 3 \times (-3) = -9$

If $\quad x = 0, \quad y = 3 \times 0 = 0$

If $\quad x = 3, \quad y = 3 \times 3 = 9$

These look neater if we write them in table form:

x	–3	0	3
y	–9	0	9

Exercise 18e

In questions **1** to **6**, draw the graphs of the given equations on the same set of axes. Use the same scale on both axes, taking values of x between –4 and 4, and values of y between –6 and 6. You should take at least three x values and record the corresponding y values in a table. Write the equation of each line somewhere on it.

1 $y = x$

2 $y = 2x$

3 $y = \frac{1}{2}x$

<u>**4**</u> $y = \frac{1}{4}x$

<u>**5**</u> $y = \frac{1}{3}x$

<u>**6**</u> $y = \frac{3}{2}x$

In questions **7** to **12**, draw the graphs of the given equations on the same set of axes.

7 $y = -x$

8 $y = -2x$

9 $y = -\frac{1}{2}x$

<u>**10**</u> $y = -\frac{1}{4}x$

<u>**11**</u> $y = -\frac{1}{3}x$

<u>**12**</u> $y = -\frac{3}{2}x$

We can conclude from these exercises that the graph of an equation of the form $y = mx$ is a straight line that:

- passes through the origin
- gets steeper as m increases
- makes an acute angle with the positive x-axis if m is positive
- makes an obtuse angle with the positive x-axis if m is negative.

Puzzle

Here is a very ingenious method of guessing the values of three dice thrown by a friend, without seeing them.

- Tell your friend to think of the first die.
- Multiply by 2. Add 5. Multiply by 5.
- Add the value of the second die.
- Multiply by 10. Add the value of the third die.
- Now ask for the total. From this total subtract 250.
- The three digits of your answer will be the values of the three dice.

As an example, if the total was 706, then 706 – 250 = 456. The three dice were therefore 4, 5 and 6. Try it and see. Why does it work?

Did you know?

A US gallon is 231 cubic inches, which is equal to the old English wine gallon.

An imperial gallon is 277.42 cubic inches, which is about 20% more than a US gallon.

In this chapter you have seen that…

- ✔ you can write down the coordinates of a point as an ordered pair of numbers
- ✔ the first number (x-coordinate) gives distance across and the second number (y-coordinate) gives distance up or down
- ✔ you can find the missing coordinate of a point on a line, given the equation of the line and one coordinate, by substituting the given coordinate into the equation and solving it
- ✔ you can draw a straight-line graph, given its equation, by finding the coordinates of three points on the line.

19 Volume and capacity

At the end of this chapter you should be able to...

1 recognise volume as a measure of space
2 measure volume using standard units
3 calculate the volume of solids given the necessary measurements
4 convert from one standard metric unit of volume to another.

Did you know?

Archimedes, one of the greatest mathematicians, lived on the island of Sicily during the 3rd century BCE. While taking a bath in a tub he discovered a law about things floating in water. He saw how this law could help the king, who thought that the man who made his crown had cheated him, know whether his crown was pure gold or not.

He was so excited that he ran out of the house naked shouting 'Eureka', which means 'I have found it'.

You need to know...

- ✔ how to change from one metric unit to another
- ✔ how to find the area of a square
- ✔ how to work with fractions and decimals.

Key words

capacity, cuboid, metric units, rectilinear solid, volume

Units of volume and capacity

In Chapter 6 we considered the *metric units* we use to measure the *volume* of a solid and the *capacity* of a container. We revise that work here.

Units of volume

1 cm = 10 mm, therefore

$$1\text{ cm}^3 = 10 \times 10 \times 10\text{ mm}^3$$
$$= 1000\text{ mm}^3$$

and 1 m = 100 cm, so

$$1\text{ m}^3 = 100 \times 100 \times 100\text{ cm}^3$$
$$= 1000\,000\text{ cm}^3$$

Units of capacity

$$1000\text{ cm}^3 = 1\text{ litre}$$
$$1000\text{ litres} = 1\text{ m}^3$$
$$1000\text{ ml} = 1\text{ litre} \quad \text{or} \quad 1\text{ ml} = 1\text{ cm}^3$$

Exercise 19a

Express in mm^3

1 4 cm^3 **2** 0.5 cm^3 **3** 2.3 cm^3 **4** 0.078 cm^3

Express in cm^3

5 3 m^3 **6** 0.25 m^3 **7** 8.2 m^3 **8** 0.073 m^3

Express in cm^3

9 1.4 litres **10** 0.75 litres **11** 54 litres **12** 0.043 litres

Express in litres

13 4500 cm^3 **14** 56000 cm^3 **15** $98\,000\text{ cm}^3$ **16** 36 cm^3

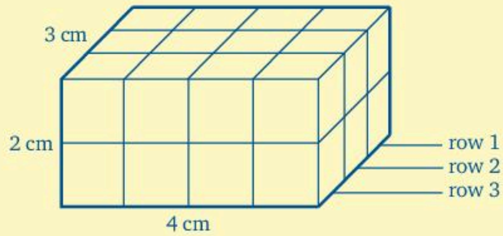
17 Express

a 400 mm^3 in cm^3 **b** 250 cm^3 in mm^3 **c** 0.07 m^3 in cm^3

Volume of a cuboid

In this chapter we investigate the volumes of *rectilinear* solids, i.e. solids whose faces are all rectangles.

A *cuboid* is the mathematical name for a rectangular block. Each face of a cuboid is a rectangle.



The diagram shows a rectangular block or cuboid measuring 4 cm by 3 cm by 2 cm. To cover the area on which the block stands we need three rows of cubes measuring 1 cm by 1 cm by 1 cm, with four cubes in each row, i.e. 12 cubes.

A second layer of 12 cubes is needed to give the volume shown, so the volume of the block is 24 cubes.

But the volume of one cube is $1\ \text{cm}^3$.

Therefore the volume of the solid is $24\ \text{cm}^3$.

This is also given when we calculate length × breadth × height,

i.e. the volume of the block $= 4 \times 3 \times 2\ \text{cm}^3$

or the volume of the block = length × breadth × height

Exercise 19b

Find the volume of a cuboid measuring 12 cm by 10 cm by 5 cm.

$$\text{Volume of cuboid} = \text{length} \times \text{breadth} \times \text{height}$$
$$= 12 \times 10 \times 5\ \text{cm}^3$$

i.e. $\text{Volume} = 600\ \text{cm}^3$

Find the volume of each of the following cuboids:

	Length	Breadth	Height		Length	Breadth	Height
1	4 cm	4 cm	3 cm	**7**	4 m	3 m	2 m
2	20 mm	10 mm	8 mm	**8**	8 m	5 m	4 m
3	45 mm	20 mm	6 mm	**9**	8 cm	3 cm	$\frac{1}{2}$ cm
4	5 mm	4 mm	0.8 mm	**10**	12 cm	1.2 cm	0.5 cm
5	6.1 m	4 m	1.3 m	**11**	4.5 m	1.2 m	0.8 m
6	3.5 cm	2.5 cm	1.2 cm	**12**	1.2 m	0.9 m	0.7 m

Find the volume of a cube with the given side:

13 4 cm

14 5 cm

15 2 m

16 $\frac{1}{2}$ cm

17 2.5 cm

18 3 km

19 8 km

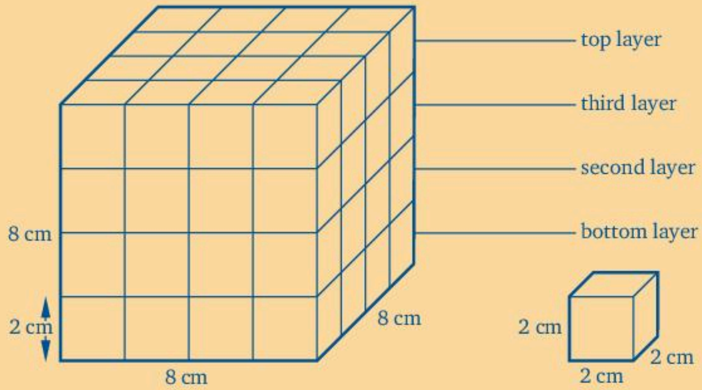
20 $1\frac{1}{2}$ km

21 3.4 m

The edges of a cube are all the same length so the volume is $(4 \times 4 \times 4)\text{cm}^3$.

Exercise 19c

Draw a cube of side 8 cm. How many cubes of side 2 cm would be needed to fill the same space?



The bottom layer requires 4×4, i.e. 16 cubes of side 2 cm, and there are four layers altogether.

Therefore 64 cubes are required.

1 Draw a cube of side 4 cm. How many cubes of side 2 cm would be needed to fill the same space?

2 Draw a cuboid measuring 6 cm by 4 cm by 2 cm. How many cubes of side 2 cm would be needed to fill the same space?

3 Draw a cube of side 6 cm. How many cubes of side 3 cm would be needed to fill the same space?

4 Draw a cuboid measuring 8 cm by 6 cm by 2 cm. How many cubes of side 2 cm would be needed to fill the same space?

Puzzle

The outer surface of the large cube in the worked example above is painted red. How many of the 64 small cubes are unpainted?

Mixed units

Before we can find the volume of a cuboid, *all* measurements must be expressed in the same unit.

Exercise 19d

Find the volume of a cuboid measuring 2 m by 70 cm by 30 cm.

Give your answer in **a** cm^3 **b** m^3.

a (All the measurements must be in centimetres so we first convert the 2 m into centimetres.)

$$\text{Length of cuboid} = 2\text{ m} = 2 \times 100\text{ cm} = 200\text{ cm}$$

$$\begin{aligned}\text{Volume of cuboid} &= \text{length} \times \text{breadth} \times \text{height}\\ &= 200 \times 70 \times 30\text{ cm}^3\\ &= 420\,000\text{ cm}^3\end{aligned}$$

b (We convert all the measurements to metres before finding the volume.)

$$\text{Breadth of cuboid} = 70\text{ cm} = \frac{70}{100}\text{ m} = 0.7\text{ m}$$

$$\text{Height of cuboid} = 30\text{ cm} = \frac{30}{100}\text{ m} = 0.3\text{ m}$$

$$\therefore \quad \begin{aligned}\text{Volume of cuboid} &= 2 \times 0.7 \times 0.3\text{ m}^3\\ &= 0.42\text{ m}^3\end{aligned}$$

Find the volumes of the following cuboids, giving your answers in the units stated in brackets:

	Length	Breadth	Height	
1	50 mm	30 mm	20 mm	(cm^3)
2	400 cm	100 cm	50 cm	(m^3)
3	1 m	4 cm	2 cm	(cm^3)
4	15 cm	80 mm	50 mm	(cm^3)
5	6 cm	12 mm	8 mm	(mm^3)
<u>6</u>	2 m	50 cm	40 mm	(cm^3)
<u>7</u>	4 cm	35 mm	2 cm	(cm^3)
<u>8</u>	20 m	80 cm	50 cm	(m^3)
<u>9</u>	3.5 cm	25 mm	20 mm	(cm^3)
<u>10</u>	$\frac{1}{2}$ m	45 mm	8 mm	(cm^3)

Problems involving cuboids

Exercise 19e

Find the volume of a rectangular block of wood measuring 8 cm by 6 cm which is 2 m long. Give your answer in cubic centimetres.

Working in centimetres:

$$\text{Length of block} = 2\text{ m} = 2 \times 100\text{ cm} = 200\text{ cm}$$

$$\begin{aligned}\text{Volume of block} &= \text{length} \times \text{breadth} \times \text{height}\\ &= 200 \times 8 \times 6\text{ cm}^3\\ &= 9600\text{ cm}^3\end{aligned}$$

A rectangular metal water tank is 3 m long, 2.5 m wide and 80 cm deep.

Find its capacity in **a** m^3 **b** litres.

To find the capacity of the tank you need to find the volume inside the tank.

a Working in metres:

$$\text{Depth of tank} = 80\text{ cm} = \frac{80}{100}\text{ m} = 0.8\text{ m}$$

$$\begin{aligned}\text{Capacity of tank} &= \text{length} \times \text{breadth} \times \text{height}\\ &= 3 \times 2.5 \times 0.8\text{ m}^3\\ &= 6\text{ m}^3\end{aligned}$$

b

$$1\text{ m}^3 = 1000\text{ litres}$$

$$\begin{aligned}\text{Capacity of tank} &= 6 \times 1000\text{ litres}\\ &= 6000\text{ litres}\end{aligned}$$

1 Find the volume of air in a room measuring 4 m by 5 m which is 3 m high.

2 Find the volume, in cm^3, of a concrete block measuring 36 cm by 18 cm by 12 cm.

3 Find the volume of a school hall which is 30 m long and 24 m wide if the ceiling is 9 m high.

4 An electric light bulb is sold in a box measuring 10 cm by 6 cm by 6 cm. If the shopkeeper receives them in a carton measuring 50 cm by 30 cm by 30 cm, how many bulbs would be packed in a carton?

5 A classroom is 10 m long, 8 m wide and 3 m high. How many pupils should it be used for if each pupil requires 5 m^3 of air space?

6 How many cubic metres of water are required to fill a rectangular swimming bath 15 m long and 10 m wide which is 2 m deep throughout? How many litres is this?

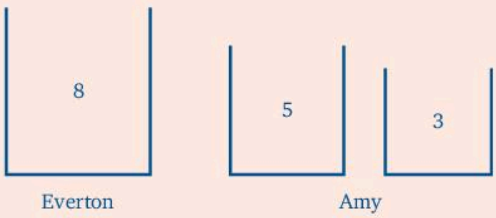
7 How many rectangular packets, measuring 8 cm by 6 cm by 4 cm, may be packed in a rectangular cardboard box measuring 30 cm by 24 cm by 16 cm?

8 A water storage tank is 3 m long, 2 m wide and $1\frac{1}{2}$ m deep. How many litres of water will it hold?

9 How many lead cubes of side 2 cm could be made from a lead cube of side 8 cm?

10 How many lead cubes of side 5 mm could be made from a rectangular block of lead measuring 10 cm by 5 cm by 4 cm?

Puzzle



Everton is returning from the local farm with an 8-pint can which is full of milk. He meets Amy who is going to the same farm for milk. She has an empty 5-pint can and an empty 3-pint can. Everton knows that there is no milk left at the farm, so being the kind boy he is, decides to share his milk equally with Amy. How do they do it using only the three containers they have?

Mixed exercises

Exercise 19f

1 Express $3.2\,m^3$ in

a cm^3

b mm^3.

2 Express 1.6 litres in cm^3.

3 Find the volume of a cube of side 4 cm.

4 Find the volume, in cm^3, of a cuboid measuring 2 m by 25 cm by 10 cm.

5 Find the volume, in mm^3, of a cuboid measuring 5 cm by 3 cm by 9 mm.

Exercise 19g

1 Express 8 cm^3 in

a mm^3 b m^3.

2 Express 3500 cm^3 in litres.

3 Find the volume of a cuboid measuring 10 cm by 5 cm by 6 cm.

4 Find, in cm^3, the volume of a cube of side 8 mm.

5 Find the volume, in cm^3, of a cuboid measuring 50 cm by 1.2 m by 20 cm.

Exercise 19h

1 Express 0.009 m^3 in

a cm^3 b mm^3.

2 Express 0.44 litres in cm^3.

3 Find the volume of a cube of side 6 cm.

4 Find the volume of a cuboid measuring 12 cm by 6 cm by 4 cm.

5 Find the capacity, in litres, of a rectangular tank measuring 2 m by 1.5 m by 80 cm.

Exercise 19i

1 Express 900 cm^3 in m^3.

2 Express 10 800 cm^3 in litres.

3 Express 0.075 m^3 in litres.

4 Find, in cm^3, the volume of a cube of side 20 mm.

5 Find, in m^3, the volume of a cuboid measuring 150 cm by 100 cm by 80 cm.

Did you know?

A Moebius strip is named after the German mathematician August Ferdinand Moebius. You can make one: take a strip of paper, twist it once and join the ends together. It has only one surface and only one edge (try it – draw a line along its surface – what happens?).

In this chapter you have seen that...

- ✔ volume and capacity are measures of space
- ✔ the volume of a cube or a cuboid is found by multiplying the length by the breadth by the height
- ✔ if the dimensions are given in different units you must change some of them so that they are all in the same unit before multiplying
- ✔ you should be familiar with the common metric units used to measure large and small volumes

  These are cubic millimetres (mm^3), cubic centimetres (cm^3), cubic metres (m^3), litres and millilitres (ml). The relationships between them are

$$1\,cm^3 = 10 \times 10 \times 10\,mm^3 = 1000\,mm^3$$

$$1\,m^3 = 100 \times 100 \times 100\,cm^3 = 1\,000\,000\,cm^3$$

$$1\text{ litre} = 1000\,cm^3 = 1000\,ml$$

REVIEW TEST 4: CHAPTERS 15–19

In questions **1** to **12** choose the letter for the correct answer.

1 If $U = \{2, 4, 6, 8, 10\}$ and $A = \{4, 8\}$, the complement of A is the set

A $\{2, 4, 6\}$ B $\{2, 6, 8\}$ C $\{4, 8, 10\}$ D $\{2, 6, 10\}$

2 If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and A', the complement of A, is the set $\{2, 4, 5, 7\}$, then A is the set

A $\{1, 2, 4, 6, 8\}$ C $\{1, 3, 5, 8\}$

B $\{3, 4, 5, 6, 7\}$ D $\{1, 3, 6, 8\}$

3 An article costing \$1600 is sold at a loss of 25%. The selling price of the article is

A \$2000 B \$1200 C \$400 D none of these

4 What type is the relation $\{(0, 0), (1, 5), (2, 6), (3, 5), (4, 6)\}$?

A $1:1$ B $1:n$ C $n:1$ D $n:n$

5 The domain and range of the relation $\{(2, 4), (4, 8), (8, 16)\}$ are the sets

A $\{2, 4, 8\}$ and $\{2, 4, 8, 16\}$

B $\{4, 8, 16\}$ and $\{2, 4, 8\}$

C $\{2, 4, 8\}$ and $\{4, 8, 16\}$

D $\{4, 8, 16\}$ and $\{2, 4, 8, 16\}$

6 A dehumidifier bought for \$12 500 is sold at a profit of 20%. The profit is

A \$2500 B \$250 C \$1500 D \$10 000

7 The points A(4, 6), B(8, 6) and C(8, 2) are the three corners of a square ABCD.

Mark the points A, B, C and D on your own set of axes.

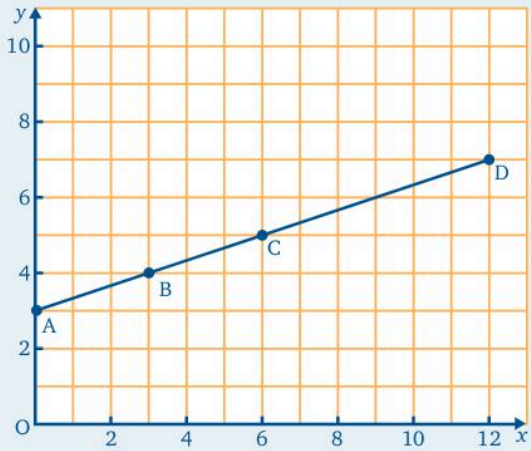
The coordinates of D are

A (4, 2) B (2, 4) C (4, 6) D (6, 4)

8 The points A, B, C and D illustrate a relation. The y-coordinate of each point is related to its x-coordinate by the equation

A $y = x + 3$ C $y = \frac{1}{3}x + 2$

B $y = \frac{1}{3}x + 1$ D $y = \frac{1}{3}x + 3$



9 Sara bought a set of garden tools for \$9500. When she sold them a year later she made a loss of 20%. What was the selling price?

A \$11 400 C \$8550

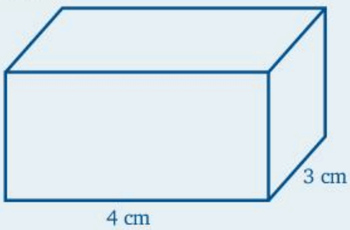
B \$9690 D \$7600

10 If US\$1 is equivalent to 180 Jamaican dollars, the equivalent of US\$550 in Jamaican dollars is

A \$990 B \$9900 C \$99 000 D \$990 000

11 The volume of this cuboid is $36\,\text{cm}^3$. The height is

A 12 cm B 6 cm C 4 cm D 3 cm



12 The capacity of a jug is 0.33 litres. This is equivalent to

A $33\,\text{cm}^3$ C $3300\,\text{cm}^3$

B $330\,\text{cm}^3$ D $33\,000\,\text{mm}^3$

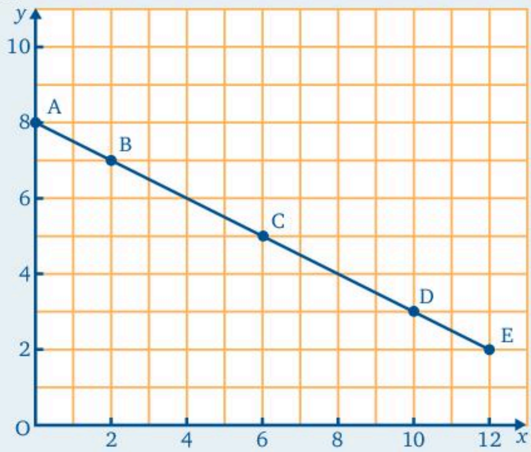
13 The points A, B, C, D and E illustrate a relation.

a Represent these points as a table.

b How is the y-coordinate of each point related to the x-coordinate?

c The points A, B, C, D and E all lie on the same straight line.
F is another point on the line. Its x-coordinate is 4.
What is its y-coordinate?

d G, H and I are also points on the line. Find the missing coordinates.
G(8,), H(5,) , I($,1\frac{1}{2}$)



14 U = {different letters in the word DODECAHEDRON}

P = {different letters in the word READER}

Q = {different letters in the word ARCHER}

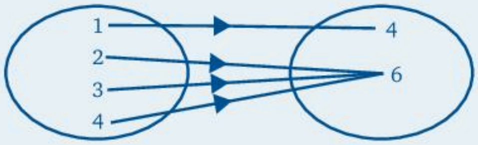
Show U, P and Q on a Venn diagram. Hence list the sets

a P' **b** Q' **c** $P' \cap Q'$ **d** $(P \cup Q)'$

15 Given A = {factors of 36} and B = {factors of 42}, list the members of

a $A \cap B$ **b** $A \cup B$

16



This diagram represents a relation. Write down the relation as a set of ordered pairs.

17 a Draw a mapping diagram to represent the relation {(2, 4), (5, 7), (8, 10), (12, 14)}.

b Represent the relation {(2, 5), (3, 6), (4, 7), (5, 8)} as a table of values of x and y.

18 A relation is given by $\{(x, y)\}$ where $y = 5 + 2x$ for $x = 2, 4, 6, 8$.

Copy and complete this table of values.

x	2	4	6	8
y		13		

19 A relation is given by $\{(x, y)\}$ where $y = x^2$ for $x = -2, -1, 0, 1, 2, 3$.

a Copy and complete this table of values.

x	–2	–1	0	1	2	3
y		1				

b Write down the domain and range.

c Represent the relation with an arrow diagram.

20 a If D = {p, q, r, s} and D' = {t, u, v}, what is U?

b Write the following statement in set notation:

Uranus is a member of the set of planets.

c Find the union of the sets A = {p, q, r, s} and B = {r, s, t, u}.

REVIEW TEST 5: CHAPTERS 1–19

In questions **1** to **12** choose the letter for the correct answer.

1 Given that 1 US dollar is equivalent to 0.75 pounds sterling and to 1.25 Canadian dollars, the equivalent value of £300 in Canadian dollars is

A $500 B $450 C $350 D $300

2 Given that 100 Jamaican dollars is equivalent to 0.74 US dollars, the equivalent value of US$450 in Jamaican dollars, correct to 3 s.f., is

A $3300 B $60 800 C $6080 D $60 900

3 The second number in each pair in the relation {(1, 2), (2, 4), (3 ,9), (5, 25)} is

A twice the first number
B the next even number
C the square of the first number
D the next odd number

4 How many lines of symmetry does a rectangle have?

A 1 B 2 C 3 D 4

5 Expressed in litres, 440 000 mm^3 is equivalent to

A 0.44 litres C 44 litres
B 4.4 litres D 440 litres

6 36 cm expressed as a percentage of 60 cm is

A 6% B 30% C 40% D 60%

7 Expressed as a percentage, correct to 1 d.p., $\frac{3}{7}$ is

A 40% B 42.9% C 42.8% D 4.29%

8 150 grams as a fraction of 1 kilogram is

A $\frac{1}{1000}$ B $\frac{3}{20}$ C $\frac{3}{20}$ D $\frac{15}{20}$

9 Given that $P = 2a + b$, the value of P when $a = 2\frac{1}{2}$ and $b = 4$ is

A 1 B $6\frac{1}{2}$ C 9 D 13

10 What percentage is 14 of 42?

A $\frac{1}{3}$% B 3% C $33\frac{1}{3}$% D 300%

11 The value of x that satisfies the equation $7 - (3 - 2x) = 10$ is

A 2 **B** 3 **C** 5 **D** 6

12 In a sale, all prices are reduced by 10%. The reduction on an item marked $2250 is

A $100 **B** $200 **C** $225 **D** $2250

13 Find the y-coordinates of points on the line $y = -2x$ that have x-coordinates

a 2 **b** -2 **c** $2\frac{1}{2}$ **d** 2.4

14 Find the x-coordinates of points on the line $y = 3x$ that have y-coordinates

a 3 **b** -6 **c** $4\frac{1}{2}$ **d** 3.6

15 Solve the following equations:

a $x - 10 = 4$

b $3x - 7 = 2$

c $3 - (x - 5) = 6$

d $3 + 2(x - 4) = 3(2x - 3)$

16 **a** I think of a number, treble it, and subtract 3. The answer is 12. What number did I think of?

b When shopping Mrs Smith spent $$x$ in the first shop, twice the amount in the second shop, $80 in the third shop and $500 in the last. The total amount she spent was $2680.
Form an equation in x and solve it to find how much she spent in the first shop.

17 **a** If n is an odd integer, what is the next odd integer above it?

b If you are m years old, what is your father's present age if he is 6 years more than three times your age?

c Solve the equation $3x - 9 = 2x - 2$.

18 **a** Arrange the following fractions in ascending order: $\frac{9}{20}, \frac{5}{8}, \frac{3}{5}, \frac{1}{2}$

b Find $0.001\,1144 \div 4$

19 The marks out of 10 in a maths test were

5, 7, 4, 5, 7, 2, 5, 3, 9, 10, 10, 5, 8, 4, 10, 1, 9, 7, 5, 8, 2, 4

a Make a frequency table to show these marks.

b Write down the modal mark.

20 A relation is given by $\{(x, y)\}$ where $y = x^2 - 2x$ for $x = 1, 2, 3, 4$.

a Copy and complete this table of values.

x	1	2	3	4
y		0		

b Write down the domain and range.

c Represent the relation with an arrow diagram.

d What type of relation is this?

21 A school hall measures 25 m by 20 m. It is to be covered with square floor tiles of side 50 cm.

a How many tiles are needed?

b The tiles are sold in packs of 20. How many packs must be bought?

c Each pack costs $6200. How much will it cost for the tiles?

22 A square and a rectangle have the same area. The side of the square is 6 cm and the length of the rectangle is 9 cm. How wide is the rectangle?

23 The rectangular base of an open cardboard box measures 14 cm by 10 cm.

The box is 8 cm deep and stands on a table. Calculate

a the surface area of the outside of the box that is visible

b the area of the inside of the box that is visible

c the capacity of the box.

24 U = {different letters in the word AUTOMOBILE}

A = {different letters in the word TOMATO}

B = {different letters in the word MOTEL}

Show U, A and B on a Venn diagram. Hence list the sets

a A' b B' c $A' \cup B'$ d $A' \cap B'$

Glossary

2D (two-dimensional)	describes a flat shape bounded by line segments
3D (three-dimensional)	describes an object that occupies three-dimensional space, for example a cube
acute	an angle whose size is less than 90°
addition	the operation that combines two or more numbers to find their sum
alternate angles	equal angles on opposite sides of a transversal, e.g. 
approximation	finding an estimate of the value of a calculation or quantity
area	the amount of surface covered
arithmetic average	the sum of a set of values divided by the number of values
associative	a property of addition and multiplication that means the way numbers are grouped does not change the result, e.g. $4 + (3 + 5) = (4 + 3) + 5$
axis (plural axes)	a fixed line from which the positions of points are measured, for example points on a graph
base	when using indices, the base is the number that is multiplied by itself the number of times shown by the index
base line	on a protractor, the straight line at or near the straight edge from which the angle is measured
bilateral symmetry	having a line of symmetry that divides an object into two equal halves
bisect	divide into two equal parts
capacity	the amount of space inside a container (three-dimensional space)
Cartesian coordinate system	a system that gives the position of a point in two-dimensions by stating its shortest distance from two fixed reference lines at right angles to each other
Cartesian plane	the plane containing the x and y axes
centimetre	a metric measure of length
centre (of a circle)	the point that is the same distance from any point on the circumference
centre of rotation	the point about which a shape is rotated
circle	a curve made by moving one point at a fixed distance from another
circumference	the edge of a circle
coefficient	the number multiplied by a variable
common denominator	a denominator that is divisible by all of the original denominators in a calculation involving fractions
common factor	a number that divides exactly into two or more other numbers
commutative	a property of addition and multiplication that means the order of the numbers does not change the result, e.g. $7 \times 6 = 6 \times 7$
compasses (a pair of)	an instrument used for drawing circles or parts of circles
complement of a set	the members of the universal set not included in the given set
composite number	a number that can be written as the product of two or more prime numbers

coordinates an ordered pair of numbers giving the position of a point on a grid

corresponding sides a pair of matching sides in two different shapes

cost price price before profit is added

cube a solid with six faces, each of which is a square, e.g. 

cubic unit a measure of volume

cuboid a solid with six faces, each of which is a rectangle, e.g. 

currency the money used by a country

data a collection of facts or figures

decimal a fraction expressed by numbers on the right of a point, e.g. $0.2 = \frac{2}{10}$

degrees Celsius a unit for measuring temperature; the boiling point of water is 100 degrees Celsius

degrees Fahrenheit a unit for measuring temperature; the boiling point of water is 212 degrees Fahrenheit

denominator the bottom of a fraction

diameter a straight line through the centre of a circle from one point on the circumference to the opposite point on the circumference

directed number the collective name for positive and negative numbers

directly proportional if two varying quantities are directly proportional, they are always in the same ratio

disjoint sets sets that have no common elements

distributive the property that means that one operation can be distributed over another, e.g. $3 \times (4 + 5) = 3 \times 4 + 3 \times 5$

divisibility rules a set of rules that use patterns in digits of numbers to decide if a given number has certain prime numbers as factors

divisible divides exactly

division the operation that undoes multiplication, splitting the number being divided into equal parts

domain the set of the first objects in the set of the ordered pairs of a relation

edge where two faces meet

element a member of a set

empty set a set with no members, shown by the symbol $\varnothing$ or by $\{\ \}$

equal set sets are equal when they contain identical members

equation two expressions connected by an equals sign

equilateral triangle a triangle whose sides are all the same length

equivalent fraction measures the same part of a quantity

equivalent set sets are equivalent when they contain the same number of elements; the elements in equivalent sets are not usually the same

even number a number that is divisible by 2

exchange rate the number of units of one currency equal to one unit of another

expression	a collection of algebraic terms connected with plus and minus signs, without an equals or inequality sign
face	a surface on a solid
factor	a number or letter that divides exactly into another number or algebraic expression
finite set	a set whose members are limited in number
foot	an imperial measure of length
fraction	part of a quantity
frequency	the number of times that a value or group of values occurs
frequency table	a table listing the number of each quantity or group of quantities
gram	a metric measure of mass
hexagon	a six-sided polygon
highest common factor (HCF)	the largest number that divides exactly into two or more other numbers
hundredweight	an imperial measure of mass
identity element	when the identity element for an operation is combined with a number using that operation, the result is the original number; for addition the identity element is 0, since $4 + 0 = 4$
image	a shape after it is reflected or translated
improper fraction	a fraction whose numerator is larger than its denominator
inch	an imperial measure of length
index (plural indices)	a superscript to a number that tells you how many of those numbers are multiplied together
infinite set	a set with an unlimited number of members
integer	a positive or negative whole number
intersection of sets	the set of elements common to two or more sets, e.g. $A \cap B$ is the set of members in both A and B
inverse element	combining an element with its inverse element under an operation 'undoes' the operation, e.g. adding 6 and -6 to any number leaves the number unchanged, so -6 is the inverse of 6 under addition
kilogram	a metric measure of mass
kilometre	a metric measure of length
kite	a quadrilateral with two pairs of adjacent sides that are equal, e.g. 
like terms	terms that contain the same combination of letters, e.g. $3xy^2$ and $7xy^2$ are like terms but xy^2 and xy are not
line	a straight or curved extent of length without thickness, start or end
line of symmetry	a line through a figure such that the parts of the figure on each side of the line are identical
line segment	a line with a beginning and an end

line symmetry	a shape has line symmetry if it is possible to fold a drawing of the shape along a line so that one half fits exactly over the other half
litre	a metric measure of capacity
loss	the amount lost by a seller when an item is sold for less than its cost price
lowest common multiple (LCM)	the lowest number that two or more other numbers divide into exactly
lowest terms	a fraction is in its lowest terms when there are no common factors between the numerator and denominator
map ratio	the ratio between the distance of two points on a map and the corresponding two points on the ground
mapping	describes the relation of the first and second elements in an ordered pair
mass	the quantity of matter in an object
mean	the sum of a set of values divided by the number of values
median	the middle item of a set of items arranged in order of size
member	an item that belongs to a set of items; the symbol $\in$ means 'is a member of' and the symbol $\notin$ means 'is not a member of' a particular set
metre	a metric measure of length
metric units	a system of units for measuring and weighing based on the metre and the kilogram; each size of unit is related to the next by a power of 10
midpoint	halfway between the two given points on a straight line
mile	an imperial measure of length
milligram	a metric measure of mass
millilitre	a metric measure of capacity
millimetre	a metric measure of length
mirror line	the line in which an object is reflected to give its image
mixed number	the sum of a whole number and a fraction
mixed operation	a calculation involving two or more of addition, subtraction, multiplication and division
mode	the most frequent item in a set
multiple	a particular number multiplied by any other number is a multiple of that particular number
multiplication	the operation where a number is added to itself a specified number of times to give a product
natural number	a counting number, i.e. 1, 2, 3, 4, …
negative number	a number less than zero
net	a flat shape that can be folded to make a solid
non-commutative	an operation is non-commutative if the order in which the numbers appear changes the result

null set	a set with no members, also called an empty set
number line	a straight line on which points are marked at equal intervals, representing the values of positive and negative numbers by their distance and direction from the origin
numerator	the top of a fraction
object	a shape before it is reflected or translated
obtuse	an angle whose size is between 90° and 180°
odd number	a whole number that is not divisible by 2
order of rotational symmetry	the number of different positions in which an object looks the same when rotated about a fixed point
ordered pair	a pair of objects in a defined order
ounce	an imperial measure of mass
parallel	two lines that are always the same distance apart
parallelogram	a four-sided figure whose opposite sides are parallel
pentagon	a five-sided polygon
percentage	out of a hundred, i.e. a fraction whose denominator is 100
perimeter	the total distance round the edge of a figure
perpendicular	at right angles to a line or surface
perpendicular bisector	a line at right angles to a line segment that divides the line segment into two equal parts
place value	the position of a digit in a number that shows its value, e.g. in 247, the digit 4 has a place value of 4 tens
plane figure	a closed shape made by lines drawn on a surface
point	a mark on a position that has no width, length or depth, usually represented by a small dot
polygon	a plane figure drawn with three or more straight lines
positive number	a number greater than zero
pound	an imperial measure of mass
prime number	a number whose only factors are 1 and itself (1 is not a prime number)
prism	a solid with two identical ends and flat faces between them
product	the result of multiplying two or more numbers together
profit	the extra amount gained by a seller when an item is sold for more than its cost price
proper fraction	a fraction whose numerator is less than its denominator
proper subset	a set that contains some, but not all, of the elements in another set
protractor	an instrument for measuring angles
pyramid	a solid with a polygon as its base and sloping sides that meet at a point
quadrilateral	a figure bounded by four straight lines

radius	the distance from the centre of a circle to the edge
range (of data)	the difference between the largest and the smallest values in a set of data
range (of a relation)	the set of the second objects in the ordered pairs in a relation
ratio	the comparison between the sizes of two quantities
rational number	a fraction whose numerator and denominator are integers
ray	a line with one end point
rectangle	a quadrilateral whose angles are each 90°
rectangular number	a number that can be shown as a rectangular array of dots
rectilinear solid	a solid whose edges are straight lines and faces are flat
recurring decimal	a decimal that never terminates but where the digits form a repeating pattern, e.g. 0.191919…
reflection	a transformation in which any two corresponding points in the object and the image are both the same distance from a fixed straight line
regular pentagon	a five-sided figure whose sides are all the same length
regular polygon	a polygon that has all angles equal and all sides of equal length
relation	a set of ordered pairs with a rule that connects the objects in each pair
relative size	the size of an object in relation or in proportion to something else
representative fraction	the scale of a map
rhombus	a four-sided figure whose sides are all the same length, e.g. ◊
right angle	one quarter of a revolution (90°)
rotation	a transformation in which an object is turned about a point called the centre of rotation
rotational symmetry	a figure has rotational symmetry when it can be turned about a point to another position and still look the same
selling price	the price at which an item is sold, which is often more than the cost price
set	a collection of items having something in common
significant figure	position of a figure in a number, e.g. in 2731 the third significant figure is 3
simplify a fraction	reduce the size of the numerator and denominator by dividing them by common factors
solid	an object that has length, breadth and depth, i.e. occupying three-dimensional space
solve	find the correct answer
square	a four-sided figure whose sides are all the same length and each of whose angles is a right angle
square centimetre	a metric measure of area
square kilometre	a metric measure of area
square metre	a metric measure of area
square millimetre	a metric measure of area

square number a number that can be shown as a square array of dots

subset a set whose members are also members of another set; in symbols $A \subset B$ means 'A is a subset of B'

subtraction the operation that tells us the difference between two numbers; subtraction is the inverse operation to addition

symmetrical a shape that has line symmetry

table of values a table giving corresponding values to two or more variables

tally a record (i.e. a mark) of an amount or a score

tessellation an arrangement of flat shapes that cover a flat surface

ton an imperial measure of mass

tonne a metric measure of mass

translation a movement of an object without turning it or reflecting it

trapezium a four-sided figure with one pair of unequal sides parallel

triangle a three-sided figure

triangular number a number that can be shown as a triangular array of dots, e.g. 6: 

twin primes the name for two prime numbers that differ by two

union of sets the set containing all the different elements of two or more sets, e.g. $A \cup B$ is the set of all members of A and B

universal set the set containing all elements, shown by the symbol U

unlike terms terms containing different combinations of letters

variable a quantity that can vary in value

Venn diagram a diagram used to show the elements in two or more sets

vertex (plural vertices) corner

volume the amount of three-dimensional space occupied by a solid

whole number any natural number (including zero)

yard an imperial measure of length

Answers

CHAPTER 1

Exercise 1a page 3

1 associative for multiplication
2 associative for addition
3 commutative
4 distributive
5 commutative
6 associative for multiplication
7 distributive
8 distributive
9 commutative
10 distributive
11 identity for addition
12 identity for multiplication
13 inverse of 10 under addition
14 −9
15 divide by 9
16 +5
17 A
18 C
19 B

Exercise 1b page 4

1	156 000	**5**	261 936 000
2	520 200	**6**	3 232 000
3	2 196 000	**7**	31 800 000
4	73 640 700	**8**	46 550 000 000

Exercise 1c page 5

1 **a** tens **c** units **e** ten thousands
b hundreds **d** thousands
2 **a** 534, 8451, 8876, 10 880
b 43 624, 734 921, 933 402, 2 000 843
3 **a** 3087 **b** 9826 **c** 211 005
4 $32 600.00
5 **a** 406 **c** 169 824 **e** 7707
b 218 **d** 22 123 **f** 458 018
6 99 704 599
7 **a** 5 **b** 9 **c** 9
8 **a** 17 **b** 5 **c** 43 **d** 10 740
9 **a** 324 **e** 180 000 **i** 22 802 000
b 1224 **f** 4 938 500 **j** 1620
c 3264 **g** 493 150 000 **k** 10 200
d 32 328 **h** 94 500 000 **l** 35 840
10 **a** 35 100 **d** 84 254 **g** 204 940 120
b 38 920 **e** 67 624 **h** 224 858 480
c 243 000 **f** 2 763 180 **i** 1 248 425 280
11 **a** 13 **e** 114 r2 **i** 2 668 200 r2
b 12 r1 **f** 235 r1 **j** 12 100 140
c 6 r6 **g** 843 000 **k** 900 900
d 74 r7 **h** 1 572 701 r1 **l** 350 001
12 **a** 221 r0 **e** 202 r22 **i** 2 r33
b 135 r24 **f** 89 r24 **j** 111 r5
c 236 r0 **g** 77 r9 **k** 90 r30
d 64 r8 **h** 469 r1 **l** 200

Exercise 1d page 7

1	80	**6**	800	**11**	31 000	**16**	46 000 000	**21**	280	**26**	360
2	150	**7**	300	**12**	876 000	**17**	7 000 000	**22**	370	**27**	250
3	630	**8**	1200	**13**	710 000	**18**	35 000 000	**23**	150	**28**	10
4	230	**9**	1400	**14**	980 000	**19**	94 000 000	**24**	250		
5	160	**10**	3800	**15**	267 000	**20**	330	**25**	250		

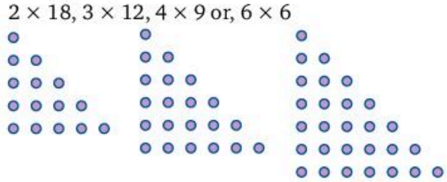
Exercise 1e page 8

1	22	**11**	8	**21**	21
2	7	**12**	22	**22**	14
3	21	**13**	13	**23**	12
4	17	**14**	17	**24**	13
5	2	**15**	6	**25**	32
6	5	**16**	8	**26**	9
7	1	**17**	10	**27**	16
8	10	**18**	8	**28**	14
9	3	**19**	5	**29**	14
10	6	**20**	9	**30**	30

Exercise 1f page 9

1	2	**7**	49	**13**	17	**19**	4	**25**	10
2	56	**8**	2	**14**	2	**20**	36	**26**	1
3	9	**9**	45	**15**	11	**21**	45	**27**	4
4	14	**10**	2	**16**	7	**22**	6	**28**	25
5	15	**11**	17	**17**	30	**23**	14	**29**	1
6	8	**12**	3	**18**	1	**24**	0	**30**	18

Exercise 1g page 11

1 $1 + 3 + 5 + 7 + 9 = 25 = 5 \times 5$
$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6 \times 6$
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7 \times 7$
a 64 **b** 400
2 $2 + 4 + 6 + 8 + 10 = 30 = 5 \times 6$
$2 + 4 + 6 + 8 + 10 + 12 = 42 = 6 \times 7$
$2 + 4 + 6 + 8 + 10 + 12 + 14 = 56 = 7 \times 8$
12
3 4, 9, 36, 169
4 8, 6, 14, 72, 91, 323 (17×19), 403 (13×31)
5 $2 \times 6, 3 \times 4$
6 $2 \times 9, 3 \times 6$
7 $2 \times 18, 3 \times 12, 4 \times 9$ or, 6×6
8

9 36, 45, 55
10 1, 4, 9, 16; square numbers
11 triangular numbers
12 **a** 1, 4, 9
b 4, 6, 8, 9, 10, 12
c 1, 3, 6, 10
13 **a** 25, 36
b 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40
c 28, 36

Exercise 1h page 13

1 $1 \times 18, 2 \times 9, 3 \times 6$
2 $1 \times 20, 2 \times 10, 4 \times 5$
3 $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6$
4 $1 \times 27, 3 \times 9$

5 $1 \times 30, 2 \times 15, 3 \times 10, 5 \times 6$
6 $1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9, 6 \times 6$
7 $1 \times 40, 2 \times 20, 4 \times 10, 5 \times 8$
8 $1 \times 45, 3 \times 15, 5 \times 9$
9 $1 \times 48, 2 \times 24, 3 \times 16, 4 \times 12, 6 \times 8$
10 $1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$
11 $1 \times 64, 2 \times 32, 4 \times 16, 8 \times 8$
12 $1 \times 72, 2 \times 36, 3 \times 24, 4 \times 18, 6 \times 12, 8 \times 9$
13 $1 \times 80, 2 \times 40, 4 \times 20, 5 \times 16, 8 \times 10$
14 $1 \times 96, 2 \times 48, 3 \times 32, 4 \times 24, 6 \times 16, 8 \times 12$
15 $1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20, 10 \times 10$
16 $1 \times 108, 2 \times 54, 3 \times 36, 4 \times 27, 6 \times 18, 9 \times 12$
17 $1 \times 120, 2 \times 60, 3 \times 40, 4 \times 30, 5 \times 24, 6 \times 20, 8 \times 15, 10 \times 12$
18 $1 \times 135, 3 \times 45, 5 \times 27, 9 \times 15$
19 $1 \times 144, 2 \times 72, 3 \times 48, 4 \times 36, 6 \times 24, 8 \times 18, 9 \times 16, 12 \times 12$
20 $1 \times 160, 2 \times 80, 4 \times 40, 5 \times 32, 8 \times 20, 10 \times 16$

Exercise 1i page 14

1 1, 2, 3, 6, 9, 18
2 1, 2, 4, 5, 10, 20
3 1, 2, 3, 4, 6, 8, 12, 24
4 1, 3, 9, 27
5 1, 2, 3, 5, 6, 10, 15, 30
6 1, 2, 3, 4, 6, 9, 12, 18, 36
7 1, 2, 4, 5, 8, 10, 20, 40
8 1, 3, 5, 9, 15, 45
9 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
10 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
11 1, 2, 4, 8, 16, 32, 64
12 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
13 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
14 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
15 1, 2, 4, 5, 10, 20, 25, 50, 100
16 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
17 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
18 1, 3, 5, 9, 15, 27, 45, 135
19 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
20 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160

Exercise 1j page 14

1 21, 24, 27, 30, 33, 36, 39
2 20, 25, 30, 35, 40, 45
3 28, 35, 42, 49, 56
4 55, 66, 77, 88, 99
5 26, 39, 52, 65

Exercise 1k page 15

1 2, 3, 5, 7, 11, 13
2 23, 29
3 31, 37, 41, 43, 47
4 5, 19, 29, 61
5 41, 101, 127
6 **a** F **b** F **c** T **d** T **e** F

Exercise 1l page 17

1	2^3	**9**	2^7	**17**	81	**25**	2^3
2	3^4	**10**	6^4	**18**	16	**26**	3^3
3	5^4	**11**	32	**19**	100	**27**	7^2
4	7^5	**12**	27	**20**	1000	**28**	5^2
5	2^5	**13**	25	**21**	10000	**29**	2^5
6	3^6	**14**	8	**22**	10	**30**	2^6
7	13^3	**15**	9	**23**	2^2		
8	19^2	**16**	49	**24**	3^2		

Exercise 1m page 18

1 **a** 6×10^1 **b** 6×10^2 **c** 6×10^0 **d** 6×10^3
2 **a** 300 **b** 2000 **c** 205 **d** 30060

3	$2^2 \times 7^2$	**8**	$2^2 \times 3 \times 11^2$
4	$3^3 \times 5^2$	**9**	$3^2 \times 5 \times 7^4$
5	$5^3 \times 13^2$	**10**	$5^2 \times 13^3$
6	$2^2 \times 3^2 \times 5^2$	**11**	$3^3 \times 5^2 \times 7^2$
7	$2^3 \times 3^2 \times 5^2$	**12**	$2^2 \times 3^2 \times 5^2$

13	108	**16**	36	**19**	153
14	225	**17**	180	**20**	371
15	112	**18**	126	**21**	370

Exercise 1n page 19

1	yes	**5**	no	**9**	yes
2	no	**6**	yes	**10**	yes
3	yes	**7**	yes	**11**	yes
4	yes	**8**	no		

Exercise 1p page 21

1	$2^3 \times 3$	**6**	$2^2 \times 3 \times 7$
2	$2^2 \times 7$	**7**	$2^3 \times 3^3$
3	$3^2 \times 7$	**8**	$2^4 \times 3 \times 11$
4	$2^3 \times 3^2$	**9**	$3^4 \times 5$
5	$2^3 \times 17$	**10**	$2^4 \times 7^2$

11 $20 = 3 + 17; 7 + 13$
$22 = 3 + 19; 5 + 17; 11 + 11$
$24 = 5 + 19; 7 + 17; 11 + 13$
$26 = 3 + 23; 7 + 19; 13 + 13$
$28 = 5 + 23; 11 + 17$
$30 = 7 + 23; 11 + 19; 13 + 17$

Exercise 1q page 21

1	3	**4**	14	**7**	21	**10**	4
2	8	**5**	25	**8**	13	**11**	15
3	12	**6**	11	**9**	5	**12**	2

Exercise 1r page 22

1	15	**4**	36	**7**	48	**10**	108
2	24	**5**	36	**8**	60	**11**	36
3	15	**6**	60	**9**	36	**12**	168

Exercise 1s page 22

1 **a** 15 **b** 8
2 **a** 4 **b** 18
3 **a** 27 **b** 48
4 **a** 21 **b** 39
5 **a** 180 **b** 576
6 **a** 168 **b** 1352
7 **a** 1452 **b** 432
8 **a** 5 **b** 420

Exercise 1t page 23

1	50 cm	**4**	50 cm	**7**	480, 20
2	$1080	**5**	78 s	**8**	18
3	120 m	**6**	30 steps; 2		

Exercise 1u page 25

1	6949	**5**	85	**9**	6
2	465578	**6**	4591000	**10**	12
3	5472	**7**	680	**11**	10
4	9	**8**	76		

12 **a** 761 **b** 1408

Exercise 1v page 26

1	D	**3**	D	**5**	C	**7**	C
2	D	**4**	B	**6**	D		

CHAPTER 2

Exercise 2a page 30

1	+10°	**9**	4° above	**17**	2°
2	−7	**10**	10° below	**18**	−2°
3	−3°	**11**	8° above	**19**	1°
4	+5°	**12**	freezing point	**20**	3°
5	−8°	**13**	10°	**21**	−7°
6	0°	**14**	12°	**22**	−2°
7	2° below	**15**	4°		
8	3° above	**16**	−3°		

23 A, B, C, D, G above sea level; E at sea level; F, H below sea level

24	−5 s	**30**	−$5000	**36**	+21 °C
25	+5 s	**31**	+5 paces	**37**	+150 m
26	+$500	**32**	−5 paces	**38**	−3 °C
27	−$500	**33**	+200 m	**39**	+$25
28	−1 min	**34**	−5 m	**40**	6 paces in front
29	+$50 000	**35**	−3 °C		

Exercise 2b page 33

1	$>$	**9**	$>$	**17**	0, −3
2	$>$	**10**	$<$	**18**	5, 8
3	$>$	**11**	$<$	**19**	−7, −11
4	$<$	**12**	$>$	**20**	16, 32
5	$>$	**13**	10, 12	**21**	$\frac{1}{6}, \frac{1}{36}$
6	$<$	**14**	−10, −12	**22**	−4, −2
7	$>$	**15**	−2, −4	**23**	−8, −16
8	$>$	**16**	2, 4	**24**	−2, −3

Exercise 2c page 34

1	−3	**9**	−12	**17**	2	**25**	4
2	3	**10**	−1	**18**	−3	**26**	6
3	−2	**11**	5	**19**	−3	**27**	3
4	−2	**12**	−2	**20**	−1	**28**	0
5	2	**13**	−2	**21**	3	**29**	−3
6	7	**14**	−1	**22**	−6	**30**	−5
7	1	**15**	4	**23**	−10		
8	2	**16**	6	**24**	−5		

Exercise 2d page 35

1	2	**8**	6	**15**	−6	**22**	13
2	−3	**9**	−14	**16**	7	**23**	−6
3	7	**10**	10	**17**	−3	**24**	8
4	3	**11**	−14	**18**	2	**25**	1
5	−9	**12**	0	**19**	−4		
6	3	**13**	0	**20**	5		
7	−3	**14**	6	**21**	13		

Exercise 2e page 36

1	1	**9**	15	**17**	−1	**25**	−4
2	−5	**10**	2	**18**	0	**26**	−4
3	9	**11**	5	**19**	2	**27**	4
4	8	**12**	−12	**20**	16	**28**	−3
5	2	**13**	5	**21**	5	**29**	−3
6	7	**14**	−9	**22**	−4	**30**	−19
7	4	**15**	1	**23**	−8	**31**	2
8	10	**16**	9	**24**	19	**32**	3

33	0	**39**	−4	**45**	2	**51**	2
34	0	**40**	3	**46**	−12	**52**	−15
35	−1	**41**	−10	**47**	3	**53**	−9
36	0	**42**	−3	**48**	18	**54**	−6
37	9	**43**	−2	**49**	−2	**55**	−8
38	−7	**44**	1	**50**	1		

Exercise 2f page 38

1	−15	**9**	−5	**17**	−24	**25**	−24
2	−8	**10**	+18	**18**	+8	**26**	−24
3	+14	**11**	+27	**19**	+3	**27**	+45
4	+4	**12**	−16	**20**	−8	**28**	−20
5	−42	**13**	−35	**21**	−6	**29**	−28
6	+12	**14**	+24	**22**	+15	**30**	+36
7	−18	**15**	−15	**23**	−18		
8	+16	**16**	−45	**24**	+20		

Exercise 2g page 39

1	−2	**8**	1	**15**	$-4\frac{2}{5}$	**22**	36	**29**	4
2	5	**9**	−3	**16**	−2	**23**	−9	**30**	$-\frac{1}{4}$
3	−4	**10**	−5	**17**	−2	**24**	5	**31**	−4
4	2	**11**	$-1\frac{1}{4}$	**18**	−9	**25**	−10	**32**	$-1\frac{1}{2}$
5	−1	**12**	$-1\frac{2}{3}$	**19**	3	**26**	0	**33**	$1\frac{1}{4}$
6	4	**13**	$3\frac{3}{5}$	**20**	$-1\frac{1}{8}$	**27**	0	**34**	36
7	−3	**14**	−1	**21**	−2	**28**	13	**35**	−25

Exercise 2h page 41

1	−5°	**6**	4	**11**	10
2	$-3 < 2, -2 > -4$	**7**	0	**12**	−14
3	2	**8**	5	**13**	−24
4	−5	**9**	−24	**14**	7
5	−2	**10**	−12		

Exercise 2i page 41

1	−2	**5**	5	**9**	−3
2	50	**6**	−18	**10**	−36
3	$\frac{1}{2}$	**7**	2	**11**	$\frac{1}{2}$
4	−1	**8**	$1\frac{1}{3}$	**12**	$30\frac{1}{2}$

CHAPTER 3

Exercise 3a page 44

1	$\frac{1}{6}$	**4**	$\frac{1}{4}$	**7**	$\frac{3}{10}$	**10**	$\frac{2}{6}$
2	$\frac{5}{6}$	**5**	$\frac{3}{4}$	**8**	$\frac{1}{4}$	**11**	$\frac{4}{8}$
3	$\frac{7}{10}$	**6**	$\frac{1}{2}$	**9**	$\frac{3}{7}$		

Exercise 3b page 46

1	36	**7**	18	**13**	1000
2	18	**8**	30	**14**	90
3	4	**9**	10	**15**	8000
4	15	**10**	10	**16**	55
5	12	**11**	100	**17**	500
6	100	**12**	8	**18**	10 000

19 **a** $\frac{12}{24}$ **b** $\frac{8}{24}$ **c** $\frac{4}{24}$ **d** $\frac{18}{24}$ **e** $\frac{10}{24}$ **f** $\frac{9}{24}$

20 **a** $\frac{6}{45}$ **b** $\frac{20}{45}$ **c** $\frac{27}{45}$ **d** $\frac{15}{45}$ **e** $\frac{42}{45}$ **f** $\frac{9}{45}$

21 a $\frac{12}{72}$ b $\frac{12}{16}$ c $\frac{12}{14}$ d $\frac{12}{15}$ e $\frac{12}{18}$ f $\frac{12}{24}$

22 b $\frac{2}{3}=\frac{6}{9}$ e $\frac{7}{10}=\frac{70}{100}$

Exercise 3c page 48

1 $\frac{1}{2}$
2 $\frac{5}{6}$
3 $\frac{4}{5}$
4 $\frac{2}{9}$
5 $\frac{3}{8}$
6 $\frac{3}{7}$
7 $\frac{5}{6}$
8 $\frac{3}{8}$
9 $\frac{6}{7}$
10 $\frac{3}{5}$
11 $\frac{3}{11}$
12 $\frac{5}{7}$
13 $\frac{5}{11}$
14 $\frac{4}{11}$
15 $\frac{2}{7}$
16 $\frac{3}{11}$
17 $\frac{7}{9}$
18 $\frac{9}{11}$
19 $\frac{2}{5}$
20 $\frac{3}{5}$
21 $<$
22 $>$
23 $<$
24 $<$
25 $>$
26 $<$
27 $>$
28 $<$
29 $>$
30 $<$
31 $\frac{7}{30}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}$
32 $\frac{4}{10}, \frac{5}{8}, \frac{13}{20}, \frac{3}{4}$
33 $\frac{1}{3}, \frac{1}{2}, \frac{7}{12}, \frac{5}{6}$
34 $\frac{3}{8}, \frac{2}{5}, \frac{1}{2}, \frac{7}{10}, \frac{17}{20}$
35 $\frac{1}{2}, \frac{17}{28}, \frac{5}{7}, \frac{3}{4}, \frac{11}{14}$
36 $\frac{2}{5}, \frac{1}{2}, \frac{14}{25}, \frac{3}{5}, \frac{7}{10}$
37 $\frac{5}{6}, \frac{7}{9}, \frac{2}{3}, \frac{11}{18}, \frac{1}{2}$
38 $\frac{3}{4}, \frac{7}{10}, \frac{13}{20}, \frac{3}{5}, \frac{1}{2}$
39 $\frac{3}{4}, \frac{17}{24}, \frac{2}{3}, \frac{7}{12}, \frac{1}{6}$
40 $\frac{4}{5}, \frac{23}{30}, \frac{11}{15}, \frac{7}{10}, \frac{2}{3}$
41 $\frac{3}{4}, \frac{5}{8}, \frac{19}{32}, \frac{1}{2}, \frac{7}{16}$
42 $\frac{5}{6}, \frac{4}{5}, \frac{3}{4}, \frac{7}{12}, \frac{1}{2}$

Exercise 3d page 51

1 $\frac{1}{3}$
2 $\frac{3}{5}$
3 $\frac{1}{3}$
4 $\frac{1}{2}$
5 $\frac{2}{3}$
6 $\frac{1}{2}$
7 $\frac{1}{4}$
8 $\frac{2}{7}$
9 $\frac{2}{7}$
10 $\frac{1}{3}$
11 $\frac{1}{2}$
12 $\frac{1}{5}$

Exercise 3e page 51

1 $\frac{3}{5}$
2 $\frac{4}{7}$
3 $\frac{11}{30}$
4 $\frac{29}{35}$
5 $\frac{29}{30}$
6 $\frac{39}{56}$
7 $\frac{25}{42}$
8 $\frac{20}{21}$
9 $\frac{19}{42}$
10 $\frac{41}{42}$
11 $\frac{82}{99}$
12 $\frac{47}{90}$
13 $\frac{19}{20}$
14 $\frac{17}{24}$
15 $\frac{19}{20}$
16 $\frac{11}{12}$
17 $\frac{6}{7}$
18 1
19 $\frac{39}{40}$
20 $\frac{13}{18}$
21 $\frac{17}{20}$
22 $\frac{17}{18}$
23 $\frac{19}{30}$
24 $\frac{2}{3}$
25 $\frac{1}{2}$
26 $\frac{5}{17}$
27 $\frac{11}{20}$
28 $\frac{2}{5}$
29 $\frac{3}{7}$
30 $\frac{3}{5}$
31 $\frac{5}{21}$
32 $\frac{5}{21}$
33 $\frac{7}{15}$
34 $\frac{1}{3}$
35 $\frac{1}{9}$
36 $\frac{3}{26}$
37 $\frac{1}{12}$
38 $\frac{9}{100}$
39 $\frac{19}{56}$
40 $\frac{4}{15}$
41 $\frac{1}{8}$
42 $\frac{1}{4}$
43 $\frac{1}{6}$
44 $\frac{4}{15}$

Exercise 3f page 53

1 $\frac{3}{8}$
2 $\frac{5}{7}$
3 $\frac{1}{16}$
4 $\frac{5}{12}$
5 $\frac{9}{50}$
6 $\frac{5}{12}$
7 $\frac{3}{5}$
8 $\frac{17}{18}$
9 $\frac{17}{50}$
10 $\frac{1}{2}$
11 $\frac{3}{4}$
12 $\frac{1}{2}$
13 $\frac{1}{18}$
14 $\frac{1}{12}$
15 $\frac{1}{5}$
16 $\frac{1}{16}$
17 $\frac{2}{9}$
18 $\frac{7}{20}$
19 $\frac{1}{8}$
20 $\frac{1}{3}$
21 $\frac{19}{100}$
22 $\frac{1}{4}$
23 $\frac{5}{18}$
24 $\frac{1}{30}$

Exercise 3g page 55

1 $2\frac{1}{4}$
2 $4\frac{3}{4}$
3 $6\frac{1}{6}$
4 $5\frac{3}{10}$
5 $5\frac{1}{8}$
6 $25\frac{2}{5}$
7 $10\frac{4}{11}$
8 $13\frac{5}{8}$

Exercise 3h page 56

1 $\frac{13}{3}$
2 $\frac{33}{4}$
3 $\frac{17}{10}$
4 $\frac{98}{9}$
5 $\frac{57}{7}$
6 $\frac{33}{5}$
7 $\frac{20}{7}$
8 $\frac{25}{6}$
9 $\frac{11}{3}$
10 $\frac{11}{2}$
11 $\frac{37}{5}$
12 $\frac{22}{9}$
13 $\frac{19}{5}$
14 $\frac{43}{9}$
15 $\frac{35}{4}$
16 $\frac{73}{7}$
17 $\frac{19}{10}$
18 $\frac{20}{3}$
19 $\frac{59}{8}$
20 $\frac{101}{10}$

Exercise 3i page 57

1 $5\frac{1}{7}$
2 $9\frac{5}{6}$
3 $4\frac{8}{11}$
4 $2\frac{1}{2}$
5 $16\frac{2}{5}$
6 $7\frac{1}{4}$
7 $13\frac{2}{3}$
8 $7\frac{1}{9}$
9 $8\frac{1}{6}$
10 $10\frac{7}{10}$
11 $7\frac{2}{5}$
12 $6\frac{1}{2}$

Exercise 3j page 57

1 $5\frac{3}{4}$
2 $3\frac{5}{6}$
3 $5\frac{23}{40}$
4 $9\frac{4}{9}$
5 $5\frac{29}{36}$
6 $4\frac{1}{6}$
7 $4\frac{9}{20}$
8 $3\frac{3}{14}$
9 $7\frac{7}{10}$
10 $13\frac{17}{21}$
11 $10\frac{13}{16}$
12 $6\frac{1}{3}$
13 $11\frac{3}{14}$
14 $8\frac{1}{16}$
15 $12\frac{1}{16}$
16 $18\frac{1}{2}$
17 $10\frac{1}{10}$
18 $11\frac{1}{10}$
19 $11\frac{1}{2}$
20 $17\frac{3}{7}$
21 $15\frac{2}{5}$
22 $15\frac{4}{5}$
23 $14\frac{51}{100}$
24 $17\frac{13}{32}$
25 $22\frac{2}{7}$

Exercise 3k page 58

1 $1\frac{5}{8}$	**7** $1\frac{5}{14}$	**13** $2\frac{25}{28}$	**19** $1\frac{3}{8}$
2 $1\frac{13}{15}$	**8** $2\frac{3}{10}$	**14** $1\frac{3}{4}$	**20** $2\frac{7}{10}$
3 $1\frac{1}{6}$	**9** $3\frac{1}{4}$	**15** $3\frac{7}{20}$	**21** $1\frac{1}{2}$
4 $\frac{3}{4}$	**10** $3\frac{3}{10}$	**16** $3\frac{9}{35}$	**22** $2\frac{5}{6}$
5 $5\frac{5}{12}$	**11** $2\frac{4}{63}$	**17** $\frac{3}{4}$	**23** $2\frac{7}{8}$
6 $1\frac{1}{2}$	**12** $3\frac{7}{24}$	**18** $1\frac{27}{35}$	**24** $3\frac{9}{10}$

Exercise 3l page 60

1 $\frac{3}{8}$	**9** $\frac{5}{24}$	**17** $\frac{1}{9}$	**25** $\frac{2}{9}$
2 $\frac{10}{21}$	**10** $\frac{14}{27}$	**18** $\frac{15}{28}$	**26** $\frac{2}{31}$
3 $\frac{2}{15}$	**11** $\frac{3}{20}$	**19** $\frac{3}{4}$	**27** $\frac{2}{3}$
4 $\frac{7}{16}$	**12** $\frac{3}{35}$	**20** $\frac{6}{7}$	**28** $\frac{3}{16}$
5 $\frac{3}{7}$	**13** $\frac{1}{6}$	**21** $\frac{5}{48}$	**29** $\frac{3}{20}$
6 $\frac{4}{63}$	**14** $\frac{4}{7}$	**22** $\frac{11}{20}$	**30** $\frac{2}{3}$
7 $\frac{6}{35}$	**15** $\frac{7}{18}$	**23** $\frac{4}{11}$	
8 $\frac{6}{25}$	**16** $\frac{2}{3}$	**24** $\frac{4}{11}$	

Exercise 3m page 61

1 $\frac{3}{5}$	**8** 30	**15** $29\frac{1}{4}$	**22** 36
2 2	**9** $16\frac{1}{2}$	**16** 23	**23** $8\frac{1}{2}$
3 $\frac{3}{4}$	**10** $7\frac{1}{2}$	**17** 30	**24** 120
4 $11\frac{1}{5}$	**11** 9	**18** $12\frac{1}{2}$	**25** $18\frac{1}{3}$
5 $\frac{1}{2}$	**12** 10	**19** $37\frac{1}{2}$	**26** 14
6 $\frac{1}{2}$	**13** 5	**20** 110	**27** 44
7 4	**14** $6\frac{1}{3}$	**21** $13\frac{1}{2}$	

Exercise 3n page 62

1 6	**9** 12 m	**17** 50 cm
2 6	**10** 25 dollars	**18** 8 cm
3 3	**11** 45 litres	**19** \$30
4 16	**12** 33 miles	**20** 12 cm
5 10	**13** 21 gallons	**21** 292 days
6 6	**14** 8 m	**22** 9 h
7 5	**15** 10 dollars	**23** 1 day
8 8	**16** 28 litres	**24** \$3

Exercise 3p page 64

1 10	**5** 30	**9** 38	**13** $10\frac{1}{2}$
2 21	**6** 99	**10** $\frac{3}{4}$	**14** $\frac{5}{6}$
3 45	**7** 39	**11** $\frac{2}{5}$	**15** $5\frac{1}{3}$
4 99	**8** 63	**12** $1\frac{1}{2}$	**16** 6
17 $2\frac{8}{11}$	**21** 4	**25** $5\frac{3}{5}$	**28** $3\frac{1}{3}$
18 $6\frac{2}{3}$	**22** 6	**26** 6	**29** $1\frac{1}{2}$
19 $4\frac{5}{6}$	**23** $2\frac{2}{3}$	**27** $1\frac{3}{7}$	**30** 12
20 $1\frac{4}{5}$	**24** 12		

Exercise 3q page 66

1 1	**14** $\frac{7}{12}$	**27** $\frac{1}{22}$	**40** $2\frac{7}{30}$
2 $2\frac{1}{2}$	**15** $\frac{1}{5}$	**28** $\frac{9}{22}$	**41** $\frac{11}{16}$
3 $1\frac{2}{3}$	**16** $\frac{3}{14}$	**29** $\frac{5}{21}$	**42** $1\frac{8}{9}$
4 $\frac{2}{3}$	**17** $\frac{13}{15}$	**30** $\frac{5}{18}$	**43** True
5 $\frac{8}{15}$	**18** $\frac{5}{24}$	**31** $\frac{2}{33}$	**44** False
6 $2\frac{2}{3}$	**19** $1\frac{5}{8}$	**32** $1\frac{2}{25}$	**45** True
7 $5\frac{1}{10}$	**20** $\frac{41}{42}$	**33** $\frac{1}{21}$	**46** True
8 $2\frac{1}{4}$	**21** $\frac{1}{16}$	**34** $1\frac{1}{4}$	**47** False
9 $1\frac{1}{2}$	**22** $\frac{1}{3}$	**35** $\frac{1}{4}$	**48** False
10 $\frac{9}{32}$	**23** $\frac{2}{21}$	**36** $\frac{1}{3}$	**49** True
11 $\frac{9}{20}$	**24** $\frac{4}{10}$	**37** $\frac{1}{9}$	**50** False
12 $\frac{4}{5}$	**25** $\frac{21}{34}$	**38** $4\frac{2}{9}$	**51** False
13 $\frac{3}{5}$	**26** $1\frac{1}{2}$	**39** $1\frac{3}{8}$	**52** True

Exercise 3r page 68

1 30 kg	**3** 3 km	**5** 22
2 $\frac{7}{20}$ litres	**4** $58\frac{1}{2}$ min	**6** $1\frac{1}{2}$

Exercise 3s page 69

1 **a** $1\frac{5}{21}$ **b** $\frac{11}{24}$ **c** $\frac{35}{72}$ **d** $2\frac{1}{6}$ **e** $\frac{11}{12}$

2 **a** $2\frac{1}{4}$ **b** $3\frac{1}{5}$

3 **a** $\frac{3}{7}$ **b** $\frac{17}{30}$

4 **a** $\frac{1}{2}, \frac{3}{5}, \frac{13}{20}, \frac{7}{10}$ **b** $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ **c** $\frac{3}{5}, \frac{7}{10}, \frac{71}{100}, \frac{17}{20}$

5 **a** $<$ **b** $>$ **c** $>$

6 **a** $\frac{3}{11}$ **b** $\frac{7}{22}$

7 6 **8** $\frac{5}{6}$ **9** $1\frac{13}{20}$ **10** $\frac{3}{5}, \frac{2}{3}, \frac{7}{10}$

11 **a** $19\frac{1}{3}$ **b** $1\frac{1}{2}$

12 $2\frac{1}{6}$ **13** 6 **14** 18 min **15** $3\frac{9}{10}$

16 **a** 27 **b** 40

17 **a** $2\frac{3}{5}$ **b** $3\frac{7}{8}$ **c** $5\frac{2}{5}$

18 **a** T **b** T **c** F

Exercise 3t page 70

1 a $\frac{2}{15}$ **b** $1\frac{7}{10}$ **c** $\frac{3}{22}$ **d** $6\frac{7}{12}$ **e** $\frac{1}{2}$ **f** $2\frac{13}{20}$

2 a $\frac{7}{8}$ **b** $1\frac{5}{6}$ **c** $\frac{12}{15}$

3 a $\frac{3}{7}$ **b** $\frac{17}{30}$

4 a > **b** < **c** <

5 a $\frac{3}{10}, \frac{7}{20}, \frac{3}{8}, \frac{2}{3}$ **b** $\frac{3}{10}, \frac{2}{5}, \frac{7}{15}, \frac{1}{2}$ **c** $\frac{17}{32}, \frac{9}{16}, \frac{5}{8}, \frac{3}{4}$

6 a $\frac{3}{11}$ **b** $\frac{7}{22}$

Exercise 3u page 71

1 a 15 **b** $11\frac{1}{3}$

2 a $1\frac{2}{3}$ **b** $4\frac{11}{18}$

3 a < **b** <

4 a $1\frac{1}{12}$ **b** 9

5 $\frac{1}{3}, \frac{2}{5}, \frac{7}{15}$

6 2

7 a $6\frac{1}{4}$ **b** $2\frac{6}{11}$

8 125 s

9 a 24 **b** 21

10 a $3\frac{1}{8}$ **b** $5\frac{4}{9}$ **c** $6\frac{1}{6}$

11 $12\frac{1}{8}$ km; $\frac{77}{97}$

12 6

CHAPTER 4

Exercise 4a page 74

	hundreds	tens	units	.	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths
1			2	.	6				
2		3	2	.	1				
3			6	.	0	3			
4			0	.	0	9			
5	1	0	1	.	3				
6			0	.	0	0	0	0	7
7			1	.	0	4	6		
8		1	2	.	0	0	1		
9			6	.	3	4			
10			0	.	6	0	4		
11		1	5	.	0	4	5		
12			0	.	0	0	9	2	

Exercise 4b page 74

1 $\frac{1}{5}$
2 $\frac{3}{50}$
3 $1\frac{3}{10}$
4 $\frac{7}{10\,000}$
5 $\frac{1}{1000}$
6 $6\frac{2}{5}$
7 $\frac{7}{10}$
8 $2\frac{1}{100}$
9 $1\frac{4}{5}$
10 $1\frac{7}{10}$
11 $15\frac{1}{2}$
12 $8\frac{3}{50}$
13 $\frac{73}{100}$
14 $\frac{81}{1000}$
15 $\frac{207}{1000}$
16 $\frac{29}{10000}$
17 $\frac{67}{100000}$
18 $\frac{17}{100}$
19 $\frac{71}{1000}$
20 $\frac{3001}{10000}$
21 $\frac{207}{10000}$
22 $\frac{63}{100}$
23 $\frac{31}{1000}$
24 $\frac{47}{100}$
25 $\frac{1}{4}$
26 $\frac{9}{125}$
27 $\frac{19}{50}$
28 $\frac{61}{2000}$
29 $\frac{3}{20}$
30 $\frac{1}{40}$
31 $\frac{7}{20}$
32 $\frac{1}{625}$
33 $\frac{11}{250}$
34 $\frac{1}{8}$
35 $\frac{12}{25}$
36 $\frac{5}{8}$

Exercise 4c page 76

1 10.8
2 7.55
3 0.039
4 3.98
5 5.83
6 14.04
7 7.6
8 12.24
9 3.68
10 9.12
11 0.2673
12 2.102
13 0.001 76
14 0.131
15 4.698
16 0.3552
17 4.6005
18 20.7
19 6.798
20 27.374
21 2.38
22 17.301
23 15.62
24 13.52
25 16.81
26 2.5
27 7.8
28 18.5
29 0.41
30 0.0321
31 16.87
32 2.241
33 0.191
34 71.4
35 6.65
36 41.45
37 6.939

Exercise 4d page 77

1 3.06
2 2.94
3 3.13
4 2.66
5 2.4
6 7.882
7 6.118
8 2.772
9 11.1974
10 0.000 197
11 0.0067
12 0.0013
13 0.005 27
14 0.059 27
15 5.27
16 5.927
17 7.24
18 729.4
19 0.729 94
20 0.13
21 57.6
22 8.3
23 0.149
24 6.81
25 10.32
26 6.92
27 2.98
28 6.6
29 100.28
30 99.72
31 0.286
32 0.234
33 39.88
34 36.52
35 202.84
36 17.76
37 0.007
38 0.382
39 6.64
40 38.82

Exercise 4e page 78

1 22.6 cm
2 5.3 m
3 $24.77
4 $21.04
5 1
6 53.2 m
7 $9.25
8 5.9 cm

Exercise 4f page 80

1 72 000
2 82.4
3 0.24
4 460
5 3278
6 430
7 6.02
8 32.06
9 72 810
10 0.000 063
11 0.703
12 374
13 2.772
14 7.626
15 0.000 024
16 0.014
17 2.7
18 0.068
19 0.026
20 0.0158
21 0.0426
22 1.34
23 0.003 74
24 0.0092
25 0.16
26 16
27 7.8

28 0.000 78
29 1420
30 6.8
31 0.0163
32 0.002
33 0.14
34 78000
35 0.24
36 63
37 3.2
38 0.079
39 0.078
40 0.24
41 11 100
42 0.000 38
43 0.0038
44 380000
45 0.000 24
46 0.000003
47 4.1
48 10.04
49 4.2 m
50 $152
51 0.138, 1380
52 0.16
53 0.1746
54 0.0038

Exercise 4g page 81

1 0.2
2 1.6
3 0.21
4 2.6
5 0.1
6 0.19
7 0.224
8 3.8
9 21.3
10 2.51
11 1.64
12 0.15
13 0.019
14 0.000 13
15 0.002 18
16 0.042
17 0.002
18 0.000 06
19 0.81
20 1.06
21 0.308
22 0.1092
23 0.0057
24 0.0453
25 0.0019
26 0.09
27 0.1043
28 0.000 015
29 0.9
30 0.0106
31 0.019
32 0.77
33 2.107
34 0.62
35 0.037
36 0.78
37 1.2
38 1.85
39 0.415
40 0.15
41 0.72
42 0.000 04
43 0.8875
44 1.75
45 4.55
46 0.000155
47 2.35
48 0.0124
49 0.125
50 0.038 75
51 0.52
52 1.905
53 2.6
54 0.05
55 0.0025
56 0.6028
57 0.853 75
58 2.45
59 0.575
60 0.055 75
61 3.65 cm
62 4.075 m
63 7.15 kg
64 3.2 cm
65 $4.50

Exercise 4h page 83

1 1.1
2 0.15
3 0.12
4 0.45
5 0.51
6 3.2
7 0.0041
8 0.036
9 0.53
10 0.26
11 0.56
12 0.7
13 0.32
14 0.26
15 0.024
16 0.000 23
17 3.2
18 0.43
19 0.21
20 0.000 713
21 0.52
22 3.12
23 0.84
24 0.005 68

Exercise 4i page 85

1 0.18
2 0.0024
3 0.018
4 0.000 56
5 0.0108
6 0.000 021
7 0.035
8 4.8
9 0.0064
10 0.0018
11 0.042
12 0.72
13 0.84
14 0.036
15 8.1
16 0.0088
17 0.077
18 0.28
19 0.1502
20 1.6
21 1.4
22 0.000 912
23 240
24 63
25 0.112
26 2.048
27 22.4
28 0.0022
29 0.03
30 0.01408
31 0.64
32 0.8
33 0.64
34 0.0008
35 6.4
36 0.08
37 0.000 000 0064
38 800
39 0.64
40 0.008
41 0.0432
42 12.4
43 6.72
44 12.48
45 0.0952
46 1253.2
47 434
48 0.4536
49 33
50 0.000 278 8
51 7476
52 118.4
53 8.97
54 198
55 64.8
56 0.111 52
57 0.002 592
58 2.56
59 2.56
60 2.56
61 0.0784
62 0.1054
63 1.722
64 17.29
65 22.96
66 0.031 02
67 $325
68 4.4 cm
69 3.8 kg
70 16.8 cm
71 4216 cents
$42.16
72 0.24
73 3.25 m
74 50.4 m

Exercise 4j page 88

1 0.25
2 0.375
3 0.6
4 0.12
5 0.04
6 2.8
7 0.625
8 0.4375
9 0.12
10 0.031 25

Exercise 4k page 89

1 0.233 0.23
2 0.002 727 0.0027
3 0.571 428 571 0.571428
4 0.143 33 0.143
5 0.004 285 714 28 0.004 28571
6 0.1222 0.12
7 0.444 0.4
8 0.666 0.6
9 0.1818 0.18
10 0.714 285 714 0.714 285
11 0.777 0.7
12 1.142 857 142 8 1.142 857

Exercise 4l page 90

1 14
2 6
3 27
4 3
5 4
6 7
7 110
8 6
9 74
10 4
11 0.363
12 0.026
13 0.007
14 0.070
15 0.001
16 0.084
17 0.084
18 0.325
19 0.033
20 4.000
21 1.8
22 42.6
23 1.01
24 0.0094
25 0.735
26 1.64
27 1.6
28 2
29 3.50
30 3.5
31 0.17
32 0.93
33 0.35
34 2.03
35 2.85
36 0.16
37 0.04
38 0.05
39 0.24
40 0.04
41 0.22
42 0.95
43 4.1
44 57.4
45 2.6
46 0.9
47 7.3
48 1.2
49 2.1
50 0.9
51 9.7
52 0.6
53 1.7
54 27.3
55 0.006
56 0.018
57 0.417
58 0.021
59 0.038
60 0.001
61 0.028
62 0.031
63 0.016
64 0.019
65 0.039
66 0.037
67 0.429
68 0.444
69 0.167
70 0.667
71 0.818
72 0.857
73 1.143
74 0.111
75 0.333
76 0.364
77 0.214
78 0.235
79 0.462
80 0.190
81 0.158
82 0.176
83 0.267
84 0.389
85 0.136
86 0.121

Exercise 4m **page 93**

1	0.2	**23**	0.004	**45**	1.03
2	0.02	**24**	60	**46**	0.71
3	8	**25**	0.8	**47**	0.02
4	20	**26**	900	**48**	2.9
5	4500	**27**	0.31	**49**	8.2
6	12	**28**	0.16	**50**	0.087
7	0.16	**29**	24.5	**51**	1.3333
8	6	**30**	3.2	**52**	32.9
9	60	**31**	1.2	**53**	20.3
10	5	**32**	41	**54**	0.032
11	13	**33**	7	**55**	283.333
12	120	**34**	1.2	**56**	1.7
13	800	**35**	9	**57**	36
14	360	**36**	0.08	**58**	3.9
15	0.012	**37**	6.33	**59**	0.167
16	0.01	**38**	8.43	**60**	1.1
17	100	**39**	16.67	**61**	2.3
18	2.3	**40**	28.17	**62**	4
19	21	**41**	0.72	**63**	0.72
20	0.012	**42**	41.67	**64**	0.2571
21	0.001 71	**43**	0.03	**65**	0.57
22	52000	**44**	0.93	**66**	2.5

Exercise 4n **page 95**

1	0.144	**8**	0.14	**15**	4
2	1.6	**9**	6.72	**16**	4
3	0.0512	**10**	4.2	**17**	10
4	128	**11**	12.24	**18**	0.12
5	2.88	**12**	84	**19**	0.125
6	5.76	**13**	0.3	**20**	0.7
7	0.000 126	**14**	0.16	**21**	12

Exercise 4p **page 95**

1	$0.2, \frac{1}{4}$	**5**	$\frac{7}{8}, \frac{8}{9}, 0.9$	**9**	$\frac{3}{7}, \frac{5}{11}, \frac{6}{13}$
2	$\frac{2}{5}, \frac{4}{9}$	**6**	$\frac{3}{4}, \frac{17}{20}$	**10**	$0.7, \frac{8}{11}$
3	$\frac{4}{9}, \frac{1}{2}$	**7**	$0.35, \frac{9}{25}, \frac{3}{8}$	**11**	$0.3, \frac{5}{12}$
4	$\frac{3}{11}, 0.3, \frac{1}{3}$	**8**	$\frac{4}{7}, 0.59, \frac{3}{5}$	**12**	$0.45, \frac{9}{19}, \frac{1}{2}$

Exercise 4q **page 96**

1	C	**4**	D
2	C	**5**	B
3	B	**6**	A

Exercise 4r **page 96**

1 $\frac{3}{50}$

2 a 0.0624 **b** 0.52

3 1.7 **4** 6.4 cm **5** 0.048 **6** 0.24

7 \$55.68

8 a 8 **b** 7.8 **c** 7.782

Exercise 4s **page 97**

1 $0.\dot{7}1428\dot{5}$

2 a 0.064 **b** 0.000 64

3 16.28 **6** 2.05

4 $\frac{31}{50}$ **7** $\frac{7}{9}$

5 7.4437 **8** 25

Exercise 4t **page 97**

1	0.16	**5**	14.63
2	9.186 (9.1857)	**6**	\$2.03
3	0.0036	**7**	2
4	$\frac{19}{2000}$	**8**	0.666 …

CHAPTER 5

Exercise 5a **page 101**

1	$\frac{1}{5}$	**5**	$\frac{1}{3}$	**9**	$\frac{13}{20}$	**13**	$\frac{5}{8}$
2	$\frac{9}{20}$	**6**	$\frac{1}{8}$	**10**	$\frac{14}{25}$	**14**	$\frac{5}{4}$
3	$\frac{1}{4}$	**7**	$\frac{1}{40}$	**11**	$\frac{37}{100}$	**15**	$\frac{7}{10}$
4	$\frac{18}{25}$	**8**	$\frac{1}{2}$	**12**	$\frac{2}{3}$	**16**	$\frac{3}{4}$
17	$\frac{12}{25}$	**25**	$\frac{41}{50}$	**33**	0.583	**41**	1.2
18	$\frac{69}{100}$	**26**	$\frac{7}{8}$	**34**	0.58	**42**	2.31
19	$\frac{3}{8}$	**27**	$\frac{1}{16}$	**35**	0.3	**43**	0.857
20	$\frac{4}{75}$	**28**	$\frac{3}{2}$	**36**	0.623	**44**	0.08
21	$\frac{7}{40}$	**29**	0.47	**37**	3.5	**45**	0.03
22	$\frac{19}{20}$	**30**	0.12	**38**	0.487	**46**	1.8
23	$\frac{3}{20}$	**31**	0.055	**39**	0.92	**47**	0.053
24	$\frac{2}{25}$	**32**	1.45	**40**	0.65	**48**	0.541

Exercise 5b **page 102**

1	50%	**11**	75%	**21**	50%	**31**	25%
2	70%	**12**	45%	**22**	22%	**32**	74%
3	65%	**13**	140%	**23**	83%	**33**	125%
4	$33\frac{1}{3}$ %	**14**	$62\frac{1}{2}$%	**24**	172%	**34**	341%
5	52.5%	**15**	$266\frac{2}{3}$%	**25**	62.5%	**35**	7.5%
6	25%	**16**	60%	**26**	90%	**36**	36%
7	15%	**17**	35%	**27**	4%	**37**	16%
8	16%	**18**	124%	**28**	55%	**38**	139%
9	37.5%	**19**	$87\frac{1}{2}$ %	**29**	264%	**39**	635%
10	$38\frac{1}{3}$ %	**20**	160%	**30**	84.5%	**40**	18.25%

Exercise 5c **page 103**

1	**a** $\frac{3}{10}$	**b** $\frac{17}{20}$	**c** $\frac{17}{40}$	**d** $\frac{21}{400}$
2	**a** 0.44	**b** 0.68	**c** 1.7	**d** 0.165
3	**a** 40%	**b** 85%	**c** $12\frac{1}{2}$ %	**d** $113\frac{1}{3}$ %

4 a 20% **b** 62% **c** $84\frac{1}{2}$ % **d** 178%

	Fraction	Percentage	Decimal
	$\frac{3}{4}$	75%	0.75
5	$\frac{4}{5}$	80%	0.8
6	$\frac{3}{5}$	60%	0.6
7	$\frac{7}{10}$	70%	0.7
8	$\frac{11}{20}$	55%	0.55
9	$\frac{11}{25}$	44%	0.44
10	$\frac{8}{25}$	32%	0.32

Exercise 5d page 104

1 52% **2** 13% **3** 36% **4** 92% **5** 88%
6 12% **7** 43% **8** 68% **9** 20% **10** 38%
11 3% **12** 252 **13** 1400
14 a 2% **b** 10% **c** 66% **d** 22%

Exercise 5e page 106

1 25% **2** 60% **3** $33\frac{1}{3}$% **4** $33\frac{1}{3}$% **5** 75% **6** 60% **7** 15% **8** 25% **9** 25%
10 37.5% **11** 20% **12** 40% **13** 60% **14** 10% **15** 20% **16** 300% **17** 50% **18** 200%
19 62.5% **20** 10% **21** $66\frac{2}{3}$% **22** 25% **23** 72% **24** $333\frac{1}{3}$% **25** 72% **26** 42%

Exercise 5f page 107

1 48 **2** 96 g **3** 55.5 cm **4** 286 km **5** 16 c **6** 3.08 kg **7** 252 **8** 989 g **9** 4.73 m **10** 206.4 cm^2 **11** 2.52 m **12** 14.4 m^2
13 333 **14** 198 kg **15** 1.44 m **16** $1.50 **17** 0.34 km **18** 1.6 litres **19** $75 **20** 198 m **21** 90 g **22** 2.94 mm **23** 18 cm **24** 9 m^2
25 320 m^2 **26** 45 km **27** 5 km **28** 149 cm^2 **29** $14 **30** $53.43 **31** 48 c **32** 6 g **33** 2.1 m **34** $10 **35** 2 kg **36** 14 mm

Exercise 5g page 108

1 40% **2** 70% **3** 20%
4 20% **5** 30% **6** 75%
7 75% **8** $66\frac{2}{3}$% **9** 65%
10 1960
11 a $46\frac{2}{3}$% **b** $53\frac{1}{3}$%

12 a 52 **b** 28
13 a 12 **b** 18
14 a 7 **b** 343
15 5760 **16** 78 **17** $7680 **18** 112

Exercise 5h page 110

1 D **2** B **3** B
4 C **5** C **6** B

Exercise 5i page 110

1 a $\frac{9}{25}$ **b** 0.36
2 a 62.5% **b** 133.3% **c** 250%
3 $12\frac{1}{2}$% **4** 289 m^2 **5** $192000

Exercise 5j page 111

1 a $12\frac{1}{2}$% **b** $37\frac{1}{2}$% **c** 50%
2 a 28.6% **b** 27.9% **c** 122.2%
3 a $\frac{1}{8}$ **b** 0.125
4 90 c **5** 54

REVIEW TEST 1 page 113

1 C **2** C **3** A
4 B **5** A **6** A
7 D **8** C **9** D
10 B **11** D **12** C
13 a 36 **b** all except 29, 31, 37, 41 **c** 28, 36
14 a $\frac{1}{9}$ **b** 10, 15 **c** 33.5
15 a $\frac{5}{12}, \frac{2}{3}, \frac{3}{4}, \frac{7}{9}$ **b** $\frac{1}{3}$
16 a $\frac{3}{7} > \frac{3}{8}$ **b** $\frac{11}{7} > 1\frac{3}{10}$
17 a 19.98 **b** 3.697 **c** 62.1 **d** 2.85
18 a $\frac{7}{8}, \frac{3}{4}, \frac{17}{24}, \frac{7}{12}$ **b** 20.64
19 a $\frac{7}{20}$ **b** 24% **c** 76%
20 a 10 **b** −3 **c** 0 **d** 30
21 a $\frac{13}{12}$ or $1\frac{1}{12}$ **b** $10\frac{1}{2}$ **c** $3\frac{17}{20}$ **d** $\frac{8}{49}$
22 a $\frac{1}{6}$ **b** $\frac{2}{3}$

CHAPTER 6

Exercise 6a page 117

1 a metres **b** centimetres **c** metres **d** kilometres **e** centimetres **f** millimetres
3 a 4 **b** 2 **c** 5 **d** 1 **e** 10
4 (to the nearest millimetre)
a 20 **b** 10 **c** 4 **d** 16 **e** 24
9 40 cm **10** 900 cm

Exercise 6b page 119

1 200 **2** 5000 **3** 30 **4** 400 **5** 12000 **6** 150 **7** 6000 **8** 100000
9 3000 **10** 2000000 **11** 500 **12** 7000 **13** 150 **14** 23 **15** 4600 **16** 3700
17 1900 **18** 3500 **19** 270 **20** 190000 **21** 38 **22** 9200 **23** 2300 **24** 840

Exercise 6c page 121

1 12000
2 3000
3 5000
4 1000000
5 1000000
6 13000
7 6000
8 2000000
9 4000
10 2000000
11 3000
12 4000
13 1500
14 2700
15 1800
16 700
17 5200000
18 600
19 11300
20 2500
21 7300
22 300000
23 500
24 800

Exercise 6d page 122

1 136
2 35
3 1050
4 48
5 207
6 3020
7 502
8 5500
9 202
10 8009
11 3500
12 2008
13 5500
14 2800
15 3250
16 1020
17 1250
18 3550
19 2050
20 1010

Exercise 6e page 123

1 30
2 6
3 1.5
4 25
5 1.6
6 0.072
7 0.12
8 8.8
9 1.25
10 2.85
11 1.5
12 3.68
13 1.5
14 5.02
15 3.8
16 0.086
17 0.56
18 0.028
19 0.19
20 0.086
21 3.45
22 8.4
23 11.002
24 2.042
25 4.4
26 5.03
27 7.005
28 4.005
29 1.0001
30 0.000085
31 5.142
32 48.171
33 9.008
34 9.088
35 12.019
36 4.111
37 1.056
38 5.003
39 0.2505
40 0.85055

Exercise 6f page 124

1 5.86
2 1.035
3 3001.36
4 3051
5 5.647
6 4.65
7 440
8 55
9 1820
10 2456
11 5059
12 1358
13 3250
14 5115
15 15100
16 2550
17 1046.68
18 308.73
19 2580
20 2362
21 2.22
22 1606.4
23 1089.6
24 5972
25 748
26 0.922
27 1150
28 73.6
29 2642
30 19850
31 35420
32 910
33 448.2
34 5

Exercise 6g page 126

1 13540
2 45792
3 13.563
4 12.55
5 32
6 10.6
7 15366
8 24.448
9 22.77
10 16240

Exercise 6h page 127

1 9.72m
2 1840g
3 748kg
4 4.11g
5 1080mm
6 4kg
7 2.2kg
8 15m
9 33.2cm
10 5.3kg

Exercise 6i page 129

1 a m^3 b mm^3 c cm^3
2 8000 mm^3
3 14000 mm^3
4 6200 mm^3
5 430 mm^3
6 92000000 mm^3
7 40 mm^3
8 3000000 cm^3
9 2500000 cm^3
10 420000 cm^3
11 6300 cm^3
12 0.022 cm^3
13 0.731 cm^3

Exercise 6j page 130

1 a cm^3 b m^3 c mm^3 d mm^3 e m^3
f cm^3 g cm^3 h mm^3 i m^3
2 2500 cm^3
3 1760 cm^3
4 540 cm^3
5 7.5 cm^3
6 35000 cm^3
7 28 cm^3
8 7 litres
9 4 litres
10 24 litres
11 0.6 litres
12 5000 litres
13 12000 litre
14 4600 litres
15 67 litres

Exercise 6k page 132

1 68in
2 14ft
3 1809yd
4 35in
5 100in
6 4320yd
7 17ft
8 123in
9 28ft
10 118in
11 3ft
12 2ft 5in
13 7ft 2in
14 3yd
15 4yd 1ft
16 1 mile 240yd
17 6ft 3in
18 33yd 1ft
19 10ft
20 17 miles 80yd

Exercise 6l page 133

1 38 oz
2 28 oz
3 67 oz
4 64 cwt
5 162 lb
6 1 lb 8 oz
7 1 lb 2 oz
8 2 lb 4 oz
9 1 ton 10 cwt
10 1 cwt 8 lb

Exercise 6m page 134

1 6lb
2 6ft
3 2kg
4 3m
5 3lb
6 15ft
7 7lb
8 $2\frac{2}{3}$m
9 8oz
10 1lb
11 16km
12 32km
13 24km
14 160km
15 120km
16 64km
17 11lb
18 2m
19 2m
20 4kg
21 1st cloth
22 37.5 miles
23 8oz
24 15cm
25 4in
26 a 25mm b 15mm
27 15cm
28 in the market

Exercise 6n page 138

1 a September b Thursday
c 17th d 13th
2 13
3 4
4 a Dennis b Johanne c 2023
5 a 17 b P. Baldrick c 2021
6 a 3 hours 10 min b 18 days 18 hours
7 a 308 seconds b 210 min
8 a $\frac{1}{3}$ b $\frac{1}{100}$
9 a 10.25 b 11.10 c 45 min
10 a 21 min b 1734
11 a 1 hour 45 min b 8 hours 40 min
c 2 hours 10 min
12 2 hours 40 min
13 a 20 min b 8.05 p.m.
14 a 6 hours 30 min b 8 hours 29 min
c 8 hours d 27 hours 3 min
15 1715
16 a 2 hours 22 min and 2 hours 17 min
b Astleton and Morgan's Hollow; shortest journey time

Exercise 6p **page 141**

1 a 12 °C b 27 °F
c the one in part **a**; it is above the freezing point of water, the other is below
2 a 105 °F b 40 °C c 20 °C d 68 °F
3 a 50 °F b 40 °F c 27 °C d 2 °C
4 $38.\dot{3}$ °C 5 18 °F
6 17.6 °F; more precise

Exercise 6q **page 143**

1 4000 m 5 3000 cm 9 0.065 kg
2 0.03 kg 6 1.25 km 10 4.29 kg
3 350 cm 7 1.5 m
4 0.25 kg 8 28 mm
11 a 8000 mm^3 b 0.000 008 m^3
12 3.5 litres

Exercise 6r **page 143**

1 2.36 m 5 4.25 km 9 4 lb
2 20 mm 6 3600 kg 10 30 cm
3 5000 g 7 2.35 kg
4 0.5 g 8 2000 mg

Exercise 6s **page 144**

1 5780 kg 5 2 years 50 days 9 80 km
2 0.35 t 6 90 minutes 10 9000 cm^3
3 0.0155 cm 7 2.05 km 11 440 cm^3
4 1.56 t 8 600 m

CHAPTER 7

Exercise 7a **page 147**

1 11
2 16
3 12
4 20
5 26
6 20
7 21
8 a A b B
9 45
10 43
11 51
12 40
13 37
14 76
15 62
16 26

Exercise 7b **page 151**

1 4 cm^2
2 64 cm^2
3 100 cm^2
4 25 cm^2
5 2.25 cm^2
6 6.25 cm^2
7 0.49 m^2
8 1.44 cm^2
9 $\frac{1}{4}$ km^2
10 $\frac{9}{16}$ m^2
11 30 cm^2
12 48 cm^2
13 27 m^2
14 280 cm^2
15 3.96 mm^2
16 1470 km^2
17 2.85 m^2
18 30.24 cm^2
19 22 800 cm^2
20 36 000 mm^2

Exercise 7c **page 151**

1 120 cm^2 5 52 m^2 9 43 m^2
2 36 m^2 6 87 cm^2 10 228 cm^2
3 149 m^2 7 544 mm^2
4 208 mm^2 8 90 cm^2

Exercise 7d **page 153**

1 8 cm 6 10 cm 11 22 cm 16 154 km
2 32 cm 7 2.8 m 12 28 cm 17 6.8 m
3 40 cm 8 4.8 cm 13 24 m 18 22.2 cm
4 20 cm 9 2 km 14 68 cm 19 670 cm
5 6 cm 10 3 m 15 8 mm 20 780 mm

Exercise 7e **page 154**

1 2 cm, 8 cm^2 5 5 cm, 22 cm 9 25 cm, 125 cm^2
2 2 cm, 10 cm^2 6 10 m, 44 m 10 80 cm, 202 cm
3 5 m, 15 m^2 7 9 km, 26 km
4 9 mm, 54 mm^2 8 9 mm, 32 mm

Exercise 7f **page 154**

1 a 24 cm b 28 cm^2
2 a 24 cm b 24 cm^2
3 a 48 mm b 80 mm^2
4 a 32 m b 15 m^2
5 a 272 cm b 1664 cm^2
6 84 cm^2
7 128 cm^2
8 78 cm^2
9 90 cm^2
10 184 cm^2
11 91 cm^2
12 198 cm^2
13 432 cm^2
14 4.84 m^2

Exercise 7g **page 157**

1 4 3 6 5 45
2 9 4 6 6 500

Exercise 7h **page 159**

1 a 30 000 b 120 000 c 75 000 d 820 000 e 85 000
2 a 1400 b 300 c 750 d 2600 e 3250
3 a 560 b 56 000
4 a 4 b 25 c 0.5 d 0.25 e 7.34
5 a 0.55 b 14 c 0.076 d 1.86 e 2970
6 a 7.5 b 0.43 c 0.05 d 0.245 e 176

Exercise 7i **page 160**

1 50 000 cm^2 5 8 m^2 9 120 000 m^2
2 1800 mm^2 6 15 000 cm^2 10 22 500 m^2
3 175 000 cm^2 7 37 500 cm^2
4 14 000 cm^2 8 180 mm^2

Exercise 7j **page 161**

1 0.4 cm 5 5 cm 9 7 m
2 5 cm 6 1.5 m 10 6 cm
3 10 m 7 1.25 cm
4 4 mm 8 3 m

Exercise 7k **page 162**

1 a 8250 m^2 b 370 m
2 a 7000 m^2 b 340 m
3 a 8400 m^2 b 380 m
4 a 312 m^2 b 76 m
5 5 m^2
6 1200
7 12 m^2, $9
8 9000 cm^2
9 100
10 96

Exercise 7l **page 162**

1 C 3 A 5 D
2 B 4 C 6 B

CHAPTER 8

Exercise 8a **page 168**

parallelogram	a 2 pairs	b the opposite side	c none
rhombus	a 4	b both pairs of opposite sides	c none
kite	a 2 pairs	b none	c none
trapezium	a none	b the shortest and longest side	c none

Exercise 8b page 169

Questions **1** to **4** Your own answers

5 Yes

Questions **6** and **7** Your own answers

Exercise 8c page 172

3 a **i** 2 **ii** 2 **iii** 3 cm by 4 cm

4 a **i** 2 **ii** 2 **iii** 4 cm by 3 cm

5 a 6

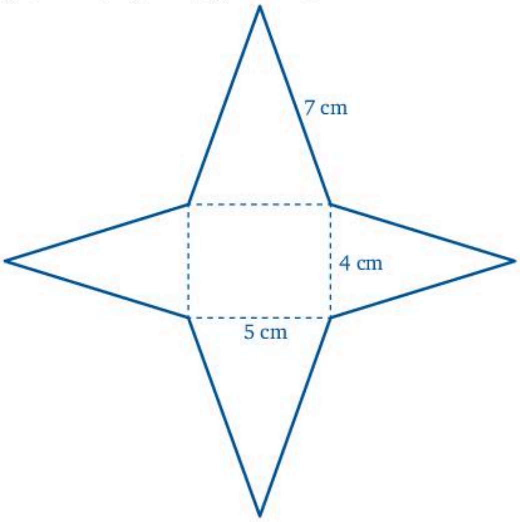
6 a 5 **b** 8 **c** 5

7 a 5 **b** 9 **c** 6

8 a a pyramid **b** 5 **c** 8 **d** 5

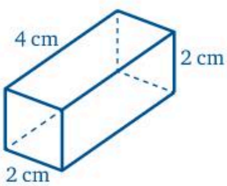
9 b prism **c** parallelogram **d** 22 cm

10 a



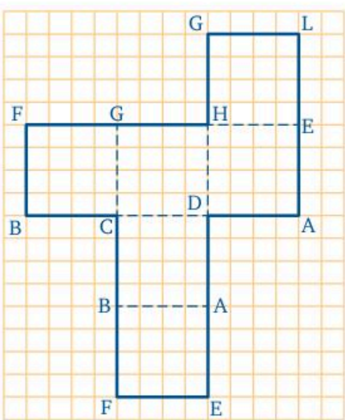
b one; BEC **c** one; EAB

11 a



b IJ **c** G, K

12



13 a There are 36 arrangements altogether.

b 11 will make a cube

CHAPTER 9

Exercise 9b page 183

1 35° **2** 60° **3** 150° **4** 20°

Exercise 9c page 186

2 c B and C

3 d equal **e** equal

4 c yes

5 e XR = XT and XS = XU

6 e they go through the same point

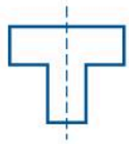
7 d A, B and C

CHAPTER 10

Exercise 10a page 192

1 A, B and C

2



3

4

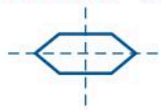
5

6

7

Exercise 10b page 193

1



2

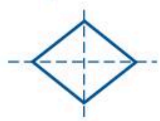
3

none

4

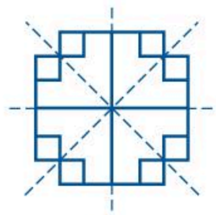
5

6



7

8



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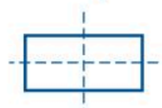

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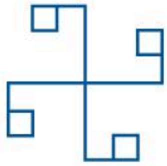
none

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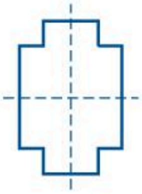

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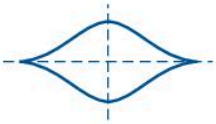

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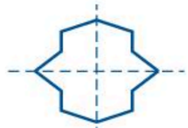

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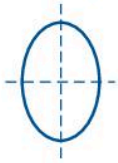
17


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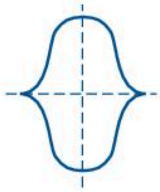
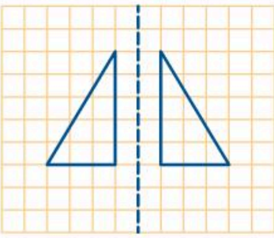


Exercise 10c page 194

1


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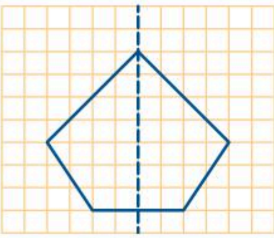

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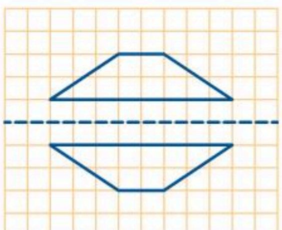

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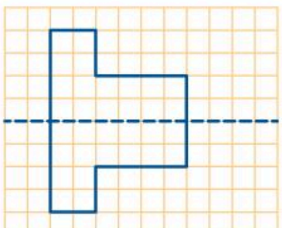

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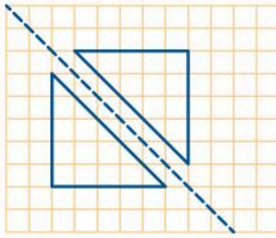

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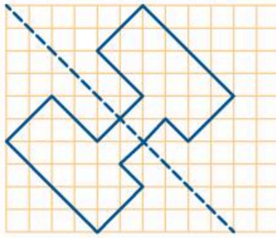

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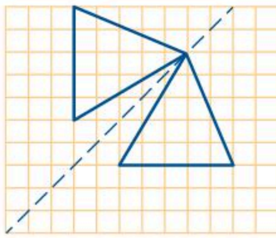

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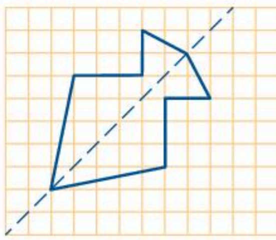

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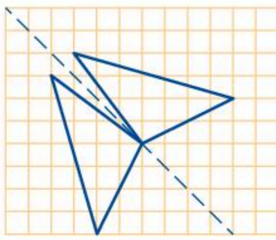

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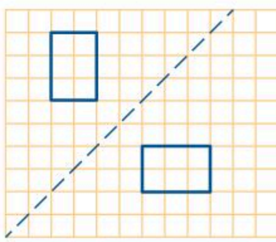

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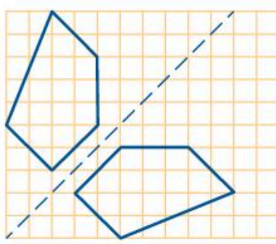

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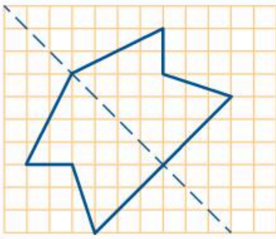
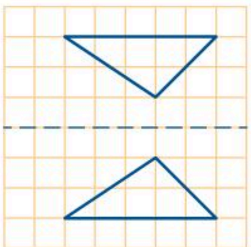


Exercise 10d page 196

1



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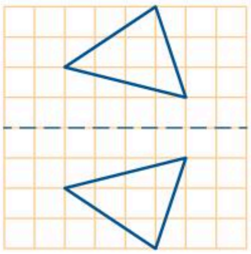

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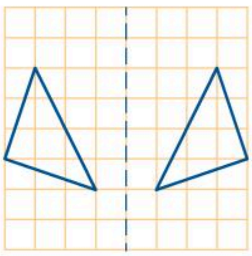

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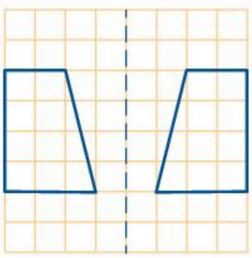

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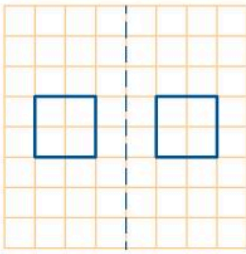

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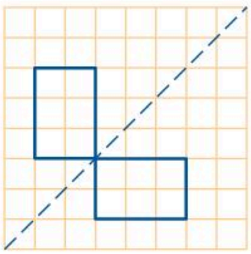

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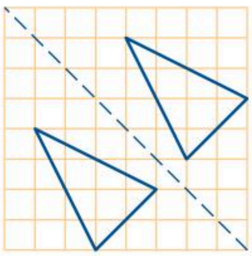
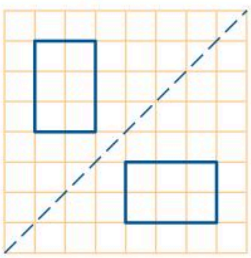
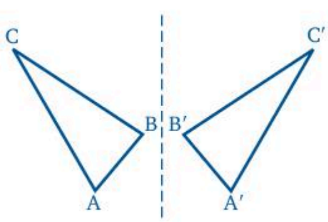


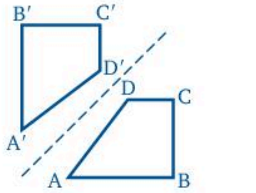
8



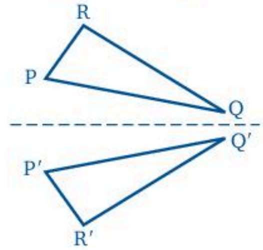
9 a



b



c



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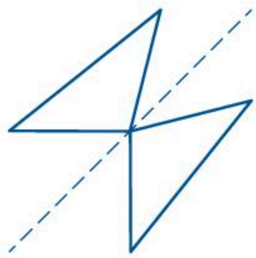

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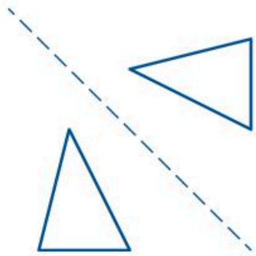

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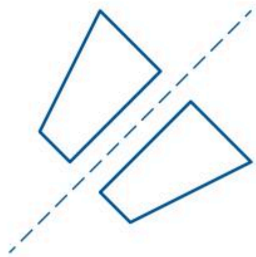

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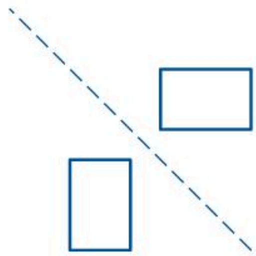
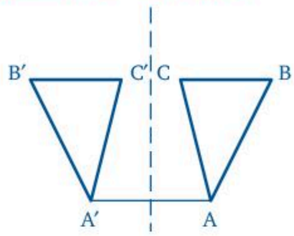
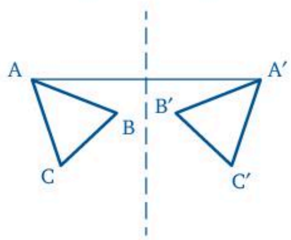


Exercise 10e page 199

1



2



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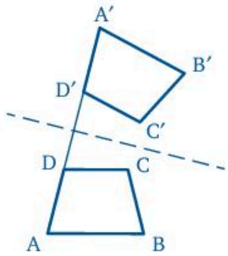
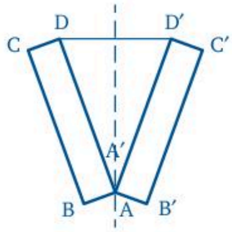


4



Exercise 10f page 201

1 a and c **2** b and c **3** d, e

4 translation b and e, reflection a and c, neither d

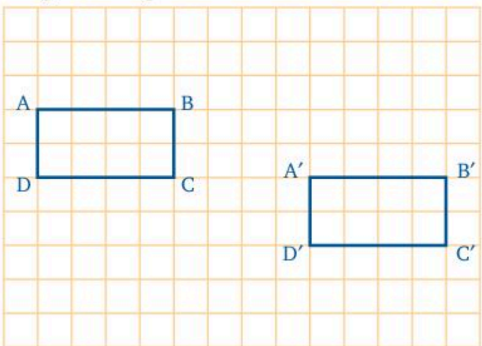
5 7 to the right and 3 down

6 6 to the left and 4 up

7 7 to the left and 2 down

8 7 right and 2 up

9



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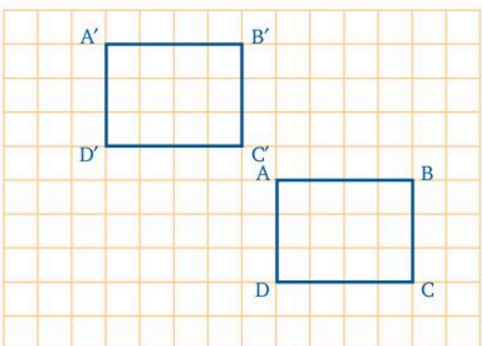

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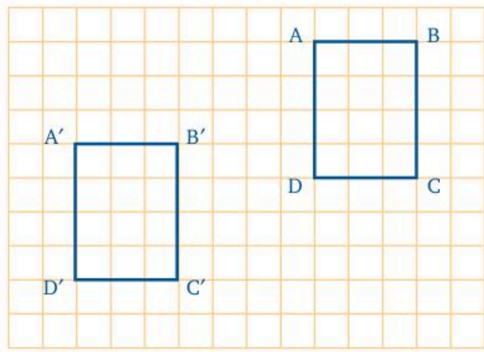

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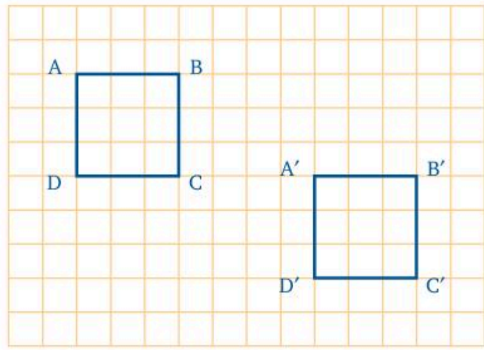

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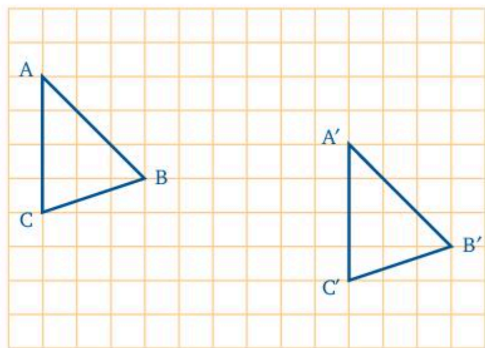

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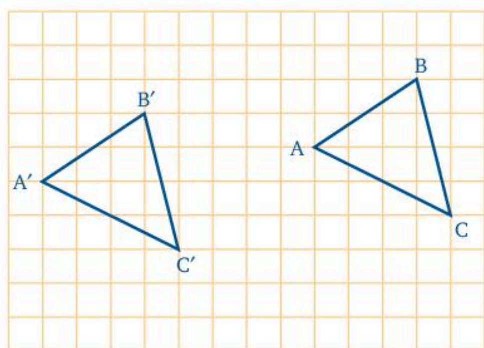


Exercise 10g page 205

1 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$

2 a, b and c

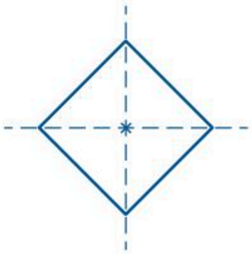
Exercise 10h page 206

1 2 **3** none **5** 1 **7** 2

2 6 **4** 5 **6** 2 **8** 4

9 3 **10** 2 **11** 2 **12** none

13

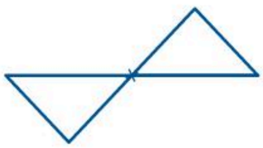


16

14

17

15



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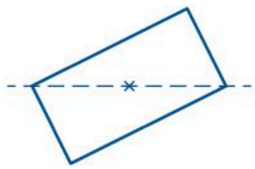


19 90°, 120°, 180°, 90°, 120°, 180°

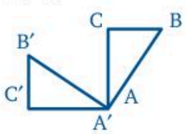
Exercise 10i page 207

1 rotational
2 rotational
3 line
4 line
5 both
6 both
7 both
8 both
9 rotational

Exercise 10j page 209

1 90° clockwise
2 90° clockwise
3 180° either way
4 90° clockwise
5 origin 180°
6 (1, 0), 90° anticlockwise
7 (1, 0), 180°
8 (2, 0), 180°
9 (2, 1), 90° clockwise
10 (3, 1), 180°

11



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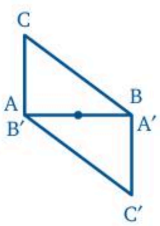

13

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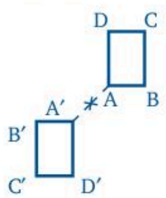

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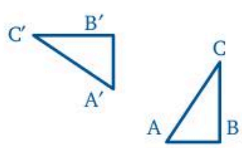

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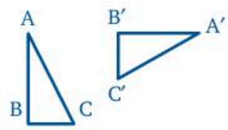

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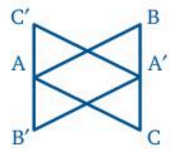

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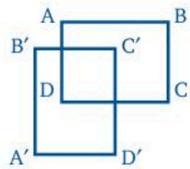


Exercise 10k page 212

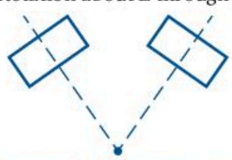
1 Reflection in BC, rotation about B through 90° clockwise

2 Reflection in vertical line through O
Translation along the horizontal line
Rotation about O through 180°

3 Reflection in OB
Translation parallel to AB
Rotation about B through 120° clockwise
Rotation about O through 120° clockwise

4 Reflection in BE
Translation parallel to AB
Rotation about B through 90° clockwise
Rotation about the midpoint of BE, through 180°
Rotation about E through 90° anticlockwise

5



centre of the turning circle

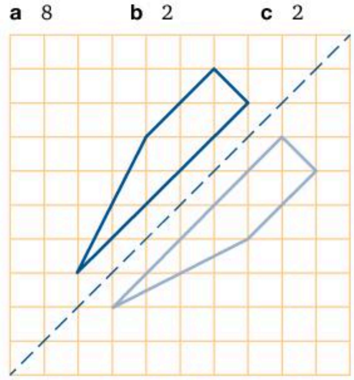
6 Rotations about different vertices, reflections, translations

REVIEW TEST 2 page 215

1 B **2** A **3** C **4** C **5** C **6** D **7** C **8** D **9** C **10** D **11** D **12** C

13 a 4 cm **b** 5 cm^2 **c** 10 000 mm^3
14 a 3490 kg **b** 105 minutes **c** 0.04 cm
15 a 60 cm^2 **b** 80 cm^3 **c** 50 mm^3
16 a 8 **b** 2 **c** 2 **d** 8 cm by 3 cm

17



18 a quadrilateral
b 2, the shorter sides are perpendicular as measured on a protractor

19

20 **a** 6 **b** 94 cm² **c** A and G
21 10 cm

CHAPTER 11

Exercise 11a page 220

1 4 : 5
2 5 : 4
3 2 : 3
4 1 : 4
5 1 : 3
6 9 : 200
7 16 : 3
8 1 : 6
9 16 : 17
10 1 : 1000
11 2 : 3 : 5
12 3 : 4 : 6
13 1 : 5 : 10
14 3 : 4 : 5
15 5 : 6 : 8
16 1 : 8 : 7
17 3 : 4 : 7
18 1 : 8 : 4
19 12 : 1 : 2
20 14 : 9 : 2

Exercise 11b page 221

1 15 : 1
2 8 : 1
3 3 : 2
4 3 : 1
5 4 : 9
6 7 : 10
7 35 : 24
8 9 : 4
9 16 : 7
10 10 : 7
11 8 : 5 : 3
12 2 : 3
13 40 : 9
14 2 : 15
15 15 : 19
16 5 : 4
17 1 : 2 : 3
18 4 : 3
19 4 : 3 : 2
20 3 : 4 : 6

Exercise 11c page 221

1 5 : 7
2 13 : 8
3 5 : 8
4 7 : 10
5 $6:8=24:32=\frac{3}{4}:1$
6 $10:24=\frac{5}{9}:\frac{4}{3}$
7 $8:64=\frac{1}{16}:\frac{1}{2}$
8 $\frac{2}{3}:3=4:18$

Exercise 11d page 222

1 3 : 2, 2 : 5
2 3 : 4, 9 : 16
3 2 : 5, 8 : 3
4 3 : 2, 2 : 3
5 2 : 1
6 **a** 3 : 2 **b** 9 : 5 **c** 18 : 13 **d** 1 : 1
7 8 : 12 : 9
8 3 : 4
9 **a** 12 : 3 : 5 **b** 2 : 3 **c** 5 : 3

Exercise 11e page 223

1 10
2 4
3 7
4 2
5 8
6 12
7 6
8 6
9 9
10 12

Exercise 11f page 224

1 $225
2 18 cm
3 98 cm
4 $10\frac{2}{3}$ cm
5 10.5 cm
6 27 cm
7 30 cm
8 12 m

Exercise 11g page 225

1 $48, $32
2 12 cm, 20 cm
3 $20, $25
4 Dick 15, Tom 25
5 $300, $450
6 16
7 $250, $1750
8 **a** 252 m² **b** 105 m²
9 12
10 **a** 264 cm **b** 110 cm,
11 **a** 210 g **b** 126 g
12 Education $2730 000, Social Services $2184 000
13 Misha $3500, John $2500
14 5 : 3
15 $6400
16 28
17 $341 000
18 $800, $1000, $800
19 6 cm, 8 cm, 10 cm
20 42 m², 14 m², 7 m²

Exercise 11h page 228

1 1 : 50000
2 1 : 500 000
3 1 : 100 000
4 1 : 500 000
5 1 : 100 000
6 1 : 2 000000
7 3 km
8 70 m
9 200 m
10 2000000 cm, 10 cm
11 1.8 cm

Exercise 11i page 229

1 12m
2 72
3 18 cm², 72
4 3.6 cm
5 105
6 2 hours
7 9 hours
8 $6120
9 $1800 000
10 $56\frac{1}{4}$ minutes, $5\frac{1}{3}$ km
11 54 minutes
12 $50000 (must buy complete lengths)
13 hardly any! (no room to work)
14 $4\frac{1}{2}$ teaspoons

Exercise 11j page 231

1 1 :10
2 7 : 6
3 2 : 5
4 9 : 7
5 1 : 500000
6 $5\frac{1}{4}$
7 $1200, $600, $800
8 510

Exercise 11k page 232

1 4 : 9
2 5 : 8
3 $4000, $5200, $800
4 **a** 2 : 3 **b** 8 : 27
5 $8\frac{1}{4}$
6 100 m
7 1 : 1
8 $6562.50

Exercise 11l page 232

1 257 : 144
2 32 : 24
3 10 kg
4 33 m
5 A : M : S = 6 : 3 : 1
6 3.2 km
7 91 : 20
8 $38 500

CHAPTER 12

Exercise 12a page 236

The answers give the frequencies for each value.

1

Shoe size:	4	5	6	7	8	9	10	11	12
Frequency:	1	2	7	15	18	14	10	4	1

$\frac{1}{4}$ of women surveyed

2 No. of goals: 0 1 2 3 4 5 6
Frequency: 2 2 8 10 10 5 1
6 games

3 No. of bedrooms: 1 2 3 4 5 6 7
Frequency: 7 16 23 7 4 2 1
1 and 4 bedrooms

Exercise 12b page 238

1 8
2 7
3 15
4 29
5 16
6 28
7 3
8 40
9 6.2
10 3.5
11 50
12 0.62
13 63
14 96
15 16.5
16 63
17 74
18 1.35
19 0.875
20 5.8
21 2 mm
22 86 kg, 81 kg
23 1837 miles
24 2583 km
25 72 mm
26 131.6 hours
27 134
28 $110
29 92
30 9
31 233, 193
32 106, 238
33 11.8 hours, 5.6 hours
34 68; reduces it to 67
35 158 cm; increases it to 159 cm
36 63 610, 12 722, 8294
37 136.4 kg
38 160.6 cm
39 55.6 kg
40 26
41 588 cm
42 2652

Exercise 12c page 242

1 12
2 9
3 1.8
4 56
5 5.9
6 26.4
7 1
8 a

Marks out of 10	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	1	0	2	2	3	1	5	7	5	4

b 8
9 155 cm
10 31, 3
11 36, 6

Exercise 12d page 243

1 5
2 42
3 17
4 16
5 3.2
6 12
7 98
8 36
9 1.885
10 15

Exercise 12e page 244

1 5
2 $5
3 5
4 a 9 b 79
5 11
6 a 21 b 5
7 a 67 b 11
8 a 32 b 121 c 7

Exercise 12f page 245

	Mean	Mode	Median	Range
1	23	21	21	16
2	71	66,67	69	16
3	45	43	45	7
4	43	13	32	80
5	28	27	27	6
6	77	72	73	15

7 a 157 cm b 157 cm c 157 cm d 10 cm
8 a 54 b 52 c 52 d 54
9 83 84 83.5 15
10 a 0 b 0 c 1.5 d 10
11 a 3 b 3.5 c 3.77 d 6

CHAPTER 13

Exercise 13a page 249

1 $x-3$
2 $x+1$
3 $x-6$
4 $x-5$
5 $2x$
6 $4x$
7 $7x$
8 $6x$
9 A number n is multiplied by 3
10 9 is added to an unknown number
11 5 is subtracted from an unknown number
12 8 is added to an unknown number
13 An unknown number is multiplied by 8
14 An unknown number is subtracted from 7
15 An unknown number is added to 12
16 An unknown number is divided by 6

Exercise 13b page 250

1 $10x$
2 $4x$
3 $2x$
4 2
5 $8y$
6 7
7 1
8 0

Exercise 13c page 251

1 $7x+7$
2 $5x+5$
3 $4x+1$
4 $5c+10a$
5 $8x+2y$
6 $8x+8y$
7 $4x+2y$
8 $4x+8y$
9 $8x+3$
10 $8x+8$
11 $3x+6$
12 $19x+3y$
13 $10x+8y$
14 $11x+3y$
15 $15x$
16 $4x+y+8z$
17 $9x+y+3$
18 11

Exercise 13d page 252

1 $2x+2$
2 $9x+6$
3 $5x+30$
4 $12x+12$
5 $8+10x$
6 $12+10a$
7 $5a+5b$
8 $16x+12$
9 $18+12x$
10 $5x+5$
11 $14+7x$
12 $24+16x$

Exercise 13e page 252

1 $6x+4$
2 $10x+18$
3 $3x+7$
4 $4x+17$
5 $5x+23$
6 $5x+5$
7 $3x+5$
8 $9x+8$

Exercise 13f page 253

1 z^3
2 a^2
3 b^5
4 y^5
5 s^3
6 z^6
7 $a \times a \times a$
8 $x \times x \times x \times x \times x$
9 $a \times a \times a \times a \times a$
10 $b \times b$
11 $x \times x \times x \times x \times x \times x \times x$
12 $z \times z \times z \times z$
13 $2a$
14 $4x^2$
15 $12a$
16 a^2b
17 $15xz^2$
18 $5a^2b^2$
19 $3 \times z \times z$
20 $2 \times a \times b \times c$
21 $4 \times z \times y \times y$
22 $6 \times a \times a \times b$
23 $2 \times x \times x \times x$
24 $3 \times a \times a \times a \times a \times b \times b$
25 $6xz$
26 $6x^3$
27 $12a^2$
28 $6a^3$
29 $2a^2bc$
30 $24x^2y$
31 z^4
32 $6z^2$
33 $24x^2$
34 $16x$
35 $4s^3$
36 x^6
37 y^2z^2
38 $10xyz$
39 a^7
40 $8x^4$
41 $axyz$
42 s^7

Exercise 13g page 254

1 a 5 b −3 c −9
2 a 7 b 0 c −4
3 a 20 b 10 c −4
4 a 1 b −1 c 7

5	**a** 10	**b** 13	**c** 1
6	**a** 2	**b** 0	**c** 10
7	**a** 10	**b** 1	**c** −4
8	**a** 5	**b** −1	**c** −7
9	**a** 8	**b** 50	**c** 8
10	**a** 8	**b** 27	**c** −8

11 $\frac{3}{4}$	**13** $3\frac{1}{4}$	**15** 15	**17** 16	**19** 20
12 24	**14** 190	**16** 30	**18** $-1\frac{1}{6}$	**20** −23

Exercise 13h page 255

1 D	**4** D
2 D	**5** D
3 B	**6** C

Exercise 13i page 256

1 $4x + 13$ **2** $60abc$ **3** $7x + 12$ **4** −24 **5** a^6

Exercise 13j page 256

1 $7 + 2x$	**4** $3x - 10$
2 $16x + 14$	**5** 18

3 x multiplied by itself five times

Exercise 13k page 256

1 $22 - x + y$	**4** 23
2 $\frac{7}{12}$	**5** $24x^2y$

3 $36a^2$

CHAPTER 14

Exercise 14a page 259

1 $x - 3 = 4, 7$	**5** $2x = 8, 4$
2 $x + 1 = 3, 2$	**6** $7x = 14, 2$
3 $3 + x = 9, 6$	**7** $3x = 15, 5$
4 $x - 5 = 2, 7$	**8** $6x = 24, 4$

9–16 student's own answers

Exercise 14b page 261

1 8	**7** 6	**13** 10	**19** 5
2 9	**8** 6	**14** 3	**20** 12
3 2	**9** 5	**15** 8	**21** 12
4 7	**10** 7	**16** 10	**22** 3
5 4	**11** 3	**17** 9	**23** 2
6 5	**12** 1	**18** 12	**24** 9

Exercise 14c page 262

1 2	**10** 4	**19** 5	**28** 23
2 9	**11** 4	**20** 11	**29** 4
3 3	**12** 8	**21** 16	**30** 7
4 13	**13** 12	**22** 12	**31** 9
5 3	**14** 10	**23** 10	**32** 9
6 3	**15** 11	**24** 9	**33** 4
7 7	**16** 10	**25** 17	**34** 4
8 0	**17** 6	**26** 5	**35** 4
9 1	**18** 11	**27** 16	**36** 2

Exercise 14d page 263

1 2	**7** $\frac{1}{3}$	**13** 6	**19** 9
2 3	**8** 3	**14** 1	**20** 2
3 $2\frac{1}{2}$	**9** $1\frac{2}{5}$	**15** $\frac{1}{6}$	**21** $\frac{3}{4}$
4 3	**10** 20	**16** 2	**22** $1\frac{1}{5}$
5 4	**11** 2	**17** $1\frac{4}{5}$	**23** 5
6 $2\frac{1}{4}$	**12** $\frac{1}{2}$	**18** $3\frac{1}{2}$	**24** $\frac{1}{7}$

Exercise 14e page 263

1 4	**3** 2	**5** $1\frac{1}{5}$
2 12	**4** 1	**6** 3
7 8	**14** $3\frac{1}{3}$	**21** 11
8 16	**15** 5	**22** 0
9 $5\frac{1}{2}$	**16** $\frac{2}{7}$	**23** 5
10 13	**17** $2\frac{2}{3}$	**24** 20
11 8	**18** 7	**25** 30
12 16	**19** 2	**26** 30
13 6	**20** $1\frac{2}{3}$	**27** $\frac{1}{5}$

Exercise 14f page 264

1 4	**10** 7	**19** 2	**28** 3
2 3	**11** 5	**20** 2	**29** $2\frac{1}{5}$
3 2	**12** 3	**21** $1\frac{2}{3}$	**30** $\frac{1}{3}$
4 6	**13** 5	**22** $\frac{1}{2}$	**31** 6
5 3	**14** 2	**23** 4	**32** $\frac{1}{4}$
6 0	**15** 3	**24** 0	**33** 5
7 6	**16** 3	**25** 2	**34** $\frac{6}{7}$
8 5	**17** 0	**26** $3\frac{1}{3}$	
9 $2\frac{2}{3}$	**18** $1\frac{4}{5}$	**27** $2\frac{3}{7}$	

Exercise 14g page 265

1 4 **2** 1 **3** 5 **4** 3 **5** −2

Exercise 14h page 266

1 4	**5** 7	**9** 7	**13** 1
2 1	**6** −2	**10** 2	**14** 2
3 3	**7** 6	**11** 1	**15** 3
4 5	**8** 5	**12** 3	**16** 2

Exercise 14i page 267

1 $4x - 8 = 20, 7$	**6** $2x + 6 = 24, 9$
2 $6x - 12 = 30, 7$	**7** $2x + 6 = 20, 7$
3 $3x + 6 = 21, 5$	**8** $2x + 10 = 24, 7$
4 $x + 8 = 10, 2$	**9** $3x - 9 = 18, 9$
5 $3x + 7 = 28, 7$	**10** $2x + 9 = 31$, 11 cm

Exercise 14j page 269

1 2	**9** $\frac{2}{7}$	**17** 8
2 0	**10** 3	**18** $-1\frac{3}{4}$
3 $1\frac{3}{8}$	**11** $\frac{1}{3}$	**19** 3
4 3	**12** 2	**20** 2
5 1	**13** 2	**21** 3
6 2	**14** $4\frac{2}{5}$	**22** 2
7 3	**15** 4	**23** 8
8 $2\frac{1}{2}$	**16** $\frac{1}{3}$	**24** 2

Exercise 14k page 271

1 11	**4** $210
2 6	**5** 16
3 9 cm	

Exercise 14l page 272

1 C	**4** B
2 C	**5** A
3 B	**6** C

Exercise 14m page 273

1 2	**4** 4
2 1	**5** 3
3 $1\frac{1}{2}$	**6** 5

Exercise 14n page 273

1 $5\frac{1}{2}$	**3** 1	**5** \$400
2 $\frac{2}{5}$	**4** 5	**6** 100°

REVIEW TEST 3 page 275

1 A	**4** A	**7** B	**10** C
2 D	**5** C	**8** C	**11** D
3 C	**6** A	**9** A	**12** A

13 **a** 12 : 7 **b** $\frac{7}{12}$ **c** \$144 and \$336 **d** 1:50 000
14 **a** $\frac{3}{5}$ **b** $\frac{5}{12}$ **c** \$210 and \$120, \$90
d Alf \$400, Bill \$600, Colin \$800, \$400
15 **a** $x = 3$ **b** 12 cm
16 **a** 25 **b** 6
17 mean 84, mode 85, median 84.5, range 15
18 **a** 5 **b** 5 **c** 6 **d** 6.48
19 **a** 5 **b** 5
20 **a** $x = 3$ **b** 2

CHAPTER 15

Exercise 15a page 279

1 **a** {rivers of the world} **c** {subjects I study at school}
b {students in my class} **d** {furniture in this room}
2 student's own answers

Exercise 15b page 280

1 {the last four letters of the alphabet}
2 {the months whose names begin with J}
3 {the 6th to 8th months of the year}
4 {the Windward Islands}
5 {three islands in the Leeward Islands}
6 {even numbers less than 13}
7 {the first six whole numbers}
8 {the first six prime numbers}
9 {the whole numbers from 45 to 50 inclusive}
10 {multiples of 5 from 15 to 35 inclusive}
11 {boys' names}
12 {outerwear}
13 {breakfast cereals}
14 {Caribbean plants}
15 {plays by Shakespeare}
16 {11, 12, 13, 14, 15}
17 {a, b, c, d, e, f, g, h}
18 {a, c, e, h, i, m, s, t}
19 {Grenada, St Vincent, St Lucia, Dominica}
20 {Antigua, St Kitts, Montserrat, Guardaloupe}
21 student's own answers
22 {Arctic, Antarctic, Pacific, Atlantic, Indian}
23 student's own answers
24 {2, 3, 5, 7, 11, 13, 17, 19}
25 {2, 4, 6, 8, 10, 12, 14, 16, 18}
26 {21, 23, 25, 27, 29}
27 {12, 15, 18, 21, 24, 27, 30}
28 {21, 28, 35, 42, 49}
29 {St George's, Kingstown, Castries, Roseau}
30 {Guyana, Trinidad, Jamaica, Barbados, Grenada, St Vincent, St Lucia, Dominica, Antigua, St Kitts, Montserrat}

Exercise 15c page 281

1 apple ∈ {fruit}
2 shirt ∈ {clothing}
3 dog ∈ {domestic animals}
4 geography ∈ {school subjects}
5 carpet ∈ {floor coverings}
6 hairdressing ∈ {occupations}

Exercise 15d page 282

1 orange ∉ {animals}
2 cat ∉ {fruit}
3 table ∉ {trees}
4 shirt ∉ {subjects I study}
5 Anne ∉ {boys' names}
6 chisel ∉ {buildings}
7 cup ∉ {bedroom furniture}
8 cherry ∉ {Japanese cars}
9 aeroplane ∉ {foreign countries}
10 curry ∉ {breeds of dogs}
11 porridge ∈ {breakfast cereals}
12 electricity ∉ {building materials}
13 water ∉ {metals}
14 spider ∈ {living things}
15 Saturday ∈ {days of the week}
16 salmon ∈ {fish}
17 August ∉ {days of the week}
18 Spain ∈ {European countries}
19 Brazil ∉ {Asian countries}
20 football is a member of the set of team games
21 shoes are not a member of the set of beverages
22 hockey is not a member of the set of electrical appliances
23 needle is a member of the set of metal objects
24 Danielle is not a member of the set of boys' names
25, 26 student's own answers

Exercise 15e page 284

1 Yes	**2** No	**3** Yes	**4** Yes	**5** Yes
6 Yes	**7** student's own answer	**8** No		

Exercise 15f page 284

2 **a, c, d**

Exercise 15g page 285

1 {x, y}, {w, x}, {y, z}, {w, y}, {x, z}, {w, z}
2 {A, L}, {A, L, K, B}, {A, L, K}, {A, L, B}
3 $A = \{1, 3, 5, 7, 9\}$ $B = \{2, 4, 6, 8, 10\}$
$C = \{2, 3, 5, 7\}$
5 **a**, **b**, **c**, **d**

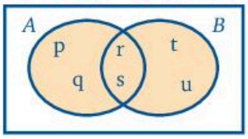
Exercise 15h page 286

1 {whole numbers}
2 {letters of the alphabet}
3 {rivers}
4 {pupils}
5 {cats}
6 {birds}
7–12 student's own answers
13 {geometrical instruments}
14 {dwellings}
15 {vehicles}
16 {footwear}
17 {sportsmen and sportswomen}

Exercise 15i page 289

1 {Peter, James, John, Andrew, Paul}
2 {3, 4, 6, 8, 9, 12, 16}
3 {a, b, c, d, e, i, o, u}
4 {a, b, c, x, y, z}
5 {p, q, r, s, t}
6 {1, 2, 3, 5, 7}
7 {5, 6, 7, 8, 10, 11, 12, 13}
8 {1, 2, 3, 4, 5, 6, 10, 12}
9 {a, c, h, l, m, o, r, s}
10 {a, b, c, e, g, h, i, l, m, r, t}

Exercise 15j page 290

1


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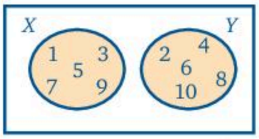

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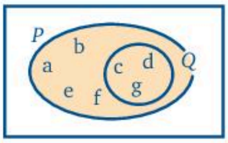

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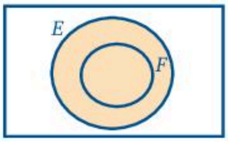

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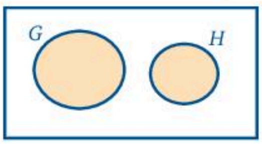

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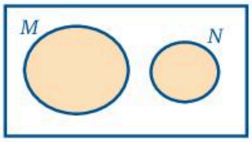

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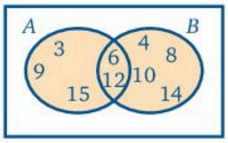

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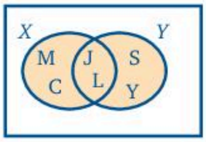
9


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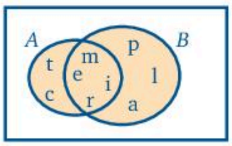


Exercise 15k page 291

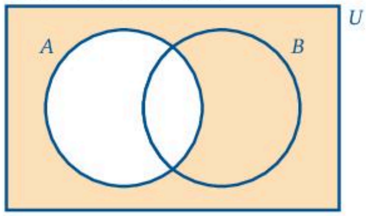
1 {6, 9}
2 {4, 12, 20}
3 {Alice, Zane}
4 {o}
5 {cabbage, tomato}
6 {3, 5, 7, 11}
7 {8, 16}
8 {2}
9 {e, t, w}
10 {m, e, i, r}

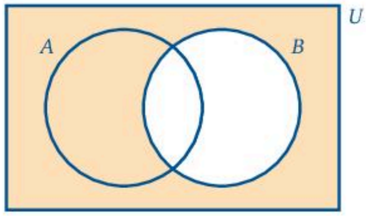
Exercise 15l page 292

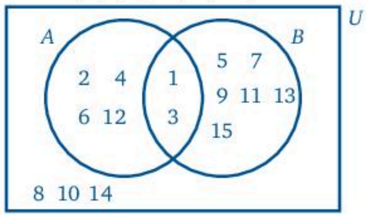
Student Venn diagrams should show the following intersections:

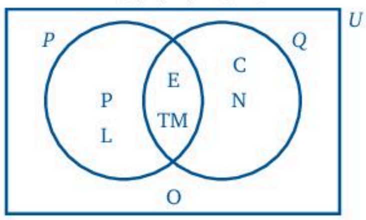
1 {3, 5, 7}
2 {Dino, John}
3 {o, u}
4 {oak, elm}
5 {boxer}
6 {8, 16}
7 {2, 4}
8 {t, i, n}
9 {r, t, m, e}
10 {3, 5, 7}

Exercise 15m page 293

1 $A' = \{10, 20\}$
2 $B' = \{5, 6, 11\}$
3 V' = {consonants}
4 P' = {vowels}
5 A' = {Tuesday, Thursday, Saturday, Sunday}
6 X' = {adults}
7 M' = {non-British motor cars}
8 S' = {female tennis players}
9 C' = {Caribbean towns not in Jamaica}
10 D' = {quadrilaterals that are not squares}
11 E' = {adults 80 and under}
12 F' = {male doctors}
13 U = {homes}
14 U = {letters of the alphabet}
15 U' = {a, b, c, d, e, f, g, h, i, j}
16 a


b


17 a $A' = \{2, 4, 6, 8, 10, 12\}$
b $B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$
c $A \cup B = \{1, 3, 5, 6, 7, 9, 11, 12\}$
d Complement of $A \cup B = \{2, 4, 8, 10\}$
18 a $A = \{5, 7, 11, 13\}$
b $A' = \{6, 8, 9, 10, 12, 14\}$
19


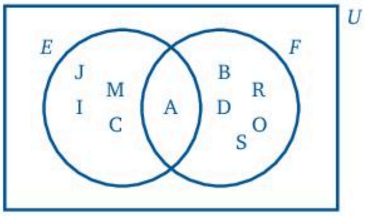
a $A' = \{5, 7, 8, 9, 10, 11, 13, 14, 15\}$
b $B' = \{2, 4, 6, 8, 10, 12, 14\}$
c $A' \cap B' = \{8, 10, 14\}$
d $A' \cup B' = \{1, 2, \ldots, 14, 15\}$
20


a $P' = \{C, N, O\}$
b $Q' = \{L, O, P\}$
c $P' \cap Q' = \{O\}$
d $P' \cup Q' = \{C, E, L, M, N, O, P\}$

Exercise 15n page 295

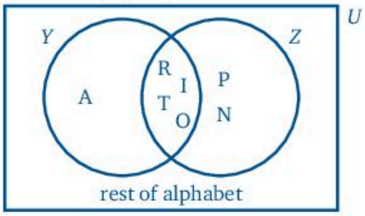
1 a Jupiter ∈ {set of planets} b saucer ∉ {motorcars}
2 no
3 a {Argentina, Brazil, Chile, Colombia, Ecuador}
b {Alberta, Manitoba, Nova Scotia, Ontario, Quebec}
4 {d, e, f, g, h, i}

5



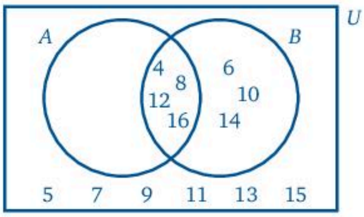
6 **a** $A \cup B = \{6, 8, 10, 12, 14, 16, 24\}$
b $A \cap B = \{8, 12\}$

7



a $Y' = \{B, C, D, E, F, G, H, J, K, L, M, N, P, Q, S, U, V, W, X, Y, Z\}$
b $Z' = \{A, B, C, D, E, F, G, H, J, K, L, M, Q, S, U, V, W, X, Y, Z\}$
c $Y \cup Z = \{A, I, O, P, R, T, N\}$
d $(Y \cup Z)' =$ {letters of the alphabet except A, I, O, P, R, T, N}

8



a $A' = \{5,6,7,9,10,11,13,14,15\}$
b $B' = \{5,7,9,11,13,15\}$
c $A \cup B = \{4,6,8,10,12,14,16\}$
d $A' \cup B' = \{5,6,7,9,10,11,13,14,15\}$
e $(A \cup B)' = \{5,7,9,11,13,15\}$

CHAPTER 16

Exercise 16a page 298

1 8 **2** 20
3 For example: $4 \times \$500$, $4 \times \$100$, $1 \times \$50$; $3 \times \$500$, $8 \times \$100$, $3 \times \$50$
4 For example: $9 \times \$100$ or $8 \times \$100$ and $2 \times \$50$

Exercise 16b page 299

1 24 **4** $58 **7** $10 152
2 12 **5** 95 **8** $4100
3 125 **6** $3960 **9** $11.50
10 **a** $3.3 **b** $330

Exercise 16c page 301

1 The 3-pack for $924 is $308 each
2 The 6-pack is better value at $66.67 each. The 4-pack is $70 each.
3 large pack is $3.33 each, the smaller pack $3.125 each so the smaller pack is the better value.
4 smaller container is $0.380/g, the larger $0.325/g so the larger container is better value
5 standard is $0.78/m, jumbo $0.75/m so the jumbo is better value.
6 The litre container costs $220 whereas two half-litre containers cost $230 so the litre is better value.
7 smaller costs $0.83/g, larger $0.79/g so the larger is better value.

Exercise 16d page 303

1 $520, profit
2 $2400, profit
3 $1120 000, loss
4 $1280, loss
5 $2840, loss
6 $4500, loss
7 $7000, loss
8 $6800, profit
9 25%
10 30%
11 25%
12 10%
13 20%
14 20%
15 15%
16 24%
17 $5600
18 $7200
19 $6090
20 $1800
21 $2700
22 $8000
23 30%
24 20%
25 10%
26 15%
27 12%
28 **a** $29 900
b $7475
29 **a** **i** $1500 **ii** $1000
b profit $500

Exercise 16e page 306

1 US$ 6.16
2 US$127.05
3 £551
4 £368.01
5 Barb$86.9
6 Barb$1226.08
7 TT$17 188.16
8 Can$254.52
9 J$7143 (nearest $)
10 J$64 935 (nearest $)
11 J$584 416
12 US$4.27 (3s.f.)
13 US$57.75 (3s.f)
14 US$653 (3s.f.)
15 J$7759
16 J$60 345
17 J$206 897
18 £19.89
19 £72.50
20 £375.84

Exercise 16f page 307

1 $14 400 **2** 40% **3** $28 200
4 **a** $90 750 **b** Can$730 (3 s.f.)
5 **a** Barb$122.45 **b** J$35 443

Exercise 16g page 308

1 **a** $4560 **b** $2040 **c** 8
2 $33\frac{1}{3}\%$
3 **a** $3680 **b** 34.8% (3 s.f.)
4 **a** $\$13\frac{1}{2}$ **b** 32 c 45
5 **a** $262.50 **b** $1518.00
6 **a** TT$348.30 **b** J$86 434

CHAPTER 17

Exercise 17a page 311

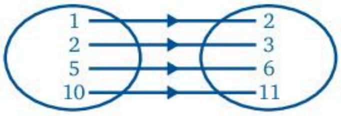
1 The second number is one more than the first.
2 The second number is two more than the first.
3 The second number is the square of the first.
4 {(Maths, Spanish), (Science, Spanish), (Science, Maths)}
5 {(4, □), (5, ⌂), (3, ▲)}
6 {(Tnd, Jma), (Bds, Jma), (St Lucia, Jma), (Bds, Tnd), (St Lucia, Tnd), (St Lucia, Bds)}
7 25, 8
8 7, 2, 17

Exercise 17b page 312

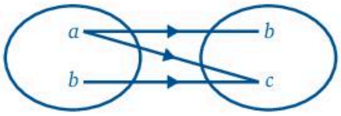
1 {1, 2, 5, 10}, {2, 3, 6, 11}
2 **a** {a, b}, {b, c} **b** {▭, △, ▱}, {□, △}
3 {4, 16, 36}
4 **a** {(Dwayne, Fred), (Scott, Fred), (Scott, Dwayne)},
b {Dwayne, Scott}, {Fred, Dwayne}
5 **a** {10°, 150°, 45°, 175°}, {acute, obtuse}
b {t, s}, {t, u, w}

Exercise 17c page 313

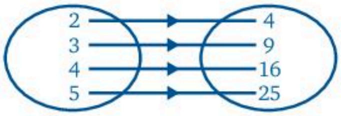
1 a



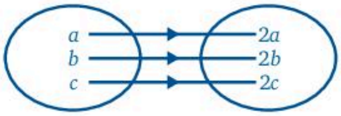
b



c

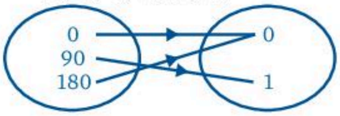


d



2 a {(1, 4), (3, 8), (5, 14)}
b {(1, 2), (2, 2), (3, 2), (4, 5)}
c {(a, b), (a, c), (b, c)}
d {(a, b), (a, d), (b, c)}

3



Exercise 17d page 316

1 a 1 : 1 **b** $n : n$ **c** 1 : 1 **d** 1 : 1
2 a 1 : 1 **b** n : 1 **c** $n : n$ **d** 1 : n
3 n : 1

Exercise 17e page 316

1 a

x	1	2	3	4
y	2	4	6	8

b

x	2	4	6	9
y	1	2	3	4.5

c

x	10	7	5	0
y	0	3	5	10

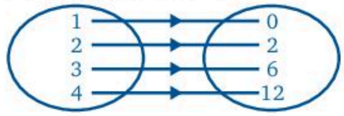
d

x	0	1	2	3	4
y	0	4	6	8	6

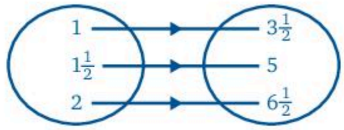
2 a 1 : 1 **b** 1 : 1 **c** 1 : 1 **d** n : 1
3 1 : n, one value of x maps to different values of y
4 a 1 : n, one value of x maps to different values of y
b $n : n$, two values of x map to one value of y and one value of x maps to two values of y
c 1 : 1, one value of x maps to one value of y

Exercise 17f page 318

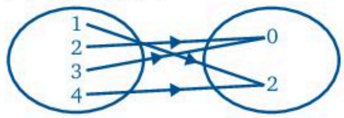
1 missing values: 3, 9
2 missing values: 3, 11
3 missing values: 8, 4, 2
4 missing values: 10, 8, 0
5 missing values: 2, 5, 17
6 missing values: 1, 7, 31
7 missing values: 2, 4, 8
8 a missing values: 0, 6, 12
b {1, 2, 3, 4}, {0, 2, 6, 12}
c



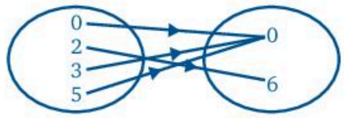
d 1 : 1
9 a missing values: 5, $6\frac{1}{2}$
b {1, $1\frac{1}{2}$, 2}, {$3\frac{1}{2}$, 5, $6\frac{1}{2}$}
c



d 1 : 1
10 a missing values: 2, 2
b {1,2, 3, 4}, {0, 2}
c



d n : 1
11 a missing values: 0, 6, 0
b {0, 2, 3, 5}, {0, 6}
c



d n : 1

Exercise 17g page 320

1 A **2** C **3** B **4** D **5** D **6** C **7** D **8** B **9** B **10** C

CHAPTER 18

Exercise 18a page 325

1 A(2, 2) B(5, 2) C(7, 6) D(4, 5) E(7, 0) F(9, 4) G(0, 8) H(5, 8)
4 square
5 isosceles triangle
6 rectangle
7 square
8 isosceles triangle
9 (2, 5)
10 (7, 1)
11 (4, 1)
12 (5, 4)
13 (3, 7)
14 (2, 3)

Exercise 18b page 328

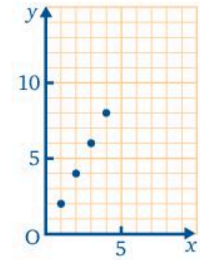
1 2, 3, 6, 1, −5, −3, 5, −3, −5, 5, 0
2 2, −2, 5, −4, 2, −2, −3, 5, −5, 5
5 square
6 isosceles triangle
7 rectangle
8 right-angled
9 (−1, 1)
10 (1, −2)
11 (−1, 3)
12 (−6, −1)
13 (−5, 1)
14 (0, −1)
15 (3, 2)
16 (−1, 2)
17 (−1, 3)
18 (1, 0)
19 (4, 2)
20 (2, −1)
21 $(-\frac{7}{2}, 3)$
22 (−3, −1)
23 (−5, −2)
24 $(4, \frac{3}{2})$

25 $(-1, 3)$
26 $(-1, 0)$
27 $(0, 0)$
28 $(-1, 0)$

Exercise 18c page 330

1 a

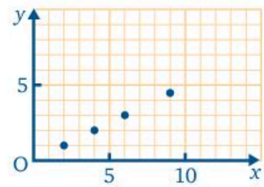
x	1	2	3	4
y	2	4	6	8



1 : 1

b

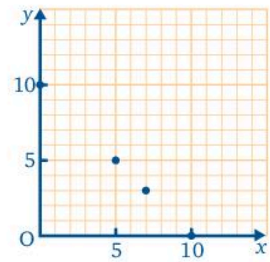
x	2	4	6	9
y	1	2	3	4.5



1 : 1

c

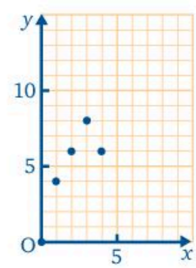
x	10	7	5	0
y	0	3	5	10



1 : 1

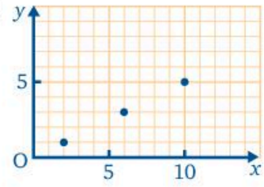
d

x	0	1	2	3	4
y	0	4	6	8	6



1 : 1

2



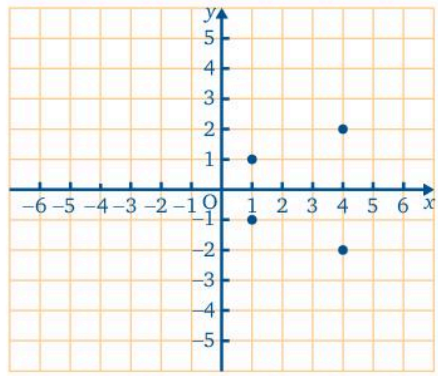
1 : 1

3 a

x	1	4	7	10	13
y	12	10	8	6	4

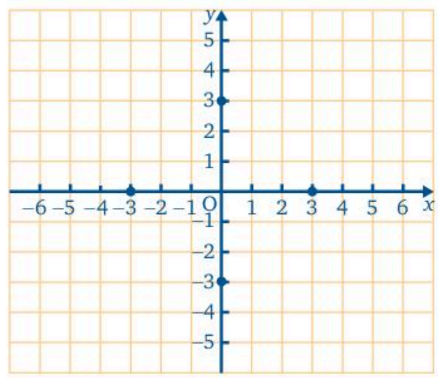
b $\{(1, 12), (4, 10), (7, 8), (10, 6), (13, 4)\}$
c 1 : 1

4 a



b no **c** $1 : n$

5 a



b no **c** $n : n$

6 a

x	0	2	4	6	10
y	1	2	3	4	6

b $\frac{1}{2}(x\text{-coordinate}) + 1$
c 5
d $3\frac{1}{2}, 9, \frac{1}{2}a + 1$

7 a

x	0	2	4	7	8
y	7	6	5	3.5	3

b $7 - \frac{1}{2}(x\text{-coordinate})$ **c** 4.5
d $6\frac{1}{2}, 1, 7 - \frac{1}{2}a$

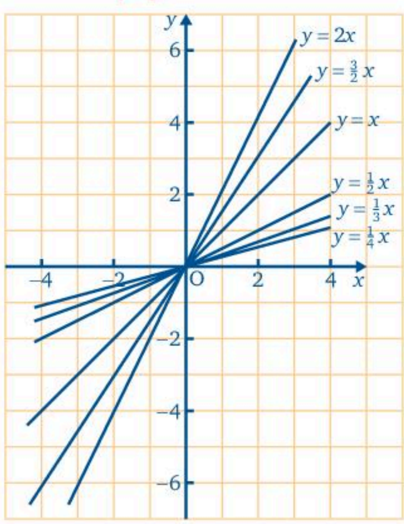
8 **a** parallelogram **d** both
c no **e** no
9 **a** square **d** both
c yes **e** yes
10 **a** trapezium **d** neither
c no **e** no
11 **a** rhombus **d** both
c no **e** yes
12 **a** rectangle **d** both
c yes **e** no
13 rectangle, square
14 rhombus, square
15 parallelogram, rectangle, rhombus, square

Exercise 18d page 334

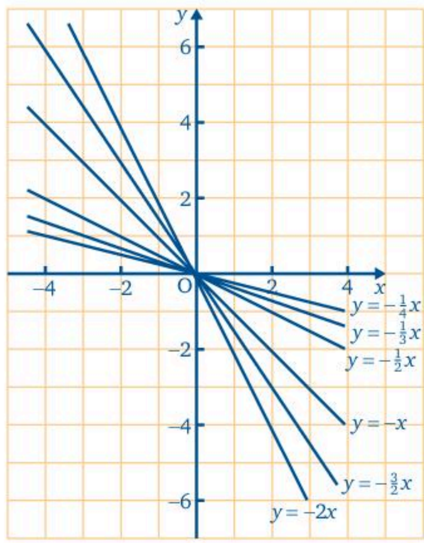
1 **a** 2 **b** 3 **c** 7 **d** 12
2 **a** −1 **b** −6 **c** −8 **d** −20
3 **a** $-3\frac{1}{2}$ **b** $4\frac{1}{2}$ **c** −6.1 **d** 8.3
4 **a** −7 **b** 2 **c** $-5\frac{1}{2}$ **d** 4.2
5 **a** 10 **b** −8 **c** 7 **d** −5.2
6 **a** −1 **b** 3 **c** −2 **d** $\frac{4}{3}$
7 **a** 12 **b** −24 **c** 1 **d** −16.4
8 **a** −2 **b** 4 **c** $-\frac{3}{2}$ **d** $\frac{3}{4}$
9 $a = -5, b = 3, c = -4$
10 $a = -2, b = 8, c = 18$
11 $y = 3x$ **13** $y = -\frac{1}{3}x$
12 $y = -2x$ **14** $y = \frac{2}{3}x$
15 (−2, −4), (6, 12)
16 (−2, 6), (1, −3}, (8, −24)
17 **a** above: (2, 2), (−2, 1), (−4.2, −2)
b below: (3, 0)

Exercise 18e page 336

1–6



7–12



CHAPTER 19

Exercise 19a page 339

1 4000 m^3
2 500 mm^3
3 2300 mm^3
4 78 mm^3
5 3000 000 cm^3
6 250 000 cm^3
7 8 200 000 cm^3
8 73 000 cm^3
9 1400 cm^3
10 750 cm^3
11 54 000 cm^3
12 43 cm^3
13 4.5 *l*
14 56 *l*
15 98 *l*
16 0.036 *l*
17 **a** 0.4 cm^3 **b** 250 000 mm^3 **c** 70 000 cm^3

Exercise 19b page 340

1 48 cm^3
2 1600 mm^3
3 5400 mm^3
4 16 mm^3
5 31.72 m^3
6 10.5 cm^3
7 24 m^3
8 160 m^3
9 12 cm^3
10 7.2 cm^3
11 4.32 m^3
12 0.756 m^3
13 64 cm^3
14 125 cm^3
15 8 m^3
16 $\frac{1}{8}$ cm^3
17 15.625 cm^3
18 27 km^3
19 512 km^3
20 $3\frac{3}{8}$ km^3
21 39.304 m^3

Exercise 19c page 341

1 8
2 6
3 8
4 12

Exercise 19d page 342

1 30 cm^3
2 2 m^3
3 800 cm^3
4 600 cm^3
5 5760 mm^3
6 40000 cm^3
7 28 cm^3
8 8m^3
9 17.5 cm^3
10 1800 cm^3

Exercise 19e page 343

1 60 m^3
2 7776 cm^3
3 6480 m^3
4 125
5 48
6 300 m^3; 300 000 litres
7 60
8 9000
9 64
10 1600

Exercise 19f page 344

1 a 3200 000 cm^3 **b** 3 200 000 000 mm^3
2 1600 cm^3
3 64 cm^3
4 50 000 cm^3
5 13 500 mm^3

Exercise 19g page 345

1 a 8000 mm^3 **b** 0.000 008 m^3
2 3.5 litres
3 300 cm^3
4 0.512 cm^3
5 120 000 cm^3

Exercise 19h page 345

1 a 9000 cm^3 **b** 9 000 000 mm^3
2 440 cm^3
3 216 cm^3
4 288 cm^3
5 2400 litres

Exercise 19i page 345

1 0.0009 m^3
2 10.8 litres
3 75 litres
4 8 cm^3
5 1.2 m^3

REVIEW TEST 4 page 347

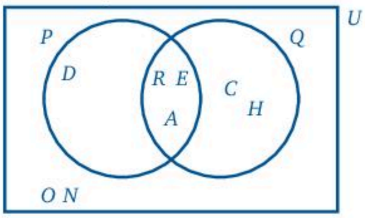
1 D **2** D **3** B **4** B **5** C **6** A **7** B **8** B **9** D **10** C **11** D **12** B

13 a

x	0	2	6	10	12
y	8	7	5	3	2

b $y = 8 - \frac{1}{2}x$ **c** 6 **d** $4, 5\frac{1}{2}, 13\frac{1}{2}$

14

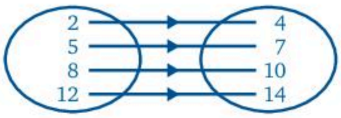


a {C, H, O, N} **b** {D, O, N} **c** {O, N} **d** {O, N}

15 a $P \cap Q = \{1, 2, 3, 6\}$
b $P \cup Q = \{1, 2, 3, 4, 6, 7, 8, 14, 18, 21, 36, 42\}$

16 {(1, 4), (2, 6), (3, 6), (5, 6)}

17 a

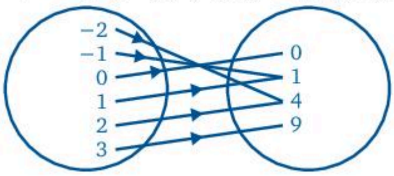


b

x	2	3	4	5
y	5	6	7	8

18 The missing values for y are 9, 17, 21

19 a The missing values for y are 4, 0, 1, 4, 9
b Domain {–2, –1, 0, 1, 2, 3}, Range {0, 1, 4, 9}
c



20 a $U = \{p, q, r, s, t, u, v\}$
b Uranus ∈ {planets]
c $A \cup B = \{p, q, r, s, t, u\}$

REVIEW TEST 5 page 350

1 A **2** B **3** C **4** B **5** A **6** D **7** B **8** B **9** C **10** C **11** B **12** C

13 a –4 **b** 4 **c** –5 **d** –4.8

14 a 1 **b** –2 **c** $1\frac{1}{2}$ **d** 1.2

15 a $x = 14$ **b** $x = 3$ **c** $x = 2$ **d** $x = 1$

16 a 5
b $3x + 580 = 2680$, she spent $700 in the first shop

17 a $n + 2$ **b** $3m + 6$ **c** $x = 7$

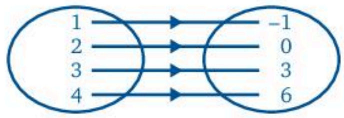
18 a $\frac{9}{20}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}$ **b** 0.000 2786

19 a

Mark	1	2	3	4	5	6	7	8	9	10
Frequency	1	2	1	3	5	0	3	2	2	3

b 5

20 a missing values are –1, 3, 6
b Domain {1, 2, 3, 4}, Range {–1, 0, 3, 6}
c



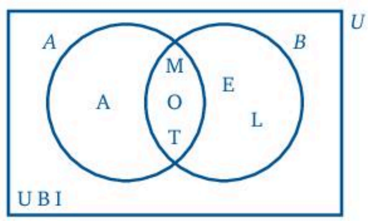
d 1 : 1

21 a 2000
b 100
c $620 000

22 4 cm

23 a 384 cm^2
b 524 cm^2
c 1120 cm^3

24



a B, E, I, L, U
b A, B, I, U
c A, B, E, I, L, U
d {U, B, I}

Index

STP Mathematics for Jamaica

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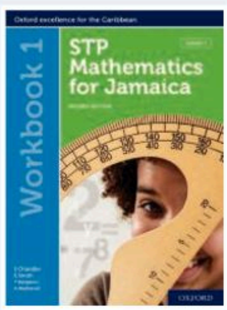
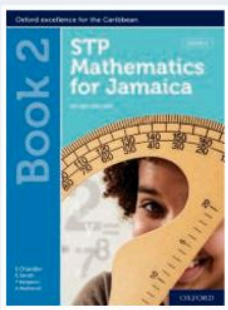
SECOND EDITION

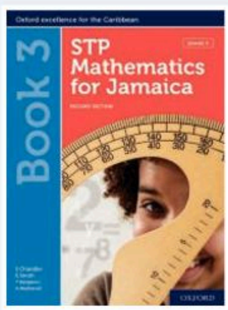
The second edition of **STP Mathematics for Jamaica** provides comprehensive coverage of the National Standards Curriculum. Maintaining its rigorous and authoritative approach, the course uses modern methods to develop problem-solving skills. It prepares students for a high level of achievement in Lower Secondary Mathematics and lays a solid foundation for study at CSEC® level. Written by a team of subject experts, this curriculum-aligned course fully supports the syllabus in Jamaica.

Features and benefits:

- Test progress and focus learning with clear objectives and end-of-chapter summaries
- 'Did you know?' questions throughout show mathematics in real-life contexts and develop problem-solving skills
- Encourage students to make synoptic links with carefully graded practice questions and varied activities to stretch and challenge comprehension
- Worked examples to illustrate and extend all topics
- Stimulate all levels of ability with investigations and puzzles
- Hone key skills and understanding with accompanying workbooks







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