29

Interference

29-1 Electromagnetic waves

In this chapter we shall discuss the subject of the preceding chapter more mathematically. We have qualitatively demonstrated that there are maxima and minima in the radiation field from two sources, and our problem now is to describe the field in mathematical detail, not just qualitatively.

We have already physically analyzed the meaning of formula (28.6) quite satisfactorily, but there are a few points to be made about it mathematically. In the first place, if a charge is accelerating up and down along a line, in a motion of very small amplitude, the field at some angle θ from the axis of the motion is in a direction at right angles to the line of sight and in the plane containing both the acceleration and the line of sight (Fig. 29–1). If the distance is called r, then at time t the electric field has the magnitude

$$E(t) = \frac{-qa(t - r/c)\sin\theta}{4\pi\epsilon_0 c^2 r},\qquad(29.1)$$

where $a(t - r/c)$ is the acceleration at the time $(t - r/c)$, called the *retarded* acceleration.

Now it would be interesting to draw a picture of the field under different conditions. The thing that is interesting, of course, is the factor $a(t - r/c)$, and to understand it we can take the simplest case, $\theta = 90°$, and plot the field graphically. What we had been thinking of before is that we stand in one position and ask how the field there changes with time. But instead of that, we are now going to see what the field looks like at different positions in space at a given instant. So what we want is a "snapshot" picture which tells us what the field is in different places. Of course it depends upon the acceleration of the charge. Suppose that the charge at first had some particular motion: it was initially standing still, and it suddenly accelerated in some manner, as shown in Fig. 29–2, and then stopped. Then, a little bit later, we measure the field at a different place. Then we may assert that the field will appear as shown in Fig. 29–3. At each point the field is determined by the acceleration of the charge at an earlier time, the amount earlier being the delay r/c. The field at farther and farther points is determined by the acceleration at earlier and earlier times. So the curve in Fig. 29–3 is really, in a sense, a "reversed" plot of the acceleration as a function of time; the distance is related to time by a constant scale factor c, which we often take as unity. This is easily seen by considering the mathematical behavior of $a(t - r/c)$. Evidently, if we add a little time Δt, we get the same value for $a(t - r/c)$ as we would have if we had subtracted a little distance: $\Delta r = -c\,\Delta t$.

Stated another way: if we add a little time Δt, we can restore $a(t - r/c)$ to its former value by *adding* a little distance $\Delta r = c\,\Delta t$. That is, as time goes on *the field moves as a wave outward from the source.* That is the reason why we sometimes say light is propagated as waves. It is equivalent to saying that the field is delayed, or to saying that the electric field is moving outward as time goes on.

An interesting special case is that where the charge q is moving up and down in an oscillatory manner. The case which we studied experimentally in the last chapter was one in which the displacement x at any time t was equal to a certain constant x_0, the magnitude of the oscillation, times $\cos\omega t$. Then the acceleration is

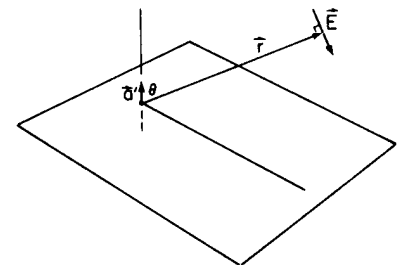$$a = -\omega^2 x_0 \cos\omega t = a_0 \cos\omega t,\qquad(29.2)$$

Fig. 29–1. The electric field **E** due to a positive charge whose retarded acceleration is a'.
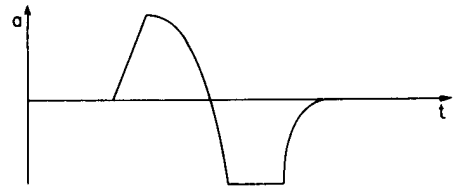

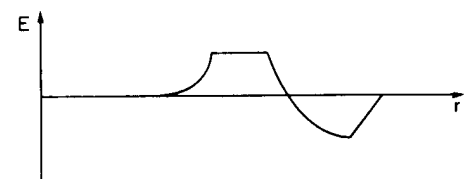Fig. 29–2. The acceleration of a certain charge as a function of time.

Fig. 29–3. The electric field as a function of position at a later time. (The $1/r$ variation is ignored.)

where a_0 is the maximum acceleration, $-\omega^2 x_0$. Putting this formula into (29.1), we find

$$E = -q \sin \theta \, \frac{a_0 \cos \omega(t - r/c)}{4\pi\epsilon_0 rc^2}.$$ (29.3)

Now, ignoring the angle θ and the constant factors, let us see what that looks like as a function of position or as a function of time.

29–2 Energy of radiation

First of all, at any particular moment or in any particular place, the strength of the field varies inversely as the distance r, as we mentioned previously. Now we must point out that the *energy* content of a wave, or the energy effects that such an electric field can have, are proportional to the *square* of the field, because if, for instance, we have some kind of a charge or an oscillator in the electric field, then if we let the field act on the oscillator, it makes it move. If this is a linear oscillator, the acceleration, velocity, and displacement produced by the electric field acting on the charge are all proportional to the field. So the kinetic energy which is developed in the charge is proportional to the *square* of the field. So we shall take it that the energy that a field can deliver to a system is proportional somehow to the square of the field.
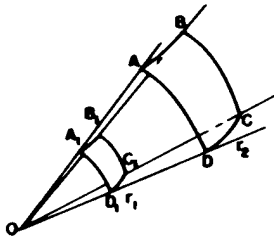
This means that the energy that the source can deliver decreases as we get farther away; in fact, it varies *inversely as the square of the distance*. But that has a very simple interpretation: if we wanted to pick up all the energy we could from the wave in a certain cone at a distance r_1 (Fig. 29–4), and we do the same at another distance r_2, we find that the amount of energy per unit area at any one place goes inversely as the square of r, but the area of the surface intercepted by the cone goes *directly* as the square of r. So the energy that we can take out of the wave within a given conical angle is the same, no matter how far away we are! In particular, the total energy that we could take out of the whole wave by putting absorbing oscillators all around is a certain fixed amount. So the fact that the amplitude of E varies as $1/r$ is the same as saying that there is an energy flux which is never lost, an energy which goes on and on, spreading over a greater and greater effective area. Thus we see that after a charge has oscillated, it has lost some energy which it can never recover; the energy keeps going farther and farther away without diminution. So if we are far enough away that our basic approximation is good enough, the charge cannot recover the energy which has been, as we say, radiated away. Of course the energy still exists somewhere, and is available to be picked up by other systems. We shall study this energy "loss" further in Chapter 32.

Let us now consider more carefully how the wave (29.3) varies as a function of time at a given place, and as a function of position at a given time. Again we ignore the $1/r$ variation and the constants.

Fig. 29–4. The energy flowing within the cone OABCD is independent of the distance r at which it is measured.

29–3 Sinusoidal waves

First let us fix the position r, and watch the field as a function of time. It is oscillatory at the angular frequency ω. The angular frequency ω can be defined as the *rate of change of phase with time* (radians per second). We have already studied such a thing, so it should be quite familiar to us by now. The *period* is the time needed for one oscillation, one complete cycle, and we have worked that out too; it is $2\pi/\omega$, because ω times the period is one cycle of the cosine.

Now we introduce a new quantity which is used a great deal in physics. This has to do with the opposite situation, in which we fix t and look at the wave as a function of distance r. Of course we notice that, as a function of r, the wave (29.3) is also oscillatory. That is, aside from $1/r$, which we are ignoring, we see that E oscillates as we change the position. So, in analogy with ω, we can define a quantity called the *wave number*, symbolized as k. This is defined as *the rate of change of phase with distance* (radians per meter). That is, as we move in space at a fixed time, the phase changes.

There is another quantity that corresponds to the period, and we might call it the period in space, but it is usually called the wavelength, symbolized λ. The wavelength is the distance occupied by one complete cycle. It is easy to see, then, that the wavelength is $2\pi/k$, because k times the wavelength would be the number of radians that the whole thing changes, being the product of the rate of change of the radians per meter, times the number of meters, and we must make a 2π change for one cycle. So $k\lambda = 2\pi$ is exactly analogous to $\omega t_0 = 2\pi$.

Now in our particular wave there is a definite relationship between the frequency and the wavelength, but the above definitions of k and ω are actually quite general. That is, the wavelength and the frequency may not be related in the same way in other physical circumstances. However, in our circumstance the rate of change of phase with distance is easily determined, because if we call $\phi = \omega(t - r/c)$ the phase, and differentiate (partially) with respect to distance r, the rate of change, $\partial\phi/\partial r$, is

$$\left| \frac{\partial\phi}{\partial r} \right| = k = \frac{\omega}{c}. \tag{29.4}$$

There are many ways to represent the same thing, such as

$$\lambda = ct_0 \tag{29.5} \qquad\qquad \lambda\nu = c \tag{29.7}$$

$$\omega = ck \tag{29.6} \qquad\qquad \omega\lambda = 2\pi c \tag{29.8}$$

Why is the wavelength equal to c times the period? That's very easy, of course, because if we sit still and wait for one period to elapse, the waves, travelling at the speed c, will move a distance ct_0, and will of course have moved over just one wavelength.

In a physical situation other than that of light, k is not necessarily related to ω in this simple way. If we call the distance along an axis x, then the formula for a cosine wave moving in a direction x with a wave number k and an angular frequency ω will be written in general as $\cos(\omega t - kx)$.

Now that we have introduced the idea of wavelength, we may say something more about the circumstances in which (29.1) is a legitimate formula. We recall that the field is made up of several pieces, one of which varies inversely as r, another part which varies inversely as r^2, and others which vary even faster. It would be worth while to know in what circumstances the $1/r$ part of the field is the most important part, and the other parts are relatively small. Naturally, the answer is "if we go 'far enough' away," because terms which vary inversely as the square ultimately become negligible compared with the $1/r$ term. How far is "far enough"? The answer is, qualitatively, that the other terms are of order λ/r smaller than the $1/r$ term. Thus, so long as we are beyond a few wavelengths, (29.1) is an excellent approximation to the field. Sometimes the region beyond a few wavelengths is called the "wave zone."

29–4 Two dipole radiators

Next let us discuss the mathematics involved in combining the effects of two oscillators to find the net field at a given point. This is very easy in the few cases that we considered in the previous chapter. We shall first describe the effects qualitatively, and then more quantitatively. Let us take the simple case, where the oscillators are situated with their centers in the same horizontal plane as the detector, and the line of vibration is vertical.

Figure 29–5(a) represents the top view of two such oscillators, and in this particular example they are half a wavelength apart in a N–S direction, and are oscillating together in the same phase, which we call zero phase. Now we would like to know the intensity of the radiation in various directions. By the intensity we mean the amount of energy that the field carries past us per second, which is proportional to the square of the field, averaged in time. So the thing to look at, when we want to know how bright the light is, is the square of the electric field, not the electric field itself. (The electric field tells the strength of the force felt by a
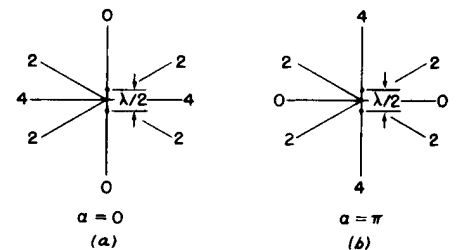

Fig. 29–5. The intensities in various directions from two dipole oscillators one-half wavelength apart. Left: in phase ($\alpha = 0$). Right: one-half period out of phase ($\alpha = \pi$).

stationary charge, but the amount of energy that is going past, in watts per square meter, is proportional to the square of the electric field. We shall derive the constant of proportionality in the next chapter.) If we look at the array from the W side, both oscillators contribute equally and in phase, so the electric field is twice as strong as it would be from a single oscillator. Therefore *the intensity is four times as strong as it would be if there were only one oscillator.* (The numbers in Fig. 29–5 represent how strong the intensity would be in this case, compared with what it would be if there were only a single oscillator of unit strength.) Now, in either the N or S direction along the line of the oscillators, since they are half a wavelength apart, the effect of one oscillator turns out to be out of phase by exactly half an oscillation from the other, and therefore the fields add to zero. At a certain particular intermediate angle (in fact, at 30°) the intensity is 2, and it falls off, 4, 2, 0, and so forth. We have to learn how to find these numbers at other angles. It is a question of adding two oscillations with different phases.

Let us quickly look at some other cases of interest. Suppose the oscillators are again one-half a wavelength apart, but the phase α of one is set half a period behind the other in its oscillation (Fig. 29–5b). In the W direction the intensity is now zero, because one oscillator is "pushing" when the other one is "pulling." But in the N direction the signal from the near one comes at a certain time, and that of the other comes half a period later. But the latter was *originally* half a period behind in timing, and therefore it is now exactly *in time* with the first one, and so the intensity in this direction is 4 units. The intensity in the direction at 30° is still 2, as we can prove later.

Now we come to an interesting case which shows up a possibly useful feature. Let us remark that one of the reasons that phase relations of oscillators are interesting is for beaming radio transmitters. For instance, if we build an antenna system and want to send a radio signal, say, to Hawaii, we set the antennas up as in Fig. 29–5(a) and we broadcast with our two antennas in phase, because Hawaii is to the west of us. Then we decide that tomorrow we are going to broadcast toward Alberta, Canada. Since that is north, not west, all we have to do is to reverse the phase of one of our antennas, and we can broadcast to the north. So we can build antenna systems with various arrangements. Ours is one of the simplest possible ones; we can make them much more complicated, and by changing the phases in the various antennas we can send the beams in various directions and send most of the power in the direction in which we wish to transmit, without ever moving the antenna! In both of the preceding cases, however, while we are broadcasting toward Alberta we are wasting a lot of power on Easter Island, and it would be interesting to ask whether it is possible to send it in only *one* direction. At first sight we might think that with a pair of antennas of this nature the result is always going to be symmetrical. So let us consider a case that comes out unsymmetrical, to show the possible variety.
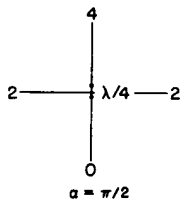
If the antennas are separated by one-quarter wavelength, and if the N one is one-fourth period behind the S one in time, then what happens (Fig. 29–6)? In the W direction we get 2, as we will see later. In the S direction we get *zero*, because the signal from S comes at a certain time; that from N comes 90° later in *time*, but it is already 90° behind in its built-in phase, therefore it arrives, altogether, 180° out of phase, and there is no effect. On the other hand, in the N direction, the N signal arrives earlier than the S signal by 90° in time, because it is a quarter wavelength closer. But its phase is set so that it is oscillating 90° *behind* in time, which just compensates the delay difference, and therefore the two signals appear *together* in phase, making the field strength twice as large, and the energy four times as great.

Thus, by using some cleverness in spacing and phasing our antennas, we can send the power all in one direction. But still it is distributed over a great range of angles. Can we arrange it so that it is focused still more sharply in a particular direction? Let us consider the case of Hawaii again, where we are sending the beam east and west but it is spread over quite an angle, because even at 30° we are still getting half the intensity—we are wasting the power. Can we do better than that? Let us take a situation in which the separation is ten wavelengths (Fig.

Fig. 29–6. A pair of dipole antennas giving maximum power in one direction.

29-7), which is more nearly comparable to the situation in which we experimented in the previous chapter, with separations of several wavelengths rather than a small fraction of a wavelength. Here the picture is quite different.

If the oscillators are ten wavelengths apart (we take the in-phase case to make it easy), we see that in the E–W direction, they are in phase, and we get a strong intensity, four times what we would get if one of them were there alone. On the other hand, at a very small angle away, the arrival times differ by 180° and the intensity is zero. To be precise, if we draw a line from each oscillator to a distant point and the difference Δ in the two distances is $\lambda/2$, half an oscillation, then they will be out of phase. So this first null occurs when that happens. (The figure is not drawn to scale; it is only a rough sketch.) This means that we do indeed have a very sharp beam in the direction we want, because if we just move over a little bit we lose all our intensity. Unfortunately for practical purposes, if we were thinking of making a radio broadcasting array and we doubled the distance Δ, then we would be a whole cycle out of phase, which is the same as being exactly *in* phase again! Thus we get many successive maxima and minima, just as we found with the $2\frac{1}{2}\lambda$ spacing in Chapter 28.

Now how can we arrange to get rid of all these extra maxima, or "lobes," as they are called? We could get rid of the unwanted lobes in a rather interesting way. Suppose that we were to place another set of antennas between the two that we already have. That is, the outside ones are still 10λ apart, but between them, say every 2λ, we have put another antenna, and we drive them all in phase. There are now six antennas, and if we looked at the intensity in the E–W direction, it would, of course, be much higher with six antennas than with one. The field would be six times and the intensity thirty-six times as great (the square of the field). We get 36 units of intensity in that direction. Now if we look at neighboring points, we find a zero as before, roughly, but if we go farther, to where we used to get a big "bump," we get a much smaller "bump" now. Let us try to see why.

The reason is that although we might expect to get a big bump when the distance Δ is exactly equal to the wavelength, it is true that dipoles 1 and 6 are then in phase and are cooperating in trying to get some strength in that direction. But numbers 3 and 4 are roughly $\frac{1}{2}$ a wavelength out of phase with 1 and 6, and although 1 and 6 push together, 3 and 4 push together too, but in opposite phase. Therefore there is very little intensity in this direction—but there is something; it does not balance exactly. This kind of thing keeps on happening; we get very little bumps, and we have the strong beam in the direction where we want it. But in this particular example, something else will happen: namely, since the distance between successive dipoles is 2λ, it is possible to find an angle where the distance δ between *successive dipoles* is exactly one wavelength, so that the effects from all of them are in phase again. Each one is delayed relative to the next one by 360°, so they all come back in phase, and we have another strong beam in that direction! It is easy to avoid this in practice because it is possible to put the dipoles closer than one wavelength apart. If we put in more antennas, closer than one wavelength apart, then this cannot happen. But the fact that this *can* happen at certain angles, if the spacing is bigger than one wavelength, is a very interesting and useful phenomenon in other applications—not radio broadcasting, but in *diffraction gratings*.

29-5 The mathematics of interference

Now we have finished our analysis of the phenomena of dipole radiators qualitatively, and we must learn how to analyze them quantitatively. To find the effect of two sources at some particular angle in the most general case, where the two oscillators have some intrinsic relative phase α from one another and the strengths A_1 and A_2 are not equal, we find that we have to add two cosines having the same frequency, but with different phases. It is very easy to find this phase difference; it is made up of a delay due to the difference in distance, and the intrinsic, built-in phase of the oscillation. Mathematically, we have to find the sum R of two waves: $R = A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2)$. How do we do it?
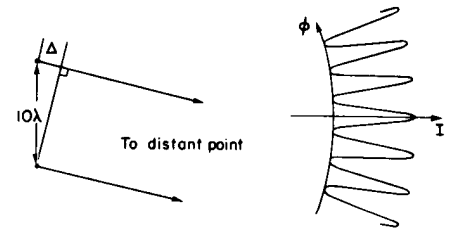

Fig. 29–7. The intensity pattern for two dipoles separated by 10λ.
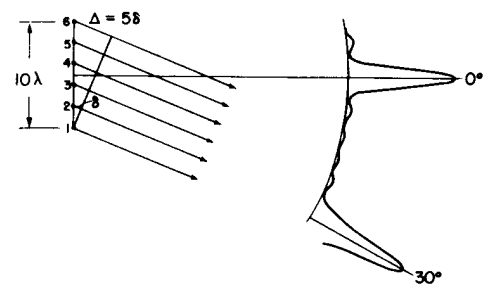

Fig. 29–8. A six-dipole antenna array and part of its intensity pattern.

It is really very easy, and we presume that we already know how to do it. However, we shall outline the procedure in some detail. First, we can, if we are clever with mathematics and know enough about cosines and sines, simply work it out. The easiest such case is the one where A_1 and A_2 are equal, let us say they are both equal to A. In those circumstances, for example (we could call this the trigonometric method of solving the problem), we have

$$R = A[\cos(\omega t + \phi_1) + \cos(\omega t + \phi_2)]. \tag{29.9}$$

Once, in our trigonometry class, we may have learned the rule that

$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A + B)\cos\tfrac{1}{2}(A - B). \tag{29.10}$$

If we know that, then we can immediately write R as

$$R = 2A\cos\tfrac{1}{2}(\phi_1 - \phi_2)\cos(\omega t + \tfrac{1}{2}\phi_1 + \tfrac{1}{2}\phi_2). \tag{29.11}$$

So we find that we have an oscillatory wave with a new phase and a new amplitude. In general, the result *will* be an oscillatory wave with a new amplitude A_R, which we may call the resultant amplitude, oscillating at the same frequency but with a phase difference ϕ_R, called the resultant phase. In view of this, our particular case has the following result: that the resultant amplitude is

$$A_R = 2A\cdot\cos\tfrac{1}{2}(\phi_1 - \phi_2), \tag{29.12}$$

and the resultant phase is the average of the two phases, and we have completely solved our problem.

Now suppose that we cannot remember that the sum of two cosines is twice the cosine of half the sum times the cosine of half the difference. Then we may use another method of analysis which is more geometrical. Any cosine function of ωt can be considered as the horizontal projection of a *rotating vector*. Suppose there were a vector $\mathbf{A}_1$ of length A_1 rotating with time, so that its angle with the horizontal axis is $\omega t + \phi_1$. (We shall leave out the ωt in a minute, and see that it makes no difference.) Suppose that we take a snapshot at the time $t = 0$, although, in fact, the picture is rotating with angular velocity ω (Fig. 29–9). The projection of A_1 along the horizontal axis is precisely $A_1\cos(\omega t + \phi_1)$. Now at $t = 0$ the second wave could be represented by another vector, $\mathbf{A}_2$, of length A_2 and at an angle ϕ_2, and also rotating. They are both rotating with the same angular velocity ω, and therefore the *relative* positions of the two are fixed. The system goes around like a rigid body. The horizontal projection of $\mathbf{A}_2$ is $A_2\cos(\omega t + \phi_2)$. But we know from the theory of vectors that if we add the two vectors in the ordinary way, by the parallelogram rule, and draw the resultant vector $\mathbf{A}_R$, the x-component of the resultant is the sum of the x-components of the other two vectors. That solves our problem. It is easy to check that this gives the correct result for the special case we treated above, where $A_1 = A_2 = A$. In this case, we see from Fig. 29–9 that $\mathbf{A}_R$ lies midway between $\mathbf{A}_1$ and $\mathbf{A}_2$ and makes an angle $\tfrac{1}{2}(\phi_2 - \phi_1)$ with each. Therefore we see that $A_R = 2A\cos\tfrac{1}{2}(\phi_2 - \phi_1)$, as before. Also, as we see from the triangle, the phase of $\mathbf{A}_R$, as it goes around, is the average angle of $\mathbf{A}_1$ and $\mathbf{A}_2$ when the two amplitudes are equal. Clearly, we can also solve for the case where the amplitudes are not equal, just as easily. We can call that the *geometrical* way of solving the problem.

There is still another way of solving the problem, and that is the *analytical* way. That is, instead of having actually to draw a picture like Fig. 29–9, we can write something down which says the same thing as the picture: instead of drawing the vectors, we write a *complex number* to represent each of the vectors. The real parts of the complex numbers are the actual physical quantities. So in our particular case the waves could be written in this way: $A_1e^{i(\omega t + \phi_1)}$ [the real part of this is $A_1\cos(\omega t + \phi_1)$] and $A_2e^{i(\omega t + \phi_2)}$. Now we can add the two:

$$R = A_1e^{i(\omega t + \phi_1)} + A_2e^{i(\omega t + \phi_2)} = (A_1e^{i\phi_1} + A_2e^{i\phi_2})e^{i\omega t} \tag{29.13}$$

or

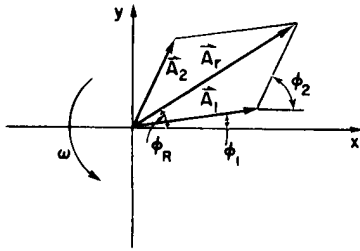$$\hat{R} = A_1e^{i\phi_1} + A_2e^{i\phi_2} = A_Re^{i\phi_R}. \tag{29.14}$$

Fig. 29–9. A geometrical method for combining two cosine waves. The entire diagram is thought of as rotating counterclockwise with angular frequency ω.

29-6

This solves the problem that we wanted to solve, because it represents the result as a complex number of magnitude A_R and phase ϕ_R.

To see how this method works, let us find the amplitude A_R which is the "length" of $\hat{R}$. To get the "length" of a complex quantity, we always multiply the quantity by its complex conjugate, which gives the length squared. The complex conjugate is the same expression, but with the sign of the i's reversed. Thus we have

$$A_R^2 = (A_1 e^{i\phi_1} + A_2 e^{i\phi_2})(A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2}). \qquad (29.15)$$

In multiplying this out, we get $A_1^2 + A_2^2$ (here the e's cancel), and for the cross terms we have

$$A_1 A_2 (e^{i(\phi_1 - \phi_2)} + e^{i(\phi_2 - \phi_1)}).$$

Now

$$e^{i\theta} + e^{-i\theta} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta.$$

That is to say, $e^{i\theta} + e^{-i\theta} = 2\cos\theta$. Our final result is therefore

$$A_R^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_2 - \phi_1). \qquad (29.16)$$

As we see, this agrees with the length of A_R in Fig. 29–9, using the rules of trigonometry.

Thus the sum of the two effects has the intensity A_1^2 we would get with one of them alone, plus the intensity A_2^2 we would get with the other one alone, plus a correction. This correction we call the *interference effect*. It is really only the difference between what we get simply by adding the intensities, and what actually happens. We call it interference whether it is positive or negative. (Interference in ordinary language usually suggests opposition or hindrance, but in physics we often do not use language the way it was originally designed!) If the interference term is positive, we call that case *constructive* interference, horrible though it may sound to anybody other than a physicist! The opposite case is called *destructive* interference.

Now let us see how to apply our general formula (29.16) for the case of two oscillators to the special situations which we have discussed qualitatively. To apply this general formula, it is only necessary to find what phase difference, $\phi_1 - \phi_2$, exists between the signals arriving at a given point. (It depends only on the phase difference, of course, and not on the phase itself.) So let us consider the case where the two oscillators, of equal amplitude, are separated by some distance d and have an intrinsic relative phase α. (When one is at phase zero, the phase of the other is α.) Then we ask what the intensity will be in some azimuth direction θ from the E–W line. [Note that this is *not* the same θ as appears in (29.1). We are torn between using an unconventional symbol like $\not\!\theta$, or the conventional symbol θ (Fig. 29–10).] The phase relationship is found by noting that the difference in distance from P to the two oscillators is $d \sin\theta$, so that the phase difference contribution from this is the number of wavelengths in $d \sin\theta$, multiplied by 2π. (Those who are more sophisticated might want to multiply the wave number k, which is the rate of change of phase with distance, by $d \sin\theta$; it is exactly the same.) The phase difference due to the distance difference is thus $2\pi d \sin\theta/\lambda$, but, due to the timing of the oscillators, there is an additional phase α. So the phase difference at arrival would be

Fig. 29–10. Two oscillators of equal amplitude, with a phase difference α between them.

$$\phi_2 - \phi_1 = \alpha + 2\pi d \sin\theta/\lambda. \qquad (29.17)$$

This takes care of all the cases. Thus all we have to do is substitute this expression into (29.16) for the case $A_1 = A_2$, and we can calculate all the various results for two antennas of equal intensity.

Now let us see what happens in our various cases. The reason we know, for example, that the intensity is 2 at 30° in Fig. 29–5 is the following: the two oscillators are $\frac{1}{2}\lambda$ apart, so at 30°, $d \sin\theta = \lambda/4$. Thus $\phi_2 - \phi_1 = 2\pi\lambda/4\lambda = \pi/2$, and so the interference term is zero. (We are adding two vectors at 90°.) The result is the hypotenuse of a 45° right-angle triangle, which is $\sqrt{2}$ times the unit amplitude; squaring it, we get twice the intensity of one oscillator alone. All the other cases c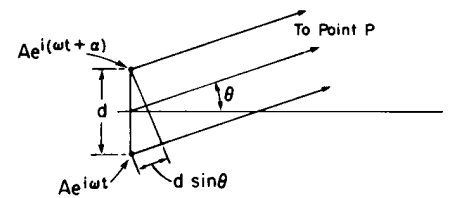an be worked out in this same way.