VII. Boundary Layer Flows

The previous chapter considered only viscous internal flows.

Viscous internal flows have the following major boundary layer characteristics:

* An entrance region where the boundary layer grows and $dP/dx \neq$ constant,
* A fully developed region where:
  - The boundary layer fills the entire flow area.
  - The velocity profiles, pressure gradient, and τ_w are constant; i.e., they are not equal to f(x),
  - The flow is either laminar or turbulent over the entire length of the flow , i.e., transition from laminar to turbulent is not considered.

However, viscous flow boundary layer characteristics for external flows are significantly different as shown below for flow over a flat plate:

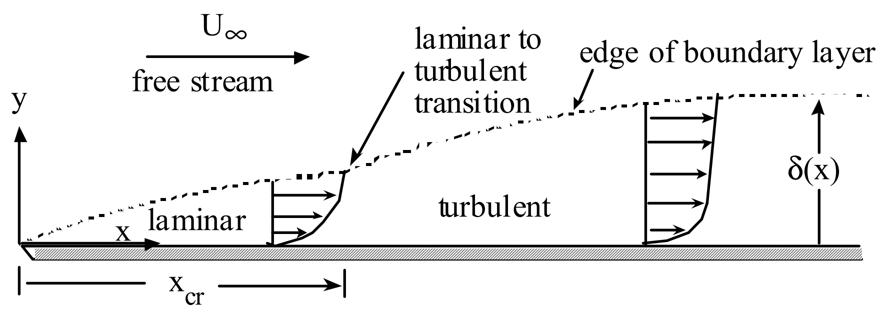


Fig. 7.1 Schematic of boundary layer flow over a flat plate

For these conditions, we note the following characteristics:

- The boundary layer thickness δ grows continuously from the start of the fluid-surface contact, e.g., the leading edge. It is a function of x, not a constant.
- Velocity profiles and shear stress τ are f(x,y).
- The flow will generally be laminar starting from x = 0.
- The flow will undergo laminar-to-turbulent transition if the streamwise dimension is greater than a distance x_{cr} corresponding to the location of the transition Reynolds number Re_{cr}.
- Outside of the boundary layer region, free stream conditions exist where velocity gradients and therefore viscous effects are typically negligible.

As it was for internal flows, the most important fluid flow parameter is the local Reynolds number defined as

$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\upsilon}$$

where

ρ = fluid density μ = fluid dynamic viscosity
ν = fluid kinematic viscosity U_∞ = characteristic flow velocity
x = characteristic flow dimension

It should be noted at this point that all external flow applications will not use a distance from the leading edge x and the characteristic flow dimension. For example, for flow over a cylinder, the diameter will be used as the characteristic dimension for the Reynolds number.

Transition from laminar to turbulent flow typically occurs at the local transition Reynolds number which for flat plate flows can be in the range of

$$500,000 \leq \mathrm{Re}_{cr} \leq 3,000,00$$

With x_{cr} = the value of x where transition from laminar to turbulent flow occurs, the typical value used for steady, incompressible flow over a flat plate is

$$\mathrm{Re}_{cr} = \frac{\rho U_\infty x_{cr}}{\mu} = 500,000$$

Thus for flat plate flows for which:

$x < x_{cr}$ the flow is laminar

$x \geq x_{cr}$ the flow is turbulent

The solution to boundary layer flows is obtained from the reduced "Navier – Stokes" equations, i.e., Navier-Stokes equations for which boundary layer assumptions and approximations have been applied.

Flat Plate Boundary Layer Theory

<u>Laminar Flow Analysis</u>

For steady, incompressible flow over a flat plate, the laminar boundary layer equations are:

Conservation of mass: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

'X' momentum: $$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

'Y' momentum: $$-\frac{\partial p}{\partial y} = 0$$

The solution to these equations was obtained in 1908 by Blasius, a student of Prandtl's. He showed that the solution to the velocity profile, shown in the table below, could be obtained as a function of a single, non-dimensional variable η defined as

$$\eta = y\left(\frac{U_\infty}{\upsilon x}\right)^{1/2}$$

with the resulting ordinary differential equation:

$$f''' + \frac{1}{2} f\, f'' = 0$$

and $$f'(\eta) = \frac{u}{U_\infty}$$

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

Boundary conditions for the differential equation are expressed as follows:

at $y = 0$, $v = 0 \rightarrow f(0) = 0$; y component of velocity is zero at $y = 0$

at $y = 0$, $u = 0 \rightarrow f'(0) = 0$; x component of velocity is zero at $y = 0$

The key result of this solution is written as follows:

$$\left.\frac{\partial^2 f}{\partial \eta^2}\right)_{y=0} = 0.332 = \frac{\tau_w}{\mu U_\infty \sqrt{U_\infty / \upsilon x}}$$

With this result and the definition of the boundary layer thickness, the following key results are obtained for the laminar flat plate boundary layer:

Local boundary layer thickness $\delta(x) = \frac{5x}{\sqrt{Re_x}}$

Local skin friction coefficient:
(defined below) $C_{f_x} = \frac{0.664}{\sqrt{Re_x}}$

Total drag coefficient for length L (integration of τ_w dA over the length of the plate, per unit area, divided by 0.5 ρ U_∞^2) $C_D = \frac{1.328}{\sqrt{Re_x}}$

where by definition $C_{f_x} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2}$ and $C_D = \frac{F_D / A}{\frac{1}{2}\rho U_\infty^2}$

With these results, we can determine local boundary layer thickness, local wall shear stress, and total drag force for laminar flow over a flat plate.

Example:

Air flows over a sharp edged flat plate with L = 1 m, a width of 3 m and U_∞ = 2 m/s . For one side of the plate, find: δ(L), C_f (L), τ_w(L), C_D, and F_D.

Air: ρ = 1.23 kg/m^3 ν = 1.46 E-5 m^2/s

First check Re: $Re_L = \frac{U_\infty L}{\upsilon} = \frac{2\,m/s * 2.15\,m}{1.46E-5\,m^2/s} = 294,520 < 500,000$

Key Point: Therefore, the flow is laminar over the entire length of the plate and calculations made for any x position from 0 - 1 m must be made using laminar flow equations.

Boundary layer thickness at x = L:

$$\delta(L)=\frac{5\,L}{\sqrt{\mathrm{Re}_L}}=\frac{5*2.15\,m}{\sqrt{294,520}}=0.0198\,m=1.98\,cm$$

Local skin friction coefficient at x = L:

$$C_f(L)=\frac{0.664}{\sqrt{\mathrm{Re}_L}}=\frac{0.664}{\sqrt{294,520}}=0.00122$$

Surface shear stress at x = L:

$$\tau_w=1/2\,\rho U_\infty^2\,C_f=0.5*1.23\,kg/m^3*2^2\,m^2/s^2*0.00122$$
$$\tau_w=0.0030\,N/m^2\,(Pa)$$

Drag coefficient over total plate, 0 – L:

$$C_D(L)=\frac{1.328}{\sqrt{\mathrm{Re}_L}}=\frac{1.328}{\sqrt{294,520}}=0.00245$$

Drag force over plate, 0 – L:

$$F_D=1/2\,\rho U_\infty^2\,C_D\,A=0.5*1.23\,kg/m^3*2^2\,m^2/s^2*0.00245*2*2.15\,m^2$$
$$F_D=0.0259\,N$$

Two key points regarding this analysis:

1. Each of these calculations can be made for any other location on the plate by simply using the appropriate x location for any $x \le L$.
2. Be careful not to confuse the calculation for C_f and C_D. C_f is a local calculation at a particular x location (including x = L) and can only be used to calculate local shear stress, not drag force. C_D is an integrated average over a specified length (including any $x \le L$) and can only be used to calculate the average shear stress and the integrated force over the length.

Turbulent Flow Equations

While the previous analysis provides an excellent representation of laminar, flat plate boundary layer flow, a similar analytical solution is not available for turbulent flow due to the complex nature of the turbulent flow structure. However, experimental results are available to provide equations for key flow field parameters.

A summary of the results for boundary layer thickness and local and average skin friction coefficient for a laminar flat plate and a comparison with experimental results for a smooth, turbulent flat plate are shown below.

Laminar

$$\delta(\mathrm{x}) = \frac{5\mathrm{x}}{\sqrt{\mathrm{Re}_{\mathrm{x}}}}$$

$$C_{f_x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$$

Turbulent

$$\delta(x) = \frac{0.37\,x}{\mathrm{Re}_x^{.2}}$$

$$C_{f_x} = \frac{0.0592}{\mathrm{Re}_x^{.2}}$$

$$C_D = \frac{0.074}{\mathrm{Re}_L^{.2}}$$

for turbulent flow over entire plate, 0 – L, i.e. assumes turbulent flow in the laminar region.

where

$$C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

local drag coefficient based on local wall shear stress (laminar or turbulent flow region).

and

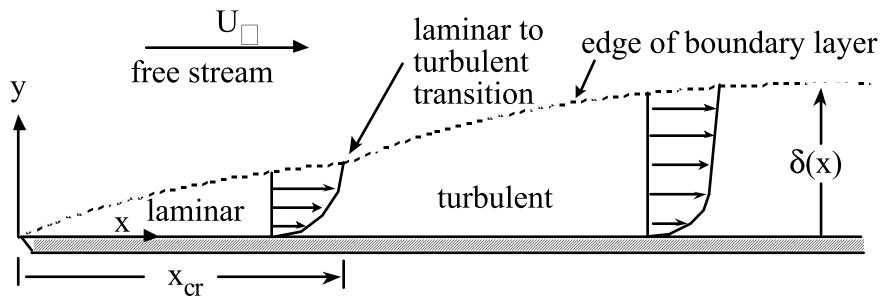
C_D = total drag coefficient based on the integrated force over the length 0 to L

$$C_D = \frac{F/A}{\frac{1}{2}\rho U_\infty^2} = \left(\tfrac{1}{2}\rho U_\infty^2\,A\right)^{-1}\int_0^L \tau_w(x)\,w\,dx$$

A careful study of these results will show that in general, boundary layer thickness grows faster for turbulent flow and wall shear and total friction drag are greater for turbulent flow than for laminar flow given the same Reynolds number.

It is noted that the expressions for turbulent flow are valid only for a flat plate with a smooth surface. Expressions including the effects of surface roughness are available in the text.

Combined Laminar and Turbulent Flow



Flat plate with both laminar and turbulent flow sections

For conditions (as shown above) where the length of the plate is sufficiently long that we have both laminar and turbulent sections:

* Local values for boundary layer thickness and wall shear stress for either the laminar or turbulent sections are obtained from the expressions for $\delta(x)$ and Cf_x for laminar or turbulent flow as appropriate for the given region.
* The result for average drag coefficient C_D and thus total frictional force over the laminar and turbulent portions of the plate is given by (assuming a transition Re of 500,000)

$$C_D = \frac{0.074}{\mathrm{Re}_L^{.2}} - \frac{1742}{\mathrm{Re}_L}$$

* Calculations assuming only turbulent flow can be made typically for two cases

  1. when some physical situation (a trip wire) has caused the flow to be leading from the leading edge or

  2. if the total length L of the plate is much greater than the length x_{cr} of the laminar section such that the total flow can be considered turbulent from x = 0 to L. Note that this will overpredict the friction drag force since turbulent drag is greater than laminar.

With these results, a detailed analysis can be obtained for laminar and/or turbulent flow over flat plates and surfaces that can be approximated as a flat plate.

Example:

Water flows over a sharp flat plate 2.55 m long, 1 m wide, with $U_\infty = 2$ m/s. Estimate the error in F_D if it is assumed that the entire plate is turbulent.

Water: ρ = 1000 kg/m^3 ν = 1.02 E- m^2/s

Reynolds number: $$\mathrm{Re}_L = \frac{U_\infty L}{\upsilon} = \frac{2\,m/s * 2.55\,m}{1.02E-6\,m^2/s} = 5E6 > 500,000$$

with $\mathrm{Re}_{cr} = 500{,}000 \Rightarrow x_{cr} = 0.255\,m$ (or 10% laminar)

a. Assume that the entire plate is turbulent

$$C_D = \frac{0.074}{\mathrm{Re}_L^{.2}} = \frac{0.074}{(5E6)^{.2}} = 0.00338$$

$$F_D = 0.5\,\rho U_\infty^2\, C_D\, A = 0.5 * 1000\frac{kg}{m^3} * 2^2\frac{m^2}{s^2} * 0.00338 * 2.55 m^2$$

$$F_D = 17.26\,N$$

This should be high since we have assumed that the entire plate is turbulent and the first 10% is actually laminar.

b. Consider the actual combined laminar and turbulent flow:

$$C_D = \frac{0.074}{\mathrm{Re}_L^{.2}} - \frac{1742}{\mathrm{Re}_L} = 0.00338 - \frac{1742}{5E6} = 0.00303$$

Note that the C_D has decreased when both the laminar and turbulent sections are considered.

$$F_D = 0.5\,\rho U_\infty^2\, C_D\, A = 0.5 * 1000\frac{kg}{m^3} * 2^2\frac{m^2}{s^2} * 0.00303 * 2.55\,m^2$$

$$F_D = 15.46\,N \quad \{\text{Lower than the fully turbulent value}\}$$

$$Error = \frac{17.26 - 15.46}{15.46} * 100 = 11.6\%\ high$$

Question: Since x_{cr} = 0.255 m, what would your answers represent if you had calculated the Re, C_D, and F_D using $x = x_{cr}$ = 0.255 m?

Answer: You would have the value of the transition Reynolds number and the drag coefficient and drag force over the laminar portion of the plate (assuming you used laminar equations). If you used turbulent equations, you would have red marks on your paper.

Von Karman Integral Momentum Analysis

While the previous results provide an excellent basis for the analysis of flat plate flows, complex geometries and boundary conditions make analytical solutions to most problems difficult.

An alternative procedure provides the basis for an approximate solution which in many cases can provide excellent results.

The key to practical results is to use a reasonable approximation to the boundary layer profile, u(x,y). This is used to obtain the following:

a. Boundary layer mass flow: $$\dot{m} = \int_0^{\delta} \rho u b \, dy$$

where b is the width of the area for which the flow rate is being obtained.

b. Wall shear stress: $$\tau_w = \mu \left.\frac{du}{dy}\right)_{y=0}$$

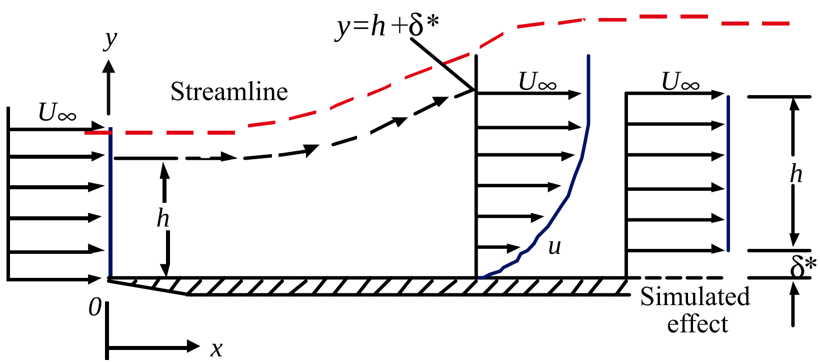
You will also need the streamwise pressure gradient $\frac{dP}{dx}$ for many problems.

The Von Karman integral momentum theory provides the basis for such an approximate analysis. The following summarizes this theory.

<u>Displacement thickness</u>:

Consider the problem indicated in the adjacent figure:

A uniform flow field with velocity U_∞ approaches a solid surface. As a result of viscous shear, a boundary layer velocity profile develops.



A viscous boundary layer is created when the flow comes in contact with the solid surface.

Key point: Compared to the uniform velocity profile approaching the solid surface, the effect of the viscous boundary layer is to displace streamlines of the flow outside the boundary layer away from the wall.

With this concept, we define δ^* = displacement thickness

δ^* = distance the solid surface would have to be displaced to maintain the same mass flow rate as for non-viscous flow.

From the development in the text, we obtain

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

Therefore, with an expression for the local velocity profile we can obtain $\delta^* = f(\delta)$

Example:

Given: $\dfrac{u}{U_\infty} = 2\left(\dfrac{y}{\delta}\right) - \left(\dfrac{y}{\delta}\right)^2$ determine an expression for $\delta^* = f(\delta)$

Note that for this assumed form for the velocity profile:

1. At $y = 0$, $u = 0$ correct for no slip condition
2. At $y = \delta$, $u = U_\infty$ correct for edge of boundary layer
3. The form is quadratic

To simplify the mathematics,

let $\eta = y/\delta$, at $y = 0$, $\eta = 0$; at $y = \delta$, $\eta = 1$; $dy = \delta\, d\eta$

Therefore: $\dfrac{u}{U_\infty} = 2\eta - \eta^2$

Substituting: $$\delta^* = \int_0^1 \left(1 - 2\eta + \eta^2\right)\delta\, d\eta = \delta\left\{\eta - \frac{2\eta^2}{2} + \frac{\eta^3}{3}\right\}_0^1$$

which yields $\delta^* = \frac{1}{3}\delta$

Therefore, for flows for which the assumed quadratic equation approximates the velocity profile, streamlines outside of the boundary layer are displaced approximately according to the equation

$$\delta^* = \frac{1}{3}\delta$$

This closely approximates flow for a flat plate.

Key Point: When assuming a form for a velocity profile to use in the Von Karman analysis, make sure that the resulting equation satisfies both surface and free stream boundary conditions as well as has a form that approximates u(y).

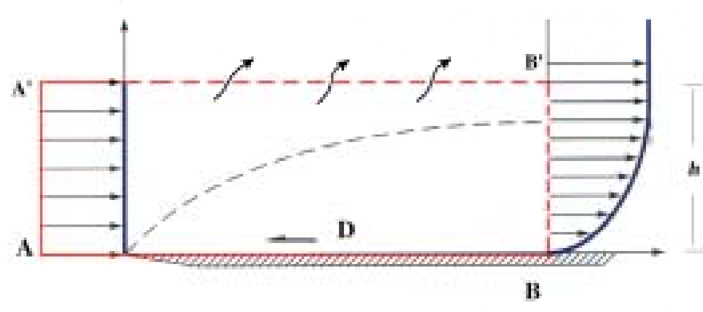
Momentum Thickness:

The second concept used in the Von Karman momentum analysis is that of

momentum thickness - θ

The concept is similar to that of displacement thickness in that θ is related to the loss of momentum due to viscous effects in the boundary layer.

Consider the viscous flow regions shown in the adjacent figure. Define a control volume as shown and integrate around the control volume to obtain the net change in momentum for the control volume.



If D = drag force on the plate due to viscous flow, we can write

$$- D = \sum (\text{momentum leaving c.v.}) - \sum (\text{momentum entering c.v.})$$

Completing an analysis shown in the text, we obtain

$$D = \rho U_\infty^2 \theta \qquad \theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Using a drag coefficient defined as $C_D = \dfrac{D/A}{\frac{1}{2}\rho U_\infty^2}$

We can also show that $C_D = \dfrac{2\,\theta(L)}{L}$

where: θ(L) is the momentum thickness evaluated over the length L.

Thus, knowledge of the boundary layer velocity distribution u = f(y) allows the drag coefficient to be determined.

Momentum integral:

The final step in the Von Karman theory applies the previous control volume analysis to a differential length of surface. Performing an analysis similar to the previous analysis for drag D we obtain

$$\frac{\tau_w}{\rho} = \delta^* U_\infty \frac{dU_\infty}{dx} + \frac{d}{dx}\left(U_\infty^2 \theta\right)$$

This is the momentum integral for 2-D, incompressible flow and is valid for laminar or turbulent flow.

where $\delta^* U_\infty \dfrac{dU_\infty}{dx} = -\dfrac{\delta^*}{\rho}\dfrac{dP}{dx}$

Therefore, this analysis also accounts for the effect of freestream pressure gradient.

For a flat plate with non-accelerating flow, we can show that

$$P = const., \quad U_\infty = const., \quad \frac{dU_\infty}{dx} = 0$$

Therefore, for a flat plate, non-accelerating flow, the Von Karman momentum integral becomes

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U_\infty^2 \theta\right) = U_\infty^2 \frac{d\theta}{dx}$$

From the previous analysis and the assumed velocity distribution of

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$$

The wall shear stress can be expressed as

$$\tau_w = \mu \left.\frac{du}{dy}\right)_w = 2U_\infty \left\{\frac{2}{\delta} - \frac{2y}{\delta^2}\right\}_{y=0} = \frac{2\mu U_\infty}{\delta} \qquad \text{(A)}$$

Also, with the assumed velocity profile, the momentum thickness θ can be evaluated as

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

or

$$\theta = \int_0^{\delta} \left(2\eta - \eta^2\right)\left(1 - 2\eta + \eta^2\right)\delta\, d\eta = \quad \frac{2\delta}{15}$$

We can now write from the previous equation for τ_w

$$\tau_w = \rho U_\infty^2 \frac{d\theta}{dx} = \frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx}$$

Equating this result to Eqn. A we obtain

$$\tau_w = \frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx} = \frac{2\mu U_\infty}{\delta}$$

or

$$\delta\, d\delta = \frac{15\mu}{\rho U_\infty} dx$$ which after integration yields

$$\delta = \left\{\frac{30\mu x}{\rho U_\infty}\right\}^{1/2} \qquad \text{or} \qquad \delta = \frac{5.48}{\sqrt{\mathrm{Re}_x}}$$

Note that the this result is within 10% of the exact result from Blasius flat plate theory.

Since for a flat plate, we only need to consider friction drag (not pressure drag), we can write

$$C_{f_x} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{2\mu U_\infty}{\delta}\frac{1}{\frac{1}{2}\rho U_\infty^2}$$

Substitute for δ to obtain

$$C_{f_x} = \frac{2\mu U_\infty}{5.48}\frac{\sqrt{\mathrm{Re}}}{\frac{1}{2}\rho U_\infty^2} = \frac{0.73}{\sqrt{\mathrm{Re}_x}}$$

Exact theory has a numerical constant of 0.664 compared with 0.73 for the previous result.

It is seen that the von Karman integral theory provides the means to determine approximate expressions for

$$\delta, \tau_w, \text{ and } C_f$$

using only an assumed velocity profile.

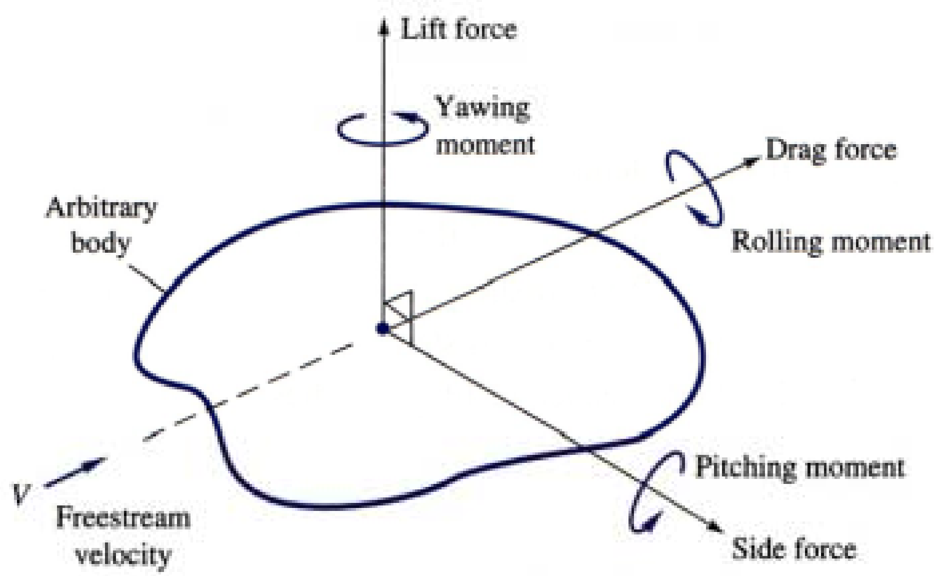
Solution summary:

1. Assume an analytical expression for the velocity profile for the problem.
2. Use the assumed velocity profile to determine the solution for the displacement thickness for the problem.
3. Use the assumed velocity profile to determine the solution for the momentum thickness for the problem.
4. Use the previous results and the von Karman integral momentum equation to determine the solution for the drag/wall shear for the problem.

Bluff Body, Viscous Flow Characteristics
(Immersed Bodies)

In general, a body immersed in a flow will experience both externally applied forces and moments as a result of the flow about its external surfaces. The typical terminology and designation of these forces and moments are given in the diagram shown below.

The orientation of the axis for the drag force is typically along the principal body axis, although in certain applications, this axis is aligned with the principal axis of the free stream, approach velocity U.



Since in many cases the drag force is aligned with the principal axis of the body shape and not necessarily aligned with the approaching wind vector. Review all data carefully to determine which coordinate system is being used: body axis coordinate system or a wind axis coordinate system.

These externally applied forces and moments are generally a function of

a. Body geometry
b. Body orientation
c. Flow conditions

These forces and moments are also generally expressed in the form of a non-dimensional force/moment coefficient, e.g. the drag coefficient:

$$C_D = \frac{F_D/A}{\frac{1}{2}\rho U_\infty^2}$$

It is noted that it is common to see one of three reference areas used depending on the application:

1. Frontal (projected) area: Used for thick, stubby, non-aerodynamic shapes, e.g., buildings, cars, etc.

2. Planform (top view, projected) area: Used for flat, thin shapes, e.g., wings, hydrofoils, etc.

3. Wetted area: The total area in contact with the fluid. Used for surface ships, barges, etc.

The previous, flat plate boundary layer results considered only the contribution of viscous surface friction to drag forces on a body. However, a second major (and usually dominant) factor is **pressure or form drag**.

Pressure drag is drag due to the integrated surface pressure distribution over the body. Therefore, in general , the total drag coefficient of a body can be expressed as

$$C_D = C_{D,press} + C_{D,friction}$$

or

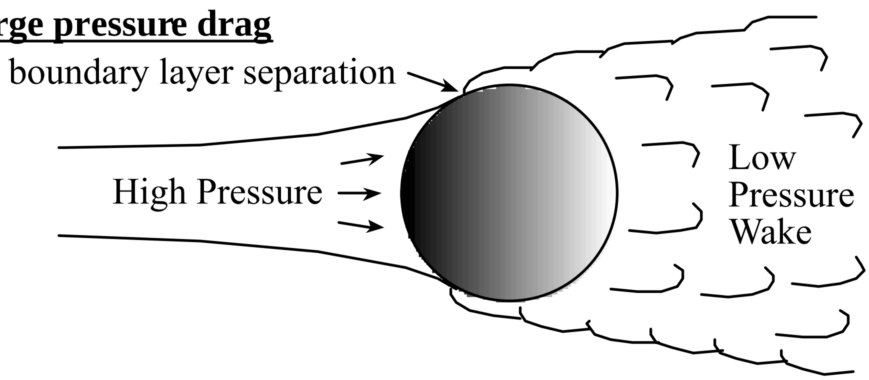
$$C_D = \frac{F_{D,total}/A}{\frac{1}{2}\rho U_\infty^2} = \frac{F_{D,press.}/A}{\frac{1}{2}\rho U_\infty^2} + \frac{F_{D,friction}/A}{\frac{1}{2}\rho U_\infty^2}$$

Which factor, pressure or friction drag, dominates depends largely on the aerodynamics (streamlining) of the shape and to a lesser extent on the flow conditions.

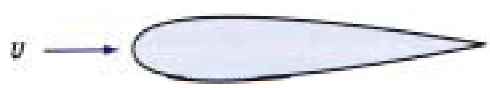
Typically the most important factor in the magnitude and significance of pressure or form drag is the boundary layer separation and resulting low pressure wake region associated with flow around non - aerodynamic shapes.

Consider the two shapes shown below:

large pressure drag



Low pressure drag no separated flow region



The flow around the streamlined airfoil remains attached, producing no boundary layer separation and comparatively small pressure drag. However, the flow around the less aerodynamic circular cylinder separates, resulting in an area of high surface pressure on the front side and low surface pressure on the back side and thus significant pressure drag.

This effect is shown very graphically in the following figures from the text.

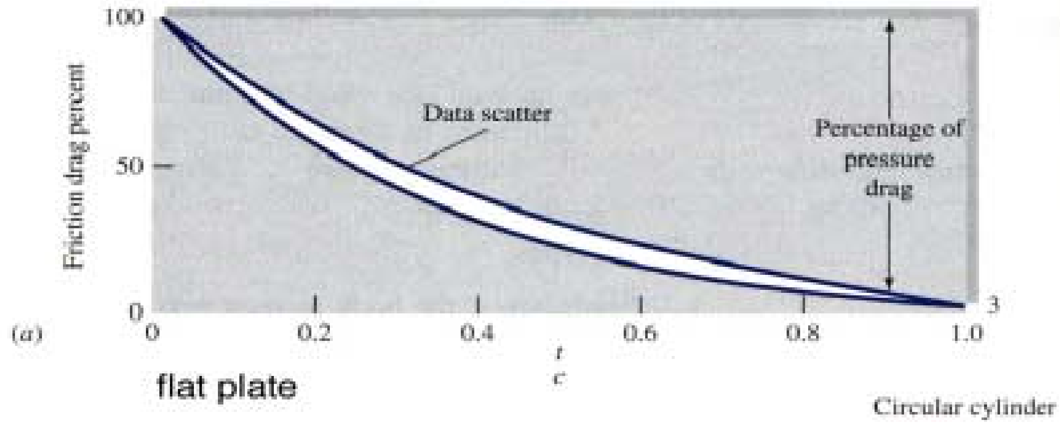


Fig. 7.12 Drag of a 2-D, streamlined cylinder

The previous figure shows the effect of streamlining and aerodynamics on the relative importance of friction and pressure drag. While for a thin flat plate (t/c = 0), all the drag is due to friction with no pressure drag, for a circular cylinder (t/c = 1), only 3% of the drag is due to friction with 97% due to pressure. Likewise for most bluff, non-aerodynamic bodies, pressure (also referred to as form drag) is the dominant contributor to the total drag.

However, the magnitude of the pressure (and therefore the total) drag can also be changed by reducing the size of the low pressure wake region. One way to do this is to change the flow conditions from laminar to turbulent. This is illustrated in the following figures from the text for a circular cylinder.

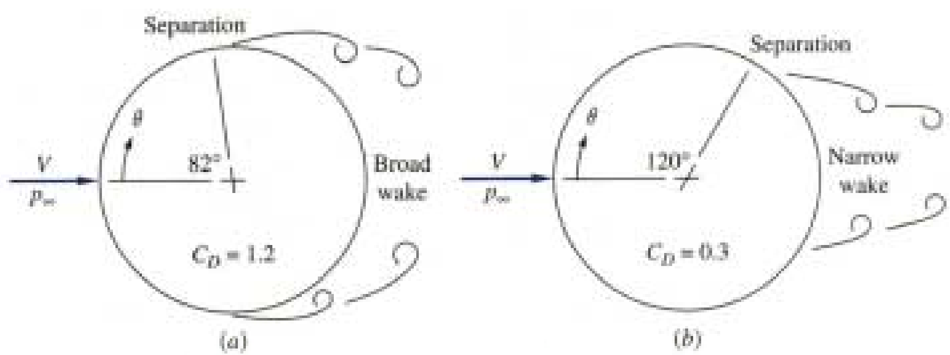


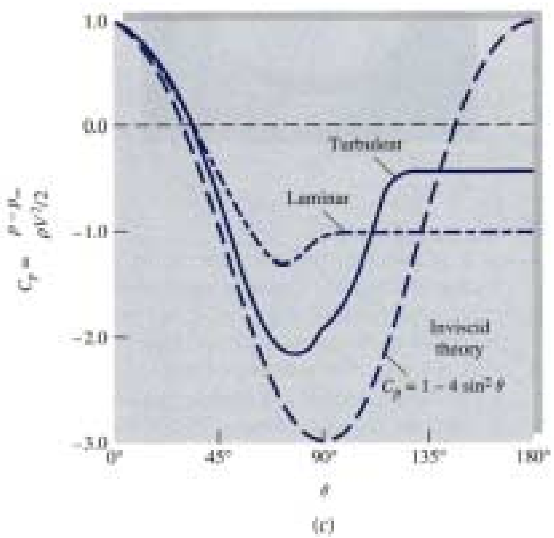
Fig. 7.13 Circular cylinder with (a) laminar separation and (b) turbulent separation

Note that for the cylinder on the left, the flow is laminar, boundary layer separation occurs at 82^{o} and the C_D is 1.2, whereas for the cylinder on the right, the flow is turbulent and separation is delayed and occurs at 120^{o}. The drag coefficient C_D is 0.3, a factor of 4 reduction due to a smaller wake region and reduced pressure drag.

It should also be pointed out that the friction drag for the cylinder on the right is probably greater (turbulent flow conditions) than for the cylinder on the left (laminar flow conditions).

However, since pressure drag dominates, the net result is a significant reduction in the total drag.

The pressure distribution for laminar and turbulent flow over a cylinder is shown in Fig. 7.13c to the right. The front-to-rear pressure difference is greater for laminar flow, thus greater drag.



Finally, the effect of streamlining on total drag is shown very graphically with the sequence of modifications in Fig. 7.15.

Two observations can be made: (1) As body shape changes from a bluff body with fixed points of separation to a more aerodynamic shape, the effect of pressure drag and the drag coefficient will decrease.

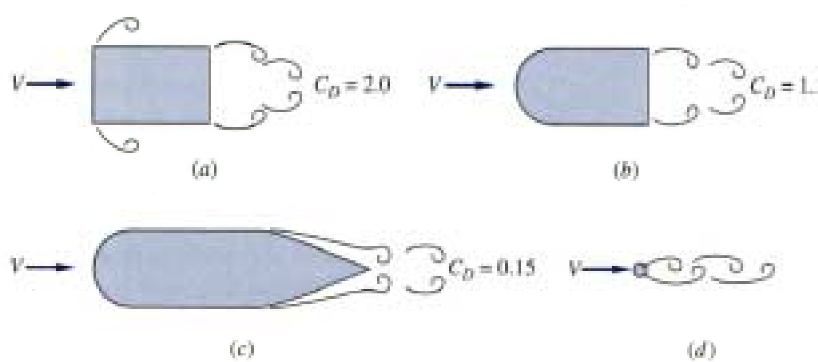


Fig. 7.15 The effect of streamlining on total drag

(2) The addition of surface area from (a) to (b) and (b) to (c) increases the friction drag, however, since pressure drag dominates, the net result is a reduction in the drag force and the C_D .

The final two figures show results for the drag coefficient for two and three dimensional shapes with various geometries.

Table 7.2 C_D for Two-Dimensional Bodies at Re $\geq 10^4$

First note that all values in Table 7.2 are for 2-D geometries, that is, the bodies are very long (compared to the cross-section dimensions) in the dimension perpendicular to the page.

Key Point: **Non – aerodynamic shapes with fixed points of separations (sharp corners) have a single value of C_D, irrespective of the value of the Reynolds number, e.g. square cylinder, half-tube, etc.**

Aerodynamic shapes generally have a reduction in C_D for a change from laminar to turbulent flow as a result of the shift in the point of boundary layer separation, e.g. elliptical cylinder.

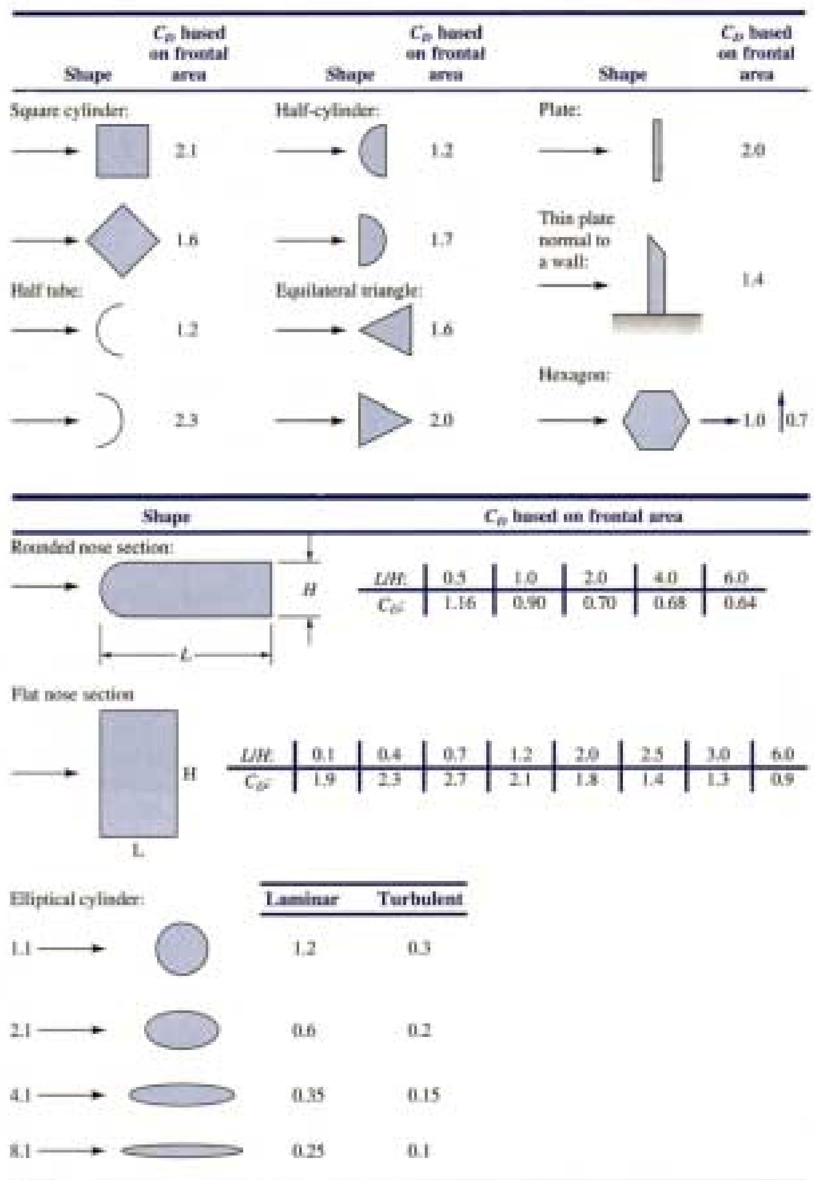


Table 7.3 Drag of three-dimensional bodies at Re ≥ 10^4

Body	C_D based on frontal area
Cube: (face to flow)	1.07
Cube: (edge to flow)	0.81
Cup: (open side downstream)	1.4
Cup: (open side upstream)	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Cone: C_D based on frontal area

θ:	10°	20°	30°	40°	60°	75°	90°
C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Short cylinder, laminar flow:

L/D:	1	2	3	5	10	20	40	∞
C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Porous parabolic dish [23]:

Porosity:	0	0.1	0.2	0.3	0.4	0.5
← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Average person: → $C_DA = 9\ ft^2$ ↑ $C_DA = 1.2\ ft^2$

Pine and spruce trees [24]:

U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Body	Ratio	C_D based on frontal area	Body	Ratio	C_D based on frontal area
Rectangular plate:	b/h 1	1.18	Flat-faced cylinder:	L/d 0.5	1.15
	5	1.2		1	0.90
	10	1.3		2	0.85
	20	1.5		4	0.87
	∞	2.0		8	0.99

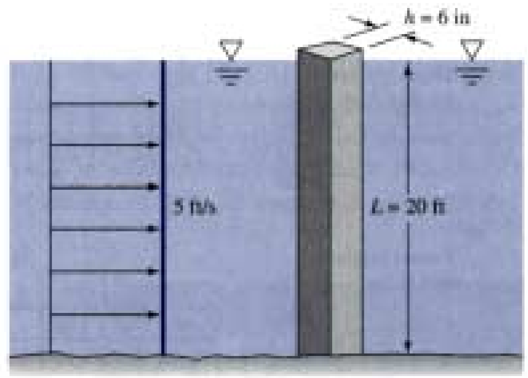
Body	Ratio	Laminar	Turbulent
Ellipsoid:	L/d 0.75	0.5	0.2
	1	0.47	0.2
	2	0.27	0.13
	4	0.25	0.1
	8	0.2	0.08

The geometries in Table 7.3 are all 3-D and thus are finite perpendicular to the page. Similar to the results from the previous table, bluff body geometries with fixed points of separation have a single C_D, whereas aerodynamic shapes such as slender bodies of revolution have individual values of C_D for laminar and turbulent flow.

In summary, one must remember that broad generalizations such as saying that turbulent flow always increases drag, drag coefficients always depend on Reynolds number, or increasing surface area increases drag are not always valid. One must consider carefully all effects (viscous and pressure drag) due to changing flow conditions and geometry.

Example:

A square 6-in piling is acted on by a water flow of 5 ft/s that is 20 ft deep. Estimate the maximum bending stress exerted by the flow on the bottom of the piling.



Water: $\rho = 1.99$ slugs/ft^3

$\nu = 1.1\ E - 5\ ft^2/s$

Assume that the piling can be treated as 2-D and thus end effects are negligible.

Thus for a width of 0.5 ft, we obtain:

$$Re = \frac{5\ ft/s .5\ ft}{1.1E-5\ ft^2/s} = 2.3\,E\,5$$

In this range, Table 7.2 applies for 2-D bodies and we read C_D = 2 .1. The frontal area is A = 20*0.5 = 10 ft^2

$$F_D = 0.5\,\rho U_\infty^2\, C_D\, A = 0.5*1.99\frac{slug}{ft^3}*5^2\,\frac{ft^2}{s^2}*2.1*10\ ft^2 = 522\,lbf$$

For uniform flow, the drag should be uniformly distributed over the total length with the net drag located at the mid-point of the piling.

Thus, relative to the bottom of the piling, the bending moment is given by

$$M_o = F * 0.5\ L = 522\ lbf * 10\ ft = 5220\ ft\text{-}lbf$$

From strength of materials, we can write

$$\sigma = \frac{M_o c}{I} = \frac{5220\ ft - lbf * 0.25\ ft}{\frac{1}{12} 0.5\ ft * 0.5^3\ ft^3} = 251{,}000\ psf = 1740\ psi$$

where c = distance to the neutral axis, I = moment of inertia = b h^3/12

Question: Since pressure acts on the piling and increases with increasing depth, why wasn't a pressure load considered?

Answer: Static pressure does act on the piling, but it acts uniformly around the piling at every depth and thus cancels. Dynamic pressure is considered in the drag coefficients of Tables 7.2 and 7.3 and does not have to be accounted for separately.