Ch. 10 Open-Channel Flow

Previous internal flow analyses have considered only closed conduits where the fluid typically fills the entire conduit and may be either a liquid or a gas.

This chapter considers only partially filled channels of liquid flow referred to as open-channel flow.

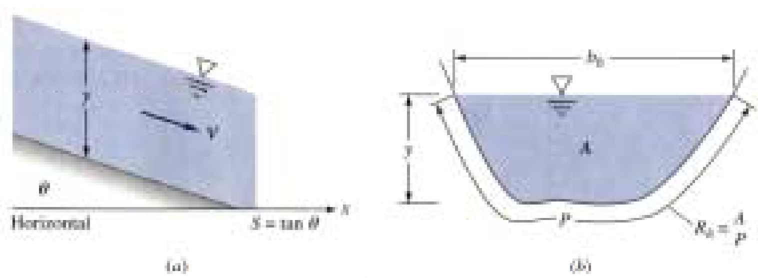
Open-Channel Flow: Flow of a liquid in a conduit with a free surface.

Open-channel flow analysis basically results in the balance of *gravity and friction forces*.

One Dimensional Approximation

While open-channel flow can, in general, be very complex (three dimensional and transient), one common approximation in basic analyses is the

One-D Approximation: The flow at any local cross section can be treated as uniform and at most varies only in the principal flow direction.



This results in the following equations.

<u>Conservation of Mass</u> (for ρ = constant)

$$Q = V(x)\,A(x) = \text{constant}$$

<u>Energy Equation</u>

$$\frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2 + h_f$$

The equation in this form is written between two points (1 – 2) on the free surface of the flow. Note that along the free surface, the pressure is a constant, is equal to local atmospheric pressure, and does not contribute to the analysis with the energy equation.

The friction head loss h_f is analogous to the head loss term in duct flow, Ch. VI, and can be represented by

$$h_f = f \frac{x_2 - x_1}{D_h} \frac{V_{avg}^2}{2g}$$

where P = wetted perimeter

D_h = hydraulic diameter = $\dfrac{4A}{P}$

Note: One of the most commonly used formulas uses the hydraulic radius:

$$R_h = \frac{1}{4} D_h = \frac{A}{P}$$

Flow Classification by Depth Variation

The most common classification method is by rate of change of free-surface depth. The classes are summarized as

1. Uniform flow (constant depth and slope)
2. Varied flow
   a. Gradually varied (one-dimensional)
   b. Rapidly varied (multidimensional)

Flow Classification by Froude Number: Surface Wave Speed

A second classification method is by the dimensionless Froude number, which is a dimensionless surface wave speed. For a rectangular or very wide channel we have

$$Fr = \frac{V}{c_o} = \frac{V}{(gy)^{1/2}}$$

where y is the water depth

and c_o = the speed of a surface wave as the wave height approaches zero.

There are three flow regimes of incompressible flow. These have analogous flow regimes in compressible flow as shown below:

Incompressible Flow		Compressible Flow	
$Fr < 1$	subcritical flow	$Ma < 1$	subsonic flow
$Fr = 1$	critical flow	$Ma = 1$	sonic flow
$Fr > 1$	supercritical flow	$Ma > 1$	supersonic flow

Hydraulic Jump

Analogous to a normal shock in compressible flow, a hydraulic jump provides a mechanism by which an incompressible flow, once having accelerated to the supercritical regime, can return to subcritical flow. This is illustrated by the following figure.

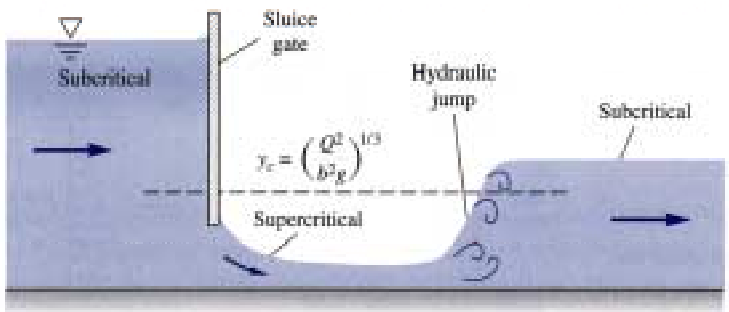


Fig. 10.5 Flow under a sluice gate accelerates from subcritical to critical to supercritical and then jumps back to subcritical flow.

The critical depth $y_c = \left(\frac{Q}{b^2 g}\right)^{1/3}$ is an important parameter in open-channel flow and is used to determine the local flow regime (Sec. 10.4).

Uniform Flow; the Chezy Formula

Uniform flow

1. Occurs in long straight runs of constant slope
2. The velocity is constant with $V = V_o$
3. Slope is constant with $S_o = \tan\theta$

From the energy equation with $V_1 = V_2 = V_o$, we have

$$h_f = Z_1 - Z_2 = S_o L$$

Since the flow is fully developed, we can write from Ch. VI

$$h_f = f \frac{L}{D_h} \frac{V_o^2}{2g} \qquad \text{and} \qquad V_o = \left(\frac{8g}{f}\right)^{1/2} R_h^{1/2} S_o^{1/2}$$

For fully developed, uniform flow, the quantity $\left(\frac{8g}{f}\right)^{1/2}$ is a constant and can be denoted by C. The equations for velocity and flow rate thus become

$$V_o = C R_h^{1/2} S_o^{1/2} \qquad \text{and} \qquad Q = C A R_h^{1/2} S_o^{1/2}$$

The quantity C is called the Chezy coefficient, and varies from 60 $ft^{1/2}/s$ for small rough channels to 160 $ft^{1/2}/s$ for large rough channels (30 to 90 $m^{1/2}/s$ in SI).

The Manning Roughness Correlation

The friction factor f in the Chezy equations can be obtained from the Moody chart of Ch. VI. However, since most flows can be considered fully rough, it is appropriate to use Eqn 6.64:

fully rough flow: $$f \approx \left(2.0 \log \frac{3.7 D_h}{\varepsilon}\right)^{-2}$$

However, most engineers use a simple correlation by Robert Manning:

S.I. Units $$V_o(m/s) \approx \frac{\alpha}{n} \left[R_h(m)\right]^{2/3} S_o^{1/2}$$

B.G. Units $$V_o(ft/s) \approx \frac{\alpha}{n} \left[R_h(ft)\right]^{2/3} S_o^{1/2}$$

where n is a roughness parameter given in Table 10.1 and is the same in both systems of units and α is a dimensional constant equal to 1.0 in S.I. units and 1.486 in B.G. units. The volume flow rate is then given by

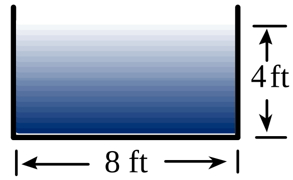
Uniform flow $$Q = V_o\, A \approx \frac{\alpha}{n} A\, R_h^{2/3}\, S_o^{1/2}$$

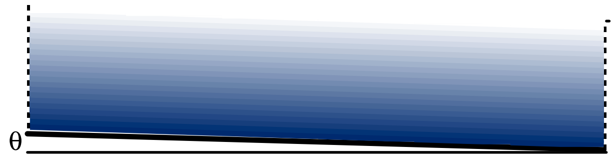
Table 10.1 Experimental Values for Manning's n Factor

		Average roughness height ϵ	
	n	ft	mm
Artificial lined channels:			
Glass	0.010 ± 0.002	0.0011	0.3
Brass	0.011 ± 0.002	0.0019	0.6
Steel, smooth	0.012 ± 0.002	0.0032	1.0
Painted	0.014 ± 0.003	0.0080	2.4
Riveted	0.015 ± 0.002	0.012	3.7
Cast iron	0.013 ± 0.003	0.0051	1.6
Cement, finished	0.012 ± 0.002	0.0032	1.0
Unfinished	0.014 ± 0.002	0.0080	2.4
Planed wood	0.012 ± 0.002	0.0032	1.0
Clay tile	0.014 ± 0.003	0.0080	2.4
Brickwork	0.015 ± 0.002	0.012	3.7
Asphalt	0.016 ± 0.003	0.018	5.4
Corrugated metal	0.022 ± 0.005	0.12	37
Rubble masonry	0.025 ± 0.005	0.26	80
Excavated earth channels:			
Clean	0.022 ± 0.004	0.12	37
Gravelly	0.025 ± 0.005	0.26	80
Weedy	0.030 ± 0.005	0.8	240
Stony, cobbles	0.035 ± 0.010	1.5	500
Natural channels:			
Clean and straight	0.030 ± 0.005	0.8	240
Sluggish, deep pools	0.040 ± 0.010	3	900
Major rivers	0.035 ± 0.010	1.5	500
Floodplains:			
Pasture, farmland	0.035 ± 0.010	1.5	500
Light brush	0.05 ± 0.02	6	2000
Heavy brush	0.075 ± 0.025	15	5000
Trees	0.15 ± 0.05	?	?

Example 10.1

Given:
Rectangular channel,
finished concrete, slope = 0.5˚
water depth: y = 4 ft, width: b = 8 ft
Find:
Volume flow rate (ft^3/s)





For the given conditions: $n = 0.012$ $S_o = \tan 0.5^\circ = 0.0873$

$$A = b\,y = (8\text{ ft})(4\text{ ft}) = 32\text{ ft}^2 \qquad P = b + 2y = 8 + 2(4) = 16\text{ ft}$$

$$R_h = \frac{A}{P} = \frac{32\text{ ft}^2}{16\text{ ft}} = 2\text{ ft} \qquad D_h = 4R_h = 8\text{ ft}$$

Using Manning's formula in BG units, we obtain for the flow rate

$$Q \approx \frac{1.486}{0.012}\left(32\text{ ft}^2\right)\left(2\text{ ft}\right)^{2/3}(0.00873)^{1.2} \approx 590\text{ ft}^3/\text{s} \quad \textbf{ans.}$$

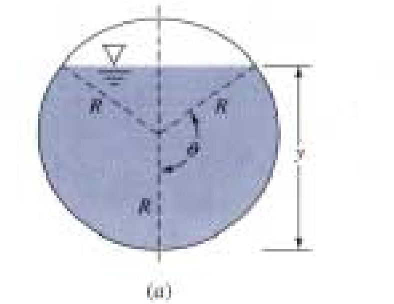
Alternative Problem

The previous uniform problem can also be formulated where the volume flow rate Q is given and the fluid depth is unknown. For these conditions, the same basic equations are used and the area A and hydraulic radius R_h are expressed in terms of the unknown water depth y_n.

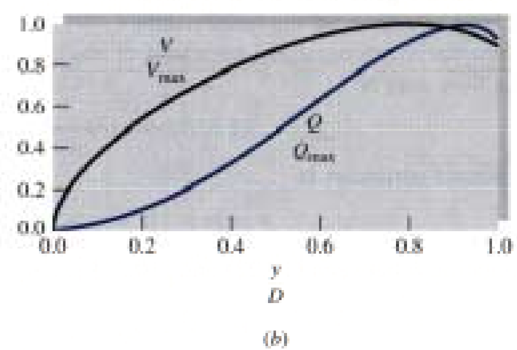
The solution is then obtained using iterative or systematic trial and error techniques that are available in several math analysis/ math solver packages such as EES (provided with the text) or Mathcad ®.

Uniform Flow in a Partly Full, Circular Pipe

Fig. 10.6 shows a partly full, circular pipe with uniform flow. Since frictional resistance increases with wetted perimeter, but volume flow rate increases with cross sectional flow area,



the maximum velocity and flow rate occur before the pipe is completely full.



For this condition, the geometric properties of the flow are given by the equations below.

Fig. 10.6 Uniform Flow in a Partly Full, Circular Channel

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad P = 2R\theta \qquad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$$

The previous Manning formulas are used to predict V_o and Q for uniform flow when the above expressions are substituted for A, P, and R_h.

$$V_o \approx \frac{\alpha}{n}\left[\frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3} S_o^{1/2} \qquad Q = V_o R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

These equations have respective maxima for V_o and Q given by

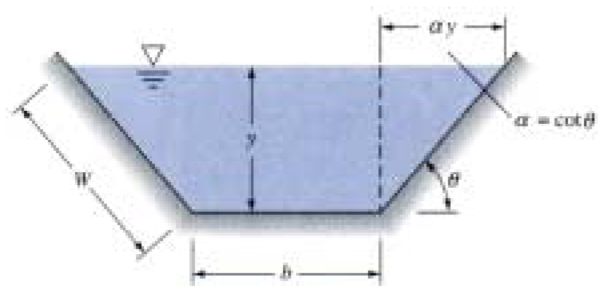
$$V_{max} = 0.718\frac{\alpha}{n}R^{2/3}S_o^{1/2} \quad \text{at} \quad \theta = 128.73° \quad \text{and} \quad y = 0.813\,D$$

$$Q_{max} = 2.129\frac{\alpha}{n}R^{8/3}S_o^{1/2} \quad \text{at} \quad \theta = 151.21° \quad \text{and} \quad y = 0.938\,D$$

Efficient Uniform Flow Channels

A common problem in channel flow is that of finding the most efficient low-resistance sections for given conditions.

This is typically obtained by maximizing R_h for a given area and flow rate. This is the same as minimizing the wetted perimeter.



Note: Minimizing the wetted perimeter for a given flow should minimize the frictional pressure drop per unit length for a given flow.

It is shown in the text that for constant value of area A and α = cot θ, the minimum value of wetted perimeter is obtained for

$$A = y^2\left[2\left(1+\alpha^2\right)^{1/2} - \alpha\right] \qquad P = 4\,y\left(1+\alpha^2\right)^{1/2} - 2\,\alpha\,y \qquad R_h = \frac{1}{2}y$$

Note: For any trapezoid angle, the most efficient cross section occurs when the hydraulic radius is one-half the depth.

For the special case of a rectangle (a = 0, q = 90˚), the most efficient cross section occurs with

$$A = 2y^2 \qquad P = 4\,y \qquad R_h = \frac{1}{2}y \qquad b = 2\,y$$

Best Trapezoid Angle

The general equations listed previously are valid for any value of α. For a given, fixed value of area A and depth y the best trapezoid angle is given by

$$\alpha = \cot\theta = \frac{1}{3^{1/2}} \qquad or \qquad \theta = 60^{\circ}$$

Example 10.3

What are the best dimensions for a rectangular brick channel designed to carry 5 m^3/s of water in uniform flow with $S_o = 0.001$?

Taking n = 0.015 from Table 10.1, $A = 2y^2$, and $R_h = 1/2\ y$; Manning's formula is written as

$$Q \approx \frac{1.0}{n} A R_h^{2/3} S_o^{1/2} \quad or \quad 5\,m^3/s = \frac{1.0}{0.015}\left(2y^2\right)\left(\frac{1}{2}y\right)^{2/3}(0.001)^{1/2}$$

This can be solved to obtain

$$y^{8/3} = 1.882\ m^{8/3} \qquad or \qquad y = 1.27\,m$$

The corresponding area and width are

$$A = 2y^2 = 3.21\,m^2 \qquad and \qquad b = \frac{A}{y} = 2.53\,m$$

Note: The text compares these results with those for two other geometries having the same area.

Specific Energy: Critical Depth

One useful parameter in channel flow is the specific energy E, where y is the local water depth.

$$E = y + \frac{V^2}{2g}$$

Defining a flow per unit channel width as q = Q/b we write

$$E = y + \frac{q^2}{2 g y^2}$$

Fig. 10.8b is a plot of the water depth y vs. the specific energy E.

The water depth for which E is a minimum is referred to as the critical depth y_c.

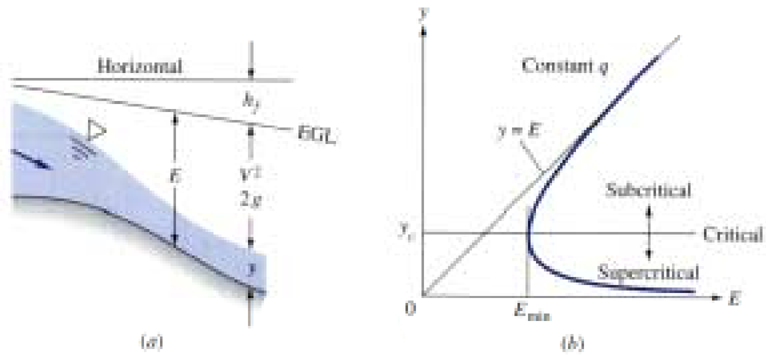


Fig. 10.8 Specific Energy Illustration

E_{min} occurs at

$$y = y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

The value of E_{min} is given by

$$E_{min} = \frac{3}{2} y_c$$

At this value of minimum energy and minimum depth we can write

$$V_c = (g y_c)^{1/2} = C_o \quad \text{and} \quad Fr = 1$$

Depending on the value of E_{min} and V, one of several flow conditions can exist.

For a given flow, if

$E < E_{min}$	No solution is possible
$E = E_{min}$	Flow is critical, $y = y_c$, $V = V_c$
$E > E_{min}$, $V < V_c$	Flow is subcritical, $y > y_c$,disturbances can propagate upstream as well as downstream
$E > E_{min}$, $V > V_c$	Flow is supercritical, $y < y_c$, disturbances can only propagate downstream within a wave angle given by

$$\mu = \sin^{-1}\frac{C_o}{V} = \sin^{-1}\frac{(gy)^{1/2}}{V}$$

Nonrectangular Channels

For flows where the local channel width varies with depth y, critical values can be expressed as

$$A_c = \left(\frac{b_o Q^2}{g}\right)^{1/3} \qquad \text{and} \qquad V_c = \frac{Q}{A_c} = \left(\frac{g A_c}{b_o}\right)^{1/2}$$

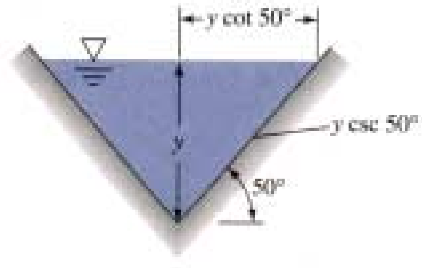
where b_o = channel width at the free surface.

These equations must be solved iteratively to determine the critical area A_c and critical velocity V_c.

For critical channel flow that is also moving with constant depth (y_c), the slope corresponds to a critical slope S_c given by

$$S_c = \frac{n^2 g A_c}{\alpha^2 b_o R_{h,c}} \quad \text{and} \quad \alpha = 1. \text{ for S I units and 2.208 for B. G. units}$$

Example 10.5



Given: a 50˚, triangular channel has a flow rate of Q = 16 m^3/s.

Compute: (a) y_c, (b) V_c,

(c) S_c for n = 0.018

a. For the given geometry, we have

P = 2 (y csc 50˚) A = 2[y (1/2 y cot 50˚)]

R_h = A/P = y/2 cos 50˚ b_o = 2 (y cot 50˚)

For critical flow, we can write

$$g A_c^3 = b_o Q^2 \quad or \quad g\left(y_c^2 \cot 50Þ\right) = \left(2 y_c \cot 50Þ\right) Q^2$$

y_c = 2.37 m **ans.**

b. With y_c, we compute

P_c = 6.18 m A_c = 4.70 m^2 $b_{o,c}$ = 3.97 m

The critical velocity is now $V_c = \frac{Q}{A} = \frac{16\, m^3/s}{4.70\, m} = 3.41\, m/s$ **ans.**

c. With n = 0.018, we compute the critical slope as

$$S_c = \frac{g n^2 P}{\alpha^2 b_o R_h^{1/3}} = \frac{9.81(0.018)^2 (6.18)}{1.0^2 (3.97)(0.76)^{1/3}} = 0.0542$$

Frictionless Flow over a Bump

Frictionless flow over a bump provides a second interesting analogy, that of compressible gas flow in a nozzle.

The flow can either increase or decrease in depth depending on whether the initial flow is subcritical or supercritical.

The height of the bump can also change the results of the downstream flow.

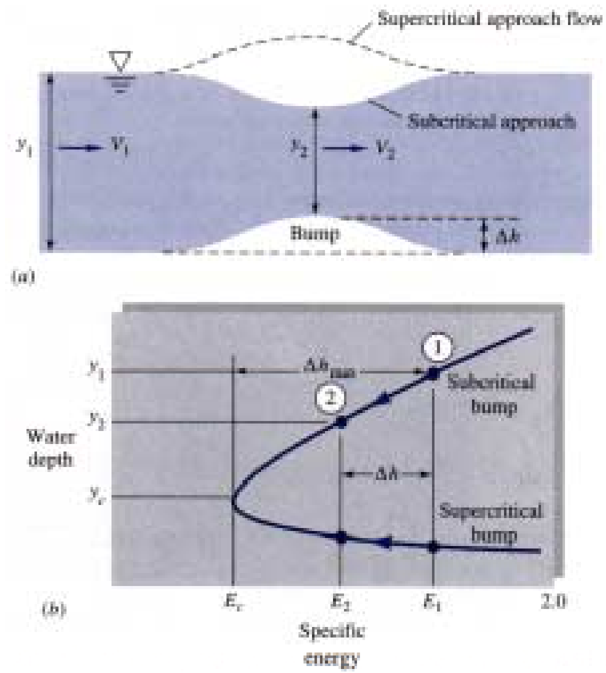


Fig. 10.9 Frictionless, 2-D flow over a bump

Writing the continuity and energy equations for two dimensional, frictionless flow between sections 1 and 2 in Fig. 10.10, we have

$$V_1 y_1 = V_2 y_2 \quad and \quad \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + \Delta h$$

Eliminating V_2, we obtain

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0 \quad where \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h$$

The problem has the following solutions depending on the initial flow condition and the height of the jump:

Key Points:

1. The specific energy E_2 is exactly Δh less than the approach energy E_1.
2. Point 2 will lie on the same leg of the curve as point 1.

3. For Fr < 1, subcritical approach	The water level will decrease at the bump. Flow at point 2 will be subcritical.
4. For Fr > 1, supercritical approach	The water level will increase at the bump. Flow at point 2 will be supercritical.
5. For bump height equal to $\Delta h_{max} = E_1 - E_c$	Flow at the crest will be exactly critical (Fr =1).
6. For $\Delta h > \Delta h_{max}$	No physically correct, frictionless solutions are possible. Instead, the channel will choke and typically result in a hydraulic jump.

Flow under a Sluice Gate

A sluice gate is a bottom opening in a wall as shown below in Fig. 10.10a. For free discharge through the gap, the flow smoothly accelerates to critical flow near the gap and the supercritical flow downstream.

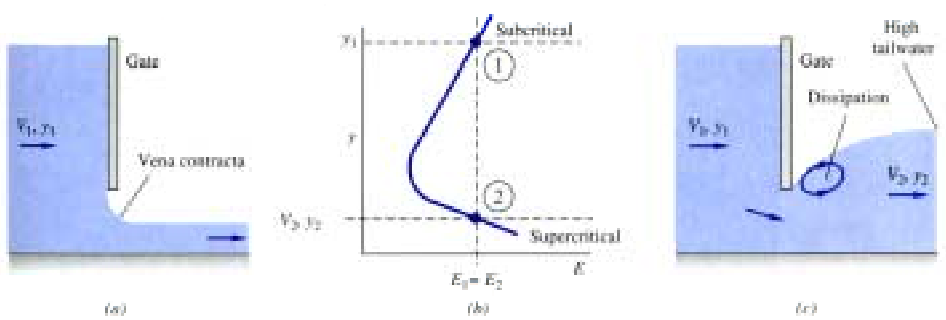


Fig. 10.10 Flow under a sluice gate

This is analogous to the compressible flow through a converging-diverging nozzle. For a free discharge, we can neglect friction. Since this flow has no bump ($\Delta h = 0$) and $E_1 = E_2$, we can write

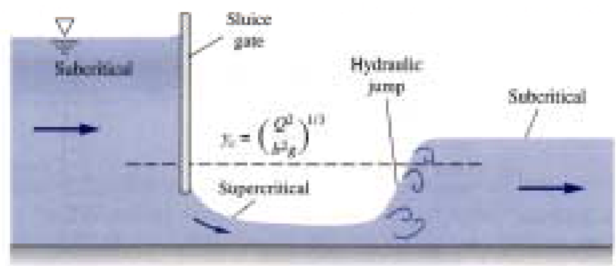
$$y_2^3 - \left(\frac{V_1^2}{2g} + y_1\right) y_2^2 + \frac{V_1^2\, y_1^2}{2g} = 0$$

This equation has the following possible solutions.

Subcritical upstream flow and low to moderate tailwater (downstream water level)	One positive, real solution. Supercritical flow at y_2 with the same specific energy $E_2 = E_1$. Flow rate varies as y_2/y_1. Maximum flow is obtained for $y_2/y_1 = 2/3$.
Subcritical upstream flow and high tailwater	The sluice gate is drowned or partially drowned (analogous to a choked condition in compressible flow). Energy dissipation will occur downstream in the form of a hydraulic jump and the flow downstream will be subcritical.

The Hydraulic Jump

The hydraulic jump is an irreversible, frictional dissipation of energy which provides a mechanism for super-critical flow to transition (jump) to subcritical flow analogous to a normal shock in compressible flow.



The development of the theory is equivalent to that for a strong fixed wave (Sec 10.1) and is summarized for a hydraulic jump in the following section.

Theory for a Hydraulic Jump

If we apply the continuity and momentum equations between points 1 and 2 across a hydraulic jump, we obtain

$$\frac{2y_2}{y_1} = -1 + \left(1 + 8\,Fr_1^2\right)^{1/2}$$ which can be solved for y_2.

We obtain V_2 from continuity: $$V_2 = \frac{V_1\,y_1}{y_2}$$

The dissipation head loss is obtained from the energy equation as

$$h_f = E_1 - E_2 = \left(\frac{V_1^2}{2g} + y_1\right) - \left(\frac{V_2^2}{2g} + y_2\right)$$

or

$$h_f = \frac{(y_2 - y_1)^3}{4\,y_1\,y_2}$$

Key points:

1. Since the dissipation loss must be positive, y_2 must be > y_1.
2. The initial Froude number Fr_1 must be > 1 (supercritical flow).
3. The downstream flow must be subcritical and $V_2 < V_1$.

Example 10.7

Water flows in a wide channel at q = 10 m^3/(s m) and y_1 = 1.25 m. If the flow undergoes a hydraulic jump, compute: (a) y_2, (b) V_2, (c) Fr_2, (d) h_f, (e) the percentage dissipation, (f) power dissipated/unit width, and (g) temperature rise.

a. The upstream velocity is $V_1 = \dfrac{q}{y_1} = \dfrac{10\,m^3/(s \cdot m)}{1.25\,m} = 8.0\,m/s$

The upstream Froude number is $Fr_1 = \dfrac{V_1}{(g\,y_1)^{1/2}} = \dfrac{8.0}{[9.81\,(1.25)]^{1/2}} = 2.285$

This is a weak jump and y_2 is given by

$$\frac{2\,y_2}{y_1} = -1 + \left(1 + 8\,(2.285)^2\right)^{1/2} = 5.54$$

and

$$y_2 = 1/2\ y_1(5.54) = 3.46\,m$$

b. The downstream velocity is $V_2 = \dfrac{V_1\,y_1}{y_2} = \dfrac{8.0(1.25)}{3.46} = 2.89\,m/s$

c. The downstream Froude number is

$$Fr_2 = \frac{V_2}{(g\,y_2)^{1/2}} = \frac{2.89}{[9.81\,(3.46)]^{1/2}} = 0.496$$

and Fr_2 is subcritical as expected.

d. The dissipation loss is given by

$$h_f = \frac{(y_2 - y_1)^3}{4\,y_1\,y_2} = \frac{(3.46 - 1.25)^3}{4(3.46)(1.25)} = 0.625\,m$$

e. The percentage dissipation is the ratio of h_f/E_1.

$$E_1 = \frac{V_1^2}{2\,g} + y_1 = 1.25 + \frac{8.0^2}{2\,(9.81)} = 4.51\,m$$

The percentage loss is thus given by

$$\% \, Loss = \frac{h_f}{E_1} 100 = \frac{0.625}{4.51} 100 = 14\%$$

f. The power dissipated per unit width is

$$\text{Power} = \rho \, Q \, g \, h_f = 9800 \text{ M/m}^3 * 10 \text{ m}^3/(\text{s m}) * 0.625 \text{ m} = 61.3 \text{ kw/m}$$

g. Using C_p = 4200 J/kg K, the temperature rise is given by

$$Power\ dissipated = \dot{m} C_p \, \Delta T$$

or

$$61{,}300 \text{ W/m} = 10{,}000 \text{ kg/s m} * 4200 \text{ J/kg K} * \Delta T$$

$$\Delta T = 0.0015^\circ\text{K}$$

negligible temperature rise