The whetstone of witte,

whiche is the seconde parte of Arithmetike: containyng thextraction of Rootes: The Coßike practise, with the rule of *Equation*: and the woorkes of *Surde Nombers*.

Though many stones doe beare greate price,
The whetstone *is for exersice*
As neadefull, and in woorke as straunge:
Dulle thinges and harde it will so chaunge,
And make them sharpe, to right good vse:
All artesmen knowe, thei can not chuse,
But vse his helpe. yet as men see,
Noe sharpenesse semeth in it to bee.
The grounde of artes *did brede this stone:*
His vse is greate, and moare then one.
Here if you list your wittes to whette,
Moche sharpenesse therby shall you gette.
Dulle wittes hereby doe greatly mende,
Sharpe wittes are fined to their fulle ende.
Now proue, and praise, as you doe finde,
And to your self be not vnkinde.

¶ These Bookes are to bee solde, at the Weste doore of Poules, by Jhon Kyngstone.

To the right worshipfull, the gouerners, Consulles, and the reste of the compainie of venturers into Moscouia, Robert Recorde Phisitian, wissheth healthe with continualle increase of commoditie, by their worthie and famous trauell.

I Wil not, nother ought I so euilly to iudge of my countrie, that learnyng here can haue no libertie: but by aide of frendeshippe, or strength of power. For as Englande did neuer wante learned wittes, so at this tyme I doubt not, but there be a great multitude, that desirously embrace all kindes of knowledge, and frendely are affected toward the furtherance of it. And therfore I dare saie, thei can not malice me, whiche am so willyng to helpe the ignoraunte, accordyng to my gifte and simple talēte. wherby also this moche praise I maie iustly craue, to haue the commendation and rewarde of a solliciter in this cause. For though my trauell can not moche profite them, that be well learned, yet doeth it excite the beste learned, to remember their duetie to their countrie: and to be a shamed, that thei hauyng so greate habilitie, shall be founde moare slacke to aide their coūtrie, then he that hath smaller knowledge, and lesse occasion otherwaies. Accordyng as men haue receiued, so are thei bounde to yeld. These excellente giftes are not lente vnto mē, to be hidden. And there are a great multitude that thurst, and long moche for soche aide. For bothe these causes I saie, that naturalle bōde to our countrie doeth chalenge it: and for that the honeste desires of so many good natures so moche requireth it, I exhorte them that be beste hable, to take from me this chargeable woorke, and to further their countrie men, as equitie would. And in the meane ceason while I see them so slacke, let them not bee offended with me, for preuentyng them. For better it is that a simple Coke doe prepare thy brekefast, then that thou shouldest goe a hungered to bedde. Yea better it is to haue some grosse repaste, then to sterue for honger. And the common sorte will finde smalle faulte of wante, as long as thei see any man serue their expectation. So that for this cause also, that my paines for a time, doeth excuse other finer wittes, thei ought to render me some thankes again. But if thei staie for feare of tauntes, and barkyng of curres, their corage is smalle. If thei misdoubte the gratefull acceptation of their studies, thei doe iniurie to their countrie. For whoe cā doubt but so ciuile a cōtrie, will thankefully receiue, and moste frendly recompense the trauelle, of soche as studie for their

benifite,

benifite, and serueth their necessarie commodities. This perswasion maketh me so bolde, that I can not thinke it neadefull, to seke any protector, for this or any like woorke. Sith euery good man will offer hymself, to defende that, wherby his natiue countrie is benifited. Excepte at some tyme, by excitation of the furies, some naughtie natures doe practice their fraude, to berefte the realme of some singulare commoditie. But as I feare no soche, so at this tyme I seke no soche aide against thē. Yet for testefiyng of frendeshippe, and gratefulle remembraunce, I would doe noe lesse, but sende this Booke to soche as I thought, not onely to deserue it, but also would gladly receiue it. And if I maie perceiue, that you doe accepte it (as I doubte not) with as good a wille, as I dooe sende it, I will for your pleasure, to your coumforte, and for your commoditie, shortly set forthe soche a booke of Nauigation, as I dare saie, shall partly satisfie and contente, not onely your expectation, but also the desire of a greate nomber beside. Wherein I will not forgett specially to touche, bothe the olde attempte for the Northlie Nauigations, and the later good aduenture, with the fortunate successe in discoueryng that voiage, whiche noe men before you durste attempte, sith the tyme of Kyng Alurede his reigne. I meane by the space of. 700. yere. Nother euer any before that tyme, had passed that voiage, excepte onely Ohthere, that dwelte in Halgolande: whoe reported that iorney to the noble Kyng Alurede: As it doeth yet remaine in auncient recorde of the olde Saxon tongue. So that if you continue with corage, as you haue well begon, you shall not onely winne greate riches to your selues, and bryng wonderfull commodities to your coũtrie. But you shall purchase therewith immortall fame, and be praised for euer, as reason would: for openyng that passage, that shall profite so many. In that Boke also I will shewe certain meanes, how without greate difficultie, you maie saile to the Northe Easte Indies. And so to Camul, Chinchital, and Balor, whiche bee coũtries of greate commodities. As for Chatai lieth so farre within the lande, toward the Southe Indian seas, that the iorneie is not to be attempted, vntill you be better acquainted with these countries, that you must first ariue at. But these thynges come in this place vntimely. I praie you accepte frendely in the meane ceason this Booke, whiche will bee a greate aide to the well vnderstandyng of the reste that is behinde. And as I shall vnderstande your desire, so will I haste the other. God prospere well your endeuoure, and sende you soche good successe, as so worthie aduenture doeth deserue: Whiche I doubte not will

insue,

insue, if cankered malice of some spitefull stomackes doe not preuaile, as thei can not cease to practice, to hinder your commoditie, and deface your trauel. But as it is euer seen, and therfore commonly knowen, that enuie doeth still repine at glorie, so ought all honeste hartes, to prosecute their good attemptes, and contempne the ballynge of dogged curres. So fare you well. And loue hym againe, that delighteth and studieth to farther your cõmoditie.

At London the .xii. daie of Nouember. 1557.

THE PREFACE to the gentle Reader.

ALthough nomber be infinite in increasyng: so that there is not in all the worlde, any thyng that can excede the quantitie of it: Nother the grasse on the grounde, nother the droppes of water in the sea, no not the small graines of Sande through the whole masse of the yearth: yet maie it seme by good reason, that noe man is so experte in *Arithmetike*, that can nōber the commodities of it. Wherefore I maie truely saie, that if any imperfection bee in nomber, it is bicause that nomber, can scarsely nomber, the commodities of it self. For the moare that any experte man, doeth weigh in his mynde the benifites of it, the more of them shall he see to remain behinde. And so shall he well perceiue, that as nomber is infinite, so are the commodities of it as infinite. And if any thyng doe or maie exceade the whole worlde, it is nomber, whiche so farre surmounteth the measure of the worlde, that if there were infinite worldes, it would at the full cōprehend them all. This nomber also hath other prerogatiues, aboue all naturalle thynges, for neither is there certaintie in any thyng without it, nother good agremente where it wanteth. Whereof no man can doubte, that hath been accustomed in the Bookes of Plato, Aristotell, and other auncient Philosophers, where he shall see, how thei searche all secrete knowledge and hid misteries, by the aide of nomber. For not onely the constitution of the whole worlde, doe thei referre to nomber, but also the composition of

The excellencie of nomber.

manne, yea and the verie substaunce of the soule. Of whiche thei professe to knowe no moare, then thei cā by the benifite of nomber attaine. Furthermore, for knowledge and certaintie in any other thynge, that mannes witte can reche vnto, there is noe possibilitie without nomber. It is confessed emongeste all men, that knowe what learnyng meaneth, that beside the Mathematicalle artes, there is noe vnfallible knowe- ledge, excepte it bee borowed of them. And emongeste them, it is sufficiently knowen, and well declared by *Nicomachus*, and diuerse other writers, that *Arithme-tike* is the fountaine of all the other, and their grounde and bonde, as he calleth it. If any man will saie, that Diuinitie, Lawe, and Physike, maie be had without it: or that thei take litle aide therby. Although I haue before this tyme aunswered thereto, yet now I saie

Diuinitie. again: that in Diuinitie there are greate hidde secre- tes in nombers. So that diuerse excellente Diuines, haue written whole Bookes of the misteries of nom- bers. And some of their Bookes intituled: *The Diui- nitie of Nombers.* But what Christen manne is igno- raunte, that betwene *Trinitie* and *vnitie*, doeth consiste the full grounde of al Diuinitie? Wherefore I neade not to allege the other hollie and sacred Nombers. Saue that. 7. will not permitte me to passe it with si- lence. In whiche is contained, not onely the secretes of the creation of all thynges: and the consummation of the whole worlde againe, with the state of eterni- tie: But also by it is the Sabbothes reste, and therby the full life and conuersation of godlie persones, re-

Lawe. presented and insinuate. In Lawe twoe kyndes of Iustice are the somme of the studie: *Iustice Distribu- tiue*, and *Iustice Commutatiue*, whiche termes I vse, as beste knowen in that arte: But what is any of thē bothe without Nomber? I haue said in an other place (as I learned of that noble Philosopher *Aristotell*) that

that if the knowledge and distinction, of *Geometricalle* and *Arithmeticall* proportion bee not well obserued, there can noe Iustice well bee executed. And how often the ministers of the Lawe use aide of Nomber, I neade not repete, bicause none but madde men doubt of it. And as for Physike, without knowledge and aide of nomber is nothynge. Wee see that nature in generation, bethe of manne and beastes, yea and of al thynges els doeth obserue nomber exactly. As well in the tyme of formation, as in the monethes of quickenyng, and of birthe. The misteries of the seuenth and nineth monethes are sufficiente testimonies therein. Beside that from the fourthe monethe til the seuenth many thynges bee permitted, that els bee not conueniente. For the use of the pulse, and for criticalle dayes, beside the proportion in degrees in simple medicines, and mixture of compounde medicines, and other infinite maters, what nomber can doe and what aide it giueth, onely the ignoraunte doe doubte. *Physike.*

But where can there bee any better testimonie for Nomber, then that the celestialle bodies doe kepe an unfallible nomber, in all their wonderfulle motions? By meanes whereof, mannes witte is habled to attaine the knowledge of them. As by the *Arithmeticalle* tables, of their motions it is easily knowen. Therefore and for that we see the yere, and all the distinction of times, beside the common use of trafike betwene menne, to depende of nomber, wee muste neades not onely confesse it to bee, as it were the onely staie of all natures workes, and of all ciuilitie: but we must also honoure and reuerence it, as often as wee duely remember the excellencie and benifite of it. Was not Nomber, thinke you, wonderfullie honoured, when noe name was thought moare meter for God, then the name of Nomber? I meane. 1. and. 3. the name of the Trinitie. But to come to moare familiare ma- *Astronomie.*

ters, I will saie, as *Plato* saieth in his Booke De summo bono. *Take awaie Arithmetike, with measure and weightes, from all other artes, and the reste that remaineth is but base, and of noe estimation.* Where although *Plato* dooe name three thinges in appearaunce, that is Nomber, Measure, and Weighte. What are Measure and Weighte, but nomber applied to seueralle vses? For *Measure* is but the nombryng of the partes of lengthe, bredthe, or depthe. And so *weighte* (as here it is taken) is the nomberyng of the heuinesse of any thyng. So that if nomber were withdrawen, no manne could either measure, or weigh any quantitie. And therfore it must followe: that nomber onely maketh all artes perfecte, and worthie estimation: seyng that without it, all artes are but base, and without commendation. This maie suffice for the iuste cōmendation of *Arithmetike*. But yet one commoditie moare, whiche all menne that studie that arte, doe fele, I can not omitte. That is the filyng, sharpenyng, and quickenyng of the witte, that by practice of *Arithmetike* doeth insue. It teacheth menne and accustometh them, so certainly to remember thynges paste: So circumspectly to consider thynges presente: And so prouidently to forsee thynges that followe: that it maie truelie bee called the *File of witte*. Yea it maie aptly bee named the *Scholehouse of reason*. The like iudgemente had *Plato* of it, as appeareth by his woordes in the seuenth booke De re publica. Where he saieth thus: *Thei that be apte of nature to Arithmetike, bee readie and quicke to attaine all kindes of learnyng. And thei that bee dulle witted, and yet bee instructed and exercised in it, though thei gette nothyng els, yet this shall thei all obtain, that thei shall bee moare sharpe witted, then thei were before.* What a benifite that onely thyng is, to haue the witte whetted and sharpened, I neade not trauell to declare, sith all menne confesse it to be as greate as maie be. Excepte any witlesse per-

Measure.
weighte.

sone

sone thinke he maie bee to wise. But he that moste feareth that, is leaste in daunger of it. Wherefore to conclude, I see moare menne to acknowledge the benifite of nomber, then I can espie willyng to studie, to attaine the benifites of it. Many praise it, but fewe dooe greately practise it: onlesse it bee for the vulgare practice, concernyng Merchaundes trade. Wherein the desire and hope of gain, maketh many willyng to sustaine some trauell. For aide of whom, I did sette forth the firste parte of *Arithmetike*. But if thei knewe how farre this seconde parte, dooeth excell the firste parte, thei would not accoumpte any tyme loste, that were imploied in it. Yea thei would not thinke any tyme well bestowed, till thei had gotten soche habilitie by it, that it might be their aide in al other studies. And if *Plato* doe require *Arithmetike*, as a specialle and a necessarie qualitie in hym, whom he would admitte as a citezein in his politike toune: How maie wee thinke of our selues, that desire to gouerne other, and yet can scante skille of common nomber? So farre are many, yea moste parte of vs from cunnyng in nomber. *Plato* thinketh noe manne hable to bee a good capitaine, excepte he bee skilfulle in this arte: And wee accoumpte it noe parte of those qualities, that bee required in any soche manne. Howbeit for the better trialle thereof, I haue in this Booke framed some of the questions in soche sorte, as thei maie approue the vse of this arte, not onely good for capitaines, but also moste necessarie for theim. So that without it, thei can not Marshall their battaile, nother vewe their enemies campe or forte. And if I shall saie as I thinke, without it a capitaine is noe capitaine. In this booke what I haue written, for the aide of all menne, and namely soche of my countrie menne, that vnderstand nothyng but Englishe, I neade not to repete perticularely, but remitte them to the booke it self, to see it at large. Onely this maie I saie: that as I haue doen in other artes, so in this I am the first venturer, in these darke maters. Wherfore I trust thei that be learned, and happen to reade this worke, wil beare the moare with me, if thei finde any thyng, that thei doe mislike: Wherein if thei will vse this curtesie, either by wri-tynge to admonishe me thereof, either theim selfes to sette forthe a moare perfecter wooorke, I will thynke them praise worthie. But if any manne will be so ha-stie, other to blame that, whiche he is not hable to a-mende, or to condempne that, whiche he did neuer vn-derstande: As some ofte tymes doe of a fonde curiosi-tie, I will wisshe hym a better witte, and moare mo-destie. And to preuente all soche seuere Judges, I thought it good to admonisshe you before, that by oc-casion of trouble vpon trouble, I was hindered from accomplisshyng this worke, as I did intende. But yet is here moare, then any manne might well looke for at my handes, if thei did knowe and consider myne estate. And this moche moare I saie: that if I maie perceiue, that this Booke bee as well receiued, as the firste parte was, I will striue moche, to stele from my troubles so moche tyme, as to set out the reste of this arte, moare completely in Englishe, then euer I sawe it in any toungue, hetherto doen: trust thereto as-suredly. And wisshe hym good, that traueleth for thy benifite.

Of the rule of Cose.

One thyng is nothyng,the prouerbe is,
which in some cases doeth not misse.
Yet here by woorkyng with one thyng,
Soche knowledge doeth from one roote spryng,
That one thyng maie with right good skille,
Compare with all thyng: And you will
The practice learne,you shall sone see,
what thynges by one thyng knowen maie bee.

To the curiouse scanner.

If you ought finde,as some men maie,
That you can mende,I shall you praie,
To take some paine so grace maie sende,
This worke to growe to perfecte ende.
But if you mende not that you blame,
I winne the praise,and you the shame.
Therfore be wise,and learne before,
Sith slaunder hurtes it self moste sore.

¶The seconde parte of Arithmetike, containyng the extraction of Rootes in diuerse kindes, with the Arte of Cossike nombers, and of Surde nombers also, in sondrie sortes.

¶The interlocutors, Master. Scholar.

The Master.

I See your desire can not bee satisfied, neither your request staied, untill I maie iustly aunswere you, that I can teache you no more: whiche aunswere maie staie your request, although it content not your desire.

Scholar. I beseche God of his mercie, to withstande all suche occasion: except it maie be more to your owne contentation and profite, then it would be pleasaunt to the louers of learning.

Master. Yet a iuste excuse maie stande for my declaration: As if ignoraunce doe inforce me to staie my trauell.

Scholar. Your owne ignorãce, I trust, you will not allege: and as for the ignorãce of other, it ought to bee no staie: sith the ignoraunte multitude doeth, but as it was euer wonte, enuie that knoweledge, whiche thei can not attaine, and wishe all men ignoraunt, like unto themself, but all gentle natures, contemneth suche malice: and despiseth theim as blinde wormes, whom nature doeth plague, to stay the poisone of their uenemous stynge.

Master. We shall not nede to stande on this talke, but trauell with knowledge to vanquishe ignorãce: And beleue that the *pricke* of knoweledge, is more of force then the stynge of ignoraunce: yea, the *poincte* in

Geometrie, and the unitie in *Arithmetike*, though bothe be undiuisible, doe make greater woorkes, & increase greater multitudes, then the brutishe bande of ignoraunce is hable to withstande.

Scholar. Our talke groweth well to our mater. I beseke you therfore, with that unitie beginne, and builde on it your worke, as a forte against ignorance

Vnitie.

Master. Unitie is of it self undiuisible, and yet is it in al partes of the worlde, and in euery thing. Yea, the worlde it self consisteth of unitie, is named of unitie, was made by unitie, and is preserued by unitie, and onely ignoraunce with her broode secluded from unitie, so that of it to repete the fulle force, would occupie muche time, and make greate volumes.

Scholar. Sith unitie is so mightie, and of suche force (as you saie) what maie be thought then of nomber, whiche containeth a multitude of unities? And is nothyng els but a collection of unities.

Nomber.

Master. Unitie is the fountaine and originalle of nomber, yea unitie by addition onely shall make a greater nomber, then any nombers can doe by multiplicacion. But this is marueilouse, that no nomber repineth against diuisiõ, till it come to an unitie: and then will it permit no farther diuisiõ. And therfore it is said, that unitie doeth neither multiplie nor diuide.

And as al nombers maie be more or lesse, so the lesser is euer a *parte* or *partes* of the greater.

A parte.
Partes.

As 5 unto 10 is a parte, named a halfe: but unto 7.5. is not a parte, but partes, and is called $\frac{5}{7}$. So 8 to. 24. is a parte that is $\frac{1}{3}$: but unto. 36. it is partes, that is $\frac{2}{9}$.

Scholar. I perceiue, you call it a parte, when the numeratour in the fraction (reduced to the smallesse) is an unitie. And when the numeratour is a nomber, then that fractiõ betokeneth partes of a nomber.

But I praie you, what varieties of nombers bee there principally to be considered in this arte?

Master.

Maſter. Nomber is diuided into diuerſe kindes, *The firſte di-*
for ſome are *whole nombers*, and thei onely of *Euclide*, *uiſion of nom*
Boetius, and other good writers are called nombers. *bers.*
Other are *broken nombers*, and are commonly called
fractions. Of theſe bothe I haue written in the firſte
and ſeconde partes of *Arithmetike*: So that I mighte
ſeme to curiouſe, to repete any parte of it again.

But now in eche kinde of theſe, there are certaine *The ſeconde*
nombers named *Abſtracte*: and other called nombers *diuiſion of*
Contracte. *nombers.*

Abſtracte nombers are thoſe, whiche haue no deno- *Abſtracte.*
mination annexed unto them. And thoſe that haue a-
ny denomination ioyned to theim, are called *Contracte* *Contracte.*
nombers.

Scholar. This I ſee to be a reaſonable diſtinctiō,
and agreable to the ſignification of the names.

For as that nomber is cōtracte, from his generall
libertie of ſignification, whiche is boūde to one deno-
mination, as in ſaiyng. 10. grotes (where. 10. is re-
ſtrained frō the libertie of ualowyng any other thing
but grotes) ſo if it had no denomination adioined, it
might then ſignifie the nomber of daies, or of miles,
or any like thyng, as well as of grotes. For when I
ſaie. 10. and doe not limitte any denominatiō, then is
that. 10. abſtracte and ſeuered frō all ſpecialties, and
ſtandeth free to any name of thing.

But this (me thinketh under your correction) can *whether bro-*
not extend to broken nombers: whiche euermore car- *ken nombers*
ry with them their denominacion: ſeyng thei conſiſte *be contracte,*
of a numerator and a denominator. *or not.*

Maſter. You ſeme to ſaie well. And the like iudge-
mēte doeth appere to be in ſome writers of this arte.
But yet ſeyng that fractions maie haue all other ar-
tificiall denominations, that whole nombers maie
receiue: and maie alſo bee without theim: therefore
muſt wee either make a more curiouſe diſtinction of

that name of denomination: or els wee must seclude fractions, frō the necessitie of that name: or els thirdly, to auoied contention, cal them nombers contracte improperly.

Scholar. I assente thereto as reason would.

Why fractiōs be not called nombers properly.

Yet one thyng more I must demaunde of you, why *Euclide*, and the other learned men, refuse to accompte fractions emongest nombers.

Master. Bicause all nombers doe consiste of a multitude of vnities: and euery proper fraction is lesse then an vnitie, and therefore can not fractions exactly be called nombers: but maie bee called rather fractions of nombers.

Scholar. In deede now that I doe waie the mater more exactly, it appereth that a fractiō is not properly a nomber, but a connexion and conference of nombers, declaryng the partes of an vnitie. For the numerator doeth signifie one nōber, and the denominator an other: The denominator declaryng into how many partes the vnitie is diuided, and the numerator signifiyng that of those partes, not all, but so many onely are to be takē, as the numerator importeth.

The diuision of nombers Abstracte.

Master. Well, then to procede, nombers abstracte are considered in. 3. principall varieties: That is, first without comparison to any other nomber or figure. And that nomber maie well be called *nomber absolute*.

Nombers Absolute.

Secondarily, some nombers bee vsed onely in relation to other, and therfore ought to bee called *nombers relatiue*.

Nombers Relatiue.

Thirdly, many nombers are referred to some figure, that maie rise by multiplicacion of their partes together, and that diuersly. And those nombers therfore maie bee called *figuralle nombers*.

Nombers Figuralle.

Scholar. If I conceiue your wordes rightly, this is your meanynge: that when I saie. 10. 25. 100. or 200. &c. these nombers stand absolute from all denomination minacion, and clere from all relatiō and comparison.

But when I saie. 6. is halfe of. 12. or. 15. is triple to 5. here the numbers beeyng compared together, are aptly called *nombers relatiue*: So if I saie, that. 16. is a *square nomber*, bicause it is made of. 4. multiplied by. 4 then is. 16. here to be called a *figuralle nomber.*

Master. You take it well. Therfore will I briefly touche the membres of euery kinde.

First of absolute nombres, some are *euen nombers*, and some are *odde.*

Scholar. All men knowe that. And farther, that *euen nombers* are those, whiche maie be diuided into equalle halfes: and so can not *odde nombers*, without a fraction. *Nombers, euen, & odde*

Master. Of this plaine easie thyng, marke what foloweth: a greater doubte dissolued. For if an odde nomber (as. 7. for example) can not bee parted into. 2. equalle nombers, eche beeyng halfe of. 7. then. 3. $\frac{1}{2}$. whiche is commonly called the halfe of. 7. is no nōber

Scholar. It can not be denied. And so (I see now) no fraction can bee a nomber. This greate doubte is plainly dissolued, by a very certaine and moste knowen principle.

Master. Now farther. Of bothe these kindes of nombers, some bee *compounde*, and some bee *simple* and *vncompounde*. *Compounde* nombers are made by multiplication of. 2. nombres together, and not by additiō, though the name might seme to serue to bothe. *Nombers compounde, and simple.*

Scholar. So I perceiue, that 5. is no *compounde* nōber, although it bee made by addition of. 2. and. 3. but 6. whiche is made by multiplication of. 2. and. 3.

Likewaies. 9. is *compounde*, bicause that. 3. multiplied by. 3. doeth make. 9.

And. 15. also is *compounde* by multiplyng. 5. and. 3. together.

And hereby I se that. 1. is not to be called a nomber *One is no nomber.*

for then all nõbers aboue it, must nedes be *compounde*, bicause thei consist all of vnities.

Master. But yet by multiplication of .1. no other nomber is *compounde*.

Scholar. By those wordes I am taught to knowe more, and speake better.

Master. *Euen nombers* are yet diuersly to be considered in their diuisions. For although the greate multitude of euen nombers bee *compounde*, yet .2. is accompted truely an euen nomber, originall, and *vncompoũde*. So that it maie make other nombers, & is made of no nõbers, but of vnities onely, as al odde nombers are.

Twoo is vncompounde.

All other euen nombers are *compounde*, and are diuersly diuided, for some are *euen nombers euenly*, and some are *euen nombers oddely*, and some are *euen nombers bothe euenly and oddely*. *Euen nombers euenly*, are suche nombers as maie bee parted continually into euen halfes, till you come to an vnitie. As for example. 32. first is diuided into. 16. as his euen halfe: and again, 16. into. 8. as his halfe: And. 8. againe by. 4. is parted into. 2. euen partes: Then. 4. also by. 2. And that. 2. is diuided into. 2. vnities, as his iuste halfes.

Euen nombers, euenly

But *euen nombers vneuenly*, are suche nombers as maie bee diuided into. 2. equalle partes: whiche are odde nombers. As. 18. is diuided into. 9. and. 9. as his halfes, and thei are odde. So. 10. is diuided by. 5. And 30. by. 15. with a greate nomber more of suche sorte.

Euẽ nombers vneuenly.

Nombers euen euenly and oddely, bee commonly called suche nombers, as maie bee diuided into. 2. equalle and euen halues: but before you come to an vnitie, the halfes will be odde nombers. As. 60. maie be first parted into. 30. and. 30. whiche are euen: And thei againe diuided by. 15. whiche is odde.

Euen nombers, euenly and oddely.

Likewaies. 24. is diuided first by 12. And that 12. by 6. & lastly. 6. is diuided by. 3. whiche is an odde nõber.

So. 28. maie bee diuided into. 2. equalle and euen

halues,

halues, that is into 14. And that. 14. into. 7. whiche is the halfe of. 14. but is an odde nomber.

Scholar. This I perceiue well. And, as I iudge, the distinction into those. 3. kindes, is not onely reasonable, but also nedefull. And yet you seme to speake doubtfully, of this laste membre. Bicause I remember not that you vse this worde commōly, but where you giue place rather to custome, then to reason.

Master. Or els to custome of the common sorte of writers, rather then to the iudgemente of the moste auncient writers.

And so in this case *Euclide* doeth not seme to admitte this thirde member. But accompteth it vnder the seconde kinde. As maie well appere in his. 9. boke, and 34. proposition, where he calleth suche a nomber, *euenly euen, and euenly odde* also, whiche place cōferred with the definitions in thesame booke, doeth approue in many wise mennes opinions, that *Euclide* minded but 2. onely kindes of those nōbers. And yet in this thing (I thinke) he did rather approue. 3. varieties by his propositiōs, then establishe onely. 2. sortes by his first definitions.

But herein I will spende no more tyme. But saie briefly that the distinctiō of. 3. kindes, serueth to good vse, and ease in teachyng.

And now for farther knowledge of nombers, some are called *nombers perfecte*, & some are *nombers imperfect*.

Perfecte nombers are suche ones, whose partes ioyned together, will make exactly the whole nomber. *Nombers perfecte.*

And therfore are. 6. and. 28. accompted perfecte nōbers: bicause the partes of eche of theim added together, doe make the ful and intere nomber, whose partes thei bee. As of. 6. the halfe is. 3. the thirde parte is 2. the sixte parte is. 1. As for a quarter, and fifte parte it hath not in whole nomber. Now put together. 1. 2. and. 3. and thei make iuste. 6. whose partes thei bee.

And therfore is.6.a perfecte nomber.

28. Likewaies. 28. hath for his halfe. 14. for his quarter.7.for his seuenth parte.4. and for his fowertenth parte.2.and for his.28. parte. 1. all whiche put together, that is.1.2.4.7.and.14.doe make.28.of this sort there are very fewe more in cōparisō. And for an exāple, I sett here, as many as are vnder.6000000000. and thei are these. 6. 28. 496. 8128. 130816. 2096128. 33550336. 536854528.

Nombers imperfecte. But now of the contrary kind, *imperfecte nombers* be suche, whose partes added together, doe make either more or lesse, then the whole nomber it self: whose partes thei bee.

And if the partes make more then the whole nomber, then is that nōber called *superfluouse*, or *abundaunt*. *Nombers superfluouse.* As 12. whose partes are.1.2.3.4,6.6. whichemake 16.

So. 20. hath for his partes. 1. 2. 4. 5. 10. whiche make.22. Likewaies. 120.hath these partes.

1. 2. 3. 4. 5. 6. 8. 10.
60. 40. 30. 24. 20. 15. 12. } whiche make 240.

And if the partes make lesse then the whole nomber, whose partes thei be, then is that nomber called *Diminute*, or *Defectiue*. *Nombers Diminute.* As.8.hath these partes.1.2.4. whiche make but.7.

So. 16. hath these partes. 8. 4. 2. 1. and thei make onely. 15.

Likewaies.32. whose partes are.1. 2. 4. 8. 16. and make but. 31.

Scholar. In all these nombers I note that you reken one, for a parte of eche one of theim: whiche before I thought you had denied.

Master. 1. canne neither multiplie nor deuide, and therfore compoundeth no nomber. But one maie increase addition, and therefore where partes be added together, there.1. maie well be called a parte.

And this shall suffice for the diuision of euen nombers

bers Abstracte.

Now to speake of *odde nombers*, some of thē are *compounde*, & some *vncompoūde*. Thei are *compounde*, whiche maie bee diuided into any other partes then unities. As. 9. whiche is cōpounde of. 3. And. 15. that is made of. 5. and. 3. Also. 21. is compounde of. 7. and. 3. And so furthe. But. 3. 5. 7. 11. 13. 17. 19. 23. 29. and suche like, bee odde nombers *vncompounde*. For thei are not made of any other then of unities. *Odde nōbers* *Compounde.* *Vncompoūde.*

Here must you understande by *composition*, the multiplication of the partes of nombers together, as you remembre, before was declared.

Scholar. I consider it so. And I remembre all that you haue taught me, for the diuisiō of nōbers *abstracte* and absolute. What saie you now of nōbers *relatiue*? *Nombers Relatiue.*

Master. Some tymes their *relation* hath regarde to their partes, namely, whether these. 2. that bee so compared, haue any common parte, that will diuide theim bothe. For if thei haue so, then are thei called *nombers commensurable*. As. 12. and. 21. bee *nombers commensurable*: for. 3. will diuide eche of theim. *Commensurable.*

Likewaies. 20. and. 36. be *commensurable*, seyng 4. is a commō diuisor for theim bothe. But if thei haue no suche common diuisor, then are thei called *incommensurable*. As 18 and 25. For 25 can bee diuided by no nomber more then by. 5. And. 18. can not be diuided by it. *Incommensurable.*

In like maner. 36. and. 49. are *incommensurable*: For 49. hath no diuisor but. 7. And 7. can not diuide. 36.

Scholar. Doe you meane then, that *incommensurable nombers*, haue no cōparison nor *proportion* together?

Master. Naie, nothyng lesse. For any. 2. nombers maie haue comparison & *proportion* together, although thei be *incommensurable*. As. 3. and. 4. are *incommensurable*, and yet are thei in a *proportion* together: as shall appeare anon.

But first I will declare unto you, the varieties of

B. i. proportion

Proportion. *proportion*, wherein there maie be double coferēce: I meane of the lesser to the greater, or of the greater to the lesser.

Whē the greater is cōpared to the lesser, it is called a *Proportion of the greater inequalitie.* As 6 to 2. or 5 to 3.

Of greater inequalitie. And when the lesser is conferred to the greater, it is called a *proportion of the lesser inequalitie*. As. 3. to. 5. or. 2. to. 6. *Of lesser inequalitie.*

Scholar. And what if I would cōpare two equalle nombers together?

Master. That is accoumpted also a proportion of many men: and is called the *proportion of equalitie.* And then ought the first diuision of proporcion to be, thus

Proporcion of
- Equalitie.
- Inequalitie.
  - The greater.
  - The lesser.

So proporciō of the greater inequalitie, is diuided into. 5. seuerall kindes: whereof. 3. be *simple*, and. 2. other *compounde*. The firste kinde is, when a greater nomber containeth the lesser diuerse times: as twise, or thrise, or oftener. So. 6. containeth. 3. twise: and it containeth. 2. thrise. This proporcion is called *Multiplex.* generally, *multiplex*, that is to saie, many folde: but specially it is named, accordyng to the tymes that it conteineth the lesser. So that if it contein hym twise, then is it named *dupla*, or double. As 2 to 1 and 4 to. 2.

And if it containe it thrise. As. 3. to. 1. and. 6. to. 2. it is called *tripla*, or triple.

If it containe it. 4. tymes, then is it *quadrupla*, or *quadruple*.

Of these and of diuerse other sortes in this kind also, here are the names briefly set doune with exāples.

Dupla

Dupla.	4.to.2:6.to.3:10. to. 5:18.to.9.	$\frac{2}{1}$	Double.
Tripla.	6.to.2:9.to.3:12.to.4: 18. to.6.	$\frac{3}{1}$	Triple.
Quadrupla.	4.to.1:8.to.2:12.to.3: 16. to.4.	$\frac{4}{1}$	Fower folde
Quintupla.	5.to.1:10.to.2:15.to.3:20.to.4.	$\frac{5}{1}$	Fiuefolde.
Sextupla.	6.to.1:12.to.3:18.to.3:24.to.4.	$\frac{6}{1}$	Sixefolde.
Septupla.	7.to.1:14.to.2:21.to.3:28.to.4.	$\frac{7}{1}$	Seuenfolde.
Octupla.	8.to.1:16.to.2:24.to.3:32.to.4.	$\frac{8}{1}$	Eightfolde.
Noncupla.	9.to.1:18.to.2:27.to.3:36.to.4.	$\frac{9}{1}$	Ninefolde.
Decupla.	10 to 1:20.to 2:30.to 3:40.to 4.	$\frac{10}{1}$	Tennefolde.
Vndecupla.	11.to.1:22.to.2:33.to.3.	$\frac{11}{1}$	Aleuenfolde
Duodecupla.	12.to.1:24.to.2:36.to.3.	$\frac{12}{1}$	Tweluefold.

And so infinitely.

Beside this there is an other kinde of proportion, when the greater containeth the lesser, more then ones, and not twise: and that maie bein 2 sortes. For if the greater containe the lesser, and any one parte of hym, that proporcion is called *Superparticulare*. For example, take. 5. to. 4. Sith. 5. doeth containe. 4. and his quarter. Likewaies. 6. to. 5. is in thesame kinde of proportion: although, not of thesame speciall sorte. For 6. comprehendeth. 5. and his fifte parte. *Superparticulare.*

So that for a more speciall distinction, eche of these and many other, haue their seueral names, accordyng to that parte, whiche thei doe containe. As if it containe the halfe more, it is named *Sesquialtera*. In whiche the proportion are these nombers followyng. *Sesquialtera*

3.to.2: 6.to.4: 9.to.6: 12.to.8: 15.to.10. | $1\frac{1}{2}$.

But if the greater comprehende the lesser, and his thirde parte, then is that named *Sesquitertia* proportion. As in these. *Sesquitertia.*

4.to.3: 8.to.6: 12.to.9: 16.to.12: 20.to.15. | $1\frac{1}{3}$.

And when the fifte, sixte, seuenth, or eight part doeth make the proporcion, or any other part els, the name is taken of that same parte. As for briefnesse I will here sette examples.

Seſquiquarta.	5. to. 4: 10. to. 8: 15. to. 12.	1 $\frac{1}{4}$	A quarter more.
Seſquiquinta.	6. to. 5: 12. to. 10: 18. to. 15.	1 $\frac{1}{5}$	a fifte more.
Seſquiſexta.	7. to. 6: 14. to. 12. 21: to. 18.	1 $\frac{1}{6}$	a ſixte more.
Seſquiſeptima.	8. to. 7: 16. to. 14: 24. to. 21.	1 $\frac{1}{7}$	a ſeuenth more.
Seſquioctaua.	9. to. 8: 18. to. 16: 27. to. 24.	1 $\frac{1}{8}$	an eight more.
Seſquinona.	10. to. 9: 20. to. 18: 30. to. 27.	1 $\frac{1}{9}$	a nineth more.
Seſquidecima.	11. to. 10: 22. to. 20: 33. to. 30.	1 $\frac{1}{10}$	a tenth more.
Seſquiundecima.	12. to. 11: 24. to. 22: 36. to. 33.	1 $\frac{1}{11}$	a leuenth more.
Seſquiduodecima.	13. to. 12: 26. to. 24: 39. to. 36.	1 $\frac{1}{12}$	a twelueth more

And ſo as farre as you liſte to procede in ſuche proportion: where one parte of the leſſer, is the iuſte difference and exceſſe, betwene it and the greater.

Superpartiēs

But if the difference be. 2. partes. 3. partes, or more partes: the proportiō is named *ſuperpartiente.* As. 5. to 3. And. 10. to. 6. For as. 5. containeth. 3. and. $\frac{2}{3}$. of it: ſo 10. holdeth. 6. and. $\frac{2}{3}$ of it.

Scholar. Now I perceiue ſome vſe alſo, of the diſtinction betwene a parte and partes in nomber: Of whiche at the beginnyng you did ſpeake. But how many kindes are there of this ſorte?

Maſter. There are infinite kindes in this ſorte of proporcion, as well as in the other. But for examples ſake, I will ſet furthe ſome of the moſte common nombers: that therby you maie gather the formes of the reſte. And theſe be thei, with their names.

Superbipartiens.	*Tertias.*	5. to. 3: 10. to. 6: 15. to. 9: 20. to. 12.	$\frac{2}{3}$
	Quintas.	7 to 5: 14. to 10: 21. to 15: 28. to. 20	$\frac{2}{5}$
	Septimas.	9 to 7: 18 to 14: 27. to 21: 36. to 28.	$\frac{2}{7}$
	Nonas.	11 to 9: 22 to 18: 33. to 27: 44. to 36	$\frac{2}{9}$
	vndecimas.	13. to 11: 26 to 22: 39 to 33: 52 to 44	$\frac{2}{11}$

ſuptripartiēs	*Quartas.*	7. to. 4: 14. to. 8: 21. to. 12: 28. to. 16.	$\frac{3}{4}$
	Quintas.	8. to. 5: 16. to. 10: 24. to. 15: 32. to 20.	$\frac{3}{5}$
	Septimas.	10 to 7: 20. to 14: 30 to 21: 40 to 28.	$\frac{3}{7}$
	Octauas.	11. to. 8: 22. to. 16: 33. to. 24.	$\frac{3}{8}$
	Decimas.	13. to. 10: 26. to. 20: 39. to. 30.	$\frac{3}{10}$
	Vndecimas.	14. to. 11: 28. to. 22: 42. to. 33.	$\frac{3}{11}$
	Decimastertias.	16. to. 13: 32. to. 26: 48. to. 39.	$\frac{3}{13}$
	Decimasquartas.	17. to. 14: 34. to. 28: 51. to. 42.	$\frac{3}{14}$
Superquadru partiens.	*Quintas.*	9. to. 5: 18. to. 10: 27. to. 15: 36. to. 20	$\frac{4}{5}$
	Septimas.	11 to 7: 22 to 14: 33. to. 21: 44. to 28.	$\frac{4}{7}$
	Nonas.	13 to 9: 26. to. 18: 39. to. 27: 52. to 36.	$\frac{4}{9}$
	Vndecimas.	15. to. 11: 30. to. 22: 45. to. 33.	$\frac{4}{11}$
	Decimastertias.	17. to. 13: 34. to. 26: 51. to. 39.	$\frac{4}{13}$
	Decimasquintas.	19. to. 15: 38. to. 30: 57. to. 45.	$\frac{4}{15}$
Superquintu partiens.	*Sextas.*	11 to 6: 22 to 12: 33. to. 18: 44. to 24.	$\frac{5}{6}$
	Septimas.	12. to. 7: 24. to. 14: 36. to. 21.	$\frac{5}{7}$
	Octauas.	13. to. 8: 26. to. 16: 39. to. 24.	$\frac{5}{8}$
	Nonas.	14. to. 9: 28. to. 18: 42. to. 27.	$\frac{5}{9}$
	Vndecimas.	16. to. 11: 32. to. 22: 48. to. 33.	$\frac{5}{11}$
	Duodecimas.	17. to. 12: 34. to. 24: 51. to. 36.	$\frac{5}{12}$
	Decimastertias.	18. to. 13: 36. to. 26: 54. to. 39.	$\frac{5}{13}$
	Decimasquartas.	19. to. 14: 38. to. 28: 57. to. 42.	$\frac{5}{14}$
	Decimasſextas.	21. to. 16: 42. to. 32: 63. to. 48.	$\frac{5}{16}$
Superſextus partiens.	*Septimas.*	13. to. 7: 26. to. 14: 39. to. 21.	$\frac{6}{7}$
	Vndecimas.	17. to. 11: 34. to. 22: 51. to. 33.	$\frac{6}{11}$
	Decimastertias.	19. to. 13: 38. to. 26: 57. to. 39.	$\frac{6}{13}$
	Decimasſeptimas	23. to. 17: 46. to. 34: 69. to. 51.	$\frac{6}{17}$
	Decimasnonas.	25. to. 19: 50. to. 38: 75. to. 57.	$\frac{6}{19}$
	Viceſimastertias.	29. to. 23: 58. to. 46: 78. to. 69.	$\frac{6}{23}$

Scholar. I vnderſtande by theſe examples, ſomewhat of their reaſons: but I perceiue, you doe not folowe their naturalle order, without interruption, in

 theſe

these of the laste kinde.

Master. To thintente you maie the better vnderstande good ground in that omission, I will set furthe here those omitted nombers: That you maie see how thei would expresse some other proportiō here named And therfore thei doe seme rather to be omitted, then in deede so to be.

Marke theim well.

Superbipartiens.	*Secundas.*	4. to. 2:	8. to. 4.	$\frac{2}{1}$
	Quartas.	6. to. 4:	12. to. 8.	$1\frac{1}{2}$
	Sextas.	8. to. 6:	16. to. 12.	$1\frac{1}{3}$
	Octauas.	10. to. 8:	20. to. 16.	$1\frac{1}{4}$
	Decimas.	12. to. 10:	24. to. 20.	$1\frac{1}{5}$

Scholar. In deede here I see, the firste is double proportion. The seconde *sesquialtera*, the thirde *sesquitertia*, the fowerth *sesquiquarta*, & the fifte *sesquiquinta*.

Master. So marke these other.

Supertripartiens	*Secundas.*	5. to. 2:	10. to. 4.	$2\frac{1}{2}$
	Tertias.	6. to. 3:	12. to. 6.	$\frac{2}{1}$
	Sextas.	9. to. 6:	18. to. 12.	$1\frac{1}{2}$
	Nonas.	12. to. 9:	24. to. 18.	$1\frac{1}{3}$
	Duodecimas.	15. to. 12:	30. to. 24.	$1\frac{1}{4}$

Scholar. The firste of these I knowe not, but all the other are named before.

Master. The firste is a compounde proportion (as anon I will declare) and is named *dupla sesquialtera*.

But now will I sette furthe all the other omitted names.

superquadrupartiens.	Secundas.	6. to. 2:12. to. 4.	$\frac{3}{1}$	Tripla.
	Tertias.	7. to. 3:14. to. 6.	$2\frac{1}{3}$	Dupla sesquitertia.
	Quartas.	8. to. 4:16. to. 8.	$\frac{2}{1}$	Dupla.
	Sextas.	10. to. 6:20. to. 12.	$1\frac{2}{3}$	supbipartiēs tertias
	Octauas.	12. to. 8:24. to. 16.	$1\frac{1}{2}$	sesquialtera.
	Decimas.	14. to 10:28. to 20	$1\frac{2}{5}$	supbipartiēs quintas
	Duodecimas.	16. to. 12:32. to. 24	$1\frac{1}{3}$	Sesquitertia.
	Decimas quartas.	18. to. 14:36. to. 28	$1\frac{2}{7}$	supbipartiēs septimas
	Decimas sextas.	20. to 16:40 to 32.	$1\frac{1}{4}$	Sesquiquarta.

superquintupartiens.	Secundas.	7 to 2 : 14 to 4.	$3\frac{1}{2}$	Tripla sesquialtera.
	Tertias.	8 to 3 : 16 to 6.	$2\frac{2}{3}$	Dupla superbipartiens tertias.
	Quartas.	9 to 4: 18 to 8.	$2\frac{1}{4}$	Dupla sesquiquarta.
	Quintas.	10 to 5: 20 to 10.	$\frac{2}{1}$	Dupla.
	Decimas.	15 to 10: 30 to 20.	$1\frac{1}{2}$	Sesquialtera.
	Decimas quintas.	20 to 15:40 to 30.	$1\frac{1}{3}$	Sesquitertia.

supersextupartiens.	Secundas.	8 to 2: 16 to 4.	$\frac{4}{1}$	Quadrupla.
	Tertias.	9 to 3: 18 to 6.	$\frac{3}{1}$	Tripla.
	Quartas.	10 to 4: 20 to 8.	$2\frac{1}{2}$	Dupla sesquialtera
	Quintas.	11 to 5: 22 to 10.	$2\frac{1}{5}$	dupla sesquiquinta.
	Sextas.	12 to 6: 24 to 12.	$\frac{2}{1}$	Dupla.
	Octauas.	14 to 8: 28 to 16.	$1\frac{3}{4}$	supertripartiens quartas.
	Nonas.	15 to 9: 30 to 18.	$1\frac{2}{3}$	superbipartiens tertias.
	Decimas.	16 to 10:32. to 20.	$1\frac{3}{5}$	supertripartiens quintas.
	Duodecimas.	18 to 12: 36 to 24.	$1\frac{1}{2}$	sesquialtera.
	Decimas quartas.	20 to 14:40 to 28	$1\frac{3}{7}$	supertripartiens septimas.
	Decimasquintas.	21 to 15: 42 to 30.	$1\frac{2}{5}$	superbipartiens quintas.
	Decimas sextas.	22 to 16:44 to 32.	$1\frac{3}{8}$	supertripartiens octauas.
	Decimas octauas.	24 to 18:48 to 36.	$1\frac{1}{3}$	sesquitertia.
	Vicesimas.	26 to 20:52 to 40.	$1\frac{3}{10}$	supertripartiens decimas.
	vicesimas secūdas.	28 to 22:56 to 44.	$1\frac{3}{11}$	supertripartiens vndecimas.

Scholar. I see well that these proportions, bee agreable with some other name: and therfore might seme superfluouse in this place.

Master.

Master. Not onely superfluousely, but also falsely should thei bee placed here: seynge thei doe belong to other places of right.

Scholar. Why doe you not name theim all by Englishe names?

Master. Bicause there are no soche names in the Englishe tongue. And if I should giue theim newe names, many would make a quarrelle against me, for obscuryng the olde Arte with newe names: as some in other cases all redy haue doen.

Scholar. Yet I praie you declare those doubtfull names of compounde proportions.

Master. As there is one kinde of proportion, that is named *multiplex*, or manyfolde, whiche doeth containe the lesser diuerse times exactly. And two other whiche doe containe the lesser ones, and some parte or partes of the same: So those kindes maie be compounded together. As when the greater number conteineth the lesser, twise, or thrise, or oftener: and yet more ouer some parte or partes of the same. So. 8. containeth 3 twise, and his $\frac{2}{3}$. And 10 comprehendeth 3. thrise and his $\frac{1}{3}$.

The firste example is generally called *multiplex superpartiens*: bicause the greater containeth the lesser certaine tymes, and some partes of it besides. But more specially it is called *dupla superbipartiens tertias*, that is, double with $\frac{2}{3}$ more.

The seconde example is generally referred to *multiplex superparticularis*: bicause in it the greater comprehendeth the lesser oftentymes, (as here thrise) and his $\frac{1}{3}$ more. And therfore specially it is called *tripla sesquitertia*.

But as I doe intende briefly to ouer runne this parte: so will I by tables set forthe the kindes of thē with their examples.

The table of proportion of the greater inequalitie.

- Proportion of the greater inequalitie.
  - Simple & vncōpoūde
    - Manyfolde.
      - Doble.
      - Triple.
      - Quadriple.
      - &c.
    - Supparticulare
      - Sesquialter.
      - Sesquiterce.
      - Sesquiquart.
      - Sesquiquint.
      - &c.
    - Superpartiente.
      - Superbipartiente.
      - Supertripartiente.
      - Superquadrupartiēte
      - Superquintupartiēte
      - &c.
  - cōpounde.
    - Manifolde. supparticulare
      - Doble.
        - Sesquialter.
        - Sesquiquart. &c.
      - Triple.
        - Sesquitierce.
        - Sesquiquinte. &c.
      - Quadriple.
        - Sesquialter.
        - Sesquisexte. &c.
    - Manifolde. Suppartiente.
      - Doble.
        - Superbipartiente.
        - Suptripartiēte &c.
      - Triple.
        - Superbipartiente.
        - Suptripartiēte. &c.
      - quadriple
        - Supquintupartiēte.
        - Supsextupartiente.
        - &c.

Examples of eche compounde kinde, mentioned in the former table.

Manifolde Superparticulare.	*Double.*	*Sesquialter.*	5 to 2.
		Sesquiquarte.	9 to 4.
	Triple.	*Sesquitierce.*	10 to 3.
		Sesquiquinte.	16 to 5.
	quadriple	*Sesquialter.*	9 to 2.
		Sesquisexte.	25 to 6.
Manifolde supparti ẽte	*Double.*	*Superbipartiente tierces.*	8 to 3.
		Superipartiente quartas	11 to 4
	Triple.	*Superbipartiente tierces.*	11 to 3.
		Suptripartiente quartes.	15 to 4.
	quadriple	*Supquĩtupartiẽte quartas*	29 to 6
		Supsextupartiẽte septimas.	34 to 7

Scholar. What more is there to bee learned of these proportions? For by these formes, I maie easely gather the valure or rate of any proportion.

Master. This maie stande for their numeration: saue that moste aptly thei ought to bee sette as fractions, in their leaste tearmes: as you haue here diuerse examples.

Scholar. You meane that double *sesquialter* must be written thus $\frac{5}{2}$, and so of the reste.

Master. Or els thus $2\frac{1}{2}$. and so *triple sesquiquinte* in this sorte: $3\frac{1}{5}$, or thus $\frac{16}{5}$ and so of all other.

And for farther woorke, you shall understande that proportions maie bee added, subtracted, multiplied and diuided: and verie straunge workes therby acchiued:

acchiued. For of the Arte of Proportios, dependenth all the subtilties, and fine workes, not onely of *Arithmetike*, but also of *Geometre* besides farther mater that as now I will not touche. But as for the workes of *Proportions*, I will omitte them til an other tyme: considerynge not onely the troublesome condition, of my vnquiete estate: but also the conuenient order of teachynge, whereby it is required that the extraction of rootes, should go orderly before the arte of Proportions: whiche without those other, cā not be wrought.

Therfore will I now onely declare these kindes of proportion, whiche yet are not spoken of: to the intente that you maie haue here, the generall diuision of numbers, somewhat sufficiently touched.

As you see that betwene any two numbers, there maie be a conference of proportion: so if any one proportion be continued in more then. 2. nombers, there maie be then a conference also of these proportions, in their seuerall termes: and that conference or comparison is named *Analogie*: whiche some delighte to call proportionalitie: As in this example, where 3 nōbers beare like proportion in their progression: 4. 6. 9. You see that 6. to 4. is in proportion *sesquialter*: and so is 9. to 6. and therfore is there a like proportion betwene the. 2. laste, as there is betwene the. 2. firste. *Analogie.*

Scholar. This I consider well by progression in *Arithmetike*.

Master. Likewaies where fower termes or more be set in order of proportion, as here 2. 6. 18. 54.

Scholar. I perceiue this wel: for here the proportiō is triple. But what saie you to this forme of comparison in Proportion? As 6. is to. 2: So. 30. is to. 20. Is it not all one kinde of *Analogie*?

Master. It is one kinde of Analogie generalle, whiche maie be called *directe Analogie*: bicause the first is compared to him that doeth folowe nexte: & so eche *Directe analogie.*

 other

other is still referred to that, that foloweth nexte. But this is the difference: that in the firste, there is a continuaunce of collation: and one terme is compared with twoo nombers: But in that forme of example, whiche you put, there is no nomber compared twise: For the first is referred to the seconde, and the second to the thirde. And so haue thei seueralle names to distincte theim a sonder.

Continuall Proportion.

Wherfore whē the first nomber is referred to the seconde, and that seconde to the thirde: the proportion is called *continualle*: and it maie consiste betwene 3. termes. As 5. 15. 45. doe procede in a continuall triple proportion. For as 5. is to 15: so is 15. to 45. as you doe see. But when I saie thus: as 5. is to 15. so 6. is to 18. Here is a triple proportion, but not continualle. For the seconde terme beynge 15. is not compared with the thirde terme, that is 6. And therfore is it called a proportion *discontinualle*.

Discontinual Proportion.

Scholar. Now I perceiue certainly their distinction: For in twoo poinctes these examples doe agree, and differ in a thirde poincte.

Firste thei agree in that (as you saied) that the formoste is referred to the other that foloweth it nexte: And secondly, thei agree in this also, that bothe are compared in a triple proportion. But in this thei differ, that the seconde terme, doeth not beare like proportion to the thirde, as the thirde doeth to the fourth or the firste to the seconde.

Master. Farther more there is to bee noted, that in discontinualle proportion, there can bee no fewer then fower termes, or numbers: and so by euen formes still, as. 6. or. 8. and so forthe. Where as in continuall proportion, your termes maie bee of any number, euen or odde: aboue. 2.

And although I might saie more of the diuersities of proportion: as of *Proportion conuersed* or *indirecte*, *Proportion*,

portion interchaunged, compounde Proportion, parted Proportion, reuersed Proportion, and *Proportion by equalitie.* Yet I thinke better to procede for this tyme, to the other kindes of nomber, and to reserue the explication of proportions to their peculiare place.

Scholar. As you knowe the best order, so it shalbe mete that you doe vse your owne iudgement therein.

Of figuralle nombers.

Master.

THE nexte kinde of nombers are called *Figuralle nombers*: bicause thei doe, or maie represente some figure: And are euer considered in relation to those formes.

Some of them haue a comparison and relatiō to length onely, and therefore are named *Linearie nombers*: whiche name, although it maie bee referred moste aptly to suche numbers, as will make no other forme onely, yet maie it also be applied to any number abstract. Sith all soche numbers maie be considered as the sides of other figuralle numbers. *Nombers linearie.*

Secondly, numbers maie be considered, according to soche formes as thei make other in progression, or in multiplication: And those maie well be named *Superficiall nombers*, or *Flatte nombers.* Whereof there bee as many varieties, as there bee diuersities of figures in *Geometrie.* As numbers *Triangulare, Quadrate, Cinkeangeled, Siseangeled* and so furthe. Also numbers *circulare, diametralle,* & *like flattes,* all whiche nombers haue bothe lengthe and breadthe: and thereof bee named *superficiall nombers.* *Superficiall nombers. Flatte nombers.*

Beside these there are other numbers, whiche are made of many multiplications, and thei are called *sounde nombers*: bicause that as by the firste multiplication, thei take lengthe and breadthe, like flatte numbers, so by the second multiplication, thei take depthe also: And thereof be thei named *bodily nombers*, or *sound nombers*.

Sounde numbers.

The leaste of them all is commonly called a *Cube*, or a *Cubike nomber*: And the other in their degrees seuerally named, as thei bee made by seueralle multiplications. For accordynge to the number of their multiplications, thei take their names. And all that haue like number of multiplications, are of one kinde, and bere one name: as well in flatte numbers, as in sounde.

But consideryng the infinite multitude of those figuralle numbers, I thinke beste to speake of theim onely in this place, whiche haue muche profitable vse in this arte. And, of those, emong infinite flatte numbers, I will take onely fower. That is to saie, *square nombers*, *longesquares*, *diametralle nombers*, and *likeflattes*.

Square nombers.

Square nombers are those, whiche maie be diuided by some one number, and haue thesame number for the quotiente: that is to saie, that a square nōber is made by the multiplication of any number into it self, as 10 multiplied by. 10. maketh. 100. That. 100. is a square number: whiche. 100. if I doe diuide by. 10. the quotiente will be. 10. also.

Scholar. So, 4. multiplied by. 4. maketh. 16: and that muste be a square number by like reason.

Master. So it is.

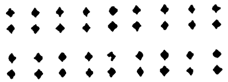
Scholar. And if I multiplie. 9. by. 4. is not that a square number? Seyng fower semeth to make all nōbers square by multiplication.

Master. Consider this well, that a square number doeth make suche a square in number, as a *iuste square* doeth make in *Geometrie*: That is suche a one

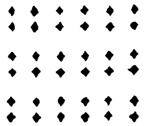
whose

whose sides are equalle. For and if the one side be longer then the other, that figure in *Geometrie* is called a *long square*, and so it is named in number, a *long square* also.

Now if I sette doune the figure of your number, as you termed it, and sette. 4. for the one side, and. 9. for the other, this will the figure shewe.



Where you se a plain longsquare: yet is the whole number that amounteth of this multiplication: truely named a square number, as here you maie see. But then is the side or roote of it, neither. 4. nor. 9. but. 6.



Scholar. Now I understande it: and the better by this figuralle example. And here also I haue learned what a *Roote* is: for you seme to expounde it, to bee the side of a figuralle number. *A roote.*

Master. Euery flatte nomber, and euery sounde number also haue their sides: But no flatte number, saue onely squares haue a roote: bicause a roote in flatte numbers, is a number multiplied by it self.

And in sounde numbers, thei onely haue rootes, whiche bee made by many multiplications, of some one nũber by it self: other by that, whiche riseth of it.

As when I saie, twoo tymes, twoo twise, maketh 8. that number is a sounde number: and is named a *Cube*. And so. 3. tymes. 3. thrise, doeth make. 27. whiche is also a *Cube*.

And generally, any number that is made by suche 2. multiplications, is called a *Cube*, or *Cubike* number. *A cube.* And the number of that multiplication, whiche commonly is named the multiplier, is in this poincte called the Cubike roote of that number. *A cubike roote.*

Wherfore, thus also maie you define a Cubike nõber, *A cubike number.*

ber: to bee ſuche a number, as beeyng diuided by his roote, ſhall haue for the quotiente the ſquare of the ſame roote.

Scholar. Hereby I perceiue, that one multiplication, of any number by it ſelfe, doeth make a ſquare number. And twoo multiplications in that ſorte, doe make a Cubike number.

What if I doe multiplie any number thriſe, or fower tymes, or oftener in that ſorte, are there proper names for ſuche numbers?

Maſter. Yes in deede: as I will declare anon.

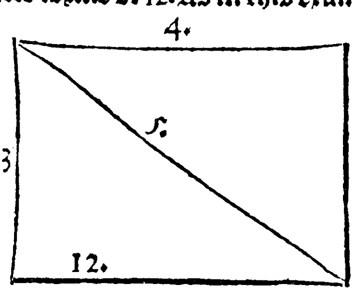
But firſte before we attempte the other ſounde nōbers, it ſhall bee mete, that we doe declare thoſe twoo ſortes of flatte numbers, whiche I named before: that is diametralle numbers, and like flattes.

A diametral nomber.

A *Diametralle nomber*, is ſuche a number as hath twoo partes of that nature: that if thei bee multiplied together, thei will make the ſaied *diametralle nomber*: And the ſquares of thoſe twoo partes, beeyng added together, will make a ſquare nōber alſo: whoſe roote is the *diameter* to that *diametralle nomber*.

A diameter.

As 12 is named a *diametralle nomber*, for that he hath twoo partes, that is. 3. and. 4, whiche beeyng multiplied together, doe make 12. that is the firſte number. And if their ſquares be added together, thei wil make a thirde ſquare: and the roote of that number will bee the *diameter* to that platte forme of 12. As in this example you ſee.



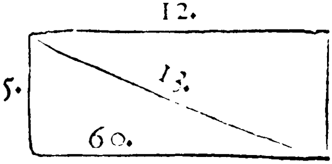
The one ſide is. 4. and the other ſide is 3 whiche bothe multiplied together, doe make 12. Then take the ſquare of fower whiche is 16 and the ſquare of. 3, whiche is. 9. and put them

together

together,and then will make.25. whose roote,beyng 5.is the *diameter* of that platte forme.

Scholar. That doe I perceiue well, bicause it is confirmed by the.33. theoreme of the pathewaie.

Master. Yet take an other example. In this platte forme of.60. you see the one side to bee.5. and the other side to bee 12. Now take the square



nomber of. 12. whiche is. 144. and the square of.5. whiche is.25.and put them together: so will it make 169. whiche is a square nomber:and hath.13. for his roote.

Likewaies.120. is to be accoumpted a *diametralle nomber*. For so muche as it hath twoo partes: that is 8. and. 15. whiche beeyng multiplied together, doe make the firste nomber. 120. And the square of those twoo partes(that is.64.for.8:and.225.for.15.)beyng bothe added together, doe make. 289. whiche is a square nōber:and hath for his roote.17. And therfore that.17. is the *diameter* to that *diametralle nomber*. 120.

Like examples infinite might I giue you. But these for explication of the name,maie suffice.

Scholar. I doe well understande the examples: saue that I knowe not how to finde the roote of the laste square nomber, whiche amounteth by the addi-tion of the former twoo squares together.

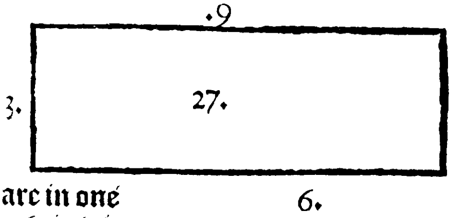
Master. That arte will I teache you anon. But we maie not forgette firste to ende all the difinitions of soche names, as I minde to write of.

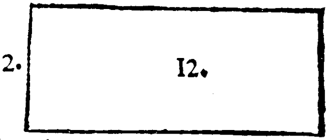
Whereof yet there resteth *like flattes*: whiche maie bee as well taken for trianguler figures, as for qua-drate figures. *Like flattes.*

So that of any of them, when the sides of one plat forme, beareth like proportion together, as the sides

 of

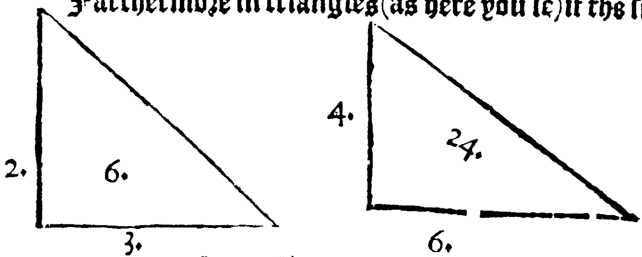
of any other flatte forme of thesame kinde doeth, then are those formes called *like flattes*. As in these. 2. longe Squares: bicause the sides of them bothe, are in one proportion (for. 6. is triple to. 2: as well as. 9. is triple to 3.) Therfore are the whole figures called *like flattes*.





And so of due conuenience, their nombers (that expresse their quantities, whiche here are. 27. and 12) be called by the like names, *like flattes*.

Farthermore in triangles (as here you se) if the si-



des of the one beare like proportion together, as the sides of the other doe: then are thei called *like flattes* also. And their nombers, that declare their quantities, in like sorte are named *like flattes*.

Scholar. I pearceiue here: As 4 is to 2: so. 6. is to 3. bothe beyng in a double proportion. And therfore 6 and. 24. are to be called *like flattes*.

Master. You vnderstande it well.

And thus haue we briefly ouer runne the diuision of nomber, into his principalle kindes: And haue set forthe the definitiōs of eche of them, with examples.

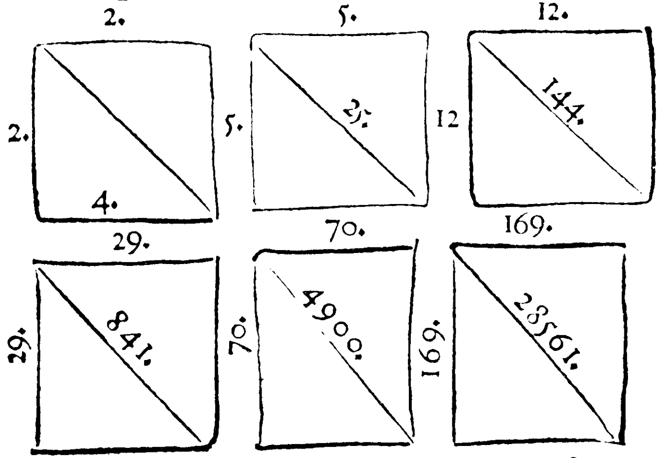
The vſe of them you ſhall ſe largely in the practiſe of this arte.

• But to the intent you maie the better obſerue and regarde theſe twoo laſte kindes of nombers: whiche are commonly neglected of artes men, I will ſhewe you ſome vſe of them, with their properties.

Firſte, all *diametralle nombers* doe ſette forthe a triangle, hauyng all three ſides knowen: whiche thyng as it doeth ſerue to many and wonderfull purpoſes: ſo can it be found in no other nombers, then onely in *diametrall nombers.* *Of diametralle nombers.*

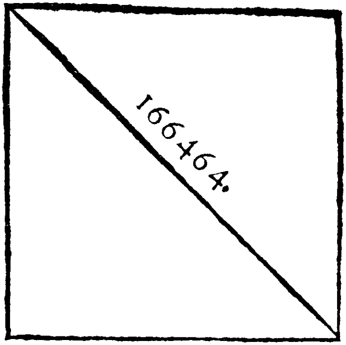
For although in figures *Geometricalle*, you maie euer more vnfallibly finde one line, that will make a ſquare, equall to the twoo ſquares of any other twoo lines (as in the pathe waie you doe ſee it taught) yet the meaſure certaine of thoſe ſides, are not knowen.

Wherfore in nomber that is not poſſible alwaies to be doen: neither can it be doen with any other nombers, then onely *diametricall nombers*. Yet maie other nombers go very nigh. As namely in theſe examples



of ſquare nombers: whoſe double, I take for the ſquares of the sides, bicause thei are equall: and thei make. 8. 50. 288. 1682. 9800. 57122. ⁊. 332928. All whiche differ onely by an unitie, from a square nomber.

For nine is a square nomber and so are these other folowyng.



	49.	289.	1681.	9801.	57121.	⁊. 332929.
whose rootes be.	7.	17.	41.	99.	239.	577.

Whiche examples if you doe consider well hereafter, thei will helpe you to gesse at the nighest rootes of nombers that be not square. And also for doblyng of squares, in a square forme: within an unspeakeable nerenesse.

For as in doblyng of this greater square. 166464. there riseth. 332928. whiche wanteth one of a iuste square. You se easely, that as that one is but a smalle portion to the whole square: So yet, that one wanteth not in the roote, but in the whole square: where by you maie perceiue, that it is a very smalle and unsensible parte of one, that wanteth in the roote.

Scholar. It must seme by reason of multiplication: that it is scarse the. 10000. parte of one.

Master. You saie truthe.

Scholar. But how shall I finde the diameter of soche nombers?

Master. That is easily doen, if you knowe firste certainly that your nomber is a diametrall nomber. And secondarily, if you knowe the true partes of it:

it: whiche you should vse in this case.

Scholar. Will not any twoo soche partes serue, whiche by multiplication will make the whole nomber?

Master. You maie by the former examples, easily se the contrary. For 12 is a *diametrall nomber*: and hath these partes (as it is sone perceiued). 2. 3. 4. 6. Yet if you take . 2 . and . 6 . for the sides of it, thei will not make a *diameter* in knowen nomber.

Scholar. That I vnderstande: for the square of 2. beyng. 4. added to. 36. whiche is the square of 6. doeth make. 40. whose roote must bee greater then. 6. and lesse then. 7. And therfore. 40. can haue no roote in whole nomber.

Master. Neither yet in broken nombers: for that is a generalle rule: that if any whole nomber haue a roote, that roote shall be a whole nomber. So that if the roote can not bee founde in whole nomber: you shall neuer finde it in broken nombers.

And for more certaintie of that I saied before, that all partes be not apte for the sides of a *diametralle nomber*, to finde out the *diameter*: marke well the seconde example, whiche is. 60. and hath these partes.

2. 3. 4. 5. 6. 10. 12. 15. 20. 30.

So that beginnyng with the two extremeste, that is. 2. and. 30. thei will by multiplication make. 60.

And likewaies any two nombers, equally distant from those extremes: As. 3. and 20. Likewaies. 4. and 15: other. 5. and. 12. And in like maner. 6. and. 10. All those couples by multiplication doe make . 60 . Yet none of them are apte sides to finde the *diameter* by, but onely 5 and. 12. For of the other sides beyng multiplied squarely (that is by thē selfes) and those squares beyng added together, there wil not rise a square nomber. As you shall better vnderstande, when you haue learned to knowe square nombers, by extractiō of their rootes.

Yet in the meane season I will set forthe certaine notes, to knowe the *diameter*, and the apte sides, in all *diametralle nombers*.

1. And firste I saie: that as thei are three nombers in all (I meane the twoo sides, and the *diameter*) so all waies if the firste or leaste side bee odde, then shall there be twoo of them odde nombers. And the *diameter* shall euer bee the other of the odde nombers: that is to saie, the greateste of them.

2. Secondarily. It is true that all *diametrall nombers* are euen nombers. And no odde nomber can bee a *diametralle nomber*.

3. Thirdly. I saie, that all odde nombers aboue one, maie be the lesser side in soche *diametralle nombers*.

But euen nombers doe not serue so generally: for thei onely maie stand in soche place, whiche be greater then. 4: As. 6. 8. 10. 12. 14. 16. 18. 20. &c. And none other euen nombers then soche as maie be diuided by 4. maie be the greater side in any *diametralle nomber*.

4. Fourthly. If the lesser side bee an odde nomber, then ordinarily, the square of it is iuste equalle with that that amounteth by the addition of the *diameter*, to the greater nomber. As in the firste example, 3. is the lesser nomber, and. 4. is the greater: vnto them bothe the *diameter* is. 5. Now. 3. hath for his square 9. and so moche is made by the addition of. 4. and. 5.

Again in the seconde example, the lesser nomber is 5. and his square is 25. The greater nomber is 12, and the *diameter*. 13. Put. 12. and. 13. together, and thei make. 25. whiche is equalle with the square of the lesser.

Likewaies. 7. and 24. multiplied together maketh 168. whiche is a *diametralle nomber*. And bicause the square of the lesser side (whiche here is. 49.) must bee equalle

equalle to the greater side, and the *diameter* added together: therfore seyng. 25. added to. 24. maketh. 49. that. 25. must nedes bee the *diameter* to the foresaied nomber.

By these rules (if you doe marke them well) you maie sone perceiue, how to make any *diametralle nomber*: if the lesser side bee giuen vnto you, and bee an odde nomber. Yet for your ease, I will giue you this plaine rule.

When any odde nomber is propounded: as the lesser side of a *diametralle nomber*, and you would finde the other side, and the *diameter* also: or els the *diametralle nomber*, that maie haue soche a side: multiplie that proponed nomber by it selfe, and it will make a square nomber, and will be an odde nomber: so that of it you shall finde no iuste halfe. Therfore take you those twoo nombers, that are nexte vnto the halfe of it: The lesser shall alwaies bee an euen nomber, and shall be the seconde side of the *diametralle nomber*: The other nomber whiche is the greater, shall alwaies be an odde nomber: and shall bee the *diameter* of that nomber whiche you desire. For example marke wel these formes that doe folowe.

If three bee propounded as the one side of a *diametralle nomber*: And you would knowe, what maie bee the other side: and what is the *diametralle nomber*: And thirdly, what is the *diameter* to that nomber: Doe, as I saied before: multiply. 3. by it self, and it will make 9. whiche is a square nomber, and an odde nomber: and therfore hath no iuste halfe. But the nighest nombers to the halfe, are. 4. and. 5.

Therfore I saie, that. 4. whiche is the lesser of the twoo, is the seconde side of the *diametralle nomber*: and 5. beyng the greater of them, is the *diameter* it self.

Scholar. Now is it light inough to perceiue that the *diametralle nomber* is. 12: seyng. 3. multiplied by

fower

4.maketh.12.

Master. So is it.

Again, if. 5. be assigned for one side of a *diametralle nomber*, and you obserue the former worke you maie easily finde the other side, and the *diameter.*

First you see, that the square of 5. is. 25. and it hath no halfe. But. 12. and. 13. are the. 2. nombers nighest his halfe: wherfore. 12. shall bee the seconde side: and 13. must be the *diameter.* And the *diametralle nōber* is. 60.

Likewaies, if. 7. be set for the lesser side, the greater side shall be. 24. and the *diameter.* 25.

Scholar. Touching this I nede no more instruction: the thyng is so manifeste.

Master. Then shewe your knowlege by an example, or twoo.

And first I appoincte 9 for the lesser side of a *diametralle nomber*, whereunto I would haue you to assigne the other side, and the *diameter.* &c.

Scholar. I followe your precepte, and multiplie 9. by it self, whereof commeth. 81. whose halfe is betwene. 40. and. 41. Therfore must. 40. be the other side: and 41. the *diameter.* And here the *diametralle nomber* is. 360.

Master. Proue the like: where. 15. is the lesser nomber.

Scholar. 15. multiplied square maketh. 225: whose nighest halfes are. 112. and. 113. of whiche the first is the seconde side, and the later is the *diameter*: and the *diametralle nomber* is. 1680.

Master. what shall be the other nombers: where 21. is the lesser side?

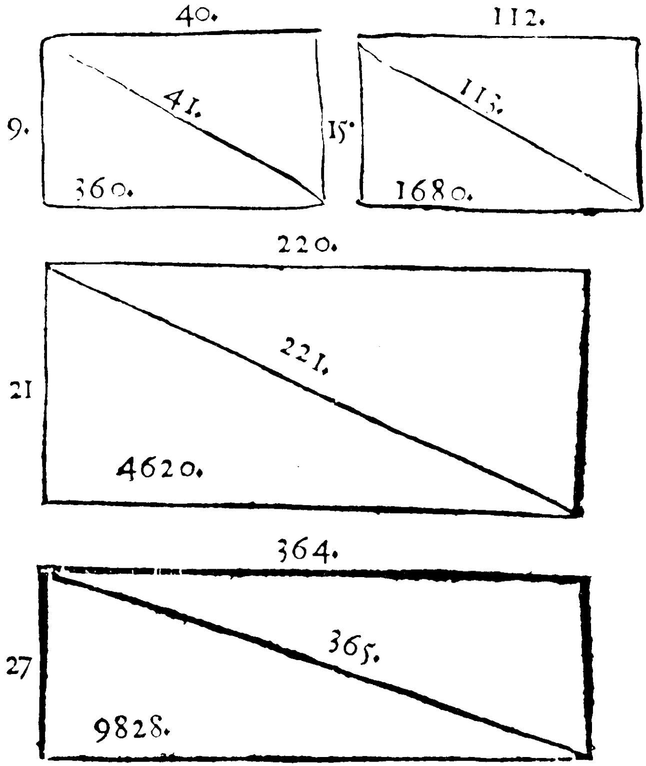
Scholar. 21. yeldeth in square. 441. whose portions nighest his halfe, are. 220. and. 221: And so appereth their offices, and the *diametralle nomber* is 4620

Master. So maie you saie that vnto. 27. being the lesser side; the greater side shall be. 364. and the *diameter*

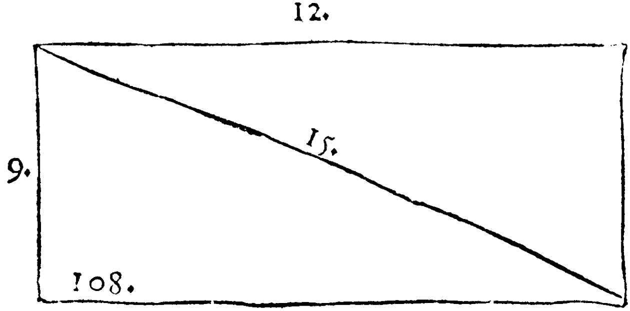
ter.365.bicause the square of.27.is.729. And the *dia-metralle nomber* is.9828.

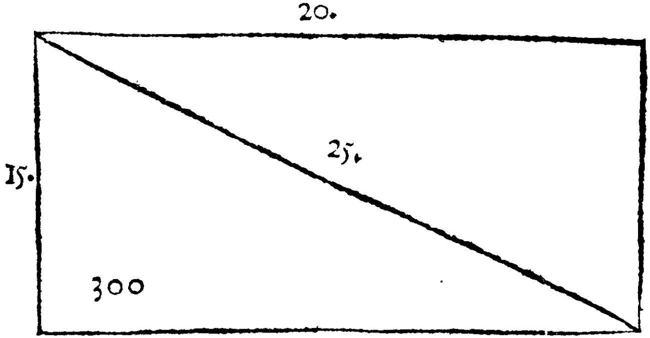
Scholar. So must it be, by your rule.

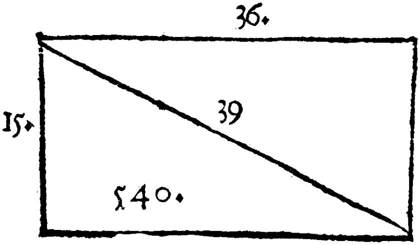
Master. Not onely the rule doth teache you that it is so, but also the nature and figure of soche *flatte nombers*. As here you see.

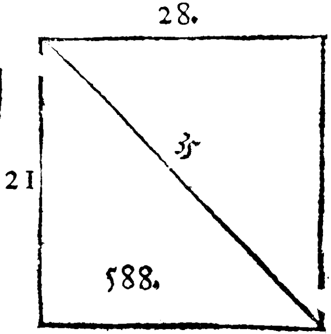


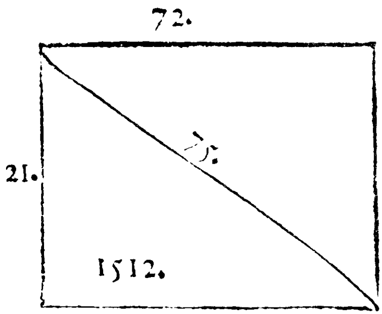
But to the intente you maie the better understand the nature of these nombers: I wil set forthe here the like sides with other nombers: whereby you maie knowe, that one side maie serue to diuerse *diametralle* nombers. Therfore marke these formes well.

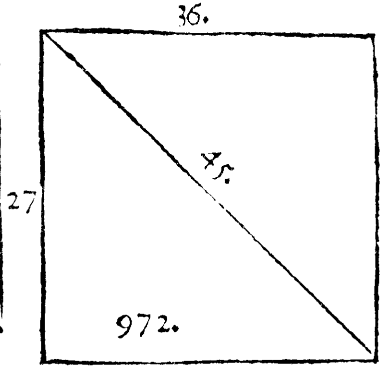


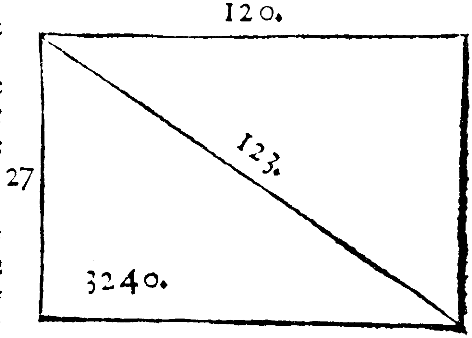












Shqlar. Here I see the same.4 nombers.9.15.21.and.27.set as the lesser sides: And their greater sides are soche as disagree frō the former rule. And in.15.21.and.27. I see twoo varieties, unlike to the former example. But seyng the sides doe disagree, I doe not maruel that the *diametralle nombers* are diuerse from the former.

Master. Examine these nombers, whether thei be true.

Scholar. I must multiplie eche side by it self, and then adde thē together: and if thei make as moche iustly, as the *diameter* beyng multiplied square, then are thei true nombers. So I see, that.9.maketh.81.and 12 doeth yelde 144 whiche bothe added doe make.225. And so moche doth 15 make, being multiplied square.

Likewaies, for the second figure 15.bryngeth forth

225. and. 20. giueth. 400. that is by addition. 625. whiche somme doeth amounte also, when. 25. is multiplied square.

The thirde figure hath. 15. also for the one side, whose square is. 225. and for the other side. 36. whiche maketh in square. 1296. And thei bothe together giue 1521. And so many commeth of 39 multiplied by it self in square.

Again for the fourthe figure. 21. maketh. 441. and 28. doeth yelde. 784. whiche bothe beyng added, doe amounte vnto. 1225. And so moche doeth there arise by. 35. multiplied into it self.

The fifte figure hath. 21. also, and his square is 441. and the seconde side beyng. 72. maketh in square 5184. So that bothe those squares doe make. 5625. And the like nomber is made by. 75. multiplied in square forme.

Now in the sixt figure 27 beyng multiplied square bryngeth forthe. 729. And. 36. likewaies multiplied doeth make. 1296. and that with the other will make by addition. 2025. whiche somme (as is well seen) doeth come of the multiplication of. 45. by it self.

In the seuenth figure. 27. multiplied square, doeth giue. 729: and the other side (whiche is. 120.) doeth bryng forthe. 14400. These bothe ioyned together doe make. 15129. And the like somme is gathered by the multiplication of. 123. squarely.

So that all those figures doe appere true.

But how thei maie agree with your former rule, I can not see.

Master. That rule did I make for nõbers vncompounde. For nombers compounde haue not onely in their owne name, the vse of that rule, but also thei folowe the forme of those nombers, of whiche thei bee compounde.

So. 9. beyng compounde of. 3. foloweth the forme

of

of.3. And therfore as.3.hath.4.for to make the seconde side with hym, so. 9.(beeyng thrise.3.) shall haue.12. (whiche is thrise.4.) for a matche side with hym.

Likewaies.15. beyng compounde of.5. and.3. shall haue their formes in the makyng of the *diametralle nō=bers*. For as.3.hath. 4. so.15.(beeyng fiue tymes.3.) shall haue. 20. (whiche is fiue tymes. 4.) for the se=conde side.

Again, as.5.hath.12. so shall.15.(beeyng three ty=mes.5.) haue. 36. (that is three tymes.12.) for his se=conde side.

Likewaies.21. beyng compounde of.3.and.7. shall haue bothe their formes.

And. 27. whiche is compounde of. 3. and. 9. shall haue all the varieties of their formes.

Scholar. I see it is euen so, and that in the *diame=ter*, as well as in the seconde side. But the *diametralle nomber* doeth varie moche in them.

Master. Yet doe those nombers agree in a mar=ueilouse good proportion. For if you doe consider the proportion of bothe the sides in one figure, to bothe the sides in an other figure: and adde those twoo pro=portions together, the addition of theim doeth make the nomber that representeth the proportiō betwene their twoo *diametralle nombers*. Whiche thynge I will now onely touche, as briefly as maie bee, to giue you occasion to marke it better hereafter: Sith this place doeth not fully serue for it. As.3. and.4. beeyng the twoo sides of a *Diametralle nomber*, doe make.12. So if 9. and 12 be the sides of a *diametralle nomber*, that nom=ber must be.9.tymes.12.that is.108. For.9.is triple to.3: and.12. is triple to. 4. And bicause the addition of proportions, is like the multiplication of fractiōs, I must multiplie. 3. by. 3. or els $\frac{3}{1}$ by $\frac{3}{1}$, whiche is all one, and that will make.9.

Likewaies, if 3.and.4. be taken for the sides of the

 lesser

lesser nomber *diametralle*, and.15.and.36. for the sides of the greater nomber: As the lesser nomber shall bee 12.so the greater must be.540.that is.45.tymes.12.

For.15. unto.3. is in a *quintuple* proportion,and is written thus.$\frac{5}{1}$:and.36. unto 4 is a *noncuple* proportion,and is written thus$\frac{9}{1}$. Now if you multiplie these nombers together, thei will make 45: whiche declareth the proportions of the twoo *diametralle nombers*. And so of all the reste,as you maie easily consider.

Scholar. I praie you,let me examine one or twoo of thẽ,in comparison to that firste *diametrall nomber*.12.

I see that 15 beyng the lesser side, and 20.the greater side,doe make.300. as their *diametralle nomber*:and that.300.is.25. tymes so moche as.12. is. Therfore by your saiyng the proportion of 15.to.3.and of.20.to 4.must make.25. And so it doeth. For eche of them is a *quintuple* proportion. And it is quickely gessed, that 5.multiplied by.5.doeth make.25.

For farther proofe, I take the *diametralle nomber* 1680.whose sides are.15.and.112.First I see, that.15. to.3.beareth a *quintuple* proportion:and.112.to. 4. is as.28.to.1. Therfore I multiplie.28.by.5.and it maketh.140.Then if I multiplie that nomber by.12.it will make.1680.

This is a sufficiente trialle for these nombers.

Of euen sides But of soche *diametralle nombers*,as haue euen nombers for their lesser side,you haue giuen no rule, nother examples,saue onely of.8. wherfore I praie you tell me, how shall I finde out the *diametralle nomber*, with his other side, and the *diameter* in soche euen nombers.

Master. You shall make it square, as you did in the other nombers,that wer odde: And of that square you shall take twoo quarters, whiche you shall alter in soche sorte,that you shall abate.1.frõ the one quarter, and put it to the other quarter. And so haue you

twoo

twoo nombers, differyng onely by .2 . and bothe be-yng odde. The lesser of them twoo, is the greater side of the *diametralle nomber*: and the other is the *diameter* to it. As.8. beyng your lesser side, the square of it is 64. whose quarter is.16.from whiche I abate.1.and there resteth.15. and that is the seconde side. Also I adde 1.to 16.and it maketh.17:whiche is the *diameter.*

Scholar. This is no thyng harde. As by example I will proue. If.12.bee the lesser side: his square is 144. and the quarter of it is.36. Then abatyng.1. I see there will bee. 35. for the other side of the *diametralle nomber.* And addyng.1.to.36.it maketh.37.to be the *diameter.* And if I multiplie.35.by.12.it bryngeth forthe.420.whiche is the *diametralle nomber.*

Now for proofe of these nombers, I multiplie.12. by it self,and it maketh. 144. Then I multiplie the other side,that is. 35. by it self, and it yeldeth. 1225. Those bothe together doe make.1369.And seyng 37 multiplied by it selfe, doeth make thesame nomber. Therfore are thei all true nombers.

An other example.10. beyng set for the lesser side, I doe multiplie it squarely: and there riseth.100. whose quarter is.25. For whiche I take (as you taught me).24.and.26. And so the whole *diametralle nomber* is.240. For proofe of the other nombers, I take.100.whiche commeth of.10.multiplied square, and to it I adde.576.whiche is the square to.24.and thei bothe doe make. 676. And so muche amounteth by the multiplication of.26.squarely.

Master. This maie suffice for this presente: if you marke that the euē nombers haue not onely one generalle forme, whiche I did expresse in the former rule, but also soche as be compounde of any other nō-bers, euen or odde: haue the like nombers in propor-tion, for the greater side, and for their *diameter* as the nombers haue, of whiche thei bee compounde. And

bicause

bicause I will not staie to long on this matter, I will here set forthe diuerse varieties of *diametrall nombers*, whereby you maie gather not onely the true vnderstandyng of the former rules: But also in theim you maie see other notable cōclusions: and straunge workes of the natures of nombers.

Marke well this table forme, with the titles ouer it: whiche declare the true meanyng of it.

And where you see one nomber in the firste columpne against twoo, three, or fower in the other columpnes, you shall vnderstande that that nomber is the side to so many seueralle nombers *diametralle*.

The table of diametralle nombers.

The lesser side.	The greater side.	The diameter.	The nõber diametrall
3.	4.	5.	12.
5.	12.	13.	60.
6.	8.	10	48.
7.	24	25	168
8.	15	17	120
9	12	15	108
	40	41	360
10	24	25	240
11	60	61	660
12.	16	20	192
	35	37	420
13	84	85	1092
14	48	50	672
15	20	25	300
	36	39	540
	112	113	1680
16	30	34	480
	63	65	1008
17	144	145	2448
18	24	30	432
	80	82	1440
19	180	181	3420
20	48	52	960
	99	101	1980
21	28	35	588
	72	75	1512
	220	221	4620
22	120	122	2640
23	264	265	6072
24	32	40	768
	45	51	1080
	70	74	1680
	143	145	3432
25	60	65	1500
	312	313	7800
26	168	170	4368
27.	36	45	972
	120	123	3240
	364	365	9828
28.	96	100	2688
	195	197	5460
29	420	421	12180
30	40	50	1200
	72	78	2160
31	480	481	14880
32	60	68	1920
	126	130	4032
	255	257	8160
33	44	55	1452
	180	183	5940
	544	545	17952
34	288	290	9792
35	84	91	2940
	120	125	4200
	612	613	21420
36	48	60	1728
	105	111	3780
	160	164	5760
	323	325	11628
37	684	685	25308
38	360	362	13680
39	52	65	2028
	252	255	9828
	760	761	29640
40	75	85	3000
	96	104	3840
	198	202	7920
	399	401	15960

F.i.

This table maie you extende infinitely. And these thinges maie you se, as thinges of greate admiratiō.

1\. There is no *diametralle nomber*, but it maie be diuided by. 12. Wherfore thei be all euen nombers euenly and oddely.

2\. Again, there is no *diametralle nomber*, but it endeth in. 0. in. 2. or in. 8.

3\. Thirdely, there is no *diametralle nomber*, that can haue any more *diameters* then one.

4\. Yet maie one nomber bee the *diameter* to diuerse other.

As you se 25. is the *diameter* to. 168. and also to. 300.
So. 65. is the *diameter* to. 1008. and also to. 1500.
Likewaies. 145. is the *diameter* to. 2448. and to 3432.

5\. Fiftely: No square nomber can bee a *diametralle nomber*.

Scholar. These properties be notable.

To knowe a diametralle nomber.

But how shall I knowe, when a nomber is proponed, whether it be a *diametralle nomber*, or not?

Master. In that thyng I finde a tediouse trauell, by any rules, in those that write of it. But I wil ease you of moche paine therein.

Firste remember the properties of those nombers.

And if you haue any other figure in the first place, then. 0. 2. or. 8. it is no *diametralle nomber*.

Secondarily, if it maie not bee diuided by. 12. although it ende in one of those. 3. figures, it is no *diametrall nomber*.

Wherfore if it haue bothe those twoo properties (whiche an infinite multitude of nombers doe want) and be no square nomber (as none be that ende in. 2. or. 8. or with odde cyphers) then sette out all the partes of it, in soche sorte, that the lesser parte doe stande directly ouer those greater partes, which beyng multiplied together, will make the whole nomber.

And

And then examine those partes, whiche seme to haue any likelihod: accordyng to the former doctrine.

As for example: if. 72. be proponed to be examined in that sorte, I sette his partes in order thus.

2.	3.	4.	6.	8.
36.	24.	18.	12.	9.

Howbeit I neded not to set doune. 2. nother. 4. for lesser partes, nother those other greater partes, that aunswere to them: For, as I said before, thei can not bee the lesser side in any *diametralle nomber*. Wherfore thei nede no examination.

Farthermore, for them that you shall nede to examine, if the lesser nomber bee an odde nomber, the square of it must contain double to that greater nomber (that is coupled with it) and one more.

And if the lesser be an euen nomber (of them twoo that you would examine) then must the square of it containe the greater nomber (that standeth by it). 4. tymes, and. 4. more. And this is not onely a shorter waie, then I see to be taughte by other artes menne: but it is also more certaine, for all nombers not compounded of other *diametralle nombers*.

Scholar. By this doctrine it appeareth quickely, that. 72. is no *diametrall nomber*.

For although it doeth ende in. 2. and maie be diuided by. 12. yet no couple of nombers here haue those properties that is required.

For vnder. 3. is. 24. whiche is to greate: and vnder 6. there is. 12. whiche is to greate also.

But vnder. 8. standeth. 9: whiche is to litle, by a greate deale.

Master. Then proue in this other nomber. 132.

Scholar. His partes will stande thus.

3.	6.	11.
44.	22.	12.

Where I see quickely that it can not bee a *diametralle nomber*. For the nombers vnder. 3. and. 6. be to greate: sith no nomber that should bee sette vnder. 3. maie be aboue. 4.

Nother vnder. 6. maie any nomber bee set greater then. 8. As it doeth sufficiently appeare by that that is taughte before.

And vnder. 11. there can bee no lesse nomber placed then. 60: and therfore. 12. is to smalle.

And herein I perceiue greate helpe by this table, whiche you haue set forthe.

Master. It is well marked of you. But yet trie this other example. 6072.

Scholar. I set doune his partes in order, thus.

3.	6.	8.	11.	12.	22.	23.	24.
2024.	1012.	759.	552.	506.	276.	264.	253.

33.	44.	46.	66.	69.
184.	138.	132.	92.	88.

And here I see a greate sorte of nombers, whiche can not serue to my purpose, bicause those that bee euen, and are lesse then. 44. make to litle a square, to be 4. times so moche as the nomber vnder any of thẽ.

And. 44. maketh to greate a square: wherfore it can be none of the euen nombers.

Again, those that be odde vnder. 23. doe make to litle a square, to bee double to the greater nomber vnder it. And those that bee odde aboue. 23. doe make to greate a square. So that. 23. doeth remain to bee the true nõber for the lesser side: and 264 the greater side.

Master. Bicause exercise is the beste instrument

in

in learnyng : therfore will I propounde to you one example more.

What saie you of. 5460: Is it a *diametralle nomber* or no?

Scholar. I will trie it, by settyng doune his partes thus.

3.	5.	6.	7.	10.	12.	13.	14.	15.
1820.	1092.	910.	780.	546.	455.	420.	390.	364.
20.	21.	28.	30.	35.	42.	52.	60.	70.
273.	260.	195.	182.	156.	130.	150.	91.	78.

And here I se diuerse and many nombers, whiche at the firste sighte, appere nothyng mete for this purpose. For. 20. is to smalle a nomber, as I maie sone iudge: and therfore all other nombers vnder it, must nedes be to smalle, of force.

Againe, I see that. 30. is to greate a nomber, and therfore, of necessitie, all other nombers aboue it, must nedes be to greate. So that. 21. other. 28. must be the true nomber, or els none.

Wherfore I examine first. 21. whose square is 441 whiche should bee one more then double, to the nomber vnder it, that is to saie, it should bee. 521. And so it is not: Therfore I refuse it, and examine. 28. whose square is. 784. And that should bee fower tymes so moche as. 195. (whiche is the nomber vnder it) and 4. more. Therfore I doe *quadriple*. 195. and it maketh. 780. And then I see that it wanteth, but fower of the other square: wherfore I take those twoo nombers, I meane. 28. and. 195. for the true sides of. 5460. whiche I finde to be a *diametralle nomber*.

Maister. By the waie, remeber that you could easily perceiue, that all nōbers vnder. 20. were to small for your purpose: and contrary waies, all aboue. 30, *A shorte meane in working.*

to be to greate. So that you neded not to sette doune so many partes of your firste nomber.

Wherfore if your nomber bee soche a one, as hath many partes, you maie chose one by gesse, which you thinke will go nigh to serue your purpose: and if you finde it to smalle, then set theim doune onely that bee greater then it, til you finde one other iuste: and then haue you your purpose. Or if you finde any to great, after that whiche was to smalle, and betwene theim none iuste, then is not your nomber a *diametrall nõber*.

But and if the parte whiche you tooke by gesse, be to great, you shall refuse all partes aboue it, and take onely lesser partes, til you finde a iuste parte for your purpose: or els one that is to litle.

And if in descendynge orderly, you finde no iuste parte, before you come to one that is to litle, then is your nomber no *diametralle nomber*.

Scholar. This is a greate ease in shortenynge of worke: whiche I will proue in this nomber. 9786.

Master. If you remembred well your former rules, you would not admitte this to be examined for a *diametralle nomber*: bicause it endeth in none of the thre peculiare terminations: that is. 0. 2. or. 8.

Scholar. I cõfesse my faulte. And therfore I take this nomber. 9780. whose. 20. parte is. 489. But seyng. 20. doeth make in square but. 400. therfore is it very moche to litle.

Then I take the. 30. parte of it, whiche is. 326. and finde it also to litle.

Thirdely, I take the. 40. parte of it, whiche is $244\frac{1}{2}$: and seyng. 40. maketh in square. 1600. I see that it is almoste. 7. tymes so moche as. $244\frac{1}{2}$: and therfore is it to greate.

So must the true nomber be betwene. 30. and. 40: or els there is none at all.

Therfore firste I take. 35. whiche is the middelle nomber,

nomber (as the moste apte for a coniecture) and it yeldeth. 279 $\frac{1}{7}$. And the square of. 35. is. 1225. whiche is farre more then the double of. 279 $\frac{3}{7}$.

Therfore, again I proue with. 32. whiche giueth 305 $\frac{5}{8}$. And seyng the square of. 32. is. 1024. it is not 4. tymes so moche as. 305 $\frac{5}{8}$. for that is. 1222 $\frac{1}{2}$.

Wherfore I take a greater nomber, betwene it and. 35. And first I take. 33. whiche bringeth forthe 296 $\frac{4}{11}$. wherby I maie see that. 33. is to greate. And seyng there is no nomber lefte betwene. 32. and. 33. therfore I iudge that firste nomber. 9780. to bee no *diametralle nomber*.

Master. Examine this nomber. 43200.

Scholar. Bicause I see it to be a greate nomber, I will begin with a greate parte of it. And therfore, I take. 100. whiche yeldeth. 432. And considerying that the square of. 100. is. 1000. whiche is farre to greate, I must seke a lesser nomber.

Master. I will ease you of your paines in that. For bicause here is more to bee considered. You remember that I tolde you before, in makyng of *diametralle nombers*, how that some nombers doe followe the rules of other, of whiche thei be compounde. And farthermore, that soche compounde *diametralle nombers*, did beare proportion to the lesser, as the proportion was of bothe their sides added together.

Scholar. That is true.

Master. Of like reason all soche *diametralle nombers*, must bee excluded from these rules, whiche bee made peculiarly for nombers that haue their owne proper formes, and depende not of other.

And yet some common rule must bee giuen, that maie extende as well to them, as to any other.

Wherfore let this be it.

That the twoo sides of all *diametralle nombers*, haue soche a proportion together, as here you see expressed

in

in ſome one of theſe formes : if thei bee continued as here thei be begon.

The firſte order.

$\frac{3}{4} : \frac{5}{12} : \frac{7}{24} : \frac{9}{40} : \frac{11}{60} : \frac{13}{84} : \frac{15}{112} : \frac{17}{144} : \frac{19}{180} : \frac{21}{220} :$

$\frac{23}{264} : \frac{25}{312} : \frac{27}{360} : \frac{29}{420} : \frac{31}{480} : \frac{33}{544} : \frac{35}{612} : \frac{37}{684} : \frac{39}{760}.$ &c.

The ſeconde order.

$\frac{8}{15} : \frac{12}{35} : \frac{16}{63} : \frac{20}{99} : \frac{24}{143} : \frac{28}{195} : \frac{32}{255} : \frac{36}{323} : \frac{40}{399} : \frac{44}{483} :$

$\frac{48}{575} : \frac{52}{2704} :$ &c.

Here haue I ſette the leſſer ſide as the numerator, and the greater ſide as the denominator. Whereby you maie perceiue the cauſe of their diſtinction.

For the firſt order is, when the leſſer ſide, or nomber, is odde.

The ſeconde order is, when that leſſer ſide is an euen nomber.

Stifelius doeth ſet them ſo, that the numerator ſtandeth for the ſeconde, or greater ſide: and the denominator for the firſte nomber, or leſſer ſide. And for the more delectable contemplation, to behold their forme of progreſſion, he ſetteth doune as many whole nombers, as the fraction will giue.

And this is his forme.

The firſte order.

$1\frac{1}{3} : 2\frac{2}{5} : 3\frac{3}{7} : 4\frac{4}{9} : 5\frac{5}{11} : 6\frac{6}{13} : 7\frac{7}{15} :$ &c.

The ſeconde order.

$1\frac{7}{8} : 2\frac{11}{12} : 3\frac{15}{16} : 4\frac{19}{20} : 5\frac{23}{24} : 6\frac{27}{28} : 7\frac{31}{32}.$ &c.

Where

Where in the first order, you se bothe in the whole nombers, and also in the numerators of the fraction, the naturalle order of nombers. And in the denominators, the naturalle progression of odde nombers.

But in the seconde order, you see that the whole nombers go in their naturalle order, and the numerators and denominators, kepe an *Arithmeticalle* progression, by equalle distaunce of. 4. saue that in the numerators, all the nombers bee odde: and in the denominators, thei be all euen.

Now by this generalle rule, if you finde any twoo partes of any nomber, in one of these former proportions, you maie bee sure that it is a *diametralle nomber*. But for the more apte conference of the partes, you shall doe beste to reduce them to their least nombers: as you haue learned in the firste parte of *Arithmetike*.

So in your last nomber, whiche was 43200. you shall finde his. 180. parte, to bee. 240. whiche beyng reduced to their smallest nombers, will bee. $\frac{3}{4}$: wherfore I am assured, that it is a *diametralle nomber*.

Yet one thyng more shall you marke.

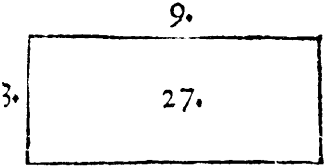
If any nomber ende in Ciphers, abate euen Ciphers, as often as you can (I meane. 2. 4. or. 6. &c, and if the reste be a *diametralle nomber*, so was the first. And therfore in this laste example. 432. is a *diametralle nomber*, as well as. 43200.

Also if any nomber beeyng diuided by any square nomber, doe make a *diametralle nomber* in the *quotiente*, then was the firste nomber a *diametralle nomber* also.

And this, for this tyme, shall suffice for *diametralle nombers*.

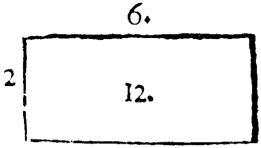
Now will I speake somewhat briefly of *like flattes*: and then procede to other *figuralle nombers*. *Of like flattes.*

Scholar. I remember you defined them before, to bee soche flatte nombers, as had one forme of proportion betwene their sides.



As here 27. and 12. be *like flattes*: becauſe their ſides be in one proporti- on. For as. 9. is to. 3. ſo 6 is to. 2. bothe beeyng in triple proportion.

Maſter. You ſaie well. And that is the cauſe why thei be called like: for the likeneſſe in the proportiō of their ſides. Although ſome menne delite



Squarelike figures. more to call them *ſquarelike figures*: becauſe thei haue ſome properties agreable with ſquare nombers (for as *Euclide* ſaieth in his. 8. booke, and. 18. propoſition:

Euery twoo nombers, beeyng like flattes, haue one meane nomber betwene theim in proporti- on. And the one flatte nomber beareth vnto the other flatte double that proportion, that their ſides doe.

For declaration of whiche propoſition, marke the twoo flatte nombers before: I meane. 27. and. 12. whoſe ſides are in proportion *Seſquialter*: And the flat nombers themſelfes be as $\frac{9}{4}$. or. 9. to. 4: that is double *Seſquiquarte*. Now doe you double the proportion *Seſquialter*, and it will make *double Seſquiquarte*.

Scholar. Thus doe I ſette them in order. $\frac{3}{2}$: $\frac{3}{2}$. And I multiplie the numerators together, and the denominators alſo. (For I remember, you tolde me before, that proportions are added, as fractions are multiplied) and then will it be. $\frac{9}{4}$: euen as you ſaied.

Maſter. Again *Euclide* ſaith in the twentieth propoſition of the ſame booke.

If any nomber ſtande as a middle nomber in proportion,

proportion, betwene other twoo nombers, those twoo are like flattes.

That is to saie: if any twoo nombers, beyng multiplied together, doe make a square nomber (for none but soche can haue a middle nomber betwene theim) then are thei *like flattes.*

As.3. and.12. multiplied together doe make.36. whiche is a square nomber: and.6. therby appeareth to bee the middell nomber betwene theim. And therfore are.3.and.12. *like flattes*

Likewaies.3.and.27. for thei make. 81. whiche is a square: and their middle nomber is.9.

And so are.2.and.8: 2.and.18: 2.and.50. 2.&.72 3.and.48: 3.and.75: 4.and.9. 4. and.16: 4. and 25. 5. and.20. 5.and.45: 6.and.24: 6.and.54.

And so of infinite other.

This exposition is confirmed by the firste and seconde proposition of the nineth boke of *Euclide*, where he saieth thus.

If twoo nombers beyng like flattes, bee multiplied together, the nomber that thei make, shall be a square nomber.

And if.2.nombers beyng multiplied together, do make a square nōber, then are thei like flattes.

By whiche rules it doeth appere, that you cā haue no progressiō *Geometricalle*, but it must be made either of square nombers, or els of *like flattes*, wherby there appeareth a greate agreablenes, betwene *like flattes*, and square nombers. And therfore saieth *Euclide* also in the.26. proposition of the eight booke.

Nombers that bee like flattes, haue soche proportion together, as one square nomber bea-

reth to an other.

This maie you proue by any of the former examples. For.12.to.3.is in like proportion,as.16. to.4. or.36.to.9.

Also.27.to.3.hath like proportion as.36. to.4: or 144.to.16. other.81.to.9.

And farther, if you deuide the one of theim by the other, the *quotiente* will be a square nomber.

Scholar. That doeth appeare euidentely at the firste vewe.

For.12.diuided by.3.doeth make.4. And.75.diuided by.3.giueth.25.

So.54.by.6.maketh.9. And.72.by.2.yeldeth.36. And so I see in the reste, that all the *quotientes* will be square nombers.

how like flattes be made. But I desire moche to knowe,how those nombers be produced. For that I knowe not yet.

Master. Take any twoo square nombers, what so euer thei bee,and multiplie them by any one nomber,that you liste: and thei will make.2.*like flattes.*

So.4.and.9.multiplied by.2.doe make.8.and.18: whiche bee *like flattes.*

Again,if you multiplie them by.5. thei make.20. and.45. whiche be also *like flattes.*

Scholar. I am perfect inough in this,if that be al.

Master. An other waie you maie make them also: If you take any twoo square nombers, that will admitte one diuisor,and diuide them bothe by it.

As for example. Seyng 9.and.36.will be bothe diuided by.3. I doe so diuide theim: and their *quotientes* are,3.and.12. whiche are *diametralle nombers.*

So in like maner,if I diuide 196 and 49 (whiche bothe are square nombers) by.7. the *quotientes* will be 28.and.7.

Again,16.and.100.beyng bothe square nombers

and

and diuided by.4.doe make.4. and.25. as their *quotientе*, and thei be *like flattes*.

Scholar. And in these I see an other straunge worke: that if those twoo *like flattes* bee multiplied together: thei will make the greater square, of whiche thei came.

For.3.tymes.12.maketh.36: and.7.tymes.28.giueth.196: And so.4.tymes.25.bryngeth forthe.100.

Master. It doeth so happen often tymes: but it is not alwaies so.

For if you diuide.16.and.100.by.2. the *quotientes* will be.8.and.50. whiche twoo nombers multiplied together, doe make.400. farre differyng from.100. So.36. and.196. beyng bothe square nombers, and diuided by.2.doe make.18.and.98.whiche be *like flattes*: and those *like flattes* multiplied together, doe yelde 1764. whiche is a square nomber, but it is.9.tymes so greate as is.196.

Scholar. Yet one doubte I haue: whether all square nombers be *like flattes*, and so bee not distincte from them?

For although in the diuision of figuralle nombers you did distincte them, yet in the examples of *like flattes*, you put certain square nombers emongest other.

Master All square nombers are *like flattes*, beyng compared together: and els not. For as any.2.square nombers maie be compared together: so maie thei be referred to their rootes, without comparison together. Or els thei maie be compared to other nombers that bee not square.

Therfore marke these two rules well. that no one nomber can bee called a *like flatte*: but in comparison to some other. For.2.by hymself is not called a *like flatte*, excepte he bee compared to.8.or to.18. other to 32.or.50.or some other soche.

So likewaies.4.whiche by nature is a square nõber, and alwaies shall bee so: yet is it not accepted as a *like flatte*, onles it bee referred to some other square nomber.

Scholar. What if it be compared with. 12. which you named before to be a *like flatte*?

Master. You remember: one of *Euclide* his rules (whiche I repeated before) is, that *like flattes* beeyng multiplied together, will make a square nōber. And so doeth not. 12. beyng multiplied by. 4.

Scholar. Now I doe vnderstande your woordes better. So. 3. and. 8. compared together, bee not *like flattes*: yet eche of them compared to other nombers, maie be *like flattes*. As. 3. compared to. 12. or to. 27: and 8. compared to. 18. or to. 50.

Of rooted nombers. Master. Now will we lette these *like flattes* alone for a tyme: And intreate more of rooted nōbers. And first I will tell you somewhat of the names and natures of soche nombers as haue rootes: Then secondarily I will teache you the order to extract their rootes: And afterwarde will I shewe some parte of the vse of theim.

A roote. Wherfore to begin, where we lefte a litle before, the explicatiō of rootes: I saie, that the roote of nomber, is a nomber also: and is of soche sorte, that by sondrie multiplications of it, by it self, or by the nomber resultyng thereof, it doeth produce that nōber, whose rooe it is. And accordyng to the nomber of times that it is multiplied, the nomber that resulteth thereof, taketh his name.

So that one multiplication maketh a *square nomber* And twoo multiplications doe make a *Cubike nomber*.

Likewaies. 3. multiplications, doe giue *a square of squares*. And. 4. multiplications doe yelde *a sursolide*.

And so infinitely.

For as multiplication hath no ende, so the nombers amountyng of them be innumerable, and their

rootes

rootes as infinite. But their names thei take certainly, of the nombers that thei doe make.

So the roote of a square nomber, is called a *Square roote*: and the roote of Cubike nomber, is named a *Cubike roote*: In like sorte that roote is called a *Squared square roote*, whiche maketh a square of squares in nomber: And that roote is a *Sursolide roote*, that yeldeth a *Sursolide nomber*: in whiche sorte of multiplication, you maie procede infinitely, as I saied.

A square roote. *A cubike roote.* *A squared square roote.* *A sursolide roote.*

Notwithstandyng for your ease, I haue set foorthe here in a table, certain of the moste notable kindes of rooted nombers.

And to the intente you maie partly conceiue the reason of their names, I will after the table, set forth a brief explication of their names, with the protracture of the figures, that thei doe resemble in multiplications *Geometricalle*: where pointes, lines, platte formes, or soundformes bee multiplied: and bringe foorthe other formes agreable to soche multiplications.

But first marke the table well: And it will giue you greate lighte, and aptnesse to understande all that foloweth, moche the better.

For examples are the lighte of teachyng.

The

	The vulgare names.	The table of rooted nombers.									The authors names.
1.	*Rootes.*	2	3	4	5	6	7	8	9	10	*Rootes.*
2.	*Squares.*	4	9	16	25	36	49	64	81	100	*Squares.*
3.	*Cubikes.*	8	27	64	125	216	343	512	729	1000	*Cubes.*
4.	*Squares of Squares.*	16	81	256	625	1296	2401	4096	6561	10000	*Longe Cubes.*
5.	*Sursolides.*	32	243	1024	3125	7776	16807	32768	59049	100000	*Squares of cubes*
6.	*Squares of cubes*	64	729	4096	15625	46656	117649	262144	531441	1000000	*Cubike Cubes.*
7.	*Seconde Sursolides.*	128	2187	16384	78125	279936	823543	2097152	4782969	1000000	*Longe Cubike Cubes.*
8.	*Squares of squaredsquares.*	256	6561	65536	390625	1679616	5764801	16777219	43046721	100000000	*Squares of Cubike Cubes.*
9.	*Cubes of Cubes.*	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000	*Cubes of Cubike Cubes.*
10	*Squares of Sursolides.*	1024	59049	1048576	9765625	60466176	282475249	107374824	3486784401	10000000000	*Longe Cubes of Cubike Cubes.*

Here you see diuerse rewes of nombers, and against euery rowe twoo names written: one on the right hande, and the other on the lefte hande, whiche serue for all the nombers in that rewe.

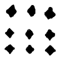
The names on the lefte hande bee those names, whiche bee commonly vsed, and attributed to those nombers.

The names on the righte hande, are names of my addition, whiche doe aptly expresse the very natures of the nombers, vnto whiche thei bee assigned: as anone I will declare.

And now concernyng the nombers, you see firste in the hedde of the table, a rewe of nombers set in order, as thei followe in common nombryng, from one forward. And thei bee called rootes, for that the multiplication of eche of them, by theimselfes, or by that, that thereof amounteth, bryngeth forthe all thother, that bee set vnder them. Of the whiche, the seconde rewe is called *Square nombers*: bicause that their length *Square* and their bredth (whiche I vnderstand by the.2.nombers of their multiplication) is equalle. *nombers.*

As.2.tymes.2.doeth make.4. whiche is a square nomber, and maie bee figured thus.

Likewaies.3. tymes.3. maketh.9. whiche is a square nomber, and is represented thus.

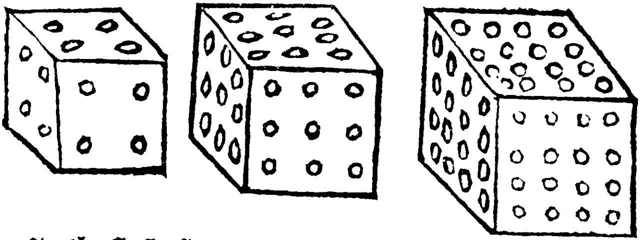


And here you se, that if you diuide the *Square nomber* by his roote, the *quotiente* will be the same nõber also.

Scholar. That must nedes be so.

Master. Then in the thirde rewe are placed *Cubike nombers*: whiche are produced by triple multiplication. As.2. tymes.2. twise, maketh.8. And.3.tymes.3.thrise, yeldeth.27. So.4.tymes.4. fower tymes, giueth. 64. These nombers can not be expressed aptly in flatte, but prospectiuely, as Dice maie be made in protracture. *Cubike nombers.*

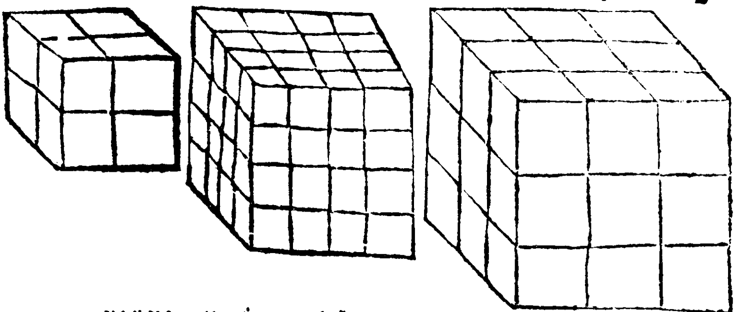
And these are their formes.



In the firste figure you see . 2 . expressed in lengthe bredthe, and depthe. And in the second forme. 3. is represented in all those. 3. dimensions. In the. 4. figure 4. is the roote, and is drawen agreably to that forme.

Scholar. This is manifeste inough to sighte.

Master. Yet reason ought to waigh it more exactly, then sight can comprehende it. For as their triple multiplication doeth resēble the nature of sounde bodies, so it might appeare more iuste expressyng of their figures, agreably as sounde bodies ought: in whiche euery parte can not appeare to sighte, sith diuerse of them loke inwardly. As by these. 3. laste figu-



res you maie partely coniecture. Of whiche at this tyme and in this place, some men will thinke it an ouersighte to speake, and moche more ouersighte to write of them any thyng largely. Saue, that we maie vse theim for the apter explication of that triple multiplication,

tiplication, wherby thei be made.

So that as it is multiplied thrise, so the nomber that doeth amounte thereof, hath gotten. 3. dimensiones, whiche properly belongeth to a bodie, or sound forme. And therfore is it called a *Cube*, or *Cubike nomber* Whiche nomber if you diuide by the roote, the *quotient* will be the square of thesame roote. As I said afore.

But to procede, if you doe multiplie that *Cubike nomber* by his roote, the nomber that riseth of it, is called a *Square of squares* commonly: bicause that not onely it is a *Square nomber*, but the roote of it also is a *Square nomber*. As you maie perceiue by examination, of all those nombers that be in the fourth rewe, whiche nombers I doe call *longe Cubes*: bicause thei make a line of Cubes. And hath in lengthe so many Cubes, as the firste roote doeth containe vnities.

Squares of Squares.

Long Cubes.

This line of Cubes, although it haue for his bredthe, and depthe also, the thickenesse of one Cube, yet bicause it hath no nomber of Cubes, in bredthe, nor in depthe (or generally no nomber of that thyng, whereof it is called a line) therfore maie it tollerably beare the similitude and name of a line. And so doe we commonly call lines, those smalle cordes, whiche are onely long, and haue litle bredthe to their length. But yet are thei not without all bredthe.

Scholar. And thereof (I thinke men call a line of Brickes, and a line of Asshelers stones, when many bee laied in a rowe, in lengthe: and but one (or fewe) in bredthe.

Master. You saie truthe. And that name doeth continue still, emongest all our countrie menne: saue that moste menne doe not call it sharply a line, but more broder (after tholde Englishe language) a laine And so men vse to saie, a laine of wine buttes, and a laine of brode clothes: and soche other like.

And vse hath so largely applied this name, that it

 maie

maie seme no greate absurditie, to name any thynge a line or laine, that hath moche more lengthe then bredthe: and is made by often addition, or multiplication of any one quantitie. But yet for auoidyng of erroure, it ought to bee limited, whereof that line is named. As in our mater to saie, *a line of vnities*: *a line of Cubes*: *a line of Cubike Cubes*: and *a line of Cubike Cubes Cubikely*· and so forthe.

In likewaies must we iudge of platte formes, that thei haue no depthe or thickenesse. When one nomber is multiplied by an other, onely twise: that is to saie, in bredthe and lengthe onely: and is not multiplied the thirde time by any nomber, to make it beare depthe.

And this must be considered generally, though the nomber so multiplied bee a *Cube*, or any other sounde nōber. For in soche case, that *Cube*, or sounde nomber, what so euer it be, standeth but as an vnitie.

Scholar. Sir, I doe very well vnderstande the meanyng, and reasonablenesse of those names, line, and square, in any thing. But I knowe not those termes, *Cubike Cubes*, and *Cubike Cubes Cubikely*· Although I se them set in the table, whiche you haue giuen me.

Master. No more then doe you vnderstande diuerse other names there, whiche I will therfore declare vnto you.

If you agree to the vse of the name, of a line and a square, in that sorte that you haue consented vnto: then if I multiplie a Cubike nomber by his roote. As to saie. 8. by. 2. or. 27. by. 3. other. 64. by. 4. then shall I haue a line of *Cubes*. whiche I doe therfore call *longe Cubes*: but commonly thei bee called *Squared Squares*, or *Squares of Squares*: and of some men thei are named *Zenzizenzikes*, as square nombers are called *Zenzikes*. Whiche name although in sounde bodies, it hath no vse, yet in practice of sounde nombers, it maie

Squares of squares.

maie and doeth expresse some properties aptly. As namely that all those nombers, whiche rise of 4 multiplications, maie be as well made by twoo multiplicatiōs. But then the roote of that multiplication shal be a square nomber also.

Scholar. So I understande that. 16. is a nomber of that sorte, which here is called *Square of squares.* And yet maie it bee called a square nomber: and is so in deede, in comparison to. 4. And therfore, I perceiue, it is set twise in the table: ones emongest square nombers, under 4 whiche then is his square roote: And again it is set emongest *squares of squares,* under 2 which in that place standeth as his squared square roote.

Likewaies. 64. is twise set in thesame table, ones emongest *squares,* under 8. whiche is his square roote: And again emongest *Cubike nombers,* under. 4. whiche is his Cubike roote.

Master. You saie truthe. Although the laste exāple be not to your purpose, concerning *Squared squares* or *Zenzizenzikes.* And if you did note it onely, forbicause it is twise set in the table: then maie you see it thrise sette in thesame table, for it is in the sixte rewe under. 2.

Scholar. So I see, wherfore I might rather haue takē. 81. whiche is a *Zenzizenzike nomber,* and so hath for his roote. 3: And also it is a square nomber, and hath. 9. for his roote.

Master. Farther to procede, if I multiplie those *squares of squares* by their roote, thei will make *Sursolide nombers.* *Sursolides.*

Scholar. I perceiue by the nombers in the table, that you meane the leaste roote of the twoo: bicause under. 16. I see. 32. in the rewe of *Sursolides.*

Master. Reason maie driue you to thinke so. For the nomber and his roote, muste beare alwaies one name. So that if I name. 16. as a square nomber, I

 must

must referre it to his square roote. And if I name it as a *Zenzizenzike nomber*: it muste bee referred to his *Zēzizenzike* roote. And in like sort of al other names.

As when I call. 64. a square nomber, & demaunde what is his roote: you muste nedes aunswere by his *Square roote*, whiche is. 8. But if I name. 64. as a *Cube*, and doe then seke for his roote: you must understande his *Cubike roote*, and that is. 4. But if I name it to bee a *Square of Cubes*, or *zenzicube*: then is. 2. his *roote*. As you maie by the table perceiue. And also by the orderly multiplication of euery rewe, or order of nombers by their roote. For therby amounteth the nexte rewe.

And so maie you increase the nombers of those rewes, or orders, accordyng to the tymes of your multiplicatiō, as moche as you list. And euery order shall beare soche names, as agreeth to the nature of their rootes.

Wherfore thei appeare to bee ouersene, that call those formers nombers *Surdesolides*, seing thei are not any waies *Surde nombers*, but haue their rootes. And yet, to confesse the truthe, I cannot well tell you the true *etymologie* of their name: except thei be so named, as it were *solide* upon *solide*. And that interpretation were to streightly racked. But the name beyng receiued and well knowen, wee maie more easily with libertie use it, then with scrupulositie, curiously scā it.

These nombers are simple nombers in their kind. For thei rise of. 5. multiplications. And if their roote bee a digite nomber, then is it the same nomber, that standeth in their firste place. And if their roote be an article, then hath that *Sursolide*. 5. tymes so many Cyphers together in the firste places, as his roote hath: and the nexte figure after those Cyphers, is the firste figure significatiue of his roote.

Scholar. I see it so in all these nombers, that bee

in

in the table.

Master. And so shall you finde it in all others.

And farther if the roote bee a nomber mixte, then the firste nomber of the *sursolide*, is the first nomber of the roote. And this I doe tell you for some helpe, in gessyng at their rootes.

This name therfore of theim, I meane *Sursolides*, in *Arithmetike*, maie serue to admonishe you of their roote. But in *Geometrie*, or in composition of sounde bodies, it serueth to no vse: and therfore I doe call thē agreable to their figure, *Squares of Cubes*: bicause thei make a square forme: but so that euery vnitie of that square, is in it self a *Cube*: As by the figures that followe, you maie well coniecture. *Squares of Cubes.*

And also thei are made by multiplication of a *Cubike nomber*, and a *Square nomber* together, bothe hauyng one roote: and the *Sursolide* hauyng thesame roote. Wherfore reason with the nature of their sounde figure, inforceth me to call thē *squares of cubes.*

Yet other menne attendyng more to the nature of their rootes, then to their owne formes and nature, doe giue that name to the nexte rewe of nombers, bicause thei maie be made of multiplication, of any *Cubike nomber* by it self, that is to saie squarely.

Scholar. It is so. For.8. whiche is a *Cubike nomber* multiplied squarely maketh.64. And that.64. is set emongeste the *Squares of Cubes.*

Master. And this commoditie commeth by that name: that it putteth menne in remembraunce of the spedie and easie extraction of their roote: As you shall learne hereafter.

But I consideryng their owne nature and makynge, as sounde nombers or bodies: doe call theim *Cubes of Cubes*, or *Cubike Cubes.*

After these nombers in the seuenth rewe, there do followe those nombers, whiche commonly are called

bsursolides,

bsursolides, or bissursolides, that is, seconde sursolides, or double sursolides. But I maie call them seconde squares of cubes, alludyng at the same name. Howbeit if I looke to their forme and nature, I shall be inforced to call thē, longe cubes of cubes, or longe cubike cubes.

Seconde sursolides.

And so by like reason, doe I cal the nexte nombers square cubes of cubes, or square cubike cubes: whiche other men doe cal zenzizenzizenzikes, that is squares of squared squares.

Squares of squared squres.

The nineth rewe of nombers, is commonly called Cubike Cubes, or Cubes of Cubes· bicause the Cubike rootes of those nombers are Cubike nombers also. But I after their true nature, doe call them Cubes of Cubes Cubikely: or Cubes of Cubike Cubes.

Cubes of Cubes.

The tenth rewe of nombers is named vulgarely, Squares of sursolides, bicause thei haue a Square roote, whiche is of it self a sursolide nomber. And for their figure Grometricalle, I name thē long cubes of cubike cubes.

Squares of Sursolides.

So that I consideryng their nature, that thei be figuralle nombers, am constrained to name theim, accordyng to their figure, I meane in this place, where I doe make explication of their natures and names.

But other men for aide of woorke, in extraction of rootes, haue giuen theim soche names, as maie beste put menne in remembraunce of redy worke therein. Whiche names I will vse also hereafter, in my writynges, bicause I will not bee an authour of vnnedefull singularitie. And yet bicause truthe in nature is as well to be regarded, as ease in woorkyng, and rather more, I could not omitte in this place, the declaration of their true nature and very formes.

And so bothe of vs hauyng good reasons, for those names, neither maie contempne other, neither contende together.

And although the names that I doe giue, maie seme to some menne (whiche are scarse apte iudges) more

A generalle reason for na

more odiouse, for the newe inuention (as thei maie mes of these
thinke) then nedefull to the practise of tharte: yet shal nombers.
you see in theim a naturall sequele, and orderly propagation.

For all those nombers are considered, in one of. 2. formes firste. That is to saie, other thei bee taken as nombers absolute, without any cõsideration of multiplication: And so thei maie be named nombers onely, without name of relation. Or els thei bee considered as nombers multiplied, and that can be but in. 3. varieties.

If thei be multiplied but ones, then doe thei make a line of nombers, or a liniarie nomber. And that nõber hath onely lengthe, without bredthe, or depthe: And therfore maie be the roote to a *Square*, or a *Cube*. But is of it self, in that consideration, nother *Square* nor *Cube*.

Secondarily, it maie bee multiplied twise, the one nomber stãdyng for the lengthe, and the other for the bredthe: and so is it a *Square nomber*, and therfore a *flat nomber*.

Thirdly, it maie bee multiplied thrise, and therby gette lengthe, bredthe, and depthe: wherby it is made a *sounde nomber*. And bicause the sides bee equalle, it is specially a *Cube* or *Cubike nomber*.

Now can there be no fowerth waie, that any multiplication maie increase: for there are no more dimẽtions in nature.

But if any manne doe multiple the fourthe tyme, then must he accoumpte that he maketh a *line of Cubes*: and the fifth multiplication maketh a *Square*, in whiche euery vnitie is a *Cube*: So the sixte multiplication maketh a *Cube* of *Cubes*, accoumptyng euery lesser *Cube* for an vnitie. And there is a staie again.

Wherfore if any man multiplie the seuenth time, he retourneth againe to the firste nature of nombers

 multiplied,

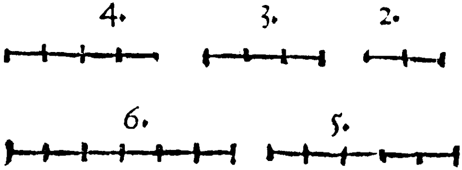
multiplied, whiche are *liniarie nombers*: And the 8. multiplication, woorketh as the seconde did, and maketh *flatte nombers*. The nineth multiplication agreably with the thirde, doeth make *Cubes*.

And ſo infinitely theſe. 3. woorkes maie bee reiterate, but a fourthe forme can neuer be deuiſed.

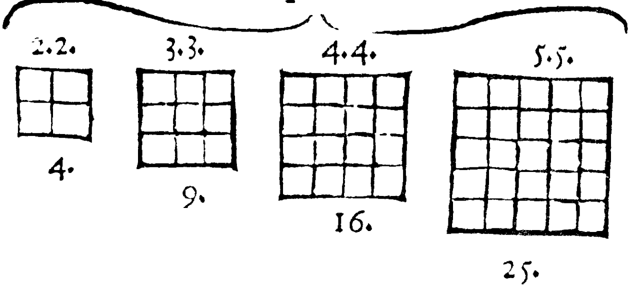
And therefore doe I, as reaſon doeth compell me, reduce all nombers to thoſe. 3. formes, as their verie originalle ſpringes and fountaines.

But to the intente that you maie the more aptly iudge of theim, and their natures, I haue here ſette foorthe the formes, whiche thei make in figures *Geometricalle*, or ſounde quantities. Admoniſhyng you to remember this well. That after any nomber is become a ſounde nomber, it is againſt reaſon, to reduce him to an abſolute flatte nomber again, and moſte of all by multiplication. But now marke theſe figures.

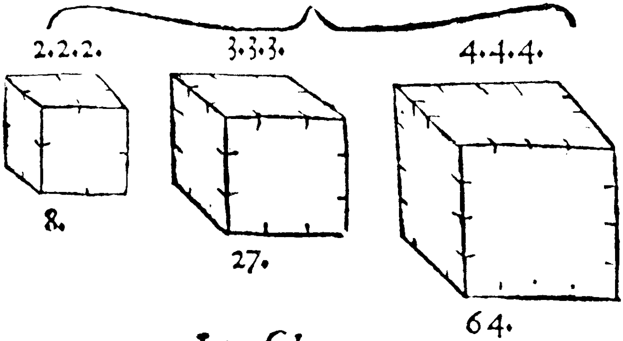
Rootes, or Lines.



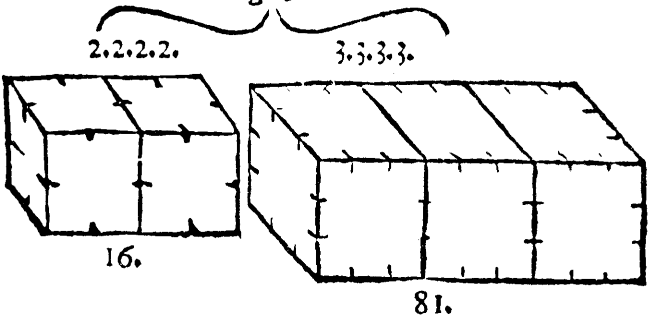
Squares.



Cubes.

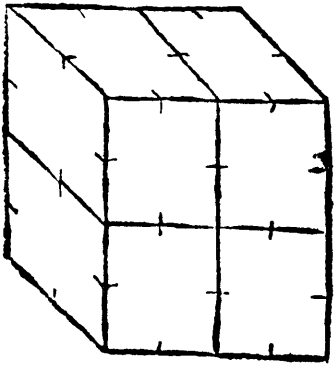


Longe Cubes.

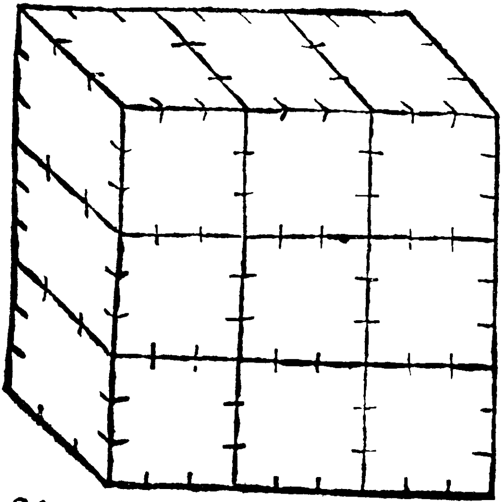


Squares of Cubes.

2.2.2.2.2.

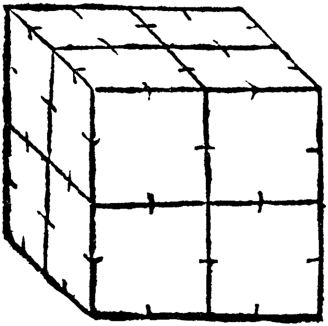


3.3.3.3.3.



Cubike Cubes.

2.2.2.2.2.2.



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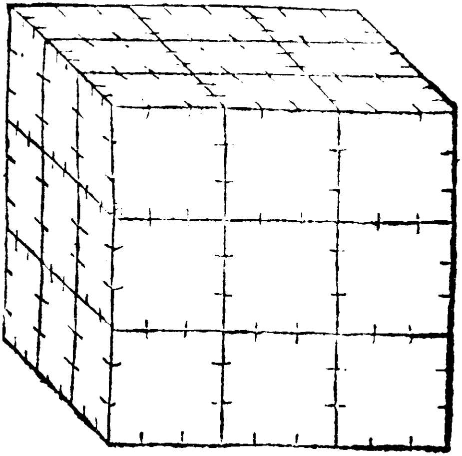


Here, as you see, I haue set first certaine lines, contaynyng soche partes as thei bee made of by multiplication: that is to saie, 2. 3. 4. or. 5. And these bee produced by the firste multiplication, where an vnitie of any thyng is multiplied by a nomber.

And so an inche multiplied by. 3. maketh. 3. inches: And a foote multiplied by. 6. maketh. 6. foote: and so of other measures and quantities, in like sorte. All whiche multiplications, doe make onely longe lines, or measures in lengthe onely, without bredth or thickenesse.

And in this multiplication, nother the nomber, nother yet the vnitie, is accoumpted or called a roote. But the line that is made therby: maie bee a roote to any of all the other kinde of nombers before rehersed, and sette forthe in the table. For if you multiplie the same line, by the nomber that his lengthe doeth include, then there will be made thereof, by this seconde multiplication, a square figure, contaynyng a square nomber in it: As you see emongest those figures, the firste fower to be, whiche are marked with these nombers. 4. 9. 16. and. 25.

Scholar. I perceiue well in eche of thē, that their lengthe is agreable with their bredthe, and so thei make square figures, but I knowe not what those nombers doe meane, that be set ouer their heddes.

Master. The quantitie of the nomber, doeth betoken the valewe of their roote. And the multitude of thesame nomber repeted, doeth declare the nomber of multiplications, for eche figure.

And therefore the lines, whiche are made by one multiplication, haue eche of theim their nomber simply set, ones onely.

The squares haue their nombers double: in token that thei haue. 2. multiplications. That is, one in lengthe, and an other in bredthe.

 The

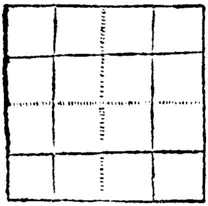
The thirde formes, whiche be *Cubes*, and are made of. 3. multiplications, haue their roote repeted thriſe.

And the like nombers did I ſette, in the ſide of the former table, againſt the like quātities. Whiche ſhall helpe you ſomewhat in the extraction of rootes.

Scholar. Now doe I perceiue not onely their names, and multiplications, moche better then I did before: but alſo I vnderſtande better the difference of your names, and their reaſons. For by thoſe figures, whiche you haue ſet in the fowerth place, and doe call theim *longe Cubes*, I ſee their forme doeth agree to that name. For thei are longer, then thei are other brode or depe. And ſaue for their depthe, I might liken theim to *longe Squares* in *Geometrie*. Howbeit, other men neglectyng their forme, and lookyng onely to their rootes, doe call them, *Squared ſquares*.

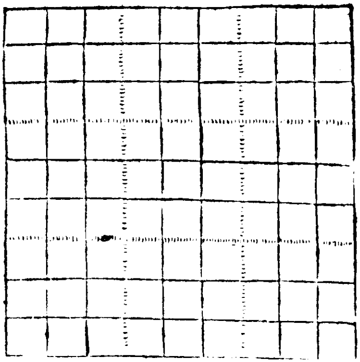
But if you will permitte me, to ſpeake in the defence of theim, as a ſimple ſcholar maie ſpeake for affection, in the defence of his maſter, it appereth to me, that thei maie well bee called ſquared ſquares; and might be figured thus.

2.2.2.2.



16.

3.3.3.3.



81.

Where the ſmalleſt ſquares, whiche be contained within the pricked lines, beyng taken as rootes, and multiplied

multiplied by thesame nomber again, whiche thei do containe (other els twise by their rootes) will make the whole greater squares.

And by this figuryng of theim, there doeth appere no inconuenience nor absurditie, in their vulgare names: but rather a iuste expressyng of their naturalle formes.

For in the first figure. 2. standyng as the side of the lesser square, and multiplied by it self, doeth make. 4. whiche is the quantitie of the lesser square. Then if I multiply that lesser square. 4: by his owne nomber, it maketh 16. whiche is the greate and whole square: and is a *Square of squares*.

So in the seconde figure. 3. standeth for the roote of the lesser square, contained within the pricked lines, and if it bee multiplied by it self, it maketh. 9. whiche is the quantitie of thesame lesser square. Then if I multiplie that. 9. by it self, it will make. 81. whiche is the quantitie of the greate Square, and is a *Square of squares*.

Master. I commende you well: not onely for so diligente excusyng of theim, whiche for their honeste trauell, deserue moche thankes, but also for that you seke to bryng manifest reason, and some shewe, at the least, of linearie demonstration for your purpose. So that you will not seme to speake, without some good grounde.

But as in deede, your figure doeth truely expresse a square of squares, so it doeth suppose the other nomber, whiche by order of multiplication, doeth go nexte before it, to be a flatte nomber also. For it is not possible that a sounde nomber (as a *Cube* is alwaies) beyng multiplied by any other nomber, maie lese the nature of a sounde nomber: But shall continue a sounde nomber still. And therfore seeyng the nexte nomber, before a *Square of squares* was a *Cube*, it is not

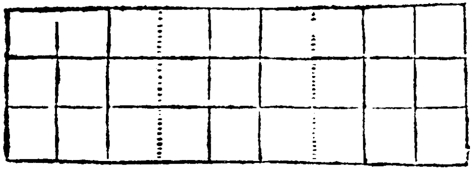
possible

possible that a *Square of squares* can be a mere *flatte nomber*, as you haue drawen it.

Wherfore yf thei had intended, that a *flatte nomber* should occupie the .4. place, then should thei haue set some *plat forme* in the third place also. Whiche might haue been made in this sorte.

And then will it be a *longe Square*, and not a *Cube*.

3.3.3.



But in as moche as thei doe not admitte this *longe Square* (whiche by that name hath no roote) therfore maie not the nomber that foloweth it, bee any other then a *sounde nomber*. For euery *Cubike* forme, beeyng multiplied by his roote, doeth make a *Square piller*. Whose length beareth vnto his bredth the same proportion, that his roote doeth vnto an vnitie.

Scholar. I am very well satisfied now: cōcernyng the names and formes of those nombers. And by this that you haue saied, I doe farther perceiue, that .5. multiplications doeth make the *square of Cubes*, whiche be set in the fifte place, emongeste the former figures. And also I vnderstande by the former table, that thei be called *Sursolides*.

Likewaies I see in the sixte place of the forsaied figures, *Cubike Cubes*, made by .6. multiplications. But commonly the nombers of those quantities, be named *Squares of Cubes*. So that for their names, thus farre I am perfecte inough.

Master.

Nowe will I shewe you, how you shal extract the roote out of any soche nomber. *The extraction of rootes*

And first I must admonishe you, that you shal allwaies vnderstande, soche a roote, as the nomber doeth admit. So that in a square nomber, you shall seke a *Square roote* onely, and no *Cubike roote*, nother any other kinde.

Likewaies a *Cubike nomber* hath no other roote, but a *Cubike roote*. Excepte the namebee compounde, as *zenzicubike*, or *Squared Cube*. For in soche there are 2. sortes of rootes, accordyng to the 2. names that thei beare. That is bothe *Square* and *Cubike roote*: as I will anon shewe you. But firste I will beginne with Square nombers, and their rootes. And this generalle order muste you obserue, before all other: That you shall haue by harte, in readie memorie all soche nombers, whose rootes are digites. For as it is superfluous to seke rules for theim, so must thei helpe in all greater nombers, whose rootes are aboue 9. And for your ease in remembraunce, I haue here sette foorthe a table for square nombers. Where in the firste columpne, you se the rootes set, and in the seconde piller, right against eche roote, there is set his square. Touchyng whiche I nede to saie no more, but that you be not in any vncertaintie of them, whē

The table of Squarerootes vncompounde.

Rootes.	*Squares.*
1.	1.
2.	4.
3.	9.
4.	16.
5.	25.
6.	36.
7.	49.
8.	64.
9.	81.

you shall nede their aied, whiche shall be continually in use of searchyng for other greater rootes.

Now for greater nombers, this is the order.

1. First set doune the nomber as it is. Then sette a pricke under euery odde place, I meane the firste, the thirde, the fifte, the seuenth, and so forthe: and so shall euery pricke haue. 2. nombers, excepte the laste, whiche some tymes hath but one.

2. Secondarily, marke the nombers that belong unto the laste pricke, toward the lefte hande: And whether he haue belongyng to it one nomber, or twoo, looke what the roote maie be of that nomber, if it bee square. And that roote sette by a croked line, as you place the *quotiente* in diuision: & cancell all that square nomber, belongyng to that pricke.

3. But and if the nomber belongyng to that pricke, bee not a Square nomber, then take the roote of the greateste square, whiche is contained in it, and place the roote as I saied before. And the square of it shall you abate from the nomber, that belongeth to that laste pricke, and let the rest be set ouer those nombers cancelled, as you doe in diuision. And so haue you ended your worke for that pricke.

Scholar. This moche is easie inough, if I understande you rightly.

Master. Then proue it in a nomber, or twoo. And first worke with this nomber. 515290O.

Scholar. I muste marke euery odde place with a pricke, thus.

515290O. And here I perceiue that unto the first pricke, there belongeth 2 Cyphers onely, and to eche of the other. 2. prickes folowyng, there are appoincted. 2. figures. But the fourthe pricke hath but one nomber, and that is. 5.

Now accordyng to the second rule, I seke the roote of 5. (for bicause there belongeth no more nombers to that

that pricke) and I see, it is no square nomber. Wherfore accordyng to the thirde rule, I take the greateste square in it, whiche is . 4. and the roote of . 4 . is . 2. Therfore I doe subtracte. 4. out of. 5. and cancell that. 5. and the. 1. that remaineth, I set ouer. 5. as here you see.

1 5 1 5 2 9 0 0 (2.

And the roote. 2. I sette behinde the *quotiente* line, as you taught me, and then the nōbers stand, as you se.

Master. You haue doen wel. Proue again in this nomber. 18766224.

Scholar. First I set theim doune and pricke theim, as here doeth appeare.

18766224.

And now I see, that the laste pricke hath twoo nombers belongyng to it, that is. 18. with whiche I must begin. And seyng it is no square nomber, I find 16. to be the greatest square in it: wherfore I subtract 16. out of. 18. and set. 2. ouer the. 8. And the roote of. 16. whiche is. 4. I sette behinde the *quotiente* line, as here is seen.

2 1 8 7 6 6 2 2 4 (4.

Master. This maie suffice for the first woorke.

Now to procede, you shall double your roote, and put that double vnder the nexte space, towarde the right hand, that is behinde the nexte pricke. Alwaies forseyng, that if the double doe contain more figures then one, that the first shall be sette vnder that place, and the seconde vnder the nexte figure, towarde the lefte hande. 4.

Then seke a *quotiente*, as you doe in diuision, whiche shall shewe how often that double nomber maie be found in that, that is ouer it, appertainyng to that place: whiche *quotiente*, you shall set before the firste roote, within the *quotiente* line. 5.

But this regarde muste you haue here specially, that you maie leaue ouer the nexte pricke, toward the right hande, as moche as the square of that *quotiente*,

 with

with which you worke, for out of that rest, the square of that *quotiente* muste bee abated. And then make bothe subtractions, and note the remainer, if any be, and place your *quotient*, and then haue you doen with that pricke also.

For the more plaines, I will giue you an example in your firste nomber, whiche stoode thus, after your worke was ended.

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  1
 5152900 (2.
 .  .  .  .
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Here I se ouer the laste pricke saue one. 11. vnder the middell figure of whiche I must set the double of the former roote. 2. that is. 4. And then I seke how often. 4. is to bee founde in. 11. And I finde that I maie haue it twoo tymes, and. 3. remainyng. Whiche. 3. with. 5. ouer the nexte pricke, doe make. 35. and that is more then the square of my *quotiente*. 2. Therfore am I bolde to sette doune that *quotiente*: And accordyng to it, to abate twise. 4. (whiche is. 8.) out of 11. and there resteth. 3. Therfore I cancell. 11. and sette. 3. ouer it.

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 13
5152900 (22.
  .  .  .
 4
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Then doe I multiplie the laste *quotiente* squarely: and it maketh. 4. whiche 4. I subtracte out of the nomber ouer the pricke, that is. 35. where. 5. maie suffice for this nomber. Therfore I abate. 4. out of. 5. and cancell that. 5. and set. 1. whiche remaineth, ouer the. 5: And then will the whole nomber stande thus.

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 131
5152900 (22.
.  .  .  .
 4
```

This woorke, whiche I haue wrought now, must be repeted as often as there bee any prickes, or pricked nombers remainyng. Wherby you maie easily gesse, that it must bee twise more repeated in this example, bicause there resteth yet. 2. prickes vntouched.

Scholar. Although I thinke, I could doe, as I haue marked you to doe, yet for more certaintie I

praie

praie you worke out this example.

Master. Then marke it well.

I shall begin againe with doublyng of all, that is within the *quotiente* line. And that double is 44. whiche I must set vnder. 312. that remaineth of the laste worke. And then will the nombers stande, as here you see.

Then I loke how often tymes maie I finde. 44. in. 312. And I see it will be abated 7 times, and 4 remain: whiche 4 with the. 9. ouer the next pricke doeth make. 49. And that will suffice to extracte the square of my *quotiente*. 7. For. 7. tymes. 7. maketh iuste. 49. Thus seyng I maie take. 7. for my *quotiente*, I worke with it, as the rule teacheth: abating first. 7. times. 44. (that is. 308) out of. 312. and there resteth. 4. ouer the space before the nexte pricke. Whiche. 4. with. 9. ouer the pricke doe make. 49. out of whiche I abate the square of my *quotiente*. 7. (that is. 49.) and so resteth nothyng, but. 2. Cyphers. And the nomber standeth thus.

```
131
5152900 (22.
 44
```

```
1314
5152900 (227
 44
```

And seyng there remaineth one pricke vntouched, I should repeate the same order of worke againe, by doublyng all the *quotiente*, whiche would bee. 454. and settyng it so that. 4. whiche is in the firste place, should be sette vnder the Cypher, that is without the pricke, and the other figures in order, toward the left hand. But all this worke were in vaine, seyng there is nothyng lefte, to serue for the subtraction.

Yet bicause there is lefte one pricked place vntouched, I must set for it a Cypher in the *quotiente*.

For this rule is generalle: that how many prickes so euer your square nomber doeth containe, your *quotiente*, or roote shall haue so many nombers. Wherfore this roote must be made vp thus. 2270.

The proofe. And so it appeareth that your nomber. 152900. is a iuste square nomber. Whiche you maie proue by the orderly proofe of extraction of rootes. That is to multiplie that *quotiente*, or roote (whiche you haue founde) by it self. And if it doe make the first nomber exactly, then haue you wrought well.

Scholar. That proofe is as certaine, as can be. And therfore I will proue, whether it will agree with this worke. Wherfore multipliyng 2270. by it self, I see that it yeldeth the firste somme. As here it doeth appeare. So is this worke approued good.

```
      2270.
      2270.
    -------
    158900.
   454
  454
  ---------
  5152900
```

And now will I attempte the like worke in the seconde example. Whiche was. 18766224.

But after the firste worke was ended, and the greatest square subtracted out of 18. it did remain in this forme.

```
  2
 18766224 (4.
```

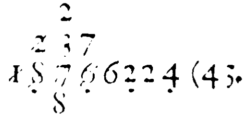
Now to continue the worke as you did, and as the rule doeth teache, I must double. 4. whiche is the roote, and standeth by the *quotiente* line: and must set it vnder. 7. that standeth in the space, betwene the laste pricke (whose worke is ended) and the nexte pricke towarde the right hande. And then will it stande thus as you see.

```
  2
 18766224 (4.
   8
```

That doen, I must seke a *quotiente*, that maie declare how often 8. maie bee subtracted out of. 27. and that *quotiente* I finde to be. 3: bicause that after I haue taken. 3. tymes 8. (that is. 24. out of. 27. there will remain. 3. whiche 3. with. 6. that standeth ouer the pricke, doe make. 36. And I see that nomber to bee greate inough, for the abatemente of the square of my *quotiente*; whiche is but. 3. tymes. 3. that is, 9.

Wherfore

Wherfore I sette doune.3. for my *quotiente*, before.4. in the *quotiente* line. And multiplying 8. by that 3. there riseth 24. whiche I doe subtract out of.27. that is ouer.8. and there will remain.3. That 3. with.6. ouer the pricke, maketh.36. out of whiche I must abate.9: whiche is the square of my *quotient*.3. and so will there reste.27. ouer that pricke.

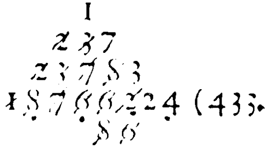


And thus haue I ended. 2. prickes, and yet. 2. more doe remain: in whiche bothe I must repeate the same forme of worke.

Therfore I double the whole *quotiente*, and it maketh.86: whiche I set vnder.276.

And then I seke the *quotiente*, declaryng how many tymes. 86. maie be abated out of. 276. whiche maie be.3. tymes. And for that cause I set.3. in the *quotiente* before the.43.

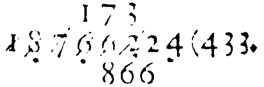
Then doe I firste multiplie.86. by that.3. saiyng.3. tymes.8. maketh.24. which I abate out of.27. and there resteth.3. And again I saie, 3. tymes.6. is. 18. whiche I abate out of.36. and there doeth remain.18.



That doen, I take the square of my *quotiente*, that is.9. whiche I doe subtract out of.12. (for the.2. ouer the pricke must borowe. 1. of. 8.) and then will there remain ouer that pricke.173.

And thus is that pricke ended.

Now, for the laste pricke in worke, though he be firste in place. The double of my *quotiente* is.866. whiche I muste sette vnder 1732. As here is doen, where I leaue out many cancelled figures, as superfluouse in



this

this place.

And then sekyng for a newe *quotiente*, I finde it to be.2. whiche I set with the other nombers in the *quotiente*. And by it I multiplie and subtract the 866. saiyng: 2. tymes. 8. is. 16. whiche I abate out of. 17. and there resteth. 1. Again. 2. times 6 is. 12 that I subtract out of. 13. and there remaineth. 1. Thirdly, I saie. 2. times. 6. giueth. 12. whiche I abate from. 12. and there is left nothyng. Saue that ouer the pricke there standeth 4 whiche is equall with the square of my *quotiēt*.

```
          1 1
         1 7 1
   1 8 7 6 6 2 2 4 (4332.
       8 6 6
```

Wherfore abatyng the square of my *quotiente* out of it, there resteth nothyng at all.

And therby I see that. 18766224. is a iuste *square nomber*. And his roote is. 4332.

The proofe. Master. Although I knowe it to bee so, yet for your better exercise, and full perswasion: I would haue you trie it, by square multiplication.

Scholar. That maie I sone doe. And so I finde it to be true. For. 4332. multiplied by it self, doeth make. 18766224. As this woorke here set, doeth shewe.

```
     4332.
     4332.
  --------
     8664.
   12996.
  12996.
 17328.
 ---------
 18766224.
```

Master. Yet bicause some other small doubtes, maie happen in workyng, that maie trouble a yong practiser, I will propounde to you one or twoo examples more. Wherein you shall finde some varietie, as well in the nomber propounded, as also in the *quotiente*.

And firste to begin, I will you to extract the roote of this nomber. 22071204.

Scholar. I must set doune the nomber, and note it with prickes in euery odde place: For that rule I

perceiue

perceiue neuer faileth.

Maſter. No more doeth any of the other, although the woorke maie varie in ſome ſmalle poinctes: whiche yet maie be greate inough to trouble a young learner.

22071204(

Scholar. Then accordyng to the firſte rule, I ſeke out the greateſt ſquare in. 22. (for I ſee it is no ſquare nomber it ſelf) and it appereth to be 16. And his roote 4. wherfore I doe ſette doune. 4. in the *quotiente*, and then I doe abate. 16. out of. 22. and the remainer is. 6. whiche I ſette ouer the pricke, and cancell the. 22. as here is ſeen.

6
22071204(4.

Now goyng on with the nexte pricke, I ſhall double the former roote in the *quotiente*, and ſette it vnder the Cypher, betwene the. 2. prickes.

Then do I ſeke how oftē that 8 (whiche is the double of the *quotiente*) maie be found in 60 and I finde it to be 7 times, and 4 remainyng to be ſet ouer the Cypher. So that for the pricke there remaineth. 47. out of whiche I ſhould abate the ſquare of my *quotiēt*. But ſeing that. 49 (whiche is the ſquare of 7) can not be taken out of. 47. there is a newe *quotiente* to be ſought.

Therfore I take 6. And ſee that it will ſerue. So I ſet. 6. in the *quotiente*: and by it I multiplie 8 whereof commeth 48. That. 48. abated out of. 60. leaueth. 12. Therefore I cancell the 60. and ſet. 12. ouer it.

19
621
22071204(46.
8

Then doe I multiplie the *quotiente*. 6. by it ſelfe: whereof riſeth. 36. And that abated out of. 127. leaueth. 91. And ſo haue I ended the ſeconde woorke.

Now for the thirde woorke, I double. 46. and it doeth yelde. 92. to bee ſette vnder. 911. as I haue put it here.

And then ſeking for a *quotient*: I ſe that I maie take

9. Wherfore I set that 9 in the *quotiente* with. 46. and by it I multiply 92 and subtract that, that riseth, in this forme.

Nine tymes. 9. maketh. 81. whiche I abate out of. 91. and there resteth 10. Then 9 tymes 2 giueth 18. whiche I must abate out of. 10. and there will remain. 83.

```
      7
     18
    1905
   62131
22071204(469.
     92
```

And now muste I multiplie that laste *quotiente*. 9. squarely, wherby will amounte. 81. that shall I subtract out of. 832. and there will remain. 751. and so that pricke with his worke is ended.

Therefore procedyng to the fourthe pricke, I double all the *quotiente*, whiche will be 938. And I set it under 7510. Then doe I seke a newe *quotiente*, whiche I finde to bee. 8. For 8. times. 9. giueth. 72. whiche I abate out of. 75. and there remaineth. 3. Again. 8. tymes. 3. is. 24. and that I deducte out of. 31. and so resteth. 7. Then saie I. 8. times. 8. is. 64. whiche beeyng subtracted from. 70. doeth leaue. 6. And that. 6. with the 4. ouer the pricke maketh. 64. out of whiche I muste withdrawe the square of. 8. that is my *quotient*, and it beyng 64. there resteth nothing. And the whole worke standeth thus.

```
      751
22071204(469
      938
```

```
      37
     7516
22071204(4698
      938
```

Wherfore I saie that the first nōber 22071204. is a square nōber: and hath for his roote. 4698 As I maie prooue also, by square multiplication. For, as in this example you see: 4698. multiplied by it self, doeth bryng forthe. 22071204.

The proofe.

```
     4698
     4698
   ------
    37584
   42282
  28188
 18792
 --------
 22071204.
```

Master.

Master. Yet one example more shall you proue: and that is this. 901740841. *Another example.*

Scholar. I set it doune, and pricke it accordyng to the rule: And then I see ouer the laste pricke, one onely nomber, that is. 9. whiche hath. 3. for his square roote. That. 3. I set within the *quotiente* line, and therfore I cancell. 9.

901740841(3

After this I should proceade with doublynge the roote. 3. and that double should I set in the next space, ouer whiche remaineth no nomber, for. 9. beyng cancelled, the Cypher is nothyng. And so am I at a staie.

Master. Seyng that you can not set the double of your *quotiente* doune there, where no nomber is (or if it so chaunce, as some times it doeth, that the nomber ouer it, is lesser then the double) then set a Cypher in the *quotiente*, and so haue you doen with that pricke. For in soche case there nedeth no multiplication, nor subtraction.

Scholar. Then am I instructed fully for that poincte: The worke is so easie. I must therfore set my nombers thus.

901740841(30
6 0

Master. And doe you not see, that the double of the *quotiente*, is greater then the nomber ouer it?

Scholar. I was so mindfull of the one halfe of the rule, that I forgate the other halfe.

But now I see, I must set an other Cypher yet in the *quotient*. And then shall I set the double of all that, in the thirde space, after this sorte.

And nowe proceadynge to searche for anewe *quotiente*, I see that. 2. shall serue me. Wherfore I sette. 2. in the *quotiente* line, with. 300. And by it shall I multiplie the double aforesaied: saiyng. 2. tymes. 6. maketh. 12.

901740841(300.
600

to bee abated out of.17. and the remainer will bee.5.

Then shall I ouerpasse the twoo Cyphers, bicause thei make nothing by multiplication: and so cõmyng to the pricke, I bate the square of my *quotiente*: whiche is 4 out of.8. and there resteth.4. Therfore I cancell.8. and set doune.4. and so haue I ended that pricke. And haue but one worke more behinde.

```
    5   4
9 0 1 7 4 0 8 4 1.(3002.
 ·    ·     ·   ·
      6 0 0
```

Wherfore I set doune the nombers, with the double of al the *quotiente*, thus. And then I loke for a new *quotiente*, whiche I finde to be.9. by it therfore I multiplie, first 6 and it maketh 54. that doeth abate the 54. ouer it. Then omit I the 2 Cyphers, and multiplie 4. by 9 whereof there commeth. 36. whiche I abate out of. 44. beyng ouer it, and there remaineth. 8. That. 8. with. 1. ouer the pricke maketh. 81. out of whiche I muste abate the square of. 9. beyng also. 81. And so is nothyng lefte, wherby it appeareth, that. 90174084 1. is a square nomber, and his roote is. 30029. The proofe of it doeth confirme the same. For 30029 multiplied by it self, doeth bringe forthe. 90174084 1.

```
    5   4
9 0 1 7 4 0 8 4 1(3002.
      6 0 0 4
```

```
     30029.
     30029.
  ---------
    270261.
   60058.
90087 · ·
  ---------
 90174084 1.
```

The nigheste roote of vnsquare nombers.

Master. This shall suffice for soche nombers as bee fully square. Other nombers there bee infinite, whiche be not square, and therfore haue thei no square rootes. Yet often tymes it happeneth, that we shall bee occasioned to searche for the nigheste nomber, that maie resemble their rootes.

Wherfore in soche case, this shall you doe. Firste

extracte

extract the roote, as if it wer a ſquare nōber. And that roote wil ſerue for the greateſt ſquare, that is in your former nomber: and there will be a remainer beſide. Of whiche remainer with the *quotient*, you ſhal make a fraction, in this ſorte.

Set the remainer ouer the line, for the nomerator, and the double of the roote (that you haue founde) ſet vnder the line, for the denominator. And this ſhall be a ſufficiente preciſeneſſe in greate nombers, for any common woorke.

Scholar. I will by an example, taken by chaunce, proue this rule. For it ſemeth to haue no difficultie. Wherfore I take. 296882.

And this, I am aſſured, can be no ſquare nomber. For, I remēber you told me before, that no ſoche nomber might be a ſquare, which had 2 for his firſt figure.

Then to ſearche his nigheſte roote, I place it, and pricke it thus.

```
 4
296882(5
 .  .  .
```

And vnder. 29. I finde the greateſte roote to bee. 5. whiche I ſet in the *quotiente* line, and cancell 29 ſettyng 4 ouer it. After that I double it, and there cometh 10. & that double I ſet in the nexte ſpace vnder 46. Then finde I a newe *quotiente*, whiche is 4 and by it I multiplie. 10. whereof amounteth 40. to be abated out of 46. And ſo remaineth. 6. Again I multiplie. 4. by it ſelf ſquarely, and there riſeth. 16. whiche I abate frō 18. (ſeyng. 8. is to ſmall) and the remainer will be. 2. So ſtandeth the whole nomber, as you ſe.

```
 452
296882(54
 10  .  .
```

Wherfore I double the *quotiente*, whiche is. 54. And it yeldeth. 108. that muſt be ſet vnder 528 as I haue here doen.

```
 452
296882(54.
  108
```

Then I looke for a *quotiente*, how often I maie abate. 108. out of. 528. And I ſee it will be but. 4.

tymes. Wherfore I set.4.in the *quotiente*, with the other nombers, and then doe I woorke with it: Firste multiplyng.4.and.1. together, whereof cõmeth but 4. whiche I abate out of.5. And there remaineth.1.

Again I multiplie.8. by.4. whereof commeth.32. that doe I subtract out of. 128. and there will remain 96. Then shall I take the square of my *quotiente*. 4. whiche is 16. And that must I abate out of 962. And so remaineth. 946. of whiche nomber set as the numerator, with the double of the roote, set for the denominator, I shall make a fraction in this sorte. $\frac{946}{1088}$. whiche is almoste. $\frac{9}{10}$.

```
   194
   5266
 296882(544.
   108
```

Master. You haue doen wel. And so you perceiue that the nighest roote of your former nomber is 544 $\frac{473}{544}$. For those fractions are all one.

And hereby also you maie understande, that if the remainer ouer your nomber bee euen, you maie take halfe of it for the numerator, and the whole *quotiente* for the denominator.

So maie you take the quarter of the remainer (if it will so bee parted) for the numerator, and the halfe of the roote for the denominator.

And in like maner generally, if the remainer and the roote in the *quotiente*, bee *nombers communicante*, diuide them so, that the diuisor of the remainer, be euer double to the diuisor of the *quotiente roote*. And so maie you easily reduce that fraction, to his least termes.

The firste proofe.

But now for proofe of this woorke, there be twoo waies: the one is certain, and the other but in a nerenesse. For as the roote of soche nombers, is not a precise roote: So if you multiplie that roote by it self, it will make a nomber, very nighe to that former nomber, but not exactly the same.

Whiche faulte some men thinke to redresse, by adding dyng of. 1. to the denominator: and yet that amende-mente sometymes increaseth the erroure.

But bicause you shall not wante a sure proofe, doe thus. Multiplie the *quotiente*, or *Roote* of whole nom-bers by it self, and unto the nomber that amounteth thereof, adde the whole remainer. And if then it make your firste nomber, your worke was well doen: els haue you missed. *The seconde proofe.*

Scholar. That maie I proue here quickly. The *quotiente* in whole nombers was. 544. whiche beyng multiplied squarely, doeth yelde. 295936. unto whi-che nomber, if I doe adde. 946. that did remain, it will amounte to. 296882. and that was the nomber proponed to me: wherfore it appereth that the worke was well doen.

```
   544.
   544.
  -----
  2176.
 2176.
2720.
------
295936.
```

Master. You shall neade no more examples, for this forme of worke.

But one other waie wil I shewe you, how you shall gesse verie nighe unto the roote. And you shall go as nighe as you will desire, in any prac-tike worke. If you desire to gesse within lesse then $\frac{1}{10}$. of one, then set before your nomber. 2. Cyphers. And if you would not erre $\frac{1}{100}$. then set doune 4. Cyphers: But and if you liste to sette doune. 6. Cyphers before your nomber, you shall not misse $\frac{1}{1000}$ of an unitie frō the true roote. And if you list to go any higher in pre-cisenesse of partes, adde still euen Cyphers. *An other waie to finde the nighest roote.*

Scholar. I would faine proue this forme, in the same example, whiche I wroughte laste: Bicause I would se the agremente betwene the bothe workes.

Master. Go to. Your consideration is reasonable And bicause the partes maie the better agree, sette doune. 6. Cyphers. And then shall your roote expresse thousande partes of the whole nomber.

Scholar. I sette doune the nomber, and pricke it thus.

thus. Whereby I perceiue that I shall haue the same order of worke, and the selfe same nombers that I had before, till that I come to the Cyphers and their prickes.

2968820000000(

Master. Truthe it is. And therfore maie you in soche a case sette doune onely the remainer, with the Cyphers. Or els cancell all the nombers, saue the remainer, and the Cyphers: and set the former whole roote, without the fraction, in the *quotiente*.

Scholar. Then will it stande thus.

946
2968820000000(544.

Now accordyng to the rule I will proceade: as if this whole nōber wer the first nomber proponed vnto me. And therfore I doe double al the *quotiente*, whiche maketh. 1088. and that doe I set vnder. 9460. And then shall I seke a *quotiente*, that maie declare how often tymes, that double is cōtained in the nomber ouer it. And I se it will bee. 8. wherfore I set doune. 8. in the *quotiente*, and by it I multiplie the double, and subtracte it, in this sorte: saiyng 8. tymes. 1. out of 9. leaueth. 1. remainyng. Again. 8. times. 8. (that is. 64.) out of 146 will leaue. 82. Then farther I abate. 8. tymes. 8. out of. 820. and there resteth. 756. And last of all, I take the square of the *quotiente*, whiche is also. 64. out of 7560. and there will remain. 7496. And so haue I doen with the firste pricke of the Cyphers.

4
78
1829
94666
2968820000000(5448
1088

A notable consideratiō. Master. Consider now that by those. 2. Cyphers you haue gotten 8 into the *quotient* more then you had before. And all your former nomber of the roote, remoued by it into one place higher, then it was before

So

So that, where by the first worke, your roote was 544. and almoste $\frac{9}{10}$: by this worke you haue founde it to bee $\frac{5448}{10}$, and $\frac{3748}{5448}$ of $\frac{1}{10}$: whiche is verie nighe the same nomber, that you had before.

Scholar. In deede, if I reduce the fractions, it wil bee. 544. $\frac{8}{10}$ and $\frac{937}{1362}$ of $\frac{1}{10}$: whiche is in one fraction, $\frac{11833}{1362}$ aboue. 544.

Master. Marke this triall. And vse the like after euery twoo Cyphers are ended: And you shall see a goodly agremente of the woorkes together.

Scholar. In the meane tyme, to procede with the former worke, I set doune the nomber with the remainer, and the doble of the *quotiente*, as here appeareth.

```
          7496
296882000000(5448
         10896
```

And searchyng for a newe *quotiente*, I finde that it will be. 6.

Therfore I sette doune. 6. in the *quotiente* with the other nombers. And by that. 6. I doe multiplie the double of the whole *quotiente*, and subtract it orderly, saiyng: 6. times. 1. beyng abated out of. 7. leueth. 1.

Likewaies. 6. tymes. 8. maketh. 48, whiche I shall abate out of. 49. and so resteth. 1. Then 6. times. 9. (whiche is. 54.) must be subtracted out of. 1016. and there will remaine. 962. Againe I shall abate. 6. tymes. 6. (that is. 36.) out of 9620. and there is lefte. 9584. Then take I the square of my *quotiente*, whiche is also 6 times 6, or 36. and that I must abate out of. 40. and there resteth. 4. And thus is the seconde pricke of the Cyphers ended.

```
          5
         968
        10120
        749644
296882000000(54486
        10896
```

And now I finde in the *quotiente* not. $\frac{8}{10}$ as I did in the

the laste woorke before this. But I finde $\frac{86}{100}$: whiche goeth more nighe to $\frac{9}{10}$. For $\frac{90}{100}$ would be $\frac{9}{10}$: and $\frac{80}{100}$ is equalle with $\frac{8}{10}$. And I maie easily se, that $\frac{86}{100}$ is more nigher to $\frac{90}{100}$ then to $\frac{80}{100}$: beside the remainer, whiche will make $\frac{47902}{54486}$ of $\frac{1}{100}$. or els $\frac{47902}{5448600}$ of one.

Master. I see, a well willyng mynde can marke diligently, and learne spedily: wherfore go forwarde with your woorke.

Scholar. I muste sette doune the double of all my *quotiente*, whiche will be. 108972. And it will stande thus.

Wherfore I doe seke for a newe *quotiente*, and I finde it to be. 8. whiche. 8. I set in the *quotiente*, with the other nombers, and by it I worke after my rule, saiyng: 8. tyme. 1. is. 8. whiche I abate from. 9. and there resteth. 1. Then take I. 8. tymes. 8. (that is. 64.) out of. 158. and the remainer will bee. 94.

```
    95802
29688200900(54486
     108972
```

Again I subtract. 8. tymes. 9. (beeyng. 72.) from. 940. and there is lefte. 868. Farthermore I take. 8. times. 7. (whiche is. 56) out of. 82. and there resteth. 26. Then doe I withdrawe. 8. tymes. 2. or. 16. out of. 60. And there remaineth. 44.

```
      8624
     19486
     958024
296882000000(544868
     108972
```

Last of al I take 8 times 8 or 64 (whiche is the square of my last *quotiente*) out of 862440 and the remainer will be. 862376. And so haue I ended all my worke.

And now I haue for the roote $\frac{544868}{1000}$ that is. 544. and $\frac{868}{1000}$ beside $\frac{431188}{544868}$ of $\frac{1}{1000}$ or in lesser termes $\frac{107797}{136217}$ of $\frac{1}{1000}$ that is $\frac{107797}{136217000}$ of one: Whiche beyng reduced into one fraction with the $\frac{868}{1000}$ will make $\frac{118344153}{136217000}$.

Master. You haue doen well.

And

And here you see, that you drawe nigher & nigher still, to the very roote, if it might haue any. For [illegible] is a nigher nomber to $\frac{9}{10}$. then is $\frac{86}{100}$ as that was nigher then. $\frac{8}{10}$.

And if you would worke with more Cyphers, you should perceiue still, that it would drawe nigher and nigher. But this maie suffice for examples sake.

Scholar. Then I praie you tell me, what is the chief vse of this rule: and for what maters it serueth.

Master. One yere will not suffice, to expresse the commodities of it. It serueth so many waies, in building: in proiection of plattes, for measuring of grou͂d Timber, or stone: And also in warre, for framyng of battailes, for makyng of diuerse engines, and generally for all woorkes of *Geometrie* and *Astronomie*. But for to satisfie you partly, I will sette forthe twoo or three questions, that depende of this worke of extraction of square rootes.

And firste of a battaile: bicause it semeth to serue leaste for that purpose. *A question of an armie.*

A capitaine generall hauyng three greate armies, would caste theim into three square battailes, but he knoweth not how many men, he shall set in the fro͂te of eche battaile.

The nombers of the three armies, are for the firste 5625: For the second 9216: And for the third 15129

Scholar. I dooe perceiue easily, that for eche of these nombers, I muste searche out the square roote, and then haue I the fronte, or flanke. Sith bothe are equalle in a square battaile.

Wherfore I set doune the first nomber thus, with his prickes. And then vnder the first pricke towarde the lefte hande, I finde the greatesse roote to bee. 7. seeyng the greatesse square is. 49. That roote doe I set within the *quotiente* line: and his square doe I abate from. 56. and so

| 5625 (

 remaineth

remaineth. 7.

Then doe I double that roote, and sette the double under. 72. and see that the newe *quotient* will bee. 5. And there will remaine. 25. whiche is the iuste square of the last *quotiente*.

7
5625 (75
14

Wherby it is euidente, that his first armie contained a square nomber, and the roote, or side of it is 75. And so many menne shall be in the fronte of the firste battaile, and as many in the flanke.

Now for the seconde battaile, I seke the square of 9216. and finde it to bee. 96. As in this example I haue wrought it.

For the firste nomber is. 9. seyng it is the greateste square roote, that can bee founde in. 92. And so is the double of it. 18. and the *quotiente* for it. 6. as it appeareth manifestly inough.

5
113
9216 (96
18

Wherfore I saie that the second battaile shal haue in euery ranke. 96. men.

And now for the thirde battaile, I sette doune the nomber, accordyng to this rule: and I finde the firste roote to be. 1. bicause. 1. tymes. 1. maketh. 1. And his double is. 2. whiche I abate twise from the nomber ouer it: and after double those bothe nombers, whiche make. 24. And finde that to be abated. 3. tymes.

1
17
15129 (123
2 4
2

And so haue I gathered that the nomber is square and the roote 123. According to whiche nomber, that thirde battaile must be marshalled.

Master. Seyng you are so redy in this poincte so sone. Tell me how many menne, shall be sette in the fronte, if all these. 3. armies be ioined into one square battaile.

Scholar. Firste I must adde all. 3. nombers together.

ther. And so will thei make. 29960. as here by example doeth appere.

5625
9216
15129
29960.

But this nomber can bee no square nomber, bicause it hath one odde Cypher in the firste place: for I remember your saiyng, that square nombers can not begin with odde Cyphers. Wherfore this nomber will not make a square battaile.

Yet wil I proue, what maie be the frõt of the greateste square battaile, that maie be made of that nõber.

And for that purpose I pricke the nombers, and finde the greateste roote in. 2. to be. 1 and the same nõber to bee the square also. Then double I that roote, and place his double vnder. 9. that is vnpricked: and serchyng for a *quotiente*, I finde it to be. 7. with whiche I worke by the rule, and so doeth remain for the nexte pricke. 10.

1 7 3
1 5 3 4 1
2 9 9 6 0 (173
2 3 4

Then doe I double that. 17. whereby commeth 34 whiche I set vnder. 106. And for it I finde. 3. to be the meteste *quotiente*: with whiche if I worke accordyngly, there will remaine. 31. as the excesse aboue the greateste square.

Wherby it appeareth that. 29929. is a square nõber: and hath. 173. for his roote. And that should bee the fronte of this greate battaile.

Master. Now will I proue you with an other question of like sorte.

A Prince hath an armie verie greate. With whiche he passeth in a Vallie, so that in marchynge the fronte can be but. 18. menne. And by that meanes the flancke containeth. 449352. *The seconde question of an armie.*

After that the armie is passed that valie, the kyng mindyng to occupie all the beste grounde, willeth the battaile to be set square. How would you doe it?

Scholar. first I multiple the flancke, by the front.

And ſo I finde the whole nomber to be.8088336.

That nomber doe I pricke as my rule teacheth me, and I finde the firſt roote to be. 2. and his ſquare.4.whiche firſt I ſubtracte out of.8.and ſo reſteth. 4. Then doe I double that *quotiente*, and finde that double. 8. tymes in the ſomme ouer it.

```
   2 2 2 3
 4 8 4 4 7 1
 8 0 8 8 3 3 6(2844.
   4 8 6 6 8
       8
```

And ſo doe I procede till I haue founde out all the 4.figures,accordyng to the.4.prickes vnder that nōber. And then the roote appeareth to be. 2844.

The thirde queſtion of an armie.

Maſter. Yet one queſtion more, for to exercise your penne, will I propounde of a like mater.

A generalle hath three armies, to the nomber of 28289.men: and none of thoſe three armies is apte to make a ſquare battaile, yet he is appoincted by his ſoueraigne,to ſette theim in three ſquare battailes.

Theſe be the.3.nombers of the.3. armies. In the firſte there are.10296. men:In the ſeconde. 9493: and in the third.8500. Now let me ſee how you can caſt them into three ſquare battailes.

Scholar. I thinke it reaſonable, to take the greateſte ſquares of the firſt and ſecond nombers, and the exceſſe of them bothe,to put to the thirde nomber.

Maſter. So are you not ſure that the third nomber,will be a true ſquare.

Scholar. Then knowe I not how to doe it.

Maſter. Take the greateſte ſquare in the thirde nomber alſo. And note thoſe three exceſſes,and their rootes alſo.

Then put one to euery roote, and marke the ſquares that will riſe of them.

Thirdly,ſubtract the firſte 3.nombers,out of thoſe 3.newe ſquares,and note the difference of eche of the firſte nombers,from thoſe ſquares:and ſo haue you.3 nombers

nombers of excesse, and. 3. other of wante.

Now compare those excesses and wantes well together: and you shall easily see from whiche you shall take any nomber, and to whiche you shall adde any.

Scholar. In the firste nōber the greatest square is 10201. and therby the excesse is. 95. and the roote 101.

In the second nomber the greatest square is. 9409 and his roote 97. So is the excesse. 84.

And in the thirde nomber, the greateste square is 8464: and the roote of it. 92. Wherfore the excesse appeareth to be. 36.

And thus haue I founde the. 3. excesses.

Now for to finde the 3 defaultes or wantes, I adde one to eche roote, and multiplie theim square: and so of. 102. I finde the square to bee. 10404. and if I subtracte the firste nomber, whiche is. 10296. out of it, there will remain. 108. for the firste wante.

Then for the seconde roote. 97. I take. 98. whose square will bee. 9604. out of whiche I abate the seconde nomber, whiche is. 9493. and there is left 111 as the wante of the seconde nomber.

Thirdly, I take 93 for the newe roote, next aboue 92. and I finde his square to bee. 8649. from whiche when the thirde nomber. 8500. is abated, the defaulte appeareth to bee. 149. And thus haue I the. 3. defaultes or wantes, and also the. 3. excesses. Whiche for ease of comparyng, I set in order thus.

	A.	B.	C.	A. B. and C. betoken the order of the 3 first nōbers.
Excesses.	95.	84.	36.	
Wantes.	108.	111.	149.	

And here I compare the excesses with the wantes, to see if any. 2. excesses will make vp the others want And I see by a lighte proofe, it will not serue.

As for the wantes, I doe not compare theim to the

excesses,

excesses, for I se that euery one want, is greater then any one excesse. And therfore. 2. wantes are farre to greate aboue any one excesse. And so am I at a staie.

Master. Therfore although that rule bee generalle, yet where it faileth, this shall you doe.

Take the. 2. wantes, of any. 2. nombers, and adde theim firste together, and then abate theim from the thirde nomber: and if the remainer be a square nomber, then haue you gotten your purpose.

Scholar. That will I proue here. And first I take the wantes, of the. 2. firste nombers, whiche make 219. And that doe I abate from the thirde nomber 8500. and there remaineth. 8281. whiche as I see, maie be a square nomber. And therfore I proue it, in my tables, and I finde it so to bee. And. 91. to bee the roote of it.

Wherfore I saie to the question, that these shall be the nombers of the 3 battailes, as here I haue set thē.

The firste battaile.	10404.	and his fronte. 102.
The second battaile.	9604.	and his fronte. 98.
The third battaile.	8281.	and his fronte. 91.
The somme of all the. 3. battailes.	28289.	

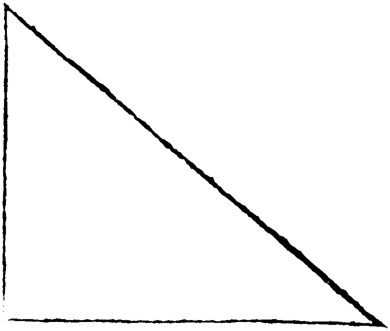
And bicause these nōbers are not onely square, but also their whole somme doeth agree, with the somme of the 3 seuerall armies, you maie be sure that thei are well parted, accordyng to the intente of the question.

But bicause soche questions, haue more difficultie then commoditie, to them that are not mete, to be trauelled in soche marshall affaires, I wil leaue that mater to marshall men, and will come to lower maters in warre.

A question of scalyng. A citie should bee scaled, beyng double diched. And the inner diche. 32. foote broade. And the walle. 21. foote high. The capitain commaundeth ladders to be made

made of that iuste lengthe, that maie reche from the vtter browe of the inner diche, to the toppe of the wal· as in this figure *C.* is partly expressed.

Where the line *A B.* standeth for the bredth of the diche. And the line. *B. C.* for the heighte of the walle. Nowe I demaunde, what shall be the lēgth of the line. *A.C:* whiche here doeth represente the ladder?



Scholar. This figure doth occasiō me to remēber the 33. theoreme of the pathewaie, whiche saith thus.

In all righte angled triangles, the square of that side, whiche lieth against the righte angle, is equalle to the twoo squares of bothe the other sides.

Wherby I vnderstand, that I must multiply those twoo sides squarely, that is, eche of them by it selfe. And then addyng those. 2. squares together, I muste extract the roote of that whole nomber: whiche roote shall be the true lengthe of the slope line.

Wherefore, firste I multiple. 32. by it self, and there riseth of it 1024.

Againe, I multiplie. 21. by it self, and it yeldeth. 441. These bothe sommes, beyng added together, doe make. 1465. whiche nomber maie bee square, bicause it begin-

32	21
32	21
64	21
96	42
1024	441

neth with. 5.

Maſter. It is no ſquare nomber, as it appeareth at the firſte ſighte. For although the firſte nomber be 5. yet in ſoche nombers it is requiſite, that the ſeconde figure ſhould be. 2. els can it not be ſquare: and here, you ſee, that the ſeconde figure is. 6. ſo that it can not be a ſquare nomber.

Wherefore you ſhall ſeke the nigheſte roote, that you can finde in it, and take that for your purpoſe.

Scholar. Here is my worke ſet forthe.

And ſo it appeareth well that the nigheſte roote is. 38. $\frac{21}{76}$, whiche is leſſe then a quarter of a foote, aboue 38. foote and that muſt be the lengthe of the ladder.

2
5 8 1
1465(38 $\frac{21}{76}$
6

Maſter. Yet one queſtion more will I propound agreable to the firſte forme.

A queſtiō of encampyng.

A capitaine generalle hauynge three armies, in three ſeueralle battailes, in the firſte. 4900. menne, in the ſeconde. 2401. And in the thirde. 2500. (ſo that the greateſte armie, is as moche as bothe the other, excepte one manne) is inforced to ioine all three battailes in one. But is in doubte, whether he maie haue good and conueniente grounde to encampe thē, in battaile forme. Wherefore conſideryng, that all. 3. battailes together, are but double to the greateſte of the. 3. alone. The capitaine deſiryng a mete grounde for his armie, ſo ioined in one ſquare battaile, is in doubte, what ſquare of grounde will ſerue his purpoſe. But ſure he is, that it muſte bee double to the grounde, that the greateſte armie of the 3. did occupie and that was ſquare euery waies. 210. foote. Wherfore his demaunde is, how many foote ſquare, ſhall the ſide of that grounde bee, that is double to the former ſquare platte, whoſe ſide was. 210. foote euery waie?

Scholar.

Scholar. Firste I must multiplie. 210. by it self, and so haue I the iust platte of grounde, of. 44100. foote, that must I double, and it will be. 88200. And out of this nomber, shall I seke the nigheste square roote. For a iuste square, I se, it is not: by reason that after the euen Cyphers, there foloweth. 2, whiche is one of those figures, that can not beginne any square nomber.

Wherefore, sekyng for the nigheste roote, I finde it to bee 296. $\frac{121}{148}$ that is almoste. 297. foote euery waies square. And so moche muste the square side of that grounde bee, whiche should serue for that whole armie.

```
   14
 1458
42124
88200(296 [illegible].
 4
   58
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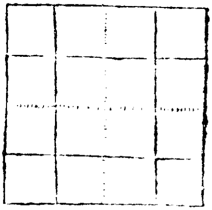
And hereby I doe perceiue, the ouersighte of many men: whiche being required to double a square platte do double the side of it, thinking the mater easily doen



But if thei marke it well, thei maie perceiue, that thei doe make, by that meanes, a square fower times so bigge as their firste square was. As by this figure, any man maie see.

For if 2. be the side of the square then is the square 4. But if I double the side, and make it. 4. the square thereof will be 16. whiche is. 4. tymes. 4. and not onely double.

So that the roote of the double platte, should bee the roote of. 8. whiche is somewhat lesse then. 3. and therfore moche lesse then. 4.

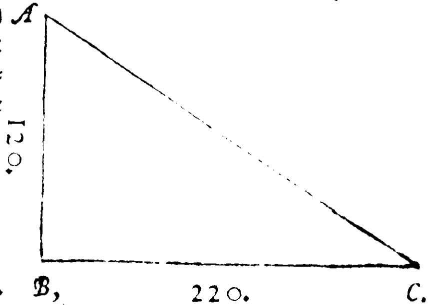
Master. You maie perceiue thesame, with the reason of it, by the 18. proposition of the. 8. booke of *Euclide*, as it is before alleged.

But now for to shewe the larger vse of this rule, *A question geographical*

I demaunde this question.

There be.2.tounes, as *Chichester* and *Yorke*, whiche lye Southe and Northe, and betwene them.220.miles. A thirde toune as *Excester*, lieth plaine Weste frō *Chichester*.120.miles. I desire to knowe the iuste distaunce of *Yorke* from *Excester*.

Scholar. I must set those.3.tounes, in forme of a Triangle, with their distaunces: As here is represented. Where *A*. stādeth for *Excester*, *B*. for *Chichester*, & *C*. for *Yorke*.



And then accordyng to the rule, I multiplie.120. squarely: and it maketh.14400. Likewaies I dooe multiplie.220. and it yeldeth.48400.

These bothe nombers I shall ioyne in one, and so haue I.62800. whose roote is very nigh.250. miles and $\frac{3}{5}$ of a mile.

And that is the true distaunce of *Yorke* and *Excester*.

By this example I gather, that this rule dooeth helpe to *Geographie*, for to drawe the true platte of any countrie.

2 3
62800(250 $\frac{3}{5}$.
4

Master. If I should stande in propoundyng exāples of this rule vnto you, vsyng but one for euery arte and science, and for euery different kinde of commodiouse practise: it would make a greate booke.

And therefore omittyng that, till occasion serue otherwaies, I will proceade to the extraction of *Cubike rootes*.

Of Cubike rootes.

When any *Cubike nomber* is propounded, whose roote you should extract After the nomber is written doune orderly: you shall set a pricke under the firste figure: and under the. 4. and so under euery third figure, omittyng still. 2. figures unpricked.

And looke how many prickes, your nomber hath, so many figures shall the roote of your nõber contain

Then to begin the searche, for the firste figure of 1.
the roote (in this order) you shall looke what maie be the roote of the nomber, belongyng to the last pricke toward the lefte hande. And that roote shall you sette by a *quotiente* line, as you did in square rootes.

And if the whole nomber ouer that pricke, be a *Cubike nomber*, you shall cancell it all. But if it bee no *Cubike nomber*, then subtracte out of it, the greateste *Cube* in it, and cancell the whole nomber, and set the reste ouer it: as you did in square rootes.

But considering, that you ought to haue in ready remembraunce, all those *Cubike rootes*, whiche be digites, with the *Cubes* that thei make: for without them you can not procede in this worke. I thinke it good to set forthe herein a table, all those rootes with their *Cubes*, that therby you maie be the more adsured in tyme of your worke. For els a litle mistakyng, might be the occasion of a greate erroure.

1.	1.
2.	8.
3.	27.
4.	64.
5.	125.
6.	216.
7.	343.
8.	512.
9.	729.

And now for this first rule I saie, as I saied of *Square rootes*, this shall be euermore the firste worke, and shall not be repeted in any one *Cubike nõber*. Whereas all the other rules folowyng, shal be so often repeated, as there are prickes in your nomber.

2. And of theim this is the firste: that you shall triple the firste roote. And that triple shall you set vnder the nexte nomber, toward the righte hande, before that pricke, whiche you did laste ende.

3. Then multiplie that triple, by thesame *quotiente*. And set it doune vnder the first triple: and that nomber shall be called your diuisor.

4. Thirdly, loke out a *quotient*, that maie declare how often the diuisor is in the nomber ouer it.

In whiche doyng, you must haue this regard, that betwene that pricke that is ended, and the nexte that standeth toward the right hande, you must subtracte 2. other nombers. That is to saie, the square of the laste *quotiente*, multiplied by the former triple. 10. tymes: and the *Cube* of thesame *quotiente*.

Scholar. This rule is very obscure in woordes.

Master. Then will I terme it thus.

2.&.3. Take the square of your whole *quotiente*, 300. tymes: and that shal be your diuisor. Then seke a newe
4. *quotiente*, declaryng how often that diuisor, maie bee founde in the nomber, that doeth belong to the nexte pricke. But so that the square of that newe *quotiente*, multiplied by the last *quotiente*. 30. tymes: and also the *Cube* of that newe *quotiente*, ioyned all in one somme, maie be taken out of thesame nomber. And if you vnderstande this, there resteth no more difficultie.

Scholar. I trust by exāple, to vnderstand it better

Master. Then take you this exāple. 2646592 whiche I shall set doune and pricke, as I taught you before: and as you maie here see. Where the. 3. prickes declare vnto me, that the roote will haue. 3. figures,

| 2646592(|

And then vnder the pricke that is nexte the lefte hande, whose nomber is. 26. I finde the greatest *Cubike nomber* to bee. 8. and his roote. 2.

For. 27. whiche is the nexte *Cube*, is to greate.

Therfore I set. 2. in the *quotiente*, and his *Cube*, beyng. 8. I doe abate out of 26. and so remaineth. 18.

That. 18. I doe sette ouer. 26. whiche I muste cancell: and then standeth the nomber, as here you doe see.

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18
26463592(2
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This is that firste worke, whiche is not repeted.

Then to procede forward, I doe triple the *quotiente* 2, and so haue I. 6. whiche I shall set vnder. 4. beyng the nexte nomber, on the righte hande of the pricke that is ended.

And that triple must I multiplie, by the first *quotiente*, wherby amounteth that nomber, that must be the diuisor: and it is in this worke 12. whiche must be set vnder the same triple: as here I haue placed it.

```
18
26463592(2.
  6
 12
```

Then shall I seke for a newe *quotiente*, declaryng how often tymes. 12. maie be founde in the nomber ouer it, that is 184. And I see it maie be in appearaunce. 15. tymes, but more then. 9. you shall neuer take for a *quotiente*: wherefore it appeareth, that I maie boldly take. 9. whiche I shall sette in the *quotiente* with the firste. 2. And then shall I multiplie. 12. whiche is the diuisor, by. 9. and thereof commeth. 108. to bee sette vnder 184. benethe the line, whiche shall euermore be drawen vnder the diuisor.

Now muste I take the square of my laste *quotiente*. 9. (whiche is 81.) and multiplie it by the triple of the former *quotiente* (that is by 6.) and so haue I. 486. to be sette one place more toward the right hande.

```
  2
18074
26463592(29
  6
 12
-----------
108
 486
  729
-----------
16389
```

Last

Laste of all, I shall multiplie the laste *quotiente Cubikely*: and that maketh.729. whiche must be set, yet one place more toward the right hand, that is to saie, vnder the nexte pricke. And then shall I adde those 3 sommes into one: wherby will rise.16389.to be subtracted out of. 18463. and so will remaine ouer that pricke.2074.

And the woorke of that pricke is doen.

This order of worke, if you marke well, you haue learned the whole arte of extraction of *Cubike rootes.*

For how greate so euer your nomber be: you shall not haue any newe kinde of woorke.

But yet bicause I did teache you before, the same woorke in other woordes, I will woorke the same example again, accordyng to these woordes.

And firste, after that the nomber is set doune, and the first *Cubike roote* taken, and the *Cube* abated. Then take the square of that roote. 300. tymes, that is in this example.4.tymes.300, whiche maketh. 1200. and that shall be your diuisor. This nomber, and all other in this woorke, shall you set doune so, that the firste nomber, shall be vnder the nexte pricke, toward the righte hande.

Then seke your *quotiente*, with the former cautele, and it will bee. 9. Wherefore multiplyng.1200.by.9.there will amounte 10800.to be set vnder the line.

```
18
26463592(29
 1200
-----------
10800
 4860
  729
-----------
16389.
```

After this, I shall take the square of.9.(whiche is the new *quotiente*) and multiplie it by.2.(whiche was the laste *quotiente* before)30.tymes. So muste I multiplie 81.by.60.and it will make.4860.whiche I place orderly.

Then set I doune the *Cube* of the *quotiente*, whiche maketh

maketh.729. And so are the.3.nombers placed, and agree with the former woorke, in all thinges, saue in 2.pointes. For here the triple of the *quotiente*, is not set doune, but kepte in memorie. And again, here are diuerse cyphers, whiche are not in the former worke.

Scholar. Sir, I perceiue, that the Cyphers dooe nothyng els, but set the nombers in their due places. And the triple of the *quotiente*, is supplied in worke by 2.multiplications. First by.300.and then by.30. So that it is all one in effecte.

And by the one worke, I vnderstande the other the better: when I compare theim bothe together. But yet I praie you, ende the woorke that you began.

Master. To continue that woorke, firste I must set doune the nombers, as thei should remaine, after 16389.is abated out of.18463. and then will thei stande thus.

```
 2074
26463592(29
```

Then shall I repeate the former worke, by settyng doune the triple of all the *quotiente*, whiche will be.87. and that must be placed vnder.45.

Nexte that I shall multiplie that.87. by.29. and there will come.2523. whiche must be the diuisor.

Wherfore I seke for a new *quotiente*, that maie shewe me how often.2523.is contained in.20745. And it will bee.8. That 8 doe I set in the *quotient* and by it I multiplie.2523. and it giueth.20184 whiche I sette doune, as here you see.

```
 2074
26463592(298.
    87
   2523
----------------
 20184
   5568
     512
----------------
 2074592.
```

Then doe I multiplie that *quotient* squarely, and that wil be 64. Whiche I shall multiplie by the triple, that is 87, and there will amounte.5568. to be set one place more toward the righte hande.

Last of all, I must take the *Cube* of. 8. that is. 512, and it shall bee sette yet one place more towarde the righte hande.

And then by additiõ, I shall bryng thẽ all into one nomber: and it will bee. 2074592. whiche is equall with the whole nomber aboue, that is vncancelled. And therfore if I abate the one out of the other, there will remain nothyng.

Wherefore I see, that the firste nomber, is a iuste *Cubike* nomber. And his roote is. 298.

Scholar. I haue marked you so well, that I truste to doe the like, without erroure.

But I praie you woorke this laste parte also, by your seconde rule, as you did woorke the other: that I maie see the due agremente of theim bothe: and also perceiue the righte vse of this woorke, the better by that other forme.

The seconde woorke.

Master. I muste in that case sette doune the nombers, as thei were set in the other woorke. And then I shall multiplie al the *quotiẽt*, whiche is. 29. by it self squarely, and it will make. 841. whiche must be multiplied by. 300. And so there amounteth. 252300. to be sette doune, as here you see.

2074
264635 92(29.

Then I shall seke out a *quotiente*, declaryng how often 252300. maie bee founde in 2074592. And that *quotiente* will bee. 8: whiche I set in the *quotient* roome, with the other nombers.

2074
264635 92(298.
252300
2018400
55680
512
2074592

And then I dooe multiplie the diuisor by the *quotiente*, and thereof riseth 2018400 whiche I set vnder a line, as you maie see.

Nexte that, I doe multiplie the newe *quotient*, by it

self,

self squarely, whereof commeth. 64. and that square of the last *quotient*, I shall multiplie by. 870. whiche is. 10. times the triple of the former *quotiente*. 29: and thereof commeth. 55680. whiche I set doune also orderly.

Laste of all, I multiplie. 8. (that is the laste *quotiente*) *Cubikely*, and it maketh. 512. whiche also I set doune in coueniēt order.

8
8
64
870
4480
512
55680.

And then shall I adde theim all together. And so haue I thesame somme, that I had before in the other former worke, and it is. 2074592.

Scholar. I neade no more instruction for this: I thinke my self so cunnyng, by occasion of your examples, whiche you haue wroughte so in double forme.

Master. That maie you proue, by this nomber 47832147.

Scholar. Firste I shall pricke it, as you taughte me, omittyng still. 2. nombers. *An other example.*

And then out of the nomber ouer the laste pricke, I shal seke out the *Cubike roote*, and abate the *Cube* thereof, out of thesame nomber, and set the remainer ouer it, cancellyng the reste.

And so in this nomber, I finde in. 47. the greateste *Cube* to bee. 27. and the roote of it 3. Wherfore I abate. 27. out of. 47. and finde the reste to be. 20. therfore I cancell. 47. and set. 20. ouer it. And the. 3. whiche is the roote, I set in the *quotient*. And so is the first worke eanded.

20
47832147(3

Then doe I triple that *quotiente*, and it maketh. 9. whiche I set doune vnder. 8.

Again I multiplie that. 9. by. 3. and it yeldeth. 27. whiche I set vnder the triple, and take it for my diuisor.

Wherefore I shall now seke a *quotiente*, that maie declare how often.27. is in.208 and I ſee, it will bee.7. tymes. Therfore I ſette doune.7. in the *quotiente*: and by it I multiplie 27 and it maketh.189. whiche I ſet vnder the line: and then I dooe multiplie.7. by it ſelf, whiche maketh.49.& that ſquare doe I multiplie by the triple of the former *quotiente*, that is, by.9. and it yeldeth. 441. whiche I ſet one place more toward the righte hande.

```
 20
47832147(37.
  9
 27
------------
189
  441
   343
------------
23653
```

Laſt of all, I take the *Cube* of.7. whiche is.343. and that doe I ſette doune, yet one place more toward the righte hande.

Theſe.3. ſommes beyng added together, doe make 23653.

Maſter. That will be hardely abated out of a leſſer ſomme.

Scholar. I ſee now my errour. I muſt take a leſſe *quotient*: whiche thyng I might haue perceiued by the ſeconde nomber. For thei twoo wer to greate, before the thirde was added.

So that I ſhould haue taken but.6. for the *quotiente* And then would the firſte nomber haue been but 162 and the ſeconde.324. and the thirde.216. but that their placyng would make theim to be of other values, ſaue the laſt of thē.

Therfore, I ſet euery one in his due roome: and adde theim together, and there amounteth 19656. to bee ſubtracted out of 20832. and the remainer will be 1176. And thus is that pricke with his woorke eanded.

```
 1
2ø176
47832147(36.
  9
 27
------------
162
 324
  216
------------
19656
```

Then for the nexte pricke, I repeate the ſame very forme

forme of worke again. First settyng doune the triple of the whole *quotiente*, whiche is. 108. so that it shall stande vnder. 11761. or vnder. 761. accoumptyng figure for figure.

That triple must I multiplie againe by the whole *quotiente*. 36. and it will make. 3888. whiche nomber I muste take for my diuisor.

Wherfore I seke how many times, I maie finde that diuisor in. 11761. and I see, it will bee. 3. tymes. Therfore I set. 3. as my *quotiente*, in his due place: and by that *quotient* I do multiplie. 3888. and so haue I for my firste nomber. 11664.

```
  1176
47832147363
   108
  3888
--------------
 11664
    972
     27
--------------
 1176147
```

Againe I doe multiplie the laste *quotiente*. 3. squarely, and so haue I. 9. whiche I shall multiplie by the triple of the former *quotient*, and it yeldeth. 972. that shall be set more nigher the right hande, by one place.

Thirdly, I take the *Cube* of. 3. whiche is. 27. and that doe I set yet one place more towarde the righte hande.

Then doe I adde those 3 sommes into one, and thei make. 1176147. whiche is equalle somme, with all the nombers ouer it, that be vncancelled.

Wherfore I saie that. 47832147. is a *Cubike nomber*, and the *Cubike roote* of it is. 363.

Master. Now doeth the order of teachynge require, that I should instructe you, how to extracte the nigheste *Cube roote*, out of any nomber, that is not a true *Cube*. As this nomber for example maie serue. 69458295I. — *The nigheste roote in a nõber not Cubike.*

Where firste I muste extracte the nigheste roote, as I taughte you, for the nigheste *Square rootes*, in nõbers that are not square: and then shall I note the re-maner: whiche I shall set for the numerator. And his denominator shall be founde, as I will tell you anon. But firste doe you worke the example, to his nigheste roote in whole nombers.

Scholar. I set it doune, and pricke it, and finde the greateste *Cube* ouer the laste pricke to bee 512. and the roote of it is. 8.

```
182
694582951(8.
```

Wherfore I set doune. 8. in the *quotiente*. And I abate. 512. out of. 694. and so resteth 182. and the former. 694. cancelled.

Then to procede, I must triple that roote. 8. and it maketh. 24. whiche. 24. I set vnder. 1825. And then I doe multiplie that again, by the *quotiente* or roote. 8 and it maketh 192. to be set vnder the saied triple. 24: as the diuisor. For whiche I seke a newe *quotient*, and it will be 8. That. 8. I set in the *quotiente* place, and by it I multiplie the diuisor. 192. and there riseth. 1536. to be set vnder the line, in conueniente order.

Nexte I multiplie the *quotiente* squarely: whiche yeldeth. 46. and that square I multiplie again by the triple, and so haue I. 1536. also. But this must stand more forwardly by one place.

Last of all I take the *Cube* of the *quotient*. 8. and that is. 512. whiche I set vnder the other twoo sommes, and that by one place more forwardly.

```
  13
182110
694582951(88.
  24
 192
------------
1536
 1536
   512
------------
169472
```

Now gatheryng all these. 3. sommes into one, thei will make 169472 whiche I shall abate out of. 182582. and so remaineth there. 13110. And that pricke with his worke eanded.

Therfore hauyng one other space to worke, I must repeate the same order of worke again: by triplyng the whole

quotiente

quotiente. 88. and that will bee. 264. And againe I must multiplie that tripled nomber, by the saied *quotiente*, and it will make. 23232. whiche shall bee the diuisor.

```
   1428
  131108826
 694582951(885.
      264
    23232
 116160
    6600
      125
 11682125
```

Wherfore I seke a newe *quotiente*, whiche is easily perceiued to be. 5. That. 5. doe I set in the *quotiente*, and by it I dooe multiplie the deuisor 23232. and there amounteth 116160. as the firste nomber, to bee set vnder the line.

Againe I shall multiplie the *quotient* squarely, whiche giueth. 25. and that square shall I multiplie by the triple. 264. and so will there rise. 6600. to bee sette, as the seconde nomber vnder the line: and one place more forwardly, towarde the righte hande.

Last of all, I shall sette vnder them bothe, and one place more towarde the righte hande, the *Cube* of. 5. whiche is. 125.

And then shall I adde all those. 3. sommes together of whiche commeth. 11682125. to bee abated out of 13110951. and so the remainer will bee. 1428826. Whereby I see, that the firste nomber that was proponed, I meane 694582951 is no *Cubike nomber*, but the greateste *Cube* in it is. 693154125. and his roote is. 885.

And so, I see, all other nombers of like kinde must bee wroughte.

But now for the remainer, how shall I dooe to bringe it vnto a fraction, that maie aptly expresse the righteste roote in that sorte?

Master. There bee as many waies, as there bee writers almoste, for euery manne deuiseth, how to

bryng

bringe it moste nighest to a true roote, if any soche were: whereof *Cardane* his rule is this.

Cardane.

Multiplie the roote squarely, and againe by 3. and that nomber shallbe the diuisor vnto the remainer,

Where he might haue vsed more plainesse in wordes, if he had saied: and that nomber shal be the denominator, to the remainer. Wherefore as here your roote is. 885 so is the square of it 783225 and the triple of that is. 2349675. So would that fraction bee $\frac{1428826}{2349675}$.

But how nigh this doeth go to the truthe, I leaue it till an other tyme.

Scheubell.

Scheubelius doeth allege an other reason, and inferreth an other order, diuerse frō this, and soche as impugneth this, saiyng:

Triple the roote, and the square of it also, and adde bothe those nombers together, and. 1. more: And so haue you a denominator for your numeratour.

The numerator euermore is vnderstād to be the remainer. By whiche meanes the fractiō in this worke would bee $\frac{1428826}{2352331}$: whiche is a lesser fraction by a good deale, then is the former fractiō, after *Cardanes* forme.

But bicause at this presente, I maie not spende so moche time, to scan their seueralle opinions, wherein eche of theim, pleaseth hymself well: the one alleging demonstration (whiche scarsely serueth) and the other namynge it a secrete, as it is worthie to bee: I will procede to a thirde waie, more certain then ether of these bothe. And that is by addition of certain Cyphers, to the remainer, in soche sorte, that thei muste all waies bee ternaries, as. 3. 6. 9. or. 12. &c. And then

searche

searche forward with the like order of worke, as you vsed before.

In this maner of practise, looke how many prickes your ciphers hath (or els how many ternaries of Ciphers, there bee set to your nomber) so many figures shall the numerator of your fraction contain. And the denominator shall euermore, contain 1. more. Wherof the laste onely shall bee an vnitie, and all the other shall bee Cyphers.

That is to saie, that if I adde but 3. Ciphers to the nomber, the fraction shall contain certain. 10. partes And if I adde. 6. Cyphers, it shall expresse. 100. partes. So. 9. Cyphers maketh the denominator to bee 1000. partes: And 12. Cyphers geueth 10000 partes.

For example. I will adde to our laste nomber that remained. 12. Cyphers. And then will the nomber be 1428826 000.000 000 000. vnto whiche I set no more prickes, then serueth for the ciphers, bicause I haue passed all the other prickes, in my former worke.

And now to continue my worke, I shall triple all the former *quotiente*, and it will be 2655. whiche nomber I shall place, as here you see it set. And then shall I multiplie that triple, by the former *quotiente*. 885. whiche will yelde. 2349675. to be set vnder the saied triple: as I haue sette it here also. And this nomber shall be the diuisor.

1428826 000000000000(
2655
2349675

Then shall I seeke for a *quotiente*, whiche can bee none other then. 6: wherefore I sette. 6. in a *quotiente* line, and by that. 6. I dooe multiplie the saied diuisor 2349675. and it giueth. 14098050. to be the firste nomber vnder the line.

After that, I take the square of the saied *quotiente*, whiche is. 36. and by it I multiplie the triple. 2655.

wherby is made 95580. to be the seconde nomber vnder the line: & set, as it ought, one place more toward ye righte hande.

18064984
1428826000000000000(8856.
2655
2349675
—————
14098050
95580
216
—————
1410761016

Last of all, for the thirde nomber I take the *Cube* of the saied *quotiente* whiche is. 216. and place it as you see, with his firste figure vnder the pricke.

Then doe I adde those. 3. nombers into one, whiche maketh. 1410761016. And that beyng subtracted out of 1428826000. doeth leaue 18064984. And so is the worke of the firste pricke eanded.

Whereby it appeareth, that the fraction is somewhat more then $\frac{6}{10}$ or $\frac{3}{5}$: as it shall bee tried better, by the workes that shall ensue.

Therefore I procede to the nexte pricke. And firste I triple that whole *quotiente*, whiche yeldeth. 26568. to bee set, as it is often before repeated, and therefore nedeth not hereafter to bee tediously rehearsed.

That triple shall I multiplie again, by the whole *quotiente* (as here I haue sette it in worke, bicause the nomber is greate, and not easily wroughte by memorie) and it doe I set in his due place, as you see.

18064984000000000(88560
26568
235286208

26568
8856
—————
159408
132840
212544
212544
—————
235286208

But then seeyng that diuisor is greater then all the nomber ouer it, I shall set a Cypher in the *quotiente*: in token that the diuisor, can not be abated ones out of the nomber ouer it.

it. And so is the woorke of that pricke canded, without any more trauell.

Wherfore to go forward, I triple all that *quotiente* and set it doune, as the rule would, and as here is seen.

```
  1594819256457
 18064984000000000 (885607.
         265680
  235286208000
 ─────────────────────────
 16470034560 0
          13018320
                343
 ─────────────────────────
 1647016474354 3
```

Then dooe I multiplie that triple, by the whole *quotiente*, whereof cōmeth. 235286208000. and that shall bee the diuisor. And the *quotiente* for it will be. 7.

So then if I multiplie that diuisor by. 7. there will amounte. 1647003456000. for the first nomber to be set vnder the line.

And for the nexte woorke, I shall multiplie. 49. (whiche is the square of the newe *quotiente*) with the triple of the former *quotiente*, and it will bryng forthe. 13018320. whiche shall bee the seconde nomber, to bee set vnder the line.

```
 265680
     49
 ───────
 2391120
106272
 ───────
13018320
```

The thirde nomber shall bee the *Cube* of. 7. whiche is. 343.

And those. 3. sommes added together, will make 1647016474354 3. whiche is to bee abated out of 18064984000000. and then shall there remain 1594819256457. And so haue I canded. 3. prickes of the Cyphers. And thereby maie saie, that the fraction is $\frac{607}{1000}$ and somewhat more: That is somewhat more then $\frac{3}{5}$.

Scholar. I see by the fraction, that it is $\frac{3}{5}$ and $\frac{7}{1000}$, beside the quantitie of the remainer. But I praie you cande the worke of that other parcke, whiche dooeth remaine.

Master. I muste triple all the *quotiente*: whereby will rise. 2656821. whiche muste bee multiplied by the saied *quotiente*: and thereof will proceade the diuisor, beyng 235289927 5347. And his *quotiente* will bee. 6.

2656821
885607
18597747
159409260
13284105
21254568
21254568
235289927 5347

Wherefore firste I set. 6. in in *quotiente* line, with the other nombers: and then doe I multiplie the diuisor by that *quotiente*, and it bryngeth foorthe 14117395652082. For the firste nomber to be sette vnder the line.

1830787347930 24	
159481925645 7000	(8856076.
2656821	
2352899275347	
14117395652082	
95645556	
216	
14117405216 63976	

And again the square of 6. beyng multiplied by the triple, will yelde. 95645556: whiche shall bee the seconde nomber vnder the line.

2656821
36
15940926
7970463
95645556.

The thirde nomber shall be. 216. bicause it is the *Cube* of. 6. And those 3. nombers beeyng added together, doe make. 14117405216 63976. to be abated out of. 159481925645 7000. And so doeth there remaine. 1830787347930 24.

Wherefore

Wherefore it doeth appeare, that beside the first. 3 nombers of the roote, that is. 885. the reste (that is 6076.) standeth for the numerator of a fraction, and the denominator unto it is. 10000.

So that the nighest roote is. $885\frac{6076}{10000}$. beside the fraction that doeth remaine: whiche would make but $\frac{1}{8}$ of $\frac{1}{10000}$.

Scholar. This is a sufficiente preciseness. And so I iudge it sufficiently taughte.

Wherefore I praie you propounde some questiōs, that doe require this arte, for their solution.

Master. I am contente. And let this be the firste.

The Grecians giuen to idle banketting, and soche like wantonnesse, did procure thereby soche mortalle sickenesses: that the quicke were scarse hable to burie the dedde. Wherefore consultynge with their Goddes, for redresse thereof, thei receiued aunswere, that when thei would double the Altare, whiche was of *Cubike* forme, thei should bee deliuered from that plague. Meanynge that learnyng is a due meane, to deliuer realmes from plagues and enormities. But to the question, what saie you? If the side of a *Cube* be. 3. foote (as that altare might bee) how many foote shall the side be of that *Cube*, whiche must be double unto it. *A question of doublyng a Cube.*

Scholar. This I cōsider. That firste I must finde the quantitie of the *Cube*, that is proponed. And then shall I double that quantitie. Thirdly, I muste extracte the *Cubike roote*, of that double nomber.

So in this question, the side of the knowen *Cube* is 3. and therfore the whole *Cube* is. 27. whose double is 54. And the *Cubike roote* is. 3. and $\frac{27}{27}$ by *Cardanes* rule: That is. 4. whiche is plainly false, for. 4. is the roote of. 64. and not of. 54. But by *Scheubelius* rule, it wil be. $3\frac{27}{37}$ that is. $3\frac{3}{4}$ almoste: whiche is moche nigher the truthe. For. $3\frac{3}{4}$ multiplied *Cubikely*, doeth make. $52\frac{47}{64}$. whiche is to litle by a good deale, that is by. $1\frac{17}{64}$.

whereas. $3\frac{27}{37}$ doeth make a lesser somme: that is to say but $51\frac{25959}{50653}$. and so wanteth. $2\frac{24694}{50653}$. And although bothe these sommes goe nigher to the truthe, then *Cardanes* rule, whiche misseth. 10. wholy: yet maie it be easily seen, that *Scheubelius* rule is not so good, as he would it were. And the worse here, for the addynge of that one more.

Master. You are lepte verie sodenly from a scholar, to a cōptroller. And yet I can not but praise your diligente obseruyng of soche thynges.

Proue now by the Cyphers, how it will frame.

Scholar. I sette doune the nomber with. 6. Cyphers, and pricke them thus.

```
27
54000000(3
```

Then doe I take the greatesse *Cubike nomber* in. 54. whiche is. 27 and that I doe abate from 54. and so resteth. 27. the roote of the *Cube* is. 3. whiche I sette in the *quotiente* line.

And then I triple. 3. whiche maketh. 9. that muste be multiplied by the *quotiente* againe, and so commeth 27. to be the diuisor. And his *quotiente* semeth to be. 9.

```
27
54000000(39
  9
 27
-----
243
 729
  729
-----
32319
```

Wherfore woorkyng with it, the firste nomber is. 243. and the seconde is. 729. that is. 81. multiplied by 9. whiche is the triple.

Againe, the *Cube* of. 9. is. 729. And all thei together, dooe make 32319 whiche seme is to greate, and therfore I must take a lesser *quotiente*. As I mighte haue perceiued well inough by the second nōber, if I had marked it in time.

```
27
54000000(38.
  9
 27
-----
216
 576
```

But now amendyng my ouersighte, I take. 8. for the *quotiente*. And woorkyng with it I see, the firste nomber vnder the line, will

be

bee.216. and the seconde.576. And here all ready I espie my ouersighte again.

Wherfore I take.7. to be the *quotiente*. And by it I multiplie the diuisor, and so haue I.189.for the firste nomber.

And for the seconde nomber, I doe worke with.49.whiche is the square of the *quotiente*, multiplied by.9. that is the triple: and it yeldeth.441.

Thirdly, I take the *Cube* of.7. whiche is.343. And then addynge al.3.nombers together, I finde the somme to bee.23653. whiche is to bee abated out of 27000.and so resteth 3347. Wherby I see, that.37. with somewhat more is the roote that I should finde.

3
27347
54000000(37.
9
27
189
441
343
23653

But for farther triall, I triple all the *quotiente*, and finde thereby.111. whiche I multiplie by thesame *quotiente* again, and so commeth 4107.to bee the diuisor. And his *quotiente* will bee.8.as it semeth: and so the first nomber will bee.32856. And the seconde shall bee.7104.but those.2.are to greate, as it is manifeste all readie.

3347000(378.
111
4107
32856
7104

Wherfore I take 7 for the *quotiente*. And by it multiplyng the diuisor, there riseth 28749.

And for the seconde somme, there is founde.5439.

And for the thirde some.343.

All whiche.3. sommes ioined in one, doe make.2929633. And that beeyng abated out of the higher somme.3347000. doeth leaue.417367.

417367
3347000(377.
111
4107
28749
5439
343
2929633

Wherfore

Wherefore I maie boldly saie, that the fraction is $\frac{77}{100}$ and more, by the portion of the remainer, whiche is nighe $\frac{1}{700}$.

And it is sone seen that $\frac{75}{100}$ are equalle to $\frac{3}{4}$: wherefore $\frac{77}{100}$ shall be more then $\frac{3}{4}$.

And so dooeth *Scheubelius* rule erre more, then I thought before.

So is your question aunswered, that the side of the double *Cube*, shall be. 3. foote and $\frac{77}{100}$ and $\frac{1}{7}$ of $\frac{1}{100}$.

Of the rootes of fractions. Master. For the rootes of fractions, I shall nede to saie no more but this: that if the numerator and denominator bothe be *Squares*, or *Cubes*. &c. then maie you finde in that fractiō the like roote. But if any of bothe doe swarue from that name, then hath that fraction no soche roote.

As $\frac{16}{27}$ is nother *Cubike* nor *Square*, bicause his partes dooe not agree in *Square* name, nor in *Cubike* name: although the numerator bee a *Square*, and the denominator a *Cube*.

Scholar. That dooeth appeare reasonable, at the the firste sighte.

Master. Then seeyng you are so readie in learnyng: aunswere me to this question.

A question of a Gonne. A Gonne of sixe inches diameter in the mouthe, dooeth shotte a bollet of twentie pound weighte: what weighte shall that bollette haue, that serueth for a gonne of. 14. inches in the mouthe?

But to helpe you in this question, and in all soche like, you shall marke well *Euclide* his saiyng, in the 18 proposition of his. 12. booke, whiche is this.

All Globes bere together triple that proportion, that their diameters doe

So in this example, the proportion of the *diameters* beyng as. 14. to. 6. Or as. 7. to. 3. I shall triple it, and then haue I the proportion of their Globes.

Wherefore

Wherefore I sette the.3.fractions thus. $\frac{7}{3}\,\frac{7}{3}\,\frac{7}{3}$ and thei make $\frac{343}{27}$. that is.12. $\frac{19}{27}$. And so is the proportion of the Globes, as well in weighte, as in bignesse.

Therfore I must multiplie. 20.that is the weight of the lesser bollette, by the numerator of the proportion, and diuide it by the denominator.

And so shall I haue. 254 $\frac{2}{27}$ for the weighte of the greater bollete.

Now prooue you the like woorke. Remembryng that *Cubes* also, as well as Globes, doe beare triple proportion, in comparison of their sides. As you learned before by the.19. proposition, of the.8. booke of *Euclide*.

```
 343        113
  20        2412
----        6860(254 2/27
6860        2777
             22
```

A *Cube* of Brasse of.4.inches square, doeth weighe 7.pounde weighte, what shall a *Cube* of Brasse of. 9. inches square, waie? *A question of.2.Cubes.*

Scholar. The proportion of the sides is as $\frac{9}{4}$ whiche I must set doune thrise, and multiplie them together, as fractions should bee. And so will it bee thus. $\frac{9}{4}\,\frac{9}{4}\,\frac{9}{4}$. that maketh. $\frac{729}{64}$.

Wherefore I multiplie the weighte of the lesser *Cube*, beyng.7.by.729.and it maketh.5103.and that doe I diuide by.64.and so finde I.79. $\frac{47}{64}$, whereby I maie knowe, that the weighte of the greater *Cube*, is 79.pounde weighte, and very nighe $\frac{3}{4}$.

Master. These.2.questions dooe teache you, rather the proportion of *Cubes*, then the vse of the rule: wherfore to make the questiõs more agreable to this rule, I propounde them thus, in backer order.

A bollette of yron of.7. inches *diameter*, doeth waie 27.pounde weighte: what shall be the *diameter* to that bollette that shall waie.125.pounde weighte?

Scholar. I praie you aunswer to it your self, that I maie see the apte forme of applyng soche questions to this rule.

Maister. As the *Cubes* are in triple proportion to the sides, so are the proportions of the sides, to bee founde by triple diuision: that is to saie, by seking the *Cubike rootes*, of the 2. termes of the proportion.

Wherefore I doe firste set doune the termes of the proportion of the bollettes, thus: $\frac{125}{27}$. And I see, that the *Cubike roote* of. 125. is. 5. and the like roote of. 27. is. 3. whiche nombers I shall set in the roome of the. 2 others, thus: $\frac{5}{3}$ And thei declare the proportion, betwene the *diameters* of the. 2. bollettes. Whereof one that is the lesser, is knowen to be. 7. Therfore I multiplie that. 7. by. 5. whereof commeth. 35. and that. 35. doe I diuide by. 3. whiche giueth. 11 $\frac{2}{3}$.

Wherfore I saie, that if. 7. inches bee the *diameter* to a bollette of. 27. pounde weighte, then. 11. inches and $\frac{2}{3}$ shall be the *diameter* to the bollete of. 125. poūde weighte.

Scholar. The proofe of this had nede bee certain, seeyng the woorke is obscure, to the common iudgemente.

The proofe. Maister. You saie well. And this is the very order of proofe for it. Multiplie bothe these rootes *Cubikely*. And if their *Cubes* be in soche proportiō as their waightes bee (that is to saie in this exāple as $\frac{125}{27}$) then is the woorke good: els not.

Scholar. That must neades bee so. And therefore will I proue it so in these nombers.

And for that eande, firste I multiplie. 7. *Cubikely*, and it giueth. 343. Then I multiplie. 11 $\frac{2}{3}$. *Cubikely*, and it maketh $\frac{42875}{27}$. But now seyng the one nomber is a fraction, I will for ease tourne the other into a fraction of the same denomination: and it will bee $\frac{9261}{27}$ in whiche. 2. fractions, the proportion muste cōsist betwene the numeratours. So that thei bothe beeyng diuided by one common nomber, muste come to this

fraction

fraction $\frac{125}{27}$.

And so I see it will be: for the lesser beyng diuided by.343. will yelde 27. And the greater diuided by the same. 343. will giue. 125. So that by triall, that worke is approued good.

Master. I will now proue your cunnynge, in a newe question, whiche Brasiers often tymes, haue occasion to vse: as thus.

I haue a dice of Brasse of. 64. vnces of Troye weighte, whose side is. 3. inches and $\frac{1}{2}$ and would haue an other dice of the same mettall of. 18. pounde weighte. *A question of weightes.*

My demaunde is: what shall be the side of the dice?

Scholar. This question must firste bee reduced to one kinde of denomination in the weightes, and then will it be more apte to be aunswered.

Wherefore I shall tourne. 18. pounde into vnces, multipliyng it by. 12. and it will be. 216.

And then I consider the proportion, that is betwene those. 2. nombers of weighte. 64. and. 216. and it is certainly. $3\frac{3}{8}$, or $\frac{27}{8}$ out of whiche proportion, I must extracte the *Cubike roote*, as I maie easily doone, seyng bothe the numerator and the denominator, are *Cubike nombers.*

And so is their roote $\frac{3}{2}$: whiche is the proportion of the sides of the twoo dice.

And seyng the side of the lesser die, is knowen to be 3. inches and $\frac{1}{2}$, the other his side must be in *Sesquialter* proportion to it, that is. $5\frac{1}{4}$: whiche is wroughte also thus. I multiplie. $3\frac{1}{2}$ by. 3. and it maketh. $10\frac{1}{2}$ whiche I shall diuide by. 2. and there commeth. $5\frac{1}{4}$.

Master. Yet one question more I will propounde to giue you occasion, to vnderstande the apte conference of masses, of diuerse stuffe.

And for that purpose, I suppose this proportion in weighte, to bee betwene masses of one biggenesse.

Examples of rates for weightes.

That if I compare Wodde and stone of one quantitie together, the stone shall weighe more then the wodde by $\frac{2}{3}$.

Likewaies yron to be heuier then stone by $\frac{1}{2}$.

And Brasse to bee heuier then Yron by $\frac{1}{3}$.

Stoffe.	Weighte.				
Wodde.	60.	1			
Stone.	100	$\frac{3}{5}$	1		
Yron.	150	$\frac{2}{5}$	$\frac{2}{3}$	1	
Brasse.	200	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Ledde.	280	$\frac{3}{14}$	$\frac{5}{14}$	$\frac{15}{28}$	$\frac{5}{7}$

Ledde to be heuier then Brasse by $\frac{2}{5}$.

All whiche rates, although thei be taken for examples, and not of truthe, yet thereby maie you learne, how to worke with true rates, set in a like table.

And now for the vse of this table, take this questiō.

A question of weighte.

I would haue . 5 . weightes of *Cubike* forme, made of these. 5. stuffes.

The weighte of the wodde shall be. 28. pounde.

The stone. 56. pounde.

The yron. 112. pounde.

The Brasse. 224. pounde.

And the Ledde. 448. pounde.

Of all these I haue but the Yron weighte: whose side, or *Cubike roote* is. 12. inches $\frac{2}{3}$.

And my desire is to knowe, of what quantitie the sides of all the other weightes shall bee.

Scholar. The question is pleasaunt: and yet some what harder then the other.

Master. The table will helpe you fully, so that you cōferre it well, with that you haue learned before

But bicause I haue litle leiser, to spende moche tyme with you (sauc that zeale to your furtheraunce doeth make me partly to forgette my owne businesse) therefore will I leaue this question to your self, to be aunswered at your laisure.

And so in all the rest, I must poste it ouer: and giue an iye to soche maters, that touche me more nighe:

and

and weighe more heuily, then all soche weightes, by 20. folde.

Wherfore, touchyng all the rootes of compounde nombers, you shall at my hand now, haue no priuate declaration. But soche as you haue learned all redie.

Of compounde rootes.

If the nomber bee compounde, other of *Square nombers*, or of *Cubike nombers*, then accordyngly as the cōposition is, so shal you drawe the roote: and without one of these two there can bee no composition.

Wherfore to begin with the smallest compounde nomber in that sorte, whiche is a *Square of squares*, you shall firste extracte the square roote, as you haue learned before. And out of that roote (whiche must nedes bee a *Square nomber*) you shall extracte his square roote also: and that roote is the *zenzizenzike roote*, of the firste *Square of squares* or *zenzizenzike nomber*. *Squares of Squares.*

For example take. 14641. whose *Square roote* is 121. and that same roote is it self, a *Square nomber*: and hath for his roote. 11.

2
14641(2916
224

Wherfore I maie saie, that. 11. is the *Squared square roote*, or the *zenzizenzike roote* of 14641.

Again 8503056. is a *Square of squares*, and therfore a *Square nomber*. And his *Square roote* is 2916. whiche is a *Square nomber* also, and hath. 54. for

4
2916(54
10

341
499893
8503056(2916.
45882
5

his roote.

So that.54.maie well bee called the *zenzizenzike roote* of.8503056.

And so shall you woorke, with all of that name.

Zenzizenzizenzikes

But and if the nomber be compounde, of.3.*zenzikes*, or.3.*Squares*, as a *Square of squared squares*, or a *zenzizenzizenzike* (whiche some men for shortnesse, call *zenzizenzenzike*). Then shall you drawe firste the *Square roote*, and then the *Square roote* of that roote, and thirdly the *Square roote* of that laste roote.

As for example.6561.is a *Square of squared squares*. And his firste roote is.81. whiche is also a *Square nomber*, and hath 9.for his roote. That.9.likewaies is a *Square nomber*, and hath.3.for his roote.

1
6561(81.
16

So that the *zenzizenzizenzike roote* of.6561.is.3.

And for these formes of nombers, I shall not nede to staie for any more explication, or examples: seeyng the mater is plaine.

Now for compounde *Cubike nombers*, you shall understande the like forme.

Cubes of cubes.

If the nomber bee a *Cube of Cubes*, you shall firste extracte the *Cubike roote*. And because that roote is a *Cubike nomber* also, therfore shall you seke the *Cubike roote* of it. And that seconde roote shall bee the *Cubicubike roote* of the firste nomber.

As for example.512.is a *Cubike nomber*, or a *Cube of Cubes*. And his *Cubike roote* is.8. whiche.8. againe is a *Cubike nomber* and hath.2.for his roote.

So that.2.is the *Cubicubike roote* of.512.

Likewaies.10077696.is a *Cubicubike nomber*, and his firste *Cubike roote* is.216. as you maie easily perceiue by these woorkes: where I haue sette forthe the order of extraction of his *Cubike roote*, whiche is.216. And that.216.is a *Cubike nomber*, you neade not to

doubte,

```
 816
10077696(216.
   63
 1323
-------------
 7938
  2268
    216
-------------
 816696
```

doubte, for that it is one of thē, which you haue, I dare saie, in perfecte memorie: Bicause his roote is a digite, and that is. 6.

```
  2816
10077696(21
 6
 12
----------
 1261
----------
 1261
```

By this you maie iudge of *Cubicubikes Cubikely*, or *Cubes of Cubicubes*, that in theim you shall firste seke their *Cubike roote*: And then the *Cubike roote* of that roote. And thirdly the *Cubike roote* of that roote againe. And so haue you the *Cubicubicubike roote* of that firste nomber. *Cubicubes Cubikely.*

The thirde waie of composition is, when *Squares* and *Cubes* be compounde together: as *Zenzicubes*, *Zenzizenzicubes*, *Zenzicubicubes*, or soche like, as it happeneth diuersely. *The thirde composition.*

In all these you shall as often abate the *Zenzike roote*, as that name is in the composition, and so likewaies of the *Cubike roote*.

So that in a *Zenzicubike*, you shall extracte firste the *Square roote*: and out of that *Square roote*, you shall extracte the *Cubike roote*. *Zēzicubike.*

As. 64. is a *Zenzicubike nomber*, whose *Square roote* is 8. and that. 8. is a *Cubike nomber*, and hath. 2. for his roote.

So. 531441. is a *Zēzizenzicube*: whose first *Square roote* is. 729. whiche nomber is a *Zenzicube*, & hath for his *Square roote*. 27. And that nōber is a *Cube*, and hath for his roote. 3. wherefore I maie iustly saie, that. 3. is the *Zenzizenzicubike roote* of. 531441. *Zenzizenzicube.*

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34
729(27
 4
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 144
 430
531441(729
 1444
  1
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But as I saied before, that I might not staie long at at this presente, so the vse of these greate nombers is rare in practise: and therefore I will ouerpasse them, for this tyme.

And yet for your aied in the meane season, I haue here drawen a table, whiche maie bee called the table of ease: in whiche you haue greate plentie of these nombers, with their rootes in diuerse kindes.

The table it self is so manifeste, that it neadeth no declaration: if you haue not forgotten, what you learned before.

And if you liste to enlarge this table, you maie easily doe it, multipliyng the nombers still by their rootes, whiche bee set ouer theim, in the hedde of the table. And so maie you make it to extende infinitely: whiche shall ease you wonderfully, in the extraction of any kinde of rootes. For which at some other time if my leisure serue me better, with quietnesse, I will giue you more specialle rules.

And also I councell you, well to examine this table, and trust not to my castynge. For haste and other troubles, maie often times cause erroure in supputation.

¶ The

The frutefulle table, whiche maie be called the table of ease.

1	*Rootes.*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	*Squares.*	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	441	484	529	576
3	*Cubes.*	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000	9261	10648	12167	13824
4	*Squares of squares.*	16	81	256	625	1296	2401	4096	6561	10000	14641	20736	28561	38416	50625	65536	83521	104976	130321	120000	194481	234256	279841	331776
5	*Sursolides.*	32	243	1024	3125	7776	16807	32768	59049	100000	161051	248832	371293	537824	759375	1048556	1419857	1889568	2476099	3600000	4084101	5153632	6436343	7962624
6	*Squares of Cubes.*	64	729	4096	15625	46656	117649	262144	531441	1000000	1771561	2985984	2826809	7529536	11390625	16777216	24137569	34012224	47045881	64000000	85766121	113379904	148035889	191102976
7	*Seconde Surselides.*	128	2187	16384	78125	279936	823543	2097152	4782969	10000000	19487171	35831808	62748517	105413504	170859375	268435456	410338673	612220032	893871739	1280000000	1801088541	2494357888	3404825447	4586471424
8	*Squares of squared sqres*	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000	214358881	429981696	815730721	1475789056	2562890625	4294967296	6975757441	11019960576	16983563041					
9	*Cubes of Cubes.*	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000	2359157691	5159780352	10604499373											
10	*Squares of Sursolides.*	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401															
11	*C. Sursoldes.*	2048	177147	4194304	48828125	362777056	1977326743	8589934592																
12	*Squares of zenzicubes.*	4096	531441	16777216	244140625	217666236																		
13	*D. Sursolides.*	8192	1594323	67108864	1220703125	13059974016																		
14	*Squares of Bsursolides.*	16384	4782969	268435456	6103515625																			
15	*Cubes of Sursolides.*	32768	14348907	1073741824																				
16	*Zezizēzizēzizēzikes*	65536	43046721	4294967296																				
17	*Esursolides.*	131072	129140163																					
18	*Squares of Cubicubes.*	262144	387420489																					
19	*Fsursolides.*	524288	1162261467																					
20	*Zenzizenzisursolides.*	1048576	3486784401																					
21	*Cubes of Bsursolides.*	2097152																						
22	*Squares of Csursolides.*	4194304																						
23	*G sursolides.*	8388608																						
24	*Zenzizenzizēzicubes*	16777216																						

1.	*Rootes.*	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
2.	*Squares.*	625	676	729	784	841	900	961	1024	1089	1156	1225	1296	1369	1444	1521	1600
3.	*Cubes.*	15625	17576	19683	21952	24389	27000	29791	32768	35937	39304	42875	46656	50623	54872	59319	64000
4.	*Squares of Squares.*	390625	456976	531441	614656	707281	810000	923521	1048576	1185921	1336336	1500625	1679616	1874161	2085136	2313441	2560000
5.	*Sursolides*	9765625	11881376	12448907	17210368	20511149	24300000	28629151	33554432	39135393	45435424	52521875	60466176	69343957	79235168	90224199	102400000
6.	*Zenzicubes.*	244140625	308915776	336120489	481890304	594823321	729000000	887503681	1073741824	1291467969	1544804416	1838265725	2176782336	2565726009	3010936384	3518743761	4096000000
7.	*Bsursolides.*	6103515625	8031810176	9075253103	13492928512	17249876309	21870090000	27512614111									

R.j.

Of nombers denominate.

Thus haue I lightly ouer run the moste common kindes of nombers *Abstracte.* And now resteth the treatice of nombers *Contracte*, or *Denominate.* Of whiche kinde there bee some called *nombers denominate vulgarely*: and other bee called nombers *denominate Coßikely.* And a thirde sorte there is of nombers *radicalle*, whiche commonly bee called *nombers irrationalle*: bicause many of theim are soche, as can not bee expressed, by common nombers *Abstracte*, nother by any certain rationalle nomber. Other men call them more aptly *Surde nombers.* *Nombers contracte.*

And although many menne would not accoumpte them, with nombers *denominate*, yet I maie iustly doe it, for that thei require a reduction to one denomination, if thei haue seueralle signes of quãtities, as you shall heare hereafter. And those nombers neuer goe alone, without some other signe, and name of rooted quantitie, annexed to theim.

Of the first kinde of nombers denominate, whiche are vulgarely denominate, as. 10. shillinges. 10. men 20. shippes, 100. shepe. 1000. yeres, and soche like, I will speake nothyng in this treatice. But of the other twoo kindes I will somewhat write, for youre learnyng and contentation.

Scholar. Sir, I am moche bounde vnto you: And therefore remit all to your owne discretion and good will. Trustynge so to applie my studie, and emploie my knowlege, that it shall neuer repente you, of your curtesie in this behalfe.

Master. Then marke well my wordes, and you shall perceiue, that I will vse as moche plainesse, as I maie, in teachyng: And therfore will beginne with *Coßike* nombers first.

The Arte Of Coßike nombers.

Nombers *Coßike*, are soche as bee contracte unto a denomination of some *Coßike* signe as 1. nomber. 1. roote. 1. square 1. Cube. &c.

But as for cōpendiousnesse in the use of theim, there bee certain figures set for to signifie them: so I thinke it good to expresse unto you those figures, before wee enter any farther, to thintente we maie procede allwaies in certentie, and knowe the thynges that wee intermedle withall: for thei are the signes of all the arte, that foloweth here to be taught.

And although there be many kindes of irrationall nombers, yet those figures that serue in *Coßike nōbers*, bee the figures also of all irrtionalle nombers, and therfore being ones well knowen, thei serue in bothe places commodiously.

These therfore be their signes, and significations briefly touched: for their nature is partly declared before.

φ. Betokeneth nomber absolute: as if it had no signe.

ꝛ. Signifieth the roote of any nomber.

ʒ. Representeth a square nomber.

ᶜ. Expresseth a Cubike nomber.

ʒʒ. Is the signe of a square of squares, or Zenzizenzike.

ſʒ. Standeth for a Sursolide.

ʒᶜ. Doeth signifie a Zenzicubike, or a square of Cubes.

bſʒ. Doeth betoken a seconde Sursolide.

ʒʒʒ. Doeth represent a square of squares squared

by,

ly, or a *zenzizenzizenzike.*

&&. Signifieth a *Cube* of *Cubes.*
ȝ·ſȝ·. Expresseth a *Square* of *Surſolides.*
cſȝ·. Betokeneth a thirde *Surſolide.*
ȝ·ȝ·&. Representeth a *Square* of *Squared Cubes*: or a *Zenzizenzicubike.*
Dſȝ·. Standeth for a fourthe *Surſolide.*
ȝ·Sbȝ·. Is the signe of a *ſquare* of seconde *Surſolides*
&ſȝ·. Signifieth a *Cube* of *Surſolides.*
ȝ·ȝ·ȝ·ȝ·. Betokeneth a *Square* oſ *ſquares*, squaredly squared.
Eſȝ·. Is the firste *Surſolide.*
ȝ·&&. Expresseth a squa.e of *Cubike Cubes.*
Fſȝ·. Is the sixte *Surſolide.*
ȝ·ȝ·ſȝ·. Doeth represente a square of squared surſolides.
&bſȝ·. Standeth for a Cube of seconde *Surſolides.*
ȝ·cſȝ·. Is a square of thirde *Surſolides.*
gſȝ·. Doeth betoken the seuenthe *Surſolide.*
ȝ·ȝ·ȝ·&. Signifieth a square of squares, of squared Cubes.

And though I maie proceade infinitely in this sorte, yet I thinke it shall be a rare chaunce, that you shall nede this moche: and therfore this maie suffice. Notwithstandynge, I will anon tell you, how you maie cõtinue these nombers, by progression, as farre as you liste.

And farther you shal understande, that many men doe euer more call square nombers *zenzikes*, as a shorter and apter name, other men call those squares the *firste quantities*, and the *cubes* thei call *seconde quantities*: squares of squares thei call *thirde quantities*, and surſolides *fourthe quantities.* And so namyng them all quantities (excepte nombers and rootes) thei doe adde to them for a difference, an ordinall name of nomber, as thei doe goe in order successiuely.

 As

As here foloweth in example.

ℨ.	Firste.	*Quantities.*	And so forthe, of as many as maie bee reckened.
ȼ.	Seconde.		
ℨℨ.	Thirde.		
ʃℨ.	Fourthe.		
ℨȼ.	Fifte.		
bʃℨ.	Sixte		
ℨℨℨ.	Seuenthe.		
ȼȼ.	Eighte.		
ℨʃℨ.	Nineth.		
cʃℨ.	Tenthe.		
ℨℨȼ.	Eleuenthe		
dʃℨ.	Twelfthe.		

But althoughe somemen accompte this the more easie waie: bicause the other names be comberouse, yet those other names before, do expresse the qualitie of the nomber, better then these later names doe.

Scholar. I thanke you double, sith you are contente to teache me double names: for so shall I be acquainted with bothe formes, as I shall chaunce on them in other mennes bookes.

Therfore now you maie proceade to numeration: whiche I thinke it nexte.

Master. There be other. 2. signes in often vse, of whiche the firste is made thus —+— and betokeneth more: the other is thus made ——— and betokeneth lesse.

And where thei come in any nomber *Cossike*, or other, that nomber is called a compounde nomber, bicause it consisteth of. 2. nombers. And where neither of theim is, the nomber is called vncompounde, although the signe be compounde. For the compounde signe, maketh not a compounde nomber. And now I will procede to numeration.

Of Numeration in nombers Cossike, vncompounde.

Master.

Nombers *Cossike* vncompounde, haue no difficultie in their numeration: for euer more the nōber representeth, so many of that *Cossike* denominatiō (be thei nōbers, rootes, squares, Cubes, squares of squa res, or any other like) as ther be vnities in that nōber *Numeration*

So. 6.℈. is.6. nombers: And. 6. ꝛ. is.6. rootes: 20.ʒ. is. 20. squares: 30.&. betokeneth. 30. Cubes.

Scholar. I see it well. For by this nōber. 20.ʒ. is not appointed any nōber absolute, of one certaintie, but onely so many *quātities* of that kinde: whiche maie bee. 80. if. 4. be one square. And if. 9. bee one square, then 20. squares make 180. And if. 25. be one of those squares thereby represented, then. 20. squares make 500. And as for the signes, you taught me thē before.

Of Addition.

Master.

This numeration is so plaine, that wee maie passe from it vnto addition: whiche is as easie also, if the quantities be of one denomination. For then nedeth no more, but to adde the nombers together, and to put that same common *Cossike* denomination, to the totall thereof. *Addition of like signes.*

Scholar. I take it thus, 20. ꝛ. added to. 30. ꝛ. will make. 50. ꝛ. And. 12. ʒ. added to. 16. ʒ. bringeth forthe. 28 ʒ.

Master. As you doe easily see al the mater of this addition, so maie you as easily conceiue, all the worke of subtractiō. For it is wrought as in vulgare nōbers *Subtraction of like signes*

Scholar. Then if I abate .6. ℭ. out of .10. ℭ. there will reste 4. ℭ. And so .9. ȝȝ. out of .25. ȝȝ. doeth leaue .16. ȝȝ.

Master. This is all for nombers of like signes *Cossike*.

Scholar. What then if I would adde .10. ꝛ. to 6. ȝ.? where the signes bee vnlike: maie it be doen? seyng thei be not of one denominatiō, nor signe *Cossike*.

Addition of vnlike signes

Master. As well as shillynges maie bee added with poundes, or pennies: and in like forme.

For thei shall stand still as thei wer, with the signe of addition, whiche is this. —+—. & betokeneth more.

So that .10. ꝛ. put to .6. ȝ. maketh .6. ȝ. —+— 10. ꝛ. that is .6. ȝ. more .10. ꝛ. or .6. ȝ. and .10. ꝛ.

Scholar. And why not .10. ꝛ —+— 6. ȝ.?

Master. Bicause it is moste orderly, to sette the greateste signe *Cossike*, formoste in order.

As you saie .20. shillynges, and .6. pennies: rather then .6. pennies and .20. shillynges.

Scholar. Then I se, if .15. ℭ. be added to .18. ȝȝ. it will make .18. ȝȝ. —+— .15. ℭ. And so .12. ſȝ. ioyned with .20. ȝℭ. dooe make .20. ȝℭ. —+— 12. ſȝ.

Of Subtraction.

Master.

Subtractiō of vnlike signes

S*ubtraction* is as easie: for it doeth depend onely of the signe of abatemente, which is this. ——, and signifieth lesse, or abatyng. And therefore if I would abate 6. ꝛ. out of .10. ȝ. I must sette it thus 10. ȝ. —— .6. ꝛ: that is to saie .10. ȝ. lesse .6. ꝛ. or abatyng .6. ꝛ.

Scholar. Then if I haue 30 ℭ. and would abate out of thē .12. φ. I must set it thus .30. ℭ. —— .12 φ. that is .30. *cubes* saue .12. nombers. And if multiplica-

tion

tion and diuision, bee as easie, thei shall neade no greate studie.

Of Multiplication.

Master.

Somewhat more laboure is there in multiplication and diuision, to finde out the newe signes as I wil tell you anon. But for findyng of the nombers, the common multiplication and diuision doeth serue. *Multiplication.* So that when. 12. ȝ. is multiplied by. 6. ℞. it maketh. 72. ∁. And if. 24. ∁. bee multiplied by. 5. ȝ. there riseth. 120. ſȝ.

Scholar. This passeth my cunnynge, for the findyng of the newe signe: although the multiplication of the nombers, be as easie as can be.

Master. If you did well remēber, what you haue learned before: the mater would not seme so harde. Doe not you knowe, that a roote multiplied by a roote, doeth make a square? And a square multiplied by his roote, doeth bryng forthe a cube?

Scholar. That I knowe right well: and therfore a *Square of Squares* multiplied by his roote, will yelde a *Surſolide*.

Master. Then by like reason, a *Cube* multiplied by a *Square*, shall make a *Surſolide*.

Scholar. In deede it is all one, to multiplie a *cube* by a *Square*, and a *Square of Squares* by a roote.

Master. Then for a generalle rule, I will sette forthe here a presidente for you: whereby you maie knowe the newe signe, in all multiplication or diuision: not onely by sight very spedily, but that you maie also commit it aptly to memorie.

Wherfore marke wel this table folowing: where you see in the higher rowe, a line of nombers, set in

naturall

naturall progreſſion : and vnder them you ſee the ſignes of *coſſike nombers*.

The table of Coſſike ſignes,
and their peculier nombers.

0.	1.	2.	3.	4.	5.	6.
φ.	ꝛ.	ʒ.	&.	ʒʒ.	ſʒ.	ʒ&
7.	8.	9.	10.	11.	12.	13.
bſʒ.	ʒʒʒ.	&&.	ʒſʒ.	cſʒ.	ʒʒ&.	dſʒ.

This table is largely ſet forthe, in the title of progreſſion, whereunto you maie haue recourſe, if your nomber be to greate for this table.

By this table maie you eaſily knowe , the ſigne that ſhall ſerue for your newe ſomme, in multiplication.

As for example, if I dooe multiple ſquares by rootes : I looke in the table , what nombers ſtande ouer them bothe, and puttyng thoſe.2.nombers together, I ſeke the totall in the ſame line, and vnder it I finde the newe denomination *coſſike*, whiche I ſhould haue

Scholar. I perceiue ouer.ꝛ.the nomber of 1.and ouer . ʒ . the nomber . 2 . whiche bothe added together make. 3. And bicauſe vnder.3.I find the figure or ſigne of.&. I muſte take that for the newe denomination.

Maſter. You ſaie truthe.

Scholar. Then if I multiplie.12.ʒ&.by.8.&. the ſomme will be.96.&&. For ouer. &. I finde 3.and ouer.ʒ&.ſtandeth.6. whiche bothe together doe make.9.and vnder.9.I ſee.&&.whiche I take for the denominator.

And if the ſame rule bee generall, I am cunnynge

inoughe

ynoughe in it.

Master. It is generall, for multiplication in this kinde.

Of Diuision.

But for diuision, you muste abate the one nomber out of the other, to finde a newe denomination. *Diuision.*

Therfore if you would diuide 96.ȼȼ by.8.ȼ. the *quotiente* will be 12.ℨ.ȼ. bicause that ouer the signe of your diuidende, standeth 9. And ouer the diuisors signe is set 3. Wherfore abatyng.3.from.9. there resteth. 6. vnder whiche is the signe.ℨȼ. that I must take, to put to my *quotiente.*

Scholar. Then for an other triall, if I would diuide.260.eſℨ. by.5.ſℨ. the *quotiēt* will be 52.ℨℨȼ. For bicause that ouer.eſℨ. I finde.17. and ouer.ſℨ. standeth.5. then subtractyng.5.frō.17. there resteth.12 vnder whiche in the table I finde. ℨℨȼ.

So diuidyng.20.ȼ.by.4.ꝛ. the *quotiente* will bee 5.ȼ: and so of other.

Master. But and if you would diuide.12.ȼ. by 5.ℨ. that must be set in forme of fraction, thus. $\frac{12.ȼ}{5.ℨ}$.

So.18.ℨ. by.7.ꝛ. maketh. $\frac{18.ℨ}{7.ꝛ}$ and 6.ℨ. by.2.ȼ. yeldeth. $\frac{6.ℨ}{2.ȼ}$. of whiche fractions, wee will speake emongeste the fractions of *Coßikes* compoūde. For thei degenerate out of this kinde.

Wherfore this maie suffice briefly, for the customable woorkes of whole *Coßike nombers.*

Of Fractions in Coßike nombers.

And as for fractions, the woorkyng is like in euery poincte, vnto the worke of nombers *Abſtracte*: remembryng onely that as those broken nombers, haue a *Coßike* denomination annexed with them, so must *Of fractions in nombers Coßike.*

that denomination followe the rules, now laste declared.

Wherefore I shall not nede to doe any more, but to set forthe onely certain examples, of euery kinde of worke in them.

Examples of Numeration.

$\frac{2}{5}$ *ze*. Signifieth $\frac{2}{5}$ of a *Roote*.
$\frac{8}{9}$ ʒ. Betokeneth $\frac{8}{9}$ of a *Square*.
$\frac{12}{17}$ ꝣ. Representeth $\frac{12}{17}$ of a *Cube*.

And so of all other formes of *Cosike* signes: whereby is intended, that the *Cosike quantitie*, is diuided into so many partes, as the denominator containeth, and there is here represented onely so many of them, as the numerator doeth importe.

Scholar. Hereby I dooe perceiue, that a fraction *Cosike*, maie signifie a nomber, and not onely a parte of an vnitie, as it did in nombers *Abstracte*.

For when I saie $\frac{2}{3}$ ʒ. if that *Square* be. 9. then that fraction signifieth. 6. But if the *Square* be. 4. then that fraction doeth represente. $2\frac{2}{3}$.

Likewaies $\frac{3}{4}$ ꝣ. if the *Cube* be. 8. then that fraction doeth signifie. 6. But if the *Cube* be. 27. then that fraction is equalle to. 20. $\frac{1}{4}$.

Master. You doe consider it well.

Of Addition.

Addition. Now for addition, take these examples.

$\frac{2}{3}$ ʒ. added to $\frac{3}{4}$ ʒ. doe make $\frac{17}{12}$ ʒ. or. 1 ʒ $\frac{5}{12}$.

$\frac{5}{7}$ ꝣ ioined with $\frac{7}{8}$ ꝣ. doe make $\frac{89}{56}$ ꝣ. or 1. ꝣ $\frac{33}{56}$.

And in vnlike signes.

$\frac{3}{4}$ ʒ. added to $\frac{4}{5}$ ꝣ. doe make $\frac{4}{5}$ ꝣ. + $\frac{3}{4}$ ʒ. or els thus by one common denominator. $\frac{16. ꝣ + 15. ʒ.}{20.}$

Of

Of whiche I will speake more in the *Binomialles*, and therefore will omitte it, till we come to them.

Scholar. As for the reste, I see it well: For the worke is all one with fractions *Abstracte*.

And here the denominatiō or *Coßike* signe is not varied, although here be vsed diuerse multiplications.

Master. And good reason: for the whole *quotiente* whiche is represented by that *Coßike* signe, is not multiplied, but certaine partes of it: and therefore oughte that *Coßike* signe, to stand vnaltered, as the quantitie represented by it, is not multiplied nor altered.

Examples of Subtraction.

$\frac{2}{3}$ ȼ. abated out of $\frac{3}{4}$ ȼ. doe leaue $\frac{1}{12}$ ȼ.

$\frac{5}{9}$ ȝ. out of $\frac{7}{8}$ ȝ. there resteth $\frac{23}{72}$ ȝ.

$\frac{8}{15}$ ȝȝ. subtracted frō $\frac{4}{5}$ ȝȝ doe leaue $\frac{20}{75}$ ȝȝ, or $\frac{4}{15}$ ȝȝ.

And in vnlike signes.

$\frac{3}{8}$ ȝȼ abated frō $\frac{6}{11}$ ȼȼ doe leue $\frac{6}{11}$ ȼȼ ——— $\frac{3}{8}$ ȝȼ.

$\frac{3}{4}$ ȝ taken out of $\frac{9}{13}$ ȼ. the reste is $\frac{9}{13}$ ȼ ——— $\frac{3}{4}$ ȝ.

Likewaies as in additiō, so in this sorte of subtraction, there maie be an other kinde of worke, whiche I will remit to the treatice of *Binomialles*.

Examples of Multiplication.

$\frac{3}{5}$ ȝ multiplied by $\frac{3}{8}$ ȝ. doe make $\frac{9}{40}$ ȝȝ.

$\frac{5}{7}$ ꝛ. multiplied by $\frac{8}{9}$ ꝛ. bryngeth forthe. $\frac{40}{63}$ ȝ.

$\frac{12}{19}$ ȝ. multiplied by $\frac{2}{3}$ ȝ. doe yelde $\frac{24}{57}$ ȝȝ, or $\frac{8}{19}$ ȝȝ.

Here the signes doe alter, as in the multiplication of whole *Coßike* nombers.

Scholar. This doeth somewhat trouble me: that the *Cossike* signes should chaunge here, rather then in addition, or subtraction: Seyng there was as moche multiplication, in any of them bothe, as there is here.

Master. Marke the mater well, and you shall bee sone satisfied.

For in addition and subtraction, the multiplicatiõ serueth onely for the reduction, of the. 2. fractions, unto one denomination: And therefore in them, you neuer multiplie the numeratores together: but you multiplie crosse waies, the numerator of the one, by the denominator of the other, where as in multiplicatiõ, you use no reduction, but doe make a plaine multiplication.

And so likewaies in diuisiõ, there is used no meane of reduction: and therefore in it the signes must alter, as before is declared.

Examples of Diuision.

$\frac{6}{7}$ ʒʒ. diuided by $\frac{6}{11}$ ʒ. doe make in the *quotiente* $\frac{66}{42}$ ʒ. or $\frac{11}{7}$ ʒ.

$\frac{7}{9}$ ℭ. diuided by $\frac{8}{15}$ ℭ. doeth yelde $\frac{105}{72}$ ℘. or els $\frac{35}{24}$.

For seyng I shall diuide. ℭ. by. ℭ. I must therefore abate. 3. from. 3. and so resteth nothing, whiche is signified by this Cipher. o. and that standeth ouer the signe of nomber: therefore the fraction, that is as the *quotiente*, must be taken as a nomber *Abstracte*.

Likewaies $\frac{8}{3}$ ʒʒ. diuided by $\frac{3}{9}$ ʒʒ. doeth make $\frac{72}{24}$ ℘. that is to saie. 3. And so $\frac{8}{15}$ ℭℭ. diuided by $\frac{9}{10}$ ℭ doeth bryng forthe $\frac{80}{135}$ ʒℭ. or $\frac{16}{27}$ ʒℭ.

Scholar. This is sufficiente for diuision. Now if you thinke good to speake of progression, I can not but remember you of your promise.

Of

Of Reduction.

Master.

Although *Reduction* should go in order before *Progression*, yet seeyng this *Reduction*, consisteth in the onely nombers, and not in the signes: and therefore agreeth with vulgare reduction of fractions (as here you maie see before in diuerse examples) therfore wil we omitte it, and go in hande with *Progression*: whiche is more straunge. *Reduction.*

Scholar. I praie you so: For I see this reduction, is but to reduce the greater fraction, to a lesser in nomber: as I learned long a gone by your other booke.

Of Progression in Cosike signes.

Master.

Progression is thus wroughte: Firste sette doune as many vulgare nōbers, in their naturall progression, as you liste to haue *Cosike* signes, that by them you maie the better know, the true places of the *Cosike* signes: so that you set in the firste place a Cipher, and vnder it. φ. And then vnder. 1. set. ℞. vnder. 2. put. ℨ. and vnder. 3. write. ℭ. As you see in the table folowyng. And by these shall you set, as many as you liste.

For all the vulgare nombers, whiche you haue set in the higher rewe, be other compounde nombers, or els vncompounde: and if the place, where you would set any *Cosike* signe, be noted with a nomber vncompounde, then must there be set one of the *Sursolides*.

For vnder the first nōber vncompounde, you must set the firste *Sursolide*, and the second vnder the second nomber vncōpounde: and the thirde vnder the thirde,

 and

and so forthe.

The nombers vncompoũde, are these in their pro-gression.

5. 7. 11. 13. 17. 19. 23. 29. 31. 37. 41.
43. 47. 53. 59. 61. 67. &c.

Vnder the. 5. must you set. /ʒ. and vnder. 7. *b*/ʒ. vnder. 11. *c*/ʒ. and vnder. 13. *d*/ʒ. and so foorthe, til you come to. 67. vnder whiche you must set. *r*/ʒ. and vnder 71 you must set *S*/ʒ. and so as farre as you list.

But for any other place, bicause the vulgare nomber is compounde, that is set (as the peculiare nomber, in the higher rewe) therefore the *Cossike* signe must nedes be compounde, other of. 2. or of. 3. or els of bothe. And if it be cõpounde of. 2. then set doune. ʒ. so often tymes, as. 2. is in the cõposition of that nomber.

As for example: 16. is cõpounde of. 2. fower tymes (not by addition, but by multiplication, as in saiyng, twise. 2. twoo tymes, twise.

Scholar. I perceiue twise. 2. to bee. 4. and twise that to be. 8. and twise that to make. 16.

Master. So maie you worke backewarde, in saiyng. 16. diuided by. 2. maketh. 8. that is ones: then. 8. by. 2. yeldeth. 4. that is twise. Again. 4. by. 2. maketh 2. that is thrise: and. 2. for himself, is the fourth: wherfore vnder. 16. I must set doune. ʒʒʒʒ.

And so vnder. 32. I muste sette. 5. ʒ. in one thus. ʒʒʒʒʒ.

And vnder. 64. I shall sette it. 6. tymes, thus. ʒʒʒʒʒʒ. Bicause. 64. is made of. 6. multiplications by. 2.

Scholar. Here by I see, that vnder. 8. I muste put 3. tymes that signe: and vnder. 4. twise the same.

Master. So must you in deede.

And now for other places, if their nombers bee cõ-

pounde

pounde of.3.onely, then must you set doune the signe of *Cube*, as oftentymes as . 3 . is multiplied, to make that nomber.

As for example. 27 . is compounde onely of.3.and not of.2.(for of all other compounde nombers herein then of soche as be cōpounde of.2.or.3. we take no regarde.) And. 3. multiplied thrise, doeth make . 27. in saiyng.3. tymes.3. thrise. And therefore under. 27. I shall set this signe of.ꝣ.three times, thus.ꝣꝣꝣ. whiche betokeneth a *Cube of Cubes Cubikely*.

But and if the nomber bee compounde bothe of. 2. and. 3 . then for euery tyme that. 2. is multiplied , to that composition, I shall sette.ʒ. and for euery tyme that.3.is multiplied, I shall set.ꝣ.remembryng still to set.ʒ.before.ꝣ.and not after hym.

As for example. Under.24.I shall set.ʒʒʒꝣ. bicause that.2.2.2. 3. that is to saie.2. tymes.2.twise thrise,doeth make.24. Or by resolution,thus.24.diuided by.2.giueth.12. For that firste.2.set.ʒ. Again 12.diuided by.2.yeldeth.6: for this seconde.2. set.ʒ. also. Then diuide.6.by.2.and it maketh.3. For the.2. I must set.ʒ. and for.3. I must put.ꝣ.and so all together maketh.ʒʒʒꝣ.in the.24.place.

Likewaies under.36.I must sette.ʒʒꝣꝣ.bicause that. 2.2.3.3. doeth make it,that is.2. tymes.2. thrise,thrise. And by resolution, thus.36.diuided by 2.giueth.18. For that.2.I set.ʒ. Againe.18.diuided by.2.maketh.9. For that.2.I sette doune againe.ʒ. Thirdly, for bicause.9. can not bee diuided by.2. but by.3,3.tymes:therefore I muste sette doune, for those twoo.3.twise.ꝣ,& so the whole signe is. ʒʒꝣꝣ.

Now if the nomber of the place,or peculiare nomber,bee compounde of one of theim twoo, with some other nomber uncōpounde, then must we ioyne their signes together.

As.10.is compounde of.2.and.5. therefore must I

set

ſet vnder. 10. the ſigne that is in the fifth place, whiche is ſʒ. and before it I muſte ſet the ſigne of. ʒ. for 2. So muſt that ſigne be. ʒſʒ.

Likewaies, bicauſe. 15. is compounde of. 3. and. 5. I ſhall ioine together the ſigne of ¢. and of. ſʒ. and make it. ¢ſʒ.

Scholar. So I vnderſtande it now, that I cannot miſſe it. Saue that for lacke of vſe, and throughe forgetfulneſſe, when I heare the name of compoſition in nombers, I doe miſtake it ſometimes for addition, els here can be no erroure. For when I doe conſider, that. 20. is compounde of. 2. 2. 5. that is twiſe. 2. and. 5 (ſith. 2. tymes. 2. maketh. 4: and. 5. tymes. 4. maketh 20.) I maie ſone conſider, to ſet. ʒ. twiſe before. ſʒ. and then it will be. ʒʒſʒ. to be put in the. 20. place.

Likewaies in the. 21. place, I ſet. ¢bſʒ. ſeyng 21 is compounde of. 3. and. 7. and. ¢. is the ſigne to the thirde place, as bſʒ. ſerueth for the. 7. place.

Maſter. What ſhall you ſet in the. 84. place?

Scholar. 84. is compounde of. 2. 2. 3. 7. therefore his ſigne muſt be. ʒʒ¢bſʒ.

Maſter. Now I ſee, you are cunnyng inough in this, and therefore take here this table, for a patrone: and then will we procede to the worke of *Coſsike nombers* compounde.

The table for progreſsion Coſsike, whiche maie increaſe it ſelf infinitely, without any diffiultie.

0.	℞.
1.	ꝛ.
2.	ʒ.
3.	&.
4.	ʒʒ.
5.	ʃʒ.
6.	ʒ&.
7.	bʃʒ.
8.	ʒʒʒ
9.	&&
10.	ʒʃʒ
11.	cʃʒ
12.	ʒʒ&
13.	Dʃʒ.
14.	ʒbʃʒ
15.	&ʃʒ
16.	ʒʒʒʒ
17.	Eʃʒ
18.	ʒ&&
19.	Fʃʒ
20.	ʒʒʃʒ
21.	&bʃʒ
22.	ʒCʃʒ
23.	Gʃʒ
24.	ʒʒʒ&
25.	ʃʒʃʒ
26.	ʒDʃʒ
27.	&&&
28.	ʒʒbʃʒ
29.	Hʃʒ
30.	ʒ&ʃʒ
31.	Iʃʒ
32.	ʒʒʒʒʒ
33.	&Cʃʒ
34.	ʒEʃʒ
35.	ʃʒbʃʒ
36.	ʒʒ&&
37.	Kʃʒ
38.	ʒEʃʒ
39.	&Dʃʒ
40.	ʒʒʒʃʒ
41.	Lʃʒ
42.	ʒ&bʃʒ
43.	Mʃʒ
44.	ʒʒcʃʒ
45.	&&ʃʒ
46.	ʒGʃʒ
47.	Nʃʒ
48.	ʒʒʒʒ&
49.	bʃʒbʃʒ
50.	ʒʃʒʃʒ
51.	&Eʃʒ
52.	ʒʒDʃʒ
53.	Oʃʒ
54.	ʒ&&&
55.	ʃʒcʃʒ
56.	ʒʒʒbʃʒ
57.	&fʃʒ
58.	ʒHʃʒ
59.	Pʃʒ
60.	ʒʒ&ʃʒ
61.	Qʃʒ
62.	ʒIʃʒ
63.	&&bʃʒ
64.	ʒʒʒʒʒʒ
65.	ʃʒDʃʒ
66.	ʒ&Cʃʒ
67.	Rʃʒ
68.	ʒʒEʃʒ
69.	Sʃʒ
70.	ʒʃʒbʃʒ
71.	Tʃʒ
72.	ʒʒʒ&&
73.	Vʃʒ
74.	ʒkʃʒ
75.	&ʃʒʃʒ
76.	ʒʒfʃʒ
77.	bʃʒCʃʒ
78.	ʒ&Dʃʒ
79.	Xʃʒ
80.	ʒʒʒʒʃʒ

In this table. ʒ. &. and ʃʒ. are the groundes: of all the reste aboue them. For of these thre, all those other bee made.

The Arte

Of Coßike nombers compounde.

Cossike nombers compounde, are made by addition of. 2. or more simple Cossike nombers together: As. 6. ze. —+— . 5. ȝ. or. 12. &. —+— 4. ze. —+— 3. φ. and so forthe in diuerse formes, whiche be infinite. Howbeit for briefnesse, we maie comprehende, vnder the same name (bicause of the like worke) all other *residualles Cossike*, whiche be made by subtraction: as. 3. &. ——— . 4. ȝ. And also those that bee made by addition and subtraction, bothe together: As. 9. ȝȝ. —+— . 4. ze. ——— 6. ȝ. In whose numeration is no hardnesse.

Scholar. Then your rules maie be the shorter.

Of Numeration.

Master.

This *Numeration* is easily vnderstāde by addition of simple *Cossikes*. For this is the forme. 6. ȝ —+— 10. φ. that is. 6. *Squares*, more. 10. nōbers. Likewaies. 8. &. —+— . 11. ze. is 8. *Cubes* and. 11. ze.

Now for *residualles*, take these examples. 9. ȝȝ. ——— . 12. &, whiche is. 9. *Squares of Squares*, saue. 12. *Cubes*. Also. 4. ʃȝ. ——— . 15. ȝ. that is. 4. *surſolides*, abatyng. 15. *squares*.

And for bothe together, this is the forme.
10. ȝȝ —+— . 6. &. ——— 30. ze. whiche signifieth 10. *Squares of Squares*, and. 6. *Cubes*, abatyng. 30. *rootes*.

Scholar. This is plaine. For so maie I vnderstande of all other As. 9. ȝ&. ——— . 3. ȝ. —+— 8. φ that is. 9. *Squares of Cubes*, lesse 3. *Squarss*, more. 8. *nōbers*.

Master.

Master. It were more orderly, to kepe the signes of more and lesse in order, then to followe the order of the *cossike* signes: bicause that addition, is orderly placed before subtraction. So were it better to set theim thus 9.ʒ&. —+— 8.ʃ. ——— 3.ʒ. Howbeit in deede all is one in these kinde of nombers, but not so in other *Surde nombers*, where the order foloweth of necessitie, as shall be declared in their place more largely.

Of Addition.

In addition, you must haue consideration of the *Cossike* signes: for noe other nomber, maie bee added into one, then soche as appertain to one signe *Cossike*.

As in vulgare denominations, you doe not adde the nōbers of shillynges, to the nombers of pennies: but you ioine shillynges to shillinges, and pennies to pennies: & poundes to poundes, so in *Cossike* nombers, *Cubes* muste bee ioined to *Cubes*, *Squares* to *Squares*, and generally, like to like.

Scholar. If this be al, I cā marke it well inough.

Master. There is somewhat more to be considered, that if there bee any signe in the one nomber, whiche is not in the other, that seueralle signe with his nomber, muste bee sette doune with his figure of —+—. or. ———. as it standeth there.

And farther, touchyng those twoo signes. —+—. ———. whiche bee the figures of more and lesse, you must giue regarde, whether thei bee like or vnlike, in those nombers that must be added: For if thei be like in nombers, of one denomination, then muste thei so remain as thei be. But if thei be vnlike, euermore abate the smaller nomber of theim, that followe those vnlike ſignes, out of the greater: and ſette doune the reſte, with the ſigne of the greater nomber.

Scholar. By examples, I ſhall better conceiue thoſe rules.

Maſter. Take theſe examples.

10.ʒ. —+— .12.ꝭ.	10.ʒ. ——— .12.ꝭ.
4.ʒ. —+— .8.ꝭ.	4.ʒ. ——— .8.ꝭ.
14.ʒ. —+— .20.ꝭ.	14ʒ. ——— .20ꝭ.

10.ʒ. ——— .8.ꝭ.	10.ʒ. —+— .8.ꝭ.
4.ʒ. ——— .12.ꝭ.	4.ʒ. —+— .12.ꝭ.
14.ʒ. ——— .20.ꝭ.	14.ʒ. —+— .20.ꝭ.

10.ʒ. —+— .12.ꝭ.	10.ʒ. ——— .12.ꝭ.
4.ʒ. ——— .8.ꝭ.	4.ʒ. —+— .8.ꝭ.
14.ʒ. —+— .4.ꝭ.	14.ʒ. ——— .4.ꝭ.

10.ʒ. —+— .8.ꝭ.	10.ʒ. ——— .8.ꝭ.
4.ʒ. ——— .12.ꝭ.	4.ʒ. —+— .12.ꝭ.
14.ʒ. ——— .4.ꝭ.	14.ʒ. —+— .4.ꝭ.

Here haue I varied one example diuerſly, to the intente you maie marke the vſe of your rules in theim.

And for the reaſon of thoſe rules, you ſhall marke thoſe

those examples well.

For where in the firste example, bothe signes are —+—, it must nedes be, that after the addition of the firste nombers, the seconde muste bee added with the signe. —+—.

In the seconde example, where bothe the signes be ——. bicause there wanteth. 21. ℞. of the first. 10. ℨ. Therfore is it reason, that bothe those wantes should be sette doune with the signe of. ——: and so in the thirde and fourthe examples.

In the fifth example, the seconde somme is not fully. 4. ℨ. but there wanteth of it. 8. ℞. and therfore if you put doune the. 4. ℨ. fully, you must abate. 8. out of the. 12. ℞. in the higher somme: and so of the other examples.

But for more practise, and better declaration of the vse of them, here are other exãples, of more varietie.

20. ℨ&. —+—. 9. ℨ. ——. 120. ʃʒ.
15. ℨ&. —+—. 5. ℨ. —+—. 16. ʃʒ.

35. ℨ&. —+—. 14. ℨ. ——. 104. ʃʒ.

16. ℨℨ. —+—. 28. ℞. ——. 16. ℨ.
12. ℨℨ. —+—. 12. ℨ. ——. 19. ℞.

28. ℨℨ. —+—. 9. ℞. ——. 4. ℨ.

In the firste example of these. 2. you see. 120. ʃʒ. with the signe of lesse, to bee added with. 16. ʃʒ. with the signe of more: and therefore, seeyng the signes of one *Coßike* denomination disagree, I doe subtracte the lesser, out of the greater: and that. 104. whiche remaineth, I doe set doune with the signe of

 lesse

lesse, bicause the remainer is of that nomber, that bare that signe.

And in the seconde exāple, the placyng of the signe —+— before ——— maketh nombers to bee sette before squares: and so the like denominations, dooe not stande one ouer an other. Yet is the worke dooen as if thei did stande eche ouer his like.

Scholar. I praie you lette me trie my cunnynge, with an example or twoo.

17.ʒʒ. —+— .10.¢. ——— .2.ꝛ.
16.ʒ¢. —+— .12.ʒ. ——— .6.ꝛ.

16.ʒ¢. —+— 17.ʒʒ. —+— 10.¢.
—+— 12.ʒ. ——— .8.ꝛ.

I set the example, as nombers came to my mynde: but I had almoste set my self on grounde: saue that I called to remembraunce, the comparison that you made, to vulgare denominations of poundes, shillinges, and pennies: and so was instructed to place euery seueralle denomination seuerally. And to sette the greateste denominatiō first, & eche other in his order.

Now will I proue an other example, or twoo.

3.ʃʒ. —+— .4.¢. ——— .20.ꝯ.
20.¢. ——— .8.ʒ. ——— .16.ꝯ.

3.ʃʒ. —+— .24.¢. ——— .8.ʒ. ——— .36.ꝯ.

13.ʒ¢. —+— .8.¢. ——— .4.ꝛ.
7.ʒ¢. ——— .6.¢. ——— .7.ꝯ.

20.ʒ¢. —+— .2.¢. ——— .4.ꝛ. ——— .7.ꝯ
6.ʒ.

6.ʒ. + 10.ꝛ. − 8.φ.
4.ʒ. + 17.φ. − 7.ꝛ.

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10. + 3.ꝛ. + 9.φ.

4.ʒȼ. + 5.ʒ. + 6.ꝛ.
8.ȼ. − 8.ʒ. − 10ꝛ.

4.ʒȼ. + 8.ȼ. − 3.ʒ. − 4.ꝛ.

Maſter. You haue doen well: And for proofe of your worke, you maie in this arte not onely proue it, by the contrary kynde, as you did in nõbers *Abſtracte*, but alſo by the reſolution of all thoſe *Coſsike* nombers into nõbers *Abſtracte*, takyng any nomber for a roote and then the *Squares* and *Cubes*. &c. accordingly. As here in this table, you maie briefly ſee, but more largely in the table at the eande of nombers figuralle.

A table for trialle by reſolution, of any woorke in this arte.

ꝛ	ʒ	ȼ	ʒʒ	ʃʒ	ʒȼ	bʃʒ
2	4	8	16	32	64	128
3	9	27	81	243	729	2187
4	16	64	256	1024	4096	46384

ʒʒʒ	ȼȼ	ʒʃʒ	
256	121	1024	
6561	19683	59049	
65536	262144	1048576	

And if this table in any parte, ſeme to ſhorte or to litle:

litle: you maie haue recourse to the table, at the ende of figuralle nombers, whiche therfore is made large and generalle: so that it maie well be called the frutefull table, or table of ease.

But now for triall of the laste example: firste there is. 4. ʒ ȼ: for whose roote I take 2. and therefore those. 4. ʒ ȼ. make. 256. whiche I sette doune in nomber *Abstracte*. Nexte is. 5. squares, whiche accordyng to that roote, must nedes be. 20. and that. 20. I sette doune also: and then. 6. rootes, whiche make 12. And all thei yelde. 288. and that is all the firste somme.

256.
20.
12.
288.

Then for the seconde somme, I see firste. 8. *Cubes*, whiche make. 64. to bee added. Then foloweth. 8. squares lesse, that is. 32. to bee abated, and also. 6. rootes lesse, that is. 20. also to bee abated: So must I abate. 52. (for theim bothe) out of. 64. and then there resteth but 12. whiche added vnto 288. of the first somme doe yelde. 300.

32.
20.
52.

64.
52.
12.

Now if the totall agree with this, then is the worke good.

288
12.
300

For triall whereof, I resolue. 4. ʒ ȼ. into nomber *Abstracte*, and thei will make. 256. then. 8. ȼ. maketh. 64: whiche bothe yelde 320. Then foloweth in thesame somme. 3. ʒ and. 4. ꝛ. to be abated. The. 3. ʒ. make. 12. and the 4. rootes yelde. 8. whiche together do amounte to. 20. and that must bee abated frō thesaid somme of 320 and then there remaineth onely 300. agreable to the former somme aboue the line.

256
64
320.

Scholar. This proofe I like well: And I perceiue that if I would worke the like, takyng for the roote 3, or any other nomber, the proofe will succede a like.

Master. Now to make an eande of Addition, bicause

cause

cause you shall the better remember the rules of it, I will giue you them in this brife forme.

In greatenesse like and signes also,
Adde like to like there nedes no mo:
And where the greatenesse disagree,
Place eche by other seuerally.
With signe of eche, as doeth require,
But if the signes vnlike appere:
Then from the more abate the lesse,
The greater his signe with the excesse.
Will make the somme,
Of that addition.
The proofe is by resoluyng,
Eche nomber into his reckenyng.

This lesson doeth containe the former rules onely in brief, and therefore neadeth no declaration: but the greatenesse doeth betoken the *Cossike* denomination, and signes betoken specially, $+$ and . $-$. the signes of more and lesse, and no other signes.

Scholar. This brief lesson will helpe memorie moche: and shall suffice for the rules of Addition.

Of Subtraction.

Master.

Then for subtraction, this shall you marke in especiall: that when your nombers are sette doune, after the cōmon maner, firste the totall, and then the deduction: you shall consider well, whether the signes bee $+$ or $-$. For in the deduction, if you haue $+$ then must that be subtracted from the like aboue. 1. *Rule.*

And if that somme in the deduction, that hath the signe 2. *Rule.*

signe ——+——, bee greater then the nomber of the like quantitie ouer hym, with the like signe ——+——. then abate the higher out of the lougher, and write the reste with this signe ———.

3. Rule. But if the like quantitie in the totall, haue the signe ———, then adde bothe nombers together and set them vnder the line with that signe ———.

4. Rule. And if the seconde somme (that is the deduction or abatement) with any nomber, haue this signe of lesse ———, it must be accoumpted for more, and must be added to the like nomber ouer it, excepte the ouer nōber haue the signe of lesse also: For then must you abate the lesser, out of the greater, and sette doune the reste, with the signe of the greater nomber: whiche thei haue at this conferēce: I meane to regarde what the signe of the seconde somme is by estimation, and not by writyng, for thei are contrary.

Scholar. I see good reason in this: For in any abatemente, the more is abated, the lesse by so moche shall remain: and the lesse is abated, the more doeth remain by so moche.

5. Rule. Master. Yet one thyng more is to bee marked, that if there be some denominations, in the one sōme that are not in the other, you shall marke in whiche somme thei bee. For if thei bee in the firste, then shall thei kepe still their owne signe. And if thei bee in the seconde somme, whiche is the deduction, then shall thei chaunge their signe to the contrary: But where soeuer thei be, thei must be set in the remainer.

Scholar. I can better vnderstande you, then remembre those rules.

Master. Then take this briefe lesson, apter to bee remembred, then to bee vnderstande, but by the letter before, and by the examples folowyng. But memorie liketh well soche aide.

A brief rule of Subtraction.

1. *When signes and greatenesse bothe agree,*
Your woorke procedeth forthe commonly.
2. *But if thabatemente greater bee,*
Thexcesse shall chaunge his signe therby.

3. *And where the signes doe dissagree,*
The higher signe must rest duely:
And though the batemente be the greater,
The reste still ioyneth bothe sommes together.

4. *If quantities doe dissagree,*
Place them with signes all seuerallie:
The totall kepeth the signe he had,
The batemente still, to chaunge is glad.

Scholar. Now some examples, will lighten these rules well.

Master. I will propounde the like, as I did in addition, to the intẽte you maie iudge the likenesse, and diuersities in bothe woorkes.

10.ℨ.—+—.12.ꝛ.	10.ℨ.—+—8.ꝛ.
4.ℨ.—+—.8.ꝛ.	4.ℨ.—+—12.ꝛ.
6.ℨ.—+—.4.ꝛ.	6.ℨ.———4.ꝛ.

10.ℨ.———.12.ꝛ.	10.ℨ.———.8.ꝛ.
4.ℨ.———.8.ꝛ.	4.ℨ.———.12.ꝛ.
6.ℨ.———.4.	6.ℨ.—+—.4.

10.ℨ.—+—.12.𝔯.	10.ℨ.—+—.8.𝔯.
4.ℨ.———.8.𝔯.	4.ℨ.———.12.𝔯.
6.ℨ.—+—.20.𝔯.	6.ℨ.—+—.20.𝔯.

10.ℨ.———.12.𝔯.	10.ℨ.———.8.𝔯.
4.ℨ.—+—.8.𝔯.	4.ℨ.—+—.12.𝔯.
6.ℨ.———.20.𝔯.	6.ℨ.———.20.𝔯.

The firste and thirde examples be very plaine: and in the seconde where, 12. should bee abated out of. 8. there is. 4. to fewe: and therefore I abate the higher, out of the lougher, and I set doune. 4. with the signe of wantyng, or abatemente.

In the fourthe example: bicause the higher nomber is the lesser, I doe subtracte him out of the nether, and sette doune the reste. 4. with a contrary signe of —+—.

But in the. 4. later examples, where the signes do disagree, the nōbers that followe the signes, are not subtracted one from an other, but are added together: and thei take still the higher signe. Bicause in value, the signe of abatemente is contrary, to that it appeareth to bee.

And for your exercise, to make you full prompte in this arte, I haue set forthe more examples.

6.ȼ.—+—.120.𝔯.	8.ℨȼ.
9.ȼ.———.40.𝔯.	9.ℨȼ.———89.𝔯.
160.𝔯.———.3.ȼ.	89.𝔯.———.1.ℨȼ.

3. ʒ. + 18. ze.	18. ze. + 3. ʒ.
12 ze —— 3. ʒ.	12. ze. —— 3. ʒ.
6. ʒ. + 6. ze.	6. ze. + 6. ʒ.

3. ʒ. + 18. ze. —— 10. ϑ.
12. ze. + 8. ϑ.

3. ʒ. + 6. ze. —— 18. ϑ.

4. Sʒ. + 10. cc. —— 6. ʒ.
5. ʒʒ. + 12. ʒ. + 3. ϑ.

4. Sʒ + 10. cc. —— 5. ʒʒ. —— 18. ʒ. —— 3 ϑ.

Here in the firste example, where I would abate 9 cc. out of. 6. cc. I maie easily perceiue, that there are 3. cc. to fewe. And therefore doe I sette doune. 3. cc. with this signe ——, whiche signifieth wante or abatemente: and the. 2. nombers that followe the unlike signes, I set doune bothe added into one: and put therto the signe of the totall or ouermoste somme.

In the seconde example, there is the like woorke: For in abatyng. 9. out of. 8. I finde. 1. to fewe: that. 1. doe I set doune with his denomination of. ʒ cc: and the signe. ——.

And the nomber 89 that foloweth the signe —— in the seconde somme, standeth in force as +, for the lesse is abated, the more must remain: therfore in the remainer, I set not the signe of more, before that nomber of. 89. but I put it in the firste place of the somme: whiche place of it self, signifieth still more.

And bicause ouer that nomber 89, there are no nõbers in the totall, therefore I muste putte doune that somme as it is, without addyng to it, or abatyng frõ it, in it self.

Scholar Those. 2. examples might be set thus, as I thinke, bicause the places doe so require.

6.ce.—+—.120.ꝗ.
9.ce.——.40.ꝗ.
——.3.ce.—+—.160.ꝗ.

8.ʒce.
9.ʒce.——.8.ꝗ.
——.1.ʒce.—+—.8.ꝗ.

Master. Remember your self well, and marke the remainer how it is written.

Scholar. I see my owne ouersighte: For no nomber maie begin, with signe of lesse: and therfore must their places be altered of necessitie, and set in order as thei were before.

Master. Then for all the reste of the examples, or any other like, I shall not neade to giue you any farther instruction: sith that by these former, you maie iudge of all other.

Proofe. And for the examination of your worke, the trialle by resolution doeth serue here, as well as els where: remembryng onely (as the order of subtraction maie admonishe you) that the somme of the totalle, whiche is the firste somme, must counteruaile the other bothe sommes: that is of the deduction, and of the remainer.

So to trie the firste example, takyng. 3. for a roote: 6.ce. make. 162. whiche I put to. 120. and it yeldeth 282. Then in the seconde somme. 9. ce. are. 243. whereof. 40. must bee abated for the signe ——, so

is

is that somme.203. Again in the remainer.3.ꝫ. are 81. whiche must bee abated out of.160. and so resteth 79. whiche with.203. doe make.282. agreable with the firste somme.

Scholar. This doe I well understande, and praie you to procede to multiplication.

Of Multiplication.

Master.

IN multiplication, there is no difficultie, so that you dooe well marke the signes —+— and ———, whiche beyng bothe like, will haue the signe —+— sette in the totalle. and beyng unlike, thei will haue in the totalle the signe ———.

And likewaies in diuision —+—. diuided by ——— or cōtrary waies ——— by —+— will alwaies haue in the totalle ———: but —+— diuided by —+—, or ——— by ———, will make alwaie —+—.

Whiche rule for ready remembraunce, I haue giuen you here in meter.

Who that will multiplie,
Or yet diuide trulie:
Shall like still to haue more,
And mislike lesse in store.
Their quantities doe kepe soche rate,
That.M.doeth adde:and.D.abate.

Scholar. So meane you, that like signes multiplied together, doe make more, or —+—: And unlike sines multiplied together, doe yelde lesse, or ———.

Master. So is the rule. But to go forward now: of the nexte difficultie, as touchyng *Coßike* quantities that chaunge their denomination, here is no more to

bee

bee saied, then was taught in multiplication of nom-
bers *Coßike* uncompounde, and in the table set forthe
for the chaunge of their names.

Scholar. I understande, that in multiplication (that is. M.) their figures must bee added. And in. D. (or division) thei muste bee abated. Therefore a fewe examples shall suffice for the reste.

Master. Take these for a presidente, of all that worke: by whiche you maie iudge of all other like.

```
   10. ce. —+— .9. ʒ. —+— .20. ze.
    5. ʒ. —+— .7. ze. ——— .8. ꝙ.
——————————————————————————————————————————
———80. ce. ———.72. ʒ. ———.160. ze.
   70. ʒʒ. —+— .63. ce. —+— .140. ʒ.
50ſʒ —+— .45ʒʒ —+— 100 ce.
——————————————————————————————————————————
50ſʒ —+— 115ʒʒ —+— 83ce —+— 68ʒ ——— 160ce
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        15. ʒce. ——— .12. ʒ.
        14. ʒ. —+— 2. ze. ——— .5. ꝙ.
——————————————————————————————————————————
              ———.75. ʒce. —+— 60. ʒ.
                   30. bſʒ ——— 24. ce.
     210. ʒʒʒ ——— 168. ʒʒ.
——————————————————————————————————————————
210. ʒʒʒ —+— 30. bſʒ ——— 75ʒce ——— 168ʒʒ
              ——— 24ce. —+— .60. ʒ.
```

Scholar. I perceiue, that these workes doe appere more hard, then thei bee in deede, and that bicause of their straunge formes: but by use I truste to bee acquainted with them well inough: and therfore I will begin with more easie examples. As these bee, that

folowe

followe here.

18.ʒ.—+—.20 ꝗ.
15.ꝛe.———.4.ꝗ.
———72.ʒ.———.80.ꝗ.
270.ℭ.—+—300ꝛe.
270.ℭ.—+—300.ꝛe.———72.ʒ.———.80.ꝗ.

16.ʒ.—+—.14.ꝛe.
8.ℭ.———.7.ꝗ.
———112.ʒ.———98.ꝛe.
128.ſʒ.—+—112ʒʒ.
128.ſʒ—+—112.ʒʒ———112.ʒ.———98.ꝛe.

And this I ſee farther now, that theſe woorkes ſeme more difficulte to looke on, then thei be in practiſe, if a manne giue good hede to the ſignes, and the quantities.

Maſter. Before we go any farther, I will ſhewe you ſomewhat of the reaſon, why the ſignes ought to chaunge. And that by twoo plaine woorkes, in nombers *Abſtracte*. As here foloweth.

16.—+—.12.
20———.4.
———64———.48.
320—+—.240
560———121
that is. 448.

Where you ſee, that when I had multiplied.16—+—12 by 20 it made.320—+—240 that is in all.560.

But bicauſe the multipliare ought not to be ſo moche by 4 therfore it is reaſon, that I ſhall multiplie the higher ſomme by.4. and abate that out of the former totall.

Whiche thyng you see here doen by. ——. 64. ——. 48. whiche bothe make. 112. to bee deducted out of 560. and so remaineth 448. The iuste somme that commeth of that multiplication.

Scholar. This I vnderstande well: and maie proue it in this sorte. 16. —+—. 12. maketh. 28: and. 20. ——. 4. is. 16. Then if I multipie. 28. by. 16. it will yelde 448. as the woorke here declareth.

28.
16.
168.
28.
442

And hereby maie I iudge, of *Coßike* nombers likewaies.

Master. Yet one example more will I propounde bicause I would put you out of all doubte. Wherfore marke this forme of woorke.

Here you maie see, that if the firste somme of 24 —— 3 wer multiplied by 15 it would make. 360. ——. 45. that is 315. But it ought not to bee so moche, but lesse by. 2. tymes 24 —— 3. that is. 48 —— 6: bicause the multiplier doeth wante. 2. of. 15.

24. ——. 3.	
15. ——. 2.	
48. —+—. 6.	
360 ——. 45.	
366 —— 93.	
that is. 273.	

And so abatyng. 42. or. 48. ——. 6. out of. 315. there resteth. 273. whiche is the iuste totall, when. 21 is multiplied by. 13. wherby the multiplication is declared to bee good.

And for bicause that —— multiplied with —— doeth make —+—: marke here, that you maie not abate fully. 48. but 48. —— 6.

Then seyng in abatemente, the signes in figure are contrary to their owne estimation and force: therfore that. 48. must be made ——. and the —— before. 6. tourned into —+—.

Scholar. I see it well, it must nedes be so.

For if thei were set, to bee subtracted, then should thei stande so. 48. ——. 6: whiche declareth that 42 should

should bee abated.

But when thesame nombers, are set emongeste other to be added: as it is here in workyng of multiplication, then must thei be written thus. —48 —+— 6 declaryng that if you abate. 48. you muste adde. 6. again, bicause you abated. 6. more then you ought.

Master. You understand it well. Wherfore here will wee make an eande of multiplication: sith there resteth nothyng but the proofe of it: whiche maie bee wrought by resolution, of all the *Coßike* nombers, into nombers *Abstracte*, as in other kindes before. Onely consideryng that the resolutions of the first and seconde sommes, must be added together.

The profe of multiplication.

And therfore if you liste to proue the firste example takyng. 2. for the roote, you shall finde the firste somme 80. —+— 36 —+— 40. that is. 156. And the seconde somme is, 20. —+— . 14. ——— . 8. that is. 26. The thirde somme is. 1600. —+— . 1840. ——— . 664. —+— 272. ——— . 320. whiche maketh. 4056. And so doeth. 156. multiplied by. 26.

Scholar. This maie I proue at any tyme: so that you shall not nede to staie aboute it.

Of Diuision.

Master.

Diuision is nexte in order, and agreable in the generall rules: and hath not more speciall, then the very nature of the woorke dooeth require. For as concernynge the signes of —+— and ———. thesame order is here, as is in multiplication. And touchyng the *Coßike* signes, it is all one with that I saied in diuision of nombers *Coßike* uncompounde.

Scholar. Then a fewe examples maie supplie the

 declaration

declaration of the vse of the rules, with the practike woorke.

Master. Take these for your purpose.

An example of the firste worke.

60.
12.ʒʒ —+— 76.𝔠. —+— 80.ʒ.(2.ʒ.
6.ʒ —+— 8.ze.

The remouyng of the diuisor, for the seconde woorke.

66.
12.ʒʒ —+— 76.𝔠. —+— 80.ʒ.(2ʒ —+— 10ze
6.ʒ. —+— 8.ze.

The proofe in nombers *Abstracte*, accoumptyng. 2. for roote.

3 480.
192. —+— 608. —+— 320.(8.
24. —+— 16.

480
192 —+— 608 —+— 320. (8 —+— 20.
24. —+— 16.

Thesame worke in vulgare forme.

ʒ
1120 (28.
440

Here I haue not onely parted the woorke, for your ease in vnderstanding: but I haue also put against it, the declaration of the same, by resoluyng the *Cossike* nõbers, into nombers *Abstracte*.

And finally, I haue putte one example of thesame nombers,

nombers, after the vulgare forme: all whiche. 3. agree together: and vouche one an other.

Scholar. Yet I praie you worke, one example more.

Master. Here is an other.

¶The firste extraction of the diuisor.

40.ʒℭ. —+— 48.ʒʒ —+— 20.ℭ —+— 24.ze. (8.ʒ.ʒ.
5.ʒ. —+— 6.9.

¶The remouynge forward of the diuisor.

40ʒℭ —+— 48ʒʒ —+— 20ℭ —+— 24ℭ (8ʒʒ —+— 4ze
5.ʒ —+— .9.

¶The comprobation of the same by resolution, accoumptyng still. 2. for a roote.

2560 —— 768 —+— 160 —+— 48.
20 —+— 6. (128.

¶The settyng forward of the diuisor.

2560 —+— 768 —+— 160 —+— 48. (128 —+— 8.
20 —+— .6.

Scholar. Yet ones again, I praie you worke the like.

For although I perswade my self, that I perceiue the worke: yet would I see more confirmation of it, before I would be to constante in my persuasion.

Master. Good aduisemēte is euer sure: but if you doubte, your councelloure is not farre absente.

Scholar. I maie iustly reioice thereof: But for euery mater to require aied, and neuer to trauell my owne witte, it might seme mere dastardlinesse. And so were it plaine babishenesse, to couet euery morsell, to be chawed before hande, and put into my mouthe.

Master. Then take this other example, in one platte complete: But with a caueat, to beware of to moche confidence, while you seme to flee doubtefulle dasterdlinesse.

16.
14.ʒ𝔠. —+— 30.ʒʒ. —+— 16.ʒ. —— 6.ſʒ. —— 8.𝔠. (7.𝔠. —+— 8.ℛ. —— 3.ʒ.
2.𝔠. —+— .2.ℛ.
2.𝔠. —+— 2.ℛ. 2.𝔠. —+— 2.ℛ.

Scholar. Now haue I, that I looked for.

Master. Softe, lette vs trie this worke, as wee haue doen the other: before we goe from it.

Scholar. I praie you let me doe it.

Master. With a good will.

64	16
14	30
256	480
64	
896	

Scholar. I kepe still the old roote 2. Then is the. ʒ𝔠. 64: whiche being multipled by. 14. maketh. 896. And so. 30. ʒʒ. doe yelde. 480. And 16. squares make. 64. All thei together yelde. 1440.

The reste of the nombers, must be abated, bicause of the signes. ——. and thei make 240. For euery. ſʒ. is. 32. and then. 6. times that, that makeh 192. whereunto I put. 48. for 6. *Cubes*: and so haue I. 240. to be abated out of. 1440. and then remaineth. 1200. for the diuidende. The diuisor is but. 20. sith. 2. 𝔠. are. 16. and. 2. rootes make. 4,

32	8
6	6
192.	48

896
480
64
1440

1440
240
1200

1200 (60
20

If I diuide now. 1200. by. 20. the *quotiente* will be. 60. agreably to the former *quotiente*. For 7. 𝔠. make. 56

And

And.8.rootes yelde.16.that is.72. From whiche I must abate.3.ℨ.that is.12. And then it is iuste.60.

Master. This is well doen.

Scholar. Yea sure, I am perfecte inough, in this feate of diuision, I trowe.

Master. You doe well to doubt.

Scholar. I thinke my self sure without doubte: As by one or twoo examples, I will declare.

And first I take this nõber 322 bſz. —+— 115 ℨℭ. —— 42.ℭ —+— 69.ℨ. —+— 30 ꝛ. to be diuided by.14.ℨ. —+— 5.ꝛ. wherefore I sette them doune thus.

322 bſz —+— 115 ℨℭ —— 42 ℭ —+— 69 ℨ —+— 30.ꝛ (23 ſz —+— 3 ꝛ
14.ℨ. —+— 5.ꝛ. | 14 ℨ —+— 5.ꝛ.
322.bſz —+— 115.ℨℭ. | 42.ℭ —+— 15 ℨ.

And finde the firste *quotiente* to bee. 23.ſz. by whiche I multiplie the diuisor, and it taketh awaie all the nombers ouer it: Wherefore I set the diuisor forward, & finde 3 ꝛ. for the *quotiente*, whiche I multiplie into the diuisor, & it maketh 42 ℭ —+— 15 ℨ. wherby I am at a staie. For although I see in the diuidende, the like nombers, yet the signe of —— declareth, that it is not possible, to abate this newe nõber thens: seyng —— 42.ℭ. is lesse then naughte.

Master. Therefore consider it, in chosyng your *quotiente*: and giue your *quotiente* the like signe.

Scholar. But then riseth an other doubte. For there will be ——.15.ℨ. whiche disagreeth in signe from the nomber ouer it.

Master. Yet maie you subtracte it well inoughe, if you haue not forgotten, your rules of subtraction.

Scholar. Now I dooe better remember my self: that by good reason, I must leaue as a remainer, not onely the whole nomber ouer it, whiche is.69.ℨ.

but

but I must adde therto. 15. ȝ. more.

So shall I cancell the. 69. and set ouer it. 84. And then doe I remoue the diuisor forward, settyng 14 ȝ. vnder. 84. ȝ. and the reste in order, whereby I perceiue, that the newe *quotiente* will be. —+—. 6. ℈.

```
                                        84.
322.bſȝ —+— 115ȝce —— 42ce —+— 69ȝ —+— 30.ce (23ſȝ — 3ze +6℈
 14.ȝ. —+— .5.ze    |—— 14ȝ. —+— .5.ze.
322bſȝ —+— 115ȝce   |—— 42ce —— 15ȝ.
                                 84.ȝ. —+— 30ze.
                                 14.ȝ. —+— .5.ze.
```

Whiche *quotiente* I doe multiplie into the diuisor, and it doeth make. 84. ȝ. —+—. 30. ze. agreable to the somme ouer it. And so there remaineth nothyng.

Master. You haue dooen well. But in chosynge your diuidende, and the diuisor, your lucke was better then your cunnyng.

Scholar. That shall I proue againe, by an other example, takyn also at all aduentures.

I would diuide this somme.

16. ȝce. —+—. 20. ȝ. —+—. 12. ze. ———. 8. ℈. by 4. ȝ. —+—. 2. ze. And therfore I set theim doune in order thus.

```
16.ȝce —+— 20ȝ —+— 12ze ——— 8℈. (4.ȝ.ȝ.
  4.ȝ. —+— 2.ze.
```

And firste I see, that. 4. is contained in. 16. fower tymes: and so maie I finde. 2. in any other nombers there. 4. tymes. Wherfore I set. 4. in the *quotiente*.

And bicause the. 4. in the diuisor are. ȝ. and the 16 to bee diuided, are. ȝce. accordyng to the former rules, I finde the newe denomination *Coßike* to be. ȝȝ.

whiche

whiche I ſet in the *quotient* with 4 and ſo is it. 4.ƺƺ.
Then ſaie I. 4.ƺƺ. multiplied by. 4.ƺ. do make 16.ƺℭ. and therfore cleareth and conſumeth al that ſome ouer it. Then farther ſaie I 4.ƺƺ. multiplied by. 2.ꝛe. doe yelde. 8.ſƺ: But I ſee noe ſoche denomination in the diuidende.

Maſter. Then maie you perceiue, that you haue miſſed.

Scholar. Why ſir, I thinke I ought to doe as you did: that is to multiplie the *quotiente* into euery parte of the diuiſor.

Maſter. That is true: but I wil detecte the faute vnto you. And that is this.

That all nombers *Coßikes* compounde, can not bee diuided orderly, by diuiſors compounde. And thoſe that can bee diuided, will not receiue any other diuiſor, of theſame kinde, but one of. 2. nombers, by multiplication of whiche, it was made: and ſo the other of thoſe. 2. ſhall be the *quotiente*: As it came to paſſe in all thoſe. 3. examples, which I ſet forthe. And therefore it is loſte laboure, to goe aboute to diuide theim in that ſorte.

Scholar. Then are there but fewe nombers of *Coßikes* compounde, that maie be diuided.

Maſter. So many men ſaie. But I ſaie thereto, that though many of them can not be diuided, by like nombers *Coßikes* cõpounde, yet are there many thouſandes, that maie be ſo diuided.

And again I ſaie, that all ſortes of theim, maie bee diuided, by an *Abſtract* nomber. And alſo any of them maie be diuided, by conuerſion into a fraction: And ſo maie your example be ſet thus.

$$\frac{16.ƺℭ. + .20.ƺ. + 12.ꝛe. — .8.9.}{4.ƺ. + .2.ꝛe.}$$

And in all other cases like, sette the diuidende ouer a line, and the diuisor vnder the same line, and so is your diuision ended: and this is the reddiest waie, and the moste indifferente, in all soche nombers.

Scholar. That is sone learned. And therfore neadeth no moare examples.

It is like in nombers *Abstracte*, when the greater nomber, doeth diuide the lesser. As. 6. diuided by. 11. maketh $\frac{6}{11}$.

Master. Somewhat like it is. Howbeit here is a worke moare like thereunto, as when we should diuide the lesser *Cossike* nomber, by the greater, for then we must set them in that forme. So. 6. ʒ. diuided by 7. ℭ. shall be set thus: $\frac{6ʒ}{7ℭ}$. And. 20. ℭ. diuided by 5. ʃʒ. must stande in this maner: $\frac{20ℭ}{5ʃʒ}$.

Scholar. Why? 20. maie be diuided by. 5.

Master. But. ℭ. can not be diuided by ʃʒ. And in *Cossike* nombers, the chief regard is to be had, to the *Cossike* signes.

Scholar. Then, as for any other forme, of regulare diuision, here is none.

Master. Noe, excepte your diuisor, bee a nomber *Abstracte*: Or at the leaste, if it haue one onely *Cossike* signe, and be vncompounde, that signe must be other equalle, or lesser then the leaste *Cossike* signe, in the diuidende.

For so. 60. ʒℭ. —+— 48 ℭ. —+— 18. ʒ. maie bee diuided by any nomber, hauyng one of these. 3. signes *Cossike*. ʒ. ꝝ. ℈.

Scholar. I vnderstand it well. For. ʒ. is the laste signe in the diuidende: And. ꝝ. and. ℈. are not onely lesse then it, but also. ℈. leaueth the nomber, as if it were a nomber *Abstracte*.

So if I would diuide your nomber, assigned by 40. ʒ. the *quotiente* would bee thus.

60. ʒ.

60.ȝcꝭ—+—48cꝭ—+—18ȝ(1½ȝȝ—+—1⅕ze—+—9/10℈.
40.ȝ.   40.ȝ.   40.ȝ.

Master. Before we ende this worke of diuision, I will admonishe you, of one easie aied, in the diuisiō of diuerse nombers. And that is, to consider, whether your diuidende, doe omit any *Coßike* denominations, betwene them, whiche it hath. For if it doe, you must yet supplie their roomes, with signes and Ciphers. As by example, you shall vnderstande.

I require to haue this nomber. 8.cꝭ.—+—64.℈. diuided by. 2ze.—+—.4.℈.

Scholar. That will I doe quickely. For I see. 4. will be the firste *quotiente* and his denomination will be. ȝ. sith. cꝭ. diuided by. ze. doe make. ȝ.

But firste I sette doune the nombers orderly. And then I multiplie the diuisor by the *quotiente*, & there riseth. 8.cꝭ.—+—16.ȝ.

8.cꝭ—+—64.℈. (4.ȝ.
2.ze—+—.4.℈.
8.cꝭ—+—16.ȝ.

Master. Stande you now amased, for all your greate confidence? You see that you can not finde any ȝ. in the diuidende. Therfore set doune the nomber as I told you before, in this sorte.

——16ȝ.
8.cꝭ—+—.0.ȝ.—+—.0.ze—+—46.℈.(4.ȝ.
2.ze—+—.4.℈.
8.cꝭ—+—.16.℈.

And then I take the same *quotient* that you did, and I finde the remainer to be.——.16.ȝ. Wherefore I doe again sette forward the diuisor: And finde the *quotiente* to bee——8.ze. by whiche I multiplie the diuisor, and it maketh. 16. ʒ. ——. 32. ʒʒ so that abatyng the. 16. ʒ. the reste, that is, ——. 32. ʒʒ. shall be the remainer with the signe —+— by the rule of subtraction.

```
——16ʒ.—+—32ʒʒ.
8ȼ—+—øʒ—+—øʒʒ—+—64ℊ(4ʒ——8ʒʒ—+—16ℊ.
2ʒʒ—+—4.ℊ.
          2ʒʒ—+—4.ℊ.
```

Then vnder that remainer, I remoue the diuisor, and finde the newe *quotiente* to bee —+— 16. ℊ. And so is the nomber clerely consumed.

Scholar. If I forgette any parte of this, I am deceiued to foule.

Master. Then haue you learned this parte, well enough, for this tyme. And therfore will we go forth vnto fractions, whiche partly were omitted before, and partly are compounde of them self.

Of fractions, and their numeration.

Fractions of this kinde appere simple: and yet are scante so to bee iudged: as $\frac{4ʒ}{3ȼ}$ betokeneth 4. ʒ. to bee diuided by. 3. ȼ. Likewaies this fractiō $\frac{12ʒʒ}{5ʒȼ}$ doeth import that 12 ʒʒ. muste bee diuided by. 5. ʒ ȼ. But $\frac{10ʒ}{19}$ betokeneth. 10. ʒ. to bee parted into. 19. portions.

And here shall you note, the doubtfull forme, that many menne in this arte vse, whiche write that laste fraction thus. $\frac{10}{19}$ ʒ. where as this fractiō doeth represent $\frac{10}{19}$ of a square: and not 10. ʒ to be diuided by. 19.

Scholar. Bicause you saie, that some doe so vse it, and

and I would gladly excuse all good writers: I maie saie for them that as in vulgare nombers, when. 10. should be diuided by 19. And is set thus $\frac{10}{19}$ it doeth importe bothe that. 10. is diuided into. 19. and also that euery portion of those. 19. is $\frac{10}{19}$ of an vnitie: so that if 10. ℓ. should be parted emongest. 19. men, euery man should haue $\frac{10}{19}$ of. 1. ℓ.

Master. Your wordes haue so moche apperance that thei maie persuade hym, that is not very precise in termes, especially seyng there is no other *quotiente* there, but the same nomber. But as the somme of 10. ℓ. beyng diuided by. 19. is farre more then $\frac{10}{19}$ of an vnitie: So. 10. ʒ. to bee diuided by. 19. differ moche from $\frac{10}{19}$ of a square. For the one is 19. tymes so moche as the other. And therfore oughte to haue a distincte forme in writyng.

Scholar. Then you would haue me to write thē so, that $\frac{10}{19}$ of a Square, should haue the signe against the line, as here is set $\frac{10}{19}$ʒ: and when I would represent. 10. ʒ. diuided by. 19. I shall write it thus. $\frac{10ʒ}{19}$. with the signe aboue the line.

Master. You maie see their agremente, and their difference by resolution, in this maner $\frac{10ʒ}{19}$ will make $\frac{40}{19}$ accoumptynge. 2. for a roote, and $\frac{10}{19}$ʒ. maketh $\frac{10}{19}$ of 4. or $\frac{40}{19}$ of. 1.

Again, accoumptyng. 3. for the roote, then $\frac{10ʒ}{19}$ yeldeth $\frac{90}{19}$: and $\frac{10}{19}$ʒ maketh $\frac{90}{19}$ of an vnitie: so thei appere to bee equall in valewe by reduction.

But now maie you see, that the one doeth betoken the firste nōber, whiche is to be diuided: and the other doeth signifie the *quotiente* of the diuision: and so are thei distincte in office and nature. But bicause by resolutiō, the one tourneth into the other, therfore many men accoumpt them as one. Howbeit, we stand to longe aboute this, consideryg the errour, is not alwaies daungerous.

 But

But their ouersighte is more daungerous, whiche misplace the signe, when it should bee sette vnder the line: as a greate clerke doeth (except I shall for his excuse, impute the faulte to the printer) for he meaning to diuide .3. by .7. ʒʒ. writeth it thus. $\frac{3}{7}$ʒʒ. where he should write it thus. $\frac{3}{7\,ʒʒ}$: and againe, myndyng to diuide .7. by .3. ʒʒ. he writeth it thus $\frac{7}{3}$ʒʒ. where he should write. $\frac{7}{3\,ʒʒ}$.

Scholar. This faulte is manifeste, and detecteth the firste negligence: For $\frac{7}{3}$ʒʒ. doeth make in nomber, after the former resolution. $\frac{112}{3}$ and. $\frac{7}{3\,ʒʒ}$ dooeth make. $\frac{7}{48}$.

Master. Well, seyng you perceiue the faulte, we will stande no longer aboute it. Therfore to procede distinctly and certainly, whether that fraction be compounde, or simple, where the numerator is a *Coßike* nomber, and the denominator, a nomber absolute, yet maie you boldly thinke, that fraction to bee compounde, whose numerator is a nomber *Coßike* and the denominator an other *Coßike* of vnlike signe: as. $\frac{8}{3}$ ℭ/ʒ and $\frac{5\,ʒℭ}{12\,ze}$.

Yet as in nombers Abstracte, it maie seme moste aptly to bee called a fraction, when the numerator, is lesser, then the denominator, so in nombers *Coßike*, moste aptly the signe of the denominator, should bee the greater. Yet bothe formes come in vse.

And forbicause easinesse in workyng, doeth oftentimes bring certaintie with it before we take in hãde the addition of fractions, I thinke it good to speake somewhat of Reduction, to an other denomination. So that you forgette not, that any .2. nombers *Coßike* compounde, with a line betwene them, maie be called a fraction. As thus. $\frac{5ℭ\ +\ 8ze\ -\ 6℞}{3ʒ\ +\ 12℞}$ | that is, 5.ℭ. + .8. ze. — .6. ℞. to bee diuided by 3.ʒ.

3ʒ. — + — .12.℈. and so of other like.

Of Reduction of fractions.

Fractions *Cossike*, not onely in their nombers, but also in their signes maie be reduced to other valutions, and namely to their leaste termes, and yet continue still in one proportion, betwene the numerator, and denominator.

So $\frac{18ʒʒ}{36ce}$ maie bee reduced to $\frac{1ʒ}{2℈}$: for so high as. ce is aboue. ℈. that is in the thirde place from it: So is ʒʒ. in the thirde place aboue. ʒ.

Againe. $\frac{27ʒ}{39ʒʒ}$. by reduction doeth make $\frac{9ce}{13℈}$: And so $\frac{12ʒce — + — 15ʒ — — 6ce}{21ʒʒ — + — 6ce}$ will bee by reduction. $\frac{4ce — + — 5ʒ — — 2℈}{7ʒ — + — 2℈}$

And so in all other fractions, where the nombers bee commensurable.

But if any one nomber, bee incōmensurable with the other, then can there be made no reduction in the nombers. Yet in the signes *Cossike*, there maie be a reduction, other to greater, or to smaller signes: For those signes be euer commensurable.

And there is no exception, but thei maie bee reduced to smaller quantities, excepte any one quantitie of theim bee. ℈. that is a nomber. For that can bee no smaller. And therfore none other maie be altered, sith euery one must be abated alike.

And looke how moche, the smalleste quantitie of that fraction, is aboue a number, so moche maie thei all bee abated: for thei are neuer reduced to the smalleste, till one of them be a nomber.

Scholar. And why maie not this reduction, serue for whole *Cossike* nombers?

Master. Bicause the whole nomber, doeth not cō

sist

ſiſt of a proportion, as the fraction doeth, and ſo maie bee expreſſed in diuerſe termes: but it importeth one ſomme certaine, whiche maie nother bee increaſed, nor decreaſed, but it will chaunge his valewe, and alter his office.

And if I ſaie: a foote is $\frac{1}{3}$ of a yarde, I maie ſaie as truely, increaſyng bothe nombers, in the like proportion, a foote is $\frac{4}{12}$ of a yarde: or in leſſer termes: a foote is $\frac{1}{3}$ of a yarde.

But when I ſaie in whole nomber, a yarde is. 3. foote, or a foote is. 12. ynches, I ſaie truely: and if I doe increaſe or abate any of thoſe nombers, my wordes will be falſe.

So although in this nomber. 8.ſʒ. —+—. 6.ℭ. ———10.ʒ. by reaſon of bothe nombers and ſignes, there might bee a reduction, yet bicauſe it is a whole nōber, it ſhould therby bee abated moche: as here you maie ſee. 4.ℭ. —+— 3.ʒℭ. ———. 5.ϙ. whiche by reſolution into vulgare nombers, 2. beyng ſette as the roote, doeth make. 32 —+— 6. ———. 5. that is. 33. and the other nomber before, doeth yelde by the like reſolution. 256 —+— 48. ———. 40. that is. 264. and is 8. tymes ſo moche as the other.

Scholar. I perceiue now good reaſon, why reduction ſerueth for fractions onely. And if there bee noe more difficultie in it, then you haue declared. I can worke it eaſily.

Reduction in ſignes onely For other the reduction cōſiſteth in the ſignes *Coſſike* onely, as $\frac{10\,ʒʒ}{13\,ℭ}$ where the nombers bee vncommenſurable, and therfore can not bee altered to any leſſer termes. But the ſignes *Coſsike* maie bee abated by. 3. denominations: ſeyng the ſmalleſte of them, is ſo many in order aboue. ϙ. And therfore it maie be reduced to $\frac{10\,ʒ}{13\,ϙ}$.

Reduction in nōbers onely Other els ſecondarily, the reduction conſiſteth in the nombers onely, when the nombers be communicante.

cante. And the signes *Cossike* bee all redy at the leaste: as when one of theim is. ρ. So $\frac{16ȼ}{12ρ}$ will bee reduced to. $\frac{4ȼ}{3ρ}$.

Or els thirdly, the reduction maie bee wroughte, bothe in signes, and also in nombers. When all the signes be aboue. ρ. and the nombers be communicant. So $\frac{15ʒ}{35ʒȼ}$ maie be reduced well unto. $\frac{1ρ}{7ze}$. *Reduction in signes and nomberses.*

Master. Yet one forme of reduction more, I will showe you, where not onely the like worke maie be, but also the nomber maie be broughte from his composition, to a more simplicitie, by abatyng some of his partes. *An other reduction.*

As this nomber $\frac{6ȼ + 18ʒ}{36ȼ + 24ʒ}$ maie bee reduced, firste by his nombers to $\frac{1ȼ + 3ʒ}{48ȼ + 12ʒ}$.

Secondarily, by his signes it maie be altered thus. $\frac{3ze + 9ρ}{48ʒ + 12ȼ}$

Thirdely, by abatynge the nombers, that followe signe of compositi (that is —+—) it maie be brought to. $\frac{1ze}{4ʒ}$. or. $\frac{3ρ}{4ȼ}$. whiche fractions, kepe the self same proportion, that the firste fraction did.

Likewaies with the signe of ———. nombers residualles, maie bee reduced. As. $\frac{6ȼ - 18ʒ}{48ȼ - 24ʒ}$. will bee reduced, as the other was to $\frac{3ρ}{4ȼ}$.

Scholar. This is unto me a maruellouse mater, that those. 2. contrary nombers, should be reduced to one fraction.

Master. The like happeneth in vulgare nombers. For. $\frac{18 + 6}{24 + 8}$. will bee reduced to $\frac{3}{4}$. For firste it maketh $\frac{24}{32}$ and then $\frac{3}{4}$. So likewaies $\frac{18 - 6}{24 - 8}$ will make firste $\frac{12}{16}$ and then $\frac{3}{4}$.

And the reason of it, doeth depende of the. 19. proposition, of the fifth booke of *Euclide*, where it is written thus.

If the proportion of the abatemente vnto abatemente be, as the whole is in proportion to the whole. Then shall the residue bee in like proportion to the residue, as the whole is to the whole.

That is in the laste example. As. 18. is vnto. 24. so is 6 vnto 8. Therfore shall 12 be to 16. as 18. is to 24.

And for to exercise you the better, loe, here are one or twoo examples more, of the like reduction.

7ℭ ——+—— 14ʒ
8ſʒ ——+—— 16ʒʒ maketh 7ℭ/8ſʒ or 7ꝯ/8ʒ. Again 192ꝯ/28ʒ ———— 48ʒℭ/7ℭ yeldeth 192ꝯ/28ʒ or 48ꝯ/7ʒ.

But this muste you farther marke, that in *Cossike* nombers, not onely the nombers, but also the *Cossike* signes must bee, accordyng to *Euclides* proposition.

Scholar. That doe I see.

For in the laste example: As. ꝯ. is to. ʒ. so. ʒℭ. is to. ℭ.

And in the nexte example before: As. ℭ. is to. ſʒ. so is. ʒ. to. ʒʒ.

Likewaies in the other examples, as ℭ is to ʒℭ so is. ʒ. to. ſʒ.

Al this is good and reasonable.

Master. Now doe you see, bothe the maner of reduction, and also some reason for it. Therfore I will procede, to declare the woorke of Addition.

Of Addition and Subtraction.

IN Addition there is nothynge moare, then you haue learned before: For as for the multiplications of the denominators together, and then crosse waies with the numerator of thother, is iuste agreable with the reductions of Abstracte fractions, to bryng theim to one common denominator

nominator.

And then the numerators added together, dooe make the newe numerator in addition.

And likewaies the lesser numerator, subtracted frō the other, doeth make the numerator in subtraction: wherfore a fewe examples maie suffice.

Examples of Addition.

$\frac{54.ʒ. —+— .28.ȼ.}{\frac{6}{7}.ʒ. \text{ to } \frac{4}{9}.ȼ.}$ 63.

$\frac{40.ʒ. —+— .42.ʒʒ.}{\frac{5}{6}.ʒ. \text{ to } \frac{7}{8}.ʒʒ.}$ 48.

That is in smaller termes. | 20.ʒ.—+—.21.ʒʒ. / 24.

Here you see hów the.2.fractions be sette betwene 2. lines: and vnder the nethermoste line, is sette the newe denominator: and ouer the higher line, are set the.2.newe numerators ioyned in one.

The firste of them, can not be reduced to any smaller termes, bicause the nombers be not all.3. commēsurable: & the denominator, also is a nomber *Abstract*.

The seconde hath also a nomber *Abstracte* for his denominator, and therfore there can be noe reduction in signes: but the nombers all.3. beyng commensurable, & diuisible by.2.maie be reduced, as there you see.

More examples of Addition.

16.ſʒ.—+—.4.ȼ.

12.ſʒ.—+—.9.ȼ. to 4.ſʒ.——.5.ȼ.

20.ʒȼ. 20.ʒȼ.

20.ʒȼ.

That is in smaller termes.

4.ȝ.—+—.1.℞.
―――――――――
5.cℭ.

Here is noe multiplication wroughte, bicause the denominators are like.

Another Example of Addition.

5.ȝcℭ.—+—.20.cℭ.———.3.ʃȝ.
―――――――――――――――――――――
5.ȝ.cℭ.—+—.3.ʃȝ.
―――――――――――
6.cℭcℭ.

to

20.cℭ.———6.ʃȝ.
―――――――――――
6.cℭcℭ.
―――――――――――――――――――――
6.cℭcℭ.

That is in lesser termes.

5.cℭ.—+—.20.℞.———.3.ȝ.
―――――――――――――――――――――
6.ȝ.cℭ.

Here is noe multiplication, nor reduction to one common denominator: sith thei bee one all ready: nother can the nombers be reduced, to any other lesser: but the quantities onely be reduced as you see.

Scholar. I praie you let me proue.

Another Example.

80.bʃȝ.—+—90.ȝcℭ—+—60.ȝcℭ———30.ʃȝ.
―――――――――――――――――――――――――――
8.cℭ—+—9.ȝ.
―――――――――
10.cℭ.

to

6.cℭ.———.3.ȝ.
―――――――――
10.ȝȝ.
―――――――――――――――――――――――――――
110.bʃȝ.

That is

Master. Marke your worke well, before you reduce it.

Scholar. I see my faulte: I haue sette.2. nombers seuerally, with one signe *Cossike*: by reason I did not foresee, that.cℭ.multiplied with.cℭ.doeth make the like

like quantitie, as. ʒʒ. multiplied by. ʒ. Therefore it should be thus.

$$\frac{80.b\!ſʒ.\ -\!\!+\!\!-\ .150.ʒȼ.\ ——\ .30.ſʒ.}{110.b\!ſʒ.}$$

Whiche maie bee reduced, by meane of the nombers, to this somme.

$$\frac{8.b\!ſʒ.\ -\!\!+\!\!-\ .15.ʒȼ.\ ——\ .3.ſʒ.}{11.b\!ſʒ.}$$

And now considering the *Coßike* signes, and workyng as I haue marked you to dooe: That is to abate the leaste signe, out of theim all: bicause. ſʒ. is here the leaste, I abate it out of. bſʒ. and there resteth. ʒ. and so doing with the other signe. ʒȼ. there remaineth. ze & then ſʒ out of ſʒ doeth leaue. 𝔑. or nōber: So will the fraction bee thus: $\frac{8ʒ.-+-15ze.——3𝔑}{11ʒ}$ by reduction in signes and nombers also.

Master. Seyng you haue so well marked the reduction of the signes (whiche followeth the forme, taught before in diuision) I thinke it not nedefull, to staie any longer aboute this.

Wherfore we will goe forward to subtraction, after that I haue admonished you of fractions, in appearaunce simple, whiche in dede by addition, bee come compounde. As this $\frac{2}{3}$ȼ. added to $\frac{3}{4}$ʒ. maie firste be added by the common signe of addition, thus. $\frac{2}{3}$ȼ. —+—. $\frac{3}{4}$ʒ. whiche by reduction, vnto one denomination, wil be thus written. $\frac{8ȼ.-+-9ʒ}{12}$

But as this is easie inough to vnderstand, so maie it helpe often times, for spedie worke, as well in additiō, as in subtractiō, by the onely addyng of the signe.

As if I would subtracte this fraction $\frac{5}{6}$ʒʒ. out of *Subtraction.*
$\frac{2}{3}$ʒȼ.

$\frac{9}{15}$ ʒ ȼ. I maie write it thus. $\frac{9}{10}$ ʒ ȼ. —— $\frac{5}{7}$ ʒʒ. And so is the Subtraction wroughte.

Yet maie you reduce theim, to one denomination, if you will, after the same forme, as you did in addition. And then will it bee. $\frac{63\text{ʒȼ} — 50\text{ʒʒ}}{70}$ whiche can not bee reduced to any smaller termes, bicause the nombers are not commensurable: and one of theim (that is to saie, the denominator) is a nomber *Abstracte*.

Scholar. I see in this, there is no difference from Addition, but in the signes. —+— and. ——. wherfore I will proue an other example, by your leaue.

I would subtracte $\frac{3}{4}$ʃʒ. out of. $\frac{4}{5}$ ʒʒ. and it will bee at the firste $\frac{4}{5}$ ʒʒ. —— $\frac{3}{4}$ʃʒ. And by reduction $\frac{16\text{ʒʒ} —— 15\text{ʃʒ}}{20}$.

Master. Your worke is well doen, accordyng to your firste meanyng: But as the numerator of this laste reduction doeth declare, it can not bee well, that 15.ʃʒ. maie bee abated out of. 16.ʒʒ. For the greater absolutely, can not well be abated out of the lesser: and therfore you might rather haue abated $\frac{4}{5}$ʒʒ. out of. $\frac{3}{4}$ʃʒ.

Scholar. I see it well now: for the ʃʒ. is alwaies double or triple, or yet more tymes greater, then the ʒʒ. Bicause the. ʃʒ. commeth by multiplication of the ʒʒ by his firste roote.

Master. Yet here in is discretion to be vsed, for in fractions, sometyme the nomber of the greater signe maie be the lesser. As for example $\frac{3}{16}$ʃʒ is lesser then $\frac{3}{4}$ʒʒ. as by resolution you maie proue, accompting 2. for the common roote.

Scholar. 2. beyng the roote. 32. is the. ʃʒ. and his $\frac{3}{16}$ maketh. 6. then. $\frac{3}{4}$ ʒʒ. beeyng. 12. dooeth appere double to it: and therefore greater by moche.

If I doe by the like resolutiō, proue the other fractions before, $\frac{3}{4}$.ʃʒ. will bee. 24: and $\frac{4}{5}$ ʒʒ. will bee $12\frac{4}{5}$: whiche is lesser moche.

So, I perceiue the greatnesse and smalnesse of the fractions, must be considered, as well in the nombers as in the *Coßike* signes. And farther, if their fractions be nigh of one greatenesse, or the fraction of the lesser signe the greater, then can not the subtraction, appeare reasonable.

Master. That is true, if those. 2. fractions stande alone: els beyng partes of other nombers, it maie appeare reasonable inough. As in this example of compounde fractions. $\frac{4}{10}\,ce.\;+\;\frac{3}{4}\,ʒ.$ maie bee abated out $\frac{3}{5}\,ce.\;+\;\frac{3}{5}\,ze.$ and yet in the abatemente after $+$ not onely the nomber $\frac{3}{4}$ is greater, then $\frac{3}{5}$ in the other, but also, the *Coßike* signe. ʒ. is greater then the other *Coßike* signe. ze.

Scholar. I consider it to be so: and yet $\frac{3}{5}\,ce.$ doeth so moche excede $\frac{4}{10}\,ce.$ that it supplieth sufficiently the other defaulte: els could it not be well doen.

But for this worke, I must craue your helpe: bicause I haue not seen the like.

Master. You maie doe in this, as I saied before, generally for all subtractions.

Set doune bothe nombers in due order, so that the abatemente dooe folowe in order: and putte betwene them the signe of subtraction: as thus.

$$\frac{3}{5}\,ce\;+\;\frac{3}{5}\,ze.\;-\;\frac{4}{10}\,ce.\;+\;\frac{3}{4}\,ʒ.$$

Howbeit, if you will firste reduce euery cōpounde fraction, into one fraction, it will seme more apte. As thus. $\frac{3}{5}\,ce.\;+\;.\frac{3}{5}\,ze.$ beyng reduced by additiō will make $\frac{15ce\;+\;15ze}{25}$. and by farther reduction of nombers. $\frac{3ʒ\;+\;3ze}{5}$. Likewaies $\frac{4}{10}.\,ce.\;+\;\frac{3}{4}\,ʒ.$ will make by the firste addition. $\frac{16ce\;+\;30ʒ}{40}$. and by farther reduction $\frac{8ce\;+\;15ʒ}{20}$.

Now ioyne theim together, with the signe of subtraction, and thei will stande thus.

$$3ce\;+\;3ze\;-\;\frac{8ce\;+\;15ʒ}{20}.$$

Scholar.

Scholar. This doeth appeare verie straunge vnto me: but by vse I shall finde it more familiare: Seeyng I see the reason of this worke, to agree with the worke of common fractions.

The proofe. But for proofe of it, I will resolue eche worke, into nombers absolute, accountyng. 2. for a roote.

Master. So shall you finde it true: But for easie woorke, take rather. 10. for the roote.

Scholar. I thanke you for your aide.

Then if. 10. be the roote, the square will be. 100. and the *Cube*. 1000. Now $\frac{3}{5}$ ℭ. that is $\frac{3}{5}$ of. 1000. is. 600. And $\frac{3}{5}$ of. 10. whiche is the roote, will bee. 6. whiche bothe put together, doe make. 606. and that is the greater nomber.

Then for the lesser $\frac{4}{10}$ ℭ. are in this example. 400. For the Cube beeyng. 1000. his $\frac{1}{10}$ is. 100. Againe the square beyng. 100. $\frac{3}{4}$ ʒ. must nedes bee 75. whiche beeyng put vnto. 400. dooeth make. 475.

Then doe I abate. 475. out of. 606. and there will reste. 131. How now.

606.
475.
131.

Master. I perceiue you staie, as beeyng astonisshed, bicause in the former worke, there is not lefte a remainer: But the. 2. firste sommes onely altered by reduction, and ioyned together, with the signe of subtraction: wherein if you had continued your worke, you should haue founde the same nombers.

For. 3. ℭ. must nedes bee. 3000. seyng. 1. ℭ. is a 1000. And also. 3 ℞. are. 30: whiche bothe added together, make. 3030. Diuide them by. 5. (as the denominator would) and it will be. 606. as the valewe of the firste fraction.

Then come to the later nomber: and you maie sone thinke that. 8. ℭ. are. 8000. And. 15. Squares are 1500. adde them together, and thei will make 9500. whiche must bee diuided by. 20. (as the denominator

minator (importeth) and there will amounte. 475. the valewe of the lesser fraction: whiche nombers appeare the same, that were before: and thereby the worke is good.

11
9800(475
2220

But if you will bryng it to a remainer, doe thus. Reduce these. 2. newe fractions, into one denomination.

Scholar. That can I doe, by multiplying the numerators together: that is. 20. by. 5. and thereof commeth. 100. whiche shall be the common numerator: then must I multiplie in crosse waies, the numerator of the firste, by the denominator of the seconde, and contrarily.

So for the firste numerator I worke thus. And thereby dooeth amounte (as you see) 60.cℭ —+— 60.ℨℨ. And for the seconde numerator, I multiplie. 8.cℭ —+— 15ℨ. by. 5. and there doeth rise. 40.cℭ —+— .75.ℨ. eche of theim hauyng one common numerator. 100.

$$\begin{array}{l} 3.c\mathfrak{C}. —+— .3.\mathfrak{Z}\mathfrak{Z}. \\ 20. \\ \hline 60.c\mathfrak{C} —+— 60.\mathfrak{Z}\mathfrak{Z}. \end{array}$$

Wherfore, seyng bothe nombers, haue one denominator, I shall abate the lesser numerator, out of the greater, as here in example is set forthe: and then the

$$\begin{array}{l} 60.c\mathfrak{C}. —+— .60.\mathfrak{Z}\mathfrak{Z}. \\ 40c\mathfrak{C}. —+— .75.\mathfrak{Z}. \\ \hline 20.c\mathfrak{C}. —+— 60.\mathfrak{Z}\mathfrak{Z}. ——— .75.\mathfrak{Z}. \end{array}$$

remainer will bee (as you see). 20.cℭ. —+— 60.ℨℨ. ——— 75.ℨ. unto whiche I muste adde the common denominator. 100. and it will be thus.

$$\frac{20.c\mathfrak{C}. —+— .60.\mathfrak{Z}\mathfrak{Z}. ——— .75.\mathfrak{Z}.}{100.}$$

Now proue whether this remainer, doe not agree to thother remainer before, in your trial: which was 131

Scholar. 20.ȼ do make. 20000. & 60.ze. yelde 600: those 2 sommes I must adde together, bicause of the signe. —+—. and it will be. 20600. then. 75.ʒ. are. 7500. whiche I must abate from the former somme of. 20600. and there will remaine. 13100. for the numerator, and 100. for the denominator, thus. $\frac{13100}{100}$.

20600.
7500
13100

Master. And what doe you thinke of it?

Scholar. By that I learned in the vulgare fractions, I knowe that it is iuste. 131. and so doeth it agree precisely, with the former proofe.

Master. Well yet for moare exactnesse in this worke, I will farther reduce that fractiō, by diuiding the numerator by the denominator: wherfore. 20.ȼ diuided by. 100. doeth yelde. $\frac{1}{5}$ȼ. And. 60.ze. diuided by. 100. doeth make $\frac{3}{5}$ze. And lastly. 75.ʒ. diuided by. 100. will yelde $\frac{3}{4}$ʒ. so is the same fraction so reduced $\frac{1}{5}$ȼ—+—$\frac{3}{5}$ze.——$\frac{3}{4}$ʒ. And now trie what that is, by the former proofe.

Scholar. I maie sone perceiue, that $\frac{1}{5}$ȼ. is. 200. when the Cube is. 1000: And so $\frac{3}{5}$ze. is. 6. whiche I must adde together, and it will be. 206. Then $\frac{3}{4}$ʒ. is 75. whiche if I dooe abate from. 206. there will remain. 131. agreably as before. And so is this woorke fully examined.

Master. Yet will I propounde one or two examples more, partly to practise your memorie, and partly to admonishe you, if you happen to see any soche misse wroughte, in some other bokes (as I haue doen) how you maie amende the erroure, and not staie at it.

Firste take this example. I would subtracte.

$\frac{48.ꝯ.}{12.ze.——3.ʒ.}$ out of $\frac{48ꝯ.}{7.ʒ.}$

Scholar.

Scholar. I muste firste multiplie the denominators together, and so it will make, as here is sette foorthe 84.ce.——21.ʒʒ.

12.ze——.3.ʒ.	
7.ʒ.	
84.ce——21.ʒʒ.	

Then I multiplie the numerator of the firste, by the denominator of the seconde, and it will bryng foorthe. 336.ʒ: whiche is the numerator for the abatemente.

48.ꝗ.
7.ʒ.
336.ʒ.

Afterward I multiplie the numerator of the seconde, by the denominator of the firste, and it will make 576.ze.——144.ʒ.

12.ze——3ʒ.	
48.ꝗ.	
96	
48	
576.ze.——144.ʒ.	

Now if I subtracte that 336.ʒ. out of. 576.ze. ——144.ʒ. it will bee 576.ze——480.ʒ. for the abatemēte that should be subtracted now, is sette after the signe —— with the former somme of. 144.

Finally, to make the remainer complete, as that laste nomber is the numerator, so unto it I must adde the common denominator. 84.ce. ——.21.ʒʒ. and it will bee. $\frac{576ze——480ʒ}{84ce——21ʒʒ}$. that is in lesser termes $\frac{192ꝗ——160ze}{28ʒ——7ce}$.

Master. Now proue your cunnyng in this sōme, $\frac{48ꝗ}{12ze——3ʒ}$, subtractyng it out of. $\frac{232ze—+—576ꝗ}{84ʒ——21ce}$.

Scholar. Firste I must reduce theim, to one common denominator: by multipliyng bothe denomina-

	84.ʒ.——.21.ce.
	12.ze.——.3 . ʒ.
	1008.ce.——.252.ʒʒ.
	63.sʒ.——.252.ʒʒ.
63.sʒ.—+—.1008.ce.——.504.ʒʒ.	

 tors

tors together. And so will it be. 63.ʃʓ ─┼─ 1008 ℭ ─── 504.ʓʓ. as by speciall worke, I haue here proued.

Then doe I multiplie the numerator of the totall, by the denominator of the abatemente, as here also I haue perticularly set forthe in worke, for my owne ease, and auoidyng of erroure: And so I finde it to be 1056.ʓ. ─┼─ .6912.ze. ─── .696.ℭ. whiche shall bee the numerator of the totalle.

232.ze. ─┼─ .576.ꝗ.
12.ze ─┼─ .3.ʓ.
___
2784ʓ. ─┼─ .6912.ze.
── 696.ℭ. ─── 1728.ʓ.
___
1056.ʓ. ─┼─ .6912.ze. ─── .696.ℭ.

Then doe I multiplie the numerator of the abatemēte, by the denominator of the totalle (whiche thing is easily dooen, bicause the one nomber, is a nomber *Abstracte*) and so haue I for the numerator of the abatemente. 4032.ʓ ─── 1008.ℭ.

And seyng these two nombers, haue one common denominator, I shall abate the lesser numerator, out

1056.ʓ. ─┼─ 6912.ze. ─── .696.ℭ.
4032.ʓ ─────── .1008.ℭ.
___
6912.ze ─┼─ .312:ℭ. ─── .2976.ʓ.

of the greater, & so will there be left for the numerator of the remainer 6912 ze ─┼─ 312 ℭ ── 2976 ʓ unto whiche, I shall adde the common denominator, and then will it be.

6912.ze. ─┼─ .312.ℭ. ─── .2976.ʓ.
___
63.ʃʓ. ─┼─ .1008.ℭ. ─── .504.ʓʓ.

That

That is in leſſer termes.

$$\frac{2304.\text{ʃ}.+.104.\text{ȝ}.-.992.\text{ꝛ}.}{21.\text{ȝȝ}+.336.\text{ȝ}.-.168.\text{ℭ}.}$$

Maſter. You haue wrought it well. And hereby I coniecture, that you are experte inough in ſubtraction. Wherfore now we will goe in hand, with multiplication and diuiſion.

Of Multiplication.

AND firſte, concernyng multiplication, here is no more to bee ſaied, then hath been taughte before. *Multiplication.*

For the nombers ſhall bee multiplied, as common fractions are wonte to bee: that is to ſaie, numerator, by numerator, and denominator, by denominator.

And for the chaunge of their denominations *Coſſike*, the rules giuen before ſhall ſuffice: ſo that a fewe examples ſhall ſufficiently inſtruct you, in the worke of it. As this for the firſte.

$$\frac{20.\text{ȝ}.+19.\text{ꝛ}.}{6.\text{ℭ}-3.\text{ʃ}.}$$

$$\frac{\begin{array}{r}120./\text{ȝ}.+114.\text{ȝȝ}.\\ -.60.\text{ȝ}.+57.\text{ꝛ}.\end{array}}{120./\text{ȝ}+114.\text{ȝȝ}-60.\text{ȝ}+57\text{ꝛ}.}$$

Where I doe multiplie.

$$\frac{20.\text{ȝ}.+19.\text{ꝛ}.}{31.\text{ℭ}.} \text{ by } \frac{6.\text{ℭ}.-.3.\text{ʃ}.}{4.\text{ȝ}.}$$

And ſecte I ſhall multiplie, numerator by numerator:

rator: where. 20. ʒ. multiplied by. 6. ce. doe make 120. ʃʒ. as the former table of multiplication, for chaunge of *Coßike* signes doeth declare. And so in all the reste, there is no difficultie, if you remember that, that you haue learned before.

Scholar. I perceiue it well. And so the whole newe numerator will bee. 120. ʃʒ. —+— 114. ʒʒ. ——. 60. ʒ. —+—. 57. ze. And the denominator will be. 124. ʃʒ.

So will the whole fraction bee.

$$\frac{\text{120. ʃʒ. —+— 114. ʒʒ. —— 60. ʒ. —+— 57. ze.}}{\text{124. ʃʒ.}}$$

That is not to bee reduced to smaller termes of nombers, bicause thei be uncommensurable, but in *Coßike* signes, it mighte bee broughte to one lesser, as.

$$\frac{\text{120. ʒʒ. —+— 114. ce —— 60. ze. —+— 57. φ.}}{\text{124. ʒʒ.}}$$

Now wil I proue an other nomber, as fortune doeth offer it to mynde. That is $\frac{\text{32ce ——— 28ʃʒ}}{\text{21ʒ ——— 57ze}}$. to bee multiplied by. $\frac{\text{12ʒ —+— 9φ}}{\text{36ze}}$.

An Absurde nomber expresseth lesse then naught

Master. It appeareth that you take theim, at all aduentures. For your firste nomber, semeth to be an *Absurde* nomber. Seyng his numerator, is lesse then naughte, in appearaunce. And then maie it not bee diuided by any nomber: and moche lesse by so greate a denominator.

Scholar. It is easie to see, now that I am admonished thereof. For it is not possible, that any *Sursolide* nomber, can bee lesse then fower tymes so moche, as the *Cube* of the same nature. Seeyng euery *Sursolide* is made, by multiplying the *Cube* by the *square* of the like *Roote*, but lesse then. 4. is there no *Square*. And therefore euery *Sursolide*, doeth excede his *Cube* fower times at the leaste.

So that. 32. ȼ. ——— 8. *ſz*. were nothyng, and so is an *Absurde* nōber. And therfore. 32. ȼ ——— 28. *ſz*. is moche lesse then nothyng, and is therby an *Absurde* nomber also.

Master. Yet maie your example serue, to teache and practise multiplication by, as well as any other.

And farthermore, I will tell you by this occasion, that I spake to you, more after the opinion of the cōmon nomber of artes men, then after my owne iudmente.

Scholar. I might thinke so, by termynge of your sentence: but yet was your saiyng true.

Master. Yet maie that fraction stand well, if you take a brokē nomber *Abstracte* for the roote. Although in whole nombers, it bee an *Absurde* nomber.

Scholar. That will I proue, by settyng. $\frac{3}{4}$. for a *Roote*. Then will the *Square* be $\frac{9}{16}$. and the *Cube*. $\frac{27}{64}$. Also the *Square of squares* will bee. $\frac{81}{256}$. And the *Sursolide* $\frac{243}{1024}$.

$\frac{3}{4}$ The *Roote*.
$\frac{9}{16}$ The *Square*.
$\frac{27}{64}$ The *Cube*.
$\frac{81}{256}$ The *zenzizenzike*
$\frac{243}{1024}$ The *Sursolide*.

And now to proue by resolution, how my nomber will rise, I take. 32. ȼ. that is. $\frac{864}{64}$, or 13. $\frac{1}{2}$. whiche I note as the firste somme. Then I take likewaies 28. *ſz*. whiche yeldeth $\frac{6804}{1024}$, that is. 6 $\frac{165}{256}$. And now I see that I maie abate it very well, out of. 13 $\frac{1}{2}$.

Master. So maie you see, that as in whole nombers, euer moare the greater *Coßike* signes, will haue the greatest nombers: So in fractions resolued by *Cossike* signes, the greatest fraction, aunswereth to the leaste signe: and the leaste fractiō, agreeth to the greateste signe.

The reason of it is this. That the moare any fraction is multiplied by a fraction, the lesser it wareth. For as whole nombers by multiplication, maie increase infinitely: so fractions by multiplication, maie decrease

decrease infinitely.

But before wee passe from multiplication, I will proue you with one example moare. I would haue $\frac{19\,ce + 3\,ze - 5\,\phi}{7\,z - 6\,\phi}$ multiplied by $\frac{24\,ce}{5\,z} - .ze$.

Scholar. I am troubled with the multiplier. For I knowe not what to make of it?

Master. You doubte (I thinke) of the numeratiō of it, bicause you had not the like example before: for it is a mixte nomber of a fraction, and a whole nomber. But seyng the signe of abatemente is set against the whole fraction, and nother againste the numerator, nor denominator, therfore must that 4.ze. be vnderstande, to be abated out of the full fraction.

Scholar. Now I perceiue the mater. For there might be. 3. diuerse formes, to place that abatemente. As here I haue set them. $\frac{24\,ce - 4\,ze}{5\,z}$ & $\frac{24\,ce}{5\,z - 4\,ze}$.

And as it was set by you, $\frac{24\,ce}{5\,z} - 4.\,ze$. whiche I will resolue into absolute nombers, to see their difference the better. And so, taking 3. for the roote, these will be their. 3. formes.

The firste. For the firste $\frac{648 - 12}{81}$. or els $\frac{636}{81}$ that is $\frac{212}{27}$.

For the seconde $\frac{648}{81 - 12}$ or els $\frac{648}{69}$ that is $\frac{216}{27}$.

And for the thirde nomber, whiche is our specialle nomber. $\frac{648}{81} - 12$. that is. $8. - .12$. and is an *Absurde* nomber. For it betokeneth lesse then naught by. 4.

Master. If you would haue it no *Absurde* nomber, you must increase the proportion of the fraction, by augmentyng the numerator, or abatyng the denominator, or els thirdly, by abatyng the nomber, after the signe of abatemente. As $\frac{40\,ce}{5\,z} - 4.\,ze$: or els secondarily, thus. $\frac{24\,ce}{4\,z}. - .4.\,ze$. or thirdely $\frac{24\,ce}{5\,z}. - 2.\,ze$.

Howbeit for examples sake, you maie woorke, as well with *Absurde* nombers, as with any other.

But

But for you ease, I will shewe you the woorke of this example, in twoo formes.

First, you shall multiplie the firste whole nomber, by the fraction of the seconde nomber, that is. $\frac{19.ℭ.—+—3.ƺ.——5.℞}{7.ʒ.——6.℞}$ by $\frac{24.ℭ.}{9.ʒ.}$ and it will bee.

$$\frac{456.ʒℭ.—+—72.ʒʒ.——.120.ℭ.}{63.ʒʒ.——.54.ʒ.}$$

As here in woorke you maie see it plaine.

19.ℭ.—+—3.ƺ.——.5.℞.
24.ℭ.
―――――――――――――
456.ʒℭ—+—.72.ʒʒ.——.120.ℭ.

7. ʒ.——.6.℞.
9. ʒ.
―――――――――
63.ʒʒ——54.ʒ.

That is in lesser termes, bothe of nombers, and of signes *Cossike.*

$$\frac{152.ʒʒ.—+—.24.ʒ.——.40.ƺ.}{21.ʒ.——.18.℞.}$$

And this is the firste parte of your somme.

Then for the nexte parte, multiplie your firste nõber, that is $\frac{19.ℭ—+—3.ƺ——5.℞}{7.ʒ——6.℞}$ by the abatement of the seconde nomber, that is by .——. 4. ƺ. and it will be.

$$\frac{20.ƺ.——.76.ʒʒ.——.12.ʒ.}{7.ʒ.——.6.℞.}$$

As by this woorke you maie see.

19.ℭ. —+— .3.ℨℨ. ——— .5.ꝛ.
——— .4.ℨℨ.

20.ℨℨ. ——— .76.ℨ ℨ. ——— .12.ℨ.

whiche being reduced to the denomination of the for-mer nomber, will be tripled (sith that denominator is triple to this) and so will it be 60.ℨℨ. ——— .228.ℨ ℨ. ——— .36.ℨ. over 21.ℨ. ——— .18.ꝛ. Now adde those two nombers together, by puttyng their bothe numerators in one, and it will be.

20.ℨℨ. ——— .76.ℨ ℨ. ——— .12.ℨ.
21.ℨ. ——— .18.ꝛ.

As here appeareth in woorke.

152.ℨ ℨ. —+— .24.ℨ. ——— .40.ℨℨ.
60.ℨℨ. ——— .228.ℨ ℨ. ——— .36.ℨ.

20.ℨℨ. ——— .76.ℨ ℨ. ——— .12.ℨ.

whiche will not bee reduced to any smaller fraction, bicause the nombers be incommensurable. and one of the *Cosike* signes is. ꝛ. And so is that the somme of the multiplication.

An other waie you maie woorke it, and all soche like, by reducynge the multiplier, into one uniforme fraction. As here in. 24.ℭ. over 9.ℨ. ——— 4.ℨℨ. you shall multiplie ——— 4.ℨℨ. by. 9.ℨ. whiche is the former denominator, and it will be ——— .36.ℭ. Then putte that to. 24.ℭ. over the line, and set the common denominator. 9.ℨ. under the line, and it will bee in one fraction reduced 24.ℭ. ——— .36.ℭ. over 9.ℨ.

Scholar. Here I maie see at the firste vewe, that this fraction is an *Absurde* nomber: for the abatement after the signe ———. is greater then the nomber be-

fore

fore it.

Master. That was cõfessed before. But yet make you worke the example by it.

Scholar. That is true: and so will the numerators, beeyng multiplied together, make exactly, 60.ꞓ.——228.ȝꞓ.——.36.ȝȝ. As here in example of worke, I haue set it, for my owne ease and certentie.

$$
\begin{array}{l}
19.ꞓ.+.3.℞.—.5.φ.\\
24.ꞓ.—.36.ꞓ.\\
\hline
456.ȝꞓ.+.72.ȝȝ.—.120.ꞓ.\\
—684.ȝꞓ.—108.ȝȝ.+.180.ꞓ.\\
\hline
60.ꞓ.—.228.ȝꞓ.—.36.ȝȝ.
\end{array}
$$

And that is the newe numerator.

And then for the seconde nomber, if the firste denominator.7 ȝ——6.φ. be multiplied by the seconde denominator, 9.ȝ. it is easily seen, that thei will make.63 ȝȝ——54 ȝ. whiche shall be the newe denominator.

And so the intere fraction shall bee.

$$
\frac{60.ꞓ.—.228.ȝꞓ.—.36.ȝȝ.}{63.ȝȝ.—.54.ȝ.}
$$

That is in the smalleste nombers and figures *Coßike*. $\frac{2c℞—13ȝ—13ȝ}{13ȝ—18φ}$: whiche somme, dooeth in all thynges fully agree, with the former nomber that you wrought.

Master. Proue theim bothe by resolution: And then shall you knowe, the reason of their agremente.

Scholar. I see that the woorke of the denominators, doeth agree. Wherfore I will take.3.for a roote to proue how the worke of the numerators wil agree And so for.19.ꞓ.I shall haue.513. And for.3.℞.

I shall haue.9.to be added to.513. And so haue I.522
out of whiche somme I must abate.5.
And then remaineth.517.to bee multi-
plied by.24.ℭ.that is by.648. And the
totalle will bee (as here in worke appea-
reth).335016. whiche somme must be a-
bated to a smaller nomber, in like rate as
the other was reduced, firste by partition
into.3. And then will it be.111672. And
again, it must bee diuided by.9. for that is the quan-
titie of a square, by whiche the former reduction, was
wroughte for the *Coßike* signes: and then will it bee.
12408. And that is the firste parte of the first worke.

	648
	517
	4536
	648
	3240
	335016

Then for the seconde parte of that worke, I shall multiplie the firste nombers, that is 517 by the abatemente of the fraction, that is by —— 4ℨ., or —— 12. (sith.3.is the roote) and thereof will come —— 6204. Whiche somme I must triple, as I did his equalle (that is.20.ℨ. —— 76.ʒʒ. —— 12.ʒ.) And so wil it bee —— 18612. Now shall I adde this somme, with the firste parte, whiche was.12408.and it will bee.12408. —— 18612.that is.6204. lesse then nothyng: and is the numerator of the firste woorke.

Wherfore I procede to the seconde woorke, where the numerator of the fraction, beeyng reduced to the common denominator, is.24.ℭ. —— 36.ℭ. whiche is —— 12.ℭ.and in nombers resolute (keping 3.still as a roote) it is —— 324.by whiche if I multiplie.517.it will yelde.167508. And that somme beyng abated, by diuision into.3. and.9. as the other was, or els diuided by.27.whiche is all one, it giueth 6204.as the former woorke did.

Master. Thus I see, you are experte inoughe in multiplication: Wherfore I will shewe you now, the order and forme of diuision.

Of Diuision.

There is noe speciall rule to be giuen, for the worke of Diuision, other then soche as are all ready taughte in other workes of diuisiõ before. Wherfore I will by one or 2. examples, shewe you the worke of it.

The firste example of Diuision.

$$\frac{14.\text{ℭ} + 9.\text{ʒ}.}{15.\text{ℊ}.}$$ to be diuided by $$\frac{5.\text{ʒ}. + 2.\text{ze}.}{3.\text{ze}.}$$

doeth yelde. $$\frac{42.\text{ʒʒ}. + 27.\text{ℭ}.}{75.\text{ʒ}. + .30.\text{ze}.}$$ that is in a lesser fraction, by bothe reductions of nombers & signes.

$$\frac{14.\text{ℭ}. + 9.\text{ʒ}.}{25.\text{ze}. + 10.\text{ℊ}.}$$

An other example.

$$\frac{12.\text{ʃʒ}. - 16.\text{ʒ}.}{2.\text{ℭ}. - .5.\text{ze}.}$$ diuided by $$\frac{19.\text{ʒ}. - .3.\text{ℊ}.}{4.\text{ʒ}. - .5.\text{ℊ}.}$$

doeth make.

$$\frac{48.\text{bʃʒ} + 60.\text{ʃʒ} - 64.\text{ʒʒ} - 80.\text{ʒ}.}{38.\text{ʃʒ} + 15.\text{ze}. - 101.\text{ℭ}.}$$

whose nombers bee incommensurable, and therfore maie not bee reduced, but by abatyng one denomination *Coßike*. And so will it be.

$$\frac{48.\text{ʒℭ} + 60.\text{ʒʒ} - 64.\text{ℭ} - 80.\text{ze}.}{38.\text{ʒʒ}. + .15.\text{ℊ}. - .101.\text{ʒ}.}$$

Scholar. I see that you multiplie crosse waies (as in vulgare fractions) the numerator of the one nomber, by the denominator of the other. And so is diuision of noe difficultie, to hym that remembreth the former rules.

Of the golden rule.

Master.

The golden rule, that is the rule of proportion, should folowe now, by the commō order. But seyng there is no difficultie in it, nother any other forme of woorke, then is in vulgare nombers, I will not staie any tyme aboute it. Saue that for your pleasure, I haue set here certaine examples, as wel in whole nombers *Coßike*, as in broken.

```
32.ʒ.  Z  4.ʃʒ.                 250.ce.  Z  20.zz.
6ce.   Z  3/4 ʒ ce.             26.ʒ.    Z  2 2/25 ce.

5.ʒ. + .3.zz.   Z  4.ce. — .5.ꝯ.
15.ce — .9.ꝯ.   Z  (60ʒce — 111ce + 45ꝯ) / (5ʒ + 3zz)

(3ce + 5ʒ) / 12zz   Z  14.zz. + . 4.ꝯ.
61ʒ — 7ꝯ            Z  (10248ʒʒ + 2928ce — 1176ʒ — 336zz) / (.3ce + 5ʒ)
```

Scholar. These fewe examples, dooe sufficiently teache the forme of the whole rule. So that here neadeth noe farther explication.

Wherfore, if in this arte, there be any forme of extraction of rootes, I praie you to procede therto.

Of extraction of rootes.

Master.

AS in nombers *Abstracte*, euery nomber is not a rooted nomber, but some certaine onely emongest theim, so in nombers *Coßike*, all nombers haue not rootes: but soche onely emongest simple *Coßike* nombers are rooted, whose nomber hath a roote, agreable to the figure of his denomination.

So that. 16. ce. is not a Square nomber, nother hath any roote. For although. 16. bee a square nomber, and hath. 4. for his roote, yet the denomination (whiche is. ce.) hath noe square roote: but. 16. ʒ. is a square nomber: and hath. 4. ꝛ, for his roote.

Likewaies. 8. ce. is a *Cubike* nomber, and his roote is. 2. ꝛ: but. 8. ʒ. hath noe roote. For bicause. 8. hath no square roote, agreable to the signe. ʒ. nother is it a *Cubike* nomber, although it haue a *Cubike* roote, bicause the roote is disagreable from the signe. ʒ.

Scholar. I perceiue that in these nombers, as wel as in all other, the roote beeyng multiplied by it self, will make the nomber, whose roote it is. And therefore can no nomber be called square, or *Cubike*, or any waies els a rooted nomber, excepte the roote of the nomber agree with his signe: Whereby I perceiue well, that. 32. ſʒ. is a rooted nomber, forbicause that 32. hath a *Sursolide* roote, agreable to the signe. So likewaies. 125. ce. is a rooted nomber, seyng 5. is the *Cubike* roote of. 125. But. 27. ʒ. is no rooted nōber.

Master. Thus you vnderstande sufficiently, the iudgemente of rooted nombers, and their knowlege, in simple *Coßike* nōbers, that be vtterly vncōpounde.

Wherfore, for extraction of their rootes, take this brief order.

Extracte

Extracte the roote of your nomber, as if it were abſolute, and put to it. ℞. for the denomination.

So. 27. *Cubes* hath for his roote. 3. ℞.

And. 49. ʒ. hath. 7. ℞. for his roote.

Again, the roote of. 216. ℭ. is. 6. ℞.

Scholar. This I perceiue. And by like reaſon, the roote of. 243. ſʒ. is. 3. ℞. But why dooe you name nõbers *Coßike* vtterly vncompounde? For as I vnderſtande, that there bee nombers compounde, in their ſignes, ſo I ſee that thei maie haue rootes alſo.

As. 16. ʒʒ. hath for his roote. 2. ℞. And likewaies. 64. ʒℭ. hath. 2. ℞. for his roote.

Maſter. And dooe you not ſee, that thoſe compounde nombers, maie haue moare rootes then one? Sith. 16. ʒʒ. hath for his ſquare roote. 4. ʒ. as wel as it hath. 2. ℞. for his *zenzizenzike* roote.

So. 4. ʒʒ. hath for his Square roote. 2. ʒ. And hath no *zenzizenzike* ℞ agreable to his whole ſigne.

Likewaies. 9. ʒℭ. hath no *zenzicubike* roote, accordyng to his whole ſigne: but it hath a ſquare roote agreable to parte of the ſigne, and that is. 3 ℭ.

Scholar. I ſee that alſo. And ſo hath. 8. ʒℭ. noe *zenzicubike* roote, but a *Cubike* roote: whiche is. 2. ʒ.

Maſter. Therfore in cõpoũde ſignes, if the ſigne maie haue ſoche a roote, as the nomber will yelde, it is a rooted nomber, els not.

Whereby you maie perceiue, that if any nomber cõpounde in ſigne, haue a roote agreable to his whole ſigne, then maie it haue alſo, as many rootes, as ther be partes in that compounde ſigne.

So 4096 ʒʒℭ hath not onely a *zenzizenzicubike* roote, whiche is. 2. ℞: but it hath a *Square* roote that is. 64. ʒℭ. And alſo it hath a *Cubike* roote, that is, 16. ʒʒ: Farther more, it hath a *zenzizenzike* roote, whiche is. 8. ℭ. And fourthly, it hath a *zenzicubike* roote, that is. 4. ʒ.

And

And so shall you iudge, of all other like.

Scholar. This shall suffice, as I will practise the mater, at moare leiser. But and if the nombers bee compounde, with signes of addition, is there then any speciall order for their rootes? As in this example. 81.ʒʒ.—+—27.ꝯ. where I haue made eche parte to be a rooted nomber.

Master. In deede. 81ʒʒ. hath bothe a Square roote, and also a *zenzizenzike* roote. But.27ꝯ. hath none of those twoo rootes, although it haue a *Cubike* roote, whiche the other nomber wanteth. And therfore is not that whole nomber, a rooted nomber.

But to the intente, that you maie be the more certein of rooted nombers, I will tell you certein notes, how it maie bee knowen, whether your nomber be a rooted nomber.

Firste, if the nomber annexed to the greatest signe of that compounde *Cosike* nomber, bee not a rooted nomber, the whole nomber can not be a rooted nõber

Secondarily, if the nomber that is ioyned with the leaste *Coßike* signe, be not a rooted nomber, the whole nomber can not be a rooted nomber.

And eche of these bothe rootes (if thei haue any) are partes of the whole roote, for the compounde *Coßike* nomber.

Thirdly, if the nomber be a rooted nomber, euery parte of it, that is not a rooted nomber, is a meane nomber, betwene the greateste and the leaste.

Fourthly, if.ze. bee any denomination in it, then is.ʃ. an other denomination in it also.

Fiftly, and generally, all rooted nõbers, other are specially framed, by orderly multiplication, or els are nombers equalle to some one rooted nomber *Abstract*.

Now specially framed are soche, as are made by multiplicatiõ of one nomber by it self, and litle or nothyng altered from that very forme.

Of square rootes.

Exãple of. 529ʒℭ + 184ʒʒ + 16ʒ. whiche is a *Square* nomber, made by multiplication of. 23.ℭ + 4.ꝛ. by it self. This nomber maie haue his Roote orderly extracted thus.

529.ʒℭ + 184ʒʒ + 16ʒ(23ℭ+4ꝛ
23 46.ℭ.

In the firste nomber, I finde the *Square* roote to bee 23. And for his denomination, I take halfe the *Cossike* signe ʒℭ, and that is. ℭ. For as. ℭ. multiplied by ℭ. doeth make. ʒℭ. So in diuision by. 2. and in extraction of *Square* rootes, I shall take the. ℭ. for the halfe of ʒℭ and the denomination of his roote: and so set it doune in the *quotiente*.

Then I shall double the nomber *Abstracte* of that *quotiente* (kepyng his *Cossike* signe vnaltered) and that double shall I set euermore vnder the nexte nomber, toward the righte hande. As here, you see, I haue set 46 (whiche is the double of 23) with his signe ℭ. vnder the seconde nomber. And there I perceiue, I maie haue it. 4. tymes, if I doe diuide (as I ought) 184. by 46. And that. 4. I sette in the *quotiente*, with the signe +, and the denomination. ꝛ: seyng. ʒʒ. diuided by. ℭ. doeth yelde. ꝛ.

Laste of all, I muste multiplie that parte of the *quotiente*. 4.ꝛ. by it self, and it will yelde. 16.ʒ. whiche beyng subtracted also (as it should) leaueth nothyng remainyng of the square nomber.

This order must you kepe in all square nombers, how greate so euer thei be. As in this seconde exãple.

— 90ʒʒ.
25ʒℭ + 80ſʒ — 26ʒʒ — 144ℭ + 81ʒ(5ℭ+8ʒ — 9ꝛ
5.ℭ. 10ℭ + 64ʒʒ.
+ 10ℭ + 16ʒ. — 9.ꝛ.

The roote of the firste nomber is.5 ȼ, whiche I set in a *quotiente*.

Then doe I double that.5, and it maketh.10, to be sette vnder.8. with his denomination, whiche is.ȼ. like to the roote.

That.10.ȼ.maie be founde in.80.ʃℨ.8.times, & therfore I set. 8. in the *quotiente*, with the signe —+— and the denomination.ℨ. And then dooe I multiplie that.8.ℨ.squaredly, whiche giueth.—+—64.ℨℨ. to be subtracted out of ———26.ℨℨ. and so remaineth ———90.ℨℨ.

After this I double all the *quotiente* again, whereof commeth —+—10.ȼ.—+—16.ℨ. And bicause there is a remainer, ouer the nomber that I wrought laste, I must set.10.ȼ. vnder the remainer, and the other nomber in order, as you see it set.

Then seke I how often tymes maie.10.ȼ.diuide 90.ℨℨ, and I finde the *quotiente* to be ———9.ꝛe. And likewaies—+—16ℨ.multiplied by———9ꝛe doeth make ——— 144.ȼ. equalle to the somme ouer it: And so subtracteth it cleane. Wherfore to ende that worke, I multiplie the laste *quotiente*, by it self square, and it yeldeth. —+—81.ℨ. whiche is to bee subtracted out of the like somme, in the square nomber: and so resteth nothyng. Wherefore I iustly affirme, that the firste nomber is a square nomber, and hath for his roote.5.ȼ.—+—8.ℨ———9.ꝛe.

Scholar. That maie I sone proue, if I multiplie

5ȼ.—+—8ℨ.———.9.ꝛe.
5ȼ.—+—8ℨ.———.9.ꝛe.

25ℨȼ —+—40ʃℨ———45ℨℨ
—+—40ʃℨ—+—64ℨℨ
81.ℨ. ———72.ȼ———45ℨℨ
———72.ȼ.

25ℨȼ—+—80.ʃℨ———26ℨℨ———144ȼ+81ℨ.

 that

that roote by it self, as hére I haue doen it. Wherby I haue not onely confirmed it to be a square nomber: but also I haue espied, that you used the nomber not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the. ʒʒ. where the particulare multiplication hath —+—64. ʒʒ. and ——90 ʒʒ. For whiche twoo nombers you sette one, that resulteth of thē bothe, that is ——26 ʒʒ.

Master. But if you would take the nōber in that sorte, the woorke would be not onely all one: but also somewhat plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forthe here, the example of that woorke. And loe, here it is.

25ʒ¢+80.ſʒ+64ʒʒ——90ʒʒ——144ʒ+81ʒ (5¢+8ʒ—9ℯ
5. ¢ 10¢+64ʒʒ 10. ¢.—+—16.

Scholar. By comparynge these bothe formes of woorke together, I dooe better understande, the reason of the firste woorke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generalle patrone, for all the varieties, in this sorte of rooted nombers. And if you examine it diligently, and marke it well, you shall neade fewe other examples, for this kinde of square nombers.

The *Square* nomber, with the woorke of extraction of his roote foloweth here.

that roote by it self, as hére I haue doen it. Wherby I haue not onely confirmed it to be a square nomber: but also I haue espied, that you vsed the nomber not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the . ƺƺ . where the particulare multiplication hath —+—64.ƺƺ. and ——90ƺƺ. For whiche twoo nombers you sette one, that resulteth of thē bothe, that is ——26ƺƺ.

Master. But if you would take the nōber in that sorte, the worke would be not onely all one: but also somewhat plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forthe here, the example of that worke. And loe, here it is.

25ƺȼ+80.ʃƺ+64ƺƺ——90ƺƺ——144ƺ+81ƺ (5ȼ+8ƺ—9℞
5. ȼ   10ȼ+64ƺƺ   10.ȼ.—+—16.

Scholar. By comparynge these bothe formes of worke together, I dooe better vnderstande, the reason of the firste worke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generalle patrone, for all the varieties, in this sorte of rooted nombers. And if you examine it diligently, and marke it well, you shall neade fewe other examples, for this kinde of square nombers.

The *Square* nomber, with the
worke of extraction
of his roote fo-
loweth
here.

The

The square nomber, with the woorke of extraction of his roote.

—24 + 12 10

—48 + 30 6 8 26 8 6

36ʒ.ſʒ. + 60ℭℭ — 23ʒʒʒ — 4bſʒ + 22ʒℭ — 12ſʒ + 35ʒʒ — 20ℭ + 10ʒ — 4ꝛ + 1φ.

6.ſʒ. 12.ſʒ. + 25ʒʒʒ.

12.ſʒ. + 10ʒʒ + 16ʒℭ

12.ſʒ. + 10ʒʒ — 8ℭ. + 9ʒʒ

12.ſʒ + 10ʒʒ — 8ℭ. + 6.ʒ. + 4.ʒ

12ſʒ. + 10ʒʒ — .8.ℭ + 6ʒ. — 4ꝛ + 1.φ

The Roote.

6.ſʒ. + 5.ʒʒ — 4.ℭ. + 3.ʒ — 2.ꝛ. + .1.φ.

The proofe by Multiplication.

6.ſʒ. + .5.ʒʒ. — .4.ℭ. + .3.ʒ. — .2.ꝛ. + 1.φ.

6.ſʒ. + .5.ʒʒ. — .4.ℭ. + .3.ʒ. — .2.ꝛ. + 1.φ.

36ʒ.ſʒ + 30ℭℭ — 24ʒʒʒ + 18bſʒ — 12ʒℭ + 6ſʒ.

+ 30.ℭℭ + 25ʒʒʒ — 20bſʒ + 15ʒℭ — 10ſʒ + 5ʒʒ.

— 24ʒʒʒ — 20bſʒ + 16ʒℭ — 12ſʒ + 8ʒʒ — .4.ℭ.

+ 18bſʒ + 15ʒℭ — 12ſʒ + 9ʒʒ — .6.ℭ + .3.ʒ

— 12ʒℭ — 10ſʒ + 8ʒʒ — .6.ℭ + .4.ʒ — 2ꝛ

+ 6.ſʒ. + 5ʒʒ — 4.ℭ. + 3.ʒ. — 2ꝛ + 1φ.

36ʒ.ſʒ + 60ℭℭ — 23ʒʒʒ — 4bſʒ + 22ʒℭ — 12ſʒ + 35ʒʒ — 20ℭ. + 10ʒ — 4ꝛ + 1φ.

Scholar. It maie appeare easily, that this example serueth for many other, it doeth contain so many varieties of signes *Coßike*, multiplied so diuersely.

And in this nomber also, as well as in the other, I see that many nombers be omitted, by reduction: namely in the thirde, fourthe, fifte, and sixthe orders of nombers. For in the .2. firste orders, and in the .5. laste, there is no varietie of the signes + and —.

Wherfore to see the varietie of woorke, I will sette doune the nombers, as thei rise in particulare multiplication, and in it will I make an experimente of my cunnyng. As here foloweth.

30 20 10 26 8 6

36ʒ.ſʒ + 60ℭℭ + 25ʒʒʒ — 48ʒʒʒ + 36bſʒ — 40bſʒ + 48ʒℭ — 24ʒℭ + 12ſʒ — 44ſʒ + 35ʒʒ — 20ℭ + 10ʒ — 4ꝛ + 1φ.

6.ſʒ. 12.ſʒ. 12.ſʒ. + 10ʒʒ

60ℭℭ + 25ʒʒʒ 48ʒʒʒ — 40bſʒ + 16ʒℭ

12.ſʒ + 10ʒʒ — .8.ℭ.

36bſʒ + — 30ʒℭ — 24ſʒ — 9ʒʒ

12ſʒ + 10ʒʒ. — 8.ℭ. — .6ʒ

— 24ʒℭ — 20ſʒ. — 16ʒʒ — 12ℭ + 4ʒ.

12.ſʒ. + 10.ʒʒ — .8.ℭ. + .6.ʒ. — 4.ꝛ. + .1.φ.

Where for myne owne ease, and aied of memorie, I haue set vnder euery doublyng of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the nombers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in nombers *Abstracte*. But sometime I must take. 2. places nexte together, and at an other tyme, I shall scippe. 2. or. 3. places.

Master. You marke it well. And yet that is a good and true rule, that some menne teache: that in these *Cossike* nombers, as well as in other *Abstracte* nõbers, you shall marke euery odde place, and vnder eche of them to finde a Square roote. But that is to be vnderstande, when the nombers are sette, in their brefeste and exacteste order.

These fewe examples maie suffice, for a declaratiõ of extractyng the roote of Square nombers, made by multiplication. And now touchyng those nombers, that bee equalle to some rooted nomber, and namely soche as be equalle to a square nomber, I will teache you how their roote maie be extracted.

The rootes of nõbers equal to be squrts.

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and nombers, the rootes maie bee coupled with the nombers onely, in. 3. formes. That is. ze.—+—.ϑ (whiche is all one with ϑ.—+—. ze) or els thus. ϑ.——.ze. Or thirdly, ze ——ϑ. And for eche of these. 3. sortes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where ze —+—ϑ is equalle to ʒ take these exãples 1 ʒ is equall to. 4. ze —+—21 ϑ or. 1 ʒ. is equalle to 35. ϑ.—+—2. ze. Likewaies 1 ʒ is equalle to. 10 ze.—+—75. ϑ. or. 1. ʒ. is equalle to 105. ϑ.—+—. 8. ze.

The firste forme.

Where for myne owne ease, and aied of memorie, I haue set vnder euery doublyng of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the nombers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in nombers *Abstracte*. But sometime I must take. 2. places nexte together, and at an other tyme, I shall scippe. 2. or. 3. places.

Master. You marke it well. And yet that is a good and true rule, that some menne teache: that in these *Coßike* nombers, as well as in other *Abstracte* nõbers, you shall marke euery odde place, and vnder eche of them to finde a Square roote. But that is to be vnderstande, when the nombers are sette, in their breefeste and exacteste order.

These fewe examples maie suffice, for a declaratiõ of extractyng the roote of Square nombers, made by multiplication. And now touchyng those nombers, that bee equalle to some rooted nomber, and namely soche as be equalle to a square nomber, I will teache you how their roote maie be extracted. *The rootes of nõbers equal to be squrts.*

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and nombers, the rootes maie bee coupled with the nombers onely, in. 3. formes. That is. ze .—+—. φ (whiche is all one with φ.—+—. ze) or els thus. φ.———. ze. Or thirdly, ze ——— φ. And for eche of these. 3. sortes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where ze —+— φ is equalle to ʒ take these exãples 1 ʒ is equall to. 4. ze —+— 21 φ or. 1 ʒ. is equalle to 35. φ.—+—2. ze. Likewaies 1 ʒ is equalle to. 10 ze .—+—75. φ. or. 1. ʒ. is equalle to 105. φ.—+—. 8. ze. *The firste forme.*

In all these exāples, and other soche like, you must first consider the nomber annexed with the signe. ze. (whiche is the middell quantitie) and the halfe of it shall you note, for with it shal you worke twise. First you shall multiplie halfe of that nomber by it self, and this is the firste worke, and to it shall you adde the other whole nomber, that is ioyned with. ℞. And that will euer more make a square nomber: out of whiche square you shall extracte the roote. And to that roote shall you adde halfe the nomber, that was annexed with the signe of. ze. (whiche was the nomber that I bade you to marke). And this is the seconde woorke. The totall that commeth of this addition, is the roote of the compounde *Coßike* nomber.

An example Example of the firste. 4. ze. —+— . 21. ℞. halfe the nomber annexed with. ze. is. 2. whose Square is. 4. that shall I put to. 21. and there riseth. 25. beeyng a square nomber, and hauyng. 5. for his roote. To that 5. I ioyne halfe the nomber annexed with. ze. and it maketh. 7. whiche is the nomber that I seke for: and is the roote to. 4. ze. —+— 21. ℞.

The proofe. For triall whereof take. 4. rootes, that is. 28. and putte to it. 21. and thereof commeth. 49. whiche is a square nomber, and hath. 7. for his roote.

An other example. Scholar. Then can I doe the like with the second exampl. 35. ℞. —+— . 2. ze. And firste the halfe of. 2. is 1. and the Square of it is. 1. whiche I put to. 35. and it maketh. 36. a Square nomber: whose roote is. 6. To that. 6. if I adde. 1. that was the halfe before reserued, it will make. 7. whiche is the roote that I doe seke.

The proofe. The proofe is this: 2. rootes maketh. 14. and. 35. giueth. 49. whose roote is. 7.

The thirde example. Likewaies for the thirde example 10 ze —+— 75 ℞ I woorke thus. Halfe. 10. is. 5. and his Square is. 25. that dooe I adde to. 75. and there riseth. 100. whose roote is. 10. to whiche roote I add. 5. and there com-

meth

meth.15.that is the roote whiche I would haue.

And that I maie proue by triall in this forte. 10. rootes giue.150.vnto whiche if I adde.75.there will amounte.225. whiche is a Square nomber: and hath 15.for his roote.

The fourthe example is.105.φ—+—8.℞. where I take firste the halfe of.8. that is.4.and it in Square giueth.16. whiche I adde to. 105. and there amounteth.121.beyng a Square nomber, and the roote of it 11.vnto whiche I shall adde. 4. for halfe the nomber of rootes: and so there riseth. 15. as the roote that I seke for. And to approue it I take.8.times.15. whiche is.120.and adde it vnto. 105. and so commeth. 225. For the square, and the roote of it is.15. *The fourthe example.* *The proofe.*

Master. The like order of worke shall you vse, in all nombers *Coßike* compounde, whē any.2.nombers with immediate denominatiōs *Coßike*, are equalle to one of the nexte denomination, in order aboue them. *Other formes in like sorte.*

As.1.ℭ.is equalle to.3.ʒ.—+—10.℞.

And again.1./ʒ.equalle to.6.ʒʒ.—+—40.ℭ.

Likewaies.1.ʒℭ.equalle to.3./ʒ.—+—28.ʒʒ.

But in al these the roote shal beare name of the greater quantie.

Scholar. By the former order of worke, I shall in the firste of these.3.examples, take halfe.3.(bicause it is the nomber of the middell quantitie). And that is $\frac{3}{2}$. and that shall I multiplie squarely, and so will there rise $\frac{9}{4}$, vnto whiche I shall adde 10 or $\frac{40}{4}$. And that maketh $\frac{49}{4}$. whiche is a square nomber, and his roote is $\frac{7}{2}$. vnto whiche I must put the firste halfe, that is $\frac{3}{2}$, and then will it be $\frac{10}{2}$, or els.5. whiche is the *Cubike* roote of that nomber.3.ʒ.—+—10.℞.beyng equalle to 1 ℭ *The firste example.*

For proofe whereof, I multiplie.5. *Cubikely*, and it maketh.125. Then doe I multiplie it squarely, and it will be.25. Now.3.ʒ.is.75.and.10.℞. maketh.50 whiche bothe added together, giue.125. *The proofe.*

The seconde example. In the seconde example, where. 1. ſʒ. is equalle to 6. ʒʒ. —+—. 40. ℭ. I shall take halfe. 6. (whiche is the nomber of the middell quantitie) and that is. 3. and the square of it is. 9. whiche I must adde unto 40 and thereof commeth. 49. whiche is a square nomber and hath. 7. for his roote, unto whiche I adde 3. and so haue I 10 for the *Sursolide* roote, of 6 ʒʒ —+— 40 ℭ

The proofe. And for proofe I saie, if. 10. bee the roote, then is 100. the square, & 1000. the *Cube*, the ʒʒ. is 10000. And the *Sursolide*. 100000. Wherfore. 6. ʒʒ. make 60000. and. 40. ℭ. yelde. 40000. And bothe thei together doe make. 100000. whiche is the quantitie of the *Sursolide*.

The thirde example. In the thirde example. 1. ʒℭ. is equalle to. 3. ſʒ. —+—. 28. ʒʒ. whose *zenzicubike* roote, I seke in this sorte.

Firste I take halfe. 3 (as the nomber of the middell quantitie) that is $\frac{3}{2}$, & that maketh in square $\frac{9}{4}$. whiche I adde unto 28 (that maketh $\frac{112}{4}$) & it yeldeth $\frac{121}{4}$ whiche is a square nomber, and his roote is $\frac{11}{2}$. unto whiche I adde $\frac{3}{2}$, and it will be $\frac{14}{2}$, or. 7. whiche is the *zenzicubike* roote unto the foresaid nomber. 3. ſʒ. —+—. 28. ʒʒ.

The proofe. For proofe whereof I multiplie. 7. *zenzicubikely*, and it maketh 117649. Then must the ſʒ. be 16807 and. 3. ſʒ. 50421. Again the ʒʒ. is. 2401. and so 28. ʒʒ. shall bee. 67228. And those bothe together yelde. 117649.

A thirde forme. Master. Yet one other forme is there, that in all thinges, saue in one poincte onely: followeth the same rule. And that is whẽ the 3 denominations doe not go immediatly together, but yet are equally distante. As ʒʒ. ʒ. and. ℞. where the distaunce is one onely quantitie. Likewaies. ʒℭ. ℭ. and. ℞. whiche differ by. 2. quantities. And in like sorte. ℭℭ. ſʒ. and ʒ. are distante by. 3. quantities. And so of other, how many so euer bee omitted, so that the difference bee

equalle

equalle. In all whiche you shall worke, as you did in the former rule, till you haue eanded all that worke. But then haue you here, one thing more to bee considered. For the laste nomber, whiche you haue founde, is not the roote, but a rooted quantitie: And his roote is the roote that you seke for.

Scholar. Doe you meane the square roote of that quantitie, or some other?

Master. It maie be any kinde of roote, in diuerse nombers, but not in one nomber. Wherfore for your certeintie marke this rule.

If the denominations of your nombers, differ onely by one, then is it a square nõber, that you doe finde by the practise of the laste rule. And therfore shall you take his square roote, for the roote of your nomber.

But if the denomination differ by . 2 . quantities, then shall you extracte a *Cubike* roote, out of your laste nomber. And if the distaunce bee. 3. quantities, the roote must bee a *zenzizenzike* roote: and for. 4. quantities distante, a *Surfolide* roote, and so forthe.

As for example. 1.ʒʒ. is equalle to. 80.ʒ. —+—. 2000.𝓅. Now for to finde the roote of. 80.ʒ. —+—. 2000.𝓅. I worke thus. Firste I take the halfe of 80. (bicause it is the nomber of the middle quantitie) and that halfe is. 40. whiche I multiplie Square, and it maketh. 1600. to it I adde. 2000. and it will bee 3600. whiche is a square nomber, &. 60. is his roote: to that. 60. I shall adde the foresaied. 40. and then will it bee. 100. whiche nomber in the firste rule, had been the true roote. But here consideryng the distãce is of one quantitie, I muste extracte his square roote, whiche is. 10. And that is the *zenzizenzike* roote, that my nomber containeth. *An example*

An other example. 1.ʒ𝒸𝒸. is equalle to. 400.𝒸𝒸. —+—57344.𝓅. I take 200. for the halfe of the middell quantities nomber, and multiplyng it square, I *The seconde example.*

 finde

finde.40000. whiche I put to.57344. and then I haue.97344: whiche is a Square nomber, and his roote is 312 vnto whiche I shall adde the halfe of 400 and so will it bee. 512. But now must I take the *Cubike* roote of this nomber (that is.8) for my roote, that I desire: Bicause the denominations in the nomber, differ by.2.quantities.

Scholar. I see very well the order of this worke: And the proofe is in like sorte, whiche I maie practise by my self at any tyme. Wherfore I praie you, procede forthe to other rules.

The seconde sort of equal nombers.

Master. This is sufficiente for the firste sorte. Now for the seconde sorte, in nombers *diminute* or *residualle*, where.ʓ.is equalle to.ϑ——— ze.the forme of worke is like vnto the other, in all poinctes saue in one. For in stedes of the laste addition, you shall vse in these nombers, Subtraction. As here for example, when I saie.1.ʓ.is equalle to.60.ϑ.———.4.ze.to finde the roote, firste I take the halfe of.4. (bicause it is the nomber of the middell signe) and that halfe beyng.2.doeth make in square.4. whiche I put to 60 and so is it.64. a square nomber, and hath.8. for his roote. From whiche roote (by the order of this rule) I must abate.2. that is the halfe of the firste nomber of rootes. And then will there remaine.6. for the verie roote of.60.ϑ.———4.ze.beyng equalle to.1.ʓ.

Example.

The proofe.

Scholar. That is sone proued. For.6.beeyng the roote, then.4.ze.maketh.24. whiche beyng abated out of.60. leaueth 36 and that is the iuste square vnto.6. as the equation saieth.

The seconde example.

Master. An other example is this.1./ʓ.is equall to.162.ce.———.9.ʓʓ.

Scholar. That can I woorke, thus: Firste I take the halfe of.9. (bicause it is the nomber of the middell signe) and it is $\frac{9}{2}$, whiche I multiplie squarely, and it will be $\frac{81}{4}$, that must bee added to.162.or $\frac{648}{4}$, and then will

will there amounte $\frac{729}{4}$. whiche is a Square nomber, and hath for his roote $\frac{27}{2}$ out of whiche, by this rule, I must abate $\frac{9}{2}$, and then riseth $\frac{18}{2}$, that is .9. whiche is the very roote to 162.¢——9.ȝȝ. beyng equall to.1.ſȝ.

And for the proofe, I multiplie. 9. *Cubikely*, and it geueth.729. so that.162.¢. doe make.118098. out of whiche I must abate.9.ȝȝ. that is.59049. (by the same roote, sith. 1ȝȝ. is. 6561). And then will there remaine.59049. whiche is the iuste quantitie of.1.ſȝ.

The proofe.

Master. Yet one example more shall you haue of a thirde sorte.

The thirde example.

When.1ȝ¢. is equalle to.275456.ϑ——26¢. I demaunde of you, what is the valewe of.1.ꝛ?

Scholar. I searche it thus. The nomber of the middell signe is.26. whose halfe I must take, and first multiplie it squarely, and there will rise 169. whiche I adde to.275456. and it will bee. 275625. whiche is a square nomber, and hath for his roote. 525. from whiche nomber I must abate halfe the nomber, of the middell signe, that is. 13. and so there will remaine 512. whose *Cubike* roote I must extract, bicause the denominations differ by. 2. quantities, and that roote will be.8. whiche is the *Cubike* roote to.512. but to the nomber propounded, it is the *zenzicubike* roote.

Master. This is inoughe for the worke of the seconde sorte. Now for the thirde sorte of equation, where.ȝ. is equalle to.ꝛ.——.ϑ. I will giue you a brief admonition onely, though it differ from bothe the other. 2. rules, in forme of woorke. For as the equalitie maie be in diuerse sortes, so some tymes you maie vse the woorke of the firste sorte, by Addition of halfe the nomber of the middle signe: and some times you shall worke by subtraction. Wherein this is the difference, from the seconde rule. That there you doe

The thirde sorte of equal nombers.

subtracte halfe the nomber of the middell signe, from the roote whiche you fonde. And in this thirde rule, you shall subtracte the roote from the halfe, and not the halfe from the roote. Forbicause that that roote, is euer lesser then that halfe.

And in this rule, this is specially to bee obserued: that the Square of halfe the nomber, of the middell signe, will euer more bee greater, then the nomber of the lesser signe: And therfore shall the nomber of the lesser signe, bee abated out of that square. And the remainer will bee a Square nomber, with whiche you shall woorke, as I haue taught you before.

And farther in this rule, it is commonly seen, that euery soche equalle nomber, hath. 2. valuations for his roote. I meane that any of those. 2. nombers, will bee as the roote in this equation. For otherwaies no nomber can haue. 2. rootes of one denomination.

Scholar. I vnderstande you thus. That no nomber can haue. 2. square rootes, or. 2. *Cubike* rootes, and so forthe: Els one nomber maie haue. 3. or. 4. rootes. As. 64. hath. 8. for his Square roote: 4. for his *Cubike* roote: and. 2. for his *zenzicubike* roote.

Master. You take it well. And farther for the easie knowledge of those. 2. nombers, or rootes: Thei must bee soche, as beeyng added together, will make the nōber of the middell signe: and beeyng multiplied together, wil make the nomber of the least signe. And so maie you finde theim without farther multiplication, or extraction of rootes.

The firste example.

For example, I sette firste. 1. ʒ. equalle to. 16. ɼ. ——— . 63. φ. where I maie espie quickely, that. 63. cā haue no more partes to his composition, but. 3. 7. 9. 21 And if I take. 3. and. 21. then their addition will bee greater then. 16. but 7. and. 9. maketh iuste 16. by addition, and. 63. by multiplication. And therefore thei shall be the. 2. rootes.

Scholar. I will proue that by examination, thus. If. 7. be the roote, then is. 49. the square. And. 16. ℞ make. 112. out of whiche I must abate. 63. and there resteth. 49. equalle with the Square: so is that a true roote. Then for. 9: his square is. 81. And. 16. ℞. doe yelde 144 frō whiche I shal abate 63. And the remainer will be. 81. equalle to the square. And so is that also a true roote.

Master. Now worke it by the other rules, that I taught you.

Scholar. Firste I take. 8. as halfe the nomber of the middell signe, and that multiplied Square, doeth giue 64 from whiche I shall abate 63 and then doeth there remain but. 1. whiche is coumpted as a Square nomber, and his roote to bee. 1. also, whiche if I adde to. 8. it will make. 9. that is one of the rootes: And if I abate it from. 8. it will leaue. 7. whiche is the other roote. And thus I see one worke cōfirmeth the other.

Master. Take this for the seconde exāple. 1. ʒ∁ is equalle to. 8. ʃʒ. ——. 12. ʒʒ. what is the roote saie you? *The seconde example.*

Scholar. To finde it, firste I loke for the partes of 12. And thei be. 2. 3. 4. 6. of whiche. 2. and. 6. doe serue my purpose, for their addition maketh. 8. and so doeth not. 3. and. 4. Wherefore I saie, that. 2. maie bee the roote, and so maie. 6. But for farther trialle of it: I worke it by the other rule, saiyng halfe. 8. is. 4. and his square is. 16. From whiche I abate. 12. and there remaineth. 4. whose roote is. 2. that I maie adde to. 4 and so haue I. 6. for one roote: or els abatyng it from 4. I shall haue. 2. for the other roote.

The proofe is manifeste for. 6. beeyng a roote, the *zenzicube* is. 46656. The *Sursolide* is. 7776. And the *zenzizenzike* is 1296. So that 8. ʃʒ. doe make 62208 And. 12. ʒʒ. are. 15552. whiche being abated out of 62208 do leaue 46656. the true quantitie of 1 ʒ∁ *The proofe.*

And so is that worke good,6.beyng a roote.

Now if.2. be sette for a roote:then is the.ʒʒ.16. the ʃʒ.32,and the.ʒ&.64.And so are.8.ʃʒ.equall to.256.And.12.ʒʒ.yelde.192.Wherfore abatyng 192.out of.256.there resteth.64, the iuste quantitie of.1.ʒ&. And so is that worke also good,and.2.a true roote.

The thirde example.

Master. Now proue this thirde example, where 1.bʃʒ is equalle to.2000.ʒʒ.——470016ꝥ.

Scholar. Halfe the nomber of the middell signe is 1000.And the square of it is. 1000000. From whiche I shall abate.470016.and there will remaine 529984.whose square roote by trialle of extraction, I finde to be728.whiche I maie other adde to.1000 and so there riseth.1728. whiche I finde to bee (as it ought)a *Cubike* nomber.And his roote to be.12.

But and if I abate728.from 1000,there will remain.272.whiche is no *Cubike* nomber.

Master. So that here semeth to be but one roote. And yet these.2.nombers.1728.and.272.kepe soche a rate, that beeyng multiplied together, thei make 470016.whiche is one of the nombers,and beeyng added together,thei make 2000.whiche is the other nomber of the same *Coßike residualle.*

The fourthe example.

But now proue in other like nõbers, whiche haue some distaunce, betwene their denominations, whether it will so happen still. As namely in this, where 1.bʃʒ.is equalle to.12.ʒʒ.——32.ꝥ.

Scholar. Halfe.12.is.6.and his Square.36.from whiche abatyng.32.there is lefte.4.whose roote is.2 And if I adde that 2.to.6.it maketh.8.whiche is a *Cubike* nomber,and hath.2.for his roote. But if I abate 2.from.6.there remaineth.4. whiche is no *Cubike* nõber,and therfore hath no soche roote. And yet these.2. nombers.4.and.8.by addition,make the middell nõber,and by multiplication,thei make the laste nõber.

Master.

Master. Proue yet ones againe in a nomber, where one quantitie onely is omitted. As when 1.ʒ. is equalle to.24.𝔯.——135.𝔡. *The fifte example.*

Scholar. 12.maketh in square.144.from whiche I shall deducte.135. and then resteth.9. whose square roote is.3. whiche if I adde to.12. it will bee. 15. and hath no square roote, as here is required. But if I abate.3.from 12.then remaineth 9 whose square roote is. 3. and serueth to the nomber, as I haue here proued in my tables. And. 9. and. 15. kepe the customable rate. For by addition thei make.24. And by multiplication, thei yelde. 135.

But in all these examples, where the denominations be are a distaunce, I can finde but one roote, and not.2. As it was in the other exāples of thesame rule.

And in some of theim, the greater nomber containeth the roote: but in other, the lesser nomber hath the roote.

Master. Bicause I can not staie now, about this varietie, I will remitte it till an other tyme. But this by the waie, I must admonishe you, that I doe folowe here, the common forme of writers, in callyng these rootes, that rise in equatiō, where as thei are not the rootes of those nombers, but are the value of a roote. For of a *Coßike* nomber, the roote must neades bee a *Coßike* nomber also. And soche as by multiplication will make the rooted nomber: But so can not those nombers doe.

And here will I make an eande, of the workes of *Cossike* ombers. And now will I applie them to practise in the rule of *equation*, that is commonly called *Algebers* rule.

The rule of equation, commonly called Algebers Rule.

The rule of equation.

Hetherto haue I taughte you, the common formes of worke, in nombers *Denominate*. Whiche rules are vsed also in nōbers *Abstracte*, & likewaies in *Surde* nombers. Although the formes of these workes be seueralle, in eche kinde of nomber. But now will I teache you that rule, that is the principall in *Coßike* woorkes: and for whiche all the other doe serue.

This Rule is called the Rule of *Algeber*, after the name of the inuentoure, as some men thinke: or by a name of singular excellencie, as other iudge. But of his vse it is rightly called, the rule of *equation*: bicause that by *equation* of nombers, it doeth dissolue doubtefull questions: And vnfolde intricate ridles. And this is the order of it.

The somme of the rule of equation:

When any question is propoūded, apperteinyng to this rule, you shall imagin a name for the nomber, that is to bee soughte, as you remember, that you learned in the rule of false position. And with that nomber shall you procede, accordyng to the question, vntil you finde a Coßike nomber, equalle to that nomber, that the question expresseth, whiche you shal

reduce

reduce euer more to the leaste nombers. And then diuide the nomber of the lesser denomination, by the nomber of the greateste denomination, and the quotient doeth aunswere to the question. Except the greater denominatiō, doe beare the signe of some rooted nōber. For then must you extract the roote of that quotiente, accordyng to that signe of denomination.

Scholar. It semeth that this rule is all one, with the rule of false position: and therefore mighte so bee called: seyng it taketh a false nōber, to worke with al.

Master. This rule doeth farre excell that other. And dooeth not take a false nomber, but a true nomber for his position, as it shall bee declared anon. Wherby it maie bee thoughte, to bee a rule of wonderfull inuention, that teacheth a manne at the firste worde, to name a true nomber, before he knoweth resolutely, what he hath named.

But bicause that name is common to many nombers (although not in one question) and therefore the name is obscure, till the worke doe detect it, I thinke this rule might well bee called, the rule of darke position, or of straunge position: but not of false position.

And for the more easie and apte worke in this arte wee dooe commonly name that darke position. 1. ꝛ. And with it doe we worke, as the question intendeth, till we come to the *equation*.

This rule of *equation*, is diuided by some men, into diuerse partes. As namely *Scheubelius* dooeth make. 3. rules of it. And in the seconde rule, he putteth. 3. seueralle cannōs. Some other men make a greater nōber of distinctiōs in this rule. But I intende (as I thinke beste for this treatice, whiche maie serue as farre *The partes of the rule.*

 as

as their woꝛkes doe extende) to distincte it onely into twoo partes. Whereof the firste is, *when one nomber is equalle vnto one other*. And the seconde is, *when one nomber is compared as equalle vnto.2.other nombers.*

Alwaies willyng you to remẽber, that you reduce your nombers, to their leaste denominations, and smalleste foꝛmes, befoꝛe you pꝛocede any farther.

And again, if your *equation* be soche, that the greateste denomination *Coßike*, be ioined to any parte of a compounde nomber, you shall tourne it so, that the nomber of the greateste signe alone, maie stande as equalle to the reste.

And this is all that neadeth to be taughte, concernyng this wooꝛke.

Howbeit, foꝛ easie alteratiõ of *equations*. I will pꝛopounde a fewe exãples, bicause the extraction of their rootes, maie the moꝛe aptly bee wꝛoughte. And to auoide the tediouse repetition of these wooꝛdes: is equalle to: I will sette as I doe often in wooꝛke vse, a paire of paralleles, oꝛ Gemowe lines of one lengthe, thus: =======, bicause noe.2. thynges, can be moare equalle. And now marke these nombers.

1. 14.ze.—+—.15.℈=====71.℈.

2. 20.ze.———.18.℈=====.102.℈.

3. 26.ʒ.—+—10ze=====9.ʒ——10ze—+—213.℈.

4. 19.ze—+—192.℈=====10ʒ.—+—108℈——19ze

5. 18.ze—+—24.℈.=====8.ʒ.—+—2.ze.

6. 34ʒ——12ze=====40ze—+—480℈——9.ʒ

1. In the firste there appeareth. 2. nombers, that is 14.ze.

14.ze.—+—15.ꝭ. equalle to one nomber, whiche is 71.ꝭ. But if you marke them well, you maie see one denominatiō, on bothe sides of the *equation*, which neuer ought to stand. Wherfore abating the lesser, that is.15.ꝭ. out of bothe the nombers, there will remain. 14.ze.=====56.ꝭ. that is, by reduction, 1.ze.=====4.ꝭ.

Scholar. I see, you abate.15.ꝭ. from them bothe. And then are thei equalle still, seyng thei wer equalle before. Accordyng to the thirde common sentence, in the patthewaie:

If you abate euen portions, from thynges that bee equalle, the partes that remain shall be equall also.

Master. You doe well remēber, the firste groundes of this arte. For all spryngeth of those principles *Geometricalle*. Wherfore call to your minde likewaies the seconde common sentence, in the same booke, and then haue you another reason, whiche will helpe you not onely, in the other formes of woorke here, but also very often in the practise of this arte.

Scholar. That is this.

If you adde equalle portions, to thynges that bee equalle, what so amounteth of them, shall be equalle.

Master. These twoo sentences doe instructe you that when you see on bothe the sides of the *equation*, any one denominatiō *Cosike*, you shall marke the signe that is annexed to the lesser of them bothe: and if it be the signe of addition.—+—. then shall you abate that lesser nomber, from bothe the partes of the *equation*. As I did in this firste example. But if the signe be of abatemente ——, then shall you adde that lesser nōber, to bothe partes. And so shall you doe, till there be noe one denomination on bothe partes, but diuerse and distincte.

So the seconde nomber will be. 20.ze.=====120ꝭ 2.
and in the leaste termes. 1.ze.=====.6.ꝭ.

Scholar. I see that you adde. 18.ꝭ. to bothe par-

3. tes of the equation. But by that reason, I doubte in the thirde somme, bicause. 10. ze. is in bothe partes of the equation: in the firste parte with +, and in the seconde parte with −, whether I shall adde 10. ze, or abate them.

Master. In soche a case, you maie dooe either of bothe, at your libertie: and all will be to one eande.

Scholar. If I adde. 10. ze. then will it be. 26. ʒ. + 20. ze. = . 9 ʒ. + 213. ꝭ.

Master. And doe you not see. ʒ. on bothe sides of the equation?

Scholar. I did loke but for one alteration onely.

Master. If there were twentie like denominations, you should alter them all. For that is the principalle and peculiare reduction, that belongeth to equations.

Scholar. Then must I abate. 9. ʒ. on bothe partes, and so will there remaine. 17. ʒ. + . 20. ze. = . 213. ꝭ.

Master. Now reduce it by abatyng. 10. ze.

Scholar. So it will bee. 17. ʒ. = . 213. ꝭ. − . 20. ze.

And now I remēber, that this is the better forme of reduction. Bicause the greater denomination, that is. ʒ, is alone with his nomber on the one side of the equation, and the. 2. lesser denominations, on the other side.

Master. How doe you reduce the other equatiōs, to their smalleste formes?

4. Scholar. In the fourthe example, there is noe denominatiō, before the signe of equation, or in the first parte, but the like is in the seconde parte also, after the signe of equation. Wherfore firste, bicause I see 19. ze. on bothe sides, I will abate it on bothe sides. And then will it be thus.

192. ꝭ. = 10. ʒ. + 108. ꝭ. − . 38. ze.

But

But bicause I see ꝛ. yet remainyng on bothe partes, I abate the lesser, that is . 108ꝛ. from bothe partes, and it will be. 84. ꝛ. ═══ .10. ȝ ——— .38. ꝣ.

Master. This equation would bee better, if the greater denomination, did stande as one parte of the equation alone. Whiche thyng you maie easily doe, by addyng. 38. ꝣ. to bothe partes: bicause so moche foloweth ———, on the one parte.

And euermore when occasion serueth, to translate nombers compounde, ——— on the one side is equalle to —+— on the other side. *Translation of nombers.*

Scholar. Then it will be thus.

84. ꝛ. —+— 38. ꝣ. ═══ .10. ȝ.

Master. It were better thus.

10. ȝ. ═══ .38. ꝣ. —+— .84. ꝛ.

And in smaller termes.

5. ȝ. ═══ .19. ꝣ. —+— .42. ꝛ.

But now procede with the examples.

Scholar. The fifthe is easily reduced, by abatyng 2. ꝣ. on bothe sides: For so will it bee. 5.

8. ȝ. ═══ .16. ꝣ. —+— .24. ꝛ.

The sixthe equation will be, by addyng. 12. ꝣ. on bothe sides. 34. ȝ. ═══ 52. ꝣ. —+— 480. ꝛ ——— 9 ȝ. 6.

But yet I must reduce it farther, by addyng. 9. ȝ. on bothe sides. And then it will stande thus.

43. ȝ. ═══ .52. ꝣ. —+— .480. ꝛ.

Master. Now will I shewe you the varieties of equatiōs, taught by *Scheubelius*, bicause you maie perceiue, how thei bee conteined in those. 2. formes, named by me. As for the manyfolde varieties, that some other doe teache, I accoumpte it but an idle bablyng, or (to speake moare fauourably of them) an unnecessary distinc- *Varieties of equations.*

distinction.

The firste equation.

The first equatiõ after *Scheubelius*, & after my meanyng also, is, when one nomber is equall to an other: meanyng that thei bothe must be simple nombers *Cossike*, and uncompounde. As. 6. ze. equalle to. 18. ℞:

4. ʒ. ══════. 12. ze. 14. &. ══════. 70. ʒ:

15. ſʒ. ══════. 90. ʒʒ: 20. ʒ & ══════ 180. ſʒ:

26. ʒſʒ. ══════. 117. & &.

In all these examples, as you see but one nomber, compared to an other: so to finde the quantitie of one roote, you shall diuide the nomber of the lesser Character, by the nomber of the greater Character, and so shall the *quotiente* bryng forthe the quantitie of. 1. ze.

Scholar. It semeth at the firste vewe, that it is against reason, to diuide the nomber of the lesser signe, by the nomber of the greater. But when I consider, that if I compare a nomber of crounes, or any like denomination, to a nomber of shillynges in equalitie, the nomber of crounes, or other soche like, must neades be lesser, then the nõber of shillinges. And so diuiding the nõber of the shillinges (or other lesser name) by the nomber of crounes (or other greater name) the *quotiente* will shewe, how many shillynges make a croune: and generally, how many of the lesser, dooe make one of the greater.

As if. 20. crounes bee equalle to. 100. shillynges, then. 5. shillynges dooeth make a croune. So when 6. ze. bee equall to. 18. ℞. then. 3. ℞. dooeth make. 1. ze. And. 4. ʒ. ══════. 12. ze. dooeth cause that. 3. ℞. must be a roote.

Master. As your examplarie proofe is good, so reduction will be a sufficiente proofe in this.

Scholar. I see it manifestly. For if. 14. &. bee equalle to. 70. ʒ. then. 1. &. is equalle to. 5. ʒ. by that reduction

reduction in nombers. And again by reduction in signes. 1.ze. is equalle to.5.φ.

Likewaies. 15.ſʒ. beyng equalle to.90.ʒʒ. reduction by signes and nombers also, will make 1.ze ===6.φ. So shall.20.ʒ&.===.180.ſʒ. be reduced to.1.ze.===.9.φ. And.26.ʒſʒ.===.104.&&. will make.1.ze.===.4.φ.

Master. And so generally, when there is noe denomination omitted, betwene those.2. that bee compared in equalitie, still the diuision of the nomber, of the lesser denomination, by the nomber of the greater denomination, will bryng forthe in the *quotiente*, the quantitie of.1.ze.

But if there bee any denominations omitted, betwene those.2. whiche be compared together in equalitie: loke how many denominations are omitted, and so many in order is the rooted quantitie, whose roote you must extract, for the aunswere to the questiō. For in soche a case, euer more you shall extracte the roote of your laste nomber. *The seconde forme of the firste equatiō*

As for example, when.6.&. be equalle to.24.ze. by the former rule, you shall finde. 4. in the *quotiente*. But here that. 4. is not the quantitie of a roote, but is a rooted nomber, whose roote I shall extracte. And seyng betwene. &. and.ze. there is no quantitie omitted, but one, that is. ʒ. Therefore I shall accoumpte.4. the firste quantitie, that is to saie, a *Square* nomber, and so take his *Square* roote, beyng. 2. for the quantitie of a roote.

Again if.7.ſʒ. be equalle to. 567.ze. the *quotinete* will be.81. and declareth a *zenzizenzike* nomber, bicause there are omitted betwene. ſʒ. and. ze. three nombers: and *zenzizenzike* is the thirde quantitie: as you did learne in the beginnyng of this treatice, of nombers denominate.

Scholar. I perceiue it. And therfore I must take

the

the *zenzizenzike* roote of.81. whiche is.3. and that is the true roote, where.7.ʃʒ. be equalle to.567.ꝛ.

Master. And if those.7ʃʒ. were accõpted equalle to.56.ʒ. the *quotiente* will be. 8. And bicause there are omitted.2.quantities, that is.ce. and.ʒʒ. therfore you shall accompte that.8. to be 1 ce. or a seconde quantitie. And his roote *Cubike* is.2. whiche standeth as the valewe of a roote, in the former equation.

And it is not possible that any other nomber, maie be placed as a roote, in that equation: or in any other forme of this firste kinde. Howbeit in one sorte of equation, of the seconde kinde, there maie be.2. diuerse rootes, when one nomber hath.2. rootes in valewe. As I taught you before in the extraction of rootes.

The seconde kinde of equation.

The secõd kinde of equatiõ, after *Scheubelius* minde and myne also, is, when one simple nomber *Cossike*, is compared as equalle to.2. other simple nõbers *Cossike*, of seueralle denominations, and like distaunce.

And in soche equation, beyng reduced as is taught before, the roote of those.2. nombers compounded, as in one (or rather the valewe thereof) shal be extracted: As I haue before taughte also. And that roote doeth aunswere to the question.

The seconde forme of the second kinde

Howbeit, here is the like obseruation, as was in the seconde forme of the firste kinde. For if those.3. denominations be not immediate, but doe omit some other betwene them, then shall you extracte the roote of that laste nomber, in all poinctes, as you did in the firste equation.

Examples of the firste sorte.

4.ʒ.═══.6.ꝛ.—+—4.ꝯ.

whiche beyng reduced, will bee:

1.ʒ.═══.1½.ꝛ.—+—.1.ꝯ. And the roote wil be.2

And.6.ʃʒ.═══.12.ʒʒ.—+—.18.ce.

That

That is by reduction.

1.ſʒ.=====2.ʒʒ.—+—3.¢. or

1.ʒ.=====2.℞.—+—.3.ꝯ. And the roote.3.

5.ſʒ=====25.ʒʒ———30.¢. Or by reduction.

1.ſʒ. =====5.ʒʒ.———6.¢. Or.

1.ʒ.=====.5.℞———.6.ꝯ. whose roote is.3.or.2.

Likewaies. 2.ʒ=====120.ꝯ.———8.℞

Or by reductiō. 1.ʒ=====60.ꝯ———4.℞. whose roote is.6.

Examples of the seconde sorte.

5.ʒʒ=====60.ʒ.—+—320.ꝯ.

That maketh by reduction.

1.ʒʒ.=====12.ʒ.—+—.64.ꝯ.

And the square roote.4.

Likewaies.8ʒ¢.=====.40.¢.—+—.30208.ꝯ.

Or by the orderly reduction.

1.ʒ¢=====5.¢.—+—3776.ꝯ. whose *Cubike* roote is. 4.

Again in residualles.

8.ʒ¢=====864.ʒ.———24.ʒʒ.

That maketh by reduction.

1.ʒ¢=====108.ʒ.———3.ʒʒ. Or els.

1.ʒʒ=====108.ꝯ———3ʒ. whose roote is.3.

So.9.bſʒ=====90.ʒʒ.———144.℞.

Or by reduction.1.ʒ¢=====10.¢.———16.ꝯ. whose roote is.8.or.2.

But now bicause *Scheubelius* dooeth make. 2. seueralle equations of these.2. formes: And giueth.3.diuerse rules, or canons for eche of them, I will declare his.6. canons to be all contained in this seconde kind of equation.

He maketh his diuiſion thus. When . 1 . nomber is compared as equalle to. 2. other, other that one nōber is of the ſmalleſte denomination. And then is it of the firſte Canon. As. 1. ʒ + 8. ꝫ ═══ 65. ℞. or els that one nomber, is of the greateſte denominatiō: As. 3. ꝫ. + 4. ℞ ═══ 1. ʒ. And then is it of the ſeconde Canon: Or els thirdely, the alone nomber is of the middle denominatiō: and then is it of the thirde Canon. As. 1. ʒ + 12. ℞. ═══ 8. ꝫ.

The like forme he uſeth, for the nombers of denominations diſtaunte.

Wherby you maie perceiue, that in my rule there is noe forme of nombers, like thē of the firſte Canon, nother yet of the thirde: but onely of the ſeconde. But then again in my rule, there are. 2. ſortes of examples whiche he hath not. And if you compare them well together, you ſhall perceiue, that thei bee agreable together.

As for exāple. In his firſte canon, this is the forme 1. ʒ + 6. ꝫ. ═══ 27. ℞. whiche equation in my rule, by trāſlation, is expreſſed thus.

1. ʒ ═══ 27. ℞ — 6. ꝫ. bicauſe I doe ſtill ſet the greateſte denomination alone.

Again in his thirde Canon, this is an example.

1. ʒ + 13. ℞ ═══ 8 ꝫ and that nomber doe I tranſlate into this forme 1 ʒ. ═══ 8. ꝫ — 15 ℞.

Now where as he giueth ſeueralle rules, for euery Cannon, I ſaie for them all: extracte the roote of that compounde nomber. For all his rules doe teache nothyng els. [illegible]

Scholar. I doe underſtande the diuerſitie, and agreemente of your rules and his. But for my exerciſe, I doone couette ſome apte queſtions, appertainyng to theſe equations.

A queſtion of ages. Maſter. Take this for the firſte queſtion.

Alexander beyng aſked how olde he was, I am. 2. yeres

yeres elder (quod he) then Ephestio. Yea, saied Ephestio. And my father was as olde as we bothe, and .4. yeres moare. And my father hauyng all those yeres, saied Alexander, was .96. yeres of age. I demaunde now of you, how olde was eche of them.

Scholar. I praie you aunswere the question your self, to teache me the forme.

Master. I will begin with the yongeste mannes age, and that will I call .1.℞. whiche is the common supposition in all soche questions. Then is Alexanders age .2. yeres moare, that is .1.℞ + 2.ϑ. And those bothe together doe make .2.℞ + .2.ϑ. Whereunto if you put .4. more, then haue you the age of Ephestio his father, that will be .2.℞ + .6.ϑ. And all these put together, that is .4.℞ + .8.ϑ. will make 96 whiche is the equation that shall open the question. *1.℞ is the common supposition.*

Wherfore I set doune the equation thus.

4.℞ + 8.ϑ = 96.ϑ. And bicause I see on bothe sides, one denomination of .ϑ. I doe abate .8.ϑ. frō bothe sides: & then there remaineth .4.℞ = 88ϑ And by reduction or diuision, 1.℞ = 22.ϑ.

Scholar. Then maie I easily saie, that Ephestio was .22. yeres olde, seyng you did putte .1.℞. for his age: and now .1℞. is founde to be .22. And therby all the other yeres be manifeste. For Alexander beyng .2 yeres elder, must be .24. And Ephestio his father had in age .22. and .24: and .4. more, that is .50. yeres. All whiche make .96. So is that question fully aunswered. *The proofe.*

22
24
50
96

Master. An other question is this. I had a somme of money owing unto me: whereof I did receiue at one tyme $\frac{1}{4}$ and afterward I receiued $\frac{2}{5}$ of that residue, whiche remained unpaied. And so remained the reste of the debte 27.l. I would knowe what was the first debte, & what wer the .2. seuerall paiementes *A question of debte.*

Scholar. This muste I obserue still, to name the firste doubtfull thyng. 1. ze. wherefore I sate that the firste debte was 1. ze. whereof I receiued $\frac{1}{4}$ And so did there remain. $\frac{3}{4}$. ze. of whiche reste, againe I receiued $\frac{2}{5}$, that is $\frac{6}{20}$. of the whole somme, or $\frac{6}{20}$ ze. And that beyng abated also, then did there remaine $\frac{9}{20}$ ze. whiche you named to be. 27. li. Then if $\frac{9}{20}$. ze. bee equalle to 27. li, diuide. 27. li. by $\frac{9}{20}$, and the *quotiente* will bee $\frac{540}{9}$, that is. 60. whiche was the whole debte: And then is it plaine, that $\frac{1}{4}$. of it is. 15. and $\frac{2}{5}$. of the residue is. 18. whiche maketh. 33. and then remaineth. 27.

Master. There is nothyng better then exercise, in attainyng any kynde of knowlege: And therfore I will proue you with diuerse questions, to make you the moare experte in this rule. And this is one.

A question of pauyng. There is a floore Paued with Square Bricke, the lengthe of that floore beyng longer then the breadth, by $\frac{1}{7}$, and the whole Pauemente containeth. 3584. brickes: I require to knowe the bredthe and lengthe.

Scholar. The lesser quātitie, whiche is the bredth I doe name. 1. ze. And then the lengthe will bee, by your proportion. $1\frac{1}{7}$. ze. Now must I multiplie the bredthe by the lengthe (for that is the orderly worke in all flatte formes, to finde out the whole platte) that is here. 1. ze. by. $\frac{8}{7}$. ze. and there will amounte the whole platte. $\frac{8}{7}$. z. whiche by your supposition is equalle to. 3584.

Wherfore accordyng to your rule, I diuide. 3584. by $\frac{8}{7}$. and the *quotiente* will be. 3136. whiche is a *Square* nomber, bicause there is one denomination omitted in this equatio. For betwene z. and φ. there is omitted. ze. And therfore must I extracte the square roote of. 3136. and it will bee the quantitie of. 1. ze. that I woorke in my tables, and finde it. 56. whiche must be the bredthe: for that I named. 1. ze. Then the length must be moare by $\frac{1}{7}$. of it: and so shall it be. 64.

Now

Now for to confirme my woorke, I multiplie.56. by.64 and it will make.3584. whiche is the nomber that you did name.

Master. That question is well aunswered: And if you had put.1.ꝛ.for the lengthe, as you might do, then the bredthe will be $\frac{7}{8}$.ꝛ. and the square $\frac{7}{8}$ʒ.and so.1.ꝛ. would bee.64. as you maie proue at leiser: but in the meane time, what saie you to this questiō? *An other woorke of that question*

There is a capitain, whiche hath a greate armie, & would gladly Marshall thē, into a square battaile, as large as mighte bee. Wherefore in his firste proofe of square forme, he had remainyng.284.to many. And prouyng again by puttyng.1. moare in the fronte, he founde wante of.25.men. How many souldiars had he, as you gesse? *A question of an armie.*

Scholar. I call the firste fronte.1.ꝛ. and then multiplying it Squarely: I shall haue for the whole battaile.1.ʒ. and so by your saiyng, there was lefte 284.men, wherefore the whole nomber of men, was 1.ʒ.—+—.284.𝓈.

Now for the seconde proofe, when the fronte was increased by.1.man: I shall set the former fronte, and 1.manne moare, that is 1.ꝛ—+—1.𝓈. And multipliynge that nomber, squarely: there will arise for the whole armie. 1.ʒ—+—2ꝛ—+—1𝓈. out of whiche I muste abate 25 that, you saie, did wante, to make vp that square battaile. And then it will be.1.ʒ.—+—2.ꝛ.———24.𝓈.

1.ꝛ.—+—.1.𝓈.
1.ꝛ.—+—.1.𝓈.
____
1.ʒ.—+—.1.ꝛ.
          1.ꝛ—+—1.𝓈
____
1.ʒ—+—2.ꝛ.—+—1.𝓈.

Now haue I one nomber of menne, expressed by.2 Cossike nombers: Of necessitie therefore must these.2. nombers be equalle: seyng thei represente one armie. Wherfore I set them thus.

1.ʒ—|—284.φ.=====1.ʒ—|—2.ze——24.φ.

And findyng.1.ʒ. on bothe partes of the equation, I doe abate it, & then standeth.284φ=====2ze——24φ. Yet again I see.φ.on bothe sides of the equation, and therfore, seing the lesser of them hath the signe of subtraction, I doe adde.24:to bothe nombers, and then will there be.308.=====2.ze.that is.154=====1.ze

So that seing.1.ze was set for the first fronte:the same front must be.154.whose Square is.23716.unto whiche I muste adde the.284.that did abounde, and then will the whole nomber be.24000.

For farther trialle whereof, I take the seconde fronte to be.155.that is.1.moare then the firste: and his Square will bee 24025. And so is there.25.moare then the iuste nomber of the armie, as the question supposed.

```
    154.
    154.
  ------
    616.
   770
  154
  ------
  23716.
    284.
  ------
  24000.
```

An other woorke of that questiō. Master. That question maie be wrought also, by namyng the seconde fronte.1.ze.and then will his square bee.1.ʒ.but seyng there wanteth.25.menne, to make that Square battaile, the nomber shall bee 1.ʒ.——25.φ.

Then for the firste front, you must set.1.man lesse, as the question importeth, & that will be.1.ze——1.φ whose square will be 1.ʒ—|—1.φ.——2.ze.

```
1.ze.——.1.φ.
1.ze.——.1.φ.
---------------------------
1.ʒ.——.1.ze.
   ——.1.ze.—|—.1.φ.
---------------------------
1.ʒ.—|—.1.φ.——.2.ze.
```

unto whiche I must adde the.284 menne that did abounde, whē that battaile was framed, and then will the

the nomber be.1.ʒ.—+—.285.φ.———.2.𝓇. And it must bee equalle to.1.ʒ.———.25.φ. Wherfore to reduce that equation, firste I adde on bothe sides 25.φ. then resteth.1.ʒ.equalle to.1.ʒ.—+—.310———2.𝓇. Then I adde.2.𝓇.because I will haue noe——in the equation: and it will be,

1.ʒ.—+—.2.𝓇.═══.1.ʒ.—+—.310.φ. Thirdely I abate.1.ʒ.on bothe sides of the equation: and then remaineth,2.𝓇.═══.310.φ.that is.1.𝓇.═══155.φ. wherby it appeareth that the seconde fronte was.155 and the firste fronte.154.& so forthe, as you wrought it before.

An other question is this.

There is a kyng with a greate armie: And his aduersarie corrupteth one of his heraultes with giftes, and maketh hym swere, that he will tell hym, how many Dukes, Erles and other souldiars there are in that armie. The Heraulte lothe to lease those giftes, and as lothe to bee untrue to his Prince, diuiseth his aunswere, whiche was true, but yet not so plain, that the aduersarie could therby understande that, whiche he desired. And that aunswere was this. *A question of an armie.*

Looke how many Dukes there are, and for eche of them, there are twise so many Erles. And under euery Erle, there are fower tymes so many soldiars, as there be Dukes in the fielde. And when the muster of the soldiars was taken, the.200.parte of them, was 9.tymes so many as the nomber of the Dukes.

This is a true declaratiō of eche nomber, quod the Heraute: and I haue discharged my othe. Now gesse you how many of eche sorte there was.

Scholar. Although the question seme harde, I see many tymes, that diligence maketh harde thynges easie, and therfore I will attempte the worke of it.

And firste for the nomber of Dukes, I sette.1.𝓇. then will the nomber of Erles bee.2.ʒ.that is.1.𝓇. by by.1.ze multiplied twise: And the nomber of soldiars are.8.cꝭ.that is. 2.ʒ. multiplied by.1.ze. fower tymes, but bicause the. 200. parte of the soldiars is.9. tymes so moche as the nomber of the Dukes, therfore must the.200.parte of.8.cꝭ be equalle to.9.ze. And so consequently.8.cꝭ ═══ 1800.ze and 1.cꝭ ═══ 225.ze .or.1.ʒ. ═══ 225.𝔑.

For if I set $\frac{8}{200}$ and. 9. as equalle together, & would by the arte of fractions, brynge the same proportion in whole nombers, I shall haue for. 9. this fraction $\frac{1800}{200}$. And seyng the denominators, be all one in $\frac{8}{200}$ and in $\frac{1800}{200}$ the proportion consisteth betwene the numerators.

Then to procede, if.225.be equalle to.1. ʒ. I shall take the square roote of.225.for.1.ze. and that is.15 whiche must be the nomber of Dukes.

And so haue I the firste nomber, wherefore the seconde nomber, that is the nomber of Erles, must bee 15.tymes.15.twise: that is.450. And the nomber of soldiars shall be.4.tymes.15. multiplied by.450. that is.27000. And for iuste trialle of this worke, I take the.200. parte of the soldiars that is. 1350. and I finde it to bee.9. tymes.15. that is.9.tymes so moche as the nomber of the Dukes. And so is that question solued, and tried.

450.
60
27000

An other question of walles.

Master. This is an other quetion. There is a grounde inclosed with.4.walles, beyng like iambes and of one heigthe. The longest.2.walles are in proportion to the shortesse, as.5.to.3 And unto the height thei bee double *Sesquialter*. Now multiplying the longesse by the shortesse, and that totalle by the heighte, there will rise.39930.foote. I demaunde then, what is the lengthe and the heighte of eche walle?

Scholar. The least quãtitie is the heighte, whiche I call. 1. ze. and unto it the longesse walle is double

Sesquialter:

Sesquialter: that is.2 $\frac{1}{2}$.ꝛ. Now that same longeste is in proportion *Superbipartiente quintas*, to the shorteste walle. So must the shorter wall be 1 $\frac{1}{2}$ ꝛ. Then must I multiplie all those.3.nōbers together, that is 1.ꝛ. by.1 $\frac{1}{2}$ ꝛ. whereof doeth come.$\frac{3}{2}$ ƺ. then shall I multiplie that totalle, by $\frac{5}{2}$ ꝛ. and it will be $\frac{15}{4}$ ꝯ. or 3 $\frac{3}{4}$ ꝯ whiche must be equalle, by the wordes of the question, to.39930.

So by reducyng them to one denomination. $\frac{15}{4}$ ꝯ. shall be equalle to $\frac{159720}{4}$ that is.15.ꝯ ═══ 159720.φ. and.1.ꝯ. ═══.10648. wherfore I shall extracte the *Cubike* roote out of.10648. and that is the quantitie of.1.ꝛ. or the heighte of the walle.

In my Tables I worke that extraction of *Cubike* roote, and finde it to be.22. And therfore must the lōgeste walle bee double *Sesquialter* to it, that is.55. And the shorteste walle will be.33.

For proofe whereof I doe multiplie.22.with.55. and it maketh.1210. whiche nomber I shall multiplie by.33.and it will be.39930.accordyng to the supposition of the question. *The proofe.*

Master. You doe chose still the leaste nomber, to be equalle to.1 ꝛ.as the easieste forme. Howbeit you maie put.1.ꝛ.for the lengthe of any of the walles.

And if you sette it for the longeste walle, then the shorteste walle will be $\frac{3}{5}$ ꝛ. and the heighte $\frac{2}{5}$. ꝛ and all those.3.nombers will make, by multiplication together $\frac{6}{25}$ ꝯ. equalle to.39930. And so will.6.ꝯ. be equalle to.998250.φ.and.1.ꝯ. ═══.166375.φ. whereof the *Cubike* roote is 55.and aunswereth to the quantitie of.1 ꝛ. *An other forme of worke.*

But if.1.ꝛ. be set for the measure of the shorteste walle, then the longeste walle will bee $\frac{5}{3}$. ꝛ. And the heighte $\frac{2}{3}$.ꝛ. And so all.3. nombers multiplied together will make $\frac{10}{9}$ ꝯ. ═══. 39930. So shall. 10.ꝯ. be equall to.359370. And.1.ꝯ. ═══.35937.where- *The thirde forme of worke.*

of the *Cubike* roote is. 33. and is the value of. 1.ꝛ. in this poſition.

Scholar. This varietie of woorke, is not onely pleaſaunte, but it maketh the reaſon of the woorke to appeare moare plainly. So that I could neuer be werie to heare ſoche queſtions.

Maſter. Then will I propounde one or 2. moare before we paſſe from this kinde of equation. Whereof one ſhall be ſomewhat like that laſt. And this it is.

A queſtion of Bricke. A Brickeleiar had a pile of Bricke, whiche he ſold by the yarde. The lengthe of it was $\frac{7}{2}$ to the bredthe, that is *Tripla ſeſquialtera*. And the heighte was fiue tymes ſo moche as the lēgthe. This pile the owner ſold for. 980. crounes. By ſoche rate that he had for euery yarde ſo many Crounes, as the Pile had yardes in bredthe. Now is the queſtion, what was the lengthe, bredthe, and heighte of this pile?

Scholar. I ſuppoſe the bredthe to bee. 1. ꝛ. then was the length $3\frac{1}{2}$ ꝛ. and the heighte. $17\frac{1}{2}$ ꝛ. Theſe 3. ſommes dooe I multiplie together, and thei make $\frac{245}{4}$ cꝫ. whiche ſtandeth as equalle to all the yardes in the whole pile. But yet what that is, I knowe not.

Wherfore to procede farther, I conſider that euery yarde coſte as many crounes, as the bredthe contained yardes. Now the bredthe being 1. ꝛ I muſt ſaie, that euery yarde did coſte. 1. ꝛ. of crounes. And then by the Golden rule: if. 1. yarde coſte. 1. ꝛ. of Crounes, what ſhall $\frac{245}{4}$ cꝫ. coſte?

1.		1. ꝛ.
$\frac{245}{4}$ cꝫ		$\frac{245}{4}$ ȝȝ.

Woorkyng by the rule, I finde that it ſhall coſt $\frac{245}{4}$ ȝȝ. And the queſtion doeth ſuppoſe that it coſte. 980. crounes. Wherfore I muſt ſaie, that. 980. crounes, are equalle to $\frac{245}{4}$ ȝȝ. And conſequently. 245. ȝȝ. ══ 3920. ꝯ. wherfore diuidynge the nomber of the leſſer name, by the other, the *quotiente* will be 16. whoſe *zenzizenzike* roote is 2

And

And that therfore must be the value of a roote, and the bredthe of the pile. So shall the lengthe be .7. yardes, and the heighte .35. yardes.

For trialle of it, I mutiplie the lengthe, by the bredthe, and that totalle by the heighte, and so haue I 490. for all the yardes of Bricke. Then considering that euery yarde coste .2. crounes, bicause .2. yardes is the bredthe of the pile: the nomber of crounes must be twise .490. that is .980. And so is the woorke good. *The proofe.*

Master. Now woorke that question, by settynge 1. ze. for the lengthe. *An other forme of woorke.*

Scholar. If the lengthe be .1. ze. the bredthe must bee $\frac{2}{7}$. ze. that is *Subtripla sesquialtera* to. 1. ze. And the heighte must bee .5. ze. All whiche sommes make by multiplication $\frac{10}{7}$ cE.

Then farther, if 1. yarde coste $\frac{2}{7}$ ze. $\frac{10}{7}$ cE. shall coste $\frac{20}{49}$. ȝ ȝ. whiche is equalle to 980. And so is .20. ȝ ȝ. equal to .48020. and by diuision 1. ȝ ȝ. ═ .2401. whose *zenzizenzike* roote is .7. And that is the lengthe of the walle, and is the value of. 1. ze.

1.		$\frac{2}{7}$ ze.
$\frac{10}{7}$ cE.		$\frac{20}{49}$ ȝ ȝ.

The reste of this woorke, is like as before.

Master. Yet proue the thirde waie.

Scholar. The heighte beeyng. 1. ze the lengthe must be the first part of it, that is $\frac{1}{5}$ ze. And the bredth $\frac{2}{35}$ ze. All these make by multiplication $\frac{2}{175}$ cE. Then for the price, if. 1. yard coste $\frac{2}{35}$ ze what shall $\frac{2}{175}$ cE. coste? By the Golden Rule there is founde, $\frac{4}{6125}$. ȝ ȝ. whiche is equalle to 980. And so shall. 4. ȝ ȝ. be equalle to. 6002500. And. 1. ȝ ȝ. ═ 1500625. whose *zenzizenzike* roote is. 35. And that is the value of. 1. ze. and the heighte of the Pile. *A thirde forme of woorke.*

1.		$\frac{2}{35}$ ze.
$\frac{2}{175}$ cE.		$\frac{4}{6125}$ ȝ ȝ

Master. One question moare will I propounde, and so eande with this equation.

A question of a Testament. A poore man died, whiche had fower children, and all his goodes came to. 72. crounes: whiche he would haue parted so, that the seconde & thirde childe should haue. 7. times so moche as the firste. And that the portions of the thirde and fourthe childe should bee. 5. tymes so moche as the secondes parte: And that the first and the fourthe, should haue twise as moche as the thirde. If you worke the solution wel, you maie seme worthy to be master of those wardes.

Scholar. I trust to obtaine moare benefite by the question, then by that office. Wherefore I will giue good hede unto it. And for the firste nōber, I set. 1. ze then muste the seconde and thirde portions make together. 7. ze. And the fourthe must bee all the reste of the. 72. that is. $72 - 8$ ze. Now the thirde must be halfe the firste & the fourthe, that is. $36 - 3\frac{1}{2}$ ze. And the third & fourthe, is. 5. tymes the second. Wherfore the seconde shall be the. 5. part of. $108 - 11\frac{1}{2}$ ze that is. $21\frac{3}{5} - 2\frac{3}{10}$ ze, whiche nomber I shall set in order with Letters, as here I haue dooen for my owne ease, and aide of memory. And then shal I adde them all together. Whereof there commeth.

A			1. ze.
B	$21\frac{3}{5}$	$-$	$2\frac{3}{10}$ ze.
C	36.	$-$	. $3\frac{1}{2}$ ze.
D	27.	$-$	. 8. ze.
	$129.\frac{3}{5}$	$-$	$12\frac{4}{5}$ ze.

$129\frac{3}{5} - 12\frac{4}{5}$ ze, whiche is equalle to 72. First therfore I do adde all that foloweth — to bothe partes of the equatiō. And so haue I $129\frac{3}{5} = 12\frac{4}{5}$ ze $+ 72$. But bicause there are nombers Absolute on bothe sides, I shall abate the lesser somme, that is. 72. from bothe partes, and then will there bee lefte, $57.\frac{3}{5} = 12\frac{4}{5}$. ze. that is. $288. = .64.$ ze. And by diuision $4\frac{1}{2}. = .1.$ ze.

The proofe. So shall the firste mannes portion bee $4\frac{1}{2}$. And the seconde and thirde mannes portion. 7. times so moche that

that is.31 $\frac{1}{2}$. Whereby it followeth, that the fourthe manne, shall haue the reste of 72. that is.36.

Then seyng the thirde manne, hath halfe so moche as the first and the fourthe, his portiō shall be 20 $\frac{1}{4}$.

A	4 $\frac{1}{2}$.	
B	11 $\frac{1}{4}$	} 31 $\frac{1}{2}$.
C	20 $\frac{1}{4}$	
D	36.	
	72.	

And then by diuerse reasons, the seconde mānes part shall bee.11 $\frac{1}{4}$. And all these partes added together, doe make iuste 72. Wherfore the woorke is good.

Master. You haue wroughte it well. And yet maie you woorke it thus. Firste sette doune. 1. ze. for the firste mannes parte. And then for the seconde and thirde ioynily. 7. ze. so shall the fourthe manne haue 72. φ. ———. 8. ze. And bicause the seconde mannes parte is $\frac{1}{5}$. of the thirde and fourthe mannes portion, if you ioyne all their. 3. partes together, the seconde mannes portion will be $\frac{1}{6}$ of that totalle. But therfore 7. ze, whiche is the partes of the second and the third vnto. 72 ——— 8 ze, whiche is the fourthe mannes parte, and the totalle will be. 72. φ ——— 1. ze. whose sixte parte is 12. φ. ——— $\frac{1}{6}$ ze, for the seconde mannes share. Whiche somme if you abate out of. 7. ze. there wil remain for the thirde mannes parte 7 $\frac{1}{6}$ ze ——— 12. φ.

An other forme of woorke.

And so haue you euery mannes portiō allotted to hym duely. As I haue here set it forthe for you. And all thei added together, doe make. 72.

A		1. ze.
B	12. φ. ———	$\frac{1}{6}$ ze
C	7 $\frac{1}{6}$ ze ———	12. φ
D	72. ———	.8. ze.
	72.	

Scholar. But here is noe equatiō yet, though the partes be diuided iustly.

Master. Now shall you see it.

The question saieth, that the thirde mannes portion is halfe the portions of the firste and fourthe man. wherefore seyng the firste and fourthe mannes portions doe make. 72 ——— 7. ze. the thirdemannes portion

tion beeyng doubled, shall make as moche. But the double of the thirde mãnes parte, is 14 1/3 ze —— 24 ꝭ. and therfore I saie, that those. 2. nombers be equalle; that is, 72. ꝭ. —— 7. ze. = 14 1/3 ze. —— 24. ꝭ. Firste adde. 7. ze. to eche parte, and it will bee 72. ꝭ. = 21 1/3 ze. —— 24. ꝭ. Then adde. 24. ꝭ. on bothe sides, and there will be. 96. ꝭ. = 21 1/3 ze. that is by reduction. 288. = . 64. ze. as you made it. And then all agreeth.

Likewaies for the equation, you maie set the third mannes portion, with the halfe of the firste & fourthe mennes partes. And so will. 7 1/6. ze. —— . 12. ꝭ. be equalle to. 36. ꝭ. —— 3 1/2 ze. And by reduction, 10 2/3 ze. = . 48. ꝭ. That is in other termes of whole nomber. 32 ze. = . 144. And by diuision it will bee 1. ze. = . 4 1/2. And thus will we ende the examples of the firste equation, for this tyme. And will shewe you some questions of the seconde equation.

Examples of the seconde equation, by questions propounded.

A question of silkes.

There are two men that haue silke to sell. The one hath. 40. elnes, and the other. 90. And the firste man his silke is not so fine as the seconde man his silke. So that he selleth in euery angell, price more by 1/3 of an elne, then the seconde mã doeth. And at the ende, bothe their moneis made but 42. angelles. Now I demaunde of you, how moche eche man solde for an angell?

Scholar. I will folowe my olde forme, in putting 1. ze. for the leaste quantitie, whiche is the seconde mannes somme, and then shall the firste mannes somme be. 1 1/3 ze.

Master. You are deceiued all readie. For you set

1. ze.

1.ze. for an elne. Seyng you name $\frac{1}{3}$ of an elne, to be $\frac{1}{3}$. ze. And so were the position neadelesse, and likewaies all the woorke.

Scholar. I see my faulte: but I knowe not how to amende it. For that. 1. ze. maie bee a parte or partes of an elne: and so maie it be moare then. 1. or. 2. elnes so that I ought not to haue set $\frac{1}{3}$ (whiche is certainly referred, in this question, to an elne) as the parte of a doubtfull quantitie, but rather as the parte of a quantitie certaine. Whereas. 1. ze. is euer put for a nomber unknowen.

Master. To helpe you herein, I will set the firste nombers, as you began theim. The seconde man his nombers of elnes, shall bee. 1. ze. as you did name it, and then shall the firste mannes portion be as moche, and $\frac{1}{3}$ of an elne moare: whiche $\frac{1}{3}$ I maie beste call $\frac{1}{3}$ ℈. And so shall it bee distaunte from 1. ze. clerely in all woorke *Arithmeticall*.

A	$1.\text{ze} + \frac{1}{3}\,℈.$
B	$1.\text{ze}.$

But now to proceade, I shall diuide eche mannes nomber of elnes, whiche he had, by the nomber of elnes, whiche he solde for an angelle, and the *quotiente* will declare how many angelles eche man had receiued. So that the firste mannes nomber of elnes, beeyng. 40. shall bee the numerator, and the somme of measure, whiche he solde for an Angelle, that is 1 ze $+ \frac{1}{3}$ ℈. shall bee the denominator. And so is the diuision ended. And that fraction is the *quotiente*.

$$\frac{40.}{1.\text{ze} + \frac{1}{3}\,℈.}$$

$$\frac{90.}{1.\text{ze}.}$$

Scholar. Now I perceiue the woorke. And by like reason: the seconde mannes somme of elnes beeyng. 90. shall bee the numerator, and. 1. ze. beyng the somme of measure, solde for one Angelle, shall be the denominator, that is in one fraction $\frac{90}{1.\text{ze}}$: accordingly as I haue sette bothe nombers here

here diſtinctly.

Maſter. It were moare eaſe for you in workyng, if you did tourne that fractiō of ⅓ into an intere vnitie.

Scholar. That wil eaſily be doen, by multiplyng euery nomber, of that whole fraction by. 3. And then will it be $\frac{120}{3ze + 1φ}$, whiche is all one in value with $\frac{40}{1ze + \frac{1}{3}φ}$. And this I conſider farther, that as theſe. 2. fractions, ſeuerally dooe expreſſe the ſommes of angelles, that eche of them receiued, ſo ioyntly bothe together, dooe declare the full ſomme, of all their angelles. Wherefore I ſhall adde theim bothe together. And thei will make. $\frac{390ze + 90φ}{3ʒ + 1ze}$ As here in woorke I haue expreſſed.

390.ze. + .90.φ.

120. / 3.ze. + .1.φ. 90. / 1.ze.

3.ʒ. + .1.ze.

And by your ſuppoſition, their bothe ſommes of Angelles made. 42. So that thoſe. 2. ſommes are equalle: and therefore am I come to an equation. In whiche I ſee a nomber abſolute, equalle to a fraction *Coßike* compounde.

Maſter. When ſo euer that, or the like dooeth chaunce, you ſhall reduce the whole nōber, to the like denomination: and then their numerators will bee equalle.

Scholar. Then ſhall I multiplie. 42. by the denominator 3ʒ + 1ze & it wil be 126ʒ + 42ze whiche muſte bee equalle to. 390.ze. + .90.φ. That is in leſſer termes.

21ʒ + 7ze. ═══ .65.ze. + .15.φ.

Where firſte I dooe abate. 7.ze. on bothe ſides: and there remaineth then. 21ʒ. ═══ .58.ze. + .15.φ.

But

But now I remember your admonitiō, that bicause the nomber annexed to the greateste signe, is moare then. 1. I shall diuide all the nombers by it, and sette their *quotientes* in their stede, with their signes. And so will the nomber of the greateste signe, euermore be 1. And this equation will be $1.ȝ. = \frac{58}{21} ꝛ. —|— \frac{15}{21} ϑ.$ Where I must extracte the square roote of the later part, according to your doctrine, and it will be. 3. As it appereth in this worke folowing, whiche I did frame in my tables.

$\frac{29}{21}$. in square doeth make $\frac{841}{441}$, vnto whiche I muste adde $\frac{315}{441}$, whiche is all one with $\frac{15}{21}$, by reduction to one denominatiō. So is the full additiō $\frac{1156}{441}$. whose square roote is $\frac{34}{21}$. vnto whiche I shall adde $\frac{29}{21}$, and it will bee $\frac{63}{21}$, that is. 3.

Master. This is well doen. Now worke the same questiō, as it was proponed, and you shall easily finde all the other nombers to bee true, and agreable to the question.

Scholar. Seyng the seconde manne solde. 3. elnes for an angell, the firste manne did sell. 3. elnes and $\frac{1}{3}$. So. 40 (whiche is the somme of elnes of the first man his silke) diuided by. $3\frac{1}{3}$. doeth yelde. 12. and sheweth how many angelles that man receiued. *The proofe.*

Again for the seconde man, whiche had. 90. elnes, diuide that. 90. by. 3. and so shall you finde. 30. for the nomber of his Angelles. And that. 30. and. 12. dooe make. 42, it neadeth not to be proued.

Master. Now againe for your exercise, suppose the firste mannes somme to be. 1. ꝛ. *An other forme of woorke.*

Scholar. Then muste the seconde manne sell for an angelle. $1.ꝛ. ——— \frac{1}{3}.ϑ.$ And their nombers of elnes, diuided by those nombers will make. $\frac{40}{1ꝛ}$. and $\frac{90}{1ꝛ ——— \frac{1}{3}}$. whiche bothe added together, will bee $\frac{390ꝛ ——— 40ϑ}{3ȝ ——— 1ꝛ}$ equalle to. 42. ϑ. That is by reduction.

$390.ꝛ. ——— .40ϑ. = .126.ȝ. ——— .42.ꝛ.$

And by addition of. 42. ze. on bothe partes.
432. ze ——— 40. 𝔯. ═══. 126. ʒ. And by diuision it will be. $\frac{24}{7}$ ze ——— $\frac{20}{63}$ 𝔯. ═══ 1. ʒ.

So that now I must extracte the roote of that compounde *Coßike* fraction, thus. $\frac{12}{7}$ squarely, dooe make $\frac{144}{49}$ out of whiche I shall abate $\frac{20}{63}$. And therfore, firste of all I doe reduce thē to one denomination, & thei make $\frac{9072}{3087}$. and $\frac{980}{3087}$. wherefore if I abate the lesser out of the greater: there will remaine $\frac{8092}{3087}$. that is in lesser termes $\frac{1156}{441}$ and is a square nomber, whose roote is. $\frac{34}{21}$ vnto whiche if I adde $\frac{12}{7}$ that is $\frac{36}{21}$. it will make $\frac{70}{21}$, or $\frac{10}{3}$. that is the valewe of. 1. ze. And is the firste mannes nomber of elnes, agreably as I tried it before. And so doe bothe workes agree.

But now commeth to my remembraūce, that this nomber, whose roote I did extract, in this laste worke is of that sorte, where. ze. ——— 𝔯. is equalle to. ʒ. And therfore hath in it. 2. rootes: thone by addition, as this, whiche I now founde: And the other by subtraction, whiche in this example, by abatyng $\frac{34}{21}$ out of $\frac{36}{21}$, will bee $\frac{2}{21}$. But how I maie frame that roote, to agree to this question, I doe not see.

Master. That varietie of rootes dooeth declare, that one equation in nomber, maie serue for. 2. seueralle questions. But the forme of the question, maie easily instruct you, whiche of those. 2. rootes, you shall take for your purpose. Howbeit sometymes you shall take bothe. As for example again, marke this question.

A question of money.

A gentilman, willyng to proue the cunnyng, of a braggyng *Arithmetician*, saied thus: I haue in bothe my handes. 8. crounes: But and if I accoumpte the somme of eche hande by it self seuerally, and put therto the squares and the *Cubes* of thē bothe, it will make in nomber. 194. Now tell me (quod he) what is in eche hande: and I will giue you all for your laboure.

Scholar.

Scholar. Soche incoragementes, would make me studie harde, and trauell very willyngly in learned exercises: though learnyng bee moste to be loued, for knowledges sake. But for to finde the true aunswere thus I doe proceade.

Firste I suppose the one nomber in one hand, to be 1.ze. And then must the other nedes be 8.φ — 1.ze. Then doe I make theim bothe Squares. And for the firste I haue. 1 z. and for the seconde. 1.z + 64 φ — 16.ze. Thirdely I multiplie them bothe *Cubikely*: and so haue I for the firste. 1.ce and for the other 24.z. + .512.φ. — 1.ce. — 192.ze. Then must I adde bothe the nõbers, with their squares, and their *Cubes*, into one somme. As here in work

1.ze. + .1.z. + .1.ce.
8.φ. — .1.ze.
1.z. + .64.φ. — .16.ze.
24.z + .512 φ — 1 ce — 192.ze.

26.z + 584.φ. — .208.ze.

it is set forthe. Where for ease I haue set. 1.ze, 1.z. and. 1.ce (whiche is the Roote, the Square and the Cube of one nomber) all in one line: and the other Roote, Square, and Cube, I haue set seuerally. And so all thei doe make. 26 z + 584 φ — 208 ze whiche is equalle to. 194. by the intente of the question. Wherefore I adde firste. 208.ze. to bothe partes, and there remaineth.

26.z. + 584.φ ═══ 208.ze + 194.φ.
Then I abate. 194. from bothe sides, and so resteth the equatiõ thus. 26.z + 390.φ ═══ 208 ze
That is by diuision. 1.z. + .15.φ. ═══ 8.ze.
And by translation of. 15.φ. to sette. 1.z. alone, it wil be. 1.z ═══ 8 ze — 15.φ. And now haue I the exacte and complete equation, where I must seke for

the value of. 1. ℨ. by extractyng the roote. Therfore firste I take halfe of. 8 and that is. 4. whose square is. 16. out of whiche I abate. 15. and the remainer is 1. whiche I maie either adde to. 4. and so haue I. 5. or ther, I maie abate it from 4 and so haue I 3. Whiche nombers also accordyng to the same rule, beyng added together dooe make. 8. that is the nomber of the middell denomination. And beyng multiplied together, thei dooe make. 15. that is the other parte of the same compounde *Coßike* nomber.

Master. And if you had marked that firste, you might easily haue found bothe those nombers, by the partes of. 15. whiche can be none other, but. 5. and 3.

And farther, seyng thei. 2. doe make. 8. and. 8. is the nomber (named in the questiõ) that thei should make, therfore you shall take them bothe. And name whiche of them you liste to be. 1. ℨ. And the other shall be of necessitie, the reste of. 8.

The proofe. Scholar. To examine theim, by the order of the question, I doe proceade thus. 3. with his Square. 9. and his *Cube*, 27. dooeth make. 39. And. 5. with his square 25 and his *Cube*. 125. doe yelde 155. And bothe thei together doe bryng forthe. 194. accordyng to the saiyng of the question: and therfore it is certain, that the woorke is good.

An other woorke for equations. Master. Before you passe any farther, I will admonishe you of one waie, whiche I oftẽ vse in reduction of soche equations, as this is, when there is noe denomination on the one side, but the like is on the other side, with a greater nomber annexed to it. Then maie you abate all the lesser nõbers, out of their greaters, and the reste shall bee accoumpted equalle to nothyng. Whiche chaunce can neuer happen: excepte there bee some nombers on the greater side, with the signe of abatemente. ——.

As here you had.

26ȝ. —+— 584&. —— 208ꝛ. ═══ 194&.

Bicause on the one side, there is noe nõber but 194& and on the other side, there is. 584.&. beeyng a greater nomber, and of thesame denomination: therfore maie you abate. 194. from bothe sides, and then remaineth. 26ȝ. —+— 390& —— 208ꝛ ═══ 0

Wherfore you maie well consider, that the nombers whiche be ioined with —+—. are equalle to the nombers that bee set with ——. And therfore the one abatyng the other iustly, dooe remaine together as equalle to nothyng.

Wherefore it is reasonable, that seyng the nombers with —— bee equalle to the nombers with —+— that I maie translate the nombers with —— from that side of the equation, and set them on the cõtrary side, with the signe of —+—. And so in this exãple it will bee. 26ȝ. —+— .390&. ═══ 208.ꝛ. And this forme shall ease you moche, in reducynge of equations.

Scholar. I thanke you moche. And I will not forget to vse it, as occasiõ shall happen. But I praie you propounde yet some moare questions, that I maie see their diuerse varieties.

Master. There were twoo seueralle men, which had certaine sommes of angelles, in soche rate, that the seconde manne his somme, was *triple sesquiquarta* to the firste: and if their. 2. sommes were multiplied together, and to that totall the 2 firste sommes added, there would amounte. 142 $\frac{1}{2}$. I demaunde of you, what was eche of their sommes in angelles? *A question of money.*

Scholar. The firste mannes somme I call. 1.ꝛ. And the seconde mannes sõme shall be. 3 $\frac{1}{4}$ ꝛ. which 2. sommes beeyng multiplied together, dooe make 3 $\frac{1}{4}$ ȝ. vnto whiche I must adde bothe the firste nombers, that is 4 $\frac{1}{4}$ ꝛ. And it will be 3 $\frac{1}{4}$ ȝ —+— 4 $\frac{1}{4}$ ꝛ equalle to. 142 $\frac{1}{2}$. All whiche nombers, I shal bring into whole nombers, if I multiplie theim by. 4. And so will it be. 13. ȝ. —+— 17. ze ═══ 570. And by reducyng the greateste denomination *Coßike*, to an vnitie. 1. ȝ. —+—. $\frac{17}{13}$ ze. ═══. 43 $\frac{11}{13}$. And laste of all, by translatyng the nomber of. ze. to set. 1. ȝ. alone, on one side of the equation, it will be. 1. ȝ ═══ 43 $\frac{11}{13}$ ϑ ——— $\frac{17}{13}$. ze. where I must extract the value of the roote thus. $\frac{17}{26}$. squarely dooe make $\frac{289}{676}$. vnto whiche I shall adde the. 43 $\frac{11}{13}$ (it beeyng firste multiplied by. 52. to bryng it to the denomination of. 676. And so makyng $\frac{29640}{676}$) And it will be $\frac{29929}{676}$ whiche is a square nomber (as I haue proued in my Tables) and his roote is $\frac{173}{26}$. from whiche roote I must abate $\frac{17}{26}$, and then wil there remain $\frac{156}{26}$, that is. 6.

And that. 6. is the value of. 1. ze, and standeth for the firste mannes nomber. So that the seconde mannes nõber must be as. $\frac{13}{4}$ to it: that is *tripla sesquiquarta*. And so shall it be. 19 $\frac{1}{2}$.

The proofe. Master. Now proue those nombers, accordyng to the question.

Scholar. 19 $\frac{1}{2}$ multiplied by. 6. doeth make. 117. vnto whiche I shall adde. 25 $\frac{1}{2}$. amountyng of their. 2 additiõs, and all will be. 142 $\frac{1}{2}$, accordyng to the purporte of the question.

An other worke of the same questiõ. Master. So is your woorke good. Yet woorke it again, by chaungyng the position.

Scholar. I maie put. 1. ze. to betoken the seconde manne his somme. And then shall the firste mannes somme bee $\frac{4}{13}$. ze. whiche bothe multiplied together doe make $\frac{4}{13}$ ȝ. And then addyng the. 2. firste sommes to it, it wil bee $\frac{4}{13}$ ȝ —+— 1 $\frac{4}{13}$ ze. And that is equalle to. 142 $\frac{1}{2}$. All whiche nombers will bee reduced to whole nombers, by multiplication conueniente. And so will it be. 8. ȝ —+— 34. ze. equalle to. 3705: that is by reduction, 1. ȝ. —+—. 4 $\frac{1}{4}$ ze ═══ 463 $\frac{1}{8}$ ϑ. and by translation of the termes.

1. ȝ.

1 ʒ. ═══ 463 $\frac{1}{8}$. φ. ——— 4 $\frac{1}{4}$. ro. out of whiche nomber I shall extract the value of the roote, in this sorte.

Firste I saie $\frac{17}{8}$ multiplied Square, doeth make $\frac{289}{64}$, vnto whiche nomber I must adde. 463 $\frac{1}{8}$, reduced as it ought, and it will bee in all $\frac{29929}{64}$. whiche is a square nomber, and hath for his roote $\frac{173}{8}$. from whiche I must abate $\frac{17}{8}$. And then will there remain $\frac{156}{8}$, that is 19 $\frac{1}{2}$, for the value of. 1. ro. And so consequently for the second mannes nõber: whiche was named in this position, 1. ro. And this maie bee proued as the other was.

Master. What saie you then to this question? *A question of iorneyng.*
There is a straunge iorneye appointed to a manne. The firste daie he must goe 1 $\frac{1}{2}$ mile, and euery daie after the firste, he must still augmente his iorney, by $\frac{1}{6}$ of a mile. So that his iorney shall procede by an *Arithmeticalle* progression. And he hath to trauell for his whole iorney. 2955. miles. I demaunde in what nõber of daies, shall he ende his iorney?

Scholar. I knowe not how to proceade in this question.

Master. Doe you not heare me name it, an *Arithmeticalle* progression? Wherby you might be assured, that it doeth appertaine to that rule. And accordyng to the canons of that rule, must you worke this question. But for your better instruction, I will helpe you in this worke.

Firste aunswere to the question, by the common position: and saie that the tyme of his iorney is. 1. ro. of daies. And then shall all the *excesses* (whiche maie also be called the *nomber of the spaces*) be. 1. ro ——— 1 φ. The *common excesse* was supposed to bee. $\frac{1}{6}$. of a mile. And therefore the *somme of all the excesses* muste bee $\frac{1\ ro \text{ ——— } 1\ φ}{6}$ that is to saie, the nomber of all the *excesses* multiplied by $\frac{1}{6}$, that is here, the sixte parte of the nomber

nomber of the *excesses*.

And bicause that *the firste nomber* is. $1\frac{1}{2}$. I must adde it vnto the *somme of the excesses*, and so haue I the *laste nomber* of that progression. Wherefore addyng. $1\frac{1}{2}$. (whiche is $\frac{3}{2}$, or in like denomination with the other, $\frac{9}{6}$) with $\frac{1ze - 1ꝰ}{6}$ it will make $\frac{1ze + 8ꝰ}{6}$. And that is *the laste nomber* of the progression.

Now you remember, that in progression *Arithmeticall*, if you adde *the firste* nomber to *the laste*: and multiplie that totalle, by the *nomber of halfe the places*, there doeth amounte the somme totalle of that progression.

And therfore in this exãple, if you adde. $1\frac{1}{2}$ (whiche is the firste nõber in the progression) vnto $\frac{1ze + 8ꝰ}{6}$ (that is the laste nomber of the progression) there wil amounte $\frac{1ze + 17ꝰ}{6}$, whiche beeyng multipilied by the nomber of halfe the places, that is $\frac{1}{2}$ ze. (For all the nomber of places is . 1 . ze) there will rise, $\frac{1ʒ + 17ze}{12}$, whiche is the totalle somme of all the miles: and therfore is equalle to. 2955.

Scholar. All the reste, and this againe can I doe now. Seyng $\frac{1ʒ + 17ze}{12}$. is equalle to. 2955. I will firste bryng the whole nomber to the like denomination, with the fraction, and it will bee. $\frac{1ʒ + 17ze}{12}$ ========= $\frac{35460}{12}$. And then omittyng the like denominations. 1. ʒ + 17. ze. ========= 35460. ꝰ. That is by translation 1. ʒ ========= 35460. ꝰ. — 17 ze. whose roote in value I shall finde out thus. I multiplie $\frac{17}{2}$ squarely, and it will be $\frac{289}{4}$. vnto whiche I shal adde. 35460. & it will make $\frac{142129}{4}$, whiche is a square nomber, and hath for his roote $\frac{377}{2}$, frõ whiche I shall abate $\frac{17}{2}$, and then remaineth $\frac{360}{2}$, that is. 180. whiche is the value of. 1 ze. And expresseth the nomber of dayes, whiche the question requireth.

The proofe. Master. The proofe in this, and the like questions, is, to set foorthe the progression with all his termes.

nies. Excepte you will for shortnesse, sette doune the firste terme, whiche in this example is. 1 $\frac{1}{2}$: and then by the nomber of the *excesses*, or distaunces (whiche is euer one lesse then the nōber of places) multiplie the quantitie of one *excesse*: and put to it the firste terme: and so haue you the laste terme. Then hauyng the firste terme and the laste, with the nomber of *excesses* you knowe how to finde the totalle.

As in this example, the nomber of *excesses* beeyng 179. And the quantitie of one *excesse* beyng $\frac{1}{6}$. their multiplication giueth $\frac{179}{6}$. unto whiche if you adde the firste nomber, that is 1 $\frac{1}{2}$, it will be $\frac{188}{6}$. And that is the laste nomber of that progression. Then to trie the totalle somme of the miles, adde the firste nomber. 1 $\frac{1}{2}$ to the laste, and thei will make $\frac{197}{6}$, that you shall multiplie by halfe the nomber of the places, whiche in our example are. 90 (sith the whole nomber is. 180) and there will amounte. 2955. accordyng as the question saieth.

Scholar. This is sufficient for this question. And at some idle time, I will not sticke to trie it out, by settyng the progression foorthe at large. In the meane tyme I praie you for better exercise, giue me some moare questions.

Master. There is a nomber, whiche I haue forgotten: and it is diuided into. 2. partes, whereof the one I haue forgotten also, but the other was. 4. And yet this I remember, that if the parte, whiche I haue forgotten, be multiplied by it self, and then also with 4. those. 2. sommes will make. 117. Now would I knowe what was the whole nomber, and also what is the parte, whiche I haue forgotten. *An other question.*

Scholar. I suppose the whole nomber to be. 1 𝔷. And bicause. 4. is his one parte, the other parte must neades bee. 1. 𝔷. —— 4. Then doe I accordyng to the question, multiplie. 1. 𝔷. —— 4. firste by it self, and it will make. 1 ȝ —+— 16.ϕ ——— 8.ze. Secondarily, I doe multiplie it (that is. 1.ze. ——— 4) by.4 And it giueth. 4.ze. ——— 16.

Then adde I bothe those nombers together, and it will be. 1 ȝ ——— 4.ze. whiche by the question shall be equalle to. 117.

1.ȝ. —+— .16.ϕ. ——— .8.ze.
4.ze ——— .16.ϕ.

1.ȝ. ——— .4.ze.

But then must I vse the accustomed translation, to bryng the greateste quantitie in denomination, to stande alone. And so will it bee.

1.ȝ. ═══. 4.ze. —+— .117.ϕ.

Where I must serche for the value of a roote. And therfore I multiplie. 2. by it self squarely, and so haue I. 4. vnto whiche I adde. 117. and it maketh. 121. whose roote is. 11. vnto whiche I muste adde. 2. and there commeth. 13. as the value of. 1 ze and the quantitie of the whole nomber.

The proofe. For proofe of this worke, I abate. 4. out of. 13. and there resteth. 9. as the other parte. Then doe I multiplie. 9. by it self, and therof riseth. 81. Also I doe multiplie. 9. by. 4. and it maketh. 36, whiche bothe together, doe make. 117. as the question would.

An other woorke. Master. Set. 1.ze. for the vnknowen parte, and then worke it, to see the diuersitie of the workes.

Scholar. If. 4. bee one parte, and. 1.ze, the other parte, then will the whole nomber be. 1.ze —+— 4ϕ Wherefore firste I multiplie. 1.ze. by it self, and it yeldeth. 1.ȝ. Then dooe I multiplie. 1.ze. by. 4. and it giueth. 4.ze. whiche bothe sommes together, dooe make. 1.ȝ. —+— .4.ze. whiche is equalle to. 117 And by translatiō. 1.ȝ. ═══. 117.ϕ ——— 4.ze.

Wherefore I doe multiplie. 2. squarely, and it giueth

ueth. 4, whiche added to. 117. maketh. 121. and the roote of that is. 11. from whiche I shall abate. 2. and there will reste. 9. as the other parte of the nomber. This is verie plain, & the profe of it as it was before.

Master. Then aunswere to this question.

There are 3 nōbers in proportion *Geometricall*. And one of the extremes is. $20\frac{1}{4}$. the other extreme, with the double of the middell terme, dooeth make 22. Now would I knowe of you, what those. 2. nombers bee? *A question of proporti*

Scholar. For trialle, I name the other extreme, 1. ze. And because it, with the double of the middle terme dooeth make. 22. the middell terme shall bee 11. ——— . $\frac{1}{2}$. ze. for his double is. 22. ——— 1 ze. whiche with. 1. ze. doeth make. 22.

Then to proceade, I knowe the propertie of those nombers in proportion *Geometricall* to bee soche, that *the multiplication of bothe the extremes is equalle to the square of the middell terme*, wherefore I multiplie the. 2. extremes together, and there will rise. $\frac{81}{4}$ ze. Then dooe I multiplie. 11 ——— $\frac{1}{2}$ ze. by it self in Square, and it will bee. 121. φ. —+— . $\frac{1}{4}$ ȝ. ——— . 11 ze, whiche must bee equalle to $\frac{81}{4}$ ze. or. $20\frac{1}{4}$ ze. Then to reduce it, I adde. 11. ze. on bothe sides, and it will be. $31\frac{1}{4}$ ze. ═══ $\frac{1}{4}$ ȝ. —+— . 121 φ. and by translation. $\frac{1}{4}$ ȝ. ═══ $31\frac{1}{4}$ ze. ——— . 121. φ. That is 1. ȝ. ═══ 125. ze. ——— 484. φ.

Now resteth nothyng, but to searche the value of 1. ze. Wherfore I take $\frac{125}{2}$ and multiplie it Square, and so haue I $\frac{15625}{4}$. from whiche I must abate. 484. that is $\frac{1936}{4}$. And there will remain $\frac{13689}{4}$ whose roote is $\frac{117}{2}$, whiche I shall abate from $\frac{125}{2}$, and there will remain $\frac{8}{2}$, that is. 4. for the other extreme.

Then for the middell terme, thus shall I doe. Multiplie. 4. and. $20\frac{1}{4}$ together, and there will rise. 81. whose roote is. 9. and is the middell nomber. That 9 doubled will make. 18. and 4. ioined therto, giueth 22 *The proofe.*

So are those.3.termes in progression *Geometricall*, accordyng to the conditions limited in the question.

Master. Proue the worke now, how it wil frame if.1.ze.be set for the middell nomber. For it wer follie, to trie whether this question, would admitte addition of the.2.laste nombers. Although the rule doe declare that in soche sorte of equations, there is double valuation to eche roote.

Scholar. Yet I beseke you, let me examine it a litle, to see the cause, why I maie not adde them, and so take the roote.

Master. I must bere with you so moche. By addition you see, there will rise $\frac{242}{2}$, that is 121. And then the middell nomber will be.49$\frac{1}{2}$. And so the proportion is $\frac{22}{9}$.that is *Dupla superquadripartiens nonas*. Where as in the other.3.nombers. 4.9.20$\frac{1}{4}$. the proportion is *Dupla sesquiquarta*.

But in the question is one cōdition, that secludeth the roote, that riseth by additiō. For the double of the middell terme, with the other vnknowen extreme, should make.22. As in.4.and.9.it doeth. But in 49$\frac{1}{2}$ and 121.it would be 220.that is 10.tymes so moche.

Scholar. And if you had saied in the question, that the double of the middell nomber, with the other extreme, would haue made.220.then I should haue taken this later roote by additiō, and not the firste roote by subtraction.

And so I perceiue the varietie of conditions in the question dooeth limite, whiche of the.2.rootes I shall of necessitie take, and leaue the other.

An other worke.

But now to varie that worke, I will set.1.ze.for the middell terme. And then the double of it, with the other terme, will make.22. The double of.1.ze. is.2.ze. So must the other terme be 22φ——2.ze.

Then to seke out an equation, I multiplie the.2. extremes together, that is.22.φ——2ze.by 20$\frac{1}{4}$.

And

And there riseth . 445 ½ . ——— 40 ½ . ℞ . And the square of . 1 . ℞ . beyng the middell terme, is sone perceiued to be . 1 . ℨ . And so the firste equation is,

1 ℨ ═══ 445 ½ ϕ . ——— . 40 ½ ℞ .

Wherefore I take halfe . 40 ½ , that is . $\frac{81}{4}$, whose Square is $\frac{6561}{16}$. And unto it I putte . 445 ½ . whereby there commeth $\frac{13689}{16}$. whose roote is $\frac{117}{4}$. from whiche roote I must abate $\frac{81}{4}$, and so remaineth $\frac{36}{4}$. that is . 9 . As the value of . 1 . ℞ . And for the middle nomber.

Then for the proofe: if . 9 . bee the middell nomber, the square of it, whiche is . 81 , shall bee equalle to the multiplications of the extremes. Wherefore if I diuide . 81 . by 20 ¼ , the *quotiente* beyng . 4 . declareth the other extreme. *The proofe.*

Master. You seme experte inough in this forme of worke. Therfore I will procede to other questiōs, that differ somewhat from these.

There are . 2 . menne talkyng together of their monies, and nother of theim willyng to expresse plainly his somme, but in this sorte. The nomber of angelles in my purse, saieth the firste manne, maie bee parted into soche 2 . nombers, whiche beyng multiplied together, will make . 24 . And their *Cubes* beeyng added together, will make . 280 . Then, quod the other man. And the like maie I saie of my money, saue that the *Cubes* of the . 2 . partes, will make . 539 . Now I desire to knowe, what monie eche of them had. *A double question.*

Scholar. The firste mannes sōme, I set to be 1 ℞ whiche I must parte into twoo soche partes, that thei bothe multiplied together, maie make . 24 .

Master. You erre verie moche. For it is not possible, that the partes of any *Cosſike* nomber multiplied together, can make an absolute nomber. Wherefore in soche cases, where you perceiue that there is required, after the firste position, any multiplication to make an absolute nomber, you shall call the firste nō-

 bers

bers, by some other name of pleasure. As here you maie call the firste mannes somme. *A*. And the seconde mannes somme. *B*. and then in their partition, use the name of. 1.ꝛ.

And as thei are twoo questions in one, so shall you make seueralle woorkes for them.

Scholar. Then shal I saie, that the firste mannes somme is. *A*. and it is diuided as he declared. Wherefore for one nomber of that diuision, I set. 1.ꝛ. And then the other shall be $\frac{24ʒ}{1ꝛ}$. for as the one nomber multiplied by the other, doeth make. 24. So. 24.ʒ. diuided by the one of them, must neades bryng forthe the other.

Master. That is well remembred of you. For as 4. and. 5. by multiplication, doe make. 20. So. 20. diuided by. 5. bringeth forthe 4. and diuided by. 4. it yeldeth. 5.

Scholar. So $\frac{20}{5}$ is but. 4. and $\frac{20}{4}$ is. 5.

Master. Go forth then with the rest of the worke.

Scholar. The *Cube* of. 1.ꝛ. is. 1.ȼ. and the *Cube* of $\frac{24ʒ}{1ꝛ}$ is $\frac{13824ʒ}{1ȼ}$ whiche. 2. nombers I maie not adde together, untill I haue reduced theim unto one denomination: whiche thyng I shall doe, by settyng. 1.ȼ. as a fraction thus $\frac{1ȼ}{1ʒ}$. And then woorkyng after the rate of fractiōs, in the firste reduction thei will stande thus. $\frac{1ȝȼ}{1ȼ} + \frac{13824ʒ}{1ȼ}$. And by farther addition thus. $\frac{1ȝȼ + 13824ʒ}{1ȼ}$

And hetherto the woorke of bothe these. 2. mennes sommes, are indifferente and agreynge. So that this one woorke serueth for theim bothe. But now thei will differ. For in the firste mannes woordes, and so in the worke for him $\frac{1ȝȼ + 13824ʒ}{1ȼ}$ is equalle to 280: but in the seconde mannes woorke, it must be accompted equalle to. 539.

But firste to goe forward with the firste man. Seyng $\frac{1ȝȼ + 13824ʒ}{1ȼ}$ is equalle to. 280. Therefore by reduction

reduction to one denomination, (1.ʒ.ce + 13824.φ) / 1.ce is equalle to 280.ce / 1.ce. And remouyng the common denominator, the numerators shal kepe the same proportion: and therfore. 1.ʒ.ce + 13824.φ. shall be equalle to. 280.ce. And by translation, to leaue the greateste denomination alone, 1.ʒ.ce = 280.ce − 13824.φ

Where I shall seke the value of. 1. ze. whiche shall not be here accoumpted the square roote, but the *zenzicubike* roote, or the *Cubike* roote of the square roote, accordyng to the greateste denomination.

Wherfore. 140. in square, maketh 19600. from whiche I must abate 13824. And there doeth remain 5776 whose square roote is. 76. whiche beyng added unto. 140. dooeth giue. 216. and beyng abated from it, it leaueth. 64. of whiche bothe I must extracte the *Cubike* roote, bicause in the equation there are. 2. quātities omitted. So that of. 216. the *Cubike* roote is 6. And of. 64. the *Cubike* roote is. 4. Here I see bothe rootes serue so my purpose, that I shall take thē both.

Master, And good reason. For as in settyng 1. ze for your position, you could not tell whether it were the greater parte, or the lesser, so maie you not now applie it to either of theim bothe, but take bothe rootes for the. 2. partes of your nomber.

Scholar. So doeth the firste mannes nomber appeare to be. 10. seyng the partes bee. 4. and. 6. whiche I maie examine thus. That thei make. 24. by multiplication, it is easily seen. And that their *Cubes* added together, doe make. 280. is sone perceiued: seyng the *Cube* of. 4. is. 64: and the *Cube* of. 6. is. 216. whiche. 2. nombers by addition, doe make. 280. *The proofe.*

Master. Now proue the seconde mannes worke. *The worke of the second parte.*

Scholar. In his woorke (1.ʒ.ce + 13824.φ) / 1.ce is equalle to 539. And by reduction to one denomination, it is equalle to 539.ce / 1.ce. So that. 1.ʒ.ce + 13824.φ. is equalle to. 539.ce. and by translation.

1.ʒ.ce.

1. ƺℭ. ═══ .539. ℭ. —— .13824. φ. whose *zenzicubike* roote I seke, thus: $\frac{539}{2}$ doth make in square $\frac{290521}{4}$, from whiche I must abate $\frac{55296}{4}$, and then remaineth $\frac{235225}{4}$, whose roote is $\frac{485}{2}$ vnto whiche I maie adde $\frac{539}{2}$. and then will it bee $\frac{1024}{2}$, that is. 512. whose *Cubike* roote is. 8. And is one parte of the seconde mannes nomber. And for the other parte, I shall abate $\frac{485}{2}$ out of $\frac{539}{2}$, and there remaineth $\frac{54}{2}$. that is, 27. whose *Cubike* roote is. 3. And is the other parte of the seconde mannes nomber. As it maie sone be tried thus. For. 3. tymes. 8. maketh. 24. and. 27. whiche is the *Cube* to. 3. added with. 512. whiche is the *Cube* to. 8, dooeth make 539, as the question intendeth.

The proofe.

A question of an armie.

Master. One other question I will propounde, of. 2. armies beyng bothe square, and of like nomber. And if you abate. 4. from the one armie, and adde. 10. to the other armie, and then multiplie them bothe together, there will amounte. 9853272. I demaunde of you, what is the fronte of those square battailes.

Scholar. I call the fronte 1 ꝛ. And then must the battaile bee. 1. ƺ. Now abatyng. 4. from the one, it will bee. 1. ƺ. —— .4. φ. Then addyng. 10. to the other, it wil make. 1. ƺ. —|— .10. φ. And if you multiplie those. 2. nombers together, there will amounte by it. 1. ƺƺ —|— 6. ƺ. —— 40. φ. whiche somme must be equalle to. 9853272.

1. ƺ. —|— .10. φ.
1. ƺ. —— .4. φ.

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1. ƺƺ. —|— .10. ƺ. —— .4. ƺ.
—— 4. ƺ. —— .40. φ.

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1. ƺƺ. —|— .6. ƺ. —— .40. φ.

And if you adde. 40. φ. to bothe partes of the equation, it will be. 1 ƺƺ —|— 6 ƺ. equalle to. 9853312

And

And by tranſlation. 1.ʒʒ. ═ 9853312. —— 6.ʒ. out of whiche laſte equation, I ſhall ſearche for the value of. 1 𝔷. by multiplyng firſt. 3. ſquarely, whereof commeth. 9. and then addyng it to. 9853312. And ſo commeth. 9853321. whoſe roote is. 3139. from whiche I muſt abate. 3. And then remaineth. 3136. whiche is the full nomber and Square of the one armie. And hath for his roote. 56.

For as here is one onely quantitie omitted, ſo the firſte nomber, whiche in other queſtiōs of immediate equations, was the verie roote, in theſe interrupte equations is a rooted nomber, and is here a ſquare nōber: whoſe roote therfore, I haue drawen accordyngly. And for triall of this woorke. 56. in ſquare maketh 3136. from whiche if you abate. 4. there will reſte 3132. Again if you adde. 10. there will riſe. 3146. And thoſe. 2. nombers multiplied together, doe make 9853272, as the queſtion intendeth. *The proofe.*

Maſter. This you ſee, what uſe is in theſe equations, yet are there many other equatiōs, whiche here be not ſpoken of: but here after you ſhall haue moare largely declared, if you ſhewe your ſelf diligente in this parte.

And one queſtion I will propounde, & aſſoyle without woorke for breefeneſſe, that you maie ſee there is moare behinde. There is a nomber whoſe Square abated by. 16. and the firſte nomber augemented by 8. and then bothe thei multiplied together, will bryng forthe. 2560. *A queſtion of ſtraunge equation.*

Scholar. I will proue the woorke of it. And therefore ſuppoſe the firſte nomber to be. 1 𝔷. Then is his ſquare 1 ʒ. whiche abated by 16. leueth. 1 ʒ. —— 16 φ. and the nōber augmēted by. 8. yeldeth 1 𝔷 —+— 8 φ. Theſe. 2. nombers multiplied together, will make 1 ℭ. —+— 8. ʒ. —— 16. 𝔷. —— 128. φ. beyng equalle to. 2560.

1. ʒ. ——— .16. ϑ.
1. ȥ. —+— .8. ϑ.

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1. ℭ. ——— .16. ȥ.
8. ʒ. ——— .128. ϑ.

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1. ℭ. —+— .8. ʒ. ——— .16. ȥ. ——— .128. ϑ.

And addyng 128. ϑ. on bothe sides of the equation, it will be. 1 ℭ —+— 8 ʒ ——— 16 ȥ ═══ 2688 ϑ
Againe addynge. 16. ȥ. on bothe sides, it will bee 1. ℭ —+— 8 ʒ ═══ 16 ȥ —+— 2688 ϑ.

Master. Where at staie you now?

Scholar. I see no shifte, but other to leaue it, as it is, 2. nombers equalle to. 2: other els to make. 1. nomber equalle to. 3. And all that is aboue my cunnyng. For hetherto I haue learned noe rule for any of them bothe. So that I can not gesse, what the firste nomber might bee.

Master. The nomber is. 12. And his Square is 144. from whiche if you abate. 16. it will bee. 128. And if you adde. 8. to. 12. it will yelde. 20. Then multiplyng. 128. by. 20. the somme will be. 2560. as the question declared.

Of other equations. But to put you out of doubte, this equation is but a trifle, to other that bee untouched. And yet I will tourne this equation a litle, to giue you some light in it, and other soche. As here.

1. ℭ. ═══ .16. ȥ. —+— .2688. ϑ. ——— 8 ʒ.

where you see. 1. ℭ. equalle to. 3. other nombers. And is it not certaine to you, that this equation is true?

Scholar. Yes, I am adsured thereof.

Master. And yet to auoide doubtfulnes the more trie it by resolution, accounmptyng. 12. for. 1. ȥ.

Scholar. Where. 12. is. 1. ȥ, there. 1. ʒ. is. 144. and. 1. ℭ. is. 1728. whiche. 1728. must bee equalle to

16. ȥ.

16.ℭ (that is.192) and to.2688. saue that you must abate.8.ʒ, that is 1152. Now if I adde 192.to 2688 it will make.2880. out of whiche abatynge.1152. there will remaine.1728. wherby I see the equation is iuste.

Master. Then you see that the equation is true. And can you doubte, that any nomber, whiche is equalle to a *Cubike* nomber, hath in it a *Cubike* roote?

Scholar. It must neades be a *Cubike* nomber, that is equalle to a *Cubike* nomber: and therefore muste neades haue a *Cubike* roote: although I knowe not how to extracte that roote.

Master. Likewaies, when I saie:

8.ʒℭ. ══ .12.ʃʒ. —|— .128.ꝭ. it is certaine, *Here the roote is.2.*
not onely that.12.ʃʒ. —|— .128.ꝭ. containeth in it as moche as.8.ʒℭ. but that the.8. parte of it is a ʒℭ.nomber, and hath a *zenzicubike* roote.

And farther it is manifeste, that as euery.ʒℭ. nomber, dooeth containe in it certaine.ʃʒ. nombers exactly, so if any nomber be annexed with those *Sursolides* (as here in this example are set 128) it is of necessitie, that that.128.must containe in it certaine *Sursolides* exactly.

So if.8.ʒℭ. bee equalle to *The roote is 5*
10.ʃʒ. —|— 20.ʒʒ. —|— 400.ℭ —|— 3125.ꝭ.
it must neades be that the.8. parte of this compounde nomber shall bee a.ʒℭ. nomber. And also that the ʒʒ. with the other nombers folowyng dooeth containe a certain nomber of.ʃʒ.nombers. And the.ℭ. in like sorte includeth a nomber of.ʒʒ.nombers. And laste of all.3125.ꝭ. doeth comprehende certain *Cubike* nombers exactly.

In like sorte, when we saie, that.1.ʃʒ.is equalle to *The roote here is.3.*
6.ℭ. —|— .8.ʒ. —|— .9.ꝭ. All this compounde nomber is a *Sursolide*, and hath a.ʃʒ.roote. And 8.ʒ. —|— .9.ꝭ. includeth certaine *Cubes*. And so

doeth.9.♁.containe exactly. 1.ʒ.or moare.

But of these and many other verie excellente and wonderfulle woorkes of equation, at an other tyme I will instructe you farther, if I see your diligence applied well in this, that I haue taughte you.

And therefore here will I make an eande of *Coßike* nombers, for this tyme.

Of Surde nombers, in diuerse sortes

And firste of Surde nombers vncompounde.

Now that you haue somewhat learned the arte of *Cossike* nombers, with the rule of equation, it semeth good tyme and apte place, to teache you the arte of *Surde* nōbers, whiche are diuerse in name, accordyng as there are diuerse natures of rootes, whiche maie giue theim name.

For generally, a *Surde* nomber is nothyng els, but soche a nomber set for a roote, as can not be expressed by any other nomber absolute. *A Surde nomber.*

As the *Square roote* of.10, or of.8. or of any nomber, that is not square. Likewaies the *Cubike roote* of.4, or of.5. or of any nomber that is not *Cubike*. So the *zenzizenzike roote* of.8. 12. or.20, or of any nomber that is no *zenzizenzike*, is called a *Surde* nomber. And in like maner, any other roote of any nomber, that hath noe soche roote, doeth cause that nomber to be a *Surde* nomber.

For if you see those signes annexed with nombers, that hath soche rootes, those nombers are not *Surde* nombers properly, but sette like *Surdes*. As the *Square roote* of.4. or of.9. or.25.&c. The *Cubike roote* of.8.27. or.125.&c. whiche sometymes is vsed for apte worke, as you shall see here after.

Of Numeration.

The numeration of thē doeth consiste, in knowlege of their figures, whiche partly be declared before. But their common and peculiare signes are these. √. ww√. w√. Although there maie be moare varieties: Yet these for this tyme maie suffice.

The firste, that is. √. is customably set, to signifie a *Square roote*. As this. √.5. betokeneth the *Square roote* of.5. And. √.12. is the *Square roote* of. 12. Howbeit many tymes it hath with it, for the moare certeintie the *Coſsike* signe. ʒ. And is written thus. √ʒ.20. the *Square roote* of.20. And. √ʒ.56. the *Square roote* of.56.

The seconde signe is annexed with *Surde Cubes*, to expresse their rootes. As this. ∛.16. whiche signifieth the *Cubike roote* of.16. And. ∛.20. betokeneth the *Cubike roote* of.20. And so forthe. But many tymes it hath the *Coſsike* signe with it also: as ∛.ȼ 25 the *Cubike roote* of.25. And. ∛.ȼ.32. the *Cubike roote* of.32.

The thirde figure doeth represente a *zenzizenzike roote*. As. ∜.12. is the *zenzizenzike roote* of.12. And ∜.35. is the *zenzizenzike roote* of.35. And likewaies if it haue with it the *Coſsike* signe. ʒʒ. As ∜ʒʒ24 the *zenzizenzike roote* of.24. and so of other.

Scholar. It were againste reason, to seke reason for those signes, whiche be set voluntarily to signifie any thyng: although some tymes there bee a certaine apte conformitie in soche thynges. And in these figures, the nomber of their minomes, seameth disagreable to their order.

Master. In that there is some reason to bee shewed: for as. √. declareth the multiplication of a nomber, ones by it self: So. ∛. representeth that multiplication *Cubike*, in whiche the roote is represented thrise. And. ∜. standeth for. √.√. that is.2. figures of *Square* multiplication: and is not expressed with. 4. minomes. For so should it seme to expresse moare then.2. *Square* multiplications. But of voluntarie signes, it is inoughe to knowe that this thei doe signifie. And if any manne can deuise other, moare easie or apter in use, thei maie well be receiued.

But concerning the numeration of *Surde* nombers this shal you marke: that when any compounde signe is putte before a nomber, whiche hath any roote, that maie bee expressed by parte of that signe, that nomber is not absolutely so to bee expressed, onlesse it bee for ease or aptnesse in worke. As. √ ʒ. ʒ. 36. whiche betokeneth the *zenzizenzike roote* of. 36. Seyng it is well knowen, that. 36. hath. 6. for his *Square roote*, it were moare apte expressynge that nomber thus. √ ʒ. 6. that is the square roote of. 6.

Otherwaies, if the nōber that foloweth the signe, haue a roote agreable to that signe: it is not *Surde* nomber. As. √ ʒ. 16. is. 4. and is not *Surde nomber*. So. √ c. 27. is. 3. and neadeth not to bee written in *Surde* forme, excepte it bee for aptnesse of woorke. And by this maie you iudge of all other, as thei come in use.

Scholar. If this bee all that is requisite to numeration, I praie you proceed. to addition. For that is nexte in order.

Master. That is the common order. Howbeit in vulgare fractions, you remember that multiplication and diuision, are set before addition and subtraction: bicause of the easier formes of woorke, in multiplication and diuisiō. And so in these *Surde nombers*, bicause the woorkes of multiplication, and of diuision, be not onely moare easie, then the woorkes of addition, and of subtraction, but also be requisite to them, therefore will I begin with them, and so come to the other.

Of Multiplication.

Multiplicatiō in *Surde nombers* uncōpounde hath noe difficultie, if thei be of one denomination: els must thei be reduced to one denomination: and that by multiplication, accordyng to their signes.

But where noe reduction neadeth, you shall multiplie

tiplie the nombers together, and sette their common signe before the nomber, that resulteth of that multiplication.

Examples of square Surdes.

If you will multiplie. √. ʒ. 15. by. √. ʒ. 26. it will make. √. ʒ. 390.

So. √. ʒ. 32. multiplied by. √. ʒ. 48. dooeth make. √. ʒ. 1536.

And. √. ʒ. 56. multiplied by. √. ʒ. 21. dooeth yelde. √. ʒ. 1176.

Howbeit some tymes it happeneth, that the nomber, whiche is made by that multiplication, is a nomber absolute, and not a *Surde nomber.*

Examples of soche as make nombers Absolute.

√. 12. √. 3. √. 36. that is. 6.	√. 48. √. 3. √. 144. that is. 12.
√. 12. $\frac{1}{2}$. √. 4. $\frac{1}{2}$. √. 56. $\frac{1}{4}$ that is. 7 $\frac{1}{2}$.	√. 28 $\frac{4}{5}$. √. 7. $\frac{1}{5}$. √. 207. $\frac{9}{25}$ that is 14 $\frac{2}{5}$.
√. 240. √. 15. √. 3600. that is 60.	√. 325. √. 13. √. 4225. that is. 65.

And generally when any nomber is multiplied by an other, if the proportion betwene those. 2. nombers bee represented by a Square nomber, as by. 4. 9. 16. 25. &c. then dooe thei make a square nomber by their multiplication.

Examples of Cubike rootes.

∛.ℭ. 91.	∛.$7\frac{1}{3}$.	∛. 256.
∛.ℭ. 12.	∛. $\frac{3}{4}$.	∛. [illegible]
∛.ℭ.1092.	∛.5.[illegible]	∛.190[illegible]

Examples of ſoche as make Abſolute nombers.

∛. 54.	∛. 686.
∛. 32.	∛. 4.
∛.1728.that is.12.	∛.2744.that is.14

∛. 486.
∛. 96.
∛. 46656. that is. 36.

Examples of zenzizenzike rootes.

∜. 15.	∜. 204.	∜. 162.
∜. 7.	∜. 26.	∜. 32.
∜.105.	∜.5304.	∜.5184.that is.√.72.

∜. $7\frac{[illegible]}{[illegible]}$.	∜. 27.
∜. [illegible].	∜. 12.
∜. 5[illegible].that is.√.2[illegible].	∜.324.that is.√.18.

Examples of zenzizenzike rootes that make abſolute nombers.

∜. 32.	∜. 128.
∜. 8.	∜. 32.
∜.256.that is.16.	∜.4096.that is.64.

√√. 288.
√√. 72.
———————
√√. 20736. that is. 12.

But here is to bee noted, that if you would multiplie any *Surde* nomber, by an absolute nomber, or any *Surde* nomber of one denominatiō, by a *Surde* nomber of an other denomination: you must firste reduce that Absolute nomber to the like denomination. And so must you reduce the. 2. *Surde* nombers to one denomination.

And bicause that this worke doeth serue often in this arte, and that in diuerse workes, I will set here the arte of reduction.

Of reduction in Surdes.

Reduction in *Surdes*, is the bringyng of sundrie denominatiōs vnto one. Whiche in absolute nōbers is this doen. You shall multiplie the absolute nomber, accordyng to the signe of the *Surde*, and then sette before it the like signe. So that if you would double. √.ʒ.8. that is to saie, if you would multiplie it by. 2. you must firste multiplie that. 2. squarely, and then multiplie those nombers together. That is to saie, you shall multiplie. √.ʒ.8. by. √.ʒ.4. and so is it doubled.

Likewaies, to *triple* any Square *Surde*, is to multiplie it by. 9. And so to *quadriple* any square *Surde*, is to multiplie it by. 16. And so forthe.

But if you double any *Cubike* nomber, you shall multiplie it by. 8. that is the *Cube* of. 2. And so if you would *triple* a *Cubike* roote, you muste multiplie it by 27. And if you would *quadriple* it, you shall multiplie it by.6 4. And so of other like woorkes.

Again, if you will double any *zenzizenzike* roote, you must multiplie it by.16. And if you will *triple* it, you shall multiplie it by.81. And so if you will *quadriple* it, you must multiplie it by 256. And in like maner euer moare, for the nomber absolute, you shall set his *zenzizenzike* nomber. Like as in Squares, for any nomber absolute, you shall set his square. And in *Cubes* you shall take his *Cube*.

Scholar. This is plaine inoughe: yet I praie you put an example or twoo, of eche kinde.

Master. Take these examples for square rootes.

√. 38.
2.
√. 152.

√.ȝ. 128.
6.
√.ȝ.4608.

√. 3264.
12.
√.469976.

Examples in Cubike rootes.

ww√. 52.
2.
ww√.416.

ww√. 163.
5.
ww√. 20375.

ww√. 4806.
8.
ww√. 2460672.

Examples in zenzizenzike nombers.

w√. 69.
2.
w√. 1104.

w√. 251.
4.
w√.64256.

w√. 1385.
5.
w√. 2250625.

Scholar. This I perceiue well. But now in *Surde* nombers of diuerse denominations, what the order of reductiõ is, I praie you to set forth with some exãples

Master. These examples with their declaration, maie sufficiently serue for a shewe, if I would multiplie.ww√.12.by. √.5. I must firste multiplie the nomber of one signe, accordynge to the signe of the other nomber, and so alter them bothe. Whiche woorke is like the reduction of fractions, to one common denomination. As here I muste multiplie. 5. *Cubikely*, and 12. must be multiplied squarely, and then shall I adde bothe signes in one, for their common signe. So shall I haue for them the. ʒ.ȼ. roote of. 144. to be multiplied by the *zenzicubike* roote of 125. And so will there come of that multiplication, the *zenzicubike* roote of. 18000. As here by example doeth appeare.

√.ʒ.ȼ.	144.
√.ʒ.ȼ.	125.
√.ʒ.ȼ.	18000.

Likewaies if I would multiplie. √√.ʒ.ʒ.250. by √√.34. I shall firste multiplie. 250. *Cubikely*, and it will bee. 15625000. And 34. must I multiplie *zenzizenzikely*, and it will yelde. 1336336. Wherefore multipliyng theim together, and addyng thereto the common denomination, it will bee the. ʒ.ʒ.ȼ. roote of. 20880250000000.

This woorke is aptly represented in figure, after this sorte. And then shall you multiplie crosse waies the nomber of the one, by the signe of the other. And so maie you dooe in all other like nombers, of diuerse denominations.

√√.ʒ.ʒ.	╳	√√.ȼ.
250.		34.

This reduction doeth serue for any other woorke, as well as for multiplication.

Of Diuision.

Diuision is as easie as multiplication. For in it there is noe regard had to the signes. But the one nomber diuided by the other as if thei were nōbers absolute. And then the firste signe added to the *quotiente*. For the more lighte and certaintie, I haue set here, examples of eche sorte.

And first examples of square rootes.

√. 72.
√. 8. (√.9. that is.3.

√. 128.
√. 4. (√.32.

√. 457 1/3.
√. 21. (√.21 7/9.

Examples of Cubike rootes.

ww√. 96.
ww√. 4. (ww√.24.

ww√. 1664.
ww√. 32. (ww√.52.

ww√. 5624.
ww√. 76. (ww√. 74 5/[illegible].

Examples of zenzizenzike rootes.

w√. 54.
w√. 6. (w√.9. that is. √.3.

w√. 286.
w√. 42. (w√.6 17/21.

w√. 5892.
w√. 54. (w√.109 1/9.

And this maie suffice for Diuision. The profe of it is by the contrary kinde. For Multiplication prouveth Diuision: and Diuision trieth Multiplication.

Scholar. All this is easie inoughe to remember.

Of Addition.

Master.

Addition is not so easie, but hath diuerse varieties of worke, as anon shall appere. Whereof the firste is as easie as can bee. For it requireth onely the signe of addition. —+—. As if I would adde. √. 12. to

The firste forme of Addition.

√.26. I shall set it thus. √.26. —+—. √.12. And so √.20. put unto. √.54. maketh. √.54. —+—. √.20. This forme serueth chiefly for rootes of diuerse names. As. √√.ℨℨ.20. —+—. √√√.ℭ.30. where √.ℭ.30. is added to √√.ℨℨ.20. And so of al other.

The seconde forme.

The seconde forme is not so easie: and yet many times it is moare certaine. And this is the order of it.

You shall sette doune your .2. nombers, that you would adde together, forseyng that thei be of one denomination. Then shall you adde in plaine forme, their nombers together, puttyng thereto the signe of the roote. And kepe that as a parte of the addition. Again you shall multiplie the .2. firste nombers together. And their totalle you shall multiplie by .4. And before that shall you sette the signe of the roote. And it shall stande as the seconde parte of that addition. So that those .2. partes, shall be added with the signe —+—. And then is the woorke ended. Example hereof. I would adde the .2. firste sommes, that is, √.12. to. √.26. wherfore I set them thus. And then doe I adde thē bothe plainly together, and thei make. √.38 whiche I set by, as one part of the addition. Then doe I multiplie √.26. by. √.12. and there riseth. √.312. whiche I must double, or multiplie by .2. And therfore seyng the woorke is in square rootes, I set the square of 2. with the signe of. √. for .2. and then multipliyng theim together, I haue. √.1248. whiche is the seconde parte of the roote. Wherfore addyng those .2. partes together, with the signe. —+—. there commeth. √.38. —+—. √.1248. as the totalle of that addition.

√.26 —+— √.12

√.38. | √.26. / √.12.

√. 312.
√. 4.

√.1248.

√.38. —+— √.1248.

Scholar. As me thinketh, the firste forme of addition,

tion serueth better for these nombers, then this seconde forme. For it is moare easie to vse, in any kinde of woorke, and moare spedily doen: and it semeth that this laste nomber, is moare obscure then the firste.

Maister. Yet is this woorke good, and very necessarie. For in these nombers, and soche other like, it serueth onely (as appereth) to alter the state of the nōbers, whereby thei maie bee commensurable, with other, then thei were before that alteration. But in some nombers, and that very many, it reduceth them to one simple forme of roote. As by the examples folowyng you shall perceiue.

A thirde forme of addition.

An example.

√.28. —+— √.7.

√. 28.
√. 7.

√. 196.
√. 4.

√. 784.

28.
7.
35.

√.35. —+— √.784.
Or. √.35. —+— 28.
That is. √. 63.

The same example other waies wroughte.

√.28. —+— √.7.

√. 28.
√. 7.

28.
7.
35.

√.196. whose roote is 14

14.
2.

√.35. —+— .28.

Where firste I haue set forthe. 2. examples of one addition, that you maie see the agremente of thē both.

And firste I would adde. √. 28. with. √. 7. wherefore I dooe ioyne. 28. and. 7. in one somme, whiche I set a parte, as the firste portion of the addition. Then I doe multiplie. 28. by. 7. And thereof commeth. 196. whiche is a square nōber, and hath. 14. for his roote. So that I maie vse now. 2. workes. For other I maie continue my woorke, as I haue doen (agreable to the firste example) in multiplying that. √. 196. by. √. 4. (whiche is but doublyng) and so there cōmeth. √. 784

whiche

whiche is a nomber absolute: bicause it hath a roote, accordyng to his signe, whiche roote is.28. and maie be set for.√.784.

Now in the seconde woorke, bicause the firste multiplication of.28. by.7. doeth make a square nomber, I doe take the roote of that nomber for it: seyng it is all one thyng to saie.√.196, and.14. for.14. is the roote of.196. And then hauyng the roote, I muste double it, accordyng to the rule, or multiplie it by.2. and there of commeth.28. whiche I shall adde with 35. And so haue I.63. whose roote containeth the addition of.√.28.and.√.7.

Scholar. This woorke semeth straunge: and farthest from common reason, of all other woorkes in this arte.

Master. I mighte easily by demonstration make you, to perceiue as moche reason in this worke, as cā be in any: for it dependeth of the.38. Theoreme of the path the waie. But haste of other businesse, maketh me to omit the demonstration at this tyme, whiche shortly you shall haue, for all the equations, and other woorkes likewaies.

But for this presente tyme, it shall be sufficiente to worke an example in *rationall* nombers, as if thei wer *Surde* nombers: that therby you maie perceiue the order, and the truthe of the woorke.

Wherfore I take these twoo nombers.√.36. and √.49. to bee added together. Where I doe firste adde the twoo nombers plainely together: And thei make 85. for the firste parte of the addition. Then doe I multiplie.49 by.36. and there riseth.1764. whiche is a square nomber. And therefore maie I vse.2. workes, as you see. In the firste I multiplie that Square nomber by.2. or by.√.4. whiche is all one: and there doeth amounte.7056. a Square nomber also, whose roote is.84.

The firste forme.		The seconde forme.
√.36.—+—.√.49.		√.36—+—√.49
√. 49	49.	√. 49.
√. 36	36.	√. 36.
294	85.	294.
147		147.
√. 1764		√. 1764.
√. 4		That is. 42.
√. 7056		2.
√.85.—+—.√.7056.		84.
Or. √.85.—+—.84		√.85—+—84.

That is. √. 169.
Or. 13.

In the seconde woorke I take the roote of. 1764. whiche is 42 and doublyng it, I haue 84. agreable to the other woorke. Then doe I sette those. 2. nombers doune with —+—, and putte to them the signe. √. in token that I muste take the roote of that compounde nomber: and not of any one parte of it.

Scholar. That haue I marked well: For 85. hath no roote, nother 84. hath any roote. But 85—+—84 that is. 169. hath. 13. for his roote.

And so I see, that the roote of. 36. whiche is. 6. And the roote of. 47. that is. 7, beeyng bothe added together will make. 13. that is the roote of. 169.

Master. Yet one other forme of easie woorke, I will shewe you, whiche is bothe pleasaunte and profitable: But is not generalle, for it serueth onely for *nombers commensurable*, I meane soche nombers, as by one common diuisor, maie bee brought into Square nombers. With whiche nombers, you shall woorke thus.

Of nombers commensurable, a fourthe forme.

Firste diuide theim by the common diuisor: and set for them their rootes. Then adde those.2. rootes together, and multiplie it squarely. And that square beyng multiplied by the common diuisor, will bryng forthe the Square of bothe the rootes. As here followeth in example.

```
      √.384 ─┼─ √.150.
────────────────────────
6.)    64        25.
        8         5.
          13.
          13.
        ───────
          169.
            6.
        ───────
          1014.
```

Where I would adde √.384 unto √.150 which nombers I doe examin, til I maie finde their commō, and leaste diuisor, whiche here is.6. Then diuidyng them by that.6. I haue for 384. a square nomber.64. And for.150. I haue an other square, that is.25. Of whiche bothe squares I set doune the rootes: and the common diuisor also. Then doe I adde bothe rootes together, and thereof commeth.13. whose Square is 169. that I doe multiplie by.6. whiche is the commō diuisor, and it will bee.1014. whose roote doeth contain bothe the rootes before named. As you shall see it proued anon by Subtraction.

Scholar. In the meane season I consider, that one of these formes, maie confirme the other. And therefore if I worke this laste example, by one of the other formes, and finde thesame totall, it must neades be that the worke is good. Whiche I proue thus.

Firste settyng doune the nombers, in forme of the easieste Addition. And then addyng theim together, I finde.534. whiche I sette a side, as one parte of the nomber, that I doe seke for.

Then dooe I multiplie the.2. nombers together, and thei make.57600. whiche I dooe multiplie again by 4. And there riseth.230400. being a square nomber, and hath.480. for his roote. Wherefore I set

$\sqrt{}.384 -\!\!+\!\!- \sqrt{}.150.$

384	384
150	150
19200	534
384	
57600	
4	
230400	

$\sqrt{}.534 -\!\!+\!\!- \sqrt{}.230400.$

Or. $\sqrt{}.534 -\!\!+\!\!- .480.$

That is. $\sqrt{}.1014.$

set.534. and.480.together. with the signe of Addition, thus. 534 —+— 480. And the roote of that nomber, is equalle to bothe the firste rootes. But considering that bothe those nombers, which bee ioyned laste of all with —+—, are nombers rationall and absolute, I maie adde thē in one, & so thei make 1014. agreably to the other woorke. Wherefore I iudge them bothe to be good

Master. You might haue wrought this woorke otherwaies, bicause the firste nomber, that riseth of the multiplication is a square nomber.

Scholar. Then I perceiue, I mighte haue taken the roote of it, whiche is. 240. and doublyng it, I should haue. 480. As I had in the other worke. And so all doe agree in one.

But my chief doubte now is, how to knowe those nombers that bee *commensurable*: For if I shall stande long in searchyng for that, I might soner woorke the other forme of worke, then to make that trialle of *commensurablenesse*.

The findyng of nombers commensurable.

Master. The easieste waie is, to diuide the greater nomber, by the lesser, as if thei were bothe nombers absolute: & the *quotiente* will declare their *Squares*. As if you doubte, whether. 384. and. 150. bee *commensurable*, diuide. 384. by. 150. and the *quotiente* will be $2\frac{14}{25}$, that is $\frac{64}{25}$. Then diuide whiche of the. 2. firste nombers you liste, by his like nomber in the *quotiente*: And the common diuisor will amounte. So if you diuide the greater nomber.384.by the greater nomber in the *quotiente*, whiche is.64,you shall finde the new *quotiente* 6. whiche. 6. is the common nomber. Or if you diuide.150.by 25.the common nomber.6.will be the *quotiente*.

But and if the *quotiente* be a whole nomber,and no fraction,and be a Square nomber,then is it the lesser square. Wherefore if you diuide the lesser nomber of the.2. by the *quotiente*, the common nomber will appeare in the seconde *quotiente*. And then if you diuide the greater of the.2.nombers, by that common nomber,his *quotiente* will shewe you the other *Square*.

And if so happen,that the *quotiente* of the firste diuision be not a square nomber, then are those nombers *incommensurable*.

So.√.32. and.√.128. bee *commensurable*: and the *quotiente* of their diuision is. 4. whiche is the lesser square. And.8. appeareth to be the common nomber. And the greater square is.16.

Howbeit by this nomber it maie easily bee espied, that some nombers maie be resolued,into more squares then one. As these.2.nombers, beyng diuided by.2 dooe giue.16.and.64. And beeyng diuided by.8,thei bryng forthe.4.and.16.

But for their addition, what Squares so euer you take,that redounde by one common diuisor,the triall will be like,and the roote one.

Scholar. I praie you let me proue that varietie.

Master. Then proue it in soche nombers, where you maie finde moare varietie. As these bee.√.288. and.√.1152.

Scholar. If I diuide.1152. by.288. the *quotiente* will bee.4. whiche I must take for the leaste Square. Then by it I diuide.288.and the *quotiente* will be.72. as the common diuisor. By whiche if I diuide.1152. there will rise.16.as the seconde square. Then set I the

the nõbers in order thus. And vnder. 1152. I set the one Square. 16. And vnder. 288. I putte the other square. 4. And vnder eche of theim his roote. Then adde I the Rootes together, whiche maketh. 6. whose square is. 36. And that beyng multiplied by 72. the common nomber, dooeth yelde. 2592. whose roote dooeth containe bothe the other. 2. rootes by addition.

	√.1152	—+—	√.288.
	16		4.
72)	4		2.
		6.	
		6.	
		36.	
		72.	
		72.	
		252.	
	√. 2592.		

But now how I shall finde any other Squares in those nõbers, to make any farther trial, I knowe not.

Maister. Diuide alwaies one of the nombers, by some square nõber, that will parte it exactly, without any remainer. And marke the *quotiente*. For by it shal you diuide the other nõber, and if the *quotiente* in that last diuision, be a square nomber, then haue you your purpose. Els muste you proue with an other Square nomber.

Scholar. I vnderstande you. And therfore in these nombers, I will make trialle with. 9. by whiche I diuide. 288. And finde the *quotient*. 32. Then by the same 32. I diuide 1152. and the *quotiente* is. 36. So haue I 9 and. 36. for the. 2. squares, and. 32. for the cõmon diuisor. Wherfore I set the nõbers in order as thei ought. And vnder them I place the. 2. square nombers with their rootes. Then addynge the rootes together, I finde. 9. whiche I multiplie square, and it yeldeth. 81. that. 81. I doe multiplie by the common nomber. 32. and there amounteth. 2592. As it did before in the other worke. Whereby I perceiue that these workes doe confirme one an other.

√.1152 —+— √.288.
36 9
32) 6 3
9
9
81
32
162
243
√ 2592

And therefore I will proue, how many varieties of this worke, I may finde in these nombers. And for my purpose, I will diuide the lesser of the .2. nombers, by as many Squares as I can, for that seameth to be the readieste waie. And firste I proue with.16. And so the *quotient* is.18. by whiche.18.I diuide.1152. and the *quotiente* is.64. whiche is a square nōber. So that I haue that varietie more.

Then again I proue with.25. But I see, that will not frame. Wherfore I assaie with.36. And finde the *quotiente* 8. by whiche I diuide the greater square, and the *quotiente* is.144. a square nomber also. And therfore I note that for an other varietie.

Thirdly, I proue with.49. but that wil not agree. Then attempte I with.64. And that serueth as euil. Nexte that I assaie.81. 100. and. 121. but none of them will diuide.288. wherefore I passe vnto. 144. whiche is twise contained in 288. by that.2. I diuide 1152. and finde the *quotiente*.576. whiche is a Square nomber also. And so haue I.3. other varieties beside the.2. former woorkes: whiche.3. varieties, for my remembraunce I set doune, thus.

√.1152.

√.1152 —|— √.288
64 16
18) 8 4
12
12
144
18
1152
144
√. 2592

√.1152 —|— √.288
144 36
8) 12 6
18
18
324
8
√. 2592

√.1152 —|— √.288
576 144
2) 24 12
36
36
1296
2
√. 2592

Maſter. Then for to gratifie you, I will ſette doune .2. other nombers with 6 varieties. Whiche maie ſeame to ſuffice for this worke, without more exāples. And bicauſe you know the order to trie thē I will ſette theim doune without any explication, other declaration. As here you ſee.

√.28800 —|— √.7200
14400 3600
2) 120 60
180
180
32400
2
√. 64800

√.28800 —|— √.7200
3600 900
8) 60 30
90
90
8100
8
√. 64800

√.28800 + √.7200

	1600		400
18)	40		20
		60	
		60	
		3600	
		18	
		28800	
		36	
√.		64800	

√.28800 + √.7200

	900		225
32)	30		15
		45	
		45	
		2025	
		32	
		4050	
		6075	
√.		64800.	

√.28800 + √.7200

	576		144
50)	24		12
		36	
		36	
		1296	
		50	
√.		64800	

√.28800 + √.7200

	400		100
72)	20		10
		30	
		30	
		900	
		72	
√.		64800	

Scholar. This varietie is pleasaunte.

Master. I will satisfie your delite better at more leisare. But yet one thyng moare will I saie, before we cease this sorte of Additiō: that if you would adde any roote to it self. As. √.6. to.√.6. or.√.10. to. √.10.&c. you shall onely *quadriple* the nomber: and so haue you doen.

Scholar. I see good reason in that: For addition of any nomber to it self, is but doublyng that nomber or multiplication by.2. And that must be doen by that *quadriplation*, as you taught before.

Addition of cubike rootes

Master. Now will I set forthe some examples of addition in *Cubike* rootes. For the worke is like vnto this laste forme in Square rootes, saue that the multiplications,

tiplications, whiche were Square in that woorke, muste be *Cubike* in this worke. And that onely in nombers *commensurable*. For nombers *incommensurable* bee added with the signe. —+—. without moare worke.

I call soche *Cubike* rootes *commensurable*, whiche beyng diuided by any common nomber, will make *Cubike* nōbers in their *quotiente*. As. w√.24. and. w√.81 whiche diuided by.3. doe make.8.and.27. bothe beyng *Cubike* nombers. So. w√.320.and. w√.135. beyng diuided by.5. doe make.27. and. 64. bothe *Cubike* nombers. Likewaies. w√.2744.and. w√.1000. be *commensurable*, bicause thei make.343.and.125. bothe *Cubike* nombers: If thei be diuided by.8. *Cubike rootes commensurable.*

Scho. I praie you make your exāples with these.

Master. There nedeth noe wordes in this worke it is so like the Addition of square rootes. And therfore marke these examples well.

```
w√. 81. —+— w√.24.
      27          8
3.)    3          2
             5
             5
         -------
           125
             3
     -----------
w√.    375

w√.320. —+— w√.135
      64          27
5.)    4           3
             7
             7
         -------
           343
             5
     -----------
w√.   1715

w√.2744 —+— w√.1000
      343        125.
8.)    7           5.
            12
            12
         -------
          1728
             8
     -----------
w√.    13824
```

Scholar. Here is noe diuersitie, from the former workes, but in settyng the *Cubike* roote, for the square roote. And in multipliyng the addition of the. 2. rootes *Cubikely*.

Master. That is all. And therefore will I stande noe longer aboute it: But will proceade to an other forme of addition, whiche serueth also for *Cubike rootes commensurable*. The rule is this. Set doune the *Cubike* rootes, with their common diuisor, and the *Cubes* that rise therby, and their rootes also. All this you did in this former worke. But now peculiarly in this rule, you shall set doune. 3. other nombers orderly, vnder those. 3. former nombers. The firste is the square of that laste *Cubike* roote: the secōde is the *triple* of that Square: and the thirde is a nomber resultyng of the multiplication of that triple by the other roote.

An other forme of addition.

Then take the. 4 extreme nombers, that is those 2 laste nombers, and the. 2. *Cubes*, and adde theim together into one somme. And that somme beyng multiplied by the common diuisor, will make a *Cubike* nomber, whose *Cubike* roote shall containe bothe the firste rootes, whiche you intended to adde. Now marke these examples: and cōferre theim well with the wordes of the rule.

ww√.384 —+— ww√.48

64   8

6)   4   2

16   4

48   12

48   96

216

6

ww√. 1296

ww√.15972 —+— ww√.2592.

1331   216

12)   11   6

121   36

363   108

1188   2178

4913

12

9826

4913

ww√. 58956

ww√.52488.

ww.52488.—+—.ww.24696.

	5832.	2744.
9)	18	14.
	324	196.
	972	588.
	10584	13608

32768.
9.

ww. 294912.

Scholar. In these examples I see, the woordes of your rule obserued. For vnder eche *Surde Cubike* roote, there is set a true *Cubike* nomber, whiche is founde by the common diuisor: then foloweth the roote of that true *Cube*: and beside it standeth the common diuisor. Then in the fourthe roome is the Square of the true *Cubike* roote. And vnder it his nomber tripled (as.48 vnder.16, and.12. vnder.4) whiche triple beyng multiplied by the roote of the other side, dooeth make the loweste nomber in that rowe. So.48. multiplied by.2. maketh.96. whiche is set vnder the roote.2. And.12. multiplied by.4. yeldeth.48. whiche is placed vnder that.4.

Then those.4. extreme nombers.64. and.48,8. & 96. doe make by addition 216. whiche somme is multipled by.6, that is the common diuisor, and so riseth 1296. whose *Cubike* roote comprehendeth bothe the firste rootes.

Master. The like maie you iudge of the other.2. examples. But bicause you maie vnderstande the certaintie of this woorke the better, I haue here sette forthe.2. examples of true *Cubike* rootes, formed like *Surde* nombers.

 ww.4096

w/. 4096. —+—. w/. 1728.

	512.	216.
8)	8.	6.
	64.	36.
	192.	108.
	864.	1152.

2744
8
w/. 21952

w/. 19683. —+—. w/. 3375.

	729	125
27)	9	5
	81	25
	243	75
	675	1215

2744
27
19208
5488
w/. 74088

Scholar. I perceiue by examination of worke in my Tables here, that 4096. is a *Cubike* nomber, and hath 16 for his roote. So 1728 is a *Cubike* nō=ber also, & his roote is. 12. those bothe rootes added together, doe make. 28. And that. 28. is the *Cubike* roote to. 21952. as the firste example would.

And for the seconde example, I see likewaies that 19683. hath. 27. for his *Cubike* roote. And. 3375. hath 15. for his roote. And thei bothe make. 41, whiche is the *Cubike* roote to. 74088. accordyng to the worke of the seconde example.

Addition of zenzizenzike rootes.

Master. Seyng you are conueniently instructed, in these nombers, wee will goe in hande with *zenzizenzike* rootes, and their additiō: wherein is no differēce of worke, but onely for the multiplicatiō, whiche must be agreable to the nature of the nombers, *zenzizenzikely*. And the reduction by the common diui=

sor,

for, in like forme, into *zenzizenzike* nombers, whē the firste nombers bee *commensurable*. But if thei be *incommensurable*, then must the addition be wroughte by the signe. —+—, without any other businesse.

Examples of *zenzizenzikes* beeyng *commensurable*.

```
√.648 —+— √.5000     | √.1280 —+— √.6480
     81         625  |      256        1296
8)    3           5  | 5)     4           6
           8         |            10
           8         |            10
        4096         |         10000
           8         |            5
√.     32768         | √.      50000
```

```
√.38416 —+— √.65536
     2401        4096
16)     7           8
           15
           15
         50625
            16
        303750
         50625
√.      810000
```

In the firste and seconde examples the nōbers are *Surdes*, but in the thirde example thei are rationall nombers, framed like unto *Surdes* to the intente that you mighte the better perceiue the forme of the worke. For 38416. is a *zenzizēzike* nomber, & hath. 14. for his roote So. 65536. is a *zenzizenzike* nomber, and hath. 16. for his roote. And these. 2. rootes do make. 30. whiche is the *zenzizenzike* roote unto. 810000. And therefore maie it bee truely saied, that. √. 810000 dooeth containe the twoo firste rootes.

Scholar. I praie you procede to Subtraction. For all this I doe well perceiue.

Of Subtraction.

Master.

Subtraction doeth differ from addition, in litle moare then the signe ———. whiche signe serueth generally, for all nombers *incommensurable.* And consideryng there is litle difficultie in Subtraction: If you remember well the arte of Addition, I will lightly passe it ouer in the same examples, that I haue wrought in Addition, bicause it maie bee a proofe of that woorke: and that woorke also a confirmation of this.

Onely this shall you obserue in this rule peculiarly: that as in the seconde forme of Addition, you must adde the rootes together, before you multiplie theim. So here you shall Subtracte the lesser roote, from the greater, before you doe multiplie theim.

Example of Subtraction, with ———.

√.12. abated out of √.26. maketh. √.26 ——— √.12. and so of other.

Examples of the seconde forme of Subtraction

√.63. ——— .√.28.		The seconde forme of that woorke.
63	63	√.63. ——— .√.28.
28	28	63
504	91	28
126		1764
√.1764		whose roote is. 42.
√. 4		42
√.7056		2
√91 ——— √.7056		84
Or. √.91 ——— 84.		√.91 ——— 84.

That is. √.7.

√.169——√.36		An other forme of that worke.
169	169	√.169.——.√.36.
36	36	169
1014	205	36
507		√. 6084
√. 6084		whose roote is.78.
√. 4		78
24336		2
√.205——√.24336.		156
or.√.205——√.156.		√.205——156.

That is.√.49.

Scholar. I see in all these examples, you take the same nombers, that you had before in Addition. And firste you set the totalle, out of whiche you abate one of the nobers, that before were added, & the remainer bringeth forthe the other. For in the firste of these.2. examples.√.28. is abated out of.√.63. and there remaineth.√.91.——84.that is.√.7.for.84.taken out of.91. leaueth.7.And in the seconde exãple.√.39 abated out of.√.169.doeth leaue remainyng.√.49.

Master. The thirde forme of Subtraction, is like the thirde forme of Addition: saue that we set ——. for —|—. And here wee muste abate the lesser roote frõ the greater (as I said) before we doe multiplie that nomber by it self. As by this exãple, you maie perceiue Where I dooe Subtracte.√.105. out √.1014, and the remainer is.√.384. Now marke the worke

√.1014——√.105.			
	169		25.
6)	13		5.
		8	
		8	
		64	
		6	
√.		384	

Here you see all thinges agree, with the forme of Addition, saue ——, for —|—. and when I begin to gather the nomber, that stãdeth in the middle, whiche I multiplie by it selfe, and I dooe not make that

nomber,

nomber, by addyng bothe rootes together: For so. 13. and. 5. would make. 18, but I abate. 5. out of 13. and so there doeth remain. 8. with whiche I procede as I did in Addition. And then commeth forthe the remainer. √. 384.

Scholar. I understande it very well. And I praie you that for a proofe, I maie varie the other examples of addition. Partly for my exercise, and partly for examinatiō of the former additions, by the cōtrary kind.

Master. With good will.

Scholar. Then will I set them, and worke them, as here foloweth.

But firste I will begin, with the worke of this last example, after the seconde forme of Subtraction: for a double confirmation of it.

√. 1014 —— √. 150

1104	1014
150	150
50700	1164

1014

√. 152100
√. 4

√. 608400

√. 1164 —— √. 608400

Or. √. 1164 —— 780

That is. √. 384.

An other forme of thesame worke.

√. 1014 —— √. 150.

1014
150

50700
1014

152100
whose roote is. 390.

390:
2

√. 1164 —— 780.

And now here are the variations of the other examples.

√. 2592.

√.2592 —— √.288.
36 4.
72) 6 2.
4
4
16
72
32
112
√. 1152

√.2592 —— √.288.
81 9.
32) 9 3.
6
6
36
32
72
108
√. 1152

√.2592 —— √.288.
144 16.
18) 12 4.
8
8
64
18
512
64
√. 1152

√.2592 —— √.288.
8) 324 36.
18 6.
12
12
144
8
√. 1152

√.2592 —— √.288.
1296 144.
2) 36 12.
24
24
576
2
√. 1152

√.2592 —— √.1152.
1296 576.
2) 36 24.
12
12
144
2
√. 288

Other examples varied, for proofe of the like .6. examples in Addition.

√.64800 —— √.7200	√.64800 —— √.7200.
32400 3600	8100 900.
2) 180 60	8) 90 30
120	60
120	60
14400	3600
2	8
√. 28800	√. 28800

√.64800 —— √.7200.	√.64800 —— √.7200.
3600 400.	2025 225.
18) 60 20	32) 45 15.
40	30
40	30
1600	900
18	32
12800	√. 28800
16	
√. 28800	

√.64800 —— √.7200.	√.64800 —— √.7200.
1296 144.	900 100.
50) 36 12	72) 30 10.
24	20
24	20
576	400
50	72
√. 28800	√. 28800

Subtraction of Cubike rootes.

Master. Like difference is there in Subtraction of *Cubike* rootes *commensurable.* And therfore I set the examples onely, without any larger declaration.

ww'.375.

w√.375 —— w√.81.
125 27.
3) 5 3
2
2
8
3
w√. 24

w√.1715 —— w√.135.
343 27.
5(7 3.
4
4
64
5
w√. 320

w√.13824 —— w√.1000
1728 125
8) 12 5
7
7
343
8
w√. 2744

An other woorke of Subtraction for Surde Cubes.

In the seconde forme of addition of *Surde Cubes*, you remember that you added 4 nombers together. But in Subtraction, you shall adde to eche roote seuerallie that, that commeth of his owne multiplication, with the other *triple*. And then shall you Subtracte the lesser nomber, out of the greater. And the remainer you shall multiplie by the common diuisor. And so shall you haue the roote that remaineth of the Subtraction. As in example.

w√.1296 —— w√.48.
216 8
6) 6 2
36 4
108 12
72 216
64
6
w√. 384

w√58956 —— w√15972
4913 1331
12) 17 11
289 121
867 363
6171 5537
216
12
w√. 2592

	ww√.294912——	ww√.24696
	32768	2744
9)	32	14
	1024	196
	3072	588
	18816	43008
	5832	
	9	
ww√.	52488	

Scholar. In all these examples I se the confirmation of the former additiō. And in these laste workes, this I see peculiare from additiō, that the *Cube* is added with the loweste nomber in that rowe (as in the firste example. 216. is added with 72. and maketh. 288: And. 8. is added with. 216. that yeldeth. 224.) And then is the lesser abated from the greater (as. 224. from 288.) And the remainer (whiche there is. 64) set in the middle under bothe the rewes of nombers. And then is multiplied by the common nomber, to make the remainer.

So in the firste example, the remainer is. ww√. 384. where. ww√. 48. is abated out of. ww√. 1296. And in the seconde example where. ww√. 15972. is subtracted out of. ww√. 58956. the remainer is ww√. 2592. Likewaies in the thirde example. ww√. 24696. is abated out of. ww√294912 & leaueth remainyng. ww√52488

Master. But now in addition there foloweth. 2. other examples, whiche by subtraction maie bee proued thus: as here you see.

	ww√21952——	ww√4096		ww√74088——	ww√19683
	2744	512		2744	729
8)	14	8	27)	14	9
	196	64		196	81
	588	192		588	243
	2688	4704		3402	5292
	216			125	
	8			27	
ww√.	1728		ww√.	3375	

Scholar. I see, in these examples of Subtraction: that the firste nomber is the totalle, or laste nomber in addition. And the seconde nomber, whiche foloweth ——, is the nomber to be abated: and then laste and loweste of all, is the remainer, whiche was one of the firste sommes in addition.

And though there remaine .3. other exāples of *zenzizenzike* nombers, I see no difficultie in theim, but that I can worke them: As here I haue set the forth.

√√.32768 —— √√.648

4096	81
8) 8	3
5	
5	
625	
8	
√√. 5000	

√√.50000 —— √√1280

10000	256
5) 10	4
6	
6	
1296	
5	
√√. 6480	

√√810000 —— √√65536

50625	4096
16) 15	8
7	
7	
2401	
16	
√√.38416	

Master. Seeyng you are experte inough in the 5. workes of these *Surdes* uncōpounde, I wil teache you the like workes in cōpounde *Surdes*.

Of reduction and extraction of rootes.

Scholar. Is there thē noe reduction, nother extraction of rootes, to bee taughte in these uncompounde *Surdes*?

Master. As for reduction, I haue taughte you all readie in multiplication, as moche as is required in these nombers.

And for extraction of rootes, you maie sone understande, that here can be none. For then were thei not *Surde* nombers. And therfore I saied unto you before, that.√.ȝ.100.is not a *Surde* nomber, although it be written like a *Surde* nomber, bicause it hath a Square roote, accordyng to his signe: and that is.10. Likewaies.√.256. is no *Surde* nomber: for his Square roote is knowen to be.16.

Scholar. I might haue cōsidered as moche, by the definition of *Surde* nombers, that their rootes can not be assigned in nombers absolute. And therfore I see that.ww√.125.is not *Surde* nōber, sith his *Cubike* roote is.5. And.w√.256.is a nomber *rationalle*, and no *Surde* nomber: for his *zenzizenzike* roote is.4.

Master. But.w√.64. is a *Surde* nomber, and yet hath.64. a *Square* roote, and a *Cubike* roote also, but not a *zenzizenzike* roote, accordyng to his signe. And therfore ought better to be written thus.√.8.

Scholar. I praie you to procede to *Surde* nombers compounde.

Of Surde nombers compounde.

Master.

S*urde* nombers compoūde, are made not onely of.2. or.3. or moare *Surde* nombers vncompounde, but also of *rationalle* or *Abstracte* nombers ioyned with *Surde* nombers. As.√.10 —+—√.12.and.8.—+—√.6.likewaies.√.20.———3.and.w√.40.—+—√.14.———.3.

Compounde Surdes.

But here shall you marke, that I call compounde nombers, not onely soche as haue the signe of.—+—, but also soche as haue the signe of——— for although in nature of the nomber √.10———√.5.be not compounde, but abated, yet in name he is compoūde, and augmented. For.———.doeth as well augmente

the

the name, as —+— doeth.

Scholar. It semeth reasonable. For when I saie, √.12.——√.7. the name is compounde, an well as if I had saied, √.12.—+—√.7. although the quantitie bee not so greate. For —— doeth euer abate the quãtitie of the nõber, though it do increase the name.

Master. Yet for a difference, the nombers that be compounde with —+— be called *Bimedialles*: and those that be compounde with ——, be named *Residualles*. And if the *Bimedialles* haue all their nombers and partes of one denominations, then bee thei called onely by their generalle name *Bimedialles*. But if their partes be of .2. denominations, then are thei named *Binomialles* properly. Howbeit, many vse to call *Binomialles* all compounde nombers that haue —+—. And so wil I let the names passe. *Bimedialles.* *Residualles.* *Binomialles.*

Euclides definitions doe not very aptly agree to this place, as at an other tyme I will shewe you, and therfore I doe omitte them for this tyme.

But touchyng our principalle intente, whiche is to declare the practike woorke of *Binomialles*, and *Residualles*, there is litle difficultie, if you marke well that whiche is taught before. For as *Binomialles* and *Residualles*, bee made of *Surdes*, or els of *rationalle* nombers with *Surdes*, so the worke of the compounde nombers dependeth of the worke of the simple nombers, and is all one with them. And concernyng the signes —+— and . ——. here is no moare to bee saied, then was taughte in *Cossike* nombers compounde.

Scholar. Yet of euery kinde, it maie please you to set forthe some examples.

Master. I thinke that mete, without many wordes els. Not forgettyng by the waie, that *vniuersalle rootes*, are not accompted emongeste these compounde *Surdes*: but are reserued to their peculiare treatice, as rootes of compounde *Surdes*.

The Arte

Of Numeration.

Numeration is moare plain, then that I neade to stande in declaryng it, other waies then by examples: As here you see.

Examples af Binomialles.

6.—+—√.8. That is 6 more the *Square* roote of 8.
√.20—+—.3. Is the *Square* roote of 20. moare. 3.
ww√.30—+—w√.9. Signifieth the *Cubike* roote of. 30. more the *zenzizenzike* roote of. 9. And so of other.

Examples of Residualles.

24. ——— √96. That is 24. abating the roote of 96
√.150.———.9. Is the *Square* roote of 150. abating 9
w√5208——√35. The *zenzizenzike* roote of. 5208. saue the *Square* roote of. 35. And so forthe.

Scholar. So I see any *Surdes* maie bee compounde with other: And any nõbers *rationalle* ioined with the.

Of Addition.

Master. Addition is as plaine. For as the partes bee, so shall the Addition bee, accordyng as you haue learned before.

Examples of Binomialles.

√.50—+—10	15.—+—√.15.	√.1264—+—8.
√.2.—+—.8.	18—+—√.60.	28—+—√316
√.72—+—18	33.—+—√.135.	36—+—√2844

w√.48.—+—√.5.	ww√.32—+—√10.
w√.243.—+—√.45.	ww√.4.—+—√19.
w√.1875—+—√.80.	ww√108—+—√29—+—√760

Examples

Of Surde nombers.

Examples of Residualles.

√.75.——.4	14——√.3.	250——√.108.
√. 3——1.	16——√.27	√.44——76.
√.108——5.	30——√.12.	174——√.275.

ww√.72.——ww√.96	w√.32.——√.5.
ww√. 9.——ww√.6.	w√.32.——√.24.
ww√243——ww√162	w√.512——√.29—+—√480

Examples of Binomialles with Residualles.

√.80.—+—6.	30.—+—√.20	561——√512
√. 5.——2.	12.——√.5.	√288—+—340
√.125—+—4.	42—+—√.5.	901——√1568

√.63.——ww√160	ww√.320——√.56.
√. 7.—+—ww√.20.	ww√.40—+—√..24.
√.112——ww√684	ww√1680——√.80——√5376

Scholar. I see that you make seueralle Additions in all these nombers. For you adde still like nombers with their matches. So that here is nothyng diuerse from the woorkes of simple *Surdes.* Although in euery thirde example, there appeare moare difficultie, then there is in deede: When I consider the like transposition in *Cosike* nombers. For the woorke addeth like nombers together.

Of Subtraction.

Master. In Subtraction there is as litle diuersitie. As these examples will sufficietly declare: whiche be set as trialles of the former Additions.

Examples of Binomialles.

√.72 —+— 18	36 —+— √2844
√.2 —+— 8.	√.1264 —+— 8.
√.50 —+— 10	28 —+— √.316.

33 —+— √.135	w√.1875 —+— √.80.
15 —+— √.15.	w√.48. —+— √.5.
18 —+— √.60	w√.243 —+— √.45.

ww√.108 —+— √.29. —+— √.760.
ww√.4 —+— √.19.
ww√.32 —+— √.10.

Examples of Residualles.

√.108 —— 5	174. —— √.275.
√.3 —— 1	√.44 —— 76
√.75 —— 4	250. —— √.108

30. —— √.12.	ww√.243 —— w√.162.
14. —— √.3.	ww√.9 —— w√.6.
16. —— √.27.	ww√.72 —— w√.96.

w√.512 —— √.29. —+— √.480.
w√.32 —— √.5.
w√.32 —— √.24.

Examples of bothe together.

√.125 —+— 4	901 —— √1568
√.5 —— 2	√.288 —+— 340
√.80 —+— 6	561 —— √.512.

42 —+— √.5. √.112 ——— ww√.648.
12 ——— √.5. √. 7 —+— ww.20.

30 —+— √.20. √.63 ——— ww.160.

ww√.1080. ——— √.80. ——— √.5376.
ww√. 40. —+— √.24.

ww√. 320. ——— √.56.

Scholar. This is as easie as Addition, saue for.3. examples, whiche I understande not. For although I see the laste example, of eche of the sortes of nombers, to bee agreable with the like examples in Addition, yet I can not so well perceiue, the order of their Subtraction, as I doe knowe the maner of their Additiō. For by the arte of simple *Surdes*, I see that.√.10 and.√.19.doe make.√.29 —+— √.760. But when √.29. —+— √.760.is set as a totalle, and.√.19.to be Subtracted out of it, how I shall worke that, and leaue.√.10.for the remainer, I see not.

So in the *rēsiduall es*, I knowe how .√.5. and √.24. doe make.√.29 —+— √.480. But I knowe not how √.5 abated out of.√.29 —+— √.480.doeth make for the remainer.√.24.

And the like doubte is in the thirde sorte of *Surdes*, whiche are mixte nombers. For where I see in Addition —+— √.24.added with ——— .√.56. And the totalle to bee. ——— .√.80. ——— √.5376. I knowe the reason of the worke, for the signes . —+— .and ——— .by that I learned in *Coßike* nombers: And the reaste is manifeste by Addition of simple *Surdes*. For it is wrought by abatyng √.24.out of.√.56. But then in Subtraction, how —+— √.24.beyng Subtracted from ——— √.80 ——— √.5376 shall leaue ——— √.56 I can not iudge. And yet by the signes I gesse (as I learned in *Coßike* nombers) that it is doen by Additiō, bicause the signes doe disagree.

Mq.it. Master.

Master. In that you remember the former rules, to conferre them aptly with these later workes, I can praise you well. But in that you can not understande the reason of that, whiche was not yet taughte you, I can not greately blame you. Although I can not praise you, for that you thinke your self to be cunnynger then you are. For in those Additions, that you thinke your self to be experte inough, I dare saie, that you bee disceiued, if you take them to bee nombers of any soche, as hetherto hath been taughte unto you.

Scholar. I take them for compounde *Surdes*.

Master. Thei are not so: Nother is their woorke agreable, with the woorke of compounde *Surdes*. But thei are the rootes of compounde *Surdes*: And therfore are called *vniuersalle rootes* of *Surdes*. And accordyng to their proper nature, thei ought to bee called rootes of *Surdes*, and not *Surde* rootes. As I will tell you anon. When I will also discusse your doubte.

But before I speake any moare of theim, I will cande the woorkes of these compounde *Surdes*: whereof. 2. kindes yet remaine behinde.

Of Multiplication.

Multiplicotion of compounde *Surdes*, is as easie as can bee. And differeth in nothyng, frō the worke of simple *Surdes*. Onely this must you marke, as reason would, that you muste multiplie euery parte of the one nōber, by euery parte of the other nōber: as you remember the worke of compounde *Coßike* nombers.

Scholar. I praie you giue me some examples.

Master. That shall you haue. And that maie suffice for this woorke. Marke them well therfore.

Examples

Examples of Binomialles.

23 + √.15.

6 + √.8.

138 + √.120.

+ √.540. + √.4232.

138 + √.4232 + √.540 + √.120.

√.120 + √.12.

√. 12 + √.7.

√1440 + √.84.

+ √.840 + 12.

12 + √.1440 + √.840 + √.84.

Examples of Residualles.

5. — √.10.

5. — √.10.

25 + 10.

— √.250 — 250.

35 — √.1000.

√. 24 — √.20.

√. 30 — √.24.

√.720 + √.480.

— 24 — √.600.

√.720 + √.480 — 24 — √.600.

Examples of bothe together.

32 + √.14.

√.124 — 6.

√.126976 + √.1736.

— . 192. — √.504.

√.126976 + √.1736 — 192 — √.504

√.52.

$\surd.\ 52 —+— 17.$
$17 ——— \surd.52.$

$\surd.15028 —+— 289.$
$——— 52 ——— \surd.15028.$

$37.$

Scholar. Multiplication, as I see, is the easieste woorke of all the other. So that I dooe marke the reduction, in gatheryng the totalle: whiche is easie inough to vnderstand, by that I haue learned in *Cosike* nombers. And Diuision be no harder, it maie sone be learned.

Of Diuision.

Master.

Diuision by one simple nomber, is no moare difficulte: as these examples doe declare. Where the diuisor is a nomber vncompounde.

$\surd.26 —+— 15$ diuided by. 5. doeth make. $\surd.1\frac{1}{25} —+— 3.$

Againe. $\surd.56 —+— \surd.24.$ diuided by. $\surd.6.$ doeth yelde. $\surd.9\frac{1}{3} —+— 2.$

And so $\surd.75 ——— \surd.48.$ diuided by. $\surd.3.$ dooeth bryng forthe. $5. ——— 4.$ that is. 1.

Likewaies. $\surd.320. —+— \surd.180.$ beyng parted by $\surd.5.$ doeth make the *quotiente*. 14.

Scholar. I see it so. For at the firste it is. $\surd.64. —+— \surd.36.$ that is. $8. —+— 6.$ whiche maketh. 14.

Master. So maie you woorke all like diuisions. But when the diuisor is a compounde nomber, then must you vse an other meane: that is to reduce that compounde nōber, to a simple nomber: whiche thing you maie easily doe, by multipliyng any *Binomialle*, by his *Residualle*, or contrary waies, the *Residualle* by his *Binomialle*.

As 6 —+— √.10. multiplied by. 6 —— √.10 doeth make. 26.

And so. √.8 —— √.5, multiplied by √.8 —+— √.5 doeth yelde. 8. —— 5. that is. 3.

Scholar. I perceiue a brief waie in this multiplication: For I neade not in the firste example, to multiplie 6. by. √.10. sith it would amounte to nothyng. In so moche as at one multiplication, it would bee —+—, and at an other. ——. And so the one would abate the other, and leaue nothyng for them bothe.

Master. That is well marked. And it is so generally. Wherefore (as you see) the diuisor by this meanes, maie lightly be tourned into a simple nomber, or a plaine absolute nomber.

And now to make the diuidēde, in the same proportion, to this newe diuisor, that it was unto the old diuisor, you shall multiplie it by the same nomber, by whiche the diuisor was multiplied. For if any nombers bee multiplied, by one common nomber, their newe totalles kepe the same proportion, that was betwene the firste nombers.

Scholar. That must neades be so. For as. 3. is *sesquialtera* unto. 2. so if you multiplie them by. 5. thei will make. 15. and. 10. whiche be in *sesquialtera* proportion and likewaies will their proportion remain, by what so euer nomber thei be multiplied. Wherefore it must neades be reasonable, that if the diuidende and the diuisor, be multiplied by any one nomber, simple or cōpounde, thei shall kepe the same proportion, that thei had before.

Master. For more certain understandyng of this rule, take these examples. The firste is, where √.68 —+— √.54. is sette to bee diuided by. √.6 —+— √.3.

Here firste I multiplie the diuisor by his contrarie, that is his *Binomi*-

√.6 —+— √.3.
√.6 —— √.3
6 —— 3.
That is. 3.

alle

alle. √.6 —— 3. And there riseth. 6 —— 3. that is. 3 whiche I shall kepe for the newe diuisor.

Then doe I multiplie the diuidēde √.68 —+— √54 by thesame *Residualle*.

√.68. —+— √54.
√. 6. —— √3.
√.408 —+— √.324.
—— √.204 —— √.162.
√.408 —+— √.324 —— √.204 —— √.162.

And there doth amoūte, as here in worke is expressed.

√.408 —+— √.324 —— √.204 —— √.162.

Whiche nomber shall be taken for the newe diuidēde: and must be diuided by. 3. that is the newe diuisor. In whose steede I set. √.9. for moare redinesse in woorke. Therfore I set thē doune in order, as here foloweth.

√.408 —+— √.324 —— √.204 —— √.162 (√.45⅓ —+— 6 —— √22⅔ —— √18.
√.9 √.9. √.9 √.9

And then doe I seke how often. √.9. maie bee founde in. √.408. whiche maie bee. 45⅓ of tymes. Wherefore I set. √.45⅓ in the *quotiente*. And then doe I reiterate the diuisor, and sette it under. √.324. where I finde it. 36. tymes: and therefore set I. 6. for it, bicause the *quotiente* els would bee. √.36. whiche is iustly. 6. Thirdly, I remoue the diuisor under √.204. where it maie bee founde. 22⅔ tymes. For whiche I sette √.22⅔ in the *quotiente*. And then set I the diuisor last of all under. 162. where it is founde. 18. tymes: and for that cause I set √.18. in the *quotiente*: And so is the whole *quotiente* √.45⅓ —+— 6 —— √.22⅔ —— √.18.

Scholar. This diuision is straunge to credite, although it be not difficulte to woorke.

Master. If you doubte of it, you maie use the accustomable trialle by the contrary kinde.

Scholar.

Scholar. So must it folowe, that if I dooe multiplie this *quotiente* by the firste diuisor, the firste diuidende will resulte thereof.

And for the proofe of that, I dooe multiplie, $\sqrt{.}45\frac{1}{3}.$ —+— .6. ——— .$\sqrt{.}22\frac{2}{3}$ ——— .$\sqrt{.}18.$ by $\sqrt{.}6.$ —+— $\sqrt{.}3.$ But for the moare case, I doe tourne all the mixte nombers into onely fractions. And then doe I multiplie them orderly.

$\sqrt{.}\frac{136}{3}$ —+— 6 ——— $\sqrt{.}\frac{68}{3}$ ——— $\sqrt{.}18.$
$\sqrt{.}$ 6 —+— $\sqrt{.}3.$

$\sqrt{.}\frac{816}{3}$ —+— $\sqrt{.}216$ ——— $\sqrt{.}\frac{408}{3}$ ——— $\sqrt{.}108.$
$\sqrt{.}\frac{408}{3}$ ——— $\sqrt{.}108$ ——— $\sqrt{.}\frac{204}{3}$ ——— $\sqrt{.}54.$

$\sqrt{.}272$ —+— $\sqrt{.}216$ —+— $\sqrt{.}136$ —+— $\sqrt{.}108.$
— $\sqrt{.}$ 68 ——— $\sqrt{.}$ 54 ——— $\sqrt{.}136$ ——— $\sqrt{.}108.$

$\sqrt{.}$ 68 —+— $\sqrt{.}$ 54.

First I multiplie $\sqrt{.}\frac{136}{3}$ by $\sqrt{.}6.$ and there commeth $\sqrt{.}\frac{816}{3}$ that is. $\sqrt{.}272.$ Again I doe multiplie. 6. or $\sqrt{.}36$ by $\sqrt{.}6.$ and it maketh $\sqrt{.}216.$ Then I multiplie $\sqrt{.}\frac{68}{3}$ by. $\sqrt{.}6.$ & it giueth. $\sqrt{.}\frac{408}{3},$ whiche is. $\sqrt{.}136.$ Fourthly $\sqrt{.}18.$ multiplied by. $\sqrt{.}6.$ dooeth make. $\sqrt{.}108.$ All whiche I set doune with their conueniente signes.

After that I multiplie. $\sqrt{.}\frac{136}{3}$ by. $\sqrt{.}3.$ and it yeldeth $\sqrt{.}\frac{408}{3}$ that is. $\sqrt{.}136.$ whiche I sette doune with his signe —+—. Then $\sqrt{.}36$ by. 3. maketh $\sqrt{.}108.$ Thirdly. $\sqrt{.}\frac{68}{3}$ by $\sqrt{}$ 3. dooeth giue. $\sqrt{.}68.$ and last of all, $\sqrt{.}18.$ multiplied by. $\sqrt{.}3.$ bryngeth forthe. $\sqrt{.}54.$

When all these be placed conueniently, I doe consider that —+— $\sqrt{.}136.$ and ——— $\sqrt{.}136.$ maie bee bothe cancelled, bicause the one dooeth abate the other. And likewaies, —+— $\sqrt{.}108.$ and ——— $\sqrt{.}108.$ eche abate other: so that thei must bothe be reiected.

Then I see, that $\sqrt{.}68.$ beyng abated out of $\sqrt{.}272$ there will remain. $\sqrt{.}68.$ And in like. $\sqrt{.}54.$ beyng abated out of. $\sqrt{.}216.$ dooeth leaue. $\sqrt{.}54.$ So that the whole multiplicatiō doth make iustly $\sqrt{}$ 68 —+— $\sqrt{}$ 54 whiche is the firste dividende. And so is that division approued good.

An other example.

Master. Yet for you exercise, you shall haue some examples moare of division.

√.456.———.√.72. is sette to bee divided by √.18—+—√.6.

Scholar. That divisor must I multiplie by his contrarie, whiche is the *Residualle*. √.18——√.6. And so, as you maie sone perceiue, there will rise. 18.———6. that is 12. whiche must be kepte for the newe divisor.

√.18.—+—√.6.
√.18.———√.6.
18———.6.
That is. 12.

Then shall I multiplie the former dividende, that is √.456——√.72 by thesame *residualle* √.18——√6

√.456———√.72.
√. 18———√. 6.
√.8208———√.1296.
√.432.———√.2736.
√.8208—+—√.432———√.2736———√.1296.

And there will rise of that multiplication, as here by example appereth √.8208—+—√.432——√.2736 ———1296. whiche nõber I shall divide by. 12. that was founde for the newe divisor. And then will the *quotiente* bee. √.57—+—√.3———√.19.———√.9. As here in woorke doeth appeare.

√.8208—+—√.432——√.2736——√.1296(√.57+√.3——
√.144.   √144   √.144   √.144.

Where I haue set. √.144. for. 12. seyng thei be all one: but that. √.144. is moare apte for this woorke. And I haue repeated it as often tymes, as the divisor should be remoued.

The proofe.

But now to trie this woorke, whether it bee well wroughte, I shall multiplie this *quotiente* by the firste divisor, & then ought the firste dividende to amounte.

As

As here in example, you see wroughte.

√.57.—+—√.3.——√.19.——√.9.
√.18.—+—√.6.

√.1026—+—√.54——√.342——√.162.
√.342—+—√.18——√.114——√.54.

√.1026—+—√.18——√.114——√.162.

Where —+—√.54. doeth cancell ——√.54. and is cancelled by it.

So —+—√.342. and ——√.342. exclude one an other, and therefore must bee bothe rejected. And then remaineth onely,

√.1026—+—√.18.——√.114——√.162.

Whiche nombers I dooe well examine: and finde that √.114. beyng abated out of. √.1026. there will remaine. √.456. Againe if —+—√.18. be subtracted out of ——√.162. there will reste ——√.72. And so is that whole multiplicatiõ onely. √.456——√72 agreable to the firste diuidende. Wherby it is manifeste, that the former diuision was good.

Master. How can you worke this example? Where. 24. is set to be diuided by. 3.—+—√.8. *The thirde example.*

Scholar. I must still obserue the generalle rule. And multiplie bothe those nombers, by the contrarie of the diuisor, that is, by the *residualle*. 3——√.8. And of the firste multiplication of it, with the diuidende. 24, there riseth. 72——√.4608. Of the seconde multiplication, where the *Binomialle* is multiplied by the *Residualle*, that is his contrary, the totalle will be. 9——8. that is but. 1. And therfore seyng. 1. doeth nother multiplie nor diuide, the former nomber.

24.
3——√.8
72——√.4608

3.—+—√.8
3.——√.8
9.——.8.
That is. 1.

That is. 72——√.4608. is the *quotiente*, when 24. is diuided by 3.—+—√.8.

The proofe. For profe whereof, I multiplie $72 - \sqrt{.4608}$ that is the *quotiente*, by. $3 + \sqrt{.8}$. And there riseth $216 + \sqrt{.41472} - \sqrt{.41472} - \sqrt{.36864}$. whereof. 2. nombers differyng but by $+$ & $-$ muste bothe bee reiected, as nombers superfluouse.

$$
\begin{array}{r}
72 - \sqrt{.4608}. \\
3 + \sqrt{.8}. \\
\hline
216 - \sqrt{.41472} \\
\sqrt{.41472} - \sqrt{.36864} \\
\hline
216 - .192. \\
\end{array}
$$

That is. 24.

Then. 36864. is a *square* nomber, and hath. 192 for his roote. Wherfore the whole nomber is, $216 - 192$ that is (as it is manifeste inough) 24. And so is the whole worke proued good.

The fourthe example. Master. You shall haue one example moare, and then will I make an ende of diuision.

When $\sqrt{.6570} + \sqrt{.254}$. is propounded to bee diuided by $\sqrt{.54} - \sqrt{.6}$. I would knowe the *quotiente*.

Scholar. I see the newe diuisor will be. $54 - 6$. that is. 48.

And then for to finde a diuidende conueniente, I shall multiplie the firste diuidende, by the contrarie of the firste diuisor, that is by $\sqrt{.54} + \sqrt{6}$ And there will rise, as you see. $\sqrt{.354780} + \sqrt{.13716} + \sqrt{.39420} + \sqrt{.1524}$.

$$
\begin{array}{l}
\sqrt{.6570} + \sqrt{.254}. \\
\sqrt{.54} + \sqrt{.6}. \\
\hline
\sqrt{.354780} + \sqrt{.13716}. \\
\sqrt{.39420} + \sqrt{.1524}. \\
\hline
\end{array}
$$

That diuidende must be diuided by. 48. or moare aptly by. $\sqrt{.2304}$. And the *quotiente* will bee.

$\sqrt{.153\frac{189}{192}} + \sqrt{.5\frac{183}{192}} + \sqrt{.17\frac{63}{576}} + \sqrt{.\frac{127}{192}}$.

As here appeareth in worke.

$$
\frac{\sqrt{.354780}}{\sqrt{.2304}} + \frac{\sqrt{.13716}}{\sqrt{.2304}} + \frac{\sqrt{39420}}{\sqrt{.2304}} + \frac{\sqrt{1524}}{\sqrt{2304}} \left(\sqrt{153\tfrac{189}{192}} + \sqrt{.5\tfrac{183}{192}} + \sqrt{17\tfrac{63}{576}} + \sqrt{\tfrac{221}{391}} \right.
$$

The proofe. And that this worke is good, I will proue it by multiplication.

multiplication. As the example folowyng dooeth declare. Where by the firste multiplication there commeth. 8. nombers, that is. 4. with. $+$. and. 4. with $-$.

$$\sqrt{.}\tfrac{29565}{192} + \sqrt{.}\tfrac{1143}{192} + \sqrt{.}\tfrac{3285}{192} + \sqrt{.}\tfrac{127}{192}.$$

$$\sqrt{.}54. - \sqrt{.}6.$$

$$\sqrt{.}\tfrac{1596510}{192} + \sqrt{.}\tfrac{61722}{192} + \sqrt{.}\tfrac{177390}{192} + \sqrt{.}\tfrac{6858}{192}.$$

$$- \sqrt{.}\tfrac{177390}{192} - \sqrt{.}\tfrac{6858}{192} - \sqrt{.}\tfrac{19710}{192} - \sqrt{.}\tfrac{762}{192}.$$

$$\sqrt{.}\tfrac{1596510}{192} + \sqrt{.}\tfrac{1722}{192} - \sqrt{.}\tfrac{19710}{192} - \sqrt{.}\tfrac{762}{192}.$$

$$\sqrt{.}6570. + \sqrt{.}254.$$

And bicause the firste nōber with $-$, is equalle to the thirde with $+$, therfore thei bothe must be reiected. Again in as moche as the seconde nōber with $-$ is equalle to the fourthe nomber with $+$, thei bothe shall bee cancelled. And then remaineth. 2. nombers with $+$, and other. 2. with $-$.

So if you abate the thirde $-$ out of the firste $+$, the *quotiente* will be. $\sqrt{.}6570$.

Likewaies if you abate the fourthe $-$ out of the seconde $+$, the *quotiente* will yelde. $\sqrt{.}254$.

And thei bothe will make the firste diuidende. $\sqrt{.}6570$. Wherby the former diuisiō is approued good.

Master. This shall suffice for diuision.

Of extraction of rootes.

The nexte woorke is extraction of rootes: whiche you maie very easilie woorke, by puttyng the signe of the roote, that you desire, before the whole nomber. As if you would haue the square roote of $\sqrt{.}10 + \sqrt{.}5$. this is it $w\sqrt{.}10 + \sqrt{.}5$. The *Cubike* roote of the same nōber is. $ww\sqrt{.}\sqrt{.}10 + \sqrt{.}5$. And the *zenzizenzike* roote of it is $w\sqrt{.}\sqrt{.}10. + \sqrt{.}5$. But if you will haue the *Square* roote of. $10 + \sqrt{.}5$

 it

it is. √.10 —+— √.5. And his *Cubike* roote is. ww√.10 —+— √.5. Likewaies his *zenzizenzike* roote is w√.10 —+— √.5.

So of. ww√.18 —— 2. the *Square* roote is √. ww√.18 —— 2. The *Cubike* roote is. ww. ww√.18 —— 2. And the *zenzizenzike* roote is. w√. ww√. —— 2.

Scholar. Hereby I perceiue that the later parte of the cōposition, is not varied at all, but onely the firste parte taketh vnto it the signe of the roote. And that signe is referred to the whole compounde nomber.

Vniuersalle rootes.

Master. These rootes therefore bee called *vniuersalle rootes*, bicause thei are the rootes, not of the seueralle partes of the compounde nōber, but of the whole compounde nomber. And that is the difference, betwene the common *Surde* nombers, and *vniuersalle rootes*. For if √.24 —+— √.144. be sette for a common *Surde* nomber, then doeth it betoken, that I must take 2. rootes, that is. √.24. and √.144, and ioyne theim together. But if it stande for an *vniuersalle roote*, it representeth the roote of this whole nomber. 24 —+— √.144. whiche is. 6. for the whole *Square* is. 36.

Scholar. I perceiue it well. For. √.144. beeyng 12, that. 12. with. 24. dooeth make. 36. And therefore must the *vniuersalle roote* of. 24 —+— √.144. bee. 6. And so √.24 —+— √.144. is iust. 6.

But if. √.24 —+— √.144. doe stande for a common *Surde* nomber compounde: then is it made of. 2. rootes, that is √.24. whiche is almoste. 5. and √.144 beyng. 12. And so the whole compounde roote, in that sorte is almoste. 17. And is nighe. 3. tymes so moche as the same nomber, beyng an *vniuersalle roote*.

Master. Bicause you maie perceiue it the better, I will put an example in *Square* nombers, made like *Surdes*. As this. √.81 —+— √.361 if it be an *vniuersalle roote*, then it is equalle to 10. For I must take first the roote of the laste nomber, whiche is. 19. And adde it

with

with.81.wherby there amounteth.100.whose roote is.10. But if it stand after the common sorte of *Surde* nombers, it betokeneth the roote of.81.and the roote of.361.(that is.9.and.19)to bee added together. And so thei make.28.whiche is farre aboue.10.

But farther now, if it stande for a common *Surde* nomber: And I would haue the *Square* roote of it, then is that. √√.81 −+− √.361. And betokeneth the *Square* roote of the square roote of.81. and the Square roote of.361. added together, that is the square roote of.28. But moste generally and moste aptly, it betokeneth the roote of the *vniuersalle roote* of.81.‡ √.361.

Scholar. Now I perceiue that in Addition, and Subtraction of *Surdes*, the last nombers that did result of that worke, were *vniuersalle rootes.*

Master. You saie truthe. But harke what meaneth that hastie knockyng at the dore?

Scholar. It is a messenger.

Master. What is the message? tel me in mine eare. Yea sir is that the mater? Then is there noe remedie, but that I must neglect all studies, and teaching, for to withstande those daungers. My fortune is not so good, to haue quiete tyme to teache.

Scholar. But my fortune and my fellowes, is moche worse, that your vnquietnes, so hindereth our knowledge. I praie God amende it.

Master. I am inforced to make an eande of this mater: But yet will I promise you, that whiche you shall chalenge of me, whē you see me at better laiser: That I will teache you the whole arte of *vniuersalle rootes*. And the extraction of rootes in all *Square Surdes*: with the demonstration of theim, and all the former workes.

If I mighte haue been quietly permitted, to reste but a litle while lōger, I had determined not to haue ceased, till I had ended all these thinges at large. But now farewell. And applie your studie diligently in this that you haue learned. And if I maie gette any quietnesse reasonable, I will not forget to performe my promise with an augementation.

Scholar. My harte is so oppressed with pēsiuenes, by this sodaine vnquietnesse, that I can not expresse my grief. But I will praie, with all theim that loue honeste knowledge, that God of his mercie, will sone ende your troubles, and graunte you soche reste, as your trauell doeth merite. And al that loue learnyng: saie therto. Amen.

Master. Amen, and Amen.

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