ARTICLE 29

The Triakis Tetrahedron and the Disdyakis Triacontahedron Embody the Fine Structure Constant and the Structural Parameter of the Heterotic Superstring

by

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Abstract

As, respectively, the first and last of the Catalan solids, the triakis tetrahedron and the disdyakis triacontahedron are unique in having numbers of edges and faces that differ by a factor of 10. This means that the numbers of geometrical elements of any particular kind surrounding their axes also differ by this factor. The ten-fold multiplicity of the properties of the disdyakis triacontahedron is evidence of its holistic character. The Godname Yah with number value 15 picks out the truncated cuboctahedron — the 15th in the families of Archimedean and Catalan solids — as the only one of them with 48 vertices and 144 triangles in its faces. Surrounding its axis are 550 vertices, edges and triangles. This is the number of Sephirothic emanations in the Cosmic Tree of Life that maps all levels of reality. The counterpart of this property in the Platonic solids is the 550 vertices, edges and triangles in their 50 faces. The Godnames Ehyeh and Adonai prescribe the numbers of geometrical elements surrounding the axes of the triakis tetrahedron and the disdyakis triacontahedron. The former is built from 137 such elements, showing how it embodies the reciprocal of the fine structure constant that determines properties of atoms. The counterpart of this in the inner Tree of Life is the 1370 yods in 137 tetractyses that are needed to construct it from tetractyses. When, instead, its internal triangles are each constructed from three tetractyses, the triakis tetrahedron has 168 elements surrounding its axis. It therefore embodies also the structural parameter of the heterotic superstring. The disdyakis triacontahedron has 840 yods surrounding its axis. They symbolize the 840 circularly polarized oscillations made by each helical whorl of the superstring as it winds 2½ around its axis. The polyhedron has 1680 geometrical elements surrounding its axis. Their counterparts in the superstring are the 1680 oscillations in a whorl. The numbers 168, 840 and 1680 have a natural representation in a tetractys pattern of the first 40 odd integers after 1.

Article 28 showed that an isomorphism exists between the disdyakis triacontahedron with 62 vertices and the two sets of the last six polygons of the inner Tree of Life whose 62 corners are unshared with its outer form. It allowed the root structure of the superstring gauge symmetry group E_8 and its subgroups E_7 and E_6 to be correlated with the icosahedron and dodecahedron within this polyhedron. This article will reveal further ways in which parameters of particle physics are embodied in it. Three ways of constructing polyhedra with triangular faces from tetractyses will be discussed (labelling of cases follows discussion in previous articles):

Case B: Internal triangles divided into three tetractyses, polygonal faces divided into three tetractyses;
Case A: internal triangles divided into three triangles; polygonal faces turned into one tetractys;
Case C: internal triangles turned into tetractyses; polygonal faces turned into tetractyses;

(The fourth possibility in which internal triangles are turned into tetractyses and faces divided into three tetractyses need not be considered for the current purpose).

Table 1. Formulae for the three types of construction of polyhedra with triangular faces.

	Case C	Case A	Case B
Surface			
Number of vertices surrounding axis =	C–2	C–2	C+F–2
Number of edges surrounding axis =	E	E	E+3F
Number of triangles surrounding axis =	F	F	3F
Subtotal =	2E	2E	2E+6F
Interior			
Number of vertices surrounding axis =	0	E	E
Number of edges surrounding axis =	C–2=E–F	C–2+3E=4E–F	C–2+3E=4E–F
Number of triangles surrounding axis =	E	3E	3E
Subtotal =	2E–F	8E–F	8E–F
Total =	4E–F	10E–F	10E+5F

Definitions
C = number of vertices of polyhedron.
E = number of polyhedral edges.
F = number of polyhedral faces.
C, E and F are related by Euler's equation:

$$C - E + F = 2.$$

m = number of triangular sectors of a face with m edges.
n(m) =number of faces with m sides
$L \equiv \sum_m mn(m)$ = number of triangular sectors of faces.

The internal triangles formed by the centre of the polyhedron and the two ends of an edge are turned into single tetractyses. Triangular sectors of polygonal faces are turned into tetractyses.

Surface

Number of vertices in faces = C + F = 2 + E.

Number of edges in faces = $E + \sum_m mn(m) = E + L$.

Number of triangles in faces = L.
Number of vertices, edges & triangles = 2 + E + E + L + L = 2E + 2L + 2.

Interior

Number of vertices = 1.
Number of edges = C.
Number of triangles = E.
Number of vertices, edges & triangles = C + E + 1 = 2E – F + 3.
Total number of vertices, edges & triangles = 4E – F + 2L +5.
The number '5' denotes the five geometrical elements making up the axis (three vertices and two edges). The number of geometrical elements in the surface of the polygon that surround the axis ≡ S = 2E + 2L. The number of internal elements surrounding the axis ≡ I = 2E – F. The total number of elements surrounding the axis ≡ N = S + I = 4E – F + 2L.

E = 18 and F = 12 for the triakis tetrahedron and E = 180 and F = 120 for the disdyakis triacontahedron, that is, these numbers are ten times as large in each case. Table 1 indicates that the numbers of geometrical elements of each kind surrounding the axis either inside or on the faces of a polyhedron with triangular faces depends only on the value of E and F. This means that, for the disdyakis triacontahedron, the number of internal or external geometrical elements of each kind is always ten times the corresponding number for the triakis tetrahedron. The ten-fold property is unique to this pair of Catalan solids because no other pair has E and F differing by a factor of 10.

Table 2 lists the values of S, I and N for the Archimedean and Catalan solids.

N	I	S	L	F	E	C	Archimedean solid
136	28	108	36	8	18	12	truncated tetrahedron
178	34	144	48	14	24	12	cuboctahedron
274	58	216	72	14	36	24	truncated cube
274	58	216	72	14	36	24	truncated octahedron
358	70	288	96	26	48	24	rhombicuboctahedron
442	82	360	120	38	60	24	snub cube
448	88	360	120	32	60	30	icosidodecahedron
550	118	432	144	26	72	48	truncated cuboctahedron
688	148	540	180	32	90	60	truncated icosahedron
688	148	540	180	32	90	60	truncated dodecahedron
898	178	720	240	62	120	60	rhombicosidodecahedron
1108	208	900	300	92	150	60	snub dodecahedron
1378	298	1080	360	62	180	120	truncated icosidodecahedron

Catalan solid	F	E	C	L	S	I	N
triakis tetrahedron	12	18	8	36	108	24	132
rhombic dodecahedron	12	24	14	48	144	36	180
triakis octahedron	24	36	14	72	216	48	264
tetrakis hexahedron	24	36	14	72	216	48	264
deltoidal icositetrahedron	24	48	26	96	288	72	360
pentagonal icositetrahedron	24	60	38	120	360	96	456
rhombic triacontahedron	30	60	32	120	360	90	450
disdyakis dodecahedron	48	72	26	144	432	96	528
triakis icosahedron	60	90	32	180	540	120	660
pentakis dodecahedron	60	90	32	180	540	120	660
deltoidal hexacontahedron	60	120	62	240	720	180	900
pentagonal hexacontahedron	60	150	92	300	900	240	1140
disdyakis triacontahedron	120	180	62	360	1080	240	1320

Starting with the truncated tetrahedron, the polyhedron with the least number of faces, and counting down these tables in a zigzag fashion across pairs of dual polyhedra listed in order of increasing C or F, the 15th polyhedron is the truncated cuboctahedron and the 26th (and last) polyhedron is the disdyakis triacontahedron (shown in Table 2 with orange cells). The Godname Yah with number value 15 picks out the only polyhedron with 48 vertices and 144 triangular sectors (L = 144). The full Godname Yahweh with number value 26 picks out the only polyhedron with 120 faces. This shows how this most famous Godname prescribes the 120 Polyhedron and the 144 Polyhedron.

Surrounding the axis of the truncated cuboctahedron are 550 vertices, edges & triangles, where

55
55 55
550 = 55 55 55
55 55 55 55

and

$$
55 = \begin{array}{c} 1 \\ 2\ \ 3 \\ 4\ \ 5\ \ 6 \\ 7\ \ 8\ \ 9\ \ 10 \end{array} .
$$

The Decad (10) — the Pythagorean measure of perfection — determines the number of geometrical elements in the truncated cuboctahedron. As discussed in Article 3,[1] the map of all levels of reality, called the 'Cosmic Tree of Life' (CTOL), consists of 91 overlapping Trees of Life, the seven lowest ones representing 26-dimensional space-time. CTOL has 550 Sephirothic emanations.[2] The truncated cuboctahedron is unique among the Archimedean and Catalan solids in being made up of the same number of geometrical elements as there are Sephirothic levels in the map of the spiritual cosmos.

The counterpart of this beautiful property in the Platonic solids is the fact that the five regular polyhedra have 550 vertices, edges and triangles in their 50 faces.[3] Its counterpart in the dodecahedron is the fact that, when constructed from tetractyses, it has 550 hexagonal yods.[4]

According to Table 2, the disdyakis triacontahedron has 1320 geometrical elements surrounding its axis. Together with the truncated cuboctahedron there are 1870 elements surrounding their axes. This is the number of yods in 187 tetractyses, where 187 is the number value of Auphanim ("wheels"), the order of angels assigned to Chokmah. As the Godname assigned to Chokmah is Yahweh, whose number value 26 selects the disdyakis triacontahedron, the Divine Name and the angelic order of this Sephirah define this polyhedron and its dual. The 144 triangles of the faces of the former have 72 vertices and 216 edges surrounding its axis. 72 is the number value of Chesed, the first Sephirah of Construction, and 216 is the number value of Geburah, the second Sephirah of Construction.

Surrounding the axis of the disdyakis triacontahedron are 1260 geometrical elements other than its 60 vertices. This is the number of yods in 126 tetractyses. Remarkably, 126 is the sum of the number values of the four types of combinations of the letters A, H and I in Ehyeh (AHIH), the Godname of Kether:

A = 1, H = 5, I = 10

1.	A + H + I	=	16
2.	AH + HI + AI + HH	=	42
3.	AHI + HIH + AHH	=	47
4.	AHIH	=	21
		Total =	126

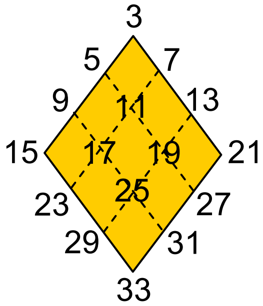
Just as remarkable is the fact that there are 1260 yods on the edges of the 360 tetractyses in the 120 faces of the disdyakis triacontahedron.[5] This illustrates how Ehyeh prescribes the disdyakis triacontahedron.

The 1320 geometrical elements surrounding the axis of this polyhedron comprise 660 (= 66×10) elements and their mirror images. As 66 is the 65th integer after 1, the Godname Adonai of Malkuth with number value 65 prescribes the disdyakis triacontahedron. Similarly, it prescribes the triakis tetrahedron with 66 such elements.

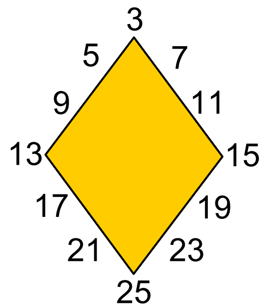
We showed in Article 27 that the number of geometrical elements of a given type surrounding the axis of the disdyakis triacontahedron is ten times the number of like elements in the triakis tetrahedron for case A and case B. Inspection of Table 2 shows that this is true also for case C. The triakis tetrahedron has 132 elements surrounding its axis, which consists of five elements. The polyhedron is therefore built from 137 geometrical elements. Three remarkable properties emerge:

Case A: 168 elements surround its axis, where 168 is the number of roots of the superstring gauge symmetry group E_8 that do not belong to its subgroup E_6;
Case B: 240 elements surround its axis, where 240 is the number of roots of E_8;
Case C: it possesses 137 elements.

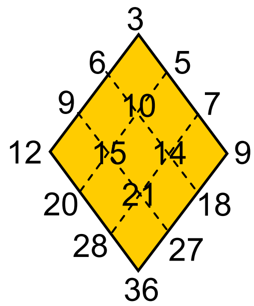
Physicists regard 137 as, perhaps, the most important number in physics because, as the measure of the strength of the electromagnetic interaction between an electron and an electromagnetic field, the fine structure constant $e^2/\hbar c \approx 1/137$ determines the very nature and size of atoms, as well as the chemistry of materials arising from their mutual bonding. The simplest construction of the triakis tetrahedron from tetractyses (case C) requires 137 geometrical elements. The Pythagorean Tetrad determines this number because 137 is the 33rd prime number, where 33 = 1! + 2! + 3! + 4!, and 33 is the 16th odd integer after 1 that can be assigned to a 4×4 array (read across the rhombus):



The next simplest construction of the triakis tetrahedron (case A) require 168 geometrical elements surrounding its axis, where 168 is the sum of the 12 odd integers after 1 that form the edges of a rhombus, four to a side:

168 = 

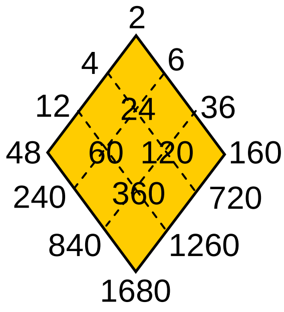
The last construction (case B) requires 240 elements surrounding its axis, where

$240 = (5^2-1)\times 10 = (3+5+7+9)(1+2+3+4) =$ 

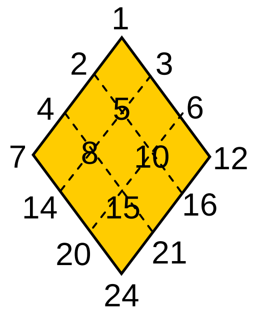
is the sum of the 16 integers that can be assigned to a 4×4 array, starting with the first four odd integers after 1 and adding multiples of them by 2, 3 & 4. It is remarkable that the largest number in the 4×4 array of the first 16 highly composite numbers* is 1680, which is the number of geometrical elements surrounding the axis of the disdyakis

* An integer is highly composite if it has more factors than all integers smaller than it.

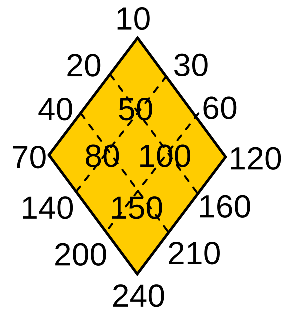
triacontahedron in case A:



If the first 16 factors of 1680 are arranged in a 4×4 array, 24 is the largest one:



which means that 240 is the largest of the first 16 divisors of 1680 multiplied by 10:



This demonstrates that the numbers 168, 240 and 1680 have an arithmetic connection as well as a geometrical one. It exists because these numbers are group-dynamical and structural parameters of the $E_8 \times E_8$ heterotic superstring.

The counterpart in the inner Tree of Life of the 240 geometrical elements surrounding the axis of the triakis tetrahedron in case B are the 240 yods in the seven separate regular polygons other than their 55 corners (Fig. 1). The inner form of ten Trees of Life

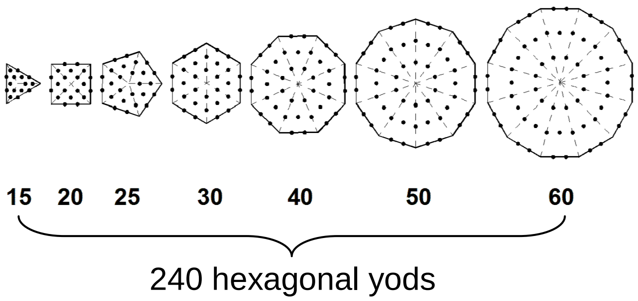
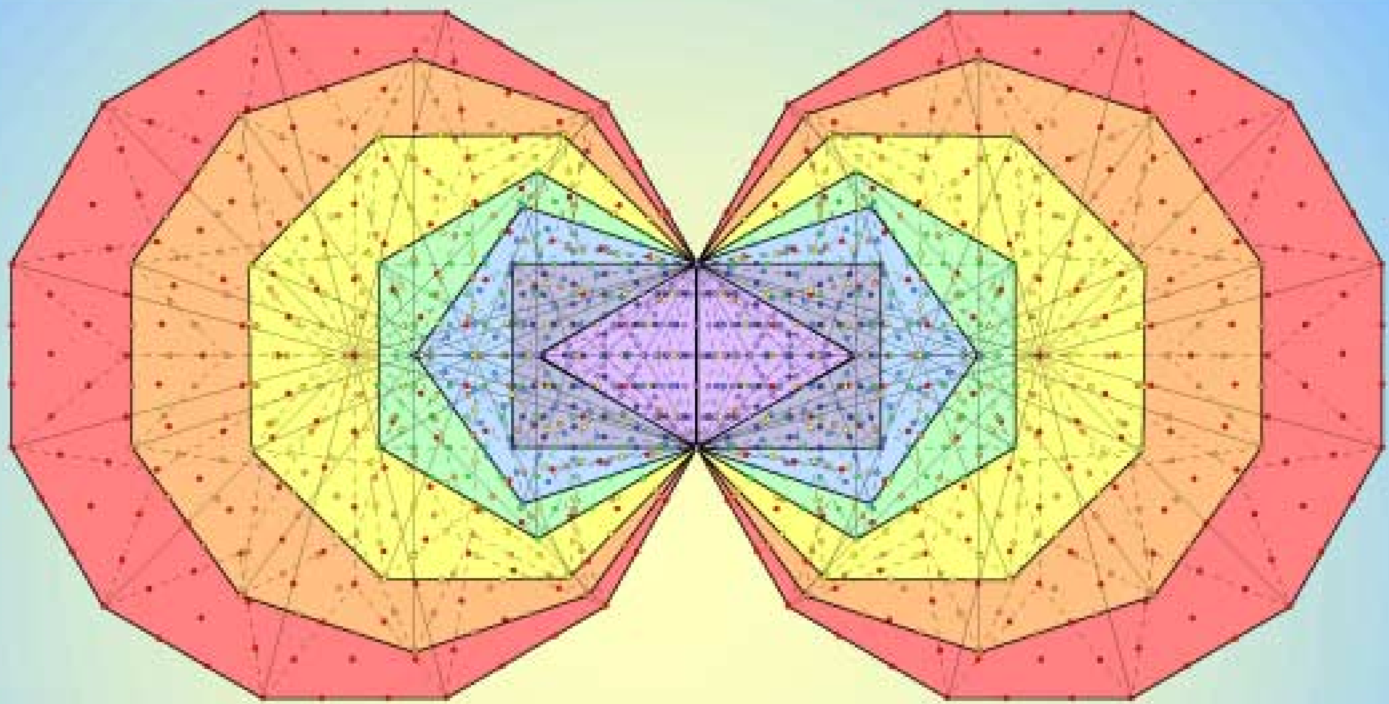


Figure 1. The seven separate polygons with 55 corners of their 48 tetractyses have 240 hexagonal yods.

consists of 70 separate polygons with 2400 hexagonal yods and 550 corners of their 480 tetractyses. The 550 elements shaping the truncated cuboctahedron in case C correspond to the 550 corners shaping the polygons. The 2400 elements shaping the disdyakis triacontahedron in case B correspond to the 2400 yods other than these corners. As

1370 =



The (7+7) enfolded, regular polygons, whose **94** sectors are each divided into 3 tetractyses, contain 1370 yods. Of these, 70 yods are corners of polygons, leaving 1300 other yods, where 1300 = **26**×**50** = $1^5 + 2^5 + 3^5 + 4^5$, **26** is the number value of the Godname Yahveh and **50** is the number value of the Godname Elohim. The number value **65** of the Godname Adonai defines the (1300÷2 = 650 = **65**x10) of such yods associated with either set of polygons:

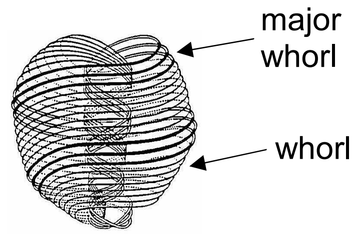
	65		$1^2\ 2^2\ 3^2\ 4^2$
	65 65		$12^2 \quad 5^2$
650 =	**65 65 65**	=	$11^2 \quad 6^2$
	65 65 65 65		$10^2\ 9^2\ 8^2\ 7^2$

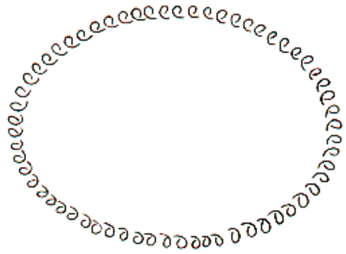
The **94** sectors have 80 corners, leaving 1290 (= **129**x10) other yods, where **129** is the number value of Yahveh Sabaoth. The Pythagorean tetrad (4) determines the number (137) of equivalent tetractyses as 137 = 33rd prime number and 33 = 1! + 2! + 3! + 4!. The number of yods in 7 enfolded polygons other than their 7 centres = 680 = $1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 13^2 +$ $\mathbf{15}^2$, where **15** is the number value of the Godname Yah.

Figure 3. 1370 yods are needed to construct the inner Tree of Life from tetractyses.

$$2400 = 49^2 - 1 = 3 + 5 + 7 + \ldots + 97,$$

2400 is the sum of the 48 odd integers after 1. Their average value is 50, the number value of Elohim, Godname of Binah. 49 is the number value of El Chai, the Godname of Yesod. As the truncated cuboctahedron, the 144 Polyhedron is the ‘inner’ or organizing aspect of the polyhedral Tree of Life, corresponding to the shape-defining corners of tetractyses, which symbolize the Supernal Triad of Kether, Chokmah and Binah. As the disdyakis triacontahedron, the 120 Polyhedron is its ‘outer’ aspect, corresponding to the formative degrees of freedom defined by the hexagonal yods of tetractyses, which symbolize the seven Sephiroth of Construction. This indicates that the 144 Polyhedron represents the ‘inner,’ or subjective aspect of Adam Kadmon, or Spiritual Man, whilst





Annie Besant

C.W. Leadbeater

Figure 2. The Theosophists Annie Besant and C.W. Leadbeater observed subatomic particles with the aid of a yogic siddhi called ‘anima.’ The basic particle of matter (identified by the author as the heterotic superstring constituent of up and down quarks) consists of ten closed curves, or ‘whorls.’ Each one is a helical coil with 1680 turns.

the 120 Polyhedron represents the objective nature of the divine prototype. This manifests in the 1680 geometrical elements surrounding its axis in case A, degrees of freedom that appear in the subatomic world as the 1680 circular turns in each whorl of the basic unit of matter described paranormally by Annie Besant and C.W. Leadbeater[6] and interpreted by the author as circularly polarised oscillations of a standing wave in a closed string (Fig. 2). Ten of these constitute the closed, $E_8 \times E_8$ heterotic superstring.

The counterpart in the inner Tree of Life of the 137 geometrical elements needed to construct the triakis tetrahedron is the fact that 1370 yods are needed to build the two sets of seven enfolded polygons when their 94 sectors are divided into three tetractyses

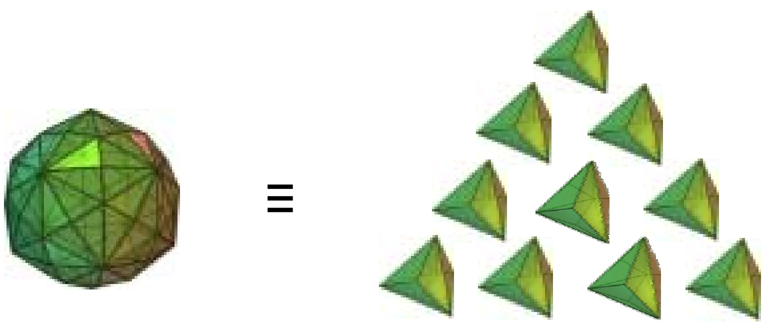


Figure 4. The tetractys nature of the disdyakis triacontahedron.

(Fig. 3). They are in 282 tetractyses, where 282 is the number value of Aralim, ("thrones"), the Angelic Order assigned to Binah. The fact that yods in 137 tetractyses fill the inner Tree of Life proves the number 137 to be archetypal, as shown by its ubiquitous presence in atomic, nuclear and particle physics. The correspondences:

1370 yods of inner Tree of Life ⟷ 1370 elements of ten triakis tetrahedra
1320 elements of disdyakis triacontahedron ⟷ 1320 elements surrounding axes of ten triakis tetrahedra

show that the disdyakis triacontahedron is equivalent to ten triakis tetrahedra (Fig. 4), whose number of geometrical elements is equal to the number of yods making up the inner Tree of Life. Each yod symbolizes one of these elements. This is further evidence that the disdyakis triacontahedron is the polyhedral counterpart of the inner Tree of Life, as discussed in Articles 22-24.[7]

Consider those Catalan solids whose faces are triangular built from tetractyses, with the interior triangles regarded as single tetractyses (case C). The formulae for the various types of geometrical elements composing the solids are shown below:

Number of vertices ≡ V = C + 1. Number of vertices surrounding axis ≡ V' = C – 2.
Number of edges ≡ e = C + E. Number of edges surrounding axis ≡ e' = C + E – 2.
Number of triangles ≡ T = E + F. Number of triangles surrounding axis = E + F.
Total number of geometrical elements ≡ N = 2C + 2E + F + 1 = C + 3E + 3.
Number of geometrical elements surrounding axis ≡ N' = C + 3E – 2.

Table 3. Numbers of geometrical elements in the Catalan solids.

Catalan solid	V	V'	e	e'	T	N	N'
triakis tetrahedron	9	6	26	24	30	65	60
rhombic dodecahedron	-	-	-	-	-	-	-
triakis octahedron	15	12	50	48	60	125	120
tetrakis hexahedron	15	12	50	48	60	125	120
deltoidal icositetrahedron	-	-	-	-	-	-	-
pentagonal icositetrahedron	-	-	-	-	-	-	-
rhombic triacontahedron	-	-	-	-	-	-	-
disdyakis dodecahedron	27	24	98	96	120	245	240
triakis icosahedron	33	30	122	120	150	305	300
pentakis dodecahedron	33	30	122	120	150	305	300
deltoidal hexacontahedron	-	-	-	-	-	-	-
pentagonal hexacontahedron	-	-	-	-	-	-	-
disdyakis triacontahedron	63	60	242	240	300	605	600

Table 3 lists the numbers of elements of each type (solids without triangular faces have dashes in cells). Table 2 indicates that, when their triangular faces are divided into three tetractyses, 240 geometrical elements inside the disdyakis triacontahedron surround its axis and 24 internal elements surround the axis of the triakis tetrahedron. Table 3 shows that, when their faces are single tetractyses (case C), the triakis tetrahedron is built from 30 tetractyses with 24 edges and the disdyakis triacontahedron is constructed from 300 tetractyses with 240 edges. For case B, the total number of geometrical elements surrounding their axes are, respectively, 240 and 2400. The counterpart of this pattern in the heterotic superstring shown in Fig. 2 are the 240 gauge charges corresponding to the 240 roots of the gauge symmetry group E_8 that are spread around each of the ten whorls, 24 to a whorl. The disdyakis triacontahedron is the polyhedral counterpart of the heterotic superstring and the triakis tetrahedron is the polyhedral counterpart of a whorl, its 24 *shape-determining* edges surrounding its axis being analogous to the 24 gauge charges carried by each 1-dimensional whorl.

This analogy can be taken further to the very dimensions of space-time. Table 3 indicates that the triakis tetrahedron has 26 edges, showing how it is prescribed by Yahweh with number value 26. Two of these edges form the central axis, leaving 24 edges surrounding it. The former correspond to the longitudinal spatial dimension (distance measured along the length of a string) and to the time dimension, whilst the latter are the counterpart of the 24 dimensions of 26-dimensional space-time that are transverse to the string. The number 26 is determined by the number 24 as

$$26 = \frac{2^2 + 4^2 + 6^2 + \ldots + 24^2}{1^3 + 2^3 + 3^3 + 4^3} .$$

Appropriately, the Godname Adonai of Malkuth (the *physical* aspect of the Tree of Life) prescribes the triakis tetrahedron as the polyhedral form of the fundamental building block of matter — the whorl — because its number value 65 is the total number of geometrical elements from which this Catalan solid is built (see Table 3). The

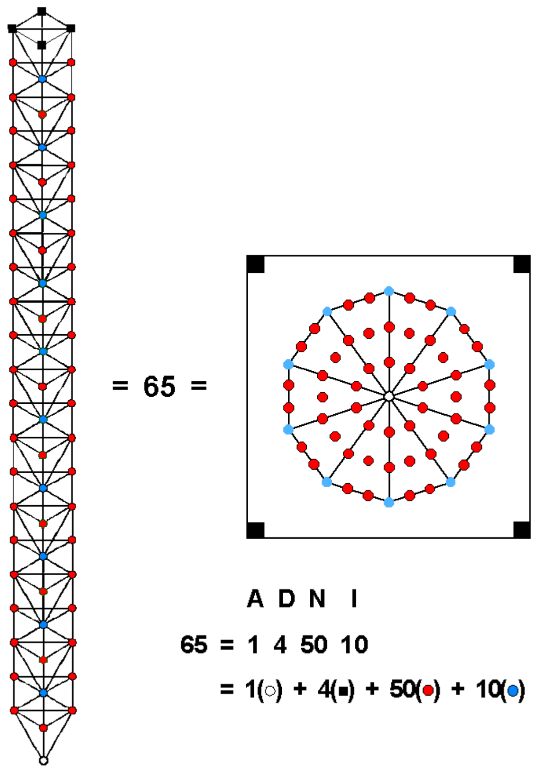
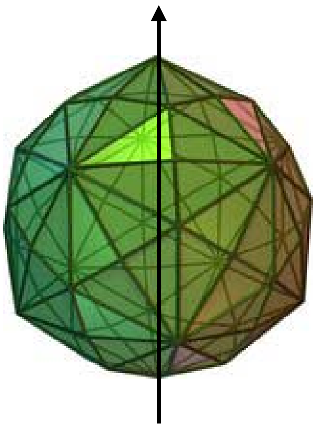


Figure 5. Equivalence of the 10-tree prescribed by the Hebrew Godname ADONAI and its tetractys representation. Its number value 65 denotes the number of Sephirothic emanations belonging to these trees.

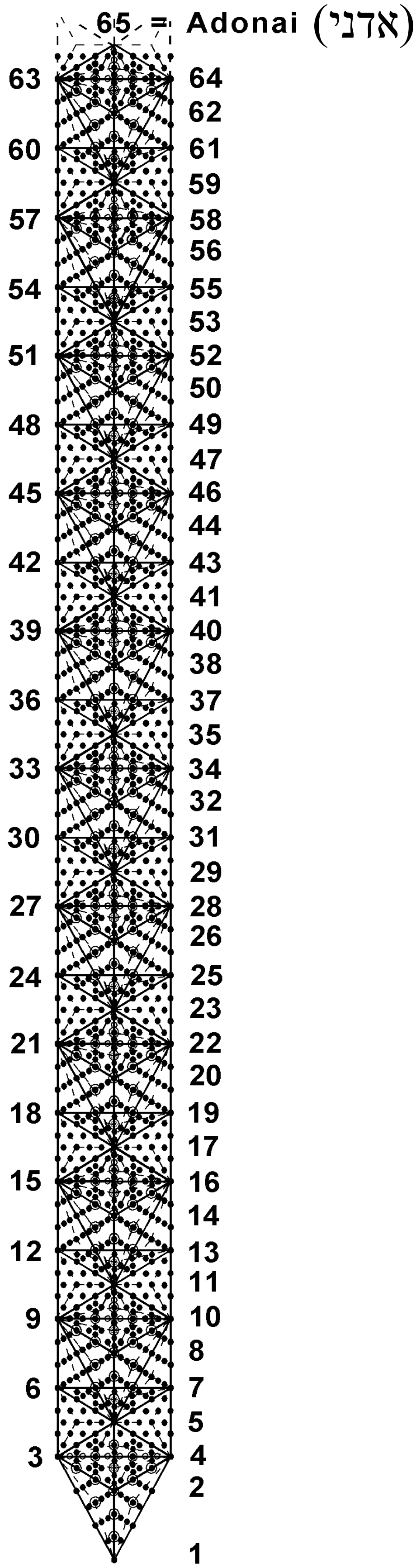
counterpart of this in the Tree of Life is the fact that there are 65 Sephirothic emanations in the lowest ten Trees of Life belonging to CTOL (Fig. 5). Below their top are 1680 yods contained in 385 tetractyses (Fig. 6), where

$$385 = \begin{matrix} 1^2 \\ 2^2 \;\; 3^2 \\ 4^2 \;\; 5^2 \;\; 6^2 \\ 7^2 \;\; 8^2 \;\; 9^2 \;\; 10^2 \end{matrix} .$$

This demonstrates how Adonai prescribes both the ten trees representing the 10-dimensional space-time of superstrings and the 1680 circularly polarised oscillations of each whorl of the heterotic superstring, their counterpart in the disdyakis



= 1680 =



240 vertices
780 edges
660 triangles

Figure 6. The number of yods below the top (65th SL) of the tenth Tree of Life prescribed by ADONAI, the Godname assigned to Malkuth, is 1680. Each yod denotes a circularly polarised oscillation in the helical whorl of the $E_8 \times E_8$ heterotic superstring, described 110 years ago by Annie Besant and C.W. Leadbeater with a yogic siddhi called 'anima.' It also symbolizes a geometrical element in the disdyakis triacontahedron that surrounds its axis.

triacontahedron being the 1680 geometrical elements surrounding its axis in case A.

The disdyakis triacontahedron in case C has 240 edges surrounding its axis. They are symbolized in the lowest Tree of Life in CTOL as the 240 extra yods generated by conversion of each of its 19 triangles into three tetractyses (Fig. 7). The number 240 expresses in both contexts the number of degrees of freedom needed to form the

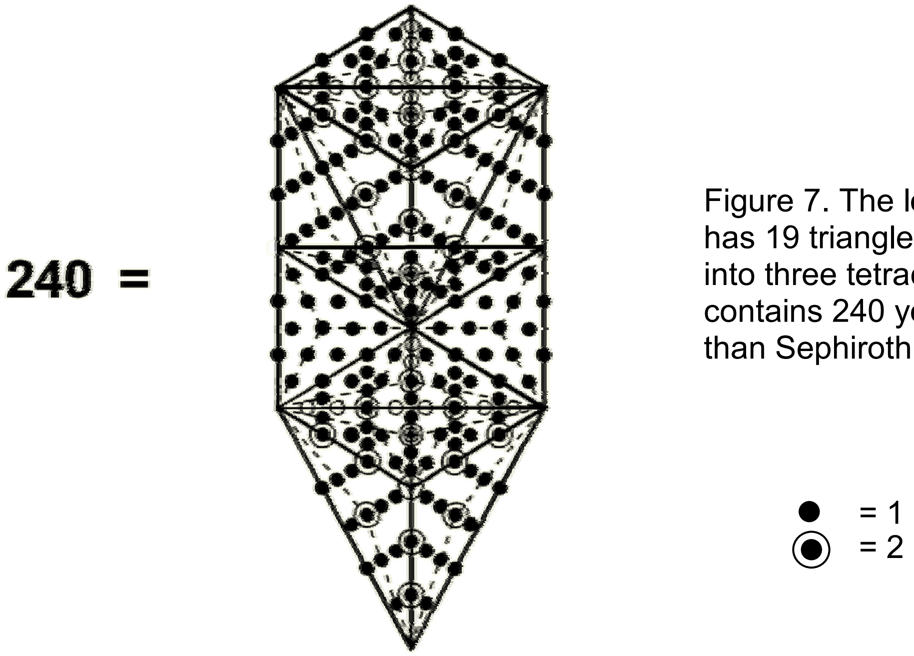


Figure 7. The lowest tree has 19 triangles divided into three tetractyses. It contains 240 yods other than Sephirothic points.

object, given in the former its essential 11 Sephirothic points and in the latter the axis of the polyhedron made up of five geometrical elements. Each of the 300 triangles in the disdyakis triacontahedron with 240 edges and 60 vertices surrounding the axis has a yod at its centre. Each edge has two hexagonal yods between its ends. The number of yods surrounding the axis of the disdyakis triacontahedron in case C = 300 + 60 + 240×2 = 840 = 84×10, where

$$84 = 1^2 + 3^2 + 5^2 + 7^2.$$

This demonstrates how the Tetrad Principle determines the yod population of this polyhedron as the number of tetractyses with the same number of yods.

The next level of differentiation of the tetractys — the 2nd-order tetractys shown in Fig. 8 — has 84 yods surrounding its centre. The meaning of this is that a holistic system patterned or structured according to the Tree of Life blueprint needs 85 degrees of

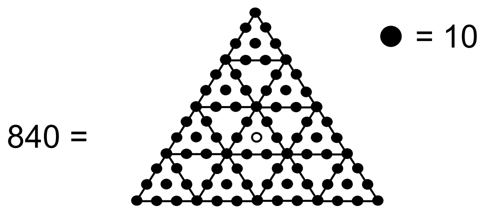


Figure 8. Weighted with the Decad (10), the 84 yods surrounding the centre of a tetractys array of tetractyses sum to 840. This is the number of yods that surround the axis of the disdyakis triacontahedron.

freedom to manifest. We saw in Article 27[8] that the polyhedral example of this is the tetrahedron, which in case B requires 30 tetractyses with 40 edges and 15 vertices, that is, 85 geometrical elements. The 15 yods at corners of the 10 tetractyses in the 2nd-order tetractys symbolize the 15 vertices and the 70 hexagonal yods symbolize the 30 tetractyses and 40 edges. Weighted with the number 10 (the Pythagorean Decad and measure of wholeness), the 84 yods surrounding the centre of the 2nd-order tetractys generate the number 840, which is the number of yods surrounding the axis of the disdyakis triacontahedron when its interior triangles and faces are turned into tetractyses. The yod at its centre symbolizes this axis.

This number manifests in the heterotic superstring as the 840 circular turns made by each helical whorl as it winds in an outer circuit of 2½ revolutions, making the same number of turns as it returns to its starting point after winding another 2½ times around the axis of the superstring (Fig. 2). As

$$29^2 – 1 = 840 = 3 + 5 + 7 + \ldots + 57,$$

the number 840 is the sum of the first 28 odd integers after 1. This means that 8400 can be thought of as the sum of ten identical sets of 28 odd integers, that is, 280 odd integers. 8400 is the number of circular turns in the ten whorls as they wind 2½ times in the inner or outer half of the superstring. The number value 280 of Sandalphon, Archangel of Malkuth, determines the number of turns in 2½ revolutions of all ten whorls and the number value 168 of Cholem Yesodeth, Mundane Chakra of Malkuth, is the number of turns in a half-revolution of each whorl. As

$$41^2 – 1 = 1680 = 3 + 5 +7 +\ldots + 81,$$

1680 is the sum of the first 40 odd integers after 1. As

$$13^2 – 1 = 168 = 3 + 5 + 7 +\ldots + 25,$$

168 is the sum of the first 12 odd integers after 1. These properties allow the number 1680 to be represented as a tetractys array of quartets of successive odd integers:

[3 5 / 7 9]
[11 13 / 15 17] [19 21 / 23 25] } 168
[27 29 / 31 33] [35 37 / 39 41] [43 45 / 47 49] } 840
[51 53 / 55 57] [59 61 / 63 65] [67 69 / 71 73] [75 77 / 79 81] } 1680

The sum of the quartet of integers at its apex is 24. This is the number of gauge charges of the superstring gauge symmetry group E_8 that are spread around each whorl of the heterotic superstring. The sum of the first three quartets of integers is 168, the number of turns in a half-revolution of a whorl, and the sum of the first seven quartets is 840, the number of turns in 2½ revolutions, that is, in half a whorl. The sum of all 10 quartets is 1680, which is the number of turns in a whorl. Such a beautiful property of the odd integers connecting them to the structure of the superstring cannot be due to coincidence. Rather, it serves to demonstrate the power of the Pythagorean insight expressed in the phrase “form is number.”

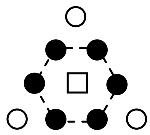
29 is the tenth prime number in the tetractys array of such numbers:

2
3 5
7 11 13
17 19 23 29 .

This illustrates once again the power of the tetractys to generate numbers that in turn determine other numbers of universal significance, such as the superstring structural parameter 1680 and the number 240 of gauge charges corresponding to the 240 roots of the gauge symmetry group E_8:

$$240 = \begin{array}{c} 24 \\ 24\;\;24 \\ 24\;\;24\;\;24 \\ 24\;\;24\;\;24\;\;24 \end{array} .$$

Indeed, the $E_8 \times E_8$ heterotic superstring is the physical manifestation of this Pythagorean symbol of holistic systems, each of its ten whorls carrying 24 gauge charges associated with these roots. The superstring exists in 10-dimensional space-time, which is symbolized by a tetractys. The three yods (○) at the corners denote the three large-scale dimensions of space, the six yods (●) at the corners of a hexagon symbolize the six compactified dimensions and the yod (□) at its centre denotes the dimension of time.



Truly, one can now understand the meaning of the oath sworn by the followers of Pythagoras to their master:

> "I swear by the discoverer of the tetractys,
> Which is the spring of all our wisdom,
> The perennial fount and root of Nature."

References

[1] Phillips, Stephen M. Article 5: "The Superstring as the Microcosm of the Spiritual Macrocosm," http://www.smphillips.8m.com.

[2] The number of Sephirothic emanations in n overlapping Trees of Life ≡ N(n) = 6n + 4. N(91) = 550.

[3] Phillips, Stephen M. Article 3: "The Sacred Geometry of the Platonic Solids," http://www.smphillips.8m.com, p. 11.

[4] Ibid., p. 10.

[5] Phillips, Stephen M. Article 27: "How the Disdyakis Triacontahedron Embodies the Structural Parameter 1680 of the $E_8 \times E_8$ Heterotic Superstring." http://www.smphillips.8m.com, p. 18.

[6] Besant, Annie and Leadbeater, C.W., "Occult Chemistry," Theosophical Publishing House, Adyar, Chennai. India, 1951.

[7] Phillips, Stephen M. Article 22: "The '120 Polyhedron' as the 3-dimensional Counterpart of the Inner Tree of Life;" Article 23: "The '120 Polyhedron' and the '144 Polyhedron' as the Exterior and Interior of the Inner Tree of Life;" Article 24: "More Evidence for the '120 Polyhedron' as the 3-dimensional Realisation of the Inner Tree of Life and its Manifestation in the $E_8 \times E_8$ Heterotic Superstring."

[8] Ref. 5, pp. 2-3.