MISSILE GUIDANCE

Missile guidance addresses the problem of steering, or guiding, a missile to a target on the basis of *a priori* known target coordinate information and/or real-time target measurements obtained from onboard and/or external sensors.

A BRIEF HISTORY: FROM 1944 TO THE PRESENT

The Missile Age

Even before World War I—when powered flight was in its first decade—forward-thinking individuals from several countries advocated the use of unmanned vehicles to deliver high-explosive weapons from afar. Although the earliest efforts to develop a practical flying bomb were undertaken in the United States and Great Britain, it was in Germany that a workable concept finally emerged. After 14 years of intense research, the Germans ushered in the missile age during World War II with their Vengeance weapons: the Luftwaffe-developed V-1 buzz bomb and the Army-developed V-2 rocket (1).

- **V-1 Buzz Bomb.** Powered by a pulse-jet engine, generating 2670 N (600 pounds) of thrust, the V-1 reached a speed of 322 km per hour (200 miles per hour) and had a range of about 241 km (150 miles). Weighing 21,138 N (4750 pounds) with an 8900 N (2000 pound) high-explosive warhead, the V-1 was launched from a long ramp with the aid of a hydrogen peroxide/potassium permanganate-propelled booster motor. A gyroscope, magnetic compass, and a barometric altimeter were used to correct deviations in altitude and direction. Despite its 0.8045 km (0.5 mile) accuracy, the V-1 proved very useful as a terror weapon against large cities. Near impact, the control surfaces would lock and spoilers would be deployed from the tail to induce a steep dive. At this point, the pulse-jet usually ceased functioning. The eerie silence that followed warned people below of the impending impact. The V-1 was launched by the thousands against London and the Belgian port of Antwerp during 1944, 1945. Well over 10,000 V-1s were launched against Great Britain, in all kinds of weather, by day and night. Although Royal Air Force pilots had some success in shooting down V-1s, the V-1s proved effective as terror weapons.
- **V-2 Rocket.** The V-2, which was developed at the secret Peenemünde rocket center, was first used in combat on September 6, 1944. Fueled by an alcohol–liquid–oxygen propellant generating 244,750 N (55,000 pounds) of thrust for about 1-minute after launch, the V-2 had a range of about 322 km (200 miles). After a 1 minute powered flight and reaching an altitude of about 113 km (70 miles), the V-2 flew an arcing free-falling (ballistic) trajectory at speeds in excess of 1.609 km/s (1 mile per second)—carrying a 7,120 N (1,600 pound) warhead. Between September 1944 and March 1945, from mobile field battery positions in France and Holland, German field units launched over 3000 V-2 missiles against Allied targets on the European continent, primarily Antwerp, Belgium, London, and Southern England. During the late 1940s and early 1950s, the U.S. Army, under Project Hermes, launched over 70 V-2s. The V-2 would became the prototype for future U. S. and Soviet rocket and strategic ballistic missile program developments.

Lark-Guided Missile

Because of the lack of success of anti-aircraft artillery in stopping Kamikaze aircraft attacks against naval vessels, the U.S. Navy initiated the development of the Lark guided missile in 1944. The first successful intercept of an unmanned aircraft occurred six years later on December 2, 1950. An account of this, as well as the development of other missiles (e.g., Sparrow and Hawk), is provided in Reference 2.

The First Ballistic Missiles

After World War II, significant improvements in inertial guidance system technology led to the Redstone missile—the first short-range U.S. ballistic missile with a highly accurate inertial guidance system. Additional progress was made with the medium-range U.S. Jupiter missile (3).

ICBMs

Additional advancements in the area of nuclear warhead design, inertial guidance system, and booster engine technology led to the development of the intercontinental ballistic missile (ICBM). The first U.S. ICBM—the Atlas—was tested in 1959. The Atlas would be used to launch satellites into orbit, launch probes to the moon and other planets, and to launch the Mercury spacecraft into orbit around the Earth. The Atlas was followed by the Titan one year later. Both Atlas and Titan were liquid-fuelled multistage rockets that needed to be fuelled just before launch. In 1961, the Minuteman ICBM was put into service. Located within dispersed hardened silos, the Minuteman used a solid propellant stored within the missile. The LGM-30 Minuteman III was deployed in 1970. This system was designed such that specially configured EC-135 airborne launch control aircraft could automatically assume command and control of an isolated missile or missiles in the event that command capability is lost between the launch control center and the remote missile launch facilities. In 1986, the LGM-118A Peacekeeper was deployed. This three-stage solid propellant system permits 10 warheads to be carried via multiple reentry independently targeted vehicles (MIRVs). At the peak of the Cold War, the Soviet Union possessed nearly 8,000 nuclear warheads on ICBMs. During the Cold war, the United States built up its strategic defense arsenal, focusing on a nuclear triad consisting of 1) long-range bombers (B-52 bombers and KC-135 tankers) with nuclear air-to-surface missiles, 2) U.S.-based ICBMs, and 3) submarine-launched ballistic missiles (SLBM) launched from nuclear-powered submarines. To complement the ground-based leg of the triad, the U.S. Navy would develop the submarine-launched Polaris, Poseidon, and Trident ICBMs. Trident I and II were

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deployed in 1979 and 1988, respectively. Both accommodate nuclear MIRVs and are deployed in Ohio-class (Trident) submarines, each carrying 24 missiles with eight 100 kiloton warheads per missile. Trident II missiles weigh roughly 65 tons and are about 44 feet long and 7 feet wide. For comparison sake, it is worth noting that the bomb dropped on Hiroshima on August 6, 1945 (designated “Little Boy”) was a 8,900 lb, 10 feet long, 2.33 feet diameter, 13–16 kiloton uranium-235 based gun-type fission weapon. Similarly, the bomb dropped on Nagasaki three days later (designated “Fat Man”) was a 10,800 lb, 10.67 feet long, 5 feet diameter, 21 kiloton plutonium-239 based implosion-type fission weapon.

Nuclear Non-Proliferation: SALT, ABM, and MAD

The first major Nuclear Non-Proliferation Treaty (NNPT) opened for signature on July 1, 1968. In addition to addressing what nations could “rightfully” possess nuclear weapons and relevant nuclear proliferation issues, it addressed disarmament and stockpile reduction as well as the peaceful use of nuclear technology (i.e., energy generation). The treaty is revisited periodically by participating states. Because of the large number of Soviet nuclear warheads during the Cold War, some in the United States felt that U.S. ICBM fields were threatened. On March 14, 1969, President Nixon announced his decision to deploy a missile defense system (called Safeguard) to protect U.S. ICBM fields from attack by Soviet missiles. This decision initiated intense strategic arms negotiations between the United States and the Soviet Union. The Strategic Arms Limitation Talks (SALT), between the United States and the Soviet Union, led to a 1971 agreement fixing the number of ICBMs that could be deployed by the two nations. The Antiballistic Missile (ABM) Treaty—signed by the U.S. and the Soviet Union on May 26, 1972—was designed to implement the doctrine of mutually assured destruction (MAD). MAD was intended to discourage the launching of a first strike by the certainty of being destroyed by retaliation. The treaty prohibits/limits deployment of certain sea, air, and space-based missiles and sensors. A key motivation behind these arrangements was to perpetuate the existing balance of power and avoid the economic chaos that would result from a full-scale arms race. In 1976, in view of technical limitations imposed by the ABM treaty, the U.S. Congress ordered the closing of Safeguard only four months after becoming operational. In 2001, the ABM treaty came under attack in the U.S. Congress as the United States and Russia (former Soviet Union) discussed how to differentiate between theater and strategic missile defenses.

BMD and SDI

In 1983, President Reagan initiated the Ballistic Missile Defense (BMD) program under the Strategic Defense Initiative (SDI). SDI would focus on space-based defense research. Because SDI deployment would contravene the ABM treaty, many critics felt SDI, with its potential offensive use, would escalate the arms race. In 1984, the Strategic Defense Initiative Organization (SDIO) was formed. In 1987, Judge Abraham D. Sofaer, State Department Legal Advisor, concluded that the ABM treaty did not preclude testing of space-based missile defense systems, including directed energy weapons. SDI research would continue. With the breakup of the Soviet Union in 1991, the need for great nuclear arsenals came into question. In 1993, the SDIO was replaced by the Ballistic Missile Defense Organization (BMDO). The national objectives of SDI were replaced by regional objectives. In 1998, emphassis shifted back to national missile defense. In 2002, BMDO was renamed the Missile Defense Agency (MDA).

Strategic Arms Reduction Treaties

In November 1994, the Strategic Arms Reduction Treaty I (START I) became effective, with the United States, Russia, Belarus, Kazakstan, and Ukraine agreeing to reduce nuclear warheads by 25%. In appreciation for the ratification, the United States appropriated $1.5 billion for assistance in dismantling nuclear weapons, properly storing weapons grade materials, and turning military factories into civilian buildings. Initiated in 1992, START II called for the removal of MIRVs and of almost three quarters of nuclear warheads over nine years, thereby reducing the U. S. and Russian arsenals to 3000–3500 strategic warheads. The U.S. Senate approved ratification on January 26, 1996, but the Russian Duma never ratified the treaty. Multiple warhead Peacekeepers were to be eliminated by 2003 under START II. On June 13, 2002, the United States withdrew from the 1972 ABM treaty. The Russian Federation followed suit by withdrawing from START II negotiations the next day. The Treaty of Moscow, also referred to as the Strategic Offensive Reductions Treaty (SORT), was signed by Presidents George W. Bush and Vladimir Putin in 2002 and took effect in 2003. SORT promises to reduce the number of operationally deployed warheads from 6000 to 2200 by 2012.

Missile Warning Systems

Although the United States has no active ABM defense system in place, an extensive warning system has been in place for many years. Air and space defense is delegated to the North American Aerospace Defense Command (NORAD)—a joint U.S.—Canadian organization that was initially founded May 12, 1958 as the North American Air Defense Command and adopted its current name in 1981. A Ballistic Missile Early Warning System (BMEWS) consisting of warning and tracking radars in Alaska, Greenland, and the United Kingdom can detect missiles 4800 km ($\sim$3000 miles) away and provides a 15 minute warning of an attack on North America. A Perimeter Acquisition Radar Characterization System (PARCS), operating within the U. S. interior, tracks incoming warheads, and determines impact areas. Phased-array radar antennas along the U.S. Atlantic, Pacific, Alaskan, and Gulf coasts provide warning of SLBM launches. With the collapse of the USSR in 1991 and the terrorist attacks on the United States of September 11, 2001, the NORAD mission has shifted considerably to the monitoring of all aircraft flying within the interior of the United States.

Persian Gulf War

In January 1991, the role of air power in modern warfare was dramatically demonstrated during the Persian Gulf War. Initial attacks by the U.S.-led multinational coalition were designed to suppress Iraqi air defenses. These attacks included Tomahawk cruise missiles launched from warships in the Persian Gulf, F-117A Stealth fighter-bombers armed with laser-guided smart bombs, and F-4G Wild Weasel aircraft carrying HARM anti-radar missiles. These attacks permitted F-14, F-15, F-16, and F/A-18 fighter bombers to achieve air superiority and to drop TV- and laser-guided precision bombs. During the ground war, A-10 Thunderbolts with armor-piercing, heat-seeking, or optically guided AGM-65 Maverick missiles provided support for ground units. The AH-64 Apache and AH-1 Cobra helicopters fired laser-guided Hellfire missiles, guided to tanks by ground observers or scout helicopters. The E-3A Airborne Warning and Control System (AWACS), a flying radar system, provided targeting information to coalition members.

Missile Defense

Although most weapon systems performed superbly during the Gulf War, little could be done to stop the Iraqi Scuds launched against Saudi Arabia and Israel. However, a Patriot surface-to-air missile (SAM) system was brought in to repel Scud attacks. Although the Patriot system had been used in 1987 to destroy another Patriot during a demonstration flight, the system was originally designed as an anti-aircraft defense system. Thus, its effectiveness against the Scuds was limited, primarily because intercepts often did not take place at sufficiently high altitudes. Part of the problem was attributed to the fact that the Patriot relied on proximity detonation rather than a hit-to-kill, which would often cause the incoming Scud to break up, leaving a free-falling warhead to detonate on the civilian population below. The many Patriot–Scud engagements were televised to a world audience and demonstrated the need for a high-altitude air defense system that could intercept (tactical) ballistic missiles far from critical military assets and civilian population centers. For this reason, much research shifted toward the development of hit-to-kill Theater High Altitude Air Defense (THAAD) systems that would focus on incoming targets situated within a 200 km ($\sim$124 miles) range and no higher than 150 km ($\sim$93 miles). In his January 1991 State of the Union Address, President George H. W. Bush formally announced a shift in SDI to a concept of Global Protection Against Limited Strikes (GPALS), and by December, he signed into law the Missile Defense Act of 1991. On January 24, 1997, a Standard Missile 2 (SM2) Block IVA successfully intercepted and destroyed a Lance missile at the White Sands Missile Range in New Mexico. During the test, the SM2 successfully transitioned from radar mid-course guidance to its heat-seeking endgame/terminal guidance system prior to destroying the target with its blast fragmentation warhead. On February 7, 1997, BMDO carried out a test in which a Patriot Advanced Capability-2 (PAC-2) missile successfully intercepted a theater ballistic target missile over the Pacific Ocean. In April 1997, BMDO established the Joint Program Office (JPO) for the National Missile Defense (NMD). On June 24, 1997, the first NMD flight test was successfully completed. During this test an Exoatmospheric Kill Vehicle (EKV) sensor was used to identify and track objects in space. In 2007, Lockheed Martin is expected to begin flight testing of a THAAD system at the Pacific Missile Range (Kauai, Hawaii). To appreciate the formidable problems associated with developing a THAAD system, it is necessary to understand the issues associated with the design of missile guidance systems. These issues will be addressed in subsequent sections.

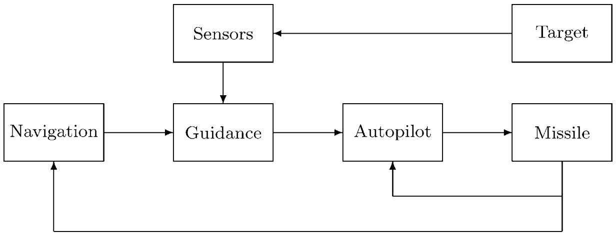

Figure 1. Information flow for missile-target engagements.

MISSILE GUIDANCE, NAVIGATION, AND CONTROL SUBSYSTEMS

We begin our technical discussion by describing the subsystems that make up a missile system. In addition to a warhead, a missile contains several key supporting subsystems. These subsystems may include 1) a target-sensing system, 2) a missile-navigation system, 3) a guidance system, 4) an autopilot or control system, and 5) the physical missile (including airframe and actuation subsystem); see Fig. 1.

Target-Sensing System

The target-sensing system provides target "information" to the missile guidance system, e.g. relative position, velocity, line-of-sight angle, and rate. Target-sensing systems may be based on several sensors, e.g., radar, laser, heat, acoustic, or optical sensors. Optical sensors, for example, may be as simple as a camera for a weapon systems officer (WSO) to visualize the target from a remote location. They may be a sophisticated imaging system (see below). For some applications, target coordinates are known *a priori* (e.g., via satellite or other intelligence) and a target sensor becomes irrelevant.

Navigation System

A navigation system provides information to the missile guidance system about the missile position in space relative to some inertial frame of reference, e.g., flat-Earth constant-gravity model for short-range flights and rotating-Earth variable-gravity model for long-range flights. To do so, it may use information obtained from a variety of sensors, which may include simple sensors such as accelerometers or a radar altimeter. It may include more sophisticated sensors such as a global positioning system (GPS) receiver or an optical terrain sensor that relies on comparisons between an image of the terrain below with a stored image and a stored desired trajectory. Optical stellar sensors rely on comparisons between an image of the stars above with a stored image and a stored desired trajectory.

Guidance System

Target and missile information are used by the guidance system to compute updated guidance commands, which when issued to the missile autopilot should ideally guide (or steer) the missile toward the target (4, 5). When target coordinates are known *a priori*, missile coordinates provided by the navigation system (e.g., GPS-based) are periodically compared with the (pre-programmed) target coordinates to compute appropriate guidance corrections. In general, the quality of the computed guidance commands depends on the quality of the gathered sensor data and the fidelity of the mathematical models used for the missile and target. Targets may be stationary, mobile, or highly maneuverable (e.g., silo, ship, fighter aircraft). Physically, guidance commands may represent quantities such as desired thrust, desired (pitch/yaw) acceleration, desired speed, desired flight path or roll angle, and desired altitude. Guidance commands issued by the guidance system to the missile autopilot are analogous to the speed commands issued by automobile drivers to the cruise control systems in their cars. In this sense, the missile guidance system is like the automobile driver and the missile autopilot is like the automobile cruise control system. Missile guidance commands are computed in accordance with a guidance algorithm. Guidance algorithms and navigational aids will be discussed below.

Autopilot

The primary function of the autopilot—sometimes referred to as the flight control system (FCS) or attitude control system (ACS)—is to ensure 1) missile attitude stability and 2) that commands issued by the guidance system are followed as closely as possible (4). The autopilot accomplishes this command-following objective by computing and issuing appropriate control commands to the missile's actuators. These actuators may include, for example, rocket thrusters, ramjets, scramjets (for hypersonic missiles), or servomotors that move aerodynamic control surfaces. More specifically, the autopilot compares commands issued by the guidance system with real-time measurements (e.g., acceleration, attitude and attitude rate, and altitude) obtained from onboard sensors (e.g., accelerometers, gyroscopes, and radar altimeters) and/or external tracking systems. This comparison, essentially a subtraction of signals, produces a feedback error signal, which is then used to compute control commands for the missile actuators. This computation, the purpose of the autopilot, may be based on a and is based on the autopilot design and hence its complexity. Autopilot design, however, is based on a very complex mathematical model that captures the following dynamical features: missile airframe, aerodynamics (depending on speed, dynamic pressure, angle-of-attack, slide-slip angle, etc.), actuators, sensors, flexible modes, and uncertainty descriptions, e.g., dynamic uncertainty, parametric uncertainty (6, 7), and disturbance/noise bounds. It should be noted that commands that are issued by the guidance system to the autopilot cannot always be followed exactly because of the presence of multiple sources of uncertainty. Sources of uncertainty may include disturbances acting on the missile, sensor noise, unmodeled or uncertain missile airframe, actuator, and sensor dynamics.

Flight Phases

The flight of a missile can be broken into three phases: 1) a launch, separation, or boost phase; 2) a mid-course or cruise phase; and 3) an endgame or terminal phase. During each phase, a missile may use distinct guidance, navigation, and control systems, specifically designed to accommodate the requirements during that phase of the flight. During each phase, the missile may very well use different sets of sensors, actuators, and power sources.

Guidance System Performance Terminology

To describe the function and performance of a guidance system, some terminology is essential. The imaginary line that connects a missile center-of-gravity (cg) to the target cg is referred to as the line-of-sight (8). The length of this line is called the range. The associated vector from missile to target is referred to as the range vector. The time derivative of the range vector is called the closing velocity. The most important measure of performance for any missile guidance system is the so-called miss distance. Miss distance is defined to be the missile-target range at that instant when the two are closest to one another (8). The objective of most guidance systems is to minimize the miss distance within an allotted time period. For some applications (hit-to-kill), zero miss distance is essential. For some applications (e.g., to minimize collateral damage), it is essential to impact the target at a specific angle. Because miss distance is sensitive to many variables and small variations from missile to missile, other quantities are used to measure performance. One of the most common measures used is circular error probability (cep). The cep for a missile attempts to provide an average miss distance for a class of missile-target engagements (i.e., Monte Carlo runs). If a missile has a cep of 10 m, then most of the time, say, 68% of the time, it will detonate within 10 m of the target.

CLASSIFICATION OF MISSILES, TARGETS, GUIDANCE SYSTEMS, NAVIGATION METHODS, AND TARGET-SENSING METHODS

The guidance system used by a missile depends on the intended use of the missile. Missiles are classified according to many categories. The most commonly used classifications are as follows: strategic, tactical, exoatmospheric, endoatmospheric, aerodynamic, ballistic, surface-to-surface, surface-to-air, air-to-surface, air-to-air, inertially guided, terrain guided, stellar guided, satellite guided, passive; active, homing, command guided, radar guided, laser guided, heat seeking, fire-and-forget, line-of-sight guided, radar terrain guided, TV guided, cruise, skid-to-turn (STD), and bank-to-turn (BTT). Each category is now briefly discussed.

Strategic Missiles

Strategic missiles are used primarily against strategic targets, that is, resources that permit an enemy to conduct large-scale military operations (e.g., battle management/command, control, and communication centers; industrial/weapons manufacturing centers; and so on). Such targets are usually located far behind the battle line. As such, strategic missiles are typically designed for long-range missions. Although such missiles are usually launched from naval vessels or from missile silos situated below ground, they are sometimes launched from aircraft (e.g., strategic bombers). Because such missiles are intended to eliminate the most significant military targets, they typically carry nuclear warheads rather than conventional warheads. Strategic missiles typically operate at orbital speeds ($\sim$5 miles per second), outside the atmosphere, and over intercontinental distances. They use rockets/thrusters/fuel and require very precise instrumentation for critical mid-course guidance. GPS has made such systems very accurate.

Tactical Missiles

Tactical missiles are used primarily against tactical targets, that is, resources that permit an enemy to conduct small-scale military operations (for example, a ship, an airfield, and a munitions bunker). Such targets are usually located near the battle line. As such, tactical missiles are typically designed for short- or medium-range missions. Such missiles have generally carried conventional explosive warheads, the size of which depends on the designated target. Tactical missiles sometimes carry nuclear warheads in an effort to deter the use of tactical nuclear/chemical/biological weapons and to engage the most hardened targets (e.g., enemy nuclear strategic missile silos). Tactical missiles typically operate at lower speeds ($<$ 1 mile per second), inside the atmosphere, and over short-to-medium distances (e.g., 150 miles). They typically use aerodynamic control surfaces (discussed below) and require adequate instrumentation for mid-course and terminal guidance. A target sensor (e.g., radar seeker) permits such missiles to engage mobile and highly maneuverable targets.

Exoatmospheric Missiles

Exoatmospheric missiles fly their missions mostly outside the Earth's atmosphere. Such missiles are used against long-range strategic targets. Because they fly outside the atmosphere, thrusters are required to change direction. Such thrusters use onboard fuel. To maximize warhead size, and because missile weight grows exponentially with fuel weight, it is important that guidance and control systems for long-range missiles (e.g., strategic and exoatmospheric) provide for minimum fuel consumption.

Endoatmospheric Missiles

Endoatmospheric missiles fly their missions inside the Earth's atmosphere. Such missiles are used against strategic and tactical targets. In contrast to exoatmospheric missiles, endoatmospheric missiles may use movable control surfaces such as fins (called aerodynamic control surfaces), which deflect air flow in order to alter the missile flight path. In such a case, the missile is called an aerodynamic missile. Endoatmospheric missiles may, in some cases, rely entirely on rocket power. In such a case, they are not aerodynamic. Exoatmospheric missiles that fly outside the Earth's atmosphere rely on rocket power and thrusters. These missiles are not aerodynamic. Examples of aerodynamic missiles are the Sidewinder and Patriot.

Ballistic Missiles

Ballistic missiles assume a free-falling (unpowered) trajectory after an internally guided, self-powered (boost and mid-course) ascent. Such missiles are usually used against long-range strategic targets. ICBMs, for example, are usually exoatmospheric strategic missiles that were developed for use against strategic targets and are typically launched from underground missile silos and submarines. Modern ICBMs contain multiple independently targeted nuclear warheads deployed via MIRVs. Examples of ICBMs are the Atlas, Titan, Minuteman, Polaris, Peacekeeper, and Trident. The Iraqi Scud, used in the Persian Gulf War, is another ballistic missile.

Surface-to-Surface Missiles (SSMs)

SSMs are typically launched from the ground, beneath the ground (e.g., from a missile silo), or from naval platforms against ground targets (e.g., tank, munitions depot, and missile silo) or naval targets (e.g., battleship and submarine). ICBMs are typically SSMs. SSMs may carry nuclear, biological, chemical, or conventional warheads. Examples of SSMs are the anti-ship Silkworm and the Tomahawk.

Surface-to-Air Missiles (SAMs)

SAMs are typically launched from the ground, beneath the ground (e.g., from a missile silo), or from naval platforms against aircraft and missiles. SAMs were developed to defend surface targets from air attacks, especially from high-altitude bombers flying well above the range of conventional anti-aircraft artillery (AAA). Most air defense SAMs employ separate radars to acquire (detect) and track enemy air threats. The separate radar is also used to guide the SAM toward the hostile target; endgame guidance may be accomplished by the missile's onboard guidance system. SSMs are typically heavier and carry larger warheads than SAMs because they are usually intended to penetrate hardened (e.g., armored) targets. Shoulder launched SAMs (e.g., Stinger) have recently become a major concern given increased terrorist activities.

Air-to-Surface Missiles (ASMs)

ASMs are launched from aircraft against ground targets (e.g., a bridge or airfield) or naval targets. Although ASMs are typically intended for tactical targets, they are used by both strategic and tactical bombers. Equipping strategic bombers with long-range ASMs extends their range, significantly reducing the range that they need to travel toward the intended target. Examples of ASMs are the anti-tank Hawk and Hellfire, the anti-radar AGM-88 HARM, the anti-ship Exocet and AGM-84D HARPOON, and the anti-armored vehicle AGM-65 Maverick. Other ASM systems include the Advanced Medium-Range Air-to-Air Missile (AIM-120 AMRAAM) and the airborne laser (ABL) system being developed by several defense contractors. The ABL system has been considered for boost-phase intercepts during which the launched missile has the largest thermal signature and is traveling at its slowest speed.

Air-to-Air Missiles (AAMs)

AAMs are launched from aircraft against aircraft, ballistic missiles, and most recently against tactical missiles. Such missiles are typically light, highly maneuverable, tactical weapons. AAMs are generally smaller, lighter, and faster than ASMs because ASMs are typically directed at hardened, less-mobile, targets. Some SAMs and ASMs are used as AAMs and vice versa. Examples of AAMs are the AIM-7 Sparrow, AIM-9 Sidewinder, AIM-54 Phoenix, and the AIM-120A AMRAAM.

Guidance Methods: Fixed Targets with Known Fixed Positions

A missile may be guided toward a target having a known fixed position using a variety of guidance methods and/or navigational aids, e.g., inertial, terrain, stellar, and satellite guidance and navigation.

- **Inertially Guided Missiles.** Inertially guided missiles use missile spatial navigation information relative to some inertial frame of reference to guide a missile to its designated target. For short-range missions, one may use a flat-Earth constant-gravity inertial frame of reference. This approach is not appropriate for long-range missions, approaching intercontinental distances, for which the Earth may not be treated as flat. For such missions, the sun or stars provide an inertial frame of reference. One can also use an Earth-centered variable-gravity frame. Position information is typically obtained by integrating acceleration information obtained from accelerometers or by pattern-matching algorithms exploiting imaging systems. Because accelerometers are sensitive to gravity, they must be mounted in a fixed position with respect to gravity. Typically, accelerometers are mounted on platforms that are stabilized by gyroscopes or star-tracking telescopes. Terrain and stellar navigation systems are examples of imaging systems. Satellite navigated missiles use satellites for navigation. Some satellite guided missiles use the Navstar GPS—a constellation of orbiting navigation satellites—to navigate and guide the missile to its target. GPS has increased accuracy (reduced miss distance) significantly.

Guidance Methods: Mobile Targets with Unknown Positions

If the target position is not known *a priori*, the aforementioned methods and aids may be used in part, but other real-time target acquisition, tracking, navigation, and guidance mechanisms are required. The most commonly used classifications for the guidance system in such cases are as follows: passive, active, and semiactive. Each of these and related topics is now discussed.

- **Passive Missiles.** Passive missiles are missiles that have a target sensor sensitive to target energy emissions (e.g., radar and thermal energy) and a guidance system that uses received target emission signals to guide the missile toward the target. Such missiles are said to have a passive guidance system. Although such systems are, in principle, simple to implement, it should be noted that they rely on a "cooperative target," i.e., targets that radiate energy at appreciable (detectable) power levels. Such systems are also susceptible to decoys.
- **Active Missiles.** Active missiles use an energy-emitting transmitter combined with a reflection–detection receiver (e.g., an active seeker) to acquire targets and guide the missile toward the target. Such missiles are said to have an active guidance system. For such systems, great care is taken to ensure that transmitted and received signals are isolated from one another. Stealthy targets are those that absorb or scatter (misdirect) the transmitted energy. Receivers can consist of a gimballed (movable) seeker antenna. Such mechanically directed antennas are slow and have a limited field of view. Fixed phase array antennas, operating on interferometric principles; offer rapid electronic scanning capability as well as a broad field of view.
- **Semiactive Missiles.** Semiactive missiles use a reflection-sensitive receiver to guide the missile to the target. The reflected energy may be provided by a ground-based, ship-based, or aircraft-based energy emission (e.g., radar or laser) system or by such a system aboard the launching platform. In either case, a human operator (e.g., WSO) illuminates the target with a radar or laser beacon and the missile automatically steers toward the source of the reflected energy. Such missiles are said to possess semiactive guidance systems. For such implementations, the illuminating power can be large.

  Passive systems, of course, are stealthier than semiactive or active systems as they do not intentionally emit energy toward the target. Anti-radar missiles typically use passive guidance systems because radars are constantly emitting energy. As an anti-radar missile approaches the intended radar, radar operators typically shut down the radar, which causes the missile to lose its critical guidance signal. In such a case, an active or semi-active guidance system must take over. It should be noted that active systems require more instrumentation than passive systems and hence are heavier and more expensive.

  - **Homing Missiles.** Homing missiles, like homing pigeons, home in on a target by steering toward energy emitted by or reflected from the target. If the missile homes in on energy emitted by the target, then it uses a passive guidance system.

If the missile transmits a signal and homes in on the reflected energy, its guidance system is active. In principle, sensor information and homing improve as the missile gets closer to the target.

- **Command Guided Missiles.** A command guided missile is a remotely controlled semi-active missile. A cooperating (ground, ship, or aircraft-based) control station uses a radar (or two) to acquire the target, track the target, and track the missile. Available computers are used to compute guidance commands (on the basis of ranges, elevations, and bearings) that are transmitted via radio uplink to the missile autopilot. Powerful computers, capable of exploiting complex target models and performance criteria, can provide precision guidance updates in real time. Such systems are limited by the distance from the tracking station to the missile and target. Noise increases, and guidance degrades, as the engagement moves further from the tracking station. Such systems are also more susceptible to electronic countermeasures (ECMs). Although command-guided missiles do not require a seeker, one can be included for terminal guidance to maximize the probability of interception at long distances from the tracking station. The Patriot is a command-guided SAM. To significantly increase ECM immunity, some short-range command-guided missiles have a wire that unspools at launch, which keeps the missile connected to the command station, e.g., all-weather optically guided anti-tank Tow missile.
    1. **Beam Rider Guidance (BRG).** BRG is a specific form of command guidance in which the missile flies along a beam (e.g., radar or laser), which in principle points continuously toward the target. If the missile stays within the beam, an intercept will occur. Guidance commands steer the missile back into the beam when it deviates. BRG causes problems at large ranges because of beam-spreading issues.
    2. **Command-to-LOS Guidance.** Command-to-LOS guidance—used by the Tow missile—is another command guidance method that improves on beam rider guidance by taking beam motion into account.
- **Energy-Guided Missiles.** Radar-guided missiles are guided to the target on the basis of radar energy. Laser-guided missiles are guided on the basis of laser energy. The Hellfire is a laser-guided anti-tank missile. Heat-seeking missiles are guided on the basis of infrared (IR, heat, or thermal) energy. The AIM-9 Sidewinder is a heat-seeking AAM. Most AAMs employ radar homing or heat-seeking devices and have replaced automatic gunfire as the main armament for fighter aircraft. The shoulder-operated Stinger is a heat-guided fire-and-forget SAM. Such a missile is called a fire-and-forget missile because it allows the user to fire, take evasive action, forget, and engage other hostile targets.
- **Degradation of Electromagnetic Energy-Based Sensors.** The performance of many electromagnetic energy-based sensors (e.g., millimeter wave radars, electro-optical thermal imagers, and laser radar) degrades under adverse weather conditions such rain, fog, dust, or smoke. This degrading occurs when the size of the weather particles are on the same order as the wavelength of the energy return from the target. Under adverse conditions, microwave radars with wavelengths in centimeters (10 GHz) are not degraded, millimeter radars with millimeter wavelengths (100 GHz) are slightly degraded, and electro-optical systems with micrometer wavelengths (10^5 GHz) are severely degraded. The AIM-120A AMRAAM is a fighter-launched fire-and-forget AAM that uses IR sensors to acquire (detect) targets at long range. It uses inertial mid-course guidance without the need for the fighter to illuminate the target. A small active seeker is used for endgame homing.
- **LOS Guidance.** When a missile is near the target, the guidance system may use line-of-sight (LOS) guidance. The guidance system of an LOS-guided missile uses target range and LOS information obtained from the target sensor (e.g., a seeker) to generate guidance commands to the missile autopilot.
- **Radar Terrain Guidance.** A radar terrain-guided missile uses a radar altimeter, an *a priori* stored path and terrain profile to navigate and guide the missile over the terrain during the mid-course phase of a flight (typically). The stored path represents a desired path over the terrain. The down-looking radar altimeter is used to measure the altitude with respect to the terrain below, which is used to determine where the missile is with respect to the desired path. Deviations from the path are corrected by adjusting guidance commands to the autopilot. The Tomahawk is an all-weather cruise missile that uses radar terrain guidance called Terrain Contour Matching (TERCOM) (9). TERCOM terrain profiles—obtained by reconnaissance satellites and other intelligence sources—become finer as the missile approaches the target. Such navigational/guidance systems permit terrain hugging. Terrain echoes (referred to as clutter) then confuse observing radars.
- **TV Guidance.** TV-guided missiles use imaging systems that permit a WSO to see the target and remotely guide the missile to the target.

Cruise Missiles

Cruise missiles are typically SSMs that use inertial and terrain following navigation/guidance systems while cruising toward the target. When near the target, endgame guidance is accomplished by either homing in on 1) target emitted/reflected energy, and 2) a target feature by exploiting a forward-looking imaging system and an onboard stored image, or by 3) using a more detailed terrain contour with a more-accurate downward-looking sensor. Cruise missiles offer the ability to destroy heavily defended targets without risking air crew. Because they are small, they are difficult to detect on radar, particularly when they hug the terrain. Examples of cruise missiles are the AGM-86, Tomahawk (9), and Harpoon. The Tomahawk uses a TERCOM guidance during the cruise-phase. For terminal guidance, a conventionally armed Tomahawk uses an electro-optical Digital Scene-Matching Area Correlator (DSMAC) guidance system that compares measured images with stored images. This technique is often referred to as an offset navigation or guidance technique. At no time during the terminal scene-matching process does the missile look at the target. Its sensor always looks down. DSMAC makes Tomahawk one of the most accurate weapon systems in service around the world.

Skid-to-Turn and Bank-to-Turn Missiles

Skid-to-turn (STT) missiles, like speed boats, skid to turn. Bank-to-turn (BTT) missiles, like airplanes, bank to turn (5, 10–16). BTT airframe designs offer higher maneuverability than conventional STT designs by use of an asymmetrical shape and/or the addition of a wing. BTT missile autopilots are more difficult to design than STT autopilots because of cross-coupling issues. STT missiles achieve velocity vector control by permitting the missile to develop angle-of-attack and side-slip angles (5). The presence of slide-slip imparts a skidding motion to the missile. BTT missiles ideally should have no side-slip. To achieve the desired orientation, a BTT missile is rolled (banked) so that the plane of maximum aerodynamic normal force is oriented to the desired direction. The magnitude of the force is controlled by adjusting the attitude (i.e., angle-of-attack) in that plane. BTT missile control is made more difficult by the high roll rates required for high performance (i.e., short response time) (4). STT missiles typically require pitch-yaw acceleration guidance commands, whereas BTT missiles require pitch-roll acceleration commands. An overview of tactical missile control design issues and approaches is provided in Reference 17.

GUIDANCE ALGORITHMS

In practice, many guidance algorithms are used (4, 8, 18–20). The purpose of a guidance algorithm is to update missile guidance commands that will be issued to the autopilot. This update is to be performed on the basis of missile and target information. The goal of any guidance algorithm is to steer the missile toward the target, which results in an intercept within an allotted time period (that is, until the fuel runs out or the target is out of range). The most common algorithms are characterized by the following terms: proportional navigation, augmented proportional navigation, and optimal (8, 20). To simplify the mathematical details of the exposition to follow, suppose that the missile-target engagement is restricted to the pitch plane of the missile. Given this, the engagement dynamics take the following simplified form (21):

$$\dot{R}(t) = V_t \cos(\lambda(t) - \gamma_t(t)) - V_m \cos(\lambda(t) - \gamma_m(t)) \tag{1}$$

$$\dot{\lambda}(t) = \frac{1}{R(t)}[-V_t \sin(\lambda(t) - \gamma_t(t)) + V_m \sin(\lambda(t) - \gamma_m(t))] \tag{2}$$

where R represents range to the target, λ represents LOS angle, and (V_m, V_t) and (γ_m, γ_t) denote missile-target speeds (assumed constant) and flight path angles.

Proportional Navigation Guidance (PNG)

For proportional navigation guidance (PNG) (8, 20), the missile is commanded to turn at a rate proportional to the closing velocity V_c (i.e., range rate) and to the angular velocity of the LOS $\dot{\lambda}$. For a PNG law, the pitch plane acceleration command $a_{c\,PNG}(t)$ takes the form

$$a_{c\mathrm{PNG}}(t) = NV_c(t)\dot{\lambda}(t) \tag{3}$$

where N is a constant of proportionality referred to as the PNG gain or constant. For tactical radar homing missiles using PNG, an active seeker provides LOS rate while a Doppler radar provides closing velocity. Traditionally, LOS rate has been obtained by filtering the output of a 2 degree-of-freedom rate gyro mounted to the inner gimbal of the seeker (22). More recently, ring laser gyros (RLGs) have been used. Unlike conventional spinning gyros, the RLG has no moving parts, no friction, and hence negligible drift. For IR missiles using PNG, the IR system provides LOS rate information, but V_c must be estimated.

PNG Optimality and Performance Issues

It can be shown that PNG minimizes the square integral criterion $\int_0^{t_f} a_c^2(\tau)d\tau$ subject to the following assumptions:

1. Zero miss distance at t_f
2. Linearized (small angle) missile-target dynamics
3. constant missile-target speeds (23)

where t_f denotes the flight time. A missile using PNG is fired not at the target, but at the expected intercept point if the target were to move at constant velocity in a straight line; i.e., the missile is fired so that, at least initially, it is on a collision triangle with the target. The initial angle between the missile velocity vector and the LOS is the missile lead angle. If the missile is not on a collision triangle with the target, then a heading error (HE) exists.

It is instructive to understand how PNG missile acceleration requirements vary with

1. Initial heading error when the target is not maneuvering
2. A constant acceleration target maneuver

These cases are now briefly discussed assuming linearized (small-angle) two-dimensional (2D) dynamics with constant missile and target speeds (V_m, V_t) missile autopilot responds instantaneously to guidance acceleration commands (i.e., no lag), and ideal sensor dynamics (8). We note that the Stinger is an example of a fire-and-forget supersonic SAM that uses PNG with passive IR/UV homing.

1. **PNG Performance: Non-maneuvering Target, Heading Error.** First, consider the impact of a heading error on PNG missile acceleration requirements when the target moves at a constant speed in a straight line. Under the simplifying assumptions given above, the resulting commanded acceleration is as follows:

   $$a_{cPNG}(t) = \frac{-V_m HEN}{t_f}\left[1 - \frac{t}{t_f}\right]^{N-2} \qquad (4)$$

   This expression shows that PNG immediately begins removing any heading error and continues doing so throughout the engagement. The acceleration requirement decreases monotonically from $a_{cPNG_{\max}} = a_{cPNG}(0) = \frac{-V_m HEN}{t_f}$ to zero as the flight progresses. A larger N results in a larger initial missile acceleration requirement, but a lesser endgame missile acceleration requirement. The larger the N, the faster the heading error is removed.
2. **PNG Performance: Target Undergoing Constant Acceleration.** Now, consider the impact of a constant target acceleration *at* on PNG missile acceleration requirements. Under the simplifying assumptions given above, the resulting commanded acceleration is as follows:

   $$a_{cPNG}(t) = \frac{N}{N-2}\left[1 - \left(1 - \frac{t}{t_f}\right)^{N-2}\right] a_t \qquad (5)$$

   In sharp contrast to the heading error case examined above, this expression shows that the PNG missile acceleration requirement for a constant target maneuver increases monotonically throughout the flight. As in the heading error case, a higher N results in a greater initial acceleration requirement and a relaxed acceleration requirement near the end of the flight ($a_{cPNG_{\max}} = a_{cPNG}(t_f) = [\frac{N}{N-2}]a_t \leq a_t$).

Zero Effort Miss (ZEM) Distance

An important concept in guidance law design is that of zero effort miss distance, denoted $ZEM(t)$ and defined as the miss distance that would result if the target would continue at a constant speed in a straight line and the missile made no further corrective maneuvers. Given this, if one defines the time-to-go as t_{go}= def $t_f - t$ and the zero effort miss distance perpendicular to the LOS as $ZEM_{PLOS}(t)$ then for PNG it can be shown that

$$a_{cPNG}(t) = N\left(\frac{ZEM_{PLOS}(t)}{t_{go}^2}\right) \qquad (6)$$

where $ZEM_{PLOS}(t) = y + \dot{y}t_{go}$, $y \approx R\lambda$ denotes the relative (small angle) vertical displacement between the missile and target, and $R \approx V_c t_{go}$. The concept of zero effort miss distance is used to derive more advanced guidance laws (8). The concept is very powerful since ZEM can be approximated in so many different ways.

PNG Miss Distance Performance: Impact of System Dynamics

For the two cases considered above, the associated relative displacement $y \approx R\lambda$ satisfies

$$y + \left(\frac{N}{t_f - t}\right) y + \left(\frac{N}{(t_f - t)^2}\right)\dot{y} = a_t \quad y(t_f) = 0 \qquad (7)$$

and we have zero miss distance. The preceding discussion on PNG assumes that guidance-control-seeker dynamics are negligible. In practice, this assumption is not satisfied and the inherent lag degrades miss distance performance. When a first-order lag with time constant τ is assumed for the combined guidance-control-seeker dynamics, one obtains small miss distances so long as τ is much smaller than t_f, e.g., $t_f > 10\tau$. In practice, of course, high-frequency dynamics impose bandwidth constraints that limit how small τ can be. Despite the above (general) rule-of-thumb, it is essential that high-frequency system dynamics be carefully modeled/analyzed to obtain reliable performance predictions. Such dynamics include those associated with control system, computational delays, A/D and D/A conversion, actuators (e.g., thrusters, canards, and tail fins), missile structure (e.g., flexible and servoelastic modes), guidance system (e.g., lead-lag compensation), and sensors (e.g., seeker radome, accelerometers, gyros). As one might expect, noise and parasitic effects place a practical upper bound on the achievable guidance system bandwidth. In practice, statistical Monte Carlo simulations [exploiting adjoint methods (8)] are used to evaluate performance before flight testing. Such simulations consider the above as well as acceleration/control saturation effects (14, 15), typical target maneuvers, and worst case target maneuvers.

TPNG and PPNG

In Reference 24, the authors distinguish between true PNG (TPNG) and pure PNG (PPNG). For missile's using TPNG, acceleration commands are issued perpendicular to the LOS (as above). For PPNG, acceleration commands are issued perpendicular to the missile velocity vector. The advantages of PPNG over traditional TPNG are highlighted in Reference 24. In contrast to PPNG, TPNG requires 1) a forward acceleration and deceleration capability (because acceleration command is perpendicular to LOS; not missile velocity), 2) unnecessarily large acceleration requirements, and 3) restrictions on the initial conditions to ensure intercept.

Tactical Missile Maneuverability

Tactical radar-guided missiles use a seeker with a radome. The radome causes a refraction or bending of the incoming radar wave, which in turn, gives a false indication of target location. This phenomenon can cause problems if the missile is highly maneuverable. One parameter that measures maneuverability is the so-called missile (pitch) turning rate frequency (or bandwidth) defined by (2)

$$\omega_\alpha \overset{\text{def}}{=} \frac{\dot{\gamma}}{\alpha} \qquad (8)$$

where $\dot{\gamma}$ denotes the time rate of change of flight path angle and α denotes angle-of-attack (AOA). ω_α measures the rate at which the missile rotates (changes flight path) by an equivalent AOA. Assuming that the missile is modeled as a "flying cylinder" (8) with length L and diameter D, it has a lift coefficient $C_L = 2\alpha[1 + 0.75 \frac{S_{\text{plan}}}{S_{\text{ref}}} \alpha]$, where $S_{\text{plan}} \approx LD,\ S_{\text{ref}} = \frac{\pi D^2}{4}$. Noting that $a_m = V_m \dot{\gamma}$ is the missile acceleration, $Q = \frac{1}{2} \rho V_m^2$ the dynamic pressure, $W = mg$ the missile weight, and ρ the density of air, it follows that

$$\omega_\alpha \overset{\text{def}}{=} \frac{\dot{\gamma}}{\alpha} = \frac{\frac{\alpha_m}{V_m}}{\alpha} = \frac{gQS_{\text{ref}}C_L}{\alpha V_m} = \frac{\rho g V_m S_{\text{ref}} \left[1 + 0.75 \frac{S_{\text{plan}}}{S_{\text{ref}}}\right]}{W} \tag{9}$$

From this, it follows that ω_α decreases with increasing missile altitude and with decreasing missile speed V_m.

Radome Effects: Homing-Robustness Trade-offs

Let ω denote the guidance-control-seeker bandwidth.

- **Homing Requirement.** If ω is too small, homing is poor and large miss distances result. Typically, we desire

  $$\omega_\alpha < \omega \tag{10}$$

  that is, the guidance-control-seeker bandwidth should be sufficiently large so that the closed-loop system "accommodates" the maneuverability capabilities of the missile, which implies that the guidance-control-seeker bandwidth ω must be large when ω_α is large (low altitude and high missile speed V_m).
- **Robustness Requirement.** If ω were too large, however, it is expected that problems can occur. This result in part, is because of radome-aerodynamic feedback of the missile acceleration a_m into $\dot{\lambda}$. Assuming n-pole dynamics, it can be shown that the missile acceleration a_m takes the form

  $$a_m = FG[\dot{\lambda} - R\dot{\theta}] = FG[\dot{\lambda} - RAa_m] = \left[\frac{FG}{1 + FGRA}\right] \dot{\lambda} \tag{11}$$

  where $G = NV_c$ represents the guidance system, $F = (\frac{\omega}{s + \omega})^n$ represents the flight control system, R is the radome slope (can be positive or negative), and $A = \frac{s + \omega_\alpha}{\omega_\alpha V_m}$ denotes the missile transfer function from a_m to pitch rate $\dot{\theta}$ For stability robustness, we require the associated open-loop transfer function

  $$L \overset{\text{def}}{=} FGRA = NV_c \left(\frac{\omega}{s + \omega}\right)^n R \left(\frac{s + \omega_\alpha}{\omega_\alpha V_m}\right) \tag{12}$$

  to satisfy an attenuation specification such as $|L(j\omega)| \approx NV_c|R| \frac{\omega}{\omega_\alpha V_m} < \varepsilon$ for some sufficiently small constant $\varepsilon > 0$. This result however, requires that the guidance-control-seeker bandwidth ω satisfies

  $$\omega < \varepsilon \left(\frac{V_m}{|R|NV_c}\right) \omega_\alpha \tag{13}$$

  for stability robustness which implies that the guidance-control-seeker bandwidth bandwidth ω must be small when V_m is small, $(|R|, N, V_c)$ are large, or ω_α is small (high altitude and low missile speed V_m).

From the above, it follows that we require the guidance-control-seeker bandwidth ω to satisfy

$$\omega_\alpha < \omega < \varepsilon \left(\frac{V_m}{|R|NV_c}\right) \omega_\alpha \tag{14}$$

The lower inequality should be satisfied for good homing. The upper inequality should be satisfied for good robustness with respect to radome effects.

- When ω_α is small (e.g., at high altitudes or low speeds), designers make the guidance-control-seeker bandwidth ω small but sufficiently large to accommodate missile maneuverability (i.e., satisfy the lower inequality). In such a case, radome effects are small and the guidance loop remains stable yielding zero miss distance after a sufficiently long flight. One can, typically, improve homing performance by increasing ω and N. If they are increased too much, radome effects become significant, miss distance can be high, and guidance loop instability can set in.
- When ω_α is large (e.g., at low altitudes or high speeds), designers would still like to make the guidance-control-seeker bandwidth ω sufficiently large to accommodate missile maneuverability (i.e., satisfy the lower inequality). This, result generally, can be accomplished provided that radome effects are not too significant. Radome effects will be significant if Vm is too small, $(|R|, N, V_c)$ are too large, or ω_α is too small (i.e., too high an altitude and/or too low a missile speed V_m).

Given the above, it therefore follows that designers are generally forced to trade off homing performance (bandwidth) for stability robustness properties. Missiles using thrust vectoring (e.g., exoatmospheric missiles) experience similar performance-stability robustness trade-offs.

Augmented Proportional Guidance (APNG)

Advanced guidance laws reduce acceleration requirements and miss distance but require more information (e.g., time-to-go and missile-target range) (19). In an attempt to take into account a constant target acceleration maneuver a_t, guidance engineers developed augmented proportional guidance (APNG). For APNG, the commanded acceleration is given by

$$a_{cA\,\text{PNG}}(t) = NV_c\dot{\lambda}(t) + \frac{1}{2}Na_t = a_{c\,\text{PNG}}(t) + \frac{1}{2}Na_t \tag{15}$$

or $a_{cA\,\text{PNG}}(t) = N(\frac{\text{ZEM}}{t_{\text{go}}^2})$, where $\text{ZEM} = y + \dot{y}t_{\text{go}} + \frac{1}{2} a_t t_{\text{go}}^2$ is the associated zero effort miss distance. Equation (15) shows that APNG is essentially PNG with an extra term to account for the maneuvering target. For this guidance law, it can be shown (under the simplifying assumptions given earlier) that

$$a_{cA\,\mathrm{PNG}}(t) = \frac{1}{2}N\left[1 - \frac{t}{t_f}\right]^{N-2} a_t \tag{16}$$

In contrast with PNG, this expression shows that the resulting APNG acceleration requirements decrease with time rather than increase. From the expression, it follows that increasing N increases the initial acceleration requirement but also reduces the time required for the acceleration requirements to decrease to negligible levels. For $N = 4$, the maximum acceleration requirement for APNG, $a_{cA\,\mathrm{PNGmax}} = \frac{1}{2}\,Na_t$, is equal to that for PNG, $a_{c\mathrm{PNGmax}} = [\frac{N}{N-2}]a_t$. For large $N = 5$, APNG requires a larger maximum acceleration but less acceleration than PNG for $t \geq 0.2632t_f$. As a result, APNG is more fuel efficient for exoatmospheric applications than PNG. Finally, it should be noted that APNG minimizes $\int_0^{t_f} a_c^2(\tau)d\tau$ subject to zero miss distance, linear dynamics, and constant target acceleration (8).

PNG Command Guidance Implementation

To implement PNG in a command guidance setting (i.e., no seeker), a differentiating filter must be used to estimate the LOS rate. As a result, command guidance is more susceptible to noise than homing guidance. This issue is exacerbated as the engagement takes place further from the tracking station, noise increases, and guidance degrades. Within Reference 25, the authors address command-guided SAMs by spreading the acceleration requirements over t_{go}. The method requires estimates for target position, velocity, acceleration, and t_{go} but takes into account nonlinear engagement geometry.

Advanced Guidance Algorithms

Classic PNG and APNG were initially based on intuition. Modern or advanced guidance algorithms exploit optimal control theory, i.e. optimizing a performance measure subject to dynamic constraints. Even simple optimal control formulations of a missile-target engagement (e.g., quadratic acceleration measures) lead to a nonlinear two-point boundary value problem requiring creative solution techniques, e.g., approximate solutions to the associated Hamilton–Jacobi–Bellman equation—a formidable nonlinear partial differential equation (23). Such a formulation remains somewhat intractable given today's computing power, even for command guidance implementations that can exploit powerful remotely situated computers. As a result, researchers have sought alternative approaches to design advanced (near-optimal) guidance laws. In Reference 20, the authors present a PNG-like control law that optimizes square-integral acceleration subject to zero miss distance in the presence of a one pole guidance-control-seeker system.

Even for advanced guidance algorithms (e.g., optimal guidance methods), the effects of guidance and control system parasitics must be carefully evaluated to ensure nominal performance and robustness (20). Advanced (optimal) guidance methods typically require additional information such as time-to-go, target acceleration, target model parameters (e.g., ballistic coefficient). As a result, Kalman filter and extended Kalman filter (EKF) techniques are often used to estimate the required information. For optimal guidance (OG) algorithms to work well, the estimates must be reliable (20). An overview of guidance and control techniques, including a comprehensive set of references, is given in Reference 18. Other approaches to guidance law design are discussed below.

Variants of PNG

Within Reference 20, the authors compare PNG, APNG, and optimal guidance (OG). The zero miss distance (stability) properties of PPNG are discussed within Reference 24. A nonlinear PPNG formulation for maneuvering targets is provided in Reference 27. Closed form expressions for PPNG are presented in Reference 28. A more complex version of PNG that is "quasi-optimal" for large maneuvers (but requires t_{go} estimates) is discussed in Reference 29. Two-dimensional miss distance analysis is conducted in Reference 21 for a guidance law that combines PNG and pursuit guidance. Within Reference 30, the authors extend PNG by using an outer LOS rate loop to control the terminal geometry of the engagement (e.g., approach angle). Generalized PNG, in which acceleration commands are issued normal to the LOS with a bias angle, is addressed in Reference 31. Three-dimensional (3D) generalized PNG is addressed within Reference 32 using a spherical coordinate system fixed to the missile to better accommodate the spherical nature of seeker measurements. Analytical solutions are presented without linearization. Generalized guidance schemes are presented in Reference 33, which result in missile acceleration commands rotating the missile perpendicular to a chosen (generalized) direction. When this direction is appropriately selected, standard laws result. Time-energy performance criteria are also examined. Capturability issues for variants of PNG are addressed in Reference 34 and the references therein. Within Reference 35, the authors present a 2-D framework that shows that many developed guidance laws are special cases of a general law. The 3-D case, using polar coordinates, is considered in Reference 36.

Optimal Guidance (OG) Laws

Weaving targets can cause large miss distances when classicl and "standard" OG laws are used. Tactical ballistic missiles, for example, can spiral or weave into resonances as they enter the atmosphere because of mass or configurational asymmetries. An OG law, based on weaving (variable amplitude) sinusoidal target maneuvers, is developed in Reference 37). An EKF is used to estimate the target maneuver weave frequency. Methods for intercepting spiraling weaving tactical ballistic targets are also presented in Reference 38, which includes an optimal weave guidance law incorporating an EKF to estimate relative position, relative velocity, target acceleration, target jerk information, and weave frequency information.

Differential Game Guidance

Differential game-theoretic concepts are addressed within Reference 23. In such formulations, a disturbance (e.g., target maneuver) "competes" with a control (e.g., missile acceleration command). The disturbance attempts to maximize a performance index (e.g., miss distance), where as the control attempts to minimize the index. Within Reference 39), the authors provide an analytical study using a zero-sum pursuit-evasion differential game formulation to develop endgame guidance laws assuming that the interceptor has two controls. Linear bi-proper transfer functions are used to represent the missile's control systems—a minimum phase transfer function for the canard system and a non-minimum phase (NMP) transfer function for the tail control system. A first-order strictly proper transfer function is used for the target dynamics. Bounds are assumed for each of the above transfer function inputs (i.e., reference commands). The optimal strategy is bang-bang in portions of the game space. A switching time exists before interception because of the NMP nature of the tail control system. This feature requires good estimates of $t_{go}.\mathcal{H}^{\infty}$ theory (7) provides a natural differential game theoretic framework for developing guidance laws as well as for control laws.

Lyapunov-Based Guidance Laws

Lyapunov methods have been very useful for deriving stabilizing control laws for nonlinear systems (40). Such methods have been used to obtain guidance laws that require target aspect angle (relative to LOS) rather than LOS rate (41) and that address maneuvering targets in 3-D (42).

Other Guidance Laws

Circular navigation guidance (CNG) steers the missile along a circular arc toward the target (43). Traditionally, the guidance and control systems are designed separately. Although this approach has worked well for years, increasing performance requirements affirm the value of an integrated guidance and control system design methodology. Integrated guidance and control issues are addressed within a polar coordinate framework in Reference 44. New advanced guidance laws may benefit from linear parameter varying (LPV) (17) and state dependent Riccati equation (SDRE) (45) concepts.

Nonlinear State Estimation: Extended Kalman Filter

As discussed OG laws often require missile-target model state/parameter estimates, e.g., relative position, velocity of target, acceleration of target, *tgo*. An extended Kalman filter (EKF) is often used to obtain the required estimates, which involves using quasi-linearized dynamics to solve the associated matrix Riccati differential equation for a covariance matrix that is used with a model based estimator—mimicking the original nonlinear dynamics—to generate quasi-optimal estimates. It is well known that poor estimates for *tgo,* for example, can result in large miss distances and significant capture region reduction (20). Estimating t_{go} as R/Vc is valid only if V_c is nearly constant. A recursive (noniterative) algorithm for t_{go} estimates, that can be used with OG laws, is provided within Reference 46. To develop useful estimation techniques, much attention has been placed on modeling the target. Initially, researchers used simple uncorrelated target acceleration models. This process however, yielded misleading results, which led to the use of simple dynamical models—point mass and more complex. Both Cartesian and spherical coordinate (47) formulations have been investigated—the latter better reflecting the radial nature of an engagement. Single and multimodeled EKFs have been used (48) to address the fact that no single model captures the dynamics that may arise. Low-observability LOS measurements make the problem particularly challenging (48). Target observability is explored in Reference 49 under PNG and noise-free angle-only measurements in 2-D. A method for obtaining required estimates for APNG (e.g., y, y, a_t, t_{go}) is presented in Reference 50. As no single (tractable) model and statistics can be used to accurately capture the large set of possible maneuvers by today's modern tactical fighters, adaptive filtering techniques have been employed. Such filters attempt to adjust the filter bandwidth to reflect the target maneuver. Some researchers have used classic Neyman–Pearson hypothesis testing to detect bias in the innovations to appropriately reinitialize the filter. Threshold levels must be judiciously selected to avoid false detections that result in switching to an inappropriate estimator.

Long-Range Exoatmospheric Missions: Weight Considerations

For long-range exoatmospheric missions approaching intercontinental ranges, orbital speeds are required (e.g., ~ 20,000 ft/sscond or 13,600 miles/hour or 4 miles/second). To study such interceptors, two new concepts are essential. Fuel-specific impulse, denoted I_{sp}, is defined as the ratio of thrust to the time rate of change of total missile weight. It corresponds to the time required to generate a weight equivalent amount of thrust. Fuel-efficient missiles have higher fuel-specific impulses. Typical tactical missile fuel-specific impulses lie in the range of 200 to 300 seconds. Fuel-mass fraction, denoted *mf*, is defined as the ratio of propellant weight W_{prop} to total weight $W_T = W_{\text{prop}} + W_{\text{structure}} + W_{\text{payload}}$. SAMs, for example, have a larger fuel-mass fraction than AAMs because SAMs must travel through the denser air at lower altitudes. For fuel-specific impulses less than 300 seconds, large fuel-mass fractions (approaching 0.9) are required for exoatmospheric applications. A consequence is that it takes considerable total booster weight to propel even small payloads to near-orbital speeds. More precisely, it can be shown (8) that the weight of the propellant required for a single-stage booster to impart a speed change AV to a payload weighing W_{payload} is given by

$$W_{\text{prop}} = W_{\text{payload}}\overline{m}f \left[\frac{\exp\left(\frac{\Delta V}{gI_{sp}}\right) - 1}{1 - (1 - \overline{m}f)\exp\left(\frac{\Delta V}{gI_{sp}}\right)} \right] \tag{17}$$

where g denotes the acceleration from gravity near the surface of the Earth and $\overline{m}f \overset{\text{def}}{=} \frac{W_{\text{prop}}}{W_{\text{prop}} + W_{\text{structure}}}$ denotes an (approximate) fuel-mass fraction that neglects the weight of the payload W_{payload}. Staging can be used to reduce total booster weight for a given fuel-specific impulse I_{sp} and (approximate) fuel-mass fraction $\overline{m}f$. Efficient propellant expenditure for exoatmospheric intercepts is addressed within Reference 51. Three-dimensional mid-course guidance for SAMs intercepting nonmaneuvering high-altitude ballistic targets is addressed within Reference 52. Neural networks are used to approximate (store) optimal vertical guidance commands and estimate *tgo*. Feedback linearization (39) is used for lateral guidance commands.

Acceleration Limitations

Endoatmospheric missile acceleration is limited by altitude, speed, structural, stall AOA, and drag constraints—stall AOA at high altitudes and structural limitations at low altitudes (see Eq. 9). Exoatmospheric interceptor acceleration is limited by thrust-to-weight ratios and flight time—the latter is because, when the fuel is exhausted, exoatmospheric missiles cannot maneuver. For the "flying cylinder" considered earlier, the lateral acceleration A in gees is given by $\frac{A}{g} = \frac{QS_{\text{ref}}C_L}{W} = \frac{0.5\rho V_m^2 S_{\text{ref}}}{W}[\alpha + 1.5\frac{S_{\text{plan}}}{S_{\text{ref}}}\alpha^2]$ (8). For $L = 20\,\text{ft},\ D = 1\,\text{ft},\ W = 100\,1\text{bs},\ V_m = 3000\,\text{ft/s}$, and $\alpha = 20$ deg, at an altitude of 25,000 ft, the resulting acceleration is $A \approx 20g$.

THAAD Systems

Recent research efforts have focused on the development of THAAD systems. Calculations show that high-altitude ballistic intercepts are best made head-on so that there is little target deceleration perpendicular to the LOS (8), because such decelerations appears as a target maneuver to the interceptor. EKF methods have been suggested for estimating target ballistic coefficients and state information to be used in OG laws. Estimating ballistic coefficients ($\beta \overset{\text{def}}{=} \frac{W}{S_{\text{ref}}C_{Da}}$ where C_{DO} is the zero lift drag coefficient) is particularly difficult at high altitudes where there is little drag $a_{\text{drag}} = \frac{1}{2\beta}\rho g V_m^2$. Also, the high closing velocity of a ballistic target engagement significantly decreases the maximum permitted guidance system bandwidth for radome slope stability. Noise issues can also significantly exacerbate the ballistic intercept problem.

FUTURE DEVELOPMENTS

Future developments will focus on theater-class ballistic missiles, guided projectiles, miniature kill vehicles, space-based sensors for missile defense and boost-phase interceptors.

The Age of Air-Breathing Hypersonic Flight

During the Gulf Wars, it often took considerable time to get a missile on a critical target (e.g., Iraqi leadership), which gave further impetus for a prompt global strike (PGS) capability—one that permits accurate strikes across thousands of miles in minutes. Many have suggested the retrofitting of Trident missiles with conventional warheads for this purpose. This idea has alarmed many who argue that such an application of Trident ICBMs could mistakingly unleash a world-impacting nuclear war. Others have proposed the development of hypersonic missiles that exploit new scramjet technology.

In 2004, NASA's scramjet powered X-43A vehicle ushered in the age of air-breathing hypersonic flight. Two history-making flights were made—one at Mach 7 and the other at Mach 10. [At sea level, 2116.2 lb/ft^2, 59°F (standard atmosphere conditions), the speed of sound is 1116.5 ft/second (761.25 mph).] These historical flights unleashed a hypersonics research revolution—one that has already begun to significantly shape the design of future aircraft, missile, and space-access systems. Like the X-43A, hypersonic missiles are expected to exploit rocket power to achieve hypersonic flight ($\sim$Mach 5) at which point the scramjet will take over.

Challenges impacting the development of hypersonic vehicles include significant operational uncertainty and aero-thermo elastic-propulsion interactions. At very high speeds, the heat generated is so severe that classic aerodynamic principles based on fluid mechanics no longer apply. In such a case, gas theory must be used to predict lift and drag properties. A consequence is significant aerodynamic uncertainty. The high temperatures induced also result in severe aero-elastic effects (e.g., servoelastic) that make control difficult. Such issues are currently being addressed by the research community. It is truly amazing how, in just over 100 years since the first powered Wright Brothers flight on December 17, 1903, we have ushered in the age of air-breathing hypersonic flight.

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ARMANDO A. RODRIGUEZ
Arizona State University