DISTRIBUTED PARAMETER SYSTEMS

Many physical systems are naturally modeled by partial differential equations (*PDEs*): Chemical reactions, fluid flow, vibrations of flexible structures, and acoustic fields are some examples that occur in engineering applications. Typical control problems might be to reduce the vibrations in a flexible structure (such as a robot arm or a satellite antenna), to reduce the noise levels in an aircraft, to damp out undesirable eddy currents in a fluid, or to attain a uniform temperature distribution for processing steel bars. Current engineering practice is to obtain a mathematically simpler linear ordinary differential equation model by approximating the original nonlinear PDE model based on the physics of the process. Then, applying the formidable arsenal of known finite-dimensional control theory and design methodology, a control scheme is formulated and subsequently tested and modified based on exhaustive computer simulation experiments. In the 1960s, certain mathematicians envisioned an alternative approach: Retain the more accurate PDE model, and formulate the control problem for this model directly (1). The obvious advantage is that the physical interpretation is retained at all stages of the control design process. However, there is also the considerable disadvantage of introducing the very complicated mathematics involved in manipulating PDEs. Moreover, the feedback couplings, which are an inevitable feature of control, introduce new types of PDEs that were not well understood in the 1960s. This has provided the motivation for intensive research into modeling and control of systems described by partial differential equations.

Although it is useful to subdivide the area of control of distributed parameter systems into

- Modeling as a controlled PDE, including well-posedness issues and identification of parameters
- Extension of finite-dimensional linear systems theory and control concepts to deterministic and stochastic PDE systems
- Nonlinear systems
- Implementation of the theory to controller design for real physical systems

We stress that all four areas are closely interrelated.

MODELING

For example, whereas the modeling aspect draws on a vast body of known theory of physics and mathematics of PDEs, the control action must be incorporated into the model. Although distributed control (the control acts over a whole region of the system) is fairly straightforward to model, control action on the boundary requires a careful analysis. The closed-loop system with boundary control action results in a nonstandard PDE, and much research has gone into developing appropriate mathematical formulations, using both PDE and semigroup approaches (2–4). Similar remarks apply to the modeling of the observation via sensors that act on the boundary or at points in the medium. More recently, the introduction of a new generation of advanced sensors and actuators (smart materials) has necessitated formulating new composite PDE models in which the actuators and sensors form an intrinsic part of this new type of PDE model (5, 6). With ever continuing technological advances in materials science and other areas of physics, we can expect a steady stream of new modeling problems for composite configurations of controlled PDEs. A consequence of physical modeling is the inclusion in the PDE model of several physical quantities (constants or operators) that are not known precisely. To complete the modeling step, these need to be estimated from measurement data. This important step has developed into a subspecialty called parameter identification (7). The techniques can be described roughly as a sophisticated deterministic least-squares data-fitting procedure in which emphasis is laid on the appropriate choice of a numerical approximation scheme to suit the PDE under consideration. This is in contrast to finite-dimensional identification techniques that use a stochastic algorithmic approach. Although a complete mathematical theory of stochastic evolution equations exists (8) and there is some stochastic identification theory in a PDE setting, these theories have not become current practice. The philosophy behind the estimation/identification step for PDE models is to exploit the given PDE structure fully, as is done in the closely related area of inverse problems.

EXTENSION OF FINITE-DIMENSIONAL THEORY

Most of the literature concerns the extension of known finite-dimensional theory in systems and control. There are two main approaches, the first using a PDE description and the second using a semigroup one. The advantage of the first approach is that it is directly applicable to PDE systems, and by using the power of PDE estimates applying to a specific type of system (for example, parabolic or hyperbolic), very sharp results on controllability or stabilizability can be obtained. For PDE systems, there are many possible concepts of controllability; the main two are exact controllability (the ability to steer exactly to a given state of the system) and the weaker concept of approximate controllability (steering arbitrarily closely to a given state). The first concept implies stabilizability of the system a feedback controller makes the system stable) as for finite-dimensional systems, but the second concept does not. For this reason, stabilizability replaces controllability as the key property needed for control design. Similar remarks hold for the dual concepts of observability and detectability. The literature on establishing that these properties hold for particular PDE systems is vast and continues unabated from the 1960s; the reason is that every new configuration of coupled PDEs with assorted control action leads to a new, very difficult mathematical problem requiring sophisticated PDE techniques (9). The other main problems studied from a PDE viewpoint are stabilization by boundary control feedback and the linear quadratic control problem and its associated operator Riccati equation (3, 10). The latter problem is the key to the main control design for distributed parameter systems. Recently, extensions of this problem to the so-called min–max versions have received attention.

On the other hand, the semigroup approach has the advantage that several different types of PDEs and delay equations can be included in the same theoretical formulation, and this formulation closely resembles that for ordinary differential equations (11, 12). The basic assumption is that the uncontrolled system can be modeled as a strongly continuous semigroup. The attractive feature of this approach is that it is more accessible to engineers (13) and the theory naturally includes frequency-domain descriptions that are so useful in robust control design. A wide range of control topics have been covered using this description: linear control, dynamic compensators, linear quadratic Gaussian and H-infinity control, Kalman filtering, model reduction, servo problems, observer theory, P.I. controllers, and adaptive control, to name just a few. The best results have been obtained for linear systems with distributed control and observation; that is, there is sensing and control distributed over the physical system. For systems that allow sensing and control at interior points or on the boundary, the mathematical technicalities increase dramatically. During the past decades, a theory for such a class of well-posed linear systems has matured (14). The key property of this class of well-posed linear systems is that they are closed under composite configurations of cascade, parallel, and closed-loop connections. Many classic control problems such as linear quadratic Gaussian and H-infinity control (15), (Riccati-) balanced realizations, tracking problems, passivity (16), and certain stabilization problems (17, 18) have been solved for this class of systems. The price one has to pay for such a broad coverage is that the step from the original PDE formulation to a semigroup one is nontrivial and the results obtained for a particular PDE example are not always the sharpest possible with dedicated PDE techniques. More recently, research has begun on a wider classes of systems, where more general types of semigroups are studied (19).

NONLINEAR PDES

Of course, many systems are nonlinear and PDE models of physical systems often entail nonlinear damping effects through the boundary conditions. Considerable literature exists on the stability analysis of nonlinear PDEs using a Lyapunov approach (20). More recently, research has been done on proving stabilization by feedback control implemented on the boundary for certain nonlinear PDE models, but as yet there is no general theory (21).

A more traditional topic is optimal control of nonlinear PDEs based on the philosophy of Pontryagin's maximum principle (22, 23). Existence results are hard to prove, but necessary conditions can be obtained and numerical schemes for implementation are available (24). Of course, the controllers are open loop, but in some applications, this suffices, for example, shape optimization (25).

IMPLEMENTATION

The implementation of controllers for PDE systems inevitably involves approximation. The usual approach is to first approximate the PDE by various finite element methods. This very high-order system is then approximated by finite-dimensional techniques (20), and the controller design is based on this reduced order model. A disadvantage of this approach is the lack of an adequate error analysis. Alternatively, one can first do a PDE design and then approximate the controller. For example, the H2 design that comprises a linear quadratic controller coupled with a deterministic observer with output gain from a dual Riccati equation. A fairly complete theory exists of numerical approximations of the operator Riccati equations involved and the effect of the approximating controller on the original PDE (10). Experience has shown that an appropriate choice of the numerical approximation is crucial: Modal approximations rarely give adequate results. An excellent overview of this design methodology and its successes in nontrivial applications to vibration control of a plate and noise attenuation in two- and three-dimensional cavities can be found in (5). Although the H2 control design usually yields good results, in some applications, a min–max modification is to be preferred. A closer analysis of the full PDE controller can often lead to useful information on the optimum placement of sensors and actuators and to choices for low-order suboptimal controllers. The above H2 methodology is now well established, but it is a linear theory. If the nonlinearities are not great, then a linearization of the nonlinear model is appropriate. However, the control of highly nonlinear systems like controlled fluid flow remains a challenge. A promising approach called "Proper Orthogonal Decomposition" exploits this new theory on local low-order approximations of PDEs to design low-order nonlinear controllers. In this approach, physical experimentation and extensive computer simulations go hand in hand with a theoretical analysis of the PDEs and their numerical approximations (see (26)).

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