# SUPERCONDUCTORS, CRYOGENIC STABILIZATION

Kamerlingh Onnes (1) discovered superconductivity in 1911 when he was measuring the electrical conductivity of mercury as a function of temperature down to the temperature of liquid helium. He found that at 4.15 K the electrical resistivity ($\rho$, $\Omega\cdot$m) became too small to measure. Although it is not possible to prove experimentally that a quantity is exactly zero, experiments to date have been able to show that the resistivity of a metal in the superconducting state is less than $10^{-22}$ $\Omega\cdot$cm (compared to the resistivity of pure copper at low temperature, $10^{-9}$ $\Omega\cdot$cm) (2). Before long, a number of other elements were also found to exhibit this same phenomenon at similar low temperatures. A list of a few of these materials, called superconductors, with their transition (or critical) temperatures, is given in Table 1 (2–4).

Even at the time of its discovery, Kamerlingh Onnes realized that the phenomenon of superconductivity could have important technological uses. However, it was soon discovered that these early superconductors (the so-called type I superconductors) remained in the superconducting state only if they were not carrying a substantial electric current ($I$, A) and if they were not in the presence of any substantial magnetic field ($H$, A/m or T: see Table 1, footnote *b*). As the external magnetic field is intensified, or as the electrical current within the superconductor is increased, its normal-mode electrical resistivity is restored at a critical value of either current or field. Hence, the superconductors were inherently unstable under certain conditions. As Silsbee suggested in 1916, these are not two unrelated phenomena, because the magnetic field generated by passing a current through the wire destroys the superconductivity at the same value as does an externally applied field (5). The temperature at which superconductivity appears in zero applied magnetic field and carrying no current is called the critical temperature ($T_c$, K). However, the practical transition temperature (i.e., the maximum temperature at which a superconductor exhibits superconductivity) is a function of both external magnetic field and the current within the wire, as shown in Fig. 1. It can be seen that practical operation of a superconductor must be made within the parameter space under the surface $F = f(I, H, T)$ shown, and thus below certain limits on temperature, current, and magnetic field.

Both the critical currents and the critical fields of the early superconductors were small. Consequently, although the discovery of superconductivity was made early in twentieth century, its practical use for producing strong magnetic fields was not realized until much later. An efficient design of superconducting magnets for large physics experiments and energy devices has had to wait until near the end of the twentieth century, (6).

By the 1950s, experimentation with intermetallic compounds and alloys had led to the discovery of materials that greatly relieved the above limitations. These materials remain in the superconducting state to somewhat higher temperatures (see Table 1) in higher magnetic fields, and also are able to transport larger electrical currents. They were given the name type II superconductors. Prior to the 21st century it was believed that the upper temperature limit of type II superconductivity was that of $Nb_3Ge$ at 23.2 K. In 2001, the discovery of superconductivity in $MgB_2$ raised that upper temperature limit of the previously called "low temperature superconductors" to approximately 40 K.

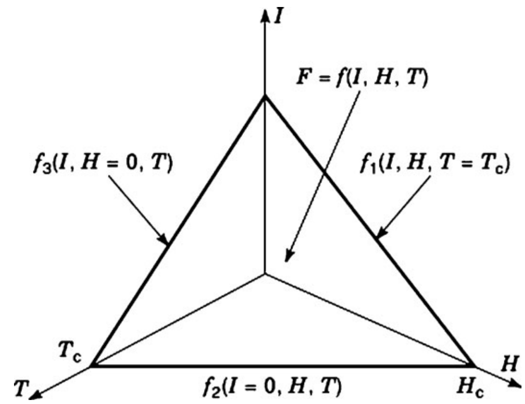

**Figure 1.** Critical linearized $F(I, H, T)$ surface of a superconductor. It should be noted that in a typical magnet-grade superconductor the critical current $I$, magnetic field $H$, and temperature $T$ are not linearly related. However, the classical theory of cryostability assumes linear relationships.

A fundamental difference in the behavior of these two types of superconductors is in the way in which a magnetic field enters the material, as illustrated in Fig. 2(a) and 2(b). In type I materials, magnetic field is excluded from the superconductor [Fig. 2(a)]. However, when any one of the limits of critical temperature ($T_c$), magnetic field ($H_c$), and current ($I_c$) is exceeded, there is an abrupt and total entry of any external field into the material as the material loses its superconductivity. In type II superconductors, above a threshold field $H_{c1}$, the field begins to enter in discrete units of field called fluxoids, and superconductivity is totally destroyed only when the field $H_{c2}$, has completely entered the superconductor [see Fig. 2(b) and 2(c)].

The important characteristic of type II superconductors (alloys such as NbTi or intermetallic compounds like $Nb_3Sn$) is their capacity to sustain high transport currents (7), which makes them suitable for use in high-current devices. However, for large enough magnetic field (and/or current) these superconductors eventually pass into the normal state as well. Consequently, these superconductors can also become unstable. For $H < H_{c1}$, a type II superconductor is in the superconducting state; for $H_{c1} < H < H_{c2}$, the superconductor is in a mixed state (the magnetic field penetrates into the regions existing in the normal state, but bulk superconductivity is not extinguished); and for $H > H_{c2}$, the superconductor is in the normal, resistive state [Fig. 2(c)]. Type I superconductors have a critical field, $H_c$, below which there is no field within the material and it is superconducting [Fig. 2(a)]. In contrast, type II superconductors have two critical fields: the lower critical field $H_{c1}$, at which the magnetic field begins to move into the superconductor, and the upper critical field $H_{c2}$, at which the penetration is complete and superconductivity is de-

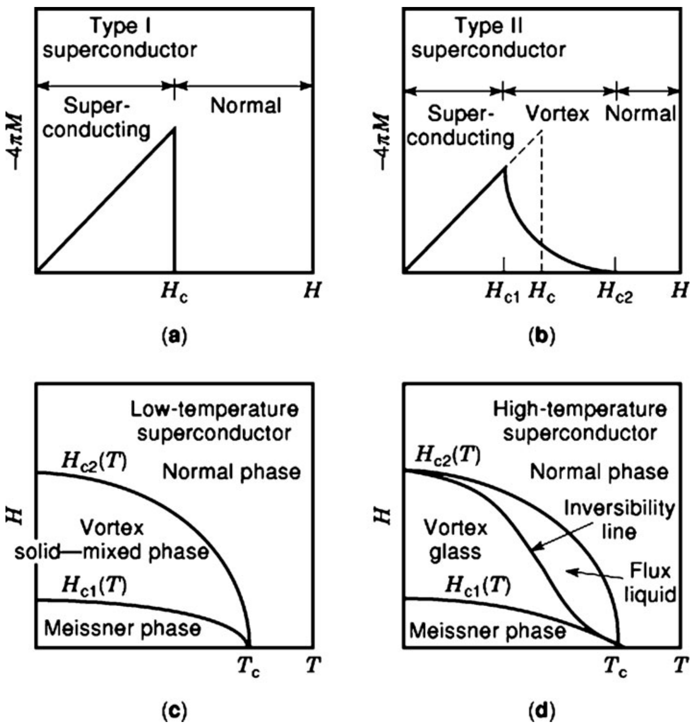


**Figure 2.** Superconductor types. (a) Magnetization (magnetic moment per unit volume, $M$; $4\pi M$ is also used) versus magnetic field for a type I superconductor. The magnetization decreases abruptly from the Meissner value to zero. (b) Magnetization versus magnetic field for a type II superconductor. The magnetization decreases monotonically from the Meissner value, hence providing three distinct regions: (1) the Meissner state (magnetic flux completely excluded), (2) the vortex state, and (3) the normal state. In the vortex state magnetic flux penetrates in the form of vortices but the material is still superconducting. The dotted line represents the comparison with the case for a type I superconductor. (c) Magnetic field versus temperature fora low-temperature superconductor. (d) Magnetic field versus temperature for a high-temperature superconductor.

stroyed; see Fig. 2(b) and (c).

An important physical property of the intermetallic compound type II superconductors is that they are brittle and their fabrication into useful shapes may require that the superconducting compound be produced by heat treatment after the wire has been formed in its final shape (3). In the case of $Nb_3Sn$, the wire is drawn to its final dimension before heat treatment causes the reaction between the Nb and the Sn to form the compound.

In the early 1980s new compounds were discovered that were able to retain superconductivity up to very much higher temperatures. The first of these compounds was $La_{2-x}Ba_xCuO_4$ with a critical temperature of 35 K (8). Since that time, other compounds with higher and higher operating temperatures have been developed. Now, practical superconductors that retain superconducting properties to temperatures near 135 K are available (4, 9). These *high-temperature superconductors* (*HTSCs*) are different from the conventional superconductors in that they are complicated oxide compounds. In addition, they are granular and exhibit their superconductivity only along certain planes within the crystal. Thus, in an application requiring a significant length, it is necessary to fabricate the finished superconductor with the grains aligned along the superconducting plane with no more than about 7° misalignment (9). A greater angle between the grains results in weak links between them, which drastically limits their ability to transmit current. Methods of addressing the alignment problem have been developed.

One type of HTSC consists of the oxides of barium, strontium, calcium, and copper, commonly referred to as *BSCCO*, one example being $Ba_2Sr_2Ca_2Cu_3O_8$, or 2223BSCCO. For the BSCCO compounds the oxide-powder-in-tube (*OPIT*) process (4) has succeeded in reducing the weak link problem to a manageable level by producing continuous conductors in lengths up to one kilometer capable of carrying acceptably high current densities. However, in a plot of critical magnetic field versus temperature, only a part of the area under the curve showing the phase space for superconductivity is available for the transport of current. Such a plot is shown in Fig. 2(d), which includes an additional line called the line of irreversibility, which varies considerably from one compound to another. This line shows a practical limit of current-carrying capability in that above it the flux is no longer pinned and the current-carrying capability vanishes, even though operation may still be within the superconducting envelope. The line of irreversibility is quite low for the highly anisotropic BSCCO: operation in magnetic fields above 1 T is only possible at temperatures below 40 K. Another HTSC is $YBa_2Cu_3O_7$, or YBCO. The more isotropic YBCO compound has a much higher line of irreversibility, making possible operation at useful current densities in magnetic fields up to 5 T at 77 K (liquid nitrogen temperature). The OPIT process does not work to reduce weak links in YCBO, and useful YBCO conductors have only been produced by deposition onto specially textured substrates (10).

Like the intermetallic compounds of the type II superconductors, the HTSCs are brittle and difficult to fabricate. A major part of the effort to produce useful HTSCs has been toward their fabrication into useful shapes (9).

## EXPERIENCE FROM EARLY MAGNET APPLICATIONS

The discovery of the type II superconductors allowed the superconducting state to be maintained in the presence of higher currents and at more elevated magnetic fields (compared to the previous situation with type I superconductors). Applications of superconductivity have been made in instrumentation devices, and a considerable amount of investigation into superconducting electric power transmission lines has been done (11, 12), but no such power lines have been built to date. The greatest application of superconductivity on a large scale has been in the building of magnets with very high field capabilities (13).

The achievement of the higher operating parameters of the type II superconductors was not in itself sufficient for the construction of high-field magnets with maximum performance. All the superconducting magnets designed until the early sixties of the twentieth century, without exception, suffered from so-called degradation, i.e., from the loss of the superconducting state before reaching the full design field. This phenomenon has been attributed to the fact that internal and/or external disturbances trigger an irreversible instability causing the appearance of normal zones followed by a so-called quench (the destruction of the magnetic field, with the stored energy being converted to heat). Degradation can be attributed to a number of sources, which may be mechanical, magnetic, or thermal in nature. Any source of heat in the system can cause local temperature increases in the superconductor to the point where the appearance of a normal resistance region is imminent. In early attempts to build large magnets it was discovered that because of effects like these, the performance of superconducting wires fell far short of the short-sample results, a phenomenon called coil degradation (14). A magnet would quench after reaching only a fraction of its design field. However, after each such quench, the next try resulted in a higher operational field. After repeated attempts, the field attained can be considerably higher than that attained at first, but not as high as the short-sample results. This phenomenon is called *training*.

At first it was thought that degradation was the result of weak spots in the long lengths of superconducting wire. However, this would not explain training, and production techniques have been shown to produce remarkably uniform properties in long lengths of superconducting wires (many kilometers, even in small magnets). It was concluded that degradation must be caused by one or more of a number of possible disturbances, such as a source of heat or the penetration or rearrangement of the magnetic field.

Such disturbances can lead to severe consequences, in which both functional and mechanical integrity of the device can be compromised: It can become unstable. An engineering application cannot tolerate such an instability. It has become obvious that a new technological solution is needed to overcome these difficulties.

## CAUSES OF INSTABILITIES (DISTURBANCES)

There are several mechanisms that can cause the generation of heat within a superconductor carrying current in a magnetic field. Because of the very low heat capacity of most materials at the low operating temperature, the generation of heat can cause a sufficient rise in temperature to create a local zone in the superconductor that is above the critical temperature and therefore has normal resistance. The normal zone then becomes an additional source of heat because of Joule heating ($I^2R$, $R$ being the electrical resistance, $\Omega$). The consequent increase in temperature can exacerbate one or more of the processes creating the heat. This positive feedback (without a stabilizing influence) can lead to a runaway situation. Regardless of the cause of instability, an uncontrolled growth of the normal zone will promote further heat generation, and in the worst case cause a quench, and all of the energy in the entire assembly, which may be hundreds of megajoules, can be released as heat, generating a pressure increase in the cooling medium and overheating the entire assembly, possibly even catastrophically. The prevention of such an occurrence is a task of stabilizing the superconductor assembly.

Disturbances to superconducting systems can be abrupt and cause local heating, so that a maximum local effect results. Other disturbances are prolonged in nature and tend to heat larger portions of the system, so that their effect is distributed more widely. Causes of disturbances have been classified as magnetic, mechanical, and thermal, as indicated in Fig. 3.

The forces generated by the magnetic field tend to stretch the magnet structure radially while causing it to contract axially (similar to the reaction to a pressure increase of a cylindrical pressure vessel with ends closed by sliding pistons). This straining of the magnet system can cause sliding friction, or even the sudden rupture of components such as epoxy used for potting. Any sudden motion like this will generate heat as the motion is stopped. Once the external magnetic field reaches $H_{c1}$, magnetic field penetration begins and the motion of the fluxoids also generates heat. Because the magnet system is operated well below ambient temperature, there is a constant input of heat, which may heat some portions of the system unequally. Any one of these disturbances can result in a normal zone within the superconductor.

### Flux Jumps

The most serious disturbance is caused by flux jumps. In the mixed state, magnetic flux is present within the superconductor. In the presence of a current passing through the superconductor the fluxoids are acted upon by the Lorentz force, which is perpendicular to the surface of the wire and is proportional to the product of the current density $J$ (A/m$^2$) and the magnetic induction $B$ (T)—in vector form, $J \times B$. The fluxoids are pinned to the crystal latice by imperfections or impurities, so that motion does not occur unless a disturbance creates a location where the Lorentz force exceeds the pinning force.

Flux-jump instability is a characteristic of the superconductor itself, not the magnet as a whole. According to Wada et al. (15), fluxoids can be defined as quantized magnetic flux lines distributed within a type II superconductor. Degradation caused by a flux jump is related to a sudden motion of the fluxoid within the superconductor (16). The collective, discontinuous motion of fluxoids in such a superconductor can be caused by mechanical, thermal, magnetic, or electrical disturbances. Breakaway of the moving fluxoid vortices is inherently associated with Joule heating. The increase of the local temperature may be sufficiently large to cause an avalanche of a fluxoid movement, thus creat-

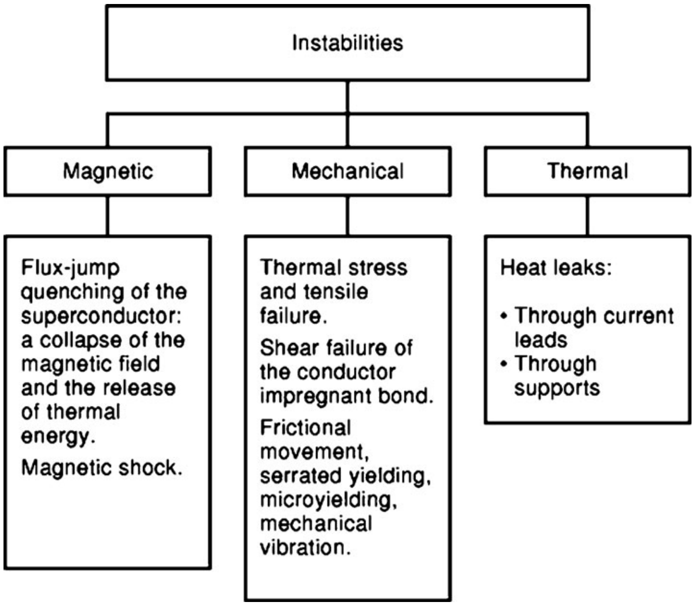


**Figure 3.** Instability sources. The instability disturbances are either magnetic (notably a flux jump), mechanical (mostly responses to thermal stresses), and thermal (such as heat inleaks).

ing additional Joule heating and a subsequent increase in temperature. This redistribution of the magnetic field in the superconductor is called a flux jump.

## Mechanical Disturbances

Mechanical influences may be responsible for a disturbance in a superconductor that can lead to instability (17). For example, during the cooldown period before the startup of operation, as well as during the subsequent period of charging, stresses of various origins are always present in superconducting windings. These may be the thermoelastic stresses caused by the difference in thermal contraction of composite components such as superconductors, normal conducting material (e.g., copper), and epoxy impregnate. Similar stresses can arise from thermal gradients within the system, and stresses may arise during the manufacture of the composite conductor (drawing) and magnetic coils (winding and banding). Stresses are also caused by magnetic forces during charging. Small conductor movements activated during field cycling may cause local heating (18).

These stresses may lead to yielding or even tensile failure of a component in the magnet system such as the potting resin, or to shear failure of the bond between an epoxy impregnant and the conductor. If thermal or magnetic stresses become great enough to cause some of the magnet material to yield, the yielding can take place in a series of discontinuous jumps (serrated yielding), which, because of their rapidity, cause rapid heating. Heat is produced as the motion is arrested. Because of the low temperature, many materials are subject to cold embrittlement. Brittle fracture can be more serious than serrated yielding because the energy is all deposited locally.

The friction between the metallic and insulated surfaces can cause local heating. If this stress results in a stick–slip process, the effect will be similar to the brittle fracture mentioned above.

Any of these mechanisms can constitute a disturbance energy that may be strong enough to initiate a quench. Studies of thermomagnetomechanical instabilities are numerous (19–22); however, the problem is open for further studies.

## Distributed Heat Sources

Some of the above disturbances can result in slower, more distributed heat sources. However, there are additional sources of heat that are distributed widely through the magnet system. Although these heat inputs may not directly cause local heating within the magnet system, the heat transfer to the coolant must be sufficient to absorb the corresponding energy input.

**Heat Leaks.** Because essentially all superconductors are operated at temperatures in the cryogenic range, an ever present source of heat is the heat that enters the system through the thermal insulation, piping, and the support system. No insulation is perfect, so that this heat source is inevitable. Where, and how much, heat enters is determined by the design of the cryogenic system. The heat leak through the thermal insulation is usually distributed uniformly. However, the heat admitted through current leads, instrumentation, and piping may be more localized.

**Hysteretic Losses.** In some cases superconductors must carry an alternating or pulsed current. In these cases, a continuous energy loss occurs and manifests itself as a heat source because of the Lorentz force on the unpinned flux lines that move in and out of the superconductor as a result of the varying current. These losses are called hysteretic losses (3). They are influenced by the roughness of the surface of the superconductor. The smoother the surface, the lower the losses. Even more important is the twist pitch of the superconducting filaments and the resistivity of the matrix material (discussed below).

## METHODS OF STABILIZATION

Because there are inevitable inputs of heat to the system as well as disturbances that can affect the temperature of the superconductor, it is necessary to provide methods to maintain the system at its operating temperature to prevent the catastrophic loss of the superconductivity. In the case of a superconducting magnet, the key problem of stability is that of sustaining the successful operation of the magnet system without loss of the magnetic field and without damage to the system. The problem is not only a possible appearance of normal zones within the superconductor, but the need to sustain conditions for the reestablishment of the superconducting state after the appearance of instability. Furthermore, the continuous operation of the device during the transition period must be assured.

*Flux-jump stabilization* suppresses an initiation of a cycle of disturbances that may cause the transition into the normal state. Thus, it is directed toward the prevention of an instability. The role of *cryostabilization,* in contrast, is to restore the superconducting mode of operation, once a disturbance has already initiated the existence of a normal zone.

There are several ways to approach the problem of stabilizing a superconducting system. First, the design of the superconductor wire can be used to minimize, or possibly even prevent, the damaging effect of a flux jump. The energy released during the passage of the flux lines through the conductor is proportional to the distance traveled across the wire. Thus the conductor is usually fabricated to consist of many very fine filaments (10 $\mu$m to 100 $\mu$m in diameter). These filaments are distributed within a matrix of a metal, which, while not a superconductor, does provide a highly conductive path for the current in case the superconductor can no longer handle the entire current. Typically the matrix metal is made of copper. However, for HTSCs silver is also used. One of the ac loss mechanisms arises from a coupling of the currents in adjacent superconductor filaments, and this loss can be minimized by twisting the superconductor composite so that the filaments are transposed along the length of the wire (3). An additional measure that can be directed toward a suppression of the ac losses is adding a more resistive metal, such as Cu–Ni, around the outer surface of the superconductor, thus decoupling the superconductor filaments and reducing the eddy currents flowing in the normal metal matrix (3).

Second, heat generated within the magnet system must be removed to maintain an acceptable operating temperature. This cooling can be provided by pool boiling in a suitable cryogenic fluid, or by the forced flow of the cryogen, either as a liquid, or as a fluid at supercritical pressure to avoid the problems of two-phase flow. In some cases where very low-temperature operation is required, use has been made of superfluid helium to obtain its excellent heat transfer capabilities. In this case an operating temperature in the vicinity of 2 K must be used.

The stabilization of the HTSCs utilizes the same methods as those for type II superconductors. In one respect, the task is eased by the fact that the specific heats of the magnet system become considerably higher as the temperature is raised. Thus the temperature increase from a given amount of heat is less than it would be at lower temperature. Another factor in favor of stabilization comes into play when the system is operated at the higher end of its temperature range. In this case, the current cannot be as high, which results in lower Lorentz forces, and thus the likelihood of flux jumps is diminished.

### Cryogenic Stabilization

According to Reid et al. (23), cryogenic stabilization is achieved if, after the release of a certain amount of Joule heat and a local rise in temperature (caused by either internal or external perturbations), efficient cooling is provided to remove that thermal energy more rapidly than it is generated. This goal assumes balancing of the following energy flows: (1) the energy brought to an intrinsically stable superconductor in the form of a thermomechanical disturbance or thermal energy from any source, including that generated by the current that is redirected into the stabilizing, resistive matrix, and (2) the energy removed from a superconductor element by convective cooling and by conduction. These processes can be highly transient in nature; indeed, their duration is usually very short (on the order of $10^{-3}$ to $10^{-2}$ s). Consequently, the use of a steady-state energy balance should lead to a very conservative stability criterion.

Cryostabilization can be full or limited. Full cryostabilization means stable operation after the entire conductor has been driven normal by a large disturbance (23). Design based on full cryostabilization is as a rule the most conservative and involves large conductors. Limited cryostabilization refers to recovery from a disturbance of limited size. By using the *stability criteria,* one can decide whether the superconductor is going to be stable (in the sense of either full or limited stabilization) or unstable. For example, full cryostability can be defined either using the so-called Stekly criterion or the Maddok–James–Norris model (see below).

All the early-developed stability criteria discussed in the following sections have been based on quasi-steady-state balances. The theory behind these static criteria will be (somewhat arbitrarily) called the classical theory of cryostabilization. The formulation of a transient problem and some of the issues involved will be given subsequently.

**The Stekly Criterion.** Stekly and his collaborators (24–26) were the first to formulate a method to prevent a catastrophic quench in a magnet. They discovered that reliable stabilization can be achieved by the simultaneous application of two measures: (1) an alternate path for the current through an adjacent material (such as copper) with high electrical conductivity (although nonsuperconducting) on the appearance of normal zones in the superconductor, and (2) very efficient cooling, such as in liquid helium. The so-called current sharing between a superconductor that abruptly loses its superconducting capability (becoming highly resistive) and the normal conductor (the matrix in which the superconductor is embedded, having a smaller resistivity than the adjacent superconductor in its normal state) secures a continuation of the magnet operation. By the intense cooling, the conductor can then be cooled back below the critical temperature for a given magnetic field and electrical current. This, in turn, leads to a reestablishment of the superconducting state in the coil. Therefore, heat transfer to a cooling stream in addition to conduction through the conductor and the assembly will remove the generated heat and provide the conditions for the superconductor to reassume the total current flow, along with a disappearance of the current flow through the normal conductor, but without losing the magnetic field. Consequently, the coil becomes stable, and no quench will result. This approach constitutes *cryogenic stabilization.*

***The Concept of Stability and the Energy Balance.*** The key modeling tool needed to define exactly the concept of stability is the energy balance of a conductor and/or magnetic device (a coil), which may be described in general as follows:

$$\text{Time rate of change of thermal energy of the conductor} \\ = \text{conduction heat transfer rate} \\ + \text{rate of thermal energy generation} \\ - \text{convection heat transfer rate}$$

This balance assumes a composite conductor material surrounded by a coolant and subject to transients as well as internal and external instabilities. The goal of a design is to keep the operating point of the conductor within the limits imposed by the critical surface $(T, I, H)_c$ but with an additional requirement formulated as follows: If the disturbance upsets this operation, the restoration of the superconducting state is still possible.

Let us first formulate the classical theory of cryostability, the concept originally introduced by Stekly and collaborators (25, 26). We will not present this approach either in its entirety or in a chronological perspective. Rather, we will discuss the main points, which depend upon an energy balance that is astonishingly simple compared to the complexity of the superconducting instability phenomena. This simplicity commends this criterion as the most conservative of several that have been developed over the years.

First, the physical background for the analysis should be emphasized. Three distinct physical situations can be distinguished in a conductor operating at a given magnetic field $H$, carrying constant current $I$, and operating at various temperatures $T$. These situations are indicated in Fig. 4(a) as a superconducting mode, a current-sharing mode, and a normal mode.

In a (completely) superconducting mode, the conductor's operating current is less than critical, for a less than critical magnetic field, at a less than critical temperature. In a current-sharing mode, current flows partly through the matrix because of the appearance in the superconductor of normal zones characterized by a resistance that is much higher than the resistance of the matrix. Finally, when the temperature increases above the critical temperature, the superconductor operates in the normal mode and the current is carried exclusively by the matrix.

In an adjacent diagram [Fig. 4(b)], the corresponding distribution of thermal energy generated by Joule heating is presented. In the superconducting-mode Joule heating is zero. In a current-sharing mode thermal energy generation per unit of area of the conductor surface is equal to $G_A = II_m(\rho/AP)_m = I(I - I_c)(\rho/AP)_m$ (W/m$^2$), where $\rho_m$ (Ω·m) is the electrical resistivity of the resistive part (matrix), and $I_m = I - I_c$ (A) is the current through the matrix. The symbols $I$ and $I_c$ represent the operating and critical currents as defined in Fig. 4(a). The quantities $A_m$ (m$^2$) and $P_m$ (m) are the cross sectional area of the matrix and the conductor perimeter, respectively. (Note that the units for electrical resistivity, area, and length, often used in practice, are Ω·cm, cm$^2$, and cm, respectively. Then the generated thermal energy per unit of conductor area is in W/cm$^2$.) In a normal mode, say for $I = I_{c,b}$, thermal energy generation has a constant value of $I^2_{c,b}(\rho/AP)_m$.

To start the analysis we introduce a series of far-reaching assumptions but still preserve the main features of the process:

- The conductor is a composite, that is, it consists of a superconducting *core* and a highly conducting (but nonsuperconducting) *matrix.*
- The heat transfer phenomena involved, including the eventual release of thermal energy caused by a disturbance, are quasisteady.
- The temperature of the conductor is uniform across its cross section, and there is no heat conduction through the conductor.
- The electrical resistance between the core and matrix of the conductor is negligible.
- The electrical resistivity of a normal zone in the superconductor is several orders of magnitude larger than the resistivity of the matrix (thus, if the conductor operates in a normal, resistive mode, the electrical current tends to flow through the matrix).
- The electrical and thermal properties of the conductor materials and kinetic properties of the processes involved are independent of temperature.

The general energy balance equation in this case should include only thermal energy generation and heat convection, that is,

$$\text{rate of thermal energy generation} \\ = \text{heat transfer rate due to convection}$$

The concept of cryostability assumes that the rate of thermal energy generation caused by instability is, in a limit, equal to the rate of heat removal by either pool boiling or forced convection of liquid helium (usually supercritical, and in some cases superfluid). This assumes that current may flow partly through the superconductor and partly through the matrix (current sharing) or it may be completely rerouted to the matrix if the superconductor becomes resistive (normal mode of operation). If the energy generated by Joule heating (both in the superconductor and in the matrix, in the current sharing mode, or entirely

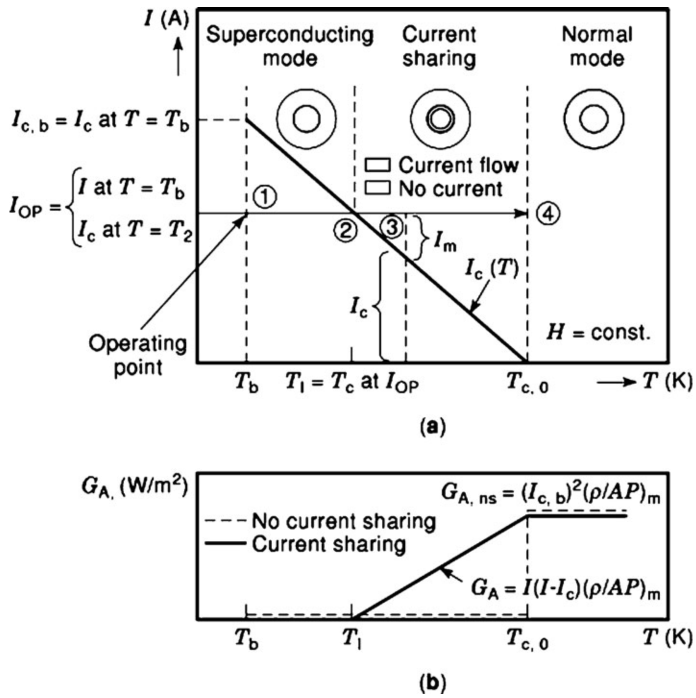


**Figure 4.** (a) *I–T–H* characteristic of a conductor: a linearized relationship $f_3(I, T, H$ fixed); see also Fig. 1. Increase in the local temperature at given operating current $I$ forces a superconductor to change from superconducting mode (state ①) to a current-sharing mode (state ③) and ultimately into the normal mode (state ④). In a current-sharing mode a part of the current flows through the resistive nonsuperconducting matrix and a part through the partially resistive superconductor. In the normal mode current flows only through the matrix, being excluded from the resistive superconducting part by its much higher electrical resistivity. (b) Thermal energy generation by Joule heating versus temperature for a given current. In the case of no current sharing, Joule heating starts abruptly at, say $T_{c,0}$ (dotted line). In the case of current sharing, thermal energy generation increases linearly with $T$ in the current-sharing zone. In the superconducting mode there is no Joule heating ($G_A$ = 0). In the normal zone the Joule heating has a constant value (for a given current).

through the matrix, in the normal mode) is more than compensated by the heat removed from the conductor, its temperature will return to below the critical temperature, and the conductor will stay stable. So the stability criterion can be expressed as follows:

$$\frac{\text{rate of thermal energy generation}}{\text{heat transfer rate due to convection}} \equiv \alpha$$

$$\alpha \begin{cases} < 1 & \text{(stable)} \\ = 1 & \text{(energy balance)} \\ > 1 & \text{(unstable)} \end{cases}$$

where $\alpha$ is the so-called Stekly stability parameter. The above-defined cryostability criterion can be written in an explicit form taking into account the energy generation under the condition that all the current is flowing through the matrix at the onset of normal mode of conductor operation. In this case the density of heat transfer rate generated within the conductor and normalized to the unit of heat transfer area between the conductor and coolant is given by

$$G_A = \frac{I_{c,b}V_L}{P} = I_{c,b}I_{c,b}\left(\frac{\rho}{AP}\right)_m = I_{c,b}^2\left(\frac{\rho}{AP}\right)_m = \frac{\rho_m J_{c,b}^2}{4} D \frac{\lambda^2}{1-\lambda} \tag{1}$$

where $V_L$ in (V·m$^{-1}$ or V·cm$^{-1}$) is the voltage drop per unit length of the composite conductor. The right-hand side in the last equality of Eq. (4) expresses the thermal energy generation in terms of the current density ($J = I/A$), the conductor diameter $D$, and the so-called filling factor $\lambda = A_{\text{superconductor}}/A_{\text{total}}$. The heat transfer rate density $Q_A$ (W/m$^2$ or W/cm$^2$) on the matrix surface is

$$\dot{Q}_A = h(T_{c,0} - T_b) \tag{2}$$

Therefore, the Stekly parameter is given as

$$\alpha = \frac{\rho_m I_{c,b}^2}{hPA_m(T_{c,0} - T_b)}$$
$$\alpha \begin{cases} < 1 & \text{(stable superconducting state)} \\ = 1 & \text{(normal-state equilibrium)} \\ > 1 & \text{(unstable state)} \end{cases} \tag{3}$$

When a composite conductor operates under current sharing, the total current $I$ is the sum of the currents flowing through the superconductor, $I_c$, and the matrix, $I_m$. In general,

$$I_m = I - I_c: \quad \begin{cases} I_m = 0, \quad I = I_c & \text{(superconducting)} \\ 0 < I_m < I & \text{(current-sharing)} \\ I_m = I & \text{(resistive)} \end{cases} \tag{4}$$

Under the current-sharing conditions, the voltage drop per unit length of the composite conductor, $V_L$, is given as follows (27):

$$V_L = I_m \frac{\rho_m}{A_m} = (I - I_c)\frac{\rho_m}{A_m} = \left[I - I_{c,b}\left(1 - \frac{T_I - T_b}{T_{c,0} - T_b}\right)\right]\frac{\rho_m}{A_m} \tag{5}$$

Here a linearity of the critical *I*–*T* curve has been assumed.

Equation (8) can be rearranged as follows:

$$\frac{V_L}{I_{c,b}(\rho_m/A_m)} = \frac{I}{I_{c,b}} - 1 + \frac{T_I - T_b}{T_{c,0} - T_b} \tag{6}$$

or

$$v = i - 1 + \theta \tag{7}$$

where $v = V_L/[I_{c,b}(\rho_m/A_m)]$, $i = I/I_{c,b}$, and $\theta = (T_I - T_b)/(T_{c,0} - T_b)$ are the reduced values of the voltage drop, current, and temperature, respectively. The modes of conductor operation can be presented formally using a current-sharing factor (27) *f* defined as follows:

$$\begin{aligned} f &= \frac{I_m}{I} = \frac{I - I_c}{I} = 1 - \frac{I_c}{I} = 1 - \frac{I_c}{I_{c,b}}\frac{I_{c,b}}{I} \\ &= 1 - (1-\theta)\frac{1}{i} = \frac{i - 1 + \theta}{i} \end{aligned} \tag{8}$$

and consequently,

$$f = \begin{cases} 1 & \text{(resistive)} \\ \dfrac{i - 1 + \theta}{i} & \text{(current-sharing)} \\ 0 & \text{(superconducting)} \end{cases} \tag{9}$$

From Eqs. (5) and (6), after rearrangement and introduction of the reduced voltage drop, current, and temperature, as well as the Stekly parameter, one can obtain

$$v = \frac{\theta}{\alpha i} \tag{10}$$

By eliminating the reduced temperature from Eqs. (10) and (13) one can obtain the reduced voltage as a function of the reduced current in the matrix. This situation describes the variation of voltage with current. An interesting insight can be gained from the *v*–*i* diagram using the relations given by Eqs. (10) and (13) and recognizing that the $v = i$ line in that diagram depicts the normal state [Fig. 5(a)]. The voltage–current relationship features are described in the legend of Fig. 5(a).

If the current is assumed to be fixed, an insight into a conductor modes of operation can be gained by using a diagram of the reduced thermal energy generation $g = G_A/[h(T_{c,0} - T_b)]$, and/or the reduced heat transfer rate Ref. $q = Q_A/[h(T_{c,0} - T_b)]$, versus reduced temperature [Fig. 5(b)], according to Gauster as reported in Ref. 27.

Finally, it should be pointed out that several definitions of the stability factor exist in the literature. In addition to the Stekly parameter $\alpha$, a stabilization parameter $\xi = 1/\alpha$ can be used (28):

$$\xi = \frac{\varphi_p P_m A_m h(T_{c,0} - T_b)}{\rho_m I_{c,b}^2} \tag{11}$$

where also a parameter $\phi_p$ has been introduced, equal to the fraction of matrix perimeter actually exposed to coolant. If the stabilization parameter as defined by Eq. (14) is to be used, note that $\xi > 1$ implies a stable system and $\xi < 1$ an unstable one. According to Ref. 17 the stabilization parameter $\xi$ can be defined either at the critical temperature and the operating current, or at the critical temperature and zero current and magnetic field. In addition, a separate parameter can be defined for the current-sharing case.

Prediction of the stability using the Stekly cryostability parameter has opened a new era in building large superconducting magnets. From Eq. (6) is obvious that, for a given superconductor and matrix material (usually copper) as well as for the defined cooling conditions, to achieve

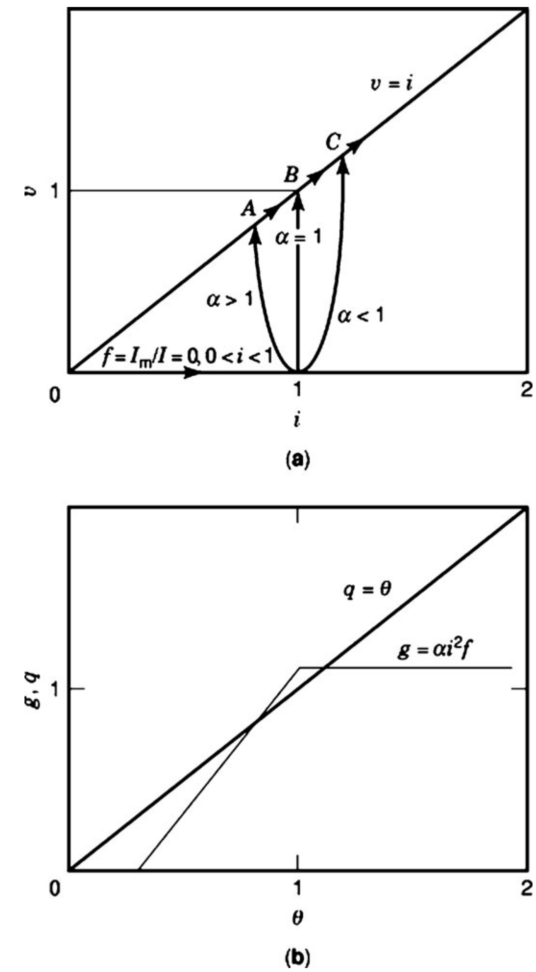


**Figure 5.** Reduced characteristics. (a) Reduced voltage versus reduced current. If the conductor is fully superconducting (i.e., $f = I_m/I = 0$), then $0 < i < 1$ (line 0–1). Current-sharing cases [$f = (\theta + i - 1)/i$, for linear $I_c$ versus *T*] correspond to the range of reduced current $1 < i < 1/\alpha^{1/2}$. Full cryostability corresponds to the 0–1–*C* curve ($\alpha < 1$) and sufficient cooling. The 0–1–*A* curve ($\alpha > 1$) corresponds to insufficient cooling. For $\alpha < 1$ the reduced voltage $v$ is a single-valued function of $i$. With an increase in $i$ from 0 to 1 the superconductor quenches at $i = 1$. With decreasing current while in normal mode (along the $v = i$ line) the conductor switches back to the superconducting mode at $i = 1/\alpha^{1/2}$. (b) Reduced thermal energy generation $g$ and reduced cooling rate $q$ versus reduced temperature $\theta$ for a fixed reduced current (for $i > 1/\alpha^{1/2}$, $\alpha > 1$). Stable conditions for $g < q$.

greater stability, one should increase the ratio of matrix to superconductor in the cross section of the composite conductor. This criterion has led to very conservative design ($\alpha$ can be smaller than 0.1, i.e., $\xi$ larger than 10), leading to very expensive solutions.

Let us consider an application of the Stekly criterion in an actual conductor design (for more details consult Ref. 27). For a typical critical current density of the order of magnitude of $10^5$ A/cm$^2$ and a standard cooling heat transfer rate of $2 \times 10^{-1}$ W/cm$^2$ (a composite conductor with a low-temperature superconductor cooled by liquid helium), a change of the critical current between $10^2$ A and $10^4$ A leads to conservative conductor designs as follows. The ranges of the conductor diameters and the copper-to-superconductor ratios are between 1 mm and 21 mm and between 7 and 35, respectively. The design constraint assumes $\alpha = 1$. These rather large matrix-to-superconductor ratios can be reduced by further reducing the operating current (the standard operation is at $i = 0.75 \pm 0.2$). A more elaborate theory, based on the same approach, takes into account thermal resistance between the matrix and the superconductor and their actual sizes. Still, the stability criterion stays conservative, $\alpha \leq 1$. It should be noted that the transition from nucleate to film boiling of the coolant (liquid helium) may cause instability when $\alpha > 1$ if the current sharing is not complete.

**Maddock–James–Norris Equal-Area Theorem.** The Stekly cryostability criterion has been developed taking into account removal of the generated thermal energy from a conductor only by convection, that is, the conduction mechanism has been neglected. This is a consequence of the assumption of the existence of a uniform temperature throughout the conductor. This assumption, however, can easily be violated, because the matrix material (usually copper) is thermally conductive. Hence, a question may be posed: How must the steady-state stability criterion be formulated if the conduction effect is to be taken into account? An elegant approach to the solution of this problem was given by Maddock et al. (29). We will summarize a one-dimensional formulation as introduced by Maddock (29), although a two-dimensional (or for that matter a three-dimensional) formulation can be readily devised.

Let us assume that a composite conductor carrying constant current $I$ and with one-dimensional geometry (i.e., a long cylindrical rod with $D/L < 1$, $D$ being its outer diameter and $L$ its length) is submerged in a liquid helium pool. Let a disturbance, caused by an instability, initiate the appearance of a normal zone in the superconducting part of the conductor, thus leading to current sharing. Thermal energy generation caused by Joule heating in the normal current-carrying parts of the conductor will cause an increase of the conductor temperature. Eventually, when all the current starts flowing through the matrix, any further increase in temperature will be accompanied by a constant thermal energy generation $G$ (the current intensity is constant). Due to the conductor surface temperature increase, the boiling heat transfer mode of liquid helium may change from nucleate (at an initially small temperature differences) to film boiling (at more pronounced temperature differences). The boiling characteristic (i.e., the curve of heat

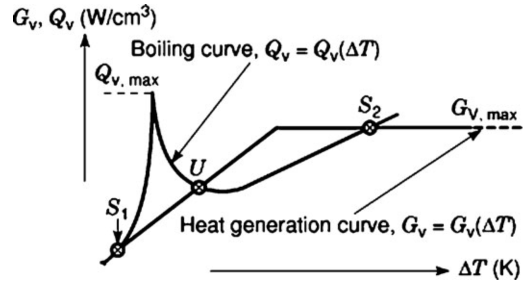


**Figure 6.** An equal-area theorem: boiling characteristic curve ($Q_V$) and thermal energy generation curve ($G_V$). The area between the $Q_V$ and $G_V$ curves on the segment between the points $S_1$ and $U$ represents an excess of heat transfer rate removed by boiling at the cold end. The area between the same curves in the zone $U$–$S_2$ represents an excess of thermal energy generated at the hot end. Equality of the two energy rates (equality of the corresponding areas) corresponds to a balanced condition $Q_V = G_V$ over the entire conductor.

transfer rate density versus temperature difference) will determine the rate of heat removal. In Fig. 6, both thermal energy generation and heat transfer rate are presented as functions of temperature difference between the conductor surface and the liquid helium pool (note that $T_b$ remains unchanged; consequently the axis in Fig. 6 may be considered as the conductor surface temperature with $T_b$ as a fixed parameter).

The area enclosed between the thermal energy generation curve and the boiling (heat transfer) curve over the segment $S_1$–$U$ is proportional to the excess thermal energy removed from the conductor. Similarly, the area enclosed over the segment $U$–$S_2$ between the thermal energy generation curve and the corresponding film boiling segment of the boiling characteristic curve is proportional to the excess of thermal energy generated within the conductor over the heat transfer rate density removed by boiling. Because of the temperature difference between the hot and cold ends, heat conduction will take place along the conductor, leading to a removal of the excess thermal energy from the hot end by the excess of boiling heat transfer from the cold end. A stable equilibrium of this heat transfer process will be reached if the two excess thermal energy rates (represented by the shaded areas between the curves in Fig. 6) are exactly the same. This straightforward phenomenological description leads directly to the required stability criterion: Equality of thermal energy rates, represented by equal areas in Fig. 6, corresponds to a limiting case of a cryogenic stability. If the cold-end thermal energy rate is larger than the hot-end thermal energy rate, the conductor temperature will ultimately return to its initial temperature, equal to the liquid bath temperature, thus leading to the disappearance of the normal zone and a restoration of the superconducting mode of the conductor. The limiting condition can be interpreted as an equality of the areas in Fig. 6 with, say, conductor temperature as abscissa. This conclusion constitutes the so-called equal-area theorem introduced by Maddock et al. (29).

Formal mathematical proof of the above described theorem is straightforward. The general energy balance equation (with the uniform temperature assumption relaxed) reads as follows:

**Heat transfer rate by conduction**

**= heat transfer rate by convection**

**− rate of thermal energy generation**

In an analytical form for a one-dimensional heat transfer problem this balance reads as follows:

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] = \frac{\varphi_P P h}{A}(T - T_b) - \frac{\rho_m(T) I^2_{c,b}}{A A_m}\left(\frac{T - T_b}{T_{c,0} - T_b}\right) \quad (12)$$

where $A$ (m$^2$) is the area of the conductor cross section, and $k$ in (W/m·K) is the thermal conductivity of the matrix. The bracketed term on the left-hand side and the two terms on the right hand side are all functions of temperature; thus, in a compact form,

$$\frac{dF(T)}{dx} = Q_V(T) - G_V(T) \quad (13)$$

where $F$, $Q_V$, and $G_V$ are the conductive heat flux, heat transfer rate per unit volume of the conductor, and thermal energy generation per unit volume of the conductor, all exclusive functions of temperature. After the formal integration of Eq. (17) one obtains

$$\int_{T_1}^{T_2} k(T)[Q_V(T) - G_V(T)]dT = \int_{F_1}^{F_2} F\,dF = 0 \quad (14)$$

where we assume that the integration boundaries ($T_1$ and $T_2$) are well apart so as to reach the zones in the conductor where the temperature gradients are equal to zero. Making the reasonable assumption that thermal conductivity $k(T)$ depends linearly on temperature in the given temperature range [$k(T) = cT$], a transformation of the dependent variable in Eq. (18), $t = cT^2/2$, will lead to

$$\int_{t_1}^{t_2} [Q_V(t) - G_V(t)]dt = 0 \quad (15)$$

A geometric interpretation of the Eq. (19) is straightforward. If one replaces the abscissa in Fig. 6 using the same transformation that led to Eq. (19), then Eq. (19) represents the equality of the areas presented in Fig. 6. If this condition is satisfied (i.e., the energy balance is preserved), any excess of thermal energy generated within the hot zone of a superconductor will be compensated by the excess of the heat transfer rate removed from the conductor in the cold zone thanks to the conduction between the two parts of the conductor.

**Wipf Minimum Propagation Zone.** The concept of the minimum propagation zone (*MPZ*) was introduced by Martinelli and Wipf (30). It has since been developed into a comprehensive stabilization theory (17). The original approach was based on a simple energy balance of a superconducting material idealized to be infinite in space, but with a localized thermal energy generation source within a preexisting normal zone of finite size. This balance, in a generalized form, is as follows:

**Net heat transfer rate due to heat conduction**

**= rate of thermal energy generation by Joule heating**

Hence, the approach assumed a balance between thermal energy generated by Joule heating (within the already established normal zone) and the rate of heat transfer from the normal zone by conduction through the superconductor. Therefore, the physical size of the normal zone that evolves into a quench has been assumed to depend on a tradeoff between Joule heating in the normal zone and the heat carried out of the normal zone by conduction. As a consequence, assuming the validity of the energy balance condition described above, the normal zone will neither grow nor collapse. The normal zone defined in such a way is called the MPZ (17).

The simplest *realistic* situation is the one that leads to a conservative stabilization limit, which requires the consideration of non-current-sharing conditions, and additional convective cooling. Let us assume that a linear superconductor, with an already developed normal zone of length $2X$, is cooled by (1) a coolant at temperature $T_b$, and (2) conduction in the axial directions (Fig. 7). In such a situation, the energy balance has an additional term [see Eq. (16)]. The conductor has a circular cross section (radius $r$), with a constant electrical resistivity $\rho$ (in the normal zone) and uniform thermal conductivity $k$ (in both normal and superconducting parts). Under these conditions, the differential energy balance per unit length of the conductor is as follows (17):

$$(\pi r^2)k\frac{d^2T}{dx^2} = (2\pi r)h(T - T_b) - (\pi r^2)\,\rho J^2 \quad (16)$$

with the following boundary conditions:

1. Superconducting zone [note that in this zone the second term on the right-hand side in Eq. (21) vanishes]:

$$\begin{aligned} &\text{at } x = \pm X, \quad T = T_1 \\ &\text{at } x = \pm\infty, \quad T = T_b \end{aligned} \quad (17)$$

2. Normal zone (with extra heating $q^2_{tr}$ at $x = 0$)

$$\begin{aligned} &\text{at } x = 0, \quad \frac{\dot{q}_{tr}}{2} = -k\frac{dT}{dx} \\ &\text{at } x = \pm X, \quad T = T_1 \end{aligned} \quad (18)$$

Note also that *at* the boundaries between the localized normal zone and both superconducting zones, the temperature gradients must be equal. Hence, Eqs. (21)–(23) (and also an additional equation representing the equality of the temperature gradients at the junctions between the zones)

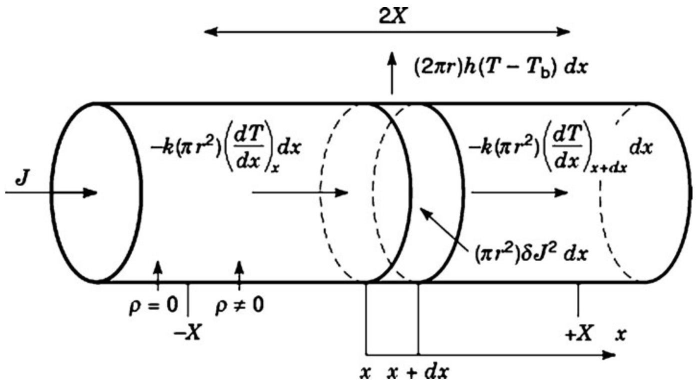


**Figure 7.** A conductor with a formed normal zone ($\rho \neq 0$) of total length $2X$. Outside the normal zone, the conductor is superconducting ($\rho = 0$). The conductor is exposed to convective cooling, and to thermal energy generation within the normal zone. The heat conduction through the conductor removes heat from the normal zone.

define the mathematical model of the temperature distribution in the conductor. The closed-form solution can be readily obtained (17). Of particular interest is an analytical relationship between the heat flux caused by heating, $q^2{}_{\rm tr}$ [defined by Eq. (23)] and the length of the normal zone. This relationship can be obtained from the first of the two equations in Eq. (23) by differentiating the temperature distribution inside the normal zone, and by subsequently applying the result at the normal zone boundary. The final result is as follows:

$$\Psi = \frac{\dot{q}}{(rk/2h)^{1/2}J^2\rho} = 2\left[\xi \exp\left(\frac{X}{(rk/2h)^{1/2}}\right) - \sinh\left(\frac{X}{(rk/2h)^{1/2}}\right)\right] \tag{19}$$

where $\xi$ is a stability parameter defined in the same manner as the one introduced by Eq. (14). The relationship given by Eq. (24) is presented graphically in Fig. 8. The abscissa in Fig. 8 represents the dimensionless length of the normal zone, $l = 2X/(rk/2h)^{1/2}$. Note that the length of the MPZ is equal to twice the normal-zone half length $X$ for the critical magnitude defined by Eq. (24), that is, for $q^2{}_{\rm tr} = 0$:

$$\mathbf{MPZ} = -(rk/2h)^{1/2}\ln(1 - 2\xi) \tag{20}$$

In Fig. 8, three distinct regions can be identified. The first region corresponds to the pairs of values of the dimensionless heating $\Psi$ and the dimensionless length of the normal zone, $l$, such that $\xi < 0.5$. The second region corresponds to the stability-parameter range $0.5 < \xi < 1$, and the third region to $\xi > 1$. In the first region, say for $\xi = 0.49$, the superconductor is in a least stable situation (15). In this region a MPZ exists for any $0 < \xi < 0.5$. At $\xi = 0.5$ the MPZ is theoretically infinitely large; see Eq. (25). The second region ($0.5 < \xi < 1$) is characterized by increasing stability. It should be noted that any $\xi$ = const curve in this region has a minimum. Hence, the left-hand branches of these lines correspond to an unstable equilibrium (between the $l = 0$ value and $l$ at $\Psi_{\min}$, for any $\xi$). The locus of these minima defines the bifurcation line between the unstable and stable equilibrium. The *maximum size of recoverable transition* (MSRT) line is defined by $X/(rk/2h)^{1/2} = -\ln(2\xi - 1)$. Finally, the third region ($\xi > 1$) corresponds to fully stable (i.e., cryostable) conditions. In that region any created normal zone is in stable equilibrium. The size of the normal zone increases monotonically with increase of the heat release per unit conductor volume. Note that $\xi \geq 1$ corresponds to the condition imposed by the conservative Stekly criterion.

The Wipf approach has been refined even further for periodic temperature distributions and for a current-sharing situation (17–31).

It is instructive to provide a simplified representation of the main features of all three stabilization criteria discussed so far (Stekly, Maddock, and Wipf). A diagram of reduced cooling rate versus reduced temperature may be used (13): see Fig. 9. There the reduced cooling rate is assumed to be a linear function of reduced temperature. The non-current-sharing case will be considered. The following situations may be distinguished. For the reduced heat generation, the one described by curve $a$, the superconductor is in a fully stable mode (a cryostable case). The limit of this type of behavior is at the condition described by the Stekly criterion (the reduced heat generation denoted by curve $b$). If current further increases (represented by an increase in reduced current $i$ in Fig. 9), the superconductor reaches the region of MRZ stability, as described by Wipf (curve $c$). Further increase in current may eventually provide the condition that corresponds to the Maddock equal-area stability criterion (curve $d$). Note that for curve $d$, the areas between the curves representing the reduced cooling rate and reduced heat generation are equal. If current increases even more, the MPZ stability zone will be reached (curve $e$).

## ADVANCED STABILITY AND THERMAL MANAGEMENT PROBLEMS FOR SUPERCONDUCTORS

The conventional theory of cryogenic stability of both low-temperature and high-temperature superconductors has not been able to address a number of issues related to the design of modern superconducting devices. Without trying to provide a comprehensive review (for the details consult the bibliography contained in Ref. 27), let us discuss briefly two characteristic topics.

As commented on in the introductory part of this article, instability phenomena are inherently transient. Consequently, the quench and recovery are also inherently transient. The existence of a normal zone, as introduced first by Wipf and others, should be considered in the light of the propagation velocity. This velocity is positive during quench and negative during recovery. Under the equilib-

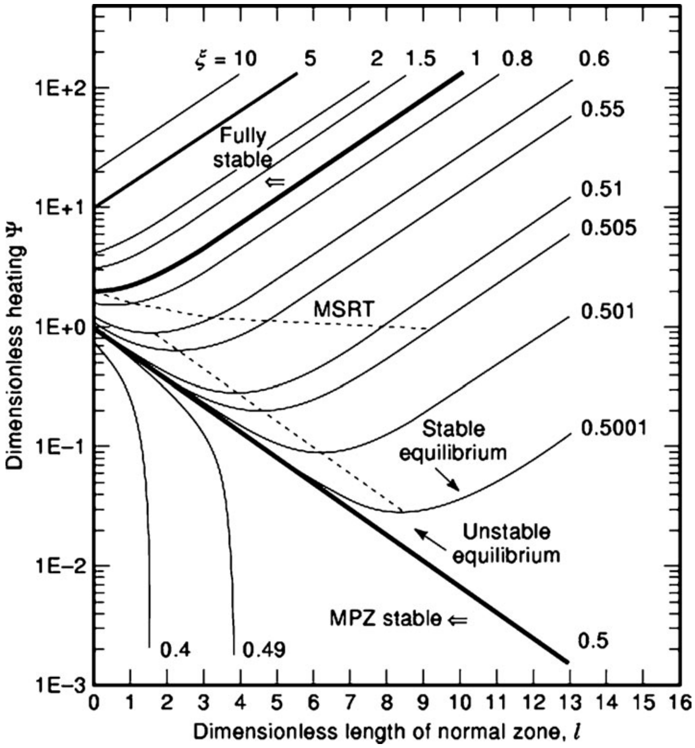


**Figure 8.** Dimensionless extra heating versus dimensionless length of the normal zone. No current sharing case. The region for $\xi \geq 1$ corresponds to a fully stable conductor (cryostable region). The region for $1 > \xi > 0.5$ respresents the so-called minimum recovery zone (MRZ) of stability. The length of the MRZ is the length of a superconducting zone in equilibrium with neighboring normal zones of infinite length. A longer superconducting zone will spread into the normal zones; a shorter one will shrink. The region for $\xi \leq 0.5$ corresponds to the so-called minimum propagation zone (MPZ) of stability. The MSRT line is the line of maximum size of recoverable transition. The locus of minima in the MRZ region provides a boundary between stable and unstable equilibrium (17).

rium conditions it is equal to zero. A good review of the early attempts to determine this velocity is provided in Ref. 27. A modern treatment of the related problems (such as the existence of the so-called traveling normal zones (*TNZ*s), the propagation velocities in the uncooled superconductors, the influence of nonlinearities introduced by temperature-dependent material properties, and the issues related to thermal and hydrodynamic management of internally cooled superconductors) is reviewed in Ref. 31. It should be added that the stability conditions defined by the conventional theory based on the presence of a continuous disturbance does not reflect the proper conditions for stability against transient heat pulses. So transient stabilization must be considered (27). An important part of these studies is an appreciation of transient heat transfer phenomena. For example (31), for very short transient phenomena (of the order of magnitude of tenths of a millisecond), the heat transfer coefficient may be in the range of

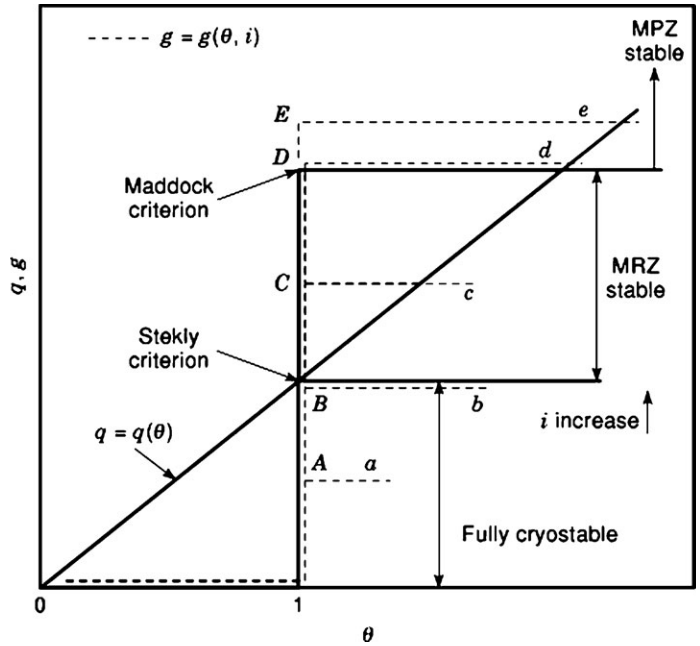


**Figure 9.** Reduced cooling rate, $q = Q_A/[h(T_{c,0} - T_b)]$, and reduced thermal generation, $g = G_A/[h(T_{c,0} - T_b)]$, versus reduced temperature, $\theta = (T_I - T_b)/(T_{c,0} - T_b)$. Curves 0–1–*A*–*a*, 0–1–*B*–*b*, 0–1–*C*–*c*, 0–1–*D*–*d*, and 0–1–*E*–*e*, (each denoted by a dotted line) correspond to various values of the reduced thermal generation *g*, each with different but fixed current. The reduced cooling curve is the same for all these cases. In a fully cryostable region (curve 0–1–*A*–*a*) the corresponding current causes a Joule heating lower than the cooling rate for any reached temperature. The stability conditions for the curve 0–1–*B*–*b* satisfy the Steckly criterion; see the curve for $\xi = 1.0$ in Fig. 8. The stability conditions for the curve 0–1–*D*–*d* satisfy the Maddock criterion (the shaded areas both below and above the cooling curve are equal to each other); see the curve for $\xi = 0.5$ in Fig. 8. This representation is valid in the absence of current sharing (compare with Fig. 8).

0.5 W/cm$^2$·K to 1.5 W/cm$^2$·K (the propagation velocity was between 5 m/s and 20 m/s). These transient heat transfer coefficients are larger by an order of magnitude than the film boiling heat transfer coefficient.

The introduction of HTSC materials brought an additional appreciation of the need to include in the analysis the temperature-dependent thermophysical properties and kinetic heat transfer characteristics over a much broader temperature range. A direct consequence of this fact in any thermal management problem for a superconductor (not necessarily related to cryostability) is a requirement to model the problem with its increasingly nonlinear character. Under these circumstances, the originally introduced assumptions must be revisited (32).

## THERMOPHYSICAL AND HEAT DATA ON CRYOGENS

Although the HTSCs can exhibit some superconducting properties even at temperatures approaching ambient, superconductivity applications under practical conditions still depend upon cooling to cryogenic temperatures. This fact does not diminish the value of the HTSCs, because of the greater ease of operation and savings in refrigeration power that are effected by operating at temperatures nearer to that of liquid nitrogen than to that of liquid helium. The theoretical power input required to produce one unit of refrigeration at cryogenic temperatures is given as $(T_a - T)/T$, where $T_a$ is ambient temperature, or the temperature at which the heat must be rejected, and $T$ is the refrigeration temperature, the temperature at which the heat is to be removed. For an ideal cryogenic refrigerator to remove heat at the temperature of liquid helium (4 K) and reject it at ambient temperature (300 K) would requires 74 W of input power per watt of refrigeration. At liquid nitrogen temperature (77 K) this ratio is reduced to a little less than 3 W power input per watt of refrigeration. As Strobridge (33) has shown, cryogenic refrigerators to date do not approach this theoretical limit very closely. The degree to which this limit is approached is not dependent upon the temperature level of refrigeration, but depends strongly upon the capacity of the unit. The larger the refrigerator, the better is the degree of approach, with the best (largest) units reaching only 35% to 40% of the limit (33).

Table 2 lists some of the thermophysical properties of cryogens (34) that might be used in the cooling of a superconducting system.

If cooling is to be done by pool boiling of a liquid, the range of temperature that is available is of interest. For a given cryogen, the boiling temperature can be fixed at any temperature between the the triple point and the critical point by maintaining the corresponding system pressure. From Table 2 it can be seen that over the range from below 4.15 K to over 150 K, there are only two gaps; from 5.25 K to 13.8 K, and from 44.4 K to 54.4 K.

Also, the application of supercritical pressure can allow forced-flow cooling without encountering two-phase flow with its consequent complications of pressure and flow oscillations and excess pressure drop in the flow channels. However, only in the case of liquid helium can this be accomplished at a pressure as low as 0.23 MPa.

The main heat transfer phenomena involved with cooling of superconducting magnets are as follows: (1) forced convection of a single-phase coolant (say, gaseous or supercritical helium), (2) phase-change heat transfer (nucleate and film boiling) in both pool boiling and channel flow conditions, and (3) heat transfer in superfluid helium. The most important mode of heat transfer for cryogenic stabilization is boiling (both nucleate and film boiling). In Table 3 typical data for (1) the critical heat flux (i.e., the maximum heat flux for nucleate boiling at the given temperature difference) and (2) the Leidenfrost-point heat flux (i.e., the minimum heat flux for film boiling) are given. It should be noted that the available heat transfer data scatter widely and depend greatly on heat transfer surface orientation, geometry, and surface conditions. Transient heat flux data for nucleate boiling of cryogens are greater by an order of magnitude than steady-state values.

## SAFETY

In working with any cryogenic fluid, safe operation requires that there be a satisfactory understanding of the hazards that can arise and also a strict compliance with safe operating principles for these fluids. These hazards can stem from the temperature of the fluid, or from its reaction with other materials that come in contact with it. The low temperatures can embrittle some structural materials, or can cause freezing of human tissue if personnel should contact the cold fluid or the exterior of a cold, uninsulated pipe. The low temperatures also cause a significant thermal contraction which, if not sufficiently compensated, can give rise to high stresses that can cause their own hazard.

Any of the cryogens that have boiling points below that of liquid oxygen can condense the atmospheric air, resulting in a condensate that is enriched in oxygen (as much as 50% oxygen) which is even more hazardous than liquid air. If this condensed air is allowed to fall on sensitive equipment, some materials can be sufficiently embrittled to crack, other equipment may no longer function properly, and, if the enriched liquid air should fall on a combustible material (such as asphalt), an explosion can result. In an improperly purged system, condensation of air, or its constituents such as water vapor or carbon dioxide, can result in the obstruction of pressure relief passages or the connections to instrumentation needed for the safe operation of the system.

The use of liquid hydrogen or liquid oxygen is less probable in the application of superconductivity. However, such an application is not impossible. Use of these fluids entails the additional hazard of combustion, or even explosion, if strict safety measures are not observed.

The most likely safety hazard to be encountered in applications of superconductivity is the excessive buildup of pressure within a cryogenic system. If a cryogenic fluid is totally confined, as in a pipe between two closed valves, the pipe must try to maintain the density of the liquid as the contained fluid becomes a gas that approaches ambient temperature. Because of the increase in the compressibility factor of the gas at high pressure, the pressure obtained is higher than what would be computed by applying the ideal gas law to the appropriate density ratios. Consequently, helium trapped in this fashion could reach a pressure of 103 MPa (15,000 psi), and nitrogen could reach a pressure of 296 MPa (43,000 psi) if the container did not rupture beforehand (35). In the case of a quench of a superconducting magnet there is a very rapid release of energy that will quickly enter the cryogenic coolant. If sufficient venting capacity is not provided, a rapid, hazardous buildup of pressure in the coolant passages or the container can result.

## ADDITIONAL READING

Several references mentioned in the bibliography deserve additional attention as useful sources for further reading. The book of Wilson (14) is a classical text and must be read as an upper-level introduction to the problems of design of superconducting magnets. A two-volume book by Collings (27) provides extensive insight into many highly technical aspects of metallurgy and physics of low-temperature superconductors, as well as a very comprehensive bibliography. The book by Dresner (31) is an advanced text that provides an insight into the modern treatment of the stability of superconducting devices. For technical calculations related to design of cryogenic devices, including cryostability aspects, the book by Iwasa (28) will be very useful. A number of useful sources can be found that deal with cryogenic and heat transfer aspects of the design of cryogenic devices. In addition to those mentioned in the bibliography, the following one may be consulted: S. W. van Sciver, *Helium Cryogenics,* New York: Plenum, 1986.

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D. P. SEKULIC
F. J. EDESKUTY
University of Kentucky,
Lexington, KY
Edeskuty Engineering, Los
Alamos, NM


**Table 1. Properties of Some Superconductors**[a]

| Material | Critical Temperature (K) | Critical Magnetic Field (T)[b] | Criti… Den… |
|---|---|---|---|
| | *Type I* | | |
| Hg | 4.15 | $H_c$ 0.04 | |
| Pb | 7.19 | 0.08 | |
| Sn | 3.72 | 0.03 | |
| | *Type II* | | |
| NbTi | 9.6 | $H_{c2}$ 12.2 | |
| $Nb_3Sn$ | 18.1 | 25 | |
| $Nb_3Ge$ | 23.2 | 38 | |
| $MgB_2$ | ≈40 | | |
| | *HTSC* | | |
| $Bi_2Sr_2CaCu_2O_8$ (2212BSCCO) | 90 | $H_{c2}$ ≈1[c] | |
| $Bi_2Sr_2Ca_2Cu_3O_{10}$ (2223BSCCO) | 110 | ≈30[c] | |
| $YBa_2Cu_3O_7$ (YBCO) | 92 | 30[c] | |

[a] Data from Refs. 2–4.
[b] It has become customary to use the tesla (T), properly a unit of magnetic induction, to express the netic field.
[c] At 77 K.

**Table 2. Pertinent Thermophysical Properties of Cryogens**[a]

| Cryogen (Liquid) | NBP[b] Temperature (K) | Triple Point Temperature (K) | Critical Temperature (K) | NBP Density (kg/m$^3$) | Critical Pressure (MPa) |
|---|---|---|---|---|---|
| He | 4.215 | –[c] | 5.25 | 125 | 0.23 |
| $H_2$[d] | 20.27 | 13.80 | 32.98 | 71 | 1.29 |
| Ne | 27.09 | 24.8 | 44.4 | 1236 | 2.37 |
| $N_2$ | 77.1 | 63.1 | 126.2 | 809 | 3.39 |
| Ar | 87.3 | 83.8 | 150.8 | 1394 | 4.86 |
| $O_2$ | 90.2 | 54.4 | 154.6 | 1140 | 5.11 |

[a] Data from Ref. 34.
[b] Normal boiling point.
[c] Helium does not have a true triple point and can only be solidified at a minimum pressure of 2.5 MPa
[d] Properties given are those of parahydrogen.

**Table 3. Typical Boiling Heat Flux Data**

| Cryogen (Liquid) | Critical Heat Flux[a] Value (W/cm$^2$) | $\Delta T$ (K) | Leidenfrost-Point Heat Flux (W/cm$^2$) |
|---|---|---|---|
| Helium | 1 | 1 | 0.3 |
| Hydrogen | 10 | 5 | 0.3 |
| Neon | 15 | 5 | 1 |
| Nitrogen | 15 | 10 | 1 |
| Oxygen | 25 | 30 | 2 |

[a] At atmospheric pressure. These numbers are only an approximate representation of a range of typical results (28).