J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

POSITION CONTROL

Position control has many applications, such as control of the elevator angle of a fighter, control of the antenna angle in a satellite tracking system, and control of robot manipulators. Often, the essence of position control is that the tracking error between the desired system output and the actual system output is used to generate a suitable control input to drive the tracking error to zero. In other words, tracking control is an important part of motion control, as it solves the problem of determining the control inputs necessary for a system to track a desired trajectory and provides a way to achieve accurate performance.

In this article, we present adaptive position control of robot manipulators and teleoperation systems. Robot manipulators are composed of links connected by joints. The joints may be electrically, hydraulically, or pneumatically actuated. The number of joints determines the number of degrees of freedom (*DOF*) of the manipulator. Position control of a robot manipulator involves control of the positions of the joints. Once given a set of desired trajectories for all the joints, the controller is designed to track these trajectories so that the end effector of the manipulator sweeps the desired positions in the workspace. The primary method of sensing the positions is with position encoders located on the joints, either on the shaft of the motor that actuates the joint or on the joint itself. At times, direct sensing of the end-effector position with the help of a camera is used to improve the accuracy of the manipulator in tracking a desired trajectory.

A teleoperation system involves two distant yet coupled robots: a local master robot and a remote slave robot. In teleoperation, the human operator controls the master robot. Motion commands are measured on the master robot and transmitted to the slave robot, which executes these commands and is expected to track the motion of the master robot. In addition, the contact force information sensed by the slave robot is reflected to the master robot for force perception. Thus, the master acts as an position input device that generates a desired trajectory. The goal of position or tracking control is to design the necessary control input that makes the slave track the motion of the master. This control problem is an example of master–slave control.

The article is organized as follows: In the following section, we present robust adaptive control schemes of robot manipulators. First, we present dynamic models of robot manipulators with time-varying parameters or unmodeled dynamics. Second, we present the controller structure and adaptive law for the time-varying parameter case and show the signal boundedness and the tracking performance of the robot system. Third, we present and analyze stable adaptive control schemes for robot manipulators with unmodeled dynamics. Some common topics of position control relevant to robot manipulators such as PD control, inverse dynamics, and path or trajectory interpolation are discussed in the fourth subsection. In the third section, we present adaptive control of teleoperation systems. Adaptive control schemes for teleoperation systems with unknown jumping parameters and with parametrizable and unparametrizable smooth time-varying parameters are presented. We also present some control issues relevant to teleoperation systems with communication time delays.

Adaptive Position Control of Manipulators

To make robot manipulators capable of handling large loads in the presence of uncertainty on the mass properties of the load or its exact position in the end effector, robust adaptive control designs for robot manipulators have been developed. In Slotine and Li (1,2) an adaptive control scheme has been proposed for the motion control of robot manipulators, which guarantees global stability and asymptotic zero tracking error between the actual joint trajectory and the desired one and needs only the measurements of the joint position and velocity. This consists of a *proportional derivative* (*PD*) feedback part and a full dynamics feedforward compensation part, with the unknown manipulator and payload parameters being estimated online. The algorithm is computationally simple, because of an effective exploitation of the particular structure of manipulator dynamics. Various modified versions of this scheme have been shown to be applicable to robot systems with unmodeled dynamics [Reed and Ioannou (3)], and joint flexibility [Spong (4)].

Recently, there has been considerable research interest in neural network control of robots, and satisfactory results have been obtained in solving some of the special issues associated with the problems of robot control. In Lewis, Jagannathan, and Yeildirek (5), neural network controllers are designed for robot manipulators in a variety of applications, including position control, force control, parallel-link mechanisms, and digital neural network control. These model-free controllers offer a powerful and robust alternative to adaptive control.

In Ge et al. (6), a comprehensive study of robot dynamics, structured network models for robots, and systematic approaches for neural-network-based adaptive controller design for rigid robots, flexible joint robots, and robots in constraint motion are presented.

In this article, we will present a robust adaptive control scheme, based on the scheme developed by Slotine and Li (1,2) with a modified controller structure and a modified adaptive law [Tao (7)], which ensures the signal boundedness in the presence of time-variations in the manipulator parameters and a mean tracking error of the order of the parameter variations, which are not required to be small. We will also show similar results for a class of unmodeled dynamics. The allowance for the existence of possible large parameter variations and unmodeled dynamics yields significant potentials for applications of the proposed robust adaptive manipulator controller.

Manipulator Models and Parametrization. In this subsection, we first present the mathematical models of robot manipulators with time-varying parameters or unmodeled dynamics, and their parametrized forms, and then use a two-link planar manipulator to illustrate the manipulator modeling and parametrization.

Manipulator Models. To derive the dynamic equations of a n-link robot manipulator (see Fig. 1, which shows an illustrative four-link manipulator) whose parameters may explicitly depend on time, we use the Euler–Lagrange equations [Spong and Vidyasagar (8); Ortega and Spong (9)] of a mechanical system:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = u \tag{1}$$

where $q = (q_1,\ldots, q_n)^{\mathrm{T}}$ is a set of position variables of n ($n > 0$) joints of the robot manipulator, $u = (u_1,\ldots, u_n)^{\mathrm{T}}$ is the applied joint torque, and L is the Lagrangian defined as $L = K - P$, the difference between the kinetic energy K and the potential energy P, in the form

$$K = \frac{1}{2}\dot{q}^{\mathrm{T}} D(q,t)\dot{q}, \qquad P = P(q,t) \tag{2}$$

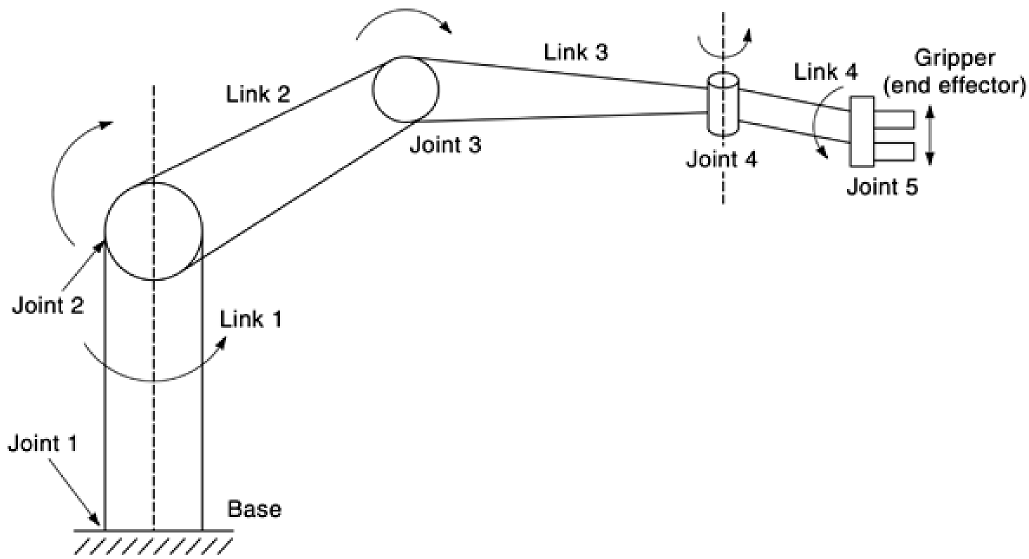

Fig. 1. Robot manipulator.

with $D(q, t) \varepsilon R^{n\times n}$ being the symmetric and positive definite manipulator inertia matrix. For Eqs. (1), (2) to represent the manipulator dynamics with time-varying parameters, the mass and the moment of inertia of each link of the manipulator should not explicitly depend on $\dot{q}$.

Letting d_{ij} be the ijth element of $D(q, t)$ and $\phi(q, t) = \partial P(q, t)/\partial q$, and substituting Eq. (2) in Eq. (1), we obtain the manipulator dynamic equation:

$$D(q,t)\ddot{q} + \frac{\partial D(q,t)}{\partial t}\dot{q} + C(q,\dot{q},t)\dot{q} + \phi(q,t) = u \qquad (3)$$

where the kjth element of $C(q, \dot{q}, t \varepsilon R^{n\times n}$ is $c_{kj} = \sum^{n}{}_{i=1}\frac{1}{2}(\partial d_{kj}/\partial q_i + \partial d_{ki}/\partial q_j - \partial d_{ij}/\partial_{qk})\,\dot{q}i$.

A key feature of the manipulator model (3) is that the inertia matrix $D(q, t)$ and the potential energy $P(q, t)$ are explicitly time-dependent, which takes into account the effect of changes in environment of the robot system or changes of the manipulator dynamics with time. Moreover, an important property of the manipulator model (3) is

$$x^{\mathrm{T}}\big[M(q,\dot{q},t) - 2C(q,\dot{q},t)\big]x = 0 \qquad \forall x \in R^n \qquad (4)$$

where $M(q, \dot{q}, t) = dD(q, t)/dt - \partial D(q, t)/\partial t$, whose ijth element is $(\partial d_{ij}/\partial q)^{\mathrm{T}}\,\dot{q}$. When $D(q, t) = D(q)$ does not explicitly depend on t, that is, $\partial D(q, t)/\partial t = 0$, Eq. (4) becomes $x^{\mathrm{T}}[dD(q)/dt - 2C(q, \dot{q})]x = 0$, which is well known in the robotics literature.

A manipulator with unmodeled dynamics may be modeled as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \phi(q) = u + H_1[g_1(\dot{q})](t) + H_2[g_2(q)](t) + H_3[g_3(u)](t) \qquad (5)$$

where H_i, $i = 1, 2, 3$, with appropriate dimensions, are linear operators with rational transfer matrices, representing the unmodeled part of the robot dynamics, and $g_1(\dot{q})$, $g_2(q)$, $g_3(u)$ are certain vector functions of $\dot{q}$, q, u. The functions $D(q)$, $C(q, \dot{q})$, $\phi(q)$ have been defined above, and, for simplicity, they are assumed not explicitly time-dependent in the unmodeled dynamics problem. The manipulator model (5) is generalized from some practical robot systems [Reed and Ioannou (3); Ortega and Spong (9)].

Control Objective and Parametrization. Our control objective is, for a given reference signal $q_d(t)$, to generate the applied torque u for the manipulator (3) or (5) with unknown parameters so that all signals in the robot system are bounded and the joint position q tracks q_d as closely as possible. To achieve such an objective we first use the transformation technique developed in Slotine and Li (1,2) to parametrize the manipulator model (3) or (5).

Let Λ be any $n \times n$ constant matrix whose eigenvalues have positive real parts; define

$$v = \dot{q}_d - \Lambda(q - q_d), \qquad s = \dot{q} - v, \qquad e = q - q_d \tag{6}$$

Clearly, it follows from Eq. (6) that

$$s = \dot{e} + \Lambda e, \qquad \dot{s} = \ddot{q} - \dot{v} \tag{7}$$

and s, v, $\dot{v}$ depend only on q, q_d, $\dot{q}$, $\dot{q}_d$, $\dot{q}_d$ and not on the joint acceleration vector $\ddot{q}(t)$.

Using Eq. (7), we express the manipulator model (3) as

$$\begin{aligned} D(q,t)\dot{s} &+ C(q,\dot{q},t)s \\ &= u - D(q,t)\dot{v} - C(q,\dot{q},t)v - \phi(q,t) - \frac{\partial D(d,t)}{\partial t}\dot{q} \\ &\triangleq u - Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d, t)\theta^*(t) - \frac{\partial D(q,t)}{\partial t}\dot{q} \end{aligned} \tag{8}$$

where Y $(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d, t)$ is an $n \times r$ matrix of known functions for some $r > 0$, and $\theta*(t)$ ε R^r contains parameters, which may be time-varying. In Eq. (8), the regressor Y $(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d, t)$ is bounded for bounded q, q_d, $\dot{q}$, $\dot{q}_d$, $\ddot{q}_d$

Let x_t be the truncated x at time t. Denote by $\|\cdot\|$ the Euclidean vector norm or the induced matrix norm, and by $\|\cdot\|_\infty$ ($\|\cdot\|_1$, $\|\cdot\|_2$) the L_∞ (L_1, L_2) vector norm or the induced operator norm [Desoer and Vidyasagar (10)], as the case may be.

We make the following assumptions about the manipulator model (8):

(1) $\|\theta* (t)\| \le \rho_0$, $\| \dot{\theta}* (t) \| \le \rho$ for some constants $\rho_0 > 0$, $\rho > 0$;
(2) $\|\partial D(q, t)/\partial t \| \le \gamma f(q)$ for some constant $\gamma > 0$ and known $f(q)$ bounded for bounded q.

Similarly, for the manipulator model (5), we obtain

$$\begin{aligned} D(q)\dot{s} + C(q,\dot{q})s = u - Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)\theta^* + H_1[g_1(\dot{q})](t) \\ + H_2[g_2(q)](t) + H_3[g_3(u)](t) \end{aligned} \tag{9}$$

where $\theta*$ ε R^r is a constant vector, and Y $(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)$ is not explicitly time-dependent.

We make the following assumptions about the manipulator model (9):

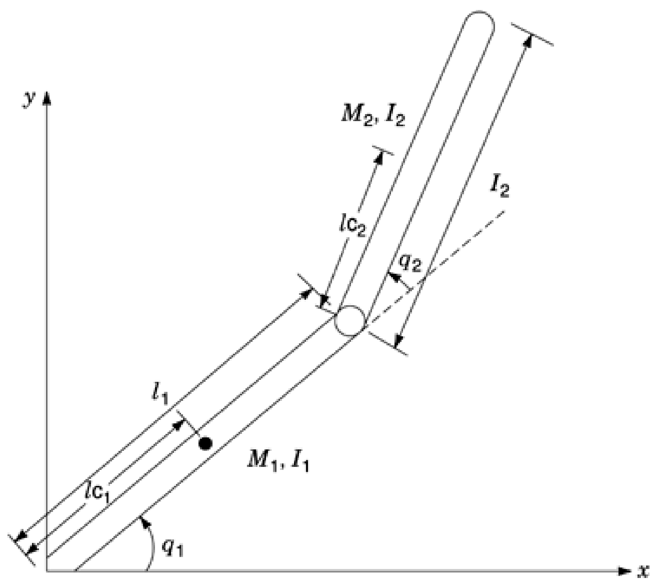

Fig. 2. Two-link planar manipulator.

(1) $\|(g_1\ (\dot{q}))_t\|_\infty \leq f_1(\dot{q}(t))$, $\|(g_2(q))_t\|_\infty \leq f_2(q(t))$ for some known $f_1(\dot{q})$, $f_2(q)$ that are bounded for bounded $\dot{q}$, q, and $\|H_1\|_\infty \leq \gamma_1$, $\|H_2\|_\infty \leq \gamma_2$ for some constants $\gamma_1 > 0$, $\gamma_2 > 0$;
(2) $\|(g_3\ (u))_t\| \leq \|u_t\ \|_\infty$, and $\|{H_3}^i\| \leq \mu_i$, where ${H_3}^i$ is the ith row of H_3, $i = 1,\ldots, n$.

We also make an assumption on the desired joint position vector $q_{\mathrm{d}}(t)$:

(1) $q_{\mathrm{d}}(t)$, $\dot{q}_{\mathrm{d}}(t)$, $\ddot{q}_{\mathrm{d}}(t)$ are bounded.

Assumption (A1) requires only the boundedness of the manipulator parameters and their derivatives, not the smallness of the time variations of the parameters. Smallness of the parameter variations is usually an assumption for the design of adaptive control schemes for time-varying plants, but it is not needed here because of the special structure of the robot manipulator dynamics. Assumption (A2) requires that $\partial D\ (q,\ t)/\partial t$ satisfy a certain relative boundedness condition. Assumption (A3) requires that the L_∞ gains of H_1, H_2 be finite and $g_1(\dot{q})$, $g_2\ (q)$ satisfy certain relative boundedness conditions. Assumption (A4) is similar to (A3), but $\mu_{\mathrm{i}} \geq 0$, $i = 1,\ldots, n$, are to be specified for the robust stability of the adaptive robot system. We note that the bounds γ, γ_1, γ_2 are not needed for the adaptive controller design.

An Illustrative Example. In this sub-subsection, we consider a two-link planar manipulator [Spong and Vidyasagar (8)], shown in Fig. 2 as an illustrative example for the robot system modeling and parametrization.

The manipulator configuration may be described as follows: there are two revolute joints with joint angles q_1, q_2, and two links with masses M_1, M_2, lengths l_1, l_2, distances l_{c1}, l_{c2} from the joints to the mass centers, and rotational inertias I_1, I_2. The inertia matrix $D(q,\ t)$ has four elements: $d_{11} = M_1 {l_{c1}}^2 + M_2\ ({l_1}^2 {l_{c2}}^2 + 2 l_1 l_{c2} \cos q_2) + I_1 + l_2$, $d_{12} = d_{21} = M_2({l_{c2}}^2 + l_1 l_{c2} \cos q_2) + I_2$, $d_{22} = M_2\ {l_{c2}}^2 + I_2$; and the potential energy is $P(q,\ t) = (M_1 l_{c1} + M_2 l_1)\ g \sin q_1 + M_2 l_{c2}\ g \sin\ (q_1 + q_2)$, where g is the gravitational acceleration. The matrix $C(q,\ \dot{q},\ t)$ in Eq. (3) has four elements: $c_{11} = h\ \dot{q}_2$, $c_{12} = (\dot{q}_1 + \dot{q}_2)h$, $c_{21} = -\ \dot{q}_1 h$, $c_{22} = 0$, where $h = -M_2 l_1 l_{c2} \sin q_2$.

The manipulator model without parameter variations and unmodeled dynamics is $D(q)\,\dot{s} + C(q, \dot{q})s = u - Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)\theta*$, where

$$\theta^* = (\theta_1^*, \ldots, \theta_9^*)^T \triangleq (M_1 l_{c1}^2, M_2 l_1^2, M_2 l_{c2}^2, M_2 l_1 l_{c2}, I_1, I_2, M_1 l_{c1} g, M_2 l_1 g, M_2 l_{c2} g)^T \tag{10}$$

$$\begin{aligned} Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d) &\triangleq Y_0(q, \dot{q}, v, \dot{v}) \\ &= \left(\begin{matrix} \dot{v}_1 & \dot{v}_1 & \dot{v}_1 + \dot{v}_2 & \cos q_2(2\dot{v}_1 + \dot{v}_2) - \sin q_2[\dot{q}_2 v_1 + (\dot{q}_1 + \dot{q}_2)v_2] \\ 0 & 0 & \dot{v}_1 + \dot{v}_2 & \cos q_2 \dot{v}_1 + \sin q_2 \dot{q}_1 v_1 \end{matrix}\right. \\ &\qquad \left.\begin{matrix} \dot{v}_1 & \dot{v}_1 + \dot{v}_2 & \cos q_1 & \cos q_1 & \cos(q_1 + q_2) \\ 0 & \dot{v}_1 + \dot{v}_2 & 0 & 0 & \cos(q_1 + q_2) \end{matrix}\right), \quad v = (v_1, v_2)^T. \end{aligned} \tag{11}$$

When $\theta* = \theta*(t)$ is time-varying, the manipulator model is Eq. (8) with $Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d, t)$ and

$$\begin{aligned} &\frac{\partial D(q,t)}{\partial t} \\ &= \begin{pmatrix} 2\dot{\theta}_4^*(t)\cos q_2 + \sum_{i=1, i\neq 4}^{6} \dot{\theta}_i^*(t) & \dot{\theta}_3^*(t) + \dot{\theta}_4^*(t)\cos q_2 + \dot{\theta}_6^*(t) \\ \dot{\theta}_3^*(t) + \dot{\theta}_4^*(t)\cos q_2 + \dot{\theta}_6^*(t) & \dot{\theta}_3^*(t) + \dot{\theta}_6^*(t) \end{pmatrix}. \end{aligned} \tag{12}$$

Assuming that $|\,\dot{\theta}_i(t)| \le \rho_i$, we obtain the bound ρ in (A1) as $\rho = \sqrt{\Sigma_{i=1}^{9} \rho_i^2}$ and the bounds γ, $f(q)$ in (A2) as $\gamma = (\rho_1{}^2 + \rho_2{}^2 + 4\rho_3{}^2 + 6\rho_4{}^2 + \rho_5{}^2 + 4\rho_6{}^2)^{1/2}$, $f(q) = 1$.

For the unmodeled dynamics problem, the manipulator model is Eq. (9). The bounds γ_1, γ_2, μ_i in (A3) and (A4) depend on the nature of the unmodeled dynamics.

Solution to the Parameter Variation Problem. In this subsection, we first present an adaptive control scheme for robot manipulators modeled by Eq. (8) and then analyze the stability and tracking properties of the proposed adaptive controller.

If the inertia $D(q, t) = D(q)$ and the potential energy $P(q, t) = P(q)$ are not explicitly time-dependent, that is, $\partial D(q, t)/\partial t = 0$, and $\theta*(t) = \theta*$ is constant in Eq. (8), then the adaptive control scheme proposed by Slotine and Li (1,2),

$$u(t) = Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)\theta(t) - K_D s(t), \qquad 0 < K_D = K_D^T \in R^{n\times n} \tag{13}$$

$$\dot{\theta}(t) = -\Gamma^{-1} Y^T(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)s(t), \qquad 0 < \Gamma = \Gamma^T \in R^{r\times r} \tag{14}$$

guarantees that the closed-loop system is globally stable and convergent in the sense that $q(t)$, $\dot{q}(t)$, $\theta(t)$ are bounded, and $\lim_{t\to\infty} e(t) = 0$, as the positive definite function

$$V(s, \tilde{\theta}) = \frac{1}{2}(s^T D s + \tilde{\theta}^T \Gamma \tilde{\theta}), \qquad \tilde{\theta}(t) = \theta(t) - \theta^*, \qquad D = D(q(t)) \tag{15}$$

has the property $\dot{V}(t) = -s^T(t) K_D\, s(t) \le 0$ [also see Spong et al. (11) for further analysis].

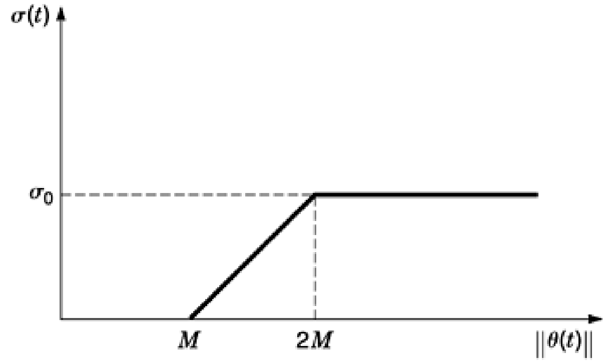

Fig. 3. The switching σ modification.

When $D(q, t)$, $P(q, t)$ are both explicitly time-dependent, we have obtained the manipulator model as Eq. (8) in which $\theta*(t)$ is time-varying and the term $\partial D(q, t)/\partial t\ \dot{q}$ appears. If the parameters in $D(q, t)$, $P(q, t)$ were known, then $\theta*(t)$ and $\partial D(q, t)/\partial t\ \dot{q}$ could be calculated so that the control law $u(t) = Y(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}}, t)\theta*(t) + [\partial D(q, t)/\partial t]\ \dot{q} - K_{\mathrm{D}}s$ could be implemented, which guarantees global stability and asymptotic tracking. For unknown $D(q, t)$, $P(q, t)$, next we present an adaptive control scheme that is robust with respect to the time variation of $\theta*(t)$ and $[\partial D(q, t)/\partial t]\ \dot{q}$.

With $Y(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}}, t)$, s, K_{D}, Γ defined before, we propose the following feedback controller structure for the manipulator (8):

$$u(t) = Y(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}}, t)\theta(t) - m(t)\psi(t) - K_{\mathrm{D}}s(t) \tag{16}$$

$$m(t) = k_0\|\dot{q}(t) + v(t)\|f(q), \qquad \psi(t) = m(t)s(t) \tag{17}$$

and the following update law for $\theta(t)$:

$$\dot{\theta}(t) = -\Gamma^{-1}\left[Y^{\mathrm{T}}(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}}, t)s + \sigma(t)\theta(t)\right] \tag{18}$$

where $\sigma(t)$ as shown in Fig. 3 is the switching signal [Ioannou and Tsakalis (12)] using *a priori* knowledge of the upper bound M on $\sup_{t\geq 0}\|\theta*(t)\|$:

$$\sigma(t) = \begin{cases} 0 & \text{if} \quad \|\theta(t)\| < M \\ \sigma_0\big(\|\theta(t)\|/M - 1\big) & \text{if} \quad M \leq \|\theta(t)\| < 2M, \quad \sigma_0 > 0 \\ \sigma_0 & \text{if} \quad \|\theta(t)\| \geq 2M \end{cases} \tag{19}$$

This adaptive control scheme has the following stability and tracking properties.

Theorem 1. All closed-loop signals are bounded, and the tracking error $e(t) = q(t) - q_{\mathrm{d}}(t)$ satisfies

$$\int_{t_1}^{t_2} \|e(t)\|^2 \, dt \le \alpha_0 \left(\frac{\gamma^2}{k_0^2} + \rho \right) (t_2 - t_1) + \beta_0 \tag{20}$$

for some constants $\alpha_0 > 0$, $\beta_0 > 0$, and any $t_2 > t_1 \ge 0$. Moreover, $e(t) \,\varepsilon\, L_2$ and $\lim_{t\to\infty} e(t) = 0$ in the absence of parameter time variations, that is, when $\dot{\theta}*(t) = 0$, $\partial D(q,\, t)/\partial t = 0$.

Proof:. Consider the positive definite function

$$V(s, \tilde{\theta}) = \frac{1}{2}\left(s^{\mathrm{T}} D s + \tilde{\theta}^{\mathrm{T}} \Gamma \tilde{\theta}\right), \qquad \tilde{\theta}(t) = \theta(t) - \theta^*(t), \quad D = D(q(t), t) \tag{21}$$

From Eqs. (4, 7, 8), (16, 17, 18) and from (A1), (A2), we have that

$$\begin{aligned}
\dot{V}(t) &= -s^{\mathrm{T}}(t) K_D s(t) - m^2(t) s^{\mathrm{T}}(t) s(t) \\
&\quad - \frac{1}{2} s^{\mathrm{T}}(t) \frac{\partial D(q,t)}{\partial t}\left[\dot{q}(t) + v(t)\right] - \sigma(t)\tilde{\theta}^{\mathrm{T}}(t)\theta(t) - \tilde{\theta}^{\mathrm{T}}(t)\Gamma\dot{\theta}^*(t) \\
&\le -s^{\mathrm{T}}(t) K_{\mathrm{D}} s(t) - \left(m(t)\|s(t)\| - \frac{\gamma}{4k_0}\right)^2 + \frac{\gamma^2}{16k_0^2} \\
&\quad -\sigma(t)\tilde{\theta}^{\mathrm{T}}(t)\theta(t) - \tilde{\theta}^{\mathrm{T}}(t)\Gamma\dot{\theta}^*(t) \\
&\le -s^{\mathrm{T}}(t) K_{\mathrm{D}} s(t) - \left(m(t)\|s(t)\| - \frac{\gamma}{4k_0}\right)^2 + \frac{\gamma^2}{16k_0^2} \\
&\quad -[\sigma(t) - \sigma_0]\tilde{\theta}^{\mathrm{T}}(t)\theta(t) - \sigma_0\left[\|\theta(t)\| - \left(\frac{\rho_0}{2} + \frac{\rho\|\Gamma\|}{2\sigma_0}\right)\right]^2 \\
&\quad + \frac{\sigma_0}{4}\left(\rho_0 + \frac{\rho\|\Gamma\|}{\sigma_0}\right)^2 + \rho_0\rho\|\Gamma\|.
\end{aligned} \tag{22}$$

Since γ, ρ, ρ_0 are constants and $\sigma(t)$ defined in Eq. (19) satisfies

$$|\,[\sigma(t) - \sigma_0]\tilde{\theta}^{\mathrm{T}}(t)\theta(t)\,| \le 12\sigma_0 M^2 \tag{23}$$

it follows from the second inequality of (2.22) that $\dot{V}(t) \le 0$ for $\theta(t)$, $s(t)$ outside a certain bounded set. Therefore $s(t)$ and $\theta(t)$ are bounded, which, in view of Eqs. (7), (16), implies that $q(t)$, $\dot{q}(t)$, $u(t)$ are also bounded.

Using the fact that $\sigma(t)\ \tilde{\theta}^{\mathrm{T}}(t)\theta(t) \ge 0$ and the first inequality of Eq. (22), we obtain

$$\dot{V}(t) \le -s^{\mathrm{T}}(t) K_{\mathrm{D}} s(t) + \frac{\gamma^2}{16k_0^2} + k_1\rho \tag{24}$$

for some constant $k_1 > 0$. Since $V(t)$ is bounded, from Eq. (24) we have

$$\int_{t_1}^{t_2} \|s(t)\|^2\,dt \le \alpha_1\left(\frac{\gamma^2}{k_0^2} + \rho\right)(t_2 - t_1) + \beta_1 \qquad (25)$$

for some constants $\alpha_1 > 0$, $\beta_1 > 0$ and any $t_2 > t_1 \ge 0$.

To show that Eq. (25) implies Eq. (20), let us consider the relation $s(t) = \dot{e}(t) + \Lambda e(t)$, where Λ is a stable matrix [see Eq. (7)], and denote by H the linear operator or the impulse-response matrix from $s(t)$ to $e(t)$ as the case may be, that is, $e(t) = H[s](t) = \int_0^t H(t-\tau)s(\tau)\,d\tau$. It follows that

$$\begin{aligned}\int_{t_1}^{t_2} \|e(t)\|^2\,dt &= \int_{t_1}^{t_2} \left\|\int_0^t H(t-\tau)s(\tau)\,d\tau\right\|^2 dt \\ &\le 2\int_{t_1}^{t_2} \left\|\int_{t_1}^t H(t-\tau)s(\tau)\,d\tau\right\|^2 dt \\ &\quad +2\int_{t_1}^{t_2} \left\|\int_0^{t_1} H(t-\tau)s(\tau)\,d\tau\right\|^2 dt \\ &\le \|H\|_2^2 \int_{t_1}^{t_2} \|s(t)\|^2\,dt + 2\int_{t_1}^{t_2} \|H(t-t_1)e(t_1)\|^2\,dt. \qquad (26)\end{aligned}$$

Since the operator H is exponentially stable and $e(t)$ is bounded, both $\|H\|_2$ and $\int_{t1}^{t2} \|H\,(t-t_1)e(t_1)\|^2\,dt$ are finite for any $t_2 > t_1 \ge 0$. Hence from Eqs. (25) and (26) we prove Eq. (20).

When $\dot{\theta}*(t) = 0$ and $\partial D(q,t)/\partial t = 0$, that is, $\rho = \gamma = 0$, it follows from Eq. (20) that $e(t)\,\varepsilon\,L_2$. This, together with the boundedness of $\dot{e}(t) = s(t) - \Lambda e(t)$, proves $\lim_{t\to\infty} e(t) = 0$.

To implement the controller (16), we need the knowledge of $f(q)$ to generate the bounding signal $m(t)$ in Eq. (17). A more sophisticated choice of $f(q)$ admits a wider class of $\partial D(q,t)/\partial t$, but may make the implementation of $m(t)$ more complicated. We also note that the above design does not need the knowledge of the bounds γ, ρ. For a chosen $f(q)$, different choices of k_0 in generating $m(t)$ may have different effects on the tracking performance, while increasing k_0 may reduce the effect of γ in the mean error (20). For the signal boundedness and the mean tracking error (20), parameter variations characterized by γ and ρ are not required to be small. This is an important feature of the robot system. With $\dot{q}$, q available for measurement, the manipulator mode (3) is equivalent to the first-order model (8), for which the adaptive controller allows "large" parameter variations to exist.

Solution to the Unmodeled Dynamics Problem. Consider the manipulator (9) with unmodeled dynamics. If the terms $H_1[g_1(\dot{q})](t)$, $H_2[g_2(q)](t)$, $H_3[g_3(u)](t)$ were available for measurement and $\theta*$ were known, then the control law $u(t) = Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)\theta* - H_1[g_1(\dot{q})](t) - H_2[g_2(q)](t) - H_3[g_3(u)](t) - K_D s(t)$ could be implemented so that $d/dt[s^T(t)D(q)s(t)] = -2s^T(t)K_D s(t)$, showing the boundedness of $s(t)$ and exponentially fast tracking. However, to ensure the boundedness of $u(t)$, one needs $\|H_3\|_\infty = \max_{i=1,\ldots,n} \mu_i < 1$.

To solve the adaptive control problem in which $H_1[g_1(\dot{q})](t)$, $H_2[g_2(q)](t)$, $H_3[g_3(u)(t)$, and $\theta*$ are unknown, with $Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d)$, K_D, $s(t) = (s_1(t),\ldots, s_n(t))^T$, $\sigma(t)$, Γ defined before, we propose the following feedback controller structure for the manipulator (9):

$$\begin{aligned} u(t) &= Y(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}})\theta(t) - m_1(t)\phi_1(t) - m_2(t)\phi_2(t) \\ &\quad - m_3(t)\phi_3(t) - K_{\mathrm{D}}s(t), \end{aligned} \tag{27}$$

$$\begin{aligned} &m_1(t) = k_{10}\sqrt{n} f_1(\dot{q}(t)), \qquad m_2(t) = k_{20}\sqrt{n} f_2(q(t)) \\ &m_3(t) = \|u_t\|_\infty \end{aligned} \tag{28}$$

$$\begin{aligned} &\phi_1(t) = m_1(t)s(t), \qquad \phi_2(t) = m_2(t)s(t), \\ &\phi_3(t) = (\phi_{31}(t), \ldots, \phi_{3n}(t))^{\mathrm{T}} \end{aligned} \tag{29}$$

$$\phi_{3i}(t) = \begin{cases} \lambda_i \operatorname{sign}[s_i(t)m_3(t)] & \text{if} \quad |s_i(t)m_3(t)| \geq \delta_i \\ \lambda_i s_i(t)m_3(t)/\delta_i & \text{if} \quad |s_i(t)m_3(t)| < \delta_i \end{cases} \tag{30}$$

$$k_{j0} > 0, \quad j = 1, 2, \qquad \delta_i > 0, \qquad 0 < \lambda_i < 1, \quad i = 1, \ldots, n \tag{31}$$

and the following update law for $\theta(t)$:

$$\dot{\theta}(t) = -\Gamma^{-1}\big[Y^{\mathrm{T}}(q, q_{\mathrm{d}}, \dot{q}, \dot{q}_{\mathrm{d}}, \ddot{q}_{\mathrm{d}})s(t) + \sigma(t)\theta(t)\big] \tag{32}$$

The stability and tracking properties of this adaptive control scheme are:

Theorem 2. All closed-loop signals are bounded for any $\mu_i \ \varepsilon\ [0, \lambda_i]$ $i = 1,\ldots,n$, and the tracking error $e(t)$ satisfies

$$\int_{t_1}^{t_2} \|e(t)\|^2 \, dt \leq \alpha_0 \left(\frac{\gamma_1^2}{k_{10}^2} + \frac{\gamma_2^2}{k_{20}^2} + \delta \right)(t_2 - t_1) + \beta_0, \qquad \delta = \max_{i=1,\ldots,n} \delta_i \tag{33}$$

for some constants $\alpha_0 > 0$, $\beta_0 > 0$. Moreover, $e(t)\ \varepsilon\ L_2$ and $\lim_{t\to\infty} e(t) = 0$ in the absence of the unmodeled dynamics, that is, when $H_1 = 0$, $H_2 = 0$, $H_3 = 0$.

Proof:. Considering $V(t)$ defined in (15), from Eqs. (9), (27), (32) and (A3), (A4) we obtain

$$\begin{aligned}\dot{V}(t) &= -s^{\mathrm{T}}(t)K_{\mathrm{D}}s(t) - \left[m_1^2(t)+m_2^2(t)\right]s^{\mathrm{T}}(t)s(t) - m_3(t)s^{\mathrm{T}}(t)\phi_3(t) - \sigma(t)\tilde{\theta}^{\mathrm{T}}(t)\theta(t) \\ &\quad + s^{\mathrm{T}}(t)H_1\left[g_1(\dot{q})\right](t) + s^{\mathrm{T}}(t)H_2\left[g_2(q)\right](t) + s^{\mathrm{T}}(t)H_3\left[g_3(u)\right](t) \\ &\leq -s^{\mathrm{T}}(t)K_{\mathrm{D}}s(t) - \left(m_1(t)\|s(t)\| - \frac{\gamma_1}{2k_{10}}\right)^2 + \frac{\gamma_1^2}{4k_{10}^2} \\ &\quad - \left(m_2(t)\|s(t)\| - \frac{\gamma_2}{2k_{20}}\right)^2 + \frac{\gamma_2^2}{4k_{20}^2} \\ &\quad - (\sigma(t)-\sigma_0)\tilde{\theta}^{\mathrm{T}}(t)\theta(t) + \frac{1}{4}\sigma_0\|\theta^*\|^2 - \sigma_0\left(\|\theta(t)\| - \frac{\|\theta^*\|}{2}\right)^2 \\ &\quad - \sum_{i=1}^{n} m_3(t)(\lambda_i-\mu_i)|s_i(t)| + \sum_{i=1}^{n} m_3(t)\left[|s_i(t)|\lambda_i - s_i(t)\phi_{3i}(t)\right] \end{aligned} \tag{34}$$

From Eq. (30), we see that

$$\left|m_3(t)\left[|s_i(t)|\lambda_i - s_i(t)\phi_{3i}(t)\right]\right| \leq 2\lambda_i\delta_i, \qquad i = 1,\ldots,n \tag{35}$$

Hence it follows from Eqs. (34), (35) that, for $0 \leq \mu_i \leq \lambda_i$, $i = 1,\ldots,n$, we have $\dot{V}(t) \leq 0$ whenever $\theta(t)$ and $s(t)$ are outside a certain bounded set, that is, $s(t)$, $\theta(t)$ are bounded, and so are $q(t)$, $\dot{q}(t)$.

From Eqs. (27, 28, 29, 30), (A3), (A4), and the boundedness of $s(t)$, $q(t)$, $\dot{q}(t)$, $\theta(t)$, we have

$$\|u_t\|_\infty \leq k_2 + \max_i \lambda_i \|u_t\|_\infty \tag{36}$$

for some constant $k_2 > 0$, which, together with Eq. (31), implies that $u(t)$ is bounded.

Using Eqs. (34), (35) and the fact that $\sigma(t)\ \tilde{\theta}^{\mathrm{T}}(t)\theta(t) \geq 0$, we obtain

$$\dot{V}(t) \leq -s^{\mathrm{T}}(t)K_{\mathrm{D}}s(t) + \frac{\gamma_1^2}{4k_{10}^2} + \frac{\gamma_2^2}{4k_{20}^2} + 2\sum_{i=1}^{n}\lambda_i\delta_i. \tag{37}$$

For $\delta = \max_{i=1,\ldots,n} \delta_i$, Eq. (37) implies that

$$\int_{t_1}^{t_2} \|s(t)\|^2\, dt \leq \alpha_1\left(\frac{\gamma_1^2}{k_{10}^2} + \frac{\gamma_2^2}{k_{20}^2} + \delta\right)(t_2 - t_1) + \beta_1 \tag{38}$$

for some constants $\alpha_1 > 0$, $\beta_1 > 0$, and any $t_2 > t_1 \geq 0$, which implies Eq. (33).

When $H_1 = 0, H_2 = 0, H_3 = 0$, the expression (34) for $\dot{V}(t)$ becomes

$$\begin{aligned}\dot{V}(t) &= -s^{\mathrm{T}}(t)K_{\mathrm{D}}s(t) - \left[m_1^2(t) + m_2^2(t)\right]s^{\mathrm{T}}(t)s(t) \\ &\quad -m_3(t)s^{\mathrm{T}}(t)\phi_3(t) - \sigma(t)\tilde{\theta}^{\mathrm{T}}(t)\theta(t).\end{aligned} \tag{39}$$

Since $s^{\mathrm{T}}(t)\phi_3(t) \geq 0$, Eq. (39) shows that $s(t) \varepsilon L_2$. Hence, from Eq. (7), it follows that $e(t) \varepsilon L_2$ and $\dot{e}(t)$ is bounded. Therefore we have $\lim_{t\to\infty} e(t) = 0$.

We have thus proved the signal boundedness of the closed-loop system in the presence of $H_1[g_1(q)](t)$, $H_2[g_2(q)](t)$, $H_3[g_3(q)](t)$. The gains of the linear operators H_1, H_2 are assumed to be finite but not small. The gain of H_3 is required to be small to ensure the boundedness of $u(t)$.

The modifying term $\sigma(t)\theta(t)$ in Eq. (32) can be replaced by $\sigma_0\theta(t)$, $\sigma_0 > 0$. The signal boundedness follows, but Eq. (33) is changed to

$$\int_{t_1}^{t_2} \|e(t)\|^2\, dt \leq \alpha_0 \left(\frac{\gamma_1^2}{k_{10}^2} + \frac{\gamma_2^2}{k_{20}^2} + \delta + \sigma_0\right)(t_2 - t_1) + \beta_0 \tag{40}$$

This scheme cannot guarantee asymptotic tracking in the absence of the unmodeled dynamics, though the scheme does not need the knowledge of the upper bound on $\|\theta*\|$.

The use of the bounding signals $m_i(t)$ defined in Eq. (28) is the key to ensuring signal boundedness in the presence of the unmodeled dynamics satisfying (A3), (A4). To generate these signals, the knowledge of the stability margin of the unmodeled dynamics is not needed. Alternative bounding signals may be used under other assumptions for the unmodeled dynamics. For example, if $\|\bar{H}_3{}^i\|_\infty \leq \mu_i$, where $\bar{H}_3{}^i$ is the ith row of $H_3(s)(s + a_0)$ (this s is the Láplace variable), $i = 1,\ldots, n$, for some known constant $a_0 > 0$, and $\|(s+a_0)^{-1}[g_3(u)])_{\mathrm{t}}\|_\infty \leq \|(s+a_0)^{-1}[u])_{\mathrm{t}}\|_\infty$, then we can choose $m_3(t) = \|(s+a_0)^{-1}[u])_t\|_\infty$ and set $0 < \lambda_{\mathrm{i}} \leq a_0$. Another choice of $m_3(t)$ is the bounding signal $[\sqrt{n}/(s+\delta_0)][\|u\|](t)$ when $\|g_3(u)\| \leq \|u\|$ and $\|\hat{H}_3{}^i\|_1 \leq \mu_i$, where $\hat{H}_3{}^i$ is the ith row of $H_3(s - \delta_0)s$ with the Laplace variable s, for some known constant $\delta_0 > 0$. For this $m_3(t)$, the condition on λ_i is $\|(\lambda_1,\ldots,\lambda_m)^{\mathrm{T}}\| \sqrt{n} < \delta_0$ and $\lambda_i > 0$ for $i = 1,\ldots,n$. We note that a similar L_1-norm condition can be established for H_1 and H_2 to design a robust adaptive controller.

When $\theta*$ is known and $H_1[g_1(\dot{q})](t)$, $H_2[g_2(q)](t)$, $H_3[g_3(u)](t)$ are present, Eq. (27) with $\theta(t) = \theta*$ becomes a robust nonadaptive controller, which results in $e(t)$ converging exponentially to a residual set whose size is of the order of $\gamma_1{}^2/k_{10}{}^2 + \gamma_2{}^2/k_{20}{}^2 + \delta$.

Next, we present the robust adaptive control design assuming that the upper bounds on the gains of the unmodeled dynamics H_1, H_2 are known:

(1) (A3a)(g_1 $\dot{q}$), $g_2(q)$ are the same as in (A3), and $\|H_j{}^i\|_\infty$: $\leq \gamma_{ji}$ for some known constants $\gamma_{ji} > 0$,where $H_j{}^i$ is the ith row of H_j, $j = 1, 2$, $i = 1,\ldots, n$.

We propose to choose $\phi_j(t) = (\phi_{j1}(t),\ldots,\phi_{jn}(t))^{\mathrm{T}}$, $j = 1,2,3$, in Eq. (27) as

$$\phi_{ji}(t) = \begin{cases} \lambda_{ji}\ \mathrm{sign}\,[s_i(t)m_j(t)] & \text{if} \quad |s_i(t)m_j(t)| \geq \delta_{ji} \\ \lambda_{ji s_i(t)m_j(t)/\delta_{ji}} & \text{if} \quad |s_i(t)m_j(t)| < \delta_{ji} \end{cases} \tag{41}$$

$$\delta_{ji} > 0, \qquad \lambda_{1i} \geq \frac{\gamma_{1i}}{k_{10}}, \qquad \lambda_{2i} \geq \frac{\gamma_{2i}}{k_{10}}, \qquad 0 < \lambda_{3i} < 1. \tag{42}$$

This scheme guarantees that all signals in the closed-loop system are bounded for $0 \leq \mu_i \leq \lambda_{3i}$, $i = 1,\ldots,n$, and the tracking error $e(t)$ satisfies

$$\int_{t_1}^{t_2} \|e(t)\|^2 \, dt \leq \alpha_0 \delta (t_2 - t_1) + \beta_0, \qquad \delta = \max_{j=1,2,3,\quad i=1,\ldots,n} \delta_{ji} \tag{43}$$

for some constants $\alpha_0 > 0$, $\beta_0 > 0$, and any $t_2 > t_1 \geq 0$. Moreover, $e(t) \, \varepsilon \, L_2$ and $\lim_{t\to\infty} e(t) = 0$ in the absence of the unmodeled dynamics H_1, H_2, and H_3.

We see from Eq. (43) that the mean tracking error explicitly depends only on the design parameter δ, not on the bounds γ_{ji} defined in (A3a). A smaller δ may result in a smaller mean tracking error (2.43). Hence, with the knowledge of the unmodeled dynamics bounds, improvements of the tracking performance may be achieved by using the control signals defined by Eq. (41).

Another interesting result is the adaptive controller with a so-called variable structure [Utkin (13)]: letting $\delta_{ji} \to 0$ in (2.41), $j = 1,2,3$, $i = 1,\ldots,n$, we obtain

$$\phi_{ji}(t) = \begin{cases} \lambda_{ji} \operatorname{sign}[s_i(t) m_j(t)] & \text{if} \quad s_i(t) m_j(t) \neq 0 \\ 0 & \text{if} \quad s_i(t) m_j(t) = 0 \end{cases} \tag{44}$$

It can be shown that for $0 \leq \mu_i \leq \lambda_{3i}$, $i = 1,\ldots,n$, all closed-loop signals are bounded and the tracking error asymptotically converges to $e(t) = 0$ with possible chatterings. For a variable structure controller, $\sigma(t) = 0$ can be used in the update law (32).

As a final remark, we note that the proposed designs can be combined to solve the problem in which both the parameter variation and unmodeled dynamics are present.

Proportional Derivative Control, Inverse Dynamics, and Path Interpolation. In this subsection, we discuss some of the general concepts related to the position control of robot manipulators.

Proportional Derivative Control. We first derive a PD control law for each joint of a manipulator based on a single-input single-output (*SISO*) model. Coupling effects among the joints are regarded as disturbances. Permanent-magnet dc motors along with gear reduction are commonly used in practice to actuate the joints of the manipulator. For such dc-motor-actuated robotic manipulator, a simplified version of the dynamics of the kth joint can be given as in Spong and Vidyasagar (8),

$$J_{\text{eff}k} \ddot{\theta}_{\text{m}k} + B_{\text{eff}k} \dot{\theta}_{\text{m}k} = \bar{K}_k V_{\text{a}k} - r_k \bar{d}_k \tag{45}$$

where $J_{\text{eff}k} = J_{\text{m}k} + r_k{}^2 \text{d}_{kk}(q)$ is the effective joint inertia of the kth actuator (motor plus gear, $J_{\text{m}k}$) and the manipulator link [$d_{kk}(q)$ is the kth diagonal element of $D(q)$ in Eq. (3)], $B_{\text{eff}k} = B_{\text{m}k} + (K_{\text{b}k}\ K_{\text{m}k}/R_k)$ is the effective damping of the kth actuator (motor plus gear, $B_{\text{m}k}$) with $K_{\text{b}k}$ the back emf constant, $K_{\text{m}k}$ the torque constant, and R_k the armature resistance; $\theta_{\text{m}k}$ is the kth motor (rotor) angular position; $V_{\text{a}k}$ is the armature voltage of the kth motor; r_k is the kth gear ratio; $\bar{K}_k = K_{\text{m}k}/R_k$; and $\bar{d}_k$ is the actuator dynamics (3) specified for the kth joint and is treated as a disturbance to simplify the problem, since in that case, we maintain the linearity of (2.45). The last can be given as

$$\bar{d}_k = \sum_{j \neq k} d_{jk}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q) \tag{46}$$

where $c_{ijk} = \frac{1}{2}(\partial d_{kj}/\partial q_i + \partial d_{ki}/\partial q_j - \partial d_{ij}/\partial q_k)$ and $\phi_k(q) = \partial P(q)/\partial q_k$ with $P(q)$ the potential energy.

The setpoint tracking problem is defined as the problem of tracking a constant step reference command $\theta_\mathrm{d} = [\theta_{\mathrm{d}1},\ldots,\theta_{\mathrm{d}n}]$ for n joints. This type of control is adequate for applications not involving very fast motion, especially in robots with large gear reduction between the actuators and the links. A PD compensator for each of the n joints can now be used to achieve setpoint tracking:

$$V_{\mathrm{a}k}(s) = K_{\mathrm{p}k}\left[\theta_{\mathrm{d}k}(s) - \theta_{\mathrm{m}k}(s)\right] - K_{\mathrm{d}k}s\theta_{\mathrm{m}k}(s) \tag{47}$$

where $K_{\mathrm{p}k}$, $K_{\mathrm{d}k}$ are the proportional and the derivative gains, respectively. The characteristic polynomial of the closed-loop system is

$$\Omega_k(s) = J_{\mathrm{eff}k}s^2 + (B_{\mathrm{eff}k} + \bar{K}_k K_{\mathrm{d}k})s + \bar{K}_k K_{\mathrm{p}k} \tag{48}$$

indicating that the closed-loop system will be stable for all positive values of $K_{\mathrm{p}k}$ and $K_{\mathrm{d}k}$ and bounded disturbances. The tracking error is given by

$$e_k(s) = \frac{J_{\mathrm{eff}k}s^2 + (B_{\mathrm{eff}k} + \bar{K}_k K_{\mathrm{d}k})s}{\Omega_k(s)}\theta_{\mathrm{d}k}(s) + \frac{r_k}{\Omega_k(s)}\bar{d}_k(s). \tag{49}$$

For a step reference input $\theta_{\mathrm{d}k}(s) = \theta_{\mathrm{d}k}/s$ and a constant disturbance $\bar{d}_k(s) = \bar{d}_k/s$, the steady-state error is $e_{k\mathrm{ss}} = - -r_k \bar{d}_k / \bar{K}_k K_{\mathrm{p}k}$ [see Spong and Vidyasagar (8)]. Thus, the steady-state error due to a constant disturbance is smaller for larger gear reduction and can be made arbitrarily small by making the position gain $K_{\mathrm{p}k}$ large. By using integral control as well (*PID*), we can achieve zero steady-state error while keeping gains small and rejecting step disturbances. However, the PD or PID compensators perform poorly for position control when there are large uncertainties in system parameters, or when varying disturbances and unmodeled dynamics are present, as is common in applications. In such situations, the adaptive control designs presented in the preceding two subsections perform much better than the PD or PID controller.

In the PD compensator presented above, the coupling effects among the joints were regarded as disturbances. In reality, the dynamic equations of a robot manipulator form a complex, nonlinear, and multivariable system. Such a dc-motor-driven n-joint actuator may be represented in a matrix equation as

$$\left[D(q) + \bar{J}\right]\ddot{q} + C(q,\dot{q})\dot{q} + \bar{B}\dot{q} + \phi(q) = u \tag{50}$$

where $D(q)$ is the time-invariant $n \times n$ inertia matrix, $C(q, \dot{q})$ and $\phi(q)$ are the time-invariant versions of $C(q, \dot{q}, t)$ and $\phi(q, t)$ in Eq. (3) respectively, $\bar{J}$ is a diagonal matrix with elements $J_{\mathrm{m}k}/r_k^2$, the input joint torque has components $u_k = (K_{\mathrm{m}k}/r_k R_k)\,V_{\mathrm{a}k}$, and B has elements $\bar{B}_k = B_{\mathrm{m}k} + K_{\mathrm{b}k}L_{\mathrm{m}k}/R_k$ (with $L_{\mathrm{m}k}$ the inductance), for $k = 1, \ldots, n$. An independent joint PD control scheme can be written for the system (50) as in Spong and

Vidyasagar (8):

$$u = \bar{K}_{\mathrm{P}}\tilde{q} - \bar{K}_{\mathrm{D}}\dot{q} \tag{51}$$

where $\tilde{q} = q_{\mathrm{d}} - q$ is the error between the desired and the actual joint displacements, and $\bar{K}_{\mathrm{P}}$, $\bar{K}_{\mathrm{D}}$ are diagonal matrices of positive proportional and derivative gains, respectively. In the absence of gravity [$\phi(q) = 0$], the PD control law (51) achieves asymptotic tracking of the desired joint positions. In presence of gravity, Eq. (51) alone cannot guarantee asymptotic tracking and has to be modified as

$$u = \bar{K}_{\mathrm{P}}\tilde{q} - \bar{K}_{\mathrm{D}}\dot{q} + \phi(q) \tag{52}$$

to cancel the steady-state error due to the effect of the gravitational terms [see Spong and Vidyasagar (8)]. For detailed analysis and performance study of PD controller for robot manipulators, please refer to Spong and Vidyasagar (8).

Inverse Dynamics. Using inverse dynamics, a more complex nonlinear control technique can be implemented for trajectory tracking of rigid manipulators [Spong and Vidyasagar (8)]. Consider the system given by Eq. (50) in a more simplified form,

$$\bar{M}(q)\ddot{q} + \bar{h}(q, \dot{q}) = u \tag{53}$$

where $\bar{M} = D + \bar{J}$, $h = C\,\dot{q} + \bar{B}\dot{q} + \phi$. The idea of inverse dynamics is to seek a nonlinear feedback control law $u = \bar{f}(q, \dot{q})$ which when substituted into Eq. (53) results in a linear closed-loop system. Since the inertia matrix M is invertible, the control law

$$u = \bar{M}(q)V_{\mathrm{a}} + \bar{h}(q, \dot{q}) \tag{54}$$

reduces the system $\ddot{q} = V_{\mathrm{a}}$ with V_{a} as the new input to the system, the armature voltages to be applied to to the actuator motors. Thus, we have a double integrator system with n uncoupled double integrators. The nonlinear control law (54) is called the inverse dynamics control and achieves a new linear and decoupled system, making it possible to design $V_{\mathrm{a}k}$ to control a simple linear second-order system and can be designed as

$$V_{\mathrm{a}} = -\chi_0 q - \chi_1\dot{q} + \bar{r} \tag{55}$$

where χ_0, χ_1 are diagonal matrices of position and velocity gains, respectively, and $\bar{r}$ is the reference. The gains could be chosen to get a joint response that is equal to the response of a critically damped linear second-order system with desired natural frequencies for each of the desired speeds of the responses of the joints. The inverse dynamics can be viewed as an input transformation that transforms the problem from one of choosing torque input commands, which is difficult, to one of choosing acceleration input commands, which is easy. There are many crucial issues of implementation and robustness that must be addressed to implement Eq. (54), and the reader is referred to Spong and Vidyasagar (8).

Path Interpolation. The simplest type of robot motion is point-to-point motion. In this approach the robot is commanded to go from an initial configuration to a final configuration without regard to the intermediate path followed by the end effector. To understand the concept of configuration, it is helpful to review some terminology used in Spong and Vidyasagar (8). Suppose a robot has $n + 1$ links numbered from 0 to n starting

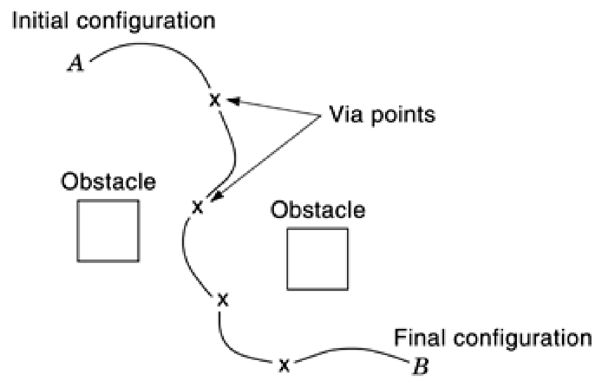

Fig. 4. Path interpolation: via points to plan motion around obstacles.

the base of the robot, which is taken as link 0. The joints are numbered 1 to n, and the ith joint is the point in space where links $i - 1$ and i are connected. The ith joint variable is denoted by q_i. A coordinate frame is attached rigidly to each link. We attach an inertial frame to the base and call it frame 0. Frames 1 to n are chosen such that frame i is rigidly attached to link i. Now the configuration is given by the transformation matrix that transforms the coordinates of a point from frame j to frame i and is denoted by $T_i{}^j$. For example, for a seven-link robot manipulator, the initial and final configurations that are of interest in point-to-point motion are the transformation matrices that transform the coordinates of frame 6 to frame 0; let them be denoted by $T_0{}^6$ init and $T_0{}^6$ final. This type of motion is suitable for materials transfer jobs where the workspace is clear of obstacles. Given the desired initial and final positions and orientation of the end effector, the inverse kinematic solution is evaluated to find the required initial and final joint variables. Suppose, $d_i{}^j$ denotes the position of frame j with respect to frame i, and $R_i{}^j$ denotes the orientation of frame j relative to frame i. For the manipulator with seven links, the motion of the first three, joints is calculated by computing the joint variables q_1, q_2, and q_3 corresponding to $d_0{}^3$ init and $d_0{}^3$ final. The motion of the final three joint variables is found by computing a set of Euler angles corresponding to $R_3{}^6$ init and $R_3{}^6$ final [Spong and Vidyasagar (8)]. For some purposes, such as obstacle avoidance, the path of the end effector can be further constrained by the addition of via points intermediate to the initial and the final configurations as shown in Fig. 4. Different techniques of generating smooth trajectories in joint space, given the initial and final joint variables, are presented in Spong and Vidyasagar (8).

Adaptive Control of Teleoperation Systems

A teleoperation system involves two distant yet coupled robots: a local master robot and a remote slave robot. An ideal teleoperation is the one in which the impedance felt by the human operator is matched to the impedance of the slave environment [Lawrence (14)]. The term "transparency" is used to describe such an ideal teleoperation. For teleoperation systems with known and time-invariant dynamics, a transparency control scheme is proposed in Lawrence (14), with a modification to handle communication time delays. For teleoperation systems with unknown time-invariant dynamics, an adaptive control scheme based on Slotine and Li (1,2) algorithm is presented in Hashtrudi-Zaad and Salcudean (15). Stability and signal boundedness of a similar adaptive control system are investigated in Lee and Chung (16). Despite recent progresses in teleoperation, transparency issues for teleoperation systems with unknown time-varying parameters, such as jumping and smoothly but rapidly changing parameters, including control designs and transparency characterizations, remain open.

Fig. 5. Structure of a teleoperation system.

In this article, we present new transparency concepts suitable for adaptive control of teleoperation systems with time-varying parameters [Shi et al. (17)]. Adaptive control schemes for teleoperation systems with jumping or rapidly time-varying parameters are developed [Shi et al. (17)]. The developed adaptive control schemes lead to stable and transparent teleoperations in the presence of unknown constant or jumping or fast-varying parameters. The teleoperation systems to be controlled are assumed to have no communication time delay. In the first subsection, we present the new concepts of weak transparency, asymptotic weak transparency, and approximate weak transparency, and formulate the transparency control problem for four types of teleoperation systems with no communication time delay [Shi et al. (17)]. In the next subsection, we present adaptive control schemes for teleoperation systems with unknown jumping parameters and with parametrizable and unparametrizable smoothly time-varying parameters [Shi et al. (17)]. In the last subsection we present some control issues relevant to teleoperation systems with communication time delays.

Teleoperation Systems. In this section, we present the general structure of a teleoperation system and its dynamic description, introduce several new concepts for transparency of teleoperation systems, and state the adaptive control objective with which the new transparency concepts are to be verified.

Dynamics of a Teleoperation System. A teleoperation system consists of five subsystems: the human operator, the master robot, the communication channels, the slave robot, and the slave environment, as shown in Fig. 5. The term *teleoperator* refers to the master and slave manipulators connected by the communication channels. Bilateral teleoperation involves velocity and force information transfer between the master and the slave. Communication time delays commonly exist in teleoperation systems due to the large distance and restrictive data transfer. These delays are assumed to be absent in the following analysis, for confinement to fundamentals and for simplicity of analysis. In Fig. 5, v_h is the velocity of the human operator's hand, v_m is the velocity of the master end effector, v_s is the velocity of slave end effector during contact, F_h is the force applied by the human operator to the master robot, F_e is the force exerted by the slave robot on its environment, and F_s is the coordinating torque. In the absence of communication time delay, $v_{sd}(t) = v_m(t)$ and $F_{md} = F_s(t)$. In the presence of communication time delay T, $v_{sd}(t) = v_m\ (t - T)$ and $F_{md} = F_s(t - T)$. In the following analysis, no communication time delay is assumed.

For analysis, a network representation of a teleoperation system is useful and Fig. 6 shows one commonly used in the literature, in which the human operator and the slave environment are represented by one-port networks, and the teleoperator by a two-port network. The blocks Z_h, Z_m, Z_s, and Z_e represent respectively the dynamics of a human operator, a master robot, a slave robot, and the slave environment; signals τ_m and τ_s denote control torques for master and slave robots; signals v_h and v_e refer to velocities of the human operator's hand and the slave environment. Note that v_h equals v_m, the velocity of master end effector, and v_e equals v_s, the velocity of slave end effector during contact. The signals F_h*, F_h, F_e represent respectively the force generated by the human operator, the force applied by the human operator to the master robot, and the force exerted by the slave robot on its environment.

As in Hashtrudi-Zaad and Salcudean (15), Lee and Chung (16), and Raju et al. (18), we consider the dynamics of the master and slave robots as

$$M_m\ddot{x}_m + B_m\dot{x}_m + K_m x_m = \tau_m + F_h \tag{56}$$

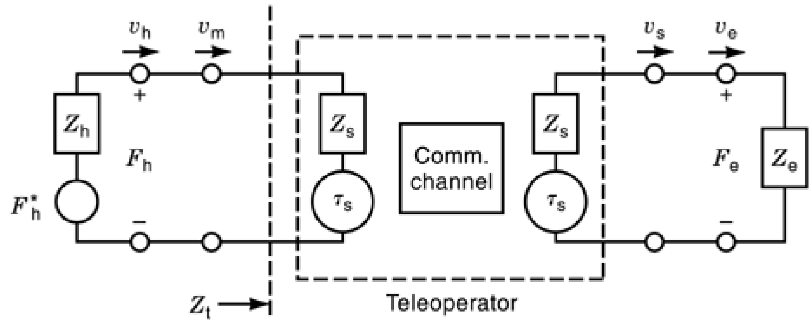

Fig. 6. A two-port network for a teleoperation system.

$$M_s\ddot{x}_s + B_s\dot{x}_s + K_s x_s = \tau_s - F_e \tag{57}$$

where M, B, and K are inertia, damping, and stiffness parameters; the signal x is the position of end effector; and the signal τ denotes the control torque with subscript m for the master and s for the slave. From Eq. (56), we see that $Z_m(s) = M_m s + B_m + K_m/s$.

Let C_m and C_s denote the master and slave feedback control, and C_i, $i = 1, \ldots, 4$, represent the data communication control for signals v_m, F_e, F_h, and v_s, respectively. Then the torques τ_m and τ_s have the following descriptions:

$$\tau_m = -C_m v_m - C_4 v_s - C_2 F_e \tag{58}$$

$$\tau_s = -C_s v_s - C_1 v_m + C_3 F_h \tag{59}$$

where the minus sign indicates the feedback signals. We also assume the human operator and the slave environment are passive, and as in Raju et al. (18), we use a generalized mass–damping–spring model to describe the human operator and the slave environment,

$$M_h\ddot{x}_m + B_h\dot{x}_m + K_h x_m = F_h^* - F_h \tag{60}$$

$$M_e\ddot{x}_s + B_e\dot{x}_s + K_e x_s = F_e \tag{61}$$

where M, B, and K are the inertia, damping, and stiffness parameters with subscript h for the human operator and e for the slave environment. Substituting Eq. (61) into Eq. (57), we get the slave system

$$M\ddot{x}_s + B\dot{x}_s + Kx_s = \tau_s \tag{62}$$

where $M = M_s + M_e$, $B = B_s + B_e$, and $K = K_s + K_e$.

Four types of teleoperation systems are usually met in applications: teleoperation systems (i) with known time-invariant dynamics, (ii) with unknown constant environment, (iii) with jumping environment parameters, and (iv) with smooth time-varying environment parameters. The transparency of adaptive teleoperation control systems is of main interest in this article, for which we will introduce new concepts suitable for adaptive control when the system parameters are unknown, for different cases of parameter uncertainties.

We first consider teleoperation systems with no communication time delay. The stability analysis is easy and simple when communication delay is not involved. A closed-loop transfer function can be obtained for the bilateral system, and the traditional tools such as the root locus technique and the Routh–Hurwitz stability criterion can be used for stability analysis. It is reasonable to assume no communication delay to develop the basic adaptive control techniques along with the stability analysis. In the last subsection below, we present some of the control issues relevant to teleoperation with communication time delays.

Transparency of a Teleoperation System. The impedance transmitted to or "felt" by human operator, Z_t (see Fig. 6), is defined by $F_h = Z_t\, v_h$, in the frequency domain.

Definition 1. *[Lawrence (14)]. A teleoperation system is transparent if*

$$Z_t = Z_e. \tag{63}$$

This means that in a transparent teleoperation, the human operator feels as if he were manipulating the slave environment directly. Note that when the slave robot is in contact with its environment, its velocity, v_s and the environment force F_e are related by the impedance Z_e as $F_e = Z_s\, v_s$ in the frequency domain. Since $v_h = v_m$, if the slave exactly reproduces the motion of the master (i.e., $v_s = v_m$, and $Z_t = Z_e$, then $F_h = F_e$, that is, the master accurately feels the slave contact force. That is, for a transparent teleoperation, the velocity tracking from the slave to the master leads to force tracking from the master to the slave.

Definition 2. *A teleoperation system is weakly transparent if*

$$F_h = F_e, \qquad v_s = v_m. \tag{64}$$

The property (64) is called weak transparency because it only needs $Z_t = Z_e$ for some specific operation frequencies at which $v_s = v_m$.

Definition 3. *A teleoperation system is asymptotic weakly transparent if*

$$F_h = F_e, \qquad \lim_{t\to\infty}\left[v_s(t) - v_m(t)\right] = 0. \tag{65}$$

This weak transparency is ensured in adaptive teleoperation control systems with parametric uncertainties.

Definition 4. *A teleoperation system is approximate weakly transparent if*

$$F_h = F_e, \qquad |v_s(t) - v_m(t)| \le c_1 e^{-\alpha t}|v_s(0) - v_m(0)| + \frac{c_2}{\beta} \tag{66}$$

for some constant $c_1 > 0$, $c_2 > 0$, $\alpha > 0$ and some design parameter $\beta > 0$.

In this case, it is expected that the design parameter $\beta > 0$ in the control system can be chosen to be large so that the tracking error $v_s(t) - v_m(t)$ can be made small.

Definition 5. *A teleoperation system is approximate weakly transparent in the mean if*

$$F_{\mathrm{h}} = F_{\mathrm{e}}, \qquad \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left[v_{\mathrm{s}}(t) - v_{\mathrm{m}}(t)\right]^2 \leq \frac{\gamma_1}{t_2 - t_1} + \gamma_2 \tag{67}$$

for some constant $\gamma_1 > 0$ and $\gamma_2 > 0$, and any $t_2 > t_1 \geq 0$.

This weak transparency is ensured in adaptive teleoperation systems with both parametric and structural uncertainties. In this case, it is expected that $\gamma_1 = \gamma_0\alpha$ for an adaptive control system, for some design parameter $\alpha > 0$ that can be made small.

Control Objective. The control objective is to develop controllers C_{m}, C_{s}, and C_i, $i = 1, \ldots, 4$, that ensure (i) closed-loop signal boundedness, (ii) $\lim_{t\to\infty} [v_{\mathrm{s}}(t) - v_{\mathrm{m}}(t)] = 0$ [$v_{\mathrm{s}}(t)$ tracks $v_{\mathrm{m}}(t)$ as closely as possible in the sense (66) or (67)], and (iii) $F_{\mathrm{h}} = F_{\mathrm{e}}$ for the slave environment with constant parameters, or with jumping parameters, or with smoothly time-varying parameters, in the presence of parameter uncertainties. This is a master–slave control problem.

Control Designs. In this subsection, we will first review two existing control designs: one for teleoperation systems with known constant parameters, and one for teleoperation systems with unknown constant parameters. We will then present new adaptive control schemes for time-varying teleoperation systems with jumping parameters or with smoothly time-varying parameters. The teleoperation systems in consideration are assumed to have no communication time delay. For the new proposed control schemes, we will analyze the system performance in terms of stability and transparency. The teleoperation system is said to be *stable* if the state variables of the system are bounded at any time.

Design for System with Known Constant Parameters. As in Lawrence (14), the forces and velocities of the teleoperator two-port as shown in Figure 6 are related by a hybrid matrix H:

$$\begin{bmatrix} F_{\mathrm{h}} \\ v_{\mathrm{h}} \end{bmatrix} = H \begin{bmatrix} v_{\mathrm{e}} \\ -F_{\mathrm{e}} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} v_{\mathrm{e}} \\ -F_{\mathrm{e}} \end{bmatrix} \tag{68}$$

where H_{ij}, $i, j = 1, 2$, representing the input–output relation of the teleoperator two-port, are $H_{11} = (Z_{\mathrm{m}} + C_{\mathrm{m}})D(Z_{\mathrm{s}} + C_{\mathrm{s}} - C_3C_4) + C_4$, $H_{12} = -(Z_{\mathrm{m}} + C_{\mathrm{m}})D(I - C_3C_2) - C_2$, $H_{21} = D(Z_{\mathrm{s}} + C_{\mathrm{s}} - C_3C_4)$, $H_{22} = -(I - C_3C_2)$ with $D = (C_1 + C_3Z_{\mathrm{m}} + C_3C_{\mathrm{m}})^{-1}$. Solving for F_{h}, v_{h} from Eq. (68), we get

$$F_{\mathrm{h}} = \underbrace{(H_{11} - H_{12}Z_{\mathrm{e}})(H_{21} - H_{22}Z_{\mathrm{e}})^{-1}}_{Z_{\mathrm{t}}} v_h.$$

To achieve $Z_{\mathrm{t}} = Z_{\mathrm{e}}$, it is needed that $H_{11} = H_{22} = 0$, $H_{12} = -1$, $H_{21} = 1$. Therefore, it can be derived that $C_1 = Z_{\mathrm{s}} + C_{\mathrm{s}}$, $C_2 = 1$, $C_3 = 1$, $C_4 = -(Z_{\mathrm{m}} + C_{\mathrm{m}})$ with C_{m}, C_{s} stable. This control scheme achieves transparency for teleoperation systems with known dynamics in the absence of communication time delay. Sufficient conditions for stability of the teleoperation system are derived in Lawrence (14). A tradeoff between stability and transparency is necessary [Lawrence (14)]. With the modified control schemes, good transparency is achieved at lower frequency. However, $Z_{\mathrm{t}} = Z_{\mathrm{e}}$ cannot be ensured for all frequencies.

In the next sub-subsections, we will consider the adaptive control problems when the slave system's parameters are unknown, to show that new transparency concepts defined in Definitions 3.3, 3.4, 3.5 are useful for adaptive teleoperation systems. To design an adaptive control law, we assume zero communication time delay and

(1) The master position signal x_m is bounded with bounded derivatives $\dot{x}_m$, 0$\ddot{2}2_m$ [Lee and Chung (16)].

Design for System with Unknown Constant Parameters. In this section, the slave environment is assumed to have unknown constant parameters. Our control objective, as specified in the sub-subsection "Transparency of a Teleoperation System" above, is to achieve (i) closed-loop signal boundedness; (ii) $\lim_{t\to\infty}[v_s(t) - v_m(t)] = 0$, and (iii) $F_h = F_e$. The last two properties imply asymptotic weak transparency of the teleoperation system. A control scheme based on Slotine and Li's (1) design is first applied to achieve signal boundedness and velocity tracking. Force matching is then designed by using the relationship of (3.13) with $H_{11} = 0$, $H_{12} = -1$.

Adaptive Control Design. The slave system as defined in Eq. (62) is

$$M\ddot{x}_s + B\dot{x}_s + Kx_s = \tau_s \tag{69}$$

where $M = M_s + M_e$, $B = B_s + B_e$, $K = K_s + K_e$, and $M_s, B_e, K_e > 0$ are unknown constants.

Let $\Lambda > 0$ be a design parameter, and define the signals $e(t) = x_s(t) - x_m(t)$, $v(t) = \dot{x}_m(t) - \Lambda e(t)$, $s(t) = \dot{x}_s(t) - v$. As in Slotine and Li (1), the control law is chosen as

$$\tau_s = Y(\dot{v}, v, x_s)\hat{\theta} - K_D s$$

where $Y(\dot{v}, v, x_s) = [\dot{v}\ v\ x_s]$ is a vector of known signals, $\theta = [M\ B\ K]^T$ is a vector of unknown parameters, $\hat{\theta}$ is the estimate of θ, and $K_D > 0$ is a design gain parameter. Choose the adaptive law as

$$\dot{\hat{\theta}} = -\Gamma Y^T(\dot{v}, v, x_s)s \tag{70}$$

where $\Gamma = \Gamma^T \varepsilon R^{3\times 3}$ is positive definite. Consider the Lyapunov function $V = \frac{1}{2}(Ms^2 + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta})$, where $\tilde{\theta} = \hat{\theta} - \theta$. Then it follows that $\dot{V} = -(K_D + B)s^2 \leq 0$, which implies that all the closed-loop signals are bounded, and that e, $\dot{e}$ converge to zero asymptotically as the time t goes to ∞ [Slotine and Li (1)].

Transparency and Stability. With velocity tracking from the slave to the master, the force tracking from the master to the slave will lead to a weak transparent teleoperation. Because F_h is related to F_e as $F_h = H_{11} v_e + H_{12}(-F_e)$, the condition for $F_h = F_e$ is

$$H_{11} = 0, \qquad H_{12} = -1. \tag{71}$$

Recall $H_{11} = (Z_m + C_m)D(Z_s + C_s - C_3C_4) + C_4$, $H_{12} = -(Z_m + C_m)D(I - C_3C_2) - C_2$, and $D = (C_1 + C_3Z_m + C_3C_m)^{-1}$. The following design [Lee and Chung (16)] will satisfy the condition (71):

$$C_m = -Z_m, \qquad C_2 = I, \qquad C_4 = 0 \tag{72}$$

Thus far asymptotic velocity tracking and force tracking are ensured, which lead to asymptotic weak transparency for a teleoperation system with unknown constant parameters. The developed teleoperation system is stable because the master, the slave, and their controllers are passive, and the human operator and the slave environment are also passive by assumption.

Design for System with Unknown Jumping Parameters. Parameter variations in this case are characterized by piecewise constant behavior. An example is the slave robot intermittently contacting with different environments. Assume that parameters B and K in the slave system (69) are unknown and piecewise constant, as modeled by

$$B(t) = B_1 f_1(t) + B_2 f_2(t) + \cdots + B_l f_l(t) \tag{73}$$

$$K(t) = K_1 f_1(t) + K_2 f_2(t) + \cdots + K_l f_l(t) \tag{74}$$

where B_i and K_i, $i = 1,\ldots, l$, are constants, which last for certain durations of time, and $f_i(t)$, $i = 1,\ldots, l$, are functions indicating which values of B_i and K_i are taken by $B(t)$ and $K(t)$ at a given time. The indicator functions, $f_i(t)$, are known functions that reveal the durations and the time instants of the parameter discontinuities, and are defined as

$$f_i(t) = \begin{cases} 1 & \text{if} \quad B = B_i, \quad K = K_i \\ 0 & \text{otherwise} \end{cases} \tag{75}$$

The function $f_i(t) = 1$ indicates the value B_i or K_i is taken by the system parameter $B(t)$ or $K(t)$. In addition, $f_i(t)$ indicates that only one value can be active at any time, that is, $f_i(t)f_j(t) = 0$ for $i \neq j$. With $\theta(t) = [M\ B(t)\ K(t)]^{\mathrm{T}}$, it follows that $\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) - \dot{\theta}(t) \neq \dot{\hat{\theta}}(t)$, so we cannot use $\dot{\hat{\theta}}(t)$ in Eq. (70) to ensure $\dot{V}(t) \leq 0$ for any $t > 0$.

In this sub-subsection, we first present an adaptive control scheme for the slave system (69) with unknown jumping parameters described by (73, 74, 75).

Adaptive Control Design. We propose the following controller structure:

$$\tau_{\mathrm{s}} = Y(\dot{v}, v, x_{\mathrm{s}})\hat{\theta} - K_{\mathrm{D}}s \tag{76}$$

where $v(t)$, $s(t)$, $Y(\dot{v}, v, x_{\mathrm{s}})$, and K_{D} are defined in the preceding sub-subsection, and $\hat{\theta} = [\hat{M}\ \hat{B}\ \hat{K}]^{\mathrm{T}}$ is the estimate of $\theta = [M\ B(t)\ K(t)]^{\mathrm{T}}$, with $\hat{B}(t) = \hat{B}_1 f_1(t) + \hat{B}_2 f_2(t) + \ldots + \hat{B}_l f_l(t)$ and $\hat{K}(t) = \hat{K}_1 f_1(t) + \hat{K}_2 f_2(t) + \ldots + \hat{K}_l f_l(t)$. Substituting (3.21) into (3.14) reveals

$$M\dot{s} + Bs = \tilde{M}\dot{v} + \left(\sum_{i=1}^{l} \tilde{B}_i f_i(t)\right) v + \left(\sum_{i=1}^{l} \tilde{K}_i f_i(t)\right) x_s - K_{\mathrm{D}}s \tag{77}$$

where the estimate errors $\tilde{M} \triangleq \hat{M} - M$, $\tilde{B}_i \triangleq \hat{B}_i - B_i$, and $\tilde{K}_i \triangleq \hat{K}_i - K_i$, $i = 1,\ldots, l$. Let us choose the positive function

$$V(t) = \frac{1}{2}\left(Ms^2 + \gamma_{\mathrm{m}}^{-1}\tilde{M}^2 + \sum_{i=1}^{l} \gamma_{\mathrm{b}i}^{-1}\tilde{B}_i^2 + \sum_{i=1}^{l} \gamma_{\mathrm{k}i}^{-1}\tilde{K}_i^2\right) \tag{78}$$

where γ_m, $\gamma_b i$, $\gamma_k i > 0$, $i = 1,\ldots, l$. We choose the adaptive laws for $\hat{M}_i$, $\hat{B}_i$ and K_i as

$$\dot{\hat{M}} = -\gamma_m \dot{v}(t)s(t) \tag{79}$$

$$\dot{\hat{B}}_i = -\gamma_{bi} f_i(t)v(t)s(t), \qquad i = 1, 2, \ldots, l \tag{80}$$

$$\dot{\hat{K}}_i = -\gamma_{ki} f_i(t)x_s(t)s(t), \qquad i = 1, 2, \ldots, l \tag{81}$$

With this adaptive design, the derivative of Eq. (78), $\dot{V}(t)$, becomes

$$\dot{V}(t) = -(K_D + B)s^2 \leq 0 \tag{82}$$

For stability analysis, we need the following lemma.

Lemma 1. *[Tao (19)]. If $\dot{f}(t)\ \varepsilon\ L^\infty$, $f(t)\ \varepsilon\ L^2$, then $\lim_{t\to\infty} f(t) = 0$.*

The fact that $\dot{V}(t) = -(K_D + B)s^2 \leq 0$ implies $s\ \varepsilon\ L^2 \cap L^\infty$, $\tilde{M}\varepsilon\ L^\infty$, $\tilde{B}_i\ \varepsilon\ L^\infty$, $\tilde{K}_i\ \varepsilon\ L^\infty$, $i = 1, 2,\ldots, l$. Since $s = \dot{e} + \Lambda e$, we conclude that e, $\dot{e}\ \varepsilon\ L^2 \cap L^\infty$. Hence x_s, $\dot{x}_s\ \varepsilon\ L^\infty$, as from assumption (A1) we have x_m, $\dot{x}_m\ \varepsilon\ L^\infty$. From Eqs. (76) and (77) it follows that τ_s, $\dot{s}\ \varepsilon\ L^\infty$; then $\ddot{e}\ \varepsilon\ L^\infty$. Applying Lemma 3.1, we conclude that the position tracking error $e(t)$ and velocity tracking error $\dot{e}(t) = v_s(t) - v_m(t)$ go to zero as t goes to ∞. In summary, we have proven the following theorem.

Theorem 3. The adaptive controller (76) with the adaptive laws (79, 80, 81), applied to the system (69) with jumping parameters (73, 74, 75), guarantees that all closed-loop signals are bounded and the tracking errors $e(t)$ and $\dot{e}(t)$ go to zero as t goes to ∞.

Transparency and Stability. With velocity tracking, controllers that ensure force tracking will also lead to asymptotic weak transparency of the teleoperation system. For such transparency, the force control (72) is also a choice for a teleoperation system with jumping parameters. Because the parameter jumping is bounded, the resulting jumping in acceleration and velocity is bounded as well. This will not change the passivity of the slave system, because its elements are still passive. Hence the system stability is guaranteed with respect to passivity.

Design for Smooth Time-Varying Parameters. Parameter variations in those systems are characterized by continuous bounded functions with bounded derivatives. The slave system is represented by

$$M(t)\ddot{x}_s + \dot{M}(t)\dot{x}_s + B(t)\dot{x}_s + K(t)x_s = \tau_s \tag{83}$$

where $m(t) > 0$, $B(t) > 0$, $K(t) > 0$ represent the time-varying mass, damping, and spring parameters. This model follows from the Euler–Lagrange equation [Spong and Vidyasagar (8)] with kinetic energy $K = \frac{1}{2}\dot{x}_s M(t) \dot{x}_s$. Transparent teleoperation designs for known and unknown time-varying parameters are considered in this section. To achieve weak transparency the key is velocity tracking and force tracking between slave and master robots.

Control Design for Known Time-Varying Parameters. A control scheme that ensures asymptotically weak transparency is proposed first for the teleoperation system with known time-varying slave system. This scheme is then extended to the time-varying slave system with bounded parameter disturbances.

Design I for Known Time-Varying Parameters.Design I for Known Time-Varying Parameters For the slave system (83) with known time-varying parameters, we propose the control scheme

$$\tau_s = Y(\dot{V}, v, x_s)\theta(t) + \dot{M}(t)\dot{x}_s - K_D s \tag{84}$$

where $\theta(t) = [M(t)\ B(t)\ K(t)]^T$, $Y(\dot{v}, v, x_s) = [\dot{v}, v, x_s]$, $v = \dot{x}_m(t) - \Lambda e(t)$, $s = \dot{x}_s(t) - v$, and $e(t) = x_s(t) - x_m(t)$, as in the sub-subsection "Design for System with Unknown Constant Parameters" above, and $K_D > 0$ is a design gain to be specified later. Substituting the controller (84) into the slave system (83) reveals

$$M(t)\dot{s} = -B(t)s - K_D s \tag{85}$$

Define the positive function

$$V(t) = \frac{1}{2}M(t)s^2 \tag{86}$$

The time derivative $\dot{V}(t)$ of $V(t)$ is

$$\dot{V}(t) = \frac{1}{2}\dot{M}(t)s^2 + M(t)s\dot{s} = -\left[-\frac{1}{2}\dot{M}(t) + B(t) + K_D\right]s^2.$$

To ensure $\dot{V}(t) \leq 0$, we choose K_D to be such that

$$K_D > \frac{1}{2}\dot{M}(t) - B(t) \tag{87}$$

The result that $\dot{V}(t) \leq 0$ implies that $s\ \varepsilon\ L^2 \cap L^\infty$. Since $s = \dot{e} + \Lambda e$, we conclude that e and $\dot{e}\ \varepsilon\ L^2 \cap L^\infty$. Hence x_s, $\dot{x}_s\ \varepsilon\ L^\infty$. From Eqs. (84) and (85) we have τ_s, $\dot{s}\ \varepsilon\ L^\infty$, and therefore $\ddot{e}\ \varepsilon\ L^\infty$. Applying Lemma 1, we conclude that the tracking errors $e(t)$ and $\dot{e}(t) = v_s(t) - v_m(t)$ go to zero as t goes to ∞.

In summary, we have the following results:

Theorem 4. All signals in the closed-loop system with the time-varying model (83) and the controller (84) where K_D satisfies (3.32) are bounded, and the tracking errors $e(t)$ and $\dot{e}(t)$ go to zero as t goes to ∞.

Design II for Time-Varying Parameters with Unknown Disturbances.Design II for Time-Varying Parameters with Unknown Disturbances In this case, the system parameters $\theta(t)$ and $M(t)$ satisfy the assumptions:

(1) The time-varying parameter vector $\theta(t)$ satisfies

$$\theta(t) = \theta_0(t) + \Delta_\theta(t) \tag{88}$$

for some known parameter $\theta_0(t)\ \varepsilon\ R^3$ and some unknown but bounded disturbance $\Delta_\theta(t)\ \varepsilon\ R^3$ such that $\|\Delta_\theta(t)\| < \rho_1$ for some constant $\rho_1 > 0$.

(2) The time-varying parameter $\dot{m}(t)$ satisfies

$$\dot{M}(t) = \dot{M}_0(t) + \Delta_M(t) \tag{89}$$

for some known function $\dot{m}_0(t)$, and some unknown but bounded disturbance $\Delta_M(t)$ such that $|\Delta_M(t)| < \rho_2$ for some constant $\rho_2 > 0$.

We propose the controller structure as

$$\tau_s = Y(\dot{v}, v, x_s)\theta_0(t) + \dot{M}_0(t)\dot{x}_s - K_D s - K_D s\|Y\|^2 - K_D s\dot{x}_s^2 \tag{90}$$

where $K_D > 0$ is the design gain. We choose the positive function $V(t)$ as defined in Eq. (86). Choose $K_D > \frac{1}{2}\dot{m}_0 + \frac{1}{2}\rho_2 + k_0$, for some design parameter $k_0 > 0$. Then $\dot{V} \leq -(B + k_0)s^2 + {\rho_1}^2/4K_D + {\rho_2}^2/4K_D$, which implies that $V(t)$ is bounded. Since $V = \frac{1}{2}M(t)s^2$, we have

$$\dot{V} \leq \frac{-2(B+k_0)}{M(t)}V + \frac{\rho_1^2}{4K_D} + \frac{\rho_2^2}{4K_D}$$

which implies

$$V \leq e^{-\alpha_1 k_0 t}\, V(0) + \frac{\rho_1^2 + \rho_2^2}{4\alpha_1 k_0 K_D}(1 - e^{-\alpha_1 k_0 t})$$

where $\alpha_1 > 0$ is a constant. We then have

$$\frac{1}{2}Ms^2 \leq e^{-\alpha_1 k_0 t}\frac{1}{2}M(0)s(0)^2 + \frac{\rho_1^2 + \rho_2^2}{4\alpha_1 k_0 K_D} \tag{91}$$

and

$$|s| \leq k_1 e^{-(\alpha_1 k_0/2)t}|s(0)| + \frac{k_2(\rho_1 + \rho_2)}{\sqrt{K_D}\sqrt{k_0}} \tag{92}$$

where $k_1 > 0$, $k_2 > 0$, and $\beta_1 > 0$ are constants, k_0 is a design parameter that can be chosen to be large, and so is $K_D > 0$. Since $s(t) = \dot{e}(t) + \lambda e(t)$ where $\lambda > 0$ is a constant, we have

$$|e| \qquad \leq c_1 e^{-(\alpha_1 k_0/2)t}|s(0)| + \frac{c_2(\rho_1 + \rho_2)}{\beta} \tag{93}$$

$$|e| \quad \leq d_1 e^{-(\alpha_1 k_0/2)t}|s(0)| + \frac{d_2(\rho_1+\rho_2)}{\beta} \tag{94}$$

where $c_1 > 0$, $c_2 > 0$, $d_1 > 0$, and $d_2 > 0$ are some constants, and $\beta_1 = \sqrt{K_D}\,\sqrt{K_0}$is a design parameter that can be chosen to large so that the errors in Eqs. (93) and (94) are small.

In summary, we have the following results.

Theorem 5. All signals in the time-varying system (83) with parameter disturbances (A2), (A3) and controller (90) are bounded, and the tracking errors $e(t)$, $\dot{e}(t)$ satisfy Eqs. (93) and (94), respectively. Moreover, $e(t)\ \varepsilon\ L^2$, $\dot{e}(t)\ \varepsilon\ L^2$, and $\lim_{t\to\infty} e(t) = 0$, $\lim_{t\to\infty} \dot{e}(t) = 0$ in the absence of parameter disturbances, that is, when $\Delta_\theta = 0$, $\delta_M = 0$.

Adaptive Control for Unknown Time-Varying Parameters. Transparent teleoperations are designed for two types of slave systems: those with unknown smooth time-varying (parametrizable) parameters, and those with unknown and disturbed time-varying (unparametrizable) parameters. An adaptive control scheme is proposed for the first type of system to achieve asymptotic weak transparency. With modification, this scheme ensures approximate weak transparency in the mean for the second type of system.

Design I for Parametrizable Parameter Variations.Design I for Parametrizable Parameter Variations We present an adaptive control design for systems satisfying the following assumptions:

(1) The unknown time-varying parameter vector $\theta(t)$ satisfies

$$\theta(t) = Y_0(t)\theta_0 \tag{95}$$

for some known function $Y_0(t)\ \varepsilon\ R^{3\times r_\theta}$ and some unknown but constant parameter $\theta_0\ \varepsilon\ Rr_\theta$, for some $r_\theta \geq 1$ [under this assumption, $Y(\dot{v}, v, x_s)\theta(t) = Y(\dot{v}, v, x_s)Y_0(t)\theta_0$, so that Slotine and Lis (1) design in the sub-subsection "Design for System with Unknown Jumping Parameters" above can be applied].

(2) The time-varying term $\frac{1}{2}\dot{m}(t)(\dot{x}_s + v)$ can be expressed as

$$\frac{1}{2}\dot{M}(t)(\dot{x}_s + v) = Z(x_s, \dot{x}_s, x_m, \dot{x}_m, t)\psi_0 \tag{96}$$

for some known function $Z(x_s, \dot{x}_s, x_m, \dot{x}_m, t)\ \varepsilon\ R^{1\times r\psi}$and some unknown but constant parameter $\psi_0\ \varepsilon\ R^{r\psi}$, for some $r_\psi \geq 1$.

We propose the adaptive controller structure

$$\tau_s = Y(\dot{V}, v, x_s)Y_0(t)\hat{\theta}_0 + Z(x_s, \dot{x}_s, x_m, \dot{x}_m, t)\hat{\psi}_0 - K_D s \tag{97}$$

where $\hat{\theta}_0$, $\hat{\Psi}_0$ are the estimates of θ_0 and ψ_0.

Including the controller (97) in the slave system (83) leads to

$$M(t)\dot{s} = -[B(t) + K_D]s - \dot{M}(t)(s + v) + Y(\hat{\theta}_0 - \theta_0) + Z\hat{\psi}_0 \tag{98}$$

Define the parameter errors $\tilde{\theta}_0 = \hat{\theta}_0 - \theta_0$, $\tilde{\psi}_0 = \hat{\psi}_0 - \psi_0$, and choose the positive function

$$V(t) = \frac{1}{2}M(t)s^2 + \frac{1}{2}\tilde{\theta}_0^{\mathrm{T}}\Gamma_\theta^{-1}\tilde{\theta}_0 + \frac{1}{2}\tilde{\psi}_0^{\mathrm{T}}\Gamma_\psi^{-1}\tilde{\psi}_0 \tag{99}$$

where $\Gamma_\theta = \Gamma_\theta^{\mathrm{T}} > 0$ and $\Gamma_\psi = \Gamma_\psi^{\mathrm{T}} > 0$ are constant matrices of the appropriate dimensions. To ensure that $\dot{V} \leq 0$, we choose the adaptive laws for $\hat{\theta}_0$ and $\hat{\psi}_0$ as

$$\dot{\hat{\theta}}_0 = -\Gamma_\theta Y_0^{\mathrm{T}} Y^{\mathrm{T}} s \tag{100}$$

$$\dot{\hat{\psi}}_0 = -\Gamma_\psi Z^{\mathrm{T}} s \tag{101}$$

With this choice of $\dot{\hat{\theta}}_0$ and $\dot{\hat{\psi}}_0$, we have $\dot{V} = -K_{\mathrm{D}}s^2 \leq 0$, which implies that $s \,\varepsilon\, L^2 \cap L^\infty$ and $\tilde{\theta}_0$, $\tilde{\psi}_0 \,\varepsilon\, L^\infty$. Since $s = \dot{e} + \Lambda e$, we conclude that $e, \dot{e} \,\varepsilon\, L^2 \cap L^\infty$. Hence $x_{\mathrm{s}}, \dot{x}_{\mathrm{s}} \,\varepsilon\, L^\infty$. From Eq. (97) it follows that $\tau_{\mathrm{s}}, \dot{s} \,\varepsilon\, L^\infty$; therefore, $\ddot{e} \,\varepsilon\, L^\infty$. Applying Lemma 3.1, we conclude that the tracking errors $e(t)$ and $\dot{e}(t) = v_{\mathrm{s}}(t) - v_{\mathrm{m}}(t)$ go to zero as t goes to ∞.

In summary, we have the following results.

Theorem 6. The adaptive controller (97) with the adaptation law (100) and (101) applied to the time-varying system (83) guarantees that all closed-loop signals are bounded and the tracking error $e(t)$ and $\dot{e}(t)$ go to zero as t goes to ∞.

Design II for Unparametrizable Parameter Variations.Design II for Unparametrizable Parameter Variations We assume the unparametric parameters having a parametric part and bounded disturbance part. They satisfy the modified assumptions:

(1) The parameter $\theta(t)$ satisfies

$$\theta(t) = Y_0(t)\theta_0 + \Delta_\theta(t) \tag{102}$$

where $Y_0(t)$ and θ_0 are the same as that defined in (A4), such that $\|\theta_0\| < M_1$ for some constant $M_1 \geq 0$, and $\|\Delta_\theta(t)\| < \rho_1$ for some constants $\rho_1 > 0$.

(2) The term $\frac{1}{2}\dot{m}(t)(\dot{x}_{\mathrm{s}} + v)$ satisfies

$$\frac{1}{2}\dot{M}(t)(\dot{x}_{\mathrm{s}} + v) = Z(x_{\mathrm{s}}, \dot{x}_{\mathrm{s}}, x_{\mathrm{m}}, \dot{x}_{\mathrm{m}}, t)\psi_0 + \Delta_\psi(t) \tag{103}$$

where $Z(x_{\mathrm{s}}, \dot{x}_{\mathrm{s}}, x_{\mathrm{m}}, \dot{x}_{\mathrm{m}}, t)$ and ψ_0 are the same as that defined in (A5) such that $|\psi_0| < M_2$ for some constant $M_2 > 0$, and $|\Delta_\psi(t)| < Y_1(t)\rho_2$ for some constant $\rho_2 > 0$ and some known function $Y_1(t)$. *Remark*: One choice of $Y_1(t)$ is $Y_1(t) = \dot{x}_{\mathrm{s}}(t) + v(t)$.

We propose the controller structure as

$$\tau_s = Y(\dot{v}, v, x_s)Y_0\hat{\theta}_0 + Z(x_s, \dot{x}_s, x_m, \dot{x}_m, t)\hat{\psi}_0 - K_D s - K_D s\|Y\|^2 - K_D s\|Y_1\|^2 \tag{104}$$

and the adaptive law for θ_0 and ψ_0 as

$$\dot{\hat{\theta}}_0 = -\Gamma_\theta Y_0^T Y^T s - \Gamma_\theta \sigma_\theta \hat{\theta}_0 \tag{105}$$

$$\dot{\hat{\psi}}_0 = -\Gamma_\psi Z^T s - \Gamma_\psi \sigma_\psi \hat{\psi}_0 \tag{106}$$

where σ_θ, σ_ψ are switching signals defined as

$$\sigma_\theta = \begin{cases} 0 & \text{if} \quad \|\hat{\theta}_0\| < M_1 \\ \sigma_{\theta_0}(\|\hat{\theta}_0\|/M_1 - 1) & \text{if} \quad M_1 \le \|\hat{\theta}_0\| < 2M_1 \\ \sigma_{\theta_0} & \text{if} \quad \|\hat{\theta}_0\| \ge 2M_1 \end{cases} \tag{107}$$

$$\sigma_\psi = \begin{cases} 0 & \text{if} \quad |\hat{\psi}_0| < M_2 \\ \sigma_{\psi_0}(|\hat{\psi}_0/M_2 - 1) & \text{if} \quad M_2 \le |\hat{\psi}_0| < 2M_2 \\ \sigma_{\psi_0} & \text{if} \quad |\hat{\psi}_0| \ge 2M_2 \end{cases} \tag{108}$$

for some constants $\sigma_{\theta 0} > 0$ and $\sigma_{\psi 0} > 0$.

The Lyapunov candidate function is same as defined in Eq. (99). Using the facts that $\sigma_\theta \tilde{\theta}_0^T \hat{\theta} \ge 0$ and $\sigma_\psi \tilde{\psi}_0^T \hat{\psi}_0 \ge 0$, that $\sigma_\theta \tilde{\theta}_0^T \hat{\theta}$ and $\sigma_\psi \tilde{\psi}_0^T \hat{\psi}_0$ go unbounded if $\hat{\theta}_0(t)$ and $\hat{\psi}_0(t)$ go unbounded [see Ioannou and Sun (20)], and that Δ_θ and ρ_2 are finite [see assumptions (A4′) and (A5′)], we have that $V(t)$ is bounded, and

$$\dot{V} \le -(B(t) + K_D)s^2 + \frac{\rho_1^2}{4K_D} + \frac{\rho_2^2}{4K_D} \tag{109}$$

Since $V(t)$ is bounded, from Eq. (109) we have

$$\int_{t_1}^{t_2} s^2(t)\, dt \le \frac{\alpha_0}{K_D}(\rho_1^2 + \rho_2^2)(t_2 - t_1) + \beta_0 \tag{110}$$

for some constants α_0, $\beta_0 > 0$, and any $t_2 > t_1 \ge 0$. Because of the relation $s(t) = \dot{e}(t) + \Lambda e(t)$ and $\Lambda > 0$ is constant, we can obtain

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} e^2(t)\, dt \le \frac{\alpha_1}{K_D}(\rho_1^2 + \rho_2^2) + \frac{1}{\beta_1} \tag{111}$$

and

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \dot{e}^2(t)\, dt \le \frac{\alpha_2}{K_D}(\rho_1^2 + \rho_2^2)(t_2 - t_1) + \frac{1}{\beta_2} \tag{112}$$

where α_1, β_1, α_2, and β_2 are some positive constants [see Tao (7)]. In this case, the design parameter $K_D > 0$ can be chosen to be large so that the mean errors are small in Eqs. (111) and (112).

In summary, we have the following results.

Theorem 7. All signals in the closed-loop system (83) with unparametric time-varying parameters, and adaptive control law (104) and adaptive law (105, 106, 107, 108, 109, 110), are bounded, and the tracking errors $e(t)$, $\dot{e}(t)$ satisfy Eqs. (111) and (112), respectively. Moreover, $e(t)\ \varepsilon\ L^2$, $\dot{e}(t)\ \varepsilon\ L^2$ and $\lim_{t\to\infty} e(t) = 0$ $\lim_{t\to\infty} \dot{e}(t) = 0$ in the absence of parameter disturbances, that is, when $\Delta_\theta = 0$, $\Delta_\psi = 0$.

Transparency and Stability. For teleoperation systems with known time-varying parameters or parametric time-varying parameters, the adaptive control schemes (84) and (97) ensure velocity tracking from the slave to the master. Therefore the force tracking design as in Eq. (72) will lead to asymptotic weak transparency (Definition 3). For time-varying systems with bounded disturbances, an arbitrary small tracking error can be obtained by increasing the design gain K_D. By using the force tracking design in Eq. (72), approximate weak transparency (Definition 4) or approximate weak transparency in the mean (Definition 5) is achieved. Stability of the resulting releoperation system is ensured by the boundedness of all the closed-loop signals.

Teleoperation with Communication Time Delay. Communication time delay in a bilateral teleoperation system reduces system stability and performance. Delay on the order of a tenth of a second were shown to destabilize the teleoperator. The stability problem becomes difficult when a communication time delay T is present, because a time delay introduce a factor e^{-sT} into the system and hence makes the system infinite-dimensional. In bilateral teleoperation, the force reflection from the slave, introduced for providing the “feeling” of the remote task, has effects on the master’s motion, which generates disturbances on the desired motion. The communication delay may worsen the situation as well. With a time delay T, the conventional communication law results in tracking of both position and force in the steady state. However, with the delay, the system is not passive, and will probably never reach a steady state. A preliminary result on a modified control scheme that provides improved tracking performance for a noncontact task in the presence of time delay and for arbitrary master trajectories has been developed in Shi et al. (21).

In the previous research, the passivity formalism and network expression were used to investigate the stability of a bilateral teleoperation system. In these methods, the human operator input is assumed to be bounded, and the human operator and the environment are assumed to be passive. In the presence of communication time delay, and with the passivity assumptions about the operator and the environment, passivity of the system depends on passivity of the communication block.

Two approaches can be used to produce a passive communication block. The first was developed by Anderson and Spong (22) using scattering transformation theory. This solution uses the transmission line equations as a basis for deriving a passive communication control law. By applying scattering theory, it is shown how conventional approaches lead to infinite gain of a scattering operator at finite frequencies, and how by implementing a set of time delay equations this instability can be overcome. The resulting system is then passive for all time delays. The proposed control law maintains steady-state force and velocity tracking.

The second approach to produce a passive communication block is developed by Niemeyer and Slotine (23). This approach uses an energy formulation to construct a teleoperation system that imitates physical systems and obeys an energy conservation law. A wave variable is utilized to characterize time delay systems and leads to a new configuration for force-reflecting teleoperation.

Since the dynamic control of the remote slave by an operator is severely restricted by the time delays in the transmission, it is important to provide consistent dynamic performance locally at the remote site in the face of uncertainties and varying operating conditions. With the development of high-speed and high-capacity computer networks, it is possible to deliver teleoperation over a public computer network. The problem of varying communication time delays arises in such a teleoperation system. Adaptivity of a teleoperation system to uncertain time delay in also desirable. The related stability, tracking, and transparency problems of a bilateral teleoperation system under uncertain environment but now with communication time delay are important issues to be addressed. Adaptive control solutions proposed in the subsection "Control Designs" need to be modified to provide adaptation mechanisms for adjusting the controller parameters to achieve desired system performance, despite system uncertainties due to the unknown slave environment, and now in the presence of communication time delays.

The passivity-based solution to the bilateral teleoperator time delay problem developed by Anderson and Spong (22) is based on the result in circuit theory that a circuit consisting of passive elements only is passive and therefore stable. However, if some elements in a circuit representing a teleoperation system are not passive, one cannot use passive network theory to conclude the stability of the teleoperation system. On the other hand, if the transfer function of a teleoperation system is positive real, then the system is passive. In Shi et al. (24), a notion of positive realness has been used to investigate the passivity of the teleoperation system proposed by Anderson and Spong (22). Shi et al. (24) have also proposed a modified control scheme that use the master accelaration information (with delayed operation, which can be obtained from the velocity information) for slave control and ensures that in the absence of slave environment torque the slave position tracks that of the master asymptotically, that is, achieves improved tracking performance for the teleoperation system.

Summary

Position control for robot manipulators and teleoperation systems involves many dimensions of control theory, such as controller design, robustness analysis, and adaptive designs, along with many practical applications. Robust adaptive control schemes have been presented to handle situations in which the robot system has bounded parameter variations or/and unmodeled dynamics of bounded gains. The distinct feature of the manipulator dynamics were used to define bounding signals in the controller structure, whose parameters are updated from a robust adaptive law to ensure signal boundedness and tracking errors of the order of parameter variations and unmodeled dynamics, which may not be small. Some common topics relevant to position control of robot manipulators, such as PD control, inverse dynamics control, and path or trajectory interpolation, were also discussed.

Adaptive motion control of teleoperation systems was addressed. Several new concepts of transparency were defined for teleoperation control systems with unknown parameters. These new transparency concepts are useful for developing adaptive control schemes for control of a teleoperation system with unknown constant parameters, or with unknown jumping parameters, or with unknown smooth but large time-varying parameters. Weak transparency properties have been established for such adaptive teleoperation control systems. Some important control issues for teleoperation systems with communication time delay were also discussed.

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