BROADBAND NETWORKS

The problem of broadband matching is one of the major concerns when working with high-frequency communication systems. All broadcasting networks such as radio and television, and all wireless communication networks, such as cellular telephones and satellite networks, are the most frequently encountered examples of such systems.

A typical high-frequency wireless communication system contains two major sites, namely, a transmitter and a receiver (Fig. 1). On the transmitter site, the generated signal must be properly transferred to the antenna, preferably over a nondissipative (lossless) network so that maximum power of the generated signal is pumped into the antenna. Similarly, on the receiver site, the received signal of the antenna is transferred over a lossless matching network and dissipated at the user end. The user end may be, for example, a radio, a TV set, or a headphone. In this case, again, the role of the matching network is to provide the maximum power transfer of the received signal to the user end. In the literature, several terms are associated with the nondissipative power transfer network, such as "impedance matching network," "equalizer," "lossless two-port," "lossless network," or "interstage-equalizer" these terms are all used interchangeably. The classical broadband matching theory deals with the proper design of the lossless matching networks between prescribed terminations.

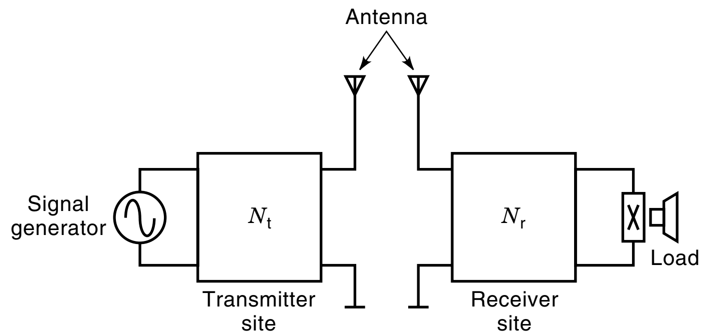


Figure 1. Generic block diagram of a high-frequency wireless communication system.

It is common that the signal-generation section of the transmitter can simply be modeled as an ideal signal generator in series with internal impedance Z_G. The transmitter antenna will behave as a typical passive load termination Z_L to the lossless power transfer two-port E (Fig. 2). Similarly, the receiver antenna can be considered as an ideal signal source with an internal impedance Z_G. The user end of the receiver site can also be considered as a dissipative load Z_L to the lossless two port E.

In the discussion above it is evident that both transmitter and receiver sites present a similar model as far as the signal flow is concerned. In both cases, the crucial issue is the maximum power, transferred from the generator Z_G to load Z_L. Therefore, once the signal generator and the load are given, the system performance can be optimized with the proper design of the nondissipative or lossless two-port E.

In all the cascaded high-frequency systems, one is faced with the problem of power transfer between cascaded sections or so called "interstages." As a principal, using Thevenin's theorem, the left site of the interstage can be modeled as an ideal signal generator E_G in series with an internal impedance Z_G. Similarly, the right site is simply regarded as a passive load Z_L, as shown in Fig. 2.

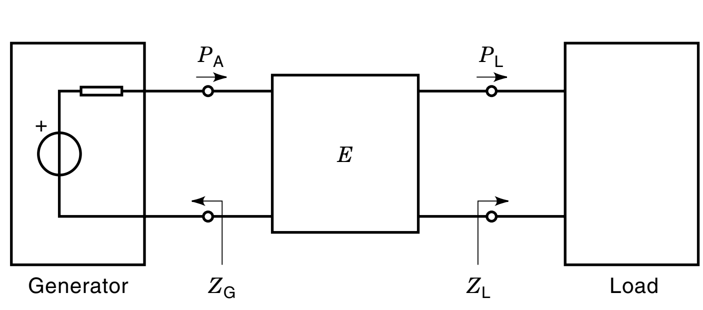


Figure 2. Power-transfer problem between a generator and a load network over a lossless equalizer.

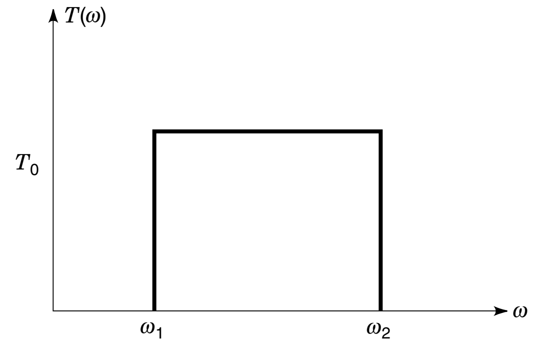


Figure 3. Rectangular flat transducer power gain characteristics with sharp roll-off over a passband (ω_1 to ω_2), which describes ideal power transfer between generator and load through a lossless matching network.

Hence, the classical broadband-matching problem is defined as one of *constructing a lossless reciprocal two-port or equalizer so that the power transfer from source (or generator) to load is maximized over a prescribed frequency band.*

The power-transfer capability of the lossless equalizer or so called "matching" network is best measured with the transducer power gain T, which is defined as the ratio of power delivered to the load P_L by the available power P_A of the generator; over a wide frequency band. That is,

$$T = \frac{P_L}{P_A} \tag{1}$$

Ideally, the designer demands the transfer of the available power of the generator to the load, which, in turn, requires the flat transducer power gain characteristic in the band of operation at a unitary gain level with sharp rectangular roll-off, as illustrated in Fig. 3. But unfortunately, the physics of the problem permit the ideal power transfer at only a single frequency. In this case, the equalizer input impedance Z_{in} is conjugately matched to the generator impedance Z_G. Therefore, the design of a matching equalizer over a wide frequency band with "high" and "flat" gain characteristics presents a very complicated theoretical problem. It is well known that the terminating impedances Z_G and Z_L impose the possible highset flat gain level over frequency band B, so called the theoretical "gain bandwidth limitation" of the matched system.

Before introducing the design methodologies it is important to classify the broadband matching problems.

Single Matching. This is a matching problem where either one of the passive terminations of the equalizer is resistive; the other is complex or frequency dependent [Fig. 4(a)].

Double Matching. This is a matching problem where both passive terminations of the equalizer are complex [Fig. 4(b)].

Active Matching. This is a matching problem of active devices. A typical example of an active matching is the design of a microwave amplifier [Fig. 4(c)].

It should be mentioned that the *filter* or the *insertion loss* problem might also be considered as a very special form of the broadband matching problem, which deals with the resistive generator and resistive load [Fig.4(d)]. In this respect, well-established filter design techniques may be employed for broadband matching problems where appropriate.

There are two main approaches to the solution of broadband matching problems, namely, (1) analytic solutions and (2) computer-aided solutions. The classical procedure is through analytic *gain-bandwidth theory* (1). Solutions of the second type are accomplished by numerical optimization and are referred to as *real frequency techniques,* after Carlin (2). In both cases it is optimal to seek the achievement of maximum level of minimum gain within the passband.

The analytic gain–bandwidth theory is essential to understanding the nature of the matching problem but, in general, is not accessible beyond simple problems. The real frequency computer-aided solutions, however, are very practical and easy to carry out for more complicated problems.

In this article, first the analytic matching theory will be briefly discussed. Then several real frequency approaches to the problem will be summarized.

Generally, the lossless matching network to be designed can be described in terms of two-port parameters (such as impedance, admittance, chain, real, or complex normalized regularized scattering or transmission parameters), or by means of driving point so-called Darlington immitance or bounded real (real normalized) reflection coefficient.

At this point it is appropriate to state the modified version of Darlington's famous theorem: (3).

Theorem. Any positive real impedance (Z) or admittance (Y) function or corresponding bounded real reflection coefficient $S = (Z - 1)/(Z + 1)$ or $S = (1 - Y)/(1 + Y)$ can be represented as a lossless two-port terminated in unit resistance. The resulting lossless two-port is called the Darlington equivalent (Fig. 5).

Based on the fundamental gain–bandwidth theory introduced by Bode (1), the analytic approach to single matching problems was first developed by Fano (4), using the concept of "Darlington equivalent" of the passive load impedance (Z_L). In Fano's approach, the problem is handled as a "pseudo-filter" or "pseudo-insertion loss" problem, since the tandem connection of the lossless equalizer E and Darlington's load equivalent L is considered as a whole lossless filter F (Fig. 6).

Later, Youla (5) proposed a rigorous solution to the problem using the concept of complex normalization. In order to solve the double-matching problem, Youla described the lossless matching network in terms of complex normalized scattering parameters with respect to frequency dependent impedances of generator and load terminations. Youla's theory provided an excellent solution to handle the single-matching problems, but was not practical to solve the double-matching problems since the realizability conditions based on the complex normalized scattering parameters of the matching equalizer was complicated to implement.

The complete analytic solution to the double-matching problem has been more simply formulated by the main theorem of Yarman and Carlin (6–8), which relates to the "real," and the "complex normalized-regularized" generator and load reflection coefficients of the doubly matched system. This theorem enables the designer to fully describe the doubly matched system in terms of the "realizable"-real normalized

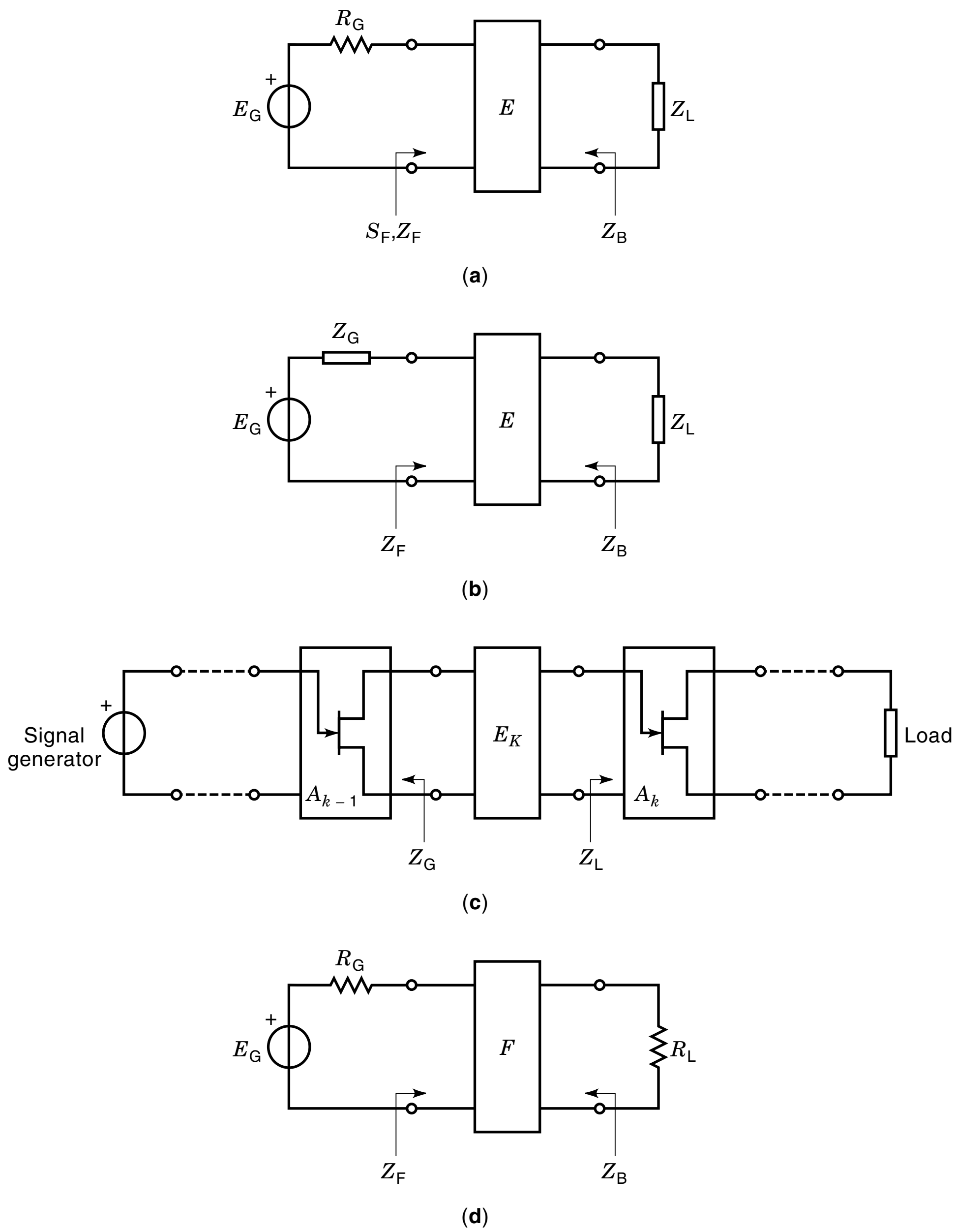


Figure 4. (a) Single-matching problem between a resistive generator and a complex load impedance. (b) Double-matching problem between a complex generator and a complex load impedance. (c) Active matching problem which involves design of interstage matching networks for multistage microwave amplifiers. (d) Filter or insertion loss problem in view of broadband matching: A special form of the matching problem between a resistive generator and a resistive load.

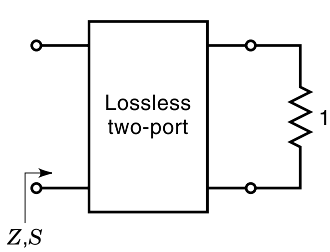


Figure 5. Darlington representation of a positive real impedance or admittance function or a bounded real reflection function.

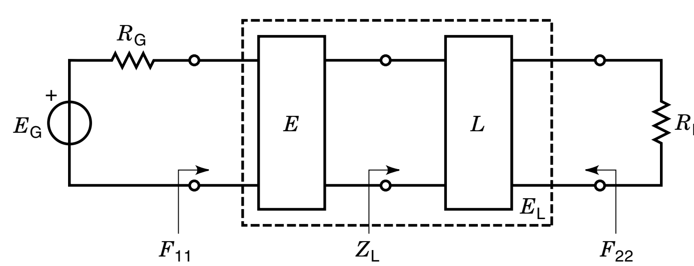


Figure 6. Single-matching problem with Darlington equivalent representation of a load impedance.

(or unit normalized) scattering parameters after replacing generator and load with their Darlington equivalents, as in the filter design theory.

Instructional accounts of gain–bandwidth theory for both single- and double-matching problems have been elaborated by Chen (9).

In the following sections, the essence of Fano's and Youla's theories will be reviewed. Subsequently, an attempt will be made to introduce analytic solutions for single- and double-matching problems under the "unified approach." Then, modern computer-aided design (CAD) or the "real frequency" techniques which are employed to construct wide band matching networks will be summarized (10). Finally, practical design techniques to construct matching networks with mixed lumped and distributed elements will be discussed.

In order to understand the analytic theory of broadband matching, it may be appropriate to first review the filter or insertion loss problem, which constitutes the heart of the unified approach and clarifies the basic properties of lossless two-ports.

FILTER OR INSERTION LOSS PROBLEM IN VIEW OF BROADBAND MATCHING

A typical filter or insertion loss problem is depicted in Fig. 4(d). In view of broadband matching, the problem is stated as follows:

PROBLEM. Given the resistive generator R_1 and the resistive load R_2, construct the reciprocal-lossless filter two-port F to transfer the maximum power of the generator to the load R_2 only over the passband ω_1 to ω_2; stop it otherwise.

In this problem, it is suitable to describe the reciprocal-lossless filter two-port F in terms of its real (or equivalently unit) normalized scattering matrix F with respect to port normalization numbers R_1 and R_2. For unit normalization

$$F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \tag{2}$$

The system performance of the filter two-port F is measured with the transducer power gain $T(\omega)$ given by

$$T(\omega) = |F_{21}(j\omega)|^2 \tag{3}$$

If the filter consists of one kind of elements (i.e., either lumped or distributed elements), the real normalized bounded real (BR) scattering parameters are given in the following, so-called "Belevitch" canonic form (11)

$$\begin{aligned} F_{11} &= \frac{h}{g}, \qquad F_{21} = \frac{f}{g} \\ F_{12} &= \eta\frac{f_*}{g}, \qquad F_{22} = -\eta\frac{h_*}{g} \end{aligned} \tag{4}$$

where $\eta = f_*/f$ and h, f, g are the real polynomials in complex variable $s = \sigma + j\omega$ for lumped element design or in $\lambda = \Sigma + j\Omega$ if F is constructed with equal length or commensurate transmission lines. Here, λ designates the Richards variable, given by $\lambda = \tanh(s)$ (12). In practice, one is mainly interested in the design of reciprocal lossless two-port filters, which require equal F_{12} and F_{21}, (i.e., $F_{21} = F_{12}$). In this case $\eta = f_*/f = \pm 1$, where the plus sign is applied if f is even; the minus sign if f is odd.

It is well known that a lossless two-port must possess a bounded real paraunitary scattering matrix. That is,

$$F_*^{\mathrm{T}} F = I \tag{5}$$

or

$$F_{11}F_{11*} + F_{21}F_{21*} = 1 \quad \text{or, on the } j\omega \text{ axis} \quad |F_{21}|^2 = 1 - |F_{11}|^2 \tag{6a}$$

$$F_{12}F_{11*} + F_{21*}F_{22} = 0 \quad \text{or, on the } j\omega \text{ axis} \quad F_{22} = -F_{11}^*F_{12}/F_{21}^* \tag{6b}$$

$$F_{22}F_{22*} + F_{12}F_{12*} = 1 \quad \text{or, on the } j\omega \text{ axis} \quad |F_{22}|^2 = 1 - |F_{12}|^2 \tag{6c}$$

$$F_{11}F_{12*} + F_{21}F_{22*} = 0 \quad \text{or, on the } j\omega \text{ axis} \quad F_{11} = -F_{22}^*F_{21}/F_{12}^* \tag{6d}$$

where I designates a 2×2 unitary matrix, superscript T indicates the transpose of a matrix, and the asterisk indicates either paraconjugate as subscript or complex conjugate as superscript.

The complex frequency variable is taken as $s = j + j\omega$ as in lumped filter design, and the equation set Eqs. (6a–6d) can be written in terms of the canonic polynomials $h(s)$, $f(s)$, and $g(s)$:

$$hh_* = gg_* - ff_* \tag{7a}$$

or in the open form

$$h(s)h(-s) = g(s)g(-s) - f(s)f(-s) \tag{7b}$$

In terms of the canonic polynomials f and g, the transducer power gain is given by

$$T(\omega^2) = \frac{f(j\omega)f(-j\omega)}{g(j\omega)g(-j\omega)} \tag{8a}$$

or in complex variable s,

$$T(-s^2) = \frac{f(s)f(-s)}{g(s)g(-s)} \tag{8b}$$

In essence, Eq. (8) dictates all the performance measures of a lossless-reciprocal filter. When the transducer gain T is other than zero, the lossless system allows the signal transmission. However, there are complex frequencies "s_l" such that $T(-s^2)$ is zero. Eq. (8b) indicates that the forward and backward signal transmission is dictated by F_{21} and F_{12}, respectively. Therefore, the function $\tilde{F}(s) = F_{21}(s)F_{12}(s)$ determines forward and backward signal transmission of the lossless reciprocal filter F. In the following the definition of transmission zeros (8) are given:

Definition. Transmission zeros of a lossless two-port are the closed right half plane (RHP) zeros of $F_{21}(s)F_{12}(s) = \tilde{F}(s)$ or, more explicitly, the closed RHP zeros of the expression

$$\tilde{F}(s) = \eta\frac{f(-s)f(s)}{g^2(s)} \tag{9}$$

where all possible common factors between the numerator and the denominator have been canceled and the zeros on the $j\omega$-axis are counted for their half multiplicity.

It should be noted that for reciprocal structures, $\tilde{F} = f^2(s)/g^2(s)$ since $F_{21}(s) = F_{12}(s)$. In this case, transmission zeros of the lossless reciprocal two-port will simply be the closed RHP zeros of transmittance parameter $F_{21}(s) = f(s)/g(s)$ with even multiplicity, which obviously overlaps with the zeros of transducer gain function $T(-s^2)$ of Eq. (8b). Transmission zeros at infinity are considered as the real frequency zeros on the $j\omega$ axis and determined as the degree difference between the polynomials $g(s)$ and $f(s)$.

Construction of Doubly Terminated Lossless-Reciprocal Filters

Based on the above theoretical overview, design steps of doubly terminated lossless reciprocal filters are straightforward.

Step 1. Choose an appropriate transducer power gain form $T(\omega^2)$ which includes all the desired transmission zeros of the doubly terminated system. Any readily available form such as Butterworth, Chybeshev, elliptic, or Bessel type of function may be suitable, depending on the application.

Step 2. Using the Belevitch notation, spectral factorization of the numerator and the denominator of the selected gain function $T(-s^2)$ is carried out to obtain the polynomials $f(s)$ and $g(s)$. At this stage it should be pointed out that the numerator $f(s)f(-s)$ must be of even multiplicity so that $F_{21} = F_{12}$. The polynomial $g(s)$ is uniquely determined by the spectral factorization of the denominator of $T(-s^2)$ since it must be strictly Hurwitz. Hence, $F_{21} = F_{12} = f/g$ is determined.

Step 3. The polynomial $h(s)$ is formed via spectral factorization of hh_* as given by Eq. (7a). However, zeros of hh_* are freely divided between the polynomials h and h_*. The sole requirement in the allocation is that each zero of one polynomial is reflected to the image location in the other polynomial, as described in Ref. 11. Thus, $F_{11} = h/g$ and $F_{22} = -\eta h_*/g$ are determined within an analytic all-pass η, which also includes RHP zeros of $f(s)$. The general solution to the factorization problem is

$$F_{11} = \eta F_{11\text{m}} \tag{10}$$

where $F_{11\text{m}}$ is the minimum phase solution.

Step 4. Finally, the filter is constructed by means of Darlington's synthesis procedure of driving point impedance $Z = (1 + F_{11})/(1 - F_{11})$ as a lossless two-port in unit termination (8).

ANALYTIC SOLUTION OF THE BROADBAND MATCHING PROBLEM

In this section basic guidelines of Fano's and Youla's approaches are given and they are linked by means of the main theorem of Yarman and Carlin which, in turn, leads to the unified approach to designing broadband matching networks (8).

Analytic Approach to Single-Matching Problems

Single matching problems deal with the construction of a broadband lossless equalizer E, which is placed between a resistive generator and a complex load, as shown in Fig. 4(a).

In Fano's theory, the frequency dependent non-Foster load is replaced with its Darlington's equivalent (Fig. 7). Let $Z_\text{L}(s)$ denote impedance of the non-Foster load, $L = \{L_{ij}\}$ scattering parameters of its lossless Darlington's equivalent and let the unit normalized scattering parameters of equalizer E be designated by $E = \{E_{ij}\}$. The cascaded connection of equalizer E and L is represented by F, whose scattering parameters are denoted by $F = \{F_{ij}\}$.

Scattering Description of the Darlington Equivalent of Load Network (6–8). Employing paraunitary properties of the lossless load equivalent network L, as stated in Eqs. (6a–6d), the unit normalized scattering parameters are given in terms of the impedance $Z_\text{L}(s) = N_\text{L}(s)/D_\text{L}(s)$:

$$L_{11} = \frac{Z_\text{L} - 1}{Z_\text{L} + 1} \tag{10a}$$

$$L_{21} = \frac{2W_\text{L}}{Z_\text{L} + 1} \tag{10b}$$

$$L_{22} = -\eta_\text{L} b_\text{L} \frac{Z_{\text{L}*} - 1}{Z_\text{L} + 1} \tag{10c}$$

where

$$b_\text{L}(s) = \frac{D_{\text{L}*}}{D_\text{L}} \tag{10d}$$

$$\eta_\text{L}(s) = \frac{n_{\text{L}*}}{n_\text{L}} \tag{10e}$$

$$W_\text{L} = n_{\text{L}*}/D_\text{L} \tag{10f}$$

n_{L*} is a polynomial formed on the closed RHP zeros of $R_\text{L}(-s^2) = W_\text{L} W_{\text{L}*}$, $R_\text{L}(-s^2)$ being the even part of $Z_\text{L}(s)$. In Belevitch form,

$$L_{11} = \frac{h_\text{L}}{g_\text{L}} \tag{11a}$$

$$L_{21} = \frac{f_\text{L}}{g_\text{L}} \tag{11b}$$

$$L_{22} = -\eta_L \frac{h_{\text{L}*}}{g_\text{L}} \tag{11c}$$

where

$$h_\text{L}(s) = N_\text{L}(s) - D_\text{L}(s) \tag{11d}$$

$$f_\text{L}(s) = 2n_\text{L}(-s) \tag{11e}$$

$$g_\text{L}(s) = N_\text{L}(s) + D_\text{L}(s) \tag{11f}$$

Transmission Zeros of the Load Network. As in Eq. (9), transmission zeros of the load network are defined as the zeros of the function

$$\tilde{F}_\text{L}(s) = L_{21}^2 = b_\text{L}(s) \frac{4R_\text{L}(-s^2)}{[Z_\text{L}(s) + 1]^2} \tag{12a}$$

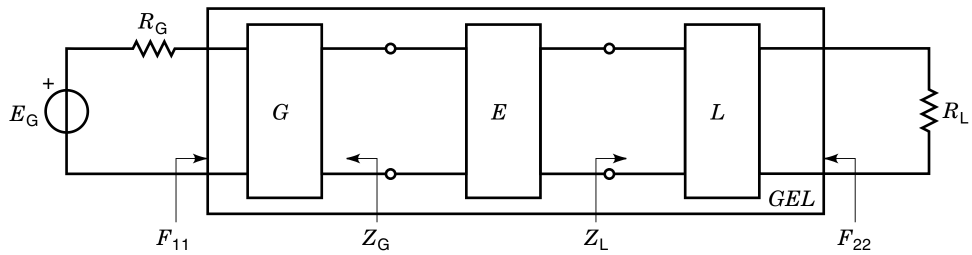


Figure 7. Double-matching problem with Darlington equivalent representation of load and generator impedances.

or

$$\tilde{F}_L(s) = \frac{4n_L^2(-s)}{g_L^2(s)} \tag{12b}$$

In Fano's work, the power performance of the singly matched system is measured in terms of the unit normalized scattering parameter F_{21} of F. In fact, the transducer power gain of the system is given as in Eq. (3):

$$T(\omega^2) = |F_{21}|^2 = 1 - |F_{22}|^2 \tag{13}$$

In terms of the unit normalized scattering parameters of E and L,

$$T(\omega^2) = \frac{|E_{21}|^2|L_{21}|^2}{|1 - E_{22}L_{11}|^2} \tag{14}$$

One may analytically handle the single-matching problem as a pseudo-filter problem as in Fano's theory, which can be stated as:

Theory. Construct the lossless two-port F with preferred gain characteristic as an insertion loss problem subject to load constraints so that the load two-port L is extracted from F yielding the desired matching network E.

In Youla's Theory, however, matching network E and the load Z_L are treated as separate entities and the transducer power gain of the singly matched system is defined in terms of the complex normalized reflectance S_{CL} at the load end. That is,

$$T(\omega^2) = 1 - |S_{CL}|^2 \tag{15}$$

where S_{CL} is given by

$$S_{CL} = \frac{Z_B - Z_{L*}}{Z_B + Z_L} \tag{16}$$

and Z_B designates the driving point Darlington impedance of E at the back end (5).

In this representation, the load impedance Z_L is regarded as the complex normalization number at the output port of E. Clearly, in the complex "s" domain, S_{CL} is not analytic due to RHP poles of $Z_L(-s)$. In order to make S_{CL} analytic, an all-pass factor $b_L(s)$ is introduced into S_{CL} to cancel the RHP poles of Eq. (16). Thus, the complex normalized regularized reflectance $S_{YL}(s)$ is, as defined in Youla sense,

$$S_{YL} = b_L(s)\frac{Z_B(s) - Z_L(-s)}{Z_B(s) + Z_L(s)} \tag{17}$$

In Youla's theory, instead of load extraction, complex normalized regularized reflectance is directly constructed from the analytic form of transducer power gain, satisfying the gain-bandwidth restriction. Then, the driving point impedance Z_B is obtained as a realizable positive real function as

$$Z_B(s) = \frac{2b_L(s)R_L(-s^2)}{b_L(s) - S_{YL}(s)} - Z_L(s) \tag{18}$$

where $R_L(-s^2)$ designates the even part of $Z_L(s)$.

Finally, employing the Darlington procedure, $Z_B(s)$ is synthesised, yielding the desired lossless matching network E in resistive termination.

Based on the definition of the complex normalized scattering parameters of the two-ports involved, extension of Youla's theory to double-matching problems is not a straightforward matter (5). Therefore, what is called the "unified approach" to broadband-matching problems is presented in the following section. The unified approach combines Youla's and Fano's works under a unique format by means of main theorem of Refs. 6 and 7 and makes the analytic theory accessible for many practical problems.

Before dealing with the main theorem, look at the definition of the bounded real (BR) analytic-complex normalized reflectance S_{YCL} in the Yarman and Carlin sense (7).

Definition: BR-Analytic-Complex Normalized Reflectance. The reflectance S_{YCL} defined by the expression

$$S_{YCL} = \frac{W_L}{W_{L*}} \cdot \frac{Z_B - Z_{L*}}{Z_B + Z_L} \tag{19}$$

is called the BR-Analytic-Complex Normalized Reflectance in the Yarman and Carlin sense.

Clearly, Eq. (19) can be related to S_{YL}, described in the Youla sense by Eq. (17):

$$S_{YCL}(s) = \eta_L(s)S_{YL}(s) \tag{20}$$

Furthermore, if the load is "simple," consisting of a few reactive elements having all the transmission zeros at finite frequencies or at infinity, all-pass product $\eta_L(s) = 1$ and $S_{YCL}(s) = S_{YL}(s)$, which is the case in many engineering applications.

Nevertheless, with proper augmentation of the load Z_L, one can make $n_L n_{L*}$ a perfect square yielding $n_L/n_{L*} = 1$. Thus, invoking superfluous factors, one can always obtain $S_{YCL}(s) = S_{YL}(s)$ as is described in Ref. (8). Therefore, in the following sections $S_{YCL}(s)$ and $S_{YL}(s)$ will be used interchangeably. Now, look at the main theorem (6–8).

Main Theorem. Referring to Fig. 6, let F_{22} be the unit normalized back end reflectance of the system F constructed by the cascade connection of the lossless two-port E and the lossless two-port L. Then, F_{22} is equal to the analytic complex-normalized reflectance defined in the Yarman–Carlin sense at the input port of L. That is, $F_{22}(s) = S_{YCL}(s)$.

The significant consequences of the main theorem may be summarized as follows:

- The load network L is directly extracted from F by applying the well established gain bandwidth restrictions of Youla on the analytic complex normalized reflectance $S_{YCL}(s)$ defined in the Yarman–Carlin sense, in a straightforward manner.
- In the course of the extraction process, the entire structure F is described in terms of its realizable, unit normalized, bounded real scattering parameters without hesitation as in Fano's theory, since $F_{22}(s) = S_{YCL}(s)$ by the main theorem.

These unique results constitute the basis of the unified approach.

One must now classify the transmission of zeros of the load. Then, introduce the modified version of the Youla's theorem on gain–bandwidth restrictions, allowing the extraction of the load network L from the combined structure.

Classification of the Transmission Zeros (8). Let s_0 denote any transmission zero of Z_L. Then, s_0 belongs to one of the following mutually exclusive classes:

- *Class A.* $\mathrm{Re}\{s_0\} > 0$: The transmission zero lies in the open RHP.
- *Class B.* $\mathrm{Re}\{s_0\} = 0$ and $Z_L(j\omega_0) \neq \infty$: The transmission zero lies on the imaginary axis at a point where the load impedance Z_L is finite (possibly zero)
- *Class C.* $\mathrm{Re}\{s_0\} = 0$, and $Z_L(j\omega_0) = \infty$: The transmission zero lies on the imaginary axis at a point where the load impedance has a pole.

The basic gain–bandwidth theorem for single-matching problems relates properties of the system reflectance to the load zeros of transmission so that extraction of the load becomes possible.

Basic Gain–Bandwidth Theorem for Single-Matching Problems (8). A function $T(\omega^2)$, such that $0 \leq T(\omega^2) \leq 1$, $\forall\omega$, can be realized as the transducer power gain of a finite lossless equalizer E, inserted between a resistive generator (whose resistance is unit normalized) and a frequency dependent load (whose impedance is a dissipative PR rational function Z_L), if and only if, at each transmission zero of the load, "s_0," of order "k," the coefficients of the Taylor expansion of

$$b_L(s) - S_{YCL}(s) = \frac{2R_L b_L}{[Z_B + Z_L]} \tag{21}$$

about "s_0" satisfy the following constraints:

- *Class A.* $\mathrm{Re}\{s_0\} > 0$:

$$b_{Lr} = S_{Lr} \qquad (r = 0, 1, 2, \ldots k-1) \tag{22a}$$

- *Class B.* Real $s_0 = 0$ and $Z_L(j\omega_0) \neq \infty$:

$$b_{Lr} = S_{Lr} \qquad (r = 0, 1, 2 \ldots 2k-2) \tag{22b}$$

and

$$\frac{[b_{L(2k-1)} - S_{L(2k-1)}]}{[2R_L b_L]_{2k}} \geq 0 \tag{22c}$$

- *Class C.* $\mathrm{Re}\{s_0\} = 0$ and $Z_L(j\omega_0) = \infty$:

$$b_{Lr} = S_{Lr} \qquad (r = 0, 1, 2, \ldots 2k-2) \tag{22d}$$

and

$$\frac{[b_{L(2k-1)} - S_{L(2k-1)}]}{[2R_L b_L]_{(2k-2)}} \leq \frac{1}{c_{-1}} \geq 0 \tag{22e}$$

In Eq. (22e), c_{-1} is the residue of Z_L at the $s_0 = j\omega_0$ pole; otherwise, the subscripts identifies a Taylor coefficient.

It should be noted that similar sets of constraints are given in the logarithmic and integral form. For details, interested readers are referred to the reading list.

Analytic Approach to Double-Matching Problems

In order to handle the double-matching problem analytically, the Darlington equivalent lossless two-ports G and L replace the generator impedance Z_G and the load impedance Z_L, respectively. Hence, the system that will be doubly matched is represented as the cascaded trio of the lossless two-ports G–E–L as shown in Fig. 7. The entire structure is combined under the lossless two-port F and described by means of the unit normalized, bounded real scattering parameters F_{ij} as in the filter theory. The load network L is described by Eqs. (10) and (11). Similarly, description of the generator network G and its transmission zeros are given as follows:

Scattering Description of the Darlington Equivalent of Generator Network. Unit normalized scattering parameters of the lossless generator network G are given in terms of the impedance $Z_G(s) = N_G(s)/D_G(s)$

$$G_{22} = \frac{Z_G - 1}{Z_G + 1} \tag{23a}$$

$$G_{21} = \frac{2W_G}{Z_G + 1} \tag{23b}$$

$$G_{11} = -\eta_G(s) \cdot b_G(s) \cdot \frac{Z_{G*} - 1}{Z_G + 1} \tag{23c}$$

where

$$b_G(s) = \frac{D_{G*}}{D_G} \quad (23d)$$

$$\eta_G(s) = \frac{n_{G*}}{n_G} \quad (23e)$$

$$W_G = \frac{n_{G*}}{D_G} \quad (23f)$$

n_{G*} is a polynomial formed on the closed RHP zeros of $R_G(-s^2) = W_G W_{G*}$, $R_G(-s^2)$ being the even part of $Z_G(s)$.

In Belevitch form,

$$G_{22} = \frac{h_G}{g_G} \quad (24a)$$

$$G_{21} = \frac{f_G}{g_G} \quad (24b)$$

$$G_{11} = -\eta_G(s) \cdot \frac{h_{G*}}{g_G} \quad (24c)$$

where

$$h_G(s) = N_G(s) - D_G(s) \quad (24d)$$

$$f_G(s) = 2n_G(-s) \quad (24e)$$

$$g_G(s) = N_G(s) + D_G(s) \quad (24f)$$

Transmission Zeros of the Generator Network. Zeros of the function $\tilde{F}_G(s) = G_{21}^2$ are the transmission zeros of $Z_G(s)$, where

$$\tilde{F}_G(s) = b_G(s) \frac{4R_G(-s^2)}{[Z_G(s)+1]^2} \quad (25a)$$

or

$$\tilde{F}_G(s) = \frac{4n_G^2(-s)}{g_G^2(s)} \quad (25b)$$

Before introducing the unified procedure to construct the broadband matching equalizer E, the theorem for double-matching problems will first be introduced.

Theorem for Double-Matching Problems (8). Let a rational transducer power gain function $T(\omega^2)$ be prescribed where $0 \le T(\omega^2) \le 1$. Let PR dissipative impedances Z_G and Z_L, for generator and load be given and zeros of transmission of these impedances be contained in T. Then, assuming no double degenerecies, the necessary and sufficient condition that T be physically realizable by the system consisting of the generator Z_G, the load Z_L, and a lossless equalizer E placed between generator and load is that the single-matching gain–bandwidth restrictions be simultaneously satisfied at the generator and load ports of the equalizer.

Based on the above presentations, it is clear that for both single- and double-matching problems, the entire matched system can be described in terms of its unit normalized BR scattering parameters $F = \{F_{ij}\}$ as in filter theory when the source (or generator) and the load are replaced with their Darlington equivalents. In order to construct the desired equalizer, Youla's gain–bandwidth restrictions are simultaneously applied at the source and the load end, and then, extractions of G and L are accomplished.

At this point it is very important to outline some basic properties of F_{ij} related to source and load networks so that an appropriate form of the transfer function $T = |F_{21}|^2$ is selected to end up with a realizable equalizer.

Essential Properties of *F*. In principal, $F_{21}(s)$ must contain all the transmission zeros of G and L with at least the same multiplicity as well as the transmission zeros of E. The following properties are either the direct results of the paraunitary condition of F or come from the definition of transmission zeros or from the main theorem.

1. If $|F_{21}|^2 = F_{21}(j\omega), F_{21}(-j\omega)$ is a transfer function with desired shape over the real frequencies.
   a. When $j\omega$ is replaced by s, $F_{21}(s)F_{21}(-s)$ must contain all the real frequency transmission zeros of G and L in the numerator polynomial $f(s) \cdot f(-s)$.
   b. RHP transmission zeros of G and L must appear in $F_{21}(s)$ as all-pass functions preserving the shape of $|F_{21}|^2$;
2. Let $\mu(s)$ be any Blaschke product of order k of $F_{21}(s)$. It should appear either in F_{11} or in F_{22} of order $2k$, or it appears simultaneously in F_{11} and F_{22} with respective orders k_{11} and k_{22} satisfying the condition $2k = k_{11} + k_{22}$.
3. In order to end up with a realizable structure, the main theorem and the paraunitary condition demand the following forms for F_{11} and F_{22}:

$$F_{11}(s) = S_{YCG} \qquad \text{where } F_{11}(s) = \epsilon\eta_G(s) \cdot [\mu(s)]^{k_{11}} \cdot \frac{h(s)}{g(s)} \quad (26a)$$

and

$$F_{22}(s) = S_{YCL} \qquad \text{where } F_{22}(s) = -\epsilon\eta_L(s) \cdot [\mu(s)]^{k_{22}} \cdot \frac{h(-s)}{g(s)} \quad (26b)$$

where ϵ is the sign term $\epsilon = \pm 1$.

The unified approach as a "step by step design procedure" will now be introduced, to construct matching equalizers for both single- and double-matching problems.

Unified Analytic Approach to Design Broadband Matching Networks

Step 1. Obtain analytic forms of the impedances to be matched and find the Darlington equivalents G and L. Determine the transmission zeros of G and L as in Eq. (12) and Eq. (25), respectively.

Step 2. Choose a desired form of the transducer power gain function as in the filter design $T(\omega^2) = |F_{21}|^2 = 1 - |F_{22}|^2$ with unknown parameters. Here it is crucial that F_{21} includes all transmission zeros G and L networks with at least the same multiplicity.

Step 3. Using the spectral factorization of $|F_{11}|^2 = |F_{22}|^2 = 1 - |F_{21}|^2$ as described in step 3 of the filter design process, construct the unit normalized reflectances F_{11} and F_{22} as equivalent BR-Analytic Complex Normalized reflectances in the Yarman and Carlin sense, as indicated below:

$$F_{11}(s) = S_{\text{YCG}} \qquad \text{where } F_{11}(s) = \epsilon\eta_{\text{G}}(s)\cdot\mu(s)\cdot\frac{h(s)}{g(s)} \tag{27a}$$

and

$$F_{22}(s) = S_{\text{YCL}} \qquad \text{where } F_{22}(s) = -\epsilon\eta_{\text{L}}(s)\cdot\mu(s)\cdot\frac{h(-s)}{g(s)} \tag{27b}$$

where ϵ is the sign term $\epsilon = \pm 1$. Polynomials $h(s)$ and $g(s)$ are obtained via spectral factorization of $F_{11}(s)\cdot F_{11}(-s) = 1 - T(-s^2)$ [or equivalently $F_{22}(s)\cdot F_{22}(-s)$]. Certainly, $g(s)$ is strictly Hurwitz. $\mu(s)$ is an arbitrary Blaschke product and it might be necessary to satisfy the GBR, otherwise it is set to 1. η_G and η_L are also Blaschke products constructed on the closed RHP zeros of even parts of generator and load impedances, respectively. It should be noted, however, that in the course of spectral factorization, unknown parameters of T are being carried into F_{11} and F_{22}.

Step 4. Apply Youla's GBR theorem for single matching at both generator and load sites simultaneously to determine the unknown parameters of T.

Step 5. Finally, synthesize the equalizer using $Z_B(s)$ as introduced by Eq. (18).

Remarks

- Implementation of the above procedure by no means is unique. First of all, one may wish to start with the expanded forms of the generator and load impedances to force resulting η_G and η_L to unity, as described earlier. Spectral factorization of $T(-s^2)$ can be carried out in a variety of fashions. For the sake of brevity details are omitted. Interested readers are referred to Refs. (9) and (13).
- The ratio $h(s)/g(s)$ may be chosen as a minimum phase function so that all RHP zeros are combined in the Blaschke product $\mu(s)$ where necessary.
- Use of an all-pass factor $\mu(s)$ in $T(-s^2)$ always penalizes the minimum gain level in the passband. Therefore one should avoid using any extra Blaschke product in step 3 if possible.
- It is interesting to observe that, whenever Z_G and Z_L possess the same transmission zeros of Class A, it is not possible to satisfy the gain–bandwidth theorem simultaneously if they are inserted into F_{11} and F_{22} as all-pass functions. In this case, a proper form of F_{21} must be selected, which naturally includes these RHP zeros of load and generator, but not as all-pass products.
- It should be emphasized that it is not an easy matter to utilize the analytic gain–bandwidth theory. It is generally the second step that imposes the most severe limitation on the practical applicability of the analytic method. Nevertheless, for simple terminations, it may be useful to determine the ideal highest flat gain level within the passband. For example, for a typical R/C load, even if infinite number of elements are employed in E, the possible highest flat gain level is given by

$$G_{\max} = 1 - \exp\left(\frac{-2\pi}{RCB}\right) \tag{28}$$

where B designates the normalized frequency bandwidth (1,4). For double-matching problems, however, ideal flat gain over a finite passband cannot be obtained (14,15).

In the above presentation, a concise discussion of network theoretical fundamentals underlying the concept of analytic solutions to matching problems is given. Based on the basic concept presented here, various alternative formulations of the problem are available in the literature. In particular, Wohlers (16) studied the problem of double matching by introducing the concept of compatible impedances. Chien (17), Chen (18–20), and Satyanaryana (21) utilized the complex normalized scattering parameters in the tradition of Youla, to obtain explicit solutions for some typical analytic load impedances. An extensive list of further studies on analytic matching theory, which includes various worked-out examples, can be found in the Reading List.

MODERN APPROACHES TO BROADBAND MATCHING PROBLEMS: CAD TECHNIQUES—REAL FREQUENCY SOLUTIONS

In the previous sections analytic solutions to broadband matching problems were presented. Analytic theory is essential to understand the gain–bandwidth limitations of the given impedances to be matched. However, its applicability is limited beyond simple problems. By simple is meant those problems of single or double matching in which the generator and load networks include at most one reactive element, either a capacitor or an inductor. For simple impedance terminations, low-pass equal ripple or flat gain prototype networks, which are obtained employing the analytic theory, may have practical use. On the other hand, if the number of elements increase in the impedance models to be matched, the theory becomes inaccessible. If it is capable of handling the problem, the resulting gain performances turn out to be suboptimal. Equalizer structures become unnecessarily complicated, and it may not even be feasible to manufacture them. Therefore, in practice, computer aided design (CAD) techniques are preferred; commercially available programs such as Super Compact, Cosmic, Ana, Touch Stones, and so forth, are employed to solve the matching problems. Readily available tools are very good in analyzing and optimizing the given structures, but they do not include network synthesis procedures in the literal sense. In other words, in designing a matching network or a microwave amplifier, a topology for the matching network with good initial element values should be supplied to a commercially available package. In this respect, many CAD packages work as fine trimming tools on the element values when the circuit is practically synthesized. Usually, a simple two element, capacitor-inductor ladder network is a practical solution for narrow bandwidth matching problems. However, if the optimum topology of the equalizer is unknown, or if substantial bandwidth is requested, the design task becomes more difficult. In this case, modern CAD techniques are strongly suggested to design matching networks.

In all single- and double-matching CAD algorithms, the goal is to optimize the transducer power gain (TPG), as high and flat as possible in the band of operation. Matching network E, generator and load are considered as separate entities. TPG is expressed in terms of these entities. The lossless equalizer E is either described in terms of its driving point "back-end impedance" Z_B (or equivalently admittance $Y_B = 1/Z_B$) or in terms of its unit normalized scattering parameters E_{ij}. The descriptive parameters of E are chosen as the unknowns of the problem and they are determined as the result of the optimization process. In this way, the analytic extraction process of generator and load networks is simply omitted. In the following, first, major ingredients such as Darlington-scattering representation of Z_B, and unit-normalized reflectance S_F of the lossless E and L chain to generate TPG, are given. Then, modern computer aided design techniques so called real frequency techniques to construct broadband matching networks, are reviewed (10).

Scattering Description of the Lossless Matching Network *E*

Based on the Darlington representation of the driving point back-end impedance $Z_B = N_B/D_B$, the scattering description of the matching network E is given in a similar manner to those of Eq. (10) and Eq. (23), as follows:

$$E_{22} = \frac{Z_B - 1}{Z_B + 1} \tag{29a}$$

$$E_{21} = \frac{2W_B}{Z_B + 1} \tag{29b}$$

$$E_{11} = \eta_B b_B \frac{Z_{B^*} - 1}{Z_B + 1} \tag{29c}$$

where

$$b_B(s) = \frac{D_{B^*}}{D_B} \tag{29d}$$

$$\eta_B(s) = \frac{n_{B^*}}{n_B} \tag{29e}$$

$$W_B = n_{B^*}/D_B \tag{29f}$$

n_{B*} is a polynomial formed on the closed RHP zeros of $R_B(-s^2) = W_{B*} \cdot W_B$, $R_B(-s^2)$ being the even part of $Z_B(s)$.

In order to construct the transducer power gain function T for single- and double-matching problems, it is useful to introduce the unit-normalized reflectance S_F at the front end of the equalizer when the other end has complex termination Z_L. This can be accomplished by means of the main theorem of Refs. 6 and 22 as follows.

Unit Normalized Input Reflectance S_F. The unit normalized input reflectance S_F of the bulk lossless section formed with E and L is given by means of the main theorem. It is straightforward to show that S_F is equal to BR-Analytic Complex, Normalized Reflectance S_{YCB} defined in the Yarman–Carlin sense at the backend of E. In this regard, the impedance Z_B is regarded as a complex termination to the lossless two-port L. Accordingly, swapping the subscripts L and B of Eq. (19), S_F is given by

$$S_F(s) = \frac{W_{B^*}}{W_B} \cdot \frac{Z_L - Z_{B^*}}{Z_L + Z_B} \tag{30}$$

where W_B is defined as in Eq. (29f).

Real Frequency Line Segment Technique

In 1977 a numerical approach known as the *real frequency* technique was introduced by Carlin for the solution of single-matching problems (2). The real frequency technique utilizes measured data, by-passing the analytic theory. Neither the equalizer topology nor the analytic form of a transfer function is assumed. They are the result of the design method. Measured data obtained from the devices to be matched are directly processed.

It is important to recognize that the "breakthrough" of the real frequency method is the recognition that the results of numerical optimization will in general always be superior to those of the analytic theory. In effect, the analytic method squanders its degrees of freedom by introducing all-pass factors to achieve special gain function properties (e.g., maximal flatness), whereas the real frequency approach directly optimizes passband gain without the artificial constraints of a specific transfer function (12).

The precise optimization method is not the crucial factor though clearly, as discussed below, some methods are more efficient and practical than others. Carlin's initial numerical method used a line segment approximation scheme and contains features often employed in later more sophisticated optimization routines. The attractive feature of the line segment scheme is its simplicity. The technique starts with the generation of a rational positive real (PR) input impedance $Z_B = R_B(\omega) + jX_B(\omega)$ looking into a lossless matching network with resistive termination [Fig.4(a)].

Let the measured load impedance be $Z_L(j\omega) = R_L(\omega) + jX_L(\omega)$; then the transducer gain $T(\omega)$ is given by

$$T(\omega) = 1 - |S_F|^2 \tag{31}$$

or by simple algebraic manipulation one obtains

$$T(\omega^2) = \frac{4R_B(\omega)R_L(\omega)}{[R_B(\omega) + R_L(\omega)]^2 + [X_B(\omega) + X_L(\omega)]^2} \tag{32}$$

In Carlin's approach, the matching problem is handled within three major steps. In the first step, $R_B(\omega)$ is represented by a set of linear combinations of unknown line segments (Fig. 8),

$$R_B(\omega) = R_0 + \sum_{i=1}^{N} a_i(\omega) \cdot r_i \tag{33a}$$

where

$$a_i = \begin{cases} 1, & \omega \geq \omega_i \\ \dfrac{\omega - \omega_{i-1}}{\omega_i - \omega_{i-1}}, & \omega_{i-1} \leq \omega \leq \omega_i \\ 0, & \omega \leq \omega_i \end{cases} \tag{33b}$$

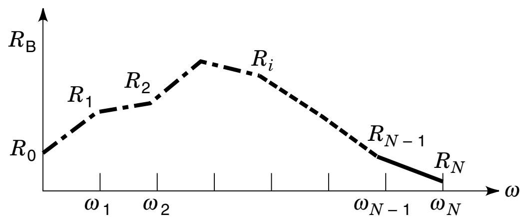


Figure 8. Line segment representation of the real part of back-end impedance $R_B(\omega)$.

ω_i is called the break frequency at the point where R_{B} takes the value R_i, that is, $R_i = R_{\mathrm{B}}(\omega_i)$. Therefore, R_i is called the break resistance, r_i is the resistance excursions of the ith segment such that $r_i = R_i - R_{i-1}$ and N designates the number of break points.

Here, $Z_{\mathrm{B}}(j\omega)$ is considered as a minimum reactance function. Therefore, $X_{\mathrm{B}}(\omega)$ is also expressed in terms of the same linear combination of the line segments using the Hilbert Transformation relation:

$$X_{\mathrm{B}}(\omega) = \sum_{i=1}^{N} b_i(\omega) R_i \tag{34a}$$

where

$$\begin{aligned} b_i(\omega) = {} & \frac{1}{\pi(\omega_i - \omega_{i-1})}\{[(\omega+\omega_i)\ln(\omega+\omega_i) + (\omega-\omega_i)\ln|\omega-\omega_i|] \\ & - [(\omega+\omega_{i-1})\ln(\omega+\omega_{i-1}) + (\omega-\omega_{i-1})\ln|\omega-\omega_{i-1}|]\} \\ & \text{for all } i = 1, 2, \ldots N \end{aligned} \tag{34b}$$

In the second step, these straight lines are then computed in such a way that TPG are optimized over the band of operation. The third step is devoted to approximate $Z_{\mathrm{B}}(j\omega)$ by a realizable rational function that fits the computed data pair (R_{B}, X_{B}). Then, Z_{B} is synthesized, using Darlington's procedure, as a lossless two-port with resistive termination.

A general realizable analytic form of R_{B} is given as

$$R_{\mathrm{B}}(\omega^2) = \frac{A_0 + A_1\omega^2 + \cdots + A_m\omega^{2m}}{B_0 + B_1\omega^2 + \cdots + B_n\omega^{2n}} \geq 0 \qquad n \geq m+1 \tag{35}$$

In many practical cases, it is appropriate to choose $R_{\mathrm{B}}(\omega)$ to yield a ladder matching network as follows:

$$R_{\mathrm{B}}(\omega^2) = \frac{A_k\omega^{2k}}{B_0 + B_1\omega^2 + \cdots + B_n\omega^{2n}} \tag{36}$$

where "k" and "n" are positive integers ($k \leq n$) and they determine the complexity of the equalizer. Equation (36) describes an L–C ladder network with all zero of transmissions at zero and infinity. More specifically, integer n designates the total number of elements of the equalizer. Integer k is the total number of transmission zeros at zero which, in turn, effects the topology of the L–C ladder. The coefficients A_k, and B_i are computed to fit the real part data, obtained from the first step of the technique, by linear regression. Afterwards, $Z_{\mathrm{B}}(s)$ is generated as a positive real analytic function from R_{B} using the Bode or Gewertz procedure as a positive real function (1,23).

$$Z_{\mathrm{B}}(s) = \frac{a_0 + a_1 s + \cdots + a_{(n-1)}s^{(n-1)}}{b_0 + b_1 s + \cdots + b_{(n-1)}s^{(n-1)} + b_n s^n} \tag{37}$$

It has been shown that the line segment technique yields superior design performances over the analytic and other CAD approaches. Almost optimum circuit topology is resolved. Furthermore, gain–bandwidth limitations of a given load may be determined by means of computer experiments. However, the following may be regarded as disadvantages of this technique.

The first two steps of the above described technique involves the computation of the unknown line segments and approximation problem, which requires the evaluation of Hilbert transformation during the optimization process. These computations may be laborious and expensive. Even though there is no longer any need to choose a circuit topology, decisions have still to be made as to whether the input impedance Z_{B} is a minimum-reactance or, equivalently, $Y_{\mathrm{B}} = 1/Z_{\mathrm{B}}$ is a minimum-susceptance function. It should be noted that if the design is restricted with minimum functions, some reactive elements can be extracted from the equalizer, leaving a minimum-reactance or minimum-susceptance input immitance. Although this process improves the flexibility of the technique, one must decide what to extract (capacitor or inductor) and how to extract (series or parallel) by trial and error, which, in turn, increases the computation time.

Despite the said drawbacks of this technique, it is reasonably satisfactory for single-matching problems. Later, the line segment technique was extended to handle double-matching problems as well, but the computational efficiency of the technique turned out to be poor. The follow-up CAD double matching design technique, namely, the "direct computational technique," overcomes some of the difficulties of the line segment approach. Details of the implementation of the line segment technique can be found in Refs. 6, 22, and 24.

Direct Computational Technique

The basic idea employed in the direct computation method is similar to that of the line segment technique. That is, referring to Fig. 4(a), the driving point impedance $Z_{\mathrm{B}} = N_{\mathrm{B}}/D_{\mathrm{B}}$ at the back end describes the lossless matching network, whereas the front end has resistive termination. In fact, the scattering description of E is given with respect to Z_{B} as has been demonstrated in the previous section.

As in the line segment approach, here Z_{B} is also considered a minimum reactance (or Y_{B} is minimum susceptance) function. Therefore, it is determined from its even part $R_{\mathrm{B}}(\omega^2)$ using the Hilbert transformation relation. For practical reasons, it is the designer's choice to start with the ladder form for $R_{\mathrm{B}}(\omega^2)$, as in Eq. (36).

The core of this method resides in the generation of the overall transducer power gain T in terms of $R_{\mathrm{B}}(\omega^2)$, which will be determined by optimization. Referring to the double-matching configuration shown in Fig. 7, considering the generator network G and describing the lossless chain E–L in terms of the unit normalized reflectance S_F, it is straightforward to show that

$$T(\omega^2) = \frac{(1 - |G_{22}|^2)(1 - |S_F|^2)}{|1 - S_F G_{22}|^2} \tag{38}$$

In the above presentation, clearly, S_F is constructed as an implicit function of R_B and it is initialized at the beginning of the optimization process. By spectral factorization of R_B, n_{B*} and D_B are computed. Then W_B is formed as in Eq. (29f) and S_F is generated as described by Eq. (30). Employing the Gewertz procedure, minimum reactance Z_B is generated. If desired, any appropriate reactive part can be introduced to Z_B as an unknown of the problem.

Once the TPG is generated, it is maximized to determine the unknown coefficients A_k and B_i of Eq. (36) (22).

In this design method, the line segment approach is simply omitted. Thus the computational efficiency is improved. Direct computational technique has all the merits of the line segment technique. However, decisions must again be made as to whether to make the input impedance Z_B minimum reactance or minimum susceptance, and so forth. For interested readers, the details can be found in the Reading List.

Parametric Approach to Matching Problems

Fettweis first introduced the parametric representation of Burne functions in 1979 (25). Pandel and Fettweis (26) applied it to single-matching problems. Later, Yarman and Fettweis elaborated this method for double-matching problems (27). In the parametric approach, the lossless equalizer E is described in terms of its minimum reactance driving point impedance Z_B as in the other techniques, and it is expressed in the form of partial fraction expansion with simple poles $p_i = \alpha_i + j\beta_i$.

$$Z_B = B_0 + \sum_{i=1}^{N} \frac{B_i}{(s - p_i)} \tag{39}$$

Here, the real parts α_i and the imaginary parts β_i are chosen as the unknowns of the matching problem. The coefficients or the residues B_i are computed in terms of the poles $p_i = \alpha_i + j\beta_i$. Once the unknowns are initialized, Z_B is explicitly generated as an analytic function. Then it is straightforward to form TPG as stated in Eq. (38). Hence, it is optimized over the band of operation which, in turn, yields the unknown poles $p_i = \alpha_i + j\beta_i$.

In the parametric approach, the Gewertz procedure, which is employed in the line segment and direct computational techniques, is simply omitted. Therefore, in the optimization scheme, neither explicit factorization of polynomials nor the solution of linear equations systems of Gewertz procedure is required. Furthermore, consideration of Z_B as a minimum reactance function, having only simple poles, does not imply any loss of generality. This is because multiple poles do not occur in impedances of practical interest, and any impedance function can be expressed as a sum of a minimum reactive function and a pure reactance, which is naturally included in the parametric form of Z_B.

For single-matching problems, gradients of TPG with respect to unknowns α_i and β_i are explicitly determined. Therefore, the parametric approach presents excellent numerical stability. The parametric approach to broadband matching problems possesses all the outstanding merits of the real frequency techniques. Furthermore, it presents improved numerical stability with less computation. Details of this method can be found in Refs. 25–29.

Simplified Real Frequency Technique: A Scattering Approach

The simplified real frequency technique (SRFT) is also a CAD procedure for double-matching problems. In this method the lossless equalizer is simply described in terms of its unit normalized scattering parameters. SRFT possesses all the outstanding merits of the other real frequency techniques. Moreover, it does not involve with any impedance or admittance computation. Therefore, the gain optimization process of the matched system is well behaved, numerically. It is faster than the other existing CAD algorithms and easier to use. It is also naturally suited to design broadband microwave amplifiers.

The basis for the scattering approach is to describe the lossless equalizer E in terms of the unit normalized reflection coefficient $E_{11}(s)$. Moreover, if $E_{11}(s)$ is described in Belevitch form, $E_{11}(s) = h(s)/g(s)$, for selected transmission zeros, the complete scattering parameters of a lossless reciprocal equalizer can be generated from the numerator polynomial $h(s)$ of $E_{11}(s)$, using the paraunitary condition given by Eq. (5) to Eq. (7). This idea constitutes the crux of the simplified real frequency technique. In other words, the TPG of the system to be matched is expressed as an implicit function of $h(s)$.

Replacing generator and load impedances by their Darlington equivalent lossless two-ports G and L, respectively, and utilizing their unit normalized scattering descriptions, as specified by Eq. (10) and Eq. (23), the transducer power gain of the doubly matched system can be given as follows:

$$T(\omega) = |G_{21}|^2 \frac{|E_{21}|^2 |L_{21}|^2}{|1 - E_{11}G_{22}|^2 |1 - \hat{E}_{22}L_{11}|^2} \tag{40}$$

where G_{ij} and L_{ij} are specified by the generator and the load measurements. However, in terms of the measured generator and load impedances,

$$G_{22} = \frac{Z_G - 1}{Z_G + 1} \qquad |G_{21}|^2 = 1 - |G_{22}|^2 \tag{41a}$$

$$L_{11} = \frac{Z_L - 1}{Z_L + 1} \qquad |L_{21}|^2 = 1 - |L_{11}|^2 \tag{41b}$$

$$\hat{E}_{22} = E_{22} + \frac{E_{21}^2 G_{22}}{1 - E_{11}G_{22}} \tag{41c}$$

Now construct TPG, once $h(s)$ is initialized.

For simplicity, assume that E is a minimum phase structure with transmission zeros only at $\omega = \infty$, $\omega = 0$. This is a convenient assumption since it assures realization without coupled coils, except possibly for an impedance level transformer. Then, the unit normalized scattering coefficients of E is given in Belevitch form as follows:

$$E_{11}(s) = \frac{h(s)}{g(s)} = \frac{h_0 + h_1 s + \cdots + h_n s^n}{g_0 + g_1 s + \cdots + g_n s^n} \tag{42a}$$

$$E_{12}(s) = E_{21}(s) = \pm \frac{s^k}{g(s)} \tag{42b}$$

$$E_{22}(s) = -(-1)^k \frac{h(-s)}{g(s)} \tag{42c}$$

where n specifies the number of reactive elements in E; $k \geq 0$ is an integer and specifies the order of the transmission zeros at zero. Since the matching network is lossless, it follows that

$$g(s)g(-s) = h(s)h(-s) + (-1)^k s^{2k} \tag{43}$$

In an SRFT algorithm the goal is to optimize the TPG over the operational frequency band (i.e., maximize the minimum of TPG in the band). The coefficients of the numerator polynomial $h(s)$ are selected as the unknowns of the matching problem. To construct the scattering parameters of E, it is sufficient to generate the Hurwitz denominator polynomial $g(s)$ from $h(s)$. It can be readily shown that once the coefficients of $h(s)$ are initialized at the start of the optimization process and the complexity of the equalizer E is specified (i.e., n and k are fixed), $g(s)$ is generated as a Hurwitz polynomial by explicit factorization of Eq. (43). Thus the physical realizibility of the scattering parameters $\{E;\ i, j = 1, 2\}$ is already built into the procedure. It should be noted that in choosing the polynomial $h(s)$ and integer k, $h(0) = 0$ and $g(0) = 0$ cannot be allowed simultaneously, since this violates the losslessness criterion of Eq. (43). Therefore, one has to pay a little attention to the initial values of the unknown coefficients. In generating the Hurwitz denominator polynomial $g(s)$ from the initialized coefficients of $h(s)$, one first constructs $g(s)g(-s)$ as in Eq. (43). That is,

$$g(s)g(-s) = G_0 + G_1 s^2 + \cdots + G_n s^{2n} \tag{44}$$

where $G_i(s)$ are given as follows:

$$\begin{aligned}
G_0 &= h_0^2 \\
G_1 &= -h_1^2 + 2h_2h_0 \\
&\vdots \\
G_i &= (-1)^i h_i^2 + 2\left(h_{2i}h_0 + \sum_{j=2}^{i}(-1)^{j-1}h_{j-1}h_{2j-i+1}\right) \\
&\vdots \\
G_k &= G_i|_{i=k} + (-1)^k \\
&\vdots \\
G_n &= (-1)^n h_n^2
\end{aligned} \tag{45}$$

Then, explicit factorization of Eq. (44) follows. Following the factorization process, polynomial $g(s)$ is formed on the left plane zeros of $g(s)g(-s)$.

Hence, the scattering parameters of E are generated as in Eq. (42) and $T(\omega)$ is computed employing Eq. (40). The objective function generated by means of TPG calls for an optimization routine. As a result of optimization, the unknown coefficients h_i are determined. Details of the numerical work can be found in the Reading List. In brief, examination of Eq. (40) together with Eq. (42) indicates that TPG is almost inverse quadratic in the unknown coefficients h_i. Furthermore, the numerical stability of the computer algorithm written for SRFT discussed above is excellent, since all the scattering parameters E_{ij} and reflection coefficients G_{22} and L_{11} are bounded real, that is, $\{|E_{ij}|, |G_{22}|, |L_{11}|\} \leq 1$. As is usually the case, an intelligent initial guess is important in efficiently running the program. It has been experienced that, for many practical problems, an ad hoc direct choice for the coefficients h_i (e.g., $h_i = 1$ or $h_i = -1$) provides satisfactory initialization to start the simplified real frequency technique algorithm.

As indicated previously, SRFT is naturally suited to design microwave amplifiers since scattering parameters for all the units to be matched are used. Several matching networks and amplifiers have been designed and built employing the SRFT. Laboratory performance measurements exhibit good agreement with theoretical computations.

Active Matching: Design of Microwave Amplifiers

One of the major problems of microwave engineers is to develop proper matching networks for active devices so that the power generated with these devices can be pumped into dissipative terminations or transferred to another device to generate more microwave power. Typical examples are the negative resistance amplifiers, constructed with impatt diodes, and single- or multi-stage amplifiers, constructed with GaAs field effect transistors (FET).

In fact, the problem of active matching is not any different from single or double matching, as explained in the first section. However, slight modifications of the techniques may be required, depending on the type of application.

Kuh and Rohrer extended analytic theory of single matching to design negative resistance amplifiers (30). Later, Ku and his coworkers applied the single-matching theory to design GaAs MESFET amplifiers using the tapered gain concept (31). These were the early works to design microwave amplifiers.

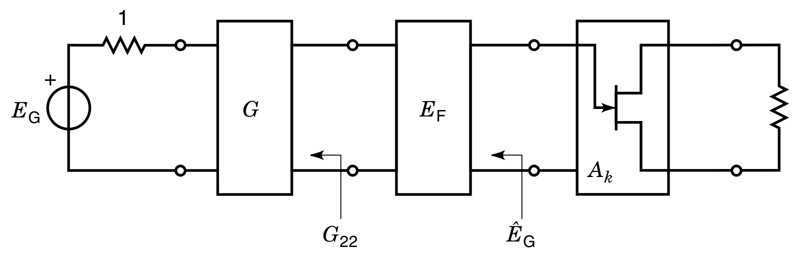
After the breakthroughs of real frequency techniques, the design of microwave amplifiers became much more practical, since the complicated gain–bandwidth restrictions were omitted and the measured device data were processed without any model. First, the line segment technique was expanded to design single-stage microwave amplifiers by Carlin and Komiak (32). Later, Yarman and Carlin developed first interstage equalizer design, in the literal sense, using the direct computational and simplified real frequency technique (33). Then, utilizing SRFT, many single- or multi-stage amplifiers were designed and different variants of the technique were developed and applied to practical problems (34–37).

Therefore, in this presentation, the application of the SRFT to design single- and multi-stage amplifiers will be outlined briefly.

Referring to Fig. 9, the design of a single-stage amplifier can be implemented within two steps. First, the front-end equalizer E_F of the active device is constructed, while the other end of the device is terminated in unit resistance. In this case, transducer power gain $T^{(1)}(\omega)$ is given by

$$T^{(1)}(\omega) = \frac{|G_{21}|^2 \cdot |E_{21F}|^2 \cdot |A_{21}|^2}{|1 - G_{22}E_{11F}|^2 \cdot |1 - \hat{E}_{22F}A_{11}|^2} \tag{46}$$

where $[G_{ij}]$, $[E_{ijF}]$ and $[A_{ij}]$ designate the unit (or real) normalized scattering parameters of the generator network, front-end equalizer, and the active device, respectively. Clearly, $T^{(1)}(\omega)$ is generated from E_{ijF}, as described in SRFT, and all the scattering parameters are determined from its numerator



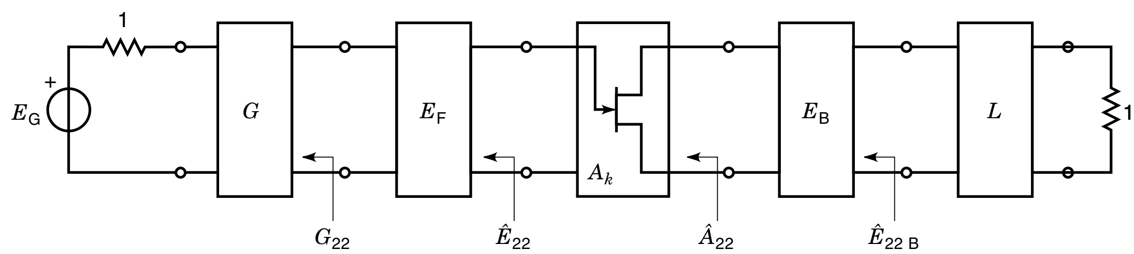


Figure 9. (a) Single-stage amplifier with front-end equalizer E_F. (b) Single-stage amplifier with front-end E_F and back-end E_B equalizers.

polynomial $h_F(s)$. Here, $\hat{E}_{22F}$ is given by

$$\hat{E}_{22F} = E_{22F} + \frac{E_{21F}^2 G_{22}}{1 - E_{11F} G_{22}} \tag{47}$$

In the second step, the back-end equalizer E_B is constructed to optimize the overall transducer power gain $T(\omega)$ given by

$$T(\omega) = T^{(1)}(\omega) \cdot \frac{|E_{21B}|^2 |L_{21}|^2}{|1 - \hat{A}_{22} E_{11B}|^2 |1 - A_{22} E_{11B}|^2 |1 - \hat{E}_{22B} L_{22}|^2} \tag{48}$$

where L_{ij} and E_{ij} are the unit normalized scattering parameters of the load network and the back-end equalizer E_B, respectively. $\hat{A}_{22}$, and $\hat{E}_{22B}$ are given as follows:

$$\hat{A}_{22} = A_{22} + \frac{A_{21} A_{12} E_{22}}{1 - \hat{E}_{22F} A_{11}} \tag{49a}$$

$$\hat{E}_{22B} = E_{22B} + \frac{E_{21B}^2 \hat{A}_{22}}{1 - E_{11B} A_{22}} \tag{49b}$$

As is customary for SRFT, the back-end equalizer is completely determined from its numerator polynomial $h_B(s)$ of the input reflectance $E_{11B}(s)$.

Employing SRFT, several single-stage amplifiers were implemented by Yarman (38). The technique is also applied to design power amplifiers. In this case, a modified version (A Dynamic CAD Technique for Designing Microwave Amplifiers) was introduced by Yarman (39).

It is straightforward to extend the SRFT to design multistage microwave amplifiers by generating the TPG in a sequential manner, as was done for single-matching amplifier design.

Design of Multi-Stage Microwave Amplifiers. Multi-stage microwave amplifiers can be designed in a similar manner, described above using a step-by-step design algorithm. Referring to Fig. 10, one can design an N-stage amplifier with field effect transistors (FET), step by step. Assume that generator G and load L are also complex. Let $A_{ij}^{(k)}$ designate the unit normalized (50 Ω normalized) scattering parameters of FETs. The design algorithm can be described as follows: First, the front-end equalizer E_1 is constructed, while the output of the first FET is terminated with its normalization resistance. In the second step, resistive termination is removed and the second equalizer E_2 and the new FET are placed into the design with resistive termination. At the kth step, insert the kth interstage equalizer with the kth active device while it is terminated in its normalization resistance at the output. As this process plays out, at the last step we introduce the back-end equalizer is introduced in between the Nth device and the load L. In other words, at each step a new interstage equalizer and an active device with resistive termination are inserted. At the last step, which corresponds to the $(N + 1)$th step, the back-end equalizer E_{N+1} is designed.

At the kth step, TPG is given by

$$T_k(\omega) = T_{k-1} E_k(\omega) \qquad k = 1, 2, \ldots, (N+1) \tag{50a}$$

where

$$E_k(\omega) = |E_{21}^{(k)}|^2 \frac{|L_{21}^{(k)}|^2}{|1 - E_{11}^{(k)} G_{22}^{(k)}|^2 |1 - \hat{E}_{22}^{(k)} L_{11}^{(k)}|^2} \tag{50b}$$

$$L_{21}^{(k)} = A_{21}^{(k)} \qquad L_{11}^{(k)} = A_{11}^{(k)} \tag{50c}$$

$$\hat{E}_{22}^{(k)} = \hat{E}_{22}^{(k)} + \frac{(E_{21}^{(k)})^2 G_{22}^{(k)}}{1 - G_{22}^{(k)} E_{11}^{(k)}} \tag{50d}$$

$$G_{22}^{(k)} = A_{22}^{(k)} + \frac{A_{12}^{(k-1)} A_{21}^{(k-1)} \hat{E}_{22}^{(k-1)}}{1 - G_{22}^{(k)} E_{11}^{(k)}} \qquad k \geq 2 \tag{50e}$$

with

$$G_{22}^{(1)} = \frac{Z_G - 1}{Z_G + 1} \tag{50f}$$

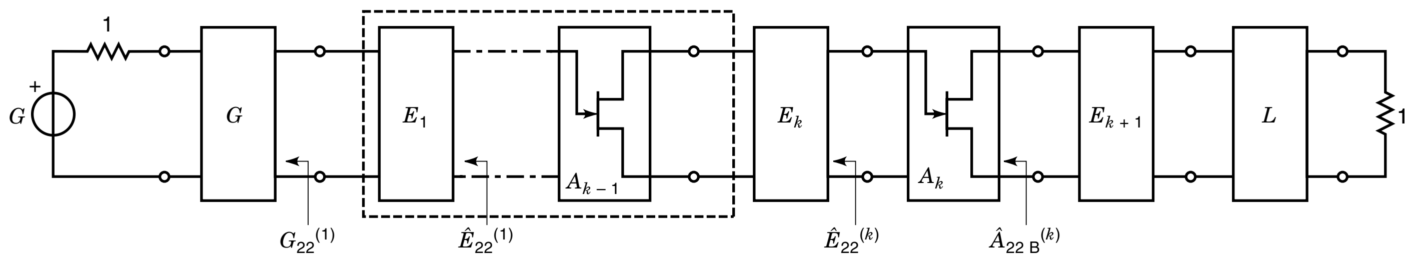


Figure 10. Multistage amplifier configuration with front-end, back-end, and interstage equalizers.

At the last step of the above process, overall transducer power gain of the multistage will be computed in a sequential manner as

$$T(\omega) = (T_1.T_2 \ldots T_N).E_{N+1} \tag{51}$$

In Eq. (51) the term $E_{(N+1)}(\omega)$ provides the impedance matching to load Z_L. In this case, parameters of Eq. (50b) are given by

$$L_{11}^{(N+1)} = \frac{Z_L - 1}{Z_L + 1} \tag{52a}$$

$$|L_{21}^{(N+1)}|^2 = 1 - |L_{11}^{(N+1)}|^2 \tag{52b}$$

At each step of the design, SRFT is accessed to construct the lossless matching networks $[E_k]$ and TPG is optimized over the band of operation. In the course of the optimization process, the gain taper of each FET is compensated at the corresponding front-end equalizer. Furthermore, nonunilateral behavior of the active devices is taken into account. Eventually, in order to improve the gain performance, the TPG of the overall system can be reoptimized.

Practical Implementation of Matching Equalizers

In the above-presented techniques, the complex variable $s = \sigma + j\omega$ was employed in the descriptive network functions, which yields lumped element circuit components (inductors and capacitors) in matching equalizers. Utilizing hybrid or monolithic integrated circuit production technologies, it is possible to build this type of lumped circuit elements up to 10 GHz. Beyond these frequencies, however, physical sizes must be included in the design process. A straightforward method to construct matching equalizers with physical sizes is to employ equal delay transmission lines throughout the design. In this case, complex variable $s = \sigma + j\omega$ is replaced with the Richard variable $\lambda = \Sigma + j\Omega$. Here, transformed frequency Ω is given by $\Omega = \tan\omega\tau$, where τ specifies the equal delay length of transmission lines employed in the designs. In this case, all the computations are carried out in the transformed frequency domain, but otherwise, all the analytic and real frequency solutions to matching problems remain unchanged. Extension of the analytic and real frequency solutions with equal length or commensurate transmission lines can be found in Refs. 34 and 40–44.

More sophisticated solutions to broadband matching problems can be given with mixed lumped and distributed elements. In these solutions, physical connection of lumped elements can be covered with transmission lines and parasitics of the discontinuities can be imbeded into lumped elements. In this case, it is necessary to carry out all the designs in at least two variables, namely, s for lumped elements and λ for equal delay transmission lines. Semi-analytic and real frequency solutions to matching problems with mixed lumped and distributed elements can be found in the works of Yarman, Aksen, and Fettweis (28,45–49).

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B. Siddik Yarman
Işık University

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BROADCAST ALGORITHMS. See GROUP COMMUNICATION.

BROADCASTING. See TRANSMITTERS FOR AMPLITUDE MODULATION BROADCASTING

BROADCASTING ANTENNAS. See ANTENNAS FOR HIGH-FREQUENCY BROADCASTING; ANTENNAS FOR MEDIUM-FREQUENCY BROADCASTING.

BROADCASTING BY DIRECT SATELLITE. See DIRECT SATELLITE TELEVISION BROADCASTING.

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