FUZZY LOGIC CONTROL

Fuzzy thinking provides a flexible way to develop an automatic controller. When process control is based on mathematical models, the degree of precision often presents difficulties in achieving adaptation and/or rigor. A high degree of understanding about the process is necessary to design effective model-based controllers that adapt to changing process conditions.

On the other hand, fuzzy logic sets up a model of how a human thinks about controlling a process rather than creating a model of the process itself. The structure of such a system is exactly as we might verbalize our understanding of the process. Rules are constructed almost as spoken by an experienced operator, that is,

If CURRENT DRAW is LOW then INCREASE FEEDRATE A LOT,
provided the SURGE BIN LEVEL is not TOO HIGH.

A set of rules like this one provides a complete means to implement control in a rapid and effective manner. Precision is not a requirement in fuzzy logic control, but a high degree of accuracy in the desired I/O map can be obtained through testing. Stability issues with fuzzy control still lack a formal mathematical proof, but stability can be a demonstrated feature of a properly tuned system through simulation.

Fuzzy logic is now an accepted technology for control systems at either the supervisory or local control level. Conventional and modern control methods demand considerable mathematical skill and knowledge to implement and tune, whereas a fuzzy controller can be set up with ease, allowing a system to mimic directly how an experienced operator achieves consistent process output. The system grows incrementally by defining rules that relate input variables to output variables in the language used by the operating personnel. Although each rule may be a simple expression of a specific I/O relationship, when the set of rules are implemented in a cooperative fashion, the combined result often represents complex, nonlinear relationships.

WHAT IS FUZZY LOGIC?

Fuzzy logic, an apparent oxymoron, evolved from the incredible figment of one man's imagination generated over

32 years ago, into an accepted figure of speech used today as a catch-phrase to sell commercial products such as rice cookers, washing machines, vacuum cleaners, and 35 mm and video cameras, and to develop complex multivariable control systems for power systems, mineral processing plants, chemical plants, pulp mills, cement kilns, Japan's famous bullet train, and even for use on the space shuttle.

If one looks up these words in a modern dictionary, one might find the following:

logic n. science of reasoning; philosophical inquiry into principles and methods of validity and proof; formal argument or reasoning of an inference or natural consequence.

fuzzy adj. 1. frayed, fluffy, blurred, indistinct; frizzed. 2. (*math.*) not precise, approximate; a set whose members lie across a spectrum of values that approximate a central value.

Although the dictionary of the times recognizes the role of fuzziness in set theory, one may wonder: "How can an inquiry into methods of proof produce imprecision?" And yet, even rigorous mathematical models can claim to achieve only an approximate representation of reality. They cannot possibly account for all intervariable relationships over all ranges of data. Clearly, when the truth or denial of an hypothesis is established beyond all reasonable doubt, there is nothing fuzzy about belief in that fact. But, what happens when doubt does exist or when the process is fraught with unknown or immeasurable inputs—into which state does one place such situations: "Are things TRUE or are they FALSE?"

Traditional logic systems have great difficulty with such cases. Often, attempts are made to define new states as mutually exclusive concepts of the original state. This redefinition can be awkward and time-consuming, and does not really mimic the way in which the human mind actually reasons. On the other hand, fuzzy logic allows one to address directly the way one thinks about problems in which one has limited fundamental knowledge or in which one does not have the time, money, or patience to conduct a detailed formal analysis.

Consider the concept *darkness.* Everyone knows the difference between day and night—at least those who are not blind can easily distinguish these states. But imagine abruptly awaking from an afternoon nap around dusk without a clock. Would you wonder if it was dawn? You might get dressed for work if it was a weekday, before realizing that it is getting darker, not lighter. So what are these terms—*dusk* and *dawn*—with respect to day and night? They are simply the boundary conditions between day and night. Neither sharp nor crisp, these regions extend over the finite and measurable time that the sun takes to rise and set each day. In addition, the degree of darkness and its rate of advance or decline during these transitions depend on the season, the latitude, and any number of environmental factors that include cloud-cover, rain, or perhaps volcano dust. A solar eclipse or the presence of a full moon might present temporary confusion about the change from day to night, or vice versa.

Dusk and dawn are classic examples of real-life fuzzy sets. As dusk begins, belief that it is night increases until, when it is completely dark, one has no doubt that night has arrived. Similarly, belief in day-time declines until, at the end of dusk, one fully accepts that it is not daytime. Figure 1 shows a mapping of these "dusk" and "dawn" fuzzy sets across the universe of discourse of the 24-hour clock.

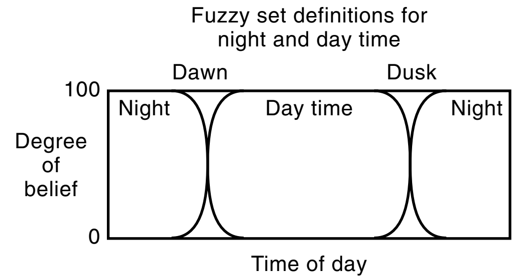

Figure 1. Fuzzy sets for "dawn" and "dusk" are located in the boundaries between night and day.

So, a fuzzy set is simply a set of elements in a universe of discourse that defines a particular state, in which each element has a rank or membership in the set on a scale from 0 to 1 (or 0 to 100%). Those elements with rank of 1 (or 100%) are full members, whose occurrence make the set TRUE. Those elements with rank of 0 are nonmembers, which make the set FALSE. Those elements with intermediate rank are partial members, whose instance suggests that there is potential movement into or out of an adjacent set, or that there is uncertainty about the validity of the set or concept. (Is it dawn or is it dusk?—one might need additional information.) There are many examples of real-life, practical fuzzy sets such as these. Here are a few others:

- An automobile changing lanes while passing
- The position of the shoreline during tidal inflow or outflow
- A door being closed or opened
- A water valve being opened or closed
- A glass of water
- The mixing together of two primary colors
- The age of a young customer in a bar
- The time it takes to drive from home to work
- The waiting time in a queue

Think of some others that one deals with in day-to-day activities.

A BRIEF HISTORY OF FUZZY LOGIC

From a mathematical viewpoint, fuzziness means multivalued or multivalent and it stems from the Heisenberg Uncertainty Principle, which deals with position and momentum. A three-valued logic was evolved by Lukasiewicz (1,2), to handle truth, falsehood, and indeterminacy or presence, absence, and ambiguity. Multivalued fuzziness corresponds to degrees of indeterminacy, ambiguity, or to the partial occurrence of an event or relationship. Consider a number of paradoxical statements:

A man says: Don't trust me. Should you trust him? If you do, then you don't:

A politician says: All politicians are liars. Is this true? If so, then he is not a liar.

A card states on one side: The sentence on the other side is false. On the other side appears: The sentence on the other side is true. How do you interpret this card?

Bertrand Russell's famous paradox: All rules have exceptions. Is this a rule? If so, then what is its exception?

These "paradoxes" all have the same form: a statement **S** and its negation **not-S**, both of which have the same truth-value **t(S):**

$$\mathbf{t}(\mathbf{S}) = \mathbf{t(not\text{-}S)} \tag{1}$$

But the two statements are both TRUE (1) and FALSE (0) at the same time, which violates the laws of noncontradiction and excluded middle in the field of bivalent logic. This approach states that negation produces the reverse truth value. Hence:

$$\mathbf{t(not\text{-}S)} = 1 - \mathbf{t(S)} \tag{2}$$

So, by combining these two expressions, one gets:

$$\mathbf{t(S)} = 1 - \mathbf{t(S)} \tag{3}$$

This is clearly contradictory for if **S** is true, then 1 =0 and if **S** is false, then 0 =1.

But a fuzzy interpretation of truth values can handle this relationship. By solving for **t(S)** and allowing **t(S)** to assume values other than the set {0,1}, one gets:

$$\mathbf{t(S)} = 0.5 \tag{4}$$

So with fuzzy logic, "paradoxes" reduce to literal half-truths. They represent, in the extreme, the uncertainty inherent in every empirical statement and in many mathematical expressions.

Quantum theorists in the 1920s and 1930s allowed for indeterminacy by including a middle truth value in the "bivalent" logic framework. The next step was to provide degrees of indeterminacy, with True and False being two limiting cases on the indeterminacy scale. In 1937, the quantum philosopher Max Black (3) applied continuous logic to sets, lists of elements, and symbols. He must be credited with the first construction of a fuzzy set membership graph. He used the term *vagueness* to describe these diagrams.

In 1965, Zadeh published the seminal paper (4) on a theory of fuzzy logic, in which the ubiquitous term "fuzzy" was introduced. This generated a second wave of interest in multivalued mathematics, with applications ranging from systems theory to topological mapping. With the emergence of commercial products and new theories in the late 1980s and early 1990s, a third wave has arisen—particularly in the hybridization of fuzzy logic and artificial neural networks (5).

At first, Zadeh believed the greatest success of fuzzy logic would be found in the area of computational linguistics (6). However, it was fuzzy control that provided the necessary springboard to take his idea from pure theory to one with numerous real-world applications (7,8).

In 1974, Mamdani and Assilian (9,10) presented the first application of fuzzy control, in which the basic paradigm of the fuzzy mechanism, in the form of a rule-based system to control a laboratory steam engine, was developed. In 1982, Holmblad and Oostergaard (11) described the first commercial application of fuzzy control of a cement kiln. For many years this was the major application area for fuzzy control, as commercialized by F. L. Smidth of Denmark. But, despite these isolated successes, for many years the second wave was a lonely ride, with much derision and denigration of Zadeh's uncertainty calculus as being illogical and not rigorous. Some "philosophers" sloughed off "fuzzy-reasoning" as being "folk art." Many mathematicians scorned the theory as "unscientific," despite the fact that most all people use their own fuzzy calculus—some without even realizing it. The tools of exact science may be decision aids but, in the end, final control is always "fuzzy."

With the exception of probability theory, the artificial intelligence (AI) community (12) almost completely shunned numerical approaches to uncertainty management. This ignorance certainly slowed acceptance of "intelligent methodologies" among the conventional scientific community. When examining material on uncertainty principles in some of the recent historical and technical books on AI, one can only wonder in dismay at the total lack of information on the subject of fuzzy logic. It is interesting to note that, at the 1998 World Congress on Expert Systems, held in Mexico City, Lofti Zadeh was presented with the Feigenbaum Award—the highest award from the AI community. AI has belatedly embraced fuzzy methods in the face of the union of fuzzy logic, artificial neural networks, and genetic algorithms into the new fields known as *soft-computing* (13) and *computational intelligence* (14).

Fuzzy expert systems are clearly superior to conventional ones, because of their intrinsic ability to deal directly with uncertainty allowing "crisp" rules to operate as a continuum across an I/O space-state map. The variety of methods to create this flexibility indicates that a fuzzy logic approach, by itself, is a "fuzzy" concept.

The similarities between fuzzy reasoning and neural network modeling suggest the marriage of these two methods, to create a *thinking machine,* able to respond dynamically to environmental stimuli; to learn and be trained; to explain its actions to others; and to understand the importance of context-reasoning which underlies the general approach to adaptive response (5,15). While artificial neural networks have an architecture and dynamic structure that can be applied to a wide variety of problems, in which "memories" are stored as distributed weight-links on myriad interconnections, fuzzy systems store information in banks of fuzzy associative memories (FAM) that connect data symbolically in the form of rules-of-thumb.

Fuzzy-neural systems are combinations of these technologies in which link-weights are used within a rule-based FAM to relate input variables to output variables in a single rule. These rules can be viewed as interacting nodes within a layered neural network structure. The link-weights can be "learned," using the backpropagation algorithm (15) or with a genetic algorithm (16).

Despite its newness, successful real-world applications of fuzzy logic have been developed in many commercial areas: subway braking systems (17), camera and video recorders (15), light-meter and image-stabilization systems, color-TV tuning (18,19), automobile transmissions (20,21) and cruise control systems (22,23), washing machine load cycles (24), automatic vacuum cleaners (24), rice cookers, security investment, traffic control (25), elevator control (26,27), cement kiln operation (11,28,29), nuclear power plant control, (30), secondary crushing plants (31), thickener operations (32), continuous casting of steel (33), electric induction motors (34), Kanji character recognition, golf club selection, and even flower arranging.

Many of the early success stories in Europe actually disguised the applications, by using terms such as “multivalued,” “continuous,” or “flexible” logic. Perhaps inspired by these efforts, in the early 1980s, the Japanese quickly assumed the lead in promoting widespread use of fuzzy control in commercial products. At first, resistance in North America was high, most likely because of our cultural abhorrence for ambiguity. Japanese society readily accepts such vagueness and so, opposition was less. But, as products began to enter the marketplace in ever-increasing quantities, the competitive forces in North America have been unable to resist any longer.

FUZZY SETS

The details below present some information to help understand the principles behind fuzzy control.

Notation

The following list contains some of the commonly used notation in set theory:

X	a whole set or the universe of discourse
x	one element in X
A	a subset
{0,1}	the set of 0 and 1
[0,1]	the interval of real numbers from 0 to 1
$a \wedge b$	the minimum of a and b
$a \vee b$	the maximum of a and b
$\forall$	for every
$\in$	belonging to
$\neg a$	the complement of ‘a’; i.e., ‘not a’

In set theory, a universe of discourse is defined as all elements which can be grouped as identifiable, labeled units, known as *sets* or *subsets* within the universe of discourse. A fuzzy subset **A** of a universe of discourse **X** is characterized by a membership function $\mu_A(\mathbf{x})$. This function assigns to each element $\mathbf{x} \in \mathbf{X}$, a number $\mu_A(\mathbf{x})$ within the closed interval [0,1] (or 0,100%), which represents the grade of membership (or degree of belief, certainty, or truth) of **x** in **A**. Two ways, among many, to denote a fuzzy set are:

$$A = \{x_1, x_2, \ldots, x_n\} \qquad \text{or} \qquad A = \{(x_i, \mu_A(x_i))\} \qquad \forall x_i \in X \tag{5}$$

The support of **A** is the set of elements of **X** which have $\mu_A(\mathbf{x})$ grades greater than zero. A cross-over point (or saddle point) of **A** is any element of **X** whose membership rank in **A** is 0.5 (or 50%). These points define the transition of the set from a tendency of being true to a tendency of being false. A

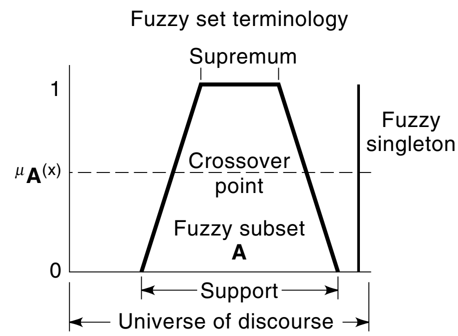

Figure 2. Fuzzy set terminology.

singleton is a fuzzy set whose support is a single element of **X**. Integers can be classified as fuzzy singletons, but linguistic terms may also be singletons. Figure 2 shows these terms graphically for a trapezoidal-shaped fuzzy set.

The supremum (or height) of a fuzzy set **A** are those values of **X** whose membership rank is 1.0 (or 100%). This characteristic can be a discrete value or a range of values, depending upon the shape of the set in question. The ratio of the supremum range to the support range is a measure of the uniqueness of a fuzzy set. As this ratio approaches 1.0, the set becomes nonfuzzy or *crisp*. But as this ratio approaches 0, the set becomes unique with respect to its supremum. A unique supremum can represent a set in a statistical sense. For example, a triangular-shaped fuzzy set with supremum position at 10 and support from 9 to 11 can be described (35) as the fuzzy number 10 with range ± 1.

The degree of fuzziness is a term that describes how much uncertainty is associated with a set over its entire support range. If all membership grades of elements of **X** consist of the set {0,1}, then the degree of fuzziness is 0. The maximum degree of fuzziness (or 1.0) occurs when all elements have membership grade of 0.5 (see Fig. 3).

When the height of a fuzzy set is 1.0 (or 100%), the set is described as *normal*. In practice, fuzzy sets are normal—at least they start out that way. But during the inferencing and defuzzification process that operate on these sets, they may transform into *subnormal* sets with supremum positions appreciably below 1.0 (or 100%).

Fuzzy Set Operations

There are several actions that can be performed on a group of fuzzy concepts or sets. These operations may compare two or more concepts, or may extract the minimum or maximum degree of belief from the group. Likewise, the implication of one

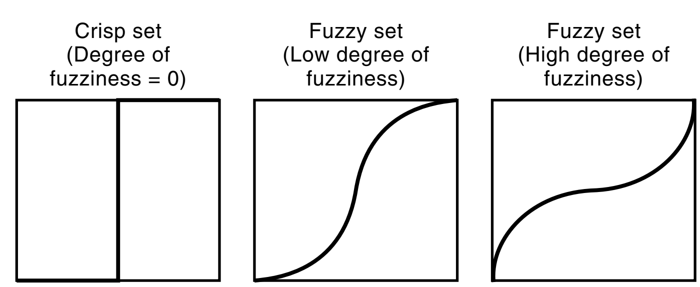

Figure 3. Fuzzy sets with different degrees of “fuzziness.”

concept based on the belief states of other fuzzy concepts is an important operation, especially with respect to expert systems. A few of the important and simple ones are given below.

Equality and Inequality. Two fuzzy subsets **A** and **B** are said to be equal if the following holds:

$$\mu_{\mathbf{A}}(x) = \mu_{\mathbf{B}}(x) \qquad \forall x \in X \tag{6}$$

If the membership grades of one set are less than or equal to those of another for all values of **x**, then the former set is described as a *subset* (or *child*) of the first. Conversely, the latter set is known as a *parent* of the former. Child and parent fuzzy sets take on important significance in the field of linguistics, where qualifiers can be used to create new fuzzy sets that are generational relations of the original set.

Union. The union of two subsets **A** and **B** is a fuzzy subset denoted as **A** ∪ **B**, with its membership function defined as:

$$\mu_{\mathbf{A}\cup\mathbf{B}}(x) = \mu_{\mathbf{A}}(x) \vee \mu_{\mathbf{B}}(x)) \qquad \forall x \in X \tag{7}$$

So the combined membership function is the maximum of the two individual sets:

$$\mu_{\mathbf{A}\cup\mathbf{B}}(x) = \max(\mu_{\mathbf{A}}(x), \mu_{\mathbf{B}}(x)) \qquad \forall x \in X \tag{8}$$

This operation is equivalent to the use of the **OR** operator for two concepts in a rule-based expert system or in a fuzzy inference. The degree of belief to be transferred from the rule premise to the rule conclusion will be the maximum of the two concepts in question.

Intersection. The intersection of two subsets **A** and **B** is a fuzzy subset denoted as **A** ∪**B** with its membership function defined by:

$$\mu_{\mathbf{A}\cap\mathbf{B}}(x) = \mu_{\mathbf{A}}(x) \wedge \mu_{\mathbf{B}}(x)) \qquad \forall x \in X \tag{9}$$

In this case, the combined membership function is the minimum of the two individual sets:

$$\mu_{\mathbf{A}\cap\mathbf{B}}(x) = \min(\mu_{\mathbf{A}}(x), \mu_{\mathbf{B}}(x)) \qquad \forall x \in X \tag{10}$$

This operation is equivalent to the use of an **AND** operator for two concepts in a rule-based expert system or in a fuzzy inference. The degree of belief to be transferred from the rule premise to the rule conclusion will be the minimum of the two concepts in question.

Complementation. The complement of a fuzzy subset **A** is denoted by ¬**A**, with its membership function defined by:

$$\mu_{\neg\mathbf{A}}(x) = 1 - \mu_{\mathbf{A}}(x) \tag{11}$$

This operation is equivalent to the **NOT** operator in a rule-based expert system. In these systems, if a statement refers to a particular concept as being "not true," then the degree of belief returned is the complementary function of the membership rank of the fuzzy concept. Complementation is often implemented in a fuzzy expert system by using the equivalent statement that a fact is "false" instead of using "not true."

Fuzzy Linguistic Hedges. There are numerous linguistic expressions used in everyday speech to "flavor" certainty in a particular concept or fact. A hedge is simply a qualifier word used with a concept to avoid total commitment or to make a vague statement. The *Random House Word Menu,* by Stephen Glazier, lists five categories of such qualifiers, which include:

1. Limitations and Conditions	325 entries
2. Approximations and Generalizations	150 entries
3. Emphasizers	85 entries
4. Maximizers and Superlatives	105 entries
5. Absolutes and Guarantees	185 entries

The English language is full of rich linguistic terms to provide "shades of gray" to a concept. Consider the following set of words: beautiful, pretty, gorgeous, voluptuous, cute, sexy, handsome, fabulous, marvelous, outstanding, remarkable, extraordinary. Each of these terms could describe the attractiveness of an individual, but the meaning of the description is quite different, depending on the phrase and context in which it is used. Notice how one's mind instantly switches context as one moves from one word to another. The term "handsome," for example, is typically reserved for males, while "gorgeous" generally refers to females, but not always.

Context identification or generalization may be a negative factor, which can introduce bias, stereotyping, or "stick-in-the-mud" attitudes into the analysis of a problem—the process is always based on experiential knowledge and must be viewed and used with caution. To provide revolutionary approaches to thinking, rules-of-thumb must always be challenged periodically, if time and money permit, or else the underlying fundamental relationships will never be discovered. Nevertheless, some simple qualifiers such as "very," "almost," "nearly," "definitely," "certainly," "more-or-less," "maybe," "somewhat," or "could be," can each be used with fuzzy concepts by applying a mathematical operation to the membership function of the original fuzzy set. In his original discussion on linguistic hedges, Zadeh (6) defined the following operators:

$$\text{concentration of A: "very":} \qquad \mu_{\mathrm{Con\{A\}}}(x) = \mu_{\mathrm{A}}(x)^2 \tag{12}$$

$$\text{dilation of A: "somewhat":} \qquad \mu_{\mathrm{Dil\{A\}}}(x) = \mu_{\mathrm{A}}(x)^{0.5} \tag{13}$$

It is interesting to note that a concentrated hedge becomes a child of the original fuzzy set while dilation produces a parent. This confirms one's intuitive sense that "very" and "somewhat" tend to make the terms they modify more exclusive and more inclusive, respectively. Similar operators can be specified for terms such as "extremely" (grade of membership is cubed) or "more or less" (membership grade is the cube root), and so on.

Linguistic hedges can be thought of as newly defined states of the universe of discourse. For example, the statement "We are 81 percent certain that it is cold," could be replaced by the more definitive statement "We are 90 percent sure it is somewhat cold," or the less definite one "We are only 66 percent sure it is very cold."

Alternatively, the belief values in the same fuzzy concept can be replaced with appropriate predicate functions, which are actually fuzzy relations such as:

90 percent certainty in cold >>>>we are very certain it is cold
81 percent certainty in cold >>>> we are kind of certain it is cold
66 percent certainty in cold >>>> we are somewhat certain it is cold

Fuzzy Relations. Fuzzy relations are used to map one or more fuzzy concepts into another. They represent rules in an Expert System, which can infer one fact from others or compare or combine two input facts in a rule premise statement.

A fuzzy relation **R** from a fuzzy set **X** to **Y** is a fuzzy subset of the Cartesian product **X** × **Y**, where the membership function in this subset is denoted by $\mu_R(x, y)$. For example, consider the sets $\mathbf{X}=\{x_1, x_2\}$ and $\mathbf{Y}=\{y_1, y_2\}$ with fuzzy subsets **A** and **B**, respectively. The fuzzy relation from **X** to **Y** is the Cartesian product **R** = **A** × **B**, of the fuzzy subsets **A** and **B** with membership function in the Cartesian product **X** × **Y**, of $\mu_R(x_n, y_m)$, where

$$\mu_R(x_n, y_m) = [\mu_A(x_n) \wedge \mu_B(x_m)], x_n \in \mathbf{X}, x_m \in \mathbf{Y} \qquad (14)$$

This relation is represented by the relation matrix **R**, where

$$\mathbf{R} = \begin{bmatrix} \mu_R(x_1, y_1) & \mu_R(x_1, y_2) \\ \mu_R(x_2, y_1) & \mu_R(x_2, y_2) \end{bmatrix} \qquad (15)$$

Now if **R** is a relation from **X** to **Y**, and **S** is a relation from **Y** to **Z**, then the fuzzy relation from **X** to **Z**, which is called the *composition* of **R** and **S** and denoted by **R** ∘ **S**, is defined by

$$\mu_{\mathbf{R}\circ\mathbf{S}}(x, z) = \vee_y[\mu_{\mathbf{R}}(x, y) \wedge \mu_{\mathbf{S}}(y, z)] \qquad (16)$$

where element [i, j] in the relation matrix **R** ∘ **S** is given by

$$\max[\min(\mu_{\mathbf{R}}(x_i, y_1), \mu_{\mathbf{S}}(y_1, z_j)), \min(\mu_{\mathbf{R}}(x_i, y_2), \mu_{\mathbf{S}}(y_2, z_j)), \ldots \min(\mu_{\mathbf{R}}(x_i, y_n), \mu_{\mathbf{S}}(y_n, z_j))] \qquad (17)$$

This relation is known as the *max–min operation.* Other examples of common binary fuzzy relations are "is much greater than," "resembles," "is relevant to," "is close to," and so forth.

MANAGING UNCERTAINTY IN FUZZY SYSTEMS

Kosco (5) suggests that well-designed fuzzy logic-based systems perform more efficiently and effectively than do conventional expert systems based on binary logic. Although these latter systems create logical decision trees of a knowledge domain, the structures are usually much wider than they are deep, and tend to exaggerate the utility of bivalent rules. Only a small portion of the stored knowlege is acted upon during any consultation, and interaction among the rules does not generally take place.

The power of a fuzzy system relates to its interaction ability. All of the inference rules within each particular fuzzy associative memory rule-set (FAM) fire on every cycle to influence the outcome. These FAMs exist as separate sections of an overall system that relate multiple input variables to a single output variable. Such rule-sets interact through a variety of combinatorial mathematics, to yield an aggregated inference on each particular output.

A typical AI rule-based system rounds off the truth value of each input to true or false, examines only those rules that can be fired from information that is true, and then chains through the knowledge base structure, using appropriate strategies such as depth-first or breadth-first, to examine the rule base and reach a unique decision. A fuzzy system also uses a preset strategy to search its rules, but uncertainty embodied in the input data is retained. All rules are examined with the uncertainty propagating through the system as it chains toward a final conclusion. Premises are used in a weighted fashion to flavor a decision based on belief in the input variables. Accumulation of these separate trains of thought are equivalent to examining a series of vague principles rather than specific hard-cold rules. Combination of these fuzzy facts and principles can be considered an act of intuition or judgement, explainable in terms of current facts and relevant principles embodied within the rule-sets. If necessary, a rule can be excluded by applying a fuzzy confidence level to the system in which a rule with a net degree of truth below this limit does not fire successfully.

The FAM rule-sets associate input data with output data with only a few FAM rules necessary for smooth control. Conventional AI systems generally need many precise rules to approximate the same performance. Adaptive fuzzy systems can use neural or statistical techniques to extract fuzzy concepts from case-studies and automatically refine the rules as new cases occur. The methods resemble our everyday processing of common-sense knowledge. Machines of the future may have the "intelligence" to match and, perhaps, exceed our ability to learn and apply fuzzy information—knowledge that is rarely expressed, but which is used to run everyday lives.

OPERATION OF A FUZZY LOGIC CONTROLLER

During operation, the "fuzziness" associated with a system is embedded, and so is hidden from the external environment. The controller receives discrete input information; maps these numbers into a series of fuzzy sets that describe the process states of each input variable; applies the degrees of belief (DoBs) in these fuzzy terms to a knowledge base that relates input states to output states according to a set of rules; infers the degrees of belief in the output fuzzy sets that describe the output variable(s); and assembles these DoBs into a discrete output value through a process known as *defuzzification.* Figure 4 presents a diagram of the three major parts

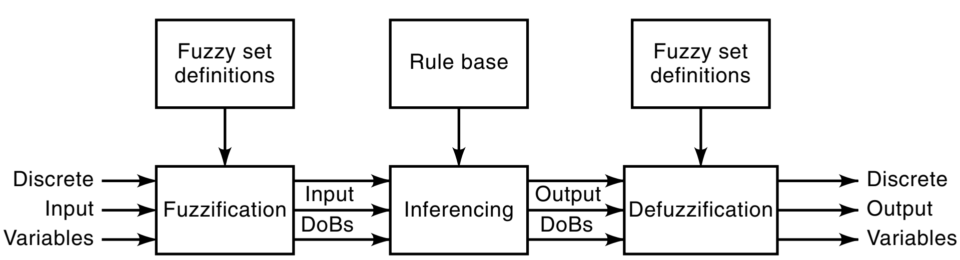

Figure 4. Major components of a fuzzy logic controller.

of a fuzzy controller—fuzzification, inferencing, and defuzzification.

Fuzzy set definitions are predetermined or may be adjusted dynamically using other rules or FAM rule sets located in the knowledge base. The rule base that links input and output fuzzy sets together is also predetermined and can be modified dynamically, as required, during operation. The methods of inferencing and defuzzification are also predefined, but as Smith (36) has demonstrated, dynamic-switching of these procedures can provide significant improvement in the degree of control and system stability.

HOW TO BUILD A FUZZY LOGIC CONTROLLER

Development and application of a fuzzy logic controller can be interesting and straightforward, or it can become a daunting project that appears to have no end point. Many people are concerned about the extreme number of data points that must be selected to "tune" a fuzzy controller. Still others, in particular those experienced with conventional control, are often unhappy with the inability to quantify measures that determine system stability. The steps required to build a fuzzy logic controller are as follows:

Define Fuzzy Sets

- Select linguistic terms to describe all I/O variable states.
- Map these terms onto discrete numerical values to create fuzzy sets.

Generate a Rule Base

- Assemble the input variable states into rule premise statements.
- Assemble the output variable states into rule conclusion statements.
- Link the appropriate input states to the appropriate output states.

Select the Inference and Defuzzification Methods

- Develop a method to "infer" the degree of belief in a conclusion statement based on the degrees of belief in the premise statements.
- Develop a method to "defuzzify" the fuzzy output states into a single discrete value.

The process begins by asking an experienced operator or individual expert to characterize the universe of discourse for each of the variables in question. Terms such as High, Low, OK, Big, Small, and No Change are defined and standardized.

The procedure involves questions like: What is the lowest value for which the term HIGH is true? What is the highest value for which the term NOT HIGH is appropriate? What is the range of values that would be considered completely OK? What is the range of values that might be considered OK? These questions formulate the support and supremum ranges for all fuzzy sets.

Selection of a fuzzy set shape is somewhat more arbitrary. Triangular and trapezoidal shapes are very popular and produce reasonable interpolation results. Bell-curves, however, yield the smoothest transition from one concept into another after defuzzification. The relative size and spread of the sets may need adjustment during testing of the controller, but it is most important that there exist at least one fuzzy set with partial belief for all values of the universe of discourse.

So, fuzzy sets such as LOW, OK, and HIGH can be used to describe possible states of an input variable. When placed within rules, the DoBs in these concepts can combine with the DoBs in the states of other variables to infer the DoBs of various output fuzzy set states such as NEGATIVE-BIG, NO CHANGE, and POSITIVE-BIG. Table 1 shows an example of a fuzzy control system in which two input variables map into a single output variable.

Construction of a two-dimensional "grid" of rules as in Table 1, is a useful way to check for completeness, consistency, and redundancy. Basically, the developer must look for evidence that set definitions have been defined for the entire universe of discourse for each input variable or concept (*completeness*). Next, the various output regions are examined to see if more than one output concept is dominant across the universe of discourse (*consistency*). Finally, rules are examined in regard to adjacent map regions to ensure efficient operation of the system (*redundancy*). If similar outputs are given for adjacent sets, then these rules can be subsumed into a common rule by defining a new input fuzzy set that is a combination of the existing ones. For example, "Low" and "Medium" could be combined into "Low-to-Medium."

At the same time, if a region produces two or more outputs related as parent or child, these rules can be subsumed by examining for exclusivity or inclusivity requirements. Again, subsuming with adjacent regions may prove expedient. The goal is to reduce the rule set to the lowest number of consistent and efficient rules without jeopardizing effectiveness.

Developing the prototype is a relatively quick operation. After completing the design phase, the controller should be run under a wide variety of input conditions to determine its performance. Discrete mapping of various input/output combinations must be done, to ensure that an acceptable relationship is achieved. These simulations provide the testing ground for proving controller reliability under different operating conditions. Simulation is the best substitute for the lack of conventional stability tests for fuzzy control systems. Interpolation across the full universe of discourse usually demonstrates a system's ability to provide tight control when near to the setpoint and very strong response when far from the target value (see Fig. 12).

Some changes in fuzzy set definitions may be helpful at this stage, but the major goal is to check on the scope of the rules linking inputs to outputs. It may be discovered that adaptive methods are useful in which input fuzzy sets are redefined dynamically. The setpoints may also have to adjust

Table 1. Feed Rate Change as a Function of Current Draw and Screen Bin Level in a Secondary Crusher

Current	Screen Bin Level		
Draw	Very Low	OK	High
High	NB	NB	NB
Medium high	NS	NS	NB
OK	NC	NC	NB
Medium low	PS	NC	NB
Low	PB	PS	NB

Where NB, NS, NC, PS, and PB represent, respectively, Negative-Big, Negative-Small, No Change, Positive-Small, Positive-Big.

to changing input conditions or changes in the external environment. This can be designed into the system with supremum and support ranges allowed to move back and forth across the universe of discourse, according to a set of overriding FAM rules.

When operated as a supervisory controller, the system should be implemented initially in a monitoring mode in parallel with a human. As decisions are made, the human operator should examine the advice and evaluate its effectiveness. If a situation exists in which the system is obviously deficient, then modifications are necessary, usually to the rule-base. Once the controller is functioning reliably without significant upsets, it can be placed into a control mode and allowed to manipulate the output variable on its own.

RULE STRUCTURE IN A FUZZY LOGIC CONTROLLER

Rules in a fuzzy logic controller are expressed in a fashion similar to that found in many expert system programs. For example, a rule to control feedrate to a crusher might be written as:

IF CURRENTDRAW is LOW
AND BIN LEVEL is_not HIGH.
THEN FEEDRATE_CHANGE is POSITIVE_BIG CF=100

Logical connections between input fuzzy set variables can be either AND or OR. An OR connection may be either inclusive or exclusive as follows:

IF CURRENTDRAW is MEDIUM-HIGH
AND BIN LEVEL is (VERY LOW OR OK) -inclusive OR
THEN FEEDRATE_CHANGE is NEGATIVE_SMALL CF=100

IF CURRENTDRAW is MEDIUM-LOW
AND BIN LEVEL is VERY_LOW
OR CURRENTDRAW is LOW -exclusive OR
AND BIN LEVEL is OK
THEN FEEDRATE_CHANGE is POSITIVE_SMALL CF=100

Note that the rule conclusion statement, which is preceded by the logical connection THEN, has an attached *certainty factor* (CF), which can be used to modify the relative importance of this particular conclusion statement.

The process of moving from the premise part of a rule to its conclusion is called *Inferencing*. Three stages are involved:

1. Determine a Net Degree of Truth (NdT) of the rule premise.
2. Calculate the Degree of Belief (DoB) in the conclusion statement.
3. Apply the DoB in the conclusion to the output fuzzy set in question.

The *net degree of truth* (NdT) is determined by combining the DoBs of the premise statements according to a chosen strategy. The conventional approach is to pick the MINIMUM DoB for ANDed statements and the MAXIMUM DoB for ORed statements. This part of the inferencing process provides a NdT value, which can be used to calculate the DoB in the conclusion statement. The conventional approach is simply to factor the NdT by the CF value attached to the conclusion statement, according to:

$$\mathrm{DoB}_{\mathrm{conc}} = \mathrm{NdT} * \mathrm{CF}/100 \tag{18}$$

The rule structure can be designed in a number of ways to accommodate a particular relationship:

- use of a single rule for each output fuzzy state;
- use of multiple rules for each fuzzy state; and
- use of fuzzy-neural rules for each fuzzy state.

Selection of a structure is a trade-off issue between desired speed and flexibility. If processing speed and system resources are most important, then the single-rule approach is preferred. Using multiple rules provides significant adaptation capability, while using fuzzy-neural rules gives the best of either option, but requires more detailed design.

Use of a Single Rule for Each Output State

The input variables are represented here by **X** and **Y**, while the output is denoted by **Z**. A single rule can be used to relate these variables. By deleting unnecessary fuzzy set relationships, a system can be constructed with one rule for each **Z** fuzzy set definition, as follows:

IF x_1 AND y_1
OR x_1 AND y_2
.
.
OR x_1 AND y_n
OR x_1 AND y_n
.
.
OR x_n AND y_n
THEN z_i cf = CF_i

Delete premise parts above as required

The degree of belief in a conclusion is calculated from a single rule, as follows:

$$\mathrm{DoB}(z_i) = \mathrm{CF}_i * \max[\min(\mathrm{DoB}(x_1), \mathrm{DoB}(y_1)), \ldots, \min(\mathrm{DoB}(x_n), \mathrm{DoB}(y_n))] \tag{19}$$

where DoB = degree of belief
CF_i = certainty factor for output i
i = output fuzzy set index
n = total number of input fuzzy sets for variables **X** and **Y**

This method provides the fastest operation of a fuzzy system. Note, however, that only one certainty factor value is available for each output fuzzy set. Multiple rules or fuzzy-neural rules possess increased flexibility by providing multiple certainty factors or link-weights, respectively, for each premise part of the above single rule.

Use of Multiple Rules

The use of multiple rules is the most common approach. By using multiple rules with one premise combination in each, a neural structure begins to emerge. The system can be built to represent the knowledge as required by deleting rules or by placing a 0 value on the CF factor in any rule conclusion.

IF x_1 AND y_1 THEN z_i cf=CF_{i11}
IF x_1 AND y_2 THEN z_i cf=CF_{i21}
.
.
IF x_1 AND y_n THEN z_i cf=CF_{i1n}
.
.
IF x_n AND y_n THEN z_i cf=CF_{inn}

Delete rules above or set CF values to 0 as required

The Degree of Belief in a conclusion is calculated from multiple rules, as follows:

$$\mathrm{DoB}(z_i) = \max[\mathrm{CF}_{i11} * \min(\mathrm{DoB}(x_1), \mathrm{DoB}(y_1)), \ldots, \mathrm{CF}_{inn} * \min(\mathrm{DoB}(x_n), \mathrm{DoB}(y_n))] \quad (20)$$

where DoB = degree of belief
CF_{ijk} = certainty factor for rule jk and output i
i = output fuzzy set index
n = total number of input fuzzy sets

Smith and Takagi (37) list a number of other combining equations that have been identified in the literature to replace the use of the max-min operator, as above. Most of these options provide a smoother transition between adjacent fuzzy sets not provided by the max–min operator. The important point to note is that each rule premise has its own unique certainty factor. The CF factors mimic the link weights of an artificial neural network and can be derived in a fashion similar to neural network training. For even more flexibility, a fuzzy-neural rule can be designed.

Use of Fuzzy-Neural Rules

This approach provides a compromise between the single-rule and multiple-rules methods. A fast system can be devised, which also possesses significant adaptation capabilities. With fuzzy-neural rules, only a single rule is necessary for each output fuzzy set description. Attached to the rule is an inference equation, which directly calculates the DoB in the output fuzzy set. An example is given below for link weights applied to each combination of input sets:

IF x_1 AND y_1
OR x_1 AND y_2
.
.
OR x_1 AND y_n
.
.
OR x_n AND y_n
THEN z_i cf = 100

Attach an inference equation to the rule as follows:

$$\mathrm{DoB}(z_i) = \frac{\sum_{j=1}^{n}\sum_{k=1}^{n}[W_{ijk} * \min(\mathrm{DoB}(x_j), \mathrm{DoB}(y_k))]}{\sum_{j=1}^{n}\sum_{k=1}^{n}[\min(\mathrm{DoB}(x_j), \mathrm{DoB}(y_k))]} \quad (21)$$

where DoB = degree of belief
W_{ijk} = link-weight for rule premise part jk and output i
i = output fuzzy set index
j = input fuzzy set index for variable **X**
k = input fuzzy set index for variable **Y**
n = total number of input fuzzy sets

Alternatively, each fuzzy set for each variable can have its own unique link weight, which makes for ultimate flexibility in dealing with complex I/O relationships. The link weights used in fuzzy-neural rules can be determined from a set of "learning rules" that receive information about the actual and desired system output. These weights can be initiated as random values; by a "best guess"; or by selections made by an expert. The error between the actual and desired output is determined and the "learning rules" would apply this error to adjust the weights using regression analysis or methods that have been developed specifically to train an artificial neural network such as backpropagation (38) or CMAC (39). Operating in a learning mode, the system iterates between these "learning" rules and the fuzzy-neural rules using a new set of data on each iteration until the overall total error is within an acceptable limit. Learning can be instigated whenever adaptation is needed due to external changes in the process environment. The link weights are stored in a data file for use by the controller during operation and learning.

SELECTION OF AN INFERENCE METHOD

The process by which a fuzzy controller changes the DoBs in the linguistic terms that describe a conclusion into a discrete output value takes place in two steps:

1. Inferencing or applying the DoB value to the output fuzzy set; and
2. Defuzzification or aggregation of all inferred output fuzzy sets into a discrete number.

Application of the DoB in a linguistic expression describing a conclusion to the specific fuzzy set prepares the system for defuzzification. There are many inferencing methods available—Smith and Takagi (35) list ten methods to combine premise DoBs. Three of the main ones are:

1. correlation-minimum;
2. correlation-product; and
3. correlation-translation.

The discussion which follows applies to the effect of inferencing methods on area-centroid weighting defuzzification which will be discussed later. The choice of inferencing method is rather subjective and context-sensitive, but it is useful to understand the impact of each method on the area and centroid of a fuzzy set used during defuzzification. Each method produces a different contribution of the output fuzzy set to the final defuzzified discrete output value.

Correlation-minimum is perhaps the most popular inferencing technique, but correlation-product is the easiest to implement. Correlation-translation was the original option proposed by Zadeh, but it is used today only under rare

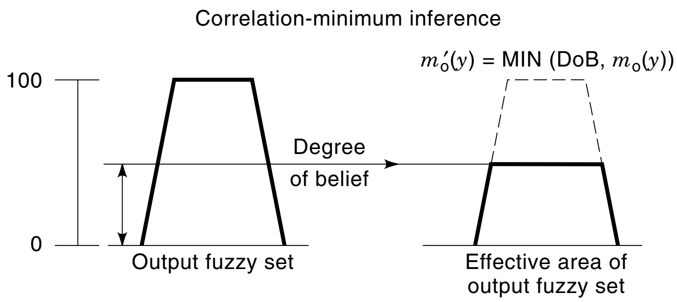

Figure 5. Correlation-minimum inferencing strategy.

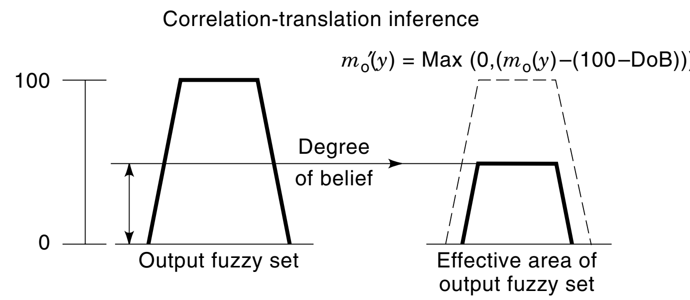

Figure 7. Correlation-translation inferencing strategy.

situations. Adaptive control can dynamically select a method by examining the current context of the situation to cause a system to change its strategy.

Correlation-Minimum Inferencing

The correlation-minimum method cuts off the top of the fuzzy set (often referred to as α-cut), using only that area of the set which lies below the current DoB. Low belief in a fuzzy concept implies that the true discrete value lies outside the supremum region. The supremum region expands as belief decreases from 100 to 0 until it equals the support of the fuzzy set (see Fig. 5).

If one wishes to retain the impact of a set as its degree of belief drops, then correlation-minimum inferencing has merit, since the percent area of the set is notably higher than its DoB (see Fig. 8).

Correlation-Product Inferencing

The correlation-product method multiplies all membership values in an output fuzzy set by the fraction of the current DoB. This method is the easiest to program and is used naturally by most expert system development tools that employ certainty factor arithmetic. In this case, the supremum and support ranges remain constant as DoB drops, implying that belief in all support values decrease in proportion to their original value (see Fig. 6). With the correlation-product method, the percent area of the fuzzy set retained for defuzzification equals its DoB (see Fig. 8).

Correlation-Translation Inferencing

If accelerated removal of the impact of a fuzzy set is desired as its DoB drops, then correlation-translation is the best choice. Correlation-translation applies that area to the defuzzification process that lies above 0 after translating the set down until the supremum position falls on top of the current DoB. Translation is accomplished by subtracting the complement of the DoB from all membership values in the fuzzy set. In this case, the supremum range remains constant as belief declines, while the support decreases until it equals the supremum at 0 belief (see Fig. 7).

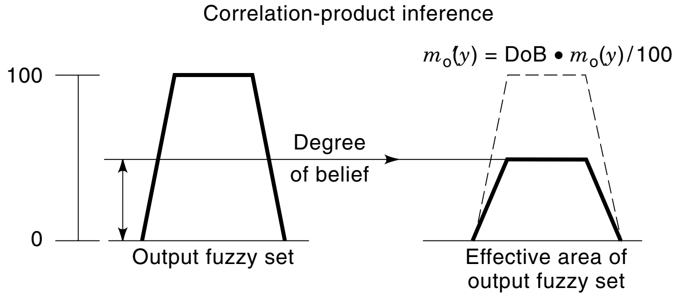

Figure 6. Correlation-product inferencing strategy.

Correlation-translation inferencing implies that only elements in the fuzzy set with membership values greater than or equal to its DoB are relevant to the defuzzification process. It is interesting to note that this method produces a complementary effect on the percent area retained to that produced by the correlation-minimum method (see Fig. 8).

Figure 8 shows how each inferencing method combines with the DoB to affect the area of a triangular fuzzy set used in a fuzzy controller. Note that at 50% DoB, correlation-minimum inferencing still retains 75% of the fuzzy set's original area, while correlation-translation inferencing only retains 25% of the original area. These two techniques can be viewed as extremes in the application of fuzzy control. Under normal circumstances, correlation-product inferencing may be best. But when risk-taking is desired, the minimum method can be used, while if a conservative approach is preferred, the translation method is preferred. The reverse could be true as well, so all three methods can be calculated with the most appropriate output for the current circumstances selected for implementation.

IMPACT OF FUZZY SET SHAPE ON DEFUZZIFICATION

Fuzzy sets can assume a variety of shapes, depending on the application and knowledge of the experts. The percent area

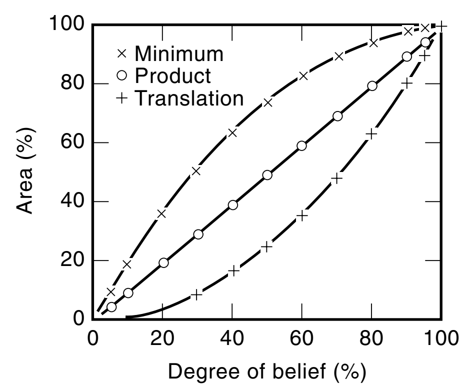

Figure 8. Influence of inferencing method and degree of belief on the retention of area for a triangular fuzzy set.

Table 2. Influence of Degree of Uniqueness on the Percent Area Applied Using the Correlation-Minimum Method

Shape	Ratio of Supremum Range to Support	Degree of Belief of 90% %Area	Degree of Belief of 50% %Area
Triangle	0.0	99	75
↑	0.2	96	65
↑	0.4	94	60
Trapezoid	0.6	93	56
↓	0.8	92	53
↓	0.9	91	51
Rectangle	1.0	90	50

applied as the DoB in a set drops, is affected by the fuzzy set shape as well as by the inferencing method. With a crisp fuzzy set shaped like a rectangle, the area applied is equivalent to the correlation-product effect shown in Fig. 8, regardless of which inferencing method is selected.

For fuzzy sets that possess nonunique supremum positions (such as trapezoid-shaped sets), the curve for the percent area applied when using correlation-minimum lies between those shown for correlation-minimum and those for correlation-product for triangular sets. The exact curve position depends on the uniqueness of the fuzzy set (the ratio of the supremum and support ranges). For correlation-translation, the applied percent area lies between the curves for triangular sets shown in Fig. 8 for the correlation-translation and correlation-product methods, again depending on the degree of uniqueness.

In Table 2, as the shape of a fuzzy set approaches that of a rectangle, the applied area approaches that determined by the correlation-product method. So, the less fuzzy the boundaries between adjacent fuzzy sets, the more likely that correlation-product is the most appropriate method, since all methods produce the same result.

EFFECT OF INFERENCING METHOD ON CENTROID POSITION

Area is not the only factor affected by the DoB of a fuzzy set. The centroid position of the set, with respect to its universe of discourse, is also affected by its DoB and the inferencing method selected. Output from the defuzzification method using area-centroid weighting is obviously dependent on the centroid position of each applied fuzzy set area.

With symmetrical fuzzy sets (whether they be triangles, trapezoids, rectangles, etc.), the centroid position is independent of DoB and inferencing method. In these cases, the centroid is always located at the midpoint of the supremum range or at the supremum position for a triangular-shaped set. So, with symmetrical fuzzy sets, the supremum position (or average) can be used instead of calculating the centroid on each cycle through the controller, that is, the fuzzy set can be considered as a singleton.

With asymmetrical fuzzy sets, the situation is somewhat different. The centroid position depends on its DoB and inferencing method. With correlation-product, the centroid remains constant as belief declines. For correlation-minimum, at full belief the centroid is located at a point on the side of the supremum where the shallower-sloped boundary exists. As DoB declines, the centroid moves away from the steeper boundary until at 0 belief, it lies in the exact middle of the *support*.

For correlation-translation, the centroid moves toward the steeper-sloped boundary until at 0 belief, it lies at the midpoint of the *supremum* range. The amount of movement in both cases is not large, unless the boundary slopes are exceeding different (see Fig. 9).

SELECTION OF A DEFUZZIFICATION METHOD

Following selection of an inferencing method that produces a composite output distribution or discrete number which represent each fuzzy set, a single output value must be calculated. For a fuzzy controller, a discrete numerical output signal is sent to a final control element or a setpoint is sent to a local control loop. For a universe of discourse containing a series of fuzzy linguistic expressions, it may be necessary to give a combined belief weight to the "best" fuzzy set output.

As with inferencing, there are alternative methods devised to accomplish defuzzification. Smith and Takagi (37) list eight methods, based on whether the individual sets are combined first and then defuzzified, or discretized first and then combined. Four of these methods are described here:

1. Weighted-average method;
2. Area-centroid weighting method;
3. Application of a fuzzy confidence level; and
4. Maximum membership method.

Weighted-Average Method. The weighted-average method is a *defuzzify-combine* approach, which is the easiest of all methods to program. The method involves multiplying the degree of belief in each set by its supremum position (or average supremum); summing the results, and dividing by the sum of all DoBs.

$$z = \frac{\sum_{i-1}^{m} [\mathrm{DoB}(z_i)\,{}^{*}\mathrm{Sup}(z_i)]}{\sum_{i-1}^{m} \mathrm{DoB}(z_i)} \tag{22}$$

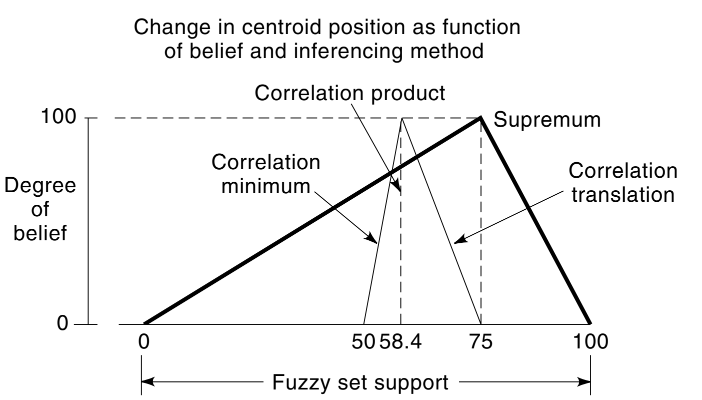

Figure 9. Change in centroid position as a function of degree of belief and inferencing method.

where z = discrete value for variable Z
$\mathrm{DoB}(z_i)$ = degree of belief in z_1
$\mathrm{Sup}(z_i)$ = supremum position (or average) for z_i
i = output fuzzy set index
m = total number of output fuzzy sets

With this method there is no need to calculate centroid positions or areas of the output fuzzy sets. The shape and support that define each fuzzy set play no role in the defuzzification process. In fact, it can be argued that, by using this method, at this point one has dispensed with fuzziness in the fuzzy set definitions, since one needs only represent each output set by a unique, discrete output-value—the supremum position, that is, the output sets are "fuzzy" singletons. Nevertheless, interpolation across the universe of discourse of the input fuzzy sets can generate complex, nonlinear, multivariable relationships, but some flexibility is lost in adjusting individual output sets to model I/O relationships at certain unique positions on the universe of discourse.

Area-Centroid Weighting. This method is the most popular defuzzification method in use today. Following application of the desired correlation-inferencing method, each fuzzy set is represented by two concepts—an output area and an output centroid position. The weighted average centroid position is then calculated by summing the product of each output area times each output centroid, and then dividing by the sum of the output areas:

$$z = \frac{\sum_{i-1}^{m} [A(z_i)\,{}^{*}C(z_i)]}{\sum_{i-1}^{m} A(z_i)} \tag{23}$$

where z = discrete value for variable Z
$A(z_i)$ = area of fuzzy set z_i
$C(z_i)$ = centroid position of the sub-normal fuzzy set z_i
i = output fuzzy set index
m = total number of output fuzzy sets

If each of the original fuzzy sets are balanced (i.e., they each begin with the same areas) and the correlation-product inferencing method is employed, the result will be the same as for the weighted-average method presented above. Area-centroid weighting is the most flexible of all methods used. It can be combined with any of the three common correlation-inferencing methods to yield complex and unique nonlinear solutions to an I/O space-state map, with as few as two fuzzy set definitions. By manipulating the relative positions of the critical points on each fuzzy set (supremum and support end-points), extremely complex changes can be modeled.

Maximum Membership Method. Some researchers believe the correct selection of a discrete value from an output distribution curve is that value with maximum belief. This may be true for crisp systems that use many sets to characterize the variables. The system must select the subnormal fuzzy output set which has the largest DoB. The maximum membership method, on its own, may produce step-changes in the I/O map with potential discontinuities. Its application should only be used in situations where the system is either very uncertain or very certain. In this way, a conservative or risk-taking approach can be implemented with ease as very little computation is required—simply choose the supremum or centroid position of the fuzzy set with maximum belief.

Application of a Confidence Level. Either of the first two methods can be modified to achieve certain specific results by applying a fuzzy confidence level to the defuzzification process. The argument supporting the use of a cut-off limit is: if you are less than 50% certain (for example) about applying a fact, then do not use this fact.

$$z = \frac{\sum_{i-1}^{m} [\mathrm{DoB}(z_\mathrm{i})\,{}^{*}\mathrm{Sup}(z_i)]}{\sum_{i-1}^{m} \mathrm{DoB}(z_i)} \qquad \text{for all } \mathrm{DoB}(z_\mathrm{i}) \geq \mathrm{FCL} \tag{24}$$

where z = discrete value for variable Z
$\mathrm{DoB}(z_i)$ = degree of belief in z_i
$\mathrm{Sup}(z_i)$ = supremum position (or average) for z_i
FCL = Fuzzy Confidence Level
i = output fuzzy set index
m = total number of output fuzzy sets

Applying a fuzzy confidence level excludes those fuzzy concepts from the calculation whose belief is less than an acceptable threshold. The fuzzy confidence level represents a factor which prevents fuzzy concepts with low DoBs from affecting the calculated discrete output value. With fuzzy control, normal defuzzification uses either the weighted-average or area-centroid weighting approach to combine the degrees of belief of all output sets into a single discrete output. There can be significant advantages in using an intermediate fuzzy confidence level to prevent those sets which are tending toward False from influencing the output value.

Systems using a FCL value of 0 apply all sets to the process of defuzzification, even those close to False. At the other extreme, if FCL is set to 100, the system is required to use only those sets that are absolutely true. Work with this technique (40) has indicated that using a fuzzy confidence level between 20% and 50% produces improvement in the response of a controller for a crushing plant simulator in terms of system stability (see Fig. 10).

When a fuzzy confidence level above 0 is used, gaps in the I/O space-state map can result. These have been referred to as vacuums of knowledge (41), as shown in Fig. 11. Our goal would be to improve on the performance generated by the I/O map shown in Fig. 12, which appears to be the desired relationship but for which an adaptable (or changeable) relationship can provide improvement. The gaps shown in Fig. 11 can produce significant stability problems for a fuzzy controller whenever inputs fall within such regions. To avoid these gaps, default values can be provided as a fall-back position. However, this can lead to a pulse discontinuity in the input/output map (see Fig. 13). By dynamically switching the defuzzification method from weighted-average (or area-centroid) to the maximum membership, these discontinuities can be removed to produce the more useful response surface shown in Fig. 14. The results show that with an FCL of 20%, the performance of the system is enhanced by about 4% (40).

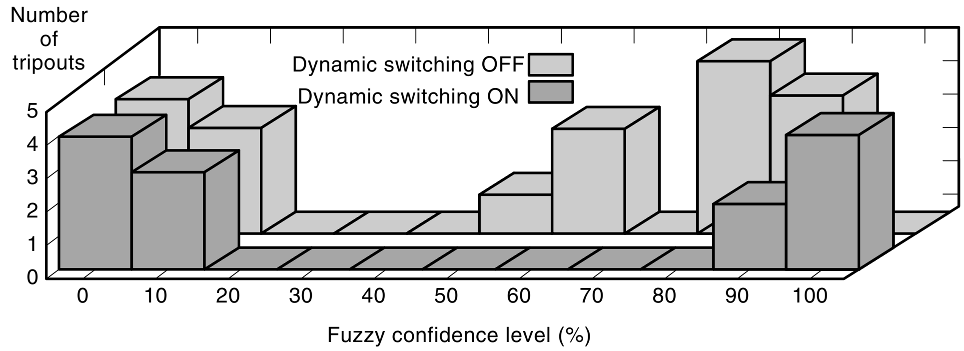

Figure 10. Effect of using a fuzzy confidence level on the stability of a fuzzy control system for a secondary crushing plant.

This dynamic switching of the defuzzification method from weighted-average (with FCL>0) to the maximum membership method has merit in improving the reliability and efficiency of the original control system. At high FCL levels, dynamic switching is absolutely necessary to ensure that the controller does not "go to sleep" because of the larger regions of vacuums created. At FCL levels below 20%, dynamic switching does not help with stability, since vacuum regions are nonexistent. The value of FCL can also be a fuzzy concept, dependent on external factors outside of the particular control system.

Dynamic switching simply involves making a temporary change in the fuzzy confidence level to the maximum DoB value of the input fuzzy sets. This restores the system to an acceptable relationship although, as Fig. 13 shows, the number of steps in the I/O graph depends on the number of fuzzy set descriptions.

COMMONLY ASKED QUESTIONS ABOUT FUZZY CONTROL—A SUMMARY

How many fuzzy sets are needed to define each variable? The number of fuzzy sets required to describe a Universe of Discourse for a variable depends on several factors:

- the expertise as defined by the expert(s);
- the speed of execution required;
- the complexity of the input/output mapping; and
- the form of data input.

The number chosen will be a compromise between these issues and others. Very complex mappings can be generated with as few as two sets (LOW and HIGH). The use of three sets provides a target range for each input variable with the provision of gain-scaling as the process state approaches the set point. Five fuzzy sets give added flexibility, by providing fine and coarse tuning rules. Some systems may need seven or nine fuzzy set definitions to accommodate certain features on the I/O map. This increases the complexity of the system and its maintenance. Many more rules are needed in such FAM modules. In most cases, five fuzzy sets are sufficient.

How should the critical points of each set be defined? To obtain useful definitions of a fuzzy linguistic term, ask the expert(s) these questions (we will use MEDIUM and HIGH power for the example):

- What is the lowest power level that you would describe as being HIGH?
- What is the highest power level that you would describe as NOT-HIGH?

For intermediate set definitions, three questions are needed:

- What is the range of discrete values for which a MEDIUM power level is TRUE?
- What is the highest value from the bottom of the universe of discourse for which MEDIUM power level is definitely FALSE?
- What is the lowest value from the top of the universe of discourse for which MEDIUM power level is definitely FALSE?

If multiple experts disagree on these critical points, this suggests the expertise is either poorly understood by some or the

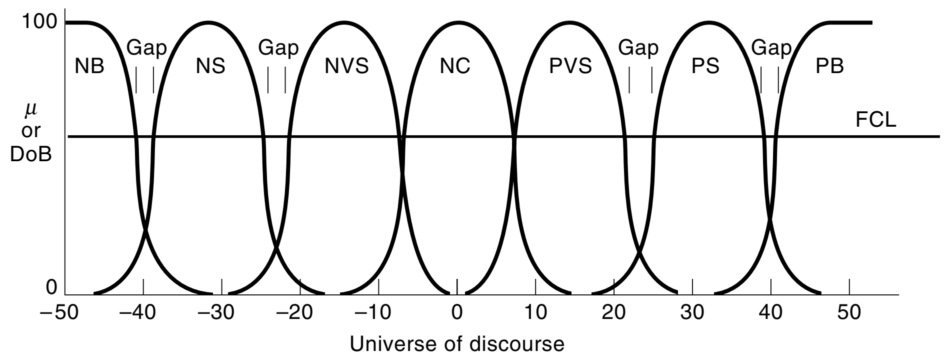

Figure 11. Vacuums of knowledge created when using a fuzzy confidence level.

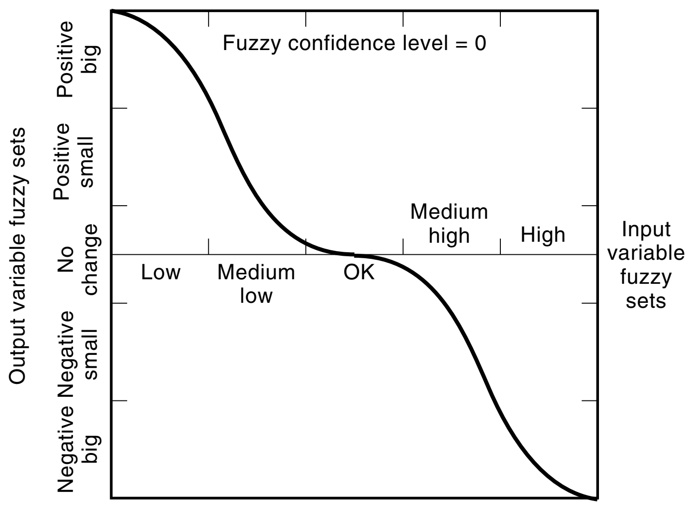

Figure 12. Possible relationship map of fuzzy output sets and fuzzy input sets.

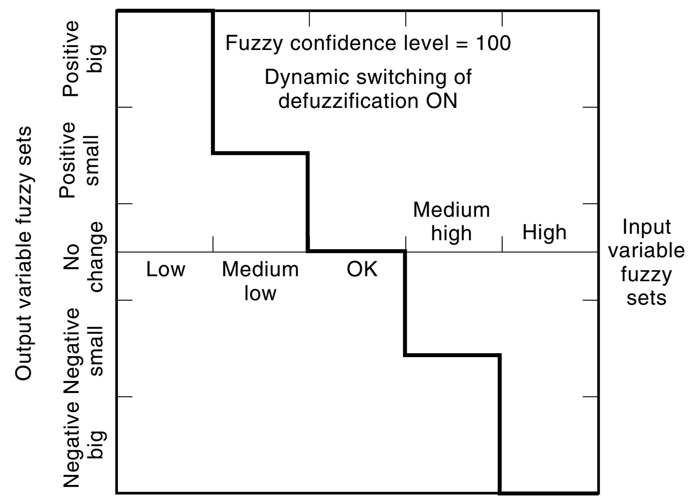

Figure 14. Influence of dynamic defuzzification on the I/O map when a fuzzy confidence level is used.

definitions do not matter. Alternatively, it may mean that there are underlying relationships still to be discovered that can be exploited to allow the set definitions to be changed dynamically during use.

How should adjacent sets overlap? Discussions with the expert(s) on the location of critical points will generally address this issue. It is very important, however, to ensure that all discrete input values be partial members of at least one set. If this is not addressed, regions of "no control" may exist on the I/O map and issues with continuity will occur. Mapping of the fuzzy sets can be a useful exercise to establish if any terms can be subsumed into parent terms. Significant overlap of adjacent sets can indicate that combining these sets into one term may be useful. The rules must be examined carefully before completing this modification.

What is the best shape to use for each set? Triangles and trapezoids are expedient shapes to use with fuzzy logic controllers. The boundaries are straight lines between the supremum and support extremities, so only three or four data points are required to define each fuzzy set. This approach reduces storage, since only the support and supremum values of the set are required. The I/O relationships generally are stepped approximations of the desired curve with these shapes, although, when adjacent sets have significantly different boundary slopes, curved relationships do result. The shape of a fuzzy set has important implications on the defuzzification process, particularly when area-centroid weighting is used.

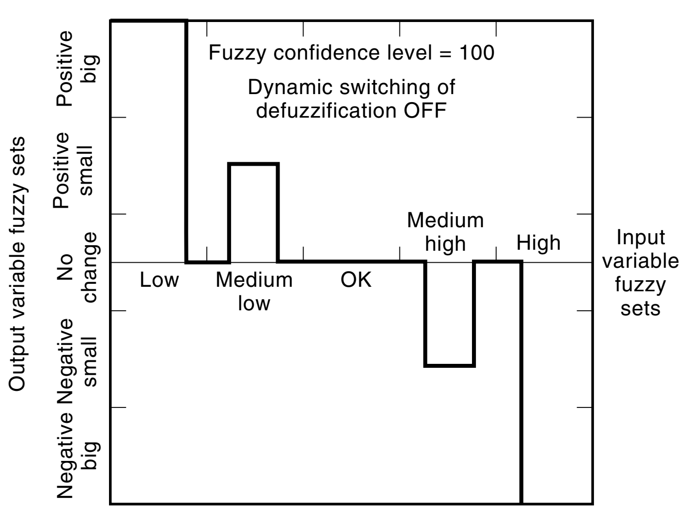

Figure 13. Formation of vacuums of knowledge when a fuzzy confidence level of 100% is used.

Which inference method is best? Smith and Takagi (37) have characterized eight different methods (there are more) to infer belief in a rule conclusion from its corresponding premise part. Each method has certain principles behind its evolution but the differences are only significant when defuzzification involves area-centroid weighting. When the weighted average technique is used to defuzzify, only the DoB of the output set together with its supremum position is important.

The max operator for ORing and the min operator for ANDing are the best ones to use initially to combine variables into the premise of a rule. There are three basic options to transfer the net degree of truth from the premise to the rule conclusion fuzzy set: Correlation-minimum, correlation-product, or correlation-translation. Dynamic switching between these methods can prove useful to adapt a system to circumstances that change from the need for a conservative approach to one that is prepared to take risks.

Which defuzzification method is best? The following methods have been described in this work:

- Weighted average method (often the same as area centroid-weighting);
- Area-centroid weighting method;
- Maximum membership method; and
- Fuzzy confidence level method.

Some authors (37) use other names like *height, best rules,* and *winning-rule* to describe the weighted-average, fuzzy confidence level and maximum membership method, respectively. Each one produces somewhat different results which are not always predictable. Weighted-average and area-centroid weighting produce similar results, particularly when correlation product inferencing is used. Often the centroid and supremum position are identical for a subnormal fuzzy set, hence the weighted-average method is usually sufficient and easiest to program.

How can the stability of the controller be measured? Conventional control systems focus considerable attention on system stability. Many mathematical techniques have been developed to deal with stability issues but few can apply to fuzzy control. As a result, the field is wide open to formulating techniques for stability analysis. In fact, it can be said that the lack of a suitable mathematical technique to handle stability studies in fuzzy control is a major impediment to developing site-critical applications for fuzzy control.

Some researchers have applied a modified version of the Lyapunov theorem for nonlinear system stability analysis with some success (42,43). Some automated techniques (44) have been developed to generate fuzzy rules sets from data, using the Lyapunov technique to ensure stability in the controller at the creation stage. Still others are working on time-domain stability criteria for nonlinear systems (45). Kosco (46) has demonstrated how feedback fuzzy systems can be proven to be stable from an analysis of their individual rule set components. A particularly good analysis of stability issues is given by Drianov et al. (47), in which fuzzy systems are examined using classical nonlinear dynamic systems theory.

Since fuzzy control implements its strategy through a rule base rather than a mathematical expression, a rigorous analysis is not straightforward. Part of this difficulty relates to representing the I/O relationship mathematically.

Process and controller simulations are the main ways to ensure sufficient rules and terminology definitions are present. If the system contains significant regions that generate vacuums of knowledge, it is likely that instability will be observed during operation of the controller. The use of MathLab (from The Math Works, Inc., Natick, MA) and Mathematica (from Wolfram Research, Inc., Champaign, IL) software tools have provided very easy-to-use programs to create models and conduct simulations quickly and effectively.

We must also consider system redundancy. This characteristic is particularly important with fuzzy systems, since the very nature of the rule-based approach contains built-in redundant features. Often the FAM maps may contain sufficient rules to accommodate a significant absence of information. The system can still provide useful control with cooperating inputs through other rules. As such certain fuzzy controllers can be considered to be a type of "soft sensor."

How does the system handle multivariable inputs? There are several alternatives to handle multivariable inputs and adapt a fuzzy control system:

- incorporation of new information into premise statements using AND and OR operators.
- dynamic adaptation of the central FAM rule set (CF factors or link weights).
- dynamic adjustment of the membership functions of the I/O fuzzy sets.
- dynamic switching of the inferencing or defuzzification methods.
- dynamic switching of separate FAM modules for new inputs.

The first method is useful when the knowledge is well understood. Most FAM modules contain at least two input variables (although often one of these is "change in the other variable," i.e., a time-series analysis).

Allowing fuzzy set definitions to change dynamically based on an analysis of conservative or risk-taking contexts can be a fast and efficient way to implement multivariable control (40). A synergy is observed when both input and output sets are allowed to change simultaneously in comparison with results obtained when each are allowed to change on their own. System stability also improves under simultaneous dynamic changing of both input and output fuzzy sets.

Smith (36) has pioneered the adaptable approach to inferencing and defuzzification, listing up to 80 separate methods that can be switched to during defuzzification. His work indicates that about seven major methods are sufficient and that an external set of performance rules can establish the best method to use under different circumstances that generally relate to the position of the process state on the I/O space-state map.

THE FUTURE OF FUZZY CONTROL

The future of fuzzy control is bright. The zenith of the field is still before us. The twenty-first century is likely to see a major proliferation of fuzzy control systems because of the ease of implementation and the confidence that comes from successful applications. Process control as a separate field is often considered of secondary importance during commissioning of new plants, since it is often difficult to build workable control solutions a priori. Fuzzy control systems, on the other hand, can be constructed based on our understanding of the principles of plant operation. This will lead to increased utilization of control in general, and provide better plant start-up performance.

Studies into methods to characterize system stability will result in ways to verify a system before implementation. The marriage of fuzzy control with artificial neural networks will provide systems that can adapt or "learn" in real time, and also explain their actions to humans, if necessary. Genetic algorithms will play an important role in yielding extremely rapid solutions to adaptable systems. "Intelligent methods" will provide widespread solutions to many real-world problems, with fuzzy logic-based control at the center of the technology.

BIBLIOGRAPHY

1. J. Lukasiewicz, Philosophical remarks on many-valued systems of propositional logic, in L. Borkowski (ed.), *Selected Works,* Studies in Logic and the Foundation of Mathematics, Amsterdam: North Holland, 1970: pp. 153–179.
2. N. Resher, *Many-Valued Logic,* New York: McGraw-Hill, 1969.
3. M. Black, Vagueness: An exercise in logical analysis, *Philosophy of Science,* **4**: 427–455, 1937.
4. L. Zadeh, Fuzzy sets, *Information and Control,* **8**: 338–353, 1965.

5. B. Kosco, *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence,* Englewood Cliffs, NJ: Prentice-Hall, 1992.
6. L. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, *IEEE Trans. Syst., Man Cybern.,* **SMC-3**: 28–44, 1973.
7. L. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. Syst., Man and Cybern.,* **2**: 28–44, 1973.
8. D. Dubois, H. Prade, and R. R. Yager (eds.), *Readings in Fuzzy Sets for Intelligent Systems,* San Mateo, CA: Morgan Kaufmann, Inc., 1993, p. 916.
9. E. H. Mamdani and S. Assilian, An experiment in linguistic synthesis with a fuzzy logic controller, *Int. J. Man-Mach. Studies,* **7**: 1–13, 1975.
10. E. H. Mamdani, Application of fuzzy algorithm for control of a simple dynamic plant, *Proc. IEEE* **121**(12): 1585–1588, 1974.
11. L. P. Holmblad and J. J. Oostergaard, Control of a cement kiln by fuzzy logic, In M. M. Gupta and E. Sanchez (eds.), *Fuzzy Information and Decision Processes,* Amsterdam: North-Holland, 1982, pp. 389–399.
12. D. Crevier, *AI: The Tumultuous History of the Search for Artificial Intelligence,* New York: Basic Books–Harper Collins, 1993, p. 386.
13. B. Bouchon, R. R. Yager, and L. A. Zadeh, *Fuzzy Logic and Soft Computing,* London: World Scientific, 1995, p. 470.
14. J. C. Bezdek, What is computational intelligence?, *Computational Intelligence—Imitating Life,* Proc. 1st World Cong. Computat. Intell.—Pt. 1, Orlando, FL, IEEE, 1994, pp. 1–12.
15. B. Kosco, *Fuzzy Engineering.* Upper Saddle River, NJ: Prentice-Hall, 1997.
16. T. Akbarzadeh et al., Genetic algorithms in learning fuzzy hierarchical control of distributed parameter systems, *Proc. IEEE Int. Conf. Syst., Man Cybern.,* **5**: 1995, 4027–4032.
17. S. Yasunobu, S. Miyamoto, and H. Ihara, Fuzzy control for automatic train operation system, *Proc. 4th IFAC/IFIP/IFORS Int. Conf. Control Trans. Syst.,* Baden-Baden, Germany, 1983, pp. 33–39.
18. M. Mancuso et al., Fuzzy logic based image processing in IQTV environment, *Dig. Tech. Papers IEEE Int. Conf. Consumer Electron.,* 1995, pp. 138–139.
19. R. Garcia-Rosa, P. D. Fernandez-Zuliani, and T. de Pedro, Robot fingers to tune TV amplifiers using fuzzy logic, *Fuzzy Sets Syst.,* **70**(2/3): 147–153, 1995.
20. H. Wang, Q. Lu, and P. Wu, Research on hydraulic energy storage transmission system for city bus, *Bus Design: Drivetrains, Electron. Safety,* SAE Special Pub., **1199**: 81–86, 1996.
21. I. Sakai et al., Shift scheduling method of automatic transmission vehicles with application of fuzzy logic, *Proc. Soc. Automotive Eng.,* 1: 1990, 343–347
22. R. Holve, P. Protzel, and K. Naab, Generating fuzzy rules for the acceleration control of an adaptive cruise control system, *New Frontiers in Fuzzy Logic and Soft Computing,* Biennial Conf. North Amer. Fuzzy Inf. Process. Soc., 1996, pp. 451–455.
23. S. Boverie et al., Contribution of fuzzy logic control to the improvement of modern car performances, *Control Eng. Practice,* **1**(2): 291–297, 1993.
24. R. K. Jurgen, Technology 1991: Consumer electronics, *IEEE Spectrum,* **28**(1): 65–68, 1991.
25. B. Krause, C. von Altrock, and M. Pozybill, Intelligent highway by fuzzy logic: Congestion detection and traffic control on multilane roads with variable road signs, *IEEE Int. Conf. Fuzzy Syst.,* **3**: 1996, 1832–1837.
26. C. B. Kim et al., Design and implementation of FEGCS: Fuzzy elevator group control system, *New Frontiers in Fuzzy Logic and Soft Computing,* Biennial Conf. North Amer. Fuzzy Inf. Process. Soc., 1996, pp. 109–113.
27. T. Tobita et al., An elevator characterized group supervisory control system, *IECON Proc.* Industrial Electronics Conference, **3**: 1991, 1972–1976.
28. G. Jaeger and K. H. Walen, Cement works automation—Current situation and range of applications, Pt. 2, *Zement Kalk Gips Int.,* **49**(2): 11 pp., 1996.
29. P. C. Bonilla, Technological changes at Cementos Norte Pacasmayo S.A., *World-Cement,* **26**(5): 4 pp., 1995.
30. A. S. Heger, N. K. Alang Rashid, and M. Jamshidi, Application of fuzzy logic in nuclear reactor control: I. An assessment of state-of-the-art, *Nucl.-Safety,* **36**(1): 109–121, 1995.
31. C. A. Harris and J. A. Meech, Fuzzy logic: A potential control technique for mineral processes, *CIM Bulletin,* **80**(905): 51–59, 1987.
32. R. Santos, J. A. Meech, and L. Ramos, Thickener operations at Carajas using a fuzzy logic controller, *Proc. 6th Int. Fuzzy Syst. Assoc. World Cong.,* IFSA-95, São Paulo, Brazil, 1995, pp. 493–496.
33. V. Rakocevic et al., Computational intelligence in a real-time fuzzy system to monitor and control continuous casting of steel billets, IFSA-95, *Proc. Int. Fuzzy Syst. Assoc. World Cong.,* São Paulo, Brazil, 1995, pp. 485–488.
34. K. Jamshidi and V. Subramanyam, Self organising fuzzy controller for CSI FED induction motor, *Proc. Int. Conf. Power Electron. Drive Systems,* **2**: 744–748, 1995.
35. A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic Theory and Applications,* New York: Van Nostrand Reinhold, 1991.
36. M. H. Smith, Parallel Dynamic Switching of Reasoning Methods in a Fuzzy System, *Proc. 2nd IEEE Int. Conf. Fuzzy Syst.* (FUZZ-IEEE'93), 1993, pp. 968–973.
37. M. H. Smith and H. Takagi, Optimization of fuzzy systems by switching reasoning methods dynamically, *Int. Conf. Fuzzy Syst.,* Seoul, Korea, 6 pp., 1993.
38. D. E. Rumelhart and J. L. McClelland, *Parallel Distributed Processing: Explorations in the Microstructure of Cognition: Foundations,* Cambridge, MA: MIT Press.
39. J. S. Albus, A new approach to manipulator control: The cerebellar model articulation controller CMAC, *Trans. ASME, J. Dynamic Syst. Meas. Control,* **97**: 220–227, 1975.
40. J. A. Meech and L. R. Jordan, Development of a self-tuning fuzzy logic controller, *Minerals Eng.,* **6**(2): 119–131, 1992.
41. J. A. Meech and S. Kumar, A HyperManual on expert systems, *Proc. 25th Canadian Mineral Processors Conf.,* Ottawa, 1993, p. 12.
42. D. Barois, A. Bigand, and R. Ikni, Adaptive fuzzy controller under stability supervisor, *Proc. IEEE Int. Conf. Syst., Man Cybern.,* **4**: 1996, 3170–3173.
43. C. Ham, Z. Qu, and J. Kaloust, New robust fuzzy control for a class of nonlinear systems, *Proc. IEEE Int. Conf. Syst., Man Cybern.,* **4**: 1996, 2619–2624.
44. C. Y. Tsai and T. H. S. Li, Design of Lyapunov function based fuzzy logic controller, *IECON Proc.* Industrial Electronics Conf., **1**: 396–401, 1996.
45. X. Wang and T. Matsumoto, New time-domain stability criterion for fuzzy control systems, *IEICE Trans. Fundamentals Electron. Commun. Comput. Sci.,* **E79-A**(10): 1700–1706, 1996.

46. B. Kosko, Stability in feedback additive fuzzy systems, *IEEE Int. Conf. Fuzzy Syst.,* **3**: 1996, 1924–1930.
47. D. Driankov, H. Hellendoorn, and M. Reinfrank, *An Introduction to Fuzzy Control,* Berlin: Springer-Verlag, 1993, p. 316.

Reading List

E. Cox, *The Fuzzy Systems Handbook,* Cambridge, MA: AP Professional, 1994, p. 615.

D. Crevier, *AI: The Tumultuous History of the Search for Artificial Intelligence,* New York: Basic Books–Harper Collins, 1993, p. 386.

C. W. de Silva, *Intelligent Control: Fuzzy Logic Applications,* Boca Raton, FL: CRC Press, 1995, p. 343.

D. Driankov, H. Hellendoorn, and M. Reinfrank, *An Introduction to Fuzzy Control,* Berlin: Springer-Verlag, 1993, p. 316.

D. Dubois, H. Prade, and R. R. Yager (eds.), *Readings in Fuzzy Sets for Intelligent Systems,* San Mateo, CA: Morgan Kaufmann, 1993, p. 916.

P. Hajek, T. Havranek, and R. Jirousek, *Uncertain Information Processing in Expert Systems,* Boca Raton, FL: CRC Press, 1992, p. 285.

A. Kandel, *Fuzzy Expert Systems,* Boca Raton, FL: CRC Press, 1991, p. 316.

G. J. Klir and T. A. Folger, *Fuzzy Sets, Uncertainty, and Information,* Englewood Cliffs, NJ: Prentice-Hall, 1988, p. 355.

B. Kosco, *Fuzzy Engineering,* Upper Saddle River, NJ: Prentice-Hall, 1997, p. 547.

B. Kosco, *Fuzzy Thinking: The New Science of Fuzzy Logic,* New York: Hyperion, 1993, p. 318.

D. McNeill and P. Freiberger, *Fuzzy Logic: The Discovery of a Revolutionary Computer Technology—And How It Is Changing Our World,* New York: Simon & Schuster, 1993, p. 319.

V. B. Rao and H. V. Rao, *C++ Neural Networks and Fuzzy Logic,* New York: MIS Press, 1993, p. 408.

J. Sibigtroth, Creating fuzzy micros, *Embedded Systems Programming,* **4**(12): 20–35, 1991.

G. Slutsker, Why fuzzy logic is good business, *Forbes Magazine,* May 13, 1991, pp. 120–122.

T. Terano, K. Asai, and M. Sugeno, *Applied Fuzzy Systems,* Cambridge, MA: AP Professional, 1994, p. 302.

C. von Altrock, *Fuzzy Logic & NeuroFuzzy Applications Explained,* Englewood Cliffs, NJ: Prentice-Hall PTR, 1995, p. 350.

S. M. Weiss and C. A. Kulikowski, *Computer Systems That Learn,* Classification and Prediction Methods from Statistics, Neural Nets, Machine Learning and Expert Systems, San Mateo, CA: Morgan Kaufmann, 1991.

T. Williams, Fuzzy logic is anything but fuzzy, *Comput. Design,* April, 113–127, 1992.

R. R. Yager and D. P. Filev, *Essentials of Fuzzy Modeling and Control,* New York: Wiley, 1994, p. 388.

L. Zadeh and J. Kacprzyk (eds.), *Fuzzy Logic for the Management of Uncertainty,* New York: Wiley, 1992, p. 676.

L. A. Zadeh, The calculus of fuzzy if/then rules, *AI Expert,* March 23–27, 1992.

H. J. Zimmermann, *Fuzzy Set Theory—and Its Applications,* 3rd ed., Norwell, MA: Kluwer, 1996.

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