DISPATCHING

INTRODUCTION

Electricity is a secondary form of energy in so far as it has to be transformed from one of the naturally-occurring primary sources of energy, such as coal, oil, natural gas, nuclear power or hydro power. Transmission of electricity is mainly a national problem but international transmission of electricity is now becoming a matter of some importance. For example, an international electricity network now links most of the European countries. Electricity has to be generated as it is consumed since it cannot be stored except in limited quantities. The rate at which electricity has to be supplied to the system from the generating stations to meet the requirements of all the consumers simultaneously is known as the system demand. The total amount of generating plant capacity to be provided, therefore, has to match the estimated maximum system demand. Due to the daily routine of the public and the seasonal effects of the weather the system demand varies, normally being high in winter and low in summer and, furthermore, the demand at night is low relative to the daytime demands. For different system demand levels, the task of dispatching is to allocate the output of units to meet the demand of electricity energy in the area served by the system at the lowest possible cost.

Economic dispatch ranks high among the major economy-security functions in power systems operations. Successful operation of power systems requires attention to: (1) safety for personnel and equipment, which have several recognized constraints imposed by the requirements of reliable service and equipment limitations and (2) provision of service to utility customers at the lowest feasible cost.

The problem of providing low-cost electrical energy is influenced by such items as efficiencies of power generating equipment, cost of installation, and fuel costs for thermal-electric plants. The factors involved in the cost of producing electrical energy can be divided into fixed costs and variable costs. Fixed costs include capital investment, interest charges on borrowed money, labor, taxes, and other expenses that continue irrespective of the load on the power system. Persons responsible for the operation of a power system have little control over these costs. Variable costs include those costs which are affected by reactive flows, the combination of hydro and thermal generation to meet daily load requirements, and purchase or sale of power. These costs are materially controlled by power system operators. The savings that can be achieved by appropriate operation of power resources are very significant and may amount to several thousand dollars a day on a large power system.

A power system operates subject to many restrictions; those usually taken into account include:

1. *Capacity restrictions of individual generators*. These constraints are determined mainly by thermal restrictions, boiler capabilities and hydro turbine rating. Also involved is the start-up time of a unit, as well as the rate at which the unit can pick up load after start-up. There is a limit on the amount of real and reactive power which a generator can deliver. These limits are imposed by both the armature winding and the field winding.
2. *Reserve requirements for system security*. This will account for such emergencies as outages on lines, generators, or transformers. See Transformers. The amount of power transferred from the generator bus to the system could be limited by stability considerations. See ***Power system stability***. Excess spinning reserve is required as a margin to account for such forced outages in addition to possible forecast errors. Some regard must also be given to geographic scheduling of reserve in the cases where transmission line outages would create the partial isolation of an area.

Power system economic operation consists of two aspects: active power regulation and reactive power dispatch. Active power regulation is also called *economic dispatch*. For any specified load condition economic dispatch determines the power output of each plant (and each *generating unit* within the plant) which will minimize the overall cost of fuel needed to serve the system load. Thus, the economic dispatch focusses upon coordinating the production costs at all power plants operating in the system. Reactive power dispatch is to control voltages of generator buses, tap settings of the on-load tap changing transformers and voltage compensators to minimize network power loss. Solving these problems is subject to a number of constraints, such as limits on bus voltages, the range of tap settings of transformers, reactive and active power capacity of power resources and transmission lines, and the number of controllable devices.

CLASSIC ECONOMIC DISPATCH

Dispatch with Transmission Losses Neglected

Economic dispatch is a computational process whereby the total generation required is allocated among the generating units available so that the constraints imposed are satisfied and the energy requirements in terms of *J/h* or *$/h* are minimized. Prior to 1930, various methods were in use such as the "base load method" and "best point loading". In these methods, as load increased, power would be supplied by the most efficient plant until the point of maximum efficiency of that plant was reached. Then, for further increase in load the next most efficient plant would start to feed power to the system and a third plant would not be called upon until the point of maximum efficiency of the second plant was reached. Even with transmission losses neglected, these methods fail to minimize the total cost. It was recognized as early as 1930, that the incremental method, later known as the equal incremental method, yielded the most economic results.

To determine the economic distribution of load between the various generating units, the variable operating costs of the unit must be expressed in terms of the power output.

Let F_i denote the input to unit i, (GJ/h), and P_i denote the output of unit i, (MW). Usually assume the output curve of unit i is quadratic, we can write

$$F_i(P_i) = \gamma_i P_i^2 + \beta_i P_i + \alpha_i \tag{1}$$

Table 1 shows the typical values of the coefficients α_i, β_i and γ_i obtained by simple least square estimation for different sized units (1).

The criterion for distribution of the load between any two units is based on whether increasing the load on one unit as the load is decreased on the other unit by the same amount results in an increase or decrease in the total cost. Thus, we are concerned with *incremental fuel* cost, which is determined by the slopes of the input-output curve in dollars per hour. The incremental fuel cost of the unit i in (GJ/MWh) is $\frac{dF_i}{dP_i}$, can be written:

$$\frac{dF_i}{dP_i} = 2\gamma_i P_i + \beta_i \tag{2}$$

Neglecting the transmission losses, the economic dispatch problem can be described as follows:

$$\min \quad F_T = \textstyle\sum_{i \in N_G} F_i(P_i) \tag{3}$$

$$\text{subject to} \quad P_D = \textstyle\sum_{i \in N_G} P_i \tag{4}$$

where P_i are the individual active power generations, N_G is the set of operational thermal units. P_D is the total load demand and F_T is the total fuel cost of generation. The objective is to find the optimal active power generations minimizing the fuel cost Eq. (3) and satisfying Eq. (4).

Such minimization problems can be solved using the *method of Lagrange multipliers* (11, 12). The new cost function L^* is formed by combining the total fuel cost and the equality constraint of Eq. (4) in the following manner:

$$L^* = F_T + \lambda\left(P_D - \sum_{i \in N_G} P_i\right) \tag{5}$$

The augmented cost function L^* is often called the *Lagrangian*, and the parameter λ is called the *Lagrange multiplier*. For finding the minimum cost we require the derivative of L^* with respect to each P_i to equal zero. This leads to

$$\frac{\partial L^*}{\partial P_i} = \frac{\partial}{\partial P_i}\left[F_T + \lambda\left(P_D - \sum_{i \in N_G} P_i\right)\right] = 0 \quad i \in N_G \tag{6}$$

Since P_D is fixed and the fuel cost of any one unit varies only if the power output of that unit is varied, Eq. (6) yields

$$\frac{\partial L^*}{\partial P_i} = \frac{\partial F_i}{\partial P_i} - \lambda = 0 \quad i \in N_G \tag{7}$$

Because F_i depends only on P_i, the partial derivative of F_i can be replaced by full derivative, and Eq. (7) then gives

$$\lambda = \frac{dF_1}{dP_1} = \cdots = \frac{dF_i}{dP_i} \tag{8}$$

for every $i \in N_G$. This equation implies that for optimality, individual units should share the load such that their incremental costs are equal. This is known as *equal incremental method*. When the fuel cost function takes the form of Eq. (1), then λ and optimal generations are:

$$\lambda = [2P_D + \sum_{i \in N_G} \frac{\beta_i}{\gamma_i}] / \sum_{i \in N_G} (\frac{1}{\gamma_i})$$

$$P_i = (\lambda - \beta_i)/(2\gamma_i).$$

Dispatch with Transmission Losses Included

Including the transmission losses in the active power balance equation, the economic dispatch problem can then be described as follows:

$$\min \quad F_T = \textstyle\sum_{i \in N_G} F_i(P_i) \tag{11}$$

$$\text{subject to} \quad P_D + P_L = \textstyle\sum_{i \in N_G} P_i \tag{12}$$

where P_L is the active power loss. The Lagrangian function for the above problem is:

$$L^* = F_T + \lambda(P_D + P_L - \sum_{i \in N_G} P_i). \tag{13}$$

The optimality conditions turn out to be:

$$\frac{\partial L^*}{\partial P_i} = \frac{\partial F_i}{\partial P_i} + \lambda\left(\frac{\partial P_L}{\partial P_i} - 1\right) = 0 \quad i \in N_G \tag{14}$$

and

$$\lambda = L_i \frac{dF_i}{dP_i} \tag{15}$$

where L_i is called the *penalty factor* of plant i and is given by

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \tag{16}$$

The result of Eq. (15) means that minimum fuel cost is obtained when the incremental fuel cost of each unit weighted by its penalty factor is the same for all generating units in the system. The penalty factor L_i depends on $\partial P_L/\partial P_i$, which is a measure of the sensitivity of the transmission-system losses to the changes in P_i alone.

Equation (15) governs the coordination of transmission losses into the problem of economic loading of units in the plants which are geographically dispersed throughout the system. Accordingly, the penalty factors of the different plants need to be determined, which requires that the total transmission losses of the system are expressed as a function of plant loading. This is formulated as follows:

$$P_L = \sum_{i \in N_G} \sum_{j \in N_G} P_i B_{ij} P_j + \sum_{i \in N_G} B_{i0} P_i + B_{00} \tag{17}$$

or in a general vector-matrix formulation

$$P_L = \mathbf{P_G^T B P_G} + \mathbf{P_G^T B_0} + B_{00}. \tag{18}$$

Table 1. Typical cost coefficients

	Fuel			Coal			Gas		
Unit size (MW)	α	β	γ	α	β	γ	α	β	γ
50	52.87	10.47	0.01160	49.92	10.06	0.0103	53.62	10.66	0.01170
200	180.68	9.039	0.00238	173.61	8.67	0.0023	182.02	9.19	0.00235
400	312.35	8.52	0.00150	300.84	8.14	0.0015	316.45	8.61	0.00150
600	483.44	8.65	0.00056	462.28	8.28	0.00053	490.02	8.73	0.00059
800	793.22	7.74	0.00107	751.39	7.48	0.00099	824.4	7.73	0.00117
1200	1194.6	7.72	0.00072	1130.8	7.47	0.00067	1240.32	7.72	0.00078

where B_{ij} are called *loss coefficients* or *B-coefficients*, **B** is an $|N_G| \times |N_G|$ square matrix, which is formed by the loss coefficients B_{ij} and known simply as the **B**-*matrix*, $\mathbf{B_0}$ is a $|N_G| \times 1$ row vector of linear loss coefficients B_{i0} and B_{00} is a constant. Derivation of the *B-coefficients* and **B**-*matrix* can be referred to Refs. (<xref target="W3316-bib-0001 W3316-bib-0003" style="unformatted">1,3</xref>).

Based on the incremental fuel cost in Eq. (2) and power loss formula, Eq. (17), the optimality condition, Eq. (14), becomes:

$$\left(\frac{2\gamma_i}{\lambda} + 2B_{ii}\right) P_i + \sum_{j\in N_G, j\neq i} 2B_{ij}P_j = (1 - B_{i0}) - \frac{\beta_i}{\lambda} \qquad (19)$$

The power balance requirement for the Eq. (12) becomes:

$$\left(\sum_{i\in N_G}\sum_{j\in N_G} P_iB_{ij}P_j + \sum_{i\in N_G} B_{i0}P_i + B_{00}\right) + P_D - \sum_{i\in N_G} P_i = 0 \qquad (20)$$

The economic dispatch strategy is concerned with solving Eq. (19) for those values of power outputs which also satisfy the power loss and load requirement of Eq. (20). There are many different ways to solve Eqs. (19) and (20) for the unknown P_i, $i \in N_G$ and λ. When an initial value of λ is chosen in Eq. (19), the set of resulting equations becomes linear. The values of P_i, $i \in N_G$ can be found using the following iterative procedure:

1. Choose initial values for the system.
2. Compute the transmission loss of Eq. (17) with the initial values of P_i.
3. Compare the quantity $(\sum_{i\in N_G} P_i - P_L)$ with P_D to check the power balance of Eq. (20). If power balance is not achieved within a specified tolerance, update λ by setting

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \qquad (21)$$

One possible formula for the increment $\Delta\lambda^{(k)}$ is

$$\Delta\lambda^{(k)} = \frac{\lambda^{(k)} - \lambda^{(k-1)}}{\sum_{i\in N_G} P_i^{(k)} - \sum_{i\in N_G} P_i^{(k-1)}}\left[P_D + P_L^{(k)} - \sum_{i\in N_G} P_i^{(k)}\right] \qquad (22)$$

where k is the iteration step of computation.

4. Return to step 2 and continue the calculations of steps 2 and 3 until the final convergence is achieved.

Dispatch with Active-Reactive Power Balance Included

The above two subsections deal with active power regulation. Actually, a detailed electric network is described by both active power and reactive power balance equations. Thus the inclusion of reactive power in the optimization process is desired. This is treated by minimizing the combined objective function

$$\tilde{F} = F_T + F_Q \qquad (23)$$

The function F_T takes the form of Eq. (3) and F_Q is concerned with the reactive capability function, which is given as:

$$F_Q = \mathbf{Q^T K Q} + \mathbf{Q^T}\tilde{\mathbf{K}} + F_{Q0} \qquad (24)$$

where **Q** is the vector of reactive power generation, the constants F_{Q0}, the vector $\tilde{\mathbf{K}}$ and the matrix **K** are assumed to be known a priori (1, 2). The network balance equations are:

$$P_D + P_L - \sum_{i\in N_G} P_i = 0 \qquad (25)$$

$$Q_D + Q_L - \sum_{i\in N_G} Q_i = 0 \qquad (26)$$

The two Lagrangian multipliers λ_p and λ_q are applied to Eq. (25) and Eq. (26), respectively. The augmented objective function is written as:

$$F_A = \tilde{F} + \lambda_p\left(P_D + P_L - \sum_{i\in N_G} P_i\right) + \lambda_q\left(Q_D + Q_L - \sum_{i\in N_G} Q_i\right) = 0 \qquad (27)$$

The decision variables are the active power generations P_i and the reactive power generations Q_i. The optimality conditions are:

$$\frac{\partial F_i}{\partial P_i} + \lambda_p\left(\frac{\partial P_L}{\partial P_i} - 1\right) + \lambda_q\left(\frac{\partial Q_L}{\partial P_i}\right) = 0 \quad i \in N_G \qquad (28)$$

$$\frac{\partial F_Q}{\partial Q_i} + \lambda_p\left(\frac{\partial P_L}{\partial Q_i}\right) + \lambda_q\left(\frac{\partial Q_L}{\partial Q_i} - 1\right) = 0 \quad i \in N_G \qquad (29)$$

along with Eqs.(25) and (26).

It should be pointed out that conventional economic dispatch uses models with far lower dimension and less sophistication. Some relevant variables such as generator voltage magnitudes are not included in the conventional optimization procedure. As a result, the constraints imposed by considering system security are not easily handled in the procedure which involves the power balance or traditional models, whilst the optimal load flow technique can include security constraints in the formulation.

Dispatch with Hydro Plants Included

This section is devoted to a treatment of the economic dispatch of electric power systems which include both thermal and hydro generation. See ***Hydrothermal power systems***. Consider a power system with hydro plants which are assumed to have reservoirs large enough to satisfy the assumption of fixed head. Assume that the fuel cost of the thermal generation is given by

$$F_T = \sum_{i \in N_G} F_i(P_{T_i}) \tag{30}$$

where P_{T_i} and N_G are the output of thermal plant i and the set of thermal plants in the system. It is desired to minimise the total fuel cost during the time interval T_f

$$J = \int_0^{T_f} F_T\, dt. \tag{31}$$

The minimization is carried out under the following two constraints:

1. The total system generation matches the power demand $P_D(t)$ and the transmission loss $P_L(t)$. The output of hydro plant i and the set of all hydro plants in the system are denoted by P_{H_i} and N_H:

$$\sum_{i \in N_G} P_{T_i}(t) + \sum_{i \in N_H} P_{H_i}(t) = P_D(t) + P_L(t) \tag{32}$$

2. The volume of water available for generation at each hydro plant is a prespecified amount b_i:

$$\int_0^{T_f} q_i(t)\, dt = b_i, \quad i \in N_H \tag{33}$$

This kind of problem stated can be solved using the variational calculus principle, which leads to the celebrated coordination equations. The volume of water constraints are included in the cost functional by using the constant multipliers v_j, thus it is to minimize

$$J_1 = \int_0^{T_f} \left(F_T + \sum_{j \in N_H} v_j q_j \right) dt \tag{34}$$

subject to satisfying Eq. (32). The latter is included in J_1 via the use of its multiplier function $\lambda(t)$. Thus the problem is transformed into an unconstrained problem of minimizing

$$J = \int_0^{T_f} \{F_T + \sum_{j \in N_H} v_j q_j + \lambda(t)[P_D(t) + P_L(t) - \sum_{i \in N_G} P_{T_i}(t) - \sum_{i \in N_H} P_{H_i}(t)]\} dt \tag{35}$$

The optimality conditions are obtained using variational calculus as

$$\frac{dF_i}{dP_{T_i}} + \lambda \left[\frac{\partial P_L}{\partial P_{T_i}} - 1 \right] = 0, \quad i \in N_G \tag{36}$$

$$v_j \frac{dq_j}{dP_{H_j}} + \lambda \left[\frac{\partial P_L}{\partial P_{H_j}} - 1 \right] = 0, \quad j \in N_H \tag{37}$$

Both Eqs. (36) and (37) together with the active power balance Eq. (32) and the volume of water constraints are the desired optimality equations.

The water conversion coefficient v can be obtained from the coordination Eqs. (36) and (37) as follows: Define the penalty factors L_{T_i} and L_{H_i} by

$$L_{T_i} = \left[1 - \frac{\partial P_L}{\partial P_{T_i}} \right]^{-1} \tag{38}$$

$$L_{H_i} = \left[1 - \frac{\partial P_L}{\partial P_{H_i}} \right]^{-1} \tag{39}$$

Hence, the coordination Eqs. (36) and (37) become

$$\lambda = L_{T_i} \left(\frac{dF_i}{dP_{T_i}} \right) \tag{40}$$

and

$$\lambda = L_{H_i} v_i \left(\frac{dq_i}{dP_{H_i}} \right) \tag{41}$$

thus

$$v_i = \frac{L_{T_i} \left(\frac{dF_i}{dP_{T_i}} \right)}{L_{H_i} \left(\frac{dq_i}{dP_{H_i}} \right)} \tag{42}$$

The coordination Eqs. (36) and (37) for a hydrothermal system are similar in form to those for an all-thermal system. This shows that it might be advantageous to consider replacing the hydro plants by their thermal equivalent. The thermal equivalent will have an incremental cost given by

$$\frac{dF_i}{dP_{T_i}} = v_i \left(\frac{\partial q_i}{\partial P_{H_i}} \right) \tag{43}$$

Once the equivalent thermal cost is determined, the problem can be solved as an all-thermal one.

REACTIVE POWER DISPATCH

As active-reactive power dispatch is a complex problem, this section discusses only reactive power dispatch to demonstrate a solution of the problem. The objective of reactive power dispatch is often to minimize the active power loss in the transmission network which can be described as follows:

$$f_Q = \sum_{k \in N_E} P_{k\text{Loss}} = \sum_{k \in N_E} g_{ij}(V_i^2 + V_j^2 - 2V_iV_j \cos\theta_{ij}) \tag{44}$$

where N_E denotes the number of network branches, g_{ij} and θ_{ij} are the conductance of the transmission line between buses i and j and the voltage angle difference between buses i and j, respectively. The minimization of the above function is subject to a number of constraints:

$$\Delta P_i = P_i - P_{Di} - V_i \sum_{j \in N_i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0; \quad i \in N_0 \tag{45}$$

$$\Delta Q_i = Q_i - Q_{Di} - V_i \sum_{j \in N_i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0; \quad i \in N_{PQ} \tag{46}$$

and

$$\begin{aligned} V_{i\min} \le V_i \le V_{i\max}, \quad & i \in N_B \\ T_{k\min} \le T_k \le T_{k\max}, \quad & k \in N_T \end{aligned} \tag{47}$$

$$\begin{aligned} Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max}, \quad & i \in \{N_{PV}, n\} \\ |Q_k| \le Q_{k\max}, \quad & k \in N_E \end{aligned} \tag{48}$$

where bus n is selected as the slack bus; G_{ij} is the transfer conductance between buses i and j; N_B, N_0, N_i and N_T are the sets of total buses, total buses excluding slack bus, the buses adjacent to bus i, and the transformer buses, respectively; N_{PQ} and N_{PV} are the numbers of PQ and PV buses; T_k is the tap position of transformer k.

OPTIMAL POWER FLOW

The optimal power flow (OPF) problem was introduced by Carpentier 5–7 in 1962 as a network constrained economic dispatch problem. Since that time, much of the work undertaken has been based on their formulation (8)–(10).

Traditionally, a power flow is characterized by the inputs to the network, and injected real and reactive powers (denoted by P and Q) at the buses. Three types of buses exist: load buses, generator buses, and swing or slack buses. At a load bus, the P and Q are given, and the voltage V and angle θ are unknown; at a generator bus, P and V are given, a range in Q is specified $Q_{\min} \le Q \le Q_{\max}$, and Q and θ are unknown; at a swing or slack bus, both the voltage and angle are specified. The objective of a power flow analysis is to determine the voltages and angles at all buses of the network from which all other quantities can be calculated. An optimal power flow is a power flow in which the fuel costs or some other quantities are minimized, with the ordinary load flow constraints at all buses and additional constraints such as bus voltage limits specified. When the fuel costs are minimized, the optimal power flow actually serves in the capacity of economic dispatch, determines the real and reactive output of all generators and that all of other VAR sources, and sets the autotransformer taps to the requested positions. The term "optimal" implies that the solution is obtained when security, economy and other operational considerations are applied.

The OPF problem aims to achieve an optimal solution of a specific power system objective function, such as fuel cost, by adjusting the power system control variables, while satisfying a set of operational and physical constraints. Carpentier's major contribution is in leading to a general formulation of the economic dispatch problem based on the Kuhn and Tucker theorem of nonlinear programming (11, 12), see Nonlinear Programming, and placing it on a firm mathematical basis.

The Kuhn-Tucker theorem states that a minimum of a given function $F(x)$ of a vector variable x under the inequality and equality constraints

$$G_i(x) \le 0 \quad i = 1, 2, \ldots, \tag{49}$$

$$H_k(x) = 0 \quad k = 1, 2, \ldots, \tag{50}$$

can be achieved, with the assumption of proper convexity for the function, under the condition that dL = 0, where

$$L = F(x) + \sum_i \mathring{\alpha}_i G_i(x) + \sum_k \mathring{\beta}_k H_k(x) \tag{51}$$

The multipliers $\mathring{\alpha}_i$ and $\mathring{\beta}_k$ are the dual variables associated with the inequality and the equality constraints such that

$$\mathring{\alpha}_i \ge 0 \text{ and } \mathring{\beta}_k \text{ arbitrary} \tag{52}$$

and

$$\mathring{\alpha}_i G_i(x) = 0. \tag{53}$$

For optimal power dispatch, it is required to minimize the cost I given by

$$I = F(P) \tag{54}$$

where P is the active power generation vector, subject to the equality and inequality constraints:

1. *Injection Relation*: The net active and reactive powers injected into the system are functions of the voltage magnitudes V_i and the corresponding phase angles θ_i. They are expressed as:

$$I_i(\theta, \mathbf{V}) = \sum_{j \in \mathcal{A}} V_i V_j Y_{ij} \cos(\theta_i - \theta_j + \theta^o_{ij}), \quad i \in N_B \tag{55}$$

$$K_i(\theta, \mathbf{V}) = \sum_{j \in \mathcal{A}} V_i V_j Y_{ij} \sin(\theta_i - \theta_j + \theta^o_{ij}), \quad i \in N_B \tag{56}$$

where the set $\mathcal{A}_i$ includes all buses connected to the ith bus, and Y_{ij} and θ^o_{ij} are the magnitude and phase angle, respectively, which are assigned as the (i, j)th element of the nodal admittance matrix. The injection relations are given by the following equations:

$$H_i = I_i(\theta, \mathbf{V}) - P_i + P_{Di} \; i \in N_B \tag{57}$$

$$R_i = K_i(\theta, \mathbf{V}) - Q_i + Q_{Di} \; i \in N_B \tag{58}$$

where P_i and Q_i are the active and reactive power generations, P_{D_i} and Q_{D_i} are the specific active and reactive power demands, respectively.

2. *Inequality Type Constraints*: They are imposed as the equipment rating limitations:

$$\pi_i = P_i^2 + Q_i^2 - (S_i^M)^2 \le 0 \tag{59}$$

$$\pi'_i = P_i^m - P_i \le 0 \tag{60}$$

$$\psi_i = Q_i - Q_i^M \le 0 \tag{61}$$

$$\psi_i' = Q_i^m - Q_i \leq 0 \tag{62}$$

$$\varepsilon_i = V_i - V_i^M \leq 0 \tag{63}$$

$$\varepsilon_i' = V_i^m - V_i \leq 0 \tag{64}$$

$$\tau_{ij} = \theta_i - \theta_j - T_{ij} \leq 0 \tag{65}$$

where S_i^M is the maximum apparent power of the ith generating node, P_i^m is the minimum active power generation, the maximum and minimum reactive power generation at the ith node are Q_i^M and Q_i^m, respectively, V_i^M and V_i^m are the maximum and minimum allowable voltage levels respectively, at all system nodes with the exception of the slack buses. The last inequality is imposed by the maximum power transfer capability of lines and transformers and is concerned with an approximation of the real power flow between nodes given by

$$P_{ij} = (V_i V_j / X_{ij}) \sin(\theta_i - \theta_j)$$

The dual variables λ_i and μ_i are applied to Eqs. (57) and (58), and the following dual variables are associated with the inequalities (59) – (65)

$$m_i,\ m_i',\ e_i,\ e_i',\ u_i,\ u_i',\ t_{ij} \geq 0$$

According to the Kuhn-Tucker theorem, the augmented Lagrangian function L takes the following form

$$\begin{aligned} L = F(P) &+ \sum_{i \in N_B} \lambda_i H_i + \sum_{i \in N_B} \mu_i R_i + \sum_{i \in N_G} m_i \pi_i + \sum_{i \in N_G} m_i' \pi_i' \\ &+ \sum_{i \in N_G} e_i \psi_i + \sum_{i \in N_G} e_i' \psi_i' + \sum_{i \in N_B} u_i \varepsilon_i + \sum_{i \in N_B} u_i' \varepsilon_i' + \sum_{i,\, j \in N_B} t_{ij} \tau_{ij} \end{aligned} \tag{66}$$

Equation (53) can then be written

$$m_i \pi_i = m_i' \pi_i' = e_i \psi_i = e_i' \psi_i' = u_i \varepsilon_i = u_i' \varepsilon_i' = t_{ij} \tau_{ij} = 0 \tag{67}$$

In order for dL to be zero, the partial derivatives with respect to P_i, Q_i, θ_i and V_i are set to zero:

$$\frac{\partial L}{\partial P_i} = \frac{\partial F}{\partial P_i} = -\lambda_i + 2m_i P_i - m_i' = 0. \tag{68}$$

that is

$$\lambda_i = \frac{\partial F}{\partial P_i} = +2m_i P_i - m_i' = 0; \tag{69}$$

and

$$\frac{\partial L}{\partial Q_i} = -\mu_i + 2m_i Q_i + e_i - e_i' = 0, \tag{70}$$

that is

$$\mu_i = 2m_i Q_i + e_i - e_i'; \tag{71}$$

Equations (70) and (73) are applicable to the generating nodes.

$$\frac{\partial L}{\partial \theta_i} = \sum_j \lambda_j \frac{\partial I_j}{\partial \theta_i} + \sum_j \mu_j \frac{\partial K_j}{\partial \theta_i} \sum_{j \neq i} (t_{ji} - t_{ij}) = 0 \tag{72}$$

that is

$$\lambda_i \frac{\partial I_i}{\partial \theta_i} + \sum_{j \neq i} \lambda_j \frac{\partial I_j}{\partial \theta_i} + \mu_i \frac{\partial K_i}{\partial \theta_i} + \sum_{j \neq i} \mu_j \frac{\partial K_j}{\partial \theta_i} + \sum_j (t_{ji} - t_{ij}) = 0; \tag{73}$$

and

$$\frac{\partial L}{\partial V_i} = \sum_j \lambda_j \frac{\partial I_j}{\partial V_i} + \sum_j \mu_j \frac{\partial K_j}{\partial V_i} + u_i - u_i' = 0 \tag{74}$$

that is

$$\lambda_i \frac{\partial I_i}{\partial V_i} + \sum_{j \neq i} \lambda_j \frac{\partial I_j}{\partial V_i} + \mu_i \frac{\partial K_i}{\partial V_i} \sum_{j \neq i} \mu_j \frac{\partial K_j}{\partial V_i} + u_i - u_i' = 0 \tag{75}$$

Equations (74)–(77) are applicable to every node or bus of the system.

In addition, the equality constraints Eqs. (57) and (58), the inequality constraints Eqs. (59) and (65) and the exclusion Eq. (69) are to be all satisfied. The solution can be obtained by employing numerical techniques, such as the Gauss-Seidel algorithm and Newton-Raphson algorithm (4). However, ensuring the convergence behavior of the algorithms has proven to be much more difficult. Many researchers have tried to develop reliable and efficient algorithms to solve the above problem. One of most important is the work of Dommel and Tinney (8), who simplified the solution procedure by dividing the variables into an unknown vector 'x' which consist of V and θ on (P, Q) buses, and θ on (P, V) buses; fixing parameters P, Q on (P, Q) buses and θ on the slack bus, denoted by parameter "p'; controlling voltage magnitudes on generator buses, generator real power P, and transformer tap ratios denoted by 'u'. Recently, Lin, Chen and Huang have developed a direct Newton-Raphson algorithm for real-time economic dispatch, eliminating the penalty factor calculation. The method results in a very fast solution and maintains high accuracy (13). In defining thermal cost models in common use, the assumption of monotonically increasing cost curves is employed. This leads to polynomial cost models which serve as the basis of most dispatch algorithms. However, this assumption is not valid everywhere because of the throttling losses near valve points. These losses introduce negative slopes into the incremental cost curves. The study of economic dispatch considering valve characteristics may be referred to references (9, 37). The problem of network security has already been included in an approximate form by Carpentier using a constraint based on the bus angles. But over the years additional security constraints have been applied which include the inequality constraints of line flow, current, or complex power, under normal conditions and contingency conditions, generator losses, regulating margins and voltage ranges during contingencies (14, 18).

To solve the OPF problem, a number of conventional optimization techniques have been applied. They include nonlinear programming (NLP) (20) (69), quadratic programming (QP) (21) (22), linear programming (LP) (23) (24), and interior point methods (25–27). All these techniques rely on convexity to find the global minimum. But due to the non-differential, nonlinearity and non-convex nature of the OPF problem, the methods based on these assumptions do not guarantee to find the global optimum. These traditional techniques also suffer from bad starting points and frequently converge to local minimum or even diverge.

GRADIENT-BASED METHODS

In conventional optimization methods, the gradients of the function are used to search for the optimum. The gradient of the power loss in the network with respect to voltage changes of *PV* buses can be given as follows:

$$\nabla f_Q|_{V_{PV}} = (\frac{\partial V_{PQ}}{\partial V_{PV}})^T \frac{\partial f_Q}{\partial V_{PQ}} + (\frac{\partial \theta}{\partial V_{PV}})^T \frac{\partial f_Q}{\partial \theta} + \frac{\partial f_Q}{\partial V_{PV}} \tag{76}$$

From Eqs. (45) and (46), the following equations hold:

$$\begin{aligned} \frac{\partial \Delta P}{\partial \theta} \Delta\theta + \frac{\partial \Delta P}{\partial V_{PQ}} \Delta V_{PQ} + \frac{\partial \Delta P}{\partial V_{PV}} \Delta V_{PV} = 0 \\ \frac{\partial \Delta Q}{\partial \theta} \Delta\theta + \frac{\partial \Delta Q}{\partial V_{PQ}} \Delta V_{PQ} + \frac{\partial \Delta Q}{\partial V_{PV}} \Delta V_{PV} = 0 \end{aligned} \tag{77}$$

Rearranging the above equations gives:

$$\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V_{PQ}} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V_{PQ}} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V_{PQ} \end{bmatrix} = -\begin{bmatrix} \frac{\partial \Delta P}{\partial V_{PV}} \\ \frac{\partial \Delta Q}{\partial V_{PV}} \end{bmatrix} \Delta V_{PV} \tag{78}$$

The deviations of angles and voltages at *PV* buses can be obtained from the above equation, and in most power system computations, the influences of voltage changes on active power and angle changes on reactive power are usually omitted, which gives:

$$\begin{aligned} \begin{bmatrix} \Delta\theta \\ \Delta V_{PQ} \end{bmatrix} &= -\begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V_{PQ}} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial V_{PQ}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial \Delta P}{\partial V_{PV}} \\ \frac{\partial \Delta Q}{\partial V_{PV}} \end{bmatrix} \Delta V_{PV} \\ &\simeq -\begin{bmatrix} (\frac{\partial \Delta Q}{\partial \theta})^{-1} & \frac{\partial \Delta P}{\partial V_{PV}} \\ (\frac{\partial \Delta Q}{\partial V_{PQ}})^{-1} & \frac{\partial \Delta Q}{\partial V_{PV}} \end{bmatrix} \Delta V_{PV} \end{aligned} \tag{79}$$

Substituting the above equations into Eq. (78), a computation formula of the gradient is obtained as follows:

$$\begin{aligned} \nabla f_Q|_{V_{PV}} = \; & \frac{\partial f_Q}{\partial V_{PV}} - (\frac{\partial \Delta P}{\partial V_{PV}})^T (\frac{\partial \Delta P^T}{\partial \theta})^{-1} \frac{\partial f_Q}{\partial \theta} \\ & - (\frac{\partial \Delta Q}{\partial V_{PV}})^T (\frac{\partial \Delta Q^T}{\partial V_{PQ}})^{-1} \frac{\partial f_Q}{\partial V_{PQ}} \end{aligned} \tag{80}$$

For simplicity and based on the same assumption that voltage and angle changes do not affect active power and reactive power respectively, in conventional optimal reactive power dispatch:

$$(\frac{\partial VP}{\partial V_{PV}})^T (\frac{\partial \Delta P^T}{\partial \theta})^{-1} \frac{\partial f_Q}{\partial \theta} \tag{81}$$

is usually omitted, which leads to:

$$\nabla f_Q|_{V_{PV}} = \frac{\partial f_Q}{\partial V_{PV}} - (\frac{\partial \Delta Q}{\partial V_{PV}})^T (\frac{\partial \Delta Q^T}{\partial V_{PQ}})^{-1} \frac{\partial f_Q}{\partial V_{PQ}} \tag{82}$$

This is used to update the controlled voltages in the following way:

$$\Delta V_{PV} = -H \nabla f_Q \tag{83}$$

where H is the Hessian matrix. In the optimal reactive power dispatch problem, updating the gradients of the objective function, Eq. (84), at each iteration involves a large amount of computation. This is not numerically reliable due to the differentiating functions and the need to invert matrices in a high dimension space.

In practical computation, minimization of the active power loss in the transmission network is equivalent to minimization of the injected active power at the slack-bus. This is because:

$$\begin{aligned} \sum_{k \in N_E} P_{k\text{Loss}} &= \sum_{i \in N_G} P_{Gi} - \sum_{i \in N_D} P_{Di} \\ &= P_s(V, \theta) - P_{\text{const}} \end{aligned} \tag{84}$$

where P_{const} includes all unchanged load and generated active power in the power system. Thus the optimization problem, Eq. (44), can be redefined as follows:

$$\min \; f_Q \Leftrightarrow \min P_s(V, \theta), \tag{85}$$

where P_s is the injected active power at slack bus. On the other hand, consideration of the constraints applied to the voltages, reactive power, and control variables in the network complicates the optimization procedure. In most nonlinear optimization problems, the constraints are considered by generalizing the objective function using penalty terms. In the reactive power dispatch problem, the *PV*- and $V\theta$-bus (slack-bus) voltages, V_{PV} and V_s, are control variables which are self-constrained. Voltages of *PQ*-buses, V_{PQ}, and injected reactive power of *PV*-buses, Q_G, are constrained by adding them as penalty terms to the objective function, Eq. (87). The above equation is generalized as follows:

$$\begin{aligned} f_Q = \; & P_s(V, \theta) + \sum_{i \in N_{V\text{lim}}} \lambda_i (V_i - V_{i\text{lim}})^2 \\ & + \sum_{i \in N_{Q\text{lim}}} \lambda_i (Q_{Gi} - Q_{Gi\text{lim}})^2 \end{aligned} \tag{86}$$

where

$$\begin{aligned} V_{i\text{lim}} &= \begin{cases} V_{i\max}; & V_i > V_{i\max} \\ V_{i\min}; & V_i < V_{i\min} \end{cases} \\ Q_{Gi\text{lim}} &= \begin{cases} Q_{Gi\max}; & Q_{Gi} > Q_{Gi\max} \\ Q_{Gi\min}; & Q_{Gi} < Q_{Gi\min}. \end{cases} \end{aligned} \tag{87}$$

It can be seen that the generalized f_Q is a nonlinear and non-smooth function. The conventional gradient-based optimization algorithms (11, 12) have been widely used to solve this problem for decades.

EVOLUTIONARY COMPUTATION

Both active power regulation and reactive power dispatch are global optimization problems which may have several local minima, and conventional optimization methods easily lead to a local optimum. On the other hand, in conventional optimization algorithms, many mathematical assumptions, such as analytic and differential properties of the objective functions and unique minima existing in problem domains, have to be given to simplify the problem. Without such assumptions, it is very difficult to calculate the gradient variables in conventional methods. Furthermore, in practical power system operation, the data acquired by the SCADA (Supervisory Control And Data Acquisition) system are contaminated by noise. Such noisy data may cause difficulties in computation of the gradients. Consequently, the optimization cannot be carried out in many situations.

In the last decade, many new search methods have been developed, such as neural networks, see (***Neural net***

architecture), simulated annealing, see Simulated Annealing, genetic algorithms, see (***Genetic Algorithms***), and evolutionary programming. These methods have been widely applied to power system dispatch problems. Interested readers may refer to references (31, 71). In the following, we introduce evolutionary computation techniques.

Evolutionary Algorithms (EAs) are inspired by natural phenomena and derived from simulating Darwinian evolutionary theory. It includes three broadly similar avenues: Genetic Algorithms (GAs), Evolution Strategies (ES), and Evolutionary Programming (EP). All these algorithms operate on a population of candidate solutions, subject these solutions to alterations, and employ a selection criterion to determine which solutions to maintain for future generations. Their characteristics make them very different from traditional optimization algorithms. The key point of evolutionary computation is that successive populations of the feasible solutions are generated in a stochastic manner following laws similar to that of natural selection. Multiple stochastic solution trajectories proceed simultaneously, allowing various interactions among them toward one or more regions of the search space, whilst nonlinear programming techniques normally follow just one deterministic trajectory, perhaps repeated many times until a satisfactory solution is reached.

In the past few decades, EAs have been applied to global optimization problems become attractive because they have better global search abilities over conventional optimization algorithms.

Genetic Algorithms

GAs are search algorithms for finding the global optimum solution for an optimization problem, in which the search is conducted using information of a population of candidate solutions so that the chance of the search being trapped in a local optimum solution can be significantly reduced. A GA carries out three basic operations: crossover, mutation and selection. An initial population of strings is randomly selected in the domain of control variables. The strings are randomly selected with their probabilities proportional to the ratio of the fitness of each string to the total fitness of the population, to form a mating pool of strings for the generation of offsprings. The strings with larger fitness get higher chances to be selected (for maximization problems). Each pair of strings in the mating pool undergoes crossover and mutation, with given crossover and mutation probabilities, to reproduce two offspring strings in the next generation. After all pairs of mates have finished crossover and mutation, a new population has been reproduced. The fitness of each new string will be computed and the new population will become the parent population and be ready to reproduce. The standard procedure of a GA is sketched as follows:

Choose an initial population
determine the fitness of each individual
perform selection
repeat
    perform crossover
    perform mutation
    determine the fitness of each individual
    perform selection
until some stopping criterion applies.

Coding structure. The coding for a solution, termed a *chromosome* in GA literature, is usually described as a string of symbols from (0,1). These components of the chromosome are then labeled as *genes*. The number of bits that must be used to describe the parameters is problem dependent. Let each solution in the population of m such solutions, x_i, $i = 1, 2, \ldots, m$, be a string of symbols (0,1) of length l. Typically, the initial population of m solutions is selected completely at random, with each bit of each solution having a 50 percent chance of taking the value 0.

Selection. There are two main selection operators in GAs: elitist selection and proportional selection. The elitist selection is that the best individual (with highest fitness) survives with probability one. It is provable that GAs using elitist selection or modified elitist selection probabilistically converge to the global optimum. But the convergence rate may be slow. When using so-called proportional selection, the population of the next generation is determined by n independent random experiments, the probability that individual b_i is selected from the tuple $(b_1, b_2, \ldots, b_n)$ to be a member of next generation at each experiment is given by

$$P\{b_i \text{ is selected}\} = \frac{f(b_i)}{\sum_{j=1}^{n} f(b_j)} > 0$$

It has been shown that GAs using *proportional selection* do not necessarily converge to the global optimum and may be trapped in the local optimum. However, many numerical experimental simulations have shown their convergence seems faster than with *elitist selection*.

Crossover. Crossover is an important random operator in GAs and the function of the crossover operator is to generate new or 'child' chromosomes from two 'parent' chromosomes by combining the information extracted from the parents. By this method, for a chromosome of a length l, a random number c between 1 and l is first generated. The first child chromosome is formed by appending the last $l - c$ elements of the first parent chromosome to the first c elements of the second parent chromosome. The second child chromosome is formed by appending the last $l - c$ elements of the second parent chromosome to the first c elements of the first parent chromosome. Typically, the probability for crossover ranges from 0.6 to 0.95.

Mutation. Mutation is a means to avoid the loss of important information at a particular position in the string of a chromosome. It operates independently on each individual by probabilistically perturbing each bit string. A usual way to mutate is to generate a random number v between 1 and l and then make a random change in the vth element of the string with probability $p_m \in (0,1)$. Typically, the probability for bit mutation ranges from 0.001 to 0.01.

The probability that string b_i resembles string b'_i after mutation can be described as:

$$P\{b_i \to b'_i\} = p_m^{H(b_i,b'_i)}(1-p_m)^{l-H(b_i,b'_i)} > 0$$

where $H(b_i, b'_i)$ denotes the *Hamming* distance between the strings b_i and b'_i.

GAs have the following features: (1) they work with a coding of the parameter sets instead of the parameters themselves; (2) they search with a population of points, not a single point; (3) they use the objective function information directly, rather than the derivatives or other auxiliary knowledge, to find a minima; (4) they process information using probabilistic transition rules, rather than deterministic rules. These features make GAs robust to computation, readily implemented with parallel processing and powerful for global optimization. Without deriving the gradients, GAs are more suitable to use in the optimization problems of large-scale systems and have been widely applied to both economic dispatch problems and reactive power dispatch problems; interested readers may refer to references (34)–(38) and (39)–(40), respectively.

Evolutionary Programming

Evolutionary programming is another efficient global optimization technique. The EP is carried out mainly with three operations: mutation, competition and reproduction. These can be described as follows.

The initial population is determined by selecting p_i, where p_i is an individual, $i = 1, 2,\ldots, k$, from the set of $U(a,b)^n$, where k is the population size and $U(a,b)^n$ denotes a uniform distribution ranging over $[a, b]$ in n dimensions. Each $p_i, i = 1, 2,\ldots, k$, is assigned a fitness score $f_i . f_i = F(p_i)$, $F: p_i \to \mathbf{R}$. F can be as complex as required and usually regarded as an objective function. Statistical methods are then used to get the maximum fitness, minimum fitness, average fitness and sum of fitnesses of the population. The mutation operation is carried out based on the statistics to double the population size from k to $2k$. Each $p_i, i = 1, 2,\ldots, k$, is mutated and assigned to p_{i+k} in the following way:

$$p_{i+k,\,j} := p_{i,\,j} + \mathcal{N}(0, \beta_j \frac{f_i}{f_\Sigma}),\ \forall\ j; = 1, \ldots, n \tag{88}$$

where $p_{i,j}$ denotes the j_{th} element of the i_{th} individual; $\mathcal{N}(\mu, \sigma^2)$ represents a Gaussian random variable with mean μ and variance σ^2; f_Σ is the sum of fitnesses; β_j is a constant of proportionality to scale $\frac{f_i}{f_\Sigma}$ and $0 < \beta_j \le 1$. Each p_{i+k}, $i = 1, 2,\ldots, k$, is again assigned a fitness score f_{i+k}. Based on the mutated population with the size of $2k$, a competition is conducted to reproduce offsprings. For each p_i, $i = 1, 2,\ldots, 2k$, a value w_i is assigned to weight the individual according to the following equation:

$$w_i = \sum_{t=1}^{s} w_t \tag{89}$$

and

$$w_t = \begin{cases} 1, & \text{if } u_1 < \frac{f_r}{f_r + f_i} \\ 0, & \text{otherwise} \end{cases}$$

where s is the number of competitors, r=int($2ku_2$ + 1), int(x) denotes the greatest integer less than x, and u_1, u_2 $\sim U(0,1)$. The individuals p_i, $i = 1, 2,\ldots, 2k$, are ranked in descending order of their corresponding value w_i. The first k individuals are transcribed along with their corresponding fitnesses f_i to be the basis of the next generation. The process will be carried out repeatedly until the given conditions are satisfied.

EP has been shown as an efficient global algorithm in solving both economic dispatch for units with non-smooth fuel cost functions (42) and optimal reactive power dispatch (71). In the next subsection, an example is given to show the potential for application of EP to optimal reactive power dispatch and voltage control of power systems.

Particle Swarm Optimizer with Passive Congregation

Particle Swarm Optimizer (PSO) is a newly proposed population based stochastic optimization algorithm which was inspired by the social behaviors of animals such as fish schooling and bird flocking (44). Compared with other stochastic optimization methods, PSO has comparable or even superior search performance for some hard optimization problems with faster convergence rates (45). It requires only few parameters to be tuned which makes it attractive from an implementation view point. However, recent studies of PSO indicated that although the PSO outperforms other evolutionary algorithms in the early iterations, it does not improve the quality of the solutions as the number of generations is increased. In (64), passive congregation, a concept from biology, was introduced to the standard PSO to improve its search performance. Experimental results show that this novel hybrid PSO outperforms standard PSO on multi-model and high dimensional optimization problems. In this paper, we present a PSO with passive congregation (PSOPC) for the solution of OPF.

The PSO is a population-based optimization algorithm. Its population is called *swarm* and each individual is called a *particle*. For the i_{th} particle at iteration k, it has the following two attributes:

1. A current position in an N-dimensional search space $X_i^k = (x_{i,1}^k, \ldots, x_{i,n}^k, \ldots, x_{i,N}^k)$, where $x_{i,n}^k \in [l_n, u_n]$, $1 \le n \le N$, l_n and u_n is the lower and upper bound for the n_{th} dimension, respectively.
2. A current velocity V_i^k, $V_i^k = (v_{1,i}^k, \ldots, v_{n,i}^k, \ldots, v_{N,i}^k)$, which is clamped to a maximum velocity $V_{\max}^k = (v_{\max,1}^k, \ldots, v_{\max,n}^k, \ldots, v_{\max,N}^k)$.

At each iteration, the swarm is updated by the following equations:

$$V_i^{k+1} = \omega V_i^k + c_1 r_1(p_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k) \tag{90}$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{91}$$

where P_i is the best previous position of the i_{th} particle (also known as *pbest*) and P_g is the global best position among all the particles in the swarm (also known as *gbest*). They are given by the following equations:

$$P_i = \{ \begin{matrix} P_i & : & f(X_i) \ge P_i \\ X_i & : & f(X_i) < P_i \end{matrix} \tag{92}$$

$$\begin{aligned} & P_g \in \{P_0, P_1, \ldots, P_m\} \mid f(P_g) \\ = \; & \min(\, f(P_0),\; f(P_1), \ldots,\; f(P_m)) \end{aligned} \tag{93}$$

where f is the objective function, m is the number of particles, r_1 and r_2 are elements from two uniform random sequence on the interval [0,1]: $r_1 \sim U(0,1)$; $r_2 \sim U(0,1)$ and ω is inertia weight (52) which is typically chosen in the range of [0,1]. A larger inertia weight facilitates the global exploration and a smaller inertia weight tends to facilitate the local exploration to fine-tune the current search area (49). Therefore the inertia weight ω is critical for the PSO's convergence behavior. A suitable value for the inertia weight ω usually provides balance between global and local exploration abilities and consequently results in a better optimum solution. c_1 and c_2 are acceleration constants (53) which also control how far a particle will move in a single iteration. The maximum velocity $V_{\max}$ is set to be half of the length of the search space.

The foundation of the development of PSO is based on the hypothesis: social sharing of information among conspecifics offers an evolutionary advantage (44). The PSO model is based on (44):

1. the autobiographical memory which remembers the best previous position of each individual (*pbest*) in the swarm and
2. the publicized knowledge which is the best solution (*gbest*) currently found by the population.

From biology point of view, the sharing of information among conspecifics is achieved by employing the publicly available information *gbest*. There is no information sharing among individuals except that *gbest* give out the information to the other individuals. Therefore, for the i_{th} particle, the search direction will only be affected by 3 factors: the inertia velocity ωV_i^k, the best previous position *pbest*, and the position of global best particle *gbest*. The population is more likely to lose diversity and confine the search around local minima. From our experimental results, the performance of standard PSO is not sufficiently good enough to solve the OPF problem due to its high-dimensional and multi-model nature.

Biologists have proposed four types of biological mechanisms that allow animals to aggregate into groups: passive aggregation, active aggregation, passive congregation, and social congregation (54). There are different information sharing mechanisms inside these forces. We found that the passive congregation model is suitable to be incorporated in the PSO model to improve the search performance. Passive congregation is an attraction of an individual to the entire group but do not display social behavior. It has been discovered that in spatially well-defined congregations, such as fish schools, individuals may have low fidelity to the group because the congregations may be composed of individuals with little to no genetic relation to each other (55). In these congregations, information may be transferred passively rather than actively (57). Such asocial types of congregations can be referred as passive congregation.

Biologists have discovered that group members in an aggregation can react without direct detection of an incoming signals from the environment, because they can get necessary information from their neighbors (54). Individuals need to monitor both environment and their immediate surroundings such as the bearing and speed of their neighbors (54). Therefore each individual in an aggregation have a multitude of potential information from other group members which may minimize the chance of missed detection and incorrect interpretations (54). Such information transfer can be employed in the model of passive congregation. Inspired by this result, and to keep the model simple and uniform with the PSO, the PSOPC is given as follows:

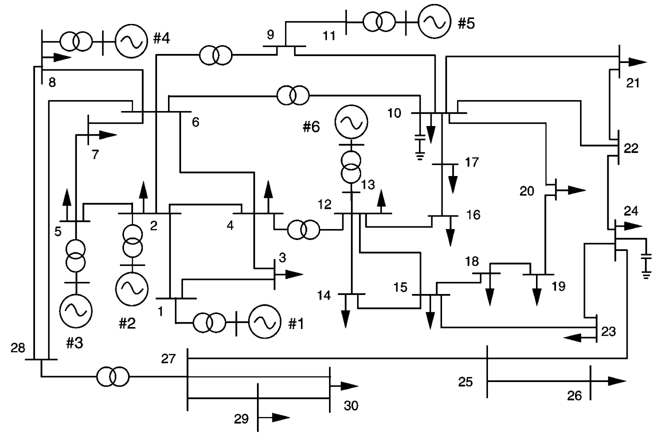

Figure 1. IEEE 30-bus power system

$$V_i^{k+1} = \omega V_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k) + c_3 r_3 (R_i^k - X_i^k) \tag{94}$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{95}$$

where R_i is a particle randomly selected from the swarm, c_3 the passive congregation coefficient and r_3 a uniform random sequence in the range (0,1): $r_3 \sim U(0,1)$.

SIMULATION STUDIES

Optimal Reactive Power Dispatch using Evolutionary Programming

The IEEE 30-bus system is shown in Fig. 1 and the system data is given in the reference (30). Six buses are selected as PV-buses and $V\,\theta$-bus as follows: PV-buses: Bus 2, 5, 8, 11, 13. $V\theta$-bus: Bus 1, The others are PQ-buses.

The network loads are given as follows:

$$P_{\text{Load}} = 2.834 \ p.u. \quad Q_{\text{Load}} = 1.0445 \ p.u. \quad \cos\varphi_{\text{Load}} = 0.938$$

Two capacitive loads at two buses are included in the Q_{Load}, which are voltage-dependent loads. Voltages of PV-buses and $V\,\theta$-bus are set to be 1.0 p.u. Based on the above initial conditions, the voltages outside the limits on three PQ-buses are calculated and given as follows:

$$V_{26} = 0.932; \quad V_{29} = 0.940; \quad V_{30} = 0.928;$$

The generated power and network power loss are obtained as follows:

$$\begin{array}{lll} P_{G\Sigma} = 2.893857 \ p.u. & Q_{G\Sigma} = 0.980199 \ p.u. & P_{\text{Loss}} = 0.059879 \ p.u. \\ \cos\varphi_{G\Sigma} = 0.947 & Q_{\text{Loss}} = -0.064327 \ p.u. & \end{array}$$

(1) Optimal solution obtained by EP

The EP method has been evaluated on the IEEE 30-bus system. The control variables of the transmission network are arranged as elements of an individual in populations during evolutionary search. The individuals are expressed as follows:

$$p_i = \{v_1, v_2, \ldots, v_j, \ldots, v_n\} \quad i = 1, 2, \ldots, k$$

where $v_j, j = 1, 2, \ldots, n$, are the PV-bus and $V\,\theta$-bus voltages. The population size, k, is chosen to be 50. The individuals in the initial population, $p_i^0 = \{\, p_{i,j}^0 | \, j = 1, 2, \ldots, n\}(i = 1, 2, \ldots, k)$, are constructed with random values assigned for their elements, $p_{i,j}^0$. The objective function with the voltage and reactive power penalty, Eq. (88), is used for reactive power dispatch. The constraints of PV-bus and $V\,\theta$-bus voltages are set to be 0.9 and 1.1 p.u. and PQ-bus voltage constraints are set to be 0.95 and 1.05 p.u. The value of the objective function, f_i, is obtained with each individual p_i, which is used for mutation, competition and reproduction according to Eqs. (88)–(93). For generating populations, the number of competitors is chosen to be 20.

After a successful search using the EP, the PV-bus voltages and the $V\,\theta$-bus voltage are obtained as follows:

$$V_1 = 1.070; \quad V_2 = 1.061; \quad V_5 = 1.039$$
$$V_8 = 1.041; \quad V_{11} = 1.072; \quad V_{13} = 1.062$$

Only the voltage of Bus 3 is slightly outside the limits, $V_3 = 1.0502$. It should be mentioned that the voltages of PQ-buses are constrained by the penalty terms in the objective function, Eq. (88).

The generated power and network power loss are obtained as follows:

$$P_{G\Sigma} = 2.884145 \; p.u. \quad Q_{G\Sigma} = 0.876552 \; p.u. \quad P_{\text{Loss}} = 0.050159 \; p.u.$$
$$Q_{\text{Loss}} = -0.139324 \; p.u. \quad \cos\varphi_{G\Sigma} = 0.955 \quad Q_{\text{Load}} = 1.015874 \; p.u.$$

Power saving is:

$$P_{save} = 0.059879 - 0.050159 = 0.00972 \, p.u.$$

and

$$P_{save}\% = \frac{0.00972}{0.059879} \times 100 = 16.23\%$$

(2) Optimal solution obtained using BFGS method

The nonlinear programming method, BFGS (Broyden, Fletcher, Goldfarb and Shanno) method (12), has also been evaluated on the IEEE 30-bus system. It is a quasi Newton method and does not require the second-order derivatives of the objective function directly and is able to approach the inverse Hessian matrix through iterations. With this method, the control variables are updated in the optimization process as follows:

$$V_{k+1} = V_k - \lambda_k H_k \nabla f_{Qk}$$
$$H_{k+1} = H_k + \frac{s_k s_k^T - H_k y_k s_k^T - s_k y_k^T H_k}{y_k^T s_k} + \frac{y_k^T H_k y_k s_k s_k^T}{(y_k^T s_k)^2}$$

where

$$s_k = V_{k+1} - V_k$$
$$y_k = \nabla f_{Qk+1} - \nabla f_{Qk}.$$

In the above equations, V is the vector including the PV-bus and $V\,\theta$-bus voltages, k indicates the iteration steps and λ is the optimum step length. ∇f_Q is obtained from Eq. (88). The process starts with $H = I$ and ends at $H_k^{-1} = \nabla^2 f_{Qk}$.

After successful optimization using the method, the PV-bus voltages and the $V\,\theta$-bus voltage are obtained as follows:

$$V_1 = 1.044: \quad V_2 = 1.046: \quad V_5 = 1.049$$
$$V_8 = 1.037: \quad V_{11} 1.084: \quad V_{13} = 1.062$$

Only the voltage of Bus 9 is slightly outside the limits, $V_9 = 1.0503$. The generated power and network power loss are obtained as follows:

$$P_{G\Sigma} = 2.888121 \; p.u. \quad Q_{G\Sigma} = 0.901599 \; p.u. \quad P_{\text{Loss}} = 0.054122 \; p.u.$$
$$Q_{\text{Loss}} = -0.114644 \; p.u. \quad \cos\varphi_{G\Sigma} = 0.955 \quad Q_{\text{Load}} = 1.016241 \; p.u.$$

Power saving is:

$$P_{save} = 0.059879 - 0.054122 = 0.005757 \, p.u.$$

and

$$P_{save}\% = \frac{0.005757}{0.059879} \times 100 = 9.61\%$$

As mentioned earlier, the generalized f_Q is a noncontinuous function, and the control variables are limited. The optimization for reactive power dispatch using the BFGS method is carried out with hard constraints, limits on control variables, and soft constraints, penalty on the voltages and reactive power outside the limits. This causes a poor convergence during the optimization procedure. By trial and error, many results have been obtained using the BFGS method. In the BFGS method, the range of optimum step length is chosen to be very small, otherwise, oscillations will occur and the algorithm will diverge. It has been noted that in gradient-based optimization methods, the convergence is sensitive to the network topology, load distribution, system initial conditions, penalty factors, a priori parameters in the algorithm and convergence criteria.

Optimal Power Flow using PSOPC

The PSOPC algorithm has also been tested on the standard IEEE 30-bus test system. The system line and bus data for 30-bus system were adopted from (20). For all problems a population of 50 individuals is used. A time decreasing inertia weight ω which starts from 0.9 and ends at 0.4 was used. The default value of acceleration constants c_1, c_2 are typically set to 2.0. However with a setting of $c_1 = c_2 = 0.5$ better results were obtained. For each problem, 100 independent runs were carried out. The maximum generation was set to 500.

Case 1: Minimization of fuel cost. The objective of this example is to minimize the total fuel cost.

$$F_T = \sum_{i \in N_G} F_i(P_i) \tag{96}$$

where $F_i(P_i)$ is the fuel cost ($/h) of the i_{th} generator:

$$F_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

α_i, β_i and γ_i are the fuel cost coefficients, P_i is the real power output generated by the i_{th} generator.

Table 2. Best values of PSOPC, PSO, Gradient-based approach and EGA for Case 1

	Case 1			
	PSOPC	PSO	Gradient (20)	EGA (19)
Fuel cost ($/h)	**802.0477**	802.41	813.74	802.6087
$\sum$ voltage deviations	**0.8089**	0.8765	1.4602	0.8073
$T_{\max}$	**0.1383**	0.1381	0.1384	0.1394

Table 3. Best values of PSOPC and PSO for Case 2

	Case 2	
	PSOPC	PSO
Fuel cost ($/h)	**804.0650**	804.1426
$\sum$ voltage deviations	**0.0954**	0.1011

Table 4. Best values of PSOPC and PSO for Case 3

	Case 3	
	PSOPC	PSO
Fuel cost ($/h)	**802.0638**	802.1190
$T_{\max}$	**0.1379**	0.1382

This problem was tackled using a gradient-based optimization method (20). The best-known result was obtained by Bakirtzis et al. (19) using an enhanced GA (EGA). The PSO was implemented based on the algorithm presented in (49). The best result of the PSOPC from 100 runs is tabulated in Table 2 in comparison to those obtained from the techniques mentioned above.

Case 2: Voltage profile improvement. This example aims at minimizing fuel cost with a flatter voltage profile. The objective function is modified to minimize the fuel cost while at the same time to improve voltage profile by minimizing the load bus voltage deviations from 1.0 per unit. The objective function can be expressed as:

$$F_T = \sum_{i \in N_G} F_i(P_i) + \omega \sum_{i \in N_{PQ}} |V_i - 1.0| \tag{97}$$

where ω is the weighting factor.

The best result of the PSOPC from 100 runs is tabulated in Table 3 in comparison to the result obtained from the standard PSO.

Case 3: Voltage stability enhancement. This example minimizes fuel cost and enhances voltage stability profile through out the whole network. T is the stability indicators at every bus of the system and $T_{\max}$ is the maximum value of T-index defined as (28):

$$T_{\max} = \max\{T_k | k = 1, \ldots, N_L\} \tag{98}$$

And T can be calculated from the following equation:

$$T_j = |1 + \frac{V_{0j}}{V_j}| = |\frac{S_j^+}{Y_{jj}^{+*} \cdot V_j^2}| \tag{99}$$

where Y_{jj}^+ is the transformed admittance, $Y_{jj}^+ = 1/Z_{jj}$; V_j is the consumer node voltage; S_j^+ is the transformed power $S_j^+ = S_j + S_j^{\mathrm{cor}}$; and S_j^{cor} is given by:

$$S_j^{\mathrm{cor}} = [\sum_{i \in \alpha} (\frac{Z_{ji}^*}{Z_{ij}^*}) \cdot (\frac{S_i}{V_i})] \cdot V_j \tag{100}$$

and α_L is the set of consumer nodes.

One way of determining T is:

$$T = \max_{j \in \alpha_L} |1 - \frac{\sum_{i \in \alpha_G} M_{ij} \cdot V_i}{V_j}| \tag{101}$$

where α_L is the set of load buses; α_G is the set of generator buses. V_j is the voltage at load bus j; V_i is the complex voltage at generator bus i; M_{ij} is the element of matrix $[M]$ determined by

$$[M] = -\frac{[Y_{LL}]}{[Y_{LG}]} \tag{102}$$

where $[Y_{LL}]$ and $[Y_{LG}]$ are sub-matrices of the Y-bus matrix.

The objective function can be expressed as:

$$F_T = \sum_{i \in N_G} F_i(P_i) + \omega T_{\max}$$

The best results of the PSOPC and the stand PSO from 100 runs are tabulated in Table 4.

In this study, the PSOPC was applied to tackle OPF problems. By introducing the passive congregation, information can be transferred among individuals which will help individuals to avoid misjudging information and trapping by poor local minima. Numerical experiments were carried out on an IEEE 30-bus for three different fuel cost minimization problems.

CONCLUSION

Power system dispatching consists of two aspects: economic dispatch and reactive power dispatch. The economic dispatch problem is to determine the power output of each unit to minimize the overall cost of fuel needed to meet the system load. Reactive power dispatch aims to control voltages of PV-buses, tap settings of the on-load tap changing transformers and voltage compensators to minimize network power loss. These two aspects have also been considered as an optimal power flow problem which have been extended in the recent years to include the problems of fuel cost, voltage profile and voltage stability. The solutions of these problems are conventionally provided using nonlinear optimization techniques which were briefly addressed in the article. A high level of research activities on applying the evolutionary algorithms to power system dispatching problems have been undertaken over the past decade. The most popular evolutionary algorithms, such as genetic algorithm, evolutionary programming and particle swarm optimizer with passive congregation, have been introduced and their applications to the above problems have been presented in this article, together with the results of the simulation studies which were obtained based on the IEEE 30-bus power system, in comparison with the conventional optimization techniques.

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