VALUE ENGINEERING

Value engineering (VE) is a proven management technique using a systematized approach to seek out the optimal functional balance between the cost, reliability, and performance of a product or project. Furthermore, VE seeks to produce the very best product at a sensible cost, and the primary focus is on improved engineering, whether it be design, construction, maintenance, materials, or any other engineering-related function.

The VE approach does not emphasize identifying errors on changes of minor significance, but rather improving practices in the areas of highest cost. The most important purpose for a VE study is to maximize the value of the product being investigated.

While value is often measured in terms of monetary benefit, it can also be measured by improved safety, better service to the users, better reliability, heightened aesthetics, or reduced environmental impact.

Quality function deployment (QFD) extends VE in that is is not restricted to a minimum essential production function (1).

Key elements of VE are:

Function analysis
Creative thinking
Job plan
Life cycle costing
Cost models
Evaluation matrix
Cost and worth
Habits and attitudes

The *function analysis* is required for each key component. This approach to problem solving is the cornerstone of VA. The function analysis used in VA consists in analyzing the functional, rather than the physical, characteristics of a system. In function analysis, the product or process under study is first converted into functions. The method requires functions to be described with only two words, a verb and a noun. The specific form used for these word pairs is called a *functive* (2).

The rules of function description are the following (1):

1. Determine the user's needs for a product or service. What are the qualities, traits, or characteristics that specify what the product must be able to do? Why is the product needed?
2. Use only one verb and one noun to describe a function. The verb should answer the question "What does it do?" The noun should answer "What does it do it to or with?" Where possible, nouns should be measurable, and verbs should be demonstrable or action-oriented.
3. Avoid passive or indirect verbs such as provides, supplies, gives, furnishes, is, and prepares. Such verbs contain very little information.
4. Avoid goallike words or phrases, such as improve, maximize, optimize, prevent, least, most, and 100%.
5. List a large number of two-word combinations, and then select the best pair. Teams can be used to derive a group definition of function.

Basic function determination logic allows functions to be ordered in a hierarchy based on cause and consequence. The function determination logic has been called the "function analysis system technique," or FAST. The functional analysis itself consists of functional decomposition.

The process of asking "how" for each higher-level function leads to lower-level functions and functional composition; the process of asking "why" for each lower-level function leads to the next higher level. For a FAST diagram (2), the four general rules are:

1. Use two words only: one verb, one noun.
2. Avoid the verb "be" or "provide."
3. The noun does not represent a part, activity, or operator.
4. Maintain the viewpoint of the user.

AREAS FOR FUTURE RESEARCH IN VALUE ENGINEERING

Influence Diagrams

The burden of problem solving in VE is now shifting to the decision maker. Unfortunately, decision making is complicated by conflicting objectives, competing alternatives, unavailable and incomplete data, and uncertain consequences. The development of increasingly complex systems has been associated with a corresponding increase in the complexity of decision problems. This has resulted in rapid growth in the development of quantitative models for decision making.

The decision analysis process is typically iterative and may be broken into the following:

1. *Problem Structuring.*
2. *Deterministic Analysis.*
3. *Probabilistic Analysis.*
4. *Evaluation.* Evaluation may also involve the *value of information* and *control analyses.* The *expected value of perfect information* is the change in expected value if the state of one or more uncertainties in the model could be observed before decisions are made. The *expected value of complete control,* on the other hand, is the change in the expected value if one or more uncertainties could be controlled in order to guarantee a particular outcome. The value of information (or control) is measured by the difference between the expected value with information (or control) and the expected value without it. While it is always tempting to insist on more information to resolve uncertainties, the concept of value of information quantifies the benefit of acquiring additional information and sets an upper bound on the value of new information.
5. *Communication.* This phase involves coherent communication of the decision analysis results in a manner that provides clear and useful insights for better decision making.

Topology of Influence Diagrams

Topologically, an *influence diagram* is a finite noncyclic graph made up of directed arcs (arrows) linking four kinds of nodes: *decision nodes, deterministic nodes, chance nodes,* and *value nodes.*

Nodes. Nodes represent variables. A node represents a choice among a set of alternatives. Each node contains a list of the possible values of the variable that the node represents. Chance or random variables are depicted by circles, decision variables by rectangles, deterministic nodes by concentric circles, and value or utility nodes by rounded rectangles. Each *chance node* contains a probability distribution for its variable X for each configuration of its predecessor nodes. The probability distributions may be obtained from subjective assessments by experts, maintenance records, statistical databases, or experimental data. Each *decision node* contains a number of decision options and represents the choices available to the decision maker. *Deterministic nodes* may be thought of as a special kind of chance node in which all the probabilities happen to be zero or one: a deterministic node has a number of states, and at any point in time, there is only one state (with an associated probability of 1) that may be assumed by the node. A *value node* may be viewed as a special kind of chance node whose value is needed to answer the question of interest to the analyst. Such a node contains a mapping that specifies the value of its variable X given values of all its predecessor nodes (3).

Arcs. Arcs linking two nodes indicate some kind of influence of one node on the other. There are two kinds of arcs: conditional and informational. Conditional arcs are arcs into chance or value nodes and indicate that there may be probabilistic dependence. Informational arcs are arcs into decision nodes and simply indicate time precedence: they indicate that information from the predecessor nodes must be available at the time of decision (3).

Evaluation of Influence Diagrams

In order to evaluate an influence diagram, there must be a question to be answered, i.e., some random variable(s) whose distribution(s) must be determined. The corresponding value node then represents the objective to be optimized (maximized or minimized) in expectation. There may be single or multiple variables associated with the value node. The variable(s) associated with nodes having arcs into the value node are the attributes of the decision maker's utility function. The random variable of the value node needs to be calculated in expectation. This expected value represents the utility of the outcome to the decision maker. If there are decisions to be made, then the expected utility may be used to compare alternatives. Given the state of information at the time of the decision, the alternative(s) selected should maximize the expected utility of the resulting outcome (3).

BELIEF FUNCTIONS

VE analysis usually involves both subjective and objective data. Some of the data are incomplete and vague. This situation is well suited for belief function (BF) analysis application.

Introduction

The BF is the central principle of the *Dempster–Shafer theory,* a mathematical theory of evidence developed by Dempster (4) and subsequently expanded by Shafer (5). BFs represents a method for assessing imprecise uncertainty. A model is uncertain but precise if a single outcome cannot be predicted, but precise statements can be made about its behavior over time. An imprecise model is one whose long term behavior cannot be predicted.

The BF approach has potential application to any system in which a number of hypotheses must be handled. One advantage of the theory over other probability theories is that as new evidence is gathered, it can be pooled with existing evidence to yield a new function (6).

The Dempster–Shafer theory is based on a *frame of discernment* (also called a universe of discernment or universe of discourse). This is a set of mutually exclusive and exhaustive alternatives. The theory allows belief to be committed to subsets within the frame of discernment, and not simply to individual members as in Bayesian probability theory. The main components of this theory may be described as follows (7):

1. All the hypotheses to be considered are grouped in a frame of discernment Θ (or universe of discernment or discourse, U). A subset of a frame of discernment is taken as a disjunction of its elements.
2. The hypotheses in Θ are assumed to be mutually exclusive and exhaustive.
3. There is a narrowing of the hypothesis set to the correct possibility as the evidence accumulates.
4. Ignorance is represented by committing all belief to the frame of discernment.
5. All belief need not be assigned to proper subsets of Θ; same belief can remain unassigned by committing it to Θ.
6. Evidence disconfirming any hypothesis in Θ can be seen as evidence confirming the remaining hypotheses. Thus, a single confirmation or disconfirmation results in a new hypothesis.
7. The set of all possible subsets of Θ is denoted by 2^{Θ}. This includes the null (empty) set $\emptyset$.

Basic Concepts

When a portion of belief is committed to one subset A of a frame of discernment Θ, that belief is also committed to any subset containing A. Thus, some of the total belief committed to A may also be committed to other proper subsets of A, with the remainder being committed to A alone (5).

Belief Functions. A belief function (Bel) is a measure of belief in each of the subsets of the frame of discernment. In general for any subset A of a frame of discernment Θ, a belief function gives a measure of the total belief in A. The belief function is derived from the basic probability assignment. The measure of the total belief committed to A is distinct from the belief committed to A alone. The total belief in A is the summation of the belief committed to all proper subsets of A (5). Thus,

$$\mathrm{Bel}(A) = \sum_{B \subset A} m(B)$$

This means that $m(A)$ must be added to $m(B)$ for all proper subsets B of A.

Hence, a function $\mathrm{Bel}: 2^{\Theta} \to [0, 1]$ is termed a belief function if it is given by

$$\mathrm{Bel}(A) = \sum_{B \subset A} m(B)$$

for a basic probability assignment $m: 2^{\Theta} \to [0, 1]$.

The following relationships can be proved (6,7):

1. Bel and m are equal for singletons, that is, $\mathrm{Bel}(A) = m(A)$ if A is a singleton.
2. $\mathrm{Bel}(A)$, where A is any other subset of Θ, is the sum of the values of m for every subset in the subhierarchy formed by using A as root. Otherwise stated, the total belief in A, $\mathrm{Bel}(A)$, is equal to the sum of all m-values for the subsets of A.
3. $\mathrm{Bel}(\Theta)$ is always equal to 1, since $\mathrm{Bel}(\Theta)$ is the sum of the values of m for every subset of Θ. By the definition of a basic probability assignment this number must equal 1.

Shafer (5) defines a function $\mathrm{Bel}: 2^{\Theta} \to [0, 1]$ as a belief function if and only if the following conditions are satisfied:

1. $\mathrm{Bel}(\emptyset) = 0$.
2. $\mathrm{Bel}(\Theta) = 1$.
3. For every positive integer n and every collection $A_1, \ldots, A_n$ of subsets of Θ,

$$\mathrm{Bel}(A_1 \cup \cdots \cup A_n) \geq \sum_{\substack{I \subset \{1,\ldots,n\} \\ I = \varnothing}} (-1)^{|I|+1} \mathrm{Bel}(A \cap A_i)$$

where $|I|$ denotes the cardinality of I.

Any such function can be defined in terms of m, a basic probability assignment. It is defined by

$$m(A) = \sum_{B \subset A} (-1)^{|A-B|} \operatorname{Bel}(B)$$

for all $A \subset \Theta$, where $|A - B|$ is the cardinality of the set $A \cap B$. Then

$$\operatorname{Bel}(B) = \sum_{A \subset B} m(A)$$

The *vacuous* belief function arises when there is no evidence. It is obtained by setting $m(\Theta) = 1$ and $m(A) = 0$ for all $A \neq \Theta$. Here, $\operatorname{Bel}(\Theta)$ is still equal to 1, but $\operatorname{Bel}(A) = 0$ for all $A \neq \Theta$.

A subset of a frame of discernment Θ is called a *focal element* of a belief function Bel over Θ if and only if

$$m(A) > 0$$

where m is the basic probability assignment associated with Bel. When Θ is the only element of a belief function, it is a vacuous belief function (8).

Plausibility. To fully describe belief in a proposition or hypothesis, an additional function is used—the *plausibility* function. This expresses the degree to which credence is lent to the hypothesis. Stated another way, whereas Bel measures total support for a possibility on the basis of the observed evidence, the plausibility (Pl) measures the maximum amount of belief possible, given the amount of evidence against the hypothesis (9). Thus, whenever Bel is a belief function over a frame of discernment Θ, the function $\operatorname{Pl} : 2^\Theta \rightarrow [0, 1]$ is defined by

$$\operatorname{Pl}(A) = 1 - \operatorname{Bel}(A^-)$$

where A^- is the negation of A. Since

$$\operatorname{Bel}(A) = 1 - \operatorname{Pl}(A^-) \qquad \text{for all} \quad A \subset \Theta$$

the functions Bel and Pl convey exactly the same information (5).

An assignment of belief to a hypothesis depends not only on the relative support suggested by present evidence, but also on a judgement of the extent to which the hypothesis has been tested, and a prediction of the likely course of further evidence. Hence, the evidence does not lead to a lone degree of belief for each hypothesis, but rather to limits being placed on the possible values that could be assigned. To further explain, in Dempster–Shafer theory the basic probability assignment m provides the distribution of belief among the subsets of Θ. This is unlike classical probability theory, which provides a precise probability to each of the elements in a set. Thus, calculation of the probabilities $P(A)$ associated with individual elements of Θ is not possible. $\operatorname{Bel}(A)$ and $\operatorname{Pl}(A)$ must be used instead. They correspond to a lower and an upper bound, respectively, on the unknown $P(A)$. Hence, the underlying probability of an event A is related to the Bel and Pl functions as follows:

$$\operatorname{Bel}(A) \leq P(A) \leq \operatorname{Pl}(A)$$

A belief function is *Bayesian* if each of the focal elements consists of a singleton. For a frame of discernment Θ, a function $\operatorname{Bel} : 2^\Theta \rightarrow [0, 1]$ is called a Bayesian belief function if

1. $\operatorname{Bel}(\emptyset) = 0$,
2. $\operatorname{Bel}(\Theta) = 1$,
3. $\operatorname{Bel}(A \cup B) = \operatorname{Bel}(A) + \operatorname{Bel}(B)$ whenever $A, B \subset \Theta$ and $A \cap B = \emptyset$.

In the case where the belief function is Bayesian, $\operatorname{Pl}(B) = \operatorname{Bel}(B)$ for all $B \subset \Theta$, and both functions are equal to the probability of the set B, $P(B)$. In the case where $m(A) > 0$ for some nonsingleton A, the implication is that there is uncertainty regarding the assignment of m among the elements of A. In Bayesian probability, there is no uncertainty about the assignments of probability (10).

There are important differences between Bayesian probability theory and Dempster–Shafer theory. In classical probability theory, for two disjoint sets A and B,

$$\operatorname{Prob}(A) + \operatorname{Prob}(B) = \operatorname{Prob}(A \cup B)$$

This is not true for belief functions, where

$$\operatorname{Bel}(A) + \operatorname{Bel}(B) \neq \operatorname{Bel}(A \cup B)$$

even if A and B are disjoint. Additionally, $\operatorname{Bel}(A) + \operatorname{Bel}(A^-) \neq 1$; this means that belief about a proposition A does not imply belief about the negation of A.

For a vacuous belief function,

$$\operatorname{Bel}(\Theta) = 1, \quad \text{and} \quad \operatorname{Bel}(A) = 0, \quad A \neq \Theta$$
$$\operatorname{Pl}(\varnothing) = 0, \quad \text{and} \quad \operatorname{Pl}(A) = 1, \quad A \neq \varnothing$$

Belief and plausibility functions have the following properties (10):

1. $\operatorname{Bel}(A) \leq \operatorname{Pl}(A)$
2. $\operatorname{Bel}(A) + \operatorname{Pl}(A^-) = 1$
3. $\operatorname{Bel}(\emptyset) = \operatorname{Pl}(\emptyset) = 0$
4. $\operatorname{Bel}(\Theta) = \operatorname{Pl}(\Theta) = 1$
5. $\operatorname{Pl}(A) = 1 - \operatorname{Bel}(A^-)$
6. $\operatorname{Bel}(A) + \operatorname{Bel}(A^-) \leq 1$
7. $\operatorname{Pl}(A) + \operatorname{Pl}(A^-) \geq 1$

Dempster's Rule of Combination (Orthogonal Summation)

Dempster's rule of combination is the most important tool of the Dempster–Shafer theory (8). Given a number of belief functions over the same frame of discernment, Dempster's rule allows for the computation of their *orthogonal sum*—a new belief function based on the combined evidence (5). Essentially, Shafer sets the following conditions for combinations:

1. If m_1 and m_2 are basic probability assignments of the belief functions Bel_1 and Bel_2 with cores of $\{A_1, \ldots, A_j\}$ and $\{B_1, \ldots, B_j\}$, respectively, then the probability masses can be represented as segments of a line of unit length. Thus, the basic probability masses of two belief

functions may be orthogonally combined to obtian a unit square.

2. If the two belief functions Bel_1 and Bel_2 are represented, with the basic probability assignments m_1 and m_2, then the square is representative of the total probability mass for the two functions, $\mathrm{Bel}_1 \oplus \mathrm{Bel}_2$.

The rule of combination as outlined by Dempster is a rule for combining a pair of belief functions. The operator of orthogonal summation of belief functions satisfies the following properties:

Commutativity:

$$m_1 \oplus m_2 = m_1 \oplus m_2$$

Associativity:

$$(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$$

These two properties allow for the combination of multiple belief functions by repeated applications of Dempster's rule. Thus, if $m_1, m_2, \ldots, m_p$ are pieces of evidence, their combination is

$$m = m_1 \oplus m_2 \oplus \cdots \oplus m_p$$

By this means, any number of belief functions may be combined, according to

$$\begin{aligned}&\mathrm{Bel}_1 \oplus \mathrm{Bel}_2\\&(\mathrm{Bel}_1 \oplus \mathrm{Bel}_2) \oplus \mathrm{Bel}_3\\&[(\mathrm{Bel}_1 \oplus \mathrm{Bel}_2) \oplus \mathrm{Bel}_3] \oplus \mathrm{Bel}_4\end{aligned}$$

etc.

The formal statement of Dempster's rule of combination is then (8) as follows: For $\mathrm{Bel}_1 \oplus \mathrm{Bel}_2$, the combined probability assignment is given by

$$m_1 \oplus m_2(A) = \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)$$

or by

$$m_1 \oplus m_2(A) = \sum_{A_i \cap B_j \neq 0} m_1(A_i) m_2(B_j)$$

if $A \neq 0$ and $m_1 \oplus m_2(\emptyset) = 0$.

Let

$$k = \sum_{\substack{i,j\\ A_i \cap B_j \neq 0}} m_1(A_i) m_2(B_j)$$

Then the renormalizing constant is $K = 1/(1 - k)$. Its reciprocal, K^{-1}, is also important in Dempster–Shafer theory as a measure of the extent of conflict between two belief functions.

SUMMARY

This article presents a new direction in VE application. VE, although very important, so far has lacked the mathematical and other analytical tools needed for an important aspect of engineering decision making. Together, influence diagrams and the belief function approach have all the properties needed to handle the challenges of VE application.

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