FAULT LOCATION

Power systems represent a vital component of the electrical utility infrastructure aimed at supplying the power to a variety of users. These systems consist of a number of different components including generators, power transformers, transmission lines, and loads. Design of the system components and overall systems is implemented under a stringent reliability requirement with a strong emphasis on continuity of the power supply.

The most common and desirable operating mode of a power system is the normal operation in which typically an alternating-current (ac) generator is used to produce and maintain supply of the sinusoidal 60 Hz waveforms of voltages and currents. Transmission lines used to connect the generators and loads are allowing the transfer of power between the generation and load sites. Power transformers are used to step-up the voltage from the generator level to the transmission-line level for more efficient transfer of power over the transmission lines connecting generators and loads. At the location of the load, power transformers are used again to step-down the voltage to the levels required by a variety of loads. All of the major components in a power system are connected using switching equipment, allowing the components to be put in and out of services as needed.

Power-system operation can be viewed as falling into one of the following states: normal, emergency, or restorative. As in any other technical system, there are circumstances under which failures in the system operation do occur. The faults on a transmission line create an emergency operating state. They are detected by special equipment called protective relays. Protective relays are designed to issue a trip command to the switching equipment (circuit breakers) to open both ends of a transmission line if a fault is detected and confirmed by the relaying algorithm as being present on that line. Eighty to ninety percent of all faults are temporary. After a fault has occurred and relays have detected the fault and disconnected the line, it is the general practice to automatically attempt to restore the line one or more times. If the fault is gone when the line is reenergized, the circuit breakers will stay closed and only a momentary loss of service has occurred. Automatic reclosing is done between 30 cycles and 30 seconds, depending upon the utility's practice. If the fault is permanent, the relays will trip the circuit breakers each time they reclose until the preset number of reclosures has occurred, at which time the circuit breaker is locked out and the line remains deenergized. In either event, it is important to determine the location of the fault. Even temporary faults leave a residue of damage which must be repaired at the earliest opportunity. If the fault is permanent, the damage must, of course, be repaired and the line returned to service.

Fault-location techniques are used to determine location of the fault on a transmission line. Once the damage caused by the fault can be located, the line can be repaired and restored as soon as possible. Since the efficiency in repairing and restoring the line is greatly dependent on the ability to locate the damaged part accurately, it is extremely important that the fault-location algorithm is very accurate, so that the maintenance crews can be dispatched to the appropriate location immediately.

Most transmission-line faults occur during severe weather conditions when lighting strikes towers or conductors, producing stresses on the insulation between the transmission conductors and supporting structures. In addition, some natural environmental conditions such as a tree growing or bird flying into a transmission line can cause a fault. The cause of the fault in this case is a foreign object connecting the cables causing the insulation breakdown. Since all of the mentioned causes are random, faults can occur at any time and at any location.

Properties of Transmission-Line Faults

The transmission line fed by an alternating-current source is built with either three-phase or single-phase conductor configuration. Our discussion will be related to a three-phase system. The three-phase system assumes that there are three conductors, each energized with currents and voltages. These conductors are mounted on towers that support the line all the way from the generating plant or a substation to another substation or a customer load. The typical span between two towers in a high-voltage transmission system is between 200 m and 500 m. The electrical relationship between the three-phase voltages or currents is represented with phasors that are of the same magnitude but 120° apart. These phasors can be defined for an electrical condition between each of the conductors, or between a conductor and a ground potential. These quantities are typically called the line and phase values, respectively.

Transmission-line faults are mostly caused by deterioration of the insulting materials due to environmental and special operating conditions. Construction of overhead transmission lines requires that the conductors carrying the current are dispensed on large supporting structures called towers or poles. Since the most common transmission principle uses three-phase systems, at least three conductors are placed on one supporting structure. To make sure that there is no insulation breakdown between the conductors and supporting structures, as well as among conductors, several insulating components and principles are used. Most commonly, ceramic or polymer insulators are used when connecting the cables to the supporting structure. In addition, adequate spacing between conductors is provided to allow for air to serve as an insulator between conductors. In some instances, a separate conductor connected to the ground at each of the supporting structures is placed on the top of the structures (the "earth" conductor). It is used to shield the other conductors from impacts of lightning that may cause an insulation breakdown and damage the insulators and conductors.

Once a fault occurs on a transmission line, it can take a variety of forms. The most common fault is the connection of a conductor to the ground. This connection can be via an electrical path of very low resistance, such as an arc caused by a lighting. In most of the ground faults that are caused by a lightning strike, the connection with the ground is established via an earth wire placed on the top of the tower and connected to the ground at the footing of each tower. Yet another possibility is that the ground connection is established via an electrical path with a higher resistance, such as the case in which a tree or a manufactured object provides the connecting path. These types of faults are called ground faults and can be established individually between any of the line conductors and the ground or between any two conductors jointly connected to the ground. In addition, all three conductors can be involved jointly in a three-phase ground fault. The other types of faults are related to various combinations of faults between the conductors without involving a ground connection. These types are called phase-to-phase faults. It is important that fault-location techniques are capable of accurately determining the fault location under a variety of different fault types.

Yet another consideration associated with the fault is the length of time required to detect the fault and disconnect the transmission line. As mentioned earlier, protective relays are used to detect a fault and issue a trip command to a breaker. There are two distinct time frames involved in fault detection and fault location. Protective relays may be required to operate in one cycle (@f = 60 Hz, 1 cycle = 16.66 ms.). To do this, relays are set to recognize whether a fault is in or out of a given zone of protection and to make the decision in the presence of electrical noise and other transient effects such as dc offset, current transformer or potential transformer inaccuracies, and so on. The exact location of the fault is not required as long as it is determined that it is within the zone of protection. This operating time requirement of 1 cycle may result in an incorrect decision and an incorrect operation. The relay, however, must be dependable and security is sacrificed. In high voltage and extra high voltage networked systems this is acceptable because the system itself is designed to be robust and maintain its integrity even with the loss of a line. At distribution and industrial voltage levels where the system is radial (i.e. only a single source), security may be a more important factor than dependability since the loss of a line will result in the loss of service to an area or a customer. In this situation the relay's operating time may be delayed beyond 1 cycle to be sure that the measurement is correct. After the relay has operated and a trip command is given to the circuit breaker, the circuit breaker will clear the fault in 2 to 3 cycles making the total clearing time 3 to 4 or more cycles. For fault location, this is the time that the current and voltage waveforms can be monitored.

Another aspect of relay operation is the provision to reclose the circuit breaker automatically after it had been opened by the relay trip action. This technique is called automatic reclosing and is commonly applied on high-voltage transmission lines. Since quite a few of the transmission-line faults are temporary in nature, the autoreclosing function provides an automatic attempt to reclose the line and keep it in service if the fault has disappeared. Furthermore, the circuit breakers can operate on all three phases simultaneously, or the construction may allow for single-phase (single pole) breaker operation. Fault-location techniques need to be able to determine the fault type correctly so that a proper autoreclosing option can be applied.

FAULT-LOCATION REQUIREMENTS

The transmission-line fault-location function needs to satisfy several major requirements as follows:

- The accuracy must be sufficient to locate the fault within a span of two towers. Typically 0.1% error is acceptable, but an accuracy of 0.01% is desirable.

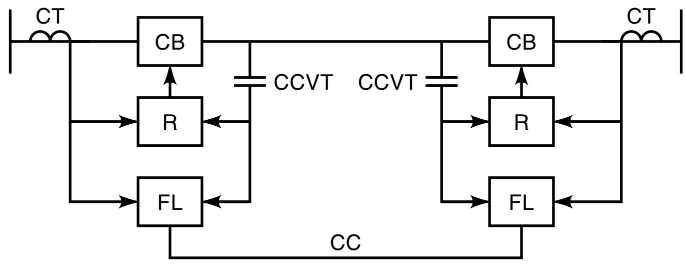

Figure 1. Fault-location equipment connection. CB is the circuit breaker, R the relay, FL the fault location, CT the current transformer, CCVT the capacitor coupling voltage transformer, and CC the communication channel (not always required).

- The accuracy should be maintained even if only a short segment of the fault data from a distorted waveform is measured. Typically, it is required that no more than a few cycles of data is sufficient for the calculation.
- The accuracy should not deteriorate if various types of faults and numerous autoreclosing requirements are considered. Typically, it is acceptable if the accuracy deteriorates under some difficult fault cases in which the fault resistance changes during the fault, but it is desirable that the accuracy be stable even under these conditions.

Fault-location application requirements are quite diverse and can be discussed using Fig. 1.

Transmission-Line Construction

Transmission-line construction imposes that an acceptable accuracy should be achieved for the following application situations:

- Long and short lines with different voltage levels (electrical properties of the line are associated with its length and voltage)
- Transposed and untransposed lines (line transposition is done by changing the relative position between conductors at a given tower altering by this the symmetrical relationship between currents and voltages)
- Lines with mutual coupling (the mutual coupling takes place between conductors through an electromagnetic field and affects electrical conditions on the conductors, in particular for the faults involving ground)
- Multiple line-per-tower construction (this is the case in which several sets of three conductors representing several lines are tied to the same tower causing mutual coupling among conductors of different lines)
- Radial lines (lines that directly connect to a single source of power)
- Series-compensated lines (lines that have capacitors connected in series with the line conductor)
- Lines with load taps (the loads are connected either directly or through a transformer to a line at any position along the line without using common switching equipment)
- Single-phase and three-phase lines (single or three conductors)

- Time-varying fault resistance (due to the breakdown of the insulation, the fault resistance changes during the fault disturbance)
- Changing prefault load conditions (the line may have distinctively different load current at a different moment of a fault)

Protective Relaying System

The fault-location application requires that full consideration is also given to the elements that constitute the relaying systems: protective relays, instrument transformers, and circuit breakers. Protective relays are supposed to detect the fault and isolate the line before the system is endangered and further damage is incurred. The fault clearing time of a typical transmission-line relay is around four cycles, which should provide sufficient measurement time to obtain the waveform data for the fault-location application. Since the relays give a determination based on the waveform measurements obtained by the current transformer and capacitor coupling voltage transformers (CT and CCTV, respectively), it is important to understand the errors introduced by the transformers. Typical distortion that may affect the current waveform is the saturation of the iron core. The CCTV are associated with low-pass filtering characteristics as well as signal oscillations in the case of voltage collapse. The instrument-transformer inaccuracies are very important in determining the overall error in the fault-location algorithm. The instrument-transformer error may significantly affect the fault location error causing it to deteriorate for an order of magnitude. Finally, the circuit breakers are initiated by the relays to clear the fault. The phenomena of breaker restrikes and ferroresonance distortion are important when using the waveform data captured before the breaker opens in calculating the fault location.

Implementation Requirements

The algorithms for fault location may be implemented using

- Fault-location devices
- Protective relays
- Digital fault recorders

Stand-alone fault locators are the most flexible option since the entire design can be optimized for fault-location application. At the same time, this is the most expensive solution since the entire device accommodates only one function, namely, the fault location. Some vendors have opted for such a solution, justifying an increased cost with a claim that their fault-location implementation guarantees unsurpassed accuracy performance (1).

The most common implementation approach is to use the transmission-line protection relays as the platform for the fault-location implementation. This approach is cost effective since the increment required to accommodate the fault-location algorithm is minimal. Almost all of the protective-relay vendors offer some form of a fault location algorithm as a standard feature of their relay designs.

Yet another option is to use a digital fault recorder (DFR) design as the platform for fault-location implementation. DFRs are commonly used in high-voltage transmission substations to record voltages and currents on the transmission lines. Again, most of the DFR vendors have implemented a fault-location algorithm and provide it as a standard feature of their product.

Even though fault-location implementation can be diverse, it should be noted that the accuracy and cost requirements are always the key consideration. Therefore, it is essential to understand the possible benefits and shortcomings of using different types of data and system-implementation approaches when designing or selecting a fault locator.

Cost/Performance Considerations

The following design considerations directly affect the cost/performance rating of a given fault-location implementation:

- One- or two-ended application
- Synchronized or unsynchronized data acquisition
- Data samples from the adjacent lines

The least expensive fault-location application is to use a single-terminal measurement of voltages and currents. In this case an existing transmission line relay or a DFR can be used. The main difference between these application approaches is the input data waveform processing requirement. Most of the protective relays use a low sampling rate to reconstruct phasors. The DFR sampling is up to 5 kHz and higher and enables recovery of other waveform components. The accuracy and complexity of the input channels have a bearing on both the cost and performance of the fault-location implementation.

A more expensive but also more accurate solution is a two-terminal implementation with which the data from the transmission line ends are collected and brought to a centralized place where the fault location is calculated. In this case a communication channel is needed to transfer the required data which increases the cost of the overall solution. A variation between these solutions is in the way the data sampling is performed. Most of the implementations do not require that the data sampling at two ends of the line is synchronized to a common time source, while the most accurate solutions require the synchronization (2).

Finally, in order to achieve even greater fault-location accuracy, data samples from the lines parallel to the faulted line, and from all ends of a multiterminal line involved in a fault, can be used. Obviously, more input channels and communication facilities are needed in this case, but accuracy can be improved significantly (3).

FAULT-LOCATION ALGORITHM FUNDAMENTALS

A fault-location algorithm defines the steps needed to obtain the fault location by using the measurements of voltages and currents from one or more ends of the line. A set of equations representing the mathematical model of the faulted transmission line is needed to define the algorithm. The quantities that appear in the equations are (1) voltages and currents, (2) transmission-line parameters, and (3) fault parameters.

The voltage and current in power systems are a combination of four kinds of signal components: fundamental, higher or lower frequency, transients, and noise. The fundamental component is a sinusoid having system frequency f_0 that is equal to 60 Hz (in the United States) or 50 Hz (in some other countries). The higher- or lower-frequency components are also sinusoids having a frequency different from the fundamental one. The transients are temporary phenomena having diverse mathematical representation. They arise whenever the voltages or currents abruptly change. An occurrence of the fault causes such an event. The noise is a random signal component usually generated by measurement errors. In the normal operation of the transmission line, the fundamental component is dominant.

Two types of transmission-line mathematical models are in use for fault-location algorithms: the distributed-parameter model and the lumped-parameter model. The distributed-parameter model is mostly suitable for long transmission lines. The lumped-parameter model is a simplification of the distributed-parameter model and is used for shorter lines only. These models are also known as the long-line and short-line model, respectively.

In the distributed-parameter model, the voltages and currents are functions of time t and position x. The model consists of two linear partial differential equations of the first order. First we consider the equations for the case of the one phase transmission line:

$$-v_x(x, t) = l i_t(x, t) + r i(x, t) \tag{1}$$

$$-i_x(x, t) = c v_t(x, t) + g v(x, t) \tag{2}$$

In these equations, line parameters l, r, c, and g are inductance, resistance, capacitance, and conductance per unit length, respectively; $v(x, t)$ is the voltage and $i(x, t)$ is the current. The subscripts x and t denote partial derivatives with respect to position and time.

A three-phase transmission line has as a model two matrix equation similar to Eqs. (1) and (2). The elements of the voltage vector are three-phase voltages, and elements of the current vector are three line currents. Transmission-line parameters are represented by matrices $\boldsymbol{R}$, $\boldsymbol{L}$, $\boldsymbol{C}$, and $\boldsymbol{G}$ and are composed of self-resistance, mutual resistance, inductance, capacitance, and conductance. The details of this model will be presented later.

The lumped-parameter model neglects the line conductance g and capacitance c. The partial derivative of the current relative to position, in Eq. (2), is equal to zero in this case. Therefore, the current does not change along the line. The integration along the transmission line from one end (the sending end) to a point at a distance x from the sending end produces the following differential equation:

$$v_x(t) - v_s(t) = x r i(t) + l x[di(t)/dt] \tag{3}$$

In Eq. (3), $v_s(t)$ is the voltage at the sending end, $v_x(t)$ is the voltage at a distance x from the sending end, and $i(t)$ is the current on the line. In the case of a multiconductor line, the model is a matrix equation similar in form to Eq. (3). The line has a matrix model containing as its elements the self-resistance, mutual resistance, and inductances.

The Fourier transformation of Eq. (3) can be made if all the line parameters are constant. Furthermore, if the currents and voltages are the fundamental components, they will appear in the equation as phasors.

Note that due to the linearity of the equations, voltages and currents in both models may be replaced by their components. For example, voltages or currents may consist of a fundamental component only or a transient component only. The classification of the existing fault-location algorithms depends on the line model and the signal component used. Most of the existing algorithms belong to two main groups:

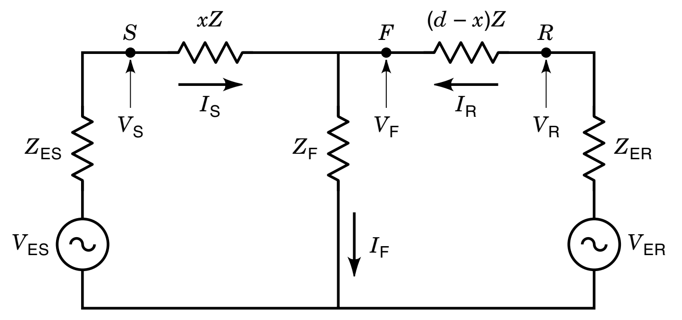

Figure 2. The circuit of a faulted transmission line. S, F, and R are the positions of sending end, fault, and receiving end, respectively. x is the distance to the fault, Z the line impedance, and d the transmission line length. V_S, V_F, and V_R are the voltages at sending end, fault, and receiving end, respectively. I_S, I_F, and I_R are the currents at the sending end, fault, and receiving end, respectively. Z_{ES} and Z_{ER} are the Thévenin equivalent impedances. V_{ES} and V_{ER} are the Thévenin equivalent voltages.

- Phasor-based algorithms use the fundamental component of the signals only. The fundamental components then appear as phasors. The line model is usually the lumped-parameter model.
- Partial differential equation–based algorithms use transient components of signals and the distributed-parameter model of the line.

We will explain the underlying principles of the two groups using their exemplary algorithms.

STANDARD APPROACHES: PHASOR-BASED ALGORITHMS

The phasor-based algorithms use a Fourier transform of Eq. (3) to model the line. The line is represented by its impedance per unit length $Z = r + j2\pi f_0 l$ and its length d. Figure 2 depicts the circuit model of the faulted line. There are three groups of quantities in Fig. 2. The phasors of voltages and currents are known since they may be calculated from the signal samples. The transmission-line impedance Z and its length are also known from the line construction data. The fault position x, the fault impedance Z_F, and the fault voltage V_F are not known.

The aim of the algorithm is to find the unknown distance x to the fault. Two main steps in a phasor-based algorithm are (1) calculation of phasors from the signal samples and (2) solution of the set of equations for the unknown fault distance.

The phasors are calculated from the corresponding voltage and current samples. An arbitrary sinusoid, say voltage $v(t)$, is represented by a phasor $\boldsymbol{V}$. A phasor is a complex number defined by its real value Re{$\boldsymbol{V}$}, its imaginary value Im{$\boldsymbol{V}$}, or alternatively by its phase θ and amplitude $|\boldsymbol{V}|$. The calculation of the phasor parameters is accomplished using Fourier analysis. The formulas for real and imaginary part of a phasor are

$$\mathrm{Re}\{\boldsymbol{V}\} = f_s \sum_{n=0}^{N-1} v(n/N f_0) \cos(2\pi n/N) \tag{4}$$

$$\mathrm{Im}\{\boldsymbol{V}\} = f_s \sum_{n=0}^{N-1} v(n/N f_0) \sin(2\pi n/N) \tag{5}$$

Here N is an integer equal to the ratio of the sampling frequency f_s and the system frequency f_0. The samples of the corresponding signal $v(t)$ are equal to $v(n/Nf_0)$. They are taken in a window of samples one cycle long. The amplitude $|\boldsymbol{V}|$ and the phase θ of the phasor are then calculated by the well-known formulas for the calculation of the amplitude and phase of a complex number from its real and imaginary values.

The preceding Fourier analysis formulas give an exact value of the phasor's real and imaginary value only if the signal is a pure sinusoid. The presence of the higher harmonics, transients, and noise introduces an error in the phasor calculation.

The phasor-based algorithms also differ depending on the location where the measurements are taken. One-end algorithms use measured data from one side of the line only. This side is conventionally named the sending end. Two-end algorithms use data from both the sending end and the other end, called the receiving end. One-end algorithms are more commonly used since they do not need the communication channel required in the two-end algorithms.

The One-End Algorithms

One of the well-known algorithms of this type was defined by Takagi et al. for the three-phase transmission line (4). The fault type considered is the line to ground fault. This is the most common type of fault. For the convenience, the fault was considered to be on phase a. The algorithm of Takagi et al. neglected mutual impedances and resistances between the phases. Therefore, the one-line diagram given in Fig. 2 can be used to represent this case with the current and voltage coming from the phase a only. Takagi et al. assumed that the impedance of the fault is a resistance equal to R_F. The equation relating the sending end voltage to the current and voltage at the fault follows from Fig. 2:

$$V_s = xZI_s + R_F I_F \tag{6}$$

This is a complex scalar equation, equivalent to two real scalar equations. However, the number of unknowns is equal to four. One unknown is x, the phase and amplitude of the fault current phasor I_F are the other two, and the fault resistance R_F is the fourth unknown. The number of unknowns exceeds the number of equations, and additional equations are needed to calculate x. The second complex equation proposed by Takagi et al. represents an assumption about the currents of the receiving and sending ends. Each of these currents is the sum of a current existing before the occurrence of the fault (prefault current) and the superimposed fault current. These two components are denoted by a prime and a double prime, respectively. The sum of the prefault sending-end and receiving-end currents is obviously equal to zero. Since I_F is the sum of the sending- end and receiving-end currents, we have

$$I_F = I_R + I_S = (I'_R + I''_R) + (I'_S + I''_S) = I''_R + I''_S \tag{7}$$

The circuit in Fig. 2 is a current divider of the fault current. Thus, the sending-end fault current I''_S is equal to

$$I''_S = \frac{[(d-x)Z + Z_{ER}]I_F}{dZ + Z_{ER} + Z_{ES}} = \frac{1}{k} I_F \tag{8}$$

Takagi et al. further assumed that all the impedances in the current divider of Eq. (8) have approximately the same phases. The consequence of this conjecture is that the fault current I_F is proportional to the sending-end fault current I''_S. This means that the current distribution coefficient k in Eq. (8) is a real number. Based on the mentioned assumptions, the modified Eq. (6) follows:

$$V_S = xZI_S + kR_F I''_S \tag{9}$$

Since the product of the current distribution factor k and resistance R_F may be seen as one unknown only, the number of unknowns is now equal to the number of equations. The fault location x is obtained by multiplying the modified equation with the conjugate of the sending-end prefault current denoted I^{*}_S, and comparing imaginary parts of the obtained equation:

$$x = \frac{\mathrm{Im}(V_S I''^{*}_S)}{\mathrm{Im}(ZI_S I''^{*}_S)} \tag{10}$$

There are several problems related to this approach. The first is a need to calculate the sending-end fault current I''_S. Since this current in the postfault period is equal to the measured postfault current less the extrapolation of the prefault current, the recordings of the prefault current must be available. The other problem is related to the algorithm basic assumption. The neglected mutual coupling with other phases may be a source of error. Next, a current distribution factor that is not a real number may be another source of error. Besides, one must know the faulted phase before the start of the calculation. These impediments may be resolved by using symmetrical components and sequence circuits that are utilized to calculate short-circuit currents in three-phase networks. A brief review of this technique is given in the next section.

One-End Algorithms Using Symmetrical Components

There are three symmetrical component phasors, zero sequence, positive sequence, and negative sequence, denoted as $\boldsymbol{V}^0$, $\boldsymbol{V}^1$, and $\boldsymbol{V}^2$, respectively, for the case of voltage. Each phase vector is a linear combination of these three components. During normal operation of the transmission line, zero and negative symmetrical components are equal to zero, and the phasor of phase a is equal to the positive-sequence phasor. Symmetrical components may be represented by a vector denoted $\boldsymbol{V}^S$. The vector of symmetrical components is obtained from the phase vectors by the following matrix equation:

$$\boldsymbol{V}^S = \boldsymbol{A}\boldsymbol{V}^P \tag{11}$$

Here $\boldsymbol{V}^{\mathrm{P}}$ is the vector having as elements the phasors pertinent to phase a, phase b, and phase c.

The matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \exp(j4\pi/3) & \exp(j2\pi/3) \\ 1 & \exp(j2\pi/3) & \exp(j4\pi/3) \end{bmatrix} \tag{12}$$

The equation defining the relation of the phase vector at the sending end $\boldsymbol{V}_{\mathrm{S}}^{\mathrm{P}}$, the phase vector at the fault $\boldsymbol{V}_{\mathrm{F}}^{\mathrm{P}}$, the phase current vector $\boldsymbol{I}_{\mathrm{S}}^{\mathrm{P}}$, and the impedance matrix $\boldsymbol{Z}^{\mathrm{P}}$ is similar in form to Eq. (6):

$$\boldsymbol{V}_{\mathrm{S}}^{\mathrm{P}} = x\boldsymbol{Z}^{\mathrm{P}}\boldsymbol{I}_{\mathrm{S}}^{\mathrm{P}} + \boldsymbol{V}_{\mathrm{F}}^{\mathrm{P}} \tag{13}$$

The impedance matrix $\boldsymbol{Z}^{\mathrm{P}}$ has mutual impedances and resistances at its off-diagonal terms. When the phasor vectors are replaced by the symmetrical component vectors, one gets

$$\boldsymbol{V}_{\mathrm{S}}^{\mathrm{S}} = x\boldsymbol{Z}^{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}}^{\mathrm{S}} + \boldsymbol{V}_{\mathrm{F}}^{\mathrm{S}} \tag{14}$$

The matrix $\boldsymbol{Z}^{\mathrm{S}}$ here is equal to

$$\boldsymbol{Z}^{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Z}^{\mathrm{P}}\boldsymbol{A} \tag{15}$$

While the matrix $\boldsymbol{Z}^{\mathrm{P}}$ has both the diagonal and off-diagonal elements, the off-diagonal elements of the matrix $\boldsymbol{Z}^{\mathrm{S}}$ are all equal to zero. Hence the matrix in Eq. (14) may be broken into three independent scalar complex equations:

$$V_{\mathrm{S}}^{k} = xZ_{kk}I^{k} + V_{\mathrm{F}}^{k}, \qquad k = 0, 1, 2 \tag{16}$$

Here Z_{kk} is the corresponding diagonal element of the matrix $\boldsymbol{Z}^{\mathrm{S}}$. The main advantage of the symmetrical component application is this decoupling. Each of the decoupled equations defines a sequence circuit. They are called the positive-, negative-, and zero-sequence circuits. Since three decoupled equations have the same form as Eq. (6), the circuit in Fig. 2 may again represent any of the sequence circuits with a suitable change in notation.

The previously mentioned obstacles of the Takagi et al. method are eliminated by using the symmetrical components in the line model (5). In this approach the negative-sequence circuit of the line is used. The decoupling feature of the symmetrical components eliminates the mutual inductance influence. Since the negative-sequence vector is equal to zero in the prefault condition, the recordings of the prefault current are not necessary as in the algorithm of Takagi et al. Moreover, according to these authors, the equivalent impedances of the negative-sequence circuit and the line impedance of the negative-sequence circuit that make up the current divider are more likely to have the same phases than in the case of the phase impedances of the line (6). This implies that the assumption that the current distribution factor is a real number is close to reality. Also, the classification of the fault type before the calculation is not necessary. However, the exclusive use of the negative-sequence representation has a drawback. In the (very rare) case of a symmetric fault, the negative-sequence phasors after the fault remain equal to zero, and the negative-sequence circuit is not suitable for fault location.

The one-end algorithms require relatively simple calculations, and their implementation is opportune, since the waveform data are necessary from one side of the line only. They assume that the fault impedance Z_{F} is a constant during the fault. Their accuracy depends on the simplifying assumptions. In the case of a high fault impedance the fault current is small; hence the fault components of the sending-end current are very small. Since the fault current for the sending end is in the denominator of Eq. (9), the system is ill-defined in this case and errors may be large.

Two-end algorithms require fewer simplifying assumptions and offer potentially more accurate calculations.

The Two-End Algorithms

Two-end algorithms fall into two subgroups: algorithms developed using synchronized samples and those developed using nonsynchronized samples. The samples are synchronous if the two data sampling clocks at the sending and receiving end ensure that the samples are taken at exactly the same moments. This may be achieved by global positioning system (GPS) of satellites using pulses emitted from a satellite to attune the two GPS receivers that synchronize the sampling clocks (2). This approach introduces additional cost to provide GPS receivers and appropriate waveform sampling interfaces. The impact of synchronization will be explained in the following paragraphs.

One must note that phasors are calculated locally. If there is a time shift between data acquisition clock pulses at the receiving and sending ends, the relative phases of the receiving and sending end phasors are not the same. The phase difference between two phasors cannot be calculated by subtracting one phase from another. Suppose that the phasor at the receiving-end voltage is calculated from two sets of samples. The first set is taken using the sample clocked by the sending end. The resulting phasor is denoted as V_{F}. The second voltage phasor denoted V_{F}' is calculated using the receiving-end clocked samples. If there is a time shift Δt between the two sets, the phases of two phasors will differ for $\delta = 2\pi f\Delta t$. This may be mathematically expressed in the following way:

$$V_{\mathrm{F}} = V_{\mathrm{F}}' e^{j\delta} \tag{17}$$

The phase shift δ restates the nonsynchronized phasor (obtained using data from the receiving end) to the frame reference of the sending end. This phase shift is the same for all voltages and currents, but it is not known in advance. Note that the phasor in both time references has the same amplitude. The two-end methods consider the phase shift δ as an additional unknown and try to solve for the fault distance x by eliminating δ. Note that the sending-end voltage phasor calculated locally is V_{S}, and the receiving-end voltage and current phasor calculated locally are V_{R}' and I_{R}', respectively.

An example of an algorithm using non-synchronized samples is presented in Ref. 5. The line model is constructed using a negative-sequence diagram. By inspecting Fig. 2 and interpreting all phasors as negative-sequence phasors and all the impedances as negative-sequence impedances, the application of the Kirchhoff's voltage law renders

$$V_{\mathrm{F}} = V_{\mathrm{S}} - xZI_{\mathrm{S}} \tag{18}$$

$$V_{\mathrm{F}}' = V_{\mathrm{R}}' - (d - x)Z_{\mathrm{R}}I_{\mathrm{R}}' \tag{19}$$

Since the absolute value of the fault voltage in both equations is the same, one gets the following scalar equation by eliminating the absolute value of the fault voltage $|V_F|$ from Eqs. (18) and (19):

$$|V_S - xZI_S| = |V'_R - (d-x)ZI'_R| \tag{20}$$

This is a quadratic equation with respect to x and it may be easily solved.

Two-end algorithms using synchronized samples start from the matrix equivalents of Eqs. (18) and (19). Since all the phasors are calculated using the samples clocked at the same time, derived from the same clock, the two equations may be combined together. When the fault voltage is eliminated from these two equations, the following matrix equation follows:

$$\boldsymbol{V}_S - \boldsymbol{V}_R - x\boldsymbol{ZI}_S + (d-x)\boldsymbol{ZI}_R = 0 \tag{21}$$

This equation is equivalent to six real scalar equations. Since there is only one unknown x, the system is overdetermined. One alternative in such a situation is to use only a sufficient number of equations as in Ref. 7. Another option is to use the minimum least squares (MLS) technique. The MLS technique is often used to identify parameters of a linear system using measurements corrupted with Gaussian noise (8).

The basic idea of the MLS method is to compensate for measurement errors by using more equations than necessary and thus decreasing the measurement-error effects by averaging. The solution attained by the MLS method should not exactly satisfy any of the equations. When the MLS solution is put into the equations, the right-hand side of each scalar equation will not be zero but rather will be equal to a quantity of the error. The solution offered by the MLS method guarantees that the sum of all the squared errors will be the smallest possible. The matrix Eq. (19) in the MLS technique is represented as:

$$\boldsymbol{A}x + \boldsymbol{B} = \boldsymbol{E} \tag{22}$$

where vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as:

$$\begin{aligned} \boldsymbol{A} &= -\boldsymbol{Z}(\boldsymbol{I}_S + \boldsymbol{I}_R) \\ \boldsymbol{B} &= \boldsymbol{V}_S - \boldsymbol{V}_R + \boldsymbol{ZI}_R \end{aligned} \tag{23}$$

Here $\boldsymbol{E}$ is the vector of errors. The solution for x provided by the MLS technique minimizes the criterion function $J = \boldsymbol{E}^T\boldsymbol{E}$, and it is given by

$$x = -(\boldsymbol{A}^T\boldsymbol{A})^{-1}(\boldsymbol{A}^T\boldsymbol{B}) \tag{24}$$

The superscript T denotes matrix transpose.

This method of the fault location applied in Ref. 7 requires more calculations but offers a consequential increase of precision if there is significant noise in the measurements.

In conclusion, all the phasor-based methods start from the fundamental assumption that all the transmission-line and fault parameters are constant during the fault and that the transmission line is homogenous between the sending end and the receiving end. These assumptions may not be satisfied in some instances. For example, the value of the fault impedance may change in time if there is an arcing fault. Also, the line may be compensated by inserting a series capacitor into the line, or there may be load taps between two line ends. In addition, neglecting the line capacitance may introduce significant errors for a longer transmission line.

However, the most important issue in the phasor-based algorithms is the need for phasor estimation. Since in reality there is usually a decaying dc component and noise in the signal, phasors calculated using the Fourier analysis–based formulas given by Eqs. (4) and (5) will differ from their true values.

The methods based on the distributed line parameters solve some of these problems. Calculation of phasors is not needed. The line capacitance is included in the model. The change of the fault impedance is not a problem, and these methods work if a series capacitor is inserted into the line.

ADVANCED APPROACHES: PARTIAL DIFFERENTIAL EQUATION–BASED METHODS

A solution of a linear partial differential equation may be found using the method of characteristics. The justification for this method may be found, for example, in Ref. 9. The partial differential equations [Eqs. (1) and (2)] of the transmission-line model have two characteristics: functions of position and time. The general solution for the voltage and current along the line is a linear combination of two arbitrary functions. Each function has one of the characteristics as its argument. The particular value of these functions is set by the boundary conditions. The boundary conditions may be the measured voltage and current signal at the same point of the line. Two arbitrary functions are selected so that the general solution at this point is equal to the measured values.

Two approaches based on the partial differential equation model have been proposed for the fault location. The first method solves partial differential equations using numerical methods with sending-end voltage and current as boundary conditions. An inspection of the voltage solution along the line reveals the fault location. The second method does not require the solution of partial differential equations, but instead it exploits a special property of the sending-end voltage and current and finds the distance by pertinent signal processing.

The Solution of Partial Differential Equations

This method was first proposed by Kohlas for the case of the one-phase transmission line (10). Kohlas neglected the conductance in Eq. (2) to obtain a hyperbolic wave equation expressed in dimensionless (per unit) quantities as follows:

$$u_x(x,t) - \chi^2 i_t(x,t) = \eta i(x,t) \tag{25}$$

$$u_t(x,t) - i_x(x,t) = 0 \tag{26}$$

In these equations, $u = -cv(x, t)$ and $\eta = rc$. This pair of equations has two characteristics: $t - \chi x$ and $t + \chi x$. These characteristics are the parallel lines in the position–time plane. Examples of two such lines are given in Fig. 3. The length along the two characteristics is denoted ρ and s, re-

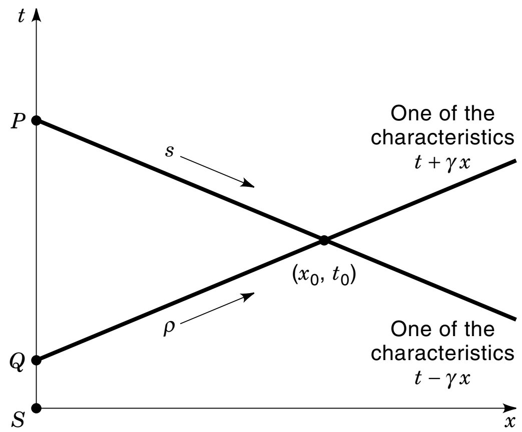

Figure 3. Characteristics in the dimensionless position–time plane.

spectively. Along a characteristic, functions u and i are related by the following two differential equations:

$$\begin{aligned} \frac{du}{ds} - \chi\frac{di}{ds} &= (1+\chi^2)^{-0.5}\,\eta i \\ \frac{du}{d\rho} + \chi\frac{di}{d\rho} &= -(1+\chi^2)^{-0.5}\,\eta i \end{aligned} \tag{27}$$

These two equations may be solved numerically using the method of meshes described in Ref. 9. The solution is obtained using the sending-end voltage and current as the boundary conditions. It is important to note that the value of the voltage $v(x_0, t_0)$ does not depend on all the values of the sending-end voltages and currents. The voltage depends only on the boundary conditions in just one segment of time. To find this segment, it is necessary to identify two characteristics passing through the point (x_0, t_0) (see Fig. 3). These two characteristics intersect the t axis at the two points P and Q. Only the values of t between these two points affect the value of $v(x_0, t_0)$. This time interval is called the zone of influence.

The fault location is found by an inspection of the voltage along the line by using a property of the voltage. If the fault resistance is zero, as in Kohlas's paper, then the value of the voltage at the fault must be equal to zero. Accordingly, the location of the fault is equal to that value of x that annihilates the voltage at any time t. When the measurements contain noise, or when the fault impedance has a low but still nonzero value, one cannot expect the exact cancellation of the voltage $v(x, t)$ but rather a minimal value in some sense. Thus, when the solution for $v(x, t)$ is found, the next task is to look for the value of x at which the voltage is minimal. The problem here is that voltage depends both on the distance x and time t. Instead of inspecting the voltage as a function of time and distance, Kohlas proposed to inspect the function of distance $F(x)$ that is defined as the square of the voltage averaged in a specific time interval determined by the zone of influence:

$$F(x) = \int_{\gamma x}^{T-\gamma x} v^2(xt)\,dt \tag{28}$$

The value of x that minimizes the function $F(x)$ is the estimate of the distance to the fault. The Kohlas idea was subsequently extended and elaborated in detail for the three-phase transmission lines in Ref. 11. In this reference, the three-phase transmission line is described by two matrix equations:

$$\boldsymbol{V}_x = \boldsymbol{L}\boldsymbol{I}_t + \boldsymbol{R}\boldsymbol{I} \tag{29}$$

$$\boldsymbol{I}_x = \boldsymbol{C}\boldsymbol{V}_x \tag{30}$$

where the subscripts x and t denote partial derivatives.

The matrices $\boldsymbol{L}$, $\boldsymbol{C}$, and $\boldsymbol{R}$ have both diagonal and off-diagonal terms. Therefore, the preceding matrix equations cannot be solved using methods described by Kohlas. In addition, the elements of these matrices depend on the transmission-line geometry and copper resistance only if the ground is not used as a return. However, if the line is grounded, the matrices depend on the soil conductivity also. This parameter may depend on the weather and type of soil and cannot be easily determined. To complicate the matter further, as a repercussion, the line parameters then become frequency dependent. Fortuitously, the two matrix partial differential equations reduce to three pairs of decoupled partial differential equations similar in form to Eq. (15) by applying modal transformation as reported in Ref. 11.

Modal transformation starts with finding three eigenvectors of the matrix product $\boldsymbol{LC}$. These vectors are columns of the transformation matrix $\boldsymbol{M}_1$. The transpose of the matrix $\boldsymbol{M}_1$ is $\boldsymbol{M}_2$. The phasor voltages and currents $\boldsymbol{V}$ and $\boldsymbol{I}$ are transformed into modal voltages and currents $\boldsymbol{V}^{(m)}$ and $\boldsymbol{I}^{(m)}$ using the following equations:

$$\boldsymbol{V}^{(m)} = \boldsymbol{M}_1^{-1}\boldsymbol{V} \tag{31}$$

$$\boldsymbol{I}^{(m)} = \boldsymbol{M}_2^{-1}\boldsymbol{I} \tag{32}$$

The matrices $\boldsymbol{R}$, $\boldsymbol{L}$, and $\boldsymbol{C}$ are also transformed to modal matrices $\boldsymbol{R}^{(m)}$, $\boldsymbol{L}^{(m)}$, and $\boldsymbol{C}^{(m)}$:

$$\begin{aligned} \boldsymbol{R}^{(m)} &= \boldsymbol{M}_1^{-1}\boldsymbol{R}\boldsymbol{M}_2 \\ \boldsymbol{L}^{(m)} &= \boldsymbol{M}_1^{-1}\boldsymbol{L}\boldsymbol{M}_2 \\ \boldsymbol{C}^{(m)} &= \boldsymbol{M}_1^{-1}\boldsymbol{C}\boldsymbol{M}_2 \end{aligned} \tag{33}$$

The particular feature of modal matrices is that their off-diagonal terms are equal to zero. Indeed, the modal transformation has the same advantage as the symmetrical component transformation. Actually, if a line is fully transposed, the symmetrical component transformation or the Clarke transformation will have the same decoupling outcome as the modal transformation. After the application of modal transformation, the transmission-line model consists of three decoupled pairs of linear partial differential equations:

$$\begin{aligned} \frac{\partial v_{kk}^{(m)}}{\partial x} + l_{kk}^{(m)}\frac{\partial i_{kk}^{(m)}}{\partial x} &= r_{kk}^{(m)}\, i_{kk}^{(m)} \\ c_{kk}^{(m)}\frac{\partial v_{kk}^{(m)}}{\partial t} + \frac{\partial i_{kk}^{(m)}}{\partial x} &= 0 \end{aligned} \tag{34}$$

Here the subscript $k = 1{,}2{,}3$ denotes three modes, and superscripts x and t denote partial derivatives. One of the modes, known as the aerial mode, has parameters that are least dependent on frequency. Usually, only the aerial mode is considered for the fault location. Once a mode is selected, the procedure for the transmission-line model solution is the same as that for the one-phase transmission line.

ADVANCED APPROACHES: TRAVELING-WAVE-BASED METHODS

Traveling-wave methods do not require the solution of partial differential equations. In this approach, the line resistance r is neglected as is the line conductance c. Such a line is known as a lossless transmission line, and the describing equation is known as the telegrapher's equation. A simplification of this kind is appropriate for long and high-voltage transmission lines. The solution of the two equations then has a rather simple form. The voltage and the current are linear combinations of two components known as forward and backward traveling waves and denoted S_F and S_B, respectively:

$$v(x, t) = [S_F(t - \chi x) + S_B(t + \chi x)]/2 \quad (35)$$

$$i(x, t) = [S_F(t - \chi x) - S_B(t + \chi x)]/2Z_0 \quad (36)$$

where $Z_0 = \sqrt{l/c}$ is the surge impedance of the line and $\eta^2 = lc$.

The forward and backward traveling waves may be calculated from the sending-end voltage $v(0, t) = v_S(t)$ and the sending-end current $i(0, t) = i_S(t)$ as follows:

$$S_F(t) = v_S(t) + Z_0 i_S(t) \quad (37)$$

$$S_B(t) = v_S(t) - Z_0 i_S(t) \quad (38)$$

Fault location uses the transient component of the traveling waves only. The transient traveling waves appear in the transmission line after any abrupt change of its voltages and currents. When a fault occurs, the voltage at the fault point drops. This generates a backward and a forward traveling wave at the place of the fault. The backward wave travels to the sending end with a speed η^{-1}, and the forward wave moves to the receiving end with the same speed.

These traveling waves do not change their shape until they reach some discontinuity in the transmission line. The discontinuities are the sending end, the receiving end, and the fault itself. When a traveling wave arrives at a discontinuity, it ceases to exist in its original form, and two new waves emerge at the discontinuity. The first is a reflection of the original wave; it has the shape of the original wave attenuated by a reflection coefficient, and it has a reverse direction. That is, a reflection of the forward wave will be a backward wave. The second wave discussed here, *through wave,* also has the shape of the original wave attenuated by another coefficient and continues motion in the same direction as the original wave. The coefficients affecting magnitudes of both new waves depend on the type of fault. Low impedance faults have high coefficients of reflection, and high impedance faults have low coefficients of reflection.

The motion of traveling waves along the transmission line and generation of new waves at points of discontinuity are represented by the lattice diagram in the Fig. 4. The initial wave arises at the fault point F. The backward wave reaches the sending end at a time t_1. Its reflection moves as a forward wave toward the fault. At the fault, it is reflected again and converted to a backward wave. It will arrive at the sending end at a time t_2. The time that elapses between the first reflection and the second reflection $\Delta t = t_2 - t_1$ depends on the distance to the fault x and the speed of travel:

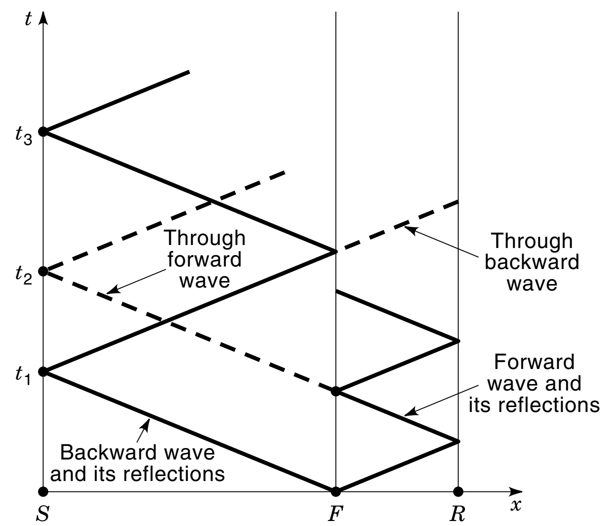

Figure 4. Lattice diagram.

$$\Delta t = 2z \times \chi \quad (39)$$

The idea to use reflections to estimate the fault location appeared in 1930 for the fault location of underground cables. A cable is energized with a short voltage impulse. The impulse and its reflection are recorded, and the travel time is found. Later, similar devices were used to measure the fault location for transmission lines. These methods are called active methods.

The calculation of the elapsed time is easy if the inserted pulse and its reflection have sufficient power. However, traveling waves caused by a fault may have a low power, especially if the fault occurs when the instantaneous voltage at the point of the fault is close to zero. In that case the calculation of this time requires special signal processing. One of the signal-processing methods most commonly used is the correlation technique (12).

The time autocorrelation of the signals $x(t)$ is defined as

$$R(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)\,dt \quad (40)$$

In real situations, the integration has to start and end with some finite time.

$$R(\tau) = \frac{1}{T} \int_{0}^{T} x(t)x(t+\tau)\,dt \quad (41)$$

For a given signal, autocorrelation is a function of the time shift τ. Consider a typical shape of a traveling wave at the sending end, as shown in the Fig. 5(a) and its time-shifted value shown in Fig. 5(b). The autocorrelation is proportional to the area of the product of two signals. This area will be largest when the first reflection is aligned with the second reflection as in Fig. 5(c). Then, the time shift is equal to the elapsed time $t_2 - t_1$). Therefore, the elapsed time may be assessed by investigating the maxima of the autocorrelation function.

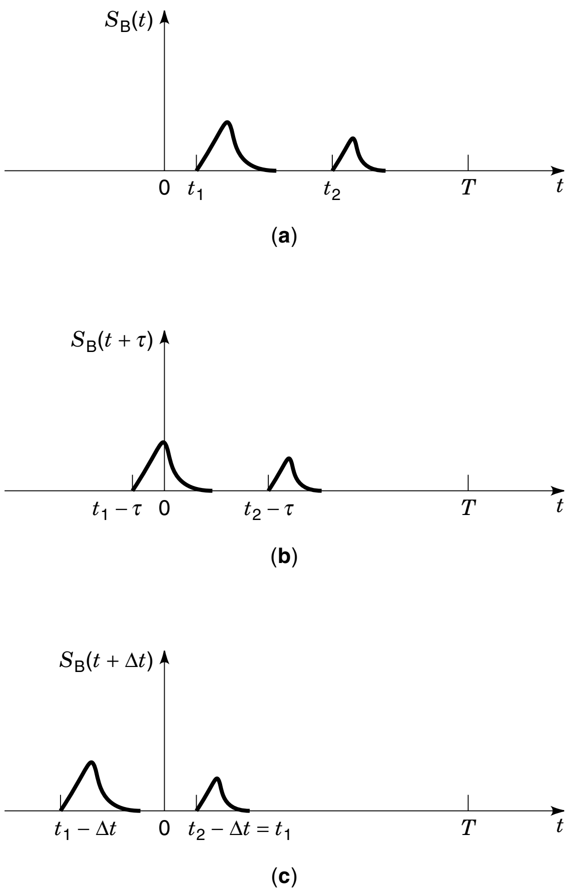

Figure 5. (a) Typical backward wave. (b) Shifted backward wave. (c) The product of $S_B(t)$ and $S_B(t + r)$ is maximum when $\tau = \Delta t$ and second and first reflections are aligned.

In fault-location algorithms, the digital version of the autocorrelation function $\Phi(k)$ is calculated using N samples of the signals taken with a frequency f_c and denoted here as $x(i)$:

$$\phi(k) = \sum_{k=1}^{N} x(i)x(i+k) \tag{42}$$

The accuracy of the fault location is very sensitive to the choice of T and N. If T is too small, the approximation is not good since an important part of the signal may be missing. On the other hand, if T is too large, the shape of the forward wave will contain multiple reflections of both the original backward and the original forward wave. For example, such a reflection will appear at time t_3 in the lattice diagram. Also, in nonsymmetrical faults, a fraction of a traveling wave in one mode may appear in another mode. Therefore, the autocorrelation will have more maxima, and the identification of the maxima corresponding to the first reflection and second reflection will be difficult. In general, the closer the fault to the sending end, the shorter the window is needed. The other important factor is the sampling frequency. In general, a very high sampling frequency (on the order of tens of kilohertz) is needed to ensure a good approximation of the autocorrelation function.

The limitations of this approach are (1) a lack of firm rules in the selection of the sample window due to its sensitivity to the fault distance, (2) the possibility of obtaining a false result due to the presence of multiple reflections, and (3) a high sampling frequency, increasing the computational burden.

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FAULT TOLERANCE. See GROUP COMMUNICATION; ROUNDOFF ERRORS.