MAGNETIC MODELING

THE CLASSICAL PREISACH FRAMEWORK

The first hysteresis model that recognized that in order to describe minor loop behavior, the model must have memory, was described by the Hungarian born physicist, Franz (Ferenc) Preisach, in his landmark paper almost 60 years ago (1). However, the novelty of Preisach's approach was not recognized and applied to describe magnetic hysteresis until the late 1950s (2). It is noted that an approach identical to Preisach's was independently discovered and developed for many years to describe adsorption hysteresis (3), without recognizing the similarities with Preisach's original approach. The unified formal mathematical treatment of hysteresis in general has been defined recently (4).

In the Preisach method, each elementary particle has a rectangular hysteresis loop and, as an isolated particle has

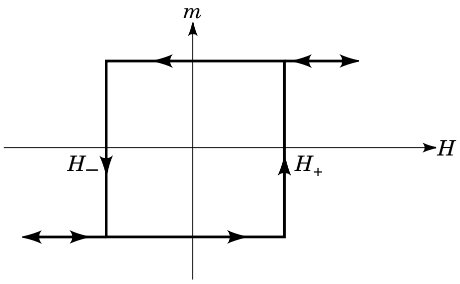

Figure 1. Classical Preisach representation of an isolated particle.

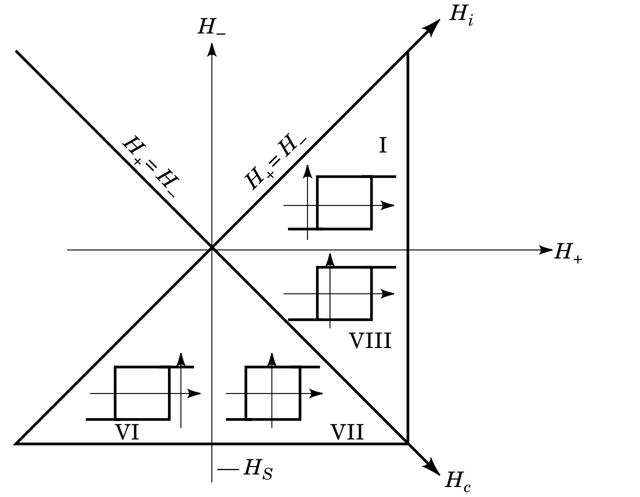

Figure 3. Representative elementary-particle hysteresis loop in each octant of the physically realizable region of the classical Preisach plane.

"up" and "down" switching fields, H_+ and H_- are equal in magnitude, as illustrated in Fig. 1. Since energy is dissipated during hysteresis, the hysteresis loop is always traversed in the counter clockwise direction; therefore, $H_+ \geq H_-$ for all particles in the medium. From a physical point of view the "up" and "down" saturated states are identical; therefore, the magnitude of the magnetization, m, in these two states is the same.

The Preisach method is a hysteresis model to describe the behavior of a collection of interacting particles with different up and down switching fields. In an assembly of particles, the field that an individual particle "sees" is not the external field but the sum of the external field and the interaction H_i due to other particles in the medium. In the classical Preisach representation, the effect of the interaction field H_i is to shift the hysteresis of the particle by H_i, as illustrated in Fig. 2.

The classical Preisach technique uses a statistical approach to describe the hysteretic many-body problem. The method assumes that the magnetic medium is composed of a continuum of "elementary particles," each of which characterizes the average behavior of an ensemble of particles. The *Preisach calculation plane* (also called the Preisach plane) takes the up and down switching fields as the coordinate axes. A point on the Preisach plane with coordinates (H_+, H_-) corresponds to the particle whose elementary hysteresis loop switches up and down at H_+ and H_-, respectively. Magnetic materials exhibit saturation-type hysteresis; therefore, it is reasonable to assume that all elementary particles switch to their up state above a positive saturating field H_S and switch down below a negative saturating field, $-H_S$. The choice of the value of H_S depends on the material that is being characterized. With these considerations, the physical region of the Preisach plane and representative elementary particles in it are shown in Fig. 3. We will call regions I, VI, VII, and VIII the first, second, third, and fourth quadrants of the Preisach plane, respectively.

The *normalized Preisach density function,* or Preisach function for short, $p(H_+, H_-)$ is a three-dimensional density function defined over the Preisach plane, as illustrated in Fig. 4. The value of $p(H_+, H_-)$ is the fraction of the magnetization contribution by the point (H_+, H_-) to the total normalized magnetization m. The density function is zero outside the Preisach plane and, from magnetic symmetry, $p(H_+, H_-) = p(-H_-, -H_+)$. Thus,

$$\iint_{H_+ \geq H_-} p(H_+, H_-)\, dH_+ dH_- = 1 \tag{1}$$

In most cases one simply illustrates the Preisach function by its contour plot or by a representative curve of the contour plot.

The normalized Preisach function can also be defined as $p(H_i, H_c)$, where H_i is the interaction field and H_c is the critical

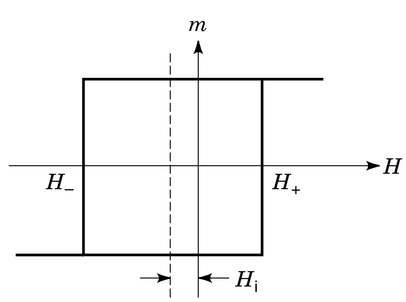

Figure 2. Classical Preisach representation of a particle in the presence of an interaction field H_i.

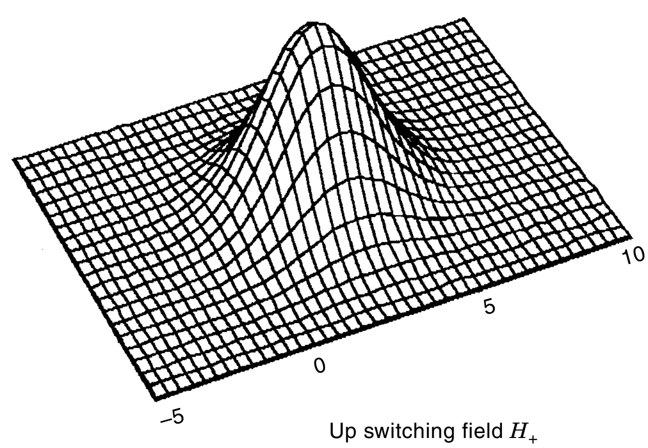

Figure 4. Illustration of the normalized Preisach function in the (H_+, H_-) coordinate system.

field, and normalization requires that

$$\int_{H_c>0}\int_{-\infty}^{\infty} p(H_i, H_c)\, dH_i dH_c = 1 \tag{2}$$

In magnetic media, the critical field distribution and the interaction field distribution are independent of each other, that is,

$$p(H_i, H_c) = p_i(H_i)p_c(H_c) \tag{3}$$

The (H_i, H_c) and (H_+, H_-) coordinate systems are related by

$$H_i = \frac{H_+ + H_-}{2}, \qquad H_c = \frac{H_+ - H_-}{2} \tag{4}$$

and

$$H_+ = H_i + H_c, \qquad H_- = H_i - H_c \tag{5}$$

It is often convenient to use one or the other of these two classical Preisach coordinate systems.

From a control point of view, the classical Preisach model can also be viewed as a parallel connection of rectangular two-state hysterons (4) with a distribution of up and down switching fields, as illustrated in Fig. 5.

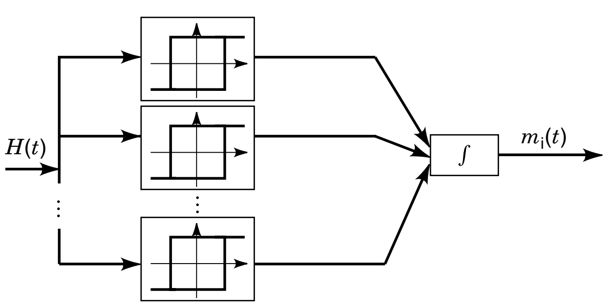

Figure 5. Classical Preisach model as a collection of parallely connected rectangular hysterons with a distribution of up and down switching fields.

CALCULATION OF THE MAGNETIZATION USING THE CLASSICAL PREISACH MODEL

We will now show how the classical Preisach model is used to compute the magnetization due to the applied field sequence illustrated in Fig. 6(a). The magnetization state of a hysteretic system depends on the magnetization history; therefore, the initial magnetization state of the system has to be known. For this example, we will assume that the initial state is negative saturation; other types of initial states will be discussed later. Using the classical Preisach representation, negative saturation means that every elementary particle on the Preisach plane is in its down state.

Let us first compute the magnetization at $H = H_*$, illustrated in Fig. 6(b). As discussed previously, the location of each elementary particle on the classical Preisach plane is determined by its up switching field H_+ and its down switching field H_-. Thus, elementary particles to the left of the vertical line that intersects the H_+ axis at H_* will switch to their up state, indicated by the + sign, since they all have up switching fields less than the applied field H_*. Elementary particles to the right of the vertical line have up switching fields greater than the applied field; therefore, they will remain in their previous magnetization state. In this example, since the initial state was negative saturation, particles in this region will remain in their negative state, as indicated by the − sign.

The case when the field reaches $H = H_1$ is illustrated in Fig. 6(c). The calculation is similar to that described previously, but since the applied field is larger, the positively magnetized region, indicated by the + sign, has grown further at the expense of the negatively magnetized region, indicated by the − sign.

As the field is reduced from H_1, let us consider the case when $H = 0$, illustrated in Fig. 6(d). At this field, all elementary particles above the horizontal line corresponding to the applied field $H = 0$, which in this case coincides with the H_+ axis, will switch to their down state, since their switching field is greater than the applied field. All particles below the line will remain in their previous magnetization state. Now, since the "medium" has been exposed to a magnetization history, the positively and negatively magnetized regions are separated by a staircase line.

The case when the field reaches $H = H_2$ is illustrated in Fig. 6(e). The calculation is similar to that described previously, but since the applied field was further reduced, the negatively magnetized region, has grown at the expense of the positively magnetized region.

The classical Preisach plane corresponding to the final state of the magnetizing process at $H = H_3$ is illustrated in Fig. 6(f). It is seen that the positively and negatively magnetized regions are separated by a staircase line. The coordinates of the two vertices of the staircase line are (H_1, H_2) and (H_3, H_2).

Thus, the normalized magnetization m, through use of the classical Preisach model, is computed by

$$nm = \iint_{R_+} p(H_+, H_-)\, dH_+ dH_- - \iint_{R_-} p(H_+, H_-)\, dH_+ dH_- \tag{6}$$

where R_+ and R_- denote the positively and negatively magnetized regions of the Preisach plane, respectively.

One may also write the preceding Preisach integral as

$$nm = \iint_{H_+ \geq H_-} Q(H_+, H_-)p(H_+, H_-)\, dH_+ dH_- \tag{7}$$

where the state variable, $Q(H_+, H_-)$, takes on the value +1 and −1 in the positively and negatively magnetized regions, respectively.

As shown in Fig. 1, the magnetization of an elementary Preisach particle switches discontinuously at the particle's up and down switching field. However, since the Preisach plane is comprised of a distribution of such elementary particles, the magnetization computed by the classical Preisach model is in general a smooth function of the applied field. The smoothness of the computed hysteresis curve depends on the number of field points at which the magnetization is computed and the resolution with which the Preisach density

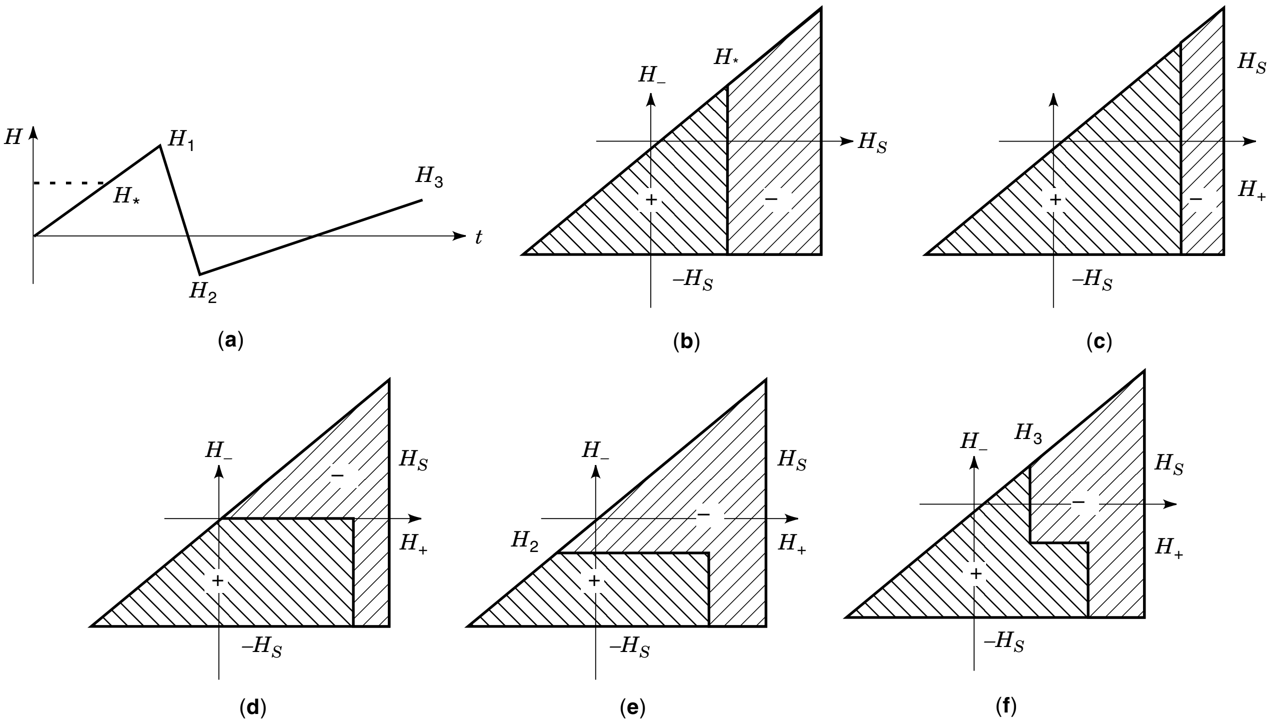

Figure 6. Applied-field sequence to illustrate magnetization calculation with the classical Preisach model, starting from (a) an initial magnetization state of negative saturation. Division of Preisach plane at (b) $H = H_*$, (c) $H = H_1$, (d) $H = 0$, (e) $H = H_2$, and (f) $H = H_3$.

function is discretized. Computational issues of the model, which are beyond the scope of this article, are discussed in Ref. 5.

The hysteresis loop computed by the classical Preisach corresponding to the applied field sequence of Fig. 6 is illustrated in Fig. 7.

PROPERTIES OF THE CLASSICAL PREISACH MODEL

Irreversible, Locally Reversible, and Apparent Reversible Magnetization

With use of Fig. 3, it is seen that the model assumes that the elementary particles comprising the medium have perfectly square hysteresis loops. This means that the magnetization of each particle is always in its saturated state at $\pm m_S$ and jumps discontinuously from one state to the other at the particle's switching fields. In other words, since the magnetization of each elementary particle is constant until its switching field is reached, the magnetization is purely irreversible.

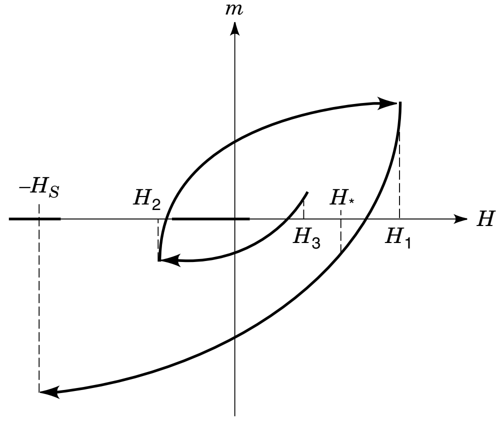

Figure 7. Illustration of the hysteresis curve corresponding to the magnetizing process of Fig. 6 using the classical Preisach model.

The locally reversible magnetization, which is defined using the stored energy in the system, is discussed in detail in Ref. 6. However, at this point it needs to be pointed out that during a locally reversible process, no energy is dissipated, that is, the locally reversible magnetization component stores the energy supplied to it by the field and returns it when the field is reversed. Furthermore, due to energy considerations, the reversible magnetization has to be zero when the applied field is zero. Since the magnetization of the elementary particles of the classical Preisach model is either constant or switches hysteretically, it does not describe locally reversible magnetizing processes.

Let us again consider the magnetizing process illustrated in Fig. 6(a), the corresponding intermediate Preisach states shown in Fig. 6(b)–(f) and hysteresis curve given by Fig. 7. We will first focus on the portion of the process where $0 < H < H_1$. A representative Preisach state of this process is illustrated in Fig. 8. It is seen that, as the applied field is reduced from H_1 towards zero, only the cross-hatched region in the first quadrant is switched, since this is the only region of the Preisach plane with elementary particles whose down switching field is positive. Thus, although each elementary Preisach particle has a rectangular hysteresis curve, the appropriate portion of the computed hysteresis curve of the collection of such particles will be descending, as illustrated in Fig. 7. Similarly, for the portion of the magnetizing process where

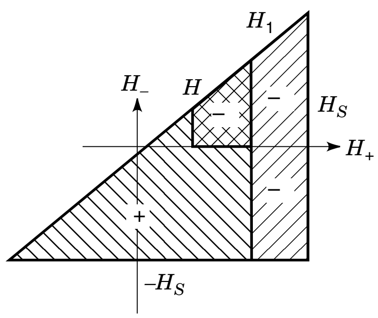

Figure 8. Preisach state of the magnetizing process in Fig. 6(a) for the case $0 < H < H_1$.

$H_2 < H < 0$, the computed hysteresis curve will be ascending due to the contribution of elementary Preisach particles in the third quadrant, as illustrated in Fig. 7.

Let us consider the two magnetizing processes illustrated by the dashed line and the solid line in Fig. 9. The final magnetization state corresponding to the solid lines was given in Fig. 6(f). Using the method presented previously to calculate the magnetization, it is easy to show that the final magnetization corresponding to dashed lines is identical to that of the solid lines. However, it is noted that at any intermediate field value, such as $H = H_*$, the magnetization corresponding to the two processes will be different. Furthermore, it also directly follows from the Preisach method of calculating the magnetization that the magnetization computed by the two processes will be identical at the local extrema H_1, H_2, and H_3. This property, referred to as rate independence, means that the calculated magnetization depends only on the local extrema of the applied field sequence.

Let us now consider the applied field sequence illustrated in Fig. 10(a). The positively and negatively magnetized regions of the Preisach plane corresponding to the local maximum H_1 is shown in Fig. 10(b). The Preisach plane corresponding to the local minimum H_2 is shown in Fig. 10(c). The Preisach plane corresponding to the local maximum H_3, whose magnitude is greater than H_1, is shown in Fig. 10(d). It is seen that this field has deleted the effect of the previous smaller local maximum. In general, it directly follows that each local maximum in the applied field deletes the vertices of the staircase line separating the positively and negatively magnetized regions of the Preisach plane, whose corresponding H_+ coordinates are below this local maximum. Similarly, each local minimum in the applied field deletes the vertices of the staircase line separating the positively and negatively magnetized regions of the Preisach plane, whose corresponding H_- coordinates are above this local minimum. This property of the classical Preisach model, which will be referred to as the deletion property, means that a larger applied field completely erases the effect of previous smaller fields. In other words, this property means that a recording system modeled by the classical Preisach model has perfect overwrite.

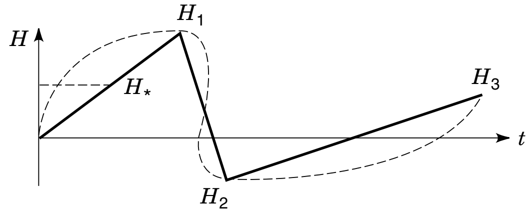

Figure 9. Illustration of rate independence of magnetization calculated by the classical Preisach model.

In general, it is now seen that at any field value H, the classical Preisach plane consists of a negatively and a positively magnetized region that are separated by a staircase line, as illustrated in Fig. 11. The vertices of this staircase line have H_+ coordinates that correspond to previously undeleted local maxima and H_- coordinates that correspond to previously undeleted local minima. The region below the staircase line is magnetized positively; the region above the staircase line is magnetized negatively. The magnetization of the region indicated by "u" will be unaffected by the magnetizing process; therefore, the magnetization of this region is determined by the *initial magnetization state* of the system. In other words, the classical Preisach model stores the magnetization history in the staircase line whose shape depends on the applied field history. Thus, we will call the staircase line corresponding to a particular value of the applied field and a given initial state the *classical Preisach state of the system.* In some cases it may be convenient to represent the classical Preisach state of the system in terms of the state variable $Q(H_+, H_-)$; however, there is obviously a one-to-one correspondence between this and the staircase line.

Let us consider the magnetizing process illustrated by the solid lines in Fig. 12(a). Starting at negative saturation, the field is increased to H_1, where it is reversed. The field is then reduced to H_A, where it is again reversed. Finally, after reaching H_B, the field is cycled between H_A and H_B, traversing a minor loop. The Preisach plane corresponding to this process is illustrated in Fig. 12(b). It is seen that, when traversing this minor loop, only the double shaded triangular region enclosed by H_A, H_B, and the H_i axis of the Preisach plane is switched. It is seen that in the classical Preisach model, a minor loop becomes stable after the first reversal at H_B. Upon subsequent cycling between H_A and H_B, the same minor loop is traversed. It is also seen that

$$m_B - m_A = 2\int\int_R p(H_+, H_-)\, dH_+ dH_- \tag{8}$$

where m_A and m_B denote the magnetization at H_A and H_B, respectively, and the region of integration, R, is the triangular region enclosed by H_A, H_B, and the H_i axis.

Let us now consider the magnetizing process illustrated by the dashed lines in Fig. 12(a). Starting at negative saturation, the field is increased to H_2, where it is reversed and is cycled between H_A and H_B, traversing a minor loop, as discussed previously. The Preisach plane corresponding to this process is illustrated in Fig. 12(c). Similarly to the previous case, only the triangular region enclosed by H_A, H_B, and the H_i axis of the Preisach plane is switched during the traversal of this minor loop. In fact, this region is identical to the corresponding region in Fig. 12(b). Furthermore, by comparing the corresponding regions of Fig. 12(b) and (c), it is seen that the identically shaded negatively magnetized regions are also identical. The positively magnetized region of Fig. 12(c) is greater than that of Fig. 12(b) and the negatively magnetized

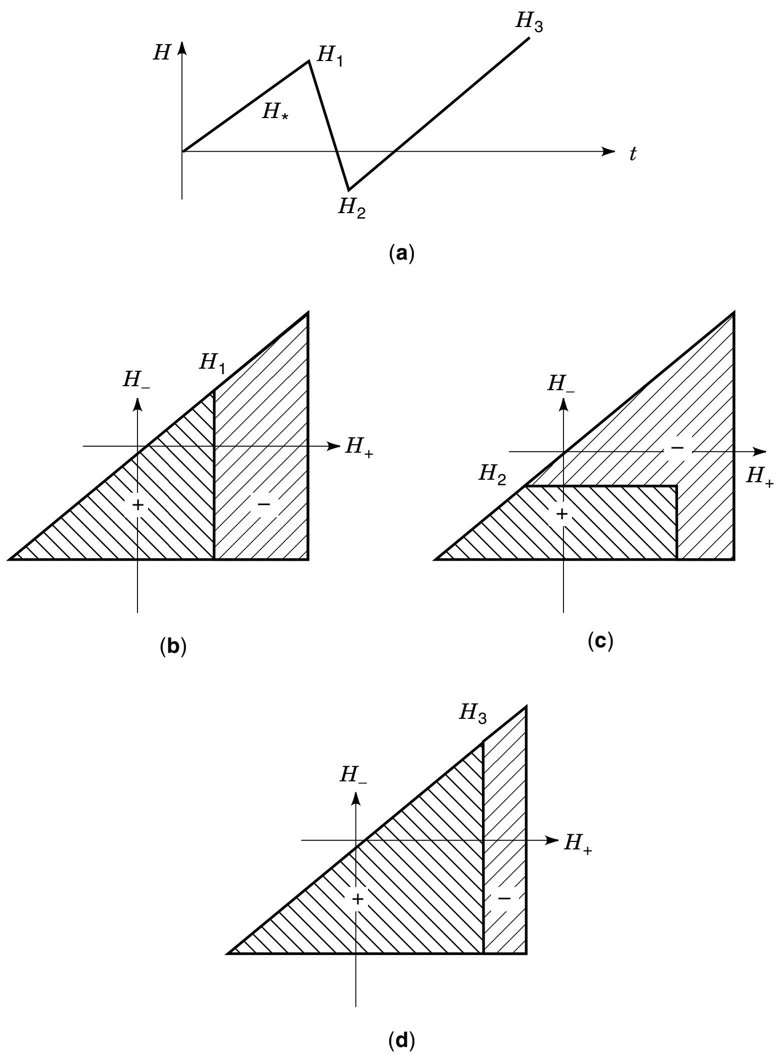

Figure 10. Applied-field sequence illustrating (a) the deletion property of the classical Preisach model. (b) Preisach plane at the local maximum H_1. (c) Preisach plane at the local minimum H_2. (d) Preisach plane at the local maximum H_3, which has deleted the effect of H_1.

region is less than that of Fig. 12(b). Thus, it is seen that the magnetization at *any* field H after the initial reversal point H_1, corresponding to the process illustrated by the solid lines will have a magnetization that is a constant less than the magnetization at the same field H, corresponding to the magnetizing process illustrated by the dashed lines. This constant difference in magnetization is given by

$$\Delta M = M(H_2) - M(H_1) \tag{9}$$

where $M(H_2)$ and $M(H_1)$ denote that magnetization at the reversal points H_2 and H_1, respectively.

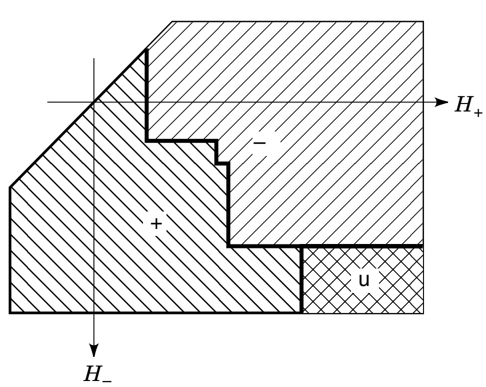

Figure 11. Illustration of positively and negatively magnetized regions and the staircase line separating these regions. The region whose magnetization is unaffected is indicated by "u."

In general, it is now seen that minor loops computed by the classical Preisach model close after the first traversal of the minor loop. In other words, minor loops computed by the classical Preisach model *do not accommodate.* We can also see that minor loops obtained by cycling between the same of pair of field extrema but originating at different initial reversal points, such as those illustrated in Fig. 12(a), will have identical shapes, in other words, minor loops computed by the classical Preisach model are *congruent* to each other.

In the previous paragraphs, we have shown that the classical Preisach model possesses the deletion property of applied-field extrema and that minor loops are congruent. It has been mathematically proven that these two properties form *the necessary and the sufficient conditions* for a process to be representable by the classical Preisach model (3).

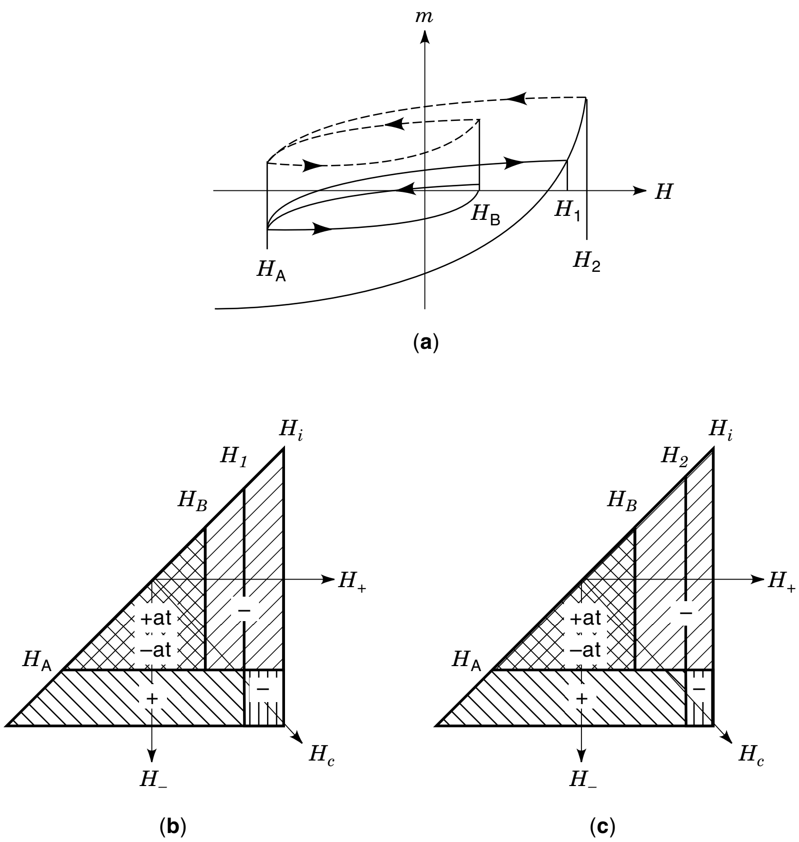

Figure 12. (a) Magnetizing process illustrating congruent minor loops computed by the classical Preisach model. (b) Preisach plane corresponding to reversal at H_1 and cycling between H_A and H_B. (c) Preisach plane corresponding to reversal at H_2 and cycling between H_A and H_B.

IMPROVED PREISACH-BASED MODELS

Since most magnetic materials do not exhibit some of the properties of the classical Preisach model, in order to develop a physically derivable model is necessary to extend it. We will only discuss physically derived extensions of the model computing static magnetic hysteresis. Physically derived dynamic extensions of the model have been developed by G. Bertotti et al. A detailed description of mathematically motivated extensions to the model are discussed in detail in Ref. 7.

A fundamental assumption of the classical Preisach model is that a constant Preisach function exists for the material to be modeled. However, since this function is a statistical description of the interaction field distribution and the critical field distribution, and the interaction field is a function of the magnetization state of the particles, the question arises whether a "stable" Preisach function exists. An analytical study of interacting Stoner–Wohlfarth particles showed that, if in addition to the up and down switching fields, the Preisach function is also a function of the magnetization, then the physically derived Preisach function is always statistically stable. Furthermore, it was shown that the standard deviation of this Preisach function is constant and its expected value is linearly proportional to the magnetization. This model, called the moving model (8) is a physically derived Preisach-based hysteresis model. As shown in Fig. 13, the moving model is a magnetization-dependent Preisach model with positive feedback. The feedback constant is the material-dependent moving constant whose value depends on the shape, orientation, and reversal mode of the particles that make up the medium.

Another fundamental limitation of the Preisach model is that since the elementary hysterons that make up the model are rectangular, the model only computes the irreversible component of the magnetization. For a detailed description of the irreversible, reversible, and locally reversible magnetization, and the energy relations in Preisach-based hysteresis models, which are beyond the scope of this article, see Ref. 6. In the complete moving hysteresis (CMH) model (9), each point on the Preisach plane is represented by a more realistic, nonrectangular hysteresis loop, as illustrated in Fig. 14. This realistic hysteron can be broken up into a rectangular classical Preisach-like hysteron computing the irreversible magnetization and to a single-valued nonlinear function computing the locally reversible component of the magnetization. In order to give the model a physical foundation, the CMH model includes the material-dependent moving parameter, as discussed previously.

SUMMARY

Hysteresis models based on the classical Preisach model have recently evolved to the point where they can be utilized to

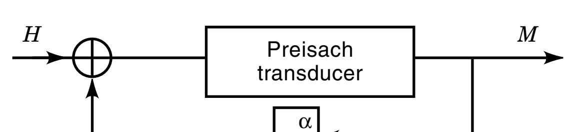

Figure 13. Functional block diagram of the moving Preisach model. The feedback is the moving parameterα, and the box denoted *Preisach transducer* is shown in detail in Fig. 5.

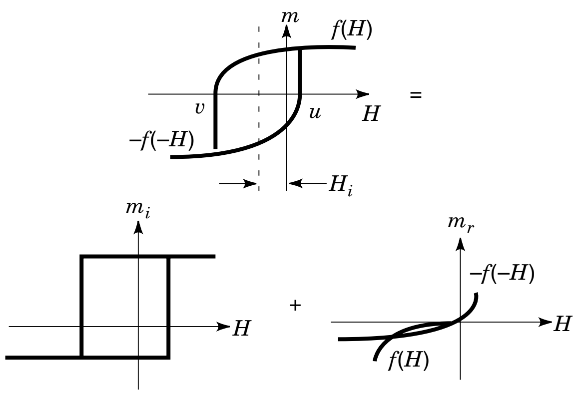

Figure 14. Elementary Preisach particle in the CMH model. Decomposition of an interacting hysteron into the sum of a purely irreversible and a purely locally reversible component.

model, predict, and explain experimental phenomena that cannot be described using any other modeling technique. This article was limited to describing the fundamentals of the model. For a detailed description of hysteresis models, including vector hysteresis modeling, modeling of accommodation and aftereffect and robust identification methods to measure the model parameters, see Ref. 10.

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