SUPERCONDUCTING ELECTROMAGNETS

The design and construction of superconducting magnets has been made possible by the development of technical superconductors. There are three principal materials: the alloy niobium-titanium (NbTi), the intermetallic compound niobium-tin (Nb_3Sn), and the collective high-temperature superconductors (HTS) based on copper oxide layers in a perovskite structure. Nb_3Sn and NbTi were developed in the early 1960s. A small solenoid made of Nb_3Sn achieved a field of 10 T in 1963, and the alloy NbTi was also first examined as a wire for a superconducting magnet in that year. In the nearly four decades that have elapsed since those first steps, superconducting magnets weighing up to many hundreds of tonnes have been built and operated.

Technical superconductors are a class having special properties that allow superconductive operation in high magnetic fields and with useful current densities. Superconductors are of two types; both can support the flow of electrical current without resistance only below a combination of maximum temperature, field, and current density, the critical parameters being T_c, B_c, and J_c (1). For Type I superconductors, typical values of T_c and B_c are 9 K and 0.1 T, respectively. Flux is excluded from the bulk of a Type I superconductor, and current flows only in a surface layer, about 10^{-4} mm thick. The current is carried by electron pairs of equal and opposite momentum. (This momentum includes terms arising from both velocity and linked flux.) Since flux is excluded from the interior of a Type I superconductor, and current flows only on the surface, a useful overall current density can only be obtained in wires of a diameter unacceptably small for magnet construction. Consequently a Type I superconductor cannot be used to construct a useful magnet, even were the critical field to be higher than 0.1 T. Examples of Type I superconductors are lead, indium, mercury, and pure annealed niobium.

By contrast, Type II superconductors allow flux to penetrate into the bulk of the lattice in the form of an array of flux quanta. A circulating supercurrent surrounds each flux quantum (2.07×10^{-15} V·s) (2). If a net transport current flows along a Type II superconducting wire, a density gradient of flux quanta exists across the wire. The flux density gradient is given by

$$\frac{dB}{dx} = -\mu_0 J_c \quad (1)$$

where J_c is a bulk critical current density and μ_0 is the permeability of free space.

However, a Type II superconductor is not necessarily a technical superconductor. The interaction of the net transport current flow with the flux quanta creates a Lorentz force that causes a migration of the flux across the superconductor. Such a migration appears as a voltage gradient along the superconductor, which causes a dissipation; that is, the superconductor appears to be resistive. For these reasons pure annealed Type II superconductors are resistive with a transport current in the presence of a field. Because of the penetration of flux, the maximum temperatures and fields below which superconductivity arises in them are up to 18 K and 25 T, respectively. The latter is the upper critical field, B_{c2}, at which the lattice is fully packed with flux quanta. The penetration of flux into the lattice begins at a lower critical field, B_{c1}.

The technical superconductor is a Type II superconductor in which this migration of flux quanta is inhibited. This is achieved by the introduction of lattice defects, in the form of crystal boundaries, impurities, vacancies, or dislocations (3). At these defects flux quanta tend to become pinned against further movement, at least until the Lorentz force increases.

To the extent that mechanical models assist in the understanding of quantum effects, a typical pinning center may be imagined as a long hole through the lattice of the superconductor. If a flux quantum, or an integral number of flux quanta, were located in this hole, the associated circulating current would lie in the material around the hole. If now the flux quantum were to move away from the hole, the current would have to become elongated so as to continue to flow in solid material. That would increase the interactive force between flux and current, so tending to force the flux back into the hole. The movement of flux quanta through an array of pinning centers is also influenced by thermal vibration. Since the temperature at which electron pairs can form is low, thermally induced lattice vibration is comparable in energy density with that of the pinning strength of the centers. Thus a flux quantum trapped at a pinning center may be dislodged by a large enough random thermal movement. That flux quantum would then migrate down the Lorentz force gradient to another pinning center. On the macroscopic scale this effect will reduce the flux density gradient and hence the Lorentz force. So, under conditions of constant transport current and external field, the random movement of flux quanta will die away until the current, field, or temperature changes. This process is known as flux creep. It is central to the manner in which technical superconductors can be used in magnets (4).

The continuous movement of flux across an annealed Type II superconductor appears as a voltage gradient along it. Similarly the process of flux creep through a field of pinning centers creates a voltage gradient, and through the interaction with the transport current, it gives rise to dissipation. The faster the flux creep, the greater is the local dissipation. Thus with flux creep is associated a local temperature rise. Since the specific heat of materials is small at low temperatures, the dissipation associated with flux creep can cause significant temperature rise. (At room temperature the energy of dissipation through flux creep would generate little temperature rise.) Thus the dissipative effect of flux creep can lead to a runaway flux movement. As the temperature rises, the critical current density decreases. According to Eq. (1), that in turn leads to an increased flux movement and thence to more dissipation. In the extreme case the temperature rises above the critical value, and superconductivity ceases. This process is known as a flux jump (5). In a magnet wound with superconducting wire, the sudden reversion to the normal state of a small length of the wire will result in the dissipation of the stored magnetic energy at high density in a volume that expands through thermal diffusion. If the stored energy is large enough and the thermal diffusion slow enough, the temperature rise at the point of initiation may exceed the melting point of the wire. The essence of the design of superconducting magnets is the suppression or control of this fundamental thermal instability.

TECHNICAL SUPERCONDUCTORS

The major properties of the three technical superconductors are typically as shown in Table 1. T_c is the highest temperature below which superconductivity can exist (electron pairs can form) in zero field and with zero current. B_{c2} is the upper critical field at 0 K at which the lattice is fully packed with flux quanta. $J_{c\,typ}$ is a typical operating current density at use-

Table 1. The Critical Properties of the Principal Technical Superconductors

	NbTi	Nb_3Sn	HTS
T_c (K)	10	18	90
B_{c2} (T)	13	27	>100
$J_{c\,typ}$ ($A \cdot mm^{-2}$)	1000	5000	1000

ful values of field and temperature. (Below the lower critical field, B_{c1}, the technical superconductor behaves as a Type I superconductor; that field is typically 0.01 T.) The two leading HTS materials are based on yttrium or on bismuth. The critical parameters refer to typical bismuth-based HTS.

Figure 1 illustrates the typical variation of critical current density with field and temperature for the two low-temperature technical superconductors. Depending on the local values of any two of the parameters, the third has a local critical value. In a magnet conductor, B and J are usually fixed, so T is the critical parameter. The distance between the operating values B, J, and T and the critical surface is the margin. In fact, since current and field are locally fixed, only the temperature can vary. Thus the temperature margin determines the susceptibility of the conductor to disturbance. The higher the margin, the more stable it will be. Since NbTi must operate at the lowest temperatures, it is the least stable. The HTS, by contrast, can operate with such high-temperature margins that they are very stable. However, that also leads to a problem of protection which must be addressed carefully in the future design of large HTS magnets.

Despite its low-temperature margin NbTi is the most widely used technical superconductor. It is a ductile alloy, compatible in metallurgical processing with copper and cupronickel. However, Nb_3Sn must be used in magnets or sections of magnets generating fields above about 12 T. HTS materials are now also beginning to be used in small magnets. Almost all magnets are wound with wire or a large cross-sectional conductor. (In principle, a magnet might be formed from a block of superconductor into which a field is frozen, e.g., by applying a field and then lowering the temperature. Instability makes that infeasible in low-temperature superconductors, but HTS may make such monolithic magnets possible.)

Composite Superconductors

The suppression of instability and the protection of the conductor in the event of an instability are the predominant

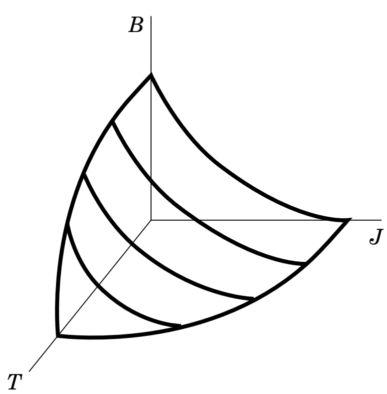

Figure 1. Variation of critical current density of a technical superconductor as a function of field and temperature.

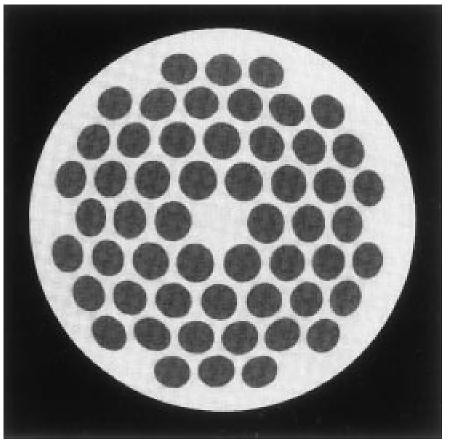

Figure 2. Cross section of a typical niobium-titanium composite superconductor. (Courtesy of Vacuumschmelze GMBH.)

problems in the design of superconducting magnets. If a magnet is to operate reliably, the tendency for a runaway magnetothermal effect to trigger a full transition from the superconductive to the normal conducting state must be suppressed. Two methods have been developed to achieve that: the elimination of flux jumping and the limiting of its consequences through cryogenic stabilization (5).

The essence of the stabilization of a technical superconductor is its subdivision into fine filaments. In the composite conductor this is achieved by embedding a large number of fine filaments in a matrix. The matrix is usually high-conductivity copper, although aluminum has also been used. The copper serves several functions. It rapidly conducts heat away from the surface of the fine filaments; since its resistivity is much lower than that of the superconductor in its normal state, it allows current to flow with relatively low dissipation when the adjacent superconductor has become normal through an instability.

Niobium-Titanium

Figure 2 shows a typical cross section through a NbTi composite superconductor. It is manufactured by a co-drawing process (6). Typical wire diameters lie in the range of 0.1 mm to 3 mm with critical currents at 4.2 K and 8 T of between 5 and 500 A. The insulation is typically Formvar (poly-vinyl-formal). The diameter of the superconducting filament is determined largely by stability but also by hysteresis loss. The latter is the integrated effect of dissipation by flux creep during the rise of current and field in a magnet. Hysteresis loss is minimized by decreasing the diameter of the filaments. Typically the number of filaments in a wire may lie between 10 and 100,000.

Niobium-Tin

Nb_3Sn is a brittle compound, and its use in a magnet winding is considerably more complex than NbTi. The predominant method of forming the compound is the diffusion of tin into niobium at a high temperature (6). The tin is contained either internally within filamentary niobium tubes or in a tin-rich bronze surrounding solid niobium filaments. The niobium has typically up to 5% of tantalum for improved critical current density. The stock wire is unreacted for winding and is insu-

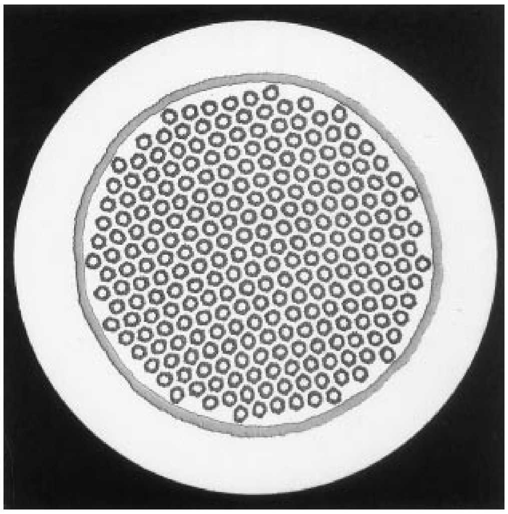

Figure 3. Cross section of a typical niobium-tin superconductor. (Courtesy of Vacuumschmelze GMBH.)

lated with a glass braid, such as silica glass. After winding, the coil is reacted at about 700°C for up to 200 h. During this reaction the tin diffuses into the niobium and forms the compound Nb_3Sn. For adequate diffusion the diameter of the niobium filament must be no greater than 0.005 mm. After reaction the wire is brittle and cannot tolerate bending. Therefore the winding must be consolidated by vacuum impregnation with epoxy resin. Figure 3 shows a typical cross section through a bronze route superconducting wire before reaction.

As in the NbTi composite, copper is required as a stabilizer and for protection. In order to preserve the low resistivity of the copper, tin must not be allowed to diffuse into it during the reaction. (That would, in any case, dilute the tin available for diffusion into the niobium.) A barrier is inserted between the bronze and the copper matrix to prevent such unwanted diffusion. This barrier is in the form of a thin wrap of tantalum or niobium.

High-Current Conductors

To form high-current conductors in either NbTi or Nb_3Sn, cables of small wires are usually embedded in a copper or aluminum matrix. In this way NbTi composite conductors with critical currents of up to 100,000 A can be constructed. Similarly high-current Nb_3Sn composites can be constructed by carefully cabling pre-reacted wires and soldering the cable into a copper matrix. Handling such pre-reacted Nb_3Sn conductors requires exacting quality control procedures (7).

An important form of high-current conductor is the so-called cable-in-conduit (CiC). A cable of composite strands is inserted into a closed sheath, usually of stainless steel. Liquid helium (sometimes in the superfluid state at a temperature below 2.17 K) is circulated through the gaps between the strands of the cable (8). This arrangement combines good support of the strands against the Lorentz forces with a low shielding loss and high current (see Section 6).

A recent development in Nb_3Sn conductors is a pre-reacted cable of very fine wires. The strands are so thin that bending radii appropriate to normal winding techniques can be tolerated (9).

HIGH-TEMPERATURE SUPERCONDUCTORS

At present only a very few superconducting magnets use these materials. These superconductors all contain copper-oxide layers in a perovskite crystal structure (10). Two major types are being actively developed for application to magnets: yttrium based and bismuth based.

Yttrium HTS

This material has the chemical composition $YBa_2Cu_3O_7$, abbreviated to YBCO. It has a critical temperature of 80 K. Its critical field at 4.2 K has not been directly measured but is believed to be about 100 T. Wire is manufactured in the form of a tape, samples of which have achieved a critical current density of 7000 $A \cdot mm^{-2}$ at 5 T and 77 K. Lengths of wire suitable for the construction of magnets have not yet been produced. Their manufacture suffers from the disadvantage of a highly anisotropic critical current density. That is, the critical current depends strongly on the orientation of the local field relative to the crystal orientation. Nevertheless, magnets operating at liquid nitrogen temperatures will probably use this material in the near future.

Bismuth HTS

This material is manufactured in two forms, with chemical compositions of $Bi_2Sr_2CaCu_2O_7$, abbreviated to 2212, or $(BiPb)_2Sr_2Ca_2Cu_3O_7$, 2223. Both of these are being manufactured in lengths of up to 1000 m, and small magnets have been wound from them. The 2212 requires lower temperatures for high-field operation than the 2223. However, it is more easily fabricated and coils of the 2212 have been tested in external fields to show that useful currents can be carried at 30 T and 4.2 K. Either of these BSCCO materials may soon be the choice for inserts in very high field solenoids.

STABILITY

Adiabatic Stability

Regrettably, the flux jump is not the only source of heating sufficient to raise the temperature above the local critical value. Movement of a wire under the influence of the Lorentz forces in the body of a winding can generate local frictional heat sufficient to raise the temperature of a wire above the local critical value, especially where the temperature margin is small. A further precaution that must therefore be exercised in a magnet designed to have no flux jumping is the consolidation of the winding so that no sudden frictional movement can occur to generate heat. This is achieved by impregnating the winding with epoxy resin, or another agent, in order to prevent sudden movement between wires, or between wires and stationary structure such as coil forms. This is the form of winding of the adiabatic magnet, which is so termed because no thermal sink exists within the winding.

Despite the use of fine filaments and winding techniques to avoid mechanical disturbances, the adiabatic winding is susceptible to sudden local transition from the superconductive to the normal state, if only through failure of the cryogenic environment. The resulting spreading normal zone is a process known as quenching. It may be precipitated by the relief of local stresses, which may merely be the repositioning of a wire, or cracking in a resin impregnant, and it may result in quenching at a current and field below design values. Upon recooling and recharging, the next premature occurrence of quenching can be at a higher current. This sequential improvement in the operating current is known as training. It is a common characteristic of large adiabatic magnets, especially of magnets with complex topology, such as particle-beam-handling magnets.

Despite the disadvantages associated with instability, quenching, and training, adiabatic magnets are the most common embodiment because they achieve the highest overall current densities in the windings. They enable the most compact magnets and the highest fields.

Cryostability

A second method of stabilization exploits the powerful cooling effect of liquid helium to maintain the temperature of a conductor below the local critical value even in the presence of flux jumping or other severe perturbations. This is achieved by paralleling the superconductor with sufficient copper or aluminum cooled by liquid helium (11). Now, even though all the transport current were to flow in the copper, the temperature would be kept below the critical value of the superconductor if sufficient surface were presented to the helium. This is the principle of cryostability. the governing expression for this mode of operation of a superconductor is

$$\frac{I^2\rho}{A} = hP \tag{2}$$

where ρ is the resistivity of the copper, A is the copper cross section, h is the heat transfer rate, and P is the wetted perimeter of the copper. If the value for h is chosen to be about 1000 $W \cdot m^{-2}$, this becomes a conservative stability criterion but leads to a low overall current density in a winding because the cross section of the stabilizing copper is large.

The earliest large magnets (mainly for bubble chambers built in the late 1960s) were designed in accordance with this principle. However, the boiling heat transfer curve for liquid helium has two branches, as shown in Fig. 4: nucleate boiling, line (a), with high heat transfer coefficient and film boiling; line (c) with lower coefficient (1000 $W \cdot m^{-2}$ is typically the lowest value of heat transfer rate in the film boiling regime). Intermediate between these is a region of unstable boiling, line (b). During an instability some or all of the current may

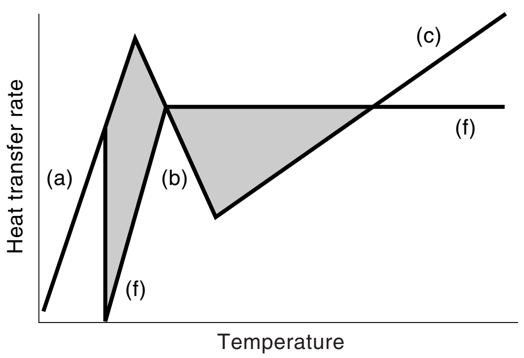

Figure 4. Boiling heat transfer curve for liquid helium and the heat generation curve for a composite superconductor.

transfer locally from the superconductor to the copper. The local generation of heat in such an extended region is line (f), rising from zero where no current flows in the copper to the full heating where all the current flows in the copper. The areas (d) and (e) then represent respectively regions where more heat is removed than generated and where more heat is generated than removed.

The sharing of current between the superconductor and copper results in the following expression for the local temperature:

$$\frac{I(I - I_s)\rho}{A} = q(\theta - \theta_b)P \tag{3}$$

where I is the total current, I_s is the current still in the superconductor, ρ is the resistivity of the copper, q is the heat transfer coefficient at the temperature difference $\theta - \theta_b$ between the copper and the helium bath. A good approximation for the current in the superconductor at a temperature θ is

$$I_s = \frac{I_c(\theta_c - \theta)}{(\theta_c - \theta_b)} \tag{4}$$

where I_c is the critical current at θ_b, the bath temperature. Equations (2), (3), and (4) are represented together with the boiling heat transfer in Fig. 4. If the cooling exceeds the heating, area (d) greater than area (e), then recovery of superconductivity will occur. The normal region will collapse from the cold ends. This type of stability, called cold end recovery, allows higher values of h to be assumed in Eq. (2), 3000 $W \cdot m^{-2}$ being commonly accepted. This in turn allows a smaller cross section of copper and a more compact winding (12).

PROTECTION

The other crucial issue in the design of superconducting magnets is protection. In both the adiabatic and cryostable types, the consequences of quenching must be anticipated. The methods by which this is achieved are quite different in the two types.

Adiabatic Protection

In this winding the conductors are in a compact array without an internal heat sink but with close thermal coupling between conductors. Therefore, when a conductor becomes normal through a perturbation, not only does the normal zone travel along the wire but the local heating rapidly causes adjacent conductors to heat up and become normal. Quenching then expands as a three-dimensional zone of increasing volume and surface area. The volume of normal conductor at any instant is approximated by

$$V_n \propto v_1 v_2 v_3 t^3 \tag{5}$$

where $v_{1,2,3}$ are effective velocities of quench propagation along the wire and transverse to it and t is the elapsed time. Thus the magnetic energy stored in the field is dissipated in a rapidly expanding volume of conductor. The velocities of propagation depend, among other things, on the local temperature margin: The greater the margin, the slower are the velocities.

Although not as high as the velocity along the wire, the dominant velocities are those between turns and between layers. Rapid propagation of quenching between turns is desired in order to spread the temperature rise over a large volume. For that reason HTS winding may be more difficult to protect than those with the smaller temperature margins of Nb_3Sn or NbTi.

Two criteria are applied to this process to gauge the vulnerability of the magnet winding to quenching: the final temperature of the wire at the point of initiation of quenching, and the voltage appearing between layers or turns of the winding. The latter criterion involves complex computation of the spatial distribution of normal and superconducting regions during the quench. However, the peak temperature generally dominates the criteria. This must clearly not exceed the allowable temperature for the insulation or impregnant nor produce unacceptable local stresses due to differential thermal expansion. A rule of thumb for acceptable peak temperature is 100 K. To achieve such limit (and simultaneously limit the voltage), sufficient copper must parallel the superconducting filaments so that the integrated heating in the copper does not exceed the acceptable peak value. Because of the rather complex time dependency of the current in a quenching adiabatic magnet, this criterion is best illustrated by reference to the protection of a cryostable magnet.

Cryostable Protection

In this type of magnet no propagation of a quench zone occurs between adjacent conductors. Furthermore the one-dimensional propagation of quench along a conductor is slow. If, for example, the level of liquid helium is low so that a length of conductor is exposed to helium vapor and has quenched, no propagation will occur, and the normal zone will be stationary. In such a case protection of the conductor against excessive temperature rise must be by external discharge of the stored magnetic energy. Interturn voltage is hardly ever a dominant effect in the protection of a cryostable magnet. The limiting criterion is therefore the temperature rise of the conductor at the point of quench initiation. Just as for the adiabatic magnet, a generally accepted criterion is a rise of not more than 100 K. The prediction of the temperature rise is now rather simple. If the magnet is discharged exponentially, the governing equation is

$$J_{\mathrm{Cu}}^2 \rho(\theta)\, dt = c(\theta)\, d\theta \tag{6}$$

The left-hand side of Eq. (6) represents the heat being generated in the resistance of the copper, and the right-hand side represents the same heat being absorbed by its enthalpy. The quantities $\rho(\theta)$ and $c(\theta)$ are the temperature-dependent resistivity and volumetric specific heat of the copper.

Equation (6) may be rewritten as

$$\int_0^\infty J_{\mathrm{Cu}}^2\, dt = \int_{4.2}^{\Theta_{\max}} \left\{ \frac{c(\theta)}{\rho(\theta)} \right\} d\theta \tag{7}$$

In this expression the right-hand side is a function of the copper, or some other conducting material, between a low temperature, generally assumed to be that of liquid helium boiling at atmospheric pressure, 4.2 K, and the allowable upper temperature, generally 100 K (13). (It is sometimes called the G-function but not in the original reference.) If the discharge of the magnet is exponential, Eq. (7) becomes

$$\tfrac{1}{2} J_{0\,\mathrm{Cu}}^2 \tau = G(\theta_{\max} - 4.2) \tag{8}$$

where $J_{0\,\mathrm{cu}}$ is the initial maximum current density in the copper. For copper at 100 K, the value of the G function is typically 6 $10^{16}\,\mathrm{A}^2 \cdot \mathrm{m}^{-4} \cdot \mathrm{s}$. In order to achieve the required time constant of discharge, the voltage across the magnet must be as large as the inductance demands. As an example, a large magnet might have a stored energy of 100 MJ, an operating current of 10,000 A, and a copper cross section of 500 mm^2. The inductance would be 2 henries and $J_{0\,\mathrm{cu}}$ would be 2 10^7 $\mathrm{A} \cdot \mathrm{m}^{-2}$. The time constant of discharge would have to be 17.32 s, and the maximum initial discharge voltage 1.154 kV.

Winding Topology

The design of a winding array is dictated by the desired field properties. The basic expression governing the field produced by a winding is the Biot–Savart law, expressed as

$$dB = \frac{I \cdot dl \sin(\theta)}{r^2} \tag{9}$$

where $I \cdot dl$ is a current element, θ is the angle between that element and the radius vector r to the field point. The direction of the field vector is perpendicular both to the current element and the radius vector. This expression is integrated in closed form for the fields of solenoids and of long linear windings. The forward computation of field produced by a given current array is straightforward. It can be performed for any configuration of currents, although the closed form calculation is easy only for circular or straight arrays. The reverse computation, of a current array to produce a given field topology, is possible only for a few simple configurations, specifically for the axially symmetric field of a current ring and the circular field of an infinite straight current. The off-axis fields of a circular current array involve elliptic integrals. Numerical methods for the solution of the off-axis field of a current arc are routinely available as commercial codes. Most field geometries are produced by combinations of straight currents and current arcs. Generally the design problem proceeds by the iterative recomputation of the fields generated by incrementally modified straight or arc current arrays.

OPERATION IN TIME-VARYING FIELDS

Although most superconducting magnets constructed to date have been designed for steady state operation, a few have been constructed for use either under ac conditions or with a rapidly changing field (14). The design of a conductor for such operation involves the reduction of dissipative mechanisms in the superconductor. One source of dissipation under conditions of changing field has been described, that is, flux creep. That loss is generally called hysteresis loss and is approximated per unit volume by the expression

$$Q \approx \tfrac{1}{4}\, \Delta B\, d_\omega J_{\mathrm{c}} \tag{10}$$

where ΔB is the field change, d_w is the wire diameter, and J_c is the critical current density. It is seen that the dissipation is smallest for thin wires.

Other sources of dissipation are found in composite superconductors. The most important of those is what is called shielding loss. It arises as follows: To reduce hysteresis loss and to improve the stability of a superconductor against flux jumping, the superconductor is finely divided into filaments in a (typically) copper matrix. However, it may be seen that the collection of filaments will still tend to exclude flux unless the filaments are transposed. Full transposition of any but a single layer of filaments is impossible within the matrix. However, a compromise is the twisting of the conductor. A short twist pitch or high resistivity of the matrix will allow flux to penetrate through the layers of filaments (15). The time constant of penetration is given roughly by

$$\tau = \left(\frac{\mu_0}{2\rho}\right)\left(\frac{L}{2\pi}\right)^2 \tag{11}$$

where ρ is the resistivity of the matrix and L is the twist pitch. External field change will generate dissipation roughly given by

$$Q = \left(\frac{B^2}{2\mu_0}\right)\left(\frac{\tau}{\tau + t}\right) \tag{12}$$

where t is the time constant of the field change B. If the change is very rapid compared with the time constant of penetration of the conductor, the dissipation approaches the energy density of the magnetic field. Some magnets have been made with superconductors that have a matrix of cupro-nickel. The resistivity of cupro-nickel is high, which leads to a short time constant for flux penetration. However, the conductor is not well stabilized, and in the event of a quench it is not well protected against local hot spot. The CiC conductor, described above, also minimizes the shielding losses.

FORCES AND STRESSES

The force on a conductor in a field is given by the expression,

$$F = BI \tag{13}$$

where I is the current in amperes, B is the component of the field in tesla perpendicular to the current vector, and F is the force per unit length in newtons which is directed perpendicular to the current and field vectors. The calculation of stresses in the conductors of a winding can be performed in closed form only for a few simple cases. For an isolated circular loop the hoop stress averaged over a section of wire (whose cross section may include superconducting filaments, copper, depleted bronze, etc.) is given by

$$\sigma = BJr \tag{14}$$

where B is the average field at the wire perpendicular to the plane of the loop, J is the current density averaged over the wire section, and r is the mean radius of the loop. If J is in $A \cdot m^{-2}$, and r is in meters, then σ is in pascals. In a multilayered solenoidal winding the stresses in the wire are influenced by the transmission of radial forces between layers and by axial forces generated by radial components of field at the ends of the solenoid. The analysis of these stresses usually is performed by standard programs. However, Eqs. (13) and (14) provide a useful rough guide to the stresses that will be encountered in a solenoidal winding.

The forces in long straight windings, such as extended dipoles, may be similarly estimated by the summation of the forces found from Eq. (13). The stresses in solenoidal windings are often supported by the conductor alone. Since the stresses are predominantly tangential, they produce tension in the components of the wires. In NbTi composites the stresses tend to be divided roughly equally between the NbTi superconductor and the copper unless the latter is annealed. In Nb_3Sn conductors most of the tensile load appears in the Nb_3Sn superconductor itself because the Young's modulus is high, 172 GPa. The other components of a Nb_3Sn composite tend to be annealed by the heat treatment.

Examples of Superconducting Magnets

Common applications of superconducting magnets include the following: magnetic resonance imaging (MRI), laboratory magnets (for NMR analysis, susceptibility measurement), particle beam handling, subatomic particle analysis, power conditioning, energy storage, open gradient magnetic separation, magnetohydrodynamic power generation (MHD). NMR and MRI magnets are described in more detail in MAGNETS FOR MAGNETIC RESONANCE ANALYSIS AND IMAGING. All superconducting magnet are wound. The most common winding topologies are solenoids and extended linear windings. The latter are in general of the form of armature windings, having straight sections with end turns of complex shape. Solenoids are used in magnets for MRI, most laboratory magnets, particle analysis, and energy storage. Extended linear windings are used in particle-beam-handling magnets, in rotating machines (rotor windings), for particle focusing (quadrupoles, hexapoles), and in MHD. Occasionally a superconducting winding has an associated ferromagnetic flux path. (In such so-called superferric magnets, the iron is usually included in the magnet's cryogenic environment, to avoid the transmission of high forces between low temperature and room temperature.)

Although most superconducting magnets are solenoids or extended dipoles and multipoles, a few magnets of more complex shape have been manufactured. These include very large "Ying-Yang" coils for the mirror fusion test facility (MFTF) (16), saddle coils for experiments on magnetohydrodynamic power generation (MHD) (17) and toroidally shaped coils for several Tokamak fusion experiments (18). (Toroidal field magnets for fusion devices are D-shaped coils which are arrayed around a circular plasma path; in the absence of orthogonal—poloidal—fields, the conductors of these coils experience pure tension as in a solenoid.)

All are large magnets, storing energy in the range of 100 MJ. The stored magnetic energy is a rough classification of the size and cost of a magnet, and not necessarily of the difficulty of design and construction. Very high field, compact magnets present more difficulty because of small operating margins, high stresses, and high-energy densities during quenching.

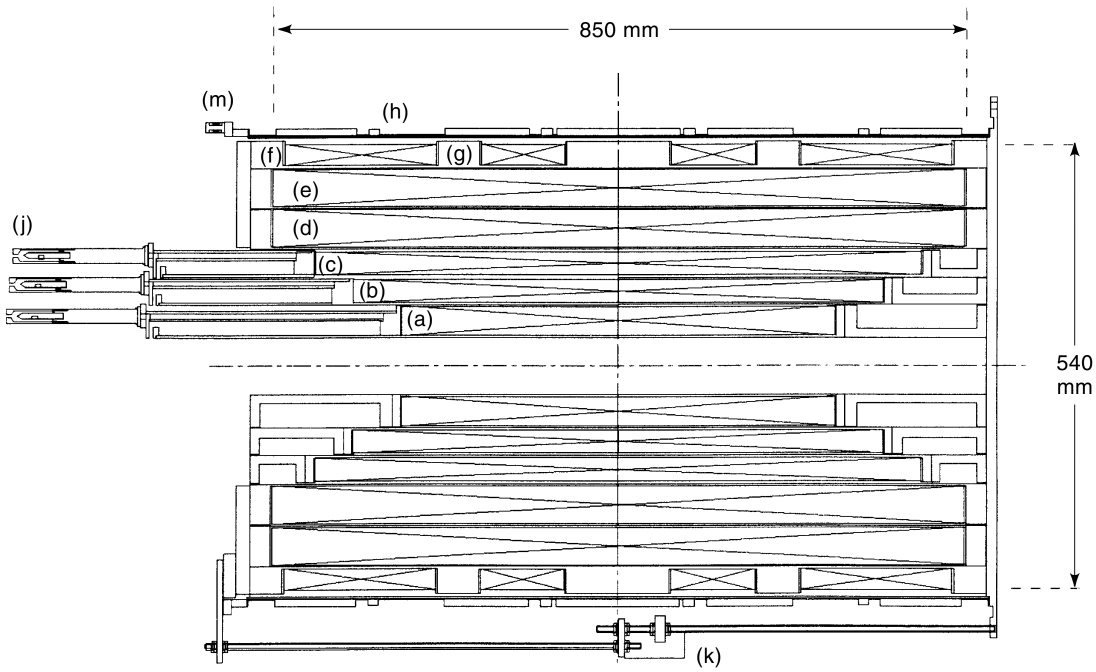

Figure 5. The winding profile of a typical commercial NMR magnet. (Courtesy of Magnex Scientific PLC.)

One of the largest superconducting magnets under construction is the model central solenoid for the International Thermonuclear Experimental Reactor (ITER). It stores 650 MJ and weighs 100 tonnes (19). The most precisely engineered have been the high-field NMR magnets. They now have reached fields of 18.8 T with homogeneity of 10^{-9} in a 10 mm spherical volume.

NMR and MRI magnets operate in persistent mode. After energizing, a superconducting switch is closed across the winding so that the current continues to flow in an essentially resistanceless circuit. These are the most commonly produced commercial magnets (20).

Superconducting particle-beam-handling magnets have made possible the very high-energy accelerators in operation at BNL, CERN, DESY, and the Fermi Laboratory. Superconducting dipole magnets are typically 10 m long or greater, generating transverse fields of up to 8 T to provide the deflection required to bend the particle beam to a circular orbit. The requirement for uniformity of field is about 10^{-4} over 25 mm width. Superconducting quadrupole magnets are used for focusing of the particle beam and typically provide field gradients of up to 25 T/m.

Specific Examples

NMR Magnet for 750 MHz Proton Frequency (21). Superconducting magnets for NMR and MRI are described separately within this encyclopedia. However, as they typify solenoidal magnets the 750 MHz magnet is described here in outline. Figure 5 gives the profile of the windings of a commercial magnet along with some of the structural details. The global characteristics are listed in Table 2.

Table 2. Major Characteristics of a 750 MHz NMR Solenoid

Type	Solenoid
Materials	NbTi and Nb_3Sn
Energy	15 MJ
Center field	17.6 T
Bore at 4.2 K	74 mm
Typical weight	250 kg

In the figure the windings are labeled (a) through (g). Windings (a), (b), and (c) are of Nb_3Sn. The wire size is largest in section a where the field is highest and the critical current density lowest. In fact section (a) consists of three grades of wire in order to maximize the local winding current density. Sections (d) and (e) are the main NbTi winding, again graded to maximize overall current density. Sections (f) and (g) com-

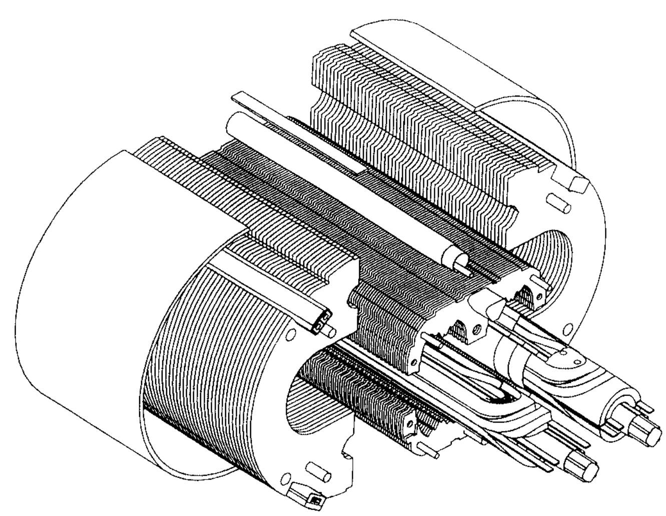

Figure 6. An illustrative view of the LHC dipole construction, showing the windings and the structural yokes. (Courtesy of CERN.)

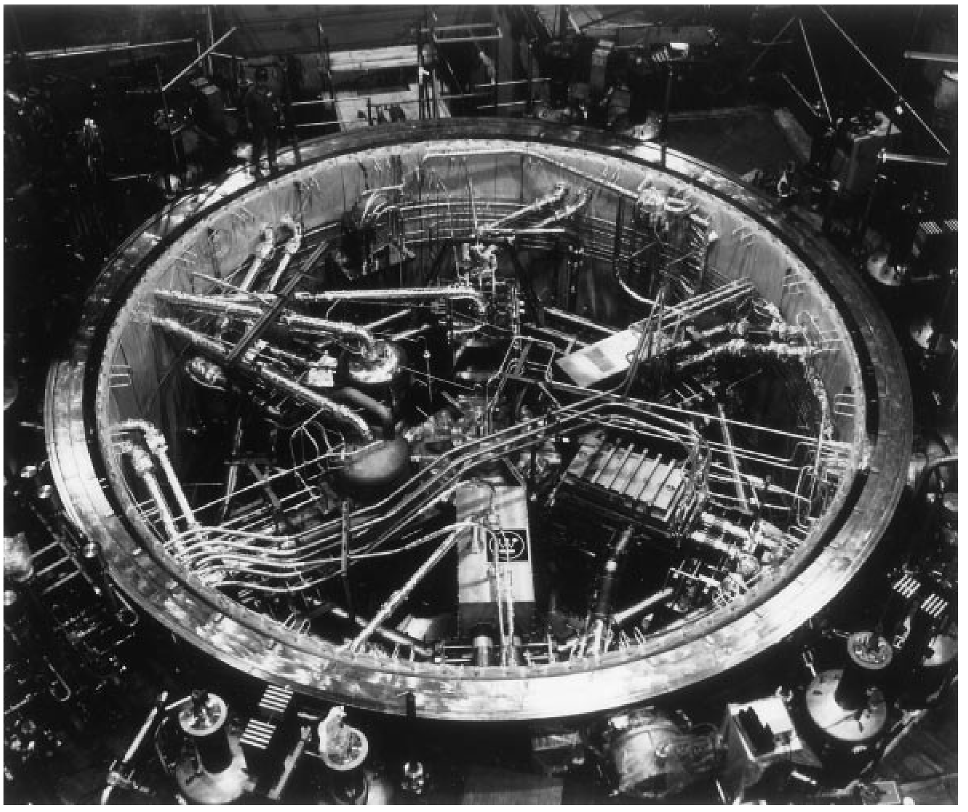

Figure 7. The six dee coils of the Large Coil Task installed in the test cryostat. (Courtesy of the Oak Ridge National Laboratory.)

pensate the axial gradients introduced by the full-length sections. All magnets used for NMR are operated in persistent mode. The windings are connected in series through superconducting joints. After energizing, the windings are closed through a superconducting switch (k). The joints (j) between the Nb_3Sn sections provide the best performance (highest critical current) in low field. Therefore they are mounted away from the windings on posts that rigidly support the brittle conductor. Surrounding the windings is an array of shim coils (h) which provides for fine adjustments. Each shim set has its own switch (m).

Particle Accelerators (22,23). The Fermilab energy doubler/saver is typical of the use of superconducting beam-bending magnets used in particle accelerators. The field generated by the magnet is oriented vertically and extends over a length of 6 m. High-energy protons are slightly deflected by the field–length product of the magnet. The transverse force on the particle beam is as predicted by the Lorentz equation (13). A large number of these dipoles (774), quadrupoles (216), and sextupoles are arranged to produce a total deflection of 360°. In the Fermilab energy doubler each dipole magnet produces a mere 0.47° deflection at maximum beam energy.

Table 3. Major Characteristics of the Coils in the Large Coil Task

Global

Design field	8 T
Total stored energy	600 MJ
Operating temperature	4.5 K
Inside height (each coil)	3.5 m
Inside span (each coil)	2.5 m
Inside radius	1.25 m

Individual

Origin	Conductor Type	Inductance (H)	Peak Field (T)	Current (kA)	Energy (MJ)
EU	NbTi FF	1.57	9.01	11.40	200
WH	Nb_3Sn FF	0.75	8.23	17.76	202
GE/ORNL	NbTi PB	1.81	8.89	10.50	193
GD	NbTi PB	1.81	7.72	10.20	136
JAERI	NbTi PB	2.00	8.81	10.22	198
CH	NbTi FF	1.00	7.86	13.00	123

A more recent project involving superconducting beam-handling magnets is the Large Hadron Collider (LHC) at CERN. This also uses a sequence of long dipoles, quadrupoles, and sextupoles in two adjacent rings in which protons, or other heavy particles, will circulate in opposite directions to collide at energies up to 14 TeV. The cross section of the winding and associated structure is shown in Fig. 6. The conductors are located so as to approximate a cosine distribution of current density. That arrangement generates a field uniform throughout the center region of each beam. The outward forces on the conductors are constrained by a stainless steel collar. This cold assembly is supported within an iron yoke which provides a return path for the flux. The yoke is assembled from transformer iron laminations, welded in a set of long steel girders.

Large Coil Task (24). Six coils were built, one each by the following groups: EURATOM (EU), The Swiss Institute for Nuclear Science (CH), The Japanese Atomic Energy Research Institute (JAERI), Westinghouse Electric, (WH), General Dynamics (GD), and General Electric/Oak Ridge National Laboratory (GE/ORNL). Each coil was a constant tension dee coil (18), but each used a different form of conductor. The Westinghouse coil alone used Nb_3Sn, in a CiC configuration; all the others used NbTi in a cryostable (helium-cooled) configuration. The cooling method was either forced-flow (FF) or pool-boiling (PB). The purpose of the program was to demonstrate stability of the coils under conditions of combined toroidal and pulsed poloidal fields. This international program has resulted in the construction of a model toroidal coil array as shown in Fig. 7. The main specifications were as listed in Table 3.

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Further Reading

More information may be found in the following books and articles:

A. C. Rose-Innes, *Introduction to Superconductivity,* 2nd. ed., Amsterdam, The Netherlands: Elsevier Science, 1994. This book describes the fundamental physics of superconductivity and its application to conductors for superconducting magnets.

Proc. IEEE, Special suppl. on superconductivity, **77**: 1110–1287, 1989. This IEEE review contains comprehensive contributions on magnets, conductors, power devices, and electronics. Despite the date of publication, the contents are still appropriate.

H. Desportes, Three decades of superconducting magnet development, *Cryogenics,* ICEC suppl., **34**: 46–56, 1994. This is one of the more recent reviews of superconducting magnet development but contains less detail than the *Proc. IEEE* issue above.

Reference 10 above is a concise review of the status of HTS conductors and their applicability to superconducting magnetic construction.

P. Schmuser, in *The Physics of Particle Accelerators: Superconducting Magnets for Particle Accelerators,* Melvin Month and Margaret Diennes (eds.), AIP Conf. Proc., **249**: 1992, pp. 1099–1103.

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