CODE DIVISION MULTIPLE ACCESS

With the advent of personal wireless communication systems in recent years, the need for instantaneous, seamless personal communication has grown. Unfortunately, this increase in demand strains a natural resource, the radio frequency (RF) spectrum. It is imperative that the design of any communication system that is intended for use in a personal communication domain be as bandwidth efficient as possible. In other words, one has to design communication systems for a band-limited scenario and to make an attempt to maximize the information throughput for the allotted bandwidth. There are two basic means by which the RF spectrum can be shared among many users: collision-free and collision-impaired multiple access. In a collision-impaired scheme, a protocol is used (which is typically common to all users) by each user to obtain access to the available spectrum. Collisions (the event in which two or more users make an attempt to use the same common resource) are possible in this scenario, and hence one must accommodate for such events (e.g., retransmission). In the collision-free scenario, it is assumed that a user is able (at least in theory) to obtain access to the channel upon request (perhaps with some delay) and that there is never any form of collision possible. In practice, often a hybrid of the two scenarios is used to provide access to the RF medium.

Although there are numerous forms of collision-free multiple access, the following means of sharing the RF spectrum have received most attention. There are time-division multiple access (TDMA), frequency-division multiple access (FDMA), and code-division multiple access (CDMA). The concepts of TDMA and FDMA may be explained as follows. In the TDMA scenario, the access to the RF spectrum is rather implicit via time-slot allocation. Namely, there is no single portion of the allotted frequency spectrum that is assigned to an individual user. Instead, users occupy the entire allotted frequency spectrum and are assigned nonoverlapping time slots for communication. In contrast, FDMA operates on the assumption that the nonoverlapping portions of the RF spectrum can be allocated to individual users and communication for each user can proceed in a continuous fashion in time.

The CDMA approach is different from TDMA and FDMA in two important aspects. First, explicit frequency assignments are not necessary. More important, communication can initiate at any time, and hence no explicit time-slot assignment must be performed prior to communication. The means by which user discrimination is achieved is through exploiting the correlation properties of binary (or perhaps higher-order) codes used to form CDMA signals. To illustrate this point, let us consider the following. In most CDMA systems, the information provided by a user is often of a bandwidth much smaller than the bandwidth allocated for CDMA communication. First, via a bandwidth spreading tactic, the information provided by a user is expanded in bandwidth to the maximum allowable bandwidth for CDMA communication. This procedure is repeated for all users involved, with each user taking advantage of a bandwidth spreading strategy that is independent of those used by others.

At the receiver, a reverse operation (i.e., a bandwidth despreading operation) is performed. Obviously, if this operation is performed successfully, the original signal is recovered. However, since all the users involved occupy the entire allotted frequency band and are allowed to communicate at all times, the bandwidth despreading operation performed on an intended signal is also affected by the presence of other interfering signals. The key assumption in CDMA is that the independence of the bandwidth spreading and then despreading operations guarantees that the bandwidth despreading of an interfering signal leads to a signal whose bandwidth remains identical to the bandwidth allowed for CDMA communication. Namely, only the intended signal is transformed to its original shape, while other signals remain as wideband signals.

Considering that a digital receiver makes an attempt to measure the useful band-limited energy of a signal, the detection of the desired signal is hampered by only a small fraction of the energies of the interfering signals, which appears as a spectrally flat noise. To elaborate, one can view a digital receiver as a narrowband filter that is designed to capture the energy (the area beneath the power spectrum) of the desired signal. Such a filter will have a bandwidth proportional to the bandwidth of the desired signal after the bandwidth despreading operation. Since the undesired signals remain wideband after such an operation, the contribution of the interfering signals to the detected energy in the desired frequency band will be small compared with the detected energy of the desired signal. This, in turn, leads to a successful recovery of the desired signal and the rejection of a large portion of the unwanted energy. To gain further insight, we proceed to formulate this problem in the next section.

SIGNAL GENERATION AND MATHEMATICAL MODELING

The preceding operation can take on a mathematical form. First, we assume that there are N CDMA users that can be active at any point in time. That is, we assume that the allotted frequency spectrum is accessible to N CDMA signals at all times. Let us begin by describing a direct-sequence CDMA signal. In particular, we are interested in representing the jth CDMA signal. For all intents and purposes, one can describe the jth CDMA signal at the transmitter as

$$x_j(t) = \mathrm{Re}\{\tilde{x}_j(t)e^{i\omega_c t}\} = \mathrm{Re}\{d_j(t)PN_j(t)e^{i\omega_c t}\} \tag{1}$$

where $\mathrm{Re}\{x\}$ denotes the real part of x, $\tilde{x}(t)$ is the complex envelope of the CDMA signal, $i = \sqrt{-1}$, ω_c denotes the carrier frequency in rad/s, and $d_j = \sum_{n=-\infty}^{\infty} d_n^{(j)} P_d(t - nT_s)$ is the data bearing portion of the jth signal, with $d_n^{(j)}$ and $P_d(t)$ denoting the complex data symbol for the jth transmitted signal in the nth signaling interval, taking on an M-ary phase-shift-keying (MPSK) signaling constellation and a unit amplitude nonreturn-to-zero (NRZ) pulse shape of duration T_s s, respectively. Moreover, PN_j (t) denotes the jth complex PN signal defined as

$$\mathrm{PN}_j(t) = \sum_{n=-\infty}^{\infty} s_{n,I}^{(j)} P_c(t - nT_c) + i \sum_{n=-\infty}^{\infty} s_{n,Q}^{(j)} P_c(t - nT_c) \tag{2}$$

where $s_{n,I}^{(j)}$ and $s_{n,Q}^{(j)}$ are the in-phase (I) and quadrature (Q) phase pseudorandom real spreading sequences for the nth chip interval of the kth user taking on $\{-1, +1\}$ according to a PN code generating device (a PN code generator typically consists of one or a combination of a number of linear feedback shift registers); $P_c(t)$ is the chip pulse shape, typically assumed to be a square root raised-cosine pulse shape; and T_c is the chip interval given by

$$T_c = \frac{T_s}{P_g} \tag{3}$$

where $P_g \gg 1$ denotes the processing gain for the CDMA system. This parameter will be explained in a different context in the ensuing discussion. We also assume that

$$\mathrm{PN}_j(t \pm kPT_c) = \mathrm{PN}_j(t) \quad \text{for } k = 1, 2, 3, \ldots$$

This implies that PN codes here are assumed to be periodic with a period of PT_c seconds. This further implies that the PN sequences have a period of P chips.

The preceding formulation implies that

$$\tilde{x}_j(t) = d_j(t)PN_j(t) \tag{4}$$

From Eq. (4), one can observe that a CDMA signal is obtained via PN code multiplication, justifying the name *direct sequence*. Before going any further, let us observe the impact of the spreading operation on the spectrum of a narrowband signal. Equation (4) sheds light on the means by which the direct-sequence spreading expands the bandwidth of $d_j(t)$. It can easily be inferred from Eq. (4) that the outcome of the correlation operation is to yield a signal whose bandwidth is identical to that of the PN code [$\mathrm{PN}_j(t)$]. Since PN code's bandwidth is far greater than that of $d_j(t)$ (i.e., $T_c^{-1} \gg T_s^{-1}$), a bandwidth spreading operation is realized. We also note that

$$P_g = \frac{B_{\mathrm{CDMA}}}{B_{\mathrm{Data}}} \tag{5}$$

where B_{CDMA} and B_{Data} denote the bandwidths of the CDMA and data signals, respectively. This can easily be verified by noting that the bandwidth of a direct-sequence CDMA signal may be shown to be $\alpha_1 T_c^{-1}$ for some α_1, while the bandwidth of the data signal is $\alpha_2 T_s^{-1}$. Since the CDMA and data signals possess identical characteristics, $\alpha_1 = \alpha_2$, using Eq. (3), we arrive at Eq. (5).

This result indicates that the processing gain for a CDMA signal is identical to the bandwidth spreading factor or P_G. In the remainder of this article, for the sake of simplicity, we deal with $\tilde{x}_j(t)$, the complex envelope of the jth CDMA signal. In the ensuing analysis, the correlation properties of the PN codes are needed to understand the means by which CDMA receivers function. For this reason, let us define

$$R_a^{(j)}(n, \tau, \hat{\tau}) \triangleq \frac{1}{2}\langle \mathrm{PN}_j(t - \tau)\mathrm{PN}_j^*(t - \hat{\tau})\rangle_{n,\hat{\tau}} \tag{6}$$

as the partial autocorrelation function of the jth PN code observed over P_G chip symbols with

$$\langle f(t)\rangle_{n,\hat{\tau}} \triangleq \frac{1}{T_s}\int_{(n-1)T_s+\hat{\tau}}^{nT_s+\hat{\tau}} f(t)\,dt$$

denoting a time-averaging operation over the interval $[(n - 1)T_s + \hat{\tau}, nT_s + \hat{\tau}]$. This function will be used in the subsequent analysis to discuss the characteristics of PN code acquisition and tracking systems. It is important to note that, in commercial CDMA systems, the period of the PN code (i.e., PT_c) is substantially greater than the processing gain, resulting in an $R_a^{(j)}(n, \tau, \hat{\tau})$ that is a function of n and represents the partial autocorrelation function of the jth PN code. (Since PN codes are often generated using linear feedback shift registers, one may assume that the resulting codes are periodic with periods that are dependent on the structural properties of the generating shift registers.) In fact, due to the pseudorandom nature of the PN code, $R_a^{(j)}(n, \tau, \hat{\tau})$ may be viewed as a random sequence. However, if one assumes a large processing gain (large number of chip symbols per integration interval), $R_a^{(j)}(n, \tau, \hat{\tau})$ does not vary substantially with n, and hence $R_a^{(j)}(n, \tau, \hat{\tau})$ may be approximated by $R_a^{(j)}(\tau, \hat{\tau})$. For the scenario where $P_G = P$ (i.e., PN code is repeated every symbol interval), $R_a^{(j)}(n, \tau, \hat{\tau})$ is not a function of n and reduces to the autocorrelation function of jth PN code.

It is also important to note that $R_a^{(j)}(\tau, \hat{\tau})$, as defined previously, is a complex function. In practice, however, the complex PN codes are designed so that the I and Q PN codes (hereafter, Re{$\mathrm{PN}_j(t)$} and Im{$\mathrm{Pn}_j(t)$}) are referred to as the I and Q PN codes, respectively) are a pair of uncorrelated sequences. That is,

$$\langle \mathrm{Re}\{\mathrm{PN}_j(t)\}\mathrm{Im}\{\mathrm{PN}_j(t)\}\rangle_{n,0} \approx 0; \text{for all } j \tag{7}$$

In that event, $R_a^{(j)}(\tau, \hat{\tau})$ is a real function that can be expressed as

$$\begin{aligned} R_a^{(j)}(\tau, \hat{\tau}) = {} & \frac{1}{2}\langle \mathrm{Re}\{\mathrm{PN}_j(t-\tau)\}\mathrm{Re}\{\mathrm{PN}_j(t-\hat{\tau})\}\rangle_{n,\hat{\tau}} \\ & + \frac{1}{2}\langle \mathrm{Im}\{\mathrm{PN}_j(t-\tau)\}\mathrm{Im}\{\mathrm{PN}_j(t-\hat{\tau})\}\rangle_{n,\hat{\tau}} \end{aligned} \tag{8}$$

If one assumes that the I and Q PN codes possess identical partial autocorrelation properties (a situation where this condition is not satisfied is of little practical interest), then

$$R_a^{(j)}(\tau, \hat{\tau}) = \langle \mathrm{Re}\{\mathrm{PN}_j(t-\tau)\}\mathrm{Re}\{\mathrm{PN}_j(t-\hat{\tau})\}\rangle_{n,\hat{\tau}} \tag{9}$$

Hence, $R_a^{(j)}(\tau, \hat{\tau})$ may be viewed as the partial autocorrelation function of the real PN sequences that form the complex PN signal. Using the preceding notation, the despreading operation may also be explained.

DESPREADING AND DETECTION

Since a binary PN spreading is used, it is fairly easy to see that

$$\begin{aligned} \frac{1}{2}\langle \tilde{x}_j(t)\mathrm{PN}_j^*(t)\rangle_{n,0} &= \frac{1}{2T_s}\int_{(n-1)T_s}^{nT_s} \tilde{x}_j(t)\mathrm{PN}_j^*(t)\,dt \\ &= \frac{1}{2T_s}\int_{(n-1)T_s}^{nT_s} d_j(t)|\mathrm{PN}_j(t)|^2\,dt = d_n^{(j)} \end{aligned} \tag{10}$$

where $\mathrm{PN}_j^*(t)$ is the complex conjugate of $\mathrm{PN}_j(t)$ and it is assumed that $\frac{1}{2}|\mathrm{PN}_j(t)|^2 = 1$. The factor $\frac{1}{2}$ is included to account for the fact that the PN code consists of real and imaginary spreading sequences. For complex spreading signals, we also observe that

$$\frac{1}{2}|\mathrm{PN}_j(t)|^2 = \frac{1}{2}(\mathrm{Re}\{\mathrm{PN}_j(t)\})^2 + \frac{1}{2}(\mathrm{Im}\{\mathrm{PN}_j(t)\})^2 \tag{11}$$

where Im{x} is the imaginary part of x. Since the real and imaginary parts of $\mathrm{PN}_j(t)$ are also binary PN codes with unit amplitudes,

$$(\mathrm{Re}\{\mathrm{PN}_j(t)\})^2 = (\mathrm{Im}\{\mathrm{PN}_j(t)\})^2 = \frac{1}{2}|\mathrm{PN}_j(t)|^2 = 1 \tag{12}$$

When $\mathrm{PN}_j(t)$ is assumed to be real, then it is fairly easy to see that $\mathrm{PN}_j^2(t) = 1$. Hereafter, the signal processing defined by Eq. (10) is referred to as a matched filtering (MF), or despreading operation. To elaborate, as can be seen in Eq. (10), the outcome of the correlation operation is the original narrowband signal $d_j(t)$.

At this junction, we need to consider the received signal when the signals described previously have been subjected to an imperfect channel. In particular, we need to be concerned with the case where the channel coherence bandwidth is smaller than the bandwidth of the CDMA signal. (Coherence bandwidth of a dispersive channel may be viewed as the maximum bandwidth that a signal can take on without getting distorted by the characteristics of the channel.) This implies a frequency selective operation for most practical applications and, in particular, for wireless communication scenarios. Hence, we need to examine the impact of channel on a CDMA signal. This also plays a critical role in selecting a detection mechanism for the problem at hand. Before doing so, however, the important problem of synchronization is addressed.

SYNCHRONIZATION

In virtually any form of digital communication, synchronization in time (symbol clock recovery) precludes communication. CDMA systems are not exempt from this requirement. The synchronization in a CDMA system, however, is somewhat different from its TDMA counterpart. In TDMA systems, one requires synchronization in frequency (and, in some cases, in phase) before a data clock can be recovered. Often, a dotting sequence (1010101. . .) is included in the preamble of a TDMA frame to provide the clock synchronization subsystem the necessary signal to lock onto.

In a CDMA scenario, since the desired signal is spread in frequency over the entire allotted CDMA band, the acquisition of PN code clock, which for most practical systems also implies data clock acquisition, must be achieved in the absence of phase and frequency synchronization. (Here, we are interested in the scenario where the PN code clock and data symbol clock are derived from a common source. Hence, an acquisition of the PN code clock leads to data symbol clock recovery.) This is due to the fact that if one chooses to achieve phase and frequency estimation in the absence of PN code acquisition, the phase and frequency synchronizers must extract synchronization information from a wideband signal. This, in general, is a formidable task due to the large bandwidth of typical CDMA signals. Hence, in a CDMA system, PN code timing acquisition precedes any other form of synchronization. Upon the recovery of the PN code phase the CDMA signal is despread and then an accurate estimate of frequency or phase is obtained.

We are then faced with a situation where PN code clock must be recovered in a noncoherent fashion. Before describing the mechanism by which PN code clock synchronization can be acquired, we need to point out that synchronization here is achieved in two phases. In phase I, an initial synchronization of the PN code phase is established via acquiring the epoch of the received PN code to within a fraction of a chip interval. This problem is identical to the estimation of the propagation delay between a transmitter/receiver pair when the propagation delay is less than the period of the PN code. In the event that the propagation delay is greater than the period of the PN code, the synchronization procedure yields an estimate of the propagation delay reduced mod PT_c. Phase I of the PN code synchronization is equivalent to estimating the state of the shift register that generates the desired PN code.

In phase II, the epoch of the desired PN code is tracked so that a real-time estimate of PN code phase can be maintained at the receiver. As noted earlier, PN code estimation is accomplished in the face of unknown channel phase and frequency. In general, one can assume that the partial autocorrelation of a PN code satisfies the following property:

$$|R_{\mathrm{a}}^{(j)}(\tau, \hat{\tau})| \ll R_{\mathrm{a}}^{(j)}(\tau, \tau) \quad \text{for} \quad |\tau - \hat{\tau}| \geqslant T_{\mathrm{c}} \tag{13}$$

This property is critical to a successful PN code acquisition since it can be exploited to realize a PN code acquisition model.

To gain further insight, let us assume that the communication channel is such that the transmission through the channel introduces a delay of τ, a phase offset of θ, a frequency error or offset of $\Delta\omega$ rad/s, and an amplitude distortion of $A(t)$. That is, let

$$\tilde{r}(t) = A(t)e^{i[\Delta\omega t+\theta]}\tilde{x}(t-\tau)$$

denote the complex envelope of the received signal at the input of a CDMA receiver in the absence of additive noise. (Since the inclusion of an additive noise results only in the presence of a noisy term at the output of the correlation operation, and hence does not provide any further insight, we may proceed with a noiseless model to illustrate the function of the PN code acquisition model.) A noncoherent PN code acquisition model computes $g(\tau, \hat{\tau})$, given by

$$g(\tau,\ \hat{\tau}) = \sum_{n=1}^{L} |<\tilde{r}(t)\mathrm{PN}_j^*(t-\hat{r})>_{n,\hat{\tau}}|^2 \tag{14}$$

where we have assumed that $\hat{\tau}$ denotes an estimate of τ, the propagation delay between transmitter and receiver, and we have collected energy over L symbol intervals. As noted earlier, if $\tau > PT_{\mathrm{c}}$, then $\hat{\tau}$ is the estimate of τ reduced mod PT_{c}. Obviously, the objective of a PN code acquisition model is to bring τ to within a fraction of T_{c} of τ. Namely, we are interested in acquiring an estimate $\hat{\tau}$ where $|\tau - \hat{\tau}| \leq T_{\mathrm{c}}/N_{\mathrm{s}}$ for $N_{\mathrm{s}} \geq 2$.

In the ensuing discussion, we further assume that the modulation is absent. This assumption is motivated by the fact that in commercial CDMA systems a pilot signal (a CDMA signal without the modulating signal) is provided by transmitter to aid synchronization. The presence of modulation further complicates the model without adding any further insight. For this reason, we proceed with a pilot-signal-aided synchronization model.

We further assume that an initial frequency estimate is obtained, and hence $\Delta\omega$ is assumed to be relatively small compared with $1/T_s$. If one assumes that the jth PN signal (without modulation) is used to generate the CDMA signal and that the amplitude distortion in the channel remains relatively constant for a symbol time, i.e., $A(t) \approx A$, then (for most channels of interest, this assumption is valid)

$$g(\tau,\ \hat{\tau}) = \sum_{n=1}^{L} |<Ae^{i[\Delta\omega t+\theta]}\mathrm{PN}_j(t-\tau)\mathrm{PN}_j^*(t-\hat{\tau})>_{n,\hat{\tau}}|^2 \tag{15}$$

Hence,

$$\begin{aligned} g(\tau,\ \hat{\tau}) &= A^2D^2(\Delta\omega T_s)\sum_{n=1}^{L} |e^{i\theta} <\mathrm{PN}_j(t-\tau)\mathrm{PN}_j^*(t-\hat{\tau})>_{n,\hat{\tau}}|^2 \\ &= A^2D^2(\Delta\omega T_s)\sum_{n=1}^{L} |<\mathrm{PN}_j(t-\tau)\mathrm{PN}_j^*(t-\hat{\tau})>_{n,\hat{\tau}}|^2 \end{aligned} \tag{16}$$

where $D(\Delta\omega T_s)$ accounts for the distortion caused by the presence of the frequency error. $D(\Delta\omega T_s)$ is a decreasing function of $\Delta\omega T_s$, and hence for $\Delta\omega T_s \ll 1$ one can expect a small level of distortion. As can be seen, the phase error is eliminated with the aid of the absolute value function. Now, let us consider the case where the I and Q PN codes are nearly orthogonal. That is, let

$$\langle \mathrm{Re}\{\mathrm{PN}_j(t)\}\mathrm{Im}\{\mathrm{PN}_j(t)\}\rangle_{n,0} \approx 0 \tag{17}$$

With some effort, it can be shown that

$$g(\tau, \hat{\tau}) \approx 4LA^2D^2(\Delta\omega T_s)(R_a^{(j)}(\tau,\ \hat{\tau}))^2 \tag{18}$$

Hence, the function of the absolute value operation is to eliminate any phase error that may be present at the receiver, while the integration operation is intended to yield $R_a^{(j)}(\tau, \hat{\tau})$. Due to Eq. (13), it is relatively easy to observe that $g(\tau, \hat{\tau})$ may be used to launch a search for a correct epoch of the code. The function of L is to provide a confidence in declaring whether or not the correct epoch of the code has been acquired when additive noise (or interference) is present.

Before discussing the acquisition model based on the above observation, let us consider the case where additive noise is present. In the presence of noise, additional terms in $g(\tau, \hat{\tau})$ that are dependent on noise must be accounted for. In that case, one can argue that

$$E\{g(\tau, \hat{\tau})\} = 4LA^2D^2(\Delta\omega T_s)(R_a^{(j)}(\tau,\ \hat{\tau}))^2 \tag{19}$$

where $E\{\}$ is the ensemble average of the enclosed. That is, the operation described by Eq. (14) yields an output whose average value provides one with the necessary function to carry out PN code acquisition. Hence $g(\tau, \hat{\tau})$ may be used as an indicator of the PN code acquisition state.

The search mechanism then consists of a chip-by-chip search that will be carried out in a serial fashion. In this scheme, $g(\tau, \hat{\tau})$ is obtained for a $\hat{\tau}$. In the event $g(\tau, \hat{\tau})$ falls below a predefined threshold, $\hat{\tau}$ is increased by T_c/N_s (N_s is the number of steps per chip interval). Once the local PN code epoch is within a chip interval of the received PN code, the output of the correlator will exceed the threshold device that is designed to yield an optimum performance. At this stage, the synchronizer declares PN code acquisition and proceeds with PN code tracking. Since noise can impair this process, the performance of this acquisition model is determined in terms of the statistics of the acquisition time (mean and standard deviation of the acquisition time), probability of acquisition, and probability of false acquisition.

Phase II of synchronization involves the tracking of the PN code. This process involves maintaining a local PN code signal whose epoch is different from the epoch of the received signal by no more than a fraction of the chip time T_c. This objective is achieved via a PN code tracking loop that generates a pair of PN signals that are delayed and advanced by a fraction of chip time with respect to the local PN code. More specifically, the following signal is formed:

$$S(\tau_e) = g\left(\tau_e - \frac{T_c}{N_s}\right) - g\left(\tau_e + \frac{T_c}{N_s}\right) \quad (20)$$

where $\tau_e = \tau - \hat{\tau}$. In arriving at Eq. (20) it is assumed that when $|\tau - \hat{\tau}| < T_c$, $g(\tau, \hat{\tau}) = g(\tau - \hat{\tau})$. This signal is then used as an error signal to adjust $\hat{\tau}$. Since a voltage-controlled-oscillator (VCO) provides the clock signal for the generation of the local PN code, the adjustment of $\hat{\tau}$ can be achieved using $S(\tau_e)$. The expected value of function $S(\tau_e)$, i.e., $E\{S(\tau_e)\}$, is often referred to as the "S-curve" of the tracking loop. Such a function determines the tracking behavior of the loop. In particular, the variance of the steady state timing error as well as the mean time to loss of lock are dependent upon this function. Using Eq. (19), we have

$$E\{S(\tau_e)\} = 4LA^2D^2(\Delta\omega T_s)\left\{\left[R_a^{(j)}\left(\tau_e - \frac{T_c}{N_s}\right)\right]^2 - \left[R_a^{(j)}\left(\tau_e + \frac{T_c}{N_s}\right)\right]^2\right\} \quad (21)$$

To gain an insight into the operation of this loop, let us consider a scenario where the I and Q PN sequences are uncorrelated and possess identical autocorrelation functions. As noted earlier, this assumption leads to

$$R_a^{(j)}(\tau_e) = \langle \mathrm{Re}\{\mathrm{PN}_j(t-\tau)\}\mathrm{Re}\{\mathrm{PN}_j(t-\tau-\tau_e)\}\rangle_{n,\tau+\tau_e} \quad \text{for} \quad |\tau - \hat{\tau}| < T_c \quad (22)$$

When $P_c(t)$ is an NRZ pulse,

$$\langle \mathrm{Re}\{\mathrm{PN}_j(t-\tau)\}\mathrm{Re}\{\mathrm{PN}_j(t-\tau-\tau_e)\}\rangle_{n,\tau+\tau_e} \approx P_g T_c \left(1 - \frac{|\tau_e|}{T_c}\right)\mathrm{rect}\left(\frac{\tau_e}{2T_c}\right);\ |\tau_e| < T_p \quad (23)$$

and

$$R_a^{(j)}(\tau + nT_P) = R_a^{(j)}(\tau) \text{ for all integer } n \quad (24)$$

In Eq. (23),

$$\mathrm{rect}(x) = \begin{cases} 1 & |x| < 0.5 \\ 0 & \text{otherwise} \end{cases}$$

This situation is commonly referred to as the time-limited case. This is due to the fact that the chip pulse shape extends over a finite time interval, and subsequently its spectrum is extended over a large frequency range. Equation (24) implies that the autocorrelation function of the PN code is a periodic function. This property is a direct consequence of the fact that PN codes are periodic functions with period $T_p = PT_c$. T_p here, then, denotes the period of the PN code. Note that Eq. (24) is a property common to all PN codes, whereas Eq. (23) is obtained with the I and Q PN sequences that make up the jth PN code satisfy the following properties:

$$\sum_{n_1=1}^{P_g} s_{n_1,I}^{(j)} s_{n_1+n,I}^{(j)} = \sum_{n_1=1}^{P_g} s_{n_1,Q}^{(j)} s_{n_1+n,Q}^{(j)} = \begin{cases} P_g & n = 0 \\ \le \lambda_a & \text{otherwise} \end{cases} \quad (25)$$

and

$$\sum_{n_1=1}^{P_g} s_{n_1,I}^{(j)} s_{n_1+n,Q}^{(j)} \le \lambda_c;\ \text{for all } n \quad (26)$$

with $\lambda_a \ll P_G$ and $\lambda_c \ll P_G$ denoting the peak out-of-phase autocorrelation function and peak cross-correlation function, respectively, of the I and Q PN sequences. For $\lambda_a = 0$ and $\lambda_c = 0$, a pair of distinct phase shifts of the I or Q PN sequences are uncorrelated. Moreover, the I and Q PN sequences may be viewed as uncorrelated sequences as well. In that event, Eq. (23) denotes the exact, and not an approximate, expression. In practice, however, one encounters λ_a and λ_c that are nonzero, and hence Eq. (23) must be used as an approximate partial autocorrelation function. The approximation due to $\lambda_a \ll P_g$ and $\lambda_c \ll P_g$ conditions, however, is a good one. We note that although one requires that $\lambda_a \ll \lambda_c$, the critical assumption for detection is that both λ_c and λ_a remain significantly smaller than PG.

Given the assumption states above, we arrive at an S-curve for the above tracking loop that is approximately a linear function of τ_e for the range $[-T_c/N_s, T_c/N_s]$. More important, the slope of the S-curve remains positive for this range. There are several aspects of this function that are of interest. First, when the timing error is zero, which implies a perfect synchronization has been achieved, $E\{S(\tau_e)\} = 0$. In this case, the input to the VCO is reduced to zero. Second, as the timing error begins to depart from 0, the signal at the input of the VCO will have a magnitude that is proportional to τ_e. Hence, $S(\tau_e)$ provides the VCO with a signal that is an odd and monotonic function of the timing error, and thus can be used to adjust $\hat{\tau}$. As noted above, the other feature of the above S-curve is that it is nearly a linear function of τ_e in the vicinity of $\tau_e = 0$. This is an important property, since the initial synchronization yields an estimate of the PN code epoch that is within $\pm T_c/N_s$ of the received PN code epoch. In this case, one can assume that the loop provides us with an error signal that is directly proportional to the timing error, and hence a linear tracking loop results.

Finally, in practice, $P_c(t)$ is chosen to be a square-root raised-cosine pulse shape. In that case, a somewhat different result emerges. That is,

$$E\{S(\tau_e)\} = 4LA^2D^2(\Delta\omega T_s)\left\{P_{RC}^2\left(\tau_e = \frac{T_c}{N_s}\right) - P_{RC}^2\left(\tau_e + \frac{T_c}{N_s}\right)\right\} \quad (27)$$

where $P_{RC}(t) = P_c(t) \circledast P_c(t)$ ($\circledast$ denotes a convolution operation) is a raised-cosine pulse shape given by Eq. (28) (note that the square-root raised cosine pulse $P_c(t)$ is implicitly defined in terms of $P_{RC}(t)$)

$$P_{RC}(t) = \frac{\sin(\pi t/T_c)}{\pi t/T_c}\frac{\cos(\pi\alpha t/T_c)}{1 - (2\alpha t/T_c)^2} \quad (28)$$

This case is referred to as the bandwidth-limited case. Note that $P_c(t)$ extends over several chip intervals, leading to a spectrum that is limited in bandwidth. Although $E\{S(\tau_e)\}$ does not yield a linear S-curve over the entire interval of $[-T_c/N_s, T_c/N_s]$, it provides us with all the necessary conditions for a successful PN code tracking. That is, $E\{S(\tau)\}$ is a linear function of τ_e in the vicinity of $\tau_e = 0$. Also, $E\{S(\tau_e)\}$ possesses a positive slope in the range $[-T_c/N_s, T_c/N_s]$. Hence, one may expect a tracking performance similar to that of the NRZ chip pulse shape case.

INTERFERENCE

Now let us consider the received signal at the input of a CDMA receiver where other CDMA signals are present. We consider two possibilities. First, it is considered that the channel is nondispersive, and hence no multipath components are present. In the second case, a more general scenario where multipath scattering is present is considered. In the event that the channel is nondispersive,

$$\tilde{r}(t) = A_j e^{i\theta_j(t)}\tilde{x}_j(t-\tau_j) + \sum_{l=1;l\neq j}^{N} A_l e^{i\theta_l(t)}\tilde{x}_l(t-\tau_l) + \tilde{z}(t) \quad (29)$$

where now $N - 1$ other CDMA signals are present. It is assumed that the lth signal encounters τ_l seconds of propagation delay, an amplitude scaling of A_l, and a random phase shift of $\theta_l(t)$. Note that any frequency errors caused by channel is represented by $d\theta_l(t)/dt$. As can be seen, the received signal is corrupted by many interfering signals and an additive noise $\tilde{z}(t)$. The term $\tilde{z}(t)$ is a complex white Gaussian noise whose real and imaginary parts are a pair of independent white Gaussian noise processes with a two-sided power spectrum density of N_0 W/Hz over the frequency range of interest. For the additive noise, we have

$$\begin{aligned} E\{\tilde{z}(t)\tilde{z}^*(t-s)\} &= 2E\{\tilde{z}_r(t)\tilde{z}_r(t-s)\} \\ &= 2E\{\tilde{z}_i(t)\tilde{z}_i(t-s)\} = 2N_0\delta(t) \end{aligned}$$

where $\delta(t)$ is a dirac-delta function and $\tilde{z}_r(t)$ and $\tilde{z}_i(t)$ denote the real and imaginary parts of $\tilde{z}(t)$, respectively. Note that $\mathrm{Re}\{\tilde{z}(t)e^{i\omega_c t}\}$ may now be considered as a band-limited Gaussian noise whose power spectrum remains flat over the frequency range of interest about ω_c rad/s at $N_0/2$ W/Hz.

To gain an insight into the means by which CDMA receivers overcome interference, let us consider the outcome of a bandwidth despreading operation. Furthermore, let us also assume a scenario where we are interested in recovering the jth signal.

Obviously, one requires that the receiver acquires an estimate of τ_j. This task remains with the PN code acquisition subsystem discussed previously. Assuming that a successful delay estimation is performed, an estimate of τ_j that is within $\pm T_c/N_s$ ($N_s \geq 2$) of τ_j can be obtained. Let such an estimate be $\hat{\tau}_j$. Also, let us assume that the frequency shift in the signal caused by the channel is compensated for and that the residual frequency error caused by estimation process is small enough so that $\theta_l(t) \approx \theta_l$ for the observation interval. That is, θ_l now denotes the residual phase error at the receiver caused by channel phase shift and imperfect estimation and compensation of frequency. This condition is typically satisfied in practice by acquiring the PN code, despreading the signal, and, with the aid of a frequency estimator, acquiring an estimate of the frequency.

Then the outcome of the bandwidth despreading operation (when the nth symbol is of interest) after frequency compensation and delay estimation is

$$\begin{aligned} &\frac{1}{2}\langle \tilde{r}(t)\mathrm{PN}_j^*(t-\hat{\tau}_j)\rangle_{n,\hat{\tau}_j} \\ &= z_n + \frac{A_j e^{i\theta_j}}{2T_s}\int_{(n-1)T_s+\hat{\tau}_j}^{nT_s+\hat{\tau}_j} d_j(t-\tau_j)\mathrm{PN}_j(t-\tau_j)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt \\ &\quad + \sum_{l=1;l\neq j}^{N}\frac{A_l e^{i\theta_l}}{2T_s}\int_{(n-1)T_s+\hat{\tau}_j}^{nT_s+\hat{\tau}_j} d_l(t-\tau_l)\mathrm{PN}_l(t-\tau_l)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt \end{aligned} \quad (30)$$

It is not immediately obvious whether or not the nth symbol can be recovered using this operation. Depending on the type of detection used to recover the transmitted data symbol, an estimate of θ_j may be needed at the receiver. To go any further, without loss of generality, let us assume that $\hat{\tau}_j \geq \tau_j$ when $|\hat{\tau}_j - \tau_j| \leq T_c$. In that event, Eq. (30) reduces to

$$\begin{aligned} &\frac{1}{2}\langle \tilde{r}(t)\mathrm{PN}_j^*(t-\hat{\tau}_j)\rangle_{n,\hat{\tau}_j} \\ &= z_n + \frac{A_j e^{i\theta_j}}{2T_s}\int_{(n-1)T_s+\hat{\tau}_j}^{nT_s+\tau_j} d_j(t-\tau_j)\mathrm{PN}_j(t-\tau_j)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt \\ &\quad + \frac{A_j e^{i\theta_j}}{2T_s}\int_{nT_s+\tau_j}^{nT_s+\hat{\tau}_j} d_j(t-\tau_j)\mathrm{PN}_j(t-\tau_j)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt \\ &\quad + \sum_{l=1;l\neq j}^{N}\frac{A_l e^{i\theta_l}}{2T_s}\int_{(n-1)T_s+\hat{\tau}_j}^{nT_s+\hat{\tau}_j} d_l(t-\tau_l)\mathrm{PN}_l(t-\tau_1)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt \end{aligned} \quad (31)$$

where

$$z_n = \frac{1}{2T_s}\int_{(n-1)T_s+\hat{\tau}_j}^{nT_s+\hat{\tau}_j} \tilde{z}(t)\mathrm{PN}_j^*(t-\hat{\tau}_j)\,dt$$

denotes a zero mean Gaussian random variable. This equation then leads to

$$\begin{aligned} &\frac{1}{2}\langle \tilde{x}(t)\mathrm{PN}_j^*(t-\hat{\tau}_j)\rangle_{n,\hat{\tau}_j} \\ &= z_n + A_j e^{i\theta_j}[R_j^{(1)}(\tau_j,\hat{\tau}_j)d_n^{(j)} + R_j^{(2)}(\tau_j,\hat{\tau}_j)d_{n+1}^{(j)}] \\ &\quad + \sum_{l=1;l\neq j}^{N} A_l e^{i\theta_l}[R_{j,l}^{(1)}(\tau_l,\hat{\tau}_j)d_{p_l}^{(l)} + R_{j,l}^{(2)}(\tau_l,\hat{\tau}_j)d_{p_l+1}^{(l)}] \end{aligned} \quad (32)$$

where

$$R_j^{(1)}(t_1,t_2) = \frac{1}{2T_s}\int_{(n-1)T_s+t_2}^{nT_s+t_1} \mathrm{PN}_j(t-t_1)\mathrm{PN}_j^*(t-t_2)\,dt$$

and

$$R_j^{(2)}(t_1,t_2) = \frac{1}{2T_s}\int_{nT_s+t_1}^{nT_s+t_2} \mathrm{PN}_j(t-t_1)\mathrm{PN}_j^*(t-t_2)\,dt$$

are partial autocorrelation functions of the jth PN code. In general, the PN codes are selected from a family of codes with identical autocorrelation properties, and hence the subscript of j may be dropped. Moreover,

$$R_{j,k}^{(1)}(t_1, t_2) = \frac{1}{2T_s} \int_{(n-1)T_s+t_2}^{nT_s+t_1} \mathrm{PN}_k(t-t_1)\mathrm{PN}_j^*(t-t_2)\,dt$$

and

$$R_{j,k}^{(2)}(t_1, t_2) = \frac{1}{2T_s} \int_{nT_s+t_1}^{nT_s+t_2} \mathrm{PN}_k(t-t_1)\mathrm{PN}_j^*(t-t_2)\,dt$$

denote the partial cross-correlation function of the jth and kth PN codes. In arriving at Eq. (32), we have assumed that the integration interval has coincided with the p_l and $p_l + 1$th signaling interval of the lth interfering signal.

From Eq. (32), it is rather obvious that the desired symbol $d_n^{(j)}$ is recovered. This recovery method, however, has yielded a number of undesirable terms. First, the presence of timing error, similar to other digital receivers, has resulted in the introduction of intersymbol interference in the detection process [note the term involving $d_{n+1}^{(j)}$]. Furthermore, the detection process is now corrupted by an interfering signal, even in the absence of additive noise. To estimate the impact of interference, the properties of the partial autocorrelation and cross-correlation functions of the PN codes must be evaluated. Before doing so, let us examine the preceding result more carefully. First, it is obvious from the definition of $R_j^{(2)}(t_1, t_2)$ that when $\hat{\tau}_j = \tau_j$, the intersymbol interference is reduced to zero. That is, $R_j^{(2)}(\hat{\tau}_j, \hat{\tau}_j) = 0$. Since $|\hat{\tau}_j - \tau_j| \le T_c/N_s$ for some $N_s \ge 2$, for a typical PN code with large processing gain, $R_j^{(1)}(\tau_j, \hat{\tau}_j) \gg R_j^{(2)}(\tau_j, \hat{\tau}_j)$ for all j. That is, the intersymbol interference may be viewed as negligible for most practical cases. The other interfering terms, however, are dependent on the partial cross-correlation functions of PN codes and thus cannot be suppressed readily.

If one assumes that the product of two PN codes results in yet another wideband code, then the bandwidth of the interfering signal remains unchanged, yielding a wideband interfering signal. That is, the despreading operation manages to despread the bandwidth of the desired signal while yielding a wideband interference. Since the integration over a symbol time is equivalent to a filtering operation over a bandwidth of $1/T_s$ Hz and the despreading interference signal possesses a bandwidth proportional to $1/T_c$, the contribution of the interference noise to the detection process is undermined by a factor proportional to the spreading gain. In other words, the interference contributes only $1/P_G$ of its total power to the detection of $d_n^{(j)}$. Stated differently, the key assumption of a CDMA receiver is that

$$\frac{|R_j^{(1)}(\tau_j, \hat{\tau}_j)|}{|R_{j,l}^{(1)}(\tau_l, \hat{\tau}_j)| + |R_{j,l}^{(2)}(\tau_l, \hat{\tau}_j)|} \approx P_G$$

for all $l \ne j$. Hence, for a large processing gain, one can expect a significant reduction in the interference level at the output of a CDMA receiver. Note that there are $(N - 1)$ interferers present, and hence one must consider a large enough P_G so that the total interference level remains small.

Finally, note that $d_n^{(j)}$ is scaled by an unknown coefficient $A_j e^{i\theta_j}$ in Eq. (32). If a phase modulation is used, then θ_j must be estimated at the receiver. In the event that θ_j remains constant over two consecutive time slots and a differential phase modulation is used, an estimate of θ_j is not required at the receiver. For other scenarios, the output of the despreader is fed to a channel estimation system so that an estimate of θ_j (and A_j in some cases) can be obtained and compensated for.

NEAR-FAR PROBLEM AND POWER CONTROL

Another important fact revealed by Eq. (32) is that the interfering signals' power levels are different from that of the desired signal. Obviously, if $A_j = \max\{A_l;\ l = 1, \ldots, N\}$, a favorable outcome results. That is, the channel has caused an attenuation in the desired signal that is smaller than those experienced by the interfering signals. Since this condition cannot be guaranteed in a mobile communication environment, the interfering signals can take on relatively large amplitudes compared with the desired signal. In this case, the interfering signals can completely suppress the desired signal, resulting in an unacceptable performance for a CDMA system. Since no fading is considered here, and assuming that all the CDMA signals are originated at the transmitter at identical power levels, the aforementioned scenario is only encountered when the distance between the desired user and the receiver is larger than all or some of the distances between the interfering users and the receiver. This problem is commonly referred to as the "near-far" problem in CDMA receivers. Considering the wide range of distances a mobile user can take on, this problem can severely hamper the performance of a wireless CDMA system.

In theory, this problem can be circumvented by regulating the power levels of all CDMA transmitters so that received signals at the receiver possess identical power levels (i.e., $A_j = A$ for all j). The mechanism by which this goal may be achieved is known as power control. Power control, in practice, is accomplished using either an open-loop or a closed-loop mechanism. For the sake of discussion, let us consider a mobile CDMA scenario. Furthermore, let $\tilde{r}(t)$ denote the received signal at a CDMA base station. Hence, N denotes the number of active mobile transmitters.

In an open-loop mechanism, the base station sends a signal (pilot signal) with known power level to all the mobile units (forward link). Mobile units measure the received power levels and, in turn, set their transmitter power levels for the reverse link in accordance with the received power level. (Typically, the power level of a mobile transmitter is increased by x dB, if the pilot signal is received at the mobile at $-x$ dB power level. For the case where the pilot is received at $+x$ dB, the transmitter power level is reduced by x dB.) If the communication channel remains the same for all users and fast fading can be ignored, this mechanism can yield favorable results.

Although reciprocity exists between reverse and forward links of a wireless channel when log-normal fading (slow fading) is of concern (log-normal shadowing effect is due to the obstruction of the direct path of communication), the forward and reverse links experience different fast fading effects. That is, the information that is obtained regarding the channel condition by observing the forward channel's power level (pilot power) may not be used to estimate the channel characteristics in the reverse link. For this reason, after initial power level setting using the open-loop mechanism, a closed-loop procedure is followed to overcome the near-far problem. In this case, A_j is a function of not only propagation distance but also channel fading characteristics. In this case, the base station makes a measurement of the power levels of the received signals from individual mobile units. This information is reported back to the mobile units using what is known as a power control bit, which indicates whether the mobile should boost or reduce its power in some fixed dB increments. This process is repeated up to 2000 times per second in some modern systems. Given the fast rate of updates, this procedure can overcome the impact of rapid fluctuations in the power level. Note that the power level adjustments of the mobile units is based on the information regarding the reverse link, and hence one can expect a more effective means of circumventing the near-far problem using the closed-loop power control mechanism.

CHANNEL EFFECTS

So far, we have considered a perfect communication channel. That is, we have assumed that the bandwidth despreading operation is performed on an exact replica of the transmitted signal at the receiver. As noted earlier, we are concerned with a dispersive channel. Let the impulse response of the channel be

$$\tilde{h}(t) = \sum_{l=1}^{N_\mathrm{p}} \tilde{c}_l(t)\delta(t-\tau_l(t)) \tag{33}$$

where $\tilde{h}(t)$ is the complex impulse response of the channel, $\delta(t)$ is the dirac-delta function, $\tau_l(t)$ denotes the propagation delay for the lth multipath between transmitter and the receiver, and $\tilde{c}_l(t)$ is a complex multiplicative distortion (MD) denoting the channel fading effect for the lth resolvable path of the multipath channel. The term $\tilde{c}_l(t)$ is often modeled as a low-pass complex Gaussian process. Moreover, N_p denotes the total number of resolvable multipaths. The set of multipath delays encountered in a channel is often referred to as the delay profile of a scattering channel. For most channels of interest and when the observation interval is short enough to render a constant delay profile, one may assume that $\tau_l(t) \approx \tau_l$. N_p and τ_l are determined by the multipath profile of the channel, whereas the characteristics of $\tilde{c}_l(t)$ is a function of the Doppler spectrum of the channel.

Due to the Gaussian property, one can fully characterize the statistics of $\tilde{c}_j(t)$ using only the second-order statistics of the process.

It is shown that the MD processes have autocorrelation functions that satisfy (assuming no log-normal shadowing)

$$E\{\tilde{c}_l(t)\tilde{c}_l(t-\tau)^*|\sigma_l^2\} = \sigma_l^2 J_0(2\pi f_\mathrm{d}^{(l)}\tau)e^{i2\pi f_\mathrm{e}\tau} \tag{34}$$

with σ_l^2, $f_\mathrm{d}^{(l)}$, and f_e denoting the mean square value of the MD for the lth path of the signal, the maximum Doppler spread experienced by the lth path of the signal, and the residual frequency error in hertz at the receiver, respectively. Moreover, $\mathrm{E}\{()|\sigma_l^2\}$ denotes the expected value of the enclosed condition on σ_l^2. Note that we have kept the discussion as general as possible to entertain the possibility of including a scenario where the desired and interfering users may be at different Doppler rates. When log-normal shadowing is present, we have

$$\sigma_l^2 = P_l 10^{\zeta/10}$$

where ζ is a normal probability density function (log-normal shadowing) with a zero mean and a standard deviation of σ_ζ (many field trials have shown σ_ζ to be in the 4 dB to 8 dB range for microcellular urban environments) and P_l is the received power in the absence of shadowing for the lth path of the signal. Hence, the average power can be calculated using

$$E\{\sigma_l^2\} = \eta P_l \tag{35}$$

where

$$\eta = E\{10^{\zeta/10}\} = \exp\left(\left(\frac{\ln(10)}{10}\right)^2 \frac{\sigma_\zeta^2}{2}\right) \quad \text{and} \quad E\{\}$$

denote the expected value of the enclosed with respect to ζ. Hence,

$$E\{\tilde{c}_l(t)\tilde{c}_l(t-\tau)^*\} = R_\mathrm{c}^{(l)}(\tau) = \eta P_l J_0(2\pi f_\mathrm{d}^{(l)}\tau)e^{i2\pi f_\mathrm{e}\tau} \tag{36}$$

Also, since uncorrelated fading is considered,

$$E\{\tilde{c}_l(t)\tilde{c}_n(t-\tau)^*\} = R_\mathrm{c}^{(l)}(\tau)\delta[l-n] \tag{37}$$

where

$$\delta[x] = \begin{cases} 1 & x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Since $\tilde{c}_l(t)$'s are all Gaussian, $\{\tilde{c}_l(t)$; for all $l\}$ is a set of mutually independent Gaussian random processes.

Finally, suppose that the channel, in addition to causing a multipath effect, adds an additive noise. That is, the complex envelope of the jth received signal in the absence of user-induced interference may now be approximated as

$$\tilde{r}(t) = \sum_{l=1}^{N_p} \tilde{c}_l(t)\tilde{x}_j(t-\tau_l(t)) + \tilde{z}(t) \tag{38}$$

Therefore, a CDMA receiver must estimate some or all of τ_l's before any form of communication can take place. Due to the unique properties of the PN codes, an estimate of τ_l is acquired via establishing PN code acquisition for each path.

INTERFERENCE-DISPERSIVE CHANNEL

Now let us consider a more realistic scenario where other CDMA signals are present and the channel suffers from multipath scattering. In that event,

$$\begin{aligned} \tilde{r}(t) = &\sum_{l=1}^{N_p} \tilde{c}_{l,j}(t)\tilde{x}_j(t-\tau_{l,j}(t)) \\ &+ \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \tilde{c}_{l,k}(t)\tilde{x}_k(t-\tau_{l,k}(t)) + \tilde{z}(t) \end{aligned} \tag{39}$$

where now $N - 1$ other CDMA signals and their respective multipath components are considered. Note that we have introduced $\tilde{c}_{l,j}(t)$ as the MD for the lth path of the jth signal. All the properties for the MD processes discussed previously can be extended to this scenario as well. That is, we consider $\tilde{c}_{l,j}(t)$ as independent, baseband complex Gaussian processes for all l and j. Moreover, $\tau_{l,j}(t)$ now denotes the delay encountered by the lth path of the jth CDMA signal and is assumed to be slow varying. The preceding may also be presented as

$$\tilde{r}(t) = \sum_{l=1}^{N_p} \tilde{c}_{l,j}(t) d_j(t - \tau_{l,j}(t)) \mathrm{PN}_j(t - \tau_{l,j}(t)) + \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \tilde{c}_{l,k}(t) d_k(t - \tau_{l,k}(t)) \mathrm{PN}_k(t - \tau_{l,k}(t)) + \tilde{z}(t) \tag{40}$$

As can be seen, the received signal is corrupted by many interfering signals. Considering a situation where path delays are slow varying, we have a simplified model given by

$$\tilde{r}(t) = \sum_{l=1}^{N_p} \tilde{c}_{l,j}(t) d_j(t - \tau_{l,j}) \mathrm{PN}_j(t - \tau_{l,j}) + \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \tilde{c}_{l,k}(t) d_k(t - \tau_{l,k}) \mathrm{PN}_k(t - \tau_{l,k}) + \tilde{z}(t) \tag{41}$$

At this stage, we assume that the observation interval (symbol time) is short enough so that the delay profile for the channel remains unchanged. That is, $\tau_{m,j}(t) \approx \tau_{m,j}$ for the observation interval. This condition is satisfied for most practical applications.

To gain an insight into the means by which CDMA receivers overcome interference in the presence of the multipath effect, let us consider the outcome of a bandwidth despreading operation. Furthermore, let us also assume a scenario where we are interested in recovering the mth path of the jth signal. This situation is encountered in practice where the strongest paths of the desired signal are acquired by the PN code acquisition subsystem. More precisely, we require that the receiver acquires an estimate of $\tau_{m,j}$. Assuming that a successful delay estimation is performed, an estimate of $\tau_{m,j}$ that is within $\pm \tau_c/N_s$ of $\tau_{m,j}$ can be obtained. Let such an estimate be $\hat{\tau}_{m,j}$. Then the outcome of the bandwidth despreading operation (when the nth symbol is of interest) is

$$\begin{aligned}
&\frac{1}{2} \langle \tilde{x}(t) PN_j^*(t - \hat{\tau}_{m,j}) \rangle_{n,\hat{\tau}_{m,j}} \\
&= z_n + \frac{1}{2} \langle \tilde{c}_{m,j}(t) d_j(t - \tau_{m,j}) \mathrm{PN}_j(t - \tau_{m,j}) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j}) \rangle_{n,\hat{\tau}_{m,j}} \\
&\quad + \frac{1}{2} \sum_{l=1;l\neq m}^{N_p} \langle \tilde{c}_{l,j}(t) d_j(t - \tau_{l,j}) \mathrm{PN}_j(t - \tau_{l,j}) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j} \rangle_{n,\hat{\tau}_{m,j}} \\
&\quad + \frac{1}{2} \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \langle \tilde{c}_{l,k}(t) d_k(t-\tau_{l,k}) \mathrm{PN}_k(t-\tau_{l,k}) \mathrm{PN}_j^*(t-\hat{\tau}_{m,j}) \rangle_{n,\hat{\tau}_{m,j}}
\end{aligned} \tag{42}$$

As seen before, it is not immediately obvious whether the desired symbol can be recovered in this case. Similar to the previous case, where fading was absent and without loss of generality, let us assume that $\hat{\tau}_{m,j} \geq \tau_{m,j}$. In that event, Eq. (42) reduces to

$$\begin{aligned}
&\frac{1}{2} \langle \tilde{x}(t) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j}) \rangle_{n,\hat{\tau}_{m,j}} \\
&= z_n + \frac{1}{2T_s} \int_{(n-1)T_s+\hat{\tau}_{m,j}}^{nT_s+\tau_{m,j}} \tilde{c}_{m,j}(t) d_j(t - \tau_{m,j}) \mathrm{PN}_j(t - \tau_{m,j}) \\
&\quad \mathrm{PN}_j^*(t - \hat{\tau}_{m,j})\, dt + \frac{1}{2T_s} \int_{nT_s+\tau_{m,j}}^{nT_s+\hat{\tau}_{m,j}} \tilde{c}_{m,j}(t) d_j(t - \tau_{m,j}) \\
&\quad \mathrm{PN}_j(t - \tau_{m,j}) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j})\, dt + \frac{1}{2T_s} \sum_{l=1;l\neq m}^{N_p} \int_{(n-1)T_s+\hat{\tau}_{m,j}}^{nT_s+\hat{\tau}_{m,j}} \\
&\quad \tilde{c}_{l,j}(t) d_j(t - \tau_{l,j}) \mathrm{PN}_j(t - \tau_{l,j}) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j})\, dt \\
&\quad + \frac{1}{2T_s} \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \int_{(n-1)T_s+\hat{\tau}_{m,j}}^{nT_s+\hat{\tau}_{m,j}} \tilde{c}_{l,k}(t) d_k(t - \tau_{l,k}) \\
&\quad \mathrm{PN}_k(t - \tau_{l,k}) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j})\, dt
\end{aligned} \tag{43}$$

This equation then leads to

$$\begin{aligned}
&\frac{1}{2} \langle \tilde{x}(t) \mathrm{PN}_j^*(t - \hat{\tau}_{m,j}) \rangle_{n,\hat{\tau}_{m,j}} \\
&= z_n + \tilde{c}_{m,j} [R_j^{(1)}(\tau_{m,j}, \hat{\tau}_{m,j}) d_n^{(j)} + R_j^{(2)}(\tau_{m,j}, \hat{\tau}_{m,j}) d_{n+1}^{(j)}] \\
&\quad + \sum_{l=1;l\neq m}^{N_p} \tilde{c}_{l,j} [R_j^{(1)}(\tau_{l,j}, \hat{\tau}_{m,j}) d_{q_{l,j}}^{(j)} + R_j^{(2)}(\tau_{l,j}, \hat{\tau}_{m,j}) d_{q_{l,j}+1}^{(j)}] \\
&\quad + \sum_{k=1;k\neq j}^{N} \sum_{l=1}^{N_p} \tilde{c}_{l,k} [R_{j,k}^{(1)}(\tau_{l,k}, \hat{\tau}_{m,j}) d_{p_{l,k}}^{(k)} + R_{j,k}^{(2)}(\tau_{l,k}, \hat{\tau}_{m,j}) d_{p_{l,k}+1}^{(k)}]
\end{aligned} \tag{44}$$

where we have assumed that the channel remains constant over a symbol time [and hence $\tilde{c}_{m,j}(t)$ is replaced with $\tilde{c}_{m,j}$]. This assumption is satisfied for many practical communication systems. Moreover, we note that the integration interval has coincided with the $q_{l,j}$th and $q_{l,j} + 1$th signaling intervals of the lth ($l \neq m$) multipath of the desired signal (jth signal). Similarly, we have assumed that the integration interval includes the $p_{l,k}$th and $p_{l,k} + 1$th signaling interval of the lth path of the kth interfering signal. Obviously, $q_{l,j}$ and $p_{l,k}$ are dependent on the delay profile of the channel and the relative delays encountered by various users, respectively.

It is rather obvious that the desired symbol $d_n^{(j)}$ is recovered. Considering the conditions imposed on the cross-correlation and autocorrelation function of PN codes in the previous sections, it is rather easy to see that the interfering signals and their respective multipath components are detected at a power level that is approximately P_G times smaller than that of the desired signal.

RAKE RECEIVER

In the previous section, we introduced CDMA signaling and its properties. We also demonstrated that the received signal at a receiver often comprises delayed and attenuated versions of the desired signal (reflections) due to the multipath effect. It was also demonstrated that the interference due to the other active CDMA users adversely affects the received signal in a typical CDMA receiver.

For the sake of clarity, in what follows we consider a scenario in which the desired signal and its multipath components only are present. From Eq. (44), it is obvious that if one despreader is used to extract the mth path of the jth signal, the other multipath components appear as interference to this form of detection. Note that with the exception of amplitude and phase distortion effects, the spreading signal for all the reflected CDMA signals is known to the receiver, and hence one can capture this useful energy using an arrangement that is analogous to a garden rake. To elaborate, without loss of generality, let us consider a single user scenario where the strongest multipath component of the received signal is $\tilde{c}_1(t)\tilde{x}(t - \tau_1(t))$. That is, the delay associated with the strongest component of the multipath signal is $\tau_1(t)$. Moreover, let us assume that the PN code acquisition and tracking subsystem has locked onto this component of the received signal. The other components of the multipath signal [i.e., $\sum_{j=2}^{N_p}\tilde{c}_j(t)\tilde{x}(t - \tau_j(t))$] may now be regarded as interference. It is obvious that with the exception of $\tilde{c}_j(t)$ and $\tau_j(t)$, the received multipath components contain the useful modulation. Let us now suppose that from N_p possible multipaths, we are only interested in capturing N_{f} signals. N_{f} is commonly referred to as the number of "fingers" in a rake receiver. Also, let $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_{N_{\mathrm{f}}}]$ denote a vector containing the N_{f} significant multipath delays in an ascending order. Moreover, let $\hat{\boldsymbol{\tau}} = [\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_{N_{\mathrm{f}}}]$ be a vector of delay estimates obtained by the PN code acquisition system.

A rake receiver, after acquiring the N_{f} possible delays, performs an MF operation. The MF operation involves correlation with $\mathrm{PN}_j^*(t - \hat{\tau}_j)$; $j = 1, 2, \ldots, N_{\mathrm{f}}$ and integration over a symbol time. That is, the receiver forms the following set of variables:

$$Y_n^{(j)} = \frac{1}{2}\langle \tilde{r}(t)\mathrm{PN}_j^*(t - \hat{\tau}_j)\rangle_{n,\hat{\tau}_j};\ j = 1, 2, \ldots, N_{\mathrm{f}} \tag{45}$$

At this stage, there are two possibilities. In one scenario, it is possible to estimate the channel MD, and hence $\hat{c}_j(t)$ [an estimate of $\tilde{c}_j(t)$]; $j = 1, 2, \ldots, N_{\mathrm{f}}$ can be obtained at the receiver. This scenario is of critical importance to a coherent modulation case where a knowledge of channel phase is necessary for a successful demodulation. In the other scenario, such estimates are not available and the modulation scheme used allows for a noncoherent detection. We consider the coherent demodulation case first. In this case, since an MF operation is required, one must recompute Y_j as follows:

$$Y_{n,\mathrm{c}}^{(j)} = \frac{1}{2}\langle \tilde{r}(t)\hat{c}_j^*(t)\mathrm{PN}_j^*(t - \hat{\tau}_j)\rangle_{n,\hat{\tau}_j};\ j = 1, 2, \ldots, N_{\mathrm{f}}$$

Then a decision variable is formed for the nth transmitted data symbol as follows:

$$D_n^{(\mathrm{c})} = \sum_{j=1}^{N_{\mathrm{f}}} Y_{n,\mathrm{c}}^{(j)}$$

This variable is passed on to a coherent demodulator for further processing. Since an MF operation is performed for each path, in the absence of timing and channel estimation errors and in the face of an additive white Gaussian noise (AWGN), no other receiver yields an energy level higher than that produced by the arrangement just suggested. For this reason, this receiver is referred to as a maximal ratio combiner (MRC).

In the other scenario, the receiver uses Eq. (45) to compute Y_j. At this stage, one must remove channel phase ambiguities before a decision can be rendered on the transmitted symbol. If a frequency-shift-keying (FSK) modulation is used, then

$$D_n = \sum_{j=1}^{N_{\mathrm{f}}} |Y_n^{(j)}|^2$$

Note that although phase ambiguities have been removed, the amplitude fluctuations due to $\tilde{c}_j(t)$ have not been compensated for. This deficiency could seriously impair the performance of the subsequent demodulation process. For yet another modulation scheme, known as differential phase-shift keying (DPSK), the desired information is stored in the difference (reduced mod 2π) between two consecutive phases of the received signal. It is also assumed that the channel remains stationary for at least two consecutive symbol intervals. To recover the desired symbol, the receiver forms the following decision variables:

$$D_n = \sum_{j=1}^{N_{\mathrm{f}}} Y_n^{(j)} Y_{n-1}^{(j)*}$$

Similar to its FSK counterpart, this receiver is impaired by the changes in the channel amplitude due to $\tilde{c}_j(t)$. Additionally, any phase changes in two consecutive symbol intervals can produce unfavorable results in this case.

BIBLIOGRAPHY

1. M. K. Simon et al., *Spread Spectrum Communications Handbook,* McGraw-Hill, New York, 1994.
2. W. C. Jakes, *Microwave Mobile Communications,* New York: Wiley, 1974.
3. J. G. Proakis, *Digital Communications,* 3rd ed., New York: McGraw-Hill, 1995.
4. A. J. Viterbi, *CDMA—Principles of Spread Spectrum Multiple Access Communication,* Reading, MA: Addison-Wesley, 1995.

KAMRAN KIASALEH
University of Texas at Dallas

CODER, VIDEO. See VIDEO COMPRESSION STANDARDS.
CODING DATA TRANSMISSION. See INFORMATION THEORY OF DATA TRANSMISSION CODES.
CODING, IMAGE. See IMAGE AND VIDEO CODING.
CODING SPEECH. See SPEECH CODING.
CODING THEORY. See ALGEBRAIC CODING THEORY.
COGNITIVE OPERATIONS ON DATA. See INFORMATION SCIENCE.