REACTIVE POWER

The power that supplies the energy stored in reactive elements is called reactive power (1). For an inductive circuit, the reactive power is the magnetizing component of power necessary to establish the magnetic flux required by the circuit. On the other hand, for a capacitive circuit the reactive power is the stored energy in the electric field required for the production of alternating electrical charge. In both cases the instantaneous reactive power is alternately positive and negative, expressing the reversible flow of energy to and from the reactive component. The letter symbol for reactive power is Q. Real power, P, and Q both have the physical dimension of Watts, but to emphasize the fact that the latter represents a nonactive or reactive power, it is measured in voltamperes reactive (var). Larger and more practical units are kilovars and megavars, related with respect to the basic unit as follows: 1 Mvar $= 10^3$ Kvar $= 10^6$ var. The square root of the sum of squares of P and Q gives the apparent power S given by:

$$|S| = \sqrt{P^2 + Q^2} \tag{1}$$

By convention, an inductive load absorbs positive Q, that is, an inductor consumes reactive power; and a capacitive load absorbs negative Q, that is, a capacitor generates reactive power.

Historically, little attention has been given to the study of reactive power and its effect on the overall electric power system operation (2). This may be attributed to the way electric utilities have billed customers for energy. There are a very few instances where an electric utility charges residents for MVARh (related to reactive power); rather operating revenue is derived primarily from selling MWh (related to real power). Less interest has been given to pricing of reactive power compared to real power pricing. One of the reasons for this is perhaps the inherent difficult in understanding the concept and role of reactive power, especially by economist (3).

Figure 1 shows the vectorial representation of reactive components in relation to real power. This use of the so-called *real axis* to represent real power and *imaginary axis* to represent reactive power is convenient but it leaves many with the impression that there is something unreal or imaginary about reactive power. This impression is further strengthened by the correct statement that reactive power does no work. However, any doubt that reactive power is real should be dispelled by the amounts of money set aside annually by utilities to supplement the reactive supplied by generators to the major portion of the system which is inductive.

REACTIVE POWER IN SINGLE PHASE AND THREE-PHASE SYSTEMS

In a single phase, two-wire circuit in which the voltage and current are both sinusoidal the reactive power Q is given by:

$$Q = |V||I|\sin(\alpha - \beta) = |V||I|\sin\theta \tag{2}$$

where $|V|$ is the effective value of the voltage phasor, $|I|$ is the effective value of the current phasor, and $\theta = \alpha - \beta$ is the phase angle by which the current lags the voltage in an inductive circuit.

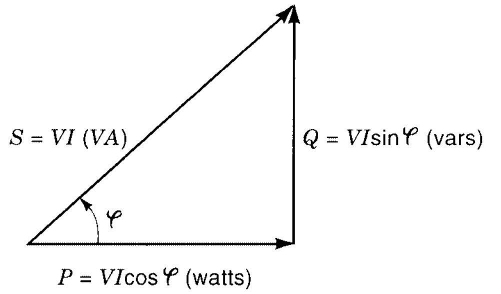

Figure 1. Vectorial representation of reactive components in relation to real power.

In a balanced three-phase circuit when the voltage and current are both sinusoidal the reactive power Q is given by:

$$Q = \sqrt{3}|V_L||I_L|\sin\theta \quad (3)$$

where $|V_L|$ and $|I_L|$ denote the effective line-to-line voltage and the line current, respectively. Sin θ is usually referred to as the reactive factor. The reactive power is also applied in power system studies on a per phase base. The per-phase reactive power is given by:

$$Q_p = |V_p||I_p|\sin\theta \quad (4)$$

where $|V_p|$ and $|I_p|$ are the effective values of the phase voltage and current, respectively.

The angle θ is related to the circuit elements as follows:

$$\theta = \tan^{-1}\frac{X_p}{R_p} = \cos^{-1}\frac{R_p}{Z_p} = \sin\frac{X_p}{Z_p} \quad (5)$$

where X_p, R_p, and Z_p are the reactance, resistance, and impedance per phase, respectively.

MEASUREMENT OF REACTIVE POWER IN THREE-PHASE SYSTEMS

In a three-phase network for which voltages and currents are balanced, a single wattmeter with its current circuit in line A and its voltage circuit connected across lines B and C will indicate the total three-phase reactive power (vars) if its reading is multiplied by $\sqrt{3}$. Figure 2 shows the connection of the wattmeter for measuring the reactive power in the balanced three-phase system. The reactive power in one of the phases is expressed as:

$$Q_a = V_{an}I_{an}\sin\theta_1 \quad (6)$$

where θ_1 is the angle between V_{an} and I_{an}, and since the system is balanced the total reactive power of the circuit will be:

$$Q_t = V_{an}I_{an}\sin\theta_1 + V_{bn}I_{bn}\sin\theta_2 + V_{cn}I_{cn}\sin\theta_3 = 3V_{an}I_{an}\sin\theta_1 \quad (7)$$

where θ_2 and θ_3 are the angles between the corresponding voltages and currents in the respective phases.

The real power in one of the phases is expressed as:

$$P_a = \sqrt{3}V_{an}I_{an}\sin\theta_1 \quad (8)$$

Hence:

$$Q_t = \sqrt{3}P_a \quad (9)$$

If the operating conditions are unbalanced and the circuit has a neutral, the total reactive power may be measured by adding three readings of three reactive-power meters connected to measure the reactive power of each of the three phases. However, where the wave forms of the circuit are sinusoidal and there is no neutral conductor, it is usual to measure reactive power by using two reactive power meters or a two-element polyphase reactive power meter connected like the two wattmeters of the two-wattmeter method (4). Figure 3 shows the connection of the two reactive voltameters for measuring the total reactive power in the unbalanced three-phase circuit. The total reactive power is given by:

$$Q_t = V_{ac}I_{an}\sin\theta_4 + V_{bc}I_{bn}\sin\theta_5 = Q_a + Q_b \quad (10)$$

where θ_4 and θ_5 are the angles between the corresponding voltages and currents in the respective phases. Q_a and Q_b represent the respective readings of the two reactive-power meters. Their sum gives the total reactive power of the load.

POWER FACTOR PENALTIES AND REACTIVE POWER

A low power factor means that more current is flowing in the network than would be ideal with a unity power factor. The excess current means that higher losses are experienced in the conductors and the transformers than in the case of higher power factor for a given load. Hence, a low power factor brings with it increased running costs and consumers whose load has a low power factor must be penalized.

By definition the power factor in an ac network is the ratio of the average power P to the apparent power S (product of

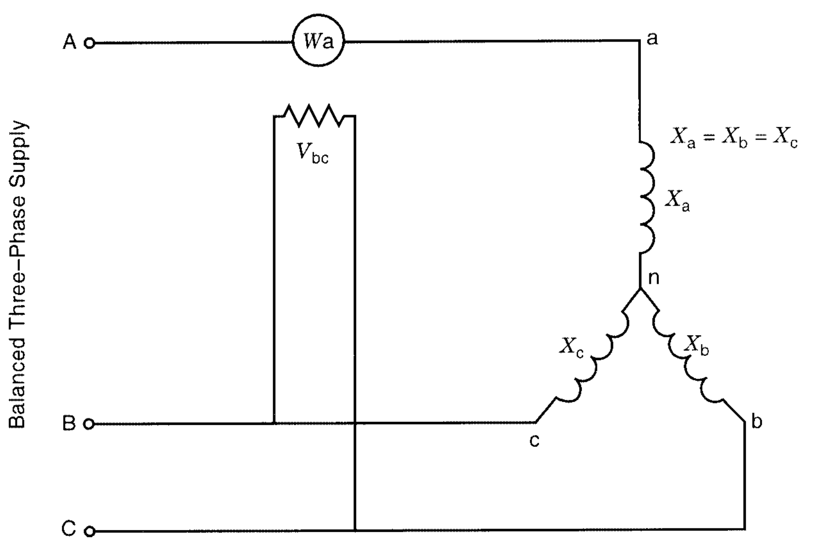

Figure 2. Measurement of total reactive power in a balanced three-phase circuit using one single phase wattmeter.

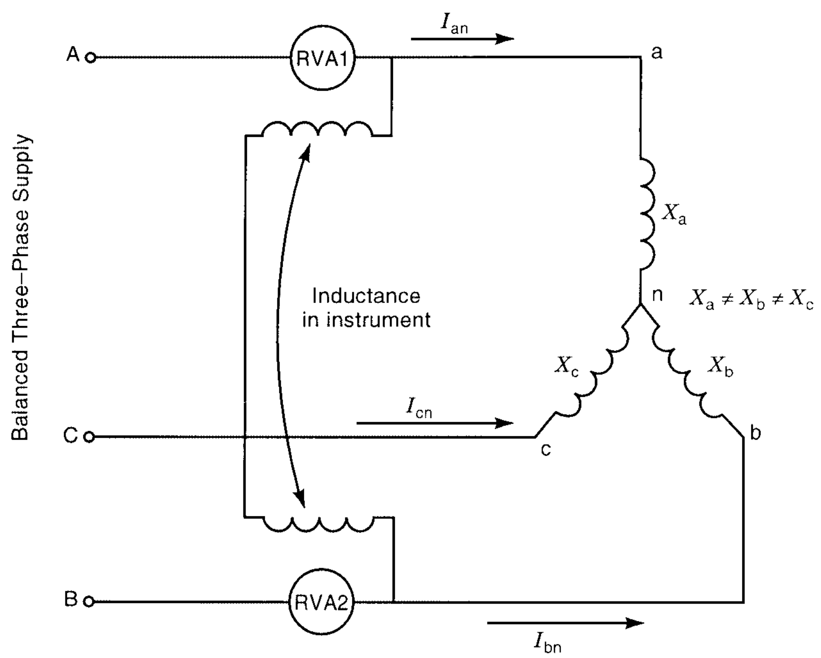

Figure 3. Measurement of total reactive power in unbalanced three-phase circuit using two reactive volt-ammeters.

rms voltage and current). Considering sinusoidal voltage and current which are under steady-state conditions having a phase difference θ, the active power is expressed as:

$$P = VI\cos\theta \tag{11}$$

where the product VI is the apparent power (volt–ampere) and $\cos\theta$ is the power factor. For nonsinusoidal voltage and current the power factor can still be defined but it is no longer equal to $\cos\theta$. The terms lagging and leading power factors imply that the current is lagging or leading the applied voltage, respectively.

Power factor penalties are ways of charging for an electricity supply by which the overall cost of energy per kwh increases as the power factor decreases (5). This can involve one of the following:

- a fixed charge per kVA of maximum demand
- a fixed charge per kW of maximum demand with a percentage penalty according to stipulated power factor values (e.g. 0.8) and
- a charge which consists of two components, one proportional to energy consumption in kWh and the other proportional to reactive consumption (in kVArh).

Since power factor is a ratio of real power to apparent power, it means use of apparatus or machinery which supply reactive power can cancel the lagging reactive component of the current in the main supply which is the cause of low lagging power factor. From this point of view, the apparatus for power factor correction can be seen as generating lagging reactive VA so as to reduce the lagging VA taken from the supply. Capacitor banks, static var systems, generators, and synchronous condensers (synchronous motors overexcited with no loads) are some of the sources which are used to deliver reactive power. The compensating apparatus reduce the line current necessary to supply the load and reduce the voltage drop in the line as the power factor is improved. This lowers the reactive requirement from generators and results in more real power output to be available. It is also possible to overcompensate, that is to make I_c (capacitor current) equal to I_l (inductive current) to compensate for the inductive reactance of the line.

REACTIVE POWER TRANSPORT (FLOW)

The reactive power flow in power systems produces losses. The reactive transmission losses (I^2X) result from the varying magnetic fields set up in and around all system equipment. The I^2X losses of the system are much larger than the I^2R losses, since the system reactance is generally several times the resistance. The flow of reactive power component of current through large system reactance is also a major source of system voltage drop. Utilities cannot claim, therefore, to be in the business of supplying kilowatts alone, but must supply kilovars as well to provide excitation currents for customers equipment, to cover transmission losses, and to maintain acceptable voltage levels.

The inductance of a transmission line is distributed throughout the line. Hence, for any transmission line, even with a unity-power-factor load, there is always a need for a capacitive input to supply the required reactive current of the transmission line. Various methods are used to supply var needed in a power system. The reactive power sources are synchronous condensers, generators, static capacitors, and high-voltage lines. The effectiveness of reactive sources in supplying reactive loads increases as the sources are located closer to the loads because of the accompanying reduction in reactive losses. Static capacitors can be connected in parallel across loads supplying leading var. Many such installations of static capacitors are automatically switched so that the capacitors are connected only when needed. The capacitors are sometimes also connected in series in a line. The series compensation reduces the series impedance of the line which is the principal cause of voltage drop and the most important factor in determining the maximum power which the line can transmit. Installations of this type are used frequently on long extra-high-voltage (EHV) transmission lines. During light loading conditions with a long line, the receiving-end voltage can exceed the sending-end voltage and may in some cases become excessive (6). In this case, shunt reactors (inductors) are installed at selected points along EHV lines from each phase to neutral. The inductors absorb reactive power and reduce overvoltages during light load conditions. Another type of shunt compensation includes thyristor-switched reactors in parallel with capacitors (7). These devices are known as static var systems. They absorb reactive power during light loads and deliver reactive power during heavy loads.

Reactive power flow is affected by the transformer winding ratios at substations of an interconnected system. There will be var flow due to unequal voltages on substations connected in parallel. Unwanted var flows can be minimized by proper selection of the transformer taps.

HARMONIC CONSIDERATIONS

Nonlinear loads generally do not cause reactive power to flow at the fundamental line frequency (8). They can draw higher rms currents and as a result add to distribution system losses for a given load. The nonlinear nature of these loads then draws nonpure sine wave currents thus causing harmonics of the fundamental to be present. Since harmonic distortion is caused by nonlinear elements connected to the power system, any device that has nonlinear characteristics will cause harmonic distortions. Some of the sources of such harmonics are

1. transformer saturation and inrush
2. variable frequency motor drives
3. inverters
4. arcing devices such as arc furnaces and
5. magneto motive force (MMF) distribution in ac rotating machine.

At present, there is no consensus among researchers on definitions and physical meanings regarding reactive power under harmonic operating conditions (9–12). Consider the case when steady-state harmonics are present:

$$v(t) = \sum_{h=1}^{\infty} v_h(t) = \sum_{h=1}^{\infty} \sqrt{2} V_h \sin(h\omega_0 t + \theta_h) \tag{12}$$

$$i(t) = \sum_{h=1}^{\infty} i_h(t) = \sum_{h=1}^{\infty} \sqrt{2} I_h \sin(h\omega_0 t + \delta_h) \tag{13}$$

A widely accepted definition of reactive power can be written as:

$$Q = \sum_{h=1}^{\infty} V_h I_h \sin(\theta_h - \delta_h) \tag{14}$$

In the presence of harmonics, static capacitors used in reactive power compensation and power factor correction can cause concern because of the possibility of system resonance. Resonance does occur when the system inductive reactance and capacitive reactance are equal. If the combination of capacitor banks and system inductance results in a parallel resonance near one of the characteristic harmonics generated by the loads, that harmonic current then feeds the resonant circuit. This results in an amplified current which oscillates between the inductance and capacitance. This oscillating current causes voltage distortion and thermal stress on the capacitors. The presence of the voltage harmonics in power system produces an extra power loss in capacitors expressed as (13):

$$\sum_{n=1}^{\infty} C(\tan\delta)\omega_n V_n^2 \tag{15}$$

where $\tan\delta = R/(1/\omega C)$ is the loss factor, $\omega_n = 2\pi fn$, and V_n is the rms voltage of nth harmonic. The total reactive power including fundamental and harmonics

$$Q = \sum_{n=1}^{\infty} Q_n \tag{16}$$

should not exceed the rated reactive power.

The power factor (pf) in the case where I is the rms current including harmonics is expressed as:

$$\text{pf} = \mu \cos\theta_1 \tag{17}$$

where μ is a current distortion factor.

OPTIMAL POWER FLOW ALGORITHMS

Optimal power flow (OPF) algorithms attempt to find the best possible setting for a list of control variables such that a desired objective is achieved. Typical objectives are minimization of losses, minimization of fuel cost, and minimization of added vars. Sometimes a composite objective function may be formed, such as minimize losses and at the same time minimize var additions. The control variables include generator bus voltage, transformer settings, real power and reactive power at generator buses, and addition of vars. In addition, an OPF includes specified constraints such as maintaining bus voltages, transmission line flows, phase angle regulator settings, as well as real and reactive power generating limits. The general constrained optimization problem can be defined mathematically as:

$$\text{minimize} \quad f(u, x) \tag{18}$$

subject to the following equality and inequality:

$$g(u, x) = 0 \tag{19}$$

$$h(u, x) \geq 0 \tag{20}$$

where u defines system controllable quantities and x defines system dependent variables.

REACTIVE POWER PRICING

For decades, power factor penalties have been used in providing reactive power pricing. Recent investigations, however, have shown that this practice is not sufficient to provide accurate price signals to customers. The area of rate structures of electricity has undergone dramatic changes as new and expanded service options are added. One of the popular new service options has been spot or real time pricing of electricity. Spot pricing means that electricity rates to the consumer follow as closely as technically practical the real cost of electricity at the time it is generated and supplied.

Caramanis, Bohn, and Shweppe (14) were the earliest to develop a spot pricing theory and analysis of spot pricing for real power. A summary of existing rates connected to real-time pricing policies is presented in ref. (15). Most findings in real-time pricing of active power are closely related to those of reactive power. Most of the rates are based on marginal cost pricing and are implemented using a modification of the OPF algorithms. A modification of the OPF models are used (3,16,17) to calculate real-time pricing of real and reactive power.

Spot pricing is also useful in defining wheeling rates. Wheeling is defined as the transmission of active and reactive power from a seller to a buyer using a transmission network belonging to a third party. Wheeling rates based on marginal cost are documented (16,17) and implemented using modification of the OPF.

Based on marginal costs, the real time-price of real power at a particular time at bus i is expressed as follows:

$$p_i = \frac{\partial}{\partial Pd_i}\text{[total cost of supplying electricity to all customers subject to operational constraints]} = Mc_{pi} \tag{21}$$

Likewise, the real-time price of reactive power based on marginal costs as bus i will be as follows:

$$q_i = \frac{\partial}{\partial Qd_i}\text{[total cost of supplying electricity to all customers subject to operational constraints]} = Mc_{qi} \tag{22}$$

where Pd_i and Qd_i are the active and reactive power demand at bus i, respectively. Mc_{pi} and Mc_{qi} are the corresponding Lagrange multipliers which yield the marginal cost of supplying real and reactive power at bus i, respectively. These multipliers, therefore, determine the pricing basis for real and reactive power, respectively.

Siddiqi and Baughman (3) present a modification of the OPF model which allows pricing of reactive and active power demand in real time. They divide the problem into two levels: namely the upper level problem which satisfies the demand functions and the lower level problem which minimizes the total operating costs subject to operational constraints. This model is solved using GRG2 (18), a program which solves nonlinear programming problems based on the generalized gradient method. The results show that the greatest impact on real-time pricing of reactive power as well as the generation and consumption patterns of reactive power by the utility and customers is due to the voltage constraints. This is because voltages are affected mainly by reactive power flows. In addition, reactive pricing results in customers paying the exact proportion as the amount of reactive power consumed. This is in contrast to using power factor penalties which cause inequitable sharing of the cost burden.

David and Li (16,17) present a modification of the OPF model, which incorporates real-time wheeling rates of both real and reactive power demand. The third party (transmission owner) is represented as a single welfare-maximizing entity that may also generate (wheeling utility) or buy electricity, and then sells both real and reactive power to independent consumers. The objective is the minimization of third-party system total cost of operating its own generating units. The solution provides a complete outline of optimal conditions and purchases as well as marginal costs of real and reactive power transport. A Han–Powell algorithm with a variable-reduction procedure is used to solve the nonlinear programming problem. The results show that wheeling rates of both real and reactive power increase as power flow is increased. Also, it is the transport of reactive power rather than real power that influences the conveyance rate of real power.

More algorithms are reported in the literature but the summary provided highlights the modification of optimal power flow algorithms in reactive power pricing. The approaches presented have merits and warrant further study because they provide a useful tool to answer the question whether to pay the reactive power charge or whether it is more profitable to make a capital investment in reactive compensation equipment. It is still, however, advisable to set a minimum power factor limit to avoid high capital cost on high rating switchgear.

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REACTIVE SYSTEMS. See REAL-TIME SYSTEMS.
REACTOR RADIATION MONITORING. See RADIATION MONITORING.
REACTORS, FUSION. See FUSION REACTORS.
REACTORS, NUCLEAR. See NUCLEAR ENGINEERING.
REACTORS, SATURABLE CORE. See SATURABLE CORE REACTORS.
REALIZATION. See LADDER FILTERS.
REAL SPACE TRANSFER DEVICES. See CHARGE INJECTION DEVICES.
REAL-TIME ANALYSIS OF POWER SYSTEMS. See POWER SYSTEM STATE ESTIMATION.