AC–DC POWER CONVERTERS

In most power systems the available power is in the form of a 50 Hz, 60 Hz, or 400 Hz ac voltage source. Many loads, such as dc motors, battery chargers, and logic electronic circuits, require dc power. *Rectifiers* provide the required conversion from ac to dc. A wide variety of rectifiers are used in different applications, which can be categorized by the forms of ac input power, the topologies, the types of switch, and the dc load characteristics. In this article some frequently used rectifier configurations are detailed. General analysis is given for rectifiers with inductive load.

CLASSIFICATION

Figure 1 shows a collection of the basic rectifier configurations, classified according to four characteristics: input power, topology, type of switches, and type of dc loads. Many variations are possible; for example, other types of load may be introduced or input power with more than three phases may be considered (in this classification converters with a higher number of phases are considered as combinations of the basic rectifiers).

Ac Input Power

The ac input power provided by the utility company is available in a single-phase or three-phase form. Multiple-phase power supplies can be obtained through a combination of single-phase or three-phase power supplies. Multiple-phase ac input power is desirable because the ripple frequency of the output dc voltage increases, easing the filtering requirements at the output. The number of phases can be increased by us-

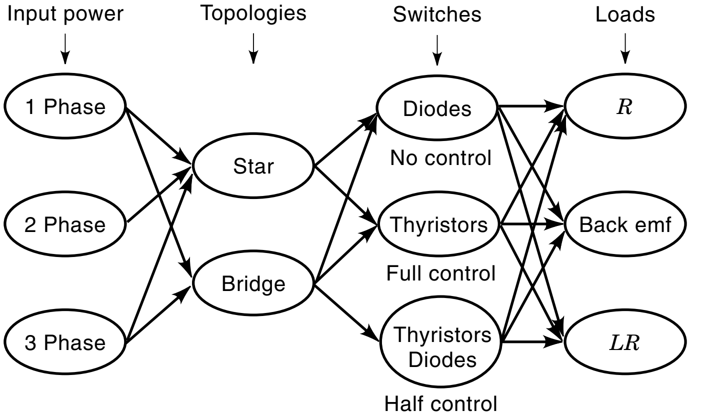

Figure 1. Basic rectifier configurations.

ing transformers with appropriate connections that generate voltages with intermediate phase shifts. For example, a six-phase input power can be obtained from a three-phase system with a delta–star and delta–delta transformer.

Rectifier Topology

A rectifier is characterized by its phase number q, which is the number of phases of input power, and by its pulse number p, which is the number of pulses in the output dc voltage per line cycle.

Rectifier topologies can be classified as stars and bridges as well as by their combinations. Stars are half-wave rectifiers, and for an ac input with q phases the pulse number p is equal to the phase number q. Typical star configurations include single phase, two phase, and three phase. Bridges are full-wave rectifiers, and for an ac input with q phases the pulse number is

$$p = q \quad \text{for } q \text{ even} \tag{1}$$

$$p = 2q \quad \text{for } q \text{ odd} \tag{2}$$

Typical bridge configurations include single-phase and three-phase bridges.

Types of Switches

Rectifiers with passive switches (i.e., diodes) do not allow control of the output dc voltage. In applications where the ouput dc voltage must be adjustable, rectifiers with active switches or a downstream dc–dc converter are used. In the past, silicon-controlled rectifiers (SCRs), also called thyristors, were the most popular choice for active switches. A limitation of SCRs is that only the turn-on transition is controllable, whereas the turn-off transition is not. Recently, rectifiers based on fully controllable switches have become popular due to improved performance; these rectifiers use devices such as GTOs (gate turn off), IGBTs (insulated gate bipolar transistors), MOSFETs (metal oxide semiconductor field effect transistor), and MCTs (MOS controlled thyristors).

Uncontrolled Rectifiers. If the load does not require a well-regulated voltage, the simplest choice is to use diode rectifiers to perform the rectifying function. Suppose the supply (ac) voltage consists of q alternating voltages $v_1, v_2 \ldots v_q$. They can be rectified by using q diodes with common cathodes (positive commutating groups), common anodes (negative commutating groups), or a combination of both. A positive commutating group is shown in Fig. 2. At any instant the diode corresponding to the most positive of voltages $v_1, v_2 \ldots v_q$ conducts and applies that voltage to the dc output. All the other diodes are reverse biased.

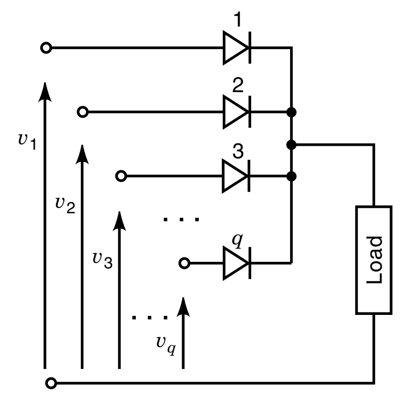

Figure 2. A positive commutating group of diodes.

Controlled Rectifiers. Controllable rectifiers are divided in two groups: naturally commutated and forced commutated.

Naturally commutated rectifiers are obtained from diode rectifiers by replacing the diodes with thyristors. In a thyristor rectifier only the turn-on instant of the thyristors is controllable and can be delayed with respect to the natural turn-on instant of the corresponding diode rectifier. As a result the average output voltage may be reduced with respect to the uncontrolled case. A thyristor is turned off when the alternating input voltage drops to zero and goes negative, causing the current to reduce to zero, or when another switch is turned on, imposing a negative bias to the previously conducting thyristor. This process is called *natural commutation* or *line commutation.*

Forced-commutated rectifiers are realized using fully controllable switches or by adding auxiliary circuits to the thyristors to turn them off in a controllable manner. Fully controllable switches such as GTOs, MOSFETs, and IGBTs are taking the place of thyristors in many applications due to their improved characteristics. Forced-commutated converters are mostly pulsewidth modulation (PWM) controlled and are operated at frequencies significantly higher than the line frequency. They provide control over the ac current waveform as well as the output dc voltage.

Half-Controlled Rectifiers. As mentioned previously, fully controlled rectifiers can be obtained from diode rectifiers by replacing all diodes with thyristors. In half-controlled rectifiers, only half of the diodes are replaced with thyristors. Advantages are lower cost and simplicity. Disadvantages include a smaller control range and lower input power factor.

Dc Load Characteristics

The dc load characteristics have a significant effect on the behavior of the rectifiers. Certain loads can be modeled as a voltage source, a current source, a resistive load, an inductor in series with a dc voltage source, and so on. Sometimes simplifying assumptions on the dc load characteristics allows simplified analysis. Some loads are shown in Fig. 3. The re-

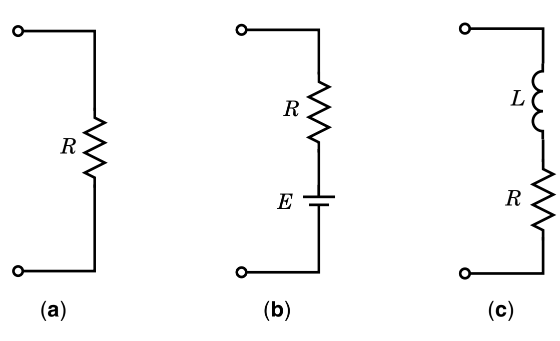

Figure 3. Different types of load: (a) resistive, (b) back emf, and (c) inductive emf.

sistive load of Fig. 3(a) may also represent a unity power factor switching converter (also called a resistance emulator). The back electromotive force (emf) load of Fig. 3(b) may represent a highly capacitive load or a battery. The inductive load of Fig. 3(c) may represent an induction heating system or an electric welding system.

BASIC RECTIFIER CONFIGURATIONS

As shown in Fig. 1, there are 36 basic rectifier configurations. If we limit our consideration to input power and topology only, Fig. 1 gives five basic rectifier configurations: single-phase star, single-phase bridge, two-phase star, three-phase star, and three-phase bridge.

Single-Phase Star Rectifiers

Single-phase rectifiers are used mostly in residential and laboratory environments for low- to medium-power applications (e.g., <5 kW). The simplest topology is a single-phase star diode rectifier (single-phase half-wave diode rectifier). An example is shown in Fig. 4 with a resistive load. When the input voltage is in its positive half cycle, the diode conducts and the load sees the positive voltage. When the input voltage is in its negative half cycle, the diode blocks the current. As a result, only half an input wave is utilized. The voltage and current at the load are unidirectional but discontinuous, with significant ripple. The waveforms of the input voltage v_s, the input current i_s, the output voltage v_o, the output current i_o, and the diode stress v_d are also shown in Fig. 4.

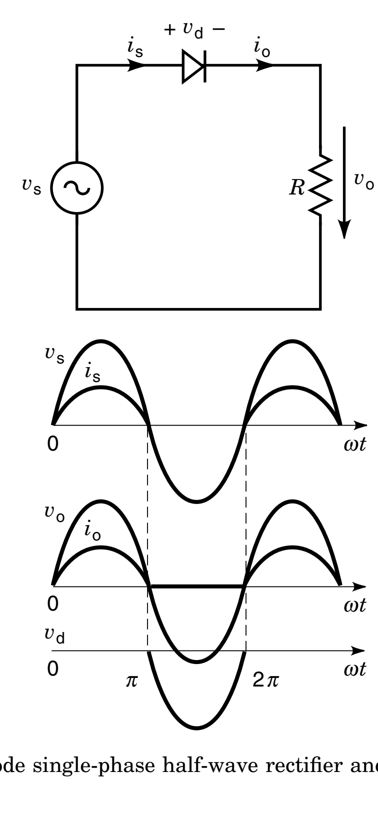

Figure 4. Diode single-phase half-wave rectifier and its waveforms.

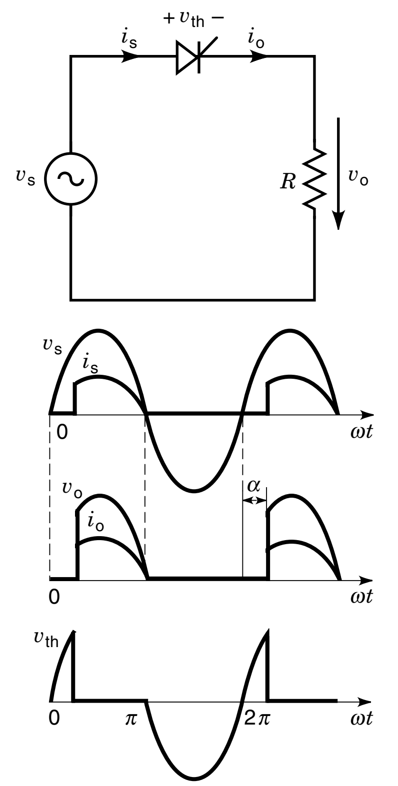

Figure 5. Thyristor single-phase half-wave rectifier and its waveforms.

The average output dc voltage is

$$V_o = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = \frac{\sqrt{2}V_s}{\pi} \tag{3}$$

When the diode is replaced with a thyristor, the output voltage becomes controllable by varying the *firing angle* α, defined as the delay angle of turn-on of the switch with respect to the "natural" turn-on instant of the corresponding diode rectifier. A thyristor star rectifier and its input voltage v_s, input current i_s, output voltage v_o, output current i_o, and thyristor voltage stress v_{th} are shown in Fig. 5. When the firing angle α is zero, the rectifier operation is identical to its corresponding diode rectifier. In the following discussion, only thyristor rectifiers are discussed, since the diode rectifier can be viewed as a special case of a thyristor rectifier when the firing angle is zero.

The average output dc voltage of a controllable half-wave rectifier is

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = \frac{\sqrt{2}V_s}{2\pi}(1+\cos\alpha) \tag{4}$$

where $0 \le \alpha \le \pi$. The output voltage is controlled by the firing angle.

Single-phase star (half-wave) rectifiers have the advantage of simplicity. However, their output ripple is large and their transformers are poorly utilized since they conduct only half a wave. Single-phase full-wave rectifiers provide alternatives that greatly reduce the output ripple and eliminate the unidirectional magnetization problem. A full-wave rectifier can be realized by using a bridge configuration or a center-tap transformer, which can be viewed topologically as a two-phase star.

Single-Phase Bridge Rectifiers

Single-phase bridges are popular for low- to medium-power applications. The operation process for different loads is detailed here.

Resistive Load. Figure 6 shows a bridge rectifier with a resistive load. Thyristors Th1, Th2, Th3, and Th4 form a bridge. The waveforms of the input voltage and current, output voltage and current, as well as the thyristor voltage stress are also illustrated in Fig. 6. When the input voltage is in its positive half cycle, thyristors Th1 and Th4 are forward biased. However, they will not conduct until the trigger pulse occurs at $\omega t = \alpha$. The current flows from the input voltage source to the load through thyristor Th1, and back to the voltage source through thyristor Th4. During this period, thyristors Th2 and Th3 are off since they are reverse biased. When the input voltage drops to zero, thyristors Th1 and Th4 turn off naturally. In the negative half cycle, thyristors Th2 and Th3 are forward biased and will conduct after the trigger pulse occurs and continue until the input voltage drops to zero, while thyristors Th1 and Th4 are off due to reverse bias. This process

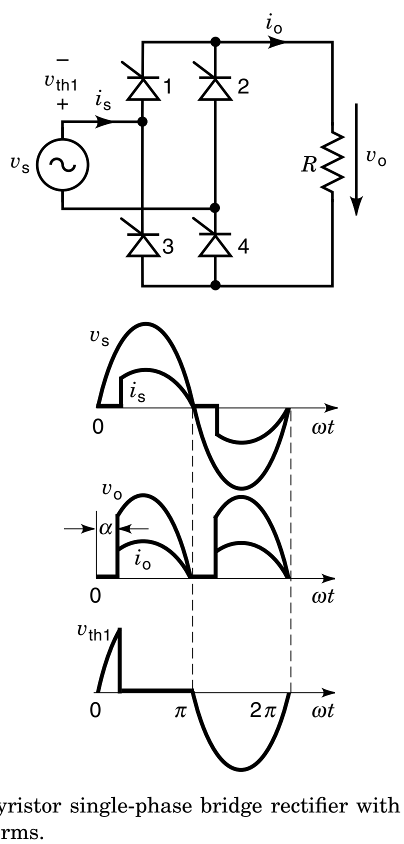

Figure 6. Thyristor single-phase bridge rectifier with resistive load and its waveforms.

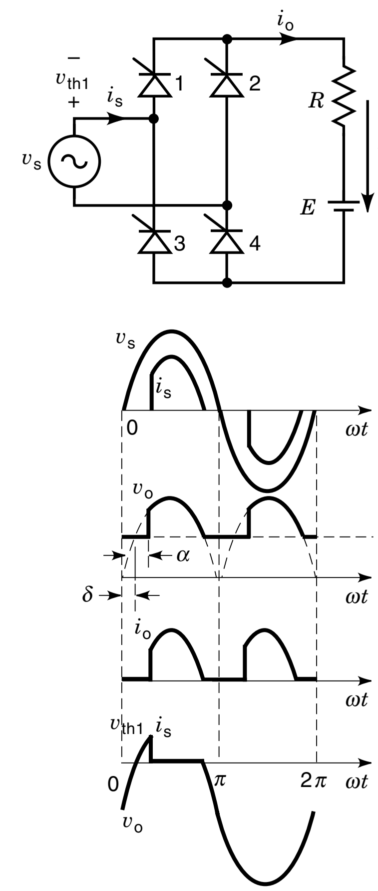

Figure 7. Thyristor single-phase bridge rectifier with back emf load and its waveforms.

repeats from cycle to cycle. In each line cycle, the load sees both pulses of the ac voltage; therefore, this configuration is called a full-wave rectifier. The output ripple is smaller than that of a half-wave rectifier. The transformer is used in both the positive and negative half cycles. The average output voltage is

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = \frac{\sqrt{2}V_s}{\pi}(1 + \cos\alpha) \tag{5}$$

where $0 \leq \alpha \leq \pi$. The output dc voltage can be controlled by adjusting the firing angle α.

Back emf Load. Some loads, such as batteries, large capacitors, and motors, impose a back emf to the rectifiers. A single-phase bridge rectifier with inductive load and its input voltage and current, output voltage and current, and thyristor voltage stress waveforms are shown in Fig. 7. In this case, current flows only when the rectified voltage is greater than the back emf. The conduction angle is reduced compared with that of a resistive load. As a result, the peak current will be higher for the same power rating. In the positive half cycle,

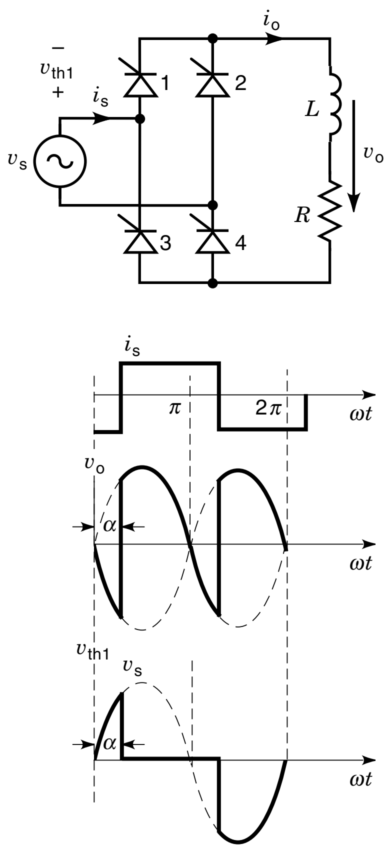

Figure 8. Thyristor single-phase bridge rectifier with inductive load and its waveforms.

thyristors Th1 and Th4 are forward biased only when the rectified ac voltage is greater than the back emf. The back emf *block angle* δ is defined as the angle at which the ac input voltage is equal to the back emf. In the case of the rectifier of Fig. 7, the block angle is $\delta = \sin^{-1} E/(\sqrt{2}V_s)$. If the trigger pulse occurs while thyristors Th1 and Th4 are forward biased ($\delta < \alpha < \pi - \delta$), the thyristors will be triggered to conduct. If the firing angle is less than the back emf block angle δ, the thyristors will not conduct when the firing pulse occurs until ωt is larger than the back emf block angle δ, assuming that the firing pulse lasts long enough. Otherwise, they may never conduct. The average output dc voltage when ($\delta < \alpha < \pi - \delta$) is

$$V_o = E + \frac{1}{\pi}\int_{\alpha}^{\pi-\delta}[\sqrt{2}V_s \sin(\omega t) - E]\,d(\omega t) = \frac{\alpha+\delta}{\pi}E + \frac{\sqrt{2}}{\pi}V_s(\cos\alpha + \cos\delta) \tag{6}$$

and, when $\alpha < \delta$,

$$V_o = E + \frac{1}{\pi}\int_{\delta}^{\pi-\delta}[\sqrt{2}V_s \sin(\omega t) - E]\,d(\omega t) = \frac{2\delta}{\pi}E + \frac{2\sqrt{2}}{\pi}V_s \cos\delta \tag{7}$$

Inductive Load. The resistive load and back emf load result in large current ripple and discontinuous conduction. In industry practice, a large inductor "choke" is often inserted in series with the load to smooth the current. Assuming that the inductor is very large such that the current is nearly constant, a current source can be used to model the load. A thyristor bridge rectifier with inductive load and its input current, output voltage, and thyristor voltage stress waveforms are shown in Fig. 8. When the input voltage is in its positive half cycle, thyristors Th1 and Th4 are forward biased. They start conducting when the trigger pulse occurs at $\omega t = \alpha$. They will continue to conduct even if the input voltage drops to zero and becomes negative, because the inductor continues to draw current from the bridge. Since Th2 and Th3 are not turned on until $\omega t = \pi + \gamma$, V_o must follow the source voltage negative. In the negative half cycle, thyristors Th2 and Th3 become forward biased. When trigger pulse occurs, they conduct forcing thyristors Th1 and Th4 to turn off. Current commutates from Th1 and Th4 to Th2 and Th3. This process repeats from cycle to cycle. The average output dc voltage is

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\sqrt{2}V_s \sin(\omega t)\,d(\omega t) = \frac{2\sqrt{2}}{\pi}V_s \cos\alpha \tag{8}$$

The average output dc voltage can be positive or negative depending on the firing angle of the trigger pulse, according to Eq. (8). The waveforms in the case of different firing angles ($\alpha = 0$, $\alpha < \pi/2$, $\alpha = \pi/2$, and $\pi/2 < \alpha < \pi$) are shown in Fig. 9. When the firing angle is zero, the thyristor bridge operates

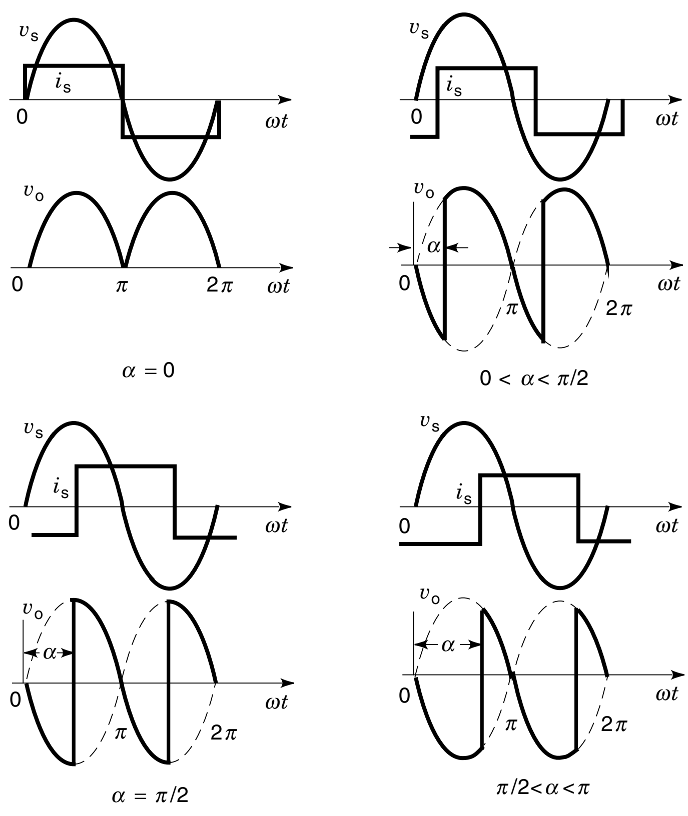

Figure 9. Thyristor single-phase bridge waveforms with different firing angles.

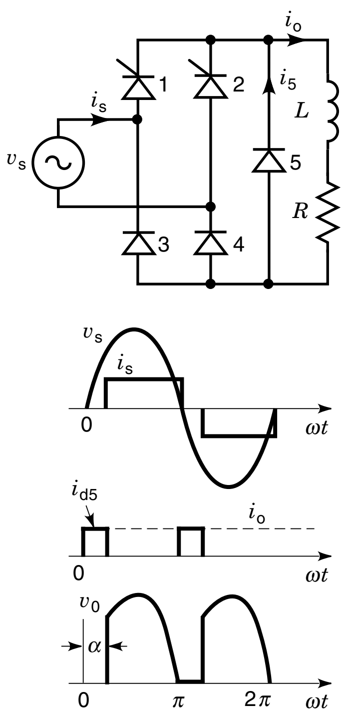

Figure 10. Thyristor single-phase bridge half-controlled rectifier with inductive load and its waveforms.

like a diode bridge. When the firing angle is between zero and $\pi/2$, a portion of the instantaneous output voltage may be negative. During the negative portion, energy in the inductor is fed back to the input voltage source. Overall, the negative portion is smaller than the positive portion; therefore, the average output voltage is still positive. When the firing angle is equal to $\pi/2$, the negative portion of the output voltage equals the positive portion. The average output voltage is zero. When the firing angle is between $\pi/2$ and π, the negative portion of the output voltage is larger than the positive portion; therefore, the net output voltage is negative, and the energy in the load is regenerated into the ac input source. This mode of operation is called synchronous inversion. It must be pointed out that when the firing angle is near π, the commutation from Th1 and Th4 to Th2 and Th3 may fail because thyristors Th1 and Th4 will be subjected to forward bias soon after the commutation and before they regain their forward-blocking capability. Therefore, a maximum firing angle limitation must be imposed, such as $\alpha < 5\pi/6$, to prevent commutation failure.

Single-Phase Bridge Half-Controlled Rectifiers

Half-controlled rectifiers provide a cost-effective alternative to fully controlled rectifiers. In a half-controlled rectifier, only two thyristors and two diodes are used; thus, the drive circuit is greatly simplified. The circuit operation is identical to that of fully controlled thyristor rectifiers when the load is resistive. However, when the load is inductive, a free-wheeling diode is used to prevent thyristors from fault conduction. In the absence of the free-wheeling diode, if the trigger signals to both thyristors are removed while one of them (say Th1) is still conducting, Th1 will not be able to turn off. The current will flow from the transformer through Th1 to the load and return to the transformer through diode D4, when the input voltage is in its positive half cycle. The current will form a loop through Th1, the load, and diode D3 when the input voltage is in its negative half cycle. Figure 10 shows a half-controlled bridge rectifier with an inductive load and its input voltage and current, output voltage and current, and free-wheeling diode current waveforms. When the input voltage is in its positive half cycle, thyristor Th1 and diode D4 will conduct after Th1 is triggered. When the input voltage drops to zero, the current will continue through the free-wheeling diode D5. In the negative half cycle, thyristor Th2 and diode D3 will conduct after Th2 is triggered. When the input voltage drops to zero, the current will continue through the free-wheeling diode D5. The output voltage waveform is the same as for the fully controlled rectifiers with resistive load.

Two-Phase Star Rectifiers

A two-phase star rectifier is often called a single-phase full-wave rectifier with a center-tap transformer because the two-phase input power is obtained from a single-phase power via a center-tap transformer.

A typical two-phase star with an inductive load and its input current, output voltage, and thyristor voltage stress waveforms are shown in Fig. 11. The two thyristors form a

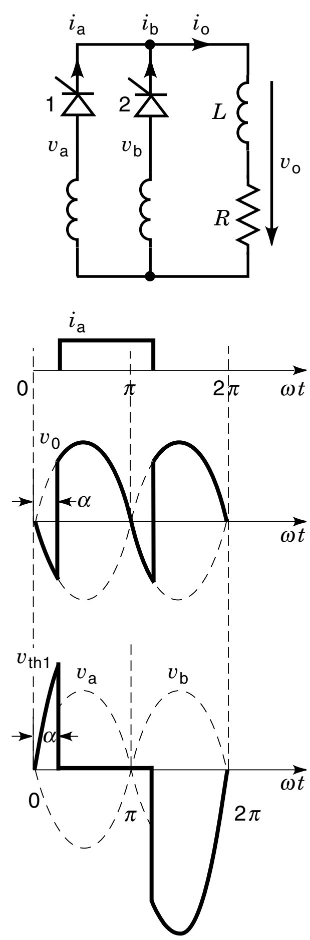

Figure 11. Two-phase star rectifier and its waveforms.

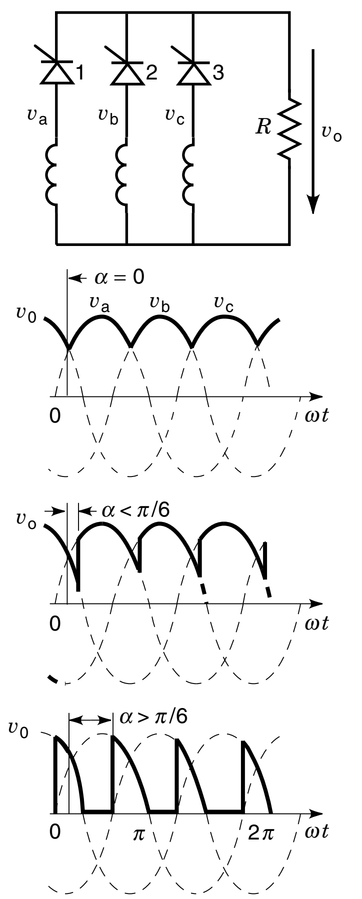

Figure 12. Thyristor star rectifier with resistive load and its waveforms.

star. When the input voltage is in its positive half cycle, thyristor Th1 is forward biased. It starts conducting when the trigger pulse occurs at $\omega t = \alpha$. It will continue to conduct even if the input voltage drops to zero and becomes negative, because the inductor voltage reverses its direction to keep the current flowing. As a result, the instantaneous output voltage can be negative. In the negative half cycle, thyristor Th2 becomes forward biased. When trigger pulse occurs, Th2 conducts, forcing thyristor Th1 to turn off. Current commutates from Th1 to Th2. This process repeats from cycle to cycle. The average output dc voltage is

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = 2\frac{\sqrt{2}V_s}{\pi} \cos\alpha \tag{9}$$

where $0 \le \alpha \le \pi$. The output dc voltage can be controlled by adjusting the firing angle α.

The output ripple of this full-wave rectifier is smaller than that of a half-wave rectifier. However, each winding of the transformer secondary conducts only for a half cycle.

Three-Phase Star Rectifier

When the required output power is large (e.g., >5 kW), three-phase rectifiers are recommended. A three-phase rectifier draws symmetric input power and has smaller ripple at the output. Commonly used topologies include three-phase star and three-phase bridge. Among them the three-phase star is the basic building block. The other configurations can be viewed as different combinations of the three-phase half-wave with series and/or parallel connection.

Resistive Load. Figure 12 shows a three-phase star with a resistive load. Thyristors Th1, Th2, and Th3 form a three-phase star. The input and output voltage for firing angle $\alpha < \pi/6$ and $\alpha > \pi/6$ are illustrated. The input voltage waveform shows that the line period is divided into three intervals. During interval $\pi/6 < \omega t < 5\pi/6$, phase a has higher voltage than phases b and c. If Th1 is triggered in this interval, it will conduct and the load will see the phase a voltage. During interval $5\pi/6 < \omega t < 3\pi/2$, phase b voltage is higher than phases a and c. If Th2 is triggered in this interval, it will conduct and Th1 will be turned off. The load will see the phase b voltage. During interval $3\pi/2 < \omega t < 2\pi + \pi/6$, phase c voltage is higher than that of phases a and b. If Th3 is triggered in this interval, it will conduct and it will turn off Th2. The load sees the phase c voltage. In one line cycle, three phases in turn supply the load. Each thyristor conducts for $2\pi/3$ radians. There are three pulses in each line cycle. At $\pi/6$, $5\pi/6$, and $3\pi/2$, two phase voltages are equal, and that marks the earliest possible time for thyristors to be triggered to conduct. These points are used as starting points for the firing angle (i.e., $\alpha = 0$).

When $\alpha \le \pi/6$, the load current is continuous, and each thyristor conducts for $2\pi/3$. In particular, when $\alpha = 0$, the rectifier is equivalent to a diode rectifier. The average output voltage is

$$V_o = \frac{3}{2\pi}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = \frac{3\sqrt{6}}{2\pi} V_s \cos\alpha \tag{10}$$

When $\alpha > \pi/6$, the dc current becomes discontinuous. The thyristor stops conducting when the ac voltage becomes negative when $\omega t = \pi$. In this case the average output voltage is

$$V_o = \frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) = \frac{3\sqrt{2}}{2\pi} V_s \left(1 + \cos\left(\frac{\pi}{6} + \alpha\right)\right) \tag{11}$$

Inductive Load. For inductive load, the load current is nearly constant. Figure 13 shows a three-phase star with an inductive load. During interval $\pi/6 < \omega t < 5\pi/6$, phase a has higher voltage than phases b and c. If Th1 is triggered in this interval, it will conduct and the current from Th3 will commute to Th1. During interval $5\pi/6 < \omega t < 3\pi/2$, phase b voltage is higher than phases a and c. If Th2 is triggered in this interval, it will conduct and current from Th1 will commute to Th2. During interval $3\pi/2 < \omega t < 2\pi + \pi/6$, phase c voltage is higher than that of phases a and b. If Th3 is triggered in this interval, it will conduct and the current from Th2 will commute to Th3. The input current, output voltage, and thyristor stress waveform are shown in Fig. 13. The average out-

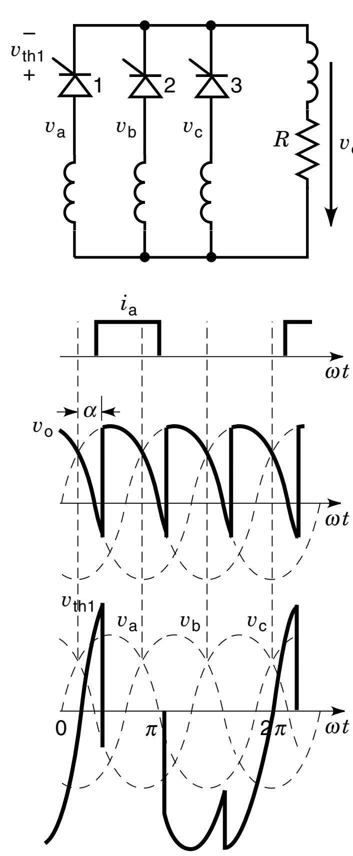

Figure 13. Three-phase star rectifier with inductive load and its waveforms.

put voltage is

$$
\begin{aligned}
V_o &= \frac{3}{2\pi}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} \sqrt{2}V_s \sin(\omega t)\, d(\omega t) \\
&= \frac{3\sqrt{6}}{2\pi} V_s \cos\alpha
\end{aligned}
\tag{12}
$$

Figures 14(a) and 14(b) show the output voltage waveform when the firing angle is less than $\pi/2$ and greater than $\pi/2$, respectively. When the firing angle is less than $\pi/2$, the output voltage is positive. Otherwise it is negative.

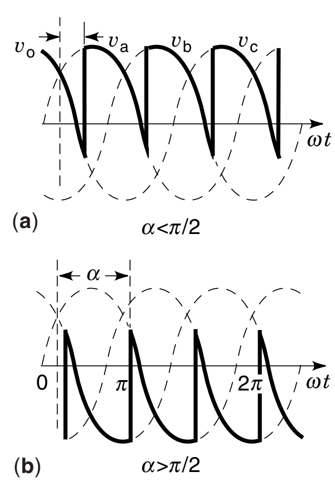

Figure 14. Thyristor star rectifier waveforms with different firing angles.

Three-Phase Bridge

Figure 15 shows a three-phase bridge and its phase current, output voltage, and the thyristor voltage stress waveforms. Thyristors Th1, Th2, Th3, Th4, Th5, and Th6 form a three-phase bridge, where Th1, Th3, and Th5 form a positive commutating three-phase star and Th2, Th4, and Th6 form a negative commutating three-phase star. The three-phase bridge can be viewed as the two three-phase stars connected in series. Assume that thyristors Th5 and Th6 are conducting. When $\omega t \geq \pi/6$, thyristor Th1 becomes forward biased since $v_{ac} = v_a - v_c$ becomes positive. Thyristor Th1 can be triggered to conduct forcing thyristor Th5 to turn off. The current from Th5 commutes to Th1. The output voltage equals v_{ab}. When $\omega t \geq \pi/2$, thyristor Th2 becomes forward biased since $v_{bc} = v_b - v_c$ becomes positive. Thyristor Th2 can be triggered to conduct, forcing Th6 to turn off. The current from Th6 commutes to Th2. The output voltage equals v_{ac}. This process continues, another commutation occurring every $\pi/3$ radians (i.e., from Th1 to Th3, from Th2 to Th4, from Th3 to Th5, from

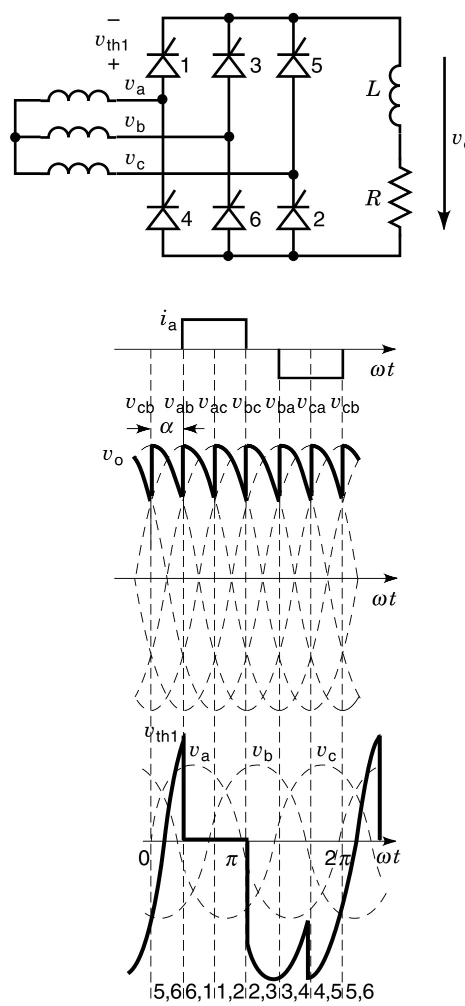

Figure 15. Three-phase bridge rectifier and its waveforms.

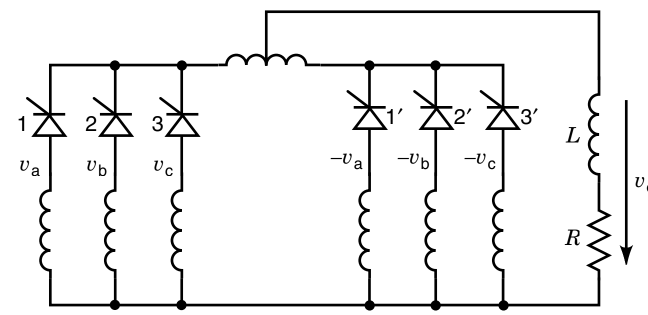

Figure 16. Thyristor double star rectifier.

Th4 to Th6, and so on). The commutations alternate between the upper thyristors and the lower thyristors. As a result, a six-pulse output voltage is obtained. The average output voltage equals twice that of the three-phase star:

$$\begin{aligned} V_o &= \frac{3}{\pi}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} \sqrt{6}V_s \sin(\omega t)\, d(\omega t) \\ &= \frac{3\sqrt{6}}{\pi}V_s \cos\alpha \end{aligned} \tag{13}$$

Three-Phase Combinations

Double Star Rectifier $p = 6$. For obtaining a six-pulse rectified voltage, the three-phase bridge rectifier is generally satisfactory. However, for low-voltage and high-current applications, the voltage drop in two series-connected diodes and the associated losses become significant. A half-wave rectifier is more desirable. But as the phase number increases, the commutation losses increase linearly, and the conduction angle of each phase reduces linearly. This causes high rms input current and the transformer is not effectively used. To keep a low phase number, two phase-shifted three-phase stars may be paralleled using a center-tapped inductor, as shown in Fig. 16. The center-tapped inductor acts as a voltage divider and allows both star rectifiers to conduct at the same time. Without the inductor only one thyristor will conduct at any given time. Moreover, the center-tapped inductor forces the two stars to share the load current (nearly) equally.

Double Parallel Bridge Rectifier. Double parallel bridge rectifiers, shown in Fig. 17, are used for high-current, low-output ripple applications. The input voltage source is six phases and the output dc ripple is 12 pulses. The six-phase source is realized by a transformer with two secondaries connected in star and delta, respectively. The average output voltage is the same as that of the three-phase bridge rectifiers.

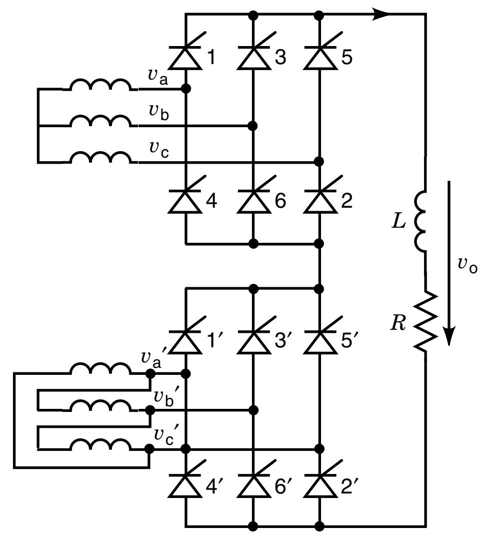

Figure 18. Thyristor double series bridge rectifier.

Double Series Bridge Rectifier. Double series bridge rectifiers, shown in Fig. 18, are used for high-voltage, low-current, low-output ripple applications. The input voltage source is six phases and the output dc ripple is 12 pulses. The six-phase source is realized by a transformer with two secondaries connected in star and delta, respectively. The average output voltage is twice that of the three-phase bridge rectifiers.

COMMUTATION, INDUCTIVE DROP, LINE NOTCHING, AND DISTORTION

In the rectifiers described so far, the ac voltage source had zero series ac side inductance. In a practical converter like the single-phase bridge converter of Fig. 19, L_{s1} represents the internal inductance of the ac source and L_{s2} represents the ac side inductance associated with the rectifier. Notice that an ac side inductor is sometimes added to the rectifier to reduce ac voltage distortion, as explained later in this section. The total ac side inductance is $L_s = L_{s1} + L_{s2}$.

In the rectifier operation, when a thyristor is fired, the load current, which was flowing through some other thyristor, commutates to the thyristor just fired. For example, in the bridge rectifier of Fig. 19, when thyristors Th2 and Th3 are fired at time t_1, the load current I_o, which was flowing through

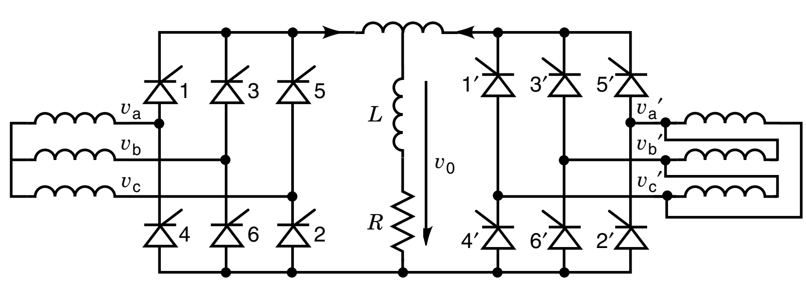

Figure 17. Thyristor double parallel bridge rectifier.

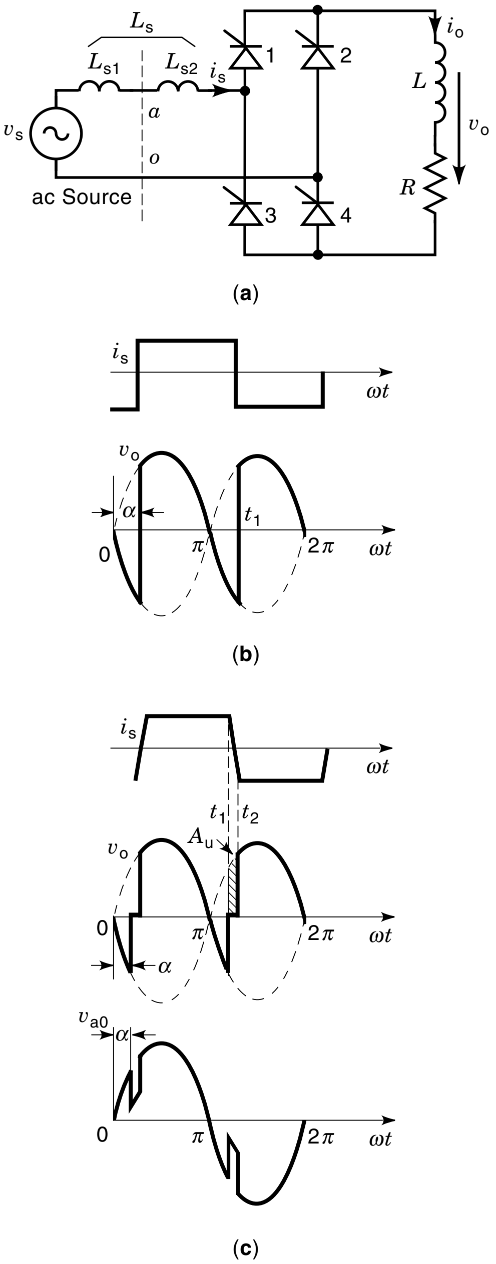

Figure 19. Single-phase bridge rectifier with ac side inductance L_s (a), waveforms for $L_s = 0$ (b), and for $L_s \neq 0$ (c).

Th1 and Th4, commutates to Th2 and Th3. In the case of $L_s = 0$, shown in Fig. 19(b), the commutation is instantaneous and the ac side current i_s changes from $+I_o$ to $-I_o$. In the case $L_s \neq 0$, shown in Fig. 19(c), the finite inductance, L_s, does not allow the current, i_s, to change instantaneously; as a result, there is a finite commutation interval $t_1 - t_2$ in which all thyristors conduct, shorting the ac source through inductance L_s. The voltage v_s is applied to L_s, reversing the current from $+I_o$ to $-I_o$. The integral of the voltage applied to L_s is given by area A_u in Fig. 19(c) and is equal to $L_s\,\Delta I$, where ΔI is the variation of inductor current. In the case of Fig. 19(c), ΔI is equal to $2I_o$ and

$$A_u = 2L_sI_o \tag{14}$$

The commutation process has various consequences. First, it puts a practical limit on the maximum delay angle α to allow sufficient time for commutation. Moreover, it reduces the average value of dc voltage, as can be seen from Fig. 19. The average dc voltage is

$$V_o = \hat{V}_o \cos\alpha - \frac{2}{\pi}\omega L_s I_o \tag{15}$$

where the first term represents the dc voltage in the ideal case and the second term represents the average voltage "lost" as a result of the commutation. Notice that the voltage drop is proportional to the dc current I_o and can be modeled by a lossless resistor $R_o = (2/\pi)\omega L_s$.

The process of commutation also causes line notching and distortion of the input ac voltage, which can affect other loads connected to the same ac source. Notice that L_{s1} is the internal impedance of the ac source, so additional loads are connected to point a of Fig. 19(a). The waveform of v_{a0} is shown in Fig. 19(c). Notice the notches in the waveform during the commutation periods. It is clear from Fig. 19(a) that the notching and distortion of voltage v_{a0} is reduced if the rectifier ac side inductance L_{s2} is increased.

GENERAL ANALYSIS

In general, rectifiers can be classified in two groups: the star configuration and the bridge configuration.

Some Definitions

Let us consider one phase of the input ac power. The input ac voltage is assumed to be purely sinusoidal, $\sqrt{2}V_s \sin(\omega t)$. The input current i_s has harmonics with rms amplitude I_{sh} and phase ϕ_h, with $h = 1, \ldots, n$.

The total harmonic distortion (THD) is

$$\mathrm{THD} = \frac{\sqrt{\sum_{h\neq 1} I_{sh}^2}}{I_{s1}} \tag{16}$$

The input real power is

$$P = V_s I_{s1} \cos(\phi_1) \tag{17}$$

The apparent power is

$$S = V_s I_s \tag{18}$$

The displacement factor is DF = cos ϕ_1. The power factor (PF) is defined as

$$\mathrm{PF} \equiv \frac{P}{S} = \frac{I_{s1}}{I_s}\cos\phi_1 \tag{19}$$

and it can be written as

$$\mathrm{PF} = \mathrm{DF}\frac{I_{s1}}{I_s} = \frac{\mathrm{DF}}{\sqrt{1+\mathrm{THD}^2}} \tag{20}$$

Unified Analysis of Rectifiers

A common assumption in the study of rectifiers, especially of the controllable type, is that the load can be modeled as a current source. In that case a simplified analysis is possible.

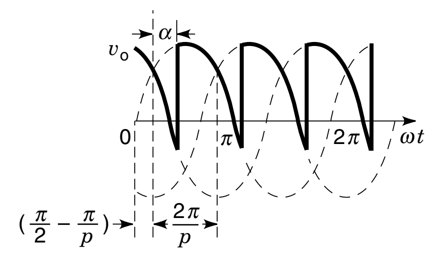

Figure 20. Output voltage waveform of generic rectifier.

Table 1. Fourier Series Coefficients of ac Phase Current for Various Phase Numbers *q*

		k							
		1	3	5	7	9	11	13	15
q	$f(q)$	a_k							
2	$4I_0/(\pi)$	1	1	1	1	1	1	1	1
3	$2\sqrt{3}I_0/(\pi)$	1	0	−1	−1	0	1	1	0
6	$2I_0/(\pi)$	1	−2	1	1	−2	1	1	−2

Dc Voltage Analysis. The dc voltage waveform depends only on a characteristic of the rectifier called the *pulse number p*, which is defined as the number of sinewave crests of the dc voltage per cycle T. Therefore, it is possible to express as a function of pulse number all quantities characterizing the dc voltage of a rectifier, such as average dc voltage, rms dc voltage, ripple factor, harmonic distortion factor, and harmonics.

The rectified voltage v_o is shown in Fig. 20. If the peak of the sinusoidal rectified voltage is V_o, the output voltage is

$$v_o(t) = \hat{V}_o \sin(\omega t) \tag{21}$$

for $(\pi/2)-\pi/p + \alpha < \omega t < (\pi/2)\pi/p + \alpha$, where α is the delay angle. In the case of a diode rectified it is $\alpha = 0$.

The average output voltage is given by

$$V_o = \frac{p}{2\pi}\int_{\pi/2-\pi/p+\alpha}^{\pi/2+\pi/p+\alpha} \hat{V}_o \sin(\omega t)\, d(\omega t) = \frac{p}{\pi}\hat{V}_o \sin\left(\frac{\pi}{p}\right)\cos\alpha \tag{22}$$

In the case of an uncontrolled rectifier, Eq. (22) becomes

$$V_o = \frac{p}{2\pi}\int_{\pi/2-\pi/p}^{\pi/2+\pi/p} \hat{V}_o \sin(\omega t)\, d(\omega t) = \frac{p}{\pi}\hat{V}_o \sin\left(\frac{\pi}{p}\right) \tag{23}$$

The rms dc voltage is

$$V_{o,\mathrm{rms}} = \hat{V}_o\left[\frac{1}{2} + \frac{p}{4\pi}\sin\left(\frac{2\pi}{p}\right)\right]^{1/2} \tag{24}$$

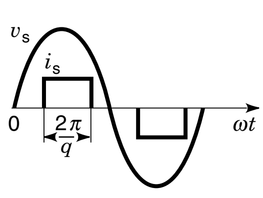

Figure 21. Ac current waveform of generic full-wave rectifier with phase number q.

The ripple factor is given by

$$K = \frac{v_{o,\max} - v_{o,\min}}{2V_o} \tag{25}$$

$$v_{o,\max} = \hat{V}_o \quad \text{for } \alpha < \frac{\pi}{p} \tag{26}$$

$$= \hat{V}_o \sin\left(\frac{\pi}{2} - \frac{\pi}{p} + \alpha\right) \quad \text{for } \alpha > \frac{\pi}{p} \tag{27}$$

$$v_{o,\min} = \hat{V}_o \sin\left(\frac{\pi}{2} + \frac{\pi}{p} + \alpha\right) \quad \text{for } \alpha < \pi - \frac{\pi}{p} \tag{28}$$

$$= -\hat{V}_o \quad \text{for } \alpha > \pi - \frac{\pi}{p} \tag{29}$$

The form factor is given by

$$\mathrm{FF} = \frac{V_{o,\mathrm{rms}}}{V_o} \tag{30}$$

The harmonic distortion factor is given by

$$\tau = (\mathrm{FF}^2 - 1)^{1/2} \tag{31}$$

Ac Current Harmonic Analysis. In general, the ac current waveform depends not only on the pulse number and phase number but on the transformer arrangement used. However, the transformer secondary current depends on the phase number only. Here, we analyze the transformer secondary phase current in the case of a full-wave rectifier. This current is shown in Fig. 21. In a full-wave rectifier with phase number q, the transformer secondary current is a square wave with amplitude I_0 and pulse duration $2\pi/q$. The current can be expressed in the form

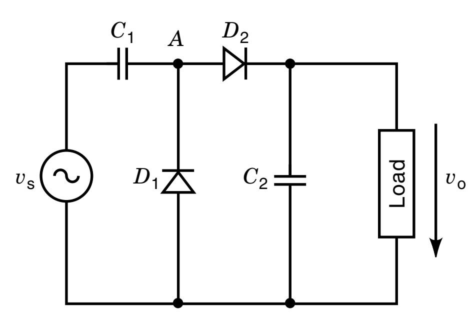

Figure 22. A voltage doubler circuit.

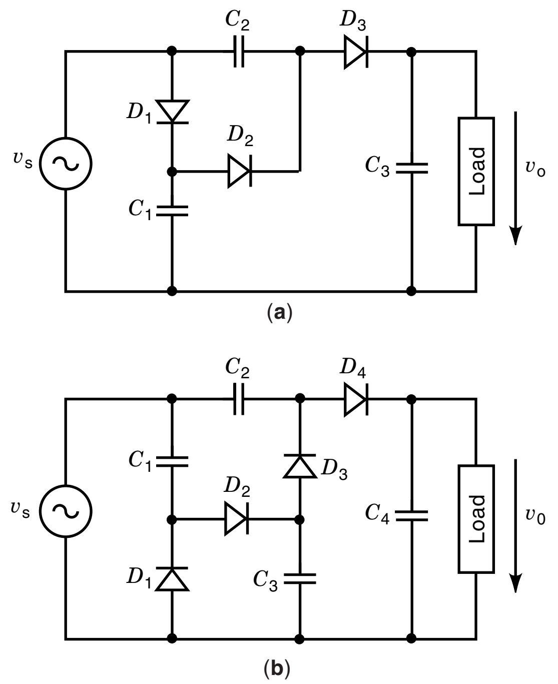

Figure 23. Voltage multiplier circuits: a voltage tripler (a) and a voltage quadrupler (b).

$$i_s(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t) \tag{32}$$

$$A_k = \frac{2I_0}{k\pi}\left[\cos\left(k\pi\frac{q+2}{2q}\right) + \cos\left(k\pi\frac{q-2}{2q}\right)\right] \tag{33}$$

The Fourier series coefficients A_k have the form

$$A_k = \frac{f(q)a_k}{k} \tag{34}$$

and for the most common phase numbers they are tabulated in Table 1.

When a controlled rectifier is operating with a firing angle α, the square wave current is shifted by an angle α and the kth harmonic component is shifted by an angle $k\alpha$.

VOLTAGE DOUBLERS AND VOLTAGE MULTIPLIERS

As the name suggests, voltage multiplier circuits provide a dc output that is approximately a multiple of the peak of the ac input voltage.

A voltage doubler is shown in Fig. 22. Diode D_1 and capacitor C_1 form a half-wave rectifier. The voltage of point A is equal to the sum of the input ac voltage and of the voltage of capacitor C_1. This voltage is half-wave rectified by diode D_2 and capacitor C_2. Variations of this circuit exist for voltage triplers, quadruplers, etc. Figure 23 shows a possible implementation of a voltage tripler and a voltage quadrupler.

PULSE-WIDTH MODULATION (PWM) RECTIFIERS

In recent years high-frequency PWM converters have been used extensively in rectifier circuits, especially at low to medium power levels. For single-phase applications the most common arrangement is a bridge diode rectifier followed by a boost switching power converter. With such an arrangement it is possible to draw high-power-factor currents from the ac line with very small distortion (THD lower than a few percentage points). If the load requires constant dc power, a large capacitor is placed at the output of the boost converter. In any case there will be some output voltage ripple at twice the line frequency.

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