ARCHITECTURAL ACOUSTICS

Room acoustics, and especially concert hall acoustics, is a subject that lives at the intersection of physical science, engineering, and art. The science of acoustics—how sound waves are propagated and reflected—is the foundation of room acoustics. Mechanical and electrical engineering govern the proper use of sound-absorbing materials, public-address systems, and artificial reverberation. Finally, the ability of people to hear and differentiate different sounds is the basis of the artistic appreciation of speech and music.

Personal preferences can be quantified by modern methods of multidimensional scaling, but to satisfy disparate musical tastes is a difficult task. This challenge is further complicated by the widespread desire to build multipurpose halls that function well for lectures, dramatic theatre, intimate musical ensembles, and large orchestras.

This article will attempt to illuminate room acoustics from the following viewpoints:

- Sound waves and acoustic resonances
- Sound rays, echoes, and reverberations
- Chaotic interference of sound waves
- Sound enhancement and artificial reverberation
- Subjective preferences
- Sound diffusion

SOUND WAVES AND ACOUSTIC RESONANCES

The propagation of sound is governed by a wave equation for the sound pressure p or, equivalently, the velocity potential ϕ:

$$\Delta\phi = c^2 \frac{\partial^2 \phi}{\partial t^2} \tag{1}$$

where the relation between p and ϕ is given by

$$p = -\rho_0 \frac{\partial \phi}{\partial t} \tag{2}$$

Δ is the Laplace operator, c is the sound velocity in air, and ρ_0 is its density. The gradient of ϕ gives the particle velocity vector

$$v = \operatorname{grad}\phi \tag{3}$$

To calculate the sound field in a given enclosure, the wave equation must be supplemented by boundary conditions. Typically, for hard walls, the normal component of the particle velocity is assumed to be zero. In general, the ratio of sound pressure to the normal particle velocity is set equal to the (often frequency-dependent) surface impedance, whose real part reflects the sound absorption.

Among the cases most easily solved is the rectangular (shoe box) enclosure, whose normal modes are given by trigonometric functions:

$$\phi(x,y,z) = \phi_0 \cos\left(\pi k \frac{x}{a}\right) \cos\left(\pi m \frac{y}{b}\right) \cos\left(\pi n \frac{z}{c}\right) \quad (4)$$

where $a \le b \le c$ are the sidelengths of the enclosure. Normal modes of vibration were first demonstrated by Ernst Chladni in 1787. The resonance frequencies f follow from the wave equation (1) and are given by

$$f = f(k,m,n) = \frac{c}{2}\sqrt{\left(\frac{k}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2} \quad (5)$$

For a cubical resonator ($a = b = c$), the resonances occur at

$$f = \frac{c}{2a}\sqrt{N} \quad (6)$$

where

$$N = k^2 + m^2 + n^2 \quad (7)$$

Interestingly, not all integers N can be represented as the sum of three squares. The so-called three-squares theorem of number theory tells us that values of N that leave a remainder of 7 when divided by 8 are "forbidden," as are all such N multiplied by powers of 4; thus, there are no resonances at $N = 7, 15, 23, 28, 31, \ldots, 135, 143, 151$, etc. Figure 1 illustrates the gap at $N = 143$ in the spectrum of resonances for a cubical enclosure.

For a flat square box ($c \ll a = b$), the resonances for $n = 0$ are given by

$$f = \frac{c}{2a}\sqrt{M} \quad (8)$$

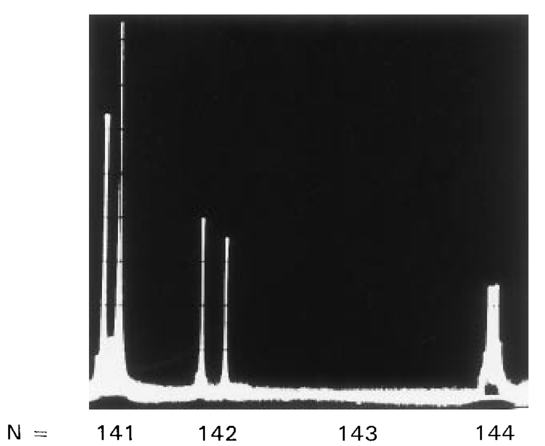

Figure 1. Resonances of a cubical enclosure. Note the missing resonance at $N = 143$, creating a gap in the frequency spectrum.

where

$$M = k^2 + m^2 \quad (9)$$

Now the two-squares theorem of number theory takes over. We must factor M into its prime factors

$$M = 2^{\alpha} \prod_{p_i} p^{\beta i} \prod_{q_i} q^{\gamma i} \quad (10)$$

where the p_i are the primes that exceed a multiple of 4 by 1 and the q_i are the primes that are 1 less than a multiple of 4. For M to lead to a resonance, all γ_i must be even. Thus, possible values are $M = 1 = 1^2 + 0^2$, $2 = 1^2 + 1^2$, $4 = 2^2 + 0^2$, $5 = 2^2 + 1^2$, $8 = 2^2 + 2^2$, $9 = 3^2 + 0^2$, $10 = 3^2 + 1^2$, $13 = 3^2 + 2^2$, etc. If $k \neq m$, as for $M = 1, 4, 5$, etc., two normal modes have the same resonance frequency, leading to the possibility of exciting circularly polarized sound waves in air! (To avoid a mode "salad," not more than two normal modes should have the same resonance frequency. This would occur at $M = 25$, which equals both $5^2 + 0^2$ and $4^2 + 3^2$, leading to *four* coinciding modes.)

The total number $\#(f)$ of normal modes in an enclosure of volume V having their resonance frequency below f is given by

$$\#(f) = \frac{4\pi}{3}\frac{V}{c^3}f^3 \quad (11)$$

neglecting terms involving f^2 and f that are insignificant for large f. While easily proved for rectangular enclosures, Hermann Weyl has shown that this formula also holds for enclosures of "arbitrary" shapes. Even for a modest-sized enclosure with $V = 130\ m^2$, the number of modes in the audio range ($f < 20{,}000$ Hz) exceeds one hundred million!

The average frequency distance Δf between adjacent modes is obtained by differentiation $\#(f)$ with respect to f, yielding

$$\Delta f = \frac{c^3}{4\pi V f^2} \quad (12)$$

Thus, for $V = 130\ m^3$ and the midaudio frequency $f = 1000$ Hz, the spacing Δf equals only 0.025 Hz.

It is clear that for audio frequencies even for midsized rooms, let alone concert halls, we will never be able to observe a single normal mode by itself. The proper treatment of normal modes is therefore *statistical* (see the section on chaotic wave interference).

SOUND RAYS, ECHOES, AND REVERBERATION

Unfortunately, the wave equation cannot be solved for any realistic shapes. Scientists therefore approximate the propagation of sound waves by sound rays. The *ray approximation* works particularly well for wavelengths that are small compared to the "obstacles" considered. Sound rays are indispen-

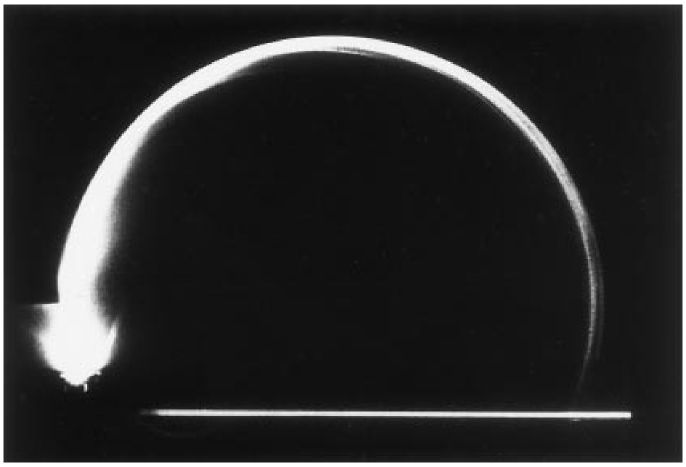

Figure 2. Sound propagation in a whispering gallery. The sound rays "cling" to the concave wall.

sible for analyzing echoes and studying the wall-hugging "whispering gallery" effect, see Fig. 2.

Reverberation theories, too, are based on sound rays. Each time a sound ray hits an absorbing wall, its energy is reduced by a factor $(1 - \alpha)$, where α is the absorption coefficient. Given that the expected rate of wall collisions equals $cS/4V$, where S is the absorber surface area, the energy as a function of time t is given by an exponential decay:

$$E(t) = E_0(1-\alpha)^{(cS/4V)t} \tag{13}$$

or, with the absorption exponent $a = -\log_e(1 - \alpha)$,

$$E(t) = E_0 \exp\left(-a\frac{cS}{4V}t\right) \tag{14}$$

Reverberation time T is traditionally defined by a 60 dB decay [i.e., the ratio $E(T)/E_0 = 10^{-6}$]. Thus, we obtain

$$T = 55.3\frac{V}{acS} \tag{15}$$

called Eyring's reverberation time formula. Approximating a by α gives the original Sabine formula.

Both formulas neglect the shape of the enclosure and the placement of the absorber, both of which can have substantial effects. Ignoring higher moments of the ray statistics and working only with the mean collision rate $cS/4V$ (or the mean free path $4V/S$) leads to additional errors. The proper way to calculate reverberation requires the solution of an integral equation.

Traditionally, reverberation times were measured by exciting the enclosure with a brief burst of sound energy (pistol shots, noise bursts, or tone pulses). The subsequent sound decay was then recorded on an oscilloscope or plotter and evaluated by curve fitting.

The statistical uncertainties of the noise excitation can be avoided by backward integration (Schroeder integration) of the sound decay. Equipment overload is circumverted by the use of number-theoretic binary maximum-length (Galois) sequences as an excitation signal and subsequent deconvolution on a computer.

Echoes and reverberation can be controlled by the proper placement of the right sound absorber. Narrow frequency ranges are best absorbed by acoustic cavities called Helmholtz resonators. Wideband absorption is achieved by lossy materials, such as mineral wool, either fully exposed or behind perforated panels. Absorption coefficient are measured in reverberation chambers using Eq. (15). Even *transparent* sound absorbers (for the German *Bundestag* in Bonn) have been realized by drilling micropores into Plexiglas. In this manner the desired "fish-bowl" architecture of the building could be maintained while solving its cocktail of acoustic problems.

CHAOTIC INTERFERENCE OF SOUND WAVES

The relation between reverberation time T and the bandwidth B of a resonance is

$$B = \frac{2.2}{T} \tag{16}$$

where the constant 2.2 equals $(3/\pi)\log_e 10$. With a reverberation time of 1.7 s, typical for midsized to large halls, the mode bandwidth equals therefore 1.3 Hz. Recalling the formula for the average mode spacing from Eq. (12), we see that for frequencies above

$$f = \frac{1}{2\pi}\sqrt{\frac{3c^3}{\log_e 10}\frac{T}{V}} \tag{17}$$

the average mode spacing will be smaller than the mode bandwidth. For $f > f_c$,

$$f_c = \frac{3}{2\pi}\sqrt{\frac{c^3}{\log_e 10}\frac{T}{V}} = 2000\sqrt{\frac{T}{V}} \tag{18}$$

on average three or more normal modes overlap, leading to a statistical response of the enclosure. Here T is measured in seconds and V in cubic meters.

More specifically, for frequencies above the Schroeder frequency f_c, the complex sound transmission function of a reverberant enclosure (with negligible direct sound) will, as a function of frequency, approach a complex Gaussian process in the frequency domain with an exponential power spectrum (if the reverberation is exponential). All other characteristics of the sound transmission in large rooms follow from this observation. For f_c to fall near the lower end of the audio band (100 Hz, say) and $T = 1.7$ s, the volume V must exceed a modest 235 m^3 to guarantee good modal overlap. Thus, the statistical theory applies even to small enclosures in the entire audio range.

The theory predicts an average spacing of $4/T$ for the response maxima, which was once considered an important objective measure of acoustic quality. The standard deviation of the statistical responses is about 6 dB, independent of reverberation time or volume. Ironically, these and other objective measures were intended to *supplement* reverberation time (which was known to be insufficient as a predictor of concert hall quality). Yet they are either numerical constants or dependent only on reverberation time.

If a public-address system is operated in a reverberant hall, high peaks in the statistical response can lead to audible acoustic feedback (howling). Inserting a frequency shifter with a small frequency shift (about 5 Hz) in the feedback loop will increase the margin of acoustic stability by several decibels.

If the Schroeder frequency f_c is expressed as a wavelength $\lambda_c = c/f_c$ and the Sabine value is substituted for the reverberation time, then

$$\lambda_c = \frac{\pi}{3}\sqrt{\frac{A}{6}} \tag{19}$$

where $A = \alpha S$ expresses the total absorption by an equivalent "open window" area. This formula is independent of the units used!

Another important room acoustical parameter is the distance r_c from an omnidirectional sound source at which direct and reverberant sound energy densities are equal:

$$r_c = \frac{1}{4}\sqrt{\frac{A}{\pi}} \tag{20}$$

It was only recently discovered that there is a close relationship between r_c and λ_c($r_c \approx 0.35\ \lambda_c$ for three-dimensional enclosures and $r_c \approx 0.16\ \lambda_c$ for two-dimensional spaces).

SOUND ENHANCEMENT AND ARTIFICIAL REVERBERATION

For optimum enjoyment, music requires a proper portion of reverberation. Think of organ music composed for a cavernous cathedral, with a reverberation time of 4 s. Or consider the romantic repertoire that sounds best with a reverberation time of 2 s, with a rise toward low frequencies for the much desired "warmth." Not infrequently, natural reverberation is scarce or completely lacking, such as when the New York Philharmonic plays in Central Park. While perhaps tolerable to the afficionados streched out on the Meadow, the lack of reverberation is jarring when listening to such a concert over the radio. Here, as in multipurpose halls, recording studies, and electronic music, *artificial* reverberation is called for. A prime example is the 6000-seat Palace of Congresses in the Kremlin, designed primarily for political events [i.e. speech(es)] for which a small reverberation time is optimum. But when the Bolshoi Theatre ran out of seating space, the Congress Hall had to take on opera as well, and the required reverberation was manufactured in reverberation chambers in the subbasement and piped into the hall via loudspeakers.

Another favorite method of creating artificial reverberation was large steel plates or springs, but they introduced a metallic twang. Simple feedback around a delay line also creates reverberation, but it discolors the sound because of its comblike frequency response (combfilter). Finally, in the late 1950s, the proper solution, "colorless" artificial reverberation, was found: *allpass* reverberators whose complex transmission functions have a frequency-independent magnitude. In the simplest case, an allpass reverberator can be realized by adding a negative-amplitude undelayed impulse to the output of a feedback-delay reverberator. Electronic allpass reverberators are now widely used, even in home entertainment. Digital reverberation networks using allpass and combfilters can even be designed to simulate sound transmission in concert halls, both existing and planned. This technique allows the pretesting of new designs before construction begins, thereby avoiding expensive architectural blunders.

An important application of artificial reverberation are multipurpose halls. Typically, these are designed for high speech intelligibility, which means short reverberation times, and the reverberation required for music is added electroacoustically, that is say through loudspeakers, as in the Palace of Congress.

Intelligibility can also be enhanced electroacoustically by "negative" reverberation (i.e., by providing extra direct sound—in other words *public-address systems* as used in many lecture halls and churches). In such systems, a multitude of loudspeakers, preferably loudspeaker *columns,* project the speaker's amplified voice directly into the audience.

For optimum intellegibility, the bass control should be turned down as far as possible because the low frequencies, which are not as effectively absorbed by hair and clothing as the higher frequencies, impede rather than increase intellegibility. This effect is the result of upward spread of masking in the inner ear of humans (i.e., low frequencies mask the higher frequencies, which carry most of the speech information).

To maintain the illusion that the sound comes from the speaker's lip, sophisticated systems exploit the "precedence" or *Hass effect* by delaying the amplified sound enough to arrive at the listeners' ears *after* the "natural" sound. Such a system was first successfully installed and operated in St. Paul's Cathedral in London.

SUBJECTIVE PREFERENCES

What kind of acoustics do people actually prefer when listening to, say, classical music? The literature abounds with the results of subjective studies—some of a questionable character. Typically, trained (or naive) listeners have to rate the hall according to various categories such as warmth, brilliance, clarity and a dozen more—on a scale from 1 to 5, say. The subjective preference scores are then averaged and correlated with the physical characteristics of the enclosures.

A better approach, however, is to abstain from such semantically loaded terms, which may mean different things to different people. Instead, the best approach is to simply ask listeners, for each pair of concert halls, which one they prefer. In order to make such comparisons possible, a selected piece of music is recorded by an orchestra in a reverberation-free environment, reproduced in the halls under investigation, and recorded with stereo-microphones embedded in an artifical head.

Such recordings can be processed to recreate, at a listener's ears listening to loudspeakers in an anechoic environment, the original sound signals. Thus, listeners can instantly switch themselves from one hall to another and, on the basis of identical musical input, make a reliable judgment.

The resulting preference scores are evaluated by multidimensional scaling, which results in a *preference space,* typically of two or three dimensions. The first dimension, which may account for some 50% of the total variance, typically represents a "consensus preference" among the listeners, while the second dimension reflects individual differences in musical taste.

When the most significant preference dimension is correlated with the objective parameters, such as reverberation time or width of the hall, it is found that the high and narrow halls of yore, such as the Vienna Grosser Musikvereinssaal, are much preferred over the low-ceiling fan-shaped halls of more modern design. Listeners also prefer "stereo" sound as opposed to the monophonic signals that are created by sound waves arriving from frontal directions. These two preferences (for narrow halls and for small interaural correlation) are actually related: high and narrow halls deliver a preponderance of *lateral* sound, giving rise to a feeling of being "bathed" in sound as opposed to a feeling of detachment. In fact, lateral sounds seem to be the main reason for the observed preference for older halls.

SOUND DIFFUSION

Unfortunately, wide halls with a low ceilings are here to stay, enforced by economic dictates: wider halls mean more seats to sell and lower ceilings engender lower building costs (the air our ancestors needed to breath now comes from air conditioning rather than the extra air volume of high halls).

But can we recover the old acoustic advantages of high and wide halls? Leaving out the ceiling, and thereby eliminating frontal sound from overhead, might be helpful for the acoustics, but it is, of course, unacceptable in most climates. How about diffusing the ceiling reflection laterally? This can indeed be done by turning the ceiling into a reflection phase grating based on number-theoretic principles, see Fig. 3. Making the depths of the troughs proportional to the quadratic residues of successive integers modulo a prime number p, say $p = 17$, such ceilings can be made to scatter sound into wide lateral angles over four musical octaves, see Fig. 4. The quadratic residues form a periodic sequence, which for $p = 17$, looks as follows: 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1, 0; 1, 4, 9, etc. Such number-theoretic diffusors (called Schroeder diffusors) are now available commercially for installation in recording studios, lecture halls, churches, and living rooms, as well as concert halls.

Other diffusors are based on primitive elements in finite fields and the number-theoretic logarithm. At low frequencies, such diffusors exhibit some sound desorption.

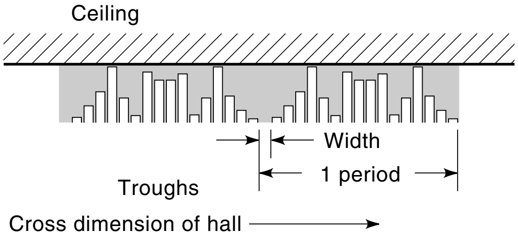

Figure 3. Number-theoretic reflection phase grating based on successive quadratic residues modulo the prime number 17. The pattern repeats with a period length of 17 and scatters frequencies over a range of 1:16, corresponding to four musical octaves.

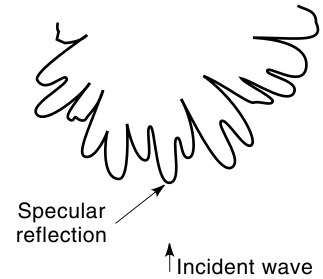

Figure 4. Reflection pattern from the phase grating shown in Fig. 3 for vertically incident sound. The wide angular scatter of sound energy also obtains for oblique incidence.

CONCLUSION

Although the proper design of halls for music, opera, drama, and lectures remains a challinging problem, especially if several of these purposes are to be combined (multipurpose halls), modern methods of realistic simulation and accurate calculation should ease the design task. With increasing reliability of digital equipment and better transducers (loudspeakers and microphones), electroacoustic means for improving and modifying room acoustics should become widely acceptable.

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ARCHITECTURE. See SOFTWARE MANAGEMENT VIA LAW-GOVERNED REGULARITIES.

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