CHAPTER 22

NAVIGATIONAL CALCULATIONS

INTRODUCTION

2200. Purpose And Scope

This chapter discusses the use of calculators and computers in navigation and summarizes the formulas the navigator depends on during voyage planning, piloting, celestial navigation, and various related tasks. To fully utilize this chapter, the navigator should be competent in basic mathematics including algebra and trigonometry (See Chapter 21, Navigational Mathematics), and possess a good working familiarity with a basic scientific calculator. The brand of calculator is not important, and no effort is made here to recommend or specify any one type. The navigator should choose a calculator based on personal needs, which may vary greatly from person to person according to individual abilities and responsibilities.

2201. Use Of Calculators In Navigation

Any common calculator can be used in navigation, even one providing only the 4 basic arithmetic functions of addition, subtraction, multiplication, and division. In general, however, the more sophisticated the calculator and the more mathematical functions it can perform, the greater will be its use in navigation. Any modern hand-held calculator labeled as a "scientific" model will have all the functions necessary for the navigator. Programmable calculators can be preset with formulas to simplify solutions even more, and special navigational calculators and computer programs reduce the navigator's task to merely collecting the data and entering it into the proper places in the program. Ephemeral (Almanac) data is included in the more sophisticated navigational calculators and computer programs.

Calculators or computers can improve celestial navigation by easily solving numerous sights to refine one's position and by reducing mathematical and tabular errors inherent in the manual sight reduction process. In other navigational tasks, they can improve accuracy in calculations and reduce the possibility of errors in computation. Errors in data entry are the most common problem in calculator and computer navigation.

While this is extremely helpful, the navigator must never forget how to do these problems by using the tables and other non-automated means. Sooner or later the calculator or computer will fail, and solutions will have to be worked out by hand and brain power. The professional navigator will regularly practice traditional methods to ensure

these skills do not fail when the calculator or computer does.

In using a calculator for any navigational task, it important to remember that the accuracy of the result, even if carried to many decimal places, is only as good as the *least* accurate entry. If a sextant observation is taken to an accuracy of only a minute, that is the best accuracy of the final solution, regardless of a calculator's ability to solve to 12 decimal places. See Chapter 23, Navigational Errors, for a discussion of the sources of error in navigation.

In addition to the 4 arithmetic functions, a basic navigational calculator should be able to perform reciprocals, roots, logarithms, trigonometry, and have at least one memory. At the other end of the scale are special navigational computer programs with almanac data and tide tables, integrated with programs for sight reduction, great circle navigation, DR, route planning, and other functions.

2202. Calculator Keys

It is not within the scope of this text to describe the steps which must be taken to solve navigational problems using any given calculator. There are far too many calculators available and far too many ways to enter the data. The purpose of this chapter is to summarize the formulas used in the solution of common navigational problems.

Despite the wide variety of calculators available from numerous manufacturers, a few basic keystrokes are common to nearly all calculators. Most scientific calculators have two or more **registers**, or active lines, known as the **xregister** and **y-register**.

- The $+,-, \times$, and \div keys perform the basic arithmetical functions.
- The Deg Rad Grad key selects degrees, radians, or grads as the method of expression of values. $360^\circ = 2\pi$ radians = 400 grads.
- The $+/-$ keys changes the sign of the number in the x-register.
- The C or CL key clears the problem from the calculator.
- The CE key clears the number just entered, but not the problem.
- The CM key clears the memory, but not the problem.
- The F, 2nd F, Arc, or Inv key activates the second

function of another key. (Some keys have more than one function.)

- The x^2 key squares the number in the x-register.
- The y^x key raises the number in the y-register to the x power.
- sin, cos, and tan keys determine the trigonometric function of angles, which must be expressed in degrees and tenths.
- $R \rightarrow P$ and $P \rightarrow R$ keys convert from rectangular to polar coordinates and vice versa.
- The p key enters the value for Pi, the circumference of a circle divided by its diameter.
- The M, STO, RCL keys enter a number into memory and recall it.
- M+, M– keys add or subtract the number in memory without displaying it.
- The ln key calculates the natural logarithm (log e) of a number.
- log calculates the common logarithm of a number.
- The $1/x$ key calculates the reciprocal of a number. (Very useful for finding the reciprocal of trigonometric functions).

Some basic calculators require the conversion of degrees, minutes and seconds (or tenths) to decimal degrees before solution. A good navigational calculator, however, should permit entry of degrees, minutes and tenths of minutes directly.

Though many non-navigational computer programs have an on-screen calculator, these are generally very simple versions with only the four basic arithmetical functions. They are thus too simple for many navigational problems. Conversely, a good navigational computer program requires no calculator, since the desired answer is calculated automatically from the entered data.

2203. Calculations Of Piloting

• **Hull speed in knots** is found by:

 $S = 1.34 \sqrt{\text{waterline length}}$ (in feet).

This is an approximate value which varies according to hull shape.

• **Nautical and U.S. survey miles** can be interconverted by the relationships:

1 nautical mile $= 1.15077945$ U.S. survey miles.

1 U.S. survey mile = 0.86897624 nautical miles.

• **The speed of a vessel over a measured mile** can becalculated by the formula:

$$
S = \frac{3600}{T}
$$

where S is the speed in knots and T is the time in seconds.

The distance traveled at a given speed is computed by the formula:

$$
D = \frac{ST}{60}
$$

where D is the distance in nautical miles, S is the speed in knots, and T is the time in minutes.

• **Distance to the visible horizon in nautical miles** can be calculated using the formula:

$$
D = 1.17 \sqrt{h_f} , or
$$

$$
D = 2.07 \sqrt{h_m}
$$

depending upon whether the height of eye of the ob-server above sea level is in feet (h_f) or in meters (h_m) .

• **Dip of the visible horizon in minutes of arc** can be calculated using the formula:

$$
D = 0.97'\sqrt{h_f} , or
$$

$$
D = 1.76'\sqrt{h_m}
$$

.

depending upon whether the height of eye of the observer above sea level is in feet (h_f) or in meters (h_m)

Distance to the radar horizon in nautical miles can be calculated using the formula:

$$
D = 1.22 \sqrt{h_f} , or
$$

$$
D = 2.21 \sqrt{h_m}
$$

depending upon whether the height of the antenna above sea level is in feet (h_f) or in meters (h_m) .

• **Dip of the sea short of the horizon** can be calculated using the formula:

$$
Ds = 60 \tan^{-1} \left(\frac{h_f}{6076.1 \text{ d}_s} + \frac{d_s}{8268} \right)
$$

where Ds is the dip short of the horizon in minutes

of arc; h_f is the height of eye of the observer above sea

level, in feet and d_s is the distance to the waterline of the object in nautical miles.

• **Distance by vertical angle between the waterline and the top of an object** is computed by solving the right triangle formed between the observer, the top of the object, and the waterline of the object by simple trigonometry. This assumes that the observer is at sea level, the earth is flat between observer and object, there is no refraction, and the object and its waterline form a right angle. For most cases of practical significance, these assumptions produce no large errors.

$$
D = \sqrt{\frac{\tan^2 a}{0.0002419^2} + \frac{H - h}{0.7349}} - \frac{\tan a}{0.0002419}
$$

where D is the distance in nautical miles, a is the corrected vertical angle, H is the height of the top of the object above sea level, and h is the observer's height of eye in feet. The constants (.0002419 and .7349) account for refraction.

2204. Tide Calculations

• **The rise and fall of a diurnal tide** can be roughly calculated from the following table, which shows the fraction of the total range the tide rises or falls during flood or ebb.

2205. Calculations Of Celestial Navigation

Unlike sight reduction by tables, sight reduction by calculator permits the use of nonintegral values of latitude of the observer, and LHA and declination of the celestial body. Interpolation is not needed, and the sights can be readily reduced from any assumed position. Simultaneous, or nearly simultaneous, observations can be reduced using a single assumed position. Using the observer's DR or MPP for the assumed longitude usually provides a better representation of the circle of equal altitude, particularly at high observed altititudes.

• **The dip correction** is computed in the *Nautical Alma-*

nac using the formula:

$$
D = 0.97\sqrt{h}
$$

where dip is in minutes of arc and h is height of eye in feet. This correction includes a factor for refraction. The *Air Almanac* uses a different formula intended for air navigation. The differences are of no significance in practical navigation.

The computed altitude (Hc) is calculated using the basic formula for solution of the undivided navigational triangle:

 $sinh = sinL sin d + cosL cosd cosLHA$,

in which h is the altitude to be computed (Hc), L is the latitude of the assumed position, d is the declination of the celestial body, and LHA is the local hour angle of the body. Meridian angle (t) can be substituted for LHA in the basic formula.

Restated in terms of the inverse trigonometric function:

 $\text{Hc} = \sin^{-1}[(\sin L \sin d) + (\cos L \cos d \cos LHA)].$

When latitude and declination are of contrary name, declination is treated as a negative quantity. No special sign convention is required for the local hour angle, as in the following azimuth angle calculations.

The azimuth angle (Z) can be calculated using the altitude azimuth formula if the altitude is known. The formula stated in terms of the inverse trigonometric function is:

$$
Z = \cos^{-1}\left(\frac{\sin d - (\sin L \sin Hc)}{(\cos L \cos Hc)}\right)
$$

If the altitude is unknown or a solution independent of altitude is required, the azimuth angle can be calculated using the time azimuth formula:

$$
Z = \tan^{-1}\left(\frac{\sin LHA}{(\cos L \tan d) - (\sin L \cos LHA)}\right)
$$

The sign conventions used in the calculations of both azimuth formulas are as follows: (1) if latitude and declination are of contrary name, declination is treated as a negative quantity; (2) if the local hour angle is greater than 180°, it is treated as a negative quantity. If the azimuth angle as calculated is negative, add 180° to obtain the desired value.

Amplitudes can be computed using the formula:

$$
A = \sin^{-1}(\sin d \sec L)
$$

this can be stated as

$$
A\ =\ sin^{-1}(\frac{\sin d}{\cos L})
$$

where A is the arc of the horizon between the prime vertical and the body, L is the latitude at the point of observation, and d is the declination of the celestial body.

2206. Calculations Of The Sailings

Plane sailing is based on the assumption that the meridian through the point of departure, the parallel through the destination, and the course line form a plane right triangle, as shown in Figure 2206.

From this:
$$
\cos C = \frac{1}{D}
$$
, $\sin C = \frac{p}{D}$, and $\tan C = \frac{p}{1}$.

From this: $1=D \cos C$, $D=1 \sec C$, and $p=D \sin C$.

From this, given course and distance (C and D), the difference of latitude (l) and departure (p) can be found, and given the latter, the former can be found, using simple trigonometry. See Chapter 24.

Traverse sailing combines plane salings with two or more courses, computing course and distance along a series of rhumb lines. See Chapter 24.

Parallel sailing consists of interconverting departure and difference of longitude. Refer to Figure 2206.

 $DLo = p \sec L$, and $p = DLo \cos L$

• **Mid-latitude sailing** combines plane and parallel sailing, with certain assumptions. The mean latitude (Lm) is half of the arithmetical sum of the latitudes of two places on the same side of the equator. For places on opposite sides of the equator, the N and S portions are solved separately.

In mid-latitude sailing:

 $DLo = p \sec Lm$, and $p = DLo \cos Lm$

Mercator Sailing problems are solved graphically on a Mercator chart. For mathematical Mercator solutions the formulas are:

$$
\tan C = \frac{DLo}{m}
$$
 or
$$
DLo = m \tan C.
$$

where m is the meridional part from Table 6. Following solution of the course angle by Mercator sailing, the distance is by the plane sailing formula:

 $D = L$ sec C.

• **Great-circle solutions for distance and initial course angle** can be calculated from the formulas:

$$
D = \cos^{-1} \left[\left(\sin L_1 \sin L_2 + \cos L_1 \cos L_2 \cos D L_0 \right) \right]
$$

$$
C = \tan^{-1}\left(\frac{\sin DLo}{(\cos L_1 \tan L_2) - (\sin L_1 \cos DLo)}\right)
$$

where D is the great-circle distance, C is the initial great-circle course angle, L_1 is the latitude of the point of departure, L_2 is the latitude of the destination, and DLo is the difference of longitude of the points of departure and destination. If the name of the latitude of the destination is contrary to that of the point of departure, it is treated as a negative quantity.

The latitude of the vertex, L_v , is always numerically equal to or greater the L_1 or L_2 . If the initial course angle C is less than 90 $^{\circ}$, the vertex is toward L_2 , but if C is greater than 90°, the nearer vertex is in the opposite direction. The ver-Figure 2206. The plane sailing triangle. the same $\frac{1}{2}$ fex nearer L_1 has the same name as L_1 .

The latitude of the vertex can be calculated from the formula:

$$
L_{v} = \cos^{-1}(\cos L_{1} \sin C)
$$

The difference of longitude of the vertex and the point of departure (DLo_v) can be calculated from the formula:

$$
D L o_V = \sin^{-1} \left(\frac{\cos C}{\sin L_v} \right)
$$

The distance from the point of departure to the vertex (D_v) can be calculated from the formula:

$$
D_v = \sin^{-1}(\cos L_1 \sin DL_{v})
$$

• **The latitudes of points on the great-circle track** can be determined for equal DLo intervals each side of the vertex (DLo_{vx}) using the formula:

$$
L_{\rm X} = \tan^{-1}(\cos D \operatorname{Lo}_{\rm VX} \tan L_{\rm V})
$$

The D_{U} and D_{V} of the nearer vertex are never greater than 90 $^{\circ}$. However, when L₁ and L₂ are of contrary name, the other vertex, 180° away, may be the better one to use in the solution for points on the great-circle track if it is nearer the mid point of the track.

The method of selecting the longitude (or DLo_{vx}), and determining the latitude at which the great-circle crosses the selected meridian, provides shorter legs in higher latitudes and longer legs in lower latitudes. Points at desired distances or desired equal intervals of distance on the great-circle from the vertex can be calculated using the formulas:

$$
L_{\rm x} = \sin^{-1} \left[\sin L_{\rm v} \cos D_{\rm vx} \right]
$$

$$
D_{\mathbf{U}_{\mathbf{V}\mathbf{X}}} = \sin^{-1}\left(\frac{\sin D_{\mathbf{V}\mathbf{X}}}{\cos L_{\mathbf{X}}}\right)
$$

A calculator which converts rectangular to polar coordinates provides easy solutions to plane sailings. However, the user must know whether the difference of latitude corresponds to the calculator's X-coordinate or to the Y-coordinate.

2207. Calculations Of Meteorology And Oceanography

• **Converting thermometer scales** between centigrade, Fahrenheit, and Kelvin scales can be done using the following formulas:

$$
C^{\circ} = \frac{5(F^{\circ} - 32^{\circ})}{9}
$$

$$
F^{\circ} = \frac{9}{5}C^{\circ} + 32^{\circ}
$$

$$
K^{\circ} = C + 273.15^{\circ}
$$

Maximum length of sea waves can be found by the formula:

 $W = 1.5 \sqrt{\text{fetch in nautical miles}}$.

- **Wave height** = 0.026 S² where S is the wind speed in knots.
- **Wave speed** in knots:

 $= 1.34 \sqrt{\text{wavelength}}$ (in feet), or

 $=$ 3.03 \times wave period (in seconds).

UNIT CONVERSION

Use the conversion tables that appear on the following pages to convert between different systems of units. Conversions followed by an asterisk are exact relationships.

344 NAVIGATIONAL CALCULATIONS

MISCELLANEOUS DATA

NAVIGATIONAL CALCULATIONS 345

Earth

 $= 28.349523125$ grams* $= 0.0625$ pound*

 $= 0.028349523125$ kilogram*

World Geodetic System (WGS) Ellipsoid of 1984

Mathematics

Meteorology

NAVIGATIONAL CALCULATIONS 349

Volume-Mass

Prefixes to Form Decimal Multiples and Sub-Multiples of International System of Units (SI)

