# CHAPTER 23

# NAVIGATIONAL ERRORS

## DEFINING NAVIGATIONAL ERRORS

### 2300. Introduction

Navigation is an increasingly exact science. Electronic positioning systems give the navigator a greater certainty than ever that his position is correct. However, the navigator makes certain assumptions which would be unacceptable in purely scientific work.

For example, when the navigator uses his latitude graduations as a mile scale to compute a great-circle course and distance, he neglects the flattening of the earth at the poles. When the navigator plots a visual bearing on a Mercator chart, he uses a rhumb line to represent a great circle. When he plots a celestial line of position, he substitutes a rhumb line for a small circle. When he interpolates in sight reduction or lattice tables, he assumes a linear (constant-rate) change between tabulated values. All of these assumptions introduce errors.

There are so many approximations in navigation that there is a natural tendency for some of them to cancel others. However, if the various small errors in a particular fix all have the same sign, the error might be significant. The navigator must recognize the limitations of his positioning systems and understand the sources of position error.

### 2301. Definitions

The following definitions apply to the discussions of this chapter:

**Error** is the difference between a specific value and the correct or standard value. As used here, it does not include mistakes, but is related to lack of perfection. Thus, an altitude determined by marine sextant is corrected for a standard amount of refraction, but if the actual refraction at the time of observation varies from the standard, the value taken from the table is in error by the difference between standard and actual refraction. This error will be compounded with others in the observed altitude. Similarly, depth determined by echo sounder is in error, among other things, by the difference between the actual speed of sound waves in the water and the speed used for calibration of the instrument. This chapter is concerned primarily with the deviation from standard values. Corrections can be applied for standard values of error. It is the deviation from standard, as well as mistakes, that produce inaccurate results in navigation.

A **mistake** is a blunder, such as an incorrect reading of an instrument, the taking of a wrong value from a table, or the plotting of a reciprocal bearing.

A **standard** is a value or quantity established by custom, agreement, or authority as a basis for comparison. Frequently, a standard is chosen as a model which approximates a mean or average condition. However, the distinction between the standard value and the actual value at any time should not be forgotten. Thus, a standard atmosphere has been established in which the temperature, pressure, and density are precisely specified for each altitude. Actual conditions, however, are generally different from those defined by the standard atmosphere. Similarly, the values for dip given in the almanacs are considered standard by those who use them, but actual dip may be appreciably different from that tabulated.

**Accuracy** is the degree of conformance with the correct value, while **precision** is a measure of refinement of a value. Thus, an altitude determined by marine sextant might be stated to the nearest 0.1', and yet be accurate only to the nearest 1.0' if the horizon is indistinct.

### 2302. Systematic And Random Errors

**Systematic errors** are those which follow some rule by which they can be predicted. **Random errors**, on the other hand, are unpredictable. The laws of probability govern random errors.

If a navigator takes several measurements that are subject to random error and graphs the results, the error values would be normally distributed around a mean, or average, value. Suppose, for example, that a navigator takes 500 celestial observations. Table 2302 shows the frequency of each error in the measurement, and Figure 2302 shows a plot of these errors. The curve's height at any point represents the percentage of observations that can be expected to have the error indicated at that point. The probability of any similar observation having any given error is the proportion of the number of observations having this error to the total number of observations. Thus, the probability of an observation having an error of -3' is:

$$\frac{40}{500} = \frac{1}{12.5} = 0.08(8\%)$$

An important characteristic of a probability distribution

| Error | No. of obs. | Percent of obs. |
|---|---|---|
| - 10′ | 0 | 0. 0 |
| - 9′ | 1 | 0. 2 |
| - 8′ | 2 | 0. 4 |
| - 7′ | 4 | 0. 8 |
| - 6′ | 9 | 1. 8 |
| - 5′ | 17 | 3. 4 |
| - 4′ | 28 | 5. 6 |
| - 3′ | 40 | 8. 0 |
| - 2′ | 53 | 10. 6 |
| - 1′ | 63 | 12. 6 |
| 0 | 66 | 13. 2 |
| + 1′ | 63 | 12. 6 |
| + 2′ | 53 | 10. 6 |
| + 3′ | 40 | 8. 0 |
| + 4′ | 28 | 5. 6 |
| + 5′ | 17 | 3. 4 |
| + 6′ | 9 | 1. 8 |
| + 7′ | 4 | 0. 8 |
| + 8′ | 2 | 0. 4 |
| + 9′ | 1 | 0. 2 |
| +10′ | 0 | 0. 0 |
| 0 | 500 | 100. 0 |

Table 2302. Normal distribution of random errors.

is the **standard deviation**. For a normal error curve, square each error, sum the squares, and divide the sum by one less than the total number of measurements. Finally, take the square root of the quotient. In the illustration, the standard deviation is:

$$\sqrt{\frac{4474}{499}} = \sqrt{8.966} = 2.99$$

One standard deviation on either side of the mean defines the area under the probability curve in which lie 67 percent of all errors. Two standard deviations encompass 95 percent of all errors, and three standard deviations encompass 99 percent of all errors.

The normalized curve of any type of random error is symmetrical about the line representing zero error. This means that in the normalized plot every positive error is matched by a negative error of the same magnitude. The average of all readings is zero. Increasing the number of readings increases the probability that the errors will fit the normalized curve.

When both systematic and random errors are present in a process, increasing the number of readings decreases the residual random error but does not decrease the systematic error. Thus, if, for example, a number of phase-difference readings are made at a fixed point, the average of all the readings should be a good approximation of the true value if there is no systematic error. But increasing the number of readings will not correct a systematic error. If a constant error is combined with a normal random error, the error curve will have the correct shape but will be offset from the zero value.

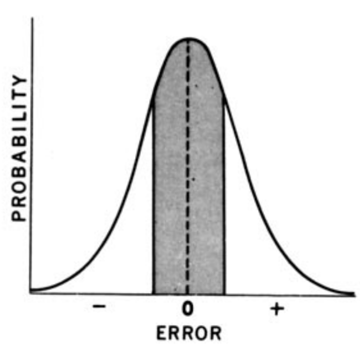


Figure 2302. Normal curve of random error with 50 percent of area shaded. Limits of shaded area indicate probable error.

**2303. Navigation System Accuracy**

In a navigation system, predictability is the measure of the accuracy with which the system can define the position in terms of geographical coordinates; repeatability is the measure of the accuracy with which the system permits the user to return to a position as defined only in terms of the coordinates peculiar to that system. Predictable accuracy, therefore, is the accuracy of positioning with respect to geographical coordinates; repeatable accuracy is the accuracy with which the user can return to a position whose coordinates have been measured previously with the same system. For example, the distance specified for the repeatable accuracy of a system, such as Loran C, is the distance between two Loran C positions established using the same stations and time-difference readings at different times. The correlation between the geographical coordinates and the system coordinates may or may not be known.

Relative accuracy is the accuracy with which a user can determine his position relative to another user of the same navigation system, at the same time. Hence, a system with high relative accuracy provides good rendezvous capability for the users of the system. The correlation between the geographical coordinates and the system coordinates is not relevant.

**2304. Most Probable Position**

Some navigators have been led by simplified definitions and explanations to conclude that the line of position is almost infallible and that a good fix has very little error.

A more realistic concept is that of the most probable position (MPP).This concept which recognizes the probability of error in all navigational information and determines position by an evaluation of all available information.

Suppose a vessel were to start from a completely accurate position and proceed on dead reckoning. If course and speed over the bottom were of equal accuracy, the uncertainty of dead reckoning positions would increase equally in all directions, with either distance or elapsed time (for any one speed these would be directly proportional, and therefore either could be used). A circle of uncertainty would grow around the dead reckoning position as the vessel proceeded. If the navigator had full knowledge of the distribution and nature of the errors of course and speed, and the necessary knowledge of statistical analysis, he could compute the radius of a circle of uncertainty, using the 50 percent, 95 percent, or other probabilities. This technique is known as fix expansion when done graphically. See Chapter 7 for a more detailed discussion of fix expansion.

In ordinary navigation, statistical computation is not practicable. However, the navigator might estimate at any time the likely error of his dead reckoning or estimated position. With practice, considerable skill in making this estimate is possible. He would take into account, too, the fact that the area of uncertainty might better be represented by an ellipse than a circle, with the major axis along the course line if the estimated error of the speed were greater than that of the course and the minor axis along the course line if the estimated error of the course were greater. He would recognize, too, that the size of the area of uncertainty would not grow in direct proportion to the distance or elapsed time, because disturbing factors, such as wind and current, could not be expected to remain of constant magnitude and direction. Also, he would know that the starting point of the dead reckoning might not be completely free from error.

The navigator can combine an LOP with either a dead reckoning or estimated position to determine an MPP. Determining the accuracy of the dead reckoning and estimated positions from which an MPP is determined is primarily a judgment call by the navigator. See Figure 2304a.

If a fix is obtained from two lines of position, the area of uncertainty is a circle if the lines are perpendicular and have equal error. If one is considered more accurate than the other, the area is an ellipse. As shown in Figure 2304b, it is also an ellipse if the likely error of each is equal and the lines cross at an oblique angle. If the errors are unequal, the major axis of the ellipse is more nearly in line with the line of position having the smaller likely error.

If a fix is obtained from three or more lines of position with a total bearing spread greater 180°, and the error of each line is normally distributed and equal to that of the others, the most probable position is the point within the figure equidistant from the sides. If the lines are of unequal error, the distance of the most probable position from each line of position varies as a function of the accuracy of each LOP.

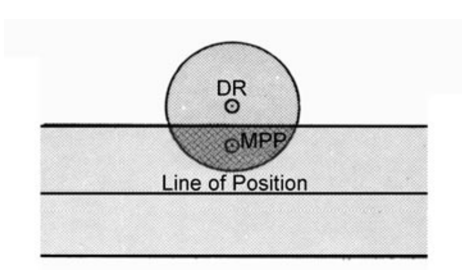


Figure 2304a. A most probable position based upon a dead reckoning position and line of position having equal probable errors.

Systematic errors are treated differently. Generally, the navigator tries to discover the errors and eliminate them or compensate for them. In the case of a position determined by three or more lines of position resulting from readings with constant error, the error might be eliminated by finding and applying that correction which will bring all lines through a common point.

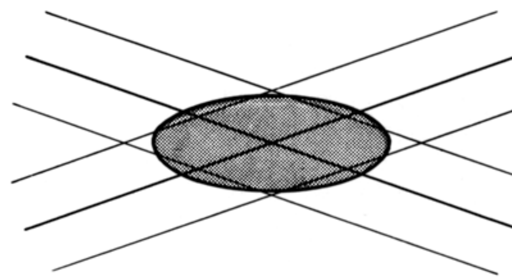

Figure 2304b. Ellipse of uncertainty with lines of positions of equal probable errors crossing at an oblique angle.

Lines of position which are known to be of uncertain accuracy might better be considered as “bands of position”, with a band with of twice the possible amount of error. Intersecting bands of position define areas of position. It is most probable that the vessel is near the center of the area, but the navigator must realize that he could be anywhere within the area, and navigate accordingly.

**2305. Mistakes**

The recognition of a mistake, as contrasted with an error, is not always easy, since a mistake may have any magnitude and may be either positive or negative. A large mistake should be readily apparent if the navigator is alert and has an understanding of the size of error to be reasonably expected. A small mistake is usually not detected unless the work is checked.

If results by two methods are compared, such as a dead reckoning position and a line of position, exact agreement is unlikely. But, if the discrepancy is unreasonably large, a mistake is a logical conclusion. If the 99.9 percent areas of the two results just touch, it is possible that no mistake has been made. However, the probability of either one having so great an error is remote if the errors are normal. The probability of both having 99.9 percent error of opposite sign at the same instant is extremely small. Perhaps a reasonable standard is that unless the most accurate result lies within the 95 percent area of the least accurate result, the possibility of a mistake should be investigated.

**2306. Conclusion**

No practical navigator need understand the mathematical theory of error probability to navigate his ship safely. However, he must understand that his systems and processes are subject to error. No matter how carefully he measures or records data, he can obtain only an approximate position. He must understand his systems' limitations and use this understanding to determine the positioning accuracy required to bring his ship safely into harbor. In making this determination, sound, professional, and conservative judgment is of paramount importance.