Structure of MATTER, *Structure of* MIND

MAN'S PLACE IN NATURE, RECONSIDERED

William L. Abler

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Man's Place in Nature, Reconsidered

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Pensoft Publishers & BainBridgeBooks
Sofia–Moscow–Philadelphia
2005

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First published 2005
ISBN 954-642-232-0 (Pensoft Publishers)
ISBN 189-169-619-X (BainBridgeBooks)

Pensoft Publishers
Geo Milev Str. 13a, 1111 Sofia, Bulgaria;
Tel: +359-2-9674070; Fax: +359-2-9674071
pensoft@mbox.infotel.bg
www.pensoft.net

BainBridgeBooks
(An imprint of Trans-Atlantic Publications Inc.)
311 Bainbridge Street, Philadelphia PA USA 19147
www.transatlanticpub.com

Printed in Bulgaria, September 2005

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PREFACE

In science, everything depends on method. Not only do the questions that we ask determine the answers that we get, but controls, i.e., comparisons, protect us from our own prejudices and imagination. We can never safely just look at a thing and hope to understand it. That is *The Blind Men and the Elephant*. Instead, we understand something only in relationship to something else. *Structure of Matter* addresses several questions, starting with foundations of mathematics, and the evolution of a mathematical sense. It examines the nature of language and the evolution of language, as well as mind and its relationship to mathematics and language. It inquires into the nature of human culture and technology – but always on the basis of method. Where a control group is not possible, a question will at least be placed into a larger context, so that we have an informed idea of what we are really asking. In the end, we will inquire into the fundamental mechanism of the brain, the limits of mathematics, and the character of foundations itself.

But *Structure of Matter* will never ask a naked question such as, "How did a mathematical sense evolve?" or, "How did language evolve?" Questions of that kind are too loaded with assumptions to be answered safely. To approach them, we will first place the question of a mathematical sense into a wider context, by asking how we know that a mathematical sense is present at all, and how evolution happens. The presence of a mathematical sense might seem obvious in every way. After all, people can do mathematics. But it is the unexpected relationship of the resulting mathematics to the underlying laws of evolution that will emerge as important. We will find a control group for the study of language, i.e., we will find a parallel system for comparison to language. We will look for natural connections of the mind to other systems. We will find definitions of a human being, mind, and language, i.e., we will insist on knowing what evolved, before we decide how this or that property evolved.

And we will push every system through to its full logical consequences, to learn whether it contains contradictions. Contradiction in theory always means that some more profound discovery is waiting in its resolution, because, whatever the apparent evidence, a theory that works by assuming the theory, or that contradicts itself, must be ruled out. Understanding in science is possible only on the basis of theory; and better evidence will emerge soon enough. Eventually, we will exchange the superficial unity of statistics for the precise formulation of specific natural law, and obtain a better understanding of what foundations means. Even at the beginning, a glimpse of the end is possible. Because writing the equation that explains the existence of equations is circular, i.e., it generates equations by assuming the existence of equations, foundations of mathematics will not look like mathematics. I hope that the mathematicians won't be disappointed.

Structure of Matter developed out of a thirty-year program intended to identify a control-group for the study of a human being. I studied linguistics in graduate school at The University of Pennsylvania, and neuropsychology as a postdoctoral fellow at Stanford University. During that time, I learned Chinese, I built a working model of the human vocal tract, was an early adviser to the Koko language project, studied aphasia, and saw the experiments designed to model language in monkeys. And, time and again, I watched Karl Pribram operate on the brains of monkeys. Then my own project began – to find a control group for the study of the human being. I spent six years studying the brain of the honey bee, and twice that time studying the behavior of dinosaurs, using their teeth as a guide to their behavior. But, in spite of a number of interesting discoveries, no biological control group turned up. The rest is the subject of this book.

As it stands, *Structure of Matter* is not the book I wanted to write. The original draft simply explained the shared-source theory, and left it at that. But evolutionary psychology and evolutionary linguistics have gripped the public intellect, both scientific and popular, so securely that simply stating a different case is not enough. Any theory that does not take evolutionary psychology/evolutionary linguistics for granted is required to explain why. This I have done, as clearly and explicitly as I know how. What the result lacks in purity, I hope it makes up in the drama of incompatible ideas in contact.

Structure of Matter does not represent progress. Nor is it intended to be up-to-the-minute with the latest research. Rather, it is a complete, permanent, first-approximation theory of foundations of arithmetic, language, and the human mind.

Numerous loyal friends sustained the *Structure of Matter* project through thirty years of effort, three completely new drafts, and countless revisions. In spirit, and with one exception, they are listed in the order I met them. Roger and Valerie Berry; Peter Tobias; Tex and Alice Freeman; John and Sherry Bowen; Rupert Wenzel; Bob and Chris Evers; Davis and Patty Gammon; Hon-chiu and Pauline Wong; Steve Sears; Ray Sauer; Karl Pribram; Karl Drake and Carol Christensen; Stan and Yingying Smerin; Jeff and Marlene Wine; Erich and Charlette Sutter; Ken Okamoto; Bill Simpson; Ken Grabowski; John Bolt; Allen Wolach; Erich Schrempp and Kathy German; Brie Taylor; Carl Degner and Nancy Streckert; Martin Schwan and Deborah Marotta; Kurt and Lyn Pearl; Ron Weber and Nancy Fagin; John Acorn and Dena Stockburger; Clive and Judith Coy; Darren Tanke; Julie Cormack; Don Brinkman; Pierre Durand and Charlotte Nyborg; Lynne and Charlie Gunn; Brian and Jan Ford; Jim Hill and Debra Rice; Joe and Susan Hammon, Geoff and Linda Tillotson, Mike Casey and the Oak Park Farmers Market Bluegrass Band; and the late, much-missed Dr. Ray Zalewski.

Sam Savage taught me, in 1956, that arithmetic is the same in any base; Richard J. Carbray gave me my Latin name; Gertrude Drake supplied the reference to *Puck of Pook's Hill*; Phil Currie gave me permission to publish on Tyrrell Museum fossils; Martin Schwan, Karl Drake, and Jeff Wine kept fire-copies of the manuscript; Lyle Schmidt and Karl Drake provided computer support; specialized photography by Erich

Schrempp Studio Chicago <www.schremppstudio.com> (312) 454-3237; anthropology and natural history books from N. Fagin Books Chicago <www.NFaginBooks.com> (312) 829-5252; Ben Stark and Michael Cummings provided advice in genetics; Hon-chiu Wong provided advice on Chinese grass characters; Jeff Wine helped with everything; Ralph Felder and Keith Mellinger provided advice in mathematics; the idea that the original use of honey is to dilute toxic sugars is due to Roy Barker; Michael Studdert-Kennedy reviewed earlier drafts; The Geology Department at The Field Museum made me a research associate; Oliver Pergams was the first practicing scientist to accept the *Structure of Matter* theory. The Chimpanzee painting and Darwin letter are used here Courtesy of The Field Museum, Library Special Collections, Mr. Ben Williams, Library Director. All mistakes remain my own. I thank my wife, Patricia, and our twins, Alice and Nicholas. Child art by Alice and Nicholas.

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Chapter 1.
Introduction. The Evolution of a Mathematical Sense

Some things never change. The questions that drove science in antiquity are very much the ones that drive science now. What is our place in nature? How did we get here? What is the world made of? Why are things the way they are?

But the answers are very different. To the ancients, the human being was a kind of chimera, with “the body which we share with the animals, and the Reason and Thought which we share with the Gods” (Epictetus 1st century A.D., Saying IX). Where the ancients were generalists who approached life on the broadest possible front, we are absolutists, what we would call “specialists”. Our absolutism expresses itself in matters of taste, esthetics, manners, customs, religion, politics, and law – as well as science.

To us moderns, the human being is one of two things. Either it is entirely a creation of God, or it is entirely animal. We believe that the ancients were wrong, and that no third possibility exists. I will not examine Creationism any further, but we must realize that our modern minds are driven by more assumptions, and more shared assumptions, than we would be happy to enumerate. It is the shared nature of what we see as irreconcilable differences, that makes new ideas so difficult to acquire.

Scientists compute their citation impact factor, and the citation impact factor of journals, to the third decimal place, as if we knew what we are doing. But Gregor Mendel was cited four times in thirty-four years after he published. His citation impact factor would have approached nil in his lifetime, yet he is an undisputed hero of science. We should look to our own ability, and hope to be cited 150 years from now, instead of jostling our neighbors for decimal points.

We know the difference between biological psychology and psychological biology, between cognitive psychology, comparative psychology, psychobiology, biopsychology, sociobiology, physiological psychology, evolutionary psychology, paleopsychology, psychophysics, biophysics, and psychological anthropology, until we actually believe that nature has the same departments that we do.

No such divisions troubled the minds of the ancients. Human language, and fire, the power behind technology, and maybe the mind, were personal gifts from the gods. Gods, and at least one goddess, had children with human beings, obliterating any boundary between mortal and divine. If we still have vestiges of that, they do not affect our science. Marsyas and Arachne could challenge the gods. Marsyas challenged Apollo to a flute-playing contest, and was flayed alive. Arachne challenged Juno to a weaving contest, and was changed into a spider. But if Marsyas came to a horrible end, Arachne's children now number some 37,000 species. The ancient gods were little more than super-aristocrats, who had our strengths and weaknesses, but bigger-than-life. They stood as clear examples of what to be, and what not to be. The ancients thought that prime numbers are magic numbers. Maybe they were a little superstitious, and maybe we know more – but they were closer to everything, and understood better.

Chapter 1 examines the evolution of a mathematical sense in our earliest pre-human ancestors, at the beginning of the formative period of human evolution. But, more importantly, it is an exercise in the use of method to protect our thinking from our own prejudices and unsuspected assumptions. We will use context and intellectual perspective to bring into focus the actual nature of a seemingly simple question.

How much do we really know? Under the modern theory of evolution, the question is, How do we get from here to there – from a dinosaur to a bird, from an okapi to a giraffe, from a worm to an insect – from a monkey to a human being? With other animals, the formula works pretty well. Sooner or later, someone will fill in the blanks. The answers are often fascinating. Zebras have stripes not so they can hide in the tall grass, but so that lions won't be able to single out one zebra from the rest as the herd thunders by. Giraffes (Spinage 1968) have long necks not so they can reach the highest leaves, but so they can keep their herds organized at great distances. Birds developed wings not so they could fly, but so they could hold themselves down while they ran up hills and sloping trees (Dial 2003). Honey bees started making honey not so they could eat during the winter, but to dilute toxic sugars down to below-toxic concentrations (This idea is due to Roy Barker). The communicative dance of the honey bee is a re-enactment, in miniature, of the foraging flight (Lindauer 1961). The migration of the monarch butterfly (Urquhart 1960, 1976) may be a re-enactment, every year, of their ancestors' pursuit of the milkweed behind the melting glacier at the end of the Ice Age (Kinsey 1926, page 512). But without a definition of the mind, or of a human being, or of language, i.e., without knowing what evolved, it is a little too soon to decide how any of those things evolved.

The modern paradigm for the solution to puzzles in evolution was established at the 1860 Oxford debate between Thomas Huxley, Darwin's self-appointed "bulldog", and Bishop "Soapy Sam" Wilberforce, who had played an important role in the abolition of slavery in England. Wilberforce took the side of Creation, while Huxley took that of evolution. Wilberforce, "with a smiling insolence, - - - begged to know, was it through his grandfather or his grandmother that he claimed descent from a monkey?" Huxley countered that "He was not ashamed to have a monkey for his ancestor; but he would be ashamed to be connected with a man who used great gifts to obscure the truth" (Thomas Huxley's son, Leonard Huxley 1900, page 197).

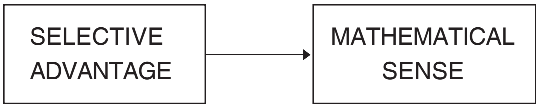
It is generally agreed that Huxley won the debate. But Wilberforce's question is funnier than Huxley's answer; and there is more. A Mr. Dingle, so obscure that his given name is lost to history, and who is omitted entirely from many of the re-tellings, won the debate. Huxley relates (page 196), "Then a Mr. Dingle got up and tried to show that Darwin would have done much better if he had taken him into consultation. He used the blackboard and began a mathematical demonstration on the question – 'Let this point A be man, and let that point B be the mawnkey'. He got no further; he was shouted down with cries of 'mawnkey.'" Mr. Dingle may have been speaking in Irish or rustic tones that the undergraduates thought comical, but his demonstration was geometric, not algebraic. If he had not been so hilariously interrupted, he would have gone on to say that our task is to discover how we get from B, the mawnkey, to A, the man.

How, then, did our earliest ancestors make the transition from zero to a mathematical sense? Such a sense must have evolved because people are animals, and because the human mind must be adapted to its environment by natural selection. That is why we can live in the world. Our task, then, is to fill in the evolutionary blanks between a pre-numerate animal, the mawnkey, and a numerate one, the man.

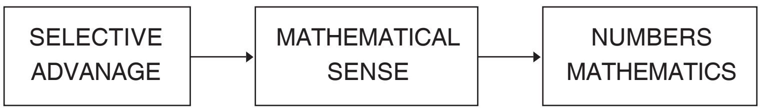
What must have happened is this. Individuals who possessed the ability to identify numbers would have had some selective advantage over individuals who didn't. A number sense would have conferred an advantage in keeping track of valuable things such as children, or arrows. It would have fostered cooperative behavior by allowing several fishermen to hold a large number of fish in a single community basket, and divvy them up later. In time, after all members of a community possessed a number sense, a fully-developed arithmetical sense would have conferred further selective advantage, allowing a fisherman, say, to decide (calculate) how many hooks to make, or how many fish to catch to feed eight people. Hunters who possessed a sense of Newtonian mechanics would be able to throw better than ones who didn't, allowing them to catch more animals, and feed their families better. Their offspring would have lived longer, and inherited their parents' Newtonian sense, allowing it to spread through the community.

The evolutionary hypothesis, then, is this. A mathematical sense in all its forms evolved because it conferred a selective advantage.

The Apparent Hypothesis:



At this stage, we may begin the process of placing the original hypothesis into perspective, i.e., into context. A mathematical sense makes its existence known through the numbers and equations that it generates, because a mathematical sense that doesn't generate numbers and mathematics isn't much of a mathematical sense, after all.



A Friendly Club brings maps and baskets of fruit to new arrivals in a town, and invites the newcomers to visit. Charles Darwin kept the financial records for the Downe Friendly Club, and, once each year, read them out to the membership gathered on his lawn, after the band stopped playing. But he was otherwise not very mathematical. Still, he has survived these 150 years or so since the publication of the *Origin* (1859) because natural selection is inherently a mathematical idea. It is a mathematical idea conceived and expressed in non-mathematical terms. There are dozens of books and websites waiting to show the mathematical nature of evolution (see, e.g., Dobzhansky 1955, page 117; or enter "hardy weinberg" into Google). But a brief exercise here will prove worthwhile

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Evolution Box 1 [The purpose of the evolution box is to show that, in principle, natural selection is expressed by an equation. Readers who are already convinced, or who don't want to look at equations, can safely skip the evolution box.]

The characteristics of living things are represented by genes, which are transmitted from one generation to the next. Evolution happens when the proportions of the genes in a population, or the kind of genes, changes from one generation to the next. Say that a plant has one gene R for red flowers, and another gene W for white ones. While most cells in a flower have two sets of chromosomes, and two genes for flower-color, the reproductive cells have only one gene, R or W, for color.

At fertilization, each R gene has a 50:50 chance of being paired off with an R or a W; and the same is true for each W gene. The gene combinations that result are:

RR and RW for the R gene, and

RW and WW for the W gene.

Another way of writing the same thing is:

$$1RR + 2RW + 1WW$$

When that expression is re-written in conventional form,

$$R^2 + 2RW + W^2,$$

we have the expression that made Gregor Mendel famous 34 years after he published it, and the reason that genetics is called "Mendelism" in his memory and honor (J. Wilson 1916; Punnett 1919; Iltis 1932; Stern et al. 1966; Olby 1985).

When Mendel's expression is set equal to 1, or 100% of a population, we have the Hardy-Weinberg equation, under which every gene has equal chances of being transmitted to the next generation in the same proportion that it occupied in the last.

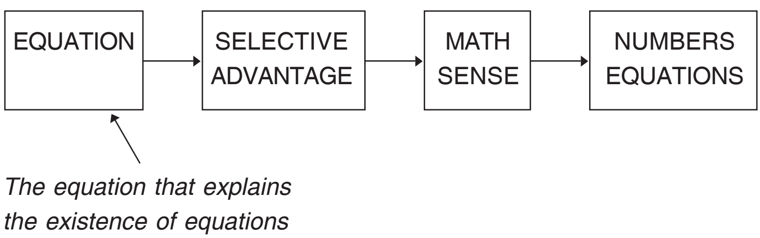
When some change in the environment, or in a gene (a mutation), upsets the balance of the Hardy-Weinberg condition, one gene will have a better chance of surviving in the next generation than some other gene – and evolution will happen. Since the Hardy-Weinberg equilibrium probably never exists in nature, evolution is probably going on all the time. In principle, the amount of imbalance is expressed as a ratio; and the ratio expresses the force or pressure of natural selection.

When the precise cause is known that pushes Hardy-Weinberg out of equilibrium, that, too, is expressed as a ratio, or equation. Eventually, we will see the equation that purports to explain the existence of sentences, and, thus, of equations.

The only point is that, at least in principle, selective advantage, i.e., evolution, is intrinsically and inherently mathematical, even if we don't know the actual equations concerned in some particular case. Evolution must be understood as a kind of analog computer that calculates genetic outcomes on the basis of equations, even if the equations are complicated or un-obvious.

However obvious the premises, and however correct the deductive sequence of the logic, the evolution of a number sense is a matter of deriving equations on the basis of an equation.

The Actual Hypothesis:



The idea that a number sense, or that a mathematical sense, evolved in biology, on the basis of selective advantage and natural selection, is circular. The system works by assuming the result. It assumes that you can use mathematics to show how our pre-human ancestors got mathematics. It contains zero information. It means that you need to have an equation in order to account for the existence of equations. In principle, the equation at the left of the above figure claims to represent the foundations of mathematics, i.e., mathematics is the explanation of mathematics. In due course, we will see that the equation has been written and published that purports to account for the existence of equations. Of course, the sequence of logic is long enough to hide the real hypothesis, but the equation is there, and we will examine it. But it never happened. A mathematical sense never evolved in biology. Equations, and things that have the same form as equations, did not emerge on the basis of an equation.

Although we do not yet have the intellectual machinery available to examine the underlying structures and properties of equations, a few of their properties are so obvious as to invite our attention now. The first of these is discreteness. All systems with emergent properties take advantage of discrete elements. For example chemical compounds may have properties that are far beyond anything known on the Periodic Table. Salt is nothing like sodium or chlorine; and its properties are not an average of the properties of those elements, or any other elements. Soap, or protein, is entirely different from the carbon, hydrogen, and oxygen that compose it; and its properties are quite beyond anything on the Periodic Table. The genes of living things are just chains of molecules. They don't have ingenious, fascinating shapes that stagger the imagination. They have a geometry so bereft of properties that it is hardly worth consideration. There is hardly anything to consider. But the genes are, somehow, instructions for the construction of biological objects that are not only very different from the gene, but very different from one another. There must be some reason for its linearity. The gene is famously discrete, or, in the language of the geneticists, particulate.

Language itself takes advantage of discreteness at every level. The speech-sounds are discrete and individual, as are the words of language. Even grammatical constructions are discrete. Discreteness seems not to be arbitrary, but necessary.

Also of a basic nature is the linear, one-at-a-time sequencing of the elements in an equation. All equations are linear in their spoken form. And the written form can always be re-organized into linear form, if only as a demonstration. The single-file structure of equations is nothing that would be studied in a school of geometry, because there is nothing intrinsically interesting about it. Yet it is shared by language and the gene, and possesses an elegant simplicity that almost exceeds the intrinsic interest of triangles and squares and hexagons, for all their fascinating tessellations.

But most fascinating of all is the equals-symbol, the "=". Equations are true when they are symmetrical about the "equals", i.e., when the material on the left of the "equals" is somehow the same as the material on the right. Bilateral symmetry is one of the most fundamental properties of geometry. All rapidly-moving animals exhibit it, none more elegantly than butterflies. I am sure that mathematicians, real mathematicians, are born with, and spend a lifetime cultivating, a sixth sense for the symmetry that makes equations true. The preservation of symmetry is what makes the steps in a proof. A step is wrong not so much because it breaks some rule, but because it breaks symmetry, and right because it preserves symmetry.

We will return to discreteness, and linearity, and symmetry, after we know more about the affiliations of mathematics to other systems. But I hope that this short introduction will show not just that a mathematical sense did not evolve in biology, and not just that we stand at the threshold of a vast and unknown sea of natural properties, but that method, rather than deduction from obvious premises, is the only means of finding our way.

Such method consists of finding a control-group for the interpretation of anything. A control-group is related to our object of interest, yet recognizably different from it. The differences and similarities can then be identified, described, and interpreted without guessing. If no control-group can be found, then at least we must place our questions into the largest possible context, so that we can see what question we are really asking. Apart from method, science is blind and helpless. Apart from method, its constructions represent imagination in its purest form, and are interesting as a reflection of our contemporary prejudices.

If we are to abandon such prejudices, we must start over. In the founding moments of an idea, there are no experts. Mathematicians may be terrific at doing mathematics, but asking a mathematician to tell you about the foundations of mathematics can be a little like asking an alligator to tell you about the foundations of zoology.

Today, airplanes fly and rocket-ships go to the moon. We have vaccines that prevent or cure diseases that once were devastating. And if we haven't exactly conquered nature, we have a better grasp than the ancients. But we will see that, in one important aspect, ancient intuition was closer to the mark than modern science: what Thomas Huxley (1896) called "*Man's Place in Nature*". Driven in part by political correctness, or a desire to avoid accusations (e.g., Pinker 1994, page 332) of species-ism, we have reduced the human being to just one more equal among equals. Where elephants are remarkable for trunks, and whales for great size; where hummingbirds are remarkable for tiny wings, and butterflies for gratuitous beauty, people are remarkable for language and mind. In the modern day, the principles are well understood, and it is just a matter of filling in the details. The only time that the modern movement in evolutionary linguistics got it right was in the founding document (Pinker and Bloom 1990, page 708, middle of column 1), where it declares that the evolutionary model is "boring" and "incredibly boring".

Chapter 2 of *Structure of Matter* will be largely about that second-class citizen of science, language. But we will use it as the control-group for the study of equations, so we must suffer through. The rest of this chapter will consist of methodological preliminaries.

A few years ago, at an important scientific meeting in one of the world's great cities, I was seated in the front row of a packed auditorium, listening to a lecture on the language-ready brain. The idea was that if deaf-and-dumb children can pick up sign-language easily, their language-ready brains are a latter-day model of the origin of language. It was clear that these children learned some form of language quickly because they had a thousand generations of ancestors who spoke normal language, and that language-ready now is not the same as language-ready then. The lecture amounted to an origin-of-language IQ test, but only two members of the audience caught it.

Worse still, a prominent and influential language-scientist, one of the organizers of the meeting, standing at the front of the auditorium, just five or six feet away from me, was so captivated by the geopolitical setting of the experiments that he blurted out, "This fits perfectly with my political agenda."

Nature and nature's laws are not concerned with our political inclinations of the moment. If we had lived in Athens we would have condemned Socrates and supported slavery. If we had lived in England, like Geoffrey Chaucer, we would have remained silent about all the pretty faces that decorated the ash-wood pikes around the Tower of London (Gardner 1977).

The forces that drive the galaxies are not concerned with our races or our religions or social justice as we see it. I don't suppose that a physicist would think that quarks fitted some political agenda better than larks. But if so, it would best be forgotten in favor of measurements and observations and refined theory. Where our own mind and being are at stake, objectivity is more difficult. But when we let our science – any science - be driven by our hopes and fears rather than by ideas, we are no longer qualified to do science at all. We are all children of our time. Pierre-Auguste Renoir says it took the privy to teach him that lesson. But we must see ourselves for what we are, and accept ourselves, and get beyond it, if we hope to solve important questions in science.

By "language", I mean naturally-occurring human language. I don't mean body language, or computer language, or the language of love, or the language of chemistry, or mathematics as a language (or set of terms) for talking about certain things, or the dance language of the honey bees, or the syntax of architecture, or "money talks", or "every living thing has a language all its own", or any other metaphor for "information" or "communication" or "system" or "structure". Just naturally-occurring human language.

Under this agreement, we will be examining equations, i.e., sentences of arithmetic, and the sentences of everyday language. We will find that equations are a subset of all sentences in the most detailed, exhaustive, and profound way. And we will find not that language and arithmetic are the same thing, a ridiculous idea on the face of it, but that language and arithmetic share a common source.

Some scientists go so far as to propose that there is no such thing as language *per se*. What we call language is actually a bundle of unrelated skills, held together temporarily by the situation, like car-driving. So, where the pedals organize your feet, and the steering-wheel organizes your hands, and the passing scene organizes your eyes, the situation as-a-whole organizes your behavior into car-driving. The moment you open the door, it vanishes.

By extension, the presence of another person, and listening to that person speak to you, organizes your behavior into phonemes and morphemes, nouns and verbs, phrases and deep-structures, sentences and discourses. The mind, too, is a Swiss-Army knife of skills tinkered together by circumstance (Jacob 1977a) in much the way that the arms and legs are borrowed from former fins, the jaws from old gill-arches, the teeth from old scales, and so forth. In principle, then, the modern view holds that there can never be a definition of mind or language or a human being.

It is all very convenient. Most problems in science are, somehow, a converging series that leads, eventually, to some clear answer, $E=mc^2$, for instance. Albert Einstein's famous equation of 1905, just recognizable on the second-to-last page of the Electrodynamics, is a good example because all the messy infrastructure is published, right out in front of God and everybody. After a few opening assumptions of the simplest and most general kind (there is no luminiferous aether, nothing can travel at the speed of light, and the laws of physics are the same no matter what the observer's frame of reference), Einstein puts on his seven-league boots and walks out into the wilderness where no one else could go without a guide. There are thickets of differential equations. But as the paper nears its end, it gets simpler.

Everything resolves itself into high-school algebra. The relationship between mass and velocity. The addition of velocities, especially near the speed of light. The answers are so simple that every bright high-school student knows them by heart. In the original, it looks more like "$W=\mu V^2$", but you can sometimes find $E=mc^2$ written on the wall in a public washroom, a cliché of our time. And "F=ma and you can't push a rope" is a standing joke among engineers, along with, "If it looks like it will stand up, chances are it will". The world expects that sort of thing. A theory in science is something with explanatory power, something you can put your finger on. Something simple. Not so with the human mind and language, which have become a diverging series that wanders off forever. The scientific public can look forward to hundreds of years of progress but, in principle, no clear answers. Researchers can expect to spend lifetimes without ever having to pin themselves down to anything.

What is more, whether we realize it or not, normative, statistical thinking lies at the center of all our ideas about the human being and the mind. We have elevated mediocrity to the level of a science, and are living by it. The structure of mind represents, in some final sense, an average of everything that ever happened to us. Over evolutionary time, the mind with its innate structures is an average of the advantages of anticipation and waiting, of better communication, and the accidents of history that befell our primate ancestors during the formative period. If there had been a few more crocodiles and a few less hippopotami, what we now think of as language would have been something different. In individual lives, it is personal experiences. John is afraid of birds (or snakes) because a bird (or a snake) frightened him when he was a child. John likes music because his mother sang songs to him when he was a child. John is interested in science because there was a fossil trilobite in the rocks behind his parents' house when he was a child. John is good at basketball because he started young. Boys and girls grow up to be men and women because of something that adults inadvertently teach them when they are little. Substantially, we can understand anything about the mind by a process of cagey definitions, rationalization, explanation, reconciliation (Chomsky with Darwin, for example), and averaging. If only we knew what Einstein's mother inadvertently did – or Bach's – the whole world could be filled with little Johanns and little Alberts. It is not possible that someone might like science because of some inner compass that finds its direction for the first time when it sees a trilobite, or the night-time sky, or a crystal of quartz.

Under the modern view, the only real definition of a human being is the trivial one of a primate that represents a certain leaf on a certain twig on a certain branch on the great branching bush of life. Indistinguishable, in perspective, from any other leaf on any other twig. But normative, statistical mechanics is exactly what doesn't lead to evolution and the emergence of new structures. The genetic material, especially, has to be discrete because a blending material would never support differentiation. A mind composed of averaging or blending influences would soon homogenize itself to its neighbors, leading to a world of identical human automatons.

Since we are not identical, and probably not automatons, science stands now, in relation to the mind, exactly where it stood in 1899 in relation to the gene, just before the public recognition of Gregor Mendel's algebraic solution to biological inheritance. When will we grow tired of watching heroic robots, driven by satellite navigation (which is neither robotics nor artificial intelligence), fail to finish obstacle courses that any human being could finish? When will we see ourselves try to model the brain by strategies based on what computers do instead of what brains do? When will we realize that machine translation continues to fail because computers are machines that do exactly and only what we tell them? When will we grow tired of seeing the same ideas, language compared to car-driving, language behavior emerging from the behavior of play, of grooming, the mechanism of GRIP or GRASP being transferred from the hand to the mouth, proposed and forgotten and proposed and forgotten. We might argue that this time we got it right, but if someone else proposed the same thing, we would tell them, "It's been done". The circularity has already begun to emerge. Eventually, we will see the need for a new theory.

Many of our exciting new ideas have been proposed before, and have been forgotten. In his *Lectures on the Science of Language* (1862), Max Muller proposes that, for building a science of language, a voyage in a time machine would be useless (page 344); a knowledge of life on other planets would be useful (page 332); all modern speakers are descended from a single pair (page 327); the study of language is a physical science (pages 86, 79). More recently, but not all that recently, Peter C. Reynolds (1976) proposes that language is a skilled activity related to the behavior of play, and (1981) that it is related to tool-use and grooming. And here is the fifty-fourth printing of George A. Dorsey's immensely successful (1925) *Why We Behave Like Human Beings*, endorsed (on the dust jacket) by Sinclair Lewis, Nobel-prize-winning author of *Main Street*, and (opposite page 512) by "Dr John B. Watson, founder of the Behavioristic School of Psychology". Dorsey explains, in 1925, that play is learned, self-rewarding, repetitive behavior, and that language is essentially the same thing (pages 353-375). The comparison between language and car-driving was popular during the late 1960s.

Interdisciplinary study is too feeble because it takes advantage of ideas without questioning them. Anthropologists can take advantage of nuclear physics to establish the age of interesting objects, but the result only makes anthropology more accurate. Neither physics nor anthropology is changed in the process. A mature science like biochemistry may be able to coast for a while, assuming that the legacy of its history will automatically generate the right questions. For a few decades, maybe. But the study of the mind is too immature and too important to trust to what history has left us, or to trust to our personal preferences. We have to let facts lead us. No compromises and, especially, no contradictions. And if our personal inclinations are offended or our failings exposed, we will just have to abandon what we know and start over. The experience could be painful, but could force us to think in new ways, and might even expand our horizons.

What is wrong with the evolutionary theory of language and mind? Apart from the fact that it is circular, nothing. That is the deceptive part. Aside from a few innocent excesses attributable to enthusiasm, every statement in support of the evolutionary theory of the mind is more-or-less true. But we have no actual knowledge of the brain's inner workings. In spite of sophisticated imaging, we know only where something is happening, not what is happening. No one has the slightest idea, for example, how the sounds of human speech are represented in the brain; and anyone who claims to know is lying. In legal terms, evolutionary theory can show motive, but not means or opportunity. And even with up-to-date technology, there is not enough evidence to convict. There is a kind of origin-of-language fever, like the polar fever that gripped the world at the turn of the 20th century. Somebody has to discover the origin of language, and the sooner the better. But like Robert Falcon Scott at the South Pole, we have only created an eloquent disaster. The leaders are simply too eager to please, and the followers too easy to please. In searching for a control group, Pinker and Bloom (1990, page 715, column 1) lament that "no one has ever invented a system that duplicates its [language's] function." But such a system does exist.

If there is need for a new theory, I will propose it now in the absolute smallest space possible. We can return to it at leisure and at length later.

1) We can't write the equation that explains the existence of equations.
2) Equations are sentences (Chicago 1956, page 30), i.e., they are sentences that have "equals" ("=") as the main verb.
3) The equation that explains the existence of sentences, must explain the existence of all sentences – equations included.
4) It is circular to derive language (i.e., sentences) on the basis of selective advantage, i.e., an equation.
5) Language did not emerge by natural selection, but is more basic than that, and shares its natural source with foundations of mathematics (i.e., because equations are sentences).
6) Since every statement in algebra has a corresponding equivalent in geometry, the property of mind has its beginnings in geometry corresponding to statements in algebra, and manifested as the mental imagery that drives both technology and sentences.

7) Language and mind are not distinct-but-intimate, like the respiratory and circulatory systems, but are aspects of the same thing, like electricity and magnetism.

The idea of evolutionary psychology or evolutionary linguistics, then, is a kind of thimblerig, or shell-game, where we don't notice the circularity because the topics keep jumping from sentence to language to equation and back again. The human brain is the pea under the last shell. The evolutionary theory of the mind is the biological equivalent of a perpetual motion machine. We can make an infinite number of true statements about it. The perfectly oiled bearings. The perfectly balanced wheels made of stainless steel plated with gold. The optimized gear-teeth made of individual rubies, designed by computer according to advanced theory. All of it is true. Only the underlying, founding assumption is circular. Once that has slipped by un-noticed, the disaster is quietly assured. Nothing is wrong, but everything is futile.

The idea of an evolution of language and mind in biology, like that of absolute rest in physics, is a contradiction, and corresponds to nothing in the natural world (see Waterman 1963, page 54). Like a judge who has decided the verdict before examining the evidence, evolutionary psychologists will find themselves defending ideas they would never accept from anyone else. They will find themselves defending contradictions, and proposing experiments that can't be controlled. Without something to compare mathematics to, a control group, we can never isolate what is primitive in it. We will drift forever on a sea of hope, based on what we want to do with mathematics, rather than on what mathematics is. That is what we are doing with the computer, which is a branch of mathematics. The same goes for language. Evolutionary psychology has done more damage to science than phrenology ever did, and I hope science can recover quickly.

Under the new system, the human being becomes interesting again, and definitions become possible. There is such a thing as language, and such a thing as a human being. Language is a communication system with forms and formulas borrowed from the forms and formulas of arithmetic and algebra. The human being is that animal that has voluntary use of language adapted from algebra, and of mind adapted from geometry corresponding to algebra. Language and mind are aspects of the same thing, and their origin is the same as the foundations of mathematics. It isn't that some equation will allow us to calculate the structure of language, or that language will allow us to discuss the structure of equations. More interesting, the two systems share a common underlying structure which can be understood only by comparing them. Each is the unique key to discovering the structure of the other; and shared structures are primitive to both systems.

We can't go back to antiquity. The human being is not exactly close to the gods. But the human mind is close to the center of nature, close to the formative properties of the universe. And if by some miracle extraterrestrial intelligences exist, we will recognize them, and they us, easily because our minds and languages will have identical foundations in geometry, while the details of our various biologies, and places on our respective branching bushes of life, will be unimportant by comparison. They will have writing; and if we can't produce the physical signals of their speech, we will be able to read their writing. The metaphor for the human mind is not the switchboard or the computer, but the particle accelerator.

Much of what follows will concern language, or about half of it will, and here I must ask the reader's indulgence. Many will feel that they have read enough about language already to last them one lifetime. But modern aversion to the study of language runs deeper than that, and with good reason. Where language in antiquity was magic, a gift from the gods, language in the modern day has become just another case of fill-in-the-evolutionary-blanks, the weak sister of science. When the physicists and the brain scientists finish working out first principles, when the mathematicians and the big-boys get the equations right, the easy stuff, like language, will fall out of the works automatically. And the self-appointed guardians of the mind have played right into their hands. One more equal among biological equals. But the word "grammar" shares its roots with "glamor", and language was enchanted until the evolutionary linguists got their hands on it. Remember the last stanza of Puck's Song from *Puck of Pook's Hill.*

"She is not any common earth,
Water or wood or air,
But Merlin's Isle of Gramarye
Where you and I will fare."
Rudyard Kipling (1905, page 2).

In the last paragraph of his *A Brief History Of Time*, physicist Stephen Hawking (1988), quoting Wittgenstein, arrogantly and accurately sums up the modern world's view of language (here paraphrased). "The philosophers, who once were going to solve the mysteries of the universe, are now reduced to the pitiful chore of solving language". But half of what follows here concerns foundations of algebra, and should justify itself on that basis alone.

More to the point, the shoe is now on the other foot. The fundamental operating mechanism of the brain will be discovered only on the basis of search-images provided by a comparison between arithmetic and language. Foundations of mathematics will be discovered only by comparison between equations and sentences. We will have to change our understanding of what it means to understand something. There will be no equation for the foundations of mathematics, or of language and mind, either. There will be aspects of natural law that can not be expressed by an equation, altering our conception of natural law. And if mathematics and naturally-occurring human language represent founding properties of the universe manifested at our own scale, where we can see them, there won't be an equation for those, either. For now, however, the question is no longer, "How did language evolve?" but, "What are the foundations of arithmetic?" not, "What is the right equation?" but, "What are equations?".

Chapter 2
The Character of the Mind

Sometimes you don't know the right question until after you have the right answer. For example, if we ask, What kinds of television programs are bad for children, we might get a long and well-informed answer. But if we ask, Is television good for children, we might get closer to the mark. If we ask how Santa Claus visits every house in one night, we might get one answer. But if we ask, what is Santa Claus, we might get something very different, and might even have a better idea of what it means to visit every house. If we ask how earth, air, fire and water combine to make the world with all its things and materials, antiquity might give us an informed, well-reasoned answer. If we ask, How did language and mind evolve, we will get a sophisticated, well-informed, and never-ending discussion. But the circularity of deriving equations from equations shows that the question is the wrong one.

The image of science as a social activity, with everyone contributing some mite of truth to the larger enterprise, is seductive. It makes us all feel safe, part of something greater than ourselves. The idea of becoming disciples of Darwin and Chomsky is comforting. Filled with emotion, we get the two old men to shake hands. We confidently start with the unarguables. Descent with modification. Variation and selection. But true statements, no matter how many or how true, do not add up to truth if they answer the wrong question. What, then, is the right question?

With no reason except that equations are sentences, we must look for the shared source of equations and sentences. That is how I eventually approached the question myself; and there is no other way. My mathematician friends, at least, are so familiar with the idea that equations are sentences that they get impatient if anyone reminds them. After we have seen the shared properties, we will, in hindsight, realize that the exercise was exactly the right one. I do not propose to write mathematics for mathematicians, or linguistics for linguists, i.e., I won't write equations and solve them, or describe languages and show their structure. There is plenty of that already. But at the place where both systems intersect, they are simpler and clearer than at their fullest development. The territory is unfamiliar to all of us. Definitely not what anyone is trying to do now. But I hope I have shown that the modern program in evolutionary psychology and linguistics (Lenneberg 1967; Harnad et al. 1976; Bellugi et al. 1980; Bickerton 1990; Pinker et al. 1990; Pinker 1994; Bickerton 1995; Dunbar 1996; Deacon 1997; Hurford et al. 1998; Knight et al. 2000 Nowak et al. 2000; Nowak et al. 2001; Nowak et al. 2002; Hauser et al. 2002) is mistaken, and needs to be replaced. There are no experts at this early stage. Other people can help you, but they can't tell you. You are on your own.

Modern evolutionary linguistics is the continuation of a trend that started in the 1920's, when Edward Sapir named the distinctive sounds of speech first "atoms", then "points", then "phonemes", establishing the modern science of linguistics by asserting its independence from chemistry and geometry, and from the rest of science.

That independence ends here. Where evolutionary psychology/evolutionary linguistics is a frankly pedestrian exercise of fill-in-the-evolutionary-blanks between animal behavior and human behavior, *Structure of Matter* uses method and a control group to restore the scientific study of language to its rightful place alongside chemistry, geometry, and physics, as a basic science. No longer a parochial, derivative specialty for those who decide to get interested, language provides the only possible search images for the foundations of mathematics, and the fundamental operating mechanism of the brain. The human mind is no longer distinguished by a few chance details of zoological classification, but is formed directly by the same dimensional forces that shape the atom and move the galaxies. Language and mind are no longer parallel, if intimate, biological systems (Vygotsky 1934; Chomsky 1968; Sokolov 1968). We will see that the mind has its beginnings in the geometry that corresponds to statements in algebra, making language and mind aspects of the same dimensional matrix. Under *Structure of Matter*, the human being emerges again as fundamentally fascinating.

Structure of Matter is about how we think, not what we think about. It won't explain why you do what you do – only how you do it. I suppose that all animals love and hate, and experience passion and affection, possessiveness and envy. This is a question of what makes us human in the first place – not what we think about, now that we *are* human. This is not Sigmund Freud (1923), and it isn't E.O. Wilson (1975). *Structure of Matter* is directed specifically at the distinctive organization that makes us human, rather than at motivation and emotion. It isn't evolutionary psychology, either (Pinker 1997, page 21).

At first glance, arithmetic and language might seem poor candidates for twins. Where language changes from neighborhood to neighborhood, even from street to street, arithmetic is universal. Where gossip and slang change almost from day to day, mathematics has a quality of the eternal that gives it an incomparable stately elegance that is completely lacking in language. But at the structural center, as we will see, language is identical to algebra. The baroque and wonderful proliferation of language happens at the edges, without touching the formal core.

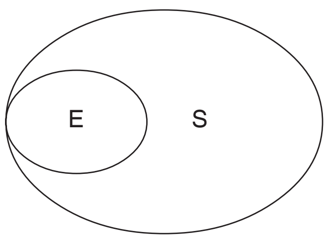
The intersection between language and arithmetic is where the common source will be found, and where we can begin to see what language, and mathematics, really are. And in the process, we will learn where the property of mind has its source, and what a human being is.

To begin, equations are sentences with a main verb and a linguistic deep-structure. For the most part, the main verb is "equals", i.e., that is what the "="-symbol stands for, although some other verbs can substitute.

2 + 2	=	4
Two plus two	equals	four
Two plus two	is	four
Two and two	are	four
Two and two	makes	four

Before civilization had equations formalized, with a neat notation, and maybe before people realized that a single kind of relation ("equals") covers every mathematical situation, equations existed almost entirely in the form of word-problems; and the "equals" was expressed (Robins et al. 1987) by wonderful phrases such as "What is the answer?" or "How many bushels will you need?" In word-problems, we begin to see that there is no clear boundary between mathematics and language, because every equation starts as a word-problem. There are even metaphor-sentences such as "John plus Mary equals romance", or "John plus cars equals trouble", where the form of the equation is adapted right out of mathematics into ordinary conversation.

Notice that the statement "Equations are sentences" is true; but "sentences are equations" is not. Later the difference will emerge as crucial. But for now, it is enough to see that equations are a subset of all sentences, and not the other way around, even if both represent some kind of infinity.



Equations (E) are a subset of all sentences (S). Later, we will see the precise genealogy of properties that are shared between the two systems, and the precise property that distinguishes one from the other.

Before we can examine the linguistic deep-structure of equations, we must first see that equations and sentences of ordinary language both have two main kinds of constituents beyond the main verb: a few constituents that have reference inside their own system, and other constituents that have reference outside it. Symbols of arithmetic represent morphemes of language.

Both arithmetic and ordinary language have a vast and potentially unlimited number of constituents that have their reference outside arithmetic or language. That doesn't mean that the constituents don't have interesting properties of their own. They do. But what defines them as a class is their reference outside their systems. For language, these constituents are vocabulary words, while for arithmetic they are numbers. Let us examine external reference briefly.

Vocabulary words are the vast majority of words in any language. To a great extent, vocabulary words are the content, even the point, of the dictionary. When you create a new object, or meet a new person, you have to learn a new vocabulary word, because people's names are vocabulary words that refer to just one person. But whatever the fascination of meaning and etymology of words, whatever the adaptations in their use, such as "the Winston Churchills of this world", or "John is a regular chameleon", they still have reference outside language. That is another of the defining properties of the constituents that have external reference: They are "productive" in the sense that people can, and constantly do, make up an unlimited number of new ones to name new things or new ideas or new acquaintances. It is true that a very few vocabulary words, such as "noun" and "verb", or "pronoun", "adjective", and "gerund", "phrase", "clause", "sentence", "discourse", refer to objects inside language, but such objects are treated as if they were outside it.

What is more, vocabulary words are constantly being invented to name new things. Every new dance, such as the Macarena, every new color, such as mauve, every new device, such as the transistor, every new everything, gets a name. If we had a number system without a base, where every new number simply was assigned a new name, more than just memory would be taxed. Each new word would have a new phonetic shape, and there are only just so many of these available in a language, even if their number far exceeds two-hundred-thousands, which it does. Under that kind of number system, it would be vocabulary that was driven out of language. Here is another intimate relationship between language and mathematics.

Numbers are the vocabulary-words of arithmetic. I don't mean this as a cheery metaphor. Numbers occupy the same role in linguistic deep-structures as any other nouns. To be sure, numbers have structure of their own, some of it fascinating. There are prime numbers that are divisible only by themselves and 1. There are squares and cubes. Squares can be sums of squares, but cubes can't be sums of cubes. There are irrational numbers that can't be expressed as the ratio of any two integers. And most mysterious of all, there are transcendental numbers that aren't the solution to any equation with integer coefficients. Christian Goldbach's wildly fascinating conjecture proposes that every even number is the sum of two prime numbers; and up to more than a million, so my mathematician friends tell me, one of the primes can be under 11 in more than ninety-nine percent of cases.

Everybody knows that complication is introduced into systems when there is a change of modality or of coordinates. Celestial navigation becomes complicated when you convert from the spherical coordinates of Earth and sky to the rectangular coordinates of flat charts and maps. The number π (Beckmann 1971) is simple when expressed in geometry, as the ratio of circumference to diameter of a circle, and becomes mysterious only when converted to digits. I will make the conjecture that every transcendental number represents some relationship that is simple in geometry, and becomes complicated only when converted into digits, which are mostly incompatible with geometry. When number and geometry coincide, it is a special case. But at the simplest level, the level where arithmetic intersects with ordinary language, the distinctive feature of numbers is that they can refer to ordinary objects such as cars or episodes or ideas, that are outside arithmetic. Albert Einstein (1922, page 28) asks, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?" External reference marks the beginning of that relationship. Later we will see that human thought is mathematics in an altered form.

The Non-Arbitrariness of Mathematics

Is mathematics an arbitrary human invention, or is it somehow given – part of the natural universe? Because if mathematics is completely arbitrary, it must be of biological origin. But if mathematics is not arbitrary, i.e., if it is more found than made, then it is the consequence of some natural law whose character we might discover. If we find that mathematics is arbitrary, then it must be a consequence of evolution, even if indirectly, and the results of the previous chapter will be refuted. But if we find that mathematics is not arbitrary, the results of the previous chapter will be confirmed, and we may be sure that mathematics is a direct consequence of some profound property of matter.

Reuben Hersh (1977), my favorite mathematician, thinks that mathematics is primarily a social activity, and, by implication, arbitrary. It works because everybody agrees to agree. In this, he is like the language scientists, who see language as a social activity. They are right, of course, but that does not address the foundations of such systems. Every now and then, the idea re-emerges that mathematics is an out-and-out invention, like Cubism in painting. In another life, it might have been Impressionism or Pointillism. No matter. The discreteness of the symbols is borrowed from the discrete rocks and animals and trees that our early ancestors saw around them. The linear ordering comes from only being able to think about one thing at a time. The remarkable correspondence that exists between mathematics and the natural world happens because we somehow tinker the math down until it fits. If we had twelve fingers instead of ten, the number system would be completely different. Pi wouldn't be 3.14159. If there were extraterrestrial intelligences, we couldn't even send a radio message to them with

- - -   - - - -   - - - - -

(the sides of a right triangle) to tell them we know geometry, because (with their different number of fingers) they wouldn't have 3-4-5 right triangles. And the real proof is in the symbols. Take the figure "5" for instance. It could as easily stand for "three" as for "five" or any other number. Nothing is more arbitrary than math.

For a lot of reasons, the arbitrarian view is nonsense. We can start at the beginning, with discreteness. The ancients, who supposedly borrowed the idea of discreteness from objects in the natural world, had clear ideas as to what is important. They had four elements that everything was made of, earth, air, fire and water. Not trees and animals and rocks. Those were derived. Earth, air, fire and water. All four are blending materials that can be combined together in the right amounts to make rocks and animals and trees. The ancients decided on discrete numbers, but not by borrowing discreteness from anything. Numbers have their discreteness property for the same reason as the gene: blending numbers would form averages that would not support arithmetic. What if $4 + 4$ always became $(4 + 4)/2 = 4$? That is how blending arithmetic would work. $3 + 4 + 5$ would become $(3 + 4 + 5)/3 = 4$. Multiplication, which is a kind of multiple addition, would collapse. 5×5, which is a short-hand form of $5 + 5 + 5 + 5 + 5$ would become $(5 + 5 + 5 + 5 + 5)/5 = 5$. Arithmetic would be so pointless and futile that no one would do it. A world ruled by such an arithmetic would collapse to a point.

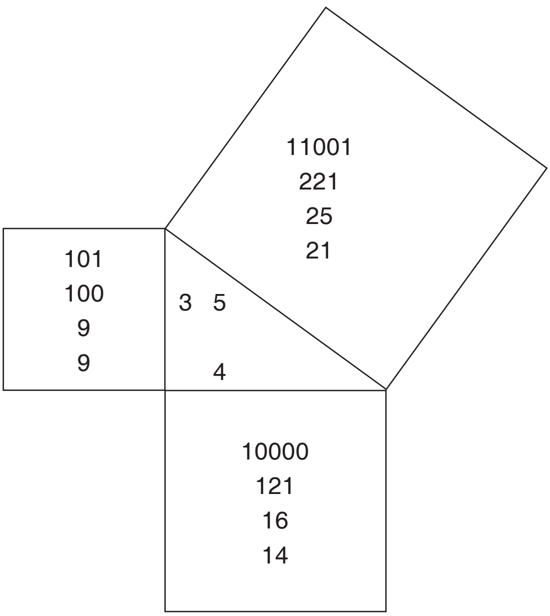
What about the number system? Under the arbitrarian program, arithmetic in base 8 or base 12 would be so different from our own arithmetic (in base 10) that there is no comparison – and only chance gave us 10 fingers instead of 8 or 12. Again, the differences between the number bases is a myth. The whole point of the New Math movement of the 1950s was to show that underlying mathematical relationships don't depend on the base. Look at the prime numbers, the ones that can only be divided by themselves and 1. 3, 5, 7, 11, 13, 17, 19, 23, to start. 21 can be divided by 3 and 7, but 23 can't be divided by anything. In base 3, or base 4 or something, everything would be different, wouldn't it?

Everything would be the same. Look at the placement of the prime numbers along the number-line.

			3		5		7				E		11				15		(12)
			3		**5**		**7**				**11**		**13**				**17**		(**10**)
			3		5		7				13		15				101		(8)
			10		12		21				102		111				122		(3)
			11		101		111				1011		1101				10001		(2)
0	*	*	^	*	^	*	^	*	*	*	^	*	^	*	*	*	^	*	base

Prime Numbers

They are the same in any base. The digits that express the number π are different in base-8 (3.11037) from what they are in base-10 (3.14159), but π still occupies the same place on the number-line. Look at the sides of a right triangle.



The geometry stays the same no matter what the number system. From the top: base-2, base-3, base-10, base-12.

Here is a puzzle, given to me by a friend. "Suppose you have a right triangle with integer-length sides. Show that one of the sides is divisible by 3." To start, you need a flag to show where the threes are. The familiar decimal system, base-10, has no such flag. But if we try base 3, every number divisible by 3 ends with 0. Now we can spot the threes. Numbers in base-3 end with

0
1
2

Squares of numbers in base-3 end with

0
1
1

Sums of squares of numbers in base-3 end with

0
1
2

So, $a^2 + b^2 = c^2$ in base-3 ends with

0 + 0 = 0
0 + 1 = 1
but not 1 + 1 = 2, because squares in base-3 don't end with 2. So, at least one side of a right triangle with integer sides ends with 0, and is divisible by 3. The geometry is the same as any, but only base-3 solves it.

While it would be impossible to make a number system without a base, where every number would have its own name and symbol, we might be able to imagine such a system. To start:

0 1 2 3 4 5 6 7 8 9 @ # $ % ^ & * (" { a b c d e f g
h i j k l m n o p q r s t u v w x y z A B C D E F G
H I J K L M N O P Q R S T U V W X Y Z α β γ δ ε
ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω Γ Δ Θ Λ Ξ Π
Σ Ψ Ω – – – –

Here,

$@^2 = \Xi$ i.e., $10^2 = 100$
$f \times 3 = \delta$ i.e., $25 \times 3 = 75$
$c + \delta = \Delta$ i.e., $22 + 75 = 97$

Again, the basic numerical and geometric relationships remain the same. Goldbach's conjecture, which opened the question of arbitrariness, would remain the same. And a little thought is enough to show that Goldbach didn't decide, "Wouldn't it be terrific to give everybody something to think about. I think I will declare that every even number is the sum of two primes". And the ancients didn't decide "Let's have some magic. Let's declare that some numbers aren't divisible by anything. We can have prime-number parties, and not tell anybody what the prime numbers are." That is backward. If 23, say, were divisible by anything, someone would have noticed by now. It isn't the kind of thing you can stop. It was only after they noticed that some numbers aren't divisible by anything that the ancients lost their heads.

There is much that we will never know, and we must accept it and live with it. As long as you live, you will never know the value of the famous number π, because π has an infinite number of digits. No one may ever know what causes the distribution of the prime numbers. There may be no rule, like the digits of a transcendental number. But the existence of prime numbers, and their distribution, reflect properties of the universe and matter. They are manifestations of natural law, even if we can't know what the law is.

At a higher level, mathematics can't be arbitrary because it is self-regulated. The two sides of an equation have to remain symmetrical.

You can't arbitrarily change one side and still have an equation. Whatever its cause, the relationship of mathematics to physics is intrinsic.

All that is left is the arbitrariness of the symbols. The precise choice of this symbol or that really is arbitrary. But the discreteness property of the symbols, and their basic two-dimensional geometry is not arbitrary. The myth of arbitrariness in mathematics is extrapolated, if I may borrow a term from mathematics itself, from the arbitrariness of the symbols, although in fact arbitrariness is confined to the shapes of the symbols only. Such mistaken ideas, perpetrated by serious adults, are at best peevish, and at worst destructive. We will abandon them immediately. Later, we will glimpse some of the properties that cause mathematics to mirror the material world.

Some of my mathematical friends dislike arithmetic because it is not conceptual enough. It doesn't express the kinds of abstract relationships that represent the fantastic New Physics. They have a point. But arithmetic won't go away. Everyone, even my mathematical friends, needs arithmetic to add up a grocery bill; and they need at least counting to measure out the flour to bake a cake. Any conception we have of actual amount is a matter of counting and arithmetic and small whole numbers. The system that gives mathematics its first and most direct connection to the world outside mathematics is, and will remain, number.

Language and Mathematics Compared

Numbers are productive, like vocabulary words. When Archimedes introduced his famous rule, stating that "A to the n-th times A to the m-th equals A to the n-plus-m-th", he was introducing a way to create large, and in his day new, numbers. "Google" and "googleplex" are the names of new, large numbers.

The vast number of vocabulary words and numbers stands in real contrast to the relatively fixed and relatively small number of constituents that have reference inside their respective systems. For arithmetic, these constituents are familiar as the operators "+", "-", “x”, “÷”. They have meanings of their own, but their shared function is nevertheless to indicate some relationship between two numbers, i.e., between two constituents that have reference outside their system. In this way, the two kinds of constituents cooperate to produce phrases that would not be possible without both kinds. For example ++ is meaningless, as is 555, in the sense of 5+5+5.

The operators are not productive, that is, we don’t ordinarily create new ones. We create “new” numbers all the time, by adding or subtracting them, or multiplying or dividing. But we don’t very often invent new arithmetical operations; and even when we do, they demand lengthy explanation, are few in number, and are short-lived.

Language, too, has a small, fixed inventory of what could be called grammar words, words that indicate relationships between vocabulary words. Like the arithmetical operators, grammar words have meanings of their own, but they indicate relationships between other words or phrases, rather than naming things or actions outside language. For English, grammar words are “so”, “if”, “of”, “and”, “the”. We can look at a few examples. Take “in spite of”. It doesn’t exactly refer to something outside language. You can’t point to one, or think of a good example of one. You can’t just say “in spite of”, and mean something, in the way that you can say “rain” or “stop’. It indicates some meaningful relation between elements of language. “In spite of” contrasts directly with “because of”, and sometimes the two are interchangeable, with opposite meanings.

John went home because of the rain.
John went home in spite of the rain.

In inflected languages, the ones with endings that students have to memorize, the difference between vocabulary elements and grammar elements is even more sharply defined, except that neither system operates as independent words. Usually, a word consists of a vocabulary item attached to one or a few grammar items. You have to memorize the vocabulary items as a list, but the grammar items as tables, because their relationships are very orderly. For example, in Latin verbs.

	Number	
	Singular	Plural
	-o	-mus
Person Tense	-s	-tis
	-t	-nt

But you can't make up new endings, at least not as easily or as successfully as vocabulary words. And new endings would be nowhere near as much fun as new vocabulary words. Imagine a new form of the first person singular, present indicative active, as compared to a new form of nincompoop. Ask any twelve-year-old. It is no contest.

The outward-looking and the inward-looking constituents of arithmetic and language combine to make meaningful phrases. For the elements of arithmetic, the phrases have two numbers connected by an operator.

25 + 25
25 x 25

The operator has the double function of separating the two numbers, preserving their identities. For example

2525+ or
2525x

is meaningless. But if a dummy symbol is introduced to separate the two numbers, the operator can be safely moved to the end of the arithmetical phrase.

25, 25 +
25, 25 x

Reverse Polish notation, made famous by Hewlett-Packard computers, takes advantage of this property of numbers and operators, as well as one other crucial property: Arithmetic has only one verb, the “equals”. So, you automatically know that there is a verb, and what the verb is. It doesn’t have to be expressed.

Phrases of ordinary language are also put together from vocabulary and grammar constituents. Like phrases of arithmetic, the constituent types don’t exactly have to alternate. Well-known words don’t need markers to signal their beginnings and ends, so there is more flexibility than with arithmetic.

The face in the mirror
Over a hundred pounds
When everybody is ready
Due to the flood
On Sunday
In a jacket and tie

Arithmetic and ordinary language share the curious properties of assertion, double-meaning (or “ambiguity”), paraphrase, ellipsis, and translation. They are associative, commutative, and distributive. We will look first at the crucial property of assertion.

Phrases express clear ideas, but make no assertions about them. To say the same thing in mathematical terms, phrases have no truth-value. They can’t be true or false. It is only when a main verb is added, linking a subject and a predicate, where truth or falsity becomes possible through the creation of a sentence that makes an assertion. For the most part, the meaning of most sentences somehow involves an assertion, or something derived from an assertion. Where some sentences make an assertion (indicative sentences),

“The cow jumped over the Moon”

other sentences ask for an assertion (interrogative sentences)

"Did the cow jump over the Moon?"

or deny an assertion (negative sentences)

"The cow didn't jump over the Moon"

or express some wish concerning an assertion (subjunctive sentences)

"If only the cow would jump over the Moon" (Optative)
"If the cow had jumped over the Moon" (Contrary-to-fact)
"Let the cow jump over the Moon" (Hortatory)

Equations, too, make an assertion, when they have a main verb. In the case of equations, you never have to wonder what the main verb is. It is always "equals", represented symbolically by the familiar "=", the "equals symbol" or "equals sign". There are a few alternate forms, such as "makes" or "is", but they ultimately amount to "equals". That is why reverse Polish notation works. If I write

"2, 2 +"

you know that, to make an assertion, something that can be true or false, there has to be a verb and, in arithmetic, it has to be "equals". So,

$2 + 2 = 4$	and	2, 2 + 4	are true, but
$2 + 2 = 6$	and	2, 2 + 6	are false.

Sentences of ordinary language can be true or false, too, but they can't be checked against themselves. Rarely are they intrinsically true or false. For example.

"John is a mathematician" (True, if he really is a mathematician)
"The cow jumped over the Empire State Building" (Wildly improbable)
"I will un-tie the knot within five minutes" (Might turn out to be true)

The truth-value of ordinary sentences has to be checked against external reality, while that of equations is intrinsic, and automatically fits with some aspects of external reality, i.e., the aspects that correspond to numbers and quantities. We will return later to the reasons for this mysterious correspondence.

The power of assertion, or of making statements that have truth-value, is not a descriptive detail of arithmetic and language, but is in some important sense the foundation of a human society. Imagine not being able to say

"I trust John"	"I don't trust John"
"That water is OK to drink"	"That water isn't OK to drink"
"John is innocent"	"John is guilty"
"I can finish this work"	"I can't finish this work"

There could be no public deliberations. No agreement or disagreement. No laws. No promises (or threats) of the kind that hold communities together. No experience beyond our own. No science, because science has to be expressed as assertions. "It is true that the sun is a star." "It is not true that snakes milk cows." No superstition, which amounts to folk-science.

"It is bad luck to see a black cat"
"If you have long, thin fingers, you will grow up to play the piano"

Apart from sentences, i.e., sentences of language and sentences of arithmetic, nothing else has the power of assertion. By directing other bees to flowers, a honey bee is not asserting, "The flowers are one-hundred yards directly into the sun." The bee is re-enacting its foraging flight. By wagging its tail (or snarling), a dog is not asserting "I am glad (or not glad) to see you." The dog is acting out its emotional state.

Later, we will be able to discover what property of sentences it is that gives them the power of assertion. But for now, we must content ourselves with noticing that all sentences have a linguistic deep-structure. There have been advances in the representation of deep-structure since Noam Chomsky published the first one in 1957 (page 27). But for several reasons, I will use the most rudimentary form. First, the advances, while genuine, seem (to me) like fine-tuning. Attaching some of the branches to words; starting the trees lower down. Nothing radical or revolutionary. And there is a conviction that we should give credit to the first expression of a good idea. But most important, the idea of treating equations as sentences forces us to start over. Back to square one. Rather than start in the middle, with advanced transformational theory that will have to be changed later anyway, we can acknowledge a new beginning by going back to the first deep-structure. In the end, it isn't such a bad one, and will reveal much about the structure of arithmetic.

It is true that Noam Chomsky has dropped the name "deep-structure" in favor of "D-structure", because people read too much into the word "deep"; and it causes misunderstanding. But "D-structure" has a history (namely "deep-structure") that will have to be explained, so the truth will out sooner or later, along with the misunderstanding. As long as we are going back to square one, we might as well take the old bad name as the new bad one.

Equations share with ordinary sentences "the so-called fundamental laws of algebra" (Mellor 1954, page 177).

Associative:
"The number of things in any group is independent of the order" (ibid). Thus,
"a + b"
is the same as "b + a"
and
"[John met] Tom and Carl"
is the same as
"[John met] Carl and Tom."

Commutative:
"The number of things in any number of groups is independent of the order" (ibid).
"(a + b) + (c + d)"
is the same as
"(d + c) + (b + a)"
and
"[John met] Tom and Carl, Peter and Bob"
is the same as
"[John met] Bob and Peter, Carl and Tom"

Distributive:
"The multiplier may be distributed over each term of the multiplicand" (ibid).
"m(a + b)"
is the same as
"ma + mb"
and
"[John met] Tom and Carl"
is the same as
"[John met] Tom, and [John met] Carl"

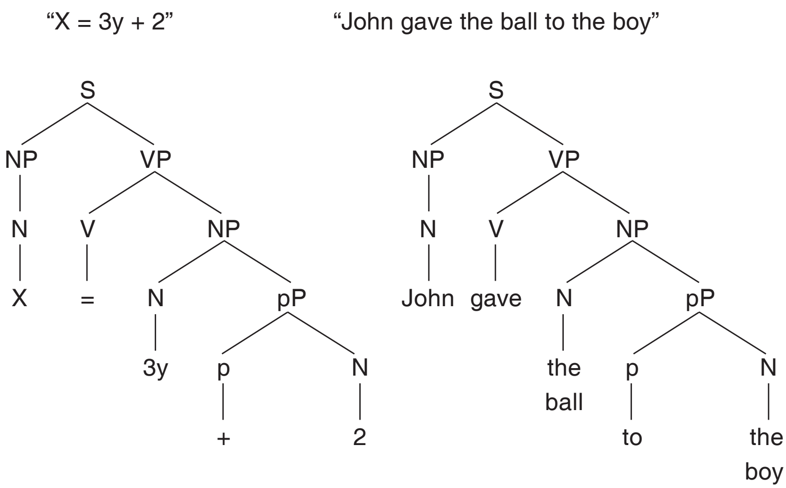
The symmetry property of equations, i.e., if A = B then B = A, is not shared by ordinary sentences, i.e., "John ordered spaghetti" is not the same as "Spaghetti ordered John". The difference is a design-feature of language, which we will examine later.

All sentences, even equations, have a linguistic deep-structure that exposes, at least to some extent, the structural relationships between the parts of the sentence. Deep-structures consist of branching trees, with symbols at the nodes where the branches divide. Everything under the symbol at a node is what the symbol says it is. So, everything under the node marked S forms a sentence. Everything under the node marked NP forms a noun phrase. Using the symbols

S – sentence
N – noun

V - verb
P – phrase
p – preposition

We can construct the linguistic deep-structure of a simple equation, and the deep-structure of a sentence of ordinary language.



To a first approximation, the sentence of arithmetic or algebra has the same deep-structure as the sentence of ordinary language. On this basis alone, we can be certain that they share a common ancestor, or common natural source. The deep-structures have identical properties of hierarchical organization, which includes the embedding of structures into structures, or "recursion".

A look at the property of double-meaning, or "ambiguity" reveals even more about the shared structure of arithmetic and language. Sentences of arithmetic and language can often be interpreted in two or more ways, a property sometimes exploited as a touchstone for grammars: If a grammar has real explanatory power, it ought to assign two different structures to the two different meanings of a sentence.

Double-meaning appears to be intrinsic to language and arithmetic, and probably most or nearly all sentences have it, if you look hard enough. But some examples are easy and obvious.

You can't read too many books.
1) Reading books is good, no matter how many you read.
2) Reading books can be bad, if you read too many.

The difference is profound. In 1) you can read all you want, while in 2) you have to stop.

"Alvarez holds the entire scientific community in awe" [This sentence is due to the late Roger Chaffee]
3) The entire scientific community is in awe of Alvarez.
4) Alvarez is in awe of the entire scientific community.
Again, the difference is profound. In 3), the entire scientific community looks at Alvarez, while in 4), Alvarez looks at the entire scientific community. Alvarez changes from object in 3) to subject in 4).

At the simplest level, double-meaning tells us that sentences are not built out of words in the way that a house is built out of bricks. The same bricks assembled in the same order will build the same house. But the same words assembled in the same order simultaneously build two very different sentences. Noam Chomsky (1957) uses different pronunciations of the same word to make the same point. Pairs like "ration" and "raytion", or "economics" and "eekonomics" show that different speech-sounds, or phonemes, can build the same word, i.e., that words are not exactly built out of their phonemes.

Equations of arithmetic or algebra also exhibit double-meaning. For example,

$$“x = 5 + 3 \times 2”$$

means both

$$11 = 5 + (3 \times 2) \text{ and } 16 = (5 + 3) \times 2.$$

I can remember seeing this kind of ambiguous equation for the first time when I was in sixth grade. I couldn't believe my eyes. The teacher explained that the mathematicians had taken steps to resolve the ambiguity. They decided that multiplication would take precedence over addition. But that just painted over the really fascinating part. Somehow, in the structure of the universe, "$a + b \times c$" has two right answers. Mathematics isn't fixed, or at least not completely fixed. You can't calculate the truth, not even in arithmetic. You still need common sense.

The situation is even worse in algebra, where every equation with x^2 in it has two answers, and every equation with x^3 in it has three answers. Ultimately, every equation with x^n in it has n answers. So, if

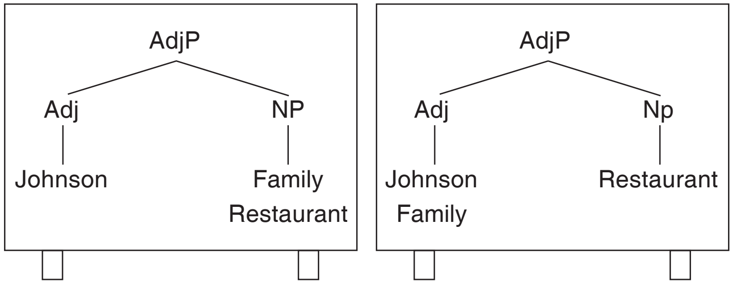
$$x^2 - 48\,x - 100 = 0$$

tells you how much fence to buy, then, in some universe of mathematics, the right answer is –2. Now, nobody would go out and buy –2 feet of fence, but mathematics won't tell you that. Only common sense tells you that the right answer is 50 feet of fence.

Now we can look at the deep-structures of two ambiguous phrases, one taken from arithmetic, the other from ordinary language.

"Johnson Family Restaurant"

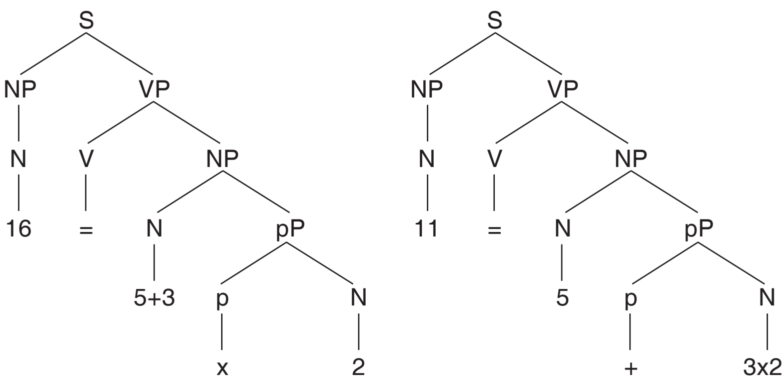
is a sign or even a billboard you might see anywhere in the United States. It means both a family restaurant owned by Johnson, and a restaurant owned by the Johnson family. You can't tell which was intended. If the billboard were written in deep-structure, though, you could tell right away, because the double meaning disappears in deep-structure.



Linguistic deep-structure works the same for arithmetic as it does for language.

"$x = 5 + 3 \times 2$"

has two answers, each with its own deep-structure.



The branching tree of linguistic deep-structure represents a new notation for the equation, and is interesting just for that reason alone. Beyond that, some equations that have double-meaning under the standard linear form, have single-meaning under linguistic deep-structure, which has two dimensions instead of one, and more information. I don't know why the deep-structure form of the equation isn't easier to work with than the standard linear form, in spite of carrying more information, but later we will be able to venture a guess.

Double-meaning in arithmetic gives us an opportunity to see what arbitrariness in mathematics really looks like. I mentioned earlier that the mathematicians used an arbitrary decision, the precedence of multiplication, to prevent equations such as

$$\text{“}x = 5 + 3 \times 2\text{”}$$

from being ambiguous. Here we can see the mathematical mind at work. The mathematicians don't want ambiguity. They are interested not in how equations work, but in finding and solving equations. If mathematicians could get rid of ambiguity in their material, they would. The persistence of ambiguity in mathematics, in spite of the mathematicians' obvious dislike for it, shows that the mathematicians did not make up their system. If they could, they would make one up without double-meaning. Mathematics emerges once again as given, more found than made.

It is important to notice that every equation can be written, and is always spoken, in linear form, i.e., one symbol at-a-time. Even an equation that looks completely tangled is really linear. For example,

$$X = \frac{- b \pm \sqrt{b^2 - 4ac}}{2a}$$

looks pretty three-dimensional. But when you read it out loud, it is linear:

"X equals minus b, plus-or-minus the square-root of b-squared minus four a-c, all over two a."

And it can be written in linear form, if necessary.

$$X = [- b \pm \sqrt{}(b^2 - 4\ ac)]\ /\ 2a$$

The ambiguity of quadratic equations can not be removed by linguistic deep-structures, and is of a more profound, structural nature.

The improved information content of the system with more dimensions, i.e., the linguistic deep-structure, makes you wonder what would happen if a three-dimensional system were possible. But it also verifies the reality of the two-dimensional deep-structure. If information were somehow lost when equations are re-represented in their deep-structure form, we would wonder whether deep-structure is the right thing. But, since they gain information, we can be sure that it is the right thing. In that case, the task of the language-user or mathematics-user is to reconstruct the underlying deep-structure by starting from the linear surface-structure. Since the information contained in the deep-structure form of sentences does not remove double-meaning entirely, sentences and equations are not built out of their deep-structure components, either. Even more interesting, both language and arithmetic have what we can call separation of levels, i.e., knowing the organization of constituents and constructions at one level of organization tells us nothing about the organization of constituents and constructions at the next level. It means that the final interpretation of a sentence or equation is not determined absolutely by the sentence or equation. The sentence (or equation) itself gives only a framework to start from. Beyond that, even a mistake in the interpretation of one level does not mean that our interpretation of a different level is also mistaken. Arithmetic and language are not an edifice in any sense.

Paraphrase is another curious property of all sentences, both ordinary and arithmetical. Under paraphrase, it is possible to say the same thing, or communicate the same meaning, by the use of symbols that look (or sound) very different. Paraphrase in ordinary language is famous. For example.

The Wright brothers were the first to make a successful aeroplane. Wilbur and Orville anticipated everyone in building a heavier-than-air flying machine.

The two sentences share one word (“a”) in common, but mean about the same thing. It is uncanny. “Wilbur and Orville” is another way of saying “the Wright brothers”. Everybody knows Wilbur and Orville. “Flying machine” is another way of saying “airplane”, and everybody knows that, too. “Anticipate” can be used to mean “first”, in the sense of “before anyone else”. Nothing in animal communication is anything like paraphrase. There is substitution. For example, honey bees have substituted the direction of gravity for the direction of the sun, when they do their communicative dance in the dark interior of their hive. But for insects, up is ordinarily the direction of the light, so the substitution is direct, gradual, and iconic. You can see a smooth developmental path from one to the other, and Martin Lindauer has demonstrated just such a path in evolution - from dancing on a level surface using the sun as itself, to dancing on a vertical surface in view of the sun, to dancing on a vertical surface with no view of the sun (Lindauer 1961; von Frisch 1967; Wilson 1971). But paraphrase is a different story, where the connection between the mutually similar sentences is outright mysterious. The path from “The Wright Brothers” to “Wilbur and Orville” is neither smooth nor developmental.

Arithmetic has paraphrase, too, i.e., you can say the same thing in ways that are physically very dis-similar. For example,

“$2 + 2$” has the same meaning as “$10^6 - 999{,}996$”

The two expressions don’t look (or sound) the same at all. One uses small, single-digit numbers with addition, while the other uses subtraction with big, multiple-digit numbers, one of them with a fancy exponent. But both expressions mean “4”, and can be substituted for it in any equation, or even in a grocery list, if you like doing arithmetic. Here, again, at the place where language and arithmetic intersect, they share a profound and mysterious property that is unknown elsewhere. Other examples are “3/4” and “0.75”, or “12:45” and “quarter-to-one”, or “$\sqrt{10}$” and “$10^{1/2}$”.

Translation might be understood as the extreme case of paraphrase. Saying the same thing in a different way in a dialect so different that the two dialects aren’t mutually comprehensible. We could imagine paraphrase between related dialects becoming so stretched and attenuated and rarefied that it eventually has to be called translation. After a couple thousand years, the one-time dialects don't even seem related, and mutual intelligibility has to be called translation.

In ordinary language, translation is so universal that it hardly needs comment. But in a large single-language place like the United States, a few remarks might be interesting. Most parts of the world have several different languages resident near each other, and most people are more sophisticated than Americans when it comes to foreign languages, and their degree of foreign-ness. In Africa, where closely-related languages are everywhere, it might take a speaker of one language three days to learn the language of a nearby town. That is called a three-day language. If you travel a lot, you may have seen, and I have participated in, a kind of language bucket-brigade, where two people want to discuss something, but don't have any languages in common. A chain of speakers can be formed, as short as possible, who speak enough languages to convert the message from one person to the other, and back. It is a game of telephone, with all the attendant dangers, but it is better than nothing. And when the subject is simple, such as the price of a dinner, it works.

We don't think of translation as applying to arithmetic, but it obviously does. The arithmetical equation

"$5 + 8 = 13$"

can be read out in English or French, or in Chinese or any other language, as far as that goes, with perfect intelligibility. Nothing else in nature resembles translation in language and arithmetic.

Ellipsis is a mechanism whereby material is not present physically, but is instead "understood". The sentence

John helped me open the window.

is an elliptical form of

John helped me [to] open the window

Ellipsis is more familiar as part of a mechanism that lets us avoid repetition, and even confusion. So the sentence

John gave Nicholas a frog, and Alice a salamander

is an elliptical form of

John gave Nicholas a frog, and [he (John) gave] Alice a salamander.

Another elliptical sentence is:

Books take double-strike quotes; scientific papers, single.

Architectural construction could never withstand ellipsis. Imagine a house with an elliptical west wall that is not present physically, but is instead "understood". Like double-meaning, ellipsis shows that sentences are not built out of their constituents in the way that a house is built out of bricks. But ellipsis is possible in arithmetic because, in arithmetic, only one verb is possible. All you have to know is where the verb goes. You already know that there is a verb, and you already know what it is. Reverse Polish notation, which we have seen before and will see again, takes advantage of the singularity of the verb in arithmetic to dispense with it. The verb is "understood".

"5, 5 x 25" is reverse Polish notation for, and an elliptical form of, "5 x 5 = 25"

Mathematics can withstand ellipsis because equations are sentences, not physical constructions. Nothing else in nature resembles ellipsis in arithmetic and language.

Last, we must ask what defines the difference between arithmetic and algebra. It is, of course, the variable "x". Algebra has it, arithmetic doesn't. In effect, "x" is the name of a number whose identity we don't know. That identity is flexible, in other words, x can take on the value of any number, depending on the equation where it finds itself. A kind of omnibus proxy. And algebra isn't implicit in arithmetic, even though they are nearly identical in form. The reader will have noticed that the pronoun of ordinary language is the same thing, an omnibus proxy that also borrows identity from its surroundings, except that its surroundings are grammatical instead of numerical. Each in its own context, the x of algebra and the pronoun of language do the same work and, ultimately, are the same thing.

Here we have seen that six profound properties, assertion, double-meaning, paraphrase, translation, ellipsis, and omnibus proxy, characterize arithmetic and language. But they characterize nothing else. They are nowhere treated as important, but when they are shared between two such basic systems as arithmetic and language, they emerge as primitive, formative, fundamental, ancestral, original. Yet there is no emergence of translation, or origin of paraphrase or evolution of ellipsis by natural selection. And ambiguity is supposed to be selected against. Yet all or nearly all sentences are ambiguous if you try hard enough. The only explanation for the survival of ambiguity is that there would be even more if it hadn't been selected against. But it is almost universal, so negative selection must have failed almost completely.

Ultimately, we must ask what ultimate ambiguity might mean. Does it mean that some single utterance, "Huh!" for example, originally stood for every meaning that any sentence might have, and that modern language represents a (mostly superfluous) elaboration of "Huh!"? Not only does separation of levels rule out such a solution, but it is preposterous to suppose that "Huh!" could mean, say, "John says he knows where to find better blueberries" and, at the same time, "Do you really expect me to start a fire with just one hand?". If such differences were communicated by context, "Huh!" had nothing to do with it.

More than that, the arithmetic/language complex emerges as something that has barely been considered, let alone understood. Recursion, the embedding of structures into structures, remains important, but is one important feature among many, not the defining, crowning property of language. After all, it is shared with arithmetic. Language does have a defining property, much more interesting than recursion, but it won't emerge until later. And separation of levels means that there is no single property, such as recursion, that can be the primary object of investigation in any meaningful sense. We are indeed back at square-one. And if arithmetic is given, rather than invented arbitrarily, then language must be given, too, rather than invented arbitrarily, because the two systems developed substantially from the same source; and one remains a subset of the other.

From where, then, does language get the formal resources to express the simple basics of meaning that characterize the majority of sentences, i.e., enough formal variety to serve as raw material for semantic variety? From arithmetic, of course. Because, very roughly, linguistic transformations are arithmetical operations. In addition to the main variations upon assertion (below), there is a kind of whole-sentence modifier in the form of the adverb clauses of time, place, manner.

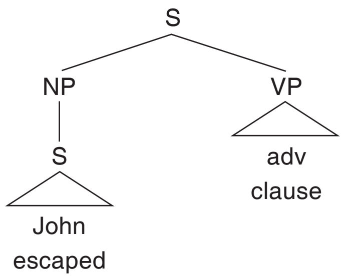
Variations upon Assertion

1) the negative
2) the future
3) the modal
4) the question
5) the subjunctive
6) the passive

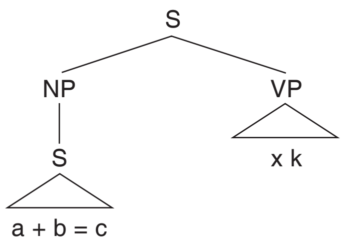
Even the imperative can be expressed as a future: "Thou shalt not kill. Thou shalt not steal. Thou shalt not covet." We can start with the adverb clause, because it is formally separate from the rest, and easy to examine by itself. The adverb clause is physically distinct from the rest of its sentence, and tends to modify the sentence as-a-whole, rather than just one word or phrase.

John escaped through the garden (place)
John escaped during lunch (time)
John escaped un-noticed (manner)

In deep-structure, the adverb clause might be represented as a kind of verb acting upon the sentence, which itself functions as a noun phrase.



Formally, the same happens when an equation is multiplied, or modified, by a constant, i.e., the linguistic deep-structure of k (a + b = c) is



The linguistic transformation that introduces the adverb clause is the same as the arithmetical operation that permits multiplication of an equation by a constant.

Where do linguistic transformations have their source? About the simplest statement worth mentioning in arithmetic is

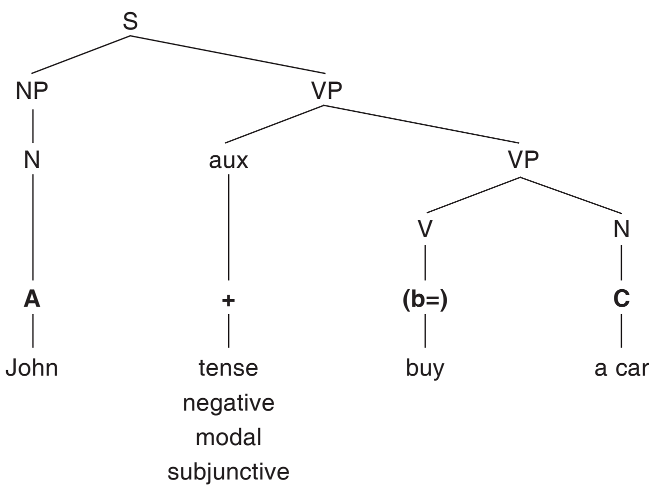
$$A + B = C,$$

so we will start there. In language, unlike arithmetic, the verb can have more than one meaning, so it has assumed a compound form, by fusing the verb function of the "equals" with the referential function of a number, thus.

(B=)

The "+" continues to function as a traffic control that indicates relationships between elements within language, rather than between elements of language and the outside world. Expressed as a sentence of ordinary language, the equation

A + B = C becomes, first,
A + (B=) C, then, in deep-structure,



So, sentences of the form

John		buys	a car	
John	will	buy	a car	(future)
John	didn't	buy	a car	(negative)
John	might	buy	a car	(maybe)
John	would	buy	a car	

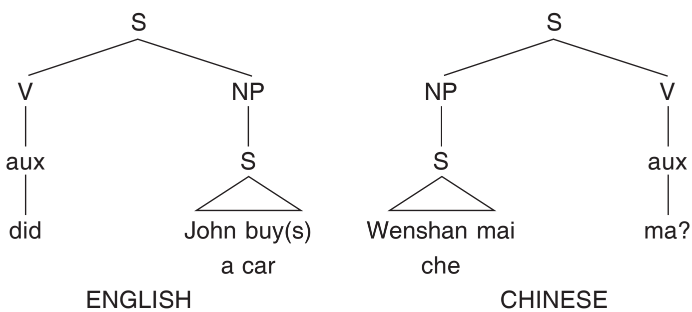
all take their form from the equation “A + B = C”. Apart from the past tense, Chinese forms the same constructions in the same way.

Wenshan		mai che	
Wenshan	jianglai	mai che	(future)
Wenshan	bu	mai che	(negative)
Wenshan	huozhe	mai che	(maybe)
(John)		(buy car)	

Language takes advantage of a flexible structural format to attach several different meanings to a similar number of formal structures. The question, which places the auxiliary at the beginning of the sentence, is close to reverse Polish notation.

$$+ \; A \; (B=) \; C$$

It retains a meaningful arithmetical structure, where the first symbol tells you what operation to carry out on the sentence as-a-whole. In effect, the question is a kind of adverb clause.

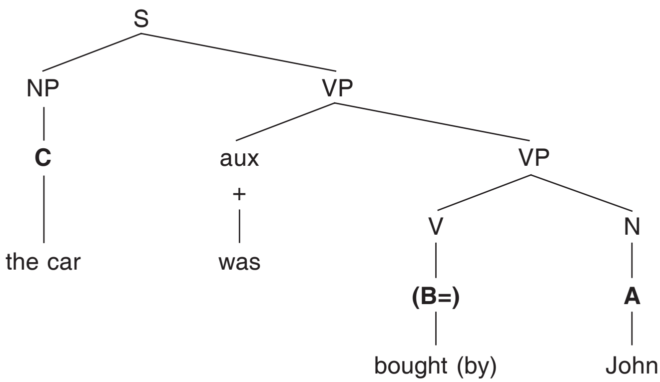


The question in English and Chinese are mirror-image adverb clause constructions. Geometrically, they are about the same thing. Chinese forms the past (with “le”) in the same way that it forms the question.

To a rough approximation, the passive is created by exchanging subject and object in language, i.e., by creating a reversal of roles, where

A + (B=) C becomes
C + (B=) A

an operation like that wouldn't work in arithmetic, with its self-referential truth and falsity. But since truth value is not intrinsic to language, such formal reversal creates no inherent contradiction. In principle, the formal relationships described here extend to the other languages of the world (Baker 2001, page 183).



A speaker of ordinary language could go a long way by attaching a few basic meanings to the forms and formulas offered by the most rudimentary arithmetic. Forms and formulas of arithmetic are the basis of linguistic transformations.

The Infinitude of Language

The vastness of language has been routinely misunderstood. But, here again, the forms and formulas of arithmetic, rather than the application of arithmetic, offer an answer. Ordinarily, the vastness of language is understood as a big number, in more-or-less the way that Archimedes understood "the grains of sand on the beach, or the blades of grass in the field", or Georg Cantor understood infinity. Since you can always embed a new clause into certain types of sentences, language is understood to be, in principle, infinite. For example,

The door swung open and shut and open and shut and - - - -
John laughed and laughed and laughed and - - - -
I know that I know that I know that - - - that the ocean is deep.
Geoff saw Mike and Joe and Peter and Kay and Bob and Jim and Mary and Vicki and - -

While such examples incontrovertibly show that sentences can, in principle, be of infinite length, and if the millionth sentence is considered to be different from the million-and-first, then an infinite number of different (?) sentences is undeniably possible. But people almost never take advantage of the serial embedding mechanism, and with good reason. Even if they did, the millionth sentence would be no different at all from the million-and-first. Even the children's poem "This is the cat that killed the rat that - - - " has a single subject, and is too limited to be valuable in the daily use of ordinary language. Run-on sentences and repeated embedding (Pinker 1994, page 86) fail to grasp the real role of infinity in language. The real infinity of language is much earthier, and much more difficult to characterize, than just a Cantorian infinity. It is also more useful and more fun, but the moral is that you can't solve the equation and calculate language. The infinity enthusiasts (Langendoen et al. 1984) have merely looked where the light is brightest. The real infinitude of language can not be captured so nicely, but it flows from the limitless variety and scope of meanings that are so highly prized and praised by humanists. We will begin there, with some examples.

John placed the red can where he could keep an eye on it.
Why would anyone want to use a chainsaw engine for an ultralight?
Even criminals are cute when they are asleep.
They aren't allowed to start the music before 8:30.
Everyone thought that John was the children's grandfather.
When he wasn't sorting letters at the Post Office, John was solving the mysteries of life.
The baptismal-font salesman's car was stolen in broad daylight.
Walking to the end of the paved road, John caught the bus to school.
Just being American doesn't mean that you understand the Constitution.

It was blackberry season, and the children had blue smudges all around their mouths.

Where do they get all that energy?

The prospector tried to find the deep holes that the old-timers missed.

They both became teen-agers on the very same day.

You can almost see the wheels turning in John's head.

Up past Iron Street and Copper Street, past syncopated houses, the bus climbed higher.

They used to make indelible ink out of some kind of South American nut.

While visiting his aunt, John almost casually solved the mystery.

Eventually, they found the birds crossing the pass just ten inches above the ground.

Did you ever meet Leopold Auer?

No one ever heard of an enzyme that has a uranium atom in it.

Since the house was in the middle of a golf course, it had bullet-proof windows.

May I be stricken dead for a liar, but your socks don't match.

Yes, there were lady pirates, and no, they weren't nice.

Everyone laughed.

The owner was a medical doctor who had studied centenarians in the Andes.

If you use owl feathers, you catch more fish.

Halfway through the season, the Club had won only half its games.

It's amazing what a year of agony with a metronome can do.

How can the lumbering amoeba catch the nimble paramecium?

The inventor tried all sorts of tricks to keep the antenna vertical.

They are a kind of industrial jewelry, accurate to a millionth of an inch.

You hold the knife like this, and break the color.

Sharp boy! Sharp boy!

John had to go all the way to Switzerland to get his steam engineer's license.

If you can't shift soundlessly into second position, John won't take you as a student.

Twenty-five miles an hour is about all you can get out of this rattle-trap.

They needed a focus for their shock, and John's siren provided it.

John felt as if he were looking up at the surface from the bottom of the ocean.
What kind of guests would arrive at four in the morning?
It can be anywhere up to twelve by ten by eighteen inches, and up to ten pounds.
In less than ten minutes, John saved the company from a disaster, a $13-million disaster.
All day long, you could hear the creek, bubbling like voices over the stones.
Commas are related to question-marks and tortoise-shells – what else?
There's the end of the line [rail terminal], and *there's* the end of the line [cemetery].
You walk for 45 minutes, sleep for fifteen, walk for 45, and keep going like that all week.
Why people would jump out of a perfectly good airplane, I'll never know.
No explanation was offered, but the diamond showed up in the mailbox.
The white-hot blade is plunged into a slurry of dry ice and acetone.
There are no secrets in *that* catalog.
Dogs must think that people are a special kind of dog.
If apples are a dollar a pound, what will plums be?
Helped by the sunlight, the tiny bee dried herself out and flew away.
Ohms are not dangerous.
The Henri Fabre sketch was so funny, I laugh out loud every time I think of it.
A peanut-butter-and-bacon sandwich goes great with coffee.
What can you do?
The world expert in mud arrived by charter flight.
They looked as if they were being carried on a phonograph turntable at 3 rpm.
It was too late to go to sleep, and too early to get up.
Can you believe a spider made that?
The letter W is in many ways the most interesting.
Around here, a photographer needs to have an explosives license.
A little bit of that will drive you two-thirds crazy and three-quarters nuts.

John had to climb out the second-story window, the snow was so deep.

If you can't tell the difference, what's the difference?

The stove has to be exactly level to give good fried eggs.

Cellini's autobiography is the model for all autobiography.

You can't get these except as antiques. They don't make them any more.

Sasquatches, sure, they had plenty of those.

Things that ordinarily have nothing to do with each other will interact unexpectedly.

It always takes four times longer than you think.

Between trips, the engine was overhauled by experts.

After that, no one cared.

To succeed on a first attempt was as unexpected as it was unprecedented.

Seventy-five dollars here and fifty dollars there, it adds up fast.

At least he died doing what he liked.

You turn the peg so the string rolls over the top.

Artichoke time comes but once a year.

Imagine seeing your own name on a road-sign on Interstate 101.

OK, well, give me a call and come up and visit.

When it's 12:00 in Chicago, John says it's 9:20 in Paris.

I'll bet you were cute when you were a baby.

It's the dumbest advantage in the world, but just being strong is a huge advantage.

Do you realize it took physicists twenty years to figure that out?

All I can think of is that my brother stole it out of my house and sold it.

Just lucky, I guess.

These shoes don't give you the kind of control you want.

First you push on the door, then you press the latch.

His worst fear was having to unload the truck, and re-pack everything into another truck.

How do you know what John likes for breakfast?

This license ran out twenty years ago.

Under the floor in his dining-room, that's where he hid them, he was so disgusted.

Antonio Stradivari was not a genius; he was a tradesman at the highest level imaginable.

Not those kind of crates, the other kind of kraits.

The label says it's for your digestion, but that's just hangovers from Prohibition.
Go four traffic-lights down, then turn right.
Those things that look like ripples really are ripples.
They could spot one broken thread in a thousand, and tie them back together in a minute.
Sometimes, nice is better than smart.
His picture is turned to the wall.
During the War, the chemists used to make "lemonade" out of sucrose and citric acid.
Children are capable of total happiness.
If you want to hunt for fossils these days, you have to join the water-skiing club.
The hill is too scary to drive, so we always walk.

There you have it. Over a hundred sentences, all different. I apologize for subjecting the reader to lists, but there is no other way than by example to show the real infinitude of language. The sentences are not exactly poetic or lyrical, but they toss our attention first here, then there in ways that are diverse and unexpected. By intuition, we sense that the list somehow exemplifies what is vast and valuable about language. Short, informative sentences on any and every subject. The kind of sentences that drive everyday life. But how do we generalize from an un-structured list to the fundamental workings of the system? How can we actually show that language is infinite in the useful sense?

The answer, again, is that equations are sentences, and the properties of ordinary sentences are shared with those of arithmetic. The great intellectual shock of the 20th century, in my opinion, was not relativity or psychoanalysis, but Kurt Gödel's famous proof (Nagel et al. 1958). The mathematician Paul Erdös is said never to have liked Gödel's proof, maybe because the proof is too iffy. Nonetheless, Gödel's proof shows that any system of axioms will generate theorems in arithmetic whose truth or falsity can not be demonstrated on the basis of the original axioms. And if you add more axioms to prove the old theorems, you will get new theorems, whose truth or falsity can not be decided on the basis of the new set of axioms, and on and on. Gödel's incompleteness theorem shows, among other things, that you can't raise the structure of arithmetic on the basis of logical deduction from a known set of founding truths. Goldbach's conjecture, which we examined earlier, has been verified by running a computer for as long as someone can stand to keep it running. But no one has proved the conjecture or disproved it, and Goldbach may represent Gödel's proof in action. Fermat's Last Theorem nearly did. The incompleteness theorem hints that there may be an infinite number of potential theorems, or statements, in arithmetic. And if the structure of language is adapted from that of arithmetic, then Gödel's proof comes as close as we may ever get to showing that there are, in principle, an infinite number of true (or anyway meaningful) statements in ordinary language.

The interesting part is that theorems of mathematics have consequences for language only because the two systems represent aspects of the same thing. So, if language and arithmetic are so similar, what makes them so different?

Here again, features of arithmetic and language that seem unimportant take their natural place in an organized system without gaps and oddments that demand to be explained away. Such features emerge as both inescapable and fundamental.

Language scientists have long felt that sentences of language somehow focus to a point. Proving it is another matter. But if we borrow from experimental science the method of the single variable factor, proof becomes possible. First, we notice that some sentences are physically symmetrical, at least to the extent that they have the same material on both sides of the main verb. Some sentences of this kind are almost idioms in their own right. Boys will be boys. A dollar is a dollar. But even these cliché expressions carry a profound meaning. Take, for example, "Fair is fair." The first time that "fair" appears in the sentence, it is just a definition. But the second time, it is a moral imperative. That is how the symmetrical sentence-structure works. The first mention of some topic only identifies or defines the topic, while the second mention mines our entire cultural knowledge of the topic, to show how it is in some way very distinguished or very un-distinguished.

Distinguished:
When our family throws a wedding, they throw a wedding. [i.e., very large and happy]
China is China. [i.e., you can't change it]
Paris is Paris. [i.e., fun-loving]
There's lost and there's lost. [i.e., well-and-truly lost]

Un-distinguished:
If I get the [news] paper, I get the paper. [i.e., I don't care either way]

Here we have sentences that are physically symmetrical. Apart from asymmetrical interpretation, sentences of this type would not be worth saying. They are possible at all because the underlying asymmetry in the interpretation of all sentences makes physically symmetrical ones meaningful. If underlying asymmetry were not already there, or given, such sentences would never be produced in the first place.

Sentences of ordinary language, then, are characterized by an underlying asymmetry that is represented (intentionally or not) in deep-structure. Equations of arithmetic or algebra, however, are different. While the truth or falsity of ordinary sentences must be checked against outside reality, equations have a property of intrinsic, self-referential truth-value. How is self-referential truth-value possible?

Equations are true when the material on one side of the "equals" is the same as the material on the other side, i.e., when they are symmetrical. Mathematicians take a very different view of truth in mathematics. To them, an equation is supposed to "model" some geometric relationship; and the equation is "true" when the model is correct. That is a practical view, almost materialism in pure mathematics. But we are interested in why there are equations in the first place, in what causes equations. Back to square one. The only way to obtain self-referential truth-value is by tautology, when the content on both sides of the "equals" is somehow the same. That is what "equals" means. So, to a mathematician,

$$x^2 - 3x + 2 = 0$$

models a parabola. But when it comes to foundations, the equation is possible at all because

$x^2 - 3x + 2$ is congruent with 0.

If there were any asymmetry between the two sides of the equation, we could not check one against the other. It is symmetry, not between the equation and some external geometry, not between the symbols on the two sides of the "equals", but symmetry between the geometry modeled by one side compared to the other, that makes equations possible.

Mathematicians like their mathematics in tautologies, because it gives them the certainty that is never possible in experimental science. Material objects can be understood as analog computers that always give the right answer in a form that can be understood only approximately. They tell nothing but the truth, and worse, they tell the whole truth, while we were looking for more limited answers. Since a true equation is a tautology, we can know exactly whether it is right or wrong, not in relation to the material universe, but one side in relation to the other. Thus our earlier example

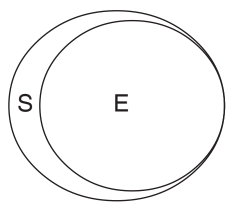
$$2 + 2 = 10^6 - 999{,}996$$

is true because the geometry underlying the two sides is somehow congruent, even if the symbols and operations are very different.

Sets and Symbols

Now the difference between equations and ordinary sentences is clear. Equations are symmetrical; ordinary sentences are not. Before we can examine the consequences of symmetry versus asymmetry, we must organize the properties that we have identified, both shared and not, of equations and sentences. Which came first, or anyway which is primary, the equation or the ordinary sentence? Equations obviously appeared later in human culture than ordinary sentences, but, since equations are found rather than made, that is beside the point. Natural selection, as we mentioned before, is, in principle, always the solution to an equation, or at least the optimization of some function or system of interacting functions. Equations didn't evolve as the solution or optimization of an equation, because that is circular.

There is no escape and no turning back. No sentences, whether sentences of arithmetic or of ordinary language, are in any way a consequence of natural selection. This solution is inorganic and non-Darwinian, but not anti-Darwinian. It affects only those aspects of biology, sentences, that have the same form as equations. It in no way reflects on the evolution of zebras or hummingbirds or blue whales or butterflies, or even the human mind, apart from the formal aspects of language and thought. But it means that the equation is primary, because if we suppose that the ordinary sentence is primary, then it must have evolved by natural selection. And if the ordinary sentence evolved by natural selection, it evolved on the basis of an equation. But equations are a subset of all sentences. We have equations evolving indirectly as a consequence of equations, and perhaps we should make this point very clear. It is true to say that equations are sentences, but it is not true to say that sentences are equations, i.e., all equations are sentences, but not all sentences are equations. Equations (E) are a subset of all sentences (S) even if there are more equations than ordinary sentences. The set-theoretical diagram for such a state of things might look like a slice through a hard-boiled egg.



When we write the equation that accounts for the existence of sentences, we have written the equation that accounts for the existence of every subset of sentences – and that includes equations.

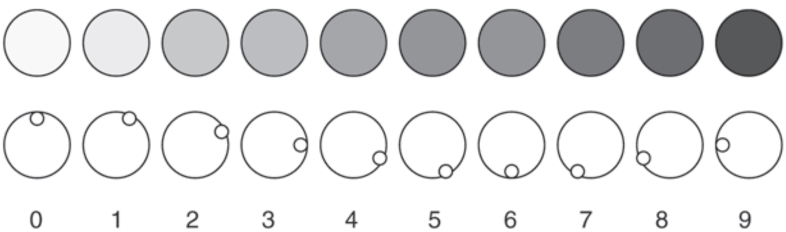
What is more, it is the equation, and not the sentence of ordinary language, that is primary because the equation is the ruling principle. Ordinary sentences are derived from sentences of arithmetic by somehow warping them out of symmetry.

The Russian mathematician L. Tarasov (1986, page 10) defines symmetry in this way: "An object is symmetrical if it can be changed somehow to obtain the same object", and attributes the definition to Hermann Weyl's 1952 classic book, *Symmetry*. The superb definition, however, is not to be found in Weyl, and must be due to Tarasov after all.

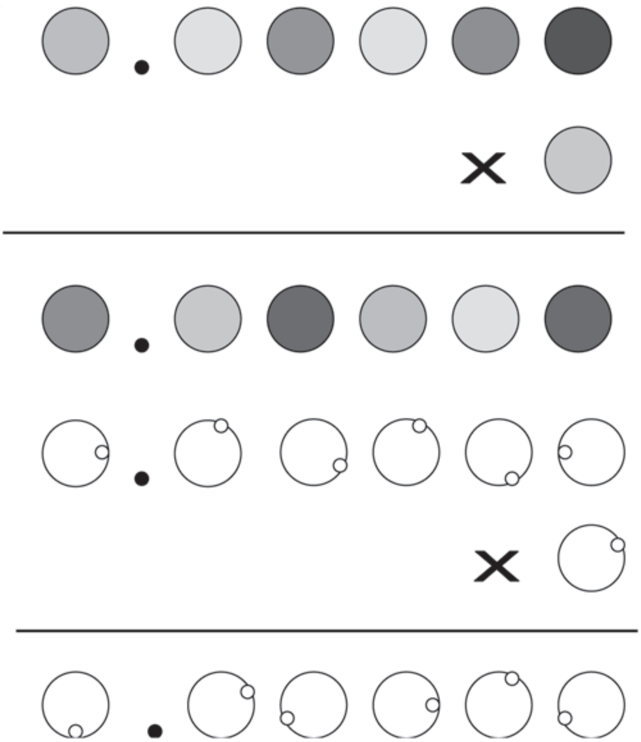
The removal of symmetry is one of the fundamental ways that living things make materials their own. If an organic molecule actually appeared in two mirror-image shapes, as many can, our metabolism would have to maintain two sets of enzymes to handle them. A living thing with left-handed molecules alongside right-handed molecules would be a kind of biochemical chimera, two mirror individuals living under the same skin. It would have to maintain two equivalent but incompatible metabolisms, which is very inefficient. So, most biological molecules exist in only one of two possible mirror-equivalent forms. After examining the genealogy of the properties of equations and sentences, we will return to the consequences of asymmetry in ordinary language. But before we can do that, we must examine the two lower levels of organization in the structure of arithmetic and language, phonemes and morphemes.

For some reason, many people don't want to hear about phonemes and morphemes, and a few are outraged at the thought. Phonemes and morphemes represent the "low" end of language, the underbelly. They are somehow not for higher thinkers, and I have seen a famous speaker at an important conference make unconscious sweeping motions with his right hand, trying to sweep them under the rug as he said, "We have those pretty much under control". But, as we will see, phonemes and morphemes lie at the basis of human mind and culture, and more than deserve our consideration. Up to now, we have been looking at arithmetic/language from the top down, but now we will have to look at it from the other direction. And here, language has much to teach us about arithmetic, so we will start with language.

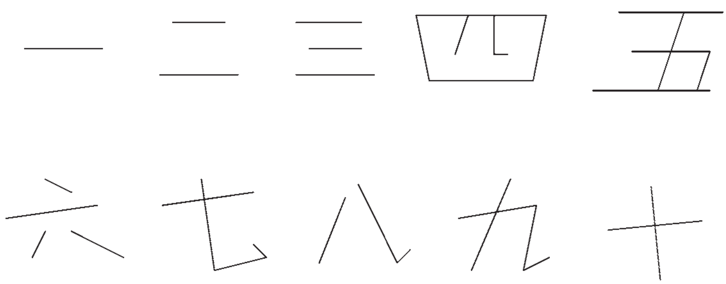
At the lowest level of its organization, naturally-occurring human language consists, roughly speaking, of discrete, distinctive speech-sounds, the phonemes. Phonemes correspond, roughly speaking, to the letters of alphabets, since the letters are also distinctive and discrete. Earlier, we calculated the consequences of a blending arithmetic. There would be no differentiation. A world ruled by a blending arithmetic would collapse to a point. If there are such worlds, they are not inhabited, because they would not support physics, not to mention natural selection. But it is also instructive to see the consequences of blending, so I have constructed two versions of the familiar decimal digit system.

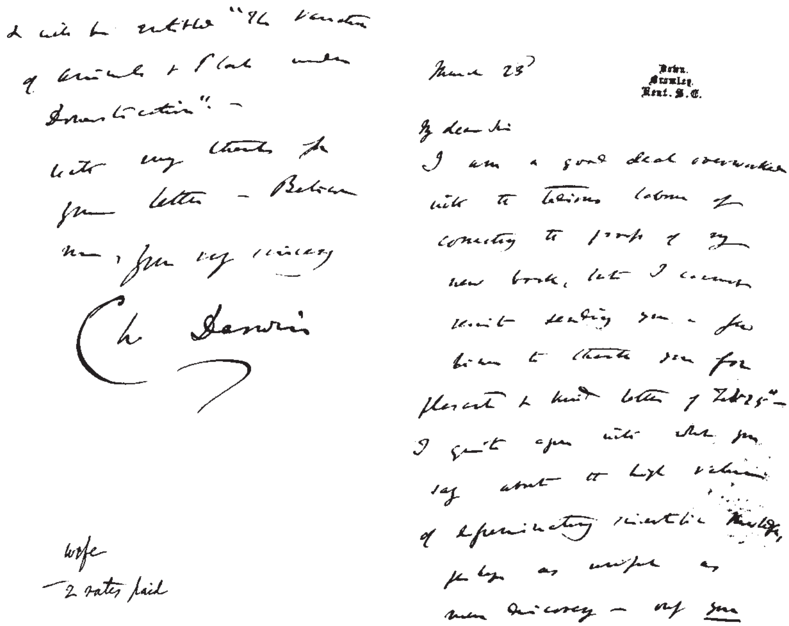


Like the familiar Hindu/Arabic digits, and like the letters of the familiar Roman and Greek alphabets, these digits are geometric and discrete. But they differ from any familiar letters or digits in that they represent degrees of variation along a one-dimensional scale. The first scale is based on degrees of darkness; the second is based on degrees of rotation. You can identify the digits, one from the others – but not very easily, and not very well. If we had to figure our income taxes with numbers like these, or if the newspaper were written with comparable letters, everyone would be illiterate and innumerate. To obtain a hint of what blending literacy (or blending numeracy) would be like, let us try a little arithmetic. To be useful, we will take the expression for the circumference of a circle, 2π.



It is no wonder that people have arrived at geometrically distinct, two-dimensional digits and letters for their writing systems. Just to show what I mean, here are the digits as they are written in Chinese, where many readers will see them as geometric shapes rather than just as numbers.




March 23d

Down. Bromley. Kent. S.E.

My dear Sir

I am a good deal overworked with the tedious labour of correcting the proofs of my new book, but I cannot resist sending you a few lines to thank you for pleasant & kind letter of Feb 25th — I quite agree with what you say about the high value of disseminating scientific knowledge, perhaps as useful as new discovery — My

& will be entitled "The Variation of Animals & Plants under Domestication". —

With many thanks for your letter — Believe me, yours very sincerely

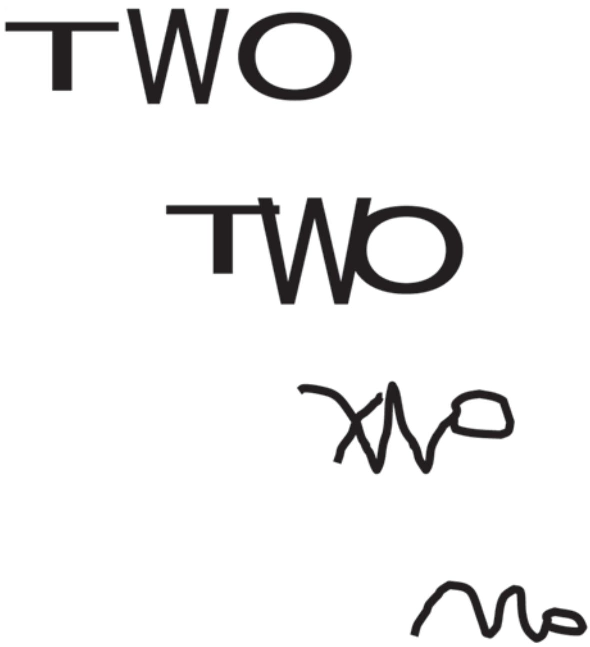
Ch. Darwin

wife
— 2 rates paid

Darwin Letter

Blending can be compared to the mixing of water with ink, where any proportions are possible, anywhere from a drop of ink in a gallon of water, to a drop of water in a gallon of ink. The result is easy to visualize.

With written symbols, the result of blending one symbol with the next is illegibility.

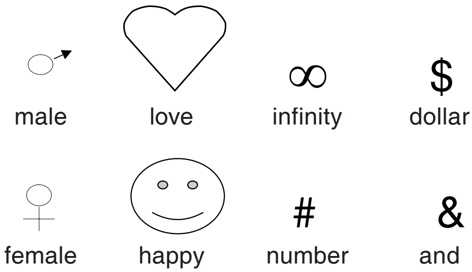


Charles Darwin's famously illegible handwriting was illegible because the letters were blended too much with their neighbors, and with a straight line (Courtesy of The Field Museum, Library Special Collections).

The discrete system that stands at the uttermost limit of intelligibility is the Chinese grass character (Hubei 1994; Abler 1977). Chinese characters have been so widely misunderstood, that the world is almost schizophrenic about them. While some scholars, trying to appear learned, call them pictograms and ideograms and logograms, others are convinced that they represent syllables. And I once watched a prominent language scientist, seated at a desk, tap his forehead on the desk, twice, to emphasize the finality of his conviction. "I'm sorry," he pronounced, tapping his head on the desk once, "they're syllables", tapping his head on the desk again. Those who use them and love them call them characters – and nobody calls them "syllabograms".

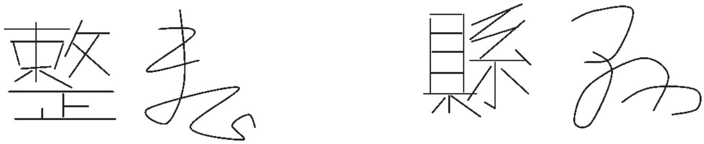
The Chinese language, having only some three-hundred-odd syllables, is filled with homonyms, and the characters offer, at least in writing, a way of distinguishing one from the next. The fact that the same syllable can be represented by several different characters with as many different meanings shows that the characters aren't syllables at all: If they were, each syllable would get only one character. English spelling does for English homonyms exactly what Chinese characters do for Chinese homonyms, by providing a written way of telling them apart: "To", "two", and "too", for example, or "pare," "pair," and "pear". The spelling tells you outright what word you are looking at, so you don't have to spend effort looking at context to help you figure it out.

Chinese characters represent morphemes, the minimal meaningful elements of language, and are possible because the morphemes of Chinese are dead-easy to identify. Unlike English, Chinese has almost no fading morpheme-like constituents such as the "sn" in "snoot", "sneeze", "snorkel" – which might mean something like "nose", or might not have any meaning in the conventional sense. Even English uses symbols that are perceptually identical to Chinese characters, when the underlying morpheme is pitifully obvious. My personal favorite is " Ψ" for "psychology".



Like a logarithm with its characteristic (its power of ten) and its mantissa (the left-overs), every Chinese character has two parts, a radical and a phonetic. For purposes of looking them up in the dictionary, each character is classified first under the number of strokes in its radical, then by the number of residual brush-strokes. Since there are exactly 214 radicals, the Chinese writing system is, in principle, a kind of ragged number system with a base of 214. The standard characters are drawn with distinct, distinctive, and clear strokes. The characters are recognizable because they form distinctive, two-dimensional geometric shapes, like the letters of alphabets. Symbols in mathematics have exactly the same perceptual status as the Chinese characters – one to every morpheme.

But the grass characters are in a class by themselves. While each grass character is inspired by the standard character for which it stands, there is no rule or direct connection. You can't get the knack. They are impressionistic sketches that you have to learn, one-at-a-time. Grass characters are like music: you have to practice them every day. They are mostly for the rich, who have the time for this superb luxury. Even great scholars can't read them, unless they already know what they mean. Since each one is unique, grass characters represent the closest thing in human culture to a number system without a base. Some are elegant, some are brutal, some are indescribably graceful. All are instructive, but I will show just two examples here, of grass characters with their corresponding standard characters, for comparison. Grass characters are as close as you can get to a blending system without becoming a blending system. Its five-thousand-or-so shapes have to be memorized outright.



Where the standard characters are a number system with base-214, the grass characters are, in effect, a number system without a base, and show the limits to such a system. As a practical matter, only a few people could remember and use a number system with a base of 3,000. But no one could handle a number system with a base of 20,000, the highest number that is engraved on slide-rules, not to mention the fantastically huge numbers that cryptologists and astronomers use routinely today. The Periodic Table of the chemical elements is a number system, but it is periodic, and the periods represent the numerical base. Nature itself seems to require a numerical base, and it averages out to about ten, just like our own. Once again, apart from this discrete symbol or that, number and mathematics emerge as given, part of nature, and not arbitrary at all.

Blending Systems

To get an even better look at the properties of blending systems, we can examine, very briefly, the three great blending systems of the world – geology, the ocean, and the weather. Geology seems to be a sharp-edged system, but viewed in perspective, and in spite of apparent diversity, it emerges as a vast blending system. The Earth, to a first approximation, can be understood as a drop of liquid in three layers: Molten metal on the inside, molten metal-oxide in the middle, and a crust of frozen oxide (rocks) on the outside. And there is a limit to how high a mountain can get. If it gets too high, gravity will pull it back. A smaller planet with weaker gravity, such as Mars, can have higher mountains than Earth (Carr 1981, page 95).

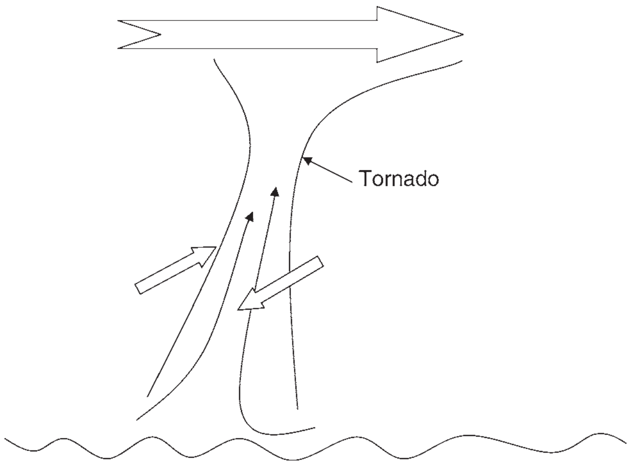
The geology of the Earth itself has only a very few tricks. Hill and valley, plain and mountain, river and continent, features in geology are fractals (Whitted 1982; Rodriguez-Iturbe et al. 1992) whose variation is confined to a single dimension, that of scale (Mandelbrot 1983). Continuous variation along one or a few dimensions is the very definition of a blending system. In spite of panoramic grandeur and local fascination, geology is a blending system. Hills, plains and valleys don't form combinations with one another to create something with properties beyond those of hills, plains, and valleys. Not the way atoms do.

The ocean is the next great blending system of the Earth. The constant, quasi-orderly motion of ocean waves, like the flames in a fire, or the shadows on a TV set, are endlessly fascinating. But the waves in the ocean are a blending mechanism that is capable of only limited differentiation. Waves on the surface of the ocean act as if they were independent entities that can pass right through one another. The wave acts as if it were a cylinder of water that lies along the surface and extends beneath it. When the wave gets close to shore, its cylinder hits the bottom and pushes the top of the wave higher. When the top gets so high that it starts to tip over and roll, that is surf. Sometimes, so I have read, wave heights will add up just so that a vertical wall of water will form at the base of a watery ramp. The addition of waveforms permits such a thing. Even a large ship caught on such a ramp will slide down it and plunge into the vertical wave ahead, possibly a hundred feet below the surface. At that depth, the ship is crushed.

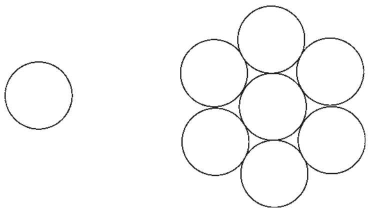
Even if such waves do form, they are still just waves. Waves do not make associations with other waves in the formation of something with properties beyond those of waves. Not the way atoms form associations with other atoms in the formation of something with properties beyond those of atoms.

The last and most differentiated of the Earth's great blending systems is the weather. Apart from warm fronts and cold fronts, weather generates the most fascinating and structured of the blending mechanisms, violent rotating storms. Gregor Mendel (Iltis 1932), better known for his interpretation of biological inheritance, was once caught in a freak tornado that swept through Brunn. Mendel, who was hiding under his bed, was nearly killed by a roof-slate that buried itself in the wall beside him. But afterward, Mendel interviewed everyone in the town, and made a map of their reports. His interpretation of the cause of the storm is substantially identical to the modern understanding.

A violent rotating storm (Bowditch 1977, page 881; Battan 1961) forms when a column of hot air rising from the surface is made to rotate by the rotation of the Earth, and a high-altitude wind draws the top of the column away from the rising air beneath it.

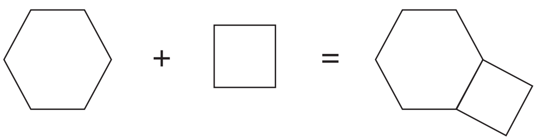


Over land or sea, violent rotating storms have a place by themselves in human experience. The Defense Mapping Agency Hydrographic Center (Bowditch 1977, page 890), not given to the use of florid language, says, "rarely does the mariner who has experienced a fully developed tropical cyclone at sea wish to encounter a second one. (page 902) The ever-stronger wind shrieks through the rigging. As the center [of the storm] approaches, the rain falls in torrents. The wind fury increases. The seas become mountainous. The tops of huge waves are blown off to mingle with the rain and fill the air with water. Objects at short distance are not visible. Even the largest and most seaworthy vessels become virtually unmanageable. Navigation virtually stops as safety of the vessel becomes the primary consideration. The awesome fury of this condition can only be experienced. Words are inadequate to describe it." But the tornado is the extreme limit of differentiation for a blending system. There are no polytornados that form living cells. There are just very sophisticated storms. Nothing comparable to the differentiation possible with combinations of atoms, with their property of discreteness that becomes the basis of molecular geometry.



An example of structures that have emergent properties is the stacking of circles into a hexagon. A circle does not have hexagons implicit in it, yet a stack of circles generates a hexagon. The properties of more differentiated structures are not somehow encoded in their lower-level constituents, but are truly emergent. This rule holds true for language and arithmetic as much as it does for circles.

The polygon also demonstrates the ability of the simplest discrete shapes to change geometry in powerful ways as the result of the simplest operations.



With a particulate, or discrete system, two elements with multiple radial, rotational, and bilateral symmetry combine to form a single, new shape with only one kind of bilateral symmetry. Here, it is the geometry of the surface, rather than the composition of the interior, that is important. The original constituents retain their original identities, and can be retrieved intact after combination. With discrete, or particulate geometry, the powerful use of simple means is canny and uncanny.

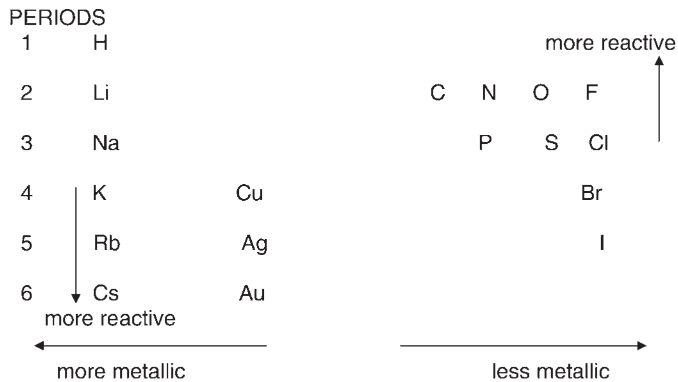
The necessity of discrete constituents is as important for language as it is for the gene. Even though many language scientists triumphantly avoid the study of writing as a mere arbitrary invention, we see that the possibility of writing is an intrinsic part of language whether writing is discovered or not. Writing is not innate, but is deeper than innate. The inevitable discrete units of language can always be represented by discrete symbols of writing.

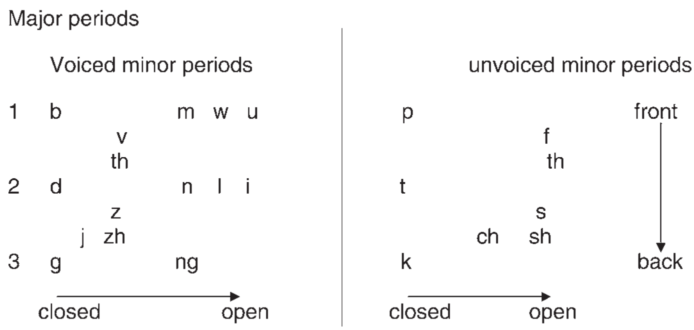
What is more, we can see that Wilhelm von Humboldt's (1836, page 70) phrase "infinite use of finite media", made famous by Noam Chomsky, has misled the imagination of millions. Remember the ten-foot-high wave on the surface of the ocean. Since it might have been ten feet plus one inch, or an infinite number of possible heights in between, since water and ink can be mixed in an infinite number of proportions, "infinite use of finite media" applies as much to blending media as to discrete ones. But blending media show variability along one or a few simple dimensions. Temperature. Speed. Brightness. Direction. That and humidity about sum up the weather. A thermometer, for example, might read zero degrees or a hundred, or any of an infinite number of possible readings in between, but its infinity is confined to temperature. It could tell nothing about the Roman Empire, say, or Anthony van Leeuwenhoek's method of grinding lenses. Just temperature. In 1836, when Humboldt published "infinite use", there was no notion of discreteness in the study of language. We will return to the question of who knew what, and when did they know it, later.

Much more important than "infinite use" is Humboldt's (1836, page 67) "properties not present per se in any of the associated constituents". Humboldt's phrase is an early mention of what we would now call emergent properties. The two phrases appear a few pages apart in Humboldt's book, and he draws no obvious connection between them. Rather than finite media, the language scientists should have concentrated their efforts toward discovering how natural systems can generate "properties not present per se in any of the associated constituents".

Now that we can see the importance of distinct, discrete constituents as the basis of systems that have emergent properties, "properties not present per se in any of the associated constituents", how can we obtain even a hint as to the representation of phonemes in the brain?

We must begin by examining the physical system that comes closest to the phonemes, i.e., the chart of the speech articulations (International Phonetic Association 1949). Here, speech sounds are organized according to their place and manner of articulation. In spite of differences between languages, the phoneme chart for one language looks approximately like the rest. The phoneme chart is two-dimensional and periodic, like the table of the chemical elements, which I include for comparison (see Mendeleeff 1897, page 20; and Macquer 1775, plates V and VI).

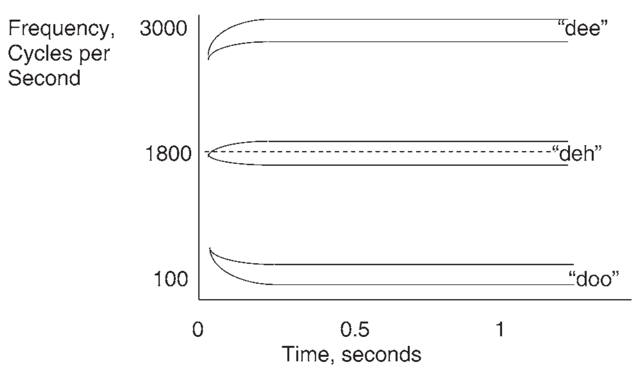




The two tables show what may be the best method of keeping large numbers of physical objects distinct from one another. The one-dimensional numerals were hard to tell apart, even with only ten of them. Imagine if there were a hundred, which is about the number of the chemical elements, and the limit for the number of phonemes in any one language, i.e., the maximum number that anyone would have to keep straight at any one time. But it is possible to keep ten discrete degrees of variation straight in one dimension, and ten in another. One hundred may be just about nature's limit to the number of distinct, discrete constituents that can be identified accurately.

The base of a number system is, for all practical purposes, the length of its periods, and corresponds roughly to periods in the periodic table and the phoneme chart. It is periods, and the two-dimensional systems they define, that enables us to differentiate up to a hundred physical items in the same system.

The resemblance between the table of the elements and that of the phonemes reflects, I believe, a single law of nature. But how are the articulations of speech related to the phonemes? The speech-sounds that are most deeply encoded are the ones that involve the most movement in their production, the stops p-t-k, b-d-g. We will take the most universal of these, the "d", and look at its sound-spectrum, which reflects the articulatory movements that produced it. The vocal system produces speech by releasing noise from the vocal cords into the mouth/throat cavity, which then echoes at some frequencies, but not others. When the tongue and the lips move, changing the shape of the cavity, the frequencies of resonation change. The process of frequency-change is what we identify as speech sounds.



In the example shown here, the acoustic event that we perceive as a consonant is a frequency-transition at the beginning of the following vowel. But the frequency transition has no fixed direction. It goes up or down, or stays the same, depending on the vowel that follows it. Worse than that, the frequency target at 1800 cycles is never actually produced in real speech. And if it is supplied in artificial speech, listeners hear a consonant corresponding to a different target. There is, then, no fixed or specific sound event that can be identified as causing a perception of "d", and even the movements of the speech muscles have no such fixed event. So, the phonemes are not carried by or present in the speech signal or the speech articulations (Liberman et al. 1967).

The speech signal is so deeply encoded that even an expert can't look at it and read it, even though everybody can understand it by ear. I once tried the experiment. It was at a fairly prominent department of phonetics. I made a sound spectrum of my own voice saying, "Bill Abler will give twenty dollars to the first person who reads this spectrogram", and pinned it up on the bulletin board, with a note that read, "Bill Abler will give twenty dollars to the first person who reads this spectrogram". There is no need to mention that no one ever read the spectrogram, and they eventually asked me to take it down.

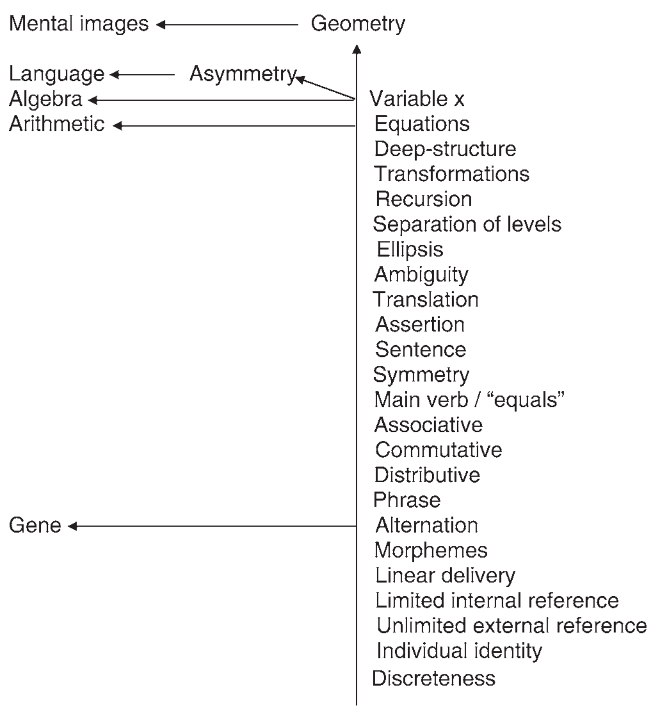
What does it all mean? First, we see that separation between the levels in the organization of language is true for the speech sounds as much as it is for words and sentences. A phoneme is no more built out of sounds than a sentence is built out of words. The phoneme chart is really an articulation chart. It is the articulations, not the phonemes, that keep their physical distance, and remain physically distinct. The phonemes themselves remain distinct and discrete, but they are not made out of the physical events of speech. What are they made of? They are abstract objects that have no property other than their identity. But the definition "abstract objects that have no property other than their identity" is a good, first working definition of numbers. To readers of this book, it should come as no surprise that phonemes are numbers. The fascination, and often the beauty, of their sound obscures their property as numbers. Who could resist the final "l" sound of "camisole", "espadrilles" "potting soil" and "knotting awl"? Oscar Wilde is said to have liked "cellar door". The astronomers have their "Aldebaran" and "Zubenelgenubi"; the geologists their "tufa", their "caliche" and their "cinnabar"; the mathematicians their "two-leaved rose lemniscate of Bernoulli", their "oui-ja board cochleoid" and their "lituus"; the turtle-ologists their "Aspideretes" and "Basylemis"; the Egyptians their "Horemheb"; the Chinese their "tsan-bow-bow"; the snake-ologists their "hamadryad"; the ecologists their "palaeoaeolian breezes". But phonemes are numbers all the same.

Since some readers may have doubts about the idea of abstractness, it is worthwhile to comment on it here. The argument is that abstract means "not material". But without material, a thing doesn't exist. So, abstract doesn't exist. A little reflection reminds us, however, that shapes and relationships may exist, not apart from, but in spite of, the material that expresses them. Circles stack to form hexagons whether they are manifested as pennies or bottle caps, or as a compass-line on paper. The orbital path of a planet, too, is meaningless in the absence of matter, but is not exactly matter, either, and will be the same whether the planet is rock or gas or ice. For purposes of this book, "abstract" will refer to the geometric or other relationships that are expressed by matter, and are properties of matter, but are not themselves matter in the usual sense of having weight.

Phonemes, then, are numbers with no properties other than their identity, i.e., their magnitude. They are not cardinal numbers, the kind that are squares and primes and cubes. Instead they are serial numbers, First, Second, Third. Twelfth isn't what you get when you multiply third times fourth, for example. Just First, Second, Third. That is what phonemes are. Words are numbers, too, compound numbers that serve as addresses for meanings. Phonemes are part of the address of words, but vocabulary remains the most mysterious part of language.

A Genealogy of Properties

We now have a complete and exhaustive comparison of the properties that are shared between arithmetic and language. By themselves, these properties are sufficient to producing an adequate, functioning arithmetic or language. They are also curiously necessary. Imagine a language without translation, or even without ambiguity. Something would be terribly wrong. Although higher-level structures in the organization of arithmetic/language are not built out of lower-level constituents, the properties nevertheless fall in a definite order that is interesting in itself. When we display the shared, emergent properties in their natural order, we can see the precise genealogical relationship between arithmetic and language, and we also get a little bonus. Both biochemists (Beadle 1966) and language scientists have long thought that some important relationship exists between the gene, with its linear organization of bases, and human language, with its linear organization of words. We will see that they were right. The gene is indeed a genuine, if distant, cousin of the arithmetic/language complex. Foundations is not indicated in the cladogram because separation of levels requires each new level of organization to inherit the requirements of the one before, and to have independent foundations of its own.



The genealogy is designed after the cladograms that are now familiar and nearly universal in biology (Wiley et al. 1991). The possibility of a neat, orderly genealogy whose elements all have their places confirms the shared-source model of arithmetic and language. The precise relationship between the gene and language/arithmetic is now identified and therefore confirmed. George Beadle was right, forty years ago, although the connection was so tempting that it was discovered independently, or has been commented on, several times (Jakobson 1970; Jacob 1977b; Pattee 1980; Abler 1989; Studdert-Kennedy 1990; Abler 1997. See also Studdert-Kennedy 1981) . We no longer have to explain why we don't have to explain assertion, ambiguity, ellipsis, paraphrase, translation. Separation of levels means that, in spite of the fairly strict ordering of properties, higher-level structures are in no sense built out of lower-level constituents. The genealogy is in no sense a construction or an edifice. It is a natural ordering that not only allows us to see the relationships between the parts, but to see the precise content of several natural systems. Evolutionary linguistics not only fails to address half of the properties necessary in order to formulate a theory of language alone, it further ignores the relationship between equations and ordinary sentences, a relationship that occupies fully half the genealogy.

The cladogram also verifies the conclusion that neither equations nor genes can be invoked to explain the existence of sentences. Such an idea is the genealogical equivalent of supposing that, say, ostriches are the genealogical source of elephants, or butterflies are the source of mammals. Systems that have already diverged from the main trunk are not the basis for the later emergence of other systems from the same trunk.

There is exactly one solution to the question of arithmetic/language. Unlike the biological solution, under which there might be any number of possible different innate language types, the cladogram shows that there is only one, and that it is given in nature, not evolved in biology or invented by geniuses during the formative period of human evolution.

Foundations of mathematics are spelled out in some detail, i.e., the genealogy tells us for the first time what the founding properties of equations actually are. Foundations will be required to account for the content of the genealogy, although we can already see some relationships. Symmetry of equations is the physical basis of assertion, i.e., the property of balance creates the first indication that two things are somehow the same. The symmetry relationship is thrown off balance by asymmetry in language, but the basis of assertion in symmetry carries across from arithmetic.

Evolution Box 2. [The reader may safely skip the Evolution Box.]

Now we can go back, and look at the equation that accounts for the existence of sentences, and, thus, equations. The evolution of language must have gone something like this. Words evolved when monkeys looked up at an eagle, say, to shout an alarm call, and down at a cobra to shout the same alarm call – but changes in larynx configuration, induced mechanically by the position of the head, inadvertently introduced differences in the acoustic signal that resulted. Baby monkeys, hearing the differences, thought that they were part of the intended signal, and imitated them voluntarily, introducing the first words, or one-word sentences, that could be quoted out-of-context. The famous tones of tone languages, such as Chinese, emerged in much the same way (Hyman 1973), confirming the hypothesis.

Later, it became more efficient to string one-word signals together than to have a different signal for every situation that might come up, so syntactic language evolved around the principle of efficiency, and memory load, and intelligibility. The sentence and its parts emerged on the model of tool-making, where the hammer became the subject, the action of striking became the verb, and the stone being hit became the direct object. Since children are not directly taught grammar (Chomsky's "poverty of the stimulus"), they can learn the precise grammar of their own language only if they are born with an innate Universal Grammar, UG, that narrows their range of choices as they search for the right one.

$$“\ dx_j/dt = \sum_{i=1}^{n} f_i(x)Q_{ij}x_i - \phi\ (x)\ x_j\ “$$ (Nowak et al. 2002)

Here, ϕ is a measure of “linguistic coherence” in a community, i.e., the similarity of universal grammars that are innate in different individuals. Q_{ij} is the probability that a parent speaking language L_i will leave a child that speaks language L_j. The expression “$-\phi(x)x_j$” causes the summation to equal 1, i.e., 100% of the population. When UG is adequately precise, linguistic coherence will evolve in the population, i.e., all speakers will share a common Universal Grammar.

The equation that, in principle, specifies the evolution of syntactic language (Nowak et al. 2002) *ipso facto* specifies the evolution of equations. Here, the circularity comes full circle.

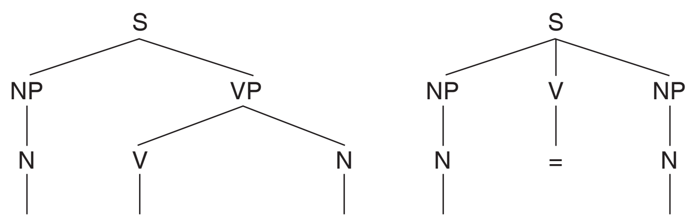
The Human Mind

What is the mind, what is its relationship to language, and where is its source? Under evolutionary psychology, language and mind are intimate-but-distinct, like the respiratory and circulatory systems. Being distinct, they must have begun separately and somehow grown together, although evolutionary psychology no more pins itself down to a definition of mind than to a definition of language. But the shared-source theory confirms itself again by providing a natural understanding without special pleading. Language and mind are not intimate. They are physical aspects of the same thing, like electricity and magnetism, or different views of the same hologram. The two did not grow together, but simply occur simultaneously.

On one hand, sentences are an altered form of equation, a linear delivery of hierarchically organized, discrete constituents that pivot upon a main verb. At the same time, sentences are driven by mental images (Abler 1973) which must be reconstructed in the mind of the listener. Under the shared-source theory, the two sides are, again, aspects of the same thing. The basic shape of the sentence is contributed by the underlying form of the equation itself. The mental image is contributed by the geometry that corresponds to every statement in algebra. At a time when our early ancestors were just beginning to speak, i.e., to discover the form of the equation/sentence, the geometry that corresponds to algebra was simply part of the system. Neither could happen by itself. Both happen together because they are the same thing.

The beginnings of technology in the human sense are taken from mental images that correspond to statements in algebra. This amounts to a definition. Making things based on behavioral routines, or even on mental images that evolved by natural selection, are not technology in the human sense, not even if they show an element of variability or learning. The beginnings of technology in the human sense are the same natural process as the beginnings of language in the human sense, and both are voluntary behavior whose structure is taken from that of algebra/geometry. These are the mental images that we experience as the mind's eye, which functions both in technology and in the construction of sentences. In technology, where the underlying geometry is largely Euclidean, mental images are taken directly from the geometry corresponding to equations. But in personal and social life, the images are altered by the same mechanism that makes ordinary sentences asymmetrical.

The linguistic deep-structure of ordinary sentences is familiar enough by now, but that of equations is more basic. It is symmetrical about the "equals".



Linguistic deep-structure of an ordinary sentence

Linguistic deep-structure of an equation

To most of us, the author of this book included, the linguistic deep-structure of equations is retro-borrowed from ordinary sentences. One side of an equation seems or "feels" different from the other, a transfer from language. To most of us, the symmetrical deep-structure of equations, above, is nothing more than an intellectual construction. We know it, but don't believe it, like picking up a male hornet with our fingers. But to a real mathematician, in whose mouth a page of equations will melt like butter, the intrinsic symmetry of equation deep-structures must be an inborn instinct. More precisely, the born mathematician has direct use and control of the equation in its primitive, symmetrical aspect.

The critical step to language, then, was the step from the symmetrical deep-structure of algebra, with its strictly iconic geometry, to the asymmetrical deep-structure of ordinary language. The property of mind, manifested in the first instance as visual images, takes its underlying geometry from both systems. From the symmetrical one of algebra, we get Euclid (geometry), and his practical twin, Archimedes (technology). The mental imagery corresponding to ordinary sentences is asymmetrical and not completely under control. It generates the varied and sometimes fantastic images we see in modern, and ancient, and primitive art. It gives us our body-image and lets us treat the human form as art. It generates dreams and nightmares. It is the property of asymmetry that enables the human mind to circumvent Gödel's proof.

Evolutionary psychology may help us understand what we think about, who does what to whom, why we eat too much, why some people kill their children and others don't, racism, freedom and captivity, mercy, altruism, beautiful and ugly, the eternal ebb and flow of good and evil. But the arithmetic/language theory tells us how we think about things in a uniquely structured way. Voluntary control of the arithmetic, language, geometry complex is the property of mind in the human sense.

Already, the numberline principle (because that is what I have always called my theory) gives us a better understanding of "*Man's*

Place In Nature". Language and mind based on arithmetic and geometry are the basis of the human being, not our primate ancestry. Language and mind are not innate in any biological sense. Innate is how spiders spin their webs. Innate means encoded into the genes over geologic time by a process of natural selection. Innate is biology. Since the idea of encoding equations on the basis of an equation is circular, language and mind are based on first principles, i.e., the principles that cause equations in the first place. But there is another feature that confirms this diagnosis. Under the cladogram, the gene is generated by the same system that generates language and mind. The gene can not be the basis of the system that produced it.

The idea of innate ideas, or innate structures of the mind, has to be revised. Language is about as innate as the orbits of planets, i.e., language is more basic than biology, and is borrowed directly from physical laws that are more universal than anything in biology. Overeating and infanticide, altruism and racism, maybe even love of the color yellow, may be innate ideas, but language and mind are closer to atoms and electrons, and the forces that rule the universe.

Here, again, unexpected details fit together without strain or special pleading or explanation. They confirm one another. The gene is not the most basic thing in biology, and it is certainly not the bedrock upon which the human mind is founded. In all likelihood, there are genes that allow us to take advantage of first principles, but they no more contain or carry first principles than the speech signal contains or carries the phonemes. The same first principles that cause language and mind also cause the structure of the gene.

While the gene will give us plenty of information about our biology, and will be very valuable for medicine, it will tell us nothing about the mind.

Further, computers will never acquire the property of mind, in spite of artificial intelligence laboratories that give appointments to theologians. A computer is a machine that does exactly and only what we tell it to do. It can run through a lot of operations in a short time, producing results that a human being might take a million years to do by hand, such as checking Goldbach to ten million. But that isn't first principles. That is human decisions carried out again and again. The machine can compute only what we already understand. And there is no way out by repeatedly correcting the computer's failures. That is still just trying to cheat God and Gödel. The machine still understands no more than we tell it.

It is our blinkered vision of ourselves that is at the heart of the self-deception. There have been psychiatric tests that you can take by computer. The computer asks "How do you feel?", and when it gets your answer, it asks, "How long have you felt that way?" Patients taking the tests don't know that there is no human being at the other end, just a computer. The reasoning goes, if you can't tell the difference, what's the difference? In effect, the question amounts to a cagey definition of a human being. But it is a know-nothing definition, and completely un-helpful. "If you can't tell the difference, what's the difference?" isn't a definition of a human being. Once we paint ourselves into some logical corner, we actually believe that there is no way out, and that we have discovered something profound.

To illustrate the limits of computing, we can look at a few extreme uses of language. The first of these is a puzzle that supposedly was actually used on an envelope, and faithfully delivered by the United States Postal Service. The address is supposed to have read

> Wood
> John
> Massachusetts,

but even if the puzzle is made up, it is a probable one. With a ZIP-code, the letter might be delivered today. The solution, found by Post Office personnel, was

> John Underwood
> Andover, Massachusetts.

Where would a computer turn for help in solving the address? It would know that the writing was an address, because it would be programmed for that, but how would it know to use the arrangement of the words to find the name of the addressee as well as the town? There is no mention of a town in the address, but the use of the word “and” to name the town is downright inspired. The puzzler has compressed words to the limit of information, but the human mind can solve it. I suggest at a minimum that a computer would have no path of probable approach.

To take another example, sardonic humor is a sophisticated figure of speech that can be very effective if used sparingly and suitably. Sardonic is saying one thing while meaning another. For example.

Sardonic Humor	Conventional Equivalent
“The vitamin G sure was nourishing.”	The gin was delicious.
“Can anybody here spell ‘Piltdown’?”	This is a hoax.
”I don’t anticipate eating lunch on Wednesday.”	I don’t want to see you.
“What universe do you live in?”	You don’t understand anything.
“sneaky Pete”	Whiskey
“both persuasions”	Male and female

The computer would have no clue as to where to begin. The meaning is not carried in the meanings of the words, or really anywhere in the sentence. Yet people respond to the sardonic with great pleasure and enthusiasm. It is the discrepancy, the cleverness of the unexpected relationship, that makes the sardonic so rich and delicious. Only the true joke is cleverer. But how would a computer realize that it was being presented with the sardonic? Even human listeners sometimes don’t realize what is happening. How will a computer figure out what vitamin G is, or why it is being asked to spell the name of a place in England, even if it “knows” that a famous hoax was perpetrated there? Why would it look behind the meaning of a perfectly transparent phrase like “I don’t anticipate eating lunch”? If it tried to look behind the phrase, what clue would it find as to the real meaning? I suggest that sardonic humor is just an example, but a good one, of why computers can't have the property of mind. The theoretical reasons are better, but the examples are more tangible.

When it comes to Sigmund Freud, people want to read the classics, *Civilization and its Discontents*, *The Ego and the Id* (*Das Ich und das Es*). But compared to Freud's great masterpiece, *Jokes and their Relation to the Unconscious* (1905, 1960), they are fluff. In one of the subtlest interpretations ever, Freud defines "the joke with its punch-line, which is formed by the interesting process of condensation accompanied by the formation of a substitute." In some important sense, Freud has described all of language, because aside from direct quotes, all language represents condensation [i.e., selecting just the relevant news] accompanied by the formation of a substitute [i.e., substituting language for actual experience].

What is the structure of Freud's punch-line? Since Freud's jokes are a little ponderous, I have selected a couple of my own favorites. The so-called explanation of the joke is no longer funny, but involves less condensation.

"I feel like the atheist at his funeral. All dressed up and no place to go"

Taken literally, the joke is meaningless. The dead don't go anywhere in the conventional sense. Yet to a listener familiar with Western culture, the joke suggests a Heavenly destination, for those who believe in it. Atheists have a soul, but without the existence of an afterlife granted by God, the soul has nowhere to go. The joke is a pretty funny comment on the intellectual corners people can paint themselves into. The explanation, then, is much less condensed than the punch-line, although it is more condensed than the theology that underlies it.

"What do you call a banjo player wearing a suit?"
"The defendant."

The "explanation" is that all banjo players are criminals who would never dress up without an overwhelming reason, and the only reason good enough is that they have, once again, been charged with some terrible crime and called into court for it. Here, again, the original punch-line is greatly condensed from the "explanation", which is in turn greatly condensed from years of personal experience with banjo players.

Of course it is the unexpected recognition of some familiar or suspected relationship hidden in the punch-line that is funny. The joke, by its nature, says less than, and other than, what is meant. But the human mind grasps the situation, fills in the missing relationships, supplies the missing connections and explanations. How would a computer know that the real point is some un-expressed, hidden explanation, not to mention navigate the subtleties of the culture to find it? Worse, some people, a very few, actually invent jokes. How do they know what is funny? How do they see the ridiculous or revealing or cruel relationships that make us laugh? How do they know precisely which substitution and which condensation will make the joke short, while leaving it understandable? How would a machine duplicate their talent? Such people are, after all, just very talented users of language.

One more case is irresistible here, where meaning substitutes for language altogether. There is an English construction that simply leaves logical conclusions un-said.

> "I have to be at the meeting before three, so."

In context, the sentence implies "so I can't see you at three", or "so I will have to see you some other time". A human listener, embedded in the situation, knows what the sentence implies, but there is no spoken language to express the idea. It is ellipsis carried almost to the ridiculous, yet some speakers seemingly couldn't get along without it. Certainly it doesn't strain or defy any rule or requirement of ordinary language. But where would a machine get the necessary information to fill in the missing idea? How would the machine know that something was missing at all?

It wouldn't. And if someone managed to program the computer to handle sentences that end with "so", some other construction would appear after a few years, and have to be programmed into the computer. Language shows every symptom of being directly subject to Gödel's proof in exactly the way that arithmetic is subject to Gödel's proof. If someone says "I don't anticipate being hungry on Wednesday", the explanation (postulate) will have to be programmed into the computer. After that, someone else will concoct, "Can anybody here spell 'Piltdown'?" and the explanation (postulate) will have to be programmed into the computer. Then, someone will say "This chair has your name on it" and the explanation (postulate) will have to be programmed into the computer. Et cetera and so on forever. There will always be a new sentence, or statement, in language that can not be understood on the basis of formal derivation from existing postulates or premises or assumptions or axioms. It is Gödel's proof in language.

If the mind were the consequence of evolutionary psychology, and the solution to some equation, the mind would be subject to Gödel's proof. But the human mind is not subject to Gödel's proof at all, i.e., it is capable not only of understanding punch-lines and sardonic sentences, it can find the "explanation", i.e., the necessary new postulate or axiom because it is made of different stuff. The structural basis that underlies the human mind is first principles – direct access to first principles. The mind is not dependent on what could be programmed into it genetically on the basis of experience, even accumulated collective experience. Any time you meet another human being, you are facing something that has voluntary access to first principles of nature. It is something to think about. Genetic programming is the solution to an equation, but language and mind are the equation itself. The human brain has somehow obtained direct access to first principles. That is how it guesses the material that is present but not mentioned. Here is the clue to formulating a search image: How could a living system obtain access to first principles?

We will try to answer that question last. But before we do, we need a better idea of the kinds of material that the mind turns out. What can we say about the mental images that drive sentences and technology? Mental images and their use was the subject of my book of thirty-odd years ago (1973), *The Sensuous Gadgeteer*. First, a skilled inventor can manipulate mental images voluntarily, going through the motions of actually building something, but without touching anything. Some procedures are impossible, so you want to catch them before you waste a lot of time and money trying them. Or a perfect device might take six months to build and cost $5,000, while a workable device that does the same thing might cost $5, and take less than a week to build. That is what happened with the testing frame that I built for studying the cutting properties of the serrations on the teeth of carnivorous dinosaurs (1992). My original idea, which I never built, was a balanced and counterweighted butcher's saw whose peripheral machinery filled an entire room. What I actually built consisted of an aluminum bar (to hold the experimental blades) attached to a frame made of brass tubes (to attach weighted strings). Built with scrap parts, the device cost nearly nothing, took less than a week to build, and gave measurements that were larger than the experimental noise introduced by a device that was not automatically self-leveling.

How is it done? First, you envision the perfect device in your mind's eye, then you remove some parts and make others smaller. I attached the brass tubes to the aluminum frame by drilling the frame and gluing the tubes in place with epoxy. No heat, no warping.

The making of molds is also fascinating and instructive. Often, we make the object we want out of wax, cover it with plaster, heat the wax and pour it out, then pour liquid metal or plastic into the space vacated by the wax. When the material hardens, we (somehow) remove the plaster to retrieve the object that we want. Casting is the closest thing to magic in technology. But the direct construction of a mold is the most stringent test of the maker's skill because there is no original object to look at. The negative mold is made directly in negative space, and can only be done by forcing the material mold to conform to a mental image. There simply is no physical object to serve as the model for a negative surface.

The anthropologist Peter C. Reynolds conducted a study (1981, page 160) in which he hired a technician to build a kite while he took notes. One of the steps was to cut notches into the ends of the sticks. Reynolds (here paraphrased) comments, "The reader will note that, before the notch is made, there is no notch. The notch is not made in response to anything in the environment. It is made, and could be made, only on the basis of an image in the mind of the kite-maker." The emergence of technology in the human sense, of making things, began when our primate ancestors began forcing the material world to conform to mental images that were generated directly from geometry corresponding to statements in algebra. Cagey, least-common-denominator definitions of "tool", or "technology" are not necessarily theories in science, and have been dangerously misleading (van-Lawik-Goodall 1971).

Whether consciously or not, many such definitions have been constructed with an eye toward demonstrating evolutionary continuity between human tools, and objects used by animals. As with the evolution of language from animal calls, such continuity is a hypothesis to be tested, not an obvious truth to be defended at the cost of contradictions. An object that is somehow held by an animal, and used to modify something else might be a tool in the way that body language is a language. But it is not obviously a tool in the human sense of forcing the world to conform to mental images based on geometry corresponding to algebra.

Reynolds also proposed that the first step in the process of tool-use is something he called GRIP. Just gripping something is an action that looks the same whether it is a mud-dauber picking up a pebble in her mandibles, or a chimpanzee picking up a stick with its hand, or Antonio Stradivari himself picking up a broken fragment of an old saber to perform magic upon a slab of wood. But GRIP is a human action, a mechanism for incorporating an object into the body image, possibly using the same underlying mechanism that incorporates clothes into the body image as a way of manipulating it. As one friend used to put it, "when you use a screwdriver, that's your brain out there on the end of the screw, boring its way into the wood." GRIP is where biology ends, and the numberline principle takes over.

Although this is probably out-of-place, I can not resist from mentioning that technology and sentences follow the identical underlying routine. When we make something, a violin top, say, we saw and cut and shave and scrape the wood to get it to the right shape and thickness, and especially to some desirable sound. The movements of sawing and cutting and scraping are carried out one-at-a-time, in such a way that neither the movements themselves, nor their linear, single-file ordering has the slightest similarity to a violin top. But their sum adds up to a duplicate, in wood, of an image that originally existed only in the mind of the violin-maker. And the same goes, more-or-less, for a sentence. A sentence is communicated by a linear series of movements that have no similarity to the meaning being conveyed, but whose sum is a duplicate, in the mind of the listener, of a mental image that originally existed only in the mind of the speaker.

The single-file, one-at-a-time, linear delivery of discrete constituents in sentences, equations, and the gene, as well as in making things, amounts to the definition of time, i.e., the counting of discrete objects or events. That is why rosaries are so popular. The one-dimensional string makes sure that you automatically count every bead without having to think about it. In the linear sequence of discrete constituents, sentences, equations, and genes encode an element of time (Newsom 1964, page 102) that indicates, again, the profoundly basic character of such systems.

What I have been calling access to first principles probably corresponds to what people intuitively call common sense. Common sense reads between the lines, so-to-speak, guessing that something is missing, and knowing where to find it. Common sense navigates what the book doesn't tell you. And Gödel's proof tells us that there will be plenty of that. Let us compare, then, the kind of common sense that gets jokes and the sardonic, and builds negative molds, and compare that to what really evolves in biology.

The Animal Mind

In much the way that the State Legislature of Indiana came within inches of voting the official value of π to be "$\pi = 16 / \sqrt{3} = 9.2376$" (Beckmann 1971, page 74), the Government of Australia came within inches of voting the chimpanzee to be an honorary *Homo sapiens*. A much better candidate would have been the honey bee, whose biology is so like ours that it is haunting. The list of corresponding features is nearly exhaustive:

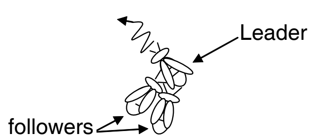
1) Tropical animals
2) That have invaded temperate regions
3) Not by biological adaptation (i.e., hibernation)
4) But by importing the climate of their tropical origin (bees keep their hive warm, even in winter)
5) Through social technology (the construction of wax combs, storing of food, and individual housing of young)
6) And the social rearing of individual young
7) Using division of labor
8) Without an anatomically fixed caste system beyond male versus female, and the queen, who suppresses the workers' ovaries through chemical means.
9) There is social sharing of food
10) And the use of technology to store food.
11) Collective, social decision-making.
12) Honey bees have social communication
13) That can send an individual to a distant location, rather than having to lead it.
14) Sexual dimorphism.
15) Only a few species (four) in its genus (Apis).

The probability that such similarity would happen by random chance is vanishing. What is wrong with this picture? It represents the kind of fantastic sophistication that can evolve in biology. And if honey bees don't have tools, a few sand wasps tamp down sand by using a pebble held in their jaws (Alfred Kinsey 1926, page 430). Why doesn't the Government of New Zealand declare the honey bee an honorary human being, and throw Edmund Hillary into jail for kidnapping them? Common sense. The biology of the honey bee is nearly identical to our own. If they are not human, it only shows that it is not our biology that makes us human. It is our mind.

The honey bee's biology is nevertheless instructive. It has evolved to cover every situation that might come up. The stinger, for example, has barbs at the end that catch in the victim's skin when the bee stings a vertebrate. As the bee walks away from the scene of the sting, a perforation in the attachment guarantees that the stinger will be torn loose and be pulled out of the bee, who will die of dehydration in a couple of days. What is more, the stinger has a scent gland on it that guides other bees to commit further altruistic acts, making the honey bee much more socially organized than her relatives the bumblebees.

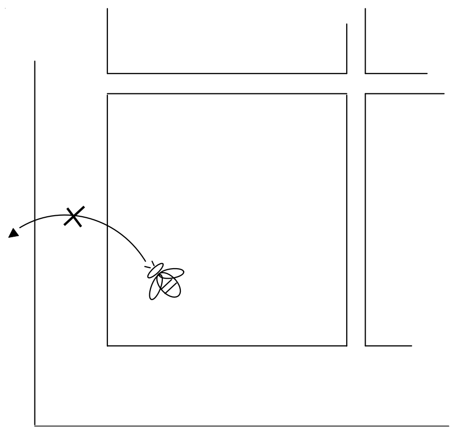
But the honey bee has a trick. If the situation doesn't justify social recruitment to suicide, she can evert her stinger just enough to leave a venom-filled scratch in the victim's skin as she walks along - teaching the victim a lesson while preserving her own life.

The idea that the honey bee can send, rather than lead, a recruit-bee to a distant destination is undoubtedly mistaken. The famous dance-language of the honey bees, which is performed on the comb surface without moving the feet at all, is a re-enactment in miniature of the foraging flight. The direction of the food in relation to the sun is represented by the direction of the dance in relation to up; and the distance to the food is represented by the speed of the dance. By the time the recruit flies out of the hive, she has already followed the leader, on foot, only to re-enact the ritual on-the-wing.



The Communicative Dance of the Honey Bee

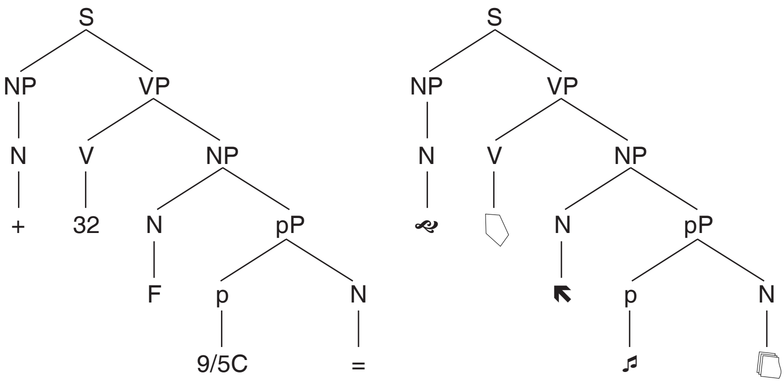
Biological evolution has provided the honey bee with a lot, but not common sense. Just half-an-inch outside her familiar routine, the bee is lost. Reach into the colony with a tweezers and lift the dancer right off the comb by the wings, as I have done many times. The two recruits following her will not get upset in the way that we might if we were following along after someone, and a giant trap suddenly lifted them up into the sky. The recruits will just wander off into the crowd of bees as if nothing ever happened. Let a honey bee loose in a room, and she will fly to the light, i.e., to the window, where she will be unable to escape, even if the window is slightly open. She will stay up against the pane, unable to walk around the wooden frame to freedom because the daylight is at the glass, not the wood. She can not see herself or her situation in perspective. She can only run through the routines that experience has taught to every honey bee through collective genetic memory. The famous dance is a matter of miniaturization and re-enactment, not "condensation accompanied by the formation of a substitute".



Honey bees even dream, sort of. At night, they will perform their communicative dances, even though no bee will fly out to find the flowers. I have seen this myself. But even then, the dream is made of the same stuff as ordinary behavior. Bees that are visiting the same flowers will cluster together on the comb at night, apparently because they have the scent of the flowers on them. And I have seen my painted experimental bees cluster together on the comb at night, apparently because they all have the same scent of paint on them. But it is still traceable to their daily biology. At first, I was trying to add lateralization of the brain to the list of similarities between humans and honey bees (Snodgrass 1925, page 230; Jawlowski 1957; Bullock et al. 1965, page 1241; Abler 1976, 1977; Pascual et al. 2004), but eventually realized that it is our mind, not lateralization, that makes us human.

Common sense derived from first principles. For the bee, if something didn't evolve in biology, as a kind of collective, accumulated lesson, taught by natural selection over the generations, it doesn't exist. Nothing sardonic. No joke with its punch-line. No sentences whose last word is "so", but that don't end there. Bees have hierarchically organized behavior, but no ellipsis, no double-meaning, no paraphrase, no translation, no assertion, no commutative, no associative, no distributive property. No molds in negative space. They get the same honeycomb every time by doing the same thing every time, not by using a mental template. We humans benefit from experience (some of us more than others), but human beings all have common sense (some of us more than others) because we have access to first principles. Our ideas are not re-enactments of real-life behavior. Apart from quotes, sentences aren't re-enactments of anything, and may be completely detached from real life even if they are about it. We might hear stories, for example, about birds without trying to fly, or about monkeys without trying to climb trees, or about Brazil without trying to speak Portuguese. But bees think about flowers in exactly the way that natural selection has taught them; and their thoughts are always part and parcel of the way bees behave toward flowers. Biological evolution doesn't have the "smell" of the source of the mind in much the way that biochemist James D. Watson (1970, page 182) thought that protein doesn't have the "smell" of the genetic material, because protein is too unstable.

Look what we would need to do in order to accept the evolutionary-genetic theory of language and mind. First, we would have to deny the exhaustive and overwhelming evidence of the cladogram. We would have to look at the linguistic deep-structure of an equation and actually convince ourselves that the equation isn't a sentence, that someone has arbitrarily pinned a few symbols onto a deep-structure and called it a sentence. Let us see what arbitrary really looks like.



The tree on the right is obviously arbitrary, and has nothing to do with nature: The symbols (?) just aren't language in any conventional sense. But the one on the left is just as arbitrary in a different way. It uses symbols that belong to the deep-structure tree, but has them in all the wrong places. "Equals" is not a noun; and "32" isn't a verb. The individual placement of the symbols makes no sense in its own right, and their relationships do not add up to any meaningful expression. But if the symbols are placed differently, they not only make sense individually, but collectively they add up to the meaningful sentence, "F equals nine-fifths C plus thirty-two". "Equals" is a verb. It has tenses (e.g., "The square-root of four equaled two when I was in school"; "The square root of four will still equal two after you graduate"), and takes verb endings. "32" is a noun, if a strange one. But we know that two 32s is 64, and that 32 can be a direct object, as in "John took 32 away from 64". "Plus" isn't a noun, in the sense that it can't be a direct object, e.g., *"John took plus away from 64". And it isn't a verb, in the sense that it can't create an assertion, e.g., "Five plus five" is neither true nor false. But "+" still creates a relationship between nouns, e.g., "5 + 5". "Plus" is most closely related to prepositions, especially "with", which can practically replace it, e.g., "Five with five equals ten". Maybe it sounds a little foreign, but it works.

Equations are sentences, and their underlying form is retained in all sentences. In order to accept the genetic-evolutionary theory, we would further have to believe that the mind somehow ignored or was oblivious to the geometry that automatically accompanies statements in algebra. We would have to explain why the mind was sensitive to algebra but not to geometry. Then we would have to believe that a second geometry evolved in the mind by natural selection, and attached itself to the already-existing algebra exactly where the geometry would have been that the mind had recently ignored. The mind would have to be sensitive to geometry that evolved by natural selection but insensitive to an identical geometry that was already granted by one of its own systems. The idea is preposterous. It didn't happen. On the other hand, if we accept the numberline theory, the arithmetical theory of the mind and language, we automatically have not just a simpler hypothesis, but unity of hypothesis.

Far beyond unity, the human mind becomes fascinating again, the way it was in antiquity. No longer is the mind a dumb response engine programmed by association learning that is preserved in the genes (the only interesting part) over geologic time. Instead, the mind is created directly by first principles, by the forces that shape the atom and rule the universe. The asymmetry that forms sentences of human language tilts the stately equilibrium of the equation out-of-balance. The parallels that once ruled the two sides of the equation no longer apply. In the human mind, they meet at infinity. The human mind at its fullest expression spins off into a kind of non-Euclidean space, finally entering the 20^{th} century, and making its way into the 21^{st}.

Like a musician who has got a little better and has to re-think every piece of music he knows, scientists will have to re-think not just what they think about language, but what they think about the brain, human evolution, foundations of mathematics, and maybe even physics, in light of the structure of language. Without a control group, we will grope along forever in two circular mazes. One maze for the origin of language and mind, the other for foundations of mathematics.

The search for extraterrestrial intelligence, SETI, offers a direct, empirical test of the numberline theory, or arithmetical theory, of language, mind, and the foundations of mathematics. The biologists say that "evolution is local and contingent". That means that natural selection happens to a specific living thing at a specific time and place for specific reasons that are contingent on local conditions. Evolution is specific, and living things carry into the future the quirks and detritus of their local-and-contingent history. The dew-claw on a dog's leg, for example, is not a design feature, but the remnant of an ancestral finger. If language is biological, and of local and contingent origin, the languages of extraterrestrial intelligences will be so loaded with evolutionary dew-claws that we won't even recognize them as language. This point is considered by many scientists to be laughably obvious. But if language and mind are fundamental at a level below that of biology, the languages of extraterrestrial intelligences will be no more different from human languages than one human language is from the next. About as different, say, as English and Chinese. The receipt of a single intelligible message sent from an extraterrestrial transmitter would immediately rule out the biological theory of language and mind, and would be completely compatible with the arithmetical theory. Just the 3-4-5 message, mentioned earlier, would show that their mathematics is the same as ours, and not an arbitrary creation that depends on local biology. A signal consisting of three dots followed by four dots followed by five dots wouldn't even be subject to biological accidents in the number of fingers.

The Emergence of the Human Mind and Culture

Allright, if language and mind did not evolve in the usual way, by natural selection, how did they come into existence? I would imagine that something approximating to the modern program in evolutionary psychology/linguistics actually started to happen. It may have been limited, but it placed sticks and stones in the hands and communicative sounds in the mouths of our primate ancestors. Once these rudimentary behaviors were established, the brain replaced them with the real thing, possibly by gaining access to structures that were already present inside brain cells, but were previously usable only by the individual cells, for their own purposes, but not collectively. In that case, the step to the human brain would have been a matter not of acquiring new information patterns, but of finding ways to make already-existing ones available above the cellular level. Such an idea is perfectly conventional in modern biology, where we realize that even amoebas and paramecia make decisions concerning what to do and where to go (Jennings 1905, pages 16, 337; Quevli 1916, pages 210, 234; Dorsey 1925, page 273; Bovee 1964, page 206; Allen 1964, page 408; Ford 1976, page 136; Hameroff 1987). Their actions, like ours, may not be controlled entirely in the periphery.

The numberline theory clears up another mystery that hangs over the evolutionary theory of language and mind. Human beings don't just have fine movement, they have really fine movement. The articulatory gestures of the speech muscles are coordinated and accurate down to five-thousandths of a second, 0.005 second. What is more, people who try to work with objects under the microscope usually discover that they can do it. That means that if you try to move something around under the microscope at 25 magnifications, with a little effort you can get accuracy down to a thousandth-of-an-inch, 0.001 inch. During the formative period of human evolution there were no microscopes, and no call for work that was accurate to a thousandth-of-an-inch. Such a skill never evolved in biology. Yet plenty of people can do it. How did it get there? If we suppose that human ability and abilities are dependent upon, and limited by, a few quirks and coincidences and biological accidents that happened to happen to our primate ancestors during the formative period of our evolution, micro-fine movement is inexplicable. It is one of those numerous details that has to be explained away. But if we suppose that at least the basis of human ability is drawn directly from first principles, we can see the geometry of an object as existing apart from its size, and the motivation for fine movement being initiated from the center.

We can see now that being able to watch the developing behavior, both vocal and manual, of our primate ancestors would be no help at all in verifying any theory of the origin of language, even if we could watch for thousands of years. We could watch relative clauses, or some other construction, emerge and never know whether it was internally generated, or whether it appeared by natural selection. That secret would be hidden inside, and would be accessible only to a theory-driven approach, not to a strategy that was based strictly on the observe-and-interpret system. A voyage in a time-machine would be worse than useless, because it would merely confirm us in what we already think, without differentiating one hypothesis from another.

If we can define the human being, mind, and language, we might be able to define culture. As with language, we can define culture any way we like – mutual learning is the prevailing definition (Whiten et al. 1999; de Waal 1999). But, as with language, even the simplest, most transparent definition can be misleading if it is crafted for the purpose of showing something else, even something as innocent as evolutionary continuity with primate behavior.

Unlike arithmetic and language, culture does not have the kind of clear and elaborate structure that allows us to identify it by its parts. But if we can catch it in a state of change, we might see something characteristic and defining. The study, by Iona and Peter Opie, of "*The Lore and Language of Schoolchildren*" (1959), is perfect for our purpose because a generation of children at school is only about five or six years. Where adult generations are only four or five to a century, schoolchild generations are fifteen or twenty to a century, giving us a kind of magnification for the study of culture in a state of change. The Opies studied cultural change by speeding it up in much the way that Galileo studied the acceleration of gravity by slowing it down.

Almost every page of their 400-page book contains examples of cultural treasures that are 50 or 150 years old; but I will content myself here with a few examples. By following the spread of specific rhymes, oaths, and games, the Opies studied culture in two aspects, its ability to persist for a long time without changing much, and its ability to spread without changing much. Both are remarkable. Here are a few examples picked almost at random. We will start with the stable persistence of items in culture over time. On page 98, The Opies comment, “It sometimes happens that a rhyme or song which seems to be recent has, in fact, been marching with history for centuries.”

On pages 25 and 26, they give some examples of what they call “tangle-talk”. One that I remember from my own childhood in the 1940s is:

“One fine day in the middle of the night,
Two dead men got up to fight,
Back to back they faced each other,
Drew their swords and shot each other.”

Another, from about 1910, is:

“One blind man to see fair play,
And two dumb men to shout hurray.”

My own grandmother used to say, about 1950:

“‘I see’, said the blind man.
‘I’ll give you a kick’, said the man without legs.”

The Opies report a tangled rhyme from 1830:

“Two dead horses ran a race,
Two blind to see all fair,
Two dead horses ran so fast
The blind began to stare.”

But the oldest they found, from a minstrel's notebook in 1480, (Bodleian Library, University of Oxford MS. Eng. poet. e. 1, fols. 11v.-17r.) is:

"I saw iij hedles playen at a ball,
An hanlas man served hem all,
Whyll iij movthles men lay & low,
iij legles a-way hem drow."

On page 83, the Opies list several child verbal traps, or "catches" such as,

"What makes more noise than a pig under a gate?" ——"Two pigs under a gate."

They comment, "These catches [have] been part of childhood lore for more than a century."

On page 121, the Opies comment that, in medieval times, a knight would offer an antagonist mercy with the word "barlay"; and children in the north-west of England still offer mercy with the same word.

And on page 251 they comment, "In the twenty-first century there will still, it seems, be many people who know how to color Easter eggs in the old ways." I can attest that they are right.

Next, we can look briefly at the way items in culture spread rapidly, yet remain stable. On page 58, The Opies observe, "A trick will sweep through a school like a disease, one child passing it on to another." They write, for example,

" 'Do you collect stamps?'
'Yes.'
'Here's another for you' (stamping on his foot)."

Items in human culture retain their identity in spite of rapid spread and endless repetition over many years and generations. Once again, in the way that James D. Watson thought that protein was too unstable to be the genetic material, mutual learning is, by itself, too unstable to have the "smell" of the cultural material. To what do items in culture owe their stability both in time and space? If cultural items were represented as a blending system, they would drift and fade like the shapes that we see in the clouds. They would be about as easy to remember as Chinese grass characters. But they are not like that at all. They represent universal riches that endure for decades, centuries, and in the case of The Ten Commandments and The Lord's Prayer, millennia. They spread to dozens and hundreds and thousands and sometimes, as in the case of the Davy Crockett song, millions of speakers, yet remain completely recognizable. They are available to the poor and even to the slow. They owe their stability to the discreteness property of phonemes and morphemes and grammatical constructions, and to the memory aids of rhyme and meter, themselves made possible largely by discreteness. Culture, in the human sense, is mutual learning of items stabilized by the discreteness property of phonemes and morphemes and linguistic deep-structures.

The stability and longevity of items in human culture gives us an opportunity to re-examine the property of discreteness. Up to now, we have only noticed that discreteness permits objects to remain stable, but it is also the beginning of something more profound. Look at a crystal of quartz, which is silicon dioxide, and try to imagine what would happen if, say, all the oxygen atoms were not exactly the same. The crystal structure would not be uniform. The wonderful and mysteriously flat faces would not exist. The crystal would be filled with faults, and the larger the crystal grew, the more obvious the faults would become. There would be, for practical purposes, no crystal of quartz.

If all atoms of oxygen were not exactly the same, the molecules of sugar and starch and wood that form trees would not be possible. The cycles of organic molecules that are the basis of our metabolism would not be possible. Imagine a world where each carbon atom had its own unique virtues that could be appreciated on an individual basis. This one is a little rounder, that one is a little fatter, the other one is a little skinnier, or cuter, or faster, or sharper than the others. No two molecules formed of such atoms would be the same. A biological molecule would not fit into the active site on an enzyme. The tens of thousands of enzyme-driven chemical reactions that take place every second in the maintenance of living things would not be possible. This is not an obscure technicality. Just the tiny differences between ordinary hydrogen and its heavy twin tritium make tritium-water unfit for many biological systems. There would be no living things. The perfect similarity of the subatomic particles is an absolute necessity for the existence of an orderly chemistry and life. I suppose that is why, as early as the 1950s, I heard the idea that there is only one proton, one neutron, and one electron in the universe, and that they travel back and forth in time, forming every material object and system everywhere. It is an ambitious idea, but it offers a solution to the perfect similarity of the subatomic particles and the perfect similarity of atoms and biological molecules.

Discreteness of constituents, then, is only the first step in the formation of systems with emergent properties. The constituents must in addition be identical, even if they are assembled according to a plan that is not implicit in them. This rule applies to the phonemes and morphemes of human language as much as it does to the atoms of biological molecules. When we sing a song, we are repeating exactly the phonemes and morphemes and sentence structures that were put into place by the composer, and that are being sung by others around us. They are the same every time we sing that song, even if it is in a different key, and fifty years later. Any direct quote - and singing a song or reciting a poem represents a direct quote - works the same way. Its phonemes and morphemes and sentence structures are exactly the same as the original. That is what makes a quote a quote. And it is our ability to produce quotes that makes culture possible, not just in songs, but in laws, because a law is a quote. It is the same every time, and has to be. Even The Ten Commandments is a quote that can be, and has been, memorized and recited as necessary. If they changed, they would retain none of the towering authority that they enjoy after millennia of use.

Laws of nature, at least as we state them in the culture of science, must also have the complete stability that is granted by absolute repeatability, based on identical phonemes, morphemes, and sentence structures. Imagine a physical world where the statement "For every action there is an equal and opposite reaction" meant something different if it were shouted, or whispered, or lisped, or spoken in baby-talk, or falsetto, or basso profundo, or spoken slowly or quickly, or angrily, or sweetly, or in a male voice or a female voice – or if it changed slowly over time. There might be physics, but there would be no science of physics, no study of physics in human culture.

The culture of science and law and poetry, whether the last is set to music or not, depends directly on absolute repeatability provided by absolutely identical constituents. It is obvious, however, that the physical mechanism of speech, the movements of the tongue and lips in producing the sound signal of speech, are a blending mechanism, with an infinite number of possible tongue-heights, for example. How does our system extract elements that are both discrete and identical from a physical signal that is never the same twice?

First, we have to realize that the speech signal is so ideally adapted to its function, both in production and perception, that it is obviously of biological origin (see Saban 1993). The curving frequency transitions that are the cue to the stops, p, t, k, b, d, g, allow the tongue to move gradually from its place of closure to the position for the following vowel. The tongue never has to jump abruptly from one position to another, a movement which, in any case, its mass and viscosity would prevent it from executing. On the perceptual side, the gradual frequency transition produced by the vocal system as the tongue moves means that the ear has extra time to listen for cues to the various phonemes. The distribution and mixing of information means that in speech with n phonemes per second, the ear has more than 1/n of a second to receive cues to each phoneme. Mixing of information in the speech signal is called coding.

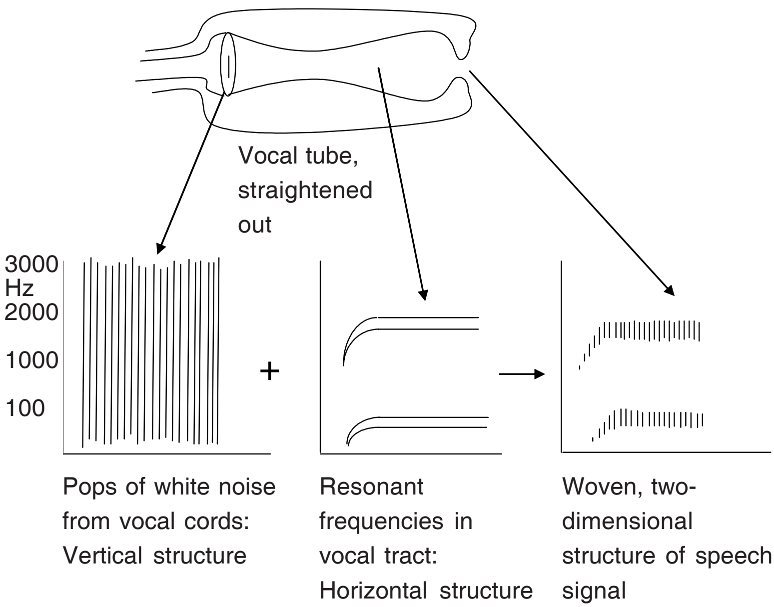
Mixing or blending of information in the speech signal is also the reason that, after all these years, there is no voice-writing machine.

Like the Xerox machine, such a device would be highly desirable even if its desirability is not obvious before the machine is available. Nevertheless, a scholar possessing such a machine could set it up, deliver a lecture, and walk away with an instant book. In a more homespun way, brothers and sisters could attach the voice-writer to the telephone and, at the end of the evening, have a family history.

If the stories are true, and I repeat them here just as I heard them at the University of Pennsylvania during the 1960s, the discreteness property of the phoneme, and formal procedures for deducing the phoneme inventory of any language, were discovered by the language-scientist Zellig S. Harris early in World War II. I myself took a course from Harris when I was in graduate school. Harris, according to the stories, divided his time between being a professor at Penn for half the year, and being a carpenter in Jerusalem for the other half. That was the story at the time, and it goes on to say that the possibility of building a translating machine for wartime use was a major goal of the Allies. If they could crack Enigma, why not language?

The story is that the Americans sent a submarine into the Mediterranean Sea and sent out a rubber boat to pick up Harris at night from the beach in Israel, and bring him back to New York City. It was the Manhattan Project of linguistics. The electronics of the radio was already being miniaturized; and the idea was to build an attachment with two dials, one that could be set to the language of an incoming radio signal, and the other to the language of a fighter-pilot. Such a machine, attached to the radio in the cockpit, would not only allow the pilot to eavesdrop on enemy conversations, it would allow the famous Polish officers who flew Spitfires for England during The Battle of Britain to communicate with English-speaking pilots and flight commanders. One of those pilots, according to the stories of the 1960s, later became one of Harris's colleagues at Penn. Then, as now, the most learned and the most powerful were waifs-in-the-woods as far as the sophistication of the human mind, but it would be twenty-five years before Alvin Liberman, Franklin S. Cooper, Donald Shankweiler and Michael Studdert Kennedy (1967) would make the first step in showing why.

The acoustic signal of human speech is more sophisticated than anyone could have guessed, and demands at least some appreciation here. The noise that escapes from the vocal cords during speech consists of consecutive bursts or pops of white noise, containing sound at all frequencies. The pulses of white noise are released in quasi-periodic fashion, and the number produced in a second is heard as the note of the singing voice. The vocal tube above the vocal cords functions as a resonating filter that lets only some frequencies through. When we move the tongue and lips and jaw during speech, changing the diameter and length of the vocal tube, we change the resonant frequencies of the speech cavity. The result, as the speech muscles move, is that the moving bands of resonating sound escaping from the mouth have an extra-dimensional structure: The vertical component amounting to pitch, the horizontal component reflecting articulation. This woven, even embroidered, extra-dimensional structure makes speech the most sophisticated sound known, and is what makes the human voice instantly recognizable.



Notice that blending or coding of acoustic information concerning neighboring phonemes is exactly what doesn't happen in Morse "Code". Morse Code is what Liberman, Cooper, Shankweiler and Studdert-Kennedy call a cipher upon the letters of the alphabet – one element of Morse Code corresponding to one element of the alphabet. But blending of information guarantees that the speech-signal has no such correspondence with the perceptual categories which are, in addition, at a lower level than the phonemes of language.

Since speech is a code not a cipher on the perceived speech-sounds, there is no possibility of creating an acoustic alphabet that can be pieced together to form words and sentences. In principle, you can't cut up a tape recording of speech and piece the sounds together in a different order to "spell" other words or sentences. At a deeper level, it means that we will never know everything about the acoustic signal of speech.

Here, I must mention that the role of missionism in the study of language can not be overestimated. In translating the Bible into the languages of the world, missionaries have learned and recorded hundreds of languages that might otherwise have been lost to science. When I was in graduate school, one of the students there had been sent by a religious sect to learn linguistics. He was their best and their brightest, and was determined to do justice to the trust that had been placed in him. He was going to learn everything, no matter what. I remember him vividly. His manner was always cheerful. His back was always straight. He always did his homework – all of it.

One day, in phonology class, the professor mentioned the speech code and its ominous portent. The Student's face fell. His back bent. He was never the same again. He knew that he would never know everything. As goes phonetics, so goes language. In the most profound and pervasive sense, the numberline principle means that we will never know everything about arithmetic, the mind and language. It just isn't in the nature of things.

How, then, does the perceptual system form a connection between the continuum of the acoustic signal, which is wholly biological, and the internal system of language, which is discrete, and of inorganic origin? It divides up the physical continuum into discrete regions, and assigns any given acoustic event to one region or another.

The boundaries between the perceptual regions are very narrow. They are called psychophysical thresholds (Lisker et al. 1964); and sounds on opposite sides of a psychophysical threshold are easily distinguished, while sounds within the same perceptual region are difficult to distinguish, even if they are physically far apart. So, all acoustic transitions that point to a high frequency are heard as "g"; to a middle-frequency as "d"; and to a low frequency, as "b". But two transitions that are physically close together on two sides of a boundary sound as completely different as if they were physically far apart. To a first approximation (but not a second approximation), perception of speech has much in common with perception of tone: The lower-level event is replaced in perception by a higher-level category. It is in this way that the physical world is given meaning by the brain. The physical signal that we experience as tone is a varying pressure-wave that travels through the air. But we are never aware of the varying pressure-wave unless we display it with instruments. All we are aware of is a tone that corresponds to a certain note on the musical scale. For purposes of language and speech, we register not the acoustic signal, but the perceived phoneme i.e. a discrete perceptual category. The same is true for the perception of words. We register the word category, not a string of speech-sounds. In this way we are attending primarily to internal categories that are simply identified, not tracked and measured and estimated, like a blending system. The tree-structures of sentences are likewise discrete. A branch of the branching deep-structure is either present or not. There are no fading branches, or moving branches, or longer branches or shorter branches or branches that curve upward or downward. The tree-structure of a sentence is a discrete structure, not a blending one, and is remembered in some exactly repeatable sense, without having to estimate degrees of attachment or of relatedness.

The basis of culture in the human sense is the long-term stability of linguistic structures provided by discreteness at every level in the organization of language. Culture in the human sense is unrelated to systems, even sophisticated ones, that are based on learning or mutual learning that is not supported by discreteness. Calling other systems of mutual learning "culture" is the same thing as calling the communicative dance of the honey bee "language". I cannot help remembering a story that is told about Abe Lincoln, who was fond of asking, "How many legs would a horse have if you call the tail a leg? - - - - Four. Calling the tail a leg doesn't make it a leg." Least-common-denominator definitions, even when innocent and well-intended, do not necessarily lead to theories in science.

Human Logic

Even structures of the mind and thought show evidence of discreteness and structured organization. We can examine religion, argumentation, and history of the law as a means of studying human thought. History of the law, especially, offers human thought meticulously recorded for later use. As ever, we are interested not in everything, but in underlying foundations that define the human being and human thought. So, we won't consider why people over-eat, or why people get angry or sad or happy. Other animals show every indication of doing the same. We want to know what makes us human. Here, a knowledge of history is the only protection against the next good research proposal, because it is only when we know explicitly the assumptions underlying our search images that we gain any control over our program of research. Eventually, we will develop search images for the study of the brain, so we have to get it right.

Since antiquity, the human mind has been treated as a truth-seeking engine. Even though many moderns would disagree with such an idea if accosted with it directly, all of association learning, and much of medical psychology, is based on the process of shaping and sculpting or gradually modifying behavior to fit external reality or truth. Think of how people are taught to lose their fear of snakes or spiders or open places, or of how they are taught to quit smoking, little by little. Natural selection itself is based on the idea of fitting the organism little by little to external truth. The idea of the infant producing random noises until these can be differentiated little by little into the sounds of speech, or of the infant's visual field as a uniform gray that eventually differentiates itself little by little into regions corresponding to experience, is also a model of the mind as a continuous, truth-seeking engine. Every time we say "I'm conditioned to getting up early" or "That reinforces my belief in getting up early", we are affirming association learning in some form. Apparent gradualism in learning misleads us now in exactly the way that apparent gradualism in biological inheritance misled the 19th century.

Even if there is no unanimity, such ideas remain influential and pervasive. The question of human thinking is a matter of our attitude toward ourselves, and is emotional lightning. I will devote extra space to human thinking just so that we can see ourselves as objects of interest, and maybe reach a reasonable understanding.

When we think about thinking we are accustomed to lifting our gaze upward to the blue of higher thinking of which there seem to be two kinds. One is creative thinking, whose purpose is to create something not dreamed of in the philosophy of others. The other is logical thinking whose purpose is to get the right answer. While the two are distinct, they nevertheless cooperate in the sense that, after creative thinking has produced an idea, logical thinking may then be mobilized to confirm it.

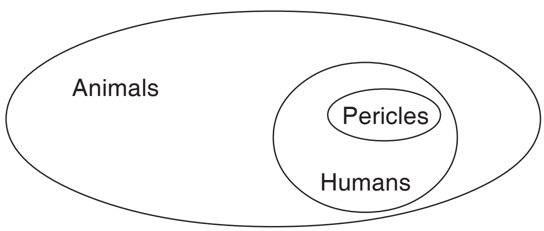
When we think of thinking we tend to think of the greats – Homer and Shakespeare; Archimedes, Newton, and Einstein; Aristotle and Russell; Praxiteles and Michaelangelo; Bach and Beethoven. As they recede into the past, the greats acquire enormous importance because they tend to become the only voices that speak for their respective centuries. Their works are in no danger of being lost today. But these people are great, poets and scientists alike, because they say for us what we could not say for ourselves. Virgil, apparently, was hired outright to write an epic for Rome, which had no Homer, no Iliad, and no excuse. Let us look more closely, then, at the content of everyday thinking.

The archetype and apotheosis of logical thinking is the deductive syllogism of Aristotle, who wrote, about 350 B.C., on the high art of the right answer. In his *Prior Analytics* (Book I), Aristotle writes that a syllogism consists of true statements together with consequences that necessarily follow from them (25.4). For example,

"Every	M is P	(All border-war is evil)
	S is M	(Border-war between Thebes and Athens is border-war)
therefore	S is P	(Therefore border-war between Thebes and Athens is evil.)"

The modern examples are more graceful, but everybody knows them already, and it is sobering to see how we often struggle to find the best words for explaining even the simplest ideas. This example is taken from the great Eleventh Edition of *The Encyclopedia Britannica*, (Chisolm 1910, Volume XVI, pages 879-880), the only serious attempt to include all human knowledge between a set of covers. Considering that it can't be done, they did pretty well. Before the Titanic carried the dream of perfect knowledge to the bottom of the sea, the Eleventh Edition steamed ahead fearlessly. With its meticulously recorded references and pellucid index, it is the only encyclopedia worth consulting.

The logical syllogism was a set-theoretic set-piece before there was set-theory, and works by sub-sets of sub-sets. To modernize, very slightly, another example from the great Eleventh Edition:



"If all humans are animals,
and if Pericles is human,
then Pericles is an animal."

If, instead of “M is P”, we state that “M might be P”, we have also stated, in effect, that “M might not be P”, i.e., we have introduced a fourth term between the first and the second. This situation, known as “the fallacy of the ambiguous middle”, invalidates the syllogism formula. Thus (42a) a syllogism contains “three mental judgments - - - the two first are the premises which are combined while the third is the conclusion which is consequent on their combination”. The truth is strictly regulated according to a rigidly structured formula.

Even the so-called “fuzzy logic” (see Kosko et al. 1993), which is intended to free us from the fallacy of the ambiguous middle by permitting regulated violations of it, never escapes entirely from the Aristotelian orbit. Fuzzy logic might state, for example, “if the radio is just a little too loud, then it might be OK to turn it down a little.” Such reasoning, applied to thermostats and loudness meters, has proven very useful for regulating machinery. But fuzzy premises and conclusions remain fundamentally syllogistic and Aristotelian, and still derive the right answer by a syllogistic formula.

Aristotle was trying to set forth the formula for all truth for all time. He was a naturalist, the kind that is interested in animals. Aristotle used observation but didn’t use experimental method. If you want to establish eternal, universal truth by pure force of genius, it is impossible to do better than Aristotle. His only flaw, and it is a forgivable one, is the “endless repetitions of the same thing” that follows from a logic-based science, about which Francis Bacon (Preface to *The Great Instauration*, 1620, page 7) bitterly complains, and which Bacon uses as the reason for establishing his new, empirically-based science. Instead of formulas for universal truth, Bacon offers formulas for pinning nature down by using what he called “fingerposts”, something you can put your finger on (1620 Book 2, aphorism xxxvi, page 191), and that we would call the single variable factor. If Geoffrey Chaucer had already made a joke of the scientific experiment two hundred years before Bacon invented science (See The Summoner’s Tale, lines 2253 ff), it was no fault of Bacon’s.

In spite of Bacon, and in spite of Chaucer, Aristotle dominated science for 2,000 years. He is still taught in introductory philosophy courses everywhere, and his grip on the modern imagination is astonishing. The previous owner of my copy of Aristotle, for example, has adoringly inscribed the back flyleaf of the book with his "PANPHILOSOPHON" that might be chiseled into the marble frieze of some celestial temple. It includes Kant and Plato, Whitehead and Husserl, but not Aristotle.

Poor Aristotle. Rejected by Francis Bacon, he couldn't even get his name inscribed in the Hall of Fame on the back page of his own book. He offered what everyone wants – perfect understanding through the right answer – and remains better-known, and better-loved, than Francis Bacon ever was (see Bowen 1963). If Aristotle had known Gödel's proof, he would have realized that you can't deduce universal truth from a fixed set of premises. All you get is "endless repetitions". Francis Bacon, in essence, enabled Aristotle to get around Gödel by providing an avenue for the introduction of new material.

From the Sphinx and Oedipus, to the Educational Testing Service, Western culture not only worships the right answer, but enforces it like a life sentence. And may God or Zeus help the student who gets the wrong answer, and spends a lifetime regretting it. With no hope of escape, you might as well try. The right answer is a discrete entity, and can be identified on that basis. Even such "philosophical" answers as "There are no right answers", or "Life is a journey", or "Why not?", are readily-identified, discrete answers.

But creative thinking is illuminating, too. Less adored in the wide world than the right answer, but nevertheless carefully studied, is the creative process itself; and there are many reasons for studying it. One is the didactic. If person A knows something that person B does not know, it should be possible, by comparing the two, to discover what the difference is, and to teach it to B. If A was born with some natural gift, the didactic process will have only limited success. But if not everyone can be great, everyone can be good; and the didactic approach will always have its place.

Another aspect of the didactic is simple copying, a method that has been used since Antiquity, and that comes as close as anything to allowing one person to enter the mind of another. Ben Franklin, for example, tells us that, until he developed a writing style of his own, he consciously copied that of Addison. And at any large museum of art, students of all ages may be seen seated before easels, colors in hand, entering the minds of the masters by copying their works.

The creative process, not only of great writers and great artists, but of great scientists, has been studied (Hadamard 1945); and a few patterns seem to repeat themselves. One of these is that a new idea becomes conscious only gradually, and that many ideas start as hunches, i.e., you have to know the right answer before you can discover how to get it. Logical thinking is separate from creative thinking. It confirms the right answer, but is helpless to obtain it in the first place. Creative ideas are often seen as a gift, as Homer does in the opening lines of the *Odyssey* (ca. 850 B.C.). The translation (1932) is by T.E. "Lawrence of Arabia" Shaw – a war-story translated by a soldier.

> "Make the tale live for us - - -
> O muse"

Homer felt, as many creative persons do, that his creative ideas were an inspiration that came from something beyond himself, a gift from the Muse. In the modern day, many creative persons feel the same way, that their creative ideas are a gift from God. But here I will treat them as a gift from first principles, whose direct use, no-doubt, is what generates consciousness and the illusion of God. The mere existence of a creative mind and thought in human life suggests direct access to first principles, not the evolution of a brain under local and contingent conditions, operating from a basis that is fixed in the accidents and coincidences of some distant time and place, and going no farther than the axioms underlying the equation that expresses its selective advantage. Pre-determined, in effect by a set of empirical postulates incorporated during the formative period, and upon which subsequent ideas must be built. Like David Hilbert's Program for the construction of arithmetic from a set of obvious truths, the evolutionary program is incompatible with the observable creative property of the mind, which is not subject to Gödel's proof. And separation of levels shows the same thing: Higher-level intellect is not built out of lower-level parts that somehow fit together.

If creative thinking generates the words and the images and the inventions that bring glory to persons and even to nations, ordinary thinking makes us part of our communities. Ordinary thinking is the stuff of daily life (Labov 1972, page xiii), of gossip and shared recipes. It decides not fashion, but what the great numbers of us wear from day to day. It decides our shared attitudes toward sex, politeness, table manners, cleanliness, loyalty, honesty, human goodness, laws and morals, what the community expects of us. Common thinking decides what is OK and what isn't OK. In some collective form, it decides who will work and who will play, who will die in war, who will succeed, and who will fail. Such thinking is simple but important. What form does it take?

As much as we feel that our personal values, and the ideas that support them, are reached on an individual basis, supported by proofs that are both numerous and obvious, they are mostly a matter of consensus and conformity, reached by long discussion and fear of disapproval. Communities remain stable when people agree on things; and people need to express an opinion in order to bring the occasional odd dissenter back into the fold, or to establish dominance as a way of maintaining unanimity, or just to look good. People use logic to pick a fight they think they can win, as Lincoln did with the riddle of the horse's tail.

When someone says, at the beginning of a conversation, "You're crazy. He walks and almost talks in complete sentences. He's no baby." And says at the end of the same conversation, "You're crazy. He doesn't care about me, or anybody but himself. He's just a baby", you have to recognize the logic of all that logical thinking.

Or someone brags "Of course I'm never cold in winter. I produce so much body-heat". Then, a few minutes later, sitting down in a seat just vacated by someone else, "What's wrong with you? You produce so much body-heat." Or a person says, "I'm Smilin' Jack", possibly alluding to the pilot-hero of the novels and comic-strips, then accuses others of being "a grinnin' @#$%&!!" we have to see the real content of the logic. Such nonsense, lacking all objective merit, is presented in the form of discrete ideas formulated as grammatical, propositional sentences. People dumb ideas down to a level where they can understand them. If "Family is the most important thing in the world" is too complicated, they resort to "Family is the only important thing in the world". And if even that is too complicated, there is always, "Family is the only thing in the world". It works.

The common, shared reasoning process appears in its most accessible form in the records of legal proceedings where logical thinking is used explicitly (Kaplan et al. 1987, page 69) as a means of establishing and maintaining the social order. In spite of Daniel Webster and Clarence Darrow, legal reasoning has little in common with Aristotelian truth. Instead, the history of the law is a record of the mechanism that regulates our communities and, increasingly, our families.

The law is common gossip elevated to a high plane by its public, formal setting, by the verbatim transcripts that are sedulously kept, and by its momentous portent. "All trials are trials for one's life" says Oscar Wilde (Ellmann 1988, page 435). Let us look at a few cases at law, and see how they retain the formal structure of logic while actually defending something else.

One case that gripped the country toward the end of the 19th century in much the way that the O.J. Simpson case gripped it toward the end of the 20th, is that of Lizzie Borden. Everybody knows the story.

> Lizzie Borden took an ax
> And hit her mother forty whacks
> And when she saw what she had done
> She hit her father forty-one.

A distant cousin of mine, moving to Fall River, Massachusetts, when Lizzie Borden was an old lady, was told by her mother, “Don’t play with Miss Borden. Miss Borden wasn’t nice to her mother and father”. But did she do it? Edgar Lustgarten’s brilliant account (pages 263-298 in London 1960) tells how the jury reached its verdict. Prosecutor Hosea Knowlton presented a litany of circumstantial evidence showing that Miss Lizzie was the only person who possessed enough minute-by-minute knowledge of the Borden household to have committed the murders undetected. He shows means, motive, and opportunity, as well as an apparent failed murder-attempt on the day before the actual murders, destruction of evidence, and a fabricated alibi.

Against Knowlton’s overwhelming evidence, defense lawyer George Robinson, former governor of the Commonwealth of Massachusetts, much like Big Jim Thompson, former Governor of Illinois in our own day, asked (page 295) “To find her guilty you must believe she is a fiend. Gentlemen does she look it?” The gentlemen of the jury didn’t think so.

What happened? Edward H. Levi, former professor in the University of Chicago Law School, former President of the University of Chicago, former Attorney General of the United States, who does not represent the lunatic fringe, tells us (1949, page 6) “Erroneous ideas, of course, have played an enormous part in shaping the law.” He continues (page 9) “The legal process does not work with the rule but on a much lower level.” It is precisely this level that interests us here.

Levi describes a succession of cases. He begins with the 1842 case of a coach-driver who was injured when the coach fell apart, but who could not collect because such a ruling (page 12) might lead to “absurd and outrageous consequences”. Levi goes on (pages 22-23) to the case, 75 years later, of a woman injured by a defective kerosene lamp. She was allowed to collect because a thing can be dangerous “even if no one thinks of it as an implement whose normal function is destruction”. The ruling, by no less than Benjamin Cardozo, represents more a change in social attitudes than an advance in logic. Cases in medical malpractice now seem to be going back to 1842.

The cases themselves have nothing to do with the rulings, which are decided "on a much lower level".

Levi (page 23) tells us that "It is traditional to think of logic as fighting with something". Since logic is fighting with red-herrings, our work is cut out for us. We must discover the precise structure of the red-herring.

A red-herring is an irrelevant statement made in denial of (or affirmation to) some other statement. The red-herring has the form

(negative red-herring):

first speaker says:	second speaker says:	audience concludes:
"X";	"but A";	"Therefore X is false"

or
(positive red-herring):

first speaker says:	second speaker says:	audience concludes:
"X";	"and A";	"Therefore X is true"

The red-herring may be represented more formulaically. Since X adopts the truth-value of A, the general formula is this.

"X"; "$\pm$A"; "$\therefore\pm$X"

The red-herring can derive conviction from several sources.

1) It has the same form as genuine objections.

2) The more truth that statement "A" contains, or the more generally believed that statement "A" is, the more it narrows attention

to itself, distracting people's attention from the crucial question of its relationship to "X".

3) The more words or ideas that "A" shares in common with "X", the greater apparent relationship that "A" has to "X".

4) If "X" is unspoken, then "A" may remain the only idea that is noticed by an audience.

5) "A" can be simply wrong, and yet succeed, if it can not be double-checked or verified.

Let us review sources 1 through 5 to determine the mechanism of each. Source 1 has the form of an ordinary denial for cause. For example.

"X": I bought this cucumber in Fond-du-Lac yesterday.
"-A": But you have not been out of Oconomowoc for a week.

If "-A" is true, then "X" is not true in any ordinary sense" This establishes a formula for legitimate objections and counterexamples.

But if the conversation concerns a different subject the result may be very different, e.g.,

"X": People ought to brush their teeth after breakfast, not before.
"-A": But you are not a dentist.

Here, statement "-A" is complex. It derives conviction from its formal similarity to a genuine denial, but makes the foolish assumption that only a dentist could have a good idea about teeth. It derives further conviction from being true, i.e., the speaker probably isn't a dentist. If delivered well, "-A" attracts an audience's attention to itself, making "X" irrelevant, and quickly forgotten. Added layers of linguistic embedding, such as "Are you a dentist?" make the red-herring even harder to deny.

An objection based on ideas that are not true, but that appeal to some hidden belief or ethic that makes them desirable, can have the same force of conviction as if it were true (Burckhardt 1860, page 170; Chubb 1940). The myth of "confusing" little children and animals is one such idea.

"X": Let's take the baby to the party.
"-A": But when it meets all those adults, it won't know who its parents are.

The foolish myth of confusion can surface any time that children or pets are involved with grandparents, adoptive parents, large families or any large groups. Objections are raised against speaking in the presence of babies because "All the unfamiliar words will only confuse it". An adopted child who visits its biological parents will become "confused", as will a dog that is brought into a large household. Children accustomed to hearing music come out of a TV or radio will only become "confused" if they hear it come out of a musical instrument. Persons who never showed any concern about anyone can win great praise by mentioning the myth of confusion at some suitably awkward moment. The myth of confusion finds its way into court rulings, where it becomes the law of the land. Apparently the myth of confusion succeeds because it possesses an appearance of kindness, helping the helpless, etc. and because persons who possess instinctive kindness never thought of it, and have no answer ready to counter it.

"X"; "But A"; "therefore –X"

The myth of quality time is a red-herring similar to that of confusion, and has more-or-less the same purpose. The reader will forgive the space that I devote to red-herrings, but they are more pervasive than is easy to believe. "It is a bad idea to breastfeed your baby because you never know how much the baby ate" is a red-herring ("But A") occasionally delivered by health care professionals. It is true that weighing babies is notoriously inaccurate, but for a healthy baby, minute-by-minute weighing is pointless.

Another common red-herring ("But A") is "Ordinary people need to be given something to do", where an un-spoken "X" might have been, "Why do you have to run other people's lives?" I lived fourteen years as the resident adviser to a fraternity at a technical college; I have worked in factories and shelters; and, while hunting dinosaur fossils, hitch-hiked all over the American and Canadian West. I have met lumberjacks and dentists, truck-drivers, ranchers and gamblers, even the Alberta Provincial Egg Inspector and his son, not to mention the egg-boss at a Hutterite colony – and can assure readers that all classes have about the same knowledge of what to do with their time. But you can still hear members of powerful families repeating the old myth of "giving ordinary people something to do".

> The Indian wars of the Wild West, which continue in only partially abated form, produced some ripe red herrings. Here, the un-spoken "X" must be The Golden Rule.

"X"	"but A"	"therefore X doesn't apply"
Golden Rule	"They weren't doing anything with the land anyway"	
	"You can always tell Indian land – they never do anything with it"	
	"They accepted the steel ax-heads didn't they?"	
	"In terms of numbers, just numbers, there are more Indians now than at any time in history"	

More insidiously, because it masquerades in learned clothing, the red-herring

"X"	"but A"	"therefore X doesn't apply"
Golden Rule	"The market is impersonal"	

is the brain-child of a well-known school of economics. Apart from learned books, it was collected for me by a friend who was handed the red-herring, along with too little cash, for a table and chairs and some furniture, by the member of that very school of economics. The market is not impersonal. I know machinists who will pay double and more for the privilege of bragging "I have no Japanese tools in my tool chest", even though many Japanese tools are superb. Against the impersonal market, you can hear, "What it comes down to is, who do you want to hand money to?", or "I just couldn't bring myself to hand him money". You can hear, "I want to pay you because the next time I need you, you'll be there", and "If you have to steal it, it's not worth having", and "I always come to you first", and "I never thought a good reputation would be worth money in the bank", and "I found out that the market is built on trust".

Against the Golden Rule, I unwillingly collected another red-herring while having breakfast at a diner in rural central Montana. The man next to me at the counter was telling me about the Hutterites, a religious sect similar to the Amish, who had a colony near there. "The Hutterites are the best citizens and the most progressive farmers in the territory. They introduced the computer to farming. They keep their animals under the cleanest conditions. They always support charity. They have no crime. If you are in trouble, the Hutterites are the first to help you out. 'Course, this place wouldn't amount to much if everybody around here was Hutterites, now would it?" I later learned that my neighbor was the most powerful rancher in the territory. "But A."

I collected another red-herring while I was studying the Chinese language in Philadelphia during the middle 1960's. Several times, strangers accosted me in the street and asked, “Are you a Communist?”, to which I answered, truthfully, “No”. But they countered, triumphantly, “But you’re studying Chinese, aren’t you?”. It was true but irrelevant. I only wanted to know a really non-indo-European language. “But A.”

Red-herrings are regularly the excuse for war. In Book 5, Chapter 7 (page 361 of the Rex Warner translation) of Thucydides’s *History of the Peloponnesian War* (about 400 B.C.), for example, the Athenians justify their attack against the Melians by explaining to them, “We have a right to our empire because we defeated the Persians”, and “You are either with us or against us”. The Athenians eventually killed all the Melian men, and sold the women and children into slavery. “But A.” In our own time, A. Hitler’s 1939 invasion of Poland (see Toland 1976, page 81) was triggered by an order that explained, in effect, that negotiation had failed. “But A.”

People believe sense, any kind of sense, no matter how foolish, if it is presented according to the right formula

“X”; “$\pm$ A”; “$\pm$ X”

provided only that “A” has something to do with “X” or with the person who said it. Aided only by the thinnest similarity between “X” and “A”, the red-herring formula is capable of forming a convincing association in people’s minds. Iona and Peter Opie, who are our best guides to the human mind, tell us, (1959, page 92), “Why the Prince of Wales should be put in jail for riding a horse without a tail, not even the teenagers know”. But we know. A rhyme is a red-herring where the truthful part is the rhyme itself.

“X”	“A”	“∴ X is justified”
The Prince of Wales Went to jail For riding a horse Without a tail	it rhymes	

> For many years, until forced by the Supreme Court to change their policy, American medical schools would admit only the youngest candidates. "Years of service" was their litany, the younger the student, the more years of service.

"X"	"A"	"∴ X is justified"
youngest candidates	years of service	

A former Peace-Corps volunteer or shelter worker might have a more keenly developed sense of human help, but "years of service" was the only thing. When one hears from doctors and medical students, "I went into medicine because I wanted to help people, and to get a little something for myself", one begins to feel that "I went into medicine because" is the official grease that lubricates the medical curriculum, and that a younger medical student is simply more naïve and more malleable.

As a matter of persuasion, our lives are illuminated not only by I.Q. tests, but by lie-detector tests. These concern some observable behavior as an index of truth, and are much easier than going out to discover facts. If someone "refuses to look me in the eye", that person is obviously lying.

"X"	"but a"	"∴ X is false"
	refuses to look me in the eye	

Shifty-eyed persons are always lying, as are those whose eyes are too close together. Any kind of moving around while talking is proof of lying. In ancient times, lack of aristocratic standing was proof of lying, which is why the extra ingredient of torture had to be added in order to guarantee truth. Today, many politicians (not all, but many) have movie-star good-looks, suggesting that voters subject candidates to some kind of public ugly-test. Indeed, in the United States, the taller Presidential candidate nearly always wins – and there are no bald Presidents. A woman candidate for high office in the state of Illinois not too many years ago was accused by her male opponent of not being good-looking enough for the job. She asked, "What is that supposed to mean?" – and lost.

The above comes nowhere near to exhausting my collection of red-herrings, but is intended to show how deeply they invade every aspect of life. Iona and Peter Opie tell us (1959, page 121) that "the schoolchild code is of barbarian simplicity", and that "even the deliberately swindled has no redress if the bargain has been concluded by a bond word". We grown-ups are not far behind. Somehow, "Tix, tax, no trade backs" embodies truth in the final sense.

Even those not interested in religion understand its overwhelming importance as a matter of history and anthropology. Today's wars, like many in history, are in real measure wars of religion, or race-wars justified in religious terms. In spite of a body of philosophy offered by every great religion, and in spite of the occasional odd joiner, we say our prayers because our parents and grandparents said their prayers. And our ancestors said their prayers because that is what God told them to do.

"X"	"A"	"∴ X is justified"
we say our prayers	because our ancestors said their prayers	

If religions really consisted of a philosophy or a body of ideas and teachings, they would be like the departments at your local university. You would sign up for the one that agrees with you the best. But religions are not like that at all. A religion is an estate in place and time. Christianity in Europe; Islam in North Africa (Walker 1918, 1952); Hinduism in India; Buddhism in China. Many times, religions are driven out of each others' territories by force of military arms (Walker 1918, 1952, pages 159-160), so each seems obviously right in the territory it now occupies.

Since religions don't regularly succeeded in persuading other religions to convert, none can claim obvious underlying logic. Religions that actually exist represent an accidental selection from an infinite range of possible religions, all with equal claim to world truth. Just recently, for example, two Protestant sects, contemplating merger, found that they could not agree on whether Jesus was personally present at the Last Supper – and called off negotiations. Where such minute distinctions are passionately held, at best the chances are infinity-to-one that any given religion is the true one. At worst, they are infinity-to-nothing. Most prayers and creeds, if recited by an isolated individual in a psychiatrist's office, would be cause for hospitalization. They are reasonable because they are normal, i.e., they are recited by thousands and millions who inherited them from the past, and who think that the other fellow's religion is cause for hospitalization – or worse.

A little child who sees an entire congregation reciting a prayer needs no further justification for it and grows up accepting it. Capital punishment is more acceptable in the American South than in the North. No matter how convinced we are of the correctness of our thinking, we believe what we believe because our neighbors believe it, and our ancestors believed it, or at least we think they did, because the most rewarded single virtue in this life is still conformity.

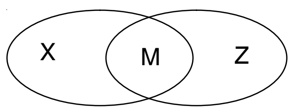
"X"	"because A"	"∴ X is right"
	our ancestors did this	

The human mind or brain is not a truth-seeking engine, but a formula-driven engine that relies on a few formal structures to decide truth. Why do we constantly have to be reminded that "To understand is not necessarily to forgive"? Why couldn't the jury convict

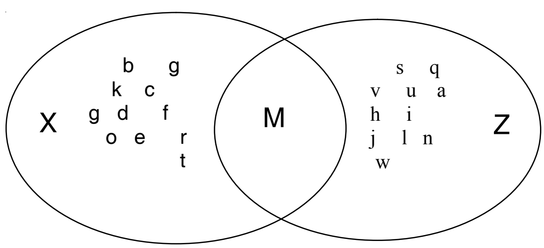
Lizzie Borden in spite of "her patrician profile"? Because our minds don't work that way.

The algebraic-geometric structure of the red-herring shows that common thinking, like syntax, is a process apart from meaning or truth. It is purely formal. The reader will have recognized that the red-herring is the argumentative arm of the more general class of cognitive mechanism called metaphor, which has the form "X is Z". A good example of a poetic metaphor might be "Life is a journey". Life is a lot of things. Life is a matter of metabolism, sensitivity, and growth. It is a matter of nutrition and reproduction, of struggle and danger, of judgement, and experience, happiness, sadness, tenderness, violence, love, hate, sex, religion, and language. We can get away with "Life is a journey" because, for the sake of conversation, we agree to focus down on one thing at-a-time. That is metaphor.

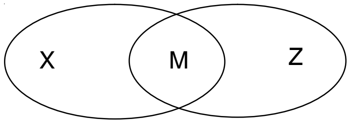
In general, metaphor becomes possible when any two items of thought share any constituent in common. Thus, metaphor has the set-theoretic formula



where we can say "X is Z" because X and Z share M. A full-disclosure formula for a metaphor might look more like this,

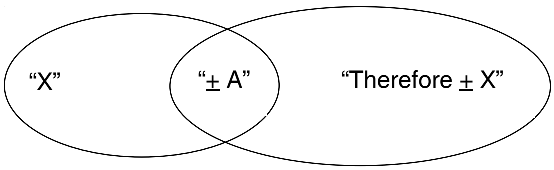


where M represents the shared feature that we are interested in. To sum up, the formal structure of the metaphor is



Therefore "x = z"

while the formal structure of the red-herring is



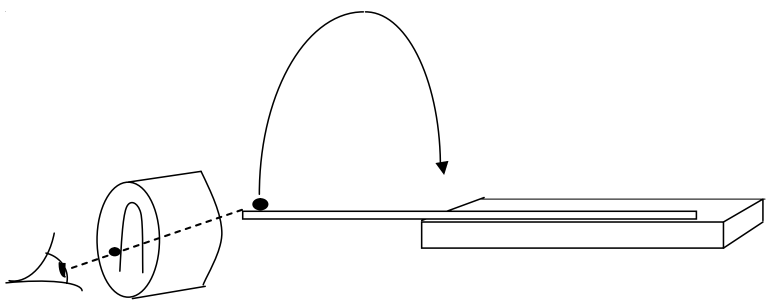
Where A shares any feature with x, or with the speaker who said x, and where A is perceived to be true.

Human thought, for the most part, is not a shapeless force that gradually molds itself into truth. Thought is not a malleable clay; and the image of gradual behavioral shaping is certainly wrong. For the most part, thought is a simple structural formula of the geometric type that characterizes arithmetic and language, and probably has the same source, first principles. The illusion of plastic deformation in thought is created in the same way as the illusion of blending inheritance in genetics: numerous tiny steps. When there are enough steps, more than ten or so, they become too difficult to detect, and appear to be a blending mechanism. Gregor Mendel is sometimes praised, and deserves to be praised, as much for selecting the right experimental species, as for doing the right experiment. The garden pea has only a few inheritable differences, i.e., few enough so they could be separated clearly by experiment. In science, the material must fit the question, and I have tried to apply the right methods to demonstrate the structures and mechanisms and operations of the mind.

Any good experiment has to be preceded by thought. I promise to propose experiments to test the numberline theory, and have already suggested one, the SETI experiment, apparently in progress already. If the SETI Institute is more interested in chemistry than in language and mind, maybe they can be persuaded to have a second look. I have tried to develop the right search image, so I could design the right experiment. But the nature of mathematics is such that it is not accessible to experiment. Its proofs are more abstract than that, giving better certainty, but not observable verification.

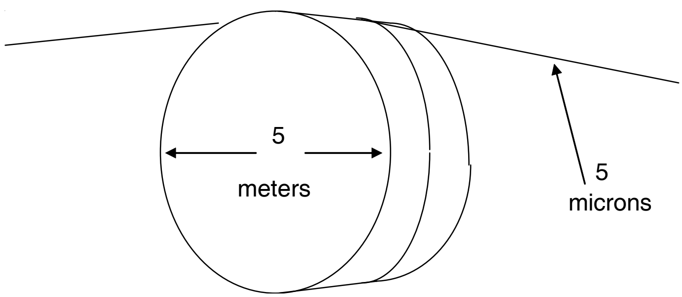
Logic Versus Experiment in Mathematics

When I was in high school, I once built a machine to test the proposition that all parabolas are the same shape. It was built in two parts. The first was a catapult made from a hacksaw blade with a little cup at the end. By pulling the blade down a fixed number of steps on a graduated scale, it was possible to launch a ballbearing a long distance, or a short distance, or any distance in-between. The other half of the device was a detector, consisting of a World War II bombsight fitted with a special reticle on which I drew a parabola in India ink. At the time, war surplus could be obtained cheaply through mail-order, so the instrument was not as exotic as it seems now. The idea was that, if all parabolas are the same shape, and if the path of a ballbearing launched into the air was a parabola, then it should be possible to adjust the distance of the bombsight so that the ballbearing would follow the India-ink line, no matter how far the ballbearing traveled.



My algebra teacher laughed and explained that math is done by logic, not by experiment, and that logic is in some important way more reliable than experiment. I now realize that he was right. If mathematics were an experimental science, we would not know about the existence of transcendental numbers, the closest to magic that actual numbers can get in the modern day. If not magic, at least mysterious. Instead of realizing that the number π has an infinite number of digits, and that we will never know them all, we would see the value of π as an engineering problem. We would keep getting more and more accurate, closer and closer to the right answer, by using measurement.

Here is a perfect example of asking the wrong question, and still getting an answer that is valuable and reasonable. "How do you measure the value of π to one more significant digit?" We would be debating how to pull a quartz filament finer and finer without breaking it. We would build atmosphere-controlled hangars to house the larger and ever-larger granite cylinders. We would develop ever-better statistical methods to correct for errors in measurement, and trigonometric formulas to compensate for the thickness of the quartz.



The mistake would be more-or-less like the modern program in evolutionary psychology and linguistics. A great and ambitious enterprise in the social pursuit of science. More and more good ideas. Dazzling expertise. Competitive grants for an important scientific goal. Announcements of almost daily progress. Vast sums of money and teams of famous scientists working together for the advancement of knowledge. Genuine engineering improvement with each new digit. Prestigious awards given to the best and the brightest. Is anything wrong? Of course not. Show me a mistake in the calculations, in the design of the machine that grinds the circular cylinders.

There are no irregularities in the thickness of the filament. We know how to compensate for the thickness of the mark where the filament crosses itself at each full revolution. Nothing is wrong anywhere. Who would listen to the voice at the door, claiming that there are transcendental numbers, and that π is one of them? The transcendence of numbers was proposed in 1733, but was proved only in 1840 (Beckmann 1971, pages 157, 167), so the story isn't even far-fetched. It could have happened. On a different stage, it is happening now in evolutionary psychology and linguistics. Truth compounded with truth does not always add up to the right theory.

We now find ourselves in a kind of methodological double-bind. For a long time, it was thought that the closest thing to linguistics is psychology. So, if you wanted to know how language works, you studied psychology. But now we see that the closest thing to language is arithmetic, not psychology. If we want to know how arithmetic works, we should study language. But we can no longer study language just by gathering observations and interpreting them, because language is an altered form of mathematics. We need some kind of example, in the form of an older, more experienced science, to show us what to do next.

Lessons from History

Under the cladogram, the closest science to language and arithmetic is genetics. If we want a guide to the study of arithmetic and language, we should look at the history of genetics. Mutatis mutandis, we can expect the future study of arithmetic/language to be a re-run of the history of genetics. If we can identify the lost opportunities and wrong turns, the sins of omission and failures of the spirit, we will know what mistakes to avoid, and where we can push through to a solution. We might develop a search image for the structures that underlie language/arithmetic in the brain, and might even find that we have to forge new intellectual tools if we hope to understand foundations.

I would love to offer a history of the cladogram model, or number-line theory. But, since the theory is new, it has no history as such. It has no experts, no heroes, and no intrinsic standard of "excellence". Instead, it represents the mining of ideas from formerly separate subjects, and modifies them as necessary into a single theory. For the most part, the theory is derived by placing each new construct into some larger context, again and again, until there are no contradictions. The early events in the history of the cladogram model concern the idea of natural particles, or units that are indivisible or discrete. And that is where we must begin.

The separate vocabularies used in different sciences to describe indivisible entities are so different as to show that a single idea may develop independently, and at different times, in different sciences, even when important investigators know one another personally. We really do believe in the scientific "fields" that we invent.

In spite of Lucretius (55 B.C.; also Bailey 1928,1979), the modern history of particles in nature begins in 1805 with the unsentimental John Dalton (Greenaway 1966), who was more moved by the volumes of gases that combined in ratios of small whole numbers, than by constancy in the midst of change. Lucretius, that best of the Romans, was well-known in Europe by 1800. His atoms were based on the completely reasonable observation that things like trees and mountains are always being worn away and washed away, yet the world remains more-or-less the same. If everything is made of unimaginably tiny objects that can not be cut into smaller pieces, i.e., "a-" [not] "-tom" [divisible], these could re-arrange themselves again, reconstructing the lost structures. Dalton introduced "atomism" into science, in opposition to the possible "continuity" or "wooliness" or "sponginess" or "permeability" of matter, questions that are occasionally raised even now by peevish scientists who can accept the idea of "discrete entities", but not that of particles.

In the 1820s, the brothers Grimm, more famous now for their collection of fairy-tales, examined not sound-changes, exactly, but what they thought of as changes in the letters used for spelling different words. The idea that Grimm's Law of language-change applies to sounds is a modern anachronism like walkie-talkies in a movie about

Nero. So, too, is the word "phoneme" that we sometimes see in translations of works by the tireless brothers. To them, their word "Sprachlaut" meant "the sounds of speech", nothing more. The word "phoneme" was ascribed to them, without their permission, by more modern enthusiasts wishing to exhibit their learning. The brothers Grimm used discreteness while taking it for granted, because the letters of the alphabet are so ever-present as to make them seem obvious, even natural.

In 1836, as we have seen, Wilhelm von Humboldt introduced the phrase "infinite use of finite media" without questioning what it might mean. He never mentions "infinite use of discrete media" or "infinite use of blending media", both of which are possible; and we may safely suppose that he never though about discreteness, any more than the brothers Grimm did. In part, when Steven Pinker, on page 84 of his oddly successful *The Language Instinct* (1994) explains that "infinite use" "captures" the idea of combining and re-combining discrete constituents to create an infinite number of possible signals, he is doing the syntactic equivalent of translating "Sprachlaut" as "phoneme". More profoundly, he is taking discreteness for granted, when it cries out to be understood. Under the phrase "discrete combinatorial system", he emphasizes discreteness and the re-arrangements of discrete elements on pages 84, 85, 89, 92, 93, 97, 127 (where he calls it "the engineering trick behind human language"), 162, 163, 179, 237, 269, 334, making it the stitching that holds the book together (although it is not listed in the index). Nothing could be more basic, especially for the founder of a movement in science.

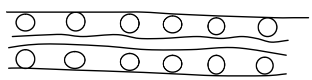
Pinker should have known better, because he lists my (1989) article "On the particulate principle of self-diversifying systems" in his bibliography as well as his end-notes. Sally Shaywitz (2003, pages 46-47), by contrast, gets the particulate principle right, and calls it "a brilliant insight" that is "elegant in its simplicity". My article is the first, as far as I know, to generalize the necessity of discreteness from the gene to language, arithmetic, and all systems with emergent properties. Pinker's reader sees only the description of an objective state of things; but Pinker has omitted the most important part: the necessity of discreteness, not just the fact of discreteness. In the last analysis, Pinker's omission, combined with the fact that no one caught it, shows that the modern agenda in evolutionary psychology/linguistics has never been thought through carefully, and is intellectually vacant. The situation is more regrettable because it could have been avoided. Since 1862, at least, some biologists have been confident that language appeared by natural selection. It is the obvious first idea to try. But biological involvement is not a biological basis. Everyone has known since at least 1803 that people are animals (Paley, page 150), so there will be biologically-based infrastructure to everything about them. But that does not, in itself, mean that language is biological in origin, any more than the calcium in our bones is biological in origin. In effect, we have an inorganic device that is carried about on a biological platform, like a teenager walking around with a radio.

In the later part of the 1860s, Gregor Mendel published his algebraic expressions detailing the number and nature of genetic combinations that might be produced by parents with a few simple genetic characters. It would be 43 years before Mendel's ideas would become generally known or accepted, but he is now recognized as the discoverer of the particulate gene. Nothing could be farther from the truth. Negative evidence is hard to prove, but if you will examine Mendel's two published papers on genetics, and the letters that he sent to Carl von Naegeli in support of his combinatorial algebra, you will find no mention of a genetic material, let alone that it was of a discrete or blending or any other nature (Mendel 1866, 1966; Wilson 1916; Fisher 1936; Stern et al. 1966; Olby 1985).

If Mendel ever thought about discreteness or the physical nature of the genetic material, he never brought his ideas to the attention of others. The modern myth of Mendel was created in 1930 by Ronald A. Fisher, the father of modern statistics. Whether you know it or not, whether you like it or not, when you think about statistics, you are thinking about Ronald A. Fisher. The idea that the genetic material is particulate, or "granular" in his words, was introduced in

1893 by August Weismann, a German biologist and chemist. Weismann was interpreting what we would now call bands on what we would now call chromosomes, and was trying to explain what he called the immortality of the germ plasm, or what we might call the transmission of the genetic material from one generation to the next. Weismann called his genetic units "ids", what we would now call genes, and even published a picture of them (page 67) which he attributes to an author named Bovari. The picture might have been of interest to Watson and Crick.

Weismann gives no reference, but Theodor Bovari is to be found on pages 48 and 121 of Franklin H. Portugal and Jack S. Cohen's (1977) *A Century of DNA*. He was a co-discoverer of meiosis. Portugal and Cohen have a keen eye for detail, and also tell us, on page 79, that Ivan Pavlov was mugged in Grand Central Station, New York City, in 1923. For one brief moment, the great Russian physiologist, whose work continued straight through the Russian Revolution, appears close, and touchingly human.

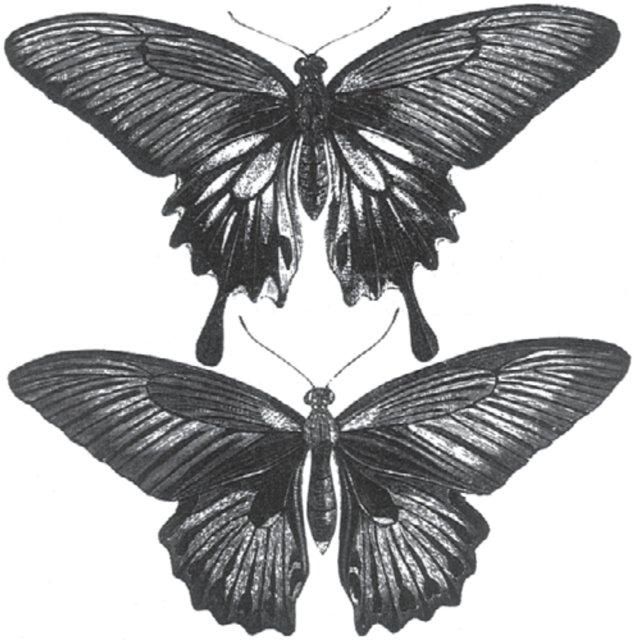


Fisher never actually states that Mendel introduced the idea of the particulate gene, but just mentions Mendel alongside the particulate gene. He trivializes Weismann by attributing to him the minor theory of mutational momentum. But in 1930, Fisher knew about Weismann's ids. A search of several volumes of Fisher's collected works (Bennett 1971) turned up not a single mention of them. But in an obscure 1926 book review, Fisher slipped. His mistake was to accept an assignment to write a book review in the first place, because it forced him to relinquish control over his editorial material. Fisher writes, "In chapter ii, in particulate theories of heredity, Professor Morgan touches on Darwin's gemmules, and Weismann's ids".

In what may be seen as an act of mathematical partisanship, Fisher attributes the particulate gene to his fellow-mathematician, Mendel, rather than to the biologist Weismann. Journals in the history of science will often reject articles based on published material, because "experts already know". But experts don't necessarily know. And a scientist of such total determination and absolute resolve as Fisher would never confide his ambitions to anyone. He was completely capable of total silence, and must have sensed that, without a letter or a diary entry, he could recruit future historians as accomplices after-the-fact.

Weismann (1909) spent the rest of his life in defending Darwinism. After a disease of the eyes forced him to abandon experimental science (Portugal et al. 1977, page 102), Weismann pursued theory, explaining that experiment can not profitably continue without it. Although its role as the genetic material was not recognized until the 1950s, DNA was already well-known in the 1890s. Then, and in the early 1900s, Weismann knew that there are only four nucleotide bases (Portugal et al. 1977, page 80). He might have guessed Chargaff's Law, which states that the amount of cytosine is equal to the amount of guanine; and the amount of adenine is equal to the amount of thymine. This knowledge, combined with Bovari's drawing, might have tipped Weismann off that DNA is the genetic material. Erwin Schrodinger, in 1946, was able to guess that the genetic material is "an aperiodic crystal" (page 61). People just couldn't believe that something so simple would be capable of producing such great variety. Weismann's omission was a tragedy for him and for genetics. In the study of the mind, this book is intended to avoid repeating it.

The understanding of genetics before 1900 was the same as the understanding of the mind now. It was a matter of larger forces overwhelming smaller ones, and driving them out. Inheritance was a mechanism with no discrete constituents and no definite answers. Carl von Naegeli rejected Gregor Mendel's combinatorial algebra, never dreaming that he would be remembered in history only for his postal correspondence with the enthusiastic gardener. Alfred Russel Wallace was Darwin's only equal, and is less-well-remembered today solely because Tom Huxley, Darwin's self-appointed "bulldog", hitched his wagon to the rich-man's star rather than the poor-man's. Wallace trav-

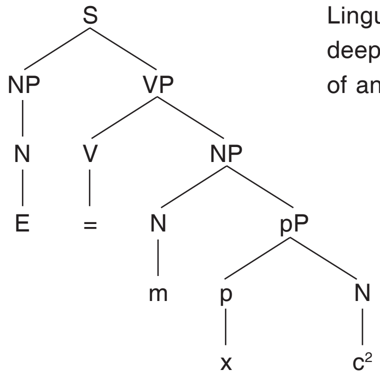


Different Females of Papilio memnon

eled all the way to the Malay Archipelago (1869, volume 1, page 202; page 140 in the ever-popular 1885 edition) to describe *Papilio memnon*, swallowtail butterflies. "A single brood of larvae produced males as well as tailed and tailless females, and there is every reason to believe that forms intermediate in character never occur." Wallace had discovered the gene, but rejected it because he couldn't believe his eyes. He immediately reverts to the idea of one character being swamped by another, after a notorious magazine article by Fleeming Jenkin (1867). Jenkin is the subject of an essay by none other and no less than Robert Louis Stevenson (1912), where he appears (page 83) under the pseudonym of "Cockshot". See also Stevenson's (1925) biography of Jenkin. Wallace might have made the same discovery closer to home, where the famous Manx "tail-less" cats produce rumpies and stumpies in the same litter.

People sometimes like to speculate as to what might have happened if Mendel, who knew that his algebra would be of importance for the theory of evolution, had contacted Darwin or Huxley. Darwin loved to correspond through the mail with other naturalists, and Huxley would probably have responded, too. But in Wallace, we have the answer. Nothing would have happened.

Another mistake that we have tried to avoid is one famously misattributed to Alfred Wegener, the modern defender of the theory of continental drift. I have been taught by a professional historian of science, and a professional geologist, both in their respective offices, that people never accepted the theory of continental drift "because Wegener never suggested a mechanism that might move the continents around". The same suggestion appears in as prestigious a forum as *Scientific American* (Hallam 1975). Wegener's 1929 book, available in English since 1966 as a Dover paperback, is not exactly a secret. Wegener's mechanism, "convective movements of a molten interior heated by radium" (pages 58, 178) would receive an "A" on any modern school exam. It is our own collective stupidity, not any fault of Wegener's, that prevented the acceptance of continental drift. Against "He never showed the linguistic deep-structure of an equation", I repeat:



Linguistic
deep-structure
of an equation

Alfred Wegener and Gregor Mendel are not just entertaining stories from the past. We have tried here to learn from history, not just enjoy it. Alfred Wegener, and Gregor Mendel, are portents for the future. As a statistical matter, there should be about one Alfred Wegener/Gregor Mendel in every century for as long as there is science.

By 1921, Edward Sapir was characterizing speech-sounds as "symbolic atoms" or "points in the pattern of [a] language" (1921, last paragraph of chapter III). Here, Sapir is using figurative vocabulary borrowed from chemistry and geometry. The images must have been real in his mind at the time. But as the emerging science of linguistics asserted its independence, largely under the guidance of Edward Sapir, he changed both his mind and his vocabulary.

By 1925, Sapir was still describing speech-sounds as "points in a pattern" (RIL page 20, column 2, bottom; page 25, column 2, top), but he also introduces the term "phoneme" (page 25, column 2, top). By the next year, Leonard Bloomfield (RIL 1926, page 27) has abandoned "atoms" and "points" in favor of "phoneme" as a name for the perceived speech-sounds. Thus, we see a progression in vocabulary

Sapir 1921	"atoms"	"points"	
Sapir 1925		"points'	"phonemes"
Bloomfield 1926			"phonemes"

that actually moved away from any comparison of language with chemistry or geometry, as "linguistics" came into its own as a science.

The independence of linguistics was only emphasized by the introduction of a special term, "psychologically real", for what in other sciences is called "naturally-occurring". One of the great questions was whether the phonemes are psychologically real, or just artificial categories dreamed up by language scientists for convenience in committing language to paper, and in describing it. The independence of language from the rest of nature is precisely the myth that makes "the evolution of language" seem obvious, and that the cladogram model, or number-line principle, is meant to banish.

By the late 1940s, Zellig Harris (1951), the same Harris who was picked up by the submarine off the coast of Israel, was talking about the discreteness property of the phoneme. Harris contrasted discreteness to continuity, not to emphasize the necessity of discreteness, but only to illustrate what he meant by it. Harris was describing, not interpreting, and there is still no reason to believe that "infinite use" was, in 1836, anything more than it is now, a dramatic, and technically true, but un-helpful slogan.

The fundamental lesson of the gene is that language/arithmetic is not a diverging series. On the contrary, a single, precise answer is possible (compare Pike 1963). Language and arithmetic share a common source in nature - and the cladogram, with its succession of properties, is that source. In effect, the history of the gene verifies the type of clean, simple, and complete solution that the cladogram offers for the mind.

A Possible Mechanism

Our last remaining task is to avoid repeating August Weismann's regrettable, if forgivable, omission. Briefly, how can we go about discovering the physical basis of the mind, as understood under the cladogram/number-line principle? The fundamental clue is first principles. If the human mind and language are directly subject to first principles, then a synaptic model is ruled out. The collective firing of synapses is isolated under the mediation of nerve cells, and already far removed from relationships that might underlie equations and basic physics. What is more, separate events cooperating together are simply not made of the same stuff as a physical system, such as the solar system, with its numerous interdependent parts that interact with each other simultaneously and seamlessly. Worse, collective action of nerve firings are the solution to an equation, and represent a circular solution to the fundamental mechanism of the brain.

Something under direct control of first principles is something small. One guess is that movements and oscillations of the DNA thread inside the cell nucleus are picked up by the cytoplasm and converted into nerve-firings. The DNA thread has been found to move inside the living nucleus. Maybe all those introns, the "junk" DNA between the genes, don't code thought directly in the base sequences, but indirectly in the effect that the sequence has on local flexibility of the molecule. Or maybe they just serve to make the molecule the right length to oscillate properly, or both. If so, the best place to look is something accessible, such as the amoeba (Jennings 1905, 1976). Amoebas are easy to work with, yet make decisions based on light gradients and chemical gradients, maybe even gravity and current. They move slowly enough so that they will not disappear from view at high magnification, like paramecium, or the dazzling, deadly didinium. It is possible that peripheral changes in their locomotion are under central control, in somewhat they way that ours are. Amoebas are small enough and transparent enough so that they can be scrutinized with light microscopy. In spite of the electron microscope, light remains the great medium because it does not damage the biological materials that we wish to study. Rapid oscillations in DNA might be stopped by stroboscope. Maybe the DNA could be stained with a fluorescent dye, and made visible in that way. If there were systematic changes in the DNA movements when paramecia were released, or when a crystal of salt or of sugar was placed in the amoeba's water, it would indicate at least central sensitivity. The control might be to observe the equivalent of amoeba dreaming. Remember that honey bees dance at night, with no stimulation from flowers. If the amoeba's DNA fiber goes through the same movements at night, with no provocation, that it goes through in the presence of germs or paramecia, or some gradient, the movements can't be a reaction to anything, and must represent some primary mechanism. Of course, we would have to use one of the small amoebas, so admired by aficionados (Bovee 1964).

If we can find the decision-making mechanism of the amoeba, we will have some idea of what to look for in a small brain, say, the brain of a flatworm, or even a rotifer, if its movements are not too rapid. Maybe brain-cells in culture might show something. Nerve firings and synapse crossings may be visible if we learn what to look for. I have already started my amoeba farm.

Even more fascinating is the possibility that, like the source of sentences in the first place, the source of sentence asymmetry has been waiting in arithmetic all along. The symmetry property of the equation is the only relationship that can produce self-checking, repeatable answers that are also "independent of experience". Apart from symmetry, assertions must be checked against experience or experiments. Mathematics would not work in asymmetrical form, and has to be the way it is.

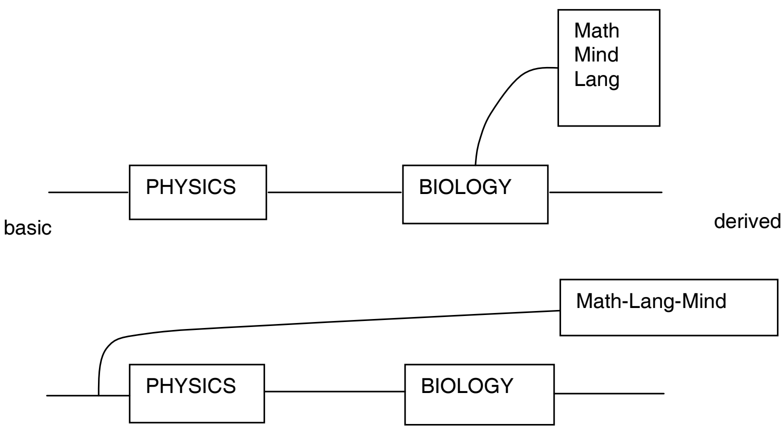
But near the speed of light, arithmetic starts to break down. Under Einstein's expression for the addition of velocities v and w (1905, page 906; 1952, page 51), v + w becomes:

$$\frac{v + w}{1 + \frac{vw}{c^2}} .$$

At ordinary speeds, walking and running, v and w are so small compared to c, the speed of light (186,000 miles per *second*) that, for all practical purposes, vw/c^2 is nil, and v+w remains v+w.

But, as speed approaches that of light, vw/c^2 approaches 1, and Einstein's expression for v+w approaches (v+w)/2, the kind of blending arithmetic that we rejected earlier as the basis for any system with emergent properties. Einstein knew better than to write his expression as part of an equation; and the gradual removal of symmetry from equations is not some remote, unimaginable process, but has been known in arithmetic for a century, and has been familiar for nearly that long.

We do not have the technology now to observe objects moving at relativistic speed inside living things – any more than the 19th century had the technology to image the genetic molecule. But, like Theodor Bovari and August Weismann, we must make an informed guess. Such objects are electrons, possibly moving along the surface of special molecules inside nerve cells, or possibly along the surfaces of the cells. It is here that we may seek the asymmetry of ordinary sentences as the "equals" relation is warped out-of-parallel in the unimaginably fast and the unimaginably small. From the symmetry of equations to the asymmetry of ordinary sentences, from the infinitude of language to non-circularity and the beginnings of the mind, the arithmetical theory is the only one that gives total unity of hypothesis based entirely in familiar properties of familiar systems. Equations as we know them are a limiting case, like Newtonian mechanics, where we tacitly assume that the transmission time of light is instantaneous.



To a first approximation, the biological theory (above) and the Structure-of-Matter theory (below) differ only in the source of their properties.

Chapter 3: DIALOG BETWEEN A SKEPTICAL READER AND THE MEDIEVAL PHILOSOPHER GUGLIEMUS PERITIOR

Skeptic: You must remember that you are asking, "Why are things the way they are?" and that you have stolen that question.

Peritior: "Why are things the way they are?" must have been the first question that was asked as soon as there were questions. Of course, I use the word "things" in the plural, exactly the way Lucretius used it in his *De Rerum Natura*, to mean nature, or even physics.

Skeptic: Do you have any idea how unlikely it is that you solved language and mind, or that they are not of biological origin?

Peritior: That is no longer the question, is it? Now that the theory is written down, it has a life of its own, independent from me. It is neither likely nor unlikely, but only right or wrong.

Skeptic: Why should I believe that the gene, arithmetic, and language share a common source just because you say so?

Peritior: I sympathize with you completely. It took me two years before I could believe my own results.

Skeptic: What happened?

Peritior: I remembered Alfred Russel Wallace reciting "Natura saltum non facit" [nature doesn't take leaps], rejecting the particulate gene when he discovered it himself in 1869, and again when it became common knowledge thirty years later. I remembered Copernicus creating a solar system by placing the sun at the center of the planets, but leaving a few epicycles around the edges. They just couldn't let go. I realized that I had to let go.

Skeptic: The numberline theory is too much about *language*. It doesn't do anything for me.

Peritior: The numberline is how nature works. It is like human anatomy. Whatever our preferences, there is nothing anyone can do about it.

Skeptic: OK. The synthesis is interesting, but not convincing.

Peritior: No idea, not even an idea in science, is ever accepted on the facts alone. Wallace is only an example from history, although an important one. But you can try the experiment at home in the autumn. Remember the male yellow jackets. They are easy to spot if you know what to look for. Their bodies are a little longer and more cylindrical than the workers. But the give-away is the antennas. They are long, and have a curl at the end. You can't miss it. You know they can't sting you because the stinger is a modified ovipositor, so males don't have one. You can safely pick one up, as I have done, and hold it in your fingers. You know why it can't sting you, and you see and feel that it isn't stinging you. No more evidence is possible. But when you see the yellow and black stripes, and the devil-markings on the face, you will expect the pain to stab you at any moment. Remember Edward H. Levi (1949). If my idea is interesting, maybe we should find out why.

Skeptic: Didn't Thomas Kuhn show that new theories come in when too many flaws accumulate in the prevailing one, like radium, and the orbits of Mercury and Encke's comet?

Peritior: You can't take those philosophers seriously. Continental drift didn't come in that way. And the non-Newtonian orbit of mercury, or radium giving off heat and light, were more counter-examples than something deep within the system that would offer some clue. Einstein eventually used contradictions in theory as a starting place.

Skeptic: OK, what is wrong with the evolutionary theory?

Peritior: First, it is circular. Remember that equations are a subset of all sentences, and that selective advantage is, in principle, the solution to an equation. So, if you use natural selection to solve language, you have indirectly derived equations on the basis of an equation. The evolutionary linguists have started mathematicising the evolution of language, as they must. But it only brings out the fundamental contradiction of the theory.

What is worse, the creative aspect of human thinking is obviously outside Gödel's proof. The mind can generate new theorems, theorems beyond the ones that are consequences of existing axioms. A mind that evolved according to equations would be limited by the postulates that are sufficient to generate those equations. The mind shows no sign of being limited in that way, especially if Goldbach can't be proved or disproved.

At a different level, psychoanalysis maybe, we see people create ideas that have vast variety, that contain no obvious hint of the remote moment when the lucky pair or isolated tribe lived who are ancestors to us all. If language and mind evolved on the basis of forces that are of local origin, and contingent upon local circumstances, you could trace the common thread in all of our thoughts through the shared manifestations of mind, back to that fateful time and place. It would have the character of destiny. It would surface in our dreams and our folktales. But that doesn't happen. There is no shared thread that is traceable through human nature to the archetypes of zebras and fever trees of our origin. Landscape with water just doesn't make human nature. If the human mind evolved according to an equation, under Gödel's proof it is The Equation At

Infinity, where Euclid's parallels meet – and Einstein's. You can't go out there and look at it.

Skeptic: Don't simple causes, repeated enough times, produce powerful consequences?

Peritior: That is the lesson of Darwin's *Wormbook* of 1881, his last book. What Darwin showed was that the humble earthworm, by leaving its tiny mound of castings at the entrance of its little burrow, can, in time, bury an entire Roman villa under the earth, and produce the known soil of entire continents.

Skeptic: Pretty hard to explain away.

Peritior: What Darwin showed was that small amounts added together enough times can amount to large amounts. He didn't show the basis of systems with emergent properties. If language emerged in the way that worm castings accumulate, it would have only one level of organization. Language just isn't an accumulation of anything. Even fractals, which have the same structure no matter what scale they are seen from, can't account for the autonomous levels in the organization of language.

Skeptic: Equations obviously appeared later in human culture than ordinary sentences. What can you mean by saying that equations are primary?

Peritior: Suppose that ordinary sentences evolved in the usual way, and that equations are derived from them. That method still derives equations on the basis of an equation, although it derives them indirectly.

Skeptic: Equations don't amount to much compared to the vast numbers of ordinary sentences.

Peritior: That isn't as obvious as it looks. Even if there were only a few equations, a thousand, say, deriving them on the basis of one of them would still be a contradiction. And the number of equations that are possible in principle must be the same order of infinity as the number of ordinary sentences. There is nowhere to hide. And since the equation is the ruling principle, equations are primary.

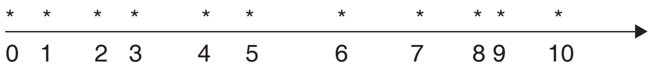
Skeptic: What I mean is that ordinary sentences appear earlier in human culture than equations.

Peritior: That isn't the point, either. Since all sentences are more found than made, they didn't exactly appear. They were found, so the order they were found in isn't a clue to their source. It is like finding a fossil horse before you find a fossil dinosaur. Finding the horse first doesn't make it older, even though you have owned it longer and know it better.

Skeptic: Why couldn't mathematical thinking, or a mathematical sense, evolve by natural selection? Behavior that is closer to the actual form of algebra would simply be favored by selection.

Peritior: The idea of an equation that gets you closer to equations is circular. But beyond that, the fit between mathematics and the natural world is too good. If mathematics were subject to the quirks of history, it wouldn't come out in the neat way that it did. We wouldn't have "F=ma", for example. The original physics would be the same, but the corresponding math wouldn't. Evolution isn't perfect. It can't be, and doesn't have to be. If mathematics, or the supposed biological engine that generates it, were subject to descent with incremental modification, we would get $F^{1.00001}=ma^{0.9999999}$, or $S=1/2gt^{2.0000001}$, or something. It would be statistical. If some biological generator had to be mobilized every time we used numbers, maybe it would be $F^{1.000001}=ma^{0.99999}$ one time, and $F^{0.99999}=ma^{1.00001}$ the next. Numbers might average out near integer boundaries, but there would be no integers. If humans had evolved a little to the south or to the north of where they did, mathematics would have been even farther off the mark. If they had evolved in the Americas, it would have been worse. It doesn't add up.

Skeptic: I still think that arithmetic is arbitrary. What if the numbers were all different sizes – like this?



Peritior: That's an old one; and there are two answers. Remember the biochemistry that has all the individual carbon atoms different?

Skeptic: Yes?

Peritior: A non-equal number system would be like that. It wouldn't add up. Nothing would fit. 5+5 wouldn't be equal to 8+2, for example. I'm not saying that such a world doesn't exist, but, if it does, it is not inhabited by living things that might try to understand it. And there may be a filter that prevents non-equal numbers from forming in the first place.

Skeptic: What is that?

Peritior: Remember Giuseppe Peano's famous axioms. Very roughly, they are,

"Zero is a number."
"Every number has one successor."
"The successor of every number is a number."

Peano's axioms (Hempel 1956, page 1623) are equivalent to Bertrand Russell's embedding method of generating numbers by placing sets inside of sets:

Peano: o * * * * * * *
Russell: <({[o]})>

Both systems generate ordinal numbers, "1^{st}", "2^{nd}", "3^{rd}", "4^{th}", - - - , that have no property other than succession. They aren't squares, or cubes, or even sums. Sixth plus sixth isn't twelfth. Third times third isn't ninth. People always complain that Peano-Russell numbers are somehow unsatisfactory, and that is why. Under Gödel's proof, arithmetical/geometric properties are added later, under new axioms. By the time numbers acquire their cardinal properties, they have already passed through a dimensionless, ordinal state. Non-equal numbers may never form in the first place.

Skeptic: But what is wrong with the idea of an innate Newtonian physics?

Peritior: If there were such a thing, it would have the character of instinct. An Aristotelian physics, where things fall down because they want to be closer to the earth, and heavier things fall faster because they have more desire to be closer to the earth, would be nearly impossible to think of, and would seem out-of-place if someone did think of it. But it gripped the public imagination for 2500 years, and was overturned only by geniuses, Copernicus, Kepler, Galileo, and Newton. It was an enormous psychological experiment, and even today, people don't always get it.

Skeptic: Don't people have a unified view of physics?

Peritior: That is the Newtonian revolution, a single theory of physics. If people had an innate Newtonian physics, there would be a single strategy for catching a fly ball. But the strategy is all *ad hoc* and piecemeal. Peter Brancazio's (1985) *Looking into Chapman's homer* was famous in its day. If the ball ascends quickly after it is hit, it will fall behind the fielder. If it ascends slowly, it will fall in front of the fielder. Then, the fielder moves so the ball has zero apparent vertical velocity as it approaches the catch.

If people had an innate Newtonian physics, they would think "F = ma", not three separate strategies. Even if it were accurate, the supposed Newtonian physics of people and animals is geometric, not algebraic. It is like cutting out a piece of paper and weighing it to get the area under a curve. It gives the same answer as calculus, but it isn't calculus. It is a geometric way of avoiding calculus. Sentences, and that includes equations, are not innate or biological.

Skeptic: You are a Pythagorean, or a Platonist, maybe. You don't believe in Creation, yet you reject evolution, too. All that is left is some ideal icon of language, floating around in space somewhere, that our primate ancestors managed to capture somehow.

Peritior: That is all that our philosophy can dream of. Of course, I accept evolution for everything in biology that doesn't have the actual form of equations. But ruling out God and Darwin doesn't leave us with Plato. That, in itself, is a kind of static, Platonic, pre-calculus view of nature. Remember abstractness. The orbits of planets are not exactly matter, yet they aren't ideal icons, either – ellipses floating around in space somewhere, waiting for the planets to capture them. They are non-material, yet are a property of matter in motion. In its foundations, language is comparable, a consequence of dimensional properties of matter.

Skeptic: Look – there were no equations during the formative period of human evolution.

Peritior: True.

Skeptic: And there were no equations during the formative period of the solar system.

Peritior: Also true.

Skeptic: But we use equations all the time to explain the origin of the solar system.

Peritior: That's true, too.

Skeptic: So, what is circular about using equations now to explain something that happened before there were equations?

Peritior: Timing isn't it. The orbits of the planets don't have the form of equations, so they are safe. But nothing that has the form of equations can be explained on the basis of equations. There is no escape.

Skeptic: Haven't they shown that some present-day jungle cultures don't have words for precise numbers (Gelman et al. 2004; Premack et al. 2005)? Doesn't it mean that number-use emerged as it was needed, for commerce and that sort of thing?

Peritior: If anything, that confirms the numberline theory. Those people aren't dumb – and if number-use emerges as needed in culture, number ability must already be there. It is like spiders who know how to spin webs, but don't. Maybe they are too polite. If such spiders exist, they have ancestors who spun webs. The cultural theory of number-use implies a Golden Age of number during the formative period of human evolution, where there was a call for number-use, and it evolved, but was forgotten. Modern cultural need re-awakens it.

Imagine a crowd of pre-Stone-Age Euclids and Leonardos, thinking thoughts of pure geometry and arithmetic, introducing an innate number sense that fell into disuse only to be re-awakened later by commerce. It is preposterous. The whole idea of psychology is in for a complete re-organization, along the lines of the gene in 1900.

Skeptic: How can you expect any biologist to accept any non-Darwinian understanding of any living thing?

Peritior: Biologists know that legs are pendulums, whose movements are ruled by Heaven, in effect. Language and mind work the same way. The numberline theory is physical, and non-Darwinian, but not anti-Darwinian, any more than the law of pendulums is anti-Darwinian for not being based on natural selection. Because only the mind and language are identical with arithmetic/geometry, the rest of biology, and that includes human biology, is un-touched by the numberline.

Skeptic: Legs are moved by pure reflex. End of story.

Peritior: Look at even a millipede walking along different surfaces. On a flat surface with good traction, its legs move in a smooth, regular way. You can see the waves advance toward the centipede's head as it moves its legs forward to take the next step. But on pebbles, it is different. The centipede feels its way along, putting a foot down, testing the traction and position before committing itself. People are the same way, striding along a sidewalk confidently on a dry, sunny day, but feeling their way along cautiously when the surface is slick with ice, or even rain. Walking can be very deliberate. But at its smoothest, walking is more a matter of pendulums than anything else – committing your weight to a stride. If a reflex is involved, it takes its cue from the pendulum.

Skeptic: What evidence might directly support the numberline theory?

Peritior: If it were found that language emerged twice, in two separate populations, universal grammar would rule out common biological inheritance. Evidence from anastomosis of the middle cerebral artery, preserved in the inner surface of the skull, might make such a determination possible (Saban 1993, pages 203, ff.). That would be true even if the two populations were related, which they would be, somehow. Besides, there already is fossil evidence of double origin. Wolpoff and his colleagues (2001) found that peripheral populations in Australia and central Europe have what they call "dual ancestry". Even if we assume a single origin, we have to explain the stability of universal grammar over a time period long enough to produce local variations. All separation has done is produce local languages, not local versions of universal grammar.

Skeptic: What about extraterrestrial civilizations?

Peritior: I won't recommend that researchers should hold their breath, but if language evolved by natural selection, and is of local origin, and contingent upon the quirks of local conditions, the languages of extraterrestrial civilizations should be utterly unlike anything known here on Earth. The way English scholars used to think Chinese was – showing no trace of grammar as we know it. That myth has melted for Chinese, and we should find a lesson there. The receipt of even a single intelligible radio transmission from an extraterrestrial civilization would rule out the biological theory. And if people really believed the biological theory, they wouldn't keep looking for signals from extraterrestrial intelligences. We are schizophrenic about the biological theory of mind.

Skeptic: How about something that we don't have to wait for.

Peritior: If voluntary action of cells in the brain were found to be under the control of first principles, it would rule out the biological theory. For example oscillations of the DNA thread in a cell nucleus would be under the control of the same forces as a chain, dividing itself into discrete regions when you twirl it like a cowboy at a rodeo. If something like that, rather than synapses, were the primary mechanism of thought, the biological model would also be ruled out, because first principles is what generates biology, not the other way around.

Skeptic: In biological terms, the numberline makes no sense. Martin Nowak's equations show that word strings are adaptive.

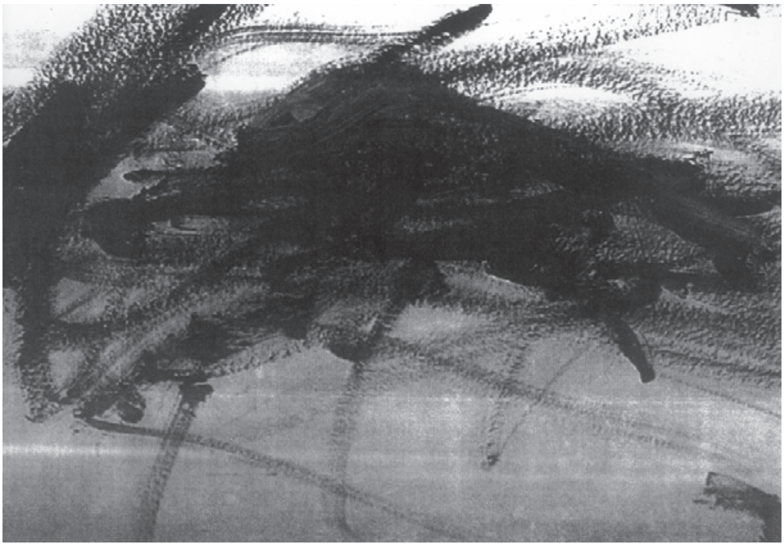
Peritior. I am completely convinced that Nowak's equations are right, but that is beside the point. Just showing that something is adaptive doesn't mean that it could happen. Maybe it would be adaptive for mosquitoes to have propellers. Language isn't biological. And even if it were, stringing words together doesn't generate the levels of organization, or the power of assertion.

Skeptic: Didn't they already solve language?

Peritior: A lot of people think so, but it isn't obvious. They thought they solved language in 1863. Until we have a clean, clear, complete solution, nothing is for sure. The idea of squeezing humans and chimpanzees ever closer together may simply be the wrong model.

Skeptic: For example?

Child Art



Chimpanzee Art

Peritior: The ape-language experiments, first suggested by the British phonetician Sir Richard Paget in 1928 (page 130), are an attempt to show continuity, continuity between the language behavior of people and another species. If you can show continuity, you have something to suggest descent with modification.

Skeptic: They showed it, didn't they?

Peritior: Here is a test. The gene, and language, are particulate, or discrete systems because they have to be. The combination or transmission of blending constituents would lack the diversity of signals necessary for language. Baby chimpanzees, at the age of six months, are compared to human children at the age of five years, maybe based on their ability to climb trees. If you compare their ability to play make-believe, or for role-switching, the comparison is very different.

But discreteness is a necessity for language, at least for transmission, which is important, too. Child art is filled with discrete structures, eyes, heads, ears, hands, the sun, butterflies, that were drawn in some order, apparently on the basis of some plan, even if a poorly-integrated one. The structures are, if anything, too discrete, because they do not show enough inter-relationship to form a convincing drawing. But chimpanzee art is something else. The example here, shown in comparison to child art, is used Courtesy of The Field Museum, Library Special Collections (Rand 1967). I don't know why the chimpanzee painted it, except that it is finger-painting. But it is not much different from a portrait of a kitten, done by Koko the gorilla, shown in Francine Patterson's book *Koko's Kitten* (1985). The ape art shows no hint of the discreteness and serial planning that is necessary both for language and arithmetic, and that is abundantly in evidence in the child art. There is nothing to suggest that apes have anything like language in the human sense.

Skeptic: But the amount of evidence in support of the evolutionary theory is enormous.

Peritior: It is circular. What do you expect?

Skeptic: If people are animals, and Albert Einstein and Charles Darwin evolved by natural selection, how can you claim that language, or mathematics, didn't evolve by natural selection?

Peritior: The system of first principles is more basic in nature than biology. First principles, in effect, is the system that generates biology, not the other way around. Part of the emergence of the human mind is that a biological structure found a way to gain voluntary control of first principles. But the first principles were already there.

Skeptic: Are you talking about *Homo neanderthalensis*, or *Homo sapiens*, or what?

Peritior: It doesn't matter. Every time a new fossil turns up, the anthropologists have to start all over again. Look what happened when they found that three-feet-tall human skeleton (Brown et al. 2004; Morwood et al. 2004; Dalton 2004; Lahr et al. 2004) where the Komodo dragons live. Everybody was ready to re-write the book on human evolution. For all practical purposes, there is no theory. Fossils themselves have never been the crucial factor in determining how fossils are understood. Before Darwin's *Origin*, fossils were proofs of Noah's flood. Remember William Buckland's *Reliquiae Diluvianae* of 1824. The sub-title was "Observations on the Organic Remains contained in Caves, Fissures, and Diluvial Gravel, and on Other Geological Phenomena, attesting the Action of an Universal Deluge." After the *Origin*, fossils were proofs of evolution. The fossils will find their place after we have the right theory, not before.

Skeptic: Even if you do show that expressions in mathematics are determined by the laws that underlie physics, language must be represented in the genes. After all, language disorders are inherited, and run in families. We have to answer the question, how did language get into the genes? And the answer has to be natural selection.

Peritior: Under the cladogram, the gene is a product of the same system that produces equations and sentences. The gene can't generate another system that developed from the same central trunk. It would be like proposing that bumblebees gave rise to bears, or something. Language disorders may run in families, but language disorders aren't language. A car might lose power if the clutch burns up, but that doesn't mean that the power came from the clutch in the first place. If we could transplant a language gene into a parrot and get the parrot to produce adverb clauses and ablative absolutes, that would be a different story.

Skeptic: Couldn't the numberline principle just set limits for the evolution of a language system, which evolved in the conventional way?

Peritior: Again, if you take the evolutionary theory, you are stuck with the evolution of equations, and the calamitous consequences of that. All we can be sure of from the genetics is that language is mediated through a biological system.

Skeptic: Isn't language just a by-product of intelligence and an enlarged brain?

Peritior: The idea of language arriving like a package in the mail is surprisingly popular, and treats language as a second-rate system. It treats language as being absolutely simple, and not needing to be understood at all, if you look at it just right. Beyond that, the idea of objects in biology being by-products has gone too far. Stephen Jay Gould's idea of biological "spandrels" has captured the popular imagination like "infinite use". It might account for the glabella, the structure that fills in the space above the eyes. But it won't explain something with emergent properties, such as arithmetic or language.

Skeptic: You only take advantage of a few simple comparisons between the gene and language, but a lot has been learned about the gene since these became known. Why haven't you continued your comparison?

Peritior: The properties shared between the gene and language/arithmetic are just the simple ones that were discovered first. They only go just so far. After that, the two systems have their own requirements and their own separate histories. That is why I show the gene diverging from the genealogy earlier than arithmetic.

Skeptic: How can there be anything more basic in biology than the gene?

Peritior: If the numberline theory is right, the gene is a manifestation of first principles, and has to follow a basic blueprint. You couldn't have a radically different kind of gene. It wouldn't have to be a double helix. It could be a flat ribbon, like the base-sequences you see in the magazines. But it would have to be a linear sequence of discrete bases, not a two-dimensional sheet like a TV screen, or a bifurcating tree, or a cube. Even if you treat the gene as the most basic thing in biology, there are forces still more basic. And those are the ones that determine the structure both of the gene and of language. It is the convergence of properties that shows the agency of some more basic law. The gene is just a manifestation of the law.

Skeptic: Language is a social activity, and the scientific study of language is a social science. How can you treat it as physics?

Peritior: The uses of language certainly are social. Even when people talk to themselves, they may perceive it as one character in a drama talking to another, or one part of the mind talking to another. Still, the uses of language aren't language. Language represents foundations of physics mobilized in the cause of social interaction.

Skeptic: Didn't Andrew Carstairs-McCarthy (1998) show that language is biological, by showing that it can take an infinite number of forms?

Peritior: Carstairs-McCarthy described a verb-less language. A dialog in his language might look like this.
"Breakfast pretty sparse. No marmalade."

"I to grocery store."
"Not today – Not on Sunday"
"Looking in cabinet. Finding what?"
"Strawberry jam here. Yipee."
The idea is that once you can demonstrate two kinds of language, it becomes impossible to show that there isn't a third, and so on. There can always be one more.

Skeptic: Sounds OK.

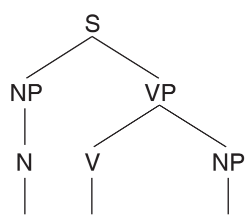
Peritior: Maybe this is off the mark, but the United States Patent Office won't grant a patent for removing a part from an existing invention.

Skeptic: What does that have to do with anything?

Peritior: A patent has to be both new and not obvious. The law is intentionally vague, but the framers of the patent law must have felt that removing a part is neither new nor not obvious. Removing the verb from English grammar would never get past the patent examiner.

Skeptic: So what?

Peritior: Let's try to see why. We have to assume that people go around with a basic linguistic deep-structure in their heads, and that it looks something like this.



When people hear a sentence, they fit the words into the spaces where they belong in the deep-structure. When there is no sentence, they fit names of items from the situation into the empty spaces in the deep-structure. For example, with the assembly of electronic components, where there are only so many things you can do, two people don't even have to speak the same language. All they have to say is "memory" and "solder" and "chip". The mind has no difficulty in making sentences out of those.

Beyond that, we use Averbalese all the time. One of Carstairs-McCarthy's examples is "A thousand pardons". We have all at least read that, and everyone knows what it means. I don't think it is new, or that it is understood in any unconventional way. I think that we supply the missing verb to the situation. That is what we mean when we say that meaning is supplied by context. To take another example, I go up to a stall at a flea market, and point to a diamond ring, and the seller tells me, "A hundred, buddy." The man hasn't invented Averbalese. He has taken advantage of context. I know he means, "I want a hundred dollars for that" because I can fill in the deep-structure. That is why we can have words like "yes" and "no". They aren't a new language; they are the old one with contextual ellipsis. There is still exactly one solution to arithmetic and language.

Skeptic: There is exactly one solution to the airfoil, but it evolved in the wings of birds. Why couldn't a mathematical sense evolve in humans? Just a mathematical sense? Individuals with the best mathematical sense survived.

Peritior: Pinker (1994, page 350) wants to compare the uniqueness of language to the uniqueness of the elephant's trunk, to show that there is nothing special about uniqueness. But it is a red herring. Shared uniqueness shouldn't be confused with shared structural foundations. Where is the linguistic deep-structure of the elephant's trunk?

The airfoil is unique, but, like the elephant's trunk, it has no linguistic deep-structure, and evolved on the basis of an equation. The idea of a mathematical sense is meaningless apart from the mathematics, the equations, that it generates. And it can't generate equations, or extensions of equations, on the basis of an equation. There is no escape from the circularity.

Skeptic: Just because you can "read out" an equation in words doesn't mean that equations are sentences.

Peritior: Certainly it isn't the whole story, but it is part of it. Suppose you are looking at the ocean, and you see a whale. You can't look at the situation and somehow read it out in words. You would have to make up a sentence. And you could paraphrase the sentence. It is the same with anything except equations. You can't look at any situation there is and read it out like a book. But you can do that, and have to do it, with equations, because symbols in algebra represent morphemes of language.

As for the rest, look at the linguistic deep-structure of equations shown in this book. You can write the linguistic deep-structure for any equation, because equations really are sentences. Of course, I am using linguistic deep-structure as a short-hand for all the properties on the genealogy. For example there is no translation for an airfoil, or an elephant's trunk, either. But you can read out an equation in any language.

Skeptic: Constituents of language are unstable, and are changed by their neighbors. But constituents of mathematics are stable. For example 3^{rd} tone changes to 2^{nd} in Chinese when another 3^{rd} tone follows. But in arithmetic, context doesn't cause 5 to change into 6. You can't compare the morphemes of language to the symbols of algebra.

Peritior: Since tones are an actual part of the Chinese language, they differentiate the meanings of words. For example the syllable "mai" means "buy" when it has 3^{rd} tone, "mai3", but means "sell" when it has 4^{th} tone, "mai4". If 3^{rd} tone really changed to 2^{nd} when followed by another 3^{rd} tone, the meaning of the first word would change, defeating communication. Take the Chinese sentence "I bought nine pens". All the words take 3^{rd} tone, Wo3 mai3 jiu3 bi3. But of course it isn't pronounced like that. It is pronounced, "Wo2 mai2 jiu2 bi3". The last word, bi3, keeps its third tone, changing the one before it, jiu3, to second. If the tone of jiu were lexically 2nd, the sentence would be pronounced, "Wo2 mai3 jiu2 bi3". But of course that can't happen, either. The underlying, lexical 3rd tones are retained, so you get a cascade effect all the way back to the first word. It tells you a lot. It shows that you know what you are going to say before you say it. And the retention of the underlying 3rd tone shows that language is more stable than it looks. Also, the pronunciation of the number nine changes from "jiu3" to "jiu2", showing that symbols in arithmetic are no more stable than symbols in language. More to the point, nobody is claiming that language and arithmetic are the same thing. If they were, there would be nothing to talk about. They share a common ancestor in nature, the way related species do.

Skeptic: Expressions in language sometimes do not have any reference, such as McCawley's favorite "the biology of unicorns" or Chomsky's famous "colorless green ideas". But expressions in arithmetic always have a reference, such as "9/5C + 32". They obviously have nothing to do with each other.

Peritior: Of course, nonsense phrases don't have to refer to material objects. Einstein showed that the phrase "absolute rest" doesn't refer to anything in the material world. But it refers to something in the mind, and fooled all of the people for an awful long time. "5x3=π" might be the arithmetical equivalent of unicorn whiskers.

Skeptic: OK, but numbers have meaning in themselves, such as "3", or "square-root-of-two". But phonemes don't. How can they be the same thing?

Peritior: Phonemes don't refer to anything in the outside world, but, to a first approximation, each number has meaning only by being different from other numbers. Without reference to anything but one another, numbers have the property of magnitude. That is what ordinal numbers are. "Twelfth" divided by "fourth" isn't "third". Arithmetic only happens when geometric properties are added to number. We have to define underlying phonemes in the same way. Again, language and arithmetic aren't the same thing, any more than lizards and turtles are the same thing. They merely share a common ancestor. We are looking here for shared vestiges of that ancestor, and there are plenty of them.

Skeptic: If phonemes are not present in the speech signal, or carried by it, how can the phonemes be numbers?

Peritior: They are numbers because they aren't present in the speech signal. If they were somehow in the acoustic events of speech, they would be sound frequencies, or patterns of frequency change. It is their abstractness that forces us to find their identity in the minimum difference possible – number.

Skeptic: Of course language is subject to arithmetic, the same as everything else. What is interesting about that?

Peritior: Language may be subject to arithmetic in certain ways, but as William Labov is fond of reminding us, that is what makes it unique among the social sciences. You can count up the phonemes. You can count up the sentences. The interesting part is that the form of the basic sentence is the same thing in arithmetic and language, a linguistic deep-structure.

Skeptic: Everybody knows that there is number; and everybody knows that there is language. Everybody knows that there is discreteness and deep-structure. You haven't introduced even a single new idea.

Peritior: I have shown that discreteness is a necessity, not an incidental descriptive detail; and that the rest of the items on the cladogram are almost certainly necessities. Why else would they be uniquely shared by such basic systems? I have shown that language and arithmetic share a common source. A lot of reviewers rejected that idea as wrong, but nobody ever rejected it as old. Maybe the pieces were already there, but I have shown their importance, and their organization, and its consequences.

Skeptic: Everybody knows that writing is an arbitrary human invention that has nothing to do with the underlying structure of the mind. Why do you waste so much space on writing?

Peritior: The discreteness property of writing, both letters of alphabets and Chinese characters, has plenty to teach us about the role of discreteness in systems with emergent properties. Let's not throw out the baby with the bath.

Skeptic: People have been trying for years to find an algebra that will generate the structures of language. Why is your attempt better than any other?

Peritior: Algebra, not "an algebra". I have not found a formal system that will generate sentences. If sentences are based on first principles, such a system may not be possible. What I have done is to show that equations in arithmetic or algebra have the same linguistic deep-structure as sentences of ordinary language, and think through the consequences.

Skeptic: It took the highest courage and originality to do what the evolutionary linguists and psychologists have done.

Peritior: Maybe they are the world's leading conformists. They took two completely acceptable, respectable ideas, by two completely acceptable, respectable authors, and stuck them together. They call it a "reconciliation". Where is the courage, or the originality?

Skeptic: You are suffering from "Don't bother me with facts; I have a theory".

Peritior: The evolutionary psychologists and linguists are suffering from exactly that. They are taking for granted the evolution of language and mind by natural selection. It is a hypothesis to be tested, not an obvious truth to be defended at the cost of contradictions.

Skeptic: Isn't the empirical method quite well regarded in science today? I mean, aren't you supposed to start with an experiment or something?

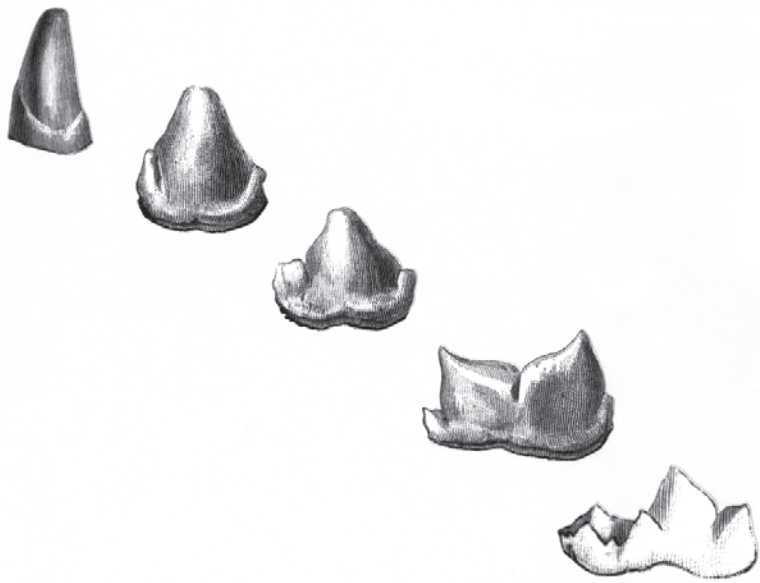
Pertitior: It's math. If you start with an experiment, you end up squaring the circle. Imagine solving universal gravitation with a telescope. You have to begin with the thought-experiment of the wagon with better and better bearings until, once you give it a push, it keeps going in a straight line at a constant speed until some outside force brings it to a stop. Sometimes there is no substitute for theory.

Skeptic: *Syntactic Structures* is almost fifty years old. A lot has happened to deep-structures since then. Why trot out a dinosaur?

Peritior: That hurts (Abler 1992, 1997, 1999, 2001; Farlow et al. 1991). The deep-structure shown in *Syntactic Structures* is surprisingly up-to-date. In the first seven printings, the tree-branches were sloping, but separated. One of them sprouted from the right, and one from the left end of each node symbol, which was a complete word. It was not until the 8th printing that Chomsky began using the more familiar bifurcated form, although even then, the branches were flat, not sloping as they are now. There was a period around 1970 when the transformationalists used trifurcations at some nodes (Langendoen 1969). Even the later changes seem to me like fine-tuning. Starting the trees lower down. Attaching some of the branches to words. If we are going back to square-one, and we are, let's just go all the way there. Besides, it is somehow justice to go back to the source of what may emerge as Chomsky's finest moment.

Skeptic: What about innate ideas?

Peritior: If the numberline theory is right, the idea of innate ideas will have to be abandoned. The word innate is a term in biology, not physics. Innate is the way that spiders spin webs. It is instinct, not first principles. If language and mind are manifestations of first principles, they are more basic than anything in biology, and not properly innate at all.



Tooth Evolution

I am sure that we have an instinct to acquire language, or anyway to develop our intrinsic ability to use it, but language acquisition or development isn't language any more than language pathology is language.

We will have to re-think our whole vocabulary of language and mind. We don't acquire language at all. Maybe it would be better to say that we elaborate it. Ideas such as over-eating and the protection of our children may be instinct, i.e., they may be innate ideas. But they do not characterize the property of mind in the human sense. Mind in the distinctively human sense is intrinsic, not instinctive.

Evolution, all evolution, is topology (cf, Thompson 1966). Evolution is always a matter of continuous, plastic deformation of a structure or an action. The result is either something new or something vestigial, but there is always continuity with something older. You can almost see evolution, frozen in the teeth of a dog, as you go from front to back: Each one is molded from the one before. The Figure is adapted from Buckland 1824: Plate 3, Figure 2; Plate 4, Figure 1; Plate 6, Figure 13. When that happens in behavior, we call it instinct. But language and mind are traceable to universal properties that are intrinsic to matter itself, not to earlier behavior.

Skeptic: Then, why was *Syntactic Structures* Chomsky's finest moment?

Peritior: Chomsky introduced a new notation, deep-structure, that shows the real subject of sentences. Beyond that, *Syntactic Structures* demonstrates the separation between the levels in the organization of language. It also contains the method of thinking through the consequences of an idea to see whether the idea is right, because if the consequences are wrong, the original idea was wrong. The mathematicians do that all the time. Like many mature scholars, Chomsky seems to have forgotten the accomplishments of his youth. I have incorporated them into my method.

Skeptic: You don't like synapses, but don't neurotransmitters trigger behavior?

Peritior: Triggering behavior isn't behavior any more than language pathology or language acquisition is language. We have to look for the central mechanism.

Skeptic: Sentences are vastly more complicated than equations, and can not be compared to them.

Peritior: Equations are not simpler than sentences. They are the simplest sentences. They have a linguistic deep-structure, just like other sentences. And you always know the verb because all of them have the same verb. And they are symmetrical, so you know the relationship between the two sides. That is to say that you know the meaning of the main verb. They are sentences of language in their simplest possible geometric state.

Skeptic: Your genealogical edifice has a definite order, and must represent a development by evolution.

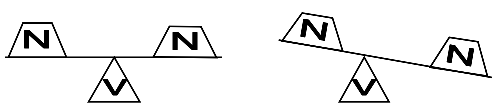
Peritior: I fear that you are using the word "edifice" in a disparaging sense. We have to keep double-meaning in mind. It shows that a single string of symbols in algebra or language can have two meanings at once. Each meaning has its own individual deep-structure. So, you can't build higher-level structures, such as words or sentences, out of lower-level ones, such as phonemes or words. The genealogy is a succession, but not an edifice. That is why evolutionary psychology, at least the part that deals with distinctly human thought, is wrong: Higher-level constructions can't be snapped together out of lower-level ones. They are made of different stuff.

Skeptic: Your idea of ambiguity may be right, but no language-scientist in the modern day would agree with it.

Peritior: We can't legislate truth by vote, not in science. Truth in science doesn't come down from the majority, but starts with one mind, and percolates out from there. Whatever the modern understanding of double-meaning, or ambiguity, that is not to deny that sentences can have more than one meaning, and that higher-level structures aren't built out of lower-level ones. Incidentally, separation of levels also means that, if you don't agree with my interpretation of one level, that does not affect the interpretations at other levels.

Skeptic: Where did the items in your "genealogy" get their properties? The whole thing looks pretty arbitrary to me.

Peritior: The properties themselves are found by lining the systems of arithmetic and language up next to one another, and picking out the shared properties by inspection. If you think about it, that is the only way to be sure we have found the right properties, and is the reason for writing this book. With no control group, we don't know what belongs or what doesn't. Equations let you identify the structures of ordinary sentences; and ordinary sentences let you identify the structures of equations. The most important structure, as far as I can tell, is the sentence. Recursion has become the Holy Grail of the language scientists, but you can't build sentences by recursion. Even little children have a sense of what a sentence is, although the concept of sentence is pretty hard to define. Under the cladogram model, a sentence in its simplest form is a symbolically symmetrical speech event. The simplest ones are equations. Symmetry, a basic property of fundamental nature, then becomes the basis for assertion. The basic assertion is that this balances that. To put it in perceptual terms, we experience sentence symmetry as assertion in much the way that we experience acoustic frequency as pitch. When the deep-structure of the basic equation is distorted into asymmetrical form, as we saw earlier, the remnants of the symmetry property allow it to retain the power of assertion in ordinary sentences, like this.



Equation Ordinary Sentence

Although the sentence is no longer balanced, it retains the basic geometry of balance, and with it the power of assertion. Separation of levels is one place where the evolutionary model fails. Even if it could generate the form of sentences by recursion, a doubtful enterprise, but even if it could, there would be no property of assertion. Assertion is the emergent property "not present per se in any of the associated constituents". When we think of sentences, we think of deep-structures, but after that forms, it is the asymmetrical balance between subject and predicate that provides the beginnings of meaning.

Skeptic: What is so great about assertion?

Peritior: You can't have a human-like society without it. You have to assert laws, whether ours or nature's. Remember that, in addition to being quotes and sentences, The Ten Commandments and Newton's Laws are assertions, and have to be. You have to assert that you did something, or that you will do something, or that somebody else did something, or that we ought to do something, or that things work in a certain way, in order to have laws or science or religion, or even customs and culture. Assertion is the basic power that makes communities human.

Skeptic: Anything else?

Peritior: Discreteness, and that includes discreteness of assertions. Governments are possible because we can pigeon-hole ideas into discrete categories. That is what laws are, or constitutions, or *Robert's Rules of Order*. They are ways of classifying sentences into discrete categories that we can then deal with under discrete rules. That is why meetings are possible, or community councils. That is what makes religious fundamentalism possible. It forms by creating a discrete mental category for Good, and another discrete mental category for everything else, and placing just one thing into the box for Good. Life is simpler that way. There is fundamentalism in science.

Skeptic: Writing uses discrete symbols. Are you telling me that writing is innate?

Peritior: It is worse than that. The nature of language requires the use of discrete phonemes and morphemes. Since you can always use a discrete symbol as a substitute for any discrete object, the way the geneticists do when they publish the base-sequence of a gene, the potential for writing is intrinsic to the nature of language.

Skeptic: Now you are using the word discrete. But your original paper called it the particulate principle of self-diversifying systems. You realize that your physics is about a hundred years behind the times. There is no such thing as particles in nature.

Peritior: My choice of vocabulary is drawing more fire than I expected. The geneticists still talk about the particulate gene, and the physicists still talk about subatomic particles, without attracting objections. I realize that after Einstein showed an equivalency between mass and energy, the idea of particles came under dispute. But as vocabulary, particulate and discrete are the same thing. If "particulate" is wrong, "discrete" is just as wrong. You are picking on my choice of words, hoping to change the subject.

Skeptic: You have completely failed to understand arithmetic or language, which are both completely arbitrary.

Peritior: That myth comes from the meaning of words, and from written symbols, which are arbitrary. If they had to refer to real-world categories, they would be a rehearsal of behavior, like the dance of the honey bee. We couldn't obtain them in any other way. But then there would be only miniaturization and re-enactment, not "condensation accompanied by the formation of a substitute". We would have behavior, not language. Arbitrariness of reference is a necessity for a human-type language, and not as arbitrary as it looks. If we want science, we have to put up with "the biology of unicorns" alongside it. Beyond the necessity of arbitrary words, the rest is fundamentally fixed in advance. You have to have discreteness because blending constituents will lose information. You have to have linearity of delivery, or you will lose your place in the signal.

Skeptic: One-at-a-time delivery of constituents of language is an epiphenomenon caused by the physiology of the vocal tract or our in-ability to do more than one thing at-a-time. Linearity has no standing whatever in the structure of language. The real topic of research in the scientific study of language is the hierarchical structure.

Peritior: To begin, if you believe in the biological theory, you believe that all formal structures become integrated into the system, whatever the reason for their introduction in the first place. So, all biological linguists must accept the idea of speech being integral to language. Here is a good example of the kind of contradiction that has to be eliminated if we want a theory of the mind. You can't accept the biological theory of language and also think that phonemes or linear delivery are outside it.

As far as linearity itself, the acoustic signal is a blending system, combining information for consecutive phonemes into the same acoustic event. We are already doing two things at the same time, and in rapid speech, more than that. We can forget about only being able to do one thing at-a-time, whether for cognitive or articulatory reasons. And the idea of a real topic of research is probably best forgotten. In the 1960s, the primary object of research was discovery procedures to identify the phonemes and morphemes of languages. Now it is the hierarchical structure. Until we know what language is, it might be better to just cast out contradictions until we have a theory that is related to nature.

Skeptic: In the evolution of language, what is wrong with the idea that sentences evolved when speakers put words together, or when single words split into two? Isn't that exactly the sort of thing that people would naturally do spontaneously?

Peritior: It is separation of levels again. You can't build sentences by putting words together. The best you could hope for might be compound words. You might put words together, but that wouldn't generate anything with emergent properties, just compound words. You might get the origin-of-language equivalent of "lightning bug" or "flint knife" or "venomous snake" or "district water pollution control research station", but not sentences. The same goes for splitting words into pieces. If it did happen, it wouldn't, in itself, generate sentences any more than recursion, in itself, would generate sentences.

Skeptic: You claim that the morphemes of language are discrete, and compare them to constituents of arithmetic. But the morphemes of sign language blend together, like the articulations of speech. There is nothing discrete about them.

Peritior: The underlying morphemes of language are exactly as abstract as the underlying phonemes. The consecutive speech gestures that signify phonemes blend together for the same reasons that the consecutive hand gestures of sign language blend together. But the morphemes don't blend any more than the phonemes do. They are at a lower level than that.

Skeptic: What can you possibly mean by saying that vocabulary is outside language?

Peritior: I hope I didn't give that impression. The vocabulary words of language have the power of referring to objects outside of language. That is all.

Skeptic: The phonemes of the world's languages occupy every imaginable place of articulation in the vocal tract. They are more a continuum than a system based on discreteness.

Peritior: Once again, it is the articulations, not the phonemes, that occupy every imaginable location in the vocal tract. But even that doesn't mean that any particular language is a blending system. The articulations possible in one language come under a rule, too. That is the one that is interesting here, because that is the one that a speaker has to deal with while talking. Even a translator only speaks one language at-a-time. The psychophysical thresholds divide up the acoustic space into perceptual regions, each with a phoneme attached to it. They amount to a kind of perceptual analog-to-digital converter. Much of language change is a matter of the speech-sounds keeping their phonetic distance from one another. There is more order than chaos.

Skeptic: If speech articulation is outside language because it is outside the numberline principle, then only botanists will be interested in phonetics.

Peritior: As we have seen, the speech signal is so perfectly adapted to its role both in production and perception that the speech mechanism will always be interesting to biologists. And the connection between the speech mechanism and the phonemes, as well as its mediation through the perceptual system, will always be interesting both to language scientists and to otorhinolaryngologists.

Skeptic: Back to syntax. Aren't you spreading yourself thin by looking at trivial properties like ambiguity and paraphrase, when the real object of inquiry is the hierarchical structure, in other words, recursion?

Peritior: Recursion is language about as much as frequency is music.

Skeptic: It is ridiculous to claim that the auxiliary with "would" has to be the subjunctive, or that the auxiliary with "will" has to be the future. These associations have to be arbitrary.

Peritior: You are right. Such a claim would have been ridiculous if I had made it. What I tried to show is that arithmetic offers opportunities for a number of different sentence forms, and that language has taken advantage of these by attaching different meanings to the different forms. Whether one sentence form is actually better for a certain meaning remains to be seen.

Skeptic: "plus" isn't a preposition, it is a conjunction, more like "and".

Peritior: That works as long as you stick to arithmetic. There are so few operators, only four, that using "and" can only mean "plus". But when you look at language, you see that prepositions like "under" or "beside" can occupy the same location. For example "John found the potatoes under the onions", or "John found the potatoes beside the onions". Either way, though, the preposition/conjunction belongs in a certain place in the sentence tree-structure.

Skeptic: Alternation can't have anything to do with language, since there are sentences that have no overt grammar-words, such as your own sentence, "John bought a car yesterday".

Peritior: People are pretty flexible. All you have to do is keep the linear constituents near enough to each other so that the tree-structure is not lost. In inflected languages, like Latin, you can mix up subject and object word order, most of the time, because the grammatical role of subject or object is attached to the words. That still gives alternation of a kind. In a word-order language like English, some sentences might not have any overt grammar words. But that doesn't mean that there is no alternation when the sentence contains grammar words.

Skeptic: Didn't Noam Chomsky himself convert to the evolutionary side (Hauser et al. 2002)?

Peritior: Chomsky's transfer of sympathies has to be understood as his capitulation to the personality and persistence of his scientific friends, who needed to score that coup. It has nothing to do with basic science.

Skeptic: Aren't chimpanzees, or anyway bonobos, almost human?

Peritior: People think that their dog is human. They tie a bandanna around its neck and get it to drink beer as a kind of initiation. The idea of chimpanzees inching closer to human is a matter of intellectualized sentimentality and political correctness. Again, it has nothing to do with basic science.

Skeptic: How can mathematics exist independently of the mind when mathematics is, ultimately, a product of the mind?

Peritior: Mathematics isn't a product of the mind. Look at the genealogy. The relationships that cause equations are the same ones that cause language and mind. Mathematics and mind are brother-and-sister, not parent-and-child. They share a common source.

Skeptic: And what might that consist of?

Peritior: Like the dimension of time encoded in every sentence, the shared source reflects dimensional properties of matter. The properties of sentences, and that includes equations, are a manifestation of dimensional properties of matter, and ultimately will serve as a guide to the discovery of dimensional properties. At this early stage, the properties appear to be discreteness, linearity, time, symmetry, the formation of groups. Physics shares a common natural source with algebra. That is why algebra is a mirror of physics. A different kind of intellect will be required for solving these problems. The items on the cladogram may have their source in higher dimensional properties in the way that an ordinary cube represents a hypercube passing through our space.

Here we can appreciate double meaning of sentences as a kind of poor-man's Gödel's proof. You can't create all true sentences by putting words together because sentences are not made by putting words together. You can't construct any level by assembling parts from the next lower level. The properties of each new level must be introduced independently from outside the system. The meaning of the sentence is only indicated by the words in the tree-structure, not prescribed by it. The program to understand the sentence by studying recursion has exactly the same footing as David Hilbert's program to build mathematics by starting with a fixed system of structures and operations.

Remember that you can't write the equation that accounts for the existence of equations. Foundations of mathematics won't look like mathematics, and will be disappointing to mathematicians because their minds love mathematics. The rest of us expect at least the assurance that a solution is possible in mathematics. But that is over.

If we want to understand our mind, we will have to discover foundations of mathematics – and be serious about it.

Skeptic: I'm pretty conservative. I instinctively distrust theories that are too sweeping.

Peritior: That pretty much brings us full circle. To me, the statement "People are animals, and animals evolve by natural selection, so language evolved by natural selection" is as sweeping as a theory can get. To me, conservatism is another name for conformity. We assume that the questions that are being asked now are the right ones because we somehow trust our scientific predecessors to bequeath us the right questions. But plenty of scientists don't know their predecessors of just ten years ago. Science is truly a dragon without a head. We hang on until the last, trying to answer old questions that no longer apply. If the evidence says that gene, language and arithmetic are manifestations of the same thing, we should accept it and move on.

Skeptic: Doesn't the evolution of the brain determine language and mind?

Peritior: The old questions no longer apply. The evolutionary psychologists and evolutionary linguists asked, How did language and mind evolve? They should have stuck to Noam Chomsky's earlier question, What is language? Even that is changed. Now the question is, what causes discreteness? What is linearity? What is hierarchy? What causes symmetry? How does the brain obtain voluntary use of these, and other properties of the cladogram?

Skeptic: Isn't everything the mind? Doesn't the mind include why we love and why we hate, why we dream, why we appreciate beautiful and ugly – as well as language and mind? Haven't you just oversimplified everything?

Peritior: That is a lawyer's argument. Truth in science doesn't work by browbeating concessions out of people, or by squeezing things closer together. You can define the mind as everything, but animals share a lot of whatever that represents. Calling love and hate, good and bad, beautiful and ugly, overeating and infanticide “mind” is the same as calling computer language “language”. You can do it, but at the cost of fatal contradictions. The interesting question is what makes us human. The rest will follow after that. We have to separate what we think about from how we think. We have to find out how we think, because what we think about is too vast and ill-defined, and is probably shared by other animals. It isn’t what makes us human in the first place.

Skeptic: Aren’t there computer experiments showing that light-sensors on the surface of skin will evolve into an eye in 40,000 generations (Nilsson et al. 1994), and that a number sense can evolve by natural selection?

Peritior: The computer only does what its programmer tells it to do. Those computer experiments are testing Intelligent Design, not natural selection.

Skeptic: In the last analysis, you are marching to the wrong drummer. You cling to a nineteenth-century ideal of simplicity, when the real answer is complexity. There is a whole science of complexity. And isn’t there a Complexity Institute, or something? Maybe you just can’t handle complexity.

Peritior: I take my cue from nature. Remember Einstein’s *Electrodynamics*. It has plenty of complexity that mediates between simplicity at the beginning and simplicity at the end. I think that that is how nature works. Whatever the complexity of the nuclear particles and forces, there are only three subatomic particles; and there are only about a hundred chemical elements, and even those are organized on a two-dimensional grid. I think it is because other atoms, nature herself, in effect, couldn’t keep track of them otherwise. Even as it is, there are too many. The lanthanides can barely be distinguished one from the next. They appear in the same minerals, and erbium, terbium, ytterbium, and yttrium are all named after the same suburb of

Stockholm, Ytterby, where the first samples were found. It is altogether right that you can barely tell the names apart. And the same goes for the speech articulations of any given language. Imagine a brain ruled by swirling storms of nerve firings. It wouldn't know what was going on inside itself. If swirling storms of nerve firings happen, they are mediating between simplicity at one end and simplicity at the other, but are not themselves the underlying representation of anything. Complexity is also introduced when you convert from nature's sometimes continuous terms to our discrete ones, the same as in celestial navigation, or transcendental numbers.

Skeptic: Your theory is so uncompromising that it is almost unbearable. Do you have to be so severe?

Peritior: If there were compromise anywhere, there would be no theory in the first place. Actually, it is a beautiful theory. To me, it looks like a crystal in the moonlight, with a place for every molecule, and every molecule perfectly in place. And the creation of language and mind by the single expedient of introducing asymmetry to equations is inspired: we get a non-Euclidean mind that circumvents Gödel.

Numberline offers a glimpse of the limits to mathematics. Time has the same form as equations – a linear sequence of discrete constituents. You can't use equations to calculate the properties of time. That is why the physicists are having so much trouble with it.

Skeptic: Why should we throw away eighty years of high-level science, maybe a billion dollars in research funding, a few million pages of published findings, the product of our finest minds, because you noticed some trivial contradiction?

Peritior: You can ignore it about as safely as you can ignore absolute rest in physics, or the transcendence of pi.

Skeptic: But there are linguistic and biological considerations. What about Subject-Object-Verb languages versus Subject-Verb-Object languages?

Peritior: That isn't the origin of language. The precise order of functions in a sentence begs the question of how they got there in the first place.

Skeptic: What about inflected languages, the ones with word-endings, like Latin?

Peritior: The word-endings are the same as grammar-words in word-order languages, except that they are attached to other words instead of occurring by themselves. But beyond that, word-endings have to emerge after words are already there. The question of inflected languages begs the question of language itself.

Skeptic: What about motherese, the way that mothers speak to their infants?

Peritior: Motherese is a modern language. And it is a reduced form of adult language. No matter how irresistibly endearing motherese is, we have no idea what its role might have been in pre-human societies. And it is too vague to be responsible for the fine detail that is obvious in even the simplest language.

Skeptic: What about game theory, and hunting? What about the behavior of play, and grooming, and tool-making, grip, hand-signals, and throwing spears? Don't those all make important contributions to our understanding of how language evolved?

Peritior: They represent mental strategies, allright, but they all take us back to square-one and drop us there. We don't see any explanation of discreteness, linearity, external and internal reference, a main verb, symmetry and asymmetry, the cladogram as-a-whole. Every new mechanism forces us to start over again. They are circular, and ultimately beg the question of mathematics, language and mind.

Skeptic: Absolutely nobody agrees with your theory. How can it be right?

Peritior: The doctors say that medicine progresses one death at-a-time. They don't mean patients, either. They mean doctors. And there may be some of that in science. You have to look in old textbooks, if you can find them, because they are misunderstood, too. The used-book dealers don't want textbooks that are "out-of-date", although old textbooks are a treasure-trove of what people thought at the time. You have to look in thrift stores and flea markets. Here is Horatio Hackett Newman, twenty-one years after the rediscovery of Mendel in 1900. Page 42 is still using the 19th century image of unit characters being "swamped out". And page 415 has a section on blends. Newman is proud of his indecision, and brags, on page 7, "In this open court of conjecture, the names of Lamarck, of Darwin, of Weismann figure prominently - - - while others, like myself, are agnostic [and] belong to no school".

And here is A.F. Shull, fully thirty-eight years after the rediscovery of Mendel, with a section on blending inheritance on pages 153-154. To be fair, this is the third printing of the third edition of a book first published in 1921. But these are very respectable authors. Newman was a professor in the University of Chicago, and his book was published by The University of Chicago Press. Shull was a professor in the University of Michigan, and his book was published by McGraw Hill. There is every reason to think that at least some of Shull's readers and students accepted the remnants of blending in 1938, and that they still accepted them 15 years later when Watson and Crick finally put an end to blending inheritance by demonstrating the particulate gene.

The old blending gene faded like the smile on the Cheshire cat, and evolutionary psychology is liable to do the same. There will be a vanishing trail of evolutionary remnants and nibbles forty years after the rediscovery of the numberline theory. And there will be reconciliation between natural selection and the numberline, in much the way that there is reconciliation now between Darwin and Chomsky among scientists, and between science and religion among the faithful. The most wonderful reconciliation I have seen comes from a completely respectable professor in the University of California at Berkeley (Le Conte 1897, page 360), publishing through the same publisher that published Darwin and Huxley in The United States: "As organic evolution reached its goal and completion in *man*, so human evolution must reach its goal and completion in the *ideal man* – i.e., the Christ" [Le Conte's emphasis]. Popular acceptance doesn't make an idea right. Even science has fads.

Skeptic: That's not fair. We have two correct theories and are closing the gap. Somebody used the word reconciliation, but they didn't intend for it to be taken seriously.

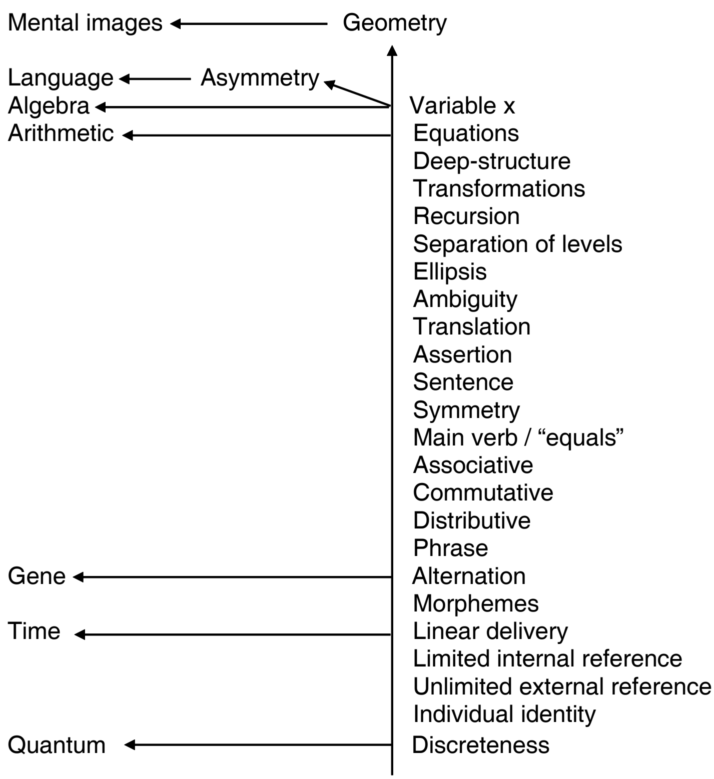
Peritior: The truth never lies somewhere in between. You have two correct theories that have nothing to do with each other. The whole image of closing the gap, of inching closer, of reconciling things, belongs to lawyers, not scientists. It is rhetorical, not factual. You can't tell yourself, "These people are saying this; and those people are saying that", and hope to get anywhere. The evolutionary linguists have one incomplete idea, transformational grammar, that covers only ten or twenty percent of the scientific territory needed for a complete theory. If Louis Pasteur taught the world anything, it is that you can't just look at a thing and interpret it. You understand something only by comparison to something else. You have to have controls to show you the right features on the basis of explicit method. Anything else is guesswork. The modern theory of language and mind is every inch The Blind Men And The Elephant come true.

Skeptic: Do you have any idea how unlikely it is that the number-line theory is true?

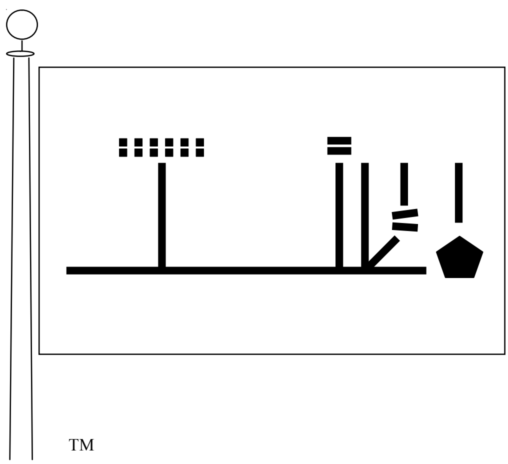
Peritior: A lot of new ideas turn out to be wrong. But plenty of abandoned theories were once standard. In our own experience, we had land bridges, polywater, and cold fusion. It is as wrong-headed to reject a theory because it is new, as it is to accept it because it is old, or even standard. Old and new aren't evidence. Going by the probabilities is just elevating mediocrity to the level of a science and living by it. Adults are supposed to have judgment.

Skeptic: Even if the numberline theory is true, it doesn't matter much anyway, does it?.

Peritior: It is pretty basic. If the quantum property of matter, its discreteness property, is the same as the discreteness property of the gene and language, and more especially of arithmetic, then the causes of the quantum property can't be calculated in mathematics. And if time is a matter of counting a series of discrete events, which is what equations are, we can't calculate the cause of time. Everybody knows there is a limit to how fast you can go, and how far. And there is a limit to how accurate a measurement can get. Now there is a limit to how much we can calculate by mathematics. Even notation as we understand it may be in question. The real cladogram includes time and the quantum.



The numberline theory offers the only definitions of language and mind and human being. The numberline principle shows that there is such a thing as language, and that there is also mind in a specific, special human sense. Mind, as we are interested in it here, concerns how we think, rather than confounding what we think about with how we think. More importantly, it shows that there is such a thing as a human being that is at the same time apart from other living things, and closer to the center of nature than any of them. The removal, that began with Copernicus, of the human race from the center of nature, is eliminated by the numberline. The cladogram of properties will be recognized by any civilization that has discovered the numberline principle, and will show that their understanding of the mind has reached a minimum of maturity. The cladogram is the symbol that unites all minds no matter what planet they inhabit, and is suitable for bumper-stickers and T-shirts, as well as inclusion on probes beyond the solar system. In effect, it is "The Union Jack of the Universal Human Mindtm".



Skeptic: You don't expect extraterrestrial beings to interpret that, do you?

Peritior: We have to take our chances, but the chances are that they will. The genealogy ought to apply as generally to intelligent beings, as the periodic table of the elements applies to the atoms. The general shape should be a schoolchild memory for any advanced civilization.

Skeptic: How will they interpret our "equals" symbol?

Peritior: Again, we have to take our chances, but they should recognize its location on the genealogy, so we would in effect be telling them what our "equals" symbol looks like. But the symbol is a good one, parallel lines, the same distance apart anywhere you look at them. And they can see that it contrasts with the asymmetrical version of the same thing where ordinary sentences go. It's as good as anything. Maybe a fulcrum would be OK, too. If not, well, Amen.

Skeptic: I just can't believe that language didn't evolve by natural selection.

Peritior: Look. Kurt Gödel couldn't have thought of Gödel's proof if his mind were limited by the axioms that underlie the equations that predicted its evolution. Andrew Wiles couldn't have solved Fermat's Last Theorem. Go ahead. Cross over. You can have a static, stodgy human mind limited by the accidents of evolution, or a fascinating, dynamic one that soars on its own in non-Euclidean space. Just on the basis of fun alone, I'll take the *Structure of Matter* vision, any day.

Skeptic. I *still* can't believe that language and mathematical instinct didn't evolve.

Peritior. OK. The gene diverges from the same cladogram as arithmetic and language. There is a gene for math or language about as much as there is a gene for genes. Imagine a gene for discreteness, another for different kinds of signals, another for linear ordering, another for alternation. There is no gene for genes because it would have to possess its own properties before it could encode them. And the gene is already pretty far along on the math/language cladogram. The only basic feature that might be coded in the genes is the asymmetry property of sentences in ordinary language.

Skeptic: Suppose it *is* right. Now what?

Peritior: If you can solve the dimensional matrix that produces the cladogram, you will find out why mathematics is “so appropriate to the objects of reality”, you will know the fundamental mechanism of the human mind; and you will have the next revolution in physics.

Skeptic: Where would you begin?

Peritior: The first thing we have to do is abandon the vocabulary of “transformations” in language. In exactly the way that the biological word “innate” obscured the inorganic basis of language and mind, “transformations” indicates that phrases in language are built out of their constituents, which they aren’t. There should be just one word, “re-attachment”, maybe, to show that constituents are re-attached to their deep-structure in a different way. The vocabulary of mathematics is a little better, although there ought to be one uniform vocabulary to cover both.

We have to believe that the next revolution in science will be to solve the dimensional matrix that generates the cladogram. Somebody has to attempt it. You practically have to be born for a solution at that level; and even then you have to work beyond your ability. I don’t know whether the intellectual machinery even exists now. And the scientific public has to be ready to accept the solution, no matter whom it comes from. By its nature, something new won’t be a linear development out of anything that is standard. It will be un-expected, so it might seem to some from out of the blue, an unexpected person as well as an unexpected idea.

We can start by organizing the structures and properties on the cladogram into families. Discrete constituents form a family: phonemes, words, phrases, sentences with their deep-structures. Reference is a class of properties; external reference precedes internal reference because internal reference needs something to refer to. The basic verb isn’t an action, but a fulcrum, or center of gravity, or even an axis of symmetry, with the property of reference added. Symmetry, and from that, assertion, represents a fundamental property of matter. Translation is an expansion of paraphrase; ellipsis is a sub-class of paraphrase; double-meaning is somehow paraphrase turned inside-out. I would separate what is primitive from what is derivative, and look at the primitive. But we will have to change our way of looking for answers, because we have reached the limit of math.

We have to remember that foundations won't be some unified infrastructure. Each new level in the organization of language and arithmetic will be introduced independently from outside the system. It isn't the way that we think of foundations. Even the word is misleading. But that is what separation of levels requires. Each new level in the cladogram will have its own separate, individual foundations. And we have to find out how the properties of matter are shared with those of numbers and arithmetic, starting with discreteness. The numberline theory offers a start.

And we have to abandon the vocabulary of "language" and "mind", and "culture". They all have two meanings that are used interchangably, and are dangerously misleading. Language means "human language", and "communication". Culture means "human culture" and "mutual learning". Mind means "human mind" and "everything we think about". Until distinct terms emerge, we will have to be content with "language in the human sense" and "culture in the human sense", and "mind in the human sense".

For the future, we will have to consciously and deliberately abandon the modern ideal of specialization. Through planned curriculum, beginning almost in kindergarten, but certainly in college, we will have to return to the Renaissance ideal of knowing something about everything. We can not guess where the next revolution will come from, foundations of mathematics in the structure of ordinary sentences, and that kind of thing. And we will have to expect new ideas from older investigators, in their fifties and sixties, instead of their twenties, simply because of the vast territories of knowledge that must be mastered. Universities will have to make allowance for that, and even require it.

Chapter 4:
The Process of Mind and The Pattern Playback Machine

To me, the scientific study of language has produced just one published paper that stands out as a permanent classic of general science. It deserves special attention and recognition as a stable milestone in a changing landscape. The conditions that led to it are heroic. "Perception of the Speech Code" was published in 1967 by Alvin M. Liberman, Franklin S. Cooper, Donald P. Shankweiler, and Michael Studdert-Kennedy, of Haskins Laboratories, and appeared on pages 431 to 461 in volume 74, issue number 6, of the *Psychological Review*. "Perception of the Speech Code" grew, slowly, out of the World War II project to build a translating machine to install in the cockpit of fighter planes. If you only read one scientific paper in your life, this is the one (Horner et al. 1993, page 227).

The first requirement for a translating machine was a speech-recognition device that could convert the acoustic signal of speech, consisting of just the sound that travels through the air, into a written form that could then be subjected to grammatical analysis. The project called for a two-pronged technological effort. One device was needed that would convert the speech signal to a printed form that could be examined and interpreted. And a second device was needed that could generate speech directly. In the end, the project took more than 30 years, and produced completely unexpected results. The eventual result, as mentioned in the main part of this book, was the discovery that there is no single sound event that corresponds to any given phoneme, and I tried to put that discovery into perspective. But the technical details of the speech synthesizer are fascinating.

It was at the height of the War, and the language scientists had to push ahead with all possible speed. They worked so fast that they never published the design of what they called the tone wheel, which was the heart of their speech synthesizer, the Pattern Playback. And it is to remedy this omission that I have included this short chapter. Franklin S. Cooper did both the theoretical work in linguistics, as well as the engineering. Many of the details were rescued for me at the last moment through the kindness of Michael Studdert-Kennedy, Director Emeritus of Haskins Laboratories, about 1995, just two months before Cooper's death. Studdert-Kennedy questioned Cooper about how he made the tone wheel. Patrick Nye, also of Haskins, was present at that conversation, and also provided valuable details.

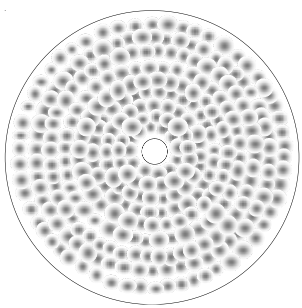
Cooper realized that they needed a device that was intuitive, something that was easy to use, and easy to understand. It also had to be versatile, so that if one acoustic pattern produced something close to the desired perceptual effect, but wasn't exactly right, they could change it quickly. And it had to be understandable to language scientists who were not engineers.

Like the vocal tract, the Pattern Playback consisted of a frequency generator and a frequency selector. Unlike the vocal tract, the frequency generator was the tone wheel, consisting of concentric circles of sinusoidally alternating dark spots and clear places. The circle closest to the center had only a few dark spots; and each successive circle had more, until the one at the outside, which had the most. By spinning the tone wheel, and directing a light through it into a magic-eye light detector, and then into a loudspeaker, it produced sound energy at all frequencies.

The frequency selector consisted of a clear plastic belt running on rollers. By painting a pattern on the plastic belt with white paint, and using the white paint as a reflector to select just the frequencies that would be converted into sound, the experimenters were able to make artificial speech that was, eventually, about ninety-eight percent understandable.

The heart of the Pattern Playback was the tone wheel, which was so important that a language scientist once told me that he built it by pouring India ink over a plate-glass wheel, drawing sine-waves onto the ink with a pencil, and scraping away the outer half of each sine wave with a razor blade. Of course, Franklin S. Cooper was responsible for the design and construction of the tone wheel, which he made by exposing a circular sheet of X-ray film mounted on the spindle of a lathe. At that time, Haskins Laboratories was located in New York City, in the shadow of the Chrysler Building. During the day, the electricity supply was so erratic that the light from the grain-of-wheat bulb used for exposing the film was too irregular for speech synthesis. Cooper had to work at night, when the electricity was steadier, and maybe to avoid interference from radio transmissions. And every darkroom enthusiast knows that it is easier to avoid unwanted light by working at night.

By attaching the grain-of-wheat bulb to an electric sine-wave generator, Cooper produced a light signal that got brighter and dimmer in a sine-wave pattern. Then he spun the wheel on the lathe, exposing successive circles of the film with the bulb, and developed the film. I visited Haskins Laboratories about 1970, and saw two failed tone-wheels hanging on the wall, as decorations.



Impressionistic sketch of the TONE WHEEL, reconstructed from descriptions. The real tone wheel is about two feet in diameter, and has many more circles than the one shown.

The Pattern Playback has taught the world so much about the nature of the human mind, and was so heroic in its conception and construction, that it is a national treasure. I understand that the Smithsonian Institution has turned it down several times, but that it is now safe at Haskins Laboratories in New Haven. I wish it could be more accessible to the public. Certainly it has more intrinsic interest than a telescope, not very different from many others, that was the one used in discovering the planet Pluto. It is more like Galileo's telescope, or the Wright flyer.

With that last example of understanding a thing by comparing it to something else, I pray the Smithsonian to accept the Pattern Playback, and the reader to accept The Numberline Theory of arithmetic, language, and the universal human mind.

The End

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