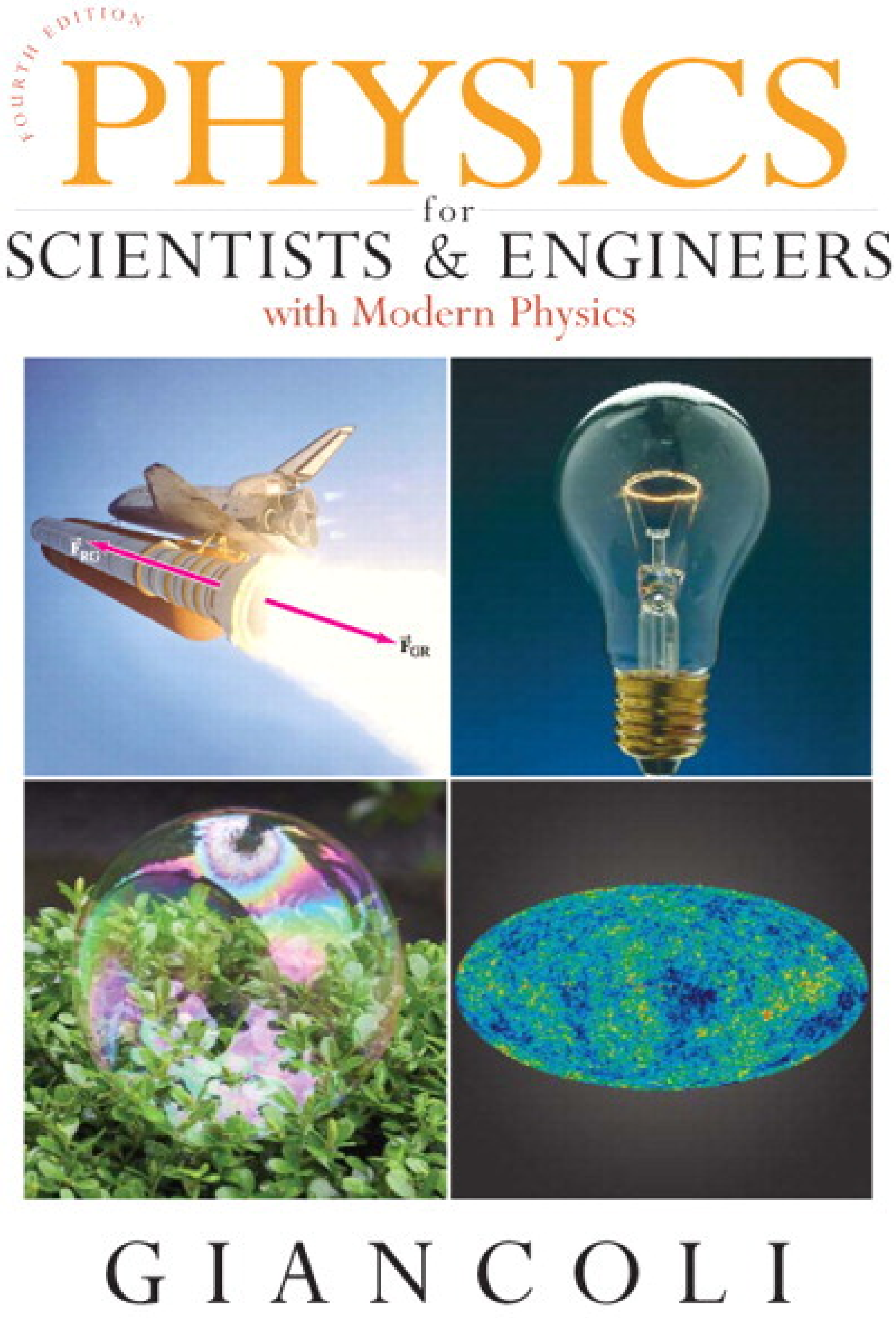
SOLUTION MANUAL FOR



FOURTH EDITION

PHYSICS

for

SCIENTISTS & ENGINEERS

with Modern Physics

GIANCOLI

CHAPTER 1: Introduction, Measurement, Estimating

Responses to Questions

1. (*a*) A particular person's foot. Merits: reproducible. Drawbacks: not accessible to the general public; not invariable (could change size with age, time of day, etc.); not indestructible.
   (*b*) Any person's foot. Merits: accessible. Drawbacks: not reproducible (different people have different size feet); not invariable (could change size with age, time of day, etc.); not indestructible.
   Neither of these options would make a good standard.

2. The number of digits you present in your answer should represent the precision with which you know a measurement; it says very little about the accuracy of the measurement. For example, if you measure the length of a table to great precision, but with a measuring instrument that is not calibrated correctly, you will not measure accurately.

3. The writers of the sign converted 3000 ft to meters without taking significant figures into account. To be consistent, the elevation should be reported as 900 m.

4. The distance in miles is given to one significant figure and the distance in kilometers is given to five significant figures! The figure in kilometers indicates more precision than really exists or than is meaningful. The last digit represents a distance on the same order of magnitude as the car's length!

5. If you are asked to measure a flower bed, and you report that it is "four," you haven't given enough information for your answer to be useful. There is a large difference between a flower bed that is 4 m long and one that is 4 ft long. Units are necessary to give meaning to the numerical answer.

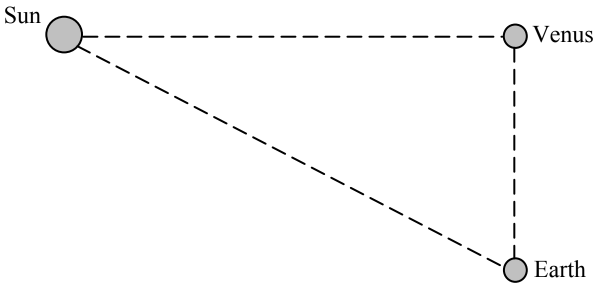
6. Imagine the jar cut into slices each about the thickness of a marble. By looking through the bottom of the jar, you can roughly count how many marbles are in one slice. Then estimate the height of the jar in slices, or in marbles. By symmetry, we assume that all marbles are the same size and shape. Therefore the total number of marbles in the jar will be the product of the number of marbles per slice and the number of slices.

7. You should report a result of 8.32 cm. Your measurement had three significant figures. When you multiply by 2, you are really multiplying by the integer 2, which is exact. The number of significant figures is determined by your measurement.

8. The correct number of significant figures is three: $\sin 30.0^\circ = 0.500$.

9. You only need to measure the other ingredients to within 10% as well.

10. Useful assumptions include the population of the city, the fraction of people who own cars, the average number of visits to a mechanic that each car makes in a year, the average number of weeks a mechanic works in a year, and the average number of cars each mechanic can see in a week.
    (*a*) There are about 800,000 people in San Francisco. Assume that half of them have cars. If each of these 400,000 cars needs servicing twice a year, then there are 800,000 visits to mechanics in a year. If mechanics typically work 50 weeks a year, then about 16,000 cars would need to be seen each week. Assume that on average, a mechanic can work on 4 cars per day, or 20 cars a week. The final estimate, then, is 800 car mechanics in San Francisco.
    (*b*) Answers will vary.



11. One common way is to observe Venus at a time when a line drawn from Earth to Venus is perpendicular to a line connecting Venus to the Sun. Then Earth, Venus, and the Sun are at the vertices of a right triangle, with Venus at the 90º angle. (This configuration will result in the greatest angular distance between Venus and the Sun, as seen from Earth.) One can then measure the distance to Venus, using radar, and measure the angular distance between Venus and the Sun. From this information you can use trigonometry to calculate the length of the leg of the triangle that is the distance from Earth to the Sun.

12. No. Length must be included as a base quantity.

Solutions to Problems

$\boxed{1.}$ (*a*) 14 billion years = $\boxed{1.4\times10^{10}\text{ years}}$

(*b*) $\left(1.4\times10^{10}\text{ y}\right)\left(3.156\times10^{7}\text{s}/1\text{ y}\right)=\boxed{4.4\times10^{17}\text{s}}$

2. (*a*) 214 $\boxed{\text{3 significant figures}}$
(*b*) 81.60 $\boxed{\text{4 significant figures}}$
(*c*) 7.03 $\boxed{\text{3 significant figures}}$
(*d*) 0.03 $\boxed{\text{1 significant figure}}$
(*e*) 0.0086 $\boxed{\text{2 significant figures}}$
(*f*) 3236 $\boxed{\text{4 significant figures}}$
(*g*) 8700 $\boxed{\text{2 significant figures}}$

3. (*a*) $1.156=\boxed{1.156\times10^{0}}$
(*b*) $21.8=\boxed{2.18\times10^{1}}$
(*c*) $0.0068=\boxed{6.8\times10^{-3}}$
(*d*) $328.65=\boxed{3.2865\times10^{2}}$
(*e*) $0.219=\boxed{2.19\times10^{-1}}$
(*f*) $444=\boxed{4.44\times10^{2}}$

4. (*a*) $8.69\times10^{4}=\boxed{86,900}$
(*b*) $9.1\times10^{3}=\boxed{9,100}$
(*c*) $8.8\times10^{-1}=\boxed{0.88}$

(*d*) $4.76\times10^{2} = \boxed{476}$

(*e*) $3.62\times10^{-5} = \boxed{0.0000362}$

5. % uncertainty $= \dfrac{0.25\text{ m}}{5.48\text{ m}}\times100\% = \boxed{4.6\%}$

6. (*a*) % uncertainty $= \dfrac{0.2\text{ s}}{5\text{ s}}\times100\% = \boxed{4\%}$

(*b*) % uncertainty $= \dfrac{0.2\text{ s}}{50\text{ s}}\times100\% = \boxed{0.4\%}$

(*c*) % uncertainty $= \dfrac{0.2\text{ s}}{300\text{ s}}\times100\% = \boxed{0.07\%}$

$\boxed{7.}$ To add values with significant figures, adjust all values to be added so that their exponents are all the same.

$$\left(9.2\times10^{3}\text{s}\right)+\left(8.3\times10^{4}\text{s}\right)+\left(0.008\times10^{6}\text{s}\right)=\left(9.2\times10^{3}\text{s}\right)+\left(83\times10^{3}\text{s}\right)+\left(8\times10^{3}\text{s}\right)$$

$$=\left(9.2+83+8\right)\times10^{3}\text{s}=100.2\times10^{3}\text{s}=\boxed{1.00\times10^{5}\text{s}}$$

When adding, keep the least accurate value, and so keep to the "ones" place in the last set of parentheses.

8. $\left(2.079\times10^{2}\text{m}\right)\left(0.082\times10^{-1}\right)=\boxed{1.7\text{ m}}$. When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.

9.

θ(radians)	sin(θ)	tan(θ)
0	0.00	0.00
0.10	0.10	0.10
0.12	0.12	0.12
0.20	0.20	0.20
0.24	0.24	0.24
0.25	0.25	0.26

Keeping 2 significant figures in the angle, and expressing the angle in radians, the largest angle that has the same sine and tangent is $\boxed{0.24\text{ radians}}$. In degrees, the largest angle (keeping 2 significant figure) is 12°. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH01.XLS," on tab "Problem 1.9."

10. To find the approximate uncertainty in the volume, calculate the volume for the minimum radius and the volume for the maximum radius. Subtract the extreme volumes. The uncertainty in the volume is then half this variation in volume.

$$V_{\text{specified}}=\tfrac{4}{3}\pi r_{\text{specified}}^{3}=\tfrac{4}{3}\pi\left(0.84\text{ m}\right)^{3}=2.483\text{ m}^{3}$$

$$V_{\text{min}}=\tfrac{4}{3}\pi r_{\text{min}}^{3}=\tfrac{4}{3}\pi\left(0.80\text{ m}\right)^{3}=2.145\text{ m}^{3}$$

$$V_{\text{max}}=\tfrac{4}{3}\pi r_{\text{max}}^{3}=\tfrac{4}{3}\pi\left(0.88\text{ m}\right)^{3}=2.855\text{ m}^{3}$$

$$\Delta V=\tfrac{1}{2}\left(V_{\text{max}}-V_{\text{min}}\right)=\tfrac{1}{2}\left(2.855\text{ m}^{3}-2.145\text{ m}^{3}\right)=0.355\text{ m}^{3}$$

The percent uncertainty is $\dfrac{\Delta V}{V_{\text{specified}}}=\dfrac{0.355\text{ m}^{3}}{2.483\text{ m}^{3}}\times100=14.3\approx\boxed{14\,\%}$.

11. (*a*) 286.6 mm $286.6\times10^{-3}\,\text{m}$ $\boxed{0.286\,6\text{ m}}$

    (*b*) $85\,\mu\text{V}$ $85\times10^{-6}\,\text{V}$ $\boxed{0.000\,085\text{ V}}$

    (*c*) 760 mg $760\times10^{-6}\,\text{kg}$ $\boxed{0.000\,76\text{ kg}}$ (if last zero is not significant)

    (*d*) 60.0 ps $60.0\times10^{-12}\,\text{s}$ $\boxed{0.000\,000\,000\,060\,0\text{ s}}$

    (*e*) 22.5 fm $22.5\times10^{-15}\,\text{m}$ $\boxed{0.000\,000\,000\,000\,022\,5\text{ m}}$

    (*f*) 2.50 gigavolts 2.5×10^{9} volts $\boxed{2{,}500{,}000{,}000\text{ volts}}$

12. (*a*) 1×10^{6} volts $\boxed{\text{1 megavolt}} = 1$ Mvolt

    (*b*) 2×10^{-6} meters $\boxed{\text{2 micrometers}} = 2\,\mu\text{m}$

    (*c*) 6×10^{3} days $\boxed{\text{6 kilodays}}$ = 6 kdays

    (*d*) 18×10^{2} bucks $\boxed{\text{18 hectobucks}}$ = 18 hbucks or 1.8 kilobucks

    (*e*) 8×10^{-8} seconds $\boxed{\text{80 nanoseconds}}$ = 80 ns

$\boxed{13.}$ Assuming a height of 5 feet 10 inches, then $5'10'' = (70\text{ in})(1\text{ m}/39.37\text{ in}) = \boxed{1.8\text{ m}}$. Assuming a weight of 165 lbs, then $(165\text{ lbs})(0.456\text{ kg}/1\text{ lb}) = \boxed{75.2\text{ kg}}$. Technically, pounds and mass measure two separate properties. To make this conversion, we have to assume that we are at a location where the acceleration due to gravity is 9.80 m/s^2.

14. (*a*) 93 million miles $= (93\times10^{6}\text{ miles})(1610\text{ m}/1\text{ mile}) = \boxed{1.5\times10^{11}\text{ m}}$

    (*b*) $1.5\times10^{11}\text{ m} = 150\times10^{9}\text{ m} = \boxed{\text{150 gigameters}}$ or $1.5\times10^{11}\text{ m} = 0.15\times10^{12}\text{ m} = \boxed{\text{0.15 terameters}}$

15. (*a*) $1\text{ ft}^2 = (1\text{ ft}^2)(1\text{ yd}/3\text{ ft})^2 = 0.111\text{ yd}^2$, and so the conversion factor is $\boxed{\dfrac{0.111\text{ yd}^2}{1\text{ ft}^2}}$.

    (*b*) $1\text{ m}^2 = (1\text{ m}^2)(3.28\text{ ft}/1\text{ m})^2 = 10.8\text{ ft}^2$, and so the conversion factor is $\boxed{\dfrac{10.8\text{ ft}^2}{1\text{ m}^2}}$.

16. Use the speed of the airplane to convert the travel distance into a time. $d = vt$, so $t = d/v$.

$$t = d/v = 1.00\text{ km}\left(\frac{1\text{ h}}{950\text{ km}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{3.8\text{ s}}$$

17. (*a*) $1.0\times10^{-10}\text{ m} = (1.0\times10^{-10}\text{ m})(39.37\text{ in}/1\text{ m}) = \boxed{3.9\times10^{-9}\text{ in}}$

    (*b*) $(1.0\text{ cm})\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)\left(\dfrac{1\text{ atom}}{1.0\times10^{-10}\text{ m}}\right) = \boxed{1.0\times10^{8}\text{ atoms}}$

18. To add values with significant figures, adjust all values to be added so that their units are all the same.

$$1.80\text{ m} + 142.5\text{ cm} + 5.34\times10^5\ \mu\text{m} = 1.80\text{ m} + 1.425\text{ m} + 0.534\text{ m} = 3.759\text{ m} = \boxed{3.76\text{ m}}$$

When adding, the final result is to be no more accurate than the least accurate number used. In this case, that is the first measurement, which is accurate to the hundredths place when expressed in meters.

$\boxed{19.}$ (*a*) $(1\,\text{km/h})\left(\dfrac{0.621\text{ mi}}{1\text{ km}}\right) = 0.621\,\text{mi/h}$, and so the conversion factor is $\boxed{\dfrac{0.621\,\text{mi/h}}{1\,\text{km/h}}}$.

(*b*) $(1\,\text{m/s})\left(\dfrac{3.28\text{ ft}}{1\text{ m}}\right) = 3.28\,\text{ft/s}$, and so the conversion factor is $\boxed{\dfrac{3.28\,\text{ft/s}}{1\,\text{m/s}}}$.

(*c*) $(1\,\text{km/h})\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 0.278\,\text{m/s}$, and so the conversion factor is $\boxed{\dfrac{0.278\,\text{m/s}}{1\,\text{km/h}}}$.

20. One mile is $1.61\times10^3\,\text{m}$. It is 110 m longer than a 1500-m race. The percentage difference is calculated here.

$$\frac{110\text{ m}}{1500\text{ m}}\times100\% = \boxed{7.3\%}$$

21. (*a*) Find the distance by multiplying the speed times the time.

$$1.00\text{ ly} = \left(2.998\times10^8\text{ m/s}\right)\left(3.156\times10^7\,\text{s}\right) = 9.462\times10^{15}\,\text{m} \approx \boxed{9.46\times10^{15}\,\text{m}}$$

(*b*) Do a unit conversion from ly to AU.

$$\left(1.00\text{ ly}\right)\left(\frac{9.462\times10^{15}\,\text{m}}{1.00\text{ ly}}\right)\left(\frac{1\text{ AU}}{1.50\times10^{11}\,\text{m}}\right) = \boxed{6.31\times10^4\,\text{AU}}$$

(*c*) $\left(2.998\times10^8\text{ m/s}\right)\left(\dfrac{1\text{ AU}}{1.50\times10^{11}\,\text{m}}\right)\left(\dfrac{3600\text{ s}}{1\text{ hr}}\right) = \boxed{7.20\,\text{AU/h}}$

22. $$\left(82\times10^9\,\text{bytes}\right)\times\frac{1\,\text{char}}{1\,\text{byte}}\times\frac{1\,\text{min}}{180\,\text{char}}\times\frac{1\,\text{hour}}{60\,\text{min}}\times\frac{1\,\text{day}}{8\,\text{hour}}\times\frac{1\,\text{year}}{365.25\,\text{days}} = 2598\text{ years} \approx \boxed{2600\,\text{years}}$$

23. The surface area of a sphere is found by $A = 4\pi r^2 = 4\pi\left(d/2\right)^2 = \pi d^2$.

(*a*) $A_{\text{Moon}} = \pi D_{\text{Moon}}^2 = \pi\left(3.48\times10^6\,\text{m}\right)^2 = \boxed{3.80\times10^{13}\,\text{m}^2}$

(*b*) $\dfrac{A_{\text{Earth}}}{A_{\text{Moon}}} = \dfrac{\pi D_{\text{Earth}}^2}{\pi D_{\text{Moon}}^2} = \left(\dfrac{D_{\text{Earth}}}{D_{\text{Moon}}}\right)^2 = \left(\dfrac{R_{\text{Earth}}}{R_{\text{Moon}}}\right)^2 = \left(\dfrac{6.38\times10^6\,\text{m}}{1.74\times10^6\,\text{m}}\right)^2 = \boxed{13.4}$

24. (*a*) $2800 = 2.8\times10^3 \approx 1\times10^3 = \boxed{10^3}$

(*b*) $86.30\times10^2 = 8.630\times10^3 \approx 10\times10^3 = \boxed{10^4}$

(*c*) $0.0076 = 7.6\times10^{-3} \approx 10\times10^{-3} = \boxed{10^{-2}}$

(*d*) $15.0\times10^8 = 1.5\times10^9 \approx 1\times10^9 = \boxed{10^9}$

$\boxed{25.}$ The textbook is approximately 25 cm deep and 5 cm wide. With books on both sides of a shelf, the shelf would need to be about 50 cm deep. If the aisle is 1.5 meter wide, then about 1/4 of the floor space is covered by shelving. The number of books on a single shelf level is then $\frac{1}{4}\left(3500\,\text{m}^2\right)\left(\dfrac{1\text{ book}}{(0.25\text{ m})(0.05\text{ m})}\right) = 7.0\times10^4\,\text{books}$. With 8 shelves of books, the total number of books stored is as follows.

$$\left(7.0\times10^4\,\frac{\text{books}}{\text{shelf level}}\right)(8\text{ shelves}) \approx \boxed{6\times10^5\text{ books}}$$

26. The distance across the United States is about 3000 miles.

$$(3000\text{ mi})(1\text{ km}/0.621\text{ mi})(1\text{ hr}/10\text{ km}) \approx \boxed{500\text{ hr}}$$

Of course, it would take more time on the clock for the runner to run across the U.S. The runner could obviously not run for 500 hours non-stop. If they could run for 5 hours a day, then it would take about 100 days for them to cross the country.

27. A commonly accepted measure is that a person should drink eight 8-oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Approximate the lifetime as 70 years.

$$(70\text{ y})(365\text{ d}/1\text{ y})(2\text{ L}/1\text{ d}) \approx \boxed{5\times10^4\text{ L}}$$

28. An NCAA-regulation football field is 360 feet long (including the end zones) and 160 feet wide, which is about 110 meters by 50 meters, or 5500 m^2. The mower has a cutting width of 0.5 meters. Thus the distance to be walked is as follows.

$$d = \frac{\text{area}}{\text{width}} = \frac{5500\,\text{m}^2}{0.5\text{ m}} = 11000\text{ m} = 11\text{ km}$$

At a speed of 1 km/hr, then it will take about $\boxed{11\text{ h}}$ to mow the field.

29. In estimating the number of dentists, the assumptions and estimates needed are:

the population of the city
the number of patients that a dentist sees in a day
the number of days that a dentist works in a year
the number of times that each person visits the dentist each year

We estimate that a dentist can see 10 patients a day, that a dentist works 225 days a year, and that each person visits the dentist twice per year.

(*a*) For San Francisco, the population as of 2001 was about 1.7 million, so we estimate the population at two million people. The number of dentists is found by the following calculation.

$$\left(2\times10^6\,\text{people}\right)\left(\frac{2\,\frac{\text{visits}}{\text{year}}}{1\text{ person}}\right)\left(\frac{1\text{ yr}}{225\text{ workdays}}\right)\left(\frac{1\text{ dentist}}{10\,\frac{\text{visits}}{\text{workday}}}\right) \approx \boxed{1800\text{ dentists}}$$

(*b*) For Marion, Indiana, the population is about 50,000. The number of dentists is found by a similar calculation to that in part (*a*), and would be $\boxed{45\text{ dentists}}$. There are about 50 dentists listed in the 2005 yellow pages.

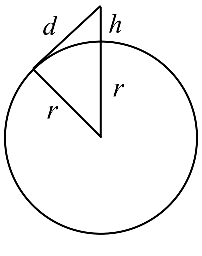
30. Assume that the tires last for 5 years, and so there is a tread wearing of 0.2 cm/year. Assume the average tire has a radius of 40 cm, and a width of 10 cm. Thus the volume of rubber that is becoming pollution each year from one tire is the surface area of the tire, times the thickness per year

that is wearing. Also assume that there are 1.5×10^8 automobiles in the country – approximately one automobile for every two people. And there are 4 tires per automobile. The mass wear per year is given by the following calculation.

$$\left(\frac{\text{mass}}{\text{year}}\right)=\left(\frac{\text{surface area}}{\text{tire}}\right)\left(\frac{\text{thickness wear}}{\text{year}}\right)(\text{density of rubber})(\#\text{ of tires})$$

$$=\left[\frac{2\pi(0.4\,\text{m})(0.1\,\text{m})}{1\text{ tire}}\right](0.002\,\text{m/y})\left(1200\,\text{kg/m}^3\right)\left(6.0\times10^8\text{tires}\right)=\boxed{4\times10^8\,\text{kg/y}}$$

$\boxed{31.}$ Consider the diagram shown (not to scale). The balloon is a distance *h* above the surface of the Earth, and the tangent line from the balloon height to the surface of the earth indicates the location of the horizon, a distance *d* away from the balloon. Use the Pythagorean theorem.



$$(r+h)^2=r^2+d^2\;\rightarrow\;r^2+2rh+h^2=r^2+d^2$$

$$2rh+h^2=d^2\;\rightarrow\;d=\sqrt{2rh+h^2}$$

$$d=\sqrt{2\left(6.4\times10^6\text{m}\right)(200\,\text{m})+(200\,\text{m})^2}=5.1\times10^4\,\text{m}\approx\boxed{5\times10^4\,\text{m}}(\approx 80\text{ mi})$$

32. At \$1,000 per day, you would earn \$30,000 in the 30 days. With the other pay method, you would get $\$0.01\left(2^{t-1}\right)$ on the t^{th} day. On the first day, you get $\$0.01\left(2^{1-1}\right)=\$0.01$. On the second day, you get $\$0.01\left(2^{2-1}\right)=\$0.02$. On the third day, you get $\$0.01\left(2^{3-1}\right)=\$0.04$. On the 30th day, you get $\$0.01\left(2^{30-1}\right)=\$5.4\times10^6$, which is over 5 million dollars. Get paid by the $\boxed{\text{second method}}$.

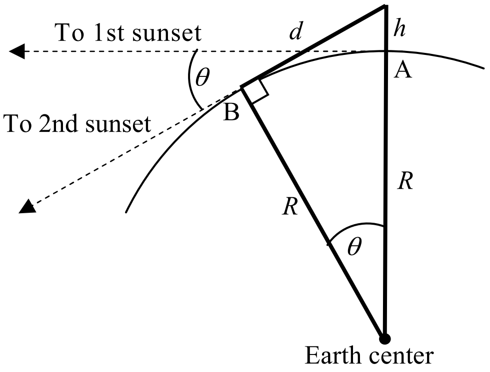
33. In the figure in the textbook, the distance *d* is perpendicular to the vertical radius. Thus there is a right triangle, with legs of *d* and *R*, and a hypotenuse of *R*+*h*. Since $h\ll R$, $h^2\ll 2Rh$.

$$d^2+R^2=(R+h)^2=R^2+2Rh+h^2\;\rightarrow\;d^2=2Rh+h^2\;\rightarrow\;d^2\approx 2Rh\;\rightarrow$$

$$R=\frac{d^2}{2h}=\frac{(4400\,\text{m})^2}{2(1.5\,\text{m})}=\boxed{6.5\times10^6\,\text{m}}$$

A better measurement gives $R=6.38\times10^6\,\text{m}$.

34. To see the Sun "disappear," your line of sight to the top of the Sun is tangent to the Earth's surface. Initially, you are lying down at point A, and you see the first sunset. Then you stand up, elevating your eyes by the height *h*. While standing, your line of sight is tangent to the Earth's surface at point B, and so that is the direction to the second sunset. The angle θ is the angle through which the Sun appears to move relative to the Earth during the time to be measured. The distance *d* is the distance from your eyes when standing to point B.



Use the Pythagorean theorem for the following relationship.

$$d^2+R^2=(R+h)^2=R^2+2Rh+h^2\;\rightarrow\;d^2=2Rh+h^2$$

The distance h is much smaller than the distance R, and so $h^2 \ll 2Rh$ which leads to $d^2 \approx 2Rh$. We also have from the same triangle that $d/R = \tan\theta$, and so $d = R\tan\theta$. Combining these two relationships gives $d^2 \approx 2Rh = R^2\tan^2\theta$, and so $R = \dfrac{2h}{\tan^2\theta}$.

The angle θ can be found from the height change and the radius of the Earth. The elapsed time between the two sightings can then be found from the angle, knowing that a full revolution takes 24 hours.

$$R = \frac{2h}{\tan^2\theta} \quad\rightarrow\quad \theta = \tan^{-1}\sqrt{\frac{2h}{R}} = \tan^{-1}\sqrt{\frac{2(1.3\,\mathrm{m})}{6.38\times10^6\,\mathrm{m}}} = \left(3.66\times10^{-2}\right)^\circ$$

$$\frac{\theta}{360^\circ} = \frac{t\,\mathrm{sec}}{24\,\mathrm{h}\times\dfrac{3600\,\mathrm{s}}{1\,\mathrm{h}}} \quad\rightarrow$$

$$t = \left(\frac{\theta}{360^\circ}\right)\left(24\,\mathrm{h}\times\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right) = \left(\frac{\left(3.66\times10^{-2}\right)^\circ}{360^\circ}\right)\left(24\,\mathrm{h}\times\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right) = \boxed{8.8\,\mathrm{s}}$$

35. $$\text{Density units} = \frac{\text{mass units}}{\text{volume units}} = \boxed{\left[\frac{M}{L^3}\right]}$$

36. (*a*) For the equation $v = At^3 - Bt$, the units of At^3 must be the same as the units of v. So the units of A must be the same as the units of v/t^3, which would be $\boxed{L/T^4}$. Also, the units of Bt must be the same as the units of v. So the units of B must be the same as the units of v/t, which would be $\boxed{L/T^2}$.

(*b*) For *A*, the SI units would be $\boxed{\mathrm{m/s^4}}$, and for B, the SI units would be $\boxed{\mathrm{m/s^2}}$.

$\boxed{37.}$ (*a*) The quantity vt^2 has units of $(\mathrm{m/s})(\mathrm{s}^2) = \mathrm{m{\bullet}s}$, which do not match with the units of meters for *x*. The quantity $2at$ has units $(\mathrm{m/s^2})(\mathrm{s}) = \mathrm{m/s}$, which also do not match with the units of meters for *x*. Thus this equation $\boxed{\text{cannot be correct}}$.

(*b*) The quantity v_0t has units of $(\mathrm{m/s})(\mathrm{s}) = \mathrm{m}$, and $\frac{1}{2}at^2$ has units of $(\mathrm{m/s^2})(\mathrm{s^2}) = \mathrm{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

(*c*) The quantity v_0t has units of $(\mathrm{m/s})(\mathrm{s}) = \mathrm{m}$, and $2at^2$ has units of $(\mathrm{m/s^2})(\mathrm{s^2}) = \mathrm{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

38. $$t_P = \sqrt{\frac{Gh}{c^5}} \quad\rightarrow\quad \sqrt{\frac{\left[\dfrac{L^3}{MT^2}\right]\left[\dfrac{ML^2}{T}\right]}{\left[\dfrac{L}{T}\right]^5}} = \sqrt{\left[\frac{L^3L^2T^5M}{MT^3L^5}\right]} = \sqrt{\left[\frac{T^5}{T^3}\right]} = \sqrt{\left[T^2\right]} = [T]$$

39. The percentage accuracy is $\dfrac{2\text{ m}}{2\times10^{7}\text{m}}\times100\% = \boxed{1\times10^{-5}\%}$. The distance of 20,000,000 m needs to be distinguishable from 20,000,002 m, which means that $\boxed{\text{8 significant figures}}$ are needed in the distance measurements.

40. Multiply the number of chips per wafer times the number of wafers that can be made from a cylinder.

$$\left(100\frac{\text{chips}}{\text{wafer}}\right)\left(\frac{1\text{ wafer}}{0.300\text{ mm}}\right)\left(\frac{250\text{ mm}}{1\text{ cylinder}}\right)=\boxed{83,000\frac{\text{chips}}{\text{cylinder}}}$$

41. (*a*) # of seconds in 1.00 y: $1.00\text{ y}=(1.00\text{ y})\left(\dfrac{3.156\times10^{7}\text{s}}{1\text{ y}}\right)=\boxed{3.16\times10^{7}\text{s}}$

(*b*) # of nanoseconds in 1.00 y: $1.00\text{ y}=(1.00\text{ y})\left(\dfrac{3.156\times10^{7}\text{s}}{1\text{ y}}\right)\left(\dfrac{1\times10^{9}\text{ns}}{1\text{ s}}\right)=\boxed{3.16\times10^{16}\text{ns}}$

(*c*) # of years in 1.00 s: $1.00\text{ s}=(1.00\text{ s})\left(\dfrac{1\text{ y}}{3.156\times10^{7}\text{s}}\right)=\boxed{3.17\times10^{-8}\text{y}}$

42. Since the meter is longer than the yard, the soccer field is longer than the football field.

$$L_{\text{soccer}}-L_{\text{football}}=100\,\text{m}\times\frac{1.09\text{ yd}}{1\text{m}}-100\text{ yd}=\boxed{9\text{ yd}}$$

$$L_{\text{soccer}}-L_{\text{football}}=100\,\text{m}-100\text{ yd}\times\frac{1\text{m}}{1.09\text{ yd}}=\boxed{8\text{ m}}$$

Since the soccer field is 109 yd compare to the 100-yd football field, the soccer field is $\boxed{9\%}$ longer than the football field.

$\boxed{43.}$ Assume that the alveoli are spherical, and that the volume of a typical human lung is about 2 liters, which is .002 m^3. The diameter can be found from the volume of a sphere, $\frac{4}{3}\pi r^{3}$.

$$\tfrac{4}{3}\pi r^{3}=\tfrac{4}{3}\pi(d/2)^{3}=\frac{\pi d^{3}}{6}$$

$$\left(3\times10^{8}\right)\pi\frac{d^{3}}{6}=2\times10^{-3}\text{m}^{3}\quad\rightarrow\quad d=\left[\frac{6\left(2\times10^{-3}\right)}{3\times10^{8}\pi}\text{m}^{3}\right]^{1/3}=\boxed{2\times10^{-4}\text{m}}$$

44. $1\text{ hectare}=(1\text{ hectare})\left(\dfrac{1.000\times10^{4}\text{m}^{2}}{1\,\text{hectare}}\right)\left(\dfrac{3.281\,\text{ft}}{1\,\text{m}}\right)^{2}\left(\dfrac{1\,\text{acre}}{4.356\times10^{4}\text{ft}^{2}}\right)=\boxed{2.471\,\text{acres}}$

45. There are about 3×10^{8} people in the United States. Assume that half of them have cars, that they each drive 12,000 miles per year, and their cars get 20 miles per gallon of gasoline.

$$\left(3\times10^{8}\text{people}\right)\left(\frac{1\text{ automobile}}{2\text{ people}}\right)\left(\frac{12,000\text{ mi/auto}}{1\text{ y}}\right)\left(\frac{1\text{ gallon}}{20\text{ mi}}\right)\approx\boxed{1\times10^{11}\text{ gal/y}}$$

46. (*a*) $$\left(\frac{10^{-15}\,\text{kg}}{1\ \text{bacterium}}\right)\left(\frac{1\ \text{proton or neutron}}{10^{-27}\,\text{kg}}\right)=\boxed{10^{12}\ \text{protons or neutrons}}$$

(*b*) $$\left(\frac{10^{-17}\,\text{kg}}{1\ \text{DNA molecule}}\right)\left(\frac{1\ \text{proton or neutron}}{10^{-27}\,\text{kg}}\right)=\boxed{10^{10}\ \text{protons or neutrons}}$$

(*c*) $$\left(\frac{10^{2}\,\text{kg}}{1\ \text{human}}\right)\left(\frac{1\ \text{proton or neutron}}{10^{-27}\,\text{kg}}\right)=\boxed{10^{29}\ \text{protons or neutrons}}$$

(*d*) $$\left(\frac{10^{41}\,\text{kg}}{1\ \text{galaxy}}\right)\left(\frac{1\ \text{proton or neutron}}{10^{-27}\,\text{kg}}\right)=\boxed{10^{68}\ \text{protons or neutrons}}$$

47. The volume of water used by the people can be calculated as follows:

$$\left(4\times10^{4}\,\text{people}\right)\left(\frac{1200\,\text{L/day}}{4\ \text{people}}\right)\left(\frac{365\,\text{day}}{1\ \text{y}}\right)\left(\frac{1000\,\text{cm}^3}{1\,\text{L}}\right)\left(\frac{1\,\text{km}}{10^{5}\text{cm}}\right)^{3}=4.38\times10^{-3}\ \text{km}^3/\text{y}$$

The depth of water is found by dividing the volume by the area.

$$d=\frac{V}{A}=\frac{4.38\times10^{-3}\ \text{km}^3/\text{y}}{50\ \text{km}^2}=\left(8.76\times10^{-5}\,\frac{\text{km}}{\text{y}}\right)\left(\frac{10^{5}\text{cm}}{1\ \text{km}}\right)=8.76\,\text{cm/y}\approx\boxed{9\,\text{cm/y}}$$

48. Approximate the gumball machine as a rectangular box with a square cross-sectional area. In counting gumballs across the bottom, there are about 10 in a row. Thus we estimate that one layer contains about 100 gumballs. In counting vertically, we see that there are about 15 rows. Thus we estimate that there are $\boxed{\text{1500 gumballs}}$ in the machine.

$\boxed{49.}$ Make the estimate that each person has 1.5 loads of laundry per week, and that there are 300 million people in the United States.

$$\left(300\times10^{6}\,\text{people}\right)\times\frac{1.5\ \text{loads/week}}{1\,\text{person}}\times\frac{52\,\text{weeks}}{1\ \text{y}}\times\frac{0.1\,\text{kg}}{1\,\text{load}}=2.34\times10^{9}\,\frac{\text{kg}}{\text{y}}\approx\boxed{2\times10^{9}\,\frac{\text{kg}}{\text{y}}}$$

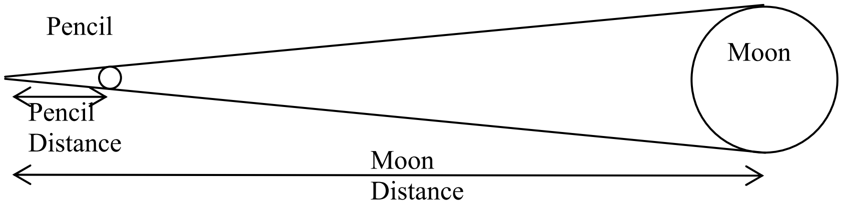
50. The volume of a sphere is given by $V=\frac{4}{3}\pi r^3$, and so the radius is $r=\left(\frac{3V}{4\pi}\right)^{1/3}$. For a 1-ton rock, the volume is calculated from the density, and then the diameter from the volume.

$$V=\left(1\ \text{T}\right)\left(\frac{2000\ \text{lb}}{1\ \text{T}}\right)\left(\frac{1\,\text{ft}^3}{186\ \text{lb}}\right)=10.8\ \text{ft}^3$$

$$d=2r=2\left(\frac{3V}{4\pi}\right)^{1/3}=2\left[\frac{3\left(10.8\ \text{ft}^3\right)}{4\pi}\right]^{1/3}=2.74\ \text{ft}\approx\boxed{3\ \text{ft}}$$

51. $$\left(783.216\times10^{6}\,\text{bytes}\right)\times\frac{8\,\text{bits}}{1\,\text{byte}}\times\frac{1\,\text{sec}}{1.4\times10^{6}\,\text{bits}}\times\frac{1\,\text{min}}{60\,\text{sec}}=74.592\,\text{min}\approx\boxed{75\,\text{min}}$$

52. A pencil has a diameter of about 0.7 cm. If held about 0.75 m from the eye, it can just block out the Moon. The ratio of pencil diameter to arm length is the same as the ratio of Moon diameter to Moon distance. From the diagram, we have the following ratios.



$$\frac{\text{Pencil diameter}}{\text{Pencil distance}}=\frac{\text{Moon diameter}}{\text{Moon distance}}\quad\rightarrow$$

$$\text{Moon diameter}=\frac{\text{Pencil diameter}}{\text{Pencil distance}}\left(\text{Moon distance}\right)=\frac{7\times10^{-3}\,\text{m}}{0.75\text{ m}}\left(3.8\times10^{5}\,\text{km}\right)\approx\boxed{3500\text{ km}}$$

The actual value is 3480 km.

53. To calculate the mass of water, we need to find the volume of water, and then convert the volume to mass. The volume of water is the area of the city $\left(40\,\text{km}^2\right)$ times the depth of the water (1.0 cm).

$$\left[\left(4\times10^{1}\text{ km}^2\right)\left(\frac{10^5\text{cm}}{1\text{ km}}\right)^2\right]\left(1.0\text{ cm}\right)\left(\frac{10^{-3}\,\text{kg}}{1\text{ cm}^3}\right)\left(\frac{1\text{ metric ton}}{10^3\,\text{kg}}\right)=\boxed{4\times10^5\,\text{metric tons}}$$

To find the number of gallons, convert the volume to gallons.

$$\left[\left(4\times10^{1}\text{ km}^2\right)\left(\frac{10^5\text{cm}}{1\text{ km}}\right)^2\right]\left(1.0\text{ cm}\right)\left(\frac{1\text{ L}}{1\times10^3\text{cm}^3}\right)\left(\frac{1\text{ gal}}{3.78\text{ L}}\right)=1.06\times10^8\,\text{gal}\approx\boxed{1\times10^8\,\text{gal}}$$

54. A cubit is about a half of a meter, by measuring several people's forearms. Thus the dimensions of Noah's ark would be $\boxed{\text{150 m long , 25 m wide, 15 m high}}$. The volume of the ark is found by multiplying the three dimensions.

$$V=\left(150\text{ m}\right)\left(25\text{ m}\right)\left(15\text{ m}\right)=5.625\times10^4\,\text{m}^3\approx\boxed{6\times10^4\,\text{m}^3}$$

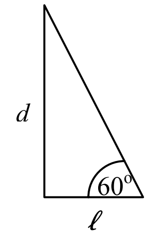
$\boxed{55.}$ The person walks $4\,\text{km/h}$, 10 hours each day. The radius of the Earth is about 6380 km, and the distance around the Earth at the equator is the circumference, $2\pi R_{\text{Earth}}$. We assume that the person can "walk on water," and so ignore the existence of the oceans.

$$2\pi\left(6380\text{ km}\right)\left(\frac{1\text{ h}}{4\text{ km}}\right)\left(\frac{1\text{ d}}{10\text{ h}}\right)=\boxed{1\times10^3\,\text{d}}$$

56. The volume of the oil will be the area times the thickness. The area is $\pi r^2=\pi\left(d/2\right)^2$, and so

$$V=\pi\left(d/2\right)^2t\quad\rightarrow\quad d=2\sqrt{\frac{V}{\pi t}}=2\sqrt{\frac{1000\,\text{cm}^3\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3}{\pi\left(2\times10^{-10}\text{m}\right)}}=\boxed{3\times10^3\text{m}}.$$

57. Consider the diagram shown. Let ℓ represent is the distance she walks upstream, which is about 120 yards. Find the distance across the river from the diagram.



$$\tan 60^\circ = \frac{d}{\ell} \quad \to \quad d = \ell \tan 60^\circ = (120\,\text{yd})\tan 60^\circ = \boxed{210\,\text{yd}}$$

$$(210\,\text{yd})\left(\frac{3\,\text{ft}}{1\,\text{yd}}\right)\left(\frac{0.305\,\text{m}}{1\,\text{ft}}\right) = \boxed{190\,\text{m}}$$

58. $$\left(\frac{8\ \text{s}}{1\ \text{y}}\right)\left(\frac{1\ \text{y}}{3.156\times10^{7}\text{s}}\right)\times100\% = \boxed{3\times10^{-5}\%}$$

59. (*a*) $$1.0\,\text{Å} = \left(1.0\,\text{Å}\right)\left(\frac{10^{-10}\text{m}}{1\,\text{Å}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\text{m}}\right) = \boxed{0.10\ \text{nm}}$$

(*b*) $$1.0\,\text{Å} = \left(1.0\,\text{Å}\right)\left(\frac{10^{-10}\text{m}}{1\,\text{Å}}\right)\left(\frac{1\ \text{fm}}{10^{-15}\text{m}}\right) = \boxed{1.0\times10^{5}\,\text{fm}}$$

(*c*) $$1.0\ \text{m} = (1.0\ \text{m})\left(\frac{1\,\text{Å}}{10^{-10}\text{m}}\right) = \boxed{1.0\times10^{10}\ \text{Å}}$$

(*d*) $$1.0\ \text{ly} = (1.0\ \text{ly})\left(\frac{9.46\times10^{15}\,\text{m}}{1\ \text{ly}}\right)\left(\frac{1\,\text{Å}}{10^{-10}\text{m}}\right) = \boxed{9.5\times10^{25}\ \text{Å}}$$

60. The volume of a sphere is found by $V = \frac{4}{3}\pi r^3$.

$$V_{\text{Moon}} = \tfrac{4}{3}\pi R_{\text{Moon}}^3 = \tfrac{4}{3}\pi\left(1.74\times10^{6}\,\text{m}\right)^3 = \boxed{2.21\times10^{19}\,\text{m}^3}$$

$$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4}{3}\pi R_{\text{Earth}}^3}{\frac{4}{3}\pi R_{\text{Moon}}^3} = \left(\frac{R_{\text{Earth}}}{R_{\text{Moon}}}\right)^3 = \left(\frac{6.38\times10^{6}\,\text{m}}{1.74\times10^{6}\,\text{m}}\right)^3 = 49.3$$

Thus it would take about $\boxed{49.3}$ Moons to create a volume equal to that of the Earth.

$\boxed{61.}$ (*a*) Note that $\sin 15.0^\circ = 0.259$ and $\sin 15.5^\circ = 0.267$, and so $\Delta\sin\theta = 0.267 - 0.259 = 0.008$.

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^\circ}{15.0^\circ}\right)100 = \boxed{3\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{8\times10^{-3}}{0.259}\right)100 = \boxed{3\%}$$

(*b*) Note that $\sin 75.0^\circ = 0.966$ and $\sin 75.5^\circ = 0.968$, and so $\Delta\sin\theta = 0.968 - 0.966 = 0.002$.

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^\circ}{75.0^\circ}\right)100 = \boxed{0.7\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{2\times10^{-3}}{0.966}\right)100 = \boxed{0.2\%}$$

A consequence of this result is that when using a protractor, and you have a fixed uncertainty in the angle ($\pm0.5^\circ$ in this case), you should measure the angles from a reference line that gives a large angle measurement rather than a small one. Note above that the angles around 75° had only a 0.2% error in sin θ, while the angles around 15° had a 3% error in sin θ.

62. Utilize the fact that walking totally around the Earth along the meridian would trace out a circle whose full 360° would equal the circumference of the Earth.

$$(1\text{ minute})\left(\frac{1^{\circ}}{60\text{ minute}}\right)\left(\frac{2\pi\left(6.38\times10^{3}\text{km}\right)}{360^{\circ}}\right)\left(\frac{0.621\text{ mi}}{1\text{ km}}\right)=\boxed{1.15\text{ mi}}$$

63. Consider the body to be a cylinder, about 170 cm tall $(\approx 5'7'')$, and about 12 cm in cross-sectional radius (which corresponds to a 30-inch waist). The volume of a cylinder is given by the area of the cross section times the height.

$$V=\pi r^{2}h=\pi(0.12\text{ m})^{2}(1.7\text{ m})=7.69\times10^{-2}\text{m}^{3}\approx\boxed{8\times10^{-2}\text{m}^{3}}$$

64. The maximum number of buses would be needed during rush hour. We assume that a bus can hold 50 passengers.

(*a*) The current population of Washington, D.C. is about half a million people. We estimate that 10% of them ride the bus during rush hour.

$$50{,}000\text{ passengers}\times\frac{1\text{ bus}}{50\text{ passengers}}\times\frac{1\text{ driver}}{1\text{ bus}}\approx\boxed{1000\text{ drivers}}$$

(*b*) For Marion, Indiana, the population is about 50,000. Because the town is so much smaller geographically, we estimate that only 5% of the current population rides the bus during rush hour.

$$2500\text{ passengers}\times\frac{1\text{ bus}}{50\text{ passengers}}\times\frac{1\text{ driver}}{1\text{ bus}}\approx\boxed{50\text{ drivers}}$$

65. The units for each term must be in liters, since the volume is in liters.

$$[\text{units of }4.1][\text{m}]=[\text{L}]\quad\rightarrow\quad\boxed{[\text{units of }4.1]=\frac{\text{L}}{\text{m}}}$$

$$[\text{units of }0.018][\text{y}]=[\text{L}]\quad\rightarrow\quad\boxed{[\text{units of }0.018]=\frac{\text{L}}{\text{y}}}$$

$$\boxed{[\text{units of }2.69]=\text{L}}$$

66. $$\text{density}=\frac{\text{mass}}{\text{volume}}=\frac{8\text{ g}}{2.8325\text{ cm}^{3}}=2.82\text{ g/cm}^{3}\approx\boxed{3\text{ g/cm}^{3}}$$

$\boxed{67.}$ (*a*) $$\frac{\text{SA}_{\text{Earth}}}{\text{SA}_{\text{Moon}}}=\frac{4\pi R_{\text{Earth}}^{2}}{4\pi R_{\text{Moon}}^{2}}=\frac{R_{\text{Earth}}^{2}}{R_{\text{Moon}}^{2}}=\frac{\left(6.38\times10^{3}\text{km}\right)^{2}}{\left(1.74\times10^{3}\text{km}\right)^{2}}=\boxed{13.4}$$

(*b*) $$\frac{\text{V}_{\text{Earth}}}{\text{V}_{\text{Moon}}}=\frac{\frac{4}{3}\pi R_{\text{Earth}}^{3}}{\frac{4}{3}\pi R_{\text{Moon}}^{3}}=\frac{R_{\text{Earth}}^{3}}{R_{\text{Moon}}^{3}}=\frac{\left(6.38\times10^{3}\text{km}\right)^{3}}{\left(1.74\times10^{3}\text{km}\right)^{3}}=\boxed{49.3}$$

68. $$\frac{\#\text{atoms}}{\text{m}^{2}}=\frac{6.02\times10^{23}\text{atoms}}{4\pi R_{\text{Earth}}^{2}}=\frac{6.02\times10^{23}\text{atoms}}{4\pi\left(6.38\times10^{6}\text{m}\right)^{2}}=\boxed{1.18\times10^{9}\,\frac{\text{atoms}}{\text{m}^{2}}}$$

69. Multiply the volume of a spherical universe times the density of matter, adjusted to ordinary matter. The volume of a sphere is $\frac{4}{3}\pi r^3$.

$$m = \rho V = \left(1\times10^{-26}\ \mathrm{kg/m^3}\right)\tfrac{4}{3}\pi\left(\left(13.7\times10^{9}\,\mathrm{ly}\right)\times\frac{9.46\times10^{15}\,\mathrm{m}}{1\,\mathrm{ly}}\right)^3\left(0.04\right)$$

$$= 3.65\times10^{51}\,\mathrm{kg} \approx \boxed{4\times10^{51}\,\mathrm{kg}}$$

CHAPTER 2: Describing Motion: Kinematics in One Dimension

Responses to Questions

$\boxed{1.}$ A car speedometer measures only speed, since it gives no indication of the direction in which the car is traveling.

2. If the velocity of an object is constant, the speed must also be constant. (A constant velocity means that the speed and direction are both constant.) If the speed of an object is constant, the velocity CAN vary. For example, a car traveling around a curve at constant speed has a varying velocity, since the direction of the velocity vector is changing.

3. When an object moves with constant velocity, the average velocity and the instantaneous velocity are the same at all times.

4. No, if one object has a greater speed than a second object, it does not necessarily have a greater acceleration. For example, consider a speeding car, traveling at constant velocity, which passes a stopped police car. The police car will accelerate from rest to try to catch the speeder. The speeding car has a greater speed than the police car (at least initially!), but has zero acceleration. The police car will have an initial speed of zero, but a large acceleration.

5. The accelerations of the motorcycle and the bicycle are the same, assuming that both objects travel in a straight line. Acceleration is the change in velocity divided by the change in time. The magnitude of the change in velocity in each case is the same, 10 km/h, so over the same time interval the accelerations will be equal.

6. Yes, for example, a car that is traveling northward and slowing down has a northward velocity and a southward acceleration.

$\boxed{7.}$ Yes. If the velocity and the acceleration have different signs (opposite directions), then the object is slowing down. For example, a ball thrown upward has a positive velocity and a negative acceleration while it is going up. A car traveling in the negative x-direction and braking has a negative velocity and a positive acceleration.

8. Both velocity and acceleration are negative in the case of a car traveling in the negative x-direction and speeding up. If the upward direction is chosen as $+y$, a falling object has negative velocity and negative acceleration.

9. Car A is going faster at this instant and is covering more distance per unit time, so car A is passing car B. (Car B is accelerating faster and will eventually overtake car A.)

10. Yes. Remember that acceleration is a *change in velocity* per unit time, or a *rate of change* in velocity. So, velocity can be increasing while the rate of increase goes down. For example, suppose a car is traveling at 40 km/h and a second later is going 50 km/h. One second after that, the car's speed is 55 km/h. The car's speed was increasing the entire time, but its acceleration in the second time interval was lower than in the first time interval.

11. If there were no air resistance, the ball's only acceleration during flight would be the acceleration due to gravity, so the ball would land in the catcher's mitt with the same speed it had when it left the bat, 120 km/h. The path of the ball as it rises and then falls would be symmetric.

12. (*a*) If air resistance is negligible, the acceleration of a freely falling object stays the same as the object falls toward the ground. (Note that the object's speed increases, but since it increases at a constant rate, the acceleration is constant.)

    (*b*) In the presence of air resistance, the acceleration decreases. (Air resistance increases as speed increases. If the object falls far enough, the acceleration will go to zero and the velocity will become constant. See Section 5-6.)

$\boxed{13.}$ Average speed is the displacement divided by the time. If the distances from A to B and from B to C are equal, then you spend more time traveling at 70 km/h than at 90 km/h, so your average speed should be less than 80 km/h. If the distance from A to B (or B to C) is x, then the total distance traveled is $2x$. The total time required to travel this distance is $x/70$ plus $x/90$. Then

$$\overline{v}=\frac{d}{t}=\frac{2x}{x/70+x/90}=\frac{2(90)(70)}{90+70}=79\text{ km/h}.$$

14. Yes. For example, a rock thrown straight up in the air has a constant, nonzero acceleration due to gravity for its entire flight. However, at the highest point it momentarily has a zero velocity. A car, at the moment it starts moving from rest, has zero velocity and nonzero acceleration.

15. Yes. Anytime the velocity is constant, the acceleration is zero. For example, a car traveling at a constant 90 km/h in a straight line has nonzero velocity and zero acceleration.

16. A rock falling from a cliff has a constant acceleration IF we neglect air resistance. An elevator moving from the second floor to the fifth floor making stops along the way does NOT have a constant acceleration. Its acceleration will change in magnitude and direction as the elevator starts and stops. The dish resting on a table has a constant acceleration (zero).

17. The time between clinks gets smaller and smaller. The bolts all start from rest and all have the same acceleration, so at any moment in time, they will all have the same speed. However, they have different distances to travel in reaching the floor and therefore will be falling for different lengths of time. The later a bolt hits, the longer it has been accelerating and therefore the faster it is moving. The time intervals between impacts decrease since the higher a bolt is on the string, the faster it is moving as it reaches the floor. In order for the clinks to occur at equal time intervals, the higher the bolt, the further it must be tied from its neighbor. Can you guess the ratio of lengths?

18. The slope of the position versus time curve is the velocity. The object starts at the origin with a constant velocity (and therefore zero acceleration), which it maintains for about 20 s. For the next 10 s, the positive curvature of the graph indicates the object has a positive acceleration; its speed is increasing. From 30 s to 45 s, the graph has a negative curvature; the object uniformly slows to a stop, changes direction, and then moves backwards with increasing speed. During this time interval its acceleration is negative, since the object is slowing down while traveling in the positive direction and then speeding up while traveling in the negative direction. For the final 5 s shown, the object continues moving in the negative direction but slows down, which gives it a positive acceleration. During the 50 s shown, the object travels from the origin to a point 20 m away, and then back 10 m to end up 10 m from the starting position.

$\boxed{19.}$ The object begins with a speed of 14 m/s and increases in speed with constant positive acceleration from $t = 0$ until $t = 45$ s. The acceleration then begins to decrease, goes to zero at $t = 50$ s, and then goes negative. The object slows down from $t = 50$ s to $t = 90$ s, and is at rest from $t = 90$ s to $t = 108$ s. At that point the acceleration becomes positive again and the velocity increases from $t = 108$ s to $t = 130$ s.

Solutions to Problems

$\boxed{1.}$ The distance of travel (displacement) can be found by rearranging Eq. 2-2 for the average velocity. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \Delta x = \bar{v}\Delta t = (110\,\text{km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)(2.0\,\text{s}) = 0.061\text{ km} = \boxed{61\text{ m}}$$

2. The average speed is given by Eq. 2-2.

$$\bar{v} = \Delta x/\Delta t = 235\text{ km}/3.25\text{ h} = \boxed{72.3\,\text{km/h}}$$

3. The average velocity is given by Eq. 2.2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8.5\,\text{cm} - 4.3\,\text{cm}}{4.5\,\text{s} - (-2.0\,\text{s})} = \frac{4.2\,\text{cm}}{6.5\,\text{s}} = \boxed{0.65\,\text{cm/s}}$$

$\boxed{\text{The average speed cannot be calculated.}}$ To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.

4. The average velocity is given by Eq. 2-2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4.2\,\text{cm} - 3.4\,\text{cm}}{5.1\,\text{s} - 3.0\,\text{s}} = \frac{-7.6\,\text{cm}}{2.1\,\text{s}} = \boxed{-3.6\,\text{cm/s}}$$

The negative sign indicates the direction.

5. The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated as follows.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left(\frac{1\,\text{mi}}{5\,\text{s}}\right)\left(\frac{1610\text{ m}}{1\text{ mi}}\right) = \boxed{300\,\text{m/s}}$$

The speed of $300\,\text{m/s}$ would imply the sound traveling a distance of 900 meters (which is approximately 1 km) in 3 seconds. So the rule could be approximated as $\boxed{\text{1 km every 3 seconds}}$.

6. The time for the first part of the trip is calculated from the initial speed and the first distance.

$$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{130\,\text{km}}{95\,\text{km/h}} = 1.37\,\text{h} = 82\,\text{min}$$

The time for the second part of the trip is now calculated.

$$\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 3.33\text{ h} - 1.37\text{ h} = 1.96\text{ h} = 118\text{ min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} \rightarrow \Delta x_2 = \bar{v}_2\Delta t_2 = (65\,\text{km/h})(1.96\,\text{h}) = 127.5\text{ km} = 1.3\times10^2\,\text{km}$$

(*a*) The total distance is then $\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 130\text{ km} + 127.5\text{ km} = 257.5\text{ km} \approx \boxed{2.6\times10^2\,\text{km}}$.

(*b*) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2-2.

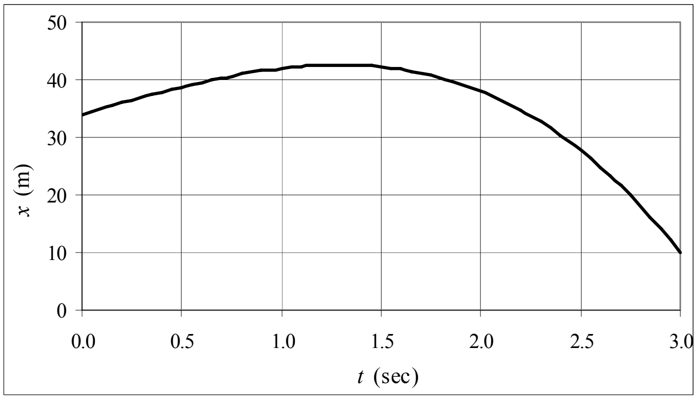
$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{257.5\text{ km}}{3.33\text{ h}} = \boxed{77\,\text{km/h}}$$

$\boxed{7.}$ The distance traveled is $116\text{ km}+\frac{1}{2}(116\text{ km})=174\text{ km}$, and the displacement is $116\text{ km}-\frac{1}{2}(116\text{ km})=58\text{ km}$. The total time is 14.0 s + 4.8 s = 18.8 s.

(*a*) Average speed $= \dfrac{\text{distance}}{\text{time elapsed}} = \dfrac{174\text{ m}}{18.8\text{ s}} = \boxed{9.26\,\text{m/s}}$

(*b*) Average velocity $= v_{\text{avg}} = \dfrac{\text{displacement}}{\text{time elapsed}} = \dfrac{58\text{ m}}{18.8\text{ s}} = \boxed{3.1\,\text{m/s}}$

8. (*a*)



The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH02.XLS”, on tab “Problem 2.8a”.

(*b*) The average velocity is the displacement divided by the elapsed time.

$$\bar{v}=\frac{x(3.0)-x(0.0)}{3.0\,\text{s}-0.0\,\text{s}}=\frac{\left[34+10(3.0)-2(3.0)^3\right]\text{m}-(34\,\text{m})}{3.0\,\text{s}}=\boxed{-8.0\,\text{m/s}}$$

(*c*) The instantaneous velocity is given by the derivative of the position function.

$$v=\frac{dx}{dt}=\left(10-6t^2\right)\text{m/s} \qquad 10-6t^2=0 \;\rightarrow\; t=\sqrt{\frac{5}{3}}\text{s}=\boxed{1.3\,\text{s}}$$

This can be seen from the graph as the “highest” point on the graph.

9. Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t=10.0\,\text{s}$, the slope is approximately $v(10)\approx\dfrac{3\,\text{m}-0}{10.0\text{ s}-0}=\boxed{0.3\,\text{m/s}}$.

(*b*) At $t=30.0\,\text{s}$, the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately $v(30)\approx\dfrac{22\,\text{m}-10\,\text{m}}{35\,\text{s}-25\,\text{s}}=\boxed{1.2\,\text{m/s}}$.

(*c*) The average velocity is given by $\bar{v}=\dfrac{x(5)-x(0)}{5.0\,\text{s}-0\,\text{s}}=\dfrac{1.5\,\text{m}-0}{5.0\,\text{s}}=\boxed{0.30\,\text{m/s}}$.

(*d*) The average velocity is given by $\bar{v}=\dfrac{x(30)-x(25)}{30.0\,\text{s}-25.0\,\text{s}}=\dfrac{16\,\text{m}-9\,\text{m}}{5.0\,\text{s}}=\boxed{1.4\,\text{m/s}}$.

(*e*) The average velocity is given by $\bar{v}=\dfrac{x(50)-x(40)}{50.0\,\text{s}-40.0\,\text{s}}=\dfrac{10\,\text{m}-19.5\,\text{m}}{10.0\,\text{s}}=\boxed{-0.95\,\text{m/s}}$.

10. (*a*) Multiply the reading rate times the bit density to find the bit reading rate.

$$N = \frac{1.2\,\text{m}}{1\,\text{s}} \times \frac{1\,\text{bit}}{0.28\times10^{-6}\text{m}} = \boxed{4.3\times10^{6}\ \text{bits/s}}$$

(*b*) The number of excess bits is $N - N_0$.

$$N - N_0 = 4.3\times10^{6}\ \text{bits/s} - 1.4\times10^{6}\ \text{bits/s} = 2.9\times10^{6}\ \text{bits/s}$$

$$\frac{N - N_0}{N} = \frac{2.9\times10^{6}\ \text{bits/s}}{4.3\times10^{6}\ \text{bits/s}} = 0.67 = \boxed{67\%}$$

11. Both objects will have the same time of travel. If the truck travels a distance Δx_{truck}, then the distance the car travels will be $\Delta x_{\text{car}} = \Delta x_{\text{truck}} + 110\,\text{m}$. Use Eq. 2-2 for average speed, $\bar{v} = \Delta x/\Delta t$, solve for time, and equate the two times.

$$\Delta t = \frac{\Delta x_{\text{truck}}}{\bar{v}_{\text{truck}}} = \frac{\Delta x_{\text{car}}}{\bar{v}_{\text{car}}} \qquad \frac{\Delta x_{\text{truck}}}{75\ \text{km/h}} = \frac{\Delta x_{\text{truck}} + 110\ \text{m}}{95\ \text{km/h}}$$

Solving for Δx_{truck} gives $\Delta x_{\text{truck}} = (110\,\text{m})\dfrac{(75\,\text{km/h})}{(95\,\text{km/h} - 75\,\text{km/h})} = 412.5\,\text{m}.$

The time of travel is $\Delta t = \dfrac{\Delta x_{\text{truck}}}{\bar{v}_{\text{truck}}} = \left(\dfrac{412.5\,\text{m}}{75000\,\text{m/h}}\right)\left(\dfrac{60\,\text{min}}{1\,\text{h}}\right) = 0.33\,\text{min} = 19.8\,\text{s} = \boxed{2.0\times10^{1}\text{s}}$.

Also note that $\Delta t = \dfrac{\Delta x_{\text{car}}}{\bar{v}_{\text{car}}} = \left(\dfrac{412.5\,\text{m} + 110\,\text{m}}{95000\,\text{m/h}}\right)\left(\dfrac{60\,\text{min}}{1\,\text{h}}\right) = 0.33\,\text{min} = 20\,\text{s}.$

ALTERNATE SOLUTION:
The speed of the car relative to the truck is $95\,\text{km/h} - 75\,\text{km/h} = 20\,\text{km/h}$. In the reference frame of the truck, the car must travel 110 m to catch it.

$$\Delta t = \frac{0.11\ \text{km}}{20\,\text{km/h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 19.8\ \text{s}$$

12. Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\bar{v} = \frac{\Delta x}{\Delta t} \to \Delta t = \frac{\Delta x}{\bar{v}} = \frac{4.25\ \text{km}}{95\,\text{km/h}} = 0.0447\ \text{h}\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 2.68\ \text{min} \approx \boxed{2.7\ \text{min}}$$

$\boxed{13.}$ (*a*) The area between the concentric circles is equal to the length times the width of the spiral path.

$$\pi R_2^2 - \pi R_1^2 = w\ell \quad \to$$

$$\ell = \frac{\pi\left(R_2^2 - R_1^2\right)}{w} = \frac{\pi\left[(0.058\,\text{m})^2 - (0.025\,\text{m})^2\right]}{1.6\times10^{-6}\text{m}} = 5.378\times10^{3}\text{m} \approx \boxed{5400\,\text{m}}$$

(*b*) $5.378\times10^{3}\text{m}\left(\dfrac{1\,\text{s}}{1.25\,\text{m}}\right)\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{72\,\text{min}}$

14. The average speed for each segment of the trip is given by $\bar{v} = \frac{\Delta x}{\Delta t}$, so $\Delta t = \frac{\Delta x}{\bar{v}}$ for each segment. For the first segment, $\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{3100\,\text{km}}{720\,\text{km/h}} = 4.306\,\text{h}$. For the second segment, $\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{2800\,\text{km}}{990\,\text{km/h}} = 2.828\text{h}.$.

    Thus the total time is $\Delta t_{\text{tot}} = \Delta t_1 + \Delta t_2 = 4.306\,\text{h} + 2.828\,\text{h} = 7.134\,\text{h} \approx \boxed{7.1\,\text{h}}$.

    The average speed of the plane for the entire trip is $\bar{v} = \frac{\Delta x_{\text{tot}}}{\Delta t_{\text{tot}}} = \frac{3100\,\text{km} + 2800\,\text{km}}{7.134\ \text{h}} = 827\,\text{km/h}$ $\approx \boxed{830\,\text{km/h}}$.

15. The distance traveled is 500 km (250 km outgoing, 250 km return, keep 2 significant figures). The displacement (Δx) is 0 because the ending point is the same as the starting point.

    (*a*) To find the average speed, we need the distance traveled (500 km) and the total time elapsed. During the outgoing portion, $\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1}$ and so $\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{250\,\text{km}}{95\,\text{km/h}} = 2.632\,\text{h}$. During the return portion, $\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2}$, and so $\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{250\,\text{km}}{55\,\text{km/h}} = 4.545\,\text{h}$. Thus the total time, including lunch, is $\Delta t_{\text{total}} = \Delta t_1 + \Delta t_{\text{lunch}} + \Delta t_2 = 8.177\,\text{h}$.

    $$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{500\,\text{km}}{8.177\,\text{h}} = \boxed{61\,\text{km/h}}$$

    (*b*) Average velocity = $\boxed{\bar{v} = \Delta x/\Delta t = 0}$

16. We are given that $x(t) = 2.0\,\text{m} - (3.6\,\text{m/s})t + (1.1\,\text{m/s}^2)t^2$.

    (*a*) $x(1.0\,\text{s}) = 2.0\,\text{m} - (3.6\,\text{m/s})(1.0\,\text{s}) + (1.1\,\text{m/s}^2)(1.0\,\text{s})^2 = \boxed{-0.5\,\text{m}}$

    $x(2.0\,\text{s}) = 2.0\,\text{m} - (3.6\,\text{m/s})(2.0\,\text{s}) + (1.1\,\text{m/s}^2)(2.0\,\text{s})^2 = \boxed{-0.8\,\text{m}}$

    $x(3.0\,\text{s}) = 2.0\,\text{m} - (3.6\,\text{m/s})(3.0\,\text{s}) + (1.1\,\text{m/s}^2)(3.0\,\text{s})^2 = \boxed{1.1\,\text{m}}$

    (*b*) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{1.1\text{m} - (-0.5\text{m})}{2.0\,\text{s}} = \boxed{0.80\,\text{m/s}}$

    (*c*) The instantaneous velocity is given by $v(t) = \frac{dx(t)}{dt} = -3.6\,\text{m/s} + (2.2\,\text{m/s}^2)t.$

    $v(2.0\,\text{s}) = -3.6\,\text{m/s} + (2.2\,\text{m/s}^2)(2.0\,\text{s}) = \boxed{0.8\,\text{m/s}}$

    $v(3.0\,\text{s}) = -3.6\,\text{m/s} + (2.2\,\text{m/s}^2)(3.0\,\text{s}) = \boxed{3.0\,\text{m/s}}$

17. The distance traveled is $120\,\text{m}+\frac{1}{2}(120\,\text{m})=180\,\text{m}$, and the displacement is $120\,\text{m}-\frac{1}{2}(120\,\text{m})=60\,\text{m}$. The total time is $8.4\,\text{s}+\frac{1}{3}(8.4\,\text{s})=11.2\,\text{s}$.

(*a*) $\text{Average speed}=\dfrac{\text{distance}}{\text{time elapsed}}=\dfrac{180\,\text{m}}{11.2\,\text{s}}=\boxed{16\,\text{m/s}}$

(*b*) $\text{Average velocity}=v_{\text{avg}}=\dfrac{\text{displacement}}{\text{time elapsed}}=\dfrac{60\,\text{m}}{11.2\text{ s}}=\boxed{+5\,\text{m/s}}$ (in original direction)(1 sig fig)

18. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{\text{car}}=v_{\text{car}}t=(95\,\text{km/h})t$ or $d_{\text{car}}=\ell_{\text{train}}+v_{\text{train}}t=1.10\text{ km}+(75\,\text{km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\,\text{km/h})t=1.10\text{ km}+(75\,\text{km/h})t\quad\rightarrow\quad t=\frac{1.10\text{ km}}{20\,\text{km/h}}=0.055\text{ h}=\boxed{3.3\text{ min}}$$

The distance the car travels during this time is $d=(95\,\text{km/h})(0.055\text{ h})=5.225\,\text{km}\approx\boxed{5.2\text{ km}}$.

If the train is traveling the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{\text{car}}=(95\,\text{km/h})t$ or $d_{\text{car}}=1.10\text{ km}-(75\,\text{km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\,\text{km/h})t=1.10\text{ km}-(75\,\text{km/h})t\quad\rightarrow\quad t=\frac{1.10\text{ km}}{170\,\text{km/h}}=6.47\times10^{-3}\text{ h}=\boxed{23.3\text{ s}}$$

The distance the car travels during this time is $d=(95\,\text{km/h})(6.47\times10^{-3}\text{ h})=\boxed{0.61\text{ km}}$.

$\boxed{19.}$ The average speed of sound is given by $v_{\text{sound}}=\Delta x/\Delta t$, and so the time for the sound to travel from the end of the lane back to the bowler is $\Delta t_{\text{sound}}=\dfrac{\Delta x}{v_{\text{sound}}}=\dfrac{16.5\text{ m}}{340\,\text{m/s}}=4.85\times10^{-2}\text{s}$. Thus the time for the ball to travel from the bowler to the end of the lane is given by $\Delta t_{\text{ball}}=\Delta t_{\text{total}}-\Delta t_{\text{sound}}=2.50\,\text{s}-4.85\times10^{-2}\text{s}=2.4515\,\text{s}$. And so the speed of the ball is as follows.

$$v_{\text{ball}}=\frac{\Delta x}{\Delta t_{\text{ball}}}=\frac{16.5\,\text{m}}{2.4515\,\text{s}}=\boxed{6.73\,\text{m/s}}.$$

20. The average acceleration is found from Eq. 2-5.

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{95\,\text{km/h}-0\,\text{km/h}}{4.5\,\text{s}}=\frac{(95\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{4.5\,\text{s}}=\boxed{5.9\,\text{m/s}^2}$$

21. The time can be found from the average acceleration, $\overline{a}=\Delta v/\Delta t$.

$$\Delta t=\frac{\Delta v}{\overline{a}}=\frac{110\,\text{km/h}-80\,\text{km/h}}{1.8\,\text{m/s}^2}=\frac{(30\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{1.8\,\text{m/s}^2}=4.630\,\text{s}\approx\boxed{5\,\text{s}}$$

22. (*a*) The average acceleration of the sprinter is $\bar{a}=\dfrac{\Delta v}{\Delta t}=\dfrac{9.00\,\text{m/s}-0.00\,\text{m/s}}{1.28\text{ s}}=\boxed{7.03\,\text{m/s}^2}$.

    (*b*) $\bar{a}=\left(7.03\,\text{m/s}^2\right)\left(\dfrac{1\text{ km}}{1000\text{ m}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)^2=\boxed{9.11\times10^4\text{ km/h}^2}$

23. Slightly different answers may be obtained since the data comes from reading the graph.

    (*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t\approx 48\text{ s}}$.

    (*b*) The indication of a constant velocity on a velocity–time graph is a slope of 0, which occurs from $\boxed{t=90\text{ s to }t\approx 108\text{ s}}$.

    (*c*) The indication of a constant acceleration on a velocity–time graph is a constant slope, which occurs from $\boxed{t=0\text{ s to }t\approx 42\text{ s}}$, again from $\boxed{t\approx 65\text{ s to }t\approx 83\text{ s}}$, and again from $\boxed{t=90\text{ s to }t\approx 108\text{ s}}$.

    (*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t\approx 65\text{ s to }t\approx 83\text{ s}}$.

24. The initial velocity of the car is the average speed of the car before it accelerates.

$$\bar{v}=\frac{\Delta x}{\Delta t}=\frac{110\text{ m}}{5.0\text{ s}}=22\,\text{m/s}=v_0$$

The final speed is $v=0$, and the time to stop is 4.0 s. Use Eq. 2-12a to find the acceleration.

$$v=v_0+at\quad\rightarrow\quad a=\frac{v-v_0}{t}=\frac{0-22\,\text{m/s}}{4.0\text{ s}}=-5.5\,\text{m/s}^2$$

Thus the magnitude of the acceleration is $\boxed{5.5\,\text{m/s}^2}$, or $\left(5.5\,\text{m/s}^2\right)\left(\dfrac{1\,g}{9.80\,\text{m/s}^2}\right)=\boxed{0.56\,g\text{'s}}$.

$\boxed{25.}$ (*a*) $\bar{v}=\dfrac{\Delta x}{\Delta t}=\dfrac{385\,\text{m}-25\,\text{m}}{20.0\,\text{s}-3.0\,\text{s}}=\boxed{21.2\,\text{m/s}}$

(*b*) $\bar{a}=\dfrac{\Delta v}{\Delta t}=\dfrac{45.0\,\text{m/s}-11.0\,\text{m/s}}{20.0\,\text{s}-3.0\,\text{s}}=\boxed{2.00\,\text{m/s}^2}$

26. Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being "in" a certain gear.

    (*a*) The average acceleration in 2nd gear is given by $\bar{a}_2=\dfrac{\Delta v_2}{\Delta t_2}=\dfrac{24\,\text{m/s}-14\,\text{m/s}}{8\,\text{s}-4\,\text{s}}=\boxed{2.5\,\text{m/s}^2}$.

    (*b*) The average acceleration in 4th gear is given by $\bar{a}_4=\dfrac{\Delta v_4}{\Delta t_4}=\dfrac{44\,\text{m/s}-37\,\text{m/s}}{27\,\text{s}-16\,\text{s}}=\boxed{0.6\,\text{m/s}^2}$.

    (*c*) The average acceleration through the first four gears is given by $\bar{a}=\dfrac{\Delta v}{\Delta t}=$

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{44\,\text{m/s}-0\,\text{m/s}}{27\,\text{s}-0\,\text{s}}=\boxed{1.6\,\text{m/s}^2}.$$

27. The acceleration is the second derivative of the position function.

$$x = 6.8t + 8.5t^2 \quad \rightarrow \quad v = \frac{dx}{dt} = 6.8 + 17.0t \quad \rightarrow \quad a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \boxed{17.0\,\mathrm{m/s^2}}$$

28. To estimate the velocity, find the average velocity over each time interval, and assume that the car had that velocity at the midpoint of the time interval. To estimate the acceleration, find the average acceleration over each time interval, and assume that the car had that acceleration at the midpoint of the time interval. A sample of each calculation is shown.

From 2.00 s to 2.50 s, for average velocity:

$$t_{\text{mid}} = \frac{2.50\text{ s} + 2.00\text{ s}}{2} = 2.25\text{ s}$$

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{13.79\text{ m} - 8.55\text{ m}}{2.50\text{ s} - 2.00\text{ s}} = \frac{5.24\text{ m}}{0.50\text{ s}} = 10.48\,\mathrm{m/s}$$

From 2.25 s to 2.75 s, for average acceleration:

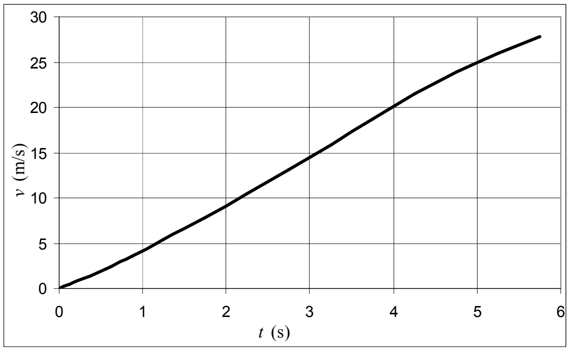
$$t_{\text{mid}} = \frac{2.25\text{ s} + 2.75\text{ s}}{2} = 2.50\text{ s}$$

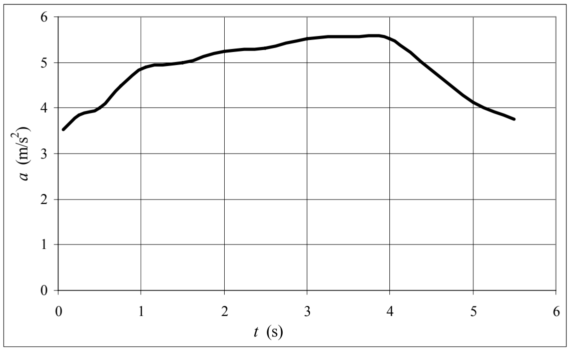
$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{13.14\,\mathrm{m/s} - 10.48\,\mathrm{m/s}}{2.75\text{ s} - 2.25\text{ s}} = \frac{2.66\,\mathrm{m/s}}{0.50\text{ s}}$$

$$= 5.32\,\mathrm{m/s^2}$$

Table of Calculations

t (s)	x (m)	t (s)	v (m/s)	t (s)	a (m/s^2)
0.00	0.00	0.00	0.00	0.063	3.52
		0.125	0.44		
0.25	0.11			0.25	3.84
		0.375	1.40		
0.50	0.46			0.50	4.00
		0.625	2.40		
0.75	1.06			0.75	4.48
		0.875	3.52		
1.00	1.94			1.06	4.91
		1.25	5.36		
1.50	4.62			1.50	5.00
		1.75	7.86		
2.00	8.55			2.00	5.24
		2.25	10.48		
2.50	13.79			2.50	5.32
		2.75	13.14		
3.00	20.36			3.00	5.52
		3.25	15.90		
3.50	28.31			3.50	5.56
		3.75	18.68		
4.00	37.65			4.00	5.52
		4.25	21.44		
4.50	48.37			4.50	4.84
		4.75	23.86		
5.00	60.30			5.00	4.12
		5.25	25.92		
5.50	73.26			5.50	3.76
		5.75	27.80		
6.00	87.16				





The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH02.XLS,” on tab “Problem 2.28.”

29. (*a*) Since the units of *A* times the units of *t* must equal meters, the units of *A* must be $\boxed{\mathrm{m/s}}$.

Since the units of *B* times the units of t^2 must equal meters, the units of *B* must be $\boxed{\mathrm{m/s^2}}$.

(*b*) The acceleration is the second derivative of the position function.

$$x = At + Bt^2 \rightarrow v = \frac{dx}{dt} = A + 2Bt \rightarrow a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \boxed{2B\,\mathrm{m/s^2}}$$

(*c*) $v = A + 2Bt \rightarrow v(5) = \boxed{(A+10B)\,\mathrm{m/s}} \qquad a = \boxed{2B\,\mathrm{m/s^2}}$

(*d*) The velocity is the derivative of the position function.

$$x = At + Bt^{-3} \rightarrow v = \frac{dx}{dt} = \boxed{A - 3Bt^{-4}}$$

30. The acceleration can be found from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (25\,\mathrm{m/s})^2}{2(85\ \mathrm{m})} = \boxed{-3.7\,\mathrm{m/s^2}}$$

$\boxed{31.}$ By definition, the acceleration is $a = \frac{v - v_0}{t} = \frac{21\,\mathrm{m/s} - 12\,\mathrm{m/s}}{6.0\ \mathrm{s}} = \boxed{1.5\,\mathrm{m/s^2}}$.

The distance of travel can be found from Eq. 2-12b.

$$x - x_0 = v_0t + \tfrac{1}{2}at^2 = (12\,\mathrm{m/s})(6.0\ \mathrm{s}) + \tfrac{1}{2}(1.5\,\mathrm{m/s^2})(6.0\ \mathrm{s})^2 = \boxed{99\,\mathrm{m}}$$

32. Assume that the plane starts from rest. The runway distance is found by solving Eq. 2-12c for $x - x_0$.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{(32\,\mathrm{m/s})^2 - 0}{2(3.0\,\mathrm{m/s^2})} = \boxed{1.7\times10^2\,\mathrm{m}}$$

33. For the baseball, $v_0 = 0$, $x - x_0 = 3.5\,\mathrm{m}$, and the final speed of the baseball (during the throwing motion) is $v = 41\,\mathrm{m/s}$. The acceleration is found from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(41\,\mathrm{m/s})^2 - 0}{2(3.5\ \mathrm{m})} = \boxed{240\,\mathrm{m/s^2}}$$

34. The average velocity is defined by Eq. 2-2, $\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$. Compare this expression to Eq. 2-12d, $\overline{v} = \tfrac{1}{2}(v + v_0)$. A relation for the velocity is found by integrating the expression for the acceleration, since the acceleration is the derivative of the velocity. Assume the velocity is v_0 at time $t = 0$.

$$a = A + Bt = \frac{dv}{dt} \rightarrow dv = (A + Bt)\,dt \rightarrow \int_{v_0}^{v} dv = \int_0^t (A + Bt)\,dt \rightarrow v = v_0 + At + \tfrac{1}{2}Bt^2$$

Find an expression for the position by integrating the velocity, assuming that $x = x_0$ at time $t = 0$.

$$v = v_0 + At + \tfrac{1}{2}Bt^2 = \frac{dx}{dt} \rightarrow dx = \left(v_0 + At + \tfrac{1}{2}Bt^2\right)dt \rightarrow$$

$$\int_{x_0}^{x} dx = \int_0^t \left(v_0 + At + \tfrac{1}{2}Bt^2\right)dt \rightarrow x - x_0 = v_0t + \tfrac{1}{2}At^2 + \tfrac{1}{6}Bt^3$$

Compare $\frac{x-x_0}{t}$ to $\frac{1}{2}(v+v_0)$.

$$\overline{v}=\frac{x-x_0}{t}=\frac{v_0t+\frac{1}{2}At^2+\frac{1}{6}Bt^3}{t}=v_0+\frac{1}{2}At+\frac{1}{6}Bt^2$$

$$\tfrac{1}{2}(v+v_0)=\frac{v_0+v_0+At+\frac{1}{2}Bt^2}{2}=v_0+\frac{1}{2}At+\frac{1}{4}Bt^2$$

They are different, so $\boxed{\overline{v}\neq\frac{1}{2}(v+v_0)}$.

35. The sprinter starts from rest. The average acceleration is found from Eq. 2-12c.

$$v^2=v_0^2+2a(x-x_0)\quad\rightarrow\quad a=\frac{v^2-v_0^2}{2(x-x_0)}=\frac{(11.5\,\text{m/s})^2-0}{2(15.0\text{ m})}=4.408\,\text{m/s}^2\approx\boxed{4.41\,\text{m/s}^2}$$

Her elapsed time is found by solving Eq. 2-12a for time.

$$v=v_0+at\quad\rightarrow\quad t=\frac{v-v_0}{a}=\frac{11.5\,\text{m/s}-0}{4.408\,\text{m/s}^2}=\boxed{2.61\text{ s}}$$

36. Calculate the distance that the car travels during the reaction time and the deceleration.

$$\Delta x_1=v_0\Delta t=(18.0\,\text{m/s})(0.200\,\text{s})=3.6\,\text{m}$$

$$v^2=v_0^2+2a\Delta x_2\quad\rightarrow\quad\Delta x_2=\frac{v^2-v_0^2}{2a}=\frac{0-(18.0\,\text{m/s})^2}{2(-3.65\,\text{m/s}^2)}=44.4\,\text{m}$$

$$\Delta x=3.6\,\text{m}+44.4\,\text{m}=48.0\,\text{m}$$

$\boxed{\text{He will NOT be able to stop in time.}}$

$\boxed{37.}$ The words “slows down uniformly” implies that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2-2 and 2-9.

$$x-x_0=\frac{v_0+v}{2}t=\left(\frac{18.0\,\text{m/s}+0\,\text{m/s}}{2}\right)(5.00\text{ sec})=\boxed{45.0\,\text{m}}$$

38. The final velocity of the car is zero. The initial velocity is found from Eq. 2-12c with $v=0$ and solving for v_0. Note that the acceleration is negative.

$$v^2=v_0^2+2a(x-x_0)\quad\rightarrow\quad v_0=\sqrt{v^2-2a(x-x_0)}=\sqrt{0-2(-4.00\,\text{m/s}^2)(85\text{ m})}=\boxed{26\,\text{m/s}}$$

39. (*a*) The final velocity of the car is 0. The distance is found from Eq. 2-12c with an acceleration of $a=-0.50\,\text{m/s}^2$ and an initial velocity of $85\,\text{km/h}$.

$$x-x_0=\frac{v^2-v_0^2}{2a}=\frac{0-\left[(85\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{2(-0.50\,\text{m/s}^2)}=557\,\text{m}\approx\boxed{560\,\text{m}}$$

(*b*) The time to stop is found from Eq. 2-12a.

$$t=\frac{v-v_0}{a}=\frac{0-\left[(85\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]}{(-0.50\,\text{m/s}^2)}=47.22\,\text{s}\approx\boxed{47\,\text{s}}$$

(*c*) Take $x_0 = x(t=0) = 0\,\text{m}$. Use Eq. 2-12b, with $a = -0.50\,\text{m/s}^2$ and an initial velocity of $85\,\text{km/h}$. The first second is from $t = 0\,\text{s}$ to $t = 1\,\text{s}$, and the fifth second is from $t = 4\,\text{s}$ to $t = 5\,\text{s}$.

$$x(0) = 0 \;\; ; \;\; x(1) = 0 + (85\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)(1\,\text{s}) + \tfrac{1}{2}\left(-0.50\,\text{m/s}^2\right)(1\,\text{s})^2 = 23.36\,\text{m} \;\rightarrow$$

$$x(1) - x(0) = \boxed{23\,\text{m}}$$

$$x(4) = 0 + (85\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)(4\,\text{s}) + \tfrac{1}{2}\left(-0.50\,\text{m/s}^2\right)(4\,\text{s})^2 = 90.44\,\text{m}$$

$$x(5) = 0 + (85\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)(5\,\text{s}) + \tfrac{1}{2}\left(-0.50\,\text{m/s}^2\right)(5\,\text{s})^2 = 111.81\,\text{m}$$

$$x(5) - x(4) = 111.81\,\text{m} - 90.44\,\text{m} = 21.37\,\text{m} \approx \boxed{21\,\text{m}}$$

40. The final velocity of the driver is zero. The acceleration is found from Eq. 2-12c with $v = 0$ and solving for a.

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - \left[(105\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{2(0.80\,\text{m})} = -531.7\,\text{m/s}^2 \approx \boxed{-5.3\times10^2\,\text{m/s}^2}$$

Converting to "*g*'s": $a = \dfrac{-531.7\,\text{m/s}^2}{\left(9.80\,\text{m/s}^2\right)/g} = \boxed{-54\,g\text{'s}}$

41. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is $(95\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$. The location where the brakes are applied is found from the equation for motion at constant velocity: $x_0 = v_0 t_R = (26.39\,\text{m/s})(1.0\,\text{s}) = 26.39\,\text{m}$. This is now the starting location for the application of the brakes. In each case, the final speed is 0.

(*a*) Solve Eq. 2-12c for the final location.

$$v^2 = v_0^2 + 2a(x - x_0) \;\rightarrow\; x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39\,\text{m} + \frac{0 - (26.39\,\text{m/s})^2}{2\left(-5.0\,\text{m/s}^2\right)} = \boxed{96\,\text{m}}$$

(*b*) Solve Eq. 2-12c for the final location with the second acceleration.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 26.39\,\text{m} + \frac{0 - (26.39\,\text{m/s})^2}{2\left(-7.0\,\text{m/s}^2\right)} = \boxed{76\,\text{m}}$$

42. Calculate the acceleration from the velocity–time data using Eq. 2-12a, and then use Eq. 2-12b to calculate the displacement at $t = 2.0\,\text{s}$ and $t = 6.0\,\text{s}$. The initial velocity is $v_0 = 65\,\text{m/s}$.

$$a = \frac{v - v_0}{t} = \frac{162\,\text{m/s} - 65\,\text{m/s}}{10.0\,\text{s}} = 9.7\,\text{m/s}^2 \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2 \;\rightarrow$$

$$x(6.0\,\text{s}) - x(2.0\,\text{s}) = \left[\left(x_0 + v_0(6.0\,\text{s}) + \tfrac{1}{2}a(6.0\,\text{s})^2\right) - \left(x_0 + v_0(2.0\,\text{s}) + \tfrac{1}{2}a(2.0\,\text{s})^2\right)\right]$$

$$= v_0\left(6.0\,\text{s} - 2.0\,\text{s}\right) + \tfrac{1}{2}a\left[\left(6.0\,\text{s}\right)^2 - \left(2.0\,\text{s}\right)^2\right] = \left(65\,\text{m/s}\right)\left(4.0\,\text{s}\right) + \tfrac{1}{2}\left(9.7\,\text{m/s}^2\right)\left(32\,\text{s}^2\right)$$

$$= 415\,\text{m} \approx \boxed{4.2\times10^2\,\text{m}}$$

$\boxed{43.}$ Use the information for the first 180 m to find the acceleration, and the information for the full motion to find the final velocity. For the first segment, the train has $v_0 = 0\,\text{m/s}$, $v_1 = 23\,\text{m/s}$, and a displacement of $x_1 - x_0 = 180\,\text{m}$. Find the acceleration from Eq. 2-12c.

$$v_1^2 = v_0^2 + 2a\left(x_1 - x_0\right) \quad\rightarrow\quad a = \frac{v_1^2 - v_0^2}{2\left(x_1 - x_0\right)} = \frac{\left(23\,\text{m/s}\right)^2 - 0}{2\left(180\ \text{m}\right)} = 1.469\,\text{m/s}^2$$

Find the speed of the train after it has traveled the total distance (total displacement of $x_2 - x_0 = 255$ m) using Eq. 2-12c.

$$v_2^2 = v_0^2 + 2a\left(x_2 - x_0\right) \quad\rightarrow\quad v_2 = \sqrt{v_0^2 + 2a\left(x_2 - x_0\right)} = \sqrt{2\left(1.469\,\text{m/s}^2\right)\left(255\ \text{m}\right)} = \boxed{27\,\text{m/s}}$$

44. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car, and find another time that both cars have the same displacement from the origin.

For the speeder, traveling with a constant speed, the displacement is given by the following.

$$\Delta x_s = v_s t = \left(135\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)(t) = \left(37.5\,t\right)\ \text{m}$$

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1 second of reaction time.

$$\Delta x_{p1} = v_{p1}\left(1.00\,\text{s}\right) = \left(95\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\left(1.00\,\text{s}\right) = 26.39\ \text{m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for $\left(t - 1.00\right)$s. So this second part of the police car displacement, using Eq. 2-12b, is given as follows.

$$\Delta x_{p2} = v_{p1}\left(t - 1.00\right) + \tfrac{1}{2}a_p\left(t - 1.00\right)^2 = \left[\left(26.39\,\text{m/s}\right)\left(t - 1.00\right) + \tfrac{1}{2}\left(2.00\,\text{m/s}^2\right)\left(t - 1.00\right)^2\right]\text{m}$$

So the total police car displacement is $\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = \left(26.39 + 26.39\left(t - 1.00\right) + \left(t - 1.00\right)^2\right)\text{m}.$

Now set the two displacements equal, and solve for the time.

$$26.39 + 26.39\left(t - 1.00\right) + \left(t - 1.00\right)^2 = 37.5\ t \quad\rightarrow\quad t^2 - 13.11t + 1.00 = 0$$

$$t = \frac{13.11 \pm \sqrt{\left(13.11\right)^2 - 4.00}}{2} = 7.67\times10^{-2}\,\text{s}\ ,\ \boxed{13.0\,\text{s}}$$

The answer that is approximately 0 s corresponds to the fact that both vehicles had the same displacement of zero when the time was 0. The reason it is not exactly zero is rounding of previous values. The answer of 13.0 s is the time for the police car to overtake the speeder.

As a check on the answer, the speeder travels $\Delta x_s = \left(37.5\,\text{m/s}\right)\left(13.0\ \text{s}\right) = 488\,\text{m}$, and the police car travels $\Delta x_p = \left[26.39 + 26.39\left(12.0\right) + \left(12.0\right)^2\right]\text{m} = 487\,\text{m}.$. The difference is due to rounding.

45. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car. Both cars have the same displacement 8.00 s after the initial passing by the speeder.

For the speeder, traveling with a constant speed, the displacement is given by $\Delta x_s = v_s t = (8.00 v_s)\,\text{m}$. For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1.00 s of reaction time.

$$\Delta x_{p1} = v_{p1}(1.00\,\text{s}) = (95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)(1.00\,\text{s}) = 26.39\text{ m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for 7.00 s. So this second part of the police car displacement, using Eq. 2-12b, is given by the following.

$$\Delta x_{p2} = v_{p1}(7.00\,\text{s}) + \tfrac{1}{2}a_p(7.00\,\text{s})^2 = (26.39\,\text{m/s})(7.00\,\text{s}) + \tfrac{1}{2}(2.00\,\text{m/s}^2)(7.00\,\text{s})^2 = 233.73\,\text{m}$$

Thus the total police car displacement is $\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = (26.39 + 233.73)\,\text{m} = 260.12\,\text{m}$.

Now set the two displacements equal, and solve for the speeder's velocity.

$$(8.00 v_s)\,\text{m} = 260.12\,\text{m} \rightarrow v_s = (32.5\,\text{m/s})\left(\frac{3.6\,\text{km/h}}{1\,\text{m/s}}\right) = \boxed{117\,\text{km/h}}$$

46. During the final part of the race, the runner must have a displacement of 1100 m in a time of 180 s (3.0 min). Assume that the starting speed for the final part is the same as the average speed thus far.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8900\text{ m}}{(27\times 60)\text{ s}} = 5.494\,\text{m/s} = v_0$$

The runner will accomplish this by accelerating from speed v_0 to speed v for t seconds, covering a distance d_1, and then running at a constant speed of v for $(180 - t)$ seconds, covering a distance d_2. We have these relationships from Eq. 2-12a and Eq. 2-12b.

$$v = v_o + at \qquad d_1 = v_o t + \tfrac{1}{2}at^2 \qquad d_2 = v(180 - t) = (v_0 + at)(180 - t)$$

$$1100\text{ m} = d_1 + d_2 = v_o t + \tfrac{1}{2}at^2 + (v_0 + at)(180 - t) \rightarrow 1100\text{ m} = 180 v_0 + 180at - \tfrac{1}{2}at^2 \rightarrow$$

$$1100\text{ m} = (180\text{ s})(5.494\,\text{m/s}) + (180\text{ s})(0.2\,\text{m/s}^2)t - \tfrac{1}{2}(0.2\,\text{m/s}^2)t^2$$

$$0.1t^2 - 36t + 111 = 0 \qquad t = 357\text{ s}, 3.11\text{ s}$$

Since we must have $t < 180$ s, the solution is $\boxed{t = 3.1\,\text{s}}$.

47. For the runners to cross the finish line side-by-side means they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2-12b.

For Sally: $22 = 5 + 5t + \tfrac{1}{2}(-.5)t^2 \rightarrow t^2 - 20t + 68 = 0 \rightarrow$

$$t = \frac{20 \pm \sqrt{20^2 - 4(68)}}{2} = 4.343\,\text{s}, 15.66\,\text{s}$$

The first time is the time she first crosses the finish line, and so is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.

For Mary: $22 = 0 + 4t + \tfrac{1}{2}at^2 \rightarrow a = \dfrac{22 - 4t}{\tfrac{1}{2}t^2} = \dfrac{22 - 4(4.343)}{\tfrac{1}{2}(4.343)^2} = \boxed{0.49\,\text{m/s}^2}$

48. Choose downward to be the positive direction, and take $y_0 = 0$ at the top of the cliff. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$. The displacement is found from Eq. 2-12b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \;\rightarrow\; y - 0 = 0 + \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)\left(3.75\ \mathrm{s}\right)^2 \;\rightarrow\; y = \boxed{68.9\ \mathrm{m}}$$

$\boxed{49.}$ Choose downward to be the positive direction. The initial velocity is $v_0 = 0$, the final velocity is $v = \left(55\,\mathrm{km/h}\right)\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 15.28\,\mathrm{m/s}$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$. The time can be found by solving Eq. 2-12a for the time.

$$v = v_0 + at \;\rightarrow\; t = \frac{v - v_0}{a} = \frac{15.28\,\mathrm{m/s} - 0}{9.80\,\mathrm{m/s^2}} = \boxed{1.6\,\mathrm{s}}$$

50. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\,\mathrm{m/s^2}$.

(*a*) The elapsed time can be found from Eq. 2-12b, with *x* replaced by *y*.

$$y - y_0 = v_0 t + \tfrac{1}{2}at^2 \;\rightarrow\; t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2\left(380\ \mathrm{m}\right)}{9.80\,\mathrm{m/s^2}}} = 8.806\,\mathrm{s} \approx \boxed{8.8\ \mathrm{s}}\,.$$

(*b*) The final velocity can be found from Eq. 2-12a.

$$v = v_0 + at = 0 + \left(9.80\,\mathrm{m/s^2}\right)\left(8.806\ \mathrm{s}\right) = \boxed{86\,\mathrm{m/s}}$$

51. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the height where the ball was hit. For the upward path, $v_0 = 20\,\mathrm{m/s}$, $v = 0$ at the top of the path, and $a = -9.80\,\mathrm{m/s^2}$.

(*a*) The displacement can be found from Eq. 2-12c, with *x* replaced by y.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \;\rightarrow\; y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(20\,\mathrm{m/s}\right)^2}{2\left(-9.80\,\mathrm{m/s^2}\right)} = \boxed{20\,\mathrm{m}}$$

(*b*) The time of flight can be found from Eq. 2-12b, with *x* replaced by y, using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \;\rightarrow\; t\left(v_0 + \tfrac{1}{2}at\right) = 0 \;\rightarrow\; t = 0\ ,\ t = \frac{2v_0}{-a} = \frac{2\left(20\,\mathrm{m/s}\right)}{9.80\,\mathrm{m/s^2}} = \boxed{4\,\mathrm{s}}$$

The result of $t = 0$ s is the time for the original displacement of zero (when the ball was hit), and the result of $t = 4$ s is the time to return to the original displacement. Thus the answer is $t = 4$ s.

52. Choose upward to be the positive direction, and take $y_0 = 0$ to be the height from which the ball was thrown. The acceleration is $a = -9.80\,\mathrm{m/s^2}$. The displacement upon catching the ball is 0, assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2-12b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \;\rightarrow$$

$$v_0 = \frac{y - y_0 - \tfrac{1}{2}at^2}{t} = -\tfrac{1}{2}at = -\tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)\left(3.2\ \mathrm{s}\right) = 15.68\,\mathrm{m/s} \approx \boxed{16\,\mathrm{m/s}}$$

The height can be calculated from Eq. 2-12c, with a final velocity of $v = 0$ at the top of the path.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(15.68\,\text{m/s}\right)^2}{2\left(-9.80\,\text{m/s}^2\right)} = 12.54\,\text{m} \approx \boxed{13\text{ m}}$$

53. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the maximum height of the kangaroo. Consider just the downward motion of the kangaroo. Then the displacement is $y = 1.65\,\text{m}$, the acceleration is $a = 9.80\,\text{m/s}^2$, and the initial velocity is $v_0 = 0\,\text{m/s}$. Use Eq. 2-12b to calculate the time for the kangaroo to fall back to the ground. The total time is then twice the falling time.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad y = \tfrac{1}{2}at^2 \quad \rightarrow \quad t_{\text{fall}} = \sqrt{\frac{2y}{a}} \quad \rightarrow$$

$$t_{\text{total}} = 2\sqrt{\frac{2y}{a}} = 2\sqrt{\frac{2\left(1.65\,\text{m}\right)}{\left(9.80\,\text{m/s}^2\right)}} = \boxed{1.16\,\text{s}}$$

54. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the floor level, where the jump starts. For the upward path, $y = 1.2\text{ m}$, $v = 0$ at the top of the path, and $a = -9.80\,\text{m/s}^2$.

(*a*) The initial speed can be found from Eq. 2-12c, with *x* replaced by y.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a\left(y - y_0\right)} = \sqrt{-2ay} = \sqrt{-2\left(-9.80\,\text{m/s}^2\right)\left(1.2\,\text{m}\right)} = 4.8497\,\text{m/s} \approx \boxed{4.8\,\text{m/s}}$$

(*b*) The time of flight can be found from Eq. 2-12b, with *x* replaced by y, using a displacement of 0 for the displacement of the jumper returning to the original height.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad t\left(v_0 + \tfrac{1}{2}at\right) = 0 \quad \rightarrow$$

$$t = 0\ ,\ t = \frac{2v_0}{-a} = \frac{2\left(4.897\,\text{m/s}\right)}{9.80\,\text{m/s}^2} = \boxed{0.99\text{ s}}$$

The result of *t* = 0 s is the time for the original displacement of zero (when the jumper started to jump), and the result of *t* = 0.99 s is the time to return to the original displacement. Thus the answer is *t* = 0.99 seconds.

$\boxed{55.}$ Choose downward to be the positive direction, and take $y_0 = 0$ to be the height where the object was released. The initial velocity is $v_0 = -5.10\,\text{m/s}$, the acceleration is $a = 9.80\,\text{m/s}^2$, and the displacement of the package will be $y = 105$ m. The time to reach the ground can be found from Eq. 2-12b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2v_0}{a}t - \frac{2y}{a} = 0 \quad \rightarrow \quad t^2 + \frac{2\left(-5.10\,\text{m/s}\right)}{9.80\,\text{m/s}^2}t - \frac{2\left(105\text{ m}\right)}{9.80\,\text{m/s}^2} = 0 \quad \rightarrow$$

$$t = 5.18\,\text{s}\ ,\ -4.14\,\text{s}$$

The correct time is the positive answer, $\boxed{t = 5.18\,\text{s}}$.

56. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height from which the object is released. The initial velocity is $v_0 = 0$, and the acceleration is $a = g$. Then we can calculate the position as a function of time from Eq. 2-12b, with *x* replaced by *y*, as $y(t) = \frac{1}{2}gt^2$. At the end of each second, the position would be as follows.

$$y(0) = 0 \;;\quad y(1) = \tfrac{1}{2}g \;;\quad y(2) = \tfrac{1}{2}g(2)^2 = 4y(1) \;;\quad y(3) = \tfrac{1}{2}g(3)^2 = 9y(1)$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$d(1) = y(1) - y(0) = y(1) \;;\quad d(2) = y(2) - y(1) = 3y(1) \;;\quad d(3) = y(3) - y(2) = 5y(1)$$

We could do this in general.

$$y(n) = \tfrac{1}{2}gn^2 \qquad y(n+1) = \tfrac{1}{2}g(n+1)^2$$

$$d(n+1) = y(n+1) - y(n) = \tfrac{1}{2}g(n+1)^2 - \tfrac{1}{2}gn^2 = \tfrac{1}{2}g\left((n+1)^2 - n^2\right)$$

$$= \tfrac{1}{2}g\left(n^2 + 2n + 1 - n^2\right) = \tfrac{1}{2}g(2n+1)$$

The value of $(2n+1)$ is always odd, in the sequence $\boxed{1, 3, 5, 7, \ldots}$.

57. Choose upward to be the positive direction, and $y_0 = 0$ to be the level from which the ball was thrown. The initial velocity is v_0,, the instantaneous velocity is $v = 14\,\mathrm{m/s}$, the acceleration is $a = -9.80\,\mathrm{m/s^2}$, and the location of the window is $y = 23\,\mathrm{m}$.

(*a*) Using Eq. 2-12c and substituting *y* for *x*, we have

$$v^2 = v_0^2 + 2a(y - y_0) \;\rightarrow$$

$$v_0 = \pm\sqrt{v^2 - 2a(y - y_0)} = \pm\sqrt{(14\,\mathrm{m/s})^2 - 2(-9.8\,\mathrm{m/s^2})(23\,\mathrm{m})} = 25.43\,\mathrm{m/s} \approx \boxed{25\,\mathrm{m/s}}$$

Choose the positive value because the initial direction is upward.

(*b*) At the top of its path, the velocity will be 0, and so we can use the initial velocity as found above, along with Eq. 2-12c.

$$v^2 = v_0^2 + 2a(y - y_0) \;\rightarrow\; y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (25.43\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = \boxed{33\,\mathrm{m}}$$

(*c*) We want the time elapsed from throwing (speed $v_0 = 25.43\,\mathrm{m/s}$) to reaching the window (speed $v = 14\,\mathrm{m/s}$). Using Eq. 2-12a, we have the following.

$$v = v_0 + at \;\rightarrow\; t = \frac{v - v_0}{a} = \frac{14\,\mathrm{m/s} - 25.43\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.166\,\mathrm{s} \approx \boxed{1.2\,\mathrm{s}}$$

(*d*) We want the time elapsed from the window (speed $v_0 = 14\,\mathrm{m/s}$) to reaching the street (speed $v = -25.43\,\mathrm{m/s}$). Using Eq. 2-12a, we have the following.

$$v = v_0 + at \;\rightarrow\; t = \frac{v - v_0}{a} = \frac{-25.43\,\mathrm{m/s} - 14\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 4.0\,\mathrm{s}$$

This is the elapsed time after passing the window. The total time of flight of the baseball from passing the window to reaching the street is $4.0\,\mathrm{s} + 1.2\,\mathrm{s} = \boxed{5.2\,\mathrm{s}}$.

58. (*a*) Choose upward to be the positive direction, and $y_0 = 0$ at the ground. The rocket has $v_0 = 0$, $a = 3.2\,\text{m/s}^2$, and $y = 950\,\text{m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq 2-12c, with *x* replaced by *y*.

$$v_{950\text{ m}}^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_{950\text{ m}} = \pm\sqrt{v_0^2 + 2a(y-y_0)} = \pm\sqrt{0 + 2(3.2\,\text{m/s}^2)(950\text{ m})} = 77.97\,\text{m/s} \approx \boxed{78\,\text{m/s}}$$

The positive root is chosen since the rocket is moving upwards when it runs out of fuel.

(*b*) The time to reach the 950 m location can be found from Eq. 2-12a.

$$v_{950\text{ m}} = v_0 + at_{950\text{ m}} \rightarrow t_{950\text{ m}} = \frac{v_{950\text{ m}} - v_0}{a} = \frac{77.97\,\text{m/s} - 0}{3.2\,\text{m/s}^2} = 24.37\text{ s} \approx \boxed{24\text{ s}}$$

(*c*) For this part of the problem, the rocket will have an initial velocity $v_0 = 77.97\,\text{m/s}$, an acceleration of $a = -9.80\,\text{m/s}^2$, and a final velocity of $v = 0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2-12c.

$$v^2 = v_{950\text{ m}}^2 + 2a(y - 950\text{ m}) \rightarrow$$

$$y_{\text{max}} = 950\text{ m} + \frac{0 - v_{950\text{ m}}^2}{2a} = 950\text{ m} + \frac{-(77.97\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 950\text{ m} + 310\text{ m} = \boxed{1260\text{ m}}$$

(*d*) The time for the "coasting" portion of the flight can be found from Eq. 2-12a.

$$v = v_{950\text{ m}} + at_{\text{coast}} \rightarrow t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 77.97\,\text{m/s}}{-9.80\,\text{m/s}^2} = 7.96\text{ s}$$

Thus the total time to reach the maximum altitude is $t = 24.37\text{ s} + 7.96\text{ s} = 32.33\text{s} \approx \boxed{32\text{ s}}$.

(*e*) For the falling motion of the rocket, $v_0 = 0\,\text{m/s}$, $a = -9.80\,\text{m/s}^2$, and the displacement is -1260 m (it falls from a height of 1260 m to the ground). Find the velocity upon reaching the Earth from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2a(y-y_0)} = \pm\sqrt{0 + 2(-9.80\,\text{m/s}^2)(-1260\text{ m})} = -157\,\text{m/s} \approx \boxed{-160\,\text{m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

(*f*) The time for the rocket to fall back to the Earth is found from Eq. 2-12a.

$$v = v_0 + at \rightarrow t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-157\,\text{m/s} - 0}{-9.80\,\text{m/s}^2} = 16.0\text{ s}$$

Thus the total time for the entire flight is $t = 32.33\text{ s} + 16.0\text{ s} = 48.33\text{s} \approx \boxed{48\text{s}}$. .

59. (*a*) Choose $y = 0$ to be the ground level, and positive to be upward. Then $y = 0\,\text{m}$, $y_0 = 15\,\text{m}$, $a = -g$, and $t = 0.83\,\text{s}$ describe the motion of the balloon. Use Eq. 2-12b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow$$

$$v_0 = \frac{y - y_0 - \frac{1}{2}at^2}{t} = \frac{0 - 15\,\text{m} - \frac{1}{2}(-9.80\,\text{m/s}^2)(0.83\,\text{s})^2}{(0.83\,\text{s})} = -14\,\text{m/s}$$

So the speed is $\boxed{14\,\text{m/s}}$.

(*b*) Consider the change in velocity from being released to being at Roger's room, using Eq. 2-12c.

$$v^2 = v_0^2 + 2a\Delta y \quad \rightarrow \quad \Delta y = \frac{v^2 - v_0^2}{2a} = \frac{-\left(-14\,\text{m/s}\right)^2}{2\left(-9.8\,\text{m/s}^2\right)} = 10\,\text{m}$$

Thus the balloons are coming from 2 floors above Roger, and so the $\boxed{\text{fifth floor}}$.

60. Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 24.0\,\text{m/s}$, $a = -9.80\,\text{m/s}^2$, and $y - y_0 = 13.0\,\text{m}$.

(*a*) The velocity can be found from Eq, 2-12c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{\left(24.0\,\text{m/s}\right)^2 + 2\left(-9.80\,\text{m/s}^2\right)\left(13.0\text{ m}\right)} = \pm 17.9\,\text{m/s}$$

Thus the speed is $\boxed{|v| = 17.9\,\text{m/s}}$.

(*b*) The time to reach that height can be found from Eq. 2-12b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2\left(24.0\,\text{m/s}\right)}{-9.80\,\text{m/s}^2}t + \frac{2\left(-13.0\text{ m}\right)}{-9.80\,\text{m/s}^2} = 0 \quad \rightarrow$$

$$t^2 - 4.898t + 2.653 = 0 \quad \rightarrow \quad \boxed{t = 4.28\text{ s}\,,\ 0.620\text{ s}}$$

(*c*) $\boxed{\text{There are two times at which the object reaches that height}}$ – once on the way up $\left(t = 0.620\,\text{s}\right)$, and once on the way down $\left(t = 4.28\,\text{s}\right)$.

$\boxed{61.}$ Choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. Call the location of the top of the window y_w, and the time for the stone to fall from release to the top of the window is t_w. Since the stone is dropped from rest, using Eq. 2-12b with *y* substituting for *x*, we have $y_w = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}gt_w^2$. The location of the bottom of the window is $y_w + 2.2\,\text{m}$, and the time for the stone to fall from release to the bottom of the window is $t_w + 0.33\,\text{s}$. Since the stone is dropped from rest, using Eq. 2-12b, we have the following: $y_w + 2.2\text{ m} = y_0 + v_0 + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}g\left(t_w + 0.33\,\text{s}\right)^2$. Substitute the first expression for y_w into the second expression.

$$\tfrac{1}{2}gt_w^2 + 2.2\text{ m} = \tfrac{1}{2}g\left(t_w + 0.33\text{ s}\right)^2 \quad \rightarrow \quad t_w = 0.515\text{ s}$$

Use this time in the first equation to get the height above the top of the window from which the stone fell.

$$y_w = \tfrac{1}{2}gt_w^2 = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(0.515\text{ s}\right)^2 = \boxed{1.3\,\text{m}}$$

62. Choose upward to be the positive direction, and $y_0 = 0$ to be the location of the nozzle. The initial velocity is v_0, the acceleration is $a = -9.80\,\text{m/s}^2$, the final location is $y = -1.5\,\text{m}$, and the time of flight is $t = 2.0\,\text{s}$. Using Eq. 2-12b and substituting *y* for *x* gives the following.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{-1.5\text{ m} - \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(2.0\text{ s}\right)^2}{2.0\text{ s}} = \boxed{9.1\,\text{m/s}}$$

63. Choose up to be the positive direction, so $a = -g$. Let the ground be the $y = 0$ location. As an intermediate result, the velocity at the bottom of the window can be found from the data given. Assume the rocket is at the bottom of the window at $t = 0$, and use Eq. 2-12b.

$$y_{\substack{\text{top of}\\\text{window}}} = y_{\substack{\text{bottom of}\\\text{window}}} + v_{\substack{\text{bottom of}\\\text{window}}} t_{\substack{\text{pass}\\\text{window}}} + \tfrac{1}{2} a t^2_{\substack{\text{pass}\\\text{window}}} \quad \rightarrow$$

$$10.0\,\text{m} = 8.0\,\text{m} + v_{\substack{\text{bottom of}\\\text{window}}}(0.15\,\text{s}) + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)(0.15\,\text{s})^2 \quad \rightarrow \quad v_{\substack{\text{bottom of}\\\text{window}}} = 14.07\,\text{m/s}$$

Now use the velocity at the bottom of the window with Eq. 2-12c to find the launch velocity, assuming the launch velocity was achieved at the ground level.

$$v^2_{\substack{\text{bottom of}\\\text{window}}} = v^2_{\text{launch}} + 2a(y - y_0) \quad \rightarrow$$

$$v_{\text{launch}} = \sqrt{v^2_{\substack{\text{bottom of}\\\text{window}}} - 2a(y - y_0)} = \sqrt{(14.07\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(8.0\,\text{m})} = 18.84\,\text{m/s}$$

$$\approx \boxed{18.8\,\text{m/s}}$$

The maximum height can also be found from Eq. 2-12c, using the launch velocity and a velocity of 0 at the maximum height.

$$v^2_{\substack{\text{maximum}\\\text{height}}} = v^2_{\text{launch}} + 2a(y_{\text{max}} - y_0) \quad \rightarrow$$

$$y_{\text{max}} = y_0 + \frac{v^2_{\substack{\text{maximum}\\\text{height}}} - v^2_{\text{launch}}}{2a} = \frac{-(18.84\,\text{m/s})^2}{2\left(-9.80\,\text{m/s}^2\right)} = \boxed{18.1\,\text{m}}$$

64. Choose up to be the positive direction. Let the bottom of the cliff be the $y = 0$ location. The equation of motion for the dropped ball is $y_{\text{ball}} = y_0 + v_0 t + \tfrac{1}{2}at^2 = 50.0\,\text{m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2$. The equation of motion for the thrown stone is $y_{\text{stone}} = y_0 + v_0 t + \tfrac{1}{2}at^2 = (24.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2$. Set the two equations equal and solve for the time of the collision. Then use that time to find the location of either object.

$$y_{\text{ball}} = y_{\text{stone}} \quad \rightarrow \quad 50.0\,\text{m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 = (24.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow$$

$$50.0\,\text{m} = (24.0\,\text{m/s})t \quad \rightarrow \quad t = \frac{50.0\,\text{m}}{24.0\,\text{m/s}} = 2.083\,\text{s}$$

$$y_{\text{ball}} = y_0 + v_0 t + \tfrac{1}{2}at^2 = 50.0\,\text{m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)(2.083\,\text{s})^2 = \boxed{28.7\,\text{m}}$$

65. For the falling rock, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is dropped. The initial velocity is $v_0 = 0\,\text{m/s}$, the acceleration is $a = g$, the displacement is $y = H$, and the time of fall is t_1. Using Eq. 2-12b with *y* substituting for *x*, we have $H = y_0 + v_0 t + \tfrac{1}{2}t^2 = 0 + 0 + \tfrac{1}{2}gt_1^2$. For the sound wave, use the constant speed equation that $v_s = \dfrac{\Delta x}{\Delta t} = \dfrac{H}{T - t_1}$, which can be rearranged to give $t_1 = T - \dfrac{H}{v_s}$, where $T = 3.4\,\text{s}$ is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for t_1 into the equation for *H* from the stone, and solve for *H*.

$$H = \tfrac{1}{2}g\left(T - \frac{H}{v_s}\right)^2 \quad\rightarrow\quad \frac{g}{2v_s^2}H^2 - \left(\frac{gT}{v_s}+1\right)H + \tfrac{1}{2}gT^2 = 0 \quad\rightarrow$$

$$4.239\times10^{-5}H^2 - 1.098H + 56.64 = 0 \quad\rightarrow\quad H = 51.7\text{ m},\ 2.59\times10^4\text{m}$$

If the larger answer is used in $t_1 = T - \dfrac{H}{v_s}$, a negative time of fall results, and so the physically correct answer is $\boxed{H = 52\,\text{m}}$.

66. (*a*) Choose up to be the positive direction. Let the throwing height of both objects be the $y = 0$ location, and so $y_0 = 0$ for both objects. The acceleration of both objects is $a = -g$. The equation of motion for the rock, using Eq. 2-12b, is $y_{\text{rock}} = y_0 + v_{0\text{ rock}}t + \tfrac{1}{2}at^2 = v_{0\text{ rock}}t - \tfrac{1}{2}gt^2$, where *t* is the time elapsed from the throwing of the rock. The equation of motion for the ball, being thrown 1.00 s later, is $y_{\text{ball}} = y_0 + v_{0\text{ ball}}(t - 1.00\,\text{s}) + \tfrac{1}{2}a(t - 1.00\,\text{s})^2 = v_{0\text{ ball}}(t - 1.00\,\text{s}) - \tfrac{1}{2}g(t - 1.00\,\text{s})^2$. Set the two equations equal (meaning the two objects are at the same place) and solve for the time of the collision.

$$y_{\text{rock}} = y_{\text{ball}} \quad\rightarrow\quad v_{0\text{ rock}}t - \tfrac{1}{2}gt^2 = v_{0\text{ ball}}(t - 1.00\,\text{s}) - \tfrac{1}{2}g(t - 1.00\,\text{s})^2 \quad\rightarrow$$

$$(12.0\,\text{m/s})t - \tfrac{1}{2}(9.80\,\text{m/s}^2)t^2 = (18.0\,\text{m/s})(t - 1.00\,\text{s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(t - 1.00\,\text{s})^2 \quad\rightarrow$$

$$(15.8\,\text{m/s})t = (22.9\,\text{m}) \quad\rightarrow\quad t = \boxed{1.45\,\text{s}}$$

(*b*) Use the time for the collision to find the position of either object.

$$y_{\text{rock}} = v_{0\text{ rock}}t - \tfrac{1}{2}gt^2 = (12.0\,\text{m/s})(1.45\,\text{s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.45\,\text{s})^2 = \boxed{7.10\,\text{m}}$$

(*c*) Now the ball is thrown first, and so $y_{\text{ball}} = v_{0\text{ ball}}t - \tfrac{1}{2}gt^2$ and $y_{\text{rock}} = v_{0\text{ rock}}(t - 1.00\,\text{s}) - \tfrac{1}{2}g(t - 1.00\,\text{s})^2$. Again set the two equations equal to find the time of collision.

$$y_{\text{ball}} = y_{\text{rock}} \quad\rightarrow\quad v_{0\text{ ball}}t - \tfrac{1}{2}gt^2 = v_{0\text{ rock}}(t - 1.00\,\text{s}) - \tfrac{1}{2}g(t - 1.00\,\text{s})^2 \quad\rightarrow$$

$$(18.0\,\text{m/s})t - \tfrac{1}{2}(9.80\,\text{m/s}^2)t^2 = (12.0\,\text{m/s})(t - 1.00\,\text{s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(t - 1.00\,\text{s})^2 \quad\rightarrow$$

$$(3.80\,\text{m/s})t = 16.9\,\text{m} \quad\rightarrow\quad t = 4.45\,\text{s}$$

But this answer can be deceptive. Where do the objects collide?

$$y_{\text{ball}} = v_{0\text{ ball}}t - \tfrac{1}{2}gt^2 = (18.0\,\text{m/s})(4.45\,\text{s}) - \tfrac{1}{2}(9.80\,\text{m/s}^2)(4.45\,\text{s})^2 = -16.9\,\text{m}$$

Thus, assuming they were thrown from ground level, they collide below ground level, which cannot happen. Thus $\boxed{\text{they never collide}}$.

$\boxed{67.}$ The displacement is found from the integral of the velocity, over the given time interval.

$$\Delta x = \int_{t_1}^{t_2} v\,dt = \int_{t=1.5\text{s}}^{t=3.1\text{s}} (25 + 18t)\,dt = \left(25t + 9t^2\right)\Big|_{t=1.5\text{s}}^{t=3.1\text{s}} = \left[25(3.1) + 9(3.1)^2\right] - \left[25(1.5) + 9(1.5)^2\right]$$

$$= \boxed{106\,\text{m}}$$

68. (*a*) The speed is the integral of the acceleration.

$$a=\frac{dv}{dt} \rightarrow dv=adt \rightarrow dv=A\sqrt{t}dt \rightarrow \int_{v_0}^{v}dv=A\int_{0}^{t}\sqrt{t}dt \rightarrow$$

$$v-v_0=\tfrac{2}{3}At^{3/2} \rightarrow v=v_0+\tfrac{2}{3}At^{3/2} \rightarrow \boxed{v=7.5\,\text{m/s}+\tfrac{2}{3}\left(2.0\,\text{m/s}^{5/2}\right)t^{3/2}}$$

(*b*) The displacement is the integral of the velocity.

$$v=\frac{dx}{dt} \rightarrow dx=vdt \rightarrow dx=\left(v_0+\tfrac{2}{3}At^{3/2}\right)dt \rightarrow$$

$$\int_{0\,\text{m}}^{x}dx=\int_{0}^{t}\left(v_0+\tfrac{2}{3}At^{3/2}\right)dt \rightarrow x=v_0t+\tfrac{2}{3}\tfrac{2}{5}At^{5/2}=\boxed{\left(7.5\,\text{m/s}\right)t+\tfrac{4}{15}\left(2.0\,\text{m/s}^{5/2}\right)t^{5/2}}$$

(*c*) $a\left(t=5.0\,\text{s}\right)=\left(2.0\,\text{m/s}^{5/2}\right)\sqrt{5.0\,\text{s}}=\boxed{4.5\,\text{m/s}^2}$

$$v\left(t=5.0\,\text{s}\right)=7.5\,\text{m/s}+\tfrac{2}{3}\left(2.0\,\text{m/s}^{5/2}\right)\left(5.0\,\text{s}\right)^{3/2}=22.41\,\text{m/s}\approx\boxed{22\,\text{m/s}}$$

$$x\left(t=5.0\,\text{s}\right)=\left(7.5\,\text{m/s}\right)\left(5.0\,\text{s}\right)+\tfrac{4}{15}\left(2.0\,\text{m/s}^{5/2}\right)\left(5.0\,\text{s}\right)^{5/2}=67.31\,\text{m}\approx\boxed{67\,\text{m}}$$

69. (*a*) The velocity is found by integrating the acceleration with respect to time. Note that with the substitution given in the hint, the initial value of u is $u_0=g-kv_0=g$.

$$a=\frac{dv}{dt} \rightarrow dv=adt \rightarrow dv=\left(g-kv\right)dt \rightarrow \frac{dv}{g-kv}=dt$$

Now make the substitution that $u\equiv g-kv$.

$$u\equiv g-kv \rightarrow dv=-\frac{du}{k} \qquad \frac{dv}{g-kv}=dt \rightarrow -\frac{du}{k}\frac{1}{u}=dt \rightarrow \frac{du}{u}=-kdt$$

$$\int_{g}^{u}\frac{du}{u}=-k\int_{0}^{t}dt \rightarrow \ln u\Big|_{g}^{u}=-kt \rightarrow \ln\frac{u}{g}=-kt \rightarrow u=ge^{-kt}=g-kv \rightarrow$$

$$\boxed{v=\frac{g}{k}\left(1-e^{-kt}\right)}$$

(*b*) As *t* goes to infinity, the value of the velocity is $v_{\text{term}}=\lim\limits_{t\to\infty}\frac{g}{k}\left(1-e^{-kt}\right)=\boxed{\frac{g}{k}}$. We also note that if the acceleration is zero (which happens at terminal velocity), then $a=g-kv=0 \rightarrow$

$$v_{\text{term}}=\frac{g}{k}.$$

70. (*a*) The train's constant speed is $v_{\text{train}}=5.0\,\text{m/s}$, and the location of the empty box car as a function of time is given by $x_{\text{train}}=v_{\text{train}}t=\left(5.0\,\text{m/s}\right)t$. The fugitive has $v_0=0\,\text{m/s}$ and $a=1.2\,\text{m/s}^2$ until his final speed is $6.0\,\text{m/s}$. The elapsed time during the acceleration is $t_{\text{acc}}=\dfrac{v-v_0}{a}=\dfrac{6.0\,\text{m/s}}{1.2\,\text{m/s}^2}=5.0$ s. Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the empty box car before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by Eq. 2-12b, $x_{\text{fugitive}}=x_0+v_0t+\tfrac{1}{2}at^2=0+0+\tfrac{1}{2}\left(1.2\,\text{m/s}^2\right)t^2$. For him to catch

the train, we must have $x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0\,\text{m/s})t = \frac{1}{2}(1.2\,\text{m/s}^2)t^2$. The solutions of this are $t = 0\,\text{s}, 8.3\,\text{s}$. Thus the fugitive cannot catch the car during his 5.0 s of acceleration.

Now the equation of motion of the fugitive changes. After the 5.0 s of acceleration, he runs with a constant speed of $6.0\,\text{m/s}$. Thus his location is now given (for times $t > 5\,\text{s}$) by the following.

$$x_{\text{fugitive}} = \tfrac{1}{2}(1.2\,\text{m/s}^2)(5.0\,\text{s})^2 + (6.0\,\text{m/s})(t - 5.0\,\text{s}) = (6.0\,\text{m/s})t - 15.0\,\text{m}$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0\,\text{m/s})t = (6.0\,\text{m/s})t - 15.0\,\text{m} \rightarrow t = \boxed{15.0\,\text{s}}$$

(*b*) The distance traveled to reach the box car is given by the following.

$$x_{\text{fugitive}}(t = 15.0\text{ s}) = (6.0\,\text{m/s})(15.0\text{ s}) - 15.0\text{ m} = \boxed{75\text{ m}}$$

71. Choose the upward direction to be positive, and $y_0 = 0$ to be the level from which the object was thrown. The initial velocity is v_0 and the velocity at the top of the path is $v = 0\,\text{m/s}$. The height at the top of the path can be found from Eq. 2-12c with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y - y_0 = \frac{-v_0^2}{2a}$$

From this we see that the displacement is inversely proportional to the acceleration, and so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, the $\boxed{\text{displacement increases by a factor of 6}}$.

72. (*a*) For the free-falling part of the motion, choose downward to be the positive direction, and $y_0 = 0$ to be the height from which the person jumped. The initial velocity is $v_0 = 0$, acceleration is $a = 9.80\,\text{m/s}^2$, and the location of the net is $y = 15.0\,\text{m}$. Find the speed upon reaching the net from Eq. 2-12c with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_{(\,)}) \rightarrow v = \pm\sqrt{0 + 2a(y - 0)} = \pm\sqrt{2(9.80\,\text{m/s}^2)(15.0\text{ m})} = 17.1\,\text{m/s}$$

The positive root is selected since the person is moving downward.

For the net-stretching part of the motion, choose downward to be the positive direction, and $y_0 = 15.0\,\text{m}$ to be the height at which the person first contacts the net. The initial velocity is $v_0 = 17.1\,\text{m/s}$, the final velocity is $v = 0$, and the location at the stretched position is $y = 16.0\,\text{m}$.. Find the acceleration from Eq. 2-12c with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0^2 - (17.1\,\text{m/s})^2}{2(1.0\text{ m})} = \boxed{-150\,\text{m/s}^2}$$

(*b*) For the acceleration to be smaller, in the above equation we see that the displacement should be larger. This means that the net should be $\boxed{\text{“loosened”}}$.

$\boxed{73.}$ The initial velocity of the car is $v_0 = (100\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 27.8\,\text{m/s}$. Choose $x_0 = 0$ to be the location at which the deceleration begins. We have $v = 0\,\text{m/s}$ and $a = -30g = -294\,\text{m/s}^2$. Find the displacement from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad x = x_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (27.8\,\text{m/s})^2}{2(-2.94\times 10^2\,\text{m/s}^2)} = 1.31\,\text{m} \approx \boxed{1.3\,\text{m}}$$

74. Choose downward to be the positive direction, and $y_0 = 0$ to be at the start of the pelican's dive. The pelican has an initial velocity is $v_0 = 0$, an acceleration of $a = g$, and a final location of $y = 16.0\,\text{m}$. Find the total time of the pelican's dive from Eq. 2-12b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}at^2 \quad \rightarrow \quad t_{\text{dive}} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(16.0\text{ m})}{9.80\,\text{m/s}^2}} = 1.81\text{ s}.$$

The fish can take evasive action if he sees the pelican at a time of 1.81 s – 0.20 s = 1.61 s into the dive. Find the location of the pelican at that time from Eq. 2-12b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at = 0 + 0 + \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.61\text{ s})^2 = 12.7\text{ m}$$

Thus the fish must spot the pelican at a minimum height from the surface of the water of $16.0\,\text{m} - 12.7\,\text{m} = \boxed{3.3\,\text{m}}$.

75. (*a*) Choose downward to be the positive direction, and $y_0 = 0$ to be the level from which the car was dropped. The initial velocity is $v_0 = 0$, the final location is $y = H$, and the acceleration is $a = g$. Find the final velocity from Eq. 2-12c, replacing *x* with *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{2gH}\ .$$

The speed is the magnitude of the velocity, $\boxed{v = \sqrt{2gH}}$.

(*b*) Solving the above equation for the height, we have that $H = \dfrac{v^2}{2g}$. Thus for a collision of

$$v = (50\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 13.89\,\text{m/s},$$ the corresponding height is as follows.

$$H = \frac{v^2}{2g} = \frac{(13.89\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 9.84\,\text{m} \approx \boxed{10\,\text{m}}$$

(*c*) For a collision of $v = (100\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 27.78\,\text{m/s}$, the corresponding height is as follow.

$$H = \frac{v^2}{2g} = \frac{(27.78\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 39.37\,\text{m} \approx \boxed{40\,\text{m}}$$

76. Choose downward to be the positive direction, and $y_0 = 0$ to be at the roof from which the stones are dropped. The first stone has an initial velocity of $v_0 = 0$ and an acceleration of $a = g$. Eqs. 2-12a and 2-12b (with *x* replaced by *y*) give the velocity and location, respectively, of the first stone as a function of time.

$$v = v_0 + at \quad \rightarrow \quad v_1 = gt_1 \qquad y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y_1 = \tfrac{1}{2}gt_1^2$$

The second stone has the same initial conditions, but its elapsed time $t - 1.50\,\text{s}$, and so has velocity and location equations as follows.

$$v_2 = g\left(t_1 - 1.50\,\text{s}\right) \qquad y_2 = \tfrac{1}{2}g\left(t_1 - 1.50\,\text{s}\right)^2$$

The second stone reaches a speed of $v_2 = 12.0\,\text{m/s}$ at a time given by the following.

$$t_1 = 1.50\,\text{s} + \frac{v_2}{g} = 1.50\,\text{s} + \frac{12.0\,\text{m/s}}{9.80\,\text{m/s}^2} = 2.72\,\text{s}$$

The location of the first stone at that time is $y_1 = \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(2.72\,\text{s}\right)^2 = 36.4\,\text{m}.$.

The location of the second stone at that time is $y_2 = \tfrac{1}{2}g\left(t_1 - 1.50\,\text{s}\right)^2 =$ $\tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(2.72 - 1.50\,\text{s}\right)^2 = 7.35\,\text{m}$. Thus the distance between the two stones is $y_1 - y_2 = 36.4\,\text{m} - 7.35\,\text{m} = \boxed{29.0\,\text{m}}$.

77. The initial velocity is $v_0 = \left(15\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 4.17\,\text{m/s}$. The final velocity is $v_0 = \left(75\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 20.83\,\text{m/s}$. The displacement is $x - x_0 = 4.0\,\text{km} = 4000\,\text{m}$. Find the average acceleration from Eq. 2-12c.

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \;\rightarrow\; a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{\left(20.83\,\text{m/s}\right)^2 - \left(4.17\,\text{m/s}\right)^2}{2\left(4000\,\text{m}\right)} = \boxed{5.2\times10^{-2}\,\text{m/s}^2}$$

78. The speed limit is $50\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 13.89\,\text{m/s}$.

(*a*) For your motion, you would need to travel $\left(10 + 15 + 50 + 15 + 70 + 15\right)\text{m} = 175\,\text{m}$ to get the front of the car all the way through the third intersection. The time to travel the 175 m is found using the distance and the constant speed.

$$\Delta x = \bar{v}\Delta t \;\rightarrow\; \Delta t = \frac{\Delta x}{\bar{v}} = \frac{175\,\text{m}}{13.89\,\text{m/s}} = 12.60\,\text{s}$$

$\boxed{\text{Yes}}$, you can make it through all three lights without stopping.

(*b*) The second car needs to travel 165 m before the third light turns red. This car accelerates from $v_0 = 0\,\text{m/s}$ to a maximum of $v = 13.89\,\text{m/s}$ with $a = 2.0\,\text{m/s}^2$. Use Eq. 2-12a to determine the duration of that acceleration.

$$v = v_0 + at \;\rightarrow\; t_{\text{acc}} = \frac{v - v_0}{a} = \frac{13.89\,\text{m/s} - 0\,\text{m/s}}{2.0\,\text{m/s}^2} = 6.94\,\text{s}$$

The distance traveled during that time is found from Eq. 2-12b.

$$\left(x - x_0\right)_{\text{acc}} = v_0 t_{\text{acc}} + \tfrac{1}{2}at_{\text{acc}}^2 = 0 + \tfrac{1}{2}\left(2.0\,\text{m/s}^2\right)\left(6.94\,\text{s}\right)^2 = 48.2\,\text{m}$$

Since 6.94 s have elapsed, there are 13 – 6.94 = 6.06 s remaining to clear the intersection. The car travels another 6.06 s at a speed of 13.89 m/s, covering a distance of $\Delta x_{\substack{\text{constant}\\\text{speed}}} = v_{\text{avg}}t =$ $\left(13.89\,\text{m/s}\right)\left(6.06\,\text{s}\right) = 84.2\,\text{m}$. Thus the total distance is 48.2 m + 84.2 m = 132.4 m. $\boxed{\text{No}}$, the car cannot make it through all three lights without stopping.

The car has to travel another 32.6 m to clear the third intersection, and is traveling at a speed of 13.89 m/s. Thus the care would enter the intersection a time $t = \dfrac{\Delta x}{v} = \dfrac{32.6\,\text{m}}{13.89\,\text{m/s}} = \boxed{2.3\,\text{s}}$ after the light turns red.

79. First consider the "uphill lie," in which the ball is being putted down the hill. Choose $x_0 = 0$ to be the ball's original location, and the direction of the ball's travel as the positive direction. The final velocity of the ball is $v = 0\,\text{m/s}$, the acceleration of the ball is $a = -1.8\,\text{m/s}^2$, and the displacement of the ball will be $x - x_0 = 6.0\,\text{m}$ for the first case and $x - x_0 = 8.0\,\text{m}$ for the second case. Find the initial velocity of the ball from Eq. 2-12c.

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad\rightarrow\quad v_0 = \sqrt{v^2 - 2a\left(x - x_0\right)} = \begin{cases} \sqrt{0 - 2\left(-1.8\,\text{m/s}^2\right)\left(6.0\,\text{m}\right)} = 4.6\,\text{m/s} \\ \sqrt{0 - 2\left(-1.8\,\text{m/s}^2\right)\left(8.0\,\text{m}\right)} = 5.4\,\text{m/s} \end{cases}$$

The range of acceptable velocities for the uphill lie is $\boxed{4.6\,\text{m/s} \text{ to } 5.4\,\text{m/s}}$, a spread of 0.8 m/s.

Now consider the "downhill lie," in which the ball is being putted up the hill. Use a very similar set-up for the problem, with the basic difference being that the acceleration of the ball is now $a = -2.8\,\text{m/s}^2$. Find the initial velocity of the ball from Eq. 2-12c.

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad\rightarrow\quad v_0 = \sqrt{v^2 - 2a\left(x - x_0\right)} = \begin{cases} \sqrt{0 - 2\left(-2.8\,\text{m/s}^2\right)\left(6.0\,\text{m}\right)} = 5.8\,\text{m/s} \\ \sqrt{0 - 2\left(-2.8\,\text{m/s}^2\right)\left(8.0\,\text{m}\right)} = 6.7\,\text{m/s} \end{cases}$$

The range of acceptable velocities for the downhill lie is $\boxed{5.8\,\text{m/s} \text{ to } 6.7\,\text{m/s}}$, a spread of 0.9 m/s.

Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, and so putting the ball downhill (the "uphill lie") is more difficult.

80. To find the distance, we divide the motion of the robot into three segments. First, the initial acceleration from rest; second, motion at constant speed; and third, deceleration back to rest.

$$d_1 = v_0 t + \tfrac{1}{2}a_1 t_1^2 = 0 + \tfrac{1}{2}\left(0.20\,\text{m/s}^2\right)\left(5.0\,\text{s}\right)^2 = 2.5\,\text{m} \qquad v_1 = a_1 t_1 = \left(0.20\,\text{m/s}^2\right)\left(5.0\,\text{s}\right) = 1.0\,\text{m/s}$$

$$d_2 = v_1 t_2 = \left(1.0\,\text{m/s}\right)\left(68\,\text{s}\right) = 68\,\text{m} \qquad v_2 = v_1 = 1.0\,\text{m/s}$$

$$d_3 = v_2 t_3 + \tfrac{1}{2}a_1 t_1^2 = \left(1.0\,\text{m/s}\right)\left(2.5\,\text{s}\right) + \tfrac{1}{2}\left(-0.40\,\text{m/s}^2\right)\left(2.5\,\text{s}\right)^2 = 1.25\,\text{m}$$

$$d = d_1 + d_2 + d_3 = 2.5\,\text{m} + 68\,\text{m} + 1.25\,\text{m} = 71.75\,\text{m} \approx \boxed{72\,\text{m}}$$

81. Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.5\,\text{m/s}$, the acceleration is $a = 9.80\,\text{m/s}^2$, and the final location is $y = 75.0\,\text{m}$.

(*a*) Using Eq. 2-12b and substituting *y* for *x*, we have the following.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow \left(4.9\,\text{m/s}^2\right)t^2 - \left(12.5\,\text{m/s}\right)t - 75.0\text{ m} = 0 \rightarrow t = -2.839\,\text{s}\,,\ 5.390\,\text{s}$$

The positive answer is the physical answer: $\boxed{t = 5.39\,\text{s}}$.

(*b*) Using Eq. 2-12a, we have $v = v_0 + at = -12.5\,\text{m/s} + \left(9.80\,\text{m/s}^2\right)\left(5.390\,\text{s}\right) = \boxed{40.3\,\text{m/s}}$.

(*c*) The total distance traveled will be the distance up plus the distance down. The distance down will be 75.0 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Using Eq. 2-12c we have the following.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(-12.5\,\mathrm{m/s}\right)^2}{2\left(9.80\,\mathrm{m/s^2}\right)} = -7.97\,\mathrm{m}$$

Thus the distance up is 7.97 m, the distance down is 82.97 m, and the total distance traveled is $\boxed{90.9\,\mathrm{m}}$.

82. (*a*) In the interval from A to B, it is $\boxed{\text{moving in the negative direction}}$, because its displacement is negative.

(*b*) In the interval from A to B, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*c*) In the interval from A to B, $\boxed{\text{the acceleration is negative}}$, because the graph is concave down, indicating that the slope is getting more negative, and thus the acceleration is negative.

(*d*) In the interval from D to E, it is $\boxed{\text{moving in the positive direction}}$, because the displacement is positive.

(*e*) In the interval from D to E, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*f*) In the interval from D to E, $\boxed{\text{the acceleration is positive}}$, because the graph is concave upward, indicating the slope is getting more positive, and thus the acceleration is positive.

(*g*) In the interval from C to D, $\boxed{\text{the object is not moving in either direction}}$.

$\boxed{\text{The velocity and acceleration are both 0.}}$

83. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant speed phase, and the deceleration phase. The maximum speed of the train is as follows.

$$\left(95\,\mathrm{km/h}\right)\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 26.39\,\mathrm{m/s}$$

In the acceleration phase, the initial velocity is $v_0 = 0\,\mathrm{m/s}$, the acceleration is $a = 1.1\,\mathrm{m/s^2}$, and the final velocity is $v = 26.39\,\mathrm{m/s}$. Find the elapsed time for the acceleration phase from Eq. 2-12a.

$$v = v_0 + at \quad \rightarrow \quad t_{\mathrm{acc}} = \frac{v - v_0}{a} = \frac{26.39\,\mathrm{m/s} - 0}{1.1\,\mathrm{m/s^2}} = 23.99\,\mathrm{s}$$

Find the displacement during the acceleration phase from Eq. 2-12b.

$$\left(x - x_0\right)_{\mathrm{acc}} = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}\left(1.1\,\mathrm{m/s^2}\right)\left(23.99\,\mathrm{s}\right)^2 = 316.5\,\mathrm{m}$$

In the deceleration phase, the initial velocity is $v_0 = 26.39\,\mathrm{m/s}$, the acceleration is $a = -2.0\,\mathrm{m/s^2}$, and the final velocity is $v = 0\,\mathrm{m/s}$. Find the elapsed time for the deceleration phase from Eq. 2-12a.

$$v = v_0 + at \quad \rightarrow \quad t_{\mathrm{dec}} = \frac{v - v_0}{a} = \frac{0 - 26.39\,\mathrm{m/s}}{-2.0\,\mathrm{m/s^2}} = 13.20\,\mathrm{s}$$

Find the distance traveled during the deceleration phase from Eq. 2-12b.

$$\left(x - x_0\right)_{\mathrm{dec}} = v_0 t + \tfrac{1}{2}at^2 = \left(26.39\,\mathrm{m/s}\right)\left(13.20\,\mathrm{s}\right) + \tfrac{1}{2}\left(-2.0\,\mathrm{m/s^2}\right)\left(13.20\,\mathrm{s}\right)^2 = 174.1\,\mathrm{m}$$

The total elapsed time and distance traveled for the acceleration / deceleration phases are:

$$t_{\text{acc}} + t_{\text{dec}} = 23.99\,\text{s} + 13.20\,\text{s} = 37.19\ \text{s}$$

$$(x - x_0)_{\text{acc}} + (x - x_0)_{\text{dec}} = 316.5\,\text{m} + 174.1\,\text{m} = 491\,\text{m}$$

(*a*) If the stations are spaced 1.80 km = 1800 m apart, then there is a total of $\frac{9000\ \text{m}}{1800\ \text{m}} = 5$ inter-station segments. A train making the entire trip would thus have a total of 5 inter-station segments and 4 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $1800\,\text{m} - 491\,\text{m} = 1309\,\text{m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39\,\text{m/s}$. The time for that 1309 m is given by $\Delta x = \bar{v}\Delta t \ \rightarrow$

$\Delta t_{\substack{\text{constant}\\ \text{speed}}} = \frac{\Delta x}{\bar{v}} = \frac{1309\,\text{m}}{26.39\,\text{m/s}} = 49.60\,\text{s}$. Thus a total inter-station segment will take 37.19 s + 49.60 s = 86.79 s. With 5 inter-station segments of 86.79 s each, and 4 stops of 22 s each, the total time is given by $t_{0.8\ \text{km}} = 5(86.79\,\text{s}) + 4(22\,\text{s}) = 522\,\text{s} = \boxed{8.7\,\text{min}}$.

(*b*) If the stations are spaced 3.0 km = 3000 m apart, then there is a total of $\frac{9000\ \text{m}}{3000\ \text{m}} = 3$ inter-station segments. A train making the entire trip would thus have a total of 3 inter-station segments and 2 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $3000\,\text{m} - 491\,\text{m} = 2509\,\text{m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39\,\text{m/s}$. The time for that 2509 m is given by $d = \bar{v}t \ \rightarrow$

$t = \frac{d}{\bar{v}} = \frac{2509\,\text{m}}{26.39\,\text{m/s}} = 95.07\,\text{s}$. Thus a total inter-station segment will take 37.19 s + 95.07 s = 132.3 s. With 3 inter-station segments of 132.3 s each, and 2 stops of 22 s each, the total time is $t_{3.0\ \text{km}} = 3(132.3\text{s}) + 2(22\,\text{s}) = 441\,\text{s} = \boxed{7.3\,\text{min}}$.

84. For the motion in the air, choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the diving board. The diver has $v_0 = 0$ (assuming the diver does not jump upward or downward), $a = g = 9.80\,\text{m/s}^2$, and $y = 4.0\ \text{m}$ when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. 2-12c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0)x \ \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{0 + 2(9.80\,\text{m/s}^2)(4.0\ \text{m})} = 8.85\,\text{m/s}$$

For the motion in the water, again choose down to be positive, but redefine $y_0 = 0$ to be at the surface of the water. For this motion, $v_0 = 8.85\,\text{m/s}$, $v = 0$, and $y - y_0 = 2.0\ \text{m}$. Find the acceleration from Eq. 2-12c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \ \rightarrow \ a = \frac{v^2 - v_0^2}{2(y - y_0)x} = \frac{0 - (8.85\,\text{m/s})^2}{2(2.0\ \text{m})} = -19.6\,\text{m/s}^2 \approx \boxed{-20\,\text{m/s}^2}$$

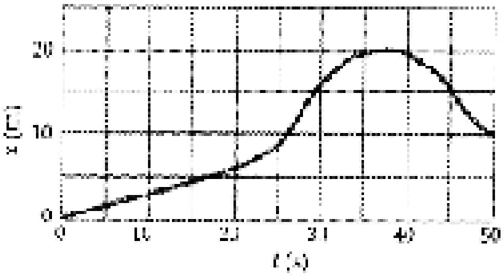
The negative sign indicates that the acceleration is directed upwards.

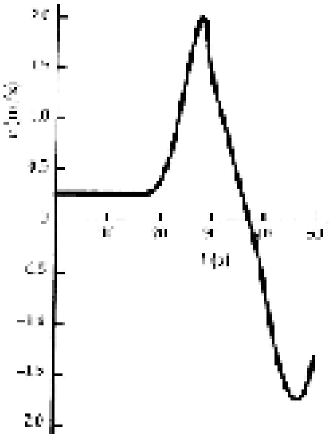
$\boxed{85.}$ Choose upward to be the positive direction, and the origin to be at the level where the ball was thrown. The velocity at the top of the ball's path will be $v = 0$, and the ball will have an acceleration of $a = -g$. If the maximum height that the ball reaches is $y = H$, then the relationship between the initial velocity and the maximum height can be found from Eq. 2-12c, with x replaced by y.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad 0 = v_0^2 + 2\left(-g\right)H \quad \rightarrow \quad H = v_0^2/2g$$

It is given that $v_{0\,\text{Bill}} = 1.5v_{0\,\text{Joe}}$, so $\dfrac{H_{\text{Bill}}}{H_{\text{Joe}}} = \dfrac{\left(v_{0\,\text{Bill}}\right)^2/2g}{\left(v_{0\,\text{Joe}}\right)^2/2g} = \dfrac{\left(v_{0\,\text{Bill}}\right)^2}{\left(v_{0\,\text{Joe}}\right)^2} = 1.5^2 = 2.25 \approx \boxed{2.3}$.

86. The v vs. t graph is found by taking the slope of the x vs. t graph. Both graphs are shown here.





87. The car's initial speed is $v_o = \left(45\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 12.5\,\text{m/s}$.

Case I: trying to stop. The constraint is, with the braking deceleration of the car $\left(a = -5.8\,\text{m/s}^2\right)$, can the car stop in a 28 m displacement? The 2.0 seconds has no relation to this part of the problem. Using Eq. 2-12c, the distance traveled during braking is as follows.

$$\left(x - x_0\right) = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(12.5\,\text{m/s}\right)^2}{2\left(-5.8\,\text{m/s}^2\right)} = 13.5\text{ m} \quad \rightarrow \quad \boxed{\text{She can stop the car in time.}}$$

Case II: crossing the intersection. The constraint is, with the given acceleration of the car $\left[a = \left(\dfrac{65\,\text{km/h} - 45\,\text{km/h}}{6.0\text{ s}}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 0.9259\,\text{m/s}^2\right]$, can she get through the intersection (travel 43 meters) in the 2.0 seconds before the light turns red? Using Eq. 2-12b, the distance traveled during the 2.0 sec is as follows.

$$\left(x - x_0\right) = v_0 t + \tfrac{1}{2}at^2 = \left(12.5\,\text{m/s}\right)\left(2.0\text{ s}\right) + \tfrac{1}{2}\left(0.927\,\text{m/s}^2\right)\left(2.0\text{ s}\right)^2 = 26.9\text{ m}$$

$\boxed{\text{She should stop.}}$

88. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 400 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are: i) the 10 m of clear room at the rear of the truck, ii) the 20 m length of the truck, iii) the 10 m of clear room at the front of the truck, and iv) the distance the truck travels. Since the truck travels at a speed of $\overline{v} = 25\,\text{m/s}$, the truck will have a displacement of $\Delta x_{\text{truck}} = \left(25\,\text{m/s}\right)t$. Thus the total displacement of the car during passing is $\Delta x_{\substack{\text{passing}\\\text{car}}} = 40\,\text{m} + \left(25\,\text{m/s}\right)t$.

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of $v_0 = 25\,\text{m/s}$ and an acceleration of $a = 1.0\,\text{m/s}^2$. Find $\Delta x_{\substack{\text{passing}\\\text{car}}}$ from Eq. 2-12b.

$$\Delta x_{\substack{\text{passing}\\\text{car}}} = x_c - x_0 = v_0 t + \tfrac{1}{2}at = (25\,\text{m/s})t + \tfrac{1}{2}(1.0\,\text{m/s}^2)t^2$$

Set the two expressions for $\Delta x_{\substack{\text{passing}\\\text{car}}}$ equal to each other in order to find the time required to pass.

$$40\,\text{m} + (25\,\text{m/s})t_{\text{pass}} = (25\,\text{m/s})t_{\text{pass}} + \tfrac{1}{2}(1.0\,\text{m/s}^2)t_{\text{pass}}^2 \rightarrow 40\,\text{m} = \tfrac{1}{2}(1.0\,\text{m/s}^2)t_{\text{pass}}^2 \rightarrow$$

$$t_{\text{pass}} = \sqrt{80\text{s}^2} = 8.94\text{ s}$$

Calculate the displacements of the two cars during this time.

$$\Delta x_{\substack{\text{passing}\\\text{car}}} = 40\,\text{m} + (25\,\text{m/s})(8.94\text{ s}) = 264\text{ m}$$

$$\Delta x_{\substack{\text{approaching}\\\text{car}}} = v_{\substack{\text{approaching}\\\text{car}}} t = (25\,\text{m/s})(8.94\text{ s}) = 224\text{ m}$$

Thus the two cars together have covered a total distance of 488 m, which is more than allowed. $\boxed{\text{The car should not pass.}}$

89. Choose downward to be the positive direction, and $y_0 = 0$ to be at the height of the bridge. Agent Bond has an initial velocity of $v_0 = 0$, an acceleration of $a = g$, and will have a displacement of $y = 13\,\text{m} - 1.5\,\text{m} = 11.5\,\text{m}$. Find the time of fall from Eq. 2-12b with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(11.5\text{ m})}{9.80\,\text{m/s}^2}} = 1.532\text{ s}$$

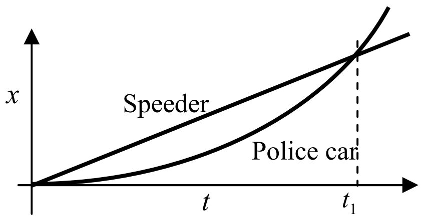
If the truck is approaching with $v = 25\,\text{m/s}$, then he needs to jump when the truck is a distance away given by $d = vt = (25\,\text{m/s})(1.532\,\text{s}) = 38.3\,\text{m}$. Convert this distance into "poles."

$$d = (38.3\text{ m})(1\text{ pole}/25\text{ m}) = 1.53\text{ poles}$$

So he should jump when the truck is about $\boxed{1.5\text{ poles}}$ away from the bridge.

90. Take the origin to be the location where the speeder passes the police car. The speeder's constant speed is $v_{\text{speeder}} = (130\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\text{ km/h}}\right) = 36.1\,\text{m/s}$, and the location of the speeder as a function of time is given by $x_{\text{speeder}} = v_{\text{speeder}} t_{\text{speeder}} = (36.1\,\text{m/s})t_{\text{speeder}}$. The police car has an initial velocity of $v_0 = 0\,\text{m/s}$ and a constant acceleration of a_{police}. The location of the police car as a function of time is given by Eq. 2-12b: $x_{\text{police}} = v_0 t + \tfrac{1}{2}at^2 = \tfrac{1}{2}a_{\text{police}}t_{\text{police}}^2$.

(*a*) The position vs. time graphs would qualitatively look like the graph shown here.



(*b*) The time to overtake the speeder occurs when the speeder has gone a distance of 750 m. The time is found using the speeder's equation from above.

$$750\text{ m} = \left(36.1\,\text{m/s}\right)t_{\text{speeder}} \quad \rightarrow \quad t_{\text{speeder}} = \frac{750\text{ m}}{36.1\,\text{m/s}} = 20.8\,\text{s} \approx \boxed{21\,\text{s}}$$

(*c*) The police car's acceleration can be calculated knowing that the police car also had gone a distance of 750 m in a time of 22.5 s.

$$750\text{ m} = \tfrac{1}{2}a_p\left(20.8\text{ s}\right)^2 \quad \rightarrow \quad a_p = \frac{2\left(750\,\text{m}\right)}{\left(20.8\text{ s}\right)^2} = 3.47\,\text{m/s}^2 \approx \boxed{3.5\,\text{m/s}^2}$$

(*d*) The speed of the police car at the overtaking point can be found from Eq. 2-12a.

$$v = v_0 + at = 0 + \left(3.47\,\text{m/s}^2\right)\left(20.8\text{ s}\right) = 72.2\,\text{m/s} \approx \boxed{72\,\text{m/s}}$$

Note that this is exactly twice the speed of the speeder.

$\boxed{91.}$ The speed of the conveyor belt is given by $d = \bar{v}\Delta t \rightarrow \bar{v} = \dfrac{d}{\Delta t} = \dfrac{1.1\text{ m}}{2.5\text{ min}} = \boxed{0.44\,\text{m/min}}$. The rate of burger production, assuming the spacing given is center to center, can be found as follows.

$$\left(\frac{1\text{ burger}}{0.15\text{ m}}\right)\left(\frac{0.44\text{ m}}{1\text{ min}}\right) = \boxed{2.9\,\frac{\text{burgers}}{\text{min}}}$$

92. Choose downward to be the positive direction, and the origin to be at the top of the building. The barometer has $y_0 = 0,\ v_0 = 0,$ and $a = g = 9.8\,\text{m/s}^2$. Use Eq. 2-12b to find the height of the building, with *x* replaced by *y*.

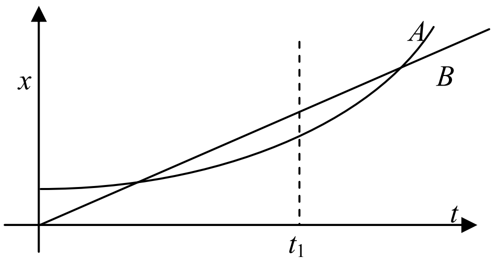
$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)t^2$$

$$y_{t=2.0} = \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(2.0\text{ s}\right)^2 = 20\text{ m} \qquad y_{t=2.3} = \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(2.3\text{ s}\right)^2 = 26\text{ m}$$

The difference in the estimates is 6 m. If we assume the height of the building is the average of the two measurements, then the % difference in the two values is $\dfrac{6\,\text{m}}{23\,\text{m}}\times 100 = \boxed{26\%}$.

93. (*a*) The two bicycles will have the same velocity at any time when the instantaneous slopes of their *x* vs. *t* graphs are the same. That occurs near the time t_1 as marked on the graph.

(*b*) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.



(*c*) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, and so is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.

(*d*) Bicycle B has the highest instantaneous velocity at all times until the time t_1, where both graphs have the same slope. For all times after t_1, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.

(*e*) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.

94. In this problem, note that $a < 0$ and $x > 0$. Take your starting position as 0. Then your position is given by Eq. 2-12b, $x_1 = v_M t + \frac{1}{2}at^2$, and the other car's position is given by $x_2 = x + v_A t$. Set the two positions equal to each other and solve for the time of collision. If this time is negative or imaginary, then there will be no collision.

$$x_1 = x_2 \quad \rightarrow \quad v_M t + \tfrac{1}{2}at^2 = x + v_A t \quad \rightarrow \quad \tfrac{1}{2}at^2 + (v_M - v_A)t - x = 0$$

$$t = \frac{(v_A - v_M) \pm \sqrt{(v_M - v_A)^2 - 4\frac{1}{2}a(-x)}}{2\frac{1}{2}a}$$

$$\text{No collision: } (v_M - v_A)^2 - 4\tfrac{1}{2}a(-x) < 0 \quad \rightarrow \quad \boxed{x > \frac{(v_M - v_A)^2}{-2a}}$$

95. The velocities were changed from km/h to m/s by multiplying the conversion factor that 1 km/hr = 1/3.6 m/s.

(*a*) The average acceleration for each interval is calculated by $a = \Delta v/\Delta t$, and taken to be the acceleration at the midpoint of the time interval. In the spreadsheet, $a_{n+\frac{1}{2}} = \dfrac{v_{n+1} - v_n}{t_{n+1} - t_n}$. The accelerations are shown in the table below.

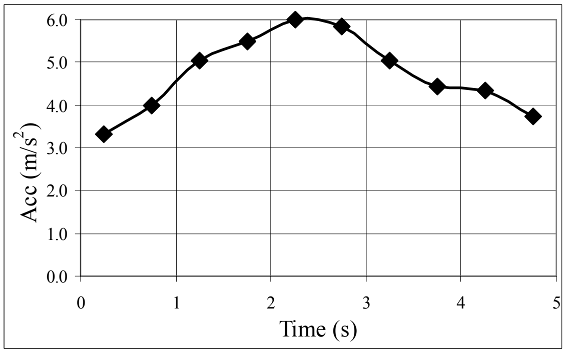
(*b*) The position at the end of each interval is calculated by $x_{n+1} = x_n + \frac{1}{2}(v_n + v_{n+1})(t_{n+1} - t_n)$. This can also be represented as $x = x_0 + \bar{v}\Delta t$. These are shown in the table below.

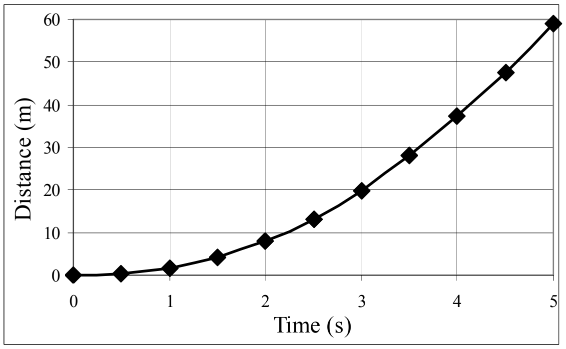
t (s)	v (km/h)	v (m/s)
0.0	0.0	0.0
0.5	6.0	1.7
1.0	13.2	3.7
1.5	22.3	6.2
2.0	32.2	8.9
2.5	43.0	11.9
3.0	53.5	14.9
3.5	62.6	17.4
4.0	70.6	19.6
4.5	78.4	21.8
5.0	85.1	23.6

t (s)	a (m/s^2)
0.25	3.33
0.75	4.00
1.25	5.06
1.75	5.50
2.25	6.00
2.75	5.83
3.25	5.06
3.75	4.44
4.25	4.33
4.75	3.72

t (s)	x (m)
0.0	0.00
0.5	0.42
1.0	1.75
1.5	4.22
2.0	8.00
2.5	13.22
3.0	19.92
3.5	27.99
4.0	37.24
4.5	47.58
5.0	58.94

(*c*) The graphs are shown below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.95c."





96. For this problem, a spreadsheet was designed. The columns of the spreadsheet are time, acceleration, velocity, and displacement. The time starts at 0 and with each interval is incremented by 1.00 s. The acceleration at each time is from the data given in the problem. The velocity at each time is found by multiplying the average of the accelerations at the current time and the previous time, by the time interval, and then adding that to the previous velocity. Thus $v_{n+1} = v_n + \frac{1}{2}\left(a_n + a_{n+1}\right)\left(t_{n+1} - t_n\right)$. The displacement from the starting position at each time interval is calculated by a constant acceleration model, where the acceleration is as given above. Thus the positions is calculated as follows.

$$x_{n+1} = x_n + v_n\left(t_{n+1} - t_n\right) + \tfrac{1}{2}\left[\tfrac{1}{2}\left(a_n + a_{n+1}\right)\right]\left(t_{n+1} - t_n\right)^2$$

The table of values is reproduced here.

(*a*) $v(17.00) = \boxed{30.3\,\mathrm{m/s}}$

(*b*) $x(17.00) = \boxed{305\,\mathrm{m}}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.96."

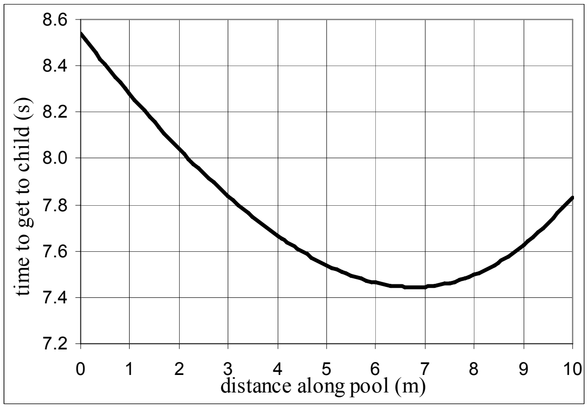
t (s)	a (m/s^2)	v (m/s)	x (m)
0.0	1.25	0.0	0
1.0	1.58	1.4	1
2.0	1.96	3.2	3
3.0	2.40	5.4	7
4.0	2.66	7.9	14
5.0	2.70	10.6	23
6.0	2.74	13.3	35
7.0	2.72	16.0	50
8.0	2.60	18.7	67
9.0	2.30	21.1	87
10.0	2.04	23.3	109
11.0	1.76	25.2	133
12.0	1.41	26.8	159
13.0	1.09	28.0	187
14.0	0.86	29.0	215
15.0	0.51	29.7	245
16.0	0.28	30.1	275
17.0	0.10	30.3	305

$\boxed{97.}$ (*a*) For each segment of the path, the time is given by the distance divided by the speed.

$$t = t_{\text{land}} + t_{\text{pool}} = \frac{d_{\text{land}}}{v_{\text{land}}} + \frac{d_{\text{pool}}}{v_{\text{pool}}}$$

$$= \frac{x}{v_R} + \frac{\sqrt{D^2 + (d - x)^2}}{v_S}$$

(*b*) The graph is shown here. The minimum time occurs at a distance along the pool of about $\boxed{x = 6.8\,\mathrm{m}}$.



An analytic differentiation to solve for the minimum point gives $x = 6.76$ m.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.97b."

CHAPTER 3: Kinematics in Two or Three Dimensions; Vectors

Responses to Questions

1. No. Velocity is a vector quantity, with a magnitude and direction. If two vectors have different directions, they cannot be equal.

2. No. The car may be traveling at a constant *speed* of 60 km/h and going around a curve, in which case it would be accelerating.

3. Automobile races that begin and end at the same place; a round-trip by car from New York to San Francisco and back; a balloon flight around the world.

4. The length of the displacement vector is the straight-line distance between the beginning point and the ending point of the trip and therefore the shortest distance between the two points. If the path is a straight line, then the length of the displacement vector is the same as the length of the path. If the path is curved or consists of different straight line segments, then the distance from beginning to end will be less than the path length. Therefore, the displacement vector can never be longer than the length of the path traveled, but it can be shorter.

5. The player and the ball have the same displacement.

6. V is the magnitude of the vector $\vec{\mathbf{V}}$; it is not necessarily larger than the magnitudes V_1 and V_2. For instance, if $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ have the same magnitude as each other and are in opposite directions, then V is zero.

7. The maximum magnitude of the sum is 7.5 km, in the case where the vectors are parallel. The minimum magnitude of the sum is 0.5 km, in the case where the vectors are antiparallel.

8. No. The only way that two vectors can add up to give the zero vector is if they have the same magnitude and point in exactly opposite directions. However, three vectors of unequal magnitudes can add up to the zero vector. As a one-dimensional example, a vector 10 units long in the positive x direction added to two vectors of 4 and 6 units each in the negative x direction will result in the zero vector. In two dimensions, consider any three vectors that when added form a triangle.

9. (*a*) Yes. In three dimensions, the magnitude of a vector is the square root of the sum of the squares of the components. If two of the components are zero, the magnitude of the vector is equal to the magnitude of the remaining component.
   (*b*) No.

10. Yes. A particle traveling around a curve while maintaining a constant speed is accelerating because its direction is changing. A particle with a constant velocity cannot be accelerating, since the velocity is not changing in magnitude or direction.

11. The odometer and the speedometer of the car both measure scalar quantities (distance and speed, respectively).

12. Launch the rock with a horizontal velocity from a known height over level ground. Use the equations for projectile motion in the y-direction to find the time the rock is in the air. (Note that the initial velocity has a zero y-component.) Use this time and the horizontal distance the rock travels in the

equation for *x*-direction projectile motion to find the speed in the *x*-direction, which is the speed the slingshot imparts. The meter stick is used to measure the initial height and the horizontal distance the rock travels.

$\boxed{13.}$ No. The arrow will fall toward the ground as it travels toward the target, so it should be aimed above the target. Generally, the farther you are from the target, the higher above the target the arrow should be aimed, up to a maximum launch angle of 45º. (The maximum range of a projectile that starts and stops at the same height occurs when the launch angle is 45º.)

14. As long as air resistance is negligible, the horizontal component of the projectile's velocity remains constant until it hits the ground. It is in the air longer than 2.0 s, so the value of the horizontal component of its velocity at 1.0 s and 2.0 s is the same.

15. A projectile has the least speed at the top of its path. At that point the vertical speed is zero. The horizontal speed remains constant throughout the flight, if we neglect the effects of air resistance.

16. If the bullet was fired from the ground, then the *y*-component of its velocity slowed considerably by the time it reached an altitude of 2.0 km, because of both acceleration due to gravity (downward) and air resistance. The *x*-component of its velocity would have slowed due to air resistance as well. Therefore, the bullet could have been traveling slowly enough to be caught!

17. (*a*) Cannonball A, because it has a larger initial vertical velocity component.
    (*b*) Cannonball A, same reason.
    (*c*) It depends. If $\theta_A < 45^\circ$, cannonball A will travel farther. If $\theta_B > 45^\circ$, cannonball B will travel farther. If $\theta_A > 45^\circ$ and $\theta_B < 45^\circ$, the cannonball whose angle is closest to 45º will travel farther.

18. (*a*) The ball lands back in her hand.
    (*b*) The ball lands behind her hand.
    (*c*) The ball lands in front of her hand.
    (*d*) The ball lands beside her hand, to the outside of the curve.
    (*e*) The ball lands behind her hand, if air resistance is not negligible.

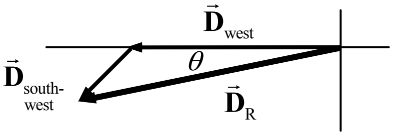
$\boxed{19.}$ This is a question of relative velocity. From the point of view of an observer on the ground, both trains are moving in the same direction (forward), but at different speeds. From your point of view on the faster train, the slower train (and the ground) will appear to be moving backward. (The ground will be moving backward faster than the slower train!)

20. The time it takes to cross the river depends on the component of velocity in the direction straight across the river. Imagine a river running to the east and rowers beginning on the south bank. Let the still water speed of both rowers be v. Then the rower who heads due north (straight across the river) has a northward velocity component v. The rower who heads upstream, though, has a northward velocity component of less than v. Therefore, the rower heading straight across reaches the opposite shore first. (However, she won't end up straight across from where she started!)

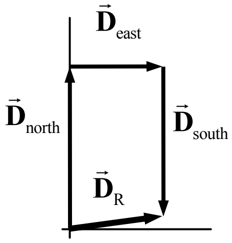
21. As you run forward, the umbrella also moves forward and stops raindrops that are at its height above the ground. Raindrops that have already passed the height of the umbrella continue to move toward the ground unimpeded. As you run, you move into the space where the raindrops are continuing to fall (below the umbrella). Some of them will hit your legs and you will get wet.

Solutions to Problems

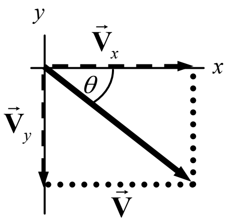
1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_{\text{R}} = \vec{\mathbf{D}}_{\text{west}} + \vec{\mathbf{D}}_{\substack{\text{south-}\\\text{west}}}$. The westward displacement is $225 + 78\cos 45° = 280.2\text{ km}$ and the south displacement is $78\sin 45° = 55.2\text{ km}$. The resultant displacement has a magnitude of $\sqrt{280.2^2 + 55.2^2} = \boxed{286\text{ km}}$. The direction is $\theta = \tan^{-1} 55.2/280.2 = \boxed{11°\text{ south of west}}$.



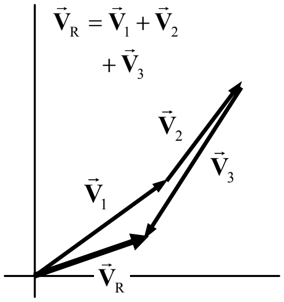
2. The truck has a displacement of $28 + (-26) = 2$ blocks north and 16 blocks east. The resultant has a magnitude of $\sqrt{2^2 + 16^2} = 16.1\text{ blocks} \approx \boxed{16\text{ blocks}}$ and a direction of $\tan^{-1} 2/16 = \boxed{7°\text{ north of east}}$.



3. Given that $V_x = 7.80$ units and $V_y = -6.40$ units, the magnitude of $\vec{\mathbf{V}}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{7.80^2 + (-6.40)^2} = \boxed{10.1\text{ units}}$. The direction is given by $\theta = \tan^{-1}\dfrac{-6.40}{7.80} = \boxed{-39.4°}$, 39.4° below the positive *x*-axis.



4. The vectors for the problem are drawn approximately to scale. The resultant has a length of $\boxed{17.5\text{ m}}$ and a direction $\boxed{19°}$ north of east. If calculations are done, the actual resultant should be 17 m at 23° north of east.

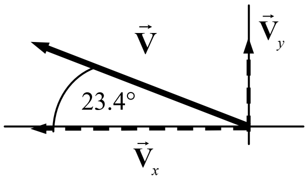


5. (*a*) See the accompanying diagram

(*b*) $V_x = -24.8\cos 23.4° = \boxed{-22.8\text{ units}}$ $V_y = 24.8\sin 23.4° = \boxed{9.85\text{ units}}$

(*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-22.8)^2 + (9.85)^2} = \boxed{24.8\text{ units}}$

$$\theta = \tan^{-1}\frac{9.85}{22.8} = \boxed{23.4°\text{above the } -x \text{ axis}}$$



6. We see from the diagram that $\vec{\mathbf{A}} = 6.8\hat{\mathbf{i}}$ and $\vec{\mathbf{B}} = -5.5\hat{\mathbf{i}}$.

(*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} + (-5.5)\hat{\mathbf{i}} = \boxed{1.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{1.3\text{ units}}$, and the direction is $\boxed{+x}$.

(*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} - (-5.5)\hat{\mathbf{i}} = \boxed{12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3\text{ units}}$, and the direction is $\boxed{+x}$.

(*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = (-5.5)\hat{\mathbf{i}} - 6.8\hat{\mathbf{i}} = \boxed{-12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3\text{ units}}$, and the direction is $\boxed{-x}$.

$\boxed{7.}$ (*a*) $v_{\text{north}} = (835\,\text{km/h})(\cos 41.5^\circ) = \boxed{625\,\text{km/h}}$ $\quad v_{\text{west}} = (835\,\text{km/h})(\sin 41.5^\circ) = \boxed{553\,\text{km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (625\ \text{km/h})(2.50\ \text{h}) = \boxed{1560\ \text{km}}$

$\Delta d_{\text{west}} = v_{\text{west}} t = (553\ \text{km/h})(2.50\ \text{h}) = \boxed{1380\ \text{km}}$

8. (*a*) $\vec{\mathbf{V}}_1 = -6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}} \quad V_1 = \sqrt{6.0^2 + 8.0^2} = \boxed{10.0} \quad \theta = \tan^{-1}\dfrac{8.0}{-6.0} = \boxed{127^\circ}$

(*b*) $\vec{\mathbf{V}}_2 = 4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}} \quad V_2 = \sqrt{4.5^2 + 5.0^2} = \boxed{6.7} \quad \theta = \tan^{-1}\dfrac{-5.0}{4.5} = \boxed{312^\circ}$

(*c*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \left(-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right) + \left(4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}\right) = -1.5\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{1.5^2 + 3.0^2} = \boxed{3.4} \quad \theta = \tan^{-1}\dfrac{3.0}{-1.5} = \boxed{117^\circ}$

(*d*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \left(4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}\right) - \left(-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right) = 10.5\hat{\mathbf{i}} - 13.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1\right| = \sqrt{10.5^2 + 13.0^2} = \boxed{16.7} \quad \theta = \tan^{-1}\dfrac{-13.0}{10.5} = \boxed{309^\circ}$

9. (*a*) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3 = \left(4.0\hat{\mathbf{i}} - 8.0\hat{\mathbf{j}}\right) + \left(1.0\hat{\mathbf{i}} + 1.0\hat{\mathbf{j}}\right) + \left(-2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right) = 3.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3\right| = \sqrt{3.0^2 + 3.0^2} = \boxed{4.2} \quad \theta = \tan^{-1}\dfrac{-3.0}{3.0} = \boxed{315^\circ}$

(*b*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3 = \left(4.0\hat{\mathbf{i}} - 8.0\hat{\mathbf{j}}\right) - \left(1.0\hat{\mathbf{i}} + 1.0\hat{\mathbf{j}}\right) + \left(-2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right) = \boxed{1.0\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}}$

$\left|\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_3\right| = \sqrt{1.0^2 + 5.0^2} = \boxed{5.1} \quad \theta = \tan^{-1}\dfrac{-5.0}{1.0} = \boxed{280^\circ}$

10. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82 \qquad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0 \qquad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}\right)_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$

$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}\right)_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$

(*b*) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}\right| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7} \qquad \theta = \tan^{-1}\dfrac{11.63}{24.03} = \boxed{25.8^\circ}$

11. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82 \qquad B_y = 26.5\sin 56.0^\circ = 21.97$

(*a*) $\left(\vec{\mathbf{B}} - \vec{\mathbf{A}}\right)_x = (-14.82) - 38.85 = -53.67 \qquad \left(\vec{\mathbf{B}} - \vec{\mathbf{A}}\right)_y = 21.97 - 20.66 = 1.31$

Note that since the *x* component is negative and the *y* component is positive, the vector is in the 2nd quadrant.

$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \boxed{-53.7\hat{\mathbf{i}} + 1.31\hat{\mathbf{j}}}$

$$\left|\vec{\mathbf{B}}-\vec{\mathbf{A}}\right| = \sqrt{(-53.67)^2+(1.31)^2} = \boxed{53.7} \qquad \theta_{B-A} = \tan^{-1}\frac{1.31}{-53.67} = \boxed{1.4^\circ \text{ above } -x \text{ axis}}$$

(*b*) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_x = 38.85-(-14.82) = 53.67 \qquad \left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_y = 20.66-21.97 = -1.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\vec{\mathbf{A}}-\vec{\mathbf{B}} = \boxed{53.7\hat{\mathbf{i}}-1.31\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(53.67)^2+(-1.31)^2} = \boxed{53.7} \qquad \theta = \tan^{-1}\frac{-1.31}{53.7} = \boxed{1.4^\circ \text{ below } + x \text{ axis}}$$

Comparing the results shows that $\vec{\mathbf{B}}-\vec{\mathbf{A}} = -\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)$.

12. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$C_x = 31.0\cos 270^\circ = 0.0 \qquad C_y = 31.0\sin 270^\circ = -31.0$

$\left(\vec{\mathbf{A}}-\vec{\mathbf{C}}\right)_x = 38.85-0.0 = 38.85 \qquad \left(\vec{\mathbf{A}}-\vec{\mathbf{C}}\right)_y = 20.66-(-31.0) = 51.66$

$$\vec{\mathbf{A}}-\vec{\mathbf{C}} = \boxed{38.8\hat{\mathbf{i}}+51.7\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{C}}\right| = \sqrt{(38.85)^2+(51.66)^2} = \boxed{64.6} \qquad \theta = \tan^{-1}\frac{51.66}{38.85} = \boxed{53.1^\circ}$$

13. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82 \qquad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0 \qquad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_x = -14.82-2(38.85) = -92.52 \qquad \left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_y = 21.97-2(20.66) = -19.35$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\vec{\mathbf{B}}-2\vec{\mathbf{A}} = \boxed{-92.5\hat{\mathbf{i}}-19.4\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right| = \sqrt{(-92.52)^2+(-19.35)^2} = \boxed{94.5} \qquad \theta = \tan^{-1}\frac{-19.35}{-92.52} = \boxed{11.8^\circ \text{below} - x \text{ axis}}$$

(*b*) $\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_x = 2(38.85)-3(-14.82)+2(0.0) = 122.16$

$\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_y = 2(20.66)-3(21.97)+2(-31.0) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}} = \boxed{122\hat{\mathbf{i}}-86.6\hat{\mathbf{j}}}$$

$$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2+(-86.59)^2} = \boxed{150} \qquad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3^\circ \text{below} + x \text{ axis}}$$

14. $A_x = 44.0\cos 28.0^\circ = 38.85 \qquad A_y = 44.0\sin 28.0^\circ = 20.66$

$B_x = -26.5\cos 56.0^\circ = -14.82 \qquad B_y = 26.5\sin 56.0^\circ = 21.97$

$C_x = 31.0\cos 270^\circ = 0.0 \qquad C_y = 31.0\sin 270^\circ = -31.0$

(*a*) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_x = 38.85-(-14.82)+0.0 = 53.67$

$$\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_y = 20.66-21.97+(-31.0) = -32.31$$

Note that since the x component is positive and the y component is negative, the vector is in the 4th quadrant.

$$\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}} = \boxed{53.7\hat{\mathbf{i}}-32.3\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{(53.67)^2+(-32.31)^2} = \boxed{62.6} \qquad \theta = \tan^{-1}\frac{-32.31}{53.67} = \boxed{31.0^\circ \text{ below } +x \text{ axis}}$$

(*b*)

$$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_x = 38.85+(-14.82)-0.0 = 24.03$$

$$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right)_y = 20.66+21.97-(-31.0) = 73.63$$

$$\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}} = \boxed{24.0\hat{\mathbf{i}}+73.6\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right| = \sqrt{(24.03)^2+(73.63)^2} = \boxed{77.5} \qquad \theta = \tan^{-1}\frac{73.63}{24.03} = \boxed{71.9^\circ}$$

(*c*)

$$\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_x = 0.0-38.85-(-14.82) = -24.03$$

$$\left(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)_y = -31.0-20.66-21.97 = -73.63$$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}} = \boxed{-24.0\hat{\mathbf{i}}-73.6\hat{\mathbf{j}}}$$

$$\left|\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(-24.03)^2+(-73.63)^2} = \boxed{77.5} \qquad \theta = \tan^{-1}\frac{-73.63}{-24.03} = \boxed{71.9^\circ \text{ below } -x \text{ axis}}$$

Note that the answer to (*c*) is the exact opposite of the answer to (*b*).

15. The x component is negative and the y component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive x axis would be 122.4°. Thus the components are found to be as follows.

$$x = 4580\cos 122.4^\circ = -2454\text{ m} \qquad y = 4580\sin 122.4^\circ = 3867\text{ m} \qquad z = 2450\text{ m}$$

$$\boxed{\vec{\mathbf{r}} = -2450\text{ m}\,\hat{\mathbf{i}}+3870\text{ m}\,\hat{\mathbf{j}}+2450\text{ m}\,\hat{\mathbf{k}}} \qquad \left|\vec{\mathbf{r}}\right| = \sqrt{(-2454)^2+(4580)^2+(2450)^2} = \boxed{5190\text{ m}}$$

16. (*a*) Use the Pythagorean theorem to find the possible x components.

$$90.0^2 = x^2+(-55.0)^2 \quad\rightarrow\quad x^2 = 5075 \quad\rightarrow\quad x = \boxed{\pm 71.2 \text{ units}}$$

(*b*) Express each vector in component form, with $\vec{\mathbf{V}}$ the vector to be determined.

$$\left(71.2\hat{\mathbf{i}}-55.0\hat{\mathbf{j}}\right)+\left(V_x\hat{\mathbf{i}}+V_y\hat{\mathbf{j}}\right) = -80.0\hat{\mathbf{i}}+0.0\hat{\mathbf{j}} \quad\rightarrow$$

$$V_x = (-80.0-71.2) = -151.2 \qquad V_y = 55.0$$

$$\vec{\mathbf{V}} = \boxed{-151.2\hat{\mathbf{i}}+55.0\hat{\mathbf{j}}}$$

17. Differentiate the position vector in order to determine the velocity, and differentiate the velocity in order to determine the acceleration.

$$\vec{\mathbf{r}} = \left(9.60t\,\hat{\mathbf{i}}+8.85\,\hat{\mathbf{j}}-1.00t^2\hat{\mathbf{k}}\right)\text{m} \quad\rightarrow\quad \vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(9.60\,\hat{\mathbf{i}}-2.00t\,\hat{\mathbf{k}}\right)\text{m/s}} \quad\rightarrow$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{-2.00\,\hat{\mathbf{k}}\text{ m/s}^2}$$

18. The average velocity is found from the displacement at the two times.

$$\begin{aligned}\vec{\mathbf{v}}_{\text{avg}} &= \frac{\vec{\mathbf{r}}(t_2)-\vec{\mathbf{r}}(t_1)}{t_2-t_1}\\ &= \frac{\left[\left(9.60(3.00)\hat{\mathbf{i}}+8.85\hat{\mathbf{j}}-(3.00)^2\hat{\mathbf{k}}\right)\text{m}\right]-\left[\left(9.60(1.00)\hat{\mathbf{i}}+8.85\hat{\mathbf{j}}-(1.00)^2\hat{\mathbf{k}}\right)\text{m}\right]}{2.00\,\text{s}}\\ &= \boxed{\left(9.60\,\hat{\mathbf{i}}-4.00\,\hat{\mathbf{k}}\right)\text{m/s}}\end{aligned}$$

The magnitude of the instantaneous velocity is found from the velocity vector.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(9.60\,\hat{\mathbf{i}}-2.00t\,\hat{\mathbf{k}}\right)\text{m/s}}$$

$$\vec{\mathbf{v}}(2.00) = \left(9.60\,\hat{\mathbf{i}}-(2.00)(2.00)\,\hat{\mathbf{k}}\right)\text{m/s} = \left(9.60\,\hat{\mathbf{i}}-4.00\,\hat{\mathbf{k}}\right)\text{m/s} \quad\rightarrow$$

$$v = \sqrt{(9.60)^2+(4.00)^2}\ \text{m/s} = \boxed{10.4\,\text{m/s}}$$

Note that, since the acceleration of this object is constant, the average velocity over the time interval is equal to the instantaneous velocity at the midpoint of the time interval.

$\boxed{19.}$ From the original position vector, we have $x = 9.60t,\ y = 8.85,\ z = -1.00t^2$. Thus

$z = -\left(\dfrac{x}{9.60}\right)^2 = -ax^2,\ y = 8.85$. This is the equation for a $\boxed{\text{parabola}}$ in the x-z plane that has its vertex at coordinate (0,8.85,0) and opens downward.

20. (*a*) Average velocity is displacement divided by elapsed time. Since the displacement is not known, the $\boxed{\text{average velocity cannot be determined}}$. A special case exists in the case of constant acceleration, where the average velocity is the numeric average of the initial and final velocities. But this is not specified as motion with constant acceleration, and so that special case cannot be assumed.

(*b*) Define east as the positive x-direction, and north as the positive y-direction. The average acceleration is the change in velocity divided by the elapsed time.

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{27.5\hat{\mathbf{i}}\,\text{m/s}-\left(-18.0\hat{\mathbf{j}}\,\text{m/s}\right)}{8.00\,\text{s}} = 3.44\hat{\mathbf{i}}\,\text{m/s}^2+2.25\hat{\mathbf{j}}\,\text{m/s}^2$$

$$\left|\vec{\mathbf{a}}_{\text{avg}}\right| = \sqrt{\left(3.44\,\text{m/s}^2\right)^2+\left(2.25\,\text{m/s}^2\right)^2} = \boxed{4.11\,\text{m/s}^2} \qquad \theta = \tan^{-1}\frac{2.25}{3.44} = \boxed{33.2^\circ}$$

(*c*) Average speed is distance traveled divided by elapsed time. Since the distance traveled is not known, the $\boxed{\text{average speed cannot be determined}}$.

21. Note that the acceleration vector is constant, and so Eqs. 3-13a and 3-13b are applicable. Also $\vec{\mathbf{v}}_0 = 0$ and $\vec{\mathbf{r}}_0 = 0$.

(*a*) $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = \left(4.0t\,\hat{\mathbf{i}}+3.0t\,\hat{\mathbf{j}}\right)\text{m/s} \quad\rightarrow\quad \boxed{v_x = 4.0t\,\text{m/s}\ ,\ v_y = 3.0t\,\text{m/s}}$

(*b*) $v = \sqrt{v_x^2+v_y^2} = \sqrt{(4.0t\,\text{m/s})^2+(3.0t\,\text{m/s})^2} = \boxed{5.0t\,\text{m/s}}$

(*c*) $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 = \boxed{\left(2.0t^2\,\hat{\mathbf{i}}+1.5t^2\,\hat{\mathbf{j}}\right)\text{m}}$

(*d*) $\boxed{v_x(2.0) = 8.0\,\text{m/s}\ ,\ v_y(2.0) = 6.0\,\text{m/s}\ ,\ v(2.0) = 10.0\,\text{m/s}\ ,\ \vec{\mathbf{r}}(2.0) = \left(8.0\,\hat{\mathbf{i}}+6.0\,\hat{\mathbf{j}}\right)\text{m}}$

22. Choose downward to be the positive *y* direction for this problem. Her acceleration is directed along the slope.

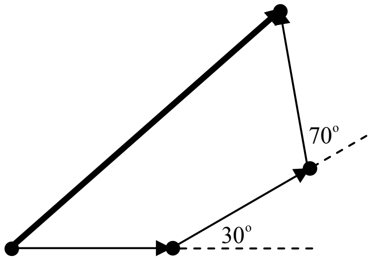
(*a*) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_y = a\sin\theta = \left(1.80\,\mathrm{m/s^2}\right)\sin 30.0^\circ = \boxed{0.900\,\mathrm{m/s^2}}$.

(*b*) The time to reach the bottom of the hill is calculated from Eq. 2-12b, with a *y* displacement of 325 m, $v_{y0} = 0$, and $a_y = 0.900\,\mathrm{m/s^2}$.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 325\ \mathrm{m} = 0 + 0 + \tfrac{1}{2}\left(0.900\,\mathrm{m/s^2}\right)(t)^2 \;\rightarrow$$

$$t = \sqrt{\frac{2(325\ \mathrm{m})}{\left(0.900\,\mathrm{m/s^2}\right)}} = \boxed{26.9\ \mathrm{s}}$$

23. The three displacements for the ant are shown in the diagram, along with the net displacement. In *x* and *y* components, they are $+10.0\,\mathrm{cm}\,\hat{\mathbf{i}}$, $\left(10.0\cos 30.0^\circ\,\hat{\mathbf{i}} + 10.0\sin 30.0^\circ\,\hat{\mathbf{j}}\right)\mathrm{cm}$, , and $\left(10.0\cos 100^\circ\,\hat{\mathbf{i}} + 10.0\sin 100^\circ\,\hat{\mathbf{j}}\right)\mathrm{cm}$. To find the average velocity, divide the net displacement by the elapsed time.



(*a*) $$\Delta\vec{\mathbf{r}} = +10.0\,\mathrm{cm}\,\hat{\mathbf{i}} + \left(10.0\cos 30.0^\circ\hat{\mathbf{i}} + 10.0\sin 30.0^\circ\hat{\mathbf{j}}\right)\mathrm{cm}$$
$$+\left(10.0\cos 100^\circ\,\hat{\mathbf{i}} + 10.0\sin 100^\circ\,\hat{\mathbf{j}}\right)\mathrm{cm} = \left(16.92\,\hat{\mathbf{i}} + 14.85\,\hat{\mathbf{j}}\right)\mathrm{cm}$$

$$\vec{\mathbf{v}}_{\mathrm{avg}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\left(16.92\,\hat{\mathbf{i}} + 14.85\,\hat{\mathbf{j}}\right)\mathrm{cm}}{2.00\,\mathrm{s} + 1.80\,\mathrm{s} + 1.55\,\mathrm{s}} = \boxed{\left(3.16\,\hat{\mathbf{i}} + 2.78\,\hat{\mathbf{j}}\right)\mathrm{cm/s}}$$

(*b*) $$\left|\vec{\mathbf{v}}_{\mathrm{avg}}\right| = \sqrt{\left(3.16\,\mathrm{cm/s}\right)^2 + \left(2.78\,\mathrm{cm/s}\right)^2} = \boxed{4.21\,\mathrm{cm/s}} \qquad \theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{2.78}{3.16} = \boxed{41.3^\circ}$$

24. Since the acceleration vector is constant, Eqs. 3-13a and 3-13b are applicable. The particle reaches its maximum *x* coordinate when the *x* velocity is 0. Note that $\vec{\mathbf{v}}_0 = 5.0\,\mathrm{m/s}\,\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_0 = 0$.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = 5.0\ \hat{\mathbf{i}}\,\mathrm{m/s} + \left(-3.0t\,\hat{\mathbf{i}} + 4.5t\,\hat{\mathbf{j}}\right)\mathrm{m/s}$$

$$v_x = (5.0 - 3.0t)\,\mathrm{m/s} \;\rightarrow\; v_x = 0 = \left(5.0 - 3.0t_{x-\max}\right)\mathrm{m/s} \;\rightarrow\; t_{x-\max} = \frac{5.0\,\mathrm{m/s}}{3.0\,\mathrm{m/s^2}} = 1.67\,\mathrm{s}$$

$$\vec{\mathbf{v}}\left(t_{x-\max}\right) = 5.0\ \hat{\mathbf{i}}\,\mathrm{m/s} + \left[-3.0(1.67)\,\hat{\mathbf{i}} + 4.5(1.67)t\,\hat{\mathbf{j}}\right]\mathrm{m/s} = \boxed{7.5\,\mathrm{m/s}\,\hat{\mathbf{j}}}$$

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 = 5.0t\,\hat{\mathbf{i}}\,\mathrm{m} + \tfrac{1}{2}\left(-3.0t^2\,\hat{\mathbf{i}} + 4.5t^2\,\hat{\mathbf{j}}\right)\mathrm{m}$$

$$\vec{\mathbf{r}}\left(t_{x-\max}\right) = 5.0(1.67)\ \hat{\mathbf{i}}\,\mathrm{m/s} + \tfrac{1}{2}\left[-3.0(1.67)^2\,\hat{\mathbf{i}} + 4.5(1.67)^2\,\hat{\mathbf{j}}\right]\mathrm{m} = \boxed{4.2\,\hat{\mathbf{i}}\,\mathrm{m} + 6.3\,\hat{\mathbf{j}}\,\mathrm{m}}$$

$\boxed{25.}$ (*a*) Differentiate the position vector, $\vec{\mathbf{r}} = \left(3.0t^2\,\hat{\mathbf{i}} - 6.0t^3\,\hat{\mathbf{j}}\right)\mathrm{m}$, with respect to time in order to find the velocity and the acceleration.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(6.0t\,\hat{\mathbf{i}} - 18.0t^2\,\hat{\mathbf{j}}\right)\mathrm{m/s}} \qquad \vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{\left(6.0\,\hat{\mathbf{i}} - 36.0t\,\hat{\mathbf{j}}\right)\mathrm{m/s^2}}$$

(*b*) $\vec{\mathbf{r}}(2.5\,\mathrm{s}) = \left[3.0(2.5)^2\,\hat{\mathbf{i}} - 6.0(2.5)^3\,\hat{\mathbf{j}}\right]\mathrm{m} = \boxed{\left(19\,\hat{\mathbf{i}} - 94\hat{\mathbf{j}}\right)\mathrm{m}}$

$$\vec{\mathbf{v}}(2.5\,\mathrm{s}) = \left[6.0(2.5)\hat{\mathbf{i}} - 18.0(2.5)^2\,\hat{\mathbf{j}}\right]\mathrm{m/s} = \boxed{\left(15\,\hat{\mathbf{i}} - 110\hat{\mathbf{j}}\right)\mathrm{m/s}}$$

26. The position vector can be found from Eq. 3-13b, since the acceleration vector is constant. The time at which the object comes to rest is found by setting the velocity vector equal to 0. Both components of the velocity must be 0 at the same time for the object to be at rest.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = \left(-14\,\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}}\right)\mathrm{m/s} + \left(6.0t\,\hat{\mathbf{i}} + 3.0t\,\hat{\mathbf{j}}\right)\mathrm{m/s} = \left[(-14 + 6.0t)\,\hat{\mathbf{i}} + (-7.0 + 3.0t)\,\hat{\mathbf{j}}\right]\mathrm{m/s}$$

$$\vec{\mathbf{v}}_{\text{rest}} = \left(0.0\,\hat{\mathbf{i}} + 0.0\hat{\mathbf{j}}\right)\mathrm{m/s} = \left[(-14 + 6.0t)\,\hat{\mathbf{i}} + (-7.0 + 3.0t)\,\hat{\mathbf{j}}\right]\mathrm{m/s} \quad \rightarrow$$

$$\left(v_x\right)_{\text{rest}} = 0.0 = -14 + 6.0t \quad \rightarrow \quad t = \frac{14}{6.0}\,\mathrm{s} = \tfrac{7}{3}\,\mathrm{s}$$

$$\left(v_y\right)_{\text{rest}} = 0.0 = -7.0 + 3.0t \quad \rightarrow \quad t = \frac{7.0}{3.0}\,\mathrm{s} = \tfrac{7}{3}\,\mathrm{s}$$

Since both components of velocity are 0 at $t = \tfrac{7}{3}\,\mathrm{s}$, the object is at rest at that time.

$$\begin{aligned}
\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 &= \left(0.0\,\hat{\mathbf{i}} + 0.0\hat{\mathbf{j}}\right)\mathrm{m} + \left(-14t\,\hat{\mathbf{i}} - 7.0t\,\hat{\mathbf{j}}\right)\mathrm{m} + \tfrac{1}{2}\left(6.0t^2\hat{\mathbf{i}} + 3.0t^2\hat{\mathbf{j}}\right)\mathrm{m} \\
&= \left(-14\left(\tfrac{7}{3}\right)\hat{\mathbf{i}} - 7.0\left(\tfrac{7}{3}\right)\hat{\mathbf{j}}\right)\mathrm{m} + \tfrac{1}{2}\left(6.0\left(\tfrac{7}{3}\right)^2\hat{\mathbf{i}} + 3.0\left(\tfrac{7}{3}\right)^2\hat{\mathbf{j}}\right)\mathrm{m} \\
&= \left(-14\left(\tfrac{7}{3}\right) + \tfrac{1}{2}6.0\left(\tfrac{7}{3}\right)^2\right)\hat{\mathbf{i}}\,\mathrm{m} + \left(-7.0\left(\tfrac{7}{3}\right) + \tfrac{1}{2}3.0\left(\tfrac{7}{3}\right)^2\right)\hat{\mathbf{j}}\,\mathrm{m} \\
&= \left(-16.3\hat{\mathbf{i}} - 8.16\hat{\mathbf{j}}\right)\mathrm{m} \approx \boxed{\left(-16.3\hat{\mathbf{i}} - 8.2\hat{\mathbf{j}}\right)\mathrm{m}}
\end{aligned}$$

27. Find the position at $t = 5.0$ s, and then subtract the initial point from that new location.

$$\vec{\mathbf{r}}(5.0) = \left[5.0(5.0) + 6.0(5.0)^2\right]\mathrm{m}\,\hat{\mathbf{i}} + \left[7.0 - 3.0(5.0)^3\right]\mathrm{m}\,\hat{\mathbf{j}} = 175\,\mathrm{m}\,\hat{\mathbf{i}} - 368\,\mathrm{m}\,\hat{\mathbf{j}}$$

$$\Delta\vec{\mathbf{r}} = \left(175.0\,\mathrm{m}\,\hat{\mathbf{i}} - 368.0\mathrm{m}\,\hat{\mathbf{j}}\right) - \left(0.0\,\mathrm{m}\,\hat{\mathbf{i}} + 7.0\mathrm{m}\,\hat{\mathbf{j}}\right) = 175\,\mathrm{m}\,\hat{\mathbf{i}} - 375\mathrm{m}\,\hat{\mathbf{j}}$$

$$\left|\Delta\vec{\mathbf{r}}\right| = \sqrt{(175\,\mathrm{m})^2 + (-375\,\mathrm{m})^2} = \boxed{414\,\mathrm{m}} \qquad \theta = \tan^{-1}\frac{-375}{175} = \boxed{-65.0^\circ}$$

28. Choose downward to be the positive y direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.2\,\mathrm{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\mathrm{m/s^2}$, $y_0 = 0$, and the final location $y = 7.5\,\mathrm{m}$. The time for the tiger to reach the ground is found from applying Eq. 2-12b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\mathrm{m} = 0 + 0 + \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(7.5\mathrm{m})}{9.80\,\mathrm{m/s^2}}} = 1.24\ \mathrm{sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = (3.2\,\mathrm{m/s})(1.24\ \mathrm{sec}) = \boxed{4.0\ \mathrm{m}}$$

29. Choose downward to be the positive y direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x0} = 2.3\,\mathrm{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\mathrm{m/s^2}$, $y_0 = 0$, and the time of flight is $t = 3.0$ s. The height of the cliff is found from applying Eq. 2-12b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)\left(3.0\,\mathrm{s}\right)^2 = \boxed{44\,\mathrm{m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t = \left(2.3\,\mathrm{m/s}\right)\left(3.0\,\mathrm{s}\right) = \boxed{6.9\,\mathrm{m}}$$

30. Apply the range formula from Example 3-10: $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$. If the launching speed and angle are held constant, the range is inversely proportional to the value of g. The acceleration due to gravity on the Moon is 1/6th that on Earth.

$$R_{\text{Earth}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Earth}}} \qquad R_{\text{Moon}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Moon}}} \quad \rightarrow \quad R_{\text{Earth}}\, g_{\text{Earth}} = R_{\text{Moon}}\, g_{\text{Moon}}$$

$$R_{\text{Moon}} = R_{\text{Earth}} \frac{g_{\text{Earth}}}{g_{\text{Moon}}} = 6R_{\text{Earth}}$$

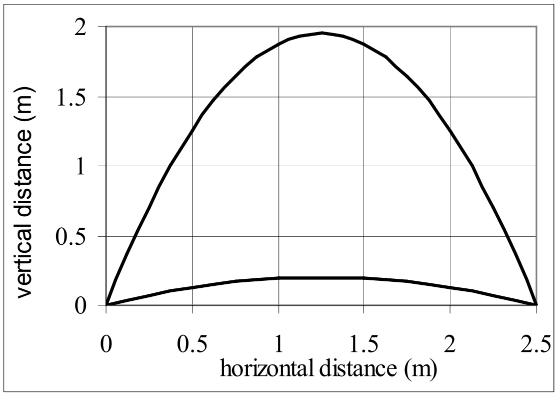
Thus on the Moon, the person can jump $\boxed{\text{6 times farther}}$.

$\boxed{31.}$ Apply the range formula from Example 3-10.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow$$

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{\left(2.5\ \mathrm{m}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\left(6.5\,\mathrm{m/s}\right)^2} = 0.5799$$

$$2\theta_0 = \sin^{-1} 0.5799 \quad \rightarrow \quad \theta_0 = \boxed{18^\circ, 72^\circ}$$



There are two angles because each angle gives the same range. If one angle is $\theta = 45^\circ + \delta$, then $\theta = 45^\circ - \delta$ is also a solution. The two paths are shown in the graph.

32. Choose downward to be the positive y direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\mathrm{m/s^2}$, $y_0 = 0$, and the displacement is 9.0 m. The time of flight is found from applying Eq. 2-12b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 9.0\ \mathrm{m} = \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\left(9.0\ \mathrm{m}\right)}{9.80\,\mathrm{m/s^2}}} = 1.355\ \mathrm{sec}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 9.5\,\mathrm{m}/1.355\,\mathrm{s} = \boxed{7.0\,\mathrm{m/s}}$$

33. Choose the point at which the football is kicked the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = \left(18.0 \sin 38.0^\circ\right)\mathrm{m/s}$, $a_y = -9.80\,\mathrm{m/s^2}$, and the final y velocity will be the opposite of the starting y velocity. Use Eq. 2-12a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{(-18.0\sin 38.0°)\,\mathrm{m/s} - (18.0\sin 38.0°)\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = \boxed{2.26\,\mathrm{s}}$$

34. Choose downward to be the positive y direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\,\mathrm{m/s^2}$. The initial horizontal velocity is 23.7 m/s and the horizontal range is 31.0 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 31.0\,\mathrm{m}/23.7\,\mathrm{m/s} = 1.308\,\mathrm{s}$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2-12b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\mathrm{m/s^2}\right)(1.308\,\mathrm{s})^2 = \boxed{8.38\,\mathrm{m}}$$

35. Choose the origin to be the point of release of the shot put. Choose upward to be the positive y direction. Then $y_0 = 0$, $v_{y0} = (14.4\sin 34.0°)\,\mathrm{m/s} = 8.05\,\mathrm{m/s}$, $a_y = -9.80\,\mathrm{m/s^2}$, and $y = -2.10$ m at the end of the motion. Use Eq. 2-12b to find the time of flight.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}a_y t^2 + v_{y0}t - y = 0 \quad \rightarrow$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(\tfrac{1}{2}a_y\right)(-y)}}{2\tfrac{1}{2}a_y} = \frac{-8.05 \pm \sqrt{(8.05)^2 - 2(-9.80)(2.10)}}{-9.80} = 1.872\,\mathrm{s}, -0.2290\,\mathrm{s}$$

Choose the positive result since the time must be greater than 0. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left[(14.4\cos 34.0°)\,\mathrm{m/s}\right](1.872\,\mathrm{s}) = \boxed{22.3\,\mathrm{m}}$$

36. Choose the origin to be the point of launch, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, $y_0 = 0$, and $v_{y0} = v_0 \sin\theta_0$. Eq. 2-12a is used to find the time required to reach the highest point, at which $v_y = 0$.

$$v_y = v_{y0} + at_{\mathrm{up}} \quad \rightarrow \quad t_{\mathrm{up}} = \frac{v_y - v_{y0}}{a} = \frac{0 - v_0\sin\theta_0}{-g} = \frac{v_0\sin\theta_0}{g}$$

Eq. 2-12c is used to find the height at this highest point.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y_{\max} - y_0\right) \quad \rightarrow \quad y_{\max} = y_0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = 0 + \frac{-v_0^2\sin^2\theta_0}{-2g} = \frac{v_0^2\sin^2\theta_0}{2g}$$

Eq. 2-12b is used to find the time for the object to fall the other part of the path, with a starting y velocity of 0 and a starting height of $y_0 = \dfrac{v_0^2\sin^2\theta_0}{2g}$.

$$y = y_o + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = \frac{v_0^2\sin^2\theta_0}{2g} + 0t_{\mathrm{down}} - \tfrac{1}{2}gt_{\mathrm{down}}^2 \quad \rightarrow \quad t_{\mathrm{down}} = \frac{v_0\sin\theta_0}{g}$$

A comparison shows that $\boxed{t_{\mathrm{up}} = t_{\mathrm{down}}}$.

$\boxed{37.}$ When shooting the gun vertically, half the time of flight is spent moving upwards. Thus the upwards flight takes 2.0 s. Choose upward as the positive y direction. Since at the top of the flight, the vertical velocity is zero, find the launching velocity from Eq. 2-12a.

$$v_y = v_{y0} + at \quad \rightarrow \quad v_{y0} = v_y - at = 0 - \left(-9.80\,\mathrm{m/s^2}\right)(2.0\,\mathrm{s}) = 19.6\,\mathrm{m/s}$$

Using this initial velocity and an angle of 45° in the range formula (from Example 3-10) will give the maximum range for the gun.

$$R=\frac{v_0^2\sin 2\theta_0}{g}=\frac{(19.6\,\mathrm{m/s})^2\sin(90°)}{9.80\,\mathrm{m/s^2}}=\boxed{39\,\mathrm{m}}$$

38. Choose the origin to be the point on the ground directly below the point where the baseball was hit. Choose upward to be the positive y direction. Then $y_0=1.0\,\mathrm{m}$, $y=13.0\,\mathrm{m}$ at the end of the motion, $v_{y0}=(27.0\sin 45.0°)\,\mathrm{m/s}=19.09\,\mathrm{m/s}$, and $a_y=-9.80\,\mathrm{m/s^2}$. Use Eq. 2-12b to find the time of flight.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_yt^2 \quad\rightarrow\quad \tfrac{1}{2}a_yt^2+v_{y0}t+(y_0-y)=0 \quad\rightarrow$$

$$t=\frac{-v_{y0}\pm\sqrt{v_{y0}^2-4\left(\tfrac{1}{2}a_y\right)(y_0-y)}}{2\tfrac{1}{2}a_y}=\frac{-19.09\pm\sqrt{(19.09)^2-2(-9.80)(-12.0)}}{-9.80}$$

$$=0.788\,\mathrm{s},\ 3.108\,\mathrm{s}$$

The smaller time is the time the baseball reached the building's height on the way up, and the larger time is the time the baseball reached the building's height on the way down. We must choose the larger result, because the baseball cannot land on the roof on the way up. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x=v_xt=\left[(27.0\cos 45.0°)\,\mathrm{m/s}\right](3.108\,\mathrm{s})=\boxed{59.3\,\mathrm{m}}$$

39. We choose the origin at the same place. With the new definition of the coordinate axes, we have the following data: $y_0=0$, $y=+1.00\,\mathrm{m}$, $v_{y0}=-12.0\,\mathrm{m/s}$, $v_{x0}=-16.0\,\mathrm{m/s}$, $a=9.80\,\mathrm{m/s^2}$.

$$y=y_0+v_{y0}t+\tfrac{1}{2}gt^2 \quad\rightarrow\quad 1.00\,\mathrm{m}=0-(12.0\,\mathrm{m/s})t+\left(4.90\,\mathrm{m/s^2}\right)t^2 \quad\rightarrow$$

$$\left(4.90\,\mathrm{m/s^2}\right)t^2-(12.0\,\mathrm{m/s})t-(1.00\,\mathrm{m})=0$$

This is the same equation as in Example 3-11, and so we know the appropriate solution is $t=2.53\,\mathrm{s}$. We use that time to calculate the horizontal distance the ball travels.

$$x=v_{x0}t=(-16.0\,\mathrm{m/s})(2.53\,\mathrm{s})=-40.5\,\mathrm{m}$$

Since the x-direction is now positive to the left, the negative value means that the ball lands $\boxed{40.5\text{ m}}$ to the right of where it departed the punter's foot.

40. The horizontal range formula from Example 3-10 can be used to find the launching velocity of the grasshopper.

$$R=\frac{v_0^2\sin 2\theta_0}{g} \quad\rightarrow\quad v_0=\sqrt{\frac{Rg}{\sin 2\theta_0}}=\sqrt{\frac{(1.0\mathrm{m})\left(9.80\,\mathrm{m/s^2}\right)}{\sin 90°}}=3.13\,\mathrm{m/s}$$

Since there is no time between jumps, the horizontal velocity of the grasshopper is the horizontal component of the launching velocity.

$$v_x=v_0\cos\theta_0=(3.13\,\mathrm{m/s})\cos 45°=\boxed{2.2\,\mathrm{m/s}}$$

41. (*a*) Take the ground to be the $y=0$ level, with upward as the positive direction. Use Eq. 2-12b to solve for the time, with an initial vertical velocity of 0.

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_yt^2 \quad\rightarrow\quad 150\,\mathrm{m}=910\,\mathrm{m}+\tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)t^2 \quad\rightarrow$$

$$t=\sqrt{\frac{2\left(150-910\right)}{\left(-9.80\,\text{m/s}^2\right)}}=12.45\,\text{s}\approx\boxed{12\,\text{s}}$$

(*b*) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$\Delta x=v_x t=\left(5.0\,\text{m/s}\right)\left(12.45\,\text{s}\right)=62.25\,\text{m}\approx\boxed{62\,\text{m}}$$

42. Consider the downward vertical component of the motion, which will occur in half the total time. Take the starting position to be $y = 0$, and the positive direction to be downward. Use Eq. 2-12b with an initial vertical velocity of 0.

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_y t^2 \;\rightarrow\; h=0+0+\tfrac{1}{2}gt_{\text{down}}^2=\tfrac{1}{2}g\left(\frac{t}{2}\right)^2=\frac{9.80}{8}t^2=1.225t^2\approx\boxed{1.2t^2}$$

$\boxed{43.}$ Choose downward to be the positive y direction. The origin is the point where the supplies are dropped. In the vertical direction, $v_{y0}=0,\ a_y=9.80\,\text{m/s}^2,\ y_0=0,$ and the final position is $y=150\,\text{m}$. The time of flight is found from applying Eq. 2-12b to the vertical motion.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 160\text{ m}=0+0+\tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad\rightarrow$$

$$t=\sqrt{\frac{2\left(150\text{ m}\right)}{9.80\,\text{m/s}^2}}=\boxed{5.5\,\text{s}}$$

Note that the horizontal speed of the airplane does not enter into this calculation.

44. (*a*) Use the "level horizontal range" formula from Example 3-10 to find her takeoff speed.

$$R=\frac{v_0^2\sin2\theta_0}{g}\quad\rightarrow\quad v_0=\sqrt{\frac{gR}{\sin2\theta_0}}=\sqrt{\frac{\left(9.80\,\text{m/s}^2\right)\left(8.0\,\text{m}\right)}{\sin90^\circ}}=8.854\,\text{m/s}\approx\boxed{8.9\,\text{m/s}}$$

(*b*) Let the launch point be at the $y = 0$ level, and choose upward to be positive. Use Eq. 2-12b to solve for the time to fall to 2.5 meters below the starting height, and then calculate the horizontal distance traveled.

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_y t^2\quad\rightarrow\quad -2.5\,\text{m}=\left(8.854\,\text{m/s}\right)\sin45^\circ t+\tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2$$

$$4.9t^2-6.261t-2.5\,\text{m}=0\quad\rightarrow$$

$$t=\frac{6.261\pm\sqrt{\left(6.261\right)^2-4\left(4.9\right)\left(-2.5\right)}}{2\left(4.9\right)}=\frac{6.261\pm9.391}{2\left(4.9\right)}=-0.319\,\text{s},\ 1.597\,\text{s}$$

Use the positive time to find the horizontal displacement during the jump.

$$\Delta x=v_{0x}t=v_0\cos45^\circ t=\left(8.854\,\text{m/s}\right)\cos45^\circ\left(1.597\,\text{s}\right)=10.0\,\text{m}$$

$\boxed{\text{She will land exactly on the opposite bank, neither long nor short.}}$

45. Choose the origin to be the location at water level directly underneath the diver when she left the board. Choose upward as the positive y direction. For the diver, $y_0=5.0\text{ m},$ the final y position is $y=0.0\text{ m}$ (water level), $a_y=-g,$ the time of flight is $t=1.3\,\text{s},$ and the horizontal displacement is $\Delta x=3.0\text{ m}.$

(*a*) The horizontal velocity is determined from the horizontal motion at constant velocity.

$$\Delta x=v_x t\quad\rightarrow\quad v_x=\frac{\Delta x}{t}=\frac{3.0\text{ m}}{1.3\text{ s}}=2.31\,\text{m/s}$$

The initial y velocity is found using Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \ \rightarrow\ 0\text{ m} = 5.0\text{ m} + v_{y0}(1.3\text{ s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(1.3\text{ s})^2 \ \rightarrow$$

$$v_{y0} = 2.52\,\text{m/s}$$

Thus the velocity in both vector and magnitude / direction format are as follows.

$$\vec{\mathbf{v}}_0 = \boxed{(2.3\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}})\,\text{m/s}} \qquad v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(2.31\,\text{m/s})^2 + (2.52\,\text{m/s})^2} = \boxed{3.4\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{2.52\,\text{m/s}}{2.31\,\text{m/s}} = \boxed{48^\circ\text{ above the horizontal}}$$

(*b*) The maximum height will be reached when the y velocity is zero. Use Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a\Delta y \quad\rightarrow\quad 0 = (2.52\,\text{m/s})^2 + 2(-9.80\,\text{m/s}^2)(y_{\max} - 5.0\text{ m}) \quad\rightarrow$$

$$y_{\max} = \boxed{5.3\,\text{m}}$$

(*c*) To find the velocity when she enters the water, the horizontal velocity is the (constant) value of $v_x = 2.31\,\text{m/s}$. The vertical velocity is found from Eq. 2-12a.

$$v_y = v_{y0} + at = 2.52\,\text{m/s} + (-9.80\,\text{m/s}^2)(1.3\text{ s}) = -10.2\,\text{m/s}$$

The velocity is as follows.

$$\vec{\mathbf{v}}_\text{f} = \boxed{(2.3\hat{\mathbf{i}} - 10.2\hat{\mathbf{j}})\,\text{m/s}}$$

$$v_\text{f} = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.31\,\text{m/s})^2 + (-10.2\,\text{m/s})^2} = 10.458\,\text{m/s} \approx \boxed{10\,\text{m/s}}$$

$$\theta_\text{f} = \tan^{-1}\frac{v_{\text{f}y}}{v_{\text{f}x}} = \tan^{-1}\frac{-10.2\,\text{m/s}}{2.31\,\text{m/s}} = \boxed{-77^\circ(\text{below the horizontal})}$$

46. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0\,\text{m/s}$, $\theta_0 = 35.0^\circ$, $a_y = -g$, $y_0 = 115\,\text{m}$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The time taken to reach the ground is found from Eq. 2-12b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = y_0 + v_0\sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{-v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 - 4(-\tfrac{1}{2}g)y_0}}{2(-\tfrac{1}{2}g)} = 9.964\,\text{s}\ ,\ -2.3655\,\text{s} = \boxed{9.96\,\text{s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0\cos\theta_0)t = (65.0\,\text{m/s})(\cos 35.0^\circ)(9.964\,\text{s}) = \boxed{531\,\text{m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0\cos\theta_0 = (65.0\,\text{m/s})\cos 35.0^\circ = \boxed{53.2\,\text{m/s}}$. The vertical component is found from Eq. 2-12a.

$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = (65.0\,\text{m/s})\sin 35.0^\circ - (9.80\,\text{m/s}^2)(9.964\,\text{s})$$

$$= \boxed{-60.4\,\text{m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part (*c*) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(53.2\,\text{m/s})^2 + (-60.4\,\text{m/s})^2} = \boxed{80.5\,\text{m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-60.4}{53.2} = -48.6°$, and so the object is moving $\boxed{48.6° \text{ below the horizon}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the *y*-velocity is 0, and is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad \to \quad 0 = v_0^2 \sin^2\theta_0 - 2gy_{\text{max}}$$

$$y_{\text{max}} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{(65.0\,\text{m/s})^2 \sin^2 35.0°}{2(9.80\,\text{m/s}^2)} = \boxed{70.9\,\text{m}}$$

47. Choose upward to be the positive *y* direction. The origin is the point from which the football is kicked. The initial speed of the football is $v_0 = 20.0\,\text{m/s}$. We have $v_{y0} = v_0 \sin 37.0° = 12.04\,\text{m/s}$, $y_0 = 0$, and $a_y = -9.80\,\text{m/s}^2$. In the horizontal direction, $v_x = v_0 \cos 37.0° = 15.97\,\text{m/s}$, and $\Delta x = 36.0$ m. The time of flight to reach the goalposts is found from the horizontal motion at constant speed.

$$\Delta x = v_x t \quad \to \quad t = \Delta x / v_x = 36.0\text{ m}/15.97\,\text{m/s} = 2.254\,\text{s}$$

Now use this time with the vertical motion data and Eq. 2-12b to find the height of the football when it reaches the horizontal location of the goalposts.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = 0 + (12.04\,\text{m/s})(2.254\text{ s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(2.254\,\text{s})^2 = 2.24\text{ m}$$

Since the ball's height is less than 3.00 m, $\boxed{\text{the football does not clear the bar}}$. It is 0.76 m too low when it reaches the horizontal location of the goalposts.

To find the distances from which a score can be made, redo the problem (with the same initial conditions) to find the times at which the ball is exactly 3.00 m above the ground. Those times would correspond with the maximum and minimum distances for making the score. Use Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \to \quad 3.00 = 0 + (12.04\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2 \quad \to$$

$$4.90t^2 - 12.04t + 3.00 = 0 \quad \to \quad t = \frac{12.04 \pm \sqrt{(12.04)^2 - 4(4.90)(3.00)}}{2(4.90)} = 2.1757\,\text{s},\ 0.2814\,\text{s}$$

$$\Delta x_1 = v_x t = 15.97\,\text{m/s}(0.2814\,\text{s}) = 4.49\,\text{m}\ ;\ \Delta x_1 = v_x t = 15.97\,\text{m/s}(2.1757\,\text{s}) = 34.746\,\text{m}$$

So the kick must be made in the range from $\boxed{4.5\,\text{m to } 34.7\,\text{m}}$.

48. The constant acceleration of the projectile is given by $\vec{\mathbf{a}} = -9.80\,\text{m/s}^2\,\hat{\mathbf{j}}$. We use Eq. 3-13a with the given velocity, the acceleration, and the time to find the initial velocity.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \quad \to \quad \vec{\mathbf{v}}_0 = \vec{\mathbf{v}} - \vec{\mathbf{a}}t = (8.6\,\hat{\mathbf{i}} + 4.8\,\hat{\mathbf{j}})\,\text{m/s} - (-9.80\,\text{m/s}^2\,\hat{\mathbf{j}})(3.0\,\text{s}) = (8.6\,\hat{\mathbf{i}} + 34.2\,\hat{\mathbf{j}})\,\text{m/s}$$

The initial speed is $v_0 = \sqrt{(8.6\,\text{m/s})^2 + (34.2\,\text{m/s})^2} = 35.26\,\text{m/s}$, and the original launch direction is given by $\theta_0 = \tan^{-1}\frac{34.2\,\text{m/s}}{8.6\,\text{m/s}} = 75.88°$. Use this information with the horizontal range formula from Example 3-10 to find the range.

(*a*) $R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(35.26\,\mathrm{m/s})^2 (\sin 151.76^\circ)}{9.80\,\mathrm{m/s^2}\ g} = \boxed{6.0\times10^1\,\mathrm{m}}$

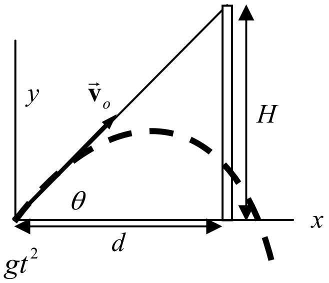
(*b*) We use the vertical information to find the maximum height. The initial vertical velocity is 34.2 m/s, and the vertical acceleration is $-9.80\,\mathrm{m/s^2}$. The vertical velocity at the maximum height is 0, and the initial height is 0. Use Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y_{\max} - y_0\right) \quad \rightarrow$$

$$y_{\max} = y_0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_{0y}^2}{2a_y} = \frac{-(34.2\,\mathrm{m/s})^2}{2\left(-9.80\,\mathrm{m/s^2}\right)} = 59.68\,\mathrm{m} \approx \boxed{6.0\times10^1\,\mathrm{m}}$$

(*c*) From the information above and the symmetry of projectile motion, we know that the final speed just before the projectile hits the ground is the same as the initial speed, and the angle is the same as the launching angle, but below the horizontal. So $v_{\text{final}} = \boxed{35\,\mathrm{m/s}}$ and $\theta_{\text{final}} = \boxed{76^\circ \text{ below the horizontal}}$.

$\boxed{49.}$ Choose the origin to be the location from which the balloon is fired, and choose upward as the positive *y* direction. Assume the boy in the tree is a distance *H* up from the point at which the balloon is fired, and that the tree is a distance *d* horizontally from the point at which the balloon is fired. The equations of motion for the balloon and boy are as follows, using constant acceleration relationships.



$$x_{\text{Balloon}} = v_0 \cos\theta_0 t \qquad y_{\text{Balloon}} = 0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \qquad y_{\text{Boy}} = H - \tfrac{1}{2}gt^2$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled *d* to the right.

$$d = v_0 \cos\theta_0 t_D \quad \rightarrow \quad t_D = \frac{d}{v_0 \cos\theta_0}$$

Where is the balloon vertically at that time?

$$y_{\text{Balloon}} = v_0 \sin\theta_0 t_D - \tfrac{1}{2}gt_D^2 = v_0 \sin\theta_0 \frac{d}{v_0\cos\theta_0} - \tfrac{1}{2}g\left(\frac{d}{v_0\cos\theta_0}\right)^2 = d\tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{v_0\cos\theta_0}\right)^2$$

Where is the boy vertically at that time? Note that $H = d\tan\theta_o$.

$$y_{\text{Boy}} = H - \tfrac{1}{2}gt_D^2 = H - \tfrac{1}{2}g\left(\frac{d}{v_0\cos\theta_0}\right)^2 = d\tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{v_0\cos\theta_0}\right)^2$$

Note that $y_{\text{Balloon}} = y_{\text{Boy}}$, and so the boy and the balloon are at the same height and the same horizontal location at the same time. Thus they collide!

50. (*a*) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. At the end of its flight over the 8 cars, the car must be at $y = -1.5\,\mathrm{m}$. Also for the car, $v_{y0} = 0$, $a_y = -g$, $v_x = v_0$, and $\Delta x = 22\,\mathrm{m}$. The time of flight is found from the horizontal motion at constant velocity: $\Delta x = v_x t \ \rightarrow\ t = \Delta x / v_0$. That expression for the time is used in Eq. 2-12b for the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(-g)\left(\Delta x / v_0\right)^2 \quad \rightarrow$$

$$v_0=\sqrt{\frac{-g\left(\Delta x\right)^2}{2\left(y\right)}}=\sqrt{\frac{-\left(9.80\,\text{m/s}^2\right)\left(22\,\text{m}\right)^2}{2\left(-1.5\text{ m}\right)}}=39.76\,\text{m/s}\approx\boxed{40\,\text{m/s}}$$

(*b*) Again choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive *y* direction. The *y* displacement of the car at the end of its flight over the 8 cars must again be $y=-1.5$ m. For the car, $v_{y0}=v_0\sin\theta_0$, $a_y=-g$, $v_x=v_0\cos\theta_0$, and $\Delta x=22\,\text{m}$. The launch angle is $\theta_0=7.0°$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x=v_xt\quad\rightarrow\quad t=\frac{\Delta x}{v_0\cos\theta_0}$$

That expression for the time is used in Eq. 2-12b for the vertical motion.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_yt^2\quad\rightarrow\quad y=v_0\sin\theta_0\frac{\Delta x}{v_0\cos\theta_0}+\tfrac{1}{2}\left(-g\right)\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2\quad\rightarrow$$

$$v_0=\sqrt{\frac{g\left(\Delta x\right)^2}{2\left(\Delta x\tan\theta_0-y\right)\cos^2\theta_0}}=\sqrt{\frac{\left(9.80\,\text{m/s}^2\right)\left(22\,\text{m}\right)^2}{2\left(\left(22\,\text{m}\right)\tan7.0°+1.5\text{ m}\right)\cos^2 7.0°}}=\boxed{24\,\text{m/s}}$$

51. The angle is in the direction of the velocity, so find the components of the velocity, and use them to define the angle. Let the positive *y*-direction be down.

$$v_x=v_0\qquad v_y=v_{y0}+a_yt=gt\qquad\theta=\tan^{-1}\frac{v_y}{v_x}=\boxed{\tan^{-1}\frac{gt}{v_0}}$$

52. Choose the origin to be where the projectile is launched, and upwards to be the positive *y* direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y=-g$, and $v_{y0}=v_0\sin\theta_0$. The range of the projectile is given by the range formula from Example 3-10, $R=\frac{v_0^2\sin2\theta_0}{g}$. The maximum height of the projectile will occur when its vertical speed is 0. Apply Eq. 2-12c.

$$v_y^2=v_{y0}^2+2a_y\left(y-y_0\right)\quad\rightarrow\quad0=v_0^2\sin^2\theta_0-2gy_{\max}\quad\rightarrow\quad y_{\max}=\frac{v_0^2\sin^2\theta_0}{2g}$$

Now find the angle for which $R=y_{\max}$.

$$R=y_{\max}\quad\rightarrow\quad\frac{v_0^2\sin2\theta_0}{g}=\frac{v_0^2\sin^2\theta_0}{2g}\quad\rightarrow\quad\sin2\theta_0=\tfrac{1}{2}\sin^2\theta_0\quad\rightarrow$$

$$2\sin\theta_0\cos\theta_0=\tfrac{1}{2}\sin^2\theta_0\quad\rightarrow\quad4\cos\theta_0=\sin\theta_0\quad\rightarrow\quad\tan\theta_0=4\quad\rightarrow\quad\theta_0=\tan^{-1}4=\boxed{76°}$$

53. Choose the origin to be where the projectile is launched, and upwards to be the positive *y* direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y=-g$, and $v_{y0}=v_0\sin\theta_0$.

(*a*) The maximum height is found from Eq. 2-12c, $v_y^2=v_{y0}^2+2a_y\left(y-y_0\right)$, with $v_y=0$ at the maximum height.

$$y_{\max}=0+\frac{v_y^2-v_{y0}^2}{2a_y}=\frac{-v_0^2\sin^2\theta_0}{-2g}=\frac{v_0^2\sin^2\theta_0}{2g}=\frac{\left(46.6\,\text{m/s}\right)^2\sin^2 42.2°}{2\left(9.80\,\text{m/s}^2\right)}=\boxed{50.0\,\text{m}}$$

(*b*) The total time in the air is found from Eq. 2-12b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{2v_0 \sin\theta_0}{g} = \frac{2(46.6\,\text{m/s})\sin 42.2^\circ}{(9.80\,\text{m/s}^2)} = \boxed{6.39\,\text{s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos\theta_0)t = (46.6\,\text{m/s})(\cos 42.2^\circ)(6.39\,\text{s}) = \boxed{221\,\text{m}}$$

(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0 \cos\theta_0 = (46.6\,\text{m/s})(\cos 42.2^\circ) = 34.5\,\text{m/s}$. The vertical velocity is found from Eq. 2-12a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = (46.6\,\text{m/s})\sin 42.2^\circ - (9.80\,\text{m/s}^2)(1.50\,\text{s}) = 16.6\,\text{m/s}$$

Thus the speed of the projectile is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{34.5^2 + 16.6^2} = \boxed{38.3\,\text{m/s}}$.

The direction above the horizontal is given by $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{16.6}{34.5} = \boxed{25.7^\circ}$.

54. (*a*) Use the "level horizontal range" formula from Example 3-10.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(7.80\text{ m})(9.80\,\text{m/s}^2)}{\sin 54.0^\circ}} = \boxed{9.72\,\text{m/s}}$$

(*b*) Now increase the speed by 5.0% and calculate the new range. The new speed would be $9.72\,\text{m/s}(1.05) = 10.2\,\text{m/s}$ and the new range would be as follows.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.2\,\text{m/s})^2 \sin 54^\circ}{9.80\,\text{m/s}^2} = 8.59\,\text{m}$$

This is an increase of $\boxed{0.79\,\text{m}\ (10\%\text{ increase})}$. .

$\boxed{55.}$ Choose the origin to be at the bottom of the hill, just where the incline starts. The equation of the line describing the hill is $y_2 = x\tan\phi$. The equations of the motion of the object are $y_1 = v_{0y}t + \tfrac{1}{2}a_y t^2$ and $x = v_{0x}t$, with $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. Solve the horizontal equation for the time of flight, and insert that into the vertical projectile motion equation.

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta} \quad \rightarrow \; y_1 = v_0 \sin\theta \frac{x}{v_0 \cos\theta} - \tfrac{1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v_0^2 \cos^2\theta}$$

Equate the *y*-expressions for the line and the parabola to find the location where the two *x*-coordinates intersect.

$$x\tan\phi = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta} \quad \rightarrow \quad \tan\theta - \tan\phi = \frac{gx}{2v_0^2\cos^2\theta} \quad \rightarrow$$

$$x = \frac{(\tan\theta - \tan\phi)}{g}2v_0^2\cos^2\theta$$

This intersection *x*-coordinate is related to the desired quantity *d* by $x = d\cos\phi$.

$$d\cos\phi=\left(\tan\theta-\tan\phi\right)\frac{2v_0^2\cos^2\theta}{g}\quad\rightarrow\quad d=\frac{2v_0^2}{g\cos\phi}\left(\sin\theta\cos\theta-\tan\phi\cos^2\theta\right)$$

To maximize the distance, set the derivative of d with respect to θ equal to 0, and solve for θ.

$$\frac{d(d)}{d\theta}=\frac{2v_0^2}{g\cos\phi}\frac{d}{d\theta}\left(\sin\theta\cos\theta-\tan\phi\cos^2\theta\right)$$

$$=\frac{2v_0^2}{g\cos\phi}\left[\sin\theta\left(-\sin\theta\right)+\cos\theta\left(\cos\theta\right)-\tan\phi\left(2\right)\cos\theta\left(-\sin\theta\right)\right]$$

$$=\frac{2v_0^2}{g\cos\phi}\left[-\sin^2\theta+\cos^2\theta+2\tan\phi\cos\theta\sin\theta\right]=\frac{2v_0^2}{g\cos\phi}\left[\cos2\theta+\sin2\theta\tan\phi\right]=0$$

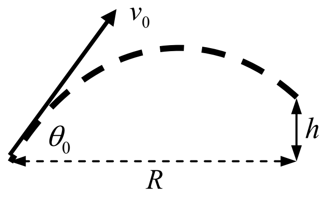
$$\cos2\theta+\sin2\theta\tan\phi=0\quad\rightarrow\quad\theta=\tfrac{1}{2}\tan^{-1}\left(-\frac{1}{\tan\phi}\right)$$

This expression can be confusing, because it would seem that a negative sign enters the solution. In order to get appropriate values, 180° or π radians must be added to the angle resulting from the inverse tangent operation, to have a positive angle. Thus a more appropriate expression would be the following:

$\theta=\tfrac{1}{2}\left[\pi+\tan^{-1}\left(-\frac{1}{\tan\phi}\right)\right]$. This can be shown to be equivalent to $\boxed{\theta=\frac{\phi}{2}+\frac{\pi}{4}}$, because

$$\tan^{-1}\left(-\frac{1}{\tan\phi}\right)=\tan^{-1}\left(-\cot\phi\right)=\cot^{-1}\cot\phi-\frac{\pi}{2}=\phi-\frac{\pi}{2}.$$

56. See the diagram. Solve for R, the horizontal range, which is the horizontal speed times the time of flight.



$$R=\left(v_0\cos\theta_0\right)t\quad\rightarrow\quad t=\frac{R}{v_0\cos\theta_0}$$

$$h=\left(v_0\sin\theta_0\right)t-\tfrac{1}{2}gt^2\quad\rightarrow\quad\tfrac{1}{2}gt^2-\left(v_0\sin\theta_0\right)t+h=0\quad\rightarrow$$

$$R^2-R\frac{2v_0^2\cos^2\theta_0\tan\theta}{g}+\frac{2hv_0^2\cos^2\theta_0}{g}=0$$

$$R=\frac{\frac{2v_0^2\cos^2\theta_0\tan\theta}{g}\pm\sqrt{\left(\frac{2v_0^2\cos^2\theta_0\tan\theta}{g}\right)^2-4\frac{2hv_0^2\cos^2\theta_0}{g}}}{2}$$

$$=\frac{v_0\cos\theta_0}{g}\left[v_0\sin\theta_0\pm\sqrt{v_0^2\sin^2\theta_0-2gh}\right]$$

Which sign is to be used? We know the result if $h=0$ from Example 3-10. Substituting $h=0$ gives $R=\frac{v_0\cos\theta_0}{g}\left[v_0\sin\theta_0\pm v_0\sin\theta_0\right]$. To agree with Example 3-10, we must choose the + sign, and so

$\boxed{R=\frac{v_0\cos\theta_0}{g}\left[v_0\sin\theta_0+\sqrt{v_0^2\sin^2\theta_0-2gh}\right]}$. We see from this result that if $h>0$, the range will shorten, and if $h<0$, the range will lengthen.

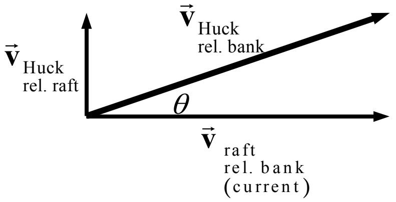
57. Call the direction of the boat relative to the water the positive direction. For the jogger moving towards the bow, we have the following:

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = 2.0\,\text{m/s}\,\hat{\mathbf{i}} + 8.5\,\text{m/s}\,\hat{\mathbf{i}} = \boxed{10.5\,\text{m/s}\,\hat{\mathbf{i}}}.$$

For the jogger moving towards the stern, we have the following.

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = -2.0\,\text{m/s}\,\hat{\mathbf{i}} + 8.5\,\text{m/s}\,\hat{\mathbf{i}} = \boxed{6.5\,\text{m/s}\,\hat{\mathbf{i}}}$$

58. Call the direction of the flow of the river the *x* direction, and the direction of Huck walking relative to the raft the *y* direction.



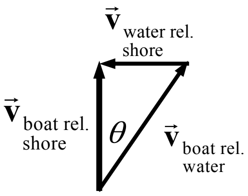
$$\vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.}\\ \text{bank}}} = 0.70\hat{\mathbf{j}}\,\text{m/s} + 1.50\hat{\mathbf{i}}\,\text{m/s}$$

$$= \left(1.50\hat{\mathbf{i}} + 0.70\hat{\mathbf{j}}\right)\text{m/s}$$

$$\text{Magnitude: } v_{\substack{\text{Huck}\\ \text{rel. bank}}} = \sqrt{1.50^2 + 0.70^2} = \boxed{1.66\,\text{m/s}}$$

$$\text{Direction: } \theta = \tan^{-1}\frac{0.70}{1.50} = \boxed{25^\circ\text{relative to river}}$$

59. From the diagram in Figure 3-33, it is seen that $v_{\substack{\text{boat rel.}\\ \text{shore}}} = v_{\substack{\text{boat rel.}\\ \text{water}}} \cos\theta =$ $\left(1.85\,\text{m/s}\right)\cos 40.4^\circ = \boxed{1.41\,\text{m/s}}$.



60. If each plane has a speed of 780 km/hr, then their relative speed of approach is 1560 km/hr. If the planes are 12.0 km apart, then the time for evasive action is found as follows.

$$\Delta d = vt \quad \rightarrow \quad t = \frac{\Delta d}{v} = \left(\frac{12.0\text{ km}}{1560\,\text{km/hr}}\right)\left(\frac{3600\text{ sec}}{1\text{ hr}}\right) = \boxed{27.7\text{ s}}$$

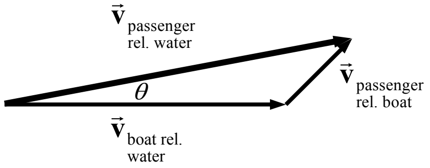
$\boxed{61.}$ The lifeguard will be carried downstream at the same rate as the child. Thus only the horizontal motion need be considered. To cover 45 meters horizontally at a rate of 2 m/s takes $\frac{45\,\text{m}}{2\,\text{m/s}} =$ $22.5\,\text{s} \approx \boxed{23\,\text{s}}$ for the lifeguard to reach the child. During this time they would both be moving downstream at 1.0 m/s, and so would travel $\left(1.0\,\text{m/s}\right)\left(22.5\,\text{s}\right) = 22.5\text{m} \approx \boxed{23\,\text{m}}$ downstream.

62. Call the direction of the boat relative to the water the *x* direction, and upward the *y* direction. Also see the diagram.

$$\vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}}$$

$$= \left(0.60\cos 45^\circ\hat{\mathbf{i}} + 0.60\sin 45^\circ\hat{\mathbf{j}}\right)\text{m/s} + 1.70\hat{\mathbf{i}}\,\text{m/s}$$

$$= \boxed{\left(2.12\hat{\mathbf{i}} + 0.42\hat{\mathbf{j}}\right)\text{m/s}}$$



63. (*a*) Call the upward direction positive for the vertical motion. Then the velocity of the ball relative to a person on the ground is the vector sum of the horizontal and vertical motions. The horizontal velocity is $v_x = 10.0\,\text{m/s}$ and the vertical velocity is $v_y = 5.0\,\text{m/s}$.

$$\vec{\mathbf{v}} = 10.0\,\text{m/s}\,\hat{\mathbf{i}} + 5.0\,\text{m/s}\,\hat{\mathbf{j}} \;\rightarrow\; v = \sqrt{(10.0\,\text{m/s})^2 + (5.0\,\text{m/s})^2} = \boxed{11.2\,\text{m/s}}$$

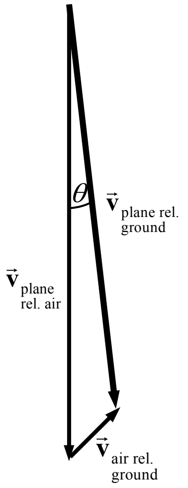
$$\theta = \tan^{-1}\frac{5.0\,\text{m/s}}{10.0\,\text{m/s}} = \boxed{27^\circ\text{ above the horizontal}}$$

(*b*) The only change is the initial vertical velocity, and so $v_y = -5.0\,\text{m/s}$.

$$\vec{\mathbf{v}} = 10.0\,\text{m/s}\,\hat{\mathbf{i}} - 5.0\,\text{m/s}\,\hat{\mathbf{j}} \;\rightarrow\; v = \sqrt{(10.0\,\text{m/s})^2 + (-5.0\,\text{m/s})^2} = \boxed{11.2\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{-5.0\,\text{m/s}}{10.0\,\text{m/s}} = \boxed{27^\circ\text{ below the horizontal}}$$

64. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.



(*a*)
$$\begin{aligned}\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} &= \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}\\ &= -580\hat{\mathbf{j}}\,\text{km/h} + \left(90.0\cos 45.0^\circ\hat{\mathbf{i}} + 90.0\sin 45.0^\circ\hat{\mathbf{j}}\right)\text{km/h}\\ &= \left(63.6\hat{\mathbf{i}} - 516\hat{\mathbf{j}}\right)\text{km/h}\end{aligned}$$

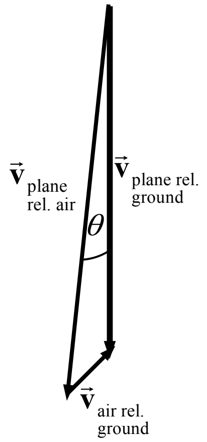
$$v_{\substack{\text{plane rel.}\\\text{ground}}} = \sqrt{(63.6\,\text{km/h})^2 + (-516\,\text{km/h})^2} = \boxed{520\,\text{km/h}}$$

$$\theta = \tan^{-1}\frac{63.6}{-516} = -7.0^\circ = \boxed{7.0^\circ\text{ east of south}}$$

(*b*) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 90.0 km/h, so after 11.0 min the plane is off course by the following amount.

$$\Delta x = v_x t = (90.0\,\text{km/h})(11.0\,\text{min})\left(\frac{1\,\text{h}}{60\,\text{min}}\right) = \boxed{16.5\,\text{km}}.$$

65. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.



$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}} \;\rightarrow$$

$$\begin{aligned}-v_{\substack{\text{plane rel.}\\\text{ground}}}\hat{\mathbf{j}} &= \left(-580\sin\theta\hat{\mathbf{i}} + 580\cos\theta\hat{\mathbf{j}}\right)\text{km/h}\\ &\quad + \left(90.0\cos 45.0^\circ\hat{\mathbf{i}} + 90.0\sin 45.0^\circ\hat{\mathbf{j}}\right)\text{km/h}\end{aligned}$$

Equate *x* components in the above equation.

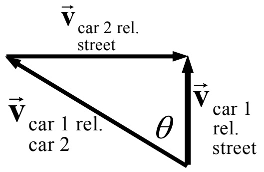
$$0 = -580\sin\theta + 90.0\cos 45.0^\circ \;\rightarrow$$

$$\theta = \sin^{-1}\frac{90.0\cos 45.0^\circ}{580} = \boxed{6.3^\circ\text{, west of south}}$$

66. Call east the positive *x* direction and north the positive *y* direction. From the first diagram, this relative velocity relationship is seen.

$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}} + \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} \quad \rightarrow$$

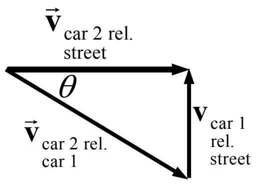
$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} - \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} = 35\hat{\mathbf{j}}\,\text{km/h} - 45\hat{\mathbf{i}}\,\text{km/h} = \left(-45\hat{\mathbf{i}} + 35\hat{\mathbf{j}}\right)\text{km/h}$$



For the other relative velocity relationship:

$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} + \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} \quad \rightarrow$$

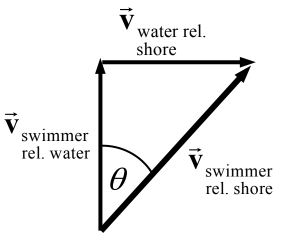
$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} - \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} = 45\hat{\mathbf{i}}\,\text{km/h} - 35\hat{\mathbf{j}}\,\text{km/h} = \left(45\hat{\mathbf{i}} - 35\hat{\mathbf{j}}\right)\text{km/h}$$



Notice that the two relative velocities are opposites of each other: $\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} = -\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}}$.

67. Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction. Call the location of the swimmer's starting point the origin.

$$\vec{\mathbf{v}}_{\substack{\text{swimmer}\\\text{rel. shore}}} = \vec{\mathbf{v}}_{\substack{\text{swimmer}\\\text{rel. water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\\text{shore}}} = 0.60\,\text{m/s}\,\hat{\mathbf{j}} + 0.50\,\text{m/s}\,\hat{\mathbf{i}}$$



(*a*) Since the swimmer starts from the origin, the distances covered in the *x* and *y* directions will be exactly proportional to the speeds in those directions.

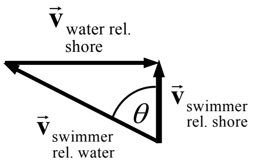
$$\frac{\Delta x}{\Delta y} = \frac{v_x t}{v_y t} = \frac{v_x}{v_y} \quad \rightarrow \quad \frac{\Delta x}{55\text{ m}} = \frac{0.50\,\text{m/s}}{0.60\,\text{m/s}} \quad \rightarrow \quad \Delta x = \boxed{46\,\text{m}}$$

(*b*) The time is found from the constant velocity relationship for either the *x* or *y* directions.

$$\Delta y = v_y t \quad \rightarrow \quad t = \frac{\Delta y}{v_y} = \frac{55\text{ m}}{0.60\,\text{m/s}} = \boxed{92\,\text{s}}$$

68. (*a*) Call the direction of the flow of the river the *x* direction, and the direction straight across the river the *y* direction.

$$\sin\theta = \frac{v_{\substack{\text{water rel.}\\\text{shore}}}}{v_{\substack{\text{swimmer}\\\text{rel. water}}}} = \frac{0.50\,\text{m/s}}{0.60\,\text{m/s}} \rightarrow \theta = \sin^{-1}\frac{0.50}{0.60} = 56.44^\circ \approx \boxed{56^\circ}$$



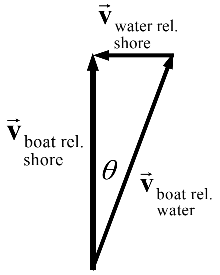
(*b*) From the diagram her speed with respect to the shore is found as follows.

$$v_{\substack{\text{swimmer}\\\text{rel. shore}}} = v_{\substack{\text{swimmer}\\\text{rel. water}}}\cos\theta = \left(0.60\,\text{m/s}\right)\cos 56.44^\circ = 0.332\,\text{m/s}$$

The time to cross the river can be found from the constant velocity relationship.

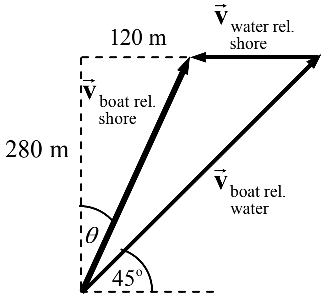
$$\Delta x = vt \quad \rightarrow \quad t = \frac{\Delta x}{v} = \frac{55\,\text{m}}{0.332\,\text{m/s}} = \boxed{170\,\text{s} = 2.8\text{ min}}$$

69. The boat is traveling directly across the stream, with a heading of $\theta = 19.5^\circ$ upstream, and speed of $v_{\substack{\text{boat rel.}\\\text{water}}} = 3.40\,\text{m/s}$.



(*a*) $v_{\substack{\text{water rel.}\\\text{shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}}\sin\theta = \left(3.40\,\text{m/s}\right)\sin 19.5^\circ = \boxed{1.13\,\text{m/s}}$

(*b*) $v_{\substack{\text{boat rel.}\\\text{shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}}\cos\theta = \left(3.40\,\text{m/s}\right)\cos 19.5^\circ = \boxed{3.20\,\text{m/s}}$



70. Call the direction of the flow of the river the x direction (to the left in the diagram), and the direction straight across the river the y direction (to the top in the diagram). From the diagram, $\theta = \tan^{-1} 120\,\text{m}/280\,\text{m}$ $= 23°$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.

$$v_{\substack{\text{boat rel.}\\ \text{shore}}} \cos\theta = v_{\substack{\text{boat rel.}\\ \text{water}}} \sin 45° \quad \rightarrow$$

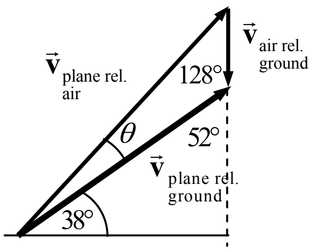
$$v_{\substack{\text{boat rel.}\\ \text{shore}}} = (2.70\,\text{m/s}) \frac{\sin 45°}{\cos 23°} = 2.07\,\text{m/s}$$

Equate the horizontal components of the velocities.

$$v_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta = v_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45° - v_{\substack{\text{water}\\ \text{rel. shore}}} \quad \rightarrow$$

$$v_{\substack{\text{water}\\ \text{rel. shore}}} = v_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45° - v_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta$$

$$= (2.70\,\text{m/s})\cos 45° - (2.07\,\text{m/s})\sin 23° = \boxed{1.10\,\text{m/s}}$$



71. Call east the positive x direction and north the positive y direction. The following is seen from the diagram. Apply the law of sines to the triangle formed by the three vectors.

$$\frac{v_{\substack{\text{plane}\\ \text{rel. air}}}}{\sin 128°} = \frac{v_{\substack{\text{air rel.}\\ \text{ground}}}}{\sin\theta} \quad \rightarrow \quad \sin\theta = \frac{v_{\substack{\text{air rel.}\\ \text{ground}}}}{v_{\substack{\text{plane}\\ \text{rel. air}}}} \sin 128° \quad \rightarrow$$

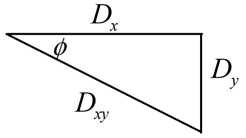
$$\theta = \sin^{-1}\left(\frac{v_{\substack{\text{air rel.}\\ \text{ground}}}}{v_{\substack{\text{plane}\\ \text{rel. air}}}} \sin 128° \right) = \sin^{-1}\left(\frac{72}{580\,\text{km/h}} \sin 128° \right) = 5.6°$$

So the plane should head in a direction of $38.0° + 5.6° = \boxed{43.6°\text{north of east}}$..

72. (*a*) For the magnitudes to add linearly, the two vectors must be parallel. $\boxed{\vec{\mathbf{V}}_1 \parallel \vec{\mathbf{V}}_2}$

(*b*) For the magnitudes to add according to the Pythagorean theorem, the two vectors must be at right angles to each other. $\boxed{\vec{\mathbf{V}}_1 \perp \vec{\mathbf{V}}_2}$

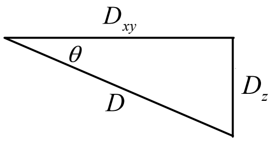
(*c*) The magnitude of $\vec{\mathbf{V}}_2$ vector 2 must be 0. $\boxed{\vec{\mathbf{V}}_2 = 0}$



$\boxed{73.}$ Let east be the positive x-direction, north be the positive y-direction, and up be the positive z-direction. Then the plumber's resultant displacement in component notation is $\boxed{\vec{\mathbf{D}} = 66\,\text{m}\,\hat{\mathbf{i}} - 35\,\text{m}\,\hat{\mathbf{j}} - 12\,\text{m}\,\hat{\mathbf{k}}}$. Since this is a 3-dimensional problem, it requires 2 angles to determine his location (similar to latitude and longitude on the surface of the Earth). For the x-y (horizontal) plane, see the first figure.

$$\phi = \tan^{-1}\frac{D_y}{D_x} = \tan^{-1}\frac{-35}{66} = -28° = 28° \text{ south of east}$$

$$D_{xy} = \sqrt{D_x^2 + D_y^2} = \sqrt{(66)^2 + (-35)^2} = 74.7\,\text{m} \approx 75\,\text{m}$$



For the vertical motion, consider another right triangle, made up of D_{xy} as one leg, and the vertical displacement D_z as the other leg. See the second figure, and the following calculations.

$$\theta_2 = \tan^{-1}\frac{D_z}{D_{xy}} = \tan^{-1}\frac{-12\text{ m}}{74.7\text{ m}} = -9° = 9° \text{ below the horizontal}$$

$$D = \sqrt{D_{xy}^2 + D_z^2} = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(66)^2 + (-35)^2 + (-12)^2} = 76\text{ m}$$

The result is that the displacement is $\boxed{76\text{m}}$, at an angle of $\boxed{28°\text{south of east}}$, and $\boxed{9°\text{ below the horizontal}}$.

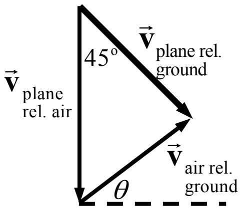
74. The deceleration is along a straight line. The starting velocity is $110\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 30.6\,\text{m/s}$, and the ending velocity is 0 m/s. The acceleration is found from Eq. 2-12a.

$$v = v_0 + at \quad\rightarrow\quad 0 = 30.6\,\text{m/s} + a(7.0\text{ s}) \quad\rightarrow\quad a = -\frac{30.6\,\text{m/s}}{7.0\text{ s}} = -4.37\,\text{m/s}^2$$

The horizontal acceleration is $a_{\text{horiz}} = a\cos\theta = -4.37\,\text{m/s}^2(\cos 26°) = \boxed{-3.9\,\text{m/s}^2}$.

The vertical acceleration is $a_{\text{vert}} = a\sin\theta = -4.37\,\text{m/s}^2(\sin 26°) = \boxed{-1.9\,\text{m/s}^2}$.

The horizontal acceleration is to the left in Figure 3-54, and the vertical acceleration is down.



75. Call east the positive *x* direction and north the positive *y* direction. Then this relative velocity relationship follows (see the accompanying diagram).

$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$

Equate the *x* components of the velocity vectors. The magnitude of $\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}}$ is given as 135 km/h.

$$(135\,\text{km/h})\cos 45° = 0 + v_{\text{wind}\,x} \quad\rightarrow\quad v_{\text{wind}\,x} = 95.5\,\text{km/h}\,.$$

From the *y* components of the above equation, we find $v_{\text{wind}\,y}$.

$$-135\sin 45° = -185 + v_{\text{wind}\,y} \quad\rightarrow\quad v_{\text{wind}\,y} = 185 - 135\sin 45° = 89.5\,\text{km/h}$$

The magnitude of the wind velocity is as follows.

$$v_{\text{wind}} = \sqrt{v_{\text{wind}\,x}^2 + v_{\text{wind}\,y}^2} = \sqrt{(95.5\,\text{km/h})^2 + (89.5\,\text{km/h})^2} = \boxed{131\,\text{km/h}}$$

The direction of the wind is $\theta = \tan^{-1}\frac{v_{\text{wind-y}}}{v_{\text{wind-x}}} = \tan^{-1}\frac{89.5}{95.5} = \boxed{43.1°\text{ north of east}}$.

76. The time of flight is found from the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = \quad\rightarrow\quad t = \Delta x/v_x = 8.0\text{ m}/9.1\,\text{m/s} = \boxed{0.88\text{ s}}$$

The *y* motion is symmetric in time – it takes half the time of flight to rise, and half to fall. Thus the time for the jumper to fall from his highest point to the ground is 0.44 sec. His vertical speed is zero at the highest point. From the time, the initial vertical speed, and the acceleration of gravity, the maximum height can be found. Call upward the positive *y* direction. The point of maximum height is the starting position y_0, the ending position is $y = 0$, the starting vertical speed is 0, and $a = -g$. Use Eq. 2-12b to find the height.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + 0 - \tfrac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(0.44\text{ s}\right)^2 \quad \rightarrow \quad y_0 = \boxed{0.95\,\text{m}}$$

77. Choose upward to be the positive *y* direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y = -9.80\,\text{m/s}^2$, the velocity at the window is $v_y = 0$, and the vertical displacement is 8.0 m. The initial *y* velocity is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right) \quad \rightarrow$$

$$v_{y0} = \sqrt{v_y^2 - 2a_y\left(y - y_0\right)} = \sqrt{0 - 2\left(-9.80\,\text{m/s}^2\right)\left(8.0\text{ m}\right)} = 12.5\,\text{m/s}$$

Find the time for the pebbles to travel to the window from Eq. 2-12a.

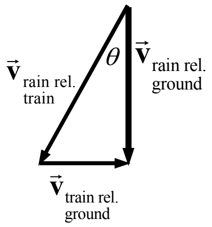
$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{0 - 12.5\,\text{m/s}}{-9.80\,\text{m/s}^2} = 1.28\,\text{s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 9.0\text{ m}/1.28\text{ s} = \boxed{7.0\,\text{m/s}}$$

This is the speed of the pebbles when they hit the window.

78. Choose the *x* direction to be the direction of train travel (the direction the passenger is facing) and choose the *y* direction to be up. This relationship exists among the velocities: $\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{rain rel.}\\\text{train}}} + \vec{\mathbf{v}}_{\substack{\text{train rel.}\\\text{ground}}}$. From the diagram, find the expression for the speed of the raindrops.



$$\tan\theta = \frac{v_{\substack{\text{train rel.}\\\text{ground}}}}{v_{\substack{\text{rain rel.}\\\text{ground}}}} = \frac{v_{\text{T}}}{v_{\substack{\text{rain rel.}\\\text{ground}}}} \quad \rightarrow \quad \boxed{v_{\substack{\text{rain rel.}\\\text{ground}}} = \frac{v_{\text{T}}}{\tan\theta}}$$

$\boxed{79.}$ Assume that the golf ball takes off and lands at the same height, so that the range formula derived in Example 3-10 can be applied. The only variable is to be the acceleration due to gravity.

$$R_{\text{Earth}} = v_0^2 \sin 2\theta_0/g_{\text{Earth}} \qquad R_{\text{Moon}} = v_0^2 \sin 2\theta_0/g_{\text{Moon}}$$

$$\frac{R_{\text{Earth}}}{R_{\text{Moon}}} = \frac{v_0^2 \sin 2\theta_0/g_{\text{Earth}}}{v_0^2 \sin 2\theta_0/g_{\text{Moon}}} = \frac{1/g_{\text{Earth}}}{1/g_{\text{Moon}}} = \frac{g_{\text{Moon}}}{g_{\text{Earth}}} = \frac{32\text{ m}}{180\text{ m}} = 0.18 \quad \rightarrow$$

$$g_{\text{Moon}} = 0.18 g_{\text{Earth}} = 0.18\left(9.80\,\text{m/s}^2\right) \approx \boxed{1.8\,\text{m/s}^2}$$

80. (*a*) Choose downward to be the positive *y* direction. The origin is the point where the bullet leaves the gun. In the vertical direction, $v_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\,\text{m/s}^2$. In the horizontal direction, $\Delta x = 68.0\text{ m}$ and $v_x = 175\,\text{m/s}$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x/v_x = 68.0\text{ m}/175\,\text{m/s} = 0.3886\text{ s}$$

This time can now be used in Eq. 2-12b to find the vertical drop of the bullet.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(0.3886\,\text{s}\right)^2 = \boxed{0.740\,\text{m}}$$

(*b*) For the bullet to hit the target at the same level, the level horizontal range formula of Example 3-10 applies. The range is 68.0 m, and the initial velocity is 175 m/s. Solving for the angle of launch results in the following.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \sin 2\theta_0 = \frac{Rg}{v_0^2} \quad \rightarrow \quad \theta_0 = \tfrac{1}{2}\sin^{-1}\frac{(68.0\text{ m})(9.80\,\text{m/s}^2)}{(175\,\text{m/s})^2} = \boxed{0.623^\circ}$$

Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be 89.4°. That is an unreasonable answer from a practical physical viewpoint – it is pointing the gun almost straight up.

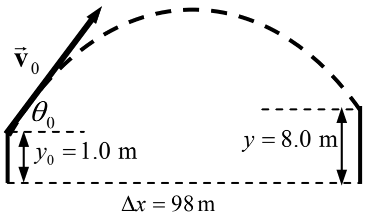
81. Choose downward to be the positive *y* direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $v_{y0} = 0$, the acceleration is $a_y = 9.80\,\text{m/s}^2$, and the displacement is 35 m. The time of flight is found from Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 35\text{ m} = 0 + 0 + \tfrac{1}{2}(9.8\,\text{m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(35\text{ m})}{9.8\,\text{m/s}^2}} = \boxed{2.7\text{ s}}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 5.0\text{ m}/2.7\text{ s} = \boxed{1.9\,\text{m/s}}$$

82. The minimum speed will be that for which the ball just clears the fence; i.e., the ball has a height of 8.0 m when it is 98 m horizontally from home plate. The origin is at home plate, with upward as the positive *y* direction. For the ball, $y_0 = 1.0\,\text{m}$, $y = 8.0\,\text{m}$,, $a_y = -g$, $v_{y0} = v_0 \sin\theta_0$, $v_x = v_0\cos\theta_0$, and $\theta_0 = 36^\circ$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x = v_x t = v_0 \cos\theta_0 t$, and so $t = \dfrac{\Delta x}{v_0\cos\theta_0}$. For the vertical motion, apply Eq. 2-12b.



$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + v_0(\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$y = y_0 + v_0 t\sin\theta_0 - \tfrac{1}{2}gt^2 \quad \rightarrow \quad y = y_0 + v_0\sin\theta_0 \frac{\Delta x}{v_0\cos\theta_0} - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \sqrt{2\left(\frac{y_0 - y + \Delta x\tan\theta_0}{g}\right)} = \sqrt{2\left(\frac{1.0\,\text{m} - 8.0\,\text{m} + (98\,\text{m})\tan 36^\circ}{9.80\,\text{m/s}^2}\right)} = 3.620\,\text{s}$$

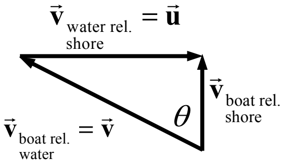
Finally, use the time with the horizontal range to find the initial speed.

$$\Delta x = v_0\cos\theta_0 t \quad \rightarrow \quad v_0 = \frac{\Delta x}{t\cos\theta_0} = \frac{98\,\text{m}}{(3.620\,\text{s})\cos 36^\circ} = \boxed{33\,\text{m/s}}$$

83. (*a*) For the upstream trip, the boat will cover a distance of $D/2$ with a net speed of $v - u$, so the time is $t_1 = \dfrac{D/2}{v-u} = \dfrac{D}{2(v-u)}$. For the downstream trip, the boat will cover a distance of $D/2$

with a net speed of $v+u$, so the time is $t_2 = \dfrac{D/2}{v+u} = \dfrac{D}{2(v+u)}$. Thus the total time for the round trip will be $t = t_1 + t_2 = \dfrac{D}{2(v-u)} + \dfrac{D}{2(v+u)} = \boxed{\dfrac{Dv}{(v^2-u^2)}}$.

(*b*) For the boat to go directly across the river, it must be angled against the current in such a way that the net velocity is straight across the river, as in the picture. This equation must be satisfied:



$$\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\\text{shore}}} = \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\\text{water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\\text{shore}}} = \vec{\mathbf{v}} + \vec{\mathbf{u}} .$$

Thus $v_{\substack{\text{boat rel.}\\\text{shore}}} = \sqrt{v^2 - u^2}$, and the time to go a distance $D/2$ across the river is $t_1 = \dfrac{D/2}{\sqrt{v^2-u^2}} = \dfrac{D}{2\sqrt{v^2-u^2}}$. The same relationship would be in effect for crossing back, so the time to come back is given by $t_2 = t_1$ and the total time is $t = t_1 + t_2 = \boxed{\dfrac{D}{\sqrt{v^2-u^2}}}$.

The speed v must be greater than the speed u. The velocity of the boat relative to the shore when going upstream is $v-u$. If $v<u$, the boat will not move upstream at all, and so the first part of the trip would be impossible. Also, in part (*b*), we see that *v* is longer than *u* in the triangle, since *v* is the hypotenuse, and so we must have $v>u$.

84. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive *y* direction. Then for the ball, $y_0 = 2.50\,\text{m}$, $v_{y0} = 0$, $a_y = -g$, and the *y* location when the ball just clears the net is $y = 0.90$ m. The time for the ball to reach the net is calculated from Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0.90\text{ m} = 2.50\text{ m} + 0 + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \;\rightarrow$$

$$t_{\substack{\text{to}\\\text{net}}} = \sqrt{\frac{2(-1.60\text{ m})}{-9.80\,\text{m/s}^2}} = 0.57143\text{ s}$$

The *x* velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \;\rightarrow\; v_x = \frac{\Delta x}{t} = \frac{15.0\text{ m}}{0.57143\text{ s}} = 26.25 \approx \boxed{26.3\,\text{m/s}}$$

This is the minimum speed required to clear the net.

To find the full time of flight of the ball, set the final *y* location to be $y = 0$, and again use Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0.0\text{ m} = 2.50\text{ m} + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \;\rightarrow$$

$$t_{\text{total}} = \sqrt{\frac{2(-2.50\text{ m})}{-9.80\,\text{m/s}^2}} = 0.7143 \approx \boxed{0.714\,\text{s}}$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (26.25\,\text{m/s})(0.7143\text{ s}) = 18.75 \approx \boxed{18.8\text{ m}}$$

Since this is between 15.0 and 22.0 m, $\boxed{\text{the ball lands in the “good” region}}$.

$\boxed{85.}$ Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of $208\,\text{km/h} - 156\,\text{km/h} = 52\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)$ $= 14.44\,\text{m/s}$. For the vertical motion, choose the level of the helicopter to be the origin, and downward to be positive. Then the package's *y* displacement is $y = 78.0$ m, $v_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 78.0\text{ m} = \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(78.0\text{ m})}{9.80\,\text{m/s}^2}} = 3.99\text{ sec}$$

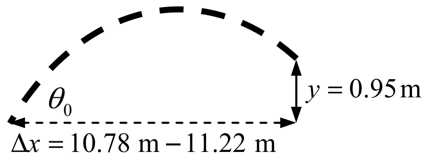
The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (14.44\,\text{m/s})(3.99\text{ s}) = 57.6\text{ m}$$

Thus the angle under the horizontal for the package release will be as follows.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0\text{ m}}{57.6\text{ m}}\right) = 53.6^\circ \approx \boxed{54^\circ}$$

86. The proper initial speeds will be those for which the ball has traveled a horizontal distance somewhere between 10.78 m and 11.22 m while it changes height from 2.10 m to 3.05 m with a shooting angle of 38.0°. Choose the origin to be at the shooting location of the basketball, with upward as the positive *y* direction. Then the vertical displacement is $y = 0.95\,\text{m}$, $a_y = -9.80\,\text{m/s}^2$, $v_{y0} = v_0 \sin\theta_0$, and the (constant) *x* velocity is $v_x = v_0\cos\theta_0$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x = v_x t = v_0 \cos\theta_0 t$ and so $t = \dfrac{\Delta x}{v_0\cos\theta_0}$. For the vertical motion, apply Eq. 2-12b.



$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = v_0\sin\theta t - \tfrac{1}{2}gt^2$$

Substitute the expression for the time of flight and solve for the initial velocity.

$$y = v_0\sin\theta t - \tfrac{1}{2}gt^2 = v_0\sin\theta\frac{\Delta x}{v_0\cos\theta_0} - \tfrac{1}{2}g\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2 = \Delta x\tan\theta - \frac{g(\Delta x)^2}{2v_0^2\cos^2\theta_0}$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2\cos^2\theta_0(-y + \Delta x\tan\theta)}}$$

For $\Delta x = 10.78$ m, the shortest shot:

$$v_0 = \sqrt{\frac{\left(9.80\,\text{m/s}^2\right)(10.78\text{ m})^2}{2\cos^2 38.0^\circ\left[(-0.95\text{ m} + (10.78\text{ m})\tan 38.0^\circ)\right]}} = \boxed{11.1\,\text{m/s}}.$$

For $\Delta x = 11.22$ m, the longest shot:

$$v_0 = \sqrt{\frac{\left(9.80\,\text{m/s}^2\right)(11.22\text{ m})^2}{2\cos^2 38.0^\circ\left[(-0.95\text{ m} + (11.22\text{ m})\tan 38.0^\circ)\right]}} = \boxed{11.3\,\text{m/s}}.$$

87. The acceleration is the derivative of the velocity.

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{3.5\,\mathrm{m/s^2}\,\hat{\mathbf{j}}}$$

Since the acceleration is constant, we can use Eq. 3-13b.

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 = \left(1.5\hat{\mathbf{i}} - 3.1\hat{\mathbf{j}}\right) + \left(-2.0\,\hat{\mathbf{i}}\right)t + \tfrac{1}{2}\left(3.5\,\hat{\mathbf{j}}\right)t^2$$

$$= \boxed{\left(1.5 - 2.0t\right)\mathrm{m}\,\hat{\mathbf{i}} + \left(-3.1 + 1.75t^2\right)\mathrm{m}\,\hat{\mathbf{j}}}$$

The shape is $\boxed{\text{parabolic}}$, with the parabola opening in the y-direction.

88. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive y direction. The y displacement of the projectile is 135 m, and the horizontal range of the projectile is 195 m. The acceleration in the y direction is $a_y = -g$, and the time of flight is 6.6 s. The horizontal velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{195\,\mathrm{m}}{6.6\,\mathrm{s}} = 29.55\,\mathrm{m/s}$$

Calculate the initial y velocity from the given data and Eq. 2-12b.

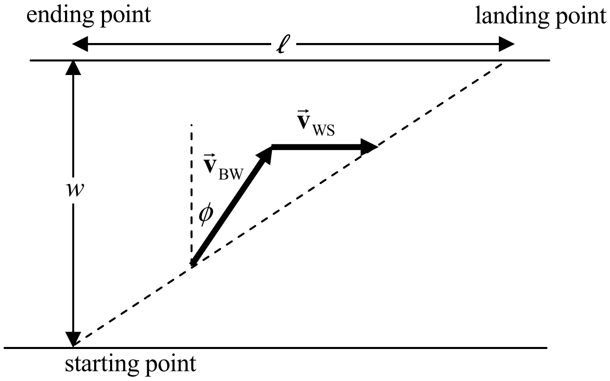
$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 135\,\mathrm{m} = v_{y0}\left(6.6\,\mathrm{s}\right) + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)\left(6.6\,\mathrm{s}\right)^2 \;\rightarrow\; v_{y0} = 52.79\,\mathrm{m/s}$$

Thus the initial velocity and direction of the projectile are as follows.

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{\left(29.55\,\mathrm{m/s}\right)^2 + \left(52.79\,\mathrm{m/s}\right)^2} = \boxed{60\,\mathrm{m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{52.79\,\mathrm{m/s}}{29.55\,\mathrm{m/s}} = \boxed{61^\circ}$$

89. We choose to initially point the boat downstream at an angle of ϕ relative to straight across the river, because then all horizontal velocity components are in the same direction, and the algebraic signs might be less confusing. If the boat should in reality be pointed upstream, the solution will give a negative angle. We use $v_{\mathrm{BW}} = 1.60\,\mathrm{m/s}$, the speed of the boat relative to the water (the rowing speed); $v_{\mathrm{WS}} = 0.80\,\mathrm{m/s}$, the speed of the water relative to the shore (the current); and $v_{\mathrm{R}} = 3.00\,\mathrm{m/s}$, his running speed. The width of the river is w = 1200 m, and the length traveled along the bank is ℓ. The time spent in the water is t_{W}, and the time running is t_{R}. The actual vector velocity of the boat is $\vec{\mathbf{v}}_{\mathrm{BS}} = \vec{\mathbf{v}}_{\mathrm{BW}} + \vec{\mathbf{v}}_{\mathrm{WS}}$. That vector addition is illustrated on the diagram (not drawn to scale).



The distance straight across the river (w) is the velocity component across the river, times the time in the water. The distance along the bank (ℓ) is the velocity component parallel to the river, times the time in the water. The distance along the bank is also his running speed times the time running. These three distances are expressed below.

$$w = \left(v_{\mathrm{BW}}\cos\phi\right)t_{\mathrm{W}} \;;\; \ell = \left(v_{\mathrm{BW}}\sin\phi + v_{WS}\right)t_{\mathrm{W}} \;;\; \ell = v_{\mathrm{R}}t_{\mathrm{R}}$$

The total time is $t = t_{\mathrm{W}} + t_{\mathrm{R}}$, and needs to be expressed as a function of ϕ. Use the distance relations above to write this function.

$$t = t_W + t_R = t_W + \frac{\ell}{v_R} = t_W + \frac{\left(v_{BW}\sin\phi + v_{WS}\right)t_W}{v_R} = t_W\left[1 + \frac{\left(v_{BW}\sin\phi + v_{WS}\right)}{v_R}\right]$$

$$= \frac{w}{v_{BW}v_R\cos\phi}\left[v_R + v_{WS} + v_{BW}\sin\phi\right] = \frac{w}{v_{BW}v_R}\left[\left(v_R + v_{WS}\right)\sec\phi + v_{BW}\tan\phi\right]$$

To find the angle corresponding to the minimum time, we set $\frac{dt}{d\phi} = 0$ and solve for the angle.

$$\frac{dt}{d\phi} = \frac{d}{d\phi}\left\{\frac{w}{v_{BW}v_R}\left[\left(v_R + v_{WS}\right)\sec\phi + v_{BW}\tan\phi\right]\right\}$$

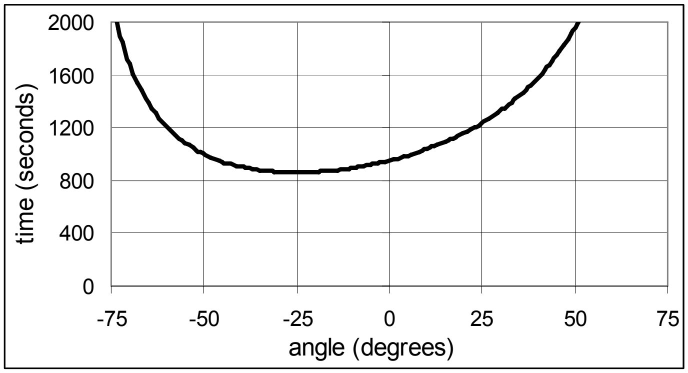
$$= \frac{w}{v_{BW}v_R}\left[\left(v_R + v_{WS}\right)\tan\phi\sec\phi + v_{BW}\sec^2\phi\right] = 0 \quad \rightarrow$$

$$\left[\left(v_R + v_{WS}\right)\tan\phi + v_{BW}\sec\phi\right]\sec\phi = 0 \quad \rightarrow \quad \sec\phi = 0\ ,\ \sin\phi = -\frac{v_{BW}}{v_R + v_{WS}}$$

The first answer is impossible, and so we must use the second solution.

$$\sin\phi = -\frac{v_{BW}}{v_R + v_{WS}} = -\frac{1.60\,\mathrm{m/s}}{3.00\,\mathrm{m/s} + 0.80\,\mathrm{m/s}} = -0.421 \quad \rightarrow \quad \phi = \sin^{-1}\left(-0.421\right) = -24.9^\circ$$

To know that this is really a minimum and not a maximum, some argument must be made. The maximum time would be infinity, if he pointed his point either directly upstream or downstream. Thus this angle should give a minimum. A second derivative test could be done, but that would be algebraically challenging. A graph of t vs. ϕ could also be examined to see that the angle is a minimum. Here is a portion of such a graph, showing a minimum time of somewhat more than 800 seconds near $\phi = -25^\circ$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.89."



The time he takes in getting to the final location can be calculated from the angle.

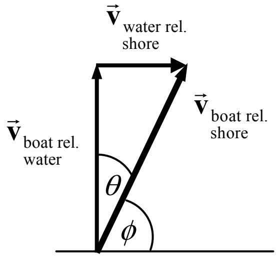
$$t_W = \frac{w}{v_{BW}\cos\phi} = \frac{1200\,\mathrm{m}}{\left(1.60\,\mathrm{m/s}\right)\cos\left(-24.9^\circ\right)} = 826.86\,\mathrm{s}$$

$$\ell = \left(v_{BW}\sin\phi + v_{WS}\right)t_W = \left[\left(1.60\,\mathrm{m/s}\right)\sin\left(-24.9^\circ\right) + 0.80\,\mathrm{m/s}\right]\left(826.86\,\mathrm{s}\right) = 104.47\,\mathrm{m}$$

$$t_R = \frac{\ell}{v_R} = \frac{104.47\,\mathrm{m}}{3.00\,\mathrm{m/s}} = 34.82\,\mathrm{s} \qquad t = t_W + t_R = 826.86\,\mathrm{s} + 34.82\,\mathrm{s} = \boxed{862\,\mathrm{s}}$$

Thus he must point the boat 24.9° upstream, taking 827 seconds to cross, and landing 104 m from the point directly across from his starting point. Then he runs the 104 m from his landing point to the point directly across from his starting point, in 35 seconds, for a total elapsed time of 862 seconds (about 14.4 minutes).

90. Call the direction of the flow of the river the x direction, and the direction the boat is headed (which is different than the direction it is moving) the y direction.



(a) $v_{\substack{\text{boat rel.}\\ \text{shore}}} = \sqrt{v^2_{\substack{\text{water rel.}\\ \text{shore}}} + v^2_{\substack{\text{boat rel.}\\ \text{water}}}} = \sqrt{1.30^2 + 2.20^2} = \boxed{2.56\,\text{m/s}}$

$\theta = \tan^{-1}\dfrac{1.30}{2.20} = 30.6°\ ,\ \phi = 90° - \theta = \boxed{59.4°\text{relative to shore}}$

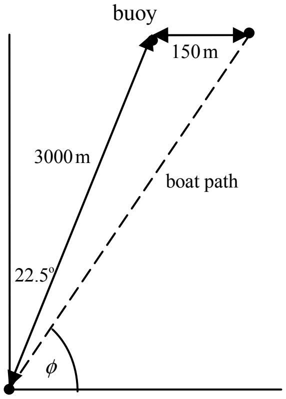
(b) The position of the boat after 3.00 seconds is given by the following.

$$\Delta d = v_{\substack{\text{boat rel.}\\ \text{shore}}} t = \left[\left(1.30\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right)\text{m/s}\right]\left(3.00\,\text{sec}\right)$$

$$= \boxed{\left(3.90\text{ m downstream}, 6.60\text{ m across the river}\right)}$$

As a magnitude and direction, it would be 7.67 m away from the starting point, at an angle of 59.4° relative to the shore.

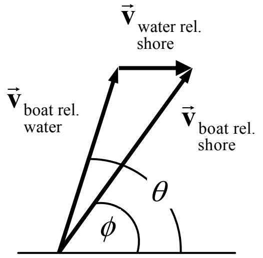
$\boxed{91.}$ First, we find the direction of the straight-line path that the boat must take to pass 150 m to the east of the buoy. See the first diagram (not to scale). We find the net displacement of the boat in the horizontal and vertical directions, and then calculate the angle.



$$\Delta x = \left(3000\,\text{m}\right)\sin 22.5° + 150\,\text{m} \qquad \Delta y = \left(3000\text{m}\right)\cos 22.5°$$

$$\phi = \tan^{-1}\frac{\Delta y}{\Delta x} = \frac{\left(3000\,\text{m}\right)\cos 22.5°}{\left(3000\,\text{m}\right)\sin 22.5° + 150\,\text{m}} = 64.905°$$

This angle gives the direction that the boat must travel, so it is the direction of the velocity of the boat with respect to the shore, $\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{shore}}}$. So $\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{shore}}} = v_{\substack{\text{boat rel.}\\ \text{shore}}}\left(\cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}\right)$. Then, using the second diagram (also not to scale), we can write the relative velocity equation relating the boat's travel and the current. The relative velocity equation gives us the following. See the second diagram.



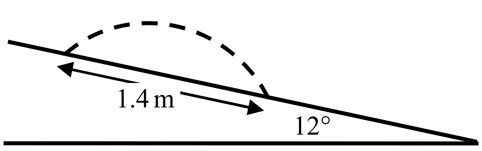
$$\vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{shore}}} = \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} \quad \rightarrow$$

$$v_{\substack{\text{boat rel.}\\ \text{shore}}}\left(\cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}\right) = 2.1\left(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}\right) + 0.2\hat{\mathbf{i}} \quad \rightarrow$$

$$v_{\substack{\text{boat rel.}\\ \text{shore}}}\cos\phi = 2.1\cos\theta + 0.2 \ ;\ v_{\substack{\text{boat rel.}\\ \text{shore}}}\sin\phi = 2.1\sin\theta$$

These two component equations can then be solved for $v_{\substack{\text{boat rel.}\\ \text{shore}}}$ and θ. One technique is to isolate the terms with θ in each equation, and then square those equations and add them. That gives a quadratic equation for $v_{\substack{\text{boat rel.}\\ \text{shore}}}$, which is solved by $v_{\substack{\text{boat rel.}\\ \text{shore}}} = 2.177\,\text{m/s}$. Then the angle is found to be $\boxed{\theta = 69.9°\text{ N of E}}$.

92. See the sketch of the geometry. We assume that the hill is sloping downward to the right. Then if we take the point where the child jumps as the origin, with the x-direction positive to the right and the y-direction positive upwards, then the equation for the hill is given by $y = -x\tan 12°$.



The path of the child (shown by the dashed line) is projectile motion. With the same origin and coordinate system, the horizontal motion of the child is given by $x = v_0 \cos 15° (t)$, and the vertical motion of the child will be given by Eq. 2-12b, $y = v_0 \sin 15° t - \frac{1}{2} g t^2$. The landing point of the child is given by $x_{\text{landing}} = 1.4 \cos 12°$ and $y_{\text{landing}} = -1.4 \sin 12°$. Use the horizontal motion and landing point to find an expression for the time the child is in the air, and then use that time to find the initial speed.

$$x = v_0 \cos 15° (t) \quad \to \quad t = \frac{x}{v_0 \cos 15°}\ ,\ t_{\text{landing}} = \frac{1.4 \cos 12°}{v_0 \cos 15°}$$

Equate the y expressions, and use the landing time. We also use the trigonometric identity that $\sin 12° \cos 15° + \sin 15° \cos 12° = \sin(12° + 15°)$.

$$y_{\text{landing}} = y_{\text{projectile}} \quad \to \quad -1.4 \sin 12° = v_0 \sin 15° t_{\text{landing}} - \tfrac{1}{2} g t_{\text{landing}}^2 \quad \to$$

$$-1.4 \sin 12° = v_0 \sin 15° \frac{1.4 \cos 12°}{v_0 \cos 15°} - \tfrac{1}{2} g \left(\frac{1.4 \cos 12°}{v_0 \cos 15°} \right)^2 \quad \to$$

$$v_0^2 = \tfrac{1}{2} g \frac{\cos^2 12°}{\sin 27°} \left(\frac{1.4}{\cos 15°} \right) \quad \to \quad v_0 = 3.8687\,\text{m/s} \approx \boxed{3.9\,\text{m/s}}$$

93. Find the time of flight from the vertical data, using Eq. 2-12b. Call the floor the $y = 0$ location, and choose upwards as positive.

$$y = y_0 + v_{0y} t + \tfrac{1}{2} a_y t^2 \quad \to \quad 3.05\,\text{m} = 2.4\,\text{m} + (12\,\text{m/s}) \sin 35° t + \tfrac{1}{2}(-9.80\,\text{m/s}^2) t^2$$

$$4.90 t^2 - 6.883 t + 0.65\,\text{m} = 0 \quad \to$$

$$t = \frac{6.883 \pm \sqrt{6.883^2 - 4(4.90)(0.65)}}{2(4.90)} = 1.303\,\text{s},\ 0.102\,\text{s}$$

(*a*) Use the larger time for the time of flight. The shorter time is the time for the ball to rise to the basket height on the way up, while the longer time is the time for the ball to be at the basket height on the way down.

$$x = v_x t = v_0 (\cos 35°) t = (12\,\text{m/s})(\cos 35°)(1.303\,\text{s}) = 12.81\,\text{m} \approx \boxed{13\,\text{m}}$$

(*b*) The angle to the horizontal is determined by the components of the velocity.

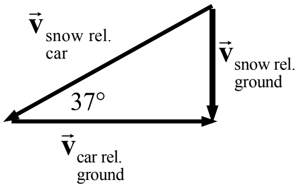
$$v_x = v_0 \cos \theta_0 = 12 \cos 35° = 9.830\,\text{m/s}$$

$$v_y = v_{y0} + at = v_0 \sin \theta_0 - gt = 12 \sin 35° - 9.80(1.303) = -5.886\,\text{m/s}$$

$$\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{-5.886}{9.830} = -30.9° \approx \boxed{-31°}$$

The negative angle means it is below the horizontal.

94. We have $v_{\substack{\text{car rel.}\\ \text{ground}}} = 25\,\text{m/s}$. Use the diagram, illustrating $\vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{car}}} + \vec{\mathbf{v}}_{\substack{\text{car rel.}\\ \text{ground}}}$, to calculate the other speeds.



$$\cos 37° = \frac{v_{\substack{\text{car rel.}\\ \text{ground}}}}{v_{\substack{\text{snow rel.}\\ \text{car}}}} \quad \to \quad v_{\substack{\text{snow rel.}\\ \text{car}}} = \frac{25\,\text{m/s}}{\cos 37°} = \boxed{31\,\text{m/s}}$$

$$\tan 37^\circ = \frac{v_{\substack{\text{snow rel.}\\ \text{ground}}}}{v_{\substack{\text{car rel.}\\ \text{ground}}}} \quad\rightarrow\quad v_{\substack{\text{snow rel.}\\ \text{ground}}} = (25\,\text{m/s})\tan 37^\circ = \boxed{19\,\text{m/s}}$$

95. Let the launch point be the origin of coordinates, with right and upwards as the positive directions. The equation of the line representing the ground is $y_{\text{gnd}} = -x$. The equations representing the motion of the rock are $x_{\text{rock}} = v_0 t$ and $y_{\text{rock}} = -\tfrac{1}{2}gt^2$, which can be combined into $y_{\text{rock}} = -\tfrac{1}{2}\frac{g}{v_0^2}x_{\text{rock}}^2$.

Find the intersection (the landing point of the rock) by equating the two expressions for *y*, and so finding where the rock meets the ground.

$$y_{\text{rock}} = y_{\text{gnd}} \quad\rightarrow\quad -\tfrac{1}{2}\frac{g}{v_0^2}x^2 = -x \quad\rightarrow\quad x = \frac{2v_0^2}{g} \quad\rightarrow\quad t = \frac{x}{v_0} = \frac{2v_0}{g} = \frac{2(25\,\text{m/s})}{9.80\,\text{m/s}^2} = \boxed{5.1\,\text{s}}$$

96. Choose the origin to be the point at ground level directly below where the ball was hit. Call upwards the positive *y* direction. For the ball, we have $v_0 = 28\,\text{m/s}$, $\theta_0 = 61^\circ$, $a_y = -g$, $y_0 = 0.9\,\text{m}$, and $y = 0.0\,\text{m}$.

(*a*) To find the horizontal displacement of the ball, the horizontal velocity and the time of flight are needed. The (constant) horizontal velocity is given by $v_x = v_0\cos\theta_0$. The time of flight is found from Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = y_0 + v_0\sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{-v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)}$$

$$= \frac{-(28\,\text{m/s})\sin 61^\circ \pm \sqrt{(28\,\text{m/s})^2\sin^2 61^\circ - 4\left(-\tfrac{1}{2}\right)\left(9.80\,\text{m/s}^2\right)(0.9\,\text{m})}}{2\left(-\tfrac{1}{2}\right)\left(9.80\,\text{m/s}^2\right)}$$

$$= 5.034\ \text{s}, -0.0365\ \text{s}$$

Choose the positive time, since the ball was hit at $t = 0$. The horizontal displacement of the ball will be found by the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = v_0\cos\theta_0 t = (28\,\text{m/s})(\cos 61^\circ)(5.034\text{s}) = 68.34\ \text{m} \approx \boxed{68\ \text{m}}$$

(*b*) The center fielder catches the ball right at ground level. He ran 105 m – 68.34 m = 36.66 m to catch the ball, so his average running speed would be as follows.

$$v_{avg} = \frac{\Delta d}{t} = \frac{36.66\ \text{m}}{5.034\,\text{s}} = 7.282\,\text{m/s} \approx \boxed{7.3\,\text{m/s}}$$

$\boxed{97.}$ Choose the origin to be the point at the top of the building from which the ball is shot, and call upwards the positive *y* direction. The initial velocity is $v_0 = 18\,\text{m/s}$ at an angle of $\theta_0 = 42^\circ$. The acceleration due to gravity is $a_y = -g$.

(*a*) $v_x = v_0\cos\theta_0 = (18\,\text{m/s})\cos 42^\circ = 13.38 \approx \boxed{13\,\text{m/s}}$

$v_{y0} = v_0\sin\theta_0 = (18\,\text{m/s})\sin 42^\circ = 12.04 \approx \boxed{12\,\text{m/s}}$

(*b*) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_x} = \frac{55\text{ m}}{13.38\,\text{m/s}} = 4.111\text{ s}$$

With that time of flight, calculate the vertical position of the ball using Eq. 2-12b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = (12.04\,\text{m/s})(4.111\text{ s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(4.111\text{ s})^2$$

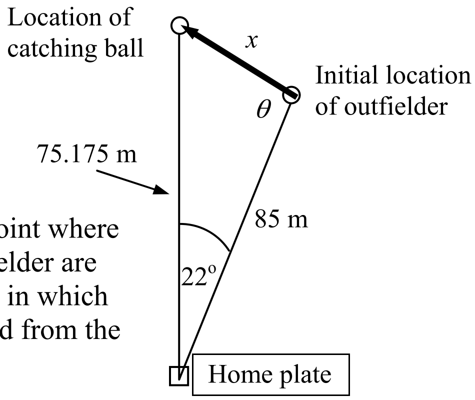
$$= -33.3 = \boxed{-33\text{ m}}$$

So the ball will strike 33 m below the top of the building.

98. Since the ball is being caught at the same height from which it was struck, use the range formula from Example 3-10 to find the horizontal distance the ball travels.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(28\,\text{m/s})^2 \sin(2\times 55^\circ)}{9.80\,\text{m/s}^2} = 75.175\text{ m}$$

Then as seen from above, the location of home plate, the point where the ball must be caught, and the initial location of the outfielder are shown in the diagram. The dark arrow shows the direction in which the outfielder must run. The length of that distance is found from the law of cosines as applied to the triangle.



$$x = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$= \sqrt{75.175^2 + 85^2 - 2(75.175)(85)\cos 22^\circ} = 32.048\,\text{m}$$

The angle θ at which the outfielder should run is found from the law of sines.

$$\frac{\sin 22^\circ}{32.048\,\text{m}} = \frac{\sin\theta}{75.175\,\text{m}} \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{75.175}{32.048}\sin 22^\circ\right) = 61.49^\circ \text{ or } 118.51^\circ$$

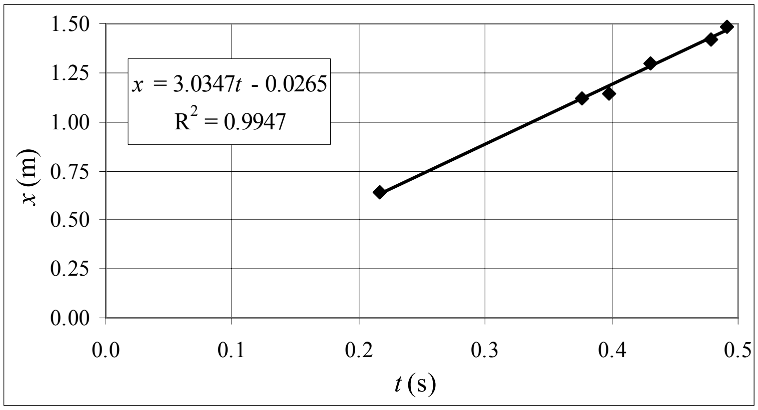
Since $75.175^2 < 85^2 + 32.048^2$, the angle must be acute, so we choose $\theta = 61.49^\circ$.

Now assume that the outfielder's time for running is the same as the time of flight of the ball. The time of flight of the ball is found from the horizontal motion of the ball at constant velocity.

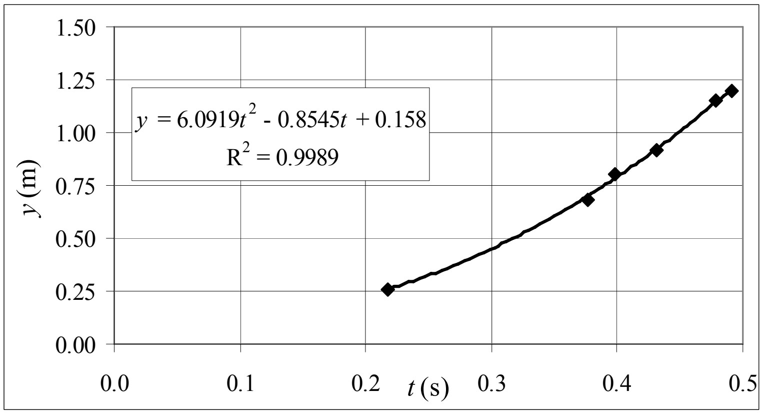
$$R = v_x t = v_0 \cos\theta_0 t \quad \rightarrow \quad t = \frac{R}{v_0\cos\theta_0} = \frac{75.175\text{ m}}{(28\,\text{m/s})\cos 55^\circ} = 4.681\,\text{s}$$

Thus the average velocity of the outfielder must be $v_{avg} = \frac{\Delta d}{t} = \frac{32.048\,\text{m}}{4.681\,\text{s}} = \boxed{6.8\,\text{m/s}}$ at an angle of $\boxed{61^\circ}$ relative to the outfielder's line of sight to home plate.

99. (*a*) To determine the best-fit straight line, the data was plotted in Excel and a linear trendline was added, giving the equation $\boxed{x = (3.03t - 0.0265)\,\text{m}}$. The initial speed of the ball is the *x*-component of the velocity, which from the equation has the value of $\boxed{3.03\,\text{m/s}}$. The graph is below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.99a."



(*b*) To determine the best-fit quadratic equation, the data was plotted in Excel and a quadratic trendline was added, giving the equation $\boxed{y = \left(0.158 - 0.855t + 6.09t^2\right)\text{m}}$. Since the quadratic term in this relationship is $\frac{1}{2}at^2$, we have the acceleration as $\boxed{12.2\,\text{m/s}^2}$. The graph is below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.99b."



100. Use the vertical motion to determine the time of flight. Let the ground be the $y = 0$ level, and choose upwards to be the positive y-direction. Use Eq. 2-12b.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; 0 = h + v_0\left(\sin\theta_0\right)t - \tfrac{1}{2}gt^2 \;\rightarrow\; \tfrac{1}{2}gt^2 - v_0\left(\sin\theta_0\right)t - h = 0$$

$$t = \frac{v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 - 4\left(\frac{1}{2}g\right)\left(-h\right)}}{2\left(\frac{1}{2}g\right)} = \frac{v_0\sin\theta_0 \pm \sqrt{v_0^2\sin^2\theta_0 + 2gh}}{g}$$

To get a positive value for the time of flight, the positive sign must be taken.

$$t = \frac{v_0\sin\theta_0 + \sqrt{v_0^2\sin^2\theta_0 + 2gh}}{g}$$

To find the horizontal range, multiply the horizontal velocity by the time of flight.

$$R = v_x t = v_0\cos\theta_0\left[\frac{v_0\sin\theta_0 + \sqrt{v_0^2\sin^2\theta_0 + 2gh}}{g}\right] = \frac{v_0^2\cos\theta_0\sin\theta_0}{g}\left[1 + \sqrt{1 + \frac{2gh}{v_0^2\sin^2\theta_0}}\right]$$

$$\boxed{R = \frac{v_0^2\sin2\theta_0}{2g}\left[1 + \sqrt{1 + \frac{2gh}{v_0^2\sin^2\theta_0}}\right]}$$

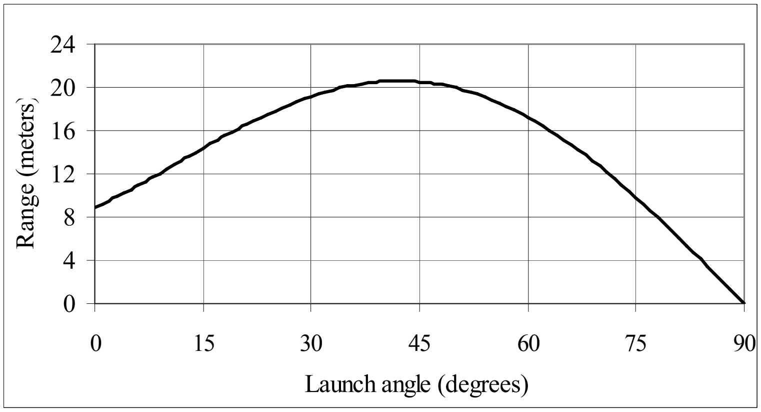
As a check, if *h* is set to 0 in the above equation, we get $R=\frac{v_0^2\sin 2\theta_0}{g}$, the level horizontal range formula.

With the values given in the problem of $v_0=13.5\,\mathrm{m/s}$, $h=2.1\,\mathrm{m}$, and $g=9.80\,\mathrm{m/s^2}$, the following relationship is obtained.

$$R=\frac{v_0^2\sin 2\theta_0}{2g}\left[1+\sqrt{1+\frac{2gh}{v_0^2\sin^2\theta_0}}\right]=\frac{(13.5)^2\sin 2\theta_0}{2(9.80)}\left[1+\sqrt{1+\frac{2(9.80)(2.1)}{(13.5)^2\sin^2\theta_0}}\right]$$

$$=9.30\sin 2\theta_0\left[1+\sqrt{1+\frac{0.226}{\sin^2\theta_0}}\right]$$

Here is a plot of that relationship. The maximum is at approximately $\boxed{42^\circ}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.100."



As a further investigation, let us find $\frac{dR}{d\theta_0}$, set it equal to 0, and solve for the angle.

$$R=\frac{v_0^2\sin 2\theta_0}{2g}\left[1+\sqrt{1+\frac{2gh}{v_0^2\sin^2\theta_0}}\right]$$

$$\frac{dR}{d\theta_0}=\frac{2v_0^2\cos 2\theta_0}{2g}\left[1+\sqrt{1+\frac{2gh}{v_0^2\sin^2\theta_0}}\right]+\frac{v_0^2\sin 2\theta_0}{2g}\left[\frac{1}{2}\left(1+\frac{2gh}{v_0^2\sin^2\theta_0}\right)^{-1/2}\left(\frac{(-2)2gh\cos\theta_0}{v_0^2\sin^3\theta_0}\right)\right]$$

$$=\frac{v_0^2}{2g}\left\{2\cos 2\theta_0\left[1+\sqrt{1+\frac{2gh}{v_0^2\sin^2\theta_0}}\right]-\sin 2\theta_0\left[\left(1+\frac{2gh}{v_0^2\sin^2\theta_0}\right)^{-1/2}\left(\frac{2gh\cos\theta_0}{v_0^2\sin^3\theta_0}\right)\right]\right\}=0$$

$$2\cos 2\theta_0\left[1+\sqrt{1+\frac{2gh}{v_0^2\sin^2\theta_0}}\right]=\sin 2\theta_0\left[\left(1+\frac{2gh}{v_0^2\sin^2\theta_0}\right)^{-1/2}\left(\frac{2gh\cos\theta_0}{v_0^2\sin^3\theta_0}\right)\right]$$

Calculate the two sides of the above equation and find where they are equal. This again happens at about 42.1°.

CHAPTER 4: Dynamics: Newton’s Laws of Motion

Responses to Questions

$\boxed{1.}$ When you give the wagon a sharp pull forward, the force of friction between the wagon and the child acts on the child to move her forward. But the force of friction acts at the contact point between the child and the wagon – either the feet, if the child is standing, or her bottom, if sitting. In either case, the lower part of the child begins to move forward, while the upper part, following Newton’s first law (the law of inertia), remains almost stationary, making it seem as if the child falls backward.

2. (*a*) Andrea, standing on the ground beside the truck, will see the box remain motionless while the truck accelerates out from under it. Since there is no friction, there is no net force on the box and it will not speed up.
   (*b*) Jim, riding on the truck, will see the box appear to accelerate backwards with respect to his frame of reference, which is not inertial. (Jim better hold on, though; if the truck bed is frictionless, he too will slide off if he is just standing!)

3. If the acceleration of an object is zero, the vector *sum* of the forces acting on the object is zero (Newton’s second law), so there can be forces on an object that has no acceleration. For example, a book resting on a table is acted on by gravity and the normal force, but it has zero acceleration, because the forces are equal in magnitude and opposite in direction.

4. Yes, the net force can be zero on a moving object. If the net force is zero, then the object’s *acceleration* is zero, but its *velocity* is not necessarily zero. [Instead of classifying objects as “moving” and “not moving,” Newtonian dynamics classifies them as “accelerating” and “not accelerating.” Both zero velocity and constant velocity fall in the “not accelerating” category.]

5. If only one force acts on an object, the object cannot have zero acceleration (Newton’s second law). It *is* possible for the object to have zero velocity, but only for an instant. For example (if we neglect air resistance), a ball thrown up into the air has only the force of gravity acting on it. Its speed will decrease while it travels upward, stop, then begin to fall back to the ground. At the instant the ball is at its highest point, its velocity is zero.

6. (*a*) Yes, there must be a force on the golf ball (Newton’s second law) to make it accelerate upward.
   (*b*) The pavement exerts the force (just like a “normal force”).

$\boxed{7.}$ As you take a step on the log, your foot exerts a force on the log in the direction opposite to the direction in which *you* want to move, which pushes the log “backwards.” (The log exerts an equal and opposite force forward on you, by Newton’s third law.) If the log had been on the ground, friction between the ground and the log would have kept the log from moving. However, the log is floating in water, which offers little resistance to the movement of the log as you push it backwards.

8. When you kick a heavy desk or a wall, your foot exerts a force on the desk or wall. The desk or wall exerts a force equal in magnitude on your foot (Newton’s third law). Ouch!

9. (*a*) The force that causes you to stop quickly is the force of friction between your shoes and the ground (plus the forces your muscles exert in moving your legs more slowly and bracing yourself).
   (*b*) If we assume the top speed of a person to be around 6 m/s (equivalent to about 12 mi/h, or a 5-minute mile), and if we assume that it take 2 s to stop, then the maximum rate of deceleration is about 3 m/s^2.

10. (*a*) When you first start riding a bicycle you need to exert a strong force to accelerate the bike and yourself. Once you are moving at a constant speed, you only need to exert a force to equal the opposite force of friction and air resistance.
    (*b*) When the bike is moving at a constant speed, the *net* force on it is zero. Since friction and air resistance are present, you would slow down if you didn't pedal to keep the net force on the bike (and you) equal to zero.

11. The father and daughter will each have the same magnitude force acting on them as they push each other away (Newton's third law). If we assume the young daughter has less mass than the father, her acceleration should be greater ($a = F/m$). Both forces, and therefore both accelerations, act over the same time interval (while the father and daughter are in contact), so the daughter's final speed will be greater than her dad's.

12. The carton would collapse (*a*). When you jump, you accelerate upward, so there must be a net upward force on you. This net upward force can only come from the normal force exerted by the carton on you and must be greater than your weight. How can you increase the normal force of a surface on you? According to Newton's third law, the carton pushes up on you just as hard as you push down on it. That means you push down with a force greater than your weight in order to accelerate upwards. If the carton can just barely support you, it will collapse when you exert this extra force.

13. If a person gives a sharp pull on the dangling thread, the thread is likely to break below the stone. In the short time interval of a sharp pull, the stone barely begins to accelerate because of its great mass (inertia), and so does not transmit the force to the upper string quickly. The stone will not move much before the lower thread breaks. If a person gives a slow and steady pull on the thread, the thread is most likely to break *above* the stone because the tension in the upper thread is the applied force *plus* the weight of the stone. Since the tension in the upper thread is greater, it is likely to break first.

14. The force of gravity on the 2-kg rock is twice as great as the force on the 1-kg rock, but the 2-kg rock has twice the mass (and twice the inertia) of the 1-kg rock. Acceleration is the ratio of force to mass ($a = F/m$, Newton's second law), so the two rocks have the same acceleration.

15. A spring responds to force, and will correctly give the force or weight in pounds, even on the Moon. Objects weigh much less on the Moon, so a spring calibrated in kilograms will give incorrect results (by a factor of 6 or so).

16. The acceleration of the box will (*c*) decrease. Newton's second law is a *vector* equation. When you pull the box at an angle θ, only the horizontal component of the force, $F\cos\theta$, will accelerate the box horizontally across the floor.

17. The Earth actually does move as seen from an inertial reference frame. But the mass of the Earth is so great, the acceleration is undetectable (Newton's second law).

18. Because the acceleration due to gravity on the Moon is less than it is on the Earth, an object with a mass of 10 kg will weigh less on the Moon than it does on the Earth. Therefore, it will be easier to lift on the Moon. (When you lift something, you exert a force to oppose its weight.) However, when throwing the object horizontally, the force needed to accelerate it to the desired horizontal speed is proportional to the object's mass, $F = ma$. Therefore, you would need to exert the same force to throw the 2-kg object on the Moon as you would on Earth.

$\boxed{19.}$ A weight of 1 N corresponds to 0.225 lb. That's about the weight of (*a*) an apple.

20. Newton's third law involves forces on *different* objects, in this case, on the two different teams. Whether or not a team moves and in what direction is determined by Newton's second law and the net force on the team. The net force on one team is the vector sum of the pull of the other team and the friction force exerted by the ground on the team. The winning team is the one that pushes hardest against the ground (and so has a greater force *on them* exerted by the ground).

21. When you stand still on the ground, two forces act on you: your weight downward, and the normal force exerted upward by the ground. You are at rest, so Newton's second law tells you that the normal force must equal your weight, *mg*. You don't rise up off the ground because the force of gravity acts downward, opposing the normal force.

22. The victim's head is not really thrown backwards during the car crash. If the victim's car was initially at rest, or even moving forward, the impact from the rear suddenly pushes the car, the seat, and the person's body forward. The head, being attached by the somewhat flexible neck to the body, can momentarily remain where it was (inertia, Newton's first law), thus lagging behind the body.

23. (*a*) The reaction force has a magnitude of 40 N.
    (*b*) It points downward.
    (*c*) It is exerted on Mary's hands and arms.
    (*d*) It is exerted by the bag of groceries.

24. No. In order to hold the backpack up, the rope must exert a vertical force equal to the backpack's weight, so that the net vertical force on the backpack is zero. The force, *F*, exerted by the rope on each side of the pack is always along the length of the rope. The vertical component of this force is *Fsinθ*, where *θ* is the angle the rope makes with the horizontal. The higher the pack goes, the smaller *θ* becomes and the larger *F* must be to hold the pack up there. No matter how hard you pull, the rope can never be horizontal because it must exert an upward (vertical) component of force to balance the pack's weight. See also Example 4-16 and Figure 4-26.

Solutions to Problems

$\boxed{1.}$ Use Newton's second law to calculate the force.

$$\sum F = ma = (55\text{ kg})(1.4\text{ m/s}^2) = \boxed{77\text{ N}}$$

2. Use Newton's second law to calculate the mass.

$$\sum F = ma \quad \rightarrow \quad m = \frac{\sum F}{a} = \frac{265\text{ N}}{2.30\text{ m/s}^2} = \boxed{115\text{ kg}}$$

3. In all cases, $W = mg$, where g changes with location.

(*a*) $W_{\text{Earth}} = mg_{\text{Earth}} = (68\text{ kg})(9.80\text{ m/s}^2) = \boxed{670\text{ N}}$

(*b*) $W_{\text{Moon}} = mg_{\text{Moon}} = (68\text{ kg})(1.7\text{ m/s}^2) = \boxed{120\text{ N}}$

(*c*) $W_{\text{Mars}} = mg_{\text{Mars}} = (68\text{ kg})(3.7\text{ m/s}^2) = \boxed{250\text{ N}}$

(*d*) $W_{\text{Space}} = mg_{\text{Space}} = (68\text{ kg})(0\text{ m/s}^2) = \boxed{0\text{ N}}$

4. Use Newton's second law to calculate the tension.

$$\sum F = F_{\mathrm{T}} = ma = (1210\,\mathrm{kg})(1.20\,\mathrm{m/s^2}) = 1452\,\mathrm{N} \approx \boxed{1.45\times10^3\,\mathrm{N}}$$

5. Find the average acceleration from Eq. 2-12c, and then find the force needed from Newton's second law. We assume the train is moving in the positive direction.

$$v = 0 \qquad v_0 = (120\,\mathrm{km/h})\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 33.33\,\mathrm{m/s} \qquad a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)}$$

$$F_{avg} = ma_{avg} = m\frac{v^2 - v_0^2}{2(x - x_0)} = (3.6\times10^5\,\mathrm{kg})\left[\frac{0 - (33.33\,\mathrm{m/s})^2}{2(150\,\mathrm{m})}\right] = -1.333\times10^6\,\mathrm{N} \approx \boxed{-1.3\times10^6\,\mathrm{N}}$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity. We compare the magnitude of this force to the weight of the train.

$$\frac{F_{avg}}{mg} = \frac{1.333\times10^6\,\mathrm{N}}{(3.6\times10^5\,\mathrm{kg})(9.80\,\mathrm{m/s^2})} = 0.3886$$

Thus the force is $\boxed{\text{39\% of the weight}}$ of the train.
By Newton's third law, the train exerts the same magnitude of force on Superman that Superman exerts on the train, but in the opposite direction. So the train exerts a force of $\boxed{1.3\times10^6\,\mathrm{N}}$ in the forward direction on Superman.

6. Find the average acceleration from Eq. 2-5. The average force on the car is found from Newton's second law.

$$v = 0 \qquad v_0 = (95\,\mathrm{km/h})\left(\frac{0.278\,\mathrm{m/s}}{1\,\mathrm{km/h}}\right) = 26.4\,\mathrm{m/s} \qquad a_{avg} = \frac{v - v_0}{t} = \frac{0 - 26.4\,\mathrm{m/s}}{8.0\ \mathrm{s}} = -3.30\,\mathrm{m/s^2}$$

$$F_{avg} = ma_{avg} = (950\,\mathrm{kg})(-3.30\,\mathrm{m/s^2}) = \boxed{-3.1\times10^3\,\mathrm{N}}$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity.

$\boxed{7.}$ Find the average acceleration from Eq. 2-12c, and then find the force needed from Newton's second law.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)} \quad \rightarrow$$

$$F_{avg} = ma_{avg} = m\frac{v^2 - v_0^2}{2(x - x_0)} = (7.0\,\mathrm{kg})\left[\frac{(13\,\mathrm{m/s})^2 - 0}{2(2.8\,\mathrm{m})}\right] = 211.25\,\mathrm{N} \approx \boxed{210\,\mathrm{N}}$$

8. The problem asks for the average force on the glove, which in a direct calculation would require knowledge about the mass of the glove and the acceleration of the glove. But no information about the glove is given. By Newton's third law, the force exerted by the ball on the glove is equal and opposite to the force exerted by the glove on the ball. So calculate the average force on the ball, and then take the opposite of that result to find the average force on the glove. The average force on the ball is its mass times its average acceleration. Use Eq. 2-12c to find the acceleration of the ball, with $v = 0,\ v_0 = 35.0\,\mathrm{m/s}$, and $x - x_0 = 0.110$ m. The initial direction of the ball is the positive direction.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (35.0\,\mathrm{m/s})^2}{2(0.110\ \mathrm{m})} = -5568\ \mathrm{m/s^2}$$

$$F_{avg} = ma_{avg} = (0.140\text{ kg})\left(-5568\text{ m/s}^2\right) = \boxed{-7.80\times10^2\text{ N}}$$

Thus the average force on the glove was 780 N, in the direction of the initial velocity of the ball.

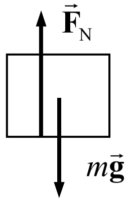
9. We assume that the fish line is pulling vertically on the fish, and that the fish is not jerking the line. A free-body diagram for the fish is shown. Write Newton's second law for the fish in the vertical direction, assuming that up is positive. The tension is at its maximum.

$$\sum F = F_{\text{T}} - mg = ma \quad \rightarrow \quad F_{\text{T}} = m(g+a) \quad \rightarrow$$

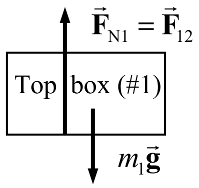
$$m = \frac{F_{\text{T}}}{g+a} = \frac{18\text{ N}}{9.80\text{ m/s}^2 + 2.5\text{ m/s}^2} = 1.5\text{ kg}$$

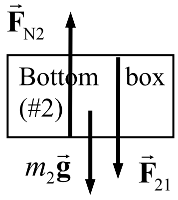
Thus a mass of 1.5 kg is the maximum that the fish line will support with the given acceleration. Since the line broke, the fish's mass is given by $\boxed{m > 1.5\text{ kg}}$ (about 3 lbs).

10. (*a*) The 20.0 kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{196\text{ N}}$. Since the box is at rest, the net force on the box must be 0, and so the normal force must also be $\boxed{196\text{ N}}$.



(*b*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's third law. $\vec{\mathbf{F}}_{\text{N2}}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must be 0. Write Newton's second law in the vertical direction for each box, taking the upward direction to be positive.





$$\sum F_1 = F_{\text{N1}} - m_1 g = 0$$

$$F_{\text{N1}} = m_1 g = (10.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{98.0\text{ N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\text{N2}} - F_{21} - m_2 g = 0$$

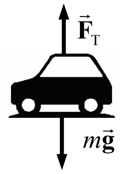
$$F_{\text{N2}} = F_{21} + m_2 g = 98.0\text{ N} + (20.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{294\text{ N}}$$

11. The average force on the pellet is its mass times its average acceleration. The average acceleration is found from Eq. 2-12c. For the pellet, $v_0 = 0$, $v = 125\text{ m/s}$, and $x - x_0 = 0.800\text{ m}$.

$$a_{avg} = \frac{v^2 - v_0^2}{2(x-x_0)} = \frac{(125\text{ m/s})^2 - 0}{2(0.800\text{ m})} = 9766\text{ m/s}^2$$

$$F_{avg} = ma_{avg} = \left(9.20\times10^{-3}\text{ kg}\right)\left(9766\text{ m/s}^2\right) = \boxed{89.8\text{ N}}$$

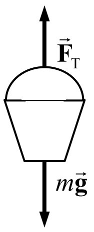
12. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the tension force.



$$\sum F = F_{\text{T}} - mg = ma \quad \rightarrow F_{\text{T}} = m(g+a)$$

$$F_{\text{T}} = (1200\text{ kg})\left(9.80\text{ m/s}^2 + 0.70\text{ m/s}^2\right) = \boxed{1.3\times10^4\text{ N}}$$

13. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the acceleration.



$$\sum F = F_T - mg = ma$$

$$a = \frac{F_T - mg}{m} = \frac{163\,\text{N} - (14.0\,\text{kg})(9.80\,\text{m/s}^2)}{14.0\,\text{kg}} = \boxed{1.8\,\text{m/s}^2}$$

Since the acceleration is positive, the bucket has an $\boxed{\text{upward}}$ acceleration.

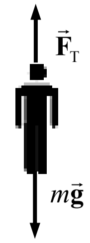
14. Use Eq. 2-12b with $v_0 = 0$ to find the acceleration.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad a = \frac{2(x - x_0)}{t^2} = \frac{2(402\ \text{m})}{(6.40\ \text{s})^2} = 19.63\,\text{m/s}^2\left(\frac{1\text{ "}g\text{"}}{9.80\,\text{m/s}^2}\right) = \boxed{2.00\,g\text{'s}}$$

The accelerating force is found by Newton's second law.

$$F = ma = (535\,\text{kg})(19.63\,\text{m/s}^2) = \boxed{1.05\times10^4\,\text{N}}$$

15. If the thief were to hang motionless on the sheets, or descend at a constant speed, the sheets would not support him, because they would have to support the full 75 kg. But if he descends with an acceleration, the sheets will not have to support the total mass. A free-body diagram of the thief in descent is shown. If the sheets can support a mass of 58 kg, then the tension force that the sheets can exert is $F_T = (58\,\text{kg})(9.80\,\text{m/s}^2) = 568\,\text{N}$.



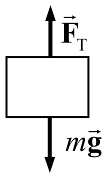
Assume that is the tension in the sheets. Then write Newton's second law for the thief, taking the upward direction to be positive.

$$\sum F = F_T - mg = ma \quad \rightarrow \quad a = \frac{F_T - mg}{m} = \frac{568\ \text{N} - (75\ \text{kg})(9.80\,\text{m/s}^2)}{75\ \text{kg}} = -2.2\,\text{m/s}^2$$

The negative sign shows that the acceleration is downward.

$\boxed{\text{If the thief descends with an acceleration of 2.2 m/s}^2\text{ or greater, the sheets will support his descent.}}$

16. In both cases, a free-body diagram for the elevator would look like the adjacent diagram. Choose up to be the positive direction. To find the MAXIMUM tension, assume that the acceleration is up. Write Newton's second law for the elevator.



$$\sum F = ma = F_T - mg \quad \rightarrow$$

$$F_T = ma + mg = m(a + g) = m(0.0680g + g) = (4850\ \text{kg})(1.0680)(9.80\,\text{m/s}^2)$$

$$= \boxed{5.08\times10^4\,\text{N}}$$

To find the MINIMUM tension, assume that the acceleration is down. Then Newton's second law for the elevator becomes the following.

$$\sum F = ma = F_T - mg \quad \rightarrow \quad F_T = ma + mg = m(a + g) = m(-0.0680g + g)$$

$$= (4850\ \text{kg})(0.9320)(9.80\,\text{m/s}^2) = \boxed{4.43\times10^4\,\text{N}}$$

17. Use Eq. 2-12c to find the acceleration. The starting speed is $35\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 9.72\,\text{m/s}$.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (9.72\,\text{m/s})^2}{2(0.017\,\text{m})} = -2779\,\text{m/s}^2 \approx \boxed{-2800\,\text{m/s}^2}$$

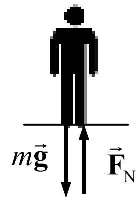
$$2779\,\text{m/s}^2\left(\frac{1g}{9.80\,\text{m/s}^2}\right)=284g\text{'s}\approx\boxed{280g\text{'s}}$$

The acceleration is negative because the car is slowing down. The required force is found by Newton's second law.

$$F=ma=(68\,\text{kg})\left(2779\,\text{m/s}^2\right)=\boxed{1.9\times10^5\,\text{N}}$$

This huge acceleration would not be possible unless the car hit some very heavy, stable object.

18. There will be two forces on the person – their weight, and the normal force of the scales pushing up on the person. A free-body diagram for the person is shown. Choose up to be the positive direction, and use Newton's second law to find the acceleration.

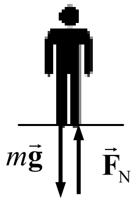


$$\sum F=F_\text{N}-mg=ma\quad\rightarrow\quad0.75mg-mg=ma\quad\rightarrow$$

$$a=-0.25g=-0.25\left(9.8\,\text{m/s}^2\right)=\boxed{-2.5\,\text{m/s}^2}$$

Due to the sign of the result, the direction of the acceleration is $\boxed{\text{down}}$. Thus the elevator must have started to move down since it had been motionless.

$\boxed{19.}$ (*a*) To calculate the time to accelerate from rest, use Eq. 2-12a.



$$v=v_0+at\quad\rightarrow\quad t=\frac{v-v_0}{a}=\frac{9.0\,\text{m/s}-0}{1.2\,\text{m/s}^2}=7.5\,\text{s}$$

The distance traveled during this acceleration is found from Eq. 2-12b.

$$x-x_0=v_0t+\tfrac{1}{2}at^2=\tfrac{1}{2}\left(1.2\,\text{m/s}^2\right)(7.5\,\text{s})^2=33.75\,\text{m}$$

To calculate the time to decelerate to rest, use Eq. 2-12a.

$$v=v_0+at\quad\rightarrow\quad t=\frac{v-v_0}{a}=\frac{0-9.0\,\text{m/s}}{-1.2\,\text{m/s}^2}=7.5\,\text{s}$$

The distance traveled during this deceleration is found from Eq. 2-12b.

$$x-x_0=v_0t+\tfrac{1}{2}at^2=(9.0\,\text{m/s})(7.5\,\text{s})+\tfrac{1}{2}\left(-1.2\,\text{m/s}^2\right)(7.5\,\text{s})^2=33.75\,\text{m}$$

To distance traveled at constant velocity is $180\,\text{m}-2(33.75\,\text{m})=112.5\,\text{m}$.

To calculate the time spent at constant velocity, use Eq. 2-8.

$$x=x_0+\bar{v}t\quad\rightarrow\quad t=\frac{x-x_0}{\bar{v}}=\frac{112.5\,\text{m/s}}{9.0\,\text{m/s}}=12.5\,\text{s}\approx13\,\text{s}$$

Thus the times for each stage are:

$$\boxed{\text{Accelerating: }7.5\,\text{s}\quad\text{Constant Velocity: }13\,\text{s}\quad\text{Decelerating: }7.5\,\text{s}}$$

(*b*) The normal force when at rest is *mg*. From the free-body diagram, if up is the positive direction, we have that $F_\text{N}-mg=ma$. Thus the change in normal force is the difference in the normal force and the weight of the person, or ma.

$$\text{Accelerating:}\quad\frac{\Delta F_N}{F_N}=\frac{ma}{mg}=\frac{a}{g}=\frac{1.2\,\text{m/s}^2}{9.80\,\text{m/s}^2}\times100=\boxed{12\%}$$

$$\text{Constant velocity:}\quad\frac{\Delta F_N}{F_N}=\frac{ma}{mg}=\frac{a}{g}=\frac{0}{9.80\,\text{m/s}^2}\times100=\boxed{0\%}$$

$$\text{Decelerating:}\quad\frac{\Delta F_N}{F_N}=\frac{ma}{mg}=\frac{a}{g}=\frac{-1.2\,\text{m/s}^2}{9.80\,\text{m/s}^2}\times100=\boxed{-12\%}$$

(*c*) The normal force is not equal to the weight during the accelerating and deceleration phases.

$$\frac{7.5\,\mathrm{s}+7.5\,\mathrm{s}}{7.5\,\mathrm{s}+12.5\,\mathrm{s}+7.5\,\mathrm{s}}=\boxed{55\%}$$

20. The ratio of accelerations is the same as the ratio of the force.

$$\frac{a_{\text{optics}}}{g}=\frac{ma_{\text{optics}}}{mg}=\frac{F_{\text{optics}}}{mg}=\frac{F_{\text{optics}}}{\rho\left(\frac{4}{3}\pi r^{3}\right)g}$$

$$=\frac{10\times10^{-12}\,\mathrm{N}}{\left(\frac{1.0\,\mathrm{g}}{1.0\,\mathrm{cm}^{3}}\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\frac{10^{6}\,\mathrm{cm}^{3}}{1\,\mathrm{m}^{3}}\right)\frac{4}{3}\pi\left(.5\times10^{-6}\,\mathrm{m}\right)^{3}\left(9.80\,\mathrm{m/s^{2}}\right)}=1949\quad\rightarrow$$

$$a\approx\boxed{2000\,g\text{'s}}$$

21. (*a*) Since the rocket is exerting a downward force on the gases, the gases will exert an upward force on the rocket, typically called the thrust. The free-body diagram for the rocket shows two forces – the thrust and the weight. Newton's second law can be used to find the acceleration of the rocket.

$$\sum F=F_{\mathrm{T}}-mg=ma\quad\rightarrow$$

$$a=\frac{F_{\mathrm{T}}-mg}{m}=\frac{3.55\times10^{7}\,\mathrm{N}-\left(2.75\times10^{6}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^{2}}\right)}{\left(2.75\times10^{6}\,\mathrm{kg}\right)}=3.109\,\mathrm{m/s^{2}}\approx\boxed{3.1\,\mathrm{m/s^{2}}}$$

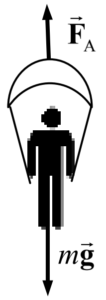
(*b*) The velocity can be found from Eq. 2-12a.

$$v=v_{0}+at=0+\left(3.109\,\mathrm{m/s^{2}}\right)\left(8.0\,\mathrm{s}\right)=24.872\,\mathrm{m/s}\approx\boxed{25\,\mathrm{m/s}}$$

(*c*) The time to reach a displacement of 9500 m can be found from Eq. 2-12b.

$$x-x_{0}=v_{0}t+\tfrac{1}{2}at^{2}\quad\rightarrow\quad t=\sqrt{\frac{2\left(x-x_{0}\right)}{a}}=\sqrt{\frac{2\left(9500\,\mathrm{m}\right)}{\left(3.109\,\mathrm{m/s^{2}}\right)}}=\boxed{78\,\mathrm{s}}$$

22. (*a*) There will be two forces on the skydivers – their combined weight, and the upward force of air resistance, $\vec{\mathbf{F}}_{\mathrm{A}}$. Choose up to be the positive direction. Write Newton's second law for the skydivers.



$$\sum F=F_{\mathrm{A}}-mg=ma\quad\rightarrow\quad0.25mg-mg=ma\quad\rightarrow$$

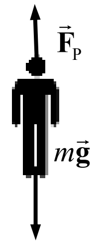
$$a=-0.75g=-0.75\left(9.80\,\mathrm{m/s^{2}}\right)=\boxed{-7.35\,\mathrm{m/s^{2}}}$$

Due to the sign of the result, the direction of the acceleration is down.

(*b*) If they are descending at constant speed, then the net force on them must be zero, and so the force of air resistance must be equal to their weight.

$$F_{\mathrm{A}}=mg=\left(132\text{ kg}\right)\left(9.80\,\mathrm{m/s^{2}}\right)=\boxed{1.29\times10^{3}\,\mathrm{N}}$$

23. The velocity that the person must have when losing contact with the ground is found from Eq. 2-12c, using the acceleration due to gravity, with the condition that their speed at the top of the jump is 0. We choose up to be the positive direction.



$$v^{2}=v_{0}^{2}+2a\left(x-x_{0}\right)\quad\rightarrow$$

$$v_{0}=\sqrt{v^{2}-2a\left(x-x_{0}\right)}=\sqrt{0-2\left(-9.80\,\mathrm{m/s^{2}}\right)\left(0.80\,\mathrm{m}\right)}=3.960\,\mathrm{m/s}$$

This velocity is the velocity that the jumper must have as a result of pushing with their legs. Use that velocity with Eq. 2-12c again to find what acceleration the jumper must have during their push on the floor, given that their starting speed is 0.

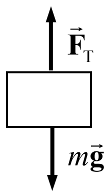
$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{\left(3.960\,\text{m/s}\right)^2 - 0}{2\left(0.20\,\text{m}\right)} = 39.20\,\text{m/s}^2$$

Finally, use this acceleration to find the pushing force against the ground.

$$\sum F = F_\text{P} - mg = ma \quad \rightarrow$$

$$F_\text{P} = m\left(g + a\right) = \left(68\,\text{kg}\right)\left(9.80\,\text{m/s}^2 + 39.20\,\text{m/s}^2\right) = \boxed{3300\,\text{N}}$$

24. Choose UP to be the positive direction. Write Newton's second law for the elevator.



$$\sum F = F_\text{T} - mg = ma \quad \rightarrow$$

$$a = \frac{F_\text{T} - mg}{m} = \frac{21{,}750\text{ N} - \left(2125\text{ kg}\right)\left(9.80\,\text{m/s}^2\right)}{2125\text{ kg}} = 0.4353\,\text{m/s}^2 \approx \boxed{0.44\,\text{m/s}^2}$$

$\boxed{25.}$ We break the race up into two portions. For the acceleration phase, we call the distance d_1 and the time t_1. For the constant speed phase, we call the distance d_2 and the time t_2. We know that $d_1 = 45\,\text{m},\ d_2 = 55\,\text{m},\text{ and } t_2 = 10.0\,\text{s} - t_1$. Eq. 2-12b is used for the acceleration phase and Eq. 2-2 is used for the constant speed phase. The speed during the constant speed phase is the final speed of the acceleration phase, found from Eq. 2-12a.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad d_1 = \tfrac{1}{2}at_1^2 \ ; \ \Delta x = vt \quad \rightarrow \quad d_2 = vt_2 = v\left(10.0\,\text{s} - t_1\right) \ ; \ v = v_0 + at_1$$

This set of equations can be solved for the acceleration and the velocity.

$$d_1 = \tfrac{1}{2}at_1^2 \ ; \ d_2 = v\left(10.0\,\text{s} - t_1\right) \ ; \ v = at_1 \quad \rightarrow \quad 2d_1 = at_1^2 \ ; \ d_2 = at_1\left(10.0 - t_1\right) \quad \rightarrow$$

$$a = \frac{2d_1}{t_1^2} \ ; \ d_2 = \frac{2d_1}{t_1^2}t_1\left(10.0 - t_1\right) = \frac{2d_1}{t_1}\left(10.0 - t_1\right) \quad \rightarrow \quad d_2 t_1 = 2d_1\left(10.0 - t_1\right) \quad \rightarrow$$

$$t_1 = \frac{20.0 d_1}{\left(d_2 + 2d_1\right)} \quad \rightarrow \quad a = \frac{2d_1}{t_1^2} = \frac{2d_1}{\left[\dfrac{20.0 d_1}{\left(d_2 + 2d_1\right)}\right]^2} = \frac{\left(d_2 + 2d_1\right)^2}{\left(200\,\text{s}^2\right)d_1}$$

$$v = at_1 = \frac{\left(d_2 + 2d_1\right)^2}{200 d_1}\frac{20.0 d_1}{\left(d_2 + 2d_1\right)} = \frac{\left(d_2 + 2d_1\right)}{10.0\,\text{s}}$$

(*a*) The horizontal force is the mass of the sprinter times their acceleration.

$$F = ma = m\frac{\left(d_2 + 2d_1\right)^2}{\left(200\,\text{s}^2\right)d_1} = \left(66\,\text{kg}\right)\frac{\left(145\,\text{m}\right)^2}{\left(200\,\text{s}^2\right)\left(45\,\text{m}\right)} = 154\,\text{N} \approx \boxed{150\,\text{N}}$$

(*b*) The velocity for the second portion of the race was found above.

$$v = \frac{\left(d_2 + 2d_1\right)}{10.0\,\text{s}} = \frac{145\,\text{m}}{10.0\,\text{s}} = \boxed{14.5\,\text{m/s}}$$

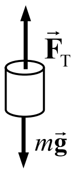
26. (*a*) Use Eq. 2-12c to find the speed of the person just before striking the ground. Take down to be the positive direction. For the person, $v_0 = 0,\ \ y - y_0 = 3.9\,\text{m},\text{ and } a = 9.80\,\text{m/s}^2$.

$$v^2 - v_0^2 = 2a\left(y - y_0\right) \quad \rightarrow \quad v = \sqrt{2a\left(y - y_0\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(3.9\text{ m}\right)} = 8.743 = \boxed{8.7\,\text{m/s}}$$

(*b*) For the deceleration, use Eq. 2-12c to find the average deceleration, choosing down to be positive.

$$v_0 = 8.743\,\text{m/s} \qquad v = 0 \qquad y - y_0 = 0.70\text{ m} \qquad v^2 - v_0^2 = 2a\left(y - y_0\right) \rightarrow$$

$$a = \frac{-v_0^2}{2\Delta y} = \frac{-\left(8.743\,\text{m/s}\right)^2}{2\left(0.70\text{ m}\right)} = -54.6\,\text{m/s}^2$$



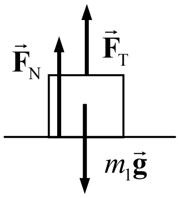
The average force on the torso $\left(F_\text{T}\right)$ due to the legs is found from Newton's second law. See the free-body diagram. Down is positive.

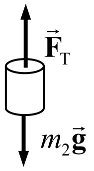
$$F_\text{net} = mg - F_\text{T} = ma \quad \rightarrow$$

$$F_\text{T} = mg - ma = m\left(g - a\right) = \left(42\,\text{kg}\right)\left(9.80\,\text{m/s}^2 - -54.6\,\text{m/s}^2\right) = \boxed{2.7\times10^3\,\text{N}}$$

The force is upward.

27. Free-body diagrams for the box and the weight are shown below. The tension exerts the same magnitude of force on both objects.





(*a*) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, and so the sum of the forces on it will be zero. For the box,

$$F_\text{N} + F_\text{T} - m_1 g = 0 \quad \rightarrow \quad F_\text{N} = m_1 g - F_\text{T} = m_1 g - m_2 g = 77.0\text{N} - 30.0\text{ N} = \boxed{47.0\text{ N}}$$

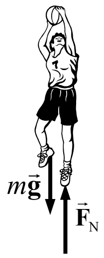
(*b*) The same analysis as for part (*a*) applies here.

$$F_\text{N} = m_1 g - m_2 g = 77.0\text{ N} - 60.0\text{ N} = \boxed{17.0\text{ N}}$$

(*c*) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be $\boxed{0\text{ N}}$.

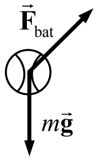
28. (*a*) Just before the player leaves the ground, the forces on the player are his weight and the floor pushing up on the player. If the player jumps straight up, then the force of the floor will be straight up – a normal force. See the first diagram. In this case, while touching the floor, $F_\text{N} > mg$.

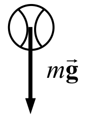
(*b*) While the player is in the air, the only force on the player is their weight. See the second diagram.



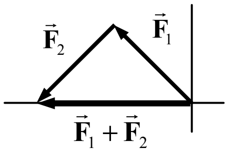
29. (*a*) Just as the ball is being hit, ignoring air resistance, there are two main forces on the ball: the weight of the ball, and the force of the bat on the ball.

(*b*) As the ball flies toward the outfield, the only force on it is its weight, if air resistance is ignored.

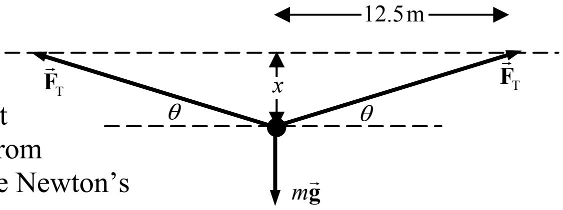




30. The two forces must be oriented so that the northerly component of the first force is exactly equal to the southerly component of the second force. Thus the second force must act $\boxed{\text{southwesterly}}$. See the diagram.



$\boxed{31.}$ (*a*) We draw a free-body diagram for the piece of the rope that is directly above the person. That piece of rope should be in equilibrium. The person's weight will be pulling down on that spot, and the rope tension will be pulling away from that spot towards the points of attachment. Write Newton's second law for that small piece of the rope.



$$\sum F_y = 2F_T \sin\theta - mg = 0 \;\rightarrow\; \theta = \sin^{-1}\frac{mg}{2F_T} = \sin^{-1}\frac{(72.0\,\text{kg})(9.80\,\text{m/s}^2)}{2(2900\,\text{N})} = 6.988^\circ$$

$$\tan\theta = \frac{x}{12.5\,\text{m}} \;\rightarrow\; x = (12.5\,\text{m})\tan 6.988^\circ = 1.532\,\text{m} \approx \boxed{1.5\,\text{m}}$$

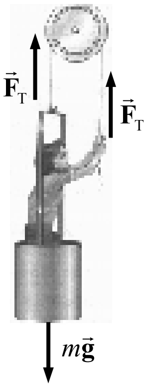
(*b*) Use the same equation to solve for the tension force with a sag of only ¼ that found above.

$$x = \tfrac{1}{4}(1.532\,\text{m}) = 0.383\,\text{m} \;;\; \theta = \tan^{-1}\frac{0.383\,\text{m}}{12.5\,\text{m}} = 1.755^\circ$$

$$F_T = \frac{mg}{2\sin\theta} = \frac{(72.0\,\text{kg})(9.80\,\text{m/s}^2)}{2(\sin 1.755^\circ)} = \boxed{11.5\,\text{kN}}$$

The $\boxed{\text{rope will not break}}$, but it exceeds the recommended tension by a factor of about 4.

32. The window washer pulls down on the rope with her hands with a tension force F_T, so the rope pulls up on her hands with a tension force F_T. The tension in the rope is also applied at the other end of the rope, where it attaches to the bucket. Thus there is another force F_T pulling up on the bucket. The bucket-washer combination thus has a net force of $2F_T$ upwards. See the adjacent free-body diagram, showing only forces <u>on</u> the bucket-washer combination, not forces exerted <u>by</u> the combination (the pull down on the rope by the person) or internal forces (normal force of bucket on person).



(*a*) Write Newton's second law in the vertical direction, with up as positive. The net force must be zero if the bucket and washer have a constant speed.

$$\sum F = F_T + F_T - mg = 0 \;\rightarrow\; 2F_T = mg \;\rightarrow$$

$$F_T = \tfrac{1}{2}mg = \tfrac{1}{2}(72\,\text{kg})(9.80\,\text{m/s}^2) = 352.8\,\text{N} \approx \boxed{350\,\text{N}}$$

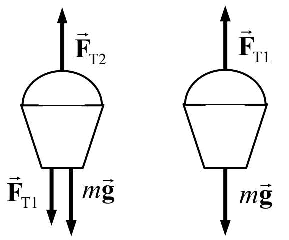
(*b*) Now the force is increased by 15%, so $F_T = 358.2\,\text{N}(1.15) = 405.72\,\text{N}$. Again write Newton's second law, but with a non-zero acceleration.

$$\sum F = F_T + F_T - mg = ma \;\rightarrow$$

$$a = \frac{2F_T - mg}{m} = \frac{2(405.72\,\text{N}) - (72\,\text{kg})(9.80\,\text{m/s}^2)}{72\,\text{kg}} = 1.47\,\text{m/s}^2 \approx \boxed{1.5\,\text{m/s}^2}$$

33. We draw free-body diagrams for each bucket.

(*a*) Since the buckets are at rest, their acceleration is 0. Write Newton's second law for each bucket, calling UP the positive direction.



Top (# 2) Bottom (# 1)

$$\sum F_1 = F_{T1} - mg = 0 \;\rightarrow$$

$$F_{T1} = mg = (3.2\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{31\,\text{N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = 0 \quad \rightarrow$$

$$F_{T2} = F_{T1} + mg = 2mg = 2(3.2\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{63\,\text{N}}$$

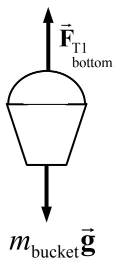
(*b*) Now repeat the analysis, but with a non-zero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{T1} - mg = ma \quad \rightarrow$$

$$F_{T1} = mg + ma = (3.2\,\text{kg})(9.80\,\text{m/s}^2 + 1.25\,\text{m/s}^2) = 35.36\,\text{N} \approx \boxed{35\,\text{N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = ma \quad \rightarrow \quad F_{T2} = F_{T1} + mg + ma = 2F_{T1} = \boxed{71\,\text{N}}$$

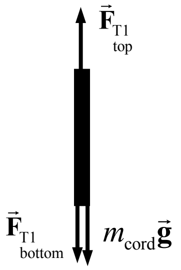
34. See the free-body diagram for the bottom bucket, and write Newton's second law to find the tension. Take the upward direction as positive.



$$\sum F = F_{\substack{\text{T1}\\\text{bottom}}} - m_{\text{bucket}}g = m_{\text{bucket}}a \quad \rightarrow$$

$$F_{\substack{\text{T1}\\\text{bottom}}} = m_{\text{bucket}}(g+a) = (3.2\,\text{kg})(9.80\,\text{m/s}^2 + 1.25\,\text{m/s}^2) = 35.36\,\text{N} \approx \boxed{35\,\text{N}}$$

Next, see the free-body for the rope between the buckets. The mass of the cord is given by $m_{\text{cord}} = \dfrac{W_{\text{cord}}}{g}$.



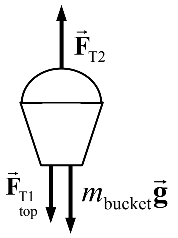
$$\sum F = F_{\substack{\text{T1}\\\text{top}}} - m_{\text{cord}}g - F_{\substack{\text{T1}\\\text{bottom}}} = m_{\text{cord}}a \quad \rightarrow$$

$$F_{\substack{\text{T1}\\\text{top}}} = F_{\substack{\text{T1}\\\text{bottom}}} + m_{\text{cord}}(g+a) = m_{\text{bucket}}(g+a) + m_{\text{cord}}(g+a)$$

$$= \left(m_{\text{bucket}} + \frac{W_{\text{cord}}}{g}\right)(g+a) = \left(3.2\,\text{kg} + \frac{2.0\,\text{N}}{9.80\,\text{m/s}^2}\right)(11.05\,\text{m/s}^2)$$

$$= 37.615\,\text{N} \approx \boxed{38\,\text{N}}$$

Note that this is the same as saying that the tension at the top is accelerating the bucket and cord together.



Now use the free-body diagram for the top bucket to find the tension at the bottom of the second cord.

$$\sum F = F_{\text{T2}} - F_{\substack{\text{T1}\\\text{top}}} - m_{\text{bucket}}g = m_{\text{bucket}}a \quad \rightarrow$$

$$F_{\text{T2}} = F_{\substack{\text{T1}\\\text{top}}} + m_{\text{bucket}}(g+a) = m_{\text{bucket}}(g+a) + m_{\text{cord}}(g+a) + m_{\text{bucket}}(g+a)$$

$$= (2m_{\text{bucket}} + m_{\text{cord}})(g+a) = \left(2m_{\text{bucket}} + \frac{W_{\text{cord}}}{g}\right)(g+a)$$

$$= \left(2(3.2\,\text{kg}) + \frac{2.0\,\text{N}}{9.80\,\text{m/s}^2}\right)(11.05\,\text{m/s}^2) = 72.98\,\text{N} \approx \boxed{73\,\text{N}}$$

Note that this is the same as saying that the tension in the top cord is accelerating the two buckets and the connecting cord.

35. Choose the *y* direction to be the "forward" direction for the motion of the snowcats, and the *x* direction to be to the right on the diagram in the textbook. Since the housing unit moves in the forward direction on a straight line, there is no acceleration in the *x* direction, and so the net force in the *x* direction must be 0. Write Newton's second law for the *x* direction.

$$\sum F_x = F_{\text{A}x} + F_{\text{B}x} = 0 \rightarrow -F_{\text{A}}\sin 48^\circ + F_{\text{B}}\sin 32^\circ = 0 \rightarrow$$

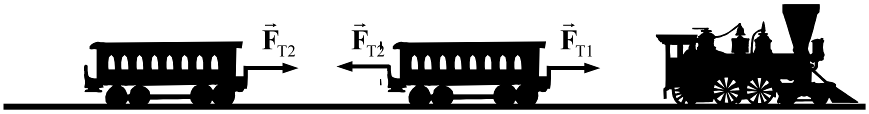
$$F_{\text{B}} = \frac{F_{\text{A}}\sin 48^\circ}{\sin 32^\circ} = \frac{(4500\,\text{N})\sin 48^\circ}{\sin 32^\circ} = 6311\,\text{N} \approx \boxed{6300\,\text{N}}$$

Since the *x* components add to 0, the magnitude of the vector sum of the two forces will just be the sum of their *y* components.

$$\sum F_y = F_{\text{A}y} + F_{\text{B}y} = F_{\text{A}}\cos 48^\circ + F_{\text{B}}\cos 32^\circ = (4500\,\text{N})\cos 48^\circ + (6311\,\text{N})\cos 32^\circ$$

$$= 8363\,\text{N} \approx \boxed{8400\,\text{N}}$$

36. Since all forces of interest in this problem are horizontal, draw the free-body diagram showing only the horizontal forces. $\vec{\mathbf{F}}_{\text{T1}}$ is the tension in the coupling between the locomotive and the first car, and it pulls to the right on the first car. $\vec{\mathbf{F}}_{\text{T2}}$ is the tension in the coupling between the first car an the second car. It pulls to the right on car 2, labeled $\vec{\mathbf{F}}_{\text{T2R}}$ and to the left on car 1, labeled $\vec{\mathbf{F}}_{\text{T2L}}$. Both cars have the same mass *m* and the same acceleration *a*. Note that $\left|\vec{\mathbf{F}}_{\text{T2R}}\right| = \left|\vec{\mathbf{F}}_{\text{T2L}}\right| = F_{T2}$ by Newton's third law.



Write a Newton's second law expression for each car.

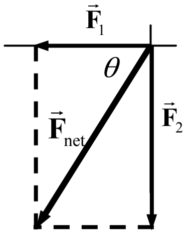
$$\sum F_1 = F_{T1} - F_{T2} = ma \qquad \sum F_2 = F_{T2} = ma$$

Substitute the expression for ma from the second expression into the first one.

$$F_{T1} - F_{T2} = ma = F_{T2} \rightarrow F_{\text{T1}} = 2F_{\text{T2}} \rightarrow \boxed{F_{\text{T1}}/F_{\text{T2}} = 2}$$

This can also be discussed in the sense that the tension between the locomotive and the first car is pulling 2 cars, while the tension between the cars is only pulling one car.

$\boxed{37.}$ The net force in each case is found by vector addition with components.



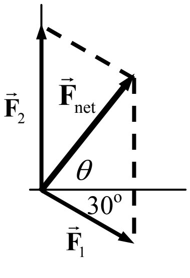
(*a*) $F_{\text{Net x}} = -F_1 = -10.2\text{ N} \quad F_{\text{Net y}} = -F_2 = -16.0\text{ N}$

$$F_{\text{Net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = 19.0\text{ N} \qquad \theta = \tan^{-1}\frac{-16.0}{-10.2} = 57.48^\circ$$

The actual angle from the *x*-axis is then 237.48°. Thus the net force is

$$F_{\text{Net}} = \boxed{19.0\text{ N at }237.5^\circ}$$

$$a = \frac{F_{\text{Net}}}{m} = \frac{19.0\,\text{N}}{18.5\,\text{kg}} = \boxed{1.03\,\text{m/s}^2\text{ at }237.5^\circ}$$

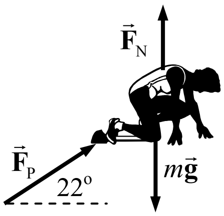


(*b*) $F_{\text{Net x}} = F_1\cos 30^\circ = 8.833\text{ N} \quad F_{\text{Net y}} = F_2 - F_1\sin 30^\circ = 10.9\,\text{N}$

$$F_{\text{Net}} = \sqrt{(8.833\text{ N})^2 + (10.9\text{ N})^2} = 14.03\text{N} \approx \boxed{14.0\text{ N}}$$

$$\theta = \tan^{-1}\frac{10.9}{8.833} = \boxed{51.0^\circ} \qquad a = \frac{F_{\text{Net}}}{m} = \frac{14.03\text{ N}}{18.5\text{ kg}} = \boxed{0.758\,\text{m/s}^2}\text{ at }\boxed{51.0^\circ}$$

38. Since the sprinter exerts a force of 720 N on the ground at an angle of 22° below the horizontal, by Newton's third law the ground will exert a force of 720 N on the sprinter at an angle of 22° above the horizontal. A free-body diagram for the sprinter is shown.



(*a*) The horizontal acceleration will be found from the net horizontal force. Using Newton's second law, we have the following.

$$\sum F_x = F_P\cos 22^\circ = ma_x \quad\rightarrow\quad a_x = \frac{F_P\cos 22^\circ}{m} = \frac{(720\text{ N})\cos 22^\circ}{65\text{ kg}}$$

$$= 10.27\,\text{m/s}^2 \approx \boxed{1.0\times 10^1\,\text{m/s}^2}$$

(*b*) Eq. 2-12a is used to find the final speed. The starting speed is 0.

$$v = v_0 + at \quad\rightarrow\quad v = 0 + at = (10.27\,\text{m/s}^2)(0.32\text{ s}) = 3.286\,\text{m/s} \approx \boxed{3.3\,\text{m/s}}$$

39. During the time while the force is F_0, the acceleration is $a = \dfrac{F_0}{m}$. Thus the distance traveled would be given by Eq. 2-12b, with a 0 starting velocity, $x - x_0 = v_0t + \frac{1}{2}at^2 = \frac{1}{2}\dfrac{F_0}{m}t_0^2$. The velocity at the end of that time is given by Eq. 2-12a, $v = v_0 + at = 0 + \left(\dfrac{F_0}{m}\right)t_0$. During the time while the force is $2F_0$, the acceleration is $a = \dfrac{2F_0}{m}$. The distance traveled during this time interval would again be given by Eq. 2-12b, with a starting velocity of $\left(\dfrac{F_0}{m}\right)t_0$.

$$x - x_0 = v_0t + \tfrac{1}{2}at^2 = \left[\left(\frac{F_0}{m}\right)t_0\right]t_0 + \tfrac{1}{2}\left(\frac{2F_0}{m}\right)t_0^2 = 2\frac{F_0}{m}t_0^2$$

The total distance traveled is $\frac{1}{2}\dfrac{F_0}{m}t_0^2 + 2\dfrac{F_0}{m}t_0^2 = \boxed{\dfrac{5}{2}\dfrac{F_0}{m}t_0^2}$.

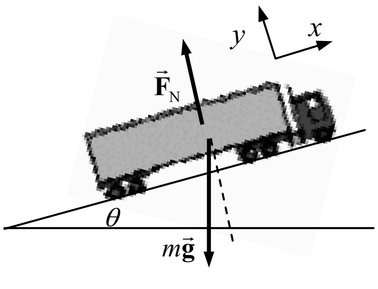
40. Find the net force by adding the force vectors. Divide that net force by the mass to find the acceleration, and then use Eq. 3-13a to find the velocity at the given time.

$$\sum\vec{\mathbf{F}} = (16\hat{\mathbf{i}} + 12\hat{\mathbf{j}})\text{N} + (-10\hat{\mathbf{i}} + 22\hat{\mathbf{j}})\text{N} = (6\hat{\mathbf{i}} + 34\hat{\mathbf{j}})\text{N} = m\vec{\mathbf{a}} = (3.0\,\text{kg})\vec{\mathbf{a}} \rightarrow$$

$$\vec{\mathbf{a}} = \frac{(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}})\text{N}}{3.0\,\text{kg}} \qquad \vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = 0 + \frac{(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}})\text{N}}{3.0\,\text{kg}}(3.0\,\text{s}) = \boxed{(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}})\,\text{m/s}}$$

In magnitude and direction, the velocity is $35\,\text{m/s}$ at an angle of 80°.

41. For a simple ramp, the decelerating force is the component of gravity along the ramp. See the free-body diagram, and use Eq. 2-12c to calculate the distance.



$$\sum F_x = -mg\sin\theta = ma \quad\rightarrow\quad a = -g\sin\theta$$

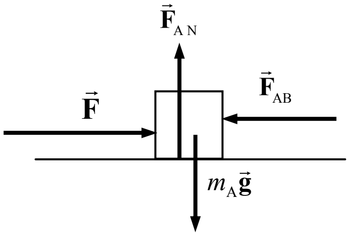
$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - v_0^2}{2(-g\sin\theta)} = \frac{v_0^2}{2g\sin\theta}$$

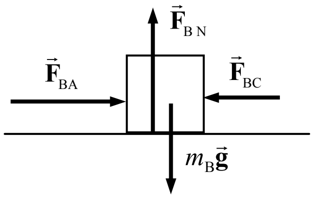
$$F_{\substack{\text{net}\\ \text{y}}} = F_{\text{T}}\cos\theta - mg = 0 \;\rightarrow\; F_{\text{T}} = \frac{mg}{\cos\theta} \;\rightarrow$$

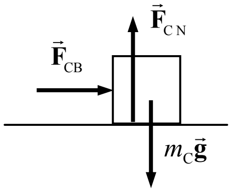
$$F_{\text{H}} = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (27\,\text{kg})\left(9.80\,\text{m/s}^2\right)\tan 2.15° = \boxed{9.9\,\text{N}}$$

$$(b)\quad F_{\text{T}} = \frac{mg}{\cos\theta} = \frac{(27\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{\cos 2.15°} = \boxed{260\,\text{N}}$$

46. (*a*) In the free-body diagrams below, $\vec{\mathbf{F}}_{\text{AB}}$ = force on block A exerted by block B, $\vec{\mathbf{F}}_{\text{BA}}$ = force on block B exerted by block A, $\vec{\mathbf{F}}_{\text{BC}}$ = force on block B exerted by block C, and $\vec{\mathbf{F}}_{\text{CB}}$ = force on block C exerted by block B. The magnitudes of $\vec{\mathbf{F}}_{\text{BA}}$ and $\vec{\mathbf{F}}_{\text{AB}}$ are equal, and the magnitudes of $\vec{\mathbf{F}}_{\text{BC}}$ and $\vec{\mathbf{F}}_{\text{CB}}$ are equal, by Newton's third law.







(*b*) All of the vertical forces on each block add up to zero, since there is no acceleration in the vertical direction. Thus for each block, $F_N = mg$. For the horizontal direction, we have the following.

$$\sum F = F - F_{\text{AB}} + F_{\text{BA}} - F_{\text{BC}} + F_{\text{CB}} = F = \left(m_{\text{A}} + m_{\text{B}} + m_{\text{C}}\right)a \;\rightarrow\; \boxed{a = \frac{F}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}}$$

(*c*) For each block, the net force must be *ma* by Newton's second law. Each block has the same acceleration since they are in contact with each other.

$$\boxed{F_{\text{A net}} = F\frac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}} \qquad \boxed{F_{\text{B net}} = F\frac{m_{\text{B}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}} \qquad \boxed{F_{3\,net} = F\frac{m_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}}$$

(*d*) From the free-body diagram, we see that for m_{C}, $F_{\text{CB}} = F_{\text{C net}} = \boxed{F\frac{m_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}}$. And by Newton's third law, $F_{\text{BC}} = F_{\text{CB}} = \boxed{F\frac{m_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}}$. Of course, $\vec{\mathbf{F}}_{23}$ and $\vec{\mathbf{F}}_{32}$ are in opposite directions. Also from the free-body diagram, we use the net force on m_{A}.

$$F - F_{\text{AB}} = F_{\text{A net}} = F\frac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}} \;\rightarrow\; F_{\text{AB}} = F - F\frac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}} \;\rightarrow$$

$$\boxed{F_{\text{AB}} = F\frac{m_{\text{B}} + m_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}}$$

By Newton's third law, $F_{\text{BC}} = F_{\text{AB}} = \boxed{F\frac{m_2 + m_3}{m_1 + m_2 + m_3}}$.

(*e*) Using the given values, $a = \dfrac{F}{m_1 + m_2 + m_3} = \dfrac{96.0\text{ N}}{30.0\text{ kg}} = \boxed{3.20\,\text{m/s}^2}$. Since all three masses are the same value, the net force on each mass is $F_{\text{net}} = ma = (10.0\text{ kg})(3.20\,\text{m/s}^2) = 32.0\text{ N}$. This is also the value of F_{CB} and F_{BC}. The value of F_{AB} and F_{BA} is found as follows.

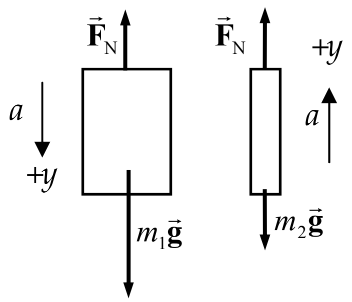
$$F_{\text{AB}} = F_{\text{BA}} = (m_2 + m_3)a = (20.0\text{ kg})(3.20\,\text{m/s}^2) = 64.0\text{ N}$$

To summarize:

$$F_{\text{A net}} = F_{\text{B net}} = F_{\text{C net}} = \boxed{32.0\text{ N}} \qquad F_{\text{AB}} = F_{\text{BA}} = \boxed{64.0\text{ N}} \qquad F_{\text{BC}} = F_{\text{CB}} = \boxed{32.0\text{ N}}$$

The values make sense in that in order of magnitude, we should have $F > F_{\text{BA}} > F_{\text{CB}}$, since F is the net force pushing the entire set of blocks, F_{AB} is the net force pushing the right two blocks, and F_{BC} is the net force pushing the right block only.

47. (*a*) Refer to the free-body diagrams shown. With the stipulation that the direction of the acceleration be in the direction of motion for both objects, we have $a_{\text{C}} = a_{\text{E}} = a$.



$$\boxed{m_{\text{E}}g - F_{\text{T}} = m_{\text{E}}a \ ; \ F_{\text{T}} - m_{\text{C}}g = m_{\text{C}}a}$$

(*b*) Add the equations together to solve them.

$$(m_{\text{E}}g - F_{\text{T}}) + (F_{\text{T}} - m_{\text{C}}g) = m_{\text{E}}a + m_{\text{C}}a \quad \rightarrow$$

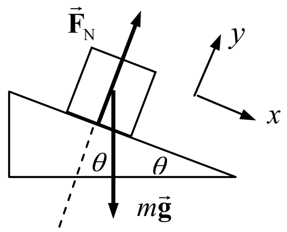
$$m_{\text{E}}g - m_{\text{C}}g = m_{\text{E}}a + m_{\text{C}}a \quad \rightarrow$$

$$a = \frac{m_{\text{E}} - m_{\text{C}}}{m_{\text{E}} + m_{\text{C}}}g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}(9.80\,\text{m/s}^2) = \boxed{0.68\,\text{m/s}^2}$$

$$F_{\text{T}} = m_{\text{C}}(g + a) = m_{\text{C}}\left(g + \frac{m_{\text{E}} - m_{\text{C}}}{m_{\text{E}} + m_{\text{C}}}g\right) = \frac{2m_{\text{C}}m_{\text{E}}}{m_{\text{E}} + m_{\text{C}}}g = \frac{2(1000\,\text{kg})(1150\,\text{kg})}{1150\,\text{kg} + 1000\,\text{kg}}(9.80\,\text{m/s}^2)$$

$$= 10{,}483\,\text{N} \approx \boxed{10{,}500\,\text{N}}$$

48. (*a*) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the y direction. Use Newton's second law for the x direction to find the acceleration.



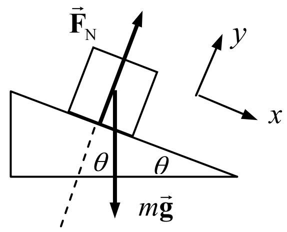
$$\sum F_x = mg\sin\theta = ma \quad \rightarrow$$

$$a = g\sin\theta = (9.80\,\text{m/s}^2)\sin 22.0^\circ = \boxed{3.67\,\text{m/s}^2}$$

(*b*) Use Eq. 2-12c with $v_0 = 0$ to find the final speed.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x - x_0)} = \sqrt{2(3.67\,\text{m/s}^2)(12.0\,\text{m})} = \boxed{9.39\,\text{m/s}}$$

$\boxed{49.}$ (*a*) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the y direction. Write Newton's second law for the x direction.



$$\sum F_x = mg\sin\theta = ma \quad \rightarrow \quad a = g\sin\theta$$

Use Eq. 2-12c with $v_0 = -4.5\,\text{m/s}$ and $v = 0\,\text{m/s}$ to find the distance that it slides before stopping.

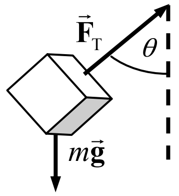
$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow$$

$$\left(x - x_0\right) = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(-4.5\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)\sin 22.0^\circ} = -2.758\,\text{m} \approx \boxed{2.8\,\text{m up the plane}}$$

(*b*) The time for a round trip can be found from Eq. 2-12a. The free-body diagram (and thus the acceleration) is the same whether the block is rising or falling. For the entire trip, $v_0 = -4.5\,\text{m/s}$ and $v = +4.5\,\text{m/s}$.

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{\left(4.5\,\text{m/s}\right) - \left(-4.5\,\text{m/s}\right)}{\left(9.80\,\text{m/s}^2\right)\sin 22^\circ} = 2.452\,\text{s} \approx \boxed{2.5\,\text{s}}$$

50. Consider a free-body diagram of the object. The car is moving to the right. The acceleration of the dice is found from Eq. 2-12a.



$$v = v_0 + = a_x t \quad \rightarrow \quad a_x = \frac{v - v_0}{t} = \frac{28\,\text{m/s} - 0}{6.0\ \text{s}} = 4.67\,\text{m/s}^2$$

Now write Newton's second law for both the vertical (*y*) and horizontal (*x*) directions.

$$\sum F_y = F_\text{T}\cos\theta - mg = 0 \quad \rightarrow \quad F_\text{T} = \frac{mg}{\cos\theta} \qquad \sum F_x = F_\text{T}\sin\theta = ma_x$$

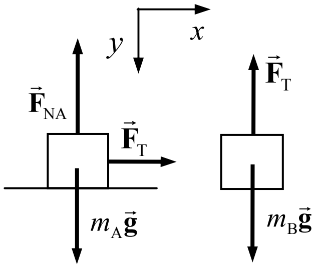
Substitute the expression for the tension from the *y* equation into the *x* equation.

$$ma_x = F_\text{T}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta \quad \rightarrow \quad a_x = g\tan\theta$$

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{4.67\,\text{m/s}^2}{9.80\,\text{m/s}^2} = 25.48^\circ \approx \boxed{25^\circ}$$

51. (*a*) See the free-body diagrams included.

(*b*) For block A, since there is no motion in the vertical direction, we have $F_\text{NA} = m_\text{A}g$. We write Newton's second law for the *x* direction: $\sum F_{\text{A}x} = F_\text{T} = m_\text{A}a_{\text{A}x}$. For block B, we only need to consider vertical forces: $\sum F_{\text{B}y} = m_\text{B}g - F_\text{T} = m_\text{B}a_{\text{B}y}$. Since the two blocks are connected, the magnitudes of their accelerations will be the same, and so let $a_{\text{A}x} = a_{\text{B}y} = a$. Combine the two force equations from above, and solve for *a* by substitution.



$$F_\text{T} = m_\text{A}a \qquad m_\text{B}g - F_\text{T} = m_\text{B}a \quad \rightarrow \quad m_\text{B}g - m_\text{A}a = m_\text{B}a \quad \rightarrow$$

$$m_\text{A}a + m_\text{B}a = m_\text{B}g \quad \rightarrow \quad \boxed{a = g\frac{m_\text{B}}{m_\text{A} + m_\text{B}} \qquad F_\text{T} = m_\text{A}a = g\frac{m_\text{A}m_\text{B}}{m_\text{A} + m_\text{B}}}$$

52. (*a*) From Problem 51, we have the acceleration of each block. Both blocks have the same acceleration.

$$a = g\frac{m_\text{B}}{m_\text{A} + m_\text{B}} = \left(9.80\,\text{m/s}^2\right)\frac{5.0\,\text{kg}}{\left(5.0\,\text{kg} + 13.0\,\text{kg}\right)} = 2.722\,\text{m/s}^2 \approx \boxed{2.7\,\text{m/s}^2}$$

(*b*) Use Eq. 2-12b to find the time.

$$x-x_0=v_0t+\tfrac{1}{2}at^2 \;\rightarrow\; t=\sqrt{\frac{2\left(x-x_0\right)}{a}}=\sqrt{\frac{2\left(1.250\,\text{m}\right)}{\left(2.722\,\text{m/s}^2\right)}}=\boxed{0.96\,\text{s}}$$

(*c*) Again use the acceleration from Problem 51.

$$a=g\frac{m_\text{B}}{m_\text{A}+m_\text{B}}=\tfrac{1}{100}g \;\rightarrow\; \frac{m_\text{B}}{m_\text{A}+m_\text{B}}=\tfrac{1}{100} \;\rightarrow\; m_\text{A}=99m_B=\boxed{99\,\text{kg}}$$

53. This problem can be solved in the same way as problem 51, with the modification that we increase mass m_A by the mass of ℓ_A and we increase mass m_B by the mass of ℓ_B. We take the result from problem 51 for the acceleration and make these modifications. We assume that the cord is uniform, and so the mass of any segment is directly proportional to the length of that segment.

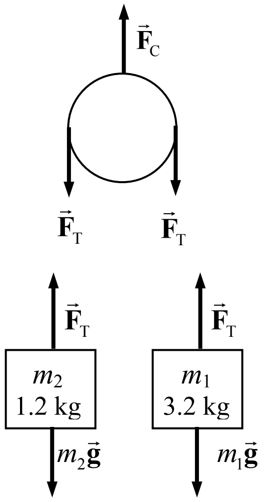
$$a=g\frac{m_\text{B}}{m_\text{A}+m_\text{B}} \;\rightarrow\; a=g\frac{m_\text{B}+\dfrac{\ell_\text{B}}{\ell_\text{A}+\ell_\text{B}}m_C}{\left(m_\text{A}+\dfrac{\ell_\text{A}}{\ell_\text{A}+\ell_\text{B}}m_C\right)+\left(m_\text{B}+\dfrac{\ell_\text{B}}{\ell_\text{A}+\ell_\text{B}}m_C\right)}=\boxed{g\frac{m_\text{B}+\dfrac{\ell_\text{B}}{\ell_\text{A}+\ell_\text{B}}m_C}{m_\text{A}+m_\text{B}+m_\text{C}}}$$

Note that this acceleration is NOT constant, because the lengths ℓ_A and ℓ_B are functions of time. Thus constant acceleration kinematics would not apply to this system.

54. We draw a free-body diagram for each mass. We choose UP to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.

$$F_\text{T}-m_1g=m_1a_1 \qquad F_\text{T}-m_2g=m_2a_2$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1=-a_2$. Substitute this into the force expressions and solve for the tension force.



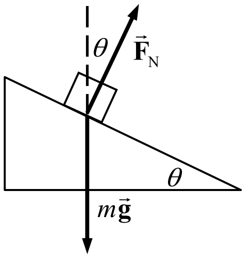
$$F_\text{T}-m_1g=-m_1a_2 \;\rightarrow\; F_\text{T}=m_1g-m_1a_2 \;\rightarrow\; a_2=\frac{m_1g-F_\text{T}}{m_1}$$

$$F_\text{T}-m_2g=m_2a_2=m_2\left(\frac{m_1g-F_\text{T}}{m_1}\right) \;\rightarrow\; F_\text{T}=\frac{2m_1m_2g}{m_1+m_2}$$

Apply Newton's second law to the stationary pulley.

$$F_\text{C}-2F_T=0 \;\rightarrow\; F_\text{C}=2F_T=\frac{4m_1m_2g}{m_1+m_2}=\frac{4\left(3.2\,\text{kg}\right)\left(1.2\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{4.4\,\text{kg}}=\boxed{34\,\text{N}}$$

$\boxed{55.}$ If *m* doesn't move on the incline, it doesn't move in the vertical direction, and so has no vertical component of acceleration. This suggests that we analyze the forces parallel and perpendicular to the floor. See the force diagram for the small block, and use Newton's second law to find the acceleration of the small block.



$$\sum F_y=F_\text{N}\cos\theta-mg=0 \;\rightarrow\; F_\text{N}=\frac{mg}{\cos\theta}$$

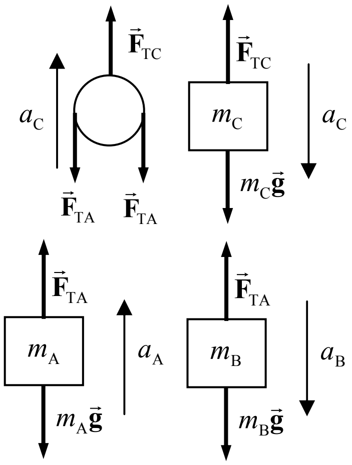
$$\sum F_x=F_\text{N}\sin\theta=ma \;\rightarrow\; a=\frac{F_\text{N}\sin\theta}{m}=\frac{mg\sin\theta}{m\cos\theta}=g\tan\theta$$

Since the small block doesn't move on the incline, the combination of both masses has the same horizontal acceleration of $g\tan\theta$. That can be used to find the applied force.

$$F_{\text{applied}} = (m+M)\,a = \boxed{(m+M)\,g\tan\theta}$$

Note that this gives the correct answer for the case of $\theta = 0$,, where it would take no applied force to keep *m* stationary. It also gives a reasonable answer for the limiting case of $\theta \to 90°$, where no force would be large enough to keep the block from falling, since there would be no upward force to counteract the force of gravity.

56. Because the pulleys are massless, the net force on them must be 0. Because the cords are massless, the tension will be the same at both ends of the cords. Use the free-body diagrams to write Newton's second law for each mass. We are using the same approach taken in problem 47, where we take the direction of acceleration to be positive in the direction of motion of the object. We assume that m_C is falling, m_B is falling relative to its pulley, and m_A is rising relative to its pulley. Also note that if the acceleration of m_A relative to the pulley above it is a_R, then $a_A = a_R + a_C$. Then, the acceleration of m_B is $a_B = a_R - a_C$, since a_C is in the opposite direction of a_B.



$$m_A:\ \sum F = F_{TA} - m_A g = m_A a_A = m_A\left(a_R + a_C\right)$$

$$m_B:\ \sum F = m_B g - F_{TA} = m_B a_B = m_B\left(a_R - a_C\right)$$

$$m_C:\ \sum F = m_C g - F_{TC} = m_C a_C$$

$$\text{pulley: } \sum F = F_{TC} - 2F_{TA} = 0 \ \to\ F_{TC} = 2F_{TA}$$

Re-write this system as three equations in three unknowns F_{TA}, a_R, a_C.

$$F_{TA} - m_A g = m_A\left(a_R + a_C\right) \ \to\ F_{TA} - m_A a_C - m_A a_R = m_A g$$

$$m_B g - F_{TA} = m_B\left(a_R - a_C\right) \ \to\ F_{TA} - m_B a_C + m_B a_R = m_B g$$

$$m_C g - 2F_{TA} = m_C a_C \ \to\ 2F_{TA} + m_C a_C = m_C g$$

This system now needs to be solved. One method to solve a system of linear equations is by determinants. We show that for a_C.

$$a_C = \frac{\begin{vmatrix} 1 & m_A & -m_A \\ 1 & m_B & m_B \\ 2 & m_C & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -m_A & -m_A \\ 1 & -m_B & m_B \\ 2 & m_C & 0 \end{vmatrix}} g = \frac{-m_B m_C + m_A\left(2m_B\right) - m_A\left(m_C - 2m_B\right)}{-m_B m_C - m_A\left(2m_B\right) - m_A\left(m_C + 2m_B\right)} g$$

$$= \frac{4m_A m_B - m_A m_C - m_B m_C}{-4m_A m_B - m_A m_C - m_B m_C} g = \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

Similar manipulations give the following results.

$$a_R = \frac{2\left(m_A m_C - m_B m_C\right)}{4m_A m_B + m_A m_C + m_B m_C} g \; ; \; F_{TA} = \frac{4m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g$$

(*a*) The accelerations of the three masses are found below.

$$a_A = a_R + a_C = \frac{2\left(m_A m_C - m_B m_C\right)}{4m_A m_B + m_A m_C + m_B m_C} g + \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

$$= \boxed{\frac{3m_A m_C - m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

$$a_B = a_R - a_C = \frac{2\left(m_A m_C - m_B m_C\right)}{4m_A m_B + m_A m_C + m_B m_C} g - \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

$$= \boxed{\frac{m_A m_C - 3m_B m_C + 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

$$a_C = \boxed{\frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

(*b*) The tensions are shown below.

$$F_{TA} = \boxed{\frac{4m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g} \; ; \; F_{TC} = 2F_{TA} = \boxed{\frac{8m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g}$$

57. Please refer to the free-body diagrams given in the textbook for this problem. Initially, treat the two boxes and the rope as a single system. Then the only accelerating force on the system is $\vec{\mathbf{F}}_P$. The mass of the system is 23.0 kg, and so using Newton's second law, the acceleration of the system is $a = \frac{F_P}{m} = \frac{35.0\text{ N}}{23.0\text{ kg}} = 1.522\,\text{m/s}^2 \approx \boxed{1.52\,\text{m/s}^2}$. This is the acceleration of each part of the system.

Now consider m_B alone. The only force on it is $\vec{\mathbf{F}}_{BT}$, and it has the acceleration found above. Thus F_{BT} can be found from Newton's second law.

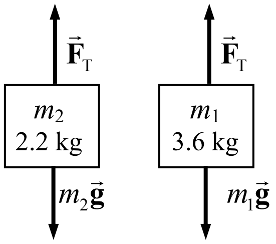
$$F_{BT} = m_B a = (12.0\text{ kg})\left(1.522\,\text{m/s}^2\right) = 18.26\text{ N} \approx \boxed{18.3\text{ N}}$$

Now consider the rope alone. The net force on it is $\vec{\mathbf{F}}_{TA} - \vec{\mathbf{F}}_{TB}$, and it also has the acceleration found above. Thus F_{TA} can be found from Newton's second law.

$$F_{TA} - F_{TB} = m_C a \;\rightarrow\; F_{TA} = F_{TB} + m_C a = 18.26\text{ N} + (1.0\text{ kg})\left(1.522\,\text{m/s}^2\right) = \boxed{19.8\text{ N}}$$

58. First, draw a free-body diagram for each mass. Notice that the same tension force is applied to each mass. Choose UP to be the positive direction. Write Newton's second law for each of the masses.

$$F_T - m_2 g = m_2 a_2 \qquad F_T - m_1 g = m_1 a_1$$



Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the acceleration by subtracting the second equation from the first.

$$F_T - m_1 g = -m_1 a_2 \quad \rightarrow \quad F_T = m_1 g - m_1 a_2$$

$$F_T - m_2 g = m_2 a_2 \quad \rightarrow \quad m_1 g - m_1 a_2 - m_2 g = m_2 a_2 \quad \rightarrow \quad m_1 g - m_2 g = m_1 a_2 + m_2 a_2$$

$$a_2 = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{3.6\text{ kg} - 2.2\text{ kg}}{3.6\text{ kg} + 2.2\text{ kg}}\left(9.80\,\text{m/s}^2\right) = 2.366\,\text{m/s}^2$$

The lighter block starts with a speed of 0, and moves a distance of 1.8 meters with the acceleration found above. Using Eq. 2-12c, the velocity of the lighter block at the end of this accelerated motion can be found.

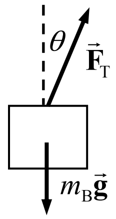
$$v^2 - v_0^2 = 2a\left(y - y_0\right) \quad \rightarrow \quad v = \sqrt{v_0^2 + 2a\left(y - y_0\right)} = \sqrt{0 + 2\left(2.366\,\text{m/s}^2\right)\left(1.8\,\text{m}\right)} = 2.918\,\text{m/s}$$

Now the lighter block has different conditions of motion. Once the heavier block hits the ground, the tension force disappears, and the lighter block is in free fall. It has an initial speed of 2.918 m/s upward as found above, with an acceleration of –9.80 m/s^2 due to gravity. At its highest point, its speed will be 0. Eq. 2-12c can again be used to find the height to which it rises.

$$v^2 - v_0^2 = 2a\left(y - y_0\right) \quad \rightarrow \quad \left(y - y_0\right) = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(2.918\,\text{m/s}\right)^2}{2\left(-9.80\,\text{m/s}^2\right)} = 0.434\,\text{m}$$

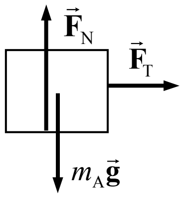
Thus the total height above the ground is 1.8 m + 1.8 m + 0.43 m = $\boxed{4.0\,\text{m}}$.

59. The force $\vec{\mathbf{F}}$ is accelerating the total mass, since it is the only force external to the system. If mass m_A does not move relative to m_C, then all the blocks have the same horizontal acceleration, and none of the blocks have vertical acceleration. We solve for the acceleration of the system and then find the magnitude of $\vec{\mathbf{F}}$ from Newton's second law. Start with free-body diagrams for m_A and m_B.



$$m_B: \quad \sum F_x = F_T \sin\theta = m_B a \;;$$

$$\sum F_y = F_T \cos\theta - m_B g = 0 \quad \rightarrow \quad F_T \cos\theta = m_B g$$



Square these two expressions and add them, to get a relationship between F_T and a.

$$F_T^2 \sin^2\theta = m_B^2 a^2 \;\; ; \;\; F_T^2 \cos^2\theta = m_B^2 g^2 \quad \rightarrow$$

$$F_T^2\left(\sin^2\theta + \cos^2\theta\right) = m_B^2\left(g^2 + a^2\right) \quad \rightarrow \quad F_T^2 = m_B^2\left(g^2 + a^2\right)$$

Now analyze m_A.

$$m_A: \quad \sum F_x = F_T = m_A a \quad \rightarrow \quad F_T^2 = m_A^2 a^2 \;\; ; \;\; \sum F_y = F_N - m_A g = 0$$

Equate the two expressions for F_T^2, solve for the acceleration and then finally the magnitude of the applied force.

$$F_T^2 = m_B^2\left(g^2 + a^2\right) = m_A^2 a^2 \quad \rightarrow \quad a^2 = \frac{m_B^2 g^2}{\left(m_A^2 - m_B^2\right)} \quad \rightarrow \quad a = \frac{m_B g}{\sqrt{\left(m_A^2 - m_B^2\right)}} \quad \rightarrow$$

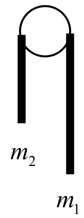
$$F = \left(m_A + m_B + m_C\right) a = \boxed{\frac{\left(m_A + m_B + m_C\right) m_B}{\sqrt{\left(m_A^2 - m_B^2\right)}} g}$$

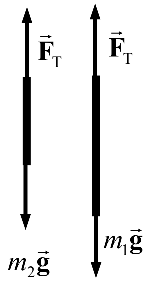
60. The velocity can be found by integrating the acceleration function, and the position can be found by integrating the position function.

$$F = ma = Ct^2 \rightarrow a = \frac{C}{m}t^2 = \frac{dv}{dt} \rightarrow dv = \frac{C}{m}t^2 dt \rightarrow \int_0^v dv = \int_0^t \frac{C}{m}t^2 dt \rightarrow \boxed{v = \frac{C}{3m}t^3}$$

$$v = \frac{C}{3m}t^3 = \frac{dx}{dt} \rightarrow dx = \frac{C}{3m}t^3 dt \rightarrow \int_0^x dx = \int_0^t \frac{C}{3m}t^3 dt \rightarrow \boxed{x = \frac{C}{12m}t^4}$$

61. We assume that the pulley is small enough that the part of the cable that is touching the surface of the pulley is negligible, and so we ignore any force on the cable due to the pulley itself. We also assume that the cable is uniform, so that the mass of a portion of the cable is proportional to the length of that portion. We then treat the cable as two masses, one on each side of the pulley. The masses are given by $m_1 = \frac{y}{\ell}M$ and $m_2 = \frac{\ell - y}{\ell}M$. Free-body diagrams for the masses are shown.





(*a*) We take downward motion of m_1 to be the positive direction for m_1, and upward motion of m_2 to be the positive direction for m_2. Newton's second law for the masses gives the following.

$$F_{\text{net }1} = m_1 g - F_\text{T} = m_1 a \; ; \; F_{\text{net }2} = F_\text{T} - m_2 g \rightarrow a = \frac{m_1 - m_2}{m_1 + m_2}g$$

$$a = \frac{\frac{y}{\ell}M - \frac{\ell - y}{\ell}M}{\frac{y}{\ell}M + \frac{\ell - y}{\ell}M}g = \frac{y - (\ell - y)}{y + (\ell - y)}g = \frac{2y - \ell}{\ell}g = \boxed{\left(\frac{2y}{\ell} - 1\right)g}$$

(*b*) Use the hint supplied with the problem to set up the equation for the velocity. The cable starts with a length y_0 (assuming $y_0 > \tfrac{1}{2}\ell$) on the right side of the pulley, and finishes with a length ℓ on the right side of the pulley.

$$a = \left(\frac{2y}{\ell} - 1\right)g = \frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = v\frac{dv}{dy} \rightarrow \left(\frac{2y}{\ell} - 1\right)g\,dy = v\,dv \rightarrow$$

$$\int_{y_0}^{\ell}\left(\frac{2y}{\ell} - 1\right)g\,dy = \int_0^{v_f} v\,dv \rightarrow g\left(\frac{y^2}{\ell} - y\right)\Bigg|_{y_0}^{\ell} = \left(\tfrac{1}{2}v^2\right)\Big|_0^{v_f} \rightarrow gy_0\left(1 - \frac{y_0}{\ell}\right) = \tfrac{1}{2}v_f^2 \rightarrow$$

$$v_f = \boxed{\sqrt{2gy_0\left(1 - \frac{y_0}{\ell}\right)}}$$

(*c*) For $y_0 = \tfrac{2}{3}\ell$, we have $v_f = \sqrt{2gy_0\left(1 - \frac{y_0}{\ell}\right)} = \sqrt{2g\left(\tfrac{2}{3}\right)\ell\left(1 - \frac{\frac{2}{3}\ell}{\ell}\right)} = \boxed{\tfrac{2}{3}\sqrt{g\ell}}$.

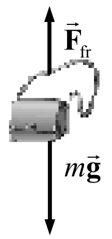
62. The acceleration of a person having a 30 "*g*" deceleration is $a = \left(30\,\text{"}g\text{"}\right)\left(\frac{9.80\,\text{m/s}^2}{\text{"}g\text{"}}\right) = 294\,\text{m/s}^2$.

The average force causing that acceleration is $F = ma = (65\,\text{kg})\left(294\,\text{m/s}^2\right) = \boxed{1.9 \times 10^4\,\text{N}}$. Since the person is undergoing a deceleration, the acceleration and force would both be directed opposite to the direction of motion. Use Eq. 2-12c to find the distance traveled during the deceleration. Take the initial velocity to be in the positive direction, so that the acceleration will have a negative value, and the final velocity will be 0.

$$v_0 = (95\,\mathrm{km/h})\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right) = 26.4\,\mathrm{m/s}$$

$$v^2 - v_0^2 = 2a(x - x_0) \rightarrow (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (26.4\,\mathrm{m/s})^2}{2(-294\,\mathrm{m/s^2})} = \boxed{1.2\,\mathrm{m}}$$

63. See the free-body diagram for the falling purse. Assume that down is the positive direction, and that the air resistance force $\vec{\mathbf{F}}_{\mathrm{fr}}$ is constant. Write Newton's second law for the vertical direction.



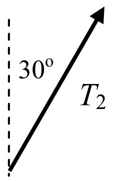
$$\sum F = mg - F_{\mathrm{fr}} = ma \rightarrow F_{\mathrm{fr}} = m(g - a)$$

Now obtain an expression for the acceleration from Eq. 2-12c with $v_0 = 0$, and substitute back into the friction force.

$$v^2 - v_0^2 = 2a(x - x_0) \rightarrow a = \frac{v^2}{2(x - x_0)}$$

$$F_f = m\left(g - \frac{v^2}{2(x - x_0)}\right) = (2.0\text{ kg})\left(9.80\,\mathrm{m/s^2} - \frac{(27\,\mathrm{m/s})^2}{2(55\text{ m})}\right) = \boxed{6.3\,\mathrm{N}}$$

64. Each rope must support 1/6 of Tom's weight, and so must have a vertical component of tension given by $T_{\mathrm{vert}} = \frac{1}{6}mg$. For the vertical ropes, their entire tension is vertical.

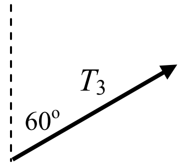


$$T_1 = \tfrac{1}{6}mg = \tfrac{1}{6}(74.0\text{ kg})(9.80\,\mathrm{m/s^2}) = 120.9\text{ N} \approx \boxed{1.21\times10^2\,\mathrm{N}}$$

For the ropes displaced 30° from the vertical, see the first diagram.

$$T_{2\,\mathrm{vert}} = T_2\cos30^\circ = \tfrac{1}{6}mg \rightarrow T_2 = \frac{mg}{6\cos30^\circ} = \frac{120.9\,\mathrm{N}}{\cos30^\circ} = \boxed{1.40\times10^2\,\mathrm{N}}$$

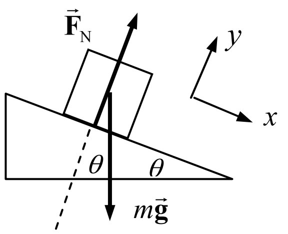
For the ropes displaced 60° from the vertical, see the second diagram.



$$T_{3\,\mathrm{vert}} = T_3\cos60^\circ = \tfrac{1}{6}mg \rightarrow T_3 = \frac{mg}{6\cos60^\circ} = \frac{120.9\,\mathrm{N}}{\cos60^\circ} = \boxed{2.42\times10^2\,\mathrm{N}}$$

The corresponding ropes on the other side of the glider will also have the same tensions as found here.

65. Consider the free-body diagram for the soap block on the frictionless surface. There is no acceleration in the *y* direction. Write Newton's second law for the *x* direction.



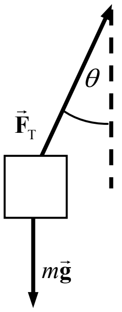
$$\sum F_x = mg\sin\theta = ma \rightarrow a = g\sin\theta$$

Use Eq. 2-12b with $v_0 = 0$ to find the time of travel.

$$x - x_0 = v_0t + \tfrac{1}{2}at^2 \rightarrow$$

$$t = \sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(x - x_0)}{g\sin\theta}} = \sqrt{\frac{2(3.0\,\mathrm{m})}{(9.80\,\mathrm{m/s^2})\sin(8.5^\circ)}} = \boxed{2.0\,\mathrm{s}}$$

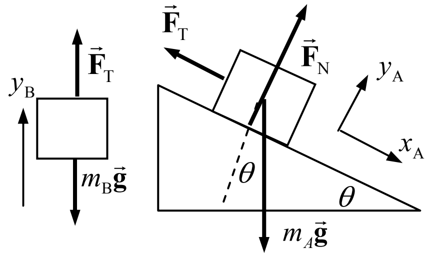
Since the mass does not enter into the calculation, the $\boxed{\text{time would be the same}}$ for the heavier bar of soap.



66. See the free-body diagram for the load. The vertical component of the tension force must be equal to the weight of the load, and the horizontal component of the tension accelerates the load. The angle is exaggerated in the picture.

$$F_{\underset{x}{\text{net}}} = F_T \sin\theta = ma \;\rightarrow\; a = \frac{F_T \sin\theta}{m} \;;\; F_{\underset{y}{\text{net}}} = F_T\cos\theta - mg = 0 \;\rightarrow$$

$$F_T = \frac{mg}{\cos\theta} \;\rightarrow\; a_H = \frac{mg}{\cos\theta}\frac{\sin\theta}{m} = g\tan\theta = \left(9.80\,\text{m/s}^2\right)\tan 5.0° = \boxed{0.86\,\text{m/s}^2}$$



$\boxed{67.}$ (*a*) Draw a free-body diagram for each block. Write Newton's second law for each block. Notice that the acceleration of block A in the y_A direction will be zero, since it has no motion in the y_A direction.

$$\sum F_{yA} = F_N - m_A g\cos\theta = 0 \;\rightarrow\; F_N = m_A g\cos\theta$$

$$\sum F_{xA} = m_A g\sin\theta - F_T = m_A a_{xA}$$

$$\sum F_{yB} = F_T - m_B g = m_B a_{yB} \;\rightarrow\; F_T = m_B\left(g + a_{yB}\right)$$

Since the blocks are connected by the cord, $a_{yB} = a_{xA} = a$. Substitute the expression for the tension force from the last equation into the *x* direction equation for block 1, and solve for the acceleration.

$$m_A g\sin\theta - m_B\left(g + a\right) = m_A a \;\rightarrow\; m_A g\sin\theta - m_B g = m_A a + m_B a$$

$$a = \boxed{g\frac{\left(m_A\sin\theta - m_B\right)}{\left(m_A + m_B\right)}}$$

(*b*) If the acceleration is to be down the plane, it must be positive. That will happen if $\boxed{m_A\sin\theta > m_B \;\left(\text{down the plane}\right)}$. The acceleration will be up the plane (negative) if $\boxed{m_A\sin\theta < m_B \;\left(\text{up the plane}\right)}$. If $m_A\sin\theta = m_B$, then the system will not accelerate. It will move with a constant speed if set in motion by a push.

68. (*a*) From problem 67, we have an expression for the acceleration.

$$a = g\frac{\left(m_A\sin\theta - m_B\right)}{\left(m_A + m_B\right)} = \left(9.80\,\text{m/s}^2\right)\frac{\left[\left(1.00\,\text{kg}\right)\sin 33.0° - 1.00\,\text{kg}\right]}{2.00\,\text{kg}} = -2.23\,\text{m/s}^2$$

$$\approx \boxed{-2.2\,\text{m/s}^2}$$

The negative sign means that m_A will be accelerating UP the plane.

(*b*) If the system is at rest, then the acceleration will be 0.

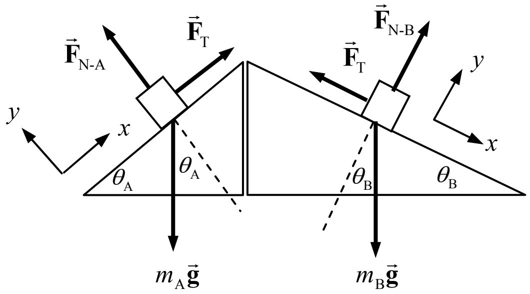
$$a = g\frac{\left(m_A\sin\theta - m_B\right)}{\left(m_A + m_B\right)} = 0 \;\rightarrow\; m_B = m_A\sin\theta = \left(1.00\,\text{kg}\right)\sin 33.0° = 0.5446\,\text{kg} \approx \boxed{0.545\,\text{kg}}$$

(*c*) Again from problem 68, we have $F_T = m_B\left(g + a\right)$.

Case (*a*): $F_T = m_B\left(g + a\right) = \left(1.00\,\text{kg}\right)\left(9.80\,\text{m/s}^2 - 2.23\,\text{m/s}^2\right) = 7.57\,\text{N} \approx \boxed{7.6\,\text{N}}$

Case (*b*): $F_T = m_B\left(g + a\right) = \left(0.5446\,\text{kg}\right)\left(9.80\,\text{m/s}^2 + 0\right) = 5.337\,\text{N} \approx \boxed{5.34\,\text{N}}$

69. (*a*) A free-body diagram is shown for each block. We define the positive *x*-direction for m_A to be up its incline, and the positive *x*-direction for m_B to be down its incline. With that definition the masses will both have the same acceleration. Write Newton's second law for each body in the *x* direction, and combine those equations to find the acceleration.



$$m_A: \ \sum F_x = F_T - m_A g\sin\theta_A = m_A a$$

$$m_B: \ \sum F_x = m_B g\sin\theta_B - F_T = m_B a \quad \text{add these two equations}$$

$$\left(F_T - m_A g\sin\theta_A\right) + \left(m_B g\sin\theta_B - F_T\right) = m_A a + m_B a \ \rightarrow \ a = \boxed{\frac{m_B\sin\theta_B - m_A\sin\theta_A}{m_A + m_B}g}$$

(*b*) For the system to be at rest, the acceleration must be 0.

$$a = \frac{m_B\sin\theta_B - m_A\sin\theta_A}{m_A + m_B}g = 0 \ \rightarrow \ m_B\sin\theta_B - m_A\sin\theta_A \ \rightarrow$$

$$m_B = m_A\frac{\sin\theta_A}{\sin\theta_B} = \left(5.0\,\text{kg}\right)\frac{\sin 32^\circ}{\sin 23^\circ} = \boxed{6.8\,\text{kg}}$$

The tension can be found from one of the Newton's second law expression from part (*a*).

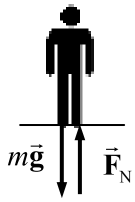
$$m_A: \ F_T - m_A g\sin\theta_A = 0 \ \rightarrow \ F_T = m_A g\sin\theta_A = \left(5.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\sin 32^\circ = \boxed{26\,\text{N}}$$

(*c*) As in part (*b*), the acceleration will be 0 for constant velocity in either direction.

$$a = \frac{m_B\sin\theta_B - m_A\sin\theta_A}{m_A + m_B}g = 0 \ \rightarrow \ m_B\sin\theta_B - m_A\sin\theta_A \ \rightarrow$$

$$\frac{m_A}{m_B} = \frac{\sin\theta_B}{\sin\theta_A} = \frac{\sin 23^\circ}{\sin 32^\circ} = \boxed{0.74}$$

70. A free-body diagram for the person in the elevator is shown. The scale reading is the magnitude of the normal force. Choosing up to be the positive direction, Newton's second law for the person says that $\sum F = F_N - mg = ma \ \rightarrow \ F_N = m\left(g + a\right)$. The kg reading of the scale is the apparent weight, F_N, divided by *g*, which gives $F_{N\text{-kg}} = \frac{F_N}{g} = \frac{m\left(g+a\right)}{g}$.



(*a*) $a = 0 \ \rightarrow \ F_N = mg = \left(75.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{7.35\times10^2\,\text{N}}$

$$F_{N\text{-kg}} = \frac{mg}{g} = m = \boxed{75.0\,\text{kg}}$$

(*b*) $a = 0 \ \rightarrow \ F_N = \boxed{7.35\times10^2\,\text{N}}$, $F_{N\text{-kg}} = \boxed{75.0\,\text{kg}}$

(*c*) $a = 0 \ \rightarrow \ F_N = \boxed{7.35\times10^2\,\text{N}}$, $F_{N\text{-kg}} = \boxed{75.0\,\text{kg}}$

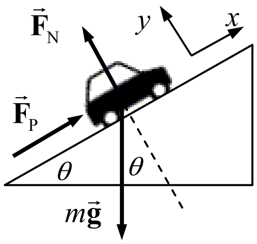
(*d*) $F_N = m\left(g + a\right) = \left(75.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2 + 3.0\,\text{m/s}^2\right)a = \boxed{9.60\times10^2\,\text{N}}$

$$F_{\text{N-kg}} = \frac{F_{\text{N}}}{g} = \frac{960\,\text{N}}{9.80\,\text{m/s}^2} = \boxed{98.0\,\text{kg}}$$

(*e*) $F_{\text{N}} = m(g+a) = (75.0\,\text{kg})(9.80\,\text{m/s}^2 - 3.0\,\text{m/s}^2)a = \boxed{5.1\times10^2\,\text{N}}$

$$F_{\text{N-kg}} = \frac{F_{\text{N}}}{g} = \frac{510\,\text{N}}{9.80\,\text{m/s}^2} = \boxed{52\,\text{kg}}$$

71. The given data can be used to calculate the force with which the road pushes against the car, which in turn is equal in magnitude to the force the car pushes against the road. The acceleration of the car on level ground is found from Eq. 2-12a.



$$v - v_0 = at \quad\rightarrow\quad a = \frac{v - v_0}{t} = \frac{21\,\text{m/s} - 0}{12.5\ \text{s}} = 1.68\,\text{m/s}^2$$

The force pushing the car in order to have this acceleration is found from Newton's second law.

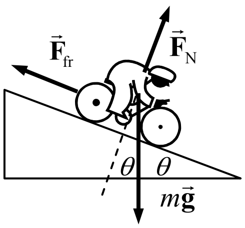
$$F_{\text{P}} = ma = (920\,\text{kg})(1.68\,\text{m/s}^2) = 1546\,\text{N}$$

We assume that this is the force pushing the car on the incline as well. Consider a free-body diagram for the car climbing the hill. We assume that the car will have a constant speed on the maximum incline. Write Newton's second law for the *x* direction, with a net force of zero since the car is not accelerating.

$$\sum F_x = F_{\text{P}} - mg\sin\theta = 0 \quad\rightarrow\quad \sin\theta = \frac{F_{\text{P}}}{mg}$$

$$\theta = \sin^{-1}\frac{F_{\text{P}}}{mg} = \sin^{-1}\frac{1546\,\text{N}}{(920\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{9.9^\circ}$$

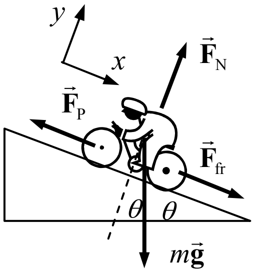
72. Consider a free-body diagram for the cyclist coasting downhill at a constant speed. Since there is no acceleration, the net force in each direction must be zero. Write Newton's second law for the *x* direction (down the plane).



$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \quad\rightarrow\quad F_{\text{fr}} = mg\sin\theta$$

This establishes the size of the air friction force at 6.0 km/h, and so can be used in the next part.

Now consider a free-body diagram for the cyclist climbing the hill. F_{P} is the force pushing the cyclist uphill. Again, write Newton's second law for the *x* direction, with a net force of 0.

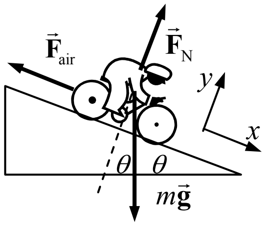


$$\sum F_x = F_{\text{fr}} + mg\sin\theta - F_{\text{P}} = 0 \quad\rightarrow$$

$$F_{\text{P}} = F_{\text{fr}} + mg\sin\theta = 2mg\sin\theta$$

$$= 2(65\ \text{kg})(9.80\,\text{m/s}^2)(\sin 6.5^\circ) = \boxed{1.4\times10^2\,\text{N}}$$

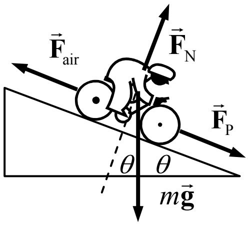
$\boxed{73.}$ (*a*) The value of the constant *c* can be found from the free-body diagram, knowing that the net force is 0 when coasting downhill at the specified speed.



$$\sum F_x = mg\sin\theta - F_{\text{air}} = 0 \quad\rightarrow\quad F_{\text{air}} = mg\sin\theta = cv \quad\rightarrow$$

$$c=\frac{mg\sin\theta}{v}=\frac{(80.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 5.0^\circ}{(6.0\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}=40.998\frac{\text{N}}{\text{m/s}}\approx\boxed{41\frac{\text{N}}{\text{m/s}}}$$

(*b*) Now consider the cyclist with an added pushing force $\vec{\mathbf{F}}_{\text{P}}$ directed along the plane. The free-body diagram changes to reflect the additional force the cyclist must exert. The same axes definitions are used as in part (*a*).



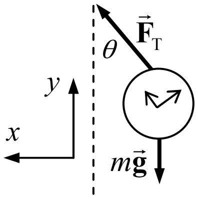
$$\sum F_x=F_{\text{P}}+mg\sin\theta-F_{\text{air}}=0\quad\rightarrow$$

$$F_{\text{P}}=F_{\text{air}}-mg\sin\theta=cv-mg\sin\theta$$

$$=\left(40.998\frac{\text{N}}{\text{m/s}}\right)\left((18.0\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right)$$

$$-(80.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 5.0^\circ=136.7\,\text{N}\approx\boxed{140\,\text{N}}$$

74. Consider the free-body diagram for the watch. Write Newton's second law for both the *x* and *y* directions. Note that the net force in the *y* direction is 0 because there is no acceleration in the *y* direction.



$$\sum F_y=F_{\text{T}}\cos\theta-mg=0\quad\rightarrow\quad F_{\text{T}}=\frac{mg}{\cos\theta}$$

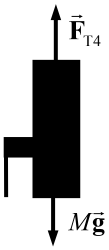
$$\sum F_x=F_{\text{T}}\sin\theta=ma\quad\rightarrow\quad\frac{mg}{\cos\theta}\sin\theta=ma$$

$$a=g\tan\theta=(9.80\,\text{m/s}^2)\tan 25^\circ=4.57\,\text{m/s}^2$$

Use Eq. 2-12a with $v_0=0$ to find the final velocity (takeoff speed).

$$v-v_0=at\quad\rightarrow\quad v=v_0+at=0+(4.57\,\text{m/s}^2)(16\,\text{s})=\boxed{73\,\text{m/s}}$$

75. (*a*) To find the minimum force, assume that the piano is moving with a constant velocity. Since the piano is not accelerating, $F_{\text{T4}}=Mg$. For the lower pulley, since the tension in a rope is the same throughout, and since the pulley is not accelerating, it is seen that $F_{\text{T1}}+F_{\text{T2}}=2F_{\text{T1}}=Mg\ \rightarrow\ F_{\text{T1}}=F_{\text{T2}}=Mg/2$.

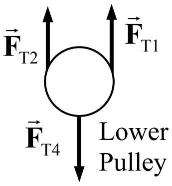


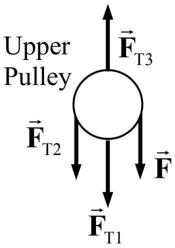
It also can be seen that since $F=F_{\text{T2}}$, that $\boxed{F=Mg/2}$.

(*b*) Draw a free-body diagram for the upper pulley. From that diagram, we see that $F_{\text{T3}}=F_{\text{T1}}+F_{\text{T2}}+F=\dfrac{3Mg}{2}$.

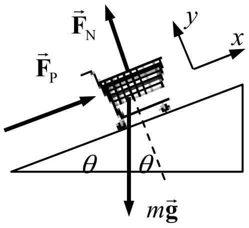
To summarize:

$$\boxed{F_{\text{T1}}=F_{\text{T2}}=Mg/2\qquad F_{\text{T3}}=3Mg/2\qquad F_{\text{T4}}=Mg}$$





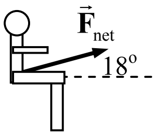
76. Consider a free-body diagram for a grocery cart being pushed up an incline. Assuming that the cart is not accelerating, we write Newton's second law for the *x* direction.

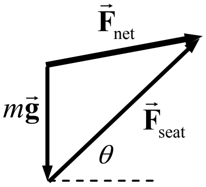


$$\sum F_x = F_P - mg\sin\theta = 0 \quad \rightarrow \quad \sin\theta = \frac{F_P}{mg}$$

$$\theta = \sin^{-1}\frac{F_P}{mg} = \sin^{-1}\frac{18\,\text{N}}{(25\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{4.2^\circ}$$

77. The acceleration of the pilot will be the same as that of the plane, since the pilot is at rest with respect to the plane. Consider first a free-body diagram of the pilot, showing only the net force. By Newton's second law, the net force MUST point in the direction of the acceleration, and its magnitude is ma. That net force is the sum of ALL forces on the pilot. If we assume that the force of gravity and the force of the cockpit seat on the pilot are the only forces on the pilot, then in terms of vectors, $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{g}} + \vec{\mathbf{F}}_{\text{seat}} = m\vec{\mathbf{a}}$. Solve this equation for the force of the seat to find $\vec{\mathbf{F}}_{\text{seat}} = \vec{\mathbf{F}}_{\text{net}} - m\vec{\mathbf{g}} = m\vec{\mathbf{a}} - m\vec{\mathbf{g}}$. A vector diagram of that equation is shown. Solve for the force of the seat on the pilot using components.





$$F_{x\text{ seat}} = F_{x\text{ net}} = ma\cos 18^\circ = (75\,\text{kg})(3.8\,\text{m/s}^2)\cos 18^\circ = 271.1\,\text{N}$$

$$F_{y\text{ seat}} = mg + F_{y\text{ net}} = mg + ma\sin 18^\circ$$
$$= (75\,\text{kg})(9.80\,\text{m/s}^2) + (75\,\text{kg})(3.8\,\text{m/s}^2)\sin 18^\circ = 823.2\,\text{N}$$

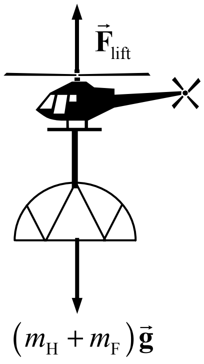
The magnitude of the cockpit seat force is as follows.

$$F = \sqrt{F_{x\text{ seat}}^2 + F_{y\text{ seat}}^2} = \sqrt{(271.1\,\text{N})^2 + (823.2\,\text{N})^2} = 866.7\,\text{N} \approx \boxed{870\,\text{N}}$$

The angle of the cockpit seat force is as follows.

$$\theta = \tan^{-1}\frac{F_{y\text{ seat}}}{F_{x\text{ seat}}} = \tan^{-1}\frac{823.2\,\text{N}}{271.1\,\text{N}} = \boxed{72^\circ} \text{ above the horizontal}$$

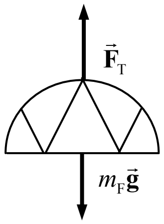
78. (*a*) The helicopter and frame will both have the same acceleration, and so can be treated as one object if no information about internal forces (like the cable tension) is needed. A free-body diagram for the helicopter-frame combination is shown. Write Newton's second law for the combination, calling UP the positive direction.



$$\sum F = F_{\text{lift}} - (m_H + m_F)g = (m_H + m_F)a \quad \rightarrow$$

$$F_{\text{lift}} = (m_H + m_F)(g + a) = (7650\text{ kg} + 1250\text{ kg})(9.80\,\text{m/s}^2 + 0.80\,\text{m/s}^2)$$
$$= \boxed{9.43\times 10^4\,\text{N}}$$

(*b*) Now draw a free-body diagram for the frame alone, in order to find the tension in the cable. Again use Newton's second law.



$$\sum F = F_T - m_F g = m_F a \quad \rightarrow$$

$$F_T = m_F(g + a) = (1250\text{ kg})(9.80\,\text{m/s}^2 + 0.80\,\text{m/s}^2) = \boxed{1.33\times 10^4\,\text{N}}$$

(*c*) The tension in the cable is the same at both ends, and so the cable exerts a force of $\boxed{1.33\times 10^4\,\text{N}}$ downward on the helicopter.

79. (*a*) We assume that the maximum horizontal force occurs when the train is moving very slowly, and so the air resistance is negligible. Thus the maximum acceleration is given by the following.

$$a_{\max}=\frac{F_{\max}}{m}=\frac{4\times10^5\,\mathrm{N}}{6.4\times10^5\,\mathrm{kg}}=0.625\,\mathrm{m/s^2}\approx\boxed{0.6\,\mathrm{m/s^2}}$$

(*b*) At top speed, we assume that the train is moving at constant velocity. Therefore the net force on the train is 0, and so the air resistance and friction forces together must be of the same magnitude as the horizontal pushing force, which is $\boxed{1.5\times10^5\,\mathrm{N}}$.

80. See the free-body diagram for the fish being pulled upward vertically. From Newton's second law, calling the upward direction positive, we have this relationship.

$$\sum F_y=F_\mathrm{T}-mg=ma\quad\rightarrow\quad F_\mathrm{T}=m(g+a)$$

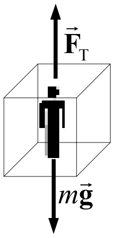
(*a*) If the fish has a constant speed, then its acceleration is zero, and so $F_\mathrm{T}=mg$. Thus the heaviest fish that could be pulled from the water in this case is $\boxed{45\ \mathrm{N}\ (10\ \mathrm{lb})}$.

(*b*) If the fish has an acceleration of 2.0 m/s^2, and F_T is at its maximum of 45 N, then solve the equation for the mass of the fish.

$$m=\frac{F_\mathrm{T}}{g+a}=\frac{45\ \mathrm{N}}{9.8\,\mathrm{m/s^2}+2.0\,\mathrm{m/s^2}}=3.8\ \mathrm{kg}\quad\rightarrow$$

$$mg=(3.8\ \mathrm{kg})(9.8\,\mathrm{m/s^2})=\boxed{37\ \mathrm{N}\ (\approx 8.4\ \mathrm{lb})}$$

(*c*) It is not possible to land a 15-lb fish using 10-lb line, if you have to lift the fish vertically. If the fish were reeled in while still in the water, and then a net used to remove the fish from the water, it might still be caught with the 10-lb line.



81. Choose downward to be positive. The elevator's acceleration is calculated by Eq. 2-12c.

$$v^2-v_0^2=2a(y-y_0)\quad\rightarrow\quad a=\frac{v^2-v_0^2}{2(y-y_0)}=\frac{0-(3.5\,\mathrm{m/s})^2}{2(2.6\,\mathrm{m})}=-2.356\,\mathrm{m/s^2}$$

See the free-body diagram of the elevator/occupant combination. Write Newton's second law for the elevator.

$$\sum F_y=mg-F_\mathrm{T}=ma$$

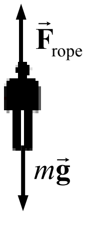
$$F_\mathrm{T}=m(g-a)=(1450\,\mathrm{kg})(9.80\,\mathrm{m/s^2}--2.356\,\mathrm{m/s^2})=\boxed{1.76\times10^4\,\mathrm{N}}$$

82. (*a*) First calculate Karen's speed from falling. Let the downward direction be positive, and use Eq. 2-12c with $v_0=0$.

$$v^2-v_0^2=2a(y-y_0)\quad\rightarrow\quad v=\sqrt{0+2a(y-y_0)}=\sqrt{2(9.8\,\mathrm{m/s^2})(2.0\ \mathrm{m})}=6.26\,\mathrm{m/s}$$

Now calculate the average acceleration as the rope stops Karen, again using Eq. 2-12c, with down as positive.

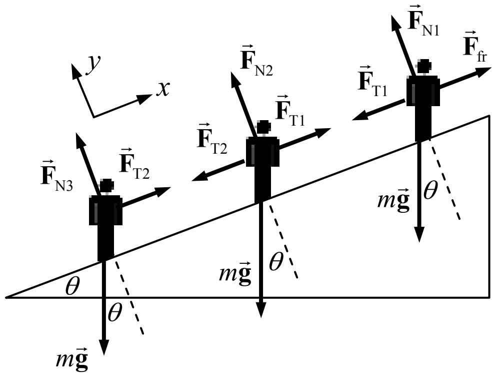
$$v^2-v_0^2=2a(y-y_0)\quad\rightarrow\quad a=\frac{v^2-v_0^2}{2(y-y_0)}=\frac{0-(6.26\,\mathrm{m/s})^2}{2(1.0\ \mathrm{m})}=-19.6\,\mathrm{m/s^2}$$



The negative sign indicates that the acceleration is upward. Since this is her acceleration, the net force on Karen is given by Newton's second law, $F_\mathrm{net}=ma$. That net force will also be upward. Now consider the free-body diagram of Karen as she decelerates. Call DOWN the positive direction. Newton's second law says that $F_{\text{net}} = ma = mg - F_{\text{rope}} \rightarrow F_{\text{rope}} = mg - ma$. The ratio of this force to Karen's weight is $\frac{F_{\text{rope}}}{mg} = \frac{mg - ma}{g} = 1.0 - \frac{a}{g} = 1.0 - \frac{-19.6\,\text{m/s}^2}{9.8\,\text{m/s}^2} = 3.0$. Thus the rope pulls upward on Karen with an average force of $\boxed{\text{3.0 times her weight}}$.

(*b*) A completely analogous calculation for Bill gives the same speed after the 2.0 m fall, but since he stops over a distance of 0.30 m, his acceleration is $-65\,\text{m/s}^2$, and the rope pulls upward on Bill with an average force of $\boxed{\text{7.7 times his weight}}$. Thus, $\boxed{\text{Bill is more likely to get hurt.}}$

83. Since the climbers are on ice, the frictional force for the lower two climbers is negligible. Consider the free-body diagram as shown. Note that all the masses are the same. Write Newton's second law in the *x* direction for the lowest climber, assuming he is at rest.



$$\sum F_x = F_{\text{T2}} - mg\sin\theta = 0$$

$$F_{\text{T2}} = mg\sin\theta = (75\,\text{kg})(9.80\,\text{m/s}^2)\sin 31.0^\circ = \boxed{380\,\text{N}}$$

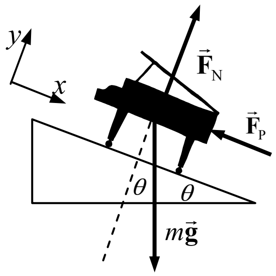
Write Newton's second law in the *x* direction for the middle climber, assuming he is at rest.

$$\sum F_x = F_{\text{T1}} - F_{\text{T2}} - mg\sin\theta = 0 \rightarrow F_{\text{T1}} = F_{\text{T2}} + mg\sin\theta = 2F_{\text{T2}}g\sin\theta = \boxed{760\,\text{N}}$$

84. Use Newton's second law.

$$F = ma = m\frac{\Delta v}{\Delta t} \rightarrow \Delta t = \frac{m\Delta v}{F} = \frac{(1.0\times10^{10}\,\text{kg})(2.0\times10^{-3}\,\text{m/s})}{(2.5\,\text{N})} = \boxed{8.0\times10^6\,\text{s}} = 93\,\text{d}$$

$\boxed{85.}$ Use the free-body diagram to find the net force in the *x* direction, and then find the acceleration. Then Eq. 2-12c can be used to find the final speed at the bottom of the ramp.

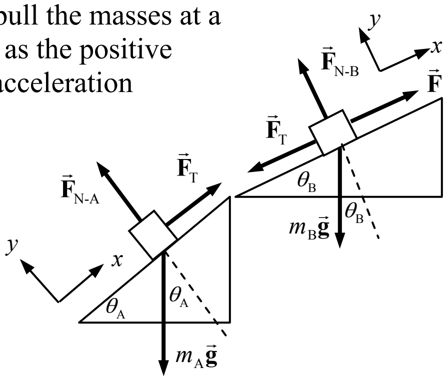


$$\sum F_x = mg\sin\theta - F_{\text{P}} = ma \rightarrow$$

$$a = \frac{mg\sin\theta - F_{\text{P}}}{m} = \frac{(450\,\text{kg})(9.80\,\text{m/s}^2)\sin 22^\circ - 1420\,\text{N}}{450\,\text{kg}} = 0.516\,\text{m/s}^2$$

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v = \sqrt{2a(x - x_0)} = \sqrt{2(0.516\,\text{m/s}^2)(11.5\,\text{m})} = \boxed{3.4\,\text{m/s}}$$

86. (*a*) We use the free-body diagram to find the force needed to pull the masses at a constant velocity. We choose the "up the plane" direction as the positive direction for both masses. Then they both have the same acceleration even if it is non-zero.



$$m_{\text{A}}: \ \sum F_x = F_{\text{T}} - m_{\text{A}} g \sin\theta_{\text{A}} = m_{\text{A}} a = 0$$

$$m_{\text{b}}: \ \sum F_x = F - F_{\text{T}} - m_{\text{B}} g \sin\theta_{\text{B}} = m_{\text{B}} a = 0$$

Add the equations to eliminate the tension force and solve for F.

$$\left(F_{\text{T}} - m_{\text{A}} g \sin\theta_{\text{A}}\right) + \left(F - F_{\text{T}} - m_{\text{B}} g \sin\theta_{\text{B}}\right) = 0 \ \rightarrow$$

$$F = g\left(m_{\text{A}} \sin\theta_{\text{A}} + m_{\text{B}} \sin\theta_{\text{B}}\right)$$

$$= \left(9.80\,\text{m/s}^2\right)\left[(9.5\,\text{kg})\sin 59^\circ + (11.5\,\text{kg})\sin 32^\circ\right] = \boxed{1.40\times10^2\,\text{N}}$$

(*b*) Since $\theta_{\text{A}} > \theta_{\text{B}}$, if there were no connecting string, m_{A} would have a larger acceleration than m_{B}. If $\theta_{\text{A}} < \theta_{\text{B}}$, there would be no tension. But, since there is a connecting string, there will be tension in the string. Use the free-body diagram from above but ignore the applied force $\vec{\mathbf{F}}$.

$$m_{\text{A}}: \ \sum F_x = F_{\text{T}} - m_{\text{A}} g \sin\theta_{\text{A}} = m_{\text{A}} a \quad ; \quad m_{\text{b}}: \ \sum F_x = -F_{\text{T}} - m_{\text{B}} g \sin\theta_{\text{B}} = m_{\text{B}} a$$

Again add the two equations to eliminate the tension force.

$$\left(F_{\text{T}} - m_{\text{A}} g \sin\theta_{\text{A}}\right) + \left(-F_{\text{T}} - m_{\text{B}} g \sin\theta_{\text{B}}\right) = m_{\text{A}} a + m_{\text{B}} a \ \rightarrow$$

$$a = -g\frac{m_{\text{A}} \sin\theta_{\text{A}} + m_{\text{B}} \sin\theta_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = -\left(9.80\,\text{m/s}^2\right)\frac{(9.5\,\text{kg})\sin 59^\circ + (11.5\,\text{kg})\sin 32^\circ}{21.0\,\text{kg}}$$

$$= -6.644\,\text{m/s}^2 \approx \boxed{6.64\,\text{m/s}^2\text{, down the planes}}$$

(*c*) Use one of the Newton's second law expressions from part (*b*) to find the string tension. It must be positive if there is a tension.

$$F_{\text{T}} - m_{\text{A}} g \sin\theta_{\text{A}} = m_{\text{A}} a \ \rightarrow$$

$$F_{\text{T}} = m_{\text{A}}\left(g \sin\theta_{\text{A}} + a\right) = (9.5\,\text{kg})\left[\left(9.80\,\text{m/s}^2\right)(\sin 59^\circ) - 6.644\,\text{m/s}^2\right] = \boxed{17\,\text{N}}$$

87. (*a*) If the 2-block system is taken as a whole system, then the net force on the system is just the force $\vec{\mathbf{F}}$, accelerating the total mass. Use Newton's second law to find the force from the mass and acceleration. Take the direction of motion caused by the force (left for the bottom block, right for the top block) as the positive direction. Then both blocks have the same acceleration.

$$\sum F_x = F = \left(m_{\text{top}} + m_{\text{bottom}}\right)a = (9.0\,\text{kg})\left(2.5\,\text{m/s}^2\right) = 22.5\,\text{N} \approx \boxed{23\,\text{N}}$$

(*b*) The tension in the connecting cord is the only force acting on the top block, and so must be causing its acceleration. Again use Newton's second law.

$$\sum F_x = F_{\text{T}} = m_{\text{top}} a = (1.5\,\text{kg})\left(2.5\,\text{m/s}^2\right) = 3.75\,\text{N} \approx \boxed{3.8\,\text{N}}$$

This could be checked by using the bottom block.

$$\sum F_x = F - F_{\text{T}} = m_{\text{bottom}} a \ \rightarrow \ F_{\text{T}} = F - m_{\text{bottom}} a = 22.5\,\text{N} - (7.5\,\text{kg})\left(2.5\,\text{m/s}^2\right) = 3.75\,\text{N}$$

88. (*a*) For this scenario, find your location at a time of 4.0 sec, using Eq. 2-12b. The acceleration is found from Newton's second law.

$$a = \frac{F_{\text{forward}}}{m} = \frac{1200\,\text{N}}{750\,\text{kg}} \ \rightarrow$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (15\,\text{m/s})(4.0\,\text{s}) + \tfrac{1}{2}\frac{1200\,\text{N}}{750\,\text{kg}}(4.0\,\text{s})^2 = 72.8\ \text{m} > 65\,\text{m}$$

$\boxed{\text{Yes}}$, you will make it through the intersection before the light turns red.

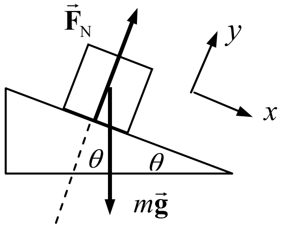
(*b*) For this scenario, find your location when the car has been fully stopped, using Eq. 2-12c. The acceleration is found from Newton's second law.

$$a = \frac{F_{\text{braking}}}{m} = -\frac{1800\,\text{N}}{750\,\text{kg}} \quad \rightarrow \quad v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15\,\text{m/s})^2}{2\left(-\dfrac{1800\,\text{N}}{750\,\text{kg}}\right)} = 46.9\,\text{m} > 45\,\text{m}$$

$\boxed{\text{No}}$, you will not stop before entering the intersection.

89. We take the mass of the crate as *m* until we insert values. A free-body diagram is shown.



(*a*) (i) Use Newton's second law to find the acceleration.

$$\sum F_x = mg\sin\theta = ma \quad \rightarrow \quad a = \boxed{g\sin\theta}$$

(ii) Use Eq. 2-12b to find the time for a displacement of ℓ.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad \ell = \tfrac{1}{2}g(\sin\theta)t^2 \quad \rightarrow$$

$$t = \boxed{\sqrt{\frac{2\ell}{g\sin\theta}}}$$

(iii) Use Eq. 2-12a to find the final velocity.

$$v = v_0 + at = g\sin\theta\left[\sqrt{\frac{2\ell}{g\sin\theta}}\right] = \boxed{\sqrt{2\ell g\sin\theta}}$$

(iv) Use Newton's second law to find the normal force.

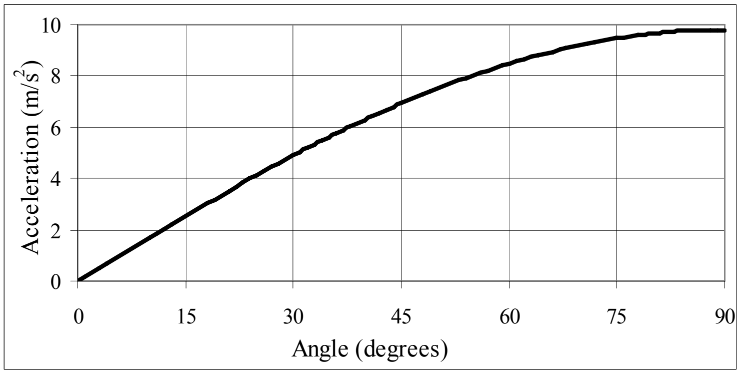
$$\sum F_y = F_\text{N} - mg\cos\theta = 0 \quad \rightarrow \quad F_\text{N} = \boxed{mg\cos\theta}$$

(*b*) Using the values of $m = 1500\,\text{kg}$, $g = 9.80\,\text{m/s}^2$, and $\ell = 100\,\text{m}$, the requested quantities become as follows.

$$a = (9.80\sin\theta)\,\text{m/s}^2 \ ;\ t = \sqrt{\frac{2(100)}{9.80\sin\theta}}\,\text{s}\ ;$$

$$v = \sqrt{2(100)(9.80)\sin\theta}\ \text{m/s}\ ;\ F_\text{N} = (1500)(9.80)\cos\theta$$

Graphs of these quantities as a function of θ are given here.



We consider the limiting cases: at an angle of $0°$, the crate does not move, and so the acceleration and final velocity would be 0. The time to travel 100 m would be infinite, and the normal force would be equal to the weight of

$$\begin{aligned} W &= mg \\ &= (1500\,\text{kg})\left(9.80\,\text{m/s}^2\right) \\ &= 1.47\times10^4\,\text{N}. \end{aligned}$$

The graphs are all consistent with those results.

For an angle of $90°$, we would expect free-fall motion. The acceleration should be $9.80\,\text{m/s}^2$. The normal force would be 0. The free-fall time for an object dropped from rest a distance of 100 m and the final velocity after that distance are calculated below.

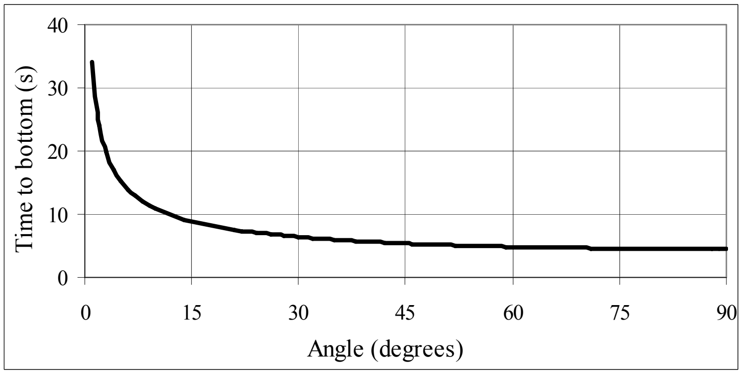
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

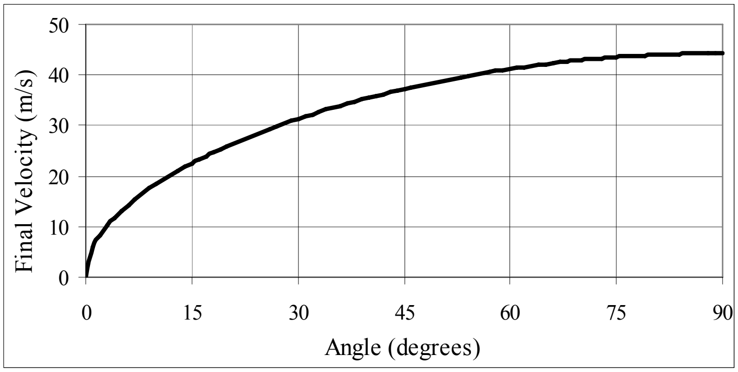
$$\ell = \tfrac{1}{2}gt^2 \quad \rightarrow$$

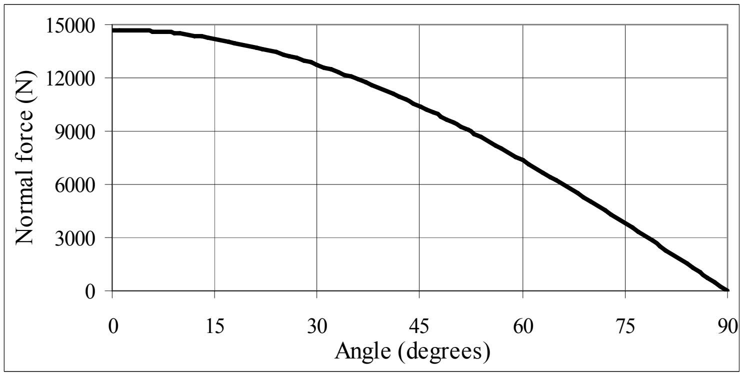
$$t = \sqrt{\frac{2\ell}{g}} = \sqrt{\frac{2(100\,\text{m})}{9.80\,\text{m/s}^2}} = 4.5\,\text{s}$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow$$

$$\begin{aligned} v &= \sqrt{2g(x - x_0)} \\ &= \sqrt{2\left(9.80\,\text{m/s}^2\right)(100\,\text{m})} \\ &= 44\,\text{m/s} \end{aligned}$$







Yes, the graphs agree with these results for the limiting cases.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH04.XLS," on tab "Problem 4.89b."

CHAPTER 5: Using Newton's Laws: Friction, Circular Motion, Drag Forces

Responses to Questions

1. Static friction between the crate and the truck bed causes the crate to accelerate.

2. The kinetic friction force is parallel to the ramp and the block's weight has a component parallel to the ramp. The parallel component of the block's weight is directed down the ramp whether the block is sliding up or down. However, the frictional force is always in the direction opposite the block's motion, so it will be down the ramp while the block is sliding up, but up the ramp while the block is sliding down. When the block is sliding up the ramp, the two forces acting on it parallel to the ramp are both acting in the *same* direction, and the magnitude of the net force is the sum of their magnitudes. But when the block is sliding down the ramp, the friction and the parallel component of the weight act in *opposite* directions, resulting in a smaller magnitude net force. A smaller net force yields a smaller (magnitude) acceleration.

3. Because the train has a larger mass. If the stopping forces on the truck and train are equal, the (negative) acceleration of the train will be much smaller than that of the truck, since acceleration is inversely proportional to mass $\left(\vec{\mathbf{a}} = \vec{\mathbf{F}}/m\right)$. The train will take longer to stop, as it has a smaller acceleration, and will travel a greater distance before stopping. The stopping force on the train may actually be greater than the stopping force on the truck, but not enough greater to compensate for the much greater mass of the train.

4. Yes. Refer to Table 5-1. The coefficient of static friction between rubber and many solid surfaces is typically between 1 and 4. The coefficient of static friction can also be greater than one if either of the surfaces is sticky.

5. When a skier is in motion, a small coefficient of kinetic friction lets the skis move easily on the snow with minimum effort. A large coefficient of static friction lets the skier rest on a slope without slipping and keeps the skier from sliding backward when going uphill.

6. When the wheels of a car are rolling without slipping, the force between each tire and the road is static friction, whereas when the wheels lock, the force is kinetic friction. The coefficient of static friction is greater than the coefficient of kinetic friction for a set of surfaces, so the force of friction between the tires and the road will be greater if the tires are rolling. Once the wheels lock, you also have no steering control over the car. It is better to apply the brakes slowly and use the friction between the brake mechanism and the wheel to stop the car while maintaining control. If the road is slick, the coefficients of friction between the road and the tires are reduced, and it is even more important to apply the brakes slowly to stay in control.

7. (*b*). If the car comes to a stop without skidding, the force that stops the car is the force of kinetic friction between the brake mechanism and the wheels. This force is designed to be large. If you slam on the brakes and skid to a stop, the force that stops the car will be the force of kinetic friction between the tires and the road. Even with a dry road, this force is likely to be less that the force of kinetic friction between the brake mechanism and the wheels. The car will come to a stop more quickly if the tires continue to roll, rather than skid. In addition, once the wheels lock, you have no steering control over the car.

8. The forces in (*a*), (*b*), and (*d*) are all equal to 400 N in magnitude.
   - (*a*) You exert a force of 400 N on the car; by Newton's third law the force exerted by the car on you also has a magnitude of 400 N.
   - (*b*) Since the car doesn't move, the friction force exerted by the road on the car must equal 400 N, too. Then, by Newton's third law, the friction force exerted by the car on the road is also 400 N.
   - (*c*) The normal force exerted by the road on you will be equal in magnitude to your weight (assuming you are standing vertically and have no vertical acceleration). This force is not required to be 400 N.
   - (*d*) The car is exerting a 400 N horizontal force on you, and since you are not accelerating, the ground must be exerting an equal and opposite horizontal force. Therefore, the magnitude of the friction force exerted by the road on you is 400 N.

9. On an icy surface, you need to put your foot straight down onto the sidewalk, with no component of velocity parallel to the surface. If you can do that, the interaction between you and the ice is through the static frictional force. If your foot has a component of velocity parallel to the surface of the ice, any resistance to motion will be caused by the kinetic frictional force, which is much smaller. You will be much more likely to slip.

10. Yes, the centripetal acceleration will be greater when the speed is greater since centripetal acceleration is proportional to the square of the speed. An object in uniform circular motion has an acceleration, since the direction of the velocity vector is changing even though the speed is constant.

11. No. The centripetal acceleration depends on 1/r, so a sharp curve, with a smaller radius, will generate a larger centripetal acceleration than a gentle curve, with a larger radius. (Note that the centripetal force in this case is provided by the static frictional force between the car and the road.)

12. The three main forces on the child are the downward force of gravity (weight), the normal force up on the child from the horse, and the static frictional force on the child from the surface of the horse. The frictional force provides the centripetal acceleration. If there are other forces, such as contact forces between the child's hands or legs and the horse, which have a radial component, they will contribute to the centripetal acceleration.

$\boxed{13.}$ As the child and sled come over the crest of the hill, they are moving in an arc. There must be a centripetal force, pointing inward toward the center of the arc. The combination of gravity (down) and the normal force (up) provides this centripetal force, which must be greater than or equal to zero. (At the top of the arc, $F_y = mg - N = mv^2/r \geq 0$.) The normal force must therefore be less than the child's weight.

14. No. The barrel of the dryer provides a centripetal force on the clothes to keep them moving in a circular path. A water droplet on the solid surface of the drum will also experience this centripetal force and move in a circle. However, as soon as the water droplet is at the location of a hole in the drum there will be no centripetal force on it and it will therefore continue moving in a path in the direction of its tangential velocity, which will take it out of the drum. There is no centrifugal force throwing the water outward; there is rather a lack of centripetal force to keep the water moving in a circular path.

15. When describing a centrifuge experiment, the force acting on the object in the centrifuge should be specified. Stating the rpm will let you calculate the speed of the object in the centrifuge. However, to find the force on an object, you will also need the distance from the axis of rotation.

16. She should let go of the string at the moment that the tangential velocity vector is directed exactly at the target.

17. The acceleration of the ball is inward, directly toward the pole, and is provided by the horizontal component of the tension in the string.

18. For objects (including astronauts) on the inner surface of the cylinder, the normal force provides a centripetal force which points inward toward the center of the cylinder. This normal force simulates the normal force we feel when on the surface of Earth.
   (*a*) Falling objects are not in contact with the floor, so when released they will continue to move with constant velocity until the floor reaches them. From the frame of reference of the astronaut inside the cylinder, it will appear that the object falls in a curve, rather than straight down.
   (*b*) The magnitude of the normal force on the astronaut's feet will depend on the radius and speed of the cylinder. If these are such that $v^2/r = g$ (so that $mv^2/r = mg$ for all objects), then the normal force will feel just like it does on the surface of Earth.
   (*c*) Because of the large size of Earth compared to humans, we cannot tell any difference between the gravitational force at our heads and at our feet. In a rotating space colony, the difference in the simulated gravity at different distances from the axis of rotation would be significant.

$\boxed{19.}$ At the top of bucket's arc, the gravitational force and normal forces from the bucket provide the centripetal force needed to keep the water moving in a circle. (If we ignore the normal forces, $mg = mv^2/r$, so the bucket must be moving with speed $v \ge \sqrt{gr}$ or the water will spill out of the bucket.) At the top of the arc, the water has a horizontal velocity. As the bucket passes the top of the arc, the velocity of the water develops a vertical component. But the bucket is traveling with the water, with the same velocity, and contains the water as it falls through the rest of its path.

20. (*a*) The normal force on the car is largest at point C. In this case, the centripetal force keeping the car in a circular path of radius R is directed upward, so the normal force must be greater than the weight to provide this net upward force.
   (*b*) The normal force is smallest at point A, the crest of the hill. At this point the centripetal force must be downward (towards the center of the circle) so the normal force must be less than the weight. (Notice that the normal force is equal to the weight at point B.)
   (*c*) The driver will feel heaviest where the normal force is greatest, or at point C.
   (*d*) The driver will feel lightest at point A, where the normal force is the least.
   (*e*) At point A, the centripetal force is weight minus normal force, or $mg - N = mv^2/r$. The point at which the car just loses contact with the road corresponds to a normal force of zero. Setting $N = 0$ gives $mg = mv^2/r$ or $v = \sqrt{gr}$.

21. Leaning in when rounding a curve on a bicycle puts the bicycle tire at an angle with respect to the ground. This increases the component of the (static) frictional force on the tire due to the road. This force component points inward toward the center of the curve, thereby increasing the centripetal force on the bicycle and making it easier to turn.

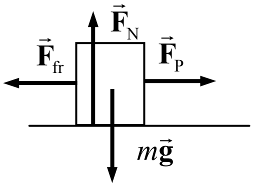
22. When an airplane is in level flight, the downward force of gravity is counteracted by the upward lift force, analogous to the upward normal force on a car driving on a level road. The lift on an airplane is perpendicular to the plane of the airplane's wings, so when the airplane banks, the lift vector has both vertical and horizontal components (similar to the vertical and horizontal components of the normal force on a car on a banked turn). The vertical component of the lift balances the weight and the horizontal component of the lift provides the centripetal force. If L = the total lift and φ = the banking angle, measured from the vertical, then $L\cos\varphi = mg$ and $L\sin\varphi = mv^2/r$ so $\varphi = \tan^{-1}\left(v^2/gr\right)$.

23. If we solve for *b*, we have $b = -F/v$. The units for *b* are N·s/m = kg·m·s/(m·s^2) = kg/s.

24. The force proportional to v^2 will dominate at high speed.

Solutions to Problems

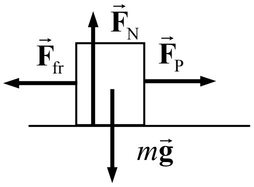
$\boxed{1.}$ A free-body diagram for the crate is shown. The crate does not accelerate vertically, and so $F_{\text{N}} = mg$. The crate does not accelerate horizontally, and so $F_{\text{P}} = F_{\text{fr}}$.



$$F_{\text{P}} = F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg = (0.30)(22\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{65\,\text{N}}$$

If the coefficient of kinetic friction is zero, then the horizontal force required is $\boxed{0\,\text{N}}$, since there is no friction to counteract. Of course, it would take a force to START the crate moving, but once it was moving, no further horizontal force would be necessary to maintain the motion.

2. A free-body diagram for the box is shown. Since the box does not accelerate vertically, $F_{\text{N}} = mg$.



(*a*) To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$. Thus we have for the starting motion,

$$\sum F_x = F_{\text{P}} - F_{\text{fr}} = 0 \;\rightarrow$$

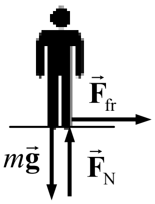
$$F_{\text{P}} = F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg \;\rightarrow\; \mu_s = \frac{F_{\text{P}}}{mg} = \frac{35.0\,\text{N}}{(6.0\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{0.60}$$

(*b*) The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

$$\sum F = F_{\text{P}} - F_{\text{fr}} = ma \;\rightarrow\; F_{\text{P}} - \mu_k F_{\text{N}} = ma \;\rightarrow\; F_{\text{P}} - \mu_k mg = ma \;\rightarrow$$

$$\mu_k = \frac{F_{\text{P}} - ma}{mg} = \frac{35.0\,\text{N} - (6.0\,\text{kg})(0.60\,\text{m/s}^2)}{(6.0\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{0.53}$$

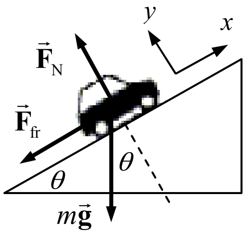
3. A free-body diagram for you as you stand on the train is shown. You do not accelerate vertically, and so $F_{\text{N}} = mg$. The maximum static frictional force is $\mu_s F_N$, and that must be greater than or equal to the force needed to accelerate you in order for you not to slip.



$$F_{\text{fr}} \ge ma \;\rightarrow\; \mu_s F_{\text{N}} \ge ma \;\rightarrow\; \mu_s mg \ge ma \;\rightarrow\; \mu_s \ge a/g = 0.20g/g = \boxed{0.20}$$

The static coefficient of friction must be at least 0.20 for you to not slide.

4. See the included free-body diagram. To find the maximum angle, assume that the car is just ready to slide, so that the force of static friction is a maximum. Write Newton's second law for both directions. Note that for both directions, the net force must be zero since the car is not accelerating.

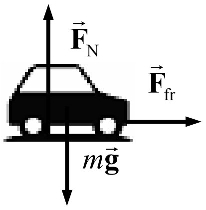


$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \;\rightarrow\; mg\sin\theta = F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg\cos\theta$$

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = 0.90 \;\rightarrow\; \theta = \tan^{-1}0.90° = \boxed{42°}$$

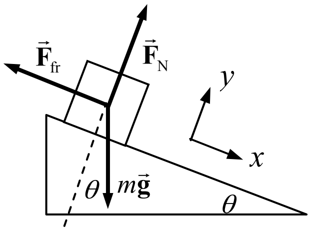
5. A free-body diagram for the accelerating car is shown. The car does not accelerate vertically, and so $F_{\text{N}} = mg$. The static frictional force is the accelerating force, and so $F_{\text{fr}} = ma$. If we assume the maximum acceleration, then we need the maximum force, and so the static frictional force would be its maximum value of $\mu_s F_{\text{N}}$. Thus we have



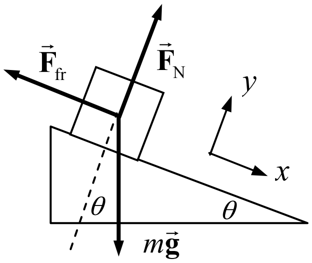
$$F_{\text{fr}} = ma \;\rightarrow\; \mu_s F_{\text{N}} = ma \;\rightarrow\; \mu_s mg = ma \;\rightarrow$$

$$a = \mu_s g = 0.90\left(9.80\,\text{m/s}^2\right) = \boxed{8.8\,\text{m/s}^2}$$

6. (*a*) Here is a free-body diagram for the box at rest on the plane. The force of friction is a STATIC frictional force, since the box is at rest.
(*b*) If the box were sliding down the plane, the only change is that the force of friction would be a KINETIC frictional force.
(*c*) If the box were sliding up the plane, the force of friction would be a KINETIC frictional force, and it would point down the plane, in the opposite direction to that shown in the diagram.
Notice that the angle is not used in this solution.



$\boxed{7.}$ Start with a free-body diagram. Write Newton's second law for each direction.



$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma_x$$
$$\sum F_y = F_{\text{N}} - mg\cos\theta = ma_y = 0$$

Notice that the sum in the *y* direction is 0, since there is no motion (and hence no acceleration) in the *y* direction. Solve for the force of friction.

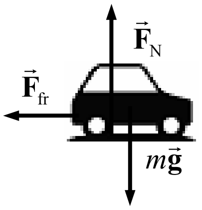
$$mg\sin\theta - F_{\text{fr}} = ma_x \;\rightarrow$$

$$F_{\text{fr}} = mg\sin\theta - ma_x = (25.0\text{ kg})\left[\left(9.80\,\text{m/s}^2\right)(\sin 27°) - 0.30\,\text{m/s}^2\right] = 103.7\text{ N} \approx \boxed{1.0\times10^2\,\text{N}}$$

Now solve for the coefficient of kinetic friction. Note that the expression for the normal force comes from the *y* direction force equation above.

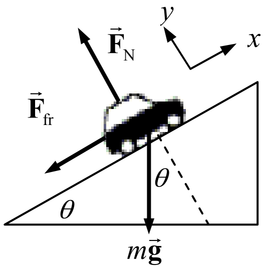
$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta \;\rightarrow\; \mu_k = \frac{F_{\text{fr}}}{mg\cos\theta} = \frac{103.7\,\text{N}}{(25.0\text{ kg})\left(9.80\,\text{m/s}^2\right)(\cos 27°)} = \boxed{0.48}$$

8. The direction of travel for the car is to the right, and that is also the positive horizontal direction. Using the free-body diagram, write Newton's second law in the *x* direction for the car on the level road. We assume that the car is just on the verge of skidding, so that the magnitude of the friction force is $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_x = -F_{\text{fr}} = ma \qquad F_{\text{fr}} = -ma = -\mu_s mg \;\rightarrow\; \mu_s = \frac{a}{g} = \frac{3.80\,\text{m/s}^2}{9.80\,\text{m/s}^2} = 0.3878$$

Now put the car on an inclined plane. Newton's second law in the x-direction for the car on the plane is used to find the acceleration. We again assume the car is on the verge of slipping, so the static frictional force is at its maximum.

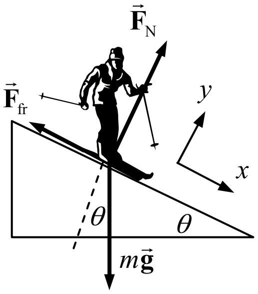


$$\sum F_x = -F_{\mathrm{fr}} - mg\sin\theta = ma \quad \rightarrow$$

$$a = \frac{-F_{\mathrm{fr}} - mg\sin\theta}{m} = \frac{-\mu_s mg\cos\theta - mg\sin\theta}{m} = -g\left(\mu_s\cos\theta + \sin\theta\right)$$

$$= -\left(9.80\,\mathrm{m/s^2}\right)\left(0.3878\cos 9.3^\circ + \sin 9.3^\circ\right) = \boxed{-5.3\,\mathrm{m/s^2}}$$

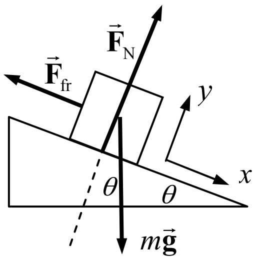
9. Since the skier is moving at a constant speed, the net force on the skier must be 0. See the free-body diagram, and write Newton's second law for both the x and y directions.



$$mg\sin\theta = F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}} = \mu_s mg\cos\theta \quad \rightarrow$$

$$\mu_s = \tan\theta = \tan 27^\circ = \boxed{0.51}$$

10. A free-body diagram for the bar of soap is shown. There is no motion in the y direction and thus no acceleration in the y direction. Write Newton's second law for both directions, and use those expressions to find the acceleration of the soap.



$$\sum F_x = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = ma$$

$$ma = mg\sin\theta - \mu_k F_{\mathrm{N}} = mg\sin\theta - \mu_k mg\cos\theta$$

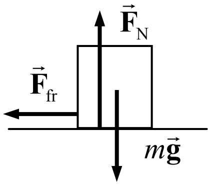
$$a = g\left(\sin\theta - \mu_k\cos\theta\right)$$

Now use Eq. 2-12b, with an initial velocity of 0, to find the final velocity.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2x}{g\left(\sin\theta - \mu_k\cos\theta\right)}} = \sqrt{\frac{2\left(9.0\,\mathrm{m}\right)}{\left(9.80\,\mathrm{m/s^2}\right)\left(\sin 8.0^\circ - \left(0.060\right)\cos 8.0^\circ\right)}} = \boxed{4.8\,\mathrm{s}}$$

11. A free-body diagram for the box is shown, assuming that it is moving to the right. The "push" is not shown on the free-body diagram because as soon as the box moves away from the source of the pushing force, the push is no longer applied to the box. It is apparent from the diagram that $F_{\mathrm{N}} = mg$ for the vertical direction. We write Newton's second law for the horizontal direction, with positive to the right, to find the acceleration of the box.



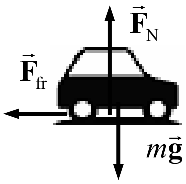
$$\sum F_x = -F_{\mathrm{fr}} = ma \quad \rightarrow \quad ma = -\mu_k F_{\mathrm{N}} = -\mu_k mg \quad \rightarrow$$

$$a = -\mu_k g = -0.15\left(9.80\,\mathrm{m/s^2}\right) = -1.47\,\mathrm{m/s^2}$$

Eq. 2-12c can be used to find the distance that the box moves before stopping. The initial speed is 4.0 m/s, and the final speed will be 0.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(3.5\,\mathrm{m/s}\right)^2}{2\left(-1.47\,\mathrm{m/s^2}\right)} = 4.17\,\mathrm{m} \approx \boxed{4.2\,\mathrm{m}}$$

12. (*a*) A free-body diagram for the car is shown, assuming that it is moving to the right. It is apparent from the diagram that $F_N = mg$ for the vertical direction. Write Newton's second law for the horizontal direction, with positive to the right, to find the acceleration of the car. Since the car is assumed to NOT be sliding, use the maximum force of static friction.



$$\sum F_x = -F_{fr} = ma \;\rightarrow\; ma = -\mu_s F_N = -\mu_s mg \;\rightarrow\; a = -\mu_s g$$

Eq. 2-12c can be used to find the distance that the car moves before stopping. The initial speed is given as *v*, and the final speed will be 0.

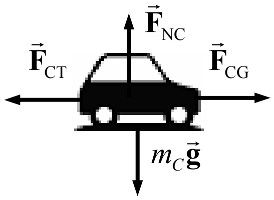
$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow\; (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - v^2}{2(-\mu_s g)} = \boxed{\frac{v^2}{2\mu_s g}}$$

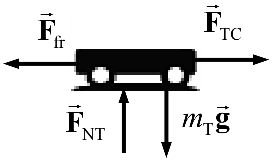
(*b*) Using the given values:

$$v = (95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.38\,\text{m/s} \qquad (x - x_0) = \frac{v^2}{2\mu_s g} = \frac{(26.38\,\text{m/s})^2}{2(0.65)(9.80\,\text{m/s}^2)} = \boxed{55\,\text{m}}$$

(*c*) From part (*a*), we see that the distance is inversely proportional to *g*, and so if *g* is reduced by a factor of 6, the distance is increased by a factor of 6 to $\boxed{330\,\text{m}}$.

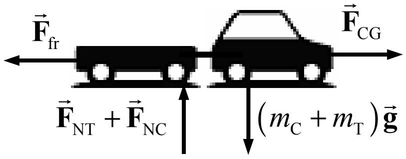
$\boxed{13.}$ We draw three free-body diagrams – one for the car, one for the trailer, and then "add" them for the combination of car and trailer. Note that since the car pushes against the ground, the ground will push against the car with an equal but oppositely directed force. $\vec{F}_{CG}$ is the force on the car due to the ground, $\vec{F}_{TC}$ is the force on the trailer due to the car, and $\vec{F}_{CT}$ is the force on the car due to the trailer. Note that by Newton's rhird law, $|\vec{F}_{CT}| = |\vec{F}_{TC}|$.





From consideration of the vertical forces in the individual free-body diagrams, it is apparent that the normal force on each object is equal to its weight. This leads to the conclusion that $F_{fr} = \mu_k F_{NT} = \mu_k m_T g = (0.15)(350\text{ kg})(9.80\,\text{m/s}^2) = 514.5\,\text{N}$.

Now consider the combined free-body diagram. Write Newton's second law for the horizontal direction, This allows the calculation of the acceleration of the system.



$$\sum F = F_{CG} - F_{fr} = (m_C + m_T)a \;\rightarrow$$

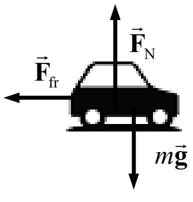
$$a = \frac{F_{CG} - F_{fr}}{m_C + m_T} = \frac{3600\,\text{N} - 514.5\,\text{N}}{1630\,\text{kg}} = 1.893\,\text{m/s}^2$$

Finally, consider the free-body diagram for the trailer alone. Again write Newton's second law for the horizontal direction, and solve for F_{TC}.

$$\sum F = F_{TC} - F_{fr} = m_T a \;\rightarrow$$

$$F_{TC} = F_{fr} + m_T a = 514.5\,\text{N} + (350\text{ kg})(1.893\,\text{m/s}^2) = 1177\,\text{N} \approx \boxed{1200\,\text{N}}$$

14. Assume that kinetic friction is the net force causing the deceleration. See the free-body diagram for the car, assuming that the right is the positive direction, and the direction of motion of the skidding car. There is no acceleration in the vertical direction, and so $F_N = mg$. Applying Newton's second law to the *x*



direction gives the following.

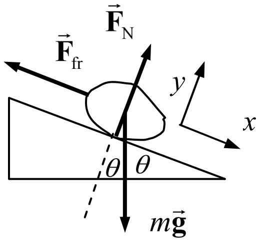
$$\sum F = -F_f = ma \quad \rightarrow \quad -\mu_k F_N = -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g$$

Use Eq. 2-12c to determine the initial speed of the car, with the final speed of the car being zero.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a\left(x-x_0\right)} = \sqrt{0 - 2\left(-\mu_k g\right)\left(x-x_0\right)} = \sqrt{2\left(0.80\right)\left(9.80\,\mathrm{m/s^2}\right)\left(72\ \mathrm{m}\right)} = \boxed{34\,\mathrm{m/s}}$$

15. (*a*) Consider the free-body diagram for the snow on the roof. If the snow is just ready to slip, then the static frictional force is at its maximum value, $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$. Write Newton's second law in both directions, with the net force equal to zero since the snow is not accelerating.



$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = 0 \quad \rightarrow$$

$$mg\sin\theta = F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}} = \mu_s mg\cos\theta \quad \rightarrow \quad \mu_s = \tan\theta = \tan 34^\circ = \boxed{0.67}$$

If $\mu_s > 0.67$, then the snow would not be on the verge of slipping.

(*b*) The same free-body diagram applies for the sliding snow. But now the force of friction is kinetic, so $F_{\mathrm{fr}} = \mu_k F_{\mathrm{N}}$, and the net force in the *x* direction is not zero. Write Newton's second law for the *x* direction again, and solve for the acceleration.

$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = ma$$

$$a = \frac{mg\sin\theta - F_{\mathrm{fr}}}{m} = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g\left(\sin\theta - \mu_k\cos\theta\right)$$

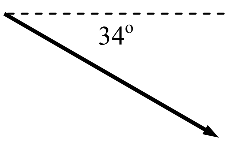
Use Eq. 2-12c with $v_i = 0$ to find the speed at the end of the roof.

$$v^2 - v_0^2 = 2a\left(x - x_0\right)$$

$$v = \sqrt{v_0 + 2a\left(x-x_0\right)} = \sqrt{2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x-x_0\right)}$$

$$= \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(\sin 34^\circ - \left(0.20\right)\cos 34^\circ\right)\left(6.0\ \mathrm{m}\right)} = 6.802\,\mathrm{m/s} \approx \boxed{6.8\,\mathrm{m/s}}$$

(*c*) Now the problem becomes a projectile motion problem. The projectile has an initial speed of 6.802 m/s, directed at an angle of 34° below the horizontal. The horizontal component of the speed, (6.802 m/s) cos 34° = 5.64 m/s, will stay constant. The vertical component will change due to gravity. Define the positive direction to be downward. Then the starting vertical velocity is (6.802 m/s) sin 34° =3.804 m/s, the vertical acceleration is 9.80 m/s^2, and the vertical displacement is 10.0 m. Use Eq. 2-12c to find the final vertical speed.



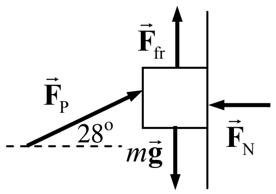
$$v_y^2 - v_{y0y}^2 = 2a\left(y - y_0\right)$$

$$v_y = \sqrt{v_{y0}^2 + 2a\left(y-y_0\right)} = \sqrt{\left(3.804\,\mathrm{m/s}\right)^2 + 2\left(9.80\,\mathrm{m/s^2}\right)\left(10.0\ \mathrm{m}\right)} = 14.5\ \mathrm{m/s}$$

To find the speed when it hits the ground, the horizontal and vertical components of velocity must again be combined, according to the Pythagorean theorem.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(5.64\,\mathrm{m/s}\right)^2 + \left(14.5\ \mathrm{m/s}\right)^2} = 15.6\,\mathrm{m/s} \approx \boxed{16\,\mathrm{m/s}}$$

16. Consider a free-body diagram for the box, showing force on the box. When $F_\text{P} = 23\,\text{N}$, the block does not move. Thus in that case, the force of friction is static friction, and must be at its maximum value, given by $F_\text{fr} = \mu_s F_\text{N}$. Write Newton's second law in both the *x* and *y* directions. The net force in each case must be 0, since the block is at rest.



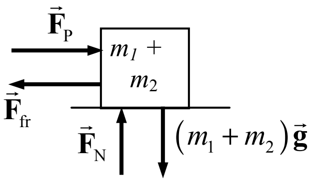
$$\sum F_x = F_\text{P}\cos\theta - F_\text{N} = 0 \;\rightarrow\; F_\text{N} = F_\text{P}\cos\theta$$

$$\sum F_y = F_\text{fr} + F_\text{P}\sin\theta - mg = 0 \;\rightarrow\; F_\text{fr} + F_\text{P}\sin\theta = mg$$

$$\mu_s F_\text{N} + F_\text{P}\sin\theta = mg \;\rightarrow\; \mu_s F_\text{P}\cos\theta + F_\text{P}\sin\theta = mg$$

$$m = \frac{F_\text{P}}{g}\left(\mu_s\cos\theta + \sin\theta\right) = \frac{23\,\text{N}}{9.80\,\text{m/s}^2}\left(0.40\cos 28^\circ + \sin 28^\circ\right) = \boxed{1.9\,\text{kg}}$$

17. (*a*) Since the two blocks are in contact, they can be treated as a single object as long as no information is needed about internal forces (like the force of one block pushing on the other block). Since there is no motion in the vertical direction, it is apparent that $F_\text{N} = (m_1 + m_2)g$, and so $F_\text{fr} = \mu_k F_\text{N} = \mu_k (m_1 + m_2) g$. Write Newton's second law for the horizontal direction.

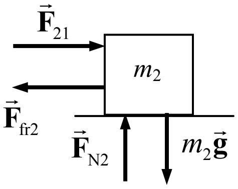


$$\sum F_x = F_\text{P} - F_\text{fr} = (m_1 + m_2)a \;\rightarrow$$

$$a = \frac{F_\text{P} - F_\text{fr}}{m_1 + m_2} = \frac{F_\text{P} - \mu_k(m_1 + m_2)g}{m_1 + m_2} = \frac{650\,\text{N} - (0.18)(190\,\text{kg})(9.80\,\text{m/s}^2)}{190\,\text{kg}}$$

$$= 1.657\,\text{m/s}^2 \approx \boxed{1.7\,\text{m/s}^2}$$

(*b*) To solve for the contact forces between the blocks, an individual block must be analyzed. Look at the free-body diagram for the second block. $\vec{\mathbf{F}}_{21}$ is the force of the first block pushing on the second block. Again, it is apparent that $F_\text{N2} = m_2 g$ and so $F_\text{fr2} = \mu_k F_\text{N2} = \mu_k m_2 g$. Write Newton's second law for the horizontal direction.

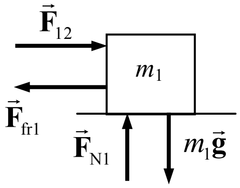


$$\sum F_x = F_{21} - F_\text{fr2} = m_2 a \;\rightarrow$$

$$F_{21} = \mu_k m_2 g + m_2 a = (0.18)(125\,\text{kg})(9.80\,\text{m/s}^2) + (125\,\text{kg})(1.657\,\text{m/s}^2) = \boxed{430\,\text{N}}$$

By Newton's third law, there will also be a 430 N force to the left on block # 1 due to block # 2.

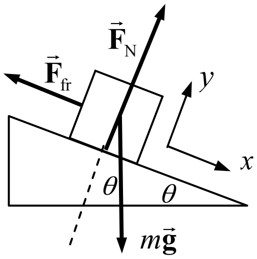
(*c*) If the crates are reversed, the acceleration of the system will remain the same – the analysis from part (*a*) still applies. We can also repeat the analysis from part (*b*) to find the force of one block on the other, if we simply change m_1 to m_2 in the free-body diagram and the resulting equations.



$$a = \boxed{1.7\,\text{m/s}^2}\;;\; \sum F_x = F_{12} - F_\text{fr1} = m_1 a \;\rightarrow$$

$$F_{12} = \mu_k m_1 g + m_1 a = (0.18)(65\,\text{kg})(9.80\,\text{m/s}^2) + (65\,\text{kg})(1.657\,\text{m/s}^2) = \boxed{220\,\text{N}}$$

18. (*a*) Consider the free-body diagram for the crate on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

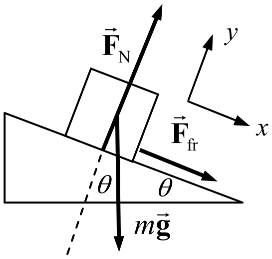
$$a = g\left(\sin\theta - \mu_k\cos\theta\right)$$

$$= \left(9.80\,\text{m/s}^2\right)\left(\sin 25.0° - 0.19\cos 25.0°\right) = 2.454\,\text{m/s}^2 \approx \boxed{2.5\,\text{m/s}^2}$$

(*b*) Now use Eq. 2-12c, with an initial velocity of 0, to find the final velocity.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad v = \sqrt{2a\left(x - x_0\right)} = \sqrt{2\left(2.454\,\text{m/s}^2\right)\left(8.15\,\text{m}\right)} = \boxed{6.3\,\text{m/s}}$$

$\boxed{19.}$ (*a*) Consider the free-body diagram for the crate on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions, and find the acceleration.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta + F_{fr} = ma$$

$$ma = mg\sin\theta + \mu_k F_N = mg\sin\theta + \mu_k mg\cos\theta$$

$$a = g\left(\sin\theta + \mu_k\cos\theta\right)$$

Now use Eq. 2-12c, with an initial velocity of $-3.0\,\text{m/s}$ and a final velocity of 0 to find the distance the crate travels up the plane.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow$$

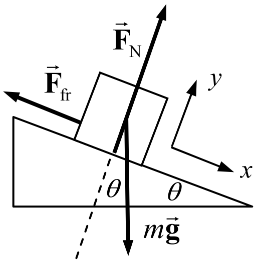
$$x - x_0 = \frac{-v_0^2}{2a} = \frac{-\left(-3.0\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)\left(\sin 25.0° + 0.17\cos 25.0°\right)} = -0.796\,\text{m}$$

The crate travels $\boxed{0.80\,\text{m}}$ up the plane.

(*b*) We use the acceleration found above with the initial velocity in Eq. 2-12a to find the time for the crate to travel up the plane.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{up}} = -\frac{v_0}{a_{up}} = -\frac{\left(-3.0\,\text{m/s}\right)}{\left(9.80\,\text{m/s}^2\right)\left(\sin 25.0° + 0.17\cos 25.0°\right)} = 0.5308\,\text{s}$$

The total time is NOT just twice the time to travel up the plane, because the acceleration of the block is different for the two parts of the motion. The second free-body diagram applies to the block sliding down the plane. A similar analysis will give the acceleration, and then Eq. 2-12b with an initial velocity of 0 is used to find the time to move down the plane.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = g\left(\sin\theta - \mu_k\cos\theta\right)$$

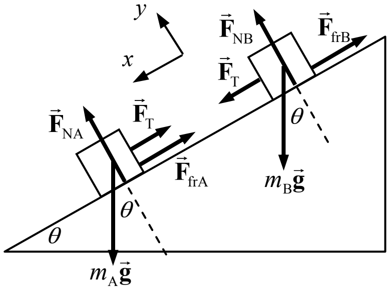
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$t_{\text{down}} = \sqrt{\frac{2(x-x_0)}{a_{\text{down}}}} = \sqrt{\frac{2(0.796\,\text{m})}{(9.80\,\text{m/s}^2)(\sin 25.0° - 0.17\cos 25.0°)}} = 0.7778\,\text{s}$$

$$t = t_{\text{up}} + t_{\text{down}} = 0.5308\,\text{s} + 0.7778\,\text{s} = \boxed{1.3\,\text{s}}$$

It is worth noting that the final speed is about 2.0 m/s, significantly less than the 3.0 m/s original speed.

20. Since the upper block has a higher coefficient of friction, that block will "drag behind" the lower block. Thus there will be tension in the cord, and the blocks will have the same acceleration. From the free-body diagrams for each block, we write Newton's second law for both the *x* and *y* directions for each block, and then combine those equations to find the acceleration and tension.



(*a*) Block A:

$$\sum F_{y\text{A}} = F_{\text{NA}} - m_{\text{A}} g\cos\theta = 0 \quad \rightarrow \quad F_{\text{NA}} = m_{\text{A}} g\cos\theta$$

$$\sum F_{x\text{A}} = m_{\text{A}} g\sin\theta - F_{\text{frA}} - F_{\text{T}} = m_{\text{A}} a$$

$$m_{\text{A}} a = m_{\text{A}} g\sin\theta - \mu_{\text{A}} F_{\text{NA}} - F_{\text{T}} = m_{\text{A}} g\sin\theta - \mu_{\text{A}} m_{\text{A}} g\cos\theta - F_{\text{T}}$$

Block B:

$$\sum F_{yB} = F_{\text{NB}} - m_{\text{B}} g\cos\theta = 0 \quad \rightarrow \quad F_{\text{NB}} = m_{\text{B}} g\cos\theta$$

$$\sum F_{xB} = m_{\text{A}} g\sin\theta - F_{\text{frA}} + F_{\text{T}} = m_{\text{B}} a$$

$$m_{\text{B}} a = m_{\text{B}} g\sin\theta - \mu_{\text{B}} F_{\text{NB}} + F_{\text{T}} = m_{\text{B}} g\sin\theta - \mu_{\text{B}} m_{\text{B}} g\cos\theta + F_{\text{T}}$$

Add the final equations together from both analyses and solve for the acceleration.

$$m_{\text{A}} a = m_{\text{A}} g\sin\theta - \mu_{\text{A}} m_{\text{A}} g\cos\theta - F_{\text{T}} \quad ; \quad m_{\text{B}} a = m_{\text{B}} g\sin\theta - \mu_{\text{B}} m_{\text{B}} g\cos\theta + F_{\text{T}}$$

$$m_{\text{A}} a + m_{\text{B}} a = m_{\text{A}} g\sin\theta - \mu_{\text{A}} m_{\text{A}} g\cos\theta - F_{\text{T}} + m_{\text{B}} g\sin\theta - \mu_{\text{B}} m_{\text{B}} g\cos\theta + F_{\text{T}} \quad \rightarrow$$

$$a = g\left[\frac{m_{\text{A}}(\sin\theta - \mu_{\text{A}}\cos\theta) + m_{\text{B}}(\sin\theta - \mu_{\text{B}}\cos\theta)}{(m_{\text{A}} + m_{\text{B}})}\right]$$

$$= (9.80\,\text{m/s}^2)\left[\frac{(5.0\,\text{kg})(\sin 32° - 0.20\cos 32°) + (5.0\,\text{kg})(\sin 32° - 0.30\cos 32°)}{(10.0\,\text{kg})}\right]$$

$$= 3.1155\,\text{m/s}^2 \approx \boxed{3.1\,\text{m/s}^2}$$

(*b*) Solve one of the equations for the tension force.

$$m_{\text{A}} a = m_{\text{A}} g\sin\theta - \mu_{\text{A}} m_{\text{A}} g\cos\theta - F_{\text{T}} \quad \rightarrow$$

$$F_{\text{T}} = m_{\text{A}}(g\sin\theta - \mu_{\text{A}} g\cos\theta - a)$$

$$= (5.0\,\text{kg})\left[(9.80\,\text{m/s}^2)(\sin 32° - 0.20\cos 32°) - 3.1155\,\text{m/s}^2\right] = \boxed{2.1\,\text{N}}$$

21. (*a*) If $\mu_A < \mu_B$, the untethered acceleration of m_A would be greater than that of m_B. If there were no cord connecting the masses, m_A would "run away" from m_B. So if they are joined together, m_A would be restrained by the tension in the cord, m_B would be pulled forward by the tension in the cord, and the two masses would have the same acceleration. This is exactly the situation for Problem 20.

(*b*) If $\mu_A > \mu_B$, the untethered acceleration of m_A would be less than that of m_B. So even if there is a cord between them, m_B will move ever closer to m_A, and there will be no tension in the cord. If the incline were long enough, eventually m_B would catch up to m_A and begin to push it down the plane.

(*c*) For $\mu_A < \mu_B$, the analysis will be exactly like Problem 20. Refer to that free-body diagram and analysis. The acceleration and tension are as follows, taken from the Problem 20 analysis.

$$a = g\left[\frac{m_A(\sin\theta - \mu_A\cos\theta) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$m_A a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T \quad \rightarrow$$

$$F_T = m_A g\sin\theta - \mu_A m_A g\cos\theta - m_A a$$

$$= m_A g\sin\theta - \mu_A m_A g\cos\theta - m_A g\left[\frac{m_A(\sin\theta - \mu_A\cos\theta) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$= \boxed{\frac{m_A m_B g\cos\theta}{(m_A + m_B)}(\mu_B - \mu_A)}$$

For $\mu_A > \mu_B$, we can follow the analysis of Problem 20 but not include the tension forces. Each block will have its own acceleration. Refer to the free-body diagram for Problem 20.

Block A:

$$\sum F_{yA} = F_{NA} - m_A g\cos\theta = 0 \quad \rightarrow \quad F_{NA} = m_A g\cos\theta$$

$$\sum F_{xA} = m_A g\sin\theta - F_{frA} = m_A a_A$$

$$m_A a_A = m_A g\sin\theta - \mu_A F_{NA} = m_A g\sin\theta - \mu_A m_A g\cos\theta \quad \rightarrow$$

$$\boxed{a_A = g(\sin\theta - \mu_A\cos\theta)}$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g\cos\theta = 0 \quad \rightarrow \quad F_{NB} = m_B g\cos\theta$$

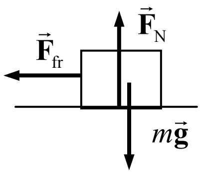
$$\sum F_{xB} = m_A g\sin\theta - F_{frA} = m_B a_B$$

$$m_B a_B = m_B g\sin\theta - \mu_B F_{NB} = m_B g\sin\theta - \mu_B m_B g\cos\theta \quad \rightarrow$$

$$\boxed{a_B = g(\sin\theta - \mu_B\cos\theta)}$$

Note that since $\mu_A > \mu_B$, $a_A > a_B$ as mentioned above. And $\boxed{F_T = 0}$.

22. The force of static friction is what decelerates the crate if it is not sliding on the truck bed. If the crate is not to slide, but the maximum deceleration is desired, then the maximum static frictional force must be exerted, and so $F_{fr} = \mu_s F_N$. The direction of travel is to the right. It is apparent that $F_N = mg$ since there is no acceleration in the y direction. Write Newton's second law for the truck in



the horizontal direction.

$$\sum F_x = -F_{\text{fr}} = ma \;\rightarrow\; -\mu_s mg = ma \;\rightarrow\; a = -\mu_s g = -(0.75)\left(9.80\,\text{m/s}^2\right) = \boxed{-7.4\,\text{m/s}^2}$$

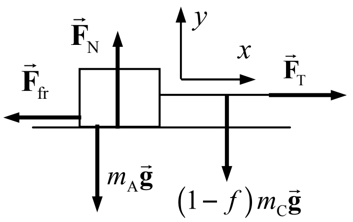
The negative sign indicates the direction of the acceleration – opposite to the direction of motion.

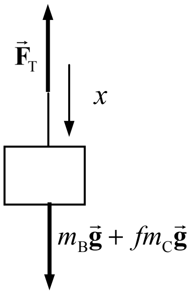
23. (*a*) For m_B to not move, the tension must be equal to $m_B g$, and so $m_B g = F_T$. For m_A to not move, the tension must be equal to the force of static friction, and so $F_S = F_T$. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_T = F_s \le \mu_s m_A g \;\rightarrow\; m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\,\text{kg}}{0.40} = 5.0\text{ kg} \;\rightarrow\; m_A \ge \boxed{5.0\text{ kg}}$$

(*b*) For m_B to move with constant velocity, the tension must be equal to $m_B g$. For m_A to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_k = \mu_k m_A g \;\rightarrow\; m_A = \frac{m_B}{\mu_k} = \frac{2.0\,\text{kg}}{0.30} = \boxed{6.7\,\text{kg}}$$





24. We define *f* to be the fraction of the cord that is handing down, between m_B and the pulley. Thus the mass of that piece of cord is fm_C. We assume that the system is moving to the right as well. We take the tension in the cord to be F_T at the pulley. We treat the hanging mass and hanging fraction of the cord as one mass, and the sliding mass and horizontal part of the cord as another mass. See the free-body diagrams. We write Newton's second law for each object.

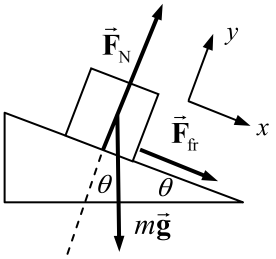
$$\sum F_{yA} = F_N - \left(m_A + (1-f)m_C\right)g = 0$$

$$\sum F_{xA} = F_T - F_{\text{fr}} = F_T - \mu_k F_N = \left(m_A + (1-f)m_C\right)a$$

$$\sum F_{xB} = \left(m_B + fm_C\right)g - F_T = \left(m_B + fm_C\right)a$$

Combine the relationships to solve for the acceleration. In particular, add the two equations for the *x*-direction, and then substitute the normal force.

$$a = \boxed{\left[\frac{m_B + fm_C - \mu_k\left(m_A + (1-f)m_C\right)}{m_A + m_B + m_C}\right]g}$$



$\boxed{25.}$ (*a*) Consider the free-body diagram for the block on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions, and find the acceleration.

$$\sum F_y = F_N - mg\cos\theta = 0 \;\rightarrow\; F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta + F_{\text{fr}} = ma$$

$$ma = mg\sin\theta + \mu_k F_N = mg\sin\theta + \mu_k mg\cos\theta$$

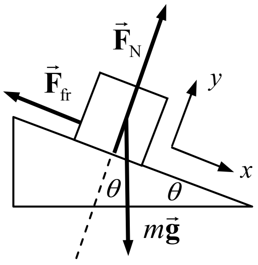
$$a = g\left(\sin\theta + \mu_k\cos\theta\right)$$

Now use Eq. 2-12c, with an initial velocity of v_0, a final velocity of 0, and a displacement of $-d$ to find the coefficient of kinetic friction.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \rightarrow 0 - v_0^2 = 2g\left(\sin\theta + \mu_k \cos\theta\right)\left(-d\right) \rightarrow$$

$$\mu_k = \boxed{\frac{v_0^2}{2gd\cos\theta} - \tan\theta}$$

(*b*) Now consider the free-body diagram for the block at the top of its motion. We use a similar force analysis, but now the magnitude of the friction force is given by $F_{\text{fr}} \le \mu_{\text{s}} F_{\text{N}}$, and the acceleration is 0.

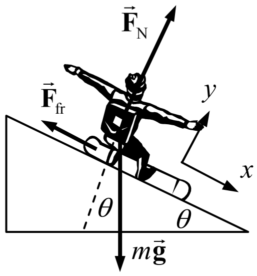


$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \rightarrow F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma = 0 \rightarrow F_{\text{fr}} = mg\sin\theta$$

$$F_{\text{fr}} \le \mu_{\text{s}} F_{\text{N}} \rightarrow mg\sin\theta \le \mu_{\text{s}} mg\cos\theta \rightarrow \boxed{\mu_{\text{s}} \ge \tan\theta}$$

26. First consider the free-body diagram for the snowboarder on the incline. Write Newton's second law for both directions, and find the acceleration.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \rightarrow F_{\text{N}} = mg\cos\theta$$

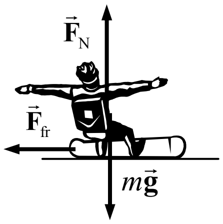
$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma$$

$$ma = mg\sin\theta - \mu_{k1}F_{\text{N}} = mg\sin\theta - \mu_{k1}mg\cos\theta$$

$$a_{\text{slope}} = g\left(\sin\theta - \mu_{k1}\cos\theta\right) = \left(9.80\,\text{m/s}^2\right)\left(\sin 28^\circ - 0.18\cos 28^\circ\right)$$

$$= 3.043\,\text{m/s}^2 \approx \boxed{3.0\,\text{m/s}^2}$$

Now consider the free-body diagram for the snowboarder on the flat surface. Again use Newton's second law to find the acceleration. Note that the normal force and the frictional force are different in this part of the problem, even though the same symbol is used.



$$\sum F_y = F_{\text{N}} - mg = 0 \rightarrow F_{\text{N}} = mg \qquad \sum F_x = -F_{\text{fr}} = ma$$

$$ma_{\text{flat}} = -F_{\text{fr}} = -\mu_{k2}F_{\text{N}} = -\mu_{k1}mg \rightarrow$$

$$a_{\text{flat}} = -\mu_{k2}g = -\left(0.15\right)\left(9.80\,\text{m/s}^2\right) = -1.47\,\text{m/s}^2 \approx \boxed{-1.5\,\text{m/s}^2}$$

Use Eq. 2-12c to find the speed at the bottom of the slope. This is the speed at the start of the flat section. Eq. 2-12c can be used again to find the distance *x*.

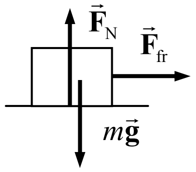
$$v^2 - v_0^2 = 2a\left(x - x_0\right) \rightarrow$$

$$v_{\substack{\text{end of}\\\text{slope}}} = \sqrt{v_0^2 + 2a_{\text{slope}}\left(x - x_0\right)} = \sqrt{\left(5.0\,\text{m/s}\right)^2 + 2\left(3.043\,\text{m/s}^2\right)\left(110\,\text{m}\right)} = 26.35\,\text{m/s}$$

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \rightarrow$$

$$\left(x - x_0\right) = \frac{v^2 - v_0^2}{2a_{\text{flat}}} = \frac{0 - \left(26.35\,\text{m/s}\right)^2}{2\left(-1.47\,\text{m/s}^2\right)} = 236\,\text{m} \approx \boxed{240\,\text{m}}$$

27. The belt is sliding underneath the box (to the right), so there will be a force of kinetic friction on the box, until the box reaches a speed of 1.5 m/s. Use the free-body diagram to calculate the acceleration of the box.



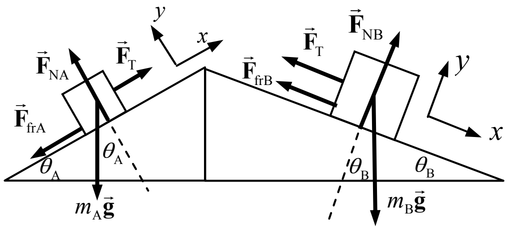
(*a*) $\sum F_x = F_{\text{fr}} = ma = \mu_{\text{k}}F_{\text{N}} = \mu_{\text{k}}mg \rightarrow a = \mu_{\text{k}}g$

$$\sum F_x = F_{\text{fr}} = ma = \mu_k F_N = \mu_k mg \rightarrow a = \mu_k g$$

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{v - 0}{\mu_k g} = \frac{1.5\,\text{m/s}}{(0.70)(9.80\,\text{m/s}^2)} = \boxed{0.22\,\text{s}}$$

$$(b) \quad x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{v^2}{2\mu_k g} = \frac{(1.5\,\text{m/s})^2}{2(0.70)(9.80\,\text{m/s}^2)} = \boxed{0.16\,\text{m}}$$

28. We define the positive x direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two x-equations to find the acceleration.



Block A:

$$\sum F_{yA} = F_{NA} - m_A g\cos\theta_A = 0 \rightarrow F_{NA} = m_A g\cos\theta_A$$

$$\sum F_{xA} = F_T - m_A g\sin\theta - F_{frA} = m_A a$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g\cos\theta_B = 0 \rightarrow F_{NB} = m_B g\cos\theta_B$$

$$\sum F_{xB} = m_B g\sin\theta - F_{frB} - F_T = m_B a$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{\text{fr}} = \mu F_N$.

$$m_A a = F_T - m_A g\sin\theta_A - \mu_A m_A g\cos\theta_A \quad ; \quad m_B a = m_B g\sin\theta_B - \mu_B m_B g\cos\theta_B - F_T$$

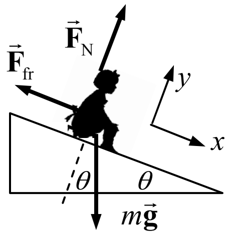
$$m_A a + m_B a = F_T - m_A g\sin\theta_A - \mu_A m_A g\cos\theta_A + m_B g\sin\theta_B - \mu_B m_B g\cos\theta_B - F_T \rightarrow$$

$$a = g\left[\frac{-m_A(\sin\theta_A + \mu_A\cos\theta_A) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\,\text{m/s}^2)\left[\frac{-(2.0\,\text{kg})(\sin 51^\circ + 0.30\cos 51^\circ) + (5.0\,\text{kg})(\sin 21^\circ - 0.30\cos 21^\circ)}{(7.0\,\text{kg})}\right]$$

$$= \boxed{-2.2\,\text{m/s}^2}$$

29. We assume that the child starts from rest at the top of the slide, and then slides a distance $x - x_0$ along the slide. A force diagram is shown for the child on the slide. First, ignore the frictional force and so consider the no-friction case. All of the motion is in the x direction, so we will only consider Newton's second law for the x direction.



$$\sum F_x = mg\sin\theta = ma \rightarrow a = g\sin\theta$$

Use Eq. 2-12c to calculate the speed at the bottom of the slide.

$$v^2 - v_0^2 = 2a(x - x_0) \rightarrow v_{(\text{No friction})} = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g\sin\theta(x - x_0)}$$

Now include kinetic friction. We must consider Newton's second law in both the x and y directions now. The net force in the y direction must be 0 since there is no acceleration in the y direction.

$$\sum F_y = F_N - mg\cos\theta = 0 \rightarrow F_N = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{\text{fr}} = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g\left(\sin\theta - \mu_k\cos\theta\right)$$

With this acceleration, we can again use Eq. 2-12c to find the speed after sliding a certain distance.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \rightarrow v_{(\text{friction})} = \sqrt{v_0^2 + 2a\left(x - x_0\right)} = \sqrt{2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right)}$$

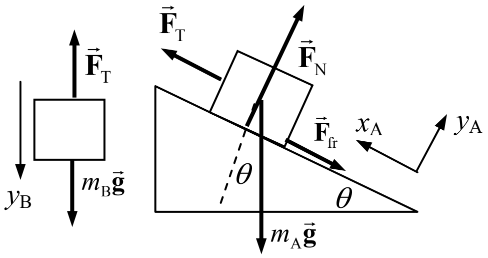
Now let the speed with friction be half the speed without friction, and solve for the coefficient of friction. Square the resulting equation and divide by $g\cos\theta$ to get the result.

$$v_{(\text{friction})} = \tfrac{1}{2}v_{(\text{No friction})} \rightarrow \sqrt{2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right)} = \tfrac{1}{2}\sqrt{2g\left(\sin\theta\right)\left(x - x_0\right)}$$

$$2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right) = \tfrac{1}{4}2g\left(\sin\theta\right)\left(x - x_0\right)$$

$$\mu_k = \tfrac{3}{4}\tan\theta = \tfrac{3}{4}\tan 34^\circ = \boxed{0.51}$$

30. (*a*) Given that m_B is moving down, m_A must be moving up the incline, and so the force of kinetic friction on m_A will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, and so $a_{yB} = a_{xA} = a$. Write Newton's second law for each mass.



$$\sum F_{yB} = m_B g - F_T = m_B a \rightarrow F_T = m_B g - m_B a$$

$$\sum F_{xA} = F_T - m_A g\sin\theta - F_{fr} = m_A a$$

$$\sum F_{yA} = F_N - m_A g\cos\theta = 0 \rightarrow F_N = m_A g\cos\theta$$

Take the information from the two *y* equations and substitute into the *x* equation to solve for the acceleration.

$$m_B g - m_B a - m_A g\sin\theta - \mu_k m_A g\cos\theta = m_A a \rightarrow$$

$$a = \frac{m_B g - m_A g\sin\theta - m_A g\mu_k g\cos\theta}{\left(m_A + m_B\right)} = \tfrac{1}{2}g\left(1 - \sin\theta - \mu_k g\cos\theta\right)$$

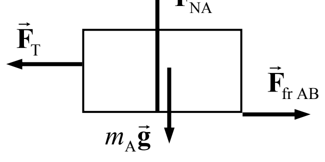
$$= \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(1 - \sin 34^\circ - 0.15\cos 34^\circ\right) = \boxed{1.6\,\text{m/s}^2}$$

(*b*) To have an acceleration of zero, the expression for the acceleration must be zero.

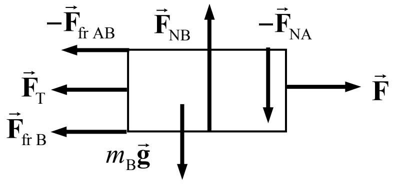
$$a = \tfrac{1}{2}g\left(1 - \sin\theta - \mu_k\cos\theta\right) = 0 \rightarrow 1 - \sin\theta - \mu_k\cos\theta = 0 \rightarrow$$

$$\mu_k = \frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin 34^\circ}{\cos 34^\circ} = \boxed{0.53}$$

$\boxed{31.}$ Draw a free-body diagram for each block.



Block A (top)



Block B (bottom)

$\vec{\mathbf{F}}_{\text{fr AB}}$ is the force of friction between the two blocks, $\vec{\mathbf{F}}_{\text{NA}}$ is the normal force of contact between the two blocks, $\vec{\mathbf{F}}_{\text{fr B}}$ is the force of friction between the bottom block and the floor, and $\vec{\mathbf{F}}_{\text{NB}}$ is the normal force of contact between the bottom block and the floor.

Neither block is accelerating vertically, and so the net vertical force on each block is zero.

$$\text{top: } F_{\text{NA}} - m_{\text{A}}g = 0 \quad \rightarrow \quad F_{\text{NA}} = m_{\text{A}}g$$

$$\text{bottom: } F_{\text{NB}} - F_{\text{NA}} - m_{\text{B}}g = 0 \quad \rightarrow \quad F_{\text{NB}} = F_{\text{NA}} + m_{\text{B}}g = \left(m_{\text{A}} + m_{\text{B}}\right)g$$

Take the positive horizontal direction to be the direction of motion of each block. Thus for the bottom block, positive is to the right, and for the top block, positive is to the left. Then, since the blocks are constrained to move together by the connecting string, both blocks will have the same acceleration. Write Newton's second law for the horizontal direction for each block.

$$\text{top: } F_{\text{T}} - F_{\text{fr AB}} = m_{\text{A}}a \qquad \text{bottom: } F - F_{\text{T}} - F_{\text{fr AB}} - F_{\text{fr B}} = m_{\text{B}}a$$

(*a*) If the two blocks are just to move, then the force of static friction will be at its maximum, and so the frictions forces are as follows.

$$F_{\text{fr AB}} = \mu_{\text{s}}F_{\text{NA}} = \mu_{\text{s}}m_{\text{A}}g \ ; \ F_{\text{fr B}} = \mu_{\text{s}}F_{\text{NB}} = \mu_{\text{s}}\left(m_{\text{A}} + m_{\text{B}}\right)g$$

Substitute into Newton's second law for the horizontal direction with $a = 0$ and solve for F.

$$\text{top: } F_{\text{T}} - \mu_{\text{s}}m_{\text{A}}g = 0 \quad \rightarrow \quad F_{\text{T}} = \mu_{\text{s}}m_{\text{A}}g$$

$$\text{bottom: } F - F_{\text{T}} - \mu_{\text{s}}m_{\text{A}}g - \mu_{\text{s}}\left(m_{\text{A}} + m_{\text{B}}\right)g = 0 \quad \rightarrow$$

$$F = F_{\text{T}} + \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}\left(m_{\text{A}} + m_{\text{B}}\right)g = \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}\left(m_{\text{A}} + m_{\text{B}}\right)g$$

$$= \mu_{\text{s}}\left(3m_{\text{A}} + m_{\text{B}}\right)g = (0.60)(14\,\text{kg})\left(9.80\,\text{m/s}^2\right) = 82.32\,\text{N} \approx \boxed{82\,\text{N}}$$

(*b*) Multiply the force by 1.1 so that $F = 1.1(82.32\,\text{N}) = 90.55\,\text{N}$. Again use Newton's second law for the horizontal direction, but with $a \neq 0$ and using the coefficient of kinetic friction.

$$\text{top: } F_{\text{T}} - \mu_{\text{k}}m_{\text{A}}g = m_{\text{A}}a$$

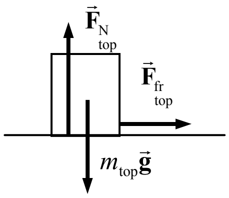
$$\text{bottom: } F - F_{\text{T}} - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}\left(m_{\text{A}} + m_{\text{B}}\right)g = m_{\text{B}}a$$

$$\text{sum: } F - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}\left(m_{\text{A}} + m_{\text{B}}\right)g = \left(m_{\text{A}} + m_{\text{B}}\right)a \quad \rightarrow$$

$$a = \frac{F - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}\left(m_{\text{A}} + m_{\text{B}}\right)g}{\left(m_{\text{A}} + m_{\text{B}}\right)} = \frac{F - \mu_{\text{k}}\left(3m_{\text{A}} + m_{\text{B}}\right)g}{\left(m_{\text{A}} + m_{\text{B}}\right)}$$

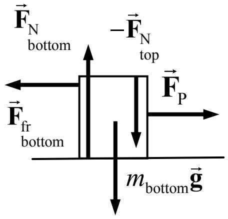
$$= \frac{90.55\,\text{N} - (0.40)(14.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{(8.0\,\text{kg})} = 4.459\,\text{m/s}^2 \approx \boxed{4.5\,\text{m/s}^2}$$

32. Free-body diagrams are shown for both blocks. There is a force of friction between the two blocks, which acts to the right on the top block, and to the left on the bottom block. They are a Newton's third law pair of forces.



(*a*) If the 4.0 kg block does not slide off, then it must have the same acceleration as the 12.0 kg block. That acceleration is caused by the force of static friction between the two blocks. To find the minimum coefficient, we use the maximum force of static friction.

$$F_{\underset{\text{top}}{\text{fr}}} = m_{\text{top}}a = \mu F_{\underset{\text{top}}{\text{N}}} = \mu m_{\text{top}}g \quad \rightarrow \quad \mu = \frac{a}{g} = \frac{5.2\,\text{m/s}^2}{9.80\,\text{m/s}^2} = 0.5306 \approx \boxed{0.53}$$



(*b*) If the coefficient of friction only has half the value, then the blocks will be sliding with respect to one another, and so the friction will be kinetic.

$$\mu = \tfrac{1}{2}(0.5306) = 0.2653 \ ; \ F_{\underset{\text{top}}{\text{fr}}} = m_{\text{top}}a = \mu F_{\underset{\text{top}}{\text{N}}} = \mu m_{\text{top}}g \quad \rightarrow$$

$$a = \mu g = (0.2653)(9.80\,\text{m/s}^2) = \boxed{2.6\,\text{m/s}^2}$$

(*c*) The bottom block is still accelerating to the right at $5.2\,\text{m/s}^2$. Since the top block has a smaller acceleration than that, it has a negative acceleration relative to the bottom block.

$$\vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{bottom}}} = \vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{ground}}} + \vec{\mathbf{a}}_{\substack{\text{ground rel}\\ \text{bottom}}} = \vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{ground}}} - \vec{\mathbf{a}}_{\substack{\text{bottom rel}\\ \text{ground}}} = 2.6\,\text{m/s}^2\,\hat{\mathbf{i}} - 5.2\,\text{m/s}^2\,\hat{\mathbf{i}} = -2.6\,\text{m/s}^2\,\hat{\mathbf{i}}$$

The top block has an acceleration of $\boxed{2.6\,\text{m/s}^2 \text{ to the left}}$ relative to the bottom block.

(*d*) No sliding:

$$F_{\substack{x\\ \text{bottom}\\ \text{net}}} = F_{\text{P}} - F_{\substack{\text{fr}\\ \text{bottom}}} = m_{\text{bottom}}a_{\text{bottom}} \quad \rightarrow$$

$$F_{\text{P}} = F_{\substack{\text{fr}\\ \text{bottom}}} + m_{\text{bottom}}a_{\text{bottom}} = F_{\substack{\text{fr}\\ \text{top}}} + m_{\text{bottom}}a_{\text{bottom}} = m_{\text{top}}a_{\text{top}} + m_{\text{bottom}}a_{\text{bottom}} = (m_{\text{top}} + m_{\text{bottom}})a$$

$$= (16.0\,\text{kg})(5.2\,\text{m/s}^2) = \boxed{83\,\text{N}}$$

This is the same as simply assuming that the external force is accelerating the total mass. The internal friction need not be considered if the blocks are not moving relative to each other.

Sliding:

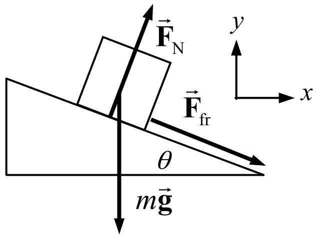
$$F_{\substack{x\\ \text{bottom}\\ \text{net}}} = F_{\text{P}} - F_{\substack{\text{fr}\\ \text{bottom}}} = m_{\text{bottom}}a_{\text{bottom}} \quad \rightarrow$$

$$F_{\text{P}} = F_{\substack{\text{fr}\\ \text{bottom}}} + m_{\text{bottom}}a_{\text{bottom}} = F_{\substack{\text{fr}\\ \text{top}}} + m_{\text{bottom}}a_{\text{bottom}} = m_{\text{top}}a_{\text{top}} + m_{\text{bottom}}a_{\text{bottom}}$$

$$= (4.0\,\text{kg})(2.6\,\text{m/s}^2) + (12.0\,\text{kg})(5.2\,\text{m/s}^2) = \boxed{73\,\text{N}}$$

Again this can be interpreted as the external force providing the acceleration for each block. The internal friction need not be considered.

33. To find the limiting value, we assume that the blocks are NOT slipping, but that the force of static friction on the smaller block is at its maximum value, so that $F_{\text{fr}} = \mu F_{\text{N}}$. For the two-block system, there is no friction on the system, and so $F = (M+m)a$ describes the horizontal motion of the system. Thus the upper block has a vertical acceleration of 0 and a horizontal acceleration of $\dfrac{F}{(M+m)}$. Write Newton's second law for the upper block, using the force diagram, and solve for the applied force *F*. Note that the static friction force will be DOWN the plane, since the block is on the verge of sliding UP the plane.

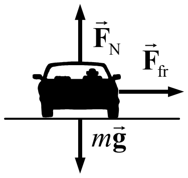


$$\sum F_y = F_{\text{N}}\cos\theta - F_{\text{fr}}\sin\theta - mg = F_{\text{N}}(\cos\theta - \mu\sin\theta) - mg = 0 \quad \rightarrow \quad F_{\text{N}} = \frac{mg}{(\cos\theta - \mu\sin\theta)}$$

$$\sum F_x = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = F_{\text{N}}(\sin\theta + \mu\cos\theta) = ma = m\frac{F}{M+m} \quad \rightarrow$$

$$F = F_{\text{N}}(\sin\theta + \mu\cos\theta)\frac{M+m}{m} = \frac{mg}{(\cos\theta - \mu\sin\theta)}(\sin\theta + \mu\cos\theta)\frac{M+m}{m}$$

$$= \boxed{(M+m)g\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$$



34. A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion.

$$F_\text{R} = F_\text{fr} \quad \rightarrow \quad m v^2/r = \mu_s F_\text{N} = \mu_s mg \quad \rightarrow$$

$$v = \sqrt{\mu_s rg} = \sqrt{(0.65)(80.0\,\text{m})(9.80\,\text{m/s}^2)} = 22.57\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

Notice that the result is $\boxed{\text{independent of the car's mass}}$.

35. (*a*) Find the centripetal acceleration from Eq. 5-1.

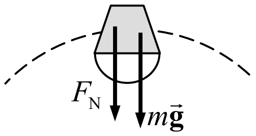
$$a_\text{R} = v^2/r = (1.30\,\text{m/s})^2/1.20\,\text{m} = 1.408\,\text{m/s}^2 \approx \boxed{1.41\,\text{m/s}^2}$$

(*b*) The net horizontal force is causing the centripetal motion, and so will be the centripetal force.

$$F_\text{R} = ma_\text{R} = (22.5\text{ kg})(1.408\,\text{m/s}^2) = 31.68\text{N} \approx \boxed{31.7\,\text{N}}$$

36. Find the centripetal acceleration from Eq. 5-1.

$$a_R = v^2/r = \frac{(525\,\text{m/s})^2}{4.80\times10^3\,\text{m}} = (57.42\,\text{m/s}^2)\left(\frac{1\,g}{9.80\,\text{m/s}^2}\right) = \boxed{5.86\,g\text{'s}}$$



37. We assume the water is rotating in a vertical circle of radius *r*. When the bucket is at the top of its motion, there would be two forces on the water (considering the water as a single mass). The weight of the water would be directed down, and the normal force of the bottom of the bucket pushing on the water would also be down. See the free-body diagram. If the water is moving in a circle, then the net downward force would be a centripetal force.

$$\sum F = F_\text{N} + mg = ma = mv^2/r \quad \rightarrow \quad F_\text{N} = m\left(v^2/r - g\right)$$

The limiting condition of the water falling out of the bucket means that the water loses contact with the bucket, and so the normal force becomes 0.

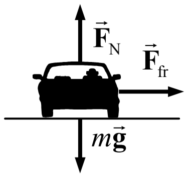
$$F_\text{N} = m\left(v^2/r - g\right) \quad \rightarrow \quad m\left(v_\text{critical}^2/r - g\right) = 0 \quad \rightarrow \quad v_\text{critical} = \sqrt{rg}$$

From this, we see that $\boxed{\text{yes}}$, it is possible to whirl the bucket of water fast enough. The minimum speed is $\boxed{\sqrt{rg}}$.

38. The centripetal acceleration of a rotating object is given by $a_\text{R} = v^2/r$.

$$v = \sqrt{a_\text{R} r} = \sqrt{(1.25\times10^5 g)r} = \sqrt{(1.25\times10^5)(9.80\,\text{m/s}^2)(8.00\times10^{-2}\,\text{m})} = 3.13\times10^2\text{ m/s}.$$

$$(3.13\times10^2\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(8.00\times10^{-2}\text{m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{3.74\times10^4\,\text{rpm}}$$



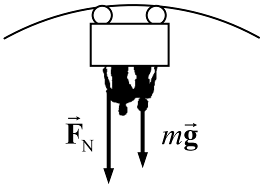
39. For an unbanked curve, the centripetal force to move the car in a circular path must be provided by the static frictional force. Also, since the roadway is level, the normal force on the car is equal to its weight. Assume the static frictional force is at its maximum value, and use the force relationships to calculate the radius of the

curve. See the free-body diagram, which assumes the center of the curve is to the right in the diagram.

$$F_{\mathrm{R}} = F_{\mathrm{fr}} \quad\rightarrow\quad mv^2/r = \mu_s F_{\mathrm{N}} = \mu_s mg \quad\rightarrow$$

$$r = v^2/\mu_s g = \frac{\left[(30\,\mathrm{km/h})\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{(0.7)\left(9.80\,\mathrm{m/s^2}\right)} = 28\,\mathrm{m} \approx \boxed{30\,\mathrm{m}}$$

40. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.

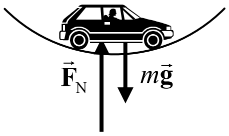


$$\sum F = F_{\mathrm{N}} + mg = ma = mv^2/r \quad\rightarrow\quad F_{\mathrm{N}} = m\left(v^2/r - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is as follows.

$$v_{\min}^2/r - g = 0 \quad\rightarrow\quad v_{\min} = \sqrt{rg} = \sqrt{\left(9.80\,\mathrm{m/s^2}\right)(7.6\,\mathrm{m})} = \boxed{8.6\,\mathrm{m/s}}$$

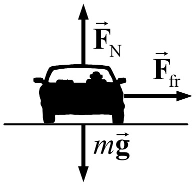
41. A free-body diagram for the car is shown. Write Newton's second law for the car in the vertical direction, assuming that up is positive. The normal force is twice the weight.



$$\sum F = F_{\mathrm{N}} - mg = ma \quad\rightarrow\quad 2mg - mg = mv^2/r \quad\rightarrow$$

$$v = \sqrt{rg} = \sqrt{(95\,\mathrm{m})\left(9.80\,\mathrm{m/s^2}\right)} = 30.51\,\mathrm{m/s} \approx \boxed{31\,\mathrm{m/s}}$$

42. In the free-body diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value.



$$F_R = F_{\mathrm{fr}} \quad\rightarrow\quad mv^2/r = \mu_s F_{\mathrm{N}} = \mu_s mg \quad\rightarrow\quad \mu_s = \frac{v^2}{rg} = \frac{\left[(95\,\mathrm{km/hr})\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/hr}}\right)\right]^2}{(85\ \mathrm{m})\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{0.84}$$

Notice that the result is independent of the car's mass.

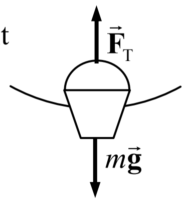
$\boxed{43.}$ The orbit radius will be the sum of the Earth's radius plus the 400 km orbit height. The orbital period is about 90 minutes. Find the centripetal acceleration from these data.

$$r = 6380\,\mathrm{km} + 400\,\mathrm{km} = 6780\,\mathrm{km} = 6.78\times10^6\,\mathrm{m} \qquad T = 90\ \mathrm{min}\left(\frac{60\ \mathrm{sec}}{1\ \mathrm{min}}\right) = 5400\ \mathrm{sec}$$

$$a_{\mathrm{R}} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(6.78\times10^6\,\mathrm{m}\right)}{(5400\ \mathrm{sec})^2} = \left(9.18\,\mathrm{m/s^2}\right)\left(\frac{1\,g}{9.80\,\mathrm{m/s^2}}\right) = 0.937 \approx \boxed{0.9\,g\text{'s}}$$

Notice how close this is to *g*, because the shuttle is not very far above the surface of the Earth, relative to the radius of the Earth.

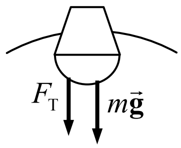
44. (*a*) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, with up as the positive direction.



$$\sum F_R = F_T - mg = ma = mv^2/r \quad \rightarrow$$

$$v = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(1.10\text{ m})\left[25.0\text{ N} - (2.00\text{ kg})(9.80\text{ m/s}^2)\right]}{2.00\text{ kg}}} = 1.723 \approx \boxed{1.7\text{ m/s}}$$

(*b*) A free-body diagram of the bucket at the top of the motion is shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, with down as the positive direction.



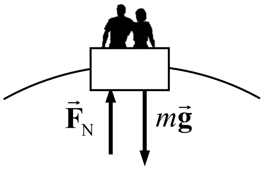
$$\sum F_R = F_T + mg = ma = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{r(F_T + mg)}{m}}$$

If the tension is to be zero, then

$$v = \sqrt{\frac{r(0 + mg)}{m}} = \sqrt{rg} = \sqrt{(1.10\text{ m})(9.80\text{ m/s}^2)} = \boxed{3.28\text{ m/s}}$$

The bucket must move faster than 3.28 m/s in order for the rope not to go slack.

45. The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive, and write Newton's second law for the passenger.



$$\sum F_R = mg - F_N = ma = mv^2/r$$

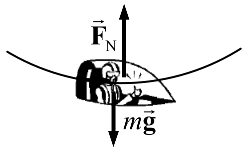
Since the passenger is to feel "weightless," they must lose contact with their seat, and so the normal force will be 0. The diameter is 22 m, so the radius is 11 m.

$$mg = mv^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(11\text{ m})} = 10.38\text{ m/s}$$

$$(10.38\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(11\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{9.0\text{ rpm}}$$

46. To describe the motion in a circle, two independent quantities are needed. The radius of the circle and the speed of the object are independent of each other, so we choose those two quantities. The radius has dimensions of $[\text{L}]$ and the speed has dimensions of $[\text{L/T}]$. These two dimensions need to be combined to get dimensions of $\left[\text{L/T}^2\right]$. The speed must be squared, which gives $\left[\text{L}^2/\text{T}^2\right]$, and then dividing by the radius gives $\left[\text{L/T}^2\right]$. So $\boxed{a_R = v^2/r}$ is a possible form for the centripetal acceleration. Note that we are unable to get numerical factors, like π or $\frac{1}{2}$, from dimensional analysis.

47. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop. We have $a_R = v^2/r = 6g$.



$$v^2/r = 6.0g \rightarrow r = \frac{v^2}{6.0g} = \frac{\left[(1200\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{6.0(9.80\,\text{m/s}^2)} = \boxed{1900\,\text{m}}$$

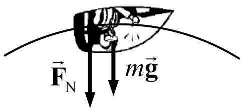
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = mv^2/r$$

The centripetal acceleration is to be $v^2/r = 6.0g$.

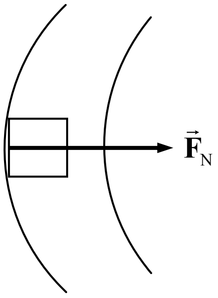
$$F_N = mg + mv^2/r = 7mg = 7(78\text{ kg})(9.80\,\text{m/s}^2) = 5350\,\text{N} = \boxed{5400\,\text{N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.



$$\sum F_R = F_N + mg = mv^2/r = 6mg \rightarrow F_N = 5mg = \boxed{3800\,\text{N}}$$

48. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall, and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's second law for the radial direction.



$$\sum F_R = F_N = ma = mv^2/r$$

If this is to have the same effect as Earth gravity, then we must also have that $F_N = mg$. Equate the two expressions for normal force and solve for the speed.

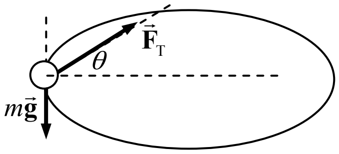
$$F_N = mv^2/r = mg \rightarrow v = \sqrt{gr} = \sqrt{(9.80\,\text{m/s}^2)(550\,\text{m})} = 73.42\,\text{m/s}$$

$$(73.42\,\text{m/s})\left(\frac{1\text{ rev}}{2\pi(550\text{ m})}\right)\left(\frac{86{,}400\text{ s}}{1\text{ d}}\right) = 1836\,\text{rev/d} \approx \boxed{1.8\times10^3\text{ rev/d}}$$

$\boxed{49.}$ The radius of either skater's motion is 0.80 m, and the period is 2.5 sec. Thus their speed is given by $v = 2\pi r/T = \frac{2\pi(0.80\text{ m})}{2.5\text{ s}} = 2.0\,\text{m/s}$. Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5-3.

$$F_R = mv^2/r = \frac{(60.0\text{ kg})(2.0\,\text{m/s})^2}{0.80\text{ m}} = \boxed{3.0\times10^2\,\text{N}}.$$

50. A free-body diagram for the ball is shown. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's second law for the vertical direction, noting that the ball is not accelerating vertically.



$$\sum F_y = F_T\sin\theta - mg = 0 \rightarrow F_T = \frac{mg}{\sin\theta}$$

The force moving the ball in a circle is the horizontal portion of the tension. Write Newton's second law for that radial motion.

$$\sum F_{\mathrm{R}} = F_{\mathrm{T}}\cos\theta = ma_{\mathrm{R}} = mv^2/r$$

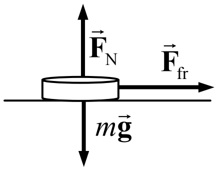
Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $v = 2\pi r/T$. Finally we recognize that if the string is of length ℓ, then the radius of the circle is $r = \ell\cos\theta$.

$$F_{\mathrm{T}}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 m\ell\cos\theta}{T^2} \quad\rightarrow$$

$$\sin\theta = \frac{gT^2}{4\pi^2\ell} \quad\rightarrow\quad \theta = \sin^{-1}\frac{gT^2}{4\pi^2\ell} = \sin^{-1}\frac{\left(9.80\,\mathrm{m/s^2}\right)\left(0.500\ \mathrm{s}\right)^2}{4\pi^2\left(0.600\ \mathrm{m}\right)} = \boxed{5.94^\circ}$$

The tension is then given by $F_{\mathrm{T}} = \dfrac{mg}{\sin\theta} = \dfrac{\left(0.150\ \mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\sin 5.94^\circ} = \boxed{14.2\ \mathrm{N}}$

51. The force of static friction is causing the circular motion – it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical acceleration. In the free-body diagram, the coin is coming out of the paper and the center of the circle is to the right of the coin, in the plane of the paper.

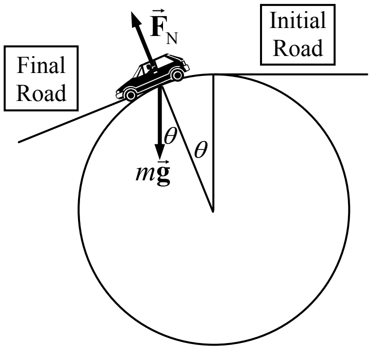


The rotational speed must be changed into a linear speed.

$$v = \left(35.0\frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)\left(\frac{2\pi\left(0.120\,\mathrm{m}\right)}{1\ \mathrm{rev}}\right) = 0.4398\,\mathrm{m/s}$$

$$F_{\mathrm{R}} = F_{\mathrm{fr}} \quad\rightarrow\quad mv^2/r = \mu_s F_{\mathrm{N}} = \mu_s mg \quad\rightarrow\quad \mu_s = \frac{v^2}{rg} = \frac{\left(0.4398\,\mathrm{m/s}\right)^2}{\left(0.120\,\mathrm{m}\right)\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{0.164}$$

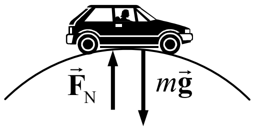
52. For the car to stay on the road, the normal force must be greater than 0. See the free-body diagram, write the net radial force, and solve for the radius.



$$F_{\mathrm{R}} = mg\cos\theta - F_{\mathrm{N}} = \frac{mv^2}{r} \quad\rightarrow\quad r = \frac{mv^2}{mg\cos\theta - F_{\mathrm{N}}}$$

For the car to be on the verge of leaving the road, the normal force would be 0, and so $r_{\text{critical}} = \dfrac{mv^2}{mg\cos\theta} = \dfrac{v^2}{g\cos\theta}$. This expression gets larger as the angle increases, and so we must evaluate at the largest angle to find a radius that is good for all angles in the range.

$$r_{\substack{\text{critical}\\ \text{maximum}}} = \frac{v^2}{g\cos\theta_{\max}} = \frac{\left[95\,\mathrm{km/h}\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{\left(9.80\,\mathrm{m/s^2}\right)\cos 22^\circ} = \boxed{77\,\mathrm{m}}$$



53. (*a*) A free-body diagram of the car at the instant it is on the top of the hill is shown. Since the car is moving in a circular path, there must be a net centripetal force downward. Write Newton's second law for the car, with down as the positive direction.

$$\sum F_\text{R} = mg - F_\text{N} = ma = mv^2/r \quad \rightarrow$$

$$F_\text{N} = m\left(g - v^2/r\right) = (975\,\text{kg})\left(9.80\,\text{m/s}^2 - \frac{(12.0\,\text{m/s})^2}{88.0\,\text{m}}\right) = \boxed{7960\,\text{N}}$$

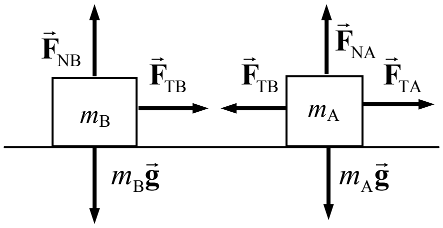
(*b*) The free-body diagram for the passengers would be the same as the one for the car, leading to the same equation for the normal force on the passengers.

$$F_\text{N} = m\left(g - v^2/r\right) = (72.0\,\text{kg})\left(9.80\,\text{m/s}^2 - \frac{(12.0\,\text{m/s})^2}{88.0\,\text{m}}\right) = \boxed{588\,\text{N}}$$

Notice that this is significantly less than the 700-N weight of the passenger. Thus the passenger will feel "light" as they drive over the hill.

(*c*) For the normal force to be zero, we must have the following.

$$F_\text{N} = m\left(g - v^2/r\right) = 0 \quad \rightarrow \quad g = v^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{\left(9.80\,\text{m/s}^2\right)(88.0\,\text{m})} = \boxed{29.4\,\text{m/s}}$$



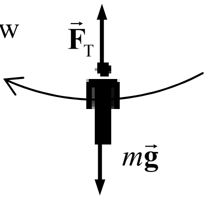
54. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.

$$\sum F_\text{RA} = F_\text{TA} - F_\text{TB} = m_\text{A}a_\text{A} = m_\text{A}v_\text{A}^2/r_\text{A} \qquad \sum F_\text{RB} = F_\text{TB} = m_\text{B}a_\text{B} = m_\text{B}v_\text{B}^2/r_\text{B}$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f\dfrac{\text{rev}}{\text{sec}}\right)\left(\dfrac{2\pi r}{1\text{ rev}}\right) = 2\pi rf$.

$$F_\text{TB} = m_\text{B}v_\text{B}^2/r_\text{B} = m_\text{B}\left(2\pi r_\text{B}f\right)^2/r_\text{B} = \boxed{4\pi^2 m_\text{B}r_\text{B}f^2}$$

$$F_\text{TA} = F_\text{TB} + m_\text{A}v_\text{A}^2/r_\text{A} = 4\pi m_\text{B}r_\text{B}f^2 + m_\text{A}\left(2\pi r_\text{A}f\right)^2/r_\text{A} = \boxed{4\pi^2 f^2\left(m_\text{A}r_\text{A} + m_\text{B}r_\text{B}\right)}$$

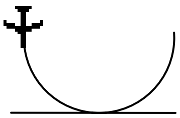


$\boxed{55.}$ A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.

$$\sum F = F_\text{T} - mg = ma = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{\left(F_\text{T} - mg\right)r}{m}}$$

The maximum speed will be obtained with the maximum tension.

$$v_\text{max} = \sqrt{\frac{\left(\vec{\mathbf{F}}_\text{T max} - mg\right)r}{m}} = \sqrt{\frac{\left(1350\text{ N} - (78\,\text{kg})\left(9.80\,\text{m/s}^2\right)\right)5.2\text{ m}}{78\,\text{kg}}} = \boxed{6.2\,\text{m/s}}$$



56. The fact that the pilot can withstand 9.0 g's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.

$$a_{\mathrm{R}} = v^2/r = 9.0g \quad \rightarrow \quad r = \frac{v^2}{9.0g} = \frac{(310\,\mathrm{m/s})^2}{9.0(9.80\,\mathrm{m/s^2})} = \boxed{1.1\times10^3\,\mathrm{m}}$$

57. (*a*) We are given that $x = (2.0\,\mathrm{m})\cos(3.0\,\mathrm{rad/s}\ t)$ and $y = (2.0\,\mathrm{m})\sin(3.0\,\mathrm{rad/s}\ t)$. Square both components and add them together.

$$x^2 + y^2 = [(2.0\,\mathrm{m})\cos(3.0\,\mathrm{rad/s}\ t)]^2 + [(2.0\,\mathrm{m})\sin(3.0\,\mathrm{rad/s}\ t)]^2$$
$$= (2.0\,\mathrm{m})^2\left[\cos^2(3.0\,\mathrm{rad/s}\ t) + \sin^2(3.0\,\mathrm{rad/s}\ t)\right] = (2.0\,\mathrm{m})^2$$

This is the equation of a circle, $x^2 + y^2 = r^2$, with a radius of 2.0 m.

(*b*) $\boxed{\vec{\mathbf{v}} = (-6.0\,\mathrm{m/s})\sin(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{i}} + (6.0\,\mathrm{m/s})\cos(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{j}}}$

$\boxed{\vec{\mathbf{a}} = (-18\,\mathrm{m/s^2})\cos(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{i}} + (-18\,\mathrm{m/s^2})\sin(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{j}}}$

(*c*) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{[(-6.0\,\mathrm{m/s})\sin(3.0\,\mathrm{rad/s}\ t)]^2 + [(6.0\,\mathrm{m/s})\cos(3.0\,\mathrm{rad/s}\ t)]^2} = \boxed{6.0\,\mathrm{m/s}}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left[(-18\,\mathrm{m/s^2})\cos(3.0\,\mathrm{rad/s}\ t)\right]^2 + \left[(-18\,\mathrm{m/s^2})\sin(3.0\,\mathrm{rad/s}\ t)\right]^2} = \boxed{18\,\mathrm{m/s^2}}$

(*d*) $\dfrac{v^2}{r} = \dfrac{(6.0\,\mathrm{m/s})^2}{2.0\,\mathrm{m}} = 18\,\mathrm{m/s^2} = a$

(*e*) $\vec{\mathbf{a}} = (-18\,\mathrm{m/s^2})\cos(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{i}} + (-18\,\mathrm{m/s^2})\sin(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{j}}$

$= (-9.0/\mathrm{s^2})\left[2.0\,\mathrm{m}\cos(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{i}} + 2.0\,\mathrm{m}\sin(3.0\,\mathrm{rad/s}\ t)\,\hat{\mathbf{j}}\right] = (9.0/\mathrm{s^2})(-\vec{\mathbf{r}})$

We see that the acceleration vector is directed oppositely of the position vector. Since the position vector points outward from the center of the circle, the acceleration vector points toward the center of the circle.

58. Since the curve is designed for 65 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5-15 in the textbook, the no-friction banking angle is given by the following.

$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(65\,\mathrm{km/h})\left(\dfrac{1.0\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{(85\,\mathrm{m})(9.80\,\mathrm{m/s^2})} = 21.4^\circ$$

Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}} = \mu_s F_{\mathrm{N}}$. Solve each equation for the normal force.

$$\sum F_y = F_{\mathrm{N}}\cos\theta - mg - F_{\mathrm{fr}}\sin\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}}\cos\theta - \mu_s F_{\mathrm{N}}\sin\theta = mg \quad \rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)}$$

$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = F_R = mv^2/r \quad\rightarrow\quad F_N \sin\theta + \mu_s F_N \cos\theta = mv^2/r \quad\rightarrow$$

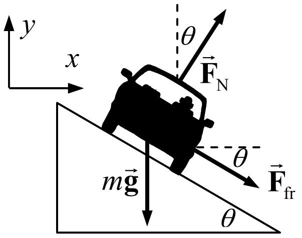
$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)}$$

Equate the two expressions for F_N, and solve for the coefficient of friction. The speed of rounding the curve is given by $v = (95\,\text{km/h})\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$.

$$\frac{mg}{(\cos\theta - \mu_s \sin\theta)} = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)} \quad\rightarrow$$

$$\mu_s = \frac{\left(\dfrac{v^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \dfrac{v^2}{r}\sin\theta\right)} = \frac{\left(\dfrac{v^2}{r} - g\tan\theta\right)}{\left(g + \dfrac{v^2}{r}\tan\theta\right)} = \frac{\left(\dfrac{(26.39\,\text{m/s})^2}{85\,\text{m}} - (9.80\,\text{m/s}^2)\tan 21.4^\circ\right)}{\left(9.80\,\text{m/s}^2 + \dfrac{(26.39\,\text{m/s})^2}{85\,\text{m}}\tan 21.4^\circ\right)} = \boxed{0.33}$$

59. Since the curve is designed for a speed of 85 km/h, traveling at that speed would mean no friction is needed to round the curve. From Example 5-15 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(85\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(68\,\text{m})(9.80\,\text{m/s}^2)} = 39.91^\circ$$

Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$.

$$\sum F_y = F_N \cos\theta - mg - F_{fr}\sin\theta = 0 \quad\rightarrow\quad F_N\cos\theta - \mu_s F_N \sin\theta = mg \quad\rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)}$$

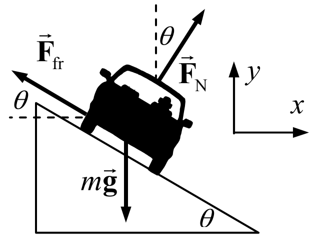
$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = m\,v^2/r \quad\rightarrow\quad F_N\sin\theta + \mu_s F_N \cos\theta = m\,v^2/r \quad\rightarrow$$

$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)} = \frac{mg}{(\cos\theta - \mu_s \sin\theta)} \quad\rightarrow$$

$$v = \sqrt{rg\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}} = \sqrt{(68\,\text{m})(9.80\,\text{m/s}^2)\frac{(\sin 39.91^\circ + 0.30\cos 39.91^\circ)}{(\cos 39.91^\circ - 0.30\sin 39.91^\circ)}} = 32\,\text{m/s}$$



Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.

$$\sum F_y = F_{\text{N}}\cos\theta - mg + F_{\text{fr}}\sin\theta = 0 \quad \rightarrow$$

$$F_{\text{N}}\cos\theta + \mu_s F_{\text{N}}\sin\theta = mg \quad \rightarrow \quad F_{\text{N}} = \frac{mg}{(\cos\theta + \mu_s\sin\theta)}$$

$$\sum F_x = F_{\text{N}}\sin\theta - F_{\text{fr}}\cos\theta = mv^2/r \quad \rightarrow \quad F_{\text{N}}\sin\theta - \mu_s F_{\text{N}}\cos\theta = mv^2/r \quad \rightarrow$$

$$F_{\text{N}} = \frac{mv^2/r}{(\sin\theta - \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta - \mu_s\cos\theta)} = \frac{mg}{(\cos\theta + \mu_s\sin\theta)} \quad \rightarrow$$

$$v = \sqrt{rg\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \sqrt{(68\,\text{m})(9.80\,\text{m/s}^2)\frac{(\sin 39.91^\circ - 0.30\cos 39.91^\circ)}{(\cos 39.91^\circ + 0.30\sin 39.91^\circ)}} = 17\,\text{m/s}$$

Thus the range is $\boxed{17\,\text{m/s} \le v \le 32\,\text{m/s}}$, which is $\boxed{61\,\text{km/h} \le v \le 115\,\text{km/h}}$.

60. (*a*) The object has a uniformly increasing speed, which means the tangential acceleration is constant, and so constant acceleration relationships can be used for the tangential motion. The object is moving in a circle of radius 2.0 meters.

$$\Delta x_{\tan} = \frac{v_{\tan} + v_{0_{\tan}}}{2}t \quad \rightarrow \quad v_{\tan} = \frac{2\Delta x_{\tan}}{t} - v_{0_{\tan}} = \frac{2\left[\frac{1}{4}(2\pi r)\right]}{t} = \frac{\pi(2.0\,\text{m})}{2.0\,\text{s}} = \boxed{\pi\,\text{m/s}}$$

(*b*) The initial location of the object is at $2.0\,\text{m}\hat{\mathbf{j}}$, and the final location is $2.0\,\text{m}\hat{\mathbf{i}}$.

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_0}{t} = \frac{2.0\,\text{m}\hat{\mathbf{i}} - 2.0\,\text{m}\hat{\mathbf{j}}}{2.0\,\text{s}} = \boxed{1.0\,\text{m/s}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)}$$

(*c*) The velocity at the end of the 2.0 seconds is pointing in the $-\hat{\mathbf{j}}$ direction.

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t} = \frac{-(\pi\,\text{m/s})\,\hat{\mathbf{j}}}{2.0\,\text{s}} = \boxed{\left(-\pi/2\,\text{m/s}^2\right)\hat{\mathbf{j}}}$$

61. Apply uniform acceleration relationships to the tangential motion to find the tangential acceleration. Use Eq. 2-12b.

$$\Delta x_{\tan} = v_{0_{\tan}}t + \tfrac{1}{2}a_{\tan}t^2 \quad \rightarrow \quad a_{\tan} = \frac{2\Delta x_{\tan}}{t^2} = \frac{2\left[\frac{1}{4}(2\pi r)\right]}{t^2} = \frac{\pi(2.0\,\text{m})}{(2.0\,\text{s})^2} = (\pi/2)\,\text{m/s}^2$$

The tangential acceleration is constant. The radial acceleration is found from $a_{\text{rad}} = \dfrac{v_{\tan}^2}{r} = \dfrac{(a_{\tan}t)^2}{r}$.

$$(a)\quad a_{\tan} = \boxed{(\pi/2)\,\text{m/s}^2},\ a_{\text{rad}} = \frac{(a_{\tan}t)^2}{r} = \frac{\left[(\pi/2)\,\text{m/s}^2(0\,\text{s})\right]^2}{2.0\,\text{m}} = \boxed{0}$$

$$(b)\quad a_{\tan} = \boxed{(\pi/2)\,\text{m/s}^2},\ a_{\text{rad}} = \frac{(a_{\tan}t)^2}{r} = \frac{\left[(\pi/2)\,\text{m/s}^2(1.0\,\text{s})\right]^2}{2.0\,\text{m}} = \boxed{(\pi^2/8)\,\text{m/s}^2}$$

$$(c)\quad a_{\tan} = \boxed{(\pi/2)\,\text{m/s}^2},\ a_{\text{rad}} = \frac{(a_{\tan}t)^2}{r} = \frac{\left[(\pi/2)\,\text{m/s}^2(2.0\,\text{s})\right]^2}{2.0\,\text{m}} = \boxed{(\pi^2/2)\,\text{m/s}^2}$$

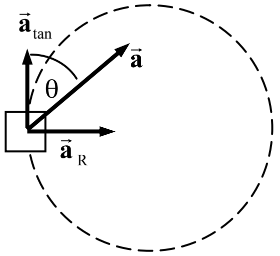
62. (*a*) The tangential acceleration is the time derivative of the speed.

$$a_{\tan} = \frac{dv_{\tan}}{dt} = \frac{d\left(3.6+1.5t^2\right)}{dt} = 3.0t \;\rightarrow\; a_{\tan}(3.0\,\text{s}) = 3.0(3.0) = \boxed{9.0\,\text{m/s}^2}$$

(*b*) The radial acceleration is given by Eq. 5-1.

$$a_{\text{rad}} = \frac{v_{\tan}^2}{r} = \frac{\left(3.6+1.5t^2\right)^2}{r} \;\rightarrow\; a_{\text{rad}}(3.0\,\text{s}) = \frac{\left(3.6+1.5(3.0)^2\right)^2}{22\,\text{m}} = \boxed{13\,\text{m/s}^2}$$

63. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially-inward component of the acceleration.



(*a*) $a_R = a\sin\theta = v^2/r \;\rightarrow$

$$v = \sqrt{ar\sin\theta} = \sqrt{\left(1.15\,\text{m/s}^2\right)(3.80\,\text{m})\sin 38.0^\circ} = \boxed{1.64\,\text{m/s}}$$

(*b*) The particle's speed change comes from the tangential acceleration, which is given by $a_{\tan} = a\cos\theta.$ If the tangential acceleration is constant, then using Eq. 2-12a,

$$v_{\tan} - v_{0\,\tan} = a_{\tan}t \;\rightarrow$$

$$v_{\tan} = v_{0\,\tan} + a_{\tan}t = 1.64\,\text{m/s} + \left(1.15\,\text{m/s}^2\right)(\cos 38.0^\circ)(2.00\text{ s}) = \boxed{3.45\,\text{m/s}}$$

64. The tangential force is simply the mass times the tangential acceleration.

$$a_T = b + ct^2 \;\rightarrow\; F_T = ma_T = \boxed{m\left(b+ct^2\right)}$$

To find the radial force, we need the tangential velocity, which is the anti-derivative of the tangential acceleration. We evaluate the constant of integration so that $v = v_0$ at $t = 0$.

$$a_T = b + ct^2 \;\rightarrow\; v_T = c + bt + \tfrac{1}{3}ct^3 \;\rightarrow\; v(0) = c = v_0 \;\rightarrow\; v_T = v_0 + bt + \tfrac{1}{3}ct^3$$

$$F_R = \frac{mv_T^2}{r} = \boxed{\frac{m}{r}\left(v_0 + bt + \tfrac{1}{3}ct^3\right)^2}$$

65. The time constant τ must have dimensions of $[\text{T}]$. The units of *m* are $[\text{M}]$. Since the expression bv is a force, we must have the dimensions of b as force units divided by speed units. So the dimensions of b are as follows: $\dfrac{\text{Force units}}{\text{speed units}} = \dfrac{[\text{M}]\left[\text{L}/\text{T}^2\right]}{[\text{L}/\text{T}]} = \left[\dfrac{\text{M}}{\text{T}}\right]$. Thus to get dimensions of $[\text{T}]$, we must have $\boxed{\tau = m/b}$.

66. (*a*) The terminal velocity is given by Eq. 5-9. This can be used to find the value of *b*.

$$v_{\text{T}}=\frac{mg}{b}\ \rightarrow\ b=\frac{mg}{v_{\text{T}}}=\frac{\left(3\times10^{-5}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(9\,\text{m/s}\right)}=3.27\times10^{-5}\,\text{kg/s}\approx\boxed{3\times10^{-5}\,\text{kg/s}}$$

(*b*) From Example 5-17, the time required for the velocity to reach 63% of terminal velocity is the time constant, $\tau=m/b$.

$$\tau=\frac{m}{b}=\frac{3\times10^{-5}\,\text{kg}}{3.27\times10^{-5}\,\text{kg/s}}=0.917\,\text{s}\approx\boxed{1\,\text{s}}$$

$\boxed{67.}$ (*a*) We choose downward as the positive direction. Then the force of gravity is in the positive direction, and the resistive force is upwards. We follow the analysis given in Example 5-17.

$$F_{\text{net}}=mg-bv=ma\ \rightarrow\ a=\frac{dv}{dt}=g-\frac{b}{m}v=-\frac{b}{m}\left(v-\frac{mg}{b}\right)\ \rightarrow$$

$$\frac{dv}{v-\frac{mg}{b}}=-\frac{b}{m}dt\ \rightarrow\ \int_{v_0}^{v}\frac{dv}{v-\frac{mg}{b}}=-\frac{b}{m}\int_0^t dt\ \rightarrow\ \ln\left[v-\frac{mg}{b}\right]_{v_0}^{v}=-\frac{b}{m}t\ \rightarrow$$

$$\ln\left[\frac{v-\frac{mg}{b}}{v_0-\frac{mg}{b}}\right]=-\frac{b}{m}t\ \rightarrow\ \frac{v-\frac{mg}{b}}{v_0-\frac{mg}{b}}=e^{-\frac{b}{m}t}\ \rightarrow\ \boxed{v=\frac{mg}{b}\left(1-e^{-\frac{b}{m}t}\right)+v_0e^{-\frac{b}{m}t}}$$

Note that this motion has a terminal velocity of $v_{\text{terminal}}=mg/b$.

(*b*) We choose upwards as the positive direction. Then both the force of gravity and the resistive force are in the negative direction.

$$F_{\text{net}}=-mg-bv=ma\ \rightarrow\ a=\frac{dv}{dt}=-g-\frac{b}{m}v=-\frac{b}{m}\left(v+\frac{mg}{b}\right)\ \rightarrow$$

$$\frac{dv}{v+\frac{mg}{b}}=-\frac{b}{m}dt\ \rightarrow\ \int_{v_0}^{v}\frac{dv}{v+\frac{mg}{b}}=-\frac{b}{m}\int_0^t dt\ \rightarrow\ \ln\left[v+\frac{mg}{b}\right]_{v_0}^{v}=-\frac{b}{m}t\ \rightarrow$$

$$\ln\left[\frac{v+\frac{mg}{b}}{v_0+\frac{mg}{b}}\right]=-\frac{b}{m}t\ \rightarrow\ \frac{v+\frac{mg}{b}}{v_0+\frac{mg}{b}}=e^{-\frac{b}{m}t}\ \rightarrow\ v=\boxed{\frac{mg}{b}\left(e^{-\frac{b}{m}t}-1\right)+v_0e^{-\frac{b}{m}t}}$$

After the object reaches its maximum height $\left[t_{\text{rise}}=\frac{m}{b}\ln\left(1+\frac{bv_0}{mg}\right)\right]$, at which point the speed will be 0, it will then start to fall. The equation from part (*a*) will then describe its falling motion.

68. The net force on the falling object, taking downward as positive, will be $\sum F=mg-bv^2=ma$.

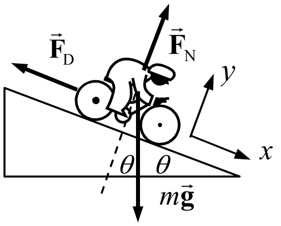
(*a*) The terminal velocity occurs when the acceleration is 0.

$$mg-bv^2=ma\ \rightarrow\ mg-bv_{\text{T}}^2=0\ \rightarrow\ \boxed{v_{\text{T}}=\sqrt{mg/b}}$$

(*b*) $$v_{\text{T}}=\sqrt{\frac{mg}{b}}\ \rightarrow\ b=\frac{mg}{v_{\text{T}}^2}=\frac{\left(75\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(60\,\text{m/s}\right)^2}=\boxed{0.2\,\text{kg/m}}$$

(*c*) The curve would be qualitatively like Fig. 5-27, because the speed would increase from 0 to the terminal velocity, asymptotically. But this curve would be ABOVE the one in Fig. 5-27, because the friction force increases more rapidly. For Fig. 5-27, if the speed doubles, the friction force doubles. But in this case, if the speed doubles, the friction force would increase by a factor of 4, bringing the friction force closer to the weight of the object in a shorter period of time.

69. (*a*) See the free-body diagram for the coasting. Since the bicyclist has a constant velocity, the net force on the bicycle must be 0. Use this to find the value of the constant *c*.

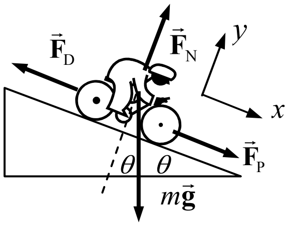


$$\sum F_x = mg\sin\theta - F_D = mg\sin\theta - cv^2 = 0 \rightarrow$$

$$c = \frac{mg\sin\theta}{v^2} = \frac{(80.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 7.0^\circ}{\left[9.5\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2} = 13.72\,\text{kg/m}$$

$$\approx \boxed{14\,\text{kg/m}}$$

(*b*) Now another force, $\vec{F}_P$, must be added down the plane to represent the additional force needed to descend at the higher speed. The velocity is still constant. See the new free-body diagram.



$$\sum F_x = mg\sin\theta + F_P - F_D = mg\sin\theta + F_P - cv^2 = 0 \rightarrow$$

$$F_P = cv^2 - mg\sin\theta$$

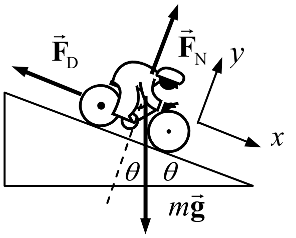
$$= (13.72\,\text{kg/m})\left[25\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 - (80.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 7.0^\circ = \boxed{570\,\text{N}}$$

70. (*a*) The rolling drag force is given as $F_{D1} \approx 4.0\,\text{N}$. The air resistance drag force is proportional to v^2, and so $F_{D2} = bv^2$. Use the data to find the proportionality constant, and then sum the two drag forces to find the total drag force.

$$F_{D2} = bv^2 \rightarrow 1.0\,\text{N} = b(2.2\,\text{m/s})^2 \rightarrow b = \frac{1.0\,\text{N}}{(2.2\,\text{m/s})^2} = 0.2066\,\text{kg/m}$$

$$F_D = F_{D1} + F_{D2} = \boxed{(4.0 + 0.21v^2)\,\text{N}}$$

(*b*) See the free-body diagram for the coasting bicycle and rider. Take the positive direction to be down the plane, parallel to the plane. The net force in that direction must be 0 for the bicycle to coast at a constant speed.



$$\sum F_x = mg\sin\theta - F_D = 0 \rightarrow mg\sin\theta = F_D \rightarrow$$

$$\theta = \sin^{-1}\frac{F_D}{mg} = \sin^{-1}\frac{(4.0 + 0.2066v^2)}{mg}$$

$$= \sin^{-1}\frac{\left(4.0\text{N} + (0.2066\,\text{kg/m})(8.0\,\text{m/s})^2\right)}{(78\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{1.3^\circ}$$

71. From Example 5-17, we have that $v = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right)$. We use this expression to find the position and acceleration expressions.

$$a = \frac{dv}{dt} = \frac{mg}{b}\left(-e^{-\frac{b}{m}t}\right)\left(-\frac{b}{m}\right) = \boxed{g e^{-\frac{b}{m}t}}$$

$$v = \frac{dx}{dt} \quad \to \quad dx = v dt \quad \to \quad \int_0^x dx = \int_0^t v\, dt = \int_0^t \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right) dt \quad \to$$

$$x = \left[\frac{mg}{b}t + \frac{mg}{b}\frac{m}{b}e^{-\frac{b}{m}t}\right]_0^t = \boxed{\frac{mg}{b}t + \frac{m^2 g}{b^2}\left(e^{-\frac{b}{m}t} - 1\right)}$$

72. We solve this problem by integrating the acceleration to find the velocity, and integrating the velocity to find the position.

$$F_{\text{net}} = -bv^{\frac{1}{2}} = ma = m\frac{dv}{dt} \quad \to \quad \frac{dv}{dt} = -\frac{b}{m}v^{\frac{1}{2}} \quad \to \quad \frac{dv}{v^{\frac{1}{2}}} = -\frac{b}{m}dt \quad \to$$

$$\int_{v_0}^{v} \frac{dv}{v^{\frac{1}{2}}} = -\frac{b}{m}\int_0^t dt \quad \to \quad 2v^{\frac{1}{2}} - 2v_0^{\frac{1}{2}} = -\frac{b}{m}t \quad \to \quad \boxed{v = \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^2}$$

$$\frac{dx}{dt} = \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^2 \quad \to \quad dx = \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^2 dt \quad \to \quad \int_0^x dx = \int_0^t \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^2 dt \quad \to$$

$$x = -\frac{2m}{3b}\left[\left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^3 - v_0^{\frac{3}{2}}\right] = \frac{2m}{3b}\left[v_0^{\frac{3}{2}} - \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^3\right]$$

$$= \frac{2m}{3b}\left(v_0^{\frac{3}{2}} - \left(v_0^{\frac{3}{2}} - 3v_0\frac{bt}{2m} + 3v_0^{\frac{1}{2}}\frac{b^2t^2}{4m^2} - \frac{b^3t^3}{8m^3}\right)\right) = \frac{2m}{3b}\left(v_0^{\frac{3}{2}} - v_0^{\frac{3}{2}} + 3v_0\frac{bt}{2m} - 3v_0^{\frac{1}{2}}\frac{b^2t^2}{4m^2} + \frac{b^3t^3}{8m^3}\right)$$

$$= \boxed{\left(v_0 t - \frac{v_0^{\frac{1}{2}}b}{2m}t^2 + \frac{b^2}{12m^2}t^3\right)}$$

73. From problem 72, we have that $v = \left(v_0^{\frac{1}{2}} - \frac{bt}{2m}\right)^2$ and $x = \left(v_0 t - \frac{v_0^{\frac{1}{2}}b}{2m}t^2 + \frac{b^2}{12m^2}t^3\right)$. The maximum distance will occur at the time when the velocity is 0. From the equation for the velocity, we see that happens at $t_{\max} = \frac{2mv_0^{\frac{1}{2}}}{b}$. Use this time in the expression for distance to find the maximum distance.

$$x\left(t = t_{\max}\right) = v_0\frac{2mv_0^{\frac{1}{2}}}{b} - \frac{v_0^{\frac{1}{2}}b}{2m}\left(\frac{2mv_0^{\frac{1}{2}}}{b}\right)^2 + \frac{b^2}{12m^2}\left(\frac{2mv_0^{\frac{1}{2}}}{b}\right)^3 = \frac{2mv_0^{\frac{3}{2}}}{b} - \frac{2mv_0^{\frac{3}{2}}}{b} + \frac{2mv_0^{\frac{3}{2}}}{3b} = \boxed{\frac{2mv_0^{\frac{3}{2}}}{3b}}$$

74. The net force is the force of gravity downward, and the drag force upwards. Let the downward direction be positive. Represent the value of $1.00 \times 10^4\ \text{kg/s}$ by the symbol *b*, as in Eq. 5-6.

$$\sum F = mg - F_{\text{d}} = mg - bv = ma = m\frac{dv}{dt} \quad \to \quad \frac{dv}{dt} = g - \frac{b}{m}v \quad \to$$

$$\frac{dv}{v-\frac{mg}{b}} = -\frac{b}{m}dt \rightarrow \int_{v_0}^{v}\frac{dv}{v-\frac{mg}{b}} = -\frac{b}{m}\int_0^t dt \rightarrow \ln\left(v-\frac{mg}{b}\right)-\ln\left(v_0-\frac{mg}{b}\right) = -\frac{b}{m}t$$

Solve for *t*, and evaluate at $v = 0.02v_0$.

$$t = \frac{\ln\left(0.02v_0-\frac{mg}{b}\right)-\ln\left(v_0-\frac{mg}{b}\right)}{-b/m}$$

$$= \frac{\ln\left(0.02(5.0\,\text{m/s})-\frac{(75\,\text{kg})(9.80\,\text{m/s}^2)}{1.00\times10^4\,\text{kg/s}}\right)-\ln\left((5.0\,\text{m/s})-\frac{(75\,\text{kg})(9.80\,\text{m/s}^2)}{1.00\times10^4\,\text{kg/s}}\right)}{-(1.00\times10^4\,\text{kg/s})/(75\,\text{kg})}$$

$$= 3.919\times10^{-2}\,\text{s} \approx \boxed{3.9\times10^{-2}\,\text{s}}$$

75. The only force accelerating the boat is the drag force, and so Newton's second law becomes $\sum F = -bv = ma$. Use this to solve for the velocity and position expressions, and then find the distance traveled under the given conditions.

$$\sum F = -bv = ma = m\frac{dv}{dt} \rightarrow \frac{dv}{dt} = -\frac{b}{m}v \rightarrow \int_{v_0}^{v}\frac{dv}{v} = -\frac{b}{m}\int_0^t dt \rightarrow \ln\frac{v}{v_0} = -\frac{b}{m}t \rightarrow$$

$$v = v_0e^{-\frac{b}{m}t}$$

Note that this velocity never changes sign. It asymptotically approaches 0 as time approaches infinity. Apply the condition that at *t* = 3.0 s the speed is $v = \frac{1}{2}v_0$.

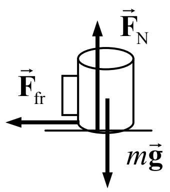
$$v(t=3.0) = v_0e^{-\frac{b}{m}(3.0)} = \tfrac{1}{2}v_0 \rightarrow \frac{b}{m} = \frac{\ln 2}{3.0\,\text{s}}$$

Now solve for the position expression. The object will reach its maximum position when it stops, which is after an infinite time.

$$v = \frac{dx}{dt} = v_0e^{-\frac{b}{m}t} \rightarrow dx = v_0e^{-\frac{b}{m}t}dt \rightarrow \int_0^x dx = \int_0^t v_0e^{-\frac{b}{m}t}dt \rightarrow$$

$$x = -v_0\frac{m}{b}\left(e^{-\frac{b}{m}t}-1\right) = v_0\frac{m}{b}\left(1-e^{-\frac{b}{m}t}\right) \rightarrow x(t=\infty) = v_0\frac{m}{b} = (2.4\,\text{m/s})\frac{3.0\,\text{s}}{\ln 2} = 10.39\,\text{m} \approx \boxed{10\,\text{m}}$$

76. A free-body diagram for the coffee cup is shown. Assume that the car is moving to the right, and so the acceleration of the car (and cup) will be to the left. The deceleration of the cup is caused by friction between the cup and the dashboard. For the cup to not slide on the dash, and to have the minimum deceleration time means the largest possible static frictional force is acting, so $F_{\text{fr}} = \mu_s F_{\text{N}}$. The normal force on the cup is equal to its weight, since there is no vertical acceleration. The horizontal acceleration of the cup is found from Eq. 2-12a, with a final velocity of zero.



$$v_0 = (45\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 12.5\,\text{m/s}$$

$$v - v_0 = at \rightarrow a = \frac{v-v_0}{t} = \frac{0-12.5\,\text{m/s}}{3.5\text{ s}} = -3.57\,\text{m/s}^2$$

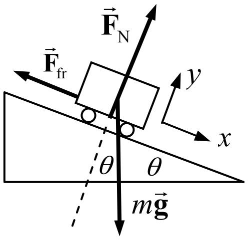
Write Newton's second law for the horizontal forces, considering to the right to be positive.

$$\sum F_x = -F_{\text{fr}} = ma \;\rightarrow\; ma = -\mu_s F_{\text{N}} = -\mu_s mg \;\rightarrow\; \mu_s = -\frac{a}{g} = -\frac{\left(-3.57\,\text{m/s}^2\right)}{9.80\,\text{m/s}^2} = \boxed{0.36}$$

77. Since the drawer moves with the applied force of 9.0 N, we assume that the maximum static frictional force is essentially 9.0 N. This force is equal to the coefficient of static friction times the normal force. The normal force is assumed to be equal to the weight, since the drawer is horizontal.

$$F_{\text{fr}} = \mu_s F_{\text{N}} = \mu_s mg \;\rightarrow\; \mu_s = \frac{F_{\text{fr}}}{mg} = \frac{9.0\,\text{N}}{(2.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)} = \boxed{0.46}$$

78. See the free-body diagram for the descending roller coaster. It starts its descent with $v_0 = (6.0\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 1.667\,\text{m/s}$. The total displacement in the *x* direction is $x - x_0 = 45.0\,\text{m}$. Write Newton's second law for both the *x* and *y* directions.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{\text{fr}} = mg\sin\theta - \mu_k F_{\text{N}} = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g\left(\sin\theta - \mu_k\cos\theta\right)$$

Now use Eq. 2-12c to solve for the final velocity.

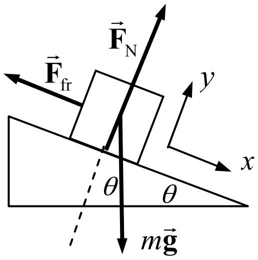
$$v^2 - v_0^2 = 2a\left(x - x_0\right) \;\rightarrow$$

$$v = \sqrt{v_0^2 + 2a\left(x - x_0\right)} = \sqrt{v_0^2 + 2g\left(\sin\theta - \mu_k\cos\theta\right)\left(x - x_0\right)}$$

$$= \sqrt{(1.667\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)\left[\sin 45^\circ - (0.12)\cos 45^\circ\right](45.0\,\text{m})}$$

$$= 23.49\,\text{m/s} \approx \boxed{23\,\text{m/s}} \approx 85\,\text{km/h}$$

$\boxed{79.}$ Consider a free-body diagram of the box. Write Newton's second law for both directions. The net force in the *y* direction is 0 because there is no acceleration in the *y* direction.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma$$

Now solve for the force of friction and the coefficient of friction.

$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma$$

$$F_{\text{fr}} = mg\sin\theta - ma = m\left(g\sin\theta - a\right) = (18.0\,\text{kg})\left[\left(9.80\,\text{m/s}^2\right)\left(\sin 37.0^\circ\right) - 0.220\,\text{m/s}^2\right]$$

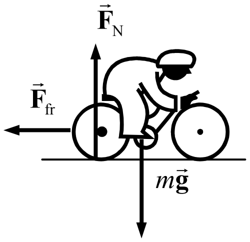
$$= 102.2\,\text{N} \approx \boxed{102\,\text{N}}$$

$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta \;\rightarrow\; \mu_k = \frac{F_{\text{fr}}}{mg\cos\theta} = \frac{102.2\,\text{N}}{(18.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)\cos 37.0^\circ} = \boxed{0.725}$$

80. Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, and so $F_T = F_R = Ma_R = M\,v^2/r$. Equate the expressions for tension and solve for the velocity.

$$M\,v^2/r = mg \quad \rightarrow \quad v = \boxed{\sqrt{mgR/M}}$$

81. Consider the free-body diagram for the cyclist in the sand, assuming that the cyclist is traveling to the right. It is apparent that $F_N = mg$ since there is no vertical acceleration. Write Newton's second law for the horizontal direction, positive to the right.



$$\sum F_x = -F_{fr} = ma \quad \rightarrow \quad -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g$$

Use Eq. 2-12c to determine the distance the cyclist could travel in the sand before coming to rest.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow \quad \left(x - x_0\right) = \frac{v^2 - v_0^2}{2a} = \frac{-v_0^2}{-2\mu_k g} = \frac{\left(20.0\,\text{m/s}\right)^2}{2\left(0.70\right)\left(9.80\,\text{m/s}^2\right)} = 29\,\text{m}$$

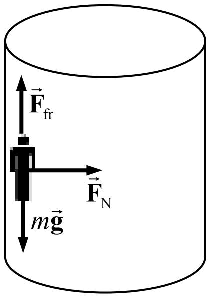
Since there is only 15 m of sand, $\boxed{\text{the cyclist will emerge from the sand}}$. The speed upon emerging is found from Eq. 2-12c.

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad \rightarrow$$

$$v = \sqrt{v_0^2 + 2a\left(x - x_0\right)} = \sqrt{v_i^2 - 2\mu_k g\left(x - x_0\right)} = \sqrt{\left(20.0\,\text{m/s}\right)^2 - 2\left(0.70\right)\left(9.80\,\text{m/s}^2\right)\left(15\,\text{m}\right)}$$

$$= \boxed{14\,\text{m/s}}$$

82. Consider the free-body diagram for a person in the "Rotor-ride." $\vec{\mathbf{F}}_N$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's second law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{fr} - mg = 0 \quad \rightarrow \quad F_{fr} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{fr} = \mu_s F_N = mg \quad \rightarrow \quad F_N = m\,g/\mu_s\,.$$

But the normal force must be the force causing the centripetal motion – it is the only force pointing to the center of rotation. Thus $F_R = F_N = m\,v^2/r$. Using $v = 2\pi r/T$, we have $F_N = \dfrac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.50 rev per sec, the period is 2.0 sec.

$$F_N = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \quad \rightarrow \quad \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{\left(9.80\,\text{m/s}^2\right)\left(2.0\,\text{s}\right)^2}{4\pi^2\left(5.5\,\text{m}\right)} = \boxed{0.18}$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder radius smaller.

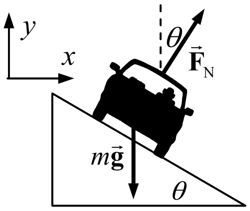
There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's third law, the riders push against the wall. This gives the sensation of being pressed into the wall.

83. The force is a centripetal force, and is of magnitude $7.45mg$. Use Eq. 5-3 for centripetal force.

$$F = m\frac{v^2}{r} = 7.45mg \rightarrow v = \sqrt{7.45rg} = \sqrt{7.45(11.0\,\mathrm{m})(9.80\,\mathrm{m/s^2})} = 28.34\,\mathrm{m/s} \approx \boxed{28.3\,\mathrm{m/s}}$$

$$(28.34\,\mathrm{m/s}) \times \frac{1\,\mathrm{rev}}{2\pi(11.0\,\mathrm{m})} = \boxed{0.410\,\mathrm{rev/s}}$$

84. The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's second law for both the *x* and *y* directions.



$$\sum F_y = F_\mathrm{N}\cos\theta - mg = 0 \rightarrow F_\mathrm{N} = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_\mathrm{R} = F_\mathrm{N}\sin\theta = ma_x$$

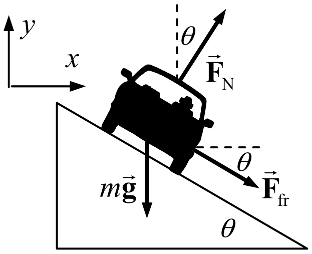
The amount of centripetal force needed for the car to round the curve is as follows.

$$F_\mathrm{R} = mv^2/r = (1250\ \mathrm{kg})\frac{\left[(85\,\mathrm{km/h})\left(\dfrac{1.0\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{72\,\mathrm{m}} = 9.679\times10^3\,\mathrm{N}$$

The actual horizontal force available from the normal force is as follows.

$$F_\mathrm{N}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1250\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\tan 14^\circ = 3.054\times10^3\,\mathrm{N}$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's second law for both directions, and again the *y* acceleration is zero.

$$\sum F_y = F_\mathrm{N}\cos\theta - mg - F_\mathrm{fr}\sin\theta = 0 \rightarrow F_\mathrm{N} = \frac{mg + F_\mathrm{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_\mathrm{N}\sin\theta + F_\mathrm{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

$$\frac{mg + F_\mathrm{fr}\sin\theta}{\cos\theta}\sin\theta + F_\mathrm{fr}\cos\theta = mv^2/r \rightarrow (mg + F_\mathrm{fr}\sin\theta)\sin\theta + F_\mathrm{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_\mathrm{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = (9.679\times10^3\,\mathrm{N})\cos14^\circ - (1250\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\sin14^\circ$$

$$= 6.428\times10^3\,\mathrm{N}$$

So a frictional force of $\boxed{6.4\times10^3\,\mathrm{N\ down\ the\ plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

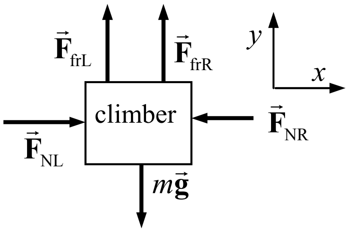
$\boxed{85.}$ The radial force is given by Eq. 5-3.

$$F_R = m\frac{v^2}{r} = (1150\,\text{kg})\frac{(27\,\text{m/s})^2}{450\,\text{m/s}} = 1863\,\text{N} \approx \boxed{1900\,\text{N}}$$

The tangential force is the mass times the tangential acceleration. The tangential acceleration is the change in tangential speed divided by the elapsed time.

$$F_T = ma_T = m\frac{\Delta v_T}{\Delta t} = (1150\,\text{kg})\frac{(27\,\text{m/s})}{(9.0\,\text{s})} = 3450\,\text{N} \approx \boxed{3500\,\text{N}}$$

86. Since the walls are vertical, the normal forces are horizontal, away from the wall faces. We assume that the frictional forces are at their maximum values, so $F_{fr} = \mu_s F_N$ applies at each wall. We assume that the rope in the diagram is not under any tension and so does not exert any forces. Consider the free-body diagram for the climber. F_{NR} is the normal force on the climber from the right wall, and F_{NL} is the normal force on the climber from the left wall. The static frictional forces are $F_{frL} = \mu_{sL}F_{NL}$ and $F_{frR} = \mu_{sR}F_{NR}$. Write Newton's second law for both the *x* and *y* directions. The net force in each direction must be zero if the climber is stationary.



$$\sum F_x = F_{NL} - F_{NR} = 0 \rightarrow F_{NL} = F_{NR} \qquad \sum F_y = F_{frL} + F_{frR} - mg = 0$$

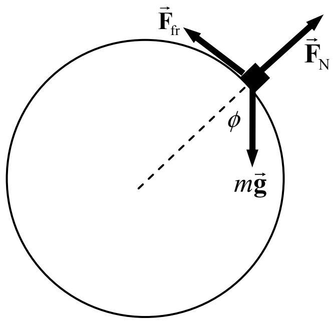
Substitute the information from the *x* equation into the *y* equation.

$$F_{frL} + F_{frR} = mg \rightarrow \mu_{sL}F_{NL} + \mu_{sR}F_{NR} = mg \rightarrow (\mu_{sL} + \mu_{sR})F_{NL} = mg$$

$$F_{NL} = \frac{mg}{(\mu_{sL} + \mu_{sR})} = \frac{(70.0\,\text{kg})(9.80\,\text{m/s}^2)}{1.40} = 4.90\times10^2\,\text{N}$$

And so $\boxed{F_{NL} = F_{NR} = 4.90\times10^2\,\text{N}}$. These normal forces arise as Newton's third law reaction forces to the climber pushing on the walls. Thus the climber must exert a force of at least 490 N against each wall.

87. The mass would start sliding when the static frictional force was not large enough to counteract the component of gravity that will be pulling the mass along the curved surface. See the free-body diagram, and assume that the static frictional force is a maximum. We also assume the block has no speed, so the radial force must be 0.



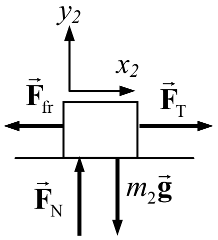
$$\sum F_{\text{radial}} = F_N - mg\cos\phi \rightarrow F_N = mg\cos\phi$$

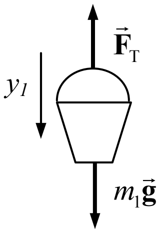
$$\sum F_{\text{tangential}} = mg\sin\phi - F_{fr} \rightarrow F_{fr} = mg\sin\phi$$

$$F_{fr} = \mu_s F_N = \mu_s mg\cos\phi = mg\sin\phi \rightarrow \mu_s = \tan\phi \rightarrow$$

$$\phi = \tan^{-1}\mu_s = \tan^{-1}0.70 = \boxed{35^\circ}$$

88. (*a*) Consider the free-body diagrams for both objects, initially stationary. As sand is added, the tension will increase, and the force of static friction on the block will increase until it reaches its maximum of $F_{fr} = \mu_s F_N$. Then the system will start to move. Write Newton's second law for each object, when the static frictional force is at its maximum, but the objects are still stationary.





$$\sum F_{y\text{ bucket}} = m_1 g - F_{\text{T}} = 0 \;\rightarrow\; F_{\text{T}} = m_1 g$$

$$\sum F_{y\text{ block}} = F_{\text{N}} - m_2 g = 0 \;\rightarrow\; F_{\text{N}} = m_2 g$$

$$\sum F_{x\text{ block}} = F_{\text{T}} - F_{\text{fr}} = 0 \;\rightarrow\; F_{\text{T}} = F_{\text{fr}}$$

Equate the two expressions for tension, and substitute in the expression for the normal force to find the masses.

$$m_1 g = F_{\text{fr}} \;\rightarrow\; m_1 g = \mu_s F_{\text{N}} = \mu_s m_2 g \;\rightarrow$$

$$m_1 = \mu_s m_2 = (0.45)(28.0\,\text{kg}) = 12.6\,\text{kg}$$

Thus $12.6\,\text{kg} - 2.00\text{ kg} = 10.6\,\text{kg} \approx \boxed{11\,\text{kg}}$ of sand was added.

(*b*) The same free-body diagrams can be used, but now the objects will accelerate. Since they are tied together, $a_{y1} = a_{x2} = a$. The frictional force is now kinetic friction, given by $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k m_2 g$. Write Newton's second laws for the objects in the direction of their acceleration.

$$\sum F_{y\text{ bucket}} = m_1 g - F_{\text{T}} = m_1 a \;\rightarrow\; F_{\text{T}} = m_1 g - m_1 a$$

$$\sum F_{x\text{ block}} = F_{\text{T}} - F_{\text{fr}} = m_2 a \;\rightarrow\; F_{\text{T}} = F_{\text{fr}} + m_2 a$$

Equate the two expressions for tension, and solve for the acceleration.

$$m_1 g - m_1 a = \mu_k m_2 g + m_2 a \;\rightarrow$$

$$a = g\frac{(m_1 - \mu_k m_2)}{(m_1 + m_2)} = \left(9.80\,\text{m/s}^2\right)\frac{\left(12.6\,\text{kg} - (0.32)(28.0\,\text{kg})\right)}{(12.6\,\text{kg} + 28.0\text{kg})} = \boxed{0.88\,\text{m/s}^2}$$

89. The acceleration that static friction can provide can be found from the minimum stopping distance, assuming that the car is just on the verge of sliding. Use Eq. 2-12c. Then, assuming an unbanked curve, the same static frictional force is used to provide the centripetal acceleration needed to make the curve. The acceleration from the stopping distance is negative, and so the centripetal acceleration is the opposite of that expression.

$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow\; a_{\text{stopping}} = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{-v_0^2}{2(x - x_0)} \;\rightarrow\; a_{\text{R}} = \frac{v_0^2}{2(x - x_0)}$$

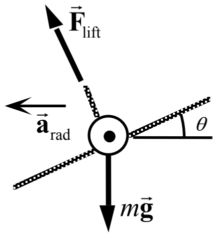
Equate the above expression to the typical expression for centripetal acceleration.

$$a_{\text{R}} = \frac{v^2}{r} = \frac{v_0^2}{2(x - x_0)} \;\rightarrow\; r = 2(x - x_0) = \boxed{132\,\text{m}}$$

Notice that we didn't need to know the mass of the car, the initial speed, or the coefficient of friction.

90. The radial acceleration is given by $a_R = v^2/r$. Substitute in the speed of the tip of the sweep hand, given by $v = 2\pi r/T$, to get $a_R = \dfrac{4\pi^2 r}{T^2}$. For the tip of the sweep hand, r = 0.015 m, and T = 60 sec.

$$a_R = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.015\text{ m})}{(60\text{ s})^2} = \boxed{1.6\times10^{-4}\text{ m/s}^2}$$



$\boxed{91.}$ (*a*) The horizontal component of the lift force will produce a centripetal acceleration. Write Newton's second law for both the horizontal and vertical directions, and combine those equations to solve for the time needed to reverse course (a half-period of the circular motion). Note that $T = \dfrac{2\pi r}{v}$.

$$\sum F_{\text{vertical}} = F_{\text{lift}} \cos\theta = mg \ ; \ \sum F_{\text{horizontal}} = F_{\text{lift}} \sin\theta = m\frac{v^2}{r}$$

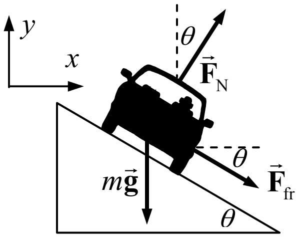
Divide these two equations.

$$\frac{F_{\text{lift}}\sin\theta}{F_{\text{lift}}\cos\theta} = \frac{mv^2}{rmg} \quad\rightarrow\quad \tan\theta = \frac{v^2}{rg} = \frac{v^2}{\frac{Tv}{2\pi}g} = \frac{2\pi v}{gT} \quad\rightarrow$$

$$\frac{T}{2} = \frac{\pi v}{g\tan\theta} = \frac{\pi\left[(480\,\text{km/h})\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]}{(9.80\,\text{m/s}^2)\tan 38^\circ} = \boxed{55\,\text{s}}$$

(*b*) The passengers will feel a change in the normal force that their seat exerts on them. Prior to the banking, the normal force was equal to their weight. During banking, the normal force will increase, so that $F_{\substack{\text{normal}\\\text{banking}}} = \dfrac{mg}{\cos\theta} = 1.27mg$. Thus they will feel "pressed down" into their seats, with about a 25% increase in their apparent weight. If the plane is banking to the left, they will feel pushed to the right by that extra 25% in their apparent weight.

92. From Example 5-15 in the textbook, the no-friction banking angle is given by $\theta = \tan^{-1}\dfrac{v_0^2}{Rg}$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg - F_{\text{fr}}\sin\theta = 0 \quad\rightarrow\quad F_{\text{N}}\cos\theta - \mu_s F_{\text{N}}\sin\theta = mg \quad\rightarrow$$

$$F_{\text{N}} = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

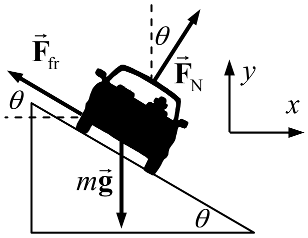
$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = mv^2/R \quad\rightarrow\quad F_{\text{N}}\sin\theta + \mu_s F_{\text{N}}\cos\theta = mv^2/R \quad\rightarrow$$

$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$\frac{mv^2/R}{(\sin\theta+\mu_s\cos\theta)}=\frac{mg}{(\cos\theta-\mu_s\sin\theta)} \quad\rightarrow$$

$$v_{\max}=\sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1+\mu_s/\tan\theta)}{(1-\mu_s\tan\theta)}}=v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}$$



Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. Assume that the car is on the verge of skidding, so that the static frictional force is given by $F_{\text{fr}}=\mu_s F_{\text{N}}$.

$$\sum F_y=F_{\text{N}}\cos\theta-mg+F_{\text{fr}}\sin\theta=0 \quad\rightarrow$$

$$F_{\text{N}}\cos\theta+\mu_s F_{\text{N}}\sin\theta=mg \quad\rightarrow\quad F_{\text{N}}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)}$$

$$\sum F_x=F_{\text{R}}=F_{\text{N}}\sin\theta-F_{\text{fr}}\cos\theta=mv^2/R \quad\rightarrow\quad F_{\text{N}}\sin\theta-\mu_s F_{\text{N}}\cos\theta=mv^2/R \quad\rightarrow$$

$$F_{\text{N}}=\frac{mv^2/R}{(\sin\theta-\mu_s\cos\theta)}$$

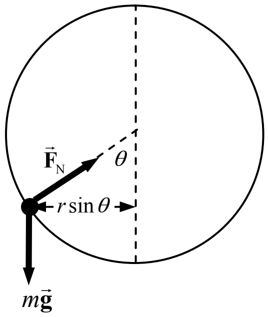
Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/R}{(\sin\theta-\mu_s\cos\theta)}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)} \quad\rightarrow$$

$$v_{\min}=\sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1-\mu_s/\tan\theta)}{(1+\mu_s\tan\theta)}}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}$$

Thus $\boxed{v_{\min}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}}$ and $\boxed{v_{\max}=v_0\sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}}$.

93. (*a*) Because there is no friction between the bead and the hoop, the hoop can only exert a normal force on the bead. See the free-body diagram for the bead at the instant shown in the textbook figure. Note that the bead moves in a horizontal circle, parallel to the floor. Thus the centripetal force is horizontal, and the net vertical force must be 0. Write Newton's second law for both the horizontal and vertical directions, and use those equations to determine the angle θ. We also use the fact that the speed and the frequency are related to each other, by $v=2\pi fr\sin\theta$.



$$\sum F_{\text{vertical}}=F_{\text{N}}\cos\theta-mg=0 \quad\rightarrow\quad F_{\text{N}}=\frac{mg}{\cos\theta}$$

$$\sum F_{\text{radial}}=F_{\text{N}}\sin\theta=m\frac{v^2}{r\sin\theta}=m\frac{4\pi^2f^2r^2\sin^2\theta}{r\sin\theta}$$

$$F_N \sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{4\pi^2 f^2 r^2 \sin^2\theta}{r\sin\theta} \quad\rightarrow\quad \theta = \boxed{\cos^{-1}\frac{g}{4\pi^2 f^2 r}}$$

(*b*) $\theta = \cos^{-1}\dfrac{g}{4\pi^2 f^2 r} = \cos^{-1}\dfrac{9.80\,\text{m/s}^2}{4\pi^2(2.00\,\text{Hz})^2(0.220\,\text{m})} = \boxed{73.6^\circ}$

(*c*) $\boxed{\text{No}}$, the bead cannot ride as high as the center of the circle. If the bead were located there, the normal force of the wire on the bead would point horizontally. There would be no force to counteract the bead's weight, and so it would have to slip back down below the horizontal to balance the force of gravity. From a mathematical standpoint, the expression $\dfrac{g}{4\pi^2 f^2 r}$ would have to be equal to 0 and that could only happen if the frequency or the radius were infinitely large.

94. An object at the Earth's equator is rotating in a circle with a radius equal to the radius of the Earth, and a period equal to one day. Use that data to find the centripetal acceleration and then compare it to *g*.

$$a_R = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \quad\rightarrow\quad \frac{a_R}{g} = \frac{\dfrac{4\pi^2(6.38\times10^6\,\text{m})}{(86{,}400\,\text{s})^2}}{(9.80\,\text{m/s}^2)} = 0.00344 \approx \boxed{\frac{3}{1000}}$$

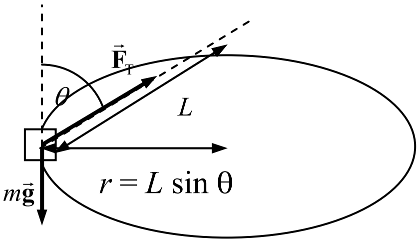
So, for example, if we were to calculate the normal force on an object at the Earth's equator, we could not say $\sum F = F_N - mg = 0$. Instead, we would have the following.

$$\sum F = F_N - mg = -m\frac{v^2}{r} \quad\rightarrow\quad F_N = mg - m\frac{v^2}{r}$$

If we then assumed that $F_N = mg_{\text{eff}} = mg - m\dfrac{v^2}{r}$, then we see that the effective value of *g* is

$$g_{\text{eff}} = g - \frac{v^2}{r} = g - 0.003g = 0.997g.$$

95. A free-body diagram for the sinker weight is shown. *L* is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r = L\sin\theta$. Write Newton's second law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.



$$\sum F_y = F_T\cos\theta - mg = 0 \quad\rightarrow\quad F_T = \frac{mg}{\cos\theta}$$

The radial force is the horizontal portion of the tension. Write Newton's second law for the radial motion.

$$\sum F_R = F_T\sin\theta = ma_R = mv^2/r$$

Substitute the tension from the vertical equation, and the relationships $r = L\sin\theta$ and $v = 2\pi r/T$.

$$F_T\sin\theta = mv^2/r \quad\rightarrow\quad \frac{mg}{\cos\theta}\sin\theta = \frac{4\pi^2 mL\sin\theta}{T^2} \quad\rightarrow\quad \cos\theta = \frac{gT^2}{4\pi^2 L}$$

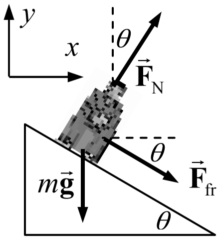
$$\theta = \cos^{-1}\frac{gT^2}{4\pi^2 L} = \cos^{-1}\frac{(9.80\,\text{m/s}^2)(0.50\text{ s})^2}{4\pi^2(0.45\text{ m})} = \boxed{82^\circ}$$

96. The speed of the train is $(160\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 44.44\,\text{m/s}$.

(*a*) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_\text{R} = mv^2/R = \frac{(75\,\text{kg})(44.44\,\text{m/s})^2}{(570\,\text{m})} = 259.9\,\text{N} \approx \boxed{2.6\times10^2\,\text{N}}$$

(*b*) For the banked case, the normal force will contribute to the radial force needed. Write Newton's second law for both the *x* and *y* directions. The *y* acceleration is zero, and the *x* acceleration is radial.



$$\sum F_y = F_\text{N}\cos\theta - mg - F_\text{fr}\sin\theta = 0 \;\rightarrow\; F_\text{N} = \frac{mg + F_\text{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_\text{N}\sin\theta + F_\text{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

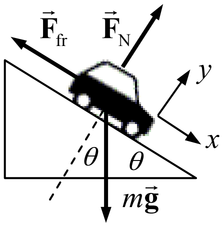
$$\frac{mg + F_\text{fr}\sin\theta}{\cos\theta}\sin\theta + F_\text{fr}\cos\theta = mv^2/r \;\rightarrow$$

$$(mg + F_\text{fr}\sin\theta)\sin\theta + F_\text{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta \;\rightarrow$$

$$F_\text{fr} = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

$$= (75\text{ kg})\left[\frac{(44.44\,\text{m/s})^2}{570\text{ m}}\cos8.0^\circ - (9.80\,\text{m/s}^2)\sin8.0^\circ\right] = 155\,\text{N} \approx \boxed{1.6\times10^2\,\text{N}}$$

$\boxed{97.}$ We include friction from the start, and then for the no-friction result, set the coefficient of friction equal to 0. Consider a free-body diagram for the car on the hill. Write Newton's second law for both directions. Note that the net force on the *y* direction will be zero, since there is no acceleration in the *y* direction.



$$\sum F_y = F_\text{N} - mg\cos\theta = 0 \;\rightarrow\; F_\text{N} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_\text{fr} = ma \;\rightarrow$$

$$a = g\sin\theta - \frac{F_\text{fr}}{m} = g\sin\theta - \frac{\mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

Use Eq. 2-12c to determine the final velocity, assuming that the car starts from rest.

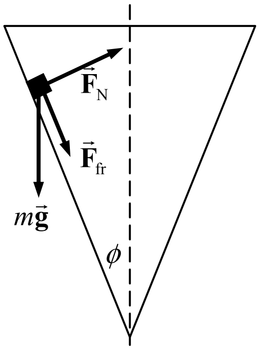
$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow\; v = \sqrt{0 + 2a(x - x_0)} = \sqrt{2g(x - x_0)(\sin\theta - \mu_k\cos\theta)}$$

The angle is given by $\sin\theta = 1/4 \;\rightarrow\; \theta = \sin^{-1}0.25 = 14.5^\circ$

(*a*) $\mu_k = 0 \;\rightarrow\; v = \sqrt{2g(x - x_0)x\sin\theta} = \sqrt{2(9.80\,\text{m/s}^2)(55\text{ m})\sin14.5^\circ} = \boxed{16\,\text{m/s}}$

(*b*) $\mu_k = 0.10 \;\rightarrow\; v = \sqrt{2(9.80\,\text{m/s}^2)(55\text{ m})(\sin14.5^\circ - 0.10\cos14.5^\circ)} = \boxed{13\,\text{m/s}}$

98. The two positions on the cone correspond to two opposite directions of the force of static friction. In one case, the frictional force points UP the cone's surface, and in the other case, it points DOWN the cone's surface. In each case the net vertical force is 0, and force of static friction is assumed to be its maximum value. The net horizontal force is producing centripetal motion.



$$\sum F_{\text{vertical}} = F_N \sin\phi - F_{fr}\cos\phi - mg = F_N\sin\phi - \mu_s F_N \cos\phi - mg = 0 \ \rightarrow$$

$$F_N = \frac{mg}{\sin\phi - \mu_s\cos\phi}$$

$$\sum F_{\text{horizontal}} = F_N\cos\phi + F_{fr}\sin\phi = F_N\cos\phi + \mu_s F_N\sin\phi$$

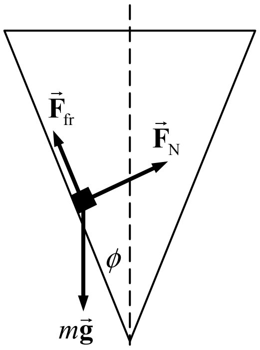
$$= F_N\left(\cos\phi + \mu_s\sin\phi\right) = m\frac{v^2}{r} = m\frac{\left(2\pi rf\right)^2}{r} = 4\pi^2 rmf^2 \rightarrow$$

$$F_N = \frac{4\pi^2 rmf^2}{\left(\cos\phi + \mu_s\sin\phi\right)}$$

Equate the two expressions for the normal force, and solve for the radius.

$$F_N = \frac{mg}{\sin\phi - \mu_s\cos\phi} = \frac{4\pi^2 rmf^2}{\left(\cos\phi + \mu_s\sin\phi\right)} \quad \rightarrow \quad \boxed{r_{\max} = \frac{g\left(\cos\phi + \mu_s\sin\phi\right)}{4\pi^2 f^2\left(\sin\phi - \mu_s\cos\phi\right)}}$$

A similar analysis will lead to the minimum radius.



$$\sum F_{\text{vertical}} = F_N \sin\phi + F_{fr}\cos\phi - mg = F_N\sin\phi + \mu_s F_N \cos\phi - mg = 0 \rightarrow$$

$$F_N = \frac{mg}{\sin\phi + \mu_s\cos\phi}$$

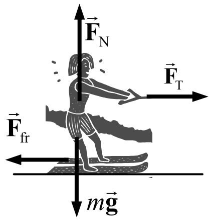
$$\sum F_{\text{horizontal}} = F_N\cos\phi - F_{fr}\sin\phi = F_N\cos\phi - \mu_s F_N\sin\phi$$

$$= F_N\left(\cos\phi - \mu_s\sin\phi\right) = m\frac{v^2}{r} = m\frac{\left(2\pi rf\right)^2}{r} = 4\pi^2 rmf^2 \rightarrow$$

$$F_N = \frac{4\pi^2 rmf^2}{\left(\cos\phi - \mu_s\sin\phi\right)}$$

$$F_N = \frac{mg}{\sin\phi + \mu_s\cos\phi} = \frac{4\pi^2 rmf^2}{\left(\cos\phi - \mu_s\sin\phi\right)} \quad \rightarrow \quad \boxed{r_{\min} = \frac{g\left(\cos\phi - \mu_s\sin\phi\right)}{4\pi^2 f^2\left(\sin\phi + \mu_s\cos\phi\right)}}$$

99. (*a*) See the free-body diagram for the skier when the tow rope is horizontal. Use Newton's second law for both the vertical and horizontal directions in order to find the acceleration.

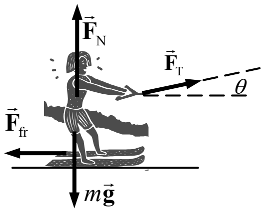


$$\sum F_y = F_N - mg = 0 \ \rightarrow \ F_N = mg$$

$$\sum F_x = F_T - F_{fr} = F_T - \mu_k F_N = F_T - \mu_k mg = ma$$

$$a = \frac{F_T - \mu_k mg}{m} = \frac{(240\,\text{N}) - 0.25(72\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{(72\,\text{kg})} = \boxed{0.88\,\text{m/s}^2}$$

(*b*) Now see the free-body diagram for the skier when the tow rope has an upward component.



$$\sum F_y = F_N + F_T \sin\theta - mg = 0 \quad \rightarrow \quad F_N = mg - F_T \sin\theta$$

$$\sum F_x = F_T \cos\theta - F_{fr} = F_T \cos\theta - \mu_k F_N$$

$$= F_T \cos\theta - \mu_k \left(mg - F_T \sin\theta\right) = ma$$

$$a = \frac{F_T\left(\cos\theta + \mu_k \sin\theta\right) - \mu_k mg}{m}$$

$$= \frac{(240\,\mathrm{N})(\cos 12^\circ + 0.25\sin 12^\circ) - 0.25(72\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{(72\,\mathrm{kg})} = \boxed{0.98\,\mathrm{m/s^2}}$$

(*c*) The acceleration is greater in part (*b*) because the upward tilt of the tow rope reduces the normal force, which then reduces the friction. The reduction in friction is greater than the reduction in horizontal applied force, and so the horizontal acceleration increases.

100. The radial acceleration is $a_R = \frac{v^2}{r}$, and so $a_R = \frac{v^2}{r} = \frac{(6.0\,\mathrm{m/s})^2}{0.80\,\mathrm{m}} = \boxed{45\,\mathrm{m/s^2}}$.

The tension force has no tangential component, and so the tangential force is seen from the diagram to be $F_{tang} = mg\cos\theta$.

$$F_{tang} = mg\cos\theta = ma_{tang} \quad \rightarrow \quad a_{tang} = g\cos\theta = (9.80\,\mathrm{m/s^2})\cos 30^\circ = \boxed{8.5\,\mathrm{m/s^2}}$$

The tension force can be found from the net radial force.

$$F_R = F_T - mg\sin\theta = m\frac{v^2}{r} \quad \rightarrow$$

$$F_T = m\left(g\sin\theta + \frac{v^2}{r}\right) = (1.0\,\mathrm{kg})\left((9.80\,\mathrm{m/s^2})\sin 30^\circ + 45\,\mathrm{m/s^2}\right) = \boxed{50\,\mathrm{N}}$$

Note that the answer has 2 significant figures.

101. (*a*) The acceleration has a magnitude given by $a = v^2/r$.

$$a = \sqrt{\left(-15.7\,\mathrm{m/s^2}\right)^2 + \left(-23.2\,\mathrm{m/s^2}\right)^2} = 28.01\,\mathrm{m/s^2} = \frac{v^2}{63.5\,\mathrm{m}} \quad \rightarrow$$

$$v = \sqrt{(28.01\,\mathrm{m/s^2})(63.5\,\mathrm{m})} = 42.17\,\mathrm{m/s} \approx \boxed{42.2\,\mathrm{m/s}}$$

(*b*) Since the acceleration points radially in and the position vector points radially out, the components of the position vector are in the same proportion as the components of the acceleration vector, but of opposite sign.

$$x = r\frac{|a_x|}{a} = (63.5\,\mathrm{m})\frac{15.7\,\mathrm{m/s^2}}{28.01\,\mathrm{m/s^2}} = \boxed{35.6\,\mathrm{m}} \qquad y = r\frac{|a_y|}{a} = (63.5\,\mathrm{m})\frac{23.2\,\mathrm{m/s^2}}{28.01\,\mathrm{m/s^2}} = \boxed{52.6\,\mathrm{m}}$$

102. (*a*) We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the rocket at later times. We take the downward direction to be positive, and the starting position to be $y = 0$.

$$F = mg - kv^2 = ma \quad \rightarrow \quad a = g - \frac{k}{m}v^2$$

For $t = 0$, $y(0) = y_0 = 0$, $v(0) = v_0 = 0$, and $a(0) = a_0 = g - \frac{k}{m}v^2 = 9.80\,\mathrm{m/s^2}$. Assume this acceleration is constant over the next interval, and so $y_1 = y_0 + v_0\Delta t + \frac{1}{2}a_0(\Delta t)^2$, $v_1 = v_0 + a_0\Delta t$, and $a_1 = -g - \frac{k}{m}v_1^2$. This continues for each successive interval. We apply this method first for a time interval of 1 s, and get the speed and position at $t = 15.0$ s. Then we reduce the interval to 0.5 s and again find the speed and position at $t = 15.0$ s. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within 2%. If not, a smaller interval is used, and the process repeated. For this problem, the results for position and velocity for time intervals of 1.0 s and 0.5 s agree to within 2%, but to get two successive acceleration values to agree to 2%, intervals of 0.05 s and 0.02 s are used. Here are the results for various intervals.

$\Delta t = 1\,\text{s}$:	$x(15\,\text{s}) = 648\,\text{m}$	$v(15\,\text{s}) = 57.5\,\text{m/s}$	$a(15\,\text{s}) = 0.109\,\text{m/s}^2$
$\Delta t = 0.5\,\text{s}$:	$x(15\,\text{s}) = 641\,\text{m}$	$v(15\,\text{s}) = 57.3\,\text{m/s}$	$a(15\,\text{s}) = 0.169\,\text{m/s}^2$
$\Delta t = 0.2\,\text{s}$:	$x(15\,\text{s}) = 636\,\text{m}$	$v(15\,\text{s}) = 57.2\,\text{m/s}$	$a(15\,\text{s}) = 0.210\,\text{m/s}^2$
$\Delta t = 0.1\,\text{s}$:	$x(15\,\text{s}) = 634.4\,\text{m}$	$v(15\,\text{s}) = 57.13\,\text{m/s}$	$a(15\,\text{s}) = 0.225\,\text{m/s}^2$
$\Delta t = 0.05\,\text{s}$:	$x(15\,\text{s}) = 633.6\,\text{m}$	$v(15\,\text{s}) = 57.11\,\text{m/s}$	$a(15\,\text{s}) = 0.232\,\text{m/s}^2$
$\Delta t = 0.02\,\text{s}$:	$x(15\,\text{s}) = 633.1\,\text{m}$	$v(15\,\text{s}) = 57.10\,\text{m/s}$	$a(15\,\text{s}) = 0.236\,\text{m/s}^2$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 102a."

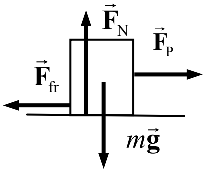
(*b*) The terminal velocity is the velocity that produces an acceleration of 0. Use the acceleration equation from above.

$$a = g - \frac{k}{m}v^2 \;\rightarrow\; v_{\text{terminal}} = \sqrt{\frac{mg}{k}} = \sqrt{\frac{(75\,\text{kg})(9.80\,\text{m/s}^2)}{0.22\,\text{kg/m}\,k}} = \boxed{58\,\text{m/s}}$$

At this velocity, the drag force is equal in magnitude to the force of gravity, so the skydiver no longer accelerates, and thus the velocity stays constant.

(*c*) From the spreadsheet, it is seen that it takes $\boxed{17.6\,\text{s}}$ to reach 99.5% of terminal velocity.

$\boxed{103.}$ Use the free body diagram to write Newton's second law for the block, and solve for the acceleration.

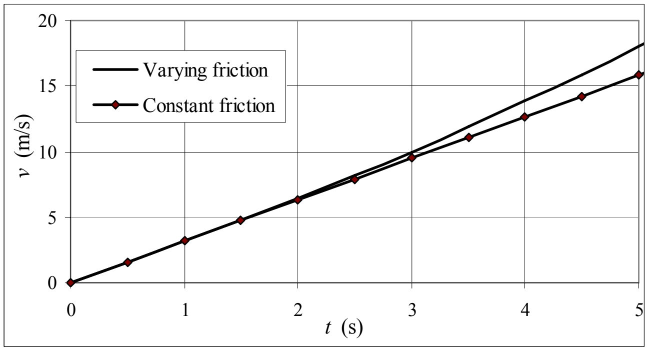


$$F = ma = F_\text{P} - F_\text{fr} = F_\text{P} - \mu_\text{k}F_\text{N} = F_\text{P} - \mu_\text{k}mg \;\rightarrow$$

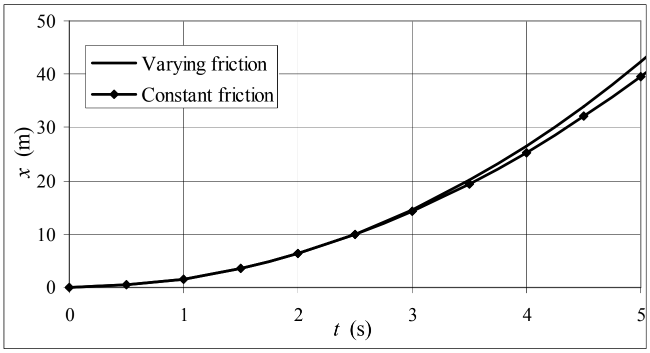
$$a = \frac{F_\text{P}}{m} - \mu_\text{k}g = \frac{41\,\text{N}}{8.0\,\text{kg}} - \frac{0.20(9.80\,\text{m/s}^2)}{(1+0.0020v^2)^2} = \left(5.125 - \frac{1.96}{(1+0.0020v^2)^2}\right)\text{m/s}^2$$

For $t = 0$, $x(0) = x_0 = 0$, $v(0) = v_0 = 0$, and $a(0) = a_0 = 3.165\,\text{m/s}^2$. Assume this acceleration is constant over the next time interval, and so $x_1 = x_0 + v_0\Delta t + \frac{1}{2}a_0(\Delta t)^2$, $v_1 = v_0 + a_0\Delta t$, and $a_1 = \left(5.125 - \frac{1.96}{(1+0.0020v_1^2)^2}\right)\text{m/s}^2$. This continues for each successive interval. We apply this method first for a time interval of 1 second, and get the speed and position at $t = 5.0$ s. Then we reduce the interval to 0.5 s and again find the speed and position at $t = 5.0$ s. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within 2%. If not, a smaller interval is used, and the process repeated. For this problem, the results for position and velocity for time intervals of 1.0 s and 0.5 s agree to within 2%.

(*a*) The speed at 5.0 s, from the numeric integration, is 18.0 m/s. The velocity–time graph is shown, along with a graph for a constant coefficient of friction, $\mu_k = 0.20$. The varying (decreasing) friction gives a higher speed than the constant friction. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.103."



(*b*) The position at 5.0 s, from the numeric integration, is 42.4 m. The position–time graph is shown, along with a graph for a constant coefficient of friction, $\mu_k = 0.20$. The varying (decreasing) friction gives a larger distance than the constant friction. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.103."



(*c*) If the coefficient of friction is constant, then $a = 3.165\,\mathrm{m/s^2}$. Constant acceleration relationships can find the speed and position at $t = 5.0$ s.

$$v = v_0 + at = 0 + at \quad \rightarrow \quad v_{\text{final}} = \left(3.165\,\mathrm{m/s^2}\right)\left(5.0\,\mathrm{s}\right) = 15.8\,\mathrm{m/s}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2 \quad \rightarrow \quad x_{\text{final}} = \tfrac{1}{2}\left(3.165\,\mathrm{m/s^2}\right)\left(5.0\,\mathrm{s}\right)^2 = 39.6\,\mathrm{m}$$

We compare the variable friction results to the constant friction results.

$$v: \quad \%\,\text{diff} = \frac{v_{\mu\text{ constant}} - v_{\mu\text{ variable}}}{v_{\mu\text{ variable}}} = \frac{15.8\,\mathrm{m/s} - 18.0\,\mathrm{m/s}}{18.0\,\mathrm{m/s}} = \boxed{-12\%}$$

$$x: \quad \%\,\text{diff} = \frac{x_{\mu\text{ constant}} - x_{\mu\text{ variable}}}{x_{\mu\text{ variable}}} = \frac{39.6\,\mathrm{m/s} - 42.4\,\mathrm{m/s}}{42.4\,\mathrm{m/s}} = \boxed{-6.6\%}$$

104. We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the rocket at later times.

t (s)	y (m)	v (m/s)	a (m/s^2)
0	0	120.0	-47.2
1	96	72.8	-23.6
2	157	49.2	-16.1
3	199	33.1	-12.6
4	225	20.5	-10.9
5	240	9.6	-10.0
6	245	-0.5	-9.8

$$F = -mg - kv^2 = ma \quad \rightarrow \quad a = -g - \frac{k}{m}v^2$$

For $t = 0$, $y(0) = 0$, $v(0) = v_0 = 120\,\mathrm{m/s}$, and $a(0) = a_0 = -g - \dfrac{k}{m}v^2 = -9.80\,\mathrm{m/s^2}$. Assume this acceleration is constant over the next time interval, and so $y_1 = y_0 + v_0\Delta t + \tfrac{1}{2}a_0\left(\Delta t\right)^2$, $v_1 = v_0 + a_0\Delta t$, and $a_1 = -g - \frac{k}{m}v_1^2$. This continues for each successive interval. Applying this method gives the results shown in the table. We estimate the maximum height reached as $y_{\text{max}} = \boxed{245\,\text{m}}$.

If air resistance is totally ignored, then the acceleration is a constant –*g* and Eq. 2-12c may be used to find the maximum height.

$$v^2 - v_0^2 = 2a\left(y - y_0\right) \quad \rightarrow$$

$$y - y_0 = \frac{v^2 - v_0^2}{2a} = \frac{-v_0^2}{-2g} = \frac{\left(120\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)} = 730\,\text{m}$$

Thus the air resistance reduces the maximum height to about 1/3 of the no-resistance value. A more detailed analysis (with smaller time intervals) gives 302 m for the maximum height, which is also the answer obtained from an analytical solution.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.104."

CHAPTER 6: Gravitation and Newton's Synthesis

Responses to Questions

$\boxed{1.}$ Whether the apple is attached to a tree or falling, it exerts a gravitational force on the Earth equal to the force the Earth exerts on it, which is the weight of the apple (Newton's third law).

2. The tides are caused by the *difference* in gravitational pull on two opposite sides of the Earth. The gravitational pull from the Sun on the side of the Earth closest to it depends on the distance from the Sun to the close side of the Earth. The pull from the Sun on the far side of the Earth depends on this distance plus the diameter of the Earth. The diameter of the Earth is a very small fraction of the total Earth–Sun distance, so these two forces, although large, are nearly equal. The diameter of the Earth is a larger fraction of the Earth–Moon distance, and so the *difference* in gravitational force from the Moon to the two opposite sides of the Earth will be greater.

3. The object will weigh more at the poles. The value of r^2 at the equator is greater, both from the Earth's center and from the bulging mass on the opposite side of the Earth. Also, the object has centripetal acceleration at the equator. The two effects do not oppose each other.

4. Since the Earth's mass is greater than the Moon's, the point at which the net gravitational pull on the spaceship is zero is closer to the Moon. A spaceship traveling from the Earth towards the Moon must therefore use fuel to overcome the net pull backwards for over half the distance of the trip. However, when the spaceship is returning to the Earth, it reaches the zero point at less than half the trip distance, and so spends more of the trip "helped" by the net gravitational pull in the direction of travel.

5. The gravitational force from the Sun provides the centripetal force to keep the Moon and the Earth going around the Sun. Since the Moon and Earth are at the same average distance from the Sun, they travel together, and the Moon is not pulled away from the Earth.

6. As the Moon revolves around the Earth, its position relative to the distant background stars changes. This phenomenon is known as "parallax." As a demonstration, hold your finger at arm's length and look at it with one eye at a time. Notice that it "lines up" with different objects on the far wall depending on which eye is open. If you bring your finger closer to your face, the shift in its position against the background increases. Similarly, the Moon's position against the background stars will shift as we view it in different places in its orbit. The distance to the Moon can be calculated by the amount of shift.

$\boxed{7.}$ At the very center of the Earth, all of the gravitational forces would cancel, and the net force on the object would be zero.

8. A satellite in a geosynchronous orbit stays over the same spot on the Earth at all times. The satellite travels in an orbit about the Earth's axis of rotation. The needed centripetal force is supplied by the component of the gravitational force perpendicular to the axis of rotation. A satellite directly over the North Pole would lie *on* the axis of rotation of the Earth. The gravitational force on the satellite in this case would be parallel to the axis of rotation, with no component to supply the centripetal force needed to keep the satellite in orbit.

9. According to Newton's third law, the force the Earth exerts on the Moon has the same magnitude as the force the Moon exerts on the Earth. The Moon has a larger acceleration, since it has a smaller mass (Newton's second law, $F = ma$).

10. The satellite needs a certain speed with respect to the center of the Earth to achieve orbit. The Earth rotates towards the east so it would require less speed (with respect to the Earth's surface) to launch a satellite towards the east (*a*). Before launch, the satellite is moving with the surface of the Earth so already has a "boost" in the right direction.

11. If the antenna becomes detached from a satellite in orbit, the antenna will continue in orbit around the Earth with the satellite. If the antenna were given a component of velocity toward the Earth (even a very small one), it would eventually spiral in and hit the Earth.

12. Ore normally has a greater density than the surrounding rock. A large ore deposit will have a larger mass than an equal amount of rock. The greater the mass of ore, the greater the acceleration due to gravity will be in its vicinity. Careful measurements of this slight increase in *g* can therefore be used to estimate the mass of ore present.

$\boxed{13.}$ Yes. At noon, the gravitational force on a person due to the Sun and the gravitational force due to the Earth are in the opposite directions. At midnight, the two forces point in the same direction. Therefore, your apparent weight at midnight is greater than your apparent weight at noon.

14. Your apparent weight will be greatest in case (*b*), when the elevator is accelerating upward. The scale reading (your apparent weight) indicates your force on the scale, which, by Newton's third law, is the same as the normal force of the scale on you. If the elevator is accelerating upward, then the net force must be upward, so the normal force (up) must be greater than your actual weight (down). When in an elevator accelerating upward, you "feel heavy."

    Your apparent weight will be least in case (*c*), when the elevator is in free fall. In this situation your apparent weight is zero since you and the elevator are both accelerating downward at the same rate and the normal force is zero.

    Your apparent weight will be the same as when you are on the ground in case (*d)*, when the elevator is moving upward at a constant speed. If the velocity is constant, acceleration is zero and $N = mg$. (Note that it doesn't matter if the elevator is moving up or down or even at rest, as long as the velocity is constant.)

15. If the Earth's mass were double what it is, the radius of the Moon's orbit would have to double (if the Moon's speed remained constant), or the Moon's speed in orbit would have to increase by a factor of the square root of 2 (if the radius remained constant). If both the radius and orbital speed were free to change, then the product rv^2 would have to double.

16. If the Earth were a perfect, nonrotating sphere, then the gravitational force on each droplet of water in the Mississippi would be the same at the headwaters and at the outlet, and the river wouldn't flow. Since the Earth is rotating, the droplets of water experience a centripetal force provided by a part of the component of the gravitational force perpendicular to the Earth's axis of rotation. The centripetal force is smaller for the headwaters, which are closer to the North pole, than for the outlet, which is closer to the equator. Since the centripetal force is equal to $mg - N$ (apparent weight) for each droplet, N is smaller at the outlet, and the river will flow. This effect is large enough to overcome smaller effects on the flow of water due to the bulge of the Earth near the equator.

17. The satellite remains in orbit because it has a velocity. The instantaneous velocity of the satellite is tangent to the orbit. The gravitational force provides the centripetal force needed to keep the satellite in orbit, acting like the tension in a string when twirling a rock on a string. A force is not needed to keep the satellite "up"; a force is needed to bend the velocity vector around in a circle.

18. Between steps, the runner is not touching the ground. Therefore there is no normal force up on the runner and so she has no apparent weight. She is momentarily in free fall since the only force is the force of gravity pulling her back toward the ground.

$\boxed{19.}$ If you were in a satellite orbiting the Earth, you would have no apparent weight (no normal force). Walking, which depends on the normal force, would not be possible. Drinking would be possible, but only from a tube or pouch, from which liquid could be sucked. Scissors would not sit on a table (no apparent weight = no normal force).

20. The centripetal acceleration of Mars in its orbit around the Sun is smaller than that of the Earth. For both planets, the centripetal force is provided by gravity, so the centripetal acceleration is inversely proportional to the square of the distance from the planet to the Sun:

$$\frac{m_p v^2}{r} = \frac{G m_s m_p}{r^2} \quad \text{so} \quad \frac{v^2}{r} = \frac{G m_s}{r^2}$$

Since Mars is at a greater distance from the Sun than Earth, it has a smaller centripetal acceleration. Note that the mass of the planet does not appear in the equation for the centripetal acceleration.

21. For Pluto's moon, we can equate the gravitational force from Pluto on the moon to the centripetal force needed to keep the moon in orbit:

$$\frac{m_m v^2}{r} = \frac{G m_p m_m}{r^2}$$

This allows us to solve for the mass of Pluto (m_p) if we know G, the radius of the moon's orbit, and the velocity of the moon, which can be determined from the period and orbital radius. Note that the mass of the moon cancels out.

22. The Earth is closer to the Sun in January. The gravitational force between the Earth and the Sun is a centripetal force. When the distance decreases, the speed increases. (Imagine whirling a rock around your head in a horizontal circle. If you pull the string through your hand to shorten the distance between your hand and the rock, the rock speeds up.)

$$\frac{m_E v^2}{r} = \frac{G m_S m_E}{r^2} \quad \text{so} \quad v = \sqrt{\frac{G m_S}{r}}$$

Since the speed is greater in January, the distance must be less. This agrees with Kepler's second law.

23. The Earth's orbit is an ellipse, not a circle. Therefore, the force of gravity on the Earth from the Sun is not perfectly perpendicular to the Earth's velocity at all points. A component of the force will be parallel to the velocity vector and will cause the planet to speed up or slow down.

24. Standing at rest, you feel an upward force on your feet. In free fall, you don't feel that force. You would, however, be aware of the acceleration during free fall, possibly due to your inner ear.

$\boxed{25.}$ If we treat $\vec{\mathbf{g}}$ as the *acceleration due to gravity*, it is the result of a force from one mass acting on another mass and causing it to accelerate. This implies action at a distance, since the two masses do not have to be in contact. If we view $\vec{\mathbf{g}}$ as a *gravitational field*, then we say that the presence of a mass changes the characteristics of the space around it by setting up a field, and the field then interacts with other masses that enter the space in which the field exists. Since the field is in contact with the mass, this conceptualization does not imply action at a distance.

Solutions to Problems

$\boxed{1.}$ The spacecraft is at 3.00 Earth radii from the center of the Earth, or three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9} m g_{\substack{\text{Earth's} \\ \text{surface}}} = \frac{(1480\,\text{kg})(9.80\,\text{m/s}^2)}{9} = \boxed{1610\,\text{N}}$$

This could also have been found using Eq. 6-1, Newton's law of universal gravitation.

2. The force of gravity on an object at the surface of a planet is given by Newton's law of universal gravitation, Eq. 6-1, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}} m}{r_{\text{Moon}}^2} = m g_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r_{\text{Moon}}^2} = (6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\,\text{kg})}{(1.74\times10^6\,\text{m})^2} = \boxed{1.62\,\text{m/s}^2}$$

3. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text{planet}} = G M_{\text{planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{M_{\text{Earth}}}{(2.3R_{\text{Earth}})^2} = \frac{1}{2.3^2}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{1}{2.3^2}g_{\text{Earth}} = \frac{9.80\,\text{m/s}^2}{2.3^2} = \boxed{1.9\,\text{m/s}^2}$$

4. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G M_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{1.80M_{\text{Earth}}}{R_{\text{Earth}}^2} = 1.80\left(G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}\right) = 1.80g_{\text{Earth}} = 1.80(9.80\,\text{m/s}^2) = \boxed{17.6\,\text{m/s}^2}$$

5. The acceleration due to gravity is determined by the mass of the Earth and the radius of the Earth.

$$g_0 = \frac{GM_0}{r_0^2} \qquad g_{\text{new}} = \frac{GM_{\text{new}}}{r_{\text{new}}^2} = \frac{G2M_0}{(3r_0)^2} = \frac{2}{9}\frac{GM_0}{r_0^2} = \tfrac{2}{9}g_0$$

So g is multiplied by a factor of $\boxed{2/9}$.

6. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = G M_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question. For this problem, $M_{\text{Planet}} = M_{\text{Earth}} = 5.97\times10^{24}\,\text{kg}$.

(*a*) $r = R_{\text{Earth}} + 6400\text{ m} = 6.38\times10^6\,\text{m} + 6400\text{ m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = (6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\,\text{kg})}{(6.38\times10^6\,\text{m} + 6400\text{ m})^2} = \boxed{9.78\,\text{m/s}^2}$$

(*b*) $r = R_{\text{Earth}} + 6400\text{ km} = 6.38\times10^{6}\text{m} + 6.4\times10^{6}\text{ m} = 12.78\times10^{6}\text{m}\ (3\text{ sig fig})$

$$g = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\text{kg}\right)}{\left(12.78\times10^{6}\text{m}\right)^2} = \boxed{2.44\text{ m/s}^2}$$

$\boxed{7.}$ The distance from the Earth's center is $r = R_{\text{Earth}} + 300\text{ km} = 6.38\times10^{6}\text{m} + 3\times10^{5}\text{m} = 6.68\times10^{6}\text{m}\ (2\text{ sig fig})$. Calculate the acceleration due to gravity at that location.

$$g = G\frac{M_{\text{Earth}}}{r^2} = G\frac{M_{\text{Earth}}}{r^2} = \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{5.97\times10^{24}\text{kg}}{\left(6.68\times10^{6}\text{m}\right)^2} = 8.924\text{ m/s}^2$$

$$= 8.924\text{ m/s}^2\left(\frac{1\text{"}g\text{"}}{9.80\text{ m/s}^2}\right) = \boxed{0.91g\text{'s}}$$

This is only about a 9% reduction from the value of *g* at the surface of the Earth.

8. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\substack{\text{Earth}\\\text{Venus}}} = (150-108)\times10^{6}\text{ km} = 4.2\times10^{10}\text{m} \qquad r_{\substack{\text{Earth}\\\text{Jupiter}}} = (778-150)\times10^{6}\text{ km} = 6.28\times10^{11}\text{m}$$

$$r_{\substack{\text{Earth}\\\text{Saturn}}} = (1430-150)\times10^{6}\text{ km} = 1.28\times10^{12}\text{m}$$

Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as positive.

$$F_{\substack{\text{Earth-}\\\text{planets}}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\substack{\text{Earth}\\\text{Jupiter}}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\substack{\text{Earth}\\\text{Saturn}}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\substack{\text{Earth}\\\text{Venus}}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{\left(6.28\times10^{11}\text{m}\right)^2} + \frac{95.1}{\left(1.28\times10^{12}\text{m}\right)^2} - \frac{0.815}{\left(4.2\times10^{10}\text{m}\right)^2}\right)$$

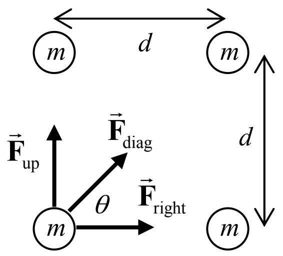
$$= \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\text{kg}\right)^2\left(4.02\times10^{-22}\text{m}^{-2}\right) = 9.56\times10^{17}\text{ N} \approx \boxed{9.6\times10^{17}\text{ N}}$$

The force of the Sun on the Earth is as follows.

$$F_{\substack{\text{Earth-}\\\text{Sun}}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\substack{\text{Earth}\\\text{Sun}}}} = \left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\text{kg}\right)\left(1.99\times10^{30}\text{kg}\right)}{\left(1.50\times10^{11}\text{m}\right)^2} = 3.52\times10^{22}\text{ N}$$

And so the ratio is $F_{\substack{\text{Earth-}\\\text{planets}}}\big/F_{\substack{\text{Earth-}\\\text{Sun}}} = 9.56\times10^{17}\text{N}/3.52\times10^{22}\text{N} = \boxed{2.7\times10^{-5}}$, which is 27 millionths.

9. Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the *x* and *y* directions will be the same. Note $\theta = 45°$.



$$F_x = F_{\text{right}} + F_{\text{dia}}\cos\theta = G\frac{m^2}{d^2} + G\frac{m^2}{\left(\sqrt{2}d\right)^2}\frac{1}{\sqrt{2}} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$$

Thus $F_y = F_x = G\dfrac{m^2}{d^2}\left(1+\dfrac{1}{2\sqrt{2}}\right)$. The net force can be found by the

Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$F=\sqrt{F_x^2+F_y^2}=\sqrt{2F_x^2}=F_x\sqrt{2}=G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\sqrt{2}=G\frac{m^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$=\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\frac{(8.5\,\mathrm{kg})^2}{(0.80\,\mathrm{m})^2}\left(\sqrt{2}+\frac{1}{2}\right)=\boxed{1.4\times10^{-8}\,\mathrm{N\ at\ }45^{\circ}}$$

The force points towards the center of the square.

10. Assume that the two objects can be treated as point masses, with $m_1=m$ and $m_2=4.00\,\mathrm{kg}-m$. The gravitational force between the two masses is given by the following.

$$F=G\frac{m_1m_2}{r^2}=G\frac{m(4.00-m)}{r^2}=\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{4.00m-m^2}{(0.25\,\mathrm{m})^2}=2.5\times10^{-10}\,\mathrm{N}$$

This can be rearranged into a quadratic form of $m^2-4.00m+0.234=0$. Use the quadratic formula to solve for *m*, resulting in two values which are the two masses.

$$\boxed{m_1=3.94\,\mathrm{kg}\,,\ m_2=0.06\,\mathrm{kg}}$$

11. The force on *m* due to 2*m* points in the $\hat{\mathbf{i}}$ direction. The force on *m* due to 4*m* points in the $\hat{\mathbf{j}}$ direction. The force on *m* due to 3*m* points in the direction given by $\theta=\tan^{-1}\frac{y_0}{x_0}$. Add the force vectors together to find the net force.

$$\vec{\mathbf{F}}=G\frac{(2m)m}{x_0^2}\hat{\mathbf{i}}+G\frac{(4m)m}{y_0^2}\hat{\mathbf{j}}+G\frac{(3m)m}{x_0^2+y_0^2}\cos\theta\,\hat{\mathbf{i}}+G\frac{(3m)m}{x_0^2+y_0^2}\sin\theta\,\hat{\mathbf{j}}$$

$$=G\frac{2m^2}{x_0^2}\hat{\mathbf{i}}+G\frac{4m^2}{y_0^2}\hat{\mathbf{j}}+G\frac{3m^2}{x_0^2+y_0^2}\frac{x_0}{\sqrt{x_0^2+y_0^2}}\hat{\mathbf{i}}+G\frac{(3m)m}{x_0^2+y_0^2}\frac{y_0}{\sqrt{x_0^2+y_0^2}}\hat{\mathbf{j}}$$

$$=Gm^2\left[\left(\frac{2}{x_0^2}+\frac{3x_0}{\left(x_0^2+y_0^2\right)^{3/2}}\right)\hat{\mathbf{i}}+\left(\frac{4}{y_0^2}+\frac{3y_0}{\left(x_0^2+y_0^2\right)^{3/2}}\right)\hat{\mathbf{j}}\right]$$

12. With the assumption that the density of Europa is the same as Earth's, the radius of Europa can be calculated.

$$\rho_{\mathrm{Europa}}=\rho_{\mathrm{Earth}}\quad\rightarrow\quad\frac{M_{\mathrm{Europa}}}{\frac{4}{3}\pi r_{\mathrm{Europa}}^3}=\frac{M_{\mathrm{Earth}}}{\frac{4}{3}\pi r_{\mathrm{Earth}}^3}\quad\rightarrow\quad r_{\mathrm{Europa}}=r_{\mathrm{Earth}}\left(\frac{M_{\mathrm{Europa}}}{M_{\mathrm{Earth}}}\right)^{1/3}$$

$$g_{\mathrm{Europa}}=\frac{GM_{\mathrm{Europa}}}{r_{\mathrm{Europa}}^2}=\frac{GM_{\mathrm{Europa}}}{\left(r_{\mathrm{Earth}}\left(\frac{M_{\mathrm{Europa}}}{M_{\mathrm{Earth}}}\right)^{1/3}\right)^2}=\frac{GM_{\mathrm{Europa}}^{1/3}M_{\mathrm{Earth}}^{2/3}}{r_{\mathrm{Earth}}^2}=\frac{GM_{\mathrm{Earth}}}{r_{\mathrm{Earth}}^2}\frac{M_{\mathrm{Europa}}^{1/3}}{M_{\mathrm{Earth}}^{1/3}}=g_{\mathrm{Earth}}\left(\frac{M_{\mathrm{Europa}}}{M_{\mathrm{Earth}}}\right)^{1/3}$$

$$=\left(9.80\,\mathrm{m/s^2}\right)\left(\frac{4.9\times10^{22}\,\mathrm{kg}}{5.98\times10^{24}\,\mathrm{kg}}\right)^{1/3}=1.98\,\mathrm{m/s^2}\approx\boxed{2.0\,\mathrm{m/s^2}}$$

$\boxed{13.}$ To find the new weight of objects at the Earth's surface, the new value of *g* at the Earth's surface needs to be calculated. Since the spherical shape is being maintained, the Earth can be treated as a point mass. Find the density of the Earth using the actual values, and use that density to find *g* under the revised conditions.

$$g_{\text{original}} = G\frac{m_E}{r_E^2} \;;\; \rho = \frac{m_E}{\frac{4}{3}\pi r_E^3} = \frac{3m_E}{4\pi r_E^3} \;\rightarrow\; r_E = \left(\frac{3m_E}{4\pi\rho}\right)^{1/3} \;\rightarrow$$

$$g_{\text{original}} = G\frac{m_E}{\left(\frac{3m_E}{4\pi\rho}\right)^{2/3}} = G\frac{(m_E)^{1/3}}{\left(\frac{3}{4\pi\rho}\right)^{2/3}} \;;\; g_{\text{new}} = G\frac{(2m_E)^{1/3}}{\left(\frac{3}{4\pi\rho}\right)^{2/3}} = 2^{1/3}G\frac{(m_E)^{1/3}}{\left(\frac{3}{4\pi\rho}\right)^{2/3}} = 2^{1/3}g$$

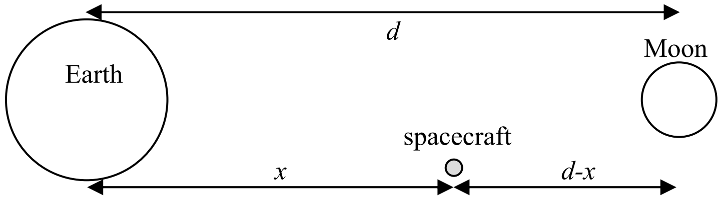
Thus *g* is multiplied by $2^{1/3}$, and so the weight would be $\boxed{\text{multiplied by } 2^{1/3}}$.

14. The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\frac{M_{\text{body}}}{R_{\text{body}}^2}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38 g_{\text{Earth}}$.

$$G\frac{M_{\text{Mars}}}{R_{\text{Mars}}^2} = 0.38\,G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} \;\rightarrow$$

$$M_{\text{Mars}} = 0.38 M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{\text{R}_{\text{Earth}}}\right)^2 = 0.38\left(5.98\times10^{24}\,\text{kg}\right)\left(\frac{3400\text{ km}}{6380\text{ km}}\right)^2 = \boxed{6.5\times10^{23}\,\text{kg}}$$

15. For the net force to be zero means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve for the distance *x*. We measure from the center of the bodies.



$$F_{\substack{\text{Earth-}\\\text{spacecraft}}} = G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} \quad ; \quad F_{\substack{\text{Moon}\\\text{spacecraft}}} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2}$$

$$G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2} \;\rightarrow\; \frac{x^2}{M_{\text{Earth}}} = \frac{(d-x)^2}{M_{\text{Moon}}} \;\rightarrow\; \frac{x}{\sqrt{M_{\text{Earth}}}} = \frac{d-x}{\sqrt{M_{\text{Moon}}}}$$

$$x = d\frac{\sqrt{M_{\text{Earth}}}}{\left(\sqrt{M_{\text{Moon}}}+\sqrt{M_{\text{Earth}}}\right)} = \left(3.84\times10^8\,\text{m}\right)\frac{\sqrt{5.97\times10^{24}\,\text{kg}}}{\left(\sqrt{7.35\times10^{22}\,\text{kg}}+\sqrt{5.97\times10^{24}\,\text{kg}}\right)} = \boxed{3.46\times10^8\,\text{m}}$$

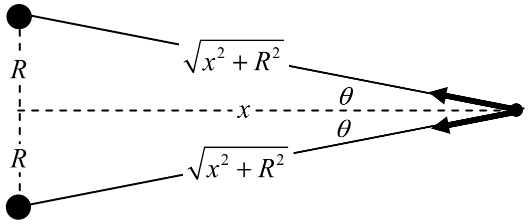
This is only about 22 Moon radii away from the Moon. Or, it is about 90% of the distance from the center of the Earth to the center of the Moon.

16. The speed of an object in an orbit of radius *r* around the Sun is given by $v = \sqrt{G M_{\text{Sun}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for M_{Sun}, using data for the Earth.

$$\sqrt{G\frac{M_{\text{Sun}}}{r}}=\frac{2\pi r}{T}\quad\rightarrow\quad M_{\text{Sun}}=\frac{4\pi^2r^3}{GT^2}=\frac{4\pi^2\left(1.50\times10^{11}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.15\times10^7\,\text{sec}\right)^2}=\boxed{2.01\times10^{30}\text{ kg}}$$

This is the same result obtained in Example 6-9 using Kepler's third law.

17. Each mass *M* will exert a gravitational force on mass *m*. The vertical components of the two forces will sum to be 0, and so the net force on *m* is directed horizontally. That net force will be twice the horizontal component of either force.

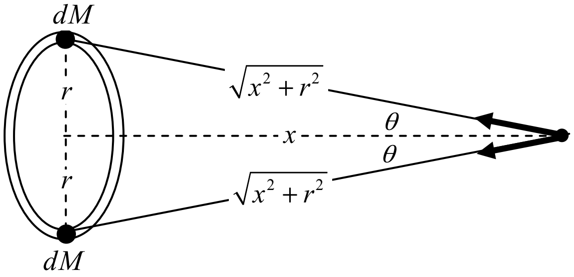


$$F_{Mm}=\frac{GMm}{\left(x^2+R^2\right)}\quad\rightarrow$$

$$F_{Mm\,x}=\frac{GMm}{\left(x^2+R^2\right)}\cos\theta=\frac{GMm}{\left(x^2+R^2\right)}\frac{x}{\sqrt{x^2+R^2}}=\frac{GMmx}{\left(x^2+R^2\right)^{3/2}}$$

$$F_{\text{net}\,x}=2F_{Mm\,x}=\boxed{\frac{2GMmx}{\left(x^2+R^2\right)^{3/2}}}$$

18. From the symmetry of the problem, we can examine diametrically opposite infinitesimal masses and see that only the horizontal components of the force will be left. Any off-axis components of force will add to zero. The infinitesimal horizontal force on *m* due to an infinitesimal mass *dM* is $dF_{dMm}=\frac{Gm}{\left(x^2+r^2\right)}dM$.



The horizontal component of that force is given by the following.

$$\left(dF_{dMm}\right)_x=\frac{Gm}{\left(x^2+r^2\right)}\cos\theta dM=\frac{Gm}{\left(x^2+r^2\right)}\frac{x}{\sqrt{\left(x^2+r^2\right)}}dM=\frac{Gmx}{\left(x^2+r^2\right)^{3/2}}dM$$

The total force is then found by integration.

$$dF_x=\frac{Gmx\,dM}{\left(x^2+r^2\right)^{3/2}}\quad\rightarrow\quad\int dF_x=\int\frac{Gmx\,dM}{\left(x^2+r^2\right)^{3/2}}\quad\rightarrow\quad F_x=\boxed{\frac{GMmx}{\left(x^2+r^2\right)^{3/2}}}$$

From the diagram we see that it points inward towards the center of the ring.

$\boxed{19.}$ The expression for *g* at the surface of the Earth is $g=G\frac{m_{\text{E}}}{r_{\text{E}}^2}$. Let $g+\Delta g$ be the value at a distance of $r_{\text{E}}+\Delta r$ from the center of Earth, which is Δr above the surface.

(*a*) $$g=G\frac{m_{\text{E}}}{r_{\text{E}}^2}\quad\rightarrow\quad g+\Delta g=G\frac{m_{\text{E}}}{\left(r_{\text{E}}+\Delta r\right)^2}=G\frac{m_{\text{E}}}{r_{\text{E}}^2\left(1+\frac{\Delta r}{r_{\text{E}}}\right)^2}=G\frac{m_{\text{E}}}{r_{\text{E}}^2}\left(1+\frac{\Delta r}{r_{\text{E}}}\right)^{-2}\approx g\left(1-2\frac{\Delta r}{r_{\text{E}}}\right)\quad\rightarrow$$

$$\boxed{\Delta g \approx -2g\frac{\Delta r}{r_E}}$$

(*b*) The minus sign indicated that the $\boxed{\text{change in } g \text{ is in the opposite direction as the change in } r}$. So, if *r* increases, *g* decreases, and vice-versa.

(*c*) Using this result:

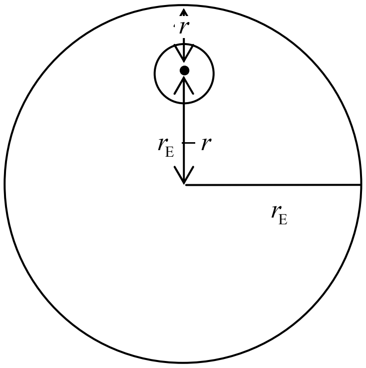
$$\Delta g \approx -2g\frac{\Delta r}{r_E} = -2\left(9.80\,\text{m/s}^2\right)\frac{1.25\times10^5\,\text{m}}{6.38\times10^6\,\text{m}} = -0.384\,\text{m/s}^2 \quad\rightarrow\quad g = \boxed{9.42\,\text{m/s}^2}$$

Direct calculation:

$$g = G\frac{m_E}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^6\,\text{m}+1.25\times10^5\,\text{m}\right)^2} = 9.43\,\text{m/s}^2$$

The difference is only about 0.1%.

20. We can find the actual *g* by taking *g* due to the uniform Earth, subtracting away *g* due to the bubble as if it contained uniform Earth matter, and adding in *g* due to the oil-filled bubble. In the diagram, *r* = 1000 m (the diameter of the bubble, and the distance from the surface to the center of the bubble). The mass of matter in the bubble is found by taking the density of the matter times the volume of the bubble.



$$g_{\substack{\text{oil}\\\text{present}}} = g_{\substack{\text{uniform}\\\text{Earth}}} - g_{\substack{\text{bubble}\\\text{(Earth}\\\text{matter)}}} + g_{\substack{\text{bubble}\\\text{(oil)}}} \quad\rightarrow$$

$$\Delta g = g_{\substack{\text{oil}\\\text{present}}} - g_{\substack{\text{uniform}\\\text{Earth}}} = g_{\substack{\text{bubble}\\\text{(oil)}}} - g_{\substack{\text{bubble}\\\text{(Earth}\\\text{matter)}}}$$

$$= \frac{GM_{\substack{\text{bubble}\\\text{(oil)}}}}{r^2} - \frac{GM_{\substack{\text{bubble}\\\text{(Earth}\\\text{matter)}}}}{r^2} = \frac{G}{r^2}\left(M_{\substack{\text{bubble}\\\text{(oil)}}} - M_{\substack{\text{bubble}\\\text{(Earth}\\\text{matter)}}}\right) = \frac{G}{r^2}\left(\rho_{\text{oil}} - \rho_{\substack{\text{Earth}\\\text{matter}}}\right)\tfrac{4}{3}\pi r_{\text{bubble}}^3$$

The density of oil is given, but we must calculate the density of a uniform Earth.

$$\rho_{\substack{\text{Earth}\\\text{matter}}} = \frac{m_E}{\frac{4}{3}\pi r_E^3} = \frac{5.98\times10^{24}\,\text{kg}}{\frac{4}{3}\pi\left(6.38\times10^6\,\text{m}\right)^3} = 5.50\times10^3\,\text{kg/m}^3$$

$$\Delta g = \frac{G}{r^2}\left(\rho_{\text{oil}} - \rho_{\substack{\text{Earth}\\\text{matter}}}\right)\tfrac{4}{3}\pi r_{\text{bubble}}^3$$

$$= \frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)}{\left(1.00\times10^3\,\text{m}\right)^2}\left(8.0\times10^2\,\text{kg/m}^3 - 5.50\times10^3\,\text{kg/m}^3\right)\tfrac{4}{3}\pi\left(5.0\times10^2\,\text{m}\right)^3$$

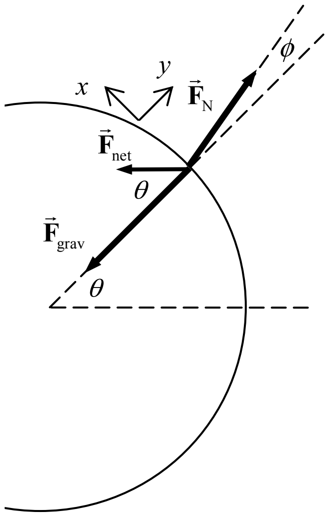
$$= -1.6414\times10^{-4}\,\text{m/s}^2 \approx -1.6\times10^{-4}\,\text{m/s}^2$$

Finally we calculate the percentage difference.

$$\frac{\Delta g}{g}(\%) = \frac{-1.6414\times10^{-4}\,\text{m/s}^2}{9.80\,\text{m/s}^2}\times100 = \boxed{-1.7\times10^{-3}\%}$$

The negative sign means that the value of *g* would decrease from the uniform Earth value.

21. For an object "at rest" on the surface of the rotating Earth, there are two force vectors that add together to form the net force: $\vec{\mathbf{F}}_{\text{grav}}$, the force of gravity, directed towards the center of the Earth; and $\vec{\mathbf{F}}_{\text{N}}$, the normal force, which is given by $\vec{\mathbf{F}}_{\text{N}} = -m\vec{\mathbf{g}}_{\text{eff}}$. The sum of these two forces must produce the centripetal force that acts on the object, causing centripetal motion. See the diagram. Notice that the component axes are parallel and perpendicular to the surface of the Earth. Write Newton's second law in vector component form for the object, and solve for $\vec{\mathbf{g}}_{\text{eff}}$. The radius of the circular motion of the object is $r = r_{\text{E}}\cos\theta$, and the speed of the circular motion is $v = \dfrac{2\pi r}{T}$, where T is the period of the rotation, one day.



$$\vec{\mathbf{F}}_{\text{grav}} + \vec{\mathbf{F}}_{\text{N}} = \vec{\mathbf{F}}_{\text{net}} \rightarrow \quad -G\frac{m_{\text{E}}m}{r_{\text{E}}^2}\hat{\mathbf{j}} + \vec{\mathbf{F}}_{\text{N}} = \frac{mv^2}{r}\sin\theta\,\hat{\mathbf{i}} - \frac{mv^2}{r}\cos\theta\,\hat{\mathbf{j}} \rightarrow$$

$$\vec{\mathbf{F}}_{\text{N}} = \frac{mv^2}{r}\sin\theta\,\hat{\mathbf{i}} + \left(G\frac{m_{\text{E}}m}{r_{\text{E}}^2} - \frac{mv^2}{r}\cos\theta\right)\hat{\mathbf{j}} = m\left[\frac{4\pi^2 r}{T^2}\sin\theta\,\hat{\mathbf{i}} + \left(G\frac{m_{\text{E}}}{r_{\text{E}}^2} - \frac{4\pi^2 r}{T^2}\cos\theta\right)\hat{\mathbf{j}}\right]$$

$$= m\left[\frac{4\pi^2 r_{\text{E}}\cos\theta}{T^2}\sin\theta\,\hat{\mathbf{i}} + \left(G\frac{m_{\text{E}}}{r_{\text{E}}^2} - \frac{4\pi^2 r_{\text{E}}\cos\theta}{T^2}\cos\theta\right)\hat{\mathbf{j}}\right]$$

$$= m\left[\frac{4\pi^2\left(6.38\times10^6\,\text{m}\right)}{\left(86{,}400\,\text{s}\right)^2}\frac{1}{2}\hat{\mathbf{i}} + \left(9.80\,\text{m/s}^2 - \frac{4\pi^2\left(6.38\times10^6\,\text{m}\right)}{\left(86{,}400\,\text{s}\right)^2}\frac{1}{2}\right)\hat{\mathbf{j}}\right]$$

$$= m\left[\left(1.687\times10^{-2}\,\text{m/s}^2\right)\hat{\mathbf{i}} + \left(9.783\,\text{m/s}^2\right)\hat{\mathbf{j}}\right]$$

From this calculation we see that $\vec{\mathbf{F}}_{\text{N}}$ points at an angle of $\phi = \tan^{-1}\dfrac{\left(1.687\times10^{-2}\,\text{m/s}^2\right)}{\left(9.783\,\text{m/s}^2\right)} = 0.0988°$ north of local "upwards" direction. Now solve $\vec{\mathbf{F}}_{\text{N}} = -m\vec{\mathbf{g}}_{\text{eff}}$ for $\vec{\mathbf{g}}_{\text{eff}}$.

$$\vec{\mathbf{F}}_{\text{N}} = m\left[\left(1.687\times10^{-2}\,\text{m/s}^2\right)\hat{\mathbf{i}} + \left(9.783\,\text{m/s}^2\right)\hat{\mathbf{j}}\right] = -m\vec{\mathbf{g}}_{\text{eff}} \rightarrow$$

$$\vec{\mathbf{g}}_{\text{eff}} = -\left[\left(1.687\times10^{-2}\,\text{m/s}^2\right)\hat{\mathbf{i}} + \left(9.783\,\text{m/s}^2\right)\hat{\mathbf{j}}\right] \rightarrow$$

$$g_{\text{eff}} = \sqrt{\left(1.687\times10^{-2}\,\text{m/s}^2\right) + \left(9.783\,\text{m/s}^2\right)} = \boxed{9.78\,\text{m/s}^2}$$

$\vec{\mathbf{g}}_{\text{eff}}$ points $\boxed{\text{0.099° south of radially inward}}$

22. Consider a distance r from the center of the Earth that satisfies $r < R_{\text{Earth}}$. Calculate the force due to the mass inside the radius r.

$$M_{\substack{\text{closer to}\\\text{center}}}(r) = \rho V = \rho\tfrac{4}{3}\pi r^3 = \frac{M_{\text{Earth}}}{\tfrac{4}{3}\pi R_{\text{Earth}}^3}\tfrac{4}{3}\pi r^3 = \frac{M_{\text{Earth}}}{R_{\text{Earth}}^3}r^3$$

$$F_{\text{gravity}} = G\frac{M_{\substack{\text{closer to}\\\text{center}}}m}{r^2} = G\frac{\left(\dfrac{M_{\text{Earth}}}{R_{\text{Earth}}^3}r^3\right)m}{r^2} = G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}m\left(\frac{r}{R_{\text{Earth}}}\right) = mg_{\text{surface}}\left(\frac{r}{R_{\text{Earth}}}\right)$$

Thus for $F_{\text{gravity}} = 0.95mg$, we must have $r = 0.95R_{\text{Earth}}$, and so we must drill down a distance equal to 5% of the Earth's radius.

$$0.05R_{\text{Earth}} = 0.05\left(6.38\times10^{6}\,\text{m}\right) = 3.19\times10^{5}\,\text{m} \approx \boxed{320\,\text{km}}$$

23. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is shown in Example 6-6 to be

$$v_{\text{orbit}} = \sqrt{G\frac{M_{\text{Earth}}}{r}}.$$

$$v = \sqrt{G\frac{M_{\text{Earth}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{\left(R_{\text{Earth}} + 680\text{ km}\right)}} = \sqrt{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^{6}\,\text{m} + 6.8\times10^{5}\,\text{m}\right)}}$$

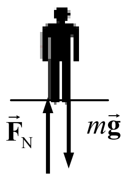
$$= \boxed{7.52\times10^{3}\text{ m/s}}$$

24. The speed of a satellite in a circular orbit around a body is shown in Example 6-6 to be $v_{\text{orbit}} = \sqrt{GM_{\text{body}}/r}$, where *r* is the distance from the satellite to the center of the body.

$$v = \sqrt{G\frac{M_{\text{body}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{R_{\text{Earth}} + 5.8\times10^{6}\,\text{m}}} = \sqrt{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\,\text{kg}\right)}{\left(12.18\times10^{6}\,\text{m}\right)}}$$

$$= \boxed{5.72\times10^{3}\text{ m/s}}$$

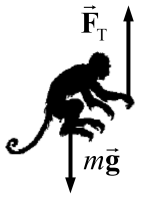
25. Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_{\text{N}}$ is the force of the scale pushing up on you, and reads the normal force. Since the scale reads 76 kg, if it were calibrated in Newtons, the normal force would be $F_{\text{N}} = \left(76\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right) = 744.8\,\text{N}$. Write Newton's second law in the vertical direction, with upward as positive.



$$\sum F = F_{\text{N}} - mg = ma \quad\rightarrow\quad a = \frac{F_{\text{N}} - mg}{m} = \frac{744.8\,\text{N} - \left(65\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{65\,\text{kg}} = \boxed{1.7\,\text{m/s}^2\text{ upward}}$$

Since the acceleration is positive, the acceleration is upward.

26. Draw a free-body diagram of the monkey. Then write Newton's second law for the vertical direction, with up as positive.



$$\sum F = F_{\text{T}} - mg = ma \quad\rightarrow\quad a = \frac{F_{\text{T}} - mg}{m}$$

For the maximum tension of 185 N,

$$a = \frac{185\,\text{N} - \left(13.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(13.0\,\text{kg}\right)} = 4.43\,\text{m/s}^2 \approx 4.4\,\text{m/s}^2$$

Thus the elevator must have an $\boxed{\text{upward acceleration greater than } a = 4.4\,\text{m/s}^2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.

27. The speed of an object in a circular orbit of radius r around mass M is given in Example 6-6 by $v=\sqrt{GM/r}$, and is also given by $v=2\pi r/T$, where T is the period of the orbiting object. Equate the two expressions for the speed and solve for T.

$$\sqrt{G\frac{M}{r}}=\frac{2\pi r}{T}\quad\rightarrow$$

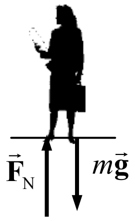
$$T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{\left(1.86\times10^6\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{m}\right)}}=\boxed{7.20\times10^3\,\text{s}\approx120\,\text{min}}$$

28. The speed of a satellite in circular orbit around the Earth is shown in Example 6-6 to be $v_{\text{orbit}}=\sqrt{G\frac{M_{\text{Earth}}}{r}}$. Thus the velocity is inversely related to the radius, and so the closer satellite will be orbiting faster.

$$\frac{v_{\text{close}}}{v_{\text{far}}}=\frac{\sqrt{\frac{GM_{\text{Earth}}}{r_{\text{close}}}}}{\sqrt{\frac{GM_{\text{Earth}}}{r_{\text{far}}}}}=\sqrt{\frac{r_{\text{far}}}{r_{\text{close}}}}=\sqrt{\frac{R_{\text{Earth}}+1.5\times10^7\,\text{m}}{R_{\text{Earth}}+5\times10^6\,\text{m}}}=\sqrt{\frac{6.38\times10^6\,\text{m}+1.5\times10^7\,\text{m}}{6.38\times10^6\,\text{m}+5\times10^6\,\text{m}}}=1.37$$

And so $\boxed{\text{the close satellite is moving 1.4 times faster}}$ than the far satellite.

29. Consider a free-body diagram for the woman in the elevator. $\vec{\mathbf{F}}_{\text{N}}$ is the upwards force the spring scale exerts, providing a normal force. Write Newton's second law for the vertical direction, with up as positive.



$$\sum F=F_{\text{N}}-mg=ma\quad\rightarrow\quad F_{\text{N}}=m\left(g+a\right)$$

(*a*, *b*) For constant speed motion in a straight line, the acceleration is 0, and so the normal force is equal to the weight.

$$F_{\text{N}}=mg=\left(53\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)=\boxed{520\,\text{N}}$$

(*c*) Here $a=+0.33g$ and so $F_{\text{N}}=1.33mg=1.33\left(53\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)=\boxed{690\,\text{N}}$.

(*d*) Here $a=-0.33g$ and so $F_{\text{N}}=0.67mg=0.67\left(53\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)=\boxed{350\,\text{N}}$.

(*e*) Here $a=-g$ and so $F_{\text{N}}=\boxed{0\,\text{N}}$.

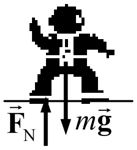
30. The speed of an object in an orbit of radius r around the Earth is given in Example 6-6 by $v=\sqrt{GM_{\text{Earth}}/r}$, and is also given by $v=2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T. Also, for a "near-Earth" orbit, $r=R_{\text{Earth}}$.

$$\sqrt{G\frac{M_{\text{Earth}}}{r}}=\frac{2\pi r}{T}\quad\rightarrow\quad T=2\pi\sqrt{\frac{r^3}{GM_{\text{Earth}}}}$$

$$T=2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}}=2\pi\sqrt{\frac{\left(6.38\times10^6\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{m}\right)}}=\boxed{5070\text{ s}=84.5\text{ min}}$$

$\boxed{\text{No}}$, the result does not depend on the mass of the satellite.

31. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, and so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\vec{\mathbf{F}}_N$. The magnitude of that force is the apparent weight of the astronaut. Take down as the positive direction.



(*a*) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0, and so the net force on the astronaut is 0.

$$\sum F = mg - F_N = 0 \quad \rightarrow$$

$$F_N = mg = G\frac{mM_{\text{Moon}}}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{(75\text{ kg})\left(7.4\times10^{22}\,\text{kg}\right)}{\left(2.5\times10^{6}\,\text{m}\right)^2} = 59.23\,\text{N}$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The astronaut will be pushed "upward" by the floor or the seat. Thus the astronaut will perceive that he has a "weight" of $\boxed{59\,\text{N, towards the Moon}}$.

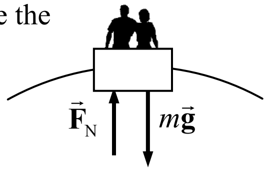
(*b*) Now the astronaut has an acceleration towards the Moon. Write Newton's second law for the astronaut, with down as the positive direction.

$$\sum F = mg - F_N = ma \quad \rightarrow \quad F_N = mg - ma = 59.23\,\text{N} - (75\,\text{kg})\left(2.3\,\text{m/s}^2\right) = -113.3\,\text{N}$$

Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram – it is pointing towards the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The astronaut will perceive being "pushed downwards," and so has an upward apparent weight of $\boxed{110\,\text{N, away from the Moon}}$.

32. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v = 2\pi r/T = 2\pi(11.0\,\text{m})/12.5\,\text{s} = 5.529\,\text{m/s}$.

(*a*) See the free-body diagram for the highest point of the motion. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

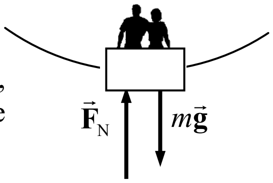


$$\sum F = F_R = mg - F_N = ma = mv^2/r \quad \rightarrow \quad F_N = mg - mv^2/r$$

The ratio of apparent weight to real weight is given by the following.

$$\frac{mg - mv^2/r}{mg} = \frac{g - v^2/r}{g} = 1 - \frac{v^2}{rg} = 1 - \frac{(5.529\,\text{m/s})^2}{(11.0\text{ m})\left(9.80\,\text{m/s}^2\right)} = \boxed{0.716}$$

(*b*) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

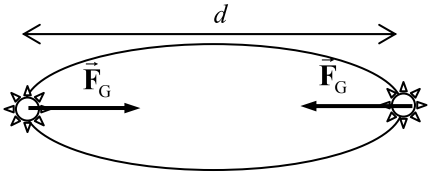


$$\sum F = F_R = F_N - mg = ma = mv^2/r \quad \rightarrow \quad F_N = mg + mv^2/r$$

The ratio of apparent weight to real weight is given by the following.

$$\frac{mg+mv^2/r}{mg}=1+\frac{v^2}{rg}=1+\frac{(5.529\,\text{m/s})^2}{(11.0\text{ m})(9.80\,\text{m/s}^2)}=\boxed{1.284}$$

33. See the diagram for the two stars.



(*a*) The two stars don't crash into each other because of their circular motion. The force on them is centripetal, and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.

(*b*) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $v=2\pi r/T=\pi d/T$, and solve for the mass.

$$F_G=G\frac{M^2}{d^2}=F_R=M\frac{v^2}{d/2}=M\frac{2(\pi d/T)^2}{d}=\frac{2\pi^2Md}{T^2}\quad\rightarrow\quad G\frac{M^2}{d^2}=\frac{2\pi^2Md}{T^2}\quad\rightarrow$$

$$M=\frac{2\pi^2d^3}{GT^2}=\frac{2\pi^2\left(8.0\times10^{11}\text{m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left(12.6\text{ y}\times\frac{3.15\times10^7\text{s}}{1\text{ y}}\right)^2}=\boxed{9.6\times10^{29}\text{kg}}$$

34. (*a*) The speed of an object in near-surface orbit around a planet is given in Example 6-6 to be $v=\sqrt{GM/R}$, where M is the planet mass and R is the planet radius. The speed is also given by $v=2\pi R/T$, where T is the period of the object in orbit. Equate the two expressions for the speed.

$$\sqrt{G\frac{M}{R}}=\frac{2\pi R}{T}\quad\rightarrow\quad G\frac{M}{R}=\frac{4\pi^2R^2}{T^2}\quad\rightarrow\quad\frac{M}{R^3}=\frac{4\pi^2}{GT^2}$$

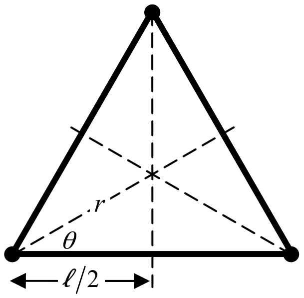
The density of a uniform spherical planet is given by $\rho=\frac{M}{\text{Volume}}=\frac{M}{\frac{4}{3}\pi R^3}$. Thus

$$\rho=\frac{3M}{4\pi R^3}=\frac{3}{4\pi}\frac{4\pi^2}{GT^2}=\boxed{\frac{3\pi}{GT^2}}$$

(*b*) For Earth, we have the following.

$$\rho=\frac{3\pi}{GT^2}=\frac{3\pi}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)\left[(85\,\text{min})(60\,\text{s/min})\right]^2}=\boxed{5.4\times10^3\text{ kg/m}^3}$$

35. Consider the lower left mass in the diagram. The center of the orbits is the intersection of the three dashed lines in the diagram. The net force on the lower left mass is the vector sum of the forces from the other two masses, and points to the center of the orbits. To find that net force, project each force to find the component that lies along the line towards the center. The angle is $\theta=30°$.



$$F=G\frac{M^2}{\ell^2}\quad\rightarrow\quad F_{\substack{\text{component}\\\text{towards}\\\text{center}}}=F\cos\theta=G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2}\quad\rightarrow$$

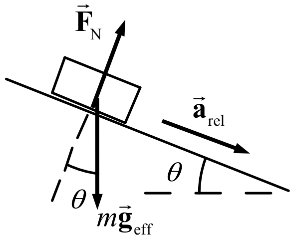
$$F_{\text{net}} = 2G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2} = \sqrt{3}G\frac{M^2}{\ell^2}$$

The net force is causing centripetal motion, and so is of the form Mv^2/r. Note that $r\cos\theta = \ell/2$.

$$F_{\text{net}} = 2G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2} = \sqrt{3}G\frac{M^2}{\ell^2} = \frac{Mv^2}{r} = \frac{Mv^2}{\ell/(2\cos\theta)} = \frac{Mv^2}{\ell/\sqrt{3}} \rightarrow \sqrt{3}G\frac{M^2}{\ell^2} = \frac{Mv^2}{\ell/\sqrt{3}} \rightarrow$$

$$v = \boxed{\sqrt{\frac{GM}{\ell}}}$$

36. The effective value of the acceleration due to gravity in the elevator is $g_{\text{eff}} = g + a_{\text{elevator}}$. We take the upwards direction to be positive. The acceleration relative to the plane is along the plane, as shown in the free-body diagram.



(*a*) The elevator acceleration is $a_{\text{elevator}} = +0.50g$.

$$g_{\text{eff}} = g + 0.50g = 1.50g \rightarrow$$

$$a_{\text{rel}} = g_{\text{eff}}\sin\theta = 1.50g\sin 32^\circ = \boxed{7.79\,\text{m/s}^2}$$

(*b*) The elevator acceleration is $a_{\text{elevator}} = -0.50g$.

$$g_{\text{eff}} = g - 0.50g = 0.50g \rightarrow a_{\text{rel}} = g_{\text{eff}}\sin\theta = 0.50g\sin 32^\circ = \boxed{2.60\,\text{m/s}^2}$$

(*c*) The elevator acceleration is $a_{\text{elevator}} = -g$.

$$g_{\text{eff}} = g - g = 0 \rightarrow a_{\text{rel}} = g_{\text{eff}}\sin\theta = 0\sin 32^\circ = \boxed{0\,\text{m/s}^2}$$

(*d*) The elevator acceleration is 0.

$$g_{\text{eff}} = g - 0 = g \rightarrow a_{\text{rel}} = g_{\text{eff}}\sin\theta = \boxed{5.19\,\text{m/s}^2}$$

$\boxed{37.}$ Use Kepler's third law for objects orbiting the Earth. The following are given.

$$T_2 = \text{period of Moon} = (27.4\text{ day})\left(\frac{86{,}400\text{ s}}{1\text{ day}}\right) = 2.367\times10^6\text{ sec}$$

$$r_2 = \text{radius of Moon's orbit} = 3.84\times10^8\,\text{m}$$

$$r_1 = \text{radius of near-Earth orbit} = R_{\text{Earth}} = 6.38\times10^6\,\text{m}$$

$$(T_1/T_2)^2 = (r_1/r_2)^3 \rightarrow$$

$$T_1 = T_2(r_1/r_2)^{3/2} = (2.367\times10^6\text{sec})\left(\frac{6.38\times10^6\,\text{m}}{3.84\times10^8\,\text{m}}\right)^{3/2} = \boxed{5.07\times10^3\text{sec}}(= 84.5\text{ min})$$

38. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $v = 2\pi r/T$. But the speed can also be calculated for any Earth satellite by $v = \sqrt{GM_{\text{Earth}}/r}$, as derived in Example 6-6. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$\sqrt{GM_{\text{Earth}}/r} = 2\pi r/T \quad \rightarrow$$

$$M_{\text{Earth}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(3.84\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[(27.4\text{ d})(86{,}400\,\text{s/d})\right]^2} = \boxed{5.98\times10^{24}\,\text{kg}}$$

39. Use Kepler's third law for objects orbiting the Sun.

$$\left(T_{\text{Neptune}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{Neptune}}/r_{\text{Earth}}\right)^3 \quad \rightarrow$$

$$T_{\text{Neptune}} = T_{\text{Earth}}\left(\frac{r_{\text{Neptune}}}{r_{\text{Earth}}}\right)^{3/2} = (1\text{ year})\left(\frac{4.5\times10^9\,\text{km}}{1.50\times10^8\,\text{km}}\right)^{3/2} = \boxed{160\,\text{years}}$$

40. As found in Example 6-6, the speed for an object orbiting a distance *r* around a mass *M* is given by $v = \sqrt{GM/r}$.

$$\frac{v_{\text{A}}}{v_{\text{B}}} = \frac{\sqrt{\dfrac{GM_{\text{star}}}{r_{\text{A}}}}}{\sqrt{\dfrac{GM_{\text{star}}}{r_{\text{B}}}}} = \sqrt{\frac{r_{\text{B}}}{r_{\text{A}}}} = \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$$

41. There are two expressions for the velocity of an object in circular motion around a mass *M*: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for *T*.

$$\sqrt{GM/r} = 2\pi r/T \quad \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{\left(\left(3\times10^4\,\text{ly}\right)\dfrac{\left(3\times10^8\,\text{m/s}\right)\left(3.16\times10^7\,\text{sec}\right)}{1\text{ ly}}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4\times10^{41}\,\text{kg}\right)}} = 5.8\times10^{15}\,\text{s} = 1.8\times10^8\,\text{y}$$

$$\approx \boxed{2\times10^8\,\text{y}}$$

42. (*a*) The relationship between satellite period *T*, mean satellite distance *r*, and planet mass *M* can be derived from the two expressions for satellite speed: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for *M*.

$$\sqrt{GM/r} = 2\pi r/T \quad \rightarrow \quad M = \frac{4\pi^2 r^3}{GT^2}$$

Substitute the values for Io to get the mass of Jupiter.

$$M_{\substack{\text{Jupiter-}\\\text{Io}}} = \frac{4\pi^2\left(4.22\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.77\text{d}\times\dfrac{24\text{ h}}{1\text{ d}}\times\dfrac{3600\text{ s}}{1\text{ h}}\right)^2} = \boxed{1.90\times10^{27}\,\text{kg}}$$

(*b*) For the other moons, we have the following.

$$M_{\substack{\text{Jupiter-}\\\text{Europa}}} = \frac{4\pi^2\left(6.71\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(3.55\times24\times3600\text{ s}\right)^2} = \boxed{1.90\times10^{27}\,\text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\\text{Ganymede}}}=\frac{4\pi^2\left(1.07\times10^9\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.16\times24\times3600\text{ s}\right)^2}=\boxed{1.89\times10^{27}\,\text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\\text{Callisto}}}=\frac{4\pi^2\left(1.883\times10^9\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(16.7\times24\times3600\text{ s}\right)^2}=\boxed{1.90\times10^{27}\,\text{kg}}$$

$\boxed{\text{Yes}}$, the results are consistent – only about 0.5% difference between them.

$\boxed{43.}$ Use Kepler's third law to find the radius of each moon of Jupiter, using Io's data for r_2 and T_2.

$$\left(r_1/r_2\right)^3=\left(T_1/T_2\right)^2 \quad\rightarrow\quad r_1=r_2\left(T_1/T_2\right)^{2/3}$$

$$r_{\text{Europa}}=r_{\text{Io}}\left(T_{\text{Europa}}/T_{\text{Io}}\right)^{2/3}=\left(422\times10^3\,\text{km}\right)\left(3.55\text{ d}/1.77\text{ d}\right)^{2/3}=\boxed{671\times10^3\,\text{km}}$$

$$r_{\text{Ganymede}}=\left(422\times10^3\,\text{km}\right)\left(7.16\text{ d}/1.77\text{ d}\right)^{2/3}=\boxed{1070\times10^3\,\text{km}}$$

$$r_{\text{Callisto}}=\left(422\times10^3\,\text{km}\right)\left(16.7\text{ d}/1.77\text{ d}\right)^{2/3}=\boxed{1880\times10^3\,\text{km}}$$

The agreement with the data in the table is excellent.

44. (*a*) Use Kepler's third law to relate the Earth and the hypothetical planet in their orbits around the Sun.

$$\left(T_{\text{planet}}/T_{\text{Earth}}\right)^2=\left(r_{\text{planet}}/r_{\text{Earth}}\right)^3 \quad\rightarrow$$

$$T_{\text{planet}}=T_{\text{Earth}}\left(r_{\text{planet}}/r_{\text{Earth}}\right)^{3/2}=\left(1\text{ y}\right)\left(3/1\right)^{3/2}=5.20\,\text{y}\approx\boxed{5\text{ y}}$$

(*b*) No mass data can be calculated from this relationship, because the relationship is mass-independent. Any object at the orbit radius of 3 times the Earth's orbit radius would have a period of 5.2 years, regardless of its mass.

45. (*a*) Use Kepler's third law to relate the orbits of the Earth and the comet around the Sun.

$$\left(\frac{r_{\text{comet}}}{r_{\text{Earth}}}\right)^3=\left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^2 \quad\rightarrow$$

$$r_{\text{comet}}=r_{\text{Earth}}\left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^{2/3}=\left(1\text{ AU}\right)\left(\frac{2400\text{ y}}{1\text{ y}}\right)^{2/3}=179.3\,\text{AU}\approx\boxed{180\,\text{AU}}$$

(*b*) The mean distance is the numeric average of the closest and farthest distances.

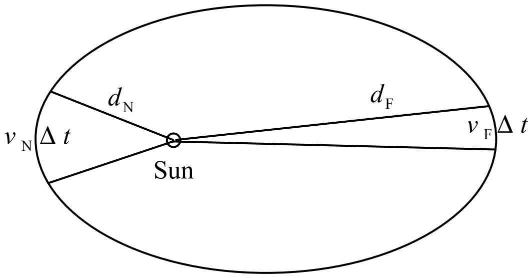
$$179.3\,\text{AU}=\frac{1.00\,\text{AU}+r_{\text{max}}}{2} \quad\rightarrow\quad r_{\text{max}}=357.6\,\text{AU}\approx\boxed{360\,\text{AU}}$$

(*c*) Refer to Figure 6-17, which illustrates Kepler's second law. If the time for each shaded region is made much shorter, then the area of each region can be approximated as a triangle. The area of each triangle is half the "base" (speed of comet multiplied by the amount of time) times the "height" (distance from Sun). So we have the following.

$$\text{Area}_{\text{min}}=\text{Area}_{\text{max}} \quad\rightarrow\quad \tfrac{1}{2}\left(v_{\text{min}}t\right)r_{\text{min}}=\tfrac{1}{2}\left(v_{\text{max}}t\right)r_{\text{max}} \quad\rightarrow$$

$$v_{\text{min}}/v_{\text{max}}=r_{\text{max}}/r_{\text{min}}=\boxed{360/1}$$

46. (*a*) In a short time Δt, the planet will travel a distance $v\Delta t$ along its orbit. That distance is essentially a straight line segment for a short time duration. The time (and distance moved) during Δt have been greatly exaggerated on the diagram. Kepler's second law states that the area swept out by a line from the Sun to the planet during the planet's motion for the Δt is the same anywhere on the orbit. Take the areas swept out at the near and far points, as shown on the diagram, and approximate them as triangles (which will be reasonable for short Δt).



$$(\text{Area})_N = (\text{Area})_F \rightarrow \tfrac{1}{2}(v_N \Delta t)d_N = \tfrac{1}{2}(v_F \Delta t)d_F \rightarrow \boxed{v_N/v_F = d_F/d_N}$$

(*b*) Since the orbit is almost circular, an average velocity can be found by assuming a circular orbit with a radius equal to the average distance.

$$v_{avg} = \frac{2\pi r}{T} = \frac{2\pi \frac{1}{2}(d_N + d_F)}{T} = \frac{2\pi \frac{1}{2}(1.47\times10^{11}\,\text{m} + 1.52\times10^{11}\,\text{m})}{3.16\times10^{7}\,\text{s}} = 2.973\times10^{4}\ \text{m/s}$$

From part (*a*) we find the ratio of near and far velocities.

$$v_N/v_F = d_F/d_N = 1.52/1.47 = 1.034$$

For this small change in velocities (3.4% increase from smallest to largest), we assume that the minimum velocity is 1.7% lower than the average velocity and the maximum velocity is 1.7% higher than the average velocity.

$$v_N = v_{avg}(1 + 0.017) = 2.973\times10^{4}\ \text{m/s}(1.017) = \boxed{3.02\times10^{4}\ \text{m/s}}$$

$$v_F = v_{avg}(1 - 0.017) = 2.973\times10^{4}\ \text{m/s}(0.983) = \boxed{2.92\times10^{4}\ \text{m/s}}$$

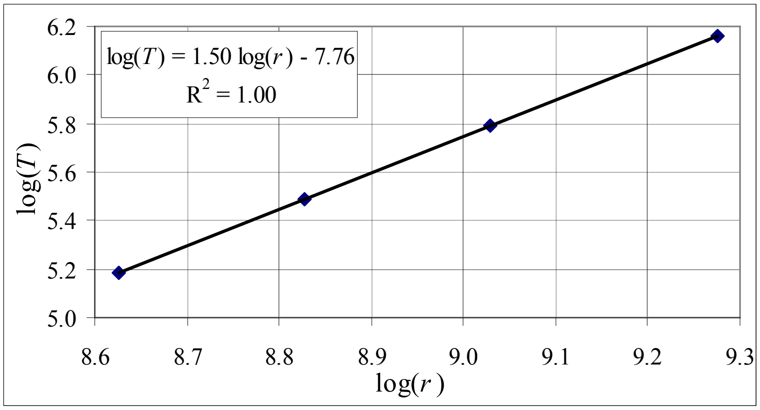
47. (*a*) Take the logarithm of both sides of the Kepler's third law expression.

$$T^2 = \left(\frac{4\pi^2}{Gm_J}\right)r^3 \rightarrow \log T^2 = \log\left(\frac{4\pi^2}{Gm_J}\right)r^3 \rightarrow 2\log T = \log\left(\frac{4\pi^2}{Gm_J}\right) + 3\log r \rightarrow$$

$$\log T = \tfrac{3}{2}\log r + \tfrac{1}{2}\log\left(\frac{4\pi^2}{Gm_J}\right)$$

This predicts a straight line graph for log(*T*) vs. log(*r*), with a $\boxed{\text{slope of 3/2}}$ and a $\boxed{y\text{-intercept of } \tfrac{1}{2}\log\left(\frac{4\pi^2}{Gm_J}\right)}$.

(*b*) The data is taken from Table 6-3, and the graph is shown here, with a straight-line fit to the data. The data need to be converted to seconds and meters before the logarithms are calculated.

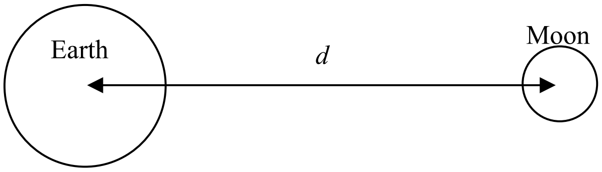


From the graph, the slope is 1.50 (as expected), and the *y*-intercept is –7.76.

$$\tfrac{1}{2}\log\left(\frac{4\pi^2}{Gm_{\text{J}}}\right)=b \;\rightarrow\; m_{\text{J}}=\frac{4\pi^2}{G\left(10^{2b}\right)}=\frac{4\pi^2}{\left(6.67\times10^{-11}\right)\left(10^{-15.52}\right)}=\boxed{1.97\times10^{27}\,\text{kg}}$$

The actual mass of Jupiter is given in problem 8 as 318 times the mass of the Earth, which is $1.90\times10^{27}\,\text{kg}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH06.XLS," on tab "Problem 6.47b."

48. We choose the line joining the Earth and Moon centers to be the *x*-axis. The field of the Earth will point towards the Earth, and the field of the Moon will point towards the Moon.



$$\vec{\mathbf{g}}=\frac{GM_{\text{Earth}}}{\left(\frac{1}{2}r_{\substack{\text{Earth-}\\\text{Moon}}}\right)^2}\left(-\hat{\mathbf{i}}\right)+\frac{GM_{\text{Moon}}}{\left(\frac{1}{2}r_{\substack{\text{Earth-}\\\text{Moon}}}\right)^2}\left(\hat{\mathbf{i}}\right)=\frac{G\left(M_{\text{Moon}}-M_{\text{Earth}}\right)}{\left(\frac{1}{2}r_{\substack{\text{Earth-}\\\text{Moon}}}\right)^2}\hat{\mathbf{i}}$$

$$=\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}-5.97\times10^{24}\,\text{kg}\right)}{\left(\frac{1}{2}\left(384\times10^{6}\,\text{m}\right)\right)^2}\hat{\mathbf{i}}=-1.07\times10^{-2}\,\text{m/s}^2\,\hat{\mathbf{i}}$$

So the magnitude is $\boxed{1.07\times10^{-2}\,\text{m/s}^2}$ and the direction is $\boxed{\text{towards the center of the Earth}}$.

$\boxed{49.}$ (*a*) The gravitational field due to a spherical mass *M*, at a distance *r* from the center of the mass, is $g=GM/r^2$.

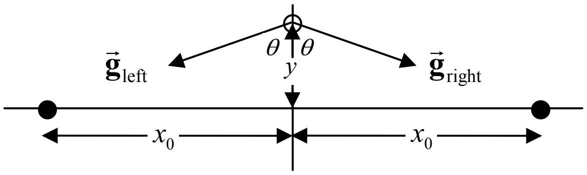
$$g_{\substack{\text{Sun at}\\\text{Earth}}}=\frac{GM_{\text{Sun}}}{r^2_{\substack{\text{Sun to}\\\text{Earth}}}}=\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.496\times10^{11}\,\text{m}\right)^2}=\boxed{5.93\times10^{-3}\,\text{m/s}^2}$$

(*b*) Compare this to the field caused by the Earth at the surface of the Earth.

$$\frac{g_{\substack{\text{Sun at}\\\text{Earth}}}}{g_{\text{Earth}}}=\frac{5.93\times10^{-3}\,\text{m/s}^2}{9.80\,\text{m/s}^2}=6.05\times10^{-4}$$

$\boxed{\text{No}}$, this is not going to affect your weight significantly. The effect is less than 0.1 %.

50. (*a*) From the symmetry of the situation, the net force on the object will be down. However, we will show that explicitly by writing the field in vector component notation.



$$\vec{\mathbf{g}}=\vec{\mathbf{g}}_{\text{left}}+\vec{\mathbf{g}}_{\text{right}}=\left[\left(-G\frac{m}{x_0^2+y^2}\sin\theta\right)\hat{\mathbf{i}}+\left(-G\frac{m}{x_0^2+y^2}\cos\theta\right)\hat{\mathbf{j}}\right]$$

$$+\left[\left(G\frac{m}{x_0^2+y^2}\sin\theta\right)\hat{\mathbf{i}}+\left(-G\frac{m}{x_0^2+y^2}\cos\theta\right)\hat{\mathbf{j}}\right]$$

$$=\left(-2G\frac{m}{x_0^2+y^2}\cos\theta\right)\hat{\mathbf{j}}=\left(-2G\frac{m}{x_0^2+y^2}\frac{y}{\sqrt{x_0^2+y^2}}\right)\hat{\mathbf{j}}=\boxed{\left(-2Gm\frac{y}{\left(x_0^2+y^2\right)^{3/2}}\right)\hat{\mathbf{j}}}$$

(*b*) If we keep y as a positive quantity, then the magnitude of the field is $g = 2Gm\dfrac{y}{\left(x_0^2 + y^2\right)^{3/2}}$.

We find locations of the maximum magnitude by setting the first derivative equal to 0. Since the expression is never negative, any extrema will be maxima.

$$g = 2Gm\frac{y}{\left(x_0^2 + y^2\right)^{3/2}} \quad \rightarrow \quad \frac{dg}{dt} = 2Gm\left[\frac{\left(x_0^2 + y^2\right)^{3/2} - y\tfrac{3}{2}\left(x_0^2 + y^2\right)^{1/2} 2y}{\left(x_0^2 + y^2\right)^{3}}\right] = 0 \quad \rightarrow$$

$$\left(x_0^2 + y^2\right)^{3/2} - y\tfrac{3}{2}\left(x_0^2 + y^2\right)^{1/2} 2y = 0 \quad \rightarrow \quad \boxed{y_{\max} = \frac{x_0}{\sqrt{2}}} \approx 0.71x_0$$

$$g_{\max} = g\left(y = \frac{x_0}{\sqrt{2}}\right) = 2Gm\frac{\dfrac{x_0}{\sqrt{2}}}{\left(x_0^2 + \left(\dfrac{x_0}{\sqrt{2}}\right)^2\right)^{3/2}} = \boxed{\frac{4Gm}{3\sqrt{3}x_0^2}} \approx 0.77\frac{Gm}{x_0^2}$$

There would also be a maximum at $y = -x_0/\sqrt{2}$.

51. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g = GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. Find the location where $g = \tfrac{1}{2}g_{\text{surface}}$.

$$\frac{GM_{\text{Earth}}}{r^2} = \frac{1}{2}\frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} \quad \rightarrow \quad r^2 = 2R_{\text{Earth}}^2 \quad \rightarrow \quad r = \sqrt{2}R_{\text{Earth}}$$

The distance above the Earth's surface is as follows.

$$r - R_{\text{Earth}} = \left(\sqrt{2} - 1\right)R_{\text{Earth}} = \left(\sqrt{2} - 1\right)\left(6.38\times10^6\,\text{m}\right) = \boxed{2.64\times10^6\,\text{m}}$$

52. (*a*) Mass is independent of location and so the mass of the ball is $\boxed{13.0\text{ kg}}$ on both the Earth and the planet.

(*b*) The weight is found by $W = mg$.

$$W_{\text{Earth}} = mg_{\text{Earth}} = (13.0\text{ kg})\left(9.80\,\text{m/s}^2\right) = \boxed{127\,\text{N}}$$

$$W_{\text{Planet}} = mg_{\text{Planet}} = (13.0\text{ kg})\left(12.0\,\text{m/s}^2\right) = \boxed{156\,\text{N}}$$

53. (*a*) The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = GM_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{sun}}}{R_{\text{Moon}}^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.74\times10^6\,\text{m}\right)^2} = \boxed{4.38\times10^7\,\text{m/s}^2}$$

(*b*) $W = mg_{\text{star}} = (65\,\text{kg})\left(4.38\times10^7\,\text{m/s}^2\right) = \boxed{2.8\times10^9\,\text{N}}$

(*c*) Use Eq. 2-12c, with an initial velocity of 0.

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \quad \rightarrow$$

$$v = \sqrt{2a\left(x - x_0\right)} = \sqrt{2\left(4.38\times10^7\,\text{m/s}^2\right)(1.0\,\text{m})} = \boxed{9.4\times10^3\,\text{m/s}}$$

54. In general, the acceleration due to gravity of the Earth is given by $g = GM_{\text{Earth}}/r^2$, where *r* is the distance from the center of the Earth to the location in question. So for the location in question, we have the following.

$$g = \tfrac{1}{10}g_{\text{surface}} \rightarrow G\frac{M_{\text{Earth}}}{r^2} = \tfrac{1}{10}G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} \rightarrow r^2 = 10\,R^2_{\text{Earth}}$$

$$r = \sqrt{10}R_{\text{Earth}} = \sqrt{10}\left(6.38\times10^6\,\text{m}\right) = \boxed{2.02\times10^7\,\text{m}}$$

$\boxed{55.}$ The speed of an object in an orbit of radius *r* around a planet is given in Example 6-6 as $v = \sqrt{GM_{\text{planet}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G\frac{M_{\text{Planet}}}{r}} = \frac{2\pi r}{T} \rightarrow T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Planet}}}}$$

For this problem, the inner orbit has radius $r_{\text{inner}} = 7.3\times10^7\,\text{m}$, and the outer orbit has radius $r_{\text{outer}} = 1.7\times10^8\,\text{m}$. Use these values to calculate the periods.

$$T_{\text{inner}} = 2\pi\sqrt{\frac{\left(7.3\times10^7\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{2.0\times10^4\,\text{s}}$$

$$T_{\text{outer}} = 2\pi\sqrt{\frac{\left(1.7\times10^8\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.7\times10^{26}\,\text{kg}\right)}} = \boxed{7.1\times10^4\,\text{s}}$$

Saturn's rotation period (day) is 10 hr 39 min, which is about 3.8×10^4sec. Thus the inner ring will appear to move across the sky "faster" than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky "slower" than the Sun (about once every two Saturn days).

56. The speed of an object in an orbit of radius *r* around the Moon is given by $v = \sqrt{GM_{\text{Moon}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{GM_{\text{Moon}}/r} = 2\pi r/T \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(R_{\text{Moon}} + 100\text{ km}\right)^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(1.74\times10^6\,\text{m} + 1\times10^5\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)}}$$

$$= \boxed{7.1\times10^3\,\text{s}\ \left(\sim 2.0\,\text{h}\right)}$$

57. Use Kepler's third law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3 = \left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \rightarrow$$

$$r_{\text{Halley}} = r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3} = \left(150\times10^6\,\text{km}\right)\left(76\text{ y}/1\text{ y}\right)^{2/3} = 2690\times10^6\,\text{km}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^6\,\text{km} = \boxed{5.4\times10^{12}\,\text{m}}$. This distance approaches the mean orbit distance of Pluto, which is $5.9\times10^{12}\,\text{m}$. $\boxed{\text{It is still in the solar system, nearest to Pluto's orbit.}}$

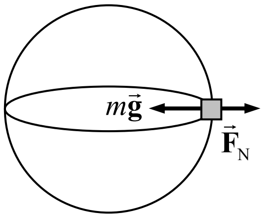
58. (*a*) The speed of a satellite orbiting the Earth is given by $v=\sqrt{GM_{\text{Earth}}/r}$. For the GPS satellites, $r=R_{\text{Earth}}+(11{,}000)(1.852\text{ km})=2.68\times10^7\,\text{m}$.

$$v=\sqrt{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times10^{24}\,\text{kg}\right)}{2.68\times10^7\,\text{m}}}=3.86\times10^3\text{ m/s}\approx\boxed{3.9\times10^3\text{ m/s}}$$

(*b*) The period can be found from the speed and the radius.

$$v=2\pi r/T \quad\to\quad T=\frac{2\pi r}{v}=\frac{2\pi\left(2.68\times10^7\,\text{m}\right)}{3.86\times10^3\text{ m/s}}=\boxed{4.4\times10^4\text{sec }\sim 12\,\text{h}}$$

59. For a body on the equator, the net motion is circular. Consider the free-body diagram as shown. F_N is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's second law with the inward direction as positive.



$$\sum F_R = mg_{\text{Jupiter}}-F_N = mv^2/R_{\text{Jupiter}} \quad\to$$

$$F_N=m\left(g_{\text{Jupiter}}-v^2/R_{\text{Jupiter}}\right)=m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}}-\frac{v^2}{R_{\text{Jupiter}}}\right)$$

Use the fact that for a rotating object, $v=2\pi r/T$.

$$F_N=m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}}-\frac{4\pi^2R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}}\right)=mg_{\text{perceived}}$$

Thus the perceived acceleration due to gravity of the object on the surface of Jupiter is as follows.

$$\begin{aligned}g_{\text{perceived}}&=G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}}-\frac{4\pi^2R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}}\\&=\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.9\times10^{27}\text{kg}\right)}{\left(7.1\times10^7\text{m}\right)^2}-\frac{4\pi^2\left(7.1\times10^7\text{m}\right)}{\left[(595\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right)\right]^2}\\&=22.94\,\text{m/s}^2\left(\frac{1\,g}{9.8\,\text{m/s}^2}\right)=\boxed{2.3\,g\text{'s}}\end{aligned}$$

Based on this result, you would not be crushed at all. You would feel "heavy," but not at all crushed.

60. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $v=\sqrt{G\frac{M_{\text{galaxy}}}{r_{\text{Sun orbit}}}}$ and so $M_{\text{galaxy}}=\frac{r_{\text{Sun orbit}}v^2}{G}$. Substitute in the relationship that $v=2\pi r_{\text{Sun orbit}}/T$.

$$M_{\text{galaxy}}=\frac{4\pi^2\left(r_{\text{Sun orbit}}\right)^3}{GT^2}=\frac{4\pi^2\left[(30{,}000)\left(9.5\times10^{15}\,\text{m}\right)\right]^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left[\left(200\times10^6\,\text{y}\right)\left(\frac{3.15\times10^7\,\text{s}}{1\,\text{y}}\right)\right]^2}$$

$$=3.452\times10^{41}\,\text{kg}\approx\boxed{3\times10^{41}\,\text{kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ stars}=\frac{M_{\text{galaxy}}}{M_{\text{Sun}}}=\frac{3.452\times10^{41}\,\text{kg}}{2.0\times10^{30}\,\text{kg}}=1.726\times10^{11}\approx\boxed{2\times10^{11}\,\text{stars}}$$

$\boxed{61.}$ In the text, it says that Eq. 6-6 is valid if the radius r is replaced with the semi-major axis s. From Fig. 6-16, the distance of closest approach r_{min} is seen to be $r_{\text{min}}=s-es=s(1-e)$, and so the semi-major axis is given by $s=\frac{r_{\text{min}}}{1-e}$.

$$\frac{T^2}{s^3}=\frac{4\pi^2}{GM_{\text{SgrA}}}\quad\rightarrow$$

$$M_{\text{SgrA}}=\frac{4\pi^2s^3}{GT^2}=\frac{4\pi^2\left(\frac{r_{\text{min}}}{1-e}\right)^3}{GT^2}=\frac{4\pi^2\left(\frac{123\,\text{AU}\times\frac{1.5\times10^{11}\,\text{m}}{1\,\text{AU}}}{1-0.87}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(15.2\text{y}\times\frac{3.156\times10^7\,\text{s}}{1\,\text{y}}\right)^2}$$

$$=7.352\times10^{36}\,\text{kg}\approx\boxed{7.4\times10^{36}\,\text{kg}}$$

$\frac{M_{\text{SgrA}}}{M_{\text{Sun}}}=\frac{7.352\times10^{36}\,\text{kg}}{1.99\times10^{30}\,\text{kg}}=\boxed{3.7\times10^6}$ and so SgrA is almost 4 million times more massive than our Sun.

62. (a) The gravitational force on the satellite is given by $F_{\text{grav}}=G\frac{M_{\text{Earth}}m}{r^2}$, where r is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text{net}}=mv^2/r$. By substituting $v=2\pi r/T$ for a circular orbit, we have $F_{\text{net}}=\frac{4\pi^2mr}{T^2}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated, and solved for the orbit radius.

$$G\frac{M_{\text{Earth}}m}{r^2}=\frac{4\pi^2mr}{T^2}\quad\rightarrow$$

$$r=\left(\frac{GM_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3}=\left[\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(6.0\times10^{24}\,\text{kg}\right)(6200\,\text{s})^2}{4\pi^2}\right]^{1/3}$$

$$=7.304\times10^6\,\text{m}\approx\boxed{7.3\times10^6\,\text{m}}$$

(*b*) From this value the gravitational force on the satellite can be calculated.

$$F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(6.0\times10^{24}\,\text{kg}\right)\left(5500\,\text{kg}\right)}{\left(7.304\times10^{6}\,\text{m}\right)^2} = 4.126\times10^{4}\,\text{N}$$

$$\approx \boxed{4.1\times10^{4}\,\text{N}}$$

(*c*) The altitude of the satellite above the Earth's surface is given by the following.

$$r - R_{\text{Earth}} = 7.304\times10^{6}\,\text{m} - 6.38\times10^{6}\,\text{m} = \boxed{9.2\times10^{5}\,\text{m}}$$

63. Your weight is given by the law of universal gravitation. The derivative of the weight with respect to time is found by taking the derivative of the weight with respect to distance from the Earth's center, and using the chain rule.

$$W = G\frac{m_{\text{E}}m}{r^2} \quad\rightarrow\quad \frac{dW}{dt} = \frac{dW}{dr}\frac{dr}{dt} = \boxed{-2G\frac{m_{\text{E}}m}{r^3}v}$$

64. The speed of an orbiting object is given in Example 6-6 as $v = \sqrt{GM/r}$, where r is the radius of the orbit, and M is the mass around which the object is orbiting. Solve the equation for M.

$$v = \sqrt{GM/r} \quad\rightarrow\quad M = \frac{rv^2}{G} = \frac{\left(5.7\times10^{17}\,\text{m}\right)\left(7.8\times10^{5}\,\text{m/s}\right)^2}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)} = \boxed{5.2\times10^{39}\,\text{kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

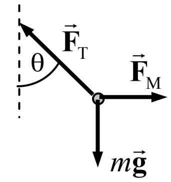
$$\#\ \text{solar masses} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{5.2\times10^{39}\,\text{kg}}{2\times10^{30}\,\text{kg}} = \boxed{2.6\times10^{9}\,\text{solar masses}}$$

65. Find the "new" Earth radius by setting the acceleration due to gravity at the Sun's surface equal to the acceleration due to gravity at the "new" Earth's surface.

$$g_{\underset{\text{new}}{\text{Earth}}} = g_{\text{Sun}} \quad\rightarrow\quad \frac{GM_{\text{Earth}}}{r^2_{\underset{\text{new}}{\text{Earth}}}} = \frac{GM_{\text{Sun}}}{r^2_{\text{Sun}}} \quad\rightarrow\quad r_{\underset{\text{new}}{\text{Earth}}} = r_{\text{Sun}}\sqrt{\frac{M_{\text{Earth}}}{M_{\text{Sun}}}} = \left(6.96\times10^{8}\,\text{m}\right)\sqrt{\frac{5.98\times10^{24}\,\text{kg}}{1.99\times10^{30}\,\text{kg}}}$$

$$= \boxed{1.21\times10^{6}\,\text{m}}\text{, about } \tfrac{1}{5} \text{ the actual Earth radius.}$$

66. (*a*) See the free-body diagram for the plumb bob. The attractive gravitational force on the plumb bob is $F_{\text{M}} = G\frac{mm_{\text{M}}}{D^2_M}$. Since the bob is not accelerating, the net force in any direction will be zero. Write the net force for both vertical and horizontal directions. Use $g = G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}}$.



$$\sum F_{\text{vertical}} = F_{\text{T}}\cos\theta - mg = 0 \quad\rightarrow\quad F_{\text{T}} = \frac{mg}{\cos\theta}$$

$$\sum F_{\text{horizontal}} = F_{\text{M}} - F_{\text{T}}\sin\theta = 0 \quad\rightarrow\quad F_{\text{M}} = F_{\text{T}}\sin\theta = mg\tan\theta$$

$$G\frac{mm_{\text{M}}}{D^2_M} = mg\tan\theta \quad\rightarrow\quad \theta = \tan^{-1}G\frac{m_{\text{M}}}{gD^2_M} = \boxed{\tan^{-1}\frac{m_{\text{M}}R^2_{\text{Earth}}}{M_{\text{Earth}}D^2_M}}$$

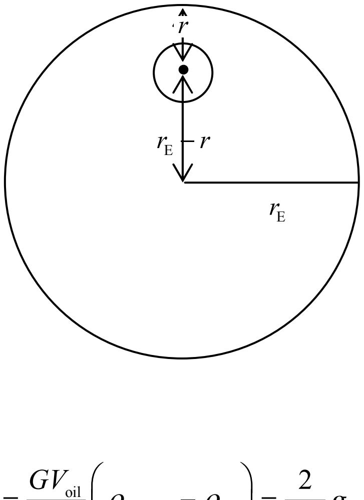
(*b*) We estimate the mass of Mt. Everest by taking its volume times its mass density. If we approximate Mt. Everest as a cone with the same size diameter as height, then its volume is $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(2000\text{ m}\right)^2\left(4000\text{ m}\right)=1.7\times10^{10}\text{m}^3$. The density is $\rho=3\times10^3\text{ kg/m}^3$. Find the mass by multiplying the volume times the density.

$$M=\rho V=\left(3\times10^3\text{ kg/m}^3\right)\left(1.7\times10^{10}\text{m}^3\right)=\boxed{5\times10^{13}\text{kg}}$$

(*c*) With *D* = 5000 m, use the relationship derived in part (a).

$$\theta=\tan^{-1}\frac{M_{\text{M}}R_{\text{Earth}}^2}{M_{\text{Earth}}D_{\text{M}}^2}=\tan^{-1}\frac{\left(5\times10^{13}\text{kg}\right)\left(6.38\times10^6\text{m}\right)^2}{\left(5.97\times10^{24}\text{kg}\right)\left(5000\text{m}\right)^2}=\boxed{8\times10^{-4}\text{degrees}}$$

$\boxed{67.}$ Since all of the masses (or mass holes) are spherical, and *g* is being measured outside of their boundaries, we can use the simple Newtonian gravitation expression. In the diagram, the distance *r* = 2000 m. The radius of the deposit is unknown.



$$g_{\text{actual}}=g_{\substack{\text{full}\\\text{Earth}}}-g_{\substack{\text{missing}\\\text{dirt mass}}}+g_{\text{oil}}=g_{\substack{\text{full}\\\text{Earth}}}-\frac{GM_{\substack{\text{missing}\\\text{dirt}}}}{r^2}+\frac{GM_{\text{oil}}}{r^2}$$

$$=g_{\substack{\text{full}\\\text{Earth}}}-\frac{G\left(M_{\substack{\text{missing}\\\text{dirt}}}-M_{\text{oil}}\right)}{r^2}$$

$$\Delta g=g_{\substack{\text{full}\\\text{Earth}}}-g_{\text{actual}}=\frac{G\left(M_{\substack{\text{missing}\\\text{dirt}}}-M_{\text{oil}}\right)}{r^2}=\frac{G}{r^2}\left(V_{\substack{\text{missing}\\\text{dirt}}}\rho_{\substack{\text{missing}\\\text{dirt}}}-V_{\text{oil}}\rho_{\text{oil}}\right)=\frac{GV_{\text{oil}}}{r^2}\left(\rho_{\substack{\text{missing}\\\text{dirt}}}-\rho_{\text{oil}}\right)=\frac{2}{10^7}g$$

$$V_{\text{oil}}=\frac{2}{10^7}g\frac{r^2}{G}\frac{1}{\left(\rho_{\substack{\text{missing}\\\text{dirt}}}-\rho_{\text{oil}}\right)}=\frac{2}{10^7}\left(9.80\text{m/s}^2\right)\frac{\left(2000\text{m}\right)^2}{\left(6.67\times10^{-11}\text{ N}\bullet\text{m}^2/\text{kg}^2\right)}\frac{1}{\left(3000-800\right)\text{kg/m}^2}$$

$$=5.34\times10^7\text{m}^3\approx\boxed{5\times10^7\text{m}^3}$$

$$r_{\text{deposit}}=\left(\frac{3V_{\text{oil}}}{4\pi}\right)^{1/3}=234\text{ m}\approx\boxed{200\text{m}}\quad;\ m_{\text{deposit}}=V_{\text{oil}}\rho_{\text{oil}}=4.27\times10^{10}\text{kg}\approx\boxed{4\times10^{10}\text{kg}}$$

68. The relationship between orbital speed and orbital radius for objects in orbit around the Earth is given in Example 6-6 as $v=\sqrt{GM_{\text{Earth}}/r}$. There are two orbital speeds involved – the one at the original radius, $v_0=\sqrt{GM_{\text{Earth}}/r_0}$, and the faster speed at the reduced radius, $v=\sqrt{GM_{\text{Earth}}/\left(r_0-\Delta r\right)}$.

(*a*) At the faster speed, 25,000 more meters will be traveled during the "catch-up" time, *t*. Note that $r_0=6.38\times10^6\text{m}+4\times10^5\text{m}=6.78\times10^6\text{m}$.

$$vt=v_0t+2.5\times10^4\text{m}\ \rightarrow\ \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t=\left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t+2.5\times10^4\text{m}\ \rightarrow$$

$$t=\frac{2.5\times10^4\,\mathrm{m}}{\sqrt{GM_{\text{Earth}}}}\left(\frac{1}{\sqrt{r_0-\Delta r}}-\frac{1}{\sqrt{r_0}}\right)^{-1}$$

$$=\frac{2.5\times10^4\,\mathrm{m}}{\sqrt{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\left(5.97\times10^{24}\,\mathrm{kg}\right)}}\left(\frac{1}{\sqrt{6.78\times10^6\,\mathrm{m}-1\times10^3\,\mathrm{m}}}-\frac{1}{\sqrt{6.78\times10^6\,\mathrm{m}}}\right)^{-1}$$

$$=4.42\times10^4\,\mathrm{s}\approx\boxed{12\,\mathrm{h}}$$

(*b*) Again, 25,000 more meters must be traveled at the faster speed in order to catch up to the satellite.

$$vt=v_0t+2.5\times10^4\,\mathrm{m}\ \rightarrow\ \left(\sqrt{G\frac{M_{\text{Earth}}}{r_0-\Delta r}}\right)t=\left(\sqrt{G\frac{M_{\text{Earth}}}{r_0}}\right)t+2.5\times10^4\,\mathrm{m}\ \rightarrow$$

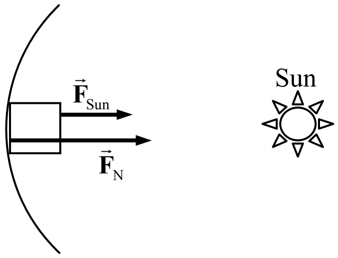
$$\sqrt{\frac{1}{r_0-\Delta r}}=\frac{1}{\sqrt{r_0}}+\frac{2.5\times10^4\,\mathrm{m}}{t\sqrt{GM_{\text{Earth}}}}\ \rightarrow\ \sqrt{r_0-\Delta r}=\left[\frac{1}{\sqrt{r_0}}+\frac{2.5\times10^4\,\mathrm{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-1}\ \rightarrow$$

$$\Delta r=r_0-\left[\frac{1}{\sqrt{r_0}}+\frac{2.5\times10^4\,\mathrm{m}}{t\sqrt{GM_{\text{Earth}}}}\right]^{-2}=\left(6.78\times10^6\,\mathrm{m}\right)$$

$$-\left[\frac{1}{\sqrt{\left(6.78\times10^6\,\mathrm{m}\right)}}+\frac{2.5\times10^4\,\mathrm{m}}{\left(25200\,\mathrm{s}\right)\sqrt{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\left(5.97\times10^{24}\,\mathrm{kg}\right)}}\right]^{-2}$$

$$=1755\ \mathrm{m}\approx\boxed{1.8\times10^3\,\mathrm{m}}$$

69. If the ring is to produce an apparent gravity equivalent to that of Earth, then the normal force of the ring on objects must be given by $F_{\mathrm{N}}=mg$. The Sun will also exert a force on objects on the ring. See the free-body diagram. Write Newton's second law for the object, with the fact that the acceleration is centripetal.



$$\sum F=F_{\mathrm{R}}=F_{\text{Sun}}+F_{\mathrm{N}}=mv^2/r$$

Substitute in the relationships that $v=2\pi r/T$, $F_{\mathrm{N}}=mg$, and $F_{\text{Sun}}=G\dfrac{M_{\text{Sun}}m}{r^2}$, and solve for the period of the rotation.

$$F_{Sun}+F_N=mv^2/r\ \rightarrow\ G\frac{M_{\text{Sun}}m}{r^2}+mg=\frac{4\pi^2mr}{T^2}\ \rightarrow\ G\frac{M_{\text{Sun}}}{r^2}+g=\frac{4\pi^2r}{T^2}$$

$$T=\sqrt{\frac{4\pi^2r}{G\frac{M_{\text{Sun}}}{r^2}+g}}=\sqrt{\frac{4\pi^2\left(1.50\times10^{11}\,\mathrm{m}\right)}{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\frac{\left(1.99\times10^{30}\,\mathrm{kg}\right)}{\left(1.50\times10^{11}\,\mathrm{m}\right)^2}+9.80\,\mathrm{m/s^2}}}$$

$$=7.77\times10^5\,\mathrm{s}=\boxed{8.99\ \mathrm{d}}$$

The force of the Sun is only about 1/1600 the size of the normal force. The force of the Sun could have been ignored in the calculation with no significant change in the result given above.

70. For an object to be apparently weightless would mean that the object would have a centripetal acceleration equal to *g*. This is really the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_C = v^2/R_{\text{Earth}}$, along with $v = 2\pi R_{\text{Earth}}/T$, and solve for *T*.

$$g = \frac{v^2}{R_{\text{Earth}}} = \frac{4\pi^2 R_{\text{Earth}}}{T^2} \rightarrow T = 2\pi\sqrt{\frac{R_{\text{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\,\text{m}}{9.80\,\text{m/s}^2}} = \boxed{5.07\times10^3\,\text{s}}\,(\sim 84.5\text{ min})$$

71. The speed of an object orbiting a mass is given in Example 6-6 as $v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$.

$$v_{\text{new}} = 1.5v \text{ and } v_{\text{new}} = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \rightarrow 1.5v = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \rightarrow 1.5\sqrt{\frac{GM_{\text{Sun}}}{r}} = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \rightarrow$$

$$r_{\text{new}} = \frac{r}{1.5^2} = \boxed{0.44r}$$

72. From the Venus data, the mass of the Sun can be determined by the following. Set the gravitational force on Venus equal to the centripetal force acting on Venus to make it orbit.

$$\frac{GM_{\text{Sun}}m_{\text{Venus}}}{r^2_{\substack{\text{Venus}\\\text{orbit}}}} = \frac{m_{\text{Venus}}v^2_{\text{Venus}}}{r_{\substack{\text{Venus}\\\text{orbit}}}} = \frac{m_{\text{Venus}}\left(\dfrac{2\pi r_{\substack{\text{Venus}\\\text{orbit}}}}{T_{\text{Venus}}}\right)^2}{r_{\substack{\text{Venus}\\\text{orbit}}}} = \frac{4\pi^2 m_{\text{Venus}} r_{\substack{\text{Venus}\\\text{orbit}}}}{GT^2_{\text{Venus}}} \rightarrow M_{\text{Sun}} = \frac{4\pi^2 r^3_{\substack{\text{Venus}\\\text{orbit}}}}{T^2_{\text{Venus}}}$$

Then likewise, for Callisto orbiting Jupiter, $M_{\text{Jupiter}} = \frac{4\pi^2 r^3_{\substack{\text{Callisto}\\\text{orbit}}}}{GT^2_{\text{Callisto}}}$, and for the Moon orbiting the Earth, $M_{\text{Earth}} = \frac{4\pi^2 r^3_{\substack{\text{Moon}\\\text{orbit}}}}{GT^2_{\text{Moon}}}$. To find the density ratios, take the mass ratios with the mass expressed as density times volume, and expressed as found above.

$$\frac{M_{\text{Jupiter}}}{M_{\text{Sun}}} = \frac{\rho_{\text{Jupiter}}\frac{4}{3}\pi r^3_{\text{Jupiter}}}{\rho_{\text{Sun}}\frac{4}{3}\pi r^3_{\text{Sun}}} = \frac{\dfrac{4\pi^2 r^3_{\substack{\text{Callisto}\\\text{orbit}}}}{GT^2_{\text{Callisto}}}}{\dfrac{4\pi^2 r^3_{\substack{\text{Venus}\\\text{orbit}}}}{GT^2_{\text{Venus}}}} \rightarrow$$

$$\frac{\rho_{\text{Jupiter}}}{\rho_{\text{Sun}}} = \frac{r^3_{\substack{\text{Callisto}\\\text{orbit}}}}{T^2_{\text{Callisto}}}\frac{T^2_{\text{Venus}}}{r^3_{\substack{\text{Venus}\\\text{orbit}}}}\frac{r^3_{\text{Sun}}}{r^3_{\text{Jupiter}}} = \frac{(0.01253)^3}{(16.69)^2}\frac{(224.7)^2}{(0.724)^3}\frac{1}{(0.0997)^3} = \boxed{0.948}$$

And likewise for the Earth–Sun combination:

$$\frac{\rho_{\text{Earth}}}{\rho_{\text{Sun}}} = \frac{r^3_{\substack{\text{Moon}\\\text{orbit}}}}{T^2_{\text{Moon}}}\frac{T^2_{\text{Venus}}}{r^3_{\substack{\text{Venus}\\\text{orbit}}}}\frac{r^3_{\text{Sun}}}{r^3_{\text{Earth}}} = \frac{(0.003069)^3}{(27.32)^2}\frac{(224.7)^2}{(0.724)^3}\frac{1}{(0.0109)^3} = \boxed{3.98}$$

$\boxed{73.}$ The initial force of 120 N can be represented as $F_{\text{grav}} = \frac{GM_{\text{planet}}}{r^2} = 120\,\text{N}$.

(*a*) The new radius is 1.5 times the original radius.

$$F_{\substack{\text{new}\\\text{radius}}} = \frac{GM_{\text{planet}}}{r_{\text{new}}^2} = \frac{GM_{\text{planet}}}{(1.5r)^2} = \frac{GM_{\text{planet}}}{2.25r^2} = \frac{1}{2.25}(120\,\text{N}) = \boxed{53\,\text{N}}$$

(*b*) With the larger radius, the period is $T = 7200$ seconds. As found in Example 6-6, orbit speed can be calculated by $v = \sqrt{\frac{GM}{r}}$.

$$v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T} \quad \rightarrow \quad M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(3.0\times10^7\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)(7200\,\text{s})^2} = \boxed{3.1\times10^{26}\,\text{kg}}$$

74. The density of the sphere is uniform, and is given by $\rho = \frac{M}{\frac{4}{3}\pi r^3}$. The mass that was removed to make the cavity is $M_{\text{cavity}} = \pi V_{\text{cavity}} = \frac{M}{\frac{4}{3}\pi r^3}\left(\frac{4}{3}\pi\left(r/2\right)^3\right) = \frac{1}{8}M$. The net force on the point mass can be found by finding the force due to the entire sphere, and then subtracting the force caused by the cavity alone.

$$F_{\text{net}} = F_{\text{sphere}} - F_{\text{cavity}} = \frac{GMm}{d^2} - \frac{G\left(\frac{1}{8}M\right)m}{\left(d - r/2\right)^2} = GMm\left(\frac{1}{d^2} - \frac{1}{8\left(d - r/2\right)^2}\right)$$

$$= \boxed{\frac{GMm}{d^2}\left(1 - \frac{1}{8\left(1 - r/2d\right)^2}\right)}$$

75. (*a*) We use the law of universal gravitation to express the force for each mass *m*. One mass is "near" the Moon, and so the distance from that mass to the center of the Moon is $R_{\text{EM}} - R_{\text{E}}$. The other mass is "far" from the Moon, and so the distance from that mass to the center of the Moon is $R_{\text{EM}} + R_{\text{E}}$.

$$F_{\substack{\text{near}\\\text{Moon}}} = \frac{GM_{\text{Moon}}m}{\left(R_{\text{EM}} - R_{\text{E}}\right)^2} \qquad F_{\substack{\text{far}\\\text{Moon}}} = \frac{GM_{\text{Moon}}m}{\left(R_{\text{EM}} + R_{\text{E}}\right)^2}$$

$$\left(\frac{F_{\text{near}}}{F_{\text{far}}}\right)_{\text{Moon}} = \frac{\dfrac{GM_{\text{Moon}}m}{\left(R_{\text{EM}} - R_{\text{E}}\right)^2}}{\dfrac{GM_{\text{Moon}}m}{\left(R_{\text{EM}} + R_{\text{E}}\right)^2}} = \left(\frac{R_{\text{EM}} + R_{\text{E}}}{R_{\text{EM}} - R_{\text{E}}}\right)^2 = \left(\frac{3.84\times10^8\,\text{m} + 6.38\times10^6\,\text{m}}{3.84\times10^8\,\text{m} - 6.38\times10^6\,\text{m}}\right)^2 = \boxed{1.0687}$$

(*b*) We use a similar analysis to part (*a*).

$$F_{\substack{\text{near}\\\text{Sun}}} = \frac{GM_{\text{Sun}}m}{\left(r_{\text{ES}} - r_{\text{E}}\right)^2} \qquad F_{\substack{\text{far}\\\text{Moon}}} = \frac{GM_{\text{Sun}}m}{\left(r_{\text{ES}} + r_{\text{E}}\right)^2}$$

$$\left(\frac{F_{\text{near}}}{F_{\text{far}}}\right)_{\text{Sun}} = \frac{\dfrac{GM_{\text{Sun}}m}{\left(r_{\text{ES}} - r_{\text{E}}\right)^2}}{\dfrac{GM_{\text{Sun}}m}{\left(r_{\text{ES}} + r_{\text{E}}\right)^2}} = \left(\frac{r_{\text{EM}} + r_{\text{E}}}{r_{\text{EM}} - r_{\text{E}}}\right)^2 = \left(\frac{1.496\times10^{11}\,\text{m} + 6.38\times10^6\,\text{m}}{1.496\times10^{11}\,\text{m} - 6.38\times10^6\,\text{m}}\right)^2 = \boxed{1.000171}$$

(*c*) For the average gravitational force on the large masses, we use the distance between their centers.

$$F_{\text{Sun}}=\frac{GM_{\text{Sun}}M_{\text{Earth}}}{r_{\text{ES}}^2}\qquad F_{\text{Moon}}=\frac{GM_{\text{Moon}}M_{\text{Earth}}}{r_{\text{EM}}^2}$$

$$\frac{F_{\text{Sun}}}{F_{\text{Moon}}}=\frac{\dfrac{GM_{\text{Sun}}M_{\text{Earth}}}{r_{\text{ES}}^2}}{\dfrac{GM_{\text{Moon}}M_{\text{Earth}}}{r_{\text{EM}}^2}}=\frac{M_{\text{Sun}}}{r_{\text{ES}}^2}\frac{r_{\text{EM}}^2}{M_{\text{Moon}}}=\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.496\times10^{11}\,\text{m}\right)^2}\frac{\left(3.84\times10^{8}\,\text{m}\right)^2}{\left(7.35\times10^{22}\,\text{kg}\right)}=\boxed{178}$$

(*d*) Apply the expression for ΔF as given in the statement of the problem.

$$\frac{\Delta F_{\text{Moon}}}{\Delta F_{\text{Sun}}}=\frac{F_{\text{Moon}}\left(\dfrac{F_{\text{near}}}{F_{\text{far}}}-1\right)_{\text{Moon}}}{F_{\text{Sun}}\left(\dfrac{F_{\text{near}}}{F_{\text{far}}}-1\right)_{\text{Sun}}}=\frac{F_{\text{Moon}}}{F_{\text{Sun}}}\frac{\left(\dfrac{F_{\text{near}}}{F_{\text{far}}}-1\right)_{\text{Moon}}}{\left(\dfrac{F_{\text{near}}}{F_{\text{far}}}-1\right)_{\text{Sun}}}=\frac{1}{178}\frac{(1.0687-1)}{(1.000171-1)}=\boxed{2.3}$$

76. The acceleration is found from the law of universal gravitation. Using the chain rule, a relationship between the acceleration expression and the velocity can be found which is integrated to find the velocity as a function of distance. The outward radial direction is taken to be positive, so the acceleration is manifestly negative.

$$F=ma=-G\frac{m_{\text{E}}m}{r^2}\;\rightarrow\;a=-\frac{Gm_{\text{E}}}{r^2}=\frac{dv}{dt}=\frac{dv}{dr}\frac{dr}{dt}=v\frac{dv}{dr}\;\rightarrow\;-\frac{Gm_{\text{E}}}{r^2}=v\frac{dv}{dr}\;\rightarrow$$

$$-Gm_{\text{E}}\frac{dr}{r^2}=vdv\;\rightarrow\;-Gm_{\text{E}}\int_{2r_{\text{E}}}^{r_{\text{E}}}\frac{dr}{r^2}=\int_0^{v_f}vdv\;\rightarrow\;\left[\frac{Gm_{\text{E}}}{r}\right]_{2r_{\text{E}}}^{r_{\text{E}}}=\tfrac{1}{2}v_f^2\;\rightarrow$$

$$\frac{Gm_{\text{E}}}{r_{\text{E}}}-\frac{Gm_{\text{E}}}{2r_{\text{E}}}=\tfrac{1}{2}v_f^2\;\rightarrow\;v_f=\pm\sqrt{\frac{Gm_{\text{E}}}{r_{\text{E}}}}\;\rightarrow\;v_f=\boxed{-\sqrt{\frac{Gm_{\text{E}}}{r_{\text{E}}}}}$$

The negative sign is chosen because the object is moving towards the center of the Earth, and the outward radial direction is positive.

77. Equate the force of gravity on a mass *m* at the surface of the Earth as expressed by the acceleration due to gravity to that as expressed by Newton's law of universal gravitation.

$$mg=\frac{GM_{\text{Earth}}m}{R_{\text{Earth}}^2}\;\rightarrow\;G=\frac{gR_{\text{Earth}}^2}{M_{\text{Earth}}}=\frac{gR_{\text{Earth}}^2}{\rho_{\text{Earth}}\frac{4}{3}\pi R_{\text{Earth}}^3}=\frac{3g}{4\pi\rho R_{\text{Earth}}}=\frac{3g}{4\pi\rho\dfrac{C_{\text{Earth}}}{2\pi}}=\frac{3g}{2\rho C_{\text{Earth}}}$$

$$=\frac{3\left(10\,\text{m/s}^2\right)}{2\left(3000\,\text{kg/m}^3\right)\left(4\times10^7\,\text{m}\right)}=1.25\times10^{-10}\,\frac{\text{m}^3}{\text{kg•s}^2}\approx\boxed{1\times10^{-10}\,\frac{\text{m}^3}{\text{kg•s}^2}}$$

This is roughly twice the size of the accepted value of *G*.

78. (*a*) From Example 6-6, the speed of an object in a circular orbit of radius r about mass M is $v=\sqrt{\frac{GM}{r}}$. Use that relationship along with the definition of density to find the speed.

$$v=\sqrt{\frac{GM}{r}}\;\rightarrow\;v^2=\frac{GM}{r}=\frac{G\rho\frac{4}{3}\pi r^3}{r}\;\rightarrow$$

$$r=\sqrt{\frac{3v^2}{4\pi G\rho}}=\sqrt{\frac{3\left(22\,\text{m/s}\right)^2}{4\pi\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(2700\,\text{kg/m}^3\right)}}=25330\,\text{m}\approx\boxed{2.5\times10^4\,\text{m}}$$

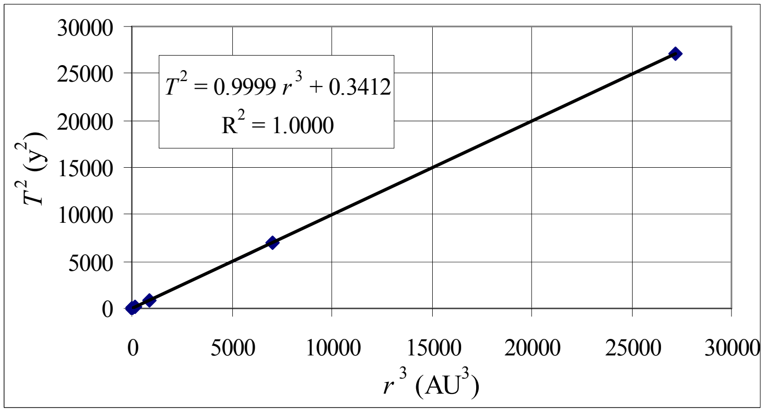
(*b*) $$v=\frac{2\pi r}{T}\rightarrow T=\frac{2\pi r}{v}=\frac{2\pi\left(25330\,\text{m}\right)}{22\,\text{m/s}}=7234\,\text{s}\approx\boxed{2.0\,\text{h}}$$

$\boxed{79.}$ (*a*) The graph is shown.

(*b*) From the graph, we get this equation.

$$T^2=0.9999r^3+0.3412$$

$$r=\left(\frac{T^2-0.3412}{0.9999}\right)^{1/3}$$



$$r\left(T=247.7\,\text{y}\right)=\left(\frac{247.7^2-0.3412}{0.9999}\right)^{1/3}=\boxed{39.44\text{ AU}}$$

A quoted value for the means distance of Pluto is 39.47 AU. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH06.XLS," on tab "Problem 6.79."

CHAPTER 7: Work and Energy

Responses to Questions

1. "Work" as used in everyday language generally means "energy expended," which is similar to the way "work" is defined in physics. However, in everyday language, "work" can involve mental or physical energy expended, and is not necessarily connected with displacement, as it is in physics. So a student could say she "worked" hard carrying boxes up the stairs to her dorm room (similar in meaning to the physics usage), or that she "worked" hard on a problem set (different in meaning from the physics usage).

2. Yes, she is doing work. The work done by her and the work done on her by the river are opposite in sign, so they cancel and she does not move with respect to the shore. When she stops swimming, the river continues to do work on her, so she floats downstream.

3. No, not if the object is moving in a circle. Work is the product of force and the displacement in the direction of the force. Therefore, a centripetal force, which is perpendicular to the direction of motion, cannot do work on an object moving in a circle.

4. You are doing no work on the wall. Your muscles are using energy generated by the cells in your body and producing byproducts which make you feel fatigued.

5. No. The magnitudes of the vectors and the angle between them are the relevant quantities, and these do not depend on the choice of coordinate system.

6. Yes. A dot product can be negative if corresponding components of the vectors involved point in opposite directions. For example, if one vector points along the positive *x*-axis, and the other along the negative *x*-axis, the angle between the vectors is 180º. Cos 180º = –1, and so the dot product of the two vectors will be negative.

7. No. For instance, imagine $\vec{\mathbf{C}}$ as a vector along the +*x* axis. $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ could be two vectors with the same magnitude and the same *x*-component but with *y*-components in opposite directions, so that one is in quadrant I and the other in quadrant IV. Then $\vec{\mathbf{A}}\bullet\vec{\mathbf{C}} = \vec{\mathbf{B}}\bullet\vec{\mathbf{C}}$ even though $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are different vectors.

8. No. The dot product of two vectors is always a scalar, with only a magnitude.

9. Yes. The normal force is the force perpendicular to the surface an object is resting on. If the object moves with a component of its displacement perpendicular to this surface, the normal force will do work. For instance, when you jump, the normal force does work on you in accelerating you vertically.

10. (*a*) If the force is the same, then $F = k_1x_1 = k_2x_2$, so $x_2 = k_1x_1/k_2$. The work done on spring 1 will be $W_1 = \frac{1}{2}k_1x_1^2$. The work done on spring 2 will be $W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2\left(k_1^2x_1^2/k_2^2\right) = W_1\left(k_1/k_2\right)$. Since $k_1 > k_2$, $W_2 > W_1$, so more work is done on spring 2.

    (*b*) If the displacement is the same, then $W_1 = \frac{1}{2}k_1x^2$ and $W_2 = \frac{1}{2}k_2x^2$. Since $k_1 > k_2$, $W_1 > W_2$, so more work is done on spring 1.

11. The kinetic energy increases by a factor of 9, since the kinetic energy is proportional to the square of the speed.

12. Until the $x = 0$ point, the spring has a positive acceleration and is accelerating the block, and therefore will remain in contact with it. After the $x = 0$ point, the spring begins to slow down, but (in the absence of friction), the block will continue to move with its maximum speed and will therefore move faster than the spring and will separate from it.

$\boxed{13.}$ The bullet with the smaller mass has a speed which is greater by a factor of $\sqrt{2} \approx 1.4$. Since their kinetic energies are equal, then $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$. If $m_2 = 2m_1$, then $\frac{1}{2}m_1v_1^2 = \frac{1}{2}\cdot 2m_1v_2^2$, so $v_1 = \sqrt{2}v_2$. They can both do the same amount of work, however, since their kinetic energies are the same. (See the work-energy principle.)

14. The net work done on a particle and the change in the kinetic energy are independent of the choice of reference frames only if the reference frames are at rest with respect to each other. The work-energy principle is also independent of the choice of reference frames if the frames are at rest with respect to each other.

If the reference frames are in relative motion, the net work done on a particle, the kinetic energy, and the change in the kinetic energy all will be different in different frames. The work-energy theorem will still be true.

15. The speed at point C will be less than twice the speed at point B. The force is constant and the displacements are the same, so the same *work* is done on the block from A to B as from B to C. Since there is no friction, the same work results in the same *change in kinetic energy*. But kinetic energy depends on the square of the speed, so the speed at point C will be greater than the speed at point B by a factor of $\sqrt{2}$, not a factor of 2.

Solutions to Problems

$\boxed{1.}$ The force and the displacement are both downwards, so the angle between them is 0°. Use Eq. 7-1.

$$W_G = mgd\cos\theta = (280\,\text{kg})(9.80\,\text{m/s}^2)(2.80\,\text{m})\cos 0^\circ = \boxed{7.7\times10^3\,\text{J}}$$

2. The rock will rise until gravity does –80.0 J of work on the rock. The displacement is upwards, but the force is downwards, so the angle between them is 180°. Use Eq. 7-1.

$$W_G = mgd\cos\theta \quad\rightarrow\quad d = \frac{W_G}{\text{mg}\cos\theta} = \frac{-80.0\,\text{J}}{(1.85\,\text{kg})(9.80\,\text{m/s}^2)(-1)} = \boxed{4.41\,\text{m}}$$

3. The minimum force required to lift the firefighter is equal to his weight. The force and the displacement are both upwards, so the angle between them is 0°. Use Eq. 7-1.

$$W_{\text{climb}} = F_{\text{climb}}d\cos\theta = mgd\cos\theta = (75.0\text{ kg})(9.80\,\text{m/s}^2)(20.0\text{m})\cos 0^\circ = \boxed{1.47\times10^4\,\text{J}}$$

4. The maximum amount of work would be the work done by gravity. Both the force and the displacement are downwards, so the angle between them is 0°. Use Eq. 7-1.

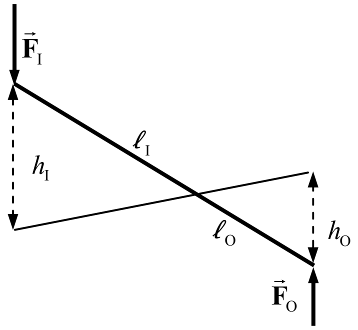
$$W_G = mgd\cos\theta = (2.0\,\text{kg})(9.80\,\text{m/s}^2)(0.50\,\text{m})\cos 0^\circ = \boxed{9.8\,\text{J}}$$

This is a small amount of energy. If the person adds a larger force to the hammer during the fall, then the hammer will have a larger amount of energy to give to the nail.

5. The distance over which the force acts is the area to be mowed divided by the width of the mower. The force is parallel to the displacement, so the angle between them is 0°. Use Eq. 7-1.

$$W = Fd\cos\theta = F\frac{A}{w}\cos\theta = (15\,\text{N})\frac{200\,\text{m}^2}{0.50\,\text{m}} = \boxed{6000\,\text{J}}$$

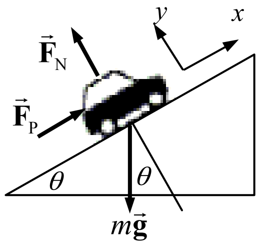
6. Consider the diagram shown. If we assume that the man pushes straight down on the end of the lever, then the work done by the man (the "input" work) is given by $W_\text{I} = F_\text{I}h_\text{I}$. The object moves a shorter distance, as seen from the diagram, and so $W_\text{O} = F_\text{O}h_\text{O}$. Equate the two amounts of work.



$$W_\text{O} = W_\text{I} \quad\rightarrow\quad F_\text{O}h_\text{O} = F_\text{I}h_\text{I} \quad\rightarrow\quad \frac{F_\text{O}}{F_\text{I}} = \frac{h_\text{I}}{h_\text{O}}$$

But by similar triangles, we see that $\dfrac{h_\text{I}}{h_\text{O}} = \dfrac{\ell_\text{I}}{\ell_\text{O}}$, and so $\boxed{\dfrac{F_\text{O}}{F_\text{I}} = \dfrac{\ell_\text{I}}{\ell_\text{O}}}$.

$\boxed{7.}$ Draw a free-body diagram of the car on the incline. The minimum work will occur when the car is moved at a constant velocity. Write Newton's second law in the *x* direction, noting that the car is unaccelerated. Only the forces parallel to the plane do work.



$$\sum F_x = F_\text{P} - mg\sin\theta = 0 \quad\rightarrow\quad F_\text{P} = mg\sin\theta$$

The work done by $\vec{\mathbf{F}}_\text{P}$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_\text{P}$) is given by Eq. 7-1.

$$W_\text{P} = F_\text{P}d\cos 0^\circ = mgd\sin\theta = (950\text{ kg})\left(9.80\,\text{m/s}^2\right)(310\,\text{m})\sin 9.0^\circ = \boxed{4.5\times10^5\,\text{J}}$$

8. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance *d*, by a force equal to its weight, *mg*. The force and the displacement are in the same direction, so the work is *mgd*. The third book will need to be moved a distance of 2*d* by the same size force, so the work is 2*mgd*. This continues through all seven books, with each needing to be raised by an additional amount of *d* by a force of *mg*. The total work done is

$$\begin{aligned} W &= mgd + 2mgd + 3mgd + 4mgd + 5mgd + 6mgd + 7mgd \\ &= 28mgd = 28(1.8\text{ kg})\left(9.8\,\text{m/s}^2\right)(0.040\text{ m}) = \boxed{2.0\times10^1\,\text{J}} \end{aligned}$$

9. Since the acceleration of the box is constant, use Eq. 2-12b to find the distance moved. Assume that the box starts from rest.

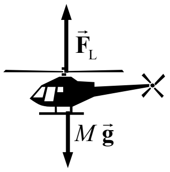
$$d = x - x_0 = v_0t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}\left(2.0\,\text{m/s}^2\right)(7.0\,\text{s})^2 = 49\,\text{m}$$

Then the work done in moving the crate is found using Eq. 7-1.

$$W = Fd\cos 0^\circ = mad = (6.0\,\text{kg})\left(2.0\,\text{m/s}^2\right)(49\,\text{m}) = \boxed{590\,\text{J}}$$

10. (*a*) Write Newton's second law for the vertical direction, with up as positive.

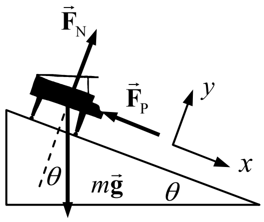
$$\sum F_y = F_{\text{L}} - Mg = Ma = M\left(0.10g\right) \quad \rightarrow \quad F_{\text{L}} = \boxed{1.10\,Mg}$$



(*b*) The work done by the lifting force in lifting the helicopter a vertical distance h is given by Eq. 7-1. The lifting force and the displacement are in the same direction.

$$W_{\text{L}} = F_{\text{L}} h \cos 0° = \boxed{1.10\,Mgh}\,.$$

11. The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_{\text{P}}$ is the force of the man pushing on the piano.



(*a*) Write Newton's second law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{P}} = 0 \quad \rightarrow$$

$$F_{\text{P}} = mg\sin\theta = mg\sin\theta$$

$$= \left(380\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(\sin 27°\right) = 1691\,\text{N} \approx \boxed{1700\,\text{N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_{\text{P}}$. The angle between $\vec{\mathbf{F}}_{\text{P}}$ and the direction of motion is 180°. Use Eq. 7-1.

$$W_{\text{P}} = F_{\text{P}} d\cos 180° = -\left(1691\,\text{N}\right)\left(3.9\,\text{m}\right) = -6595\,\text{J} \approx \boxed{-6600\,\text{J}}\,.$$

(*c*) The angle between the force of gravity and the direction of motion is 63°. Calculate the work done by gravity.

$$W_G = F_G d\cos 63° = mgd\cos 63° = \left(380\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3.9\,\text{m}\right)\cos 63°$$

$$= 6594\,\text{N} \approx \boxed{6600\,\text{J}}$$

(*d*) Since the piano is not accelerating, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the two work amounts calculated.

$$W_{\text{net}} = W_{\text{P}} + W_{\text{G}} = -6.6\times10^3\,\text{J} + 6.6\times10^3\,\text{J} = \boxed{0\,\text{J}}$$

12. (*a*) The motor must exert a force equal and opposite to the force of gravity on the gondola and passengers in order to lift it. The force is in the same direction as the displacement. Use Eq. 7-1 to calculate the work.

$$W_{\text{motor}} = F_{\text{motor}} d\cos 0° = mgd = \left(2250\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3345\,\text{m} - 2150\,\text{m}\right) = \boxed{2.63\times10^7\,\text{J}}$$

(*b*) Gravity would do the exact opposite amount of work as the motor, because the force and displacement are of the same magnitude, but the angle between the gravity force and the displacement is 180°.

$$W_{\text{G}} = F_{\text{G}} d\cos 180° = -mgd = -\left(2250\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3345\,\text{m} - 2150\,\text{m}\right) = \boxed{-2.63\times10^7\,\text{J}}$$

(*c*) If the motor is generating 10% more work, than it must be able to exert a force that is 10% larger than the force of gravity. The net force then would be as follows, with up the positive direction.

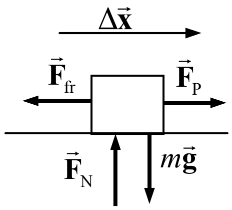
$$F_{\text{net}} = F_{\text{motor}} - F_{\text{G}} = 1.1mg - mg = 0.1mg = ma \quad \rightarrow \quad a = 0.1g = \boxed{0.98\,\text{m/s}^2}$$

13. (*a*) The gases exert a force on the jet in the same direction as the displacement of the jet. From the graph we see the displacement of the jet during launch is 85 m. Use Eq. 7-1 to find the work.

$$W_{\text{gas}} = F_{\text{gas}} d \cos 0° = \left(130\times10^3\,\text{N}\right)\left(85\,\text{m}\right) = \boxed{1.1\times10^7\,\text{J}}$$

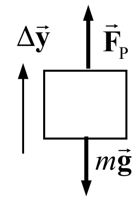
(*b*) The work done by catapult is the area underneath the graph in Figure 7-22. That area is a trapezoid.

$$W_{\text{catapult}} = \tfrac{1}{2}\left(1100\times10^3\,\text{N} + 65\times10^3\,\text{N}\right)\left(85\,\text{m}\right) = \boxed{5.0\times10^7\,\text{J}}$$



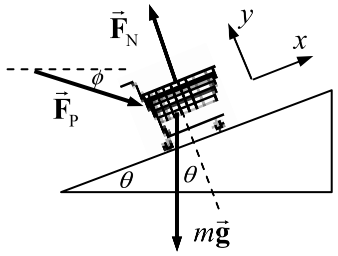
14. (*a*) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_{\text{P}} = F_{\text{fr}} = 230\,\text{N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is 0°. Use Eq. 7-1.

$$W_{\text{P}} = F_{\text{P}} d \cos 0° = \left(230\ \text{N}\right)\left(4.0\ \text{m}\right)\left(1\right) = \boxed{920\,\text{J}}$$



(*b*) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.

$$W_{\text{P}} = F_{\text{P}} d \cos 0^{\circ} = mgd = \left(2200\,\text{N}\right)\left(4.0\,\text{m}\right) = \boxed{8800\,\text{J}}$$



15. Consider a free-body diagram for the grocery cart being pushed up the ramp. If the cart is not accelerating, then the net force is 0 in all directions. This can be used to find the size of the pushing force. The angles are $\phi = 17°$ and $\theta = 12°$. The displacement is in the *x*-direction. The work done by the normal force is 0 since the normal force is perpendicular to the displacement. The angle between the force of gravity and the displacement is $90° + \theta = 102°$. The angle between the normal force and the displacement is $90°$. The angle between the pushing force and the displacement is total work done is $\phi + \theta = 29°$.

$$\sum F_x = F_P \cos\left(\phi+\theta\right) - mg\sin\theta = 0 \quad \rightarrow \quad F_P = \frac{mg\sin\theta}{\cos\left(\phi+\theta\right)}$$

$$W_{mg} = mgd\cos 112° = \left(16\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(15\,\text{m}\right)\cos 102° = \boxed{-490\,\text{J}}$$

$$W_{\text{normal}} = F_{\text{N}} d \cos 90° = \boxed{0}$$

$$W_P = F_{\text{P}} d\cos 29° = \left(\frac{mg\sin 12°}{\cos 29°}\right) d\cos 29° = mgd\sin 12°$$

$$= \left(16\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(15\,\text{m}\right)\sin 12° = \boxed{490\,\text{J}}$$

16. Use Eq. 7.4 to calculate the dot product.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z = \left(2.0x^2\right)\left(11.0\right) + \left(-4.0x\right)\left(2.5x\right) + \left(5.0\right)\left(0\right) = 22x^2 - 10x^2$$

$$= \boxed{12x^2}$$

17. Use Eq. 7.4 to calculate the dot product. Note that $\hat{\mathbf{i}} = 1\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, $\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, and $\hat{\mathbf{k}} = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 1\hat{\mathbf{k}}$.

$$\hat{\mathbf{i}}\bullet\vec{\mathbf{V}} = (1)V_x + (0)V_y + (0)V_z = V_x \qquad \hat{\mathbf{j}}\bullet\vec{\mathbf{V}} = (0)V_x + (1)V_y + (0)V_z = V_y$$

$$\hat{\mathbf{k}}\bullet\vec{\mathbf{V}} = (0)V_x + (0)V_y + (1)V_z = V_z$$

18. Use Eq. 7.4 and Eq. 7.2 to calculate the dot product, and then solve for the angle.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z = (6.8)(8.2) + (-3.4)(2.3) + (-6.2)(-7.0) = 91.34$$

$$A = \sqrt{(6.8^2) + (-3.4)^2 + (-6.2)^2} = 9.81 \qquad B = \sqrt{(8.2^2) + (2.3)^2 + (-7.0)^2} = 11.0$$

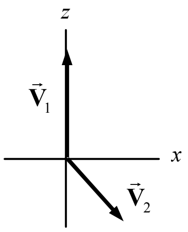
$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta \;\rightarrow\; \theta = \cos^{-1}\frac{\vec{\mathbf{A}}\bullet\vec{\mathbf{B}}}{AB} = \cos^{-1}\frac{91.34}{(9.81)(11.0)} = \boxed{32^\circ}$$

$\boxed{19.}$ We utilize the fact that if $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, then $-\vec{\mathbf{B}} = (-B_x)\hat{\mathbf{i}} + (-B_y)\hat{\mathbf{j}} + (-B_z)\hat{\mathbf{k}}$.

$$\begin{aligned}\vec{\mathbf{A}}\bullet(-\vec{\mathbf{B}}) &= A_x(-B_x) + A_y(-B_y) + A_z(-B_z)\\ &= (-A_x)(B_x) + (-A_y)(B_y) + (-A_z)(B_z) = -\vec{\mathbf{A}}\bullet\vec{\mathbf{B}}\end{aligned}$$

20. See the diagram to visualize the geometric relationship between the two vectors. The angle between the two vectors is 138°.

$$\vec{\mathbf{V}}_1\bullet\vec{\mathbf{V}}_2 = V_1V_2\cos\theta = (75)(58)\cos138^\circ = \boxed{-3200}$$



21. If $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}}$, then $\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = 0$. Use this to find $\vec{\mathbf{B}}$.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_xB_x + A_yB_y = (3.0)B_x + (1.5)B_y = 0 \;\rightarrow\; B_y = -2.0B_x$$

Any vector $\vec{\mathbf{B}}$ that satisfies $B_y = -2.0B_x$ will be perpendicular to $\vec{\mathbf{A}}$. For example, $\vec{\mathbf{B}} = \boxed{1.5\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}}}$.

22. Both vectors are in the first quadrant, so to find the angle between them, we can simply subtract the angles of each of them.

$$\vec{\mathbf{F}} = (2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})\,\text{N} \;\rightarrow\; F = \sqrt{(2.0\,\text{N})^2 + (4.0\,\text{N})^2} = (\sqrt{20})\,\text{N} \;;\; \phi_F = \tan^{-1}\frac{4.0}{2.0} = \tan^{-1}2.0$$

$$\vec{\mathbf{d}} = (1.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}})\,\text{m} \;\rightarrow\; d = \sqrt{(1.0\,\text{m})^2 + (5.0\,\text{m})^2} = (\sqrt{26})\,\text{m} \;;\; \phi_d = \tan^{-1}\frac{5.0}{1.0} = \tan^{-1}5.0$$

(*a*) $W = Fd\cos\theta = \left[(\sqrt{20})\,\text{N}\right]\left[(\sqrt{26})\,\text{m}\right]\cos\left[\tan^{-1}5.0 - \tan^{-1}2.0\right] = \boxed{22\,\text{J}}$

(*b*) $W = F_xd_x + F_yd_y = (2.0\,\text{N})(1.0\,\text{m}) + (4.0\,\text{N})(5.0\,\text{m}) = \boxed{22\,\text{J}}$

23. (*a*) $\vec{\mathbf{A}}\bullet(\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (9.0\hat{\mathbf{i}} - 8.5\hat{\mathbf{j}})\bullet\left[(-8.0\hat{\mathbf{i}} + 7.1\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}}) + (6.8\hat{\mathbf{i}} - 9.2\hat{\mathbf{j}})\right]$

$$= (9.0\hat{\mathbf{i}} - 8.5\hat{\mathbf{j}})\bullet(-1.2\hat{\mathbf{i}} - 2.1\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}}) = (9.0)(-1.2) + (-8.5)(-2.1) + (0)(4.2) = 7.05 \approx \boxed{7.1}$$

(*b*) $(\vec{\mathbf{A}} + \vec{\mathbf{C}})\bullet\vec{\mathbf{B}} = \left[(9.0\hat{\mathbf{i}} - 8.5\hat{\mathbf{j}}) + (6.8\hat{\mathbf{i}} - 9.2\hat{\mathbf{j}})\right]\bullet(-8.0\hat{\mathbf{i}} + 7.1\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}})$

$$= \left(15.8\hat{\mathbf{i}} - 17.7\hat{\mathbf{j}}\right)\bullet\left(-8.0\hat{\mathbf{i}} + 7.1\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}}\right) = (15.8)(-8.0) + (-17.7)(7.1) + (0)(4.2)$$

$$= -252 \approx \boxed{-250}$$

(*c*) $$\left(\vec{\mathbf{B}} + \vec{\mathbf{A}}\right)\bullet\vec{\mathbf{C}} = \left[\left(-8.0\hat{\mathbf{i}} + 7.1\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}}\right) + \left(9.0\hat{\mathbf{i}} - 8.5\hat{\mathbf{j}}\right)\right]\bullet\left(6.8\hat{\mathbf{i}} - 9.2\hat{\mathbf{j}}\right)$$

$$= \left(1.0\hat{\mathbf{i}} - 1.4\hat{\mathbf{j}} + 4.2\hat{\mathbf{k}}\right)\bullet\left(6.8\hat{\mathbf{i}} - 9.2\hat{\mathbf{j}}\right) = (1.0)(6.8) + (-1.4)(-9.2) + (4.2)(0)$$

$$= 19.68 \approx \boxed{20}$$

24. We assume that the dot product of two vectors is given by Eq. 7-2. Note that for two unit vectors, this gives the following.

$$\hat{\mathbf{i}}\bullet\hat{\mathbf{i}} = (1)(1)\cos 0° = 1 = \hat{\mathbf{j}}\bullet\hat{\mathbf{j}} = \hat{\mathbf{k}}\bullet\hat{\mathbf{k}} \text{ and } \hat{\mathbf{i}}\bullet\hat{\mathbf{j}} = (1)(1)\cos 90° = 0 = \hat{\mathbf{i}}\bullet\hat{\mathbf{k}} = \hat{\mathbf{j}}\bullet\hat{\mathbf{i}} = \hat{\mathbf{j}}\bullet\hat{\mathbf{k}} = \hat{\mathbf{k}}\bullet\hat{\mathbf{i}} = \hat{\mathbf{k}}\bullet\hat{\mathbf{j}}$$

Apply these results to $\vec{\mathbf{A}}\bullet\vec{\mathbf{B}}$.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\bullet\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$= A_xB_x\hat{\mathbf{i}}\bullet\hat{\mathbf{i}} + A_xB_y\hat{\mathbf{i}}\bullet\hat{\mathbf{j}} + A_xB_z\hat{\mathbf{i}}\bullet\hat{\mathbf{k}} + A_yB_x\hat{\mathbf{j}}\bullet\hat{\mathbf{i}} + A_yB_y\hat{\mathbf{j}}\bullet\hat{\mathbf{j}} + A_yB_z\hat{\mathbf{j}}\bullet\hat{\mathbf{k}} + A_zB_x\hat{\mathbf{k}}\bullet\hat{\mathbf{i}} + A_zB_y\hat{\mathbf{k}}\bullet\hat{\mathbf{j}} + A_zB_z\hat{\mathbf{k}}\bullet\hat{\mathbf{k}}$$

$$= A_xB_x(1) + A_xB_y(0) + A_xB_z(0) + A_yB_x(0) + A_yB_y(1)$$
$$+ A_yB_z(0) + A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

$$= A_xB_x + A_yB_y + A_zB_z$$

$\boxed{25.}$ If $\vec{\mathbf{C}}$ is perpendicular to $\vec{\mathbf{B}}$, then $\vec{\mathbf{C}}\bullet\vec{\mathbf{B}} = 0$. Use this along with the value of $\vec{\mathbf{C}}\bullet\vec{\mathbf{A}}$ to find $\vec{\mathbf{C}}$. We also know that $\vec{\mathbf{C}}$ has no *z*-component.

$$\vec{\mathbf{C}} = C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} \;;\; \vec{\mathbf{C}}\bullet\vec{\mathbf{B}} = C_xB_x + C_yB_y = 0 \;;\; \vec{\mathbf{C}}\bullet\vec{\mathbf{A}} = C_xA_x + C_yA_y = 20.0 \;\rightarrow$$

$$9.6C_x + 6.7C_y = 0 \;;\; -4.8C_x + 6.8C_y = 20.0$$

This set of two equations in two unknowns can be solved for the components of $\vec{\mathbf{C}}$.

$$9.6C_x + 6.7C_y = 0 \;;\; -4.8C_x + 6.8C_y = 20.0 \;\rightarrow\; C_x = -1.4 \,,\, C_y = 2.0 \;\rightarrow$$

$$\boxed{\vec{\mathbf{C}} = -1.4\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}}$$

26. We are given that the magnitudes of the two vectors are the same, so $A_x^2 + A_y^2 + A_z^2 = B_x^2 + B_y^2 + B_z^2$. If the sum and difference vectors are perpendicular, their dot product must be zero.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = (A_x - B_x)\hat{\mathbf{i}} + (A_y - B_y)\hat{\mathbf{j}} + (A_z - B_z)\hat{\mathbf{k}}$$

$$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right)\bullet\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = (A_x + B_x)(A_x - B_x) + (A_y + B_y)(A_y - B_y) + (A_z + B_z)(A_z - B_z)$$

$$= A_x^2 - B_x^2 + A_y^2 - B_y^2 + A_z^2 - B_z^2 = \left(A_x^2 + A_y^2 + A_z^2\right) - \left(B_x^2 + B_y^2 + B_z^2\right) = 0$$

27. Note that by Eq. 7-2, the dot product of a vector $\vec{\mathbf{A}}$ with a unit vector $\vec{\mathbf{B}}$ would give the magnitude of $\vec{\mathbf{A}}$ times the cosine of the angle between the unit vector and $\vec{\mathbf{A}}$. Thus if the unit vector lies along one of the coordinate axes, we can find the angle between the vector and the coordinate axis. We also use Eq. 7-4 to give a second evaluation of the dot product.

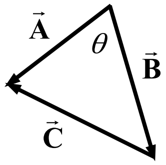
$$\vec{\mathbf{V}}\bullet\hat{\mathbf{i}} = V\cos\theta_x = V_x \;\rightarrow$$

$$\theta_x = \cos^{-1}\frac{V_x}{V} = \cos^{-1}\frac{V_x}{\sqrt{V_x^2+V_y^2+V_z^2}} = \cos^{-1}\frac{20.0}{\sqrt{(20.0)^2+(22.0)^2+(-14.0)^2}} = \boxed{52.5°}$$

$$\theta_y = \cos^{-1}\frac{V_y}{V} = \cos^{-1}\frac{22.0}{\sqrt{(20.0)^2+(22.0)^2+(-14.0)^2}} = \boxed{48.0°}$$

$$\theta_z = \cos^{-1}\frac{V_z}{V} = \cos^{-1}\frac{-14.0}{\sqrt{(20.0)^2+(22.0)^2+(-14.0)^2}} = \boxed{115°}$$

28. For the diagram shown, $\vec{\mathbf{B}}+\vec{\mathbf{C}}=\vec{\mathbf{A}}$, or $\vec{\mathbf{C}}=\vec{\mathbf{A}}-\vec{\mathbf{B}}$. Let the magnitude of each vector be represented by the corresponding lowercase letter, so $|\vec{\mathbf{C}}|=c$, for example. The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is θ. Take the dot product $\vec{\mathbf{C}}\bullet\vec{\mathbf{C}}$.



$$\vec{\mathbf{C}}\bullet\vec{\mathbf{C}} = (\vec{\mathbf{A}}-\vec{\mathbf{B}})\bullet(\vec{\mathbf{A}}-\vec{\mathbf{B}}) = \vec{\mathbf{A}}\bullet\vec{\mathbf{A}}+\vec{\mathbf{B}}\bullet\vec{\mathbf{B}}-2\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} \rightarrow c^2 = a^2+b^2-2ab\cos\theta$$

29. The scalar product is positive, so the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ must be acute. But the direction of the angle from $\vec{\mathbf{A}}$ to $\vec{\mathbf{B}}$ could be either counterclockwise or clockwise.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta = (12.0)(24.0)\cos\theta = 20.0 \rightarrow \theta = \cos^{-1}\frac{20.0}{(12.0)(24.0)} = 86.0°$$

So this angle could be either added or subtracted to the angle of $\vec{\mathbf{A}}$ to find the angle of $\vec{\mathbf{B}}$.

$$\theta_B = \theta_A \pm \theta = 27.4° \pm 86.0° = \boxed{113.4° \text{ or } -58.6°(301.4°)}$$

30. We can represent the vectors as $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}} = A\cos\alpha\hat{\mathbf{i}}+A\sin\alpha\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}$ $= B\cos\beta\hat{\mathbf{i}}+B\sin\beta\hat{\mathbf{j}}$. The angle between the two vectors is $\alpha-\beta$. Use Eqs. 7-2 and 7-4 to express the dot product.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos(\alpha-\beta) = A_xB_x+A_yB_y = A\cos\alpha B\cos\beta+A\sin\alpha B\sin\beta \rightarrow$$

$$AB\cos(\alpha-\beta) = AB\cos\alpha\cos\beta+AB\sin\alpha\sin\beta \rightarrow \boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta}$$

$\boxed{31.}$ (*a*) Use the two expressions for dot product, Eqs. 7-2 and 7-4, to find the angle between the two vectors.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta = A_xB_x+A_yB_y+A_zB_z \rightarrow$$

$$\theta = \cos^{-1}\frac{A_xB_x+A_yB_y+A_zB_z}{AB}$$

$$= \cos^{-1}\frac{(1.0)(-1.0)+(1.0)(1.0)+(-2.0)(2.0)}{\left[(1.0)^2+(1.0)^2+(-2.0)^2\right]^{1/2}\left[(-1.0)^2+(1.0)^2+(2.0)^2\right]^{1/2}}$$

$$= \cos^{-1}\left(-\tfrac{2}{3}\right) = 132° \approx \boxed{130°}$$

(*b*) The negative sign in the argument of the inverse cosine means that the angle between the two vectors is obtuse.

32. To be perpendicular to the given vector means that the dot product will be 0. Let the unknown vector be given as $\hat{\mathbf{u}} = u_x\hat{\mathbf{i}} + u_y\hat{\mathbf{j}}$.

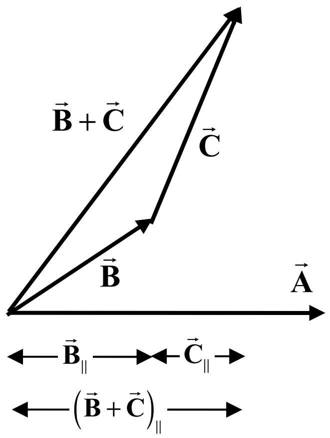
$$\hat{\mathbf{u}}\bullet\left(3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right) = 3.0u_x + 4.0u_y \quad\rightarrow\quad u_y = -0.75u_x \ ;\ \text{unit length} \rightarrow u_x^2 + u_y^2 = 1 \quad\rightarrow$$

$$u_x^2 + u_y^2 = u_x^2 + \left(-0.75u_x\right)^2 = 1.5625u_x^2 = 1 \quad\rightarrow\quad u_x = \pm\frac{1}{\sqrt{1.5625}} = \pm 0.8 \ ,\ u_y = \mp 0.6$$

So the two possible vectors are $\boxed{\hat{\mathbf{u}} = 0.8\hat{\mathbf{i}} - 0.6\hat{\mathbf{j}}}$ and $\boxed{\hat{\mathbf{u}} = -0.8\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}}}$.

Note that it is very easy to get a non-unit vector perpendicular to another vector in two dimensions, simply by interchanging the coordinates and negating one of them. So a non-unit vector perpendicular to $\left(3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right)$ could be either $\left(4.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}}\right)$ or $\left(-4.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}\right)$. Then divide each of those vectors by its magnitude (5.0) to get the possible unit vectors.

33. From Figure 7-6, we see a graphical interpretation of the scalar product as the magnitude of one vector times the projection of the other vector onto the first vector. So to show that $\vec{\mathbf{A}}\bullet\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\bullet\vec{\mathbf{B}} + \vec{\mathbf{A}}\bullet\vec{\mathbf{C}}$ is the same as showing that $A\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right)_{\parallel} = A\left(\vec{\mathbf{B}}\right)_{\parallel} + A\left(\vec{\mathbf{C}}\right)_{\parallel}$, where the subscript is implying the component of the vector that is parallel to vector $\vec{\mathbf{A}}$. From the diagram, we see that $\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right)_{\parallel} = \left(\vec{\mathbf{B}}\right)_{\parallel} + \left(\vec{\mathbf{C}}\right)_{\parallel}$. Multiplying this equation by the magnitude of vector $\vec{\mathbf{A}}$ gives $A\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right)_{\parallel} = A\left(\vec{\mathbf{B}}\right)_{\parallel} + A\left(\vec{\mathbf{C}}\right)_{\parallel}$. But from Figure 7-6, this is the same as $\vec{\mathbf{A}}\bullet\left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\bullet\vec{\mathbf{B}} + \vec{\mathbf{A}}\bullet\vec{\mathbf{C}}$. So we have proven the statement.



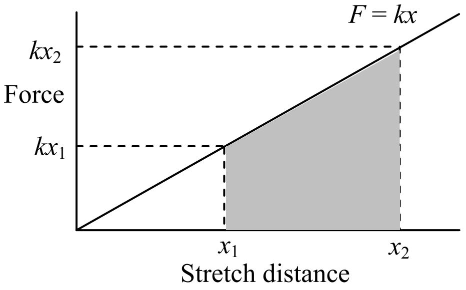
34. The downward force is 450 N, and the downward displacement would be a diameter of the pedal circle. Use Eq. 7-1.

$$W = Fd\cos\theta = \left(450\,\text{N}\right)\left(0.36\,\text{m}\right)\cos 0^\circ = \boxed{160\,\text{J}}$$

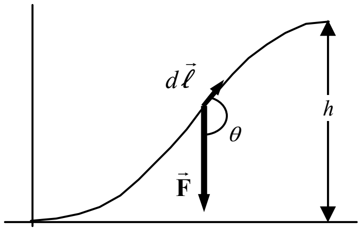
35. The force exerted to stretch a spring is given by $F_{\text{stretch}} = kx$ (the opposite of the force exerted by the spring, which is given by $F = -kx$. A graph of F_{stretch} vs. x will be a straight line of slope k through the origin. The stretch from x_1 to x_2, as shown on the graph, outlines a trapezoidal area. This area represents the work.

$$W = \tfrac{1}{2}\left(kx_1 + kx_2\right)\left(x_2 - x_1\right) = \tfrac{1}{2}k\left(x_1 + x_2\right)\left(x_2 - x_1\right)$$

$$= \tfrac{1}{2}\left(65\,\text{N/m}\right)\left(0.095\,\text{m}\right)\left(0.035\,\text{m}\right) = \boxed{0.11\,\text{J}}$$



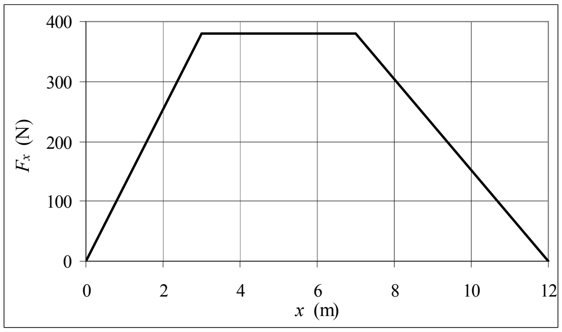
36. For a non-linear path, the work is found by considering the path to be an infinite number of infinitesimal (or differential) steps, each of which can be considered to be in a specific direction, namely, the direction tangential to the path. From the diagram, for each step we have $dW = \vec{\mathbf{F}}\bullet d\vec{\ell} = Fd\ell\cos\theta$. But $d\ell\cos\theta = -dy$, the projection of the path in the direction of the force, and $F = mg$, the force of



gravity. Find the work done by gravity.

$$W_g = \int \vec{\mathbf{F}} \cdot d\vec{\ell} = \int mg\cos\theta d\ell = mg\int(-dy) = -mgh$$

This argument could even be extended to going part way up the hill, and then part way back down, and following any kind of path. The work done by gravity will only depend on the height of the path.



$\boxed{37.}$ See the graph of force vs. distance. The work done is the area under the graph. It can be found from the formula for a trapezoid.

$$W = \tfrac{1}{2}(12.0\text{ m} + 4.0\text{ m})(380\text{ N})$$

$$= 3040\text{ J} \approx \boxed{3.0\times10^3\text{ J}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.37."

38. The work required to stretch a spring from equilibrium is proportional to the length of stretch, squared. So if we stretch the spring to 3 times its original distance, a total of 9 times as much work is required for the total stretch. Thus it would take 45.0 J to stretch the spring to a total of 6.0 cm. Since 5.0 J of work was done to stretch the first 2.0 cm, $\boxed{40.0\text{ J}}$ of work is required to stretch it the additional 4.0 cm.

This could also be done by calculating the spring constant from the data for the 2.0 cm stretch, and then using that spring constant to find the work done in stretching the extra distance.

39. The x-axis is portioned into 7 segments, so each segment is 1/7 of the full 20.0-m width. The force on each segment can be approximated by the force at the middle of the segment. Thus we are performing a simple Riemann sum to find the area under the curve. The value of the mass does not come into the calculation.

$$W = \sum_{i=1}^{7} F_i\Delta x_i = \Delta x\sum_{i=1}^{7} F_i = \tfrac{1}{7}(20.0\text{ m})(180\text{ N} + 200\text{ N} + 175\text{ N} + 125\text{ N} + 110\text{ N} + 100\text{ N} + 95\text{ N})$$

$$= \tfrac{1}{7}(20.0\text{ m})(985\text{ N}) \approx \boxed{2800\text{ J}}$$

Another method is to treat the area as a trapezoid, with sides of 180 N and 100 N, and a base of 20.0 m. Then the work is $W = \tfrac{1}{2}(20.0\text{ m})(180\text{ N} + 100\text{ N}) \approx \boxed{2800\text{ J}}$.

40. The work done will be the area under the F_x vs. x graph.

(a) From $x = 0.0$ to $x = 10.0\text{ m}$, the shape under the graph is trapezoidal. The area is

$$W_a = (400\text{ N})\tfrac{1}{2}(10\text{ m} + 4\text{ m}) = \boxed{2800\text{ J}}.$$

(b) From $x = 10.0\text{ m}$ to $x = 15.0\text{ m}$, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$W_a = (-200\text{ N})\tfrac{1}{2}(5\text{ m} + 2\text{ m}) = -700\text{ J}.$$

Thus the total work from $x = 0.0$ to $x = 15.0\text{ m}$ is $2800\text{ J} - 700\text{ J} = \boxed{2100\text{ J}}$.

41. Apply Eq. 7-1 to each segment of the motion.

$$\begin{aligned} W &= W_1 + W_2 + W_3 = F_1 d_1 \cos\theta_1 + F_2 d_2 \cos\theta_2 + F_3 d_3 \cos\theta_3 \\ &= (22\,\mathrm{N})(9.0\,\mathrm{m})\cos 0° + (38\,\mathrm{N})(5.0\,\mathrm{m})\cos 12° + (22\,\mathrm{N})(13.0\,\mathrm{m})\cos 0° = \boxed{670\,\mathrm{J}} \end{aligned}$$

42. Since the force only has an x-component, only the x-displacement is relevant. The object moves from $x = 0$ to $x = d$.

$$W = \int_0^d F_x dx = \int_0^d kx^4 dx = \boxed{\tfrac{1}{5}kd^5}$$

43. Since we are compressing the spring, the force and the displacement are in the same direction.

$$W = \int_0^X F_x dx = \int_0^X \left(kx + ax^3 + bx^4\right) dx = \boxed{\tfrac{1}{2}kX^2 + \tfrac{1}{4}aX^4 + \tfrac{1}{5}bX^5}$$

44. Integrate the force over the distance the force acts to find the work. We assume the displacement is all in the x-direction.

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_0^{0.20\,\mathrm{m}} \left(150x - 190x^2\right) dx = \left(75x^2 - \frac{190}{3}x^3\right)_0^{0.20\,\mathrm{m}} = \boxed{2.49\,\mathrm{J}}$$

45. Integrate the force over the distance the force acts to find the work.

$$W = \int_0^{1.0\,\mathrm{m}} F_1 dx = \int_0^{1.0\,\mathrm{m}} \frac{A}{\sqrt{x}} dx = 2A\sqrt{x}\Big|_0^{1.0\,\mathrm{m}} = 2\left(2.0\,\mathrm{N}{\bullet}\mathrm{m}^{1/2}\right)(1.0\,\mathrm{m})^{1/2} = \boxed{4.0\,\mathrm{J}}$$

Note that the work done is finite.

46. Because the object moves along a straight line, we know that the x-coordinate increases linearly from 0 to 10.0 m, and the y-coordinate increases linearly from 0 to 20.0 m. Use the relationship developed at the top of page 170.

$$\begin{aligned} W &= \int_{x_a}^{x_b} F_x dx + \int_{y_a}^{y_b} F_y dy = \int_0^{10.0\,\mathrm{m}} 3.0x\,dx + \int_0^{20.0\,\mathrm{m}} 4.0y\,dy = \tfrac{1}{2}\left(3.0x^2\right)_0^{10.0} + \tfrac{1}{2}\left(4.0y^2\right)_0^{20.0} = 150\,\mathrm{J} + 800\,\mathrm{J} \\ &= \boxed{950\,\mathrm{J}} \end{aligned}$$

47. Since the force is of constant magnitude and always directed at 30° to the displacement, we have a simple expression for the work done as the object moves.

$$W = \int_{\text{start}}^{\text{finish}} \vec{\mathbf{F}}{\bullet}d\vec{\ell} = \int_{\text{start}}^{\text{finish}} F\cos 30° d\ell = F\cos 30° \int_{\text{start}}^{\text{finish}} d\ell = F\cos 30° \pi R = \boxed{\frac{\sqrt{3}\pi FR}{2}}$$

48. The force on the object is given by Newton's law of universal gravitation, $F = G\dfrac{mm_E}{r^2}$. The force is a function of distance, so to find the work, we must integrate. The directions are tricky. To use Eq. 7-7, we have $\vec{\mathbf{F}} = -G\dfrac{mm_E}{r^2}\hat{\mathbf{r}}$ and $d\vec{\ell} = dr\,\hat{\mathbf{r}}$. It is tempting to put a negative sign with the $d\vec{\ell}$ relationship since the object moves inward, but since r is measured outward away from the center of the Earth, we must not include that negative sign. Note that we move from a large radius to a small radius.

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\ell} = \int_{\text{far}}^{\text{near}} -G\frac{mm_{\text{E}}}{r^2}\hat{\mathbf{r}} \cdot (dr\,\hat{\mathbf{r}}) = -\int_{r_{\text{E}}+3300\,\text{km}}^{r_{\text{E}}} G\frac{mm_{\text{E}}}{r^2}dr = G\frac{mm_{\text{E}}}{r}\Bigg|_{r_{\text{E}}+3300\,\text{km}}^{r_{\text{E}}}$$

$$= Gmm_{\text{E}}\left(\frac{1}{r_{\text{E}}} - \frac{1}{r_{\text{E}} + 3300\,\text{km}}\right)$$

$$= \left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(2800\,\text{kg})\left(5.97\times10^{24}\,\text{kg}\right)\left(\frac{1}{6.38\times10^6\,\text{m}} - \frac{1}{(6.38+3.30)\times10^6\,\text{m}}\right)$$

$$= \boxed{6.0\times10^{10}\,\text{J}}$$

$\boxed{49.}$ Let y represent the length of chain hanging over the table, and let λ represent the weight per unit length of the chain. Then the force of gravity (weight) of the hanging chain is $F_{\text{G}} = \lambda y$. As the next small length of chain dy comes over the table edge, gravity does an infinitesimal amount of work on the hanging chain given by the force times the distance, $F_{\text{G}}dy = \lambda y dy$. To find the total amount of work that gravity does on the chain, integrate that work expression, with the limits of integration representing the amount of chain hanging over the table.

$$W = \int_{y_{\text{initial}}}^{y_{\text{final}}} F_{\text{G}}dy = \int_{1.0\,\text{m}}^{3.0\,\text{m}} \lambda y dy = \tfrac{1}{2}\lambda y^2\Big|_{1.0\,\text{m}}^{3.0\,\text{m}} = \tfrac{1}{2}(18\,\text{N/m})\left(9.0\,\text{m}^2 - 1.0\,\text{m}^2\right) = \boxed{72\,\text{J}}$$

50. Find the velocity from the kinetic energy, using Eq. 7-10.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(6.21\times10^{-21}\,\text{J}\right)}{5.31\times10^{-26}}} = \boxed{484\,\text{m/s}}$$

51. (*a*) Since $K = \tfrac{1}{2}mv^2$, then $v = \sqrt{2K/m}$ and so $v \propto \sqrt{K}$. Thus if the kinetic energy is tripled, the speed will be multiplied by a factor of $\boxed{\sqrt{3}}$.

(*b*) Since $K = \tfrac{1}{2}mv^2$, then $K \propto v^2$. Thus if the speed is halved, the kinetic energy will be multiplied by a factor of $\boxed{1/4}$.

52. The work done on the electron is equal to the change in its kinetic energy.

$$W = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.40\times10^6\,\text{m/s}\right)^2 = \boxed{-8.93\times10^{-19}\,\text{J}}$$

Note that the work is negative since the electron is slowing down.

53. The work done on the car is equal to the change in its kinetic energy.

$$W = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(1300\,\text{kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = \boxed{-4.5\times10^5\,\text{J}}$$

Note that the work is negative since the car is slowing down.

54. We assume the train is moving 20 m/s (which is about 45 miles per hour), and that the distance of "a few city blocks" is perhaps a half-mile, which is about 800 meters. First find the kinetic energy of the train, and then find out how much work the web must do to stop the train. Note that the web does negative work, since the force is in the OPPOSITE direction of the displacement.

$$W_{\substack{\text{to stop}\\ \text{train}}} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}\left(10^4\,\text{kg}\right)\left(20\,\text{m/s}\right)^2 = -2\times10^6\,\text{J}$$

$$W_{\text{web}} = -\tfrac{1}{2}kx^2 = -2\times10^6\,\text{J} \;\rightarrow\; k = \frac{2\left(2\times10^6\,\text{J}\right)}{\left(800\,\text{m}^2\right)} = \boxed{6\,\text{N/m}}$$

Note that this is not a very stiff "spring," but it does stretch a long distance.

$\boxed{55.}$ The force of the ball on the glove will be the opposite of the force of the glove on the ball, by Newton's third law. Both objects have the same displacement, and so the work done on the glove is opposite the work done on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball.

$$W_{\text{on ball}} = \left(K_2 - K_1\right)_{\text{ball}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}\left(0.145\,\text{kg}\right)\left(32\,\text{m/s}\right)^2 = -74.24\,\text{J}$$

So $W_{\text{on glove}} = 74.24\,\text{J}$. But $W_{\text{on glove}} = F_{\text{on glove}}d\cos 0°$, because the force on the glove is in the same direction as the motion of the glove.

$$74.24\,\text{J} = F_{\text{on glove}}\left(0.25\,\text{m}\right) \;\rightarrow\; F_{\text{on glove}} = \frac{74.24\,\text{J}}{0.25\,\text{m}} = \boxed{3.0\times10^2\,\text{N}}$$, in the direction of the original velocity of the ball.

56. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W = Fd\cos 0^\circ = Fd = \left(105\,\text{N}\right)\left(0.75\,\text{m}\right) = 78.75\,\text{J}$. But that work changes the kinetic energy of the arrow, by the work-energy theorem. Thus

$$Fd = W = K_2 - K_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \;\rightarrow\; v_2 = \sqrt{\frac{2Fd}{m} + v_1^2} = \sqrt{\frac{2\left(78.75\,\text{J}\right)}{0.085\,\text{kg}} + 0} = \boxed{43\,\text{m/s}}$$

57. (*a*) The spring constant is found by the magnitudes of the initial force and displacement, and so $k = F/x$. As the spring compresses, it will do the same amount of work on the block as was done on the spring to stretch it. The work done is positive because the force of the spring is parallel to the displacement of the block. Use the work-energy theorem to determine the speed of the block.

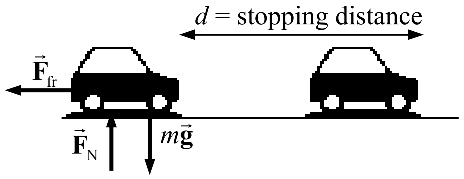
$$W_{\substack{\text{on block}\\ \text{during}\\ \text{compression}}} = \Delta K_{\text{block}} = W_{\substack{\text{on spring}\\ \text{during}\\ \text{stretching}}} \;\rightarrow\; \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}kx^2 = \tfrac{1}{2}\frac{F}{x}x^2 \;\rightarrow\; v_f = \boxed{\sqrt{\frac{Fx}{m}}}$$

(*b*) Now we must find how much work was done on the spring to stretch it from $x/2$ to x. This will be the work done on the block as the spring pulls it back from x to $x/2$.

$$W_{\substack{\text{on spring}\\ \text{during}\\ \text{stretching}}} = \int_{x/2}^{x} F\,dx = \int_{x/2}^{x} kx\,dx = \tfrac{1}{2}kx^2\Big|_{x/2}^{x} = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k\left(x/2\right)^2 = \tfrac{3}{8}kx^2$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{3}{8}kx^2 \;\rightarrow\; v_f = \boxed{\sqrt{\frac{3Fx}{4m}}}$$

58. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car. Assume the maximum possible frictional force, which results in the minimum braking



distance. Thus $F_{\text{fr}} = \mu_s F_{\text{N}}$. The normal force is equal to the car's weight if it is on a level surface, and so $F_{\text{fr}} = \mu_s mg$. In the diagram, the car is traveling to the right.

$$W = \Delta K \;\rightarrow\; F_{\text{fr}} d \cos 180^\circ = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 \;\rightarrow\; -\mu_s mgd = -\tfrac{1}{2} m v_1^2 \;\rightarrow\; d = \frac{v_1^2}{2g\mu_s}$$

Since $d \propto v_1^2$, if v_1 increases by 50%, or is multiplied by 1.5, then d will be multiplied by a factor of $(1.5)^2$, or $\boxed{2.25}$.

59. The net work done on the car must be its change in kinetic energy. By applying Newton's third law, the negative work done on the car by the spring must be the opposite of the work done in compressing the spring.

$$W = \Delta K = -W_{\text{spring}} \;\rightarrow\; \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 = -\tfrac{1}{2} k x^2 \;\rightarrow$$

$$k = m\frac{v_1^2}{x^2} = (1200\,\text{kg})\frac{\left[66\,\text{km/k}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(2.2\,\text{m})^2} = \boxed{8.3\times10^4\ \text{N/m}}$$

60. The first car mentioned will be called car 1. So we have these statements:

$$K_1 = \tfrac{1}{2} K_2 \;\rightarrow\; \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_2 v_2^2\right) \;;\; K_{1,\text{fast}} = K_{2,\text{fast}} \;\rightarrow \tfrac{1}{2} m_1 (v_1 + 7.0)^2 = \tfrac{1}{2} m_2 (v_2 + 7.0)^2$$

Now use the mass information, that $m_1 = 2m_2$.

$$\tfrac{1}{2} 2 m_2 v_1^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_2 v_2^2\right) \;;\; \tfrac{1}{2} 2 m_2 (v_1 + 7.0)^2 = \tfrac{1}{2} m_2 (v_2 + 7.0)^2 \;\rightarrow$$

$$2v_1 = v_2 \;;\; 2(v_1 + 7.0)^2 = (v_2 + 7.0)^2 \;\rightarrow\; 2(v_1 + 7.0)^2 = (2v_1 + 7.0)^2 \;\rightarrow$$

$$\sqrt{2}(v_1 + 7.0) = (2v_1 + 7.0) \;\rightarrow\; v_1 = \frac{7.0}{\sqrt{2}} = 4.9497\,\text{m/s} \;;\; v_2 = 2v_1 = 9.8994\,\text{m/s}$$

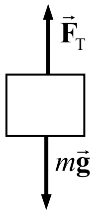
$$\boxed{v_1 = 4.9\,\text{m/s} \;;\; v_2 = 9.9\,\text{m/s}}$$

$\boxed{61.}$ The work done by the net force is the change in kinetic energy.

$$W = \Delta K = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

$$= \tfrac{1}{2}(4.5\,\text{kg})\left[(15.0\,\text{m/s})^2 + (30.0\,\text{m/s})^2\right] - \tfrac{1}{2}(4.5\,\text{kg})\left[(10.0\,\text{m/s})^2 + (20.0\,\text{m/s})^2\right] = \boxed{1400\,\text{J}}$$

62. (*a*) From the free-body diagram for the load being lifted, write Newton's second law for the vertical direction, with up being positive.



$$\sum F = F_{\text{T}} - mg = ma = 0.150mg \;\rightarrow$$

$$F_{\text{T}} = 1.150mg = 1.150(265\ \text{kg})(9.80\,\text{m/s}^2) = \boxed{2.99\times10^3\,\text{N}}$$

(*b*) The net work done on the load is found from the net force.

$$W_{\text{net}} = F_{\text{net}} d \cos 0^\circ = (0.150mg)d = 0.150(265\,\text{kg})(9.80\,\text{m/s}^2)(23.0\,\text{m})$$

$$= \boxed{8.96\times10^3\,\text{J}}$$

(*c*) The work done by the cable on the load is as follows.

$$W_{\text{cable}} = F_{\text{T}} d \cos 0^\circ = (1.150mg)d = 1.15(265\,\text{kg})(9.80\,\text{m/s}^2)(23.0\,\text{m}) = \boxed{6.87\times10^4\,\text{J}}$$

(*d*) The work done by gravity on the load is as follows.

$$W_G = mgd\cos 180^\circ = -mgd = -(265\,\text{kg})(9.80\,\text{m/s}^2)(23.0\,\text{m}) = \boxed{-5.97\times10^4\,\text{J}}$$

(*e*) Use the work-energy theorem to find the final speed, with an initial speed of 0.

$$W_{\text{net}} = K_2 - K_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \;\rightarrow\; v_2 = \sqrt{\frac{2W_{net}}{m} + v_1^2} = \sqrt{\frac{2(8.96\times10^3\,\text{J})}{265\,\text{kg}} + 0} = \boxed{8.22\,\text{m/s}}$$

63. (*a*) The angle between the pushing force and the displacement is 32°.

$$W_P = F_P d\cos\theta = (150\,\text{N})(5.0\,\text{m})\cos 32^\circ = 636.0\,\text{J} \approx \boxed{640\,\text{J}}$$

(*b*) The angle between the force of gravity and the displacement is 122°.

$$W_G = F_G d\cos\theta = mgd\cos\theta = (18\,\text{kg})(9.80\,\text{m/s}^2)(5.0\,\text{m})\cos 122^\circ = -467.4\,\text{J} \approx \boxed{-470\,\text{J}}$$

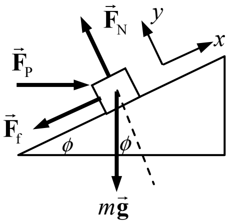
(*c*) Because the normal force is perpendicular to the displacement, the work done by the normal force is $\boxed{0}$.

(*d*) The net work done is the change in kinetic energy.

$$W = W_P + W_g + W_N = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \;\rightarrow$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(636.0\,\text{J} - 467.4\,\text{J})}{(18\,\text{kg})}} = \boxed{4.3\,\text{m/s}}$$

64. See the free-body diagram help in the determination of the frictional force.



$$\sum F_y = F_N - F_P\sin\phi - mg\cos\phi = 0 \;\rightarrow\; F_N = F_P\sin\phi + mg\cos\phi$$

$$F_f = \mu_k F_N = \mu_k\left(F_P\sin\phi + mg\cos\phi\right)$$

(*a*) The angle between the pushing force and the displacement is 32°.

$$W_P = F_P d\cos\theta = (150\,\text{N})(5.0\,\text{m})\cos 32^\circ = 636.0\,\text{J} \approx \boxed{640\,\text{J}}$$

(*b*) The angle between the force of gravity and the displacement is 122°.

$$W_G = F_G d\cos\theta = mgd\cos\theta = (18\,\text{kg})(9.80\,\text{m/s}^2)(5.0\,\text{m})\cos 122^\circ = -467.4\,\text{J} \approx \boxed{-470\,\text{J}}$$

(*c*) Because the normal force is perpendicular to the displacement, the work done by the normal force is $\boxed{0}$.

(*d*) To find the net work, we need the work done by the friction force. The angle between the friction force and the displacement is 180°.

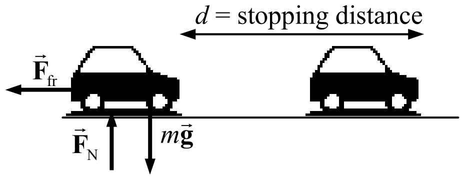
$$W_f = F_f d\cos\theta = \mu_k\left(F_P\sin\phi + mg\cos\phi\right)d\cos\theta$$

$$= (0.10)\left[(150\,\text{N})\sin 32^\circ + (18\text{kg})(9.80\,\text{m/s}^2)\cos 32^\circ\right](5.0\,\text{m})\cos 180^\circ = -114.5\,\text{J}$$

$$W = W_P + W_g + W_N + W_f = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \;\rightarrow$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(636.0\,\text{J} - 467.4\,\text{J} - 114.5\,\text{J})}{(18\,\text{kg})}} = \boxed{2.5\,\text{m/s}}$$

65. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{fr} = \mu_k F_N$. Since the car is on a level surface, the normal force is equal to the



car's weight, and so $F_{\text{fr}} = \mu_k mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.

$$W = \Delta K \quad \rightarrow \quad F_{\text{fr}} d \cos 180^\circ = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad \rightarrow \quad -\mu_k mgd = 0 - \tfrac{1}{2}mv_1^2 \quad \rightarrow$$

$$v_1 = \sqrt{2\mu_k gd} = \sqrt{2(0.38)(9.80\,\text{m/s}^2)(98\,\text{m})} = \boxed{27\,\text{m/s}}$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

66. For the first part of the motion, the net force doing work is the 225 N force. For the second part of the motion, both the 225 N force and the force of friction do work. The friction force is the coefficient of friction times the normal force, and the normal force is equal to the weight. The work-energy theorem is then used to find the final speed.

$$W_{\text{total}} = W_1 + W_2 = F_{\text{pull}} d_1 \cos 0^\circ + F_{\text{pull}} d_2 \cos 0^\circ + F_{\text{f}} d_2 \cos 180^\circ = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \quad \rightarrow$$

$$v_f = \sqrt{\frac{2\left[F_{\text{pull}}\left(d_1 + d_2\right) - \mu_{\text{k}} mgd_2\right]}{m}}$$

$$= \sqrt{\frac{2\left[(225\,\text{N})(21.0\,\text{m}) - (0.20)(46.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)(10.0\,\text{m})\right]}{(46.0\,\text{kg})}} = \boxed{13\,\text{m/s}}$$

$\boxed{67.}$ (*a*) In the Earth frame of reference, the ball changes from a speed of v_1 to a speed of $v_1 + v_2$.

$$\Delta K_{\text{Earth}} = \tfrac{1}{2}m\left(v_1 + v_2\right)^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m\left(v_1^2 + 2v_1v_2 + v_2^2\right) - \tfrac{1}{2}mv_1^2 = mv_1v_2 + \tfrac{1}{2}mv_2^2$$

$$= \boxed{\tfrac{1}{2}mv_2^2\left(1 + 2\frac{v_1}{v_2}\right)}$$

(*b*) In the train frame of reference, the ball changes from a speed of 0 to a speed of v_2.

$$\Delta K_{\text{train}} = \tfrac{1}{2}mv_2^2 - 0 = \boxed{\tfrac{1}{2}mv_2^2}$$

(*c*) The work done is the change of kinetic energy, in each case.

$$W_{\text{Earth}} = \boxed{\tfrac{1}{2}mv_2^2\left(1 + 2\frac{v_1}{v_2}\right)} \ ; \ W_{\text{train}} = \boxed{\tfrac{1}{2}mv_2^2}$$

(*d*) The difference can be seen as due to the definition of work as force exerted through a distance. In both cases, the force on the ball is the same, but relative to the Earth, the ball moves further during the throwing process than it does relative to the train. Thus more work is done in the Earth frame of reference. Another way to say it is that kinetic energy is very dependent on reference frame, and so since work is the change in kinetic energy, the amount of work done will be very dependent on reference frame as well.

68. The kinetic energy of the spring would be found by adding together the kinetic energy of each infinitesimal part of the spring. The mass of an infinitesimal part is given by $dm = \frac{M_{\text{S}}}{D}dx$, and the speed of an infinitesimal part is $v = \frac{x}{D}v_0$. Calculate the kinetic energy of the mass + spring.

$$K_{\substack{\text{speed}\\ v_0}} = K_{\text{mass}} + K_{\text{spring}} = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\int_{\text{mass}} v^2 dm = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\int_0^D\left(v_0\frac{x}{D}\right)^2\frac{M_{\text{S}}}{D}dx = \tfrac{1}{2}mv_0^2 + \frac{v_0^2 M_{\text{S}}}{D^3}\tfrac{1}{2}\int_0^D x^2 dx$$

$$= \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\frac{v_0^2 M_S}{D^3}\frac{D^3}{3} = \tfrac{1}{2}v_0^2\left(m + \tfrac{1}{3}M_S\right)$$

So for a generic speed *v*, we have $K_{\underset{v}{\text{speed}}} = \boxed{\tfrac{1}{2}\left(m + \tfrac{1}{3}M_S\right)v^2}$.

69. (*a*) The work done by gravity as the elevator falls is the weight times the displacement. They are in the same direction.

$$W_G = mgd\cos 0° = (925\,\text{kg})\left(9.80\,\text{m/s}^2\right)(22.5\,\text{m}) = 2.0396\times10^5\,\text{J} \approx \boxed{2.04\times10^5\,\text{J}}$$

(*b*) The work done by gravity on the elevator is the net work done on the elevator while falling, and so the work done by gravity is equal to the change in kinetic energy.

$$W_G = \Delta K = \tfrac{1}{2}mv^2 - 0 \rightarrow v = \sqrt{\frac{2W_G}{m}} = \sqrt{\frac{2\left(2.0396\times10^5\,\text{J}\right)}{(925\,\text{kg})}} = \boxed{21.0\,\text{m/s}}$$

(*c*) The elevator starts and ends at rest. Therefore, by the work-energy theorem, the net work done must be 0. Gravity does positive work as it falls a distance of $(22.5 + x)\,\text{m}$, and the spring will do negative work at the spring is compressed. The work done on the spring is $\tfrac{1}{2}kx^2$, and so the work done by the spring is $-\tfrac{1}{2}kx^2$.

$$W = W_G + W_{\text{spring}} = mg(d+x) - \tfrac{1}{2}kx^2 = 0 \rightarrow \tfrac{1}{2}kx^2 - mgx - mgd = 0 \rightarrow$$

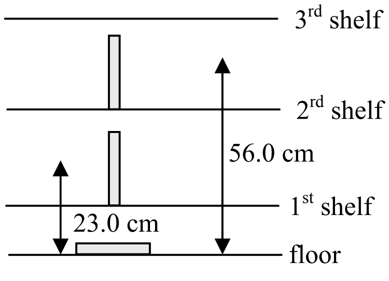
$$x = \frac{mg \pm \sqrt{m^2g^2 - 4\left(\tfrac{1}{2}k\right)(-mgd)}}{2\left(\tfrac{1}{2}k\right)}$$

The positive root must be taken since we have assumed $x > 0$ in calculating the work done by gravity. Using the values given in the problem gives $x = \boxed{2.37\,\text{m}}$.

70. (*a*) $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(3.0\times10^{-3}\,\text{kg}\right)(3.0\,\text{m/s})^2 = 1.35\times10^{-2}\,\text{J} \approx \boxed{1.4\times10^{-2}\,\text{J}}$

(*b*) $K_{\text{actual}} = 0.35E_{\text{required}} \rightarrow E_{\text{required}} = \dfrac{K_{\text{actual}}}{0.35} = \dfrac{1.35\times10^{-2}\,\text{J}}{0.35} = \boxed{3.9\times10^{-2}\,\text{J}}$

71. The minimum work required to shelve a book is equal to the weight of the book times the vertical distance the book is moved. See the diagram. Each book that is placed on the lowest shelf has its center moved upwards by 23.0 cm (the height of the bottom of the first shelf, plus half the height of a book). So the work to move 28 books to the lowest shelf is $W_1 = 28mg(0.230\,\text{m})$. Each book that is placed on the second shelf has its center of mass moved upwards by 56.0 cm (23.0 cm + 33.0 cm), so the work to move 28 books to the second shelf is $W_2 = 28mg(0.560\,\text{m})$.



Similarly, $W_3 = 28mg(0.890\,\text{m})$, $W_4 = 28mg(1.220\,\text{m})$, and $W_5 = 28mg(1.550\,\text{m})$. The total work done is the sum of the five work expressions.

$$W = 28mg(0.230\,\text{m} + .560\,\text{m} + .890\,\text{m} + 1.220\,\text{m} + 1.550\,\text{m})$$

$$= 28(1.40\,\text{kg})\left(9.80\,\text{m/s}^2\right)(4.450\,\text{m}) = \boxed{1710\,\text{J}}$$

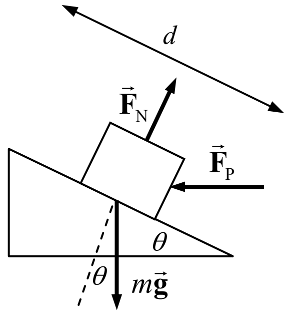
72. There are two forces on the meteorite – gravity and the force from the mud. Take down to be the positive direction, and then the net force is $F_{\text{net}} = mg - 640x^3$. Use this (variable) force to find the work done on the meteorite as it moves in the mud, and then use the work-energy theorem to find the initial velocity of the meteorite.

$$W = \int_{x=0}^{x=5.0} \left(mg - 640x^3\right)dx = \left(mgx - 160x^4\right)_{x=0}^{x=5.0} = (75\,\text{kg})(9.80\,\text{m/s}^2)(5.0\,\text{m}) - 160(5.0\,\text{m})^4$$

$$= -9.625\times10^4\,\text{J}$$

$$W = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \rightarrow v_i = \sqrt{\frac{-2W}{m}} = \sqrt{\frac{-2\left(-9.625\times10^4\,\text{J}\right)}{(75\,\text{kg})}} = \boxed{51\,\text{m/s}}$$

$\boxed{73.}$ Consider the free-body diagram for the block as it moves up the plane.



(*a*) $K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(6.10\,\text{kg})(3.25\,\text{m/s})^2 = 32.22\text{ J} \approx \boxed{32.2\,\text{J}}$

(*b*) $W_\text{P} = F_\text{P}d\cos 37^\circ = (75.0\,\text{N})(9.25\,\text{m})\cos 37.0^\circ = 554.05\,\text{J}$

$\approx \boxed{554\,\text{J}}$

(*c*) $W_G = mgd\cos 127.0^\circ = (6.10\,\text{kg})(9.80\,\text{m/s}^2)(9.25\,\text{m})\cos 127.0^\circ$

$= -332.78\text{ J} \approx \boxed{-333\,\text{J}}$

(*d*) $W_\text{N} = F_\text{N}d\cos 90^\circ = \boxed{0\,\text{J}}$

(*e*) Apply the work-energy theorem.

$W_{\text{total}} = K_2 - K_1 \rightarrow$

$KE_2 = W_{\text{total}} + K_1 = W_\text{P} + W_\text{G} + W_\text{N} + K_1 = (554.05 - 332.78 + 0 + 32.22)\,\text{J} \approx \boxed{253\,\text{J}}$

74. The dot product can be used to find the angle between the vectors.

$$\vec{\mathbf{d}}_{1-2} = \left[\left(0.230\hat{\mathbf{i}} + 0.133\hat{\mathbf{j}}\right)\times10^{-9}\,\text{m}\right] \; ; \; \vec{\mathbf{d}}_{1-3} = \left[\left(0.077\hat{\mathbf{i}} + 0.133\hat{\mathbf{j}} + 0.247\hat{\mathbf{k}}\right)\times10^{-9}\,\text{m}\right]$$

$$\vec{\mathbf{d}}_{1-2}\bullet\vec{\mathbf{d}}_{1-3} = \left[\left(0.230\hat{\mathbf{i}} + 0.133\hat{\mathbf{j}}\right)\times10^{-9}\,\text{m}\right]\bullet\left[\left(0.077\hat{\mathbf{i}} + 0.133\hat{\mathbf{j}} + 0.247\hat{\mathbf{k}}\right)\times10^{-9}\,\text{m}\right]$$

$$= \left[3.540\times10^{-2}\right]\times10^{-18}\,\text{m}^2$$

$$d_{1-2} = \sqrt{(0.230)^2 + (0.133)^2}\times10^{-9}\,\text{m} = 0.2657\times10^{-9}\,\text{m}$$

$$d_{1-3} = \sqrt{(0.077)^2 + (0.133)^2 + (0.247)^2}\times10^{-9}\,\text{m} = 0.2909\times10^{-9}\,\text{m}$$

$$\vec{\mathbf{d}}_{1-2}\bullet\vec{\mathbf{d}}_{1-3} = d_{1-2}d_{1-3}\cos\theta \rightarrow$$

$$\theta = \cos^{-1}\frac{\vec{\mathbf{d}}_{1-2}\bullet\vec{\mathbf{d}}_{1-3}}{d_{1-2}d_{1-3}} = \cos^{-1}\frac{\left[3.540\times10^{-2}\right]\times10^{-18}\,\text{m}^2}{\left(0.2657\times10^{-9}\,\text{m}\right)\left(0.2909\times10^{-9}\,\text{m}\right)} = \boxed{62.7^\circ}$$

75. Since the forces are constant, we may use Eq. 7-3 to calculate the work done.

$$W_{\text{net}} = \left(\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2\right)\bullet\vec{\mathbf{d}} = \left[\left(1.50\hat{\mathbf{i}} - 0.80\hat{\mathbf{j}} + 0.70\hat{\mathbf{k}}\right)\text{N} + \left(-0.70\hat{\mathbf{i}} + 1.20\hat{\mathbf{j}}\right)\text{N}\right]\bullet\left[\left(8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}}\right)\text{m}\right]$$

$$= \left[\left(0.80\hat{\mathbf{i}} + 0.40\hat{\mathbf{j}} + 0.70\hat{\mathbf{k}}\right)\text{N}\right]\bullet\left[\left(8.0\hat{\mathbf{i}} + 6.0\hat{\mathbf{j}} + 5.0\hat{\mathbf{k}}\right)\text{m}\right] = (6.4 + 2.4 + 3.5)\,\text{J} = \boxed{12.3\,\text{J}}$$

76. The work done by the explosive force is equal to the change in kinetic energy of the shells. The starting speed is 0. The force is in the same direction as the displacement of the shell.

$$W = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 \ ; \ W = Fd\cos\theta \ \rightarrow \ \tfrac{1}{2}mv_f^2 = Fd\cos\theta \ \rightarrow$$

$$F = \frac{mv_f^2}{2d\cos\theta} = \frac{(1250\,\text{kg})(750\,\text{m/s})^2}{2(15\,\text{m})} = 2.344\times10^7\,\text{N} \approx \boxed{2.3\times10^7\,\text{N}}$$

$$2.344\times10^7\,\text{N}\left(\frac{1\,\text{lb}}{4.45\,\text{N}}\right) = \boxed{5.3\times10^6\,\text{lbs}}$$

77. We assume the force is in the x-direction, so that the angle between the force and the displacement is 0. The work is found from Eq. 7-7.

$$W = \int_{x=0.10\,\text{m}}^{x=\infty} Ae^{-kx}dx = -\frac{A}{k}e^{-kx}\Big|_{x=0.10}^{x=\infty} = \boxed{\frac{A}{k}e^{-0.10k}}$$

78. The force exerted by the spring will be the same magnitude as the force to compress the spring. The spring will do positive work on the ball by exerting a force in the direction of the displacement. This work is equal to the change in kinetic energy of the ball. The initial speed of the ball is 0.

$$W = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 \ ; \ W = \int_{x=0}^{x=2.0\,\text{m}} \left(150x + 12x^3\right)dx = \left(75x^2 + 3x^4\right)_{x=0}^{x=2.0} = 348\,\text{J}$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(348\,\text{J})}{3.0\,\text{kg}}} = \boxed{15\,\text{m/s}}$$

$\boxed{79.}$ The force is constant, and so we may calculate the force by Eq. 7-3. We may also use that to calculate the angle between the two vectors.

$$W = \vec{\mathbf{F}}\bullet\vec{\mathbf{d}} = \left[\left(10.0\hat{\mathbf{i}} + 9.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}\right)\text{kN}\right]\bullet\left[\left(5.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right)\text{m}\right] = \boxed{86\,\text{kJ}}$$

$$F = \left[(10.0)^2 + (9.0)^2 + (12.0)^2\right]^{1/2}\text{kN} = 18.0\text{kN} \ ; \ d = \left[(5.0)^2 + (4.0)^2\right]^{1/2}\text{m} = 6.40\,\text{m}$$

$$W = Fd\cos\theta \ \rightarrow \ \theta = \cos^{-1}\frac{W}{Fd} = \cos^{-1}\frac{8.6\times10^4\,\text{J}}{(1.80\times10^4\,\text{N})(6.40\,\text{m})} = \boxed{42^\circ}$$

80. (*a*) The force and displacement are in the same direction.

$$W = Fd\cos\theta \ ; \ W = \Delta K \ \rightarrow$$

$$F = \frac{\Delta K}{d} = \frac{\tfrac{1}{2}m\left(v_f^2 - v_i^2\right)}{d} = \frac{\tfrac{1}{2}(0.033\,\text{kg})(85\,\text{m/s})^2}{0.32\,\text{m}} = 372.5\,\text{N} \approx \boxed{370\,\text{N}}$$

(*b*) Combine Newton's second law with Eq. 2-12c for constant acceleration.

$$F = ma = \frac{m\left(v_f^2 - v_i^2\right)}{2\Delta x} = \frac{(0.033\,\text{kg})(85\,\text{m/s})^2}{2(0.32\,\text{m})} = 372.5\,\text{N} \approx \boxed{370\,\text{N}}$$

81. The original speed of the softball is $(110\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 30.56\,\text{m/s}$. The final speed is 90% of this, or 27.50 m/s. The work done by air friction causes a change in the kinetic energy of the ball, and thus the speed change. In calculating the work, notice that the force of friction is directed oppositely to the direction of motion of the ball.

$$W_{\text{fr}} = F_{\text{fr}} d \cos 180^\circ = K_2 - K_1 = \tfrac{1}{2} m \left(v_2^2 - v_1^2 \right) \rightarrow$$

$$F_{\text{fr}} = \frac{m\left(v_2^2 - v_1^2\right)}{-2d} = \frac{m v_1^2 \left(0.9^2 - 1\right)}{-2d} = \frac{(0.25\text{ kg})(30.56\,\text{m/s})^2\left(0.9^2 - 1\right)}{-2(15\text{ m})} = \boxed{1.5\,\text{N}}$$

82. (*a*) The pilot's initial speed when he hit the snow was 45 m/s. The work done on him as he fell the 1.1 m into the snow changed his kinetic energy. Both gravity and the snow did work on the pilot during that 1.1-meter motion. Gravity did positive work (the force was in the same direction as the displacement), and the snow did negative work (the force was in the opposite direction as the displacement).

$$W_{\text{gravity}} + W_{\text{snow}} = \Delta K \rightarrow mgd + W_{\text{snow}} = -\tfrac{1}{2} m v_i^2 \rightarrow$$

$$W_{\text{snow}} = -\tfrac{1}{2} m v_i^2 - mgd = -m\left(\tfrac{1}{2} v_i^2 + gd\right) = -(88\,\text{kg})\left[\tfrac{1}{2}(45\,\text{m/s})^2 + \left(9.80\,\text{m/s}^2\right)(1.1\,\text{m})\right]$$

$$= -9.005\times10^4\,\text{J} \approx \boxed{-9.0\times10^4\,\text{J}}$$

(*b*) The work done by the snowbank is done by an upward force, while the pilot moves down.

$$W_{\text{snow}} = F_{\text{snow}} d \cos 180^\circ = -F_{\text{snow}} d \rightarrow$$

$$F_{\text{snow}} = -\frac{W_{\text{snow}}}{d} = -\frac{-9.005\times10^4\text{ J}}{1.1\text{ m}} = 8.186\times10^4\,\text{N} \approx \boxed{8.2\times10^4\,\text{N}}$$

(*c*) During the pilot's fall in the air, positive work was done by gravity, and negative work by air resistance. The net work was equal to his change in kinetic energy while falling. We assume he started from rest when he jumped from the aircraft.

$$W_{\text{gravity}} + W_{\text{air}} = \Delta K \rightarrow mgh + W_{\text{air}} = \tfrac{1}{2} m v_f^2 - 0 \rightarrow$$

$$W_{\text{air}} = \tfrac{1}{2} m v_f^2 - mgh = m\left(\tfrac{1}{2} v_f^2 - gh\right) = (88\,\text{kg})\left[\tfrac{1}{2}(45\,\text{m/s})^2 - \left(9.80\,\text{m/s}^2\right)(370\,\text{m})\right]$$

$$= \boxed{-2.3\times10^5\,\text{J}}$$

83. The (negative) work done by the bumper on the rest of the car must equal the change in the car's kinetic energy. The work is negative because the force on the car is in the opposite direction to the car's displacement.

$$W_{\text{bumper}} = \Delta K = \rightarrow -\tfrac{1}{2} k x^2 = 0 - \tfrac{1}{2} m v_i^2 \rightarrow$$

$$k = m\frac{v_i^2}{x^2} = (1050\,\text{kg})\frac{\left[(8\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(0.015\,\text{m})^2} = \boxed{2\times10^7\ \text{N/m}}$$

84. The spring must be compressed a distance such that the work done by the spring is equal to the change in kinetic energy of the car. The distance of compression can then be used to find the spring constant. Note that the work done by the spring will be negative, since the force exerted by the spring is in the opposite direction to the displacement of the spring.

$$W_{\text{spring}} = \Delta K = \rightarrow -\tfrac{1}{2} k x^2 = 0 - \tfrac{1}{2} m v_i^2 \rightarrow x = v_i\sqrt{\frac{m}{k}}$$

$$F = ma = -kx \rightarrow m(-5.0g) = -k v_i \sqrt{\frac{m}{k}} \rightarrow$$

$$k = m\left(\frac{5.0g}{v_i}\right)^2 = (1300\,\text{kg})(25)\frac{(9.80\,\text{m/s}^2)^2}{\left[90\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2} = \boxed{5.0\times10^3\,\text{N/m}}$$

$\boxed{85.}$ If the rider is riding at a constant speed, then the positive work input by the rider to the (bicycle + rider) combination must be equal to the negative work done by gravity as he moves up the incline. The net work must be 0 if there is no change in kinetic energy.

(*a*) If the rider's force is directed downwards, then the rider will do an amount of work equal to the force times the distance parallel to the force. The distance parallel to the downward force would be the diameter of the circle in which the pedals move. Then consider that by using 2 feet, the rider does twice that amount of work when the pedals make one complete revolution. So in one revolution of the pedals, the rider does the work calculated below.

$$W_{\text{rider}} = 2(0.90m_{\text{rider}}g)\,d_{\substack{\text{pedal}\\\text{motion}}}$$

In one revolution of the front sprocket, the rear sprocket will make $42/19$ revolutions, and so the back wheel (and the entire bicycle and rider as well) will move a distance of $(42/19)(2\pi r_{\text{wheel}})$. That is a distance along the plane, and so the height that the bicycle and rider will move is $h = (42/19)(2\pi r_{\text{wheel}})\sin\theta$. Finally, the work done by gravity in moving that height is calculated.

$$W_{\text{G}} = (m_{\text{rider}} + m_{\text{bike}})gh\cos 180^\circ = -(m_{\text{rider}} + m_{\text{bike}})gh = -(m_{\text{rider}} + m_{\text{bike}})g(42/19)(2\pi r_{\text{wheel}})\sin\theta$$

Set the total work equal to 0, and solve for the angle of the incline.

$$W_{\text{rider}} + W_{\text{G}} = 0 \;\rightarrow\; 2[0.90m_{\text{rider}}g]d_{\substack{\text{pedal}\\\text{motion}}} - (m_{\text{rider}} + m_{\text{bike}})g(42/19)(2\pi r_{\text{wheel}})\sin\theta = 0 \;\rightarrow$$

$$\theta = \sin^{-1}\frac{(0.90m_{\text{rider}})d_{\substack{\text{pedal}\\\text{motion}}}}{(m_{\text{rider}} + m_{\text{bike}})(42/19)(\pi r_{\text{wheel}})} = \sin^{-1}\frac{0.90(65\,\text{kg})(0.36\,\text{m})}{(77\,\text{kg})(42/19)\pi(0.34\,\text{m})} = \boxed{6.7^\circ}$$

(*b*) If the force is tangential to the pedal motion, then the distance that one foot moves while exerting a force is now half of the circumference of the circle in which the pedals move. The rest of the analysis is the same.

$$W_{\text{rider}} = 2(0.90m_{\text{rider}}g)\left(\pi r_{\substack{\text{pedal}\\\text{motion}}}\right) \;;\; W_{\text{rider}} + W_{\text{G}} = 0 \;\rightarrow$$

$$\theta = \sin^{-1}\frac{(0.90m_{\text{rider}})\pi r_{\substack{\text{pedal}\\\text{motion}}}}{(m_{\text{rider}} + m_{\text{bike}})(42/19)(\pi r_{\text{wheel}})} = \sin^{-1}\frac{0.90(65\,\text{kg})(0.18\,\text{m})}{(77\,\text{kg})(42/19)(0.34\,\text{m})} = 10.5^\circ \approx \boxed{10^\circ}$$

86. Because the acceleration is essentially 0, the net force on the mass is 0. The magnitude of $\vec{\mathbf{F}}$ is found with the help of the free-body diagram in the textbook.

$$\sum F_y = F_{\text{T}}\cos\theta - mg = 0 \;\rightarrow\; F_{\text{T}} = \frac{mg}{\cos\theta}$$

$$\sum F_x = F - F_{\text{T}}\sin\theta = 0 \;\rightarrow\; F = F_{\text{T}}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

(*a*) A small displacement of the object along the circular path is given by $dr = \ell\, d\theta$, based on the definition of radian measure. The force $\vec{\mathbf{F}}$ is at an angle θ to the direction of motion. We use the symbol $d\vec{\mathbf{r}}$ for the infinitesimal displacement, since the symbol ℓ is already in use as the length of the pendulum.

$$W_F=\int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}=\int_{\theta=0}^{\theta=\theta_0} F\cos\theta\ell d\theta=\ell\int_{\theta=0}^{\theta=\theta_0}\left(mg\tan\theta\right)\cos\theta d\theta=mg\ell\int_{\theta=0}^{\theta=\theta_0}\sin\theta d\theta$$

$$=-mg\ell\cos\theta\Big|_0^{\theta_0}=\boxed{mg\ell\left(1-\cos\theta_0\right)}$$

(*b*) The angle between $m\vec{\mathbf{g}}$ and the direction of motion is $\left(90+\theta\right)$.

$$W_G=\int m\vec{\mathbf{g}}\cdot d\vec{\mathbf{r}}=mg\ell\int_{\theta=0}^{\theta=\theta_0}\cos\left(90^\circ+\theta\right)d\theta=-mg\ell\int_{\theta=0}^{\theta=\theta_0}\sin\theta d\theta$$

$$=mg\ell\cos\theta\Big|_0^{\theta_0}=\boxed{mg\ell\left(\cos\theta_0-1\right)}$$

Alternatively, it is proven in problem 36 that the shape of the path does not determine the work done by gravity – only the height change. Since this object is rising, gravity will do negative work.

$$W_G=mgd\cos\phi=mg\left(\text{height}\right)\cos 180^\circ=-mgy_{\text{final}}=-mg\left(\ell-\ell\cos\theta_0\right)$$

$$=\boxed{mg\ell\left(\cos\theta_0-1\right)}$$

Since $\vec{\mathbf{F}}_T$ is perpendicular to the direction of motion, it does $\boxed{0}$ work on the bob.

Note that the total work done is 0, since the object's kinetic energy does not change.

87. (*a*) The work done by the arms of the parent will change the kinetic energy of the child. The force is in the opposite direction of the displacement.

$$W_{\text{parent}}=\Delta K_{\text{child}}=K_f-K_i=0-\tfrac{1}{2}mv_i^2\ ;\ W_{\text{parent}}=F_{\text{parent}}d\cos 180^\circ\ \rightarrow$$

$$-\tfrac{1}{2}mv_i^2=-F_{\text{parent}}d\ \rightarrow\ F_{\text{parent}}=\frac{mv_i^2}{2d}=\frac{\left(18\,\text{kg}\right)\left(25\,\text{m/s}\right)^2}{2\left(45\,\text{m}\right)}=125\,\text{N}\approx\boxed{130\,\text{N}}\approx 28\,\text{lbs}$$

This force is achievable by an average parent.

(*b*) The same relationship may be used for the shorter distance.

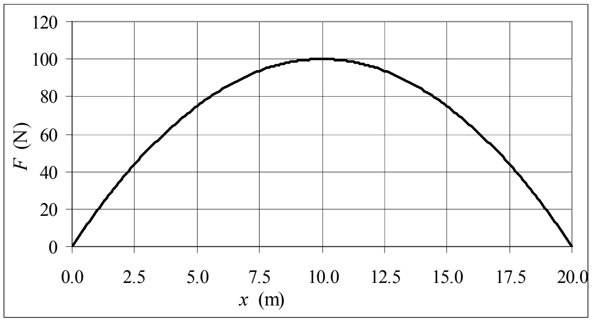
$$F_{\text{parent}}=\frac{mv_i^2}{2d}=\frac{\left(18\,\text{kg}\right)\left(25\,\text{m/s}\right)^2}{2\left(12\,\text{m}\right)}=469\,\text{N}\approx\boxed{470\,\text{N}}\approx 110\,\text{lbs}$$

This force may not be achievable by an average parent. Many people might have difficulty with a 110-pound bench press exercise, for example.

88. (*a*) From the graph, the shape of the force function is roughly that of a triangle. The work can be estimated using the formula for the area of a triangle of base 20 m and height 100 N.

$$W\approx\tfrac{1}{2}"b"\ "h"=\tfrac{1}{2}\left(20.0\,\text{m}\right)\left(100\,\text{N}\right)$$

$$=\boxed{1000\,\text{J}}$$



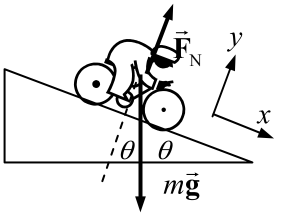
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.88a."

(*b*) Integrate the force function to find the exact work done.

$$W=\int_{x_i}^{x_f}F dx=\int_{0.0\,\text{m}}^{20.0\,\text{m}}\left[100-\left(x-10\right)^2\right]dx$$

$$= \int_{0.0\,\text{m}}^{20.0\,\text{m}} \left(20x - x^2\right)dx = \left[10x^2 - \tfrac{1}{3}x^3\right]_{0.0\,\text{m}}^{20.0\,\text{m}} = 1333\,\text{J} \approx \boxed{1330\,\text{J}}$$

89. (*a*) The work done by gravity is given by Eq. 7-1.



$$W_G = mgd\cos(90-\theta) = (85\,\text{kg})\left(9.80\,\text{m/s}^2\right)(250\,\text{m})\cos 86.0^\circ$$

$$= 1.453\times10^4\,\text{J} \approx \boxed{1.5\times10^4\,\text{J}}$$

(*b*) The work is the change in kinetic energy. The initial kinetic energy is 0.

$$W_G = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 \quad\rightarrow\quad v_f = \sqrt{\frac{2W_G}{m}} = \sqrt{\frac{2\left(1.453\times10^4\,\text{J}\right)}{85\,\text{kg}}} = \boxed{18\,\text{m/s}}$$

90. (*a*) The work-energy principle says the net work done is the change in kinetic energy. The climber both begins and ends the fall at rest, so the change in kinetic energy is 0. Thus the total work done (by gravity and by the rope) must be 0. This is used to find x. Note that the force of gravity is parallel to the displacement, so the work done by gravity is positive, but the force exerted by the rope is in the opposite direction to the displacement, so the work done by the rope is negative.

$$W_{\text{net}} = W_{\text{grav}} + W_{\text{rope}} = mg(2\ell + x) - \tfrac{1}{2}kx^2 = 0 \quad\rightarrow\quad \tfrac{1}{2}kx^2 - mgx - 2\ell mg = 0 \quad\rightarrow$$

$$x = \frac{mg \pm \sqrt{m^2g^2 - 4\left(\frac{1}{2}k\right)(-2\ell mg)}}{2\left(\frac{1}{2}k\right)} = \frac{mg \pm \sqrt{m^2g^2 + 4k\ell mg}}{k} = \frac{mg}{k}\left(1 \pm \sqrt{1 + \frac{4k\ell}{mg}}\right)$$

We have assumed that x is positive in the expression for the work done by gravity, and so the "plus" sign must be taken in the above expression.

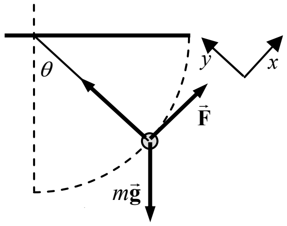
$$\text{Thus } x = \boxed{\frac{mg}{k}\left(1 + \sqrt{1 + \frac{4k\ell}{mg}}\right)}.$$

(*b*) Use the values given to calculate $\dfrac{x}{\ell}$ and $\dfrac{kx}{mg}$.

$$x = \frac{mg}{k}\left(1 + \sqrt{1 + \frac{4k\ell}{mg}}\right) = \frac{(85\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{(850\,\text{N/m})}\left(1 + \sqrt{1 + \frac{4(850\,\text{N/m})(8.0\,\text{m})}{(85\,\text{kg})\left(9.80\,\text{m/s}^2\right)}}\right) = 6.665\,\text{m}$$

$$\frac{x}{\ell} = \frac{6.665\,\text{m}}{8.0\,\text{m}} = \boxed{0.83}\ ;\quad \frac{kx}{mg} = \frac{(850\,\text{N/m})(6.665\,\text{m})}{(85\,\text{kg})\left(9.80\,\text{m/s}^2\right)} = \boxed{6.8}$$

$\boxed{91.}$ Refer to the free body diagram. The coordinates are defined simply to help analyze the components of the force. At any angle θ, since the mass is not accelerating, we have the following.

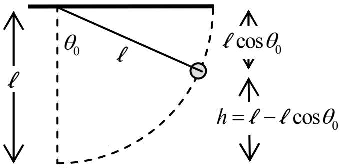


$$\sum F_x = F - mg\sin\theta = 0 \quad\rightarrow\quad F = mg\sin\theta$$

Find the work done in moving the mass from $\theta = 0$ to $\theta = \theta_0$.

$$W_F = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = \int_{\theta=0}^{\theta=\theta_0} F\cos 0^\circ \ell\, d\theta = mg\ell \int_{\theta=0}^{\theta=\theta_0} \sin\theta\, d\theta$$

$$= -mg\ell\cos\theta\Big|_0^{\theta_0} = mg\ell\left(1 - \cos\theta_0\right)$$



See the second diagram to find the height that the mass has risen. We see that $h=\ell-\ell\cos\theta_0=\ell(1-\cos\theta_0)$, and so

$\boxed{W_F=mg\ell(1-\cos\theta_0)=mgh}$.

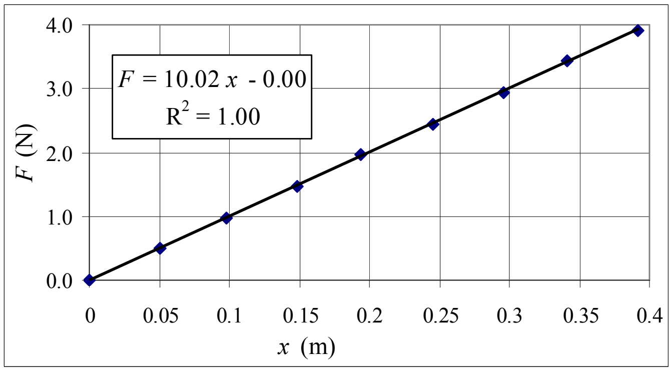
92. For each interval, the average force for that interval was calculated as the numeric average of the forces at the beginning and end of the interval. Then this force was multiplied by 10.0 cm (0.0100 m) to find the work done on that interval. The total work is the sum of those work amounts. That process is expressed in a formula below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.92."

$$W_{\text{applied}}=\sum_{i=1}^{n-1}\tfrac{1}{2}(F_i+F_{i+1})\Delta x=102.03\,\text{J}\approx\boxed{102\,\text{J}}$$

93. (*a*) See the adjacent graph. The best-fit straight line is as follows.

$$\boxed{F_{\text{applied}}=(10.0\,\text{N/m})\,x}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.93a."



(*b*) Since $F_{\text{applied}}=kx$ for the stretched spring, the slope is the spring constant.

$$k=\boxed{10.0\,\text{N/m}}$$

(*c*) Use the best-fit equation from the graph.

$$F=kx=(10.0\,\text{N/m})(0.200\text{m})=\boxed{2.00\text{N}}$$

CHAPTER 8: Conservation of Energy

Responses to Questions

1. Friction is not conservative; it dissipates energy in the form of heat, sound, and light. Air resistance is not conservative; it dissipates energy in the form of heat and the kinetic energy of fluids. “Human” forces, for example, the forces produced by your muscles, are also not conservative. They dissipate energy in the form of heat and also through chemical processes.

2. The two forces on the book are the applied force upward (nonconservative) and the downward force of gravity (conservative). If air resistance is non-negligible, it is nonconservative.

3. (*a*) If the net force is conservative, the change in the potential energy is equal to the negative of the change in the kinetic energy, so $\Delta U = -300$ J.
   (*b*) If the force is conservative, the total mechanical energy is conserved, so $\Delta E = 0$.

4. No. The maximum height on the rebound cannot be greater than the initial height if the ball is dropped. Initially, the dropped ball’s total energy is gravitational potential energy. This energy is changed to other forms (kinetic as it drops, and elastic potential during the collision with the floor) and eventually back into gravitational potential energy as the ball rises back up. The final energy cannot be greater than the initial (unless there is an outside energy source) so the final height cannot be greater than the initial height. Note that if you *throw* the ball down, it initially has kinetic energy as well as potential so it may rebound to a greater height.

5. (*a*) No. If there is no friction, then gravity is the only force doing work on the sled, and the system is conservative. All of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy. The speed at the bottom of the hill depends only on the initial height *h*, and not on the angle of the hill. $K_{\mathrm{f}} = \frac{1}{2}mv^2 = mgh,$ and $v = (2gh)^{1/2}$.
   (*b*) Yes. If friction is present, then the net force doing work on the sled is not conservative. Only part of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy; the rest will be dissipated by the frictional force. The frictional force is proportional to the normal force on the sled, which will depend on the angle θ of the hill. $K_{\mathrm{f}} = \frac{1}{2}mv^2 = mgh - fx = mgh - \mu mgh\cos\theta/\sin\theta = mgh(1-\mu/\tan\theta),$ and $v = [2gh(1-\mu/\tan\theta)]^{1/2},$ which does depend on the angle of the hill and will be smaller for smaller angles.

6. No work is done *on the wall* (since the wall does not undergo displacement) but internally your muscles are converting chemical energy to other forms of energy, which makes you tired.

7. At the top of the pendulum’s swing, all of its energy is gravitational potential energy; at the bottom of the swing, all of the energy is kinetic.
   (*a*) If we can ignore friction, then energy is transformed back and forth between potential and kinetic as the pendulum swings.
   (*b*) If friction is present, then during each swing energy is lost to friction at the pivot point and also to air resistance. During each swing, the kinetic energy and the potential energy decrease, and the pendulum’s amplitude decreases. When a grandfather clock is wound up, the energy lost to friction and air resistance is replaced by energy stored as potential energy (either elastic or gravitational, depending on the clock mechanism).

8. The drawing shows water falling over a waterfall and then flowing back to the top of the waterfall. The top of the waterfall is above the bottom, with greater gravitational potential energy. The optical illusion of thhe diagram implies that water is flowing freely from the bottom of the waterfall back to the top. Since water won't move uphill unless work is done on it to increase its gravitational potential energy (for example, work done by a pump), the water from the bottom of the waterfall would NOT be able to make it back to the top.

9. For each of the water balloons, the initial energy (kinetic plus potential) will equal the final energy (all kinetic). Since the initial energy depends only on the speed and not on the direction of the initial velocity, and all balloons have the same initial speed and height, the final speeds will all be the same. $\left[E_i = \tfrac{1}{2}mv_i^2 + mgh = E_f = \tfrac{1}{2}mv_f^2\right]$

10. Yes, the spring can leave the table. When you push down on the spring, you do work on it and it gains elastic potential energy, and loses a little gravitational potential energy, since the center of mass of the spring is lowered. When you remove your hand, the spring expands, and the elastic potential energy is converted into kinetic energy and into gravitational potential energy. If enough elastic potential energy was stored, the center of mass of the spring will rise above its original position, and the spring will leave the table.

11. The initial potential energy of the water is converted first into the kinetic energy of the water as it falls. When the falling water hits the pool, it does work on the water already in the pool, creating splashes and waves. Additionally, some energy is converted into heat and sound.

12. Stepping on top of a log and jumping down the other side requires you to raise your center of mass farther than just stepping over a log does. Raising your center of mass farther requires you to do more work, or use more energy.

13. (*a*) As a car accelerates uniformly from rest, the potential energy stored in the fuel is converted into kinetic energy in the engine and transmitted through the transmission into the turning of the wheels, which causes the car to accelerate (if friction is present between the road and the tires).
    (*b*) If there is a friction force present between the road and the tires, then when the wheels turn, the car moves forward and gains kinetic energy. If the static friction force is large enough, then the point of contact between the tire and the road is instantaneously at rest – it serves as an instantaneous axis of rotation. If the static friction force is not large enough, the tire will begin to slip, or skid, and the wheel will turn without the car moving forward as fast. If the static friction force is very small, the wheel may spin without moving the car forward at all, and the car will not gain any kinetic energy (except the kinetic energy of the spinning tires).

14. The gravitational potential energy is the greatest when the Earth is farthest from the Sun, or when the Northern Hemisphere has summer. (Note that the Earth moves fastest in its orbit, and therefore has the greatest *kinetic energy*, when it is closest to the Sun.)

15. Yes. If the potential energy U is negative (which it can be defined to be), and the absolute value of the potential energy is greater than the kinetic energy K, then the total mechanical energy E will be negative.

16. In order to escape the Earth's gravitational field, the rocket needs a certain minimum speed with respect to the center of the Earth. If you launch the rocket from any location except the poles, then the rocket will have a tangential velocity due to the rotation of the Earth. This velocity is towards the east and is greatest at the equator, where the surface of the Earth is farthest from the axis of rotation. In order to use the minimum amount of fuel, you need to maximize the contribution of this tangential

velocity to the needed escape velocity, so launch the rocket towards the east from a point as close as possible to the equator. (As an added bonus, the weight of the rocket will be slightly less at the equator because the Earth is not a perfect sphere and the surface is farthest from the center at the equator.)

17. For every meter the load is raised, two meters of rope must be pulled up. The work done on the piano must be equal to the work done by you. Since you are pulling with half the force (the tension in the rope is equal to half of the weight of the piano), you must pull through twice the distance to do the same amount of work.

18. The faster arrow has the same mass and twice the speed of the slower arrow, so will have four times the kinetic energy $\left(K = \frac{1}{2}mv^2\right)$. Therefore, four times as much work must be done on the faster arrow to bring it to rest. If the force on the arrows is constant, the faster arrow will travel four times the distance of the slower arrow in the hay.

$\boxed{19.}$ When the ball is released, its potential energy will be converted into kinetic energy and then back into potential energy as the ball swings. If the ball is not pushed, it will lose a little energy to friction and air resistance, and so will return almost to the initial position, but will not hit the instructor. If the ball is pushed, it will have an initial kinetic energy, and will, when it returns, still have some kinetic energy when it reaches the initial position, so it will hit the instructor in the nose. (Ouch!)

20. Neglecting any air resistance or friction in the pivot, the pendulum bob will have the same speed at the lowest point for both launches. In both cases, the initial energy is equal to potential energy *mgh* plus kinetic energy $\frac{1}{2}mv^2$, with $v = 3.0$ m/s. (Notice that the direction of the velocity doesn't matter.) Since the total energy at any point in the swing is constant, the pendulum will have the same energy at the lowest point, and therefore the same speed, for both launches.

21. When a child hops around on a pogo stick, gravitational potential energy (at the top of the hop) is transformed into kinetic energy as the child moves downward, and then stored as spring potential energy as the spring in the pogo stick compresses. As the spring begins to expand, the energy is converted back to kinetic and gravitational potential energy, and the cycle repeats. Since energy is lost due to friction, the child must add energy to the system by pushing down on the pogo stick while it is on the ground to get a greater spring compression.

22. At the top of the hill, the skier has gravitational potential energy. If the friction between her skis and the snow is negligible, the gravitational potential energy is changed into kinetic energy as she glides down the hill and she gains speed as she loses elevation. When she runs into the snow bank, work is done by the friction between her skis and the snow and the energy changes from kinetic energy of the skier to kinetic energy of the snow as it moves and to thermal energy.

23. The work done on the suitcase depends only on (*c*) the height of the table and (*d*) the weight of the suitcase.

24. Power is the rate of doing work. Both (*c*) and (*d*) will affect the total amount of work needed, and hence the power. (*b*), the time the lifting takes, will also affect the power. The length of the path (*a*) will only affect the power if different paths take different times to traverse.

$\boxed{25.}$ When you climb a mountain by going straight up, the force needed is large (and the distance traveled is small), and the power needed (work per unit time) is also large. If you take a zigzag trail, you will use a smaller force (over a longer distance, so that the work done is the same) and less power, since the time to climb the mountain will be longer. A smaller force and smaller power output make the climb seem easier.

26. (*a*) The force is proportional to the negative of the slope of the potential energy curve, so the magnitude of the force will be greatest where the curve is steepest, at point C.
    (*b*) The force acts to the left at points A, E, and F, to the right at point C, and is zero at points B, D, and G.
    (*c*) Equilibrium exists at points B, D, and G. B is a point of neutral equilibrium, D is a point of stable equilibrium, and G is a point of unstable equilibrium.

27. (*a*) If the particle has E_3 at x_6, then it has both potential and kinetic energy at that point. As the particle moves toward x_0 , it gains kinetic energy as its speed increases. Its speed will be a maximum at x_0. As the particle moves to x_4, its speed will decrease, but will be larger than its initial speed. As the particle moves to x_5, its speed will increase, then decrease to zero. The process is reversed on the way back to x_6. At each point on the return trip the speed of the particle is the same as it was on the forward trip, but the direction of the velocity is opposite.
    (*b*) The kinetic energy is greatest at point x_0, and least at x_5.

28. A is a point of unstable equilibrium, B is a point of stable equilibrium, and C is a point of neutral equilibrium.

Solutions to Problems

$\boxed{1.}$ The potential energy of the spring is given by $U_{\text{el}} = \frac{1}{2}kx^2$ where x is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2U_{\text{el}}}{k}} = \sqrt{\frac{2(35.0\,\text{J})}{82.0\,\text{N/m}}} = \boxed{0.924\,\text{m}}$$

2. Subtract the initial gravitational potential energy from the final gravitational potential energy.

$$\Delta U_{\text{grav}} = mgy_2 - mgy_1 = mg(y_2 - y_1) = (6.0\ \text{kg})(9.80\,\text{m/s}^2)(1.3\,\text{m}) = \boxed{76\,\text{J}}$$

3. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$mg = k(\Delta x) \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(2.5\ \text{kg})(9.80\,\text{m/s}^2)}{63\,\text{N/m}} = 0.39\,\text{m}$$

Thus the ruler reading will be $\boxed{39\,\text{cm} + 15\,\text{cm} = 54\,\text{cm}}$.

4. (*a*) The change in gravitational potential energy is given by the following.

$$\Delta U_{\text{grav}} = mg(y_2 - y_1) = (56.5\,\text{kg})(9.80\,\text{m/s}^2)(2660\,\text{m} - 1270\,\text{m}) = \boxed{7.7\times10^5\,\text{J}}$$

(*b*) The minimum work required by the hiker would equal the change in potential energy, which is $\boxed{7.7\times10^5\,\text{J}}$.

(*c*) $\boxed{\text{Yes}}$. The actual work may be more than this, because the climber almost certainly had to overcome some dissipative forces such as air friction. Also, as the person steps up and down, they do not get the full amount of work back from each up-down event. For example, there will be friction in their joints and muscles.

5. (*a*) Relative to the ground, the potential energy is given by the following.

$$U_{\text{grav}} = mg\left(y_{\text{book}} - y_{\text{ground}}\right) = (1.95\,\text{kg})\left(9.80\,\text{m/s}^2\right)(2.20\,\text{m}) = \boxed{42.0\text{ J}}$$

(*b*) Relative to the top of the person's head, the potential energy is given by the following.

$$U_{\text{grav}} = mg\left(y_{\text{book}} - y_{\text{head}}\right) = (1.95\,\text{kg})\left(9.80\,\text{m/s}^2\right)(2.20\,\text{m} - 1.60\,\text{m}) = 11.47\,\text{J} \approx \boxed{11\,\text{J}}$$

(*c*) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part (*a*), $\boxed{42.0\text{ J}}$. In part (*a*), the potential energy is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part (*b*), because the potential energy is not calculated relative to the starting location of the application of the force on the book.

6. Assume that all of the kinetic energy of the car becomes potential energy of the compressed spring.

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kx_{\text{final}}^2 \;\rightarrow\; k = \frac{mv_0^2}{x_{\text{final}}^2} = \frac{(1200\,\text{kg})\left[(75\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(2.2\text{ m})^2} = \boxed{1.1\times10^5\text{ N/m}}$$

$\boxed{7.}$ (*a*) This force is conservative, because the work done by the force on an object moving from an initial position (x_1) to a final position (x_2) depends only on the endpoints.

$$W = \int_{x_1}^{x_2} \vec{\mathbf{F}}\bullet d\vec{\mathbf{l}} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2}\left(-kx + ax^3 + bx^4\right)dx = \left(-\tfrac{1}{2}kx^2 + \tfrac{1}{4}ax^4 + \tfrac{1}{5}bx^5\right)_{x_1}^{x_2}$$

$$= \left(-\tfrac{1}{2}kx_2^2 + \tfrac{1}{4}ax_2^4 + \tfrac{1}{5}bx_2^5\right) - \left(-\tfrac{1}{2}kx_1^2 + \tfrac{1}{4}ax_1^4 + \tfrac{1}{5}bx_1^5\right)$$

The expression for the work only depends on the endpoints.

(*b*) Since the force is conservative, there is a potential energy function *U* such that $F_x = -\dfrac{\partial U}{\partial x}$.

$$F_x = \left(-kx + ax^3 + bx^4\right) = -\frac{\partial U}{\partial x} \;\rightarrow\; \boxed{U(x) = \tfrac{1}{2}kx^2 - \tfrac{1}{4}ax^4 - \tfrac{1}{5}bx^5 + C}$$

8. The force is found from the relations on page 189.

$$F_x = -\frac{\partial U}{\partial x} = -(6x + 2y) \qquad F_y = -\frac{\partial U}{\partial y} = -(2x + 8yz) \qquad F_z = -\frac{\partial U}{\partial z} = -4y^2$$

$$\boxed{\vec{\mathbf{F}} = \hat{\mathbf{i}}(-6x - 2y) + \hat{\mathbf{j}}(-2x - 8yz) + \vec{\mathbf{k}}\left(-4y^2\right)}$$

9. Use Eq. 8-6 to find the potential energy function.

$$U(x) = -\int F(x)\,dx + C = -\int -\frac{k}{x^3}dx + C = -\frac{k}{2x^2} + C$$

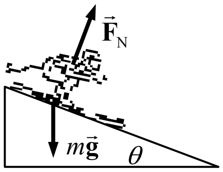
$$U(2.0\,\text{m}) = -\frac{k}{2(2.0\,\text{m})^2} + C = 0 \;\rightarrow\; C = \frac{k}{8\text{m}^2} \;\rightarrow\; \boxed{U(x) = -\frac{k}{2x^2} + \frac{k}{8\text{m}^2}}$$

10. Use Eq. 8-6 to find the potential energy function.

$$U(x) = -\int F(x)\,dx + C = -\int A\sin(kx)\,dx + C = \frac{A}{k}\cos(kx) + C$$

$$U(0)=\frac{A}{k}+C=0 \ \rightarrow\ C=-\frac{A}{k} \ \rightarrow\ \boxed{U(x)=\frac{A}{k}\left[\cos(kx)-1\right]}$$

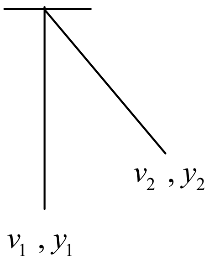
11. The forces on the skier are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the skier's mechanical energy is conserved. Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at the bottom of the hill. The ground is the zero location for gravitational potential energy $(y=0)$. We have $v_1=0$, $y_1=125\,\text{m}$, and $y_2=0$ (bottom of the hill). Solve for v_2, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ 0+mgy_1=\tfrac{1}{2}mv_2^2+0 \ \rightarrow$$

$$v_2=\sqrt{2gy_1}=\sqrt{2\left(9.80\,\text{m/s}^2\right)(125\,\text{m})}=\boxed{49\,\text{m/s}}\left(\approx 110\,\text{mi/h}\right)$$

12. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work – the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for gravitational potential energy $(y=0)$. We have $v_1=5.0\,\text{m/s}$, $y_1=0$, and $v_2=0$ (top of swing). Solve for y_2, the height of her swing.



$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+0=0+mgy_2 \ \rightarrow$$

$$y_2=\frac{v_1^2}{2g}=\frac{(5.0\,\text{m/s})^2}{2\left(9.80\,\text{m/s}^2\right)}=1.276\,\text{m}\approx\boxed{1.3\,\text{m}}$$

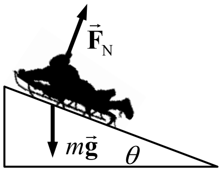
$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.65 m long. If that were the case, she could not rise 1.3 m high.

$\boxed{13.}$ We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for gravitational potential energy $(y=0)$. We have $y_1=0$, $v_2=0.70\,\text{m/s}$, and $y_2=2.10\,\text{m}$. Solve for v_1, the speed at the bottom.

$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+0=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow$$

$$v_1=\sqrt{v_2^2+2gy_2}=\sqrt{(0.70\,\text{m/s})^2+2\left(9.80\,\text{m/s}^2\right)(2.10\ \text{m})}=6.454\,\text{m/s}\approx\boxed{6.5\,\text{m/s}}$$

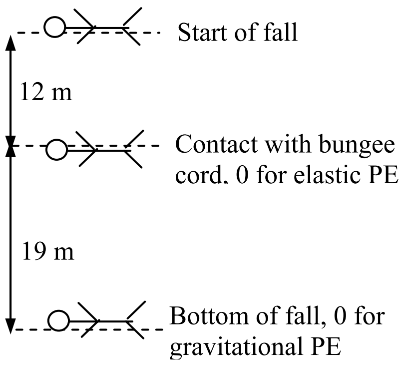
14. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for gravitational potential energy $(y=0)$. We have $y_1=0$, $v_2=0$, and $y_2=1.12\,\text{m}$. Solve for v_1, the speed at the bottom. Note that the angle is not used.



$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+0=0+mgy_2 \ \rightarrow$$

$$v_1=\sqrt{2gy_2}=\sqrt{2\left(9.80\,\text{m/s}^2\right)(1.12\,\text{m})}=\boxed{4.69\,\text{m/s}}$$

15. Consider this diagram for the jumper's fall.

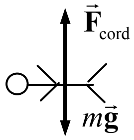


(*a*) The mechanical energy of the jumper is conserved. Use *y* for the distance from the 0 of gravitational potential energy and *x* for the amount of bungee cord "stretch" from its unstretched length. Subscript 1 represents the jumper at the start of the fall, and subscript 2 represents the jumper at the lowest point of the fall. The bottom of the fall is the zero location for gravitational potential energy $(y=0)$, and the location where the bungee cord just starts to be stretched is the zero location for elastic potential energy $(x=0)$. We have $v_1=0,\ y_1=31\,\text{m},\ x_1=0,\ v_2=0,$ $y_2=0,$ and $x_2=19\,\text{m}$. Apply conservation of energy.

$$E_1=E_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+mgy_1+\tfrac{1}{2}kx_1^2=\tfrac{1}{2}mv_2^2+mgy_2+\tfrac{1}{2}kx_2^2 \ \rightarrow\ mgy_1=\tfrac{1}{2}kx_2^2 \ \rightarrow$$

$$k=\frac{2mgy_1}{x_2^2}=\frac{2(55\text{ kg})(9.80\,\text{m/s}^2)(31\text{ m})}{(19\text{ m})^2}=92.57\,\text{N/m}\approx\boxed{93\,\text{N/m}}$$

(*b*) The maximum acceleration occurs at the location of the maximum force, which occurs when the bungee cord has its maximum stretch, at the bottom of the fall. Write Newton's second law for the force on the jumper, with upward as positive.



$$F_{\text{net}}=F_{\text{cord}}-mg=kx_2-mg=ma \ \rightarrow$$

$$a=\frac{kx_2}{m}-g=\frac{(92.57\,\text{N/m})(19\text{ m})}{(55\text{ kg})}-9.80\,\text{m/s}^2=22.2\,\text{m/s}^2\approx\boxed{22\,\text{m/s}^2}$$

16. (*a*) Since there are no dissipative forces present, the mechanical energy of the person–trampoline–Earth combination will be conserved. We take the level of the unstretched trampoline as the zero level for both elastic and gravitational potential energy. Call up the positive direction. Subscript 1 represents the jumper at the start of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic potential energy involved in this part of the problem. We have $v_1=4.5\,\text{m/s}$, $y_1=2.0$ m, and $y_2=0$. Solve for v_2, the speed upon arriving at the trampoline.

$$E_1=E_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+0 \ \rightarrow$$

$$v_2=\pm\sqrt{v_1^2+2gy_1}=\pm\sqrt{(4.5\,\text{m/s})^2+2(9.80\,\text{m/s}^2)(2.0\text{ m})}=\pm7.710\,\text{m/s}\approx\boxed{7.7\,\text{m/s}}$$

The speed is the absolute value of v_2.

(*b*) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline, and *x* represent the amount of stretch of the trampoline. We have $v_2=-7.710\,\text{m/s}$, $y_2=0$, $x_2=0$, $v_3=0$, and $x_3=y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational potential energy at position 3 is negative, and so $y_3<0$. A quadratic relationship results from the conservation of energy condition.

$$E_2=E_3 \ \rightarrow\ \tfrac{1}{2}mv_2^2+mgy_2+\tfrac{1}{2}kx_2^2=\tfrac{1}{2}mv_3^2+mgy_3+\tfrac{1}{2}kx_3^2 \ \rightarrow$$

$$\tfrac{1}{2}mv_2^2+0+0=0+mgy_3+\tfrac{1}{2}ky_3^2 \ \rightarrow\ \tfrac{1}{2}ky_3^2+mgy_3-\tfrac{1}{2}mv_2^2=0 \ \rightarrow$$

$$y_3=\frac{-mg\pm\sqrt{m^2g^2-4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_2^2\right)}}{2\left(\tfrac{1}{2}k\right)}=\frac{-mg\pm\sqrt{m^2g^2+kmv_2^2}}{k}$$

$$=\frac{-(72\,\text{kg})(9.80\,\text{m/s}^2)\pm\sqrt{(72\,\text{kg})^2(9.80\,\text{m/s}^2)^2+(5.8\times10^4\,\text{N/m})(72\,\text{kg})(7.71\,\text{m/s})^2}}{(5.8\times10^4\,\text{N/m})}$$

$$=-0.284\,\text{m}\,,\;0.260\,\text{m}$$

Since $y_3<0$, $y_3=\boxed{-0.28\,\text{m}}$. .

The first term under the quadratic is about 500 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational potential energy for the final position. If that approximation were made, the result would have been found by taking the negative result from the following solution.

$$E_2=E_3\;\rightarrow\;\tfrac{1}{2}mv_2^2=\tfrac{1}{2}ky_3^2\;\rightarrow\;y_3=v_2\sqrt{\frac{m}{k}}=(7.71\,\text{m/s})\sqrt{\frac{72\,\text{kg}}{5.8\times10^4\,\text{N/m}}}=\pm0.27\,\text{m}$$

17. Take specific derivatives with respect to position, and note that E is constant.

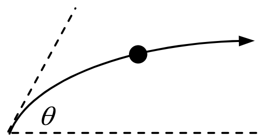
$$E=\tfrac{1}{2}mv^2+U\;\rightarrow\;\frac{dE}{dx}=\tfrac{1}{2}m\left(2v\frac{dv}{dx}\right)+\frac{dU}{dx}=mv\frac{dv}{dx}+\frac{dU}{dx}=0$$

Use the chain rule to change $v\dfrac{dv}{dx}$ to $\dfrac{dx}{dt}\dfrac{dv}{dx}=\dfrac{dv}{dt}$.

$$mv\frac{dv}{dx}+\frac{dU}{dx}=0\;\rightarrow\;m\frac{dv}{dt}=-\frac{dU}{dx}\;\rightarrow\;\boxed{ma=F}$$

The last statement is Newton's second law.

18. (*a*) See the diagram for the thrown ball. The speed at the top of the path will be the horizontal component of the original velocity.



$$v_{\text{top}}=v_0\cos\theta=(8.5\,\text{m/s})\cos36^\circ=\boxed{6.9\,\text{m/s}}$$

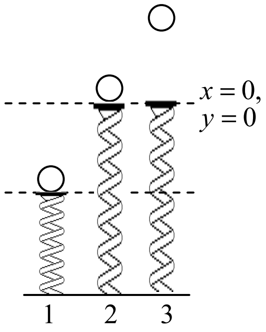
(*b*) Since there are no dissipative forces in the problem, the mechanical energy of the ball is conserved. Subscript 1 represents the ball at the release point, and subscript 2 represents the ball at the top of the path. The ball's release point is the zero location for gravitational potential energy $(y=0)$. We have $v_1=8.5\,\text{m/s},\;y_1=0,$ and $v_2=v_1\cos\theta.$ Solve for y_2.

$$E_1=E_2\;\rightarrow\;\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2\;\rightarrow\;\tfrac{1}{2}mv_1^2+0=\tfrac{1}{2}mv_1^2\cos^2\theta+mgy_2\;\rightarrow$$

$$y_2=\frac{v_1^2\left(1-\cos^2\theta\right)}{2g}=\frac{(8.5\,\text{m/s})^2\left(1-\cos^2 36^\circ\right)}{2(9.80\,\text{m/s}^2)}=\boxed{1.3\text{ m}}$$

This is the height above its throwing level.

$\boxed{19.}$ Use conservation of energy. The level of the ball on the uncompressed spring is taken as the zero location for both gravitational potential energy $(y=0)$ and elastic potential energy $(x=0)$. It is diagram 2 in the figure. Take "up" to be positive for both x and y.



(*a*) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_1=0,\;x_1=y_1=-0.160\,\text{m},$ and $x_2=y_2=0.$ Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0 + 0 \quad \rightarrow \quad v_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

$$v_2 = \sqrt{\frac{(875\,\text{N/m})(0.160\text{ m})^2 + 2(0.380\text{ kg})(9.80\,\text{m/s}^2)(-0.160\text{ m})}{(0.380\text{ kg})}} = \boxed{7.47\,\text{m/s}}$$

(*b*) Subscript 3 represents the ball at its highest point. We have $v_1 = 0,\ x_1 = y_1 = -0.160\,\text{m},\ v_3 = 0,$ and $x_3 = 0.$ Solve for y_3.

$$E_1 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \quad \rightarrow \quad y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{(875\,\text{N/m})(0.160\text{ m})^2}{2(0.380\text{ kg})(9.80\,\text{m/s}^2)} = \boxed{3.01\,\text{m}}$$

20. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_1 = 0$ and $y_1 = 32\,\text{m}$.

Point 2: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \ ;\ y_2 = 0 \ \rightarrow\ mgy_1 = \tfrac{1}{2}mv_2^2 \ \rightarrow$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\,\text{m/s}^2)(32\,\text{m})} = \boxed{25\,\text{m/s}}$$

Point 3: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3 \ ;\ y_3 = 26\,\text{m} \ \rightarrow\ mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3 \ \rightarrow$

$$v_3 = \sqrt{2g(y_1 - y_3)} = \sqrt{2(9.80\,\text{m/s}^2)(6\,\text{m})} = \boxed{11\,\text{m/s}}$$

Point 4: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_4 \ ;\ y_4 = 14\,\text{m} \ \rightarrow\ mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_1 \ \rightarrow$

$$v_4 = \sqrt{2g(y_1 - y_4)} = \sqrt{2(9.80\,\text{m/s}^2)(18\,\text{m})} = \boxed{19\,\text{m/s}}$$

21. With the mass at rest on the spring, the upward force due to the spring must be the same as the weight of the mass.

$$kd = mg \quad \rightarrow \quad d = \frac{mg}{k}$$

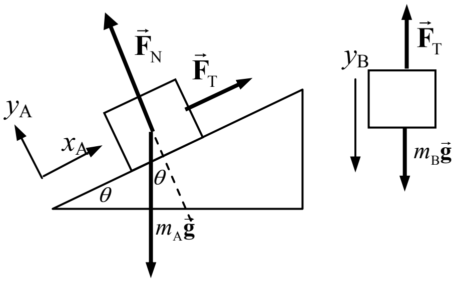
The distance *D* is found using conservation of energy. Subscript 1 represents the mass at the top of the uncompressed spring, and subscript 2 represents the mass at the bottom of its motion, where the spring is compressed by *D*. Take the top of the uncompressed spring to be the zero location for both gravitational and elastic potential energy $(y = 0)$. Choose up to be the positive direction. We have $v_1 = v_2 = 0,\ y_1 = 0,$ and $y_2 = -D.$ Solve for *D*.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}ky_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad \rightarrow$$

$$0 + 0 + 0 = 0 - mgD + \tfrac{1}{2}kD^2 \quad \rightarrow \quad D = \frac{2mg}{k}$$

We see that $\boxed{D = 2d}$, and so $D \neq d$. The reason that the two distances are not equal is that putting the mass at rest at the compressed position requires that other work be done in addition to the work done by gravity and the spring. That other work is not done by a conservative force, but done instead by an external agent such as your hand.

22. (*a*) Draw a free-body diagram for each block. Write Newton's second law for each block. Notice that the acceleration of block A in the y_A is 0 zero.



$$\sum F_{y1} = F_N - m_A g\cos\theta = 0 \;\rightarrow\; F_N = m_A g\cos\theta$$
$$\sum F_{x1} = F_T - m_A g\sin\theta = m_A a_{xA}$$
$$\sum F_{y2} = m_B g - F_T = m_B a_{yB} \;\rightarrow\; F_T = m_B\left(g + a_{yB}\right)$$

Since the blocks are connected by the cord, $a_{yB} = a_{xA} = a$. Substitute the expression for the tension force from the last equation into the *x* direction equation for block 1, and solve for the acceleration.

$$m_B\left(g + a\right) - m_A g\sin\theta = m_A a \;\rightarrow\; m_B g - m_A g\sin\theta = m_A a + m_B a$$
$$a = g\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)} = \left(9.80\,\mathrm{m/s^2}\right)\frac{\left(5.0\,\mathrm{kg} - 4.0\,\mathrm{kg}\sin 32^\circ\right)}{9.0\,\mathrm{kg}} = \boxed{3.1\,\mathrm{m/s^2}}$$

(*b*) Find the final speed of m_B (which is also the final speed of m_A) using constant acceleration relationships.

$$v_f^2 = v_0^2 + 2a\Delta y \;\rightarrow\; v_f^2 = 2g\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)}h \;\rightarrow$$
$$v_f = \sqrt{2gh\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)}} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.75\,\mathrm{m}\right)\frac{\left(5.0\,\mathrm{kg} - 4.0\,\mathrm{kg}\sin 32^\circ\right)}{9.0\,\mathrm{kg}}} = \boxed{2.2\,\mathrm{m/s}}$$

(*c*) Since there are no dissipative forces in the problem, the mechanical energy of the system is conserved. Subscript 1 represents the blocks at the release point, and subscript 2 represents the blocks when m_B reaches the floor. The ground is the zero location for gravitational potential energy for m_B, and the starting location for m_A is its zero location for gravitational potential energy. Since m_B falls a distance *h*, m_A moves a distance *h* along the plane, and so rises a distance $h\sin\theta$. The starting speed is 0.

$$E_1 = E_2 \;\rightarrow\; 0 + m_A gh = \tfrac{1}{2}\left(m_A + m_B\right)v_2^2 + m_B gh\sin\theta \;\rightarrow$$
$$\boxed{v_2 = \sqrt{2gh\left(\frac{m_A - m_B\sin\theta}{m_A + m_B}\right)}}$$

This is the same expression found in part (*b*), and so gives the same numeric result.

23. At the release point the mass has both kinetic energy and elastic potential energy. The total energy is $\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2$. If friction is to be ignored, then that total energy is constant.

(*a*) The mass has its maximum speed at a displacement of 0, and so only has kinetic energy at that point.

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_{\max}^2 \;\rightarrow\; v_{\max} = \boxed{\sqrt{v_0^2 + \frac{k}{m}x_0^2}}$$

(*b*) The mass has a speed of 0 at its maximum stretch from equilibrium, and so only has potential energy at that point.

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}kx_{\max}^2 \;\rightarrow\; x_{\max} = \boxed{\sqrt{x_0^2 + \frac{m}{k}v_0^2}}$$

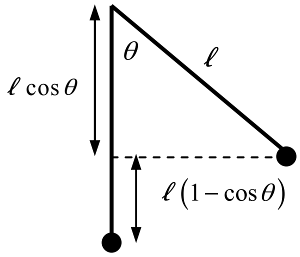
24. (*a*) The work done against gravity is the change in potential energy.

$$W_{\substack{\text{against}\\\text{gravity}}} = \Delta U = mg\left(y_2 - y_1\right) = \left(75.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(125\,\text{m}\right) = \boxed{9.19\times10^4\,\text{J}}$$

(*b*) The work done by the force on the pedals in one revolution is equal to the average tangential force times the circumference of the circular path of the pedals. That work is also equal to the potential energy change of the bicycle during that revolution, assuming that the speed of the bicycle is constant. Note that a vertical rise on the incline is related to the distance along the incline by $\text{rise} = \text{distance}\times(\sin\theta)$.

$$W_{\substack{\text{pedal}\\\text{force}}} = F_{\text{tan}}2\pi r = \Delta U_{\substack{\text{grav}\\\text{1 rev}}} = mg\left(\Delta y\right)_{\text{1 rev}} = mgd_{\text{1 rev}}\sin\theta \;\rightarrow$$

$$F_{\text{tan}} = \frac{mgd_{\text{1 rev}}\sin\theta}{2\pi r} = \frac{\left(75.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(5.10\,\text{m}\right)\sin 9.50^\circ}{2\pi\left(0.180\,\text{m}\right)} = \boxed{547\,\text{N}}$$



$\boxed{25.}$ Since there are no dissipative forces in the problem, the mechanical energy of the pendulum bob is conserved. Subscript 1 represents the bob at the release point, and subscript 2 represents the ball at some subsequent position. The lowest point in the swing of the pendulum is the zero location for potential energy $(y = 0)$. We have $v_1 = 0$ and $y_1 = \ell\left(1-\cos\theta\right)$. The "second" point for the energy conservation will vary from part to part of the problem.

(*a*) The second point is at the bottom of the swing, so $y_2 = 0$.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; mg\ell\left(1-\cos 30.0^\circ\right) = \tfrac{1}{2}mv_2^2 \;\rightarrow$$

$$v_2 = \sqrt{2g\ell\left(1-\cos 30.0^\circ\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(2.00\,\text{m}\right)\left(1-\cos 30.0^\circ\right)} = \boxed{2.29\,\text{m/s}}$$

(*b*) The second point is displaced from equilibrium by 15.0°, so $y_2 = \ell\left(1-\cos 15.0^\circ\right)$.

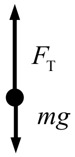
$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow$$

$$mg\ell\left(1-\cos 30.0^\circ\right) = \tfrac{1}{2}mv_2^2 + mg\ell\left(1-\cos 15.0^\circ\right) \;\rightarrow\; v_2 = \sqrt{2g\ell\left(\cos 15.0^\circ - \cos 30.0^\circ\right)}$$

$$= \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(2.00\,\text{m}\right)\left(\cos 15.0^\circ - \cos 30.0^\circ\right)} = \boxed{1.98\,\text{m/s}}$$

(*c*) The second point is displaced from equilibrium by -15.0°. The pendulum bob is at the same height at -15.0° as it was at 15.0°, and so the speed is the same. Also, since $\cos\left(-\theta\right) = \cos\left(\theta\right)$, the mathematics is identical. Thus $v_2 = \boxed{1.98\,\text{m/s}}$.

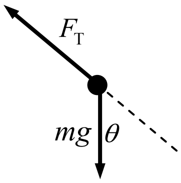
(*d*) The tension always pulls radially on the pendulum bob, and so is related to the centripetal force on the bob. The net centripetal force is always mv^2/r. Consider the free body diagram for the pendulum bob at each position.



$$(a)\quad F_{\text{T}} - mg = \frac{mv^2}{r} \;\rightarrow\; F_{\text{T}} = m\left(g + \frac{v^2}{\ell}\right) = m\left(g + \frac{2g\ell\left(1-\cos 30.0^\circ\right)}{\ell}\right)$$

$$= mg\left(3 - 2\cos 30.0^\circ\right) = \left(0.0700\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3 - 2\cos 30.0^\circ\right) = \boxed{0.870\,\text{N}}$$

$$(b)\quad F_{\text{T}} - mg\cos\theta = \frac{mv^2}{r} \;\rightarrow\; F_{\text{T}} = m\left(g\cos\theta + \frac{v^2}{\ell}\right)$$



$$= m\left(g\cos 15.0^\circ + \frac{2g\ell\left(\cos 15.0^\circ - \cos 30.0^\circ\right)}{\ell}\right)$$
$$= mg\left(3\cos 15.0^\circ - 2\cos 30.0^\circ\right)$$
$$= (0.0700\,\text{kg})\left(9.80\,\text{m/s}^2\right)\left(3\cos 15.0^\circ - 2\cos 30.0^\circ\right) = \boxed{0.800\,\text{N}}$$

(*c*) Again, as earlier, since the cosine and the speed are the same for -15.0° as for 15.0°, the tension will be the same, $\boxed{0.800\,\text{N}}$.

(*e*) Again use conservation of energy, but now we have $v_1 = v_0 = 1.20\,\text{m/s}$.

(*a*) The second point is at the bottom of the swing, so $y_2 = 0$.

$$\tfrac{1}{2}mv_1^2 + mg\ell\left(1-\cos 30.0^\circ\right) = \tfrac{1}{2}mv_2^2 \quad\rightarrow\quad v_2 = \sqrt{v_1^2 + 2g\ell\left(1-\cos 30.0^\circ\right)}$$
$$= \sqrt{(1.20\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(2.00\,\text{m})\left(1-\cos 30.0^\circ\right)} = \boxed{2.59\,\text{m/s}}$$

(*b*) The second point is displaced from equilibrium by 15.0°, so $y_2 = \ell\left(1-\cos 15.0^\circ\right)$.

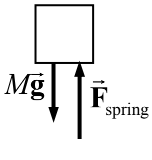
$$\tfrac{1}{2}mv_1^2 + mg\ell\left(1-\cos 30.0^\circ\right) = \tfrac{1}{2}mv_2^2 + mg\ell\left(1-\cos 15.0^\circ\right) \quad\rightarrow$$
$$v_2 = \sqrt{v_1^2 + 2g\ell\left(\cos 15.0^\circ - \cos 30.0^\circ\right)}$$
$$= \sqrt{(1.20\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(2.00\,\text{m})\left(\cos 15.0^\circ - \cos 30.0^\circ\right)} = \boxed{2.31\,\text{m/s}}$$

(*c*) As before, the pendulum bob is at the same height at -15.0° as it was at 15.0°, and so the speed is the same. Thus $v_2 = \boxed{2.31\,\text{m/s}}$.

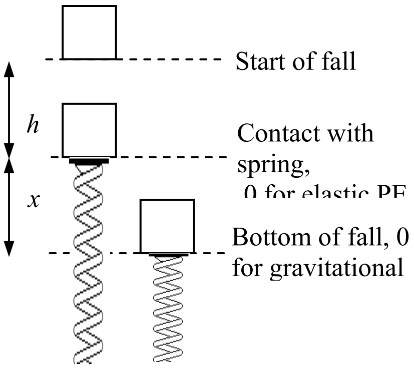
26. The maximum acceleration of 5.0 *g* occurs where the force is at a maximum. The maximum force occurs at the maximum displacement from the equilibrium of the spring. The acceleration and the displacement are related by Newton's second law and the spring law, $F_{\text{net}} = F_{\text{spring}} \rightarrow ma = -kx \rightarrow x = -\frac{m}{k}a$. Also, by conservation of energy, the initial kinetic energy of the car will become the final potential energy stored in the spring.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}k\left(\frac{m}{k}a_{\text{max}}\right)^2 = \tfrac{1}{2}\frac{m^2}{k}(5.0g)^2 \quad\rightarrow$$
$$k = \frac{m(5.0g)^2}{v_0^2} = \frac{(1200\,\text{kg})25\left(9.80\,\text{m/s}^2\right)^2}{\left[95\,\text{km/h}\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2} = \boxed{4100\,\text{N/m}}$$

27. The maximum acceleration of 5.0 *g* occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's second law for the elevator at the bottom of the motion, with up as the positive direction.



$$F_{\text{net}} = F_{\text{spring}} - Mg = Ma = 5.0Mg \quad\rightarrow\quad F_{\text{spring}} = 6.0Mg$$



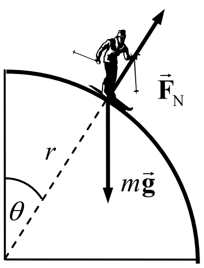
Now consider the diagram for the elevator at various points in its motion. If there are no non-conservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational potential energy $(y = 0)$. There is also a point at the top of the spring that is the zero location for elastic potential energy ($x = 0$). We have $v_1 = 0,\ y_1 = x + h,\ x_1 = 0,\ v_2 = 0,\ y_2 = 0,$ and $x_2 = x$. Apply conservation of energy.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}Mv_1^2 + Mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}Mv_2^2 + Mgy_2 + \tfrac{1}{2}kx_2^2 \rightarrow$$

$$0 + Mg(x+h) + 0 = 0 + 0 + \tfrac{1}{2}kx^2 \rightarrow Mg(x+h) = \tfrac{1}{2}kx^2$$

$$F_{\text{spring}} = 6.0Mg = kx \rightarrow x = \frac{6.0Mg}{k} \rightarrow Mg\left(\frac{6Mg}{k} + h\right) = \tfrac{1}{2}k\left(\frac{6Mg}{k}\right)^2 \rightarrow \boxed{k = \frac{12Mg}{h}}$$

28. (a) The skier, while in contact with the sphere, is moving in a circular path, and so must have some component of the net force towards the center of the circle. See the free body diagram.



$$F_{\text{radial}} = mg\cos\theta - F_N = m\frac{v^2}{r}$$

If the skier loses contact with the sphere, the normal force is 0. Use that relationship to find the critical angle and speed.

$$mg\cos\theta_{\text{crit}} = m\frac{v_{\text{crit}}^2}{r} \rightarrow \cos\theta_{\text{crit}} = \frac{v_{\text{crit}}^2}{rg}$$

Using conservation of mechanical energy, the velocity can be found as a function of angle. Let subscript 1 represent the skier at the top of the sphere, and subscript 2 represent the skier at angle θ. The top of the sphere is the zero location for gravitational potential energy $(y = 0)$. There is also a point at the top of the spring that is the zero location for elastic potential energy ($x = 0$). We have $v_1 = 0,\ y_1 = 0,$ and $y_2 = -(r - r\cos\theta)$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow 0 = \tfrac{1}{2}mv_2^2 - mg(r - r\cos\theta) \rightarrow$$

$$v_2 = \sqrt{2g(r - r\cos\theta)}$$

Combine the two relationships to find the critical angle.

$$\cos\theta_{\text{crit}} = \frac{v_{\text{crit}}^2}{rg} = \frac{2g(r - r\cos\theta_{\text{crit}})}{rg} = 2 - 2\cos\theta_{\text{crit}} \rightarrow \theta_{\text{crit}} = \boxed{\cos^{-1}\tfrac{2}{3}} \approx 48°$$

(*b*) If friction is present, another force will be present, tangential to the surface of the sphere. The friction force will not affect the centripetal relationship of $\cos\theta_{\text{crit}} = \frac{v_{\text{crit}}^2}{rg}$. But the friction will reduce the speed at any given angle, and so the skier will be at a $\boxed{\text{greater}}$ angle before the critical speed is reached.

29. Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

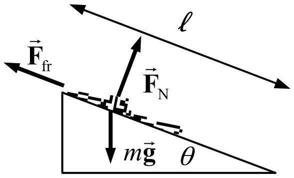
$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 = E_{\text{thermal}} = \tfrac{1}{2}(2)(56{,}000\,\text{kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = \boxed{3.9\times10^7\,\text{J}}$$

30. Apply the conservation of energy to the child, considering work done by gravity and thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for potential energy $(y=0)$. We have $v_1 = 0$, $y_1 = 2.2$ m, $v_2 = 1.25\,\text{m/s}$, and $y_2 = 0$. Solve for the work changed into thermal energy.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + E_{\text{thermal}} \quad \rightarrow$$

$$E_{\text{thermal}} = mgy_1 - \tfrac{1}{2}mv_2^2 = (16.0\text{ kg})(9.80\,\text{m/s}^2)(2.20\,\text{m}) - \tfrac{1}{2}(16.0\text{ kg})(1.25\,\text{m/s})^2 = \boxed{332\,\text{J}}$$

$\boxed{31.}$ (*a*) See the free-body diagram for the ski. Write Newton's second law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.



$$\sum F_\perp = F_{\text{N}} - mg\cos\theta \quad \rightarrow \quad F_{\text{N}} = mg\cos\theta \quad \rightarrow$$

$$F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta$$

Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational potential energy $(y=0)$. We have $v_1 = 0$, $y_1 = \ell\sin\theta$, and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg\cos\theta.$

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}\ell \quad \rightarrow \quad mg\ell\sin\theta = \tfrac{1}{2}mv_2^2 + \mu_k mg\ell\cos\theta \quad \rightarrow$$

$$v_2 = \sqrt{2g\ell(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\,\text{m/s}^2)(85\text{ m})(\sin 28^\circ - 0.090\cos 28^\circ)}$$

$$= 25.49\,\text{m/s} \approx \boxed{25\,\text{m/s}}$$

(*b*) Now, on the level ground, $F_{\text{fr}} = \mu_k mg$, and there is no change in potential energy. We again use conservation of energy, including the non-conservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance ℓ_3 on the level. We have $v_2 = 25.49\,\text{m/s}$, $y_2 = 0$, $v_3 = 0$, and $y_3 = 0$.

$$\tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 + F_{\text{fr}}\ell_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 = \mu_k mg\ell_3 \quad \rightarrow$$

$$\ell_3 = \frac{v_2^2}{2g\mu_k} = \frac{(25.49\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)(0.090)} = 368.3\,\text{m} \approx \boxed{370\,\text{m}}$$

32. (*a*) Apply energy conservation with no non-conservative work. Subscript 1 represents the ball as it is dropped, and subscript 2 represents the ball as it reaches the ground. The ground is the zero location for gravitational potential energy. We have $v_1 = 0$, $y_1 = 14.0\,\text{m}$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

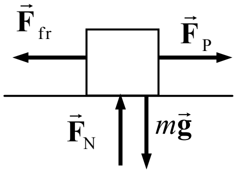
$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\,\text{m/s}^2)(14.0\text{ m})} = \boxed{16.6\,\text{m/s}}$$

(*b*) Apply energy conservation, but with non-conservative work due to friction included. The energy dissipated will be given by $F_{\mathrm{fr}}d$. The distance d over which the frictional force acts will be the 14.0 m distance of fall. With the same parameters as above, and $v_2 = 8.00\,\mathrm{m/s}$, solve for the force of friction.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\mathrm{fr}}d \;\rightarrow\; mgy_1 = \tfrac{1}{2}mv_2^2 + F_{\mathrm{fr}}d \;\rightarrow$$

$$F_{\mathrm{fr}} = m\left(g\frac{y_1}{d} - \frac{v_2^2}{2d}\right) = (0.145\text{ kg})\left(9.80\,\mathrm{m/s^2} - \frac{(8.00\,\mathrm{m/s})^2}{2(14.0\text{ m})}\right) = \boxed{1.09\text{ N, upwards}}$$

33. We apply the work-energy theorem. There is no need to use potential energy since the crate moves along the level floor, and there are no springs in the problem. There are two forces doing work in this problem – the pulling force and friction. The starting speed is $v_0 = 0$. Note that the two forces do work over different distances.



$$W_{\mathrm{net}} = W_{\mathrm{P}} + W_{\mathrm{fr}} = F_{\mathrm{P}}d_{\mathrm{P}}\cos 0^\circ + F_{\mathrm{fr}}d_{\mathrm{fr}}\cos 180^\circ = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \;\rightarrow$$

$$F_{\mathrm{P}}d_{\mathrm{P}} - \mu_k mgd_{\mathrm{fr}} = \tfrac{1}{2}mv_f^2 \;\rightarrow\; v_f = \sqrt{\frac{2}{m}\left(F_{\mathrm{P}}d_{\mathrm{P}} - \mu_k mgd_{\mathrm{fr}}\right)}$$

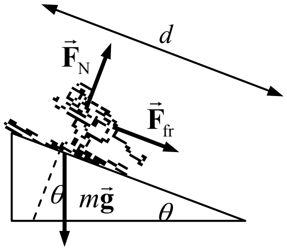
$$= \sqrt{\frac{2}{(96\,\mathrm{kg})}\left[(350\,\mathrm{N})(30\,\mathrm{m}) - (0.25)(96\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(15\,\mathrm{m})\right]} = \boxed{12\,\mathrm{m/s}}$$

34. Since there is a non-conservative force, apply energy conservation with the dissipative friction term. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational potential energy. We have $v_1 = 1.70\,\mathrm{m/s}$, $y_1 = 32\,\mathrm{m}$, and $y_2 = 0$. Solve for v_2. Note that the dissipated energy is given by $F_{\mathrm{fr}}d = 0.23mgd$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + 0.23mgd \;\rightarrow\; v_2 = \sqrt{-0.46gd + v_1^2 + 2gy_1}$$

$$= \sqrt{-0.46(9.80\,\mathrm{m/s^2})(45.0\text{ m}) + (1.70\,\mathrm{m/s})^2 + 2(9.80\,\mathrm{m/s^2})(32\,\mathrm{m})} = 20.67\,\mathrm{m/s} \approx \boxed{21\,\mathrm{m/s}}$$

35. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's second law for the direction perpendicular to the plane, with an acceleration of 0.



$$\sum F_\perp = F_{\mathrm{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\mathrm{N}} = mg\cos\theta \;\rightarrow$$

$$F_{\mathrm{fr}} = \mu_k F_{\mathrm{N}} = \mu_k mg\cos\theta$$

Apply conservation of energy to the skier, including the dissipative friction force. Subscript 1 represents the skier at the bottom of the slope, and subscript 2 represents the skier at the point furthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 9.0\,\mathrm{m/s}$, $y_1 = 0$, $v_2 = 0$, and $y_2 = d\sin\theta$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\mathrm{fr}}d \;\rightarrow\; \tfrac{1}{2}mv_1^2 + 0 = 0 + mgd\sin\theta + \mu_k mgd\cos\theta \;\rightarrow$$

$$\mu_k = \frac{\tfrac{1}{2}v_1^2 - gd\sin\theta}{gd\cos\theta} = \frac{v_1^2}{2gd\cos\theta} - \tan\theta = \frac{(9.0\,\mathrm{m/s})^2}{2(9.80\,\mathrm{m/s^2})(12\,\mathrm{m})\cos 19^\circ} - \tan 19^\circ = \boxed{0.020}$$

36. (*a*) Use conservation of energy to equate the potential energy at the top of the circular track to the kinetic energy at the bottom of the circular track. Take the bottom of the track to the be 0 level for gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgr = \tfrac{1}{2} m v_{\text{bottom}}^2 \quad \rightarrow$$

$$v_{\text{bottom}} = \sqrt{2gr} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(2.0\,\text{m}\right)} = 6.261\,\text{m/s} \approx \boxed{6.3\,\text{m/s}}$$

(*b*) The thermal energy produced is the opposite of the work done by the friction force. In this situation, the force of friction is the weight of the object times the coefficient of kinetic friction.

$$E_{\text{thermal}} = -W_{\text{friction}} = -\vec{\mathbf{F}}_{\text{friction}} \bullet \Delta\vec{\mathbf{x}} = -F_{\text{friction}} \Delta x \cos\theta = -\mu_{\text{k}} mg \Delta x \left(\cos 180^\circ\right) = \mu_{\text{k}} mg \Delta x$$

$$= \left(0.25\right)\left(1.0\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(3.0\text{m}\right) = 7.35\,\text{J} \approx \boxed{7.4\,\text{J}}$$

(*c*) The work done by friction is the change in kinetic energy of the block as it moves from point B to point C.

$$W_{\text{friction}} = \Delta K = K_{\text{C}} - K_{\text{B}} = \tfrac{1}{2} m \left(v_{\text{C}}^2 - v_{\text{B}}^2\right) \quad \rightarrow$$

$$v_{\text{C}} = \sqrt{\frac{2W_{\text{friction}}}{m} + v_{\text{B}}^2} = \sqrt{\frac{2\left(-7.35\,\text{J}\right)}{\left(1.0\,\text{kg}\right)} + \left(6.261\,\text{m/s}\right)^2} = 4.9498\,\text{m/s} \approx \boxed{4.9\,\text{m/s}}$$

(*d*) Use conservation of energy to equate the kinetic energy when the block just contacts the spring with the potential energy when the spring is fully compressed and the block has no speed. There is no friction on the block while compressing the spring.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2} m v_{\text{contact}}^2 = \tfrac{1}{2} k x_{\text{max}}^2 \quad \rightarrow$$

$$k = m \frac{v_{\text{contact}}^2}{x_{\text{max}}^2} = \left(1.0\,\text{kg}\right) \frac{\left(4.9498\,\text{m/s}\right)^2}{\left(0.20\,\text{m}\right)^2} = 612.5\,\text{N/m} \approx \boxed{610\,\text{N/m}}$$

$\boxed{37.}$ Use conservation of energy, including the non-conservative frictional force, as developed in Eq. 8-15. The block is on a level surface, so there is no gravitational potential energy change to consider. The frictional force is given by $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg$, since the normal force is equal to the weight. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic potential energy ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.050$ m, $v_2 = 0$, and $x_2 = 0.023$ m.

$$E_1 = E_2 + F_{\text{fr}} \ell \quad \rightarrow \quad \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} k x_1^2 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2} k x_2^2 + F_{\text{fr}} \left(x_2 - x_1\right) \quad \rightarrow$$

$$\tfrac{1}{2} k x_1^2 = \tfrac{1}{2} k x_2^2 + \mu_k mg \left(x_2 - x_1\right) \quad \rightarrow$$

$$\mu_k = \frac{k\left(x_1^2 - x_2^2\right)}{2mg\left(x_2 - x_1\right)} = \frac{-k\left(x_2 + x_1\right)}{2mg} = \frac{-\left(180\,\text{N/m}\right)\left[\left(-0.050\text{m}\right) + \left(0.023\text{m}\right)\right]}{2\left(0.620\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{0.40}$$

38. Use conservation of energy, including the non-conservative frictional force, as developed in Eq. 8-15. The block is on a level surface, so there is no gravitational potential energy change to consider. Since the normal force is equal to the weight, the frictional force is $F_{\text{fr}} = \mu_k F_{\text{N}} = \mu_k mg$. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic potential energy ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.18$ m, and $v_2 = 0$. The value of the spring constant is found from the fact that

a 25-N force compresses the spring 18 cm, and so $k = F/x = 25\,\text{N}/0.18\text{ m} = 138.9\,\text{N/m}$. The value of x_2 must be positive.

$$E_1 = E_2 + F_{\text{fr}}\ell \;\rightarrow\; \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 + F_{\text{fr}}\left(x_2 - x_1\right) \;\rightarrow$$

$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 + \mu_k mg\left(x_2 - x_1\right) \;\rightarrow\; x_2^2 + \frac{2\mu_k mg}{k}x_2 - \left(\frac{2\mu_k mg}{k}x_1 + x_1^2\right) = 0 \;\rightarrow$$

$$x_2^2 + \frac{2(0.30)(0.18)(9.80)}{138.9}x_2 - \left(\frac{2(0.30)(0.18)(9.80)}{138.9}(-0.18) + (-0.18)^2\right) = 0 \;\rightarrow$$

$$x_2^2 + 0.00762x_2 - 0.03103 = 0 \;\rightarrow\; x_2 = 0.1724\,\text{m}, -0.1800\,\text{m} \;\rightarrow\; x_2 = \boxed{0.17\,\text{m}}$$

39. (*a*) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy lost. The energy at the two heights is all gravitational potential energy, since the ball has no kinetic energy at those maximum heights.

$$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = mgy_{\text{initial}} - mgy_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{mgy_{\text{initial}} - mgy_{\text{final}}}{mgy_{\text{initial}}} = \frac{y_{\text{initial}} - y_{\text{final}}}{y_{\text{initial}}} = \frac{2.0\text{ m} - 1.5\text{ m}}{2.0\text{ m}} = 0.25 = \boxed{25\%}$$

(*b*) The ball's speed just before the bounce is found from the initial gravitational potential energy, and the ball's speed just after the bounce is found from the ball's final gravitational potential energy.

$$U_{\text{initial}} = K_{\text{before}} \;\rightarrow\; mgy_{\text{initial}} = \tfrac{1}{2}mv_{\text{before}}^2 \;\rightarrow$$

$$v_{\text{before}} = \sqrt{2gy_{\text{initial}}} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(2.0\,\text{m})} = \boxed{6.3\,\text{m/s}}$$

$$U_{\text{final}} = K_{\text{after}} \;\rightarrow\; mgy_{\text{final}} = \tfrac{1}{2}mv_{\text{after}}^2 \;\rightarrow$$

$$v_{\text{after}} = \sqrt{2gy_{\text{final}}} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(1.5\text{ m})} = \boxed{5.4\,\text{m/s}}$$

(*c*) The energy "lost" was changed $\boxed{\text{primarily into heat energy}}$ – the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.

40. Since there is friction in this problem, there will be energy dissipated by friction.

$$E_{\text{friction}} + \Delta K + \Delta U = 0 \;\rightarrow\; E_{\text{friction}} = -\Delta K - \Delta U = \tfrac{1}{2}m\left(v_1^2 - v_2^2\right) + mg\left(y_1 - y_2\right)$$

$$= \tfrac{1}{2}(56\,\text{kg})\left[0 - (11.0\,\text{m/s})^2\right] + (56\,\text{kg})\left(9.80\,\text{m/s}^2\right)(230\,\text{m}) = \boxed{1.2\times10^5\,\text{J}}$$

41. The change in gravitational potential energy is given by $\Delta U = mg\Delta y$. Assume a mass of 75 kg.

$$\Delta U = mg\Delta y = (75\,\text{kg})\left(9.80\,\text{m/s}^2\right)(1.0\,\text{m}) = \boxed{740\,\text{J}}$$

42. (*a*) Use conservation of energy. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at its maximum position up the slope. The initial location of the block at the bottom of the plane is taken to be the zero location for gravitational potential energy ($y = 0$). The variable x will represent the amount of spring compression or stretch. We have $v_1 = 0,\; x_1 = 0.50\,\text{m},\; y_1 = 0,\; v_2 = 0,$ and $x_2 = 0.$ The distance the block moves up the plane is given by $d = \dfrac{y}{\sin\theta}$, so $y_2 = d\sin\theta$. Solve for d.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgy_2 = mgd\sin\theta \quad \rightarrow \quad d = \frac{kx_1^2}{2mg\sin\theta} = \frac{(75\,\mathrm{N/m})(0.50\,\mathrm{m})^2}{2(2.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})\sin 41^\circ} = \boxed{0.73\,\mathrm{m}}$$

(*b*) Now the spring will be stretched at the turning point of the motion. The first half-meter of the block's motion returns the block to the equilibrium position of the spring. After that, the block beings to stretch the spring. Accordingly, we have the same conditions as before except that $x_2 = d - 0.5\,\mathrm{m}$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgd\sin\theta + \tfrac{1}{2}k\left(d - 0.5\,\mathrm{m}\right)$$

This is a quadratic relation in *d*. Solving it gives $d = \boxed{0.66\,\mathrm{m}}$.

(*c*) The block now moves $d = 0.50\,\mathrm{m}$, and stops at the equilibrium point of the spring. Accordingly, $x_2 = 0$. Apply the method of Section 8-6.

$$\Delta K + \Delta U + F_{\mathrm{fr}}\ell = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right) + \tfrac{1}{2}k\left(x_2^2 - x_1^2\right) + mg\left(y_2 - y_1\right) + \mu_k mgd\cos\theta \quad \rightarrow$$

$$\mu_k = \frac{-\tfrac{1}{2}kx_1^2 + mgd\sin\theta}{-mgd\cos\theta} = \frac{kx_1^2}{2mgd\cos\theta} - \tan\theta$$

$$= \frac{(75\,\mathrm{N/m})(0.50\,\mathrm{m})^2}{2(2.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(0.50\,\mathrm{m})\cos 41^\circ} - \tan 41^\circ = \boxed{0.40}$$

$\boxed{43.}$ Because friction does work, Eq. 8-15 applies.

(*a*) The spring is initially uncompressed, so $x_0 = 0$. The block is stopped at the maximum compression, so $v_f = 0$.

$$\Delta K + \Delta U + F_{\mathrm{fr}}\ell = \tfrac{1}{2}m\left(v_f^2 - v_0^2\right) + \tfrac{1}{2}k\left(x_f^2 - x_0^2\right) + mg\mu_{\mathrm{k}}\left(x_f - x_0\right) = 0 \quad \rightarrow$$

$$\tfrac{1}{2}kx_f^2 + mg\mu_{\mathrm{k}}x_f - \tfrac{1}{2}mv_0^2 = 0 \quad \rightarrow$$

$$x_f = \frac{-mg\mu_{\mathrm{k}} \pm \sqrt{(mg\mu_{\mathrm{k}})^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}mv_0^2\right)}}{2\left(\tfrac{1}{2}k\right)} = \frac{-mg\mu_{\mathrm{k}} \pm \sqrt{(mg\mu_{\mathrm{k}})^2 + kmv_0^2}}{k}$$

$$= \frac{mg\mu_{\mathrm{k}}}{k}\left(-1 \pm \sqrt{1 + \frac{kmv_0^2}{(mg\mu_{\mathrm{k}})^2}}\right)$$

$$= \frac{(2.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(0.30)}{(120\,\mathrm{N/m})}\left(-1 \pm \sqrt{1 + \frac{(120\,\mathrm{N/m})(2.0\,\mathrm{kg})(1.3\,\mathrm{m/s})^2}{(2.0\,\mathrm{kg})^2(9.80\,\mathrm{m/s^2})^2(0.30)^2}}\right)$$

$$= 0.1258\,\mathrm{m} \approx \boxed{0.13\,\mathrm{m}}$$

(*b*) To remain at the compressed position with the minimum coefficient of static friction, the magnitude of the force exerted by the spring must be the same as the magnitude of the maximum force of static friction.

$$kx_f = \mu_{\mathrm{s}}mg \quad \rightarrow \quad \mu_{\mathrm{s}} = \frac{kx_f}{mg} = \frac{(120\,\mathrm{N/m})(0.1258\,\mathrm{m})}{(2.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})} = 0.7702 \approx \boxed{0.77}$$

(*c*) If static friction is not large enough to hold the block in place, the spring will push the block back towards the equilibrium position. The block will detach from the decompressing spring at the equilibrium position because at that point the spring will begin to slow down while the block continues moving. Use Eq. 8-15 to relate the block at the maximum compression position to the equilibrium position. The block is initially at rest, so $v_0 = 0$. The spring is relaxed at the equilibrium position, so $x_f = 0$.

$$\Delta K + \Delta U + F_{\text{fr}}\ell = \tfrac{1}{2}m\left(v_f^2 - v_0^2\right) + \tfrac{1}{2}k\left(x_f^2 - x_0^2\right) + mg\mu_{\text{k}}\left(x_f - x_0\right) = 0 \quad \rightarrow$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}kx_0^2 + mg\mu_{\text{k}}x_0 = 0 \quad \rightarrow$$

$$v_f = \sqrt{\frac{k}{m}x_0^2 - 2g\mu_{\text{k}}x_0} = \sqrt{\frac{(120\,\text{N/m})}{(2.0\,\text{kg})}(0.1258\,\text{m})^2 - 2\left(9.80\,\text{m/s}^2\right)(0.30)(0.1258\,\text{m})}$$

$$= 0.458\,\text{m/s} \approx \boxed{0.5\,\text{m/s}}$$

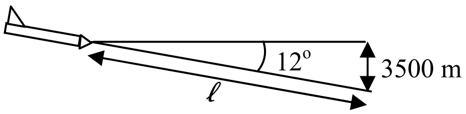
44. (*a*) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represents the glider when at launch, and subscript 2 represents the glider at landing. The landing location is the zero location for elastic potential energy (*y* = 0). We have $y_1 = 3500$ m, $y_2 = 0$, and $v_1 = 480\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 133.3\,\text{m/s}$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(133.3\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(3500\text{ m})} = 293.8\,\text{m/s}\left(\frac{3.6\,\text{km/h}}{1\,\text{m/s}}\right)$$

$$= 1058\,\text{km/h} \approx \boxed{1100\,\text{km/h}}$$

(*b*) Now include the work done by the non-conservative frictional force. Consider the diagram of the glider. The distance over which the friction acts is given by $\ell = \frac{3500\text{ m}}{\sin 12°}$. Use the same subscript representations as above, with y_1, v_1, and y_2 as before, and $v_2 = 210\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 58.33\,\text{m/s}$. Write the energy conservation equation and solve for the frictional force.



$$E_1 = E_2 + F_{\text{fr}}\ell \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}\ell \quad \rightarrow \quad F_f = \frac{m\left(v_1^2 - v_2^2 + 2gy_1\right)}{2\ell}$$

$$= \frac{(980\text{ kg})\left[(133.3\,\text{m/s})^2 - (58.33\,\text{m/s})^2 + 2\left(9.80\,\text{m/s}^2\right)(3500\text{ m})\right]}{2\left(\dfrac{3500\text{ m}}{\sin 12°}\right)} = 2415\,\text{N} \approx \boxed{2400\,\text{N}}$$

45. (*a*) Equate the gravitational force to the expression for centripetal force, since the orbit is circular. Let M_E represent the mass of the Earth.

$$\frac{m_s v_s^2}{r_s} = \frac{GM_\text{E}m_s}{r_s^2} \quad\rightarrow\quad m_s v_s^2 = \frac{GM_\text{E}m_s}{r_s} \quad\rightarrow\quad \tfrac{1}{2}m_s v_s^2 = \boxed{K = \frac{GM_\text{E}m_s}{2r_s}}$$

(*b*) The potential energy is given by Eq. 8-17, $\boxed{U = -GM_\text{E}m_s/r_s}$.

(*c*)
$$\frac{K}{U} = \frac{\dfrac{GM_\text{E}m_s}{2r_s}}{-\dfrac{GM_\text{E}m_s}{r_s}} = \boxed{-\frac{1}{2}}$$

46. Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the rocket at launch, and subscript 2 represents the rocket at its highest altitude. We have $v_1 = 850\,\text{m/s}$, $v_2 = 0$, and we take the final altitude to be a distance *h* above the surface of the Earth.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + \left(-\frac{GM_\text{E}m}{r_\text{E}}\right) = \tfrac{1}{2}mv_2^2 + \left(-\frac{GM_\text{E}m}{r_\text{E}+h}\right) \quad\rightarrow$$

$$h = \left(\frac{1}{r_\text{E}} - \frac{v_1^2}{2GM_\text{E}}\right)^{-1} - r_\text{E} = r_\text{E}\left(\frac{2GM_\text{E}}{r_\text{E}v_0^2} - 1\right)^{-1}$$

$$= \left(6.38\times10^6\,\text{m}\right)\left(\frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^6\,\text{m}\right)\left(850\,\text{m/s}\right)^2} - 1\right)^{-1} = 3.708\times10^4\,\text{m} \approx \boxed{3.7\times10^4\,\text{m}}$$

If we would solve this problem with the approximate gravitation potential energy of *mgh*, we would get an answer of $3.686\times10^4\,\text{m}$, which agrees to 2 significant figures.

47. The escape velocity is given by Eq. 8-19.

$$\underset{\text{A}}{v_\text{esc}} = \sqrt{\frac{2M_\text{A}G}{r_\text{A}}} \qquad \underset{\text{B}}{v_\text{esc}} = \sqrt{\frac{2M_\text{B}G}{r_\text{B}}} \qquad \underset{\text{A}}{v_\text{esc}} = 2\underset{\text{B}}{v_\text{esc}} \quad\rightarrow\quad \sqrt{\frac{2M_\text{A}G}{r_\text{A}}} = 2\sqrt{\frac{2M_\text{B}G}{r_\text{B}}} \quad\rightarrow$$

$$\frac{2M_\text{A}G}{r_\text{A}} = 2\left(\frac{2M_\text{B}G}{r_\text{B}}\right) \quad\rightarrow\quad \boxed{\frac{r_\text{A}}{r_\text{B}} = \frac{1}{4}}$$

48. Note that the difference in the two distances from the center of the Earth, $r_2 - r_1$, is the same as the height change in the two positions, $y_2 - y_1$. Also, if the two distances are both near the surface of the Earth, then $r_1r_2 \approx r_\text{E}^2$.

$$\Delta U = \left(-\frac{GM_\text{E}m}{r_2}\right) - \left(-\frac{GM_\text{E}m}{r_1}\right) = \frac{GM_\text{E}m}{r_1} - \frac{GM_\text{E}m}{r_2} = GM_\text{E}m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{GM_\text{E}m}{r_1r_2}\left(r_2 - r_1\right)$$

$$\approx \frac{GM_\text{E}m}{r_\text{E}^2}\left(y_2 - y_1\right) = m\frac{GM_\text{E}}{r_\text{E}^2}\left(y_2 - y_1\right) = \boxed{mg\left(y_2 - y_1\right)}$$

49. The escape velocity for an object located a distance *r* from a mass *M* is given by Eq. 8-19, $v_\text{esc} = \sqrt{\dfrac{2MG}{r}}$. The orbit speed for an object located a distance *r* from a mass *M* is $v_\text{orb} = \sqrt{\dfrac{MG}{r}}$.

(*a*) $$v_{\substack{\text{esc at}\\ \text{Sun's}\\ \text{surface}}} = \sqrt{\frac{2M_{\text{Sun}}G}{r_{\text{Sun}}}} = \sqrt{\frac{2\left(2.0\times10^{30}\,\text{kg}\right)\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)}{7.0\times10^{8}\,\text{m}}} = \boxed{6.2\times10^{5}\ \text{m/s}}$$

(*b*) $$v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} = \sqrt{\frac{2M_{\text{Sun}}G}{r_{\text{Earth orbit}}}} = \sqrt{\frac{2\left(2.0\times10^{30}\,\text{kg}\right)\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)}{1.50\times10^{-11}\,\text{m}}} = \boxed{4.2\times10^{4}\ \text{m/s}}$$

$$\frac{v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}}}{v_{\substack{\text{Earth}\\ \text{orbit}}}} = \frac{\sqrt{\dfrac{2M_{\text{Sun}}G}{r_{\text{Earth orbit}}}}}{\sqrt{\dfrac{M_{\text{Sun}}G}{r_{\text{Earth orbit}}}}} = \sqrt{2} \quad\rightarrow\quad \boxed{v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} = \sqrt{2}\,v_{\substack{\text{Earth}\\ \text{orbit}}}}$$

Since $v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} \approx 1.4 v_{\substack{\text{Earth}\\ \text{orbit}}}$, the orbiting object will not escape the orbit.

50. (*a*) The potential energy is given by Eq. 8-17.

$$U_{\text{A}} = -\frac{GmM_{\text{E}}}{r_{\text{A}}} = -\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(950\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^{6}\,\text{m} + 4.20\times10^{6}\,\text{m}\right)}$$

$$= -3.5815\times10^{10}\,\text{J} \approx \boxed{-3.6\times10^{10}\,\text{J}}$$

$$U_{\text{B}} = -\frac{GmM_{\text{E}}}{r_{\text{B}}} = -\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(950\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^{6}\,\text{m} + 1.26\times10^{7}\,\text{m}\right)}$$

$$= -1.9964\times10^{10}\,\text{J} \approx \boxed{-2.0\times10^{10}\,\text{J}}$$

(*b*) An expression for the kinetic energy is found by equating the gravitational force to the expression for centripetal force, since the satellites are in circular orbits.

$$\frac{mv^2}{r} = \frac{GmM_{\text{E}}}{r^2} \quad\rightarrow\quad \tfrac{1}{2}mv^2 = K = \frac{GmM_{\text{E}}}{2r} = -\tfrac{1}{2}U$$

$$K_{\text{A}} = \frac{GmM_{\text{E}}}{2r_{\text{A}}} = -\tfrac{1}{2}\left(-3.5815\times10^{10}\,\text{J}\right) = 1.7908\times10^{10}\,\text{J} \approx \boxed{1.8\times10^{10}\,\text{J}}$$

$$K_{\text{B}} = \frac{GmM_{\text{E}}}{2r_{\text{B}}} = -\tfrac{1}{2}\left(-1.9964\times10^{10}\,\text{J}\right) = 0.9982\times10^{10}\,\text{J} \approx \boxed{1.0\times10^{10}\,\text{J}}$$

(*c*) We use the work-energy theorem to calculate the work done to change the orbit.

$$W_{\text{Net}} = \Delta K = W_{\substack{\text{orbit}\\ \text{change}}} + W_{\text{gravity}} = W_{\substack{\text{orbit}\\ \text{change}}} - \Delta U_{\text{gravity}} \quad\rightarrow\quad W_{\substack{\text{orbit}\\ \text{change}}} = \Delta K + \Delta U_{\text{gravity}} \quad\rightarrow$$

$$W_{\substack{\text{orbit}\\ \text{change}}} = \Delta K + \Delta U_{\text{gravity}} = \left(K_{\text{B}} - K_{\text{A}}\right) + \left(U_{\text{B}} - U_{\text{A}}\right) = \left(-\tfrac{1}{2}U_{\text{B}} + \tfrac{1}{2}U_{\text{A}}\right) + \left(U_{\text{B}} - U_{\text{A}}\right)$$

$$= \tfrac{1}{2}\left(U_{\text{B}} - U_{\text{A}}\right) = \tfrac{1}{2}\left(-1.9964\times10^{10}\,\text{J} - -3.5815\times10^{10}\,\text{J}\right) = \boxed{7.9\times10^{9}\,\text{J}}$$

51. For a circular orbit, the gravitational force is a centripetal force. The escape velocity is given by Eq. 8-19.

$$\frac{GMm}{r^2} = \frac{mv_{\text{orbit}}^2}{r} \quad\rightarrow\quad v_{\text{orbit}} = \sqrt{\frac{MG}{r}} \qquad v_{\text{esc}} = \sqrt{\frac{2MG}{r}} = \sqrt{2}\sqrt{\frac{MG}{r}} = \sqrt{2}\,v_{\text{orbit}}$$

52. (*a*) With the condition that $U=0$ at $r=\infty$, the potential energy is given by $U=-\dfrac{GM_Em}{r}$. The kinetic energy is found from the fact that for a circular orbit, the gravitational force is a centripetal force.

$$\frac{GM_Em}{r^2}=\frac{mv_{\text{orbit}}^2}{r} \rightarrow mv_{\text{orbit}}^2=\frac{GM_Em}{r} \rightarrow K=\tfrac{1}{2}mv_{\text{orbit}}^2=\tfrac{1}{2}\frac{GM_Em}{r}$$

$$E=K+U=\tfrac{1}{2}\frac{GM_Em}{r}-\frac{GM_Em}{r}=\boxed{-\tfrac{1}{2}\frac{GM_Em}{r}}$$

(*b*) As the value of *E* decreases, since *E* is negative, the radius *r* must get smaller. But as the radius gets smaller, the kinetic energy increases, since $K\propto\dfrac{1}{r}$. If the total energy decreases by 1 Joule, the potential energy decreases by 2 Joules and the kinetic energy increases by 1 Joule.

53. The speed of the surface of the Earth at the equator (relative to the center of the Earth) is given by the following. It is an eastward velocity. Call east the *x*-direction, and up the *y*-direction.

$$v=\frac{2\pi r_E}{T}=\frac{2\pi\left(6.38\times10^6\,\text{m}\right)}{86{,}400\,\text{s}}=464\,\text{m/s}$$

The escape velocity from the Earth (relative to the center of the Earth) is given in Eq. 8-19.

$$v_{\text{esc}}=\sqrt{\frac{2GM_E}{r_E}}=\sqrt{\frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}{6.38\times10^6\,\text{m}}}=11{,}182\,\text{m/s}$$

(*a*) With the surface of the Earth traveling east and the rocket velocity to the east, the rocket velocity and surface velocity will add linearly to give the escape velocity.

$$v_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}+464\,\text{m/s}=11{,}182\,\text{m/s} \rightarrow v_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}=\boxed{10{,}700\,\text{m/s}}$$

(*b*) With the surface of the Earth traveling east and the rocket velocity to the west, the rocket velocity will have to be higher than the nominal escape velocity.

$$v_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}+464\,\text{m/s}=-11{,}182\,\text{m/s} \rightarrow v_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}=11{,}646\,\text{m/s}\approx\boxed{11{,}600\,\text{m/s}}$$

(*c*) When fired vertically upward, the rocket velocity and the Earth's velocity are at right angles to each other, and so add according to the Pythagorean theorem to give the escape velocity.

$$v^2_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}+\left(464\,\text{m/s}\right)^2=\left(11{,}182\,\text{m/s}\right)^2 \rightarrow v_{\substack{\text{rocket relative}\\\text{to surface of}\\\text{Earth}}}=11{,}172\,\text{m/s}\approx\boxed{11{,}200\,\text{m/s}}$$

54. (*a*) Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the rocket at launch, and subscript 2 represents the rocket at its highest altitude. We have $v_1=v_0$, $v_2=0$, $r_1=r_E$, and $r_2=r_E+h$ where we take the final altitude to be a distance *h* above the surface of the Earth.

$$E_1=E_2 \rightarrow \tfrac{1}{2}mv_0^2+\left(-\frac{GM_Em}{r_E}\right)=\tfrac{1}{2}mv_2^2+\left(-\frac{GM_Em}{r_E+h}\right)=\left(-\frac{GM_Em}{r_E+h}\right) \rightarrow$$

$$h=\left(\frac{1}{r_E}-\frac{v_0^2}{2GM_E}\right)^{-1}-r_E=\boxed{r_E\left(\frac{2GM_E}{r_Ev_0^2}-1\right)^{-1}}$$

(*b*) $$h=r_{\mathrm{E}}\left(\frac{2GM_{\mathrm{E}}}{r_{\mathrm{E}}v_0^2}-1\right)^{-1}$$

$$=\left(6.38\times10^6\,\mathrm{m}\right)\left(\frac{2\left(6.67\times10^{-11}\,\mathrm{N\bullet m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(6.38\times10^6\,\mathrm{m}\right)\left(8350\,\mathrm{m/s}\right)^2}-1\right)^{-1}=\boxed{8.0\times10^6\,\mathrm{m}}$$

$\boxed{55.}$ (*a*) From Eq. 8-19, the escape velocity at a distance $r\geq r_{\mathrm{E}}$ from the center of the Earth is

$$v_{\mathrm{esc}}=\sqrt{\frac{2GM_{\mathrm{E}}}{r}}.$$

$$v_{\mathrm{esc}}=\sqrt{\frac{2GM_{\mathrm{E}}}{r}}=r^{-1/2}\sqrt{2GM_{\mathrm{E}}}\rightarrow\frac{dv_{\mathrm{esc}}}{dr}=-\tfrac{1}{2}r^{-3/2}\sqrt{2GM_{\mathrm{E}}}=\boxed{-\sqrt{\frac{GM_{\mathrm{E}}}{2r^3}}}$$

(*b*) $$\Delta v_{\mathrm{esc}}\approx\frac{dv_{\mathrm{esc}}}{dr}\Delta r=-\sqrt{\frac{GM_{\mathrm{E}}}{2r^3}}\Delta r=-\sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\bullet m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{2\left(6.38\times10^6\,\mathrm{m}\right)^3}}\left(3.2\times10^5\,\mathrm{m}\right)$$

$$=-280\,\mathrm{m/s}$$

The escape velocity has decreased by 280 m/s, and so is $v_{\mathrm{esc}}=1.12\times10^4\,\mathrm{m/s}-280\,\mathrm{m/s}=$ $\boxed{1.09\times10^4\,\mathrm{m/s}}$.

56. (*a*) Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the meteorite at the high altitude, and subscript 2 represents the meteorite just before it hits the sand. We have $v_1=90.0\,\mathrm{m/s}$, $r_1=r_{\mathrm{E}}+h=r_{\mathrm{E}}+850\,\mathrm{km}$, and $r_2=r_{\mathrm{E}}$.

$$E_1=E_2\rightarrow\tfrac{1}{2}mv_1^2+\left(-\frac{GM_{\mathrm{E}}m}{r_{\mathrm{E}}+h}\right)=\tfrac{1}{2}mv_2^2+\left(-\frac{GM_{\mathrm{E}}m}{r_{\mathrm{E}}}\right)\rightarrow$$

$$v_2=\sqrt{v_1^2+2GM_{\mathrm{E}}\left(\frac{1}{r_{\mathrm{E}}}-\frac{1}{r_{\mathrm{E}}+h}\right)}=3835.1\,\mathrm{m/s}\approx\boxed{3840\,\mathrm{m/s}}$$

(*b*) We use the work-energy theorem, where work is done both by gravity (over a short distance) and the sand. The initial speed is 3835.1 m/s, and the final speed is 0.

$$W_{\mathrm{net}}=W_{\mathrm{G}}+W_{\mathrm{fr}}=mgd+W_{\mathrm{fr}}=\Delta K=\tfrac{1}{2}m\left(v_f^2-v_i^2\right)\rightarrow$$

$$W_{\mathrm{fr}}=-\tfrac{1}{2}mv_i^2-mgd=-\tfrac{1}{2}\left(575\,\mathrm{kg}\right)\left(3835.1\,\mathrm{m/s}\right)^2-\left(575\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(3.25\,\mathrm{m}\right)$$

$$=\boxed{-4.23\times10^9\,\mathrm{J}}$$

(*c*) The average force is the magnitude of the work done, divided by the distance moved in the sand.

$$F_{\mathrm{sand}}=\frac{\left|W_{\mathrm{sand}}\right|}{d_{\mathrm{sand}}}=\frac{4.23\times10^9\,\mathrm{J}}{3.25\,\mathrm{m}}=\boxed{1.30\times10^9\,\mathrm{N}}$$

(*d*) The work done by the sand shows up as thermal energy, so $\boxed{4.23\times10^9\,\mathrm{J}}$ of thermal energy is produced.

57. The external work required $\left(W_{\text{other}}\right)$ is the change in the mechanical energy of the satellite. Note the following, from the work-energy theorem.

$$W_{\text{total}} = W_{\text{gravity}} + W_{\text{other}} \rightarrow \Delta K = -\Delta U + W_{\text{other}} \rightarrow W_{\text{other}} = \Delta K + \Delta U = \Delta\left(K+U\right) = \Delta E_{\text{mech}}$$

From problem 52, we know that the mechanical energy is given by $E = -\frac{1}{2}\frac{GMm}{r}$.

$$E = -\tfrac{1}{2}\frac{GMm}{r} \rightarrow \Delta E = \left(-\tfrac{1}{2}\frac{GMm}{r}\right)_{\text{final}} - \left(-\tfrac{1}{2}\frac{GMm}{r}\right)_{\text{initial}} = \left(\tfrac{1}{2}\frac{GMm}{r}\right)_{\text{initial}} - \left(\tfrac{1}{2}\frac{GMm}{r}\right)_{\text{final}}$$

$$= \left(\tfrac{1}{2}\frac{GMm}{2r_{\text{E}}}\right)_{\text{initial}} - \left(\tfrac{1}{2}\frac{GMm}{3r_{\text{E}}}\right)_{\text{final}} = \boxed{\frac{GMm}{12r_{\text{E}}}}$$

58. (*a*) The work to put m_1 in place is 0, because it is still infinitely distant from the other two masses. The work to put m_2 in place is the potential energy of the 2-mass system, $-\frac{Gm_1m_2}{r_{12}}$. The work to put m_3 in place is the potential energy of the $m_1 - m_3$ combination, $-\frac{Gm_1m_3}{r_{13}}$, and the potential energy of the $m_2 - m_3$ combination, $-\frac{Gm_2m_3}{r_{23}}$. The total work is the sum of all of these potential energies, and so $W = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_2m_3}{r_{23}} \rightarrow$ $\boxed{W = -G\left(\frac{m_1m_2}{r_{12}} + \frac{m_1m_3}{r_{13}} + \frac{m_2m_3}{r_{23}}\right)}$. Notice that the work is negative, which is a result of the masses being gravitationally attracted towards each other.

(*b*) This formula gives the potential energy of the entire system. Potential energy does not "belong" to a single object, but rather to the entire system of objects that interact to give the potential energy.

(*c*) Actually, $\left|W\right| = G\left(\frac{m_1m_2}{r_{12}} + \frac{m_1m_3}{r_{13}} + \frac{m_2m_3}{r_{23}}\right)$ is the binding energy of the system. It would take that much work (a positive quantity) to separate the masses infinitely far from each other.

59. Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the asteroid at high altitude, and subscript 2 represents the asteroid at the Earth's surface. We have $v_1 = 660\,\text{m/s}$, $r_1 = r_{\text{E}} + 5.0\times10^9\,\text{m}$, and $r_2 = r_{\text{E}}$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + \left(-\frac{GM_{\text{E}}m}{r_1}\right) = \tfrac{1}{2}mv_2^2 + \left(-\frac{GM_{\text{E}}m}{r_2}\right) \rightarrow v_2 = \sqrt{v_1^2 + 2GM_{\text{E}}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$= \sqrt{\left(660\,\text{m/s}\right)^2 + \left\{\begin{matrix} 2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)\bullet \\ \left(\dfrac{1}{6.38\times10^6\,\text{m} + 5.0\times10^9\,\text{m}} - \dfrac{1}{6.38\times10^6\,\text{m}}\right)\end{matrix}\right\}} = \boxed{1.12\times10^4\,\text{m/s}}$$

60. Calculate the density of the shell. Use that density to calculate the potential due to a full sphere of radius r_1, and then subtract the potential due to a mass of radius r_2.

$$\rho = \frac{M}{\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)} \qquad M_{\substack{\text{full}\\\text{sphere}}} = \frac{M}{\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)}\tfrac{4}{3}\pi r_1^3 \qquad M_{\substack{\text{inner}\\\text{sphere}}} = \frac{M}{\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)}\tfrac{4}{3}\pi r_2^3$$

$$U_{\text{shell}} = U_{\substack{\text{full}\\\text{sphere}}} - U_{\substack{\text{inner}\\\text{sphere}}} = -\frac{GM_{\substack{\text{full}\\\text{sphere}}}m}{r} - \left(-\frac{GM_{\substack{\text{inner}\\\text{sphere}}}m}{r}\right) = -\frac{Gm}{r}\left(M_{\substack{\text{full}\\\text{sphere}}} - M_{\substack{\text{inner}\\\text{sphere}}}\right)$$

$$= -\frac{Gm}{r}\left(\frac{M}{\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)}\tfrac{4}{3}\pi r_1^3 - \frac{M}{\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)}\tfrac{4}{3}\pi r_2^3\right) = -\frac{GmM}{r}\left(\frac{r_1^3}{\left(r_1^3 - r_2^3\right)} - \frac{r_2^3}{\left(r_1^3 - r_2^3\right)}\right)$$

$$= -GmM/r$$

$\boxed{61.}$ (*a*) The escape speed from the surface of the Earth is $v_E = \sqrt{2GM_E/r_E}$. The escape velocity from the gravitational field of the sun, is $v_S = \sqrt{2GM_S/r_{SE}}$. In the reference frame of the Earth, if the spacecraft leaves the surface of the Earth with speed *v* (assumed to be greater than the escape velocity of Earth), then the speed v' at a distance far from Earth, relative to the Earth, is found from energy conservation.

$$\tfrac{1}{2}mv^2 - \frac{GM_E m}{r_E^2} = \tfrac{1}{2}mv'^2 \quad\rightarrow\quad v'^2 = v^2 - \frac{2GM_E}{r_E^2} = v^2 - v_E^2 \quad\rightarrow\quad v^2 = v'^2 + v_E^2$$

The reference frame of the Earth is orbiting the sun with speed v_0. If the rocket is moving with speed v' relative to the Earth, and the Earth is moving with speed v_0 relative to the Sun, then the speed of the rocket relative to the Sun is $v' + v_0$ (assuming that both speeds are in the same direction). This is to be the escape velocity from the Sun, and so $v_S = v' + v_0$, or $v' = v_S - v_0$. Combine this with the relationship from above.

$$v^2 = v'^2 + v_E^2 = \left(v_S - v_0\right)^2 + v_E^2 \quad\rightarrow\quad \boxed{v = \sqrt{\left(v_S - v_0\right)^2 + v_E^2}}$$

$$v_E = \sqrt{\frac{2GM_E}{r_E}} = \sqrt{\frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}{6.38\times10^6\,\text{m}}} = 1.118\times10^4\,\text{m/s}$$

$$v_S = \sqrt{\frac{2GM_S}{r_{SE}}} = \sqrt{\frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\,\text{kg}\right)}{1.496\times10^{11}\,\text{m}}} = 4.212\times10^4\,\text{m/s}$$

$$v_0 = \frac{2\pi r_{SE}}{T_{SE}} = \frac{2\pi\left(1.496\times10^{11}\,\text{m}\right)}{\left(3.156\times10^7\,\text{s}\right)} = 2.978\times10^4\,\text{m/s}$$

$$v = \sqrt{\left(v_S - v_0\right)^2 + v_E^2} = \sqrt{\left(4.212\times10^4\,\text{m/s} - 2.978\times10^4\,\text{m/s}\right)^2 + \left(1.118\times10^4\,\text{m/s}\right)^2}$$

$$= 1.665\times10^4\,\text{m/s} \approx \boxed{16.7\,\text{km/s}}$$

(*b*) Calculate the kinetic energy for a 1.00 kg mass moving with a speed of 1.665×10^4 m/s. This is the energy required per kilogram of spacecraft mass.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(1.00\,\text{kg}\right)\left(1.665\times10^4\,\text{m/s}\right)^2 = \boxed{1.39\times10^8\,\text{J}}$$

62. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd\cos 0° = mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \rightarrow t = \frac{mgh}{P} = \frac{(335\text{ kg})(9.80\,\text{m/s}^2)(16.0\text{ m})}{1750\text{ W}} = \boxed{30.0\,\text{s}}$$

63. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is Eq. 8-21 with the force and velocity in the same direction, $P = Fv$. Thus the force to propel the car forward is found by $F = P/v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F = P/v$.

$$F = \frac{P}{v} = \frac{(18\text{ hp})(746\text{ W/1 hp})}{(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)} = \boxed{510\,\text{N}}$$

64. (*a*) $K = \frac{1}{2}mv^2 = \frac{1}{2}(85\,\text{kg})(5.0\,\text{m/s})^2 = 1062.5\,\text{J} \approx \boxed{1100\,\text{J}}$

(*b*) The power required to stop him is the change in energy of the player, divided by the time to carry out the energy change.

$$P = \frac{1062.5\,\text{J}}{1.0\,\text{s}} = 1062.5\,\text{W} \approx \boxed{1100\,\text{W}}$$

65. The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$v_1 = 95\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s} \qquad v_2 = 65\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 18.06\,\text{m/s}$$

$$\Delta KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1080\text{ kg})\left[(18.06\,\text{m/s})^2 - (26.39\,\text{m/s})^2\right] = -1.999\times10^5\,\text{J}$$

$$P = \frac{W}{t} = \frac{1.999\times10^5\,\text{J}}{7.0\text{ s}} = 2.856\times10^4\,\text{W}\text{ , or } (2.856\times10^4\,\text{W})\frac{1\text{ hp}}{746\text{ W}} = 38.29\text{ hp}$$

So $\boxed{2.9\times10^4\,\text{W}}$ or $\boxed{38\,\text{hp}}$ is needed from the engine.

66. Since $P = \frac{W}{t}$, we have $W = Pt = 3.0\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right)(1\text{ hr})\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{8.1\times10^6\,\text{J}}$.

$\boxed{67.}$ The power is the force that the motor can provide times the velocity, as given in Eq. 8-21. The force provided by the motor is parallel to the velocity of the boat. The force resisting the boat will be the same magnitude as the force provided by the motor, since the boat is not accelerating, but in the opposite direction to the velocity.

$$P = \vec{\mathbf{F}}\bullet\vec{\mathbf{v}} = Fv \rightarrow F = \frac{P}{v} = \frac{(55\,\text{hp})(746\text{ W/1 hp})}{(35\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)} = 4220\,\text{N} \approx 4200\,\text{N}$$

So the force resisting the boat is $\boxed{4200\,\text{N, opposing the velocity}}$.

68. The average power is the energy transformed per unit time. The energy transformed is the change in kinetic energy of the car.

$$P=\frac{\text{energy transformed}}{\text{time}}=\frac{\Delta K}{t}=\frac{\frac{1}{2}m\left(v_2^2-v_1^2\right)}{t}=\frac{(1400\text{ kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{2(7.4\text{ s})}$$

$$=\boxed{6.6\times10^4\,\text{W}}\approx 88\text{ hp}$$

69. The minimum force needed to lift the football player vertically is equal to his weight, *mg*. The distance over which that force would do work would be the change in height, $\Delta y=(78\,\text{m})\sin 33^\circ$. So the work done in raising the player is $W=mg\Delta y$ and the power output required is the work done per unit time.

$$P=\frac{W}{t}=\frac{mg\Delta y}{t}=\frac{(92\,\text{kg})(9.80\,\text{m/s}^2)(78\,\text{m})\sin 33^\circ}{75\text{ sec}}=\boxed{510\,\text{W}}$$

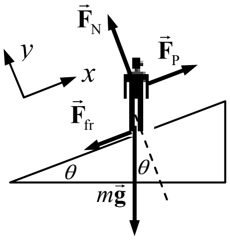
70. The force to lift the water is equal to its weight, and so the work to lift the water is equal to the weight times the vertical displacement. The power is the work done per unit time.

$$P=\frac{W}{t}=\frac{mgh}{t}=\frac{(21.0\text{ kg})(9.80\,\text{m/s}^2)(3.50\text{ m})}{60\text{ sec}}=\boxed{12.0\text{ W}}$$

71. The force to lift a person is equal to the person's weight, so the work to lift a person up a vertical distance *h* is equal to *mgh*. The work needed to lift *N* people is *Nmgh*, and so the power needed is the total work divided by the total time. We assume the mass of the average person to be 70 kg.

$$P=\frac{W}{t}=\frac{Nmgh}{t}=\frac{47000(70\,\text{kg})(9.80\,\text{m/s}^2)(200\,\text{m})}{3600\text{ s}}=1.79\times10^6\,\text{W}\approx\boxed{2\times10^6\,\text{W}}.$$

72. We represent all 30 skiers as one person on the free-body diagram. The engine must supply the pulling force. The skiers are moving with constant velocity, and so their net force must be 0.



$$\sum F_y=F_\text{N}-mg\cos\theta=0\ \rightarrow\ F_\text{N}=mg\cos\theta$$

$$\sum F_x=F_\text{P}-mg\sin\theta-F_\text{fr}=0\ \rightarrow$$

$$F_\text{P}=mg\sin\theta+F_\text{fr}=mg\sin\theta+\mu_\text{k}mg\cos\theta$$

The work done by F_P in pulling the skiers a distance *d* is $F_\text{P}d$ since the force is parallel to the displacement. Finally, the power needed is the work done divided by the time to move the skiers up the incline.

$$P=\frac{W}{t}=\frac{F_\text{P}d}{t}=\frac{mg\left(\sin\theta+\mu_\text{k}\cos\theta\right)d}{t}$$

$$=\frac{30(75\,\text{kg})(9.80\,\text{m/s}^2)(\sin 23^\circ+0.10\cos 23^\circ)(220\,\text{m})}{120\,\text{s}}=19516\,\text{W}\left(\frac{1\,\text{hp}}{746\text{ W}}\right)=\boxed{26\,\text{hp}}$$

$\boxed{73.}$ The net rate of work done is the power, which can be found by $P=Fv=mav$. The velocity is given by $v=\frac{dx}{dt}=15.0t^2-16.0t-44$ and $a=\frac{dv}{dt}=30.0t-16.0.$

(*a*) $P = mav = (0.28\,\text{kg})\left([30.0(2.0)-16.0]\,\text{m/s}^2\right)\left[15.0(2.0)^2-16.0(2.0)-44\right]\text{m/s}$

$= -197.1\,\text{W} \approx \boxed{-2.0\times10^2\,\text{W}}$

(*b*) $P = mav = (0.28\,\text{kg})\left([30.0(4.0)-16.0]\,\text{m/s}^2\right)\left[15.0(4.0)^2-16.0(4.0)-44\right]\text{m/s}$

$= 3844\,\text{W} \approx \boxed{3800\,\text{W}}$

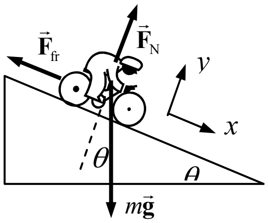
The average net power input is the work done divided by the elapsed time. The work done is the change in kinetic energy. Note $v(0) = -44\,\text{m/s}$, $v(2.0) = 15.0(2.0)^2 - 16.0(2.0) - 44 = -16\,\text{m/s}$, and $v(4.0) = 15.0(4.0)^2 - 16.0(4.0) - 44 = 132\,\text{m/s}$.

(*c*) $$P_{\substack{\text{avg}\\ \text{0 to 2.0}}} = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}m\left(v_f^2 - v_i^2\right)}{\Delta t} = \frac{\frac{1}{2}(0.28\,\text{kg})\left[(-16\,\text{m/s})^2 - (-44\,\text{m/s})^2\right]}{2.0\,\text{s}} = \boxed{-120\,\text{W}}$$

(*d*) $$P_{\substack{\text{avg}\\ \text{2.0 to 4.0}}} = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}m\left(v_f^2 - v_i^2\right)}{\Delta t} = \frac{\frac{1}{2}(0.28\,\text{kg})\left[(132\,\text{m/s})^2 - (16\,\text{m/s})^2\right]}{2.0\,\text{s}} = \boxed{1200\,\text{W}}$$

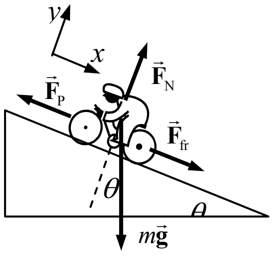
74. First, consider a free-body diagram for the cyclist going down hill. Write Newton's second law for the *x* direction, with an acceleration of 0 since the cyclist has a constant speed.

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \;\rightarrow\; F_{\text{fr}} = mg\sin\theta$$



Now consider the diagram for the cyclist going up the hill. Again, write Newton's second law for the *x* direction, with an acceleration of 0.

$$\sum F_x = F_{\text{fr}} - F_{\text{P}} + mg\sin\theta = 0 \;\rightarrow\; F_{\text{P}} = F_{\text{fr}} + mg\sin\theta$$



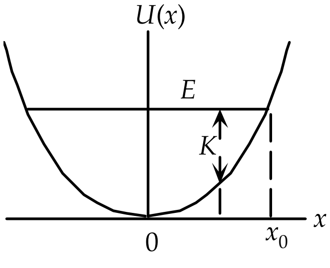
Assume that the friction force is the same when the speed is the same, so the friction force when going uphill is the same magnitude as when going downhill.

$$F_{\text{P}} = F_{\text{fr}} + mg\sin\theta = 2mg\sin\theta$$

The power output due to this force is given by Eq. 8-21, with the force and velocity parallel.

$$P = F_{\text{P}}v = 2mgv\sin\theta = 2(75\text{ kg})\left(9.80\,\text{m/s}^2\right)(4.0\,\text{m/s})\sin6.0^\circ = \boxed{610\,\text{W}}$$

75. The potential energy is given by $U(x) = \frac{1}{2}kx^2$ and so has a parabolic shape. The total energy of the object is $E = \frac{1}{2}kx_0^2$. The object, when released, will gain kinetic energy and lose potential energy until it reaches the equilibrium at $x = 0$, where it will have its maximum kinetic energy and maximum speed. Then it continues to move to the left, losing kinetic energy and gaining potential energy, until it reaches its extreme point of $x = x_0$. Then the motion reverses, until the object reaches its original position. Then it will continue this oscillatory motion between $x = 0$ and $x = x_0$.



76. (*a*) The total energy is $E = \frac{1}{2}kx_0^2 = \frac{1}{2}(160\,\text{N/m})(1.0\,\text{m})^2 = \boxed{80\,\text{J}}$. The answer has 2 significant figures.

(*b*) The kinetic energy is the total energy minus the potential energy.

$$K = E - U = E - \tfrac{1}{2}kx^2 = 80\,\text{J} - \tfrac{1}{2}(160\,\text{N/m})(0.50\,\text{m})^2 = \boxed{60\,\text{J}}$$

The answer has 2 significant figures.

(*c*) The maximum kinetic energy is the total energy, $\boxed{80\,\text{J}}$.

(*d*) The maximum speed occurs at $\boxed{x = 0}$, the equilibrium position at the center of the motion. Use the maximum kinetic energy (which is equal to the total energy) to find the maximum speed.

$$K_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m}} = \sqrt{\frac{2(80\,\text{J})}{5.0\,\text{kg}}} = \boxed{5.7\,\text{m/s}}$$

(*e*) The maximum acceleration occurs at the maximum displacement, $\boxed{x = 1.0\,\text{m}}$, since

$$F = ma = -kx \quad \rightarrow \quad |a| = \frac{k|x|}{m}.$$

$$|a_{\text{max}}| = \frac{k|x_{\text{max}}|}{m} = \frac{(160\,\text{N/m})(1.0\text{m})}{5.0\,\text{kg}} = \boxed{32\,\text{m/s}^2}$$

77. (*a*) To find possible minima and maxima, set the first derivative of the function equal to 0 and solve for the values of *r*.

$$U(r) = -\frac{a}{r^6} + \frac{b}{r^{12}} = \frac{1}{r^{12}}\left(b - ar^6\right) \quad \rightarrow \quad \frac{dU}{dr} = 6\frac{a}{r^7} - 12\frac{b}{r^{13}}$$

$$\frac{dU}{dr} = 0 \quad \rightarrow \quad \frac{a}{r^7} = 2\frac{b}{r^{13}} \quad \rightarrow \quad r_{\text{crit}} = \left(\frac{2b}{a}\right)^{1/6}, \infty$$

The second derivative test is used to determine the actual type of critical points found.

$$\frac{d^2U}{dr^2} = -42\frac{a}{r^8} + 156\frac{b}{r^{14}} = \frac{1}{r^{14}}\left(156b - 42ar^6\right)$$

$$\left.\frac{d^2U}{dr^2}\right|_{\left(\frac{2b}{a}\right)^{1/6}} = \frac{1}{\left(\frac{2b}{a}\right)^{14/6}}\left(156b - 42a\frac{2b}{a}\right) = \frac{1}{\left(\frac{2b}{a}\right)^{14/6}}(156b - 84b) > 0 \quad \rightarrow r_{\text{crit}} = \left(\frac{2b}{a}\right)^{1/6}$$

Thus there is a $\boxed{\text{minimum at } r = \left(\frac{2b}{a}\right)^{1/6}}$. We also must check the endpoints of the function.

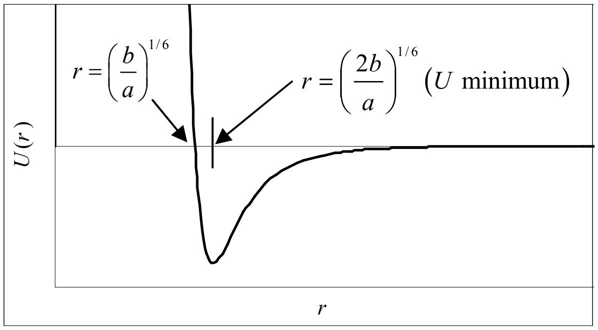
We see from the form $U(r) = \frac{1}{r^{12}}\left(b - ar^6\right)$ that as $r \rightarrow 0$, $U(r) \rightarrow \infty$, and so there is a $\boxed{\text{maximum at } r = 0}$.

(*b*) Solve $U(r) = 0$ for the distance.

$$U(r) = -\frac{a}{r^6} + \frac{b}{r^{12}} = \frac{1}{r^{12}}\left(b - ar^6\right) = 0 \quad \rightarrow \quad \frac{1}{r^{12}} = 0 \text{ or } \left(b - ar^6\right) = 0 \quad \rightarrow$$

$$\boxed{r = \infty \; ; \; r = \left(\frac{b}{a}\right)^{1/6}}$$

(*c*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH08.XLS," on tab "Problem 8.77c."



(*d*) For $E<0$, there will be bound oscillatory motion between two turning points. This could represent a chemical bond type of situation. For $E>0$, the motion will be unbounded, and so the atoms will not stay together.

(*e*) The force is the opposite of the slope of the potential energy graph.

$$\boxed{F>0 \text{ for } r<\left(\frac{2b}{a}\right)^{1/6}} \ ; \ \boxed{F<0 \ \text{ for } \left(\frac{2b}{a}\right)^{1/6}<r<\infty} \ ; \ \boxed{F=0 \ \text{ for } r=\left(\frac{2b}{a}\right)^{1/6}, r=\infty}$$

(*f*) $F(r)=-\frac{dU}{dr}=\boxed{\frac{12b}{r^{13}}-\frac{6a}{r^{7}}}$

78. The binding energy will be $U(\infty)-U(r_{\text{U min}})$. The value of *r* for which *U*(*r*) has a minimum is found in problem 77 to be $r=\left(\frac{2b}{a}\right)^{1/6}$.

$$U(\infty)-U(r_{\text{U min}})=0-U\left(r=\left(\frac{2b}{a}\right)^{1/6}\right)=0-\left[-\frac{a}{\left(\frac{2b}{a}\right)}+\frac{b}{\left(\frac{2b}{a}\right)^{2}}\right]=0-\left[-\frac{a^2}{2b}+\frac{a^2b}{4b^2}\right]=\boxed{\frac{a^2}{4b}}$$

Notice that this is just the depth of the potential well.

$\boxed{79.}$ The power must exert a force equal to the weight of the elevator, through the vertical height, in the given time.

$$P=\frac{mgh}{t}=\frac{(885\,\text{kg})(9.80\,\text{m/s}^2)(32.0\,\text{m})}{(11.0\,\text{s})}=\boxed{2.52\times10^4\,\text{W}}$$

80. Since there are no non-conservative forces, the mechanical energy of the projectile will be conserved. Subscript 1 represents the projectile at launch and subscript 2 represents the projectile as it strikes the ground. The ground is the zero location for potential energy $(y=0)$. We have $v_1=165\,\text{m/s}$, $y_1=135\,\text{m}$, and $y_2=0$. Solve for v_2.

$$E_1=E_2 \ \rightarrow \ \tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow \ \tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2 \ \rightarrow$$

$$v_2=\sqrt{v_1^2+2gy_1}=\sqrt{(165\,\text{m/s})^2+2(9.80\,\text{m/s}^2)(135\,\text{m})}=\boxed{173\,\text{m/s}}$$

Notice that the launch angle does not enter the problem, and so does not influence the final speed.

81. (*a*) Use conservation of mechanical energy, assuming there are no non-conservative forces. Subscript 1 represents the water at the top of the dam, and subscript 2 represents the water as it strikes the turbine blades. The level of the turbine blades is the zero location for potential energy $(y=0)$. Assume that the water goes over the dam with an approximate speed of 0. We have $v_1=0,\ y_1=80\text{ m},$ and $y_2=0.$ Solve for v_2.

$$E_1=E_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ mgy_1=\tfrac{1}{2}mv_2^2 \ \rightarrow$$

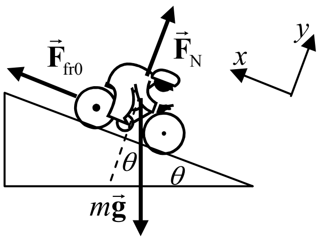
$$v_2=\sqrt{2gy_1}=\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(88\,\text{m}\right)}=41.53\,\text{m/s}\approx\boxed{42\,\text{m/s}}$$

(*b*) The energy of the water at the level of the turbine blades is all kinetic energy, and so is given by $\tfrac{1}{2}mv_2^2$. 55% of that energy gets transferred to the turbine blades. The rate of energy transfer to the turbine blades is the power developed by the water.

$$P=0.55\left(\tfrac{1}{2}\frac{m}{t}v_2^2\right)=\frac{(0.55)(550\,\text{kg/s})(41.53\,\text{m/s})^2}{2}=\boxed{2.6\times10^5\,\text{W}}$$

82. First, define three speeds:

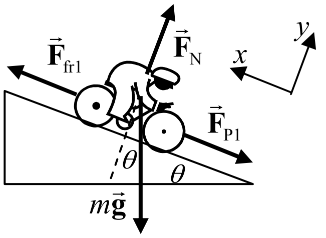
$v_0=12\,\text{km/h}=$ speed when coasting downhill.
$v_1=32\,\text{km/h}=$ speed when pedaling downhill.
$v_2=$ Speed when climbing the hill.



For coasting downhill at a constant speed, consider the first free-body diagram shown. The net force on the bicyclist must be 0. Write Newton's second law for the *x* direction.

$$\sum F_x=F_{\text{fr0}}-mg\sin\theta=0 \ \rightarrow\ F_{\text{fr0}}=mg\sin\theta$$

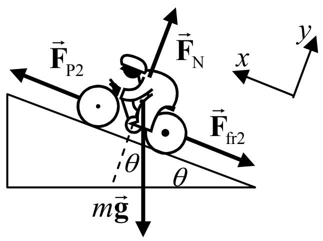
Note that this occurs at $v=v_0$.



When pumping hard downhill, the speed is $v_1=\tfrac{32}{12}v_o=\tfrac{8}{3}v_o$. Since the frictional force is proportional to v^2, the frictional force increases by a factor of $\left(\tfrac{8}{3}\right)^2$: $F_{\text{fr1}}=\left(\tfrac{8}{3}\right)^2F_{\text{fr0}}=\tfrac{64}{9}mg\sin\theta$. See the second free-body diagram. There is a new force, $\vec{\mathbf{F}}_{\text{P1}}$, created by the bicyclist. Since the cyclist is moving at a constant speed, the net force in the *x* direction must still be 0. Solve for F_{P1}, and calculate the power associated with the force.

$$\sum F_x=F_{\text{fr1}}-mg\sin\theta-F_{\text{P1}}=0 \ \rightarrow\ F_{\text{P1}}=F_{\text{fr1}}-mg\sin\theta=\left(\tfrac{64}{9}-1\right)mg\sin\theta=\tfrac{55}{9}mg\sin\theta$$

$$P_1=F_{\text{fr1}}v_1=\tfrac{55}{9}mgv_1\sin\theta$$



Now consider the cyclist going uphill. The speed of the cyclist going up the hill is v_2. Since the frictional force is proportional to v^2, the frictional force is given by $F_{\text{fr2}}=\left(v_2/v_0\right)^2mg\sin\theta$. See the third free-body diagram. There is a new force, $\vec{\mathbf{F}}_{\text{P2}}$, created by the bicyclist. Since the cyclist is moving at a constant speed, the net force in the *x* direction must still be 0.

$$\sum F_x=F_{\text{P2}}-mg\sin\theta-F_{\text{fr2}}=0$$

The power output of the cyclist while pedaling uphill is the same as when pedaling going downhill.

$$P_2 = P_1 = \tfrac{55}{9} mgv_1 \sin\theta \rightarrow F_{P2}v_2 = \tfrac{55}{9} mgv_1 \sin\theta \rightarrow F_{P2} = \tfrac{55}{9} mg\left(v_1/v_2\right)\sin\theta$$

Combine this information with Newton's second law equation for the bicyclist going uphill.

$$F_{P2} - mg\sin\theta - F_{fr2} = \tfrac{55}{9} mg\left(v_1/v_2\right)\sin\theta - mg\sin\theta - \left(v_2/v_0\right)^2 mg\sin\theta = 0$$

This simplifies to the following cubic equation: $v_2^3 + v_2v_0^2 - \frac{55}{9}v_1v_0^2 = 0$. Note that since every term has speed to the third power, there is no need to do unit conversions. Numerically, this equation is $v_2^3 + 144v_2 - 28160 = 0$, when the speed is in km/h. Solving this cubic equation (with a spreadsheet, for example) gives $v_2 = 28.847\,\text{km/h} \approx \boxed{29\,\text{km/h}}$.

83. (*a*) The speed v_B can be found from conservation of mechanical energy. Subscript A represents the skier at the top of the jump, and subscript B represents the skier at the end of the ramp. Point B is taken as the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = 40.6\,\text{m}$, and $y_2 = 0$. Solve for v_2.

$$E_A = E_B \rightarrow \tfrac{1}{2}mv_A^2 + mgy_A = \tfrac{1}{2}mv_B^2 + mgy_B \rightarrow mgy_A = \tfrac{1}{2}mv_B^2 \rightarrow$$

$$v_B = \sqrt{2gy_A} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(40.6\,\text{m}\right)} = 28.209\,\text{m/s} \approx \boxed{28.2\,\text{m/s}}$$

(*b*) Now we use projectile motion. We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text{slope}} = -x\tan 30^\circ$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction. The initial y-velocity is 0, and the x-velocity is v_B as found above.

$$x = v_Bt \; ; \; y_{\text{proj}} = y_0 - \tfrac{1}{2}gt^2 = y_0 - \tfrac{1}{2}g\left(x/v_B\right)^2$$

The skier lands at the intersection of the two paths, so $y_{\text{slope}} = y_{\text{proj}}$.

$$y_{\text{slope}} = y_{\text{proj}} \rightarrow -x\tan 30^\circ = y_0 - \tfrac{1}{2}g\left(\frac{x}{v_B}\right)^2 \rightarrow gx^2 - x\left(2v_B^2\tan 30^\circ\right) - 2y_0v_B^2 = 0 \rightarrow$$

$$x = \frac{\left(2v_B^2\tan 30^\circ\right) \pm \sqrt{\left(2v_B^2\tan 30^\circ\right)^2 + 8gy_0v_B^2}}{2g} = \frac{\left(v_B^2\tan 30^\circ\right) \pm \sqrt{\left(v_B^2\tan 30^\circ\right)^2 + 2gy_0v_B^2}}{g}$$

Solving this with the given values gives $x = -7.09\,\text{m}, 100.8\,\text{m}$. The positive root is taken.

Finally, $s\cos 30.0^\circ = x \rightarrow s = \dfrac{x}{\cos 30.0^\circ} = \dfrac{100.8\,\text{m}}{\cos 30.0^\circ} = \boxed{116\,\text{m}}$.

84. (*a*) The slant of the jump at point B does not affect the energy conservation calculations from problem 83, and so this part of the problem is solved exactly as in problem 83, and the answer is exactly the same as in problem 83: $v_B = 28.209\,\text{m/s} \approx \boxed{28.2\,\text{m/s}}$.

(*b*) The projectile motion is now different because the velocity at point B is not purely horizontal. We have that $v_B = 28.209\,\text{m/s}$ and $v_{By} = 3.0\,\text{m/s}$. Use the Pythagorean theorem to find v_{Bx}.

$$v_{Bx} = \sqrt{v_B^2 - v_{By}^2} = \sqrt{\left(28.209\,\text{m/s}\right)^2 - \left(3.0\,\text{m/s}\right)^2} = 28.049\,\text{m/s}$$

We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text{slope}} = -x\tan 30^\circ$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction.

$$x = v_{Bx}t \;\; ; \; y_{\text{proj}} = y_0 + v_{By}t - \tfrac{1}{2}gt^2 = y_0 + v_{By}\left(\frac{x}{v_{Bx}}\right) - \tfrac{1}{2}g\left(\frac{x}{v_{Bx}}\right)^2$$

The skier lands at the intersection of the two paths, so $y_{\text{slope}} = y_{\text{proj}}$.

$$y_{\text{slope}} = y_{\text{proj}} \;\rightarrow\; -x\tan 30° = y_0 + v_{By}\left(\frac{x}{v_{Bx}}\right) - \tfrac{1}{2}g\left(\frac{x}{v_{Bx}}\right)^2 \;\rightarrow$$

$$gx^2 - x\left[2v_{Bx}\left(v_{Bx}\tan 30° + v_{By}\right)\right] - 2y_0 v_{Bx}^2 = 0 \;\rightarrow$$

$$x = \frac{\left[2v_{Bx}\left(v_{Bx}\tan 30° + v_{By}\right)\right] \pm \sqrt{\left[2v_{Bx}\left(v_{Bx}\tan 30° + v_{By}\right)\right]^2 - 8gy_0 v_{Bx}^2}}{2g}$$

Solving this with the given values gives $x = -6.09\,\text{m},\, 116.0\,\text{m}$. The positive root is taken.

Finally, $s\cos 30.0° = x \;\rightarrow\; s = \dfrac{x}{\cos 30.0°} = \dfrac{116.0\,\text{m}}{\cos 30.0°} = \boxed{134\,\text{m}}$.

$\boxed{85.}$ (*a*) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus only gravity does work on the ball, and so the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = \ell$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; mg\ell = \tfrac{1}{2}mv_2^2 \;\rightarrow\; v_2 = \boxed{\sqrt{2g\ell}}$$

(*b*) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_2 = \sqrt{2g\ell}$, $y_2 = 0$, and $y_3 = 2(\ell - h) = 2(\ell - 0.80\ell) = 0.40\ell$. Solve for v_3.

$$E_2 = E_3 \;\rightarrow\; \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \;\rightarrow\; \tfrac{1}{2}m(2g\ell) = \tfrac{1}{2}mv_3^2 + mg(0.40\ell) \;\rightarrow$$

$$\boxed{v_3 = \sqrt{1.2g\ell}}$$

86. The ball is moving in a circle of radius $(\ell - h)$. If the ball is to complete the circle with the string just going slack at the top of the circle, the force of gravity must supply the centripetal force at the top of the circle. This tells the critical (slowest) speed for the ball to have at the top of the circle.

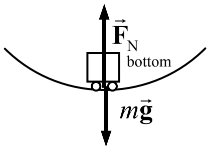
$$mg = \frac{mv_{\text{crit}}^2}{r} \;\rightarrow\; v_{\text{crit}}^2 = gr = g(\ell - h)$$

To find another expression for the speed, we use energy conservation. Subscript 1 refers to the ball at the launch point, and subscript 2 refers to the ball at the top of the circular path about the peg. The zero for gravitational potential energy is taken to be the lowest point of the ball's path. Let the speed at point 2 be the critical speed found above.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; mg\ell = \tfrac{1}{2}mg(\ell - h) + 2mg(\ell - h) \;\rightarrow$$

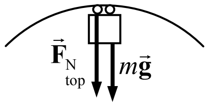
$$\boxed{h = 0.6\ell}$$

If *h* is any smaller than this, then the ball would be moving slower than the critical speed when it reaches the top of the circular path, and would not stay in centripetal motion.



87. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.

$$\sum F_{\text{bottom}} = F_{\substack{\text{N}\\\text{bottom}}} - mg = m v_{\text{bottom}}^2/R \quad\rightarrow\quad F_{\substack{\text{N}\\\text{bottom}}} = mg + m v_{\text{bottom}}^2/R$$



Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.

$$\sum F_{\text{top}} = F_{\substack{\text{N}\\\text{top}}} + mg = m v_{\text{top}}^2/R \quad\rightarrow\quad F_{\substack{N\\\text{top}}} = m v_{\text{top}}^2/R - mg\,.$$

Assume that the speed at the top is large enough that $F_{\substack{N\\\text{top}}} > 0$, and so $v_{\text{top}} > \sqrt{Rg}$.

Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for potential energy $(y = 0)$. We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR$$

The difference in apparent weights is the difference in the normal forces.

$$F_{\substack{\text{N}\\\text{bottom}}} - F_{\substack{\text{N}\\\text{top}}} = \left(mg + m v_{\text{bottom}}^2/R\right) - \left(m v_{\text{top}}^2/R - mg\right) = 2mg + m\left(v_{\text{bottom}}^2 - v_{\text{top}}^2\right)/R$$

$$= 2mg + m\left(4gR\right)/R = \boxed{6mg}$$

Notice that the result does not depend on either v or R.

88. The spring constant for the scale can be found from the 0.5 mm compression due to the 760 N force. $k = \dfrac{F}{x} = \dfrac{760\,\text{N}}{5.0\times10^{-4}\,\text{m}} = 1.52\times10^6\ \text{N/m}$. Use conservation of energy for the jump. Subscript 1 represents the initial location, and subscript 2 represents the location at maximum compression of the scale spring. Assume that the location of the uncompressed scale spring is the 0 location for gravitational potential energy. We have $v_1 = v_2 = 0$ and $y_1 = 1.0\,\text{m}$. Solve for y_2, which must be negative.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad\rightarrow$$

$$mgy_1 = mgy_2 + \tfrac{1}{2}ky_2^2 \quad\rightarrow\quad y_2^2 + 2\frac{mg}{k}y_2 - 2\frac{mg}{k}y_1 = y_2^2 + 1.00\times10^{-3}y_2 - 1.00\times10^{-3} = 0$$

$$y_2 = -3.21\times10^{-2}\,\text{m},\ 3.11\times10^{-2}\,\text{m}$$

$$F_{\text{scale}} = k|x| = \left(1.52\times10^6\ \text{N/m}\right)\left(3.21\times10^{-2}\,\text{m}\right) = \boxed{4.9\times10^4\,\text{N}}$$

89. (*a*) The work done by the hiker against gravity is the change in gravitational potential energy.

$$W_{\text{G}} = mg\Delta y = (65\ \text{kg})\left(9.80\,\text{m/s}^2\right)\left(4200\,\text{m} - 2800\,\text{m}\right) = 8.918\times10^5\,\text{J} \approx \boxed{8.9\times10^5\,\text{J}}$$

(*b*) The average power output is found by dividing the work by the time taken.

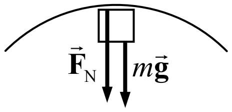
$$P_{\text{output}} = \frac{W_{\text{grav}}}{t} = \frac{8.918\times10^5\,\text{J}}{(5.0\,\text{h})(3600\ \text{s/1 h})} = 49.54\,\text{W} \approx \boxed{5.0\times10^1\,\text{W}}$$

$$49.54\,\text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{6.6\times10^{-2}\,\text{hp}}$$

(*c*) The output power is the efficiency times the input power.

$$P_{\text{output}} = 0.15P_{\text{input}} \quad\rightarrow\quad P_{\text{input}} = \frac{P_{\text{output}}}{0.15} = \frac{49.54\,\text{W}}{0.15} = \boxed{330\,\text{W}} = \boxed{0.44\,\text{hp}}$$

90. (*a*) Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's second law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_N = 0$.



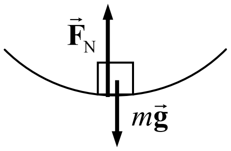
$$\sum F_R = F_N + mg = mv^2/r \;\rightarrow\; mg = mv_{top}^2/r \;\rightarrow\; v_{top} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for potential energy $(y = 0)$. We have $v_1 = 0,\; y_1 = h,\; v_2 = \sqrt{gr},$ and $y_2 = 2r$. Solve for *h*.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; 0 + mgh = \tfrac{1}{2}mgr + 2mgr \;\rightarrow$$

$$h = \boxed{2.5r}$$

(*b*) See the free-body diagram for the block at the bottom of the loop. The net force is again centripetal, and must be upwards.



$$\sum F_R = F_N - mg = mv^2/r \;\rightarrow\; F_N = mg + mv_{bottom}^2/r$$

The speed at the bottom of the loop can be found from energy conservation, similar to what was done in part (*a*) above, by equating the energy at the release point (subscript 1) and the bottom of the loop (subscript 2). We now have $v_1 = 0$, $y_1 = 2h = 5r$, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; 0 + 5mgr = \tfrac{1}{2}mv_{bottom}^2 + 0 \;\rightarrow$$

$$v_{bottom}^2 = 10gr \;\rightarrow\; F_N = mg + mv_{bottom}^2/r = mg + 10mg = \boxed{11mg}$$

(*c*) Again we use the free body diagram for the top of the loop, but now the normal force does not vanish. We again use energy conservation, with $v_1 = 0$, $y_1 = 3r$, and $y_2 = 0$. Solve for v_2.

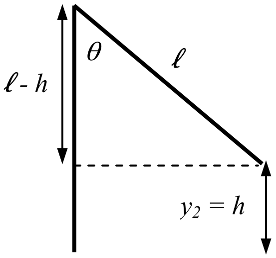
$$\sum F_R = F_N + mg = mv^2/r \;\rightarrow\; F_N = mv_{top}^2/r - mg$$

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; 0 + 3mgr = \tfrac{1}{2}mv_{top}^2 + 0 \;\rightarrow$$

$$v_{top}^2 = 6gr \;\rightarrow\; F_N = mv_{top}^2/r - mg = 6mg - mg = \boxed{5mg}$$

(*d*) On the flat section, there is no centripetal force, and $F_N = \boxed{mg}$.

$\boxed{91.}$ (*a*) Use conservation of energy for the swinging motion. Subscript 1 represents the student initially grabbing the rope, and subscript 2 represents the student at the top of the swing. The location where the student initially grabs the rope is the zero location for potential energy $(y = 0)$. We have $v_1 = 5.0\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$. Solve for y_2.



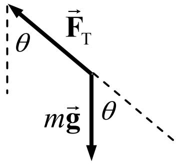
$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow$$

$$\tfrac{1}{2}mv_1^2 = mgy_2 \;\rightarrow\; y_2 = \frac{v_1^2}{2g} = h$$

Calculate the angle from the relationship in the diagram.

$$\cos\theta = \frac{\ell - h}{\ell} = 1 - \frac{h}{\ell} = 1 - \frac{v_1^2}{2g\ell} \;\rightarrow$$

$$\theta = \cos^{-1}\left(1-\frac{v_1^2}{2g\ell}\right) = \cos^{-1}\left(1-\frac{(5.0\,\mathrm{m/s})^2}{2\left(9.80\,\mathrm{m/s^2}\right)(10.0\,\mathrm{m})}\right) = \boxed{29^\circ}$$



(*b*) At the release point, the speed is 0, and so there is no radial acceleration, since $a_\mathrm{R} = v^2/r$. Thus the centripetal force must be 0. Use the free-body diagram to write Newton's second law for the radial direction.

$$\sum F_\mathrm{R} = F_\mathrm{T} - mg\cos\theta = 0 \quad\rightarrow$$

$$F_\mathrm{T} = mg\cos\theta = (56\text{ kg})\left(9.80\,\mathrm{m/s^2}\right)\cos 29^\circ = \boxed{480\,\mathrm{N}}$$

(*c*) Write Newton's second law for the radial direction for any angle, and solve for the tension.

$$\sum F_\mathrm{R} = F_\mathrm{T} - mg\cos\theta = mv^2/r \quad\rightarrow\quad F_\mathrm{T} = mg\cos\theta + mv^2/r$$

As the angle decreases, the tension increases, and as the speed increases, the tension increases. Both effects are greatest at the bottom of the swing, and so that is where the tension will be at its maximum.

$$F_{\underset{\max}{\mathrm{T}}} = mg\cos 0 + mv_1^2/r = (56\text{ kg})\left(9.80\,\mathrm{m/s^2}\right) + \frac{(56\,\mathrm{kg})(5.0\,\mathrm{m/s})^2}{10.0\,\mathrm{m}} = \boxed{690\,\mathrm{N}}$$

92. (*a*) $$F(r) = -\frac{dU(r)}{dr} = -\left[(-U_0)\left(-\frac{r_0}{r^2}\right)e^{-r/r_0} + \left(-U_0\frac{r_0}{r}\right)\left(-\frac{1}{r_0}\right)e^{-r/r_0}\right] = \boxed{-U_0\frac{r_0}{r}e^{-r/r_0}\left(\frac{1}{r}+\frac{1}{r_0}\right)}$$

(*b*) $$F(3r_0)/F(r_0) = \frac{-U_0\dfrac{r_0}{3r_0}e^{-3r_0/r_0}\left(\dfrac{1}{3r_0}+\dfrac{1}{r_0}\right)}{-U_0\dfrac{r_0}{r_0}e^{-r_0/r_0}\left(\dfrac{1}{r_0}+\dfrac{1}{r_0}\right)} = \boxed{\tfrac{2}{9}e^{-2} \approx 0.03}$$

(*c*) $$F(r) = -\frac{dU(r)}{dr} = -\left[(-C)\left(-\frac{1}{r^2}\right)\right] = -\frac{C}{r^2} \quad;\quad F(3r_0)/F(r_0) = \frac{-C\dfrac{1}{(3r_0)^2}}{-C\dfrac{1}{(r_0)^2}} = \boxed{\tfrac{1}{9} \approx 0.1}$$

The Yukawa potential is said to be "short range" because as the above examples illustrate, the Yukawa force "drops off" more quickly then the electrostatic force. The Yukawa force drops by about 97% when the distance is tripled, while the electrostatic force only drops by about 89%.

93. Energy conservation can be used to find the speed that the water must leave the ground. We take the ground to be the 0 level for gravitational potential energy. The speed at the top will be 0.

$$E_\text{ground} = E_\text{top} \quad\rightarrow\quad \tfrac{1}{2}mv_\text{ground}^2 = mgy_\text{top} \quad\rightarrow\quad v_\text{ground} = \sqrt{2gy_\text{top}} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)(33\,\mathrm{m})} = 25.43\,\mathrm{m/s}$$

The area of the water stream times the velocity gives a volume flow rate of water. If that is multiplied by the density, then we have a mass flow rate. That is verified by dimensional analysis.

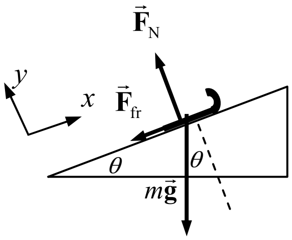
$$Av\rho \quad\rightarrow\quad \left[\mathrm{m^2}\right][\mathrm{m/s}]\left[\mathrm{kg/m^3}\right] = [\mathrm{kg/s}]$$

Another way to think about it is that $Av\rho$ is the mass that flows out of the hose per second. It takes a minimum force of $Av\rho g$ to lift that mass, and so the work done per second to lift that mass to a height of y_top is $Av\rho g y_\text{top}$. That is the power required.

$$P = Av\rho g y_{\text{top}} = \pi\left(1.5\times10^{-2}\,\text{m}\right)^2\left(25.43\,\text{m/s}\right)\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(33\,\text{m}\right) = 5813\,\text{W}$$

$$\approx \boxed{5800\,\text{W or } 7.8\,\text{hp}}$$

94. A free-body diagram for the sled is shown as it moves up the hill. From this we get an expression for the friction force.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \;\rightarrow\; F_{\text{N}} = mg\cos\theta \;\rightarrow\; F_{\text{fr}} = \mu_{\text{k}} mg\cos\theta$$

(*a*) We apply conservation of energy with a frictional force as given in Eq. 8-15. Subscript 1 refers to the sled at the start of its motion, and subscript 2 refers to the sled at the top of its motion. Take the starting position of the sled to be the 0 for gravitational potential energy. We have $v_1 = 2.4\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$. The relationship between the distance traveled along the incline and the height the sled rises is $y_2 = d\sin\theta$. Solve for *d*.

$$E_1 = E_2 + F_{\text{fr}}\ell \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}d \;\rightarrow$$

$$\tfrac{1}{2}mv_1^2 = mgd\sin\theta + \mu_{\text{k}}mgd\cos\theta \;\rightarrow$$

$$d = \frac{v_1^2}{2g\left(\sin\theta + \mu_{\text{k}}\cos\theta\right)} = \frac{\left(2.4\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)\left(\sin 28^\circ + 0.25\cos 28^\circ\right)} = 0.4258\,\text{m} \approx \boxed{0.43\,\text{m}}$$

(*b*) For the sled to slide back down, the friction force will now point UP the hill in the free-body diagram. In order for the sled to slide down, the component of gravity along the hill must be large than the maximum force of static friction.

$$mg\sin\theta > F_{\text{fr}} \;\rightarrow\; mg\sin\theta > \mu_{\text{s}}mg\cos\theta \;\rightarrow\; \mu_{\text{s}} < \tan 28^\circ \;\rightarrow\; \boxed{\mu_{\text{s}} < 0.53}$$

(*c*) We again apply conservation of energy including work done by friction. Subscript 1 refers to the sled at the top of the incline, and subscript 2 refers to the sled at the bottom of the incline. We have $v_1 = 0$, $y_1 = d\sin\theta$, and $y_2 = 0$.

$$E_1 = E_2 + F_{\text{fr}}\ell \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}d \;\rightarrow$$

$$mgd\sin\theta = \tfrac{1}{2}mv_2^2 + \mu_{\text{k}}mgd\cos\theta \;\rightarrow$$

$$v_2 = \sqrt{2gd\left(\sin\theta - \mu_{\text{k}}\cos\theta\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.4258\,\text{m}\right)\left(\sin 28^\circ - 0.25\cos 28^\circ\right)}$$

$$= \boxed{1.4\,\text{m/s}}$$

95. We apply conservation of mechanical energy. We take the surface of the Moon to be the 0 level for gravitational potential energy. Subscript 1 refers to the location where the engine is shut off, and subscript 2 refers to the surface of the Moon. Up is the positive *y*-direction.

(*a*) We have $v_1 = 0$, $y_1 = h$, $v_2 = 3.0\,\text{m/s}$, and $y_2 = 0$.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; mgh = \tfrac{1}{2}mv_2^2 \;\rightarrow$$

$$h = \frac{v_2^2}{2g} = \frac{\left(3.0\,\text{m/s}\right)^2}{2\left(1.62\,\text{m/s}^2\right)} = \boxed{2.8\,\text{m}}$$

(*b*) We have the same conditions except $v_1 = -2.0\,\text{m/s}$.

$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgh = \tfrac{1}{2}mv_2^2 \;\rightarrow$$

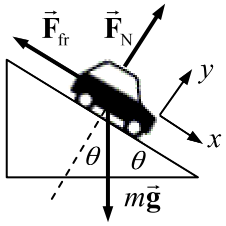
$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{\left(3.0\,\text{m/s}\right)^2 - \left(-2.0\,\text{m/s}\right)^2}{2\left(1.62\,\text{m/s}^2\right)} = \boxed{1.5\,\text{m}}$$

(*c*) We have the same conditions except $v_1 = 2.0\,\mathrm{m/s}$. And since the speeds, not the velocities, are used in the energy conservation calculation, this is the same as part (*b*), and so $h = \boxed{1.5\,\mathrm{m}}$.

$$E_1 = E_2 \quad\to\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\to\quad \tfrac{1}{2}mv_1^2 + mgh = \tfrac{1}{2}mv_2^2 \quad\to$$

$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{(3.0\,\mathrm{m/s})^2 - (-2.0\,\mathrm{m/s})^2}{2\left(1.62\,\mathrm{m/s^2}\right)} = \boxed{1.5\,\mathrm{m}}$$

96. A free-body diagram for the car is shown. We apply conservation of energy with a frictional force as given in Eq. 8-15. Subscript 1 refers to the car at the start of its motion, and subscript 2 refers to the sled at the end of the motion. Take the ending position of the car to be the 0 for gravitational potential energy. We have $v_1 = 95\,\mathrm{km/h}$, $y_2 = 0$, and $v_2 = 35\,\mathrm{km/h}$. The relationship between the distance traveled along the incline and the initial height of the car is $y_1 = d\sin\theta$.



$$E_1 = E_2 + E_{\mathrm{fr}} \quad\to\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + E_{\mathrm{fr}} \quad\to$$

$$E_{\mathrm{fr}} = \tfrac{1}{2}m\left(v_1^2 - v_2^2\right) + mgy_1 = \tfrac{1}{2}m\left[\left(v_1^2 - v_2^2\right) + 2gd\sin\theta\right]$$

$$= \tfrac{1}{2}(1500\,\mathrm{kg})\left[\left((95\,\mathrm{km/h})^2 - (35\,\mathrm{km/h})^2\right)\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)^2 + 2\left(9.80\,\mathrm{m/s^2}\right)\left(3.0\times10^2\,\mathrm{m}\right)\sin 17^\circ\right]$$

$$= \boxed{1.7\times10^6\,\mathrm{J}}$$

$\boxed{97.}$ The energy to be stored is the power multiplied by the time: $E = Pt$. The energy will be stored as the gravitational potential energy increase in the water: $E = \Delta U = mg\Delta y = \rho Vg\Delta y$, where ρ is the density of the water, and V is the volume of the water.

$$Pt = \rho Vg\Delta y \quad\to\quad V = \frac{Pt}{\rho g\Delta y} = \frac{\left(180\times10^6\,\mathrm{W}\right)(3600\ \mathrm{s})}{\left(1.00\times10^3\,\mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)(380\ \mathrm{m})} = \boxed{1.7\times10^5\,\mathrm{m^3}}$$

98. It is shown in problem 52 that the total mechanical energy for a satellite orbiting in a circular orbit of radius *r* is $E = -\tfrac{1}{2}\dfrac{GmM_{\mathrm{E}}}{r}$. That energy must be equal to the energy of the satellite at the surface of the Earth plus the energy required by fuel.

(*a*) If launched from the equator, the satellite has both kinetic and potential energy initially. The kinetic energy is from the speed of the equator of the Earth relative to the center of the Earth. In problem 53 that speed is calculated to be 464 m/s.

$$E_{\mathrm{surface}} + E_{\mathrm{fuel}} = E_{\mathrm{orbit}} \quad\to\quad \tfrac{1}{2}mv_0^2 - \frac{GmM_{\mathrm{E}}}{R_{\mathrm{E}}} + E_{\mathrm{fuel}} = -\tfrac{1}{2}\frac{GmM_{\mathrm{E}}}{r} \quad\to$$

$$E_{\mathrm{fuel}} = GmM_{\mathrm{E}}\left(\frac{1}{R_{\mathrm{E}}} - \frac{1}{2r}\right) - \tfrac{1}{2}mv_0^2 = \left\{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)(1465\,\mathrm{kg})\left(5.98\times10^{24}\,\mathrm{kg}\right)\right.$$

$$\left.\bullet\left(\frac{1}{6.38\times10^6\,\mathrm{m}} - \frac{1}{2\left(6.38\times10^6\,\mathrm{m} + 1.375\times10^6\,\mathrm{m}\right)}\right)\right\} - \tfrac{1}{2}(1465\,\mathrm{kg})(464\,\mathrm{m/s})^2$$

$$= \boxed{5.38\times10^{10}\,\mathrm{J}}$$

(*b*) If launched from the North Pole, the satellite has only potential energy initially. There is no initial velocity from the rotation of the Earth.

$$E_{\text{surface}} + E_{\text{fuel}} = E_{\text{orbit}} \quad\rightarrow\quad -\frac{GmM_{\text{E}}}{R_{\text{E}}} + E_{\text{fuel}} = -\tfrac{1}{2}\frac{GmM_{\text{E}}}{r} \quad\rightarrow$$

$$E_{\text{fuel}} = GmM_{\text{E}}\left(\frac{1}{R_{\text{E}}} - \frac{1}{2r}\right) = \left\{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1465\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)\right.$$

$$\left.\bullet\left(\frac{1}{6.38\times10^{6}\,\text{m}} - \frac{1}{2\left(6.38\times10^{6}\,\text{m} + 1.375\times10^{6}\,\text{m}\right)}\right)\right\}$$

$$= \boxed{5.39\times10^{10}\,\text{J}}$$

99. (*a*) Use energy conservation and equate the energies at A and B. The distance from the center of the Earth to location B is found by the Pythagorean theorem.

$$r_{\text{B}} = \sqrt{\left(13{,}900\,\text{km}\right)^2 + \left(8230\,\text{km}\right)^2} = 16{,}150\,\text{km}$$

$$E_A = E_B \quad\rightarrow\quad \tfrac{1}{2}mv_A^2 + \left(-\frac{GM_{\text{E}}m}{r_{\text{A}}}\right) = \tfrac{1}{2}mv_B^2 + \left(-\frac{GM_{\text{E}}m}{r_{\text{B}}}\right) \quad\rightarrow$$

$$v_B = \sqrt{v_A^2 + 2GM_{\text{E}}\left(\frac{1}{r_{\text{B}}} - \frac{1}{r_{\text{A}}}\right)} = \sqrt{\left(8650\,\text{m/s}\right)^2 + \left\{\begin{array}{l}2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)\bullet \\ \left(\dfrac{1}{1.615\times10^{7}\,\text{m}} - \dfrac{1}{8.23\times10^{6}\,\text{m}}\right)\end{array}\right\}}$$

$$= \boxed{5220\,\text{m/s}}$$

(*b*) Use energy conservation and equate the energies at A and C.

$$r_{\text{C}} = 16{,}460\,\text{km} + 8230\,\text{km} = 24{,}690\,\text{km}$$

$$E_A = E_B \quad\rightarrow\quad \tfrac{1}{2}mv_A^2 + \left(-\frac{GM_{\text{E}}m}{r_{\text{A}}}\right) = \tfrac{1}{2}mv_B^2 + \left(-\frac{GM_{\text{E}}m}{r_{\text{B}}}\right) \quad\rightarrow$$

$$v_B = \sqrt{v_A^2 + 2GM_{\text{E}}\left(\frac{1}{r_{\text{B}}} - \frac{1}{r_{\text{A}}}\right)} = \sqrt{\left(8650\,\text{m/s}\right)^2 + \left\{\begin{array}{l}2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)\bullet \\ \left(\dfrac{1}{2.469\times10^{7}\,\text{m}} - \dfrac{1}{8.23\times10^{6}\,\text{m}}\right)\end{array}\right\}}$$

$$= \boxed{3190\,\text{m/s}}$$

100. (*a*) The force is found from the potential function by Eq. 8-7.

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}e^{-\alpha r}\right) = GMm\frac{d}{dr}\left(\frac{e^{-\alpha r}}{r}\right) = GMm\left(\frac{r\left(-\alpha e^{-\alpha r}\right) - e^{-\alpha r}}{r^2}\right)$$

$$= \boxed{-\frac{GMm}{r^2}e^{-\alpha r}\left(1 + \alpha r\right)}$$

(*b*) Find the escape velocity by using conservation of energy to equate the energy at the surface of the Earth to the energy at infinity with a speed of 0.

$$E_{R_{\text{E}}} = E_{\infty} \quad\rightarrow\quad \tfrac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R_{\text{E}}}e^{-\alpha R_{\text{E}}} = 0 + 0 \quad\rightarrow\quad v_{\text{esc}} = \boxed{\sqrt{\frac{2GM}{R_{\text{E}}}}e^{-\frac{1}{2}\alpha R_{\text{E}}}}$$

Notice that this escape velocity is smaller than the Newtonian escape velocity by a factor of $e^{-\frac{1}{2}\alpha R_E}$.

101. (*a*) Assume that the energy of the candy bar is completely converted into a change of potential energy.

$$E_{\substack{\text{candy}\\\text{bar}}} = \Delta U = mg\Delta y \quad\rightarrow\quad \Delta y = \frac{E_{\substack{\text{candy}\\\text{bar}}}}{mg} = \frac{1.1\times10^6\,\text{J}}{(76\text{ kg})(9.8\,\text{m/s}^2)} = \boxed{1500\,\text{m}}$$

(*b*) If the person jumped to the ground, the same energy is all converted into kinetic energy.

$$E_{\substack{\text{candy}\\\text{bar}}} = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{\frac{2E_{\substack{\text{candy}\\\text{bar}}}}{m}} = \sqrt{\frac{2\left(1.1\times10^6\,\text{J}\right)}{(76\text{ kg})}} = \boxed{170\,\text{m/s}}$$

102. (*a*) $1\text{ kW}\bullet\text{h} = 1\text{ kW}\bullet\text{h}\left(\dfrac{1000\text{ W}}{1\text{ kW}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)\left(\dfrac{1\text{ J/s}}{1\text{ W}}\right) = \boxed{3.6\times10^6\,\text{J}}$

(*b*) $(580\,\text{W})(1\,\text{month}) = (580\,\text{W})(1\,\text{month})\left(\dfrac{1\,\text{kW}}{1000\,\text{W}}\right)\left(\dfrac{30\,\text{d}}{1\,\text{month}}\right)\left(\dfrac{24\,\text{h}}{1\,\text{d}}\right) = 417.6\,\text{kW}\bullet\text{h}$

$\approx \boxed{420\,\text{kW}\bullet\text{h}}$

(*c*) $417.6\,\text{kW}\bullet\text{h} = 417.6\,\text{kW}\bullet\text{h}\left(\dfrac{3.6\times10^6\,\text{J}}{1\text{ kW}\bullet\text{h}}\right) = 1.503\times10^9\,\text{J} \approx \boxed{1.5\times10^9\,\text{J}}$

(*d*) $(417.6\,\text{kW}\bullet\text{h})\left(\dfrac{\$0.12}{1\text{ kW}\bullet\text{h}}\right) = \$50.11 \approx \boxed{\$50}$

Kilowatt-hours is a measure of energy, not power, and so $\boxed{\text{no}}$, the actual rate at which the energy is used does not figure into the bill. They could use the energy at a constant rate, or at a widely varying rate, and as long as the total used is about 420 kilowatt-hours, the price would be about \$50.

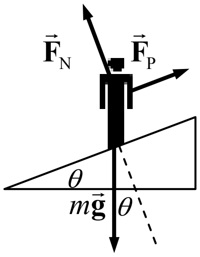
$\boxed{103.}$ The only forces acting on the bungee jumper are gravity and the elastic force from the bungee cord, so the jumper's mechanical energy is conserved. Subscript 1 represents the jumper at the bridge, and subscript 2 represents the jumper at the bottom of the jump. Let the lowest point of the jumper's motion be the zero location for gravitational potential energy $(y = 0)$. The zero location for elastic potential energy is the point at which the bungee cord begins to stretch. See the diagram in the textbook. We have $v_1 = v_2 = 0,\ y_1 = h,\ y_2 = 0,$ and the amount of stretch of the cord $x_2 = h - 15.$ Solve for *h*.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad\rightarrow\quad mgh = \tfrac{1}{2}k(h-15)^2 \quad\rightarrow$$

$$h^2 - \left(30 + 2\frac{mg}{k}\right)h + 225 = 0 \quad\rightarrow\quad h^2 - 59.4h + 225 = 0 \quad\rightarrow$$

$$h = \frac{59.4 \pm \sqrt{59.4^2 - 4(225)}}{2} = 55\,\text{m}, 4\,\text{m} \quad\rightarrow\quad h = \boxed{60\,\text{m}}$$

The larger answer must be taken because *h* > 15 m. And only 1 significant figure is justified.



104. See the free-body diagram for the patient on the treadmill. We assume that there are no dissipative forces. Since the patient has a constant velocity, the net force parallel to the plane must be 0. Write Newton's second law for forces parallel to the plane, and then calculate the power output of force $\vec{\mathbf{F}}_P$.

$$\sum F_{\text{parallel}} = F_P - mg\sin\theta = 0 \quad \rightarrow \quad F_P = mg\sin\theta$$

$$P = F_P v = mgv\sin\theta = (75\,\text{kg})(9.8\,\text{m/s}^2)(3.3\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\sin 12^\circ$$

$$= 140.1\,\text{W} \approx \boxed{140\,\text{W}}$$

This is 1.5 to 2 times the wattage of typical household light bulbs (60–100 W).

105. (*a*) Assume that there are no non-conservative forces on the rock, and so its mechanical energy is conserved. Subscript 1 represents the rock as it leaves the volcano, and subscript 2 represents the rock at its highest point. The location as the rock leaves the volcano is the zero location for PE $(y = 0)$. We have $y_1 = 0,\ y_2 = 500\,\text{m},$ and $v_2 = 0$. Solve for v_1.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 = mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80\,\text{m/s}^2)(320\,\text{m})} = 79.20\,\text{m/s} \approx \boxed{79\,\text{m/s}}$$

(*b*) The power output is the energy transferred to the launched rocks per unit time. The launching energy of a single rock is $\tfrac{1}{2}mv_1^2$, and so the energy of 1000 rocks is $1000\left(\tfrac{1}{2}mv_1^2\right)$. Divide this energy by the time it takes to launch 1000 rocks (1 minute) to find the power output needed to launch the rocks.

$$P = \frac{1000\left(\tfrac{1}{2}mv_1^2\right)}{t} = \frac{500(450\,\text{kg})(79.20\,\text{m/s})^2}{60\ \text{sec}} = \boxed{2.4\times10^7\,\text{W}}$$

106. Assume that there are no non-conservative forces doing work, so the mechanical energy of the jumper will be conserved. Subscript 1 represents the jumper at the launch point of the jump, and subscript 2 represents the jumper at the highest point. The starting height of the jump is the zero location for potential energy $(y = 0)$. We have $y_1 = 0,\ y_2 = 1.1\,\text{m},$ and $v_2 = 6.5\,\text{m/s}$. Solve for v_1.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(6.5\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(1.1\ \text{m})} = \boxed{8.0\,\text{m/s}}$$

107. (*a*) The work done by gravity as the elevator falls is the opposite of the change in gravitational potential energy.

$$W_{\text{grav}} = -\Delta U_{\text{grav}} = U_1 - U_2 = mg(y_1 - y_2) = (920\,\text{kg})(9.8\,\text{m/s}^2)(24\,\text{m})$$

$$= 2.164\times10^5\,\text{J} \approx \boxed{2.2\times10^5\,\text{J}}$$

Gravity is the only force doing work on the elevator as it falls (ignoring friction), so this result is also the net work done on the elevator as it falls.

(*b*) The net work done on the elevator is equal to its change in kinetic energy. The net work done just before striking the spring is the work done by gravity found above.

$$W_G = K_2 - K_1 \quad \rightarrow \quad mg(y_1 - y_2) = \tfrac{1}{2}mv_2 - 0 \quad \rightarrow$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\,\text{m/s}^2)(24\,\text{m})} = 21.69\,\text{m/s} \approx \boxed{22\,\text{m/s}}$$

(*c*) Use conservation of energy. Subscript 1 represents the elevator just before striking the spring, and subscript 2 represents the elevator at the bottom of its motion. The level of the elevator just before striking the spring is the zero location for both gravitational potential energy and elastic potential energy. We have $v_1 = 21.69\,\mathrm{m/s}$, $y_1 = 0$, and $v_2 = 0$. We assume that $y_2 < 0$.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}ky_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad\rightarrow$$

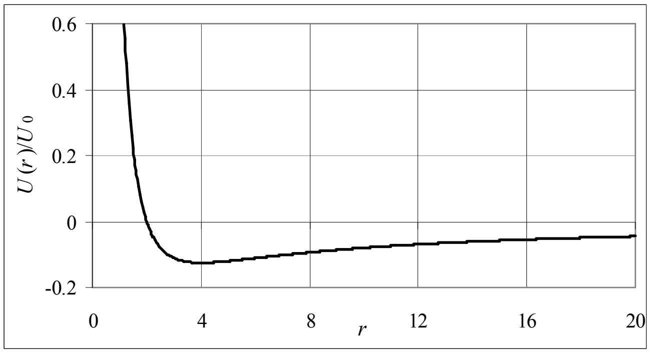
$$y_2^2 + 2\frac{mg}{k}y_2 - \frac{m}{k}v_1^2 = 0 \quad\rightarrow\quad y_2 = \frac{-\dfrac{2mg}{k} \pm \sqrt{\dfrac{4m^2g^2}{k^2} + 4\dfrac{mv_1^2}{k}}}{2} = \frac{-mg \pm \sqrt{m^2g^2 + mkv_1^2}}{k}$$

We must choose the negative root so that y_2 is negative. Thus

$$y_2 = \frac{-(920\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) - \sqrt{(920\,\mathrm{kg})^2(9.80\,\mathrm{m/s^2})^2 + (920\,\mathrm{kg})(2.2\times10^5\,\mathrm{N/m})(21.69\,\mathrm{m/s})^2}}{2.2\times10^5\,\mathrm{N/m}}$$

$$= \boxed{-1.4\,\mathrm{m}}$$

108. (*a*) The plot is included here. To find the crossing point, solve $U(r) = 0$ for *r*.



$$U(r) = U_0\left[\frac{2}{r^2} - \frac{1}{r}\right] = 0 \quad\rightarrow$$

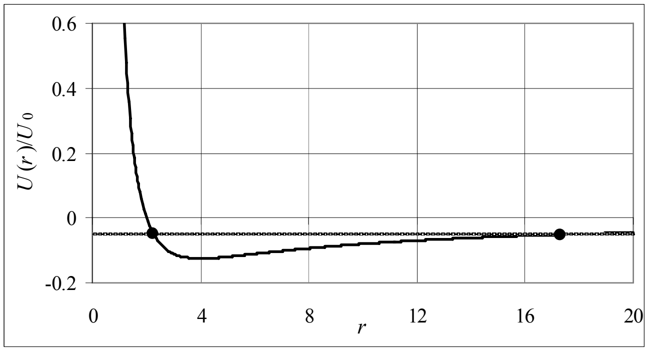
$$\frac{2}{r^2} - \frac{1}{r} = 0 \quad\rightarrow\quad \boxed{r = 2}$$

To find the minimum value, set $\dfrac{dU}{dr} = 0$ and solve for *r*.

$$\frac{dU}{dr} = U_0\left[-\frac{4}{r^3} + \frac{1}{r^2}\right] = 0 \quad\rightarrow\quad -\frac{4}{r^3} + \frac{1}{r^2} = 0 \quad\rightarrow\quad \boxed{r = 4}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH08.XLS,” on tab “Problem 8.108a.”

(*b*) The graph is redrawn with the energy value included. The approximate turning points are indicated by the small dots. An analytic solution to the relationship $U(r) = -0.050U_0$ gives $r \approx 2.3$, 17.7. The maximum kinetic energy of the particle occurs at the minimum of the potential energy, and is found from $E = K + U$.



$$E = K + U \quad\rightarrow$$

$$-0.050U_0 = K + U(r=4) = K + U_0\left(\tfrac{2}{16} - \tfrac{1}{4}\right) \quad\rightarrow\quad K = \tfrac{1}{8}U_0 - 0.050U_0 = \boxed{0.075U_0}$$

$\boxed{109.}$ A point of stable equilibrium will have $\frac{dU}{dx}=0$ and $\frac{d^2U}{dx^2}>0,$ indicating a minimum in the potential equilibrium function.

$$U(x)=\frac{a}{x}+bx \qquad \frac{dU}{dx}=-\frac{a}{x^2}+b=0 \;\rightarrow\; x^2=\frac{a}{b} \;\rightarrow\; x=\pm\sqrt{a/b}$$

But since the problem restricts us to $x>0$, the point of must be $x=\boxed{\sqrt{a/b}}$.

$\left.\frac{d^2U}{dx^2}\right|_{x=\sqrt{a/b}}=\left.\frac{2a}{x^3}\right|_{x=\sqrt{a/b}}=\frac{2a}{(a/b)^{3/2}}=\frac{2}{ab^{3/2}}>0,$ and so the point $x=\sqrt{a/b}$ gives a minimum in the potential energy function.

110. (*a*) $U=-\int F dr+C=-\int F_0\left[2\left(\frac{\sigma}{r}\right)^{13}-\left(\frac{\sigma}{r}\right)^{7}\right]dr+C=\boxed{\frac{F_0\sigma}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]+C}$

(*b*) The equilibrium distance occurs at the location where the force is 0.

$$F=F_0\left[2\left(\frac{\sigma}{r_0}\right)^{13}-\left(\frac{\sigma}{r_0}\right)^{7}\right]=0 \;\rightarrow\; r_0=2^{1/6}\sigma=2^{1/6}\left(3.50\times10^{-11}\,\text{m}\right)=\boxed{3.93\times10^{-11}\,\text{m}}$$

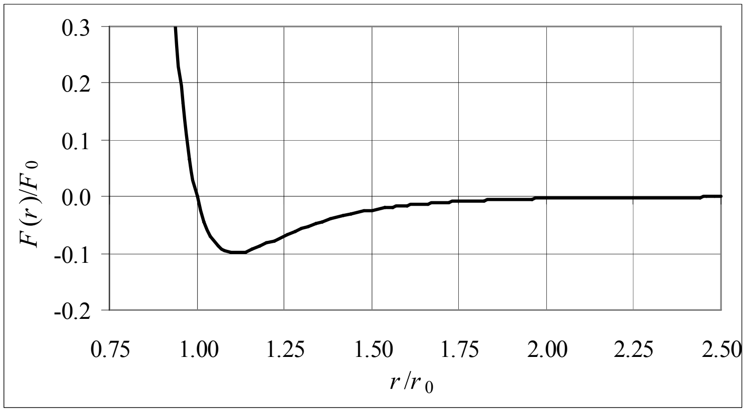
(*c*) In order to draw the graphs in terms of r_0, and to scale them to the given constants, the functions have been parameterized as follows.

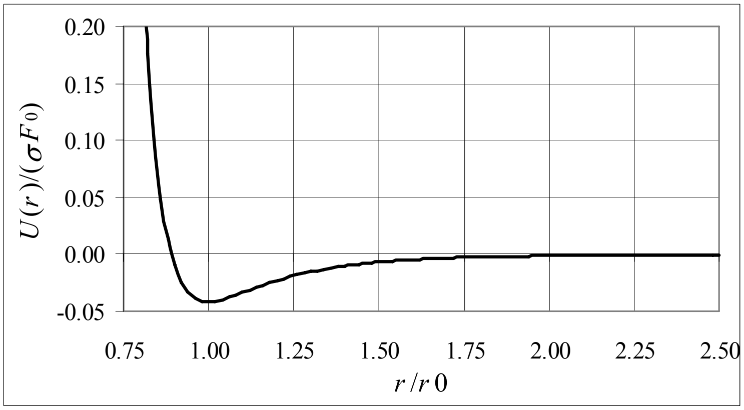
$$F(r)=F_0\left[2\left(\frac{\sigma}{r}\right)^{13}-\left(\frac{\sigma}{r}\right)^{7}\right]=F_0\left[2\left(\frac{\sigma}{r_0}\right)^{13}\left(\frac{r_0}{r}\right)^{13}-\left(\frac{\sigma}{r_0}\right)^{7}\left(\frac{r_0}{r}\right)^{7}\right] \;\rightarrow$$

$$\frac{F(r)}{F_0}=\left[2\left(\frac{\sigma}{r_0}\right)^{13}\left(\frac{r}{r_0}\right)^{-13}-\left(\frac{\sigma}{r_0}\right)^{7}\left(\frac{r}{r_0}\right)^{-7}\right]$$

$$U(r)=\frac{F_0\sigma}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]=\frac{F_0\sigma}{6}\left[\left(\frac{\sigma}{r_0}\right)^{12}\left(\frac{r}{r_0}\right)^{-12}-\left(\frac{\sigma}{r_0}\right)^{6}\left(\frac{r}{r_0}\right)^{-6}\right] \;\rightarrow$$

$$\frac{U(r)}{F_0\sigma}=\frac{1}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]=\frac{1}{6}\left[\left(\frac{\sigma}{r_0}\right)^{12}\left(\frac{r}{r_0}\right)^{-12}-\left(\frac{\sigma}{r_0}\right)^{6}\left(\frac{r}{r_0}\right)^{-6}\right]$$





The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH08.XLS," on tab "Problem 8.110c."

CHAPTER 9: Linear Momentum

Responses to Questions

1. Momentum is conserved if the sum of the external forces acting on an object is zero. In the case of moving objects sliding to a stop, the sum of the external forces is not zero; friction is an unbalanced force. Momentum will not be conserved in that case.

2. With the spring stretched, the system of two blocks and spring has elastic potential energy. When the blocks are released, the spring pulls them back together, converting the potential energy into kinetic energy. The blocks will continue past the equilibrium position and compress the spring, eventually coming to rest as the kinetic energy changes back into potential energy. If no thermal energy is lost, the blocks will continue to oscillate. The center of mass of the system will stay stationary. Since momentum is conserved, and the blocks started at rest, $m_1v_1 = -m_2v_2$ at all times, if we assume a massless spring.

3. The heavy object will have a greater momentum. If a light object m_1 and a heavy object m_2 have the same kinetic energy, then the light object must have a larger velocity than the heavy object. If $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$, where $m_1 < m_2$, then $v_1 = v_2\sqrt{\frac{m_2}{m_1}}$. The momentum of the light object is $m_1v_1 = m_1v_2\sqrt{\frac{m_2}{m_1}} = m_2v_2\sqrt{\frac{m_1}{m_2}}$. Since the ratio $\frac{m_1}{m_2}$ is less than 1, the momentum of the light object will be a fraction of the momentum of the heavy object.

4. The momentum of the person is changed (to zero) by the force of the ground acting on the person. This change in momentum is equal to the impulse on the person, or the average force times the time over which it acts.

5. As the fish swishes its tail back and forth, it moves water backward, away from the fish. If we consider the system to be the fish and the water, then, from conservation of momentum, the fish must move forward.

6. (*d*) The girl moves in the opposite direction at 2.0 m/s. Since there are no external forces on the pair, momentum is conserved. The initial momentum of the system (boy and girl) is zero. The final momentum of the girl must be the same in magnitude and opposite in direction to the final momentum of the boy, so that the net final momentum is also zero.

7. (*d*) The truck and the car will have the same change in the magnitude of momentum because momentum is conserved. (The sum of the changes in momentum must be zero.)

8. Yes. In a perfectly elastic collision, kinetic energy is conserved. In the Earth/ball system, the kinetic energy of the Earth after the collision is negligible, so the ball has the same kinetic energy leaving the floor as it had hitting the floor. The height from which the ball is released determines its potential energy, which is converted to kinetic energy as the ball falls. If it leaves the floor with this same amount of kinetic energy and a velocity upward, it will rise to the same height as it originally had as the kinetic energy is converted back into potential energy.

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9. In order to conserve momentum, when the boy dives off the back of the rowboat the boat will move forward.

10. He could have thrown the coins in the direction opposite the shore he was trying to reach. Since the lake is frictionless, momentum would be conserved and he would “recoil” from the throw with a momentum equal in magnitude and opposite in direction to the coins. Since his mass is greater than the mass of the coins, his speed would be less than the speed of the coins, but, since there is no friction, he would maintain this small speed until he hit the shore.

11. When the tennis ball rebounds from a stationary racket, it reverses its component of velocity perpendicular to the racket with very little energy loss. If the ball is hit straight on, and the racket is actually moving forward, the ball can be returned with an energy (and a speed) equal to the energy it had when it was served.

12. Yes. Impulse is the product of the force and the time over which it acts. A small force acting over a longer time could impart a greater impulse than a large force acting over a shorter time.

13. If the force is non-constant, and reverses itself over time, it can give a zero impulse. For example, the spring force would give a zero impulse over one period of oscillation.

14. The collision in which the two cars rebound would probably be more damaging. In the case of the cars rebounding, the change in momentum of each car is greater than in the case in which they stick together, because each car is not only brought to rest but also sent back in the direction from which it came. A greater impulse results from a greater force, and so most likely more damage would occur.

15. (*a*) No. The ball has external forces acting on it at all points of its path.
    (*b*) If the system is the ball and the Earth, momentum is conserved for the entire path. The forces acting on the ball-Earth system are all internal to the system.
    (*c*) For a piece of putty falling and sticking to a steel plate, if the system is the putty and the Earth, momentum is conserved for the entire path.

16. The impulse imparted to a car during a collision is equal to the change in momentum from its initial speed times mass to zero, assuming the car is brought to rest. The impulse is also equal to the force exerted on the car times the time over which the force acts. For a given change in momentum, therefore, a longer time results in a smaller average force required to stop the car. The “crumple zone” extends the time it takes to bring the car to rest, thereby reducing the force.

17. For maximum power, the turbine blades should be designed so that the water rebounds. The water has a greater change in momentum if it rebounds than if it just stops at the turbine blade. If the water has a greater change in momentum, then, by conservation of momentum, the turbine blades also have a greater change in momentum, and will therefore spin faster.

18. (*a*) The direction of the change in momentum of the ball is perpendicular to the wall and away from it, or outward.
    (*b*) The direction of the force *on the ball* is the same as the direction of its change in momentum. Therefore, by Newton’s third law, the direction of the force *on the wall* will be perpendicular to the wall and towards it, or inward.

19. When a ball is thrown into the air, it has only a vertical component of velocity. When the batter hits the ball, usually in or close to the horizontal direction, the ball acquires a component of velocity in the horizontal direction from the bat. If the ball is pitched, then when it is hit by the bat it reverses its

horizontal component of velocity (as it would if it bounced off of a stationary wall) and acquires an additional contribution to its horizontal component of velocity from the bat. Therefore, a pitched ball can be hit farther than one tossed into the air.

20. A perfectly inelastic collision between two objects that initially had momenta equal in magnitude but opposite in direction would result in all the kinetic energy being lost. For instance, imagine sliding two clay balls with equal masses and speeds toward each other across a frictionless surface. Since the initial momentum of the system is zero, the final momentum must be zero as well. The balls stick together, so the only way the final momentum can be zero is if they are brought to rest. In this case, all the kinetic energy would be lost.

21. (*b*) Elastic collisions conserve both momentum and kinetic energy; inelastic collisions only conserve momentum.

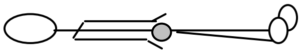
22. Passengers may be told to sit in certain seats in order to balance the plane. If they move during the flight, they could change the position of the center of mass of the plane and affect its flight.

23. You lean backward in order to keep your center of mass over your feet. If, due to the heavy load, your center of mass is in front of your feet, you will fall forward.

24. A piece of pipe is typically uniform, so that its center of mass is at its geometric center. Your arm and leg are not uniform. For instance, the thigh is bigger than the calf, so the center of mass of a leg will be higher than the midpoint.

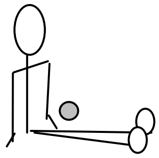
25.

Lying down



CM is within the body, approximately half-way between the head and feet.

Sitting up



CM is outside the body.

26. Draw a line from each vertex to the midpoint of the opposite side. The center of mass will be the point at which these lines intersect.

27. When you stand next to a door in the position described, your center of mass is over your heels. If you try to stand on your toes, your center of mass will not be over your area of support, and you will fall over backward.

28. If the car were on a frictionless surface, then the internal force of the engine could not accelerate the car. However, there is friction, which is an external force, between the car tires and the road, so the car can be accelerated.

29. The center of mass of the system of pieces will continue to follow the original parabolic path.

30. Far out in space there are no external forces acting on the rocket, so momentum is conserved. Therefore, to change directions, the rocket needs to expel something (like gas exhaust) in one direction so that the rest of it will move in the opposite direction and conserve momentum.

31. If there were only two particles involved in the decay, then by conservation of momentum, the momenta of the particles would have to be equal in magnitude and opposite in direction, so that the momenta would be required to lie along a line. If the momenta of the recoil nucleus and the electron do not lie along a line, then some other particle must be carrying off some of the momentum.

32. Consider Bob, Jim, and the rope as a system. The center of mass of the system is closer to Bob, because he has more mass. Because there is no net external force on the system, the center of mass will stay stationary. As the two men pull hand-over-hand on the rope they will move toward each other, eventually colliding at the center of mass. Since the CM is on Bob's side of the midline, Jim will cross the midline and lose.

33. The ball that rebounds off the cylinder will give the cylinder a larger impulse and will be more likely to knock it over.

Solutions to Problems

1. The force on the gas can be found from its change in momentum. The speed of 1300 kg of the gas changes from rest to $4.5\times10^4\ \mathrm{m/s}$, over the course of one second.

$$F=\frac{\Delta p}{\Delta t}=\frac{m\Delta v}{\Delta t}=\Delta v\frac{m}{\Delta t}=\left(4.5\times10^4\ \mathrm{m/s}\right)\left(1300\ \mathrm{kg/s}\right)=5.9\times10^7\ \mathrm{N},\ \text{opposite to the velocity}$$

The force on the rocket is the Newton's third law pair (equal and opposite) to the force on the gas, and so is $\boxed{5.9\times10^7\ \mathrm{N}\ \text{in the direction of the velocity}}$.

2. For a constant force, Eq. 9-2 can be written as $\Delta\vec{\mathbf{p}}=\vec{\mathbf{F}}\Delta t.$ For a constant mass object, $\Delta\vec{\mathbf{p}}=m\Delta\vec{\mathbf{v}}.$ Equate the two expressions for $\Delta\vec{\mathbf{p}}.$

$$\vec{\mathbf{F}}\Delta t=m\Delta\vec{\mathbf{v}}\quad\rightarrow\quad\Delta\vec{\mathbf{v}}=\frac{\vec{\mathbf{F}}\Delta t}{m}$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$\Delta v=-\frac{F\Delta t}{m}=-\frac{(25\ \mathrm{N})(15\ \mathrm{s})}{65\ \mathrm{kg}}=\boxed{-5.8\ \mathrm{m/s}}$$

The skier loses $5.8\ \mathrm{m/s}$ of speed.

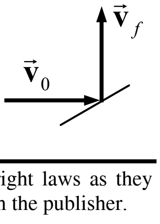
3. The force is the derivative of the momentum with respect to time.

$$\vec{\mathbf{F}}=\frac{d\vec{\mathbf{p}}}{dt}=\frac{d\left(4.8t^2\hat{\mathbf{i}}-8.0\hat{\mathbf{j}}-8.9t\,\hat{\mathbf{k}}\right)}{dt}=\boxed{\left(9.6t\,\hat{\mathbf{i}}-8.9\,\hat{\mathbf{k}}\right)\mathrm{N}}$$

4. The change in momentum is the integral of the force, since the force is the derivative of the momentum.

$$\vec{\mathbf{F}}=\frac{d\vec{\mathbf{p}}}{dt}\quad\rightarrow\quad\vec{\mathbf{p}}=\int_{t_1}^{t_2}\vec{\mathbf{F}}dt=\int_{t=1.0\,\mathrm{s}}^{t=2.0\,\mathrm{s}}\left(26\hat{\mathbf{i}}-12t^2\hat{\mathbf{j}}\right)dt=\left(26t\hat{\mathbf{i}}-4t^3\hat{\mathbf{j}}\right)_{t=1.0\,\mathrm{s}}^{t=2.0\,\mathrm{s}}=\boxed{\left(26\hat{\mathbf{i}}-28\hat{\mathbf{j}}\right)\mathrm{kg{\bullet}m/s}}$$

5. The change is momentum is due to the change in direction.



$$\Delta\vec{\mathbf{p}}=m\left(\vec{\mathbf{v}}_f-\vec{\mathbf{v}}_0\right)=(0.145\ \mathrm{kg})\left(30.0\ \mathrm{m/s}\,\hat{\mathbf{j}}-30.0\ \mathrm{m/s}\,\hat{\mathbf{i}}\right)=\boxed{4.35\ \mathrm{kg{\bullet}m/s}\left(\hat{\mathbf{j}}-\hat{\mathbf{i}}\right)}$$

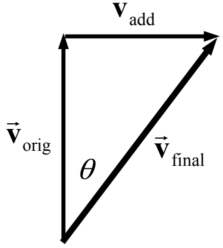
6. The average force is the change in momentum divided by the elapsed time. Call the direction from the batter to the pitcher the positive x direction, and call upwards the positive y direction. The initial momentum is in the negative x direction, and the final momentum is in the positive y direction. The final y-velocity can be found using the height to which the ball rises, with conservation of mechanical energy during the rising motion.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; \tfrac{1}{2}mv_y^2 = mgh \;\rightarrow\; v_y = \sqrt{2gh} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(36.5\,\text{m})} = 26.75\,\text{m/s}$$

$$\vec{\mathbf{F}}_{\text{avg}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{m}{\Delta t}\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_0\right) = (0.145\text{ kg})\left(\frac{26.75\hat{\mathbf{j}}\,\text{m/s} - \left(-32.0\hat{\mathbf{i}}\,\text{m/s}\right)}{2.5\times10^{-3}\text{s}}\right) = \left(1856\hat{\mathbf{i}} + 1552\hat{\mathbf{j}}\right)\text{N}$$

$$F_{\text{avg}} = \sqrt{(1856\,\text{N})^2 + (1552\,\text{N})^2} = \boxed{2400\,\text{N}} \;\;;\;\; \theta = \tan^{-1}\frac{1552\,\text{N}}{1856\,\text{N}} = \boxed{39.9^\circ}$$

7. To alter the course by 35.0°, a velocity perpendicular to the original velocity must be added. Call the direction of the added velocity, $\vec{\mathbf{v}}_{\text{add}}$, the positive direction. From the diagram, we see that $v_{\text{add}} = v_{\text{orig}}\tan\theta$. The momentum in the perpendicular direction will be conserved, considering that the gases are given perpendicular momentum in the opposite direction of $\vec{\mathbf{v}}_{\text{add}}$. The gas is expelled oppositely to $\vec{\mathbf{v}}_{\text{add}}$, and so a negative value is used for $v_{\perp\,\text{gas}}$.



$$\underset{\text{before}}{p_\perp} = \underset{\text{after}}{p_\perp} \;\rightarrow\; 0 = m_{\text{gas}}v_{\perp\,\text{gas}} + \left(m_{\text{rocket}} - m_{\text{gas}}\right)v_{\text{add}} \;\rightarrow$$

$$m_{\text{gas}} = \frac{m_{\text{rocket}}v_{\text{add}}}{\left(v_{\text{add}} - v_{\perp\,\text{gas}}\right)} = \frac{(3180\text{ kg})(115\,\text{m/s})\tan 35.0^\circ}{\left[(115\,\text{m/s})\tan 35.0^\circ - (-1750\,\text{m/s})\right]} = \boxed{1.40\times10^2\,\text{kg}}$$

8. The air is moving with an initial speed of $120\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 33.33\,\text{m/s}$. Thus, in one second, a volume of air measuring 45 m x 65 m x 33.33 m will have been brought to rest. By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air. The mass of the stopped air is its volume times its density.

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{V\rho\Delta v}{\Delta t} = \frac{(45\,\text{m})(65\,\text{m})(33.33\,\text{m})\left(1.3\,\text{kg/m}^3\right)(33.33\,\text{m/s} - 0)}{1\,\text{s}} = \boxed{4.2\times10^6\,\text{N}}$$

9. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let A represent the first car and B represent the second car. Momentum will be conserved in the collision. Note that $v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = \left(m_{\text{A}} + m_{\text{B}}\right)v' \;\rightarrow$$

$$m_{\text{B}} = \frac{m_{\text{A}}\left(v_{\text{A}} - v'\right)}{v'} = \frac{(7700\,\text{kg})(18\,\text{m/s} - 5.0\,\text{m/s})}{5.0\,\text{m/s}} = \boxed{2.0\times10^4\,\text{kg}}$$

10. Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let A represent the car and B represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = \left(m_{\text{A}} + m_{\text{B}}\right)v' \;\rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(9150\,\text{kg})(15.0\,\text{m/s}) + 0}{(9150\,\text{kg}) + (4350\,\text{kg})} = \boxed{10.2\,\text{m/s}}$$

11. Consider the motion in one dimension, with the positive direction being the direction of motion of the alpha particle. Let A represent the alpha particle, with a mass of m_A, and let B represent the daughter nucleus, with a mass of $57m_A$. The total momentum must be 0 since the nucleus decayed at rest.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad 0 = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_B' = -\frac{m_A v_A'}{m_B} = -\frac{m_A\left(2.8\times10^5\,\text{m/s}\right)}{57m_A} \quad \rightarrow \quad |v_B'| = \boxed{4900\,\text{m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

12. The tackle will be analyzed as a one-dimensional momentum conserving situation. Let A represent the halfback and B represent the tackler. We take the direction of the halfback to be the positive direction, so $v_A > 0$ and $v_B < 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = (m_A + m_B)v' \quad \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(82\,\text{kg})(5.0\,\text{m/s}) + (130\,\text{kg})(-2.5\,\text{m/s})}{82\,\text{kg} + 130\,\text{kg}} = 0.401\,\text{m/s} \approx \boxed{0.4\,\text{m/s}}$$

They will be moving it the direction that the halfback was running before the tackle.

$\boxed{13.}$ The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B)v = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_A' = -\frac{m_B v_B'}{m_A} = -\frac{(5.70\,\text{kg})(10.0\,\text{m/s})}{(24.0\,\text{kg} + 35.0\,\text{kg})} = \boxed{-0.966\,\text{m/s}}$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.

14. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let A represent the alpha particle, with a mass of 4 u, and let B represent the new nucleus, with a mass of 218 u. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B)v = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_A' = \frac{(m_A + m_B)v - m_B v_B'}{m_A} = \frac{(222\,\text{u})(420\,\text{m/s}) - (218\,\text{u})(350\,\text{m/s})}{4.0\,\text{u}} = \boxed{4200\,\text{m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

15. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad 0 = m_A v_A' + m_B v_B' \quad \rightarrow \quad v_B' = -\frac{m_A v_A'}{m_B}$$

$$K_{\mathrm{A}} = 2K_{\mathrm{B}} \rightarrow \tfrac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}'^2 = 2\left(\tfrac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}'^2\right) = m_{\mathrm{B}}\left(-\frac{m_{\mathrm{A}} v_{\mathrm{A}}'}{m_{\mathrm{B}}}\right)^2 \rightarrow \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}} = \boxed{\frac{1}{2}}$$

The fragment with the larger kinetic energy has half the mass of the other fragment.

16. Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let A represent the bullet and B represent the block. Since there is no net force outside of the block-bullet system (like friction with the table), the momentum of the block and bullet combination is conserved. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}} + m_{\mathrm{B}} v_{\mathrm{B}} = m_{\mathrm{A}} v_{\mathrm{A}}' + m_{\mathrm{B}} v_{\mathrm{B}}' \rightarrow$$

$$v_{\mathrm{B}}' = \frac{m_{\mathrm{A}}\left(v_{\mathrm{A}} - v_{\mathrm{A}}'\right)}{m_{\mathrm{B}}} = \frac{(0.022\,\mathrm{kg})(210\,\mathrm{m/s} - 150\,\mathrm{m/s})}{2.0\,\mathrm{kg}} = \boxed{0.66\,\mathrm{m/s}}$$

17. Momentum will be conserved in two dimensions. The fuel was ejected in the *y* direction as seen by an observer at rest, and so the fuel had no *x*-component of velocity in that reference frame.

$$p_x:\quad m_{\text{rocket}} v_0 = \left(m_{\text{rocket}} - m_{\text{fuel}}\right) v_x' + m_{\text{fuel}} 0 = \tfrac{2}{3} m_{\text{rocket}} v_x' \rightarrow v_x' = \tfrac{3}{2} v_0$$

$$p_y:\quad 0 = m_{\text{fuel}} v_{\text{fuel}} + \left(m_{\text{rocket}} - m_{\text{fuel}}\right) v_y' = \tfrac{1}{3} m_{\text{rocket}} \left(2v_0\right) + \tfrac{2}{3} m_{\text{rocket}} v_y' \rightarrow v_y' = -v_0$$

Thus $\vec{\mathbf{v}}' = \boxed{\tfrac{3}{2} v_0 \hat{\mathbf{i}} - v_0 \hat{\mathbf{j}}}$.

18. Since the neutron is initially at rest, the total momentum of the three particles after the decay must also be zero. Thus $0 = \vec{\mathbf{p}}_{\text{proton}} + \vec{\mathbf{p}}_{\text{electron}} + \vec{\mathbf{p}}_{\text{neutrino}}$. Solve for any one of the three in terms of the other two: $\vec{\mathbf{p}}_{\text{proton}} = -\left(\vec{\mathbf{p}}_{\text{electron}} + \vec{\mathbf{p}}_{\text{neutrino}}\right)$. Any two vectors are always coplanar, since they can be translated so that they share initial points. So in this case the common initial point and their two terminal points of the electron and neutrino momenta define a plane, which contains their sum. Then, since the proton momentum is just the opposite of the sum of the other two momenta, it is in the same plane.

$\boxed{19.}$ Since no outside force acts on the two masses, their total momentum is conserved.

$$m_1 \vec{\mathbf{v}}_1 = m_1 \vec{\mathbf{v}}_1' + m_2 \vec{\mathbf{v}}_2' \rightarrow$$

$$\vec{\mathbf{v}}_2' = \frac{m_1}{m_2}\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_1'\right) = \frac{2.0\,\mathrm{kg}}{3.0\,\mathrm{kg}}\left[\left(4.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}} - 2.0\hat{\mathbf{k}}\right)\mathrm{m/s} - \left(-2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{k}}\right)\mathrm{m/s}\right]$$

$$= \frac{2.0\,\mathrm{kg}}{3.0\,\mathrm{kg}}\left[\left(6.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}}\right)\mathrm{m/s}\right]$$

$$= \boxed{\left(4.0\hat{\mathbf{i}} + 3.3\hat{\mathbf{j}} - 3.3\hat{\mathbf{k}}\right)\mathrm{m/s}}$$

20. (*a*) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let A represent the upper stage (that moves away faster) and B represent the lower stage. It is given that $m_{\mathrm{A}} = m_{\mathrm{B}}$, $v_{\mathrm{A}} = v_{\mathrm{B}} = v$, and $v_{\mathrm{B}}' = v_{\mathrm{A}}' - v_{\text{rel}}$. Momentum conservation gives the following.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow \left(m_{\mathrm{A}} + m_{\mathrm{B}}\right) v = m_{\mathrm{A}} v_{\mathrm{A}}' + m_{\mathrm{B}} v_{\mathrm{B}}' = m_{\mathrm{A}} v_{\mathrm{A}}' + m_{\mathrm{B}}\left(v_{\mathrm{A}}' - v_{\text{rel}}\right) \rightarrow$$

$$v_{\mathrm{A}}' = \frac{\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right) v + m_{\mathrm{B}} v_{\text{rel}}}{\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right)} = \frac{(925\,\mathrm{kg})\left(6.60\times10^3\,\mathrm{m/s}\right) + \tfrac{1}{2}(925\,\mathrm{kg})\left(2.80\times10^3\,\mathrm{m/s}\right)}{(925\,\mathrm{kg})}$$

$$= \boxed{8.00\times10^{3}\ \text{m/s}\ \text{, away from Earth}}$$

$$v'_{\text{B}} = v'_{\text{A}} - v_{\text{rel}} = 8.00\times10^{3}\ \text{m/s} - 2.80\times10^{3}\ \text{m/s} = \boxed{5.20\times10^{3}\ \text{m/s, away from Earth}}$$

(*b*) The change in kinetic energy was supplied by the explosion.

$$\Delta K = K_{\text{final}} - K_{\text{initial}} = \left(\tfrac{1}{2}m_{\text{A}}v'^{2}_{\text{A}} + \tfrac{1}{2}m_{\text{B}}v'^{2}_{\text{B}}\right) - \tfrac{1}{2}\left(m_{\text{A}} + m_{\text{B}}\right)v^{2}$$

$$= \tfrac{1}{2}(462.5\,\text{kg})\left[\left(8.00\times10^{3}\ \text{m/s}\right)^{2} + \left(5.20\times10^{3}\ \text{m/s}\right)^{2}\right] - \tfrac{1}{2}(925\ \text{kg})\left(6.60\times10^{3}\ \text{m/s}\right)^{2}$$

$$= 9.065\times10^{8}\,\text{J} \approx \boxed{9\times10^{8}\,\text{J}}$$

21. (*a*) For the initial projectile motion, the horizontal velocity is constant. The velocity at the highest point, immediately before the explosion, is exactly that horizontal velocity, $v_x = v_0\cos\theta$. The explosion is an internal force, and so the momentum is conserved during the explosion. Let $\vec{\mathbf{v}}_3$ represent the velocity of the third fragment.

$$\vec{\mathbf{p}}_{\text{before}} = \vec{\mathbf{p}}_{\text{after}} \quad\rightarrow\quad mv_0\cos\theta\hat{\mathbf{i}} = \tfrac{1}{3}mv_0\cos\theta\hat{\mathbf{i}} + \tfrac{1}{3}mv_0\cos\theta\left(-\hat{\mathbf{j}}\right) + \tfrac{1}{3}m\vec{\mathbf{v}}_3 \quad\rightarrow$$

$$\vec{\mathbf{v}}_3 = 2v_0\cos\theta\hat{\mathbf{i}} + v_0\cos\theta\hat{\mathbf{j}} = 2(116\,\text{m/s})\cos60.0^\circ\hat{\mathbf{i}} + (116\,\text{m/s})\cos60.0^\circ\hat{\mathbf{j}}$$

$$= \boxed{(116\,\text{m/s})\,\hat{\mathbf{i}} + (58.0\,\text{m/s})\,\hat{\mathbf{j}}}$$

This is 130 ms at an angle of 26.6° above the horizontal.

(*b*) The energy released in the explosion is $K_{\text{after}} - K_{\text{before}}$. Note that $v_3^2 = \left(2v_0\cos\theta\right)^2 + \left(v_0\cos\theta\right)^2 = 5v_0^2\cos^2\theta.$

$$K_{\text{after}} - K_{\text{before}} = \left[\tfrac{1}{2}\left(\tfrac{1}{3}m\right)\left(v_0\cos\theta\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{3}m\right)\left(v_0\cos\theta\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{3}m\right)v_3^2\right] - \tfrac{1}{2}m\left(v_0\cos\theta\right)^2$$

$$= \tfrac{1}{2}m\left\{\left[\tfrac{1}{3}v_0^2\cos^2\theta + \tfrac{1}{3}v_0^2\cos^2\theta + \tfrac{1}{3}\left(5v_0^2\cos^2\theta\right)\right] - v_0^2\cos^2\theta\right\}$$

$$= \tfrac{1}{2}\tfrac{4}{3}mv_0^2\cos^2\theta = \tfrac{2}{3}(224\,\text{kg})(116\,\text{m/s})^2\cos^2 60.0^\circ = \boxed{5.02\times10^{5}\,\text{J}}$$

22. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$\Delta p = F_{\text{avg}}\Delta t = m\Delta v \quad\rightarrow$$

$$F_{\text{avg}} = m\frac{\Delta v}{\Delta t} = (0.145\ \text{kg})\left(\frac{56.0\,\text{m/s} - (-35.0\,\text{m/s})}{5.00\times10^{-3}\text{s}}\right) = \boxed{2640\,\text{N, towards the pitcher}}$$

23. (*a*) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta v = \left(4.5\times10^{-2}\text{kg}\right)(45\,\text{m/s} - 0) = \boxed{2.0\ \text{kg}\bullet\text{m/s}}\text{, in the }\boxed{\text{forward}}\text{ direction}$$

(*b*) The average force is the impulse divided by the interaction time.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2.0\ \text{kg}\bullet\text{m/s}}{3.5\times10^{-3}\text{s}} = \boxed{580\,\text{N}}\text{, in the }\boxed{\text{forward}}\text{ direction}$$

24. (*a*) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} = mv_i - mv_f = (12\,\text{kg})(8.5\,\text{m/s}) - 0 = \boxed{1.0\times10^{2}\ \text{kg}\bullet\text{m/s}}$$

(*b*) The average force is the impulse divided by the time of contact.

$$F_{\text{avg}}=\frac{\Delta p}{\Delta t}=\frac{1.0\times10^{2}\ \text{kg}\bullet\text{m/s}}{8.0\times10^{-3}\,\text{s}}=\boxed{1.3\times10^{4}\,\text{N}}$$

$\boxed{25.}$ The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta p_{\perp}=mv_{\underset{\text{final}}{\perp}}-mv_{\underset{\text{initial}}{\perp}}=m\left(v\sin45^{\circ}--v\sin45^{\circ}\right)=2mv\sin45^{\circ}$$

$$=2\left(6.0\times10^{-2}\text{km}\right)\left(25\,\text{m/s}\right)\sin45^{\circ}=\boxed{2.1\,\text{kg}\bullet\text{m/s, to the left}}$$

26. (*a*) The momentum of the astronaut–space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_{\text{A}}=v_{\text{B}}=0$. We also have $v'_{\text{A}}=2.50\,\text{m/s}$.

$$p_{\text{initial}}=p_{\text{final}}\quad\rightarrow\quad m_{\text{A}}v_{\text{A}}+m_{\text{B}}v_{B}=0=m_{\text{A}}v'_{\text{A}}+m_{\text{B}}v'_{\text{B}}\quad\rightarrow$$

$$v'_{\text{B}}=-v'_{\text{A}}\frac{m_{\text{A}}}{m_{\text{B}}}=-\left(2.50\,\text{m/s}\right)\frac{130\ \text{kg}}{1700\ \text{kg}}=-0.1912\,\text{m/s}\approx\boxed{-0.19\,\text{m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$F_{\text{avg}}=\frac{\Delta p}{\Delta t}=\frac{m\left(v'_{\text{A}}-v_{\text{A}}\right)}{\Delta t}=\frac{\left(130\ \text{kg}\right)\left(2.50\,\text{m/s}-0\right)}{0.500\ \text{s}}=\boxed{6.5\times10^{2}\,\text{N}}$$

(*c*) $K_{\text{astronaut}}=\frac{1}{2}\left(130\,\text{kg}\right)\left(2.50\,\text{m/s}\right)^{2}=\boxed{4.0\times10^{2}\,\text{J}}$

$K_{\text{capsule}}=\frac{1}{2}\left(1700\,\text{kg}\right)\left(-0.1912\,\text{m/s}\right)^{2}=\boxed{31\,\text{J}}$

27. If the rain does not rebound, then the final speed of the rain is 0. By Newton's third law, the force on the pan due to the rain is equal in magnitude to the force on the rain due to the pan. The force on the rain can be found from the change in momentum of the rain. The mass striking the pan is calculated as volume times density.

$$F_{\text{avg}}=\frac{\Delta p}{\Delta t}=\frac{\left(mv_{f}-mv_{0}\right)}{\Delta t}=-\frac{m}{\Delta t}\left(v_{f}-v_{0}\right)=\frac{\rho V}{\Delta t}v_{0}=\frac{\rho Ah}{\Delta t}v_{0}=\frac{h}{\Delta t}\rho Av_{0}$$

$$=\frac{\left(5.0\times10^{-2}\,\text{m}\right)}{1\,\text{h}\left(\dfrac{3600\,\text{s}}{1\,\text{h}}\right)}\left(1.00\times10^{3}\ \text{kg/m}^{3}\right)\left(1.0\,\text{m}^{2}\right)\left(8.0\,\text{m/s}\right)=\boxed{0.11\,\text{N}}$$

28. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.04 sec. A width slightly smaller than the base was chosen to compensate for the "inward" concavity of the force graph.

$$\Delta p=\tfrac{1}{2}\left(250\ \text{N}\right)\left(0.04\ \text{s}\right)=\boxed{5\,\text{N}\bullet\text{s}}$$

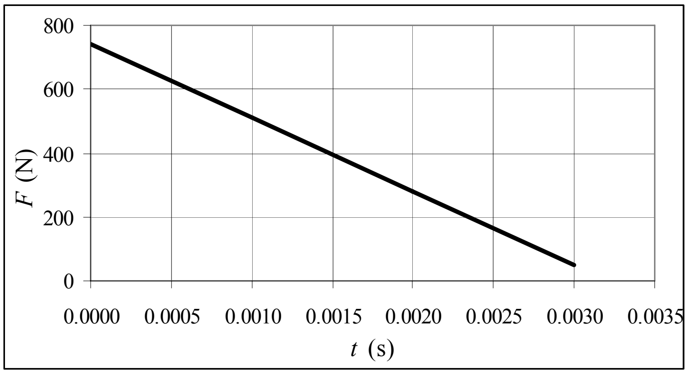
(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta v = m\left(v_f - v_\text{i}\right) \rightarrow v_f = v_i + \frac{\Delta p}{m} = 0 + \frac{5\,\text{N}\bullet\text{s}}{6.0\times10^{-2}\,\text{kg}} = \boxed{80\,\text{m/s}}$$

29. Impuse is the change of momentum, Eq. 9-6. This is a one-dimensional configuration.

$$J = \Delta p = m\left(v_\text{final} - v_0\right) = \left(0.50\,\text{kg}\right)\left(3.0\,\text{m/s}\right) = \boxed{1.5\,\text{kg}\bullet\text{m/s}}$$

30. (*a*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH09.XLS," on tab "Problem 9.30a."



(*b*) The area is trapezoidal. We estimate values rather than calculate them.

$$J \approx \tfrac{1}{2}\left(750\,\text{N} + 50\,\text{N}\right)\left(0.0030\,\text{s}\right) = \boxed{1.2\,\text{N}\bullet\text{s}}$$

(*c*) $J = \int F dt = \int_0^{0.0030} \left[740 - \left(2.3\times10^5\right)t\right]dt = \left[740t - \left(1.15\times10^5\right)t^2\right]_0^{0.0030\,\text{s}}$

$$= \left(740\,\text{N}\right)\left(0.0030\,\text{s}\right) - \left(1.15\times10^5\,\text{N/s}\right)\left(0.0030\,\text{s}\right)^2 = 1.185\,\text{N}\bullet\text{s} \approx \boxed{1.2\,\text{N}\bullet\text{s}}$$

(*d*) The impulse found above is the change in the bullet's momentum

$$J = \Delta p = m\Delta v \rightarrow m = \frac{J}{\Delta v} = \frac{1.185\text{N}\bullet\text{s}}{260\,\text{m/s}} = 4.558\times10^{-3}\text{kg} \approx \boxed{4.6\,\text{g}}$$

(*e*) The momentum of the bullet–gun combination is conserved during the firing of the bullet. Use this to find the recoil speed of the gun, calling the direction of the bullet's motion the positive direction. The momentum before firing is 0.

$$p_\text{initial} = p_\text{final} \rightarrow 0 = m_\text{bullet}v_\text{bullet} - m_\text{gun}v_\text{gun} \rightarrow$$

$$v_\text{gun} = \frac{m_\text{bullet}v_\text{bullet}}{m_\text{gun}} = \frac{\left(4.558\times10^{-3}\,\text{kg}\right)\left(260\,\text{m/s}\right)}{4.5\,\text{kg}} = \boxed{0.26\,\text{m/s}}$$

$\boxed{31.}$ (*a*) Since the velocity changes direction, the momentum changes. Take the final velocity to be in the positive direction. Then the initial velocity is in the negative direction. The average force is the change in momentum divided by the time.

$$F_\text{avg} = \frac{\Delta p}{\Delta t} = \frac{\left(mv - -mv\right)}{\Delta t} = \boxed{2\frac{mv}{\Delta t}}$$

(*b*) Now, instead of the actual time of interaction, use the time between collisions in order to get the average force over a long time.

$$F_\text{avg} = \frac{\Delta p}{t} = \frac{\left(mv - -mv\right)}{t} = \boxed{2\frac{mv}{t}}$$

32. (*a*) The impulse is the change in momentum. Take upwards to be the positive direction. The velocity just before reaching the ground is found from conservation of mechanical energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow mgh = \tfrac{1}{2}mv_y^2 \rightarrow v_y = -\sqrt{2gh} = -\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(3.0\,\text{m}\right)} = 7.668\,\text{m/s}$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_0\right) = \left(65\,\text{kg}\right)\left(-7.668\,\text{m/s}\right) = 498\,\text{kg}\bullet\text{m/s} \approx \boxed{5.0\times10^2\,\text{kg}\bullet\text{m/s, upwards}}$$

(*b*) The net force on the person is the sum of the upward force from the ground, plus the downward force of gravity.

$$F_{\text{net}} = F_{\text{ground}} - mg = ma \rightarrow$$

$$F_{\text{ground}} = m\left(g+a\right) = m\left(g + \frac{\left(v_f^2 - v_0^2\right)}{2\Delta x}\right) = \left(65\,\text{kg}\right)\left(\left(9.80\,\text{m/s}^2\right) + \frac{0-\left(-7.668\,\text{m/s}\right)^2}{2\left(-0.010\,\text{m}\right)}\right)$$

$$= \boxed{1.9\times10^5\,\text{m/s, upwards}}$$

This is about 300 times the jumper's weight.

(*c*) We do this the same as part (*b*).

$$F_{\text{ground}} = m\left(g + \frac{\left(v_f^2 - v_0^2\right)}{2\Delta x}\right) = \left(65\,\text{kg}\right)\left(\left(9.80\,\text{m/s}^2\right) + \frac{0-\left(-7.668\,\text{m/s}\right)^2}{2\left(-0.50\,\text{m}\right)}\right)$$

$$= \boxed{4.5\times10^3\,\text{m/s, upwards}}$$

This is about 7 times the jumper's weight.

33. Take the upwards direction as positive.

(*a*) The scale reading as a function of time will be due to two components – the weight of the (stationary) water already in the pan, and the force needed to stop the falling water. The weight of the water in the pan is just the rate of mass being added to the pan, times the acceleration due to gravity, times the elapsed time.

$$W_{\substack{\text{water}\\ \text{in pan}}} = \frac{\Delta m}{\Delta t}\left(g\right)\left(t\right) = \left(0.14\,\text{kg/s}\right)\left(9.80\,\text{m/s}^2\right)t = \left(1.372t\right)\text{N} \approx \left(1.4t\right)\text{N}$$

The force needed to stop the falling water is the momentum change per unit time of the water striking the pan, $F_{\substack{\text{to stop}\\ \text{moving}\\ \text{water}}} = \dfrac{\Delta p}{\Delta t}$. The speed of the falling water when it reaches the pan can be found from energy conservation. We assume the water leaves the faucet with a speed of 0, and that there is no appreciable friction during the fall.

$$E_{\substack{\text{water}\\ \text{at faucet}}} = E_{\substack{\text{water}\\ \text{at pan}}} \rightarrow mgh = \tfrac{1}{2}mv^2 \rightarrow v_{\substack{\text{at}\\ \text{pan}}} = -\sqrt{2gh}$$

The negative sign is because the water is moving downwards.

$$F_{\substack{\text{to stop}\\ \text{moving}\\ \text{water}}} = \frac{\Delta p}{\Delta t} = \frac{m_{\text{falling}}}{\Delta t}\Delta v = \left(0.14\,\text{kg/s}\right)\left(0 - -\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(2.5\,\text{m}\right)}\right) = 0.98\,\text{N}$$

This force is constant, as the water constantly is hitting the pan. And we assume the water level is not riding. So the scale reading is the sum of these two terms.

$$F_{\text{scale}} = F_{\substack{\text{to stop}\\ \text{moving}\\ \text{water}}} + W_{\substack{\text{water}\\ \text{in pan}}} = \boxed{\left(0.98 + 1.4t\right)\text{N}}$$

(*b*) After 9.0 s, the reading is $F_{\text{scale}} = \left(0.98 + 1.372\left(0.9\,\text{s}\right)\right)\text{N} = \boxed{13.3\,\text{N}}$.

(*c*) In this case, the level of the water rises over time. The height of the water in the cylinder is the volume of water divided by the area of the cylinder.

$$h_{\substack{\text{in}\\\text{tube}}} = \frac{V_{\substack{\text{water}\\\text{in tube}}}}{A_{\text{tube}}} = \frac{\left[(0.14t)\,\text{kg}\right]\left(\dfrac{1\text{m}^3}{1.0\times10^3\,\text{kg}}\right)}{\left(20\times10^{-4}\,\text{m}^2\right)} = 0.070t\ \text{m}$$

The height that the water falls is now $h' = (2.5 - 0.070t)\,\text{m}$. Following the same analysis as above, the speed of the water when it strikes the surface of the already-fallen water is now $v' = -\sqrt{2gh'}$, and so the force to stop the falling water is given by the following.

$$F_{\substack{\text{to stop}\\\text{moving}\\\text{water}}} = (0.14\,\text{kg/s})\left(0 - -\sqrt{2\left(9.80\,\text{m/s}^2\right)(2.5 - .070t)\,\text{m}}\right) = 0.6198\sqrt{(2.5 - .070t)}\text{N}$$

The scale reading is again the sum of two terms.

$$F_{\text{scale}} = F_{\substack{\text{to stop}\\\text{moving}\\\text{water}}} + W_{\substack{\text{water}\\\text{in cylinder}}} = \left(0.6198\sqrt{(2.5 - .070t)} + 1.372t\right)\text{N}$$

$$\approx \boxed{\left(0.62\sqrt{(2.5 - .070t)} + 1.4t\right)\text{N}}$$

At $t = 9.0$ s, the scale reading is as follows.

$$F_{\text{scale}} == \left(0.6198\sqrt{(2.5 - .070(9.0))} + 1.372(9.0)\right)\text{N} = 13.196\,\text{N} \approx \boxed{13.2\,\text{N}}$$

34. Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $v_{\text{A}} = 4.50\,\text{m/s}$ and $v_{\text{B}} = 3.00\,\text{m/s}$. Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -\left(v'_{\text{A}} - v'_{\text{B}}\right) \quad \rightarrow \quad v'_{\text{B}} = 1.50\,\text{m/s} + v'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}\left(1.50\,\text{m/s} + v'_{\text{A}}\right) \quad \rightarrow$$

$$v'_{\text{A}} = \frac{m_{\text{A}}v_{\text{A}} + m_{\text{B}}\left(v_{\text{B}} - 1.50\,\text{m/s}\right)}{m_{\text{A}} + m_{\text{B}}} = \frac{(0.060\text{ kg})(4.50\,\text{m/s}) + (0.090\text{ kg})(3.00\,\text{m/s} - 1.50\,\text{m/s})}{0.150\text{ kg}}$$

$$= \boxed{2.7\,\text{m/s}}$$

$$v'_{\text{B}} = 1.50\,\text{m/s} + v'_{\text{A}} = \boxed{4.2\,\text{m/s}}$$

Both balls move in the direction of the tennis ball's initial motion.

35. Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $v_{\text{A}} = 4.80\,\text{m/s}$ and $v_{\text{B}} = 0$. Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_{\text{A}} - v_{\text{B}} = -\left(v'_{\text{A}} - v'_{\text{B}}\right) \quad \rightarrow \quad v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}\left(v_{\text{A}} + v'_{\text{A}}\right) \quad \rightarrow$$

$$v'_{\text{A}} = \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}v_{\text{A}} = \frac{-0.450\text{ kg}}{1.350\text{ kg}}(4.80\,\text{m/s}) = -1.60\,\text{m/s} = \boxed{1.60\,\text{m/s}\,(\text{west})}$$

$$v'_{\text{B}} = v_{\text{A}} + v'_{\text{A}} = 4.80\,\text{m/s} - 1.60\,\text{m/s} = \boxed{3.20\,\text{m/s}\,(\text{east})}$$

36. (*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_B = 0$ and $v'_B = \frac{1}{2}v_A$. Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \rightarrow v'_A = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow m_A v_A = -\tfrac{1}{2}m_A v_A + m_B \tfrac{1}{2}v_A \rightarrow$$

$$m_B = 3m_A = 3\left(0.280\text{ kg}\right) = \boxed{0.840\text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows.

$$\frac{K'_B}{K_A} = \frac{\frac{1}{2}m_B v'^2_B}{\frac{1}{2}m_A v_A^2} = \frac{3m_A\left(\frac{1}{2}v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$

$\boxed{37.}$ Let A represent the moving ball, and let B represent the ball initially at rest. The initial direction of the ball is the positive direction. We have $v_A = 7.5\,\text{m/s}$, $v_B = 0$, and $v'_A = -3.8\,\text{m/s}$.

(*a*) Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \rightarrow v'_B = v_A - v_B + v'_A = 7.5\,\text{m/s} - 0 - 3.8\,\text{m/s} = \boxed{3.7\,\text{m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \rightarrow$$

$$m_B = m_A\frac{\left(v_A - v'_A\right)}{\left(v'_B - v_B\right)} = \left(0.220\text{ kg}\right)\frac{\left(7.5\,\text{m/s} - -3.8\,\text{m/s}\right)}{3.7\,\text{m/s}} = \boxed{0.67\text{ kg}}$$

38. Use the relationships developed in Example 9-8 for this scenario.

$$v'_A = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) \rightarrow$$

$$m_B = \left(\frac{v_A - v'_A}{v'_A + v_A}\right)m_A = \left(\frac{v_A - \left(-0.350\right)v_A}{\left(-0.350\right)v_A + v_A}\right)m_A = \left(\frac{1.350}{0.650}\right)m_A = \boxed{2.08m}$$

39. The one-dimensional stationary target elastic collision is analyzed in Example 9-8. The fraction of kinetic energy lost is found as follows.

$$\frac{K_{A_{\text{inital}}} - K_{A_{\text{final}}}}{K_{A_{\text{inital}}}} = \frac{K_{B_{\text{final}}}}{K_{A_{\text{inital}}}} = \frac{\frac{1}{2}m_B v'^2_B}{\frac{1}{2}m_A v_A^2} = \frac{m_B\left[v_A\left(\dfrac{2m_A}{m_A + m_B}\right)\right]^2}{m_A v_A^2} = \frac{4m_A m_B}{\left(m_A + m_B\right)^2}$$

(*a*) $\dfrac{4m_A m_B}{\left(m_A + m_B\right)^2} = \dfrac{4\left(1.01\right)\left(1.01\right)}{\left(1.01 + 1.01\right)^2} = \boxed{1.00}$

All the initial kinetic energy is lost by the neutron, as expected for the target mass equal to the incoming mass.

(*b*) $\dfrac{4m_A m_B}{\left(m_A + m_B\right)^2} = \dfrac{4\left(1.01\right)\left(2.01\right)}{\left(1.01 + 2.01\right)^2} = \boxed{0.890}$

(*c*) $\dfrac{4m_A m_B}{\left(m_A + m_B\right)^2} = \dfrac{4\left(1.01\right)\left(12.00\right)}{\left(1.01 + 12.00\right)^2} = \boxed{0.286}$

(*d*) $\dfrac{4m_{\mathrm{A}}m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^2}=\dfrac{4(1.01)(208)}{(1.01+208)^2}=\boxed{0.0192}$

Since the target is quite heavy, almost no kinetic energy is lost. The incoming particle "bounces off" the heavy target, much as a rubber ball bounces off a wall with approximately no loss in speed.

40. Both momentum and kinetic energy are conserved in this one-dimensional collision. We start with Eq. 9-3 (for a one-dimensional setting) and Eq. 9-8.

$$m_{\mathrm{A}}v_{\mathrm{A}}+m_{\mathrm{B}}v_{\mathrm{B}}=m_{\mathrm{A}}v'_{\mathrm{A}}+m_{\mathrm{B}}v'_{\mathrm{B}}\ ;\ v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v'_{\mathrm{A}}-v'_{\mathrm{B}}\right)\ \rightarrow\ v'_{\mathrm{B}}=v_{\mathrm{A}}-v_{\mathrm{B}}+v'_{\mathrm{A}}$$

Insert the last result above back into the momentum conservation equation.

$$m_{\mathrm{A}}v_{\mathrm{A}}+m_{\mathrm{B}}v_{\mathrm{B}}=m_{\mathrm{A}}v'_{\mathrm{A}}+m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}+v'_{\mathrm{A}}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{A}}+m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}\right)\ \rightarrow$$

$$m_{\mathrm{A}}v_{\mathrm{A}}+m_{\mathrm{B}}v_{\mathrm{B}}-m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{A}}\ \rightarrow\ \left(m_{\mathrm{A}}-m_{\mathrm{B}}\right)v_{\mathrm{A}}+2m_{\mathrm{B}}v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{A}}\ \rightarrow$$

$$\boxed{v'_{\mathrm{A}}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)+v_{\mathrm{B}}\left(\frac{2m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)}$$

Do a similar derivation by solving Eq. 9-8 for v'_{A}, which gives $v'_{\mathrm{A}}=v'_{\mathrm{B}}-v_{\mathrm{A}}+v_{\mathrm{B}}$.

$$m_{\mathrm{A}}v_{\mathrm{A}}+m_{\mathrm{B}}v_{\mathrm{B}}=m_{\mathrm{A}}\left(v'_{\mathrm{B}}-v_{\mathrm{A}}+v_{\mathrm{B}}\right)+m_{\mathrm{B}}v'_{\mathrm{B}}=m_{\mathrm{A}}\left(-v_{\mathrm{A}}+v_{\mathrm{B}}\right)+\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{B}}\ \rightarrow$$

$$m_{\mathrm{A}}v_{\mathrm{A}}+m_{\mathrm{B}}v_{\mathrm{B}}-m_{\mathrm{A}}\left(-v_{\mathrm{A}}+v_{\mathrm{B}}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{B}}\ \rightarrow\ 2m_{\mathrm{A}}v_{\mathrm{A}}+\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'_{\mathrm{B}}\ \rightarrow$$

$$\boxed{v'_{\mathrm{B}}=v_{\mathrm{A}}\left(\frac{2m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)+v_{\mathrm{B}}\left(\frac{m_{\mathrm{B}}-m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)}$$

41. (*a*) At the maximum compression of the spring, the blocks will not be moving relative to each other, and so they both have the same forward speed. All of the interaction between the blocks is internal to the mass-spring system, and so momentum conservation can be used to find that common speed. Mechanical energy is also conserved, and so with that common speed, we can find the energy stored in the spring and then the compression of the spring. Let A represent the 3.0 kg block, let B represent the 4.5 kg block, and let x represent the amount of compression of the spring.

$$p_{\text{initial}}=p_{\text{final}}\ \rightarrow\ m_{\mathrm{A}}v_{\mathrm{A}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'\ \rightarrow\ v'=\frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}v_{\mathrm{A}}$$

$$E_{\text{initial}}=E_{\text{final}}\ \rightarrow\ \tfrac{1}{2}m_{\mathrm{A}}v_{\mathrm{A}}^2=\tfrac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'^2+\tfrac{1}{2}kx^2\ \rightarrow$$

$$x=\sqrt{\frac{1}{k}\left[m_{\mathrm{A}}v_{\mathrm{A}}^2-\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)v'^2\right]}=\sqrt{\frac{1}{k}\frac{m_{\mathrm{A}}m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}v_{\mathrm{A}}^2}$$

$$=\sqrt{\left(\frac{1}{850\,\mathrm{N/m}}\right)\frac{(3.0\,\mathrm{kg})(4.5\,\mathrm{kg})}{(7.5\,\mathrm{kg})}(8.0\,\mathrm{m/s})^2}=\boxed{0.37\,\mathrm{m}}$$

(*b*) This is a stationary target elastic collision in one dimension, and so the results of Example 9-8 may be used.

$$v'_{\mathrm{A}}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(8.0\,\mathrm{m/s})\left(\frac{-1.5\,\mathrm{kg}}{7.5\,\mathrm{kg}}\right)=\boxed{-1.6\,\mathrm{m/s}}$$

$$v'_{\mathrm{B}}=v_{\mathrm{A}}\left(\frac{2m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(8.0\,\mathrm{m/s})\left(\frac{6.0\,\mathrm{kg}}{7.5\,\mathrm{kg}}\right)=\boxed{6.4\,\mathrm{m/s}}$$

(*c*) $\boxed{\text{Yes}}$, the collision is elastic. All forces involved in the collision are conservative forces.

42. From the analysis in Example 9-11, the initial projectile speed is given by $v = \frac{m+M}{m}\sqrt{2gh}$. Compare the two speeds with the same masses.

$$\frac{v_2}{v_1} = \frac{\frac{m+M}{m}\sqrt{2gh_2}}{\frac{m+M}{m}\sqrt{2gh_1}} = \frac{\sqrt{h_2}}{\sqrt{h_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{5.2}{2.6}} = \sqrt{2} \rightarrow \boxed{v_2 = \sqrt{2}v_1}$$

$\boxed{43.}$ (*a*) In Example 9-11, $K_i = \frac{1}{2}mv^2$ and $K_f = \frac{1}{2}(m+M)v'^2$. The speeds are related by

$$v' = \frac{m}{m+M}v.$$

$$\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2 - mv^2}{mv^2}$$

$$= \frac{\frac{m^2v^2}{m+M} - mv^2}{mv^2} = \frac{m}{m+M} - 1 = \boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M} = \frac{-380\text{ g}}{396\text{ g}} = \boxed{-0.96}$. Thus 96% of the energy is lost.

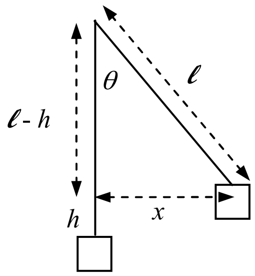
44. From the analysis in the Example 9-11, we know that

$$v = \frac{m+M}{m}\sqrt{2gh} \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2\left(9.80\,\text{m/s}^2\right)}\left(\frac{(0.028\text{ kg})(210\,\text{m/s})}{0.028\text{ kg} + 3.6\text{kg}}\right)^2$$

$$= 0.134\,\text{m} \approx \boxed{1.3\times10^{-1}\,\text{m}}$$

From the diagram we see the following.



$$\ell^2 = (\ell - h)^2 + x^2$$

$$x = \sqrt{\ell^2 - (\ell - h)^2} = \sqrt{(2.8\,\text{m})^2 - (2.8\,\text{m} - 0.134\,\text{m})^2} = \boxed{0.86\,\text{m}}$$

45. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle, and B represent the lighter particle. We have $m_\text{A} = 1.5m_\text{B}$, and $v_\text{A} = v_\text{B} = 0$.

$$p_\text{initial} = p_\text{final} \rightarrow 0 = m_\text{A}v'_\text{A} + m_\text{B}v'_\text{B} \rightarrow v'_\text{A} = -\frac{m_\text{B}v'_\text{B}}{m_\text{A}} = -\tfrac{2}{3}v'_\text{B}$$

The negative sign indicates direction. Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy added.

$$E_{\text{added}} = K'_A + K'_B = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 = \tfrac{1}{2}\left(1.5m_B\right)\left(\tfrac{2}{3}v'_B\right)^2 + \tfrac{1}{2}m_B v_B'^2 = \tfrac{5}{3}\left(\tfrac{1}{2}m_B v_B'^2\right) = \tfrac{5}{3}K'_B$$

$$K'_B = \tfrac{3}{5}E_{\text{added}} = \tfrac{3}{5}\left(7500\,\text{J}\right) = 4500\,\text{J} \qquad K'_A = E_{\text{added}} - K'_B = 7500\,\text{J} - 4500\,\text{J} = 3000\,\text{J}$$

Thus $\boxed{K'_A = 3.0\times10^3\,\text{J} \quad K'_B = 4.5\times10^3\,\text{J}}$.

46. Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive *x* direction. Let A represent the sports car, and B represent the SUV. We have $v_B = 0$ and $v'_\text{A} = v'_\text{B}$. Solve for v_A.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_\text{A}v_\text{A} + 0 = \left(m_\text{A} + m_\text{B}\right)v'_\text{A} \quad\rightarrow\quad v_\text{A} = \frac{m_\text{A} + m_\text{B}}{m_\text{A}}v'_\text{A}$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for v'_A, and use that to find v_A.

$$W_{\text{fr}} = \left(K_{\text{final}} - K_{\text{initial}}\right)_{\substack{\text{after}\\\text{collision}}} = 0 - \tfrac{1}{2}\left(m_\text{A} + m_\text{B}\right)v_\text{A}'^2$$

$$W_{\text{fr}} = F_{\text{fr}}\Delta x\cos180^\circ = -\mu_k\left(m_\text{A} + m_\text{B}\right)g\Delta x$$

$$-\tfrac{1}{2}\left(m_\text{A} + m_\text{B}\right)v_\text{A}'^2 = -\mu_k\left(m_\text{A} + m_\text{B}\right)g\Delta x \quad\rightarrow\quad v'_\text{A} = \sqrt{2\mu_k g\Delta x}$$

$$v_\text{A} = \frac{m_\text{A} + m_\text{B}}{m_\text{A}}v'_\text{A} = \frac{m_\text{A} + m_\text{B}}{m_\text{A}}\sqrt{2\mu_k g\Delta x} = \frac{920\,\text{kg} + 2300\,\text{kg}}{920\,\text{kg}}\sqrt{2\left(0.80\right)\left(9.8\,\text{m/s}^2\right)\left(2.8\,\text{m}\right)}$$

$$= 23.191\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

47. The impulse on the ball is its change in momentum. Call upwards the positive direction, so that the final velocity is positive, and the initial velocity is negative. The speeds immediately before and immediately after the collision can be found from conservation of energy. Take the floor to be the zero level for gravitational potential energy.

Falling: $K_{\text{bottom}} = U_{\text{top}} \quad\rightarrow\quad \tfrac{1}{2}mv_{\text{down}}^2 = mgh_{\text{down}} \quad\rightarrow\quad v_{\text{down}} = -\sqrt{2gh_{\text{down}}}$

Rising: $K_{\text{bottom}} = U_{\text{top}} \quad\rightarrow\quad \tfrac{1}{2}mv_{\text{up}}^2 = mgh_{\text{up}} \quad\rightarrow\quad v_{\text{up}} = \sqrt{2gh_{\text{up}}}$

$$J = \Delta p = m\Delta v = m\left(v_{\text{up}} - v_{\text{down}}\right) = m\left(\sqrt{2gh_{\text{up}}} - -\sqrt{2gh_{\text{down}}}\right) = m\sqrt{2g}\left(\sqrt{h_{\text{up}}} + \sqrt{h_{\text{down}}}\right)$$

$$= \left(0.012\,\text{kg}\right)\sqrt{2\left(9.80\,\text{m/s}^2\right)}\left(\sqrt{0.75\,\text{m}} + \sqrt{1.5\,\text{m}}\right) = 0.11\,\text{kg}\cdot\text{m/s}$$

The direction of the impulse is upwards, so the complete specification of the impulse is $\boxed{0.11\,\text{kg}\cdot\text{m/s, upwards}}$.

48. $$\text{Fraction}\,K\text{ lost} = \frac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}} = \frac{\tfrac{1}{2}m_\text{A}v_\text{A}^2 - \tfrac{1}{2}m_\text{B}v_\text{B}'^2}{\tfrac{1}{2}m_\text{A}v_\text{A}^2} = \frac{v_\text{A}^2 - v_\text{B}'^2}{v_\text{A}^2} = \frac{\left(35\,\text{m/s}\right)^2 - \left(25\,\text{m/s}\right)^2}{\left(35\,\text{m/s}\right)^2} = \boxed{0.49}$$

$\boxed{49.}$ (*a*) For a perfectly elastic collision, Eq. 9-8 says $v_A - v_B = -\left(v'_A - v'_B\right)$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = -\frac{\left(v_A - v_B\right)}{v_B - v_A} = 1.$$

For a completely inelastic collision, $v'_A = v'_B$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v'_A - v'_B}{v_B - v_A} = 0$$

(*b*) Let A represent the falling object and B represent the heavy steel plate. The speeds of the steel plate are $v_B = 0$ and $v'_B = 0$. Thus $e = -v'_A/v_A$. Consider energy conservation during the falling or rising path. The potential energy of body A at height *h* is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$mgh = \tfrac{1}{2}mv_{\text{A}}^2 \quad\rightarrow\quad v_{\text{A}} = \sqrt{2gh}$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$mgh' = \tfrac{1}{2}mv_{\text{A}}'^2 \quad\rightarrow\quad v'_{\text{A}} = -\sqrt{2gh'}$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$e = -v'_A/v_A = -\frac{-\sqrt{2gh'}}{\sqrt{2gh}} \quad\rightarrow\quad \boxed{e = \sqrt{h'/h}}$$

50. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$K_{\text{bottom}} = U_{\text{top}} \quad\rightarrow\quad \tfrac{1}{2}\left(m+M\right)V_{\text{bottom}}^2 = \left(m+M\right)g\left(2L\right) \quad\rightarrow\quad V_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad mv = \left(m+M\right)V_{\text{bottom}} \quad\rightarrow\quad v = \frac{m+M}{m} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

51. (*a*) The collision is assumed to happen fast enough that the bullet–block system does not move during the collision. So the totally inelastic collision is described by momentum conservation. The conservation of energy (including the non-conservative work done by friction) can be used to relate the initial kinetic energy of the bullet–block system to the spring compression and the dissipated energy. Let *m* represent the mass of the bullet, *M* represent the mass of the block, and *x* represent the distance the combination moves after the collision

$$\text{collision: } mv = \left(m+M\right)v' \quad\rightarrow\quad v = \frac{m+M}{m}v'$$

$$\text{after collision: } \tfrac{1}{2}\left(m+M\right)v'^2 = \tfrac{1}{2}kx^2 + \mu\left(m+M\right)gx \quad\rightarrow\quad v' = \sqrt{\frac{kx^2}{m+M} + 2\mu gx}$$

$$v = \frac{m+M}{m}\sqrt{\frac{kx^2}{m+M} + 2\mu gx}$$

$$= \frac{1.000\,\text{kg}}{1.0\times10^{-3}\,\text{kg}}\sqrt{\frac{(120\,\text{N/m})(0.050\,\text{m})^2}{1.000\,\text{kg}} + 2(0.50)\left(9.80\,\text{m/s}^2\right)(0.050\,\text{m})} = 888.8\,\text{m/s}$$

$$\approx \boxed{890\,\text{m/s}}$$

(*b*) The fraction of kinetic energy dissipated in the collision is $\frac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}}$, where the kinetic energies are calculated immediately before and after the collision.

$$\frac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}(m+M)v'^2}{\frac{1}{2}mv^2} = 1 - \frac{(m+M)v'^2}{mv^2} = 1 - \frac{(m+M)v'^2}{m\left(\frac{m+M}{m}v'\right)^2}$$

$$= 1 - \frac{m}{m+M} = 1 - \frac{0.0010\,\text{kg}}{1.00\,\text{kg}} = \boxed{0.999}$$

52. (*a*) Momentum is conserved in the one-dimensional collision. Let A represent the baseball and let B represent the brick.

$$m_A v_A = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_A = \frac{m_A v_A - m_B v'_B}{m_A} = \frac{(0.144\,\text{kg})(28.0\,\text{m/s}) - (5.25\,\text{kg})(1.10\,\text{m/s})}{0.144\,\text{kg}} = -12.10\,\text{m/s}$$

So the baseball's speed in the reverse direction is $\boxed{12.1\,\text{m/s}}$.

(*b*) $K_{\text{before}} = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(0.144\,\text{kg})(28.0\,\text{m/s})^2 = \boxed{56.4\,\text{J}}$

$K_{\text{after}} = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_A v'^2_B = \frac{1}{2}(0.144\,\text{kg})(1.21\,\text{m/s})^2 + \frac{1}{2}(5.25\,\text{kg})(1.10\,\text{m/s})^2 = \boxed{13.7\,\text{J}}$

53. In each case, use momentum conservation. Let A represent the 6.0-kg object and let B represent the 10.0-kg object. We have $v_A = 5.5\,\text{m/s}$ and $v_B = -4.0\,\text{m/s}$..

(*a*) In this totally inelastic case, $v'_A = v'_B$.

$$m_A v_A + m_B v_B = (m_A + m_B)v'_A \quad \rightarrow$$

$$v'_B = v'_A = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(6.0\,\text{kg})(5.5\,\text{m/s}) + (8.0\,\text{kg})(-4.0\,\text{m/s})}{14.0\,\text{kg}} = \boxed{7.1\times10^{-2}\,\text{m/s}}$$

(*b*) In this case, use Eq. 9-8 to find a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A - v_B + v'_A$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B = m_A v'_A + m_B(v_A + v_B + v'_A) \quad \rightarrow$$

$$v'_A = \frac{(m_A - m_B)v_A + 2m_B v_B}{m_A + m_B} = \frac{(-2.0\,\text{kg})(5.5\,\text{m/s}) + 2(8.0\,\text{kg})(-4.0\,\text{m/s})}{14.0\,\text{kg}} = \boxed{-5.4\,\text{m/s}}$$

$$v'_B = v_A - v_B + v'_A = 5.5\,\text{m/s} - (-4.0\,\text{m/s}) - 5.4\,\text{m/s} = \boxed{4.1\,\text{m/s}}$$

(*c*) In this case, $v'_A = 0$.

$$m_A v_A + m_B v_B = m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{m_A v_A + m_B v_B}{m_B} = \frac{(6.0\,\text{kg})(5.5\,\text{m/s}) + (8.0\,\text{kg})(-4.0\,\text{m/s})}{8.0\,\text{kg}} = \boxed{0.13\,\text{m/s}}$$

To check for "reasonableness," first note the final directions of motion. A has stopped, and B has gone in the opposite direction. This is reasonable. Secondly, since both objects are moving slower than their original speeds, there has been a loss of kinetic energy. Since the system has lost kinetic energy and the directions are possible, this interaction is $\boxed{\text{"reasonable."}}$

(*d*) In this case, $v_B' = 0$.

$$m_A v_A + m_B v_B = m_A v_A' \quad \rightarrow$$

$$v_A' = \frac{m_A v_A + m_B v_B}{m_A} = \frac{(6.0\,\text{kg})(5.5\,\text{m/s}) + (8.0\,\text{kg})(-4.0\,\text{m/s})}{6.0\,\text{kg}} = \boxed{0.17\,\text{m/s}}$$

This answer is $\boxed{\text{not reasonable}}$ because A is still moving in its original direction while B has stopped. Thus A has somehow passed through B. If B has stopped, A should have rebounded in the negative direction.

(*e*) In this case, $v_A' = -4.0\,\text{m/s}$.

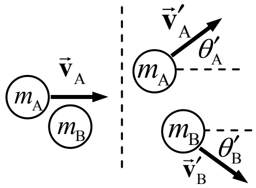
$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$v_B' = \frac{(6.0\,\text{kg})(5.5\,\text{m/s} - -4.0\,\text{m/s}) + (8.0\,\text{kg})(-4.0\,\text{m/s})}{8.0\,\text{kg}} = \boxed{3.1\,\text{m/s}}$$

The directions are reasonable, in that each object rebounds. Secondly, since both objects are moving slower than their original speeds, there has been a loss of kinetic energy. Since the system has lost kinetic energy and the directions are possible, this interaction is $\boxed{\text{"reasonable."}}$

54. (*a*) $p_x: \quad m_A v_A = m_A v_A' \cos\theta_A' + m_B v_B' \cos\theta_B'$

$p_y: \quad 0 = m_A v_A' \sin\theta_A' - m_B v_B' \sin\theta_B'$



(*b*) Solve the *x* equation for $\cos\theta_B'$ and the *y* equation for $\sin\theta_B'$, and then find the angle from the tangent function.

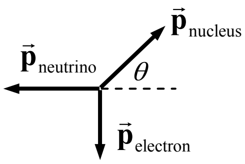
$$\tan\theta_B' = \frac{\sin\theta_B'}{\cos\theta_B'} = \frac{\dfrac{m_A v_A' \sin\theta_A'}{m_B v_B'}}{\dfrac{m_A\left(v_A - v_A'\cos\theta_A'\right)}{m_B v_B'}} = \frac{v_A' \sin\theta_A'}{\left(v_A - v_A'\cos\theta_A'\right)}$$

$$\theta_B' = \tan^{-1}\frac{v_A' \sin\theta_A'}{v_A - v_A'\cos\theta_A'} = \tan^{-1}\frac{(2.10\,\text{m/s})\sin 30.0^\circ}{2.80\,\text{m/s} - (2.10\,\text{m/s})\cos 30.0^\circ} = \boxed{46.9^\circ}$$

With the value of the angle, solve the *y* equation for the velocity.

$$v_B' = \frac{m_A v_A' \sin\theta_A'}{m_B \sin\theta_B'} = \frac{(0.120\,\text{kg})(2.10\,\text{m/s})\sin 30.0^\circ}{(0.140\,\text{kg})\sin 46.9^\circ} = \boxed{1.23\,\text{m/s}}$$

$\boxed{55.}$ Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the *x* and *y* directions.



$$\left(p_{\text{nucleus}}\right)_x = p_{\text{neutrino}} \qquad \left(p_{\text{nucleus}}\right)_y = p_{\text{electron}}$$

$$p_{\text{nucleus}} = \sqrt{\left(p_{\text{nucleus}}\right)_x^2 + \left(p_{\text{nucleus}}\right)_y^2} = \sqrt{\left(p_{\text{neutrino}}\right)^2 + \left(p_{\text{electron}}\right)^2}$$

$$= \sqrt{\left(6.2\times10^{-23}\,\text{kg}\bullet\text{m/s}\right)^2 + \left(9.6\times10^{-23}\,\text{kg}\bullet\text{m/s}\right)^2} = \boxed{1.14\times10^{-22}\,\text{kg}\bullet\text{m/s}}$$

$$\theta = \tan^{-1}\frac{\left(p_{\text{nucleus}}\right)_y}{\left(p_{\text{nucleus}}\right)_x} = \tan^{-1}\frac{\left(p_{\text{electron}}\right)}{\left(p_{\text{neutrino}}\right)} = \tan^{-1}\frac{\left(9.6\times 10^{-23}\ \text{kg}\bullet\text{m/s}\right)}{\left(6.2\times 10^{-23}\ \text{kg}\bullet\text{m/s}\right)} = 57^\circ$$

The second nucleus' momentum is $\boxed{147^\circ \text{ from the electron's momentum}}$, and is $\boxed{123^\circ \text{ from the neutrino's momentum}}$.

56. Write momentum conservation in the *x* and *y* directions, and kinetic energy conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}'_{\text{A}}$ to have both *x* and *y* components.

$$p_x: \quad mv_{\text{B}} = mv'_{\text{A}x} \;\rightarrow\; v_{\text{B}} = v'_{\text{A}x}$$

$$p_y: \quad mv_{\text{A}} = mv'_{\text{A}y} + mv'_{\text{B}} \;\rightarrow\; v_{\text{A}} = v'_{\text{A}y} + v'_{\text{B}}$$

$$K: \quad \tfrac{1}{2}mv_{\text{A}}^2 + \tfrac{1}{2}mv_{\text{B}}^2 = \tfrac{1}{2}mv_{\text{A}}'^2 + \tfrac{1}{2}mv_{\text{B}}'^2 \;\rightarrow\; v_{\text{A}}^2 + v_{\text{B}}^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2$$

Substitute the results from the momentum equations into the kinetic energy equation.

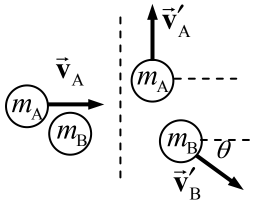
$$\left(v'_{\text{A}y} + v'_{\text{B}}\right)^2 + \left(v'_{\text{A}x}\right)^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \;\rightarrow\; v_{\text{A}y}'^2 + 2v_{\text{A}y}'^2 v'_{\text{B}} + v_{\text{B}}'^2 + v_{\text{A}y}'^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \;\rightarrow$$

$$v_{\text{A}}'^2 + 2v_{\text{A}y}'^2 v'_{\text{B}} + v_{\text{B}}'^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \;\rightarrow\; 2v_{\text{A}y}'^2 v'_{\text{B}} = 0 \;\rightarrow\; v'_{\text{A}y} = 0 \text{ or } v'_{\text{B}} = 0$$

Since we are given that $v'_{\text{B}} \neq 0,$ we must have $v'_{\text{A}y} = 0.$ This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$v'_{\text{A}} = v'_{\text{A}x} = v_{\text{B}} = \boxed{3.7\,\text{m/s}} \qquad v'_{\text{B}} = v_{\text{A}} = \boxed{2.0\,\text{m/s}}$$

57. (*a*) Let A represent the incoming nucleus, and B represent the target particle. Take the *x* direction to be in the direction of the initial velocity of mass A (to the right in the diagram), and the *y* direction to be up in the diagram. Momentum is conserved in two dimensions, and gives the following relationships.



$$p_x: \quad m_{\text{A}}v_{\text{A}} = m_{\text{B}}v'_{\text{B}}\cos\theta \;\rightarrow\; v = 2v'_{\text{B}}\cos\theta$$

$$p_y: \quad 0 = m_{\text{A}}v'_{\text{A}} - m_{\text{B}}v'_{\text{B}}\sin\theta \;\rightarrow\; v'_{\text{A}} = 2v'_{\text{B}}\sin\theta$$

The collision is elastic, and so kinetic energy is also conserved.

$$K: \quad \tfrac{1}{2}m_{\text{A}}v_{\text{A}}^2 = \tfrac{1}{2}m_{\text{A}}v_{\text{A}}'^2 + \tfrac{1}{2}m_{\text{B}}v_{\text{B}}'^2 \;\rightarrow\; v^2 = v_{\text{A}}'^2 + 2v_{\text{B}}'^2 \;\rightarrow\; v^2 - v_{\text{A}}'^2 = 2v_{\text{B}}'^2$$

Square the two momentum equations and add them together.

$$v = 2v'_{\text{B}}\cos\theta\ ;\ v'_{\text{A}} = 2v'_{\text{B}}\sin\theta \;\rightarrow\; v^2 = 4v_{\text{B}}'^2\cos^2\theta\ ;\ v_{\text{A}}'^2 = 4v_{\text{B}}'^2\sin^2\theta \;\rightarrow\; v^2 + v_{\text{A}}'^2 = 4v_{\text{B}}'^2$$

Add these two results together and use them in the *x* momentum expression to find the angle.

$$v^2 - v_{\text{A}}'^2 = 2v_{\text{B}}'^2\ ;\ v^2 + v_{\text{A}}'^2 = 4v_{\text{B}}'^2 \;\rightarrow\; 2v^2 = 6v_{\text{B}}'^2 \;\rightarrow\; v'_{\text{B}} = \frac{v}{\sqrt{3}}$$

$$\cos\theta = \frac{v}{2v'_{\text{B}}} = \frac{v}{2\dfrac{v}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \;\rightarrow\; \boxed{\theta = 30^\circ}$$

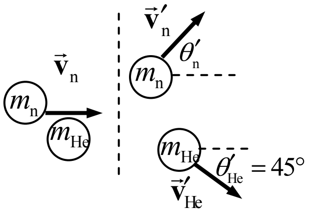
(*b*) From above, we already have $\boxed{v'_{\text{B}} = \dfrac{v}{\sqrt{3}}}$. Use that in the *y* momentum equation.

$$v'_{\text{A}} = 2v'_{\text{B}}\sin\theta = 2\frac{v}{\sqrt{3}}\sin 30^\circ = \boxed{v'_{\text{A}} = \frac{v}{\sqrt{3}}}$$

(*c*) The fraction transferred is the final energy of the target particle divided by the original kinetic energy.

$$\frac{K_{\text{target}}}{K_{\text{original}}}=\frac{\frac{1}{2}m_{\text{B}}v_B'^2}{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2}=\frac{\frac{1}{2}\left(2m_{\text{A}}\right)\left(v^2/3\right)}{\frac{1}{2}m_{\text{A}}v^2}=\boxed{\frac{2}{3}}$$

58. Let n represent the incoming neutron, and let He represent the helium nucleus. We take $m_{\text{He}}=4m_{\text{n}}$. Take the *x* direction to be the direction of the initial velocity of the neutron (to the right in the diagram), and the *y* direction to be up in the diagram. Momentum is conserved in two dimensions, and gives the following relationships.



$$p_x:\quad m_{\text{n}}v_{\text{n}}=m_{\text{n}}v'_{\text{n}}\cos\theta'_{\text{n}}+m_{\text{He}}v'_{\text{He}}\cos\theta'_{\text{He}}\quad\rightarrow$$

$$v_{\text{n}}-4v'_{\text{He}}\cos\theta'_{\text{He}}=v'_{\text{n}}\cos\theta'_{\text{n}}$$

$$p_y:\quad 0=m_{\text{n}}v'_{\text{n}}\sin\theta'_{\text{n}}-m_{\text{He}}v'_{\text{He}}\sin\theta'_{\text{He}}\quad\rightarrow\quad 4v'_{\text{He}}\sin\theta'_{\text{He}}=v'_{\text{n}}\sin\theta'_{\text{n}}$$

The collision is elastic, and so kinetic energy is also conserved.

$$K:\quad \tfrac{1}{2}m_{\text{n}}v_{\text{n}}^2=\tfrac{1}{2}m_{\text{n}}v_{\text{n}}'^2+\tfrac{1}{2}m_{\text{He}}v_{\text{He}}'^2\quad\rightarrow\quad v_{\text{n}}^2=v_{\text{n}}'^2+4v_{\text{He}}'^2\quad\rightarrow\quad v_{\text{n}}'^2=v_{\text{n}}^2-4v_{\text{He}}'^2$$

This is a set of three equations in the three unknowns v'_{n}, v'_{He}, and θ'_{n}. We can eliminate θ'_{n} by squaring and adding the momentum equations. That can be combined with the kinetic energy equation to solve for one of the unknown speeds.

$$\left(v_{\text{n}}-4v'_{\text{He}}\cos\theta'_{\text{He}}\right)^2=\left(v'_{\text{n}}\cos\theta'_{\text{n}}\right)^2\ ;\quad\left(4v'_{\text{He}}\sin\theta'_{\text{He}}\right)^2=\left(v'_{\text{n}}\sin\theta'_{\text{n}}\right)^2\quad\rightarrow$$

$$v_{\text{n}}^2-8v_{\text{n}}v'_{\text{He}}\cos\theta'_{\text{He}}+16v_{\text{He}}'^2\cos^2\theta'_{\text{He}}+16v_{\text{He}}'^2\sin^2\theta'_{\text{He}}=v_{\text{n}}'^2\cos^2\theta'_{\text{n}}+v_{\text{n}}'^2\sin^2\theta'_{\text{n}}\quad\rightarrow$$

$$v_{\text{n}}^2-8v_{\text{n}}v'_{\text{He}}\cos\theta'_{\text{He}}+16v_{\text{He}}'^2=v_{\text{n}}'^2=v_{\text{n}}^2-4v_{\text{He}}'^2\quad\rightarrow$$

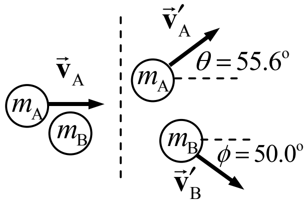
$$v'_{\text{He}}=0.4v_{\text{n}}\cos\theta'_{\text{He}}=0.4\left(6.2\times10^5\text{ m/s}\right)\cos45°=1.754\times10^5\text{ m/s}$$

$$v_{\text{n}}'^2=v_{\text{n}}^2-4v_{\text{He}}'^2\quad\rightarrow\quad v'_{\text{n}}=\sqrt{v_{\text{n}}^2-4v_{\text{He}}'^2}=\sqrt{\left(6.2\times10^5\text{ m/s}\right)^2-4\left(1.754\times10^5\text{ m/s}\right)}=5.112\times10^5\text{ m/s}$$

$$4v'_{\text{He}}\sin\theta'_{\text{He}}=v'_{\text{n}}\sin\theta'_{\text{n}}\quad\rightarrow\quad\theta'_{\text{n}}=\sin^{-1}\left(4\frac{v'_{\text{He}}}{v'_{\text{n}}}\sin\theta'_{\text{He}}\right)=\sin^{-1}\left(4\frac{\left(1.754\times10^5\text{ m/s}\right)}{\left(5.112\times10^5\text{ m/s}\right)}\sin45°\right)=76°$$

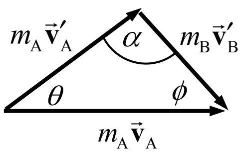
To summarize: $\boxed{v'_{\text{n}}=5.1\times10^5\text{ m/s}}$, $\boxed{v'_{\text{He}}=1.8\times10^5\text{ m/s}}$, $\boxed{\theta'_{\text{n}}=76°}$.

59. Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_{\text{A}}\vec{\mathbf{v}}_{\text{A}}=m_{\text{A}}\vec{\mathbf{v}}'_{\text{A}}+m_{\text{B}}\vec{\mathbf{v}}'_{\text{B}}$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.



$$\frac{m_{\text{A}}v'_{\text{A}}}{m_{\text{A}}v_{\text{A}}}=\frac{\sin\phi}{\sin\alpha}\quad\rightarrow\quad v'_{\text{A}}=v_{\text{A}}\frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_{\text{B}}v'_{\text{B}}}{m_{\text{A}}v_{\text{A}}}=\frac{\sin\theta}{\sin\alpha}\quad\rightarrow\quad v'_{\text{B}}=v_{\text{A}}\frac{m_{\text{A}}}{m_{\text{B}}}\frac{\sin\theta}{\sin\alpha}$$

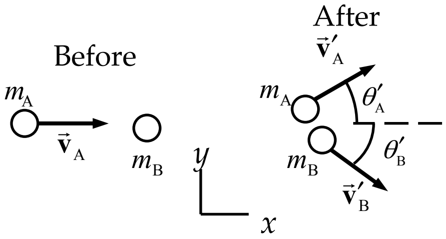


The collision is elastic, so write the kinetic energy conservation equation, and substitute the results from above. Also note that $\alpha=180.0-55.6°-50.0°=74.4°$.

$$\tfrac{1}{2}m_{\text{A}}v_{\text{A}}^2=\tfrac{1}{2}m_{\text{A}}v_{\text{A}}'^2+\tfrac{1}{2}m_{\text{B}}v_{\text{B}}'^2\quad\rightarrow\quad m_{\text{A}}v_{\text{A}}^2=m_{\text{A}}\left(v_{\text{A}}\frac{\sin\phi}{\sin\alpha}\right)^2+m_{\text{B}}\left(v_{\text{A}}\frac{m_{\text{A}}}{m_{\text{B}}}\frac{\sin\theta}{\sin\alpha}\right)^2\quad\rightarrow$$

$$m_B = \frac{m_A \sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\text{ u})\sin^2 55.6^\circ}{\sin^2 74.4 - \sin^2 50.0^\circ} = \boxed{39.9\text{ u}}$$

60. Use the coordinate system indicated in the diagram. We start with the conditions for momentum and kinetic energy conservation.



$$p_x:\quad m_A v_A = m_A v'_A\cos\theta'_A + m_B v'_B\cos\theta'_B \quad\rightarrow$$
$$m_A v_A - m_A v'_A\cos\theta'_A = m_B v'_B\cos\theta'_B$$
$$p_y:\quad 0 = m_A v'_A\sin\theta'_A - m_B v'_B\sin\theta'_B \quad\rightarrow$$
$$m_A v'_A\sin\theta'_A = m_B v'_B\sin\theta'_B$$
$$K:\quad \tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 \;\rightarrow\; m_A\left(v_A^2 - v_A'^2\right) = m_B v_B'^2 \;\rightarrow\; m_A m_B\left(v_A^2 - v_A'^2\right) = m_B^2 v_B'^2$$

Note that from the kinetic energy relationship, since the right side of the equation is positive, we must have $v_A \ge v'_A \ge 0$.

Now we may eliminate θ'_B by squaring the two momentum relationships and adding them.

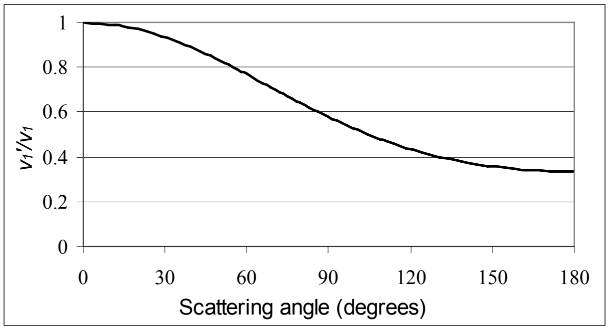
$$\left(m_A v_A - m_A v'_A\cos\theta'_A\right)^2 + \left(m_A v'_A\sin\theta'_A\right)^2 = \left(m_B v'_B\cos\theta'_B\right)^2 + \left(m_B v'_B\sin\theta'_B\right)^2 \quad\rightarrow$$
$$\left(m_A v_A\right)^2 - \left(2m_A^2 v_A v'_A\cos\theta'_A\right) + \left(m_A v'_A\right)^2 = \left(m_B v'_B\right)^2$$

Combining the previous result with the conservation of energy result gives the following.

$$\left(m_A v_A\right)^2 - \left(2m_A^2 v_A v'_A\cos\theta'_A\right) + \left(m_A v'_A\right)^2 = \left(m_B v'_B\right)^2 = m_A m_B\left(v_A^2 - v_A'^2\right) \quad\rightarrow$$
$$\cos\theta'_A = \tfrac{1}{2}\left[\left(1 - \frac{m_B}{m_A}\right)\frac{v_A}{v'_A} + \left(1 + \frac{m_B}{m_A}\right)\frac{v'_A}{v_A}\right]\text{ ; still with } v_A \ge v'_A \ge 0$$

(*a*) Consider $m_A < m_B$. If $v'_A = v_A$, its maximum value, then

$\cos\theta'_A = \tfrac{1}{2}\left[\left(1 - \frac{m_B}{m_A}\right)\frac{v_A}{v'_A} + \left(1 + \frac{m_B}{m_A}\right)\frac{v'_A}{v_A}\right] = 1 \;\rightarrow\; \theta'_A = 0.$ As v'_A decreases towards 0, eventually the first term in the expression for $\cos\theta'_A$ will dominate, since it has $\frac{v_A}{v'_A}$ as a factor. That term will also be negative because $m_A < m_B$. The expression for $\cos\theta'_A$ will eventually become negative and approach $-\infty$ in a continuous fashion. Thus $\cos\theta'_A$ will for some value of $\frac{v_A}{v'_A}$ have the value of -1, indicating that there is some allowable value of v'_A that causes $\theta'_A = 180^\circ$, and so all scattering angles are possible.



A plot of $\frac{v'_A}{v_A}$ vs. θ'_A is helpful in seeing this. Here is such a plot for $m_A = 0.5m_B$. Note that it indicates that the speed of the incident particle will range from a minimum of about $0.35v_A$ for a complete backscatter (a one-dimensional collision) to $1.00v_A$, which essentially means a "miss" – no collision. We also see that the graph is monotonically decreasing, which means that there are no analytical extrema to consider in the analysis.

(*b*) Now consider $m_A > m_B$. If $v'_A = v_A$, its maximum value, then again we will have

$$\cos\theta'_A = \tfrac{1}{2}\left[\left(1-\frac{m_B}{m_A}\right)\frac{v_A}{v'_A} + \left(1+\frac{m_B}{m_A}\right)\frac{v'_A}{v_A}\right] = 1 \;\rightarrow\; \theta'_A = 0.$$

As v'_A decreases towards 0, eventually the first term in the expression for $\cos\theta'_A$ will dominate, since it has $\frac{v_A}{v'_A}$ as a factor. But both terms in the expression are positive, since $m_A > m_B$. So the expression for $\cos\theta'_A$ will eventually approach $+\infty$ in a continuous fashion, and will never be negative. Thus there will not be any scattering angles bigger than $90°$ in any case. But is there a maximum angle, corresponding to a minimum value of $\cos\theta'_A$? We look for such a point by calculating the derivative $\frac{d}{dv'_A}\cos\theta'_A$.

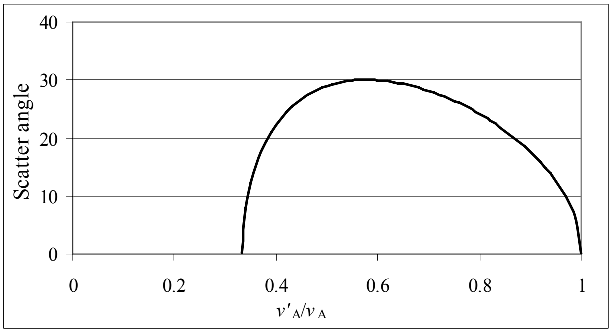
$$\frac{d}{dv'_A}\cos\theta'_A = \tfrac{1}{2}\left[-\left(1-\frac{m_B}{m_A}\right)\frac{v_A}{v'^2_A} + \left(1+\frac{m_B}{m_A}\right)\frac{1}{v_A}\right] = 0 \;\rightarrow\; v'_A = v_A\left[\frac{\left(1-\frac{m_B}{m_A}\right)}{\left(1+\frac{m_B}{m_A}\right)}\right]^{1/2}$$

Using this critical value gives the following value for $\cos\theta'_1$, which we label as $\cos\phi$.

$$\cos\phi = \tfrac{1}{2}\left[\left(1-\frac{m_B}{m_A}\right)\left[\frac{\left(1+\frac{m_B}{m_A}\right)}{\left(1-\frac{m_B}{m_A}\right)}\right]^{1/2} + \left(1+\frac{m_B}{m_A}\right)\left[\frac{\left(1-\frac{m_B}{m_A}\right)}{\left(1+\frac{m_B}{m_A}\right)}\right]^{1/2}\right] = \left(1-\left(\frac{m_B}{m_A}\right)^2\right)^{1/2} \;\rightarrow$$

$$\boxed{\cos^2\phi = 1-\left(\frac{m_B}{m_A}\right)^2}$$

This gives the largest possible scattering angle for the given mass ratio. Again, a plot is instructive. Here is such a plot for $m_A = 2m_B$. We find the maximum scattering angle according to the equation above.



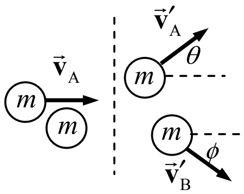
$$\cos^2\phi = 1-\left(\frac{m_B}{m_A}\right)^2 \;\rightarrow$$

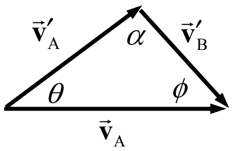
$$\phi = \cos^{-1}\sqrt{\left[1-\left(\frac{m_B}{m_A}\right)^2\right]}$$

$$= \cos^{-1}\sqrt{\left[1-(0.5)^2\right]} = 30°$$

The equation and the graph agree. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH09.XLS," on tab "Problem 9.60b."

$\boxed{61.}$ To do this problem with only algebraic manipulations is complicated. We use a geometric approach instead. See the diagram of the geometry.





Momentum conservation: $m\vec{\mathbf{v}}_A = m\vec{\mathbf{v}}'_A + m\vec{\mathbf{v}}'_B \;\rightarrow\; \vec{\mathbf{v}}_A = \vec{\mathbf{v}}'_A + \vec{\mathbf{v}}'_B$

Kinetic energy conservation: $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_A'^2 + \frac{1}{2}mv_B'^2 \;\rightarrow\; v_A^2 = v_A'^2 + v_B'^2$

The momentum equation can be illustrated as a vector summation diagram, and the kinetic energy equation relates the magnitudes of the vectors in that summation diagram. Examination of the energy equation shows that it is identical to the Pythagorean theorem. The only way that the Pythagorean theorem can hold true is if the angle α in the diagram is a right angle. If α is a right angle, then $\theta + \phi = 90°$, and so the angle between the final velocity vectors must be $90°$.

62. Find the CM relative to the front of the car.

$$x_{CM} = \frac{m_{car}x_{car} + m_{front}x_{front} + m_{back}x_{back}}{m_{car} + m_{front} + m_{back}}$$

$$= \frac{(1250\text{ kg})(2.50\text{ m}) + 2(70.0\text{ kg})(2.80\text{ m}) + 3(70.0\text{ kg})(3.90\text{ m})}{1250\text{ kg} + 2(70.0\text{ kg}) + 3(70.0\text{ kg})} = \boxed{2.71\text{ m}}$$

63. Choose the carbon atom as the origin of coordinates.

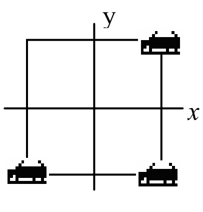
$$x_{CM} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\text{ u})(0) + (16\text{ u})(1.13\times10^{-10}\text{m})}{12\text{ u} + 16\text{ u}} = \boxed{6.5\times10^{-11}\text{m}} \text{ from the C atom.}$$

64. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, and so $m_1 = \rho(\ell_0)^3$, $m_2 = \rho(2\ell_0)^3$, $m_3 = \rho(3\ell_0)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}\ell_0$, $x_2 = 2\ell_0$, $x_3 = 4.5\ell_0$. Use Eq. 9-10 to calculate the CM of the system.

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho\ell_0^3\left(\frac{1}{2}\ell_0\right) + 8\rho\ell_0^3(2\ell_0) + 27\rho\ell_0^3(4.5\ell_0)}{\rho\ell_0^3 + 8\rho\ell_0^3 + 27\rho\ell_0^3}$$

$$= \boxed{3.8\,\ell_0 \text{ from the left edge of the smallest cube}}$$

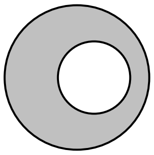
65. Consider this diagram of the cars on the raft. Notice that the origin of coordinates is located at the CM of the raft. Reference all distances to that location.



$$x_{CM} = \frac{(1350\text{ kg})(9\text{ m}) + (1350\text{ kg})(9\text{ m}) + (1350\text{ kg})(-9\text{ m})}{3(1350\text{ kg}) + 6200\text{ kg}} = \boxed{1.2\text{ m}}$$

$$y_{CM} = \frac{(1350\text{ kg})(9\text{ m}) + (1350\text{ kg})(-9\text{ m}) + (1350\text{ kg})(-9\text{ m})}{3(1350\text{ kg}) + 6200\text{ kg}} = \boxed{-1.2\text{ m}}$$

66. Consider the following. We start with a full circle of radius 2*R*, with its CM at the origin. Then we draw a circle of radius *R*, with its CM at the coordinates $\left(0.80R, 0\right)$. The full circle can now be labeled as a "gray" part and a "white" part. The *y* coordinate of the CM of the entire circle, the CM of the gray part, and the CM of the white part are all at $y = 0$ by the symmetry of the system. The *x* coordinate of the entire circle is at $x_{\text{CM}} = 0$, and can be calculated by $x_{\text{CM}} = \dfrac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}}$. Rearrange this equation to solve for the CM of the "gray" part.



$$x_{\text{CM}} = \frac{m_{\text{gray}}x_{\text{gray}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}} \quad \rightarrow$$

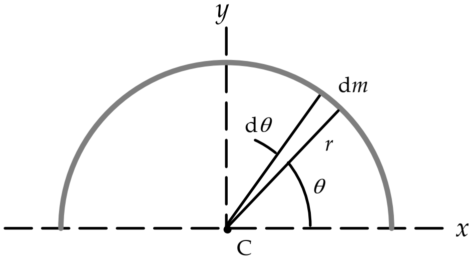
$$x_{\text{gray}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{gray}}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}}$$

This is functionally the same as treating the white part of the figure as a hole of negative mass. The mass of each part can be found by multiplying the area of the part times the uniform density of the plate.

$$x_{\text{gray}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-\rho\pi R^2\left(0.80R\right)}{\rho\pi\left(2R\right)^2 - \rho\pi R^2} = \frac{-0.80R}{3} = \boxed{-0.27R}$$

The negative sign indicates that the CM of the "gray" part is to the left of the center of the circle of radius 2*R*.

$\boxed{67.}$ From the symmetry of the wire, we know that $x_{\text{CM}} = 0$. Consider an infinitesimal piece of the wire, with mass *dm*, and coordinates $\left(x, y\right) = \left(r\cos\theta, r\sin\theta\right)$. If the length of that piece of wire is $d\ell$, then since the wire is uniform, we have $dm = \dfrac{M}{\pi r}d\ell$. And from the diagram and the definition of radian angle measure, we have $d\ell = rd\theta$. Thus $dm = \dfrac{M}{\pi r}rd\theta = \dfrac{M}{\pi}d\theta$. Now apply Eq. 9-13.



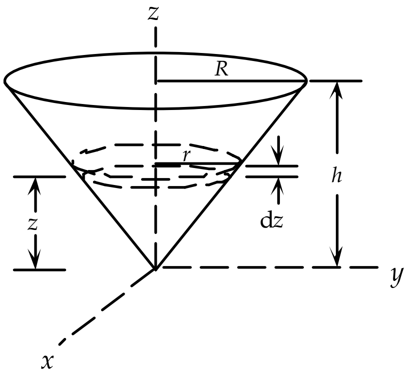
$$y_{\text{CM}} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^\pi r\sin\theta\frac{M}{\pi}d\theta = \frac{r}{\pi}\int_0^\pi \sin\theta d\theta = \frac{2r}{\pi}$$

Thus the coordinates of the center of mass are $\left(x_{\text{CM}}, y_{\text{CM}}\right) = \boxed{\left(0, \frac{2r}{\pi}\right)}$.

68. From the symmetry of the hydrogen equilateral triangle, and the fact that the nitrogen atom is above the center of that triangle, the center of mass will be perpendicular to the plane of the hydrogen atoms, on a line from the center of the hydrogen triangle to the nitrogen atom. We find the height of the center of mass above the triangle from the heights of the individual atoms. The masses can be expressed in any consistent units, and so atomic mass units from the periodic table will be used.

$$z_{\text{CM}} = \frac{3m_{\text{H}}z_{\text{H}} + m_{\text{N}}z_{\text{N}}}{m_{\text{total}}} = \frac{3\left(1.008\,\text{u}\right)\left(0\right) + \left(14.007\,\text{u}\right)\left(0.037\,\text{nm}\right)}{3\left(1.008\,\text{u}\right) + \left(14.007\,\text{u}\right)} = 0.030\,\text{nm}$$

And so the center of mass is $\boxed{\text{0.030 nm above the center of the hydrogen triangle}}$.



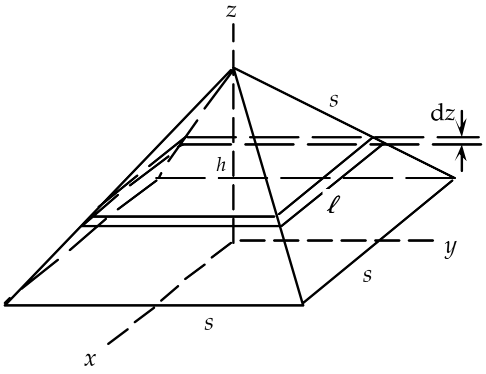
69. Let the tip of the cone be at the origin, and the symmetry axis of the cone be vertical. From the symmetry of the cone, we know that $x_{\text{CM}} = y_{\text{CM}} = 0,$ and so the center of mass lies on the z axis. We have from Eq. 9-13 that $z_{\text{CM}} = \frac{1}{M}\int z\,dm.$ The mass can be expressed as $M = \int dm,$ and so $z_{\text{CM}} = \frac{\int z\,dm}{\int dm}.$ Since the object is uniform, we can express the mass as the uniform density ρ times the volume, for any part of the cone. That results in the following.

$$z_{\text{CM}} = \frac{\int z\,dm}{\int dm} = \frac{\int z\,\rho\,dV}{\int \rho\,dV}$$

From the diagram, a disk of radius r and thickness dz has a volume of $dV = \pi r^2 dz.$ Finally, the geometry of the cone is such that $r/z = R/h,$ and so $r = zR/h.$ Combine these relationships and integrate over the z dimension to find the center of mass.

$$z_{\text{CM}} = \frac{\int z\,\rho\,dV}{\int \rho\,dV} = \frac{\rho\int z\pi r^2 dz}{\rho\int \pi r^2 dz} = \frac{\rho\pi\int z\left(zR/h\right)^2 dz}{\rho\pi\int \left(zR/h\right)^2 dz} = \frac{\rho\pi\left(R/h\right)^2\int z^3 dz}{\rho\pi\left(R/h\right)^2\int z^2 dz} = \frac{\int_0^h z^3 dz}{\int_0^h z^2 dz} = \frac{h^4/4}{h^3/3} = \tfrac{3}{4}h$$

Thus the center of mass is at $\boxed{\left(0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + \tfrac{3}{4}h\hat{\mathbf{k}}\right)}.$



70. Let the peak of the pyramid be directly above the origin, and the base edges of the pyramid be parallel to the x and y axes. From the symmetry of the pyramid, we know that $x_{\text{CM}} = y_{\text{CM}} = 0,$ and so the center of mass lies on the z axis. We have from Eq. 9-13 that $z_{\text{CM}} = \frac{1}{M}\int z\,dm.$ The mass can be expressed as $M = \int dm,$ and so $z_{\text{CM}} = \frac{\int z\,dm}{\int dm}.$ Since the object is uniform, we can express the mass as the uniform density ρ times the volume, for any part of the pyramid. That results in the following.

$$z_{\text{CM}} = \frac{\int z\,dm}{\int dm} = \frac{\int z\,\rho\,dV}{\int \rho\,dV}$$

From the diagram, for the differential volume we use a square disk of side ℓ and thickness dz, which has a volume of $dV = \ell^2 dz.$ The geometry of the pyramid is such that $\ell = \frac{s}{h}\left(h - z\right).$ That can be checked from the fact that ℓ is a linear function of z, $\ell = s$ for $z = 0,$ and $\ell = 0$ for $z = h.$ We can relate s to h by expressing the length of an edge in terms of the coordinates of the endpoints of an edge. One endpoint of each edge is at $\left(x = \pm s/2,\ y = \pm s/2,\ z = 0\right)$, and the other endpoint of each edge is at $\left(x = 0,\ y = 0,\ z = h\right)$. Using the Pythagorean theorem and knowing the edge length is s gives the following relationship.

$$s^2 = \left(s/2\right)^2 + \left(s/2\right)^2 + h^2 \quad \rightarrow \quad h = s/\sqrt{2}$$

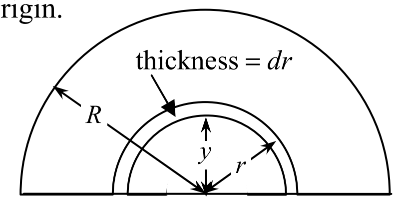
We combine these relationships and integrate over the z dimension to find the center of mass.

$$z_{\text{CM}} = \frac{\int z\,dm}{\int dm} = \frac{\int z\rho\,dV}{\int \rho\,dV} = \frac{\rho\int z\,\ell^2 dz}{\rho\int \ell^2 dz} = \frac{\rho\int_0^h z\left[\frac{s}{h}(h-z)\right]^2 dz}{\rho\int_0^h \left[\frac{s}{h}(h-z)\right]^2 dz} = \frac{\int_0^h z\left[(h-z)\right]^2 dz}{\int_0^h \left[(h-z)\right]^2 dz}$$

$$= \frac{\int_0^h \left(h^2 z - 2hz^2 + z^3\right)dz}{\int_0^h \left(h^2 - 2hz + z^2\right)dz} = \frac{\left(\frac{1}{2}h^2 z^2 - \frac{2}{3}hz^3 + \frac{1}{4}z^4\right)_0^h}{\left(h^2 z - hz^2 + \frac{1}{3}z^3\right)_0^h} = \frac{1}{4}h = \frac{1}{4}\frac{s}{\sqrt{2}} = \frac{s}{4\sqrt{2}}$$

Thus the center of mass is at $\boxed{\left(0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + \frac{s}{4\sqrt{2}}\hat{\mathbf{k}}\right)}$.

71. Let the radius of the semicircular plate be R, with the center at the origin. From the symmetry of the semicircle, we know that $x_{\text{CM}} = 0$, and so the center of mass lies on the y axis. We have from Eq. 9-13 that $y_{\text{CM}} = \frac{1}{M}\int y\,dm$. The mass can be expressed as $M = \int dm$, and so $y_{\text{CM}} = \frac{\int y\,dm}{\int dm}$. Since the object is uniform, we can express the mass as a uniform density σ times the area, for any part of the semicircle. That results in the following.



$$y_{\text{CM}} = \frac{\int y\,dm}{\int dm} = \frac{\int y\,\sigma\,dA}{\int \sigma\,dA}$$

From the diagram, for the differential area we use a semicircular strip of width *dr* and length πr, which has a differential area of $dA = \pi r dr$. And from problem 67, the y coordinate of the center of mass of that strip is $\frac{2r}{\pi}$. (Note the discussion immediately before Example 9-17 which mentions using the center of mass of individual objects to find the center of mass of an extended object.) We combine these relationships and integrate over the z dimension to find the center of mass.

$$y_{\text{CM}} = \frac{\int y\,dm}{\int dm} = \frac{\int y\,\sigma\,dA}{\int \sigma\,dA} = \frac{\sigma\int_0^R \frac{2r}{\pi}\pi r dr}{\sigma\int_0^R \pi r dr} = \frac{2\int_0^R r^2 dr}{\pi\int_0^R r dr} = \frac{\frac{2}{3}R^3}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}$$

Thus the center of mass is at $\boxed{\left(0\hat{\mathbf{i}} + \frac{4R}{3\pi}\hat{\mathbf{j}}\right)}$.

72. From Eq. 9-15, we see that $\vec{\mathbf{v}}_{\mathrm{CM}}=\frac{1}{M}\sum m_i\vec{\mathbf{v}}_i$.

$$\vec{\mathbf{v}}_{\mathrm{CM}}=\frac{(35\,\mathrm{kg})(12\hat{\mathbf{i}}-16\hat{\mathbf{j}})\,\mathrm{m/s}+(25\,\mathrm{kg})(-20\hat{\mathbf{i}}+14\hat{\mathbf{j}})\,\mathrm{m/s}}{(35\,\mathrm{kg}+25\,\mathrm{kg})}$$

$$=\frac{[(35)(12)-(25)(20)]\hat{\mathbf{i}}\,\mathrm{kg{\bullet}m/s}+[(35)(-12)+(25)(24)]\hat{\mathbf{j}}\,\mathrm{kg{\bullet}m/s}}{(60\,\mathrm{kg})}$$

$$=\frac{-80\hat{\mathbf{i}}\,\mathrm{kg{\bullet}m/s}-210\hat{\mathbf{j}}\,\mathrm{kg{\bullet}m/s}}{(60\,\mathrm{kg})}=\boxed{-1.3\hat{\mathbf{i}}\,\mathrm{m/s}-3.5\hat{\mathbf{j}}\,\mathrm{m/s}}$$

73. (*a*) Find the CM relative to the center of the Earth.

$$x_{CM}=\frac{m_Ex_E+m_Mx_M}{m_E+m_M}=\frac{(5.98\times10^{24}\,\mathrm{kg})(0)+(7.35\times10^{22}\,\mathrm{kg})(3.84\times10^{8}\,\mathrm{m})}{5.98\times10^{24}\,\mathrm{kg}+7.35\times10^{22}\,\mathrm{kg}}$$

$$=\boxed{4.66\times10^{6}\,\mathrm{m\ from\ the\ center\ of\ the\ Earth}}$$

This is actually inside the volume of the Earth, since $R_E=6.38\times10^{6}\,\mathrm{m}$.

(*b*) It is this Earth–Moon CM location that actually traces out the orbit as discussed in an earlier chapter. The Earth and Moon will orbit about this orbit path in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: i) the motion of the Moon about the Earth–Moon CM; and ii) the motion of the Earth–Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth–Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun). The Earth's motion would be similar, but since the center of mass of that Earth-Moon motion is inside the Earth, the Earth would be observed to "wobble" about that CM.

74. The point that will follow a parabolic trajectory is the center of mass of the mallet. Find the CM relative to the bottom of the mallet. Each part of the hammer (handle and head) can be treated as a point mass located at the CM of the respective piece. So the CM of the handle is 12.0 cm from the bottom of the handle, and the CM of the head is 28.0 cm from the bottom of the handle.

$$x_{\mathrm{CM}}=\frac{m_{\mathrm{handle}}x_{\mathrm{handle}}+m_{\mathrm{head}}x_{\mathrm{head}}}{m_{\mathrm{handle}}+m_{\mathrm{head}}}=\frac{(0.500\ \mathrm{kg})(12.0\ \mathrm{cm})+(2.80\ \mathrm{kg})(28.0\ \mathrm{cm})}{3.30\ \mathrm{kg}}=\boxed{25.6\,\mathrm{cm}}$$

Note that this is inside the head of the mallet. The mallet will rotate about this point as it flies through the air, giving it a wobbling kind of motion.

75. (*a*) Measure all distances from the original position of the woman.

$$x_{\mathrm{CM}}=\frac{m_{\mathrm{W}}x_{\mathrm{W}}+m_{\mathrm{M}}x_{\mathrm{M}}}{m_{\mathrm{W}}+m_{\mathrm{M}}}=\frac{(55\ \mathrm{kg})(0)+(72\ \mathrm{kg})(10.0\ \mathrm{m})}{127\ \mathrm{kg}}=5.669\,\mathrm{m}$$

$$\approx\boxed{5.7\,\mathrm{m\ from\ the\ woman}}$$

(*b*) Since there is no force external to the man–woman system, the CM will not move, relative to the original position of the woman. The woman's distance will no longer be 0, and the man's distance has changed to 7.5 m.

$$x_{\mathrm{CM}}=\frac{m_{\mathrm{W}}x_{\mathrm{W}}+m_{\mathrm{M}}x_{\mathrm{M}}}{m_{\mathrm{W}}+m_{\mathrm{M}}}=\frac{(55\ \mathrm{kg})x_{\mathrm{W}}+(72\ \mathrm{kg})(7.5\ \mathrm{m})}{127\ \mathrm{kg}}=5.669\ \mathrm{m}\ \rightarrow$$

$$x_{\mathrm{W}} = \frac{(5.669\ \mathrm{m})(127\ \mathrm{kg}) - (72\ \mathrm{kg})(7.5\ \mathrm{m})}{55\ \mathrm{kg}} = 3.272\ \mathrm{m}$$

$$x_{\mathrm{M}} - x_{\mathrm{W}} = 7.5\ \mathrm{m} - 3.272\ \mathrm{m} = 4.228\,\mathrm{m} \approx \boxed{4.2\ \mathrm{m}}$$

(*c*) When the man collides with the woman, he will be at the original location of the center of mass.

$$\underset{\text{final}}{x_{\mathrm{M}}} - \underset{\text{initial}}{x_{\mathrm{M}}} = 5.669\,\mathrm{m} - 10.0\ \mathrm{m} = -4.331\ \mathrm{m}$$

He has moved $\boxed{4.3\ \mathrm{m}}$ from his original position.

76. (*a*) As in Example 9-18, the CM of the system follows the parabolic trajectory. Part I will again fall vertically, the CM will "land" a distance *d* from part I (as in Fig. 9-32), and part II will land a distance *x* to the right of the CM. We measure horizontal distances from the point underneath the explosion.

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{I}}x_{\mathrm{I}} + m_{\mathrm{II}}x_{\mathrm{II}}}{m_{\mathrm{I}} + m_{\mathrm{II}}} \rightarrow x_{\mathrm{II}} = \frac{x_{\mathrm{CM}}(m_{\mathrm{I}} + m_{\mathrm{II}}) - m_{\mathrm{I}}x_{\mathrm{I}}}{m_{\mathrm{II}}} = \frac{d(m_{\mathrm{I}} + 3m_{\mathrm{I}}) - m_{\mathrm{I}}(0)}{3m_{\mathrm{I}}} = \tfrac{4}{3}d$$

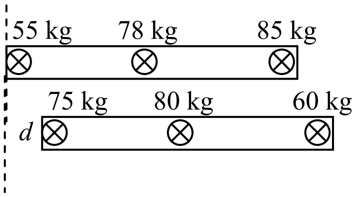
Therefore part II lands a total distance $\boxed{\tfrac{7}{3}d}$ from the starting point.

(*b*) Use a similar analysis for this case, but with $m_{\mathrm{I}} = 3m_{\mathrm{II}}$.

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{I}}x_{\mathrm{I}} + m_{\mathrm{II}}x_{\mathrm{II}}}{m_{\mathrm{I}} + m_{\mathrm{II}}} \rightarrow x_{\mathrm{II}} = \frac{x_{\mathrm{CM}}(m_{\mathrm{I}} + m_{\mathrm{II}}) - m_{\mathrm{I}}x_{\mathrm{I}}}{m_{\mathrm{II}}} = \frac{d(3m_{\mathrm{II}} + m_{\mathrm{II}}) - 3m_{\mathrm{II}}(0)}{m_{\mathrm{II}}} = 4d$$

Therefore part II lands a total distance $\boxed{5d}$ from the starting point.

77. Calculate the CM relative to the 55-kg person's seat, at one end of the boat. See the first diagram. Be sure to include the boat's mass.



$$x_{\mathrm{CM}} = \frac{m_{\mathrm{A}}x_{\mathrm{A}} + m_{\mathrm{B}}x_{\mathrm{B}} + m_{\mathrm{C}}x_{\mathrm{C}}}{m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}} = \frac{(55\,\mathrm{kg})(0) + (78\,\mathrm{kg})(1.5\ \mathrm{m}) + (85\,\mathrm{kg})(3.0\,\mathrm{m})}{218\,\mathrm{kg}} = 1.706\,\mathrm{m}$$

Now, when the passengers exchange positions, the boat will move some distance "*d*" as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{A}}x_{\mathrm{A}} + m_{\mathrm{B}}x_{\mathrm{B}} + m_{\mathrm{C}}x_{\mathrm{C}}}{m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}}$$

$$1.706\ \mathrm{m} = \frac{(85\ \mathrm{kg})(d) + (78\ \mathrm{kg})(1.5\ \mathrm{m} + d) + (55\ \mathrm{kg})(3.0\ \mathrm{m} + d)}{218\ \mathrm{kg}} = \frac{218d\ \mathrm{kg{\bullet}m} + 282\ \mathrm{kg{\bullet}m}}{218\ \mathrm{kg}}$$

$$d = 0.412\ \mathrm{m}$$

Thus the boat will move $\boxed{\text{0.41 m towards the initial position of the 85 kg person}}$.

78. Because the interaction between the worker and the flatcar is internal to the worker–flatcar system, their total momentum will be conserved, and the center of mass of the system will move with a constant velocity relative to the ground. The velocity of the center of mass is 6.0 m/s. Once the worker starts to move, the velocity of the flatcar relative to the ground will be taken as v_{car} and the velocity of the worker relative to the ground will then be $v_{\mathrm{car}} + 2.0\,\mathrm{m/s}$. Apply Eq. 9-15, in one dimension. Letter A represents the worker, and letter B represents the flatcar.

$$v_{\text{CM}} = \frac{m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \frac{m_{\text{A}}\left(v_{\text{car}} + 2.0\,\text{m/s}\right) + m_{\text{B}}v_{\text{car}}}{m_{\text{A}} + m_{\text{B}}} \rightarrow$$

$$v_{\text{car}} = v_{\text{CM}} - \frac{m_{\text{A}}}{\left(m_{\text{A}} + m_{\text{B}}\right)}\left(2.0\,\text{m/s}\right) = 6.0\,\text{m/s} - \frac{95\,\text{kg}}{375\,\text{kg}}\left(2.0\,\text{m/s}\right) = 5.493\,\text{m/s}$$

The flatcar moves this speed while the worker is walking. The worker walks 25 m along the flatcar at a relative speed of 2.0 m/s, and so he walks for 12.5 s.

$$\Delta x_{\text{car}} = v_{\text{car}}\Delta t = \left(5.493\,\text{m/s}\right)\left(12.5\,\text{s}\right) = 68.66\,\text{m} \approx \boxed{69\,\text{m}}$$

$\boxed{79.}$ Call the origin of coordinates the CM of the balloon, gondola, and person at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0. The man climbing the rope cannot change the total momentum of the system, and so the CM must stay at rest. Call the upward direction positive. Then the velocity of the man with respect to the balloon is $-v$. Call the velocity of the balloon with respect to the ground v_{BG}. Then the velocity of the man with respect to the ground is $v_{\text{MG}} = -v + v_{\text{BG}}$. Apply conservation of linear momentum in one dimension.

$$0 = mv_{\text{MG}} + Mv_{\text{BG}} = m\left(-v + v_{\text{BG}}\right) + Mv_{\text{BG}} \rightarrow \boxed{v_{\text{BG}} = v\frac{m}{m+M}\text{ , upward}}$$

If the passenger stops, $\boxed{\text{the balloon also stops}}$, and the CM of the system remains at rest.

80. Use Eq. 9-19a. Call upwards the positive direction. The external force is gravity, acting downwards. The exhaust is in the negative direction, and the rate of change of mass is negative.

$$\sum\vec{\mathbf{F}}_{\text{ext}} = M\frac{d\vec{\mathbf{v}}}{dt} - \vec{\mathbf{v}}_{\text{rel}}\frac{dM}{dt} \rightarrow -Mg = Ma + v_{\text{exhaust}}\frac{dM}{dt} \rightarrow$$

$$v_{\text{exhaust}} = \frac{-4.0Mg}{dM/dt} = \frac{-4.0\left(3500\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{-27\,\text{kg/s}} = \boxed{5100\,\text{m/s}}$$

81. The external force on the belt is the force supplied by the motor and the oppositely-directed force of friction. Use Eq. 9-19 in one dimension. The belt is to move at a constant speed, so the acceleration of the loaded belt is 0.

$$M\frac{dv}{dt} = F_{\text{ext}} + v_{\text{rel}}\frac{dM}{dt} \rightarrow M\left(0\right) = F_{\text{motor}} + F_{\text{friction}} + \left(-v\right)\frac{dM}{dt} \rightarrow$$

$$F_{\text{motor}} = \left(v\right)\frac{dM}{dt} - F_{\text{friction}} = \left(2.20\,\text{m/s}\right)\left(75.0\,\text{kg/s}\right) - \left(-150\text{N}\right) = 315\,\text{N}$$

The required power output from the motor is then found as the product of the force and the velocity.

$$P_{\text{motor}} = F_{\text{motor}}v = \left(315\,\text{N}\right)\left(2.20\,\text{m/s}\right) = 693\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = \boxed{0.93\,\text{hp}}$$

When the gravel drops from the conveyor belt, it is not accelerated in the horizontal direction by the belt and so has no further force interaction with the belt. The "new" gravel dropping on the belt must still be accelerated, so the power required is constant.

82. The thrust is, in general, given as $v_{\text{rel}}\dfrac{dM}{dt}$.

(*a*) The mass is ejected at a rate of 4.2 kg/s, with a relative speed of 550 m/s opposite to the direction of travel.

$$F_{\substack{\text{thrust}\\ \text{fuel}}} = v_{\text{rel}}\frac{dM_{\text{fuel}}}{dt} = (-550\,\text{m/s})(-4.2\,\text{kg/s}) = 2310\,\text{N} \approx \boxed{2300\,\text{N}}$$

(*b*) The mass is first added at a rate of 120 kg/s, with a relative speed of 270 m/s opposite to the direction of travel, and then ejected at a rate of 120 kg/s, with a relative speed of 550 m/s opposite to the direction of travel.

$$F_{\substack{\text{thrust}\\ \text{air}}} = v_{\text{rel}}\frac{dM_{\text{air}}}{dt} = (-270\,\text{m/s})(120\,\text{kg/s}) + (-550\,\text{m/s})(-120\,\text{kg/s}) = 33600\,\text{N}$$

$$\approx \boxed{3.4\times 10^{4}\,\text{N}}$$

(*c*) The power developed is the force of thrust times the velocity of the airplane.

$$P = \left(F_{\substack{\text{thrust}\\ \text{fuel}}} + F_{\substack{\text{thrust}\\ \text{air}}}\right)v = (2310\,\text{N} + 33600\,\text{N})(270\,\text{m/s}) = 9.696\times 10^{6}\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right)$$

$$= \boxed{1.3\times 10^{4}\,\text{hp}}$$

83. We apply Eq. 9-19b in one dimension, with "away" from the Earth as the positive direction, and "towards" the Earth as the negative direction. The external force is the force of gravity at that particular altitude, found from Eq. 6-1.

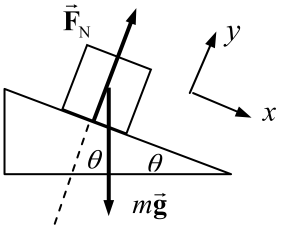
$$M\frac{dv}{dt} = F_{\text{ext}} + v_{\text{rel}}\frac{dM}{dt} \quad \rightarrow$$

$$\frac{dM}{dt} = \frac{1}{v_{\text{rel}}}\left(M\frac{dv}{dt} - F_{\text{ext}}\right) = \frac{1}{v_{\text{rel}}}\left(M\frac{dv}{dt} - -\frac{GM_{\text{Earth}}M}{r^2}\right)$$

$$= \frac{(25000\,\text{kg})}{(-1300\,\text{m/s})}\left[1.5\,\text{m/s}^2 + \frac{(6.67\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\,\text{kg})}{(6.38\times 10^{6}\,\text{m} + 6.4\times 10^{6}\,\text{m})^2}\right] = \boxed{-76\,\text{kg/s}}$$

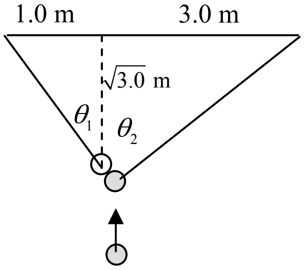
The negative sign means that the mass is being ejected rather than absorbed.

84. Because the sand is leaking out of the hole, rather than being pushed out the hole, there is no relative velocity of the leaking sand with respect to the sled (during the leaking process). Thus there is no "thrust" in this situation, and so the problem is the same as if there were no hole in the sled. From the free body diagram, we see that the acceleration down the plane will be $a = g\sin\theta$, as analyzed several times in Chapter 4. Use the constant acceleration relationships to find the time.



$$x = x_0 + v_{y0}t + \tfrac{1}{2}a_x t^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2(120\,\text{m})}{(9.80\,\text{m/s}^2)(\sin 32^\circ)}} = \boxed{6.8\,\text{s}}$$

$\boxed{85.}$ It is proven in the solution to problem 61 that in an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. For this specific circumstance, see the diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left' triangle: $\theta_1 = \tan^{-1}\frac{1.0}{\sqrt{3.0}} = 30^\circ$. Find θ_2 from



the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\dfrac{3.0}{\sqrt{3.0}} = 60°$. Since the balls will separate at a $90°$ angle, if the target ball goes in the pocket, this does appear to be a $\boxed{\text{good possibility of a scratch shot}}$.

86. The force stopping the wind is exerted by the person, so the force on the person would be equal in magnitude and opposite in direction to the force stopping the wind. Calculate the force from Eq. 9-2, in magnitude only.

$$\frac{m_{\text{wind}}}{\Delta t} = \frac{45\,\text{kg/s}}{\text{m}^2}(1.60\,\text{m})(0.50\,\text{m}) = 36\,\text{kg/s} \qquad \Delta v_{\text{wind}} = 120\,\text{km/h}\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 33.33\,\text{m/s}$$

$$F_{\text{on person}} = F_{\text{on wind}} = \frac{\Delta p_{\text{wind}}}{\Delta t} = \frac{m_{\text{wind}}\Delta v_{\text{wind}}}{\Delta t} = \frac{m_{\text{wind}}}{\Delta t}\Delta v_{\text{wind}} = (36\,\text{kg/s})(33.33\,\text{m/s})$$

$$= \boxed{1200\,\text{N}}$$

The typical maximum frictional force is $F_{\text{fr}} = \mu_s mg = (1.0)(75\,\text{kg})(9.80\,\text{m/s}^2) = 740\,\text{N}$, and so we see that $\boxed{F_{\text{on person}} > F_{\text{fr}}}$. The wind is literally strong enough to blow a person off his feet.

87. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $mgh = \frac{1}{2}mv^2$ for any particular bounce cycle, and so for an interaction with the floor, the ratio of the energies before and after the bounce is $\dfrac{K_{\text{after}}}{K_{\text{before}}} = \dfrac{mgh'}{mgh} = \dfrac{1.20\text{ m}}{1.50\text{ m}} = 0.80$. We assume that each bounce will further reduce the energy to 80% of its pre-bounce amount. The number of bounces to lose 90% of the energy can be expressed as follows.

$$(0.8)^n = 0.1 \;\rightarrow\; n = \frac{\log 0.1}{\log 0.8} = 10.3$$

Thus after $\boxed{\text{11 bounces}}$, more than 90% of the energy is lost.

As an alternate method, after each bounce, 80% of the available energy is left. So after 1 bounce, 80% of the original energy is left. After the second bounce, only 80% of 80%, or 64% of the available energy is left. After the third bounce, 51 %. After the fourth bounce, 41%. After the fifth bounce, 33 %. After the sixth bounce, 26%. After the seventh bounce, 21%. After the eight bounce, 17%. After the ninth bounce, 13%. After the tenth bounce, 11%. After the eleventh bounce, 9% is left. So again, it takes 11 bounces.

88. Since the collision is elastic, both momentum (in two dimensions) and kinetic energy are conserved. Write the three conservation equations and use them to solve for the desired quantities. The positive *x* direction in the diagram is taken to the right, and the positive *y* direction is taken towards the top of the picture.

$$p_{\underset{\text{initial}}{x}} = p_{\underset{\text{final}}{x}} \;\rightarrow\; 0 = mv_{\text{pin}}\sin 75° - Mv_{\text{ball}}\sin\theta \;\rightarrow\; v_{\text{pin}}\sin 75° = 5v_{\text{ball}}\sin\theta$$

$$p_{\underset{\text{initial}}{y}} = p_{\underset{\text{final}}{y}} \;\rightarrow\; M(13.0\,\text{m/s}) = mv_{\text{pin}}\cos 75° + Mv_{\text{ball}}\cos\theta \;\rightarrow$$

$$65.0\,\text{m/s} - v_{\text{pin}}\cos 75° = 5v_{\text{ball}}\cos\theta$$

$$K_{\text{initial}} = K_{\text{final}} \rightarrow \tfrac{1}{2}M\left(13.0\,\text{m/s}\right)^2 = \tfrac{1}{2}mv_{\text{pin}}^2 + \tfrac{1}{2}Mv_{\text{ball}}^2 \rightarrow 845\,\text{m}^2/\text{s}^2 = v_{\text{pin}}^2 + 5v_{\text{ball}}^2 \rightarrow$$
$$845\,\text{m}^2/\text{s}^2 - v_{\text{pin}}^2 = 5v_{\text{ball}}^2$$

Square the two momentum equations and add them to eliminate the dependence on θ.

$$v_{\text{pin}}^2\sin^2 75^\circ = 25v_{\text{ball}}^2\sin^2\theta \;;\; \left(65.0\right)^2 - 2\left(65.0\right)v_{\text{pin}}\cos 75^\circ + v_{\text{pin}}^2\cos^2 75^\circ = 25v_{\text{ball}}^2\cos^2\theta \rightarrow$$
$$v_{\text{pin}}^2\sin^2 75^\circ + \left(65.0\right)^2 - 2\left(65.0\right)v_{\text{pin}}\cos 75^\circ + v_{\text{pin}}^2\cos^2 75^\circ = 25v_{\text{ball}}^2\sin^2\theta + 25v_{\text{ball}}^2\cos^2\theta + \rightarrow$$
$$\left(65.0\right)^2 - 130v_{\text{pin}}\cos 75^\circ + v_{\text{pin}}^2 = 25v_{\text{ball}}^2 = 5\left(5v_{\text{ball}}^2\right)$$

Substitute from the kinetic energy equation.

$$\left(65.0\right)^2 - 130v_{\text{pin}}\cos 75^\circ + v_{\text{pin}}^2 = 5\left(845 - v_{\text{pin}}^2\right) \rightarrow 4225 - 130v_{\text{pin}}\cos 75^\circ + v_{\text{pin}}^2 = 4225 - 5v_{\text{pin}}^2$$
$$6v_{\text{pin}}^2 = 130v_{\text{pin}}\cos 75^\circ \rightarrow v_{\text{pin}} = 5.608\,\text{m/s}$$
$$845 - v_{\text{pin}}^2 = 5v_{\text{ball}}^2 \rightarrow v_{\text{ball}} = \sqrt{\tfrac{1}{5}\left(845 - v_{\text{pin}}^2\right)} = \sqrt{\tfrac{1}{5}\left(845 - \left(5.608\right)^2\right)} = 12.756\,\text{m/s}$$
$$v_{\text{pin}}\sin 75^\circ = 5v_{\text{ball}}\sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{v_{\text{pin}}\sin 75^\circ}{5v_{\text{ball}}}\right) = \sin^{-1}\left(\frac{\left(5.608\right)\sin 75^\circ}{5\left(12.756\right)}\right) = 4.87^\circ$$

So the final answers are as follows.

(*a*) $v_{\text{pin}} = 5.608\,\text{m/s} \approx \boxed{5.6\,\text{m/s}}$

(*b*) $v_{\text{ball}} = 12.756\,\text{m/s} \approx \boxed{13\,\text{m/s}}$

(*c*) $\theta = 4.87^\circ \approx \boxed{4.9^\circ}$

89. This is a ballistic "pendulum" of sorts, similar to Example 9-11 in the textbook. There is no difference in the fact that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$v = \frac{m+M}{m}\sqrt{2gh} \rightarrow$$
$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2\left(9.80\,\text{m/s}^2\right)}\left(\frac{\left(0.0240\,\text{kg}\right)\left(310\,\text{m/s}\right)}{0.0240\,\text{kg} + 1.40\text{ kg}}\right)^2 = \boxed{1.4\,\text{m}}$$

90. The initial momentum is 0, and the net external force on the puck is 0. Thus momentum will be conserved in two dimensions.

$$\vec{\mathbf{p}}_{\text{initial}} = \vec{\mathbf{p}}_{\text{initial}} \rightarrow 0 = mv\hat{i} + 2m\left(2v\right)\hat{\mathbf{j}} + m\vec{\mathbf{v}}_3 \rightarrow \vec{\mathbf{v}}_3 = \boxed{-v\hat{i} - 4v\hat{\mathbf{j}}}$$
$$v_3 = \sqrt{\left(-v\right)^2 + \left(-4v\right)^2} = \boxed{\sqrt{17}v} \qquad \theta_3 = \tan^{-1}\frac{-4v}{-v} = \boxed{256^\circ}$$

91. The fraction of energy transformed is $\dfrac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}}$.

$$\frac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}} = \frac{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2 - \frac{1}{2}\left(m_{\text{A}} + m_{\text{B}}\right)v'^2}{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2} = \frac{m_{\text{A}}v_{\text{A}}^2 - \left(m_{\text{A}} + m_{\text{B}}\right)\left(\dfrac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}}}\right)^2 v_{\text{A}}^2}{m_{\text{A}}v_{\text{A}}^2}$$

$$= 1 - \frac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}}} = \frac{m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \boxed{\frac{1}{2}}$$

92. Momentum will be conserved in the horizontal direction. Let A represent the railroad car, and B represent the snow. For the horizontal motion, $v_B = 0$ and $v'_B = v'_A$. Momentum conservation in the horizontal direction gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}v_{\text{A}} = \left(mt + mt\right)v'_{\text{A}}$$

$$v'_{\text{A}} = \frac{m_{\text{A}}v_{\text{A}}}{m_{\text{A}} + m_{\text{B}}} = \frac{\left(4800\,\text{kg}\right)\left(8.60\,\text{m/s}\right)}{4800\,\text{kg} + \left(\dfrac{3.80\text{ kg}}{\text{min}}\right)\left(60.0\,\text{min}\right)} = 8.210\,\text{m/s} \approx \boxed{8.2\,\text{m/s}}$$

93. (*a*) We consider only the horizontal direction (the direction of motion of the railroad car). There is no external force in the horizontal direction. In Eq. 9-19b, the relative velocity (in the horizontal direction) of the added mass is the opposite of the horizontal velocity of the moving mass, since the added mass is moving straight down.

$$M\frac{dv}{dt} = F_{\text{ext}} + v_{\text{rel}}\frac{dM}{dt} \quad \rightarrow \quad M\frac{dv}{dt} = -v\frac{dM}{dt} \quad \rightarrow \quad \frac{dv}{v} = -\frac{dM}{M} \quad \rightarrow \quad \int_{v_0}^{v_f}\frac{dv}{v} = -\int_{M_0}^{M_f}\frac{dM}{M} \quad \rightarrow$$

$$\ln\frac{v_f}{v_0} = -\ln\frac{M_f}{M_0} = \ln\frac{M_0}{M_{\text{f}}} \quad \rightarrow$$

$$v_{\text{f}} = v_0\frac{M_0}{M_f} = \boxed{v_0\frac{M_0}{M_0 + \dfrac{dM}{dt}t}}$$

(*b*) Evaluate the speed at $t = 60.0\,\text{min}$.

$$v\left(t = 60.0\right) = v_0\frac{M_0}{M_0 + \dfrac{dM}{dt}t} = \frac{4800\,\text{kg}}{4800\,\text{kg} + \left(3.80\,\text{kg/min}\right)\left(60.0\,\text{min}\right)} = \boxed{8.2\,\text{m/s}}$$

This agrees with the previous problem.

94. (*a*) No, there is no net external force on the system. In particular, the spring force is internal to the system.

(*b*) Use conservation of momentum to determine the ratio of speeds. Note that the two masses will be moving in opposite directions. The initial momentum, when the masses are released, is 0.

$$p_{\text{initial}} = p_{\text{later}} \quad \rightarrow \quad 0 = m_{\text{A}}v_{\text{A}} - m_{\text{B}}v_{\text{B}} \quad \rightarrow \quad v_{\text{A}}/v_{\text{B}} = \boxed{m_{\text{B}}/m_{\text{A}}}$$

(*c*) $$\frac{K_{\text{A}}}{K_{\text{B}}} = \frac{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2}{\frac{1}{2}m_{\text{B}}v_{\text{B}}^2} = \frac{m_{\text{A}}}{m_{\text{B}}}\left(\frac{v_{\text{A}}}{v_{\text{B}}}\right)^2 = \frac{m_{\text{A}}}{m_{\text{B}}}\left(\frac{m_{\text{B}}}{m_{\text{A}}}\right)^2 = \boxed{m_{\text{B}}/m_{\text{A}}}$$

(*d*) The center of mass was initially at rest. Since there is no net external force on the system, the center of mass does not move, and so stays at rest.

(*e*) With friction present, there could be a net external force on the system, because the forces of friction on the two masses would not necessarily be equal in magnitude. If the two friction forces are not equal in magnitude, the ratios found above would not be valid. Likewise, the center of mass would not necessarily be at rest with friction present.

95. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows.

$$W_{\text{fr}} = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^\circ\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \quad \rightarrow$$

$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right)$$

For post-collision sliding, $v_f = 0$ and v_i is the speed immediately after the collision, v'. Use this relationship to find the speed of each car immediately after the collision.

$$\text{Car A:} \quad -\mu_k g\Delta x'_{\text{A}} = -\tfrac{1}{2}v'^2_{\text{A}} \;\rightarrow\; v'_{\text{A}} = \sqrt{2\mu_k g\Delta x'_{\text{A}}} = \sqrt{2(0.60)\left(9.80\,\text{m/s}^2\right)(18\text{ m})} = 14.55\,\text{m/s}$$

$$\text{Car B:} \quad -\mu_k g\Delta x'_{\text{B}} = -\tfrac{1}{2}v'^2_{\text{B}} \;\rightarrow\; v'_{\text{B}} = \sqrt{2\mu_k g\Delta x'_{\text{B}}} = \sqrt{2(0.60)\left(9.80\,\text{m/s}^2\right)(30\text{ m})} = 18.78\,\text{m/s}$$

During the collision, momentum is conserved in one dimension. Note that $v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; m_{\text{A}}v_{\text{A}} = m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}$$

$$v_A = \frac{m_{\text{A}}v'_{\text{A}} + m_{\text{B}}v'_{\text{B}}}{m_{\text{A}}} = \frac{(1500\,\text{kg})(14.55\,\text{m/s}) + (1100\,\text{kg})(18.78\,\text{m/s})}{1500\,\text{kg}} = 28.32\,\text{m/s}$$

For pre-collision sliding, again apply the friction–energy relationship, with $v_f = v_{\text{A}}$ and v_i is the speed when the brakes were first applied.

$$-\mu_k g\Delta x_{\text{A}} = \tfrac{1}{2}\left(v_{\text{A}}^2 - v_i^2\right) \;\rightarrow\; v_i = \sqrt{v_A^2 + 2\mu_k g\Delta x_A} = \sqrt{(28.32\,\text{m/s})^2 + 2(0.60)\left(9.80\,\text{m/s}^2\right)(15\,\text{m})}$$

$$= 31.23\,\text{m/s}\left(\frac{1\,\text{mi/h}}{0.447\,\text{m/s}}\right) = \boxed{70\,\text{mi/h}}$$

This is definitely over the speed limit.

96. (*a*) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, and so $v_{\text{A}} = 0$. Write momentum conservation for the collision.

$$m_{\text{B}}v_{\text{B}} = \left(m_{\text{A}} + m_{\text{B}}\right)v' \;\rightarrow$$

$$v' = v_{\text{B}}\frac{m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \left(2.5\times10^4\text{ m/s}\right)\frac{2.0\times10^8\,\text{kg}}{6.0\times10^{24}\,\text{kg} + 2.0\times10^8\,\text{kg}} = \boxed{8.3\times10^{-13}\text{ m/s}}$$

This is so small as to be considered 0.

(*b*) The fraction of the meteor's kinetic energy transferred to the Earth is the final kinetic energy of the Earth divided by the initial kinetic energy of the meteor.

$$\frac{K_{\substack{\text{final}\\\text{Earth}}}}{K_{\substack{\text{initial}\\\text{meteor}}}} = \frac{\tfrac{1}{2}m_{\text{A}}v'^2}{\tfrac{1}{2}m_{\text{B}}v_{\text{B}}^2} = \frac{\tfrac{1}{2}\left(6.0\times10^{24}\,\text{kg}\right)\left(8.3\times10^{-13}\text{ m/s}\right)^2}{\tfrac{1}{2}\left(2.0\times10^8\,\text{kg}\right)\left(2.5\times10^4\text{ m/s}\right)^2} = \boxed{3.3\times10^{-17}}$$

(*c*) The Earth's change in kinetic energy can be calculated directly.

$$\Delta K_{\text{Earth}} = K_{\substack{\text{final}\\\text{Earth}}} - K_{\substack{\text{initial}\\\text{Earth}}} = \tfrac{1}{2}m_{\text{A}}v'^2 - 0 = \tfrac{1}{2}\left(6.0\times10^{24}\,\text{kg}\right)\left(8.3\times10^{-13}\,\text{m/s}\right)^2 = \boxed{2.1\text{ J}}$$

$\boxed{97.}$ Since the only forces on the astronauts are internal to the 2-astronaut system, their CM will not change. Call the CM location the origin of coordinates. That is also the original location of the two astronauts.

$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} \quad\rightarrow\quad 0 = \frac{(60\text{ kg})(12\text{ m}) + (80\text{ kg})x_{\text{B}}}{140\text{ kg}} \quad\rightarrow\quad x = -9\text{ m}$$

Their distance apart is $x_{\text{A}} - x_{\text{B}} = 12\,\text{m} - (-9\,\text{m}) = \boxed{21\,\text{m}}$.

98. This is a ballistic "pendulum" of sorts, similar to Example 9-11 in the textbook. The mass of the bullet is *m*, and the mass of the block of wood is *M*. The speed of the bullet before the collision is v, and the speed of the combination after the collision is v'. Momentum is conserved in the totally inelastic collision, and so $mv = (m+M)v'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}} = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)_{\substack{\text{after}\\\text{collision}}} \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos180^\circ\theta = -\mu_k F_{\text{N}}\Delta x = -\mu_k mg\Delta x \quad\rightarrow$$

$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}v'^2 \quad\rightarrow\quad v' = \sqrt{2\mu_k g\Delta x}$$

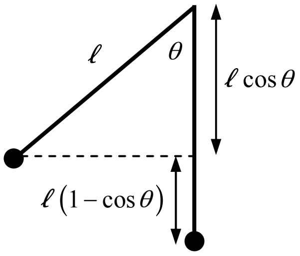
Use this expression for v' in the momentum conservation equation in one dimension in order to solve for v.

$$mv = (m+M)v' = (m+M)\sqrt{2\mu_k g\Delta x} \quad\rightarrow$$

$$v = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g\Delta x} = \left(\frac{0.022\text{ kg} + 1.35\text{ kg}}{0.022\text{ kg}}\right)\sqrt{2(0.28)\left(9.80\,\text{m/s}^2\right)(8.5\text{ m})}$$

$$= \boxed{4.3\times10^2\text{ m/s}}$$

99. (*a*) Conservation of mechanical energy can be used to find the velocity of the lighter ball before impact. The potential energy of the ball at the highest point is equal to the kinetic energy of the ball just before impact. Take the lowest point in the swing as the zero location for gravitational potential energy.



$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad m_{\text{A}}g\ell(1-\cos\theta) = \tfrac{1}{2}m_{\text{A}}v_{\text{A}}^2 \quad\rightarrow$$

$$v_{\text{A}} = \sqrt{2g\ell(1-\cos\theta)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(0.30\,\text{m})(1-\cos66^\circ)}$$

$$= 1.868\,\text{m/s} \approx \boxed{1.9\,\text{m/s}}$$

(*b*) This is an elastic collision with a stationary target. Accordingly, the relationships developed in Example 9-8 are applicable.

$$v'_{\text{A}} = v_{\text{A}}\left(\frac{m_{\text{A}} - m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}}\right) = (1.868\,\text{m/s})\left(\frac{0.045\,\text{kg} - 0.065\,\text{kg}}{0.045\,\text{kg} + 0.065\,\text{kg}}\right) = -0.3396\,\text{m/s} = \boxed{-0.34\,\text{m/s}}$$

$$v'_{\text{B}} = v_{\text{A}}\left(\frac{2m_{\text{A}}}{m_{\text{A}} + m_{\text{B}}}\right) = (1.868\,\text{m/s})\left(\frac{2(0.045\,\text{kg})}{0.045\,\text{kg} + 0.065\,\text{kg}}\right) = 1.528\,\text{m/s} = \boxed{1.5\,\text{m/s}}$$

(*c*) We can again use conservation of energy for each ball after the collision. The kinetic energy of each ball immediately after the collision will become gravitational potential energy as each ball rises.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \tfrac{1}{2}mv^2 = mgh \rightarrow h = \frac{v^2}{2g}$$

$$h_{\text{A}} = \frac{v_{\text{A}}^2}{2g} = \frac{(-0.3396\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \boxed{5.9\times10^{-3}\,\text{m}} \;;\; h_{\text{B}} = \frac{v_{\text{B}}^2}{2g} = \frac{(1.528\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \boxed{0.12\,\text{m}}$$

100. (*a*) Use conservation of energy to find the speed of mass *m* before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$mgh_{\text{A}} = \tfrac{1}{2}mv_{\text{A}}^2 \rightarrow v_{\text{A}} = \sqrt{2gh_{\text{A}}} = \sqrt{2(9.80\,\text{m/s}^2)(3.60\text{ m})} = 8.40\,\text{m/s}$$

Use Eq. 9-8 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \rightarrow v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}}$$

Apply momentum conservation for the collision, and substitute the result from Eq. 9-8.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = mv'_{\text{A}} + M\left(v_{\text{A}} + v'_{\text{A}}\right) \rightarrow$$

$$v'_{\text{A}} = \frac{m-M}{m+M}v_{\text{A}} = \left(\frac{2.20\text{ kg} - 7.00\text{ kg}}{9.20\text{ kg}}\right)(8.4\,\text{m/s}) = -4.38\,\text{m/s} \approx \boxed{-4.4\,\text{m/s}}$$

$$v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}} = -4.4\,\text{m/s} + 8.4\,\text{m/s} = \boxed{4.0\,\text{m/s}}$$

(*b*) Again use energy conservation to find the height to which mass *m* rises after the collision. The kinetic energy of *m* immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$\tfrac{1}{2}mv_{\text{A}}'^2 = mgh'_{\text{A}} \rightarrow h'_{\text{A}} = \frac{v_{\text{A}}'^2}{2g}$$

$$d'_{\text{A}} = \frac{h'_{\text{A}}}{\sin 30^\circ} = \frac{v_{\text{A}}'^2}{2g\sin 30^\circ} = \frac{(-4.38\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)g\sin 30^\circ} = 1.96\text{ m} \approx \boxed{2.0\text{ m}}$$

101. Let A represent mass *m* and B represent mass *M*. Use Eq. 9-8 to obtain a relationship between the velocities, noting that $v_{\text{B}} = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \rightarrow v'_{\text{A}} = v'_{\text{B}} - v_{\text{A}}.$$

After the collision, $v'_{\text{A}} < 0$ since *m* is moving in the negative direction. For there to be a second collision, then after *m* moves up the ramp and comes back down, with a positive velocity at the bottom of the incline of $-v'_{\text{A}}$, the speed of *m* must be greater than the speed of *M* so that *m* can catch *M*. Thus $-v'_{\text{A}} > v'_{\text{B}}$, or $v'_{\text{A}} < -v'_{\text{B}}$. Substitute the result from Eq. 9-8 into the inequality.

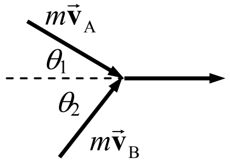
$$v'_{\text{B}} - v_{\text{A}} < -v'_{\text{B}} \rightarrow v'_{\text{B}} < \tfrac{1}{2}v_{\text{A}}$$

Now write momentum conservation for the original collision, and substitute the result from Eq. 9-8.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = m\left(v'_{\text{B}} - v_{\text{A}}\right) + Mv'_{\text{B}} \rightarrow v'_{\text{B}} = \frac{2m}{m+M}v_{\text{A}}$$

Finally, combine the above result with the inequality from above.

$$\frac{2m}{m+M}v_{\text{A}} < \tfrac{1}{2}v_{\text{A}} \rightarrow 4m < m+M \rightarrow \boxed{m < \tfrac{1}{3}M = 2.33\,\text{kg}}$$



102. Call the final direction of the joined objects the positive *x* axis. A diagram of the collision is shown. Momentum will be conserved in both the *x* and *y* directions. Note that $v_A = v_B = v$ and $v' = v/3$.

$$p_y: \quad -mv\sin\theta_1 + mv\sin\theta_2 = 0 \quad\rightarrow\quad \sin\theta_1 = \sin\theta_2 \quad\rightarrow\quad \theta_1 = \theta_2$$

$$p_x: \quad mv\cos\theta_1 + mv\cos\theta_2 = (2m)(v/3) \quad\rightarrow\quad \cos\theta_1 + \cos\theta_2 = \tfrac{2}{3}$$

$$\cos\theta_1 + \cos\theta_2 = 2\cos\theta_1 = \tfrac{2}{3} \quad\rightarrow\quad \theta_1 = \cos^{-1}\tfrac{1}{3} = 70.5^\circ = \theta_2$$

$$\theta_1 + \theta_2 = \boxed{141^\circ}$$

$\boxed{103.}$ The original horizontal distance can be found from the range formula from Example 3-10.

$$R = v_0^2\sin 2\theta_0/g = (25\,\text{m/s})^2(\sin 56^\circ)/(9.8\,\text{m/s}^2) = 52.87\,\text{m}$$

The height at which the objects collide can be found from Eq. 2-12c for the vertical motion, with $v_y = 0$ at the top of the path. Take up to be positive.

$$v_y^2 = v_{y0}^2 + 2a(y - y_0) \quad\rightarrow\quad (y - y_0) = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0 - [(25\,\text{m/s})\sin 28^\circ]^2}{2(-9.80\,\text{m/s}^2)} = 7.028\,\text{m}$$

Let *m* represent the bullet and *M* the skeet. When the objects collide, the skeet is moving horizontally at $v_0\cos\theta = (25\,\text{m/s})\cos 28^\circ = 22.07\,\text{m/s} = v_x$, and the bullet is moving vertically at $v_y = 230\,\text{m/s}$. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$p_x: \quad Mv_x = (M+m)v_x' \quad\rightarrow\quad v_x' = \frac{Mv_x}{M+m} = \frac{(0.25\text{ kg})(22.07\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 20.82\,\text{m/s}$$

$$p_y: \quad mv_y = (M+m)v_y' \quad\rightarrow\quad v_y' = \frac{mv_y}{M+m} = \frac{(0.015\text{ kg})(230\,\text{m/s})}{(0.25+0.015)\,\text{kg}} = 13.02\,\text{m/s}$$

(*a*) The speed v_y' can be used as the starting vertical speed in Eq. 2-12c to find the height that the skeet–bullet combination rises above the point of collision.

$$v_y^2 = v_{y0}^2 + 2a(y - y_0)_{\text{extra}} \quad\rightarrow$$

$$(y - y_0)_{\text{extra}} = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0 - (13.02\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 8.649\text{m} \approx \boxed{8.6\,\text{m}}$$

(*b*) From Eq. 2-12b applied to the vertical motion after the collision, we can find the time for the skeet–bullet combination to reach the ground.

$$y = y_0 + v_y't + \tfrac{1}{2}at^2 \quad\rightarrow\quad 0 = 8.649\,\text{m} + (13.02\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2 \quad\rightarrow$$

$$4.9t^2 - 13.02t - 8.649 = 0 \quad\rightarrow\quad t = 3.207\,\text{s}\,,\ -0.550\,\text{s}$$

The positive time root is used to find the horizontal distance traveled by the combination after the collision.

$$x_{\text{after}} = v_x't = (20.82\,\text{m/s})(3.207\,\text{s}) = 66.77\,\text{m}$$

If the collision would not have happened, the skeet would have gone $\tfrac{1}{2}R$ horizontally from this point.

$$\Delta x = x_{\text{after}} - \tfrac{1}{2}R = 66.77\,\text{m} - \tfrac{1}{2}(52.87\,\text{m}) = 40.33\,\text{m} \approx \boxed{40\,\text{m}}$$

Note that the answer is correct to 2 significant figures.

104. In this interaction, energy is conserved (initial potential energy of mass - compressed spring system = final kinetic energy of moving blocks) and momentum is conserved, since the net external force is 0. Use these two relationships to find the final speeds.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = mv_m - 3mv_{3m} \rightarrow v_m = 3v_{3m}$$

$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\substack{\text{spring}\\\text{initial}}} = K_{\text{final}} \rightarrow \tfrac{1}{2}kD^2 = \tfrac{1}{2}mv_m^2 + \tfrac{1}{2}3mv_{3m}^2 = \tfrac{1}{2}m\left(3v_{3m}\right)^2 + \tfrac{1}{2}3mv_{3m}^2 = 6mv_{3m}^2$$

$$\tfrac{1}{2}kD^2 = 6mv_{3m}^2 \rightarrow \boxed{v_{3m} = D\sqrt{\frac{k}{12m}} \;;\; v_m = 3D\sqrt{\frac{k}{12m}}}$$

105. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a 1-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 9-8, with $v_A = 10.4\,\text{km/s}$ and $v_B = v_B' = -9.6\,\text{km/s}$.

$$v_A - v_B = -v_A' + v_B' \rightarrow v_A' = 2v_B - v_A = 2\left(-9.6\,\text{km/s}\right) - 10.4\,\text{km/s} = \boxed{-29.6\,\text{km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed, and it reverses direction.

106. Let the original direction of the cars be the positive direction. We have $v_A = 4.50\,\text{m/s}$ and $v_B = 3.70\,\text{m/s}$.

(*a*) Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v_A' - v_B'\right) \rightarrow v_B' = v_A - v_B + v_A' = 0.80\,\text{m/s} + v_A'$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \rightarrow m_A v_A + m_B v_B = m_A v_A' + m_B\left(0.80\,\text{m/s} + v_A'\right) \rightarrow$$

$$v_A' = \frac{m_A v_A + m_B\left(v_B - 0.80\,\text{m/s}\right)}{m_A + m_B} = \frac{\left(450\,\text{kg}\right)\left(4.50\,\text{m/s}\right) + \left(490\,\text{kg}\right)\left(2.90\,\text{m/s}\right)}{940\,\text{kg}} = 3.666\,\text{m/s}$$

$$\approx \boxed{3.67\,\text{m/s}} \;;\; v_B' = 0.80\,\text{m/s} + v_A' = 4.466\,\text{m/s} \approx \boxed{4.47\,\text{m/s}}$$

(*b*) Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_A = m_A v_A' - m_A v_A = \left(450\,\text{kg}\right)\left(3.666\,\text{m/s} - 4.50\,\text{m/s}\right) = -3.753\times10^2\,\text{kg}\bullet\text{m/s}$$

$$\approx \boxed{-380\,\text{kg}\bullet\text{m/s}}$$

$$\Delta p_B = m_B v_B' - m_B v_B = \left(490\,\text{kg}\right)\left(4.466\,\text{m/s} - 3.70\,\text{m/s}\right) = 3.753\times10^2\,\text{kg}\bullet\text{m/s}$$

$$\approx \boxed{380\,\text{kg}\bullet\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved.

107. Let A represent the cube of mass *M* and B represent the cube of mass *m*. Find the speed of A immediately before the collision, v_A, by using energy conservation.

$$Mgh = \tfrac{1}{2}Mv_A^2 \rightarrow v_A = \sqrt{2gh} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.35\text{ m}\right)} = 2.619\,\text{m/s}$$

Use Eq. 9-8 for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_B = 0$ and $M = 2m$.

$$v_A - v_B = -\left(v_A' - v_B'\right) \rightarrow v_B' = v_A + v_A'$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \;\rightarrow\; m_A v_A = m_A v_A' + m_B\left(v_A + v_A'\right) \;\rightarrow$$

$$2mv_A = 2mv_A' + m\left(v_A + v_A'\right) \;\rightarrow\; v_A' = \frac{v_A}{3} = \frac{\sqrt{2gh}}{3} = \frac{\sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.35\,\text{m}\right)}}{3} = 0.873\,\text{m/s}$$

$$v_B' = v_A + v_A' = \tfrac{4}{3}v_A = 3.492\,\text{m/s}$$

Each mass is moving horizontally initially after the collision, and so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. 2-12b with down as positive and the table top as the vertical origin to find the time of fall.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \;\rightarrow\; H = 0 + 0 + \tfrac{1}{2}gt^2 \;\rightarrow\; t = \sqrt{2H/g}$$

Each cube then travels a horizontal distance found by $\Delta x = v_x \Delta t$.

$$\Delta x_m = v_A'\Delta t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{2}{3}\sqrt{hH} = \tfrac{2}{3}\sqrt{\left(0.35\,\text{m}\right)\left(0.95\,\text{m}\right)} = 0.3844\,\text{m} \approx \boxed{0.38\,\text{m}}$$

$$\Delta x_M = v_B'\Delta t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{8}{3}\sqrt{hH} = \tfrac{8}{3}\sqrt{\left(0.35\,\text{m}\right)\left(0.95\,\text{m}\right)} = 1.538\,\text{m} \approx \boxed{1.5\,\text{m}}$$

108. (*a*) Momentum is conserved in the *z* direction. The initial *z*-momentum is 0.

$$\underset{\text{before}}{p_z} = \underset{\text{after}}{p_z} \;\rightarrow\; 0 = m_{\text{satellite}} v_{z\ \text{satellite}} + m_{\text{shuttle}} v_{z\ \text{shuttle}} \;\rightarrow$$

$$v_{z\ \text{shuttle}} = -\frac{m_{\text{satellite}} v_{z\ \text{satellite}}}{m_{\text{shuttle}}} = -\frac{850\,\text{kg}}{92{,}000\,\text{kg}}\left(0.30\,\text{m/s}\right) = -2.8\times10^{-3}\,\text{m/s}$$

And so the component in the minus *z* direction is $\boxed{2.8\times10^{-3}\,\text{m/s}}$.

(*b*) The average force is the change in momentum per unit time. The force on the satellite is in the positive *z* direction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{\left(850\,\text{kg}\right)\left(0.30\,\text{m/s}\right)}{4.0\,\text{s}} = \boxed{64\,\text{N}}$$

$\boxed{109.}$ (*a*) The average force is the momentum change divided by the elapsed time.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{\left(1500\,\text{kg}\right)\left(0 - 45\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)}{0.15\,\text{s}} = -1.25\times10^{5}\,\text{N} \approx \boxed{-1.3\times10^{5}\,\text{N}}$$

The negative sign indicates direction – that the force is in the opposite direction to the original direction of motion.

(*b*) Use Newton’s second law.

$$F_{\text{avg}} = ma_{\text{avg}} \;\rightarrow\; a_{\text{avg}} = \frac{F_{\text{avg}}}{m} = \frac{-1.25\times10^{5}\,\text{N}}{1500\,\text{kg}} = -83.33\,\text{m/s}^2 \approx \boxed{-83\,\text{m/s}^2}$$

110. (*a*) In the reference frame of the Earth, the final speed of the Earth–asteroid system is essentially 0, because the mass of the Earth is so much greater than the mass of the asteroid. It is like throwing a ball of mud at the wall of a large building – the smaller mass stops, and the larger mass doesn’t move appreciably. Thus all of the asteroid’s original kinetic energy can be released as destructive energy.

$$K_{\text{orig}} = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}\left[\left(3200\,\text{kg/m}^3\right)\tfrac{4}{3}\pi\left(1.0\times10^3\,\text{m}\right)^3\right]\left(1.5\times10^4\,\text{m/s}\right)^2 = 1.507\times10^{21}\,\text{J}$$

$$\approx \boxed{1.5\times 10^{21}\,\text{J}}$$

(*b*) $1.507\times 10^{21}\,\text{J}\left(\dfrac{1\,\text{bomb}}{4.0\times 10^{16}\,\text{J}}\right) = \boxed{38,000\,\text{bombs}}$

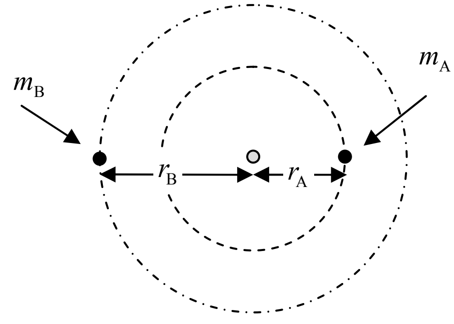
111. We apply Eq. 9-19b, with no external forces. We also assume that the motion is all in one dimension.

$$M\frac{d\vec{\mathbf{v}}}{dt} = \vec{\mathbf{v}}_{\text{rel}}\frac{dM}{dt} \;\rightarrow\; Mdv = v_{\text{rel}}dM \;\rightarrow\; \frac{1}{v_{\text{rel}}}dv = \frac{1}{M}dM \;\rightarrow$$

$$\frac{1}{v_{\text{rel}}}\int_0^{v_{\text{final}}} dv = \int_{M_0}^{M_{\text{final}}}\frac{1}{M}dM \;\rightarrow\; \frac{v_{\text{final}}}{v_{\text{rel}}} = \ln\frac{M_{\text{final}}}{M_0} \;\rightarrow\; M_{\text{final}} = M_0 e^{v_{\text{final}}/v_{\text{rel}}} \;\rightarrow$$

$$M_{\text{ejected}} = M_0 - M_{\text{final}} = M_0\left(1-e^{v_{\text{final}}/v_{\text{rel}}}\right) = (210\,\text{kg})\left(1-e^{2.0/(-35)}\right) = 11.66\,\text{kg} \approx \boxed{12\,\text{kg}}$$

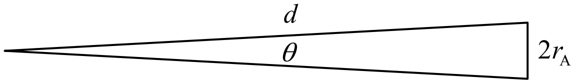
112. (*a*) We take the CM of the system as the origin of coordinates. Then at any time, we consider the *x* axis to be along the line connecting the star and the planet. Use the definition of center of mass:



$$x_{\text{CM}} = \frac{m_{\text{A}}r_{\text{A}} + m_{\text{B}}(-r_{\text{B}})}{m_{\text{A}}+m_{\text{B}}} = 0 \;\rightarrow\; \boxed{r_{\text{A}} = \frac{m_{\text{B}}}{m_{\text{A}}}r_{\text{B}}}$$

(*b*) $r_{\text{A}} = \dfrac{m_{\text{B}}}{m_{\text{A}}}r_{\text{B}} = \dfrac{1.0\times 10^{-3}m_{\text{A}}}{m_{\text{A}}}\left(8.0\times 10^{11}\,\text{m}\right) = \boxed{8.0\times 10^{8}\,\text{m}}$

(*c*) The geometry of this situation is illustrated in the adjacent diagram. For small angles in radian measure, $\theta \approx \tan\theta \approx \sin\theta$.



$$\theta \approx \tan\theta \approx \frac{2r_{\text{A}}}{d} \;\rightarrow\; d = \frac{2r_{\text{A}}}{\theta} = \frac{2\left(8.0\times 10^{8}\,\text{m}\right)}{\left(\frac{1}{1000}\right)\left(\frac{1}{3600}\right)\frac{\pi}{180}} = 3.30\times 10^{17}\,\text{m}\left(\frac{1\text{ ly}}{9.46\times 10^{15}\,\text{m}}\right) = \boxed{35\,\text{ly}}$$

(*d*) We assume that stars are distributed uniformly, with an average interstellar distance of 4 ly. If we think about each star having a spherical "volume" associated with it, that volume would have a radius of 2 ly (half the distance to an adjacent star). Each star would have a volume of $\frac{4}{3}\pi r^3_{\substack{\text{star}\\ \text{to}\\ \text{star}}} = \frac{4}{3}\pi(2\,\text{ly})^3$. If wobble can be detected from a distance of 35 ly, the volume over which wobble can be detected is $\frac{4}{3}\pi r^3_{\substack{\text{detectable}\\ \text{wobble}}} = \frac{4}{3}\pi(35\,\text{ly})^3$.

$$\#\text{ stars} = \frac{\frac{4}{3}\pi r^3_{\substack{\text{detectable}\\ \text{wobble}}}}{\frac{4}{3}\pi r^3_{\substack{\text{star}\\ \text{to star}}}} = \frac{(35\,\text{ly})^3}{(2\,\text{ly})^3} \approx \boxed{5400\text{ stars}}$$

113. This is a totally inelastic collision in one dimension. Call the direction of the Asteroid A the positive direction.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = (m_{\text{A}}+m_{\text{B}})v' \;\rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{\left(7.5\times10^{12}\,\text{kg}\right)\left(3.3\,\text{km/s}\right) + \left(1.45\times10^{13}\,\text{kg}\right)\left(-1.4\,\text{km/s}\right)}{7.5\times10^{12}\,\text{kg} + 1.45\times10^{13}\,\text{kg}}$$

$$= \boxed{0.2\,\text{km/s, in the original direction of asteroid A}}$$

114. (*a*) The elastic, stationary-target one-dimensional collision is analyzed in Example 9-8. We can use the relationships derived there to find the final velocity of the target.

$$v'_B = v_A\left(\frac{2m_A}{m_A + m_B}\right) = \frac{2m_A v_A}{m_A + m_B} = \boxed{\frac{2v_A}{1 + m_B/m_A}}$$

Note that since $m_B < m_A$, $v'_B > v_A$.

(*b*) In this scenario, the first collision would follow the same calculation as above, giving v'_C. Then particle C is incident on particle B, and using the same calculation as above, would give v'_B.

$$v'_C = v_A\left(\frac{2m_A}{m_A + m_C}\right)$$

$$v'_B = v'_C\left(\frac{2m_C}{m_B + m_C}\right) = v_A\left(\frac{2m_A}{m_A + m_C}\right)\left(\frac{2m_C}{m_B + m_C}\right) = \boxed{4v_A\frac{m_A m_C}{\left(m_A + m_C\right)\left(m_B + m_C\right)}}$$

(*c*) To find the value of m_C that gives the maximum v'_B, set $\frac{dv'_B}{dm_C} = 0$ and solve for m_C.

$$\frac{dv'_B}{dm_C} = 4v_A m_A \frac{\left[\left(m_A + m_C\right)\left(m_B + m_C\right) - m_C\left(m_A + m_B + 2m_C\right)\right]}{\left(m_A + m_C\right)^2\left(m_B + m_C\right)^2} = 0 \;\rightarrow$$

$$\left(m_A + m_C\right)\left(m_B + m_C\right) - m_C\left(m_A + m_B + 2m_C\right) = 0 \;\rightarrow$$

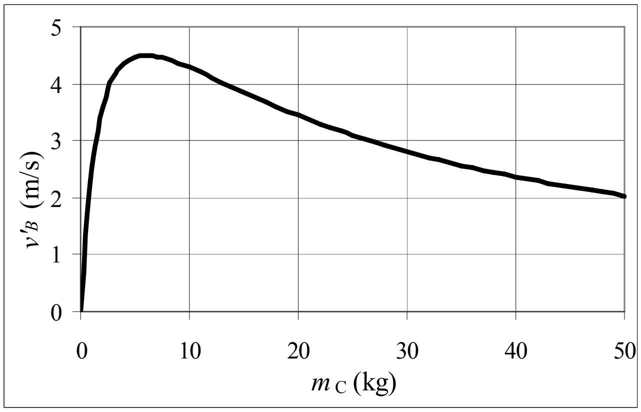
$$m_A m_B - m_C^2 = 0 \;\rightarrow\; \boxed{m_C = \sqrt{m_A m_B}}$$

(*d*) The graph is shown here. The numeric maximum of the graph has $\boxed{v'_B = 4.5\,\text{m/s}}$ and occurs at $\boxed{m_C = 6.0\,\text{kg}}$. According to the analysis from part (*c*), the value of $m_C = \sqrt{m_A m_B} = \sqrt{\left(18.0\,\text{kg}\right)\left(2.0\,\text{kg}\right)} = 6.0\,\text{kg}$, and gives a speed of $v'_B = \frac{4v_A m_A m_C}{\left(m_A + m_C\right)\left(m_B + m_C\right)}$

$$= \frac{4\left(2.0\,\text{m/s}\right)\left(18.0\,\text{kg}\right)\left(6.0\,\text{kg}\right)}{\left(24.0\,\text{kg}\right)\left(8.0\,\text{kg}\right)}.$$

$$= 4.5\,\text{m/s}.$$

The numeric results agree with the analytical results. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH09.XLS,” on tab “Problem 9.114d.”



CHAPTER 10: Rotational Motion

Responses to Questions

1. The odometer will register a distance greater than the distance actually traveled. The odometer counts the number of revolutions and the calibration gives the distance traveled per revolution (2πr). The smaller tire will have a smaller radius, and a smaller actual distance traveled per revolution.

2. A point on the rim of a disk rotating with constant angular velocity has no tangential acceleration since the tangential speed is constant. It does have radial acceleration. Although the point's speed is not changing, its velocity is, since the velocity vector is changing direction. The point has a centripetal acceleration, which is directed radially inward. If the disk's angular velocity increases uniformly, the point on the rim will have both radial and tangential acceleration, since it is both moving in a circle and speeding up. The magnitude of the radial component of acceleration will increase in the case of the disk with a uniformly increasing angular velocity, although the tangential component will be constant. In the case of the disk rotating with constant angular velocity, neither component of linear acceleration will change.

3. No. The relationship between the parts of a non-rigid object can change. Different parts of the object may have different values of ω.

4. Yes. The magnitude of the torque exerted depends not only on the magnitude of the force but also on the lever arm, which involves both the distance from the force to the axis of rotation and the angle at which the force is applied. A small force applied with a large lever arm could create a greater torque than a larger force with a smaller lever arm.

5. When you do a sit-up, you are rotating your trunk about a horizontal axis through your hips. When your hands are behind your head, your moment of inertia is larger than when your hands are stretched out in front of you. The sit-up with your hands behind your head will require more torque, and therefore will be "harder" to do.

6. Running involves rotating the leg about the point where it is attached to the rest of the body. Therefore, running fast requires the ability to change the leg's rotation easily. The smaller the moment of inertia of an object, the smaller the resistance to a change in its rotational motion. The closer the mass is to the axis of rotation, the smaller the moment of inertia. Concentrating flesh and muscle high and close to the body minimizes the moment of inertia and increases the angular acceleration possible for a given torque, improving the ability to run fast.

7. No. If two equal and opposite forces act on an object, the net force will be zero. If the forces are not co-linear, the two forces will produce a torque. No. If an unbalanced force acts through the axis of rotation, there will be a net force on the object, but no net torque.

8. The speed of the ball will be the same on both inclines. At the top of the incline, the ball has gravitational potential energy. This energy becomes converted to translational and rotational kinetic energy as the ball rolls down the incline. Since the inclines have the same height, the ball will have the same initial potential energy and therefore the same final kinetic energy and the same speed in both cases.

9. Roll the spheres down an incline. The hollow sphere will have a great moment of inertia and will take longer to reach the bottom of the incline.

10. The two spheres will reach the bottom at the same time with the same speed. The larger, more massive sphere will have the greater total kinetic energy at the bottom, since the total kinetic energy can be stated in terms of mass and speed.

11. A tightrope walker carries a long, narrow beam in order to increase his or her moment of inertia, making rotation (and falling off the wire) more difficult. The greater moment of inertia increases the resistance to change in angular motion, giving the walker more time to compensate for small shifts in position.

12. The moment of inertia of a solid sphere is given by $\frac{2}{5}MR^2$ and that of a solid cylinder is given by $\frac{1}{2}MR^2$. The solid sphere, with a smaller moment of inertia and therefore a smaller resistance to change in rotational motion, will reach the bottom of the incline first and have the greatest speed. Since both objects begin at the same height and have the same mass, they have the same initial potential energy. Since the potential energy is completely converted to kinetic energy at the bottom of the incline, the two objects will have the same total kinetic energy. However, the cylinder will have a greater rotational kinetic energy because its greater moment of inertia more than compensates for its lower velocity. At the bottom, $v_{\text{sphere}} = \sqrt{\frac{10}{7}gh}$ and $v_{\text{cylinder}} = \sqrt{\frac{4}{3}gh}$. Since rotational kinetic energy is $K_{\text{rot}} = \frac{1}{2}I\omega^2$, then $K_{\substack{\text{rot}\\ \text{sphere}}} = \frac{2}{7}mgh$ and $K_{\substack{\text{rot}\\ \text{cylinder}}} = \frac{1}{3}mgh$.

$\boxed{13.}$ The moment of inertia will be least about an axis parallel to the spine of the book, passing through the center of the book. For this choice, the mass distribution for the book will be closest to the axis.

14. Larger. The moment of inertia depends on the distribution of mass. Imagine the disk as a collection of many little bits of mass. Moving the axis of rotation to the edge of the disk increases the average distance of the bits of mass to the axis, and therefore increases the moment of inertia. (See the Parallel Axis theorem.)

15. If the angular velocity vector of a wheel on an axle points west, the wheel is rotating such that the linear velocity vector of a point at the top of the wheel points north. If the angular acceleration vector points east (opposite the angular velocity vector), then the wheel is slowing down and the linear acceleration vector for the point on the top of the wheel points south. The angular speed of the wheel is decreasing.

Solutions to Problems

$\boxed{1.}$ (*a*) $(45.0^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{\pi/4\,\text{rad}} = \boxed{0.785\,\text{rad}}$

(*b*) $(60.0^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{\pi/3\,\text{rad}} = \boxed{1.05\,\text{rad}}$

(*c*) $(90.0^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{\pi/2\,\text{rad}} = \boxed{1.57\,\text{rad}}$

(*d*) $(360.0^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{2\pi\,\text{rad}} = \boxed{6.283\,\text{rad}}$

(*e*) $(445^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{89\pi/36\,\text{rad}} = \boxed{7.77\,\text{rad}}$

2. The subtended angle (in radians) is the diameter of the Sun divided by the Earth – Sun distance.

$$\theta = \frac{\text{diameter of Sun}}{r_{\text{Earth-Sun}}} \rightarrow$$

$$\text{radius of Sun} = \tfrac{1}{2}\theta r_{\text{Earth-Sun}} = \tfrac{1}{2}(0.5^\circ)\left(\frac{\pi\,\text{rad}}{180^\circ}\right)\left(1.5\times10^{11}\,\text{m}\right) = 6.545\times10^{8}\,\text{m} \approx \boxed{7\times10^{8}\,\text{m}}$$

3. We find the diameter of the spot from the definition of radian angle measure.

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \rightarrow \text{diameter} = \theta r_{\text{Earth-Moon}} = \left(1.4\times10^{-5}\,\text{rad}\right)\left(3.8\times10^{8}\,\text{m}\right) = \boxed{5300\,\text{m}}$$

4. The initial angular velocity is $\omega_o = \left(6500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = 681\,\text{rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681\,\text{rad/s}}{4.0\ \text{s}} = \boxed{-170\,\text{rad/s}^2}$$

5. (*a*) $\omega = \left(\frac{2500\,\text{rev}}{1\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 261.8\,\text{rad/sec} \approx \boxed{260\,\text{rad/sec}}$

(*b*) $v = \omega r = (261.8\,\text{rad/sec})(0.175\ \text{m}) = \boxed{46\,\text{m/s}}$

$a_{\text{R}} = \omega^2 r = (261.8\,\text{rad/sec})^2\,(0.175\ \text{m}) = \boxed{1.2\times10^{4}\ \text{m/s}^2}$

6. In each revolution, the wheel moves forward a distance equal to its circumference, πd.

$$\Delta x = N_{\text{rev}}(\pi d) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{7200\,\text{m}}{\pi(0.68\ \text{m})} = \boxed{3400\,\text{rev}}$$

$\boxed{7.}$ The angular velocity is expressed in radians per second. The second hand makes 1 revolution every 60 seconds, the minute hand makes 1 revolution every 60 minutes, and the hour hand makes 1 revolution every 12 hours.

(*a*) Second hand: $\omega = \left(\frac{1\ \text{rev}}{60\,\text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = \boxed{\frac{\pi}{30}\text{rad/sec}} \approx \boxed{1.05\times10^{-1}\,\frac{\text{rad}}{\text{sec}}}$

(*b*) Minute hand: $\omega = \left(\frac{1\ \text{rev}}{60\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\,\text{min}}{60\ \text{s}}\right) = \boxed{\frac{\pi}{1800}\frac{\text{rad}}{\text{sec}}} \approx \boxed{1.75\times10^{-3}\,\frac{\text{rad}}{\text{sec}}}$

(*c*) Hour hand: $\omega = \left(\frac{1\ \text{rev}}{12\,\text{h}}\right)\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\,\text{h}}{3600\ \text{s}}\right) = \boxed{\frac{\pi}{21{,}600}\frac{\text{rad}}{\text{sec}}} \approx \boxed{1.45\times10^{-4}\,\frac{\text{rad}}{\text{sec}}}$

(*d*) The angular acceleration in each case is $\boxed{0}$, since the angular velocity is constant.

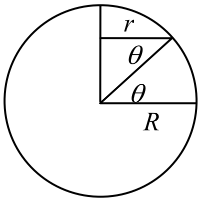
8. The angular speed of the merry-go-round is $2\pi\,\text{rad}/4.0\,\text{s} = 1.57\,\text{rad/s}$.

(*a*) $v = \omega r = (1.57\,\text{rad/sec})(1.2\ \text{m}) = \boxed{1.9\,\text{m/s}}$

(*b*) The acceleration is radial. There is no tangential acceleration.

$$a_R = \omega^2 r = (1.57\,\text{rad/sec})^2(1.2\,\text{m}) = \boxed{3.0\,\text{m/s}^2\text{ towards the center}}$$

9. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R\cos\theta$, where R is the radius of the Earth, and r is the radius at latitude θ.



(*a*) $$v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)(6.38\times10^6\,\text{m}) = \boxed{464\,\text{m/s}}$$

(*b*) $$v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)(6.38\times10^6\,\text{m})\cos 66.5^\circ = \boxed{185\,\text{m/s}}$$

(*c*) $$v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)(6.38\times10^6\,\text{m})\cos 45.0^\circ = \boxed{328\,\text{m/s}}$$

10. (*a*) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\,\text{rad}}{1\,\text{year}}\right)\left(\frac{1\text{ year}}{3.16\times10^7\,\text{s}}\right) = \boxed{1.99\times10^{-7}\text{ rad/s}}$$

(*b*) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86{,}400\,\text{s}}\right) = \boxed{7.27\times10^{-5}\text{ rad/s}}$$

11. The centripetal acceleration is given by $a = \omega^2 r$. Solve for the angular velocity.

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{(100{,}000)(9.80\,\text{m/s}^2)}{0.070\text{ m}}} = 3741\frac{\text{rad}}{\text{s}}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{1\text{ min}}\right) = \boxed{3.6\times10^4\,\text{rpm}}$$

12. Convert the rpm values to angular velocities.

$$\omega_0 = \left(130\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right) = 13.6\,\text{rad/s}$$

$$\omega = \left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right) = 29.3\,\text{rad/s}$$

(*a*) The angular acceleration is found from Eq. 10-3a.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{29.3\,\text{rad/s} - 13.6\,\text{rad/s}}{4.0\text{ s}} = 3.93\,\text{rad/s}^2 \approx \boxed{3.9\,\text{rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 13.6\,\text{rad/s} + (3.93\,\text{rad/s}^2)(2.0\,\text{s}) = 21.5\,\text{rad/s}$$

The instantaneous radial acceleration is given by $a_R = \omega^2 r$.

$$a_R = \omega^2 r = (21.5\,\text{rad/s})^2(0.35\,\text{m}) = \boxed{160\,\text{m/s}^2}$$

The tangential acceleration is given by $a_{\tan} = \alpha r$.

$$a_{\tan} = \alpha r = (3.93\,\text{rad/s}^2)(0.35\,\text{m}) = \boxed{1.4\,\text{m/s}^2}$$

$\boxed{13.}$ (*a*) The angular rotation can be found from Eq. 10-3a. The initial angular frequency is 0 and the final frequency is 1 rpm.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(1.0\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\text{ rev}}\right)\left(\frac{1.0\,\text{min}}{60\text{ s}}\right) - 0}{720\text{ s}} = 1.454\times10^{-4}\,\text{rad/s}^2 \approx \boxed{1.5\times10^{-4}\,\text{rad/s}^2}$$

(*b*) After 7.0 min (420 s), the angular speed is as follows.

$$\omega = \omega_0 + \alpha t = 0 + \left(1.454\times10^{-4}\,\text{rad/s}^2\right)\left(420\,\text{s}\right) = 6.107\times10^{-2}\,\text{rad/s}$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$a_{\text{tan}} = \alpha R = \left(1.454\times10^{-4}\,\text{rad/s}^2\right)\left(4.25\,\text{m}\right) = \boxed{6.2\times10^{-4}\,\text{m/s}^2}$$

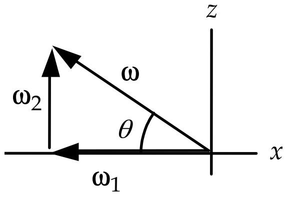
$$a_{\text{rad}} = \omega^2 R = \left(6.107\times10^{-2}\,\text{rad/s}\right)^2\left(4.25\,\text{m}\right) = \boxed{1.6\times10^{-2}\,\text{m/s}^2}$$

14. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$v_1 = v_2 \quad\rightarrow\quad \omega_1 R_1 = \omega_2 R_2 \quad\rightarrow\quad \boxed{\omega_1/\omega_2 = R_2/R_1}$$

15. (*a*) The direction of ω_1 is along the axle of the wheel, to the left. That is the $\boxed{-\hat{\mathbf{i}}}$ direction. The direction of ω_2 is also along its axis of rotation, so it is straight up. That is the $\boxed{+\hat{\mathbf{k}}}$ direction. That is also the angular velocity of the axis of the wheel.

(*b*) At the instant shown in the textbook, we have the vector relationship as shown in the diagram.



$$\omega = \sqrt{\omega_1^2 + \omega_2^2} = \sqrt{\left(44.0\,\text{rad/s}\right)^2 + \left(35.0\,\text{rad/s}\right)^2} = \boxed{56.2\,\text{rad/s}}$$

$$\theta = \tan^{-1}\frac{\omega_2}{\omega_2} = \tan^{-1}\frac{35.0}{44.0} = \boxed{38.5^\circ}$$

(*c*) Angular acceleration is given by $\vec{\boldsymbol{\alpha}} = \frac{d\vec{\boldsymbol{\omega}}}{dt}$. Since $\vec{\boldsymbol{\omega}} = \vec{\boldsymbol{\omega}}_1 + \vec{\boldsymbol{\omega}}_2$, and $\vec{\boldsymbol{\omega}}_2$ is a constant $35.0\hat{\mathbf{k}}\,\text{rad/s}$, $\vec{\boldsymbol{\alpha}} = \frac{d\vec{\boldsymbol{\omega}}_1}{dt}$. $\vec{\boldsymbol{\omega}}_1$ is rotating counterclockwise about the *z* axis with the angular velocity of ω_2, and so if the figure is at $t = 0$, then $\vec{\boldsymbol{\omega}}_1 = \omega_1\left(-\cos\omega_2 t\hat{\mathbf{i}} - \sin\omega_2 t\hat{\mathbf{j}}\right)$.

$$\vec{\boldsymbol{\alpha}} = \frac{d\vec{\boldsymbol{\omega}}}{dt} = \frac{d\left(\vec{\boldsymbol{\omega}}_1 + \vec{\boldsymbol{\omega}}_2\right)}{dt} = \frac{d\vec{\boldsymbol{\omega}}_1}{dt} = \frac{d\left[\omega_1\left(-\cos\omega_2 t\hat{\mathbf{i}} - \sin\omega_2 t\hat{\mathbf{j}}\right)\right]}{dt} = \omega_1\omega_2\left(\sin\omega_2 t\hat{\mathbf{i}} - \cos\omega_2 t\hat{\mathbf{j}}\right)$$

$$\vec{\boldsymbol{\alpha}}\left(t = 0\right) = \omega_1\omega_2\left(-\hat{\mathbf{j}}\right) = -\left(44.0\,\text{rad/s}\right)\left(35.0\,\text{rad/s}\right)\hat{\mathbf{j}} = \boxed{-1540\,\text{rad/s}^2\,\hat{\mathbf{j}}}$$

16. (*a*) For constant angular acceleration:

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{1200\,\text{rev/min} - 3500\,\text{rev/min}}{2.5\text{ s}} = \frac{-2300\,\text{rev/min}}{2.5\,\text{s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= -96.34\,\text{rad/s}^2 \approx \boxed{-96\,\text{rad/s}^2}$$

(*b*) For the angular displacement, given constant angular acceleration:

$$\theta = \tfrac{1}{2}(\omega_o + \omega)t = \tfrac{1}{2}(3500\,\text{rev/min} + 1200\,\text{rev/min})(2.5\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{98\,\text{rev}}$$

17. The angular displacement can be found from Eq. 10-9d.

$$\theta = \overline{\omega}t = \tfrac{1}{2}(\omega_o + \omega)t = \tfrac{1}{2}(0 + 15000\,\text{rev/min})(220\,\text{s})(1\,\text{min}/60\,\text{s}) = \boxed{2.8\times 10^4\,\text{rev}}$$

18. (*a*) The angular acceleration can be found from Eq. 10-9b with $\omega_o = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2(20\,\text{rev})}{(1.0\,\text{min})^2} = \boxed{4.0\times 10^1\,\text{rev/min}^2}$$

(*b*) The final angular speed can be found from $\theta = \tfrac{1}{2}(\omega_o + \omega)t$, with $\omega_o = 0$.

$$\omega = \frac{2\theta}{t} - \omega_o = \frac{2(20\text{ rev})}{1.0\text{ min}} = \boxed{4.0\times 10^1\text{ rpm}}$$

$\boxed{19.}$ (*a*) The angular acceleration can be found from Eq. 10-9c.

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{0 - (850\,\text{rev/min})^2}{2(1350\text{ rev})} = \left(-267.6\frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = \boxed{-0.47\frac{\text{rad}}{\text{s}^2}}$$

(*b*) The time to come to a stop can be found from $\theta = \tfrac{1}{2}(\omega_o + \omega)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(1350\,\text{rev})}{850\,\text{rev/min}}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{190\,\text{s}}$$

20. We start with $\alpha = \frac{d\omega}{dt}$. We also assume that α is constant, that the angular speed at time $t = 0$ is ω_0, and that the angular displacement at time $t = 0$ is 0.

$$\alpha = \frac{d\omega}{dt} \rightarrow d\omega = \alpha dt \rightarrow \int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha dt \rightarrow \omega - \omega_0 = \alpha t \rightarrow \boxed{\omega = \omega_0 + \alpha t}$$

$$\omega = \omega_0 + \alpha t = \frac{d\theta}{dt} \rightarrow d\theta = (\omega_0 + \alpha t)dt \rightarrow \int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)dt \rightarrow \boxed{\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2}$$

21. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.

(*a*) $a_{\substack{\text{tan}\\\text{small}}} = a_{\substack{\text{tan}\\\text{large}}} \rightarrow \alpha_{\text{small}} r_{\text{small}} = \alpha_{\text{large}} r_{\text{large}} \rightarrow$

$$\alpha_{\text{large}} = \alpha_{\text{small}}\frac{r_{\text{small}}}{r_{\text{large}}} = (7.2\,\text{rad/s}^2)\left(\frac{2.0\text{ cm}}{21.0\text{ cm}}\right) = 0.6857\,\text{rad/s}^2 \approx \boxed{0.69\,\text{rad/s}^2}$$

(*b*) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 10-9a.

$$\omega = \left(65\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 6.807\,\text{rad/s}$$

$$\omega = \omega_0 + \alpha t \rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{6.807\,\text{rad/s}}{0.6857\,\text{rad/s}^2} = \boxed{9.9\,\text{s}}$$

22. We are given that $\theta = 8.5t - 15.0t^2 + 1.6t^4$.

(*a*) $\omega = \frac{d\theta}{dt} = \boxed{8.5 - 30.0t + 6.4t^3}$, where ω is in rad/sec and *t* is in sec.

(*b*) $\alpha = \frac{d\omega}{dt} = \boxed{-30.0 + 19.2t^2}$, where α is in rad/sec^2 and *t* is in sec.

(*c*) $\omega(3.0) = 8.5 - 30.0(3.0) + 6.4(3.0)^3 = \boxed{91\,\text{rad/s}}$

$\alpha(3.0) = -30.0 + 19.2(3.0)^2 = \boxed{140\,\text{rad/s}^2}$

(*d*) The average angular velocity is the angular displacement divided by the elapsed time.

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(3.0) - \theta(2.0)}{3.0\,\text{s} - 2.0\,\text{s}}$$

$$= \frac{\left[8.5(3.0) - 15.0(3.0)^2 + 1.6(3.0)^4\right] - \left[8.5(2.0) - 15.0(2.0)^2 + 1.6(2.0)^4\right]}{1.0\,\text{s}}$$

$$= \boxed{38\,\text{rad/s}}$$

(*e*) The average angular acceleration is the change in angular velocity divided by the elapsed time.

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\omega(3.0) - \omega(2.0)}{3.0\,\text{s} - 2.0\,\text{s}}$$

$$= \frac{\left[8.5 - 30.0(3.0) + 6.4(3.0)^3\right] - \left[8.5 - 30.0(2.0) + 6.4(2.0)^3\right]}{1.0\,\text{s}} = \boxed{92\,\text{rad/s}^2}$$

23. (*a*) The angular velocity is found by integrating the angular acceleration function.

$$\alpha = \frac{d\omega}{dt} \rightarrow d\omega = \alpha dt \rightarrow \int_0^\omega d\omega = \int_0^t \alpha dt = \int_0^t \left(5.0t^2 - 8.5t\right)dt \rightarrow \boxed{\omega = \tfrac{1}{3}5.0t^3 - \tfrac{1}{2}8.5t^2}$$

(*b*) The angular position is found by integrating the angular velocity function.

$$\omega = \frac{d\theta}{dt} \rightarrow d\theta = \omega dt \rightarrow \int_0^\theta d\theta = \int_0^t \omega dt = \int_0^t \left(\tfrac{1}{3}5.0t^3 - \tfrac{1}{2}8.5t^2\right)dt \rightarrow$$

$$\boxed{\theta = \tfrac{1}{12}5.0t^4 - \tfrac{1}{6}8.5t^3}$$

(*c*) $\omega(2.0\,\text{s}) = \tfrac{1}{3}5.0(2.0)^3 - \tfrac{1}{2}8.5(2.0)^2 = -3.7\,\text{rad/s} \approx \boxed{-4\,\text{rad/s}}$

$\theta(2.0\,\text{s}) = \tfrac{1}{12}5.0(2.0)^4 - \tfrac{1}{6}8.5(2.0)^3 = -4.67\,\text{rad} \approx \boxed{-5\,\text{rad}}$

24. (*a*) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$\tau = r_\perp F = r_\perp mg = (0.17\,\text{m})(62\,\text{kg})\left(9.80\,\text{m/s}^2\right) = \boxed{1.0\times10^2\ \text{m}\bullet\text{N}}$$

(*b*) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upwards on the handle bars as she pedals, which will increase the downward force of her legs.

25. Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by the following.

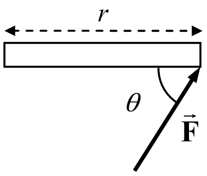
$$\tau_{\substack{\text{applied}\\\text{forces}}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) = -1.8\text{ m•N}$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, and so the frictional torque is counterclockwise. Thus the net torque is as follows.

$$\tau_{\text{net}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) + 0.40\text{ m•N} = -1.4\text{ m•N}$$

$$= \boxed{1.4\text{ m•N , clockwise}}$$

26. The torque is calculated by $\tau = rF\sin\theta$. See the diagram, from the top view.



(*a*) For the first case, $\theta = 90°$.

$$\tau = rF\sin\theta = (0.96\,\text{m})(32\,\text{N})\sin 90° = \boxed{31\,\text{m•N}}$$

(*b*) For the second case, $\theta = 60.0°$.

$$\tau = rF\sin\theta = (0.96\,\text{m})(32\,\text{N})\sin 60.0° = \boxed{27\,\text{m•N}}$$

27. There is a counterclockwise torque due to the force of gravity on the left block, and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$\sum\tau = mg\ell_2 - mg\ell_1 = \boxed{mg(\ell_2 - \ell_1)\text{ , clockwise}}$$

28. The lever arm to the point of application of the force is along the *x* axis. Thus the perpendicular part of the force is the *y* component. Use Eq. 10-10b.

$$\tau = RF_\perp = (0.135\,\text{m})(43.4\,\text{N}) = \boxed{5.86\,\text{m•N, counterclockwise}}$$

29. The force required to produce the torque can be found from $\tau = rF\sin\theta$. The force is applied perpendicularly to the wrench, so $\theta = 90°$.

$$F = \frac{\tau}{r} = \frac{75\,\text{m•N}}{0.28\text{ m}} = \boxed{270\,\text{N}}$$

The net torque still must be $75\,\text{m•N}$. This is produced by 6 forces, one at each of the 6 points. We assume that those forces are also perpendicular to their lever arms.

$$\tau_{\text{net}} = (6F_{\text{point}})r_{\text{point}} \rightarrow F_{\text{point}} = \frac{\tau}{6r} = \frac{75\text{ m•N}}{6(0.0075\,\text{m})} = \boxed{1700\,\text{N}}$$

30. For each torque, use Eq. 10-10c. Take counterclockwise torques to be positive.

(*a*) Each force has a lever arm of 1.0 m.

$$\tau_{\substack{\text{about}\\\text{C}}} = -(1.0\,\text{m})(56\,\text{N})\sin 30° + (1.0\,\text{m})(52\,\text{N})\sin 60° = \boxed{17\text{m•N}}$$

(*b*) The force at C has a lever arm of 1.0 m, and the force at the top has a lever arm of 2.0 m.

$$\tau_{\substack{\text{about}\\\text{P}}} = -(2.0\,\text{m})(56\,\text{N})\sin 30° + (1.0\,\text{m})(65\,\text{N})\sin 45° = \boxed{-10\,\text{m•N}}\ \text{(2 sig fig)}$$

The negative sign indicates a clockwise torque.

31. For a sphere rotating about an axis through its center, the moment of inertia is as follows.

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(10.8\text{ kg})(0.648\text{ m})^2 = \boxed{1.81\text{ kg•m}^2}$$

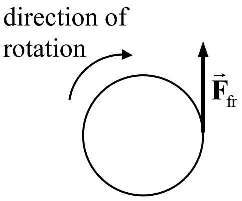
32. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by $I = MR^2$.

$$I = MR^2 = (1.1\,\text{kg})\left(\tfrac{1}{2}(0.67\,\text{m})\right)^2 = \boxed{0.12\,\text{kg}\bullet\text{m}^2}$$

The hub mass can be ignored because its distance from the axis of rotation is very small, and so it has a very small rotational inertia.

33. (*a*) The torque exerted by the frictional force is $\tau = rF_{\text{fr}}\sin\theta$. The force of friction is assumed to be tangential to the clay, and so $\theta = 90°$.

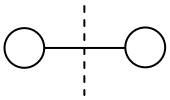
$$\tau_{\text{total}} = rF_{\text{fr}}\sin\theta = \left(\tfrac{1}{2}(0.12\,\text{m})\right)(1.5\,\text{N})\sin 90° = \boxed{0.090\,\text{m}\bullet\text{N}}$$



(*b*) The time to stop is found from $\omega = \omega_o + \alpha t$, with a final angular velocity of 0. The angular acceleration can be found from $\tau_{\text{total}} = I\alpha$. The net torque (and angular acceleration) is negative since the object is slowing.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{\omega - \omega_o}{\tau/I} = \frac{0 - (1.6\,\text{rev/s})(2\pi\,\text{rad/rev})}{(-0.090\,\text{m}\bullet\text{N})/(0.11\,\text{kg}\bullet\text{m}^2)} = \boxed{12\,\text{s}}$$

34. The oxygen molecule has a "dumbbell" geometry, rotating about the dashed line, as shown in the diagram. If the total mass is *M*, then each atom has a mass of *M*/2. If the distance between them is *d*, then the distance from the axis of rotation to each atom is *d*/2. Treat each atom as a particle for calculating the moment of inertia.



$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \tfrac{1}{4}Md^2 \quad\rightarrow$$
$$d = \sqrt{4I/M} = \sqrt{4\left(1.9\times10^{-46}\,\text{kg}\bullet\text{m}^2\right)/\left(5.3\times10^{-26}\,\text{kg}\right)} = \boxed{1.2\times10^{-10}\,\text{m}}$$

35. The torque can be calculated from $\tau = I\alpha$. The rotational inertia of a rod about its end is given by $I = \tfrac{1}{3}ML^2$.

$$\tau = I\alpha = \tfrac{1}{3}ML^2\frac{\Delta\omega}{\Delta t} = \tfrac{1}{3}(2.2\,\text{kg})(0.95\,\text{m})^2\frac{(2.7\,\text{rev/s})(2\pi\,\text{rad/rev})}{0.20\,\text{s}} = \boxed{56\,\text{m}\bullet\text{N}}$$

36. (*a*) The moment of inertia of a cylinder is $\tfrac{1}{2}MR^2$.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.380\,\text{kg})(0.0850\,\text{m})^2 = 1.373\times10^{-3}\,\text{kg}\bullet\text{m}^2 \approx \boxed{1.37\times10^{-3}\,\text{kg}\bullet\text{m}^2}$$

(*b*) The wheel slows down "on its own" from 1500 rpm to rest in 55.0s. This is used to calculate the frictional torque.

$$\tau_{\text{fr}} = I\alpha_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} = \left(1.373\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)\frac{(0-1500\,\text{rev/min})(2\pi\ \text{rad/rev})(1\ \text{min}/60\ \text{s})}{55.0\ \text{s}}$$
$$= -3.921\times10^{-3}\,\text{m}\bullet\text{N}$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$\sum\tau = \tau_{\text{applied}} + \tau_{\text{fr}} = I\alpha \quad\rightarrow\quad \tau_{\text{applied}} = I\alpha - \tau_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} - \tau_{\text{fr}}$$
$$= \left(1.373\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)\frac{(1750\,\text{rev/min})(2\pi\,\text{rad/rev})(1\ \text{min}/60\ \text{s})}{5.00\ \text{s}} - \left(-3.921\times10^{-3}\,\text{m}\bullet\text{N}\right)$$
$$= \boxed{5.42\times10^{-2}\,\text{m}\bullet\text{N}}$$

$\boxed{37.}$ (*a*) The small ball can be treated as a particle for calculating its moment of inertia.

$$I = MR^2 = (0.650\,\text{kg})(1.2\,\text{m})^2 = \boxed{0.94\,\text{kg}\bullet\text{m}^2}$$

(*b*) To keep a constant angular velocity, the net torque must be zero, and so the torque needed is the same magnitude as the torque caused by friction.

$$\sum\tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \;\rightarrow\; \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}}r = (0.020\,\text{N})(1.2\,\text{m}) = \boxed{2.4\times10^{-2}\,\text{m}\bullet\text{N}}$$

38. (*a*) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\tau = I\alpha = MR^2\alpha = MR^2\frac{a_{\text{tan}}}{R} = MRa_{\text{tan}} = (3.6\,\text{kg})(0.31\,\text{m})(7.0\,\text{m/s}^2)$$
$$= 7.812\,\text{m}\bullet\text{N} \approx \boxed{7.8\,\text{m}\bullet\text{N}}$$

(*b*) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_{\perp} \;\rightarrow\; F = \tau/r_{\perp} = 7.812\,\text{m}\bullet\text{N}\big/(2.5\times10^{-2}\,\text{m}) = \boxed{310\,\text{N}}$$

39. (*a*) The angular acceleration can be found from the following.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = \frac{v/r}{t} = \frac{(8.5\,\text{m/s})/(0.31\,\text{m})}{0.35\text{ s}} = 78.34\,\text{rad/s}^2 \approx \boxed{78\,\text{rad/s}^2}$$

(*b*) The force required can be found from the torque, since $\tau = Fr\sin\theta$. In this situation the force is perpendicular to the lever arm, and so $\theta = 90°$. The torque is also given by $\tau = I\alpha$, where I is the moment of inertia of the arm-ball combination. Equate the two expressions for the torque, and solve for the force.

$$Fr\sin\theta = I\alpha$$

$$F = \frac{I\alpha}{r\sin\theta} = \frac{m_{\text{ball}}d_{\text{ball}}^2 + \tfrac{1}{3}m_{\text{arm}}L_{\text{arm}}^2}{r\sin 90^{\circ}}\alpha$$
$$= \frac{(1.00\,\text{kg})(0.31\,\text{m})^2 + \tfrac{1}{3}(3.7\,\text{kg})(0.31\text{ m})^2}{(0.025\,\text{m})}(78.34\,\text{rad/s}^2) = \boxed{670\,\text{N}}$$

40. (*a*) To calculate the moment of inertia about the y axis (vertical), use the following.

$$I = \sum M_iR_{ix}^2 = m(0.50\,\text{m})^2 + M(0.50\,\text{m})^2 + m(1.00\,\text{m})^2 + M(1.00\,\text{m})^2$$
$$= (m+M)\left[(0.50\,\text{m})^2 + (1.00\,\text{m})^2\right] = (5.3\,\text{kg})\left[(0.50\,\text{m})^2 + (1.00\,\text{m})^2\right] = \boxed{6.6\,\text{kg}\bullet\text{m}^2}$$

(*b*) To calculate the moment of inertia about the x-axis (horizontal), use the following.

$$I = \sum M_iR_{iy}^2 = (2m + 2M)(0.25\,\text{m})^2 = \boxed{0.66\,\text{kg}\bullet\text{m}^2}$$

(*c*) Because of the larger I value, it is ten times harder to accelerate the array about the $\boxed{\text{vertical axis}}$.

41. The torque required is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 10-9a. Use the moment of inertia of a solid cylinder.

$$\omega = \omega_0 + \alpha t \;\rightarrow\; \alpha = \omega/t$$

$$\tau = I\alpha = \left(\tfrac{1}{2}MR_0^2\right)\left(\frac{\omega}{t}\right) = \frac{MR_0^2\omega}{2t} = \frac{(31000\,\text{kg})(7.0\,\text{m})^2(0.68\,\text{rad/s})}{2(24\,\text{s})} = \boxed{2.2\times10^4\,\text{m}\bullet\text{N}}$$

42. The torque supplied is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 10-9b, with $\omega_0 = 0$. Use the moment of inertia of a sphere.

$$\theta = \omega_0 + \tfrac{1}{2}\alpha t^2 \rightarrow \alpha = \frac{2\theta}{t^2} \;;\; \tau = I\alpha = \left(\tfrac{2}{5}Mr_0^2\right)\left(\frac{2\theta}{t^2}\right) \rightarrow$$

$$M = \frac{5\tau t^2}{4r_0^2\theta} = \frac{5(10.8\,\text{m}\bullet\text{N})(15.0\,\text{s})^2}{4(0.36\,\text{m})^2(360\pi\,\text{rad})} = \boxed{21\,\text{kg}}$$

43. The applied force causes torque, which gives the pulley an angular acceleration. Since the applied force varies with time, so will the angular acceleration. The variable acceleration will be integrated to find the angular velocity. Finally, the speed of a point on the rim is the tangential velocity of the rim of the wheel.

$$\sum \tau = R_0 F_T = I\alpha \rightarrow \alpha = \frac{R_0 F_\text{T}}{I} = \frac{d\omega}{dt} \rightarrow d\omega = \frac{R_0 F_\text{T}}{I}dt \rightarrow \int_{\omega_0}^{\omega} d\omega = \int_0^t \frac{R_0 F_\text{T}}{I}dt \rightarrow$$

$$\omega = \frac{v}{R_0} = \omega_0 + \frac{R_0}{I}\int_0^t F_\text{T}dt = \frac{R_0}{I}\int_0^t F_\text{T}dt \rightarrow$$

$$v_\text{T} = \omega R_0 = \frac{R_0^2}{I}\int_0^t F_\text{T}dt = \frac{R_0^2}{I}\int_0^t \left(3.00t - 0.20t^2\right)dt = \frac{R_0^2}{I}\left[\left(\tfrac{3}{2}t^2 - \tfrac{0.20}{3}t^3\right)\text{N}\bullet\text{s}\right]$$

$$v(t = 8.0\,\text{s}) = \frac{(0.330\,\text{m})^2}{\left(0.385\,\text{kg}\bullet\text{m}^2\right)}\left[\left(\tfrac{3}{2}(8.0\,\text{s})^2 - \tfrac{0.20}{3}(8.0\,\text{s})^3\right)\text{N}\bullet\text{s}\right] = 17.499\,\text{m/s} \approx \boxed{17\,\text{m/s}}$$

44. The torque needed is the moment of inertia of the system (merry-go-round and children) times the angular acceleration of the system. Let the subscript "mgr" represent the merry-go-round.

$$\tau = I\alpha = \left(I_\text{mgr} + I_\text{children}\right)\frac{\Delta\omega}{\Delta t} = \left(\tfrac{1}{2}M_\text{mgr}R^2 + 2m_\text{child}R^2\right)\frac{\omega - \omega_0}{t}$$

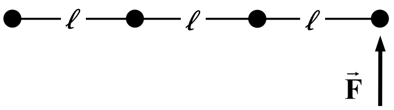
$$= \left[\tfrac{1}{2}(760\,\text{kg}) + 2(25\,\text{kg})\right](2.5\text{ m})^2\,\frac{(15\,\text{rev/min})(2\pi\,\text{rad/rev})(1\,\text{min}/60\text{ s})}{10.0\,\text{s}}$$

$$= 422.15\,\text{m}\bullet\text{N} \approx \boxed{420\,\text{m}\bullet\text{N}}$$

The force needed is calculated from the torque and the radius. We are told that the force is directed perpendicularly to the radius.

$$\tau = F_\perp R\sin\theta \rightarrow F_\perp = \tau/R = 422.15\,\text{m}\bullet\text{N}/2.5\text{ m} = \boxed{170\,\text{N}}$$

45. Each mass is treated as a point particle. The first mass is at the axis of rotation; the second mass is a distance ℓ from the axis of rotation; the third mass is 2ℓ from the axis, and the fourth mass is 3ℓ from the axis.



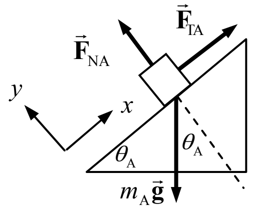
(*a*) $I = M\ell^2 + M(2\ell)^2 + M(3\ell)^2 = \boxed{14M\ell^2}$

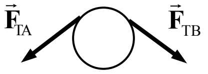
(*b*) The torque to rotate the rod is the perpendicular component of force times the lever arm, and is also the moment of inertia times the angular acceleration.

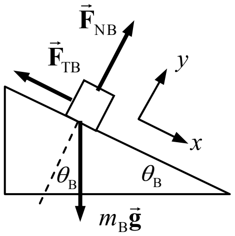
$$\tau = I\alpha = F_\perp r \rightarrow F_\perp = \frac{I\alpha}{r} = \frac{14M\ell^2\alpha}{3\ell} = \boxed{\tfrac{14}{3}M\ell\alpha}$$

(*c*) The force must be perpendicular to the rod connecting the masses, and perpendicular to the axis of rotation. An appropriate direction is shown in the diagram.

46. (*a*) The free body diagrams are shown. Note that only the forces producing torque are shown on the pulley. There would also be a gravity force on the pulley (since it has mass) and a normal force from the pulley's suspension, but they are not shown.







(*b*) Write Newton's second law for the two blocks, taking the positive *x* direction as shown in the free body diagrams.

$$m_A: \ \sum F_x = F_{TA} - m_A g\sin\theta_A = m_A a \ \rightarrow$$

$$F_{TA} = m_A\left(g\sin\theta_A + a\right)$$

$$= (8.0\,\text{kg})\left[\left(9.80\,\text{m/s}^2\right)\sin 32^\circ + 1.00\,\text{m/s}^2\right] = 49.55\,\text{N}$$

$$\approx \boxed{50\,\text{N}} \ (2\text{ sig fig})$$

$$m_B: \ \sum F_x = m_B g\sin\theta_B - F_{TB} = m_B a \ \rightarrow$$

$$F_{TB} = m_B\left(g\sin\theta_B - a\right)$$

$$= (10.0\,\text{kg})\left[\left(9.80\,\text{m/s}^2\right)\sin 61^\circ - 1.00\,\text{m/s}^2\right] = 75.71\,\text{N}$$

$$\approx \boxed{76\,\text{N}}$$

(*c*) The net torque on the pulley is caused by the two tensions. We take clockwise torques as positive.

$$\sum\tau = \left(F_{TB} - F_{TB}\right)R = (75.71\,\text{N} - 49.55\,\text{N})(0.15\,\text{m}) = 3.924\,\text{m}\bullet\text{N} \approx \boxed{3.9\,\text{m}\bullet\text{N}}$$

Use Newton's second law to find the rotational inertia of the pulley. The tangential acceleration of the pulley's rim is the same as the linear acceleration of the blocks, assuming that the string doesn't slip.

$$\sum\tau = I\alpha = I\frac{a}{R} = \left(F_{TB} - F_{TB}\right)R \ \rightarrow$$

$$I = \frac{\left(F_{TB} - F_{TB}\right)R^2}{a} = \frac{(75.71\,\text{N} - 49.55\,\text{N})(0.15\,\text{m})^2}{1.00\,\text{m/s}^2} = \boxed{0.59\,\text{kg}\bullet\text{m}^2}$$

47. (*a*) The moment of inertia of a thin rod, rotating about its end, is $\frac{1}{3}ML^2$. There are three blades to add together.

$$I_{\text{total}} = 3\left(\tfrac{1}{3}M\ell^2\right) = M\ell^2 = (135\,\text{kg})(3.75\,\text{m})^2 = 1898\,\text{kg}\bullet\text{m}^2 \approx \boxed{1.90\times10^3\,\text{kg}\bullet\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}}\frac{\omega - \omega_0}{t} = \left(1898\,\text{kg}\bullet\text{m}^2\right)\frac{(5.0\,\text{rev/sec})(2\pi\,\text{rad/rev})}{8.0\,\text{s}} = \boxed{7500\,\text{m}\bullet\text{N}}$$

48. The torque on the rotor will cause an angular acceleration given by $\alpha = \tau/I$. The torque and angular acceleration will have the opposite sign of the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into the equation $\omega^2 = \omega_o^2 + 2\alpha\theta$, and solve for the angular displacement.

$$\omega^2 = \omega_o^2 + 2\alpha\theta \quad \rightarrow \quad \theta = \frac{\omega^2 - \omega_o^2}{2\alpha} = \frac{0 - \omega_o^2}{2\left(\tau/I\right)} = \frac{-\omega_o^2}{2\left(\tau/\frac{1}{2}MR^2\right)} = \frac{-MR^2\omega_o^2}{4\tau}$$

$$= \frac{-\left(3.80\,\text{kg}\right)\left(0.0710\,\text{m}\right)^2\left[\left(10{,}300\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\right]^2}{4\left(-1.20\,\text{N}\bullet\text{m}\right)} = 4643\,\text{rad}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$

$$= \boxed{739\,\text{rev}}$$

The time can be found from $\theta = \frac{1}{2}\left(\omega_o + \omega\right)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2\left(739\,\text{rev}\right)}{10{,}300\,\text{rev/min}}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{8.61\,\text{s}}$$

$\boxed{49.}$ (*a*) Thin hoop, radius R_0 $\quad I = Mk^2 = MR_0^2 \rightarrow k = \boxed{R_0}$

(*b*) Thin hoop, radius R_0, width *w* $\quad I = Mk^2 = \frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2 \rightarrow k = \boxed{\sqrt{\frac{1}{2}R_0^2 + \frac{1}{12}w^2}}$

(*c*) Solid cylinder $\quad I = Mk^2 = \frac{1}{2}MR_0^2 \rightarrow k = \boxed{\sqrt{\frac{1}{2}}\,R_0}$

(*d*) Hollow cylinder $\quad I = Mk^2 = \frac{1}{2}M\left(R_1^2 + R_2^2\right) \rightarrow k = \boxed{\sqrt{\frac{1}{2}\left(R_1^2 + R_2^2\right)}}$

(*e*) Uniform sphere $\quad I = Mk^2 = \frac{2}{5}Mr_0^2 \rightarrow k = \boxed{\sqrt{\frac{2}{5}}\,r_0}$

(*f*) Long rod, through center $\quad I = Mk^2 = \frac{1}{12}M\ell^2 \rightarrow k = \boxed{\sqrt{\frac{1}{12}}\,\ell}$

(*g*) Long rod, through end $\quad I = Mk^2 = \frac{1}{3}M\ell^2 \rightarrow k = \boxed{\sqrt{\frac{1}{3}}\,\ell}$

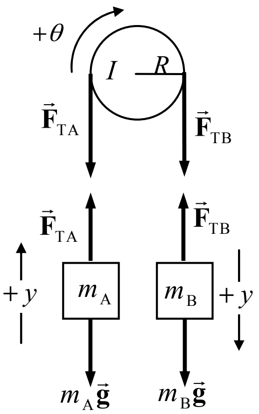
(*h*) Rectangular thin plate $\quad I = Mk^2 = \frac{1}{12}M\left(\ell^2 + w^2\right) \rightarrow k = \boxed{\sqrt{\frac{1}{12}\left(\ell^2 + w^2\right)}}$

50. The firing force of the rockets will create a net torque, but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus $\tau_{\text{net}} = 4FR$. This torque will cause an angular acceleration according to $\tau = I\alpha$, where $I = \frac{1}{2}MR^2 + 4mR^2$, combining a cylinder of mass *M* and radius *R* with 4 point masses of mass *m* and lever arm *R* each. The angular acceleration can be found from the kinematics by $\alpha = \frac{\Delta\omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

$$4FR = I\alpha = \left(\tfrac{1}{2}M + 4m\right)R^2\frac{\Delta\omega}{\Delta\tau} \quad \rightarrow \quad F = \frac{\left(\frac{1}{2}M + 4m\right)R\Delta\omega}{4\Delta t}$$

$$= \frac{\left(\frac{1}{2}\left(3600\text{kg}\right) + 4\left(250\,\text{kg}\right)\right)\left(4.0\,\text{m}\right)\left(32\,\text{rev/min}\right)\left(2\pi\ \text{rad/rev}\right)\left(1\ \text{min}/60\ \text{s}\right)}{4\left(5.0\,\text{min}\right)\left(60\,\text{s/min}\right)} = 31.28\,\text{N}$$

$$\approx \boxed{31\,\text{N}}$$



51. We assume that $m_{\text{B}} > m_{\text{A}}$, and so m_{B} will accelerate down, m_{A} will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley will have that same acceleration since the cord is making it rotate, and so $\alpha_{\text{pulley}} = a/R$. From the free-body diagrams for each object, we have the following.

$$\sum F_{y\text{A}} = F_{\text{TA}} - m_{\text{A}}g = m_{\text{A}}a \;\rightarrow\; F_{\text{TA}} = m_{\text{A}}g + m_{\text{A}}a$$

$$\sum F_{y\text{B}} = m_{\text{B}}g - F_{\text{TB}} = m_{\text{B}}a \;\rightarrow\; F_{\text{TB}} = m_{\text{B}}g - m_{\text{B}}a$$

$$\sum \tau = F_{\text{TB}}r - F_{\text{TA}}r = I\alpha = I\frac{a}{R}$$

Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{\text{TB}}R - F_{\text{TA}}R = I\frac{a}{R} \;\rightarrow\; \left(m_{\text{B}}g - m_{\text{B}}a\right)R - \left(m_{\text{A}}g + m_{\text{A}}a\right)R = I\frac{a}{R} \;\rightarrow$$

$$\boxed{a = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}} + I/R^2\right)}g}$$

If the moment of inertia is ignored, then from the torque equation we see that $F_{\text{TB}} = F_{\text{TA}}$, and the acceleration will be $\boxed{a_{I=0} = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}}\right)}g}$. We see that the acceleration with the moment of inertia included will be smaller than if the moment of inertia is ignored.

52. (*a*) The free body diagram and analysis from problem 51 are applicable here, for the no-friction case.

$$a = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}} + I/r^2\right)}g = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}} + \frac{1}{2}m_{\text{P}}r^2/r^2\right)}g = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}} + \frac{1}{2}m_{\text{P}}\right)}g$$

$$= \frac{\left(3.80\,\text{kg} - 3.15\,\text{kg}\right)}{\left(3.80\,\text{kg} + 3.15\,\text{kg} + 0.40\,\text{kg}\right)}\left(9.80\,\text{m/s}^2\right) = 0.8667\,\text{m/s}^2 \approx \boxed{0.87\,\text{m/s}^2}$$

(*b*) With a frictional torque present, the torque equation from problem 51 would be modified, and the analysis proceeds as follows.

$$\sum \tau = F_{\text{TB}}r - F_{\text{TA}}r - \tau_{\text{fr}} = I\alpha = I\frac{a}{r} \;\rightarrow\; \left(m_{\text{B}}g - m_{\text{B}}a\right)r - \left(m_{\text{A}}g + m_{\text{A}}a\right)r - \tau_{\text{fr}} = I\frac{a}{r} \;\rightarrow$$

$$\tau_{\text{fr}} = r\left[\left(m_{\text{B}} - m_{\text{A}}\right)g - \left(m_{\text{B}} + m_{\text{A}} + \frac{I}{r^2}\right)a\right] = r\left[\left(m_{\text{B}} - m_{\text{A}}\right)g - \left(m_{\text{B}} + m_{\text{A}} + \frac{1}{2}m_{\text{p}}\right)a\right]$$

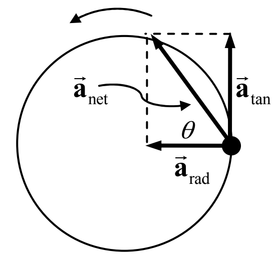
The acceleration can be found from the kinematical data and Eq. 2-12a.

$$v = v_0 + at \;\rightarrow\; a = \frac{v - v_0}{t} = \frac{0 - 0.20\,\text{m/s}}{6.2\,\text{s}} = -0.03226\,\text{m/s}^2$$

$$\tau_{\text{fr}} = r\left[\left(m_{\text{B}} - m_{\text{A}}\right)g - \left(m_{\text{B}} + m_{\text{A}} + \frac{1}{2}m_{\text{p}}\right)a\right]$$

$$= \left(0.040\,\text{m}\right)\left[\left(0.65\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right) - \left(7.35\,\text{kg}\right)\left(-0.03226\,\text{m/s}^2\right)\right] = \boxed{0.26\,\text{m}\bullet\text{N}}$$

53. A top view diagram of the hammer is shown, just at the instant of release, along with the acceleration vectors.



(*a*) The angular acceleration is found from Eq. 10-9c.

$$\omega^2=\omega_0^2+2\alpha\Delta\theta \;\rightarrow\; \alpha=\frac{\omega^2-\omega_0^2}{2\Delta\theta}=\frac{(v/r)^2-0}{2\Delta\theta}$$

$$=\frac{\left[(26.5\,\text{m/s})/(1.20\,\text{m})\right]^2}{2(8\pi\,\text{rad})}=9.702\,\text{rad/s}^2\approx\boxed{9.70\,\text{rad/s}^2}$$

(*b*) The tangential acceleration is found from the angular acceleration and the radius.

$$a_{\text{tan}}=\alpha r=\left(9.702\,\text{rad/s}^2\right)(1.20\text{ m})=11.64\,\text{m/s}^2\approx\boxed{11.6\,\text{m/s}^2}$$

(*c*) The centripetal acceleration is found from the speed and the radius.

$$a_{\text{rad}}=v^2/r=(26.5\,\text{m/s})^2/(1.20\text{ m})=585.2\,\text{m/s}^2\approx\boxed{585\,\text{m/s}^2}$$

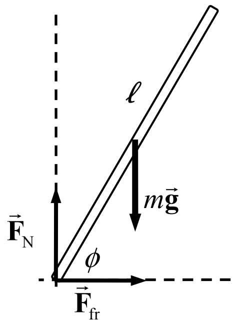
(*d*) The net force is the mass times the net acceleration. It is in the same direction as the net acceleration.

$$F_{\text{net}}=ma_{\text{net}}=m\sqrt{a_{\text{tan}}^2+a_{\text{rad}}^2}=(7.30\text{ kg})\sqrt{\left(11.64\,\text{m/s}^2\right)^2+\left(585.2\,\text{m/s}^2\right)^2}=\boxed{4270\,\text{N}}$$

(*e*) Find the angle from the two acceleration vectors.

$$\theta=\tan^{-1}\frac{a_{\text{tan}}}{a_{\text{rad}}}=\tan^{-1}\frac{11.64\,\text{m/s}^2}{585.2\,\text{m/s}^2}=\boxed{1.14^\circ}$$

54. (*a*) See the free body diagram for the falling rod. The axis of rotation would be coming out of the paper at the point of contact with the floor. There are contact forces between the rod and the table (the friction force and the normal force), but they act through the axis of rotation and so cause no torque. Thus only gravity causes torque. Write Newton's second law for the rotation of the rod. Take counterclockwise to be the positive direction for rotational quantities. Thus in the diagram, the angle is positive, but the torque is negative.



$$\sum\tau=I\alpha=-mg\left(\tfrac{1}{2}\ell\cos\phi\right)=\tfrac{1}{3}m\ell^2\frac{d\omega}{dt}\;\rightarrow$$

$$-\frac{3g}{2\ell}\cos\phi=\frac{d\omega}{dt}=\frac{d\omega}{d\phi}\frac{d\phi}{dt}=\frac{d\omega}{d\phi}\phi\;\rightarrow\;\frac{3g}{2\ell}\cos\phi\,d\phi=-\omega\,d\omega\;\rightarrow$$

$$\frac{3g}{2\ell}\int_{\pi/2}^{\phi}\cos\phi\,d\phi=-\int_0^{\omega}\omega\,d\omega\;\rightarrow\;\frac{3g}{2\ell}(\sin\phi-1)=-\tfrac{1}{2}\omega^2\;\rightarrow\;\boxed{\omega=\sqrt{\frac{3g}{\ell}(1-\sin\phi)}}$$

(*b*) The speed of the tip is the tangential speed of the tip, since the rod is rotating. At the tabletop, $\phi=0$.

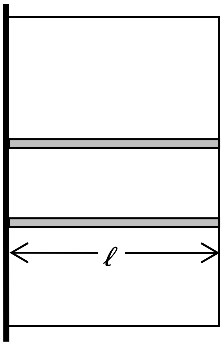
$$v=\omega\ell=\sqrt{3g\ell(1-\sin\phi)}\;\rightarrow\;v(0)=\boxed{\sqrt{3g\ell}}$$

$\boxed{55.}$ The parallel axis theorem is given in Eq. 10-17. The distance from the center of mass of the rod to the end of the rod is $h=\tfrac{1}{2}\ell$.

$$I=I_{\text{CM}}+Mh^2=\tfrac{1}{12}M\ell^2+M\left(\tfrac{1}{2}\ell\right)^2=\left(\tfrac{1}{12}+\tfrac{1}{4}\right)M\ell^2=\boxed{\tfrac{1}{3}M\ell^2}$$

56. We can consider the door to be made of a large number of thin horizontal rods, each of length $\ell = 1.0\,\text{m}$, and rotating about one end. Two such rods are shown in the diagram. The moment of inertia of one of these rods is $\frac{1}{3}m_i\ell^2$, where m_i is the mass of a single rod. For a collection of identical rods, then, the moment of inertia would be $I = \sum_i \frac{1}{3}m_i\ell^2 = \frac{1}{3}M\ell^2$. The height of the door does not enter into the calculation directly.

$$I = \tfrac{1}{3}M\ell^2 = \tfrac{1}{3}(19.0\,\text{kg})(1.0\,\text{m})^2 = \boxed{6.3\,\text{kg}\bullet\text{m}^2}$$



57. (*a*) The parallel axis theorem (Eq. 10-17) is to be applied to each sphere. The distance from the center of mass of each sphere to the axis of rotation is $h = 1.5r_0$.

$$I_{\substack{\text{for one}\\\text{sphere}}} = I_{\text{CM}} + Mh^2 = \tfrac{2}{5}Mr_0^2 + M(1.5r_0)^2 = 2.65Mr_0^2 \;\rightarrow\; I_{\text{total}} = \boxed{5.3Mr_0^2}$$

(*b*) Treating each mass as a point mass, the point mass would be a distance of $1.5r_0$ from the axis of rotation.

$$I_{\text{approx}} = 2\left[M(1.5r_0)^2\right] = 4.5Mr_0^2$$

$$\%\text{ error} = \left(\frac{I_{\text{approx}} - I_{\text{exact}}}{I_{\text{exact}}}\right)(100) = \left[\frac{4.5Mr_0^2 - 5.3Mr_0^2}{5.3Mr_0^2}\right](100) = \left[\frac{4.5-5.3}{5.3}\right](100)$$

$$= \boxed{-15\%}$$

The negative sign means that the approximation is smaller than the exact value, by about 15%.

58. (*a*) Treating the ball as a point mass, the moment of inertia about AB is $I = \boxed{MR_0^2}$.

(*b*) The parallel axis theorem is given in Eq. 10-17. The distance from the center of mass of the ball to the axis of rotation is $h = R_0$.

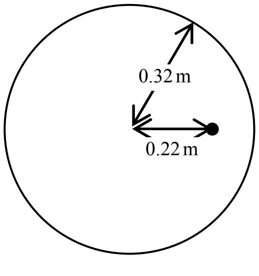
$$I = I_{\text{CM}} + Mh^2 = \boxed{\tfrac{2}{5}Mr_1^2 + MR_0^2}$$

(*c*)
$$\%\text{ error} = \left(\frac{I_{\text{approx}} - I_{\text{exact}}}{I_{\text{exact}}}\right)(100) = \left[\frac{MR_0^2 - \left(\frac{2}{5}Mr_1^2 + MR_0^2\right)}{\frac{2}{5}Mr_1^2 + MR_0^2}\right](100) = \frac{-\frac{2}{5}Mr_1^2}{\frac{2}{5}Mr_1^2 + MR_0^2}(100)$$

$$= \frac{-1}{1 + \frac{5}{2}(R_0/r_1)^2}(100) = -\frac{-1}{1 + \frac{5}{2}(1.0/0.090)^2}(100) = -0.32295 \approx \boxed{-0.32}$$

The negative sign means that the approximation is smaller than the exact value, by about 0.32%.

59. The 1.50-kg weight is treated as a point mass. The origin is placed at the center of the wheel, with the *x* direction to the right. Let A represent the wheel and B represent the weight.



(*a*)
$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \frac{(7.0\,\text{kg})(0) + (1.50\,\text{kg})(0.22\,\text{m})}{8.50\,\text{kg}}$$

$$= 3.88\times10^{-2}\,\text{m} \approx \boxed{0.039\,\text{m}}$$

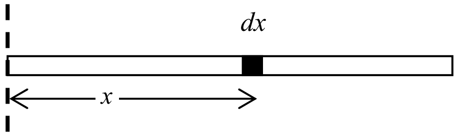
(*b*) The moment of inertia of the wheel is found from the parallel axis theorem.

$$I = I_{\text{wheel}} + I_{\text{weight}} = I_{\substack{\text{wheel}\\ \text{CM}}} + M_{\text{wheel}}x_{\text{CM}}^2 + M_{\text{weight}}\left(x_{\text{weight}} - x_{\text{CM}}\right)^2$$

$$= \tfrac{1}{2}M_{\text{wheel}}R^2 + M_{\text{wheel}}x_{\text{CM}}^2 + M_{\text{weight}}\left(x_{\text{weight}} - x_{\text{CM}}\right)^2$$

$$= \left(7.0\,\text{kg}\right)\left(\tfrac{1}{2}\left(0.32\,\text{m}\right)^2 + \left(0.0388\,\text{m}\right)^2\right) + \left(1.50\,\text{kg}\right)\left(0.22\,\text{m} - 0.0388\,\text{m}\right)^2 = \boxed{0.42\,\text{kg}\bullet\text{m}^2}$$

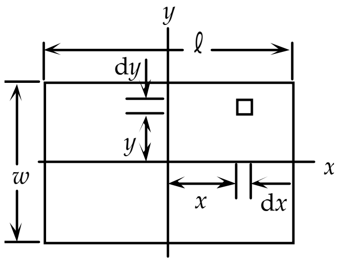
60. We calculate the moment of inertia about one end, and then use the parallel axis theorem to find the moment of inertia about the center. Let the mass of the rod be *M*, and use Eq. 10-16. A small mass *dM* can be found as a small length *dx* times the mass per unit length of the rod.



$$I_{\text{end}} = \int R^2 dM = \int_0^{\ell} x^2 \frac{M}{\ell}dx = \frac{M}{\ell}\frac{\ell^3}{3} = \tfrac{1}{3}M\ell^2$$

$$I_{\text{end}} = I_{\text{CM}} + M\left(\tfrac{1}{2}\ell\right)^2 \quad \rightarrow \quad I_{\text{CM}} = I_{\text{end}} - M\left(\tfrac{1}{2}\ell\right)^2 = \tfrac{1}{3}M\ell^2 - \tfrac{1}{4}M\ell^2 = \boxed{\tfrac{1}{12}M\ell^2}$$

$\boxed{61.}$ (*a*) We choose coordinates so that the center of the plate is at the origin. Divide the plate up into differential rectangular elements, each with an area of $dA = dxdy$. The mass of an element is $dm = \left(\frac{M}{\ell w}\right)dxdy$. The distance of that element from the axis of rotation is $R = \sqrt{x^2 + y^2}$. Use Eq. 10-16 to calculate the moment of inertia.

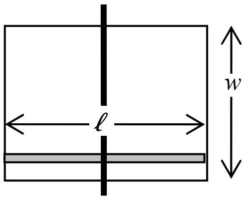


$$I_{\text{center}} = \int R^2 dM = \int_{-w/2}^{w/2}\int_{-\ell/2}^{\ell/2}\left(x^2 + y^2\right)\frac{M}{\ell w}dxdy = \frac{4M}{\ell w}\int_0^{w/2}\int_0^{\ell/2}\left(x^2 + y^2\right)dxdy$$

$$= \frac{4M}{\ell w}\int_0^{w/2}\left[\tfrac{1}{3}\left(\tfrac{1}{2}\ell\right)^3 + \left(\tfrac{1}{2}\ell\right)y^2\right]dy = \frac{2M}{w}\int_0^{w/2}\left[\tfrac{1}{12}\ell^2 + y^2\right]dy$$

$$= \frac{2M}{w}\left[\tfrac{1}{12}\ell^2\left(\tfrac{1}{2}w\right) + \tfrac{1}{3}\left(\tfrac{1}{2}w\right)^3\right] = \boxed{\tfrac{1}{12}M\left(\ell^2 + w^2\right)}$$

(*b*) For the axis of rotation parallel to the *w* dimension (so the rotation axis is in the *y* direction), we can consider the plate to be made of a large number of thin rods, each of length ℓ, rotating about an axis through their center. The moment of inertia of one of these rods is $\tfrac{1}{12}m_i\ell^2$, where m_i is the mass of a single rod. For a collection of identical rods, then, the moment of inertia would be $I_y = \sum_i \tfrac{1}{12}m_i\ell^2 = \boxed{\tfrac{1}{12}M\ell^2}$. A similar argument would give $I_x = \boxed{\tfrac{1}{12}Mw^2}$. This illustrates the perpendicular axis theorem, Eq. 10-18, $I_z = I_x + I_y$.



62. Work can be expressed in rotational quantities as $W = \tau\,\Delta\theta,$ and so power can be expressed in rotational quantities as $P = \dfrac{W}{\Delta t} = \tau\dfrac{\Delta\theta}{\Delta t} = \tau\omega.$

$$P=\tau\omega=\left(255\,\text{m}\bullet\text{N}\right)\left(3750\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1\,\text{hp}}{746\,\text{W}}\right)=\boxed{134\,\text{hp}}$$

63. The energy required to bring the rotor up to speed from rest is equal to the final rotational kinetic energy of the rotor.

$$K_{\text{rot}}=\tfrac{1}{2}I\omega^2=\tfrac{1}{2}\left(4.25\times10^{-2}\,\text{kg}\bullet\text{m}^2\right)\left[9750\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\right]^2=\boxed{2.22\times10^4\,\text{J}}$$

64. To maintain a constant angular speed ω_{steady} will require a torque τ_{motor} to oppose the frictional torque. The power required by the motor is $P=\tau_{\text{motor}}\omega_{\text{steady}}=-\tau_{\text{friction}}\omega_{\text{steady}}.$

$$\tau_{\text{friction}}=I\alpha_{\text{friction}}=\tfrac{1}{2}MR^2\left(\frac{\omega_f-\omega_0}{t}\right)\;\rightarrow$$

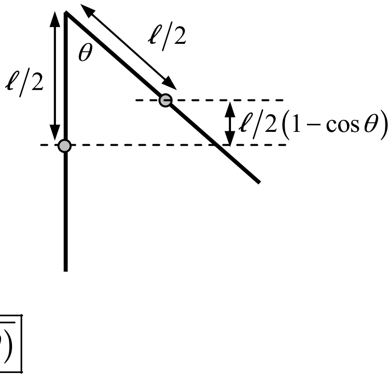
$$P_{\text{motor}}=\tfrac{1}{2}MR^2\left(\frac{\omega_0-\omega_f}{t}\right)\omega_{\text{steady}}=\tfrac{1}{2}\left(220\,\text{kg}\right)\left(5.5\,\text{m}\right)^2\frac{\left[\left(3.8\,\text{rev/s}\right)\left(\dfrac{2\pi\,\text{rad}}{\text{rev}}\right)\right]^2}{16\,\text{s}}=1.186\times10^5\,\text{W}$$

$$=1.186\times10^5\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right)=158.9\,\text{hp}\approx\boxed{160\,\text{hp}}$$

65. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W=\Delta K_{\text{rot}}=\tfrac{1}{2}I\omega_f^2-\tfrac{1}{2}I\omega_i^2=\tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2=\tfrac{1}{4}\left(1640\,\text{kg}\right)\left(7.50\,\text{m}\right)^2\left(\frac{2\pi\,\text{rad}}{8.00\,\text{s}}\right)^2=\boxed{1.42\times10^4\,\text{J}}$$

66. Mechanical energy will be conserved. The rotation is about a fixed axis, so $K_{\text{tot}}=K_{\text{rot}}=\tfrac{1}{2}I\omega^2$. For gravitational potential energy, we can treat the object as if all of its mass were at its center of mass. Take the lowest point of the center of mass as the zero location for gravitational potential energy.

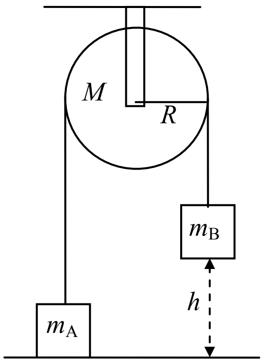


$$E_{\text{initial}}=E_{\text{final}}\;\rightarrow\;U_{\text{initial}}=K_{\text{final}}\;\rightarrow$$

$$Mg\tfrac{1}{2}\ell\left(1-\cos\theta\right)=\tfrac{1}{2}I\omega_{\text{bottom}}^2=\tfrac{1}{2}\left(\tfrac{1}{3}M\ell^2\right)\omega_{\text{bottom}}^2\;\rightarrow$$

$$\omega_{\text{bottom}}=\boxed{\sqrt{\frac{3g}{\ell}\left(1-\cos\theta\right)}}\;;\;v_{\text{bottom}}=\omega_{\text{bottom}}\ell=\boxed{\sqrt{3g\ell\left(1-\cos\theta\right)}}$$

$\boxed{67.}$ The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with m_{A} on the ground and all objects at rest. The final state of the system has m_{B} just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f \quad \rightarrow$$

$$\tfrac{1}{2}m_A v_i^2 + \tfrac{1}{2}m_B v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_A g y_{1i} + m_B g y_{2i} = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_A g y_{1f} + m_B g y_{2f}$$

$$m_B gh = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_A gh$$

$$v_f = \sqrt{\frac{2(m_B - m_A)gh}{\left(m_A + m_B + \tfrac{1}{2}M\right)}} = \sqrt{\frac{2(38.0\,\text{kg} - 35.0\,\text{kg})(9.80\,\text{m/s}^2)(2.5\,\text{m})}{\left(38.0\text{ kg} + 35.0\text{ kg} + \left(\tfrac{1}{2}\right)3.1\text{ kg}\right)}} = \boxed{1.4\,\text{m/s}}$$

68. (*a*) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass, and B the lighter mass.

$$K = \tfrac{1}{2}I_A\omega_A^2 + \tfrac{1}{2}I_B\omega_B^2 = \tfrac{1}{2}m_A r_A^2\omega_A^2 + \tfrac{1}{2}m_B r_B^2\omega_A^2 = \tfrac{1}{2}r^2\omega^2(m_A + m_B)$$
$$= \tfrac{1}{2}(0.210\,\text{m})^2(5.60\,\text{rad/s})^2(7.00\,\text{kg}) = \boxed{4.84\,\text{J}}$$

(*b*) The net force on each object produces centripetal motion, and so can be expressed as $mr\omega^2$.

$$F_A = m_A r_A \omega_A^2 = (4.00\,\text{kg})(0.210\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{26.3\,\text{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\text{kg})(0.210\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{19.8\,\text{N}}$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod, so that there is no net vertical force on either mass.

(*c*) Take the 4.00 kg mass to be the origin of coordinates for determining the center of mass.

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{(4.00\,\text{kg})(0) + (3.00\,\text{kg})(0.420\,\text{m})}{7.00\,\text{kg}} = 0.180\,\text{m from mass A}$$

So the distance from mass A to the axis of rotation is now 0.180 m, and the distance from mass B to the axis of rotation is now 0.24 m. Re-do the above calculations with these values.

$$K = \tfrac{1}{2}I_A\omega_A^2 + \tfrac{1}{2}I_B\omega_B^2 = \tfrac{1}{2}m_A r_A^2\omega_A^2 + \tfrac{1}{2}m_B r_B^2\omega_A^2 = \tfrac{1}{2}\omega^2\left(m_A r_A^2 + m_B r_B^2\right)$$
$$= \tfrac{1}{2}(5.60\,\text{rad/s})^2\left[(4.00\,\text{kg})(0.180\,\text{m})^2 + (3.00\,\text{kg})(0.240\,\text{m})^2\right] = \boxed{4.74\,\text{J}}$$

$$F_A = m_A r_A \omega_A^2 = (4.00\,\text{kg})(0.180\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\text{kg})(0.240\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

Note that the horizontal forces are now equal, and so there will be no horizontal force on the rod or axle.

69. Since the lower end of the pole does not slip on the ground, the friction does no work, and so mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as if it were at the CM. The final energy is rotational kinetic energy, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity

divided by the length.

$$E_{\text{intial}} = E_{\text{final}} \quad\rightarrow\quad U_{\text{initial}} = K_{\text{final}} \quad\rightarrow\quad mgh = \tfrac{1}{2}I\omega^2 \quad\rightarrow\quad mg\,L/2 = \tfrac{1}{2}\left(\tfrac{1}{3}mL^2\right)\left(v_{\text{end}}/L\right)^2 \quad\rightarrow$$

$$v_{\text{end}} = \sqrt{3gL} = \sqrt{3\left(9.80\,\text{m/s}^2\right)\left(2.30\text{ m}\right)} = \boxed{8.22\,\text{m/s}}$$

70. Apply conservation of mechanical energy. Take the bottom of the incline to be the zero location for gravitational potential energy. The energy at the top of the incline is then all gravitational potential energy, and at the bottom of the incline, there is both rotational and translational kinetic energy. Since the cylinder rolls without slipping, the angular velocity is given by $\omega = v/R$.

$$E_{\text{top}} = E_{\text{bottom}} \quad\rightarrow\quad Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\tfrac{1}{2}MR^2\frac{v^2}{R^2} = \tfrac{3}{4}Mv^2 \quad\rightarrow$$

$$v = \sqrt{\tfrac{4}{3}gh} = \sqrt{\tfrac{4}{3}\left(9.80\,\text{m/s}^2\right)\left(7.20\,\text{m}\right)} = \boxed{9.70\,\text{m/s}}$$

71. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = v/R$. The rotational inertia of a sphere about an axis through its center is $I = \tfrac{2}{5}mR^2$.

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\tfrac{2}{5}mR^2\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= 0.7\left(7.3\text{ kg}\right)\left(3.7\,\text{m/s}\right)^2 = \boxed{7.0\times10^1\text{ J}}$$

72. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$K_{\text{daily}} = \tfrac{1}{2}I\omega_{\text{daily}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}^2$$

$$= \tfrac{1}{5}\left(6.0\times10^{24}\text{kg}\right)\left(6.4\times10^6\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{1\text{day}}\right)\left(\frac{1\text{day}}{86{,}400\text{s}}\right)\right]^2 = \boxed{2.6\times10^{29}\text{J}}$$

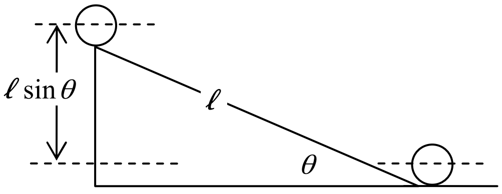
(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$K_{\text{yearly}} = \tfrac{1}{2}I\omega_{\text{yearly}}^2 = \tfrac{1}{2}\left(MR_{\substack{\text{Sun-}\\\text{Earth}}}^2\right)\omega_{\text{yearly}}^2$$

$$= \tfrac{1}{2}\left(6.0\times10^{24}\text{kg}\right)\left(1.5\times10^{11}\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{365\text{day}}\right)\left(\frac{1\text{day}}{86{,}400\text{s}}\right)\right]^2 = \boxed{2.7\times10^{33}\text{J}}$$

Thus the total kinetic energy is $K_{\text{daily}} + K_{\text{yearly}} = 2.6\times10^{29}\text{J} + 2.7\times10^{33}\text{J} = \boxed{2.7\times10^{33}\text{J}}$. The kinetic energy due to the daily motion is about 10,000 times smaller than that due to the yearly motion.

73. (*a*) Mechanical energy is conserved as the sphere rolls without slipping down the plane. Take the zero level of gravitational potential energy to the level of the center of mass of the sphere when it is on the level surface at the bottom of the plane. All of the energy is potential energy at the top, and all is kinetic energy (of both translation and rotation) at the bottom.



$$E_{\text{intial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} = K_{\text{CM}} + K_{\text{rot}} \rightarrow$$

$$mgh = mg\ell\sin\theta = \tfrac{1}{2}mv_{\text{bottom}}^2 + \tfrac{1}{2}I\omega_{\text{bottom}}^2 = \tfrac{1}{2}mv_{\text{bottom}}^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr_0^2\right)\left(\frac{v_{\text{bottom}}}{r_0}\right)^2 \rightarrow$$

$$v_{\text{bottom}} = \sqrt{\tfrac{10}{7}gh} = \sqrt{\tfrac{10}{7}g\ell\sin\theta} = \sqrt{\tfrac{10}{7}\left(9.80\,\text{m/s}^2\right)\left(10.0\,\text{m}\right)\sin 30.0^\circ} = 8.367\,\text{m/s}$$

$$\approx \boxed{8.37\,\text{m/s}}$$

$$\omega_{\text{bottom}} = \frac{v_{\text{bottom}}}{r_0} = \frac{8.367\,\text{m/s}}{0.254\,\text{m}} = \boxed{32.9\,\text{rad/s}}$$

(*b*) $$\frac{K_{\text{CM}}}{K_{\text{rot}}} = \frac{\tfrac{1}{2}mv_{\text{bottom}}^2}{\tfrac{1}{2}I\omega_{\text{bottom}}^2} = \frac{\tfrac{1}{2}mv_{\text{bottom}}^2}{\tfrac{1}{2}\left(\tfrac{2}{5}mr_0^2\right)\left(\frac{v_{\text{bottom}}}{r_0}\right)^2} = \boxed{\frac{5}{2}}$$

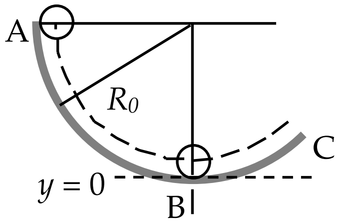
(*c*) The translational speed at the bottom, and the ratio of kinetic energies, are both independent of the radius and the mass. The rotational speed at the bottom depends on the radius.

74. (*a*) Since the center of mass of the spool is stationary, the net force must be 0. Thus the force on the thread must be equal to the weight of the spool and so $\boxed{F_{\text{thread}} = Mg}$.

(*b*) By the work–energy theorem, the work done is the change in kinetic energy of the spool The spool has rotational kinetic energy.

$$W = K_{\text{final}} - K_{\text{initial}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 = \boxed{\tfrac{1}{4}MR^2\omega^2}$$

75. Use conservation of mechanical energy to equate the energy at points A and B . Call the zero level for gravitational potential energy to be the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega = v/r_0$.

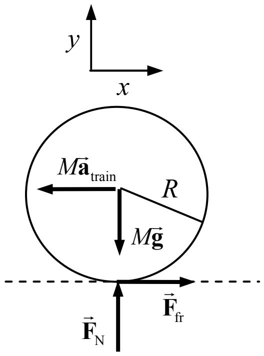


$$E_{\text{A}} = E_{\text{B}} \rightarrow U_{\text{A}} = U_{\text{B}} + K_{\text{B final}} = U_{\text{B}} + K_{\text{B CM}} + K_{\text{B rot}} \rightarrow$$

$$mgR_0 = mgr_0 + \tfrac{1}{2}mv_{\text{B}}^2 + \tfrac{1}{2}I\omega_{\text{B}}^2$$

$$= mgr_0 + \tfrac{1}{2}mv_{\text{B}}^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr_0^2\right)\left(\frac{v_{\text{B}}}{r_0}\right)^2 \rightarrow v_{\text{B}} = \boxed{\sqrt{\tfrac{10}{7}g\left(R_0 - r_0\right)}}$$

76. (*a*) We work in the accelerating reference frame of the car. In the accelerating frame, we must add a fictitious force of magnitude $Ma_{\substack{\text{train rel}\\\text{ground}}}$, in the opposite direction to the acceleration of the train. This is discussed in detail in section 11-8 of the textbook. Since the ball is rolling without slipping, $\alpha = a_{\substack{\text{ball rel}\\\text{train}}}/R$. See the free-body diagram for the ball in the accelerating reference frame. Write Newton's second law for the horizontal direction and for torques, with clockwise torques as positive. Combine these relationships to find $a_{\substack{\text{ball rel}\\\text{train}}}$, the acceleration of the ball in the accelerated frame.



$$\sum\tau = -F_{\text{fr}}R = I\alpha = I\frac{a_{\substack{\text{ball rel}\\\text{train}}}}{R} \rightarrow F_{\text{fr}} = -\tfrac{2}{5}Ma_{\substack{\text{ball rel}\\\text{train}}}$$

$$\sum F_x = F_{\text{fr}} - Ma_{\substack{\text{train rel}\\\text{ground}}} = Ma_{\substack{\text{ball rel}\\\text{train}}} \rightarrow -\tfrac{2}{5}Ma_{\substack{\text{ball rel}\\\text{train}}} - Ma_{\substack{\text{train rel}\\\text{ground}}} = Ma_{\substack{\text{ball rel}\\\text{train}}} \rightarrow$$

$$\boxed{a_{\substack{\text{ball rel}\\\text{train}}} = -\tfrac{5}{7}a_{\substack{\text{train rel}\\\text{ground}}}}$$

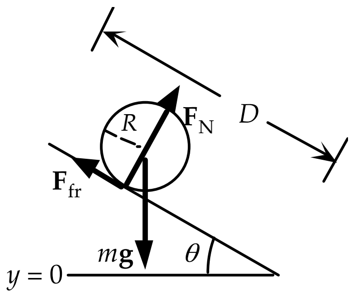
And so as seen from inside the train, the ball is accelerating backwards.

(*b*) Use the relative acceleration relationship.

$$a_{\substack{\text{ball rel}\\\text{ground}}} = a_{\substack{\text{ball rel}\\\text{train}}} + a_{\substack{\text{train rel}\\\text{ground}}} = -\tfrac{5}{7}a_{\substack{\text{train rel}\\\text{ground}}} + a_{\substack{\text{train rel}\\\text{ground}}} = \boxed{\tfrac{2}{7}a_{\text{train}}}$$

And so as seen from outside the train, the ball is accelerating forwards, but with a smaller acceleration than the train.

77. (*a*) Use conservation of mechanical energy. Call the zero level for gravitational potential energy to be the lowest point on which the pipe rolls. Since the pipe rolls without slipping, $\omega = v/R$. See the attached diagram.



$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} = K_{\text{CM}} + K_{\text{rot}}$$

$$mgD\sin\theta = \tfrac{1}{2}mv^2_{\text{bottom}} + \tfrac{1}{2}I\omega^2_{\text{bottom}}$$

$$= \tfrac{1}{2}mv^2_{\text{bottom}} + \tfrac{1}{2}\left(mR^2\right)\left(\frac{v^2_{\text{bottom}}}{R^2}\right) = mv^2_{\text{bottom}} \rightarrow$$

$$v_{\text{bottom}} = \sqrt{gD\sin\theta} = \sqrt{\left(9.80\,\text{m/s}^2\right)\left(5.60\,\text{m}\right)\sin 17.5^\circ} = \boxed{4.06\,\text{m/s}}$$

(*b*) The total kinetic energy at the base of the incline is the same as the initial potential energy.

$$K_{\text{final}} = U_{\text{initial}} = mgD\sin\theta = \left(0.545\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(5.60\,\text{m}\right)\sin 17.5^\circ = \boxed{8.99\,\text{J}}$$

(*c*) The frictional force supplies the torque for the object to roll without slipping, and the frictional force has a maximum value. Since the object rolls without slipping, $\alpha = a/R$. Use Newton's second law for the directions parallel and perpendicular to the plane, and for the torque, to solve for the coefficient of friction.

$$\sum\tau = F_{\text{fr}}R = I\alpha = mR^2\frac{a}{R} = maR \rightarrow F_{\text{fr}} = ma$$

$$\sum F_{\perp} = F_{\text{N}} - mg\cos\theta \rightarrow F_{\text{N}} = mg\cos\theta$$

$$\sum F_{\parallel} = mg\sin\theta - F_{\text{fr}} = ma \rightarrow F_{\text{fr}} = \tfrac{1}{2}mg\sin\theta$$

$$F_{\text{fr}} \le F_{\substack{\text{static}\\\text{max}}} \rightarrow \tfrac{1}{2}mg\sin\theta \le \mu_{\text{s}}F_N = \mu_{\text{s}}mg\cos\theta \rightarrow \mu_{\text{s}} \ge \tfrac{1}{2}\tan\theta \rightarrow$$

$$\mu_{\substack{\text{s}\\\text{min}}} = \tfrac{1}{2}\tan\theta = \tfrac{1}{2}\tan 17.5^\circ = \boxed{0.158}$$

78. (*a*) While the ball is slipping, the acceleration of the center of mass is constant, and so constant acceleration relationships may be used. Use Eq. 2-12b with results from Example 10-20.

$$x - x_0 = v_0t + \tfrac{1}{2}at^2 = v_0\left(\frac{2v_0}{7\mu_k g}\right) + \tfrac{1}{2}\left(-\mu_k g\right)\left(\frac{2v_0}{7\mu_k g}\right)^2 = \boxed{\frac{12v_0^2}{49\mu_k g}}$$

(*b*) Again make use of the fact that the acceleration is constant. Once the final speed is reached, the angular velocity is given by $\omega = v/r_0$.

$$v = v_0 + at = v_0 + (-\mu_k g)\left(\frac{2v_0}{7\mu_k g}\right) = \boxed{\tfrac{5}{7}v_0} \;;\; \omega = \boxed{\tfrac{5}{7}v_0/r_0}$$

$\boxed{79.}$ (*a*) The total kinetic energy included the translational kinetic energy of the car's total mass, and the rotational kinetic energy of the car's wheels. The wheels can be treated as one cylinder. We assume the wheels are rolling without slipping, so that $v_{\text{CM}} = \omega/R_{\text{wheels}}$.

$$K_{\text{tot}} = K_{\text{CM}} + K_{\text{rot}} = \tfrac{1}{2}M_{\text{tot}}v_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{wheels}}\omega^2 = \tfrac{1}{2}M_{\text{tot}}v_{\text{CM}}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_{\text{wheels}}R_{\text{wheels}}^2\right)\frac{v_{\text{CM}}^2}{R_{\text{wheels}}^2}$$

$$= \tfrac{1}{2}\left(M_{\text{tot}} + \tfrac{1}{2}M_{\text{wheels}}\right)v_{\text{CM}}^2 = \tfrac{1}{2}(1170\,\text{kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = 4.074\times10^5\,\text{J}$$

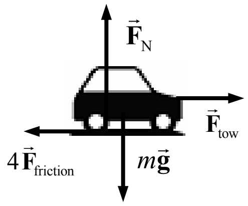
$$\approx \boxed{4.1\times10^5\,\text{J}}$$

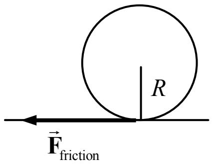
(*b*) The fraction of kinetic energy in the tires and wheels is $\dfrac{K_{\text{rot}} + K_{\substack{\text{trans}\\\text{wheels}}}}{K_{\text{tot}}}$.

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{\tfrac{1}{2}I_{\text{wheels}}\omega^2 + \tfrac{1}{2}M_{\text{wheels}}v_{\text{CM}}^2}{\tfrac{1}{2}M_{\text{tot}}v_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{wheels}}\omega^2} = \frac{\tfrac{1}{2}\left(\tfrac{1}{2}M_{\text{wheels}} + M_{\text{wheels}}\right)v_{\text{CM}}^2}{\tfrac{1}{2}\left(M_{\text{tot}} + \tfrac{1}{2}M_{\text{wheels}}\right)v_{\text{CM}}^2} = \frac{\left(\tfrac{3}{2}M_{\text{wheels}}\right)}{\left(M_{\text{tot}} + \tfrac{1}{2}M_{\text{wheels}}\right)}$$

$$= \frac{210\,\text{kg}}{1170\,\text{kg}} = \boxed{0.18}$$

(*c*) A free body diagram for the car is shown, with the frictional force of $\vec{\mathbf{F}}_{\text{fr}}$ at each wheel to cause the wheels to roll. A separate diagram of one wheel is also shown. Write Newton's second law for the horizontal motion of the car as a whole, and the rotational motion of one wheel. Take clockwise torques as positive. Since the wheels are rolling without slipping, $a_{\text{CM}} = \alpha/R_{\text{wheels}}$.





$$\sum\tau = 4F_{\text{fr}}R = I_{\text{wheels}}\alpha = \tfrac{1}{2}M_{\text{wheels}}R_{\text{wheels}}^2\frac{a_{\text{CM}}}{R_{\text{wheels}}} \quad\rightarrow$$

$$F_{\text{fr}} = \tfrac{1}{8}M_{\text{wheels}}a_{\text{CM}}$$

$$\sum F_x = F_{\text{tow}} - 4F_{\text{fr}} = M_{\text{tot}}a_{\text{CM}} \quad\rightarrow$$

$$F_{\text{tow}} - 4\left(\tfrac{1}{8}M_{\text{wheels}}a_{\text{CM}}\right) = M_{\text{tot}}a_{\text{CM}} \quad\rightarrow$$

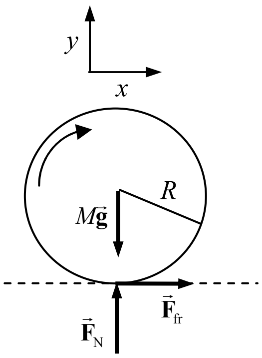
$$a_{\text{CM}} = \frac{F_{\text{tow}}}{\left(M_{\text{tot}} + \tfrac{1}{2}M_{\text{wheels}}\right)} = \frac{1500\,\text{N}}{(1170\,\text{kg})} = 1.282\,\text{m/s}^2 \approx \boxed{1.3\,\text{m/s}^2}$$

(*d*) If the rotational inertia were ignored, we would have the following.

$$\sum F_x = F_{\text{tow}} = M_{\text{tot}}a_{\text{CM}} \quad\rightarrow\quad a_{\text{CM}} = \frac{F_{\text{tow}}}{M_{\text{tot}}} = \frac{1500\,\text{N}}{1100\,\text{kg}} = 1.364\,\text{m/s}^2$$

$$\%\ \text{error} = \frac{\Delta a_{\text{CM}}}{a_{\text{CM}}}\times100 = \frac{1.364\,\text{m/s}^2 - 1.282\,\text{m/s}^2}{1.282\,\text{m/s}^2}\times100 = \boxed{6\%}$$

80. (*a*) The friction force accelerates the center of mass of the wheel. If the wheel is spinning (and slipping) clockwise in the diagram, then the surface of the wheel that touches the ground is moving to the left, and the friction force is to the right or forward. It acts in the direction of motion of the velocity of the center of mass of the wheel.



(*b*) Write Newton's second law for the *x* direction, the *y* direction, and the rotation. Take clockwise torques (about the center of mass) as positive.

$$\sum F_y = F_{\text{N}} - Mg = 0 \quad\rightarrow\quad F_{\text{N}} = Mg$$

$$\sum F_x = F_{\text{fr}} = Ma \quad\rightarrow\quad a = \frac{F_{\text{fr}}}{M} = \frac{\mu_{\text{k}}F_{\text{N}}}{M} = \frac{\mu_{\text{k}}Mg}{M} = \mu_{\text{k}}g$$

$$\sum \tau = -F_{\text{fr}}R = I\alpha \quad\rightarrow\quad \alpha = -\frac{F_{\text{fr}}R}{\frac{1}{2}MR^2} = -\frac{2Mg}{MR} = -\frac{2\mu_{\text{k}}g}{R}$$

Both the acceleration and angular acceleration are constant, and so constant acceleration kinematics may be used to express the velocity and angular velocity.

$$v = v_0 + at = \mu_{\text{k}}gt \;\;;\;\; \omega = \omega_0 + \alpha t = \omega_0 - \frac{2\mu_{\text{k}}g}{R}t$$

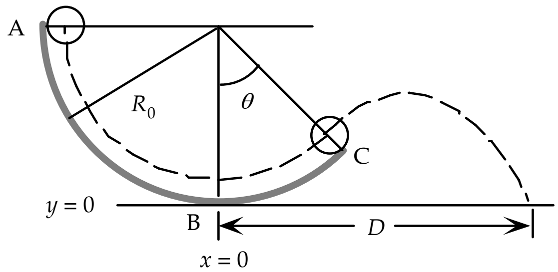
Note that the velocity starts at 0 and increases, while the angular velocity starts at ω_0 and decreases. Thus at some specific time T, the velocity and angular velocity will be $\omega = v/R$, and the ball will roll without slipping. Solve for the value of T needed to make that true.

$$\omega = v/R \quad\rightarrow\quad \omega_0 - \frac{2\mu_{\text{k}}g}{R}T = \mu_{\text{k}}gT/R \quad\rightarrow\quad T = \boxed{\frac{\omega_0 R}{3\mu_{\text{k}}g}}$$

(*c*) Once the ball starts rolling without slipping, there is no more frictional sliding force, and so the velocity will remain constant.

$$v_{\text{final}} = \mu_{\text{k}}gT = \mu_{\text{k}}g\frac{\omega_0 R}{3\mu_{\text{k}}g} = \boxed{\tfrac{1}{3}R\omega_0}$$

81. (a) Use conservation of mechanical energy to equate the energy at point A to the energy at point C. Call the zero level for gravitational potential energy to be the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega = v/r_0$. All locations given for the ball are for its center of mass.



$$E_{\text{A}} = E_{\text{C}} \quad\rightarrow$$

$$U_{\text{A}} = U_{\text{C}} + K_{\text{C}} = U_{\text{C}} + K_{\underset{\text{CM}}{\text{C}}} + K_{\underset{\text{rot}}{\text{C}}} \quad\rightarrow$$

$$mgR_0 = mg\left[R_0 - \left(R_0 - r_0\right)\cos\theta\right] + \tfrac{1}{2}mv_{\text{C}}^2 + \tfrac{1}{2}I\omega_{\text{C}}^2$$

$$= mg\left[R_0 - \left(R_0 - r_0\right)\cos\theta\right] + \tfrac{1}{2}mv_{\text{C}}^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr_0^2\right)\frac{v_{\text{C}}^2}{r_0^2} \quad\rightarrow$$

$$v_{\text{C}} = \sqrt{\tfrac{10}{7}g\left(R_0 - r_0\right)\cos\theta} = \sqrt{\tfrac{10}{7}\left(9.80\,\text{m/s}^2\right)\left(0.245\,\text{m}\right)\cos 45^\circ} = 1.557\,\text{m/s} \approx \boxed{1.6\,\text{m/s}}$$

(*b*) Once the ball leaves the ramp, it will move as a projectile under the influence of gravity, and the constant acceleration equations may be used to find the distance. The initial location of the ball is given by $x_0 = \left(R_0 - r_0\right)\sin 45^\circ$ and $y_0 = R_0 - \left(R_0 - r_0\right)\cos 45^\circ$. The initial velocity of the ball

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is given by $v_{0x} = v_\text{C}\cos 45°$ and $v_{0y} = v_\text{C}\sin 45°$. The ball lands when $y = r_0 = 0.015\,\text{m}$. Find the time of flight from the vertical motion, and then find D from the horizontal motion. Take the upward direction as positive for the vertical motion.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2 = R_0 - (R_0 - r_0)\cos 45° + v_\text{C}\sin 45° t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$4.90t^2 - 1.101t - 0.07178 = 0 \quad \rightarrow \quad t = 0.277\,\text{s},\ -0.0528\,\text{s}$$

We use the positive time.

$$D = x = x_0 + v_{0x}t = (R_0 - r_0)\sin 45° + v_\text{C}\cos 45° t$$

$$= (0.245\,\text{m})\sin 45° + (1.557\,\text{m/s})\cos 45°(0.277\,\text{s}) = 0.4782\,\text{m} \approx \boxed{0.48\,\text{m}}$$

82. Write the rotational version of Newton's second law, with counterclockwise torques as positive.

$$\tau_\text{net} = \tau_\text{N} - \tau_\text{fr} = F_\text{N}\ell - FR = I_\text{CN}\alpha_\text{CM} = \tfrac{2}{5}MR^2\alpha_\text{CM}$$

Newton's second law for the translational motion, with left as the positive direction, gives the following.

$$F_\text{net} = F = ma \quad \rightarrow \quad a = \frac{F}{m}$$

If the sphere is rolling without slipping, we have $\alpha_\text{CM} = a/R$. Combine these relationships to analyze the relationship between the torques.

$$F_\text{N}\ell = FR + \tfrac{2}{5}MR^2\alpha_\text{CM} = FR + \tfrac{2}{5}MR^2\frac{a}{R} = FR + \tfrac{2}{5}MaR = FR + \tfrac{2}{5}FR = \tfrac{7}{5}FR \quad \rightarrow$$

$$\boxed{\tau_\text{N} = \tfrac{7}{5}\tau_\text{fr}}$$

And since the torque due to the normal force is larger than the torque due to friction, the sphere has a counterclockwise angular acceleration, and thus the rotational velocity will decrease.

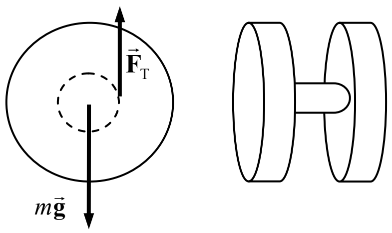
83. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $v = r\omega = v_\text{CM}$ relative to the center of the spool, where v_CM is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2v_\text{CM}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance ℓ, in the same time the center of the spool, traveling with half the speed, moves forward a distance $\boxed{\ell/2}$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\boxed{\ell/2}$ also.

84. The linear speed is related to the angular velocity by $v = \omega R$, and the angular velocity (rad / sec) is related to the frequency (rev / sec) by Eq. 10-7, $\omega = 2\pi f$. Combine these relationships to find values for the frequency.

$$\omega = 2\pi f = \frac{v}{R} \quad \rightarrow \quad f = \frac{v}{2\pi R} \quad ; \ f_1 = \frac{v}{2\pi R_1} = \frac{1.25\,\text{m/s}}{2\pi(0.025\,\text{m})}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{480\,\text{rpm}}$$

$$f_2 = \frac{v}{2\pi R_2} = \frac{1.25\,\text{m/s}}{2\pi(0.058\,\text{m})}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{210\,\text{rpm}}$$

$\boxed{85.}$ (*a*) There are two forces on the yo-yo: gravity and string tension. If the top of the string is held fixed, then the tension does no work, and so mechanical energy is conserved. The initial gravitational potential energy is converted into rotational and translational kinetic energy. Since the yo-yo rolls without slipping at the point of contact of the string, the velocity of the CM is related to the angular velocity of the yo-yo by $v_{\text{CM}} = r\omega$, where *r* is the radius of the inner hub. Let *m* be the mass of the inner hub, and *M* and *R* be the mass and radius of each outer disk. Calculate the rotational inertia of the yo-yo about its CM, and then use conservation of energy to find the linear speed of the CM. We take the 0 of gravitational potential energy to be at the bottom of its fall.



$$I_{\text{CM}} = \tfrac{1}{2}mr^2 + 2\left(\tfrac{1}{2}MR^2\right) = \tfrac{1}{2}mr^2 + MR^2$$

$$= \tfrac{1}{2}\left(5.0\times10^{-3}\,\text{kg}\right)\left(5.0\times10^{-3}\,\text{m}\right)^2 + \left(5.0\times10^{-2}\,\text{kg}\right)\left(3.75\times10^{-2}\,\text{m}\right)^2 = 7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2$$

$$m_{\text{total}} = m + 2M = 5.0\times10^{-3}\,\text{kg} + 2\left(5.0\times10^{-2}\,\text{kg}\right) = 0.105\ \text{kg}$$

$$U_{\text{initial}} = K_{\text{final}} \quad \rightarrow$$

$$m_{\text{total}}gh = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}\frac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2 = \left(\tfrac{1}{2}m_{\text{total}} + \tfrac{1}{2}\frac{I_{\text{CM}}}{r^2}\right)v_{\text{CM}}^2 \quad \rightarrow$$

$$v_{\text{CM}} = \sqrt{\frac{m_{\text{total}}gh}{\tfrac{1}{2}\left(m_{\text{total}} + \dfrac{I_{\text{CM}}}{r^2}\right)}} = \sqrt{\frac{(0.105\ \text{kg})\left(9.80\,\text{m/s}^2\right)(1.0\ \text{m})}{\tfrac{1}{2}\left[(0.105\ \text{kg}) + \dfrac{\left(7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2\right)}{\left(5.0\times10^{-3}\,\text{m}\right)^2}\right]}} = 0.8395 = \boxed{0.84\,\text{m/s}}$$

(*b*) Calculate the ratio $K_{\text{rot}}/K_{\text{tot}}$.

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{K_{\text{rot}}}{U_{\text{initial}}} = \frac{\tfrac{1}{2}I_{\text{CM}}\omega^2}{m_{\text{total}}gh} = \frac{\tfrac{1}{2}\dfrac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2}{m_{\text{total}}gh} = \frac{I_{\text{CM}}v_{\text{CM}}^2}{2r^2m_{\text{total}}gh}$$

$$= \frac{\left(7.038\times10^{-5}\,\text{kg}\bullet\text{m}^2\right)\left(0.8395\,\text{m/s}\right)^2}{2\left(5.0\times10^{-3}\,\text{m}\right)^2(0.105\ \text{kg})\left(9.8\,\text{m/s}^2\right)(1.0\ \text{m})} = 0.96 = \boxed{96\%}$$

86. As discussed in the text, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $v = r\omega$, where v is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of v to the right relative to the axle, so it has a velocity of $2v$ to the right relative to the ground.

$$\vec{\mathbf{v}}_{\substack{\text{top rel}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{top rel}\\ \text{center}}} + \vec{\mathbf{v}}_{\substack{\text{center rel}\\ \text{ground}}} = (v\ \text{to the right}) + (v\ \text{to the right}) = 2v\ \text{to the right}$$

$$v_{\substack{\text{top rel}\\ \text{ground}}} = 2v = 2\left(v_0 + at\right) = 2at = 2\left(1.00\,\text{m/s}^2\right)(2.5\,\text{s}) = \boxed{5.0\,\text{m/s}}$$

87. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$\tau = I\alpha = \left(mr^2\right)\left(\frac{\omega - \omega_0}{t}\right) = \frac{(0.50\,\text{kg})(1.5\,\text{m})^2}{5.0\,\text{s}}\left[\left(85\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\right] = \boxed{2.0\,\text{m}\bullet\text{N}}$$

That torque comes from the arm swinging the sling, and so comes from the arm muscles.

88. The torque is found from $\tau = I\alpha$. The angular acceleration can be found from $\omega = \omega_o + \alpha t$, with an initial angular velocity of 0. The rotational inertia is that of a cylinder.

$$\tau = I\alpha = \tfrac{1}{2}MR^2\left(\frac{\omega-\omega_o}{t}\right) = 0.5(1.4\,\text{kg})(0.20\,\text{m})^2\frac{(1800\,\text{rev/s})(2\pi\,\text{rad/rev})}{6.0\,\text{s}} = \boxed{53\,\text{m}\bullet\text{N}}$$

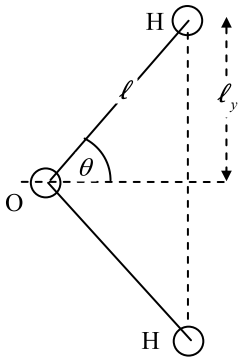
89. (*a*) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $v = \omega R$, and so $\omega_R R_R = \omega_F R_F$. If the spacing of the teeth on the sprockets is a distance *d*, then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$Nd = 2\pi R \text{ and so } R = \frac{Nd}{2\pi}. \text{ Thus } \omega_R\frac{N_R d}{2\pi} = \omega_F\frac{N_F d}{2\pi} \quad\rightarrow\quad \boxed{\frac{\omega_R}{\omega_F} = \frac{N_F}{N_R}}$$

(*b*) $\boxed{\omega_R/\omega_F = 52/13 = 4.0}$

(*c*) $\boxed{\omega_R/\omega_F = 42/28 = 1.5}$

90. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is 1.66×10^{-27} kg. Since the axis passes through the oxygen atom, it will have no rotational inertia.



(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance ℓ from the axis of rotation.

$$I_{\text{perp}} = 2m_H\ell^2 = 2(1.01)(1.66\times10^{-27}\,\text{kg})(0.96\times10^{-9}\,\text{m})^2$$
$$= \boxed{3.1\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H-O-H bonds, each hydrogen atom is a distance of $\ell_y = \ell\sin\theta = (9.6\times10^{-10}\,\text{m})\sin 52^\circ$

$= 7.564\times10^{-10}\,\text{m}$. Thus the moment of inertia is as follows.

$$I_{\text{plane}} = 2m_H\ell_y^2 = 2(1.01)(1.66\times10^{-27}\,\text{kg})(7.564\times10^{-10}\,\text{m})^2 = \boxed{1.9\times10^{-45}\,\text{kg}\bullet\text{m}^2}$$

$\boxed{91.}$ (*a*) The initial energy of the flywheel is used for two purposes – to give the car translational kinetic energy 20 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$W_{\text{fr}} = K_{\text{final}} - K_{\text{initial}} \quad\rightarrow\quad F_{\text{fr}}\Delta x\cos180^\circ = \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2 - K_{\text{flywheel}}$$

$$K_{\text{flywheel}} = F_{\text{fr}}\Delta x + \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2$$

$$= (450\,\text{N})(3.5\times10^5\,\text{m}) + (20)\tfrac{1}{2}(1100\,\text{kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2$$

$$= 1.652\times10^8\,\text{J} \approx \boxed{1.7\times10^8\,\text{J}}$$

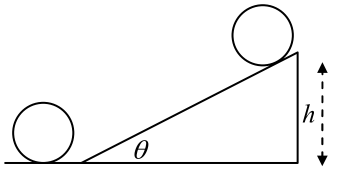
(*b*) $K_{\text{flywheel}} = \tfrac{1}{2}I\omega^2$

$$\omega = \sqrt{\frac{2\,KE}{I}} = \sqrt{\frac{2\,KE}{\tfrac{1}{2}M_{\text{flywheel}}R_{\text{flywheel}}^2}} = \sqrt{\frac{2(1.652\times10^8\,\text{J})}{\tfrac{1}{2}(240\,\text{kg})(0.75\,\text{m})^2}} = \boxed{2200\,\text{rad/s}}$$

(*c*) To find the time, use the relationship that $\text{power} = \dfrac{\text{work}}{\text{time}}$, where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$P = \frac{W}{t} \quad \rightarrow \quad t = \frac{W}{P} = \frac{\left(1.652\times10^{8}\,\text{J}\right)}{\left(150\,\text{hp}\right)\left(746\ \text{W/hp}\right)} = 1.476\times10^{3}\,\text{s} \approx \boxed{25\,\text{min}}$$

92. (*a*) Assuming that there are no dissipative forces doing work, conservation of mechanical energy may be used to find the final height *h* of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = v/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh \quad \rightarrow \quad \tfrac{1}{2}mv^2 + \tfrac{1}{2}mR^2\frac{v^2}{R^2} = mgh \quad \rightarrow$$

$$h = \frac{v^2}{g} = \frac{\left(3.3\,\text{m/s}\right)^2}{9.80\,\text{m/s}^2} = 1.111\ \text{m}$$

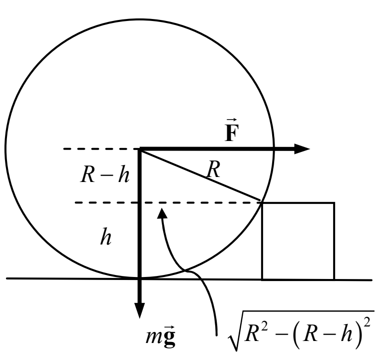
The distance along the plane is given by $d = \dfrac{h}{\sin\theta} = \dfrac{1.111\ \text{m}}{\sin 15^\circ} = 4.293\ \text{m} \approx \boxed{4.3\ \text{m}}$

(*b*) The time can be found from the constant acceleration linear motion.

$$\Delta x = \tfrac{1}{2}\left(v + v_o\right)t \quad \rightarrow \quad t = \frac{2\Delta x}{v + v_o} = \frac{2\left(4.293\ \text{m}\right)}{0 + 3.3\,\text{m/s}} = 2.602\ \text{s}$$

This is the time to go up the plane. The time to come back down the plane is the same, and so the total time is $\boxed{5.2\,\text{s}}$.

93. The wheel is rolling about the point of contact with the step, and so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel – the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum \tau = F\left(R - h\right) - mg\sqrt{R^2 - \left(R - h\right)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2 - \left(R - h\right)^2}}{R - h} = \boxed{\frac{Mg\sqrt{2Rh - h^2}}{R - h}}$$

94. Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of gravitational potential energy to be the bottom of the track, so that the bottom of the ball is initially a height *h* above the 0 position of gravitational potential energy. We also assume that the

marble is rolling without slipping, so $\omega = v/r$, and that the marble is released from rest. The marble has both translational and rotational kinetic energy.

(*a*) Since $r \ll R$, the marble's CM is very close to the surface of the track. While the marble is on the loop, we then approximate that its CM will be moving in a circle of radius *R*. When the marble is at the top of the loop, we approximate that its CM is a distance of 2*R* above the 0 position of gravitational potential energy. For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$\frac{mv^2_{\substack{\text{top of}\\\text{loop}}}}{R} = mg \quad\rightarrow\quad v^2_{\substack{\text{top of}\\\text{loop}}} = gR$$

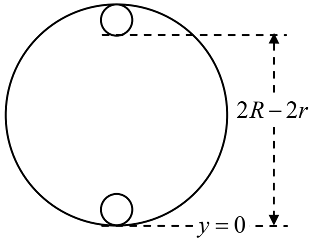
Use energy conservation to relate the release point to the point at the top of the loop.

$$E_{\text{release}} = E_{\substack{\text{top of}\\\text{loop}}} \quad\rightarrow\quad K_{\text{release}} + U_{\text{release}} = K_{\substack{\text{top of}\\\text{loop}}} + U_{\substack{\text{top of}\\\text{loop}}}$$

$$0 + mgh = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}I\omega^2_{\substack{\text{top of}\\\text{loop}}} + mg2R = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2_{\substack{\text{top of}\\\text{loop}}}}{r^2} + 2mgR$$

$$mgh = \tfrac{7}{10}mv^2_{\substack{\text{top of}\\\text{loop}}} + 2mgR = \tfrac{7}{10}mgR + 2mgR = 2.7mgR \quad\rightarrow\quad \boxed{h = 2.7R}$$

(*b*) Since we are not to assume that $r \ll R$, then while the marble is on the loop portion of the track, it is moving in a circle of radius $R - r$, and when at the top of the loop, the bottom of the marble is a height of $2\left(R - r\right)$ above the 0 position of gravitational potential energy (see the diagram). For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.



$$\frac{mv^2_{\substack{\text{top of}\\\text{loop}}}}{R - r} = mg \quad\rightarrow\quad v^2_{\substack{\text{top of}\\\text{loop}}} = g\left(R - r\right)$$

Use energy conservation to equate the energy at the release point to the energy at the top of the loop.

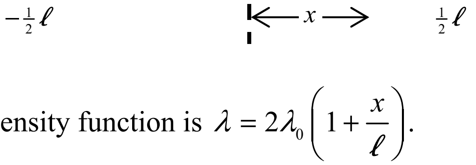
$$E_{\text{release}} = E_{\substack{\text{top of}\\\text{loop}}} \quad\rightarrow\quad K_{\text{release}} + U_{\text{release}} = K_{\substack{\text{top of}\\\text{loop}}} + U_{\substack{\text{top of}\\\text{loop}}}$$

$$0 + mgh = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}I\omega^2_{\substack{\text{top of}\\\text{loop}}} + mg2\left(R - r\right) = \tfrac{1}{2}mv^2_{\substack{\text{top of}\\\text{loop}}} + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2_{\substack{\text{top of}\\\text{loop}}}}{r^2} + 2mg\left(R - r\right)$$

$$mgh = \tfrac{7}{10}mv^2_{\substack{\text{top of}\\\text{loop}}} + 2mg\left(R - r\right) = \tfrac{7}{10}mg\left(R - r\right) + 2mg\left(R - r\right) = 2.7mg\left(R - r\right)$$

$$\boxed{h = 2.7\left(R - r\right)}$$

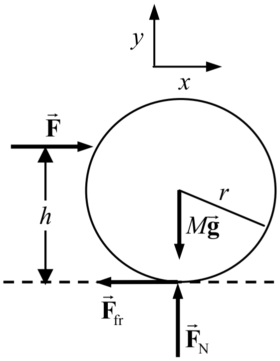
95. We calculate the moment of inertia about an axis through the geometric center of the rod. Select a differential element of the rod of length dx, a distance x from the center of the rod. Because the mass density changes uniformly from λ_0 at $x = -\tfrac{1}{2}\ell$ to $3\lambda_0$ at $x = \tfrac{1}{2}\ell$, the mass density function is $\lambda = 2\lambda_0\left(1 + \dfrac{x}{\ell}\right)$.



The mass of the differential element is then $dM = \lambda dx = 2\lambda_0\left(1+\frac{x}{\ell}\right)dx$. Use Eq. 10-16 to calculate the moment of inertia.

$$I_{\text{end}} = \int R^2 dM = \int_{-\ell/2}^{\ell/2} x^2 2\lambda_0\left(1+\frac{x}{\ell}\right)dx = 2\lambda_0\int_{-\ell/2}^{\ell/2}\left(x^2+\frac{x^3}{\ell}\right)dx = 2\lambda_0\left(\tfrac{1}{3}x^3+\tfrac{1}{4}\frac{x^4}{\ell}\right)\Bigg|_{-\ell/2}^{\ell/2} = \boxed{\tfrac{1}{6}\lambda_0\ell^3}$$

96. A free body diagram for the ball while the stick is in contact is shown. Write Newton's second law for the *x* direction, the *y* direction, and the rotation. Take clockwise torques (about the center of mass) as positive.



$$\sum F_y = F_{\text{N}} - Mg = 0 \;\rightarrow\; F_{\text{N}} = Mg$$

$$\sum F_x = F - F_{\text{fr}} = F - \mu_k F_{\text{N}} = F - \mu_{\text{k}}Mg = Ma \;\rightarrow\; a = \frac{F}{M} - \mu_{\text{k}}g$$

$$\sum \tau = F(h-r) + F_{\text{fr}}r = F(h-r) + \mu_{\text{k}}Mgr = I\alpha \;\rightarrow$$

$$\alpha = \frac{F(h-r)+\mu_{\text{k}}Mgr}{I}$$

The acceleration and angular acceleration are constant, and so constant acceleration kinematics may be used to find the velocity and angular velocity as functions of time. The object starts from rest.

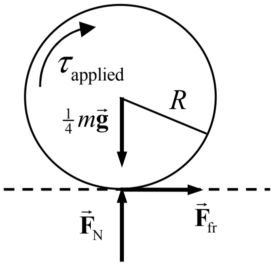
$$v_{\text{CM}} = v_0 + at = \left(\frac{F}{M}-\mu_{\text{k}}g\right)t \;\;;\;\; \omega = \omega_0 + \alpha t = \left(\frac{F(h-r)+\mu_{\text{k}}Mgr}{I}\right)t$$

At a specific time t_{release}, when the ball loses contact with the pushing stick, the ball is rolling without slipping, and so at that time $\omega = v_{\text{CM}}/r$. Solve for the value of *h* needed to make that true. The moment of inertia is $I = \frac{2}{5}Mr^2$.

$$\omega = v_{\text{CM}}/r \;\rightarrow\; \left(\frac{F(h-r)+\mu_{\text{k}}Mgr}{I}\right)t_{\text{release}} = \frac{1}{r}\left(\frac{F}{M}-\mu_{\text{k}}g\right)t_{\text{release}} \;\rightarrow$$

$$h = \frac{1}{F}\left[\frac{I}{r}\left(\frac{F}{M}-\mu_{\text{k}}g\right) - \mu_{\text{k}}Mgr + Fr\right] = \boxed{\tfrac{7}{5}\frac{r}{F}\left(F-\mu_{\text{k}}Mg\right)}$$

97. Each wheel supports ¼ of the weight of the car. For rolling without slipping, there will be static friction between the wheel and the pavement. So for the wheel to be on the verge of slipping, there must be an applied torque that is equal to the torque supplied by the static frictional force. We take counterclockwise torques to the right in the diagram. The bottom wheel would be moving to the left relative to the pavement if it started to slip, so the frictional force is to the right. See the free-body diagram.



$$\tau_{\substack{\text{applied}\\\text{min}}} = \tau_{\substack{\text{static}\\\text{friction}}} = RF_{\text{fr}} = R\mu_{\text{s}}F_{\text{N}} = R\mu_{\text{s}}\tfrac{1}{4}mg$$

$$= \tfrac{1}{4}(0.33\,\text{m})(0.65)(950\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{5.0\times10^2\,\text{m}\bullet\text{N}}$$

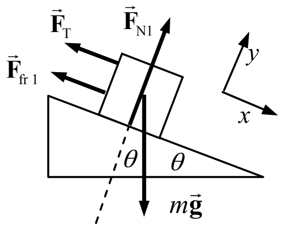
98. (*a*) If there is no friction, then conservation of mechanical energy can be used to find the speed of the block. We assume the cord unrolls from the cylinder without slipping, and so $v_{\text{block}} = v_{\text{cord}} = \omega_{\text{cord}}R$. We take the zero position of gravitational potential energy to be the

bottom of the motion of the block. Since the cylinder does not move vertically, we do not have to consider its gravitational potential energy.

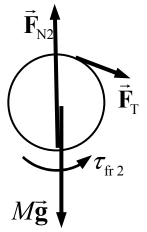
$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad U_{\text{initial}} = K_{\text{final}} = K_{\text{block}} + K_{\text{cylinder}} \quad \rightarrow$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \quad \rightarrow \quad mgD\sin\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right) \quad \rightarrow$$

$$v = \sqrt{\frac{2mgD\sin\theta}{\left(m+\frac{1}{2}M\right)}} = \sqrt{\frac{2(3.0\,\text{kg})(9.80\,\text{m/s}^2)(1.80\,\text{m})\sin 27^\circ}{(19.5\,\text{kg})}} = 1.570\,\text{m/s} \approx \boxed{1.6\,\text{m/s}}$$



(*b*) The first printing of the textbook has $\mu = 0.055$, while later printings will have $\mu = 0.035$. The results are fundamentally different in the two cases. Consider the free body diagrams for both the block and the cylinder. We make the following observations and assumptions. Note that for the block to move down the plane from rest, $F_{\text{T}} < mg$. Also note that $mg < 0.1Mg$ due to the difference in masses. Thus $F_{\text{T}} < 0.1Mg$. Accordingly, we will ignore F_{T} when finding the net vertical and horizontal forces on the cylinder, knowing that we will make less than a 10% error. Instead of trying to assign a specific direction for the force of friction between the cylinder and the depression $\left(F_{\text{fr 2}}\right)$, we show a torque in the counterclockwise direction (since the cylinder will rotate clockwise). Finally, we assume that $F_{\text{fr 2}} = \mu F_{\text{N2}} = \mu Mg$.



Write Newton's second law to analyze the linear motion of the block and the rotational motion of the cylinder, and solve for the acceleration of the block. We assume the cord unrolls without slipping.

$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{T}} - F_{\text{fr 1}} = mg\sin\theta - F_{\text{T}} - \mu mg\cos\theta = ma$$

$$\sum \tau = F_{\text{T}}R - \tau_{\text{fr 2}} = F_{\text{T}}R - \mu F_{\text{N2}}R = F_{\text{T}}R - \mu MgR = I\alpha = I\frac{a}{R} = \tfrac{1}{2}MRa \quad \rightarrow$$

$$F_{\text{T}} - \mu Mg = \tfrac{1}{2}Ma$$

Add the x equation to the torque equation.

$$mg\sin\theta - F_{\text{T}} - \mu mg\cos\theta = ma \;\; ; \;\; F_{\text{T}} - \mu Mg = \tfrac{1}{2}Ma \quad \rightarrow$$

$$mg\sin\theta - \mu Mg - \mu mg\cos\theta = ma + \tfrac{1}{2}Ma \quad \rightarrow$$

$$a = g\frac{m(\sin\theta - \mu\cos\theta) - \mu M}{\left(m+\frac{1}{2}M\right)}$$

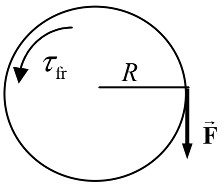
If $\mu = 0.055$, $a = g\dfrac{(3.0\,\text{kg})(\sin 27^\circ - 0.055\cos 27^\circ) - (0.055)(33\,\text{kg})}{(19.5\,\text{kg})} = -0.302\,\text{m/s}^2$. But the object cannot accelerate UP the plane from rest. So the conclusion is that object will not move with $\mu = 0.055$. The small block is not heavy enough to move itself, rotate the cylinder, and overcome friction.

If $\mu = 0.035$, $a = g\dfrac{(3.0\,\text{kg})(\sin 27^\circ - 0.035\cos 27^\circ) - (0.035)(33\,\text{kg})}{(19.5\,\text{kg})} = 0.057\,\text{m/s}^2$.

Use Eq. 2-12c to find the speed after moving 1.80 m.

$$v^2 = v_0^2 + 2a\Delta x \quad \rightarrow \quad v = \sqrt{2\left(0.057\,\text{m/s}^2\right)\left(1.80\,\text{m}\right)} = \boxed{0.45\,\text{m/s}}\,.$$

99. (*a*) See the free body diagram. Take clockwise torques as positive. Write Newton's second law for the rotational motion. The angular acceleration is constant, and so constant acceleration relationships can be used. We also use the definition of radian angles, $\Delta\theta = \dfrac{\Delta s}{R}$.



$$\sum \tau = FR - \tau_{\text{fr}} = I\alpha_1 \;\;;\;\; \Delta\theta_1 = \omega_0 t_1 + \tfrac{1}{2}\alpha_1 t_1^2 = \tfrac{1}{2}\alpha_1 t_1^2 \;\;;\;\; \Delta s_1 = R\Delta\theta_1$$

Combine the relationships to find the length unrolled, Δs_1.

$$\Delta s_1 = R\Delta\theta_1 = R\left(\tfrac{1}{2}\alpha_1 t_1^2\right) = \frac{Rt_1^2}{2I}\left(FR - \tau_{\text{fr}}\right)$$

$$= \frac{\left(0.076\,\text{m}\right)\left(1.3\,\text{s}\right)^2}{2\left(3.3\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)}\left[\left(2.5\,\text{N}\right)\left(0.076\,\text{m}\right) - \left(0.11\,\text{m}\bullet\text{N}\right)\right] = 1.557\,\text{m} \approx \boxed{1.6\,\text{m}}$$

(*b*) Now the external force is removed, but the frictional torque is still present. The analysis is very similar to that in part (*a*), except that the initial angular velocity is needed. That angular velocity is the final angular velocity from the motion in part (*a*).

$$\omega_1 = \omega_0 + \alpha_1 t_1 = \left(\frac{FR - \tau_{\text{fr}}}{I}\right)t_1 = \frac{\left[\left(2.5\,\text{N}\right)\left(0.076\,\text{m}\right) - \left(0.11\,\text{m}\bullet\text{N}\right)\right]}{\left(3.3\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)}\left(1.3\,\text{s}\right) = 31.515\,\text{rad/s}$$

$$\sum \tau = -\tau_{\text{fr}} = I\alpha_2 \;\;;\;\; \omega_2^2 - \omega_1^2 = 2\alpha_2\Delta\theta_2 = -\omega_1^2 \;\;;\;\; \Delta s_2 = R\Delta\theta_2$$

Combine the relationships to find the length unrolled, Δs_2.

$$\Delta s_2 = R\Delta\theta_2 = R\left(\frac{-\omega_1^2}{2\alpha_2}\right) = R\left(\frac{-\omega_1^2 I}{-2\tau_{\text{fr}}}\right) = \frac{\left(0.076\,\text{m}\right)\left(31.515\,\text{rad/s}\right)^2\left(3.3\times10^{-3}\,\text{kg}\bullet\text{m}^2\right)}{2\left(0.11\,\text{m}\bullet\text{N}\right)}$$

$$= 1.13\,\text{m} \approx \boxed{1.1\,\text{m}}$$

100. (*a*) The disk starts from rest, and so the velocity of the center of mass is in the direction of the net force: $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \;\rightarrow\; \vec{\mathbf{v}} = \vec{\mathbf{F}}_{\text{net}}\dfrac{t}{m}$. Thus the center of mass moves $\boxed{\text{to the right}}$.

(*b*) For the linear motion of the center of mass, we may apply constant acceleration equations, where the acceleration is $\dfrac{F}{m}$.

$$v^2 = v_0^2 + 2a\Delta x \quad\rightarrow\quad v = \sqrt{2\frac{F}{m}\Delta x} = \sqrt{2\frac{\left(35\,\text{N}\right)}{\left(21.0\,\text{kg}\right)}\left(5.5\,\text{m}\right)} = 4.282\,\text{m/s} \approx \boxed{4.3\,\text{m/s}}$$

(*c*) The only torque is a constant torque caused by the constant string tension. That can be used to find the angular velocity.

$$\tau = I\alpha = I\left(\frac{\omega - \omega_0}{t}\right) = \frac{I\omega}{t} = Fr \quad\rightarrow\quad \omega = \frac{Frt}{I} = \frac{Frt}{\tfrac{1}{2}mr^2} = \frac{2Ft}{mr}$$

The time can be found from the center of mass motion under constant acceleration.

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 = \tfrac{1}{2}\frac{F}{m}t^2 \quad\rightarrow\quad t = \sqrt{\frac{2m\Delta x}{F}}$$

$$\omega = \frac{2Ft}{mr} = \frac{2F}{mr}\sqrt{\frac{2m\Delta x}{F}} = \frac{2}{r}\sqrt{\frac{2F\Delta x}{m}} = \frac{2}{(0.850\,\text{m})}\sqrt{\frac{2(35.0\,\text{N})(5.5\,\text{m})}{(21.0\,\text{kg})}}$$

$$= 10.07\,\text{rad/s} \approx \boxed{10\,\text{rad/s}}\ (2\text{ sig fig})$$

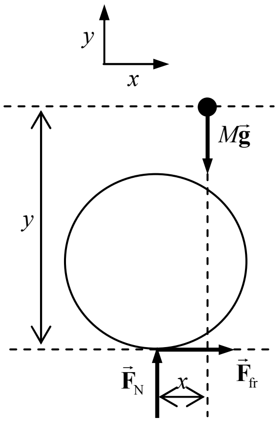
Note that $v \neq \omega r$ since the disk is NOT rolling without slipping.

(*d*) The amount of string that has unwrapped is related to the angle through which the disk has turned, by the definition of radian measure, $s = r\Delta\theta$. The angular displacement is found from constant acceleration relationships.

$$\Delta\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}\omega t = \tfrac{1}{2}\left(\frac{2Ft}{mr}\right)t = \frac{Ft^2}{mr} = \frac{F\dfrac{2m\Delta x}{F}}{mr} = \frac{2\Delta x}{r}$$

$$s = r\Delta\theta = r\frac{2\Delta x}{r} = 2\Delta x = \boxed{11\,\text{m}}$$

101. (*a*) We assume that the front wheel is barely lifted off the ground, so that the only forces that act on the system are the normal force on the bike's rear wheel, the static frictional force on the bike's wheel, and the total weight of the system. We assume that the upward acceleration is zero and the angular acceleration about the center of mass is also zero. Write Newton's second law for the *x* direction, the *y* direction, and rotation. Take positive torques to be clockwise.



$$\sum F_y = F_N - Mg = 0 \quad\rightarrow\quad F_N = Mg$$

$$\sum F_x = F_{fr} = Ma \quad\rightarrow\quad a = \frac{F_{fr}}{M}$$

$$\sum \tau_{CM} = F_N x - F_{fr} y = 0$$

Combine these equations to solve for the acceleration.

$$F_N x - F_{fr} y = 0 \quad\rightarrow\quad Mgx = May \quad\rightarrow\quad a = \boxed{\frac{x}{y}g}$$

(*b*) Based on the form of the solution for the acceleration, $a = \frac{x}{y}g$, to minimize the acceleration $\boxed{x \text{ should be as small as possible}}$ and $\boxed{y \text{ should be as large as possible}}$. $\boxed{\text{The rider should move upwards and towards the rear of the bicycle.}}$

(*c*) $a = \frac{x}{y}g = \frac{0.35\,\text{m}}{0.95\,\text{m}}(9.80\,\text{m/s}^2) = \boxed{3.6\,\text{m/s}^2}$

102. We follow the hint given in the problem. The mass of the cutout piece is proportional by area to the mass of the entire piece.

$$I_{\text{total}} = \tfrac{1}{2}MR_0^2 = I_{\text{remainder}} + I_{\text{cutout}} \quad\rightarrow\quad I_{\text{remainder}} = \tfrac{1}{2}MR_0^2 - I_{\text{cutout}}$$

$$I_{\text{cutout}} = \tfrac{1}{2}m_{\text{cutout}}R_1^2 + m_{\text{cutout}}h^2 \ ;\ m_{\text{cutout}} = \frac{M}{\pi R_0^2}\pi R_1^2 = M\frac{R_1^2}{R_0^2} \quad\rightarrow$$

$$I_{\text{remainder}} = \tfrac{1}{2}MR_0^2 - \left(\tfrac{1}{2}m_{\text{cutout}}R_1^2 + m_{\text{cutout}}h^2\right) = \tfrac{1}{2}MR_0^2 - M\frac{R_1^2}{R_0^2}\left(\tfrac{1}{2}R_1^2 + h^2\right)$$

$$= \boxed{\tfrac{1}{2}\frac{M}{R_0^2}\left(R_0^4 - R_1^4 - 2R_1^2h^2\right)}$$

103. Since there is no friction at the table, there are no horizontal forces on the rod, and so the center of mass will fall straight down. The moment of inertia of the rod about its center of mass is $\tfrac{1}{12}M\ell^2$. Since there are no dissipative forces, energy will be conserved during the fall. Take the zero level of gravitational potential energy to be at the tabletop. The angular velocity and the center of mass velocity are related by $\omega_{\text{CM}} = \dfrac{v_{\text{CM}}}{\left(\frac{1}{2}\ell\right)}$.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; U_{\text{release}} = K_{\text{final}} \;\rightarrow\; Mg\left(\tfrac{1}{2}\ell\right) = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I\omega_{\text{CM}}^2 \;\rightarrow$$

o

$$Mg\left(\tfrac{1}{2}\ell\right) = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}\left(\tfrac{1}{12}M\ell^2\right)\left[\frac{v_{\text{CM}}}{\left(\frac{1}{2}\ell\right)}\right]^2 \;\rightarrow\; g\ell = \tfrac{4}{3}v_{\text{CM}}^2 \;\rightarrow\; v_{\text{CM}} = \boxed{\sqrt{\tfrac{3}{4}g\ell}}$$

104. (*a*) The acceleration is found in Example 10-19 to be a constant value, $a = \tfrac{2}{3}g$, and so constant acceleration kinematics can be used. Take downward to be the positive direction.

$$v_y^2 = v_{y0}^2 + 2a_y\Delta y \;\rightarrow\; v_y = \sqrt{2a_y\Delta y} = \sqrt{2\tfrac{2}{3}gh} = \boxed{\sqrt{\tfrac{4}{3}gh}}$$

(*b*) We take the zero level for gravitational potential energy to be the starting height of the yo-yo. Then the final gravitational potential energy is negative.

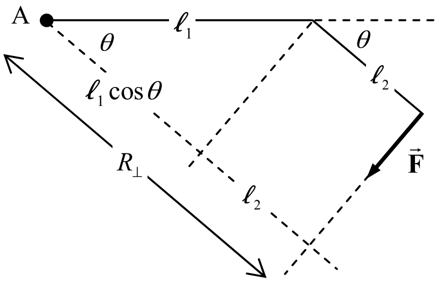
$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; 0 = U_{\text{final}} + K_{\text{final}} = -Mgh + \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I\omega_{\text{CM}}^2 \;\rightarrow$$

$$Mgh = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_{\text{CM}}}{R}\right)^2 \;\rightarrow\; v_{\text{CM}} = \boxed{\sqrt{\tfrac{4}{3}gh}}$$

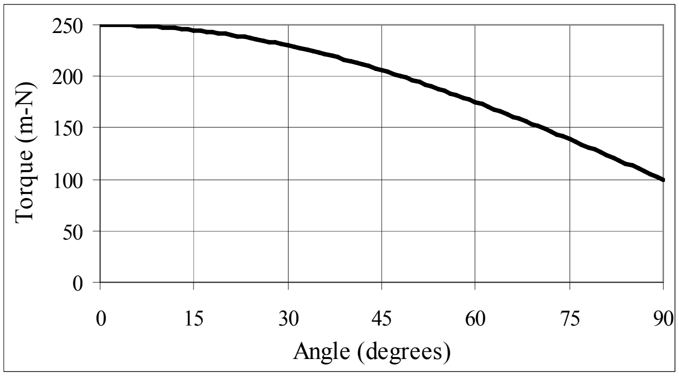
105. From the diagram, we see that the torque about the support A is as follows.

$$\tau = R_{\perp}F = \left(\ell_1\cos\theta + \ell_2\right)F$$

$$= \boxed{\left[\left(0.300\,\text{m}\right)\cos\theta + 0.200\,\text{m}\right]\left(500\,\text{N}\right)}$$



The graph of torque as a function of angle is shown.

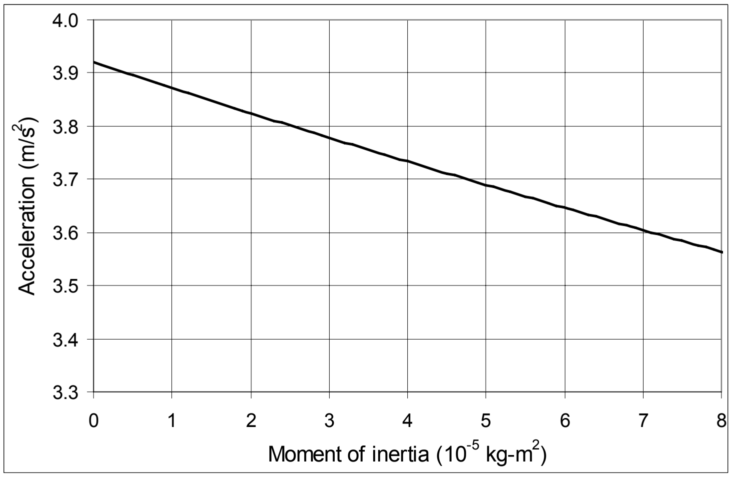


The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH10.XLS," on tab "Problem 10.105."

106. From problem 51, the acceleration is as follows.

$$a = \frac{\left(m_{\text{B}} - m_{\text{A}}\right)}{\left(m_{\text{A}} + m_{\text{B}} + I/R^2\right)} g = \frac{\left(0.200\,\text{kg}\right)}{\left(0.500\,\text{kg} + I/\left(0.040\,\text{m}\right)^2\right)}\left(9.80\,\text{m/s}^2\right)$$

(*a*) The graph is shown here. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH10.XLS," on tab "Problem 10.106a."



(*b*) The value of the acceleration with a zero moment of inertia is found as follows.

$$a = \frac{\left(0.200\,\text{kg}\right)}{\left(0.500\,\text{kg}\right)}\left(9.80\,\text{m/s}^2\right) = \boxed{3.92\,\text{m/s}^2}$$

(*c*) A 2.0% decrease in the acceleration means the acceleration is as follows. $a = 3.92\,\text{m/s}^2\left(0.98\right) = 3.84\,\text{m/s}^2$. Looking at the graph, that would occur roughly for a moment of inertia of $\boxed{1.6\times10^{-5}\,\text{kg}\bullet\text{m}^2}$.

(*d*) Using the value above gives the following pulley mass.

$$I = \tfrac{1}{2}mr^2 = 1.6\times10^{-5}\,\text{kg}\bullet\text{m}^2 \quad\rightarrow\quad m = \frac{2I}{R^2} = 2\left(\frac{1.6\times10^{-5}\,\text{kg}\bullet\text{m}^2}{\left(0.040\,\text{m}\right)^2}\right) = 0.020\,\text{kg} = \boxed{\text{20 grams}}$$

CHAPTER 11: Angular Momentum; General Rotation

Responses to Questions

1. (*a*) With more people at the equator, more mass would be farther from the axis of rotation, and the moment of inertia of the Earth would increase. Due to conservation of angular momentum, the Earth's angular velocity would decrease. The length of the day would increase.

2. No. Once the diver is in the air, there will be no net torque on her and therefore angular momentum will be conserved. If she leaves the board with no initial rotation, her initial angular momentum will be zero. Conservation of angular momentum requires that her final angular momentum will also be zero, so she will not be able to do a somersault.

3. Your angular velocity will stay the same. The angular momentum of the system of you and the stool and the masses is conserved. The masses carry off their angular momentum (until they hit something); you and the stool continue to rotate as before.

4. Once the motorcycle leaves the ground, there is no net torque on it and angular momentum must be conserved. If the throttle is on, the rear wheel will spin faster as it leaves the ground because there is no torque from the ground acting on it. The front of the motorcycle must rise up, or rotate in the direction opposite the rear wheel, in order to conserve angular momentum.

5. As you walk toward the center, the moment of inertia of the system of you + the turntable will decrease. No external torque is acting on the system, so angular momentum must be conserved, and the angular speed of the turntable will increase.

6. When the player is in the air, there is no net torque on him so his total angular momentum must be conserved. If his upper body rotates one direction, his lower body will rotate the other direction to conserve angular momentum.

7. The cross product remains the same. $\vec{\mathbf{V}}_1 \times \vec{\mathbf{V}}_2 = (-\vec{\mathbf{V}}_1) \times (-\vec{\mathbf{V}}_2)$

8. The cross product of the two vectors will be zero if the magnitude of either vector is zero or if the vectors are parallel or anti-parallel to each other.

9. The torque about the CM, which is the cross product between **r** and **F**, depends on *x* and *z*, but not on *y*.

10. The angular momentum will remain constant. If the particle is moving in a straight line at constant speed, there is no net torque acting on it and therefore its angular momentum must be conserved.

11. No. If two equal and opposite forces act on an object, the net force will be zero. If the forces are not co-linear, the two forces will produce a torque. No. If an unbalanced force acts through the axis of rotation, there will be a net force on the object, but no net torque.

12. At the forward peak of the swinging motion, the child leans forward, increasing the gravitational torque about the axis of rotation by moving her center of mass forward. This increases the angular momentum of the system. At the back peak of the swinging motion, the child leans backward, increasing the gravitational torque about the axis of rotation by moving her center of mass backward.

This again increases the angular momentum of the system. As the angular momentum of the system increases, the swing goes higher.

13. A force directed to the left will produce a torque that will cause the axis of the rotating wheel to move directly upward.

14. In both cases, angular momentum must be conserved. Assuming that the astronaut starts with zero angular momentum, she must move her limbs so that her total angular momentum remains zero. The angular momentum of her limbs must be opposite the angular momentum of the rest of her body.
    (*a*) In order to turn her body upside down, the astronaut could hold her arms straight out from her sides and rotate them from the shoulder in vertical circles. If she rotates them forward, her body will rotate backwards.
    (*b*) To turn her body about-face, she could hold her arms straight out from her sides and then pull one across the front of her body while she pulls the other behind her back. If she moves her arms counterclockwise, her body will twist clockwise.

15. Once the helicopter has left the ground, no external torques act on it and angular momentum must be conserved. If there were only one propeller, then when the angular velocity of the propeller changed, the body of the helicopter would begin to rotate in a direction so as to conserve angular momentum. The second propeller can be in the same plane as the first, but spinning in the opposite direction, or perpendicular to the plane of the first. Either case will stabilize the helicopter.

16. The rotational speed of the wheel will not change. Angular momentum of the entire system is conserved, since no net torque operates on the wheel. The small parts of the wheel that fly off will carry angular momentum with them. The remaining wheel will have a lower angular momentum and a lower rotational kinetic energy since it will have the same angular velocity but a smaller mass, and therefore a smaller moment of inertia. The kinetic energy of the total system is not conserved.

17. (*a*) Displacement, velocity, acceleration, and momentum are independent of the choice of origin.
    (*b*) Displacement, acceleration, and torque are independent of the velocity of the coordinate system.

18. Turning the steering wheel changes the axis of rotation of the tires, and makes the car turn. The torque is supplied by the friction between the tires and the pavement. (Notice that if the road is slippery or the tire tread is worn, the car will not be able to make a sharp turn.)

19. The Sun will pull on the bulge closer to it more than it pulls on the opposite bulge, due to the inverse-square law of gravity. These forces, and those from the Moon, create a torque which causes the precession of the axis of rotation of the Earth. The precession is about an axis perpendicular to the plane of the orbit. During the equinox, no torque exists, since the forces on the bulges lie along a line.

20. Because of the rotation of the Earth, the plumb bob will be slightly deflected by the Coriolis force, which is a “pseudoforce.”

21. Newton’s third law is not valid in a rotating reference frame, since there is no reaction to the pseudoforce.

22. In the Northern Hemisphere, the shots would be deflected to the right, with respect to the surface of the Earth, due to the Coriolis effect. In the Southern Hemisphere, the deflection of the shots would be to the left. The gunners had experience in the Northern Hemisphere and so miscalculated the necessary launch direction.

Solutions to Problems

$\boxed{1.}$ The angular momentum is given by Eq. 11-1.

$$L = I\omega = MR^2\omega = (0.210\,\text{kg})(1.35\,\text{m})^2(10.4\,\text{rad/s}) = \boxed{3.98\,\text{kg}\bullet\text{m}^2/\text{s}}$$

2. (*a*) The angular momentum is given by Eq. 11-1.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.8\,\text{kg})(0.18\text{ m})^2\left[\left(\frac{1300\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 6.175\,\text{kg}\bullet\text{m}^2/\text{s} \approx \boxed{6.2\,\text{kg}\bullet\text{m}^2/\text{s}}$$

(*b*) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 6.175\,\text{kg}\bullet\text{m}^2/\text{s}}{6.0\text{ s}} = \boxed{-1.0\,\text{m}\bullet\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

3. (*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. $\boxed{\text{Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.}}$

(*b*) $L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f \;\rightarrow\; I_f = I_i\dfrac{\omega_i}{\omega_f} = I_i\dfrac{0.90\,\text{rev/s}}{0.70\,\text{rev/s}} = 1.286\,I_i \approx 1.3\,I_i$

The rotational inertia has increased by a factor of $\boxed{1.3}$.

4. The skater's angular momentum is constant, since no external torques are applied to her.

$$L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f \;\rightarrow\; I_f = I_i\frac{\omega_i}{\omega_f} = (4.6\text{ kg}\bullet\text{m}^2)\frac{1.0\,\text{rev}/1.5\,\text{s}}{2.5\,\text{rev/s}} = \boxed{1.2\text{ kg}\bullet\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then $\boxed{\text{pulling her arms in to the center of her body}}$ (final angular velocity).

5. There is no net torque on the diver because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \;\rightarrow\; I_1\omega_1 = I_2\omega_2 \;\rightarrow\; \omega_2 = \omega_1\frac{I_1}{I_2} = \left(\frac{2\text{ rev}}{1.5\text{ sec}}\right)\left(\frac{1}{3.5}\right) = \boxed{0.38\,\text{rev/s}}$$

6. The angular momentum is the total moment of inertia times the angular velocity.

$$L = I\omega = \left[\tfrac{1}{12}M\ell^2 + 2m\left(\tfrac{1}{2}\ell\right)^2\right]\omega = \boxed{\left(\tfrac{1}{12}M + \tfrac{1}{2}m\right)\ell^2\omega}$$

$\boxed{7.}$ (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}$$

$$=\tfrac{2}{5}\left(6.0\times10^{24}\,\text{kg}\right)\left(6.4\times10^{6}\,\text{m}\right)^{2}\left[\left(\frac{2\pi\ \text{rad}}{1\ \text{day}}\right)\left(\frac{1\ \text{day}}{86{,}400\ \text{s}}\right)\right]=\boxed{7.1\times10^{33}\ \text{kg}\bullet\text{m}^{2}/\text{s}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}}=I\omega_{\text{daily}}=\left(MR^{2}_{\substack{\text{Sun-}\\\text{Earth}}}\right)\omega_{\text{daily}}$$

$$=\left(6.0\times10^{24}\,\text{kg}\right)\left(1.5\times10^{11}\,\text{m}\right)^{2}\left[\left(\frac{2\pi\ \text{rad}}{365\ \text{day}}\right)\left(\frac{1\ \text{day}}{86{,}400\ \text{s}}\right)\right]=\boxed{2.7\times10^{40}\ \text{kg}\bullet\text{m}^{2}/\text{s}}$$

8. (*a*) $L=I\omega=\tfrac{1}{2}MR^{2}\omega=\tfrac{1}{2}\left(48\,\text{kg}\right)\left(0.15\,\text{m}\right)^{2}\left(2.8\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)=9.50\,\text{kg}\bullet\text{m}^{2}/\text{s}\approx\boxed{9.5\,\text{kg}\bullet\text{m}^{2}/\text{s}}$

(*b*) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau=\frac{\Delta L}{\Delta t}=\frac{I\omega_{\text{final}}-I\omega_{0}}{\Delta t}=\frac{0-9.50\,\text{kg}\bullet\text{m}^{2}/\text{s}}{5.0\ \text{s}}=\boxed{-1.9\,\text{m}\bullet\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

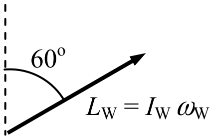
9. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, and so any change that the person–wheel–platform undergoes must result in no net angular momentum along the vertical axis.

(*a*) If the wheel is moved so that its angular momentum points upwards, then the person and platform must get an equal but opposite angular momentum, which will point downwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_{i}=L_{f}\ \rightarrow\ 0=I_{\text{W}}\omega_{\text{W}}+I_{\text{P}}\omega_{\text{P}}\ \rightarrow\ \boxed{\omega_{\text{P}}=-\frac{I_{\text{W}}}{I_{\text{P}}}\omega_{\text{W}}}$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.

(*b*) If the wheel is pointing at a 60° angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_{\text{W}}\omega_{\text{W}}\cos60^{\circ}$. See the diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.



$$L_{i}=L_{f}\ \rightarrow\ 0=I_{\text{W}}\omega_{\text{W}}\cos60^{\circ}+I_{\text{P}}\omega_{\text{P}}\ \rightarrow\ \boxed{\omega_{\text{P}}=-\frac{I_{\text{W}}}{2I_{\text{P}}}\omega_{\text{W}}}$$

Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.

(*c*) If the wheel is moved so that its angular momentum points downwards, then the person and platform must get an equal but opposite angular momentum, which will point upwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_{i}=L_{f}\ \rightarrow\ 0=-I_{\text{W}}\omega_{\text{W}}+I_{\text{P}}\omega_{\text{P}}\ \rightarrow\ \boxed{\omega_{\text{P}}=\omega_{\text{W}}\,I_{\text{W}}/I_{\text{P}}}$$

The platform is rotating in the same direction as the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is also spinning counterclockwise.

(*d*) Since the total angular momentum is 0, if the wheel is stopped from rotating, the platform will also stop. Thus $\boxed{\omega_{\mathrm{P}} = 0}$.

10. The angular momentum of the disk–rod combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1 = L_2 \quad \rightarrow \quad I_1\omega_1 = I_2\omega_2 \quad \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}} = \omega_1 \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M(2R)^2} \right] = (3.7\,\text{rev/s})\left(\frac{3}{5}\right) = \boxed{2.2\,\text{rev/s}}$$

11. Since the person is walking radially, no torques will be exerted on the person–platform system, and so angular momentum will be conserved. The person will be treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(*a*) $L_i = L_f \quad \rightarrow \quad I_{\text{platform}}\omega_i = \left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f$

$$\omega_f = \frac{I_{\text{platform}}}{I_{\text{platform}} + mR^2}\omega_i = \frac{920\,\text{kg}\bullet\text{m}^2}{920\,\text{kg}\bullet\text{m}^2 + (75\,\text{kg})(3.0\,\text{m})^2}(0.95\,\text{rad/s}) = 0.548\,\text{rad/s} \approx \boxed{0.55\,\text{rad/s}}$$

(*b*) $KE_i = \frac{1}{2}I_{\text{platform}}\omega_i^2 = \frac{1}{2}\left(920\,\text{kg}\bullet\text{m}^2\right)(0.95\,\text{rad/s})^2 = \boxed{420\,\text{J}}$

$$KE_f = \tfrac{1}{2}\left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f^2 = \tfrac{1}{2}\left(I_{\text{platform}} + m_{\text{person}}r_{\text{person}}^2\right)\omega_f^2$$

$$= \tfrac{1}{2}\left[920\,\text{kg}\bullet\text{m}^2 + (75\,\text{kg})(3.0\,\text{m})^2\right](0.548\,\text{rad/s})^2 = 239\,\text{J} \approx \boxed{240\,\text{J}}$$

12. Because there is no external torque applied to the wheel–clay system, the angular momentum will be conserved. We assume that the clay is thrown with no angular momentum so that its initial angular momentum is 0. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the clay and the wheel. Subscript 1 represents before the clay is thrown, and subscript 2 represents after the clay is thrown.

$$L_1 = L_2 \quad \rightarrow \quad I_1\omega_1 = I_2\omega_2 \quad \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \frac{I_{\text{wheel}}}{I_{\text{wheel}} + I_{\text{clay}}} = \omega_1 \left(\frac{\frac{1}{2}M_{\text{wheel}}R_{\text{wheel}}^2}{\frac{1}{2}M_{\text{wheel}}R_{\text{wheel}}^2 + \frac{1}{2}M_{\text{clay}}R_{\text{clay}}^2} \right) = \omega_1 \left(\frac{M_{\text{wheel}}R_{\text{wheel}}^2}{M_{\text{wheel}}R_{\text{wheel}}^2 + M_{\text{clay}}R_{\text{clay}}^2} \right)$$

$$= (1.5\,\text{rev/s})\left[\frac{(5.0\text{ kg})(0.20\text{ m})^2}{(5.0\text{ kg})(0.20\text{ m})^2 + (2.6\text{ kg})\left(8.0\times10^{-2}\,\text{m}\right)^2} \right] = 1.385\,\text{rev/s} \approx \boxed{1.4\,\text{rev/s}}$$

$\boxed{13.}$ The angular momentum of the merry-go-round and people combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$L_1 = L_2 \quad \rightarrow \quad I_1\omega_1 = I_2\omega_2 \quad \rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + I_{\text{people}}} = \omega_1 \left[\frac{I_{\text{m-g-r}}}{I_{\text{m-g-r}} + 4M_{\text{person}} R^2} \right]$$

$$= (0.80\,\text{rad/s}) \left[\frac{1760\,\text{kg}\bullet\text{m}^2}{1760\,\text{kg}\bullet\text{m}^2 + 4(65\,\text{kg})(2.1\,\text{m})^2} \right] = \boxed{0.48\,\text{rad/s}}$$

If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round, and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $\boxed{0.80\,\text{rad/s}}$.

14. (*a*) The angular momentum of the system will be conserved as the woman walks. The woman's distance from the axis of rotation is $r = R - vt$.

$$L_i = L_f \;\rightarrow\; \left(I_{\text{platform}} + I_{\underset{\text{woman}}{0}} \right) \omega_0 = \left(I_{\text{platform}} + I_{\text{woman}} \right) \omega \;\rightarrow$$

$$\left(\tfrac{1}{2} MR^2 + mR^2 \right) \omega_0 = \left(\tfrac{1}{2} MR^2 + m(R - vt)^2 \right) \omega \;\rightarrow$$

$$\omega = \frac{\left(\tfrac{1}{2} MR^2 + mR^2 \right) \omega_0}{\left(\tfrac{1}{2} MR^2 + m(R - vt)^2 \right)} = \boxed{\frac{\left(\tfrac{1}{2} M + m \right) \omega_0}{\tfrac{1}{2} M + m \left(1 - \dfrac{vt}{R} \right)^2}}$$

(*b*) Evaulate at $r = R - vt = 0 \;\rightarrow\; R = vt$.

$$\omega = \frac{\left(\tfrac{1}{2} M + m \right) \omega_0}{\tfrac{1}{2} M} = \boxed{\left(1 + \frac{2m}{M} \right) \omega_0}$$

15. Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$L_i = L_f \;\rightarrow\; I\omega + I(0) = 2I\omega_f \;\rightarrow\; \boxed{\omega_f = \tfrac{1}{2}\omega}$$

16. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \;\rightarrow\; I_i \omega_i = I_f \omega_f \;\rightarrow\; \frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\tfrac{2}{5} M_i R_i^2}{\tfrac{2}{5} M_f R_f^2} = \frac{M_i R_i^2}{(0.5M_i)(0.01R_f)^2} = 2.0 \times 10^4$$

$$\omega_f = 2.0 \times 10^4 \omega_i = 2.0 \times 10^4 \left(\frac{2\pi\,\text{rad}}{30\,\text{day}} \right) \left(\frac{1\,\text{d}}{86400\,\text{s}} \right) = 4.848 \times 10^{-2}\,\text{rad/s} \approx \boxed{5 \times 10^{-2}\,\text{rad/s}}$$

The period would be a factor of 20,000 smaller, which would make it about 130 seconds. The ratio of angular kinetic energies of the spinning mass would be as follows.

$$\frac{K_{\text{final}}}{K_{\text{initial}}} = \frac{\tfrac{1}{2} I_f \omega_f^2}{\tfrac{1}{2} I_i \omega_i^2} = \frac{\tfrac{1}{2} \left[\tfrac{2}{5} (0.5M_i)(0.01R_i)^2 \right] \left(2.0 \times 10^4 \omega_i \right)^2}{\tfrac{1}{2} \left(\tfrac{2}{5} M_i R_i^2 \right) \omega_i^2} = 2.0 \times 10^4 \;\rightarrow$$

$$\boxed{K_{\text{final}} = 2 \times 10^4 K_{\text{initial}}}$$

17. For our crude estimate, we model the hurricane as a rigid cylinder of air. Since the "cylinder" is rigid, each part of it has the same angular velocity. The mass of the air is the product of the density of air times the volume of the air cylinder.

$$M = \rho V = \rho\pi R^2 h = \left(1.3\,\text{kg/m}^3\right)\pi\left(8.5\times10^4\,\text{m}\right)^2\left(4.5\times10^3\,\text{m}\right) = 1.328\times10^{14}\,\text{kg}$$

(*a*) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right)^2 = \frac{1}{4}Mv_{\text{edge}}^2$

$$= \tfrac{1}{4}\left(1.328\times10^{14}\,\text{kg}\right)\left[\left(120\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = 3.688\times10^{16}\,\text{J} \approx \boxed{3.7\times10^{16}\,\text{J}}$$

(*b*) $L = I\omega = \left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right) = \frac{1}{2}MRv_{\text{edge}}$

$$= \tfrac{1}{2}\left(1.328\times10^{14}\,\text{kg}\right)\left(8.5\times10^4\,\text{m}\right)\left[\left(120\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right] = 2.213\times10^{20}\,\text{kg}\cdot\text{m}^2/\text{s}$$

$$\approx \boxed{1.9\times10^{20}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

18. Angular momentum will be conserved in the Earth–asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text{asteroid}} = v_{\text{asteroid}}/R_{\text{Earth}}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere, and the asteroid as a point mass.

$$L_i = L_f \quad\rightarrow\quad I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = \left(I_{\text{Earth}} + I_{\text{asteroid}}\right)\omega_f$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, and so the percent change in Earth's angular velocity is found as follows.

$$I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = I_{\text{Earth}}\omega_f \quad\rightarrow\quad \frac{\left(\omega_f - \omega_{\text{Earth}}\right)}{\omega_{\text{Earth}}} = \frac{I_{\text{asteroid}}}{I_{\text{Earth}}}\frac{\omega_{\text{asteroid}}}{\omega_{\text{Earth}}}$$

$$\%\text{ change} = \frac{\left(\omega_f - \omega_{\text{Earth}}\right)}{\omega_{\text{Earth}}}(100) = \frac{m_{\text{asteroid}}R_{\text{Earth}}^2}{\frac{2}{5}M_{\text{Earth}}R_{\text{Earth}}^2}\frac{\frac{v_{\text{asteroid}}}{R_{\text{Earth}}}}{\omega_{\text{Earth}}} = \frac{m_{\text{asteroid}}}{\frac{2}{5}M_{\text{Earth}}}\frac{v_{\text{asteroid}}}{\omega_{\text{Earth}}R_{\text{Earth}}}(100)$$

$$= \frac{\left(1.0\times10^5\,\text{kg}\right)\left(3.5\times10^4\,\text{m/s}\right)}{\left(0.4\right)\left(5.97\times10^{24}\,\text{kg}\right)\left(\dfrac{2\pi\text{ rad}}{86400\,\text{s}}\right)\left(6.38\times10^6\,\text{m}\right)}(100) = \boxed{3.2\times10^{-16}\,\%}$$

$\boxed{19.}$ The angular momentum of the person–turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $v + v_{\text{T}}$, where v is the person's speed relative to the turntable, and v_{T} is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_{\text{T}} = v_{\text{T}}/R$, and the person's angular speed relative to the ground is $\omega_{\text{P}} = \dfrac{v + v_{\text{T}}}{R} = \dfrac{v}{R} + \omega_{\text{T}}$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_i = L_f \quad\rightarrow\quad 0 = I_{\text{T}}\omega_{\text{T}} + I_{\text{P}}\omega_{\text{P}} = I_{\text{T}}\omega_{\text{T}} + mR^2\left(\omega_{\text{T}} + \frac{v}{R}\right) \quad\rightarrow$$

$$\omega_{\mathrm{T}} = -\frac{mRv}{I_{\mathrm{T}} + mR^2} = -\frac{(65\text{ kg})(3.25\text{ m})(3.8\text{ m/s})}{1850\text{ kg}\bullet\text{m}^2 + (65\text{ kg})(3.25\text{ m})^2} = \boxed{-0.32\text{ rad/s}}$$

20. We use the determinant rule, Eq. 11-3b.

(a) $$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -A & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = \hat{\mathbf{i}}[(0)(B)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(-A)(B)] + \hat{\mathbf{k}}[(-A)(0)-(0)(0)]$$

$$= AB\hat{\mathbf{j}}$$

So the direction of $\vec{\mathbf{A}}\times\vec{\mathbf{B}}$ is in the $\boxed{\hat{\mathbf{j}}}$ direction.

(*b*) Based on Eq. 11-4b, we see that interchanging the two vectors in a cross product reverses the direction. So the direction of $\vec{\mathbf{B}}\times\vec{\mathbf{A}}$ is in the $\boxed{-\hat{\mathbf{j}}}$ direction.

(*c*) Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular, we have $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{B}}\times\vec{\mathbf{A}}\right| = AB\sin 90^\circ = \boxed{AB}$.

21. (*a*) For all three expressions, use the fact that $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta$. If both vectors in the cross product point in the same direction, then the angle between them is $\theta = 0^\circ$. Since $\sin 0^\circ = 0$, a vector crossed into itself will always give 0. Thus $\boxed{\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0}$.

(*b*) We use the determinant rule (Eq. 11-3b) to evaluate the other expressions.

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{\mathbf{i}}[(0)(0)-(0)(1)] + \hat{\mathbf{j}}[(0)(0)-(1)(0)] + \hat{\mathbf{k}}[(1)(1)-(0)(0)] = \hat{\mathbf{k}}$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{k}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}}[(0)(1)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(1)(1)] + \hat{\mathbf{k}}[(1)(0)-(0)(0)] = -\hat{\mathbf{j}}$$

$$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}}[(1)(1)-(0)(0)] + \hat{\mathbf{j}}[(0)(0)-(0)(1)] + \hat{\mathbf{k}}[(0)(0)-(0)(1)] = \hat{\mathbf{i}}$$

22. (*a*) East cross south is $\boxed{\text{into the ground}}$.
(*b*) East cross straight down is $\boxed{\text{north}}$.
(*c*) Straight up cross north is $\boxed{\text{west}}$.
(*d*) Straight up cross straight down is $\boxed{0}$ (the vectors are anti-parallel).

23. Use the definitions of cross product and dot product, in terms of the angle between the two vectors.

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}\bullet\vec{\mathbf{B}} \quad\to\quad AB\left|\sin\theta\right| = AB\cos\theta \quad\to\quad \left|\sin\theta\right| = \cos\theta$$

This is true only for angles with positive cosines, and so the angle must be in the first or fourth quadrant. Thus the solutions are $\theta = 45^\circ, 315^\circ$. But the angle between two vectors is always taken to be the smallest angle possible, and so $\theta = \boxed{45^\circ}$.

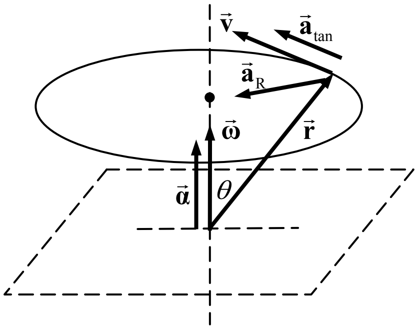
24. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4.0 & 3.5 & 6.0 \\ 0 & 9.0 & -4.0 \end{vmatrix} \text{m}\bullet\text{N}$$

$$= \left\{ \hat{\mathbf{i}}\left[(3.5)(-4.0) - (6)(9)\right] + \hat{\mathbf{j}}\left[(6)(0) - (-4)(4)\right] + \hat{\mathbf{k}}\left[(4)(9) - (3)(5)\right] \right\} \text{m}\bullet\text{N}$$

$$= \left(-68\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + 36\hat{\mathbf{k}}\right) \text{m}\bullet\text{N}$$

25. We choose coordinates so that the plane in which the particle rotates is the *x*-*y* plane, and so the angular velocity is in the *z* direction. The object is rotating in a circle of radius $r\sin\theta$, where θ is the angle between the position vector and the axis of rotation. Since the object is rigid and rotates about a fixed axis, the linear and angular velocities of the particle are related by $v = \omega r \sin\theta$. The magnitude of the tangential acceleration is $a_{\tan} = \alpha r \sin\theta$. The radial acceleration is given by $a_{\text{R}} = \dfrac{v^2}{r\sin\theta} = v\dfrac{v}{r\sin\theta} = v\omega$. We assume the object is gaining speed. See the diagram showing the various vectors involved.



The velocity and tangential acceleration are parallel to each other, and the angular velocity and angular acceleration are parallel to each other. The radial acceleration is perpendicular to the velocity, and the velocity is perpendicular to the angular velocity.

We see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_{\text{R}}$ is in the direction of $\vec{\boldsymbol{\omega}} \times \vec{\mathbf{v}}$. Also, since $\vec{\boldsymbol{\omega}}$ and $\vec{\mathbf{v}}$ are perpendicular, we have $\left|\vec{\boldsymbol{\omega}} \times \vec{\mathbf{v}}\right| = \omega v$ which from above is $v\omega = a_{\text{R}}$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_{\text{R}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{v}}}$.

We also see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_{\tan}$ is in the direction of $\vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}$. The magnitude of $\vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}$ is $\left|\vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}\right| = \alpha r \sin\theta$, which from above is $\alpha r \sin\theta = a_{\tan}$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}}$.

26. (*a*) We use the distributive property, Eq. 11-4c, to obtain 9 single-term cross products.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) \times \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$= A_xB_x\left(\hat{\mathbf{i}} \times \hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}} \times \hat{\mathbf{k}}\right) + A_yB_x\left(\hat{\mathbf{j}} \times \hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}} \times \hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}} \times \hat{\mathbf{k}}\right)$$

$$+ A_zB_x\left(\hat{\mathbf{k}} \times \hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}} \times \hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}} \times \hat{\mathbf{k}}\right)$$

Each of these cross products of unit vectors is evaluated using the results of Problem 21 and Eq. 11-4b.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = A_xB_x(0) + A_xB_y\hat{\mathbf{k}} + A_xB_z\left(-\hat{\mathbf{j}}\right) + A_yB_x\left(-\hat{\mathbf{k}}\right) + A_yB_y(0) + A_yB_z\hat{\mathbf{i}}$$

$$+ A_zB_x\hat{\mathbf{j}} + A_zB_y\left(-\hat{\mathbf{i}}\right) + A_zB_z(0)$$

$$= A_xB_y\hat{\mathbf{k}} - A_xB_z\hat{\mathbf{j}} - A_yB_x\hat{\mathbf{k}} + A_yB_z\hat{\mathbf{i}} + A_zB_x\hat{\mathbf{j}} - A_zB_y\hat{\mathbf{i}}$$

$$=\boxed{\left(A_yB_z-A_zB_y\right)\hat{\mathbf{i}}+\left(A_zB_x-A_xB_z\right)\hat{\mathbf{j}}+\left(A_xB_y-A_yB_x\right)\hat{\mathbf{k}}}$$

(*b*) The rules for evaluating a literal determinant of a 3 x 3 matrix are as follows. The indices on the matrix elements identify the row and column of the element, respectively.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\left(a_{22}a_{33}-a_{23}a_{32}\right)+a_{12}\left(a_{23}a_{31}-a_{21}a_{33}\right)+a_{13}\left(a_{21}a_{32}-a_{22}a_{31}\right)$$

Apply this as a pattern for finding the cross product of two vectors.

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}=\boxed{\hat{\mathbf{i}}\left(A_yB_z-A_zB_y\right)+\hat{\mathbf{j}}\left(A_zB_x-A_xB_z\right)+\hat{\mathbf{k}}\left(A_xB_y-A_yB_x\right)}$$

This is the same expression as found in part (*a*).

27. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$\vec{\boldsymbol{\tau}}=\vec{\mathbf{r}}\times\vec{\mathbf{F}}=\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 8.0 & 6.0 \\ \pm 2.4 & -4.1 & 0 \end{vmatrix}\text{m}\bullet\text{kN}$$

$$=\left\{\hat{\mathbf{i}}\left[-(6.0)(-4.1)\right]+\hat{\mathbf{j}}\left[(6.0)(\pm 2.4)\right]+\hat{\mathbf{k}}\left[-(8.0)(\pm 2.4)\right]\right\}\text{m}\bullet\text{kN}$$

$$=\left(24.6\hat{\mathbf{i}}\pm 14.4\hat{\mathbf{j}}\mp 19.2\hat{\mathbf{k}}\right)\text{m}\bullet\text{kN}\approx\boxed{\left(2.5\hat{\mathbf{i}}\pm 1.4\hat{\mathbf{j}}\mp 1.9\hat{\mathbf{k}}\right)\times 10^4\,\text{m}\bullet\text{N}}$$

The magnitude of this maximum torque is also found.

$$\left|\vec{\boldsymbol{\tau}}\right|=\boxed{\sqrt{(2.46)^2+(1.44)^2+(1.92)^2}\times 10^4\text{m}\bullet\text{N}}=\boxed{3.4\times 10^4\text{m}\bullet\text{N}}$$

28. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$\vec{\boldsymbol{\tau}}=\vec{\mathbf{r}}\times\vec{\mathbf{F}}=\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0.280 & 0.335 & 0 \\ 215\cos 33.0^\circ & 215\sin 33.0^\circ & 0 \end{vmatrix}\text{m}\bullet\text{N}$$

$$=\hat{\mathbf{k}}\left[(0.280)(215\sin 33.0^\circ)-(0.335)(215\cos 33.0^\circ)\right]\text{m}\bullet\text{N}$$

$$=-27.6\,\text{m}\bullet\text{N}\,\hat{\mathbf{k}}=\boxed{27.6\,\text{m}\bullet\text{N in the }-z\text{ direction}}$$

This could also be calculated by finding the magnitude and direction of $\vec{\mathbf{r}}$, and then using Eq. 11-3a and the right-hand rule.

29. (*a*) We use the determinant rule, Eq. 11-3b, to evaluate the cross product.

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 5.4 & -3.5 & 0 \\ -8.5 & 5.6 & 2.0 \end{vmatrix}=-7.0\hat{\mathbf{i}}-10.8\hat{\mathbf{j}}+0.49\hat{\mathbf{k}}\approx\boxed{-7.0\hat{\mathbf{i}}-11\hat{\mathbf{j}}+0.5\hat{\mathbf{k}}}$$

(*b*) Now use Eq. 11-3a to find the angle between the two vectors.

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=\sqrt{(-7.0)^2+(-10.8)^2+(0.49)^2}=12.88$$

$$A=\sqrt{(5.4)^2+(3.5)^2}=6.435\ ;\ B\sqrt{(-8.5)^2+(5.6)^2+(2.0)^2}=10.37$$

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=AB\sin\theta \quad\to\quad \theta=\sin^{-1}\frac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}=\sin^{-1}\frac{12.88}{(6.435)(10.37)}=11.1^\circ \text{ or } 168.9^\circ$$

Use the dot product to resolve the ambiguity.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}}=(5.4)(-8.5)+(3.5)(5.6)+0(2.0)=-26.3$$

Since the dot product is negative, the angle between the vectors must be obtuse, and so $\theta=168.9^\circ\approx\boxed{170^\circ}$.

30. We choose the z axis to be the axis of rotation, and so $\vec{\boldsymbol{\omega}}=\omega\hat{\mathbf{k}}$. We describe the location of the point as $\vec{\mathbf{r}}=R\cos\omega t\hat{\mathbf{i}}+R\sin\omega t\hat{\mathbf{j}}+z_0\hat{\mathbf{k}}$. In this description, the point is moving counterclockwise in a circle of radius R centered on the point $(0,0,z_0)$, and is located at $(R,0,z_0)$ at $t=0$.

$$\vec{\mathbf{v}}=\frac{d\vec{\mathbf{r}}}{dt}=-R\omega\sin\omega t\hat{\mathbf{i}}+R\omega\cos\omega t\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 0 & 0 & \omega\\ R\cos\omega t & R\sin\omega t & z_0\end{vmatrix}=-R\omega\sin\omega t\hat{\mathbf{i}}+R\omega\cos\omega t\hat{\mathbf{j}}=\vec{\mathbf{v}}$$

And so we see that $\vec{\mathbf{v}}=\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}$.

If the origin were moved to a different location on the axis of rotation (the z axis) that would simply change the value of the z coordinate of the point to some other value, say z_1. Changing that value will still lead to $\vec{\mathbf{v}}=\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}$.

But if the origin is moved from the original point to something off the rotation axis, then the position vector will change. If the new origin is moved to (x_2,y_2,z_2), then the position vector will change to $\vec{\mathbf{r}}=(R\cos\omega t-x_2)\hat{\mathbf{i}}+(R\sin\omega t-y_2)\hat{\mathbf{j}}+(z_0-z_2)\hat{\mathbf{k}}$. See how that affects the relationships.

$$\vec{\mathbf{v}}=\frac{d\vec{\mathbf{r}}}{dt}=-R\omega\sin\omega t\hat{\mathbf{i}}+R\omega\cos\omega t\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 0 & 0 & \omega\\ R\cos\omega t-x_2 & R\sin\omega t-y_2 & z_0-z_2\end{vmatrix}=(-R\omega\sin\omega t+\omega y_2)\hat{\mathbf{i}}+(R\omega\cos\omega t-\omega x_2)\hat{\mathbf{j}}=\vec{\mathbf{v}}$$

We see that with this new off-axis origin, $\vec{\mathbf{v}}\neq\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}$.

$\boxed{31.}$ Calculate the three "triple products" as requested.

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\boxed{\hat{\mathbf{i}}(A_yB_z-A_zB_y)+\hat{\mathbf{j}}(A_zB_x-A_xB_z)+\hat{\mathbf{k}}(A_xB_y-A_yB_x)}$$

$$\vec{\mathbf{B}}\times\vec{\mathbf{C}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}=\boxed{\hat{\mathbf{i}}(B_yC_z-B_zC_y)+\hat{\mathbf{j}}(B_zC_x-B_xC_z)+\hat{\mathbf{k}}(B_xC_y-B_yC_x)}$$

$$\vec{\mathbf{C}}\times\vec{\mathbf{A}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ C_x & C_y & C_z\\ A_x & A_y & A_z\end{vmatrix}=\boxed{\hat{\mathbf{i}}\left(C_yA_z-C_zA_y\right)+\hat{\mathbf{j}}\left(C_zA_x-C_xA_z\right)+\hat{\mathbf{k}}\left(C_xA_y-C_yA_x\right)}$$

$$\begin{aligned}\vec{\mathbf{A}}\bullet\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right)&=\left(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}+A_z\hat{\mathbf{k}}\right)\bullet\left[\hat{\mathbf{i}}\left(B_yC_z-B_zC_y\right)+\hat{\mathbf{j}}\left(B_zC_x-B_xC_z\right)+\hat{\mathbf{k}}\left(B_xC_y-B_yC_x\right)\right]\\&=A_x\left(B_yC_z-B_zC_y\right)+A_y\left(B_zC_x-B_xC_z\right)+A_z\left(B_xC_y-B_yC_x\right)\\&=A_xB_yC_z-A_xB_zC_y+A_yB_zC_x-A_yB_xC_z+A_zB_xC_y-A_zB_yC_x\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{B}}\bullet\left(\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right)&=\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}+B_z\hat{\mathbf{k}}\right)\bullet\left[\hat{\mathbf{i}}\left(C_yA_z-C_zA_y\right)+\hat{\mathbf{j}}\left(C_zA_x-C_xA_z\right)+\hat{\mathbf{k}}\left(C_xA_y-C_yA_x\right)\right]\\&=B_x\left(C_yA_z-C_zA_y\right)+B_y\left(C_zA_x-C_xA_z\right)+B_z\left(C_xA_y-C_yA_x\right)\\&=B_xC_yA_z-B_xC_zA_y+B_yC_zA_x-B_yC_xA_z+B_zC_xA_y-B_zC_yA_x\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{C}}\bullet\left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right)&=\left(C_x\hat{\mathbf{i}}+C_y\hat{\mathbf{j}}+C_z\hat{\mathbf{k}}\right)\bullet\left[\hat{\mathbf{i}}\left(A_yB_z-A_zB_y\right)+\hat{\mathbf{j}}\left(A_zB_x-A_xB_z\right)+\hat{\mathbf{k}}\left(A_xB_y-A_yB_x\right)\right]\\&=C_x\left(A_yB_z-A_zB_y\right)+C_y\left(A_zB_x-A_xB_z\right)+C_z\left(A_xB_y-A_yB_x\right)\\&=C_xA_yB_z-C_xA_zB_y+C_yA_zB_x-C_yA_xB_z+C_zA_xB_y-C_zA_yB_x\end{aligned}$$

A comparison of three results shows that they are all the same.

32. We use the determinant rule, Eq. 11-3b, to evaluate the angular momentum.

$$\vec{\mathbf{L}}=\vec{\mathbf{r}}\times\vec{\mathbf{p}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}=\boxed{\left(yp_z-zp_y\right)\hat{\mathbf{i}}+\left(zp_x-xp_z\right)\hat{\mathbf{j}}+\left(xp_y-yp_x\right)\hat{\mathbf{k}}}$$

33. The position vector and velocity vectors are at right angles to each other for circular motion. The angular momentum for a particle moving in a circle is $L=rp\sin\theta=rmv\sin 90^\circ=mrv$. The moment of inertia is $I=mr^2$.

$$\frac{L^2}{2I}=\frac{\left(mrv\right)^2}{2mr^2}=\frac{m^2r^2v^2}{2mr^2}=\frac{mv^2}{2}=\tfrac{1}{2}mv^2=K$$

This is analogous to $K=\dfrac{p^2}{2m}$ relating kinetic energy, linear momentum, and mass.

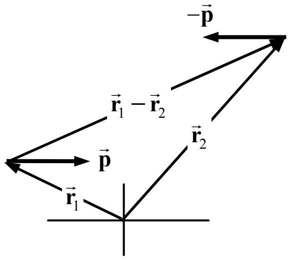
34. (*a*) See Figure 11-33 in the textbook. We have that $L=r_\perp p=dmv$. The direction is into the plane of the page.

(*b*) Since the velocity (and momentum) vectors pass through O′, $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ are parallel, and so $\vec{\mathbf{L}}=\vec{\mathbf{r}}\times\vec{\mathbf{p}}=0$. Or, $r_\perp=0$, and so $L=0$.

35. See the diagram. Calculate the total angular momentum about the origin.

$$\vec{\mathbf{L}}=\vec{\mathbf{r}}_1\times\vec{\mathbf{p}}+\vec{\mathbf{r}}_2\times\left(-\vec{\mathbf{p}}\right)=\left(\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2\right)\times\vec{\mathbf{p}}$$

The position dependence of the total angular momentum only depends on the difference in the two position vectors. That difference is the same no matter where the origin is chosen, because it is the relative distance between the two particles.



36. Use Eq. 11-6 to calculate the angular momentum.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = m\left(\vec{\mathbf{r}} \times \vec{\mathbf{v}}\right) = \left(0.075\,\mathrm{kg}\right)\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4.4 & -6.0 & 0 \\ 3.2 & 0 & -8.0 \end{vmatrix}\mathrm{m^2/s}$$

$$= \left(0.075\right)\left(48\hat{\mathbf{i}} + 35.2\hat{\mathbf{j}} + 19.2\hat{\mathbf{k}}\right)\mathrm{kg{\bullet}m^2/s} = \boxed{\left(3.6\hat{\mathbf{i}} + 2.6\hat{\mathbf{j}} + 1.4\hat{\mathbf{k}}\right)\mathrm{kg{\bullet}m^2/s}}$$

$\boxed{37.}$ Use Eq. 11-6 to calculate the angular momentum.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = m\left(\vec{\mathbf{r}} \times \vec{\mathbf{v}}\right) = \left(3.8\,\mathrm{kg}\right)\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1.0 & 2.0 & 3.0 \\ -5.0 & 2.8 & -3.1 \end{vmatrix}\mathrm{m^2/s}$$

$$= \left(3.8\right)\left(-14.6\hat{\mathbf{i}} - 11.9\hat{\mathbf{j}} + 12.8\hat{\mathbf{k}}\right)\mathrm{kg{\bullet}m^2/s} = \boxed{\left(-55\hat{\mathbf{i}} - 45\hat{\mathbf{j}} + 49\hat{\mathbf{k}}\right)\mathrm{kg{\bullet}m^2/s}}$$

38. (*a*) From Example 11-8, $a = \dfrac{\left(m_\mathrm{B} - m_\mathrm{A}\right)g}{\left(m_\mathrm{A} + m_\mathrm{B} + I/R_0^2\right)}$.

$$a = \frac{\left(m_\mathrm{B} - m_\mathrm{A}\right)g}{\left(m_\mathrm{A} + m_\mathrm{B} + I/R_0^2\right)} = \frac{\left(m_\mathrm{B} - m_\mathrm{A}\right)g}{\left(m_\mathrm{A} + m_\mathrm{B}\right) + \frac{1}{2}mR_0^2/R_0^2} = \frac{\left(m_\mathrm{B} - m_\mathrm{A}\right)g}{m_\mathrm{A} + m_\mathrm{B} + \frac{1}{2}m}$$

$$= \frac{\left(1.2\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{15.6\,\mathrm{kg}} = 0.7538\,\mathrm{m/s^2} \approx \boxed{0.75\,\mathrm{m/s^2}}$$

(*b*) If the mass of the pulley is ignored, then we have the following.

$$a = \frac{\left(m_\mathrm{B} - m_\mathrm{A}\right)g}{\left(m_\mathrm{A} + m_\mathrm{B}\right)} = \frac{\left(1.2\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{15.2\,\mathrm{kg}} = 0.7737\,\mathrm{m/s^2}$$

$$\%\text{ error} = \left(\frac{0.7737\,\mathrm{m/s^2} - 0.7538\,\mathrm{m/s^2}}{0.7538\,\mathrm{m/s^2}}\right) \times 100 = \boxed{2.6\%}$$

39. The rotational inertia of the compound object is the sum of the individual moments of inertia.

$$I = I_\text{particles} + I_\text{rod} = m\left(0\right)^2 + m\left(\tfrac{1}{3}\ell\right)^2 + m\left(\tfrac{2}{3}\ell\right)^2 + m\ell^2 + \tfrac{1}{3}M\ell^2 = \left(\tfrac{14}{9}m + \tfrac{1}{3}M\right)\ell^2$$

(*a*) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{14}{9}m + \frac{1}{3}M\right)\ell^2\omega^2 = \boxed{\left(\frac{7}{9}m + \frac{1}{6}M\right)\ell^2\omega^2}$

(*b*) $L = I\omega = \boxed{\left(\frac{14}{9}m + \frac{1}{3}M\right)\ell^2\omega}$

40. (*a*) We calculate the full angular momentum vector about the center of mass of the system. We take the instant shown in the diagram, with the positive x axis to the right, the positive y axis up along the axle, and the positive z axis out of the plane of the diagram towards the viewer. We take the upper mass as mass A and the lower mass as mass B. If we assume that the system is rotating counterclockwise when viewed from above along the rod, then the velocity of mass A is in the positive z direction, and the velocity of mass B is in the negative z direction. The speed is given by $v = \omega r = \left(4.5\,\mathrm{rad/s}\right)\left(0.24\,\mathrm{m}\right) = 1.08\,\mathrm{m/s}$.

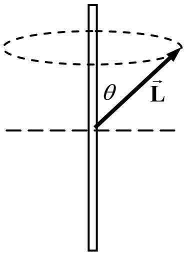
$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_\mathrm{A} \times \vec{\mathbf{p}}_\mathrm{A} + \vec{\mathbf{r}}_\mathrm{B} \times \vec{\mathbf{p}}_\mathrm{B} = m\left\{\vec{\mathbf{r}}_\mathrm{A} \times \vec{\mathbf{v}}_\mathrm{A} + \vec{\mathbf{r}}_\mathrm{B} \times \vec{\mathbf{v}}_\mathrm{B}\right\}$$

$$= m\left\{\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -0.24\,\text{m} & 0.21\,\text{m} & 0 \\ 0 & 0 & v \end{vmatrix} + \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0.24\,\text{m} & -0.21\,\text{m} & 0 \\ 0 & 0 & -v \end{vmatrix}\right\}$$

$$= m\left\{2\hat{\mathbf{i}}(0.21\,\text{m})v + 2\hat{\mathbf{j}}(0.24\,\text{m})v\right\} = 2mv\left\{\hat{\mathbf{i}}(0.21\,\text{m}) + \hat{\mathbf{j}}(0.24\,\text{m})\right\}$$

$$= 2(0.48\,\text{kg})(1.08\,\text{m/s})\left\{\hat{\mathbf{i}}(0.21\,\text{m}) + \hat{\mathbf{j}}(0.24\,\text{m})\right\} = \left[\hat{\mathbf{i}}(0.2177) + \hat{\mathbf{j}}(0.2488)\right]\text{kg}\cdot\text{m}^2/\text{s}$$

The component along the axis is the $\hat{\mathbf{j}}$ component, $\boxed{0.25\,\text{kg}\cdot\text{m}^2/\text{s}}$.



(*b*) The angular momentum vector will precess about the axle. The tip of the angular momentum vector traces out the dashed circle in the diagram.

$$\theta = \tan^{-1}\frac{L_x}{L_y} = \tan^{-1}\frac{0.2177\,\text{kg}\cdot\text{m}^2/\text{s}}{0.2488\,\text{kg}\cdot\text{m}^2/\text{s}} = \boxed{41^\circ}$$

41. (*a*) We assume the system is moving such that mass B is moving down, mass A is moving to the left, and the pulley is rotating counterclockwise. We take those as positive directions. The angular momentum of masses A and B is the same as that of a point mass. We assume the rope is moving without slipping, so $v = \omega_{\text{pulley}}R_0$.

$$L = L_A + L_B + L_{\text{pulley}} = M_A v R_0 + M_B v R_0 + I\omega = M_A v R_0 + M_B v R_0 + I\frac{v}{R_0}$$

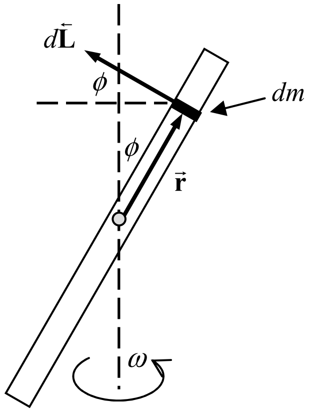
$$= \boxed{\left((M_A + M_B)R_0 + \frac{I}{R_0}\right)v}$$

(*b*) The net torque about the axis of the pulley is that provided by gravity, $M_B g R_0$. Use Eq. 11-9, which is applicable since the axis is fixed.

$$\sum\tau = \frac{dL}{dt} \;\rightarrow\; M_B g R_0 = \frac{d}{dt}\left((M_A + M_B)R_0 + \frac{I}{R_0}\right)v = \left((M_A + M_B)R_0 + \frac{I}{R_0}\right)a \;\rightarrow$$

$$a = \frac{M_B g R_0}{\left((M_A + M_B)R_0 + \dfrac{I}{R_0}\right)} = \boxed{\frac{M_B g}{M_A + M_B + \dfrac{I}{R_0^2}}}$$

42. Take the origin of coordinates to be at the rod's center, and the axis of rotation to be in the z direction. Consider a differential element $dm = \frac{M}{\ell}dr$ of the rod, a distance r from the center. That element rotates in a circle of radius $r\sin\phi$, at a height of $r\cos\phi$. The position and velocity of this point are given by the following.



$$\vec{\mathbf{r}} = r\sin\phi\cos\omega t\,\hat{\mathbf{i}} + r\sin\phi\sin\omega t\,\hat{\mathbf{j}} + r\cos\phi\,\hat{\mathbf{k}}$$

$$= r\left[\sin\phi\cos\omega t\,\hat{\mathbf{i}} + \sin\phi\sin\omega t\,\hat{\mathbf{j}} + \cos\phi\,\hat{\mathbf{k}}\right]$$

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -r\omega\sin\phi\sin\omega t\,\hat{\mathbf{i}} + r\omega\sin\phi\cos\omega t\,\hat{\mathbf{j}}$$

$$= r\omega\left[-\sin\phi\sin\omega t\,\hat{\mathbf{i}} + \sin\phi\cos\omega t\,\hat{\mathbf{j}}\right]$$

Calculate the angular momentum of this element.

$$d\vec{\mathbf{L}} = dm\left(\vec{\mathbf{r}}\times\vec{\mathbf{v}}\right) = r^2\omega\frac{M}{\ell}dr\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ \sin\phi\cos\omega t & \sin\phi\sin\omega t & \cos\phi\\ -\sin\phi\sin\omega t & \sin\phi\cos\omega t & 0\end{vmatrix}$$

$$= \frac{M}{\ell}dr\left[\left(-\sin\phi\cos\phi\cos\omega t\right)\hat{\mathbf{i}} + \left(-\sin\phi\cos\phi\sin\omega t\right)\hat{\mathbf{j}} + \left(\sin^2\phi\cos^2\omega t + \sin^2\phi\sin^2\omega t\right)\hat{\mathbf{k}}\right]$$

$$= \frac{Mr^2\omega\sin\phi}{\ell}dr\left[\left(-\cos\phi\cos\omega t\right)\hat{\mathbf{i}} + \left(-\cos\phi\sin\omega t\right)\hat{\mathbf{j}} + \sin\phi\hat{\mathbf{k}}\right]$$

Note that the directional portion has no *r* dependence. Thus $d\vec{\mathbf{L}}$ for every piece of mass has the same direction. What is that direction? Consider the dot product $\vec{\mathbf{r}}\bullet d\vec{\mathbf{L}}$.

$$\vec{\mathbf{r}}\bullet d\vec{\mathbf{L}} = r\left[\sin\phi\cos\omega t\,\hat{\mathbf{i}} + \sin\phi\sin\omega t\,\hat{\mathbf{j}} + \cos\phi\,\hat{\mathbf{k}}\right]$$

$$\bullet\left[\frac{Mr^2\omega\sin\phi\,dr}{\ell}\left[\left(-\cos\phi\cos\omega t\right)\hat{\mathbf{i}} + \left(-\cos\phi\sin\omega t\right)\hat{\mathbf{j}} + \sin\phi\hat{\mathbf{k}}\right]\right]$$

$$= \frac{Mr^3\omega\sin\phi\,dr}{\ell}\left[\sin\phi\cos\omega t\left(-\cos\phi\cos\omega t\right) + \sin\phi\sin\omega t\left(-\cos\phi\sin\omega t\right) + \cos\phi\sin\phi\right] = 0$$

Thus $d\vec{\mathbf{L}} \perp \vec{\mathbf{r}}$ for every point on the rod. Also, if ϕ is an acute angle, the *z* component of $d\vec{\mathbf{L}}$ is positive. The direction of $d\vec{\mathbf{L}}$ is illustrated in the diagram.

Integrate over the length of the rod to find the total angular momentum. And since the direction of $d\vec{\mathbf{L}}$ is not dependent on *r*, the direction of $\vec{\mathbf{L}}$ is the same as the direction of $d\vec{\mathbf{L}}$.

$$\vec{\mathbf{L}} = \int d\vec{\mathbf{L}} = \frac{M\omega\sin\phi}{\ell}\left[\left(-\cos\phi\cos\omega t\right)\hat{\mathbf{i}} + \left(-\cos\phi\sin\omega t\right)\hat{\mathbf{j}} + \sin\phi\hat{\mathbf{k}}\right]\int_{-\ell/2}^{\ell/2} r^2dr$$

$$= \frac{M\omega\ell^2\sin\phi}{12}\left[\left(-\cos\phi\cos\omega t\right)\hat{\mathbf{i}} + \left(-\cos\phi\sin\omega t\right)\hat{\mathbf{j}} + \sin\phi\hat{\mathbf{k}}\right]$$

Find the magnitude using the Pythagorean theorem.

$$L = \frac{M\omega\ell^2\sin\phi}{12}\left[\left(-\cos\phi\cos\omega t\right)^2 + \left(-\cos\phi\sin\omega t\right)^2 + \sin^2\phi\right]^{1/2} = \boxed{\frac{M\omega\ell^2\sin\phi}{12}}$$

$\vec{\mathbf{L}}$ is inclined upwards an angle of ϕ from the *x*-*y* plane, perpendicular to the rod.

$\boxed{43.}$ We follow the notation and derivation of Eq. 11-9b. Start with the general definition of angular momentum, $\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$. Then express position and velocity with respect to the center of mass.

$\vec{\mathbf{r}}_i = \vec{\mathbf{r}}_{\text{CM}} + \vec{\mathbf{r}}_i^*$, where $\vec{\mathbf{r}}_i^*$ is the position of the i^{th} particle with respect to the center of mass

$\vec{\mathbf{v}}_i = \vec{\mathbf{v}}_{\text{CM}} + \vec{\mathbf{v}}_i^*$, which comes from differentiating the above relationship for position

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i = \sum_i \vec{\mathbf{r}}_i \times m_i\vec{\mathbf{v}}_i = \sum_i \left(\vec{\mathbf{r}}_{\text{CM}} + \vec{\mathbf{r}}_i^*\right)\times m_i\left(\vec{\mathbf{v}}_{\text{CM}} + \vec{\mathbf{v}}_i^*\right)$$

$$= \sum_i m_i \vec{\mathbf{r}}_{\text{CM}} \times \vec{\mathbf{v}}_{\text{CM}} + \sum_i m_i \vec{\mathbf{r}}_{\text{CM}} \times \vec{\mathbf{v}}_i^* + \sum_i m_i \vec{\mathbf{r}}_i^* \times \vec{\mathbf{v}}_{\text{CM}} + \sum_i m_i \vec{\mathbf{r}}_i^* \times \vec{\mathbf{v}}_i^*$$

Note that the center of mass quantities are not dependent on the summation subscript, and so they may be taken outside the summation process.

$$\vec{\mathbf{L}} = \left(\vec{\mathbf{r}}_{\text{CM}} \times \vec{\mathbf{v}}_{\text{CM}}\right) \sum_i m_i + \vec{\mathbf{r}}_{\text{CM}} \times \sum_i m_i \vec{\mathbf{v}}_i^* + \left(\sum_i m_i \vec{\mathbf{r}}_i^*\right) \times \vec{\mathbf{v}}_{\text{CM}} + \sum_i m_i \vec{\mathbf{r}}_i^* \times \vec{\mathbf{v}}_i^*$$

In the first term, $\sum_i m_i = M$. In the second term, we have the following.

$$\sum_i m_i \vec{\mathbf{v}}_i^* = \sum_i m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\text{CM}}\right) = \sum_i m_i \vec{\mathbf{v}}_i - \sum_i m_i \vec{\mathbf{v}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{v}}_i - M\vec{\mathbf{v}}_{\text{CM}} = 0$$

This is true from the definition of center of mass velocity: $\vec{\mathbf{v}}_{\text{CM}} = \frac{1}{M} \sum_i m_i \vec{\mathbf{v}}_i$.

Likewise, in the third term, we have the following.

$$\sum_i m_i \vec{\mathbf{r}}_i^* = \sum_i m_i \left(\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_{\text{CM}}\right) = \sum_i m_i \vec{\mathbf{r}}_i - \sum_i m_i \vec{\mathbf{r}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{r}}_i - M\vec{\mathbf{r}}_{\text{CM}} = 0$$

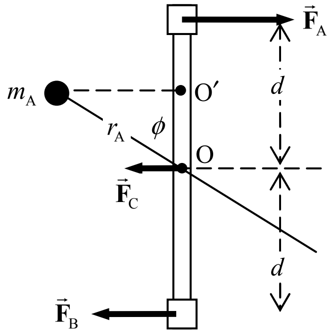
This is true from the definition of center of mass: $\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \sum_i m_i \vec{\mathbf{r}}_i$.

Thus $\vec{\mathbf{L}} = M\left(\vec{\mathbf{r}}_{\text{CM}} \times \vec{\mathbf{v}}_{\text{CM}}\right) + \sum_i m_i \vec{\mathbf{r}}_i^* \times \vec{\mathbf{v}}_i^* = \boxed{\vec{\mathbf{L}}^* + \left(\vec{\mathbf{r}}_{\text{CM}} \times M\vec{\mathbf{v}}_{\text{CM}}\right)}$ as desired.

44. The net torque to maintain the rotation is supplied by the forces at the bearings. From Figure 11-18 we see that the net torque is $2Fd$, where d is the distance from the bearings to the center of the axle. The net torque is derived in Example 11-10.

$$\tau_{\text{net}} = \frac{I\omega^2}{\tan\phi} = 2Fd \quad \rightarrow \quad F = \frac{I\omega^2}{2d\tan\phi} = \frac{\left(m_{\text{A}} r_{\text{A}}^2 + m_{\text{B}} r_{\text{B}}^2\right)\left(\omega^2 \sin^2\phi\right)}{2d\tan\phi}$$

45. As in problem 44, the bearings are taken to be a distance d from point O. We choose the center of the circle in which m_{A} moves as the origin, and label it O′ in the diagram. This choice of origin makes the position vector and the velocity vector always perpendicular to each other, and so makes $\vec{\mathbf{L}}$ point along the axis of rotation at all times. So $\vec{\mathbf{L}}$ is parallel to $\vec{\boldsymbol{\omega}}$. The magnitude of the angular momentum is as follows.



$$L = m_{\text{A}} r_{\text{A}\perp} v = m_{\text{A}}\left(r_{\text{A}}\sin\phi\right)\left(\omega r_{\text{A}}\sin\phi\right) = m_{\text{A}} r_{\text{A}}^2 \omega \sin^2\phi$$

$\vec{\mathbf{L}}$ is constant in both magnitude and direction, and so $\frac{d\vec{\mathbf{L}}}{dt} = \vec{\boldsymbol{\tau}}_{\text{net}} = 0$.

Be careful to take torques about the same point used for the angular momentum.

$$\tau_{\text{net}} = 0 = F_{\text{A}}\left(d - r_{\text{A}}\cos\phi\right) + F_{\text{B}}\left(d + r_{\text{A}}\cos\phi\right) = 0 \quad \rightarrow \quad F_{\text{B}} = -F_{\text{A}} \frac{\left(d - r_{\text{A}}\cos\phi\right)}{\left(d + r_{\text{A}}\cos\phi\right)}$$

The mass is moving in a circle and so must have a net centripetal force pulling in on the mass (if shown, it would point to the right in the diagram). This force is given by $F_{\text{C}} = m_{\text{A}}\omega^2 r_{\text{A}}\sin\phi$. By Newton's third law, there must be an equal but opposite force (to the left) on the rod and axle due to the mass. But the rod and axle are massless, and so the net force on it must be 0.

$$F_{\substack{\text{net}\\\text{on axle}}} = F_A - F_B - F_C = F_A - \left(-F_A\frac{(d - r_A\cos\phi)}{(d + r_A\cos\phi)}\right) - m_A\omega^2 r_A\sin\phi = 0 \rightarrow$$

$$\boxed{F_A = \frac{m_A\omega^2 r_A\sin\phi(d + r_A\cos\phi)}{2d}}$$

$$F_B = -F_A\frac{(d - r_A\cos\phi)}{(d + r_A\cos\phi)} = \boxed{-\frac{m_A\omega^2 r_A\sin\phi(d - r_A\cos\phi)}{2d}}$$

We see that $\vec{\mathbf{F}}_B$ points in the opposite direction as shown in the free-body diagram.

46. We use the result from Problem 44,

$$F = \frac{(m_A r_A^2 + m_B r_B^2)(\omega^2\sin^2\phi)}{2d\tan\phi} = \frac{mr^2\omega^2\sin^2\phi}{d\tan\phi} = \frac{(0.60\,\text{kg})(0.30\,\text{m})^2(11.0\,\text{rad/s})^2\sin^2 34.0°}{(0.115\,\text{m})\tan 34.0°}$$

$$= \boxed{26\,\text{N}}$$

47. This is a variation on the ballistic pendulum problem. Angular momentum is conserved about the pivot at the upper end of the rod during the collision, and this is used to find the angular velocity of the system immediately after the collision. Mechanical energy is then conserved during the upward swing. Take the 0 position for gravitational potential energy to be the original location of the center of mass of the rod. The bottom of the rod will rise twice the distance of the center of mass of the system, since it is twice as far from the pivot.

$$L_{\substack{\text{before}\\\text{collision}}} = L_{\substack{\text{after}\\\text{collision}}} \rightarrow m\left(\tfrac{1}{2}\ell\right)v = \left(I_{\text{rod}} + I_{\text{putty}}\right)\omega \rightarrow \omega = \frac{m\ell v}{2\left(I_{\text{rod}} + I_{\text{putty}}\right)}$$

$$E_{\substack{\text{after}\\\text{collision}}} = E_{\substack{\text{top of}\\\text{swing}}} \rightarrow K_{\substack{\text{after}\\\text{collision}}} = U_{\substack{\text{top of}\\\text{swing}}} \rightarrow \tfrac{1}{2}\left(I_{\text{rod}} + I_{\text{putty}}\right)\omega^2 = (m + M)gh \rightarrow$$

$$h_{\text{CM}} = \frac{\left(I_{\text{rod}} + I_{\text{putty}}\right)\omega^2}{2(m + M)g} = \frac{\left(I_{\text{rod}} + I_{\text{putty}}\right)}{2(m + M)g}\left[\frac{m\ell v}{2\left(I_{\text{rod}} + I_{\text{putty}}\right)}\right]^2 = \frac{m^2\ell^2 v^2}{8g(m + M)\left(I_{\text{rod}} + I_{\text{putty}}\right)}$$

$$= \frac{m^2\ell^2 v^2}{8g(m + M)\left(\tfrac{1}{3}M\ell^2 + m\left(\tfrac{1}{2}\ell\right)^2\right)} = \frac{m^2 v^2}{2g(m + M)\left(\tfrac{4}{3}M + m\right)}$$

$$h_{\text{bottom}} = 2h_{\text{CM}} = \boxed{\frac{m^2 v^2}{g(m + M)\left(\tfrac{4}{3}M + m\right)}}$$

48. Angular momentum about the pivot is conserved during this collision. Note that both objects have angular momentum after the collision.

$$L_{\substack{\text{before}\\\text{collision}}} = L_{\substack{\text{after}\\\text{collision}}} \rightarrow L_{\substack{\text{bullet}\\\text{initial}}} = L_{\substack{\text{stick}\\\text{final}}} + L_{\substack{\text{bullet}\\\text{final}}} \rightarrow m_{\text{bullet}}v_0\left(\tfrac{1}{4}\ell\right) = I_{\text{stick}}\omega + m_{\text{bullet}}v_f\left(\tfrac{1}{4}\ell\right) \rightarrow$$

$$\omega = \frac{m_{\text{bullet}}(v_0 - v_f)\left(\tfrac{1}{4}\ell\right)}{I_{\text{stick}}} = \frac{m_{\text{bullet}}(v_0 - v_f)\left(\tfrac{1}{4}\ell\right)}{\tfrac{1}{12}M_{\text{stick}}\ell_{\text{stick}}^2} = \frac{3m_{\text{bullet}}(v_0 - v_f)}{M_{\text{stick}}\ell_{\text{stick}}} = \frac{3(0.0030\,\text{kg})(110\,\text{m/s})}{(0.27\,\text{kg})(1.0\,\text{m})}$$

$$= \boxed{3.7\,\text{rad/s}}$$

$\boxed{49.}$ The angular momentum of the Earth–meteorite system is conserved in the collision. The Earth is spinning counterclockwise as viewed in the diagram. We take that direction as the positive direction for rotation about the Earth's axis, and so the initial angular momentum of the meteorite is negative.

$$L_{\text{initial}} = L_{\text{final}} \rightarrow I_{\text{Earth}}\omega_0 - mR_{\text{E}}v\sin 45^\circ = \left(I_{\text{Earth}} + I_{\text{meteorite}}\right)\omega \rightarrow$$

$$\omega = \frac{I_{\text{Earth}}\omega_0 - mR_{\text{E}}v\sin 45^\circ}{\left(I_{\text{Earth}} + I_{\text{meteorite}}\right)} = \frac{\frac{2}{5}M_{\text{E}}R_{\text{E}}^2\omega_0 - mR_{\text{E}}v\sin 45^\circ}{\left(\frac{2}{5}M_{\text{E}}R_{\text{E}}^2 + mR_{\text{E}}^2\right)}$$

$$\frac{\omega}{\omega_0} = \frac{\frac{2}{5}M_{\text{E}}R_{\text{E}}^2 - mR_{\text{E}}\frac{v}{\omega_0}\frac{1}{\sqrt{2}}}{R_{\text{E}}^2\left(\frac{2}{5}M_{\text{E}} + m\right)} = \frac{R_{\text{E}}^2\left(\frac{2}{5}M_{\text{E}} - \frac{mv}{\sqrt{2}\omega_0 R_{\text{E}}}\right)}{R_{\text{E}}^2\left(m + \frac{2}{5}M_{\text{E}}\right)} = \frac{\left(\frac{2}{5}M_{\text{E}} - \frac{mv}{\sqrt{2}\omega_0 R_{\text{E}}}\right)}{\left(m + \frac{2}{5}M_{\text{E}}\right)}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1 = \frac{\left(\frac{2}{5}M_{\text{E}} - \frac{mv}{\sqrt{2}\omega_0 R_{\text{E}}}\right)}{\left(m + \frac{2}{5}M_{\text{E}}\right)} - 1 = \frac{-\left(\frac{v}{\sqrt{2}\omega_0 R_{\text{E}}} + 1\right)}{\left(1 + \frac{2}{5}\frac{M_{\text{E}}}{m}\right)}$$

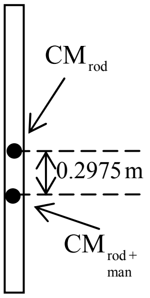
$$= -\frac{\left(\frac{2.2\times10^4\text{ m/s}}{\sqrt{2}\left(\frac{2\pi}{86{,}400}\text{ rad/s}\right)\left(6.38\times10^6\text{ m}\right)} + 1\right)}{\left(1 + \frac{2}{5}\frac{5.97\times10^{24}\text{ kg}}{5.8\times10^{10}\text{ kg}}\right)} = -8.387\times10^{-13} \approx \boxed{-8.4\times10^{-13}}$$

50. (*a*) Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_{\text{beam}}v_0 = \left(m_{\text{beam}} + m_{\text{man}}\right)v_{\text{final}} \rightarrow$$

$$v_{\text{final}} = \frac{m_{\text{beam}}v_0}{\left(m_{\text{beam}} + m_{\text{man}}\right)} = \frac{(230\text{ kg})(18\text{ m/s})}{(295\text{ kg})} = \boxed{14\text{ m/s}}$$

(*b*) Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking down as the positive direction. See the diagram.



$$y_{\text{CM}} = \frac{m_{\text{beam}}(0) + m_{\text{man}}\left(\frac{1}{2}\ell\right)}{\left(m_{\text{beam}} + m_{\text{man}}\right)} = \frac{(65\text{ kg})(1.35\text{ m})}{(295\text{ kg})}$$

$$= 0.2975\text{ m below center of rod}$$

We need the moment of inertia of the beam about the center of mass of the entire system. Use the parallel axis theorem.

$$I_{\text{beam}} = \tfrac{1}{12}m_{\text{beam}}\ell^2 + m_{\text{beam}}r_{\text{beam}}^2 \ ; \ I_{\text{man}} = m_{\text{man}}\left(\tfrac{1}{2}\ell - r_{\text{beam}}\right)^2$$

$$L_{\text{initial}} = L_{\text{final}} \rightarrow m_{\text{beam}}v_0 r_{\text{beam}} = \left(I_{\text{beam}} + I_{\text{man}}\right)\omega_{\text{final}} \rightarrow$$

$$\omega_{\text{final}} = \frac{m_{\text{beam}}v_0 r_{\text{beam}}}{\left(I_{\text{beam}} + I_{\text{man}}\right)} = \frac{m_{\text{beam}}v_0 r_{\text{beam}}}{\frac{1}{12}m_{\text{beam}}\left(\frac{1}{2}\ell\right)^2 + m_{\text{beam}}r_{\text{beam}}^2 + m_{\text{man}}\left(\frac{1}{2}\ell - r_{\text{beam}}\right)^2}$$

$$= \frac{(230\text{ kg})(18\text{ m/s})(0.2975\text{ m})}{\frac{1}{12}(230\text{ kg})(2.7\text{ m})^2 + (230\text{ kg})(0.2975\text{ m})^2 + (65\text{ kg})(1.0525\text{ m})^2}$$

$$= 5.307\text{ rad/s} \approx \boxed{5.3\text{ rad/s}}$$

51. Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow mv = (m+M)v_{\substack{\text{CM}\\\text{final}}} \rightarrow \boxed{v_{\substack{\text{CM}\\\text{final}}} = \frac{mv}{m+M}}$$

Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking up as the positive direction. See the diagram.

$$y_{\text{CM}} = \frac{m\left(\frac{1}{4}\ell\right) + M(0)}{(m+M)} = \frac{m\ell}{4(m+M)}$$

The distance of the stuck clay ball from the system's center of mass is found.

$$y_{\text{clay}} = \tfrac{1}{4}\ell - y_{\text{CM}} = \tfrac{1}{4}\ell - \frac{m\ell}{4(m+M)} = \frac{M\ell}{4(m+M)}$$

We need the moment of inertia of the rod about the center of mass of the entire system. Use the parallel axis theorem. Treat the clay as a point mass.

$$I_{\text{rod}} = \tfrac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2$$

Now express the conservation of angular momentum about the system's center of mass.

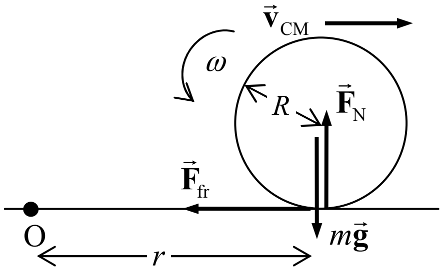
$$L_{\text{initial}} = L_{\text{final}} \rightarrow mvy_{\text{clay}} = \left(I_{\text{rod}} + I_{\text{clay}}\right)\omega_{\text{final}} \rightarrow$$

$$\omega_{\text{final}} = \frac{mvy_{\text{clay}}}{\left(I_{\text{rod}} + I_{\text{clay}}\right)} = \frac{mvy_{\text{clay}}}{\left(\frac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2 + my_{\text{clay}}^2\right)}$$

$$= \frac{mv\dfrac{M\ell}{4(m+M)}}{\left(\frac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2 + m\left[\frac{M\ell}{4(m+M)}\right]^2\right)} = \frac{12mv(m+M)}{\ell\left(7m^2 + 11mM + 4M^2\right)}$$

$$= \boxed{\frac{12mv}{\ell(7m+4M)}}$$

52. (*a*) See the free-body diagram for the ball, after it has moved away from the initial point. There are three forces on the ball. $\vec{\mathbf{F}}_{\text{N}}$ and $m\vec{\mathbf{g}}$ are in opposite directions and each has the same lever arm about an axis passing through point O perpendicular to the plane of the paper. Thus they cause no net torque. $\vec{\mathbf{F}}_{\text{fr}}$ has a 0 lever arm about an axis through O, and so also produces no torque. Thus the net torque on the ball is 0. Since we are calculating torques about a point fixed in an inertial reference frame, we may say that $\sum\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt} = 0$ and so $\vec{\mathbf{L}}$ is constant. Note that the ball is initially slipping while it rolls, and so we may NOT say that $v_0 = R\omega_0$ at the initial motion of the ball.



(*b*) We follow the hint, and express the total angular momentum as a sum of two terms. We take clockwise as the positive rotational direction.

$$\vec{\mathbf{L}} = \vec{\mathbf{L}}_{v_{\text{CM}}} + \vec{\mathbf{L}}_{\omega} = mRv_{\text{CM}} - I\omega$$

The angular momentum is constant. We equate the angular momentum at the initial motion, with $v_{\text{CM}} = v_0$ and $\omega = \omega_0 = \omega_{\text{C}}$, to the final angular momentum, with $v_{\text{CM}} = 0$ and $\omega = 0$.

$$\vec{\mathbf{L}}_{\text{initial}} = \vec{\mathbf{L}}_{\text{final}} \rightarrow \; = mRv_0 - I\omega_{\text{C}} = mR(0) - I(0) = 0 \;\rightarrow\; \omega_C = \frac{mRv_0}{I_{\text{CM}}} = \frac{mRv_0}{\frac{2}{5}mR^2} = \boxed{\frac{5v_0}{2R}}$$

(*c*) Angular momentum is again conserved. In the initial motion, $v_{\text{CM}} = v_0$ and $\omega_0 = 0.90\omega_{\text{C}}$. Note that in the final state, $\omega = v_{\text{CM}}/R$, and the final angular momenta add to each other.

$$L_{\text{initial}} = L_{\text{final}} \;\rightarrow\; mRv_0 - 0.90I\omega_{\text{C}} = mRv_{\text{CM}} + I\frac{v_{\text{CM}}}{R} \;\rightarrow$$

$$mRv_0 - 0.90\left(\tfrac{2}{5}mR^2\right)\left(\frac{5v_0}{2R}\right) = mRv_{\text{CM}} + \left(\tfrac{2}{5}mR^2\right)\frac{v_{\text{CM}}}{R} \;\rightarrow\; \tfrac{1}{10}v_0 = \tfrac{7}{5}v_{\text{CM}} \;\rightarrow$$

$$\boxed{v_{\text{CM}} = \tfrac{1}{14}v_0}$$

This answer is reasonable. There is not enough "backspin" since $\omega_0 < \omega_{\text{C}}$, and so the ball's final state is rolling forwards.

(*d*) Angular momentum is again conserved. In the initial motion, $v_{\text{CM}} = v_0$ and $\omega_0 = 1.10\omega_{\text{C}}$. Note that in the final state, $\omega = v_{\text{CM}}/R$, and the final angular momenta add to each other.

$$L_{\text{initial}} = L_{\text{final}} \;\rightarrow\; mRv_0 - 1.10I\omega_{\text{C}} = mRv_{\text{CM}} + I\frac{v_{\text{CM}}}{R} \;\rightarrow$$

$$mRv_0 - 1.10\left(\tfrac{2}{5}mR^2\right)\left(\frac{5v_0}{2R}\right) = mRv_{\text{CM}} + \left(\tfrac{2}{5}mR^2\right)\frac{v_{\text{CM}}}{R} \;\rightarrow\; -\tfrac{1}{10}v_0 = \tfrac{7}{5}v_{\text{CM}} \;\rightarrow$$

$$\boxed{v_{\text{CM}} = -\tfrac{1}{14}v_0}$$

This answer is reasonable. There is more than enough "backspin" since $\omega_0 > \omega_{\text{C}}$, and so the ball's final state is rolling backwards.

53. Use Eq. 11-13c for the precessional angular velocity.

$$\Omega = \frac{Mgr}{I\omega} \;\rightarrow\; I = \frac{Mgr}{\Omega\omega} = \frac{(0.22\,\text{kg})(9.80\,\text{m/s}^2)(0.035\,\text{m})}{\left[\dfrac{1\,\text{rev}}{6.5\,\text{s}}\left(\dfrac{2\pi\,\text{rad}}{\text{rev}}\right)\right]\left[\dfrac{15\,\text{rev}}{1\,\text{s}}\left(\dfrac{2\pi\,\text{rad}}{\text{rev}}\right)\right]} = \boxed{8.3\times10^{-4}\,\text{kg}\bullet\text{m}^2}$$

54. (*a*) The period of precession is related to the reciprocal of the angular precessional frequency.

$$T = \frac{2\pi}{\Omega} = \frac{2\pi I\omega}{Mgr} = \frac{2\pi\left[\frac{1}{2}Mr_{\text{disk}}^2\right]2\pi f}{Mgr} = \frac{2\pi^2 f r_{\text{disk}}^2}{gr} = \frac{2\pi^2(45\,\text{rev/s})(0.055\,\text{m})^2}{(9.80\,\text{m/s}^2)(0.105\,\text{m})}$$

$$= 2.611\,\text{s} \approx \boxed{2.6\,\text{s}}$$

(*b*) Use the relationship $T = \frac{2\pi^2 f r_{\text{disk}}^2}{gr}$ derived above to see the effect on the period.

$$\frac{T_{\text{new}}}{T_{\text{original}}} = \frac{\dfrac{2\pi^2 f r^2_{\substack{\text{disk}\\\text{new}}}}{g r_{\text{new}}}}{\dfrac{2\pi^2 f r^2_{\text{disk}}}{gr}} = \frac{\dfrac{r^2_{\substack{\text{disk}\\\text{new}}}}{r_{\text{new}}}}{\dfrac{r^2_{\text{disk}}}{r}} = \left(\frac{r_{\substack{\text{disk}\\\text{new}}}}{r_{\text{disk}}}\right)^2 \frac{r}{r_{\text{new}}} = \left(\frac{2}{1}\right)^2 \frac{1}{2} = 2$$

So the period would double, and thus be $T_{\text{new}} = 2T_{\text{original}} = 2(2.611\,\text{s}) = 5.222\,\text{s} \approx \boxed{5.2\,\text{s}}$.

$\boxed{55.}$ Use Eq. 11-13c for the precessional angular velocity.

$$\Omega = \frac{Mgr}{I\omega} = \frac{Mg\left(\frac{1}{2}\ell_{\text{axle}}\right)}{\frac{1}{2}Mr^2_{\text{wheel}}\omega} = \frac{g\ell_{\text{axle}}}{r^2_{\text{wheel}}\omega} = \frac{(9.80\,\text{m/s}^2)(0.25\,\text{m})}{(0.060\,\text{m})^2(85\,\text{rad/s})} = \boxed{8.0\,\text{rad/s}} \quad (1.3\,\text{rev/s})$$

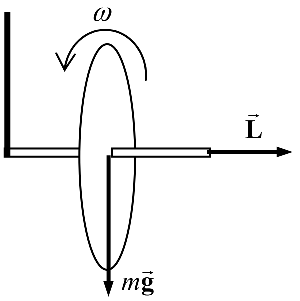
56. The mass is placed on the axis of rotation and so does not change the moment of inertia. The addition of the mass does change the center of mass position *r*, and it does change the total mass, *M*, to $\frac{3}{2}M$.

$$r_{\text{new}} = \frac{M\left(\frac{1}{2}\ell_{\text{axle}}\right) + \frac{1}{2}M\ell_{\text{axle}}}{M + \frac{1}{2}M} = \frac{M\ell_{\text{axle}}}{\frac{3}{2}M} = \frac{2}{3}\ell_{\text{axle}}$$

$$\frac{\Omega_{\text{new}}}{\Omega_{\text{original}}} = \frac{\dfrac{M_{\text{new}}gr_{\text{new}}}{I\omega}}{\dfrac{M_{\text{original}}gr_{\text{original}}}{I\omega}} = \frac{\frac{3}{2}M\left(\frac{2}{3}\ell_{\text{axle}}\right)}{M\left(\frac{1}{2}\ell_{\text{axle}}\right)} = 2 \quad \rightarrow$$

$$\Omega_{\text{new}} = 2\Omega_{\text{original}} = 2(8.0\,\text{rad/s}) = \boxed{16\,\text{rad/s}}$$

57. The spinning bicycle wheel is a gyroscope. The angular frequency of precession is given by Eq. 11-13c.

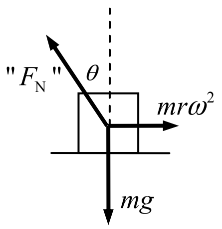


$$\Omega = \frac{Mgr}{I\omega} = \frac{Mgr}{Mr^2_{\text{wheel}}\omega} = \frac{gr}{r^2_{\text{wheel}}\omega} = \frac{(9.80\,\text{m/s}^2)(0.20\,\text{m})}{(0.325\,\text{m})^2(4.0\pi\,\text{rad/s})}$$

$$= 1.477\,\text{rad/s}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{14\,\text{rev/min}}$$

In the figure, the torque from gravity is directed back into the paper. This gives the direction of precession. When viewed from above, the wheel will precess $\boxed{\text{counterclockwise}}$.

58. We assume that the plant grows in the direction of the local "normal" force. In the rotating frame of the platform, there is an outward fictitious force of magnitude $m\frac{v^2}{r} = mr\omega^2$. See the free body diagram for the rotating frame of reference. Since the object is not accelerated in that frame of reference, the "normal" force must be the vector sum of the other two forces. Write Newton's second law in this frame of reference.



$$\sum F_{\text{vertical}} = F_{\text{N}} \cos\theta - mg = 0 \quad \to \quad F_{\text{N}} = \frac{mg}{\cos\theta}$$

$$\sum F_{\text{horizontal}} = F_{\text{N}} \sin\theta - mr\omega^2 = 0 \quad \to \quad F_{\text{N}} = \frac{mr\omega^2}{\sin\theta}$$

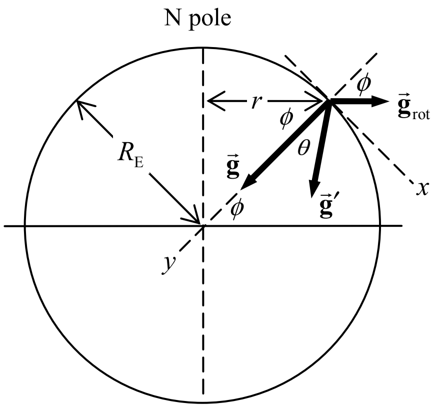
$$\frac{mg}{\cos\theta} = \frac{mr\omega^2}{\sin\theta} \quad \to \quad \tan\theta = \frac{r\omega^2}{g} \quad \to \quad \theta = \boxed{\tan^{-1}\frac{r\omega^2}{g}}$$

In the inertial frame of reference, the "normal" force still must point inward. The horizontal component of that force is providing the centripetal acceleration, which points inward.

59. (*a*) At the North Pole, the factor $m\omega^2 r$ is zero, and so there is no effect from the rotating reference frame.

$$g' = g - \omega^2 r = g - 0 = \boxed{9.80\,\text{m/s}^2\text{, inward along a radial line}}$$

(*b*) To find the direction relative to a radial line, we orient the coordinate system along the tangential (x) and radial (y, with inward as positive) directions. See the diagram. At a specific latitude ϕ, the "true" gravity will point purely in the positive y direction, $\vec{\mathbf{g}} = g\,\hat{\mathbf{j}}$. We label the effect of the rotating reference frame as $\vec{\mathbf{g}}_{\text{rot}}$. The effect of $\vec{\mathbf{g}}_{\text{rot}}$ can be found by decomposing it along the axes. Note that the radius of rotation is not the radius of the Earth, but $r = R_{\text{E}} \cos\phi$.



$$\begin{aligned}\vec{\mathbf{g}}_{\text{rot}} &= r\omega^2 \sin\phi\,\hat{\mathbf{i}} - r\omega^2 \cos\phi\,\hat{\mathbf{j}} \\ &= R_{\text{E}}\omega^2 \cos\phi \sin\phi\,\hat{\mathbf{i}} - R_{\text{E}}\omega^2 \cos^2\phi\,\hat{\mathbf{j}}\end{aligned}$$

$$\vec{\mathbf{g}}' = \vec{\mathbf{g}} + \vec{\mathbf{g}}_{\text{rot}} = R_{\text{E}}\omega^2 \cos\phi \sin\phi\,\hat{\mathbf{i}} + \left(g - R_{\text{E}}\omega^2 \cos^2\phi\right)\hat{\mathbf{j}}$$

The angle of deflection from the vertical (θ) can be found from the components of $\vec{\mathbf{g}}'$.

$$\theta = \tan^{-1}\frac{g'_x}{g'_y} = \tan^{-1}\frac{R_{\text{E}}\omega^2 \cos\phi \sin\phi}{g - R_{\text{E}}\omega^2 \cos^2\phi}$$

$$= \tan^{-1}\frac{\left(6.38\times10^6\,\text{m}\right)\left(\dfrac{2\pi\,\text{rad}}{86{,}400\,\text{s}}\right)^2 \cos 45^\circ \sin 45^\circ}{9.80\,\text{m/s}^2 - \left(6.38\times10^6\,\text{m}\right)\left(\dfrac{2\pi\,\text{rad}}{86{,}400\,\text{s}}\right)^2 \cos^2 45^\circ} = \tan^{-1}\frac{1.687\times10^{-2}\,\text{m/s}^2}{9.783\,\text{m/s}^2} = 0.988^\circ$$

The magnitude of $\vec{\mathbf{g}}'$ is found from the Pythagorean theorem.

$$g' = \sqrt{g_x'^2 + g_y'^2} = \sqrt{\left(1.687\times10^{-2}\,\text{m/s}^2\right)^2 + \left(9.783\,\text{m/s}^2\right)^2} = 9.78\,\text{m/s}^2$$

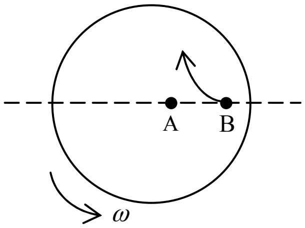
And so $\boxed{g' = 9.78\,\text{m/s}^2\text{, }0.0988^\circ\text{ south from an inward radial line}}$.

(*c*) At the equator, the effect of the rotating reference frame is directly opposite to the "true" acceleration due to gravity. Thus the values simply subtract.

$$g' = g - \omega^2 r = g - \omega^2 R_{\text{Earth}} = 9.80\,\text{m/s}^2 - \left(\frac{2\pi\,\text{rad}}{86.400\,\text{s}}\right)^2 \left(6.38\times10^6\,\text{m}\right)$$

$$= \boxed{9.77\,\text{m/s}^2\text{, inward along a radial line}}$$

60. (*a*) In the inertial frame, the ball has the tangential speed of point B, $v_B = r_B\omega$. This is greater than the tangential speed of the women at A, $v_A = r_A\omega$, so the ball passes in front of the women. The ball deflects $\boxed{\text{to the right}}$ of the intended motion. See the diagram.



(*b*) We follow a similar derivation to that given in section 11-9. In the inertial frame, the ball is given an inward radial velocity *v* by the man at B. The ball moves radially inward a distance $r_B - r_A$ during a short time *t*, and so $r_B - r_A = vt$. During this time, the ball moves sideways a distance $s_B = v_B t$, while the woman moves a distance $s_A = v_A t$. The ball will pass in front of the woman a distance given by the following.

$$s = s_B - s_A = \left(v_B - v_A\right)t = \left(r_B - r_A\right)\omega t = v\omega t^2$$

This is the sideways displacement as seen from the noninertial frame, and so the deflection is $\boxed{v\omega t^2}$. This has the same form as motion at constant acceleration, with $s = v\omega t^2 = \frac{1}{2}a_{\text{Cor}}t^2$.

Thus the Coriolis acceleration is $a_{\text{Cor}} = \boxed{2v\omega}$.

$\boxed{61.}$ The footnote on page 302 gives the Coriolis acceleration as $\vec{\mathbf{a}}_{\text{Cor}} = 2\vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}$. The angular velocity vector is parallel to the axis of rotation of the Earth. For the Coriolis acceleration to be 0, then, the velocity must be parallel to the axis of rotation of the Earth. At the equator this means moving either $\boxed{\text{due north or due south}}$.

62. The Coriolis acceleration of the ball is modified to $a_{\text{Cor}} = 2\omega v_\perp = 2\omega v\cos\lambda$, where *v* is the vertical speed of the ball. The vertical speed is not constant as the ball falls, but is given by $v = v_0 + gt$. Assuming the ball starts from rest, then $a_{\text{Cor}} = 2\omega gt\cos\lambda$. That is not a constant acceleration, and so to find the deflection due to this acceleration, we must integrate twice.

$$a_{\text{Cor}} = 2\omega gt\cos\lambda = \frac{dv_{\text{Cor}}}{dt} \rightarrow dv_{\text{Cor}} = 2\omega gt\cos\lambda dt \rightarrow \int_0^{v_{\text{Cor}}} dv_{\text{Cor}} = 2\omega g\cos\lambda\int_0^t tdt \rightarrow$$

$$v_{\text{Cor}} = \omega gt^2\cos\lambda = \frac{dx_{\text{Cor}}}{dt} \rightarrow dx_{\text{Cor}} = \omega gt^2\cos\lambda dt \rightarrow \int_0^{x_{\text{Cor}}} dx_{\text{Cor}} = \int_0^t \omega gt^2\cos\lambda dt \rightarrow$$

$$x_{\text{Cor}} = \tfrac{1}{3}\omega gt^3\cos\lambda$$

So to find the Coriolis deflection, we need the time of flight. The vertical motion is just uniform acceleration, for an object dropped from rest. Use that to find the time.

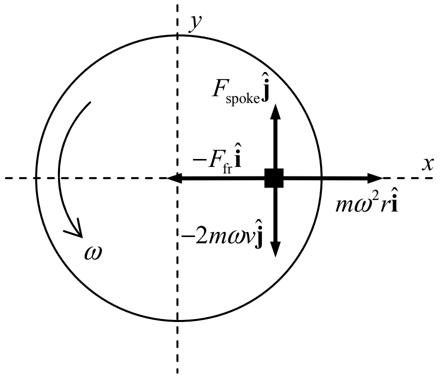
$$y = y_0 + v_{0y}t + \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2\left(y - y_0\right)}{g}} = \sqrt{\frac{2h}{g}}$$

$$x_{\text{Cor}} = \tfrac{1}{3}\omega g t^3 \cos\lambda = \tfrac{1}{3}\omega g \cos\lambda \left(\frac{2h}{g}\right)^{3/2} = \tfrac{1}{3}\omega \cos\lambda \left(\frac{8h^3}{g}\right)^{1/2}$$

$$= \tfrac{1}{3}\left(\frac{2\pi\,\text{rad}}{86{,}400\,\text{s}}\right)(\cos 44^\circ)\left(\frac{8(110\,\text{m})^3}{9.80\,\text{m/s}^2}\right)^{1/2} = \boxed{0.018\,\text{m}}$$

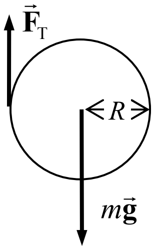
The ball is deflected by about 2 cm in falling 110 meters.

63. The diagram is a view from above the wheel. The ant is moving in a curved path, and so there is a fictitious outward radial force of $m\omega^2 r\hat{\mathbf{i}}$. The ant is moving away from the axis of rotation, and so there is a fictitious Coriolis force of $-2m\omega v\hat{\mathbf{j}}$. The ant is moving with a constant speed, and so in the rotating reference frame the net force is 0. Thus there must be forces that oppose these fictitious forces. The ant is in contact with the spoke, and so there can be components of that contact force in each of the coordinate axes. The force opposite to the local direction of motion is friction, and so is $-F_{\text{fr}}\hat{\mathbf{i}}$. The spoke is also pushing in the opposite direction to the Coriolis force, and so we have $F_{\text{spoke}}\hat{\mathbf{j}}$. Finally, in the vertical direction, there is gravity $\left(-mg\hat{\mathbf{k}}\right)$ and the usual normal force $\left(F_{\text{N}}\hat{\mathbf{k}}\right)$. These forces are not shown on the diagram, since it is viewed from above.



$$\boxed{\vec{\mathbf{F}}_{\substack{\text{rotating}\\ \text{frame}}} = \left(m\omega^2 r - F_{\text{Fr}}\right)\hat{\mathbf{i}} + \left(F_{\text{spoke}} - 2m\omega v\right)\hat{\mathbf{j}} + \left(F_{\text{N}} - mg\right)\hat{\mathbf{k}}}$$

64. (*a*) Because the hoop is rolling without slipping, the acceleration of the center of the center of mass is related to the angular acceleration by $a_{\text{CM}} = \alpha R$. From the free-body diagram, write Newton's second law for the vertical direction and for rotation. We call down and clockwise the positive directions. Combine those equations to find the angular acceleration.



$$\sum F_{\text{vertical}} = Mg - F_{\text{T}} = Ma_{\text{CM}} \rightarrow F_{\text{T}} = M\left(g - a_{\text{CM}}\right)$$

$$\sum \tau = F_{\text{T}}R = I\alpha = MR^2\frac{a_{\text{CM}}}{R} = MRa_{\text{CM}}$$

$$M\left(g - a_{\text{CM}}\right)R = MRa_{\text{CM}} \rightarrow \left(g - a_{\text{CM}}\right) = a_{\text{CM}} \rightarrow a_{\text{CM}} = \tfrac{1}{2}g \rightarrow \alpha = \tfrac{1}{2}\frac{g}{R}$$

$$\tau = I\alpha = MR^2\,\tfrac{1}{2}\frac{g}{R} = \tfrac{1}{2}MRg = \frac{dL}{dt} \rightarrow \boxed{L = \tfrac{1}{2}MRgt}$$

(*b*) $F_{\text{T}} = M\left(g - a_{\text{CM}}\right) = \boxed{\tfrac{1}{2}Mg}$, and is constant in time.

65. (*a*) Use Eq. 11-6 to find the angular momentum.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = m\left(\vec{\mathbf{r}} \times \vec{\mathbf{v}}\right) = (1.00\,\text{kg})\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0 & 4.0 \\ 7.0 & 6.0 & 0 \end{vmatrix}\text{kg}\bullet\text{m}^2/\text{s} = \boxed{\left(-24\hat{\mathbf{i}} + 28\hat{\mathbf{j}} - 14\hat{\mathbf{k}}\right)\text{kg}\bullet\text{m}^2/\text{s}}$$

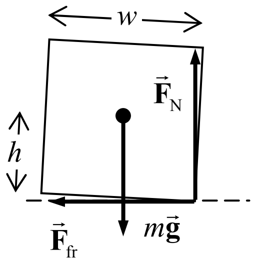
(*b*) $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0 & 4.0 \\ 4.0 & 0 & 0 \end{vmatrix} \text{m}\bullet\text{N} = \boxed{\left(16\hat{\mathbf{j}} - 8.0\hat{\mathbf{k}}\right)\text{m}\bullet\text{N}}$

66. Angular momentum is conserved in the interaction between the child and the merry-go-round.

$$L_{\text{initial}} = L_{\text{final}} \rightarrow L_{\underset{\text{mgr}}{0}} = L_{\underset{\text{child}}{\text{f}}} + L_{\underset{\text{mgr}}{\text{f}}} \rightarrow I_{\text{mgr}}\omega_0 = \left(I_{\text{mgr}} + I_{\text{child}}\right)\omega = \left(I_{\text{mgr}} + m_{\text{child}}R^2_{\text{mgr}}\right)\omega \rightarrow$$

$$m_{\text{child}} = \frac{I_{\text{mgr}}\left(\omega_0 - \omega\right)}{R^2_{\text{mgr}}\omega} = \frac{\left(1260\,\text{kg}\bullet\text{m}^2\right)\left(0.45\,\text{rad/s}\right)}{\left(2.5\,\text{m}\right)^2\left(1.25\,\text{rad/s}\right)} = \boxed{73\,\text{kg}}$$

$\boxed{67.}$ (*a*) See the free-body diagram for the vehicle, tilted up on 2 wheels, on the verge of rolling over. The center of the curve is to the left in the diagram, and so the center of mass is accelerating to the left. The force of gravity acts through the center of mass, and so causes no torque about the center of mass, but the normal force and friction cause opposing torques about the center of mass. The amount of tilt is exaggerated. Write Newton's second laws for the horizontal and vertical directions and for torques, taking left, up, and counterclockwise as positive.



$$\sum F_{\text{vertical}} = F_{\text{N}} - Mg = 0 \rightarrow F_{\text{N}} = Mg$$

$$\sum F_{\text{horizontal}} = F_{\text{fr}} = M\frac{v_{\text{C}}^2}{R}$$

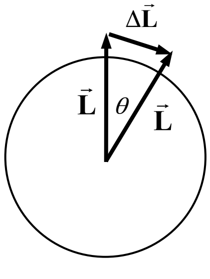
$$\sum \tau = F_{\text{N}}\left(\tfrac{1}{2}w\right) - F_{\text{fr}}h = 0 \rightarrow F_{\text{N}}\left(\tfrac{1}{2}w\right) = F_{\text{fr}}h$$

$$Mg\left(\tfrac{1}{2}w\right) = M\frac{v_{\text{C}}^2}{R}h \rightarrow \boxed{v_{\text{C}} = \sqrt{Rg\left(\frac{w}{2h}\right)}}$$

(*b*) From the above result, we see that $R = \frac{v_{\text{C}}^2}{g}\frac{2h}{w} = \frac{v_{\text{C}}^2}{g\left(\text{SSF}\right)}$.

$$\frac{R_{\text{car}}}{R_{\text{SUV}}} = \frac{\dfrac{v_{\text{C}}^2}{g\left(\text{SSF}\right)_{\text{car}}}}{\dfrac{v_{\text{C}}^2}{g\left(\text{SSF}\right)_{\text{SUV}}}} = \frac{\left(\text{SSF}\right)_{\text{SUV}}}{\left(\text{SSF}\right)_{\text{car}}} = \frac{1.05}{1.40} = \boxed{0.750}$$

68. The force applied by the spaceship puts a torque on the asteroid which changes its angular momentum. We assume that the rocket ship's direction is adjusted to always be tangential to the surface. Thus the torque is always perpendicular to the angular momentum, and so will not change the magnitude of the angular momentum, but only its direction, similar to the action of a centripetal force on an object in circular motion. From the diagram, we make an approximation.



$$\tau = \frac{dL}{dt} \approx \frac{\Delta L}{\Delta t} \approx \frac{L\Delta\theta}{\Delta t} \rightarrow$$

$$\Delta t = \frac{L\Delta\theta}{\tau} = \frac{I\omega\Delta\theta}{Fr} = \frac{\frac{2}{5}mr^2\omega\Delta\theta}{Fr} = \frac{2mr\omega\Delta\theta}{5F}$$

$$= \frac{2\left(2.25\times10^{10}\,\text{kg}\right)\left(123\,\text{m}\right)\left[\left(\frac{4\,\text{rev}}{1\,\text{day}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)\right]\left[10.0^\circ\left(\frac{2\pi\,\text{rad}}{360^\circ}\right)\right]}{5\left(265\,\text{N}\right)}$$

$$= \left(2.12\times10^{5}\,\text{s}\right)\frac{1\,\text{hr}}{3600\,\text{s}} = \boxed{58.9\,\text{hr}}$$

Note that, in the diagram in the book, the original angular momentum is "up" and the torque is into the page. Thus the planet's axis would actually tilt backwards into the plane of the paper, not rotate clockwise as shown in the figure above.

69. The velocity is the derivative of the position.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[R\cos(\omega t)\,\hat{\mathbf{i}} + R\sin(\omega t)\,\hat{\mathbf{j}}\right] = -\omega R\sin(\omega t)\,\hat{\mathbf{i}} + \omega R\cos(\omega t)\,\hat{\mathbf{j}}$$

$$= \boxed{v\left[-\sin(\omega t)\,\hat{\mathbf{i}} + \cos(\omega t)\,\hat{\mathbf{j}}\right]}$$

From the right hand rule, a counterclockwise rotation in the x - y plane produces an angular velocity in the $+\hat{\mathbf{k}}$ -direction. Thus $\boxed{\vec{\boldsymbol{\omega}} = \left(\frac{v}{R}\right)\hat{\mathbf{k}}}$. Now take the cross product $\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}$.

$$\vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}} = \left[\frac{v}{R}\hat{\mathbf{k}}\right]\times\left[R\cos(\omega t)\,\hat{\mathbf{i}} + R\sin(\omega t)\,\hat{\mathbf{j}}\right] = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 0 & \dfrac{v}{R} \\ R\cos(\omega t) & R\sin(\omega t) & 0 \end{vmatrix}$$

$$= -v\sin(\omega t)\,\hat{\mathbf{i}} + v\cos(\omega t)\,\hat{\mathbf{j}} = \vec{\mathbf{v}}$$

Thus we see that $\boxed{\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}}\times\vec{\mathbf{r}}}$.

70. Note that $z = v_z t$, and so $\frac{dz}{dt} = v_z$. To find the angular momentum, use Eq. 11-6, $\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}}$.

$$\vec{\mathbf{r}} = R\cos\left(\frac{2\pi z}{d}\right)\hat{\mathbf{i}} + R\sin\left(\frac{2\pi z}{d}\right)\hat{\mathbf{j}} + z\hat{\mathbf{k}} = R\cos\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{i}} + R\sin\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{j}} + v_z t\hat{\mathbf{k}}$$

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -R\frac{2\pi v_z}{d}\sin\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{i}} + R\frac{2\pi v_z}{d}\cos\left(\frac{2\pi v_z t}{d}\right)\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

To simplify the notation, let $\alpha \equiv \frac{2\pi v_z}{d}$. Then the kinematical expressions are as follows.

$$\vec{\mathbf{r}} = R\cos(\alpha t)\,\hat{\mathbf{i}} + R\sin(\alpha t)\,\hat{\mathbf{j}} + v_z t\hat{\mathbf{k}}\ ;\ \vec{\mathbf{v}} = -\alpha R\sin(\alpha t)\,\hat{\mathbf{i}} + \alpha R\cos(\alpha)\,\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = m\vec{\mathbf{r}}\times\vec{\mathbf{v}} = m\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ R\cos(\alpha t) & R\sin(\alpha t) & v_z t \\ -R\alpha\sin(\omega t) & R\alpha\cos(\alpha t) & v_z \end{vmatrix}$$

$$= m\left[Rv_z\sin(\alpha t) - R\alpha v_z t\cos(\alpha t)\right]\hat{\mathbf{i}} + m\left[-R\alpha v_z t\sin(\alpha t) - Rv_z\cos(\alpha t)\right]\hat{\mathbf{j}}$$

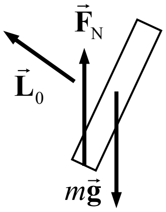
$$+ m\left[R^2\alpha\cos^2(\alpha t) + R^2\alpha\sin^2(\alpha t)\right]\hat{\mathbf{k}}$$

$$= mRv_z\left[\sin(\alpha t) - \alpha t\cos(\alpha t)\right]\hat{\mathbf{i}} + mRv_z\left[-\alpha t\sin(\alpha t) - \cos(\alpha t)\right]\hat{\mathbf{j}} + mR^2\alpha\hat{\mathbf{k}}$$

$$= mRv_z\left\{\left[\sin(\alpha t) - \alpha t\cos(\alpha t)\right]\hat{\mathbf{i}} + \left[-\alpha t\sin(\alpha t) - \cos(\alpha t)\right]\hat{\mathbf{j}} + \frac{R\alpha}{v_z}\hat{\mathbf{k}}\right\}$$

$$= \boxed{mRv_z\left\{\left[\sin\left(\frac{2\pi z}{d}\right) - \frac{2\pi z}{d}\cos\left(\frac{2\pi z}{d}\right)\right]\hat{\mathbf{i}} + \left[-\frac{2\pi z}{d}\sin\left(\frac{2\pi z}{d}\right) - \cos\left(\frac{2\pi z}{d}\right)\right]\hat{\mathbf{j}} + \frac{2\pi R}{d}\mathbf{k}\right\}}$$

71. (*a*) From the free-body diagram, we see that the normal force will produce a torque about the center of mass. That torque, $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}_{\mathrm{N}}$, is clockwise in the diagram and so points into the paper, and will cause a change $\Delta\vec{\mathbf{L}} = \vec{\boldsymbol{\tau}}\Delta t$ in the tire's original angular momentum. $\Delta\vec{\mathbf{L}}$ also points into the page, and so the angular momentum will change to have a component into the page. That means that the tire will $\boxed{\text{turn to the right}}$ in the diagram.

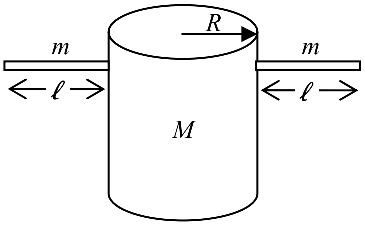


(*b*) The original momentum is the moment of inertia times the angular velocity. We assume the wheel is rolling without slipping.

$$\Delta L = \vec{\boldsymbol{\tau}}\Delta t = \left(rF_{\mathrm{N}}\sin\theta\right)\Delta t = rmg\sin\theta\,\Delta t \;\; ; \;\; L_0 = I\omega = I\,v/r$$

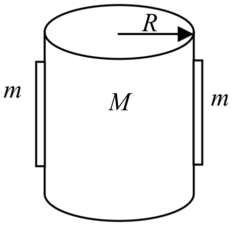
$$\frac{\Delta L}{L_0} = \frac{r^2 mg\sin\theta\,\Delta t}{Iv} = \frac{(0.32\,\mathrm{m})^2(8.0\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)\sin 12^\circ(0.20\,\mathrm{s})}{\left(0.83\,\mathrm{kg{\bullet}m^2}\right)(2.1\,\mathrm{m/s})} = \boxed{0.19}$$

72. (*a*) See the diagram. The parallel axis theorem is used to find the moment of inertia of the arms.



$$I_a = I_{\text{body}} + I_{\text{arms}}$$
$$= \tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2\left[\tfrac{1}{12}M_{\text{arm}}\ell_{\text{arm}}^2 + M_{\text{arm}}\left(R_{\text{body}} + \tfrac{1}{2}\ell_{\text{arm}}\right)^2\right]$$
$$= \tfrac{1}{2}(60\,\mathrm{kg})(0.12\,\mathrm{m})^2$$
$$+ 2\left[\tfrac{1}{12}(5.0\,\mathrm{kg})(0.60\,\mathrm{m})^2 + (5.0\,\mathrm{kg})(0.42\,\mathrm{m})^2\right] = 2.496\,\mathrm{kg{\bullet}m^2} \approx \boxed{2.5\,\mathrm{kg{\bullet}m^2}}$$

(*b*) Now the arms can be treated like particles, since all of the mass of the arms is the same distance from the axis of rotation.



$$I_b = I_{\text{body}} + I_{\text{arms}} = \tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2M_{\text{arm}}R_{\text{body}}^2$$
$$= \tfrac{1}{2}(60\,\mathrm{kg})(0.12\,\mathrm{m})^2 + 2(5.0\,\mathrm{kg})(0.12\,\mathrm{m})^2 = 0.576\,\mathrm{kg{\bullet}m^2}$$
$$\approx \boxed{0.58\,\mathrm{kg{\bullet}m^2}}$$

(*c*) Angular momentum is conserved through the change in posture.

$$L_{\text{initial}} = L_{\text{final}} \;\rightarrow\; I_a\omega_a = I_b\omega_b \;\rightarrow\; I_a\frac{2\pi}{T_a} = I_b\frac{2\pi}{T_b} \;\rightarrow$$

$$T_b = \frac{I_b}{I_a}T_a = \frac{0.576\,\mathrm{kg{\bullet}m^2}}{2.496\,\mathrm{kg{\bullet}m^2}}(1.5\,\mathrm{s}) = 0.3462\,\mathrm{s} \approx \boxed{0.35\,\mathrm{s}}$$

(*d*) The change in kinetic energy is the final kinetic energy (arms horizontal) minus the initial kinetic energy (arms at sides).

$$\Delta K = K_a - K_b = \tfrac{1}{2}I_a\omega_a^2 - \tfrac{1}{2}I_b\omega_b^2 = \tfrac{1}{2}\left(2.496\,\text{kg}\bullet\text{m}^2\right)\left(\frac{2\pi}{1.5\,\text{s}}\right)^2 - \tfrac{1}{2}\left(0.576\,\text{kg}\bullet\text{m}^2\right)\left(\frac{2\pi}{0.3462\,\text{s}}\right)^2$$

$$= \boxed{-73\,\text{J}}$$

(*e*) Because of the decrease in kinetic energy, it is easier to lift the arms when rotating. There is no corresponding change in kinetic energy if the person is at rest. In the rotating system, the arms tend to move away from the center of rotation. Another way to express this is that it takes work to bring the arms into the sides when rotating.

$\boxed{73.}$ (*a*) The angular momentum delivered to the waterwheel is that lost by the water.

$$\Delta L_{\text{wheel}} = -\Delta L_{\text{water}} = L_{\substack{\text{initial}\\\text{water}}} - L_{\substack{\text{final}\\\text{water}}} = mv_1R - mv_2R \quad\rightarrow$$

$$\frac{\Delta L_{\text{wheel}}}{\Delta t} = \frac{mv_1R - mv_2R}{\Delta t} = \frac{mR}{\Delta t}\left(v_1 - v_2\right) = \left(85\,\text{kg/s}\right)\left(3.0\,\text{m}\right)\left(3.2\,\text{m/s}\right) = 816\,\text{kg}\bullet\text{m}^2/\text{s}^2$$

$$\approx \boxed{820\,\text{kg}\bullet\text{m}^2/\text{s}^2}$$

(*b*) The torque is the rate of change of angular momentum, from Eq. 11-9.

$$\tau_{\substack{\text{on}\\\text{wheel}}} = \frac{\Delta L_{\text{wheel}}}{\Delta t} = 816\,\text{kg}\bullet\text{m}^2/\text{s}^2 = 816\,\text{m}\bullet\text{N} \approx \boxed{820\,\text{m}\bullet\text{N}}$$

(*c*) Power is given by Eq. 10-21, $P = \tau\omega$.

$$P = \tau\omega = \left(816\,\text{m}\bullet\text{N}\right)\left(\frac{2\pi\,\text{rev}}{5.5\,\text{s}}\right) = \boxed{930\,\text{W}}$$

74. Due to the behavior of the Moon, the period for the Moon's rotation about its own axis is the same as the period for the Moon's rotation about the Earth. Thus the angular velocity is the same in both cases.

$$\frac{L_{\text{spin}}}{L_{\text{orbit}}} = \frac{I_{\text{spin}}\omega}{I_{\text{orbit}}\omega} = \frac{I_{\text{spin}}}{I_{\text{orbit}}} = \left(\frac{\tfrac{2}{5}MR_{\text{Moon}}^2}{MR_{\text{orbit}}^2}\right) = \frac{2R_{\text{Moon}}^2}{5R_{\text{orbit}}^2} = \frac{2\left(1.74\times10^6\,\text{m}\right)^2}{5\left(384\times10^6\,\text{m}\right)^2} = \boxed{8.21\times10^{-6}}$$

75. From problem 25, we have that $\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}$. For this object, rotating counterclockwise and gaining angular speed, the angular acceleration is $\vec{\boldsymbol{\alpha}} = \alpha\hat{\mathbf{k}}$.

$$\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 0 & \alpha \\ R\cos\theta & R\sin\theta & 0 \end{vmatrix} = \boxed{-\alpha R\sin\theta\hat{\mathbf{i}} + \alpha R\cos\theta\hat{\mathbf{j}}}$$

(*a*) We need the acceleration in order to calculate $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}$. The force consists of two components, a radial (centripetal) component and a tangential component. There is no torque associated with the radial component since the angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}_{\text{centrip}}$ is 180°. Thus $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}_{\tan}$ $= \vec{\mathbf{r}}\times m\vec{\mathbf{a}}_{\tan} = m\vec{\mathbf{r}}\times\vec{\mathbf{a}}_{\tan}$.

$$\vec{\boldsymbol{\tau}} = m\vec{\mathbf{r}}\times\vec{\mathbf{a}}_{\text{tan}} = m\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ R\cos\theta & R\sin\theta & 0 \\ -\alpha R\sin\theta & \alpha R\cos\theta & 0 \end{vmatrix} = m\left(R^2\alpha\cos^2\theta + R^2\alpha\sin^2\theta\right)\hat{\mathbf{k}} = \boxed{mR^2\alpha\hat{\mathbf{k}}}$$

(*b*) The moment of inertia of the particle is $I = mR^2$.

$$\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \boxed{mR^2\alpha\hat{\mathbf{k}}}$$

76. (*a*) The acceleration is needed since $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

$$\vec{\mathbf{r}} = \left(v_{x0}t\right)\hat{\mathbf{i}} + \left(v_{y0}t - \tfrac{1}{2}gt^2\right)\hat{\mathbf{j}} \ ; \ \vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = v_{x0}\hat{\mathbf{i}} + \left(v_{y0} - gt\right)\hat{\mathbf{j}} \ ; \ \vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = -g\hat{\mathbf{j}} \text{ (as expected)}$$

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}} = \vec{\mathbf{r}}\times m\vec{\mathbf{a}} = m\vec{\mathbf{r}}\times\vec{\mathbf{a}} = m\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_{x0}t & v_{y0}t - \tfrac{1}{2}gt^2 & 0 \\ 0 & -g & 0 \end{vmatrix} = \boxed{-gv_{x0}t\hat{\mathbf{k}}}$$

(*b*) Find the angular momentum from $\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = m\left(\vec{\mathbf{r}}\times\vec{\mathbf{v}}\right)$, and then differentiate with respect to time.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = m\left(\vec{\mathbf{r}}\times\vec{\mathbf{v}}\right) = m\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_{x0}t & v_{y0}t - \tfrac{1}{2}gt^2 & 0 \\ v_{x0} & v_{y0} - gt & 0 \end{vmatrix} = \left[v_{x0}t\left(v_{y0} - gt\right) - v_{x0}\left(v_{y0}t - \tfrac{1}{2}gt^2\right)\right]\hat{\mathbf{k}}$$

$$= -\tfrac{1}{2}v_{x0}gt^2\hat{\mathbf{k}}$$

$$\frac{d\vec{\mathbf{L}}}{dt} = \frac{d}{dt}\left(-\tfrac{1}{2}v_{x0}gt^2\hat{\mathbf{k}}\right) = \boxed{-v_{x0}gt\hat{\mathbf{k}}}$$

77. We calculate spin angular momentum for the Sun, and orbital angular momentum for the planets, treating them as particles relative to the size of their orbits. The angular velocities are calculated by $\omega = \dfrac{2\pi}{T}$.

$$L_{\text{Sun}} = I_{\text{Sun}}\omega_{\text{Sun}} = \tfrac{2}{5}M_{\text{Sun}}R_{\text{Sun}}^2\frac{2\pi}{T_{\text{Sun}}} = \tfrac{2}{5}\left(1.99\times10^{30}\,\text{kg}\right)\left(6.96\times10^{8}\,\text{m}\right)^2\frac{2\pi}{\left(25\text{ days}\right)}\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)$$

$$= 1.1217\times10^{42}\ \text{kg}\bullet\text{m/s}$$

$$L_{\text{Jupiter}} = M_{\text{Jupiter}}R_{\substack{\text{Jupiter}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Jupiter}}} = \left(190\times10^{25}\text{kg}\right)\left(778\times10^{9}\,\text{m}\right)^2\frac{2\pi}{11.9\,\text{y}}\left(\frac{1\text{ y}}{3.156\times10^{7}\,\text{s}}\right)$$

$$= 1.9240\times10^{43}\ \text{kg}\bullet\text{m/s}$$

In a similar fashion, we calculate the other planetary orbital angular momenta.

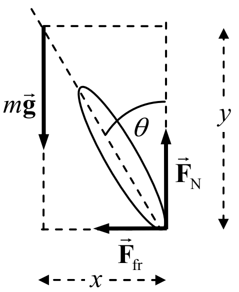
$$L_{\text{Saturn}} = M_{\text{Saturn}}R_{\substack{\text{Saturn}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Saturn}}} = 7.806\times10^{42}\ \text{kg}\bullet\text{m/s}$$

$$L_{\text{Uranus}} = M_{\text{Uranus}}R_{\substack{\text{Uranus}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Uranus}}} = 1.695\times10^{42}\ \text{kg}\bullet\text{m/s}$$

$$L_{\text{Neptune}} = M_{\text{Neptune}}R_{\substack{\text{Neptune}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Neptune}}} = 2.492\times10^{42}\ \text{kg}\bullet\text{m/s}$$

$$f=\frac{L_{\text{planets}}}{L_{\text{planets}}+L_{\text{Sun}}}=\frac{(19.240+7.806+1.695+2.492)\times10^{42}\ \text{kg}\cdot\text{m/s}}{(19.240+7.806+1.695+2.492+1.122)\times10^{42}\ \text{kg}\cdot\text{m/s}}=\boxed{0.965}$$

78. (*a*) In order not to fall over, the net torque on the cyclist about an axis through the CM and parallel to the ground must be zero. Consider the free-body diagram shown. Sum torques about the CM, with counterclockwise as positive, and set the sum equal to zero.



$$\sum\tau=F_{\text{N}}x-F_{\text{fr}}y=0\quad\rightarrow\quad\frac{F_{\text{fr}}}{F_{\text{N}}}=\frac{x}{y}=\tan\theta$$

(*b*) The cyclist is not accelerating vertically, so $F_{\text{N}}=mg$. The cyclist is accelerating horizontally, because he is traveling in a circle. Thus the frictional force must be supplying the centripetal force, so $F_{\text{fr}}=mv^2/r$.

$$\tan\theta=\frac{F_{\text{fr}}}{F_{\text{N}}}=\frac{mv^2/r}{mg}=\frac{v^2}{rg}\quad\rightarrow\quad\theta=\tan^{-1}\frac{v^2}{rg}=\tan^{-1}\frac{(9.2\,\text{m/s})^2}{(12\ \text{m})(9.80\,\text{m/s}^2)}=35.74^\circ\approx\boxed{36^\circ}$$

(*c*) From $F_{\text{fr}}=mv^2/r$, the smallest turning radius results in the maximum force. The maximum static frictional force is $F_{\text{fr}}=\mu F_{\text{N}}$. Use this to calculate the radius.

$$mv^2/r_{\text{min}}=\mu_{\text{s}}F_{\text{N}}=\mu_{\text{s}}mg\quad\rightarrow\quad r_{\text{min}}=\frac{v^2}{\mu_{\text{s}}g}=\frac{(9.2\,\text{m/s})^2}{(0.65)(9.80\,\text{m/s}^2)}=\boxed{13\,\text{m}}$$

$\boxed{79.}$ (*a*) During the jump (while airborne), the only force on the skater is gravity, which acts through the skater's center of mass. Accordingly, there is $\boxed{\text{no torque about the center of mass}}$, and so angular momentum is conserved during the jump.

(*b*) For a single axel, the skater must have 1.5 total revolutions. The number of revolutions during each phase of the motion is the rotational frequency times the elapsed time. Note that the rate of rotation is the same for both occurrences of the "open" position.

$$(1.2\,\text{rev/s})(0.10\,\text{s})+f_{\text{single}}(0.50\,\text{s})+(1.2\,\text{rev/s})(0.10\,\text{s})=1.5\,\text{rev}\quad\rightarrow$$

$$f_{\text{single}}=\frac{1.5\,\text{rev}-2(1.2\,\text{rev/s})(0.10\,\text{s})}{(0.50\,\text{s})}=2.52\,\text{rev/s}\approx\boxed{2.5\,\text{rev/s}}$$

The calculation is similar for the triple axel.

$$(1.2\,\text{rev/s})(0.10\,\text{s})+f_{\text{triple}}(0.50\,\text{s})+(1.2\,\text{rev/s})(0.10\,\text{s})=3.5\,\text{rev}\quad\rightarrow$$

$$f_{\text{triple}}=\frac{3.5\,\text{rev}-2(1.2\,\text{rev/s})(0.10\,\text{s})}{(0.50\,\text{s})}=6.52\,\text{rev/s}\approx\boxed{6.5\,\text{rev/s}}$$

(*c*) Apply angular momentum conservation to relate the moments of inertia.

$$L_{\substack{\text{single}\\\text{open}}}=L_{\substack{\text{single}\\\text{closed}}}\quad\rightarrow\quad I_{\substack{\text{single}\\\text{open}}}\omega_{\substack{\text{single}\\\text{open}}}=I_{\substack{\text{single}\\\text{closed}}}\omega_{\substack{\text{single}\\\text{closed}}}\quad\rightarrow$$

$$\frac{I_{\substack{\text{single}\\\text{closed}}}}{I_{\substack{\text{single}\\\text{open}}}}=\frac{\omega_{\substack{\text{single}\\\text{open}}}}{\omega_{\substack{\text{single}\\\text{closed}}}}=\frac{f_{\substack{\text{single}\\\text{open}}}}{f_{\substack{\text{single}\\\text{closed}}}}=\frac{1.2\,\text{rev/s}}{2.52\,\text{rev/s}}=0.476\approx\boxed{\tfrac{1}{2}}$$

Thus the single axel moment of inertia must be reduced by a factor of about 2. For the triple axel, the calculation is similar.

$$\frac{I_{\substack{\text{triple}\\\text{closed}}}}{I_{\substack{\text{triple}\\\text{open}}}} = \frac{f_{\substack{\text{single}\\\text{open}}}}{f_{\substack{\text{single}\\\text{closed}}}} = \frac{1.2\,\text{rev/s}}{6.52\,\text{rev/s}} = 0.184 \approx \boxed{\tfrac{1}{5}}$$

Thus the triple axel moment of inertia must be reduced by a factor of about 5.

80. We assume that the tensions in the two unbroken cables immediately become zero, and so they have no effect on the motion. The forces on the tower are the forces at the base joint, and the weight. The axis of rotation is through the point of attachment to the ground. Since that axis is fixed in an inertial system, we may use Eq. 11-9 in one dimension, $\sum \tau = \frac{dL}{dt}$. See the free-body diagram in the text to express the torque.

$$\sum \tau = \frac{dL}{dt} \rightarrow mg\left(\tfrac{1}{2}\ell\right)\sin\theta = \frac{d(I\omega)}{dt} = \tfrac{1}{3}m\ell^2\frac{d\omega}{dt} \rightarrow \tfrac{1}{2}g\sin\theta = \tfrac{1}{3}\ell\frac{d\omega}{dt} = \tfrac{1}{3}\ell\frac{d^2\theta}{dt^2}$$

This equation could be considered, but it would yield θ as a function of time. Use the chain rule to eliminate the dependence on time.

$$\tfrac{1}{2}g\sin\theta = \tfrac{1}{3}\ell\frac{d\omega}{dt} = \tfrac{1}{3}\ell\frac{d\omega}{d\theta}\frac{d\theta}{dt} = \tfrac{1}{3}\ell\omega\frac{d\omega}{d\theta} \rightarrow \tfrac{3}{2}\frac{g}{\ell}\sin\theta\,d\theta = \omega\,d\omega \rightarrow$$

$$\tfrac{3}{2}\frac{g}{\ell}\int_0^\theta \sin\theta\,d\theta = \int_0^\omega \omega\,d\omega \rightarrow \tfrac{3}{2}\frac{g}{\ell}(1-\cos\theta) = \tfrac{1}{2}\omega^2 \rightarrow \omega = \sqrt{3\frac{g}{\ell}(1-\cos\theta)} = \frac{v}{\ell} \rightarrow$$

$$v = \sqrt{3g\ell(1-\cos\theta)} = \sqrt{3\left(9.80\,\text{m/s}^2\right)(12\,\text{m})(1-\cos\theta)} = 19\sqrt{1-\cos\theta}$$

Note that the same result can be obtained from conservation of energy, since the forces at the ground do no work.

81. (*a*) We assume that no angular momentum is in the thrown-off mass, so the final angular momentum of the neutron star is equal to the angular momentum before collapse.

$$L_0 = L_{\text{f}} \rightarrow I_0\omega_0 = I_{\text{f}}\omega_{\text{f}} \rightarrow \left[\tfrac{2}{5}\left(8.0M_{\text{Sun}}\right)R_{\text{Sun}}^2\right]\omega_0 = \left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_{\text{f}}^2\right]\omega_f \rightarrow$$

$$\omega_f = \frac{\left[\tfrac{2}{5}\left(8.0M_{\text{Sun}}\right)R_{\text{Sun}}^2\right]}{\left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_{\text{f}}^2\right]}\omega_0 = \frac{4R_{\text{Sun}}^2}{R_{\text{f}}^2}\omega_0 = \frac{4\left(6.96\times10^8\,\text{m}\right)^2}{\left(12\times10^3\,\text{m}\right)^2}\left(\frac{1.0\,\text{rev}}{9.0\,\text{days}}\right)$$

$$= \left(1.495\times10^9\,\text{rev/day}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right) = 1.730\times10^4\,\text{rev/s} \approx \boxed{17{,}000\,\text{rev/s}}$$

(*b*) Now we assume that the final angular momentum of the neutron star is only ¼ of the angular momentum before collapse. Since the rotation speed is directly proportional to angular momentum, the final rotation speed will be ¼ of that found in part (*a*).

$$\omega_f = \tfrac{1}{4}\left(1.730\times10^4\,\text{rev/s}\right) = \boxed{4300\,\text{rev/s}}$$

82. The desired motion is pure rotation about the handle grip. Since the grip is not to have any linear motion, an axis through the grip qualifies as an axis fixed in an inertial reference frame. The pure rotation condition is expressed by $a_{\text{CM}} = \alpha_{\text{bat}}\left(d_{\text{CM}} - d_{\text{grip}}\right)$, where d_{grip} is the 0.050 m distance from the end of the bat to the grip. Apply Newton's second law for both the translational motion of the center of mass, and rotational motion about the handle grip.

$$\sum F = F = ma_{\text{CM}}\ ;\ \sum \tau = Fd = I_{\text{grip}}\alpha \rightarrow ma_{\text{CM}}d = I_{\text{grip}}\alpha \rightarrow$$

$$m\alpha\left(d_{\text{CM}}-d_{\text{grip}}\right)d = I_{\text{grip}}\alpha \quad\rightarrow\quad d=\frac{I_{\text{grip}}}{m\left(d_{\text{CM}}-d_{\text{grip}}\right)}$$

So we must calculate the moment of inertia of the bat about an axis through the grip, the mass of the bat, and the location of the center of mass. An infinitesimal element of mass is given by $dm=\lambda dx$, where λ is the linear mass density.

$$I_{\text{grip}}=\int r^2dm=\int_0^{0.84\,\text{m}}\left(x-d_{\text{grip}}\right)^2\lambda dx=\int_0^{0.84\,\text{m}}\left(x-0.050\right)^2\left(0.61+3.3x^2\right)dx$$

$$=\int_0^{0.84\,\text{m}}\left(3.3x^4-0.33x^3+0.61825x^2-0.061x+0.001525\right)dx$$

$$=\left(\tfrac{1}{5}3.3x^5-\tfrac{1}{4}0.33x^4+\tfrac{1}{3}0.61825x^3-\tfrac{1}{2}0.061x^2+0.001525x\right)_0^{0.84}=0.33685\,\text{kg}\bullet\text{m}^2$$

$$m=\int dm=\int\lambda dx=\int_0^{0.84\,\text{m}}\left[\left(0.61+3.3x^2\right)\text{kg/m}\right]dx=\left(0.61x+1.1x^3\right)_0^{0.84}=1.1644\,\text{kg}$$

$$x_{\text{CM}}=\frac{1}{m}\int xdm=\frac{1}{m}\int x\lambda dx=\frac{1}{m}\int_0^{0.84\,\text{m}}\left[\left(0.61x+3.3x^3\right)\text{kg/m}\right]dx=\frac{\left(\tfrac{1}{2}0.61x^2+\tfrac{1}{4}3.3x^4\right)_0^{0.84}}{1.1644\,\text{kg}}$$

$$=0.53757\,\text{m}$$

$$d=\frac{0.33685\,\text{kg}\bullet\text{m}^2}{\left(1.1644\,\text{kg}\right)\left(0.53757\,\text{m}-0.050\,\text{m}\right)}=0.59333\,\text{m}\approx 0.593\,\text{m}$$

So the distance from the end of the bat to the "sweet spot" is $d+0.050\,\text{m}=0.643\,\text{m}\approx\boxed{0.64\,\text{m}}$.

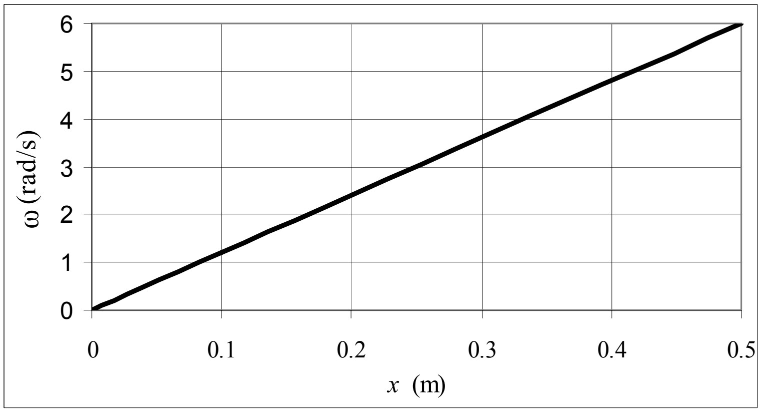
83. (*a*) Angular momentum about the pivot is conserved during this collision. Note that both objects have angular momentum after the collision.

$$L_{\underset{\text{collision}}{\text{before}}}=L_{\underset{\text{collision}}{\text{after}}}\quad\rightarrow\quad L_{\underset{\text{initial}}{\text{bullet}}}=L_{\underset{\text{final}}{\text{stick}}}+L_{\underset{\text{final}}{\text{bullet}}}\quad\rightarrow\quad m_{\text{bullet}}v_0x=I_{\text{stick}}\omega+m_{\text{bullet}}v_{\text{f}}x\quad\rightarrow$$

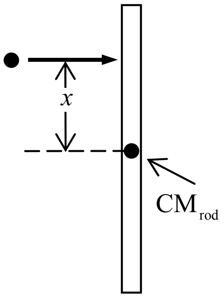
$$\omega=\frac{m_{\text{bullet}}\left(v_0-v_{\text{f}}\right)x}{I_{\text{stick}}}=\frac{m_{\text{bullet}}\left(v_0-v_{\text{f}}\right)x}{\frac{1}{12}M_{\text{stick}}\ell_{\text{stick}}^2}=\frac{12m_{\text{bullet}}\left(v_0-v_{\text{f}}\right)x}{M_{\text{stick}}\ell_{\text{stick}}^2}=\frac{12\left(0.0030\,\text{kg}\right)\left(110\,\text{m/s}\right)}{\left(0.33\,\text{kg}\right)\left(1.00\,\text{m}\right)^2}x$$

$$=\boxed{\left(12\,\frac{\text{rad/s}}{\text{m}}\right)x}$$

(*b*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH11.XLS," on tab "Problem 11.83b."



84. (*a*) Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking up as the positive direction. See the diagram.



$$x_{\text{CM}} = \frac{mx + M(0)}{(m+M)} = \frac{mx}{m+M}$$

The distance of the stuck clay ball from the system's center of mass is found.

$$x_{\substack{\text{clay}\\ \text{from CM}}} = x - x_{\text{CM}} = x - \frac{mx}{m+M} = \frac{Mx}{m+M}$$

Calculate the moment of inertia of the rod about the center of mass of the entire system. Use the parallel axis theorem. Treat the clay as a point mass.

$$I_{\text{rod}} = \tfrac{1}{12}M\ell^2 + M\left(\frac{mx}{m+M}\right)^2$$

Now express the conservation of angular momentum about the system's center of mass.

$$L_{\text{initial}} = L_{\text{final}} \;\rightarrow\; mvx_{\text{clay}} = \left(I_{\text{rod}} + I_{\text{clay}}\right)\omega_{\text{final}} \;\rightarrow$$

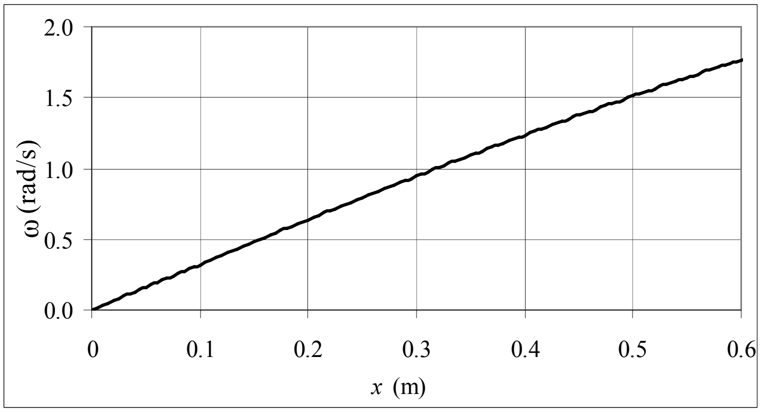
$$\omega_{\text{final}} = \frac{mvx_{\text{clay}}}{\left(I_{\text{rod}} + I_{\text{clay}}\right)} = \frac{mvx_{\text{clay}}}{\left(\tfrac{1}{12}M\ell^2 + M\left(\frac{mx}{m+M}\right)^2 + mx_{\text{clay}}^2\right)}$$

$$= \frac{mv\dfrac{Mx}{m+M}}{\left(\tfrac{1}{12}M\ell^2 + M\left(\dfrac{mx}{m+M}\right)^2 + m\left(\dfrac{Mx}{m+M}\right)^2\right)} = \boxed{\frac{vx}{\tfrac{1}{12}\left(1+\dfrac{M}{m}\right)\ell^2 + x^2}}$$

(*b*) Graph this function with the given values, from $x =$ 9 to $x = 0.60$ m.

$$\omega_{\text{final}} = \frac{vx}{\tfrac{1}{12}\left(1+\dfrac{M}{m}\right)\ell^2 + x^2}$$

$$= \frac{12x}{3.72 + x^2}\,\text{rad/s}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH11.XLS," on tab "Problem 11.84b."

(*c*) Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; mv = (m+M)v_{\substack{\text{CM}\\ \text{final}}} \;\rightarrow\; \boxed{v_{\substack{\text{CM}\\ \text{final}}} = \frac{mv}{m+M}}$$

We see that the translational motion (the velocity of the center of mass) is $\boxed{\text{NOT dependent on } x}$.

CHAPTER 12: Static Equilibrium; Elasticity and Fracture

Responses to Questions

1. Equilibrium requires both the net force and net torque on an object to be zero. One example is a meter stick with equal and opposite forces acting at opposite ends. The net force is zero but the net torque is not zero, because the forces are not co-linear. The meter stick will rotate about its center.

2. No. An object in equilibrium has zero *acceleration*. At the bottom of the dive, the bungee jumper momentarily has zero velocity, but not zero acceleration. There is a net upward force on the bungee jumper so he is not in equilibrium.

3. The meter stick is originally supported by both fingers. As you start to slide your fingers together, more of the weight of the meter stick is supported by the finger that is closest to the center of gravity, so that the torques produced by the fingers are equal and the stick is in equilibrium. The other finger feels a smaller normal force, and therefore a smaller frictional force, and so slides more easily and moves closer to the center of gravity. The roles switch back and forth between the fingers as they alternately move closer to the center of gravity. The fingers will eventually meet at the center of gravity.

4. The sliding weights on the movable scale arm are positioned much farther from the pivot point than is the force exerted by your weight. In this way, they can create a torque to balance the torque caused by your weight, even though they weigh less. When the torques are equal in magnitude and opposite in direction, the arm will be in rotational equilibrium.

5. (*a*) The wall remains upright if the counterclockwise and clockwise torques about the lower left corner of the wall are equal. The counterclockwise torque is produced by $\vec{\mathbf{F}}$. The clockwise torque is the sum of the torques produced by the normal force from the ground on the left side of the wall and the weight of the wall. $\vec{\mathbf{F}}$ and its lever arm are larger than the force and lever arm for the torque from the ground on the left. The lever arm for the torque generated by the weight is small, so the torque will be small, even if the wall is very heavy. Case (*a*) is likely to be an unstable situation.
   (*b*) In this case, the clockwise torque produced by the weight of the ground above the horizontal section of the wall and clockwise torque produced by the larger weight of the wall and its lever arm balance the counterclockwise torque produced by $\vec{\mathbf{F}}$.

6. Yes. For example, consider a meter stick lying along the x-axis. If you exert equal forces downward (in the negative y-direction) on the two ends of the stick, the torques about the center of the stick will be equal and opposite, so the net torque will be zero. However, the net force will not be zero; it will be in the negative y-direction. Also, any force through the pivot point will supply zero torque.

7. The ladder is more likely to slip when a person stands near the top of the ladder. The torque produced by the weight of the person about the bottom of the ladder increases as the person climbs the ladder, because the lever arm increases.

8. The mass of the meter stick is equal to the mass of the rock. Since the meter stick is uniform, its center of mass is at the 50-cm mark, and in terms of rotational motion about a pivot at the 25-cm mark, it can be treated as though its entire mass is concentrated at the center of mass. The meter stick's mass at the 50-cm mark (25 cm from the pivot) balances the rock at the 0-cm mark (also 25 cm from the pivot) so the masses must be equal.

9. You lean backward in order to keep your center of mass over your feet. If, due to the heavy load, your center of mass is in front of your feet, you will fall forward.

10. (*a*) The cone will be in stable equilibrium if it is placed flat on its base. If it is tilted slightly from this position and then released, it will return to the original position. (*b*) The cone will be in unstable equilibrium if it is balanced on its tip. A slight displacement in this case will cause the cone to topple over. (*c*) If the cone is placed on its side (as shown in Figure 12-42) it will be in neutral equilibrium. If the cone is displaced slightly while on its side, it will remain in its new position.

11. When you stand next to a door in the position described, your center of mass is over your heels. If you try to stand on your toes, your center of mass will not be over your area of support, and you will fall over backward.

12. Once you leave the chair, you are supported only by your feet. In order to keep from falling backward, your center of mass must be over your area of support, so you must lean forward so that your center of mass is over your feet.

13. When you do a sit-up, you generate a torque with your abdominal muscles to rotate the upper part of your body off the floor while keeping the lower part of your body on the floor. The weight of your legs helps produce the torque about your hips. When your legs are stretched out, they have a longer lever arm, and so produce a larger torque, than when they are bent at the knee. When your knees are bent, your abdominal muscles must work harder to do the sit-up.

14. Configuration (*b*) is likely to be more stable. Because of the symmetry of the bricks, the center of mass of the entire system (the two bricks) is the midpoint between the individual centers of mass shown on the diagram. In figure (*a*), the center of mass of the entire system is not supported by the table.

15. A is a point of unstable equilibrium, B is a point of stable equilibrium, and C is a point of neutral equilibrium.

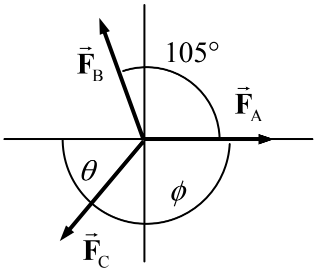
16. The Young’s modulus for the bungee cord will be smaller than that for an ordinary rope. The Young’s modulus for a material is the ratio of stress to strain. For a given stress (force per unit area), the bungee cord will have a greater strain (change in length divided by original length) than the rope, and therefore a smaller Young’s modulus.

17. An object under shear stress has equal and opposite forces applied across its opposite faces. This is exactly what happens with a pair of scissors. One blade of the scissors pushes down on the cardboard, while the other blade pushes up with an equal and opposite force, at a slight displacement. This produces a shear stress in the cardboard, which causes it to fail.

18. Concrete or stone should definitely *not* be used for the support on the left. The left-hand support pulls downward on the beam, so the beam must pull upward on the support. Therefore, the support will be under tension and should not be made of ordinary concrete or stone, since these materials are weak under tension. The right-hand support pushes up on the beam and so the beam pushes down on it; it will therefore be under a compression force. Making this support of concrete or stone would be acceptable.

Solutions to Problems

$\boxed{1.}$ If the tree is not accelerating, then the net force in all directions is 0.



$$\sum F_x = F_A + F_B\cos 105° + F_{Cx} = 0 \quad \rightarrow$$

$$F_{Cx} = -F_A - F_B\cos 105° = -385\text{ N} - (475\text{ N})\cos 105° = -262.1\text{ N}$$

$$\sum F_y = F_B\sin 105° + F_{Cy} = 0 \quad \rightarrow$$

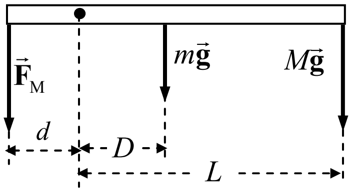
$$F_{Cy} = -F_B\sin 105° = -(475\text{ N})\sin 105° = -458.8\text{ N}$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(-262.1\text{ N})^2 + (-458.8\text{ N})^2} = 528.4\text{ N} \approx \boxed{528\text{ N}}$$

$$\theta = \tan^{-1}\frac{F_{Cy}}{F_{Cx}} = \tan^{-1}\frac{-458.8\text{ N}}{-262.1\text{ N}} = 60.3°\ ,\ \phi = 180° - 60.3° = \boxed{120°}$$

And so $\vec{\mathbf{F}}_C$ is 528 N, at an angle of 120° clockwise from $\vec{\mathbf{F}}_A$. The angle has 3 sig. fig.

2. Calculate the torques about the elbow joint (the dot in the free body diagram). The arm is in equilibrium. Counterclockwise torques are positive.

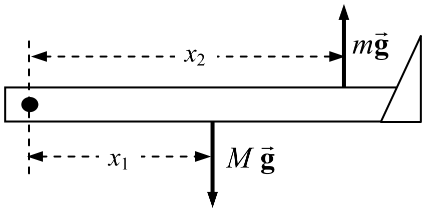


$$\sum \tau = F_M d - mgD - MgL = 0$$

$$F_M = \frac{mD + ML}{d}g$$

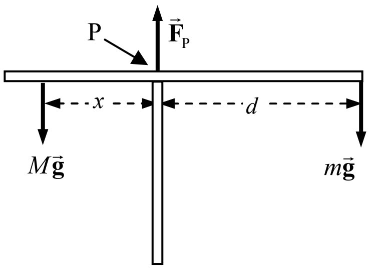
$$= \frac{(2.3\text{kg})(0.12\text{m}) + (7.3\text{kg})(0.300\text{m})}{0.025\text{m}}(9.80\text{m/s}^2) = \boxed{970\text{N}}$$

3. Because the mass m is stationary, the tension in the rope pulling up on the sling must be mg, and so the force of the sling on the leg must be mg, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.



$$\sum \tau = mgx_2 - Mgx_1 = 0 \quad \rightarrow \quad m = M\frac{x_1}{x_2} = (15.0\text{kg})\frac{(35.0\text{cm})}{(78.0\text{cm})} = \boxed{6.73\text{kg}}$$

4. (*a*) See the free-body diagram. Calculate torques about the pivot point P labeled in the diagram. The upward force at the pivot will not have any torque. The total torque is zero since the crane is in equilibrium.



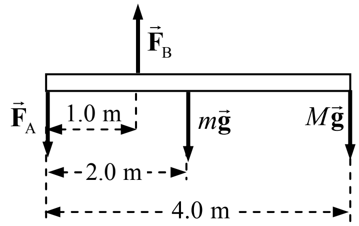
$$\sum \tau = Mgx - mgd = 0 \quad \rightarrow$$

$$x = \frac{md}{M} = \frac{(2800\text{kg})(7.7\text{m})}{(9500\text{kg})} = \boxed{2.3\text{m}}$$

(*b*) Again we sum torques about the pivot point. Mass *m* is the unknown in this case, and the counterweight is at its maximum distance from the pivot.

$$\sum\tau = Mgx_{\text{max}} - m_{\text{max}}gd = 0 \;\rightarrow\; m_{\text{max}} = \frac{Mx_{\text{max}}}{d} = \frac{(9500\,\text{kg})(3.4\,\text{m})}{(7.7\,\text{kg})} = \boxed{4200\,\text{kg}}$$

5. (*a*) Let $m = 0$. Calculate the net torque about the left end of the diving board, with counterclockwise torques positive. Since the board is in equilibrium, the net torque is zero.



$$\sum\tau = F_B(1.0\text{ m}) - Mg(4.0\text{ m}) = 0 \;\rightarrow$$

$$F_B = 4Mg = 4(52\text{ kg})\left(9.80\,\text{m/s}^2\right) = 2038\,\text{N} \approx \boxed{2.0\times10^3\,\text{N, up}}$$

Use Newton's second law in the vertical direction to find F_A.

$$\sum F_y = F_B - Mg - F_A = 0 \;\rightarrow$$

$$F_A = F_B - Mg = 4Mg - Mg = 3Mg = 3(52\,\text{kg})\left(9.80\,\text{m/s}^2\right) = 1529\,\text{N} \approx \boxed{1500\,\text{N, down}}$$

(*b*) Repeat the basic process, but with $m = 28$ kg. The weight of the board will add more clockwise torque.

$$\sum\tau = F_B(1.0\text{ m}) - mg(2.0\text{ m}) - Mg(4.0\text{ m}) = 0 \;\rightarrow$$

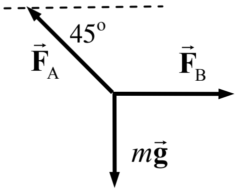
$$F_B = 4Mg + 2mg = \left[4(52\text{ kg}) + 2(28\text{ kg})\right]\left(9.80\,\text{m/s}^2\right) = 2587\,\text{N} \approx \boxed{2600\,\text{N, up}}$$

$$\sum F_y = F_B - Mg - mg - F_A \;\rightarrow$$

$$F_A = F_B - Mg - mg = 4Mg + 2mg - Mg - mg = 3Mg + mg$$

$$= \left[3(52\,\text{kg}) + 28\,\text{kg}\right]\left(9.80\,\text{m/s}^2\right) = 1803\,\text{N} \approx \boxed{1800\,\text{N, down}}$$

6. Write Newton's second law for the junction, in both the *x* and *y* directions.



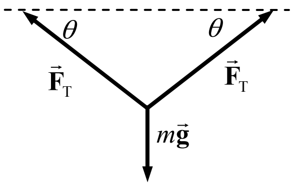
$$\sum F_x = F_B - F_A\cos45^\circ = 0$$

From this, we see that $F_A > F_B$. Thus set $F_A = 1660\text{ N}$.

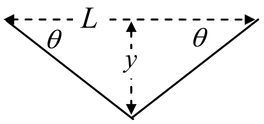
$$\sum F_y = F_A\sin45^\circ - mg = 0$$

$$mg = F_A\sin45^\circ = (1660\text{ N})\sin45^\circ = 1174\,\text{N} \approx \boxed{1200\,\text{N}}$$

$\boxed{7.}$ Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.



(*a*) $\theta = \tan^{-1}\dfrac{y}{L/2} = \tan^{-1}\dfrac{1.5\text{ m}}{3.3\text{ m}} = 24.4^\circ$



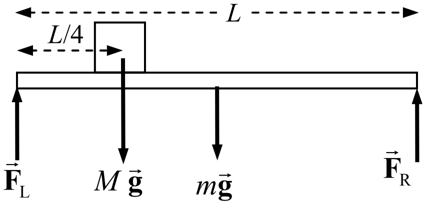
$$\sum F_y = 2F_T\sin\theta_1 - mg = 0 \;\rightarrow$$

$$F_T = \frac{mg}{2\sin\theta_1} = \frac{(19\text{ kg})\left(9.80\,\text{m/s}^2\right)}{2\sin24.4^\circ} = 225.4\,\text{N} \approx \boxed{230\,\text{N}}$$

(*b*) $\theta = \tan^{-1}\dfrac{y}{L/2} = \tan^{-1}\dfrac{0.15\text{ m}}{3.3\text{ m}} = 2.60°$

$$F_T = \frac{mg}{2\sin\theta_1} = \frac{(19\text{ kg})(9.80\text{ m/s}^2)}{2\sin 2.60°} = 2052\text{ N} \approx \boxed{2100\text{ N}}$$

8. Let *m* be the mass of the beam, and *M* be the mass of the piano. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

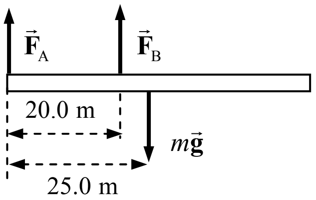


$$\sum\tau = F_R L - mg\left(\tfrac{1}{2}L\right) - Mg\left(\tfrac{1}{4}L\right) = 0$$
$$F_R = \left(\tfrac{1}{2}m + \tfrac{1}{4}M\right)g = \left[\tfrac{1}{2}(110\text{ kg}) + \tfrac{1}{4}(320\text{ kg})\right](9.80\text{ m/s}^2) = 1.32\times10^3\text{ N}$$
$$\sum F_y = F_L + F_R - mg - Mg = 0$$
$$F_L = (m+M)g - F_R = (430\text{ kg})(9.80\text{ m/s}^2) - 1.32\times10^3\text{ N} = 2.89\times10^3\text{ N}$$

The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$\boxed{F_R = 1300\text{ N down}}$ $\boxed{F_L = 2900\text{ N down}}$

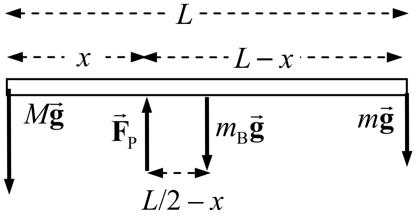
9. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum\tau = F_B(20.0\text{ m}) - mg(25.0\text{ m}) = 0 \rightarrow$$
$$F_B = \frac{25.0}{20.0}mg = (1.25)(1200\text{ kg})(9.80\text{ m/s}^2) = \boxed{1.5\times10^4\text{ N}}$$
$$\sum F_y = F_A + F_B - mg = 0$$
$$F_A = mg - F_B = mg - 1.25mg = -0.25mg = -(0.25)(1200\text{ kg})(9.80\text{ m/s}^2) = \boxed{-2900\text{ N}}$$

Notice that $\vec{\mathbf{F}}_A$ points down.

10. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques positive. The upward force $\vec{\mathbf{F}}_P$ at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the child is *m*, the mass of the adult is *M*, the mass of the board is m_B, and the center of gravity is at the middle of the board.



(*a*) Ignore the force $m_B g$.

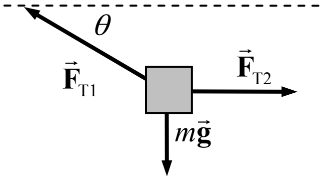
$$\sum\tau = Mgx - mg(L-x) = 0 \rightarrow$$
$$x = \frac{m}{m+M}L = \frac{(25\text{ kg})}{(25\text{ kg} + 75\text{ kg})}(9.0\text{ m}) = 2.25\text{ m} \approx \boxed{2.3\text{ m from adult}}$$

(*b*) Include the force $m_B g$.

$$\sum \tau = Mgx - mg\left(L-x\right) - m_{\mathrm{B}}g\left(L/2 - x\right) = 0$$

$$x = \frac{\left(m + m_{\mathrm{B}}/2\right)}{\left(M + m + m_{\mathrm{B}}\right)}L = \frac{\left(25\,\mathrm{kg} + 7.5\,\mathrm{kg}\right)}{\left(75\,\mathrm{kg} + 25\,\mathrm{kg} + 15\,\mathrm{kg}\right)}\left(9.0\text{ m}\right) = 2.54\,\mathrm{m} \approx \boxed{2.5\text{ m from adult}}$$

11. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.



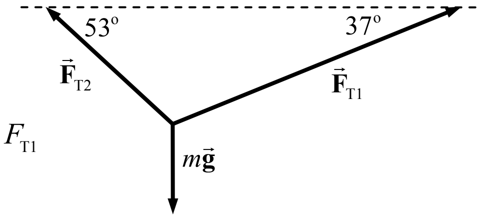
$$\sum F_x = F_{\mathrm{T2}} - F_{\mathrm{T1}}\cos\theta = 0 \;\rightarrow\; F_{\mathrm{T2}} = F_{\mathrm{T1}}\cos\theta$$

$$\sum F_y = F_{\mathrm{T1}}\sin\theta - mg = 0 \;\rightarrow\; F_{\mathrm{T1}} = \frac{mg}{\sin\theta}$$

$$F_{\mathrm{T2}} = F_{\mathrm{T1}}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \frac{\left(190\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\tan 33^\circ} = 2867\,\mathrm{N} \approx \boxed{2900\,\mathrm{N}}$$

$$F_{\mathrm{T1}} = \frac{mg}{\sin\theta} = \frac{\left(190\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\sin 33^\circ} = 3418\,\mathrm{N} \approx \boxed{3400\,\mathrm{N}}$$

12. Draw a free-body diagram of the junction of the three wires. The tensions can be found from the conditions for force equilibrium.



$$\sum F_x = F_{\mathrm{T1}}\cos 37^\circ - F_{\mathrm{T2}}\cos 53^\circ = 0 \;\rightarrow\; F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}$$

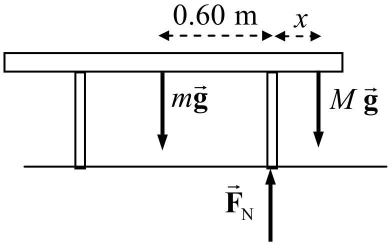
$$\sum F_y = F_{\mathrm{T1}}\sin 37^\circ + F_{\mathrm{T2}}\sin 53^\circ - mg = 0$$

$$F_{\mathrm{T1}}\sin 37^\circ + \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}\sin 53^\circ - mg = 0 \;\rightarrow$$

$$F_{\mathrm{T1}} = \frac{\left(33\text{ kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\sin 37^\circ + \dfrac{\cos 37^\circ}{\cos 53^\circ}\sin 53^\circ} = 194.6\,\mathrm{N} \approx \boxed{190\,\mathrm{N}}$$

$$F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}} = \frac{\cos 37^\circ}{\cos 53^\circ}\left(1.946\times 10^2\,\mathrm{N}\right) = 258.3\,\mathrm{N} \approx \boxed{260\,\mathrm{N}}$$

$\boxed{13.}$ The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass *M*) is on the right side of the table, and that the table (mass *m*) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.



$$\sum \tau = mg\left(0.60\text{ m}\right) - Mgx = 0 \;\rightarrow\; x = \left(0.60\,\mathrm{m}\right)\frac{m}{M} = \left(0.60\,\mathrm{m}\right)\frac{24.0\,\mathrm{kg}}{66.0\,\mathrm{kg}} = 0.218\,\mathrm{m}$$

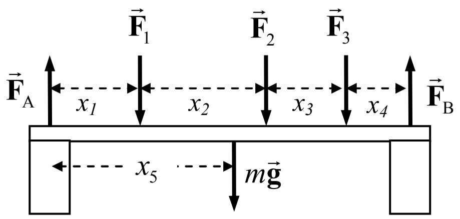
Thus the distance from the edge of the table is $0.50\,\mathrm{m} - 0.218\,\mathrm{m} = \boxed{0.28\,\mathrm{m}}$.

14. The cork screw will pull upward on the cork with a force of magnitude F_{cork}, and so there is a downward force on the opener of magnitude F_{cork}. We assume that there is no net torque on the opener, so that it does not have an angular acceleration. Calculate torques about the rim of the bottle where the opener is resting on the rim.

$$\sum \tau = F(79\,\text{mm}) - F_{\text{cork}}(9\,\text{mm}) = 0 \quad \rightarrow$$

$$F = \frac{9}{70}F_{\text{cork}} = \frac{9}{79}(200\,\text{N}) \text{ to } \frac{9}{79}(400\,\text{N}) = 22.8\,\text{N to } 45.6\,\text{N} \approx \boxed{20\,\text{N to } 50\,\text{N}}$$

15. The beam is in equilibrium, and so both the net torque and net force on it must be zero. From the free-body diagram, calculate the net torque about the center of the left support, with counterclockwise torques as positive. Calculate the net force, with upward as positive. Use those two equations to find F_{A} and F_{B}.



$$\sum \tau = F_{\text{B}}(x_1 + x_2 + x_3 + x_4) - F_1x_1 - F_2(x_1 + x_2) - F_3(x_1 + x_2 + x_3) - mgx_5$$

$$F_{\text{B}} = \frac{F_1x_1 + F_2(x_1 + x_2) + F_3(x_1 + x_2 + x_3) + mgx_5}{(x_1 + x_2 + x_3 + x_4)}$$

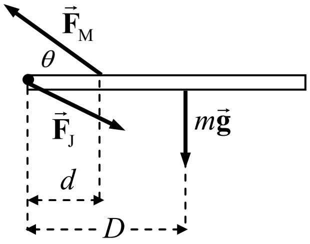
$$= \frac{(4300\,\text{N})(2.0\,\text{m}) + (3100\,\text{N})(6.0\,\text{m}) + (2200\,\text{N})(9.0\,\text{m}) + (280\,\text{kg})(9.80\,\text{m/s}^2)(5.0\,\text{m})}{10.0\text{ m}}$$

$$= 6072\,\text{N} \approx \boxed{6100\,\text{N}}$$

$$\sum F = F_{\text{A}} + F_{\text{B}} - F_1 - F_2 - F_3 - mg = 0$$

$$F_{\text{A}} = F_1 + F_2 + F_3 + mg - F_{\text{B}} = 9600\,\text{N} + (280\,\text{kg})(9.80\,\text{m/s}^2) - 6072\,\text{N} = 6272\,\text{N} \approx \boxed{6300\,\text{N}}$$

16. (*a*) Calculate the torques about the elbow joint (the dot in the free-body diagram). The arm is in equilibrium. Take counterclockwise torques as positive.



$$\sum \tau = (F_{\text{M}} \sin\theta)d - mgD = 0 \quad \rightarrow$$

$$F_{\text{M}} = \frac{mgD}{d\sin\theta} = \frac{(3.3\text{ kg})(9.80\,\text{m/s}^2)(0.24\text{ m})}{(0.12\text{ m})\sin 15^\circ} = 249.9\,\text{N}$$

$$\approx \boxed{250\,\text{N}}$$

(*b*) To find the components of F_{J}, write Newton's second law for both the *x* and *y* directions. Then combine them to find the magnitude.

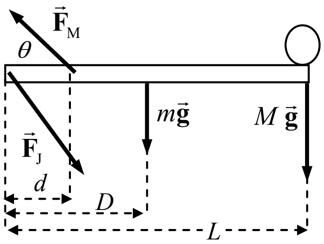
$$\sum F_x = F_{\text{J}x} - F_{\text{M}}\cos\theta = 0 \quad \rightarrow \quad F_{\text{J}x} = F_{\text{M}}\cos\theta = (249.9\,\text{N})\cos 15^\circ = 241.4\,\text{N}$$

$$\sum F_y = F_{\text{M}}\sin\theta - mg - F_{\text{J}y} = 0 \quad \rightarrow$$

$$F_{\text{J}y} = F_{\text{M}}\sin\theta - mg = (249.9\,\text{N})\sin 15^\circ - (3.3\,\text{kg})(9.80\,\text{m/s}^2) = 32.3\,\text{N}$$

$$\text{F}_{\text{J}} = \sqrt{F_{\text{J}x}^2 + F_{\text{J}y}^2} = \sqrt{(241.4\,\text{N})^2 + (32.3\,\text{N})^2} = 243.6\,\text{N} \approx \boxed{240\,\text{N}}$$

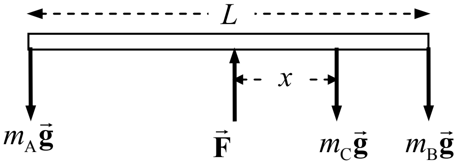
17. Calculate the torques about the shoulder joint, which is at the left end of the free-body diagram of the arm. Since the arm is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques to be positive. The force due to the shoulder joint is drawn, but it does not exert any torque about the shoulder joint.



$$\sum \tau = F_m d\sin\theta - mgD - MgL = 0$$

$$F_m = \frac{mD+ML}{d\sin\theta}g = \frac{(3.3\,\text{kg})(0.24\,\text{cm})+(8.5\,\text{kg})(0.52\,\text{m})}{(0.12\,\text{m})\sin 15^\circ}\left(9.80\,\text{m/s}^2\right) = \boxed{1600\,\text{N}}$$

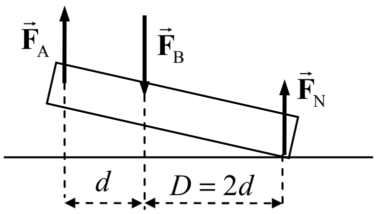
18. From the free-body diagram, the conditions of equilibrium are used to find the location of the girl (mass m_{C}). The 45-kg boy is represented by m_{A}, and the 35-kg girl by m_{B}. Calculate torques about the center of the see-saw, and take counterclockwise torques to be positive. The upward force of the fulcrum on the see-saw $\left(\vec{\mathbf{F}}\right)$ causes no torque about the center.



$$\sum \tau = m_{\text{A}}g\left(\tfrac{1}{2}L\right) - m_{\text{C}}gx - m_{\text{B}}g\left(\tfrac{1}{2}L\right) = 0$$

$$x = \frac{(m_{\text{A}}-m_{\text{B}})}{m_{\text{C}}}\left(\tfrac{1}{2}L\right) = \frac{(45\,\text{kg}-35\,\text{kg})}{25\,\text{kg}}\tfrac{1}{2}(3.2\,\text{m}) = \boxed{0.64\,\text{m}}$$

19. There will be a normal force upwards at the ball of the foot, equal to the person's weight $\left(F_{\text{N}} = mg\right)$. Calculate torques about a point on the floor directly below the leg bone (and so in line with the leg bone force, $\vec{\mathbf{F}}_{\text{B}}$). Since the foot is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques as positive.



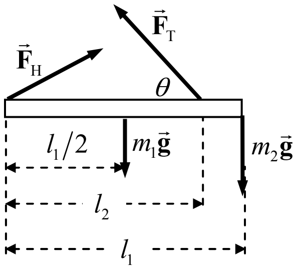
$$\sum \tau = F_{\text{N}}(2d) - F_{\text{A}}d = 0 \;\rightarrow$$

$$F_{\text{A}} = 2F_{\text{N}} = 2mg = 2(72\text{ kg})\left(9.80\,\text{m/s}^2\right) = \boxed{1400\,\text{N}}$$

The net force in the y direction must be zero. Use that to find F_{B}.

$$\sum F_y = F_{\text{N}} + F_{\text{A}} - F_{\text{B}} = 0 \;\rightarrow\; F_{\text{B}} = F_{\text{N}} + F_{\text{A}} = 2mg + mg = 3mg = \boxed{2100\,\text{N}}$$

20. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.



$$\sum \tau = (F_{\text{T}}\sin\theta)l_2 - m_1 g\, l_1/2 - m_2 g l_1 = 0 \;\rightarrow$$

$$F_{\text{T}} = \frac{\tfrac{1}{2}m_1 g l_1 + m_2 g l_1}{l_2\sin\theta} = \frac{\tfrac{1}{2}(155\,\text{N})(1.70\,\text{m}) + (215\,\text{N})(1.70\,\text{m})}{(1.35\,\text{m})(\sin 35.0^\circ)}$$

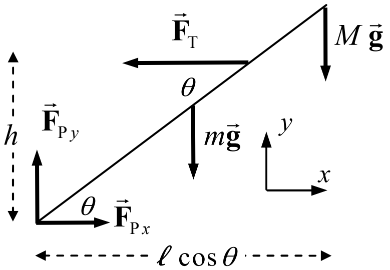
$$= 642.2\,\text{N} \approx \boxed{642\,\text{N}}$$

$$\sum F_x = F_{\text{H}x} - F_{\text{T}}\cos\theta = 0 \;\rightarrow\; F_{\text{H}x} = F_{\text{T}}\cos\theta = (642.2\,\text{N})\cos 35.0^\circ = 526.1\,\text{N} \approx \boxed{526\,\text{N}}$$

$$\sum F_y = F_{\mathrm{H}y} + F_{\mathrm{T}}\sin\theta - m_1 g - m_2 g = 0 \ \rightarrow$$

$$F_{\mathrm{H}y} = m_1 g + m_2 g - F_{\mathrm{T}}\sin\theta = 155\,\mathrm{N} + 215\,\mathrm{N} - (642.2\,\mathrm{N})\sin 35.0^\circ = 1.649\,\mathrm{N} \approx \boxed{2\,\mathrm{N}}$$

21. (*a*) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is ℓ.



$$\sum \tau = F_{\mathrm{T}} h - mg(\ell/2)\cos\theta - Mg\ell\cos\theta = 0$$

$$F_{\mathrm{T}} = \frac{(m/2 + M)g\ell\cos\theta}{h}$$

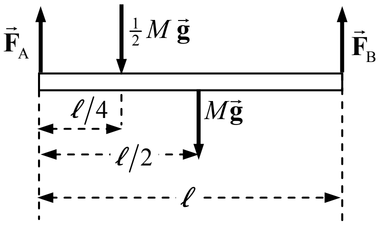
$$= \frac{(6.0\,\mathrm{kg} + 21.5\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(7.20\,\mathrm{m})\cos 37^\circ}{3.80\,\mathrm{m}} = 407.8\,\mathrm{N} \approx \boxed{410\,\mathrm{N}}$$

(*b*) The net force on the pole is also zero since it is in equilibrium. Write Newton's second law in both the *x* and *y* directions to solve for the forces at the pivot.

$$\sum F_x = F_{\mathrm{P}x} - F_{\mathrm{T}} = 0 \ \rightarrow \ F_{\mathrm{P}x} = F_{\mathrm{T}} = \boxed{410\,\mathrm{N}}$$

$$\sum F_y = F_{\mathrm{P}y} - mg - Mg = 0 \ \rightarrow \ F_{\mathrm{P}y} = (m + M)g = (33.5\ \mathrm{kg})(9.80\,\mathrm{m/s^2}) = \boxed{328\,\mathrm{N}}$$

22. The center of gravity of each beam is at its geometric center. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



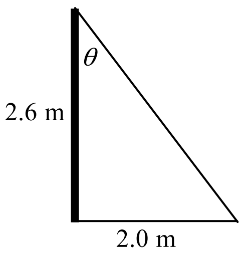
$$\sum \tau = F_{\mathrm{B}}\ell - Mg(\ell/2) - \tfrac{1}{2}Mg(\ell/4) = 0 \ \rightarrow$$

$$F_{\mathrm{B}} = \tfrac{5}{8}Mg = \tfrac{5}{8}(940\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 5758\,\mathrm{N} \approx \boxed{5800\,\mathrm{N}}$$

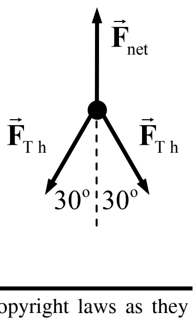
$$\sum F_y = F_{\mathrm{A}} + F_{\mathrm{B}} - Mg - \tfrac{1}{2}Mg = 0 \ \rightarrow$$

$$F_{\mathrm{A}} = \tfrac{3}{2}Mg - F_{\mathrm{B}} = \tfrac{7}{8}Mg = \tfrac{7}{8}(940\ \mathrm{kg})(9.80\,\mathrm{m/s^2}) = 8061\,\mathrm{N} \approx \boxed{8100\,\mathrm{N}}$$

23. First consider the triangle made by the pole and one of the wires (first diagram). It has a vertical leg of 2.6 m, and a horizontal leg of 2.0 m. The angle that the tension (along the wire) makes with the vertical is $\theta = \tan^{-1}\dfrac{2.0}{2.6} = 37.6^\circ$. The part of the tension that is parallel to the ground is therefore $F_{\mathrm{T\,h}} = F_{\mathrm{T}}\sin\theta$.



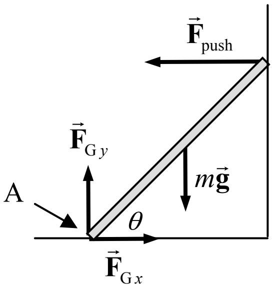
Now consider a top view of the pole, showing only force parallel to the ground (second diagram). The horizontal parts of the tension lie as the sides of an equilateral triangle, and so each make a 30° angle with the tension force of the net. Write the equilibrium equation for the forces along the direction of the tension in the net.



$$\sum F = F_{\mathrm{net}} - 2F_{\mathrm{T\,h}}\cos 30^\circ = 0 \ \rightarrow$$

$$F_{\mathrm{net}} = 2F_{\mathrm{T}}\sin\theta\cos 30^\circ = 2(115\,\mathrm{N})\sin 37.6^\circ\cos 30^\circ = 121.5\,\mathrm{N} \approx \boxed{120\,\mathrm{N}}$$

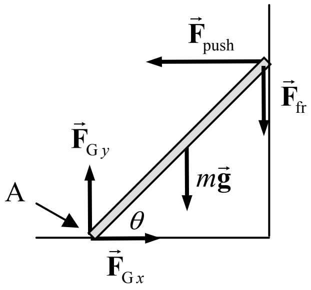
24. See the free-body diagram. We assume that the board is at the edge of the door opposite the hinges, and that you are pushing at that same edge of the door. Then the width of the door does not enter into the problem. Force $\vec{\mathbf{F}}_{\text{push}}$ is the force of the door on the board, and is the same as the force the person exerts on the door. Take torques about the point A in the free-body diagram, where the board rests on the ground. The board is of length ℓ.



$$\sum \tau = F_{\text{push}}\ell\sin\theta - mg\left(\tfrac{1}{2}\ell\right)\cos\theta = 0 \quad\rightarrow$$

$$F_{\text{push}} = \frac{mg}{2\tan\theta} = \frac{(62.0\,\text{kg})(9.80\,\text{m/s}^2)}{2\tan 45^\circ} = 303.8\,\text{N} \approx \boxed{3.0\times10^2\,\text{N}}$$

$\boxed{25.}$ Because the board is firmly set against the ground, the top of the board would move upwards as the door opened. Thus the frictional force on the board at the door must be down. We also assume that the static frictional force is a maximum, and so is given by $F_{\text{fr}} = \mu F_{\text{N}} = \mu F_{\text{push}}$. Take torques about the point A in the free-body diagram, where the board rests on the ground. The board is of length ℓ.

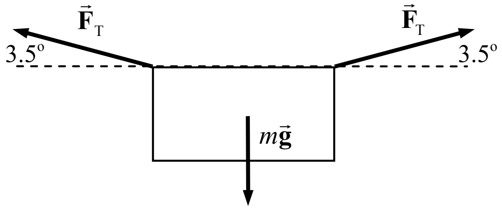


$$\sum \tau = F_{\text{push}}\ell\sin\theta - mg\left(\tfrac{1}{2}\ell\right)\cos\theta - F_{\text{fr}}\ell\cos\theta = 0 \quad\rightarrow$$

$$F_{\text{push}}\ell\sin\theta - mg\left(\tfrac{1}{2}\ell\right)\cos\theta - \mu F_{\text{push}}\ell\cos\theta = 0 \quad\rightarrow$$

$$F_{\text{push}} = \frac{mg}{2(\tan\theta - \mu)} = \frac{mg}{2(\tan\theta - \mu)} = \frac{(62.0\,\text{kg})(9.80\,\text{m/s}^2)}{2(\tan 45^\circ - 0.45)} = 552.4\,\text{N} \approx \boxed{550\,\text{N}}$$

26. Draw the free-body diagram for the sheet, and write Newton's second law for the vertical direction. Note that the tension is the same in both parts of the clothesline.



$$\sum F_y = F_{\text{T}}\sin 3.5^\circ + F_{\text{T}}\sin 3.5^\circ - mg = 0 \quad\rightarrow$$

$$F_{\text{T}} = \frac{mg}{2(\sin 3.5^\circ)} = \frac{(0.75\,\text{kg})(9.80\,\text{m/s}^2)}{2(\sin 3.5^\circ)}$$

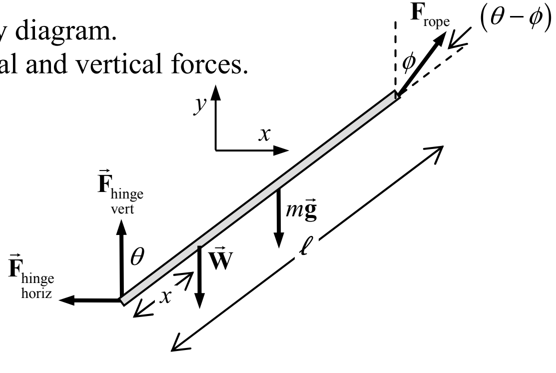
$$= \boxed{60\,\text{N}} \quad (2\text{ sig. fig.})$$

The 60-N tension is much higher than the ~ 7.5-N weight of the sheet because of the small angle. Only the vertical components of the tension are supporting the sheet, and since the angle is small, the tension has to be large to have a large enough vertical component to hold up the sheet.

27. (*a*) Choose the coordinates as shown in the free-body diagram.
(*b*) Write the equilibrium conditions for the horizontal and vertical forces.

$$\sum F_x = F_{\text{rope}}\sin\phi - F_{\substack{\text{hinge}\\ \text{horiz}}} = 0 \quad\rightarrow$$

$$F_{\substack{\text{hinge}\\ \text{horiz}}} = F_{\text{rope}}\sin\phi = (85\,\text{N})\sin 37^\circ = \boxed{51\,\text{N}}$$



$$\sum F_y = F_{\text{rope}}\cos\phi + F_{\substack{\text{hinge}\\\text{vert}}} - mg - W = 0 \quad \rightarrow$$

$$F_{\substack{\text{hinge}\\\text{vert}}} = mg + W - F_{\text{rope}}\cos\phi = (3.8\,\text{kg})\left(9.80\,\text{m/s}^2\right) + 22\,\text{N} - (85\,\text{N})\cos 37^\circ$$

$$= -8.6\,\text{N} \approx \boxed{-9\,\text{N}}$$

And so the vertical hinge force actually points downward.

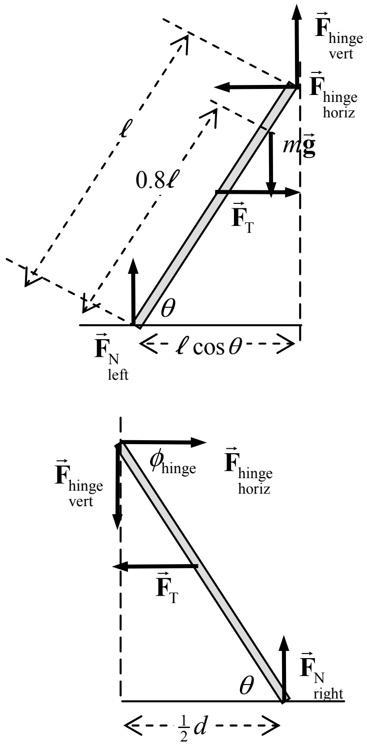
(*c*) We take torques about the hinge point, with clockwise torques as positive.

$$\sum\tau = Wx\sin\theta + mg\left(\tfrac{1}{2}\ell\right)\sin\theta - F_{\text{rope}}\ell\sin(\theta-\phi) = 0 \quad \rightarrow$$

$$x = \frac{F_{\text{rope}}\ell\sin(\theta-\phi) - mg\left(\frac{1}{2}\ell\right)\sin\theta}{W\sin\theta}$$

$$= \frac{(85\,\text{N})(5.0\,\text{m})\sin 16^\circ - (3.8\,\text{kg})\left(9.80\,\text{m/s}^2\right)(2.5\,\text{m})\sin 53^\circ}{(22\,\text{N})\sin 53^\circ} = 2.436\,\text{m} \approx \boxed{2.4\,\text{m}}$$

28. (a) Consider the free-body diagram for each side of the ladder. Because the two sides are not identical, we must have both horizontal and vertical components to the hinge force of one side of the ladder on the other.



First determine the angle from $\cos\theta = \dfrac{\frac{1}{2}d}{\ell} = \dfrac{d}{2\ell}$.

$$\theta = \cos^{-1}\frac{\frac{1}{2}d}{\ell} = \cos^{-1}\frac{0.9\,\text{m}}{2.5\text{m}} = 68.9^\circ$$

Write equilibrium equations for the following conditions:
Vertical forces on total ladder:

$$\sum F_{\text{vert}} = F_{\substack{\text{N}\\\text{left}}} - mg + F_{\substack{\text{hinge}\\\text{vert}}} - F_{\substack{\text{hinge}\\\text{vert}}} + F_{\substack{\text{N}\\\text{right}}} = 0 \quad \rightarrow$$

$$F_{\substack{\text{N}\\\text{left}}} + F_{\substack{\text{N}\\\text{right}}} = mg$$

Torques on left side, about top, clockwise positive.

$$\sum\tau = F_{\substack{\text{N}\\\text{left}}}(\ell\cos\theta) - mg(0.2\ell)\cos\theta - F_T\left(\tfrac{1}{2}\ell\right)\sin\theta = 0$$

Torques on right side, about top, clockwise positive.

$$\sum\tau = -F_{\substack{\text{N}\\\text{right}}}(\ell\cos\theta) + F_T\left(\tfrac{1}{2}\ell\right)\sin\theta = 0$$

Subtract the second torque equation from the first.

$$\left(F_{\substack{\text{N}\\\text{left}}} + F_{\substack{\text{N}\\\text{right}}}\right)(\ell\cos\theta) - mg(0.2\ell)\cos\theta - 2F_T\left(\tfrac{1}{2}\ell\right)\sin\theta = 0$$

Substitute in from the vertical forces equation, and solve for the tension.

$$mg(\ell\cos\theta) - mg(0.2\ell)\cos\theta - 2F_T\left(\tfrac{1}{2}\ell\right)\sin\theta = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{\sin\theta}(0.8\cos\theta) = \frac{0.8mg}{\tan\theta} = \frac{0.8(56.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{\tan 68.9^\circ} = 169.4\,\text{N} \approx \boxed{170\,\text{N}}$$

(*b*) To find the normal force on the right side, use the torque equation for the right side.

$$-F_{\substack{\text{N}\\\text{right}}}(\ell\cos\theta)+F_{\text{T}}\left(\tfrac{1}{2}\ell\right)\sin\theta=0 \;\rightarrow$$

$$F_{\substack{\text{N}\\\text{right}}}=\tfrac{1}{2}F_{\text{T}}\tan\theta=\tfrac{1}{2}(169.4\,\text{N})\tan 68.9^\circ=219.5\,\text{N}\approx\boxed{220\,\text{N}}$$

To find the normal force on the left side, use the vertical force equation for the entire ladder.

$$F_{\substack{\text{N}\\\text{left}}}+F_{\substack{\text{N}\\\text{right}}}=mg \;\rightarrow$$

$$F_{\substack{\text{N}\\\text{left}}}=mg-F_{\substack{\text{N}\\\text{right}}}=(56.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)-219.5\,\text{N}=329.3\,\text{N}\approx\boxed{330\,\text{N}}$$

(*c*) We find the hinge force components from the free-body diagram for the right side.

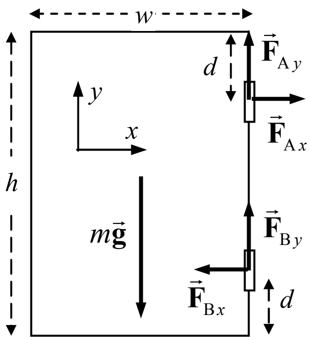
$$\sum F_{\text{vert}}=F_{\substack{\text{N}\\\text{right}}}-F_{\substack{\text{hinge}\\\text{vert}}}=0 \;\rightarrow\; F_{\substack{\text{hinge}\\\text{vert}}}=F_{\substack{\text{N}\\\text{right}}}=219.5\,\text{N}$$

$$\sum F_{\text{horiz}}=F_{\substack{\text{hinge}\\\text{horiz}}}-F_{\text{T}}=0 \;\rightarrow\; F_{\substack{\text{hinge}\\\text{horiz}}}=F_{\text{T}}=169.4\,\text{N}$$

$$F_{\text{hinge}}=\sqrt{F^2_{\substack{\text{hinge}\\\text{horiz}}}+F^2_{\substack{\text{hinge}\\\text{vert}}}}=\sqrt{(169.4\,\text{N})^2+(219.5\,\text{N})^2}=277.3\,\text{N}\approx\boxed{280\,\text{N}}$$

$$\phi_{\text{hinge}}=\tan^{-1}\frac{F_{\substack{\text{hinge}\\\text{vert}}}}{F_{\substack{\text{hinge}\\\text{horiz}}}}=\tan^{-1}\frac{219.5\,\text{N}}{169.4\,\text{N}}=\boxed{52^\circ}$$

29. The forces on the door are due to gravity and the hinges. Since the door is in equilibrium, the net torque and net force must be zero. Write the three equations of equilibrium. Calculate torques about the bottom hinge, with counterclockwise torques as positive. From the statement of the problem, $F_{\text{A}y}=F_{\text{B}y}=\frac{1}{2}mg$.



$$\sum\tau=mg\frac{w}{2}-F_{Ax}(h-2d)=0$$

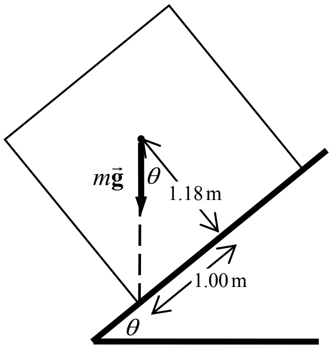
$$F_{Ax}=\frac{mgw}{2(h-2d)}=\frac{(13.0\text{ kg})\left(9.80\,\text{m/s}^2\right)(1.30\text{ m})}{2(2.30\text{ m}-0.80\text{ m})}=\boxed{55.2\text{ N}}$$

$$\sum F_x=F_{Ax}-F_{Bx}=0 \;\rightarrow\; F_{Bx}=F_{Ax}=\boxed{55.2\text{ N}}$$

$$\sum F_y=F_{Ay}+F_{By}-mg=0 \;\rightarrow\; F_{Ay}=F_{By}=\tfrac{1}{2}mg=\tfrac{1}{2}(13.0\text{ kg})\left(9.80\,\text{m/s}^2\right)=\boxed{63.7\text{ N}}$$

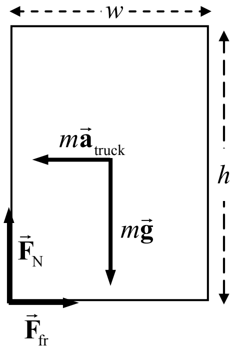
30. See the free–body diagram for the crate on the verge of tipping. From the textbook Figure 12-12 and the associated discussion, if a vertical line projected downward from the center of gravity falls outside the base of support, then the object will topple. So the limiting case is for the vertical line to intersect the edge of the base of support. Any more tilting and the gravity force would cause the block to tip over, with the axis of rotation through the lower corner of the crate.

$$\tan\theta=\frac{1.00}{1.18} \;\rightarrow\; \theta=\tan^{-1}\frac{1.00}{1.18}=\boxed{40^\circ}\;(2\text{ sig fig})$$



The other forces on the block, the normal force and the frictional force, would be acting at the lower corner and so would not cause any torque about the lower corner. The gravity force causes the tipping. It wouldn't matter if the block were static or sliding, since the magnitude of the frictional force does not enter into the calculation.

31. We assume the truck is accelerating to the right. We want the refrigerator to not tip in the non-inertial reference frame of the truck. Accordingly, to analyze the refrigerator in the non-inertial reference frame, we must add a pseudoforce in the opposite direction of the actual acceleration. The free-body diagram is for a side view of the refrigerator, just ready to tip so that the normal force and frictional force are at the lower back corner of the refrigerator. The center of mass is in the geometric center of the refrigerator. Write the conditions for equilibrium, taking torques about an axis through the center of mass, perpendicular to the plane of the paper. The normal force and frictional force cause no torque about that axis.



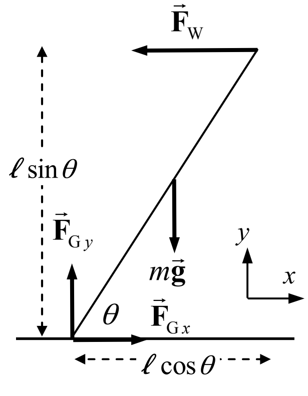
$$\sum F_{\text{horiz}} = F_{\text{fr}} - ma_{\text{truck}} = 0 \;\rightarrow\; F_{\text{fr}} = ma_{\text{truck}}$$

$$\sum F_{\text{vert}} = F_{\text{N}} - mg = 0 \;\rightarrow\; F_{\text{N}} = mg$$

$$\sum \tau = F_{\text{N}}\left(\tfrac{1}{2}w\right) - F_{\text{fr}}\left(\tfrac{1}{2}h\right) = 0 \;\rightarrow\; \frac{F_{\text{N}}}{F_{\text{fr}}} = \frac{h}{w}$$

$$\frac{F_{\text{N}}}{F_{\text{fr}}} = \frac{h}{w} = \frac{mg}{ma_{\text{truck}}} \;\rightarrow\; a_{\text{truck}} = g\frac{w}{h} = \left(9.80\,\text{m/s}^2\right)\frac{1.0\,\text{m}}{1.9\,\text{m}} = \boxed{5.2\,\text{m/s}^2}$$

32. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.



$$\sum \tau = F_{\text{W}}\ell\sin\theta - mg\left(\tfrac{1}{2}\ell\cos\theta\right) = 0 \;\rightarrow\; F_{\text{W}} = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{\text{G}x} - F_{\text{W}} = 0 \;\rightarrow\; F_{\text{G}x} = F_{\text{W}} = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

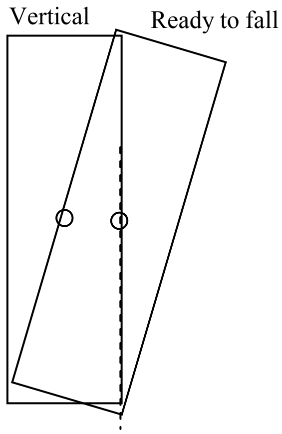
$$\sum F_y = F_{\text{G}y} - mg = 0 \;\rightarrow\; F_{\text{G}y} = mg$$

For the ladder to not slip, the force at the ground $F_{\text{G}x}$ must be less than or equal to the maximum force of static friction.

$$F_{\text{G}x} \le \mu F_{\text{N}} = \mu F_{\text{G}y} \;\rightarrow\; \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu mg \;\rightarrow\; \frac{1}{2\mu} \le \tan\theta \;\rightarrow\; \theta \ge \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Thus the minimum angle is $\boxed{\theta_{\text{min}} = \tan^{-1}\left(\frac{1}{2\mu}\right)}$.

33. The tower can lean until a line projected downward through its center of gravity will fall outside its base of support. Since we are assuming that the tower is uniform, its center of gravity (or center of mass) will be at its geometric center. The center of mass can move a total of 3.5 m off of center and still be over the support base. It has currently moved 2.25 m off of center. So it can lean over another $\boxed{\text{1.25 m at the center}}$, or $\boxed{\text{2.5 m at the top}}$. Note that the diagram is NOT to scale. The tower should be twice as tall as shown to be to scale.



34. The amount of stretch can be found using the elastic modulus in Eq. 12-4.

$$\Delta\ell = \frac{1}{E}\frac{F}{A}\ell_0 = \frac{1}{5\times10^9\,\text{N/m}^2}\frac{275\text{ N}}{\pi\left(5.00\times10^{-4}\right)^2}(0.300\text{ m}) = \boxed{2.10\times10^{-2}\,\text{m}}$$

35. (*a*) $$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(25000\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{1.4\,\text{m}^2} = 175{,}000\,\text{N/m}^2 \approx \boxed{1.8\times10^5\,\text{N/m}^2}$$

(*b*) $$\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{175{,}000\times10^5\,\text{N/m}^2}{50\times10^9\,\text{N/m}^2} = \boxed{3.5\times10^{-6}}$$

36. The change in length is found from the strain.

$$\text{Strain} = \frac{\Delta\ell}{\ell_0} \quad\rightarrow\quad \Delta\ell = \ell_0(\text{Strain}) = (8.6\,\text{m})\left(3.5\times10^{-6}\right) = \boxed{3.0\times10^{-5}\,\text{m}}$$

$\boxed{37.}$ (*a*) $$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(1700\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{0.012\,\text{m}^2} = 1.388\times10^6\,\text{N/m}^2 \approx \boxed{1.4\times10^6\,\text{N/m}^2}$$

(*b*) $$\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{1.388\times10^6\,\text{N/m}^2}{200\times10^9\,\text{N/m}^2} = 6.94\times10^{-6} \approx \boxed{6.9\times10^{-6}}$$

(*c*) $$\Delta\ell = (\text{Strain})(\ell_0) = \left(6.94\times10^{-6}\right)(9.50\,\text{m}) = 6.593\times10^{-5}\,\text{m} \approx \boxed{6.6\times10^{-5}\,\text{m}}$$

38. The relationship between pressure change and volume change is given by Eq. 12-7.

$$\Delta V = -V_0\frac{\Delta P}{B} \quad\rightarrow\quad \Delta P = -\frac{\Delta V}{V_0}B = -\left(0.10\times10^{-2}\right)\left(90\times10^9\,\text{N/m}^2\right) = \boxed{9.0\times10^7\,\text{N/m}^2}$$

$$\frac{\Delta P}{P_{\text{atm}}} = \frac{9.0\times10^7\,\text{N/m}^2}{1.0\times10^5\,\text{N/m}^2} = \boxed{9.0\times10^2}\text{ , or 900 atmospheres}$$

39. The Young's Modulus is the stress divided by the strain.

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta\ell/\ell_0} = \frac{(13.4\text{ N})\Big/\left[\pi\left(\tfrac{1}{2}\times8.5\times10^{-3}\text{m}\right)^2\right]}{\left(3.7\times10^{-3}\text{m}\right)\Big/\left(15\times10^{-2}\text{m}\right)} = \boxed{9.6\times10^6\,\text{N/m}^2}$$

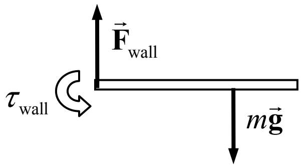
40. The percentage change in volume is found by multiplying the relative change in volume by 100. The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure. Use Eq. 12-7.

$$100\frac{\Delta V}{V_o}=-100\frac{\Delta P}{B}=-100\frac{199\left(1.0\times10^5\ \mathrm{N/m^2}\right)}{90\times10^9\ \mathrm{N/m^2}}=\boxed{-2\times10^{-2}\%}$$

The negative sign indicates that the interior space got smaller.

41. (*a*) The torque due to the sign is the product of the weight of the sign and the distance of the sign from the wall.

$$\tau=mgd=(6.1\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)(2.2\,\mathrm{m})=\boxed{130\,\mathrm{m\bullet N}\text{ , clockwise}}$$



(*b*) Since the wall is the only other object that can put force on the pole (ignoring the weight of the pole), then $\boxed{\text{the wall must put a torque on the pole}}$. The torque due to the hanging sign is clockwise, so the torque due to the wall must be counterclockwise. See the diagram. Also note that the wall must put a net upward force on the pole as well, so that the net force on the pole will be zero.

(*c*) The torque on the rod can be considered as the wall pulling horizontally to the left on the top left corner of the rod and pushing horizontally to the right at the bottom left corner of the rod. The reaction forces to these put a shear on the wall at the point of contact. Also, since the wall is pulling upwards on the rod, the rod is pulling down on the wall at the top surface of contact, causing tension. Likewise the rod is pushing down on the wall at the bottom surface of contact, causing compression. Thus $\boxed{\text{all three are present}}$.

42. Set the compressive strength of the bone equal to the stress of the bone.

$$\text{Compressive Strength}=\frac{F_{\text{max}}}{A}\ \rightarrow\ F_{\text{max}}=\left(170\times10^6\ \mathrm{N/m^2}\right)\left(3.0\times10^{-4}\mathrm{m^2}\right)=\boxed{5.1\times10^4\,\mathrm{N}}$$

43. (*a*) The maximum tension can be found from the ultimate tensile strength of the material.

$$\text{Tensile Strength}=\frac{F_{\text{max}}}{A}\ \rightarrow$$

$$F_{\text{max}}=(\text{Tensile Strength})A=\left(500\times10^6\ \mathrm{N/m^2}\right)\pi\left(5.00\times10^{-4}\mathrm{m}\right)^2=\boxed{393\ \mathrm{N}}$$

(*b*) To prevent breakage, $\boxed{\text{thicker strings}}$ should be used, which will increase the cross-sectional area of the strings, and thus increase the maximum force. Breakage occurs because when the strings are hit by the ball, they stretch, increasing the tension. The strings are reasonably tight in the normal racket configuration, so when the tension is increased by a particularly hard hit, the tension may exceed the maximum force.

44. (*a*) Compare the stress on the bone to the compressive strength to see if the bone breaks.

$$\begin{aligned}\text{Stress}&=\frac{F}{A}=\frac{3.3\times10^4\,\mathrm{N}}{3.6\times10^{-4}\,\mathrm{m^2}}\\&=9.167\times10^7\ \mathrm{N/m^2}<1.7\times10^8\ \mathrm{N/m^2}\ (\text{Compressive Strength of bone})\end{aligned}$$

$\boxed{\text{The bone will not break.}}$

(*b*) The change in length is calculated from Eq. 12-4.

$$\Delta\ell=\frac{\ell_0}{E}\frac{F}{A}=\left(\frac{0.22\,\mathrm{m}}{15\times10^9\ \mathrm{N/m^2}}\right)\left(9.167\times10^7\ \mathrm{N/m^2}\right)=\boxed{1.3\times10^{-3}\mathrm{m}}$$

45. (*a*) The area can be found from the ultimate tensile strength of the material.

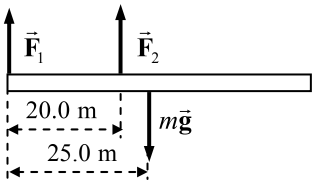
$$\frac{\text{Tensile Strength}}{\text{Safety Factor}}=\frac{F}{A}\quad\rightarrow\quad A=F\left(\frac{\text{Safety Factor}}{\text{Tensile Strength}}\right)\quad\rightarrow$$

$$A=(270\,\text{kg})(9.80\,\text{m/s}^2)\frac{7.0}{500\times10^6\ \text{N/m}^2}=3.704\times10^{-5}\text{m}^2\approx\boxed{3.7\times10^{-5}\text{m}^2}$$

(*b*) The change in length can be found from the stress-strain relationship, Eq. 12-5.

$$\frac{F}{A}=E\frac{\Delta\ell}{\ell_0}\quad\rightarrow\quad\Delta\ell=\frac{\ell_0F}{AE}=\frac{(7.5\,\text{m})(320\,\text{kg})(9.80\,\text{m/s}^2)}{(3.704\times10^{-5}\text{m}^2)(200\times10^9\ \text{N/m}^2)}=\boxed{2.7\times10^{-3}\text{m}}$$

46. For each support, to find the minimum cross-sectional area with a safety factor means that $\frac{F}{A}=\frac{\text{Strength}}{\text{Safety Factor}}$, where either the tensile or compressive strength is used, as appropriate for each force. To find the force on each support, use the conditions of equilibrium for the beam. Take torques about the left end of the beam, calling counterclockwise torques positive, and also sum the vertical forces, taking upward forces as positive.



$$\sum\tau=F_2(20.0\text{ m})-mg(25.0\text{ m})=0\quad\rightarrow\quad F_2=\tfrac{25.0}{20.0}mg=1.25mg$$

$$\sum F_y=F_1+F_2-mg=0\quad\rightarrow\quad F_1=mg-F_2=mg-1.25mg=-0.25mg$$

Notice that the forces on the supports are the opposite of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. So the force on support # 1 is directed upwards, which means that support # 1 is in tension. The force on support # 2 is directed downwards, so support # 2 is in compression.

$$\frac{F_1}{A_1}=\frac{\text{Tensile Strength}}{9.0}\quad\rightarrow$$

$$A_1=9.0\frac{(0.25mg)}{\text{Tensile Strength}}=9.0\frac{(0.25)(2.9\times10^3\text{kg})(9.80\,\text{m/s}^2)}{40\times10^6\ \text{N/m}^2}=\boxed{1.6\times10^{-3}\text{m}^2}$$

$$\frac{F_2}{A_2}=\frac{\text{Compressive Strength}}{9.0}\quad\rightarrow$$

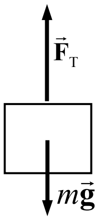
$$A_1=9.0\frac{(1.25mg)}{\text{Compressive Strength}}=9.0\frac{(1.25)(2.9\times10^3\text{kg})(9.80\,\text{m/s}^2)}{35\times10^6\ \text{N/m}^2}=\boxed{9.1\times10^{-3}\text{m}^2}$$

47. The maximum shear stress is to be 1/7th of the shear strength for iron. The maximum stress will occur for the minimum area, and thus the minimum diameter.

$$\text{stress}_{\text{max}}=\frac{F}{A_{\text{min}}}=\frac{\text{shear strength}}{7.0}\quad\rightarrow\quad A_1=\pi\left(\tfrac{1}{2}d\right)^2=\frac{7.0F}{\text{shear strength}}\quad\rightarrow$$

$$d=\sqrt{\frac{4(7.0)F}{\pi(\text{shear strength})}}=\sqrt{\frac{28(3300\,\text{N})}{\pi(170\times10^6\ \text{N/m}^2)}}=1.3\times10^{-2}\text{m}=\boxed{1.3\,\text{cm}}$$

48. From the free-body diagram, write Newton's second law for the vertical direction. Solve for the maximum tension required in the cable, which will occur for an upwards acceleration.



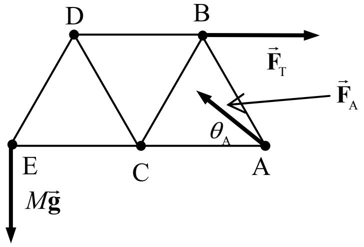
$$\sum F_y = F_T - mg = ma \;\rightarrow\; F_T = m(g+a)$$

The maximum stress is to be 1/8th of the tensile strength for steel. The maximum stress will occur for the minimum area, and thus the minimum diameter.

$$\text{stress}_{\max} = \frac{F_T}{A_{\min}} = \frac{\text{tensile strength}}{8.0} \;\rightarrow\; A_1 = \pi\left(\tfrac{1}{2}d\right)^2 = \frac{8.0F_T}{\text{tensile strength}} \;\rightarrow$$

$$d = \sqrt{\frac{4(8.0)m(g+a)}{\pi(\text{tensile strength})}} = \sqrt{\frac{32(3100\,\text{kg})(11.0\,\text{m/s}^2)}{\pi(500\times10^6\,\text{N/m}^2)}} = 2.6\times10^{-2}\,\text{m} = \boxed{2.6\,\text{cm}}$$

$\boxed{49.}$ (*a*) The three forces on the truss as a whole are the tension force at point B, the load at point E, and the force at point A. Since the truss is in equilibrium, these three forces must add to be 0 and must cause no net torque. Take torques about point A, calling clockwise torques positive. Each member is 3.0 m in length.



$$\sum \tau = F_T(3.0\,\text{m})\sin 60^\circ - Mg(6.0\,\text{m}) = 0 \;\rightarrow$$

$$F_T = \frac{Mg(6.0\,\text{m})}{(3.0\,\text{m})\sin 60^\circ} = \frac{(66.0\,\text{kN})(6.0\,\text{m})}{(3.0\,\text{m})\sin 60^\circ} = 152\,\text{kN} \approx \boxed{150\,\text{kN}}$$

The components of $\vec{\mathbf{F}}_A$ are found from the force equilibrium equations, and then the magnitude and direction can be found.

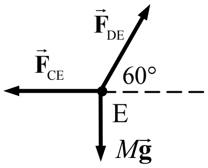
$$\sum F_{\text{horiz}} = F_T - F_{\text{A horiz}} = 0 \;\rightarrow\; F_{\text{A horiz}} = F_T = 152\,\text{kN}$$

$$\sum F_{\text{vert}} = F_{\text{A vert}} - Mg = 0 \;\rightarrow\; F_{\text{A vert}} = Mg = 66.0\,\text{kN}$$

$$F_A = \sqrt{F_{\text{A horiz}}^2 + F_{\text{A vert}}^2} = \sqrt{(152\,\text{kN})^2 + (66.0\,\text{kN})^2} = 166\,\text{kN} \approx \boxed{170\,\text{kN}}$$

$$\theta_A = \tan^{-1}\frac{F_{\text{A vert}}}{F_{\text{A horiz}}} = \tan^{-1}\frac{66.0\,\text{kN}}{152\,\text{kN}} = 23.47^\circ \approx \boxed{23^\circ\text{ above AC}}$$

(*b*) Analyze the forces on the pin at point E. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



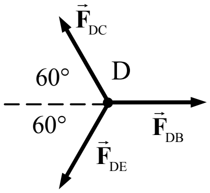
$$\sum F_{\text{vert}} = F_{DE}\sin 60^\circ - Mg = 0 \;\rightarrow$$

$$F_{DE} = \frac{Mg}{\sin 60^\circ} = \frac{66.0\,\text{kN}}{\sin 60^\circ} = 76.2\,\text{kN} \approx \boxed{76\,\text{kN, in tension}}$$

$$\sum F_{\text{horiz}} = F_{DE}\cos 60^\circ - F_{CE} = 0 \;\rightarrow$$

$$F_{CE} = F_{DE}\cos 60^\circ = (76.2\,\text{kN})\cos 60^\circ = 38.1\,\text{kN} \approx \boxed{38\,\text{kN, in compression}}$$

Analyze the forces on the pin at point D. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{vert}} = F_{DC}\sin 60^\circ - F_{DE}\sin 60^\circ = 0 \;\rightarrow$$

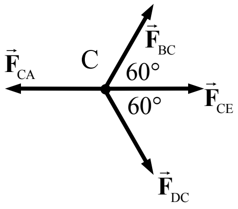
$$F_{DC} = F_{DE} = 76.2\,\text{kN} \approx \boxed{76\,\text{kN, in compression}}$$

$$\sum F_{\text{horiz}} = F_{\text{DB}} - F_{\text{DE}}\cos 60° - F_{\text{DC}}\cos 60° = 0 \;\rightarrow$$

$$F_{\text{DB}} = \left(F_{\text{DE}} + F_{\text{DC}}\right)\cos 60° = 2\left(76.2\,\text{kN}\right)\cos 60° = 76.2\,\text{kN}$$

$$\approx \boxed{76\,\text{kN, in tension}}$$

Analyze the forces on the pin at point C. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{vert}} = F_{\text{BC}}\sin 60° - F_{\text{DC}}\sin 60° = 0 \;\rightarrow$$

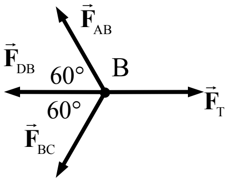
$$F_{\text{BC}} = F_{\text{DC}} = 76.2\,\text{kN} \approx \boxed{76\,\text{kN, in tension}}$$

$$\sum F_{\text{horiz}} = F_{\text{CE}} + F_{\text{BC}}\cos 60° + F_{\text{DC}}\cos 60° - F_{\text{CA}} = 0 \;\rightarrow$$

$$F_{\text{CA}} = F_{\text{CE}} + \left(F_{\text{BC}} + F_{\text{DC}}\right)\cos 60° = 38.1\,\text{kN} + 2\left(76.2\,\text{kN}\right)\cos 60°$$

$$= 114.3\,\text{kN} \approx \boxed{114\,\text{kN, in compression}}$$

Analyze the forces on the pin at point B. See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{vert}} = F_{\text{AB}}\sin 60° - F_{\text{BC}}\sin 60° = 0 \;\rightarrow$$

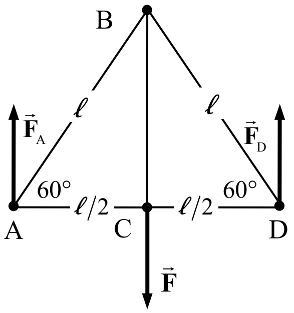
$$F_{\text{AB}} = F_{\text{BC}} = 76.2\,\text{kN} \approx \boxed{76\,\text{kN, in compression}}$$

$$\sum F_{\text{horiz}} = F_{\text{T}} - F_{\text{BC}}\cos 60° - F_{\text{AB}}\cos 60° - F_{\text{DB}} = 0 \;\rightarrow$$

$$F_{\text{T}} = \left(F_{\text{BC}} + F_{\text{AB}}\right)\cos 60° + F_{\text{DB}} = 2\left(76.2\,\text{kN}\right)\cos 60° + 76.2\text{kN} = 152\,\text{kN}$$

This final result confirms the earlier calculation, so the results are consistent. We could also analyze point A to check for consistency.

50. There are upward forces at each support (points A and D) and a downward applied force at point C. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive. Let each side of the equilateral triangle be of length ℓ.

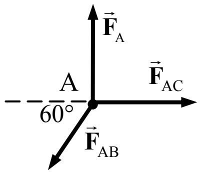


$$\sum F_{\text{vert}} = F_{\text{A}} + F_{\text{D}} - F = 0$$

$$\sum \tau = F\left(\tfrac{1}{2}\ell\right) - F_{\text{D}}\ell = 0 \;\rightarrow\; F_{\text{D}} = \tfrac{1}{2}F = \tfrac{1}{2}\left(1.35\times 10^{4}\,\text{N}\right) = 6750\,\text{N}$$

$$F_{\text{A}} = F - F_{\text{D}} = 1.35\times 10^{4}\,\text{N} - 6750\,\text{N} = 6750\,\text{N}$$

(*a*) Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{vert}} = F_{\text{A}} - F_{\text{AB}}\sin 60° = 0 \;\rightarrow$$

$$F_{\text{AB}} = \frac{F_{\text{A}}}{\sin 60°} = \frac{6750\,\text{N}}{\sin 60°} = 7794\,\text{N} \approx \boxed{7790\,\text{N, compression}}$$

$$\sum F_{\text{horiz}} = F_{\text{AC}} - F_{\text{AB}}\cos 60° = 0 \;\rightarrow$$

$$F_{\text{AC}} = F_{\text{AB}}\cos 60° = \left(7794\,\text{N}\right)\cos 60° = 3897\,\text{N} \approx \boxed{3900\,\text{N, tension}}$$

By the symmetry of the structure, we also know that $F_{\text{DB}} = 7794\,\text{N} \approx \boxed{7790\,\text{N, compression}}$ and $F_{\text{DC}} = 3897\,\text{N} \approx \boxed{3900\,\text{N, tension}}$. Finally, from consideration of the vertical forces on pin C, we see that $F_{\mathrm{BC}} = \boxed{1.35\times10^{4}\,\mathrm{N,\ tension}}$.

(*b*) As listed above, we have struts $\boxed{\text{AB and DB under compression}}$, and struts $\boxed{\text{AC, DC, and BC under compression}}$.

51. (*a*) We assume that all of the trusses are of the same cross-sectional area, and so to find the minimum area needed, we use the truss that has the highest force in it. That is $F_{\mathrm{AB}} = \frac{1}{\sqrt{3}}Mg.$

Apply the safety condition to find the area.

$$\frac{F_{\mathrm{AB}}}{A} = \frac{\text{Ultimate strength}}{7.0} \quad \rightarrow$$

$$A = \frac{7.0F_{\mathrm{AB}}}{\text{Ultimate strength}} = \frac{7.0Mg}{\sqrt{3}\left(\text{Ultimate strength}\right)} = \frac{7.0\left(7.0\times10^{5}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\sqrt{3}\left(500\times10^{6}\,\mathrm{N/m^2}\right)}$$

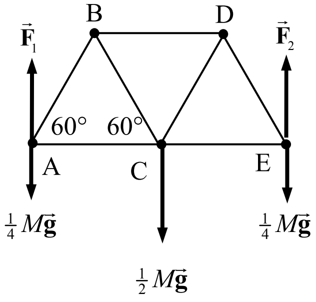
$$= \boxed{5.5\times10^{-2}\,\mathrm{m^2}}$$

(*b*) Recall that each truss must carry half the load, and so we need to add in an additional mass equal to 30 trucks. As in Example 12-11, we will assume that the mass of the trucks acts entirely at the center, so the analysis of that example is still valid. Let *m* represent the mass of a truck.

$$A = \frac{7.0\left(M+30m\right)g}{\sqrt{3}\left(\text{Ultimate strength}\right)} = \frac{7.0\left[7.0\times10^{5}\,\mathrm{kg}+30\left(1.3\times10^{4}\,\mathrm{kg}\right)\right]\left(9.80\,\mathrm{m/s^2}\right)}{\sqrt{3}\left(500\times10^{6}\,\mathrm{N/m^2}\right)}$$

$$= \boxed{8.6\times10^{-2}\,\mathrm{m^2}}$$

52. See the free-body diagram from Figure 12-29, as modified to indicate the changes in the roadway mass distribution. As in that example, if the roadway mass is $1.40\times10^{6}\,\mathrm{kg}$, then for one truss, we should use $M = 7.0\times10^{5}\,\mathrm{kg}$. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.



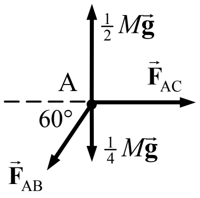
$$\sum F_{\mathrm{vert}} = F_1 + F_2 - Mg = 0$$

$$\sum \tau = \tfrac{1}{2}Mg\left(32\,\mathrm{m}\right) + \tfrac{1}{4}Mg\left(64\,\mathrm{m}\right) - F_2\left(64\,\mathrm{m}\right) = 0 \quad \rightarrow \quad F_2 = \tfrac{1}{2}Mg$$

$$F_1 = Mg - F_2 = \tfrac{1}{2}Mg$$

Note that the problem is still symmetric about a vertical line through pin C. Also note that the forces at the ends each bear half of the weight of that side of the structure.

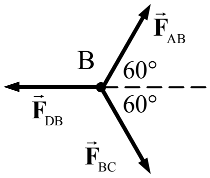
Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\mathrm{vert}} = \tfrac{1}{2}Mg - \tfrac{1}{4}Mg - F_{\mathrm{AB}}\sin60^\circ = 0 \quad \rightarrow$$

$$F_{\mathrm{AB}} = \frac{\frac{1}{4}Mg}{\sin60^\circ} = \frac{\frac{1}{4}Mg}{\frac{1}{2}\sqrt{3}} = \boxed{\frac{Mg}{2\sqrt{3}},\ \text{in compression}}$$

$$\sum F_{\mathrm{horiz}} = F_{\mathrm{AC}} - F_{\mathrm{AB}}\cos60^\circ = 0 \quad \rightarrow \quad F_{\mathrm{AC}} = F_{\mathrm{AB}}\cos60^\circ = \frac{Mg}{2\sqrt{3}}\frac{1}{2} = \boxed{\frac{Mg}{4\sqrt{3}},\ \text{in tension}}$$



Analyze the forces on the pin at point B. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$\sum F_{\text{vert}} = F_{\text{AB}} \sin 60° - F_{\text{BC}} \sin 60° = 0 \quad \rightarrow$$

$$F_{\text{BC}} = F_{\text{AB}} = \boxed{\frac{Mg}{2\sqrt{3}}, \text{ in tension}}$$

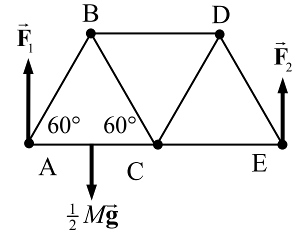
$$\sum F_{\text{horiz}} = F_{\text{AB}} \cos 60° + F_{\text{BC}} \cos 60° - F_{\text{DB}} = 0 \quad \rightarrow$$

$$F_{\text{DB}} = \left(F_{\text{AB}} + F_{\text{BC}}\right)\cos 60° = 2\left(\frac{Mg}{2\sqrt{3}}\right)\cos 60° = \boxed{\frac{Mg}{2\sqrt{3}}, \text{ in compression}}$$

By the symmetry of the geometry, we can determine the other forces.

$$F_{\text{DE}} = F_{\text{AB}} = \boxed{\frac{Mg}{2\sqrt{3}}, \text{ in compression}}, \; F_{\text{DC}} = F_{\text{BC}} = \boxed{\frac{Mg}{2\sqrt{3}}, \text{ in tension}}, \; F_{\text{CE}} = F_{\text{AC}} = \boxed{\frac{Mg}{4\sqrt{3}}, \text{ in tension}}.$$

Note that each force is reduced by a factor of 2 from the original solution given in Example 12-11.

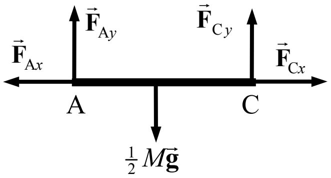


53. See the free-body diagram from Figure 12-29. *M* represents the mass of the train, and each member has a length of ℓ. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.

$$\sum F_{\text{vert}} = F_1 + F_2 - \tfrac{1}{2}Mg = 0$$

$$\sum \tau = \tfrac{1}{2}Mg\left(\tfrac{1}{2}\ell\right) - F_2\left(2\ell\right) = 0 \quad \rightarrow \quad F_2 = \tfrac{1}{8}Mg$$

$$F_1 = \tfrac{1}{2}Mg - F_2 = \tfrac{3}{8}Mg$$



Analyze the forces on strut AC, using the free-body diagram given in Figure 12-29b. Note that the forces at the pins are broken up into components. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions, and for torques about point A.

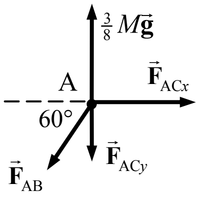
$$\sum F_{\text{vert}} = F_{\text{A}y} + F_{\text{C}y} - \tfrac{1}{2}Mg = 0$$

$$\sum F_{\text{horiz}} = -F_{\text{A}x} + F_{\text{C}x} = 0 \quad \rightarrow \quad F_{\text{C}x} = F_{\text{A}x}$$

$$\sum \tau = \tfrac{1}{2}Mg\left(\tfrac{1}{2}\ell\right) - F_{\text{C}y}\left(\ell\right) = \quad \rightarrow \quad F_{\text{C}y} = \tfrac{1}{4}Mg$$

$$F_{\text{A}y} = \tfrac{1}{2}Mg - F_{\text{C}y} = \tfrac{1}{4}Mg$$

Since their *x* components are equal and their *y* components are equal, $F_{\text{A}} = F_{\text{C}} = F_{\text{AC}}$.



Analyze the forces on the pin at point A. The components found above are forces of the pin on the strut, so we put in the opposite forces, which are the forces of the strut on the pin. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$\sum F_{\text{vert}} = \tfrac{3}{8}Mg - F_{\text{AC}y} - F_{\text{AB}} \sin 60° = 0 \quad \rightarrow$$

$$F_{\text{AB}} = \frac{\tfrac{3}{8}Mg - F_{\text{AC}y}}{\sin 60°} = \frac{\tfrac{3}{8}Mg - \tfrac{1}{4}Mg}{\tfrac{1}{2}\sqrt{3}} = \frac{Mg}{4\sqrt{3}} = \frac{\left(53\times10^3\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{4\sqrt{3}}$$

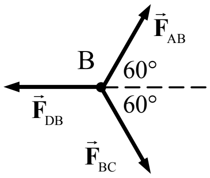
$$= 7.497\times10^4\,\text{N} \approx \boxed{7.5\times10^4\,\text{N}, \text{ in compression}}$$

$$\sum F_{\text{horiz}} = F_{\text{AC}x} - F_{\text{AB}}\cos 60^\circ = 0 \;\rightarrow$$

$$F_{\text{AC}x} = \tfrac{1}{2}F_{\text{AB}} = \frac{Mg}{8\sqrt{3}} = 3.7485\times10^4\,\text{N} \approx \boxed{3.7\times10^4\,\text{N, in tension}}$$

The actual force F_{AC} has both a tension component $F_{\text{AC}x}$ and a shearing component $F_{\text{AC}y}$. Since the problem asks for just the compressive or tension force, only $F_{\text{AC}x}$ is included in the answer.

Analyze the forces on the pin at point B. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



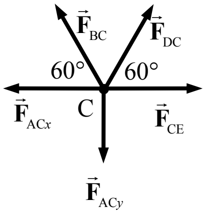
$$\sum F_{\text{vert}} = F_{\text{AB}}\sin 60^\circ - F_{\text{BC}}\sin 60^\circ = 0 \;\rightarrow$$

$$F_{\text{BC}} = F_{\text{AB}} = \frac{Mg}{4\sqrt{3}} = \boxed{7.5\times10^4\,\text{N, in tension}}$$

$$\sum F_{\text{horiz}} = F_{\text{AB}}\cos 60^\circ + F_{\text{BC}}\cos 60^\circ - F_{\text{DB}} = 0 \;\rightarrow$$

$$F_{\text{DB}} = \left(F_{\text{AB}} + F_{\text{BC}}\right)\cos 60^\circ = 2\left(\frac{Mg}{4\sqrt{3}}\right)\cos 60^\circ = \frac{Mg}{4\sqrt{3}} = \boxed{7.5\times10^4\,\text{N, in compression}}$$

Analyze the forces on the pin at point C. See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{vert}} = F_{\text{BC}}\sin 60^\circ + F_{\text{DC}}\sin 60^\circ - F_{\text{AC}y} = 0 \;\rightarrow$$

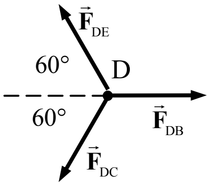
$$F_{\text{DC}} = \frac{F_{\text{AC}y}}{\sin 60^\circ} - F_{\text{BC}} = \frac{\tfrac{1}{4}Mg}{\tfrac{1}{2}\sqrt{3}} - \frac{Mg}{4\sqrt{3}} = \frac{\tfrac{1}{4}Mg}{\tfrac{1}{2}\sqrt{3}} - \frac{Mg}{4\sqrt{3}}$$

$$= \frac{Mg}{4\sqrt{3}} \approx \boxed{7.5\times10^4\,\text{N, in tension}}$$

$$\sum F_{\text{horiz}} = F_{\text{CE}} + F_{\text{DC}}\cos 60^\circ - F_{\text{BC}}\cos 60^\circ - F_{\text{AC}x} = 0 \;\rightarrow$$

$$F_{\text{CE}} = F_{\text{AC}x} + \left(F_{\text{BC}} - F_{\text{DC}}\right)\cos 60^\circ = \frac{Mg}{8\sqrt{3}} + 0 \approx \boxed{3.7\times10^4\,\text{N, in tension}}$$

Analyze the forces on the pin at point D. See the sixth free-body diagram. Write the equilibrium equation for the vertical direction.

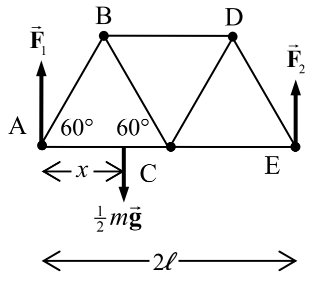


$$\sum F_{\text{vert}} = F_{\text{DE}}\sin 60^\circ - F_{\text{DC}}\sin 60^\circ = 0 \;\rightarrow$$

$$F_{\text{DE}} = F_{\text{DC}} = \frac{Mg}{4\sqrt{3}} \approx \boxed{7.5\times10^4\,\text{N, in compression}}$$

This could be checked by considering the forces on pin E.

54. See the free-body diagram from Figure 12-29. We let *m* be the mass of the truck, *x* be the distance of the truck from the left end of the bridge, and 2ℓ be the length of the bridge. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive. And we use half of the mass of the truck, because there are 2 trusses.



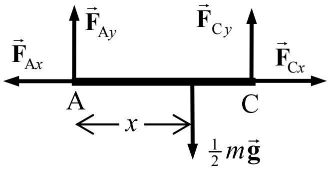
$$\sum F_{\text{vert}} = F_1 + F_2 - \tfrac{1}{2}mg = 0$$

$$\sum \tau = \tfrac{1}{2}mgx - F_2\left(2\ell\right) = 0 \;\rightarrow$$

$$F_2 = \frac{mgx}{4\ell} = \frac{\left(23000\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(22\,\text{m}\right)}{4\left(32\,\text{m}\right)} = 38740\,\text{N}$$

$$F_1 = \tfrac{1}{2}mg - F_2 = \tfrac{1}{2}(23000\,\text{kg})(9.80\,\text{m/s}^2) - 38740 = 73960\,\text{N}$$

Analyze the forces on strut AC, using the free-body diagram given in Figure 12-29b. Note that the forces at the pins are broken up into components. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions, and for torques about point A.



$$\sum F_{\text{vert}} = F_{Ay} + F_{Cy} - \tfrac{1}{2}mg = 0$$

$$\sum F_{\text{horiz}} = -F_{Ax} + F_{Cx} = 0 \;\rightarrow\; F_{Cx} = F_{Ax}$$

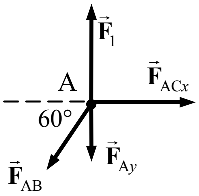
$$\sum \tau = \tfrac{1}{2}mgx - F_{Cy}(\ell) = \;\rightarrow$$

$$F_{Cy} = \tfrac{1}{2}mg\frac{x}{\ell} = \tfrac{1}{2}(23000\,\text{kg})(9.80\,\text{m/s}^2)\frac{22\,\text{m}}{32\,\text{m}} = 77480\,\text{N} \approx \boxed{77{,}000\,\text{N}}$$

$$F_{Ay} = \tfrac{1}{2}mg - F_{Cy} = \tfrac{1}{2}(23000\,\text{kg})(9.80\,\text{m/s}^2) - 77480\,\text{N} = 35220\,\text{N} \approx \boxed{35{,}000\,\text{N}}$$

Since their x components are equal, $F_{AC} = F_{CA}$ for tension or compression along the beams.

Analyze the forces on the pin at point A. The components found above are forces of the pin on the strut, so we put in the opposite forces, which are the forces of the strut on the pin. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



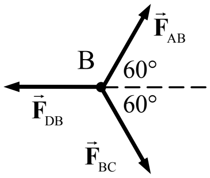
$$\sum F_{\text{vert}} = F_1 - F_{Ay} - F_{AB}\sin 60^\circ = 0 \;\rightarrow$$

$$F_{AB} = \frac{F_1 - F_{Ay}}{\sin 60^\circ} = \frac{73960\,\text{N} - 35220\,\text{N}}{\sin 60^\circ} = 44730\,\text{N} \approx \boxed{4.5\times10^4\,\text{N, in compression}}$$

$$\sum F_{\text{horiz}} = F_{ACx} - F_{AB}\cos 60^\circ = 0 \;\rightarrow$$

$$F_{ACx} = F_{AB}\cos 60^\circ = (44730\,\text{N})\cos 60^\circ = 22365\,\text{N} \approx \boxed{2.2\times10^4\,\text{N, in tension}}$$

Analyze the forces on the pin at point B. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



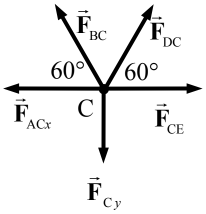
$$\sum F_{\text{vert}} = F_{AB}\sin 60^\circ - F_{BC}\sin 60^\circ = 0 \;\rightarrow$$

$$F_{BC} = F_{AB} = \boxed{4.5\times10^4\,\text{N, in tension}}$$

$$\sum F_{\text{horiz}} = F_{AB}\cos 60^\circ + F_{BC}\cos 60^\circ - F_{DB} = 0 \;\rightarrow$$

$$F_{DB} = (F_{AB} + F_{BC})\cos 60^\circ = F_{AB} = \boxed{4.5\times10^4\,\text{N, in compression}}$$

Analyze the forces on the pin at point C. See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

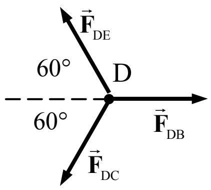


$$\sum F_{\text{vert}} = F_{BC}\sin 60^\circ + F_{DC}\sin 60^\circ - F_{Cy} = 0 \;\rightarrow$$

$$F_{DC} = \frac{F_{Cy}}{\sin 60^\circ} - F_{BC} = \frac{77480\,\text{N}}{\sin 60^\circ} - 44730\text{N} = 44740\,\text{N} \approx \boxed{4.5\times10^4\,\text{N, in tension}}$$

$$\sum F_{\text{horiz}} = F_{CE} + F_{DC}\cos 60^\circ - F_{BC}\cos 60^\circ - F_{ACx} = 0 \;\rightarrow$$

$$F_{CE} = F_{ACx} + (F_{BC} - F_{DC})\cos 60^\circ = F_{ACx} \approx \boxed{2.2\times10^4\,\text{N, in tension}}$$

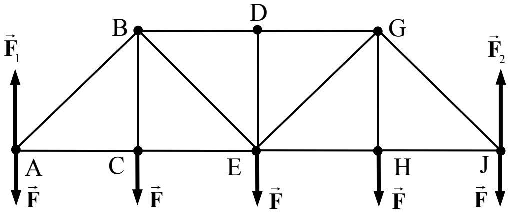


Analyze the forces on the pin at point D. See the sixth free-body diagram. Write the equilibrium equation for the vertical direction.

$$\sum F_{\text{vert}} = F_{DE}\sin 60° - F_{DC}\sin 60° = 0 \;\rightarrow$$

$$F_{DE} = F_{DC} \approx \boxed{4.5\times10^4\,\text{N, in compression}}$$

This could be checked by considering the forces on pin E.

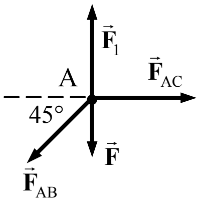


$\boxed{55.}$ We first show a free-body diagram for the entire structure. All acute angles in the structure are 45°. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.

$$\sum F_{\text{vert}} = F_1 + F_2 - 5F = 0$$

$$\sum \tau = Fa + F(2a) + F(3a) + F(4a) - F_2(4a) = 0$$

$$F_2 = F\frac{10a}{4a} = 2.5F \;\; ; \;\; F_1 = 5F - F_2 = 2.5F$$

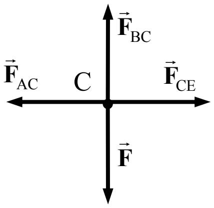


Note that the forces at the ends each support half of the load. Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$\sum F_{\text{vert}} = F_1 - F - F_{AB}\sin 45° = 0 \;\rightarrow$$

$$F_{AB} = \frac{F_1 - F}{\sin 45°} = \frac{\frac{3}{2}F}{\frac{1}{2}\sqrt{2}} = \boxed{\frac{3F}{\sqrt{2}}\text{, in compression}}$$

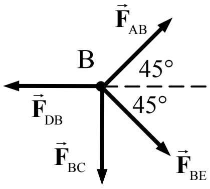
$$\sum F_{\text{horiz}} = F_{AC} - F_{AB}\cos 45° = 0 \;\rightarrow\; F_{AC} = F_{AB}\cos 45° = \frac{3F}{\sqrt{2}}\frac{\sqrt{2}}{2} = \boxed{\tfrac{3}{2}F\text{, in tension}}$$



Analyze the forces on the pin at point C. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$\sum F_{\text{vert}} = F_{BC} - F = 0 \;\rightarrow\; F_{BC} = \boxed{F\text{, tension}}$$

$$\sum F_{\text{horiz}} = F_{CE} - F_{AC} = 0 \;\rightarrow\; F_{CE} = F_{AC} = \boxed{\tfrac{3}{2}F\text{, in tension}}$$



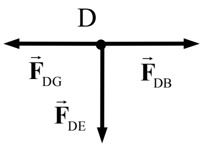
Analyze the forces on the pin at point B. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$\sum F_{\text{vert}} = F_{AB}\sin 45° - F_{BE}\sin 45° - F_{BC} = 0 \;\rightarrow$$

$$F_{BE} = F_{AB} - \frac{F_{BC}}{\sin 45°} = \frac{3F}{\sqrt{2}} - \frac{F}{\frac{1}{2}\sqrt{2}} = \boxed{\frac{F}{\sqrt{2}}\text{, tension}}$$

$$\sum F_{\text{horiz}} = F_{AB}\cos 45° + F_{BE}\cos 45° - F_{DB} = 0 \;\rightarrow$$

$$F_{DB} = \left(F_{AB} + F_{BE}\right)\cos 45° = \left(\frac{3F}{\sqrt{2}} + \frac{F}{\sqrt{2}}\right)\frac{\sqrt{2}}{2} = \boxed{2F\text{, in compression}}$$



Analyze the forces on the pin at point D. See the fifth free-body diagram. Write equilibrium equations for the vertical direction.

$$\sum F_{\text{vert}} = -F_{DE} \;\rightarrow\; F_{DE} = \boxed{0}$$

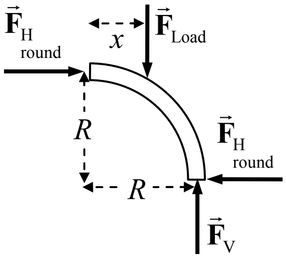
All of the other forces can be found from the equilibrium of the structure.

$$F_{\text{DG}} = F_{\text{DB}} = \boxed{2F\text{, in compression}}\ ,\ F_{\text{GE}} = F_{\text{BE}} = \boxed{\frac{F}{\sqrt{2}}\text{, tension}}\ ,$$

$$F_{\text{EH}} = F_{\text{CE}} = \boxed{\tfrac{3}{2}F\text{, in tension}}\ ,\ F_{\text{GH}} = F_{\text{BC}} = \boxed{F\text{, tension}}\ ,\ F_{\text{HJ}} = F_{\text{AC}} = \boxed{\tfrac{3}{2}F\text{, in tension}}\ ,$$

$$F_{\text{GJ}} = F_{\text{AB}} = \boxed{\frac{3F}{\sqrt{2}}\text{, in compression}}$$

56. Draw free-body diagrams similar to Figures 12-36(a) and 12-36(b) for the forces on the right half of a round arch and a pointed arch. The load force is placed at the same horizontal position on each arch. For each half-arch, take torques about the lower right hand corner, with counterclockwise as positive.

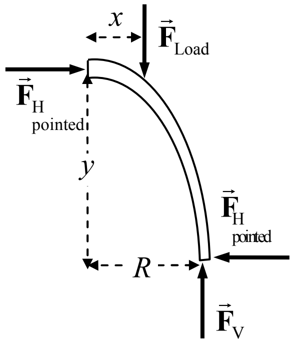


For the round arch:

$$\sum\tau = F_{\text{Load}}\left(R-x\right) - F_{\underset{\text{round}}{\text{H}}}R = 0 \ \rightarrow\ F_{\underset{\text{round}}{\text{H}}} = F_{\text{Load}}\frac{R-x}{R}$$

For the pointed arch:

$$\sum\tau = F_{\text{Load}}\left(R-x\right) - F_{\underset{\text{pointed}}{\text{H}}}y = 0 \ \rightarrow\ F_{\underset{\text{pointed}}{\text{H}}} = F_{\text{Load}}\frac{R-x}{y}$$



Solve for y, given that $F_{\underset{\text{pointed}}{\text{H}}} = \tfrac{1}{3}F_{\underset{\text{round}}{\text{H}}}$.

$$F_{\underset{\text{pointed}}{\text{H}}} = \tfrac{1}{3}F_{\underset{\text{round}}{\text{H}}} \ \rightarrow\ F_{\text{Load}}\frac{R-x}{y} = \tfrac{1}{3}F_{\text{Load}}\frac{R-x}{R} \ \rightarrow$$

$$y = 3R = 3\left(\tfrac{1}{2}8.0\text{ m}\right) = \boxed{12\text{ m}}$$

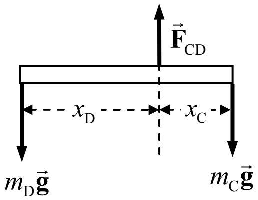
57. Each crossbar in the mobile is in equilibrium, and so the net torque about the suspension point for each crossbar must be 0. Counterclockwise torques will be taken as positive. The suspension point is used so that the tension in the suspension string need not be known initially. The net vertical force must also be 0.

The bottom bar:

$$\sum\tau = m_{\text{D}}gx_{\text{D}} - m_{\text{C}}gx_{\text{C}} = 0 \ \rightarrow$$

$$m_{\text{C}} = m_{\text{D}}\frac{x_{\text{D}}}{x_{\text{C}}} = m_{\text{D}}\frac{17.50\,\text{cm}}{5.00\,\text{cm}} = 3.50m_{\text{D}}$$

$$\sum F_y = F_{\text{CD}} - m_{\text{C}}g - m_{\text{D}}g = 0 \ \rightarrow\ F_{\text{CD}} = \left(m_{\text{C}} + m_{\text{D}}\right)g = 4.50m_{\text{D}}g$$



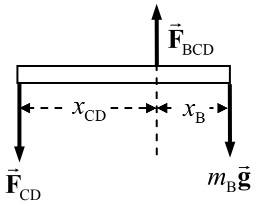
The middle bar:

$$\sum\tau = F_{\text{CD}}x_{\text{CD}} - m_{\text{B}}gx_{\text{B}} = 0 \ \rightarrow\ F_{\text{CD}} = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}} \ \rightarrow\ 4.50m_{\text{D}}g = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}}$$

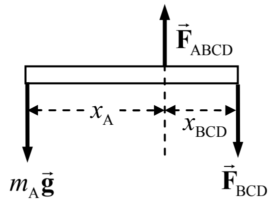
$$m_{\text{D}} = \frac{m_{\text{B}}}{4.50}\frac{x_{\text{B}}}{x_{\text{CD}}} = \frac{\left(0.748\,\text{kg}\right)\left(5.00\,\text{cm}\right)}{\left(4.50\right)\left(15.00\,\text{cm}\right)} = 0.05541 \approx \boxed{5.54\times10^{-2}\,\text{kg}}$$

$$m_{\text{C}} = 3.50m_{\text{D}} = \left(3.50\right)\left(0.05541\,\text{kg}\right) = \boxed{0.194\,\text{kg}}$$

$$\sum F_y = F_{\text{BCD}} - F_{\text{CD}}c - m_{\text{B}}g = 0 \ \rightarrow\ F_{\text{BCD}} = F_{\text{CD}} + m_{\text{B}}g = \left(4.50m_{\text{D}} + m_{\text{B}}\right)g$$



The top bar:

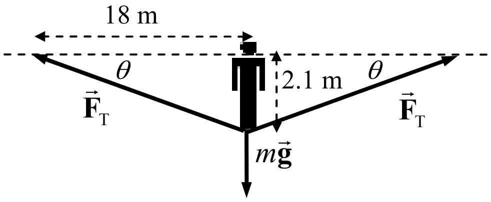


$$\sum \tau = m_A g x_A - F_{BCD} x_{BCD} = 0 \rightarrow$$

$$m_A = \frac{(4.50 m_D + m_B) g x_{BCD}}{g x_A} = (4.50 m_D + m_B)\frac{x_{BCD}}{x_A}$$

$$= [(4.50)(0.05541\text{ kg}) + 0.748\text{ kg}]\frac{7.50\text{ cm}}{30.00\text{ cm}} = \boxed{0.249\text{ kg}}$$

58. From the free-body diagram (not to scale), write the force equilibrium condition for the vertical direction.



$$\sum F_y = 2F_T \sin\theta - mg = 0$$

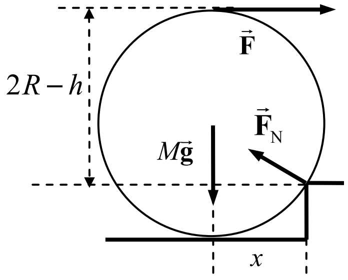
$$F_T = \frac{mg}{2\sin\theta} \approx \frac{mg}{2\tan\theta} = \frac{(60.0\text{ kg})(9.80\text{ m/s}^2)}{2\left(\frac{2.1\text{ m}}{18\text{ m}}\right)}$$

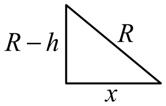
$$= \boxed{2500\text{ N}}$$

Note that the angle is small enough (about 7°) that we have made the substitution of $\sin\theta \approx \tan\theta$.

$\boxed{\text{It is not possible to increase the tension so that there is no sag.}}$ There must always be a vertical component of the tension to balance the gravity force. The larger the tension gets, the smaller the sag angle will be, however.

59. (*a*) If the wheel is just lifted off the lowest level, then the only forces on the wheel are the horizontal pull, its weight, and the contact force $\vec{F}_N$ at the corner. Take torques about the corner point, for the wheel just barely off the ground, being held in equilibrium. The contact force at the corner exerts no torque and so does not enter the calculation. The pulling force has a lever arm of $R + R - h = 2R - h$, and gravity has a lever arm of x, found from the triangle shown.



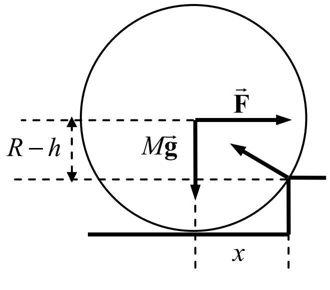


$$x = \sqrt{R^2 - (R-h)^2} = \sqrt{h(2R-h)}$$

$$\sum \tau = Mgx - F(2R-h) = 0 \rightarrow$$

$$F = \frac{Mgx}{2R-h} = Mg\frac{\sqrt{h(2R-h)}}{2R-h} = \boxed{Mg\sqrt{\frac{h}{2R-h}}}$$

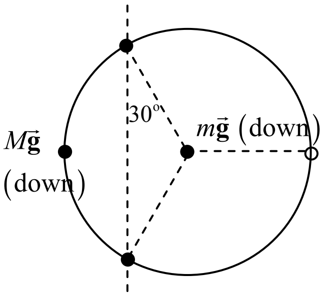
. (*b*) The only difference is that now the pulling force has a lever arm of $R - h$.

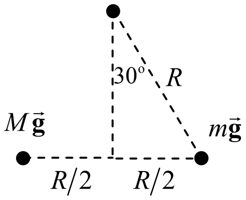


$$\sum \tau = Mgx - F(R-h) = 0 \rightarrow$$

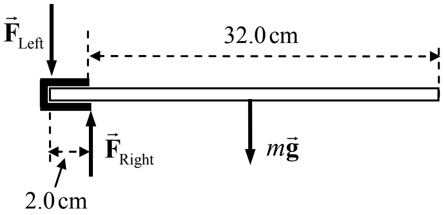
$$F = \frac{Mgx}{R-h} = \boxed{Mg\frac{\sqrt{h(2R-h)}}{R-h}}$$

60. The mass is to be placed symmetrically between two legs of the table. When enough mass is added, the table will rise up off of the third leg, and then the normal force on the table will all be on just two legs. Since the table legs are equally spaced, the angle marked in the diagram is 30°. Take torques about a line connecting the two legs that remain on the floor, so that the normal forces cause no torque. It is seen from the second diagram (a portion of the first diagram but enlarged) that the two forces are equidistant from the line joining the two legs on the floor. Since the lever arms are equal, then the torques will be equal if the forces are equal. Thus, to be in equilibrium, the two forces must be the same. If the force on the edge of the table is any bigger than the weight of the table, it will tip. Thus $\boxed{M > 28\,\text{kg}}$ will cause the table to tip.





61. (*a*) The weight of the shelf exerts a downward force and a clockwise torque about the point where the shelf touches the wall. Thus there must be an upward force and a counterclockwise torque exerted by the slot for the shelf to be in equilibrium. Since any force exerted by the slot will have a short lever arm relative to the point where the shelf touches the wall, the upward force must be larger than the gravity force. Accordingly, there then must be a downward force exerted by the slot at its left edge, exerting no torque, but balancing the vertical forces.



(*b*) Calculate the values of the three forces by first taking torques about the left end of the shelf, with the net torque being zero, and then sum the vertical forces, with the sum being zero.

$$\sum\tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) - mg\left(17.0\times10^{-2}\,\text{m}\right) = 0 \quad \rightarrow$$

$$F_{\text{Right}} = (6.6\,\text{kg})\left(9.80\,\text{m/s}^2\right)\left(\frac{17.0\times10^{-2}\,\text{m}}{2.0\times10^{-2}\,\text{m}}\right) = 549.8\,\text{N} \approx \boxed{550\,\text{N}}$$

$$\sum F_y = F_{\text{Right}} - F_{\text{Left}} - mg \quad \rightarrow$$

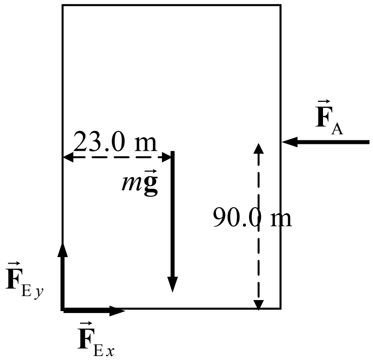
$$F_{\text{Left}} = F_{\text{Right}} - mg = 549.8\,\text{N} - (6.6\,\text{kg})\left(9.80\,\text{m/s}^2\right) = \boxed{490\,\text{N}}$$

$$mg = (6.6\,\text{kg})\left(9.80\,\text{m/s}^2\right) = \boxed{65\,\text{N}}$$

(*c*) The torque exerted by the support about the left end of the rod is

$$\tau = F_{\text{Right}}\left(2.0\times10^{-2}\,\text{m}\right) = (549.8\,\text{N})\left(2.0\times10^{-2}\,\text{m}\right) = \boxed{11\,\text{m}\bullet\text{N}}$$

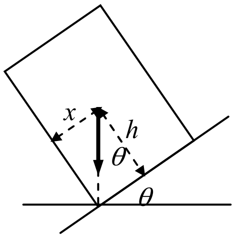
62. Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.



$$\begin{aligned}\sum\tau &= F_A(90.0\text{ m}) - mg(23.0\text{ m})\\ &= \left[\left(950\,\text{N/m}^2\right)(180.0\text{ m})(76.0\text{ m})\right](90.0\text{ m}) - \left(1.8\times10^7\text{kg}\right)\left(9.80\,\text{m/s}^2\right)(23.0\text{ m})\\ &= -2.9\times10^9\,\text{m}\bullet\text{N}\end{aligned}$$

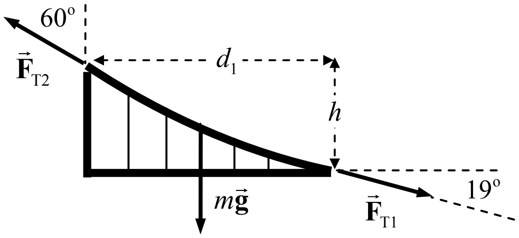
Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus $\boxed{\text{the building will not topple}}$.

63. The truck will not tip as long as a vertical line down from the CG is between the wheels. When that vertical line is at the wheel, it is in unstable equilibrium and will tip if the road is inclined any more. See the diagram for the truck at the tipping angle, showing the truck's weight vector.



$$\tan\theta = \frac{x}{h} \quad\rightarrow\quad \theta = \tan^{-1}\frac{x}{h} = \tan^{-1}\frac{1.2\text{ m}}{2.2\text{ m}} = \boxed{29^\circ}$$

64. Draw a force diagram for the cable that is supporting the right-hand section. The forces will be the tension at the left end, $\vec{\mathbf{F}}_{T2}$, the tension at the right end, $\vec{\mathbf{F}}_{T1}$, and the weight of the section, $m\vec{\mathbf{g}}$. The weight acts at the midpoint of the horizontal span of the cable. The system is in equilibrium. Write Newton's second law in both the x and y directions to find the tensions.



$$\sum F_x = F_{T1}\cos19^\circ - F_{T2}\sin60^\circ = 0 \quad\rightarrow$$

$$F_{T2} = F_{T1}\frac{\cos19^\circ}{\sin60^\circ}$$

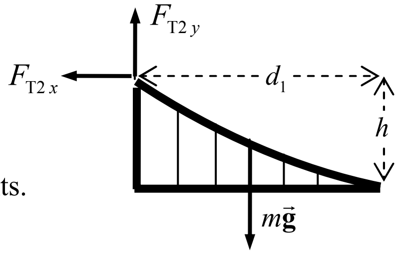
$$\sum F_y = F_{T2}\cos60^\circ - F_{T1}\sin19^\circ - mg = 0 \quad\rightarrow$$

$$F_{T1} = \frac{F_{T2}\cos60^\circ - mg}{\sin19^\circ} = \frac{F_{T1}\dfrac{\cos19^\circ}{\sin60^\circ}\cos60^\circ - mg}{\sin19^\circ} \quad\rightarrow$$

$$F_{T1} = mg\frac{\sin60^\circ}{\left(\cos19^\circ\cos60^\circ - \sin19^\circ\sin60^\circ\right)} = 4.539\,mg \approx \boxed{4.5\,mg}$$

$$F_{T2} = F_{T1}\frac{\cos19^\circ}{\sin60^\circ} = 4.539\frac{\cos19^\circ}{\sin60^\circ}mg = 4.956\,mg \approx \boxed{5.0\,mg}$$

To find the height of the tower, take torques about the point where the roadway meets the ground, at the right side of the roadway. Note that then $\vec{\mathbf{F}}_{T1}$ will exert no torque. Take counterclockwise torques as positive. For purposes of calculating the torque due to $\vec{\mathbf{F}}_{T2}$, split it into x and y components.

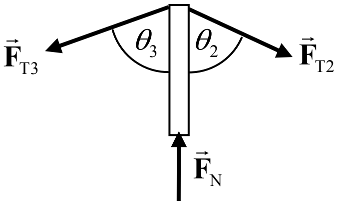


$$\sum\tau = mg\left(\tfrac{1}{2}d_1\right) + F_{T2x}h - F_{T2y}d_1 = 0 \quad\rightarrow$$

$$h = \frac{\left(F_{T2y} - \frac{1}{2}mg\right)}{F_{T2x}}d_1 = \frac{\left(F_{T2}\cos60^\circ - \frac{1}{2}mg\right)}{F_{T2}\sin60^\circ}d_1 = \frac{\left(4.956\,mg\cos60^\circ - 0.50\,mg\right)}{4.956\,mg\sin60^\circ}(343\text{ m})$$

$$= \boxed{158\text{ m}}$$

65. We consider the right half of the bridge in the diagram in the book. We divide it into two segments of length d_1 and $\frac{1}{2}d_2$, and let the mass of those two segments be *M*. Since the roadway is uniform, the mass of each segment will be in proportion to the length of the section, as follows.

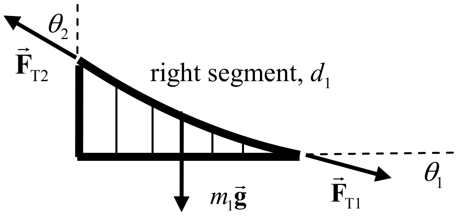


$$\frac{m_2}{m_1}=\frac{\frac{1}{2}d_2}{d_1} \rightarrow \frac{d_2}{d_1}=2\frac{m_2}{m_1}$$

The net horizontal force on the right tower is to be 0. From the force diagram for the tower, we write this.

$$F_{T3}\sin\theta_3 = F_{T2}\sin\theta_2$$

From the force diagram for each segment of the cable, write Newton's second law for both the vertical and horizontal directions.



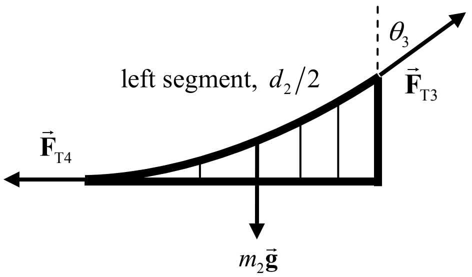
Right segment:

$$\sum F_x = F_{T1}\cos\theta_1 - F_{T2}\sin\theta_2 = 0 \rightarrow$$

$$F_{T1}\cos\theta_1 = F_{T2}\sin\theta_2$$

$$\sum F_y = F_{T2}\cos\theta_2 - F_{T1}\sin\theta_1 - m_1 g = 0 \rightarrow$$

$$m_1 g = F_{T2}\cos\theta_2 - F_{T1}\sin\theta_1$$



Left segment:

$$\sum F_x = F_{T3}\sin\theta_3 - F_{T4} = 0 \rightarrow F_{T3}\sin\theta_3 = F_{T4}$$

$$\sum F_y = F_{T3}\cos\theta_3 - m_2 g = 0 \rightarrow$$

$$m_2 g = F_{T3}\cos\theta_3$$

We manipulate the relationships to solve for the ratio of the masses, which will give the ratio of the lengths.

$$F_{T1}\cos\theta_1 = F_{T2}\sin\theta_2 \rightarrow F_{T1} = F_{T2}\frac{\sin\theta_2}{\cos\theta_1}$$

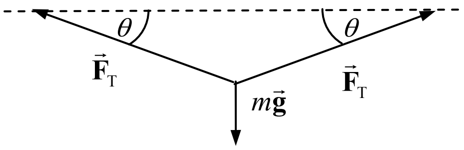
$$m_1 g = F_{T2}\cos\theta_2 - F_{T1}\sin\theta_1 = F_{T2}\cos\theta_2 - F_{T2}\frac{\sin\theta_2}{\cos\theta_1}\sin\theta_1 = F_{T2}\left(\cos\theta_2 - \frac{\sin\theta_2}{\cos\theta_1}\sin\theta_1\right)$$

$$F_{T3}\sin\theta_3 = F_{T2}\sin\theta_2 \rightarrow F_{T3} = F_{T2}\frac{\sin\theta_2}{\sin\theta_3} \rightarrow m_2 g = F_{T3}\cos\theta_3 = F_{T2}\frac{\sin\theta_2}{\sin\theta_3}\cos\theta_3$$

$$\frac{d_2}{d_1} = 2\frac{m_2}{m_1} = 2\frac{m_2 g}{m_1 g} = \frac{2F_{T2}\frac{\sin\theta_2}{\sin\theta_3}\cos\theta_3}{F_{T2}\left(\cos\theta_2 - \frac{\sin\theta_2}{\cos\theta_1}\sin\theta_1\right)} = \frac{2\sin\theta_2\cos\theta_3\cos\theta_1}{\left(\cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1\right)\sin\theta_3}$$

$$= \frac{2\sin\theta_2\cos\theta_1}{\cos\left(\theta_1+\theta_2\right)\tan\theta_3} = \frac{2\sin 60^\circ\cos 19^\circ}{\cos 79^\circ\tan 66^\circ} = 3.821 \approx \boxed{3.8}$$

66. The radius of the wire can be determined from the relationship between stress and strain, expressed by Eq. 12-5.

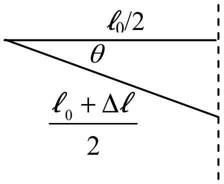


$$\frac{F}{A} = E\frac{\Delta\ell}{\ell_0} \rightarrow A = \frac{F\ell_0}{E\Delta\ell} = \pi r^2 \rightarrow r = \sqrt{\frac{1}{\pi}\frac{F}{E}\frac{\ell_0}{\Delta\ell}}$$

Use the free-body diagram for the point of connection of the mass to the wire to determine the tension force in the wire.

$$\sum F_y = 2F_\text{T}\sin\theta - mg = 0 \quad\rightarrow\quad F_\text{T} = \frac{mg}{2\sin\theta} = \frac{(25\text{ kg})(9.80\,\text{m/s}^2)}{2\sin 12^\circ} = 589.2\text{ N}$$

The fractional change in the length of the wire can be found from the geometry of the problem.

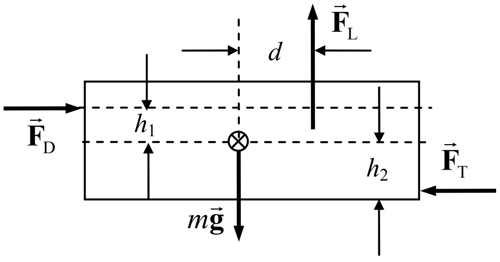


$$\cos\theta = \frac{\ell_0/2}{\frac{\ell_0+\Delta\ell}{2}} \quad\rightarrow\quad \frac{\Delta\ell}{\ell_0} = \frac{1}{\cos\theta} - 1 = \frac{1}{\cos 12^\circ} - 1 = 2.234\times10^{-2}$$

Thus the radius is

$$r = \sqrt{\frac{1}{\pi}\frac{F_\text{T}}{E}\frac{\ell_0}{\Delta\ell}} = \sqrt{\frac{1}{\pi}\frac{589.2\text{ N}}{70\times10^9\text{ N/m}^2}\frac{1}{\left(2.234\times10^{-2}\right)}} = \boxed{3.5\times10^{-4}\,\text{m}}$$

67. The airplane is in equilibrium, and so the net force in each direction and the net torque are all equal to zero. First write Newton's second law for both the horizontal and vertical directions, to find the values of the forces.



$$\sum F_x = F_D - F_T = 0 \quad\rightarrow\quad F_D = F_T = \boxed{5.0\times10^5\text{N}}$$

$$\sum F_y = F_L - mg = 0$$

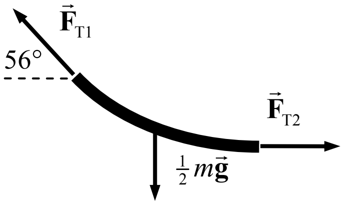
$$F_L = mg = \left(7.7\times10^4\text{kg}\right)\left(9.80\,\text{m/s}^2\right) = 7.546\times10^5\text{N}$$

Calculate the torques about the CM, calling counterclockwise torques positive.

$$\sum\tau = F_L d - F_D h_1 - F_T h_2 = 0$$

$$h_1 = \frac{F_L d - F_T h_2}{F_D} = \frac{\left(7.546\times10^5\text{N}\right)(3.2\text{ m}) - \left(5.0\times10^5\text{N}\right)(1.6\text{ m})}{\left(5.0\times10^5\text{N}\right)} = \boxed{3.2\,\text{m}}$$

68. Draw a free-body diagram for half of the cable. Write Newton's second law for both the vertical and horizontal directions, with the net force equal to 0 in each direction.



$$\sum F_y = F_\text{T1}\sin 56^\circ - \tfrac{1}{2}mg = 0 \quad\rightarrow\quad F_\text{T1} = \tfrac{1}{2}\frac{mg}{\sin 56^\circ} = 0.603\,mg$$

$$\sum F_x = F_\text{T2} - F_\text{T1}\cos 56^\circ = 0 \quad\rightarrow$$

$$F_\text{T2} = 0.603\,mg\left(\cos 56^\circ\right) = 0.337\,mg$$

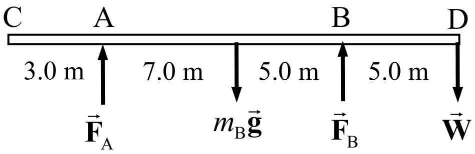
So the results are:

(*a*) $F_\text{T2} = \boxed{0.34mg}$

(*b*) $F_\text{T1} = \boxed{0.60mg}$

(*c*) The direction of the tension force is tangent to the cable at all points on the cable. Thus the direction of the tension force is $\boxed{\text{horizontal at the lowest point}}$, and is $\boxed{56^\circ\text{ above the horizontal at the attachment point}}$.

69. (*a*) For the extreme case of the beam being ready to tip, there would be no normal force at point A from the support. Use the free-body diagram to write the equation of rotational equilibrium under that condition to find the weight of the person, with $F_A = 0$. Take torques about the location of support B, and call counterclockwise torques positive. $\vec{\mathbf{W}}$ is the weight of the person.



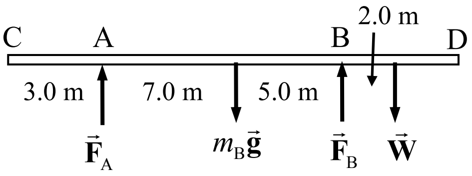
$$\sum \tau = m_B g(5.0\,\text{m}) - W(5.0\,\text{m}) = 0 \quad \rightarrow$$

$$W = m_B g = \boxed{650\,\text{N}}$$

(*b*) With the person standing at point D, we have already assumed that $\boxed{F_A = 0}$. The net force in the vertical direction must also be zero.

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W = 650\,\text{N} + 650\,\text{N} = \boxed{1300\,\text{N}}$$

(*c*) Now the person moves to a different spot, so the free-body diagram changes as shown. Again use the net torque about support B and then use the net vertical force.



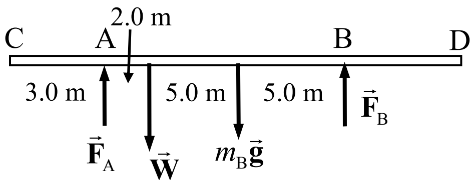
$$\sum \tau = m_B g(5.0\,\text{m}) - W(2.0\,\text{m}) - F_A(12.0\,\text{m}) = 0$$

$$F_A = \frac{m_B g(5.0\,\text{m}) - W(2.0\,\text{m})}{12.0\,\text{m}} = \frac{(650\,\text{N})(3.0\,\text{m})}{12.0\,\text{m}}$$

$$= 162.5\,\text{N} \approx \boxed{160\,\text{N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1300\ \text{N} - 160\ \text{N} = \boxed{1140\ \text{N}}$$

(*d*) Again the person moves to a different spot, so the free-body diagram changes again as shown. Again use the net torque about support B and then use the net vertical force.

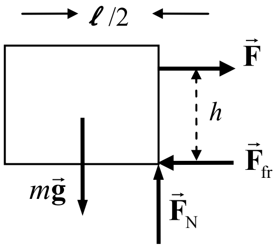


$$\sum \tau = m_B g(5.0\ \text{m}) + W(10.0\ \text{m}) - F_A(12.0\ \text{m}) = 0$$

$$F_A = \frac{m_B g(5.0\ \text{m}) + W(10.0\ \text{m})}{12.0\ \text{m}} = \frac{(650\ \text{N})(5.0\ \text{m}) + (650\ \text{N})(10.0\ \text{m})}{12.0\ \text{m}} = \boxed{810\ \text{N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1300\ \text{N} - 810\ \text{N} = \boxed{490\ \text{N}}$$

70. If the block is on the verge of tipping, the normal force will be acting at the lower right corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the *x* and *y* directions and for torque with the conditions as stated above.



$$\sum F_y = F_{\text{N}} - mg = 0 \quad \rightarrow \quad F_{\text{N}} = mg$$

$$\sum F_x = F - F_{\text{fr}} = 0 \quad \rightarrow \quad F = F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}} = \mu_{\text{s}} mg$$

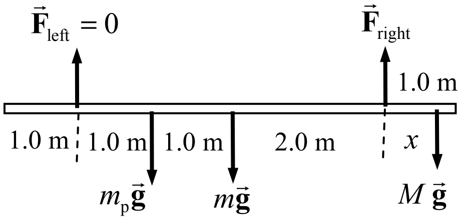
$$\sum \tau = mg\frac{\ell}{2} - Fh = 0 \quad \rightarrow \quad \frac{mg\ell}{2} = Fh = \mu_{\text{s}} mgh$$

Solve for the coefficient of friction in this limiting case, to find $\mu_s = \dfrac{\ell}{2h}$.

(*a*) If $\boxed{\mu_s < \ell/2h}$, then sliding will happen before tipping.

(*b*) If $\boxed{\mu_s > \ell/2h}$, then tipping will happen before sliding.

71. The limiting condition for the safety of the painter is the tension in the ropes. The ropes can only exert an upward tension on the scaffold. The tension will be least in the rope that is farther from the painter. The mass of the pail is m_p, the mass of the scaffold is m, and the mass of the painter is M.



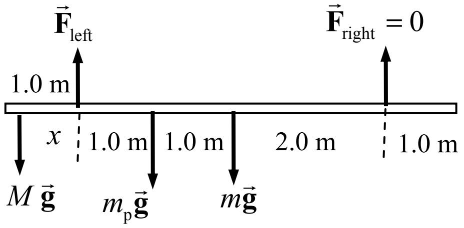
Find the distance to the right that the painter can walk before the tension in the left rope becomes zero. Take torques about the point where the right-side rope is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$\sum \tau = mg(2.0\,\text{m}) + m_p g(3.0\,\text{m}) - Mgx = 0 \quad \rightarrow$$

$$x = \frac{m(2.0\,\text{m}) + m_p(3.0\,\text{m})}{M} = \frac{(25\,\text{kg})(2.0\,\text{m}) + (4.0\,\text{kg})(3.0\,\text{m})}{65.0\,\text{kg}} = 0.9538\,\text{m} \approx 0.95\,\text{m}$$

The painter can walk to $\boxed{\text{within 5 cm of the right edge}}$ of the scaffold.

Now find the distance to the left that the painter can walk before the tension in the right rope becomes zero. Take torques about the point where the left-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

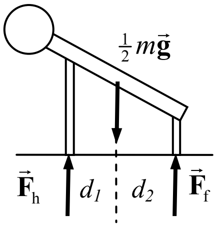


$$\sum \tau = Mgx - m_p g(1.0\,\text{m}) - mg(2.0\,\text{m}) = 0 \quad \rightarrow$$

$$x = \frac{m(2.0\,\text{m}) + m_p(1.0\,\text{m})}{M} = \frac{(25\,\text{kg})(2.0\,\text{m}) + (4.0\,\text{kg})(1.0\,\text{m})}{65.0\,\text{kg}} = 0.8308\,\text{m} \approx 0.83\,\text{m}$$

The painter can walk to $\boxed{\text{within 17 cm of the left edge}}$ of the scaffold. We found that both ends are dangerous.

72. (*a*) The man is in equilibrium, so the net force and the net torque on him must be zero. We use half of his weight, and then consider the force just on one hand and one foot, considering him to be symmetric. Take torques about the point where the foot touches the ground, with counterclockwise as positive.



$$\sum \tau = \tfrac{1}{2}mgd_2 - F_h(d_1 + d_2) = 0$$

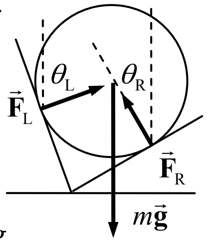
$$F_h = \frac{mgd_2}{2(d_1 + d_2)} = \frac{(68\,\text{kg})(9.80\,\text{m/s}^2)(0.95\,\text{m})}{2(1.37\,\text{m})} = 231\,\text{N} \approx \boxed{230\,\text{N}}$$

(*b*) Use Newton's second law for vertical forces to find the force on the feet.

$$\sum F_y = 2F_h + 2F_f - mg = 0$$

$$F_f = \tfrac{1}{2}mg - F_h = \tfrac{1}{2}(68\,\text{kg})(9.80\,\text{m/s}^2) - 231\,\text{N} = 103\,\text{N} \approx \boxed{100\,\text{N}}$$

The value of 100 N has 2 significant figures.



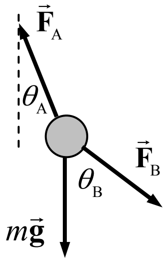
$\boxed{73.}$ The force on the sphere from each plane is a normal force, and so is perpendicular to the plane at the point of contact. Use Newton's second law in both the horizontal and vertical directions to determine the magnitudes of the forces.

$$\sum F_x = F_L \sin\theta_L - F_R \sin\theta_R = 0 \rightarrow F_R = F_L \frac{\sin\theta_L}{\sin\theta_R} = F_L \frac{\sin 67^\circ}{\sin 32^\circ}$$

$$\sum F_y = F_L \cos\theta_L + F_R \cos\theta_R - mg = 0 \rightarrow F_L \left(\cos 67^\circ + \frac{\sin 67^\circ}{\sin 32^\circ}\cos 32^\circ\right) = mg$$

$$F_L = \frac{mg}{\left(\cos 67^\circ + \frac{\sin 67^\circ}{\sin 32^\circ}\cos 32^\circ\right)} = \frac{(23\,\text{kg})(9.80\,\text{m/s}^2)}{\left(\cos 67^\circ + \frac{\sin 67^\circ}{\sin 32^\circ}\cos 32^\circ\right)} = 120.9\,\text{N} \approx \boxed{120\,\text{N}}$$

$$F_R = F_L \frac{\sin 67^\circ}{\sin 32^\circ} = (120.9\,\text{N})\frac{\sin 67^\circ}{\sin 32^\circ} = 210.0\,\text{N} \approx \boxed{210\,\text{N}}$$



74. See the free-body diagram. The ball is at rest, and so is in equilibrium. Write Newton's second law for the horizontal and vertical directions, and solve for the forces.

$$\sum F_{\text{horiz}} = F_B \sin\theta_B - F_A \sin\theta_A = 0 \rightarrow F_B = F_A \frac{\sin\theta_A}{\sin\theta_B}$$

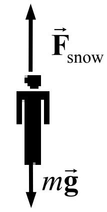
$$\sum F_{\text{vert}} = F_A \cos\theta_A - F_B \cos\theta_B - mg = 0 \rightarrow F_A \cos\theta_A = F_B \cos\theta_B + mg \rightarrow$$

$$F_A \cos\theta_A = F_A \frac{\sin\theta_A}{\sin\theta_B}\cos\theta_B + mg \rightarrow F_A\left(\cos\theta_A - \frac{\sin\theta_A}{\sin\theta_B}\cos\theta_B\right) = mg \rightarrow$$

$$F_A = mg\frac{\sin\theta_B}{(\cos\theta_A \sin\theta_B - \sin\theta_A \cos\theta_B)} = mg\frac{\sin\theta_B}{\sin(\theta_B - \theta_A)} = (15.0\,\text{kg})(9.80\,\text{m/s}^2)\frac{\sin 53^\circ}{\sin 31^\circ}$$

$$= 228\text{N} \approx \boxed{230\text{N}}$$

$$F_B = F_A \frac{\sin\theta_A}{\sin\theta_B} = (228\,\text{N})\frac{\sin 22^\circ}{\sin 53^\circ} = 107\,\text{N} \approx \boxed{110\,\text{N}}$$



75. Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2-12c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area to the strength of body tissue.

$$v^2 = v_0^2 - 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (55\,\text{m/s})^2}{2(-1.0\text{ m})} = 1513\,\text{m/s}^2$$

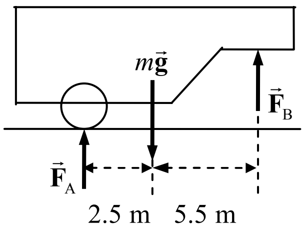
$$\frac{F}{A} = \frac{ma}{A} = \frac{(75\,\text{kg})(1513\,\text{m/s}^2)}{0.30\text{m}^2} = 3.78\times10^5\ \text{N/m}^2 < \text{ Tissue strength} = 5\times10^5\ \text{N/m}^2$$

Since $\boxed{\text{the average force on the person is less than the strength of body tissue}}$, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and so still sustain injury.

76. The mass can be calculated from the equation for the relationship between stress and strain. The force causing the strain is the weight of the mass suspended from the wire. Use Eq. 12-4.

$$\frac{\Delta\ell}{\ell_0} = \frac{1}{E}\frac{F}{A} = \frac{mg}{EA} \rightarrow m = \frac{EA}{g}\frac{\Delta\ell}{\ell_0} = \left(200\times10^9\,\mathrm{N/m^2}\right)\frac{\pi\left(1.15\times10^{-3}\,\mathrm{m}\right)^2}{\left(9.80\,\mathrm{m/s^2}\right)}\frac{0.030}{100} = \boxed{25\,\mathrm{kg}}$$

77. To find the normal force exerted on the road by the trailer tires, take the torques about point B, with counterclockwise torques as positive.



$$\sum\tau = mg\left(5.5\text{ m}\right) - F_\mathrm{A}\left(8.0\text{ m}\right) = 0 \rightarrow$$

$$F_\mathrm{A} = mg\left(\frac{5.5\,\mathrm{m}}{8.0\,\mathrm{m}}\right) = \left(2500\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(\frac{5.5\,\mathrm{m}}{8.0\,\mathrm{m}}\right) = 16{,}844\,\mathrm{N}$$

$$\approx \boxed{1.7\times10^4\,\mathrm{N}}$$

The net force in the vertical direction must be zero.

$$\sum F_y = F_\mathrm{B} + F_\mathrm{A} - mg = 0 \rightarrow$$

$$F_\mathrm{B} = mg - F_\mathrm{A} = \left(2500\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right) - 16{,}844\,\mathrm{N} = 7656\,\mathrm{N} \approx \boxed{7.7\times10^3\,\mathrm{N}}$$

78. The number of supports can be found from the compressive strength of the wood. Since the wood will be oriented longitudinally, the stress will be parallel to the grain.

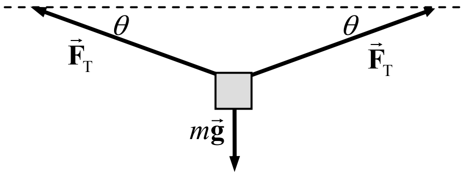
$$\frac{\text{Compressive Strength}}{\text{Safety Factor}} = \frac{\text{Load force on supports}}{\text{Area of supports}} = \frac{\text{Weight of roof}}{\left(\#\text{ supports}\right)\left(\text{area per support}\right)}$$

$$\left(\#\text{ supports}\right) = \frac{\text{Weight of roof}}{\left(\text{area per support}\right)}\frac{\text{Safety Factor}}{\text{Compressive Strength}}$$

$$= \frac{\left(1.36\times10^4\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\left(0.040\,\mathrm{m}\right)\left(0.090\,\mathrm{m}\right)}\frac{12}{\left(35\times10^6\,\mathrm{N/m^2}\right)} = 12.69\text{ supports}$$

Since there are to be more than 12 supports, and to have the same number of supports on each side, there will be 14 supports, or $\boxed{\text{7 supports on each side}}$. That means there will be 6 support-to-support spans, each of which would be given by $\text{Spacing} = \frac{10.0\text{ m}}{6\text{ gaps}} = \boxed{1.66\,\mathrm{m/gap}}$.

$\boxed{79.}$ The tension in the string when it breaks is found from the ultimate strength of nylon under tension, from Table 12-2.

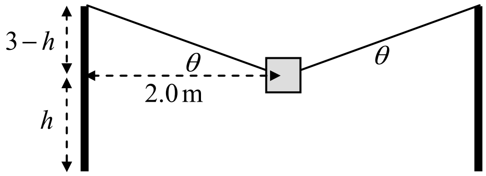


$$\frac{F_\mathrm{T}}{A} = \text{Tensile Strength} \rightarrow$$

$$F_\mathrm{T} = A\left(\text{Tensile Strength}\right)$$

$$= \pi\left[\tfrac{1}{2}\left(1.15\times10^{-3}\,\mathrm{m}\right)\right]^2\left(500\times10^6\,\mathrm{N/m^2}\right) = 519.3\,\mathrm{N}$$

From the force diagram for the box, we calculate the angle of the rope relative to the horizontal from Newton's second law in the vertical direction. Note that since the tension is the same throughout the string, the angles must be the same so that the object does not accelerate horizontally.



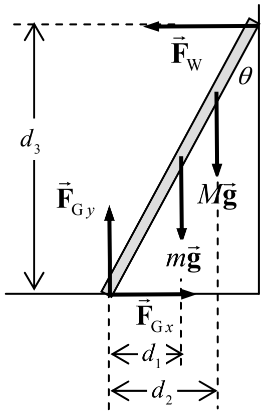
$$\sum F_y = 2F_{\mathrm{T}}\sin\theta - mg = 0 \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{mg}{2F_{\mathrm{T}}} = \sin^{-1}\frac{(25\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{2(519.3\,\mathrm{N})} = 13.64^\circ$$

To find the height above the ground, consider the second diagram.

$$\tan\theta = \frac{3.00\,\mathrm{m} - h}{2.00\,\mathrm{m}} \quad \rightarrow \quad h = 3.00\,\mathrm{m} - 2.00\,\mathrm{m}(\tan\theta) = 3.00\,\mathrm{m} - 2.00\,\mathrm{m}(\tan 13.64^\circ) = \boxed{2.5\,\mathrm{m}}$$

80. See the free-body diagram. Assume that the ladder is just ready to slip, so the force of static friction is $F_{\mathrm{fr}} = \mu F_{\mathrm{N}}$. The ladder is of length ℓ, and so $d_1 = \frac{1}{2}\ell\sin\theta$, $d_2 = \frac{3}{4}\ell\sin\theta$, and $d_3 = \ell\cos\theta$. The ladder is in equilibrium, so the net vertical and horizontal forces are 0, and the net torque is 0. We express those three equilibrium conditions, along with the friction condition. Take torques about the point where the ladder rests on the ground, calling clockwise torques positive.



$$\sum F_{\mathrm{vert}} = F_{\mathrm{G}y} - mg - Mg = 0 \quad \rightarrow \quad F_{\mathrm{G}y} = (m+M)g$$

$$\sum F_{\mathrm{horiz}} = F_{\mathrm{G}x} - F_{\mathrm{W}} = 0 \quad \rightarrow \quad F_{\mathrm{G}x} = F_{\mathrm{W}}$$

$$\sum \tau = mgd_1 + Mgd_2 - F_{\mathrm{W}}d_3 = 0 \quad \rightarrow \quad F_{\mathrm{W}} = \frac{mgd_1 + Mgd_2}{d_3}$$

$$F_{\mathrm{fr}} = \mu F_{\mathrm{N}} \quad \rightarrow \quad F_{\mathrm{G}x} = \mu F_{\mathrm{G}y}$$

These four equations may be solved for the coefficient of friction.

$$\mu = \frac{F_{\mathrm{G}x}}{F_{\mathrm{G}y}} = \frac{F_{\mathrm{W}}}{(m+M)g} = \frac{\dfrac{mgd_1 + Mgd_2}{d_3}}{(m+M)g} = \frac{md_1 + Md_2}{d_3(m+M)} = \frac{m\left(\frac{1}{2}\ell\sin\theta\right) + M\left(\frac{3}{4}\ell\sin\theta\right)}{(\ell\cos\theta)(m+M)}$$

$$= \frac{\left(\frac{1}{2}m + \frac{3}{4}M\right)\tan\theta}{(m+M)} = \frac{\left[\frac{1}{2}(16.0\,\mathrm{kg}) + \frac{3}{4}(76.0\,\mathrm{kg})\right]\tan 20.0^\circ}{(92.0\,\mathrm{kg})} = \boxed{0.257}$$

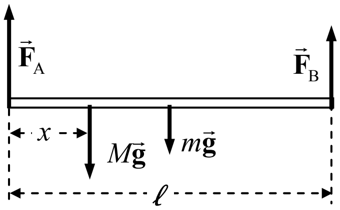
81. The maximum compressive force in a column will occur at the bottom. The bottom layer supports the entire weight of the column, and so the compressive force on that layer is mg. For the column to be on the verge of buckling, the weight divided by the area of the column will be the compressive strength of the material. The mass of the column is its volume (area x height) times its density.

$$\frac{mg}{A} = \text{Compressive Strength} = \frac{hA\rho g}{A} \quad \rightarrow \quad h = \frac{\text{Compressive Strength}}{\rho g}$$

Note that the area of the column cancels out of the expression, and so the height does not depend on the cross-sectional area of the column.

(*a*) $$h_{\mathrm{steel}} = \frac{\text{Compressive Strength}}{\rho g} = \frac{500\times10^6\,\mathrm{N/m^2}}{(7.8\times10^3\,\mathrm{kg/m^3})(9.80\,\mathrm{m/s^2})} = \boxed{6500\,\mathrm{m}}$$

(*b*) $$h_{\mathrm{granite}} = \frac{\text{Compressive Strength}}{\rho g} = \frac{170\times10^6\,\mathrm{N/m^2}}{(2.7\times10^3\,\mathrm{kg/m^3})(9.80\,\mathrm{m/s^2})} = \boxed{6400\,\mathrm{m}}$$



82. See the free–body diagram. Let M represent the mass of the train, and m represent the mass of the bridge. Write the equilibrium conditions for torques, taken about the left end, and for vertical forces. These two equations can be solved for the forces. Take counterclockwise torques as positive. Note that the position of the train is given by $x = vt$.

$$\sum \tau = Mgx + mg\left(\tfrac{1}{2}\ell\right) - F_B\ell = 0 \ \rightarrow$$

$$F_B = \left(Mg\frac{x}{\ell} + \tfrac{1}{2}mg\right) = \left(\frac{Mgv}{\ell}t + \tfrac{1}{2}mg\right)$$

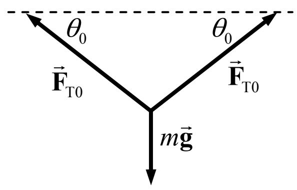
$$= \left(\frac{(95000\,\text{kg})(9.80\,\text{m/s}^2)\left((80.0\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right)}{280\,\text{m}}t + \tfrac{1}{2}(23000\,\text{kg})(9.80\,\text{m/s}^2)\right)$$

$$= \left(7.388\times10^4\,\text{N/s}\right)t + 1.127\times10^5\,\text{N} \approx \boxed{\left(7.4\times10^4\,\text{N/s}\right)t + 1.1\times10^5\,\text{N}}$$

$$\sum F_{\text{vert}} = F_A + F_B - Mg - mg = 0 \ \rightarrow$$

$$F_A = (M+m)g - F_B = \left(1.18\times10^5\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right) - \left[\left(7.388\times10^4\,\text{N/s}\right)t + 1.127\times10^5\,\text{N}\right]$$

$$= -\left(7.388\times10^4\,\text{N/s}\right)t + 1.044\times10^6\,\text{N} \approx \boxed{-\left(7.4\times10^4\,\text{N/s}\right)t + 1.0\times10^6\,\text{N}}$$



83. Since the backpack is midway between the two trees, the angles in the free-body diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the original downward vertical force.

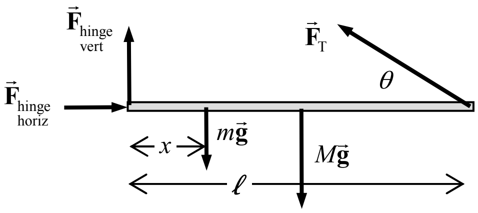
$$\sum F_y = 2F_{T0}\sin\theta_0 - mg = 0 \ \rightarrow \ F_{T0} = \frac{mg}{2\sin\theta_0}$$

Now assume the bear pulls down with an additional force, F_{bear}. The force equation would be modified as follows.

$$\sum F_y = 2F_{\text{T final}}\sin\theta_{\text{final}} - mg - F_{\text{bear}} = 0 \ \rightarrow$$

$$F_{\text{bear}} = 2F_{\text{T final}}\sin\theta_{\text{final}} - mg = 2\left(2F_{T0}\right)\sin\theta_{\text{final}} - mg = 4\left(\frac{mg}{2\sin\theta_0}\right)\sin\theta_{\text{final}} - mg$$

$$= mg\left(\frac{2\sin\theta_{\text{final}}}{\sin\theta_0} - 1\right) = (23.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)\left(\frac{2\sin 27^\circ}{\sin 15^\circ} - 1\right) = 565.3\,\text{N} \approx \boxed{570\,\text{N}}$$



84. (*a*) See the free-body diagram. To find the tension in the wire, take torques about the left edge of the beam, with counterclockwise as positive. The net torque must be 0 for the beam to be in equilibrium.

$$\sum \tau = mgx + Mg\left(\tfrac{1}{2}\ell\right) - F_T\sin\theta\ell = 0 \ \rightarrow$$

$$F_T = \boxed{\frac{g(2mx + M\ell)}{2\ell\sin\theta}} = \frac{mg}{\ell\sin\theta}x + \frac{Mg}{2\sin\theta}$$

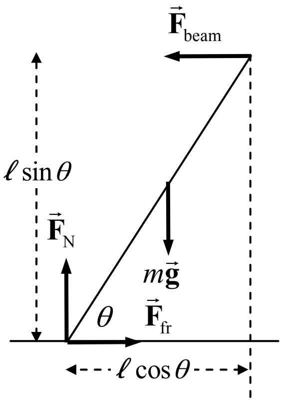
We see that the tension force is linear in x.

(*b*) Write the equilibrium condition for vertical and horizontal forces.

$$\sum F_x = F_{\substack{\text{hinge}\\ \text{horiz}}} - F_\text{T}\cos\theta = 0 \ \rightarrow \ F_{\substack{\text{hinge}\\ \text{horiz}}} = F_\text{T}\cos\theta = \frac{g\left(2mx+M\ell\right)}{2\ell\sin\theta}\cos\theta = \boxed{\frac{g\left(2mx+M\ell\right)}{2\ell\tan\theta}}$$

$$\sum F_y = F_{\substack{\text{hinge}\\ \text{vert}}} + F_\text{T}\sin\theta - \left(m+M\right)g = 0 \ \rightarrow$$

$$F_{\substack{\text{hinge}\\ \text{vert}}} = \left(m+M\right)g - F_\text{T}\sin\theta = \left(m+M\right)g - \frac{g\left(2mx+M\ell\right)}{2\ell\sin\theta}\sin\theta = \boxed{mg\left(1-\frac{x}{\ell}\right)+\tfrac{1}{2}Mg}$$



$\boxed{85.}$ Draw a free-body diagram for one of the beams. By Newton's third law, if the right beam pushes down on the left beam, then the left beam pushes up on the right beam. But the geometry is symmetric for the two beams, and so the beam contact force must be horizontal. For the beam to be in equilibrium, $F_\text{N} = mg$ and so $F_\text{fr} = \mu_\text{s}F_\text{N} = \mu mg$ is the maximum friction force. Take torques about the top of the beam, so that $\vec{\mathbf{F}}_\text{beam}$ exerts no torque. Let clockwise torques be positive.

$$\sum\tau = F_\text{N}\ell\cos\theta - mg\left(\tfrac{1}{2}\ell\right)\cos\theta - F_\text{fr}\ell\sin\theta = 0 \ \rightarrow$$

$$\theta = \tan^{-1}\frac{1}{2\mu_\text{s}} = \tan^{-1}\frac{1}{2\left(0.5\right)} = \boxed{45^\circ}$$

86. Take torques about the elbow joint. Let clockwise torques be positive. Since the arm is in equilibrium, the total torque will be 0.

$$\sum\tau = \left(2.0\,\text{kg}\right)g\left(0.15\,\text{m}\right) + \left(35\,\text{kg}\right)g\left(0.35\,\text{m}\right) - F_\text{max}\left(0.050\,\text{m}\right)\sin 105^\circ = 0 \ \rightarrow$$

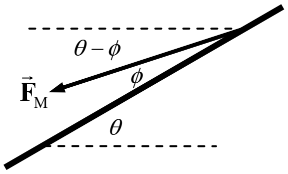
$$F_\text{max} = \frac{\left(2.0\,\text{kg}\right)g\left(0.15\,\text{m}\right) + \left(35\,\text{kg}\right)g\left(0.35\,\text{m}\right)}{\left(0.050\,\text{m}\right)\sin 105^\circ} = 2547\,\text{N} \approx \boxed{2500\,\text{N}}$$

87. (*a*) Use the free-body diagram in the textbook. To find the magnitude of $\vec{\mathbf{F}}_\text{M}$, take torques about an axis through point S and perpendicular to the paper. The upper body is in equilibrium, so the net torque must be 0. Take clockwise torques as positive.

$$\sum\tau = \left[w_\text{T}\left(0.36\,\text{m}\right) + w_\text{A}\left(0.48\,\text{m}\right) + w_\text{H}\left(0.72\,\text{m}\right)\right]\cos 30^\circ - F_\text{M}\left(0.48\text{m}\right)\sin 12^\circ = 0 \ \rightarrow$$

$$F_\text{M} = \frac{\left[w_\text{T}\left(0.36\,\text{m}\right) + w_\text{A}\left(0.48\,\text{m}\right) + w_\text{H}\left(0.72\,\text{m}\right)\right]\cos 30^\circ}{\left(0.48\text{m}\right)\sin 12^\circ}$$

$$= \frac{w\left[\left(0.46\right)\left(0.36\,\text{m}\right) + \left(0.12\right)\left(0.48\,\text{m}\right) + \left(0.07\right)\left(0.72\,\text{m}\right)\right]\cos 30^\circ}{\left(0.48\text{m}\right)\sin 12^\circ} = 2.374w \approx \boxed{2.4w}$$



(*b*) Write equilibrium conditions for the horizontal and vertical forces. Use those conditions to solve for the components of $\vec{\mathbf{F}}_\text{V}$, and then find the magnitude and direction. Note the free–body diagram for determining the components of $\vec{\mathbf{F}}_\text{M}$. The two dashed lines are parallel, and so both make an angle of θ with the heavy line representing the back.

$$\sum F_{\text{horiz}} = F_{\text{V horiz}} - F_{\text{M}}\cos(30^\circ - 12^\circ) = 0 \quad\rightarrow$$

$$F_{\text{V horiz}} = F_{\text{M}}\cos 18^\circ = (2.374w)\cos 18^\circ = 2.258w$$

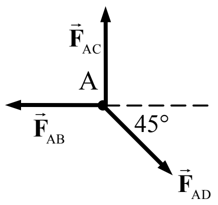
$$\sum F_{\text{vert}} = F_{\text{V vert}} - F_{\text{M}}\sin(30^\circ - 12^\circ) - w_{\text{T}} - w_{\text{A}} - w_{\text{H}} = 0 \quad\rightarrow$$

$$F_{\text{V vert}} = F_{\text{M}}\sin 18^\circ + w_{\text{T}} + w_{\text{A}} + w_{\text{H}} = (2.374w)\sin 18^\circ + 0.65w = 1.384w$$

$$F_{\text{V}} = \sqrt{F_{\text{V horiz}}^2 + F_{\text{V vert}}^2} = \sqrt{(2.258w)^2 + (1.384w)^2} = 2.648w \approx \boxed{2.6w}$$

$$\theta_{\text{V}} = \tan^{-1}\frac{F_{\text{V vert}}}{F_{\text{V horiz}}} = \tan^{-1}\frac{1.384w}{2.258w} = 31.51^\circ \approx \boxed{32^\circ \text{ above the horizontal}}$$

88. We are given that rod AB is under a compressive force F. Analyze the forces on the pin at point A. See the first free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{\text{horiz}} = F_{\text{AD}}\cos 45^\circ - F_{\text{AB}} = 0 \quad\rightarrow\quad F_{\text{AD}} = \frac{F_{\text{AB}}}{\cos 45^\circ} = \boxed{\sqrt{2}F,\ \text{in tension}}$$

$$\sum F_{\text{vert}} = F_{\text{AC}} - F_{\text{AD}}\sin 45^\circ = 0 \quad\rightarrow$$

$$F_{\text{AC}} = F_{\text{AD}}\sin 45^\circ = \sqrt{2}F\frac{\sqrt{2}}{r} = \boxed{F,\ \text{in compression}}$$

By symmetry, the other outer forces must all be the same magnitude as F_{AB}, and the other diagonal force must be the same magnitude as F_{AB}.

$$F_{\text{AC}} = F_{\text{AB}} = F_{\text{BD}} = F_{\text{CD}} = \boxed{F,\ \text{in compression}}\ ;\ F_{\text{AD}} = F_{\text{BC}} = \boxed{\sqrt{2}F,\ \text{in tension}}$$

89. (*a*) The fractional decrease in the rod's length is the strain Use Eq. 12-5. The force applied is the weight of the man.

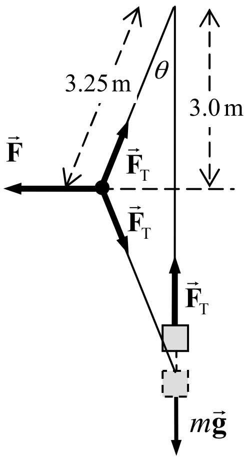
$$\frac{\Delta\ell}{\ell_0} = \frac{F}{AE} = \frac{mg}{\pi r^2 E} = \frac{(65\,\text{kg})(9.80\,\text{m/s}^2)}{\pi(0.15)^2(200\times10^9\,\text{N/m}^2)} = 4.506\times10^{-8} = \boxed{(4.5\times10^{-6})\%}$$

(*b*) The fractional change is the same for the atoms as for the macroscopic material. Let *d* represent the interatomic spacing.

$$\frac{\Delta d}{d_0} = \frac{\Delta\ell}{\ell_0} = 4.506\times10^{-8} \quad\rightarrow$$

$$\Delta d = (4.506\times10^{-8})d_0 = (4.506\times10^{-8})(2.0\times10^{-10}\,\text{m}) = \boxed{9.0\times10^{-18}\,\text{m}}$$

90. (*a*) See the free-body diagram for the system, showing forces on the engine and the forces at the point on the rope where the mechanic is pulling (the point of analysis). Let *m* represent the mass of the engine. The fact that the engine was raised a half-meter means that the part of the rope from the tree branch to the mechanic is 3.25 m, as well as the part from the mechanic to the bumper. From the free-body diagram for the engine, we know that the tension in the rope is equal to the weight of the engine. Use this, along with the equations of equilibrium at the point where the mechanic is pulling, to find the pulling force by the mechanic.



$$\text{Angle: } \theta = \cos^{-1}\frac{3.0\,\text{m}}{3.25\,\text{m}} = 22.62°$$

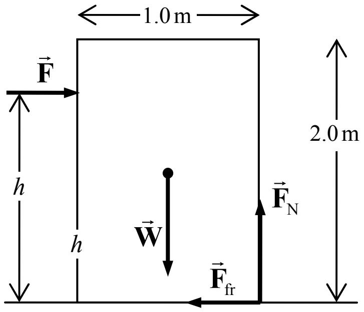
$$\text{Engine: } \sum F_y = F_\text{T} - mg = 0 \;\rightarrow\; F_\text{T} = mg$$

$$\text{Point: } \sum F_x = F - 2F_\text{T}\sin\theta = 0 \;\rightarrow$$

$$F = 2mg\sin\theta = 2(280\,\text{kg})(9.80\,\text{m/s}^2)\sin 22.62° = 2111\,\text{N} \approx \boxed{2100\,\text{N}}$$

(*b*) $$\text{Mechanical advantage} = \frac{\text{Load force}}{\text{Applied force}} = \frac{mg}{F} = \frac{(280\,\text{kg})(9.80\,\text{m/s}^2)}{2111\,\text{N}} = \boxed{1.3\,\text{N}}$$

$\boxed{91.}$ Consider the free-body diagram for the box. The box is assumed to be in equilibrium, but just on the verge of both sliding and tipping. Since it is on the verge of sliding, the static frictional force is at its maximum value. Use the equations of equilibrium. Take torques about the lower right corner where the box touches the floor, and take clockwise torques as positive. We also assume that the box is just barely tipped up on its corner, so that the forces are still parallel and perpendicular to the edges of the box.

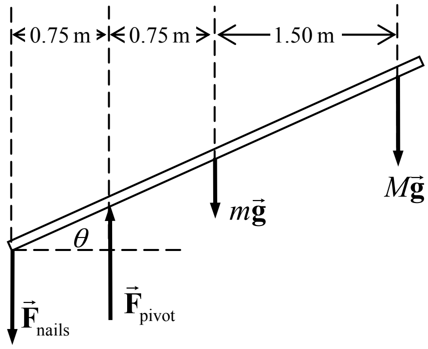


$$\sum F_y = F_\text{N} - W = 0 \;\rightarrow\; F_\text{N} = W$$

$$\sum F_x = F - F_\text{fr} = 0 \;\rightarrow\; F = F_\text{fr} = \mu W = (0.60)(250\,\text{N}) = \boxed{150\,\text{N}}$$

$$\sum \tau = Fh - W(0.5\,\text{m}) = 0 \;\rightarrow\; h = (0.5\,\text{m})\frac{W}{F} = (0.5\,\text{m})\frac{250\,\text{N}}{150\,\text{N}} = \boxed{0.83\,\text{m}}$$

92. See the free-body diagram. Take torques about the pivot point, with clockwise torques as positive. The plank is in equilibrium. Let *m* represent the mass of the plank, and *M* represent the mass of the person. The minimum nail force would occur if there was no normal force pushing up on the left end of the board.



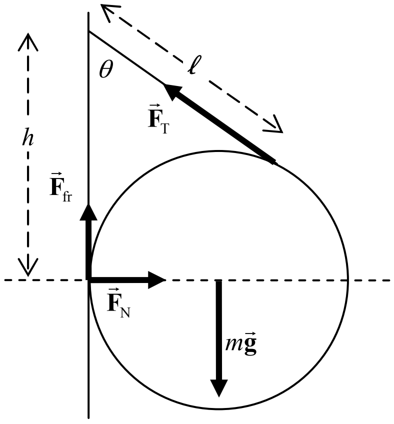
$$\sum \tau = mg(0.75\,\text{m})\cos\theta + Mg(2.25\,\text{m})\cos\theta$$

$$- F_\text{nails}(0.75\,\text{m})\cos\theta = 0 \;\rightarrow$$

$$F_\text{nails} = \frac{mg(0.75\,\text{m}) + Mg(2.25\,\text{m})}{(0.75\,\text{m})} = mg + 3Mg$$

$$= (45\text{kg} + 3(65\,\text{kg}))(9.80\,\text{m/s}^2) = 2352\,\text{N} \approx \boxed{2400\,\text{N}}$$

93. (*a*) Note that since the friction is static friction, we may NOT use $F_\text{fr} = \mu F_\text{N}$. It could be that $F_\text{fr} < \mu F_\text{N}$. So, we must determine F_fr by the equilibrium equations. Take an axis of rotation to be out of the paper, through the point of contact of the rope with the wall. Then neither F_T nor F_fr can cause any torque. The torque equilibrium equation is as follows.



$$F_\text{N}h = mgr_0 \;\rightarrow\; F_\text{N} = \frac{mgr_0}{h}$$

Take the sum of the forces in the horizontal direction.

$$F_N = F_T \sin\theta \;\rightarrow\; F_T = \frac{F_N}{\sin\theta} = \frac{mgr_0}{h\sin\theta}$$

Take the sum of the forces in the vertical direction.

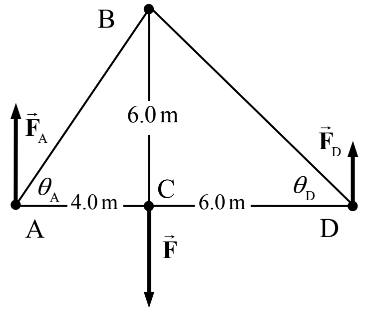
$$F_T\cos\theta + F_{fr} = mg \;\rightarrow$$

$$F_{fr} = mg - F_T\cos\theta = mg - \frac{mgr_0\cos\theta}{h\sin\theta} = \boxed{mg\left(1 - \frac{r_0}{h}\cot\theta\right)}$$

(*b*) Since the sphere is on the verge of slipping, we know that $F_{fr} = \mu F_N$.

$$F_{fr} = \mu F_N \;\rightarrow\; mg\left(1 - \frac{r_0}{h}\cot\theta\right) = \mu\frac{mgr_0}{h} \;\rightarrow\; \left(\frac{h}{r_0} - \cot\theta\right) = \mu = \boxed{\frac{h}{r_0} - \cot\theta}$$

94. There are upward forces at each support (points A and D) and a downward applied force at point C. To find the angles of members AB and BD, see the free-body diagram for the whole truss.



$$\theta_A = \tan^{-1}\frac{6.0}{4.0} = 56.3° \;;\; \theta_B = \tan^{-1}\frac{6.0}{6.0} = 45°$$

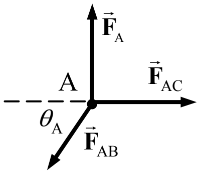
Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.

$$\sum F_{vert} = F_A + F_D - F = 0$$

$$\sum\tau = F(4.0\,\text{m}) - F_D(10.0\,\text{m}) = 0 \;\rightarrow\; F_D = F\left(\frac{4.0}{10.0}\right) = (12{,}000\,\text{N})\left(\frac{4.0}{10.0}\right) = 4800\,\text{N}$$

$$F_A = F - F_D = 12{,}000\,\text{N} - 4800\,\text{N} = 7200\,\text{N}$$

Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



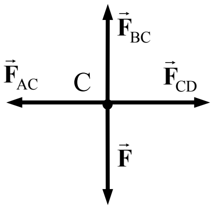
$$\sum F_{vert} = F_A - F_{AB}\sin\theta_A = 0 \;\rightarrow$$

$$F_{AB} = \frac{F_A}{\sin\theta_A} = \frac{7200\,\text{N}}{\sin 56.3°} = 8654\,\text{N} \approx \boxed{8700\,\text{N, compression}}$$

$$\sum F_{horiz} = F_{AC} - F_{AB}\cos\theta_A = 0 \;\rightarrow$$

$$F_{AC} = F_{AB}\cos\theta_A = (8654\,\text{N})\cos 56.3° = 4802\,\text{N} \approx \boxed{4800\,\text{N, tension}}$$

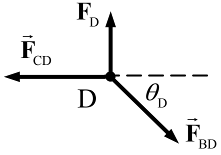
Analyze the forces on the pin at point C. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.



$$\sum F_{vert} = F_{BC} - F = 0 \;\rightarrow\; F_{BC} = F = \boxed{12{,}000\,\text{N, tension}}$$

$$\sum F_{horiz} = F_{CD} - F_{AC} = 0 \;\rightarrow\; F_{CD} = F_{AC} = \boxed{4800\,\text{N, tension}}$$

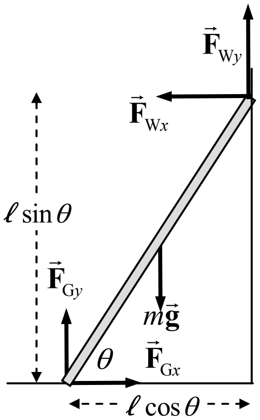
Analyze the forces on the pin at point D. See the fourth free-body diagram. Write the equilibrium equation for the horizontal direction.



$$\sum F_{vert} = F_{BD}\cos\theta_D - F_{CD} = 0 \;\rightarrow$$

$$F_{BD} = \frac{F_{CD}}{\cos\theta_D} = \frac{4800\,\text{N}}{\cos 45°} = 6788\,\text{N} \approx \boxed{6800\,\text{N, compression}}$$

95. (*a*) See the free-body diagram. We write the equilibrium conditions for horizontal and vertical forces, and for rotation. We also assume that both static frictional forces are at their maximum values. Take clockwise torques as positive. We solve for the smallest angle that makes the ladder be in equilibrium.



$$\sum F_{\text{horiz}} = F_{\mathrm{G}x} - F_{\mathrm{W}x} = 0 \;\rightarrow\; F_{\mathrm{G}x} = F_{\mathrm{W}x}$$

$$\sum F_{\text{vert}} = F_{\mathrm{G}y} + F_{\mathrm{W}y} - mg = 0 \;\rightarrow\; F_{\mathrm{G}y} + F_{\mathrm{W}y} = mg$$

$$\sum \tau = mg\left(\tfrac{1}{2}\ell\cos\theta\right) - F_{\mathrm{W}x}\ell\sin\theta - F_{\mathrm{W}y}\ell\cos\theta = 0$$

$$F_{\mathrm{G}x} = \mu_{\mathrm{G}}F_{\mathrm{G}y} \;\;;\;\; F_{\mathrm{W}y} = \mu_{\mathrm{W}}F_{\mathrm{W}x}$$

Substitute the first equation above into the fourth equation, and simplify the third equation, to give this set of equations.

$$F_{\mathrm{G}y} + F_{\mathrm{W}y} = mg \;;\; mg = 2\left(F_{\mathrm{W}x}\tan\theta + F_{\mathrm{W}y}\right) \;;\; F_{\mathrm{W}x} = \mu_{\mathrm{G}}F_{\mathrm{G}y} \;;\; F_{\mathrm{W}y} = \mu_{\mathrm{W}}F_{\mathrm{W}x}$$

Substitute the third equation into the second and fourth equations.

$$F_{\mathrm{G}y} + F_{\mathrm{W}y} = mg \;;\; mg = 2\left(\mu_{\mathrm{G}}F_{\mathrm{G}y}\tan\theta + F_{\mathrm{W}y}\right) \;;\; F_{\mathrm{W}y} = \mu_{\mathrm{W}}\mu_{\mathrm{G}}F_{\mathrm{G}y}$$

Substitute the third equation into the first two equations.

$$F_{\mathrm{G}y} + \mu_{\mathrm{W}}\mu_{\mathrm{G}}F_{\mathrm{G}y} = mg \;;\; mg = 2\left(\mu_{\mathrm{G}}F_{\mathrm{G}y}\tan\theta + \mu_{\mathrm{W}}\mu_{\mathrm{G}}F_{\mathrm{G}y}\right)$$

Now equate the two expressions for *mg*, and simplify.

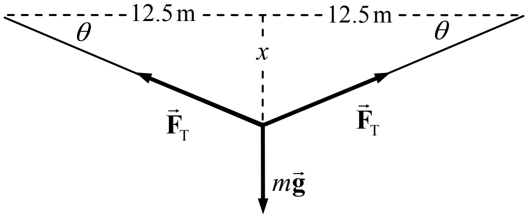
$$F_{\mathrm{G}y} + \mu_{\mathrm{W}}\mu_{\mathrm{G}}F_{\mathrm{G}y} = 2\left(\mu_{\mathrm{G}}F_{\mathrm{G}y}\tan\theta + \mu_{\mathrm{W}}\mu_{\mathrm{G}}F_{\mathrm{G}y}\right) \;\rightarrow\; \boxed{\tan\theta_{\min} = \frac{1-\mu_{\mathrm{W}}\mu_{\mathrm{G}}}{2\mu_{\mathrm{G}}}}$$

(*b*) For a frictional wall: $\theta_{\min} = \tan^{-1}\dfrac{1-\mu_{\mathrm{W}}\mu_{\mathrm{G}}}{2\mu_{\mathrm{G}}} = \tan^{-1}\dfrac{1-(0.40)^2}{2(0.40)} = 46.4^\circ \approx \boxed{46^\circ}$

For a frictionless wall: $\theta_{\min} = \tan^{-1}\dfrac{1-\mu_{\mathrm{W}}\mu_{\mathrm{G}}}{2\mu_{\mathrm{G}}} = \tan^{-1}\dfrac{1-(0)^2}{2(0.40)} = 51.3^\circ \approx \boxed{51^\circ}$

$$\%\,\text{diff} = \left(\frac{51.3^\circ - 46.4^\circ}{46.4^\circ}\right)100 = 10.6\% \approx \boxed{11\%}$$

96. (*a*) See the free-body diagram for the Tyrolean traverse technique. We analyze the point on the rope that is at the bottom of the "sag." To include the safety factor, the tension must be no more than 2900 N.



$$\sum F_{\text{vert}} = 2F_{\mathrm{T}}\sin\theta - mg = 0 \;\rightarrow$$

$$\theta_{\min} = \sin^{-1}\frac{mg}{2F_{\mathrm{T}_{\max}}} = \sin^{-1}\frac{(75\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{2(2900\,\mathrm{N})} = 7.280^\circ$$

$$\tan\theta_{\min} = \frac{x_{\min}}{12.5\,\mathrm{m}} \;\rightarrow\; x_{\min} = (12.5\,\mathrm{m})\tan(7.280^\circ) = 1.597\,\mathrm{m} \approx \boxed{1.6\,\mathrm{m}}$$

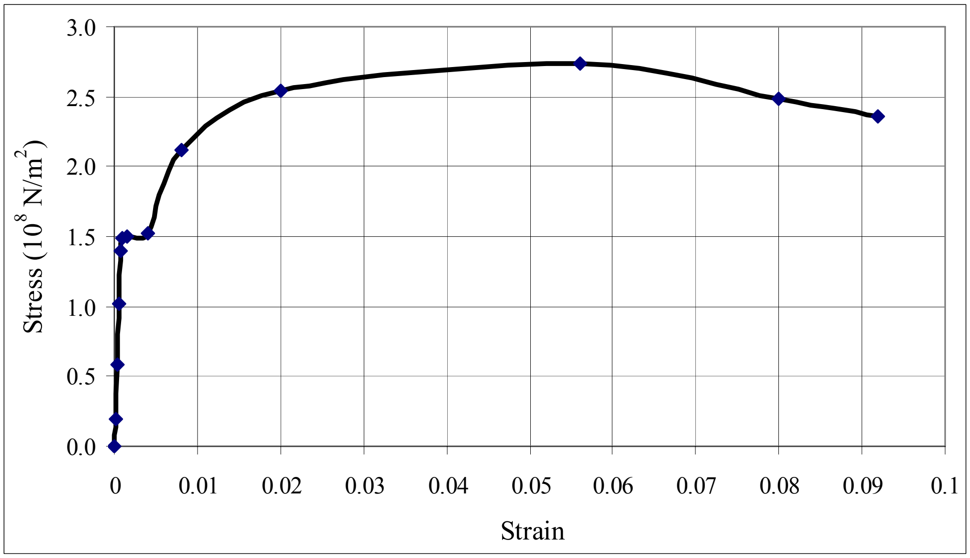
(*b*) Now the sag amount is $x = \tfrac{1}{4}x_{\min} = \tfrac{1}{4}(1.597\,\mathrm{m}) = 0.3992\,\mathrm{m}$. Use that distance to find the tension in the rope.

$$\theta = \tan^{-1}\frac{x}{12.5\,\text{m}} = \tan^{-1}\frac{0.3992\,\text{m}}{12.5\,\text{m}} = 1.829^\circ$$

$$F_\text{T} = \frac{mg}{2\sin\theta} = \frac{(75\,\text{kg})(9.80\,\text{m/s}^2)}{2\sin 1.829^\circ} = 11{,}512\,\text{N} \approx \boxed{12{,}000\,\text{N}}$$

The rope will not break, but the safety factor will only be about 4 instead of 10.

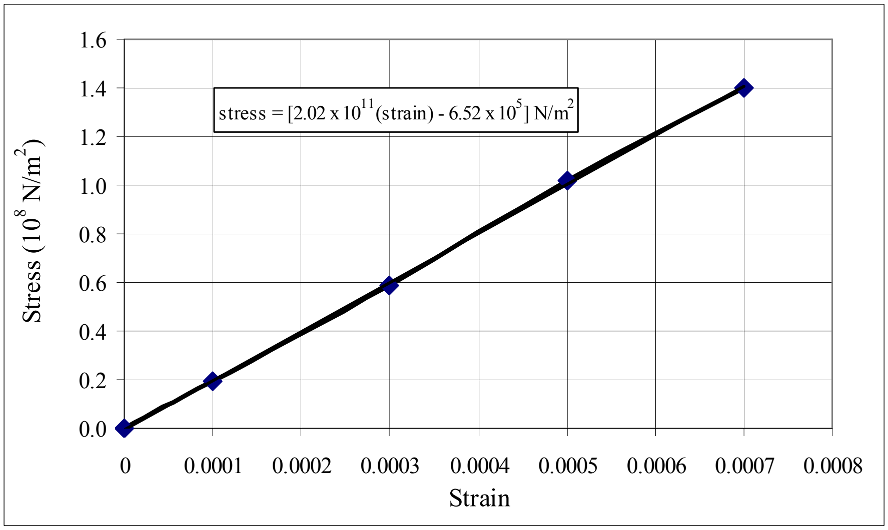
$\boxed{97.}$ (*a*) The stress is given by $\frac{F}{A}$, the applied force divided by the cross-sectional area, and the strain is given by $\frac{\Delta\ell}{\ell_0}$, the elongation over the original length.



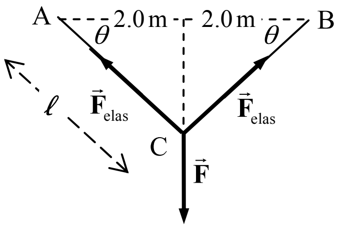
The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH12.XLS,” on tab “Problem 12.97a.”

(*b*) The elastic region is shown in the graph.

The slope of the stress vs. strain graph is the elastic modulus, and is $\boxed{2.02\times10^{11}\,\text{N/m}^2}$.



The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH12.XLS,” on tab “Problem 12.97b.”



98. See the free-body diagram. We assume that point C is not accelerating, and so the net force at point C is 0. That net force is the vector sum of applied force $\vec{\mathbf{F}}$ and two identical spring forces $\vec{\mathbf{F}}_{\text{elas}}$. The elastic forces are given by $F_{\text{elas}} = k(\text{amount of stretch})$. If the springs are unstretched for $\theta = 0$, then 2.0 m must be subtracted from the length of AC and BC to find the amount the springs have been stretched. Write Newton's second law for the vertical direction in order to obtain a relationship between F and θ. Note that $\cos\theta = \dfrac{2.0\,\text{m}}{\ell}$.

$$\sum F_{\text{vert}} = 2F_{\text{elas}}\sin\theta - F = 0 \;\rightarrow\; F = 2F_{\text{elas}}\sin\theta$$

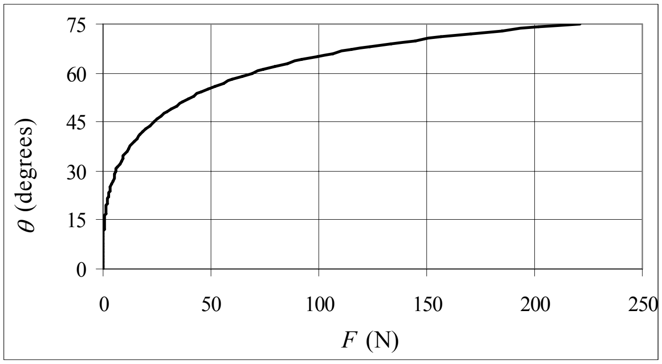
$$F_{\text{elas}} = k(\ell - 2.0\,\text{m}) = k\left(\frac{2.0\,\text{m}}{\cos\theta} - 2.0\,\text{m}\right) \;\rightarrow$$

$$F = 2F_{\text{elas}}\sin\theta = 2k\left(\frac{2.0\,\text{m}}{\cos\theta} - 2.0\,\text{m}\right)\sin\theta = 2(20.0\,\text{N/m})(2.0\,\text{m})\left(\frac{1}{\cos\theta} - 1\right)\sin\theta$$

$$= 80\,\text{N}(\tan\theta - \sin\theta)$$

This gives F as a function of θ, but we require a graph of θ as a function of F. To graph this, we calculate F for $0 \le \theta \le 75°$, and then simply interchange the axes in the graph.

The spreadsheets used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH12.XLS", on tab "Problem 12.98".



CHAPTER 13: Fluids

Responses to Questions

1. No. If one material has a higher density than another, then the molecules of the first could be heavier than those of the second, or the molecules of the first could be more closely packed together than the molecules of the second.

2. The cabin of an airplane is maintained at a pressure lower than sea-level atmospheric pressure, and the baggage compartment is not pressurized. Atmospheric pressure is lower at higher altitudes, so when an airplane flies up to a high altitude, the air pressure outside a cosmetics bottle drops, compared to the pressure inside. The higher pressure inside the bottle forces fluid to leak out around the cap.

3. In the case of the two non-cylindrical containers, perpendicular forces from the sides of the containers on the fluid will contribute to the net force on the base. For the middle container, the forces from the sides (perpendicular to the sides) will have an upward component, which helps support the water and keeps the force on the base the same as the container on the left. For the container on the right, the forces from the sides will have a downward component, increasing the force on the base so that it is the same as the container on the left.

4. The pressure is what determines whether or not your skin will be cut. You can push both the pen and the pin with the same force, but the pressure exerted by the point of the pin will be much greater than the pressure exerted by the blunt end of the pen, because the area of the pin point is much smaller.

5. As the water boils, steam displaces some of the air in the can. When the lid is put on, and the water and the can cool, the steam that is trapped in the can condenses back into liquid water. This reduces the pressure in the can to less than atmospheric pressure, and the greater force from the outside air pressure crushes the can.

6. If the cuff is held below the level of the heart, the measured pressure will be the actual blood pressure from the pumping of the heart plus the pressure due to the height of blood above the cuff. This reading will be too high. Likewise, if the cuff is held above the level of the heart, the reported pressure measurement will be too low.

7. Ice floats in water, so ice is less dense than water. When ice floats, it displaces a volume of water that is equal to the weight of the ice. Since ice is less dense than water, the volume of water displaced is smaller than the volume of the ice, and some of the ice extends above the top of the water. When the ice melts and turns back into water, it will fill a volume exactly equal to the original volume of water displaced. The water will not overflow the glass as the ice melts.

8. No. Alcohol is less dense than ice, so the ice cube would sink. In order to float, the ice cube would need to displace a weight of alcohol equal to its own weight. Since alcohol is less dense than ice, this is impossible.

9. All carbonated drinks have gas dissolved in them, which reduces their density to less than that of water. However, Coke has a significant amount of sugar dissolved in it, making its density greater than that of water, so the can of Coke sinks. Diet Coke has no sugar, leaving its density, including the can, less that the density of water. The can of Diet Coke floats.

10. In order to float, a ship must displace an amount of water with a weight equal to its own weight. An iron block would sink, because it does not have enough volume to displace an amount of water equal to its weight. However, the iron of a ship is shaped more like a bowl, so it is able to displace more water. If you were to find the average density of the ship and all its contents, including the air it holds, you would find that this density would be less than the density of water.

11. The liquid in the vertical part of the tube over the lower container will fall into the container through the action of gravity. This action reduces the pressure in the top of the tube and draws liquid through the tube, and into the tube from the upper container. As noted, the tube must be full of liquid initially for this to work.

12. Sand must be added to the barge. If sand is removed, the barge will not need to displace as much water since its weight will be less, and it will rise up in the water, making it even less likely to fit under the bridge. If sand is added, the barge will sink lower into the water, making it more likely to fit under the bridge.

13. As the weather balloon rises into the upper atmosphere, atmospheric pressure on it decreases, allowing the balloon to expand as the gas inside it expands. If the balloon were filled to maximum capacity on the ground, then the balloon fabric would burst shortly after take-off, as the balloon fabric would be unable to expand any additional amount. Filling the balloon to a minimum value on take-off allows plenty of room for expansion as the balloon rises.

14. The water level will fall in all three cases.
    - (*a*) The boat, when floating in the pool, displaces water, causing an increase in the overall level of water in the pool. Therefore, when the boat is removed, the water returns to its original (lower) level.
    - (*b*) The boat and anchor together must displace an amount of water equal to their combined weight. If the anchor is removed, this water is no longer displaced and the water level in the pool will go down.
    - (*c*) If the anchor is removed and dropped in the pool, so that it rests on the bottom of the pool, the water level will again go down, but not by as much as when the anchor is removed from the boat and pool altogether. When the anchor is in the boat, the combination must displace an amount of water equal to their weight because they are floating. When the anchor is dropped overboard, it can only displace an amount of water equal to its volume, which is less than the amount of water equal to its weight. Less water is displaced so the water level in the pool goes down.

15. No. If the balloon is inflated, then the air inside the balloon is slightly compressed by the balloon fabric, making it more dense than the outside air. The increase in the buoyant force, present because the balloon is filled with air, is more than offset by the increase in weight due to the denser air filling the balloon. The apparent weight of the filled balloon will be slightly greater than that of the empty balloon.

16. In order to float, you must displace an amount of water equal to your own weight. Salt water is more dense than fresh water, so the volume of salt water you must displace is less than the volume of fresh water. You will float higher in the salt water because you are displacing a lower volume of water.

17. The papers will move toward each other. When you blow between the sheets of paper, you reduce the air pressure between them (Bernoulli's principle). The greater air pressure on the other side of each sheet will push the sheets toward each other.

18. As the water falls, it speeds up because of the acceleration due to gravity. Because the volume flow rate must remain constant, the faster-moving water must have a smaller cross-sectional area (equation of continuity). Therefore the water farther from the faucet will have a narrower stream than the water nearer the faucet.

$\boxed{19.}$ As a high-speed train travels, it pulls some of the surrounding air with it, due to the viscosity of the air. The moving air reduces the air pressure around the train (Bernoulli's principle), which in turn creates a force toward the train from the surrounding higher air pressure. This force is large enough that it could push a light-weight child toward the train.

20. No. Both the cup and the water in it are in free fall and are accelerating downward because of gravity. There is no "extra" force on the water so it will not accelerate any faster than the cup; both will fall together and water will not flow out of the holes in the cup.

21. Taking off into the wind increases the velocity of the plane relative to the air, an important factor in the creation of lift. The plane will be able to take off with a slower ground speed, and a shorter runway distance.

22. As the ships move, they drag water with them. The moving water has a lower pressure than stationary water, as shown by Bernoulli's principle. If the ships are moving in parallel paths fairly close together, the water between them will have a lower pressure than the water to the outside of either one, since it is being dragged by both ships. The ships are in danger of colliding because the higher pressure of the water on the outsides will tend to push them towards each other.

23. Air traveling over the top of the car is moving quite fast when the car is traveling at high speed, and, due to Bernoulli's principle, will have a lower pressure than the air inside the car, which is stationary with respect to the car. The greater air pressure inside the car will cause the canvas top to bulge out.

24. The air pressure inside and outside a house is typically the same. During a hurricane or tornado, the outside air pressure may drop suddenly because of the high wind speeds, as shown by Bernoulli's principle. The greater air pressure inside the house may then push the roof off.

Solutions to Problems

$\boxed{1.}$ The mass is found from the density of granite (found in Table 13-1) and the volume of granite.

$$m = \rho V = \left(2.7\times10^3\ \mathrm{kg/m^3}\right)\left(10^8\ \mathrm{m^3}\right) = 2.7\times10^{11}\ \mathrm{kg} \approx \boxed{3\times10^{11}\ \mathrm{kg}}$$

2. The mass is found from the density of air (found in Table 13-1) and the volume of air.

$$m = \rho V = \left(1.29\ \mathrm{kg/m^3}\right)(5.6\ \mathrm{m})(3.8\ \mathrm{m})(2.8\ \mathrm{m}) = \boxed{77\ \mathrm{kg}}$$

3. The mass is found from the density of gold (found in Table 13-1) and the volume of gold.

$$m = \rho V = \left(19.3\times10^3\ \mathrm{kg/m^3}\right)(0.56\ \mathrm{m})(0.28\ \mathrm{m})(0.22\ \mathrm{m}) = \boxed{670\ \mathrm{kg}} \quad (\approx 1500\ \mathrm{lb})$$

4. Assume that your density is that of water, and that your mass is 75 kg.

$$V = \frac{m}{\rho} = \frac{75\ \mathrm{kg}}{1.00\times10^3\ \mathrm{kg/m^3}} = \boxed{7.5\times10^{-2}\ \mathrm{m^3}} = 75\ \mathrm{L}$$

5. To find the specific gravity of the fluid, take the ratio of the density of the fluid to that of water, noting that the same volume is used for both liquids.

$$SJ_{\text{fluid}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{(m/V)_{\text{fluid}}}{(m/V)_{\text{water}}} = \frac{m_{\text{fluid}}}{m_{\text{water}}} = \frac{89.22\,\text{g} - 35.00\,\text{g}}{98.44\,\text{g} - 35.00\,\text{g}} = \boxed{0.8547}$$

6. The specific gravity of the mixture is the ratio of the density of the mixture to that of water. To find the density of the mixture, the mass of antifreeze and the mass of water must be known.

$$m_{\text{antifreeze}} = \rho_{\text{antifreeze}} V_{\text{antifreeze}} = SG_{\text{antifreeze}} \rho_{\text{water}} V_{\text{antifreeze}} \qquad m_{\text{water}} = \rho_{\text{water}} V_{\text{water}}$$

$$SG_{\text{mixture}} = \frac{\rho_{\text{mixture}}}{\rho_{\text{water}}} = \frac{m_{\text{mixture}}/V_{\text{mixture}}}{\rho_{\text{water}}} = \frac{m_{\text{antifreeze}} + m_{\text{water}}}{\rho_{\text{water}} V_{\text{mixture}}} = \frac{SG_{\text{antifreeze}} \rho_{\text{water}} V_{\text{antifreeze}} + \rho_{\text{water}} V_{\text{water}}}{\rho_{\text{water}} V_{\text{mixture}}}$$

$$= \frac{SG_{\text{antifreeze}} V_{\text{antifreeze}} + V_{\text{water}}}{V_{\text{mixture}}} = \frac{(0.80)(5.0\text{ L}) + 4.0\text{ L}}{9.0\text{ L}} = \boxed{0.89}$$

$\boxed{7.}$ (*a*) The density from the three-part model is found from the total mass divided by the total volume. Let subscript 1 represent the inner core, subscript 2 represent the outer core, and subscript 3 represent the mantle. The radii are then the outer boundaries of the labeled region.

$$\rho_{\substack{\text{three}\\\text{layers}}} = \frac{m_1 + m_2 + m_3}{V_1 + V_2 + V_3} = \frac{\rho_1 m_1 + \rho_2 m_2 + \rho_3 m_3}{V_1 + V_2 + V_3} = \frac{\rho_1 \frac{4}{3}\pi r_1^3 + \rho_2 \frac{4}{3}\pi\left(r_2^3 - r_1^3\right) + \rho_3 \frac{4}{3}\pi\left(r_3^3 - r_2^3\right)}{\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi\left(r_2^3 - r_1^3\right) + \frac{4}{3}\pi\left(r_3^3 - r_2^3\right)}$$

$$= \frac{\rho_1 r_1^3 + \rho_2\left(r_2^3 - r_1^3\right) + \rho_3\left(r_3^3 - r_2^3\right)}{r_3^3} = \frac{r_1^3\left(\rho_1 - \rho_2\right) + r_2^3\left(\rho_2 - \rho_3\right) + r_3^3 \rho_3}{r_3^3}$$

$$= \frac{(1220\,\text{km})^3\left(1900\,\text{kg/m}^3\right) + (3480\,\text{km})^3\left(6700\,\text{kg/m}^3\right) + (6371\,\text{km})^3\left(4400\,\text{kg/m}^3\right)}{(6371\,\text{km})^3}$$

$$= 5505.3\,\text{kg/m}^3 \approx \boxed{5510\,\text{kg/m}^3}$$

(*b*)

$$\rho_{\substack{\text{one}\\\text{density}}} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{5.98\times10^{24}\,\text{kg}}{\frac{4}{3}\pi\left(6371\times10^3\,\text{m}\right)^3} = 5521\,\text{kg/m}^3 \approx \boxed{5520\,\text{kg/m}^3}$$

$$\%\text{diff} = 100\left(\frac{\rho_{\substack{\text{one}\\\text{density}}} - \rho_{\substack{\text{three}\\\text{layers}}}}{\rho_{\substack{\text{three}\\\text{layers}}}}\right) = 100\left(\frac{5521\,\text{kg/m}^3 - 5505\,\text{kg/m}^3}{5505\,\text{kg/m}^3}\right) = 0.2906 \approx \boxed{0.3\%}$$

8. The pressure is given by Eq. 13-3.

$$P = \rho g h = (1000)\left(9.80\,\text{m/s}^2\right)(35\,\text{m}) = \boxed{3.4\times10^5\ \text{N/m}^2} \approx 3.4\,\text{atm}$$

9. (*a*) The pressure exerted on the floor by the chair leg is caused by the chair pushing down on the floor. That downward push is the reaction to the normal force of the floor on the leg, and the normal force on one leg is assumed to be one-fourth of the weight of the chair.

$$P_{\text{chair}} = \frac{W_{\text{leg}}}{A} = \frac{\frac{1}{4}(66\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{\left(0.020\,\text{cm}^2\right)\left(\dfrac{1\,\text{m}}{100\,\text{cm}}\right)^2} = 8.085\times10^7\ \text{N/m}^2 \approx \boxed{8.1\times10^7\ \text{N/m}^2}.$$

(*b*) The pressure exerted by the elephant is found in the same way, but with ALL of the weight being used, since the elephant is standing on one foot.

$$P_{\text{elephant}}=\frac{W_{\text{elephant}}}{A}=\frac{(1300\,\text{kg})(9.80\,\text{m/s}^2)}{(800\,\text{cm}^2)\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^2}=1.59\times10^5\text{ N/m}^2\approx\boxed{2\times10^5\text{ N/m}^2}.$$

Note that the chair pressure is larger than the elephant pressure by a factor of about 400.

10. Use Eq. 13-3 to find the pressure difference. The density is found in Table 13-1.

$$P=\rho gh\ \rightarrow\ \Delta P=\rho g\Delta h=(1.05\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)(1.70\,\text{m})$$
$$=1.749\times10^4\text{ N/m}^2\left(\frac{1\text{ mm-Hg}}{133\,\text{N/m}^2}\right)=\boxed{132\,\text{mm-Hg}}$$

11. The height is found from Eq. 13-3, using normal atmospheric pressure. The density is found in Table 13-1.

$$P=\rho gh\ \rightarrow\ h=\frac{P}{\rho g}=\frac{1.013\times10^5\text{ N/m}^2}{(0.79\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)}=\boxed{13\,\text{m}}$$

That is so tall as to be impractical in many cases.

12. The pressure difference on the lungs is the pressure change from the depth of water.

$$\Delta P=\rho g\Delta h\ \rightarrow\ \Delta h=\frac{\Delta P}{\rho g}=\frac{(85\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\text{ mm-Hg}}\right)}{(1.00\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)}=1.154\,\text{m}\approx\boxed{1.2\,\text{m}}$$

$\boxed{13.}$ The force exerted by the gauge pressure will be equal to the weight of the vehicle.

$$mg=PA=P(\pi r^2)\ \rightarrow$$
$$m=\frac{P\pi r^2}{g}=\frac{(17.0\,\text{atm})\left(\dfrac{1.013\times10^5\text{ N/m}^2}{1\text{ atm}}\right)\pi\left[\tfrac{1}{2}(0.225\,\text{m})\right]^2}{(9.80\,\text{m/s}^2)}=\boxed{6990\,\text{kg}}$$

14. The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$mg=4PA\ \rightarrow\ m=\frac{4PA}{g}=\frac{4(2.40\times10^5\text{ N/m}^2)(220\,\text{cm}^2)\left(\dfrac{1\text{ m}^2}{10^4\text{ cm}^2}\right)}{(9.80\,\text{m/s}^2)}=\boxed{2200\,\text{kg}}$$

15. (*a*) The absolute pressure is given by Eq. 13-6b, and the total force is the absolute pressure times the area of the bottom of the pool.

$$P=P_0+\rho gh=1.013\times10^5\text{ N/m}^2+(1.00\times10^3\text{ kg/m}^3)(9.80\,\text{m/s}^2)(1.8\,\text{m})$$
$$=1.189\times10^5\text{ N/m}^2\approx\boxed{1.2\times10^5\text{ N/m}^2}$$
$$F=PA=(1.189\times10^5\text{ N/m}^2)(28.0\,\text{m})(8.5\,\text{m})=\boxed{2.8\times10^7\text{N}}$$

(*b*) The pressure against the side of the pool, near the bottom, will be the same as the pressure at the bottom. Pressure is not directional. $P=\boxed{1.2\times10^5\text{ N/m}^2}$

16. (*a*) The gauge pressure is given by Eq. 13-3. The height is the height from the bottom of the hill to the top of the water tank.

$$P_G = \rho g h = \left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left[5.0\ \text{m} + \left(110\ \text{m}\right)\sin 58^\circ\right] = \boxed{9.6\times10^5\ \text{N/m}^2}$$

(*b*) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$h = 5.0\,\text{m} + \left(110\,\text{m}\right)\sin 58^\circ = \boxed{98\,\text{m}}$$

17. The pressure at points a and b are equal since they are the same height in the same fluid. If they were unequal, the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_a = P_b \;\rightarrow\; P_0 + \rho_{\text{oil}} g h_{\text{oil}} = P_0 + \rho_{\text{water}} g h_{\text{water}} \;\rightarrow\; \rho_{\text{oil}} h_{\text{oil}} = \rho_{\text{water}} h_{\text{water}} \;\rightarrow$$

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(0.272\,\text{m} - 0.0862\,\text{m}\right)}{\left(0.272\,\text{m}\right)} = \boxed{683\,\text{kg/m}^3}$$

18. (*a*) The mass of water in the tube is the volume of the tube times the density of water.

$$m = \rho V = \rho \pi r^2 h = \left(1.00\times10^3\ \text{kg/m}^3\right)\pi\left(0.30\times10^{-2}\,\text{m}\right)^2\left(12\ \text{m}\right) = 0.3393\,\text{kg} \approx \boxed{0.34\,\text{kg}}$$

(*b*) The net force exerted on the lid is the gauge pressure of the water times the area of the lid. The gauge pressure is found from Eq. 13-3.

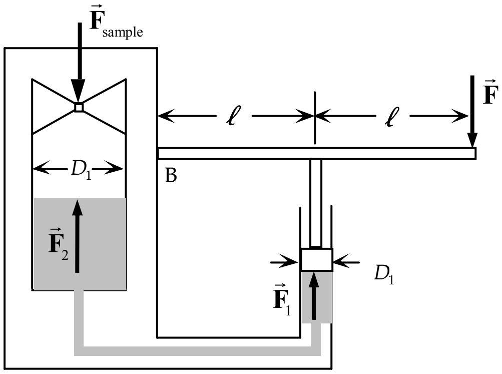
$$F = P_{\text{gauge}} A = \rho g h \pi R^2 = \left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(12\,\text{m}\right)\pi\left(0.21\,\text{m}\right)^2 = \boxed{1.6\times10^4\,\text{N}}$$

$\boxed{19.}$ We use the relationship developed in Example 13-5.

$$P = P_0 e^{-(\rho_0 g/P_0)y} = \left(1.013\times10^5\ \text{N/m}^2\right)e^{-\left(1.25\times10^{-4}\,\text{m}^{-1}\right)\left(8850\,\text{m}\right)} = \boxed{3.35\times10^4\ \text{N/m}^2} \approx 0.331\,\text{atm}$$

Note that if we used the constant density approximation, $P = P_0 + \rho g h$, a negative pressure would result.

20. Consider the lever (handle) of the press. The net torque on that handle is 0. Use that to find the force exerted by the hydraulic fluid upwards on the small cylinder (and the lever). Then Pascal's principle can be used to find the upwards force on the large cylinder, which is the same as the force on the sample.



$$\sum \tau = F\left(2\ell\right) - F_1 \ell = 0 \;\rightarrow\; F_1 = 2F$$

$$P_1 = P_2 \;\rightarrow\; \frac{F_1}{\pi\left(\frac{1}{2}d_1\right)^2} = \frac{F_2}{\pi\left(\frac{1}{2}d_2\right)^2} \;\rightarrow$$

$$F_2 = F_1\left(d_2/d_1\right)^2 = 2F\left(d_2/d_1\right)^2 = F_{\text{sample}} \;\rightarrow$$

$$P_{\text{sample}} = \frac{F_{\text{sample}}}{A_{\text{sample}}} = \frac{2F\left(d_2/d_1\right)^2}{A_{\text{sample}}} = \frac{2\left(350\,\text{N}\right)\left(5\right)^2}{4.0\times10^{-4}\text{m}^2} = \boxed{4.4\times10^7\ \text{N/m}^2} \approx 430\,\text{atm}$$

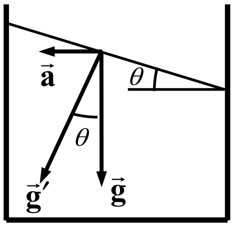
21. The pressure in the tank is atmospheric pressure plus the pressure difference due to the column of mercury, as given in Eq. 13-6b.

(*a*) $P = P_0 + \rho g h = 1.04\,\text{bar} + \rho_{\text{Hg}} g h$

$$= (1.04\,\text{bar})\left(\frac{1.00\times10^5\,\text{N/m}^2}{1\,\text{bar}}\right) + \left(13.6\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(0.210\,\text{m}\right) = \boxed{1.32\times10^5\,\text{N/m}^2}$$

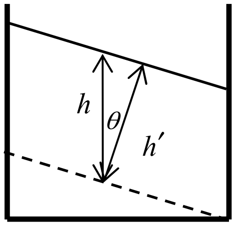
(*b*) $$P = (1.04\,\text{bar})\left(\frac{1.00\times10^5\,\text{N/m}^2}{1\,\text{bar}}\right) + \left(13.6\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(-0.052\,\text{m}\right) = \boxed{9.7\times10^4\,\text{N/m}^2}$$

22. (*a*) See the diagram. In the accelerated frame of the beaker, there is a pseudoforce opposite to the direction of the acceleration, and so there is a pseudo acceleration as shown on the diagram. The effective acceleration, $\vec{\mathbf{g}}'$, is given by $\vec{\mathbf{g}}' = \vec{\mathbf{g}} + \vec{\mathbf{a}}$. The surface of the water will be perpendicular to the effective acceleration, and thus makes an angle $\boxed{\theta = \tan^{-1}\frac{a}{g}}$.



(*b*) The left edge of the water surface, opposite to the direction of the acceleration, will be higher.

(*c*) Constant pressure lines will be parallel to the surface. From the second diagram, we see that a vertical depth of h corresponds to a depth of h' perpendicular to the surface, where $h' = h\cos\theta$, and so we have the following.



$$P = P_0 + \rho g'h' = P_0 + \rho\sqrt{g^2+a^2}\,(h\cos\theta)$$

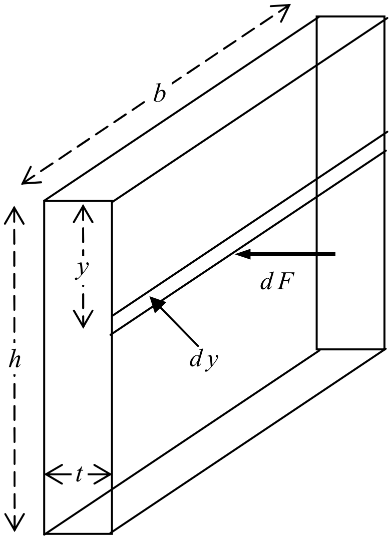
$$= P_0 + \rho\sqrt{g^2+a^2}\left(h\frac{g}{\sqrt{g^2+a^2}}\right) = P_0 + \rho hg$$

And so $\boxed{P = P_0 + \rho hg}$, as in the unaccelerated case.

23. (*a*) Because the pressure varies with depth, the force on the wall will also vary with depth. So to find the total force on the wall, we will have to integrate. Measure vertical distance y downward from the top level of the water behind the dam. Then at a depth y, choose an infinitesimal area of width b and height dy. The pressure due to the water at that depth is $P = \rho gy$.

$$P = \rho gy\ ;\ dF = PdA = (\rho gy)(b\,dy) \quad \rightarrow$$

$$F = \int dF = \int_0^h (\rho gy)(b\,dy) = \boxed{\tfrac{1}{2}\rho gbh^2}$$



(*b*) The lever arm for the force dF about the bottom of the dam is $h-y$, and so the torque caused by that force is $d\tau = (h-y)\,dF$. Integrate to find the total torque.

$$\tau = \int d\tau = \int_0^h (h-y)(\rho gy)(b\,dy) = \rho gb\int_0^h \left(hy - y^2\right)dy$$

$$= \rho gb\left(\tfrac{1}{2}hy^2 - \tfrac{1}{3}y^3\right)_0^h = \tfrac{1}{6}\rho gbh^3$$

Consider that torque as caused by the total force, applied at a single distance from the bottom d.

$$\tau = \tfrac{1}{6}\rho gbh^3 = Fd = \tfrac{1}{2}\rho gbh^2 d \quad \rightarrow \quad \boxed{d = \tfrac{1}{3}h}$$

(*c*) To prevent overturning, the torque caused by gravity about the lower right front corner in the diagram must be at least as big as the torque caused by the water. The lever arm for gravity is half the thickness of the dam.

$$mg\left(\tfrac{1}{2}t\right) \ge \tfrac{1}{6}\rho g b h^3 \;\rightarrow\; \rho_{\text{concrete}}\left(hbt\right) g\left(\tfrac{1}{2}t\right) \ge \tfrac{1}{6}\rho_{\text{water}} g b h^3 \;\rightarrow$$

$$\frac{t}{h} \ge \sqrt{\frac{1}{3}\frac{\rho_{\text{water}}}{\rho_{\text{concrete}}}} = \sqrt{\frac{1}{3}\frac{1.00\times10^3\,\text{kg/m}^3}{2.3\times10^3\,\text{kg/m}^3}} = 0.38$$

So we must have $\boxed{t \ge 0.38h}$ to prevent overturning. Atmospheric pressure need not be added in because it is exerted on BOTH sides of the dam, and so causes no net force or torque. In part (*a*), the actual pressure at a depth *y* is $P = P_0 + \rho g y$, and of course air pressure acts on the exposed side of the dam as well.

24. From section 9-5, the change in volume due to pressure change is $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta P}{B}$, where *B* is the bulk modulus of the water, given in Table 12-1. The pressure increase with depth for a fluid of constant density is given by $\Delta P = \rho g \Delta h$, where Δh is the depth of descent. If the density change is small, then we can use the initial value of the density to calculate the pressure change, and so $\Delta P \approx \rho_0 g \Delta h$. Finally, consider a constant mass of water. That constant mass will relate the volume and density at the two locations by $M = \rho V = \rho_0 V_0$. Combine these relationships and solve for the density deep in the sea, ρ.

$$\rho V = \rho_0 V_0 \;\rightarrow$$

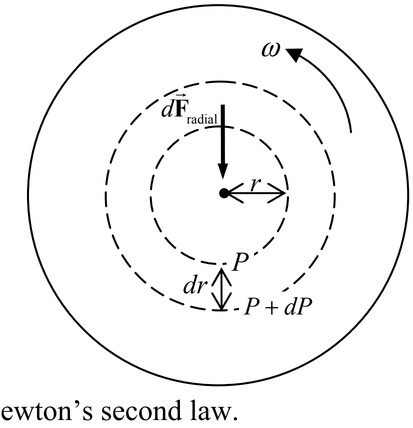
$$\rho = \frac{\rho_0 V_0}{V} = \frac{\rho_0 V_0}{V_0 + \Delta V} = \frac{\rho_0 V_0}{V_0 + \left(-V_0 \dfrac{\Delta P}{B}\right)} = \frac{\rho_0}{1 - \dfrac{\rho_0 g h}{B}} = \frac{1025\,\text{kg/m}^3}{1 - \dfrac{\left(1025\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(5.4\times10^3\,\text{m}\right)}{2.0\times10^9\,\text{N/m}^2}}$$

$$= 1054\,\text{kg/m}^3 \approx \boxed{1.05\times10^3\,\text{kg/m}^3}$$

$$\rho/\rho_0 = \frac{1054}{1025} = 1.028$$

The density at the 6 km depth is about $\boxed{3\%\ \text{larger}}$ than the density at the surface.

$\boxed{25.}$ Consider a layer of liquid of (small) height Δh, and ignore the pressure variation due to height in that layer. Take a cylindrical ring of water of height Δh, radius *r*, and thickness *dr*. See the diagram (the height is not shown). The volume of the ring of liquid is $\left(2\pi r \Delta h\right) dr$, and so has a mass of $dm = \left(2\pi \rho r \Delta h\right) dr$. That mass of water has a net centripetal force on it of magnitude $dF_{\text{radial}} = \omega^2 r\left(dm\right) = \omega^2 r \rho\left(2\pi r \Delta h\right) dr$. That force comes from a pressure difference across the surface area of the liquid. Let the pressure at the inside surface be P, which causes an outward force, and the pressure at the outside surface be $P + dP$, which causes an inward force. The surface area over which these pressures act is $2\pi r \Delta h$, the "walls" of the cylindrical ring. Use Newton's second law.



$$dF_{\text{radial}} = dF_{\substack{\text{outer}\\\text{wall}}} - dF_{\substack{\text{inner}\\\text{wall}}} \;\rightarrow\; \omega^2 r \rho\left(2\pi r \Delta h\right) dr = \left(P + dP\right) 2\pi r \Delta h - \left(P\right) 2\pi r \Delta h \;\rightarrow$$

$$dP = \omega^2 r\rho dr \;\rightarrow\; \int_{P_0}^{P} dP = \int_0^r \omega^2 r\rho dr \;\rightarrow\; P - P_0 = \tfrac{1}{2}\rho\omega^2 r^2 \;\rightarrow\; \boxed{P = P_0 + \tfrac{1}{2}\rho\omega^2 r^2}$$

26. If the iron is floating, then the net force on it is zero. The buoyant force on the iron must be equal to its weight. The buoyant force is equal to the weight of the mercury displaced by the submerged iron.

$$F_{\text{buoyant}} = m_{\text{Fe}}g \;\rightarrow\; \rho_{\text{Hg}}gV_{\text{submerged}} = \rho_{\text{Fe}}gV_{\text{total}} \;\rightarrow$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{Fe}}}{\rho_{\text{Hg}}} = \frac{7.8\times10^3\,\text{kg/m}^3}{13.6\times10^3\,\text{kg/m}^3} = \boxed{0.57} \approx 57\%$$

27. The difference in the actual mass and the apparent mass is the mass of the water displaced by the rock. The mass of the water displaced is the volume of the rock times the density of water, and the volume of the rock is the mass of the rock divided by its density. Combining these relationships yields an expression for the density of the rock.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{water}}V_{\text{rock}} = \rho_{\text{water}}\frac{m_{\text{rock}}}{\rho_{\text{rock}}} \;\rightarrow$$

$$\rho_{\text{rock}} = \rho_{\text{water}}\frac{m_{\text{rock}}}{\Delta m} = \left(1.00\times10^3\,\text{kg/m}^3\right)\frac{9.28\,\text{kg}}{9.28\,\text{kg} - 6.18\,\text{kg}} = \boxed{2990\,\text{kg/m}^3}$$

28. (*a*) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, and so their sum is equal to the weight of the hull, if the hill is not accelerated as it is lifted. The buoyant force is the weight of the water displaced.

$$T + F_{\text{buoyant}} = mg \;\rightarrow$$

$$T = mg - F_{\text{buoyant}} = m_{\text{hull}}g - \rho_{\text{water}}V_{\text{sub}}g = m_{\text{hull}}g - \rho_{\text{water}}\frac{m_{\text{hull}}}{\rho_{\text{hull}}}g = m_{\text{hull}}g\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{hull}}}\right)$$

$$= \left(1.6\times10^4\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(1 - \frac{1.00\times10^3\,\text{kg/m}^3}{7.8\times10^3\,\text{kg/m}^3}\right) = 1.367\times10^5\,\text{N} \approx \boxed{1.4\times10^5\,\text{N}}$$

(*b*) When the hull is completely out of the water, the tension in the crane's cable must be equal to the weight of the hull.

$$T = mg = \left(1.6\times10^4\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right) = 1.568\times10^5\,\text{N} \approx \boxed{1.6\times10^5\,\text{N}}$$

29. The buoyant force of the balloon must equal the weight of the balloon plus the weight of the helium in the balloon plus the weight of the load. For calculating the weight of the helium, we assume it is at 0°C and 1 atm pressure. The buoyant force is the weight of the air displaced by the volume of the balloon.

$$F_{\text{buoyant}} = \rho_{\text{air}}V_{\text{balloon}}g = m_{\text{He}}g + m_{\text{balloon}}g + m_{\text{cargo}}g \;\rightarrow$$

$$m_{\text{cargo}} = \rho_{\text{air}}V_{\text{balloon}} - m_{\text{He}} - m_{\text{balloon}} = \rho_{\text{air}}V_{\text{balloon}} - \rho_{\text{He}}V_{\text{balloon}} - m_{\text{balloon}} = \left(\rho_{\text{air}} - \rho_{\text{He}}\right)V_{\text{balloon}} - m_{\text{balloon}}$$

$$= \left(1.29\,\text{kg/m}^3 - 0.179\,\text{kg/m}^3\right)\tfrac{4}{3}\pi\left(7.35\,\text{m}\right)^3 - 930\,\text{kg} = \boxed{920\,\text{kg}} = 9.0\times10^3\,\text{N}$$

30. The difference in the actual mass and the apparent mass is the mass of the water displaced by the legs. The mass of the water displaced is the volume of the legs times the density of water, and the volume of the legs is the mass of the legs divided by their density. The density of the legs is assumed to be the same as that of water. Combining these relationships yields an expression for the mass of the legs.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{water}} V_{\text{legs}} = \rho_{\text{water}} \frac{m_{\text{legs}}}{\rho_{\text{legs}}} = 2m_{\text{leg}} \quad \rightarrow$$

$$m_{\text{leg}} = \tfrac{1}{2}\Delta m = \tfrac{1}{2}\left(74\,\text{kg} - 54\,\text{kg}\right) = \boxed{10\,\text{kg}} \;\; (2 \text{ sig. fig.})$$

31. The apparent weight is the actual weight minus the buoyant force. The buoyant force is weight of a mass of water occupying the volume of the metal sample.

$$m_{\text{apparent}} g = m_{\text{metal}} g - F_{\text{B}} = m_{\text{metal}} g - V_{\text{metal}} \rho_{\text{H}_2\text{O}} g = m_{\text{metal}} g - \frac{m_{\text{metal}}}{\rho_{\text{metal}}} \rho_{\text{H}_2\text{O}} g \quad \rightarrow$$

$$m_{\text{apparent}} = m_{\text{metal}} - \frac{m_{\text{metal}}}{\rho_{\text{metal}}} \rho_{\text{H}_2\text{O}} \quad \rightarrow$$

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}}{\left(m_{\text{metal}} - m_{\text{apparent}}\right)} \rho_{\text{H}_2\text{O}} = \frac{63.5\,\text{g}}{\left(63.5\,\text{g} - 55.4\,\text{g}\right)} \left(1000\,\text{kg/m}^3\right) = 7840\,\text{kg/m}^3$$

Based on the density value, the metal is probably $\boxed{\text{iron or steel}}$.

32. The difference in the actual mass and the apparent mass of the aluminum is the mass of the air displaced by the aluminum. The mass of the air displaced is the volume of the aluminum times the density of air, and the volume of the aluminum is the actual mass of the aluminum divided by the density of aluminum. Combining these relationships yields an expression for the actual mass.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{air}} V_{\text{Al}} = \rho_{\text{air}} \frac{m_{\text{actual}}}{\rho_{\text{Al}}} \quad \rightarrow$$

$$m_{\text{actual}} = \frac{m_{\text{apparent}}}{1 - \dfrac{\rho_{\text{air}}}{\rho_{\text{Al}}}} = \frac{3.0000\text{ kg}}{1 - \dfrac{1.29\,\text{kg/m}^3}{2.70 \times 10^3\text{ kg/m}^3}} = \boxed{3.0014\,\text{kg}}$$

33. The buoyant force on the drum must be equal to the weight of the steel plus the weight of the gasoline. The weight of each component is its respective volume times density. The buoyant force is the weight of total volume of displaced water. We assume that the drum just "barely" floats – in other words, the volume of water displaced is equal to the total volume of gasoline and steel.

$$F_{\text{B}} = W_{\text{steel}} + W_{\text{gasoline}} \quad \rightarrow \quad \left(V_{\text{gasoline}} + V_{\text{steel}}\right) \rho_{\text{water}} g = V_{\text{steel}} \rho_{\text{steel}} g + V_{\text{gasoline}} \rho_{\text{gasoline}} g \quad \rightarrow$$

$$V_{\text{gasoline}} \rho_{\text{water}} + V_{\text{steel}} \rho_{\text{water}} = V_{\text{steel}} \rho_{\text{steel}} + V_{\text{gasoline}} \rho_{\text{gasoline}} \quad \rightarrow$$

$$V_{\text{steel}} = V_{\text{gasoline}} \left(\frac{\rho_{\text{water}} - \rho_{\text{gasoline}}}{\rho_{\text{steel}} - \rho_{\text{water}}} \right) = \left(230\,\text{L}\right) \left(\frac{1000\,\text{kg/m}^3 - 680\,\text{kg/m}^3}{7800\,\text{kg/m}^3 - 1000\,\text{kg/m}^3} \right) = 10.82\,\text{L} \approx \boxed{1.1 \times 10^{-2}\,\text{m}^3}$$

34. (*a*) The buoyant force is the weight of the water displaced, using the density of sea water.

$$F_{\text{buoyant}} = m_{\substack{\text{water}\\\text{displaced}}} g = \rho_{\text{water}} V_{\text{displaced}} g$$

$$= \left(1.025 \times 10^3\text{ kg/m}^3\right)\left(65.0\text{ L}\right)\left(\frac{1 \times 10^{-3}\,\text{m}^3}{1\text{ L}}\right)\left(9.80\,\text{m/s}^2\right) = \boxed{653\,\text{N}}$$

(*b*) The weight of the diver is $m_{\text{diver}} g = \left(68.0\text{ kg}\right)\left(9.80\,\text{m/s}^2\right) = 666\text{ N}$. Since the buoyant force is not as large as her weight, $\boxed{\text{she will sink}}$, although it will be very gradual since the two forces are almost the same.

35. The buoyant force on the ice is equal to the weight of the ice, since it floats.

$$F_{\text{buoyant}} = W_{\text{ice}} \rightarrow m_{\substack{\text{seawater}\\\text{submerged}}} g = m_{\text{ice}} g \rightarrow m_{\substack{\text{seawater}\\\text{submerged}}} = m_{\text{ice}} \rightarrow$$

$$\rho_{\text{seawater}} V_{\text{seawater}} = \rho_{\text{ice}} V_{\text{ice}} \rightarrow (SG)_{\text{seawater}} \rho_{\text{water}} V_{\substack{\text{submerged}\\\text{ice}}} = (SG)_{\text{ice}} \rho_{\text{water}} V_{\text{ice}} \rightarrow$$

$$(SG)_{\text{seawater}} V_{\substack{\text{submerged}\\\text{ice}}} = (SG)_{\text{ice}} V_{\text{ice}} \rightarrow$$

$$V_{\substack{\text{submerged}\\\text{ice}}} = \frac{(SG)_{\text{ice}}}{(SG)_{\text{seawater}}} V_{\text{ice}} = \frac{0.917}{1.025} V_{\text{ice}} = 0.895 V_{\text{ice}}$$

Thus the fraction above the water is $V_{\text{above}} = V_{\text{ice}} - V_{\text{submerged}} = 0.105\ V_{\text{ice}}$ or $\boxed{10.5\%}$

36. (*a*) The difference in the actual mass and the apparent mass of the aluminum ball is the mass of the liquid displaced by the ball. The mass of the liquid displaced is the volume of the ball times the density of the liquid, and the volume of the ball is the mass of the ball divided by its density. Combining these relationships yields an expression for the density of the liquid.

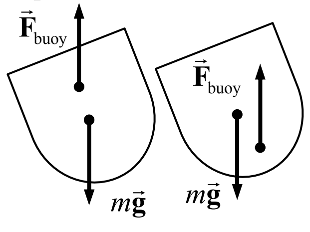
$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{liquid}} V_{\text{ball}} = \rho_{\text{liquid}} \frac{m_{\text{ball}}}{\rho_{\text{Al}}} \rightarrow$$

$$\rho_{\text{liquid}} = \frac{\Delta m}{m_{\text{ball}}} \rho_{\text{Al}} = \frac{(3.80\,\text{kg} - 2.10\,\text{kg})}{3.80\,\text{kg}} \left(2.70\times10^3\ \text{kg/m}^3\right) = \boxed{1210\,\text{kg/m}^3}$$

(*b*) Generalizing the relation from above, we have $\boxed{\rho_{\text{liquid}} = \left(\frac{m_{\text{object}} - m_{\text{apparent}}}{m_{\text{object}}}\right) \rho_{\text{object}}}$.

37. (*a*) The buoyant force on the object is equal to the weight of the fluid displaced. The force of gravity of the fluid can be considered to act at the center of gravity of the fluid (see section 9-8). If the object were removed from the fluid and that space re-filled with an equal volume of fluid, that fluid would be in equilibrium. Since there are only two forces on that volume of fluid, gravity and the buoyant force, they must be equal in magnitude <u>and</u> act at the same point. Otherwise they would be a couple (see Figure 12-4), exert a non-zero torque, and cause rotation of the fluid. Since the fluid does not rotate, we may conclude that the buoyant force acts at the center of gravity.

(*b*) From the diagram, if the center of buoyancy (the point where the buoyancy force acts) is above the center of gravity (the point where gravity acts) of the entire ship, when the ship tilts, the net torque about the center of mass will tend to reduce the tilt. If the center of buoyancy is below the center of gravity of the entire ship, when the ship tilts, the net torque about the center of mass will tend to increase the tilt. Stability is achieved when the center of buoyancy is $\boxed{\text{above the center of gravity}}$.



38. The weight of the object must be balanced by the two buoyant forces, one from the water and one from the oil. The buoyant force is the density of the liquid, times the volume in the liquid, times the acceleration due to gravity. We represent the edge length of the cube by ℓ.

$$mg = F_{\substack{\text{B}\\\text{oil}}} + F_{\substack{\text{B}\\\text{water}}} = \rho_{\text{oil}} V_{\text{oil}} g + \rho_{\text{water}} V_{\text{water}} g = \rho_{\text{oil}} \ell^2 (0.28\ell) g + \rho_{\text{water}} \ell^2 (0.72\ell) g \rightarrow$$

$$m = \ell^3 (0.28\rho_{\text{oil}} + 0.72\rho_{\text{water}}) = (0.100\,\text{m})^3 \left[0.28\left(810\,\text{kg/m}^2\right) + 0.72\left(1000\,\text{kg/m}^2\right)\right]$$

$$= 0.9468\,\text{kg} \approx \boxed{0.95\,\text{kg}}$$

The buoyant force is the weight of the object, $mg = (0.9468\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{9.3\,\text{N}}$

39. The buoyant force must be equal to the combined weight of the helium balloons and the person. We ignore the buoyant force due to the volume of the person, and we ignore the mass of the balloon material.

$$F_\text{B} = (m_\text{person} + m_\text{He})g \;\rightarrow\; \rho_\text{air}V_\text{He}g = (m_\text{person} + \rho_\text{He}V_\text{He})g \;\rightarrow\; V_\text{He} = N\tfrac{4}{3}\pi r^3 = \frac{m_\text{person}}{(\rho_\text{air} - \rho_\text{He})} \;\rightarrow$$

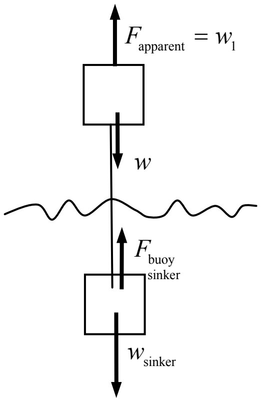
$$N = \frac{3m_\text{person}}{4\pi r^3(\rho_\text{air} - \rho_\text{He})} = \frac{3(75\,\text{kg})}{4\pi(0.165\,\text{m})^3(1.29\,\text{kg/m}^3 - 0.179\,\text{kg/m}^3)} = 3587 \approx \boxed{3600\,\text{balloons}}$$

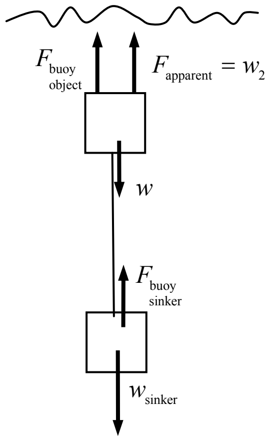
40. There will be a downward gravity force and an upward buoyant force on the fully submerged tank. The buoyant force is constant, but the gravity force will decrease as the air is removed. Take upwards to be positive.

$$F_\text{full} = F_\text{B} - m_\text{total}g = \rho_\text{water}V_\text{tank}g - (m_\text{tank} + m_\text{air})g$$
$$= \left[(1025\,\text{kg/m}^3)(0.0157\,\text{m}^3) - 17.0\,\text{kg}\right](9.80\,\text{m/s}^2) = -8.89\,\text{N} \approx \boxed{9\text{N downward}}$$

$$F_\text{empty} = F_\text{B} - m_\text{total}g = \rho_\text{water}V_\text{tank}g - (m_\text{tank} + m_\text{air})g$$
$$= \left[(1025\,\text{kg/m}^3)(0.0157\,\text{m}^3) - 14.0\,\text{kg}\right](9.80\,\text{m/s}^2) = 20.51\,\text{N} \approx \boxed{21\text{N upward}}$$

41. The apparent weight is the force required to hold the system in equilibrium. In the first case, the object is held above the water. In the second case, the object is allowed to be pulled under the water. Consider the free-body diagram for each case.





Case 1: $\sum F = w_1 - w + F_{\substack{\text{buoy}\\\text{sinker}}} - w_\text{sinker} = 0$

Case 2: $\sum F = w_2 + F_{\substack{\text{buoy}\\\text{object}}} - w + F_{\substack{\text{buoy}\\\text{sinker}}} - w_\text{sinker} = 0$

Since both add to 0, equate them. Also note that the specific gravity can be expressed in terms of the buoyancy force.

$$F_{\substack{\text{buoy}\\\text{object}}} = V_\text{object}\rho_\text{water}g = \frac{m_\text{object}}{\rho_\text{object}}\rho_\text{water}g = m_\text{object}g\frac{\rho_\text{water}}{\rho_\text{object}} = \frac{w}{\text{S.G.}}$$

$$w_1 - w + F_{\substack{\text{buoy}\\\text{sinker}}} - w_\text{sinker} = 0 = w_2 + F_{\substack{\text{buoy}\\\text{object}}} - w + F_{\substack{\text{buoy}\\\text{sinker}}} - w_\text{sinker} \;\rightarrow$$

$$w_1 = w_2 + F_{\substack{\text{buoy}\\\text{object}}} = w_2 + \frac{w}{\text{S.G.}} \;\rightarrow\; \text{S.G.} = \boxed{\frac{w}{(w_1 - w_2)}}$$

42. For the combination to just barely sink, the total weight of the wood and lead must be equal to the total buoyant force on the wood and the lead.

$$F_\text{weight} = F_\text{buoyant} \;\rightarrow\; m_\text{wood}g + m_\text{Pb}g = V_\text{wood}\rho_\text{water}g + V_\text{Pb}\rho_\text{water}g \;\rightarrow$$

$$m_\text{wood} + m_\text{Pb} = \frac{m_\text{wood}}{\rho_\text{wood}}\rho_\text{water} + \frac{m_\text{Pb}}{\rho_\text{Pb}}\rho_\text{water} \;\rightarrow\; m_\text{Pb}\left(1 - \frac{\rho_\text{water}}{\rho_\text{Pb}}\right) = m_\text{wood}\left(\frac{\rho_\text{water}}{\rho_\text{wood}} - 1\right) \;\rightarrow$$

$$m_{\text{Pb}} = m_{\text{wood}}\frac{\left(\dfrac{\rho_{\text{water}}}{\rho_{\text{wood}}}-1\right)}{\left(1-\dfrac{\rho_{\text{water}}}{\rho_{\text{Pb}}}\right)} = m_{\text{wood}}\frac{\left(\dfrac{1}{SG_{\text{wood}}}-1\right)}{\left(1-\dfrac{1}{SG_{\text{Pb}}}\right)} = (3.25\,\text{kg})\frac{\left(\dfrac{1}{0.50}-1\right)}{\left(1-\dfrac{1}{11.3}\right)} = \boxed{3.57\,\text{kg}}$$

$\boxed{43.}$ We apply the equation of continuity at constant density, Eq. 13-7b.

Flow rate out of duct = Flow rate into room

$$A_{\text{duct}}v_{\text{duct}} = \pi r^2 v_{\text{duct}} = \frac{V_{\text{room}}}{t_{\substack{\text{to fill}\\ \text{room}}}} \rightarrow v_{\text{duct}} = \frac{V_{\text{room}}}{\pi r^2 t_{\substack{\text{to fill}\\ \text{room}}}} = \frac{(8.2\,\text{m})(5.0\,\text{m})(3.5\,\text{m})}{\pi(0.15\,\text{m})^2(12\,\text{min})\left(\dfrac{60\,\text{s}}{1\,\text{min}}\right)} = \boxed{2.8\,\text{m/s}}$$

44. Use Eq. 13-7b, the equation of continuity for an incompressible fluid, to compare blood flow in the aorta and in the major arteries.

$$(Av)_{\text{aorta}} = (Av)_{\text{arteries}} \rightarrow$$

$$v_{\text{arteries}} = \frac{A_{\text{aorta}}}{A_{\text{arteries}}}v_{\text{aorta}} = \frac{\pi(1.2\,\text{cm})^2}{2.0\,\text{cm}^2}(40\,\text{cm/s}) = 90.5\,\text{cm/s} \approx \boxed{0.9\,\text{m/s}}$$

45. We may apply Torricelli's theorem, Eq. 13-9.

$$v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2(9.80\,\text{m/s}^2)(5.3\,\text{m})} = 10.2\,\text{m/s} \approx \boxed{10\,\text{m/s}}\ \ (2\text{ sig. fig.})$$

46. The flow speed is the speed of the water in the input tube. The entire volume of the water in the tank is to be processed in 4.0 h. The volume of water passing through the input tube per unit time is the volume rate of flow, as expressed in the text immediately after Eq. 13-7b.

$$\frac{V}{\Delta t} = Av \rightarrow v = \frac{V}{A\Delta t} = \frac{\ell wh}{\pi r^2 \Delta t} = \frac{(0.36\,\text{m})(1.0\,\text{m})(0.60\,\text{m})}{\pi(0.015\,\text{m})^2(4.0\,\text{h})\left(\dfrac{3600\,\text{s}}{1\,\text{h}}\right)} = 0.02122\,\text{m/s} \approx \boxed{2.1\,\text{cm/s}}$$

47. Apply Bernoulli's equation with point 1 being the water main, and point 2 being the top of the spray. The velocity of the water will be zero at both points. The pressure at point 2 will be atmospheric pressure. Measure heights from the level of point 1.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow$$

$$P_1 - P_{\text{atm}} = \rho g y_2 = (1.00\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(18\,\text{m}) = \boxed{1.8\times10^5\,\text{N/m}^2}$$

48. The volume flow rate of water from the hose, multiplied times the time of filling, must equal the volume of the pool.

$$(Av)_{\text{hose}} = \frac{V_{\text{pool}}}{t} \rightarrow t = \frac{V_{\text{pool}}}{A_{\text{hose}}v_{\text{hose}}} = \frac{\pi(3.05\,\text{m})^2(1.2\,\text{m})}{\pi\left[\tfrac{1}{2}\left(\tfrac{5}{8}\right)''\left(\dfrac{1\,\text{m}}{39.37''}\right)\right]^2(0.40\,\text{m/s})} = 4.429\times10^5\,\text{s}$$

$$4.429\times10^5\,\text{s}\left(\frac{1\,\text{day}}{60\times60\times24\,\text{s}}\right) = \boxed{5.1\text{ days}}$$

$\boxed{49.}$ We assume that there is no appreciable height difference between the two sides of the roof. Then the net force on the roof due to the air is the difference in pressure on the two sides of the roof, times the area of the roof. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 A_{\text{roof}} = \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left[\left(180\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 \left(6.2\,\text{m}\right)\left(12.4\,\text{m}\right)$$

$$= \boxed{1.2\times10^5\,\text{N}}$$

50. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the pressure conditions at the two locations. We assume that the two locations are at the same height. Express the pressures as atmospheric pressure plus gauge pressure. Use subscript "1" for the larger diameter, and "2" for the smaller diameter.

$$A_1 v_1 = A_2 v_2 \rightarrow v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \frac{r_1^2}{r_2^2}$$

$$P_0 + P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 = P_2 + \tfrac{1}{2}\rho v_1^2 \frac{r_1^4}{r_2^4} \rightarrow v_1 = \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} \rightarrow$$

$$A_1 v_1 = \pi r_1^2 \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} = \pi\left(3.0\times10^{-2}\,\text{m}\right)^2 \sqrt{\frac{2\left(32.0\times10^3\,\text{Pa} - 24.0\times10^3\,\text{Pa}\right)}{\left(1.0\times10^3\,\text{kg/m}^3\right)\left(\frac{\left(3.0\times10^{-2}\,\text{m}\right)^4}{\left(2.25\times10^{-2}\,\text{m}\right)^4} - 1\right)}}$$

$$= \boxed{7.7\times10^{-3}\,\text{m}^3/\text{s}}$$

51. The air pressure inside the hurricane can be estimated using Bernoulli's equation. Assume the pressure outside the hurricane is air pressure, the speed of the wind outside the hurricane is 0, and that the two pressure measurements are made at the same height.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \rightarrow$$

$$P_{\text{inside}} = P_{\text{outside}} - \tfrac{1}{2}\rho_{\text{air}} v_{\text{inside}}^2$$

$$= 1.013\times10^5\,\text{Pa} - \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left[\left(300\,\text{km/h}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\right]^2$$

$$= \boxed{9.7\times10^4\,\text{Pa}} \approx 0.96\,\text{atm}$$

52. The lift force would be the difference in pressure between the two wing surfaces, times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1, and the top surface point 2.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$F_{\text{lift}} = \left(P_1 - P_2\right)\left(\text{Area of wing}\right) = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$$

$$= \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left[\left(280\,\text{m/s}\right)^2 - \left(150\,\text{m/s}\right)^2\right]\left(88\,\text{m}^2\right) = \boxed{3.2\times10^6\,\text{N}}$$

53. Consider the volume of fluid in the pipe. At each end of the pipe there is a force towards the contained fluid, given by $F = PA$. Since the area of the pipe is constant, we have that $F_{\text{net}} = \left(P_1 - P_2\right)A$. Then, since the power required is the force on the fluid times its velocity, and $AV = Q$ = volume rate of flow, we have $P = F_{\text{net}}v = \left(P_1 - P_2\right)Av = \boxed{\left(P_1 - P_2\right)Q}$.

54. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the conditions at the street to those at the top floor. Express the pressures as atmospheric pressure plus gauge pressure.

$$A_{\text{street}}v_{\text{street}} = A_{\text{top}}v_{\text{top}} \quad \rightarrow$$

$$v_{\text{top}} = v_{\text{street}}\frac{A_{\text{street}}}{A_{\text{top}}} = \left(0.68\,\text{m/s}\right)\frac{\pi\left[\tfrac{1}{2}\left(5.0\times10^{-2}\,\text{m}\right)\right]^2}{\pi\left[\tfrac{1}{2}\left(2.8\times10^{-2}\,\text{m}\right)\right]^2} = 2.168\,\text{m/s} \approx \boxed{2.2\,\text{m/s}}$$

$$P_0 + P_{\substack{\text{gauge}\\\text{street}}} + \tfrac{1}{2}\rho v_{\text{street}}^2 + \rho g y_{\text{street}} = P_0 + P_{\substack{\text{gauge}\\\text{top}}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} \quad \rightarrow$$

$$P_{\substack{\text{gauge}\\\text{top}}} = P_{\substack{\text{gauge}\\\text{street}}} + \tfrac{1}{2}\rho\left(v_{\text{street}}^2 - v_{\text{top}}^2\right) + \rho g y\left(y_{\text{street}} - y_{\text{top}}\right)$$

$$= \left(3.8\,\text{atm}\right)\left(\frac{1.013\times10^5\,\text{Pa}}{\text{atm}}\right) + \tfrac{1}{2}\left(1.00\times10^3\,\text{kg/m}^3\right)\left[\left(0.68\,\text{m/s}\right)^2 - \left(2.168\,\text{m/s}\right)^2\right]$$

$$+ \left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(-18\,\text{m}\right)$$

$$= 2.064\times10^5\,\text{Pa}\left(\frac{1\,\text{atm}}{1.013\times10^5\,\text{Pa}}\right) \approx \boxed{2.0\,\text{atm}}$$

$\boxed{55.}$ Apply both Bernoulli's equation and the equation of continuity between the two openings of the tank. Note that the pressure at each opening will be atmospheric pressure.

$$A_2v_2 = A_1v_1 \quad \rightarrow \quad v_2 = v_1\frac{A_1}{A_2}$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad v_1^2 - v_2^2 = 2g\left(y_2 - y_1\right) = 2gh$$

$$v_1^2 - \left(v_1\frac{A_1}{A_2}\right) = 2gh \quad \rightarrow \quad v_1^2\left(1 - \frac{A_1^2}{A_2^2}\right) = 2gh \quad \rightarrow \quad \boxed{v_1 = \sqrt{\frac{2gh}{\left(1 - A_1^2/A_2^2\right)}}}$$

56. (*a*) Relate the conditions at the top surface and at the opening by Bernoulli's equation.

$$P_{\text{top}} + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_{\text{opening}} + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 \quad \rightarrow \quad P_2 + P_0 + \rho g\left(y_2 - y_1\right) = P_0 + \tfrac{1}{2}\rho v_1^2 \quad \rightarrow$$

$$\boxed{v_1 = \sqrt{\frac{2P_2}{\rho} + 2g\left(y_2 - y_1\right)}}$$

(*b*) $$v_1=\sqrt{\frac{2P_2}{\rho}+2g\left(y_2-y_1\right)}=\sqrt{\frac{2\left(0.85\,\text{atm}\right)\left(\frac{1.013\times10^5\,\text{Pa}}{\text{atm}}\right)}{\left(1.00\times10^3\,\text{kg/m}^3\right)}+2\left(9.80\,\text{m/s}^2\right)\left(2.4\,\text{m}\right)}=\boxed{15\,\text{m/s}}$$

57. We assume that the water is launched from the same level at which it lands. Then the level range formula, derived in Example 3-10, applies. That formula is $R=\frac{v_0^2\sin2\theta_0}{g}$. If the range has increased by a factor of 4, then the initial speed has increased by a factor of 2. The equation of continuity is then applied to determine the change in the hose opening. The water will have the same volume rate of flow, whether the opening is large or small.

$$\left(Av\right)_{\substack{\text{fully}\\\text{open}}}=\left(Av\right)_{\substack{\text{partly}\\\text{open}}}\rightarrow A_{\substack{\text{partly}\\\text{open}}}=A_{\substack{\text{fully}\\\text{open}}}\frac{v_{\substack{\text{fully}\\\text{open}}}}{v_{\substack{\text{partly}\\\text{open}}}}=A_{\substack{\text{fully}\\\text{open}}}\left(\frac{1}{2}\right)$$

Thus $\boxed{1/2}$ of the hose opening was blocked.

58. Use Bernoulli's equation to find the speed of the liquid as it leaves the opening, assuming that the speed of the liquid at the top is 0, and that the pressure at each opening is air pressure.

$$P_1+\tfrac{1}{2}\rho v_1^2+\rho gy_1=P_2+\tfrac{1}{2}\rho v_2^2+\rho gy_2\rightarrow v_1=\sqrt{2g\left(h_2-h_1\right)}$$

(*a*) Since the liquid is launched horizontally, the initial vertical speed is zero. Use Eq. 2-12b for constant acceleration to find the time of fall, with upward as the positive direction. Then multiply the time of fall times v_1, the (constant) horizontal speed.

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_yt^2\rightarrow 0=h_1+0-\tfrac{1}{2}gt^2\rightarrow t=\sqrt{\frac{2h_1}{g}}$$

$$\Delta x=v_1t=\sqrt{2g\left(h_2-h_1\right)}\sqrt{\frac{2h_1}{g}}=\boxed{2\sqrt{\left(h_2-h_1\right)h_1}}$$

(*b*) We seek some height h_1' such that $2\sqrt{\left(h_2-h_1\right)h_1}=2\sqrt{\left(h_2-h_1'\right)h_1'}$.

$$2\sqrt{\left(h_2-h_1\right)h_1}=2\sqrt{\left(h_2-h_1'\right)h_1'}\rightarrow\left(h_2-h_1\right)h_1=\left(h_2-h_1'\right)h_1'\rightarrow$$

$$h_1'^2-h_2h_1'+\left(h_2-h_1\right)h_1=0\rightarrow$$

$$h_1'=\frac{h_2\pm\sqrt{h_2^2-4\left(h_2-h_1\right)h_1}}{2}=\frac{h_2\pm\sqrt{h_2^2-4h_1h_2+4h_1^2}}{2}=\frac{h_2\pm\left(h_2-2h_1\right)}{2}=\frac{2h_2-2h_1}{2},\frac{2h_1}{2}$$

$$\boxed{h_1'=h_2-h_1}$$

59. (*a*) Apply Bernoulli's equation to point 1, the exit hole, and point 2, the top surface of the liquid in the tank. Note that both points are open to the air and so the pressure is atmospheric pressure. Also apply the equation of continuity $\left(A_1v_1=A_2v_2\right)$ to the same two points.

$$P_1+\tfrac{1}{2}\rho v_1^2+\rho gy_1=P_2+\tfrac{1}{2}\rho v_2^2+\rho gy_2\rightarrow P_{\text{atm}}+\tfrac{1}{2}\rho\left(v_1^2-v_2^2\right)=P_{\text{atm}}+\rho g\left(y_2-y_1\right)\rightarrow$$

$$\tfrac{1}{2}\rho\left(v_1^2-v_2^2\right)=\rho gh\rightarrow\left(v_1^2-v_2^2\right)=2gh\rightarrow\left(\frac{A_2^2}{A_1^2}-1\right)v_2^2=2gh\rightarrow$$

$$v_2=\sqrt{\frac{2gh}{\left(\frac{A_2^2}{A_1^2}-1\right)}}=\sqrt{\frac{2ghA_1^2}{\left(A_2^2-A_1^2\right)}}$$

Note that since the water level is decreasing, we have $v_2=-\frac{dh}{dt}$, and so $\boxed{\frac{dh}{dt}=-\sqrt{\frac{2ghA_1^2}{\left(A_2^2-A_1^2\right)}}}$.

(*b*) Integrate to find the height as a function of time.

$$\frac{dh}{dt}=-\sqrt{\frac{2ghA_1^2}{\left(A_2^2-A_1^2\right)}} \quad\to\quad \frac{dh}{\sqrt{h}}=-\sqrt{\frac{2gA_1^2}{\left(A_2^2-A_1^2\right)}}dt \quad\to\quad \int_{h_0}^{h}\frac{dh}{\sqrt{h}}=-\sqrt{\frac{2gA_1^2}{\left(A_2^2-A_1^2\right)}}\int_0^t dt \quad\to$$

$$2\left(\sqrt{h}-\sqrt{h_0}\right)=-\sqrt{\frac{2gA_1^2}{\left(A_2^2-A_1^2\right)}}\,t \quad\to\quad \boxed{h=\left[\sqrt{h_0}-t\sqrt{\frac{gA_1^2}{2\left(A_2^2-A_1^2\right)}}\right]^2}$$

(*c*) We solve for the time at which $h=0$, given the other parameters. In particular,

$$A_1=\pi\left(0.25\times10^{-2}\,\text{m}\right)^2=1.963\times10^{-5}\,\text{m}^2\ ;\ A_2=\frac{1.3\times10^{-3}\,\text{m}^3}{0.106\,\text{m}}=1.226\times10^{-2}\,\text{m}^2$$

$$\left[\sqrt{h_0}-t\sqrt{\frac{gA_1^2}{2\left(A_2^2-A_1^2\right)}}\right]^2=0 \quad\to$$

$$t=\sqrt{\frac{2h_0\left(A_2^2-A_1^2\right)}{gA_1^2}}=\sqrt{\frac{2\left(0.106\,\text{m}\right)\left[\left(1.226\times10^{-2}\,\text{m}^2\right)^2-\left(1.963\times10^{-5}\,\text{m}^2\right)^2\right]}{\left(9.80\,\text{m/s}^2\right)\left(1.963\times10^{-5}\,\text{m}^2\right)^2}}=\boxed{92\,\text{s}}$$

60. (*a*) Apply the equation of continuity and Bernoulli's equation at the same height to the wide and narrow portions of the tube.

$$A_2v_2=A_1v_1 \quad\to\quad v_2=v_1\frac{A_1}{A_2}$$

$$P_1+\tfrac{1}{2}\rho v_1^2=P_2+\tfrac{1}{2}\rho v_2^2 \quad\to\quad \frac{2\left(P_1-P_2\right)}{\rho}=v_2^2-v_1^2 \quad\to$$

$$\left(v_1\frac{A_1}{A_2}\right)^2-v_1^2=\frac{2\left(P_1-P_2\right)}{\rho} \quad\to\quad v_1^2\left(\frac{A_1^2}{A_2^2}-\frac{A_2^2}{A_2^2}\right)=\frac{2\left(P_1-P_2\right)}{\rho} \quad\to$$

$$v_1^2=\frac{2A_2^2\left(P_1-P_2\right)}{\rho\left(A_1^2-A_2^2\right)} \quad\to\quad \boxed{v_1=A_2\sqrt{\frac{2\left(P_1-P_2\right)}{\rho\left(A_1^2-A_2^2\right)}}}$$

(*b*) $v_1=A_2\sqrt{\frac{2\left(P_1-P_2\right)}{\rho\left(A_1^2-A_2^2\right)}}$

$$=\pi\left[\tfrac{1}{2}\left(0.010\,\text{m}\right)\right]^2\sqrt{\frac{2\left(18\,\text{mm Hg}\right)\left(\frac{133\,\text{N/m}^2}{\text{mm Hg}}\right)}{\left(1000\,\text{kg/m}^3\right)\left(\pi^2\left[\tfrac{1}{2}\left(0.030\,\text{m}\right)\right]^4-\pi^2\left[\tfrac{1}{2}\left(0.010\,\text{m}\right)\right]^4\right)}}=\boxed{0.24\,\text{m/s}}$$

61. (*a*) Relate the conditions inside the rocket and just outside the exit orifice by means of Bernoulli's equation and the equation of continuity. We ignore any height difference between the two locations.

$$P_{\text{in}} + \tfrac{1}{2}\rho v_{\text{in}}^2 + \rho g y_{\text{in}} = P_{\text{out}} + \tfrac{1}{2}\rho v_{\text{out}}^2 + \rho g y_{\text{out}} \rightarrow P + \tfrac{1}{2}\rho v_{\text{in}}^2 = P_0 + \tfrac{1}{2}\rho v_{\text{out}}^2 \rightarrow$$

$$\frac{2\left(P - P_0\right)}{\rho} = v_{\text{out}}^2 - v_{\text{in}}^2 = v_{\text{out}}^2\left[1 - \left(\frac{v_{\text{in}}}{v_{\text{out}}}\right)^2\right]$$

$$A_{\text{in}}v_{\text{in}} = A_{\text{out}}v_{\text{out}} \rightarrow Av_{\text{in}} = A_0 v_{\text{out}} \rightarrow \frac{v_{\text{in}}}{v_{\text{out}}} = \frac{A_0}{A} \ll 1 \rightarrow$$

$$\frac{2\left(P - P_0\right)}{\rho} = v_{\text{out}}^2\left[1 - \left(\frac{v_{\text{in}}}{v_{\text{out}}}\right)^2\right] \approx v_{\text{out}}^2 \rightarrow \boxed{v_{\text{out}} = v = \sqrt{\frac{2\left(P - P_0\right)}{\rho}}}$$

(*b*) Thrust is defined in section 9-10, by $F_{\text{thrust}} = v_{\text{rel}}\dfrac{dm}{dt}$, and is interpreted as the force on the rocket due to the ejection of mass.

$$F_{\text{thrust}} = v_{\text{rel}}\frac{dm}{dt} = v_{\text{out}}\frac{d\left(\rho V\right)}{dt} = v_{\text{out}}\rho\frac{dV}{dt} = v_{\text{out}}\rho\left(v_{\text{out}}A_{\text{out}}\right) = \rho v^2 A_0 = \rho\frac{2\left(P - P_0\right)}{\rho}A_0$$

$$= \boxed{2\left(P - P_0\right)A_0}$$

62. There is a forward force on the exiting water, and so by Newton's third law there is an equal force pushing backwards on the hose. To keep the hose stationary, you push forward on the hose, and so the hose pushes backwards on you. So the force on the exiting water is the same magnitude as the force on the person holding the hose. Use Newton's second law and the equation of continuity to find the force. Note that the 450 L/min flow rate is the volume of water being accelerated per unit time. Also, the flow rate is the product of the cross-sectional area of the moving fluid, times the speed of the fluid, and so $\dfrac{V}{t} = A_1 v_1 = A_2 v_2$.

$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_2 - v_1}{t} = \rho\left(\frac{V}{t}\right)\left(v_2 - v_1\right) = \rho\left(\frac{V}{t}\right)\left(\frac{A_2 v_2}{A_2} - \frac{A_1 v_1}{A_1}\right) = \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$$= \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{\pi r_2^2} - \frac{1}{\pi r_1^2}\right)$$

$$= \left(1.00\times10^3\,\text{kg/m}^3\right)\left(\frac{450\,\text{L}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\times\frac{1\,\text{m}^3}{1000\,\text{L}}\right)^2\left(\frac{1}{\pi\frac{1}{2}\left(0.75\times10^{-2}\,\text{m}\right)^2} - \frac{1}{\pi\frac{1}{2}\left(7.0\times10^{-2}\,\text{m}\right)^2}\right)$$

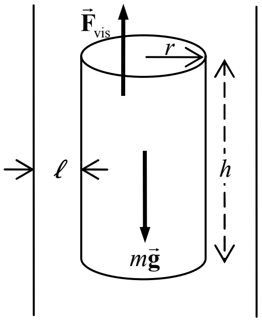
$$= 1259\,\text{N} \approx \boxed{1300\,\text{N}}$$

63. Apply Eq. 13-11 for the viscosity force. Use the average radius to calculate the plate area.

$$F = \eta A\frac{v}{\ell} \rightarrow \eta = \frac{F\ell}{Av} = \frac{\left(\dfrac{\tau}{r_{\text{inner}}}\right)\left(r_{\text{outer}} - r_{\text{inner}}\right)}{\left(2\pi r_{\text{avg}}h\right)\left(\omega r_{\text{inner}}\right)}$$

$$= \frac{\left(\frac{0.024\,\text{m}\bullet\text{N}}{0.0510\,\text{m}}\right)\left(0.20\times10^{-2}\,\text{m}\right)}{2\pi\left(0.0520\,\text{m}\right)\left(0.120\,\text{m}\right)\left(57\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\text{ s}}\right)\left(0.0510\,\text{m}\right)} = \boxed{7.9\times10^{-2}\,\text{Pa}\bullet\text{s}}$$

64. The relationship between velocity and the force of viscosity is given by Eq. 13-11, $F_{\text{vis}} = \eta A\frac{v}{\ell}$. The variable A is the area of contact between the moving surface and the liquid. For a cylinder, $A = 2\pi rh$. The variable ℓ is the thickness of the fluid layer between the two surfaces. See the diagram. If the object falls with terminal velocity, then the net force must be 0, and so the viscous force will equal the weight. Note that $\ell = \frac{1}{2}\left(1.00\,\text{cm} - 0.900\,\text{cm}\right) = 0.05\,\text{cm}$.



$$F_{\text{weight}} = F_{\text{vis}} \rightarrow mg = \eta A\frac{v}{\ell} \rightarrow$$

$$v = \frac{mg\ell}{\eta A} = \frac{mg\ell}{2\pi rh\eta} = \frac{\left(0.15\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(0.050\times10^{-2}\,\text{m}\right)}{2\pi\left(0.450\times10^{-2}\,\text{m}\right)\left(0.300\,\text{m}\right)\left(200\times10^{-3}\,\text{N}\bullet\text{s/m}^2\right)}$$

$$= \boxed{0.43\,\text{m/s}}$$

65. Use Poiseuille's equation (Eq. 13-12) to find the pressure difference.

$$Q = \frac{\pi R^4\left(P_2 - P_1\right)}{8\eta\ell} \rightarrow$$

$$\left(P_2 - P_1\right) = \frac{8Q\eta\ell}{\pi R^4} = \frac{8\left[\frac{6.2\times10^{-3}\,\text{L}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\times\frac{1\times10^{-3}\,\text{m}^3}{1\,\text{L}}\right]\left(0.2\,\text{Pa}\bullet\text{s}\right)\left(8.6\times10^{-2}\,\text{m}\right)}{\pi\left(0.9\times10^{-3}\,\text{m}\right)^4}$$

$$= \boxed{6900\,\text{Pa}}$$

66. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus $Q/R^4 = \frac{V}{t}\frac{1}{R^4}$ is constant. If the volume of water used to water the garden is to be the same in both cases, then tR^4 is constant.

$$t_1R_1^4 = t_2R_2^4 \rightarrow t_2 = t_1\left(\frac{R_1}{R_2}\right)^4 = t_1\left(\frac{3/8}{5/8}\right)^4 = 0.13t_1$$

Thus the $\boxed{\text{time has been cut by 87\%}}$.

$\boxed{67.}$ Use Poiseuille's equation to find the radius, and then double the radius to the diameter.

$$Q = \frac{\pi R^4\left(P_2 - P_1\right)}{8\eta\ell} \rightarrow$$

$$d = 2R = 2\left[\frac{8\eta\ell Q}{\pi\left(P_2 - P_1\right)}\right]^{1/4} = 2\left[\frac{8\left(1.8\times10^{-5}\,\text{Pa}\bullet\text{s}\right)\left(15.5\,\text{m}\right)\left(\dfrac{8.0\times14.0\times4.0\,\text{m}^3}{720\text{ s}}\right)}{\pi\left(0.71\times10^{-3}\text{atm}\right)\left(1.013\times10^5\,\text{Pa/atm}\right)}\right]^{1/4} = \boxed{0.10\,\text{m}}$$

68. Use Poiseuille's equation to find the pressure difference.

$$Q = \frac{\pi R^4\left(P_2 - P_1\right)}{8\eta\ell} \quad\rightarrow$$

$$\left(P_2 - P_1\right) = \frac{8Q\eta\ell}{\pi R^4} = \frac{8\left(650\,\text{cm}^3/\text{s}\right)\left(10^{-6}\,\text{m}^3/\text{cm}^3\right)\left(0.20\,\text{Pa}\bullet\text{s}\right)\left(1.9\times10^3\,\text{m}\right)}{\pi\left(0.145\,\text{m}\right)^4}$$

$$= 1423\,\text{Pa} \approx \boxed{1400\text{ Pa}}$$

69. (*a*) $Re = \dfrac{2\overline{v}r\rho}{\eta} = \dfrac{2\left(0.35\,\text{m/s}\right)\left(0.80\times10^{-2}\,\text{m}\right)\left(1.05\times10^3\,\text{kg/m}^3\right)}{4\times10^{-3}\,\text{Pa}\bullet\text{s}} = 1470$

The flow is $\boxed{\text{laminar}}$ at this speed.

(*b*) Since the velocity is doubled the Reynolds number will double to 2940. The flow is $\boxed{\text{turbulent}}$ at this speed.

70. From Poiseuille's equation, Eq. 13-12, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus Q/R^4 is constant.

$$\frac{Q_{\text{final}}}{R^4_{\text{final}}} = \frac{Q_{\text{initial}}}{R^4_{\text{initial}}} \quad\rightarrow\quad R_{\text{final}} = \left(\frac{Q_{\text{final}}}{Q_{\text{initial}}}\right)^{1/4} R_{\text{initial}} = \left(0.15\right)^{1/4} R_{\text{initial}} = 0.622 R_{\text{initial}}\text{, a } \boxed{38\%} \text{ reduction.}$$

71. The fluid pressure must be 78 torr higher than air pressure as it exits the needle, so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 78 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 13-6b to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q = \frac{\pi R^4\left(P_2 - P_1\right)}{8\eta_{\text{blood}}\ell} \quad\rightarrow\quad P_2 = P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4} = \rho_{\text{blood}} g\Delta h \quad\rightarrow$$

$$\Delta h = \frac{1}{\rho_{\text{blood}} g}\left(P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4}\right)$$

$$= \frac{1}{\left(1.05\times10^3\,\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)}\left(\begin{array}{l}\left(78\,\text{mm-Hg}\right)\left(\dfrac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right) + \\ \dfrac{8\left(4\times10^{-3}\,\text{Pa}\bullet\text{s}\right)\left(2.5\times10^{-2}\,\text{m}\right)\left(\dfrac{2.0\times10^{-6}\,\text{m}^3}{60\text{ s}}\right)}{\pi\left(0.4\times10^{-3}\,\text{m}\right)^4}\end{array}\right)$$

$$= 1.04\,\text{m} \approx \boxed{1.0\,\text{m}}$$

72. In Figure 13-35, we have $\gamma = F/2\ell$. Use this to calculate the force.

$$\gamma = \frac{F}{2\ell} = \frac{3.4\times10^{-3}\,\mathrm{N}}{2(0.070\,\mathrm{m})} = \boxed{2.4\times10^{-2}\,\mathrm{N/m}}$$

73. In Figure 13-35, we have $\gamma = F/2\ell$. Use this relationship to calculate the force.

$$\gamma = F/2\ell \;\rightarrow\; F = 2\gamma\ell = 2(0.025\,\mathrm{N/m})(0.245\,\mathrm{m}) = \boxed{1.2\times10^{-2}\,\mathrm{N}}$$

74. (*a*) We assume that the weight of the platinum ring is negligible. Then the surface tension is the force to lift the ring, divided by the length of surface that is being pulled. Surface tension will act at both edges of the ring, as in Figure 13-35b. Thus $\boxed{\gamma = \frac{F}{2(2\pi r)} = \frac{F}{4\pi r}}$

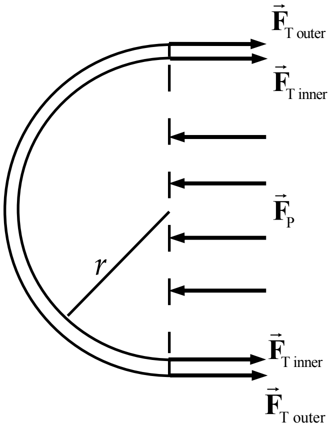
(*b*) $\gamma = \frac{F}{4\pi r} = \frac{5.80\times10^{-3}\,\mathrm{N}}{4\pi(2.8\times10^{-2}\,\mathrm{m})} = \boxed{1.6\times10^{-2}\,\mathrm{N/m}}$

75. As an estimate, we assume that the surface tension force acts vertically. We assume that the free-body diagram for the cylinder is similar to Figure 13-37(a) in the text. The weight must equal the total surface tension force. The needle is of length ℓ.

$$mg = 2F_{\mathrm{T}} \;\rightarrow\; \rho_{\mathrm{needle}}\pi\left(\tfrac{1}{2}d_{\mathrm{needle}}\right)^2\ell g = 2\gamma\ell \;\rightarrow$$

$$d_{\mathrm{needle}} = \sqrt{\frac{8\gamma}{\rho_{\mathrm{needle}}\pi g}} = \sqrt{\frac{8(0.072\,\mathrm{N/m})}{(7800\,\mathrm{kg/m^3})\pi(9.80\,\mathrm{m/s^2})}} = 1.55\times10^{-3}\,\mathrm{m} \approx \boxed{1.5\,\mathrm{mm}}$$

76. Consider half of the soap bubble – a hemisphere. The forces on the hemisphere will be the surface tensions on the two circles and the net force from the excess pressure between the inside and the outside of the bubble. This net force is the sum of all the forces perpendicular to the surface of the hemisphere, but must be parallel to the surface tension. Therefore we can find it by finding the force on the circle that is the base of the hemisphere. The total force must be zero. Note that the forces $\vec{\mathbf{F}}_{\mathrm{T\,outer}}$ and $\vec{\mathbf{F}}_{\mathrm{T\,inner}}$ act over the entire length of the circles to which they are applied. The diagram may look like there are 4 tension forces, but there are only 2. Likewise, there is only 1 pressure force, $\vec{\mathbf{F}}_{\mathrm{P}}$, but it acts over the area of the hemisphere.



$$2F_{\mathrm{T}} = F_{\mathrm{P}} \;\rightarrow\; 2(2\pi r\gamma) = \pi r^2\Delta P \;\rightarrow\; \boxed{\Delta P = \frac{4\gamma}{r}}$$

77. The mass of liquid that rises in the tube will have the force of gravity acting down on it, and the force of surface tension acting upwards. The two forces must be equal for the liquid to be in equilibrium. The surface tension force is the surface tension times the circumference of the tube, since the tube circumference is the length of the "cut" in the liquid surface. The mass of the risen liquid is the density times the volume.

$$F_{\mathrm{T}} = mg \;\rightarrow\; \gamma 2\pi r = \rho\pi r^2 hg \;\rightarrow\; \boxed{h = 2\gamma/\rho g r}$$

78. (*a*) The fluid in the needle is confined, and so Pascal's principle may be applied.

$$P_{\text{plunger}} = P_{\text{needle}} \quad \rightarrow \quad \frac{F_{\text{plunger}}}{A_{\text{plunger}}} = \frac{F_{\text{needle}}}{A_{\text{needle}}} \quad \rightarrow$$

$$F_{\text{needle}} = F_{\text{plunger}}\frac{A_{\text{needle}}}{A_{\text{plunger}}} = F_{\text{plunger}}\frac{\pi r^2_{\text{needle}}}{\pi r^2_{\text{plunger}}} = F_{\text{plunger}}\frac{r^2_{\text{needle}}}{r^2_{\text{plunger}}} = (2.8\,\text{N})\frac{\left(0.10\times10^{-3}\,\text{m}\right)^2}{\left(0.65\times10^{-2}\,\text{m}\right)^2}$$

$$= 6.627\times10^{-4}\,\text{N} \approx \boxed{6.6\times10^{-4}\,\text{N}}$$

(*b*) $F_{\text{plunger}} = P_{\text{plunger}}A_{\text{plunger}} = (75\,\text{mm-Hg})\left(\frac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)\pi\left(0.65\times10^{-2}\,\text{m}\right)^2 = \boxed{1.3\,\text{N}}$

79. The pressures for parts (*a*) and (*b*) stated in this problem are gauge pressures, relative to atmospheric pressure. The pressure change due to depth in a fluid is given by $\Delta P = \rho g \Delta h$.

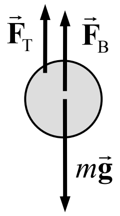
(*a*) $$\Delta h = \frac{\Delta P}{\rho g} = \frac{(55\,\text{mm-Hg})\left(\frac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)}{\left(1.00\frac{\text{g}}{\text{cm}^3}\times\frac{1\,\text{kg}}{1000\,\text{g}}\times\frac{10^6\,\text{cm}^3}{1\,\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{0.75\,\text{m}}$$

(*b*) $$\Delta h = \frac{\Delta P}{\rho g} = \frac{(650\,\text{mm-H}_2\text{O})\left(\frac{9.81\,\text{N/m}^2}{1\,\text{mm-H}_2\text{O}}\right)}{\left(1.00\frac{\text{g}}{\text{cm}^3}\times\frac{1\,\text{kg}}{1000\,\text{g}}\times\frac{10^6\,\text{cm}^3}{1\,\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{0.65\,\text{m}}$$

(*c*) For the fluid to just barely enter the vein, the fluid pressure must be the same as the blood pressure.

$$\Delta h = \frac{\Delta P}{\rho g} = \frac{(78\,\text{mm-Hg})\left(\frac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)}{\left(1.00\frac{\text{g}}{\text{cm}^3}\times\frac{1\,\text{kg}}{1000\,\text{g}}\times\frac{10^6\,\text{cm}^3}{1\,\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)} = 1.059\,\text{m} \approx \boxed{1.1\,\text{m}}$$

80. The ball has three vertical forces on it – string tension, buoyant force, and gravity. See the free-body diagram for the ball. The net force must be 0.



$$F_{\text{net}} = F_\text{T} + F_\text{B} - mg = 0 \quad \rightarrow$$

$$F_\text{T} = mg - F_\text{B} = \tfrac{4}{3}\pi r^3 \rho_{\text{Cu}} g - \tfrac{4}{3}\pi r^3 \rho_{\text{water}} g = \tfrac{4}{3}\pi r^3 g\left(\rho_{\text{Cu}} - \rho_{\text{water}}\right)$$

$$= \tfrac{4}{3}\pi(0.013\,\text{m})^3\left(9.80\,\text{m/s}^2\right)\left(8900\,\text{kg/m}^3 - 1000\,\text{kg/m}^3\right) = 0.7125\,\text{N} \approx \boxed{0.71\,\text{N}}$$

Since the water pushes up on the ball via the buoyant force, there is a downward force on the water due to the ball, equal in magnitude to the buoyant force. That mass-equivalent of that force (indicated by $m_\text{B} = F_\text{B}/g$) will show up as an increase in the balance reading.

$$F_\text{B} = \tfrac{4}{3}\pi r^3 \rho_{\text{water}} g \quad \rightarrow$$

$$m_\text{B} = \frac{F_\text{B}}{g} = \tfrac{4}{3}\pi r^3 \rho_{\text{water}} = \tfrac{4}{3}\pi(0.013\,\text{m})^3\left(1000\,\text{kg/m}^3\right) = 9.203\times10^{-3}\,\text{kg} = 9.203\,\text{g}$$

Balance reading = $998.0\,\text{g} + 9.2\,\text{g} = \boxed{1007.2\,\text{g}}$

81. The change in pressure with height is given by $\Delta P = \rho g \Delta h$.

$$\Delta P = \rho g \Delta h \rightarrow \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{(1.29\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(380\,\text{m})}{1.013\times10^5\,\text{Pa}} = 0.047 \rightarrow$$

$$\boxed{\Delta P = 0.047\,\text{atm}}$$

82. (*a*) The input pressure is equal to the output pressure.

$$P_{\text{input}} = P_{\text{output}} \rightarrow \frac{F_{\text{input}}}{A_{\text{input}}} = \frac{F_{\text{output}}}{A_{\text{output}}} \rightarrow$$

$$A_{\text{input}} = A_{\text{output}}\frac{F_{\text{input}}}{F_{\text{output}}} = \pi\left(9.0\times10^{-2}\,\text{m}\right)^2 \frac{350\,\text{N}}{(920\,\text{kg})(9.80\,\text{m/s}^2)} = 9.878\times10^{-4}\,\text{m}^2$$

$$\approx \boxed{9.9\times10^{-4}\,\text{m}^2}$$

(*b*) The work is the force needed to lift the car (its weight) times the vertical distance lifted.

$$W = mgh = (920\,\text{kg})(9.80\,\text{m/s}^2)(0.42\,\text{m}) = 3787\,\text{J} \approx \boxed{3800\,\text{J}}$$

(*c*) The work done by the input piston is equal to the work done in lifting the car.

$$W_{\text{input}} = W_{\text{output}} \rightarrow F_{\text{input}} d_{\text{input}} = F_{\text{output}} d_{\text{ouptut}} = mgh \rightarrow$$

$$h = \frac{F_{\text{input}} d_{\text{input}}}{mg} = \frac{(350\,\text{N})(0.13\,\text{m})}{(920\,\text{kg})(9.80\,\text{m/s}^2)} = 5.047\times10^{-3}\,\text{m} \approx \boxed{5.0\times10^{-3}\,\text{m}}$$

(*d*) The number of strokes is the full distance divided by the distance per stroke.

$$h_{\text{full}} = N h_{\text{stroke}} \rightarrow N = \frac{h_{\text{full}}}{h_{\text{stroke}}} = \frac{0.42\,\text{m}}{5.047\times10^{-3}\,\text{m}} = \boxed{83\,\text{strokes}}$$

(*e*) The work input is the input force times the total distance moved by the input piston.

$$W_{\text{input}} = N F_{\text{input}} d_{\text{input}} \rightarrow 83(350\,\text{N})(0.13\,\text{m}) = 3777\,\text{J} \approx \boxed{3800\,\text{J}}$$

Since the work input is equal to the work output, energy is conserved.

83. The pressure change due to a change in height is given by $\Delta P = \rho g \Delta h$. That pressure is the excess force on the eardrum, divided by the area of the eardrum.

$$\Delta P = \rho g \Delta h = \frac{F}{A} \rightarrow$$

$$F = \rho g \Delta h A = (1.29\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(950\,\text{m})(0.20\times10^{-4}\,\text{m}^2) = \boxed{0.24\,\text{N}}$$

84. The change in pressure with height is given by $\Delta P = \rho g \Delta h$.

$$\Delta P = \rho g \Delta h \rightarrow \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{(1.05\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(6\,\text{m})}{1.013\times10^5\,\text{Pa}} = 0.609 \rightarrow$$

$$\boxed{\Delta P = 0.6\,\text{atm}}$$

$\boxed{85.}$ The pressure difference due to the lungs is the pressure change in the column of water.

$$\Delta P=\rho g\Delta h \quad\rightarrow\quad \Delta h=\frac{\Delta P}{\rho g}=\frac{(75\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\ \text{mm-Hg}}\right)}{(1.00\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)}=1.018\ \text{m}\approx\boxed{1.0\ \text{m}}$$

86. We use the relationship developed in Example 13-5.

$$P=P_0e^{-(\rho_0 g/P_0)y}=(1.0\,\text{atm})\,e^{-(1.25\times10^{-4}\,\text{m}^{-1})(2400\,\text{m})}=\boxed{0.74\,\text{atm}}$$

87. The buoyant force, equal to the weight of mantle displaced, must be equal to the weight of the continent. Let *h* represent the full height of the continent, and *y* represent the height of the continent above the surrounding rock.

$$W_{\text{continent}}=W_{\substack{\text{displaced}\\ \text{mantle}}} \quad\rightarrow\quad Ah\rho_{\text{continent}}g=A(h-y)\rho_{\text{mantle}}g \quad\rightarrow$$

$$y=h\left(1-\frac{\rho_{\text{continent}}}{\rho_{\text{mantle}}}\right)=(35\,\text{km})\left(1-\frac{2800\,\text{kg/m}^3}{3300\,\text{kg/m}^3}\right)=\boxed{5.3\,\text{km}}$$

88. The "extra" buoyant force on the ship, due to the loaded fresh water, is the weight of "extra" displaced seawater, as indicated by the ship floating lower in the sea. This buoyant force is given by $F_{\text{buoyant}}=V_{\text{displaced}}\rho_{\substack{\text{sea}\\ \text{water}}}g$. But this "extra" buoyant force is what holds up the fresh water, and so must also be equal to the weight of the fresh water.

$$F_{\text{buoyant}}=V_{\text{displaced}}\rho_{\substack{\text{sea}\\ \text{water}}}g=m_{\text{fresh}}g \quad\rightarrow\quad m_{\text{fresh}}=(2240\,\text{m}^2)(8.50\,\text{m})(1025\,\text{kg/m}^3)=\boxed{1.95\times10^7\,\text{kg}}$$

This can also be expressed as a volume.

$$V_{\text{fresh}}=\frac{m_{\text{fresh}}}{\rho_{\text{fresh}}}=\frac{1.95\times10^7\,\text{kg}}{1.00\times10^3\,\text{kg/m}^3}=\boxed{1.95\times10^4\,\text{m}^3}=\boxed{1.95\times10^7\,\text{L}}$$

89. (*a*) We assume that the one descending is close enough to the surface of the Earth that constant density may be assumed. Take Eq. 13-6b, modify it for rising, and differentiate it with respect to time.

$$P=P_0-\rho g y \quad\rightarrow$$

$$\frac{dP}{dt}=-\rho g\frac{dy}{dt}=-(1.29\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(-7.0\,\text{m/s})=88.49\,\text{Pa/s}\approx\boxed{88\,\text{Pa/s}}$$

(*b*) $\Delta y=vt \quad\rightarrow\quad t=\dfrac{\Delta y}{v}=\dfrac{350\,\text{m}}{7.0\,\text{m/s}}=\boxed{50\,\text{s}}\ (2\text{ sig. fig.})$

90. The buoyant force must be equal to the weight of the water displaced by the full volume of the logs, and must also be equal to the full weight of the raft plus the passengers. Let *N* represent the number of passengers.

weight of water displaced by logs = weight of people + weight of logs

$$12\left(V_{\log}\rho_{\text{water}}g\right)=Nm_{\text{person}}g+12\left(V_{\log}\rho_{\log}g\right) \quad\rightarrow$$

$$N=\frac{12V_{\log}\left(\rho_{\text{water}}-\rho_{\log}\right)}{m_{\text{person}}}=\frac{12\pi r_{\log}^2 l_{\log}\left(\rho_{\text{water}}-SG_{\log}\rho_{\text{water}}\right)}{m_{\text{person}}}=\frac{12\pi r_{\log}^2 l_{\log}\rho_{\text{water}}\left(1-SG_{\log}\right)}{m_{\text{person}}}$$

$$= \frac{12\pi\left(0.225\,\text{m}\right)^2\left(6.1\,\text{m}\right)\left(1000\,\text{kg/m}^3\right)\left(1-0.60\right)}{68\,\text{kg}} = 68.48$$

Thus $\boxed{68}$ people can stand on the raft without getting wet. When the 69th person gets on, the raft will go under the surface.

$\boxed{91.}$ We assume that the air pressure is due to the weight of the atmosphere, with the area equal to the surface area of the Earth.

$$P = \frac{F}{A} \;\rightarrow\; F = PA = mg \;\rightarrow$$

$$m = \frac{PA}{g} = \frac{4\pi R_{\text{Earth}}^2 P}{g} = \frac{4\pi\left(6.38\times10^6\,\text{m}\right)^2\left(1.013\times10^5\,\text{N/m}^2\right)}{9.80\,\text{m/s}^2} = 5.29\times10^{18}\,\text{kg} \approx \boxed{5\times10^{18}\,\text{kg}}$$

92. The work done during each heartbeat is the force on the fluid times the distance that the fluid moves in the direction of the force.

$$W = F\Delta l = PA\Delta l = PV \;\rightarrow$$

$$\text{Power} = \frac{W}{t} = \frac{PV}{t} = \frac{\left(105\,\text{mm-Hg}\right)\left(\dfrac{133\,\text{N/m}^2}{1\text{ mm-Hg}}\right)\left(70\times10^{-6}\,\text{m}^3\right)}{\left(\dfrac{1}{70}\text{min}\right)\left(\dfrac{60\text{ s}}{\text{min}}\right)} = 1.1\,\text{W} \approx \boxed{1\,\text{W}}$$

93. (*a*) We assume that the water is launched at ground level. Since it also lands at ground level, the level range formula from Example 3-10 may be used.

$$R = \frac{v_0^2\sin 2\theta}{g} \;\rightarrow\; v_0 = \sqrt{\frac{Rg}{\sin 2\theta}} = \sqrt{\frac{\left(7.0\,\text{m}\right)\left(9.80\,\text{m/s}^2\right)}{\sin 70^\circ}} = 8.544\,\text{m/s} \approx \boxed{8.5\,\text{m/s}}$$

(*b*) The volume rate of flow is the area of the flow times the speed of the flow. Multiply by 4 for the 4 heads.

$$\text{Volume flow rate} = Av = 4\pi r^2 v = 4\pi\left(1.5\times10^{-3}\,\text{m}\right)^2\left(8.544\,\text{m/s}\right)$$

$$= 2.416\times10^{-4}\,\text{m}^3\text{/s}\left(\frac{1\text{ L}}{1.0\times10^{-3}\,\text{m}^3}\right) \approx \boxed{0.24\,\text{L/s}}$$

(*c*) Use the equation of continuity to calculate the flow rate in the supply pipe.

$$\left(Av\right)_{\text{supply}} = \left(Av\right)_{\text{heads}} \;\rightarrow\; v_{\text{supply}} = \frac{\left(Av\right)_{\text{heads}}}{A_{\text{supply}}} = \frac{2.416\times10^{-4}\,\text{m}^3\text{/s}}{\pi\left(0.95\times10^{-2}\,\text{m}\right)^2} = \boxed{0.85\,\text{m/s}}$$

94. The buoyant force on the rock is the force that would be on a mass of water with the same volume as the rock. Since the equivalent mass of water is accelerating upward, that same acceleration must be taken into account in the calculation of the buoyant force.

$$F_{\text{buoyant}} - m_{\text{water}}g = m_{\text{water}}a \;\rightarrow$$

$$F_{\text{buoyant}} = m_{\text{water}}\left(g+a\right) = V_{\text{water}}\rho_{\text{water}}\left(g+a\right) = V_{\text{rock}}\rho_{\text{water}}\left(g+a\right) = \frac{m_{\text{rock}}}{\rho_{\text{rock}}}\rho_{\text{water}}\left(g+a\right)$$

$$= \frac{m_{\text{rock}}}{SG_{\text{rock}}}\left(g+1.8g\right) = \frac{\left(3.0\,\text{kg}\right)2.8\left(9.80\,\text{m/s}^2\right)}{2.7} = 30.49\text{ N} \approx \boxed{30\,\text{N}}\ \left(2\text{ sig. fig.}\right)$$

For the rock to not sink, the upward buoyant force on the rock minus the weight of the rock must be equal to the net force on the rock.

$$F_{\text{buoyant}} - m_{\text{rock}}g = m_{\text{rock}}a \quad \rightarrow \quad F_{\text{buoyant}} = m_{\text{rock}}(g+a) = (3.0\,\text{kg})2.8(9.80\,\text{m/s}^2) = 82\,\text{N}$$

$\boxed{\text{The rock will sink}}$, because the buoyant force is not large enough to “float” the rock.

95. Apply both Bernoulli’s equation and the equation of continuity at the two locations of the stream, with the faucet being location 0 and the lower position being location 1. The pressure will be air pressure at both locations. The lower location has $y_1 = 0$ and the faucet is at height $y_0 = y$.

$$A_0v_0 = A_1v_1 \quad \rightarrow \quad v_1 = v_0\frac{A_0}{A_1} = v_0\frac{\pi(d_0/2)^2}{\pi(d_1/2)^2} = v_0\frac{d_0^2}{d_1^2} \quad \rightarrow$$

$$P_0 + \tfrac{1}{2}\rho v_0^2 + \rho g y_0 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 \quad \rightarrow \quad v_0^2 + 2gy = v_1^2 = v_0^2\frac{d_0^4}{d_1^4} \quad \rightarrow$$

$$\boxed{d_1 = d_0\left(\frac{v_0^2}{v_0^2 + 2gy}\right)^{1/4}}$$

96. (*a*) Apply Bernoulli’s equation between the surface of the water in the sink and the lower end of the siphon tube. Note that both are open to the air, and so the pressure at both is air pressure.

$$P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} = P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} \quad \rightarrow$$

$$v_{\text{bottom}} = \sqrt{2g(y_{\text{top}} - y_{\text{bottom}})} = \sqrt{2(9.80\,\text{m/s}^2)(0.44\,\text{m})} = 2.937\,\text{m/s} \approx \boxed{2.9\,\text{m/s}}$$

(*b*) The volume flow rate (at the lower end of the tube) times the elapsed time must equal the volume of water in the sink.

$$(Av)_{\text{lower}}\Delta t = V_{\text{sink}} \quad \rightarrow \quad \Delta t = \frac{V_{\text{sink}}}{(Av)_{\text{lower}}} = \frac{(0.38\,\text{m}^2)(4.0\times10^{-2}\,\text{m})}{\pi(1.0\times10^{-2}\,\text{m})^2(2.937\,\text{m/s})} = 16.47\,\text{s} \approx \boxed{16\,\text{s}}$$

$\boxed{97.}$ The upward force due to air pressure on the bottom of the wing must be equal to the weight of the airplane plus the downward force due to air pressure on the top of the wing. Bernoulli’s equation can be used to relate the forces due to air pressure. We assume that there is no appreciable height difference between the top and the bottom of the wing.

$$P_{\text{top}}A + mg = P_{\text{bottom}}A \quad \rightarrow \quad (P_{\text{bottom}} - P_{\text{top}}) = \frac{mg}{A}$$

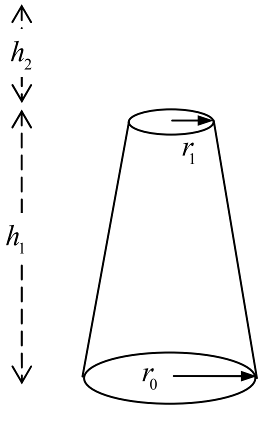
$$P_0 + P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} = P_0 + P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}}$$

$$v_{\text{top}}^2 = \frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2 \quad \rightarrow$$

$$v_{\text{top}} = \sqrt{\frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2} = \sqrt{\frac{2mg}{\rho A} + v_{\text{bottom}}^2} = \sqrt{\frac{2(1.7\times10^6\,\text{kg})(9.80\,\text{m/s}^2)}{(1.29\,\text{kg/m}^3)(1200\,\text{m}^2)} + (95\,\text{m/s})^2}$$

$$= 174.8\,\text{m/s} \approx \boxed{170\,\text{m/s}}$$

98. We label three vertical levels. Level 0 is at the pump, and the supply tube has a radius of r_0 at that location. Level 1 is at the nozzle, and the nozzle has a radius of r_1. Level 1 is a height h_1 above level 0. Level 2 is the highest point reached by the water. Level 2 is a height h_2 above level 1. We may write Bernoulli's equation relating any 2 of the levels, and we may write the equation of continuity relating any 2 of the levels. The desired result is the gauge pressure of the pump, which would be $P_0 - P_{\text{atm}}$. Start by using Bernoulli's equation to relate level 0 to level 1.



$$P_0 + \rho g h_0 + \tfrac{1}{2}\rho v_0^2 = P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2$$

Since level 1 is open to the air, $P_1 = P_{\text{atm}}$. Use that in the above equation.

$$P_0 - P_{\text{atm}} = \rho g h_1 + \tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho v_0^2$$

Use the equation of continuity to relate level 0 to level 1, and then use that result in the Bernoulli expression above.

$$A_0 v_0 = A_1 v_1 \rightarrow \pi r_0^2 v_0 = \pi r_1^2 v_1 \rightarrow v_0 = \frac{r_1^2}{r_0^2} v_1 = \frac{d_1^2}{d_0^2} v_1$$

$$P_0 - P_{\text{atm}} = \rho g h_1 + \tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho \left(\frac{d_1^2}{d_0^2} v_1 \right)^2 = \rho g h_1 + \tfrac{1}{2}\rho v_1^2 \left(1 - \frac{d_1^4}{d_0^4} \right)$$

Use Bernoulli's equation to relate levels 1 and 2. Since both levels are open to the air, the pressures are the same. Also note that the speed at level 2 is zero. Use that result in the Bernoulli expression above.

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g \left(h_1 + h_2 \right) + \tfrac{1}{2}\rho v_2^2 \rightarrow v_1^2 = 2 g h_2$$

$$P_0 - P_{\text{atm}} = \rho g h_1 + \tfrac{1}{2}\rho v_1^2 \left(1 - \frac{d_1^4}{d_0^4} \right) = \rho g \left[h_1 + h_2 \left(1 - \frac{d_1^4}{d_0^4} \right) \right]$$

$$= \left(1.00 \times 10^3 \,\text{kg/m}^3 \right) \left(9.80 \,\text{m/s}^2 \right) \left[1.1 \,\text{m} + \left(0.14 \,\text{m} \right) \left(1 - 0.5^4 \right) \right]$$

$$= 12066 \,\text{N/m}^2 \approx \boxed{1.2 \times 10^4 \,\text{N/m}^2}$$

99. We assume that there is no appreciable height difference to be considered between the two sides of the window. Then the net force on the window due to the air is the difference in pressure on the two sides of the window, times the area of the window. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 A_{\text{roof}} = \tfrac{1}{2}\left(1.29 \,\text{kg/m}^3 \right) \left[\left(200 \,\text{km/h} \right) \left(\frac{1 \,\text{m/s}}{3.6 \,\text{km/h}} \right) \right]^2 \left(6.0 \,\text{m}^2 \right) = \boxed{1.2 \times 10^4 \,\text{N}}$$

100. From Poiseuille's equation, the viscosity can be found from the volume flow rate, the geometry of the tube, and the pressure difference. The pressure difference over the length of the tube is the same as the pressure difference due to the height of the reservoir, assuming that the open end of the needle is at atmospheric pressure.

$$Q=\frac{\pi R^4\left(P_2-P_1\right)}{8\eta\ell}\ ;\ P_2-P_1=\rho_{\text{blood}}gh\ \rightarrow$$

$$\eta=\frac{\pi R^4\left(P_2-P_1\right)}{8Q\ell}=\frac{\pi R^4\rho_{\text{blood}}gh}{8Q\ell}=\frac{\pi\left(0.20\times10^{-3}\,\text{m}\right)^4\left(1.05\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(1.30\,\text{m}\right)}{8\left[4.1\frac{\text{cm}^3}{\text{min}}\times\frac{1\text{ min}}{60\text{ s}}\times\frac{10^{-6}\text{m}^3}{\text{cm}^3}\right]\left(3.8\times10^{-2}\,\text{m}\right)}$$

$$=\boxed{3.2\times10^{-3}\,\text{Pa}\bullet\text{s}}$$

101. The net force is 0 if the balloon is moving at terminal velocity. Therefore the upwards buoyancy force (equal to the weight of the displaced air) must be equal to the net downwards force of the weight of the balloon material plus the weight of the helium plus the drag force at terminal velocity. Find the terminal velocity, and use that to find the time to rise 12 m.

$$F_{\text{B}}=m_{\text{balloon}}g+m_{\text{Helium}}g+F_{\text{D}}\ \rightarrow\ \tfrac{4}{3}\pi r^3\rho_{\text{air}}g=m_{\text{balloon}}g+\tfrac{4}{3}\pi r^3\rho_{\text{He}}g+\tfrac{1}{2}C_{\text{D}}\rho_{\text{air}}\pi r^2v_{\text{T}}^2\ \rightarrow$$

$$v_{\text{T}}=\sqrt{\frac{2\left[\frac{4}{3}\pi r^3\left(\rho_{\text{air}}-\rho_{\text{He}}\right)-m\right]g}{C_{\text{D}}\rho_{\text{air}}\pi r^2}}=\frac{h}{t}\ \rightarrow\ t=h\sqrt{\frac{C_{\text{D}}\rho_{\text{air}}\pi r^2}{2\left[\frac{4}{3}\pi r^3\left(\rho_{\text{air}}-\rho_{\text{He}}\right)-m\right]g}}\ \rightarrow$$

$$t=\left(12\,\text{m}\right)\sqrt{\frac{(0.47)\left(1.29\,\text{kg/m}^3\right)\pi\left(0.15\,\text{m}\right)^2}{2\left[\frac{4}{3}\pi\left(0.15\,\text{m}\right)^3\left(1.29\,\text{kg/m}^3-0.179\text{kg/m}^3\right)-\left(0.0028\,\text{kg}\right)\right]\left(9.80\,\text{m/s}^2\right)}}=\boxed{4.9\,\text{s}}$$

102. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus Q/R^4 is constant. Also, if the diameter is reduced by 15%, so is the radius.

$$\frac{Q_{\text{final}}}{R_{\text{final}}^4}=\frac{Q_{\text{initial}}}{R_{\text{initial}}^4}\ \rightarrow\ \frac{Q_{\text{final}}}{Q_{\text{initial}}}=\frac{R_{\text{final}}^4}{R_{\text{initial}}^4}=\left(0.85\right)^4=0.52$$

$\boxed{\text{The flow rate is 52\% of the original value.}}$

$\boxed{103.}$ Use the definition of density and specific gravity, and then solve for the fat fraction, *f*.

$$m_{\text{fat}}=mf=V_{\text{fat}}\rho_{\text{fat}}\ ;\ m_{\substack{\text{fat}\\ \text{free}}}=m\left(1-f\right)=V_{\substack{\text{fat}\\ \text{free}}}\rho_{\substack{\text{fat}\\ \text{free}}}$$

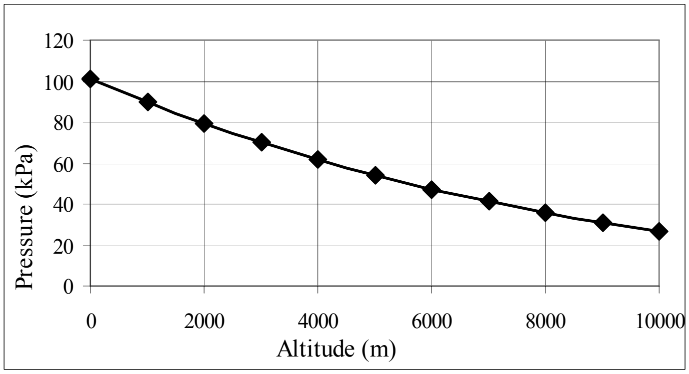
$$\rho_{\text{body}}=X\rho_{\text{water}}=\frac{m_{\text{total}}}{V_{\text{total}}}=\frac{m_{\text{fat}}+m_{\substack{\text{fat}\\ \text{free}}}}{V_{\text{fat}}+V_{\substack{\text{fat}\\ \text{free}}}}=\frac{m}{\frac{mf}{\rho_{\text{fat}}}+\frac{m\left(1-f\right)}{\rho_{\substack{\text{fat}\\ \text{free}}}}}=\frac{1}{\frac{f}{\rho_{\text{fat}}}+\frac{\left(1-f\right)}{\rho_{\substack{\text{fat}\\ \text{free}}}}}\ \rightarrow$$

$$f=\frac{\rho_{\text{fat}}\rho_{\substack{\text{fat}\\ \text{free}}}}{X\rho_{\text{water}}\left(\rho_{\substack{\text{fat}\\ \text{free}}}-\rho_{\text{fat}}\right)}-\frac{\rho_{\text{fat}}}{\left(\rho_{\substack{\text{fat}\\ \text{free}}}-\rho_{\text{fat}}\right)}=\frac{\left(0.90\,\text{g/cm}^3\right)\left(1.10\,\text{g/cm}^3\right)}{X\left(1.0\,\text{g/cm}^3\right)\left(0.20\,\text{g/cm}^3\right)}-\frac{\left(0.90\,\text{g/cm}^3\right)}{\left(0.20\,\text{g/cm}^3\right)}$$

$$=\frac{4.95}{X}-4.5\ \rightarrow\ \%\text{ Body fat}=100f=100\left(\frac{4.95}{X}-4.5\right)=\boxed{\frac{495}{X}-450}$$

104. The graph is shown. The best-fit equations as calculated by Excel are also shown. Let P represent the pressure in kPa and y the altitude in m.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH13.XLS," on tab "Problem 13.104."



(*a*) Quadratic fit: $P_{\text{quad}} = \left(3.9409\times10^{-7}\right)y^2 - \left(1.1344\times10^{-2}\right)y + 100.91$,

(*b*) Exponential fit: $P_{\exp} = 103.81\,e^{-\left(1.3390\times10^{-4}\right)y}$

(*c*) $P_{\text{quad}} = \left(3.9409\times10^{-7}\right)(8611)^2 - \left(1.1344\times10^{-2}\right)(8611) + 100.91 = \boxed{32.45\,\text{kPa}}$

$P_{\exp} = 103.81e^{-\left(1.3390\times10^{-4}\right)(8611)} = \boxed{32.77\,\text{kPa}}$

$$\%\,\text{diff} = \frac{100\left(P_{\exp} - P_{\text{quad}}\right)}{\tfrac{1}{2}\left(P_{\exp} + P_{\text{quad}}\right)} = \frac{200\left(P_{\exp} - P_{\text{quad}}\right)}{\left(P_{\exp} + P_{\text{quad}}\right)} = \frac{200\left(32.77\,\text{kPa} - 32.45\,\text{kPa}\right)}{\left(32.77\,\text{kPa} + 32.45\,\text{kPa}\right)} = \boxed{0.98\,\%}$$

CHAPTER 14: Oscillations

Responses to Questions

1. Examples are: a child's swing (SHM, for small oscillations), stereo speakers (complicated motion, the addition of many SHMs), the blade on a jigsaw (approximately SHM), the string on a guitar (complicated motion, the addition of many SHMs).

2. The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.

3. When the engine is running at constant speed, the piston will have a constant period. The piston has zero velocity at the top and bottom of its path. Both of these properties are also properties of SHM. In addition, there is a large force exerted on the piston at one extreme of its motion, from the combustion of the fuel–air mixture, and in SHM the largest forces occur at the extremes of the motion.

4. The true period will be larger and the true frequency will be smaller. The spring needs to accelerate not only the mass attached to its end, but also its own mass. As a mass on a spring oscillates, potential energy is converted into kinetic energy. The maximum potential energy depends on the displacement of the mass. This maximum potential energy is converted into the maximum kinetic energy, but if the mass being accelerated is larger then the velocity will be smaller for the same amount of energy. A smaller velocity translates into a longer period and a smaller frequency.

5. The maximum speed of a simple harmonic oscillator is given by $v = A\sqrt{\frac{k}{m}}$. The maximum speed can be doubled by doubling the amplitude, A.

6. Before the trout is released, the scale reading is zero. When the trout is released, it will fall downward, stretching the spring to beyond its equilibrium point so that the scale reads something over 5 kg. Then the spring force will pull the trout back up, again to a point beyond the equilibrium point, so that the scale will read something less than 5 kg. The spring will undergo damped oscillations about equilibrium and eventually come to rest at equilibrium. The corresponding scale readings will oscillate about the 5-kg mark, and eventually come to rest at 5 kg.

7. At high altitude, g is slightly smaller than it is at sea level. If g is smaller, then the period T of the pendulum clock will be longer, and the clock will run slow (or lose time).

8. The tire swing is a good approximation of a simple pendulum. Pull the tire back a short distance and release it, so that it oscillates as a pendulum in simple harmonic motion with a small amplitude. Measure the period of the oscillations and calculate the length of the pendulum from the expression $T = 2\pi\sqrt{\frac{\ell}{g}}$. The length, ℓ, is the distance from the center of the tire to the branch. The height of the branch is ℓ plus the height of the center of the tire above the ground.

9. The displacement and velocity vectors are in the same direction while the oscillator is moving away from its equilibrium position. The displacement and acceleration vectors are never in the same direction.

10. The period will be unchanged, so the time will be (*c*), two seconds. The period of a simple pendulum oscillating with a small amplitude does not depend on the mass.

11. The two masses reach the equilibrium point simultaneously. The angular frequency is independent of amplitude and will be the same for both systems.

12. Empty. The period of the oscillation of a spring increases with increasing mass, so when the car is empty the period of the harmonic motion of the springs will be shorter, and the car will bounce faster.

$\boxed{13.}$ When walking at a normal pace, about 1 s (timed). The faster you walk, the shorter the period. The shorter your legs, the shorter the period.

14. When you rise to a standing position, you raise your center of mass and effectively shorten the length of the swing. The period of the swing will decrease.

15. The frequency will decrease. For a physical pendulum, the period is proportional to the square root of the moment of inertia divided by the mass. When the small sphere is added to the end of the rod, both the moment of inertia and the mass of the pendulum increase. However, the increase in the moment of inertia will be greater because the added mass is located far from the axis of rotation. Therefore, the period will increase and the frequency will decrease.

16. When the 264-Hz fork is set into vibration, the sound waves generated are close enough in frequency to the resonance frequency of the 260-Hz fork to cause it to vibrate. The 420-Hz fork has a resonance frequency far from 264 Hz and far from the harmonic at 528 Hz, so it will not begin to vibrate.

17. If you shake the pan at a resonant frequency, standing waves will be set up in the water and it will slosh back and forth. Shaking the pan at other frequencies will not create large waves. The individual water molecules will move but not in a coherent way.

18. Examples of resonance are: pushing a child on a swing (if you push at one of the limits of the oscillation), blowing across the top of a bottle, producing a note from a flute or organ pipe.

$\boxed{19.}$ Yes. Rattles which occur only when driving at certain speeds are most likely resonance phenomena.

20. Building with lighter materials doesn't necessarily make it easier to set up resonance vibrations, but it does shift the fundamental frequency and decrease the ability of the building to dampen oscillations. Resonance vibrations will be more noticeable and more likely to cause damage to the structure.

Solutions to Problems

$\boxed{1.}$ The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A$. So the total distance $= 4A = 4\left(0.18\,\text{m}\right) = \boxed{0.72\,\text{m}}$.

2. The spring constant is the ratio of external applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{180\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.65\text{ m}} = \frac{105\text{ N}}{0.20\text{ m}} = 525\,\text{N/m} \approx \boxed{530\,\text{N/m}}$$

3. The spring constant is found from the ratio of applied force to displacement.

$$k=\frac{F_{\text{ext}}}{x}=\frac{mg}{x}=\frac{(68\text{ kg})(9.80\,\text{m/s}^2)}{5.0\times10^{-3}\text{m}}=1.333\times10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{1.333\times10^5\text{ N/m}}{1568\text{ kg}}}=1.467\text{ Hz}\approx\boxed{1.5\text{ Hz}}$$

4. (*a*) The motion starts at the maximum extension, and so is a cosine. The amplitude is the displacement at the start of the motion.

$$x=A\cos(\omega t)=A\cos\left(\frac{2\pi}{T}t\right)=(8.8\,\text{cm})\cos\left(\frac{2\pi}{0.66}t\right)=(8.8\,\text{cm})\cos(9.520t)$$

$$\approx\boxed{(8.8\,\text{cm})\cos(9.5t)}$$

(*b*) Evaluate the position function at $t = 1.8$ s.

$$x=(8.8\,\text{cm})\cos\left(9.520\text{s}^{-1}(1.8\,\text{s})\right)=-1.252\,\text{cm}\approx\boxed{-1.3\,\text{cm}}$$

5. The period is 2.0 seconds, and the mass is 35 kg. The spring constant can be calculated from Eq. 14-7b.

$$T=2\pi\sqrt{\frac{m}{k}}\;\rightarrow\;T^2=4\pi^2\frac{m}{k}\;\rightarrow\;k=4\pi^2\frac{m}{T^2}=4\pi^2\frac{35\,\text{kg}}{(2.0\,\text{s})^2}=\boxed{350\,\text{N/m}}$$

6. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k=\frac{F_{\text{ext}}}{x}=\frac{mg}{x}=\frac{(2.4\,\text{kg})(9.80\,\text{m/s}^2)}{0.036\,\text{m}}=653\,\text{N/m}\approx\boxed{650\,\text{N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A=\boxed{2.5\,\text{cm}}$

The frequency of oscillation is found from the oscillating mass and the spring constant.

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{653\,\text{N/m}}{2.4\,\text{kg}}}=2.625\,\text{Hz}\approx\boxed{2.6\,\text{Hz}}$$

$\boxed{7.}$ The maximum velocity is given by Eq. 14-9a.

$$v_{\max}=\omega A=\frac{2\pi A}{T}=\frac{2\pi(0.15\,\text{m})}{7.0\,\text{s}}=\boxed{0.13\,\text{m/s}}$$

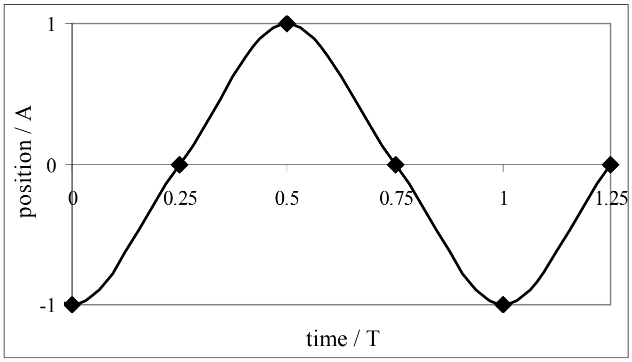
The maximum acceleration is given by Eq. 14-9b.

$$a_{\max}=\omega^2A=\frac{4\pi^2A}{T^2}=\frac{4\pi^2(0.15\,\text{m})}{(7.0\,\text{s})^2}=0.1209\,\text{m/s}^2\approx\boxed{0.12\,\text{m/s}^2}$$

$$\frac{a_{\max}}{g}=\frac{0.1209\,\text{m/s}^2}{9.80\,\text{m/s}^2}=1.2\times10^{-2}=\boxed{1.2\%}$$

8. The table of data is shown, along with the smoothed graph. Every quarter of a period, the mass moves from an extreme point to the equilibrium. The graph resembles a $\boxed{\text{cosine}}$ wave (actually, the opposite of a cosine wave).

time	position
0	- A
T/4	0
T/2	A
3T/4	0
T	- A
5T/4	0



9. The relationship between frequency, mass, and spring constant is Eq. 14-7a, $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

$$(a)\quad f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\quad\rightarrow\quad k=4\pi^2 f^2 m=4\pi^2\left(4.0\text{ Hz}\right)^2\left(2.5\times10^{-4}\text{kg}\right)=0.1579\,\text{N/m}\approx\boxed{0.16\,\text{N/m}}$$

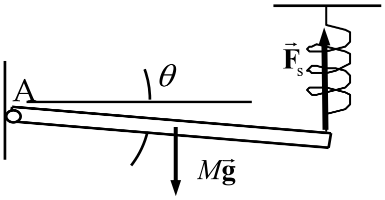
$$(b)\quad f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{0.1579\,\text{N/m}}{5.0\times10^{-4}\text{kg}}}=\boxed{2.8\text{ Hz}}$$

10. The spring constant is the same regardless of what mass is attached to the spring.

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\quad\rightarrow\quad\frac{k}{4\pi^2}=mf^2=\text{constant}\quad\rightarrow\quad m_1f_1^2=m_2f_1^2\quad\rightarrow$$

$$\left(m\text{ kg}\right)\left(0.83\,\text{Hz}\right)^2=\left(m\text{ kg}+0.68\text{ kg}\right)\left(0.60\text{ Hz}\right)^2\quad\rightarrow\quad m=\frac{\left(0.68\,\text{kg}\right)\left(0.60\,\text{Hz}\right)^2}{\left(0.83\,\text{Hz}\right)^2-\left(0.60\,\text{Hz}\right)^2}=\boxed{0.74\,\text{kg}}$$

11. We assume that the spring is stretched some distance y_0 while the rod is in equilibrium and horizontal. Calculate the net torque about point A while the object is in equilibrium, with clockwise torques as positive.



$$\sum\tau=Mg\left(\tfrac{1}{2}\ell\right)-F_s\ell=\tfrac{1}{2}Mg\ell-ky_0\ell=0$$

Now consider the rod being displaced an additional distance y below the horizontal, so that the rod makes a small angle of θ as shown in the free-body diagram. Again write the net torque about point A. If the angle is small, then there has been no appreciable horizontal displacement of the rod.

$$\sum\tau=Mg\left(\tfrac{1}{2}\ell\right)-F_s\ell=\tfrac{1}{2}Mg\ell-k\left(y+y_0\right)\ell=I\alpha=\tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2}$$

Include the equilibrium condition, and the approximation that $y=\ell\sin\theta\approx\ell\theta$.

$$\tfrac{1}{2}Mg\ell-ky\ell-ky_0\ell=\tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2}\quad\rightarrow\quad\tfrac{1}{2}Mg\ell-ky\ell-\tfrac{1}{2}Mg\ell=\tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2}\quad\rightarrow$$

$$-k\ell^2\theta=\tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2}\quad\rightarrow\quad\frac{d^2\theta}{dt^2}+\frac{3k}{M}\theta=0$$

This is the equation for simple harmonic motion, corresponding to Eq. 14-3, with $\omega^2=\dfrac{3k}{M}$.

$$\omega^2 = 4\pi^2 f^2 = \frac{3k}{M} \rightarrow f = \boxed{\frac{1}{2\pi}\sqrt{\frac{3k}{M}}}$$

12. (a) We find the effective spring constant from the mass and the frequency of oscillation.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow$$

$$k = 4\pi^2 m f^2 = 4\pi^2 (0.055\,\text{kg})(3.0\,\text{Hz})^2 = 19.54\,\text{N/m} \approx \boxed{20\,\text{N/m}}\,(2\text{ sig fig})$$

(b) Since the objects are the same size and shape, we anticipate that the spring constant is the same.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{19.54\,\text{N/m}}{0.25\,\text{kg}}} = \boxed{1.4\,\text{Hz}}$$

$\boxed{13.}$ (*a*) For A, the amplitude is $A_A = \boxed{2.5\,\text{m}}$. For B, the amplitude is $A_B = \boxed{3.5\,\text{m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_A = \boxed{0.25\,\text{Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_B = \boxed{0.50\,\text{Hz}}$.

(*c*) For C, the period is $T_A = \boxed{4.0\,\text{s}}$. For B, the period is $T_B = \boxed{2.0\,\text{s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_A = A_A \sin(2\pi f_A t) \rightarrow \boxed{x_A = (2.5\,\text{m})\sin\left(\tfrac{1}{2}\pi t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_B = A_B \cos(2\pi f_B t) \rightarrow \boxed{x_B = (3.5\,\text{m})\cos(\pi t)}$$

14. Eq. 14-4 is $x = A\cos(\omega t + \phi)$.

(*a*) If $x(0) = -A$, then $-A = A\cos\phi \rightarrow \phi = \cos^{-1}(-1) \rightarrow \boxed{\phi = \pi}$.

(*b*) If $x(0) = 0$, then $0 = A\cos\phi \rightarrow \phi = \cos^{-1}(0) \rightarrow \boxed{\phi = \pm\tfrac{1}{2}\pi}$.

(*c*) If $x(0) = A$, then $A = A\cos\phi \rightarrow \phi = \cos^{-1}(1) \rightarrow \boxed{\phi = 0}$.

(*d*) If $x(0) = \tfrac{1}{2}A$, then $\tfrac{1}{2}A = A\cos\phi \rightarrow \phi = \cos^{-1}\left(\tfrac{1}{2}\right) \rightarrow \boxed{\phi = \pm\tfrac{1}{3}\pi}$.

(*e*) If $x(0) = -\tfrac{1}{2}A$, then $-\tfrac{1}{2}A = A\cos\phi \rightarrow \phi = \cos^{-1}\left(-\tfrac{1}{2}\right) \rightarrow \boxed{\phi = \pm\tfrac{2}{3}\pi}$.

(*f*) If $x(0) = A/\sqrt{2}$, then $A/\sqrt{2} = A\cos\phi \rightarrow \phi = \cos^{-1}\left(\tfrac{1}{\sqrt{2}}\right) \rightarrow \boxed{\phi = \pm\tfrac{1}{4}\pi}$.

The ambiguity in the answers is due to not knowing the direction of motion at *t* = 0.

15. We assume that downward is the positive direction of motion. For this motion, we have $k = 305\,\text{N/m}$, $A = 0.280$ m, $m = 0.260$ kg, and $\omega = \sqrt{k/m} = \sqrt{(305\,\text{N/m})/0.260\text{ kg}} = 34.250\,\text{rad/s}$.

(*a*) Since the mass has a zero displacement and a positive velocity at *t* = 0, the equation is a sine function.

$$\boxed{y(t) = (0.280\text{ m})\sin[(34.3\,\text{rad/s})t]}$$

(*b*) The period of oscillation is given by $T=\frac{2\pi}{\omega}=\frac{2\pi}{34.25\,\text{rad/s}}=0.18345\,\text{s}$. The spring will have its maximum extension at times given by the following.

$$t_{\max}=\frac{T}{4}+nT=\boxed{4.59\times10^{-2}\text{s}+n(0.183\text{ s}),\ n=0,1,2,\cdots}$$

The spring will have its minimum extension at times given by the following.

$$t_{\min}=\frac{3T}{4}+nT=\boxed{1.38\times10^{-1}\text{s}+n(0.183\text{ s}),\ n=0,1,2,\cdots}$$

16. (*a*) From the graph, the period is 0.69 s. The period and the mass can be used to find the spring constant.

$$T=2\pi\sqrt{\frac{m}{k}}\quad\rightarrow\quad k=4\pi^2\frac{m}{T^2}=4\pi^2\frac{0.0095\,\text{kg}}{(0.69\,\text{s})^2}=0.7877\,\text{N/m}\approx\boxed{0.79\,\text{N/m}}$$

(*b*) From the graph, the amplitude is 0.82 cm. The phase constant can be found from the initial conditions.

$$x=A\cos\left(\frac{2\pi}{T}t+\phi\right)=(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t+\phi\right)$$

$$x(0)=(0.82\,\text{cm})\cos\phi=0.43\,\text{cm}\quad\rightarrow\quad\phi=\cos^{-1}\frac{0.43}{0.82}=\pm1.02\,\text{rad}$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$x=\boxed{(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t-1.0\right)}\ \text{or}\ (0.82\,\text{cm})\cos(9.1t-1.0)$$

17. (*a*) The period and frequency are found from the angular frequency.

$$\omega=2\pi f\quad\rightarrow\quad f=\frac{1}{2\pi}\omega=\frac{1}{2\pi}\frac{5\pi}{4}=\boxed{\frac{5}{8}\text{Hz}}\qquad T=\frac{1}{f}=\boxed{1.6\text{ s}}$$

(*b*) The velocity is the derivative of the position.

$$x=(3.8\,\text{m})\cos\left(\frac{5\pi}{4}t+\frac{\pi}{6}\right)\qquad v=\frac{dx}{dt}=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}t+\frac{\pi}{6}\right)$$

$$x(0)=(3.8\,\text{m})\cos\left(\frac{\pi}{6}\right)=\boxed{3.3\,\text{m}}\qquad v(0)=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)=\boxed{-7.5\,\text{m/s}}$$

(*c*) The acceleration is the derivative of the velocity.

$$v=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}t+\frac{\pi}{6}\right)\qquad a=\frac{dv}{dt}=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)^2\cos\left(\frac{5\pi}{4}t+\frac{\pi}{6}\right)$$

$$v(2.0)=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}(2.0)+\frac{\pi}{6}\right)=\boxed{-13\,\text{m/s}}$$

$$a(2.0)=-(3.8\,\text{m})\left(\frac{5\pi}{4}\right)^2\cos\left(\frac{5\pi}{4}(2.0)+\frac{\pi}{6}\right)=\boxed{29\,\text{m/s}^2}$$

18. (*a*) The maximum speed is given by Eq. 14-9a.

$$v_{\max} = 2\pi f A = 2\pi\left(441\,\text{Hz}\right)\left(1.5\times10^{-3}\,\text{m}\right) = \boxed{4.2\,\text{m/s}}\,.$$

(*b*) The maximum acceleration is given by Eq. 14-9b.

$$a_{\max} = 4\pi^2 f^2 A = 4\pi^2\left(441\,\text{Hz}\right)^2\left(1.5\times10^{-3}\,\text{m}\right) = \boxed{1.2\times10^{4}\,\text{m/s}^2}\,.$$

$\boxed{19.}$ When the object is at rest, the magnitude of the spring force is equal to the force of gravity. This determines the spring constant. The period can then be found.

$$\sum F_{\text{vertical}} = kx_0 - mg \;\rightarrow\; k = \frac{mg}{x_0}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{mg}{x_0}}} = 2\pi\sqrt{\frac{x_0}{g}} = 2\pi\sqrt{\frac{0.14\text{m}}{9.80\,\text{m/s}^2}} = \boxed{0.75\,\text{s}}$$

20. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{\left(1.62\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{0.215\,\text{m}} = 73.84\,\text{N/m}$$

After being stretched further and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position.

$$\tfrac{1}{4}T = \tfrac{1}{4}2\pi\sqrt{m/k} = \frac{\pi}{2}\sqrt{\frac{1.62\text{ kg}}{73.84\,\text{N/m}}} = \boxed{0.233\,\text{s}}$$

21. Each object will pass through the origin at the times when the argument of its sine function is a multiple of π.

A: $2.0t_{\text{A}} = n_{\text{A}}\pi \;\rightarrow\; t_{\text{A}} = \tfrac{1}{2}n_{\text{A}}\pi,\ n_{\text{A}} = 1,2,3,\ldots$ so $t_{\text{A}} = \tfrac{1}{2}\pi,\pi,\tfrac{3}{2}\pi,2\pi,\tfrac{5}{2}\pi,3\pi,\tfrac{7}{2}\pi,4\pi,\ldots$

B: $3.0t_{\text{B}} = n_{\text{B}}\pi \;\rightarrow\; t_{\text{B}} = \tfrac{1}{3}n_{\text{B}}\pi,\ n_{\text{B}} = 1,2,3,\ldots$ so $t_{\text{B}} = \tfrac{1}{3}\pi,\tfrac{2}{3}\pi,\pi,\tfrac{4}{3}\pi,\tfrac{5}{3}\pi,2\pi,\tfrac{7}{3}\pi,\tfrac{8}{3}\pi,3\pi,\ldots$

Thus we see the first three times are $\boxed{\pi\,\text{s},\,2\pi\,\text{s},\,3\pi\,\text{s}}$ or $\boxed{3.1\,\text{s},\,6.3\,\text{s},\,9.4\,\text{s}}$.

22. (*a*) The object starts at the maximum displacement in the positive direction, and so will be represented by a cosine function. The mass, period, and amplitude are given.

$$A = 0.16\,\text{m}\,;\ \omega = \frac{2\pi}{T} = \frac{2\pi}{0.55\,\text{s}} = 11.4\,\text{rad/s} \;\rightarrow\; \boxed{y = \left(0.16\,\text{m}\right)\cos\left(11t\right)}$$

(*b*) The time to reach the equilibrium is one-quarter of a period, so $\tfrac{1}{4}\left(0.55\,\text{s}\right) = \boxed{0.14\,\text{s}}$.

(*c*) The maximum speed is given by Eq. 14-9a.

$$v_{\max} = \omega A = \left(11.4\,\text{rad/s}\right)\left(0.16\,\text{m}\right) = \boxed{1.8\,\text{m/s}}$$

(*d*) The maximum acceleration is given by Eq. 14-9b.

$$a_{\max} = \omega^2 A = \left(11.4\,\text{rad/s}\right)^2\left(0.16\,\text{m}\right) = \boxed{2.1\,\text{m/s}^2}$$

The maximum acceleration occurs at the endpoints of the motion, and is first attained at the $\boxed{\text{release point}}$.

23. The period of the jumper's motion is $T=\dfrac{43.0\text{ s}}{8\text{ cycles}}=5.375\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

$$T=2\pi\sqrt{\frac{m}{k}}\ \rightarrow\ k=\frac{4\pi^2 m}{T^2}=\frac{4\pi^2(65.0\,\text{kg})}{(5.375\,\text{s})^2}=88.821\,\text{N/m}\approx\boxed{88.8\,\text{N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

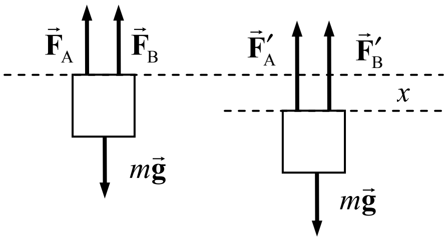
$$k\Delta x=mg\ \rightarrow\ \Delta x=\frac{mg}{k}=\frac{(65.0\text{ kg})(9.80\,\text{m/s}^2)}{88.821\,\text{N/m}}=7.17\,\text{m}$$

Thus the unstretched bungee cord must be $25.0\text{ m}-7.17\text{ m}=\boxed{17.8\,\text{m}}$.

24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives this.

$$\sum F_y=F_{\text{A}}+F_{\text{B}}-mg=0\ \rightarrow\ F_{\text{A}}+F_{\text{B}}=mg$$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x<0$. Again write Newton's second law for vertical forces.



$$\sum F_y=F_{net}=F'_{\text{A}}+F'_{\text{B}}-mg=F_{\text{A}}-kx+F_{\text{B}}-kx-mg=-2kx+\left(F_{\text{A}}+F_{\text{B}}-mg\right)=-2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f=\frac{1}{2\pi}\sqrt{k_{\text{effective}}/m}=\boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

$\boxed{25.}$ (*a*) If the block is displaced a distance *x* to the right in Figure 14-32a, then the length of spring # 1 will be increased by a distance x_1 and the length of spring # 2 will be increased by a distance x_2, where $x=x_1+x_2$. The force on the block can be written $F=-k_{\text{eff}}x.$ Because the springs are massless, they act similar to a rope under tension, and the same force F is exerted by each spring. Thus $F=-k_{\text{eff}}x=-k_1x_1=-k_2x_2$.

$$x=x_1+x_2=-\frac{F}{k_1}-\frac{F}{k_2}=-F\left(\frac{1}{k_1}+\frac{1}{k_2}\right)=-\frac{F}{k_{\text{eff}}}\ \rightarrow\ \frac{1}{k_{\text{eff}}}=\frac{1}{k_1}+\frac{1}{k_2}$$

$$T=2\pi\sqrt{\frac{m}{k_{\text{eff}}}}=\boxed{2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}}$$

(*b*) The block will be in equilibrium when it is stationary, and so the net force at that location is zero. Then, if the block is displaced a distance *x* to the right in the diagram, then spring # 1 will exert an additional force of $F_1=-k_1x$, in the opposite direction to *x*. Likewise, spring # 2 will exert an additional force $F_2=-k_2x$, in the same direction as F_1. Thus the net force on the displaced block is $F = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. The effective spring constant is thus $k = k_1 + k_2$, and the period is given by $T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{m}{k_1 + k_2}}}$.

26. The impulse, which acts for a very short time, changes the momentum of the mass, giving it an initial velocity v_0. Because this occurs at the equilibrium position, this is the maximum velocity of the mass. Since the motion starts at the equilibrium position, we represent the motion by a sine function.

$$J = \Delta p = m\Delta v = mv_0 - 0 = mv_0 \quad \rightarrow \quad v_0 = \frac{J}{m} = v_{\max} = A\omega = A\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$\frac{J}{m} = A\sqrt{\frac{k}{m}} \quad \rightarrow \quad A = \frac{J}{\sqrt{km}} \quad \rightarrow \quad x = A\sin\omega t = \boxed{\frac{J}{\sqrt{km}}\sin\left(\sqrt{\frac{k}{m}}\,t\right)}$$

27. The various values can be found from the equation of motion, $x = A\cos\omega t = 0.650\cos 7.40t$.

(*a*) The amplitude is the maximum value of *x*, and so $A = \boxed{0.650\,\text{m}}$.

(*b*) The frequency is $f = \frac{\omega}{2\pi} = \frac{7.40\,\text{rad/s}}{2\pi\,\text{rad}} = \boxed{1.18\,\text{Hz}}$.

(*c*) The total energy can be found from the maximum potential energy.

$$E = U_{\max} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}m\omega^2A^2 = \tfrac{1}{2}(1.15\,\text{kg})(7.40\,\text{rad/s})^2(0.650\,\text{m})^2 = 13.303\,\text{J} \approx \boxed{13.3\,\text{J}}$$

(*d*) The potential energy can be found from $U = \tfrac{1}{2}kx^2$, and the kinetic energy from $E = U + K$.

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(1.15\,\text{kg})(7.40\,\text{rad/s})^2(0.260\,\text{m})^2 = \boxed{2.1\,\text{J}}$$

$$K = E - U = 13.3\,\text{J} - 2.1\,\text{J} = \boxed{11.2\,\text{J}}$$

28. (*a*) The total energy is the maximum potential energy.

$$U = \tfrac{1}{2}E \quad \rightarrow \quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad \rightarrow \quad \boxed{x = A/\sqrt{2} \approx 0.707A}$$

(b) Now we are given that $x = \tfrac{1}{3}A$.

$$\frac{U}{E} = \frac{\tfrac{1}{2}kx^2}{\tfrac{1}{2}kA^2} = \frac{x^2}{A^2} = \frac{1}{9}$$

Thus the energy is divided up into $\boxed{\tfrac{1}{9}\text{ potential and }\tfrac{8}{9}\text{ kinetic}}$.

29. The total energy can be found from the spring constant and the amplitude.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(95\,\text{N/m})(0.020\,\text{m})^2 = 0.019\,\text{J}$$

That is represented by the horizontal line on the graph.

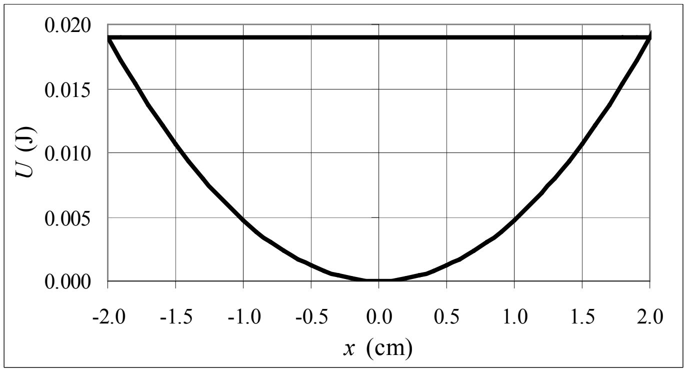
(*a*) From the graph, at $x = 1.5\,\text{cm}$, we have $U \approx \boxed{0.011\,\text{J}}$.

(*b*) From energy conservation, at $x = 1.5\,\text{cm}$, we have $K = E - U = \boxed{0.008\,\text{J}}$.

(*c*) Find the speed from the estimated kinetic energy.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.008\,\text{J})}{0.055\,\text{kg}}} = \boxed{0.5\,\text{m/s}}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.29."

30. (*a*) At equilibrium, the velocity is its maximum. Use Eq. 14-9a, and realize that the object can be moving in either direction.

$$v_{\text{max}} = \omega A = 2\pi f A = 2\pi(2.5\,\text{Hz})(0.15\,\text{m}) = 2.356\,\text{m/s} \quad \rightarrow \quad v_{\text{equib}} \approx \boxed{\pm 2.4\,\text{m/s}}$$

(*b*) From Eq. 14-11b, we find the velocity at any position.

$$v = \pm v_{\text{max}}\sqrt{1-\frac{x^2}{A^2}} = \pm(2.356\,\text{m/s})\sqrt{1-\frac{(0.10\,\text{m})^2}{(0.15\,\text{m})^2}} = \pm 1.756\,\text{m/s} \approx \boxed{\pm 1.8\,\text{m/s}}$$

(*c*) $E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}(0.35\,\text{kg})(2.356\,\text{m/s})^2 = 0.9714\,\text{J} \approx \boxed{0.97\,\text{J}}$

(*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function.

$$x = (0.15\,\text{m})\cos(2\pi(2.5\text{ Hz})t) \quad \rightarrow \quad \boxed{x = (0.15\,\text{m})\cos(5.0\pi t)}$$

$\boxed{31.}$ The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{95.0\text{ N}}{0.175\text{ m}} = 542.9\,\text{N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_i = E_f \quad \rightarrow \quad \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = x_{\text{max}}\sqrt{\frac{k}{m}} = (0.175\,\text{m})\sqrt{\frac{542.9\,\text{N/m}}{0.160\text{ kg}}} = \boxed{10.2\,\text{m/s}}$$

32. The energy of the oscillator will be conserved after the collision.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m+M)v_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = A\sqrt{k/(m+M)}$$

This speed is the speed that the block and bullet have immediately after the collision. Linear momentum in one dimension will have been conserved during the (assumed short time) collision, and so the initial speed of the bullet can be found.

$$p_{\text{before}} = p_{\text{after}} \quad \rightarrow \quad mv_o = (m+M)v_{\text{max}}$$

$$v_o = \frac{m+M}{m}A\sqrt{\frac{k}{m+M}} = \frac{0.2525\,\text{kg}}{0.0125\,\text{kg}}(0.124\,\text{m})\sqrt{\frac{2250\,\text{N/m}}{0.2525\,\text{kg}}} = \boxed{236\,\text{m/s}}$$

33. To compare the total energies, we can compare the maximum potential energies. Since the frequencies and the masses are the same, the spring constants are the same.

$$\frac{E_{\substack{\text{high}\\\text{energy}}}}{E_{\substack{\text{low}\\\text{energy}}}} = \frac{\frac{1}{2}kA^2_{\substack{\text{high}\\\text{energy}}}}{\frac{1}{2}kA^2_{\substack{\text{low}\\\text{energy}}}} = \frac{A^2_{\substack{\text{high}\\\text{energy}}}}{A^2_{\substack{\text{low}\\\text{energy}}}} = 5 \rightarrow \boxed{\frac{A_{\substack{\text{high}\\\text{energy}}}}{A_{\substack{\text{low}\\\text{energy}}}} = \sqrt{5}}$$

34. (*a*) The spring constant can be found from the mass and the frequency of oscillation.

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \rightarrow k = 4\pi^2 f^2 m = 4\pi^2 (3.0\,\text{Hz})^2 (0.24\,\text{kg}) = 85.27\,\text{N/m} \approx \boxed{85\,\text{N/m}}$$

(*b*) The energy can be found from the maximum potential energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(85.27\,\text{N/m})(0.045\,\text{m})^2 = 8.634\times10^{-2}\,\text{J} \approx \boxed{0.086\,\text{J}}$$

35. (*a*) The work done in compressing the spring is stored as potential energy. The compressed location corresponds to the maximum potential energy and the amplitude of the ensuing motion.

$$W = \tfrac{1}{2}kA^2 \rightarrow k = \frac{2W}{A^2} = \frac{2(3.6\,\text{J})}{(0.13\,\text{m})^2} = 426\,\text{N/m} \approx \boxed{430\,\text{N/m}}$$

(*b*) The maximum acceleration occurs at the compressed location, where the spring is exerting the maximum force. If the compression distance is positive, then the acceleration is negative.

$$F = -kx = ma \rightarrow m = -\frac{kx}{a} = -\frac{(426\,\text{N/m})(0.13\,\text{m})}{15\,\text{m/s}^2} = \boxed{3.7\,\text{kg}}$$

36. (*a*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \rightarrow$$

$$A = \sqrt{\frac{m}{k}v^2 + x^2} = \sqrt{\frac{2.7\text{ kg}}{280\,\text{N/m}}(0.55\,\text{m/s})^2 + (0.020\text{ m})^2} = 5.759\times10^{-2}\,\text{m} \approx \boxed{5.8\times10^{-2}\,\text{m}}$$

(*b*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2_{\text{max}} \rightarrow$$

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = (5.759\times10^{-2}\,\text{m})\sqrt{\frac{280\,\text{N/m}}{2.7\text{ kg}}} = 0.5865\,\text{m/s} \approx \boxed{0.59\,\text{m/s}}$$

$\boxed{37.}$ We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\text{before}} = p_{\text{after}} \rightarrow mv_0 = (m+M)v_{\text{max}} \rightarrow v_{\text{max}} = \frac{m}{m+M}v_0$$

$$\tfrac{1}{2}(m+M)v^2_{\text{max}} = \tfrac{1}{2}kA^2 \rightarrow \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 = \tfrac{1}{2}kA^2 \rightarrow$$

$$v_0 = \frac{A}{m}\sqrt{k\left(m+M\right)} = \frac{\left(9.460\times10^{-2}\,\text{m}\right)}{\left(7.870\times10^{-3}\,\text{kg}\right)}\sqrt{\left(142.7\,\text{N/m}\right)\left(7.870\times10^{-3}\,\text{kg}+4.648\,\text{kg}\right)}$$

$$= \boxed{309.8\,\text{m/s}}$$

38. The hint says to integrate Eq. 14-11a, which comes from the conservation of energy. Let the initial position of the oscillator be x_0.

$$v = \pm\sqrt{\frac{k}{m}\left(A^2-x^2\right)} = \frac{dx}{dt} \quad\to\quad \frac{dx}{\sqrt{\left(A^2-x^2\right)}} = \pm\sqrt{\frac{k}{m}}dt \quad\to\quad \int_{x_0}^{x}\frac{dx}{\sqrt{\left(A^2-x^2\right)}} = \pm\sqrt{\frac{k}{m}}\int_0^t dt \quad\to$$

$$-\cos^{-1}\left(\frac{x}{A}\right)_{x_0}^{x} = -\cos^{-1}\frac{x}{A}+\cos^{-1}\frac{x_0}{A} = \pm\sqrt{\frac{k}{m}}t$$

Make these definitions: $\sqrt{\frac{k}{m}} \equiv \omega$; $\cos^{-1}\frac{x_0}{A} \equiv \phi$. Then we have the following.

$$-\cos^{-1}\frac{x}{A}+\cos^{-1}\frac{x_0}{A} = \pm\sqrt{\frac{k}{m}}t \quad\to\quad -\cos^{-1}\frac{x}{A}+\phi = \pm\omega t \quad\to\quad \boxed{x = A\cos\left(\pm\omega t+\phi\right)}$$

The phase angle definition could be changed so that the function is a sine instead of a cosine. And the ± sign can be resolved if the initial velocity is known.

39. (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.785\,\text{kg}}{184\,\text{N/m}}} = 0.4104\,\text{s} \approx \boxed{0.410\,\text{s}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{184\,\text{N/m}}{0.785\,\text{kg}}} = 2.437\,\text{Hz} \approx \boxed{2.44\,\text{Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$v_{\text{max}} = A\sqrt{k/m} \quad\to$$

$$A = v_{\text{max}}\sqrt{m/k} = \left(2.26\,\text{m/s}\right)\sqrt{0.785\,\text{kg}/\left(184\,\text{N/m}\right)} = 0.1476\,\text{m} \approx \boxed{0.148\,\text{m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude

$$a_{\text{max}} = Ak/m = \left(0.1476\,\text{m}\right)\left(184\,\text{N/m}\right)/\left(0.785\,\text{kg}\right) = \boxed{34.6\,\text{m/s}^2}$$

(*d*) Because the mass started at the equilibrium position of $x = 0$, the position function will be proportional to the sine function.

$$x = \left(0.148\,\text{m}\right)\sin\left[2\pi\left(2.437\,\text{Hz}\right)t\right] \quad\to\quad \boxed{x = \left(0.148\,\text{m}\right)\sin\left(4.87\pi\,t\right)}$$

(*e*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}\left(0.785\,\text{kg}\right)\left(2.26\,\text{m/s}\right)^2 = \boxed{2.00\,\text{J}}$$

(*f*) Use the conservation of mechanical energy for the oscillator.

$$E = \tfrac{1}{2}kx^2+\tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2 \quad\to\quad \tfrac{1}{2}k\left(0.40A\right)^2+K = \tfrac{1}{2}kA^2 \quad\to$$

$$K = \tfrac{1}{2}kA^2\left(1-0.40^2\right) = \left(2.00\,\text{J}\right)\left(0.84\right) = \boxed{1.68\,\text{J}}$$

40. We solve this using conservation of energy, equating the energy at the compressed point with the energy as the ball leaves the launcher. Take the 0 location for gravitational potential energy to be at the level where the ball is on the compressed spring. The 0 location for elastic potential energy is the uncompressed position of the spring. Initially, the ball has only elastic potential energy. At the point where the spring is uncompressed and the ball just leaves the spring, there will be gravitational potential energy, translational kinetic energy, and rotational kinetic energy. The ball is rolling without slipping.

$$E_{\mathrm{i}} = E_{\mathrm{f}} \quad \rightarrow \quad \tfrac{1}{2}kx^2 = mgh + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgx\sin\theta + \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}\right)mr^2\frac{v^2}{r^2} \quad \rightarrow$$

$$k = 2\frac{m}{x^2}\left(gx\sin\theta + \tfrac{7}{10}v^2\right) = 2\frac{0.025\,\mathrm{kg}}{(0.060\,\mathrm{m})^2}\left[\left(9.80\,\mathrm{m/s^2}\right)(0.060\,\mathrm{m})\sin 15^\circ + \tfrac{7}{10}(3.0\,\mathrm{m/s})^2\right]$$

$$= 89.61\,\mathrm{N/m} \approx \boxed{90\,\mathrm{N/m}}\ (2\text{ sig. fig.})$$

41. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\mathrm{Mars}}}{T_{\mathrm{Earth}}} = \frac{2\pi\sqrt{L/g_{\mathrm{Mars}}}}{2\pi\sqrt{L/g_{\mathrm{Earth}}}} = \sqrt{\frac{g_{\mathrm{Earth}}}{g_{\mathrm{Mars}}}} \quad \rightarrow$$

$$T_{\mathrm{Mars}} = T_{\mathrm{Earth}}\sqrt{\frac{g_{\mathrm{Earth}}}{g_{\mathrm{Mars}}}} = (1.35\,\mathrm{s})\sqrt{\frac{1}{0.37}} = \boxed{2.2\,\mathrm{s}}$$

42. (*a*) The period is given by $T = \dfrac{50\,\mathrm{s}}{32\,\mathrm{cycles}} = \boxed{1.6\,\mathrm{s}}$.

(*b*) The frequency is given by $f = \dfrac{32\,\mathrm{cycles}}{50\,\mathrm{s}} = \boxed{0.64\,\mathrm{Hz}}$.

$\boxed{43.}$ We consider this a simple pendulum. Since the motion starts at the amplitude position at $t = 0$, we may describe it by a cosine function with no phase angle, $\theta = \theta_{\max}\cos\omega t$. The angular velocity can be written as a function of the length, $\theta = \theta_{\max}\cos\left(\sqrt{\dfrac{g}{\ell}}\,t\right)$.

(*a*) $$\theta(t = 0.35\,\mathrm{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\mathrm{m/s^2}}{0.30\,\mathrm{m}}}(0.35\,\mathrm{s})\right) = \boxed{-5.4^\circ}$$

(*b*) $$\theta(t = 3.45\,\mathrm{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\mathrm{m/s^2}}{0.30\,\mathrm{m}}}(3.45\,\mathrm{s})\right) = \boxed{8.4^\circ}$$

(*c*) $$\theta(t = 6.00\,\mathrm{s}) = 13^\circ\cos\left(\sqrt{\frac{9.80\,\mathrm{m/s^2}}{0.30\,\mathrm{m}}}(6.00\,\mathrm{s})\right) = \boxed{-13^\circ}$$

44. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$.

(*a*) $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\dfrac{0.53\,\text{m}}{9.80\,\text{m/s}^2}} = \boxed{1.5\,\text{s}}$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob, and so no restoring force to cause oscillations. Thus there will be no period – the pendulum will not oscillate and so no period can be defined.

45. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta = \theta_0 \cos\omega t = \theta_0 \cos\dfrac{2\pi t}{T}$. Solve for the time for the position to decrease to half the amplitude.

$$\theta_{1/2} = \tfrac{1}{2}\theta_0 = \theta_0 \cos\frac{2\pi t_{1/2}}{T} \;\rightarrow\; \frac{2\pi t_{1/2}}{T} = \cos^{-1}\tfrac{1}{2} = \frac{\pi}{3} \;\rightarrow\; t_{1/2} = \tfrac{1}{6}T$$

It takes $\tfrac{1}{6}T$ for the position to change from $+10°$ to $+5°$. It takes $\tfrac{1}{4}T$ for the position to change from $+10°$ to 0. Thus it takes $\tfrac{1}{4}T - \tfrac{1}{6}T = \tfrac{1}{12}T$ for the position to change from $+5°$ to 0. Due to the symmetric nature of the cosine function, it will also take $\tfrac{1}{12}T$ for the position to change from 0 to $-5°$, and so from $+5°$ to $-5°$ takes $\tfrac{1}{6}T$. The second half of the cycle will be identical to the first, and so the total time spent between $+5°$ and $-5°$ is $\tfrac{1}{3}T$. So the pendulum spends $\boxed{\text{one-third}}$ of its time between $+5°$ and $-5°$.

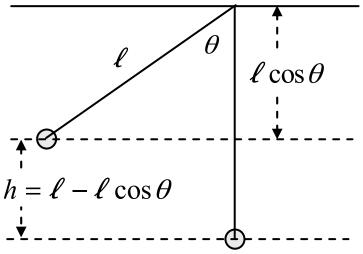
46. There are $(24\,\text{h})(60\,\text{min/h})(60\,\text{s/min}) = 86{,}400\,\text{s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), and so the clock should make 43,200 cycles per day. After one day, the clock in question is 26 seconds slow, which means that it has made 13 less cycles than required for precise timekeeping. Thus the clock is only making 43,187 cycles in a day. Accordingly, the period of the clock must be decreased by a factor of $\dfrac{43{,}187}{43{,}200}$.

$$T_{\text{new}} = \frac{43{,}187}{43{,}200}T_{\text{old}} \;\rightarrow\; 2\pi\sqrt{\ell_{\text{new}}/g} = \left(\frac{43{,}187}{43{,}200}\right)2\pi\sqrt{\ell_{\text{old}}/g} \;\rightarrow$$

$$\ell_{\text{new}} = \left(\frac{43{,}187}{43{,}200}\right)^2 \ell_{\text{old}} = \left(\frac{43{,}187}{43{,}200}\right)^2 (0.9930\,\text{m}) = 0.9924\,\text{m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.6 mm}}$.

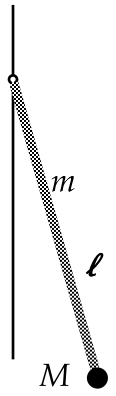
47. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \;\rightarrow\; K_{\text{top}} + U_{\text{top}} = K_{\text{bottom}} + U_{\text{bottom}} \;\rightarrow$$

$$0 + mgh = \tfrac{1}{2}mv_{\max}^2 \;\rightarrow\; v_{\max} = \sqrt{2gh} = \boxed{\sqrt{2g\ell(1-\cos\theta)}}$$

48. (*a*) For a physical pendulum with the small angle approximation, we may apply Eq. 14-14. We need the moment of inertia and the distance from the suspension point to the center of mass. We approximate the cord as a rod, and find the center of mass relative to the stationary end of the cord.



$$I = I_{\text{bob}} + I_{\text{cord}} = M\ell^2 + \tfrac{1}{3}m\ell^2 = \left(M + \tfrac{1}{3}m\right)\ell^2$$

$$h = x_{\text{CM}} = \frac{M\ell + m\left(\frac{1}{2}\ell\right)}{M+m} = \left(\frac{M + \frac{1}{2}m}{M+m}\right)\ell$$

$$T = 2\pi\sqrt{\frac{I}{m_{\text{total}}gh}} = 2\pi\sqrt{\frac{\left(M+\frac{1}{3}m\right)\ell^2}{\left(M+m\right)g\left(\frac{M+\frac{1}{2}m}{M+m}\right)\ell}} = \boxed{2\pi\sqrt{\frac{\left(M+\frac{1}{3}m\right)\ell}{\left(M+\frac{1}{2}m\right)g}}}$$

(*b*) If we use the expression for a simple pendulum we would have $T_{\text{simple}} = 2\pi\sqrt{\ell/g}$. Find the fractional error.

$$\text{error} = \frac{T - T_{\text{simple}}}{T} = \frac{2\pi\sqrt{\frac{\left(M+\frac{1}{3}m\right)\ell}{\left(M+\frac{1}{2}m\right)g}} - 2\pi\sqrt{\frac{\ell}{g}}}{2\pi\sqrt{\frac{\left(M+\frac{1}{3}m\right)\ell}{\left(M+\frac{1}{2}m\right)g}}} = \frac{\sqrt{\frac{\left(M+\frac{1}{3}m\right)}{\left(M+\frac{1}{2}m\right)}} - 1}{\sqrt{\frac{\left(M+\frac{1}{3}m\right)}{\left(M+\frac{1}{2}m\right)}}} = \boxed{1 - \sqrt{\frac{\left(M+\frac{1}{2}m\right)}{\left(M+\frac{1}{3}m\right)}}}$$

Note that this is negative, indicating that the simple pendulum approximation is too large.

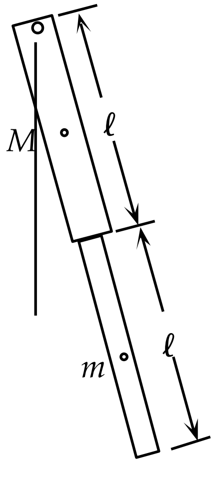
49. The balance wheel of the watch is a torsion pendulum, described by $\tau = -K\theta$. A specific torque and angular displacement are given, and so the torsional constant can be determined. The angular frequency is given by $\omega = \sqrt{K/I}$. Use these relationships to find the mass.

$$\tau = -K\theta \quad \rightarrow \quad K = \left|\frac{\theta}{\tau}\right| = \frac{1.1\times10^{-5}\,\text{m}\bullet\text{N}}{\pi/4\,\text{rad}}$$

$$\omega = 2\pi f = \sqrt{\frac{K}{I}} = \sqrt{\frac{K}{mr^2}} \quad \rightarrow$$

$$m = \frac{K}{4\pi^2 f^2 r^2} = \frac{\frac{1.1\times10^{-5}\,\text{m}\bullet\text{N}}{\pi/4\,\text{rad}}}{4\pi^2\left(3.10\,\text{Hz}\right)^2\left(0.95\times10^{-2}\,\text{m}\right)^2} = 4.1\times10^{-4}\,\text{kg} = \boxed{0.41\,\text{g}}$$

50. (*a*) We call the upper mass *M* and the lower mass *m*. Both masses have length ℓ. The period of the physical pendulum is given by Eq. 14-14. Note that we must find both the moment of inertia of the system about the uppermost point, and the center of mass of the system. The parallel axis theorem is used to find the moment of inertia.



$$I = I_{\text{upper}} + I_{\text{lower}} = \tfrac{1}{3}M\ell^2 + \tfrac{1}{12}m\ell^2 + m\left(\tfrac{3}{2}\ell\right)^2 = \left(\tfrac{1}{3}M + \tfrac{7}{3}m\right)\ell^2$$

$$h = x_{\text{CM}} = \frac{M\left(\frac{1}{2}\ell\right) + m\left(\frac{3}{2}\ell\right)}{M+m} = \left(\frac{\frac{1}{2}M + \frac{3}{2}m}{M+m}\right)\ell$$

$$T = 2\pi\sqrt{\frac{I}{m_{\text{total}}gh}} = 2\pi\sqrt{\frac{\left(\frac{1}{3}M+\frac{7}{3}m\right)\ell^2}{\left(M+m\right)g\left(\frac{\frac{1}{2}M+\frac{3}{2}m}{M+m}\right)\ell}} = 2\pi\sqrt{\frac{\left(\frac{1}{3}M+\frac{7}{3}m\right)\ell}{\left(\frac{1}{2}M+\frac{3}{2}m\right)g}}$$

$$=2\pi\sqrt{\frac{\left[\frac{1}{3}(7.0\,\text{kg})+\frac{7}{3}(4.0\,\text{kg})\right](0.55\,\text{m})}{\left[\frac{1}{2}(7.0\,\text{kg})+\frac{3}{2}(4.0\,\text{kg})\right]\left(9.80\,\text{m/s}^2\right)}}=1.6495\,\text{s}\approx\boxed{1.6\,\text{s}}$$

(*b*) It took 7.2 seconds for 5 swings, which gives a period of 1.4 seconds. That is reasonable qualitative agreement.

51. (*a*) In the text, we are given that $\tau=-K\theta$. Newton's second law for rotation, Eq. 10-14, says that $\sum\tau=I\alpha=I\dfrac{d^2\theta}{dt^2}$. We assume that the torque applied by the twisting of the wire is the only torque.

$$\sum\tau=I\alpha=I\frac{d^2\theta}{dt^2}=-K\theta\ \rightarrow\ \frac{d^2\theta}{dt^2}=-\frac{K}{I}\theta=-\omega^2\theta$$

This is the same form as Eq. 14-3, which is the differential equation for simple harmonic oscillation. We exchange variables with Eq. 14-4, and write the equation for the angular motion.

$$x=A\cos(\omega t+\phi)\ \rightarrow\ \boxed{\theta=\theta_0\cos(\omega t+\phi),\ \omega^2=\frac{K}{I}}$$

(*b*) The period of the motion is found from the angular velocity ω.

$$\omega^2=\frac{K}{I}\ \rightarrow\ \omega=\sqrt{\frac{K}{I}}=\frac{2\pi}{T}\ \rightarrow\ \boxed{T=2\pi\sqrt{\frac{I}{K}}}$$

52. The meter stick used as a pendulum is a physical pendulum. The period is given by Eq. 14-14, $T=2\pi\sqrt{\dfrac{I}{mgh}}$. Use the parallel axis theorem to find the moment of inertia about the pin. Express the distances from the center of mass.

$$I=I_{\text{CM}}+mh^2=\tfrac{1}{12}m\ell^2+mh^2\ \rightarrow\ T=2\pi\sqrt{\frac{I}{mgh}}=2\pi\sqrt{\frac{\frac{1}{12}m\ell^2+mh^2}{mgh}}=\frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{\ell^2}{h}+h\right)^{1/2}$$

$$\frac{dT}{dh}=2\pi\left(\tfrac{1}{2}\right)\left(\tfrac{1}{12}\frac{\ell^2}{h}+h\right)^{-1/2}\left(-\tfrac{1}{12}\frac{\ell^2}{h^2}+1\right)=0\ \rightarrow\ h=\sqrt{\tfrac{1}{12}}\ell=0.2887\,\text{m}$$

$$x=\tfrac{1}{2}\ell-h=0.500-0.2887\approx\boxed{0.211\,\text{m}}\ \text{from the end}$$

Use the distance for *h* to calculate the period.

$$T=\frac{2\pi}{\sqrt{g}}\left(\tfrac{1}{12}\frac{\ell^2}{h}+h\right)^{1/2}=\frac{2\pi}{\sqrt{9.80\,\text{m/s}^2}}\left(\tfrac{1}{12}\frac{(1.00\,\text{m})^2}{0.2887\,\text{m}}+0.2887\,\text{m}\right)^{1/2}=\boxed{1.53\,\text{s}}$$

53. This is a torsion pendulum. The angular frequency is given in the text as $\omega=\sqrt{K/I}$, where *K* is the torsion constant (a property of the wire, and so a constant in this problem). The rotational inertia of a rod about its center is $\frac{1}{12}M\ell^2$.

$$\omega=\sqrt{\frac{K}{I}}=\frac{2\pi}{T}\ \rightarrow\ T=2\pi\sqrt{\frac{I}{K}}\ \rightarrow$$

$$\frac{T}{T_0}=\frac{2\pi\sqrt{\frac{I}{K}}}{2\pi\sqrt{\frac{I_0}{K}}}=\sqrt{\frac{I}{I_0}}=\sqrt{\frac{\frac{1}{12}M\ell^2}{\frac{1}{12}M_0\ell_0^2}}=\sqrt{\frac{(0.700M_0)(0.700\ell_0)^2}{M_0\ell_0^2}}=0.58566$$

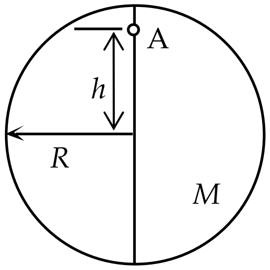
$$T=(0.58566)T_0=(0.58566)(5.0\,\mathrm{s})=\boxed{2.9\,\mathrm{s}}$$

54. The torsional constant is related to the period through the relationship given in problem 51. The rotational inertia of a disk in this configuration is $I=\frac{1}{2}MR^2$.

$$T=2\pi\sqrt{\frac{I}{K}}\ \rightarrow\ K=\frac{4\pi^2I}{T^2}=\frac{4\pi^2\frac{1}{2}MR^2}{T^2}=2\pi^2MR^2f^2=2\pi^2(0.375\,\mathrm{kg})(0.0625\,\mathrm{m})^2(0.331\,\mathrm{Hz})^2$$

$$=\boxed{3.17\times10^{-3}\,\mathrm{m\bullet N/rad}}$$

$\boxed{55.}$ This is a physical pendulum. Use the parallel axis theorem to find the moment of inertia about the pin at point A, and then use Eq. 14-14 to find the period.



$$I_{\mathrm{pin}}=I_{\mathrm{CM}}+Mh^2=\tfrac{1}{2}MR^2+Mh^2=M\left(\tfrac{1}{2}R^2+h^2\right)$$

$$T=2\pi\sqrt{\frac{I}{Mgh}}=2\pi\sqrt{\frac{M\left(\frac{1}{2}R^2+h^2\right)}{Mgh}}=2\pi\sqrt{\frac{\left(\frac{1}{2}R^2+h^2\right)}{gh}}$$

$$=2\pi\sqrt{\frac{\frac{1}{2}(0.200\,\mathrm{m})^2+(0.180\,\mathrm{m})^2}{(9.80\,\mathrm{m/s^2})(0.180\,\mathrm{m})}}=\boxed{1.08\,\mathrm{s}}$$

56. (*a*) The period of the motion can be found from Eq. 14-18, giving the angular frequency for the damped motion.

$$\omega'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}=\sqrt{\frac{(41.0\,\mathrm{N/m})}{(0.835\,\mathrm{kg})}-\frac{(0.662\,\mathrm{N\bullet s/m})^2}{4(0.835\,\mathrm{kg})^2}}=6.996\,\mathrm{rad/s}$$

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{6.996\,\mathrm{rad/s}}=\boxed{0.898\,\mathrm{s}}$$

(*b*) If the amplitude at some time is A, then one cycle later, the amplitude will be $Ae^{-\gamma T}$. Use this to find the fractional change.

$$\text{fractional change}=\frac{Ae^{-\gamma T}-A}{A}=e^{-\gamma T}-1=e^{-\frac{b}{2m}T}-1=e^{-\frac{(0.662\,\mathrm{N\bullet s/m})}{2(0.835\,\mathrm{kg})}(0.898\,\mathrm{s})}-1=\boxed{-0.300}$$

And so the amplitude decreases by 30% from the previous amplitude, every cycle.

(*c*) Since the object is at the origin at $t=0$, we will use a sine function to express the equation of motion.

$$x=Ae^{-\gamma t}\sin(\omega't)\ \rightarrow\ 0.120\,\mathrm{m}=Ae^{-\frac{(0.662\,\mathrm{N\bullet s/m})}{2(0.835\,\mathrm{kg})}(1.00\,\mathrm{s})}\sin(6.996\,\mathrm{rad})\ \rightarrow$$

$$A=\frac{0.120\,\mathrm{m}}{e^{-\frac{(0.662\,\mathrm{N\bullet s/m})}{2(0.835\,\mathrm{kg})}(1.00\,\mathrm{s})}\sin(6.996\,\mathrm{rad})}=0.273\,\mathrm{m}\ ;\ \gamma=\frac{b}{2m}=\frac{(0.662\,\mathrm{N\bullet s/m})}{2(0.835\,\mathrm{kg})}=0.396\,\mathrm{s^{-1}}$$

$$\boxed{x=(0.273\,\text{m})e^{-\left(0.396\,\text{s}^{-1}\right)t}\sin\left[(7.00\,\text{rad/s})t\right]}$$

57. We assume that initially, the system is critically damped, so $b_{\text{critical}}^2 = 4mk$. Then, after aging, we assume that after 3 cycles, the car's oscillatory amplitude has dropped to 5% of its original amplitude. That is expressed by $A = A_0 e^{-\frac{bt}{2m}}$.

$$A=A_0e^{-\frac{bt}{2m}} \rightarrow 0.05A_0 = Ae^{-\frac{b(3T)}{2m}} = Ae^{-\frac{b}{2m}\frac{3}{f}} \rightarrow \ln(0.05) = -\frac{3b}{2m}\frac{1}{f} \rightarrow$$

$$\ln(0.05) = -\frac{3b}{2m}\frac{1}{\frac{1}{2\pi}\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}} = -\frac{3b}{2m}\frac{2\pi}{\sqrt{\frac{b_{\text{critical}}^2}{4m^2}-\frac{b^2}{4m^2}}} = -\frac{6\pi b}{\sqrt{b_{\text{critical}}^2-b^2}} \rightarrow$$

$$\frac{b}{b_{\text{critical}}} = \left(1+\frac{36\pi^2}{\left[\ln(0.05)\right]^2}\right)^{-1/2} = 0.16$$

And so b has decreased to about 16% of its original value, or $\boxed{\text{decreased by a factor of 6}}$. If we used 2% instead of 5%, we would have found that b decreased to about 20% of its original value. And if we used 10% instead of 5%, we would have found that b decreased to about 6% of its original value.

58. (*a*) Since the angular displacement is given as $\theta = Ae^{-\gamma t}\cos(\omega' t)$, we see that the displacement at $t = 0$ is the initial amplitude, so $A = 15°$. We evaluate the amplitude 8.0 seconds later.

$$5.5° = 15°e^{-\gamma(8.0\text{s})} \rightarrow \gamma = \frac{-1}{8.0\,\text{s}}\ln\left(\frac{5.5}{15}\right) = 0.1254\,\text{s}^{-1} \approx \boxed{0.13\,\text{s}^{-1}}$$

(*b*) The approximate period can be found from the damped angular frequency. The undamped angular frequency is also needed for the calculation.

$$\omega_0 = \sqrt{\frac{mgh}{I}} = \sqrt{\frac{mg\left(\frac{1}{2}\ell\right)}{\frac{1}{3}m\ell^2}} = \sqrt{\frac{3g}{2\ell}}$$

$$\omega' = \sqrt{\omega_0^2-\gamma^2} = \sqrt{\frac{3g}{2\ell}-\gamma^2} = \sqrt{\frac{3\left(9.80\,\text{m/s}^2\right)}{2(0.85\,\text{m})}-\left(0.1254\,\text{s}^{-1}\right)^2} = 4.157\,\text{rad/s}$$

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi\,\text{rad}}{4.157\,\text{rad/s}} = \boxed{1.5\,\text{s}}$$

(*c*) We solve the equation of motion for the time when the amplitude is half the original amplitude.

$$7.5° = 15°e^{-\gamma t} \rightarrow t_{1/2} = \frac{\ln 2}{\gamma} = \frac{\ln 2}{0.1254\,\text{s}^{-1}} = \boxed{5.5\,\text{s}}$$

59. (*a*) The energy of the oscillator is all potential energy when the cosine (or sine) factor is 1, and so $E = \frac{1}{2}kA^2 = \frac{1}{2}kA_0^2e^{-\frac{bt}{m}}$. The oscillator is losing 6.0% of its energy per cycle. Use this to find the actual frequency, and then compare to the natural frequency.

$$E(t+T) = 0.94E(t) \rightarrow \tfrac{1}{2}kA_0^2e^{-\frac{b(t+T)}{m}} = 0.94\left(\tfrac{1}{2}kA_0^2e^{-\frac{bt}{m}}\right) \rightarrow e^{-\frac{bT}{m}} = 0.94 \rightarrow$$

$$\frac{b}{2m}=-\frac{1}{2T}\ln(0.94)=-\frac{\omega_0}{4\pi}\ln(0.94)$$

$$\frac{f'-f_0}{f_0}=\frac{\frac{1}{2\pi}\sqrt{\omega_0^2-\frac{b^2}{4m^2}}-\frac{\omega_0}{2\pi}}{\frac{\omega_0}{2\pi}}=\sqrt{1-\frac{b^2}{4m^2\omega_0^2}}-1=\sqrt{1-\frac{[\ln(0.94)]^2}{16\pi^2}}-1\approx 1-\tfrac{1}{2}\frac{[\ln(0.94)]^2}{16\pi^2}-1$$

$$=-\tfrac{1}{2}\frac{[\ln(0.94)]^2}{16\pi^2}=-1.2\times10^{-5}\ \rightarrow\ \%\,\text{diff}=\left(\frac{f'-f_0}{f_0}\right)100=\boxed{\left(-1.2\times10^{-3}\right)\%}$$

(*b*) The amplitude's decrease in time is given by $A=A_0e^{-\frac{bt}{2m}}$. Find the decrease at a time of nT, and solve for n. The value of $\frac{b}{2m}$ was found in part (*a*).

$$A=A_0e^{-\frac{bt}{2m}}\ \rightarrow\ A_0e^{-1}=A_0e^{-\frac{bnT}{2m}}\ \rightarrow\ 1=\frac{b}{2m}nT=-\frac{1}{2T}\ln(0.94)nT\ \rightarrow$$

$$n=-\frac{2}{\ln(0.94)}=32.32\approx\boxed{32\text{ periods}}$$

60. The amplitude of a damped oscillator decreases according to $A=A_0e^{-\gamma t}=A_0e^{-\frac{bt}{2m}}$. The data can be used to find the damping constant.

$$A=A_0e^{-\frac{bt}{2m}}\ \rightarrow\ b=\frac{2m}{t}\ln\left(\frac{A_0}{A}\right)=\frac{2(0.075\,\text{kg})}{(3.5\,\text{s})}\ln\left(\frac{5.0}{2.0}\right)=\boxed{0.039\,\text{kg/s}}$$

61. (*a*) For the "lightly damped" harmonic oscillator, we have $b^2\ll 4mk\ \rightarrow\ \frac{b^2}{4m^2}\ll\frac{k}{m}\ \rightarrow\ \omega'\approx\omega_0$. We also assume that the object starts to move from maximum displacement, and so

$$x=A_0e^{-\frac{bt}{2m}}\cos\omega' t\ \text{ and }\ v=\frac{dx}{dt}=-\frac{b}{2m}A_0e^{-\frac{bt}{2m}}\cos\omega' t-\omega' A_0e^{-\frac{bt}{2m}}\sin\omega' t\approx-\omega_0A_0e^{-\frac{bt}{2m}}\sin\omega' t.$$

$$E=\tfrac{1}{2}kx^2+\tfrac{1}{2}mv^2=\tfrac{1}{2}kA_0^2e^{-\frac{bt}{m}}\cos^2\omega' t+\tfrac{1}{2}m\omega_0^2A_0^2e^{-\frac{bt}{m}}\sin^2\omega' t$$

$$=\tfrac{1}{2}kA_0^2e^{-\frac{bt}{m}}\cos^2\omega' t+\tfrac{1}{2}kA_0^2e^{-\frac{bt}{m}}\sin^2\omega' t=\tfrac{1}{2}kA_0^2e^{-\frac{bt}{m}}=\boxed{E_0e^{-\frac{bt}{m}}}$$

(*b*) The fractional loss of energy during one period is as follows. Note that we use the approximation that $\frac{b}{2m}\ll\omega_0=\frac{2\pi}{T}\ \rightarrow\ \frac{bT}{m}\ll 4\pi\ \rightarrow\ \frac{bT}{m}\ll 1$.

$$\Delta E=E(t)-E(t+T)=E_0e^{-\frac{bt}{m}}-E_0e^{-\frac{b(t+T)}{m}}=E_0e^{-\frac{bt}{m}}\left(1-e^{-\frac{bT}{m}}\right)\ \rightarrow$$

$$\frac{\Delta E}{E}=\frac{E_0e^{-\frac{bt}{m}}\left(1-e^{-\frac{bT}{m}}\right)}{E_0e^{-\frac{bt}{m}}}=1-e^{-\frac{bT}{m}}\approx 1-\left(1-\frac{bT}{m}\right)=\frac{bT}{m}=\frac{b2\pi}{m\omega_0}=\boxed{\frac{2\pi}{Q}}$$

62. (*a*) From problem 25 (b), we can calculate the frequency of the undamped motion.

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} = 2\pi\sqrt{\frac{m}{2k}} \quad \rightarrow$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \sqrt{\frac{k}{2\pi^2 m}} = \sqrt{\frac{125\,\mathrm{N/s}}{2\pi^2\left(0.215\,\mathrm{kg}\right)}} = \boxed{5.43\,\mathrm{Hz}}$$

(*b*) Eq. 14-16 says $x = Ae^{-\gamma t}\cos\omega' t$, which says the amplitude follows the relationship $x_{\max} = Ae^{-\gamma t}$. Use the fact that $x_{\max} = \frac{1}{2}A$ after 55 periods have elapsed, and assume that the damping is light enough that the damped frequency is the same as the natural frequency.

$$\tfrac{1}{2}A = Ae^{-\gamma(55T)} \quad \rightarrow \quad \gamma = \frac{\ln 2}{55T} = \frac{f}{55}\ln 2 = \frac{5.43\,\mathrm{Hz}}{55}\ln 2 = 0.06843\,\mathrm{s}^{-1} \approx \boxed{0.0684\,\mathrm{s}^{-1}}$$

(*c*) Again use $x_{\max} = Ae^{-\gamma t}$.

$$x_{\max} = Ae^{-\gamma t} \quad \rightarrow \quad \tfrac{1}{4}A = Ae^{-\gamma t} \quad \rightarrow \quad t = \frac{\ln 4}{\gamma} = \frac{\ln 4}{0.06843\,\mathrm{s}^{-1}} = \boxed{20.3\,\mathrm{s}}$$

This is the time for 110 oscillations, since 55 oscillations corresponds to a "half-life."

63. (*a*) Eq. 14-24 is used to calculate ϕ_0.

$$\phi_0 = \tan^{-1}\frac{\omega_0^2 - \omega^2}{\omega(b/m)} \quad \rightarrow \quad \text{if } \omega = \omega, \quad \phi_0 = \tan^{-1}\frac{\omega_0^2 - \omega_0^2}{\omega(b/m)} = \boxed{0}$$

(*b*) With $\omega = \omega_0$, we have $F_{\text{ext}} = F_0\cos\omega_0 t$ and $x = A_0\sin\omega_0 t$. The displacement and the driving force are one-quarter cycle $\left(\frac{1}{2}\pi\,\text{rad or } 90^\circ\right)$ out of phase with each other. $\boxed{\text{The displacement is 0 when the driving force is a maximum, and the displacement is a maximum (}+A\text{ or }-A\text{) when the driving force is 0.}}$

(*c*) As mentioned above, the phase difference is $\boxed{90^\circ}$.

64. Eq. 14-23 gives the amplitude A_0 as a function of driving frequency ω. To find the frequency for maximum amplitude, we set $\dfrac{dA_0}{d\omega} = 0$ and solve for ω.

$$A_0 = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + b^2\omega^2/m^2}} = \frac{F_0}{m}\left[\left(\omega^2 - \omega_0^2\right)^2 + b^2\omega^2/m^2\right]^{-1/2}$$

$$\frac{dA_0}{d\omega} = \frac{F_0}{m}\left(-\tfrac{1}{2}\right)\left[\left(\omega^2 - \omega_0^2\right)^2 + b^2\omega^2/m^2\right]^{-3/2}\left[2\left(\omega^2 - \omega_0^2\right)2\omega + 2b^2\omega/m^2\right] = 0 \quad \rightarrow$$

$$-\tfrac{1}{2}\frac{F_0}{m}\frac{\left[2\left(\omega^2 - \omega_0^2\right)2\omega + 2b^2\omega/m^2\right]}{\left[\left(\omega^2 - \omega_0^2\right)^2 + b^2\omega^2/m^2\right]^{3/2}} = 0 \quad \rightarrow \quad 2\left(\omega^2 - \omega_0^2\right)2\omega + 2b^2\omega/m^2 = 0 \quad \rightarrow$$

$$\omega^2 = \omega_0^2 - \frac{b^2}{2m^2} \quad \rightarrow \quad \boxed{\omega = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}}$$

65. We approximate that each spring of the car will effectively support one-fourth of the mass. The rotation of the improperly-balanced car tire will force the spring into oscillation. The shaking will be most prevalent at resonance, where the frequency of the tire matches the frequency of the spring. At resonance, the angular velocity of the car tire, $\omega=\frac{v}{r}$, will be the same as the angular frequency of the spring system, $\omega=\sqrt{\frac{k}{m}}$.

$$\omega=\frac{v}{r}=\sqrt{\frac{k}{m}} \quad\rightarrow\quad v=r\sqrt{\frac{k}{m}}=(0.42\,\text{m})\sqrt{\frac{16{,}000\,\text{N/m}}{\frac{1}{4}(1150\,\text{kg})}}=\boxed{3.1\,\text{m/s}}$$

66. First, we put Eq. 14-23 into a form that explicitly shows A_0 as a function of Q and has the ratio ω/ω_0.

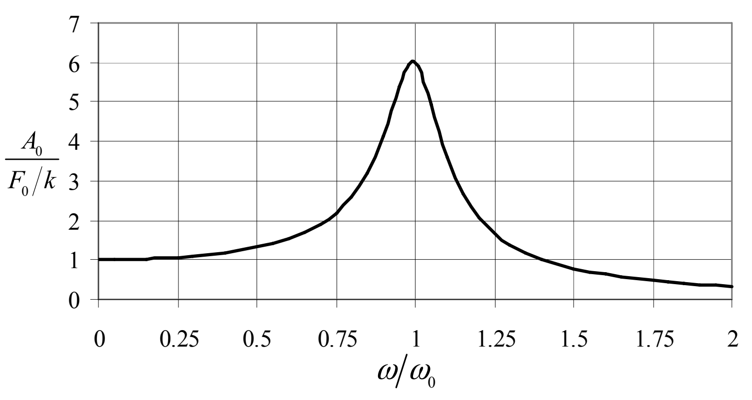
$$A_0=\frac{F_0}{m\sqrt{\left(\omega^2-\omega_0^2\right)^2+b^2\omega^2/m^2}}=\frac{F_0}{m\sqrt{\left(\omega^2\frac{\omega_0^2}{\omega_0^2}-\omega_0^2\right)^2+\frac{b^2\omega^2}{m^2}\frac{\omega_0^2}{\omega_0^2}}}$$

$$=\frac{F_0}{m\sqrt{\omega_0^4\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\omega_0^4\frac{b^2}{m^2\omega_0^2}\frac{\omega^2}{\omega_0^2}}}=\frac{F_0}{m\omega_0^2\sqrt{\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}}}=\frac{F_0/\left(m\omega_0^2\right)}{\sqrt{\left(\frac{\omega^2}{\omega_0^2}-1\right)^2+\frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}}}$$

$$=\frac{F_0/k}{\sqrt{\left(\omega^2/\omega_0^2-1\right)^2+\frac{1}{Q^2}\omega^2/\omega_0^2}} \quad\rightarrow\quad \boxed{\frac{A_0}{F_0/k}=\frac{1}{\sqrt{\left((\omega/\omega_0)^2-1\right)^2+(\omega/\omega_0)^2\frac{1}{Q^2}}}}$$

For a value of $Q=6.0$, the following graph is obtained.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.66."



$\boxed{67.}$ Apply the resonance condition, $\omega=\omega_0$, to Eq. 14-23, along with the given condition of $A_0=23.7\frac{F_0}{m}$. Note that for this condition to be true, the value of 23.7 must have units of s^2.

$$A_0=\frac{F_0}{m\sqrt{\left(\omega^2-\omega_0^2\right)^2+b^2\omega^2/m^2}} \quad\rightarrow$$

$$A_0\left(\omega=\omega_0\right)=\frac{F_0}{m\sqrt{b^2\omega_0^2/m^2}}=\frac{F_0}{m\dfrac{b\omega_0}{m}}=\frac{F_0}{m\dfrac{b\omega_0^2}{m\omega_0}}=\frac{F_0}{\dfrac{m\omega_0^2}{Q}}=Q\frac{F_0}{k}=23.7\frac{F_0}{k}\quad\rightarrow\quad Q=\boxed{23.7}$$

68. We are to show that $x=A_0\sin\left(\omega t+\phi_0\right)$ is a solution of $m\frac{dx^2}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos\omega t$ by direct substitution.

$$x=A_0\sin\left(\omega t+\phi_0\right)\ ;\ \frac{dx}{dt}=\omega A_0\cos\left(\omega t+\phi_0\right)\ ;\ \frac{d^2x}{dt^2}=-\omega^2A_0\sin\left(\omega t+\phi_0\right)$$

$$m\frac{dx^2}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos\omega t\quad\rightarrow$$

$$m\left[-\omega^2A_0\sin\left(\omega t+\phi_0\right)\right]+b\left[\omega A_0\cos\left(\omega t+\phi_0\right)\right]+k\left[A_0\sin\left(\omega t+\phi_0\right)\right]=F_0\cos\omega t$$

Expand the trig functions.

$$\left(kA_0-m\omega^2A_0\right)\left[\sin\omega t\cos\phi_0+\cos\omega t\sin\phi_0\right]+b\omega A_0\left[\cos\omega t\cos\phi_0-\sin\omega t\sin\phi_0\right]=F_0\cos\omega t$$

Group by function of time.

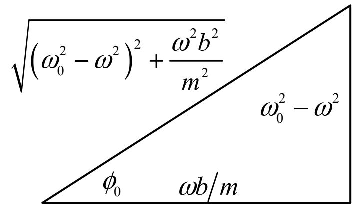
$$\left[\left(kA_0-m\omega^2A_0\right)\cos\phi_0-b\omega A_0\sin\phi_0\right]\sin\omega t+\left[\left(kA_0-m\omega^2A_0\right)\sin\phi_0+b\omega A_0\cos\phi_0\right]\cos\omega t$$
$$=F_0\cos\omega t$$

The equation has to be valid for all times, which means that the coefficients of the functions of time must be the same on both sides of the equation. Since there is no $\sin\omega t$ on the right side of the equation, the coefficient of $\sin\omega t$ must be 0.

$$\left(kA_0-m\omega^2A_0\right)\cos\phi_0-b\omega A_0\sin\phi_0=0\quad\rightarrow$$

$$\frac{\sin\phi_0}{\cos\phi_0}=\frac{kA_0-m\omega^2A_0}{b\omega A_0}=\frac{k-m\omega^2}{b\omega}=\frac{m\omega_0^2-m\omega^2}{b\omega}=\frac{\omega_0^2-\omega^2}{\omega b/m}=\tan\phi_0\quad\rightarrow\quad\boxed{\phi_0=\tan^{-1}\frac{\omega_0^2-\omega^2}{\omega b/m}}$$

Thus we see that Eq. 14-24 is necessary for $x=A_0\sin\left(\omega t+\phi_0\right)$ to be the solution. This can be illustrated with the diagram shown.



Equate the coefficients of $\cos\omega t$.

$$\left(kA_0-m\omega^2A_0\right)\sin\phi_0+b\omega A_0\cos\phi_0=F_0\quad\rightarrow$$

$$A_0\left[\left(k-m\omega^2\right)\frac{\left(\omega_0^2-\omega^2\right)}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\dfrac{\omega^2b^2}{m^2}}}+b\omega\frac{\dfrac{\omega b}{m}}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\dfrac{\omega^2b^2}{m^2}}}\right]=F_0\quad\rightarrow$$

$$A_0m\left[\frac{\left(\omega_0^2-\omega^2\right)^2}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\dfrac{\omega^2b^2}{m^2}}}+\frac{\dfrac{\omega^2b^2}{m^2}}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\dfrac{\omega^2b^2}{m^2}}}\right]=F_0\quad\rightarrow\quad\boxed{A_0=\frac{F_0}{m\left[\sqrt{\left(\omega_0^2-\omega^2\right)^2+\dfrac{\omega^2b^2}{m^2}}\right]}}$$

Thus we see that Eq. 14-23 is also necessary for $x=A_0\sin\left(\omega t+\phi_0\right)$ to be the solution.

69. (*a*) For the damped oscillator, the amplitude decays according to $A = A_0 e^{-\frac{bt}{2m}}$. We are also given the Q value, and $Q = \dfrac{m\omega_0}{b}$. We use these relationships to find the time for the amplitude to decrease to one-third of its original value.

$$Q = \frac{m\omega_0}{b} \;\rightarrow\; \frac{b}{m} = \frac{\omega_0}{Q} = \frac{\sqrt{g/\ell}}{Q} \;\; ; \; A = A_0 e^{-\frac{bt_{1/3}}{2m}} = \tfrac{1}{3}A_0 \;\rightarrow$$

$$t_{1/3} = \frac{2m}{b}\ln 3 = \frac{2Q}{\omega_0}\ln 3 = \frac{2Q}{\sqrt{g/\ell}}\ln 3 = \frac{2(350)}{\sqrt{\left(9.80\,\mathrm{m/s^2}\right)/(0.50\,\mathrm{m})}}\ln 3 = 173.7\,\mathrm{s} \approx \boxed{170\,\mathrm{s}}$$

(*b*) The energy is all potential energy when the displacement is at its maximum value, which is the amplitude. We assume that the actual angular frequency is very nearly the same as the natural angular frequency.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}\omega^2 m\left(A_0 e^{-\frac{bt}{2m}}\right)^2 = \frac{mg}{2\ell}A_0^2 e^{-\frac{bt}{m}} \;\; ; \;\; \frac{dE}{dt} = -\frac{b}{m}\frac{mg}{2\ell}A_0^2 e^{-\frac{bt}{m}} \;\rightarrow$$

$$\left.\frac{dE}{dt}\right|_{t=0} = -\frac{\sqrt{g/\ell}}{Q}\frac{mg}{2\ell} = \frac{mA_0^2}{2Q}\left(\frac{g}{\ell}\right)^{3/2} = \frac{(0.27\,\mathrm{kg})(0.020\,\mathrm{m})^2}{2(350)}\left(\frac{9.80\,\mathrm{m/s^2}}{0.50\,\mathrm{m}}\right)^{3/2} = \boxed{1.3\times10^{-5}\,\mathrm{W}}$$

(*c*) Use Eq. 14-26 to find the frequency spread.

$$\frac{\Delta\omega}{\omega_0} = Q \;\rightarrow\; \frac{\Delta 2\pi f}{2\pi f_0} = \frac{\Delta f}{f_0} = \frac{1}{Q} \;\rightarrow$$

$$\Delta f = \frac{f_0}{Q} = \frac{\omega_0}{2\pi Q} = \frac{\sqrt{g/\ell}}{2\pi Q} = \frac{\sqrt{\left(9.80\,\mathrm{m/s^2}\right)/(0.50\,\mathrm{m})}}{2\pi(350)} = 2.0\times10^{-3}\,\mathrm{Hz}$$

Since this is the total spread about the resonance frequency, the driving frequency must be within $\boxed{1.0\times10^{-3}\,\mathrm{Hz}}$ on either side of the resonance frequency.

70. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. The amount of stretch of the fire net is given by x, measured positively in the downward direction. The vertical displacement for gravitational potential energy is given by the variable y, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height and the lowest location of the person. The person has no kinetic energy at either location.

$$E_{\text{top}} = E_{\text{bottom}} \;\rightarrow\; mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}kx_{\text{bottom}}^2$$

$$k = 2mg\frac{\left(y_{top} - y_{\text{bottom}}\right)}{x_{\text{bottom}}^2} = 2(62\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)\frac{\left[20.0\,\mathrm{m} - (-1.1\,\mathrm{m})\right]}{(1.1\,\mathrm{m})^2} = 2.119\times10^4\,\mathrm{N/m}$$

(*a*) If the person were to lie on the fire net, they would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$F_{\text{ext}} = kx = mg \;\rightarrow\; x = \frac{mg}{k} = \frac{(62\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)}{2.1198\times10^4\,\mathrm{N/m}} = \boxed{2.9\times10^{-2}\,\mathrm{m}}$$

(*b*) To find the amount of stretch given a starting height of 38 m, again use conservation of energy. Note that $y_{\text{bottom}} = -x$, and there is no kinetic energy at the top or bottom positions.

$$E_{\text{top}} = E_{\text{bottom}} \rightarrow mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}kx^2 \rightarrow x^2 - 2\frac{mg}{k}x - 2\frac{mg}{k}y_{top} = 0$$

$$x^2 - 2\frac{(62\,\text{kg})(9.80\,\text{m/s}^2)}{2.1198\times10^4\,\text{N/m}}x - 2\frac{(62\,\text{kg})(9.80\,\text{m/s}^2)}{2.1198\times10^4\,\text{N/m}}(38\,\text{m}) = 0 \rightarrow$$

$$x^2 - 0.057326\,x - 2.1784 = 0 \rightarrow x = 1.5049\text{ m},\ -1.4476\text{ m}$$

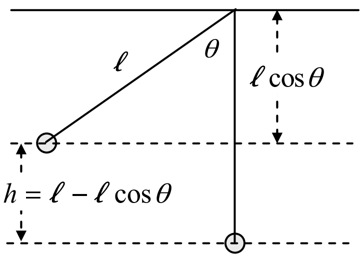
This is a quadratic equation. The solution is the positive root, since the net must be below the unstretched position. The result is $\boxed{1.5\,\text{m}}$.

71. Apply the conservation of mechanical energy to the car, calling condition # 1 to be before the collision and condition # 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_1 = 0$ and $v_2 = 0$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \rightarrow \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \rightarrow$$

$$x_2 = \sqrt{\frac{m}{k}}v_1 = \sqrt{\frac{1300\text{ kg}}{430\times10^3\text{ N/m}}}(2.0\,\text{m/s}) = \boxed{0.11\,\text{m}}$$

72. (*a*) The frequency can be found from the length of the pendulum, and the acceleration due to gravity.



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{9.80\,\text{m/s}^2}{0.63\,\text{m}}} = 0.6277\,\text{Hz} \approx \boxed{0.63\,\text{Hz}}$$

(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \rightarrow KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}}$$

$$0 + mg(L - L\cos\theta) = \tfrac{1}{2}mv_{\text{bottom}}^2 + 0$$

$$v_{\text{bottom}} = \sqrt{2gL(1-\cos\theta)} = \sqrt{2(9.80\,\text{m/s}^2)(0.63\,\text{m})(1-\cos15^\circ)} = 0.6487\,\text{m/s} \approx \boxed{0.65\,\text{m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{bottom}}^2 = \tfrac{1}{2}(0.295\,\text{kg})(0.6487\,\text{m/s})^2 = \boxed{6.2\times10^{-2}\,\text{J}}$$

$\boxed{73.}$ The frequency of a simple pendulum is given by $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$. The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.

(*a*) $$f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{g+a}{L}} = \frac{1}{2\pi}\sqrt{\frac{1.50g}{L}} = \sqrt{1.50}\,\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{1.50}f = \boxed{1.22f}$$

(*b*) $$f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{g+a}{L}} = \frac{1}{2\pi}\sqrt{\frac{0.5g}{L}} = \sqrt{0.5}\,\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{0.5}f = \boxed{0.71f}$$

74. The equation of motion is $x = 0.25\sin 5.50t = A\sin\omega t$.

(*a*) The amplitude is $A = x_{\max} = \boxed{0.25\,\text{m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 5.50\,\mathrm{s}^{-1} \rightarrow f = \dfrac{5.50\,\mathrm{s}^{-1}}{2\pi} = \boxed{0.875\,\mathrm{Hz}}$

(*c*) The period is the reciprocal of the frequency. $T = 1/f = \dfrac{2\pi}{5.50\mathrm{s}^{-1}} = \boxed{1.14\,\mathrm{s}}$.

(*d*) The total energy is given by

$$E_{\text{total}} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.650\,\mathrm{kg})\left[(5.50\,\mathrm{s}^{-1})(0.25\,\mathrm{m})\right]^2 = 0.6145\,\mathrm{J} \approx \boxed{0.61\,\mathrm{J}}.$$

(*e*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.650\,\mathrm{kg})(5.50\,\mathrm{s}^{-1})^2(0.15\,\mathrm{m})^2 = 0.2212\,\mathrm{J} \approx \boxed{0.22\,\mathrm{J}}.$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 0.6145\,\mathrm{J} - 0.2212\,\mathrm{J} = 0.3933\,\mathrm{J} \approx \boxed{0.39\,\mathrm{J}}.$$

75. (*a*) The car on the end of the cable produces tension in the cable, and stretches the cable according to Equation (12-4), $\Delta\ell = \dfrac{1}{E}\dfrac{F}{A}\ell_o$, where E is Young's modulus. Rearrange this equation to see that the tension force is proportional to the amount of stretch, $F = \dfrac{EA}{\ell_o}\Delta\ell$, and so the effective spring constant is $k = \dfrac{EA}{\ell_o}$. The period of the bouncing can be found from the spring constant and the mass on the end of the cable.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m\ell_o}{EA}} = 2\pi\sqrt{\frac{(1350\,\mathrm{kg})(20.0\,\mathrm{m})}{(200\times10^9\,\mathrm{N/m^2})\pi(3.2\times10^{-3}\,\mathrm{m})^2}} = 0.407\,\mathrm{s} \approx \boxed{0.41\,\mathrm{s}}$$

(*b*) The cable will stretch some due to the load of the car, and then the amplitude of the bouncing will make it stretch even farther. The total stretch is to be used in finding the maximum amplitude. The tensile strength is found in Table 12-2.

$$\frac{F}{A} = \frac{k(x_{\text{static}} + x_{\text{amplitude}})}{\pi r^2} = \text{tensile strength (abbrev T.S.)} \rightarrow$$

$$x_{\text{amplitude}} = \frac{(\mathrm{T.S.})\pi r^2}{k} - \Delta\ell = \frac{(\mathrm{T.S.})\pi r^2}{\dfrac{E\pi r^2}{\ell_0}} - \frac{mg\ell_0}{E\pi r^2} = \frac{\ell_0}{E}\left[(\mathrm{T.S.}) - \frac{mg}{\pi r^2}\right]$$

$$= \frac{(20.0\,\mathrm{m})}{(200\times10^9\,\mathrm{N/m^2})}\left[500\times10^6\,\mathrm{N/m^2} - \frac{(1350\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{\pi(0.0032\,\mathrm{m})^2}\right] = \boxed{9\times10^{-3}\,\mathrm{m}} = 9\,\mathrm{mm}$$

76. The spring constant does not change, but the mass does, and so the frequency will change. Use Eq. 14-7a to relate the spring constant, the mass, and the frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2} = f^2m = \text{constant} \rightarrow f_{\mathrm{O}}^2m_{\mathrm{O}} = f_{\mathrm{S}}^2m_{\mathrm{S}} \rightarrow$$

$$f_{\mathrm{S}} = f_{\mathrm{O}}\sqrt{\frac{m_{\mathrm{O}}}{m_{\mathrm{S}}}} = (3.7\times10^{13}\,\mathrm{Hz})\sqrt{\frac{16.0}{32.0}} = \boxed{2.6\times10^{13}\,\mathrm{Hz}}$$

77. The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$, and so the length is $\ell = \dfrac{T^2 g}{4\pi^2}$.

(*a*) $$\ell_{\text{Austin}} = \frac{T^2 g_{\text{Austin}}}{4\pi^2} = \frac{(2.000\text{ s})^2 (9.793\,\text{m/s}^2)}{4\pi^2} = 0.992238\,\text{m} \approx \boxed{0.9922\text{ m}}$$

(*b*) $$\ell_{\text{Paris}} = \frac{T^2 g_{\text{Paris}}}{4\pi^2} = \frac{(2.000\text{ s})^2 (9.809\,\text{m/s}^2)}{4\pi^2} = 0.993859\,\text{m} \approx 0.9939\text{ m}$$

$$\ell_{\text{Paris}} - \ell_{\text{Austin}} = 0.993859\text{ m} - 0.992238\text{ m} = 0.001621\text{ m} \approx \boxed{1.6\text{ mm}}$$

(*c*) $$\ell_{\text{Moon}} = \frac{T^2 g_{\text{Moon}}}{4\pi^2} = \frac{(2.00\text{ s})^2 (1.62\,\text{m/s}^2)}{4\pi^2} = \boxed{0.164\text{ m}}$$

78. The force of the man's weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of the force put on the raft by the man. This is analogous to pulling down on a mass–spring system that is in equilibrium, by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft–water system acts like a spring, with a spring constant found as follows.

$$k = \frac{F_{\text{ext}}}{x} = \frac{(75\,\text{kg})(9.80\,\text{m/s}^2)}{3.5\times10^{-2}\,\text{m}} = 2.1\times10^4\text{ N/m}$$

(*a*) The frequency of vibration is determined by the "spring constant" and the mass of the raft.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.1\times10^4\text{ N/m}}{320\text{ kg}}} = 1.289\,\text{Hz} \approx \boxed{1.3\,\text{Hz}}$$

(*b*) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus

$$E_{\text{total}} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(2.1\times10^4\text{ N/m})(3.5\times10^{-2}\text{m})^2 = 12.86\,\text{J} \approx \boxed{13\,\text{J}}.$$

$\boxed{79.}$ The relationship between the velocity and the position of a SHO is given by Eq. 14-11b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$v = \pm v_{\text{max}}\sqrt{1 - x^2/A^2} = \tfrac{1}{2}v_{\text{max}} \quad\rightarrow\quad \pm\sqrt{1-x^2/A^2} = \tfrac{1}{2} \quad\rightarrow\quad 1 - x^2/A^2 = \tfrac{1}{4} \quad\rightarrow\quad x^2/A^2 = \tfrac{3}{4} \quad\rightarrow$$

$$\boxed{x = \pm\sqrt{3}A/2 \approx \pm 0.866A}$$

80. For the pebble to lose contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be *g* downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus if the board's downward acceleration exceeds *g*, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\text{max}} = 4\pi^2 f^2 A$.

$$a_{\text{max}} = 4\pi^2 f^2 A \le g \quad\rightarrow\quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.80\,\text{m/s}^2}{4\pi^2(2.5\text{ Hz})^2} \le \boxed{4.0\times10^{-2}\text{m}}$$

81. Assume the block has a cross-sectional area of A. In the equilibrium position, the net force on the block is zero, and so $F_{\text{buoy}} = mg$. When the block is pushed into the water (downward) an additional distance Δx, there is an increase in the buoyancy force (F_{extra}) equal to the weight of the additional water displaced. The weight of the extra water displaced is the density of water times the volume displaced.

$$F_{\text{extra}} = m_{\substack{\text{add.}\\\text{water}}} g = \rho_{\text{water}} V_{\substack{\text{add.}\\\text{water}}} g = \rho_{\text{water}} gA\Delta x = \left(\rho_{\text{water}} gA\right)\Delta x$$

This is the net force on the displaced block. Note that if the block is pushed down, the additional force is upwards. And if the block were to be displaced upwards by a distance Δx, the buoyancy force would actually be less than the weight of the block by the amount F_{extra}, and so there would be a net force downwards of magnitude F_{extra}. So in both upward and downward displacement, there is a net force of magnitude $\left(\rho_{\text{water}} gA\right)\Delta x$ but opposite to the direction of displacement. As a vector, we can write the following.

$$\vec{\mathbf{F}}_{\text{net}} = -\left(\rho_{\text{water}} gA\right)\Delta\vec{\mathbf{x}}$$

This is the equation of simple harmonic motion, with a "spring constant" of $\boxed{k = \rho_{\text{water}} gA}$

82. (*a*) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \quad\rightarrow$$

$$k = m\frac{v_1^2}{x_2^2} = (950\,\text{kg})\frac{(25\,\text{m/s})^2}{(5.0\,\text{m})^2} = 2.375\times10^4\ \text{N/m} \approx \boxed{2.4\times10^4\ \text{N/m}}$$

(*b*) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$\tfrac{1}{2}T = \tfrac{1}{2}2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{(950\,\text{kg})}{2.375\times10^4\ \text{N/m}}} = \boxed{0.63\,\text{s}}$$

83. (*a*) The effective spring constant is found from the final displacement caused by the additional mass on the table. The weight of the mass will equal the upward force exerted by the compressed springs.

$$F_{\text{grav}} = F_{\text{springs}} \quad\rightarrow\quad mg = k\Delta y \quad\rightarrow$$

$$k = \frac{mg}{\Delta y} = \frac{(0.80\,\text{kg})(9.80\,\text{m/s}^2)}{(0.060\,\text{m})} = 130.67\,\text{N/m} \approx \boxed{130\,\text{N/m}}$$

(*b*) We assume the collision takes place in such a short time that the springs do not compress a significant amount during the collision. Use momentum conservation to find the speed immediately after the collision.

$$p_{\text{before}} = p_{\text{after}} \quad\rightarrow\quad m_{\text{clay}} v_{\text{clay}} = \left(m_{\text{clay}} + m_{\text{table}}\right) v_{\text{after}} \quad\rightarrow$$

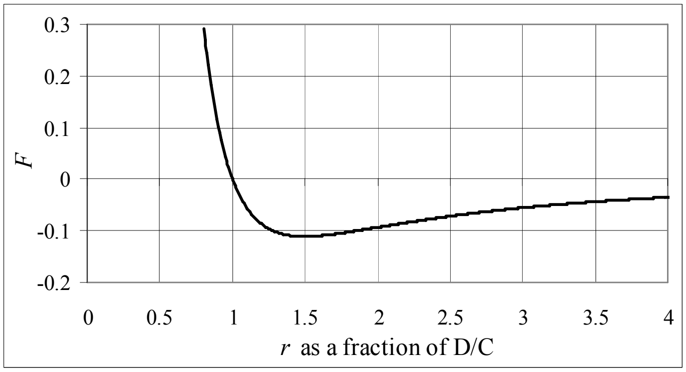
$$v_{\text{after}} = \frac{m_{\text{clay}}}{\left(m_{\text{clay}} + m_{\text{table}}\right)} v_{\text{clay}} = \frac{0.80\,\text{kg}}{2.40\,\text{kg}}(1.65\,\text{m/s}) = 0.55\,\text{m/s}$$

As discussed in the text, if we measure displacements from the new equilibrium position, we may use an energy analysis of the spring motion without including the effects of gravity. The total elastic and kinetic energy immediately after the collision will be the maximum elastic energy, at the amplitude location.

$$E_1 = E_2 \rightarrow \tfrac{1}{2} m_{\text{total}} v_{\text{after}}^2 + \tfrac{1}{2} k x_{\text{after}}^2 = \tfrac{1}{2} k A^2 \rightarrow$$

$$A = \sqrt{\frac{m_{\text{total}}}{k} v_{\text{after}}^2 + x_{\text{after}}^2} = \sqrt{\left(\frac{2.40\,\text{kg}}{130.67\,\text{N/m}}\right)(0.55\,\text{m/s})^2 + (0.060\,\text{m})^2} = 0.096\,\text{m} = \boxed{9.6\,\text{cm}}$$

84. (*a*) The graph is shown. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.84a."



(*b*) Equilibrium occurs at the location where the force is 0. Set the force equal to 0 and solve for the separation distance *r*.

$$F(r_0) = -\frac{C}{r_0^2} + \frac{D}{r_0^3} = 0 \rightarrow$$

$$\frac{C}{r_0^2} = \frac{D}{r_0^3} \rightarrow Cr_0^3 = Dr_0^2 \rightarrow \boxed{r_0 = \frac{D}{C}}$$

This does match with the graph, which shows $F = 0$ at $r = D/C$.

(*c*) We find the net force at $r = r_0 + \Delta r$. Use the binomial expansion.

$$F(r_0 + \Delta r) = -C(r_0 + \Delta r)^{-2} + D(r_0 + \Delta r)^{-3} = -Cr_0^{-2}\left(1 + \frac{\Delta r}{r_0}\right)^{-2} + Dr_0^{-3}\left(1 + \frac{\Delta r}{r_0}\right)^{-3}$$

$$\approx -\frac{C}{r_0^2}\left(1 - 2\frac{\Delta r}{r_0}\right) + \frac{D}{r_0^3}\left(1 - 3\frac{\Delta r}{r_0}\right) = \frac{C}{r_0^3}\left[-r_0\left(1 - 2\frac{\Delta r}{r_0}\right) + \frac{D}{C}\left(1 - 3\frac{\Delta r}{r_0}\right)\right]$$

$$= \frac{C}{r_0^3}\left[-r_0 + 2\Delta r + r_0 - 3\Delta r\right] = \frac{C}{r_0^3}\left[-\Delta r\right] \rightarrow F(r_0 + \Delta r) = -\frac{C}{r_0^3}\Delta r$$

We see that the net force is proportional to the displacement and in the opposite direction to the displacement. Thus the motion is simple harmonic.

(*d*) Since for simple harmonic motion, the general form is $F = -kx$, we see that for this situation, the spring constant is given by $k = \dfrac{C}{r_0^3} = \boxed{\dfrac{C^4}{D^3}}$.

(*e*) The period of the motion can be found from Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k}} = \boxed{2\pi\sqrt{\frac{mD^3}{C^4}}}$$

85. (*a*) The relationship between the velocity and the position of a SHO is given by Eq. 14-11b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$v = \pm v_{\max}\sqrt{1 - x^2/x_0^2} = \tfrac{1}{2} v_{\max} \rightarrow \pm\sqrt{1 - x^2/x_0^2} = \tfrac{1}{2} \rightarrow 1 - x^2/x_0^2 = \tfrac{1}{4} \rightarrow$$

$$x^2/x_0^2 = \tfrac{3}{4} \rightarrow \boxed{x = \pm\sqrt{3}x_0/2 \approx \pm 0.866 x_0}$$

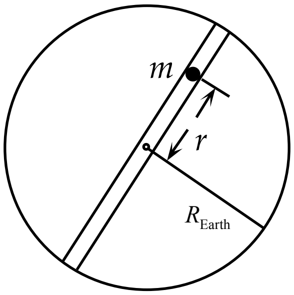
(*b*) Since $F = -kx = ma$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a = -x\,k/m$. Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\boxed{\pm\frac{1}{2}x_0}$

86. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 14-7a.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k = 4\pi^2 f^2 m = 4\pi^2\left(2.83\times10^{13}\,\text{Hz}\right)\left(16.00\,\text{u}\right)\left(\frac{1.66\times10^{-27}\,\text{kg}}{1\,\text{u}}\right) = \boxed{840\,\text{N/m}}\ \left(3\text{ sig. fig.}\right)$$

87. We quote from the next to last paragraph of Appendix D: "… we see that at points within a solid sphere, say 100 km below the Earth's surface, only the mass up to that radius contributes to the net force. The outer shells beyond the point in question contribute zero net gravitational effect." So when the mass is a distance *r* from the center of the Earth, there will be a force toward the center, opposite to *r*, due only to the mass within a sphere of radius *r*. We call that mass m_r. It is the density of the (assumed uniform) Earth, times the volume within a sphere of radius *r*.



$$m_r = \rho V_r = \frac{M_{\text{Earth}}}{V_{\text{Earth}}}V_r = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}^3}\tfrac{4}{3}\pi r^3 = M_{\text{Earth}}\frac{r^3}{R_{\text{Earth}}^3}$$

$$F = -\frac{Gmm_r}{r^2} = -\frac{GmM_{\text{Earth}}\dfrac{r^3}{R_{\text{Earth}}^3}}{r^2} = -\frac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3}r$$

The force on the object is opposite to and proportional to the displacement, and so will execute simple harmonic motion, with a "spring constant" of $k = \dfrac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3}$. The time for the apple to return is the period, found from the "spring constant."

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\dfrac{GmM_{\text{Earth}}}{R_{\text{Earth}}^3}}} = 2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}} = 2\pi\sqrt{\frac{\left(6.38\times10^6\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}}$$

$$= \boxed{507\,\text{s or }84.5\,\text{min}}$$

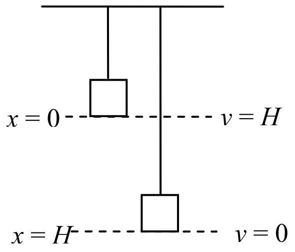
88. (*a*) The rod is a physical pendulum. Use Eq. 14-14 for the period of a physical pendulum.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}m\ell^2}{mg\left(\frac{1}{2}\ell\right)}} = 2\pi\sqrt{\frac{2\ell}{3g}} = 2\pi\sqrt{\frac{2\left(1.00\,\text{m}\right)}{3\left(9.80\,\text{m/s}^2\right)}} = \boxed{1.64\,\text{s}}$$

(*b*) The simple pendulum has a period given by $T = 2\pi\sqrt{\ell/g}$. Use this to find the length.

$$T = 2\pi\sqrt{\frac{\ell_{\text{simple}}}{g}} = 2\pi\sqrt{\frac{2\ell}{3g}} \quad \rightarrow \quad \ell_{\text{simple}} = \tfrac{2}{3}\ell = \tfrac{2}{3}\left(1.00\,\text{m}\right) = \boxed{0.667\,\text{m}}$$

89. Consider energy conservation for the mass over the range of motion from "letting go" (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no kinetic energy at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable "*x*" represents the amount that the spring is stretched from the equilibrium position.



$$E_{\text{top}} = E_{\text{bottom}} \rightarrow \tfrac{1}{2}mv_{\text{top}}^2 + mgy_{\text{top}} + \tfrac{1}{2}kx_{\text{top}}^2 = \tfrac{1}{2}mv_{\text{bottom}}^2 + mgy_{\text{bottom}} + \tfrac{1}{2}kx_{\text{bottom}}^2$$

$$\tfrac{1}{2}mv_{\text{top}}^2 + mgy_{\text{top}} + \tfrac{1}{2}kx_{\text{top}}^2 = \tfrac{1}{2}mv_{\text{bottom}}^2 + mgy_{\text{bottom}} + \tfrac{1}{2}kx_{\text{bottom}}^2$$

$$0 + mgH + 0 = 0 + 0 + \tfrac{1}{2}kH^2 \rightarrow \frac{k}{m} = \frac{2g}{H} = \omega^2 \rightarrow \omega = \sqrt{\frac{2g}{H}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2g}{H}} = \frac{1}{2\pi}\sqrt{\frac{2\left(9.80\,\text{m/s}^2\right)}{0.320\,\text{m}}} = \boxed{1.25\,\text{Hz}}$$

90. For there to be no slippage, the child must have the same acceleration as the slab. This will only happen if the force of static friction is big enough to provide the child with an acceleration at least as large as the maximum acceleration of the slab. The maximum force of static friction is given by $F_{\underset{\text{max}}{\text{fr}}} = \mu_s F_N$. Since the motion is horizontal and there are not other vertical forces besides gravity and the normal force, we know that $F_N = mg$. Finally, the maximum acceleration of the slab will occur at the endpoints, and is given by Eq. 14-9b. The mass to use in Eq. 14-9b is the mass of the oscillating system, $m + M$.

$$a_{\underset{\text{max}}{\text{fr}}} \geq a_{\underset{\text{max}}{\text{elastic}}} \rightarrow \frac{\mu_s F_N}{m} = \frac{\mu_s mg}{m} = \mu_s g \geq \frac{k}{m+M}A \rightarrow$$

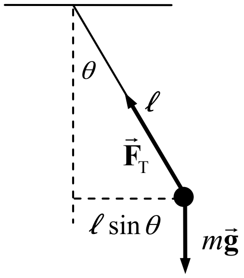
$$m \geq \frac{k}{\mu_s g}A - M = \frac{430\,\text{N/m}}{(0.40)\left(9.80\,\text{m/s}^2\right)}(0.50\,\text{m}) - 35\,\text{kg} = 19.8\text{kg} \approx \boxed{20\,\text{kg}}\ (2\text{ sig. fig.})$$

And so the child must have a minimum mass of 20 kg (about 44 lbs) in order to ride safely.

$\boxed{91.}$ We must make several assumptions. Consider a static displacement of the trampoline, by someone sitting on the trampoline mat. The upward elastic force of the trampoline must equal the downward force of gravity. We estimate that a 75-kg person will depress the trampoline about 25 cm at its midpoint.

$$kx = mg \rightarrow k = \frac{mg}{x} = \frac{(75\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{0.25\,\text{m}} = 2940\,\text{N/m} \approx \boxed{3000\,\text{N/m}}$$

92. We may use Eq. 10-14, $\sum \tau = I\alpha$, as long as the axis of rotation is fixed in an inertial frame. We choose the axis to be at the point of support, perpendicular to the plane of motion of the pendulum. There are two forces on the pendulum bob, but only gravity causes any torque. Note that if the pendulum is displaced in the counterclockwise direction (as shown in Fig. 14-46), then the torque caused by gravity will be in the clockwise direction, and vice versa. See the free-body diagram in order to write Newton's second law for rotation, with counterclockwise as the positive rotational direction.



$$\sum \tau = -mg\ell \sin\theta = I\alpha = I\frac{d^2\theta}{dt^2}$$

If the angular displacement is limited to about 15°, then $\sin\theta \approx \theta$.

$$-mg\ell\theta = I\frac{d^2\theta}{dt^2} \quad \rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{mg\ell}{I}\theta = -\frac{mg\ell}{m\ell^2}\theta = -\frac{g}{\ell}\theta$$

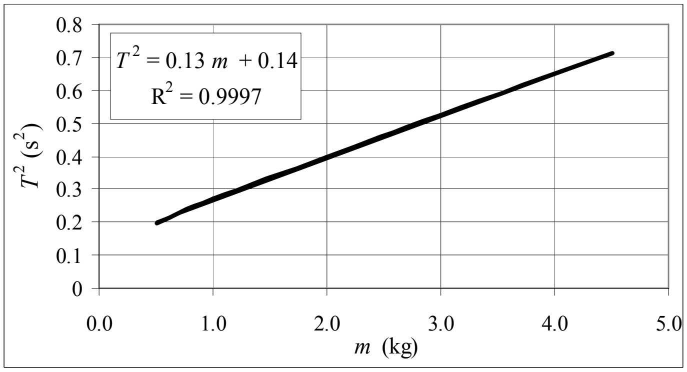
This is the equation of simple harmonic motion, with $\omega^2 = g/\ell$. Thus we can write the displacement of the pendulum as follows, imitating Eq. 14-4.

$$\theta = \theta_{\max}\cos(\omega t + \phi) \quad \rightarrow \quad \boxed{\theta = \theta_{\max}\cos\left(\sqrt{\frac{g}{\ell}}\,t + \phi\right)}$$

93. (*a*) Start with Eq. 14-7b, $T = 2\pi\sqrt{\frac{m}{k}} \rightarrow T^2 = \frac{4\pi^2}{k}m.$ This fits the straight-line equation form of $y = (\text{slope})x + (y - \text{intercept})$, if we plot T^2 vs. m. The slope is $4\pi^2/k$, and so $\boxed{k = \frac{4\pi^2}{\text{slope}}}$. The y-intercept is expected to be $\boxed{0}$.

(*b*) The graph is included on the next page. The slope is $0.1278\,\text{s}^2/\text{kg} \approx \boxed{0.13\,\text{s}^2/\text{kg}}$, and the y-intercept is $0.1390\,\text{s}^2 \approx \boxed{0.14\,\text{s}^2}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.93b."

(*c*) Start with the modified Eq. 14-7b.



$$T = 2\pi\sqrt{\frac{m + m_0}{k}} \quad \rightarrow$$

$$T^2 = \frac{4\pi^2}{k}m + \frac{4\pi^2 m_0}{k}$$

The spring constant is still given by $k = \frac{4\pi^2}{\text{slope}}$ and the y-intercept is expected to be $\frac{4\pi^2 m_0}{k}$.

$$k = \frac{4\pi^2}{0.1278\,\text{s}^2/\text{kg}} = 308.9\,\text{N/m} \approx \boxed{310\,\text{N/m}}$$

$$\frac{4\pi^2 m_0}{k} = y_0 = y - \text{intercept} \quad \rightarrow \quad m_0 = \frac{ky_0}{4\pi^2} = \frac{y_0}{\text{slope}} = \frac{0.1390\,\text{s}^2}{0.1278\,\text{s}^2/\text{kg}} = 1.088\,\text{kg} \approx \boxed{1.1\,\text{kg}}$$

(*d*) The mass m_0 can be interpreted as the effective mass of the spring. The mass of the spring does oscillate, but not all of the mass has the same amplitude of oscillation, and so m_0 is likely less than the mass of the spring. One straightforward analysis predicts that $m_0 = \frac{1}{3}M_{\text{spring}}$.

94. There is a subtle point in the modeling of this problem. It would be easy to assume that the net force on the spring is given by $F_{\text{net}} = -kx - cv^2 = ma$. But then the damping force would always be in the negative direction, since $cv^2 \geq 0$. So to model a damping force that is in the opposite direction of the velocity, we instead must use $F_{\text{net}} = -kx - cv|v| = ma$. Then the damping force will be in the

opposite direction of the velocity, and have a magnitude of cv^2. We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the oscillator at later times. We take the downward direction to be positive, and the starting position to be $y = 0$.

$$F = -kx - cv\left|v\right| = ma \quad \rightarrow \quad a = -\frac{k}{m}x - \frac{c}{m}v\left|v\right|$$

From Example 14-5, we have $x(0) = x_0 = -0.100\,\text{m}$ and $v(0) = v_0 = 0$. We calculate the initial acceleration, $a_0 = -\frac{k}{m}x_0 - \frac{c}{m}v_0\left|v_0\right|$, and assume that acceleration is constant over the next time interval. Then $x_1 = x_0 + v_0\Delta t + \frac{1}{2}a_0(\Delta t)^2$, $v_1 = v_0 + a_0\Delta t$, and $a_1 = -\frac{k}{m}x_1 - \frac{c}{m}v_1\left|v_1\right|$. This continues for each successive interval. We apply this method first for a time interval of 0.01 s, and record the position, velocity, and acceleration $t = 2.00$ s. Then we reduce the interval to 0.005 s and again find the position, velocity, and acceleration at $t = 2.00$ s. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within 2%. If not, a smaller interval is used, and the process repeated. For this problem, the results for position, velocity, and acceleration for time intervals of 0.001 s and 0.0005 s agree to within 2%. Here are the results for various intervals.

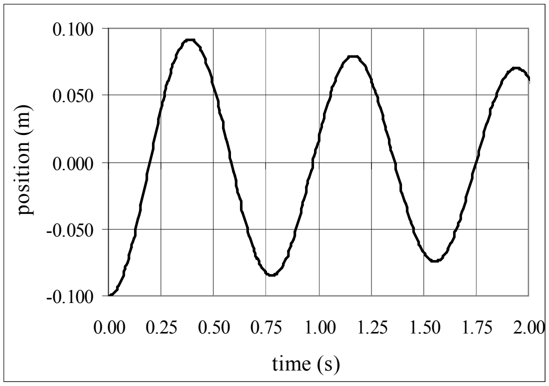
$\Delta t = 0.01\,\text{s}$: $x(2.00\,\text{s}) = 0.0713\,\text{m}$ $v(2.00\,\text{s}) = -0.291\,\text{m/s}$ $a(2.00\,\text{s}) = -4.58\,\text{m/s}^2$

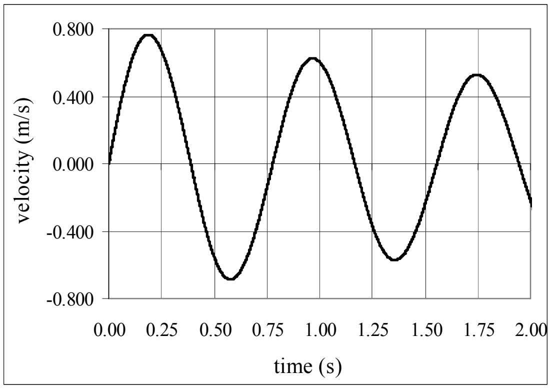
$\Delta t = 0.005\,\text{s}$: $x(2.00\,\text{s}) = 0.0632\,\text{m}$ $v(2.00\,\text{s}) = -0.251\,\text{m/s}$ $a(2.00\,\text{s}) = -4.07\,\text{m/s}^2$

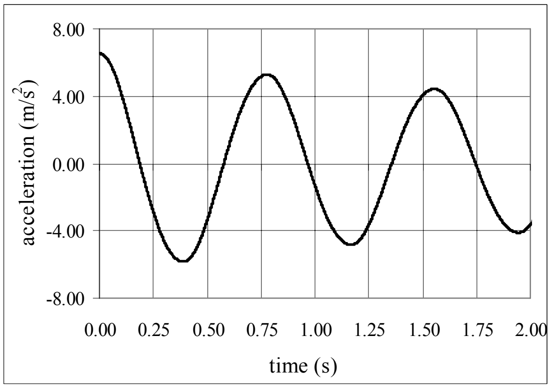
$\Delta t = 0.001\,\text{s}$: $x(2.00\,\text{s}) = 0.0574\,\text{m}$ $v(2.00\,\text{s}) = -0.222\,\text{m/s}$ $a(2.00\,\text{s}) = -3.71\,\text{m/s}^2$

$\Delta t = 0.0005\,\text{s}$: $x(2.00\,\text{s}) = 0.0567\,\text{m}$ $v(2.00\,\text{s}) = -0.218\,\text{m/s}$ $a(2.00\,\text{s}) = -3.66\,\text{m/s}^2$

The graphs of position, velocity, and acceleration are shown below. The spreadsheet used can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS", on tab "Problem 14.94".







CHAPTER 15: Wave Motion

Responses to Questions

$\boxed{1.}$ Yes. A simple periodic wave travels through a medium, which must be in contact with or connected to the source for the wave to be generated. If the medium changes, the wave speed and wavelength can change but the frequency remains constant.

2. The speed of the transverse wave is the speed at which the wave disturbance propagates down the cord. The individual tiny pieces of cord will move perpendicular to the cord with an average speed of four times the amplitude divided by the period. The average velocity of the individual pieces of cord is zero, but the average speed is not the same as the wave speed.

3. The maximum climb distance (4.3 m) occurs when the tall boat is at a crest and the short boat is in a trough. If we define the height difference of the boats on level seas as Δh and the wave amplitude as A, then $\Delta h + 2A = 4.3$ m. The minimum climb distance (2.5 m) occurs when the tall boat is in a trough and the short boat is at a crest. Then $\Delta h - 2A = 2.5$ m. Solving these two equations for A gives a wave amplitude of 0.45 m.

4. (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
   (*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

5. The speed of sound in air obeys the equation $v = \sqrt{B/\rho}$. If the bulk modulus is approximately constant and the density of air decreases with temperature, then the speed of sound in air should increase with increasing temperature.

6. First, estimate the number of wave crests that pass a given point per second. This is the frequency of the wave. Then, estimate the distance between two successive crests, which is the wavelength. The product of the frequency and the wavelength is the speed of the wave.

$\boxed{7.}$ The speed of sound is defined as $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of the material. The bulk modulus of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.

8. One reason is that the wave energy is spread out over a larger area as the wave travels farther from the source, as can be seen by the increasing diameter of the circular wave. The wave does not gain energy as it travels, so if the energy is spread over a larger area, the amplitude of the wave must be smaller. Secondly, the energy of the wave dissipates due to damping, and the amplitude decreases.

9. If two waves have the same speed but one has half the wavelength of the other, the wave with the shorter wavelength must have twice the frequency of the other. The energy transmitted by a wave depends on the wave speed and the square of the frequency. The wave with the shorter wavelength will transmit four times the energy transmitted by the other wave.

10. Yes. Any function of $(x - vt)$ will represent wave motion because it will satisfy the wave equation, Eq. 15-16.

11. The frequency does not change at the boundary because the two sections of cord are tied to each other and they must oscillate together. The wavelength and wave speed can be different, but the frequency must remain constant across the boundary.

12. The transmitted wave has a shorter wavelength. If the wave is inverted upon reflection at the boundary between the two sections of rope, then the second section of rope must be heavier. Therefore, the transmitted wave (traveling in the heavier rope) will have a lower velocity than the incident wave or the reflected wave. The frequency does not change at the boundary, so the wavelength of the transmitted wave must also be smaller.

$\boxed{13.}$ Yes, total energy is always conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity, and therefore kinetic energy.

14. Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

15. No. The energy of the incident and reflected wave is distributed around the antinodes, which exhibit large oscillations. The energy is a property of the wave as a whole, not of one particular point on the wave.

16. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large amplitude oscillations, even when the generating oscillations from the hand are small.

17. When a hand or mechanical oscillator vibrates a string, the motion of the hand or oscillator is not exactly the same for each vibration. This variation in the generation of the wave leads to nodes which are not quite “true” nodes. In addition, real cords have damping forces which tend to reduce the energy of the wave. The reflected wave will have a smaller amplitude than the incident wave, so the two waves will not completely cancel, and the node will not be a true node.

18. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave in comparison to the size of the obstacle. A hill is much larger than the wavelength of FM waves, and so there will be a “shadow” region behind the hill. However, the hill is not large compared to the wavelength of AM signals, so the AM radio waves will bend around the hill.

$\boxed{19.}$ Waves exhibit diffraction. If a barrier is placed between the energy source and the energy receiver, and energy is still received, it is a good indication that the energy is being carried by waves. If placement of the barrier stops the energy transfer, it may be because the energy is being transferred by particles or that the energy is being transferred by waves with wavelengths smaller than the barrier.

Solutions to Problems

$\boxed{1.}$ The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$$v = \lambda f = \lambda/T = (8.0\,\text{m})/(3.0\,\text{s}) = \boxed{2.7\,\text{m/s}}$$

2. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = 343\,\text{m/s}/262\ \text{Hz} = \boxed{1.31\ \text{m}}$$

3. The elastic and bulk moduli are taken from Table 12-1. The densities are taken from Table 13-1.

(*a*) For water: $v = \sqrt{B/\rho} = \sqrt{\dfrac{2.0\times10^9\ \text{N/m}^2}{1.00\times10^3\ \text{kg/m}^3}} = \boxed{1400\,\text{m/s}}$

(*b*) For granite: $v = \sqrt{E/\rho} = \sqrt{\dfrac{45\times10^9\ \text{N/m}^2}{2.7\times10^3\ \text{kg/m}^3}} = \boxed{4100\,\text{m/s}}$

(*c*) For steel: $v = \sqrt{E/\rho} = \sqrt{\dfrac{200\times10^9\ \text{N/m}^2}{7.8\times10^3\ \text{kg/m}^3}} = \boxed{5100\,\text{m/s}}$

4. To find the wavelength, use $\lambda = v/f$.

AM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^8\ \text{m/s}}{550\times10^3\,\text{Hz}} = 545\ \text{m}$ $\lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^8\ \text{m/s}}{1600\times10^3\,\text{Hz}} = 188\ \text{m}$ $\boxed{\text{AM: 190 m to 550 m}}$

FM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^8\ \text{m/s}}{88\times10^6\,\text{Hz}} = 3.41\,\text{m}$ $\lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^8\ \text{m/s}}{108\times10^6\,\text{Hz}} = 2.78\,\text{m}$ $\boxed{\text{FM: 2.8 m to 3.4 m}}$

5. The speed of the longitudinal wave is given by Eq. 15-3, $v = \sqrt{E/\rho}$. The speed and the frequency are used to find the wavelength. The bulk modulus is found in Table 12-1, and the density is found in Table 13-1.

$$\lambda = \frac{v}{f} = \frac{\sqrt{\dfrac{E}{\rho}}}{f} = \frac{\sqrt{\dfrac{100\times10^9\ \text{N/m}^2}{7.8\times10^3\ \text{kg/m}^3}}}{5800\,\text{Hz}} = \boxed{0.62\,\text{m}}$$

6. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v = \sqrt{F_T/\mu}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} \quad\rightarrow\quad \Delta t = \frac{\Delta x}{\sqrt{\dfrac{F_T}{\mu}}} = \frac{8.0\,\text{m}}{\sqrt{\dfrac{140\,\text{N}}{(0.65\,\text{kg})/(8.0\,\text{m})}}} = \boxed{0.19\,\text{s}}$$

$\boxed{7.}$ For a cord under tension, we have from Eq. 15-2 that $v = \sqrt{F_T/\mu}$. The speed is also the displacement divided by the elapsed time, $v = \dfrac{\Delta x}{\Delta t}$. The displacement is the length of the cord.

$$v = \sqrt{\frac{F_T}{\mu}} = \frac{\Delta x}{\Delta t} \quad\rightarrow\quad F_T = \mu\frac{\ell^2}{(\Delta t)^2} = \frac{m}{\ell}\frac{\ell^2}{(\Delta t)^2} = \frac{m\ell}{(\Delta t)^2} = \frac{(0.40\,\text{kg})(7.8\,\text{m})}{(0.85\,\text{s})^2} = \boxed{4.3\,\text{N}}$$

8. The speed of the water wave is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 12-1, and ρ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v=\frac{2\ell}{t}\quad\rightarrow\quad \ell=\frac{vt}{2}=\frac{t}{2}\sqrt{\frac{B}{\rho}}=\frac{2.8\,\text{s}}{2}\sqrt{\frac{2.0\times10^{9}\,\text{N/m}^2}{1.025\times10^{3}\,\text{kg/m}^3}}=\boxed{2.0\times10^{3}\,\text{m}}$$

9. (*a*) The speed of the pulse is given by

$$v=\frac{\Delta x}{\Delta t}=\frac{2\left(660\,\text{m}\right)}{17\,\text{s}}=77.65\,\text{m/s}\approx\boxed{78\,\text{m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v=\sqrt{F_{\text{T}}/\mu}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho=\frac{m}{V}=\frac{m}{\pi\left(d/2\right)^2\ell}\quad\rightarrow$$

$$\mu=\frac{m}{\ell}=\pi\rho\left(\frac{d}{2}\right)^2=\pi\left(7.8\times10^{3}\,\text{kg/m}^3\right)\left(\frac{1.5\times10^{-2}\,\text{m}}{2}\right)^2=1.378\,\text{kg/m}$$

$$v=\sqrt{F_{\text{T}}/\mu}\quad\rightarrow\quad F_{\text{T}}=v^2\mu=\left(77.65\,\text{m/s}\right)^2\left(1.378\,\text{kg/m}\right)=\boxed{8300\,\text{N}}$$

10. (*a*) Both waves travel the same distance, so $\Delta x=v_1t_1=v_2t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

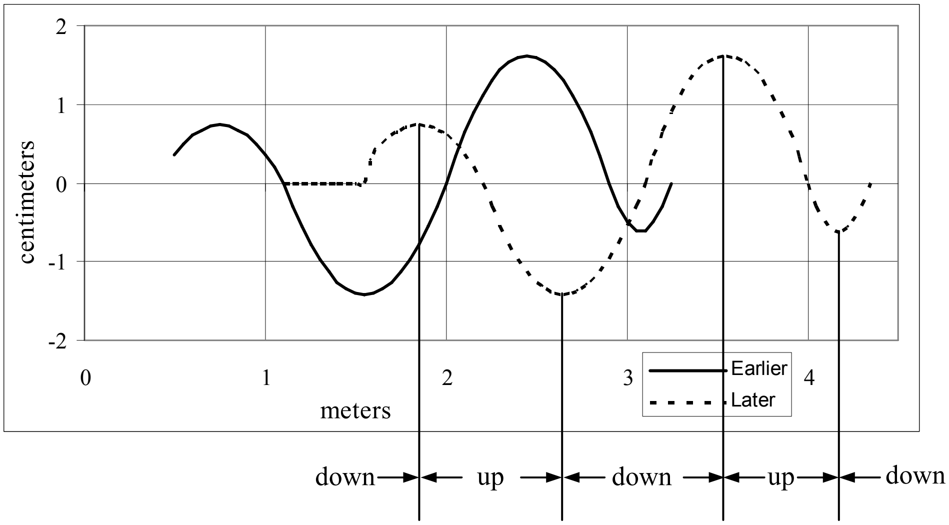
$$t_1=t_2+1.7\,\text{min}=t_2+102\,\text{s}\quad\rightarrow\quad v_1\left(t_2+102\,\text{s}\right)=v_2t_2\quad\rightarrow$$

$$t_2=\frac{v_1}{v_2-v_1}\left(102\,\text{s}\right)=\frac{5.5\,\text{km/s}}{8.5\,\text{km/s}-5.5\,\text{km/s}}\left(102\,\text{s}\right)=187\,\text{s}$$

$$\Delta x=v_2t_2=\left(8.5\,\text{km/s}\right)\left(187\,\text{s}\right)=\boxed{1600\,\text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^{3}\,\text{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

11. (*a*) The shape will not change. The wave will move 1.10 meters to the right in 1.00 seconds. See the graph. The parts of the string that are moving up or down are indicated.



(*b*) At the instant shown, the string at point A will be moving down. As the wave moves to the right, the string at point A will move down by 1 cm in the time it takes the "valley" between 1 m and 2 m to move to the right by about 0.25 m.

$$v=\frac{\Delta y}{\Delta t}=\frac{-1\,\text{cm}}{0.25\,\text{m}/1.10\,\text{m/s}}\approx\boxed{-4\,\text{cm/s}}$$

This answer will vary depending on the values read from the graph.

12. We assume that the wave will be transverse. The speed is given by Eq. 15-2. The tension in the wire is equal to the weight of the hanging mass. The linear mass density is the volume mass density times the cross-sectional area of the wire. The volume mass density is found in Table 13-1.

$$v=\sqrt{\frac{F_T}{\mu}}=\sqrt{\frac{m_{\text{ball}}g}{\frac{\rho V}{\ell}}}=\sqrt{\frac{m_{\text{ball}}g}{\rho\frac{A\ell}{\ell}}}=\sqrt{\frac{(5.0\,\text{kg})(9.80\,\text{m/s}^2)}{(7800\,\text{kg/m}^3)\pi(0.50\times10^{-3}\,\text{m})^2}}=\boxed{89\,\text{m/s}}$$

$\boxed{13.}$ The speed of the waves on the cord can be found from Eq. 15-2, $v=\sqrt{F_{\text{T}}/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x=v\Delta t=\Delta t\sqrt{\frac{F_{\text{T}}}{m/\Delta x}}\quad\rightarrow\quad\Delta x=(\Delta t)^2\frac{F_{\text{T}}}{m}=(0.50\,\text{s})^2\frac{35\,\text{N}}{0.50\,\text{kg}}=\boxed{18\,\text{m}}$$

14. (*a*) We are told that the speed of the waves only depends on the acceleration due to gravity and the wavelength.

$$v=kg^{\alpha}\lambda^{\gamma}\quad\rightarrow\quad\left[\frac{L}{T}\right]=\left[\frac{L}{T^2}\right]^{\alpha}[L]^{\gamma}\qquad T:-1=-2\alpha\quad\rightarrow\quad\alpha=1/2$$

$$L:1=\alpha+\gamma\quad\rightarrow\quad\gamma=1-\alpha=1/2\qquad\boxed{v=k\sqrt{g\lambda}}$$

(*b*) Here the speed of the waves depends only on the acceleration due to gravity and the depth of the water.

$$v=kg^{\alpha}h^{\beta}\quad\rightarrow\quad\left[\frac{L}{T}\right]=\left[\frac{L}{T^2}\right]^{\alpha}[L]^{\beta}\qquad T:-1=-2\alpha\quad\rightarrow\quad\alpha=1/2$$

$$L:1=\alpha+\beta\quad\rightarrow\quad\beta=1-\alpha=1/2\qquad\boxed{v=k\sqrt{gh}}$$

15. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1=E_2/E_1=A_2^2/A_1^2=3\quad\rightarrow\quad A_2/A_1=\sqrt{3}=\boxed{1.73}$$

The more energetic wave has the larger amplitude.

16. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (15-8ab) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\,\text{km}}/I_{15\,\text{km}}=(15\,\text{km})^2/(45\,\text{km})^2=\boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\,\text{km}}/A_{15\,\text{km}} = 15\,\text{km}/45\,\text{km} = \boxed{0.33}$$

17. We assume that all of the wave motion is outward along the surface of the water – no waves are propagated downwards. Consider two concentric circles on the surface of the water, centered on the place where the circular waves are generated. If there is no damping, then the power (energy per unit time) being transferred across the boundary of each of those circles must be the same. Or, the power associated with the wave must be the same at each circular boundary. The intensity depends on the amplitude squared, so for the power we have this.

$$P = I\left(2\pi r\right) = kA^2 2\pi r = \text{constant} \rightarrow A^2 = \frac{\text{constant}}{2\pi rk} \rightarrow \boxed{A \propto \frac{1}{\sqrt{r}}}$$

18. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus Ir^2 will be constant.

$$I_{\text{near}} r_{\text{near}}^2 = I_{\text{far}} r_{\text{far}}^2 \rightarrow$$

$$I_{\text{near}} = I_{\text{far}} \frac{r_{\text{far}}^2}{r_{\text{near}}^2} = \left(3.0\times10^6\ \text{W/m}^2\right)\frac{\left(48\,\text{km}\right)^2}{\left(1.0\,\text{km}\right)^2} = 6.912\times10^9\ \text{W/m}^2 \approx \boxed{6.9\times10^9\ \text{W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = \left(6.912\times10^9\ \text{W/m}^2\right)\left(2.0\,\text{m}^2\right) = \boxed{1.4\times10^{10}\,\text{W}}$$

$\boxed{19.}$ (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\overline{P} = 2\pi^2 \rho S v f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_{\text{T}}}{\mu}} f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_{\text{T}}}{\rho S}} f^2 A^2 = 2\pi^2 f^2 A^2 \sqrt{S\rho F_{\text{T}}}$$

$$= 2\pi^2 \left(60.0\,\text{Hz}\right)^2 \left(0.0050\,\text{m}\right)^2 \sqrt{\pi\left(5.0\times10^{-3}\text{m}\right)^2 \left(7800\,\text{kg/m}^3\right)\left(7.5\,\text{N}\right)} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

20. Consider a wave traveling through an area *S* with speed *v*, much like Figure 15-11. Start with Eq. 15-7, and use Eq. 15-6.

$$I = \frac{\overline{P}}{S} = \frac{E}{St} = \frac{E\ell}{S\ell t} = \frac{E}{S\ell}\frac{\ell}{t} = \frac{\text{energy}}{\text{volume}}\times v$$

21. (*a*) We start with Eq. 15-6. The linear mass density is the mass of a given volume of the cord divided by the cross-sectional area of the cord.

$$\overline{P} = 2\pi^2 \rho S v f^2 A^2 \ ;\ \mu = \frac{m}{\ell} = \frac{\rho V}{\ell} = \frac{\rho S \ell}{\ell} = \rho S \rightarrow \overline{P} = 2\pi^2 \mu v f^2 A^2$$

(*b*) The speed of the wave is found from the given tension and mass density, according to Eq. 15-2.

$$\overline{P} = 2\pi^2 \mu v f^2 A^2 = 2\pi^2 f^2 A^2 \mu \sqrt{F_{\text{T}}/\mu} = 2\pi^2 f^2 A^2 \sqrt{\mu F_{\text{T}}}$$

$$= 2\pi^2 \left(120\,\text{Hz}\right)^2 \left(0.020\,\text{m}\right)^2 \sqrt{\left(0.10\,\text{kg/m}\right)\left(135\,\text{N}\right)} = \boxed{420\,\text{W}}$$

22. (*a*) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x + 1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

23. To represent a wave traveling to the left, we replace x by $x + vt$. The resulting expression can be given in various forms.

$$\boxed{\begin{aligned} D &= A\sin[2\pi(x+vt)/\lambda + \phi] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{vt}{\lambda}\right)+\phi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)+\phi\right] \\ &= A\sin(kx + \omega t + \phi) \end{aligned}}$$

24. The traveling wave is given by $D = 0.22\sin(5.6x + 34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \rightarrow f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(*c*) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}}\frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(*d*) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(*e*) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{\text{max}} = D\omega = 2\pi fD$.

$$v_{\text{max}} = 2\pi fD = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{\text{min}} = \boxed{0}$$

$\boxed{25.}$ The traveling wave is given by $D(x,t) = (0.026\,\text{m})\sin\left[(45\text{m}^{-1})x - (1570\text{s}^{-1})t + 0.66\right]$.

(*a*) $$v_x = \frac{\partial D(x,t)}{\partial t} = -(1570\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \rightarrow$$

$$(v_x)_{\text{max}} = (1570\text{s}^{-1})(0.026\text{m}) = \boxed{41\,\text{m/s}}$$

(*b*) $$a_x = \frac{\partial^2 D(x,t)}{\partial t^2} = -(1570\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \rightarrow$$

$$(a_x)_{\text{max}} = (1570\text{s}^{-1})^2(0.026\,\text{m}) = \boxed{6.4\times10^4\,\text{m/s}^2}$$

(*c*) $$v_x(1.00\,\text{m},2.50\,\text{s}) = -(1570\,\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right] = \boxed{35\,\text{m/s}}$$

$$a_x(1.00\,\text{m},2.50\,\text{s}) = -(1570\,\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right] = \boxed{3.2\times10^4\,\text{m/s}^2}$$

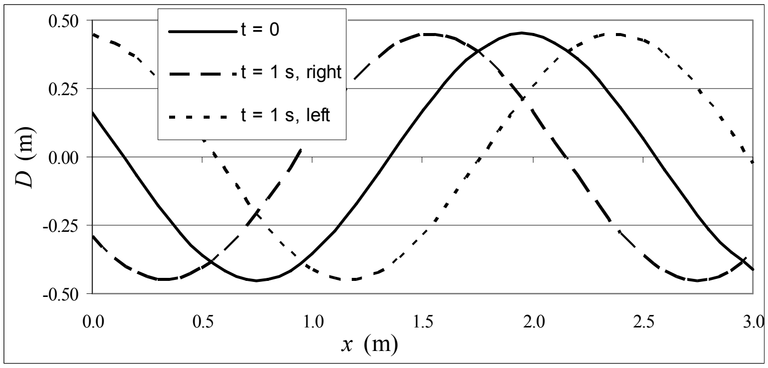
26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\frac{\partial D}{\partial t}$.

$$D(0.60\,\text{m},0.20\,\text{s}) = (0.12\,\text{m})\sin\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m},0.20\,\text{s}) = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

27. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.27a."



(*b*) For motion to the right, replace x by $x - vt$.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x - 2.0t) + 1.2]}$$

(*c*) See the graph above.

(*d*) For motion to the left, replace x by $x + vt$. Also see the graph above.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x + 2.0t) + 1.2]}$$

28. (*a*) The wavelength is the speed divided by the frequency.

$$\lambda = \frac{v}{f} = \frac{345\,\text{m/s}}{524\,\text{Hz}} = \boxed{0.658\,\text{m}}$$

(*b*) In general, the phase change in degrees due to a time difference is given by $\frac{\Delta\phi}{360^\circ} = \frac{\Delta t}{T}$.

$$\frac{\Delta\phi}{360^\circ} = \frac{\Delta t}{T} = f\Delta t \;\rightarrow\; \Delta t = \frac{1}{f}\frac{\Delta\phi}{360^\circ} = \frac{1}{524\,\text{Hz}}\left(\frac{90^\circ}{360^\circ}\right) = \boxed{4.77\times10^{-4}\,\text{s}}$$

(*c*) In general, the phase change in degrees due to a position difference is given by $\frac{\Delta\phi}{360^\circ}=\frac{\Delta x}{\lambda}$.

$$\frac{\Delta\phi}{360^\circ}=\frac{\Delta x}{\lambda} \rightarrow \Delta\phi=\frac{\Delta x}{\lambda}(360^\circ)=\frac{0.044\,\text{m}}{0.658\,\text{m}}(360^\circ)=\boxed{24.1^\circ}$$

29. The amplitude is 0.020 cm, the wavelength is 0.658 m, and the frequency is 524 Hz. The displacement is at its most negative value at $x = 0$, $t = 0$, and so the wave can be represented by a cosine that is phase shifted by half of a cycle.

$$D(x,t)=A\cos(kx-\omega t+\phi)$$

$$A=0.020\,\text{cm}\,;\ k=\frac{2\pi}{\lambda}=\frac{2\pi f}{v}=\frac{2\pi(524\,\text{Hz})}{345\,\text{m/s}}=9.54\,\text{m}^{-1}\ ;\ \omega=2\pi f=2\pi(524\,\text{Hz})=3290\,\text{rad/s}$$

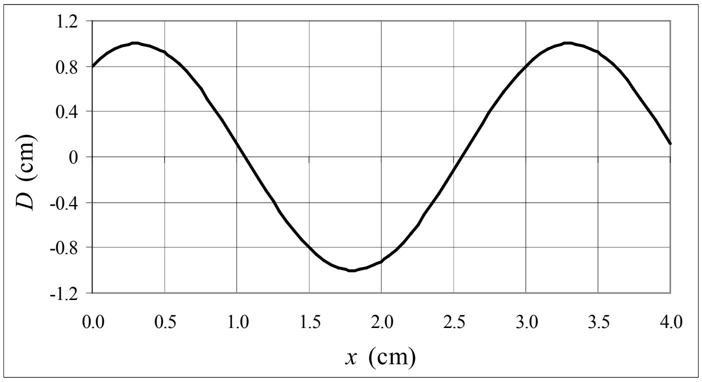
$$\boxed{D(x,t)=(0.020\,\text{cm})\cos\left[\left(9.54\,\text{m}^{-1}\right)x-(3290\,\text{rad/s})t+\pi\right]\text{ , } x \text{ in m, } t \text{ in s}}$$

Other equivalent expressions include the following.

$$D(x,t)=-(0.020\,\text{cm})\cos\left[\left(9.54\,\text{m}^{-1}\right)x-(3290\,\text{rad/s})t\right]$$

$$D(x,t)=(0.020\,\text{cm})\sin\left[\left(9.54\,\text{m}^{-1}\right)x-(3290\,\text{rad/s})t+\tfrac{3}{2}\pi\right]$$

30. (*a*) For the particle of string at $x = 0$, the displacement is not at the full amplitude at $t = 0$. The particle is moving upwards, and so a maximum is approaching from the right. The general form of the wave is given by $D(x,t)=A\sin(kx+\omega t+\phi)$. At $x = 0$ and $t = 0$, $D(0,0)=A\sin\phi$ and so we can find the phase angle.



$$D(0,0)=A\sin\phi \rightarrow 0.80\,\text{cm}=(1.00\,\text{cm})\sin\phi \rightarrow \phi=\sin^{-1}(0.80)=0.93$$

So we have $D(x,0)=A\sin\left(\frac{2\pi}{3.0}x+0.93\right)$, x in cm. See the graph. It matches the description given earlier. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.30a."

(*b*) We use the given data to write the wave function. Note that the wave is moving to the right, and that the phase angle has already been determined.

$$D(x,t)=A\sin(kx+\omega t+\phi)$$

$$A=1.00\,\text{cm}\,;\ k=\frac{2\pi}{3.00\,\text{cm}}=2.09\,\text{cm}^{-1}\ ;\ \omega=2\pi f=2\pi(245\,\text{Hz})=1540\,\text{rad/s}$$

$$\boxed{D(x,t)=(1.00\,\text{cm})\sin\left[\left(2.09\,\text{cm}^{-1}\right)x+(1540\,\text{rad/s})t+0.93\right]\text{ , } x \text{ in cm, } t \text{ in s}}$$

$\boxed{31.}$ To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = A\sin kx\cos\omega t$$

$$\frac{\partial D}{\partial x}=kA\cos kx\cos\omega t \;;\; \frac{\partial^2 D}{\partial x^2}=-k^2A\sin kx\cos\omega t$$

$$\frac{\partial D}{\partial t}=-\omega A\sin kx\sin\omega t \;;\; \frac{\partial^2 D}{\partial t^2}=-\omega^2A\sin kx\cos\omega t$$

This gives $\frac{\partial^2 D}{\partial x^2}=\frac{k^2}{\omega^2}\frac{\partial^2 D}{\partial t^2}$, and since $v=\frac{\omega}{k}$ from Eq. 15-12, we have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. $\boxed{\text{Yes, the function is a solution.}}$

32. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

(*a*) $D=A\ln\left(x+vt\right)$

$$\frac{\partial D}{\partial x}=\frac{A}{x+vt} \;;\; \frac{\partial^2 D}{\partial x^2}=-\frac{A}{\left(x+vt\right)^2} \;;\; \frac{\partial D}{\partial t}=\frac{Av}{x+vt} \;;\; \frac{\partial^2 D}{\partial t^2}=-\frac{Av^2}{\left(x+vt\right)^2}$$

This gives $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so $\boxed{\text{yes, the function is a solution.}}$

(*b*) $D=\left(x-vt\right)^4$

$$\frac{\partial D}{\partial x}=4\left(x-vt\right)^3 \;;\; \frac{\partial^2 D}{\partial x^2}=12\left(x-vt\right)^2 \;;\; \frac{\partial D}{\partial t}=-4v\left(x-vt\right)^3 \;;\; \frac{\partial^2 D}{\partial t^2}=12v^2\left(x-vt\right)^2$$

This gives $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so $\boxed{\text{yes, the function is a solution.}}$

33. We find the various derivatives for the function from Eq. 15-13c.

$$D\left(x,t\right)=A\sin\left(kx+\omega t\right) \;;\; \frac{\partial D}{\partial x}=Ak\cos\left(kx+\omega t\right) \;;\; \frac{\partial^2 D}{\partial x^2}=-Ak^2\sin\left(kx+\omega t\right);$$

$$\frac{\partial D}{\partial t}=A\omega\cos\left(kx+\omega t\right) \;;\; \frac{\partial^2 D}{\partial t^2}=-A\omega^2\sin\left(kx+\omega t\right)$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \;\rightarrow\; -Ak^2\sin\left(kx+\omega t\right)=\frac{1}{v^2}\left(-A\omega^2\sin\left(kx+\omega t\right)\right) \;\rightarrow\; k^2=\frac{\omega^2}{v^2}$$

Since $v=\omega/k$, $\boxed{\text{the wave equation is satisfied}}$.

We find the various derivatives for the function from Eq. 15-15. Make the substitution that $u=x+vt$, and then use the chain rule.

$$D(x,t)=D(x+vt)=D(u)\ \ ;\ \ \frac{\partial D}{\partial x}=\frac{dD}{du}\frac{\partial u}{\partial x}=\frac{dD}{du}\ \ ;\ \ \frac{\partial^2 D}{\partial x^2}=\frac{\partial}{\partial x}\frac{dD}{du}=\left(\frac{d}{dx}\frac{dD}{du}\right)\frac{\partial u}{\partial x}=\frac{d^2D}{du^2}$$

$$\frac{\partial D}{\partial t}=\frac{dD}{du}\frac{\partial u}{\partial t}=v\frac{dD}{du}\ \ ;\ \ \frac{\partial^2 D}{\partial t^2}=\frac{\partial}{\partial t}\left(v\frac{dD}{du}\right)=v\frac{\partial}{\partial t}\frac{dD}{du}=v\left(\frac{d}{du}\frac{dD}{du}\right)\frac{\partial u}{\partial t}=v\frac{d^2D}{du^2}v=v^2\frac{d^2D}{du^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}\ \ \rightarrow\ \ \frac{d^2D}{du^2}=\frac{1}{v^2}v^2\frac{d^2D}{du^2}=\frac{d^2D}{du^2}$$

Since we have an identity, $\boxed{\text{the wave equation is satisfied}}$.

34. Find the various derivatives for the linear combination.

$$D(x,t)=C_1D_1+C_2D_2=C_1f_1(x,t)+C_2f_2(x,t)$$

$$\frac{\partial D}{\partial x}=C_1\frac{\partial f_1}{\partial x}+C_2\frac{\partial f_2}{\partial x}\ \ ;\ \ \frac{\partial^2 D}{\partial x^2}=C_1\frac{\partial^2 f_1}{\partial x^2}+C_2\frac{\partial^2 f_2}{\partial x^2}$$

$$\frac{\partial D}{\partial t}=C_1\frac{\partial f_1}{\partial t}+C_2\frac{\partial f_2}{\partial t}\ \ ;\ \ \frac{\partial^2 D}{\partial t^2}=C_1\frac{\partial^2 f_1}{\partial t^2}+C_2\frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both f_1 and f_2 satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2}=C_1\frac{\partial^2 f_1}{\partial x^2}+C_2\frac{\partial^2 f_2}{\partial x^2}=C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right]+C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right]=\frac{1}{v^2}\left[C_1\frac{\partial^2 f_1}{\partial t^2}+C_2\frac{\partial^2 f_2}{\partial t^2}\right]=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so *D* satisfies the wave equation.

35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D=e^{-(kx-\omega t)^2}\ \ ;\ \ \frac{\partial D}{\partial x}=-2k(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2}=-2k(kx-\omega t)\left[-2k(kx-\omega t)e^{-(kx-\omega t)^2}\right]+\left(-2k^2\right)e^{-(kx-\omega t)^2}=2k^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial D}{\partial t}=2\omega(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial t^2}=2\omega(kx-\omega t)\left[2\omega(kx-\omega t)e^{-(kx-\omega t)^2}\right]+\left(-2\omega^2\right)e^{-(kx-\omega t)^2}=2\omega^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}\ \ \rightarrow\ \ 2k^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}=\frac{1}{v^2}2\omega^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}\ \ \rightarrow$$

$$k^2=\frac{\omega^2}{v^2}$$

Since $v=\frac{\omega}{k}$, we have an identity. $\boxed{\text{Yes, the function is a solution.}}$

36. We assume that $A \ll \lambda$ for the wave given by $D = A\sin\left(kx - \omega t\right)$.

$$D = A\sin\left(kx-\omega t\right) \;\rightarrow\; v' = \frac{\partial D}{\partial t} = -\omega A\cos\left(kx-\omega t\right) \;\rightarrow\; v'_{\text{max}} = \omega A$$

$$A \ll \lambda \;\rightarrow\; \frac{v'_{\text{max}}}{\omega} \ll \lambda \;\rightarrow\; v'_{\text{max}} \ll \omega\lambda = v_{\text{wave}} \;\rightarrow\; \boxed{v'_{\text{max}} \ll v_{\text{wave}}}$$

$$\frac{v'_{\text{max}}}{v} = \frac{\omega A}{v} = \frac{2\pi f A}{v} = \frac{2\pi f\dfrac{\lambda}{100}}{f\lambda} = \boxed{\frac{\pi}{50} \approx 0.063}$$

$\boxed{37.}$ (*a*) For the wave in the lighter cord, $D\left(x,t\right) = \left(0.050\,\text{m}\right)\sin\left[\left(7.5\,\text{m}^{-1}\right)x - \left(12.0\,\text{s}^{-1}\right)t\right]$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(7.5\,\text{m}^{-1}\right)} = \boxed{0.84\,\text{m}}$$

(*b*) The tension is found from the velocity, using Eq. 15-2.

$$v = \sqrt{\frac{F_{\text{T}}}{\mu}} \;\rightarrow\; F_{\text{T}} = \mu v^2 = \mu\frac{\omega^2}{k^2} = \left(0.10\,\text{kg/m}\right)\frac{\left(12.0\,\text{s}^{-1}\right)^2}{\left(7.5\,\text{m}^{-1}\right)^2} = \boxed{0.26\,\text{N}}$$

(*c*) The tension and the frequency do not change from one section to the other.

$$F_{\text{T1}} = F_{\text{T2}} \;\rightarrow\; = \mu_1\frac{\omega_1^2}{k_1^2} = \mu_2\frac{\omega_2^2}{k_2^2} \;\rightarrow\; \lambda_2 = \lambda_1\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{k_1}\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{\left(7.5\,\text{m}^{-1}\right)}\sqrt{0.5} = \boxed{0.59\,\text{m}}$$

38. (*a*) The speed of the wave in a stretched cord is given by Eq. 15-2, $v = \sqrt{F_{\text{T}}/\mu}$. The tensions must be the same in both parts of the cord. If they were not the same, then the net longitudinal force on the joint between the two parts would not be zero, and the joint would have to accelerate along the length of the cord.

$$v = \sqrt{F_{\text{T}}/\mu} \;\rightarrow\; \frac{v_{\text{H}}}{v_{\text{L}}} = \frac{\sqrt{F_{\text{T}}/\mu_{\text{H}}}}{\sqrt{F_{\text{T}}/\mu_{\text{L}}}} = \boxed{\sqrt{\frac{\mu_{\text{L}}}{\mu_{\text{H}}}}}$$

(*b*) The frequency must be the same in both sections. If it were not, then the joint between the two sections would not be able to keep the two sections together. The ends could not stay in phase with each other if the frequencies were different.

$$f = \frac{v}{\lambda} \;\rightarrow\; \frac{v_{\text{H}}}{\lambda_{\text{H}}} = \frac{v_{\text{L}}}{\lambda_{\text{L}}} \;\rightarrow\; \frac{\lambda_{\text{H}}}{\lambda_{\text{L}}} = \frac{v_{\text{H}}}{v_{\text{L}}} = \boxed{\sqrt{\frac{\mu_{\text{L}}}{\mu_{\text{H}}}}}$$

(*c*) The ratio under the square root sign is less than 1, and so the $\boxed{\text{lighter cord}}$ has the greater wavelength.

39. (*a*) The distance traveled by the reflected sound wave is found from the Pythagorean theorem.

$$d = 2\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2} = vt \;\rightarrow\; \boxed{t = \frac{2}{v}\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2}}$$

(*b*) Solve for t^2.

$$t^2 = \frac{4}{v^2}\left[D^2 + \left(\tfrac{1}{2}x\right)^2\right] = \frac{x^2}{v^2} + \frac{4}{v^2}D^2$$

A plot of t^2 vs x^2 would have a slope of $1/v^2$, which can be used to determine the value of *v*. The *y* intercept of that plot is $\frac{4}{v^2}D^2$. Knowing the *y* intercept and the value of *v*, the value of *D* can be determined.

40. The tension and the frequency do not change from one side of the knot to the other.

(*a*) We force the cord to be continuous at $x = 0$ for all times. This is done by setting the initial wave plus the reflected wave (the displacement of a point infinitesimally to the LEFT of $x = 0$) equal to the transmitted wave (the displacement of a point infinitesimally to the RIGHT of $x = 0$) for all times. We also use the facts that $\sin(-\theta) = -\sin\theta$ and $k_1v_1 = k_2v_2$.

$$D(0,t) + D_{\mathrm{R}}(0,t) = D_{\mathrm{T}}(0,t) \rightarrow A\sin(-k_1v_1t) + A_{\mathrm{R}}\sin(k_1v_1t) = A_{\mathrm{T}}\sin(-k_2v_2t) \rightarrow$$
$$-A\sin(k_1v_1t) + A_{\mathrm{R}}\sin(k_1v_1t) = -A_{\mathrm{T}}\sin(k_2v_2t) = -A_{\mathrm{T}}\sin(k_1v_1t) \rightarrow$$
$$-A + A_{\mathrm{R}} = -A_{\mathrm{T}} \rightarrow \boxed{A = A_{\mathrm{T}} + A_{\mathrm{R}}}$$

(*b*) To make the slopes match for all times, we must have $\frac{\partial}{\partial x}\left[D(x,t) + D_{\mathrm{R}}(x,t)\right] = \frac{\partial}{\partial x}\left[D_{\mathrm{T}}(x,t)\right]$ when evaluated at the origin. We also use the result of the above derivation, and the facts that $\cos(-\theta) = \cos\theta$ and $k_1v_1 = k_2v_2$.

$$\left.\frac{\partial}{\partial x}\left[D(x,t) + D_{\mathrm{R}}(x,t)\right]\right|_{x=0} = \left.\frac{\partial}{\partial x}\left[D_{\mathrm{T}}(x,t)\right]\right|_{x=0} \rightarrow$$
$$k_1A\cos(-k_1v_1t) + k_1A_{\mathrm{R}}\cos(k_1v_1t) = k_2A_{\mathrm{T}}\cos(-k_2v_2t) \rightarrow$$
$$k_1A\cos(k_1v_1t) + k_1A_{\mathrm{R}}\cos(k_1v_1t) = k_2A_{\mathrm{T}}\cos(k_2v_2t) \rightarrow$$
$$k_1A + k_1A_{\mathrm{R}} = k_2A_{\mathrm{T}} = k_2(A - A_{\mathrm{R}}) \rightarrow \boxed{A_{\mathrm{R}} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A}$$

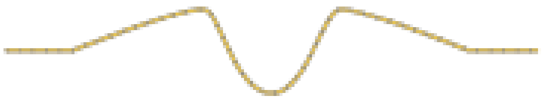
Use $k_2 = k_1\frac{v_1}{v_2}$.

$$A_{\mathrm{R}} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A = \left(\frac{k_1\frac{v_1}{v_2} - k_1}{k_1\frac{v_1}{v_2} + k_1}\right)A = \frac{k_1}{k_1}\left(\frac{\frac{v_1}{v_2} - 1}{\frac{v_1}{v_2} + 1}\right)A = \left(\frac{\frac{v_1}{v_2} - \frac{v_2}{v_2}}{\frac{v_1}{v_2} + \frac{v_2}{v_2}}\right)A = \boxed{\left(\frac{v_1 - v_2}{v_1 + v_2}\right)A}$$

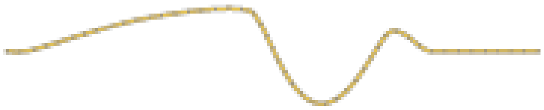
(*c*) Combine the results from the previous two parts.

$$A_{\mathrm{T}} = A - A_{\mathrm{R}} = A - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A = A\left[1 - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\right] = A\left[\left(\frac{k_2 + k_1}{k_2 + k_1}\right) - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\right] = \boxed{\left(\frac{2k_1}{k_2 + k_1}\right)A}$$
$$= \left(\frac{2k_1}{k_1\frac{v_1}{v_2} + k_1}\right)A = \boxed{\left(\frac{2v_2}{v_1 + v_2}\right)A}$$

41. (*a*)



(*b*)



(*c*) The energy is $\boxed{\text{all kinetic energy}}$ at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

42. (*a*) The resultant wave is the algebraic sum of the two component waves.

$$D = D_1 + D_2 = A\sin(kx-\omega t) + A\sin(kx-\omega t+\phi) = A[\sin(kx-\omega t) + A\sin(kx-\omega t+\phi)]$$

$$= A\{2\sin\tfrac{1}{2}[(kx-\omega t)+(kx-\omega t+\phi)]\}\{\cos\tfrac{1}{2}[(kx-\omega t)-(kx-\omega t+\phi)]\}$$

$$= 2A\{\sin\tfrac{1}{2}(2kx-2\omega t+\phi)\}\{\cos\tfrac{1}{2}(\phi)\} = \boxed{\left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)}$$

(*b*) The amplitude is the absolute value of the coefficient of the sine function, $\boxed{\left|2A\cos\frac{\phi}{2}\right|}$. The wave is $\boxed{\text{purely sinusoidal}}$ because the dependence on x and t is $\sin\left(kx-\omega t+\frac{\phi}{2}\right)$.

(*c*) If $\phi = 0, 2\pi, 4\pi, \cdots, 2n\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{2n\pi}{2}\right| = \left|2A\cos n\pi\right| = \left|2A(\pm 1)\right|$ $= 2A$, which is constructive interference. If $\phi = \pi, 3\pi, 5\pi, \cdots, (2n+1)\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{(2n+1)\pi}{2}\right| = \left|2A\cos\left[(n+\tfrac{1}{2})\pi\right]\right| = 0$, which is destructive interference.

(*d*) If $\phi = \frac{\pi}{2}$, then the resultant wave is as follows.

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right) = \left(2A\cos\frac{\pi}{4}\right)\sin\left(kx-\omega t+\frac{\pi}{4}\right) = \sqrt{2}A\sin\left(kx-\omega t+\frac{\pi}{4}\right)$$

This wave has an amplitude of $\sqrt{2}A$, is traveling in the positive x direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\frac{1}{2}A$ at the origin at $t = 0$.

$\boxed{43.}$ The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \frac{v}{2\ell} = 441\,\text{Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be as follows.

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}\ell\right)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right)f_{\text{unfingered}} = \left(\frac{3}{2}\right)(441\,\text{Hz}) = \boxed{662\,\text{Hz}}$$

44. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 294\,\text{Hz}\ ,f_2 = 588\,\text{Hz}\ ,f_3 = 882\,\text{Hz}\ ,f_4 = 1176\,\text{Hz}}$.

45. The oscillation corresponds to the fundamental. The frequency of that oscillation is $f_1 = \frac{1}{T} = \frac{1}{1.5\,\text{s}} = \frac{2}{3}\,\text{Hz}$. The bridge, with both ends fixed, is similar to a vibrating string, and so $\boxed{f_n = nf_1 = \frac{2n}{3}\,\text{Hz},\ n = 1,2,3\ldots}$. The periods are the reciprocals of the frequency, and so $\boxed{T_n = \frac{1.5\,\text{s}}{n},\ n = 1,2,3\ldots}$.

46. Four loops is the standing wave pattern for the 4th harmonic, with a frequency given by $f_4 = 4f_1 = 280\ \text{Hz}$. Thus $\boxed{f_1 = 70\ \text{Hz}\ ,f_2 = 140\ \text{Hz}\ ,\ f_3 = 210\ \text{Hz, and } f_5 = 350\ \text{Hz}}$ are all other resonant frequencies.

47. Each half of the cord has a single node, at the center of the cord. Thus each half of the cord is a half of a wavelength, assuming that the ends of the cord are also nodes. The tension is the same in both halves of the cord, and the wavelengths are the same based on the location of the node. Let subscript 1 represent the lighter density, and subscript 2 represent the heavier density.

$$v_1 = \sqrt{\frac{F_{\text{T1}}}{\mu_1}} = \lambda_1 f_1\ ;\ v_2 = \sqrt{\frac{F_{\text{T2}}}{\mu_2}} = \lambda_2 f_2\ ;\ \lambda_1 = \lambda_2\ ;\ F_{\text{T1}} = F_{\text{T2}}$$

$$\frac{f_1}{f_2} = \frac{\frac{1}{\lambda_1}\sqrt{\frac{F_{\text{T1}}}{\mu_1}}}{\frac{1}{\lambda_2}\sqrt{\frac{F_{\text{T2}}}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}} = \boxed{\sqrt{2}}$$

The frequency is higher on the lighter portion.

48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$\lambda = \frac{v}{f} \quad\to\quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{96\,\text{m/s}}{2(445\,\text{Hz})} = \boxed{0.11\,\text{m}}$$

$\boxed{49.}$ Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 320\,\text{Hz} - 240\,\text{Hz} = \boxed{80\,\text{Hz}}$$

50. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_{\text{T}}/\mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 15-17b, $f_n = \frac{nv}{2\ell_{\text{vib}}}$, where ℓ_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{\mu}} \qquad f_1 = \frac{1}{2(0.600\,\text{m})}\sqrt{\frac{520\,\text{N}}{(3.16\times10^{-3}\,\text{kg})/(0.900\,\text{m})}} = 320.7\,\text{Hz}$$

$$f_2 = 2f_1 = 641.4\,\text{Hz} \quad f_3 = 3f_1 = 962.1\,\text{Hz}$$

So the three frequencies are $\boxed{320\,\text{Hz}, 640\,\text{Hz}, 960\,\text{Hz}}$, to 2 significant figures.

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2\ell$. Thus the frequency of the fundamental is $f_1 = \dfrac{v}{\lambda_1} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\dfrac{n}{2\ell}\sqrt{\dfrac{F_T}{\mu}}}$, $n = 1,2,3,\ldots$.

52. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 120 Hz, the same as the vibrator. That frequency is also expressed by Eq. 15-17b, $f_n = \dfrac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$, ignoring the mass of the string itself. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $$f_n = \frac{nv}{2\ell} = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell}\sqrt{\frac{mg}{\mu}} \rightarrow m = \frac{4\ell^2 f_n^2 \mu}{n^2 g}$$

$$m_1 = \frac{4(1.50\,\text{m})^2(120\,\text{Hz})^2(6.6\times10^{-4}\,\text{kg/m})}{1^2(9.80\,\text{m/s}^2)} = 8.728\,\text{kg} \approx \boxed{8.7\,\text{kg}}$$

(*b*) $$m_2 = \frac{m_1}{2^2} = \frac{8.728\,\text{kg}}{4} = \boxed{2.2\,\text{kg}}$$

(*c*) $$m_5 = \frac{m_1}{5^2} = \frac{8.728\,\text{kg}}{25} = \boxed{0.35\,\text{kg}}$$

53. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $v = \sqrt{\dfrac{F_T}{\mu}} = \sqrt{\dfrac{mg}{\mu}}$, and the frequency is given as $f = 120\,\text{Hz}$. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{1}{120\,\text{Hz}}\sqrt{\frac{(0.070\,\text{kg})(9.80\,\text{m/s}^2)}{(6.6\times10^{-4}\,\text{kg/m})}} = 0.2687\,\text{m}.$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $\ell = \lambda/2, \lambda, 3\lambda/2, \cdots n\lambda/2$. The number of standing wave patterns is given by the number of integers that satisfy $0.10\,\text{m} < n\lambda/2 < 1.5\,\text{m}$.

$$0.10\,\text{m} < n\lambda/2 \quad \rightarrow \quad n > \frac{2(0.10\,\text{m})}{\lambda} = \frac{2(0.10\,\text{m})}{0.2687\,\text{m}} = 0.74$$

$$n\lambda/2 < 1.5\,\text{m} \rightarrow n < \frac{2(1.5\,\text{m})}{\lambda} = \frac{2(1.5\,\text{m})}{0.2687\,\text{m}} = 11.1$$

Thus we see that we must have *n* from 1 to 11, and so there are $\boxed{11}$ standing wave patterns that may be achieved.

54. The standing wave is given by $D = (2.4\,\text{cm})\sin(0.60x)\cos(42t)$.

(*a*) The distance between nodes is half of a wavelength.

$$d = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{2\pi}{k} = \frac{\pi}{0.60\,\text{cm}^{-1}} = 5.236\,\text{cm} \approx \boxed{5.2\,\text{cm}}$$

(*b*) The component waves travel in opposite directions. Each has the same frequency and speed, and each has half the amplitude of the standing wave.

$$A = \tfrac{1}{2}(2.4\,\text{cm}) = \boxed{1.2\,\text{cm}}\ ;\ f = \frac{\omega}{2\pi} = \frac{42\,\text{s}^{-1}}{2\pi} = 6.685\,\text{Hz} \approx \boxed{6.7\,\text{Hz}}\ ;$$

$$v = \lambda f = 2d_{\text{node}}f = 2(5.236\,\text{cm})(6.685\,\text{Hz}) = 70.01\,\text{cm/s} \approx \boxed{70\,\text{cm/s}}\ (2\text{ sig. fig.})$$

(*c*) The speed of a particle is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}[(2.4\,\text{cm})\sin(0.60x)\cos(42t)] = (-42\,\text{rad/s})(2.4\,\text{cm})\sin(0.60x)\sin(42t)$$

$$\frac{\partial D}{\partial t}(3.20\text{cm, }2.5\text{s}) = (-42\,\text{rad/s})(2.4\,\text{cm})\sin\left[(0.60\,\text{cm}^{-1})(3.20\,\text{cm})\right]\sin[(42\,\text{rad/s})(2.5\,\text{s})]$$

$$= \boxed{92\,\text{cm/s}}$$

$\boxed{55.}$ (*a*) The given wave is $D_1 = 4.2\sin(0.84x - 47t + 2.1)$. To produce a standing wave, we simply need to add a wave of the same characteristics but traveling in the opposite direction. This is the appropriate wave.

$$\boxed{D_2 = 4.2\sin(0.84x + 47t + 2.1)}$$

(*b*) The standing wave is the sum of the two component waves. We use the trigonometric identity that $\sin\theta_1 + \sin\theta_2 = 2\sin\tfrac{1}{2}(\theta_1+\theta_2)\cos\tfrac{1}{2}(\theta_1-\theta_2)$.

$$\begin{aligned} D = D_1 + D_2 &= 4.2\sin(0.84x - 47t + 2.1) + 4.2\sin(0.84x + 47t + 2.1) \\ &= 4.2(2)\left\{\sin\tfrac{1}{2}[(0.84x - 47t + 2.1) + (0.84x + 47t + 2.1)]\right\} \\ &\qquad \left\{\cos\tfrac{1}{2}[(0.84x - 47t + 2.1) - (0.84x + 47t + 2.1)]\right\} \\ &= 8.4\sin(0.84x + 2.1)\cos(-47t) = \boxed{8.4\sin(0.84x + 2.1)\cos(47t)} \end{aligned}$$

We note that the origin is NOT a node.

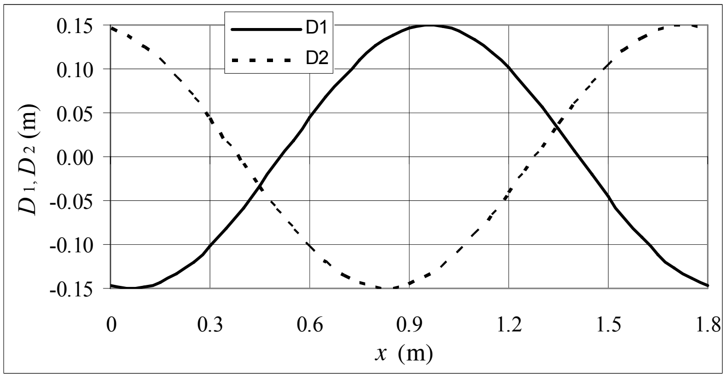
56. From the description of the water's behavior, there is an antinode at each end of the tub, and a node in the middle. Thus one wavelength is twice the tub length.

$$v = \lambda f = (2\ell_{\text{tub}})f = 2(0.45\,\text{m})(0.85\text{ Hz}) = \boxed{0.77\,\text{m/s}}$$

57. The frequency is given by $f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F}{\mu}}$. The wavelength and the mass density do not change when the string is tightened.

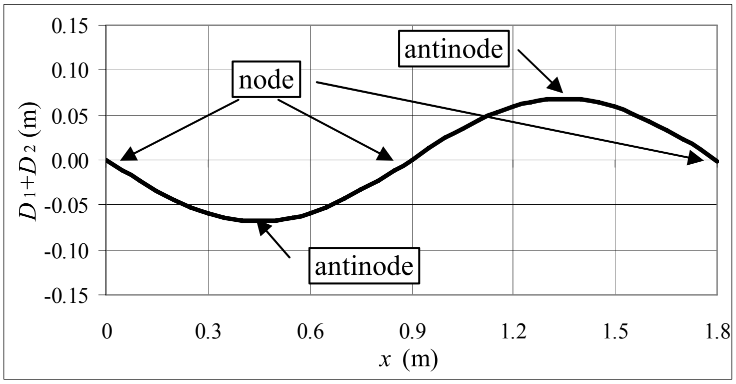
$$f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F}{\mu}} \rightarrow \frac{f_2}{f_1}=\frac{\frac{1}{\lambda}\sqrt{\frac{F_2}{\mu}}}{\frac{1}{\lambda}\sqrt{\frac{F_1}{\mu}}}=\sqrt{\frac{F_2}{F_1}} \rightarrow f_2=f_1\sqrt{\frac{F_2}{F_1}}=(294\,\text{Hz})\sqrt{1.15}=\boxed{315\,\text{Hz}}$$

58. (*a*) Plotting one full wavelength means from $x=0$ to $x=\lambda=\frac{2\pi}{k}=\frac{2\pi}{3.5\,\text{m}^{-1}}=1.795\,\text{m}$ $\approx 1.8\,\text{m}$. The functions to be plotted are given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."



$$D_1=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right] \text{ and } D_2=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x+1.8\right]$$

(*b*) The sum D_1+D_2 is plotted, and the nodes and antinodes are indicated. The analytic result is given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."



$$D_1+D_2=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right]+(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right]$$
$$=(0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos(1.8)$$

This expression should have nodes and antinodes at positions given by the following.

$$3.5\,\text{m}^{-1}x_{\text{node}}=n\pi, n=0,1,2\ldots \rightarrow x=\frac{n\pi}{3.5}=0,\ 0.90\,\text{m},\ 1.80\text{ m}$$

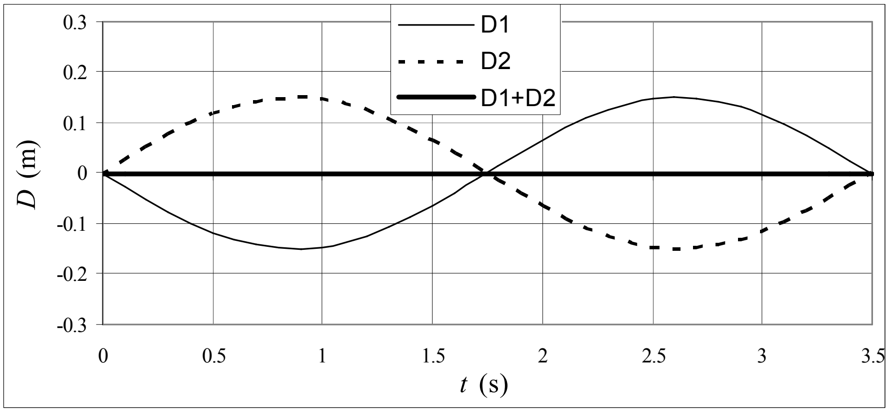
$$3.5\,\text{m}^{-1}x_{\text{antinode}}=\left(n+\tfrac{1}{2}\right)\pi, n=0,1,2\ldots \rightarrow x=\frac{\left(n+\frac{1}{2}\right)\pi}{3.5}=0.45\,\text{m},\ 1.35\text{ m}$$

The graph agrees with the calculations.

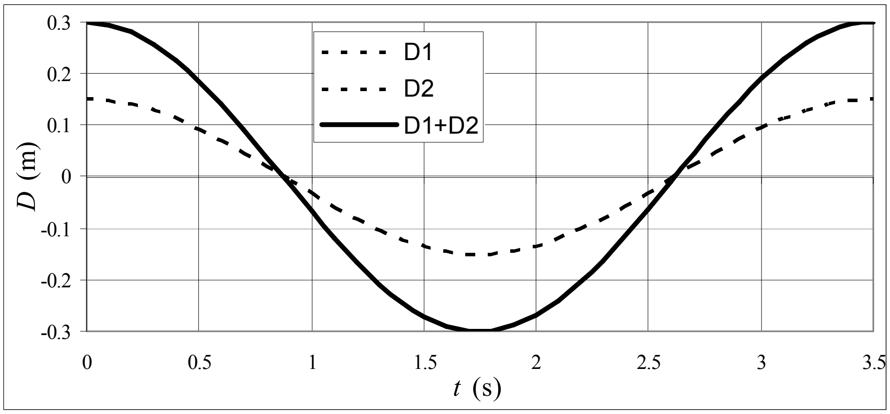
59. The standing wave formed from the two individual waves is given below. The period is given by $T = 2\pi/\omega = 2\pi/1.8\,\text{s}^{-1} = 3.5\,\text{s}$.

$$D_1 + D_2 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right] + (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right]$$
$$= (0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos\left(1.8\,\text{s}^{-1}t\right)$$

(*a*) For the point $x = 0$, we see that the sum of the two waves is identically 0. This means that the point $x = 0$ is a node of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."



(*b*) For the point $x = \lambda/4$, we see that the amplitude of that point is twice the amplitude of either wave. Thus this point is an antinode of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."



60. (*a*) The maximum swing is twice the amplitude of the standing wave. Three loops is 1.5 wavelengths, and the frequency is given.

$$A = \tfrac{1}{2}(8.00\,\text{cm}) = 4.00\,\text{cm} \;;\; \omega = 2\pi f = 2\pi(120\,\text{Hz}) = 750\,\text{rad/s} \;;$$

$$k = \frac{2\pi}{\lambda} \;\rightarrow\;;\; \tfrac{3}{2}\lambda = 1.64\,\text{m} \;\rightarrow\; \lambda = 1.09\,\text{m} \;;\; k = \frac{2\pi}{1.09\,\text{m}} = 5.75\,\text{m}^{-1}$$

$$D = A\sin(kx)\cos(\omega t) = \boxed{(4.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x\right]\cos\left[(750\,\text{rad/s})t\right]}$$

(*b*) Each component wave has the same wavelength, the same frequency, and half the amplitude of the standing wave.

$$\boxed{\begin{array}{l} D_1 = (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x - (750\,\text{rad/s})t\right] \\ D_2 = (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x + (750\,\text{rad/s})t\right] \end{array}}$$

$\boxed{61.}$ Any harmonic with a node directly above the pickup will NOT be "picked up" by the pickup. The pickup location is exactly 1/4 of the string length from the end of the string, so a standing wave with a frequency corresponding to 4 (or 8 or 12 etc.) loops will not excite the pickup. So $\boxed{n = 4,\ 8,\ \text{and}\ 12}$ will not excite the pickup.

62. The gap between resonant frequencies is the fundamental frequency (which is thus 300 Hz for this problem), and the wavelength of the fundamental is twice the string length.

$$v = \lambda f = (2\ell)(f_{n+1} - f_n) = 2(0.65\,\text{m})(300\,\text{Hz}) = \boxed{390\,\text{m/s}}$$

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \ ; \ \cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

$$\begin{aligned} D = D_1 + D_2 &= A\cos(kx - \omega t) + A\cos(kx + \omega t) = A[\cos(kx - \omega t) + \cos(kx + \omega t)] \\ &= A[\cos kx \cos \omega t + \sin kx \sin \omega t + \cos kx \cos \omega t - \sin kx \sin \omega t] \\ &= 2A \cos kx \cos \omega t \end{aligned}$$

Thus the standing wave is $D = 2A\cos kx \cos \omega t$. The nodes occur where the position term forces $D = 2A\cos kx \cos \omega t = 0$ for all time. Thus $\cos kx = 0 \ \rightarrow \ kx = \pm(2n+1)\dfrac{\pi}{2},\ n = 0, 1, 2, \cdots$. Thus, since $k = 2.0\,\text{m}^{-1}$, we have $\boxed{x = \pm\left(n + \tfrac{1}{2}\right)\dfrac{\pi}{2}\,\text{m},\ n = 0, 1, 2, \cdots}$.

64. The frequency for each string must be the same, to ensure continuity of the string at its junction. Each string will obey these relationships: $\lambda f = v$, $v = \sqrt{\dfrac{F_T}{\mu}}$, $\lambda = \dfrac{2\ell}{n}$. Combine these to find the nodes. Note that n is the number of "loops" in the string segment, and that n loops requires $n+1$ nodes.

$$\lambda f = v\ ,\ v = \sqrt{\frac{F_T}{\mu}}\ ,\ \lambda = \frac{2\ell}{n} \ \rightarrow \ \frac{2\ell}{n} f = \sqrt{\frac{F_T}{\mu}} \ \rightarrow f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

$$\frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_T}{\mu_{\text{Al}}}} = \frac{n_{\text{Fe}}}{2\ell_{\text{Fe}}}\sqrt{\frac{F_T}{\mu_{\text{Fe}}}} \quad \rightarrow \quad \frac{n_{\text{Al}}}{n_{\text{Fe}}} = \frac{\ell_{\text{Al}}}{\ell_{\text{Fe}}}\sqrt{\frac{\mu_{\text{Al}}}{\mu_{\text{Fe}}}} = \frac{0.600\,\text{m}}{0.882\,\text{m}}\sqrt{\frac{2.70\,\text{g/m}}{7.80\,\text{g/m}}} = 0.400 = \frac{2}{5}$$

Thus there are 3 nodes on the aluminum, since $n_{\text{Al}} = 2$, and 6 nodes on the steel, since $n_{\text{Fe}} = 5$, but one node is shared so there are $\boxed{8\ \text{total nodes}}$. Use the formula derived above to find the lower frequency.

$$f = \frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_{\text{Al}}}{\mu_{\text{Al}}}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{135\,\text{N}}{2.70\times10^{-3}\,\text{kg/m}}} = \boxed{373\,\text{Hz}}$$

65. The speed in the second medium can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \quad \rightarrow \quad v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\,\text{km/s})\left(\frac{\sin 31^\circ}{\sin 52^\circ}\right) = \boxed{5.2\,\text{km/s}}$$

66. The angle of refraction can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \quad \rightarrow \quad \sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \sin 35^\circ \frac{2.5\,\text{m/s}}{2.8\,\text{m/s}} = 0.512 \quad \rightarrow \quad \theta_2 = \sin^{-1} 0.419 = \boxed{31^\circ}$$

67. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from the relationship given in the problem.

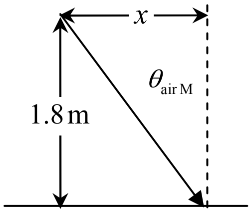
$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{331+0.60T_2}{331+0.60T_1}\quad\rightarrow\quad\sin\theta_2=\sin 33^\circ\frac{331+0.60(-15)}{331+0.60(25)}=\sin 33^\circ\frac{322}{346}=0.5069$$

$$\theta_2=\sin^{-1}0.5069=\boxed{30^\circ}\ (2\text{ sig. fig.})$$

68. (*a*) Eq. 15-19 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text{water}}=90^\circ$ or $\sin\theta_{\text{water}}=1$. The air is the "incident" media. Thus the incident angle is given by the following.

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}}=\frac{v_{\text{air}}}{v_{\text{water}}}\ ;\ \theta_{\text{air}}=\theta_{\text{i}}=\sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right]\ \rightarrow\ \boxed{\theta_{\text{iM}}=\sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right]=\sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.



$$\theta_{\text{air M}}=\sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}}=\sin^{-1}\frac{343\text{ m/s}}{1440\text{ m/s}}=13.8^\circ$$

$$\tan\theta_{\text{air M}}=\frac{x}{1.8\text{ m}}\quad\rightarrow\quad x=(1.8\text{ m})\tan 13.8^\circ=\boxed{0.44\text{ m}}$$

69. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from Eq. 15-3.

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{SG_1\rho_{\text{water}}}{SG_2\rho_{\text{water}}}}=\sqrt{\frac{SG_1}{SG_2}}$$

$$\sin\theta_2=\sin\theta_1\sqrt{\frac{SG_1}{SG_2}}=\sin 38^\circ\sqrt{\frac{3.6}{2.8}}=0.70\quad\rightarrow\quad\theta_2=\sin^{-1}0.70=\boxed{44^\circ}$$

70. The error of 2^o is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta\approx\lambda/\ell$ is satisfied.

$$\theta\approx\frac{\lambda}{\ell}\quad\rightarrow\quad\lambda=\ell\theta=(0.5\text{ m})\left(2^\text{o}\times\frac{\pi\text{ rad}}{180^\text{o}}\right)=1.745\times10^{-2}\text{m}\approx\boxed{2\times10^{-2}\text{m}}$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.

71. The frequency is 880 Hz and the phase velocity is 440 m/s, so the wavelength is

$$\lambda=\frac{v}{f}=\frac{440\text{ m/s}}{880\text{ Hz}}=0.50\text{ m}.$$

(*a*) Use the ratio of distance to wavelength to define the phase difference.

$$\frac{x}{\lambda}=\frac{\pi/6}{2\pi}\quad\rightarrow\quad x=\frac{\lambda}{12}=\frac{0.50\text{ m}}{12}=\boxed{0.042\text{ m}}$$

(b) Use the ratio of time to period to define the phase difference.

$$\frac{t}{T}=\frac{\phi}{2\pi}\quad\rightarrow\quad\phi=\frac{2\pi t}{T}=2\pi tf=2\pi\left(1.0\times10^{-4}\text{s}\right)(880\text{ Hz})=\boxed{0.55\text{ rad}}$$

72. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda = 16$ cm. The wave speed can be calculated from the frequency and the wavelength.

$$v = \lambda f = (16\text{ cm})(1\text{ Hz}) = \boxed{16\text{ cm/s}}$$

$\boxed{73.}$ The speed of a longitudinal wave in a solid is given by Eq. 15-3, $v = \sqrt{E/\rho}$. Let the density of the less dense material be ρ_1, and the density of the more dense material be ρ_2. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$\frac{v_1}{v_2} = \frac{\sqrt{E/\rho_1}}{\sqrt{E/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{2.5} \approx \boxed{1.6}$$

74. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = P_2/P_1 = A_2^2/A_1^2 = 2.5 \rightarrow A_2/A_1 = \sqrt{2.5} = \boxed{1.6}$$

The more energetic wave has the larger amplitude.

75. (*a*) The amplitude is half the peak-to-peak distance, so $\boxed{0.05\text{m}}$.

(*b*) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$.

Compare the two kinetic energy maxima.

$$\frac{K_{2\text{ max}}}{K_{1\text{ max}}} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{0.075\text{m}}{0.05\text{m}}\right)^2 = \boxed{2.25}$$

76. From Eq. 15-17b, $f_n = \dfrac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From Eq. 15-2, $v = \sqrt{F_{\text{T}}/\mu}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{\text{T}2}}{F_{\text{T}1}}} = \frac{f_2}{f_1} \rightarrow F_{\text{T}2} = \left(\frac{f_2}{f_1}\right)^2 F_{\text{T}1} = \left(\frac{247\text{ Hz}}{255\text{ Hz}}\right)^2 F_{\text{T}1} = 0.938 F_{\text{T}1}$$

Thus the tension should be $\boxed{\text{decreased by }6.2\%}$.

77. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it – the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of *g* downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than *g*. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$a_{\text{max}} = \omega^2 A > g \rightarrow A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.80\text{ m/s}^2}{4\pi^2 (0.60\text{ Hz})^2} = \boxed{0.69\text{m}}$$

78. (*a*) The speed of the wave at a point *h* above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

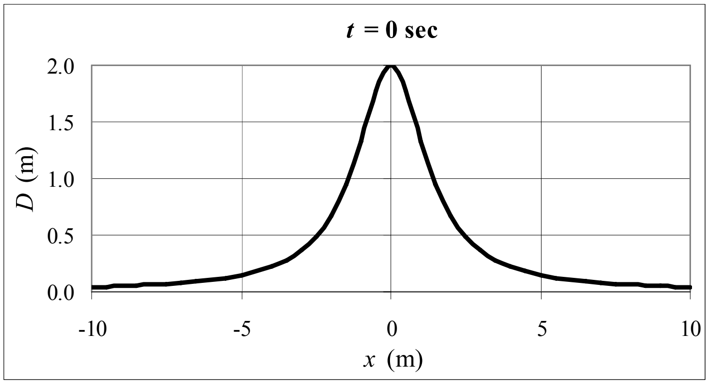
$$F_T = m_{\text{segment}}g = \frac{h}{\ell}mg \ ; \ v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\frac{h}{\ell}mg}{\frac{m}{\ell}}} = \boxed{\sqrt{hg}}$$

(*b*) We treat *h* as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v = \sqrt{hg}$. The distance a wave would travel up the cord during a time *dt* is then $dh = vdt = \sqrt{hg}\,dt$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$dh = vdt = \sqrt{hg}dt \ \rightarrow \ dt = \frac{dh}{\sqrt{hg}} \ \rightarrow \ \int_0^{t_{\text{total}}} dt = \int_0^L \frac{dh}{\sqrt{hg}} \ \rightarrow$$

$$t_{\text{total}} = \int_0^L \frac{dh}{\sqrt{hg}} = 2\sqrt{\frac{h}{g}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

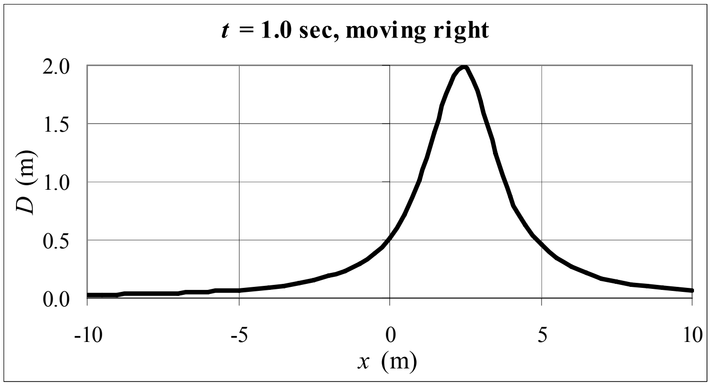
$\boxed{79.}$ (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."



(*b*) The wave function is found by replacing *x* in the pulse by $x - vt$.

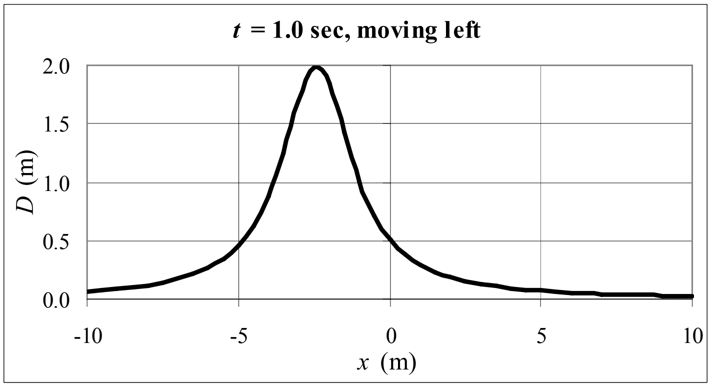
$$\boxed{D(x,t) = \frac{4.0\,\text{m}^3}{\left[x - (2.4\,\text{m/s})t\right]^2 + 2.0\,\text{m}^2}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."



(*d*) The wave function is found by replacing x in the pulse by $x+vt$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."

$$\boxed{D=\frac{4.0\,\mathrm{m}^3}{\left[x+(2.4\,\mathrm{m/s})t\right]^2+2.0\,\mathrm{m}^2}}$$



80. (*a*) The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}}\rightarrow\frac{df}{dF_T}=\frac{1}{2\lambda}\sqrt{\frac{1}{\mu F_T}}=\frac{1}{2F_T}\frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}}=\frac{f}{2F_T}$$

$$\frac{\Delta f}{\Delta F_T}\approx\frac{f}{2F_T}\rightarrow\boxed{\Delta f\approx\tfrac{1}{2}\left(\frac{\Delta F_T}{F_T}\right)f}$$

(*b*) $$\frac{\Delta f}{\Delta F_T}\approx\frac{f}{2F_T}\rightarrow\frac{\Delta F_T}{F_T}\approx2\frac{\Delta f}{f}=2\left(\frac{6}{436}\right)=0.0275=\boxed{3\%}$$

(*c*) The only change in the expression $\frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}}$ as the overtone changes is the wavelength, and the wavelength does not influence the final result. So $\boxed{\text{yes}}$, the formula still applies.

81. (*a*) The overtones are given by $f_n=nf_1, n=2,3,4\ldots$

$$\mathrm{G}:\ f_2=2(392\,\mathrm{Hz})=\boxed{784\,\mathrm{Hz}}\qquad f_3=3(392\,\mathrm{Hz})=1176\,\mathrm{Hz}\approx\boxed{1180\,\mathrm{Hz}}$$

$$\mathrm{B}:\ f_2=2(494\,\mathrm{Hz})=\boxed{988\,\mathrm{Hz}}\qquad f_3=3(440\,\mathrm{Hz})=1482\,\mathrm{Hz}\approx\boxed{1480\,\mathrm{Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$\frac{f_G}{f_A}=\frac{v_G/\lambda}{v_A/\lambda}=\frac{v_G}{v_A}=\frac{\sqrt{\dfrac{F_T}{m_G/\ell}}}{\sqrt{\dfrac{F_T}{m_A/\ell}}}=\sqrt{\frac{m_A}{m_G}}\rightarrow\frac{m_G}{m_A}=\left(\frac{f_A}{f_G}\right)^2=\left(\frac{494}{392}\right)^2=\boxed{1.59}$$

(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_B}=\frac{v/\lambda_G}{v/\lambda_B}=\frac{\lambda_B}{\lambda_G}=\frac{2\ell_B}{2\ell_G}\rightarrow\frac{\ell_G}{\ell_B}=\frac{f_B}{f_G}=\frac{494}{392}=\boxed{1.26}$$

(*d*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_B}{f_A}=\frac{v_B/\lambda}{v_A/\lambda}=\frac{v_B}{v_A}=\frac{\sqrt{\frac{F_{TB}}{m/L}}}{\sqrt{\frac{F_{TA}}{m/L}}}=\sqrt{\frac{F_{TB}}{F_{TA}}}\rightarrow\frac{F_{TB}}{F_{TA}}=\left(\frac{f_B}{f_A}\right)^2=\left(\frac{392}{494}\right)^2=\boxed{0.630}$$

82. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$v=\left(33\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\text{ s}}\right)\left(\frac{2\pi(0.108\text{ m})}{1\text{ rev}}\right)=0.3732\,\text{m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f=\frac{v}{\lambda}=\frac{0.3732\,\text{m/s}}{1.55\times10^{-3}\text{m}}=240.77\text{ Hz}\approx\boxed{240\,\text{Hz}}$$

83. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$v=\sqrt{\frac{F_T}{\mu}}=\sqrt{\frac{255\,\text{N}}{0.152\,\text{kg}/10.0\,\text{m}}}=129.52\,\text{m/s}$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms.

$$\ell=d_1+d_2=vt_1+vt_2=vt_1+v\left(t_1-2.00\times10^{-2}\right)$$

$$t_1=\frac{\ell+2.00\times10^{-2}v}{2v}=\frac{(10.0\,\text{m})+2.00\times10^{-2}(129.52\,\text{m/s})}{2(129.52\,\text{m/s})}=4.8604\times10^{-2}\text{s}$$

$$d_1=vt_1=(129.52\,\text{m/s})\left(4.8604\times10^{-2}\text{s}\right)=6.30\,\text{m}$$

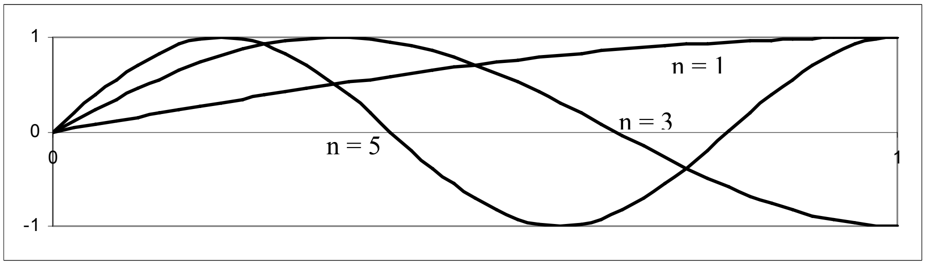
The two pulses meet $\boxed{6.30\,\text{m}}$ from the end where the first pulse originated.

84. We take the wave function to be $D(x,t)=A\sin(kx-\omega t)$. The wave speed is given by $v=\frac{\omega}{k}=\frac{\lambda}{f}$, while the speed of particles on the cord is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t}=-\omega A\cos(kx-\omega t)\rightarrow\left(\frac{\partial D}{\partial t}\right)_{\max}=\omega A$$

$$\omega A=v=\frac{\omega}{k}\rightarrow A=\frac{1}{k}=\frac{\lambda}{2\pi}=\frac{10.0\,\text{cm}}{2\pi}=\boxed{1.59\,\text{cm}}$$

$\boxed{85.}$ For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at



one end and an antinode at the other end include $3\lambda/4$, $5\lambda/4$, … . See the diagrams. The general relationship is $\ell=(2n-1)\lambda/4$, $n=1,2,3,\cdots$. Solving for the wavelength gives

$$\boxed{\lambda=\frac{4\ell}{2n-1}\ ,\ n=1,2,3,\cdots}\ .$$

86. The addition of the support will force the bridge to have its lowest mode of oscillation to have a node at the center of the span, which would be the first overtone of the fundamental frequency. If the wave speed in the bridge material remains constant, then the resonant frequency will double, to $\boxed{6.0\text{ Hz}}$. Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.

87. From the figure, we can see that the amplitude is 3.5 cm, and the wavelength is 20 cm. The maximum of the wave at $x = 0$ has moved to $x = 12$ cm at $t = 0.80$ s, which is used to find the velocity. The wave is moving to the right. Finally, since the displacement is a maximum at $x = 0$ and $t = 0$, we can use a cosine function without a phase angle.

$$A=3.5\,\text{cm};\ \lambda=20\,\text{cm}\ \rightarrow\ k=\frac{2\pi}{\lambda}=0.10\pi\,\text{cm}^{-1};\ v=\frac{12\,\text{cm}}{0.80\,\text{s}}=15\,\text{cm/s};\ \omega=vk=1.5\pi\,\text{rad/s}$$

$$D(x,t)=A\cos(kx-\omega t)=\boxed{(3.5\,\text{cm})\cos(0.10\pi x-1.5\pi t),\,x\text{ in cm, }t\text{ in s}}$$

88. From the given data, $A=0.50\,\text{m}$ and $v=2.5\,\text{m}/4.0\,\text{s}=0.625\,\text{m/s}$. We use Eq. 15-6 for the average power, with the density of sea water from Table 13-1. We estimate the area of the chest as $(0.30\,\text{m})^2$. Answers may vary according to the approximation used for the area of the chest.

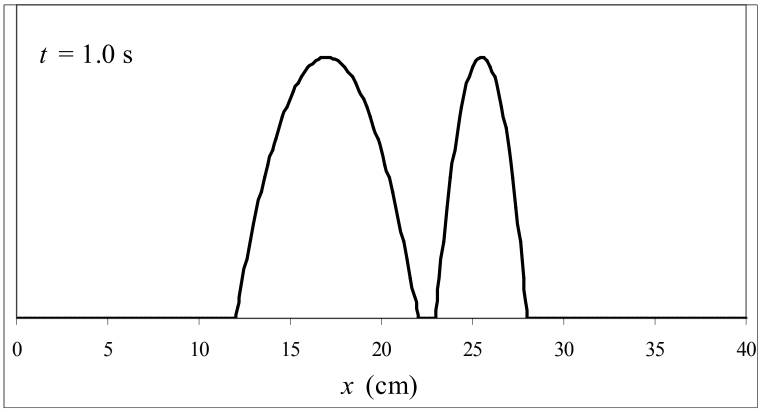
$$\overline{P}=2\pi^2\rho Svf^2A^2=2\pi^2\left(1025\,\text{kg/m}^3\right)(0.30\,\text{m})^2(0.625\,\text{m/s})(0.25\,\text{Hz})^2(0.50\,\text{m})^2$$

$$=\boxed{18\,\text{W}}$$

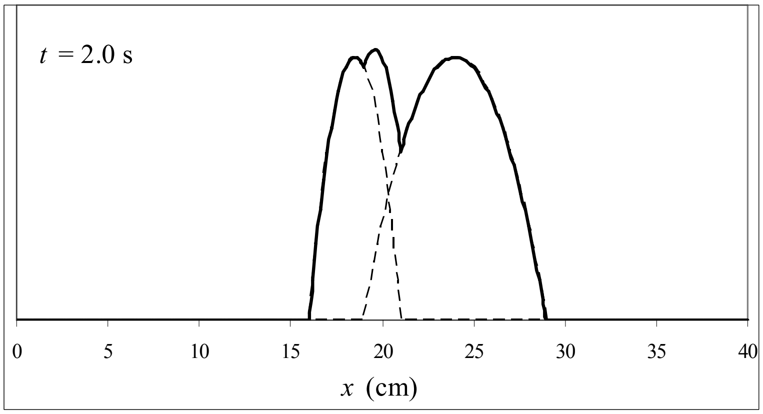
89. The unusual decrease of water corresponds to a trough in Figure 15-4. The crest or peak of the wave is then one-half wavelength from the shore. The peak is 107.5 km away, traveling at 550 km/hr.

$$\Delta x=vt\ \rightarrow\ t=\frac{\Delta x}{v}=\frac{\frac{1}{2}(215\,\text{km})}{550\,\text{km/hr}}\left(\frac{60\,\text{min}}{1\,\text{hr}}\right)=11.7\,\text{min}\approx\boxed{12\,\text{min}}$$

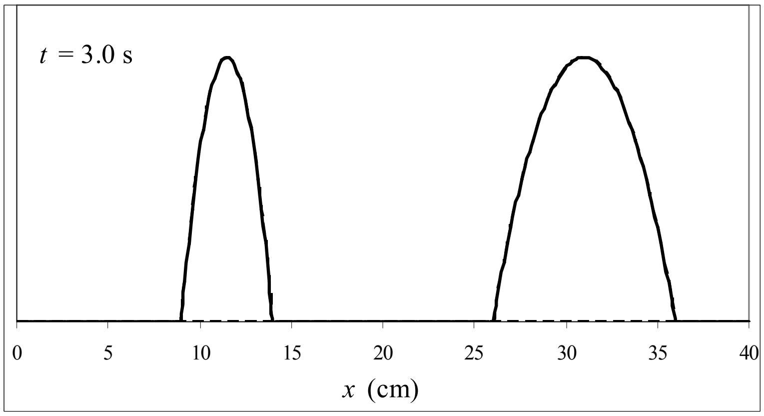
90. At $t = 1.0$ s, the leading edge of each wave is 1.0 cm from the other wave. They have not yet interfered. The leading edge of the wider wave is at 22 cm, and the leading edge of the narrower wave is at 23 cm.



At $t = 2.0$ s, the waves are overlapping. The diagram uses dashed lines to show the parts of the original waves that are undergoing interference.



At $t = 3.0$ s, the waves have "passed through" each other, and are no longer interfering.



91. Because the radiation is uniform, the same energy must pass through every spherical surface, which has the surface area $4\pi r^2$. Thus the intensity must decrease as $1/r^2$. Since the intensity is proportional to the square of the amplitude, the amplitude will decrease as $1/r$. The radial motion will be sinusoidal, and so we have $D = \left(\frac{A}{r}\right)\sin(kr - \omega t)$.

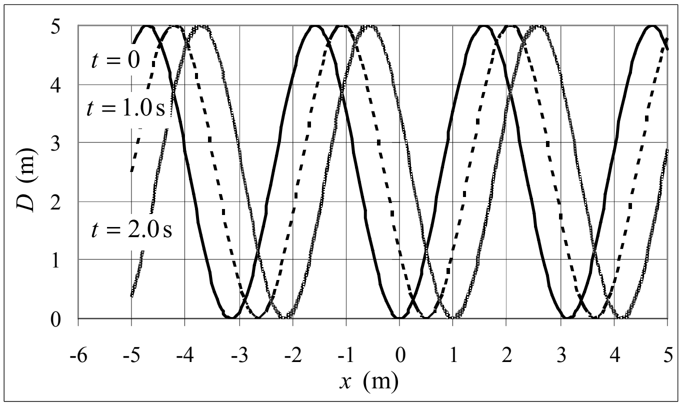
92. The wavelength is to be 1.0 m. Use Eq. 15-1.

$$v = f\lambda \;\rightarrow\; f = \frac{v}{\lambda} = \frac{344\,\text{m/s}}{1.0\,\text{m}} = \boxed{340\,\text{Hz}}$$

There will be significant diffraction only for wavelengths larger than the width of the window, and so waves with frequencies $\boxed{\text{lower than 340 Hz}}$ would diffract when passing through this window.

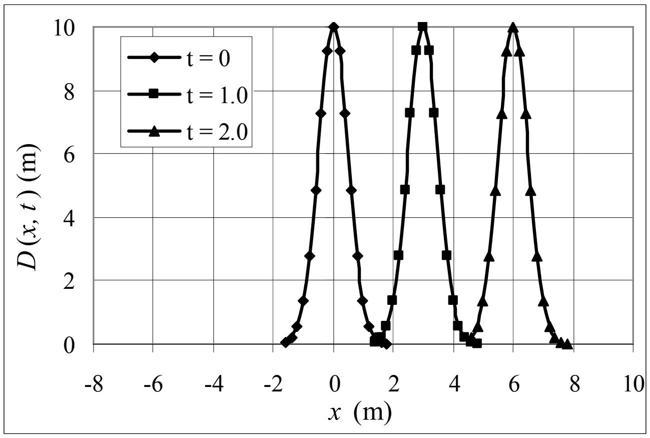
93. The value of k was taken to be $1.0\,\text{m}^{-1}$ for this problem. The peak of the wave moves to the right by 0.50 m during each second that elapses. This can be seen qualitatively from the graph, and quantitatively from the spreadsheet data. Thus the wave speed is given by the constant c, $\boxed{0.50\,\text{m/s}}$. The direction of motion is in the $\boxed{\text{positive } x \text{ direction}}$. The wavelength is seen to be $\boxed{\lambda = \pi\,\text{m}}$. Note that this doesn't agree with the relationship $\lambda = \frac{2\pi}{k}$. The period of the function $\sin^2\theta$ is π, not 2π as is the case for $\sin\theta$. In a similar fashion the period of this function is $\boxed{T = 2\pi\,\text{s}}$. Note that this

doesn't agree with the relationship $kv = \omega = \frac{2\pi}{T}$, again because of the behavior of the $\sin^2\theta$ function. But the relationship $\frac{\lambda}{T} = v$ is still true for this wave function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.93."

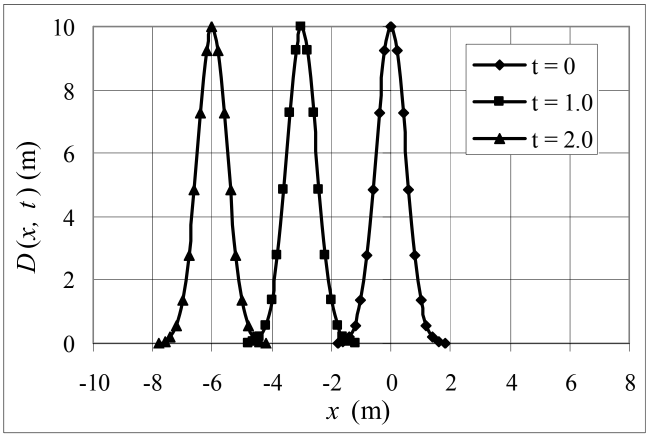


Further insight is gained by re-writing the function using the trigonometric identity $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, because function $\cos 2\theta$ has a period of π.

94. (*a*) The graph shows the wave moving 3.0 m to the right each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the right. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94a."



(*b*) The graph shows the wave moving 3.0 m to the left each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the left. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94b."



CHAPTER 16: Sound

Responses to Questions

$\boxed{1.}$ Sound exhibits diffraction, refraction, and interference effects that are characteristic of waves. Sound also requires a medium, a characteristic of mechanical waves.

2. Sound can cause objects to vibrate, which is evidence that sound is a form of energy. In extreme cases, sound waves can even break objects. (See Figure 14-24 showing a goblet shattering from the sound of a trumpet.)

3. Sound waves generated in the first cup cause the bottom of the cup to vibrate. These vibrations excite vibrations in the stretched string which are transmitted down the string to the second cup, where they cause the bottom of the second cup to vibrate, generating sound waves which are heard by the second child.

4. The wavelength will change. The frequency cannot change at the boundary since the media on both sides of the boundary are oscillating together. If the frequency were to somehow change, there would be a "pile-up" of wave crests on one side of the boundary.

5. If the speed of sound in air depended significantly on frequency, then the sounds that we hear would be separated in time according to frequency. For example, if a chord were played by an orchestra, we would hear the high notes at one time, the middle notes at another, and the lower notes at still another. This effect is not heard for a large range of distances, indicating that the speed of sound in air does not depend significantly on frequency.

6. Helium is much less dense than air, so the speed of sound in the helium is higher than in air. The wavelength of the sound produced does not change, because it is determined by the length of the vocal cords and other properties of the resonating cavity. The frequency therefore increases, increasing the pitch of the voice.

$\boxed{7.}$ The speed of sound in a medium is equal to $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of the medium. The bulk moduli of air and hydrogen are very nearly the same. The density of hydrogen is less than the density of air. The reduced density is the main reason why sound travels faster in hydrogen than in air.

8. The intensity of a sound wave is proportional to the square of the frequency, so the higher-frequency tuning fork will produce more intense sound.

9. Variations in temperature will cause changes in the speed of sound and in the length of the pipes. As the temperature rises, the speed of sound in air increases, increasing the resonance frequency of the pipes, and raising the pitch of the sound. But the pipes get slightly longer, increasing the resonance wavelength and decreasing the resonance frequency of the pipes and lowering the pitch. As the temperature decreases, the speed of sound decreases, decreasing the resonance frequency of the pipes, and lowering the pitch of the sound. But the pipes contract, decreasing the resonance wavelength and increasing the resonance frequency of the pipes and raising the pitch. These effects compete, but the effect of temperature change on the speed of sound dominates.

10. A tube will have certain resonance frequencies associated with it, depending on the length of the tube and the temperature of the air in the tube. Sounds at frequencies far from the resonance

frequencies will not undergo resonance and will not persist. By choosing a length for the tube that isn't resonant for specific frequencies you can reduce the amplitude of those frequencies.

11. As you press on frets closer to the bridge, you are generating higher frequency (and shorter wavelength) sounds. The difference in the wavelength of the resonant standing waves decreases as the wavelengths decrease, so the frets must be closer together as you move toward the bridge.

12. Sound waves can diffract around obstacles such as buildings if the wavelength of the wave is large enough in comparison to the size of the obstacle. Higher frequency corresponds to shorter wavelength. When the truck is behind the building, the lower frequency (longer wavelength) waves bend around the building and reach you, but the higher frequency (shorter wavelength) waves do not. Once the truck has emerged from behind the building, all the different frequencies can reach you.

13. Standing waves are generated by a wave and its reflection. The two waves have a constant phase relationship with each other. The interference depends only on where you are along the string, on your position in space. Beats are generated by two waves whose frequencies are close but not equal. The two waves have a varying phase relationship, and the interference varies with time rather than position.

14. The points would move farther apart. A lower frequency corresponds to a longer wavelength, so the distance between points where destructive and constructive interference occur would increase.

15. According to the principle of superposition, adding a wave and its inverse produces zero displacement of the medium. Adding a sound wave and its inverse effectively cancels out the sound wave and substantially reduces the sound level heard by the worker.

16. (a) The closer the two component frequencies are to each other, the longer the wavelength of the beat. If the two frequencies are very close together, then the waves very nearly overlap, and the distance between a point where the waves interfere constructively and a point where they interfere destructively will be very large.

17. No. The Doppler shift is caused by relative motion between the source and observer.

18. No. The Doppler shift is caused by relative motion between the source and observer. If the wind is blowing, both the wavelength and the velocity of the sound will change, but the frequency of the sound will not.

19. The child will hear the highest frequency at position C, where her speed toward the whistle is the greatest.

20. The human ear can detect frequencies from about 20 Hz to about 20,000 Hz. One octave corresponds to a doubling of frequency. Beginning with 20 Hz, it takes about 10 doublings to reach 20,000 Hz. So, there are approximately 10 octaves in the human audible range.

21. If the frequency of the sound is halved, then the ratio of the frequency of the sound as the car recedes to the frequency of the sound as the car approaches is equal to ½. Substituting the appropriate Doppler shift equations in for the frequencies yields a speed for the car of $^{1}/_{3}$ the speed of sound.

Solutions to Problems

In these solutions, we usually treat frequencies as if they are significant to the whole number of units. For example, 20 Hz is taken as to the nearest Hz, and 20 kHz is taken as to the nearest kHz. We also treat all decibel values as good to whole number of decibels. So 120 dB is good to the nearest decibel.

$\boxed{1.}$ The round trip time for sound is 2.0 seconds, so the time for sound to travel the length of the lake is 1.0 seconds. Use the time and the speed of sound to determine the length of the lake.

$$d = vt = (343\,\text{m/s})(1.0\text{ s}) = 343\text{ m} \approx \boxed{340\,\text{m}}$$

2. The round trip time for sound is 2.5 seconds, so the time for sound to travel the length of the lake is 1.25 seconds. Use the time and the speed of sound in water to determine the depth of the lake.

$$d = vt = (1560\,\text{m/s})(1.25\text{ s}) = 1950\text{ m} = \boxed{2.0\times10^3\,\text{m}}$$

3. (*a*) $\lambda_{20\text{ Hz}} = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{20\text{ Hz}} = \boxed{17\text{ m}} \qquad \lambda_{20\text{ kHz}} = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{2.0\times10^4\,\text{Hz}} = \boxed{1.7\times10^{-2}\,\text{m}}$

So the range is from 1.7 cm to 17 m.

(*b*) $\lambda = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{15\times10^6\,\text{Hz}} = \boxed{2.3\times10^{-5}\,\text{m}}$

4. The distance that the sounds travels is the same on both days. That distance is equal to the speed of sound times the elapsed time. Use the temperature-dependent relationships for the speed of sound in air.

$$d = v_1t_1 = v_2t_2 \quad\rightarrow\quad \left[(331+0.6(27))\,\text{m/s}\right](4.70\,\text{s}) = \left[(331+0.6(T_2))\,\text{m/s}\right](5.20\,\text{s}) \quad\rightarrow$$

$$T_2 = \boxed{-29^\circ\text{C}}$$

5. (*a*) The ultrasonic pulse travels at the speed of sound, and the round trip distance is twice the distance *d* to the object.

$$2d_{\text{min}} = vt_{\text{min}} \quad\rightarrow\quad d_{\text{min}} = \tfrac{1}{2}vt_{\text{min}} = \tfrac{1}{2}(343\,\text{m/s})(1.0\times10^{-3}\,\text{s}) = \boxed{0.17\,\text{m}}$$

(*b*) The measurement must take no longer than 1/15 s. Again, the round trip distance is twice the distance to the object.

$$2d_{\text{max}} = vt_{\text{max}} \quad\rightarrow\quad d_{\text{max}} = \tfrac{1}{2}vt_{\text{max}} = \tfrac{1}{2}(343\,\text{m/s})(\tfrac{1}{15}\,\text{s}) = \boxed{11\,\text{m}}$$

(*c*) The distance is proportional to the speed of sound. So the percentage error in distance is the same as the percentage error in the speed of sound. We assume the device is calibrated to work at 20°C.

$$\frac{\Delta d}{d} = \frac{\Delta v}{v} = \frac{v_{23^\circ\text{C}} - v_{20^\circ\text{C}}}{v_{20^\circ\text{C}}} = \frac{\left[331+0.60(23)\right]\text{m/s} - 343\,\text{m/s}}{343\,\text{m/s}} = 0.005248 \approx \boxed{0.5\%}$$

6. (*a*) For the fish, the speed of sound in seawater must be used.

$$d = vt \quad\rightarrow\quad t = \frac{d}{v} = \frac{1350\,\text{m}}{1560\,\text{m/s}} = \boxed{0.865\,\text{s}}$$

(*b*) For the fishermen, the speed of sound in air must be used.

$$d = vt \quad\rightarrow\quad t = \frac{d}{v} = \frac{1350\,\text{m}}{343\,\text{m/s}} = \boxed{3.94\,\text{s}}$$

$\boxed{7.}$ The total time *T* is the time for the stone to fall $\left(t_{\text{down}}\right)$ plus the time for the sound to come back to the top of the cliff $\left(t_{\text{up}}\right)$: $T = t_{\text{up}} + t_{\text{down}}$. Use constant acceleration relationships for an object dropped from rest that falls a distance *h* in order to find t_{down}, with down as the positive direction. Use the constant speed of sound to find t_{up} for the sound to travel a distance *h*.

$$\text{down: } y = y_0 + v_0 t_{\text{down}} + \tfrac{1}{2} a t_{\text{down}}^2 \rightarrow h = \tfrac{1}{2} g t_{\text{down}}^2 \qquad \text{up: } h = v_{\text{snd}} t_{\text{up}} \rightarrow t_{\text{up}} = \frac{h}{v_{\text{snd}}}$$

$$h = \tfrac{1}{2} g t_{\text{down}}^2 = \tfrac{1}{2} g \left(T - t_{\text{up}}\right)^2 = \tfrac{1}{2} g \left(T - \frac{h}{v_{\text{snd}}}\right)^2 \rightarrow h^2 - 2v_{\text{snd}}\left(\frac{v_{\text{snd}}}{g} + T\right) h + T^2 v_{\text{snd}}^2 = 0$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$h^2 - 2\left(343\,\text{m/s}\right)\left(\frac{343\,\text{m/s}}{9.80\,\text{m/s}^2} + 3.0\,\text{s}\right) h + \left(3.0\,\text{s}\right)^2 \left(343\,\text{m/s}\right)^2 = 0 \rightarrow$$

$$h^2 - \left(26068\,\text{m}\right) h + 1.0588 \times 10^6\,\text{m}^2 = 0 \rightarrow h = 26028\,\text{m}\,,\ 41\,\text{m}$$

The larger root is impossible since it takes more than 3.0 sec for the rock to fall that distance, so the correct result is $h = \boxed{41\,\text{m}}$.

8. The two sound waves travel the same distance. The sound will travel faster in the concrete, and thus take a shorter time.

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{concrete}} t_{\text{concrete}} = v_{\text{concrete}} \left(t_{\text{air}} - 0.75\,\text{s}\right) \rightarrow t_{\text{air}} = \frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}} 0.75\,\text{s}$$

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{air}} \left(\frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}} 0.75\,\text{s}\right)$$

The speed of sound in concrete is obtained from Table 16-1 as 3000 m/s.

$$d = \left(343\,\text{m/s}\right)\left(\frac{3000\,\text{m/s}}{3000\,\text{m/s} - 343\,\text{m/s}}\left(0.75\,\text{s}\right)\right) = \boxed{290\,\text{m}}$$

9. The "5 second rule" says that for every 5 seconds between seeing a lightning strike and hearing the associated sound, the lightning is 1 mile distant. We assume that there are 5 seconds between seeing the lightning and hearing the sound.

(*a*) At 30°C, the speed of sound is $\left[331 + 0.60\left(30\right)\right]\text{m/s} = 349\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(349\,\text{m/s}\right)\left(5\,\text{s}\right) = 1745\ \text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1745 - 1610}{1745}\left(100\right) \approx \boxed{8\%}$$

(*b*) At 10°C, the speed of sound is $\left[331 + 0.60\left(10\right)\right]\text{m/s} = 337\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(337\,\text{m/s}\right)\left(5\,\text{s}\right) = 1685\ \text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1685 - 1610}{1685}\left(100\right) \approx \boxed{4\%}$$

10. The relationship between the pressure and displacement amplitudes is given by Eq. 16-5.

(*a*) $$\Delta P_{\text{M}} = 2\pi\rho vAf \quad\to\quad A = \frac{\Delta P_{\text{M}}}{2\pi\rho vf} = \frac{3.0\times10^{-3}\,\text{Pa}}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(150\,\text{Hz}\right)} = \boxed{7.5\times10^{-9}\,\text{m}}$$

(*b*) $$A = \frac{\Delta P_{\text{M}}}{2\pi\rho vf} = \frac{3.0\times10^{-3}\,\text{Pa}}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(15\times10^{3}\,\text{Hz}\right)} = \boxed{7.5\times10^{-11}\,\text{m}}$$

11. The pressure amplitude is found from Eq. 16-5. The density of air is $1.29\,\text{kg/m}^3$.

(*a*) $$\Delta P_{\text{M}} = 2\pi\rho vAf = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(3.0\times10^{-10}\,\text{m}\right)\left(55\,\text{Hz}\right) = \boxed{4.4\times10^{-5}\,\text{Pa}}$$

(*b*) $$\Delta P_{\text{M}} = 2\pi\rho vAf = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(3.0\times10^{-10}\,\text{m}\right)\left(5500\,\text{Hz}\right) = \boxed{4.4\times10^{-3}\,\text{Pa}}$$

12. The pressure wave can be written as Eq. 16-4.

(*a*) $\Delta P = -\Delta P_{\text{M}}\cos\left(kx-\omega t\right)$

$$\Delta P_{\text{M}} = 4.4\times10^{-5}\,\text{Pa}\ ;\ \omega = 2\pi f = 2\pi\left(55\,\text{Hz}\right) = 110\pi\,\text{rad/s}\ ;\ k = \frac{\omega}{v} = \frac{110\pi\,\text{rad/s}}{331\,\text{m/s}} = 0.33\pi\,\text{m}^{-1}$$

$$\boxed{\Delta P = -\left(4.4\times10^{-5}\,\text{Pa}\right)\cos\left[\left(0.33\pi\,\text{m}^{-1}\right)x-\left(110\pi\,\text{rad/s}\right)t\right]}$$

(*b*) All is the same except for the amplitude and $\omega = 2\pi f = 2\pi\left(5500\ \text{Hz}\right) = 1.1\times10^{4}\pi\,\text{rad/s}$.

$$\boxed{\Delta P = -\left(4.4\times10^{-3}\,\text{Pa}\right)\cos\left[\left(0.33\pi\,\text{m}^{-1}\right)x-\left(1.1\times10^{4}\pi\,\text{rad/s}\right)t\right]}$$

$\boxed{13.}$ The pressure wave is $\Delta P = \left(0.0035\,\text{Pa}\right)\sin\left[\left(0.38\pi\,\text{m}^{-1}\right)x-\left(1350\pi\,\text{s}^{-1}\right)t\right]$.

(*a*) $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.38\pi\,\text{m}^{-1}} = \boxed{5.3\,\text{m}}$$

(*b*) $$f = \frac{\omega}{2\pi} = \frac{1350\pi\,\text{s}^{-1}}{2\pi} = \boxed{675\,\text{Hz}}$$

(*c*) $$v = \frac{\omega}{k} = \frac{1350\pi\,\text{s}^{-1}}{0.38\pi\,\text{m}^{-1}} = 3553\,\text{m/s} \approx \boxed{3600\,\text{m/s}}$$

(*d*) Use Eq. 16-5 to find the displacement amplitude.

$$\Delta P_{\text{M}} = 2\pi\rho vAf \quad\to$$

$$A = \frac{\Delta P_{\text{M}}}{2\pi\rho vf} = \frac{\left(0.0035\,\text{Pa}\right)}{2\pi\left(2300\,\text{kg/m}^3\right)\left(3553\,\text{m/s}\right)\left(675\,\text{Hz}\right)} = \boxed{1.0\times10^{-13}\,\text{m}}$$

14. $$120\ \text{dB} = 10\log\frac{I_{120}}{I_0} \quad\to\quad I_{120} = 10^{12}I_0 = 10^{12}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = \boxed{1.0\,\text{W/m}^2}$$

$$20\ \text{dB} = 10\log\frac{I_{20}}{I_0} \quad\to\quad I_{20} = 10^{2}I_0 = 10^{2}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = \boxed{1.0\times10^{-10}\ \text{W/m}^2}$$

The pain level is 10^{10} times more intense than the whisper.

15. $\beta = 10\log\dfrac{I}{I_0} = 10\log\dfrac{2.0\times10^{-6}\ \mathrm{W/m^2}}{1.0\times10^{-12}\ \mathrm{W/m^2}} = \boxed{63\ \mathrm{dB}}$

16. From Figure 16-6, at 40 dB the low frequency threshold of hearing is about $\boxed{70-80\,\mathrm{Hz}}$. There is no intersection of the threshold of hearing with the 40 dB level on the high frequency side of the chart, and so a 40 dB signal can be heard all the way up to the highest frequency that a human can hear, $\boxed{20{,}000\ \mathrm{Hz}}$.

17. (*a*) From Figure 16-6, at 100 Hz, the threshold of hearing (the lowest detectable intensity by the ear) is approximately $5\times10^{-9}\ \mathrm{W/m^2}$. The threshold of pain is about $5\,\mathrm{W/m^2}$. The ratio of highest to lowest intensity is thus $\dfrac{5\,\mathrm{W/m^2}}{5\times10^{-9}\ \mathrm{W/m^2}} = \boxed{10^9}$.

(*b*) At 5000 Hz, the threshold of hearing is about $10^{-13}\ \mathrm{W/m^2}$, and the threshold of pain is about $10^{-1}\ \mathrm{W/m^2}$. The ratio of highest to lowest intensity is $\dfrac{10^{-1}\ \mathrm{W/m^2}}{10^{-13}\ \mathrm{W/m^2}} = \boxed{10^{12}}$.

Answers may vary due to estimation in the reading of the graph.

18. Compare the two power output ratings using the definition of decibels.

$$\beta = 10\log\frac{P_{150}}{P_{100}} = 10\log\frac{150\,\mathrm{W}}{100\,\mathrm{W}} = \boxed{1.8\,\mathrm{dB}}$$

This would barely be perceptible.

$\boxed{19.}$ The intensity can be found from the decibel value.

$$\beta = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{\beta/10}I_0 = 10^{12}\left(10^{-12}\ \mathrm{W/m^2}\right) = 1.0\,\mathrm{W/m^2}$$

Consider a square perpendicular to the direction of travel of the sound wave. The intensity is the energy transported by the wave across a unit area perpendicular to the direction of travel, per unit time. So $I = \dfrac{\Delta E}{S\Delta t}$, where *S* is the area of the square. Since the energy is "moving" with the wave, the "speed" of the energy is *v*, the wave speed. In a time Δt, a volume equal to $\Delta V = Sv\Delta t$ would contain all of the energy that had been transported across the area *S*. Combine these relationships to find the energy in the volume.

$$I = \frac{\Delta E}{S\Delta t} \quad\rightarrow\quad \Delta E = IS\Delta t = \frac{I\Delta V}{v} = \frac{\left(1.0\,\mathrm{W/m^2}\right)\left(0.010\,\mathrm{m}\right)^3}{343\,\mathrm{m/s}} = \boxed{2.9\times10^{-9}\,\mathrm{J}}$$

20. From Example 12-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus the sound level for one firecracker will be $95\ \mathrm{dB} - 3\ \mathrm{dB} = \boxed{92\ \mathrm{dB}}$.

21. From Example 16-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, if two engines are shut down, the intensity will be cut in half, and the sound level will be 127 dB. Then, if one more engine is shut down, the intensity will be cut in half again, and the sound level will drop by 3 more dB, to a final value of $\boxed{124\ \mathrm{dB}}$.

22. $$62\,\text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \rightarrow \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{6.2} = \boxed{1.6\times10^{6}}$$

$$98\,\text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \rightarrow \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{9.8} = \boxed{6.3\times10^{9}}$$

23. (*a*) According to Table 16-2, the intensity in normal conversation, when about 50 cm from the speaker, is about $3\times10^{-6}\ \text{W/m}^2$. The intensity is the power output per unit area, and so the power output can be found. The area is that of a sphere.

$$I = \frac{P}{A} \rightarrow P = IA = I\left(4\pi r^2\right) = \left(3\times10^{-6}\ \text{W/m}^2\right)4\pi\left(0.50\ \text{m}\right)^2 = 9.425\times10^{-6}\,\text{W} \approx \boxed{9.4\times10^{-6}\,\text{W}}$$

(*b*) $$75\,\text{W}\left(\frac{1\ \text{person}}{9.425\times10^{-6}\,\text{W}}\right) = 7.96\times10^{6} \approx \boxed{8.0\times10^{6}\,\text{people}}$$

24. (*a*) The energy absorbed per second is the power of the wave, which is the intensity times the area.

$$50\ \text{dB} = 10\log\frac{I}{I_0} \rightarrow I = 10^{5}I_0 = 10^{5}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 1.0\times10^{-7}\ \text{W/m}^2$$

$$P = IA = \left(1.0\times10^{-7}\ \text{W/m}^2\right)\left(5.0\times10^{-5}\,\text{m}^2\right) = \boxed{5.0\times10^{-12}\,\text{W}}$$

(*b*) $$1\ \text{J}\left(\frac{1\ \text{s}}{5.0\times10^{-12}\,\text{J}}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\,\text{s}}\right) = \boxed{6.3\times10^{3}\,\text{yr}}$$

25. The intensity of the sound is defined to be the power per unit area. We assume that the sound spreads out spherically from the loudspeaker.

(*a*) $$I_{250} = \frac{250\,\text{W}}{4\pi\left(3.5\,\text{m}\right)^2} = 1.624\,\text{W/m}^2 \qquad \beta_{250} = 10\log\frac{I_{250}}{I_0} = 10\log\frac{1.624\,\text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2} = \boxed{122\,\text{dB}}$$

$$I_{45} = \frac{45\,\text{W}}{4\pi\left(3.5\text{m}\right)^2} = 0.2923\,\text{W/m}^2 \qquad \beta_{45} = 10\log\frac{I_{45}}{I_0} = 10\log\frac{0.2923\,\text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2} = \boxed{115\,\text{dB}}$$

(*b*) According to the textbook, for a sound to be perceived as twice as loud as another means that the intensities need to differ by a factor of 10. That is not the case here – they differ only by a factor of $\dfrac{1.624}{0.2598} \approx 6$. $\boxed{\text{The expensive amp will not sound twice as loud as the cheaper one.}}$

26. (*a*) Find the intensity from the 130 dB value, and then find the power output corresponding to that intensity at that distance from the speaker.

$$\beta = 130\ \text{dB} = 10\log\frac{I_{2.8\text{m}}}{I_0} \rightarrow I_{2.8\text{m}} = 10^{13}I_0 = 10^{13}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 10\,\text{W/m}^2$$

$$P = IA = 4\pi r^2 I = 4\pi\left(2.2\,\text{m}\right)^2\left(10\,\text{W/m}^2\right) = 608\,\text{W} \approx \boxed{610\,\text{W}}$$

(*b*) Find the intensity from the 85 dB value, and then from the power output, find the distance corresponding to that intensity.

$$\beta = 85\,\text{dB} = 10\log\frac{I}{I_0} \rightarrow I = 10^{8.5}I_0 = 10^{8.5}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 3.16\times10^{-4}\ \text{W/m}^2$$

$$P = 4\pi r^2 I \rightarrow r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{608\,\text{W}}{4\pi\left(3.16\times10^{-4}\ \text{W/m}^2\right)}} = \boxed{390\,\text{m}}$$

27. The first person is a distance of $r_1 = 100\,\text{m}$ from the explosion, while the second person is a distance $r_2 = \sqrt{5}\,(100\,\text{m})$ from the explosion. The intensity detected away from the explosion is inversely proportional to the square of the distance from the explosion.

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left[\frac{\sqrt{5}\,(100\,\text{m})}{100\,\text{m}}\right]^2 = 5 \;;\; \beta = 10\log\frac{I_1}{I_2} = 10\log 5 = \boxed{7.0\,\text{dB}}$$

28. (*a*) The intensity is proportional to the square of the amplitude, so if the amplitude is 2.5 times greater, the intensity will $\boxed{\text{increase by a factor of } 6.25 \approx 6.3}$.

(*b*) $\beta = 10\log I/I_0 = 10\log 6.25 = \boxed{8\,\text{dB}}$

29. (*a*) The pressure amplitude is seen in Eq. 16-5 to be proportional to the displacement amplitude and to the frequency. Thus $\boxed{\text{the higher frequency wave has the larger pressure amplitude, by a factor of 2.6.}}$

(*b*) The intensity is proportional to the square of the frequency. Thus the ratio of the intensities is the square of the frequency ratio.

$$\frac{I_{2.6f}}{I_f} = \frac{(2.6f)^2}{f^2} = 6.76 \approx \boxed{6.8}$$

30. The intensity is given by Eq. 15-7, $I = 2\pi^2 v\rho f^2 A^2$, using the density of air and the speed of sound in air.

$$I = 2\rho v\pi^2 f^2 A^2 = 2\left(1.29\,\text{kg/m}^3\right)(343\,\text{m/s})\pi^2(380\,\text{Hz})^2\left(1.3\times10^{-4}\,\text{m}\right)^2 = 21.31\,\text{W/m}^2$$

$$\beta = 10\log\frac{I}{I_0} = 10\log\frac{21.31\,\text{W/m}^2}{1.0\times10^{-12}\,\text{W/m}^2} = 133\,\text{dB} \approx \boxed{130\,\text{dB}}$$

Note that this is above the threshold of pain.

$\boxed{31.}$ (*a*) We find the intensity of the sound from the decibel value, and then calculate the displacement amplitude from Eq. 15-7.

$$\beta = 10\log\frac{I}{I_0} \;\rightarrow\; I = 10^{\beta/10} I_0 = 10^{12}\left(10^{-12}\,\text{W/m}^2\right) = 1.0\,\text{W/m}^2$$

$$I = 2\pi^2 v\rho f^2 A^2 \;\rightarrow$$

$$A = \frac{1}{\pi f}\sqrt{\frac{I}{2\rho v}} = \frac{1}{\pi(330\,\text{Hz})}\sqrt{\frac{1.0\,\text{W/m}^2}{2\left(1.29\,\text{kg/m}^3\right)(343\,\text{m/s})}} = \boxed{3.2\times10^{-5}\,\text{m}}$$

(*b*) The pressure amplitude can be found from Eq. 16-7.

$$I = \frac{(\Delta P_{\text{M}})^2}{2v\rho} \;\rightarrow$$

$$\Delta P_{\text{M}} = \sqrt{2v\rho I} = \sqrt{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)\left(1.0\,\text{W/m}^2\right)} = \boxed{30\,\text{Pa}\,(2\text{ sig. fig.})}$$

32. (*a*) We assume that there has been no appreciable absorption in this 25 meter distance. The intensity is the power divide by the area of a sphere of radius 25 meters. We express the sound level in dB.

$$I=\frac{P}{4\pi r^2}\ ;\ \beta=10\log\frac{I}{I_0}=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(25\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=\boxed{138\,\text{dB}}$$

(*b*) We find the intensity level at the new distance, and subtract due to absorption.

$$\beta=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(1000\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=106\,\text{dB}$$

$$\beta_{\substack{\text{with}\\ \text{absorption}}}=106\,\text{dB}-\left(1.00\,\text{km}\right)\left(7.0\,\text{dB/km}\right)=\boxed{99\,\text{dB}}$$

(*c*) We find the intensity level at the new distance, and subtract due to absorption.

$$\beta=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(7500\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=88.5\,\text{dB}$$

$$\beta_{\substack{\text{with}\\ \text{absorption}}}=88.5\,\text{dB}-\left(7.50\,\text{km}\right)\left(7.0\,\text{dB/km}\right)=\boxed{36\,\text{dB}}$$

33. For a closed tube, Figure 16-12 indicates that $f_1=\frac{v}{4\ell}$. We assume the bass clarinet is at room temperature.

$$f_1=\frac{v}{4\ell}\ \rightarrow\ \ell=\frac{v}{4f_1}=\frac{343\,\text{m/s}}{4\left(69.3\,\text{Hz}\right)}=\boxed{1.24\,\text{m}}$$

34. For a vibrating string, the frequency of the fundamental mode is given by $f=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{F_\text{T}}{m/L}}$.

$$f=\frac{1}{2L}\sqrt{\frac{F_\text{T}}{m/L}}\ \rightarrow\ F_\text{T}=4Lf^2m=4\left(0.32\,\text{m}\right)\left(440\,\text{Hz}\right)^2\left(3.5\times10^{-4}\,\text{kg}\right)=\boxed{87\,\text{N}}$$

35. (*a*) If the pipe is closed at one end, only the odd harmonic frequencies are present, and are given by $f_n=\frac{nv}{4L}=nf_1, n=1,3,5\cdots$.

$$f_1=\frac{v}{4L}=\frac{343\,\text{m/s}}{4\left(1.24\ \text{m}\right)}=\boxed{69.2\ \text{Hz}}$$

$$f_3=3f_1=\boxed{207\,\text{Hz}}\quad f_5=5f_1=\boxed{346\,\text{Hz}}\quad f_7=7f_1=\boxed{484\,\text{Hz}}$$

(*b*) If the pipe is open at both ends, all the harmonic frequencies are present, and are given by $f_n=\frac{nv}{2\ell}=nf_1$.

$$f_1=\frac{v}{2\ell}=\frac{343\,\text{m/s}}{2\left(1.24\,\text{m}\right)}=138.3\,\text{Hz}\approx\boxed{138\,\text{Hz}}$$

$$f_2=2f_1=\frac{v}{\ell}=\boxed{277\,\text{Hz}}\quad f_3=3f_1=\frac{3v}{2\ell}=\boxed{415\,\text{Hz}}\quad f_4=4f_1=\frac{2v}{\ell}=\boxed{553\,\text{Hz}}$$

36. (*a*) The length of the tube is one-fourth of a wavelength for this (one end closed) tube, and so the wavelength is four times the length of the tube.

$$f=\frac{v}{\lambda}=\frac{343\,\text{m/s}}{4(0.21\,\text{m})}=\boxed{410\,\text{Hz}}$$

(*b*) If the bottle is one-third full, then the effective length of the air column is reduced to 14 cm.

$$f=\frac{v}{\lambda}=\frac{343\,\text{m/s}}{4(0.14\,\text{m})}=\boxed{610\,\text{Hz}}$$

$\boxed{37.}$ For a pipe open at both ends, the fundamental frequency is given by $f_1=\dfrac{v}{2\ell}$, and so the length for a given fundamental frequency is $\ell=\dfrac{v}{2f_1}$.

$$\ell_{20\text{ Hz}}=\frac{343\,\text{m/s}}{2(20\text{ Hz})}=\boxed{8.6\text{ m}}\qquad \ell_{20\text{ kHz}}=\frac{343\,\text{m/s}}{2(20{,}000\text{ Hz})}=\boxed{8.6\times10^{-3}\,\text{m}}$$

38. We approximate the shell as a closed tube of length 20 cm, and calculate the fundamental frequency.

$$f=\frac{v}{4\ell}=\frac{343\,\text{m/s}}{4(0.20\,\text{m})}=429\,\text{Hz}\approx\boxed{430\,\text{Hz}}$$

39. (*a*) We assume that the speed of waves on the guitar string does not change when the string is fretted. The fundamental frequency is given by $f=\dfrac{v}{2\ell}$, and so the frequency is inversely proportional to the length.

$$f\propto\frac{1}{\ell}\quad\to\quad f\ell=\text{constant}$$

$$f_E\ell_E=f_A\ell_A\quad\to\quad \ell_A=\ell_E\frac{f_E}{f_A}=(0.73\text{ m})\left(\frac{330\text{ Hz}}{440\text{ Hz}}\right)=0.5475\text{ m}$$

The string should be fretted a distance $0.73\text{ m}-0.5475\text{ m}=0.1825\,\text{m}\approx\boxed{0.18\text{ m}}$ from the nut of the guitar.

(*b*) The string is fixed at both ends and is vibrating in its fundamental mode. Thus the wavelength is twice the length of the string (see Fig. 16-7).

$$\lambda=2\ell=2(0.5475\text{ m})=1.095\,\text{m}\approx\boxed{1.1\text{ m}}$$

(*c*) The frequency of the sound will be the same as that of the string, $\boxed{440\text{ Hz}}$. The wavelength is given by the following.

$$\lambda=\frac{v}{f}=\frac{343\,\text{m/s}}{440\text{ Hz}}=\boxed{0.78\text{ m}}$$

40. (*a*) At $T=15^\circ\text{C}$, the speed of sound is given by $v=(331+0.60(15))\,\text{m/s}=340\,\text{m/s}$ (with 3 significant figures). For an open pipe, the fundamental frequency is given by $f=\dfrac{v}{2\ell}$.

$$f=\frac{v}{2\ell}\quad\to\quad\ell=\frac{v}{2f}=\frac{340\,\text{m/s}}{2(262\text{ Hz})}=\boxed{0.649\,\text{m}}$$

(*b*) The frequency of the standing wave in the tube is $\boxed{262\text{ Hz}}$. The wavelength is twice the length of the pipe, $\boxed{1.30\,\text{m}}$.

(*c*) The wavelength and frequency are the same in the air, because it is air that is resonating in the organ pipe. The frequency is $\boxed{262\text{ Hz}}$ and the wavelength is $\boxed{1.30\,\text{m}}$.

41. The speed of sound will change as the temperature changes, and that will change the frequency of the organ. Assume that the length of the pipe (and thus the resonant wavelength) does not change.

$$f_{22} = \frac{v_{22}}{\lambda} \qquad f_{5.0} = \frac{v_{5.0}}{\lambda} \qquad \Delta f = f_{5.0} - f_{22} = \frac{v_{5.0} - v_{22}}{\lambda}$$

$$\frac{\Delta f}{f} = \frac{\dfrac{v_{5.0} - v_{22}}{\lambda}}{\dfrac{v_{22}}{\lambda}} = \frac{v_{5.0}}{v_{22}} - 1 = \frac{331 + 0.60(5.0)}{331 + 0.60(22)} - 1 = -2.96\times10^{-2} = \boxed{-3.0\%}$$

42. A flute is a tube that is open at both ends, and so the fundamental frequency is given by $f = \dfrac{v}{2\ell}$, where ℓ is the distance from the mouthpiece (antinode) to the first open side hole in the flute tube (antinode).

$$f = \frac{v}{2\ell} \quad \rightarrow \quad \ell = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(349\,\text{Hz})} = \boxed{0.491\,\text{m}}$$

$\boxed{43.}$ For a tube open at both ends, all harmonics are allowed, with $f_n = nf_1$. Thus consecutive harmonics differ by the fundamental frequency. The four consecutive harmonics give the following values for the fundamental frequency.

$$f_1 = 523\,\text{Hz} - 392\,\text{Hz} = 131\,\text{Hz},\ \ 659\,\text{Hz} - 523\,\text{Hz} = 136\,\text{Hz},\ \ 784\,\text{Hz} - 659\,\text{Hz} = 125\,\text{Hz}$$

The average of these is $f_1 = \frac{1}{3}(131\,\text{Hz} + 136\,\text{Hz} + 125\,\text{Hz}) \approx 131\,\text{Hz}$. We use that for the fundamental frequency.

(*a*) $f_1 = \dfrac{v}{2\ell} \quad \rightarrow \quad \ell = \dfrac{v}{2f_1} = \dfrac{343\,\text{m/s}}{2(131\,\text{Hz})} = \boxed{1.31\,\text{m}}$

Note that the bugle is coiled like a trumpet so that the full length fits in a smaller distance.

(*b*) $f_n = nf_1 \quad \rightarrow \quad n_{\text{G4}} = \dfrac{f_{\text{G4}}}{f_1} = \dfrac{392\,\text{Hz}}{131\,\text{Hz}} = 2.99\ ;\ n_{\text{C5}} = \dfrac{f_{\text{C5}}}{f_1} = \dfrac{523\,\text{Hz}}{131\,\text{Hz}} = 3.99\ ;$

$$n_{\text{E5}} = \frac{f_{\text{E5}}}{f_1} = \frac{659\,\text{Hz}}{131\,\text{Hz}} = 5.03\ ;\ n_{\text{G5}} = \frac{f_{\text{G5}}}{f_1} = \frac{784\,\text{Hz}}{131\,\text{Hz}} = 5.98$$

The harmonics are $\boxed{3,\ 4,\ 5,\ \text{and}\ 6}$.

44. (*a*) The difference between successive overtones for this pipe is 176 Hz. The difference between successive overtones for an open pipe is the fundamental frequency, and each overtone is an integer multiple of the fundamental. Since 264 Hz is not a multiple of 176 Hz, 176 Hz cannot be the fundamental, and so the pipe cannot be open. Thus it must be a $\boxed{\text{closed}}$ pipe.

(*b*) For a closed pipe, the successive overtones differ by twice the fundamental frequency. Thus 176 Hz must be twice the fundamental, so the fundamental is $\boxed{88\,\text{Hz}}$. This is verified since 264 Hz is 3 times the fundamental, 440 Hz is 5 times the fundamental, and 616 Hz is 7 times the fundamental.

45. The tension and mass density of the string do not change, so the wave speed is constant. The frequency ratio for two adjacent notes is to be $2^{1/12}$. The frequency is given by $f = \dfrac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \rightarrow \frac{f_{\substack{\text{1st}\\\text{fret}}}}{f_{\text{unfingered}}} = \frac{\dfrac{v}{2\ell_{\substack{\text{1st}\\\text{fret}}}}}{\dfrac{v}{2\ell_{\text{unfingered}}}} = 2^{1/12} \rightarrow \ell_{\substack{\text{1st}\\\text{fret}}} = \frac{\ell_{\text{unfingered}}}{2^{1/12}} = \frac{65.0\,\text{cm}}{2^{1/12}} = 61.35\,\text{cm}$$

$$\ell_{\substack{\text{2nd}\\\text{fret}}} = \frac{\ell_{\substack{\text{1st}\\\text{fret}}}}{2^{1/12}} = \frac{\ell_{\text{unfingered}}}{2^{2/12}} \rightarrow \ell_{\substack{\text{nth}\\\text{fret}}} = \frac{\ell_{\text{unfingered}}}{2^{n/12}} \;;\; x_{\substack{\text{nth}\\\text{fret}}} = \ell_{\text{unfingered}} - \ell_{\substack{\text{nth}\\\text{fret}}} = \ell_{\text{unfingered}}\left(1 - 2^{-n/12}\right)$$

$$x_1 = (65.0\,\text{cm})\left(1 - 2^{-1/12}\right) = \boxed{3.6\,\text{cm}} \;;\; x_2 = (65.0\,\text{cm})\left(1 - 2^{-2/12}\right) = \boxed{7.1\,\text{cm}}$$

$$x_3 = (65.0\,\text{cm})\left(1 - 2^{-3/12}\right) = \boxed{10.3\,\text{cm}} \;;\; x_4 = (65.0\,\text{cm})\left(1 - 2^{-4/12}\right) = \boxed{13.4\,\text{cm}}$$

$$x_5 = (65.0\,\text{cm})\left(1 - 2^{-5/12}\right) = \boxed{16.3\,\text{cm}} \;;\; x_6 = (65.0\,\text{cm})\left(1 - 2^{-6/12}\right) = \boxed{19.0\,\text{cm}}$$

46. (*a*) The difference between successive overtones for an open pipe is the fundamental frequency.

$$f_1 = 330\,\text{Hz} - 275\,\text{Hz} = \boxed{55\text{ Hz}}$$

(*b*) The fundamental frequency is given by $f_1 = \dfrac{v}{2\ell}$. Solve this for the speed of sound.

$$v = 2\ell f_1 = 2(1.80\text{ m})(55\text{ Hz}) = 198\,\text{m/s} \approx \boxed{2.0\times10^2\text{ m/s}}$$

47. The difference in frequency for two successive harmonics is 40 Hz. For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz, with 240 Hz being the 6th harmonic and 280 Hz being the 7th harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz. But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz. So the pipe must be an $\boxed{\text{open pipe}}$.

$$f = \frac{v}{2\ell} \rightarrow \ell = \frac{v}{2f} = \frac{[331 + 0.60(23.0)]\,\text{m/s}}{2(40\,\text{Hz})} = \boxed{4.3\,\text{m}}$$

48. (*a*) The harmonics for the open pipe are $f_n = \dfrac{nv}{2\ell}$. To be audible, they must be below 20 kHz.

$$\frac{nv}{2\ell} < 2\times10^4\,\text{Hz} \rightarrow n < \frac{2(2.48\,\text{m})\left(2\times10^4\,\text{Hz}\right)}{343\,\text{m/s}} = 289.2$$

Since there are 289 harmonics, there are $\boxed{\text{288 overtones}}$.

(*b*) The harmonics for the closed pipe are $f_n = \dfrac{nv}{4\ell}$, *n* odd. Again, they must be below 20 kHz.

$$\frac{nv}{4\ell} < 2\times10^4\,\text{Hz} \rightarrow n < \frac{4(2.48\,\text{m})\left(2\times10^4\,\text{Hz}\right)}{343\,\text{m/s}} = 578.4$$

The values of *n* must be odd, so *n* = 1, 3, 5, …, 577. There are 289 harmonics, and so there are $\boxed{\text{288 overtones}}$.

49. A tube closed at both ends will have standing waves with displacement nodes at each end, and so has the same harmonic structure as a string that is fastened at both ends. Thus the wavelength of the fundamental frequency is twice the length of the hallway, $\lambda_1 = 2\ell = 16.0\,\text{m}$.

$$f_1 = \frac{v}{\lambda_1} = \frac{343\,\text{m/s}}{16.0\,\text{m}} = \boxed{21.4\,\text{Hz}} \;\; ; \; f_2 = 2f_1 = \boxed{42.8\,\text{Hz}}$$

50. To operate with the first harmonic, we see from the figure that the thickness must be half of a wavelength, so the wavelength is twice the thickness. The speed of sound in the quartz is given by $v = \sqrt{G/\rho}$, analogous to Eqs. 15-3 and 15-4.

$$t = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{v}{f} = \tfrac{1}{2}\frac{\sqrt{G/\rho}}{f} = \tfrac{1}{2}\frac{\sqrt{\left(2.95\times10^{10}\,\text{N/m}^2\right)/\left(2650\,\text{kg/m}^2\right)}}{12.0\times10^6\,\text{Hz}} = \boxed{1.39\times10^{-4}\,\text{m}}$$

51. The ear canal can be modeled as a closed pipe of length 2.5 cm. The resonant frequencies are given by $f_n = \frac{nv}{4\ell}$, n odd. The first several frequencies are calculated here.

$$f_n = \frac{nv}{4\ell} = \frac{n\left(343\,\text{m/s}\right)}{4\left(2.5\times10^{-2}\,\text{m}\right)} = n\left(3430\text{ Hz}\right),\, n\text{ odd}$$

$$\boxed{f_1 = 3430\text{ Hz} \quad f_3 = 10{,}300\text{ Hz} \quad f_5 = 17{,}200\text{ Hz}}$$

In the graph, the most sensitive frequency is between 3000 and 4000 Hz. This corresponds to the fundamental resonant frequency of the ear canal. The sensitivity decrease above 4000 Hz, but is seen to “flatten out” around 10,000 Hz again, indicating higher sensitivity near 10,000 Hz than at surrounding frequencies. This 10,000 Hz relatively sensitive region corresponds to the first overtone resonant frequency of the ear canal.

52. From Eq. 15-7, the intensity is proportional to the square of the amplitude and the square of the frequency. From Fig. 16-14, the relative amplitudes are $\frac{A_2}{A_1} \approx 0.4$ and $\frac{A_3}{A_1} \approx 0.15$.

$$I = 2\pi^2 v\rho f^2 A^2 \;\rightarrow\; \frac{I_2}{I_1} = \frac{2\pi^2 v\rho f_2^2 A_2^2}{2\pi^2 v\rho f_1^2 A_1^2} = \frac{f_2^2 A_2^2}{f_1^2 A_1^2} = \left(\frac{f_2}{f_1}\right)^2\left(\frac{A_2}{A_1}\right)^2 = 2^2\left(0.4\right)^2 = \boxed{0.64}$$

$$\frac{I_3}{I_1} = \left(\frac{f_3}{f_1}\right)^2\left(\frac{A_3}{A_1}\right)^2 = 3^2\left(0.15\right)^2 = \boxed{0.20}$$

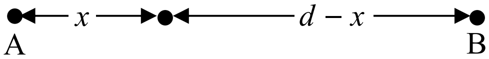
$$\beta_{2-1} = 10\log\frac{I_2}{I_1} = 10\log 0.64 = \boxed{-2\,\text{dB}} \;\; ; \; \beta_{3-1} = 10\log\frac{I_3}{I_1} = 10\log 0.24 = \boxed{-7\,\text{dB}}$$

53. The beat period is 2.0 seconds, so the beat frequency is the reciprocal of that, 0.50 Hz. Thus the other string is off in frequency by $\boxed{\pm0.50\text{ Hz}}$. The beating does not tell the tuner whether the second string is too high or too low.

54. The beat frequency is the difference in the two frequencies, or $277\text{ Hz} - 262\text{ Hz} = \boxed{15\text{ Hz}}$. If the frequencies are both reduced by a factor of 4, then the difference between the two frequencies will also be reduced by a factor of 4, and so the beat frequency will be $\tfrac{1}{4}\left(15\text{ Hz}\right) = 3.75\,\text{Hz} \approx \boxed{3.8\text{ Hz}}$.

$\boxed{55.}$ Since there are 4 beats/s when sounded with the 350 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 354 Hz. Since there are 9 beats/s when sounded with the 355 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 364 Hz. The common value is $\boxed{346\text{ Hz}}$.

56. (*a*) Since the sounds are initially 180° out of phase, another 180° of phase must be added by a path length difference. Thus the difference of the distances from the speakers to the point of constructive interference must be half of a wavelength. See the diagram.



$$\left(d-x\right)-x=\tfrac{1}{2}\lambda \;\rightarrow\; d=2x+\tfrac{1}{2}\lambda \;\rightarrow\; d_{\text{min}}=\tfrac{1}{2}\lambda=\frac{v}{2f}=\frac{343\text{ m/s}}{2\left(294\text{ Hz}\right)}=\boxed{0.583\text{ m}}$$

This minimum distance occurs when the observer is right at one of the speakers. If the speakers are separated by more than 0.583 m, the location of constructive interference will be moved away from the speakers, along the line between the speakers.

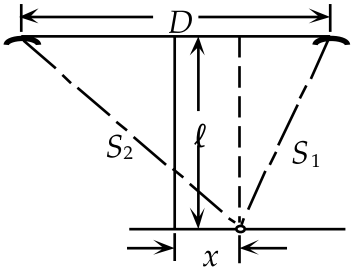
(*b*) Since the sounds are already 180° out of phase, as long as the listener is equidistant from the speakers, there will be completely destructive interference. So even if the speakers have a tiny separation, the point midway between them will be a point of completely destructive interference. The minimum separation between the speakers is $\boxed{0}$.

57. Beats will be heard because the difference in the speed of sound for the two flutes will result in two different frequencies.

$$f_1=\frac{v_1}{2\ell}=\frac{\left[331+0.60\left(28\right)\right]\text{m/s}}{2\left(0.66\text{ m}\right)}=263.4\text{ Hz}$$

$$f_2=\frac{v_2}{2\ell}=\frac{\left[331+0.60\left(5.0\right)\right]\text{m/s}}{2\left(0.66\text{ m}\right)}=253.0\text{ Hz} \qquad \Delta f=263.4\text{ Hz}-253.0\text{ Hz}=\boxed{10\text{ beats/sec}}$$

58. (*a*) The microphone must be moved to the right until the difference in distances from the two sources is half a wavelength. See the diagram. We square the expression, collect terms, isolate the remaining square root, and square again.



$$S_2-S_1=\tfrac{1}{2}\lambda \;\rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D+x\right)^2+\ell^2}-\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2}=\tfrac{1}{2}\lambda \;\rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D+x\right)^2+\ell^2}=\tfrac{1}{2}\lambda+\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2} \;\rightarrow$$

$$\left(\tfrac{1}{2}D+x\right)^2+\ell^2=\tfrac{1}{4}\lambda^2+2\left(\tfrac{1}{2}\lambda\right)\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2}+\left(\tfrac{1}{2}D-x\right)^2+\ell^2 \;\rightarrow$$

$$2Dx-\tfrac{1}{4}\lambda^2=\lambda\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2} \;\rightarrow\; 4D^2x^2-2\left(2Dx\right)\tfrac{1}{4}\lambda^2+\tfrac{1}{16}\lambda^4=\lambda^2\left[\left(\tfrac{1}{2}D-x\right)^2+\ell^2\right]$$

$$4D^2x^2-Dx\lambda^2+\tfrac{1}{16}\lambda^4=\tfrac{1}{4}D^2\lambda^2-Dx\lambda^2+x^2\lambda^2+\lambda^2\ell^2 \;\rightarrow\; x=\lambda\sqrt{\frac{\left(\tfrac{1}{4}D^2+\ell^2-\tfrac{1}{16}\lambda^2\right)}{\left(4D^2-\lambda^2\right)}}$$

The values are $D=3.00\text{ m}$, $\ell=3.20\text{ m}$, and $\lambda=v/f=\left(343\text{ m/s}\right)/\left(494\text{ Hz}\right)=0.694\text{ m}$.

$$x=\left(0.694\text{ m}\right)\sqrt{\frac{\tfrac{1}{4}\left(3.00\text{ m}\right)^2+\left(3.20\text{ m}\right)^2-\tfrac{1}{16}\left(0.694\text{ m}\right)^2}{4\left(3.00\text{ m}\right)^2-\left(0.694\text{ m}\right)^2}}=\boxed{0.411\text{ m}}$$

(*b*) When the speakers are exactly out of phase, the maxima and minima will be interchanged. The intensity $\boxed{\text{maxima are 0.411 m to the left or right of the midpoint}}$, and the intensity $\boxed{\text{minimum is at the midpoint}}$.

59. The beat frequency is 3 beats per 2 seconds, or 1.5 Hz. We assume the strings are the same length and the same mass density.

(*a*) The other string must be either $220.0\,\text{Hz} - 1.5\,\text{Hz} = \boxed{218.5\,\text{Hz}}$ or $220.0\,\text{Hz} + 1.5\,\text{Hz} = \boxed{221.5\text{ Hz}}$.

(*b*) Since $f = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_\text{T}}{\mu}}$, we have $f \propto \sqrt{F_\text{T}} \;\rightarrow\; \dfrac{f}{\sqrt{F_\text{T}}} = \dfrac{f'}{\sqrt{F'_\text{T}}} \;\rightarrow\; F'_\text{T} = F_\text{T}\left(\dfrac{f'}{f}\right)^2$.

To change 218.5 Hz to 220.0 Hz: $F' = F_\text{T}\left(\dfrac{220.0}{218.5}\right)^2 = 1.014$, $\boxed{\text{1.4\% increase}}$.

To change 221.5 Hz to 220.0 Hz: $F' = F_\text{T}\left(\dfrac{220.0}{221.5}\right)^2 = 0.9865$, $\boxed{\text{1.3\% decrease}}$.

60. (*a*) To find the beat frequency, calculate the frequency of each sound, and then subtract the two frequencies.

$$f_\text{beat} = \left|f_1 - f_2\right| = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right| = \left(343\,\text{m/s}\right)\left|\frac{1}{2.64\,\text{m}} - \frac{1}{2.72\,\text{m}}\right| = 3.821\,\text{Hz} \approx \boxed{4\,\text{Hz}}$$

(*b*) The speed of sound is 343 m/s, and the beat frequency is 3.821 Hz. The regions of maximum intensity are one "beat wavelength" apart.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{3.821\,\text{Hz}} = 89.79\,\text{m} \approx \boxed{90\,\text{m}}\left(2\text{ sig. fig.}\right)$$

$\boxed{61.}$ (*a*) Observer moving towards stationary source.

$$f' = \left(1 + \frac{v_\text{obs}}{v_\text{snd}}\right)f = \left(1 + \frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)\left(1350\text{ Hz}\right) = \boxed{1470\text{ Hz}}$$

(*b*) Observer moving away from stationary source.

$$f' = \left(1 - \frac{v_\text{obs}}{v_\text{snd}}\right)f = \left(1 - \frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)\left(1350\text{ Hz}\right) = \boxed{1230\text{ Hz}}$$

62. The moving object can be treated as a moving "observer" for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$f'_\text{object} = f_\text{bat}\left(1 - \frac{v_\text{object}}{v_\text{snd}}\right)$$

Then the object can be treated as a moving source emitting the frequency f'_object, and the bat as a stationary observer.

$$f''_\text{bat} = \frac{f'_\text{object}}{\left(1 + \dfrac{v_\text{object}}{v_\text{snd}}\right)} = f_\text{bat}\frac{\left(1 - \dfrac{v_\text{object}}{v_\text{snd}}\right)}{\left(1 + \dfrac{v_\text{object}}{v_\text{snd}}\right)} = f_\text{bat}\frac{\left(v_\text{snd} - v_\text{object}\right)}{\left(v_\text{snd} + v_\text{object}\right)}$$

$$= \left(5.00\times10^4\,\text{Hz}\right)\frac{343\,\text{m/s}-30.0\,\text{m/s}}{343\,\text{m/s}+30.0\,\text{m/s}} = \boxed{4.20\times10^4\,\text{Hz}}$$

63. (*a*) For the 18 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\text{ Hz}\right)\frac{1}{\left(1-\dfrac{18\,\text{m/s}}{343\,\text{m/s}}\right)} = 2427\,\text{Hz} \approx \boxed{2430\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\text{ Hz}\right)\left(1+\frac{18\,\text{m/s}}{343\,\text{m/s}}\right) = 2421\,\text{Hz} \approx \boxed{2420\,\text{Hz}}$$

The frequency shifts are slightly different, with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$. The two frequencies are close, but they are not identical. As a means of comparison, calculate the spread in frequencies divided by the original frequency.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{2427\,\text{Hz}-2421\,\text{Hz}}{2300\,\text{Hz}} = 0.0026 = 0.26\%$$

(*b*) For the 160 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\text{ Hz}\right)\frac{1}{\left(1-\dfrac{160\,\text{m/s}}{343\,\text{m/s}}\right)} = 4311\,\text{Hz} \approx \boxed{4310\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\text{ Hz}\right)\left(1+\frac{160\,\text{m/s}}{343\,\text{m/s}}\right) = 3372\text{ Hz} \approx \boxed{3370\,\text{Hz}}$$

The difference in the frequency shifts is much larger this time, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{4311\,\text{Hz}-3372\,\text{Hz}}{2300\,\text{Hz}} = 0.4083 = 41\%$$

(*c*) For the 320 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\text{ Hz}\right)\frac{1}{\left(1-\dfrac{320\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{34{,}300\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\text{ Hz}\right)\left(1+\frac{320\,\text{m/s}}{343\,\text{m/s}}\right) = 4446\,\text{Hz} \approx \boxed{4450\,\text{Hz}}$$

The difference in the frequency shifts is quite large, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{34{,}300\,\text{Hz}-4446\,\text{Hz}}{2300\,\text{Hz}} = 12.98 = 1300\%$$

(*d*) The Doppler formulas are asymmetric, with a larger shift for the moving source than for the moving observer, when the two are getting closer to each other. In the following derivation, assume $v_{\text{src}} \ll v_{\text{snd}}$, and use the binomial expansion.

$$f'_{\substack{\text{source}\\ \text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = f\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)^{-1} \approx f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = f'_{\substack{\text{observer}\\ \text{moving}}}$$

64. The frequency received by the stationary car is higher than the frequency emitted by the stationary car, by $\Delta f = 4.5\text{ Hz}$.

$$f_{\text{obs}} = f_{\text{source}} + \Delta f = \frac{f_{\text{source}}}{\left(1-\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} \quad \rightarrow$$

$$f_{\text{source}} = \Delta f\left(\frac{v_{\text{snd}}}{v_{\text{source}}}-1\right) = (4.5\text{ Hz})\left(\frac{343\text{ m/s}}{15\text{ m/s}}-1\right) = \boxed{98\text{ Hz}}$$

65. (*a*) The observer is stationary, and the source is moving. First the source is approaching, then the source is receding.

$$120.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s}$$

$$f'_{\substack{\text{source}\\ \text{moving}\\ \text{towards}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = (1280\text{ Hz})\frac{1}{\left(1-\dfrac{33.33\text{ m/s}}{343\text{ m/s}}\right)} = \boxed{1420\text{ Hz}}$$

$$f'_{\substack{\text{source}\\ \text{moving}\\ \text{away}}} = f\frac{1}{\left(1+\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = (1280\text{ Hz})\frac{1}{\left(1+\dfrac{33.33\text{ m/s}}{343\text{ m/s}}\right)} = \boxed{1170\text{ Hz}}$$

(*b*) Both the observer and the source are moving, and so use Eq. 16-11.

$$90.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 25\text{ m/s}$$

$$f'_{\text{approaching}} = f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}+25\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})} = \boxed{1520\text{ Hz}}$$

$$f'_{\text{receding}} = f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}-25\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})} = \boxed{1080\text{ Hz}}$$

(*c*) Both the observer and the source are moving, and so again use Eq. 16-11.

$$80.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 22.22\text{ m/s}$$

$$f'_{\substack{\text{police}\\ \text{car}\\ \text{approaching}}} = f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}-22.22\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})} = \boxed{1330\text{ Hz}}$$

$$f'_{\substack{\text{police}\\ \text{car}\\ \text{receding}}} = f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}+22.22\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})} = \boxed{1240\text{ Hz}}$$

66. The wall can be treated as a stationary "observer" for calculating the frequency it receives. The bat is flying toward the wall.

$$f'_{\text{wall}} = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}$$

Then the wall can be treated as a stationary source emitting the frequency f'_{wall}, and the bat as a moving observer, flying toward the wall.

$$f''_{\text{bat}} = f'_{\text{wall}}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{(v_{\text{snd}} + v_{\text{bat}})}{(v_{\text{snd}} - v_{\text{bat}})}$$

$$= \left(3.00\times10^4\,\text{Hz}\right)\frac{343\,\text{m/s} + 7.0\,\text{m/s}}{343\,\text{m/s} - 7.0\,\text{m/s}} = \boxed{3.13\times10^4\,\text{Hz}}$$

$\boxed{67.}$ We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$f_{\text{obs}} = \frac{f_{\text{source}}}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{75\,\text{Hz}}{\left(1 - \frac{12.0\,\text{m/s}}{343\,\text{m/s}}\right)} = 78\,\text{Hz} \qquad f_{\text{beat}} = 78\,\text{Hz} - 75\,\text{Hz} = \boxed{3\,\text{Hz}}.$$

68. For the sound to be shifted up by one note, we must have $f'_{\substack{\text{source}\\\text{moving}}} = f\left(2^{1/12}\right)$.

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1 - \frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = f\left(2^{1/12}\right) \rightarrow$$

$$v_{\text{src}} = \left(1 - \frac{1}{2^{1/12}}\right)v_{\text{snd}} = \left(1 - \frac{1}{2^{1/12}}\right)(343\,\text{m/s}) = 19.25\,\text{m/s}\left(\frac{3.6\,\text{km/h}}{\text{m/s}}\right) = \boxed{69.3\,\text{km/h}}$$

69. The ocean wave has $\lambda = 44\,\text{m}$ and $v = 18\,\text{m/s}$ relative to the ocean floor. The frequency of the ocean wave is then $f = \frac{v}{\lambda} = \frac{18\,\text{m/s}}{44\,\text{m}} = 0.409\,\text{Hz}.$

(*a*) For the boat traveling west, the boat will encounter a Doppler shifted frequency, for an observer moving towards a stationary source. The speed $v = 18\,\text{m/s}$ represents the speed of the waves in the stationary medium, and so corresponds to the speed of sound in the Doppler formula. The time between encountering waves is the period of the Doppler shifted frequency.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 + \frac{15\,\text{m/s}}{18\,\text{m/s}}\right)(0.409\,\text{Hz}) = 0.750\,\text{Hz} \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.750\,\text{Hz}} = \boxed{1.3\,\text{s}}$$

(*b*) For the boat traveling east, the boat will encounter a Doppler shifted frequency, for an observer moving away from a stationary source.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1-\frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1-\frac{15\,\text{m/s}}{18\,\text{m/s}}\right)(0.409\,\text{Hz}) = 0.0682\,\text{Hz} \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.0682\,\text{Hz}} = \boxed{15\,\text{s}}$$

70. The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other and so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, as if the speed of sound were different, but the frequency of the waves doesn't change. We do a detailed analysis of this claim in part (*a*).

(*a*) The wind velocity is a movement of the medium, and so adds or subtracts from the speed of sound in the medium. Because the wind is blowing away from the observer, the effective speed of sound is $v_{\text{snd}} - v_{\text{wind}}$. The wavelength of the waves traveling towards the observer is $\lambda_a = (v_{\text{snd}} - v_{\text{wind}})/f_0$, where f_0 is the frequency emitted by the factory whistle. This wavelength approaches the observer at a relative speed of $v_{\text{snd}} - v_{\text{wind}}$. Thus the observer hears the frequency calculated here.

$$f_a = \frac{v_{\text{snd}} - v_{\text{wind}}}{\lambda_a} = \frac{v_{\text{snd}} - v_{\text{wind}}}{\dfrac{v_{\text{snd}} - v_{\text{wind}}}{f_0}} = f_0 = \boxed{720\,\text{Hz}}$$

(*b*) Because the wind is blowing towards the observer, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The same kind of analysis as applied in part (a) gives that $f_b = \boxed{720\,\text{Hz}}$.

(*c*) Because the wind is blowing perpendicular to the line towards the observer, the effective speed of sound along that line is v_{snd}. Since there is no relative motion of the whistle and observer, there will be no change in frequency, and so $f_c = \boxed{720\,\text{Hz}}$.

(*d*) This is just like part (*c*), and so $f_d = \boxed{720\,\text{Hz}}$.

(*e*) Because the wind is blowing toward the cyclist, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The wavelength traveling toward the cyclist is $\lambda_e = (v_{\text{snd}} + v_{\text{wind}})/f_0$. This wavelength approaches the cyclist at a relative speed of $v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}}$. The cyclist will hear the following frequency.

$$f_e = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{\lambda_e} = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{(v_{\text{snd}} + v_{\text{wind}})}f_0 = \frac{(343+15.0+12.0)\,\text{m/s}}{(343+15.0)}(720\,\text{Hz})$$
$$= \boxed{744\,\text{Hz}}$$

(*f*) Since the wind is not changing the speed of the sound waves moving towards the cyclist, the speed of sound is 343 m/s. The observer is moving towards a stationary source with a speed of 12.0 m/s.

$$f' = f\left(1+\frac{v_{\text{obs}}}{v_{\text{sns}}}\right) = (720\text{ Hz})\left(1+\frac{12.0\,\text{m/s}}{343\,\text{m/s}}\right) = \boxed{745\,\text{Hz}}$$

71. The maximum Doppler shift occurs when the heart has its maximum velocity. Assume that the heart is moving away from the original source of sound. The beats arise from the combining of the original 2.25 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the heart receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{heart}} = f_{\text{original}}\left(1-\frac{v_{\text{heart}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{heart}}}{\left(1+\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1-\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)}{\left(1+\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{heart}}\right)}{\left(v_{\text{snd}}+v_{\text{heart}}\right)}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{\left(v_{\text{snd}}-v_{\text{blood}}\right)}{\left(v_{\text{snd}}+v_{\text{blood}}\right)} = f_{\text{original}}\frac{2v_{\text{blood}}}{\left(v_{\text{snd}}+v_{\text{blood}}\right)} \quad \rightarrow$$

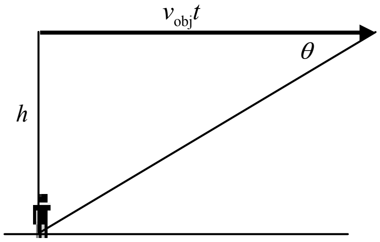
$$v_{\text{blood}} = v_{\text{snd}}\frac{\Delta f}{2f_{\text{original}}-\Delta f} = \left(1.54\times10^{3}\ \text{m/s}\right)\frac{260\,\text{Hz}}{2\left(2.25\times10^{6}\,\text{Hz}\right)-260\,\text{Hz}} = \boxed{8.9\times10^{-2}\ \text{m/s}}$$

If instead we had assumed that the heart was moving towards the original source of sound, we would get $v_{\text{blood}} = v_{\text{snd}}\dfrac{\Delta f}{2f_{\text{original}}+\Delta f}$. Since the beat frequency is much smaller than the original frequency, the Δf term in the denominator does not significantly affect the answer.

72. (*a*) The angle of the shock wave front relative to the direction of motion is given by Eq. 16-12.

$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}} = \frac{v_{\text{snd}}}{2.0v_{\text{snd}}} = \frac{1}{2.0} \quad\rightarrow\quad \theta = \sin^{-1}\frac{1}{2.0} = \boxed{30^{\circ}}\ \text{(2 sig. fig.)}$$

(*b*) The displacement of the plane $\left(v_{\text{obj}}t\right)$ from the time it passes overhead to the time the shock wave reaches the observer is shown, along with the shock wave front. From the displacement and height of the plane, the time is found.



$$\tan\theta = \frac{h}{v_{\text{obj}}t} \quad\rightarrow\quad t = \frac{h}{v_{\text{obj}}\tan\theta}$$

$$= \frac{6500\ \text{m}}{(2.0)(310\,\text{m/s})\tan 30^{\circ}} = \boxed{18\,\text{s}}$$

$\boxed{73.}$ (*a*) The Mach number is the ratio of the object's speed to the speed of sound.

$$M = \frac{v_{\text{obs}}}{v_{\text{sound}}} = \frac{\left(1.5\times10^{4}\ \text{km/hr}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right)}{45\,\text{m/s}} = 92.59 \approx \boxed{93}$$

(*b*) Use Eq. 16-125 to find the angle.

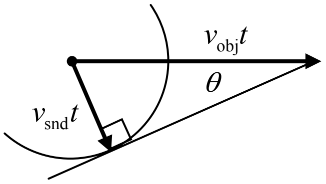
$$\theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{1}{M} = \sin^{-1}\frac{1}{92.59} = \boxed{0.62^{\circ}}$$

74. From Eq. 16-12, $\sin\theta = \dfrac{v_{\text{snd}}}{v_{\text{obj}}}$.

(*a*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{343\,\text{m/s}}{8800\,\text{m/s}} = \boxed{2.2^\circ}$

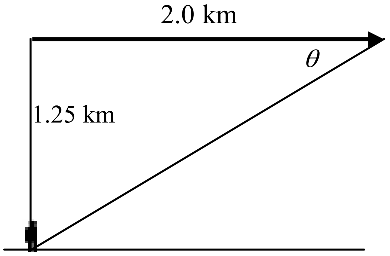
(*b*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{1560\,\text{m/s}}{8800\,\text{m/s}} = \boxed{10^\circ}$ (2 sig. fig.)

75. Consider one particular wave as shown in the diagram, created at the location of the black dot. After a time *t* has elapsed from the creation of that wave, the supersonic source has moved a distance $v_{\text{obj}}t$, and the wave front has moved a distance $v_{\text{snd}}t$. The line from the position of the source at time *t* is tangent to all of the wave fronts, showing the location of the shock wave. A tangent to a circle at a point is perpendicular to the radius connecting that point to the center, and so a right angle is formed. From the right triangle, the angle θ can be defined.



$$\boxed{\sin\theta = \frac{v_{\text{snd}}t}{v_{\text{obj}}t} = \frac{v_{\text{snd}}}{v_{\text{obj}}}}$$

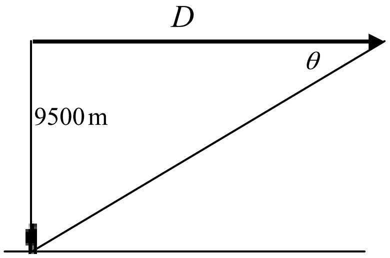
76. (*a*) The displacement of the plane from the time it passes overhead to the time the shock wave reaches the listener is shown, along with the shock wave front. From the displacement and height of the plane, the angle of the shock wave front relative to the direction of motion can be found. Then use Eq. 16-12.



$$\tan\theta = \frac{1.25\text{ km}}{2.0\text{ km}} \rightarrow \theta = \tan^{-1}\frac{1.25}{2.0} = \boxed{32^\circ}$$

(*b*) $M = \dfrac{v_{\text{obj}}}{v_{\text{snd}}} = \dfrac{1}{\sin\theta} = \dfrac{1}{\sin 32^\circ} = \boxed{1.9}$

77. Find the angle of the shock wave, and then find the distance the plane has traveled when the shock wave reaches the observer. Use Eq. 16-12.



$$\theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{v_{\text{snd}}}{2.2v_{\text{snd}}} = \sin^{-1}\frac{1}{2.2} = 27^\circ$$

$$\tan\theta = \frac{9500\,\text{m}}{D} \rightarrow D = \frac{9500\,\text{m}}{\tan 27^\circ} = 18616\,\text{m} = \boxed{19\,\text{km}}$$

78. The minimum time between pulses would be the time for a pulse to travel from the boat to the maximum distance and back again. The total distance traveled by the pulse will be 150 m, at the speed of sound in fresh water, 1440 m/s.

$$d = vt \rightarrow t = \frac{d}{v} = \frac{150\,\text{m}}{1440\,\text{m/s}} = \boxed{0.10\,\text{s}}$$

$\boxed{79.}$ Assume that only the fundamental frequency is heard. The fundamental frequency of an open pipe is given by $f=\dfrac{v}{2L}$.

(*a*) $f_{3.0}=\dfrac{v}{2L}=\dfrac{343\,\text{m/s}}{2(3.0\text{ m})}=\boxed{57\text{ Hz}}$ $\qquad f_{2.5}=\dfrac{v}{2L}=\dfrac{343\,\text{m/s}}{2(2.5\text{ m})}=\boxed{69\text{ Hz}}$

$f_{2.0}=\dfrac{v}{2L}=\dfrac{343\,\text{m/s}}{2(2.0\text{ m})}=\boxed{86\text{ Hz}}$ $\qquad f_{1.5}=\dfrac{v}{2L}=\dfrac{343\,\text{m/s}}{2(1.5\text{ m})}=114.3\,\text{Hz}\approx\boxed{110\text{ Hz}}$

$f_{1.0}=\dfrac{v}{2L}=\dfrac{343\,\text{m/s}}{2(1.0\text{ m})}=171.5\text{ Hz}\approx\boxed{170\text{ Hz}}$

(*b*) On a noisy day, there are a large number of component frequencies to the sounds that are being made – more people walking, more people talking, etc. Thus it is more likely that the frequencies listed above will be a component of the overall sound, and then the resonance will be more prominent to the hearer. If the day is quiet, there might be very little sound at the desired frequencies, and then the tubes will not have any standing waves in them to detect.

80. The single mosquito creates a sound intensity of $I_0=1\times10^{-12}\text{ W/m}^2$. Thus 100 mosquitoes will create a sound intensity of 100 times that of a single mosquito.

$$I=100I_0 \qquad \beta=10\log\frac{100I_0}{I_0}=10\log100=\boxed{20\,\text{dB}}\,.$$

81. The two sound level values must be converted to intensities, then the intensities added, and then converted back to sound level.

$$I_{82}:\ \ 82\,\text{dB}=10\log\frac{I_{82}}{I_0}\ \rightarrow\ I_{82}=10^{8.2}I_0=1.585\times10^8I_0$$

$$I_{89}:\ \ 89\,\text{dB}=10\log\frac{I_{87}}{I_0}\ \rightarrow\ I_{89}=10^{8.9}I_0=7.943\times10^8I_0$$

$$I_{\text{total}}=I_{82}+I_{89}=\left(9.528\times10^8\right)I_0\ \rightarrow$$

$$\beta_{\text{total}}=10\log\frac{9.528\times10^8I_0}{I_0}=10\log6.597\times10^8=89.8\,\text{dB}\approx\boxed{90\,\text{dB}}\ \left(2\text{ sig. fig.}\right)$$

82. The power output is found from the intensity, which is the power radiated per unit area.

$$115\text{ dB}=10\log\frac{I}{I_0}\ \rightarrow\ I=10^{11.5}I_0=10^{11.5}\left(1.0\times10^{-12}\text{ W/m}^2\right)=3.162\times10^{-1}\text{ W/m}^2$$

$$I=\frac{P}{A}=\frac{P}{4\pi r^2}\ \rightarrow\ P=4\pi r^2I=4\pi(9.00\text{ m})^2\left(3.162\times10^{-1}\text{ W/m}^2\right)=\boxed{322\,\text{W}}$$

83. Relative to the 1000 Hz output, the 15 kHz output is –12 dB.

$$-12\text{ dB}=10\log\frac{P_{15\,\text{kHz}}}{175\,\text{W}}\ \rightarrow\ -1.2=\log\frac{P_{15\,\text{kHz}}}{175\text{ W}}\ \rightarrow\ 10^{-1.2}=\frac{P_{15\,\text{kHz}}}{175\,\text{W}}\ \rightarrow\ P_{15\,\text{kHz}}=\boxed{11\,\text{W}}$$

84. The 130 dB level is used to find the intensity, and the intensity is used to find the power. It is assumed that the jet airplane engine radiates equally in all directions.

$$\beta = 130\,\text{dB} = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{13}I_0 = 10^{13}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 1.0\times10^{1}\ \text{W/m}^2$$

$$P = IA = I\pi r^2 = \left(1.0\times10^{1}\ \text{W/m}^2\right)\pi\left(2.0\times10^{-2}\right)^2 = \boxed{0.013\,\text{W}}$$

$\boxed{85.}$ The gain is given by $\beta = 10\log\frac{P_{\text{out}}}{P_{\text{in}}} = 10\log\frac{125\ \text{W}}{1.0\times10^{-3}\,\text{W}} = \boxed{51\,\text{dB}}$.

86. It is desired that the sound from the speaker arrives at a listener 30 ms after the sound from the singer arrives. The fact that the speakers are 3.0 m behind the singer adds in a delay of $\frac{3.0\,\text{m}}{343\,\text{m/s}} = 8.7\times10^{-3}\text{s}$, or about 9 ms. Thus there must be $\boxed{21\text{ ms}}$ of delay added into the electronic circuitry.

87. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f = \frac{v}{2\ell}\ ;\ v = \sqrt{\frac{F_{\text{T}}}{\mu}} \quad\rightarrow\quad f = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\rho\pi r^2}} \quad\rightarrow\quad F_{\text{T}} = 4\ell^2\rho f^2\pi r^2 \quad\rightarrow$$

$$\frac{F_{\text{T high}}}{F_{\text{T low}}} = \frac{4\ell^2\rho f^2\pi r_{\text{high}}^2}{4\ell^2\rho f^2\pi r_{\text{low}}^2} = \left(\frac{r_{\text{high}}}{r_{\text{low}}}\right)^2 = \left(\frac{\frac{1}{2}d_{\text{high}}}{\frac{1}{2}d_{\text{low}}}\right)^2 = \left(\frac{0.724\,\text{mm}}{0.699\,\text{mm}}\right)^2 = \boxed{1.07}$$

88. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f = \frac{v}{2\ell}\ ;\ v = \sqrt{\frac{F_{\text{T}}}{\mu}} \quad\rightarrow\quad f = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\rho\pi r^2}} \quad\rightarrow\quad F_{\text{T}} = 4\ell^2\rho f^2\pi r^2 \quad\rightarrow$$

$$\frac{F_{\text{T acoustic}}}{F_{\text{T electric}}} = \frac{4\ell^2\rho_{\text{acoustic}}f^2\pi r_{\text{acoustic}}^2}{4\ell^2\rho_{\text{electric}}f^2\pi r_{\text{electric}}^2} = \frac{\rho_{\text{acoustic}}r_{\text{acoustic}}^2}{\rho_{\text{electric}}r_{\text{electric}}^2} = \left(\frac{\rho_{\text{acoustic}}}{\rho_{\text{electric}}}\right)\left(\frac{d_{\text{acoustic}}}{d_{\text{electric}}}\right)^2$$

$$= \left(\frac{7760\,\text{kg/m}^3}{7990\,\text{kg/m}^3}\right)\left(\frac{0.33\,\text{m}}{0.25\,\text{m}}\right)^2 = \boxed{1.7}$$

89. (*a*) The wave speed on the string can be found from the length and the fundamental frequency.

$$f = \frac{v}{2\ell} \quad\rightarrow\quad v = 2\ell f = 2\left(0.32\ \text{m}\right)\left(440\ \text{Hz}\right) = 281.6\,\text{m/s} \approx \boxed{280\,\text{m/s}}$$

The tension is found from the wave speed and the mass per unit length.

$$v = \sqrt{\frac{F_{\text{T}}}{\mu}} \quad\rightarrow\quad F_{\text{T}} = \mu v^2 = \left(7.21\times10^{-4}\ \text{kg/m}\right)\left(281.6\,\text{m/s}\right)^2 = \boxed{57\,\text{N}}$$

(*b*) The length of the pipe can be found from the fundamental frequency and the speed of sound.

$$f = \frac{v}{4\ell} \quad\rightarrow\quad \ell = \frac{v}{4f} = \frac{343\,\text{m/s}}{4\left(440\ \text{Hz}\right)} = 0.1949\,\text{m} \approx \boxed{0.19\,\text{m}}$$

(*c*) The first overtone for the string is twice the fundamental. $\boxed{880\text{ Hz}}$

The first overtone for the open pipe is 3 times the fundamental. $\boxed{1320\text{ Hz}}$

90. The apparatus is a closed tube. The water level is the closed end, and so is a node of air displacement. As the water level lowers, the distance from one resonance level to the next corresponds to the distance between adjacent nodes, which is one-half wavelength.

$$\Delta\ell = \tfrac{1}{2}\lambda \rightarrow \lambda = 2\Delta\ell = 2\left(0.395\,\text{m} - 0.125\,\text{m}\right) = 0.540\text{ m}$$

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{0.540\text{ m}} = \boxed{635\text{ Hz}}$$

$\boxed{91.}$ The fundamental frequency of a tube closed at one end is given by $f_1 = \dfrac{v}{4\ell}$. The change in air temperature will change the speed of sound, resulting in two different frequencies.

$$\frac{f_{30.0°\text{C}}}{f_{25.0°\text{C}}} = \frac{\dfrac{v_{30.0°\text{C}}}{4\ell}}{\dfrac{v_{25.0°\text{C}}}{4\ell}} = \frac{v_{30.0°\text{C}}}{v_{25.0°\text{C}}} \rightarrow f_{30.0°\text{C}} = f_{25.0°\text{C}}\left(\frac{v_{30.0°\text{C}}}{v_{25.0°\text{C}}}\right)$$

$$\Delta f = f_{30.0°\text{C}} - f_{25.0°\text{C}} = f_{25.0°\text{C}}\left(\frac{v_{30.0°\text{C}}}{v_{25.0°\text{C}}} - 1\right) = \left(349\,\text{Hz}\right)\left(\frac{331 + 0.60\left(30.0\right)}{331 + 0.60\left(25.0\right)} - 1\right) = \boxed{3\,\text{Hz}}$$

92. Call the frequencies of four strings of the violin f_A, f_B, f_C, f_D with f_A the lowest pitch. The mass per unit length will be named μ. All strings are the same length and have the same tension. For a string with both ends fixed, the fundamental frequency is given by $f_1 = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$.

$$f_B = 1.5 f_A \rightarrow \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_B}} = 1.5\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \rightarrow \mu_B = \frac{\mu_A}{\left(1.5\right)^2} = \boxed{0.44\mu_A}$$

$$f_C = 1.5 f_B = \left(1.5\right)^2 f_A \rightarrow \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_C}} = \left(1.5\right)^2\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \rightarrow \mu_C = \frac{\mu_A}{\left(1.5\right)^4} = \boxed{0.20\mu_A}$$

$$f_D = 1.5 f_C = \left(1.5\right)^3 f_A \rightarrow \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_D}} = \left(1.5\right)^3\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \rightarrow \mu_D = \frac{\mu_A}{\left(1.5\right)^6} = \boxed{0.088\mu_A}$$

93. The effective length of the tube is $\ell_{\text{eff.}} = \ell + \tfrac{1}{3}D = 0.60\,\text{m} + \tfrac{1}{3}\left(0.030\,\text{m}\right) = 0.61\,\text{m}$.

Uncorrected frequencies: $f_n = \dfrac{\left(2n-1\right)v}{4\ell}, n = 1, 2, 3\ldots \rightarrow$

$$f_{1-4} = \left(2n-1\right)\frac{343\,\text{m/s}}{4\left(0.60\,\text{m}\right)} = 143\,\text{Hz, }429\,\text{Hz, }715\,\text{Hz, }1000\,\text{Hz}$$

Corrected frequencies: $f_n = \dfrac{\left(2n-1\right)v}{4\ell_{\text{eff}}}, n = 1, 2, 3\ldots \rightarrow$

$$f_{1-4} = \left(2n-1\right)\frac{343\,\text{m/s}}{4\left(0.61\,\text{m}\right)} = \boxed{141\,\text{Hz, }422\,\text{Hz, }703\,\text{Hz, }984\,\text{Hz}}$$

94. Since the sound is loudest at points equidistant from the two sources, the two sources must be in phase. The difference in distance from the two sources must be an odd number of half-wavelengths for destructive interference.

$$0.28\,\text{m} = \lambda/2 \;\rightarrow\; \lambda = 0.56\,\text{m} \qquad f = v/\lambda = 343\,\text{m/s}/0.56\,\text{m} = \boxed{610\,\text{Hz}}$$

$$0.28\,\text{m} = 3\lambda/2 \;\rightarrow\; \lambda = 0.187\,\text{m} \qquad f = v/\lambda = 343\,\text{m/s}/0.187\,\text{m} = 1840\,\text{Hz}\;(\text{out of range})$$

95. As the train approaches, the observed frequency is given by $f'_{\text{approach}} = f\Big/\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. As the train recedes, the observed frequency is given by $f'_{\text{recede}} = f\Big/\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. Solve each expression for f, equate them, and then solve for v_{train}.

$$f'_{\text{approach}}\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right) = f'_{\text{recede}}\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right) \rightarrow$$

$$v_{\text{train}} = v_{\text{snd}}\frac{\left(f'_{\text{approach}} - f'_{\text{recede}}\right)}{\left(f'_{\text{approach}} + f'_{\text{recede}}\right)} = (343\,\text{m/s})\frac{(552\,\text{Hz} - 486\,\text{Hz})}{(552\,\text{Hz} + 486\,\text{Hz})} = \boxed{22\,\text{m/s}}$$

96. The Doppler shift is 3.5 Hz, and the emitted frequency from both trains is 516 Hz. Thus the frequency received by the conductor on the stationary train is 519.5 Hz. Use this to find the moving train's speed.

$$f' = f\frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{source}})} \;\rightarrow\; v_{\text{source}} = \left(1-\frac{f}{f'}\right)v_{\text{snd}} = \left(1-\frac{516\text{ Hz}}{519.5\text{ Hz}}\right)(343\,\text{m/s}) = \boxed{2.31\,\text{m/s}}$$

$\boxed{97.}$ (*a*) Since both speakers are moving towards the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears $\boxed{\text{no beats}}$.

(*b*) The observer will detect an increased frequency from the speaker moving towards him and a decreased frequency from the speaker moving away. The difference in those two frequencies will be the beat frequency that is heard.

$$f'_{\text{towards}} = f\frac{1}{\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} \qquad f'_{\text{away}} = f\frac{1}{\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)}$$

$$f'_{\text{towards}} - f'_{\text{away}} = f\frac{1}{\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} - f\frac{1}{\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} = f\left[\frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{train}})} - \frac{v_{\text{snd}}}{(v_{\text{snd}} + v_{\text{train}})}\right]$$

$$(348\text{ Hz})\left[\frac{343\,\text{m/s}}{(343\,\text{m/s} - 10.0\,\text{m/s})} - \frac{343\,\text{m/s}}{(343\,\text{m/s} + 10.0\,\text{m/s})}\right] = \boxed{20\,\text{Hz}}\,(2\text{ sig. fig.})$$

(*c*) Since both speakers are moving away from the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears $\boxed{\text{no beats}}$.

98. For each pipe, the fundamental frequency is given by $f=\frac{v}{2\ell}$. Find the frequency of the shortest pipe.

$$f=\frac{v}{2\ell}=\frac{343\,\mathrm{m/s}}{2(2.40\,\mathrm{m})}=71.46\,\mathrm{Hz}$$

The longer pipe has a lower frequency. Since the beat frequency is 8.0 Hz, the frequency of the longer pipe must be 63.46 Hz. Use that frequency to find the length of the longer pipe.

$$f=\frac{v}{2\ell}\quad\rightarrow\quad\ell=\frac{v}{2f}=\frac{343\,\mathrm{m/s}}{2(63.46\,\mathrm{Hz})}=\boxed{2.70\,\mathrm{m}}$$

99. Use Eq. 16-11, which applies when both source and observer are in motion. There will be two Doppler shifts in this problem – first for the emitted sound with the bat as the source and the moth as the observer, and then the reflected sound with the moth as the source and the bat as the observer.

$$f'_{\text{moth}}=f_{\text{bat}}\frac{(v_{\text{snd}}+v_{\text{moth}})}{(v_{\text{snd}}-v_{\text{bat}})}\qquad f''_{\text{bat}}=f'_{\text{moth}}\frac{(v_{\text{snd}}+v_{\text{bat}})}{(v_{\text{snd}}-v_{\text{moth}})}=f_{\text{bat}}\frac{(v_{\text{snd}}+v_{\text{moth}})}{(v_{\text{snd}}-v_{\text{bat}})}\frac{(v_{\text{snd}}+v_{\text{bat}})}{(v_{\text{snd}}-v_{\text{moth}})}$$

$$=(51.35\,\mathrm{kHz})\frac{(343+5.0)}{(343-7.5)}\frac{(343+7.5)}{(343-5.0)}=\boxed{55.23\,\mathrm{kHz}}$$

100. The beats arise from the combining of the original 3.80 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the blood cells receiving the original frequency (observer moving away from stationary source) and one for the detector receiving the reflected frequency (source moving away from stationary observer).

$$f'_{\text{blood}}=f_{\text{original}}\left(1-\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)\qquad f''_{\text{detector}}=\frac{f'_{\text{blood}}}{\left(1+\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}=f_{\text{original}}\frac{\left(1-\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1+\frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}=f_{\text{original}}\frac{(v_{\text{snd}}-v_{\text{blood}})}{(v_{\text{snd}}+v_{\text{blood}})}$$

$$\Delta f=f_{\text{original}}-f''_{\text{detector}}=f_{\text{original}}-f_{\text{original}}\frac{(v_{\text{snd}}-v_{\text{blood}})}{(v_{\text{snd}}+v_{\text{blood}})}=f_{\text{original}}\frac{2v_{\text{blood}}}{(v_{\text{snd}}+v_{\text{blood}})}$$

$$=(3.80\times10^{6}\,\mathrm{Hz})\frac{2(0.32\,\mathrm{m/s})}{(1.54\times10^{3}\,\mathrm{m/s}+0.32\,\mathrm{m/s})}=\boxed{1600\,\mathrm{Hz}}$$

101. It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus the time for the pulse to travel to the moth and back again is 67.0 ms. The distance to the moth is half the distance that the sound can travel in 67.0 ms, since the sound must reach the moth and return during the 67.0 ms.

$$d=v_{\text{snd}}t=(343\,\mathrm{m/s})\tfrac{1}{2}(67.0\times10^{-3}\,\mathrm{s})=\boxed{11.5\ \mathrm{m}}$$

102. (*a*) We assume that $v_{\text{src}}\ll v_{\text{snd}}$, and use the binomial expansion.

$$f'_{\substack{\text{source}\\\text{moving}}}=f\frac{1}{\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)}=f\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)^{-1}\approx f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right)=f'_{\substack{\text{observer}\\\text{moving}}}$$

(*b*) We calculate the percent error in general, and then substitute in the given relative velocity.

$$\%\,\text{error} = \left(\frac{\text{approx.} - \text{exact}}{\text{exact}}\right)100 = 100\left(\frac{f\left(1+\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right) - f\dfrac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}}{f\dfrac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}}\right)$$

$$= 100\left[\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right)\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)-1\right] = -100\left(\frac{v_{\text{src}}}{v_{\text{snd}}}\right)^2 = -100\left(\frac{18.0\,\text{m/s}}{343\,\text{m/s}}\right)^2 = \boxed{-0.28\%}$$

The negative sign indicates that the approximate value is less than the exact value.

103. The person will hear a frequency $f'_{\text{towards}} = f\left(1+\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk towards.

The person will hear a frequency $f'_{\text{away}} = f\left(1-\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk away from.

The beat frequency is the difference in those two frequencies.

$$f'_{\text{towards}} - f'_{\text{away}} = f\left(1+\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) - f\left(1-\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) = 2f\frac{v_{\text{walk}}}{v_{\text{snd}}} = 2\left(282\,\text{Hz}\right)\frac{1.4\,\text{m/s}}{343\,\text{m/s}} = \boxed{2.3\,\text{Hz}}$$

104. There will be two Doppler shifts in this problem – first for a stationary source with a moving "observer" (the blood cells), and then for a moving source (the blood cells) and a stationary "observer" (the receiver). Note that the velocity component of the blood parallel to the sound transmission is $v_{\text{blood}}\cos 45^\circ = \frac{1}{\sqrt{2}}v_{\text{blood}}$. It is that component that causes the Doppler shift.

$$f'_{\text{blood}} = f_{\text{original}}\left(1-\frac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)$$

$$f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1+\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1-\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1+\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}}-\frac{1}{\sqrt{2}}v_{\text{blood}}\right)}{\left(v_{\text{snd}}+\frac{1}{\sqrt{2}}v_{\text{blood}}\right)} \quad \rightarrow$$

$$v_{\text{blood}} = \sqrt{2}\,\frac{\left(f_{\text{original}} - f''_{\text{detector}}\right)}{\left(f''_{\text{detector}} + f_{\text{original}}\right)}v_{\text{snd}}$$

Since the cells are moving away from the transmitter / receiver combination, the final frequency received is less than the original frequency, by 780 Hz. Thus $f''_{\text{detector}} = f_{\text{original}} - 780\,\text{Hz}$.

$$v_{\text{blood}} = \sqrt{2}\,\frac{\left(f_{\text{original}} - f''_{\text{detector}}\right)}{\left(f''_{\text{detector}} + f_{\text{original}}\right)}v_{\text{snd}} = \sqrt{2}\,\frac{\left(780\,\text{Hz}\right)}{\left(2f_{\text{original}} - 780\,\text{Hz}\right)}v_{\text{snd}}$$

$$= \sqrt{2}\,\frac{\left(780\,\text{Hz}\right)}{\left[2\left(5.0\times10^{6}\,\text{Hz}\right) - 780\,\text{Hz}\right]}\left(1540\,\text{m/s}\right) = \boxed{0.17\,\text{m/s}}$$

105. The apex angle is 15°, so the shock wave angle is 7.5°. The angle of the shock wave is also given by $\sin\theta = v_{\text{wave}}/v_{\text{object}}$.

$$\sin\theta = v_{\text{wave}}/v_{\text{object}} \quad \rightarrow \quad v_{\text{object}} = v_{\text{wave}}/\sin\theta = 2.2\,\text{km/h}/\sin 7.5^\circ = \boxed{17\,\text{km/h}}$$

106. First, find the path difference in the original configuration. Then move the obstacle to the right by Δd so that the path difference increases by $\frac{1}{2}\lambda$. Note that the path difference change must be on the same order as the wavelength, and so $\Delta d \ll d,\ell$ since $\lambda \ll \ell, d$.

$$(\Delta D)_{\text{initial}} = 2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell \ ; \ (\Delta D)_{\text{final}} = 2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell$$

$$(\Delta D)_{\text{final}} - (\Delta D)_{\text{initial}} = \tfrac{1}{2}\lambda = \left(2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell\right) - \left(2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell\right) \rightarrow$$

$$2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} = \tfrac{1}{2}\lambda + 2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2}$$

Square the last equation above.

$$4\left[d^2 + 2d\Delta d + (\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2\right] = \tfrac{1}{4}\lambda^2 + 2\left(\tfrac{1}{2}\lambda\right)2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} + 4\left[d^2 + \left(\tfrac{1}{2}\ell\right)^2\right]$$

We delete terms that are second order in the small quantities Δd and λ.

$$8d\Delta d = 2\lambda\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} \quad \rightarrow \quad \boxed{\Delta d = \frac{\lambda}{4d}\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2}}$$

107. (*a*) The "singing" rod is manifesting standing waves. By holding the rod at its midpoint, it has a node at its midpoint, and antinodes at its ends. Thus the length of the rod is a half wavelength. The speed of sound in aluminum is found in Table 16-1.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{5100\,\text{m/s}}{1.50\text{ m}} = \boxed{3400\text{ Hz}}$$

(*b*) The wavelength of sound in the rod is twice the length of the rod, $\boxed{1.50\,\text{m}}$.

(*c*) The wavelength of the sound in air is determined by the frequency and the speed of sound in air.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{3400\,\text{Hz}} = \boxed{0.10\,\text{m}}$$

108. The displacement amplitude is related to the intensity by Eq. 15-7. The intensity can be calculated from the decibel value. The medium is air.

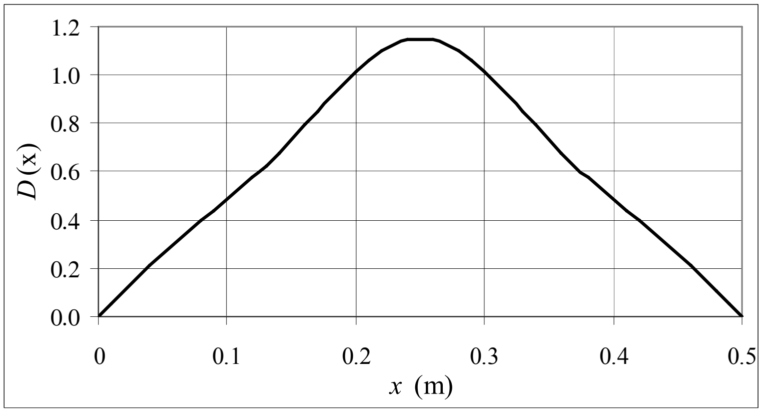
$$\beta = 10\log\frac{I}{I_0} \quad \rightarrow \quad I = \left(10^{\beta/10}\right)I_0 = 10^{10.5}\left(10^{-12}\text{ W/m}^2\right) = 0.0316\,\text{W/m}^2$$

(*a*) $I = 2\pi^2 v\rho f^2 A^2 \quad \rightarrow$

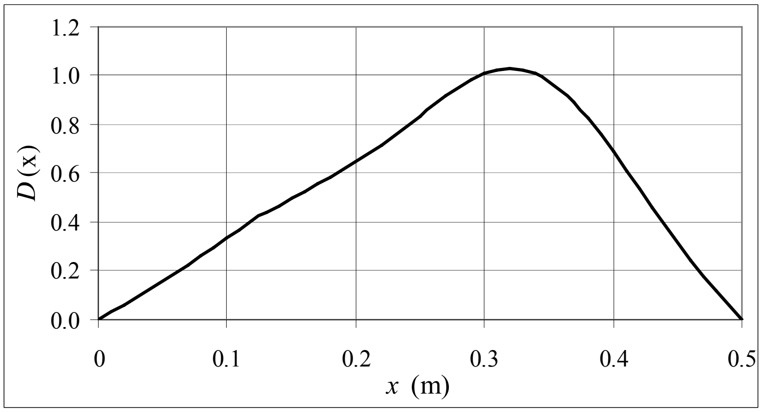
$$A = \frac{1}{\pi f}\sqrt{\frac{I}{2v\rho}} = \frac{1}{\pi\left(8.0\times10^3\,\text{Hz}\right)}\sqrt{\frac{0.0316\,\text{W/m}^2}{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)}} = \boxed{2.4\times10^{-7}\text{m}}$$

(*b*) $$A = \frac{1}{\pi f}\sqrt{\frac{I}{2v\rho}} = \frac{1}{\pi(35\,\text{Hz})}\sqrt{\frac{0.0316\,\text{W/m}^2}{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)}} = \boxed{5.4\times10^{-5}\text{m}}$$

109. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH16.XLS," on tab "Problem 16.109a."



(*b*) The spreadsheet used for this problem can be found on the Media Manager, with filename \ "PSE4_ISM_CH16.XLS," on tab "Problem 16.109b."



CHAPTER 17: Temperature, Thermal Expansion, and the Ideal Gas Law

Responses to Questions

1. 1 kg of aluminum will have more atoms. Aluminum has an atomic mass less than iron. Since each Al atom is less massive than each Fe atom, there will be more Al atoms than Fe atoms in 1 kg.

2. Properties of materials that could be exploited in making a thermometer include:
    a. thermal expansion, both linear and volume
    b. the proportionality between temperature and pressure (for an ideal gas when volume is held constant)
    c. temperature dependence of resistivity
    d. frequency of emitted radiation from a heated object (blackbody radiation curve).
   (Note: resistivity and blackbody radiation are defined in Volume II.)

3. 1C° is larger. Between the freezing and boiling points of water there are 100 Celsius degrees and 180 Fahrenheit degrees, so the Celsius degrees must be larger.

4. A and B have the same temperature; the temperature of C is different.

5. No. We can only infer that the temperature of C is different from that of A and B. We cannot infer anything about the relationship of the temperatures of A and B.

6. The initial length should be ℓ_0. However, since α is very small, the absolute value of $\Delta\ell$ will be about the same whether the initial or final length is used.

7. Aluminum. Al has a larger coefficient of linear expansion than Fe, and so will expand more than Fe when heated and will be on the outside of the curve.

8. As the pipe changes temperature due to the presence or absence of steam it will expand and contract. The bend allows the pipe to increase or decrease slightly in length without applying too much stress to the fixed ends.

9. Lower. Mercury has a larger coefficient of volume expansion than lead. When the temperature rises, mercury will expand more than lead. The density of mercury will decrease more than the density of lead will decrease, and so the lead will need to displace more mercury in order to balance its weight.

10. The bimetallic strip is made of two types of metal joined together. The metal of the outside strip has a higher coefficient of linear expansion than that of the inside strip, so it will expand and contract more dramatically. If the temperature goes above the thermostat setting, the outer strip will expand more than the inner, causing the spiral to wind more tightly and tilt the glass vessel so that the liquid mercury flows away from the contact wires and the heater turns off. If the temperature goes below the thermostat setting, the vessel tilts back as the outer strip contracts more than the inner and the spiral opens, and the heater turns on. Moving the temperature setting lever changes the initial position of the glass vessel. For instance, if the lever is set at 50, the vessel is tilted with the mercury far from the contact wires. The outer strip has to shrink significantly to uncurl the spiral enough to tilt the vessel back.

11. If water is added quickly to an overheated engine, it comes into contact with the very hot metal parts of the engine. Some areas of the metal parts will cool off very rapidly; others will not. Some of the water will quickly turn to steam and will expand rapidly. The net result can be a cracked engine block or radiator, due to the thermal stress, and/or the emission of high temperature steam from the radiator. Water should always be added slowly, with the engine running. The water will mix with the hotter water already in the system, and will circulate through the engine, gradually cooling all parts at about the same rate.

12. No. Whatever units are used for the initial length (ℓ_0) will be the units of the change in length $(\Delta\ell)$. The ratio $\Delta\ell/\ell_0$ does not depend on the units used.

$\boxed{13.}$ When the cold thermometer is placed in the hot water, the glass part of the thermometer will expand first, as heat is transferred to it first. This will cause the mercury level in the thermometer to decrease. As heat is transferred to the mercury inside the thermometer, the mercury will expand at a rate greater than the glass, and the level of mercury in the thermometer will rise.

14. Since Pyrex glass has a smaller coefficient of linear expansion than ordinary glass, it will expand less than ordinary glass when heated, making it less likely to crack from internal stresses. Pyrex glass is therefore more suitable for applications involving heating and cooling. An ordinary glass mug may expand to the point of cracking if boiling water is poured in it, whereas a Pyrex mug will not.

15. Slow. On a hot day, the brass rod holding the pendulum will expand and lengthen, increasing the length and therefore the period of the pendulum, causing the clock to run slow.

16. Soda is essentially water, and water (unlike most other substances) expands when it freezes. If the can is full while the soda is liquid, then as the soda freezes and expands, it will push on the inside surfaces of the can and the ends will bulge out.

17. The coefficient of volume expansion is much greater for alcohol than for mercury. A given temperature change will therefore result in a greater change in volume for alcohol than for mercury. This means that smaller temperature changes can be measured with an alcohol thermometer.

18. Decrease. As the temperature changes, both the aluminum sphere and the water will expand, decreasing in density. The coefficient of volume expansion of water is greater than that of aluminum, so the density of the water will decrease more than the density of the aluminum will decrease. Even though the sphere displaces a greater volume of water at a higher temperature, the weight of the water displaced (the buoyant force) will decrease because of the greater decrease in the density of the water.

$\boxed{19.}$ Helium. If we take the atomic mass of $6.7\times10^{-27}\,\text{kg}$ and divide by the conversion factor from kg to u (atomic mass units), which is $1.66\times10^{-27}\,\text{kg/u}$, we get 4.03 u. This corresponds to the atomic mass of helium.

20. Not really, as long as the pressure is very low. At low pressure, most gases will behave like an ideal gas. Some practical considerations would be the volatility of the gas and its corrosive properties. Light monatomic or diatomic gases are best.

21. Fresh water is less dense than sea water, requiring the ship to displace more water for the same buoyant force. (The buoyant force has to equal the weight of the ship for the ship to float.) The ship sat lower in the fresh water than in sea water, and was therefore more likely to be swamped by waves in a storm and sink.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

$\boxed{1.}$ The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic masses of gold and silver from the periodic table.

$$\frac{N_{\text{Au}}}{N_{\text{Ag}}}=\frac{\dfrac{2.15\times10^{-2}\,\text{kg}}{(196.96655\,\text{u/atom})(1.66\times10^{-27}\,\text{kg/u})}}{\dfrac{2.15\times10^{-2}\,\text{kg}}{(107.8682\,\text{u/atom})(1.66\times10^{-27}\,\text{kg/u})}}=\frac{107.8682}{196.96655}=0.548 \quad\rightarrow\quad \boxed{N_{\text{Au}}=0.548N_{\text{Ag}}}$$

Because a gold atom is heavier than a silver atom, there are fewer gold atoms in the given mass.

2. The number of atoms is found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of copper from the periodic table.

$$N_{\text{Cu}}=\frac{3.4\times10^{-3}\,\text{kg}}{(63.546\,\text{u/atom})(1.66\times10^{-27}\,\text{kg/u})}=\boxed{3.2\times10^{22}\,\text{atoms of Cu}}$$

3. (*a*) $T(°\text{C})=\frac{5}{9}\left[T(°\text{F})-32\right]=\frac{5}{9}[68-32]=\boxed{20°\text{C}}$

(*b*) $T(°\text{F})=\frac{9}{5}T(°\text{C})+32=\frac{9}{5}(1900)+32=3452°\text{F}\approx\boxed{3500°\text{F}}$

4. High: $T(°\text{C})=\frac{5}{9}\left[T(°\text{F})-32\right]=\frac{5}{9}[136-32]=\boxed{57.8°\text{C}}$

Low: $T(°\text{C})=\frac{5}{9}\left[T(°\text{F})-32\right]=\frac{5}{9}[-129-32]=\boxed{-89.4°\text{C}}$

5. $T(°\text{F})=\frac{9}{5}T(°\text{C})+32=\frac{9}{5}(39.4°\text{C})+32=\boxed{102.9°\text{F}}$

6. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows.

$$\frac{\Delta T}{\Delta \ell}=\frac{100.0°\text{C}-0.0°\text{C}}{21.85\text{ cm}-11.82\text{ cm}}=9.970\,\text{C}°/\text{cm}$$

Then the temperature corresponding to length L is $T(\ell)=0.0°\text{C}+(\ell-11.82\text{ cm})(9.970\,\text{C}°/\text{cm})$.

(*a*) $T(18.70\text{ cm})=0.0°\text{C}+(18.70\text{ cm}-11.82\text{ cm})(9.970\,\text{C}°/\text{cm})=\boxed{68.6°\text{C}}$

(*b*) $T(14.60\text{ cm})=0.0°\text{C}+(14.60\text{ cm}-11.82\text{ cm})(9.970\,\text{C}°/\text{cm})=\boxed{27.7°\text{C}}$

7. Take the 300 m height to be the height in January. Then the increase in the height of the tower is given by Eq. 17-1a.

$$\Delta\ell = \alpha\ell_0\Delta T = \left(12\times10^{-6}/\mathrm{C}^\circ\right)(300\ \mathrm{m})\left(25^\circ\mathrm{C}-2^\circ\mathrm{C}\right) = \boxed{0.08\,\mathrm{m}}$$

8. When the concrete cools in the winter, it will contract, and there will be no danger of buckling. Thus the low temperature in the winter is not a factor in the design of the highway. But when the concrete warms in the summer, it will expand. A crack must be left between the slabs equal to the increase in length of the concrete as it heats from 15°C to 50°C.

$$\Delta\ell = \alpha\ell_0\Delta T = \left(12\times10^{-6}/\mathrm{C}^\circ\right)(12\ \mathrm{m})\left(50^\circ\mathrm{C}-15^\circ\mathrm{C}\right) = \boxed{5.0\times10^{-3}\mathrm{m}}$$

9. The increase in length of the table is given by Eq. 17-1a.

$$\Delta\ell = \alpha\ell_0\Delta T = \left(0.2\times10^{-6}/\mathrm{C}^\circ\right)(1.6\,\mathrm{m})\left(5.0\,\mathrm{C}^\circ\right) = \boxed{1.6\times10^{-6}\mathrm{m}}$$

For steel, $\Delta\ell = \alpha\ell_0\Delta T = \left(12\times10^{-6}/\mathrm{C}^\circ\right)(1.6\,\mathrm{m})\left(5.0\,\mathrm{C}^\circ\right) = \boxed{9.6\times10^{-5}\mathrm{m}}$.

The change for Super Invar is only $\frac{1}{60}$ of the change for steel.

10. The increase in length of the rod is given by Eq. 17-1a.

$$\Delta\ell = \alpha\ell_0\Delta T \quad\rightarrow\quad \Delta T = \frac{\Delta\ell}{\alpha\ell_0} \quad\rightarrow\quad T_{\mathrm{f}} = T_i + \frac{\Delta\ell}{\alpha\ell_0} = 25^\circ\mathrm{C} + \frac{0.010}{19\times10^{-6}/\mathrm{C}^\circ} = 551.3^\circ\mathrm{C} \approx \boxed{550^\circ\mathrm{C}}$$

11. The density at 4°C is $\rho = \dfrac{M}{V} = \dfrac{1.00\times10^3\,\mathrm{kg}}{1.00\,\mathrm{m}^3}$. When the water is warmed, the mass will stay the same, but the volume will increase according to Eq. 17-2.

$$\Delta V = \beta V_0\Delta T = \left(210\times10^{-6}/\mathrm{C}^\circ\right)\left(1.00\,\mathrm{m}^3\right)\left(94^\circ\mathrm{C}-4^\circ\mathrm{C}\right) = 1.89\times10^{-2}\,\mathrm{m}^3$$

The density at the higher temperature is $\rho = \dfrac{M}{V} = \dfrac{1.00\times10^3\,\mathrm{kg}}{1.00\,\mathrm{m}^3 + 1.89\times10^{-2}\,\mathrm{m}^3} = \boxed{981\,\mathrm{kg}/\mathrm{m}^3}$

12. We assume that all of the expansion of the water is in the thickness of the mixed layer. We also assume that the volume of the water can be modeled as a shell of constant radius, equal to the radius of the earth, and so the volume of the shell is the surface area of the shell times its thickness. $V = 4\pi R_{\mathrm{E}}^2 d$. Use Eq. 17-2.

$$V = 4\pi R_{\mathrm{E}}^2 d \quad\rightarrow\quad \Delta V = 4\pi R_{\mathrm{E}}^2\Delta d \ ; \ \Delta V = \beta V\Delta T = \beta 4\pi R_{\mathrm{E}}^2 d\Delta T$$

$$4\pi R_{\mathrm{E}}^2\Delta d = \beta 4\pi R_{\mathrm{E}}^2 d\Delta T \quad\rightarrow$$

$$\Delta d = \beta d\Delta T = \left(210\times10^{-6}/\mathrm{C}^\circ\right)(50\,\mathrm{m})\left(0.5^\circ\mathrm{C}\right) = 0.00525\,\mathrm{m} \approx \boxed{5\,\mathrm{mm}}$$

13. The rivet must be cooled so that its diameter becomes the same as the diameter of the hole.

$$\Delta\ell = \alpha\ell_0\Delta T \quad\rightarrow\quad \ell-\ell_0 = \alpha L_0\left(T-T_0\right)$$

$$T = T_0 + \frac{\ell-\ell_0}{\alpha\ell_0} = 20^\circ\mathrm{C} + \frac{1.870\ \mathrm{cm}-1.872\ \mathrm{cm}}{\left(12\times10^{-6}/\mathrm{C}^\circ\right)(1.872\ \mathrm{cm})} = \boxed{-69^\circ\mathrm{C}}$$

The temperature of "dry ice" is about $-80^\circ\mathrm{C}$, so this process will be successful.

14. Assume that each dimension of the plate changes according to Eq. 17-1a.

$$\Delta A = A - A_0 = (\ell + \Delta\ell)(w + \Delta w) - \ell w = \ell w + \ell\Delta w + w\Delta\ell + \Delta\ell\Delta w - \ell w$$
$$= \ell\Delta w + w\Delta\ell + \Delta\ell\Delta w$$

Neglect the very small quantity $\Delta\ell\Delta w$.

$$\Delta A = \ell\Delta w + w\Delta\ell = \ell(\alpha w\Delta T) + w(\alpha\ell\Delta T) = \boxed{2\alpha\ell w\Delta T}$$

15. The change in volume of the aluminum is given by the volume expansion formula, Eq. 17-2.

$$\Delta V = \beta V_0\Delta T = \left(75\times10^{-6}/\text{C}^\circ\right)\left(\tfrac{4}{3}\pi\left(\frac{8.75\text{ cm}}{2}\right)^3\right)(180^\circ\text{C} - 30^\circ\text{C}) = \boxed{3.9\,\text{cm}^3}$$

16. Since the coefficient of volume expansion is much larger for the coolant than for the aluminum and the steel, the coolant will expand more than the aluminum and steel, and so coolant will overflow the cooling system. Use Eq. 17-2.

$$\Delta V = \Delta V_{\text{coolant}} - \Delta V_{\text{aluminum}} - \Delta V_{\text{steel}} = \beta_{\text{coolant}}V_{\text{coolant}}\Delta T - \beta_{\text{aluminum}}V_{\text{aluminum}}\Delta T - \beta_{\text{steel}}V_{\text{steel}}\Delta T$$
$$= (\beta_{\text{coolant}}V_{\text{coolant}} - \beta_{\text{aluminum}}V_{\text{aluminum}} - \beta_{\text{steel}}V_{\text{steel}})\Delta T$$
$$= \left[\left(410\times10^{-6}/\text{C}^\circ\right)(17\,\text{L}) - \left(75\times10^{-6}/\text{C}^\circ\right)(3.5\,\text{L}) - \left(35\times10^{-6}/\text{C}^\circ\right)(13.5\,\text{L})\right](12\text{C}^\circ)$$
$$= 0.0748\,\text{L} \approx \boxed{75\,\text{mL}}$$

17. (*a*) The amount of water lost is the final volume of the water minus the final volume of the container. Also note that the original volumes of the water and the container are the same.

$$V_{\text{lost}} = (V_0 + \Delta V)_{\text{H}_2\text{O}} - (V_0 + \Delta V)_{\text{container}} = \Delta V_{\text{H}_2\text{O}} - \Delta V_{\text{container}} = \beta_{\text{H}_2\text{O}}V_0\Delta T - \beta_{\text{container}}V_0\Delta T$$

$$\beta_{\text{container}} = \beta_{\text{H}_2\text{O}} - \frac{V_{\text{lost}}}{V_0\Delta T} = 210\times10^{-6}/\text{C}^\circ - \frac{(0.35\text{g})\left(\dfrac{1\,\text{mL}}{0.98324\text{ g}}\right)}{(55.50\text{ mL})(60^\circ\text{C} - 20^\circ\text{C})} = \boxed{5.0\times10^{-5}/\text{C}^\circ}$$

(*b*) From Table 17-1, the most likely material is $\boxed{\text{copper}}$.

18. (*a*) The sum of the original diameter plus the expansion must be the same for both the plug and the ring.

$$(\ell_0 + \Delta\ell)_{\text{iron}} = (\ell_0 + \Delta\ell)_{\text{brass}} \quad\rightarrow\quad \ell_{\text{iron}} + \alpha_{\text{iron}}\ell_{\text{iron}}\Delta T = \ell_{\text{brass}} + \alpha_{\text{brass}}\ell_{\text{brass}}\Delta T$$

$$\Delta T = \frac{\ell_{\text{brass}} - \ell_{\text{iron}}}{\alpha_{\text{iron}}\ell_{\text{iron}} - \alpha_{\text{brass}}\ell_{\text{brass}}} = \frac{8.753\text{ cm} - 8.743\text{ cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)(8.743\text{ cm}) - \left(19\times10^{-6}/\text{C}^\circ\right)(8.753\text{ cm})}$$
$$= -163\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 15^\circ\text{C} \quad\rightarrow\quad T_{\text{final}} = -148^\circ\text{C} \approx \boxed{-150^\circ\text{C}}$$

(*b*) Simply switch the initial values in the above calculation.

$$\Delta T = \frac{\ell_{\text{brass}} - \ell_{\text{iron}}}{\alpha_{\text{iron}}\ell_{\text{iron}} - \alpha_{\text{brass}}\ell_{\text{brass}}} = \frac{8.743\text{ cm} - 8.753\text{ cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)(8.753\text{ cm}) - \left(19\times10^{-6}/\text{C}^\circ\right)(8.743\text{ cm})} =$$
$$= 164\text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 15^\circ\text{C} \quad\rightarrow\quad T_{\text{final}} = 179^\circ\text{C} \approx \boxed{180^\circ\text{C}}$$

$\boxed{19.}$ We model the vessel as having a constant cross-sectional area A. Then a volume V_0 of fluid will occupy a length ℓ_0 of the tube, given that $V_0 = A\ell_0$. Likewise $V = A\ell$.

$$\Delta V = V - V_0 = A\ell - A\ell_0 = A\Delta\ell \text{ and } \Delta V = \beta V_0\Delta T = \beta A\ell_0\Delta T.$$

Equate the two expressions for ΔV, and get $A\Delta\ell = \beta A\ell_0\Delta T \rightarrow \Delta\ell = \beta\ell_0\Delta T$. But $\Delta\ell = \alpha\ell_0\Delta T$, so we see that under the conditions of the problem, $\boxed{\alpha = \beta}$.

20. (*a*) When a substance changes temperature, its volume will change by an amount given by Eq. 17-2. This causes the density to change.

$$\Delta\rho = \rho_{\text{f}} - \rho = \frac{M}{V} - \frac{M}{V_0} = \frac{M}{V_0 + \Delta V} - \frac{M}{V_0} = \frac{M}{V_0 + \beta V_0\Delta T} - \frac{M}{V_0} = \frac{M}{V_0}\left(\frac{1}{1+\beta\Delta T} - 1\right)$$
$$= \rho\left(\frac{1}{1+\beta\Delta T} - \frac{1+\beta\Delta T}{1+\beta\Delta T}\right) = \rho\left(\frac{-\beta\Delta T}{1+\beta\Delta T}\right)$$

If we assume that $\beta\Delta T \ll 1$, then the denominator is approximately 1, so $\boxed{\Delta\rho = -\rho\beta\Delta T}$.

(*b*) The fractional change in density is

$$\frac{\Delta\rho}{\rho} = \frac{-\rho\beta\Delta T}{\rho} = -\beta\Delta T = -\left(87\times10^{-6}/{}^\circ\text{C}\right)\left(-55^\circ\text{C} - 25^\circ\text{C}\right) = 6.96\times10^{-3}$$

This is a $\boxed{\text{0.70\% increase}}$.

21. As the wine contracts or expands, its volume changes. We assume that the volume change can only occur by a corresponding change in the headspace. Note that if the volume increases, the headspace decreases, so their changes are of opposite signs. Use Eq. 17-2.

(*a*) The temperature decreases, so the headspace should increase.

$$\Delta V = \beta V_0\Delta T = -\pi r^2\Delta H \quad\rightarrow$$

$$\Delta H = -\frac{\beta V_0\Delta T}{\pi r^2} = \frac{\left(420\times10^{-6}/\text{C}^\circ\right)\left(0.750\,\text{L}\right)\left(\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right)\left(10\,\text{C}^\circ\right)}{\pi\left(0.00925\,\text{m}\right)^2} = 0.0117\,\text{m}$$

$$H = 1.5\,\text{cm} + 1.17\,\text{cm} = 2.67\,\text{cm} \approx \boxed{2.7\,\text{cm}}$$

(*b*) The temperature increases, so the headspace should decrease.

$$\Delta H = -\frac{\beta V_0\Delta T}{\pi r^2} = \frac{\left(420\times10^{-6}/\text{C}^\circ\right)\left(0.750\,\text{L}\right)\left(\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right)\left(-10\,\text{C}^\circ\right)}{\pi\left(0.00925\,\text{m}\right)^2} = -0.0117\,\text{m}$$

$$H = 1.5\,\text{cm} - 1.17\,\text{cm} = 0.33\,\text{cm} \approx \boxed{0.3\,\text{cm}}$$

22. (*a*) The original surface area of the sphere is given by $A = 4\pi r^2$. The radius will expand with temperature according to Eq. 17-1b, $r_{\text{new}} = r\left(1+\alpha\Delta T\right)$. The final surface area is $A_{\text{new}} = 4\pi r_{\text{new}}^2$, and so the change in area is $\Delta A = A_{\text{new}} - A$.

$$\Delta A = A_{\text{new}} - A = 4\pi r^2\left(1+\alpha\Delta T\right)^2 - 4\pi r^2 = 4\pi r^2\left[\left(1+\alpha\Delta T\right)^2 - 1\right]$$
$$= 4\pi r^2\left[1 + 2\alpha\Delta T + \alpha^2\left(\Delta T\right)^2 - 1\right] = 4\pi r^2\left(2\alpha\Delta T\right)\left[1+\tfrac{1}{2}\alpha\Delta T\right]$$

If the temperature change is not large, $\tfrac{1}{2}\alpha\Delta T \ll 1$, and so $\boxed{\Delta A = 8\pi r^2\alpha\Delta T}$

(*b*) Evaluate the above expression for the solid iron sphere.

$$\Delta A = 8\pi r^2\alpha\Delta T = 8\pi\left(60.0\times10^{-2}\,\text{m}\right)^2\left(12\times10^{-6}/{}^\circ\text{C}\right)\left(275^\circ\text{C} - 15^\circ\text{C}\right) = \boxed{2.8\times10^{-2}\,\text{m}^2}$$

23. The pendulum has a period of $\tau_0 = 2\pi\sqrt{\ell_0/g}$ at 17°C, and a period of $\tau = 2\pi\sqrt{\ell/g}$ at 28°C. Notice that $\tau > \tau_0$ since $\ell > \ell_0$. With every swing of the clock, the clock face will indicate that a time τ_0 has passed, but the actual amount of time that has passed is τ. Thus the clock face is "losing time" by an amount of $\Delta\tau = \tau - \tau_0$ every swing. The fractional loss is given by $\dfrac{\Delta\tau}{\tau_0}$, and the length at the higher temperature is given by

$$\frac{\Delta\tau}{\tau_0} = \frac{\tau-\tau_0}{\tau_0} = \frac{2\pi\sqrt{\ell/g}-2\pi\sqrt{\ell_0/g}}{2\pi\sqrt{\ell_0/g}} = \frac{\sqrt{\ell}-\sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0+\Delta\ell}-\sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0+\alpha\ell_0\Delta T}-\sqrt{\ell_0}}{\sqrt{\ell_0}}$$

$$= \sqrt{1+\alpha\Delta T}-1 = \sqrt{1+\left(19\times10^{-6}/\text{C}^\circ\right)\left(11\,\text{C}^\circ\right)}-1 = 1.04\times10^{-4}$$

Thus the amount of time lost in any time period τ_0 is $\Delta\tau = \left(1.04\times10^{-4}\right)\tau_0$. For one year, we have the following.

$$\Delta\tau = \left(1.04\times10^{-4}\right)\left(3.16\times10^{7}\,\text{s}\right) = 3286\,\text{s} \approx \boxed{55\,\text{min}}$$

24. The change in radius with heating does not cause a torque on the rotating wheel, and so the wheel's angular momentum does not change. Also recall that for a cylindrical wheel rotating about its axis, the moment of inertia is $I = \frac{1}{2}mr^2$.

$$L_0 = L_{\text{final}} \quad\rightarrow\quad I_0\omega_0 = I_{\text{final}}\omega_{\text{final}} \quad\rightarrow\quad \omega_{\text{final}} = \frac{I_0\omega_0}{I_{\text{final}}} = \frac{\frac{1}{2}mr_0^2\omega_0}{\frac{1}{2}mr^2} = \frac{r_0^2\omega_0}{r^2}$$

$$\frac{\Delta\omega}{\omega} = \frac{\omega_{\text{final}}-\omega_0}{\omega_0} = \frac{\frac{r_0^2\omega_0}{r^2}-\omega_0}{\omega_0} = \frac{r_0^2}{r^2}-1 = \frac{r_0^2}{\left(r_0+\Delta r\right)^2}-1 = \frac{r_0^2}{\left(r_0+\alpha r_0\Delta T\right)^2}-1 = \frac{r_0^2}{\left(r_0+\alpha r_0\Delta T\right)^2}-1$$

$$= \frac{1}{\left(1+\alpha\Delta T\right)^2}-1 = \frac{1-\left(1+2\alpha\Delta T+\left(\alpha\Delta T\right)^2\right)}{\left(1+\alpha\Delta T\right)^2} = \frac{-2\alpha\Delta T-\left(\alpha\Delta T\right)^2}{\left(1+\alpha\Delta T\right)^2} = -\alpha\Delta T\frac{2+\alpha\Delta T}{\left(1+\alpha\Delta T\right)^2}$$

Now assume that $\alpha\Delta T \ll 1$, and so $\dfrac{\Delta\omega}{\omega} = -\alpha\Delta T\dfrac{2+\alpha\Delta T}{\left(1+\alpha\Delta T\right)^2} \approx -2\alpha\Delta T$. Evaluate at the given values.

$$-2\alpha\Delta T = -2\left(25\times10^{-6}/\text{C}^\circ\right)\left(75.0\,\text{C}^\circ\right) = \boxed{-3.8\times10^{-3}}$$

$\boxed{25.}$ The thermal stress must compensate for the thermal expansion. E is Young's modulus for the aluminum.

$$\text{Stress} = F/A = \alpha E\Delta T = \left(25\times10^{-6}/\text{C}^\circ\right)\left(70\times10^{9}\,\text{N/m}^2\right)\left(35^\circ\text{C}-18^\circ\text{C}\right) = \boxed{3.0\times10^{7}\,\text{N/m}^2}$$

26. (*a*) Since the beam cannot shrink while cooling, the tensile stress must compensate in order to keep the length constant.

$$\text{Stress} = F/A = \alpha E\Delta T = \left(12\times10^{-6}/\text{C}^\circ\right)\left(200\times10^{9}\,\text{N/m}^2\right)\left(50\,\text{C}^\circ\right) = \boxed{1.2\times10^{8}\,\text{N/m}^2}$$

(*b*) The ultimate tensile strength of steel (from Table 12-2) is $5\times10^8\,\text{N/m}^2$, and so $\boxed{\text{the ultimate strength is not exceeded}}$. There would only be a safety factor of about 4.2.

(*c*) For concrete, repeat the calculation with the expansion coefficient and elastic modulus for concrete.

$$\text{Stress} = F/A = \alpha E \Delta T = \left(12\times10^{-6}/\text{C}^\circ\right)\left(20\times10^{9}\ \text{N/m}^2\right)\left(50\,\text{C}^\circ\right) = \boxed{1.2\times10^{7}\ \text{N/m}^2}$$

The ultimate tensile strength of concrete is $2\times10^{6}\ \text{N/m}^2$, and so $\boxed{\text{the concrete will fracture}}$.

27. (*a*) Calculate the change in temperature needed to increase the diameter of the iron band so that it fits over the barrel. Assume that the barrel does not change in dimensions.

$$\Delta\ell = \alpha\ell_0\Delta T \quad\rightarrow\quad \ell-\ell_0 = \alpha\ell_0\left(T-T_0\right)$$

$$T = T_0 + \frac{\ell-\ell_0}{\alpha\ell_0} = 20^\circ\text{C} + \frac{134.122\ \text{cm} - 134.110\ \text{cm}}{\left(12\times10^{-6}/\text{C}^\circ\right)\left(134.110\ \text{cm}\right)} = 27.457^\circ\text{C} \approx \boxed{27^\circ\text{C}}$$

(*b*) Since the band cannot shrink while cooling, the thermal stress must compensate in order to keep the length at a constant 132.122 cm. E is Young's modulus for the material.

$$\text{Stress} = F/A = \alpha E\Delta T \quad\rightarrow\quad F = AE\frac{\Delta\ell}{\ell_0} = AE\alpha\Delta T$$

$$= \left(9.4\times10^{-2}\text{m}\right)\left(6.5\times10^{-3}\text{m}\right)\left(100\times10^{9}\ \text{N/m}^2\right)\left(12\times10^{-6}/\text{C}^\circ\right)\left(7.457\,\text{C}^\circ\right) = \boxed{5500\,\text{N}}$$

28. Use the relationships $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15$ and $T\left(\text{K}\right) = \frac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] + 273.15$.

(*a*) $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15 = 66 + 273.15 = \boxed{339\ \text{K}}$

(*b*) $T\left(\text{K}\right) = \frac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] + 273.15 = \frac{5}{9}\left[92 - 32\right] + 273.15 = \boxed{306\,\text{K}}$

(*c*) $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15 = -55 + 273.15 = \boxed{218\ \text{K}}$

(*d*) $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15 = 5500 + 273.15 = 5773.15\,\text{K} \approx \boxed{5800\,\text{K}}$

29. Use the relationship that $T\left(\text{K}\right) = \frac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] + 273.15$.

$$T\left(\text{K}\right) = \tfrac{5}{9}\left[T\left(^\circ\text{F}\right) - 32\right] + 273.15 \quad\rightarrow$$

$$T\left(^\circ\text{F}\right) = \tfrac{9}{5}\left[T\left(\text{K}\right) - 273.15\right] + 32 = \tfrac{9}{5}\left[0 - 273.15\right] + 32 = \boxed{-459.67^\circ\text{F}}$$

30. Use the relationship that $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15$.

(*a*) $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15 = 4270\ \text{K} \approx \boxed{4300\ \text{K}}$; $T\left(\text{K}\right) = T\left(^\circ\text{C}\right) + 273.15 = \boxed{15\times10^{6}\ \text{K}}$

(*b*) $\%\ \text{error} = \dfrac{\Delta T}{T\left(\text{K}\right)}\times100 = \dfrac{273}{T\left(\text{K}\right)}\times100$

$$4000^\circ\text{C:}\ \frac{273}{4000}\times100 \approx \boxed{7\%} \qquad 15\times10^{6}\,^\circ\text{C:}\ \frac{273}{15\times10^{6}}\times100 \approx \boxed{2\times10^{-3}\%}$$

$\boxed{31.}$ Assume the gas is ideal. Since the amount of gas is constant, the value of $\dfrac{PV}{T}$ is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad\rightarrow\quad V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1} = \left(3.80\,\text{m}^3\right)\left(\frac{1.00\ \text{atm}}{3.20\ \text{atm}}\right)\frac{\left(273+38.0\right)\text{K}}{273\ \text{K}} = \boxed{1.35\,\text{m}^3}$$

32. Assume the air is an ideal gas. Since the amount of air is constant, the value of $\frac{PV}{T}$ is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (293\text{ K})\left(\frac{40\text{ atm}}{1\text{ atm}}\right)\left(\frac{1}{8}\right) = 1465\text{ K} = 1192°\text{C} \approx \boxed{1200°\text{C}}$$

33. Assume the nitrogen is an ideal gas. From Example 17-10, the volume of one mole of nitrogen gas at STP is $22.4\times10^{-3}\,\text{m}^3$. The mass of one mole of nitrogen, with a molecular mass of 28.0 u, is 28.0 grams. Use these values to calculate the density of the oxygen gas.

$$\rho = \frac{M}{V} = \frac{28.0\times10^{-3}\text{kg}}{22.4\times10^{-3}\text{m}^3} = \boxed{1.25\,\text{kg/m}^3}$$

34. (*a*) Assume that the helium is an ideal gas, and then use the ideal gas law to calculate the volume. Absolute pressure must be used, even though gauge pressure is given.

$$PV = nRT \rightarrow V = \frac{nRT}{P} = \frac{(14.00\text{ mol})(8.314\,\text{J/mol•K})(283.15\,\text{K})}{(1.350\,\text{atm})(1.013\times10^5\text{ Pa/atm})} = \boxed{0.2410\,\text{m}^3}$$

(*b*) Since the amount of gas is not changed, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (283.15\text{ K})\left(\frac{2.00\text{ atm}}{1.350\text{ atm}}\right)\left(\frac{1}{2}\right) = 210\text{ K} = \boxed{-63^\circ\text{C}}$$

35. We ignore the weight of the stopper. Initially there is a net force (due to air pressure) on the stopper of 0, because the pressure is the same both above and below the stopper. With the increase in temperature, the pressure inside the tube will increase, and so there will be a net upward force given by $F_{\text{net}} = (P_{\text{in}} - P_{\text{out}})A.$ The inside pressure can be expressed in terms of the inside temperature by means of the ideal gas law for a constant volume and constant mass of gas.

$$\frac{P_{\text{in}}V_{\text{tube}}}{T_{\text{in}}} = \frac{P_0V_{\text{tube}}}{T_0} \rightarrow P_{\text{in}} = P_0\frac{T_{\text{in}}}{T_0}$$

$$F_{\text{net}} = (P_{\text{in}} - P_{\text{out}})A = \left(P_0\frac{T_{\text{in}}}{T_0} - P_0\right)A = P_0\left(\frac{T_{\text{in}}}{T_0} - 1\right)A \rightarrow$$

$$T_{\text{in}} = \left(\frac{F_{\text{net}}}{P_0A} + 1\right)T_0 = \left[\frac{(10.0\,\text{N})}{(1.013\times10^5\text{Pa})\pi(0.0075\,\text{m})^2} + 1\right](273\,\text{K} + 18\,\text{K}) = 454\,\text{K} = \boxed{181°\text{C}}$$

36. Assume that the nitrogen and carbon dioxide are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\frac{P}{n} = \frac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered.

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \rightarrow$$

$$P_2 = P_1\frac{n_2}{n_1} = (3.85\,\text{atm})\left(\frac{\dfrac{21.6\text{ kg CO}_2}{44.01\times10^{-3}\text{ kg CO}_2/\text{mol}}}{\dfrac{21.6\text{ kg N}_2}{28.01\times10^{-3}\text{ kg N}_2/\text{mol}}}\right) = (3.85\,\text{atm})\left(\frac{28.01}{44.01}\right) = \boxed{2.45\,\text{atm}}$$

$\boxed{37.}$ (*a*) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n=(28.5\text{ kg})\frac{1\text{ mole N}_2}{28.01\times10^{-3}\text{kg}}=1017\,\text{mol}$$

$$PV=nRT\ \rightarrow\ V=\frac{nRT}{P}=\frac{(1017\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(273\text{ K})}{1.013\times10^{5}\,\text{Pa}}=22.79\text{ m}^3\approx\boxed{22.8\,\text{m}^3}$$

(*b*) Hold the volume and temperature constant, and again use the ideal gas law.

$$n=(28.5\text{ kg}+25.0\text{ kg})\frac{1\text{ mole N}_2}{28.01\times10^{-3}\text{kg}}=1910\,\text{mol}$$

$$PV=nRT\ \rightarrow$$

$$P=\frac{nRT}{V}=\frac{(1910\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(273\text{ K})}{22.79\,\text{m}^3}=\boxed{1.90\times10^5\,\text{Pa}=1.88\,\text{atm}}$$

38. We assume that the mass of air is unchanged, and the volume of air is unchanged (since the tank is rigid). Use the ideal gas law.

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}\ \rightarrow\ T_2=T_1\frac{P_2}{P_1}=[(273+29)\,\text{K}]\left(\frac{194\,\text{atm}}{204\,\text{atm}}\right)=287\,\text{K}=\boxed{14^\circ\text{C}}$$

39. Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n=(105.0\text{ kg})\frac{1\text{ mole Ar}}{39.95\times10^{-3}\text{kg}}=2628\text{ mol}$$

$$PV=nRT\ \rightarrow\ P=\frac{nRT}{V}=\frac{(2628\text{ mol})(8.314\,\text{J/mol}\bullet\text{k})(293.15\,\text{K})}{(38.0\text{ L})(1.00\times10^{-3}\text{ m}^3/\text{L})}=\boxed{1.69\times10^8\,\text{Pa}}$$

This is 1660 atm.

40. Assume that the oxygen and helium are ideal gases, and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\frac{P}{n}=\frac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered. Finally, gauge pressure must be changed to absolute pressure.

$$\frac{P_1}{n_1}=\frac{P_2}{n_2}\ \rightarrow\ n_2=n_1\frac{P_2}{P_1}=(30.0\text{ kg O}_2)\left(\frac{1\text{ mole O}_2}{32\times10^{-3}\text{kg}}\right)\frac{(8.00\text{ atm})}{(9.20\text{ atm})}=8.152\times10^2\,\text{moles}$$

$$(8.152\times10^2\text{moles})\left(\frac{4.0\times10^{-3}\text{kg}}{1\text{ mole He}}\right)=\boxed{3.26\,\text{kg He}}$$

41. We assume that the gas is ideal, that the amount of gas is constant, and that the volume of the gas is constant.

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}\ \rightarrow\ T_2=T_1\frac{P_2}{P_1}=[(273.15+20.0)\,\text{K}]\left(\frac{2.00\,\text{atm}}{1.00\,\text{atm}}\right)=586.3\,\text{K}=313.15^\circ\text{C}\approx\boxed{313^\circ\text{C}}$$

42. Assume that the air is an ideal gas. The pressure and volume are held constant. From the ideal gas law, the value of $\frac{PV}{R} = nT$ is held constant.

$$n_1T_1 = n_2T_2 \rightarrow \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{(273+38)\,\text{K}}{(273+15)\,\text{K}} = \frac{311}{288} = 1.080$$

Thus $\boxed{8.0\%}$ must be removed.

$\boxed{43.}$ Assume the oxygen is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow P_2 = P_1\frac{V_1}{V_2}\frac{T_2}{T_1} = (2.45\text{ atm})\left(\frac{61.5\text{ L}}{48.8\text{ L}}\right)\frac{(273+56.0)\,\text{K}}{(273+18.0)\,\text{K}} = \boxed{3.49\,\text{atm}}$$

44. Assume the helium is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease 30%.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}\frac{T_2}{T_1} = \left(\frac{1.0\,\text{atm}}{0.68\,\text{atm}}\right)\frac{(273+5.0)\,\text{K}}{(273+20.0)\,\text{K}} = \boxed{1.4\text{ times the original volume}}$$

45. Since the container can withstand a pressure difference of 0.50 atm, we find the temperature for which the inside pressure has dropped from 1.0 atm to 0.50 atm. We assume the mass of contained gas and the volume of the container are constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow T_2 = T_1\frac{P_2}{P_1} = [(273.15+18)\,\text{K}]\left(\frac{0.50\,\text{atm}}{1.0\,\text{atm}}\right) = 146.6\,\text{K} \approx \boxed{-130^\circ\text{C}}$$

46. The pressure inside the bag will change to the surrounding air pressure as the volume of the bag changes. We assume the amount of gas and temperature of the gas are constant. Use the ideal gas equation.

$$P_1V_1 = P_2V_2 \rightarrow V_2 = V_1\frac{P_1}{P_2} = V_1\left(\frac{1.0\,\text{atm}}{0.75\,\text{atm}}\right) = 1.3V_1$$

Thus the bag has expanded by $\boxed{30\%}$.

47. We assume that all of the gas in this problem is at the same temperature. Use the ideal gas equation. First, find the initial number of moles in the tank at 34 atm, n_1. Then find the final number of moles in the tank at 204 atm, n_2. The difference in those two values is the number of moles needed to add to the tank.

$$P_1V_1 = n_1RT \rightarrow n_1 = \frac{P_1V_1}{RT} = \frac{(34\,\text{atm})(12\,\text{L})}{RT}$$

$$P_2V_2 = n_2RT \rightarrow n_2 = \frac{P_2V_2}{RT} = \frac{(204\,\text{atm})(12\,\text{L})}{RT}$$

$$n_{\text{add}} = n_2 - n_1 = \frac{(170\,\text{atm})(12\,\text{L})}{RT}$$

Use this number of moles at atmospheric pressure to find the volume of air needed to add, and then find the time needed to add it.

$$P_{\text{add}}V_{\text{add}} = n_{\text{add}}RT \quad\rightarrow\quad V_{\text{add}} = \frac{n_{\text{add}}RT}{P_{\text{add}}} = \frac{\dfrac{(170\,\text{atm})(12\,\text{L})}{RT}}{1\,\text{atm}} = 2040\,\text{L}$$

$$2040\,\text{L}\left(\frac{1\,\text{min}}{290\,\text{L}}\right) = \boxed{7.0\,\text{min}}$$

48. From the ideal gas equation, we have $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$, since the amount of gas is constant. Use these relationships along with the given conditions to find the original pressure and temperature.

$$P_1V_1 = nRT_1 \;\;;\;\; P_2V_2 = nRT_2 \quad\rightarrow\quad P_1V_1 - P_2V_2 = nRT_1 - nRT_2 = nR(T_1 - T_2) \quad\rightarrow$$

$$P_1V_1 - (P_1 + 450\,\text{Pa})V_2 = nR(T_1 - T_2) \quad\rightarrow\quad P_1(V_1 - V_2) - V_2(450\,\text{Pa}) = nR(9.0\,\text{K}) \quad\rightarrow$$

$$P_1 = \frac{nR(9.0\,\text{K}) + V_2(450\,\text{Pa})}{V_1 - V_2} = \frac{(4.0\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(9.0\,\text{K}) + (0.018\,\text{m}^3)(450\,\text{Pa})}{0.002\,\text{m}^3}$$

$$= 1.537\times10^5\,\text{Pa} \approx \boxed{1.5\times10^5\,\text{Pa}}$$

$$T_1 = \frac{P_1V_1}{nR} = T_1 = \frac{(1.537\times10^5\,\text{Pa})(0.020\,\text{m}^3)}{(4.0\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})} = \boxed{92.4\,\text{K}} \approx -181^\circ\text{C}$$

$\boxed{49.}$ We calculate the density of water vapor, with a molecular mass of 18.0 grams per mole, from the ideal gas law.

$$PV = nRT \quad\rightarrow\quad \frac{n}{V} = \frac{P}{RT} \quad\rightarrow$$

$$\rho = \frac{m}{V} = \frac{Mn}{V} = \frac{MP}{RT} = \frac{(0.0180\,\text{kg/mol})(1.013\times10^5\,\text{Pa})}{(8.314\,\text{J/mol}\bullet\text{K})(373\,\text{K})} = \boxed{0.588\,\text{m}^3}$$

The density from Table 13-1 is $0.598\,\text{m}^3$. Because this gas is very "near" a phase change state (water can also exist as a liquid at this temperature and pressure), we would not expect it to act like an ideal gas. It is reasonable to expect that the molecules will have other interactions besides purely elastic collisions. That is evidenced by the fact that steam can form droplets, indicating an attractive force between the molecules.

50. The ideal gas law can be used to relate the volume at the surface to the submerged volume of the bubble. We assume the amount of gas in the bubble doesn't change as it rises. The pressure at the submerged location is found from Eq. 13-6b.

$$PV = nRT \quad\rightarrow\quad \frac{PV}{T} = nR = \text{constant} \quad\rightarrow\quad \frac{P_{\text{surface}}V_{\text{surface}}}{T_{\text{surface}}} = \frac{P_{\text{submerged}}V_{\text{submerged}}}{T_{\text{submerged}}} \quad\rightarrow$$

$$V_{\text{surface}} = V_{\text{submerged}}\frac{P_{\text{submerged}}}{P_{\text{surface}}}\frac{T_{\text{surface}}}{T_{\text{submerged}}} = V_{\text{submerged}}\frac{P_{\text{atm}} + \rho gh}{P_{\text{atm}}}\frac{T_{\text{surface}}}{T_{\text{submerged}}}$$

$$=\left(1.00\,\text{cm}^3\right)\frac{\left[1.013\times10^5+\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(37.0\,\text{m}\right)\right]}{\left(1.013\times10^5\right)\text{Pa}}\frac{\left(273.15+18.5\right)\text{K}}{\left(273.15+5.5\right)\text{K}}$$

$$=\boxed{4.79\,\text{cm}^3}$$

51. At STP, 1 mole of ideal gas occupies 22.4 L.

$$\frac{1\text{ mole}}{22.4\text{ L}}\left(\frac{6.02\times10^{23}\,\text{molecules}}{\text{mole}}\right)\left(\frac{1\text{ L}}{10^{-3}\text{m}^3}\right)=\boxed{2.69\times10^{25}\text{ molecules/m}^3}$$

52. We assume that the water is at 4°C so that its density is $1000\,\text{kg/m}^3$.

$$1.000\text{ L}\left(\frac{10^{-3}\text{m}^3}{1\text{ L}}\right)\left(\frac{1000\text{ kg}}{1\text{ m}^3}\right)\left(\frac{1\text{ mol}}{\left(15.9994+2\times1.00794\right)\times10^{-3}\text{kg}}\right)=\boxed{55.51\text{ mol}}$$

$$55.51\text{ mol}\left(\frac{6.022\times10^{23}\,\text{molecules}}{1\text{ mol}}\right)=\boxed{3.343\times10^{25}\,\text{molecules}}$$

53. We use Eq. 17-4.

$$PV=NkT\ \rightarrow\ P=\frac{N}{V}kT=\left(\frac{1\,\text{molecule}}{1\,\text{cm}^3}\right)\left(\frac{100^3\,\text{cm}^3}{\text{m}^3}\right)\left(1.38\times10^{-23}\,\frac{\text{J}}{\text{K}}\right)\left(3\,\text{K}\right)=\boxed{4\times10^{-17}\,\text{Pa}}$$

54. (*a*) Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\text{Earth}}^2$ and a thickness Δy. Then use the density of sea water to find the mass, and the molecular weight of water to find the number of moles.

$$\text{Volume}=0.75\left(4\pi R_{\text{Earth}}^2\right)\Delta y=0.75\left(4\pi\right)\left(6.38\times10^6\,\text{m}\right)^2\left(3\times10^3\,\text{m}\right)=1.15\times10^{18}\,\text{m}^3$$

$$1.15\times10^{18}\,\text{m}^3\left(\frac{1025\text{ kg}}{\text{m}^3}\right)\left(\frac{1\text{ mol}}{18\times10^{-3}\text{kg}}\right)=6.55\times10^{22}\,\text{moles}\approx\boxed{7\times10^{22}\,\text{moles}}$$

(*b*) $6.55\times10^{22}\,\text{moles}\left(6.02\times10^{23}\,\text{molecules/1 mol}\right)\approx\boxed{4\times10^{46}\,\text{molecules}}$

$\boxed{55.}$ Assume the gas is ideal at those low pressures, and use the ideal gas law.

$$PV=NkT\ \rightarrow\ \frac{N}{V}=\frac{P}{kT}=\frac{1\times10^{-12}\,\text{N/m}^2}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\text{ K}\right)}=\left(3\times10^8\,\frac{\text{molecules}}{\text{m}^3}\right)\left(\frac{10^{-6}\,\text{m}^3}{1\text{ cm}^3}\right)$$

$$=\boxed{300\,\text{molecules/cm}^3}$$

56. We assume an ideal gas at STP. Example 17-10 shows that the molar volume of this gas is 22.4 L. We calculate the actual volume of one mole of gas particles, assuming a volume of ℓ_0^3, and then find the ratio of the actual volume of the particles to the volume of the gas.

$$\frac{V_{\text{molecules}}}{V_{\text{gas}}}=\frac{\left(6.02\times10^{23}\,\text{molecules}\right)\left(\left(3.0\times10^{-10}\,\text{m}\right)^3/\text{molecule}\right)}{\left(22.4\,\text{L}\right)\left(1\times10^{-3}\,\text{m}^3/1\,\text{L}\right)}=\boxed{7.3\times10^{-4}}$$

The molecules take up less than 0.1% of the volume of the gas.

57. We assume that the last breath Galileo took has been spread uniformly throughout the atmosphere since his death. Calculate the number of molecules in Galileo's last breath, and divide it by the volume of the atmosphere, to get "Galileo molecules/m^3". Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breaths.

$$PV = NkT \quad \rightarrow \quad N = \frac{PV}{kT} = \frac{\left(1.01\times10^{5}\,\text{Pa}\right)\left(2.0\times10^{-3}\,\text{m}^3\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)} = 4.9\times10^{22}\,\text{molecules}$$

$$\text{Atmospheric volume} = 4\pi R_{\text{Earth}}^2 h = 4\pi\left(6.38\times10^{6}\,\text{m}\right)^2\left(1.0\times10^{4}\,\text{m}\right) = 5.1\times10^{18}\,\text{m}^3$$

$$\frac{\text{Galileo molecules}}{\text{m}^3} = \frac{4.9\times10^{22}\,\text{molecules}}{5.8\times10^{18}\,\text{m}^3} = 9.6\times10^{3}\,\text{molecules/m}^3$$

$$\frac{\#\text{ Galileo molecules}}{\text{breath}} = 9.6\times10^{3}\,\frac{\text{molecules}}{\text{m}^3}\left(\frac{2.0\times10^{-3}\,\text{m}^3}{1\text{ breath}}\right) = \boxed{19\,\frac{\text{molecules}}{\text{breath}}}$$

58. Use Eq. 17-5a for the constant-volume gas thermometer to relate the boiling point to the triple point.

$$T = \left(273.16\,\text{K}\right)\frac{P}{P_{\text{tp}}} \quad \rightarrow \quad \frac{P_{\text{bp}}}{P_{\text{tp}}} = \frac{T_{\text{bp}}}{273.16\,\text{K}} = \frac{\left(273.15+100\right)\text{K}}{273.16\,\text{K}} = \boxed{1.3660}$$

59. (*a*) For the constant-volume gas thermometer, we use Eq. 17-5a.

$$T = \left(273.16\,\text{K}\right)\frac{P}{P_{\text{tp}}} \quad \rightarrow \quad P_{\text{tp}} = P\frac{273.16\,\text{K}}{T} = \left(187\,\text{torr}\right)\frac{273.16\,\text{K}}{\left(273.15+444.6\right)\text{K}} = \boxed{71.2\,\text{torr}}$$

(*b*) We again use Eq. 17-5a.

$$T = \left(273.16\,\text{K}\right)\frac{P}{P_{\text{tp}}} = \left(273.16\,\text{K}\right)\frac{118\,\text{torr}}{71.2\,\text{torr}} = \boxed{453\,\text{K}} = \boxed{180^\circ\text{C}}$$

60. From Fig. 17-17, we estimate a temperature of 373.35 K from the oxygen curve at a pressure of 268 torr. The boiling point of water is 373.15 K.

(*a*) The inaccuracy is $\Delta T = 373.35\,\text{K} - 373.15\,\text{K} = \boxed{0.20\,\text{K}}$

(*b*) As a percentage, we have the following.

$$\frac{\Delta T}{T}\left(100\right) = \frac{0.20\,\text{K}}{373.15\,\text{K}}\left(100\right) = \boxed{0.054\%}$$

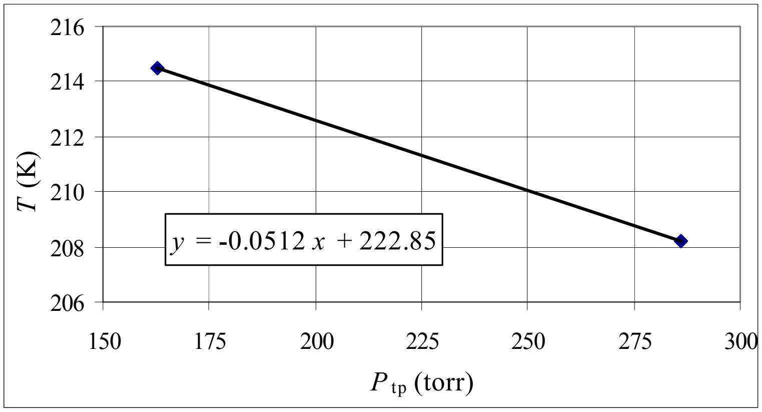
The answers may vary due to differences in reading the graph.

$\boxed{61.}$ Since the volume is constant, the temperature of the gas is proportional to the pressure of the gas. First we calculate the two temperatures of the different amounts of gas.

$$T_1 = \left(273.16\,\text{K}\right)\frac{P_{1\,\text{melt}}}{P_{1\,\text{tp}}} = \left(273.16\,\text{K}\right)\frac{218}{286} = 208.21\,\text{K}$$

$$T_2 = \left(273.16\,\text{K}\right)\frac{P_{2\,\text{melt}}}{P_{2\,\text{tp}}} = \left(273.16\,\text{K}\right)\frac{128}{163} = 214.51\,\text{K}$$

Assume that there is a linear relationship between the melting-point temperature and the triple-point pressure, as shown in Fig. 17-17. The actual melting point is the *y*-intercept of that linear relationship. We use Excel to find that *y*-intercept. The graph is shown below.



We see the melting temperature is $222.85\,\text{K} \approx \boxed{223\,\text{K}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH17.XLS," on tab "Problem 17.61."

62. Since the glass does not expand, the measuring cup will contain 350 mL of hot water. Find the volume of water after it cools.

$$\Delta V = V_0\beta\Delta T = (350\text{ mL})\left(210\times10^{-6}/\text{C}^\circ\right)(20^\circ\text{C}-95^\circ\text{C}) = \boxed{-5.5\,\text{mL}}$$

The volume of cool water is about 5.5 mL less than the desired volume of 350 mL.

63. (*a*) At 36°C, the tape will expand from its calibration, and so will $\boxed{\text{read low}}$.

(*b*) $$\frac{\Delta\ell}{\ell_0} = \alpha\Delta T = \left(12\times10^{-6}/^\circ\text{C}\right)(36^\circ\text{C}-15^\circ\text{C}) = 2.52\times10^{-4} \approx \boxed{2.5\times10^{-2}\,\%}$$

64. The net force on each side of the box will be the pressure difference between the inside and outside of the box, times the area of a side of the box. The outside pressure is 1 atmosphere. The ideal gas law is used to find the pressure inside the box, assuming that the mass of gas and the volume are constant.

$$\frac{P}{T} = \frac{nR}{V} = \text{constant} \rightarrow \frac{P_2}{T_2} = \frac{P_1}{T_1} \rightarrow P_2 = P_1\frac{T_2}{T_1} = (1.00\text{ atm})\frac{(273+185)\,\text{K}}{(273+15)\,\text{K}} = 1.590\text{ atm}$$

The area of a side of the box is given by the following.

$$\text{Area} = \ell^2 = \left[(\text{Volume of box})^{1/3}\right]^2 = \left(6.15\times10^{-2}\,\text{m}^2\right)^{2/3} = 1.5581\times10^{-1}\,\text{m}^2$$

The net force on a side of the box is the pressure difference times the area.

$$F = (\Delta\,\text{Pressure})(\text{Area}) = (0.590\,\text{atm})\left(1.01\times10^5\,\text{Pa}\right)\left(1.5581\times10^{-1}\,\text{m}^2\right) = \boxed{9300\,\text{N}}$$

65. Assume the helium is an ideal gas. The volume of the cylinder is constant, and we assume that the temperature of the gas is also constant in the cylinder. From the ideal gas law, $PV = nRT$, under these conditions the amount of gas is proportional to the absolute pressure.

$$PV = nRT \rightarrow \frac{P}{n} = \frac{RT}{V} = \text{constant} \rightarrow \frac{P_1}{n_1} = \frac{P_2}{n_2} \rightarrow \frac{n_2}{n_1} = \frac{P_2}{P_1} = \frac{5\,\text{atm}+1\,\text{atm}}{32\,\text{atm}+1\,\text{atm}} = \frac{6}{33}$$

Thus $\boxed{6/33 = 0.182 \approx 20\%}$ of the original gas remains in the cylinder.

66. When the rod has a length ℓ, then a small (differential) change in temperature will cause a small (differential) change in length according to Eq. 17-1a, expressed as $d\ell = \alpha\ell\Delta T$.

(*a*) $$d\ell = \alpha\ell\Delta T \rightarrow \frac{d\ell}{\ell} = \alpha\Delta T \rightarrow \int_{\ell_1}^{\ell_2}\frac{d\ell}{\ell} = \int_{T_1}^{T_2}\alpha dT \rightarrow \ln\frac{\ell_2}{\ell_1} = \alpha\left(T_2 - T_1\right) \rightarrow$$

$$\boxed{\ell_2 = \ell_1 e^{\alpha\left(T_2 - T_1\right)}}$$

(*b*) $$\int_{\ell_1}^{\ell_2}\frac{d\ell}{\ell} = \int_{T_1}^{T_2}\alpha dT \rightarrow \ln\frac{\ell_2}{\ell_1} = \int_{T_1}^{T_2}\alpha dT \rightarrow \boxed{\ell_2 = \ell_1 e^{\int_{T_1}^{T_2}\alpha dT}}$$

(*c*) $$d\ell = \alpha\ell\Delta T \rightarrow \frac{d\ell}{\ell} = \alpha\Delta T \rightarrow \int_{\ell_1}^{\ell_2}\frac{d\ell}{\ell} = \int_{T_1}^{T_2}\alpha dT = \int_{T_1}^{T_2}\left(\alpha_0 + bT\right)dT \rightarrow$$

$$\ln\frac{\ell_2}{\ell_1} = \alpha_0\left(T_2 - T_1\right) + \tfrac{1}{2}b\left(T_2^2 - T_1^2\right) \rightarrow \boxed{\ell_2 = \ell_1 e^{\left[\alpha_0\left(T_2 - T_1\right) + \frac{1}{2}b\left(T_2^2 - T_1^2\right)\right]}}$$

67. Assume that the air in the lungs is an ideal gas, that the amount of gas is constant, and that the temperature is constant. The ideal gas law then says that the value of PV is constant. The pressure a distance *h* below the surface of a fluid is given by Eq. 13-6b, $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the fluid. We assume that the diver is in sea water.

$$\left(PV\right)_{\text{surface}} = \left(PV\right)_{\text{submerged}} \rightarrow V_{\text{surface}} = V_{\text{submerged}}\frac{P_{\text{submerged}}}{P_{\text{surface}}} = V_{\text{submerged}}\frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}}$$

$$= \left(5.5\,\text{L}\right)\frac{1.01\times10^5\,\text{Pa} + \left(1025\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(8.0\,\text{m}\right)}{1.01\times10^5\,\text{Pa}} = \boxed{9.9\,\text{L}}$$

This is obviously very dangerous, to have the lungs attempt to inflate to twice their volume. Thus it is $\boxed{\text{not advisable}}$ to quickly rise to the surface.

68. (*a*) Assume the pressure and amount of gas are held constant, and so $P_0V_0 = nRT_0$ and $P_0V = nRT$. From these two expressions calculate the change in volume and relate it to the change in temperature.

$$V = V_0 + \Delta V \rightarrow \Delta V = V - V_0 = \frac{nRT}{P_0} - \frac{nRT_0}{P_0} = \frac{nR}{P_0}\left(T - T_0\right) = \frac{V_0}{T_0}\Delta T$$

But $\Delta V = \beta V_0 \Delta T$, and so $\Delta V = \frac{V_0}{T_0}\Delta T = \beta V_0 \Delta T \rightarrow \beta = \frac{1}{T_0}$

For $T_0 = 293\text{ K}$, $\beta = \frac{1}{T_0} = \frac{1}{293\text{ K}} = \boxed{3.4\times10^{-3}/\text{K}}$, which agrees well with Table 17-1.

(*b*) Assume the temperature and amount of gas are held constant, and so $P_0V_0 = nRT_0 = PV$. From these two expressions calculate change in volume and relate it to the change in pressure.

$$V = V_0 + \Delta V \rightarrow$$

$$\Delta V = V - V_0 = \frac{nRT_0}{P} - \frac{nRT_0}{P_0} = nRT_0\left(\frac{1}{P} - \frac{1}{P_0}\right) = \frac{nRT_0}{P_0}\left(\frac{P_0 - P}{P}\right) = V_0\frac{1}{P}\left(-\Delta P\right)$$

But from Eq. 12-7, $\Delta V = -V_0\frac{1}{B}\Delta P$ and so $\Delta V = V_0\frac{1}{P}\left(-\Delta P\right) = -V_0\frac{1}{B}\Delta P \rightarrow \boxed{B = P}$

69. To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N_2 (molecular weight 28) and 30% O_2 (molecular weight 32), then the average molecular weight is $0.70(28)+0.30(32)=29.2.$

(*a*) Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$PV = nRT \quad \rightarrow \quad n=\frac{PV}{RT}=\frac{\left(1.013\times10^{5}\,\text{Pa}\right)\left(870\,\text{m}^{3}\right)}{\left(8.314\,\text{J/mol•k}\right)\left(293\text{ K}\right)}=3.6178\times10^{4}\,\text{moles}$$

$$m=\left(3.6178\times10^{4}\,\text{moles}\right)\left(29.2\times10^{-3}\text{ kg/mol}\right)=1056.4\,\text{kg}\approx\boxed{1100\,\text{kg}}$$

(*b*) Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$n=\frac{PV}{RT}=\frac{\left(1.013\times10^{5}\,\text{Pa}\right)\left(870\,\text{m}^{3}\right)}{\left(8.314\,\text{J/mol•k}\right)\left(263\text{ K}\right)}=4.0305\times10^{4}\,\text{moles}$$

$$m=\left(4.0305\times10^{4}\,\text{moles}\right)\left(29.2\times10^{-3}\text{ kg/mol}\right)=1176.9\,\text{kg}$$

The mass entering the house is $1176.9\,\text{kg}-1056.4\,\text{kg}=120.5\,\text{kg}\approx\boxed{100\,\text{kg}}$.

70. We are given that $P\propto\dfrac{1}{V^{2}}$ for constant temperature and $V\propto T^{2/3}$ for constant pressure. We also assume that $V\propto n$ for constant pressure and temperature. Combining these relationships gives the following.

$$\boxed{PV^{2}=n^{2}RT^{4/3}}$$

$$R=\frac{PV^{2}}{n^{2}T^{4/3}}=\frac{\left(1.00\,\text{atm}\right)\left(22.4\,\text{L}\right)^{2}}{\left(1.00\,\text{mol}\right)^{2}\left(273.15\,\text{K}\right)^{4/3}}=\frac{\left(1.00\,\text{atm}\right)\left(22.4\,\text{L}\right)^{2}}{\left(1.00\,\text{mol}\right)^{2}\left(273.15\,\text{K}\right)^{4/3}}=\boxed{0.283\frac{\text{L}^{2}\text{•atm}}{\text{mol}^{2}\text{•K}^{4/3}}}$$

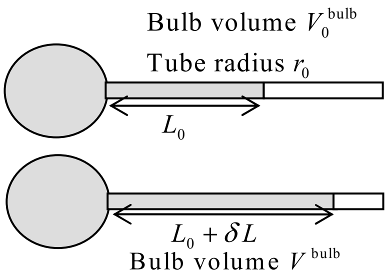
71. (*a*) The iron floats in the mercury because $\rho_{\text{Hg}}>\rho_{\text{Fe}}$. As the substances are heated, the density of both substances will decrease due to volume expansion. The density of the mercury decreases more upon heating than the density of the iron, because $\beta_{\text{Hg}}>\beta_{\text{Fe}}$. The net effect is that the densities get closer together, and so relatively more mercury will have to be displaced to hold up the iron, and the iron will float $\boxed{\text{lower}}$ in the mercury.

(*b*) The fraction of the volume submerged is $V_{\underset{\text{displaced}}{\text{Hg}}}\big/V_{\text{Fe}}$. Both volumes expand as heated. The subscript "displaced" is dropped for convenience.

$$\text{fractional change}=\frac{V_{\text{Hg}}/V_{\text{Fe}}-V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}=\frac{\dfrac{V_{0\,\text{Hg}}\left(1+\beta_{\text{Hg}}\Delta T\right)}{V_{0\,\text{Fe}}\left(1+\beta_{\text{Fe}}\Delta T\right)}-V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}=\frac{\left(1+\beta_{\text{Hg}}\Delta T\right)}{\left(1+\beta_{\text{Fe}}\Delta T\right)}-1$$

$$=\frac{1+\left(180\times10^{-6}/\text{C}^{\circ}\right)\left(25\text{C}^{\circ}\right)}{1+\left(35\times10^{-6}\right)\left(25\text{C}^{\circ}\right)}-1=\frac{1.0045}{1.000875}-1=3.6\times10^{-3}\quad;\quad\%\text{ change}=\boxed{0.36\%}$$

72. (*a*) Consider the adjacent diagrams. The mercury expands due to the heat, as does the bulb volume. The volume of filled glass is equal to the volume of mercury at both temperatures. The value δL is the amount the thread of mercury moves. The additional length of the mercury column in the tube multiplied by the tube cross sectional area will be equal to the expansion of the volume of mercury, minus the expansion of the volume of the glass bulb. Since the tube volume is so much smaller than the bulb volume we can ignore any changes in the tube dimensions and in the mercury initially in the tube volume.



Original volume for glass bulb and Hg in bulb: V_0^{bulb}

Change in glass bulb volume: $\Delta V_{\text{glass}} = V_0^{\text{bulb}} \beta_{\text{glass}} \Delta T$

Change in Hg volume in glass bulb: $\Delta V_{\text{Hg}} = V_0^{\text{bulb}} \beta_{\text{Hg}} \Delta T$

Now find the additional volume of Hg, and use that to find the change in length of Hg in the tube.

$$(\delta L)\pi r_0^2 = \Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = V_0^{\text{bulb}} \beta_{\text{Hg}} \Delta T - V_0^{\text{bulb}} \beta_{\text{glass}} \Delta T \quad \rightarrow$$

$$\delta L = \frac{V_0^{\text{bulb}}}{\pi r_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right) = \frac{V_0^{\text{bulb}}}{\pi \left(d_0/2\right)^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right) = \frac{4V_0^{\text{bulb}}}{\pi d_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)$$

$$= \frac{4\left(0.275\,\text{cm}^3\right)}{\pi\left(1.40\times10^{-2}\,\text{cm}\right)^2} \left(33.0°\text{C} - 10.5°\text{C}\right)\left[\left(180-9\right)\times10^{-6}/\text{C}°\right] = \boxed{6.87\,\text{cm}}$$

(*b*) The formula is quoted above: $\delta L = \dfrac{4V_0^{\text{bulb}}}{\pi d_0^2} \Delta T \left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)$.

$\boxed{73.}$ Since the pressure is force per unit area, if the pressure is multiplied by the surface area of the Earth, the force of the air is found. If we assume that the force of the air is due to its weight, then the mass of the air can be found. The number of molecules can then be found using the molecular mass of air (calculated in problem 71) and Avogadro's number.

$$P = \frac{F}{A} \quad \rightarrow \quad F = PA \quad \rightarrow \quad Mg = P4\pi R_{\text{Earth}}^2 \quad \rightarrow$$

$$M = \frac{4\pi R_{\text{Earth}}^2 P}{g} = \frac{4\pi\left(6.38\times10^6\,\text{m}\right)^2\left(1.01\times10^5\,\text{Pa}\right)}{9.80\,\text{m/s}^2} = 5.27\times10^{18}\,\text{kg}$$

$$N = 5.27\times10^{18}\,\text{kg}\left(\frac{1\text{ mole}}{29\times10^{-3}\,\text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{molecules}}{1\text{ mole}}\right) = \boxed{1.1\times10^{44}\,\text{molecules}}$$

74. The density is the mass divided by the volume. Let the original volume of the mass of iron be V_0, the original density $\rho_0 = M/V_0$. The volume of that same mass deep in the Earth is $V = V_0 + \Delta V$, and so the density deep in the Earth is $\rho = M/V = M/\left(V_0 + \Delta V\right)$. The change in volume is due to two effects: the increase in volume due to a higher temperature, $\Delta V_{\text{temp}} = \beta V_0 \Delta T$, and the decrease in volume due to a higher pressure, $\Delta V_{\text{pressure}} = -V_0 \Delta P/B$. So $\Delta V = \Delta V_{\text{temp}} + \Delta V_{\text{pressure}}$. The new density is then calculated by $\rho = M/\left(V_0 + \Delta V\right)$.

$$\rho = M/V = \frac{M}{V_0 + \Delta V} = \frac{M}{V_0 + \Delta V_{\text{temp}} + \Delta V_{\text{pressure}}} = \frac{M}{V_0 + \beta V_0 \Delta T - V_0\,\Delta P/B} = \frac{M}{V_0}\frac{1}{\left(1+\beta\Delta T - \Delta P/B\right)}$$

$$= \frac{\rho_0}{\left(1+\beta\Delta T - \Delta P/B\right)}$$

$$= \frac{\rho_0}{\left[1+\left(35\times10^{-6}/\text{C}^\circ\right)\left(2000\text{C}^\circ\right)-\left(5000\,\text{atm}\right)\left(1.01\times10^5\,\text{Pa/atm}\right)\big/\left(90\times10^9\ \text{N/m}^2\right)\right]}$$

$$= \frac{\rho_0}{\left[1+0.07-.00561\right]} = 0.9395\rho_0 \quad\rightarrow\quad \boxed{6\%\ \text{decrease}}$$

75. One mole of gas at STP occupies 22.4 L, as found in Example 17-10. We find the volume of the gas per particle for a mole of gas at STP. We then assume that each molecule occupies of cube of side *a*, and then solve for *a* as the average distance between molecules.

$$\left(22.4\frac{\text{L}}{\text{mol}}\right)\left(\frac{1\,\text{mol}}{6.02\times10^{23}\,\text{molecules}}\right)\left(\frac{10^{-3}\text{m}^3}{1\,\text{L}}\right) = 3.72\times10^{-26}\ \text{m}^3/\text{molecule} = a^3$$

$$a = \left(3.72\times10^{-26}\text{m}^2\right)^{1/3} = \boxed{3.34\times10^{-9}\text{m}}$$

76. We find the number of moles of helium in the balloon from the ideal gas equation.

$$PV = nRT \quad\rightarrow\quad n = \frac{PV}{RT} = \frac{(1.06)\left(1.013\times10^5\,\text{Pa}\right)\frac{4}{3}\pi\left(0.220\,\text{m}\right)^3}{\left(8.314\,\text{J/mol}\bullet\text{k}\right)\left(293\,\text{K}\right)} = 1.966\,\text{mol} \approx \boxed{1.97\,\text{mol}}$$

$$1.966\,\text{mol}\left(\frac{4.00\text{g}}{1\ \text{mol}}\right) = \boxed{7.86\,\text{g}}$$

77. We assume the temperature is constant. As the oxygen pressure drops to atmospheric pressure, we can find the volume that it occupies at atmospheric pressure. We assume the final pressure inside the cylinder is atmospheric pressure. The gas would quit flowing at that pressure.

$$P_1V_1 = P_2V_2 \quad\rightarrow\quad V_2 = V_1\frac{P_1}{P_2} = \left(14\,\text{L}\right)\frac{\left(1.38\times10^7\,\text{Pa}+1.013\times10^5\,\text{Pa}\right)}{1.013\times10^5\,\text{Pa}} = 1921\,\text{L}$$

14 L of that gas is not available — it is left in the container. So there is a total of 1907 L available.

$$\left(1907\,\text{L}\right)\frac{1\,\text{min}}{2.4\,\text{L}} = 794.6\,\text{min} \approx \boxed{79\,\text{min}} \approx \boxed{13\,\text{h}}$$

78. The gap will be the radius of the lid minus the radius of the jar. Also note that the original radii of the lid and the jar are the same.

$$r_{\text{gap}} = \left(r_0+\Delta r\right)_{\text{lid}} - \left(r_0+\Delta r\right)_{\text{jar}} = \Delta r_{\text{lid}} - \Delta r_{\text{jar}} = \left(\alpha_{\text{brass}}-\alpha_{\text{glass}}\right)r_0\Delta T$$

$$= \left(19\times10^{-6}/\text{C}^\circ - 9\times10^{-6}/\text{C}^\circ\right)\left(4.0\ \text{cm}\right)\left(60\,\text{C}^\circ\right) = \boxed{2.4\times10^{-3}\text{cm}}$$

$\boxed{79.}$ (*a*) Assume that a mass M of gasoline with volume V_0 at 0°C is under consideration, and so its density is $\rho_0 = M/V_0$. At a temperature of 35°C, the same mass has a volume $V = V_0\left(1+\beta\Delta T\right)$.

$$\rho=\frac{M}{V}=\frac{M}{V_0\left(1+\beta\Delta T\right)}=\frac{\rho_0}{1+\beta\Delta T}=\frac{0.68\times10^3\,\mathrm{kg/m^3}}{1+\left(950\times10^{-6}/\mathrm{C}^\circ\right)\left(35\mathrm{C}^\circ\right)}=0.6581\times10^3\,\mathrm{kg/m^3}$$

$$\approx\boxed{660\,\mathrm{kg/m^3}}$$

(*b*) Calculate the percentage change in the density.

$$\%\text{ change}=\frac{\left(0.6581-0.68\right)\times10^3\,\mathrm{kg/m^3}}{0.68\times10^3\,\mathrm{kg/m^3}\,V}\times100=\boxed{-3\%}$$

80. (*a*) From Example 13-5, we have that the pressure of the atmosphere varies as $P_{\text{air}}=\left(P_{\text{air}}\right)_0 e^{-cy}$, where $c=\dfrac{\rho_0 g}{P_0}$, with the subscript indicating to use the value at $y=0$. We assume that the helium is an ideal gas, that the helium pressure is 1.05 times the atmospheric pressure, and that the helium temperature is the same as the surrounding air.

$$\frac{\left(P_{\text{He}}\right)_0\left(V_{\text{He}}\right)_0}{T_0}=\frac{P_{\text{He}}V_{\text{He}}}{T_1}\rightarrow\frac{\left(1.05P_{\text{air}}\right)_0\left(V_{\text{He}}\right)_0}{T_0}=\frac{1.05P_{\text{air}}V_{\text{He}}}{T_1}\rightarrow$$

$$\frac{\left(P_{\text{air}}\right)_0\left(V_{\text{He}}\right)_0}{T_0}=\frac{\left(P_{\text{air}}\right)_0 e^{-cy}V_{\text{He}}}{T_1}\rightarrow\frac{\left(V_{\text{He}}\right)_0}{T_0}=\frac{e^{-cy}V_{\text{He}}}{T_1}\rightarrow\boxed{V=V_0\frac{T_1}{T_0}e^{+cy}}$$

$$c=\frac{\rho_0 g}{P_0}=\frac{\left(1.29\,\mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)}{\left(1.013\times10^5\,\mathrm{Pa}\right)}=\boxed{1.25\times10^{-4}\,\mathrm{m}^{-1}}$$

(*b*) The buoyant force is the weight of the air displaced by the balloon, which would be the density of the air, times the volume of the balloon, times the acceleration due to gravity. The density of the air displaced by the balloon can be found from the ideal gas equation, applied at any particular location.

$$F_{\text{buoy}}=\rho_{\text{air}}V_{\text{balloon}}g\ ;$$

$$P_{\text{air}}V_{\text{air}}=n_{\text{air}}RT=\frac{m_{\text{air}}}{\left(\text{mol. mass}\right)_{\text{air}}}RT\rightarrow\frac{P_{\text{air}}\left(\text{mol. mass}\right)_{\text{air}}}{RT}=\frac{m_{\text{air}}}{V_{\text{air}}}=\rho_{\text{air}}$$

$$F_{\text{buoy}}=\rho_{\text{air}}V_{\text{balloon}}g=\frac{P_{\text{air}}\left(\text{mol. mass}\right)_{\text{air}}}{RT}V_{\text{balloon}}g=\frac{P_{\text{air}}V_{\text{balloon}}}{RT}g\left(\text{mol. mass}\right)_{\text{air}}$$

$$=\frac{\left(\dfrac{P_{\text{He}}}{1.05}\right)V_{\text{balloon}}}{RT}g\left(\text{mol. mass}\right)_{\text{air}}=\frac{P_{\text{He}}V_{\text{balloon}}}{RT}\frac{g\left(\text{mol. mass}\right)_{\text{air}}}{1.05}=n_{\text{He}}\frac{g\left(\text{mol. mass}\right)_{\text{air}}}{1.05}$$

The final expression is constant since the number of moles of helium in the balloon is constant.

81. The change in length is to be restricted to $\Delta\ell<1.0\times10^{-6}\,\mathrm{m}$.

$$\Delta\ell=\alpha\ell_0\Delta T\le1.0\times10^{-6}\,\mathrm{m}\rightarrow\Delta T\le\frac{1.0\times10^{-6}\,\mathrm{m}}{\left(9\times10^{-6}/\mathrm{C}^\circ\right)\left(1.0\ \mathrm{m}\right)}\le0.11\mathrm{C}^\circ$$

Thus the temperature would have to be controlled to within $\boxed{\pm0.11\mathrm{C}^\circ}$

82. (*a*) Treat the air as an ideal gas. Since the amount and temperature of the air are the same in both cases, the ideal gas law says $PV = nRT$ is a constant.

$$P_2V_2 = P_1V_1 \quad\rightarrow\quad V_2 = V_1\frac{P_1}{P_2} = (11.3\text{ L})\frac{180\,\text{atm}}{1.00\,\text{atm}} = 2034\,\text{L} \approx \boxed{2030\,\text{L}}$$

(*b*) Before entering the water, the air coming out of the tank will be at 1.00 atm pressure, and so the person will be able to breathe $2034\,\text{L}$ of air.

$$t = 2034\,\text{L}\left(\frac{1\text{ breath}}{2.0\text{ L}}\right)\left(\frac{1\text{ min}}{12\text{ breaths}}\right) = 84.75\,\text{min} \approx \boxed{85\,\text{min}}$$

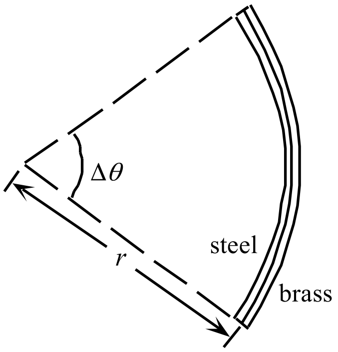
(*c*) When the person is underwater, the temperature and pressure will be different. Use the ideal gas equation to relate the original tank conditions to the underwater breathing conditions. The amount of gas will be constant, so $PV/T = nR$ will be constant. The pressure a distance h below the surface of the water is given in Eq. 13-6b, $P = P_0 + \rho g h$, where P_0 is atmospheric pressure and ρ is the density of the sea water.

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \quad\rightarrow\quad V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1}$$

$$V_2 = (11.3\text{ L})\left[\frac{180\text{ atm}\left(1.01\times10^5\,\text{Pa/atm}\right)}{1.01\times10^5\,\text{Pa} + \left(1.025\times10^3\text{ kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)(20.0\text{ m})}\right]\left(\frac{283\text{ K}}{293\text{ K}}\right)$$

$$= 6.572\times10^2\text{ L} \qquad t = 6.572\times10^2\text{ L}\left(\frac{1\text{ breath}}{2.0\text{ L}}\right)\left(\frac{1\text{ min}}{12\text{ breaths}}\right) = 27.38\,\text{min} \approx \boxed{27\,\text{min}}$$

83. We will take the average radius of curvature as being the radius to the boundary between the two materials, and so is equal to the radius of the inside curve of the steel, plus the thickness of the steel. Each strip, when curved, subtends the same angle $\Delta\theta$.



$$\Delta\theta = \frac{r_{\text{steel}}}{\ell_{\text{steel}}} = \frac{r_{\text{brass}}}{\ell_{\text{brass}}} \quad\rightarrow\quad \frac{r_{\text{steel}}}{\ell_0 + \Delta\ell_1} = \frac{r_{\text{brass}}}{\ell_0 + \Delta\ell_2} \quad\rightarrow$$

$$\frac{r_{\text{steel}}}{\ell_0 + \alpha_{\text{steel}}\ell_0\Delta T} = \frac{r_{\text{brass}}}{\ell_0 + \alpha_{\text{brass}}\ell_0\Delta T} \quad\rightarrow\quad \frac{r_{\text{steel}}}{1 + \alpha_{\text{steel}}\Delta T} = \frac{r_{\text{brass}}}{1 + \alpha_{\text{brass}}\Delta T}$$

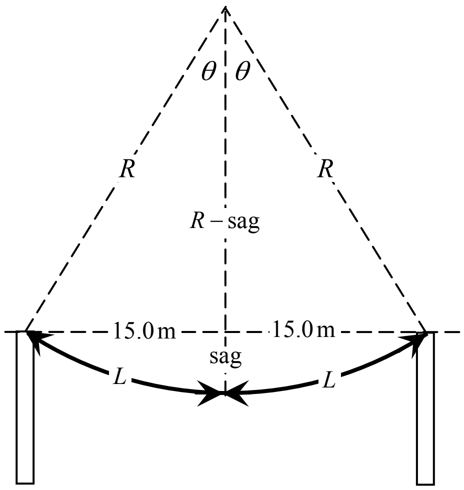
Use the relationship that the radius of the inside curve of the brass is equal to the radius of the inside curve of the steel, plus the thickness of the steel, so $r_{\text{brass}} = r_{\text{steel}} + t$.

$$\frac{r_{\text{steel}}}{1 + \alpha_{\text{steel}}\Delta T} = \frac{r_{\text{steel}} + t}{1 + \alpha_{\text{brass}}\Delta T} \quad\rightarrow\quad r_{\text{steel}} = \left(\frac{1 + \alpha_{\text{steel}}\Delta T}{1 + \alpha_{\text{brass}}\Delta T}\right)\left(r_{\text{steel}} + t\right) \quad\rightarrow$$

$$r_{\text{steel}} = \frac{t}{\left(\dfrac{1 + \alpha_{\text{brass}}\Delta T}{1 + \alpha_{\text{steel}}\Delta T} - 1\right)} = \frac{0.20\,\text{cm}}{\left[\dfrac{1 + \left(19\times10^{-6}/\text{°C}\right)(80\,\text{C°})}{1 + \left(12\times10^{-6}/\text{°C}\right)(80\,\text{C°})} - 1\right]} = 357.49\,\text{cm}$$

$$r = r_{\text{steel}} + t = 357.49\,\text{cm} + 0.20\,\text{cm} = 357.69\,\text{cm} \approx \boxed{3.6\,\text{m}}$$

84. Consider this basic geometry for the problem, with the assumption that the shape of the sagging wire is an arc of a circle. The amount of sag is greatly exaggerated in the figure. A subscript of "0" will be used for the original (low temperature) configuration, and no subscript will be used for the final (high temperature) configuration. The variable "*s*" will be used for the amount of "sag." Note that "*L*" refers to half the length of the sagging wire.



$$(15.0\,\text{m})^2 + (R_0 - s_0)^2 = R_0^2 \quad \rightarrow$$

$$R_0 = \frac{(15.0\,\text{m})^2 + s_0^2}{2s_0} = \frac{(15.0\,\text{m})^2 + (0.500\,\text{m})^2}{2(0.500\,\text{m})} = 225.25\,\text{m}$$

$$\theta_0 = \sin^{-1}\frac{15.0\,\text{m}}{225.25\,\text{m}} = 6.6642\times10^{-2}\,\text{rad}$$

$$L_0 = R_0\theta_0 = (225.25\,\text{m})(6.6642\times10^{-2}\,\text{rad}) = 15.011\,\text{m}$$

Now let the wire expand due to heating.

$$L = L_0(1+\alpha\Delta T) = 15.011\,\text{m}\left(\left[1+\left(17\times10^{-6}\,(\text{C}^\circ)^{-1}\right)(50\,\text{C}^\circ)\right]\right) = 15.024\,\text{m}$$

$$\sin\theta = \frac{15.0}{R} \quad \rightarrow \quad \theta = \sin^{-1}\frac{15.0}{R} \quad ; \quad \theta = \frac{L}{R} = \frac{15.024}{R}$$

These two expressions for θ cannot be solved analytically. When solved numerically, the result is $R = 153.42\,\text{m}$. Use this value to find the new "sag." Note that we ignore s^2 since $s \ll R$.

$$(15.0\,\text{m})^2 + (R-s)^2 = R^2 \quad \rightarrow \quad (15.0\,\text{m})^2 - 2Rs + s^2 = 0 \ ;$$

$$s \approx \frac{(15.0\,\text{m})^2}{2R} = \frac{(15.0\,\text{m})^2}{2(153.42\,\text{m})} = \boxed{73.3\,\text{cm}}$$

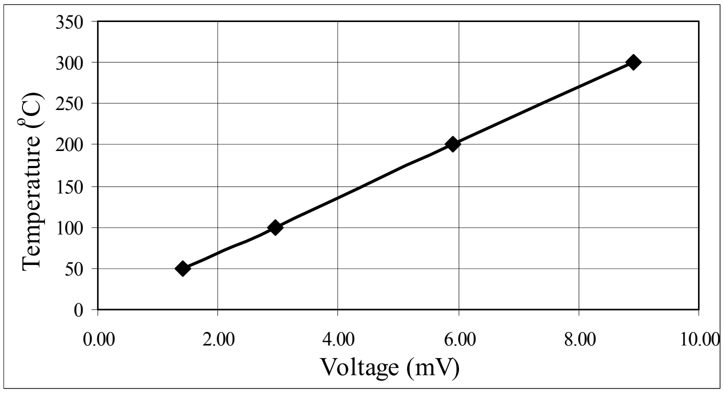
$\boxed{85.}$ We assume ideal gas behavior for the air in the lungs, and a constant temperature for the air in the lungs. When underwater, we assume the relaxed lung of the diver is at the same pressure as the surrounding water, which is given by Eq. 13-6b, $P = P_0 + \rho gh$. In order for air to flow through the snorkel from the atmospheric air above the water's surface (assume to be at atmospheric pressure), the diver must reduce the pressure in his lungs to atmospheric pressure or below, by increasing the volume of the lungs. We assume that the diver is in sea water.

$$P_{\text{relaxed}}V_{\text{relaxed}} = P_{\text{inhaling}}V_{\text{inhaling}} \quad \rightarrow \quad \frac{V_{\text{inhaling}}}{V_{\text{relaxed}}} = \frac{P_{\text{relaxed}}}{P_{\text{inhaling}}} = \frac{P_{\text{underwater}}}{P_{\text{atmospheric}}} = \frac{P_0 + \rho gh}{P_0} = 1 + \frac{\rho gh}{P_0}$$

$$\frac{\Delta V}{V_{\text{relaxed}}} = \frac{V_{\text{inhaling}} - V_{\text{relaxed}}}{V_{\text{relaxed}}} = \frac{V_{\text{inhaling}}}{V_{\text{relaxed}}} - 1 = \frac{\rho gh}{P_0} = \frac{(1025\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(0.30\text{m})}{1.013\times10^5\,\text{Pa}} = \boxed{0.030}$$

This is a 3% increase.

86. Since the problem is asking to find the temperature for a given voltage, we will graph temperature vs. voltage. The graph is shown here.



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH17.XLS," on tab "Problem 17.86."

When a cubic equation is used to fit this data, this equation results.

$$T = \left(-7.22\times10^{-2}\right)V^3 + \left(1.132\right)V^2 + \left(28.39\right)V + 7.926$$

The equation assumes that the voltage is in mV and the temperature is in °C.

$$T\left(3.21\,\text{mV}\right) = \left(-7.22\times10^{-2}\right)\left(3.21\right)^3 + \left(1.132\right)\left(3.21\right)^2 + \left(28.39\right)\left(3.21\right) + 7.926 = \boxed{108^\circ\text{C}}$$

When a quadratic equation is used to fit this data, the following equation results.

$$T = \left(8.996\times10^{-3}\right)V^2 + \left(33.30\right)V + 2.452$$

The equation assumes that the voltage is in mV and the temperature is in °C.

$$T\left(3.21\,\text{mV}\right) = \left(8.996\times10^{-3}\right)\left(3.21\right)^2 + \left(33.30\right)\left(3.21\right) + 2.452 = \boxed{109^\circ\text{C}}$$

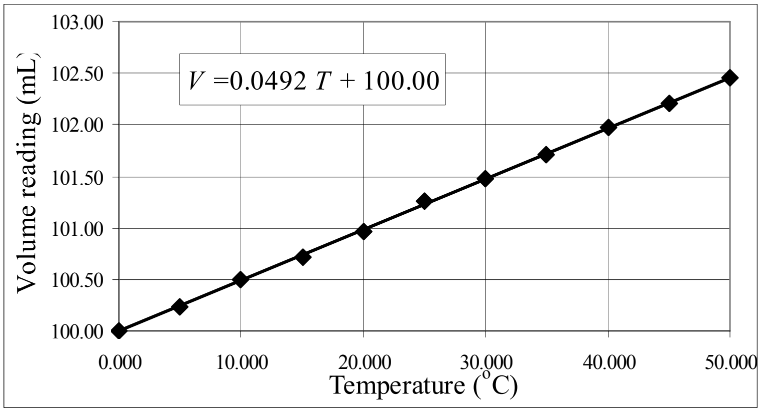
87. Both the glass and the liquid expand. The expansion of the liquid would cause the volume reading to increase, but the expansion of the glass would cause the volume reading to decrease. So the actual change in reading is the difference in those two volume changes. We use the subscript "ℓ" for the liquid and "g" for the glass. We see from the data that the volume readings are increasing with temperature, and so the volume increase of the liquid is more than the volume increase of the glass.

$$\Delta V_{\text{reading}} = V_\ell - V_{\text{g}} = V_0\left(1+\beta_\ell\Delta T\right) - V_0\left(1+\beta_{\text{g}}\Delta T\right) = V_0\left(\beta_\ell - \beta_{\text{g}}\right)\Delta T$$

From this expression, if the graph of V vs. T is linear, it should have a slope of $m = V_0\left(\beta_\ell - \beta_{\text{g}}\right)$.

Thus we can find the coefficient of expansion of the liquid from $\beta_\ell = \dfrac{m}{V_0} + \beta_{\text{g}}$.

From the graph, the slope is seen to be $\boxed{0.0492\,\text{mL/C}^\circ}$. The effective coefficient of volume expansion is



$$\left(\beta_\ell - \beta_{\text{g}}\right) = \frac{m}{V_0} = \frac{0.0492\,\text{mL/C}^\circ}{100.00\,\text{mL}}$$

$$= \boxed{4.92\times10^{-4}/\text{C}^\circ}.$$

$$\beta_\ell = \frac{0.0492\,\text{mL/C}^\circ}{100.00\,\text{mL}} + 9\times10^{-6}/\text{C}^\circ$$

$$= \boxed{5.01\times10^{-4}/\text{C}^\circ}\ ;\ \boxed{\text{glycerin}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH17.XLS," on tab "Problem 17.87."

CHAPTER 18: Kinetic Theory of Gases

Responses to Questions

1. One of the fundamental assumptions for the derivation of the ideal gas law is that the average separation of the gas molecules is much greater than the diameter of the molecules. This assumption eliminates the need to consider the different sizes of the molecules.

2. The change in temperature when a gas is compressed or when it expands against a piston is due to the increase or decrease in the average speed of the molecules. The increase or decrease in speed comes about when the gas molecules collide elastically with the moving piston. In the case of compression, the piston is moving toward the molecules. The net result is an increase in the momentum of the gas molecules. When the gas expands, the piston is moving away from the gas molecules. In this case the net result of collisions between the molecules and the piston is a decrease in the momentum of the molecules. (See Section 9-5.)

3. If the walls are at the same temperature as the gas, then the gas molecules will not lose (or gain) energy in collisions with the walls, and so it is not necessary to specify that the collisions must be elastic.

4. Charles's law states that if pressure is held constant, volume is proportional to temperature. The average kinetic energy of the gas molecules is also proportional to temperature. According to kinetic theory, pressure is proportional to the average kinetic energy of the gas molecules per unit volume. Therefore, if the temperature increases, the average kinetic energy also increases by the same factor. In order to keep the pressure constant, the volume must also increase, again by the same factor.

5. Gay-Lussac's law states that if volume is constant, the pressure in a gas is proportional to the temperature. Kinetic theory tells us that temperature and the product of pressure and volume are proportional to the kinetic energy of the gas molecules. If volume is held constant, then temperature and pressure are both proportional to the kinetic energy, and so are proportional to each other.

6. Near the surface of the Earth, the N_2 molecules and the O_2 molecules are all at the same temperature and therefore have the same average kinetic energies. Since N_2 molecules are lighter than O_2 molecules, the N_2 molecules will have a higher average speed, which allows them to travel higher (on average) in the atmosphere than the O_2 molecules.

7. For an absolute vacuum, no. But for most "vacuums," there are still a few molecules in the containers, and the temperature can be determined from the (very low) pressure.

8. Temperature is a macroscopic variable, measured for a whole system. It is related to the average molecular kinetic energy, which is a microscopic variable.

9. At both temperatures (310 K and 273 K), the lower limit for molecular speed is zero. However, the higher temperature gas (310 K) will have more molecules with higher speeds. Since the total number of molecules is the same, the higher temperature gas must have fewer molecules at the peak speed. (Kinetic theory predicts that the *relative* number of molecules with higher speeds increases with increasing temperature.)

10. (*a*) Because the escape velocity for the Moon is 1/5 that of the Earth, molecules with lower speeds will be able to escape. The Moon may have started with an atmosphere, but over time most of the molecules of gas have escaped.

(*b*) Hydrogen is the lightest gas. For a given kinetic energy (temperature) it has the highest speed and will be most likely to escape.

11. Velocity is a vector quantity. When the velocity is averaged, the direction must be taken into account. Since the molecules travel in random paths, with no net displacement (the container is at rest), the average velocity will have to be zero. Speed is a scalar quantity, so only the (positive) magnitude is considered in the averaging process. The molecules are not at rest, so the average speed will not be zero.

12. (*a*) If the pressure is doubled while the volume is held constant, the temperature also doubles. v_{rms} is proportional to the square root of temperature, so it will increase by a factor of the square root of two.
    (*b*) The average velocity is also proportional to the square root of the temperature, so it will increase by a factor of the square root of two as well.

$\boxed{13.}$ Evaporation. Only molecules in a liquid that are traveling fast enough will be able to escape the surface of the liquid and evaporate.

14. No. Boiling occurs when the saturated vapor pressure and the external pressure are equal. At that point bubbles will be able to form in the liquid. For water at 100ºC, the saturated vapor pressure is 1 atm. If the external pressure is also 1 atm, then the water will boil at 100ºC. If the water is at 100ºC and the external pressure is greater than 1 atm (such as in a pressure cooker), the saturated vapor pressure will still be 1 atm, but bubbles will not be able to form and the water will not boil because the external pressure is higher. The saturated vapor pressure does not depend on the external pressure, but the temperature of boiling does.

15. If alcohol evaporates more quickly than water at room temperature, then it must be easier for the alcohol molecules to escape from the surface of the liquid. Alcohol molecules are more massive than water molecules, and so will not be moving as fast at the same temperature. We can therefore infer that the attractive intermolecular forces between the alcohol molecules are less than the forces between the water molecules.

16. On a hot day, cooling occurs through evaporation of perspiration. If the day is hot and *dry*, then the partial pressure of water vapor in the air will be low and evaporation will readily occur, since the saturated vapor pressure for water will be higher than the external pressure. If the day is hot and *humid*, then the partial pressure of water vapor in the air will be much higher and the air will be holding all or nearly all the water vapor it can. In this case evaporation will not occur as readily, resulting in less cooling.

17. Yes. If you place the water and its container in a vessel that can be evacuated (depressurized), and pump the air out of the vessel, the water will boil at room temperature.

18. Boiling occurs when the saturated vapor pressure equals the external pressure. When we say the oxygen "boils" at –183ºC, we mean that the saturated vapor pressure for oxygen will be 1 atm (the same as atmospheric pressure) at a temperature of –183ºC. At this temperature and pressure, liquid oxygen will vaporize.

$\boxed{19.}$ The freezing point of water decreases slightly with higher pressure. The wire exerts a large pressure on the ice (due to the weights hung at each end). The ice under the wire will melt, allowing the wire to move lower into the block. Once the wire has passed a given position, the water now above the wire will have only atmospheric pressure on it and will refreeze. This process allows the wire to pass all the way through the block and yet leave a solid block of ice.

20. The humid air will be more dense than the dry air at the same temperature because it will have more water vapor suspended in it.

21. (*a*) A pressure cooker, by definition, increases the pressure on what is inside it. An increased pressure yields a higher boiling point. The water in which the food is usually prepared will boil at a higher temperature than normal, thereby cooking the food faster. (*b*) At high altitudes, the atmospheric pressure is less than it is at sea level. If atmospheric pressure decreases, the boiling point of water will decrease. Boiling occurs at a lower temperature. Food (including pasta and rice) will need to cook longer at this lower temperature to be properly prepared. (*c*) It is actually easier to boil water at higher altitude, because it boils at a lower temperature.

22. Both “vapor” and “gas” refer to a substance in the gaseous state. They differ in that a vapor is below the critical temperature and a gas is above the critical temperature for the substance.

23. (*a*) Yes. As an example, think of ice skating. The pressure from the weight of the skater melts the ice, and the skater glides on a thin layer of water.
    (*b*) No. See Figure 18-6. The solid–liquid interface has a positive slope, and so it is not possible to melt carbon dioxide simply by applying pressure.

24. Dry ice is carbon dioxide in the solid state. As shown in Figure 18-6, carbon dioxide at room temperature will be a vapor unless it is at a pressure several times atmospheric pressure. When brought to room temperature, the dry ice sublimates and therefore does not last long.

$\boxed{25.}$ Liquid CO_2 can exist at temperatures between –56.6ºC and 31ºC and pressures between 5.11 atm and 73 atm. (See Figure 18-6.) CO_2 can exist as a liquid at normal room temperature, if the pressure is between 56 and 73 atm.

26. Exhaled air contains a large amount of water vapor and is initially at a temperature equal to body temperature. When the exhaled air comes into contact with the external air on a cold day it cools rapidly and reaches the dew point. At the dew point temperature, the air can no longer hold all the water vapor and water condenses into little droplets, forming a cloud.

27. A sound wave can be described as a pressure wave or a displacement wave. Transmission of the wave depends on the collisions of the gas molecules and their displacements away from an equilibrium position. If the wavelength of a sound wave is less than or equal to the mean free path of the molecules in a gas, then there is no net displacement from the equilibrium position and the sound wave will be “lost” in the movement of the molecules. The forces between the molecules will not be large enough to transmit the sound wave.

28. Ways to reduce the mean free path in a gas include increasing the size of the gas molecules and increasing the density of the gas. Gas density can be increased either by increasing the number of molecules or by decreasing the volume.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

$\boxed{1.}$ (*a*) The average translational kinetic energy of a gas molecule is $\frac{3}{2}kT$.

$$K_{\text{avg}} = \tfrac{3}{2}kT = \tfrac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(273\,\text{K}\right) = \boxed{5.65\times10^{-21}\,\text{J}}$$

(*b*) The total translational kinetic energy is the average kinetic energy per molecule, times the number of molecules.

$$KE_{\text{total}} = N\left(KE_{\text{avg}}\right) = \left(1.0\,\text{mol}\right)\left(\frac{6.02\times10^{23}\,\text{molecules}}{1}\right)\tfrac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(298\,\text{K}\right)$$

$$= \boxed{3700\,\text{J}}$$

2. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. Helium has an atomic mass of 4.0.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(6000\ \text{K}\right)}{4.0\left(1.66\times10^{-27}\,\text{kg}\right)}} = 6116\,\text{m/s} \approx \boxed{6\times10^{3}\ \text{m/s}}$$

3. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature must be in Kelvins.

$$\frac{\left(v_{\text{rms}}\right)_2}{\left(v_{\text{rms}}\right)_1} = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{453\ \text{K}}{273\ \text{K}}} = \boxed{1.29}$$

4. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. Since the rms speed is proportional to the square root of the absolute temperature, to triple the rms speed without changing the mass, the absolute temperature must be multiplied by a factor of 9.

$$T_{\text{fast}} = 4T_{\text{slow}} = 9\left(273+20\right)\text{K} = 2637\ \text{K} = \boxed{2364°\text{C}}$$

5. The average kinetic molecular energy is $\frac{3}{2}kT$. Set this equal to the kinetic energy of the paper clip.

$$\tfrac{1}{2}mv^2 = \tfrac{3}{2}kT \quad\rightarrow\quad v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(288\,\text{K}\right)}{1.0\times10^{-3}\text{kg}}} = \boxed{3.5\times10^{-9}\ \text{m/s}}$$

6. (*a*) The average molecular kinetic energy is $\frac{3}{2}kT$, so the total kinetic energy for a mole would be Avogadro's number times $\frac{3}{2}kT$.

$$K = N_0\left(\tfrac{3}{2}kT\right) = \tfrac{3}{2}RT = \tfrac{3}{2}\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(273\,\text{K}+15\,\text{K}\right) = \boxed{3740\ \text{J}}$$

(*b*) $K = \frac{1}{2}mv^2 = 3704\,\text{J} \rightarrow v = \sqrt{\frac{2\left(3740\,\text{J}\right)}{65\,\text{kg}}} = \boxed{11\,\text{m/s}}$

$\boxed{7.}$ The mean (average) speed is as follows.

$$v_{\text{avg}} = \frac{6.0+2.0+4.0+6.0+0.0+4.0+1.0+8.0+5.0+3.0+7.0+8.0}{12} = \frac{54.0}{12} = \boxed{4.5}.$$

The rms speed is the square root of the mean (average) of the squares of the speeds.

$$v_{\text{rms}} = \sqrt{\frac{6.0^2+2.0^2+4.0^2+6.0^2+0.0^2+4.0^2+1.0^2+8.0^2+5.0^2+3.0^2+7.0^2+8.0^2}{12}}$$

$$= \sqrt{\frac{320}{12}} = \boxed{5.2}$$

8. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = 1.020 = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} \quad \rightarrow$$

$$T_2 = T_1(1.020)^2 = (293.15\,\text{K})(1.020)^2 = 305.0\,\text{K} = \boxed{31.8^\circ\text{C}}$$

9. From the ideal gas law, $PV = nRT$, if the volume and amount of gas are held constant, the temperature is proportional to the pressure, $PV = nRT \;\rightarrow\; P = \frac{nR}{V}T = (\text{constant})T$. Thus the temperature will be tripled. Since the rms speed is proportional to the square root of the temperature, $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$, v_{rms} will be multiplied by a factor of $\boxed{\sqrt{3}} \approx 1.73$.

10. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \;\rightarrow\; kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = \frac{M}{V}$. Combining these relationships gives the following.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

11. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} \quad \rightarrow \quad \boxed{\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \sqrt{\frac{m_1}{m_2}}}$$

12. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3k}{m}\frac{PV}{nR}} = \sqrt{\frac{3(1.38\times10^{-23}\,\text{J/K})}{28(1.66\times10^{-27}\,\text{kg})}\frac{(3.1\,\text{atm})(1.013\times10^5\,\text{Pa/atm})(8.5\,\text{m}^3)}{(1800\text{ mol})(8.314\,\text{J/mol}\bullet\text{K})}}$$

$$= 398.6\,\text{m/s} \approx \boxed{4.0\times10^2\,\text{m/s}}$$

13. From Eq. 18-5, we have $v_{\text{rms}} = \sqrt{3kT/m}$.

(*a*) $$\frac{dv_{\text{rms}}}{dT} = \frac{d}{dT}\left(\frac{3kT}{m}\right)^{1/2} = \frac{1}{2}\left(\frac{3k}{m}\right)^{1/2}\frac{1}{T^{1/2}} = \frac{1}{2}\left(\frac{3kT}{m}\right)^{1/2}\frac{1}{T} = \boxed{\frac{1}{2}\frac{v_{\text{rms}}}{T}}$$

$$\Delta v_{\text{rms}} \approx \frac{dv_{\text{rms}}}{dT}\Delta T = \frac{1}{2}\frac{v_{\text{rms}}}{T}\Delta T \quad\rightarrow\quad \frac{\Delta v_{\text{rms}}}{v_{\text{rms}}} \approx \boxed{\frac{1}{2}\frac{\Delta T}{T}}$$

(*b*) The temperature must be calculated in Kelvin for the formula to be applicable. We calculate the percent change relative to the winter temperature.

$$\frac{\Delta v_{\text{rms}}}{v_{\text{rms}}} \approx \frac{1}{2}\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{30\,\text{K}}{268\,\text{K}}\right) = 0.056 = \boxed{5.6\%}$$

14. Assume that oxygen is an ideal gas, and that each molecule occupies the same cubical volume of ℓ^3. Find the volume per molecule from the ideal gas law, and then the side length of that cubical molecular volume will be an estimate of the average distance between molecules.

$$PV = NkT \quad\rightarrow\quad \frac{V}{N} = \frac{kT}{P} = \frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\text{ K}\right)}{1.01\times10^{5}\,\text{Pa}} = 3.73\times10^{-26}\,\text{m}^3/\text{molecule}$$

$$\ell = \left(\frac{kT}{P}\right)^{1/3} = \left(\frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\text{ K}\right)}{1.01\times10^{5}\,\text{Pa}}\right)^{1/3} = \boxed{3.34\times10^{-9}\,\text{m}}$$

15. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{\left(v_{\text{rms}}\right)_{^{235}\text{UF}_6}}{\left(v_{\text{rms}}\right)_{^{235}\text{UF}_6}} = \frac{\sqrt{3kT/m_{^{235}\text{UF}_6}}}{\sqrt{3kT/m_{^{238}\text{UF}_6}}} = \sqrt{\frac{m_{^{238}\text{UF}_6}}{m_{^{235}\text{UF}_6}}} = \sqrt{\frac{238+6(19)}{235+6(19)}} = \sqrt{\frac{352}{349}} = \boxed{1.004}$$

16. Gas molecules will rush into the vacuum from all directions. An estimate for the time for air to refill this vacuum region is the radius of the region divided by the rms speed of the molecules.

$$\Delta t = \frac{\Delta d}{v_{\text{rms}}} = \frac{\Delta d}{\sqrt{3kT/m}} = \frac{0.01\,\text{m}}{\sqrt{\dfrac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\text{ K}\right)}{29\left(1.66\times10^{-27}\,\text{kg}\right)}}} = \boxed{2\times10^{-5}\,\text{s}}$$

17. (*a*) The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\text{ K}\right)}{32\left(1.66\times10^{-27}\,\text{kg}\right)}} = \boxed{461\,\text{m/s}}$$

(*b*) Assuming that the particle has no preferred direction, then we have the following:

$$v_{\text{rms}}^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2 \quad\rightarrow\quad v_x = v_{\text{rms}}/\sqrt{3}.$$

The time for one crossing of the room is then given by $t = d/v_x = \sqrt{3}d/v_{\text{rms}}$, and so the time for a round trip is $2\sqrt{3}d/v_{\text{rms}}$. Thus the number of back and forth round trips per second is the reciprocal of this time, $\dfrac{v_{\text{rms}}}{2\sqrt{3}d}$.

$$\#\text{ round trips per sec} = \frac{v_{\text{rms}}}{2\sqrt{3}d} = \frac{461\,\text{m/s}}{2\sqrt{3}\left(5.0\text{ m}\right)} = 26.6 \approx \boxed{26\text{ round trips per sec}}$$

18. (*a*) The average time for a molecule to travel from one side of the box to the other and back again is simply the round-trip distance, say in the *x* direction, divided by the average *x* speed of the molecule. The frequency of collisions for that molecule is the reciprocal of that round-trip time. The overall frequency of collisions is *N* times the frequency for a single particle. Use the ideal gas law to relate the number of particles in the room to the gas parameters of pressure, volume, and temperature.

$$t_{\substack{\text{round}\\ \text{trip}}} = \frac{\Delta x}{\overline{v}_x} = \frac{2\ell}{\overline{v}_x}\ ;\ PV = NkT \ \rightarrow\ N = \frac{PV}{kT} = \frac{P\ell^3}{kT}$$

$$f = N\frac{1}{t_{\substack{\text{round}\\ \text{trip}}}} = \frac{N\overline{v}_x}{2\ell} = \frac{\dfrac{P\ell^3}{kT}\overline{v}_x}{2\ell} = \boxed{\frac{\overline{v}_x}{2}\frac{P}{kT}\ell^2}$$

(*b*) We approximate that $\overline{v}_x \approx \sqrt{\overline{v_x^2}}$. From section 18-1, we have that $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$ and Eq. 18-4, $\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$. Combine these results with the result from part (*a*).

$$f = \frac{\overline{v}_x}{2}\frac{P}{kT}\ell^2 \approx \frac{\sqrt{\overline{v_x^2}}}{2}\frac{P}{kT}\ell^2 = \frac{\sqrt{\frac{1}{3}\overline{v^2}}}{2}\frac{P}{kT}\ell^2 = \frac{\sqrt{\frac{1}{3}3\dfrac{kT}{m}}}{2}\frac{P}{kT}\ell^2 = \boxed{\frac{P\ell^2}{\sqrt{4mkT}}}$$

(*c*) We assume the pressure is at one atmosphere, and we take the molecular mass of air to be 29 u, as given in problem 16.

$$f = \frac{P\ell^2}{\sqrt{4mkT}} = \frac{\left(1.013\times10^{5}\,\text{Pa}\right)\left(3\,\text{m}\right)^2}{\sqrt{4\left(29\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\text{ K}\right)}} = 3.27\times10^{28}\,\text{Hz}$$

$$\approx \boxed{3.27\times10^{28}\,\text{Hz}}$$

$\boxed{19.}$ In the Maxwell distribution, Eq. 18-6, we see that the mass and temperature always occur as a ratio. Thus if the mass has been doubled, $\boxed{\text{doubling the temperature}}$ will keep the velocity distribution constant.

20. (*a*) We find the average by adding the speed of every particle and then dividing by the number of particles.

$$\overline{v} = \frac{1}{N}\sum_i n_i v_i = \frac{1}{25}\left[\begin{array}{c} 2\left(10\,\text{m/s}\right) + 7\left(15\,\text{m/s}\right) + 4\left(20\,\text{m/s}\right) + 3\left(25\,\text{m/s}\right) \\ +\,6\left(30\,\text{m/s}\right) + 1\left(35\,\text{m/s}\right) + 2\left(40\,\text{m/s}\right) \end{array}\right] = \boxed{23\,\text{m/s}}$$

(*b*) We find the rms speed by taking the square root of the average squared speed.

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_i n_i v_i^2} = \sqrt{\frac{1}{25}\left[\begin{array}{c} 2\left(10\,\text{m/s}\right)^2 + 7\left(15\,\text{m/s}\right)^2 + 4\left(20\,\text{m/s}\right)^2 + 3\left(25\,\text{m/s}\right)^2 \\ +\,6\left(30\,\text{m/s}\right)^2 + 1\left(35\,\text{m/s}\right)^2 + 2\left(40\,\text{m/s}\right)^2 \end{array}\right]}$$

$$= 24.56\,\text{m/s} \approx \boxed{25\,\text{m/s}}$$

(*c*) The most probable speed is that one that occurs most frequently, $\boxed{15\,\text{m/s}}$.

21. (*a*) We find the rms speed by taking the square root of the average squared speed.

$$v_{\text{rms}}=\sqrt{\frac{1}{N}\sum_i n_i v_i^2}=\sqrt{\frac{1}{15{,}200}\begin{bmatrix}1600(220\,\text{m/s})^2+4100(440\,\text{m/s})^2+4700(660\,\text{m/s})^2\\+3100(880\,\text{m/s})^2+1300(1100\,\text{m/s})^2+400(1320\,\text{m/s})^2\end{bmatrix}}$$

$$=706.6\,\text{m/s}\approx\boxed{710\,\text{m/s}}$$

(*b*) The temperature is related to the rms speed by Eq. 18-5.

$$v_{\text{rms}}=\sqrt{\frac{3kT}{m}}\quad\to\quad T=\frac{mv_{\text{rms}}^2}{3k}=\frac{(2.00\times10^{-26}\,\text{kg})(706.6\,\text{m/s})^2}{3(1.38\times10^{-23}\,\text{J/K})}=241.2\,\text{K}\approx\boxed{240\,\text{K}}$$

(*c*) Find the average speed, and then use a result from Example 18-5.

$$\bar{v}=\frac{1}{N}\sum_i n_i v_i=\frac{1}{15{,}200}\begin{bmatrix}1600(220\,\text{m/s})+4100(440\,\text{m/s})+4700(660\,\text{m/s})\\+3100(880\,\text{m/s})+1300(1100\,\text{m/s})+400(1320\,\text{m/s})\end{bmatrix}$$

$$=654.2\,\text{m/s}\approx\boxed{650\,\text{m/s}}$$

$$\bar{v}=\sqrt{\frac{8}{\pi}\frac{kT}{m}}\quad\to\quad T=\frac{\pi m\bar{v}^2}{8k}=\frac{\pi(2.00\times10^{-26}\,\text{kg})(654.2\,\text{m/s})^2}{8(1.38\times10^{-23}\,\text{J/K})}=243.6\,\text{K}\approx\boxed{240\,\text{K}}$$

Yes, the temperatures are consistent.

22. (*a*) Show that $\int_0^\infty f(v)\,dv=N$. We use a change of variable, and we make use of an integral from Appendix B-5; specifically, $\int_0^\infty x^2e^{-ax^2}dx=\sqrt{\frac{\pi}{16a^3}}$.

$$\int_0^\infty f(v)\,dv=\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-\frac{1}{2}\frac{mv^2}{kT}}dv$$

$$x=\sqrt{\frac{mv^2}{2kT}}\quad\to\quad x^2=\frac{mv^2}{2kT}\quad\to\quad v^2=\frac{2kT}{m}x^2\quad\to\quad v=\sqrt{\frac{2kT}{m}}x\quad\to\quad dv=\sqrt{\frac{2kT}{m}}dx$$

$$\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-\frac{1}{2}\frac{mv^2}{kT}}dv=\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{2kT}{m}x^2e^{-x^2}\sqrt{\frac{2kT}{m}}dx$$

$$=4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{2kT}{m}\sqrt{\frac{2kT}{m}}\int_0^\infty x^2e^{-x^2}dx=4\pi N\left(\frac{1}{\pi}\right)^{3/2}\frac{\sqrt{\pi}}{4}=\boxed{N}$$

(*b*) Show that $\int_0^\infty v^2f(v)\,dv\Big/N=\frac{3kT}{m}$. We use the same change of variable as above, and we make use of an integral from Appendix B-5; specifically, $\int_0^\infty x^4e^{-ax^2}dx=\frac{3}{8}\sqrt{\pi}$.

$$\int_0^\infty v^2f(v)\,dv=\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^4e^{-\frac{1}{2}\frac{mv^2}{kT}}dv$$

$$x=\sqrt{\frac{mv^2}{2kT}}\quad\to\quad x^2=\frac{mv^2}{2kT}\quad\to\quad v^2=\frac{2kT}{m}x^2\quad\to\quad v=\sqrt{\frac{2kT}{m}}x\quad\to\quad dv=\sqrt{\frac{2kT}{m}}dx$$

$$\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^4 e^{-\frac{1}{2}\frac{mv^2}{kT}}\,dv = \int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{4k^2T^2}{m^2} x^4 e^{-x^2}\sqrt{\frac{2kT}{m}}\,dx$$

$$= 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{4k^2T^2}{m^2}\sqrt{\frac{2kT}{m}}\int_0^\infty x^4 e^{-x^2}\,dx = N\frac{3kT}{m} \quad \rightarrow$$

$$\int_0^\infty v^2 f(v)\,dv = N\frac{3kT}{m} \quad \rightarrow \quad \boxed{\int_0^\infty v^2 f(v)\,dv \Big/ N = \frac{3kT}{m}}$$

23. From Fig. 18-6, we see that CO_2 is a $\boxed{\text{vapor}}$ at 30 atm and 30°C.

24. (*a*) From Fig. 18-6, at atmospheric pressure, CO_2 can exist as $\boxed{\text{solid or vapor}}$.
    (*b*) From Fig. 18-6, for CO_2 to exist as a liquid, $\boxed{5.11\text{ atm} \le P \le 73\text{ atm}}$ and $\boxed{-56.6^\circ\text{C} \le T \le 31^\circ\text{C}}$.

25. (*a*) From Fig. 18-5, water is $\boxed{\text{vapor}}$ when the pressure is 0.01 atm and the temperature is 90°C.
    (*b*) From Fig. 18-5, water is $\boxed{\text{solid}}$ when the pressure is 0.01 atm and the temperature is –20°C.

26. (*a*) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a vapor at some pressure between 1.0 atm and 0.006 atm. It would still be a vapor at 0.004 atm.
    (*b*) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a solid at a pressure of 1.0 atm, and then becomes a vapor at some pressure lower than 0.006 atm. It would be a vapor at 0.004 atm.

27. From Table 18-2, the saturated vapor pressure at 30°C is 4240 Pa. Since the relative humidity is 85%, the partial pressure of water is as follows.

$$P_{\text{water}} = 0.85P_{\text{saturated}} = 0.85(4240\text{ Pa}) = \boxed{3600\,\text{Pa}}$$

28. From Table 18-2, the saturated vapor pressure at 25°C is 3170 Pa. Since the relative humidity is 55%, the partial pressure of water is as follows.

$$P_{\text{water}} = 0.55P_{\text{saturated}} = 0.55(3170\text{ Pa}) = \boxed{1700\,\text{Pa}}$$

29. At the boiling temperature, the external air pressure equals the saturated vapor pressure. Thus from Table 18-2, for 80°C the saturated air pressure is $\boxed{355\,\text{torr}}$ or $\boxed{4.73\times10^4\,\text{Pa}}$ or $\boxed{0.466\,\text{atm}}$.

30. From Table 18-2, if the temperature is 25°C, the saturated vapor pressure is 23.8 torr. If the relative humidity is 75%, then the partial pressure of water is 75% of the saturated vapor pressure, or 17.85 torr. The dew point is the temperature at which the saturated vapor pressure is 17.85 torr, and from Table 18-2 that is between 20°C and 25°C. Since there is no entry for 17.85 torr, the temperature can be estimated by a linear interpolation. Between 20°C and 25°C, the temperature change per torr is as follows:

$$\frac{(25-20)\,\text{C}^\circ}{(23.8-17.5)\,\text{torr}} = 0.7937\,\text{C}^\circ\!/\text{torr}\,.$$

Thus the temperature corresponding to 17.85 torr is

$$20°\text{C}+\left[(17.85-17.5)\,\text{torr}\right](0.7937\,\text{C}°/\text{torr})=20.28°\text{C}\approx\boxed{20°\text{C}}.$$

$\boxed{31.}$ At the boiling temperature, the air pressure equals the saturated vapor pressure. The pressure of 0.75 atm is equal to $7.60\times10^4\,\text{Pa}$. From Table 18-2, the temperature is between 90°C and 100°C. Since there is no entry for $7.60\times10^4\,\text{Pa}$, the temperature can be estimated by a linear interpolation. Between 90°C and 100°C, the temperature change per Pa is as follows:

$$\frac{(100-90)\,\text{C}^\circ}{(10.1-7.01)\times10^4\text{Pa}}=3.236\times10^{-4}\,\text{C}^\circ/\text{Pa}.$$

Thus the temperature corresponding to $7.60\times10^4\,\text{Pa}$ is

$$90°\text{C}+\left[(7.60-7.01)\times10^4\text{Pa}\right]\left(3.236\times10^{-4}\,\text{C}^\circ/\text{Pa}\right)=91.9°\text{C}\approx\boxed{92°\text{C}}.$$

32. The volume, temperature, and pressure of the water vapor are known. We use the ideal gas law to calculate the mass. The pressure must be interpolated from Table 18-2. Between 20°C and 25°C, the pressure change per temperature change per C° is as follows.

$$\frac{(3170-2330)\,\text{Pa}}{(25-20)\,\text{C}^\circ}=168\,\text{Pa}/\text{C}^\circ$$

Thus the saturated vapor pressure at 20°C is $2330\,\text{Pa}+\left(168\,\text{Pa}/\text{C}^\circ\right)4\text{C}^\circ=3000\,\text{Pa}$.

$$PV=nRT\quad\rightarrow\quad n=\frac{PV}{RT}=\frac{(0.65)(3000\,\text{Pa})(5.0\,\text{m})(6.0\,\text{m})(2.4\,\text{m})}{(8.314\,\text{J/mol•K})(273.15\,\text{K}+24.0\,\text{K})}=56.8\text{ mol}$$

$$m_{\text{H}_2\text{O}}=(56.8\text{ mol})(0.018\,\text{kg/mol})=\boxed{1.0\,\text{kg}}$$

33. Since the water is boiling at 120°C, the saturated vapor pressure is the same as the pressure inside the pressure cooker. From Table 18-2, the pressure is $\boxed{1.99\times10^5\,\text{Pa}=1.97\text{ atm}}$.

34. The total amount of water vapor that can be in the air can be found from the saturated vapor pressure in Table 18-2, using the ideal gas law. At 25°C, that pressure is $3.17\times10^3\,\text{Pa}$.

$$PV=nRT\quad\rightarrow\quad n=\frac{PV}{RT}=\frac{\left(3.17\times10^3\text{Pa}\right)\left(440\,\text{m}^3\right)}{(8.314\,\text{J/mol•K})(273+25)\,\text{K}}=563\,\text{moles}$$

Since the relative humidity is only 65%, only 65% of the total possible water is in the air. Thus 35% of the total possible water can still evaporate into the air.

$$m_{\text{evaporate}}=0.35(563\,\text{moles})\left(\frac{18\times10^{-3}\text{kg}}{1\text{ mole}}\right)=\boxed{3.5\,\text{kg}}$$

35. For boiling to occur at 120°C, the pressure inside the cooker must be the saturated vapor pressure of water at that temperature. That value can be found in Table 18-2. For the mass to stay in place and contain the steam inside the cooker, the weight of the mass must be greater than the force exerted by the gauge pressure from the gas inside the cooker. The limiting case, to hold the temperature right at 120°C, would be with the mass equal to that force.

$$mg=F_{\substack{\text{gauge}\\\text{pressure}}}=\left(P_{\text{inside}}-P_{\text{atm}}\right)A=\left(P_{\text{inside}}-P_{\text{atm}}\right)\pi r^2\quad\rightarrow$$

$$m=\frac{\left(P_{\text{inside}}-P_{\text{atm}}\right)\pi r^2}{g}=\frac{\left(1.99\times10^5\,\text{Pa}-1.013\times10^5\,\text{Pa}\right)\pi\left(1.5\times10^{-3}\,\text{m}\right)^2}{9.80\,\text{m/s}^2}=0.0705\,\text{kg}$$

$$\approx\boxed{70\,\text{g}}\ (2\text{ sig. fig.})$$

36. (*a*) The true atmospheric pressure will be greater than the reading from the barometer. In Figure 13-11, if there is a vapor pressure at the top of the tube, then $P_{\text{atm}}-\rho gh=P_{\text{vapor}}$. The reading from the barometer will be $\rho gh=P_{\text{atm}}-P_{\text{vapor}}<P_{\text{atm}}$.

(*b*) The percent error is found from the atmospheric pressure and the vapor pressure.

$$\%\text{ diff}=\left(\frac{\rho gh-P_{\text{atm}}}{P_{\text{atm}}}\right)\times100=\left(-\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right)\times100=\left(-\frac{0.0015\,\text{mm-Hg}}{760\,\text{mm-Hg}}\right)\times100$$

$$=\boxed{\left(-2.0\times10^{-4}\right)\%}$$

(*c*) From Table 18-2, the saturated water vapor pressure at STP is 611 Pa.

$$\%\text{ diff}=\left(-\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right)\times100=\left(-\frac{611\,\text{Pa}}{1.013\times10^5\,\text{Pa}}\right)\times100=\boxed{0.603\,\%}$$

$\boxed{37.}$ At 30.0°C, the saturated vapor pressure as found in Table 18-2 is 4240 Pa. We can find the partial pressure of the water vapor by using the equation given immediately before Example 18-6.

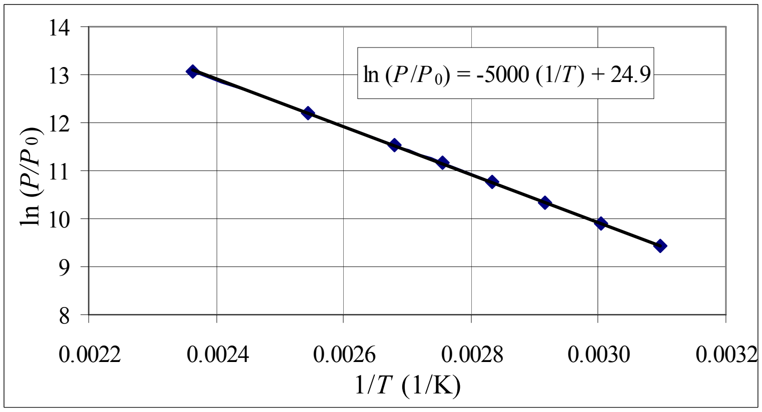
$$\text{Rel. Hum.}=\frac{P_{\text{partial}}}{P_{\text{saturated}}}\times100\ \rightarrow\ P_{\text{partial}}=P_{\text{saturated}}\frac{\text{Rel. Hum.}}{100}=\left(4240\,\text{Pa}\right)\left(0.45\right)=1908\,\text{Pa}$$

The dew point is that temperature at which 1908 Pa is the saturated vapor pressure. From Table 18-2, we see that will be between 15°C and 20°C.

$$T_{1920}=T_{1710}+\frac{1908\,\text{Pa}-1710\,\text{Pa}}{2330\,\text{Pa}-1710\,\text{Pa}}\left(5\text{C}^\circ\right)=\boxed{16.6^\circ\text{C}}$$

38. The outside air is at the dew point, and so its water vapor pressure is the saturated vapor pressure at 5.0°C, which comes from Table 18-2 and is 872 Pa. Consider a fixed number of moles that moves from outside to inside at constant pressure. Because the pressure is constant, the partial pressure of water vapor is 872 Pa inside as well. The saturated pressure at the higher temperature is 2300 Pa. So the relative humidity is $872\,\text{Pa}/2330\,\text{Pa}=0.374=\boxed{37.4\%}$.

39. (*a*) The plot is shown, with an accompanying linear fit. The slope of the line is $\boxed{-5000\,\text{K}}$, and the *y*-intercept is $\boxed{24.91}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH18.XLS," on tab "Problem 18.39a."



(*b*) The straight line results can be expressed as follows.

$$y = mx + b \quad \rightarrow \quad \ln\left(P/P_0\right) = m\left(\frac{1}{T}\right) + b \quad \rightarrow \quad P = P_0 e^{\frac{m}{T}+b} = P_0 e^b e^{\frac{m}{T}}$$

We define $B = P_0 e^b = \left(1\,\text{Pa}\right)e^{24.91} = 6.58\times10^{10}\,\text{Pa} \approx 7\times10^{10}\,\text{Pa}$ and $A = -m = 5000\,\text{K}$. Then we have the following.

$$P = P_0 e^b e^{m/T} = \boxed{Be^{-A/T}} = \left(7\times10^{10}\,\text{Pa}\right)e^{-5000/T}$$

40. For one mole of gas, the "lost" volume (the volume occupied by the molecules) is the value of *b*. We assume spherical molecules.

$$b = N_0 \tfrac{4}{3}\pi\left(\tfrac{1}{2}d\right)^3 \quad \rightarrow$$

$$d = 2\left(\frac{3b}{4\pi N_0}\right)^{1/3} = 2\left(\frac{3\left(3.2\times10^{-5}\,\text{m}^3/\text{mol}\right)}{4\pi\left(6.02\times10^{23}\,\text{molecules/mol}\right)}\right)^{1/3} = \boxed{4.7\times10^{-10}\,\text{m}}$$

41. (*a*) Use the van der Waals equation.

$$P = \frac{RT}{\left(V/n\right) - b} - \frac{a}{\left(V/n\right)^2}$$

$$= \frac{\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(273\,\text{K}\right)}{\left(0.70\times10^{-3}\,\text{m}^3/\text{mol}\right) - \left(3.2\times10^{-5}\,\text{m}^3/\text{mol}\right)} - \frac{0.13\,\text{m/s}}{\left(0.70\times10^{-3}\,\text{m}^3/\text{mol}\right)^2} = \boxed{3.1\times10^6\,\text{Pa}}$$

(*b*) Use the ideal gas law.

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{\left(1.0\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(273\,\text{K}\right)}{0.70\times10^{-3}\,\text{m}^3} = \boxed{3.2\times10^6\,\text{Pa}}$$

42. The van der Waals pressure can be either higher or lower than the ideal gas pressure, depending on the volume. Accordingly, we use a parameter "*c*," which is the ratio of the van der Waals pressure to the ideal pressure.

$$P_V = cP_I \quad \rightarrow \quad \frac{RT}{\left(V/n\right) - b} - \frac{a}{\left(V/n\right)^2} = c\frac{nRT}{V} \quad \rightarrow \quad RT\left(c-1\right)V^2 + \left(an - bcnRT\right)V - ban^2 = 0 \quad \rightarrow$$

$$V = \frac{-\left(an - bcnRT\right) \pm n\sqrt{\left(a - bcRT\right)^2 + 4RT\left(c-1\right)ba}}{2RT\left(c-1\right)}$$

$$RT\left(c-1\right)V^2 + \left(an - bcnRT\right)V - ban^2 = 0$$

For $c = 0.95$, the van der Waals pressure being lower than the ideal gas pressure, we get volumes of $\boxed{4.16\times10^{-5}\,\text{m}^3}$ and $\boxed{2.16\times10^{-4}\,\text{m}^3}$. For $c = 1.05$, the van der Waals pressure being higher than the ideal gas pressure, we get a volume of $\boxed{3.46\times10^{-5}\,\text{m}^3}$. Note that the pressures are equal for a volume of $3.72\times10^{-5}\,\text{m}^3$.

$\boxed{43.}$ (*a*) The Van der Waals equation of state is given by Eq. 18-9, $P=\dfrac{RT}{(V/n)-b}-\dfrac{a}{(V/n)^2}$. At the critical point, both the first and second derivatives of *P* with respect to *V* are 0. Use those conditions to find the critical volume, and then evaluate the critical temperature and critical pressure.

$$P=\frac{RT}{(V/n)-b}-\frac{a}{(V/n)^2}=\frac{nRT}{V-nb}-\frac{an^2}{V^2}$$

$$\frac{dP}{dV}=-\frac{nRT}{(V-nb)^2}+\frac{2an^2}{V^3}\ ;\ \frac{dP}{dV}=0\ \to\ T_{\text{crit}}=\frac{2an\left(V_{\text{crit}}-nb\right)^2}{RV_{\text{crit}}^3}$$

$$\frac{d^2P}{dV^2}=\frac{2nRT}{(V-nb)^3}-\frac{6an^2}{V^4}\ ;\ \frac{d^2P}{dV^2}=0\ \to\ T_{\text{crit}}=\frac{3an\left(V_{\text{crit}}-nb\right)^3}{RV_{\text{crit}}^4}$$

Set the two expressions for the critical temperature equal to each other, and solve for the critical volume. Then use that expression to find the critical temperature, and finally the critical pressure.

$$\frac{2an\left(V_{\text{crit}}-nb\right)^2}{RV_{\text{crit}}^3}=\frac{3an\left(V_{\text{crit}}-nb\right)^3}{RV_{\text{crit}}^4}\ \to\ V_{\text{crit}}=3nb$$

$$T_{\text{crit}}=\frac{2an\left(V_{\text{crit}}-nb\right)^2}{RV_{\text{crit}}^3}=\frac{2an(3nb-nb)^2}{R(3nb)^3}=\boxed{\frac{8a}{27bR}}$$

$$P_{\text{crit}}=\frac{nRT_{\text{crit}}}{V_{\text{crit}}-nb}-\frac{an^2}{V_{\text{crit}}^2}=\boxed{\frac{a}{27b^2}}$$

(*b*) To evaluate the constants, use the ratios $\dfrac{T_{\text{crit}}^2}{P_{\text{crit}}}$ and $\dfrac{T_{\text{crit}}}{P_{\text{crit}}}$.

$$\frac{T_{\text{crit}}^2}{P_{\text{crit}}}=\frac{\left(\dfrac{8a}{27bR}\right)^2}{\dfrac{a}{27b^2}}=\frac{64a}{27R^2}\ \to$$

$$a=\frac{27R^2T_{\text{crit}}^2}{64P_{\text{crit}}}=\frac{27\left(8.314\dfrac{\text{J}}{\text{mol}\bullet\text{K}}\right)^2(304\,\text{K})^2}{64(72.8)\left(1.013\times10^5\,\text{Pa}\right)}=\boxed{0.365\frac{\text{N}\bullet\text{m}^4}{\text{mol}^2}}$$

$$\frac{T_{\text{crit}}}{P_{\text{crit}}}=\frac{\dfrac{8a}{27bR}}{\dfrac{a}{27b^2}}=\frac{8b}{R}\ \to\ b=\frac{RT_{\text{crit}}}{8P_{\text{crit}}}=\frac{\left(8.314\dfrac{\text{J}}{\text{mol}\bullet\text{K}}\right)(304\,\text{K})}{8(72.8)\left(1.013\times10^5\,\text{Pa}\right)}=\boxed{4.28\times10^{-5}\,\text{m}^3/\text{mol}}$$

44. (*a*) We use the ideal gas law as applied to the air before it was put into the tank.

$$PV=nRT\ \to\ n=\frac{\left(1.013\times10^5\,\text{Pa}\right)\left(2.3\,\text{m}^3\right)}{(8.314\,\text{J/mol}\bullet\text{K})(293\,\text{K})}=95.64\,\text{mol}\approx\boxed{96\,\text{mol}}$$

(*b*) Use the ideal gas law.

$$PV = nRT \quad \rightarrow$$

$$P = \frac{nRT}{V} = \frac{(96\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(293\,\text{K})}{(0.012\,\text{m}^3)} = 1.9488\times10^7\,\text{Pa} \approx \boxed{1.9\times10^7\,\text{Pa}} \approx 190\,\text{atm}$$

(*c*) Use the van der Waals equation.

$$P = \frac{RT}{(V/n)-b} - \frac{a}{(V/n)^2} = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

$$= \frac{(96\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(293\,\text{K})}{(0.012\,\text{m}^3)-(96\,\text{mol})(3.72\times10^{-5}\,\text{m}^3/\text{mol})} - \frac{(0.1373\,\text{N}\bullet\text{m}^4/\text{mol}^2)(96\,\text{mol})^2}{(0.012\,\text{m}^3)^2}$$

$$= 1.8958\times10^7\,\text{Pa} \approx \boxed{1.9\times10^7\,\text{Pa}}$$

(*d*) $$\%\,\text{error} = \left(\frac{P_{\text{ideal}} - P_{\substack{\text{van der}\\\text{Waals}}}}{P_{\substack{\text{van der}\\\text{Waals}}}}\right)\times100 = \left(\frac{1.9488\times10^7\,\text{Pa} - 1.8958\times10^7\,\text{Pa}}{1.8958\times10^7\,\text{Pa}}\right)\times100 = 2.796\% \approx \boxed{3\%}$$

45. The mean free path is given by Eq. 18-10b. Combine this with the ideal gas law to find the mean free path–pressure relationship.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT}\ ;\ \ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^2(N/V)} = \frac{kT}{4\pi\sqrt{2}r^2P} \quad \rightarrow \quad P = \frac{kT}{4\pi\sqrt{2}r^2\ell_{\text{M}}}$$

(*a*) $$P = \frac{kT}{4\pi\sqrt{2}r^2\ell_{\text{M}}} = \frac{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}{4\pi\sqrt{2}(1.5\times10^{-10}\,\text{m})^2(0.10\,\text{m})} = \boxed{0.10\,\text{Pa}}$$

(*b*) $$P = \frac{kT}{4\pi\sqrt{2}r^2\ell_{\text{M}}} = \frac{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}{4\pi\sqrt{2}(1.5\times10^{-10}\,\text{m})^2(3\times10^{-10}\,\text{m})} = \boxed{3\times10^7\,\text{Pa}} \approx 300\,\text{atm}$$

46. We want the mean free path to be 1.0 m. Use Eq. 18-10b with the ideal gas law.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT}\ ;\ \ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^2(N/V)} = \frac{kT}{4\pi\sqrt{2}r^2P} \quad \rightarrow$$

$$P = \frac{kT}{4\pi\sqrt{2}r^2\ell_{\text{M}}} = \frac{(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})}{4\pi\sqrt{2}(1.5\times10^{-10}\,\text{m})^2(1.0\,\text{m})} = \boxed{0.010\,\text{Pa}}$$

47. First, we compare the rms speed of the hydrogen to the rms speed of the air, by Eq. 18-5.

$$\frac{(v_{\text{rms}})_{\text{H}_2}}{(v_{\text{rms}})_{\text{air}}} = \frac{\sqrt{3kT/m_{\text{H}_2}}}{\sqrt{kT/m_{\text{air}}}} = \sqrt{\frac{m_{\text{air}}}{m_{\text{H}_2}}} = \sqrt{\frac{29}{2}} = 3.8$$

Since the hydrogen is moving about 4 times faster than the air, we will use a stationary target approximation. We also assume that the inter-molecular distance for a collision would be the sum of the radii of the hydrogen and air molecules. The size of the air molecules are given in problem 45, and based on problem 71, we assume that the radius of the hydrogen molecule is the same as the diameter of the hydrogen atom. We use these assumptions to calculate the mean free path, similar to Eq. 18-10a. Use the ideal gas law.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} \ ;$$

$$\ell_{\mathrm{M}} = \frac{1}{\pi\left(r_{\mathrm{H_2}} + r_{\mathrm{air}}\right)^2 (N/V)} = \frac{kT}{\pi r^2 P} = \frac{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(288\,\mathrm{K})}{\pi\left[(1.0+1.5)\times10^{-10}\,\mathrm{m}\right]^2\left(1.013\times10^5\,\mathrm{Pa}\right)}$$

$$= 1.998\times10^{-7}\,\mathrm{m} \approx \boxed{2\times10^{-7}\,\mathrm{m}}$$

48. The mean free path is given by Eq. 18-10b. Combine this with the ideal gas law to find the mean free path–diameter relationship.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} \ ; \ \ell_{\mathrm{M}} = \frac{1}{4\pi\sqrt{2}r^2 (N/V)} = \frac{kT}{4\pi\sqrt{2}\left(\tfrac{1}{2}d\right)^2 P} \quad \rightarrow \quad d = \sqrt{\frac{kT}{\sqrt{2}\pi\ell_{\mathrm{M}}P}}$$

(*a*) $$d = \sqrt{\frac{kT}{\sqrt{2}\pi\ell_{\mathrm{M}}P}} = \sqrt{\frac{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(273\,\mathrm{K})}{\sqrt{2}\pi\left(5.6\times10^{-8}\,\mathrm{m}\right)\left(1.013\times10^5\,\mathrm{Pa}\right)}} = \boxed{3.9\times10^{-10}\,\mathrm{m}}$$

(*b*) $$d = \sqrt{\frac{kT}{\sqrt{2}\pi\ell_{\mathrm{M}}P}} = \sqrt{\frac{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(273\,\mathrm{K})}{\sqrt{2}\pi\left(25\times10^{-8}\,\mathrm{m}\right)\left(1.013\times10^5\,\mathrm{Pa}\right)}} = \boxed{1.8\times10^{-10}\,\mathrm{m}}$$

$\boxed{49.}$ (*a*) If the average speed of a molecule is $\bar{v}$, then the average time between collisions (seconds per collision) is the mean free path divided by the average speed. The reciprocal of that average time (collisions per second) is the frequency of collisions. Use Eq. 18-10b for the mean free path. The typical size of an air molecule is given in problem 45, which can be used for the size of the nitrogen molecule.

$$\Delta t_{\mathrm{avg}} = \frac{\ell_{\mathrm{M}}}{\bar{v}} \quad \rightarrow \quad f = \frac{1}{\Delta t_{\mathrm{avg}}} = \frac{\bar{v}}{\ell_{\mathrm{M}}} = \boxed{4\sqrt{2}\pi r^2 \bar{v}\frac{N}{V}}$$

(*b*) From Eq. 18-7b, $\bar{v} = \sqrt{\frac{8kT}{\pi m}}$, and from the ideal gas law, $PV = NkT \ \rightarrow \ \frac{N}{V} = \frac{P}{kT}$. Use these relationships to calculate the collision frequency.

$$f = 4\sqrt{2}\pi r^2 \bar{v}\frac{N}{V} = 4\sqrt{2}\pi r^2\sqrt{\frac{8kT}{\pi m}}\frac{P}{kT} = 16Pr^2\sqrt{\frac{\pi}{mkT}}$$

$$= 16(0.010)\left(1.013\times10^5\,\mathrm{Pa}\right)\left(1.5\times10^{-10}\,\mathrm{m}\right)^2 \times$$

$$\sqrt{\frac{\pi}{28\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(293\,\mathrm{K})}} = \boxed{4.7\times10^7\ \mathrm{collisions/s}}$$

50. The collision frequency is derived in problem 49 as $f = 16Pr^2\sqrt{\frac{\pi}{mkT}}$. Only one significant figure was given for the mean free path, so only one significant figure should be in the answer.

$$f = 16Pr^2\sqrt{\frac{\pi}{mkT}}$$

$$= 16\left(1.013\times10^{5}\,\text{Pa}\right)\left(1.5\times10^{-10}\,\text{m}\right)^{2}\sqrt{\frac{\pi}{29\left(1.66\times10^{-27}\,\text{kg}\right)\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\,\text{K}\right)}}$$

$$= 4.8\times10^{9}\,\text{Hz} \approx \boxed{5\times10^{9}\,\text{Hz}}$$

51. Use the ideal gas law to evaluate the mean free path. Then compare the mean free path to the dimensions of the box in order to estimate the collision ratio. The size of air molecules is given in problem 45. The number of collisions per second is the reciprocal of the average time between collisions.

$$PV = NkT \;\;\rightarrow\;\; \frac{N}{V} = \frac{P}{kT}\;;$$

$$\ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^{2}\left(N/V\right)} = \frac{kT}{4\pi\sqrt{2}r^{2}P} = \frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\,\text{K}\right)}{4\pi\sqrt{2}\left(1.5\times10^{-10}\,\text{m}\right)^{2}\left(1\times10^{-6}\,\text{torr}\right)\left(\dfrac{133\,\text{Pa}}{1\,\text{torr}}\right)} = 70.8\,\text{m}$$

$$\bar{v} = \frac{\ell_{\text{M}}}{t_{\substack{\text{molecular}\\\text{collisions}}}} = \frac{\ell_{\text{box}}}{t_{\substack{\text{wall}\\\text{collisions}}}} \;\;\rightarrow\;\; \frac{N_{\substack{\text{wall}\\\text{collisions}}}}{N_{\substack{\text{molecular}\\\text{collisions}}}} = \frac{t_{\substack{\text{molecular}\\\text{collisions}}}}{t_{\text{wall}}} = \frac{\ell_{\text{M}}}{\ell_{\text{box}}} = \frac{70.8\,\text{m}}{1.80\,\text{m}} = 39.3$$

The wall collisions are about 40 times more frequent than the inter-molecular collisions. So the particles make about $\boxed{1/40}$ of a collision with each other for each collision with a wall.

52. We estimate that only 2% of the electrons will have a collision in 32 cm or less, and so approximate that 2% of the electrons will have a collision in every 32 cm length. Thus 50% of the electrons should have a collision in a length of 25 times 32 cm, which is 8.00 m. So we want the mean free path to be 8.00 m. We also assume that the electrons are moving much faster than the air molecules, so that we model the air molecules as stationary. Finally, a collision will occur if an electron comes within a distance of r from a gas molecule (the radius of the gas molecule), not $2r$ as in the derivation in section 18-6. Combine this with the ideal gas law. We assume room temperature.

$$PV = NkT \;\;\rightarrow\;\; \frac{N}{V} = \frac{P}{kT}\;;\;\; \ell_{\text{M}} = \frac{1}{\pi r^{2}\left(N/V\right)} = \frac{kT}{\pi r^{2}P} \;\;\rightarrow$$

$$P = \frac{kT}{\pi r^{2}\ell_{\text{M}}} = \frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}{\pi\left(1.5\times10^{-10}\,\text{m}\right)^{2}\left(8.00\,\text{m}\right)} = \boxed{7.3\times10^{-3}\,\text{Pa}} \approx 7\times10^{-6}\,\text{atm}$$

53. We use the equation derived in Eq. 18-9.

$$t = \frac{\bar{C}}{\Delta C}\frac{\left(\Delta x\right)^{2}}{D} = \frac{1}{2}\frac{\left(1.0\,\text{m}\right)^{2}}{\left(4\times10^{-5}\,\text{m}^{2}/\text{s}\right)} = 12{,}500\,\text{s} \approx \boxed{3.5\,\text{h}}$$

Because this time is so long, we see that $\boxed{\text{convection is much more important than diffusion}}$.

54. From Example 18-9, we have an expression for the time to diffuse a given distance. Divide the distance by the time to get the average speed.

$$t = \frac{\bar{C}}{\Delta C}\frac{\left(\Delta x\right)^{2}}{D} = \frac{\frac{1}{2}\left(1.00+0.50\right)\text{mol/m}^{3}}{\left(1.00-0.50\right)\text{mol/m}^{3}}\frac{\left(15\times10^{-6}\,\text{m}\right)^{2}}{\left(95\times10^{-11}\,\text{m}^{2}/\text{s}\right)} = 0.3553\,\text{s} \approx \boxed{0.36\,\text{s}}$$

$$v_{\text{diffuse}} = \frac{\Delta x}{t} = \frac{15\times10^{-6}\,\text{m}}{0.3553\ \text{s}} = \boxed{4.2\times10^{-5}\ \text{m/s}}$$

The rms thermal speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(293\ \text{K}\right)}{75\left(1.66\times10^{-27}\,\text{kg}\right)}} = \boxed{3.1\times10^{2}\ \text{m/s}}$$

$$\frac{v_{\text{diffuse}}}{v_{\text{rms}}} = \frac{4.2\times10^{-5}\ \text{m/s}}{3.1\times10^{2}\ \text{m/s}} = 1.4\times10^{-7}$$

The diffusion speed is about seven orders of magnitude smaller than the thermal speed.

$\boxed{55.}$ (*a*) Use the ideal gas law to find the concentration of the oxygen. We assume that the air pressure is 1.00 atm, and so the pressure caused by the oxygen is 0.21 atm.

$$PV = nRT \quad \rightarrow$$

$$\frac{n}{V} = \frac{P}{RT} = \frac{\left(0.21\,\text{atm}\right)\left(1.013\times10^{5}\ \text{Pa/atm}\right)}{\left(8.315\,\text{J/mol•K}\right)\left(293\ \text{K}\right)} = 8.732\,\text{mol/m}^3 \approx \boxed{8.7\,\text{mol/m}^3}$$

(*b*) Use Eq. 18-11 to calculate the diffusion rate.

$$J = DA\frac{dC}{dx} \approx DA\frac{C_1 - C_2}{\Delta x} = \left(1\times10^{-5}\ \text{m}^2/\text{s}\right)\left(2\times10^{-9}\,\text{m}^2\right)\left(\frac{8.732\,\text{mol/m}^3 - 4.366\,\text{mol/m}^3}{2\times10^{-3}\,\text{m}}\right)$$

$$= 4.366\times10^{-11}\ \text{mol/s} \approx \boxed{4\times10^{-11}\ \text{mol/s}}$$

(*c*) From Example 18-9, we have an expression for the time to diffuse a given distance.

$$t = \frac{\bar{C}}{\Delta C}\frac{\left(\Delta x\right)^2}{D} = \frac{\frac{1}{2}\left(8.732\,\text{mol/m}^3 + 4.366\,\text{mol/m}^3\right)}{\left(8.732\,\text{mol/m}^3 - 4.366\,\text{mol/m}^3\right)}\frac{\left(2\times10^{-3}\,\text{m}\right)^2}{1\times10^{-5}\ \text{m}^2/\text{s}} = \boxed{0.6\ \text{s}}$$

56. We use the ideal gas law to find the length.

$$PV = P\ell^3 = NkT \quad \rightarrow \quad \ell = \left(\frac{NkT}{P}\right)^{1/3} = \left[\frac{\left(1\times10^{6}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)\left(273\,\text{K}\right)}{\left(1.013\times10^{5}\text{Pa}\right)}\right]^{1/3} = \boxed{3\times10^{-7}\text{m}}$$

57. The rms speed is given by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. Hydrogen atoms have a mass of 1 atomic mass unit.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(2.7\ \text{K}\right)}{1\left(1.66\times10^{-27}\,\text{kg}\right)}} = \boxed{260\,\text{m/s}}$$

The pressure is found from the ideal gas law, $PV = NkT$.

$$PV = NkT \quad \rightarrow \quad P = \frac{NkT}{V} = \frac{\left(1\right)\left(1.38\times10^{-23}\ \text{J/K}\right)\left(2.7\ \text{K}\right)}{1\ \text{cm}^3\left(\frac{1\times10^{-6}\text{m}^3}{1\text{cm}^3}\right)} = 3.726\times10^{-17}\,\text{Pa}\left(\frac{1\ \text{atm}}{1.01\times10^{5}\text{Pa}}\right)$$

$$= 3.689\times10^{-22}\text{atm} \approx \boxed{3.7\times10^{-22}\text{atm}}$$

58. We assume that each molecule will have an average kinetic energy of $\frac{3}{2}kT$. Find the total number of molecules from the mass of the bacterium.

$$\begin{aligned}
N &= N_{\mathrm{H_2O}} + N_{\text{other}} \\
&= 0.70\left(2.0\times10^{-15}\,\text{kg}\right)\left(\frac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}}\right)\left(\frac{1\,\text{molecule}}{18\,\text{u}}\right) \\
&\quad + 0.30\left(2.0\times10^{-15}\,\text{kg}\right)\left(\frac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}}\right)\left(\frac{1\,\text{molecule}}{10^5\,\text{u}}\right) \\
&= \left(2.0\times10^{-15}\,\text{kg}\right)\left(\frac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}}\right)\left[0.70\left(\frac{1\,\text{molecule}}{18\,\text{u}}\right)+0.30\left(\frac{1\,\text{molecule}}{10^5\,\text{u}}\right)\right] \\
&= 4.69\times10^{10}\,\text{molecules}
\end{aligned}$$

$$K = N\left(\tfrac{3}{2}kT\right) = \left(4.69\times10^{10}\,\text{molecules}\right)\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)\left(310\,\text{K}\right) = \boxed{3\times10^{-10}\,\text{J}}$$

59. The rms speed is given by Eq. 18-5.

(*a*) $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(310\,\text{K}\right)}{\left(89\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)}} = 294.7\,\text{m/s} \approx \boxed{290\,\text{m/s}}$$

(*b*) $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(310\,\text{K}\right)}{\left(8.5\times10^{4}\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)}} = 9.537\,\text{m/s} \approx \boxed{9.5\,\text{m/s}}$$

60. The mean (average) speed is given in E. 18-7b, $\bar{v} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$. Using the escape velocity as $\bar{v}$, solve for the temperature.

(*a*) For oxygen molecules: $$T = \frac{\pi m\bar{v}^2}{8k} = \frac{\pi\left(32.0\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left(1.12\times10^{4}\,\text{m/s}\right)^2}{8\left(1.38\times10^{-23}\,\text{J/K}\right)} = \boxed{1.90\times10^{5}\,\text{K}}$$

(*b*) For helium atoms: $$T = \frac{\pi m\bar{v}^2}{8k} = \frac{\pi\left(4.00\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left(1.12\times10^{4}\,\text{m/s}\right)^2}{8\left(1.38\times10^{-23}\,\text{J/K}\right)} = \boxed{2.37\times10^{4}\,\text{K}}$$

(*c*) Because the "escape temperature" is so high for oxygen, very few oxygen molecules ever escape the atmosphere. But helium, with one-eighth the mass, can escape at a much lower temperature. While the temperature of the Earth is not close to $2.37\times10^{4}\,\text{K}$ today, during the Earth's formation its temperature was possibly much hotter — presumably hot enough that helium was able to escape the atmosphere.

$\boxed{61.}$ Calculate the volume per molecule from the ideal gas law, and assume the molecular volume is spherical.

$$PV = NkT \;\rightarrow\; \frac{V}{N} = \frac{kT}{P} = \tfrac{4}{3}\pi r^3 \;\rightarrow$$

$$r_{\text{volume}} = \left(\frac{3kT}{4\pi P}\right)^{1/3} = \left(\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\,\text{K}\right)}{4\pi\left(1.01\times10^{5}\,\text{Pa}\right)}\right)^{1/3} = 2.07\times10^{-9}\,\text{m}$$

The intermolecular distance would be twice this "radius," so about $4\times10^{-9}\,\text{m}$. This is about 130 times larger than the molecular diameter: $\dfrac{d_{\text{volume}}}{d_{\text{molecule}}}\approx\dfrac{4\times10^{-9}\,\text{m}}{3\times10^{-10}\,\text{m}}=13.3$. So if we say the molecular diameter is 4 cm, then the intermolecular distance would be 13.3 times that, or about $\boxed{50\text{ cm}}$.

62. (*a*) The mean speed is given by Eq. 18-7b. The atomic weight of cesium is 133.

$$\bar{v}=\sqrt{\frac{8kT}{\pi m}}=\sqrt{\frac{8\left(1.38\times10^{-23}\,\text{J/K}\right)\left(673\,\text{K}\right)}{\pi\left(133\right)\left(1.66\times10^{-27}\,\text{kg}\right)}}=327.3\,\text{m/s}\approx\boxed{330\,\text{m/s}}$$

(*b*) The collision frequency is the mean speed divided by the mean free path as given by Eq. 18-10b. We also use the ideal gas law.

$$PV=NkT\quad\rightarrow\quad\frac{N}{V}=\frac{P}{kT}\;;$$

$$\ell_{\text{M}}=\frac{1}{4\pi\sqrt{2}r^2\left(N/V\right)}=\frac{kT}{4\pi\sqrt{2}r^2P}=\frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(673\,\text{K}\right)}{4\pi\sqrt{2}\left(1.65\times10^{-10}\,\text{m}\right)^2\left(17\,\text{mm-Hg}\right)\left(\dfrac{133\,\text{Pa}}{1\,\text{mm-Hg}}\right)}$$

$$=8.49\times10^{-6}\,\text{m}$$

$$f=\frac{\bar{v}}{\ell_{\text{M}}}=\frac{327.3\,\text{m/s}}{8.49\times10^{-6}\,\text{m}}=3.855\times10^{7}\text{ collisions/s}\approx\boxed{3.9\times10^{7}\text{ collisions/s}}$$

(*c*) The total number of collisions per second in the gas is the number of collisions per second for a single atom times half the number of atoms in the gas, because each collision involves 2 of the gas atoms.

$$f_{\text{total}}=\tfrac{1}{2}Nf_{\text{single}}=\frac{PV}{2kT}f_{\text{single}}$$

$$=\frac{\left(17\,\text{mm-Hg}\right)\left(\dfrac{133\,\text{Pa}}{1\,\text{mm-Hg}}\right)\left(55\,\text{cm}^3\right)\left(\dfrac{1\,\text{m}^3}{10^6\,\text{cm}^3}\right)}{2\left(1.38\times10^{-23}\,\text{J/K}\right)\left(673\,\text{K}\right)}\left(3.855\times10^{7}\text{ collisions/s}\right)$$

$$=\boxed{2.6\times10^{26}\text{ collisions/s}}$$

63. The gravitational potential energy is given by $U=mgh$, and the average kinetic energy is $K=\tfrac{1}{2}mv_{\text{rms}}^2=\tfrac{3}{2}kT$. We find the ratio of potential energy to kinetic energy. The molecular mass of oxygen molecules is 32 u.

$$\frac{U}{K}=\frac{mgh}{\tfrac{3}{2}kT}=\frac{\left(32.0\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(1.00\,\text{m}\right)}{\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\,\text{K}\right)}=\boxed{8.58\times10^{-5}}$$

$\boxed{\text{Yes}}$, it is reasonable to neglect the gravitational potential energy.

64. Assume that the water vapor behaves like an ideal gas. At 20°C, the saturated vapor pressure is $2.33\times10^{3}\,\text{Pa}$. Using the ideal gas law, find the number of moles of water in the air at both 95% and 40%. Subtract those mole amounts to find the amount of water that must be removed.

$$PV=nRT\quad\rightarrow\quad n=\frac{PV}{RT}\quad\rightarrow$$

$$n_1 - n_2 = \frac{V}{RT}\left(P_1 - P_2\right) = \frac{\left(115\,\text{m}^2\right)\left(2.8\,\text{m}\right)}{\left(8.314\,\text{J/mol}\bullet\text{k}\right)\left(293\text{ K}\right)}\left(2.33\times10^3\,\text{Pa}\right)\left(0.95-0.40\right) = 169.4\text{ mol}$$

$$169.4\,\text{mol}\left(\frac{18\times10^{-3}\,\text{kg}}{1\text{ mol}}\right) = \boxed{3.0\,\text{kg}}$$

65. Find the volume "allotted" per molecule in the ideal gas law for a room at 1 atm and 23°C, and compare this to the volume of an actual molecule, modeled by a cubical volume.

$$PV = NkT \;\rightarrow\; V_{\text{allotted}} = \frac{V}{N} = \frac{kT}{P}\;\;;\;\; V_{\text{molecule}} \approx d^3$$

$$\frac{V_{\text{molecule}}}{V_{\text{allotted}}} = \frac{d^3}{\frac{kT}{P}} = \frac{Pd^3}{kT} = \frac{\left(1.013\times10^5\,\text{Pa}\right)\left(0.3\times10^{-9}\,\text{m}\right)^3}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)} = 6.6\times10^{-4} = \left(6.6\times10^{-2}\right)\% \approx \boxed{0.07\%}$$

66. (*a*) The volume of each gas is half of the tank volume. Use the ideal gas law, with a pressure of 13 atm, to find the number of molecules.

$$PV = NkT \;\rightarrow\; N = \frac{PV}{kT} = \frac{13\left(1.013\times10^5\,\text{Pa}\right)\frac{1}{2}\left(3.1\times10^{-3}\,\text{m}^3\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\,\text{K}\right)} = 5.048\times10^{23}\,\text{molecules}$$

$$\approx \boxed{5.0\times10^{23}\,\text{molecules}}$$

Both gases have the same number of molecules. The identity of the gas does not enter into the ideal gas law.

(*b*) The average kinetic energy of a molecule is $\frac{3}{2}kT$. Since both gases are the same temperature, the ratio of the average kinetic energies is $\boxed{1:1}$.

(*c*) The average kinetic energy of a molecule is also given by $\frac{1}{2}mv_{\text{rms}}^2$. Use this to find the ratio of the rms speeds.

$$\left(\tfrac{1}{2}mv_{\text{rms}}^2\right)_{\text{He}} = \left(\tfrac{1}{2}mv_{\text{rms}}^2\right)_{\text{O}_2} = \tfrac{3}{2}kT \;\rightarrow\; \frac{\left(v_{\text{rms}}\right)_{\text{He}}}{\left(v_{\text{rms}}\right)_{\text{O}_2}} = \sqrt{\frac{m_{\text{O}_2}}{m_{\text{He}}}} = \sqrt{\frac{32}{4}} = \sqrt{8} = \boxed{2.8}$$

$\boxed{67.}$ The temperature can be found from the rms speed by Eq. 18-5, $v_{\text{rms}} = \sqrt{3kT/m}$. The molecular mass of nitrogen molecules is 28.

$$v_{\text{rms}} = \sqrt{3kT/m} \;\rightarrow$$

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{\left(28\right)\left(1.66\times10^{-27}\,\text{kg}\right)\left[\left(4.2\times10^4\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{3\left(1.38\times10^{-23}\,\text{J/K}\right)} = \boxed{1.5\times10^5\,\text{K}}$$

68. We assume that the energy required to evaporate the water is the kinetic energy of the evaporating molecules. The rms speed is given by Eq. 18-5.

$$E_{\text{evap}} = \tfrac{1}{2}m_{\text{evap}}v_{\text{evap}}^2 \;\rightarrow\; v_{\text{evap}} = \sqrt{\frac{2E_{\text{evap}}}{m_{\text{evap}}}} = \sqrt{\frac{2\left(2450\,\text{J}\right)}{\left(1.00\times10^{-3}\,\text{kg}\right)}} = \boxed{2210\,\text{m/s}}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\,\text{K}\right)}{18.0\left(1.66\times10^{-27}\,\text{kg}\right)}} = 637\,\text{m/s} \quad\rightarrow\quad \frac{v_{\text{evap}}}{v_{\text{rms}}} = \frac{2210\,\text{m/s}}{637\,\text{m/s}} = \boxed{3.47}$$

69. (*a*) At a temperature of 30°C, the saturated vapor pressure, from Table 18-2, is $4240\,\text{Pa}$. If the relative humidity is 65%, then the water vapor pressure is 50% of the saturated vapor pressure.

$$0.65\left(4240\,\text{Pa}\right) = 2756\,\text{Pa} \approx \boxed{2800\,\text{Pa}}$$

(*b*) At a temperature of 5°C, the saturated vapor pressure, from Table 18-2, is $872\,\text{Pa}$. If the relative humidity is 75%, then the water vapor pressure is 75% of the saturated vapor pressure.

$$0.75\left(872\,\text{Pa}\right) = 654\,\text{Pa} \approx \boxed{650\,\text{Pa}}$$

70. First we find the pressure from the ideal gas equation.

$$P = \frac{nRT}{V} = \frac{\left(8.50\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{k}\right)\left(300\,\text{K}\right)}{0.220\,\text{m}^3} = 9.6367\times10^4\,\text{Pa} \approx \boxed{9.64\times10^4\,\text{Pa}}$$

Now find the pressure from the van der Waals equation.

$$P = \frac{RT}{\left(V/n\right)-b} - \frac{a}{\left(V/n\right)^2} = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

$$= \frac{\left(8.50\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{k}\right)\left(300\,\text{K}\right)}{\left(0.220\,\text{m}^3\right)-\left(8.50\,\text{mol}\right)\left(4.5\times10^{-5}\,\text{m}^3/\text{mol}\right)} - \frac{\left(0.36\,\text{N}\bullet\text{m}^4/\text{mol}^2\right)\left(8.50\,\text{mol}\right)^2}{\left(0.220\,\text{m}^3\right)^2}$$

$$= 9.5997\times10^4\,\text{Pa} \approx \boxed{9.60\times10^4\,\text{Pa}}$$

$$\%\,\text{error} = \left(\frac{P_{\text{ideal}} - P_{\substack{\text{van der}\\ \text{Waals}}}}{P_{\substack{\text{van der}\\ \text{Waals}}}}\right)\times100 = \left(\frac{9.6367\times10^4\,\text{Pa} - 9.5997\times10^4\,\text{Pa}}{9.5997\times10^4\,\text{Pa}}\right)\times100 \approx \boxed{0.39\%}$$

71. The mean free path is given by Eq. 18-10b.

$$\ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^2\left(N/V\right)} = \frac{1}{4\pi\sqrt{2}\left(0.5\times10^{-10}\,\text{m}\right)^2\left(1\,\text{atom/cm}^3\right)\left(10^6\,\text{cm}^3/\text{m}^3\right)} = \boxed{2\times10^{13}\,\text{m}}$$

72. We combine the ideal gas law with Eq. 18-10b for the mean free path. From problem 45, we se that the diameter of the average air molecule is $3\times10^{-10}\,\text{m}$. Since air is mostly nitrogen molecules, this is a good approximation for the size of a nitrogen molecule.

$$PV = NkT \quad\rightarrow\quad \frac{N}{V} = \frac{P}{kT};$$

$$\ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^2\left(N/V\right)} = \frac{1}{4\pi\sqrt{2}r^2\left(P/kT\right)} = \boxed{\frac{kT}{4\pi\sqrt{2}r^2P}}$$

$$\ell_{\text{M}} = \frac{kT}{4\pi\sqrt{2}r^2P} = \frac{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}{4\pi\sqrt{2}\left(1.5\times10^{-10}\,\text{m}\right)^2 7.5\left(1.013\times10^5\,\text{Pa}\right)} = \boxed{1.4\times10^{-8}\,\text{m}}$$

Note that this is about 100 times the radius of the molecules.

73. Assume that the water is an ideal gas, and that the temperature is constant. From Table 18-2, saturated vapor pressure at 90°C is $7.01\times10^4\,\text{Pa}$, and so to have a relative humidity of 10%, the vapor pressure will be $7.01\times10^3\,\text{Pa}$. Use the ideal gas law to calculate the amount of water.

$$PV = nRT \quad \rightarrow$$

$$n = \frac{PV}{RT} = \frac{\left(7.01\times10^3\,\text{Pa}\right)\left(8.5\,\text{m}^3\right)}{\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(273+90\right)\text{K}} = 19.74\text{ moles}\left(\frac{18\times10^{-3}\,\text{kg}}{1\text{ mole}}\right) = \boxed{0.36\,\text{kg}}$$

74. Following the development of the kinetic molecular theory in the textbook, the tennis balls hitting the trash can lid are similar to the particles colliding with the walls of a container causing pressure. Quoting from the text, "the average force — averaged over many collisions — will be equal to the momentum change during one collision divided by the time between collisions." That average force must be the weight of the trash can lid in order to suspend it.

$$F_{\text{avg}} = M_{\text{lid}}g \;\; ; \;\; F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{2m_{\text{ball}}v_{\text{ball}}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{2m_{\text{ball}}v_{\text{ball}}}{M_{\text{lid}}g}$$

The above expression is "seconds per ball," so its reciprocal will be "balls per second."

$$\text{balls/s} = \frac{1}{\Delta t} = \frac{M_{\text{lid}}g}{2m_{\text{ball}}v_{\text{ball}}} = \frac{\left(0.50\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{2\left(0.060\,\text{kg}\right)\left(12\,\text{m/s}\right)} = \boxed{3.4\text{ balls/s}}$$

75. (*a*) The average time between collisions can be approximated as the mean free path divided by the mean speed. The highest frequency for a sound wave is the inverse of this average collision time. Combine the ideal gas law with Eq. 18-10b for the mean free path, and Eq. 18-7b for the mean speed.

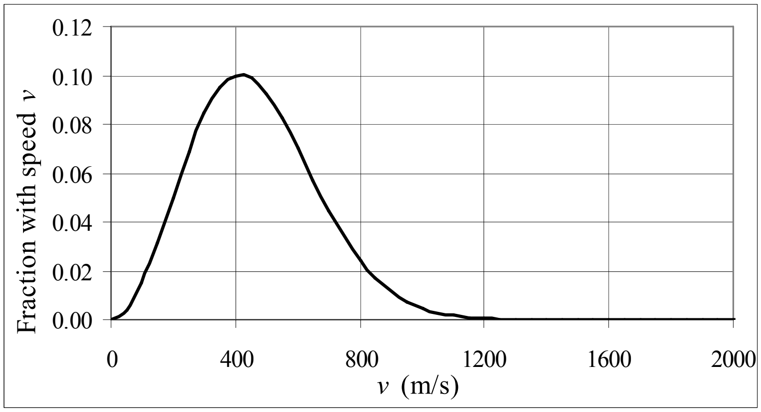
$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} \;\; ; \;\; \ell_{\text{M}} = \frac{1}{4\pi\sqrt{2}r^2\left(N/V\right)} = \frac{1}{4\pi\sqrt{2}r^2\left(P/kT\right)} = \frac{kT}{4\pi\sqrt{2}r^2P} \;\; ;$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} \quad \rightarrow \quad f_{\max} = \frac{\bar{v}}{\ell_{\text{M}}} = \frac{\sqrt{\dfrac{8kT}{\pi m}}}{\dfrac{kT}{4\pi\sqrt{2}r^2P}} = \boxed{16Pr^2\sqrt{\frac{\pi}{mkT}}}$$

(*b*) We have estimated the molecular mass of air to be 29 u in problem 16, and the average molecular diameter to be $3\times10^{-10}\,\text{m}$ in problem 48.

$$\begin{aligned} f_{\max} &= 16Pr^2\sqrt{\frac{\pi}{mkT}} \\ &= 16\left(1.013\times10^5\,\text{Pa}\right)\left(1.5\times10^{-10}\,\text{m}\right)^2\sqrt{\frac{\pi}{29\left(1.66\times10^{-27}\,\text{kg}\right)\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\,\text{K}\right)}} \\ &= \boxed{4.6\times10^9\,\text{Hz}} \end{aligned}$$

This frequency is about $\dfrac{4.6\times10^9\,\text{Hz}}{2.0\times10^4\,\text{Hz}} = \boxed{2.3\times10^5}$ larger than the highest frequency in the human audio range.



76. From section 18-2, the quantity $f(v)\,dv$ represents the number of molecules that have speeds between v and $v + dv$. So for a finite velocity range Δv, the number of molecules with speeds between v and $v+\Delta v$ is approximately $f(v)\Delta v$. If there are N total molecules, then the fraction with speeds between v and $v+\Delta v$ is $\frac{f(v)\Delta v}{N}$. We assume that we have air molecules with a molecular mass of 29, as given in problem 16.

$$f(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}} \quad\rightarrow\quad \frac{f(v)\Delta v}{N} = 4\pi\Delta v\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH18.XLS," on tab "Problem 18.76."

77. From section 18-2, the quantity $f(v)\,dv$ represents the number of molecules that have speeds between v and $v + dv$. The number of molecules with speeds greater than 1.5 times the most probable speed $v_{\rm p} = \sqrt{\frac{2kT}{m}}$ is $\int_{1.5v_{\rm p}}^{\infty} f(v)\,dv$. If there are N total molecules, then the fraction with speeds greater than $1.5v_{\rm p}$ is $\frac{1}{N}\int_{1.5v_{\rm p}}^{\infty} f(v)\,dv$. Since $\frac{1}{N}\int_{0}^{\infty} f(v)\,dv = 1$ (see problem 22a), we calculate the desired fraction as follows.

$$\frac{1}{N}\int_0^{\infty} f(v)\,dv = 1 = \frac{1}{N}\int_0^{1.5v_{\rm p}} f(v)\,dv + \frac{1}{N}\int_{1.5v_{\rm p}}^{\infty} f(v)\,dv \quad\rightarrow\quad \frac{1}{N}\int_{1.5v_{\rm p}}^{\infty} f(v)\,dv = 1 - \frac{1}{N}\int_0^{1.5v_{\rm p}} f(v)\,dv$$

We use a substation of variables to simplify the constants.

$$f(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}$$

$$x = \frac{v}{v_{\rm p}} = \frac{v}{\sqrt{2kT/m}} \quad\rightarrow\quad v^2 = \frac{2kT}{m}x^2 \quad\rightarrow\quad dv = \sqrt{\frac{2kT}{m}}dx$$

$$\frac{1}{N}\int_0^{1.5v_{\rm p}} f(v)\,dv = \frac{1}{N}\int_0^{1.5v_{\rm p}} 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}\,dv = \int_0^{1.5} 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{2kT}{m}x^2 e^{-x^2}\sqrt{\frac{2kT}{m}}dx$$

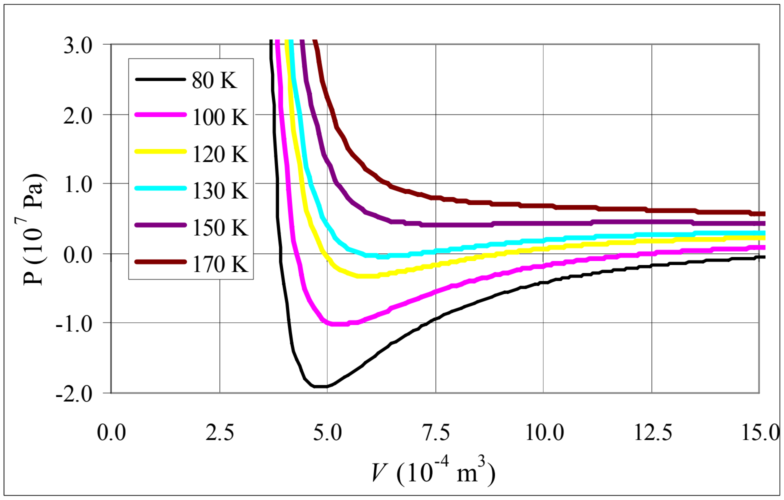
$$= \frac{4}{\sqrt{\pi}}\int_0^{1.5} x^2 e^{-x^2}\,dx$$

To do the integral, we approximate it as this sum: $\frac{4}{\sqrt{\pi}}\int_0^{1.5} x^2 e^{-x^2}\,dx \approx \frac{4}{\sqrt{\pi}}\sum_{i=1}^{n} x_i^2 e^{-x_i^2}\Delta x$, where $n = \frac{1.5}{\Delta x}$ is the number of intervals used to approximate the integral. We start with $\Delta x = 0.15$ and then try smaller intervals until the answers agree to within 2%. Here are the results of the numeric integration.

Δx	n	$\frac{4}{\sqrt{\pi}}\sum_{i=1}^{n} x_i^2 e^{-x_i^2}\Delta x$	% diff. from previous answer
0.15	10	0.8262	--
0.075	20	0.8074	2.3 %
0.03	50	0.7957	1.4 %

We approximate $\frac{1}{N}\int_0^{1.5v_p} f(v)\,dv = \frac{4}{\sqrt{\pi}}\int_0^{1.5} x^2 e^{-x^2}\,dx \approx 0.7957$, and so $\frac{1}{N}\int_{1.5v_p}^{\infty} f(v)\,dv = 1 - 0.7975$ $= 0.2025 \approx \boxed{0.20}$. Using more sophisticated software gives an answer of 0.21. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH18.XLS," on tab "Problem 18.77."

78. For each temperature, a graph of pressure vs. volume was plotted, based on Eq. 18-9, $P = \frac{RT}{(V/n) - b} - \frac{a}{(V/n)^2}$. From the graphs, it would appear that the critical temperature for oxygen is approximately $\boxed{150\text{ K}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH18.XLS," on tab "Problem 18.78."



CHAPTER 19: Heat and the First Law of Thermodynamics

Responses to Questions

1. When a jar of orange juice is vigorously shaken, the work done on it goes into heating the juice (increasing the kinetic energy of the molecules), mixing the components of the juice (liquid and pulp), and dissolving air in the juice (froth).

2. No. *Energy* is exchanged between them, not *temperature*. Once the objects have reached thermal equilibrium, they will have the same temperature. However, their temperature *changes* will not necessarily be the same.

3. (*a*) No. Because the internal energies of solids and liquids are complicated and include potential energies associated with the bonds between atoms and molecules, two objects may have different internal energies but the same temperature. Internal energy will also vary with the mass of the object. If two objects that are at different temperatures are placed in contact, there will be a net energy transfer from the hotter object to the colder one, regardless of their internal energies.
   (*b*) Yes. Just as in (*a*), the transfer of energy depends on the temperature difference between the two objects, which may not be directly related to the difference in internal energies.

4. Plants are damaged if the water inside their cells freezes. The latent heat of water is large, so if the cells are plump with water, rather than dry, it will take more time for them to lose enough heat to freeze. Well-hydrated plants are therefore less likely to be damaged if the temperature dips below freezing for a short time.

5. Because the specific heat of water is quite large, water can absorb a large amount of energy with a small increase in temperature. Water can be heated, then easily transported throughout a building, and will give off a large amount of energy as it cools. This makes water particularly useful in radiator systems.

6. The water on the cloth jacket will evaporate. Evaporation is a cooling process since energy is required to change the liquid water to vapor. If, for instance, radiant energy from the sun falls on the canteen, the energy will evaporate the water from the cloth cover instead of heating the water inside the canteen.

7. When water at 100ºC comes in contact with the skin, energy is transferred to the skin and the water begins to cool. When steam at 100ºC comes in contact with the skin, energy is transferred to the skin and the steam begins to condense to water at 100ºC. Steam burns are often more severe than water burns due to the energy given off by the steam as it condenses, before it begins to cool.

8. Energy is needed to convert water in the liquid state to the gaseous state (latent heat). Some of the energy needed to evaporate molecules on the surface comes from the internal energy of the water, thus decreasing the water temperature.

9. No. The water temperature cannot go above 100ºC, no matter how vigorously it is boiling. The rate at which potatoes cook depends on the temperature at which they are cooking.

10. Whether an animal freezes or not depends more on internal energy of the mass of air surrounding it than on the temperature. Temperature is a measure of the average kinetic energy of the molecules in a substance. If a mass of air in the upper atmosphere has a low density of fast moving molecules, it will have a high temperature but a low internal (or thermal) energy. Even though the molecules are moving quickly, there will be few collisions, and little energy transferred to the animal. The animal will also be radiating thermal energy, and so will quickly deplete its internal energy.

11. Energy is transferred from the water vapor to the glass as the water vapor condenses on the glass and then cools from the temperature of the surrounding air to the temperature of the glass. The glass (and the cold water inside) will heat up. No work is done, but heat is exchanged.

12. When a gas is compressed, work must be done on it by some outside force, such as a person pushing a piston. This work becomes the increase in internal energy of the gas, and if no gas is allowed to escape, an increase in internal energy results in faster average molecular speeds and a higher temperature. When a gas expands, it does work on the piston. If the gas is insulated so that no heat enters from the outside, then the energy for this work comes from the internal energy of the gas. A decrease in internal energy with no change in the number of molecules translates into a decrease in average molecular speed and therefore temperature.

$\boxed{13.}$ In an isothermal process, the temperature, and therefore the internal energy, of the ideal gas is constant. From the first law of thermodynamics, we know that if the change in the internal energy is zero, then the heat added to the system is equal to the work done by the system. Therefore, 3700 J of heat must have been added to the system.

14. Snow consists of crystals with tiny air pockets in between the flakes. Air is a good insulator, so when the Arctic explorers covered themselves with snow they were using its low thermal conductivity to keep heat from leaving their bodies. (In a similar fashion, down comforters keep you warm because of all the air trapped in between the feathers.) Snow would also protect the explorers from the very cold wind and prevent heat loss by convection.

15. Wet sand has been cooled by conduction (ocean water is usually cooler than the beach) and continues to be cooled by evaporation, and so will be cooler than dry sand. Wet sand will also feel cooler because of the thermal conductivity of water. The water in the sand will also cool your feet by evaporation.

16. Hot air furnaces often depend on natural convection. If the return air vent is blocked, convective currents in the room will not occur and the room will not be heated uniformly.

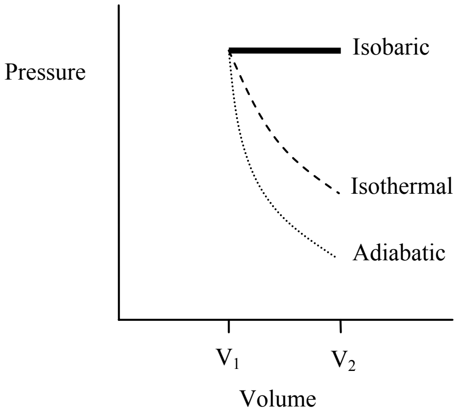
17. Yes. This is the case for any isothermal process or a process in which a substance changes state (melting/freezing, or condensing/evaporating).

18. Metabolism is a biochemical process by which living organisms get energy from food. If a body is doing work and losing heat, then its internal energy would drop drastically if there were no other source of energy. Metabolism (food) supplies this other source of energy. The first law of thermodynamics applies because the energy contributions from metabolism are included in Q.

$\boxed{19.}$ When a gas is heated at constant volume, all of the energy added goes into increasing the internal energy, since no work is done. When a gas is heated at constant pressure, some of the added energy is used for the work needed to expand the gas, and less is available for increasing the internal energy. It takes more energy to raise the temperature of a gas by a given amount at constant pressure than at constant volume.

20. An adiabatic compression is one that takes place with no exchange of heat with the surroundings. During the compression, work is done on the gas. Since no heat leaves the gas, then the work results in an increase in the gas's internal energy by the same amount, and therefore an increase in its temperature. $\Delta E_{\text{int}} = Q - W$, so if $Q = 0$ then $\Delta E_{\text{int}} = -W$.

21. ΔE_{int} is proportional to the change in temperature. The change in the internal energy is zero for the isothermal process, largest for the isobaric process, and least (negative) for the adiabatic process. The work done, W, is the area under the curve and is greatest for the isobaric process and least for the adiabatic process. From the first law of thermodynamics, Q is the sum of ΔE_{int} and W and is zero for the adiabatic process and maximum for the isobaric process.



22. In general, cooler air will be nearer the floor and warmer air nearer the ceiling. The fan operating in either direction redistributes the air by creating convection currents. Set the fan so that it will blow air down in the summer, creating a breeze, which has a cooling effect by increasing evaporation. In the winter, set the fan so that it pulls air up. This will cause convection currents which will help mix warm and cool air without creating a direct breeze.

23. The actual insulating value comes from the air trapped between the down feathers. The more air is trapped, the greater the loft, and the lower the rate of thermal conduction. So loft determines the warmth of the sleeping bag or parka.

24. The use of "fins" increases the surface area of the heat sink. The greater the surface area, the more heat can be given off from the chip to the surroundings.

$\boxed{25.}$ On a sunny day, the land heats faster than the water. The air over the land is also heated and it rises due to a decrease in density. The cooler air over the water is then pulled in to replace the rising air, creating an onshore or sea breeze.

26. At night, the Earth cools primarily through radiation of heat back into space. Clouds reflect energy back to the Earth and so the surface cools less on a cloudy night than on a clear one.

27. In direct sunlight, the solar radiation will heat the thermometer to a temperature greater than the surrounding air.

28. A premature baby will not have a well developed metabolism and will not produce much heat but will radiate heat to its surroundings. The surface of the incubator must be warmed so that the surface radiates sufficient heat back to the baby. In addition, a premature baby's skin is underdeveloped and the baby tends to lose moisture by evaporation. Since evaporation is a cooling process, this may dangerously cool the baby even in a warm incubator. The air in the incubator needs to be humid as well as warm. Finally, premature babies tend to have very little fat under the skin, so they are not well insulated, and have trouble maintaining body temperature without assistance.

29. If a house is built directly on the ground or on a slab, it can only lose energy through the floor by conduction. If air can circulate under the house, then energy loss also can occur due to convection and evaporation, especially if air is moving through the space.

30. A thermos bottle is designed to minimize heat transfer between the liquid contents and the outside air, even when the temperature difference is large. Heat transfer by radiation is minimized by the silvered lining. Shiny surfaces have very low emissivity, *e*, and thus the net rate of energy flow by radiation between the contents of the thermos and the outside air will be small. Heat transfer by conduction and convection will be minimized by the vacuum between the inner and outer walls of the thermos, since both these methods require a medium to transport heat.

31. Water has a greater thermal conductivity than air. Water at 22ºC will feel cooler than air at the same temperature because the rate of heat transfer away from the body will be greater.

32. The south-facing windows will allow radiant heat from the sun to enter the room to contribute to the heating, so that less heat will need to be provided internally.

33. (*a*) (1) Ventilation around the edges: convection; (2) through the frame: conduction; (3) through the glass panes: conduction and radiation.
    (*b*) Heavy curtains help prevent all three mechanisms for heat loss. They physically block convection currents and they are opaque and insulating and therefore reduce heat loss by radiation and conduction.

34. When the sun reaches the slope of the mountain early in the day, the ground is warmed by radiation. The air above the ground is also warmed and rises by convection. This rising air will move up the slope. When the slope is in shadow, the air cools and the convection currents reverse.

35. Wood has a much lower thermal conductivity than metals and so will feel cooler because the rate of heat transfer away from the hand will be less.

36. Shiny surfaces have low values of *e*, the emissivity. Thus, the net rate of heat flow from the person to the surroundings (outside the blanket) will be low, since most of the heat is reflected by blanket back to the person, and the person will stay warmer. The blanket will also prevent energy loss due to wind (convection).

37. The temperature of the air around cities near oceans is moderated by the presence of large bodies of water which act like a heat reservoir. Water has a high heat capacity. It will absorb energy in the summer with only a small temperature increase, and radiate energy in the winter, with a small temperature decrease.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. The kcal is the heat needed to raise 1 kg of water by 1C°. Use this relation to find the change in the temperature.

$$(8700\,\text{J})\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\frac{(1\,\text{kg})(1\text{C}^\circ)}{1\text{ kcal}}\left(\frac{1}{3.0\,\text{kg}}\right)=0.69\,\text{C}^\circ$$

Thus the final temperature is $10.0^\circ\text{C}+0.69^\circ\text{C}=\boxed{10.7^\circ\text{C}}$

2. Find the mass of warmed water from the volume of water and its density of $1000\,\text{kg}/\text{m}^3$. Then use the fact that 1 kcal of energy raises 1 kg of water by 1C°, and that the water warms by 25C°.

$$V = At = \frac{m}{\rho} \quad \rightarrow \quad m = \rho At = \left(1025\,\text{kg}/\text{m}^3\right)\left(1.0\,\text{m}^2\right)\left(0.50\times10^{-3}\,\text{m}\right) = 0.5125\,\text{kg}$$

$$\left(0.5125\,\text{kg}\right)\left(25\text{C}^\circ\right)\frac{\left(1\,\text{kcal}\right)}{\left(1\,\text{kg}\right)\left(1\text{C}^\circ\right)} = 12.8\,\text{kcal} \;;\; 12.8\,\text{kcal}\left(\frac{1\,\text{bar}}{300\,\text{kcal}}\right) = \boxed{0.043\,\text{bars}}$$

3. (*a*) $$2500\ \text{Cal}\left(\frac{4.186\times10^3\,\text{J}}{1\ \text{Cal}}\right) = \boxed{1.0\times10^7\,\text{J}}$$

(*b*) $$2500\ \text{Cal}\left(\frac{1\ \text{kWh}}{860\ \text{Cal}}\right) = \boxed{2.9\ \text{kWh}}$$

(*c*) At 10 cents per day, the food energy costs $\boxed{\$0.29\text{ per day}}$. It would be practically impossible to feed yourself in the United States on this amount of money.

4. Assume that we are at the surface of the Earth so that 1 lb is equivalent to 0.454 kg.

$$1\,\text{Btu} = \left(1\ \text{lb}\right)\left(1\text{F}^\circ\right)\left(\frac{0.454\ \text{kg}}{1\ \text{lb}}\right)\left(\frac{5/9\,\text{C}^\circ}{1\text{F}^\circ}\right)\frac{1\ \text{kcal}}{\left(1\ \text{kg}\right)\left(1\text{C}^\circ\right)} = 0.2522\,\text{kcal} \approx \boxed{0.252\ \text{kcal}}$$

$$0.2522\ \text{kcal}\left(\frac{4186\ \text{J}}{1\ \text{kcal}}\right) = \boxed{1056\,\text{J}}$$

5. The energy generated by using the brakes must equal the car's initial kinetic energy, since its final kinetic energy is 0.

$$Q = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}\left(1.2\times10^3\,\text{kg}\right)\left[\left(95\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = 4.178\times10^5\,\text{J} \approx \boxed{4.2\times10^5\,\text{J}}$$

$$\left(4.178\times10^5\,\text{J}\right)\left(\frac{1\ \text{kcal}}{4186\ \text{J}}\right) = 99.81\,\text{kcal} \approx \boxed{1.0\times10^2\,\text{kcal}}$$

6. The wattage rating is 350 Joules per second. Note that 1 L of water has a mass of 1 kg.

$$\left[\left(2.5\times10^{-1}\,\text{L}\right)\left(\frac{1\ \text{kg}}{1\ \text{L}}\right)\left(60\text{C}^\circ\right)\right]\frac{1\ \text{kcal}}{\left(1\ \text{kg}\right)\left(1\text{C}^\circ\right)}\left(\frac{4186\ \text{J}}{\text{kcal}}\right)\left(\frac{1\ \text{s}}{350\ \text{J}}\right) = \boxed{180\,\text{s} = 3.0\,\text{min}}$$

$\boxed{7.}$ The heat absorbed can be calculated from Eq. 19-2. Note that 1 L of water has a mass of 1 kg.

$$Q = mc\Delta T = \left[\left(18\,\text{L}\right)\left(\frac{1\times10^{-3}\,\text{m}^3}{1\ \text{L}}\right)\left(\frac{1.0\times10^3\,\text{kg}}{1\ \text{m}^3}\right)\right]\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(95^\circ\text{C} - 15^\circ\text{C}\right) = \boxed{6.0\times10^6\,\text{J}}$$

8. The specific heat can be calculated from Eq. 19-2.

$$Q = mc\Delta T \quad \rightarrow \quad c = \frac{Q}{m\Delta T} = \frac{1.35\times10^5\,\text{J}}{\left(5.1\ \text{kg}\right)\left(37.2^\circ\text{C} - 18.0^\circ\text{C}\right)} = 1379\,\text{J/kg}\bullet\text{C}^\circ \approx \boxed{1400\,\text{J/kg}\bullet\text{C}^\circ}$$

9. (*a*) The heat absorbed can be calculated from Eq. 19-2. Note that 1 L of water has a mass of 1 kg.

$$Q = mc\Delta T = \left[(1.0\,\text{L})\left(\frac{1\times10^{-3}\,\text{m}^3}{1\,\text{L}}\right)\left(\frac{1.0\times10^3\,\text{kg}}{1\,\text{m}^3}\right)\right](4186\,\text{J/kg}\bullet\text{C}^\circ)(100^\circ\text{C}-20^\circ\text{C})$$

$$= 3.349\times10^5\,\text{J} \approx \boxed{3.3\times10^5\,\text{J}}$$

(*b*) Power is the rate of energy usage.

$$P = \frac{\Delta E}{\Delta t} = \frac{Q}{\Delta t} \quad\rightarrow\quad \Delta t = \frac{Q}{P} = \frac{3.349\times10^5\,\text{J}}{100\,\text{W}} \approx \boxed{3300\,\text{s}} \approx 56\,\text{min}$$

10. The heat absorbed by all three substances is given by Eq. 19-2, $Q = mc\Delta T$. Thus the amount of mass can be found as $m = \dfrac{Q}{c\Delta T}$. The heat and temperature change are the same for all three substances.

$$m_{\text{Cu}} : m_{\text{Al}} : m_{\text{H}_2\text{O}} = \frac{Q}{c_{\text{Cu}}\Delta T} : \frac{Q}{c_{\text{Al}}\Delta T} : \frac{Q}{c_{\text{H}_2\text{O}}\Delta T} = \frac{1}{c_{\text{Cu}}} : \frac{1}{c_{\text{Al}}} : \frac{1}{c_{\text{H}_2\text{O}}} = \frac{1}{390} : \frac{1}{900} : \frac{1}{4186}$$

$$= \frac{4186}{390} : \frac{4186}{900} : \frac{4186}{4186} = 10.7 : 4.65 : 1 \approx \boxed{11 : 4.7 : 1}$$

11. The heat must warm both the water and the pot to 100°C. The heat is also the power times the time.

$$Q = Pt = \left(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right)\Delta T_{\text{H}_2\text{O}} \quad\rightarrow$$

$$t = \frac{\left(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right)\Delta T_{\text{H}_2\text{O}}}{P} = \frac{\left[(0.28\,\text{kg})(900\,\text{J/kg}\bullet\text{C}^\circ) + (0.75\,\text{kg})(4186\,\text{J/kg}\bullet\text{C}^\circ)\right](92\,\text{C}^\circ)}{750\,\text{W}}$$

$$= 416\,\text{s} \approx \boxed{420\,\text{s or } 6.9\,\text{min}}$$

12. The heat lost by the horseshoe must be equal to the heat gained by the iron pot and the water. Note that 1 L of water has a mass of 1 kg.

$$m_{\text{shoe}}c_{\text{Fe}}\left(T_{\text{shoe}} - T_{\text{eq}}\right) = m_{\text{pot}}c_{\text{Fe}}\left(T_{\text{eq}} - T_{\text{pot}}\right) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{\text{eq}} - T_{\text{H}_2\text{O}}\right)$$

$$(0.40\,\text{kg})(450\,\text{J/kg}\bullet\text{C}^\circ)(T_{\text{shoe}} - 25.0^\circ\text{C}) = (0.30\,\text{kg})(450\,\text{J/kg}\bullet\text{C}^\circ)(25.0^\circ\text{C} - 20.0^\circ\text{C})$$
$$+ (1.05\,\text{kg})(4186\,\text{J/kg}\bullet\text{C}^\circ)(25.0^\circ\text{C} - 20.0^\circ\text{C})$$

$$T_{\text{shoe}} = 150.8^\circ\text{C} \approx \boxed{150^\circ\text{C}}$$

$\boxed{13.}$ The heat gained by the glass thermometer must be equal to the heat lost by the water.

$$m_{\text{glass}}c_{\text{glass}}\left(T_{\text{eq}} - T_{\text{glass}}\right) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{\text{H}_2\text{O}} - T_{\text{eq}}\right)$$

$$(31.5\,\text{g})(0.20\,\text{cal/g}\bullet\text{C}^\circ)(39.2^\circ\text{C} - 23.6^\circ\text{C}) = (135\,\text{g})(1.00\,\text{cal/g}\bullet\text{C}^\circ)\left(T_{\text{H}_2\text{O}} - 39.2^\circ\text{C}\right)$$

$$T_{\text{H}_2\text{O}} = \boxed{39.9^\circ\text{C}}$$

14. The heat released by the 15 grams of candy in the burning is equal to the heat absorbed by the aluminum and water.

$$Q_{\underset{\text{candy}}{15\,\text{g}}} = \left(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\right)\Delta T$$

$$=\left[\left(0.325\text{ kg}+0.624\text{ kg}\right)\left(0.22\,\text{kcal/kg}\bullet\text{C}^\circ\right)+\left(2.00\text{ kg}\right)\left(1.00\,\text{kcal/kg}\bullet\text{C}^\circ\right)\right]\left(53.5^\circ\text{C}-15.0^\circ\text{C}\right)$$
$$=85.04\,\text{kcal}$$

The heat released by 65 grams of the candy would be 65/15 times that released by the 15 grams.

$$Q_{\substack{65\text{g}\\ \text{candy}}}=\tfrac{65}{15}Q_{\substack{15\text{g}\\ \text{candy}}}=\tfrac{65}{15}\left(85.04\text{ kcal}\right)=369\,\text{kcal}\approx\boxed{370\,\text{Cal}}$$

15. The heat lost by the iron must be the heat gained by the aluminum and the glycerin.

$$m_{\text{Fe}}c_{\text{Fe}}\left(T_{i\,\text{Fe}}-T_{\text{eq}}\right)=m_{\text{Al}}c_{\text{Al}}\left(T_{\text{eq}}-T_{i\,\text{Al}}\right)+m_{\text{gly}}c_{\text{gly}}\left(T_{\text{eq}}-T_{i\,\text{gly}}\right)$$
$$\left(0.290\text{ kg}\right)\left(450\,\text{J/kg}\bullet\text{C}^\circ\right)\left(142\text{C}^\circ\right)=\left(0.095\text{ kg}\right)\left(900\,\text{J/kg}\bullet\text{C}^\circ\right)\left(28\text{C}^\circ\right)+\left(0.250\text{ kg}\right)c_{\text{gly}}\left(28\text{C}^\circ\right)$$
$$c_{\text{gly}}=2305\,\text{J/kg}\bullet\text{C}^\circ\approx\boxed{2300\,\text{J/kg}\bullet\text{C}^\circ}$$

16. (*a*) Since $Q=mc\Delta T$ and $Q=C\Delta T$, equate these two expressions for Q and solve for C.

$$Q=mc\Delta T=C\Delta T\quad\rightarrow\quad\boxed{C=mc}$$

(*b*) For 1.0 kg of water: $C=mc=\left(1.0\text{ kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)=\boxed{4200\,\text{J/C}^{\text{o}}}$

(*c*) For 35 kg of water: $C=mc=\left(35\text{ kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^{\text{o}}\right)=\boxed{1.5\times10^5\text{ J/C}^{\text{o}}}$

17. We assume that all of the kinetic energy of the hammer goes into heating the nail.

$$KE=Q\quad\rightarrow\quad10\left(\tfrac{1}{2}m_{\text{hammer}}v_{\text{hammer}}^2\right)=m_{\text{nail}}c_{\text{Fe}}\Delta T\quad\rightarrow$$
$$\Delta T=\frac{10\left(\tfrac{1}{2}m_{\text{hammer}}v_{\text{hammer}}^2\right)}{m_{\text{nail}}c_{\text{Fe}}}=\frac{5\left(1.20\text{ kg}\right)\left(7.5\,\text{m/s}\right)^2}{\left(0.014\text{ kg}\right)\left(450\,\text{J/kg}\bullet\text{C}^\circ\right)}=53.57\,\text{C}^\circ\approx\boxed{54\,\text{C}^\circ}$$

18. The silver must be heated to the melting temperature and then melted.

$$Q=Q_{\text{heat}}+Q_{\text{melt}}=mc\Delta T+mL_{\text{fusion}}$$
$$=\left(26.50\text{ kg}\right)\left(230\,\text{J/kg}\bullet\text{C}^\circ\right)\left(961^\circ\text{C}-25^\circ\text{C}\right)+\left(26.50\text{ kg}\right)\left(0.88\times10^5\text{ J/kg}\right)=\boxed{8.0\times10^6\,\text{J}}$$

19. Assume that the heat from the person is only used to evaporate the water. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the person's temperature is closer to room temperature than 100°C.

$$Q=mL_{\text{vap}}\quad\rightarrow\quad m=\frac{Q}{L_{\text{vap}}}=\frac{180\,\text{kcal}}{585\,\text{kcal/kg}}=0.308\,\text{kg}\approx\boxed{0.31\,\text{kg}}=310\,\text{mL}$$

20. Assume that all of the heat lost by the ice cube in cooling to the temperature of the liquid nitrogen is used to boil the nitrogen, and so none is used to raise the temperature of the nitrogen. The boiling point of the nitrogen is $77\,\text{K}=-196^{\text{o}}\text{C}$.

$$m_{\text{ice}}c_{\text{ice}}\left(T_{\substack{\text{ice}\\ \text{initial}}}-T_{\substack{\text{ice}\\ \text{final}}}\right)=m_{\text{nitrogen}}L_{\text{vap}}\quad\rightarrow$$
$$m_{\text{nitrogen}}=\frac{m_{\text{ice}}c_{\text{ice}}\left(T_{\substack{\text{ice}\\ \text{initial}}}-T_{\substack{\text{ice}\\ \text{final}}}\right)}{L_{\text{vap}}}=\frac{\left(3.5\times10^{-2}\text{kg}\right)\left(2100\,\text{J/kg}\bullet\text{C}^\circ\right)\left(0^\circ\text{C}--196^\circ\text{C}\right)}{200\times10^3\text{ J/kg}}=\boxed{7.2\times10^{-2}\text{kg}}$$

21. (*a*) The energy absorbed from the body must warm the snow to the melting temperature, melt the snow, and then warm the melted snow to the final temperature.

$$Q_a = Q_{\underset{\text{snow}}{\text{warm}}} + Q_{\text{melt}} + Q_{\underset{\text{liquid}}{\text{warm}}} = mc_{\text{snow}}\Delta T_1 + mL_{\text{fusion}} + mc_{\text{liquid}}\Delta T_2 = m\left[c_{\text{snow}}\Delta T_1 + L_{\text{fusion}} + c_{\text{liquid}}\Delta T_2\right]$$

$$= (1.0\text{ kg})\left[(2100\,\text{J/kg}\bullet\text{C}^\circ)(10\,\text{C}^\circ) + (3.33\times10^5\,\text{J/kg}) + (4186\,\text{J/kg}\bullet\text{C}^\circ)(37\,\text{C}^\circ)\right]$$

$$= \boxed{5.1\times10^5\,\text{J}}$$

(*b*) The energy absorbed from the body only has to warm the melted snow to the final temperature.

$$Q_b = Q_{\underset{\text{liquid}}{\text{heat}}} = mc_{\text{liquid}}\Delta T_2 = (1.0\text{ kg})(4186\,\text{J/kg}\bullet\text{C}^\circ)(35\,\text{C}^\circ) = \boxed{1.5\times10^5\,\text{J}}$$

22. (*a*) The heater must heat both the boiler and the water at the same time.

$$Q_1 = Pt_1 = \left(m_{\text{Fe}}c_{\text{Fe}} + m_{H_2O}c_{H_2O}\right)\Delta T \quad\rightarrow$$

$$t_1 = \frac{\left(m_{\text{Fe}}c_{\text{Fe}} + m_{H_2O}c_{H_2O}\right)\Delta T}{P} = \frac{\left[(180\,\text{kg})(450\,\text{J/kg}\bullet\text{C}^\circ) + (730\,\text{kg})(4186\,\text{J/kg}\bullet\text{C}^\circ)\right](82\,\text{C}^\circ)}{5.2\times10^7\,\text{J/h}}$$

$$= 4.946\,\text{h} \approx \boxed{4.9\,\text{h}}$$

(*b*) Assume that after the water starts to boil, all the heat energy goes into boiling the water, and none goes to raising the temperature of the iron or the steam.

$$Q_2 = Pt_2 = m_{H_2O}L_{\text{vap}} \quad\rightarrow\quad t_2 = \frac{m_{H_2O}L_{\text{vap}}}{P} = \frac{(730\,\text{kg})(22.6\times10^5\,\text{J/kg})}{5.2\times10^7\,\text{J/h}} = 31.727\,\text{h}$$

Thus the total time is $t_1 + t_2 = 4.946\,\text{h} + 31.727\,\text{h} = 36.673\,\text{h} \approx \boxed{37\,\text{h}}$

23. We assume that the cyclist's energy is only going to evaporation, not any heating. Then the energy needed is equal to the mass of the water times the latent heat of vaporization for water. Note that 1 L of water has a mass of 1 kg. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the cyclist's temperature is closer to room temperature than 100°C.

$$Q = m_{H_2O}L_{\text{vap}} = (8.0\,\text{kg})(585\,\text{kcal/kg}) = \boxed{4700\,\text{kcal}}$$

24. The heat lost by the aluminum and the water must equal the heat needed to melt the mercury and to warm the mercury to the equilibrium temperature.

$$m_{\text{Al}}c_{\text{Al}}\left(T_{\text{Al}} - T_{\text{eq}}\right) + m_{H_2O}c_{H_2O}\left(T_{H_2O} - T_{\text{eq}}\right) = m_{\text{Hg}}\left[L_{\text{fusion}} + c_{\text{Hg}}\left(T_{\text{eq}} - T_{\text{melt}}\right)\right]$$

$$L_{\text{fusion}} = \frac{m_{\text{Al}}c_{\text{Al}}\left(T_{\text{Al}} - T_{\text{eq}}\right) + m_{H_2O}c_{H_2O}\left(T_{H_2O} - T_{\text{eq}}\right)}{m_{\text{Hg}}} - c_{\text{Hg}}\left(T_{\text{eq}} - T_{\text{melt}}\right)$$

$$= \frac{\left[(0.620\text{ kg})(900\,\text{J/kg}\bullet\text{C}^\circ) + (0.400\text{ kg})(4186\,\text{J/kg}\bullet\text{C}^\circ)\right](12.80^\circ\text{C} - 5.06^\circ\text{C})}{1.00\text{ kg}}$$

$$-(138\,\text{J/kg}\bullet\text{C}^\circ)\left[5.06^\circ\text{C} - (-39.0^\circ\text{C})\right]$$

$$= \boxed{1.12\times10^4\,\text{J/kg}}$$

25. The kinetic energy of the bullet is assumed to warm the bullet and melt it.

$$\tfrac{1}{2}mv^2 = Q = mc_{\text{Pb}}\left(T_{\text{melt}} - T_{\text{initial}}\right) + mL_{\text{fusion}} \quad \rightarrow$$

$$v = \sqrt{2\left[c_{\text{Pb}}\left(T_{\text{melt}} - T_{\text{initial}}\right) + L_{\text{fusion}}\right]} = \sqrt{2\left[\left(130\,\text{J/kg}\bullet\text{C}^\circ\right)\left(327^\circ\text{C} - 20^\circ\text{C}\right) + \left(0.25\times10^5\,\text{J/kg}\right)\right]}$$

$$= \boxed{360\,\text{m/s}}$$

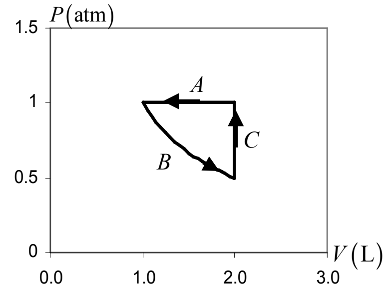
26. Assume that all of the melted ice stays at 0°C, so that all the heat is used in melting ice, and none in warming water. The available heat is half of the original kinetic energy

$$\tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{skater}}v^2\right) = Q = m_{\text{ice}}L_{\text{fusion}} \quad \rightarrow$$

$$m_{\text{ice}} = \frac{\frac{1}{4}m_{\text{skater}}v^2}{L_{\text{fusion}}} = \frac{\frac{1}{4}\left(58\,\text{kg}\right)\left(7.5\,\text{m/s}\right)^2}{3.33\times10^5\,\text{J/kg}} = \boxed{2.4\times10^{-3}\,\text{kg}} = 2.4\,\text{g}$$

27. Segment *A* is the compression at constant pressure. Since the process is at a constant pressure, the path on the diagram is horizontal from 2.0 L to 1.0 L.

Segment *B* is the isothermal expansion. Since the temperature is constant, the ideal gas law says that the product *PV* is constant. Since the volume is doubled, the pressure must be halved, and so the final point on this segment is at a pressure of 0.5 atm. The path is a piece of a hyperbola.



Segment *C* is the pressure increase at constant volume. Since the process is at a constant volume, the path on the diagram is vertical from 0.5 atm to 1.0 atm.

28. (*a*) The work done by a gas at constant pressure is found from Eq. 19-9a.

$$W = P\Delta V = \left(1\text{ atm}\right)\left(\frac{1.01\times10^5\,\text{Pa}}{1\text{ atm}}\right)\left(18.2\text{ m}^3 - 12.0\text{ m}^3\right) = 6.262\times10^5\,\text{J} \approx \boxed{6.3\times10^5\,\text{J}}$$

(*b*) The change in internal energy is calculated from the first law of thermodynamics

$$\Delta E_{\text{int}} = Q - W = \left(1250\text{ kcal}\right)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right) - 6.262\times10^5\,\text{J} = \boxed{4.60\times10^6\,\text{J}}$$

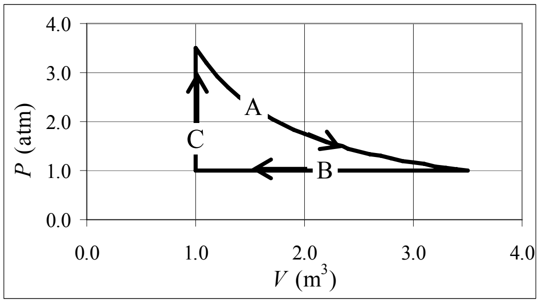
29. (*a*) Since the container has rigid walls, there is no change in volume.

$$W = P\Delta V = \boxed{0\text{ J}}$$

(*b*) Use the first law of thermodynamics to find the change in internal energy.

$$\Delta E_{\text{int}} = Q - W = \left(-365\text{ kJ}\right) - 0 = \boxed{-365\text{ kJ}}$$

30. Segment *A* is the isothermal expansion. The temperature and the amount of gas are constant, so $PV = nRT$ is constant. Since the pressure is reduced by a factor of 3.5, the volume increases by a factor of 3.5, to a final volume of 3.5 L. Segment *B* is the compression at constant pressure, and segment *C* is the pressure increase at constant volume. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH19.XLS," on tab "Problem 19.30."



$\boxed{31.}$ (*a*) No work is done during the first step, since the volume is constant. The work in the second step is given by $W = P\Delta V$.

$$W = P\Delta V = (1.4\text{ atm})\left(\frac{1.01\times10^5\text{Pa}}{1\text{ atm}}\right)(9.3\text{L} - 5.9\text{L})\left(\frac{1\times10^{-3}\text{m}^3}{1\text{ L}}\right) = \boxed{480\text{J}}$$

(*b*) Since there is no overall change in temperature, $\Delta E_{\text{int}} = \boxed{0\text{J}}$

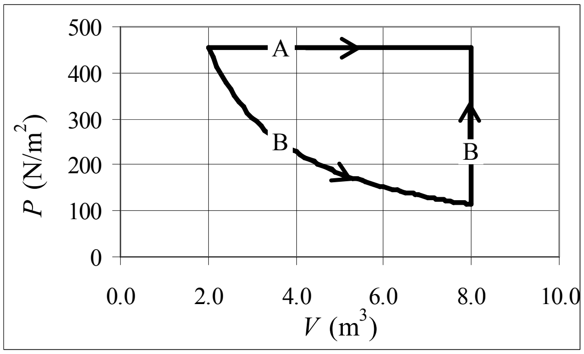
(*c*) The heat flow can be found from the first law of thermodynamics.

$$\Delta E_{\text{int}} = Q - W \quad\rightarrow\quad Q = \Delta E_{\text{int}} + W = 0 + 480\text{J} = \boxed{480\text{J (into the gas)}}$$

32. (*a*) See the diagram. The isobaric expansion is just a horizontal line on the graph.

(*b*) The work done is found from Eq. 19-9a.

$$W = P\Delta V = (455\text{N/m}^2)(8.00\text{m}^3 - 2.00\text{m}^3) = \boxed{2730\text{J}}$$



The change in internal energy depends on the temperature change, which can be related to the ideal gas law, $PV = nRT$.

$$\Delta E_{\text{int}} = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(nRT_2 - nRT_1)$$

$$= \tfrac{3}{2}\left[(PV)_2 - (PV)_1\right] = \tfrac{3}{2}P\Delta V = \tfrac{3}{2}W = \tfrac{3}{2}(2730\text{J}) = \boxed{4.10\times10^3\text{J}}$$

(*c*) For the isothermal expansion, since the volume expands by a factor of 4, the pressure drops by a factor of 4 to 114N/m^2. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH19.XLS," on tab "Problem 19.32."

(*d*) The change in internal energy only depends on the initial and final temperatures. Since those temperatures are the same for process (B) as they are for process (A), the internal energy change is the same for process (B) as for process (A), $\boxed{4.10\times10^3\text{J}}$.

33. (*a*) The work done by an ideal gas during an isothermal volume change is given by Eq. 19-8.

$$W = nRT\ln\frac{V_2}{V_1} = (2.60\text{mol})(8.314\text{J/mol}\bullet\text{K})(290\text{K})\ln\frac{7.00\text{m}^3}{3.50\text{m}^3} = 4345.2\text{J} \approx \boxed{4350\text{J}}$$

(*b*) Since the process is isothermal, there is no internal energy change. Apply the first law of thermodynamics.

$$\Delta E_{\text{int}} = Q - W = 0 \quad\rightarrow\quad Q = W = \boxed{4350\text{J}}$$

(*c*) Since the process is isothermal, there is no internal energy change, and so $\boxed{\Delta E_{\text{int}} = 0}$.

34. (*a*) Since the process is adiabatic, $Q = \boxed{0\text{ J}}$

(*b*) Use the first law of thermodynamics to find the change in internal energy. The work is done on the gas, and so is negative.

$$\Delta E_{\text{int}} = Q - W = 0 - (-2850\text{J}) = \boxed{2850\text{ J}}$$

(*c*) Since the internal energy is proportional to the temperature, a rise in internal energy means a $\boxed{\text{rise}}$ in temperature.

35. Since the expansion is adiabatic, there is no heat flow into or out of the gas. Use the first law of thermodynamics to calculate the temperature change.

$$\Delta E_{\text{int}} = Q - W \;\rightarrow\; \tfrac{3}{2}nR\Delta T = 0 - W \;\rightarrow$$

$$\Delta T = -\tfrac{2}{3}\frac{W}{nR} = -\frac{2(7500\text{ J})}{3(1.5\text{ mol})(8.315\,\text{J/mol}\bullet\text{K})} = -401\,\text{K} = \boxed{-4.0\times10^{2}\,\text{K}}$$

36. (*a*) The initial volume of the water is found from its mass and density. The final volume is found from the ideal gas law. The work done at constant pressure is given by Eq. 19-9a.

$$V_1 = \frac{m}{\rho} = \frac{(1.00\,\text{kg})}{(1.00\times10^{3}\,\text{kg/m}^3)} = 1.00\times10^{-3}\,\text{m}^3$$

$$V_2 = \frac{nRT}{P_2} = \frac{(1.00\,\text{kg})\left(\dfrac{1\,\text{mol}}{0.018\,\text{kg}}\right)(8.315\,\text{J/mol}\bullet\text{K})(373\,\text{K})}{1.013\times10^{5}\,\text{Pa}} = 1.70\,\text{m}^3$$

Note that the initial volume is negligible. We might have assumed that since the original state was liquid, that the gas volume was 0 to begin with, without significant error.

$$W = P\Delta V = (1.013\times10^{5}\,\text{Pa})(1.70\,\text{m}^3) = 1.722\times10^{5}\,\text{J} \approx \boxed{1.72\times10^{5}\,\text{J}}$$

(b) The heat added to the system is calculated from the latent heat of vaporization. Then the first law of thermodynamics will give the internal energy change.

$$Q = mL_{\text{V}} = (1.00\,\text{kg})(2260\,\text{kJ/kg}) = 2260\,\text{kJ} = 2.26\times10^{6}\,\text{J}$$

$$\Delta E_{\text{int}} = Q - W = 2.26\times10^{6}\,\text{J} - 1.72\times10^{5}\,\text{J} = \boxed{2.09\times10^{6}\,\text{J}}$$

$\boxed{37.}$ The work done by an ideal gas during an isothermal volume change is given by Eq. 19-8.

$$W = nRT\ln\frac{V_{\text{B}}}{V_{\text{A}}} = P_{\text{A}}V_{\text{A}}\ln\frac{V_{\text{B}}}{V_{\text{A}}} = (1.013\times10^{5}\,\text{Pa})(3.50\times10^{-3}\,\text{m}^3)\left(\ln\frac{1.80\,\text{L}}{3.50\,\text{L}}\right) = -236\,\text{J}$$

The work done by an external agent is the opposite of the work done by the gas, $\boxed{236\,\text{J}}$.

38. For the path ac, use the first law of thermodynamics to find the change in internal energy.

$$\Delta E_{\underset{\text{ac}}{\text{int}}} = Q_{\text{ac}} - W_{\text{ac}} = -63\text{ J} - (-35\text{ J}) = -28\text{ J}$$

Since internal energy only depends on the initial and final temperatures, this ΔE_{int} applies to any path that starts at a and ends at c. And for any path that starts at c and ends at a, $\Delta E_{\underset{\text{ca}}{\text{int}}} = -\Delta E_{\underset{\text{ac}}{\text{int}}} = 28\text{ J}.$

(*a*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta E_{\underset{\text{abc}}{\text{int}}} = Q_{\text{abc}} - W_{\text{abc}} \;\rightarrow\; Q_{\text{abc}} = \Delta E_{\underset{\text{abc}}{\text{int}}} + W_{\text{abc}} = -28\,\text{J} + (-54\,\text{J}) = \boxed{-82\,\text{J}}$$

(*b*) Since the work along path bc is 0, $W_{\text{abc}} = W_{\text{ab}} = P_{\text{b}}\Delta V_{\text{ab}} = P_{\text{b}}(V_{\text{b}} - V_{\text{a}})$. Also note that the work along path da is 0.

$$W_{\text{cda}} = W_{\text{cd}} = P_{\text{c}}\Delta V_{\text{cd}} = P_{\text{c}}(V_{\text{d}} - V_{\text{c}}) = \tfrac{1}{2}P_{\text{b}}(V_{\text{a}} - V_{\text{b}}) = -\tfrac{1}{2}W_{\text{abc}} = -\tfrac{1}{2}(-54\,\text{J}) = \boxed{27\,\text{J}}$$

(*c*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta E_{\substack{\text{int}\\\text{cda}}} = Q_{\text{cda}} - W_{\text{cda}} \rightarrow Q_{\text{cda}} = \Delta E_{\substack{\text{int}\\\text{cda}}} + W_{\text{cda}} = 28\text{ J} + 27\text{ J} = \boxed{55\text{ J}}$$

(*d*) As found above, $E_{\text{int,a}} - E_{\text{int,c}} = \Delta E_{\substack{\text{int}\\\text{ca}}} = -\Delta E_{\substack{\text{int}\\\text{ac}}} = \boxed{28\text{ J}}$

(*e*) Since $E_{\text{int,d}} - E_{\text{int,c}} = 12\text{ J}$, $E_{\text{int,d}} = E_{\text{int,c}} + 12\text{ J}$ and so $\Delta E_{\substack{\text{int}\\\text{da}}} = E_{\text{int,a}} - E_{\text{int,d}} = E_{\text{int,a}} - \left(E_{\text{int,c}} + 12\text{ J}\right)$ which then gives $\Delta E_{\substack{\text{int}\\\text{da}}} = \Delta E_{\substack{\text{int}\\\text{ca}}} - 12\text{ J} = 28\text{ J} - 12\text{ J} = 16\text{ J}$. Use the first law of thermodynamics to find Q_{da}.

$$\Delta E_{\substack{\text{int}\\\text{da}}} = Q_{\text{da}} - W_{\text{da}} \rightarrow Q_{\text{da}} = \Delta E_{\substack{\text{int}\\\text{da}}} + W_{\text{da}} = 16\text{ J} + 0 = \boxed{16\text{ J}}$$

39. We are given that $Q_{\text{ac}} = -85\text{ J}$, $W_{\text{ac}} = -55\text{ J}$, $W_{\text{cda}} = 38\text{ J}$, $E_{\text{int,a}} - E_{\text{int,b}} = \Delta E_{\substack{\text{int}\\\text{ba}}} = 15\text{ J}$, and $P_{\text{a}} = 2.2P_{\text{d}}$.

(*a*) Use the first law of thermodynamics to find $E_{\text{int,a}} - E_{\text{int,c}} = \Delta E_{\text{ca}}$.

$$\Delta E_{\substack{\text{int}\\\text{ca}}} = -\Delta E_{\substack{\text{int}\\\text{ac}}} = -\left(Q_{\text{ac}} - W_{\text{ac}}\right) = -\left(-85\text{ J} - -55\text{ J}\right) = \boxed{30\text{ J}}$$

(*b*) Use the first law of thermodynamics to find Q_{cda}.

$$\Delta E_{\substack{\text{int}\\\text{cda}}} = Q_{\text{cda}} - W_{\text{cda}} \rightarrow Q_{\text{cda}} = \Delta E_{\substack{\text{int}\\\text{cda}}} + W_{\text{cda}} = \Delta E_{\substack{\text{int}\\\text{ca}}} + W_{\text{cda}} = 30\text{ J} + 38\text{ J} = \boxed{68\text{ J}}$$

(*c*) Since the work along path bc is 0, $W_{\text{abc}} = W_{\text{ab}} = P_{\text{a}}\Delta V_{\text{ab}} = P_{\text{a}}\left(V_{\text{b}} - V_{\text{a}}\right)$.

$$W_{\text{abc}} = W_{\text{ab}} = P_{\text{a}}\Delta V_{\text{ab}} = P_{\text{a}}\left(V_{\text{b}} - V_{\text{a}}\right) = 2.2P_{\text{d}}\left(V_{\text{c}} - V_{\text{d}}\right) = -2.2W_{\text{cda}} = -2.2\left(38\text{ J}\right) = \boxed{-84\text{ J}}$$

(*d*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta E_{\substack{\text{int}\\\text{abc}}} = Q_{\text{abc}} - W_{\text{abc}} \rightarrow Q_{\text{abc}} = \Delta E_{\substack{\text{int}\\\text{abc}}} + W_{\text{abc}} = \Delta E_{\substack{\text{int}\\\text{ac}}} + W_{\text{abc}} = -30\text{ J} - 84\text{ J} = \boxed{-114\text{ J}}$$

(*e*) Since $E_{\text{int,a}} - E_{\text{int,b}} = 15\text{ J} \rightarrow E_{\text{int,b}} = E_{\text{int,a}} - 15\text{ J}$, we have the following.

$$\Delta E_{\substack{\text{int}\\\text{bc}}} = E_{\text{int,c}} - E_{\text{int,b}} = E_{\text{int,c}} - \left(E_{\text{int,a}} - 15\text{ J}\right) = \Delta E_{\substack{\text{int}\\\text{ac}}} + 15\text{ J} = -30\text{ J} + 15\text{ J} = -15\text{ J}.$$

Use the first law of thermodynamics to find Q_{bc}.

$$\Delta E_{\substack{\text{int}\\\text{bc}}} = Q_{\text{bc}} - W_{\text{bc}} \rightarrow Q_{\text{bc}} = \Delta E_{\substack{\text{int}\\\text{bc}}} + W_{\text{bc}} = -15\text{ J} + 0 = \boxed{-15\text{ J}}$$

40. (*a*) Leg ba is an isobaric expansion, and so the work done is positive.
Leg ad is an isovolumetric reduction in pressure, and so the work done on that leg is 0.
Leg dc is an isobaric compression, and so the work done is negative.
Leg cb is an isovolumetric expansion in pressure, and so the work done on that leg is 0.

(*b*) From problem 38, $W_{\text{cda}} = W_{\text{cd}} + W_{\text{da}} = 38\text{ J}$, so $W_{\text{adc}} = W_{\text{ad}} + W_{\text{dc}} = -38\text{ J}$. Also from problem 38, $W_{\text{abc}} = -84\text{ J}$, and so $W_{\text{cba}} = W_{\text{cb}} + W_{\text{ba}} = 84\text{ J}$. So the net work done during the cycle is as follows.

$$W_{\text{net}} = W_{\text{ba}} + W_{\text{ad}} + W_{\text{dc}} + W_{\text{cb}} = 84\text{ J} - 38\text{ J} = \boxed{46\text{ J}}$$

(*c*) Since the process is a cycle, the initial and final states are the same, and so the internal energy does not change.

$$\Delta E_{\text{int}} = \boxed{0}$$

(*d*) Use the first law of thermodynamics, applied to the entire cycle.

$$\Delta E_{\substack{\text{int}\\ \text{tot}}} = Q_{\text{net}} - W_{\text{net}} \rightarrow Q_{\text{net}} = \Delta E_{\text{int}} + W_{\text{net}} = 0 + 46\,\text{J} = \boxed{46\,\text{J}}$$

(*e*) From problem 39(*b*), we have $Q_{\text{adc}} = -68\,\text{J}$. This is the exhaust heat. So the input heat is found as follows.

$$Q_{\text{net}} = Q_{\text{acd}} + Q_{\text{dca}} = -68\,\text{J} + Q_{\text{dca}} = 46\,\text{J} \rightarrow Q_{\text{dca}} = 114\,\text{J} = Q_{\text{input}}$$

$$\text{efficiency} = \frac{W_{\text{net}}}{Q_{\text{input}}} \times 100 = \frac{46\,\text{J}}{114\,\text{J}} \times 100 = \boxed{40\%} \ (2 \text{ sig. fig.})$$

41. The work is given by $W = \int_{V_1}^{V_2} P\,dV$. The pressure is given by the van der Waals expression, Eq. 18-9, with *n* = 1.00. The temperature is held constant. We will keep the moles as *n* until the last step.

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \left(\frac{RT}{\frac{V}{n} - b} - \frac{a}{\left(\frac{V}{n}\right)^2} \right) dV = \int_{V_1}^{V_2} \left(\frac{nRT}{V - bn} - \frac{an^2}{V^2} \right) dV = \left[nRT \ln(V - bn) + \frac{an^2}{V} \right]_{V_1}^{V_2}$$

$$= \left[nRT \ln(V_2 - bn) + \frac{an^2}{V_2} \right] - \left[nRT \ln(V_1 - bn) + \frac{an^2}{V_1} \right] = nRT \ln\frac{(V_2 - bn)}{(V_1 - bn)} + an^2 \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

We evaluate for *n* = 1.00 mol, to get $W = \boxed{RT \ln\frac{(V_2 - b)}{(V_1 - b)} + a\left(\frac{1}{V_2} - \frac{1}{V_1} \right)}$.

42. For a diatomic gas with all degrees of freedom active, the internal energy is given by $E_{\text{int}} = \frac{7}{2} nRT$.

$$E_{\text{int}} = \frac{7}{2} nRT = \frac{7}{2}(4.50\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(645\,\text{K}) = \boxed{8.45 \times 10^4\,\text{J}}$$

43. If there are no heat losses or mass losses, then the heating occurs at constant volume, and so Eq. 19-10a applies, $Q = nC_{\text{V}}\Delta T$. Air is primarily made of diatomic molecules, and for an ideal diatomic gas, $C_{\text{V}} = \frac{5}{2}R$.

$$Q = nC_{\text{V}}\Delta T = \frac{5}{2} nR\Delta T = \frac{5}{2} \frac{P_0 V_0}{T_0} \Delta T \rightarrow$$

$$\Delta T = \frac{2QT}{5PV} = \frac{2(1.8 \times 10^6\,\text{J})(293\,\text{K})}{5(1.013 \times 10^5\,\text{Pa})(3.5\,\text{m})(4.6\,\text{m})(3.0\,\text{m})} = 43.12\,\text{K} \approx \boxed{43\,\text{C}^\circ}$$

44. For one mole of gas, each degree of freedom has an average energy of $\frac{1}{2}RT$, so the internal energy of a mole of the gas is as follows.

$$E_{\text{int}} = n\left(\tfrac{1}{2}RT\right) = C_V T \rightarrow C_V = \boxed{\tfrac{1}{2} nR}$$

$$C_P = C_V + R = \frac{n}{2}R + R = \frac{n}{2}R + \frac{2}{2}R = \boxed{\tfrac{1}{2}(n+2)R}$$

45. Since the gas is monatomic, the molar specific heat is given by $C_V = \tfrac{3}{2}R$. The molar specific heat is also given by $C_V = Mc_V$, where M is the molecular mass. Equate these two expressions to find the molecular mass and the identity of the gas.

$$C_V = \tfrac{3}{2}R = Mc_V \quad \rightarrow \quad M = \frac{3R}{2c_V} = \frac{3\left(8.314\,\text{J/mol}\bullet\text{K}\right)}{2\left(0.0356\,\text{kcal/kg}\bullet\text{C}^\circ\right)}\frac{1\,\text{kcal}}{4186\,\text{J}}\frac{1000\,\text{g}}{1\,\text{kg}} = \boxed{83.7\,\text{g/mol}}$$

From the periodic table, we see that the gas is $\boxed{\text{krypton}}$, which has a molecular mass of 83.6 g/mol.

46. The process is adiabatic, and so the heat transfer is 0. Apply the first law of thermodynamics.

$$\Delta E_{\text{int}} = Q - W = 0 - W \quad \rightarrow \quad W = -\Delta E_{\text{int}} = -nC_V\Delta T = nC_V\left(T_1 - T_2\right)$$

47. If there are no heat losses, and no work being done, then the heat due to the people will increase the internal energy of the air, as given in Eq. 19-12. Note that air is basically diatomic. Use the ideal gas equation to estimate the number of moles of air, assuming the room is initially at 293 K.

$$Q = \Delta E_{\text{int}} = nC_V\Delta T \;\; ; \;\; n = \frac{PV}{RT} \quad \rightarrow$$

$$\Delta T = \frac{Q}{nC_V} = \frac{Q}{\dfrac{P_0V_0}{RT_0}C_V} = \frac{RT_0Q}{P_0V_0C_V} = \frac{RT_0Q}{P_0V_0\tfrac{5}{2}R} = \frac{2T_0Q}{5P_0V_0}$$

$$= \frac{2\left(293\,\text{K}\right)\left[\left(1800\,\text{people}\right)\left(\dfrac{70\,\text{W}}{\text{person}}\right)\left(7200\,\text{s}\right)\right]}{5\left(1.013\times10^5\,\text{Pa}\right)\left(2.2\times10^4\,\text{m}^3\right)} = 47.7\,\text{K} \approx \boxed{48\,\text{C}^\circ}$$

48. (*a*) First find the molar specific heat at constant volume, then the molar specific heat at constant pressure, and then finally the specific heat at constant pressure.

$$C_V = Mc_V = \left(0.034\,\text{kg/mol}\right)\left(0.182\,\text{kcal/kg}\bullet\text{K}\right)\left(\frac{10^3\,\text{cal}}{1\,\text{kcal}}\right) = 6.188\,\text{cal/mol}\bullet\text{K}$$

$$c_P = \frac{C_P}{M} = \frac{C_V + R}{M} = \frac{\left(6.188\,\text{cal/mol}\bullet\text{K}\right) + \left(1.99\,\text{cal/mol}\bullet\text{K}\right)}{\left(0.034\,\text{kg/mol}\right)}\left(\frac{1\,\text{kcal}}{10^3\,\text{cal}}\right) = \boxed{0.241\,\text{kcal/kg}\bullet\text{K}}$$

(*b*) From the value for $C_V = 6.188\,\text{cal/mol}\bullet\text{K}$, we see from Table 19-4 that this gas is probably $\boxed{\text{triatomic}}$.

$\boxed{49.}$ (*a*) The change in internal energy is given by Eq. 19-12. The nitrogen is diatomic.

$$\Delta E_{\text{int}} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = \tfrac{5}{2}\left(2.00\,\text{mol}\right)\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)\left(150\,\text{K}\right) = 6236\,\text{J} \approx \boxed{6240\,\text{J}} \text{ or}$$

$$\Delta E_{\text{int}} = nC_V\Delta T = \left(2.00\,\text{mol}\right)\left(4.96\frac{\text{cal}}{\text{mol}\bullet\text{K}}\right)\left(4.186\frac{\text{J}}{\text{cal}}\right)\left(150\,\text{K}\right) = 6229\,\text{J} \approx \boxed{6230\,\text{J}}$$

(*b*) The work is done at constant pressure.

$$W = P\Delta V = nR\Delta T = \left(2.00\,\text{mol}\right)\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)\left(150\,\text{K}\right) = 2494\,\text{J} \approx \boxed{2490\,\text{J}}$$

(*c*) The heat is added at constant pressure, and so Eq. 19-10b applies.

$$Q = nC_\text{P}\Delta T = n\left(\tfrac{7}{2}R\right)\Delta T = \tfrac{7}{2}\left(2.00\,\text{mol}\right)\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)\left(150\,\text{K}\right) = \boxed{8730\,\text{J}} \text{ or}$$

$$\Delta E_{\text{int}} = Q - W \quad \rightarrow \quad Q = \Delta E_{\text{int}} + W = 6229\,\text{J} + 2494\,\text{J} = \boxed{8720\,\text{J}}$$

50. (*a*) The change in internal energy is given by Eq. 19-12.

$$\Delta E_{\text{int}} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = \tfrac{5}{2}\left(1.00\,\text{mol}\right)\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)\left(300\,\text{K}\right) = \boxed{6240\,\text{J}}$$

(*b*) The work is given by $W = \int PdV$. The pressure is a linear function of temperature, so $P = P_0 + aT$. Use the given data to find the constants P_0 and a. Use the ideal gas equation to express the integral in terms of pressure.

$$1.60\,\text{atm} = P_0 + a\left(720\,\text{K}\right)\ ;\ 1.00\,\text{atm} = P_0 + a\left(420\,\text{K}\right) \quad \rightarrow$$

$$\left(1.60\,\text{atm} - 1.00\,\text{atm}\right) = \left[P_0 + a\left(720\,\text{K}\right)\right] - P_0 + a\left(420\,\text{K}\right) = a\left(300\,\text{K}\right) \quad \rightarrow$$

$$a = \frac{0.60\,\text{atm}}{300\,\text{K}} = 2.0\times10^{-3}\,\text{atm/K}\ ;\ P_0 = 1.60\,\text{atm} - \left(2.0\times10^{-3}\,\text{atm/K}\right)\left(720\text{K}\right) = -0.16\,\text{atm}$$

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{nR}{P}\frac{\left(P - P_0\right)}{a} = \frac{nR}{a}\left(1 - \frac{P_0}{P}\right) \quad \rightarrow \quad dV = \frac{nR}{a}\frac{P_0}{P^2}dP$$

$$W = \int_{V_1}^{V_2} PdV = \int_{P_1}^{P_2} P\frac{nR}{a}\frac{P_0}{P^2}dP = \frac{nRP_0}{a}\int_{P_1}^{P_2}\frac{1}{P}dP = \frac{nRP_0}{a}\ln\left(\frac{P_2}{P_1}\right)$$

$$= \frac{\left(1.00\,\text{mol}\right)\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)\left(-0.16\,\text{atm}\right)}{2.0\times10^{-3}\,\text{atm/K}}\ln\left(\frac{1.60\,\text{atm}}{1.00\,\text{atm}}\right) = \boxed{-310\text{J}}$$

(*c*) Use the first law of thermodynamics.

$$\Delta E_{\text{int}} = Q - W \quad \rightarrow \quad Q = \Delta E_{\text{int}} + W = 6240\,\text{J} - 310\,\text{J} = \boxed{5930\,\text{J}}$$

51. For a diatomic gas with no vibrational modes excited, we find the γ parameter.

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{5}{2}R + R}{\frac{5}{2}R} = \frac{7}{5}$$

For an adiabatic process, we have $PV^\gamma = \text{constant}$. Use this to find the final pressure.

$$P_1V_1^\gamma = P_2V_2^\gamma \quad \rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = \left(1.00\,\text{atm}\right)\left(\frac{1}{1.75}\right)^{1.4} = 0.4568\,\text{atm} \approx \boxed{0.457\,\text{atm}}$$

Use the ideal gas law to find the final temperature.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad T_2 = T_1\left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right) = \left(293\,\text{K}\right)\left(0.4568\right)\left(1.75\right) = 234\,\text{K} = \boxed{-39^\circ\text{C}}$$

52. The work is given by $W = \int_{V_1}^{V_2} PdV$. The pressure is given by the adiabatic condition, $PV^\gamma = c$, where c is a constant. Note that for the two states given, $c = P_1V_1^\gamma = P_2V_2^\gamma$

$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2}\frac{c}{V^\gamma}dV = \left.\frac{cV^{1-\gamma}}{1-\gamma}\right|_{V_1}^{V_2} = \frac{\left(cV_2^{1-\gamma} - cV_1^{1-\gamma}\right)}{1-\gamma} = \frac{\left(P_2V_2^\gamma V_2^{1-\gamma} - P_1V_1^\gamma V_1^{1-\gamma}\right)}{1-\gamma} = \boxed{\frac{\left(P_1V_1 - P_2V_2\right)}{\gamma - 1}}$$

53. (*a*) We first find the final pressure from the adiabatic relationship, and then use the ideal gas law to find the temperatures. For a diatomic gas, $\gamma = 1.4$.

$$P_1V_1^\gamma = P_2V_2^\gamma \quad \rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = (1.00\,\text{atm})\left(\frac{0.1210\,\text{m}^3}{0.750\,\text{m}^3}\right)^{1.4} = 7.772\times10^{-2}\,\text{atm}$$

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR} \qquad T_1 = \frac{P_1V_1}{nR} = \frac{(1.013\times10^5\,\text{Pa})(0.1210\,\text{m}^3)}{(3.65\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)} = 403.9\,\text{K} \approx \boxed{404\,\text{K}}$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{(7.772\times10^{-2})(1.013\times10^5\,\text{Pa})(0.750\,\text{m}^3)}{(3.65\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)} = 194.6\,\text{K} \approx \boxed{195\,\text{K}}$$

(*b*) $\Delta E_{\text{int}} = nC_C\Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = (3.65\,\text{mol})\tfrac{5}{2}\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(194.6\,\text{K} - 403.9\,\text{K}) = \boxed{-1.59\times10^4\,\text{J}}$

(*c*) Since the process is adiabatic, no heat is transferred. $\boxed{Q = 0}$

(*d*) Use the first law of thermodynamics to find the work done by the gas. The work done ON the gas is the opposite of the work done BY the gas.

$$\Delta E_{\text{int}} = Q - W \quad \rightarrow \quad W = Q - \Delta E_{\text{int}} = 0 - \left(-1.59\times10^4\,\text{J}\right) = 1.59\times10^4\,\text{J}$$

$$W_{\substack{\text{on}\\\text{gas}}} = \boxed{-1.59\times10^4\,\text{J}}$$

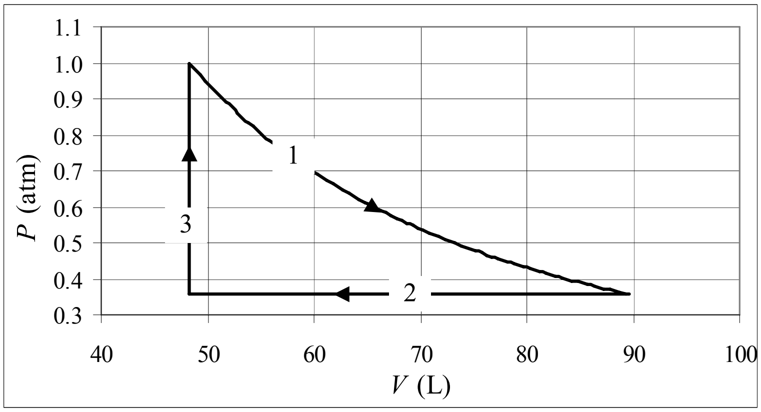
54. Combine the ideal gas equation with Eq. 19-15 for adiabatic processes.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad \frac{P_1}{P_2} = \frac{V_2}{V_1}\frac{T_1}{T_2} \quad ; \quad P_1V_1^\gamma = P_2V_2^\gamma \quad \rightarrow \quad \frac{P_1}{P_2} = \frac{V_2^\gamma}{V_1^\gamma}$$

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{\tfrac{3}{2}R + R}{\tfrac{3}{2}R} = \tfrac{5}{3}$$

$$\frac{V_2}{V_1}\frac{T_1}{T_2} = \frac{V_2^\gamma}{V_1^\gamma} \quad \rightarrow \quad V_2^{\gamma-1} = V_1^{\gamma-1}\left(\frac{T_1}{T_2}\right) \quad \rightarrow \quad V_2 = V_1\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = (0.086\,\text{m}^3)\left(\frac{273\,\text{K} + 25\,\text{K}}{273\,\text{K} - 68\,\text{K}}\right)^{3/2} = \boxed{0.15\,\text{m}^3}$$

$\boxed{55.}$ (*a*) To plot the graph accurately, data points must be calculated. V_1 is found from the ideal gas equation, and P_2 and V_2 are found from the fact that the first expansion is adiabatic. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH19.XLS," on tab "Problem 19.55a."



$$P_1V_1 = nRT_1 \quad \rightarrow \quad V_1 = \frac{nRT_1}{P_1} = \frac{(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(588\,\text{K})}{1.013\times10^5\,\text{Pa}} = 48.26\times10^{-3}\,\text{m}^3 = 48.26\,\text{L}$$

$$P_2V_2 = nRT_2 \;,\; P_2V_2^{\gamma} = P_1V_1^{\gamma} \quad \rightarrow \quad V_2^{\gamma-1} = \frac{P_1V_1^{\gamma}}{nRT_2} \quad \rightarrow$$

$$V_2 = \left(\frac{P_1V_1^{5/3}}{nRT_2}\right)^{3/2} = \left(\frac{(1.013\times10^5\,\text{Pa})(48.26\times10^{-3}\,\text{m}^3)^{5/3}}{(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(389\,\text{K})}\right)^{3/2} = 89.68\times10^{-3}\,\text{m}^3 = 89.68\,\text{L}$$

$$P_2 = \frac{nRT_2}{V_2} = \frac{(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(389\,\text{K})}{89.68\times10^{-3}\,\text{m}^3} = 3.606\times10^4\,\text{Pa}\left(\frac{1\,\text{atm}}{1.013\times10^5\,\text{Pa}}\right) = 0.356\,\text{atm}$$

(*b*) Both the pressure and the volume are known at the lower left corner of the graph.

$$P_3 = P_2 \;,\; V_3 = V_1 \quad \rightarrow \quad P_3V_3 = nRT_3 \quad \rightarrow$$

$$T_3 = \frac{P_3V_3}{nR} = \frac{P_2V_1}{nR} = \frac{(3.606\times10^4\,\text{Pa})(48.26\times10^{-3}\,\text{m}^3)}{(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)} = \boxed{209\,\text{K}}$$

(*c*) For the adiabatic process, state 1 to state 2:

$$\Delta E_{\text{int}} = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(389\,\text{K} - 588\,\text{K}) = -2482\,\text{J} \approx \boxed{-2480\,\text{J}}$$

$$Q = \boxed{0} \;\; (\text{adiabatic}) \;\; ; \;\; W = Q - \Delta E_{\text{int}} = \boxed{2480\,\text{J}}$$

For the constant pressure process, state 2 to state 3:

$$\Delta E_{\text{int}} = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(209\,\text{K} - 389\,\text{K}) = -2244.7\,\text{J} \approx \boxed{-2240\,\text{J}}$$

$$W = P\Delta V = (3.606\times10^4\,\text{Pa})(48.26\times10^{-3}\,\text{m}^3 - 89.68\times10^{-3}\,\text{m}^3) = -1494\,\text{J} \approx \boxed{-1490\,\text{J}}$$

$$Q = W + \Delta E_{\text{int}} = -1494\,\text{J} - 2244.7\,\text{J} = -3739\,\text{J} \approx \boxed{-3740\,\text{J}}$$

For the constant volume process, state 3 to state 1:

$$\Delta E_{\text{int}} = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(1.00\,\text{mol})\left(8.314\frac{\text{J}}{\text{mol}\bullet\text{K}}\right)(588\,\text{K} - 209\,\text{K}) = 4727\,\text{J} \approx \boxed{4730\,\text{J}}$$

$$W = P\Delta V = 0 \;\; ; \;\; Q = W + \Delta E_{\text{int}} = \boxed{4730\,\text{J}}$$

(*d*) For the complete cycle, by definition $\Delta E_{\text{int}} = \boxed{0}$. If the values from above are added, we get $\Delta E_{\text{int}} = 4727\,\text{J} - 2482\,\text{J} - 2245\,\text{J} = 0$. Add the separate values for the work done and the heat added.

$$W = 2482\,\text{J} - 1494\,\text{J} = 998\,\text{J} \approx \boxed{990\,\text{J}} \;\; ; \;\; Q = -3739\,\text{J} + 4727\,\text{J} = 998\,\text{J} \approx \boxed{990\,\text{J}}$$

Notice that the first law of thermodynamics is satisfied.

56. (*a*) Combine the ideal gas law with Eq. 19-15 for adiabatic processes.

$$PV = nRT \;\rightarrow\; V = \frac{nRT}{P} \;;\; PV^{\gamma} = \text{constant} = P\left(\frac{nRT}{P}\right)^{\gamma} = P^{1-\gamma}T^{\gamma}\left(nR\right)^{\gamma} \;\rightarrow$$

$$P^{1-\gamma}T^{\gamma} = \frac{\text{constant}}{\left(nR\right)^{\gamma}}$$

But since *n* and *R* are constant for a fixed amount of gas, we have $\boxed{P^{1-\gamma}T^{\gamma} = \text{constant}}$.

Take the derivative of the above expression with respect to *y*, using the product rule and chain rule.

$$P^{1-\gamma}T^{\gamma} = \text{constant} \;\rightarrow\; \frac{d}{dy}\left(P^{1-\gamma}T^{\gamma}\right) = \frac{d}{dy}\left(\text{constant}\right) = 0 \;\rightarrow$$

$$T^{\gamma}\left(1-\gamma\right)P^{-\gamma}\frac{dP}{dy} + P^{1-\gamma}\gamma T^{\gamma-1}\frac{dT}{dy} = 0$$

Multiply this equation by P^{γ}/T^{γ}.

$$\frac{P^{\gamma}}{T^{\gamma}}\left[T^{\gamma}\left(1-\gamma\right)P^{-\gamma}\frac{dP}{dy} + P^{1-\gamma}\gamma T^{\gamma-1}\frac{dT}{dy}\right] = \boxed{\left(1-\gamma\right)\frac{dP}{dy} + \gamma\frac{P}{T}\frac{dT}{dy} = 0}$$

We also assume that $\frac{dP}{dy} = -\rho g$, and so get the following.

$$\boxed{\left(1-\gamma\right)\left(-\rho g\right) + \gamma\frac{P}{T}\frac{dT}{dy} = 0}, \text{ or } \frac{dT}{dy} = \frac{\left(1-\gamma\right)}{\gamma}\frac{T}{P}\rho g$$

(*b*) Use the ideal gas law. We let *M* represent the total mass of the gas and *m* represent the mass of one molecule, so $N = \frac{M}{m}$. The density of the gas is $\rho = \frac{M}{V}$.

$$PV = NkT \;\rightarrow\; \frac{T}{P} = \frac{V}{Nk} = \frac{Vm}{Mk} = \frac{m}{\rho k} \;\rightarrow$$

$$\frac{dT}{dy} = \frac{\left(1-\gamma\right)}{\gamma}\frac{T}{P}\rho g = \frac{\left(1-\gamma\right)}{\gamma}\left(\frac{m}{\rho k}\right)\rho g = \boxed{\frac{\left(1-\gamma\right)}{\gamma}\frac{mg}{k}}$$

(*c*) For a diatomic ideal gas, $\gamma = \frac{7}{5}$.

$$\frac{dT}{dy} = \frac{\left(1-\gamma\right)}{\gamma}\frac{mg}{k} = \frac{\left(1-\frac{7}{5}\right)}{\frac{7}{5}}\frac{29\left(1.66\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{1.38\times10^{-23}\,\text{J/K}} = -9.77\times10^{-3}\,\text{K/m}$$

$$= \left(-9.77\times10^{-3}\,\text{K/m}\right)\left(\frac{10^3\,\text{m}}{1\,\text{km}}\right)\left(\frac{1\,\text{C}^\circ}{1\,\text{K}}\right) = -9.77\,\text{C}^\circ/\text{km} \approx \boxed{-9.8\,\text{C}^\circ/\text{km}}$$

(*d*) $\Delta T = \frac{dT}{dy}\Delta y = \left(-9.77\,\text{C}^\circ/\text{km}\right)\left(-0.1\,\text{km} - 4.0\,\text{km}\right) = 40\text{C}^\circ = T_f - \left(-5^\circ\text{C}\right) \;\rightarrow\; T_f = \boxed{35^\circ\text{C}} \approx 95^\circ\text{F}$

57. (*a*) The power radiated is given by Eq. 19-17. The temperature of the tungsten is $273\,\text{K} + 25\,\text{K} = 298\,\text{K}$.

$$\frac{\Delta Q}{\Delta t} = e\sigma AT^4 = \left(0.35\right)\left(5.67\times10^{-8}\,\text{W/m}^2\bullet\text{K}^4\right)4\pi\left(0.16\,\text{m}\right)^2\left(298\,\text{K}\right)^4 = \boxed{50\,\text{W}} \;\left(2\text{ sig. fig.}\right)$$

(*b*) The net power is given by Eq. 19-18. The temperature of the surroundings is 268 K.

$$\frac{\Delta Q}{\Delta t} = e\sigma A\left(T_1^4 - T_1^4\right) = (0.35)\left(5.67\times10^{-8}\ \mathrm{W/m^2{\bullet}K^4}\right)4\pi(0.16\,\mathrm{m})^2\left[(298\,\mathrm{K})^4 - (268\,\mathrm{K})^4\right]$$

$$= \boxed{17\,\mathrm{W}}$$

58. The heat conduction rate is given by Eq. 19-16a.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (380\,\mathrm{J/s{\bullet}m{\bullet}C^\circ})\pi(0.010\,\mathrm{m})^2\frac{(460^\circ\mathrm{C} - 22^\circ\mathrm{C})}{0.45\,\mathrm{m}} = 116\,\mathrm{W} \approx \boxed{120\,\mathrm{W}}$$

59. Eq. 19-19 gives the heat absorption rate for an object facing the Sun. The heat required to melt the ice is the mass of the ice times the latent heat of fusion for the ice. The mass is found by multiplying the volume of ice times its density.

$$\Delta Q = mL_f = \rho VL_f = \rho A(\Delta x)L_f \qquad \frac{\Delta Q}{\Delta t} = \left(1000\,\mathrm{W/m^2}\right)eA\cos\theta \ \rightarrow$$

$$\Delta t = \frac{\rho A(\Delta x)L_f}{\left(1000\,\mathrm{W/m^2}\right)eA\cos\theta} = \frac{\rho(\Delta x)L_f}{\left(1000\,\mathrm{W/m^2}\right)e\cos\theta}$$

$$= \frac{\left(9.17\times10^2\,\mathrm{kg/m^3}\right)\left(1.0\times10^{-2}\,\mathrm{m}\right)\left(3.33\times10^5\,\mathrm{J/kg}\right)}{\left(1000\,\mathrm{W/m^2}\right)(0.050)\cos35^\circ} = \boxed{7.5\times10^4\,\mathrm{s}} \approx 21\,\mathrm{h}$$

60. The distance can be calculated from the heat conduction rate, given by Eq. 19-16a. The rate is given as a power (150 W = 150 J/s).

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} \ \rightarrow\ \ell = kA\frac{T_1 - T_2}{P} = (0.2\,\mathrm{J/s{\bullet}m{\bullet}C^\circ})\left(1.5\,\mathrm{m^2}\right)\frac{0.50\,\mathrm{C^\circ}}{150\,\mathrm{W}} = \boxed{1.0\times10^{-3}\,\mathrm{m}}$$

$\boxed{61.}$ (*a*) The rate of heat transfer due to radiation is given by Eq. 19-17. We assume that each teapot is a sphere that holds 0.55 L. The radius and then the surface area can be found from that.

$$V = \tfrac{4}{3}\pi r^3 \ \rightarrow\ r = \left(\frac{3V}{4\pi}\right)^{1/3} \ \rightarrow\ S.A. = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$$

$$\frac{\Delta Q}{\Delta t} = \varepsilon\sigma A\left(T_1^4 - T_2^4\right) = 4\pi\varepsilon\sigma\left(\frac{3V}{4\pi}\right)^{2/3}\left(T_1^4 - T_2^4\right)$$

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\text{ceramic}} = 4\pi(0.70)\left(5.67\times10^{-8}\frac{\mathrm{W}}{\mathrm{m^2{\bullet}K^4}}\right)\left(\frac{3\left(0.55\times10^{-3}\,\mathrm{m^2}\right)}{4\pi}\right)^{2/3}\left[(368\,\mathrm{K})^4 - (293\,\mathrm{K})^4\right]$$

$$= 14.13\mathrm{W} \approx \boxed{14\,\mathrm{W}}$$

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\text{shiny}} = \left(\frac{\Delta Q}{\Delta t}\right)_{\text{ceramic}}\left(\frac{0.10}{0.70}\right) = 2.019\,\mathrm{W} \approx \boxed{2.0\,\mathrm{W}}$$

(*b*) We assume that the heat capacity comes primarily from the water in the teapots, and ignore the heat capacity of the teapots themselves. We apply Eq. 19-2, along with the results from part (*a*). The mass is that of 0.55 L of water, which would be 0.55 kg.

$$\Delta Q = mc\Delta T \ \rightarrow\ \Delta T = \frac{1}{mc}\left(\frac{\Delta Q}{\Delta t}\right)_{\text{radiation}}\Delta t_{\text{elapsed}}$$

$$(\Delta T)_{\text{ceramic}} = \frac{14.13\,\text{W}}{(0.55\,\text{kg})\left(4186\dfrac{\text{J}}{\text{kg}\bullet\text{C}°}\right)}(1800\,\text{s}) = \boxed{11\,\text{C}°}$$

$$(\Delta T)_{\text{shiny}} = \frac{1}{7}(\Delta T)_{\text{ceramic}} = \boxed{1.6\,\text{C}°}$$

62. For the temperature at the joint to remain constant, the heat flow in both rods must be the same. Note that the cross-sectional areas and lengths are the same. Use Eq. 19-16a for heat conduction.

$$\left(\frac{Q}{t}\right)_{\text{Cu}} = \left(\frac{Q}{t}\right)_{\text{Al}} \rightarrow k_{\text{Cu}}A\frac{T_{\text{hot}} - T_{\text{middle}}}{\ell} = k_{\text{Al}}A\frac{T_{\text{middle}} - T_{\text{cool}}}{\ell} \rightarrow$$

$$T_{\text{middle}} = \frac{k_{\text{Cu}}T_{\text{hot}} + k_{\text{Al}}T_{\text{cool}}}{k_{\text{Cu}} + k_{\text{Al}}} = \frac{(380\,\text{J/s}\bullet\text{m}\bullet\text{C}°)(225°\text{C}) + (200\,\text{J/s}\bullet\text{m}\bullet\text{C}°)(0.0°\text{C})}{380\,\text{J/s}\bullet\text{m}\bullet\text{C}° + 200\,\text{J/s}\bullet\text{m}\bullet\text{C}°} = \boxed{147°\text{C}}$$

63. (*a*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, and so has an area of πR^2_{Earth}. Multiply this area times the solar constant to get the rate at which the Earth is receiving solar energy.

$$\frac{Q}{t} = \pi R^2_{\text{Earth}}(\text{solar constant}) = \pi\left(6.38\times10^6\,\text{m}\right)^2\left(1350\,\text{W/m}^2\right) = \boxed{1.73\times10^{17}\,\text{W}}$$

(*b*) Use Eq. 19-18 to calculate the rate of heat output by radiation, and assume that the temperature of space is 0 K. The whole sphere is radiating heat back into space, and so we use the full surface area of the Earth, $4\pi R^2_{\text{Earth}}$.

$$\frac{Q}{t} = e\sigma AT^4 \rightarrow T = \left(\frac{Q}{t}\frac{1}{\varepsilon\sigma A}\right)^{1/4}$$

$$= \left[\left(1.73\times10^{17}\,\text{J/s}\right)\frac{1}{(1.0)\left(5.67\times10^{-8}\,\text{W/m}^2\bullet\text{K}^4\right)4\pi\left(6.38\times10^6\,\text{m}\right)^2}\right]^{1/4} = \boxed{278\text{ K} = 5°\text{C}}$$

64. This is an example of heat conduction. The temperature difference can be calculated by Eq. 19-16a.

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} \rightarrow \Delta T = \frac{P\ell}{kA} = \frac{(95\text{ W})\left(5.0\times10^{-4}\text{m}\right)}{\left(0.84\,\text{J/s}\bullet\text{m}\bullet\text{C}°\right)4\pi\left(3.0\times10^{-2}\text{m}\right)^2} = \boxed{5.0\,\text{C}°}$$

65. We model the heat loss as conductive, so that, using Eq. 19-16a, $\frac{Q}{t} = \frac{kA}{\ell}\Delta T \rightarrow Q = \alpha t\Delta T$, where α describes the average heat conductivity properties of the house, such as insulation materials and surface area of the conducting surfaces. It could have units of J/h•°C. We see that the heat loss is proportional to the product of elapsed time and the temperature difference. We assume that the proportionality constant α does not vary during the day, so that, for example, heating by direct sunlight through windows is not considered. We also assume that α is independent of temperature, and so is the same during both the day and the night.

$$Q_{\substack{\text{turning}\\\text{down}}} = (\alpha\,\text{J/h}\bullet\text{C}°)(15\,\text{h})(22°\text{C} - 8°\text{C}) + (\alpha\,\text{J/h}\bullet\text{C}°)(9\,\text{h})(12°\text{C} - 0°\text{C}) = 318\alpha\,\text{J}$$

$$Q_{\substack{\text{not turning}\\\text{down}}} = (\alpha\,\text{J/h}\bullet\text{C}°)(15\text{ h})(22°\text{C} - 8°\text{C}) + (\alpha\,\text{J/h}\bullet\text{C}°)(9\,\text{h})(22°\text{C} - 0°\text{C}) = 408\alpha\,\text{J}$$

$$\frac{\Delta Q}{Q_{\substack{\text{turning}\\\text{down}}}}=\frac{408\alpha\,\text{J}-318\alpha\,\text{J}}{318\alpha\,\text{J}}=0.28=\boxed{28\%}$$

To keep the thermostat "up" requires about 28% more heat in this model than turning it down.

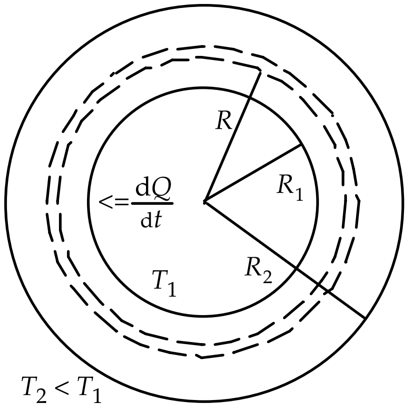
66. This is an example of heat conduction. The heat conducted is the heat released by the melting ice, $Q=m_{\text{ice}}L_{\text{fusion}}$. The area through which the heat is conducted is the total area of the six surfaces of the box, and the length of the conducting material is the thickness of the Styrofoam. We assume that all of the heat conducted into the box goes into melting the ice, and none into raising the temperature inside the box. The time can then be calculated by Eq. 19-16a.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{\ell}\quad\rightarrow\quad t=\frac{Q\ell}{kA\Delta T}=\frac{m_{\text{ice}}L_{\text{fusion}}\ell}{kA\Delta T}$$

$$=\frac{(9.5\text{ kg})(3.33\times10^5\text{ J/kg})(0.015\text{ m})}{2(0.023\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)[2(0.25\text{ m})(0.35\text{ m})+2(0.25\text{ m})(0.55\text{ m})+2(0.35\text{ m})(0.55\text{ m})](34\text{ C}^\circ)}$$

$$=\boxed{3.6\times10^4\text{ s}}\approx10\text{ h}$$

67. (*a*) Choose a cylindrical shell of length ℓ, radius R, and thickness dR. Apply Eq. 19-16b, modified for the radial geometry. See the figure. Note that $\frac{dQ}{dt}$ must be a constant, so that all of the heat energy that enters a shell also exits that shell.



$$\frac{dQ}{dt}=-kA\frac{dT}{dR}=-k2\pi R\ell\frac{dT}{dR}\quad\rightarrow$$

$$\frac{dR}{R}=-\frac{2\pi k\ell}{dQ/dt}dT\quad\rightarrow\quad\int_{R_1}^{R_2}\frac{dR}{R}=-\frac{2\pi k\ell}{dQ/dt}\int_{T_1}^{T_2}dT\quad\rightarrow$$

$$\ln\frac{R_2}{R_1}=\frac{2\pi k\ell}{dQ/dt}(T_1-T_2)\quad\rightarrow\quad\boxed{\frac{dQ}{dt}=\frac{2\pi k(T_1-T_2)\ell}{\ln(R_2/R_1)}}$$

(*b*) For still water, the initial heat flow outward from the water is described by Eq. 19-2.

$$\Delta Q=mc\Delta T\quad\rightarrow\quad\frac{\Delta Q}{\Delta t}=mc\frac{\Delta T}{\Delta t}=\frac{2\pi k(T_1-T_2)\ell}{\ln(R_2/R_1)}\quad\rightarrow$$

$$\frac{\Delta T}{\Delta t}=\frac{2\pi k(T_1-T_2)\ell}{mc\ln(R_2/R_1)}=\frac{2\pi k(T_1-T_2)\ell}{\rho_{\text{H}_2\text{O}}\pi R_1^2\ell c\ln(R_2/R_1)}=\frac{2k(T_1-T_2)}{\rho_{\text{H}_2\text{O}}R_1^2c\ln(R_2/R_1)}$$

$$=\frac{2\left(40\frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)(71^\circ\text{C}-18^\circ\text{C})}{(1.0\times10^3\text{ kg/m}^3)(0.033\text{m})^2\left(4186\frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)\ln\left(\frac{4.0}{3.3}\right)}=4.835\text{C}^\circ/\text{s}\approx\boxed{4.8\text{C}^\circ/\text{s}}$$

Note that this will lower the value of T_1, which would lower the rate of temperature change as time elapses.

(*c*) The water at the entrance is losing temperature at a rate of $4.835\,\text{C}^\circ/\text{s}$. In one second, it will have traveled 8 cm, and so the temperature drop per cm is $\left(4.835\dfrac{\text{C}^\circ}{\text{s}}\right)\left(\dfrac{1\,\text{s}}{8\,\text{cm}}\right)=\boxed{0.60\,\text{C}^\circ/\text{cm}}$.

68. The conduction rates through the two materials must be equal. If they were not, the temperatures in the materials would be changing. Call the temperature at the boundary between the materials T_x.

$$\frac{Q}{t}=k_1A\frac{T_1-T_x}{\ell_1}=k_2A\frac{T_x-T_2}{\ell_2}\quad\rightarrow\quad\frac{Q}{t}\frac{\ell_1}{k_1A}=T_1-T_x\ ;\ \frac{Q}{t}\frac{\ell_2}{k_2A}=T_x-T_2$$

Add these two equations together, and solve for the heat conduction rate.

$$\frac{Q}{t}\frac{\ell_1}{k_1A}+\frac{Q}{t}\frac{\ell_2}{k_2A}=T_1-T_x+T_x-T_2\quad\rightarrow\quad\frac{Q}{t}\left(\frac{\ell_1}{k_1}+\frac{\ell_2}{k_2}\right)\frac{1}{A}=T_1-T_2\quad\rightarrow$$

$$\frac{Q}{t}=A\frac{(T_1-T_2)}{\left(\dfrac{\ell_1}{k_1}+\dfrac{\ell_2}{k_2}\right)}=A\frac{(T_1-T_2)}{(R_1+R_2)}$$

The *R*-value for the brick needs to be calculated, using the definition of *R* given on page 517.

$$R=\frac{\ell}{k}=\frac{4\,\text{in}\left(\dfrac{1\,\text{ft}}{12\,\text{in}}\right)}{0.84\dfrac{\text{J}}{\text{s}\bullet\text{m}\bullet\text{C}^\circ}\left(\dfrac{1\,\text{Btu}}{1055\,\text{J}}\right)\left(\dfrac{1\,\text{m}}{3.281\,\text{ft}}\right)\left(\dfrac{5\,\text{C}^\circ}{9\,\text{F}^\circ}\right)\left(\dfrac{3600\,\text{s}}{1\,\text{h}}\right)}=0.69\,\text{ft}^2\bullet\text{h}\bullet\text{F}^\circ/\text{Btu}$$

$$\frac{Q}{t}=A\frac{(T_1-T_2)}{(R_1+R_2)}=\left(195\,\text{ft}^2\right)\frac{(12\,\text{F}^\circ)}{(0.69+19)\,\text{ft}^2\bullet\text{h}\bullet\text{F}^\circ/\text{Btu}}=119\,\text{Btu/h}\approx\boxed{120\,\text{Btu/h}}$$

This is about 35 Watts.

69. The heat needed to warm the liquid can be calculated by Eq. 19-2.

$$Q=mc\Delta T=(0.20\text{ kg})(1.00\,\text{kcal/kg}\bullet\text{C}^\circ)(37^\circ\text{C}-5^\circ\text{C})=6.4\text{ kcal}=\boxed{6.4\text{ Cal}}$$

70. (*a*) Use Eq. 19-17 for total power radiated.

$$\frac{Q}{t}=e\sigma AT^4=e\sigma4\pi R_{\text{Sun}}^2T^4=(1.0)\left(5.67\times10^{-8}\text{ W/m}^2\bullet\text{K}^4\right)4\pi\left(7.0\times10^8\text{m}\right)^2(5500\text{ K})^4$$

$$=3.195\times10^{26}\,\text{W}\approx\boxed{3.2\times10^{26}\,\text{W}}$$

(*b*) Assume that the energy from the Sun is distributed symmetrically over a spherical surface with the Sun at the center.

$$\frac{P}{A}=\frac{Q/t}{4\pi R_{\text{Sun-Earth}}^2}=\frac{3.195\times10^{26}\,\text{W}}{4\pi\left(1.5\times10^{11}\text{m}\right)^2}=1.130\times10^3\text{ W/m}^2\approx\boxed{1.1\times10^3\text{ W/m}^2}$$

71. The heat released can be calculated by Eq. 19-2. To find the mass of the water, use the density (of pure water).

$$Q=mc\Delta T=\rho Vc\Delta T=\left(1.0\times10^3\text{ kg/m}^3\right)\left(1.0\times10^3\text{m}\right)^3(4186\,\text{J/kg}\bullet\text{C}^\circ)(1\text{C}^\circ)=\boxed{4\times10^{15}\text{J}}$$

72. Use the heat conduction rate equation, Eq. 19-16a.

(*a*) $$\frac{Q}{t}=kA\frac{T_1-T_2}{\ell}=\left(0.025\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(0.95\,\text{m}^2\right)\frac{\left[34^\circ\text{C}-\left(-18^\circ\text{C}\right)\right]}{3.5\times10^{-2}\,\text{m}}=35.3\,\text{W}\approx\boxed{35\,\text{W}}$$

(*b*) $$\frac{Q}{t}=kA\frac{T_1-T_2}{\ell}=\left(0.56\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(0.95\,\text{m}^2\right)\frac{\left[34^\circ\text{C}-\left(-18^\circ\text{C}\right)\right]}{5.0\times10^{-3}\,\text{m}}=5533\,\text{W}\approx\boxed{5500\,\text{W}}$$

73. The temperature rise can be calculated from Eq. 19-2.

$$Q=mc\Delta T\quad\rightarrow\quad\Delta T=\frac{Q}{mc}=\frac{\left(0.80\right)\left(200\,\text{kcal/h}\right)\left(0.5\,\text{h}\right)}{\left(70\text{ kg}\right)\left(0.83\,\text{kcal/kg}\bullet\text{C}^\circ\right)}=1.38\,\text{C}^\circ\approx\boxed{1\text{C}^\circ}$$

74. For an estimate of the heat conduction rate, use Eq. 19-16a.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{\ell}=\left(0.2\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(1.5\,\text{m}^2\right)\frac{\left(37^\circ\text{C}-34^\circ\text{C}\right)}{4.0\times10^{-2}\,\text{m}}=22.5\,\text{W}\approx\boxed{20\,\text{W}}$$

This is only about 10% of the cooling capacity that is needed for the body. Thus convection cooling is clearly necessary.

75. We assume that all of the heat provided by metabolism goes into evaporating the water. For the energy required for the evaporation of water, we use the heat of vaporization at room temperature (585 kcal/kg), since the runner's temperature is closer to room temperature than 100$^\text{o}$C.

$$2.2\,\text{h}\left(\frac{950\,\text{kcal}}{1\,\text{h}}\right)\left(\frac{\text{kg H}_2\text{O}}{585\,\text{kcal}}\right)=\boxed{3.6\,\text{kg}}$$

76. (*a*) To calculate heat transfer by conduction, use Eq. 19-16a for all three areas – walls, roof, and windows. Each area has the same temperature difference.

$$\frac{Q_{\text{conduction}}}{t}=\left[\left(\frac{kA}{\ell}\right)_{\text{walls}}+\left(\frac{kA}{\ell}\right)_{\text{roof}}+\left(\frac{kA}{\ell}\right)_{\text{windows}}\right]\left(T_1-T_2\right)$$

$$=\left[\begin{array}{l}\dfrac{\left(0.023\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(410\,\text{m}^2\right)}{0.195\,\text{m}}+\dfrac{\left(0.1\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(280\,\text{m}^2\right)}{0.055\,\text{m}}\\+\dfrac{\left(0.84\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)\left(33\,\text{m}^2\right)}{6.5\times10^{-3}\,\text{m}}\end{array}\right]\left(38\,\text{C}^\circ\right)$$

$$=1.832\times10^5\,\text{W}\approx\boxed{1.8\times10^5\,\text{W}}$$

(*b*) The energy being added must both heat the air and replace the energy lost by conduction, as considered above. The heat required to raise the temperature is given by Eq. 19-2, $Q_{\substack{\text{raise}\\\text{temp}}}=m_{\text{air}}c_{\text{air}}\left(\Delta T\right)_{\text{warming}}$. The mass of the air can be found from the density of the air times its volume. The conduction heat loss is proportional to the temperature difference between the inside and outside, which varies from 27C$^\text{o}$ to 38C$^\text{o}$. We will estimate the average temperature difference as 32.5$^\text{o}$C and scale the answer from part (a) accordingly.

$$Q_{\text{added}}=Q_{\substack{\text{raise}\\\text{temp}}}+Q_{\text{conduction}}=\rho_{\text{air}}Vc_{\text{air}}\left(\Delta T\right)_{\text{warming}}+\left(\frac{Q_{\text{conduction}}}{t}\right)\left(1800\text{ s}\right)$$

$$=\left(1.29\frac{\text{kg}}{\text{m}^3}\right)\left(750\text{m}^3\right)\left(0.24\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}\right)\left(\frac{4186\text{ J}}{\text{kcal}}\right)\left(11^\circ\text{C}\right)$$

$$+\left(1.832\times10^{5}\,\frac{\text{J}}{\text{s}}\right)\left(\frac{32.5^{\circ}\text{C}}{38^{\circ}\text{C}}\right)(1800\,\text{s}) = \boxed{2.5\times10^{8}\,\text{J}}$$

(*c*) We assume a month is 30 days.

$$0.9Q_{\text{gas}} = \left(\frac{Q}{t}\right)_{\text{conduction}} t_{\text{month}} \rightarrow$$

$$Q_{\text{gas}} = \frac{1}{0.9}\left(\frac{Q}{t}\right)_{\text{conduction}} t_{\text{month}} = \frac{1}{0.9}\left(1.832\times10^{5}\,\text{J/s}\right)(30\,\text{d})\left(\frac{24\,\text{h}}{1\,\text{d}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = 5.276\times10^{11}\,\text{J}$$

$$5.276\times10^{11}\,\text{J}\left(\frac{1\,\text{kg}}{5.4\times10^{7}\,\text{J}}\right)\left(\frac{\$0.080}{\text{kg}}\right) = \$781.65 \approx \boxed{\$780}$$

77. Assume that the loss of kinetic energy is all turned into heat which changes the temperature of the squash ball.

$$KE_{\text{lost}} = Q \;\rightarrow\; \tfrac{1}{2}m\left(v_i^2 - v_f^2\right) = mc\Delta T \;\rightarrow\; \Delta T = \frac{v_i^2 - v_f^2}{2c} = \frac{(22\,\text{m/s})^2 - (12\,\text{m/s})^2}{2\left(1200\,\text{J/kg}\bullet\text{C}^{\circ}\right)} = \boxed{0.14\,\text{C}^{\circ}}$$

78. Since the handle is pushed in very quickly, we approximate the process as adiabatic, since there will be little time for heat transfer to or from the air in the pump. We combine the ideal gas law and Eq. 19-15 for adiabatic processes.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \;\rightarrow\; \frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{V_2}{V_1} \;;\; P_1V_1^{\gamma} = P_2V_2^{\gamma} \;\rightarrow\; \frac{P_2}{P_1} = \frac{V_1^{\gamma}}{V_2^{\gamma}}$$

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{5}{2}R + R}{\frac{5}{2}R} = \tfrac{7}{5}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{V_2}{V_1} = \frac{V_1^{\gamma}}{V_2^{\gamma}}\frac{V_2}{V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \;\rightarrow\; T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (293\,\text{K})\left(2^{0.4}\right) = 386.6\,\text{K} = \boxed{114^{\circ}\text{C}}$$

$\boxed{79.}$ (*a*) The energy required to raise the temperature of the water is given by Eq. 19-2.

$$Q = mc\Delta T \;\rightarrow$$

$$\frac{Q}{\Delta t} = mc\frac{\Delta T}{\Delta t} = (0.250\,\text{kg})\left(4186\,\frac{\text{J}}{\text{kg}\bullet\text{C}^{\circ}}\right)\left(\frac{80\,\text{C}^{\circ}}{105\,\text{s}}\right) = 797\,\text{W} \approx \boxed{800\,\text{W}\,(2\text{ sig. fig.})}$$

(*b*) After 105 s, the water is at 100°C. So for the remaining 15 s the energy input will boil the water. Use the heat of vaporization.

$$Q = mL_{\text{V}} \;\rightarrow\; m = \frac{Q}{L_{\text{V}}} = \frac{(\text{Power})\Delta t}{L_{\text{V}}} = \frac{(797\,\text{W})(15\,\text{s})}{2260\,\text{J/g}} = \boxed{5.3\,\text{g}}$$

80. (*a*) We consider just the 30 m of crust immediately below the surface of the Earth, assuming that all the heat from the interior gets transferred to the surface, and so it all passes through this 30 m layer. This is a heat conduction problem, and so Eq. 19-16a is appropriate. The radius of the Earth is about $6.38\times10^{6}\,\text{m}$.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{l} \;\rightarrow\; Q_{\text{interior}} = kA\frac{T_1 - T_2}{l}t = \left(0.80\,\text{J/s}\bullet\text{m}\bullet\text{C}^{\circ}\right)4\pi R_{\text{Earth}}^2\,\frac{1.0\,\text{C}^{\circ}}{30\text{ m}}\left[1\,\text{hour}\left(\frac{3600\,\text{s}}{\text{hour}}\right)\right]$$

$$= 4.910\times10^{16}\,\text{J} \approx \boxed{4.9\times10^{16}\,\text{J}}$$

(*b*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, and so has an area of πR_{Earth}^2. Multiply this area times the solar constant of $1350\,\text{W/m}^2$ to get the amount of energy incident on the Earth from the Sun per second, and then convert to energy per day.

$$Q_{\text{Sun}} = \pi R_{\text{Earth}}^2 \left(1350\,\text{W/m}^2\right)\left[1\,\text{hour}\left(\frac{3600\,\text{s}}{\text{day}}\right)\right]$$

$$\frac{Q_{\text{Sun}}}{Q_{\text{interior}}} = \frac{\pi R_{\text{Earth}}^2 \left(1350\,\text{W/m}^2\right)\left[1\,\text{hour}\left(\frac{3600\,\text{s}}{\text{day}}\right)\right]}{\left(0.80\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)4\pi R_{\text{Earth}}^2\frac{1.0\,\text{C}^\circ}{30\text{ m}}\left[1\,\text{hour}\left(\frac{3600\,\text{s}}{\text{hour}}\right)\right]} = \frac{\left(1350\,\text{W/m}^2\right)}{\left(0.80\,\text{J/s}\bullet\text{m}\bullet\text{C}^\circ\right)4\frac{1.0\,\text{C}^\circ}{30\text{ m}}}$$

$$= 1.3\times10^4$$

And so $\boxed{Q_{\text{Sun}} = 1.3\times10^4 Q_{\text{interior}}}$.

81. Consider a rectangular block of ice. The surface area of the top of the block is A, and this surface is exposed to the air. The surface area of the bottom of the block is A, and this surface is exposed to the water. The thickness of this block is *x*. The heat of fusion from the ice forming at the bottom surface must be conducted through the ice to the air. For the conduction, from Eq. 19-16a, we have $\frac{dQ}{dt} = \frac{k_{\text{ice}}A\left(T_{\text{bottom}} - T_{\text{top}}\right)}{x}$. This is the rate of heat energy leaving the bottom surface and escaping to the air, through the ice block. For the freezing process, if a layer of ice of thickness *dx*, with mass *dm*, is formed at the bottom surface, the energy released is $dQ = L_{\text{f}}dm = L_{\text{f}}\rho_{\text{ice}}Adx$. The rate of release of this energy is $\frac{dQ}{dt} = L_{\text{f}}\rho_{\text{ice}}A\frac{dx}{dt}$. The rate of energy release must be equal to the rate of energy conduction, if the temperature at the bottom surface is to remain constant.

$$\frac{dQ}{dt} = \frac{k_{\text{ice}}A\left(T_{\text{bottom}} - T_{\text{top}}\right)}{x} = L_{\text{f}}\rho_{\text{ice}}A\frac{dx}{dt} \rightarrow \frac{k_{\text{ice}}\left(T_{\text{bottom}} - T_{\text{top}}\right)}{L_{\text{f}}\rho_{\text{ice}}}dt = x\,dx \rightarrow$$

$$\int_0^{t_{15}} \frac{k_{\text{ice}}\left(T_{\text{bottom}} - T_{\text{top}}\right)}{L_{\text{f}}\rho_{\text{ice}}}dt = \int_0^{0.15\text{m}} x\,dx \rightarrow \frac{k_{\text{ice}}\left(T_{\text{bottom}} - T_{\text{top}}\right)}{L_{\text{f}}\rho_{\text{ice}}}t_{15} = \tfrac{1}{2}\left(0.15\,\text{m}\right)^2 \rightarrow$$

$$t_{15} = \frac{\tfrac{1}{2}\left(0.15\,\text{m}\right)^2 L_{\text{f}}\rho_{\text{ice}}}{k_{\text{ice}}\left(T_{\text{bottom}} - T_{\text{top}}\right)} = \frac{\tfrac{1}{2}\left(0.15\,\text{m}\right)^2\left(3.33\times10^5\,\text{J/kg}\right)\left(917\,\text{kg/m}^3\right)}{\left(2\frac{\text{J}}{\text{s}\bullet\text{m}\bullet\text{C}^\circ}\right)\left(18\,\text{C}^\circ\right)} = \boxed{9.5\times10^4\,\text{s} \approx 1.1\,\text{d}}$$

82. Assume that the final speed of the meteorite, as it completely melts, is 0, and that all of its initial kinetic energy was used in heating the iron to the melting point and then melting the iron.

$$\tfrac{1}{2}mv_i^2 = mc_{\text{Fe}}\left(T_{\text{melt}} - T_i\right) + mL_{\text{fusion}} \rightarrow$$

$$v_i = \sqrt{2\left[c_{\text{Fe}}\left(T_{\text{melt}} - T_i\right) + L_{\text{fusion}}\right]} = \sqrt{2\left[\left(450\,\text{J/kg}\bullet\text{C}^\circ\right)\left(1808^\circ\text{C} - \left(-105^\circ\text{C}\right)\right) + 2.89\times10^5\,\text{J/kg}\right]}$$

$$= \boxed{1520\,\text{m/s}}$$

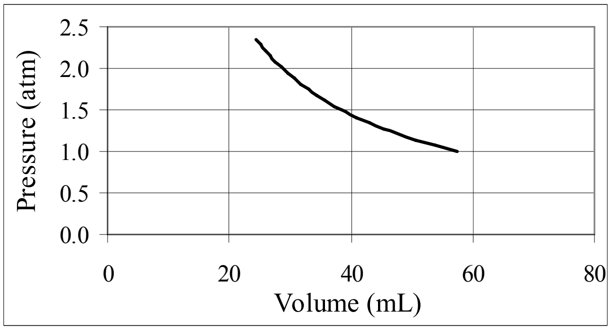
83. (*a*) The pressure varies with depth according to Eq. 13-6b. Use Eq. 13-6b to find the pressure at the original depth, and then use the ideal gas equation with a constant temperature to find the size of the bubble at the surface.

$$P_1 = P_{\text{atm}} + \rho g h \;;\; P_2 = P_{\text{atm}} \;;\; P_1V_1 = P_2V_2 \;\rightarrow\; \left(P_{\text{atm}} + \rho g h\right)\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_1\right)^3 = P_{\text{atm}}\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_2\right)^3 \;\rightarrow$$

$$d_2 = d_1\left(\frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}}\right)^{1/3} = d_1\left(1 + \frac{\rho g h}{P_{\text{atm}}}\right)^{1/3}$$

$$= \left(3.60\,\text{cm}\right)\left[1 + \frac{\left(1000\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(14.0\,\text{m}\right)}{1.013\times10^5\,\text{Pa}}\right]^{1/3} = \boxed{4.79\,\text{cm}}$$

(*b*) See the accompanying graph. The path is an isotherm, so the product *PV* is constant. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH19.XLS," on tab "Problem 19.83b."



(*c*) Since the process is isothermal, Eq. 19-8 is used to calculate the work.

$$W = nRT\ln\frac{V_2}{V_1} = P_2V_2\ln\frac{V_2}{V_1}$$

$$= P_2\left[\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_2\right)^3\right]\ln\frac{\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_2\right)^3}{\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_1\right)^3}$$

$$= P_2\left[\tfrac{1}{2}\pi d_2^3\right]\ln\left(\frac{d_2}{d_1}\right) = \left(1.013\times10^5\,\text{Pa}\right)\left[\tfrac{1}{2}\pi\left(0.0479\,\text{m}\right)^3\right]\ln\left(\frac{0.0479\,\text{m}}{0.0360\,\text{m}}\right) = \boxed{4.99\,\text{J}}$$

Since the process is isothermal, $\boxed{\Delta E_{\text{int}} = 0}$. Then, by the first law of thermodynamics,

$\Delta E_{\text{int}} = Q - W \;\rightarrow\; Q = \Delta E_{\text{int}} + W = \boxed{4.99\,\text{J}}$. The heat is added.

84. Use the first law of thermodynamics to find the rate of change of internal energy. Heat is being removed, so the heat term is negative. Work is being done on the gas, so the work term is also negative.

$$\Delta E_{\text{int}} = Q - W \;\rightarrow\; \frac{dE_{\text{int}}}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = -1.5\times10^3\,\text{W} - \left(7.5\times10^3\,\text{W}\right) = 6.0\times10^3\,\text{W}$$

This internal energy increase will happen during the compression stroke of a cycle. We assume that the time for compression is half of a cycle.

$$\Delta t_{\text{compression}} = \tfrac{1}{2}\left(\frac{1}{150\text{ cycles/min}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 0.20\,\text{s}$$

Assume an ideal diatomic gas and use Eq. 19-12 in order to find the temperature change.

$$\Delta E_{\text{int}} = nC_V\Delta T \;\rightarrow\; \frac{dE_{\text{int}}}{dt} = nC_V\left(\frac{\Delta T}{\Delta t}\right)_{\text{compression}} \;\rightarrow$$

$$\Delta T_{\text{compression}} = \frac{dE_{\text{int}}}{dt}\frac{\Delta t_{\text{compression}}}{nC_V} = \left(6.0\times10^3\,\text{W}\right)\frac{0.20\,\text{s}}{\left(1.00\,\text{mol}\right)\left(5.00\,\text{cal/mol}\bullet\text{K}\right)\left(4.186\,\text{J/cal}\right)} = \boxed{57\text{C}^\circ}$$

85. We assume that the light bulb emits energy by radiation, and so Eq. 19-18 applies. Use the data for the 75-W bulb to calculate the product $e\sigma A$ for the bulb, and then calculate the temperature of the 150-W bulb.

$$(Q/t)_{60\text{ W}} = e\sigma A\left(T^4_{60\text{ W}} - T^4_{\text{room}}\right) \;\rightarrow$$

$$e\sigma A = \frac{(Q/t)_{60\text{ W}}}{\left(T^4_{60\text{ W}} - T^4_{\text{room}}\right)} = \frac{(0.90)(75\text{ W})}{[(273+75)\text{ K}]^4 - [(273+18)\text{ K}]^4} = 9.006\times10^{-9}\text{ W/K}^4$$

$$(Q/t)_{150\text{ W}} = e\sigma A\left(T^4_{150\text{ W}} - T^4_{\text{room}}\right) \;\rightarrow$$

$$T_{150\text{ W}} = \left[\frac{(Q/t)_{150\text{ W}}}{e\sigma A} + T^4_{\text{room}}\right]^{1/4} = \left[\frac{(0.90)(150\text{ W})}{\left(9.006\times10^{-9}\text{ W/K}^4\right)} + (291\text{ K})^4\right]^{1/4}$$

$$= 386\text{ K} = 113^\circ\text{C} \approx \boxed{110^\circ\text{C}}$$

86. Let the subscript 1 refer to the original state, subscript 2 refer to the compressed state, and subscript 3 refer to the final state. The first process is best described by the ideal gas law for constant temperature, and the second process by the adiabatic relationship in Eq. 19-15. We use the ideal gas equation to relate the initial and final temperatures. Note that the initial volume is 3 times the molar volume at STP, and so is 67.2 L.

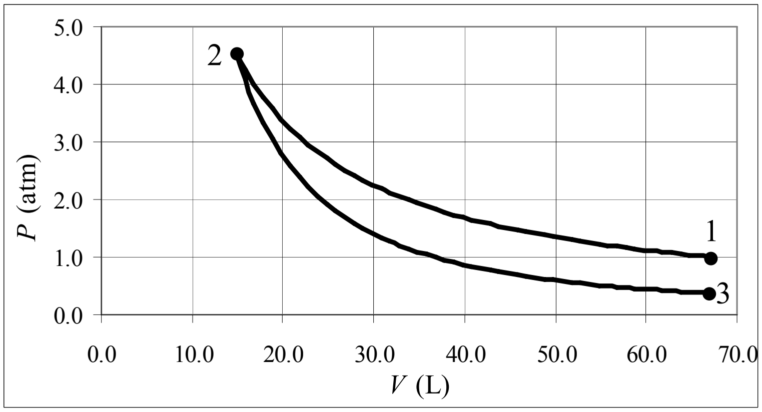
Process 1: $$P_1V_1 = P_2V_2 \;\rightarrow\; P_2 = P_1\left(\frac{V_1}{V_2}\right) = (1\text{atm})\left(\frac{1}{0.22}\right) = 4.545\text{atm} \;;\; T_2 = T_1 = 273\text{K}$$

Process 2: $$P_2V_2^\gamma = P_3V_3^\gamma \;\rightarrow\; P_3 = P_2\left(\frac{V_2}{V_3}\right)^\gamma = \left(\frac{1\text{atm}}{0.22}\right)\left(\frac{0.22}{1}\right)^{5/3} = 0.3644\text{atm}$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \;\rightarrow\; T_3 = T_1\left(\frac{P_3}{P_1}\right)\left(\frac{V_3}{V_1}\right) = (273\text{K})(0.3644)(1) = 99.48\text{K}$$

$$\boxed{\begin{array}{l} T_{\text{max}} = 273\text{K} = T_1 = T_2 \\ T_{\text{min}} = 99\text{K} = T_3 \\ P_{\text{max}} = 4.5\text{atm} = P_2 \\ P_{\text{min}} = 0.36\text{atm} = P_3 \end{array}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH19.XLS," on tab "Problem 19.86."



87. Since the specific heat is a function of temperature, we need to integrate the infinitesimal version of Eq. 19-2, and use moles instead of grams.

$$dQ = nCdT \;\rightarrow$$

$$\Delta Q = \int_{T_1}^{T_2} nk\frac{T^3}{T_0^3}dT = \frac{nk}{4T_0^3}\left(T_2^4 - T_1^4\right) = \frac{(2.75\text{mol})(1940\text{ J/mol}\bullet\text{K})}{4(281\text{K})^3}\left[(48.0\text{K})^4 - (22.0\text{K})^4\right] = \boxed{305\text{J}}$$

88. Combine the ideal gas relationship for a fixed amount of gas, $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$, with the adiabatic relationship, $P_1V_1^\gamma = P_2V_2^\gamma$.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \rightarrow \frac{V_1}{V_2}=\frac{P_2}{P_1}\frac{T_1}{T_2} \;;\; P_1V_1^\gamma = P_2V_2^\gamma \rightarrow \frac{P_2}{P_1}=\frac{V_1^\gamma}{V_2^\gamma}$$

$$\frac{V_1}{V_2}=\frac{V_1^\gamma}{V_2^\gamma}\frac{T_1}{T_2} \rightarrow \left(\frac{V_1}{V_2}\right)^{\gamma-1}=\frac{T_2}{T_1} \rightarrow \frac{V_1}{V_2}=\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}=\left(\frac{273\,\mathrm{K}+560\,\mathrm{K}}{280\,\mathrm{K}}\right)^{5/2}=\boxed{15.3}$$

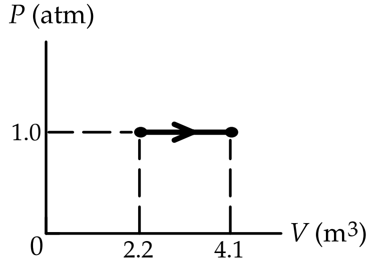
89. (*a*) Since the pressure is constant, the work is found using Eq. 19-9a.

$$W=P\Delta V=\left(1.013\times10^5\,\mathrm{Pa}\right)\left(4.1\,\mathrm{m}^3-2.2\,\mathrm{m}^3\right)=1.9247\times10^5\,\mathrm{J}\approx\boxed{1.9\times10^5\,\mathrm{J}}$$

(*b*) Use the first law of thermodynamics.

$$\Delta E_{\mathrm{int}}=Q-W=6.30\times10^5\,\mathrm{J}-1.92\times10^5\,\mathrm{J}=4.38\times10^5\,\mathrm{J}\approx\boxed{4.4\times10^5\,\mathrm{J}}$$

(*c*) See the accompanying graph.



90. The body's metabolism (blood circulation in particular) provides cooling by convection. If the metabolism has stopped, then heat loss will be by conduction and radiation, at a rate of 200 W, as given. The change in temperature is related to the body's heat loss by Eq. 19-2, $Q=mc\Delta T$.

$$\frac{Q}{t}=P=\frac{mc\Delta T}{t} \rightarrow$$

$$t=\frac{mc\Delta T}{P}=\frac{(70\text{ kg})(3470\,\mathrm{J/kg{\bullet}C^\circ})(36.6^\circ\mathrm{C}-35.6^\circ\mathrm{C})}{200\text{ W}}=\boxed{1200\text{ s}}=20\text{ min}$$

$\boxed{91.}$ The work is given by $W=\int_{V_1}^{V_2}PdV$. The pressure is found from the van der Waals expression, Eq. 18-9. The temperature is a constant. The analytic integration is done in problem 41.

$$W=nRT\ln\frac{(V_2-bn)}{(V_1-bn)}+an^2\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$$

$$=(1.0\,\mathrm{mol})(8.314\,\mathrm{J/mol{\bullet}K})(373\,\mathrm{K})\ln\frac{\left[1.00\,\mathrm{m}^3-\left(3.0\times10^{-5}\,\mathrm{m}^3/\mathrm{mol}\right)(1.0\,\mathrm{mol})\right]}{\left[0.50\,\mathrm{m}^3-\left(3.0\times10^{-5}\,\mathrm{m}^3/\mathrm{mol}\right)(1.0\,\mathrm{mol})\right]}$$

$$+\left(0.55\,\mathrm{N{\bullet}m^4/mol^2}\right)(1.0\,\mathrm{mol})^2\left(\frac{1}{1.00\,\mathrm{m}^3}-\frac{1}{0.50\,\mathrm{m}^3}\right)$$

$$=3101.122\ln\frac{\left[1.0-\left(3.0\times10^{-5}\right)\right]}{\left[0.5-\left(3.0\times10^{-5}\right)\right]}-0.55=2149\,\mathrm{J}$$

An acceptable answer from the numeric integration would need to be in the range of 2100 – 2200 J.

To do the numeric integration, we partition the volume range. For each volume value, the pressure is calculated using Eq. 18-9. The pressure is assumed constant over each segment of the volume partition, and then the work for that segment is the (constant) pressure times the small change in volume. Even with a relatively crude partition size of 0.1m^3, good agreement is found. The result from the numeric integration is 2158 J, which rounds to $\boxed{2200\text{ J}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH19.XLS,” on tab “Problem 19.91.”

CHAPTER 20: Second Law of Thermodynamics

Responses to Questions

1. Yes, mechanical energy can be transformed completely into heat or internal energy, as when an object moving over a surface is brought to rest by friction. All of the original mechanical energy is converted into heat. No, the reverse cannot happen (second law of thermodynamics) except in very special cases (reversible adiabatic expansion of an ideal gas). For example, in an explosion, a large amount of internal energy is converted into mechanical energy, but some internal energy is lost to heat or remains as internal energy of the explosion fragments.

2. Yes, you can warm a kitchen in winter by leaving the oven door open. The oven converts electrical energy to heat and leaving the door open will allow this heat to enter the kitchen. However, you cannot cool a kitchen in the summer by leaving the refrigerator door open. The refrigerator is a heat engine which (with an input of work) takes heat from the low-temperature reservoir (inside the refrigerator) and exhausts heat to the high-temperature reservoir (the room). As shown by the second law of thermodynamics, there is no "perfect refrigerator," so more heat will be exhausted into the room than removed from the inside of the refrigerator. Thus, leaving the refrigerator door open will actually warm the kitchen.

3. No. The definition of heat engine efficiency as $e = W/Q_L$ does not account for Q_H, the energy needed to produce the work. Efficiency should relate the input energy and the output work.

4. (*a*) In an internal-combustion engine, the high-temperature reservoir is the ignited gas–air mixture in the cylinder. The low-temperature reservoir is the outside; the burned gases leave through the exhaust pipe.
   (*b*) In the steam engine, the high-temperature reservoir is the steam-water mixture in the boiler and the low-temperature reservoir is the condensed water in the condenser. In the cases of both these engines, these areas are not technically heat "reservoirs," because each one is not at a constant temperature.

5. A 10ºC decrease in the low-temperature reservoir will give a greater improvement in the efficiency of a Carnot engine. By definition, T_L is less than T_H, so a 10ºC change will be a larger percentage change in T_L than in T_H, yielding a greater improvement in efficiency.

6. A heat engine operates when heat is allowed to flow from a high-temperature area to a lower temperature area and some of the heat is converted into work in the process. In order to obtain useful work from the thermal energy in the oceans, the oceans would have to represent the high-temperature area. This is not practical since the oceans are not at a high temperature compared to their surroundings and so there is no low-temperature reservoir available for the heat engine.

7. The two main factors which keep real engines from Carnot efficiency are friction and heat loss to the environment.

8. The expansion value allows the fluid to move from an area of higher pressure to an area of lower pressure. When the fluid moves to the low pressure area, it expands rapidly (adiabatic expansion). Expansion requires energy, which is taken from the internal energy of the fluid, thus lowering its temperature.

9. Water freezing on the surface of a body of water when the temperature is 0ºC is nearly a reversible process.

10. (*a*) Consider a gas enclosed in a cylinder with a movable (and frictionless) piston. If the gas and cylinder are kept at a constant temperature by contact with a thermal reservoir, then heat can be added very slowly allowing the gas to do work on the piston and expand. This isothermal expansion is reversible. If work is done very slowly on the gas, by pushing down on the piston, while the gas is still in contact with the thermal reservoir, then the same amount of heat will leave during the isothermal compression. Other processes which are not isothermal are possible, as long as they are gradual and temperature change occurs for all parts of the system at once.
    (*b*) A stove burner could not be used to add heat to a system reversibly since the heat added by a burner would not be distributed uniformly throughout the system and because the energy needed to heat the burner would not be recoverable.

11. The isothermal process will result in a greater change in entropy. The entropy change for a reversible process is the integral of dQ/T. $Q = 0$ for an adiabatic process, so the change in entropy is also 0.

12. Three examples of naturally occurring processes in which order goes to disorder are a landslide, fallen leaves decaying, and a cup of coffee cooling as it sits on a table. The reverse processes are not observed.

$\boxed{13.}$ 1 kg of liquid iron will have greater entropy, since it is less ordered than solid iron and its molecules have more thermal motion. In addition, heat must be added to solid iron to melt it; the addition of heat will increase the entropy of the iron.

14. (*a*) When the lid on a bottle of chlorine gas is removed, the gas mixes with the air in the room around the bottle so that eventually both the room and the bottle contain a mixture of air and chlorine.
    (*b*) The reverse process, in which the individual chlorine atoms reorganize so that they occupy only the bottle, violates the second law of thermodynamics and does not occur naturally.
    (*c*) Adding a drop of food coloring to a glass of water is another example of an irreversible process; the food coloring will eventually disperse throughout the water but will not ever gather into a drop again. Sliding a book across the floor is another example. Friction between the book and the floor will cause the kinetic energy to be dissipated as thermal energy and the book will eventually come to rest. There is no way to reverse this process and take thermal energy from the book and the floor and turn it into organized kinetic energy of the book.

15. The machine is clearly doing work to remove heat from some of the air in the room. The waste heat is dumped back into the room, and the heat generated in the process of doing work is also dumped into the room. The net result is the addition of heat into the room by the machine.

16. Some processes that would obey the first law of thermodynamics but not the second, if they actually occurred, include: a cup of tea warming itself by gaining thermal energy from the cooler air molecules around it, a ball sitting on a soccer field gathering energy from the grass and beginning to roll, and a bowl of popcorn placed in the refrigerator and unpopping as it cools.

17. No. While you have reduced the entropy of the papers, you have increased your own entropy by doing work, for which your muscles have consumed energy. The entropy of the universe has increased as a result of your actions.

18. The first law of thermodynamics is essentially a statement of the conservation of energy. “You can’t get something for nothing” is similar to the statement that “energy can be neither created nor destroyed.” If the “something” you want to get is useful work, then input energy is required. The second law concerns the direction of energy transfers. The Kelvin-Planck statement of the second

law says that "no device is possible whose sole effect is to transform a given amount of heat completely into work." In other words, 100 joules of heat will result is something less than 100 joules of work, so "you can't even break even."

19. No. Even if the powdered milk is added very slowly, it cannot be re-extracted from the water without very large investments of energy. This is not a reversible process.

20. Entropy is a state variable, so the change in entropy for the system will be the same for the two different irreversible processes that take the system from state a to state b. However, the change in entropy for the environment will not necessarily be the same. The total change in entropy (system plus environment) will be positive for irreversible processes, but the amount may be different for different irreversible processes.

21. For a reversible process, $\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{environment}} = 0$. Neither the process, nor its reverse process, would cause an increase in entropy. A "nearly reversible" process would have only a small increase in entropy, and the reverse process would require only a small input of energy. The greater ΔS_{total}, the more energy would be needed to reverse the process.

22. For reversible processes, the change in entropy is proportional to Q. Therefore, for a reversible, *adiabatic* process $\Delta S_{\text{total}} = 0$ because $Q = 0$. For an irreversible process, ΔS_{system} can be calculated by finding a series of reversible processes (not all of which will be adiabatic) that take the system between the same two states. The change in entropy for the system will be positive, since not all of the reversible processes will be adiabatic, and the change in the entropy of the environment will be zero. The total change in entropy for the irreversible adiabatic process will be positive.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. The efficiency of a heat engine is given by Eq. 20-1a. We also invoke energy conservation.

$$e = \frac{W}{Q_H} = \frac{W}{W + Q_L} = \frac{2600\text{ J}}{2600\text{ J} + 7800\text{ J}} = 0.25 = \boxed{25\%}$$

2. The efficiency of a heat engine is given by Eq. 20-1a. We also invoke energy conservation.

$$e = \frac{W}{Q_H} = \frac{W}{W + Q_L} \rightarrow Q_L = W\left(1/e - 1\right) \rightarrow$$

$$Q_L/t = W/t\left(1/e - 1\right) = \left(580\text{ MW}\right)\left(1/0.35 - 1\right) = 1.077\times10^3\,\text{MW} \approx \boxed{1.1\times10^3\,\text{MW}}$$

3. We calculate both the energy per second (power) delivered by the gasoline, and the energy per second (power) needed to overcome the drag forces. The ratio of these is the efficiency.

$$\frac{W}{t} = P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}} = Fv = \left(350\,\text{N}\right)\left(55\frac{\text{mi}}{\text{h}}\right)\left(\frac{1609\,\text{m}}{1\,\text{mi}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 8604\,\text{W}$$

$$\frac{Q_{\text{H}}}{t} = P_{\substack{\text{input}\\(\text{from}\\\text{gasoline})}} = \left(3.2\times10^{7}\,\frac{\text{J}}{\text{L}}\right)\left(\frac{3.8\,\text{L}}{1\,\text{gal}}\right)\left(\frac{1\,\text{gal}}{35\,\text{mi}}\right)\left(55\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 53079\,\text{W}$$

$$e = \frac{W}{Q_{\text{H}}} = \frac{P_{\substack{\text{output}\\(\text{to move}\\\text{car})}}}{P_{\substack{\text{input}\\(\text{from}\\\text{gasoline})}}} = \frac{8604\,\text{W}}{53079\,\text{W}} = \boxed{0.16}$$

4. (*a*) The work done per second is found from the engine specifications.

$$\frac{W}{t} = \left(180\frac{\text{J}}{\text{cycle•cylinder}}\right)\left(4\,\text{cylinders}\right)\left(25\frac{\text{cycles}}{\text{s}}\right) = \boxed{1.8\times10^{4}\,\text{J/s}}$$

(*b*) The efficiency is given by Eq. 20-1a.

$$e = \frac{W}{Q_{\text{H}}} = \;\rightarrow\; Q_{\text{H}} = \frac{W}{e} \;\rightarrow\; Q_{\text{H}}/t = \frac{W/t}{e} = \frac{1.8\times10^{4}\,\text{J/s}}{0.22} = 8.182\times10^{4}\,\text{J/s} \approx \boxed{8.2\times10^{4}\,\text{J/s}}$$

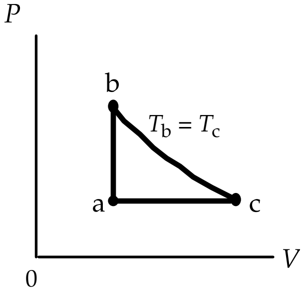
(*c*) Divide the energy in a gallon of gasoline by the rate at which that energy gets used.

$$\frac{130\times10^{6}\,\text{J/gal}}{8.182\times10^{4}\,\text{J/s}} = 1589\,\text{s} \approx \boxed{26\,\text{min}}$$

5. The efficiency is the work done by the engine, divided by the heat input to the engine from the burning of the gasoline. Both energy terms are expressed in terms of a rate. The gasoline provides the input energy, and the horsepower of the engine represents the output work.

$$e = \frac{W}{Q_{\text{H}}} = \frac{W/t}{Q_{\text{H}}/t} = \frac{\left(25\,\text{hp}\right)\left(\frac{746\,\text{W}}{1\,\text{hp}}\right)\left(\frac{1\,\text{J/s}}{1\,\text{W}}\right)}{\left(3.0\times10^{4}\,\frac{\text{kcal}}{\text{gal}}\right)\left(\frac{1\,\text{gal}}{38\,\text{km}}\right)\left(95\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\left(\frac{4186\,\text{J}}{\text{kcal}}\right)} = \boxed{0.21\text{ or }21\%}$$

6. (*a*) For the net work done by the engine to be positive, the path must be carried out $\boxed{\text{clockwise}}$. Then the work done by process bc is positive, the work done by process ca is negative, and the work done by process ab is 0. From the shape of the graph, we see that $\left|W_{\text{bc}}\right| > \left|W_{\text{ca}}\right|$.



(*b*) The efficiency of the engine is given by Eq. 20-1a. So we need to find the work done and the heat input. At first glance we might assume that we need to find the pressure, volume, and temperature at the three points on the graph. But as shown here, only the temperatures and the first law of thermodynamics are needed, along with ratios that are obtained from the ideal gas law.

$$\text{ab:}\quad W_{\text{ab}} = P\Delta V = 0\;;\; Q_{\text{ab}} = \Delta E_{\substack{\text{int}\\\text{ab}}} = nC_{V}\left(T_{\text{b}} - T_{\text{a}}\right) = \tfrac{3}{2}nR\left(T_{\text{b}} - T_{\text{a}}\right) > 0$$

$$\text{bc:}\quad \Delta E_{\substack{\text{int}\\\text{bc}}} = nC_{V}\left(T_{\text{c}} - T_{\text{b}}\right) = 0\;;\; Q_{\text{bc}} = W_{\text{bc}} = nRT_{\text{b}}\ln\frac{V_{\text{c}}}{V_{\text{b}}} = nRT_{\text{b}}\ln\frac{V_{\text{c}}}{V_{\text{a}}} = nRT_{\text{b}}\ln\frac{T_{\text{c}}}{T_{\text{a}}} > 0$$

$$\text{ca:}\quad W_{\text{ca}} = P_{\text{a}}\left(V_{\text{a}} - V_{\text{c}}\right) = \frac{nRT_{\text{a}}}{V_{\text{a}}}\left(V_{\text{a}} - V_{\text{c}}\right) = nRT_{\text{a}}\left(1 - \frac{V_{\text{c}}}{V_{\text{a}}}\right) = nRT_{\text{a}}\left(1 - \frac{T_{\text{c}}}{T_{\text{a}}}\right) = nR\left(T_{\text{a}} - T_{\text{c}}\right)$$

$$Q_{\text{ca}} = nC_{P}\left(T_{\text{a}} - T_{\text{c}}\right) = \tfrac{5}{2}nR\left(T_{\text{a}} - T_{\text{c}}\right) < 0$$

$$e=\frac{W}{Q_{\text{input}}}=\frac{W_{\text{bc}}+W_{\text{ca}}}{Q_{\text{ab}}+Q_{\text{bc}}}=\frac{nRT_{\text{b}}\ln\frac{T_{\text{c}}}{T_{\text{a}}}+nR\left(T_{\text{a}}-T_{\text{c}}\right)}{\frac{3}{2}nR\left(T_{\text{b}}-T_{\text{a}}\right)+nRT_{\text{b}}\ln\frac{T_{\text{c}}}{T_{\text{a}}}}=\frac{T_{\text{b}}\ln\frac{T_{\text{c}}}{T_{\text{a}}}+\left(T_{\text{a}}-T_{\text{c}}\right)}{\frac{3}{2}\left(T_{\text{b}}-T_{\text{a}}\right)+T_{\text{b}}\ln\frac{T_{\text{c}}}{T_{\text{a}}}}=\frac{T_{\text{b}}\ln\frac{T_{\text{b}}}{T_{\text{a}}}+\left(T_{\text{a}}-T_{\text{b}}\right)}{\frac{3}{2}\left(T_{\text{b}}-T_{\text{a}}\right)+T_{\text{b}}\ln\frac{T_{\text{b}}}{T_{\text{a}}}}$$

$$=\frac{(423\,\text{K})\ln\frac{423\,\text{K}}{273\,\text{K}}+(273\,\text{K}-423\,\text{K})}{\frac{3}{2}(423\,\text{K}-273\,\text{K})+(423\,\text{K})\ln\frac{423\,\text{K}}{273\,\text{K}}}=0.0859=\boxed{8.59\%}$$

Of course, individual values could have been found for the work and heat on each process, and used in the efficiency equation instead of referring everything to the temperatures.

$\boxed{7.}$ (*a*) To find the efficiency, we need the heat input and the heat output. The heat input occurs at constant pressure, and the heat output occurs at constant volume. Start with Eq. 20-1b.

$$e=1-\frac{Q_{\text{L}}}{Q_{\text{H}}}=1-\frac{nC_{\text{V}}\left(T_{\text{d}}-T_{\text{a}}\right)}{nC_{\text{P}}\left(T_{\text{c}}-T_{\text{b}}\right)}=1-\frac{\left(T_{\text{d}}-T_{\text{a}}\right)}{\gamma\left(T_{\text{c}}-T_{\text{b}}\right)}$$

So we need to express the temperatures in terms of the corresponding volume, and use the ideal gas law and the adiabatic relationship between pressure and volume to get those expressions. Note that $P_{\text{c}}=P_{\text{b}}$ and $V_{\text{d}}=V_{\text{a}}$.

$$PV=nRT\ \rightarrow\ T=\frac{PV}{nR}\ \rightarrow$$

$$e=1-\frac{\left(\frac{P_{\text{d}}V_{\text{d}}}{nR}-\frac{P_{\text{a}}V_{\text{a}}}{nR}\right)}{\gamma\left(\frac{P_{\text{c}}V_{\text{c}}}{nR}-\frac{P_{\text{b}}V_{\text{b}}}{nR}\right)}=1-\frac{\left(P_{\text{d}}V_{\text{d}}-P_{\text{a}}V_{\text{a}}\right)}{\gamma\left(P_{\text{c}}V_{\text{c}}-P_{\text{b}}V_{\text{b}}\right)}=1-\frac{V_{\text{a}}\left(P_{\text{d}}-P_{\text{a}}\right)}{\gamma P_{\text{b}}\left(V_{\text{c}}-V_{\text{b}}\right)}=1-\frac{\left(\frac{P_{\text{d}}}{P_{\text{c}}}-\frac{P_{\text{a}}}{P_{\text{b}}}\right)}{\gamma\left(\frac{V_{\text{c}}}{V_{\text{a}}}-\frac{V_{\text{b}}}{V_{\text{a}}}\right)}$$

Use the adiabatic relationship between pressure and volume on the two adiabatic paths.

$$P_{\text{d}}V_{\text{d}}^{\gamma}=P_{\text{c}}V_{\text{c}}^{\gamma}\ \rightarrow\ \frac{P_{\text{d}}}{P_{\text{c}}}=\left(\frac{V_{\text{c}}}{V_{\text{d}}}\right)^{\gamma}=\left(\frac{V_{\text{c}}}{V_{\text{a}}}\right)^{\gamma}\ ;\ P_{\text{a}}V_{\text{a}}^{\gamma}=P_{\text{b}}V_{\text{b}}^{\gamma}\ \rightarrow\ \frac{P_{\text{a}}}{P_{\text{b}}}=\left(\frac{V_{\text{b}}}{V_{\text{a}}}\right)^{\gamma}$$

$$e=1-\frac{\left(\frac{P_{\text{d}}}{P_{\text{c}}}-\frac{P_{\text{a}}}{P_{\text{b}}}\right)}{\gamma\left(\frac{V_{\text{c}}}{V_{\text{a}}}-\frac{V_{\text{b}}}{V_{\text{a}}}\right)}=1-\frac{\left(\frac{V_{\text{c}}}{V_{\text{a}}}\right)^{\gamma}-\left(\frac{V_{\text{b}}}{V_{\text{a}}}\right)^{\gamma}}{\gamma\left[\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{-1}-\left(\frac{V_{\text{a}}}{V_{\text{b}}}\right)^{-1}\right]}=\boxed{1-\frac{\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{-\gamma}-\left(\frac{V_{\text{a}}}{V_{\text{b}}}\right)^{-\gamma}}{\gamma\left[\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{-1}-\left(\frac{V_{\text{a}}}{V_{\text{b}}}\right)^{-1}\right]}}$$

(*b*) For a diatomic ideal gas, $\gamma=\frac{7}{5}=1.4$.

$$e=1-\frac{\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{-\gamma}-\left(\frac{V_{\text{a}}}{V_{\text{b}}}\right)^{-\gamma}}{\gamma\left[\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{-1}-\left(\frac{V_{\text{a}}}{V_{\text{b}}}\right)^{-1}\right]}=1-\frac{(4.5)^{-1.4}-(16)^{-1.4}}{1.4\left[(4.5)^{-1}-(16)^{-1}\right]}=\boxed{0.55}$$

8. The maximum efficiency is the Carnot efficiency, given in Eq. 20-3.

$$e = 1 - \frac{T_L}{T_H} = 1 - \frac{(365+273)\,\text{K}}{(550+273)\,\text{K}} = \boxed{0.225 \text{ or } 22.5\%}$$

We assume that both temperatures are measured to the same precision – the nearest degree.

9. Calculate the Carnot efficiency for the given temperatures.

$$e_{\text{ideal}} = 1 - \frac{T_L}{T_H} = 1 - \frac{77\text{ K}}{293\text{ K}} = 0.7372 \approx \boxed{74\%}$$

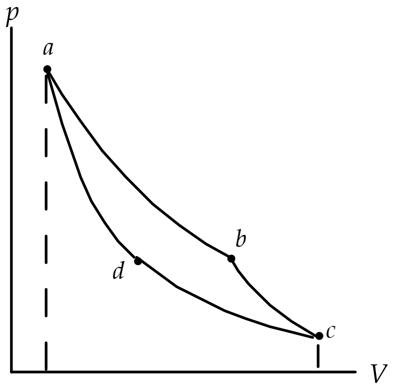
10. Find the intake temperature from the original Carnot efficiency, and then recalculate the exhaust temperature for the new Carnot efficiency, using the same intake temperature.

$$e_1 = 1 - \frac{T_{L1}}{T_H} \rightarrow T_H = \frac{T_{L1}}{1-e_1} = \frac{(340+273)\,\text{K}}{1-0.38} = 989\text{ K}$$

$$e_2 = 1 - \frac{T_{L2}}{T_H} \rightarrow T_{L2} = T_H(1-e_2) = (989\,\text{K})(1-0.45) = 544\,\text{K} = 271°\text{C} \approx \boxed{270°\text{C}}$$

11. (*a*) The work done during any process is given by Eq. 19-7. So the net work done is as follows.

$$W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$
$$= \int_a^b PdV + \int_b^c PdV + \int_c^d PdV + \int_d^a PdV$$
$$= \left(\int_a^b PdV + \int_b^c PdV\right) - \left(\int_a^d PdV + \int_d^c PdV\right)$$



The sum of the first two terms is the area under the abc path (the "upper" path), and the sum of the last two terms is the area under the adc path (the "lower" path). So the net work is the area enclosed by the cycle.

(*b*) Any reversible cycle can be represented as a closed loop in the *P*-*V* plane. If we select the two points on the loop with the maximum and minimum volumes, we can apply the reasoning from above to find the net work.

12. This is a perfect Carnot engine, and so its efficiency is given by Eq. 20-1a and Eq. 20-3. Use these two expressions to solve for the rate of heat output.

$$e = 1 - \frac{T_L}{T_H} = 1 - \frac{(45+273)\,\text{K}}{(210+273)\,\text{K}} = 0.3416 \qquad e = \frac{W}{Q_H} = \frac{W}{W+Q_L} \rightarrow Q_L = W(1/e - 1)$$

$$Q_L/t = W/t(1/e-1) = (950\text{ W})(1/0.3416 - 1) = 1831\,\text{W} \approx \boxed{1800\,\text{W}}$$

$\boxed{13.}$ The maximum (or Carnot) efficiency is given by Eq. 20-3, with temperatures in Kelvins.

$$e = 1 - \frac{T_L}{T_H} = 1 - \frac{(330+273)\,\text{K}}{(665+273)\,\text{K}} = 0.354$$

Thus the total power generated can be found as follows.

$$\text{Actual Power} = (\text{Total Power})(\text{max. eff.})(\text{operating eff.}) \rightarrow$$

$$\text{Total Power} = \frac{\text{Actual Power}}{(\text{max. eff.})(\text{operating eff.})} = \frac{1.2\,\text{GW}}{(0.354)(0.65)} = 5.215\,\text{GW}$$

$$\text{Exhaust Power} = \text{Total Power} - \text{Actual Power} = 5.215\ \text{GW} - 1.2\ \text{GW} = 4.015\ \text{GW}$$

$$= \left(4.015\times10^{9}\ \text{J/s}\right)\left(3600\,\text{s/h}\right) = \boxed{1.4\times10^{13}\ \text{J/h}}$$

14. This is a perfect Carnot engine, and so its efficiency is given by Eq. 20-1a and Eq. 20-3. Equate these two expressions for the efficiency.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = \frac{W}{Q_{\text{H}}} \quad \rightarrow$$

$$T_{\text{L}} = T_{\text{H}}\left(1 - \frac{W}{Q_{\text{H}}}\right) = T_{\text{H}}\left(1 - \frac{W/t}{Q_{\text{H}}/t}\right) = \left[(560+273)\,\text{K}\right]\left(1 - \frac{5.2\times10^{5}\,\text{J/s}}{(950\,\text{kcal/s})(4186\,\text{J/kcal})}\right)$$

$$= 724\ \text{K} = 451^\circ\text{C} \approx \boxed{450^\circ\text{C}}$$

15. We assume the efficiency of the person's metabolism is that of a reversible engine. Then we take the work from that "engine" and assume that it is all used to increase the person's potential energy by climbing higher.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = \frac{W}{Q_{\text{H}}} = \frac{mgh}{Q_{\text{H}}} \quad \rightarrow$$

$$h = \frac{Q_{\text{H}}}{mg}\left(1 - \frac{T_{\text{L}}}{T_{\text{H}}}\right) = \frac{4.0\times10^{3}\,\text{kcal}\left(\frac{4186\,\text{J}}{1\,\text{kcal}}\right)}{(65\,\text{kg})(9.80\,\text{m/s}^2)}\left(1 - \frac{(273+20)\,\text{K}}{(273+37)\,\text{K}}\right) = 1441\,\text{m} \approx \boxed{1400\,\text{m}}$$

16. The minimum value for T_{H} would occur if the engine were a Carnot engine. We calculate the efficiency of the engine from the given data, and use this as a Carnot efficiency to calculate T_{H}.

$$\frac{W}{t} = P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}} = 7000\,\text{W}\ ;\ \frac{Q_{\text{H}}}{t} = P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}} = \left(3.2\times10^{7}\,\frac{\text{J}}{\text{L}}\right)\left(\frac{1\,\text{L}}{17000\,\text{m}}\right)\left(\frac{20\,\text{m}}{1\,\text{s}}\right) = 37647\,\text{W}$$

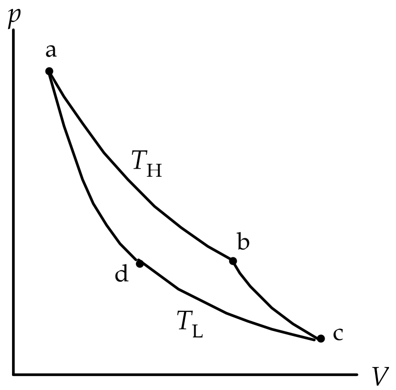
$$e = \frac{W}{Q_{\text{H}}} = \frac{P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}}}{P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}}} = \frac{7000\,\text{W}}{37647\,\text{W}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} \quad \rightarrow \quad T_{\text{H}} = \frac{T_{\text{L}}}{(1-e)} = \frac{(273+25)\,\text{K}}{\left(1 - \frac{7000\,\text{W}}{37647\,\text{W}}\right)} = 366\,\text{K} = \boxed{93^\circ\text{C}}$$

17. Find the exhaust temperature from the original Carnot efficiency, and then recalculate the intake temperature for the new Carnot efficiency, using the same exhaust temperature.

$$e_1 = 1 - \frac{T_{\text{L}}}{T_{\text{H1}}} \quad \rightarrow \quad T_{\text{L}} = T_{\text{H1}}(1-e) = (580\,\text{K} + 273\,\text{K})(1-0.32) = 580.0\,\text{K}$$

$$e_2 = 1 - \frac{T_{\text{L}}}{T_{\text{H2}}} \quad \rightarrow \quad T_{\text{H2}} = \frac{T_{\text{L}}}{1-e_2} = \frac{580.0\ \text{K}}{1-0.38} = 936\ \text{K} = 663^{\circ}\text{C} \approx \boxed{660^{\circ}\text{C}}$$

18. The heat input must come during the isothermal expansion. From section 20-3, page 534, we have $Q_H = nRT_H \ln\frac{V_b}{V_a} = nRT_H \ln 2$.



Since this is a Carnot cycle, we may use Eq. 20-3 combined with Eq. 20-1.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} \rightarrow$$

$$W = Q_H\left(\frac{T_H - T_L}{T_H}\right) = (nRT_H \ln 2)\left(\frac{T_H - T_L}{T_H}\right) = nR(T_H - T_L)\ln 2$$

The adiabatic relationship between points b and c and the ideal gas law are used to express the temperature ratio in terms of the volume ratio.

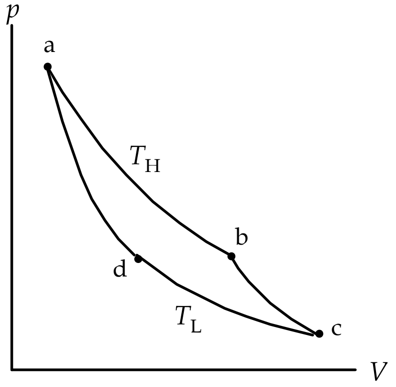
$$P_bV_b^\gamma = P_cV_c^\gamma \rightarrow \frac{nRT_H}{V_b}V_b^\gamma = \frac{nRT_L}{V_c}V_c^\gamma \rightarrow T_H = T_L\left(\frac{V_c}{V_b}\right)^{\gamma-1} = T_L\left(5.7^{2/3}\right)$$

$$W = nR(T_H - T_L)\ln 2 = nRT_L\left(5.7^{2/3} - 1\right)\ln 2 \rightarrow$$

$$T_L = \frac{W}{nR\ln 2\left[5.7^{2/3} - 1\right]} = \frac{920\,\text{J}}{(1.00\,\text{mol})(8.314\,\text{J/mol•K})\ln 2\left[5.7^{2/3} - 1\right]} = 72.87\,\text{K} \approx \boxed{73\,\text{K}}$$

$$T_H = (72.87\,\text{K})\left(5.7^{2/3}\right) = 232.52\,\text{K} \approx \boxed{233\,\text{K}}$$

$\boxed{19.}$ (*a*) The pressures can be found from the ideal gas equation.



$$PV = nRT \rightarrow P = \frac{nRT}{V} \rightarrow$$

$$P_a = \frac{nRT_a}{V_a} = \frac{(0.50\,\text{mol})(8.314\,\text{J/mol•K})(743\,\text{K})}{7.5\times10^{-3}\,\text{m}^3}$$

$$= 4.118\times10^5\,\text{Pa} \approx \boxed{4.1\times10^5\,\text{Pa}}$$

$$P_b = \frac{nRT_b}{V_b} = \frac{(0.50\,\text{mol})(8.314\,\text{J/mol•K})(743\,\text{K})}{15.0\times10^{-3}\,\text{m}^3}$$

$$= 2.059\times10^5\,\text{Pa} \approx \boxed{2.1\times10^5\,\text{Pa}}$$

(*b*) The volumes can be found from combining the ideal gas law and the relationship between pressure and volume for an adiabatic process.

$$P_cV_c^\gamma = P_bV_b^\gamma \rightarrow \frac{nRT_c}{V_c}V_c^\gamma = \frac{nRT_b}{V_b}V_b^\gamma \rightarrow T_cV_c^{\gamma-1} = T_bV_b^{\gamma-1} \rightarrow$$

$$V_c = \left(\frac{T_b}{T_c}\right)^{\frac{1}{\gamma-1}} V_b = \left(\frac{743\,\text{K}}{533\,\text{K}}\right)^{2.5}(15.0\,\text{L}) = \boxed{34.4\,\text{L}}$$

$$V_d = \left(\frac{T_a}{T_d}\right)^{\frac{1}{\gamma-1}} V_a = \left(\frac{743\,\text{K}}{533\,\text{K}}\right)^{2.5}(7.5\,\text{L}) = 17.2\,\text{L} \approx \boxed{17\,\text{L}}$$

(*c*) The work done at a constant temperature is given by Eq. 19-8.

$$W = nRT\ln\left(\frac{V_b}{V_a}\right) = (0.50\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(743\,\text{K})\ln\left(\frac{15.0}{7.5}\right) = 2141\,\text{J} \approx \boxed{2100\,\text{J}}$$

Note that this is also the heat input during that process.

(*d*) Along process cd, there is no change in internal energy since the process is isothermic. Thus by the first law of thermodynamics, the heat exhausted is equal to the work done during that process.

$$Q = W = nRT\ln\left(\frac{V_d}{V_c}\right) = (0.50\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(533\,\text{K})\ln\left(\frac{17.2}{34.4}\right) = -1536\,\text{J} \approx \boxed{-1500\,\text{J}}$$

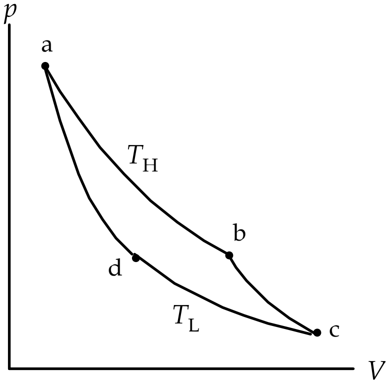
So 2100 J of heat was exhausted during process cd.

(*e*) From the first law of thermodynamics, for a closed cycle, the net work done is equal to the net heat input.

$$W_{net} = Q_{net} = 2141\,\text{J} - 1536\,\text{J} = 605\,\text{J} \approx \boxed{600\,\text{J}}$$

(*f*) $$e = \frac{W}{Q_H} = \frac{605\,\text{J}}{2141\,\text{J}} = \boxed{0.28}\ ;\ e = 1 - \frac{T_L}{T_H} = 1 - \frac{533\,\text{K}}{743\,\text{K}} = 0.28$$

20. (*a*) We use the ideal gas law and the adiabatic process relationship to find the values of the pressure and volume at each of the four points.



$$P_a = \boxed{8.8\,\text{atm}}\ ;\ T_a = 623\,\text{K};$$

$$V_a = \frac{nRT_a}{P_a} = \frac{(1.00\,\text{mol})(0.0821\,\text{L}\bullet\text{atm/mol}\bullet\text{K})(623\,\text{K})}{8.8\,\text{atm}} = 5.81\,\text{L} \approx \boxed{5.8\,\text{L}}$$

$$T_b = 623\,\text{K};\ V_b = 2V_a = 2(5.81\,\text{L}) = 11.62\,\text{L} \approx \boxed{11.6\,\text{L}}$$

$$P_b = P_a\frac{V_a}{V_b} = \tfrac{1}{2}P_a = \boxed{4.4\,\text{atm}}$$

$$T_c = 483\,\text{K};\ P_bV_b^\gamma = P_cV_c^\gamma \rightarrow \frac{nRT_b}{V_b}V_b^\gamma = \frac{nRT_c}{V_c}V_c^\gamma \rightarrow$$

$$V_c = V_b\left(\frac{T_b}{T_c}\right)^{\frac{1}{\gamma-1}} = (11.62\,\text{L})\left(\frac{623\,\text{K}}{483\,\text{K}}\right)^{3/2} = 17.02\,\text{L} \approx \boxed{17.0\,\text{L}}$$

$$P_c = \frac{nRT_c}{V_c} = \frac{(1.00\,\text{mol})(0.0821\,\text{L}\bullet\text{atm/mol}\bullet\text{K})(483\,\text{K})}{17.02\,\text{L}} = 2.33\,\text{atm} \approx \boxed{2.3\,\text{atm}}$$

$$T_d = 483\,\text{K};\ V_d = V_a\left(\frac{T_a}{T_d}\right)^{\frac{1}{\gamma-1}} = (5.81\,\text{L})\left(\frac{623\,\text{K}}{483\,\text{K}}\right)^{3/2} = 8.51\,\text{L} \approx \boxed{8.5\,\text{L}}$$

$$P_d = \frac{nRT_d}{V_d} = \frac{(1.00\,\text{mol})(0.0821\,\text{L}\bullet\text{atm/mol}\bullet\text{K})(483\,\text{K})}{8.51\,\text{L}} = 4.66\,\text{atm} \approx \boxed{4.7\,\text{atm}}$$

To summarize:

$$P_a = \boxed{8.8\,\text{atm}}\ ;\ V_a = \boxed{5.8\,\text{L}}\ ;\ P_b = \boxed{4.4\,\text{atm}}\ ;\ V_b = \boxed{11.6\,\text{L}}$$

$$P_c = \boxed{2.3\,\text{atm}}\ ;\ V_c = \boxed{17.0\,\text{L}}\ ;\ P_d = \boxed{4.7\,\text{atm}}\ ;\ V_d = \boxed{8.5\,\text{L}}$$

(*b*) Isotherm ab: $\Delta E_{\underset{\text{ab}}{\text{int}}} = \boxed{0}$;

$$Q_{\text{ab}} = W_{\text{ab}} = nRT_a \ln\frac{V_{\text{b}}}{V_{\text{a}}} = (1.00\,\text{mol})(8.314\,\text{J/mol•K})(623\,\text{K})\ln 2 = 3590\,\text{J} \approx \boxed{3600\,\text{J}}$$

Adiabat bc: $Q_{\text{bc}} = \boxed{0}$;

$$\Delta E_{\underset{\text{bc}}{\text{int}}} = nC_V(T_{\text{c}} - T_{\text{b}}) = \tfrac{3}{2}nR(T_{\text{c}} - T_{\text{b}}) = \tfrac{3}{2}(1.00\,\text{mol})(8.314\,\text{J/mol•K})(-140\,\text{K})$$

$$= -1746\,\text{J} \approx \boxed{-1700\,\text{J}} \;;\; W_{\text{bc}} = Q_{\text{bc}} - \Delta E_{\underset{\text{bc}}{\text{int}}} = 1746\,\text{J} \approx \boxed{1700\,\text{J}}$$

Isotherm cd: $\Delta E_{\underset{\text{cd}}{\text{int}}} = \boxed{0}$;

$$Q_{\text{cd}} = W_{\text{cd}} = nRT_c \ln\frac{V_{\text{d}}}{V_{\text{c}}} = (1.00\,\text{mol})(8.314\,\text{J/mol•K})(483\,\text{K})\ln\tfrac{1}{2} = -2783\,\text{J} \approx \boxed{-2800\,\text{J}}$$

Adiabat da: $Q_{\text{da}} = \boxed{0}$;

$$\Delta E_{\underset{\text{bc}}{\text{int}}} = nC_V(T_{\text{c}} - T_{\text{b}}) = \tfrac{3}{2}nR(T_{\text{c}} - T_{\text{b}}) = \tfrac{3}{2}(1.00\,\text{mol})(8.314\,\text{J/mol•K})(140\,\text{K})$$

$$= 1746\,\text{J} \approx \boxed{1700\,\text{J}} \;;\; W_{\text{bc}} = Q_{\text{bc}} - \Delta E_{\underset{\text{bc}}{\text{int}}} = 1746\,\text{J} \approx \boxed{-1700\,\text{J}}$$

To summarize:

ab : $\Delta E_{\text{int}} = 0$; $Q = 3600\,\text{J}$; $W = 3600\,\text{J}$

bc: $\Delta E_{\text{int}} = -1700\,\text{J}$; $Q = 0$; $W = 1700\,\text{J}$

cd: $\Delta E_{\text{int}} = 0$; $Q = -2800\,\text{J}$; $W = -2800\,\text{J}$

da: $\Delta E_{\text{int}} = 1700\,\text{J}$; $Q = 0$; $W = -1700\,\text{J}$

(*c*) Using Eq. 20-1: $$e = \frac{W}{Q_{\text{input}}} = \frac{3590\,\text{J} + 1746\,\text{J} - 2783\,\text{J} - 1746\,\text{J}}{3590\,\text{J}} = \frac{807\,\text{J}}{3590\,\text{J}} = 0.2248 \approx \boxed{0.22}$$

Using Eq. 20-3: $$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{(273 + 210)\,\text{K}}{(273 + 350)\,\text{K}} = 0.2247 \approx \boxed{0.22}$$

The slight disagreement is due to rounding of various calculations.

21. The adiabatic compression takes place between temperatures of 25°C and 430°C. Use the adiabatic relationship and the ideal gas law to express the volumes in terms of the temperatures.

$$P_{\text{a}}V_{\text{a}}^{\gamma} = P_{\text{b}}V_{\text{b}}^{\gamma} \;;\; PV = nRT \;\rightarrow\; P_{\text{a}} = \frac{nRT_{\text{a}}}{V_{\text{a}}} \;,\; P_{\text{b}} = \frac{nRT_{\text{b}}}{V_{\text{b}}} \;\rightarrow$$

$$\frac{nRT_{\text{a}}}{V_{\text{a}}}V_{\text{a}}^{\gamma} = \frac{nRT_{\text{b}}}{V_{\text{b}}}V_{\text{b}}^{\gamma} \;\rightarrow\; T_{\text{a}}V_{\text{a}}^{\gamma-1} = T_{\text{b}}V_{\text{b}}^{\gamma-1} \;\rightarrow\; \frac{V_{\text{a}}}{V_{\text{b}}} = \left(\frac{T_{\text{b}}}{T_{\text{a}}}\right)^{\frac{1}{\gamma-1}} = \left(\frac{273\,\text{K} + 430\,\text{K}}{273\,\text{K} + 25\,\text{K}}\right)^{\frac{1}{1.4-1}} = \boxed{8.55}$$

22. The ideal coefficient of performance is given by Eq. 20-4c.

$$\text{COP}_{\text{ideal}} = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}} = \frac{[273 + 3.0]\,\text{K}}{[22 - 3.0]\,\text{K}} = 14.53 \approx \boxed{15}$$

23. The ideal coefficient of performance for a refrigerator is given by Eq. 20-4c.

$$\mathrm{COP}=\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}-T_{\mathrm{L}}}=\frac{(-15+273)\,\mathrm{K}}{(33+273)\,\mathrm{K}-(-15+273)\,\mathrm{K}}=\boxed{5.4}$$

24. The COP for a heat pump is $\mathrm{COP}=\frac{Q_{\mathrm{H}}}{W}$ and the efficiency is $e=\frac{W}{Q_{\mathrm{H}}}$. Thus they are reciprocals of each other. So if the efficiency is 0.38, the COP is $\frac{1}{0.38}=\boxed{2.6}$.

$\boxed{25.}$ (*a*) The total rate of adding heat to the house by the heat pump must equal the rate of heat lost by conduction.

$$\frac{Q_{\mathrm{L}}+W}{\Delta t}=(650\,\mathrm{W/C^\circ})(T_{\mathrm{in}}-T_{\mathrm{out}})$$

Since the heat pump is ideal, we have the following.

$$1-\frac{T_{\mathrm{out}}}{T_{\mathrm{in}}}=1-\frac{Q_{\mathrm{L}}}{Q_{\mathrm{H}}}=1-\frac{Q_{\mathrm{L}}}{Q_{\mathrm{L}}+W}=\frac{W}{Q_{\mathrm{L}}+W}\;\rightarrow\;Q_{\mathrm{L}}+W=W\frac{T_{\mathrm{in}}}{T_{\mathrm{in}}-T_{\mathrm{out}}}$$

Combine these two expressions, and solve for T_{out}.

$$\frac{Q_{\mathrm{L}}+W}{\Delta t}=(650\,\mathrm{W/C^\circ})(T_{\mathrm{in}}-T_{\mathrm{out}})=\frac{W}{\Delta t}\frac{T_{\mathrm{in}}}{(T_{\mathrm{in}}-T_{\mathrm{out}})}\;\rightarrow\;(T_{\mathrm{in}}-T_{\mathrm{out}})^2=\frac{W}{\Delta t}\frac{T_{\mathrm{in}}}{(650\,\mathrm{W/C^\circ})}\;\rightarrow$$

$$T_{\mathrm{out}}=T_{\mathrm{in}}-\sqrt{\frac{W}{\Delta t}\frac{T_{\mathrm{in}}}{(650\,\mathrm{W/C^\circ})}}=295\,\mathrm{K}-\sqrt{(1500\,\mathrm{W})\frac{295\,\mathrm{K}}{(650\,\mathrm{W/C^\circ})}}=269\,\mathrm{K}=\boxed{-4^\circ\mathrm{C}}$$

(*b*) If the outside temperature is 8°C, then the rate of heat loss by conduction is found to be $(650\,\mathrm{W/C^\circ})(14\,\mathrm{C^\circ})=9100\,\mathrm{W}$. The heat pump must provide this much power to the house in order for the house to stay at a constant temperature. That total power is $(Q_{\mathrm{L}}+W)/\Delta t$. Use this to solve for rate at which the pump must do work.

$$(Q_{\mathrm{L}}+W)/\Delta t=\frac{W}{\Delta t}\left(\frac{T_{\mathrm{in}}}{T_{\mathrm{in}}-T_{\mathrm{out}}}\right)=9100\,\mathrm{W}\;\rightarrow$$

$$\frac{W}{\Delta t}=9100\,\mathrm{W}\left(\frac{T_{\mathrm{in}}-T_{\mathrm{out}}}{T_{\mathrm{in}}}\right)=9100\,\mathrm{W}\left(\frac{14\,\mathrm{K}}{295\,\mathrm{K}}\right)=432\,\mathrm{W}$$

Since the maximum power the pump can provide is 1500 W, the pump must work $\frac{432\,\mathrm{W}}{1500\,\mathrm{W}}=0.29$ or $\boxed{29\%}$ of the time.

26. The coefficient of performance for an ideal refrigerator is given by Eq. 20-4c, with temperatures in Kelvins. Use that expression to find the temperature inside the refrigerator.

$$\mathrm{COP}=\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}-T_{\mathrm{L}}}\;\rightarrow\;T_{\mathrm{L}}=T_{\mathrm{H}}\frac{\mathrm{COP}}{1+\mathrm{COP}}=\left[(32+273)\,\mathrm{K}\right]\frac{5.0}{6.0}=254\ \mathrm{K}=\boxed{-19^\circ\mathrm{C}}$$

27. The efficiency of a perfect Carnot engine is given by Eq. 20-1a and Eq. 20-3. Equate these two expressions to solve for the work required.

$$e=1-\frac{T_L}{T_H}= \; ; \; e=\frac{W}{Q_H} \rightarrow 1-\frac{T_L}{T_H}=\frac{W}{Q_H} \rightarrow W=Q_H\left(1-\frac{T_L}{T_H}\right)$$

(*a*) $W=Q_H\left(1-\frac{T_L}{T_H}\right)=(3100\,\mathrm{J})\left(1-\frac{0+273}{22+273}\right)=231.2\,\mathrm{J}\approx\boxed{230\,\mathrm{J}}$

(*b*) $W=Q_H\left(1-\frac{T_L}{T_H}\right)=(3100\,\mathrm{J})\left(1-\frac{-15+273}{22+273}\right)=388.8\,\mathrm{J}\approx\boxed{390\,\mathrm{J}}$

28. (*a*) Use Eq. 20-2.

$$\mathrm{COP}_{\mathrm{ideal}}=\frac{Q_L}{W}=\frac{Q_L}{Q_H-Q_L}=\frac{\frac{Q_L}{Q_H}}{\frac{Q_H}{Q_H}-\frac{Q_L}{Q_H}}=\frac{\frac{T_L}{T_H}}{1-\frac{T_L}{T_H}}=\frac{\frac{T_L}{T_H}}{\frac{T_H}{T_H}-\frac{T_L}{T_H}}=\boxed{\frac{T_L}{T_H-T_L}}$$

(*b*) Use Eq. 20-3.

$$e=1-\frac{T_L}{T_H} \rightarrow \frac{T_L}{T_H}=1-e \; ; \; \mathrm{COP}_{\mathrm{ideal}}=\frac{\frac{T_L}{T_H}}{\frac{T_H}{T_H}-\frac{T_L}{T_H}}=\frac{1-e}{1-(1-e)}=\boxed{\frac{1-e}{e}}$$

(*c*) $\mathrm{COP}_{\mathrm{ideal}}=\frac{T_L}{T_H-T_L}=\frac{(273-18)\,\mathrm{K}}{42\,\mathrm{K}}=\boxed{6.1}$

29. (*a*) Use the coefficient of performance and the heat that is to be removed (Q_L) to calculate the work done. The heat that is to be removed is the amount of heat released by cooling the water, freezing the water, and cooling the ice. We calculate that heat as a positive quantity.

$$\mathrm{COP}_{\mathrm{ideal}}=\frac{Q_L}{W}=\frac{T_L}{T_H-T_L} \rightarrow$$

$$W=\frac{Q_L(T_H-T_L)}{T_L}=\frac{\left[mc_{H_2O}\Delta T_{\mathrm{liquid}}+mL_{\mathrm{fusion}}+mc_{\mathrm{ice}}\Delta T_{\mathrm{ice}}\right](T_H-T_L)}{T_L}$$

$$=\frac{(0.40\,\mathrm{kg})\left[(4186\,\mathrm{J/kg{\bullet}^\circ C})(25\mathrm{C^\circ})+3.33\times10^5\,\mathrm{J/kg}+(2100\,\mathrm{J/kg{\bullet}^\circ C})(17\mathrm{C^\circ})\right](42\,\mathrm{K})}{(273\,\mathrm{K}-17\,\mathrm{K})}$$

$$=3.106\times10^4\,\mathrm{J}\approx\boxed{3.1\times10^4\,\mathrm{J}}$$

(*b*) Now the compressor power (W/t) is given, and is to be related to the rate of removing heat from the freezer, Q_L/t.

$$\mathrm{COP}_{\mathrm{ideal}}=\frac{Q_L}{W}=\frac{Q_L/t}{W/t}=\frac{Q_L/t}{P}=\frac{T_L}{T_H-T_L} \rightarrow$$

$$t=\frac{Q_L(T_H-T_L)}{PT_L}=\frac{Q_L(T_H-T_L)}{PT_L}=\frac{\left[mc_{H_2O}\Delta T_{\mathrm{liquid}}+mL_{\mathrm{fusion}}\right](T_H-T_L)}{PT_L}$$

$$=\frac{(0.40\,\text{kg})\left[(4186\,\text{J/kg}\bullet{}^\circ\text{C})(25\text{C}^\circ)+3.33\times10^5\,\text{J/kg}\right](42\,\text{K})}{(180\,\text{W})(273\,\text{K}-17\,\text{K})}=159.6\,\text{s}\approx\boxed{2.7\,\text{min}}$$

30. (*a*) The ideal COP is given by Eq. 20-4c.

$$\text{COP}=\frac{T_\text{L}}{T_\text{H}-T_\text{L}}\ \rightarrow\ \text{COP}_\text{eff}=(0.20)\frac{T_\text{L}}{T_\text{H}-T_\text{L}}=(0.20)\frac{(273+24)\,\text{K}}{14\,\text{K}}=4.243\approx\boxed{4.2}$$

(*b*) The compressor power can be found from Eq. 20-4a.

$$\text{COP}=\frac{Q_\text{L}}{W}\ \rightarrow\ W=\frac{Q_\text{L}}{\text{COP}_\text{eff}}\ \rightarrow$$

$$W/t=\frac{Q_\text{L}/t}{\text{COP}_\text{eff}}=\frac{\left(33{,}000\frac{\text{Btu}}{\text{h}}\right)\left(\frac{1055\,\text{J}}{\text{Btu}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)}{4.243}\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)=2.279\,\text{kW}\approx\boxed{2.3\,\text{kW}}$$

(*c*) $2279\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right)=\boxed{3.1\,\text{hp}}$

$\boxed{31.}$ The coefficient of performance is the heat removed from the low-temperature area divided by the work done to remove the heat. In this case, the heat removed is the latent heat released by the freezing ice, and the work done is 1.2 kW times the elapsed time. The mass of water frozen is its density times its volume.

$$\text{COP}=\frac{Q_\text{L}}{W}=\frac{mL_\text{f}}{W}=\frac{\rho VL_\text{f}}{Pt}\ \rightarrow$$

$$V=\frac{(\text{COP})Pt}{\rho L_\text{f}}=\frac{(7.0)(1200\,\text{W})(3600\,\text{s})}{(1.0\times10^3\,\text{kg/m}^3)(3.33\times10^5\,\text{J/kg})}=0.0908\text{m}^3\approx\boxed{91\,\text{L}}$$

32. Heat energy is taken away from the water, so the change in entropy will be negative. The heat transfer is the mass of the steam times the latent heat of vaporization.

$$\Delta S=\frac{Q}{T}=-\frac{mL_\text{vap}}{T}=-\frac{(0.25\,\text{kg})(22.6\times10^5\,\text{J/kg})}{(273+100)\,\text{K}}=\boxed{-1500\,\text{J/K}}$$

33. Energy has been made "unavailable" in the frictional stopping of the sliding box. We take that "lost" kinetic energy as the heat term of the entropy calculation.

$$\Delta S=Q/T=\tfrac{1}{2}mv_i^2/T=\tfrac{1}{2}(7.5\,\text{kg})(4.0\,\text{m/s})^2/293\,\text{K}=\boxed{0.20\,\text{J/K}}$$

Since this is a decrease in "availability," the entropy of the universe has increased.

34. Heat energy is taken away from the water, so the change in entropy will be negative. The heat taken away from the water is found from $\Delta Q=mL_\text{fusion}$. Note that 1.00 m^3 of water has a mass of $1.00\times10^3\,\text{kg}$.

$$\Delta S=\frac{Q}{T}=-\frac{mL_\text{fusion}}{T}=-\frac{(1.00\times10^3\,\text{kg})(3.33\times10^5\,\text{J/kg})}{273\,\text{K}}=\boxed{-1.22\times10^6\,\text{J/K}}$$

35. Because the temperature change is small, we can approximate any entropy integrals by $\Delta S = Q/T_{\text{avg}}$. There are three terms of entropy to consider. First, there is a loss of entropy from the water for the freezing process, ΔS_1. Second, there is a loss of entropy from that newly-formed ice as it cools to -10°C, ΔS_2. That process has an "average" temperature of -5°C. Finally, there is a gain of entropy by the "great deal of ice," ΔS_3, as the heat lost from the original mass of water in steps 1 and 2 goes into that great deal of ice. Since it is a large quantity of ice, we assume that its temperature does not change during the processes.

$$\Delta S_1 = \frac{Q_1}{T_1} = -\frac{mL_{\text{fusion}}}{T_1} = -\frac{\left(1.00\times10^3\,\text{kg}\right)\left(3.33\times10^5\,\text{J/kg}\right)}{273\,\text{K}} = -1.2198\times10^6\,\text{J/K}$$

$$\Delta S_2 = \frac{Q_2}{T_2} = -\frac{mc_{\text{ice}}\Delta T_2}{T_2} = -\frac{\left(1.00\times10^3\,\text{kg}\right)\left(2100\,\text{J/kg}\bullet\text{C}^{\circ}\right)\left(10\text{C}^{\circ}\right)}{\left(-5+273\right)\text{K}} = -7.8358\times10^4\,\text{J/K}$$

$$\Delta S_3 = \frac{Q_3}{T_3} = \frac{-Q_1-Q_2}{T_3} = \frac{mL_{\text{fusion}} + mc_{\text{ice}}\Delta T_2}{T_3}$$

$$= \frac{\left(1.00\times10^3\,\text{kg}\right)\left[\left(3.33\times10^5\,\text{J/kg}\right)+\left(2100\,\text{J/kg}\bullet\text{C}^{\circ}\right)\left(10\text{C}^{\circ}\right)\right]}{\left(-10+273\right)\text{K}} = 1.3460\times10^6\,\text{J/K}$$

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = -1.2198\times10^6\,\text{J/K} - 7.8358\times10^4\,\text{J/K} + 1.3460\times10^6\,\text{J/K}$$

$$= 4.784\times10^4\,\text{J/K} \approx \boxed{5\times10^4\,\text{J/K}}$$

36. (*a*) $\Delta S_{\text{water}} = \dfrac{Q_{\text{water}}}{T_{\text{water}}} = \dfrac{m_{\text{water}}L_{\text{vaporization}}}{T_{\text{water}}} = \dfrac{\left(0.45\,\text{kg}\right)\left(2.26\times10^6\,\text{J/kg}\right)}{373\,\text{K}} = 2727\,\text{J/K} \approx \boxed{2700\,\text{J/K}}$

(*b*) Because the heat to vaporize the water comes from the surroundings, and we assume that the temperature of the surroundings does not change, we have $\Delta S_{\text{surroundings}} = -\Delta S_{\text{water}} = \boxed{-2700\,\text{J/K}}$.

(*c*) The entropy change of the universe for a reversible process is $\Delta S_{\text{universe}} = \boxed{0}$.

(*d*) If the process were irreversible, we would have $\left|\Delta S_{\text{surroundings}}\right| < \Delta S_{\text{water}}$, because the temperature of the surroundings would increase, and so $\boxed{\Delta S_{\text{universe}} > 0}$.

$\boxed{37.}$ The same amount of heat that leaves the high temperature heat source enters the low temperature body of water. The temperatures of the heat source and body of water are constant, so the entropy is calculated without integration.

$$\Delta S = \Delta S_1 + \Delta S_2 = -\frac{Q}{T_{\text{high}}} + \frac{Q}{T_{\text{low}}} = Q\left(\frac{1}{T_{\text{low}}} - \frac{1}{T_{\text{high}}}\right) \rightarrow$$

$$\frac{\Delta S}{t} = \frac{Q}{t}\left(\frac{1}{T_{\text{low}}} - \frac{1}{T_{\text{high}}}\right) = \left(9.50\,\text{cal/s}\right)\left(\frac{4.186\,\text{J}}{1\,\text{cal}}\right)\left(\frac{1}{\left(22+273\right)\text{K}} - \frac{1}{\left(225+273\right)\,\text{K}}\right)$$

$$= \boxed{5.49\times10^{-2}\,\frac{\text{J/K}}{\text{s}}}$$

38. The equilibrium temperature is found using calorimetry, from Chapter 19. The heat lost by the aluminum is equal to the heat gained by the water. We assume the Styrofoam insulates the mixture.

$$m_{\text{Al}}c_{\text{Al}}\left(T_{i\text{Al}}-T_f\right)=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_f-T_{i\text{H}_2\text{O}}\right)\quad\rightarrow$$

$$T_f=\frac{m_{\text{Al}}c_{\text{Al}}T_{i\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}T_{i\text{H}_2\text{O}}}{m_{\text{Al}}c_{\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}}$$

$$=\frac{(2.8\text{ kg})(900\text{ J/kg}\bullet\text{C}^\circ)(43.0^\circ\text{C})+(1.0\text{ kg})(4186\text{ J/kg}\bullet\text{C}^\circ)(20^\circ\text{C})}{(2.8\text{ kg})(900\text{ J/kg}\bullet\text{C}^\circ)+(1.0\text{ kg})(4186\text{ J/kg}\bullet\text{C}^\circ)}=28.64^\circ\text{C}$$

$$\Delta S=\Delta S_{\text{Al}}+\Delta S_{\text{H}_2\text{O}}=\int_{T_{\text{Al}}}^{T_{\text{final}}}\frac{dQ_{\text{Al}}}{T}+\int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}}\frac{dQ_{\text{H}_2\text{O}}}{T}=m_{\text{Al}}c_{\text{Al}}\int_{T_{\text{Al}}}^{T_{\text{final}}}\frac{dT}{T}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}}\frac{dT}{T}$$

$$=m_{\text{Al}}c_{\text{Al}}\ln\frac{T_{\text{final}}}{T_{\text{Al}}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\ln\frac{T_{\text{final}}}{T_{\text{H}_2\text{O}}}$$

$$=(2.8\text{kg})(900\text{ J/kg}\bullet\text{K})\ln\frac{(273.15+28.6)\text{ K}}{(273.15+43.0)\text{ K}}+(1.0\text{kg})(4186\text{ J/kg}\bullet\text{K})\ln\frac{(273.15+28.6)\text{ K}}{(273.15+20.0)\text{ K}}$$

$$=\boxed{4.4\text{ J/K}}$$

39. Because the process happens at a constant temperature, we have $\Delta S=Q/T$. The heat flow can be found from the first law of thermodynamics, the work for expansion at a constant temperature, and the ideal gas equation

$$\Delta E_{\text{int}}=Q-W=0\quad\rightarrow\quad Q=W=nRT\ln\frac{V_2}{V_1}=PV\ln\frac{P_1}{P_2}\quad\rightarrow$$

$$\Delta S=\frac{Q}{T}=\frac{PV}{T}\ln\frac{P_1}{P_2}=\frac{(7.5)(1.013\times10^5\text{Pa})(2.50\times10^{-3}\text{m}^3)}{410\text{K}}\ln\frac{7.5\text{atm}}{1.0\text{atm}}=\boxed{9.3\text{ J/K}}$$

40. (*a*) We find the final temperature of the system using calorimetry, and then approximate each part of the system as having stayed at an average constant temperature. We write both heat terms as positive and then set them equal to each other.

$$Q_{\text{lost}}=Q_{\text{gained}}\quad\rightarrow\quad m_{\text{hot}}c\left(T_{\text{hot}}-T_{\text{final}}\right)=m_{\text{cool}}c\left(T_{\text{final}}-T_{\text{cool}}\right)\quad\rightarrow$$

$$T_{\text{final}}=\frac{T_{\text{hot}}m_{\text{hot}}+T_{\text{cool}}m_{\text{cool}}}{m_{\text{cool}}+m_{\text{hot}}}=\frac{(38.0^\circ\text{C})(3.0\text{kg})+(12.0^\circ\text{C})(2.0\text{kg})}{5.0\text{kg}}=27.6^\circ\text{C}$$

To calculate the entropy we must use the correctly signed heat terms.

$$\Delta S_{\text{cool}}=\frac{Q_{\text{cool}}}{T_{\substack{\text{cool}\\\text{avg}}}}=\frac{m_{\text{cool}}c\left(T_{\text{final}}-T_{\text{cool}}\right)}{\frac{1}{2}\left(T_{\text{cool}}+T_{\text{final}}\right)}=\frac{(2.0\text{kg})(4186\text{ J/kg}\bullet\text{K})(27.6^\circ\text{C}-12.0^\circ\text{C})}{\frac{1}{2}\left[(273\text{K}+12.0\text{K})+(273\text{K}+27.6\text{K})\right]}=446.0\text{ J/kg}$$

$$\Delta S_{\text{hot}}=\frac{Q_{\text{hot}}}{T_{\substack{\text{hot}\\\text{avg}}}}=\frac{m_{\text{hot}}c\left(T_{\text{hot}}-T_{\text{final}}\right)}{\frac{1}{2}\left(T_{\text{hot}}+T_{\text{final}}\right)}=\frac{(3.0\text{kg})(4186\text{ J/kg}\bullet\text{K})(27.6-38.0^\circ\text{C})}{\frac{1}{2}\left[(273\text{K}+38.0\text{K})+(273\text{K}+27.6\text{K})\right]}=-427.1\text{ J/kg}$$

$$\Delta S=\Delta S_{\text{cool}}+\Delta S_{\text{hot}}=446.0\text{ J/kg}-427.1\text{ J/kg}=18.9\text{ J/kg}\approx\boxed{19\text{ J/kg}}$$

(*b*) We use the final temperature as found above, and then calculate the entropy changes by integration.

$$\Delta S_{\text{cool}}=\int\frac{dQ_{\text{cool}}}{T}=\int_{T_{\text{cool}}}^{T_{\text{final}}}\frac{m_{\text{cool}}c\,dT}{T}=m_{\text{cool}}c\ln\frac{T_{\text{final}}}{T_{\text{cool}}}$$

$$=(2.0\,\text{kg})(4186\,\text{J/kg}\bullet\text{K})\ln\left(\frac{273\,\text{K}+27.6\,\text{K}}{273\,\text{K}+12.0\,\text{K}}\right)=446.2\,\text{J/kg}$$

$$\Delta S_{\text{hot}}=\int\frac{dQ_{\text{hot}}}{T}=\int_{T_{\text{hot}}}^{T_{\text{final}}}\frac{m_{\text{hot}}c\,dT}{T}=m_{\text{hot}}c\ln\frac{T_{\text{final}}}{T_{\text{hot}}}$$

$$=(3.0\,\text{kg})(4186\,\text{J/kg}\bullet\text{K})\ln\left(\frac{273\,\text{K}+27.6\,\text{K}}{273\,\text{K}+38.0\,\text{K}}\right)=-427.1\,\text{J/kg}$$

$$\Delta S=\Delta S_{\text{cool}}+\Delta S_{\text{hot}}=446.2\,\text{J/kg}-427.1\,\text{J/kg}=19.1\,\text{J/kg}\approx\boxed{19\,\text{J/kg}}$$

Using the integrals only changed the answer by about 1%.

41. (*a*) The same amount of heat that leaves the room enters the ice and water.

$$\Delta S_{\text{melt}}=\frac{Q_{\text{melt}}}{T_{\text{melt}}}=\frac{mL_{\text{fusion}}}{T_{\text{melt}}}=m\frac{3.33\times10^5\,\text{J/kg}}{273\,\text{K}}=(1219.78m)\,\text{J/K}$$

$$\Delta S_{\text{warming}}=\int\frac{dQ_{\text{warming}}}{T}=\int_{T_{\text{melt}}}^{T_{\text{room}}}\frac{mcdT}{T}=mc\ln\frac{T_{\text{room}}}{T_{\text{melt}}}=m(4186\,\text{J/kg}\bullet\text{K})\ln\frac{293\,\text{K}}{273\,\text{K}}=(295.95m)\,\text{J/K}$$

$$\Delta S_{\text{room}}=\frac{-mL-mc(T_{\text{room}}-T_{\text{melt}})}{T_{\text{room}}}=-m\left[\frac{3.33\times10^5\,\text{J/kg}+(4186\,\text{J/kg}\bullet\text{K})(20\,\text{K})}{293\,\text{K}}\right]$$

$$=(-1422.25m)\,\text{J/K}$$

$$\Delta S_{\text{total}}=\Delta S_{\text{melt}}+\Delta S_{\text{warming}}+\Delta S_{\text{room}}=(1219.78m)\,\text{J/K}+(295.95m)\,\text{J/K}-(1422.25m)\,\text{J/K}$$

$$=93.48m\,\text{J/K}\approx\boxed{(93m)\,\text{J/K}}$$

This process $\boxed{\text{will}}$ occur naturally. Note that we are assuming that *m* is in kg.

(*b*) For this situation, every heat exchange is exactly the opposite as in part (a). Thus we have $\Delta S_{\text{total}}\approx\boxed{(-93m)\,\text{J/K}}$. This $\boxed{\text{will not}}$ occur naturally.

42. Since the process is at a constant volume, $dQ=nC_VdT$. For a diatomic gas in the temperature range of this problem, $C_V=\frac{5}{2}R$.

$$\Delta S=\int\frac{dQ}{T}=\int_{T_1}^{T_2}\frac{nC_VdT}{T}=\tfrac{5}{2}nR\ln\frac{T_2}{T_1}=\tfrac{5}{2}(2.0\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})\ln\frac{(273+55)\,\text{K}}{(273+25)\,\text{K}}=\boxed{4.0\,\text{J/K}}$$

$\boxed{43.}$ (*a*) To approximate, we use the average temperature of the water.

$$\Delta S=\int\frac{dQ}{T}\approx\frac{\Delta Q}{T_{\text{avg}}}=\frac{mc\Delta T_{\text{water}}}{T_{\text{avg}}}=\frac{(1.00\,\text{kg})\left(4186\frac{\text{J}}{\text{kg}\bullet\text{K}}\right)(75\,\text{C}^\circ)}{(273+\frac{1}{2}75)\,\text{K}}=1011\,\text{J/K}\approx\boxed{1010\,\text{J/K}}$$

(*b*) The heat input is given by $Q = mc\Delta T$, so $dQ = mcdT$.

$$\Delta S = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc\ln\frac{T_2}{T_1} = (1.00\,\text{kg})\left(4186\frac{\text{J}}{\text{kg}\bullet\text{K}}\right)\ln\left[\frac{(273+75)\,\text{K}}{(273)\,\text{K}}\right]$$

$$= 1016\,\text{J/K} \approx \boxed{1020\,\text{J/K}}$$

The approximation is only about 1% different.

(*c*) We assume that the temperature of the surroundings is constant at 75°C (the water was moved from a cold environment to a hot environment).

$$\Delta S = \int \frac{dQ}{T} = \frac{\Delta Q}{T} = \frac{-mc\Delta T_{\text{water}}}{T_{\text{surroundings}}} \; \frac{-(1.00\,\text{kg})\left(4186\frac{\text{J}}{\text{kg}\bullet\text{K}}\right)(-75\,\text{C}^\circ)}{(273+75)\,\text{K}} = \boxed{-900\,\text{J/K}}\,(2 \text{ sig. fig.})$$

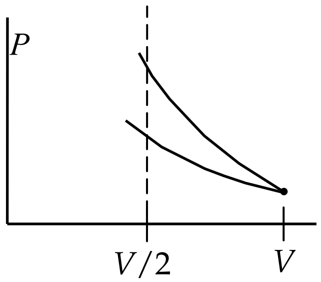
If instead the heating of the water were done reversibly, the entropy of the surroundings would decrease by 1020 J/K. For a general non-reversible case, the entropy of the surroundings would decrease, but by less than 1020 J/K (as in the calculation here).

44. Entropy is a state variable, and so the entropy difference between two states is the same for any path. Since we are told that states a and b have the same temperature, we may find the entropy change by calculating the change in entropy for an isothermal process connecting the same two states. We also use the first law of thermodynamics.

$$\Delta E_{\text{int}} = nC_V\Delta T = 0 = Q - W \;\rightarrow\; Q = W = nRT\ln\left(V_{\text{b}}/V_{\text{a}}\right)$$

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{nRT\ln\left(V_{\text{b}}/V_{\text{a}}\right)}{T} = \boxed{nR\ln\left(V_{\text{b}}/V_{\text{a}}\right)}$$

45. (*a*) The figure shows two processes that start at the same state. The top process is adiabatic, and the bottom process is isothermic. We see from the figure that at a volume of *V*/2, the pressure is greater for the $\boxed{\text{adiabatic}}$ process. We also prove it analytically.



Isothermal: $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2} \;\rightarrow\; P_2 = P_1\dfrac{V_1}{V_2}\dfrac{T_2}{T_1} = P_1\left(\dfrac{V}{\frac{1}{2}V}\right)(1) = 2P_1$

Adiabatic: $P_1V_1^\gamma = P_2V_2^\gamma \;\rightarrow\; P_2 = P_1\left(\dfrac{V_1}{V_2}\right)^\gamma = P_1\left(\dfrac{V}{\frac{1}{2}V}\right)^\gamma = 2^\gamma P_1$

Since $\gamma > 1$, we see that $(P_2)_{\text{adiabatic}} > (P_2)_{\text{isothermic}}$. The ratio is $\dfrac{(P_2)_{\text{adiabatic}}}{(P_2)_{\text{isothermic}}} = \dfrac{2^\gamma P_1}{2P_1} = 2^{\gamma-1}$.

(*b*) For the adiabatic process: No heat is transferred to or from the gas, so $\Delta S_{\text{adiabatic}} = \int \dfrac{dQ}{T} = \boxed{0}$.

For the isothermal process: $\Delta E_{\substack{\text{int}\\ \text{isothermal}}} = 0 \;\rightarrow\; Q_{\text{isothermal}} = W_{\text{isothermal}} = nRT\ln\left(\dfrac{V_2}{V_1}\right)$

$$\Delta S_{\text{isothermal}} = \int \frac{dQ_{\text{isothermal}}}{T} = \frac{1}{T}\int dQ_{\text{isothermal}} = \frac{\Delta Q_{\text{isothermal}}}{T} = \frac{nRT\ln\left(V_2/V_1\right)}{T}$$

$$= nR\ln\left(V_2/V_1\right) = nR\ln\left(\tfrac{1}{2}\right) = \boxed{-nR\ln 2}$$

(*c*) Since each process is reversible, the energy change of the universe is 0, and so $\Delta S_{\text{surroundings}} = -\Delta S_{\text{system}}$. For the adiabatic process, $\Delta S_{\text{surroundings}} = \boxed{0}$. For the isothermal process, $\Delta S_{\text{surroundings}} = \boxed{nR\ln 2}$.

46. (*a*) The equilibrium temperature is found using calorimetry, from Chapter 19. The heat lost by the water is equal to the heat gained by the aluminum.

$$m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{\text{H}_2\text{O}} - T_f\right) = m_{\text{Al}}c_{\text{Al}}\left(T_f - T_{\text{Al}}\right) \rightarrow$$

$$T_f = \frac{m_{\text{Al}}c_{\text{Al}}T_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}T_{\text{H}_2\text{O}}}{m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}}$$

$$= \frac{(0.150\,\text{kg})(900\,\text{J/kg•C°})(15°\text{C}) + (0.215\,\text{kg})(4186\,\text{J/kg•C°})(100°\text{C})}{(0.150\,\text{kg})(900\,\text{J/kg•C°}) + (0.215\,\text{kg})(4186\,\text{J/kg•C°})}$$

$$= 88.91°\text{C} = \boxed{89°\text{C}}$$

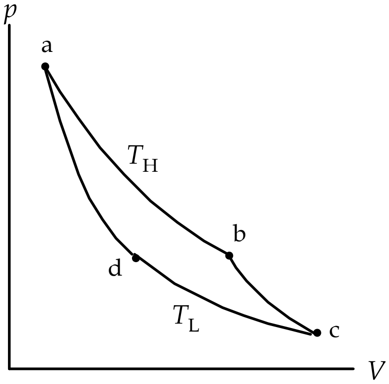
(*b*) $$\Delta S = \Delta S_{\text{Al}} + \Delta S_{\text{H}_2\text{O}} = \int_{T_{\text{Al}}}^{T_{\text{final}}} \frac{dQ_{\text{Al}}}{T} + \int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}} \frac{dQ_{\text{H}_2\text{O}}}{T} = m_{\text{Al}}c_{\text{Al}}\int_{T_{\text{Al}}}^{T_{\text{final}}} \frac{dT}{T} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}} \frac{dT}{T}$$

$$= m_{\text{Al}}c_{\text{Al}}\ln\frac{T_{\text{final}}}{T_{\text{Al}}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\ln\frac{T_{\text{final}}}{T_{\text{H}_2\text{O}}}$$

$$= (0.150\,\text{kg})(900\,\text{J/kg•K})\ln\frac{(273.15+88.91)\,\text{K}}{(273.15+15)\,\text{K}}$$

$$+ (0.215\,\text{kg})(4186\,\text{J/kg•K})\ln\frac{(273.15+88.91)\,\text{K}}{(273.15+100)\,\text{K}} = \boxed{3.7\,\text{J/K}}$$

47. (*a*) Entropy is a state function, which means that its value only depends on the state of the sample under consideration, not on its history of how it arrived at that state. A cyclical process starts and ends at the same state. Since the state is the same, the entropy is the same, and thus the change in entropy for the system is 0. Then, because all of the processes involved are reversible, the entropy change for the universe is 0, and so the entropy change for the surroundings must also be 0.



(*b*) For the two adiabatic processes, *Q* is constant. Thus $dS = \frac{dQ}{T} = 0$ for every infinitesimal part of an adiabatic path, and so $\Delta S_{\text{bc}} = \Delta S_{\text{da}} = 0$. For the two isothermic processes, we have the following, based on the first law of thermodynamics and Eq. 19-8.

$$\Delta E_{\text{int}} = Q - W = 0 \rightarrow Q_{\text{ab}} = W_{\text{ab}} = nRT_{\text{H}}\ln\frac{V_{\text{b}}}{V_{\text{a}}} \rightarrow$$

$$\Delta S_{\text{ab}} = \int_{\text{state a}}^{\text{state b}} \frac{dQ}{T_{\text{H}}} = \frac{1}{T_{\text{H}}}\int_{\text{state a}}^{\text{state b}} dQ = \frac{Q_{\text{ab}}}{T_{\text{H}}} = \frac{nRT_{\text{H}}\ln\frac{V_{\text{b}}}{V_{\text{a}}}}{T_{\text{H}}} = nR\ln\frac{V_{\text{b}}}{V_{\text{a}}} \;;\; \Delta S_{\text{cd}} = nR\ln\frac{V_{\text{d}}}{V_{\text{c}}}$$

$$\Delta S_{\text{cycle}} = \Delta S_{\text{ab}} + \Delta S_{\text{cd}} = nR\ln\frac{V_{\text{b}}}{V_{\text{a}}} + nR\ln\frac{V_{\text{d}}}{V_{\text{c}}} = nR\ln\left(\frac{V_{\text{b}}}{V_{\text{a}}}\frac{V_{\text{d}}}{V_{\text{c}}}\right)$$

From the discussion on page 534, we see that $\frac{V_{\text{b}}}{V_{\text{a}}} = \frac{V_{\text{c}}}{V_{\text{d}}} \rightarrow \frac{V_{\text{b}}}{V_{\text{a}}}\frac{V_{\text{d}}}{V_{\text{c}}} = 1$. Thus

$$\Delta S_{\text{cycle}} = nR\ln\left(\frac{V_{\text{b}}}{V_{\text{a}}}\frac{V_{\text{d}}}{V_{\text{c}}}\right) = nR\ln 1 = 0.$$

48. (*a*) The gases do not interact since they are ideal, and so each gas expands to twice its volume with no change in temperature. Even though the actual process is not reversible, the entropy change can be calculated for a reversible process that has the same initial and final states. This is discussed in Example 20-7.

$$\Delta S_{\text{N}_2} = \Delta S_{\text{Ar}} = nR\ln\frac{V_2}{V_1} = nR\ln 2$$

$$\Delta S_{\text{total}} = \Delta S_{\text{N}_2} + \Delta S_{\text{Ar}} = 2nR\ln 2 = 2(1.00\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})\ln 2 = \boxed{11.5\,\text{J/K}}$$

(*b*) Because the containers are insulated, no heat is transferred to or from the environment. Thus

$$\Delta S_{\text{surroundings}} = \int\frac{dQ}{T} = \boxed{0}.$$

(*c*) Let us assume that the argon container is twice the size of the nitrogen container. Then the final nitrogen volume is 3 times the original volume, and the final argon volume is 1.5 times the original volume.

$$\Delta S_{\text{N}_2} = nR\ln\left(\frac{V_2}{V_1}\right)_{\text{N}_2} = nR\ln 3 \;;\; \Delta S_{\text{Ar}} = nR\ln\left(\frac{V_2}{V_1}\right)_{\text{Ar}} = nR\ln 1.5$$

$$\Delta S_{\text{total}} = \Delta S_{\text{N}_2} + \Delta S_{\text{Ar}} = nR\ln 3 + nR\ln 1.5 = nR\ln 4.5 = (1.00\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})\ln 4.5$$
$$= \boxed{12.5\,\text{J/K}}$$

$\boxed{49.}$ For a system with constant volume, the heat input is given by Eq. 19-10a, $Q = nC_V\Delta T$. At temperature *T*, an infinitesimal amount of heat would result in an infinitesimal temperature change, related by $dQ = nC_V dT$. Use this with the definition of entropy.

$$dS = \frac{dQ}{T} = \frac{nC_V dT}{T} \rightarrow \frac{dT}{dS} = \frac{T}{nC_V}$$

This is exactly the definition of the slope of a process shown on a *T*–*S* graph, so the slope is $\boxed{\frac{T}{nC_V}}$.

A function with this property would be $T = T_0 e^{S/nC_V}$.

50. We assume that the process is reversible, so that the entropy change is given by Eq. 20-8. The heat transfer is given by $dQ = nC_V dT$.

$$S = \int_{T_1}^{T_2}\frac{dQ}{T} = \int_{T_1}^{T_2}\frac{nC_V dT}{T} = \int_{T_1}^{T_2}\frac{n\left(aT + bT^3\right)dT}{T} = \int_{T_1}^{T_2} n\left(a + bT^2\right)dT = n\left(aT + \tfrac{1}{3}bT^3\right)_{T_1}^{T_2}$$

$$= (0.15\,\text{mol})\left[\left(2.08\,\text{mJ/mol}\bullet\text{K}^2\right)(1.0\,\text{K} - 3.0\,\text{K}) + \tfrac{1}{3}\left(2.57\,\text{mJ/mol}\bullet\text{K}^4\right)\left[(1.0\,\text{K})^3 - (3.0\,\text{K})^3\right]\right]$$

$$= \boxed{-4.0\,\text{mJ/K}}$$

51. (*a*) Express the first law of thermodynamics in differential form, as given in Section 19-6.

$$dE_{\text{int}} = dQ - dW$$

For a reversible process, $dQ = TdS$ (Eq. 20-7), and for any process, $dW = PdV$. Also, since $\Delta E_{\text{int}} = nC_V\Delta T$, we have $dE_{\text{int}} = nC_V dT$. Finally, for an ideal gas, $P = \frac{nRT}{V}$.

$$dE_{\text{int}} = dQ - dW \rightarrow nC_V dT = TdS - PdV = TdS - \frac{nRT}{V}dV \rightarrow$$

$$TdS = nC_V dT + \frac{nRT}{V}dV \rightarrow \boxed{dS = nC_V\frac{dT}{T} + nR\frac{dV}{V}}$$

(*b*) Use the ideal gas law, the differentiation product rule, and Eq. 19-11, with the above result.

$$PV = nRT \rightarrow PdV + VdP = nRdT \rightarrow \frac{PdV}{nRT} + \frac{VdP}{nRT} = \frac{nRdT}{nRT} \rightarrow$$

$$\frac{PdV}{PV} + \frac{VdP}{PV} = \frac{dT}{T} \rightarrow \frac{dT}{T} = \frac{dV}{V} + \frac{dP}{P}$$

$$dS = nC_V\frac{dT}{T} + nR\frac{dV}{V} = nC_V\left(\frac{dV}{V} + \frac{dP}{P}\right) + nR\frac{dV}{V} = nC_V\frac{dP}{P} + n\left(C_V + R\right)\frac{dV}{V} \rightarrow$$

$$\boxed{dS = nC_V\frac{dP}{P} + nC_P\frac{dV}{V}}$$

(*c*) Let $dS = 0$ in the above result.

$$dS = nC_V\frac{dP}{P} + nC_P\frac{dV}{V} = 0 \rightarrow nC_V\frac{dP}{P} = -nC_P\frac{dV}{V} \rightarrow \frac{dP}{P} = -\frac{C_P}{C_V}\frac{dV}{V} = -\gamma\frac{dV}{V}$$

$$\int\frac{dP}{P} = -\gamma\int\frac{dV}{V} \rightarrow \ln P = -\gamma\ln V + \text{C} = \ln V^{-\gamma} + \text{C} \rightarrow e^{\ln P} = e^{\ln V^{-\gamma} + C} = e^{\ln V^{-\gamma}}e^{C} \rightarrow$$

$$P = V^{-\gamma}e^{C} \rightarrow \boxed{PV^{\gamma} = \text{constant}} = e^{C}$$

52. (*a*) The kinetic energy the rock loses when it hits the ground becomes a heat flow to the ground. That energy is then unavailable. We assume the temperature of the ground, T_{L}, does not change when the rock hits it.

$$\Delta S = \frac{\Delta Q}{T} = \frac{K}{T_{\text{L}}} \rightarrow E_{\text{lost}} = T_{\text{L}}\Delta S = T_{\text{L}}\left(\frac{K}{T_{\text{L}}}\right) = K$$

(*b*) The work done in a free expansion (which is isothermic if it is insulated) becomes unavailable as the gas expands. From Example 20-7, $\Delta S = nR\ln\frac{V_2}{V_1}$. The work done in an isothermal expansion is given in Eq. 19-8, as $W = nRT\ln\frac{V_2}{V_1}$. Since it is isothermal, $T = T_{\text{L}}$.

$$E_{\text{lost}} = T_{\text{L}}\Delta S = T_{\text{L}}nR\ln\left(V_2/V_1\right) = W$$

(*c*) Assume an amount of heat Q_H is transferred from a high temperature reservoir T_H to a lower temperature reservoir T_L. The entropy change during that process is $\Delta S = \frac{Q_H}{T_L} - \frac{Q_H}{T_H}$. The energy "lost" is $T_L \Delta S = T_L\left(\frac{Q_H}{T_L} - \frac{Q_H}{T_H}\right) = Q_H\left(1 - \frac{T_L}{T_H}\right) = Q_H e_{\text{Carnot}} = W$. The work done during the process is no longer available to do any other work, and so has become unavailable to do work in some other process following the first process.

53. The total energy stored in the copper block is found from the heat flow that initially raised its temperature above the temperature of the surroundings.

$$Q = mc\Delta T = (3.5\,\text{kg})(390\,\text{J/kg}\bullet\text{K})(200\,\text{K}) = 2.73\times10^5\,\text{J}$$

We find the entropy change assuming that amount of energy leaves the copper in a reversible process, and that amount of energy enters the surroundings. The temperature of the surroundings is assumed to be constant.

$$\Delta S_{\text{Cu}} = \int \frac{dQ}{T} = \int_{490\,\text{K}}^{290\,\text{K}} \frac{mcdT}{T} = mc\ln\left(\frac{290\,\text{K}}{490\,\text{K}}\right) = (3.5\,\text{kg})(390\,\text{J/kg}\bullet\text{K})\ln\left(\frac{290\,\text{K}}{490\,\text{K}}\right) = -716\,\text{J/K}$$

$$\Delta S_{\text{surroundings}} = \frac{Q}{T_{\text{surroundings}}} = \frac{mc\Delta T}{T_{\text{surroundings}}} = \frac{2.73\times10^5\,\text{J}}{290\,\text{K}} = 941\,\text{J/K} \;;\; \Delta S = (941 - 716)\,\text{J/K} = 225\,\text{J/K}$$

$$E_{\text{lost}} = T_L \Delta S = (290\,\text{K})(225\,\text{J/K}) = 6.5\times10^4\,\text{J}$$

$$W_{\text{available}} = Q - E_{\text{lost}} = 2.73\times10^5\,\text{J} - 6.5\times10^4\,\text{J} = \boxed{2.1\times10^5\,\text{J}}$$

54. For four heads: $W = 1 \;\rightarrow\; \Delta S = k\ln W = (1.38\times10^{-23}\,\text{J/K})\ln 1 = \boxed{0}$

For 3 heads, 1 tail: $W = 4 \;\rightarrow\; \Delta S = k\ln W = (1.38\times10^{-23}\,\text{J/K})\ln 4 = \boxed{1.91\times10^{-23}\,\text{J/K}}$

For 2 heads, 2 tails: $W = 6 \;\rightarrow\; \Delta S = k\ln W = (1.38\times10^{-23}\,\text{J/K})\ln 6 = \boxed{2.47\times10^{-23}\,\text{J/K}}$

For 1 head, 3 tails: $W = 4 \;\rightarrow\; \Delta S = k\ln W = (1.38\times10^{-23}\,\text{J/K})\ln 4 = \boxed{1.91\times10^{-23}\,\text{J/K}}$

For four tails: $W = 1 \;\rightarrow\; \Delta S = k\ln W = (1.38\times10^{-23}\,\text{J/K})\ln 1 = \boxed{0}$

$\boxed{55.}$ From the table below, we see that there are a total of $2^6 = 64$ microstates.

Macrostate	Possible Microstates (H = heads, T = tails)						Number of Microstates
6 heads, 0 tails	H H H H H H						1
5 heads, 1 tails	H H H H H T	H H H H T H	H H H T H H	H H T H H H	H T H H H H	T H H H H H	6
4 heads, 2 tails	H H H H T T	H H H T H T	H H T H H T	H T H H H T	T H H H H T		15
	H H H T T H	H H T H T H	H T H H T H	T H H H T H	H H T T H H		
	H T H T H H	T H H T H H	H T T H H H	T H T H H H	T T H H H H		
3 heads, 3 tails	H H H T T T	H H T H T T	H T H H T T	T H H H T T	H H T T H T		20
	H T H T H T	T H H T H T	H T T H H T	T H T H H T	T T H H H T		
	T T T H H H	T T H T H H	T H T T H H	H T T T H H	T T H H T H		
	T H T H T H	H T T H T H	T H H T T H	H T H T T H	H H T T T H		
2 heads, 4 tails	T T T T H H	T T T H T H	T T H T T H	T H T T T H	H T T T T H		15
	T T T H H T	T T H T H T	T H T T H T	H T T T H T	T T H H T T		
	T H T H T T	H T T H T T	T H H T T T	H T H T T T	H H T T T T		
1 heads, 5 tails	T T T T T H	T T T T H T	T T T H T T	T T H T T T	T H T T T T	H T T T T T	6
0 heads, 6 tails	T T T T T T						1

(*a*) The probability of obtaining three heads and three tails is $\boxed{20/64}$ or $\boxed{5/16}$.

(*b*) The probability of obtaining six heads is $\boxed{1/64}$.

56. When throwing two dice, there are 36 possible microstates.

(*a*) The possible microstates that give a total of 7 are: (1)(6) , (2)(5) , (3)(4), (4)(3), (5)(2), and (6)(1). Thus the probability of getting a 7 is $6/36=\boxed{1/6}$.

(*b*) The possible microstates that give a total of 11 are: (5)(6) and (6)(5). Thus the probability of getting an 11 is $2/36=\boxed{1/18}$.

(*c*) The possible microstates that give a total of 4 are: (1)(3) , (2)(2) , and (3)(1). Thus the probability of getting a 5 is $3/36=\boxed{1/12}$.

57. (*a*) There is only one microstate for 4 tails: TTTT. There are 6 microstates with 2 heads and 2 tails: HHTT, HTHT, HTTH, THHT, THTH, and TTHH. Use Eq. 20-14 to calculate the entropy change.

$$\Delta S=k\ln W_2-k\ln W_1=k\ln\frac{W_2}{W_1}=\left(1.38\times10^{-23}\,\text{J/K}\right)\ln 6=\boxed{2.47\times10^{-23}\,\text{J/K}}$$

(*b*) Apply Eq. 20-14 again. There is only 1 final microstate, and about 1.0×10^{29} initial microstates.

$$\Delta S=k\ln W_2-k\ln W_1=k\ln\frac{W_2}{W_1}=\left(1.38\times10^{-23}\,\text{J/K}\right)\ln\left(\frac{1}{1.0\times10^{29}}\right)=\boxed{-9.2\times10^{-22}\,\text{J/K}}$$

(*c*) These changes are much smaller than those for ordinary thermodynamic entropy changes. For ordinary processes, there are many orders of magnitude more particles than we have considered in this problem. That leads to many more microstates and larger entropy values.

58. The number of microstates for macrostate A is $W_{\text{A}}=\frac{10!}{10!0!}=1$. The number of microstates for macrostate B is $W_{\text{B}}=\frac{10!}{5!5!}=252$.

(*a*) $\Delta S=k\ln W_B-k\ln W_A=k\ln\frac{W_B}{W_A}=\left(1.38\times10^{-23}\,\text{J/K}\right)\ln 252=\boxed{7.63\times10^{-23}\,\text{J/K}}$

Since $\Delta S>0$, this $\boxed{\text{can occur naturally}}$.

(*b*) $\Delta S=k\ln W_A-k\ln W_B=-k\ln\frac{W_B}{W_A}=-\left(1.38\times10^{-23}\,\text{J/K}\right)\ln 252=\boxed{-7.63\times10^{-23}\,\text{J/K}}$

Since $\Delta S<0$, this $\boxed{\text{cannot occur naturally}}$.

59. (*a*) Assume that there are no dissipative forces present, and so the energy required to pump the water to the lake is just the gravitational potential energy of the water.

$$U_{\text{grav}}=mgh=\left(1.35\times10^{5}\,\text{kg/s}\right)\left(10.0\,\text{h}\right)\left(9.80\,\text{m/s}^2\right)\left(135\,\text{m}\right)=1.786\times10^{9}\,\text{W}\bullet\text{h}$$

$$\approx\boxed{1.79\times10^{6}\,\text{kWh}}$$

(*b*) $$\frac{\left(1.786\times10^{6}\,\text{kW}\bullet\text{h}\right)\left(0.75\right)}{14\,\text{h}}=\boxed{9.6\times10^{4}\,\text{kW}}$$

60. The required area is $\left(22\frac{10^3\,\text{W}\bullet\text{h}}{\text{day}}\right)\left(\frac{1\,\text{day}}{9\,\text{h}\,\text{Sun}}\right)\left(\frac{1\,\text{m}^2}{40\,\text{W}}\right)=61\,\text{m}^2\approx\boxed{60\,\text{m}^2}$. A small house with 1000 ft^2 of floor space, and a roof tilted at 30°, would have a roof area of $\left(1000\,\text{ft}^2\right)\left(\frac{1}{\cos 30^\circ}\right)\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right)^2=$ $110\,\text{m}^2$, which is about twice the area needed, and so $\boxed{\text{the cells would fit on the house}}$. But not all parts of the roof would have 9 hours of sunlight, so more than the minimum number of cells would be needed.

$\boxed{61.}$ We assume that the electrical energy comes from the 100% effective conversion of the gravitational potential energy of the water.

$$W = mgh \quad\rightarrow$$

$$P=\frac{W}{t}=\frac{m}{t}gh=\rho\frac{V}{t}gh=\left(1.00\times10^3\,\text{kg/m}^3\right)\left(32\,\text{m}^3/\text{s}\right)\left(9.80\,\text{m/s}^2\right)\left(38\,\text{m}\right)=\boxed{1.2\times10^7\,\text{W}}$$

62. (*a*) Calculate the Carnot efficiency for an engine operated between the given temperatures.

$$e_{\text{ideal}}=1-\frac{T_{\text{L}}}{T_{\text{H}}}=1-\frac{(273+4)\,\text{K}}{(273+27)\,\text{K}}=0.077=\boxed{7.7\%}$$

(*b*) Such an engine might be feasible in spite of the low efficiency because of the large volume of "fuel" (ocean water) available. Ocean water would appear to be an "inexhaustible" source of heat energy. And the oceans are quite accessible.

(*c*) The pumping of water between radically different depths would probably move smaller sea-dwelling creatures from their natural location, perhaps killing them in the transport process. Mixing the water at different temperatures will also disturb the environment of sea-dwelling creatures. There is a significant dynamic of energy exchange between the ocean and the atmosphere, and so any changing of surface temperature water might affect at least the local climate, and perhaps also cause larger-scale climate changes.

63. The gas is diatomic, and so $\gamma = 1.4$ and $C_V=\frac{5}{2}R.$

(*a*) Find the number of moles by applying the ideal gas law to state a.

$$P_{\text{a}}V_{\text{a}}=nRT_{\text{a}}\quad\rightarrow\quad n=\frac{P_{\text{a}}V_{\text{a}}}{RT_{\text{a}}}=\frac{\left(1.013\times10^5\,\text{Pa}\right)\left(0.010\,\text{m}^3\right)}{\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(300\,\text{K}\right)}=0.406\,\text{mol}\approx\boxed{0.41\,\text{mol}}$$

(*b*) Find T_{c} using the adiabatic relationship with the ideal gas law.

$$P_{\text{c}}V_{\text{c}}^{\gamma}=P_{\text{a}}V_{\text{a}}^{\gamma}\quad\rightarrow\quad\frac{nRT_{\text{c}}}{V_{\text{c}}}V_{\text{c}}^{\gamma}=\frac{nRT_{\text{a}}}{V_{\text{a}}}V_{\text{a}}^{\gamma}\quad\rightarrow\quad T_{\text{c}}V_{\text{c}}^{\gamma-1}=T_{\text{a}}V_{\text{a}}^{\gamma-1}\quad\rightarrow$$

$$T_{\text{c}}=T_{\text{a}}\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{\gamma-1}=\left(300\,\text{K}\right)\left(2\right)^{0.4}=396\,\text{K}\approx\boxed{400\,\text{K}}\left(2\text{ sig. fig.}\right)$$

(*c*) This is a constant volume process.

$$Q_{\text{bc}}=nC_V\left(T_{\text{c}}-T_{\text{b}}\right)=\tfrac{5}{2}nR\left(T_{\text{c}}-T_{\text{b}}\right)=\tfrac{5}{2}\left(0.406\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(96\,\text{K}\right)$$
$$=810.1\,\text{J}\approx\boxed{810\,\text{J}}$$

(*d*) The work done by an isothermal process is given by Eq. 19-8.

$$W_{\text{ab}}=nRT\ln\frac{V_{\text{b}}}{V_{\text{a}}}=\left(0.406\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(300\,\text{K}\right)\ln\left(\frac{0.0050}{0.010}\right)=-702\,\text{J}\approx\boxed{-700\,\text{J}}$$

(*e*) Use the first law of thermodynamics, and the fact that $Q_{\text{ca}} = 0$.

$$\Delta E_{\underset{\text{ca}}{\text{int}}} = Q_{\text{ca}} - W_{\text{ca}} = -W_{\text{ca}} \quad \rightarrow$$

$$W_{\text{ca}} = -\Delta E_{\underset{\text{ca}}{\text{int}}} = -nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\left(T_{\text{c}} - T_{\text{a}}\right) = \tfrac{5}{2}\left(0.406\,\text{mol}\right)\left(8.314\,\text{J/mol}\bullet\text{K}\right)\left(396\,\text{K} - 300\,\text{K}\right)$$

$$= 810.1\,\text{J} \approx \boxed{810\,\text{J}}$$

(*f*) Heat is input to the gas only along path bc.

$$e = \frac{W}{Q_{\text{in}}} = \frac{W_{\text{ca}} + W_{\text{ab}}}{Q_{\text{bc}}} = \frac{810.1\,\text{J} - 702\,\text{J}}{810.1\,\text{J}} = \boxed{0.13}$$

(*g*) $$e_{\text{Carnot}} = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = \frac{396\,\text{K} - 300\,\text{K}}{396\,\text{K}} = \boxed{0.24}$$

64. (*a*) The equilibrium temperature is found using calorimetry, from Chapter 19. The heat lost by the water is equal to the heat gained by the aluminum.

$$m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\left(T_{\text{H}_2\text{O}} - T_f\right) = m_{\text{Al}}c_{\text{Al}}\left(T_f - T_{\text{Al}}\right) \quad \rightarrow$$

$$T_f = \frac{m_{\text{Al}}c_{\text{Al}}T_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}T_{\text{H}_2\text{O}}}{m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}}$$

$$= \frac{\left(0.1265\,\text{kg}\right)\left(900\,\text{J/kg}\bullet\text{C}^\circ\right)\left(18.00^\circ\text{C}\right) + \left(0.1325\,\text{kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(46.25^\circ\text{C}\right)}{\left(0.1265\,\text{kg}\right)\left(900\,\text{J/kg}\bullet\text{C}^\circ\right) + \left(0.1325\,\text{kg}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)} = \boxed{41.44^\circ\text{C}}$$

(*b*) $$\Delta S = \Delta S_{\text{Al}} + \Delta S_{\text{H}_2\text{O}} = \int_{T_{\text{Al}}}^{T_{\text{final}}} \frac{dQ_{\text{Al}}}{T} + \int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}} \frac{dQ_{\text{H}_2\text{O}}}{T} = m_{\text{Al}}c_{\text{Al}}\int_{T_{\text{Al}}}^{T_{\text{final}}} \frac{dT}{T} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\int_{T_{\text{H}_2\text{O}}}^{T_{\text{final}}} \frac{dT}{T}$$

$$= m_{\text{Al}}c_{\text{Al}}\ln\frac{T_{\text{final}}}{T_{\text{Al}}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}\ln\frac{T_{\text{final}}}{T_{\text{H}_2\text{O}}}$$

$$= \left(0.1265\,\text{kg}\right)\left(900\,\text{J/kg}\bullet\text{K}\right)\ln\frac{\left(273.15 + 41.44\right)\text{K}}{\left(273.15 + 18.00\right)\text{K}}$$

$$+ \left(0.1325\,\text{kg}\right)\left(4186\,\text{J/kg}\bullet\text{K}\right)\ln\frac{\left(273.15 + 41.44\right)\text{K}}{\left(273.15 + 46.25\right)\text{K}} = \boxed{0.399\,\text{J/K}}$$

65. (*a*) For each engine, the efficiency is given by $e = 0.65e_{\text{Carnot}}$. Thus

$$e_1 = 0.65e_{C-1} = 0.65\left(1 - \frac{T_{\text{L1}}}{T_{\text{H1}}}\right) = 0.65\left[1 - \frac{\left(430 + 273\right)\text{K}}{\left(710 + 273\right)\text{K}}\right] = 0.185$$

$$e_2 = 0.65e_{C-2} = 0.65\left(1 - \frac{T_{\text{L2}}}{T_{\text{H2}}}\right) = 0.65\left[1 - \frac{\left(270 + 273\right)\text{K}}{\left(415 + 273\right)\text{K}}\right] = 0.137$$

For the first engine, the input heat is from the coal.

$$W_1 = e_1Q_{\text{H1}} = e_1Q_{\text{coal}} \text{ and } Q_{\text{L1}} = Q_{\text{H1}} - W_1 = \left(1 - e_1\right)Q_{\text{coal}}.$$

For the second energy, the input heat is the output heat from the first engine.

$$W_2 = e_2Q_{\text{H2}} = e_2Q_{\text{L1}} = e_2\left(1 - e_1\right)Q_{\text{coal}}$$

Add the two work expressions together, and solve for Q_{coal}.

$$W_1 + W_2 = e_1 Q_{\text{coal}} + e_2\left(1 - e_1\right) Q_{\text{coal}} = \left(e_1 + e_2 - e_1 e_2\right) Q_{\text{coal}}$$

$$Q_{\text{coal}} = \frac{W_1 + W_2}{e_1 + e_2 - e_1 e_2} \quad \rightarrow \quad Q_{\text{coal}}/t = \frac{\left(W_1 + W_2\right)/t}{e_1 + e_2 - e_1 e_2}$$

Calculate the rate of coal use from the required rate of input energy, Q_{coal}/t.

$$Q_{\text{coal}}/t = \frac{950\times10^6\,\text{W}}{0.185 + 0.137 - \left(0.185\right)\left(0.137\right)} = 3.202\times10^9\ \text{J/s}$$

$$\left(3.202\times10^9\ \text{J/s}\right)\left(\frac{1\ \text{kg}}{2.8\times10^7\,\text{J}}\right) = 114.4\ \text{kg/s} \approx \boxed{110\,\text{kg/s}}$$

(*b*) The heat exhausted into the water will make the water temperature rise according to Eq. 19-2. The heat exhausted into water is the heat from the coal, minus the useful work.

$$Q_{\text{exhaust}} = Q_{\text{coal}} - W\ ;\ Q_{\text{exhaust}} = m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} \Delta T_{\text{H}_2\text{O}} \quad \rightarrow \quad m_{\text{H}_2\text{O}} = \frac{Q_{\text{exhaust}}}{c_{\text{H}_2\text{O}} \Delta T_{\text{H}_2\text{O}}} = \frac{Q_{\text{coal}} - W}{c_{\text{H}_2\text{O}} \Delta T_{\text{H}_2\text{O}}} \quad \rightarrow$$

$$\frac{m_{\text{H}_2\text{O}}}{t} = \frac{\left(Q_{\text{coal}}/t\right) - \left(W/t\right)}{c_{\text{H}_2\text{O}} \Delta T_{\text{H}_2\text{O}}} = \frac{\left(3.202\times10^9\ \text{J/s}\right) - \left(9.50\times10^8\right)\text{J/s}}{\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(5.5\text{C}^\circ\right)} = 9.782\times10^4\ \text{kg/s}$$

$$= \left(9.782\times10^4\,\frac{\text{kg}}{\text{s}}\right)\left(3600\frac{\text{s}}{\text{h}}\right)\left(\frac{1\,\text{m}^3}{1000\,\text{kg}}\right)\left(\frac{1\,\text{L}}{10^{-3}\text{m}^3}\right)\left(\frac{1\,\text{gal}}{3.785\,\text{L}}\right) = \boxed{9.3\times10^7\ \text{gal/h}}$$

66. We start with Eq. 20-4a for the COP of a refrigerator. The heat involved is the latent heat of fusion for water.

$$\text{COP} = \frac{Q_{\text{L}}}{W} \quad \rightarrow \quad W = \frac{Q_{\text{L}}}{\text{COP}} \quad \rightarrow$$

$$W/t = \frac{Q_{\text{L}}/t}{\text{COP}} = \frac{5\,\text{tons}}{0.15\,\text{COP}_{\text{ideal}}} = \frac{5\left(909\,\text{kg/d}\right)\left(3.33\times10^5\ \text{J/kg}\right)}{0.15\left(\dfrac{273\,\text{K}{+}22\,\text{K}}{13\,\text{K}}\right)} = 4.446\times10^8\ \text{J/d}$$

$$\text{cost/h} = \left(4.446\times10^8\ \text{J/d}\right)\left(\frac{1\,\text{d}}{24\,\text{h}}\right)\left(\frac{1\,\text{kWh}}{3.600\times10^6\,\text{J}}\right)\left(\frac{\$0.10}{\text{kWh}}\right) = \boxed{\$0.51/\text{h}}$$

$\boxed{67.}$ (*a*) The exhaust heating rate is found from the delivered power and the efficiency. Use the output energy with the relationship $Q = mc\Delta T = \rho Vc\Delta T$ to calculate the volume of air that is heated.

$$e = W/Q_{\text{H}} = W/\left(Q_{\text{L}} + W\right) \quad \rightarrow \quad Q_{\text{L}} = W\left(1/e - 1\right) \quad \rightarrow$$

$$Q_{\text{L}}/t = W/t\left(1/e - 1\right) = \left(9.2\times10^8\,\text{W}\right)\left(1/0.35 - 1\right) = 1.709\times10^9\,\text{W}$$

$$Q_{\text{L}} = mc\Delta T \quad \rightarrow \quad Q_{\text{L}}/t = \frac{mc\Delta T}{t} = \frac{\rho Vc\Delta T}{t} \quad \rightarrow \quad V/t = \frac{\left(Q_{\text{L}}/t\right)}{\rho c\Delta T}$$

The change in air temperature is $7.0\,\text{C}^\circ$. The heated air is at a constant pressure of 1 atm.

$$V/t = \frac{\left(Q_{\text{L}}/t\right)t}{\rho c\Delta T} = \frac{\left(1.709\times10^9\,\text{W}\right)\left(8.64\times10^4\ \text{s/day}\right)}{\left(1.2\,\text{kg/m}^3\right)\left(1.0\times10^3\ \text{J/kg}\bullet\text{C}^\circ\right)\left(7.0\text{C}^\circ\right)}$$

$$=1.757\times10^{10}\ \mathrm{m^3/day}\left(\frac{10^{-9}\mathrm{km^3}}{1\ \mathrm{m^3}}\right)=17.57\,\mathrm{km^3/day}\approx\boxed{18\,\mathrm{km^3/day}}$$

(*b*) If the air is 200 m thick, find the area by dividing the volume by the thickness.

$$A=\frac{\text{Volume}}{\text{thickness}}=\frac{17.57\,\mathrm{km^3}}{0.15\ \mathrm{km}}=117\,\mathrm{km^2}\approx\boxed{120\,\mathrm{km^2}}$$

This would be a square of approximately 6 miles to a side. Thus the local climate for a few miles around the power plant might be heated significantly.

68. The COP for an ideal heat pump is found from Eq. 20-5.

(*a*) $$\mathrm{COP}=\frac{Q_\mathrm{H}}{W}=\frac{Q_\mathrm{H}}{Q_\mathrm{H}-Q_\mathrm{L}}=\frac{T_\mathrm{H}}{T_\mathrm{H}-T_\mathrm{L}}=\frac{(24+273)\,\mathrm{K}}{(24-11)\,\mathrm{K}}=22.85\approx\boxed{23}$$

(*b*) From Figure 20-11, The heat delivered from a heat pump is Q_H.

$$\mathrm{COP}=\frac{Q_\mathrm{H}}{W}\ \rightarrow$$

$$Q_\mathrm{H}=(W/t)(t)(\mathrm{COP})=(1400\,\mathrm{W})(3600\,\mathrm{s})(22.85)=1.152\times10^8\,\mathrm{J}\approx\boxed{1.2\times10^8\,\mathrm{J}}$$

69. All of the processes are either constant pressure or constant volume, and so the heat input and output can be calculated with specific heats at constant pressure or constant volume. This tells us that heat is input when the temperature increases, and heat is exhausted when the temperature decreases. The lowest temperature will be the temperature at point b. We use the ideal gas law to find the temperatures.

$$PV=nRT\ \rightarrow\ T=\frac{PV}{nR}\ \rightarrow$$

$$T_\mathrm{b}=\frac{P_0V_0}{nR},\ T_\mathrm{a}=\frac{P_0(2V_0)}{nR}=2T_\mathrm{b},\ T_\mathrm{c}=\frac{(3P_0)V_0}{nR}=3T_\mathrm{b},\ T_\mathrm{d}=\frac{(3P_0)(2V_0)}{nR}=6T_\mathrm{b}$$

(*a*) process ab: $W_\mathrm{ab}=P\Delta V=P_0(-V_0)=-P_0V_0\ ;\ Q_\mathrm{ab}<0$

process bc: $W_\mathrm{bc}=P\Delta V=0\ ;\ Q_\mathrm{bc}=nC_V\Delta T=\frac{3}{2}nR(T_\mathrm{c}-T_\mathrm{b})=\frac{3}{2}nR(2T_\mathrm{b})=\frac{3}{2}nR\left(2\frac{P_0V_0}{nR}\right)=3P_0V_0$

process cd: $W_\mathrm{bc}=P\Delta V=3P_0V_0\ ;$

$$Q_\mathrm{cd}=nC_P\Delta T=\tfrac{5}{2}nR(T_\mathrm{d}-T_\mathrm{c})=\tfrac{5}{2}nR(3T_\mathrm{b})=\tfrac{5}{2}nR\left(3\frac{P_0V_0}{nR}\right)=\tfrac{15}{2}P_0V_0$$

process da: $W_\mathrm{da}=P\Delta V=0\ ;\ Q_\mathrm{da}<0$

$$e_\text{rectangle}=\frac{W}{Q_\mathrm{H}}=\frac{3P_0V_0-P_0V_0}{3P_0V_0+\frac{15}{2}P_0V_0}=\frac{2}{\frac{21}{2}}=0.1905\approx\boxed{0.19}$$

(*b*) $$e_\text{Carnot}=\frac{T_\mathrm{H}-T_\mathrm{L}}{T_\mathrm{H}}=\frac{6T_\mathrm{b}-T_\mathrm{b}}{6T_\mathrm{b}}=0.8333\ ;\ \frac{e_\text{rectangle}}{e_\text{Carnot}}=\frac{0.1905}{0.8333}=\boxed{0.23}$$

70. (*a*) Calculate the Carnot efficiency by $e=1-T_\mathrm{L}/T_\mathrm{H}$ and compare it to the 15% actual efficiency.

$$e_\text{Carnot}=1-T_\mathrm{L}/T_\mathrm{H}=1-(95+273)\,\mathrm{K}/(495+273)\,\mathrm{K}=0.521=52.1\%$$

Thus the engine's relative efficiency is $e_\text{actual}/e_\text{Carnot}=0.15/0.521=0.288\approx\boxed{29\%}$

(*b*) Take the stated 155 hp as the useful power obtained from the engine. Use the efficiency to calculate the exhaust heat.

$$P=\frac{W}{t}=\left(155\text{ hp}\right)\left(\frac{746\,\text{W}}{1\text{ hp}}\right)=1.156\times10^{5}\,\text{W}\approx\boxed{1.16\times10^{5}\,\text{W}}\text{ (for moving the car)}$$

$$e=\frac{W}{Q_{\text{H}}}=\frac{W}{Q_L+W}\quad\rightarrow$$

$$Q_L=W\left(\frac{1}{e}-1\right)=Pt\left(\frac{1}{e}-1\right)=\left(1.156\times10^{5}\,\text{J/s}\right)\left(1\text{ h}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1}{0.15}-1\right)$$

$$=2.36\times10^{9}\,\text{J}\approx\boxed{2.4\times10^{9}\,\text{J}}=\left(2.36\times10^{9}\,\text{J}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)=\boxed{5.6\times10^{5}\text{kcal}}$$

71. (*a*) The exhaust heating rate can be found from the delivered power *P* and the Carnot efficiency. Then use the relationship between energy and temperature change, $Q=mc\Delta T$, to calculate the temperature change of the cooling water.

$$e=1-\frac{T_{\text{L}}}{T_{\text{H}}}=\frac{W}{Q_{\text{H}}}=\frac{W}{Q_{\text{L}}+W}\quad\rightarrow\quad Q_{\text{L}}=W\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}\quad\rightarrow\quad Q_{\text{L}}/t=W/t\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}=P\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}$$

$$Q_{\text{L}}=mc\Delta T\quad\rightarrow\quad Q_{\text{L}}/t=\frac{m}{t}c\Delta T=\rho\frac{V}{t}c\Delta T$$

Equate the two expressions for Q_{L}/t, and solve for ΔT.

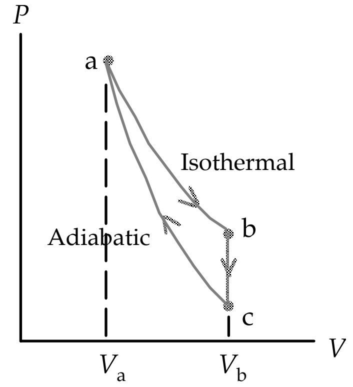
$$P\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}=\rho\frac{V}{t}c\Delta T\quad\rightarrow\quad\Delta T=\frac{P}{\rho\frac{V}{t}c}\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}$$

$$=\frac{8.5\times10^{8}\,\text{W}}{\left(1.0\times10^{3}\,\text{kg/m}^{3}\right)\left(34\,\text{m}^{3}/\text{s}\right)\left(4186\,\text{J/kg}\bullet\text{C}^{\circ}\right)}\frac{285\text{ K}}{\left(625\text{ K}-285\text{ K}\right)}=5.006\,\text{K}=\boxed{5.0\,\text{C}^{\circ}}$$

(*b*) The addition of heat per kilogram for the downstream water is $Q_{\text{L}}/t=c\Delta T$.

$$\frac{\Delta S}{m}=\int\frac{dS}{m}=\int\frac{dQ}{mT}=\int\frac{cdT}{T}=c\int_{285\,\text{K}}^{290\,\text{K}}\frac{dT}{T}=\left(4186\,\text{J/kg}\bullet\text{C}^{\circ}\right)\left(\ln\frac{290\,\text{K}}{285\,\text{K}}\right)=\boxed{72.8\,\text{J/kg}\bullet\text{K}}$$

72. We have a monatomic gas, so $\gamma=\frac{5}{3}$. Also the pressure, volume, and temperature for state a are known. We use the ideal gas law, the adiabatic relationship, and the first law of thermodynamics.



(*a*) Use the ideal gas equation to relate states a and b. Use the adiabatic relationship to relate states a and c.

$$\frac{P_{\text{b}}V_{\text{b}}}{T_{\text{b}}}=\frac{P_{\text{a}}V_{\text{a}}}{T_{\text{a}}}\quad\rightarrow$$

$$P_{\text{b}}=P_{\text{a}}\frac{V_{\text{a}}}{V_{\text{b}}}\frac{T_{\text{b}}}{T_{\text{a}}}=\left(1.00\,\text{atm}\right)\left(\frac{22.4\,\text{L}}{56.0\,\text{L}}\right)\left(\frac{273\,\text{K}}{273\,\text{K}}\right)=\boxed{0.400\,\text{atm}}$$

$$P_{\text{a}}V_{\text{a}}^{\gamma}=P_{\text{c}}V_{\text{c}}^{\gamma}\quad\rightarrow$$

$$P_{\text{c}}=P_{\text{a}}\left(\frac{V_{\text{a}}}{V_{\text{c}}}\right)^{\gamma}=\left(1.00\,\text{atm}\right)\left(\frac{22.4\,\text{L}}{56.0\,\text{L}}\right)^{5/3}=0.2172\,\text{atm}\approx\boxed{0.217\,\text{atm}}$$

(*b*) Use the ideal gas equation to calculate the temperature at c.

$$\frac{P_bV_b}{T_b}=\frac{P_cV_c}{T_c}= \;\rightarrow\; T_c=T_b\frac{P_c}{P_b}\frac{V_c}{V_b}=(273\,\mathrm{K})\left(\frac{0.2172\,\mathrm{atm}}{0.400\,\mathrm{atm}}\right)(1)=\boxed{148\,\mathrm{K}}$$

(*c*) Process ab: $\Delta \underset{\mathrm{ab}}{E_{\mathrm{int}}}=nC_V\Delta T=\boxed{0}$;

$$Q_{\mathrm{ab}}=W_{\mathrm{ab}}=nRT\ln\frac{V_b}{V_a}=(1.00\,\mathrm{mol})(8.314\,\mathrm{J/mol{\bullet}K})(273\,\mathrm{K})\ln 2.5$$

$$=2079.7\,\mathrm{J}\approx\boxed{2080\,\mathrm{J}}$$

$$\Delta S_{\mathrm{ab}}=\frac{Q_{\mathrm{ab}}}{T_{\mathrm{ab}}}=\frac{2079.7\,\mathrm{J}}{273\,\mathrm{K}}=\boxed{7.62\,\mathrm{J/K}}$$

Process bc: $W_{\mathrm{bc}}=\boxed{0}$;

$$\Delta \underset{\mathrm{bc}}{E_{\mathrm{int}}}=Q_{\mathrm{bc}}=nC_V\Delta T=(1.00\,\mathrm{mol})\tfrac{3}{2}(8.314\,\mathrm{J/mol{\bullet}K})(148\,\mathrm{K}-273\,\mathrm{K})$$

$$=-1559\,\mathrm{J}\approx\boxed{-1560\,\mathrm{J}}$$

$$\Delta S_{\mathrm{bc}}=\int_b^c\frac{dQ}{T}=\int_{T_b}^{T_c}\frac{nC_VdT}{T}=nC_V\ln\frac{T_c}{T_b}=(1.00\,\mathrm{mol})\tfrac{3}{2}(8.314\,\mathrm{J/mol{\bullet}K})\ln\frac{148\,\mathrm{K}}{273\,\mathrm{K}}$$

$$=\boxed{-7.64\,\mathrm{J/K}}$$

Process ca: $Q_{\mathrm{ca}}=\boxed{0}$; $\Delta S_{\mathrm{bc}}=\boxed{0}$ (adiabatic) ;

$$\Delta \underset{\mathrm{ca}}{E_{\mathrm{int}}}=-W=-\Delta \underset{\mathrm{ab}}{E_{\mathrm{int}}}-\Delta \underset{\mathrm{bc}}{E_{\mathrm{int}}}=-0-(-1560\,\mathrm{J})\;\rightarrow$$

$$\Delta \underset{\mathrm{ca}}{E_{\mathrm{int}}}=\boxed{1560\,\mathrm{J}}\;;\;W_{\mathrm{ca}}=\boxed{-1560\,\mathrm{J}}$$

(*d*) $e=\dfrac{W}{Q_{\mathrm{input}}}=\dfrac{2080\,\mathrm{J}-1560\,\mathrm{J}}{2080\,\mathrm{J}}=\boxed{0.25}$

$\boxed{73.}$ Take the energy transfer to use as the initial kinetic energy of the cars, because this energy becomes "unusable" after the collision – it is transferred to the environment.

$$\Delta S=\frac{Q}{T}=\frac{2\left(\frac{1}{2}mv_i^2\right)}{T}=\frac{(1100\,\mathrm{kg})\left[(75\,\mathrm{km/h})\left(\dfrac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{(15+273)\,\mathrm{K}}=\boxed{1700\,\mathrm{J/K}}$$

74. (*a*) Multiply the power times the time times the mass per Joule relationship for the fat.

$$(95\,\mathrm{J/s})(3600\,\mathrm{s/h})(24\,\mathrm{h/d})(1.0\text{ kg fat}/3.7\times10^7\,\mathrm{J})=0.2218\,\mathrm{kg/d}\approx\boxed{0.22\,\mathrm{kg/d}}$$

(*b*) $1.0\text{ kg}(1\mathrm{d}/0.2218\text{ kg})=\boxed{4.5\mathrm{d}}$

75. Heat will enter the freezer due to conductivity, at a rate given by 19-16b. This is the heat that must be removed from the freezer to keep it at a constant temperature, and so is the value of Q_L in the equation for the COP, Eq. 20-4a. The work in the COP is the work input by the cooling motor. The motor must remove the heat in 15% of the time that it takes for the heat to enter the freezer, so that it

only runs 15% of the time. To find the minimum power requirement, we assume the freezer is ideal in its operation.

$$\frac{Q_\text{L}}{t} = kA\frac{\Delta T}{\Delta x} \;;\; \text{COP} = \frac{Q_\text{L}}{W} = \frac{Q_\text{L}/t}{W/(0.15t)} = \frac{T_\text{L}}{T_\text{H} - T_\text{L}} \rightarrow$$

$$W/t = \frac{Q_\text{L}/t}{(0.15)}\left(\frac{T_\text{H}-T_\text{L}}{T_\text{L}}\right) = \frac{kA\frac{\Delta T}{\Delta x}}{(0.15)}\left(\frac{T_\text{H}-T_\text{L}}{T_\text{L}}\right) = \boxed{57\,\text{W}} \approx 0.076\,\text{hp}$$

76. The radiant energy is the heat to be removed at the low temperature. It can be related to the work necessary through the efficiency.

$$e = 1 - \frac{T_\text{L}}{T_\text{H}} = \frac{W}{Q_\text{H}} = \frac{W}{W + Q_\text{L}} \rightarrow W = Q_\text{L}\left(\frac{T_\text{H}}{T_\text{L}} - 1\right) \rightarrow W/t = Q_\text{L}/t\left(\frac{T_\text{H}}{T_\text{L}} - 1\right)$$

$$(W/t)_{3300} = (3300\text{ W})\left(\frac{T_\text{H}}{T_\text{L}} - 1\right) \qquad (W/t)_{500} = (500\text{ W})\left(\frac{T_\text{H}}{T_\text{L}} - 1\right)$$

$$(W/t)_\text{savings} = (W/t)_{3300} - (W/t)_{500} = (3300\,\text{W} - 500\,\text{W})\left(\frac{(273+32)\,\text{K}}{(273+21)\,\text{K}} - 1\right) = 104.8\text{W}$$

$$\approx \boxed{100\,\text{W}}\,(2\text{ sig. fig.})$$

77. We need to find the efficiency in terms of the given parameters, T_H, T_L, V_a, and V_b. So we must find the net work done and the heat input to the system. The work done during an isothermal process is given by Eq. 19-8. The work done during an isovolumentric process is 0. We also use the first law of thermodynamics.

ab (isothermal): $\Delta E_{\underset{\text{ab}}{\text{int}}} = 0 = Q_\text{ab} - W_\text{ab} \rightarrow Q_\text{ab} = W_\text{ab} = nRT_\text{H}\ln\frac{V_\text{b}}{V_\text{a}} > 0$

bc (isovolumetric): $\Delta E_{\underset{\text{bc}}{\text{int}}} = Q_\text{bc} - 0 \rightarrow Q_\text{bc} = \Delta E_{\underset{\text{bc}}{\text{int}}} = nC_V(T_\text{L} - T_\text{H}) = \tfrac{3}{2}nR(T_\text{L} - T_\text{H}) < 0$

cd (isothermal): $\Delta E_{\underset{\text{cd}}{\text{int}}} = 0 = Q_\text{cd} - W_\text{cd} \rightarrow Q_\text{cd} = W_\text{cd} = nRT_\text{L}\ln\frac{V_\text{a}}{V_\text{b}} = -nRT_\text{L}\ln\frac{V_\text{b}}{V_\text{a}} < 0$

da (isovolumetric): $\Delta E_{\underset{\text{da}}{\text{int}}} = Q_\text{da} - 0 \rightarrow Q_\text{da} = \Delta E_{\underset{\text{da}}{\text{int}}} = nC_V(T_\text{H} - T_\text{L}) = \tfrac{3}{2}nR(T_\text{H} - T_\text{L}) > 0$

$$W = W_\text{ab} + W_\text{cd} = nRT_\text{H}\ln\frac{V_\text{b}}{V_\text{a}} - nRT_\text{L}\ln\frac{V_\text{b}}{V_\text{a}} = nR(T_\text{H} - T_\text{L})\ln\frac{V_\text{b}}{V_\text{a}}$$

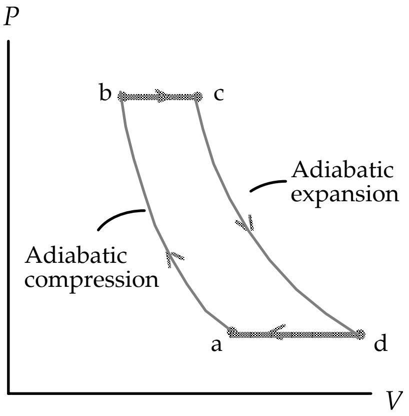
$$Q_\text{in} = Q_\text{ab} + Q_\text{da} = nRT_\text{H}\ln\frac{V_\text{b}}{V_\text{a}} + \tfrac{3}{2}nR(T_\text{H} - T_\text{L})$$

$$e_\text{Sterling} = \frac{W}{Q_\text{in}} = \frac{(T_\text{H} - T_\text{L})\ln\frac{V_\text{b}}{V_\text{a}}}{T_\text{H}\ln\frac{V_\text{b}}{V_\text{a}} + \tfrac{3}{2}(T_\text{H} - T_\text{L})} = \boxed{\left(\frac{T_\text{H} - T_\text{L}}{T_\text{H}}\right)\left[\frac{\ln\frac{V_\text{b}}{V_\text{a}}}{\ln\frac{V_\text{b}}{V_\text{a}} + \frac{3}{2}\left(\frac{T_\text{H} - T_\text{L}}{T_\text{H}}\right)}\right]}$$

$$= e_{\text{Carnot}}\left[\frac{\ln\frac{V_b}{V_a}}{\ln\frac{V_b}{V_a}+\frac{3}{2}\left(\frac{T_H-T_L}{T_H}\right)}\right]$$

Since the factor in [ ] above is less than 1, we see that $\boxed{e_{\text{Sterling}} < e_{\text{Carnot}}}$.

78. Since two of the processes are adiabatic, no heat transfer occurs in those processes. Thus the heat transfer must occur along the isobaric processes.



$$Q_H = Q_{bc} = nC_P(T_c - T_b) \;;\; Q_L = Q_{da} = nC_P(T_d - T_a)$$

$$e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{nC_P(T_d-T_a)}{nC_P(T_c-T_b)} = 1 - \frac{(T_d-T_a)}{(T_c-T_b)}$$

Use the ideal gas relationship, which says that $PV = nRT$.

$$e = 1 - \frac{(T_d-T_a)}{(T_c-T_b)} = 1 - \frac{\left(\frac{P_dV_d}{nR}-\frac{P_aV_a}{nR}\right)}{\left(\frac{P_cV_c}{nR}-\frac{P_bV_b}{nR}\right)} = 1 - \frac{(P_dV_d - P_aV_a)}{(P_cV_c - P_bV_b)}$$

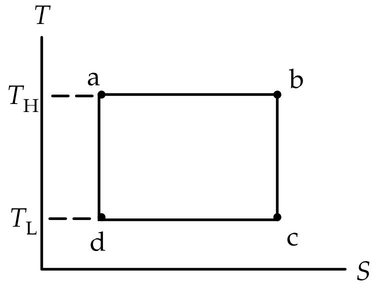
$$= 1 - \frac{P_a(V_d-V_a)}{P_b(V_c-V_b)}$$

Because process ab is adiabatic, we have $P_aV_a^\gamma = P_bV_b^\gamma \;\rightarrow\; V_a = V_b\left(\frac{P_b}{P_a}\right)^{1/\gamma}$. Because process cd is adiabatic, we have $P_bV_c^\gamma = P_aV_d^\gamma \;\rightarrow\; V_d = V_c\left(\frac{P_b}{P_a}\right)^{1/\gamma}$. Substitute these into the efficiency expression.

$$e = 1 - \frac{P_a(V_d-V_a)}{P_b(V_c-V_b)} = 1 - \frac{P_a\left(V_c\left(\frac{P_b}{P_a}\right)^{1/\gamma} - V_b\left(\frac{P_b}{P_a}\right)^{1/\gamma}\right)}{P_b(V_c-V_b)} = 1 - \frac{P_a\left(\frac{P_b}{P_a}\right)^{1/\gamma}(V_c-V_b)}{P_b(V_c-V_b)}$$

$$= 1 - \left(\frac{P_b}{P_a}\right)^{\frac{1}{\gamma}-1} = \boxed{1 - \left(\frac{P_b}{P_a}\right)^{\frac{1-\gamma}{\gamma}}}$$

$\boxed{79.}$ (*a*) For the Carnot cycle, two of the processes are reversible adiabats, which are constant entropy processes. The other two processes are isotherms, at the low and high temperatures. See the adjacent diagram.



(*b*) The area underneath any path on the T-S diagram would be written as $\int T\,dS$. This integral is the heat involved in the process.

$$\int T\,dS = \int T\frac{dQ}{T} = \int dQ = Q_{\text{net}}$$

For a closed cycle such as the Carnot cycle shown, since there is no internal energy change, the first law of thermodynamics says that $\int T\,dS = Q_{\text{net}} = \boxed{W_{\text{net}}}$, the same as $\int P\,dV$.

80. First we find the equilibrium temperature from calorimetry (section 19-4), and then calculate the entropy change of the system. The heat lost by the warm water must be equal to the heat gained by the cold water. Since the amounts of mass are the same, the equilibrium temperature is just the average of the two starting temperatures, 25°C.

$$\Delta S = \Delta S_{\substack{\text{cool}\\\text{water}}} + \Delta S_{\substack{\text{warm}\\\text{water}}} = \int_{T_{\substack{\text{initial}\\\text{cool}}}}^{T_{\text{final}}} \frac{dQ}{T} + \int_{T_{\substack{\text{initial}\\\text{warm}}}}^{T_{\text{final}}} \frac{dQ}{T} = \int_{273\,\text{K}}^{298\,\text{K}} \frac{mc\,dT}{T} + \int_{323\,\text{K}}^{298\,\text{K}} \frac{mc\,dT}{T} = mc\ln\left(\frac{298\,\text{K}}{273\,\text{K}}\right) + mc\ln\left(\frac{298\,\text{K}}{323\,\text{K}}\right)$$

$$= \left(4186\,\text{J/kg}\bullet{}^\circ\text{C}\right)\left[\ln\left(\frac{298\,\text{K}}{273\,\text{K}}\right) + \ln\left(\frac{298\,\text{K}}{323\,\text{K}}\right)\right] = \boxed{13\,\text{J/kg}}$$

81. To find the mass of water removed, find the energy that is removed from the low temperature reservoir from the work input and the Carnot efficiency. Then use the latent heat of vaporization to determine the mass of water from the energy required for the condensation. Note that the heat of vaporization used is that given in section 19-5 for evaporation at 20°C.

$$e = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = \frac{W}{Q_{\text{H}}} = \frac{W}{W + Q_{\text{L}}} \quad \to \quad Q_{\text{L}} = W\frac{T_{\text{L}}}{\left(T_{\text{H}} - T_{\text{L}}\right)} = mL_{\text{vapor}}$$

$$m = \frac{W}{L_{\text{vapor}}}\frac{T_{\text{L}}}{\left(T_{\text{H}} - T_{\text{L}}\right)} = \frac{(650\,\text{W})(3600\,\text{s})}{\left(2.45\times10^{6}\,\text{J/kg}\right)}\frac{(273+8)\,\text{K}}{(25-8)\,\text{K}} = 15.79\,\text{kg} \approx \boxed{16\,\text{kg}}$$

82. (*a*) From the table below, we see that there are 10 macrostates, and a total of 27 microstates.

Macrostate	Microstates (r = red, o = orange, g = green)			Number of Microstates
3 red, 0 orange, 0 green	r r r			1
2 red, 1 orange, 0 green	r r o	r o r	o r r	3
2 red, 0 orange, 1 green	r r g	r g r	g r r	3
1 red, 2 orange, 0 green	r o o	o r o	o o r	3
1 red, 0 orange, 2 green	r g g	g r g	g g r	3
1 red, 1 orange, 1 green	r o g o g r	r g o g r o	o r g g o r	6
0 red, 3 orange, 0 green	o o o			1
0 red, 2 orange, 1 green	g o o	o g o	o o g	3
0 red, 1 orange, 2 green	o g g	g o g	g g o	3
0 red, 0 orange, 3 green	g g g			1

(*b*) The probability of obtaining all 3 beans red is $\boxed{1/27}$.

(*c*) The probability of obtaining 2 greens and 1 orange is $\boxed{3/27}$ or $\boxed{1/9}$.

83. To do the numeric integration, first a value of ΔT is chosen. The temperature range is then partitioned into a series of individual temperatures, starting with 4 K, and each subsequent temperature an amount ΔT larger than the previous. So if $\Delta T = 1\,\text{K}$, then the temperatures used are 4 K, 5 K, 6 K, … 40 K. For each temperature above 4 K, an entropy change from the previous

temperature is calculated by $dS \approx nC_V \Delta T$. The total entropy change ΔS is then the sum of the individual dS terms. The process could be written as $\Delta S = \sum_i \frac{nC_V \Delta T}{T_i}$, where $T_{i+1} = T_i + \Delta T$. For a value of $\Delta T = 1\,\text{K}$, a value of $\Delta S = 3.75\times10^{-3}\,\text{J/kg}$ was calculated, which is 3.7% larger than the analytic answer. So a smaller ΔT was chosen. For a value of $\Delta T = 0.5\,\text{K}$, a value of $\Delta S = 3.68\times10^{-3}\,\text{J/kg}$ was calculated, which is 1.9% larger than the analytic answer.

Here is the analytic calculation of the entropy change.

$$\Delta S = \int \frac{dQ}{T} = \int_{T_\text{L}}^{T_\text{H}} \frac{nC_V dT}{T} = \int_{T_\text{L}}^{T_\text{H}} \frac{n\left(1800\,\text{J}\bullet\text{mol}^{-1}\bullet\text{K}^{-1}\right)\left(T/T_\text{D}\right)^3 dT}{T}$$

$$= \left(1.00\,\text{mol}\right)\frac{\left(1800\,\text{J}\bullet\text{mol}^{-1}\bullet\text{K}^{-1}\right)}{T_\text{D}^3}\int_{4\,\text{K}}^{40\,\text{K}} T^2 dT = \frac{\left(1800\,\text{J}\bullet\text{K}^{-1}\right)}{\left(2230\,\text{K}\right)^3}\frac{1}{3}\left[\left(40\,\text{K}\right)^3 - \left(4\,\text{K}\right)^3\right]$$

$$= \boxed{3.61\times10^{-3}\,\text{J/kg}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH20.XLS," on tab "Problem 20.83."

CHAPTER 21: Electric Charges and Electric Field

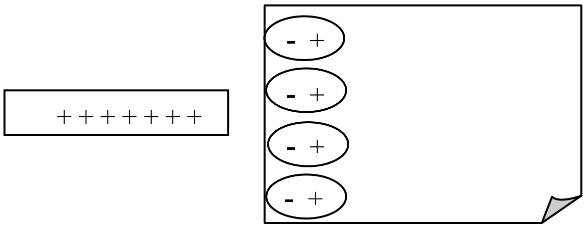
Responses to Questions

1. Rub a glass rod with silk and use it to charge an electroscope. The electroscope will end up with a net positive charge. Bring the pocket comb close to the electroscope. If the electroscope leaves move farther apart, then the charge on the comb is positive, the same as the charge on the electroscope. If the leaves move together, then the charge on the comb is negative, opposite the charge on the electroscope.

2. The shirt or blouse becomes charged as a result of being tossed about in the dryer and rubbing against the dryer sides and other clothes. When you put on the charged object (shirt), it causes charge separation within the molecules of your skin (see Figure 21-9), which results in attraction between the shirt and your skin.

3. Fog or rain droplets tend to form around ions because water is a polar molecule, with a positive region and a negative region. The charge centers on the water molecule will be attracted to the ions (positive to negative).

4. See also Figure 21-9 in the text. The negatively charged electrons in the paper are attracted to the positively charged rod and move towards it within their molecules. The attraction occurs because the negative charges in the paper are closer to the positive rod than are the positive charges in the paper, and therefore the attraction between the unlike charges is greater than the repulsion between the like charges.



5. A plastic ruler that has been rubbed with a cloth is charged. When brought near small pieces of paper, it will cause separation of charge in the bits of paper, which will cause the paper to be attracted to the ruler. On a humid day, polar water molecules will be attracted to the ruler and to the separated charge on the bits of paper, neutralizing the charges and thus eliminating the attraction.

6. The *net charge* on a conductor is the difference between the total positive charge and the total negative charge in the conductor. The “free charges” in a conductor are the electrons that can move about freely within the material because they are only loosely bound to their atoms. The “free electrons” are also referred to as “conduction electrons.” A conductor may have a zero net charge but still have substantial free charges.

7. Most of the electrons are strongly bound to nuclei in the metal ions. Only a few electrons per atom (usually one or two) are free to move about throughout the metal. These are called the “conduction electrons.” The rest are bound more tightly to the nucleus and are not free to move. Furthermore, in the cases shown in Figures 21-7 and 21-8, not all of the conduction electrons will move. In Figure 21-7, electrons will move until the attractive force on the remaining conduction electrons due to the incoming charged rod is balanced by the repulsive force from electrons that have already gathered at the left end of the neutral rod. In Figure 21-8, conduction electrons will be repelled by the incoming rod and will leave the stationary rod through the ground connection until the repulsive force on the remaining conduction electrons due to the incoming charged rod is balanced by the attractive force from the net positive charge on the stationary rod.

8. The electroscope leaves are connected together at the top. The horizontal component of this tension force balances the electric force of repulsion. (Note: The vertical component of the tension force balances the weight of the leaves.)

9. Coulomb's law and Newton's law are very similar in form. The electrostatic force can be either attractive or repulsive; the gravitational force can only be attractive. The electrostatic force constant is also much larger than the gravitational force constant. Both the electric charge and the gravitational mass are properties of the material. Charge can be positive or negative, but the gravitational mass only has one form.

10. The gravitational force between everyday objects on the surface of the Earth is extremely small. (Recall the value of G: $6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$.) Consider two objects sitting on the floor near each other. They are attracted to each other, but the force of static fiction for each is much greater than the gravitational force each experiences from the other. Even in an absolutely frictionless environment, the acceleration resulting from the gravitational force would be so small that it would not be noticeable in a short time frame. We are aware of the gravitational force between objects if at least one of them is very massive, as in the case of the Earth and satellites or the Earth and you.

    The electric force between two objects is typically zero or close to zero because ordinary objects are typically neutral or close to neutral. We are aware of electric forces between objects when the objects are charged. An example is the electrostatic force (static cling) between pieces of clothing when you pull the clothes out of the dryer.

11. Yes, the electric force is a conservative force. Energy is conserved when a particle moves under the influence of the electric force, and the work done by the electric force in moving an object between two points in space is independent of the path taken.

12. Coulomb observed experimentally that the force between two charged objects is directly proportional to the charge on each one. For example, if the charge on either object is tripled, then the force is tripled. This is not in agreement with a force that is proportional to the *sum* of the charges instead of to the *product* of the charges. Also, a charged object is not attracted to or repelled from a neutral object, which would be the case if the numerator in Coulomb's law were proportional to the sum of the charges.

13. When a charged ruler attracts small pieces of paper, the charge on the ruler causes a separation of charge in the paper. For example, if the ruler is negatively charged, it will force the electrons in the paper to the edge of the paper farthest from the ruler, leaving the near edge positively charged. If the paper touches the ruler, electrons will be transferred from the ruler to the paper, neutralizing the positive charge. This action leaves the paper with a net negative charge, which will cause it to be repelled by the negatively charged ruler.

14. The test charges used to measure electric fields are small in order to minimize their contribution to the field. Large test charges would substantially change the field being investigated.

15. When determining an electric field, it is best, but not required, to use a positive test charge. A negative test charge would be fine for determining the magnitude of the field. But the direction of the electrostatic force on a negative test charge will be opposite to the direction of the electric field. The electrostatic force on a positive test charge will be in the same direction as the electric field. In order to avoid confusion, it is better to use a positive test charge.

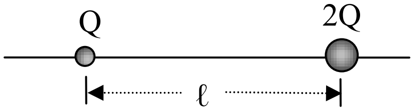
16. See Figure 21-34b. A diagram of the electric field lines around two negative charges would be just like this diagram except that the arrows on the field lines would point towards the charges instead of away from them. The distance between the charges is *l*.

17. The electric field will be strongest to the right of the positive charge (between the two charges) and weakest to the left of the positive charge. To the right of the positive charge, the contributions to the field from the two charges point in the same direction, and therefore add. To the left of the positive charge, the contributions to the field from the two charges point in opposite directions, and therefore subtract. Note that this is confirmed by the density of field lines in Figure 21-34a.
18. At point C, the positive test charge would experience zero net force. At points A and B, the direction of the force on the positive test charge would be the same as the direction of the field. This direction is indicated by the arrows on the field lines. The strongest field is at point A, followed (in order of decreasing field strength) by B and then C.

19. Electric field lines can never cross because they give the direction of the electrostatic force on a positive test charge. If they were to cross, then the force on a test charge at a given location would be in more than one direction. This is not possible.

20. The field lines must be directed radially toward or away from the point charge (see rule 1). The spacing of the lines indicates the strength of the field (see rule 2). Since the magnitude of the field due to the point charge depends only on the distance from the point charge, the lines must be distributed symmetrically.

21. The two charges are located along a line as shown in the diagram.



    (*a*) If the signs of the charges are opposite then the point on the line where $E = 0$ will lie to the left of Q. In that region the electric fields from the two charges will point in opposite directions, and the point will be closer to the smaller charge.
    (*b*) If the two charges have the same sign, then the point on the line where $E = 0$ will lie between the two charges, closer to the smaller charge. In this region, the electric fields from the two charges will point in opposite directions.

22. The electric field at point P would point in the negative *x*-direction. The magnitude of the field would be the same as that calculated for a positive distribution of charge on the ring:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Qx}{\left(x^2 + a^2\right)^{3/2}}$$

23. The velocity of the test charge will depend on its initial velocity. The field line gives the direction of the change in velocity, not the direction of the velocity. The acceleration of the test charge will be along the electric field line.

24. The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.

$\boxed{25.}$ The motion of the electron in Example 21-16 is projectile motion. In the case of the gravitational force, the acceleration of the projectile is in the same direction as the field and has a value of *g*; in the case of an electron in an electric field, the direction of the acceleration of the electron and the field direction are opposite, and the value of the acceleration varies.

26. Initially, the dipole will spin clockwise. It will "overshoot" the equilibrium position (parallel to the field lines), come momentarily to rest and then spin counterclockwise. The dipole will continue to oscillate back and forth if no damping forces are present. If there are damping forces, the amplitude will decrease with each oscillation until the dipole comes to rest aligned with the field.

27. If an electric dipole is placed in a nonuniform electric field, the charges of the dipole will experience forces of different magnitudes whose directions also may not be exactly opposite. The addition of these forces will leave a net force on the dipole.

Solutions to Problems

$\boxed{1.}$ Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\text{C}\right)\left(26\times1.602\times10^{-19}\text{C}\right)}{\left(1.5\times10^{-12}\text{m}\right)^2} = \boxed{2.7\times10^{-3}\,\text{N}}$$

2. Use the charge per electron to find the number of electrons.

$$\left(-38.0\times10^{-6}\text{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\text{C}}\right) = \boxed{2.37\times10^{14}\,\text{electrons}}$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(25\times10^{-6}\text{C}\right)\left(2.5\times10^{-3}\text{C}\right)}{\left(0.28\,\text{m}\right)^2} = \boxed{7200\,\text{N}}$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\text{C}\right)^2}{\left(4.0\times10^{-15}\text{m}\right)^2} = \boxed{14\,\text{N}}$$

5. The charge on the plastic comb is negative, so the comb has gained electrons.

$$\frac{\Delta m}{m} = \frac{\left(3.0\times10^{-6}\text{C}\right)\left(\dfrac{1\text{e}^-}{1.602\times10^{-19}\text{C}}\right)\left(\dfrac{9.109\times10^{-31}\text{kg}}{1\text{e}^-}\right)}{0.035\,\text{kg}} = 4.9\times10^{-16} = \boxed{4.9\times10^{-14}\%}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64\left(3.2\times10^{-2}\text{N}\right) = \boxed{2.0\ \text{N}}$$

$\boxed{7.}$ Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45\ \text{cm}}{\sqrt{3}} = \boxed{4.88\ \text{cm}}$$

8. Use the charge per electron and the mass per electron.

$$\left(-46\times10^{-6}\,\text{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\text{C}}\right) = 2.871\times10^{14} \approx \boxed{2.9\times10^{14}\,\text{electrons}}$$

$$\left(2.871\times10^{14}\,\text{e}^-\right)\left(\frac{9.109\times10^{-31}\,\text{kg}}{1\ \text{e}^-}\right) = \boxed{2.6\times10^{-16}\,\text{kg}}$$

9. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$15\,\text{kg Au} = \left(15\,\text{kg Au}\right)\left(\frac{1\,\text{mole Al}}{0.197\,\text{kg}}\right)\left(\frac{6.022\times10^{23}\,\text{atoms}}{1\ \text{mole}}\right)\left(\frac{79\,\text{electrons}}{1\,\text{molecule}}\right)\left(\frac{-1.602\times10^{-19}\,\text{C}}{\text{electron}}\right)$$

$$= \boxed{-5.8\times10^{8}\,\text{C}}$$

The net charge of the bar is $\boxed{0\,\text{C}}$, since there are equal numbers of protons and electrons.

10. Take the ratio of the electric force divided by the gravitational force.

$$\frac{F_\text{E}}{F_\text{G}} = \frac{k\frac{Q_1Q_2}{r^2}}{G\frac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.67\times10^{-27}\,\text{kg}\right)} = \boxed{2.3\times10^{39}}$$

The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario.

11. (*a*) Let one of the charges be q, and then the other charge is $Q_\text{T} - q$. The force between the charges is $F_\text{E} = k\frac{q\left(Q_\text{T}-q\right)}{r^2} = \frac{k}{r^2}\left(qQ_\text{T} - q^2\right)$. To find the maximum and minimum force, set the first derivative equal to 0. Use the second derivative test as well.

$$F_\text{E} = \frac{k}{r^2}\left(qQ_\text{T} - q^2\right)\ ;\ \frac{dF_\text{E}}{dq} = \frac{k}{r^2}\left(Q_\text{T} - 2q\right) = 0 \ \rightarrow\ q = \tfrac{1}{2}Q_\text{T}$$

$$\frac{d^2F_\text{E}}{dq^2} = -\frac{2k}{r^2} < 0 \ \rightarrow\ q = \tfrac{1}{2}Q_\text{T}\ \text{gives}\ \left(F_\text{E}\right)_\text{max}$$

So $\boxed{q_1 = q_2 = \tfrac{1}{2}Q_\text{T}}$ gives the maximum force.

(*b*) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0, which is the minimum possible force. So $\boxed{q_1 = 0,\ q_2 = Q_\text{T}}$ gives the minimum force.

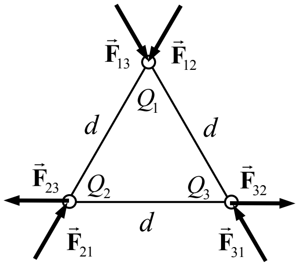
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^9\ \mathrm{N\cdot m^2/C^2}$.

$$\vec{\mathbf{F}}_{+75} = -k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(75\mu\mathrm{C})(85\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} = -147.2\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-150\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{+48} = k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(48\mu\mathrm{C})(85\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = 563.5\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{560\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{-85} = -k\frac{(85\mu\mathrm{C})(75\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} - k\frac{(85\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = -416.3\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-420\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$\boxed{13.}$ The forces on each charge lie along a line connecting the charges. Let the variable *d* represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.



$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(7.0\times10^{-6}\mathrm{C})(8.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2}$$
$$= 0.3495\,\mathrm{N}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(7.0\times10^{-6}\mathrm{C})(6.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2}$$
$$= 0.2622\,\mathrm{N}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(8.0\times10^{-6}\mathrm{C})(6.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2} = 0.2996\,\mathrm{N} = F_{32}$$

Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -(0.3495\,\mathrm{N})\cos60^\circ + (0.2622\,\mathrm{N})\cos60^\circ = -4.365\times10^{-2}\,\mathrm{N}$$
$$F_{1y} = F_{12y} + F_{13y} = -(0.3495\,\mathrm{N})\sin60^\circ - (0.2622\,\mathrm{N})\sin60^\circ = -5.297\times10^{-1}\,\mathrm{N}$$
$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.53\,\mathrm{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-5.297\times10^{-1}\,\mathrm{N}}{-4.365\times10^{-2}\,\mathrm{N}} = \boxed{265^\circ}$$
$$F_{2x} = F_{21x} + F_{23x} = (0.3495\,\mathrm{N})\cos60^\circ - (0.2996\,\mathrm{N}) = -1.249\times10^{-1}\,\mathrm{N}$$
$$F_{2y} = F_{21y} + F_{23y} = (0.3495\,\mathrm{N})\sin60^\circ + 0 = 3.027\times10^{-1}\,\mathrm{N}$$
$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.33\,\mathrm{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{3.027\times10^{-1}\,\mathrm{N}}{-1.249\times10^{-1}\,\mathrm{N}} = \boxed{112^\circ}$$
$$F_{3x} = F_{31x} + F_{32x} = -(0.2622\,\mathrm{N})\cos60^\circ + (0.2996\,\mathrm{N}) = 1.685\times10^{-1}\,\mathrm{N}$$
$$F_{3y} = F_{31y} + F_{32y} = (0.2622\,\mathrm{N})\sin60^\circ + 0 = 2.271\times10^{-1}\,\mathrm{N}$$
$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\mathrm{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{2.271\times10^{-1}\,\mathrm{N}}{1.685\times10^{-1}\,\mathrm{N}} = \boxed{53^\circ}$$

14. (*a*) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge Q.

$$Q_1+Q_2=Q \qquad F=\frac{kQ_1Q_2}{d^2}=\frac{kQ_1\left(Q-Q_1\right)}{d^2} \;\rightarrow\; Q_1^2-QQ_1+\frac{Fd^2}{k}=0$$

$$Q_1=\frac{Q\pm\sqrt{Q^2-4\frac{Fd^2}{k}}}{2}=\frac{Q\pm\sqrt{Q^2-4\frac{Fd^2}{k}}}{2}$$

$$=\tfrac{1}{2}\left[\left(90.0\times10^{-6}\text{C}\right)\pm\sqrt{\left(90.0\times10^{-6}\text{C}\right)^2-4\frac{\left(12.0\text{N}\right)\left(1.16\,\text{m}\right)^2}{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}\,\right]$$

$$=\boxed{60.1\times10^{-6}\text{C}\ ,\ 29.9\times10^{-6}\text{C}}$$

(*b*) If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (*a*).

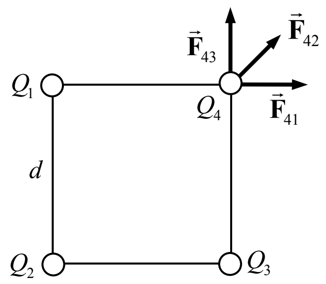
$$Q_1+Q_2=Q \qquad F=\frac{kQ_1Q_2}{d^2}=\frac{kQ_1\left(Q-Q_1\right)}{d^2} \;\rightarrow\; Q_1^2-QQ_1+\frac{Fd^2}{k}=0$$

$$Q_1=\frac{Q\pm\sqrt{Q^2-4\frac{Fd^2}{k}}}{2}=\frac{Q\pm\sqrt{Q^2-4\frac{Fd^2}{k}}}{2}$$

$$=\tfrac{1}{2}\left[\left(90.0\times10^{-6}\text{C}\right)\pm\sqrt{\left(90.0\times10^{-6}\text{C}\right)^2-4\frac{\left(-12.0\text{N}\right)\left(1.16\,\text{m}\right)^2}{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}\,\right]$$

$$=\boxed{106.8\times10^{-6}\text{C}\ ,\ -16.8\times10^{-6}\text{C}}$$

15. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.



$$F_{41}=k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x}=k\frac{Q^2}{d^2}\ ,\ F_{41y}=0$$

$$F_{42}=k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x}=k\frac{Q^2}{2d^2}\cos45^\circ=k\frac{\sqrt{2}Q^2}{4d^2}\ ,\ F_{42y}=k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43}=k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x}=0\ ,\ F_{43y}=k\frac{Q^2}{d^2}$$

Add the *x* and *y* components together to find the total force, noting that $F_{4x}=F_{4y}$.

$$F_{4x}=F_{41x}+F_{42x}+F_{43x}=k\frac{Q^2}{d^2}+k\frac{\sqrt{2}Q^2}{4d^2}+0=k\frac{Q^2}{d^2}\left(1+\frac{\sqrt{2}}{4}\right)=F_{4y}$$

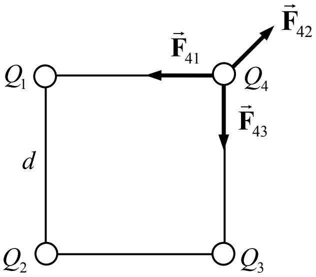
$$F_4=\sqrt{F_{4x}^2+F_{4y}^2}=k\frac{Q^2}{d^2}\left(1+\frac{\sqrt{2}}{4}\right)\sqrt{2}=k\frac{Q^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right)=\boxed{2.96\times10^7\,\text{N}}$$

$\theta=\tan^{-1}\dfrac{F_{4y}}{F_{4x}}=\boxed{45^\circ}$ above the *x*-direction.

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.

16. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.



The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.

$$F_{41}=k\frac{Q^2}{d^2}\ \rightarrow\ F_{41x}=-k\frac{Q^2}{d^2}\ ,\ F_{41y}=0$$

$$F_{42}=k\frac{Q^2}{2d^2}\ \rightarrow\ F_{42x}=k\frac{Q^2}{2d^2}\cos45^\circ=k\frac{\sqrt{2}Q^2}{4d^2}\ ,\ F_{42y}=k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43}=k\frac{Q^2}{d^2}\ \rightarrow\ F_{43x}=0\ ,\ F_{43y}=-k\frac{Q^2}{d^2}$$

Add the *x* and *y* components together to find the total force, noting that $F_{4x}=F_{4y}$.

$$F_{4x}=F_{41x}+F_{42x}+F_{43x}=-k\frac{Q^2}{d^2}+k\frac{\sqrt{2}Q^2}{4d^2}+0=k\frac{Q^2}{d^2}\left(-1+\frac{\sqrt{2}}{4}\right)=-0.64645k\frac{Q^2}{d^2}=F_{4y}$$

$$F_4=\sqrt{F_{4x}^2+F_{4y}^2}=k\frac{Q^2}{d^2}\left(0.64645\right)\sqrt{2}=k\frac{Q^2}{d^2}\left(0.9142\right)$$

$$=\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(0.9142\right)=\boxed{1.42\times10^7\,\text{N}}$$

$\theta=\tan^{-1}\dfrac{F_{4y}}{F_{4x}}=\boxed{225^\circ}$ from the *x*-direction, or exactly towards the center of the square.

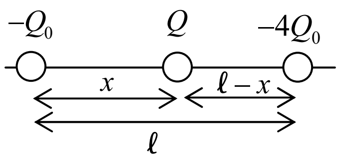
For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $\boxed{1.42\times10^7\,\text{N}}$ and will lie along the line from the charge inwards towards the center of the square.

17. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude Q of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$F=k\frac{Q_1Q_2}{r^2}=k\frac{Q^2}{r^2}\ \rightarrow\ Q=r\sqrt{\frac{F}{k}}=\left(0.12\ \text{m}\right)\sqrt{\frac{1.7\times10^{-2}\,\text{N}}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$=1.650\times10^{-7}\text{C}\left(\frac{1\ \text{electron}}{1.602\times10^{-19}\,\text{C}}\right)=\boxed{1.0\times10^{12}\,\text{electrons}}$$

18. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables. For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < \ell$.



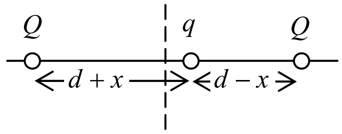
$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \qquad \text{right: } k\frac{4Q_0Q}{(\ell - x)^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0Q}{(\ell - x)^2} \quad \rightarrow \quad x = \tfrac{1}{3}\ell$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow \quad Q = 4Q_0\frac{x^2}{\ell^2} = Q_0\frac{4}{(3)^2} = \tfrac{4}{9}Q_0$$

Thus the charge should be of magnitude $\boxed{\tfrac{4}{9}Q_0}$, and a distance $\boxed{\tfrac{1}{3}\ell \text{ from } -Q_0 \text{ towards } -4Q_0}$.

19. (*a*) The charge will experience a force that is always pointing towards the origin. In the diagram, there is a greater force of $\dfrac{Qq}{4\pi\varepsilon_0(d-x)^2}$ to the left, and a lesser force of $\dfrac{Qq}{4\pi\varepsilon_0(d+x)^2}$ to the right. So the net force is towards the origin. The same would be true if the mass were to the left of the origin. Calculate the net force.



$$F_{\text{net}} = \frac{Qq}{4\pi\varepsilon_0(d+x)^2} - \frac{Qq}{4\pi\varepsilon_0(d-x)^2} = \frac{Qq}{4\pi\varepsilon_0(d+x)^2(d-x)^2}\left[(d-x)^2 - (d+x)^2\right]$$

$$= \frac{-4Qqd}{4\pi\varepsilon_0(d+x)^2(d-x)^2}x = \frac{-Qqd}{\pi\varepsilon_0(d+x)^2(d-x)^2}x$$

We assume that $x \ll d$.

$$F_{\text{net}} = \frac{-Qqd}{\pi\varepsilon_0(d+x)^2(d-x)^2}x \approx \frac{-Qq}{\pi\varepsilon_0 d^3}x$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{\pi\varepsilon_0 d^3}$.

The spring constant can be used to find the period. See Eq. 14-7b.

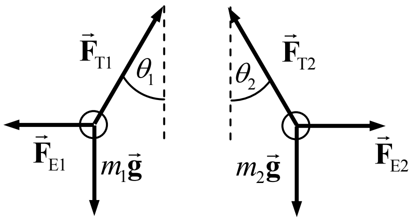
$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\dfrac{Qq}{\pi\varepsilon_0 d^3}}} = \boxed{2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}}}$$

(*b*) Sodium has an atomic mass of 23.

$$T = 2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}} = 2\pi\sqrt{\frac{(29)\left(1.66\times10^{-27}\text{kg}\right)\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3\times10^{-10}\text{m}\right)^3}{\left(1.60\times10^{-19}\text{C}\right)^2}}$$

$$= 2.4\times10^{-13}\text{s}\left(\frac{10^{12}\text{ps}}{1\,\text{s}}\right) = 0.24\,\text{ps} \approx \boxed{0.2\,\text{ps}}$$

20. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal. Likewise, the small angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force. Take to the right to be the positive horizontal direction, and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.



(*a*) $\sum F_{1x} = F_{T1}\sin\theta_1 - F_{E1} = 0 \;\rightarrow\; F_{E1} = F_{T1}\sin\theta_1$

$$\sum F_{1y} = F_{T1}\cos\theta_1 - m_1 g \;\rightarrow\; F_{T1} = \frac{m_1 g}{\cos\theta_1} \;\rightarrow\; F_{E1} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

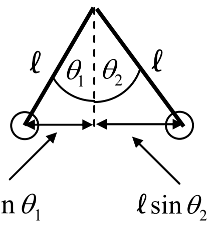
A completely parallel analysis would give $F_{E2} = m_2 g\theta_2$. Since the electric forces are a Newton's third law pair, they can be set equal to each other in magnitude.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

(*b*) The same analysis can be done for this case.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_1/m_1 = \boxed{2}$$

(*c*) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_E = mg\theta$ to solve for the distance.



Case 1: $d = \ell(\theta_1 + \theta_2) = 2\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{d}{2\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2\ell} \;\rightarrow\; \boxed{d = \left(\frac{4\ell kQ^2}{mg}\right)^{1/3}}$$

Case 2: $d = \ell(\theta_1 + \theta_2) = \dfrac{3}{2}\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{2d}{3\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{2d}{3\ell} \;\rightarrow\; \boxed{d = \left(\frac{3\ell kQ^2}{mg}\right)^{1/3}}$$

21. Use Eq. 21–3 to calculate the force. Take east to be the positive *x* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \;\rightarrow\; \vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-1.602\times10^{-19}\,\mathrm{C}\right)\left(1920\,\mathrm{N/C}\,\hat{\mathbf{i}}\right) = -3.08\times10^{-16}\,\mathrm{N}\,\hat{\mathbf{i}} = \boxed{3.08\times10^{-16}\,\mathrm{N\ west}}$$

22. Use Eq. 21–3 to calculate the electric field. Take north to be the positive *y* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{-2.18\times10^{-14}\,\mathrm{N}\,\hat{\mathbf{j}}}{1.602\times10^{-19}\,\mathrm{C}} = -1.36\times10^{5}\,\mathrm{N/C}\,\hat{\mathbf{j}} = \boxed{1.36\times10^{5}\,\mathrm{N/C\ south}}$$

23. Use Eq. 21–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = \left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\frac{33.0\times10^{-6}\,\mathrm{C}}{(0.164\,\mathrm{m})^2} = \boxed{1.10\times10^{7}\,\mathrm{N/C\ up}}$$

Note that the electric field points away from the positive charge.

24. Use Eq. 21–3 to calculate the electric field.

$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{8.4\,\text{N down}}{-8.8\times10^{-6}\,\text{C}}=\boxed{9.5\times10^{5}\ \text{N/C up}}$$

25. Use the definition of the electric field, Eq. 21-3.

$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{\left(7.22\times10^{-4}\,\text{N}\,\hat{\mathbf{j}}\right)}{4.20\times10^{-6}\,\text{C}}=\boxed{172\,\text{N/C}\,\hat{\mathbf{j}}}$$

26. Use the definition of the electric field, Eq. 21-3.

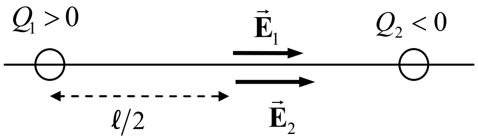
$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{\left(3.0\hat{\mathbf{i}}-3.9\hat{\mathbf{j}}\right)\times10^{-3}\,\text{N}}{1.25\times10^{-6}\,\text{C}}=\boxed{\left(2400\,\hat{\mathbf{i}}-3100\,\hat{\mathbf{j}}\right)\text{N/C}}$$

27. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}}=m\vec{\mathbf{a}}=q\vec{\mathbf{E}}\ \rightarrow\ a=\frac{|q|}{m}E=\frac{\left(1.602\times10^{-19}\,\text{C}\right)}{\left(9.109\times10^{-31}\,\text{kg}\right)}\left(576\,\text{N/C}\right)=\boxed{1.01\times10^{14}\ \text{m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

28. The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the negative charge, and so can be added.

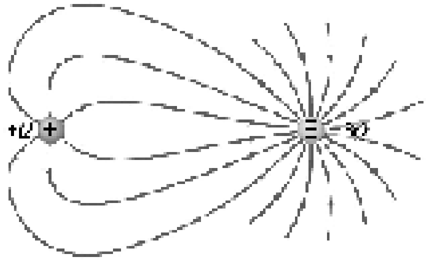


$$E=|E_1|+|E_2|=k\frac{|Q_1|}{r_1^2}+k\frac{|Q_2|}{r_2^2}=k\frac{|Q_1|}{(\ell/2)^2}+k\frac{|Q_2|}{(\ell/2)^2}=\frac{4k}{\ell^2}\left(|Q_1|+|Q_2|\right)$$

$$=\frac{4\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(0.080\,\text{m}\right)^2}\left(8.0\times10^{-6}\,\text{C}+5.8\times10^{-6}\,\text{C}\right)=\boxed{7.8\times10^{7}\ \text{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

29.



30. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the electric field strength.

$$F_{net}=ma=qE\ \rightarrow\ E=\frac{ma}{q}=\frac{\left(1.673\times10^{-27}\,\text{kg}\right)\left(1.8\times10^{6}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)}=\boxed{0.18\,\text{N/C}}$$

31. The field at the point in question is the vector sum of the two fields shown in Figure 21-56. Use the results of Example 21-11 to find the field of the long line of charge.

$$\vec{\mathbf{E}}_{\text{thread}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} \ ; \ \vec{\mathbf{E}}_Q = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\left(-\cos\theta\hat{\mathbf{i}} - \sin\theta\hat{\mathbf{j}}\right) \ \rightarrow$$

$$\vec{\mathbf{E}} = \left(-\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta\right)\hat{\mathbf{i}} + \left(\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta\right)\hat{\mathbf{j}}$$

$$d^2 = (0.070\,\text{m})^2 + (0.120\,\text{m})^2 = 0.0193\,\text{m}^2 \ ; \ y = 0.070\,\text{m} \ ; \ \theta = \tan^{-1}\frac{12.0\,\text{cm}}{7.0\,\text{cm}} = 59.7^\circ$$

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta = -\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\cos 59.7^\circ = -4.699\times10^{11}\,\text{N/C}$$

$$E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda}{y} - \frac{|Q|}{d^2}\sin\theta\right)$$

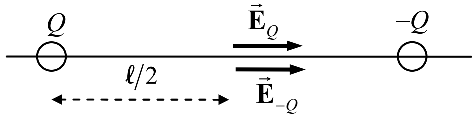
$$= \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{2(2.5\,\text{C/m})}{0.070\,\text{cm}} - \frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\sin 59.7^\circ\right] = -1.622\times10^{11}\,\text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{\left(-4.7\times10^{11}\,\text{N/C}\right)\hat{\mathbf{i}} + \left(-1.6\times10^{11}\,\text{N/C}\right)\hat{\mathbf{j}}}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(-4.699\times10^{11}\,\text{N/C}\right)^2 + \left(-1.622\times10^{11}\,\text{N/C}\right)^2} = \boxed{5.0\times10^{11}\,\text{N/C}}$$

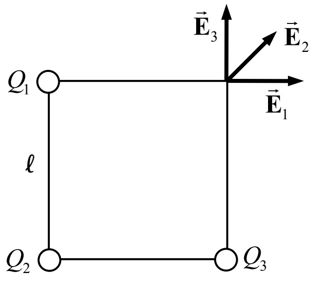
$$\theta_E = \tan^{-1}\frac{\left(-1.622\times10^{11}\,\text{N/C}\right)}{\left(-4.699\times10^{11}\,\text{N/C}\right)} = \boxed{199^\circ}$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.



$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(\ell/2)^2} = \frac{8kQ}{\ell^2} \ \rightarrow \ Q = \frac{E\ell^2}{8k} = \frac{(586\,\text{N/C})(0.160\,\text{m})^2}{8\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{2.09\times10^{-10}\,\text{C}}$$

33. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable ℓ represent the 1.0 m length of a side of the square, and let the variable Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{\ell^2} \ \rightarrow \ E_{1x} = k\frac{Q}{\ell^2} \ , E_{1y} = 0$$

$$E_2 = k\frac{Q}{2\ell^2} \ \rightarrow \ E_{2x} = k\frac{Q}{2\ell^2}\cos 45^\circ = k\frac{\sqrt{2}Q}{4\ell^2} \ , E_{2y} = k\frac{\sqrt{2}Q}{4\ell^2}$$

$$E_3 = k\frac{Q}{\ell^2} \ \rightarrow \ E_{3x} = 0 \ , E_{1y} = k\frac{Q}{\ell^2}$$

Add the *x* and *y* components together to find the total electric field, noting that $E_x = E_y$.

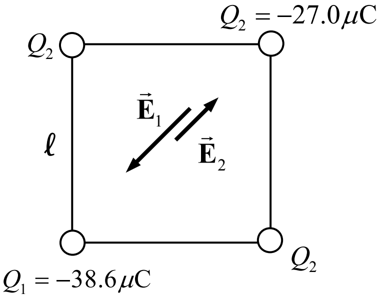
$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{\ell^2} + k\frac{\sqrt{2}Q}{4\ell^2} + 0 = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{\ell^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(2.25\times10^{-6}\,\mathrm{C}\right)}{\left(1.22\,\mathrm{m}\right)^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{2.60\times10^4\,\mathrm{N/C}}$$

$\theta = \tan^{-1}\dfrac{E_y}{E_x} = \boxed{45.0^\circ}$ from the x-direction.

34. The field at the center due to the two $-27.0\mu\mathrm{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.



$$E = E_1 - E_2 = k\frac{|Q_1|}{\ell^2/2} - k\frac{|Q_2|}{\ell^2/2} = k\frac{|Q_1| - |Q_2|}{\ell^2/2}$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(38.6 - 27.0\right)\times10^{-6}\,\mathrm{C}}{\left(0.525\,\mathrm{m}\right)^2/2}$$

$$= \boxed{7.57\times10^6\,\mathrm{N/C},\text{ towards the } -38.6\mu\mathrm{C}\text{ charge}}$$

35. Choose the rightward direction to be positive. Then the field due to +*Q* will be positive, and the field due to –*Q* will be negative.

$$E = k\frac{Q}{\left(x+a\right)^2} - k\frac{Q}{\left(x-a\right)^2} = kQ\left(\frac{1}{\left(x+a\right)^2} - \frac{1}{\left(x-a\right)^2}\right) = \boxed{\frac{-4kQxa}{\left(x^2-a^2\right)^2}}$$

The negative sign means the field points to the $\boxed{\text{left}}$.

36. For the net field to be zero at point P, the magnitudes of the fields created by Q_1 and Q_2 must be equal. Also, the distance x will be taken as positive to the left of Q_1. That is the only region where the total field due to the two charges can be zero. Let the variable ℓ represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \quad\rightarrow\quad k\frac{|Q_1|}{x^2} = k\frac{Q_2}{\left(x+\ell\right)^2} \quad\rightarrow$$

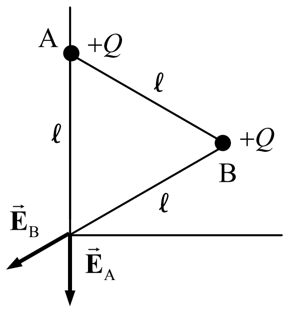
$$x = \ell\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2} - \sqrt{|Q_1|}\right)} = \left(12\,\mathrm{cm}\right)\frac{\sqrt{25\mu\mathrm{C}}}{\left(\sqrt{45\mu\mathrm{C}} - \sqrt{25\mu\mathrm{C}}\right)} = \boxed{35\,\mathrm{cm}}$$

$\boxed{37.}$ Make use of Example 21-11. From that, we see that the electric field due to the line charge along the y axis is $\vec{\mathbf{E}}_1 = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{x}\hat{\mathbf{i}}$. In particular, the field due to that line of charge has no y dependence. In a similar fashion, the electric field due to the line charge along the x axis is $\vec{\mathbf{E}}_2 = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{y}\hat{\mathbf{j}}$. Then the total field at (x, y) is the vector sum of the two fields.

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{\mathbf{i}} + \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x}\hat{\mathbf{i}} + \frac{1}{y}\hat{\mathbf{j}}\right)$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\sqrt{\frac{1}{x^2} + \frac{1}{y^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 xy}\sqrt{x^2 + y^2}} \ ; \ \theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}}{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}} = \boxed{\tan^{-1}\frac{x}{y}}$$

38. (*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, 30° below the negative x axis.



$$E_A = k\frac{Q}{\ell^2} \ \to\ E_{Ax} = 0\ ,\ E_{Ax} = -k\frac{Q}{\ell^2}$$

$$E_B = k\frac{Q}{\ell^2} \ \to\ E_{Bx} = -k\frac{Q}{\ell^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2\ell^2},$$

$$E_{By} = -k\frac{Q}{\ell^2}\sin 30^\circ = -k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2\ell^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{9k^2Q^2}{4\ell^4}} = \sqrt{\frac{12k^2Q^2}{4\ell^4}} = \boxed{\frac{\sqrt{3}kQ}{\ell^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2\ell^2}}{-k\frac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

(*b*) Now reverse the direction of $\vec{\mathbf{E}}_A$

$$E_A = k\frac{Q}{\ell^2} \ \to\ E_{Ax} = 0\ ,\ E_{Ax} = -k\frac{Q}{\ell^2}$$

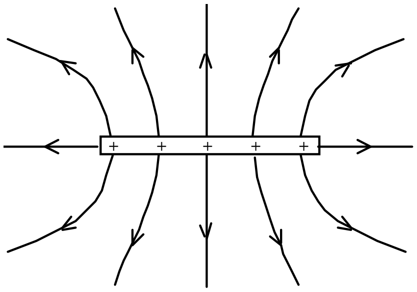
$$E_B = k\frac{Q}{\ell^2} \ \to\ E_{Bx} = k\frac{Q}{\ell^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2\ell^2}\ ,\ E_{By} = k\frac{Q}{\ell^2}\sin 30^\circ = k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2\ell^2}$$

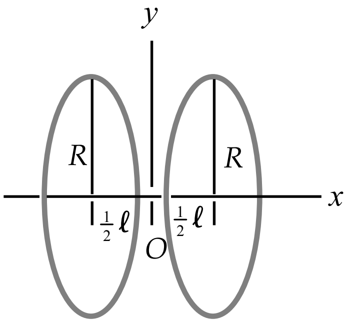
$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{k^2Q^2}{4\ell^4}} = \sqrt{\frac{4k^2Q^2}{4\ell^4}} = \boxed{\frac{kQ}{\ell^2}}$$

$$\theta=\tan^{-1}\frac{E_y}{E_x}=\tan^{-1}\frac{k\dfrac{Q}{2\ell^2}}{-k\dfrac{\sqrt{3}Q}{2\ell^2}}=\tan^{-1}\frac{1}{-\sqrt{3}}=\boxed{330^{\circ}}$$

39. Near the plate, the lines should come from it almost vertically, because it is almost like an infinite line of charge when the observation point is close. When the observation point is far away, it will look like a point charge.



40. Consider Example 21-9. We use the result from this example, but shift the center of the ring to be at $x=\frac{1}{2}\ell$ for the ring on the right, and at $x=-\frac{1}{2}\ell$ for the ring on the left. The fact that the original expression has a factor of *x* results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.



$$\vec{\mathbf{E}}=\vec{\mathbf{E}}_{\text{right}}+\vec{\mathbf{E}}_{\text{left}}$$

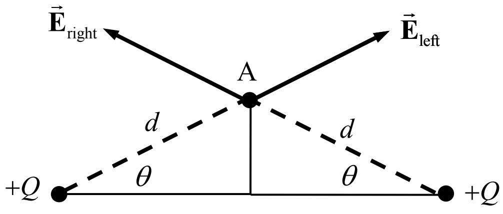
$$=\frac{1}{4\pi\varepsilon_0}\frac{Q\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}}+\frac{1}{4\pi\varepsilon_0}\frac{Q\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}}$$

$$=\hat{\mathbf{i}}\frac{Q}{4\pi\varepsilon_0}\left\{\frac{\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}+\frac{\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\right\}$$

41. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$E_1=E_2\ \rightarrow\ k\frac{Q_1}{(\ell/3)^2}=k\frac{Q_2}{(2\ell/3)^2}\ \rightarrow\ 9Q_1=\frac{9Q_2}{4}\ \rightarrow\ \frac{Q_1}{Q_2}=\boxed{\frac{1}{4}}$$

42. In each case, find the vector sum of the field caused by the charge on the left $\left(\vec{\mathbf{E}}_{\text{left}}\right)$ and the field caused by the charge on the right $\left(\vec{\mathbf{E}}_{\text{right}}\right)$



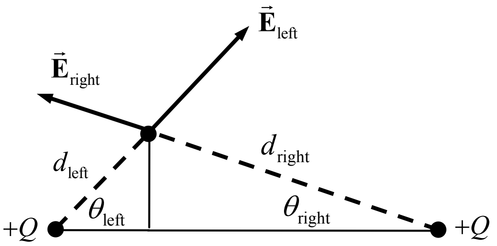
Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta=\tan^{-1}\frac{5.0}{10.0}=26.6^{\circ}$$

$$d=\sqrt{(5.0\,\text{cm})^2+(10.0\,\text{cm})^2}=0.1118\,\text{m}$$

$$E_{\mathrm{A}}=2\frac{kQ}{d^2}\sin\theta=2\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.7\times10^{-6}\,\mathrm{C}}{\left(0.1118\,\mathrm{m}\right)^2}\sin26.6^\circ=\boxed{3.7\times10^6\,\mathrm{N/C}}\qquad\theta_{\mathrm{A}}=\boxed{90^\circ}$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.



$$\theta_{\mathrm{left}}=\tan^{-1}\frac{5.0}{5.0}=45^\circ\qquad\theta_{\mathrm{right}}=\tan^{-1}\frac{5.0}{15.0}=18.4^\circ$$

$$d_{\mathrm{left}}=\sqrt{\left(5.0\,\mathrm{cm}\right)^2+\left(5.0\,\mathrm{cm}\right)^2}=0.0707\,\mathrm{m}$$

$$d_{\mathrm{right}}=\sqrt{\left(5.0\,\mathrm{cm}\right)^2+\left(15.0\,\mathrm{cm}\right)^2}=0.1581\,\mathrm{m}$$

$$E_x=\left(\vec{\mathbf{E}}_{\mathrm{left}}\right)_x+\left(\vec{\mathbf{E}}_{\mathrm{right}}\right)_x=k\frac{Q}{d^2_{\mathrm{left}}}\cos\theta_{\mathrm{left}}-k\frac{Q}{d^2_{\mathrm{right}}}\cos\theta_{\mathrm{right}}$$

$$=\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(5.7\times10^{-6}\,\mathrm{C}\right)\left[\frac{\cos45^\circ}{\left(0.0707\,\mathrm{m}\right)^2}-\frac{\cos18.4^\circ}{\left(0.1581\,\mathrm{m}\right)^2}\right]=5.30\times10^6\,\mathrm{N/C}$$

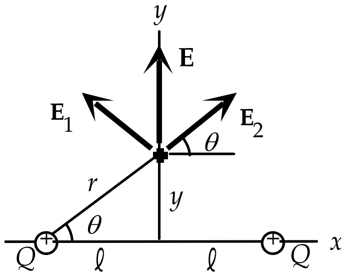
$$E_y=\left(\vec{\mathbf{E}}_{\mathrm{left}}\right)_y+\left(\vec{\mathbf{E}}_{\mathrm{right}}\right)_y=k\frac{Q}{d^2_{\mathrm{left}}}\sin\theta_{\mathrm{left}}+k\frac{Q}{d^2_{\mathrm{right}}}\sin\theta_{\mathrm{right}}$$

$$=\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(5.7\times10^{-6}\,\mathrm{C}\right)\left[\frac{\sin45^\circ}{\left(0.0707\,\mathrm{m}\right)^2}+\frac{\sin18.4^\circ}{\left(0.1581\,\mathrm{m}\right)^2}\right]=7.89\times10^6\,\mathrm{N/C}$$

$$E_{\mathrm{B}}=\sqrt{E_x^2+E_y^2}=\boxed{9.5\times10^6\,\mathrm{N/C}}\qquad\theta_B=\tan^{-1}\frac{E_y}{E_x}=\boxed{56^\circ}$$

The results are consistent with Figure 21-34b. In the figure, the field at Point A points straight up, matching the calculations. The field at Point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at Point B than at Point A, indicating that the field is stronger there, matching the calculations.

$\boxed{43.}$ (*a*) See the diagram. From the symmetry of the charges, we see that the net electric field points along the *y* axis.



$$\vec{\mathbf{E}}=2\frac{Q}{4\pi\varepsilon_0\left(\ell^2+y^2\right)}\sin\theta\,\hat{\mathbf{j}}=\boxed{\frac{Qy}{2\pi\varepsilon_0\left(\ell^2+y^2\right)^{3/2}}\hat{\mathbf{j}}}$$

(*b*) To find the position where the magnitude is a maximum, set the first derivative with respect to *y* equal to 0, and solve for the *y* value.

$$E=\frac{Qy}{2\pi\varepsilon_0\left(\ell^2+y^2\right)^{3/2}}\;\rightarrow$$

$$\frac{dE}{dy}=\frac{Q}{2\pi\varepsilon_0\left(\ell^2+y^2\right)^{3/2}}+\left(-\tfrac{3}{2}\right)\frac{Qy}{2\pi\varepsilon_0\left(\ell^2+y^2\right)^{5/2}}\left(2y\right)=0\;\rightarrow$$

$$\frac{1}{\left(\ell^2+y^2\right)^{3/2}}=\frac{3y^2}{\left(\ell^2+y^2\right)^{5/2}}\;\rightarrow\;y^2=\tfrac{1}{2}\ell^2\;\rightarrow\;y=\boxed{\pm\ell/\sqrt{2}}$$

This has to be a maximum, because the magnitude is positive, the field is 0 midway between the charges, and $E \to 0$ as $y \to \infty$.

44. From Example 21-9, the electric field along the x-axis is $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$. To find the position where the magnitude is a maximum, we differentiate and set the first derivative equal to zero.

$$\frac{dE}{dx} = \frac{Q}{4\pi\varepsilon_0}\frac{\left(x^2+a^2\right)^{\frac{3}{2}} - x\frac{3}{2}\left(x^2+a^2\right)^{\frac{1}{2}}2x}{\left(x^2+a^2\right)^3} = \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[\left(x^2+a^2\right)-3x^2\right]$$

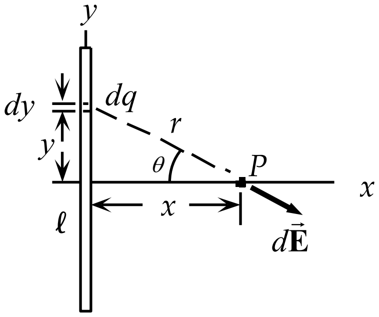
$$= \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[a^2-2x^2\right] = 0 \;\to\; \boxed{x_M = \pm\frac{a}{\sqrt{2}}}$$

Note that $E=0$ at $x=0$ and $x=\infty$, and that $|E|>0$ for $0<|x|<\infty$. Thus the value of the magnitude of E at $x = x_M$ must be a maximum. We could also show that the value is a maximum by using the second derivative test.

45. Because the distance from the wire is much smaller than the length of the wire, we can approximate the electric field by the field of an infinite wire, which is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} = \left(8.988\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{2\left(\dfrac{4.75\times10^{-6}\,\text{C}}{2.0\,\text{m}}\right)}{\left(2.4\times10^{-2}\,\text{m}\right)} = \boxed{\begin{array}{l}1.8\times10^6\,\text{N/C},\\ \text{away from the wire}\end{array}}$$

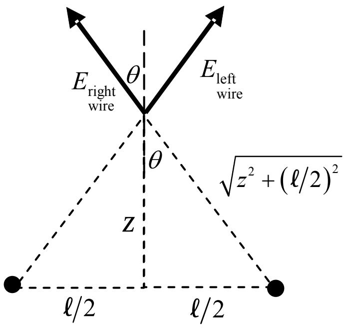
46. This is essentially Example 21-11 again, but with different limits of integration. From the diagram here, we see that the maximum angle is given by $\sin\theta = \frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}$. We evaluate the results at that angle.



$$E = \frac{\lambda}{4\pi\varepsilon_0 x}\left(\sin\theta\right)\Bigg|_{\sin\theta = \frac{-\ell/2}{\sqrt{x^2+(\ell/2)^2}}}^{\sin\theta = \frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left[\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}} - \left(-\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}\right)\right] = \frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+(\ell/2)^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{x\left(4x^2+\ell^2\right)^{1/2}}}$$

47. If we consider just one wire, then from the answer to problem 46, we would have the following. Note that the distance from the wire to the point in question is $x = \sqrt{z^2+(\ell/2)^2}$.



$$E_{\text{wire}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+(\ell/2)^2}\left(4\left[z^2+(\ell/2)^2\right]+\ell^2\right)^{1/2}}$$

But the total field is not simply four times the above expression, because the fields due to the four wires are not parallel to each other.

Consider a side view of the problem. The two dots represent two parallel wires, on opposite sides of the square. Note that only the vertical component of the field due to each wire will actually contribute to the total field. The horizontal components will cancel.

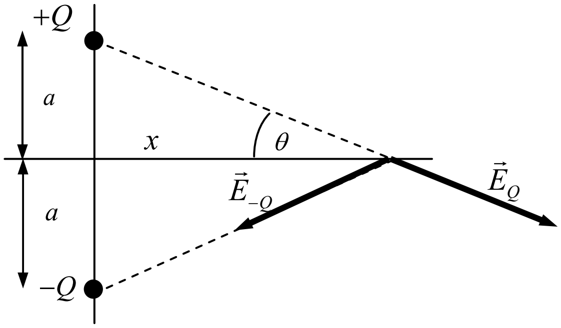
$$E_{\text{wire}} = 4\left(E_{\text{wire}}\right)\cos\theta = 4\left(E_{\text{wire}}\right)\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$E_{\text{wire}} = 4\left[\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+\left(\ell/2\right)^2}\left(4\left[z^2+\left(\ell/2\right)^2\right]+\ell^2\right)^{1/2}}\right]\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$= \boxed{\frac{8\lambda\ell z}{\pi\varepsilon_0\left(4z^2+\ell^2\right)\left(4z^2+2\ell^2\right)^{1/2}}}$$

The direction is $\boxed{\text{vertical}}$, perpendicular to the loop.

48. From the diagram, we see that the x components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative y-direction, and will be twice the y-component of either electric field vector.

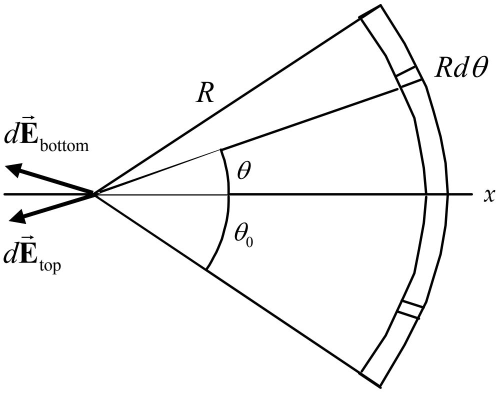


$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \frac{2kQ}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$= \boxed{\frac{2kQa}{\left(x^2+a^2\right)^{3/2}}\text{ in the negative } y \text{ direction}}$$

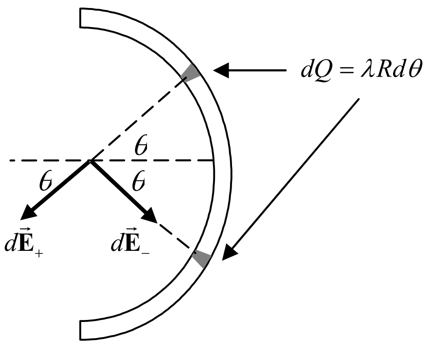
$\boxed{49.}$ Select a differential element of the arc which makes an angle of θ with the x axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda Rd\theta$. The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda Rd\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the x axis, we see that the total field will only have an x component. The vertical components of the field due to symmetric portions of the arc will cancel each other. So we have the following.



$$dE_{\text{horizontal}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda Rd\theta}{R^2}\cos\theta$$

$$E_{\text{horizontal}} = \int_{-\theta_0}^{\theta_0}\frac{1}{4\pi\varepsilon_0}\cos\theta\frac{\lambda Rd\theta}{R^2} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_{-\theta_0}^{\theta_0}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\left[\sin\theta_0 - \sin\left(-\theta_0\right)\right] = \frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}$$

The field points in the negative x direction, so $E = \boxed{-\frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}\hat{\mathbf{i}}}$



50. (*a*) Select a differential element of the arc which makes an angle of θ with the *x* axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda R d\theta$. The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the *x* axis, we see that the total field will only have a *y* component, because the magnitudes of the fields due to those two pieces are the same. From the diagram we see that the field will point down. The horizontal components of the field cancel.

$$dE_{\text{vertical}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\sin\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta d\theta$$

$$E_{\text{vertical}} = \int_{-\pi/2}^{\pi/2}\frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2}\sin^2\theta d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)_{-\pi/2}^{\pi/2}$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\pi\right) = \frac{\lambda_0}{8\varepsilon_0 R} \quad\rightarrow\quad \vec{\mathbf{E}} = \boxed{-\frac{\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}}}$$

(*b*) The force on the electron is given by Eq. 21-3. The acceleration is found from the force.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -\frac{q\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}} \quad\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{q\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{e\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{-6}\,\text{C/m}\right)}{8\left(9.11\times10^{-31}\text{kg}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.010\,\text{m}\right)}\hat{\mathbf{j}}$$

$$= \boxed{2.5\times10^{17}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

51. (*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_0^{\ell}\frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_0^{\ell}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \boxed{\frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}}$$

$$E_y = -\int_0^{\ell} \frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\ell}\frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+\ell^2}}-\frac{1}{x}\right) = \boxed{\frac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}$$

Note that $E_y < 0,$ and so the electric field points to the right and down.

(*b*) The angle that the electric field makes with the *x* axis is given as follows.

$$\tan\theta = \frac{E_y}{E_x} = \frac{\dfrac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}{\dfrac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}} = \frac{x-\sqrt{x^2+\ell^2}}{\ell} = \frac{x}{\ell}-\sqrt{1+\frac{x^2}{\ell^2}}$$

As $\ell \to \infty,$ the expression becomes $\tan\theta = -1$, and so the field makes an angle of $\boxed{45^\circ \text{ below the } x \text{ axis}}$.

52. Please note: the first printing of the textbook gave the length of the charged wire as 6.00 m, but it should have been 6.50 m. That error has been corrected in later printings, and the following solution uses a length of 6.50 m.

(*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\,dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_{y_{\min}}^{y_{\max}}\frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_{y_{\min}}^{y_{\max}}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left(\frac{y_{\max}}{\sqrt{x^2+y_{\max}^2}}-\frac{y_{\min}}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)/\left(6.50\,\text{m}\right)}{\left(0.250\,\text{m}\right)}$$

$$\left(\frac{2.50\,\text{m}}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(2.50\,\text{m}\right)^2}}-\frac{\left(-4.00\,\text{m}\right)}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(-4.00\,\text{m}\right)^2}}\right)$$

$$= 3.473\times10^4\,\text{N/C} \approx \boxed{3.5\times10^4\,\text{N/C}}$$

$$E_y = -\int_{y_{\min}}^{y_{\max}} \frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\int_{y_{\min}}^{y_{\max}}\frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_{\max}^2}}-\frac{1}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\text{C}\right)}{(6.50\,\text{m})}$$

$$\left(\frac{1}{\sqrt{(0.250\,\text{m})^2+(2.50\,\text{m})^2}}-\frac{1}{\sqrt{(0.250\,\text{m})^2+(-4.00\,\text{m})^2}}\right)$$

$$= 647\,\text{N/C} \approx \boxed{650\,\text{N/C}}$$

(*b*) We calculate the infinite line of charge result, and calculate the errors.

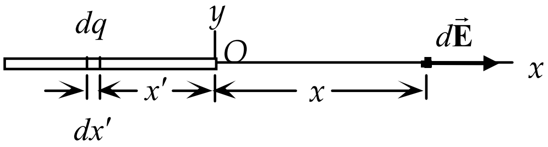
$$E = \frac{\lambda}{2\pi\varepsilon_0 x} = \frac{2\lambda}{4\pi\varepsilon_0 x} = 2\left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\text{C}\right)}{(6.50\,\text{m})(0.250\,\text{m})} = 3.485\times10^4\,\text{N/m}$$

$$\frac{E_x - E}{E} = \frac{\left(3.473\times10^4\,\text{N/C}\right)-\left(3.485\times10^4\,\text{N/m}\right)}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{-0.0034}$$

$$\frac{E_y}{E} = \frac{(647\,\text{N/C})}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{0.019}$$

And so we see that E_x is only about 0.3% away from the value obtained from the infinite line of charge, and E_y is only about 2% of the value obtained from the infinite line of charge. The field of an infinite line of charge result would be a good approximation for the field due to this wire segment.

53. Choose a differential element of the rod dx' a distance x' from the origin, as shown in the diagram. The charge on that differential element is $dq = \frac{Q}{\ell}dx'$. The variable x' is treated as positive,



so that the field due to this differential element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x+x')^2} = \frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{(x+x')^2}$. Integrate along the rod to find the total field.

$$E = \int dE = \int_0^{\ell}\frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{(x+x')^2} = \frac{Q}{4\pi\varepsilon_0\ell}\int_0^{\ell}\frac{dx'}{(x+x')^2} = \frac{Q}{4\pi\varepsilon_0\ell}\left(-\frac{1}{x+x'}\right)_0^{\ell} = \frac{Q}{4\pi\varepsilon_0\ell}\left(\frac{1}{x}-\frac{1}{x+\ell}\right)$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 x(x+\ell)}}$$

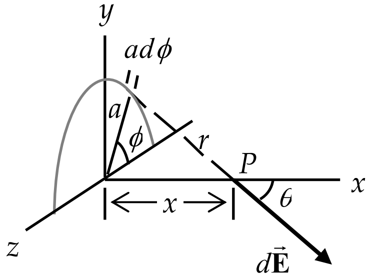
54. As suggested, we divide the plane into long narrow strips of width dy and length ℓ. The charge on the strip is the area of the strip times the charge per unit area: $dq = \sigma \ell dy$. The charge per unit length on the strip is $\lambda = \frac{dq}{\ell} = \sigma dy$. From Example 21-11, the field due to that narrow strip is $dE = \frac{\lambda}{2\pi\varepsilon_0\sqrt{y^2+z^2}} = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}}$. From Figure 21-68 in the textbook, we see that this field does not point vertically. From the symmetry of the plate, there is another long narrow strip a distance y on the other side of the origin, which would create the same magnitude electric field. The horizontal components of those two fields would cancel each other, and so we only need calculate the vertical component of the field. Then we integrate along the y direction to find the total field.

$$dE = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}} \quad ; \; dE_z = dE\cos\theta = \frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)}$$

$$E = E_z = \int_{-\infty}^{\infty}\frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dy}{\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\frac{1}{z}\left(\tan^{-1}\frac{y}{z}\right)_{-\infty}^{\infty}$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = \frac{\sigma}{2\pi\varepsilon_0}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

$\boxed{55.}$ Take Figure 21-28 and add the angle ϕ, measured from the $-z$ axis, as indicated in the diagram. Consider an infinitesimal length of the ring $ad\phi$. The charge on that infinitesimal length is $dq = \lambda\left(ad\phi\right) = \frac{Q}{\pi a}\left(ad\phi\right) = \frac{Q}{\pi}d\phi$. The charge creates an infinitesimal electric field, $d\vec{\mathbf{E}}$, with magnitude $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{\pi}d\phi}{x^2+a^2}$. From the symmetry of the figure, we see that the z component of $d\vec{\mathbf{E}}$ will be cancelled by the z component due to the piece of the ring that is on the opposite side of the y axis. The trigonometric relationships give $dE_x = dE\cos\theta$ and $dE_y = -dE\sin\theta\sin\phi$. The factor of $\sin\phi$ can be justified by noting that $dE_y = 0$ when $\phi = 0$, and $dE_y = -dE\sin\theta$ when $\phi = \pi/2$.



$$dE_x = dE\cos\theta = \frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{Qx}{4\pi^2\varepsilon_0}\frac{d\phi}{\left(x^2+a^2\right)^{3/2}}$$

$$E_x = \frac{Qx}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi}d\phi = \boxed{\frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

$$dE_y = -dE\sin\theta\sin\phi = -\frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{a}{\sqrt{x^2+a^2}}\sin\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\sin\phi d\phi$$

$$E_y = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi}\sin\phi d\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left[\left(-\cos\pi\right) - \left(-\cos 0\right)\right]$$

$$= \boxed{-\frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

We can write the electric field in vector notation.

$$\vec{\mathbf{E}} = \frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{i}} - \frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{j}} = \boxed{\frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left(x\hat{\mathbf{i}} - \frac{2a}{\pi}\hat{\mathbf{j}}\right)}$$

56. (*a*) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use constant acceleration relationships with a final velocity of 0.

$$F = ma = qE = -eE \;\rightarrow\; a = -\frac{eE}{m} \;;\; v^2 = v_0^2 + 2a\Delta x = 0 \;\rightarrow$$

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\dfrac{eE}{m}\right)} = \frac{mv_0^2}{2eE} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{0.189\,\text{m}}$$

(*b*) Find the elapsed time from constant acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity.

$$v = v_0 + at \;\rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-2v_0}{\left(-\dfrac{eE}{m}\right)} = \frac{2mv_0}{eE} = \frac{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{2.75\times10^{-8}\,\text{s}}$$

57. (*a*) The acceleration is produced by the electric force.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}} \;\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{e}{m}\vec{\mathbf{E}} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}\left[\left(2.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right)\times10^4\,\text{N/C}\right] = \left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2$$

$$\approx \boxed{-3.5\times10^{15}\,\text{m/s}^2\,\hat{\mathbf{i}} - 1.4\times10^{16}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

(*b*) The direction is found from the components of the velocity.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t = \left(8.0\times10^4\,\text{m/s}\right)\hat{\mathbf{j}} + \left[\left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2\right]\left(1.0\times10^{-9}\,\text{s}\right)$$

$$= \left(-3.513\times10^6\hat{\mathbf{i}} - 1.397\times10^7\hat{\mathbf{j}}\right)\text{m/s}$$

$$\tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-1.397\times10^7\,\text{m/s}}{-3.513\times10^6\,\text{m/s}}\right) = 256^\circ\,\text{or} - 104^\circ$$

This is the direction relative to the *x* axis. The direction of motion relative to the initial direction is measured from the *y* axis, and so is $\boxed{\theta = 166^\circ\text{ counter-clockwise}}$ from the initial direction.

58. (*a*) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the $\boxed{\text{right}}$.

(*b*) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \;\rightarrow\; a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \;\rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.109\times10^{-31}\,\text{kg}\right)\left(7.5\times10^5\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(0.040\,\text{m}\right)} = \boxed{40\,\text{N/C}}\ \left(2\text{ sig. fig.}\right)$$

59. The angle is determined by the velocity. The x component of the velocity is constant. The time to pass through the plates can be found from the x motion. Then the y velocity can be found using constant acceleration relationships.

$$x = v_0 t \;\rightarrow\; t = \frac{x}{v_0} \;;\; v_y = v_{y0} + a_y t = -\frac{eE}{m}\frac{x}{v_0}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-\dfrac{eE}{m}\dfrac{x}{v_0}}{v_0} = -\frac{eEx}{mv_0^2} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(5.0\times10^3\,\text{N/C}\right)\left(0.049\,\text{m}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.00\times10^7\,\text{m/s}\right)^2} = -.4303 \;\rightarrow$$

$$\theta = \tan^{-1}\left(-0.4303\right) = \boxed{-23^\circ}$$

60. Since the field is constant, the force on the electron is constant, and so the acceleration is constant. Thus constant acceleration relationships can be used. The initial conditions are $x_0 = 0,\ y_0 = 0,$ $v_{x0} = 1.90\,\text{m/s}$, and $v_{y0} = 0.$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \;\rightarrow\; \vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}} = -\frac{e}{m}\vec{\mathbf{E}} \;;\; a_x = -\frac{e}{m}E_x \;,\; a_y = -\frac{e}{m}E_y$$

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t - \frac{eE_x}{2m}t^2$$

$$= \left(1.90\,\text{m/s}\right)\left(2.0\,\text{s}\right) - \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.00\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{-3.2\,\text{m}}$$

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = -\frac{eE_y}{2m}t^2 = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(-1.20\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{4.2\,\text{m}}$$

$\boxed{61.}$ (*a*) The field along the axis of the ring is given in Example 21-9, with the opposite sign because this ring is negatively charged. The force on the charge is the field times the charge q. Note that if x is positive, the force is to the left, and if x is negative, the force is to the right. Assume that $x \ll R$.

$$F = qE = \frac{q}{4\pi\varepsilon_0}\frac{(-Q)x}{\left(x^2 + R^2\right)^{3/2}} = \frac{-qQx}{4\pi\varepsilon_0}\frac{1}{\left(x^2 + R^2\right)^{3/2}} \approx \frac{-qQx}{4\pi\varepsilon_0 R^3}$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{4\pi\varepsilon_0 R^3}$.

(*b*) The spring constant can be used to find the period. See Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\dfrac{Qq}{4\pi\varepsilon_0 R^3}}} = 2\pi\sqrt{\frac{m4\pi\varepsilon_0 R^3}{Qq}} = 4\pi\sqrt{\frac{m\pi\varepsilon_0 R^3}{Qq}}$$

62. (*a*) The dipole moment is given by the product of the positive charge and the separation distance.

$$p = Q\ell = \left(1.60\times10^{-19}\,\text{C}\right)\left(0.68\times10^{-9}\,\text{m}\right) = 1.088\times10^{-28}\,\text{C•m} \approx \boxed{1.1\times10^{-28}\,\text{C•m}}$$

(*b*) The torque on the dipole is given by Eq. 21-9a.

$$\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C•m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left(\sin 90^\circ\right) = \boxed{2.4\times10^{-24}\,\text{C•m}}$$

(*c*) $\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C•m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left(\sin 45^\circ\right) = \boxed{1.7\times10^{-24}\,\text{N•m}}$

(*d*) The work done by an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(1.088\times10^{-28}\,\text{C•m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left[1-(-1)\right] = \boxed{4.8\times10^{-24}\,\text{J}}$$

63. (*a*) The dipole moment is the effective charge of each atom times the separation distance.

$$p = Q\ell \ \rightarrow\ Q = \frac{p}{\ell} = \frac{3.4\times10^{-30}\,\text{C•m}}{1.0\times10^{-10}\,\text{m}} = \boxed{3.4\times10^{-20}\,\text{C}}$$

(*b*) $\dfrac{Q}{e} = \dfrac{3.4\times10^{-20}\,\text{C}}{1.60\times10^{-19}\,\text{C}} = 0.21$ $\boxed{\text{No}}$, the net charge on each atom is not an integer multiple of *e*. This is an indication that the H and Cl atoms are not ionized – they haven't fully gained or lost an electron. But rather, the electrons spend more time near the Cl atom than the H atom, giving the molecule a net dipole moment. The electrons are not distributed symmetrically about the two nuclei.

(*c*) The torque is given by Eq. 21-9a.

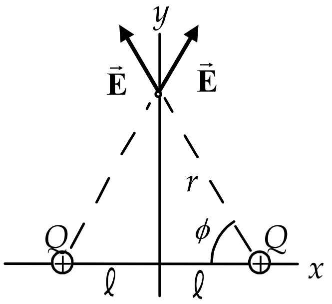
$$\tau = pE\sin\theta \ \rightarrow\ \tau_{\max} = pE = \left(3.4\times10^{-30}\,\text{C•m}\right)\left(2.5\times10^{4}\ \text{N/C}\right) = \boxed{8.5\times10^{-26}\,\text{N•m}}$$

(*d*) The energy needed from an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(3.4\times10^{-30}\,\text{C•m}\right)\left(2.5\times10^{4}\ \text{N/C}\right)\left[1-\cos 45^\circ\right] = \boxed{2.5\times10^{-26}\,\text{J}}$$

64. (*a*) From the symmetry in the diagram, we see that the resultant field will be in the *y* direction. The vertical components of the two fields add together, while the horizontal components cancel.



$$E_{\text{net}} = 2E\sin\phi = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}}$$

$$= \frac{2Qr}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2}} \approx \frac{2Qr}{4\pi\varepsilon_0\left(r^3\right)} = \boxed{\frac{2Q}{4\pi\varepsilon_0 r^2}}$$

(*b*) Both charges are the same sign. A long distance away from the charges, they will look like a single charge of magnitude $2Q$, and so $E = k\dfrac{q}{r^2} = \dfrac{2Q}{4\pi\varepsilon_0 r^2}$.

65. (*a*) There will be a torque on the dipole, in a direction to decrease θ. That torque will give the dipole an angular acceleration, in the opposite direction of θ.

$$\tau = -pE\sin\theta = I\alpha \ \rightarrow\ \alpha = \frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta$$

If $\boxed{\theta \text{ is small}}$, so that $\sin\theta \approx \theta$, then the equation is in the same form as Eq. 14-3, the equation of motion for the simple harmonic oscillator.

$$\frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta \approx -\frac{pE}{I}\theta \quad \to \quad \frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$$

(*b*) The frequency can be found from the coefficient of θ in the equation of motion.

$$\omega^2 = \frac{pE}{I} \quad \to \quad f = \boxed{\frac{1}{2\pi}\sqrt{\frac{pE}{I}}}$$

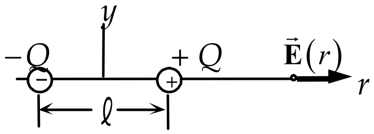
66. If the dipole is of very small extent, then the potential energy is a function of position, and so Eq. 21-10 gives $U(x) = -\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)$. Since the potential energy is known, we can use Eq. 8-7.

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[-\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)\right] = \vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}$$

Since the field does not depend on the *y* or *z* coordinates, all other components of the force will be 0.

Thus $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} = \boxed{\left(\vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}\right)\hat{\mathbf{i}}}$.

$\boxed{67.}$ (*a*) Along the *x* axis the fields from the two charges are parallel so the magnitude is found as follows.



$$E_{\text{net}} = E_{+Q} + E_{-Q} = \frac{Q}{4\pi\varepsilon_0\left(r-\frac{1}{2}\ell\right)^2} + \frac{(-Q)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q\left[\left(r+\frac{1}{2}\ell\right)^2 - \left(r-\frac{1}{2}\ell\right)^2\right]}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q(2r\ell)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2} \approx \frac{Q(2r\ell)}{4\pi\varepsilon_0 r^4} = \frac{2Q\ell}{4\pi\varepsilon_0 r^3} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

The same result is obtained if the point is to the left of −*Q*.

(*b*) The electric field points in the $\boxed{\text{same direction as the dipole}}$ moment vector.

68. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\text{E}} = F_{\text{G}} \quad \to \quad k\frac{e^2}{r^2} = mg \quad \to$$

$$r = e\sqrt{\frac{k}{mg}} = \left(1.602\times10^{-19}\,\text{C}\right)\sqrt{\frac{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}} = \boxed{5.08\,\text{m}}$$

69. Water has an atomic mass of 18, so 1 mole of water molecules has a mass of 18 grams. Each water molecule contains 10 protons.

$$65\text{kg}\left(\frac{6.02\times10^{23}\,\text{H}_2\text{O molecules}}{0.018\,\text{kg}}\right)\left(\frac{10\,\text{protons}}{1\text{ molecule}}\right)\left(\frac{1.60\times10^{-19}\,\text{C}}{\text{proton}}\right) = \boxed{3.5\times10^9\,\text{C}}$$

70. Calculate the total charge on all electrons in 3.0 g of copper, and compare $38\mu\text{C}$ to that value.

$$\text{Total electron charge} = 3.0\,\text{g}\left(\frac{1\,\text{mole}}{63.5\,\text{g}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{\text{mole}}\right)\left(\frac{29\,\text{e}}{\text{atoms}}\right)\left(\frac{1.602\times10^{-19}\,\text{C}}{1\,\text{e}}\right) = 1.32\times10^{5}\,\text{C}$$

$$\text{Fraction lost} = \frac{38\times10^{-6}\,\text{C}}{1.32\times10^{5}\,\text{C}} = \boxed{2.9\times10^{-10}}$$

71. Use Eq. 21-4a to calculate the magnitude of the electric charge on the Earth.

$$E = k\frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{(150\ \text{N/C})(6.38\times10^{6}\,\text{m})^2}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\,\text{C}}$$

Since the electric field is pointing towards the Earth's center, the charge must be $\boxed{\text{negative}}$.

72. (*a*) From problem 71, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_{\text{E}} = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\,\text{kg})} = 2.638\times10^{13}\ \text{m/s}^2 \approx \boxed{2.6\times10^{13}\ \text{m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downwards force of magnitude $F_{\text{E}} = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(1.67\times10^{-27}\,\text{kg})} = 1.439\times10^{10}\ \text{m/s}^2 \approx \boxed{1.4\times10^{10}\ \text{m/s}^2\text{, down}}$$

(*c*) Electron: $\frac{a}{g} = \frac{2.638\times10^{13}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{2.7\times10^{12}}$; Proton: $\frac{a}{g} = \frac{1.439\times10^{10}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{1.5\times10^{9}}$

$\boxed{73.}$ For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_{\text{E}} = |q|E = mg \rightarrow neE = \tfrac{4}{3}\pi r^3\rho g \rightarrow$$

$$n = \frac{4\pi r^3\rho g}{3eE} = \frac{4\pi(1.8\times10^{-5}\,\text{m})^3(1.00\times10^{3}\ \text{kg/m}^3)(9.80\,\text{m/s}^2)}{3(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\,\text{electrons}}$$

74. There are four forces to calculate. Call the rightward direction the positive direction. The value of *k* is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of *e* is $1.602\times10^{-19}\,\text{C}$.

$$F_{\text{net}} = F_{\text{CH}} + F_{\text{CN}} + F_{\text{OH}} + F_{\text{ON}} = \frac{k(0.40e)(0.20e)}{(1\times10^{-9}\,\text{m})^2}\left[-\frac{1}{(0.30)^2}+\frac{1}{(0.40)^2}+\frac{1}{(0.18)^2}-\frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

75. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_{\mathrm{E}} = F_{\mathrm{G}} \quad \rightarrow \quad k\frac{Q^2}{r_{\mathrm{orbit}}^2} = G\frac{M_{\mathrm{Moon}}M_{\mathrm{Earth}}}{r_{\mathrm{orbit}}^2} \quad \rightarrow$$

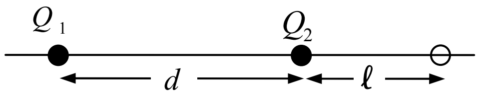
$$Q = \sqrt{\frac{GM_{\mathrm{Moon}}M_{\mathrm{Earth}}}{k}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\left(7.35\times10^{22}\,\mathrm{kg}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)}} = \boxed{5.71\times10^{13}\,\mathrm{C}}$$

76. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\mathrm{E}} = F_{\mathrm{radial}} \quad \rightarrow \quad k\frac{Q^2}{r_{\mathrm{orbit}}^2} = \frac{mv^2}{r_{\mathrm{orbit}}} \quad \rightarrow$$

$$r_{\mathrm{orbit}} = k\frac{Q^2}{mv^2} = \left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{\left(9.109\times10^{-31}\,\mathrm{kg}\right)\left(2.2\times10^{6}\,\mathrm{m/s}\right)^2} = \boxed{5.2\times10^{-11}\,\mathrm{m}}$$

77. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (Q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that ℓ must be positive.



$$E = -k\frac{|Q_2|}{\ell^2} + k\frac{Q_1}{(\ell+d)^2} = 0 \quad \rightarrow \quad |Q_2|(\ell+d)^2 = Q_1\ell^2 \quad \rightarrow$$

$$\ell = \frac{\sqrt{|Q_2|}}{\sqrt{Q_1}-\sqrt{|Q_2|}}d = \frac{\sqrt{5.0\times10^{-6}\,\mathrm{C}}}{\sqrt{2.5\times10^{-5}\,\mathrm{C}}-\sqrt{5.0\times10^{-6}\,\mathrm{C}}}(2.0\,\mathrm{m}) = \boxed{\begin{array}{l}1.6\,\mathrm{m\ from\ }Q_2,\\ 3.6\,\mathrm{m\ from\ }Q_1\end{array}}$$

78. We consider that the sock is only acted on by two forces – the force of gravity, acting downward, and the electrostatic force, acting upwards. If charge Q is on the sweater, then it will create an electric field of $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{Q/A}{2\varepsilon_0}$, where A is the surface area of one side of the sweater. The same magnitude of charge will be on the sock, and so the attractive force between the sweater and sock is $F_E = QE = \dfrac{Q^2}{2\varepsilon_0 A}$. This must be equal to the weight of the sweater. We estimate the sweater area as 0.10 m^2, which is roughly a square foot.

$$F_E = QE = \frac{Q^2}{2\varepsilon_0 A} = mg \quad \rightarrow$$

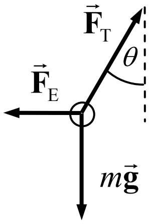
$$Q = \sqrt{2\varepsilon_0 Amg} = \sqrt{2\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(0.10\,\mathrm{m^2}\right)\left(0.040\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{8\times10^{-7}\,\mathrm{C}}$$

$\boxed{79.}$ The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega = \sqrt{k/m} = \sqrt{126\,\mathrm{N/m}/0.650\,\mathrm{kg}} = 13.92\,\mathrm{rad/s}$. The distance of the sphere from the table is given by $r = \left[0.150 - 0.0500\cos(13.92t)\right]\mathrm{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E = k\frac{|Q|}{r^2} = \frac{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-6}\,\mathrm{C}\right)}{\left[0.150-0.0500\cos\left(13.92t\right)\right]^2\,\mathrm{m}^2} = \frac{2.70\times10^4}{\left[0.150-0.0500\cos\left(13.92t\right)\right]^2}\,\mathrm{N/C}$$

$$= \boxed{\frac{1.08\times10^7}{\left[3.00-\cos\left(13.9t\right)\right]^2}\,\mathrm{N/C},\ \text{upwards}}$$

80. The wires form two sides of an isosceles triangle, and so the two charges are separated by a distance $\ell = 2\left(78\,\mathrm{cm}\right)\sin 26^\circ = 68.4\,\mathrm{cm}$ and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.



$$\sum F_y = F_\mathrm{T}\cos\theta - mg = 0 \ \rightarrow\ F_\mathrm{T} = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_\mathrm{T}\sin\theta - F_\mathrm{E} = 0 \ \rightarrow\ F_\mathrm{E} = F_\mathrm{T}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$F_\mathrm{E} = k\frac{\left(Q/2\right)^2}{\ell^2} = mg\tan\theta \ \rightarrow\ Q = 2\ell\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\left(0.684\,\mathrm{m}\right)\sqrt{\frac{\left(24\times10^{-3}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan 26^\circ}{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)}} = 4.887\times10^{-6}\,\mathrm{C} \approx \boxed{4.9\times10^{-6}\,\mathrm{C}}$$

81. The electric field at the surface of the pea is given by Eq. 21-4a. Solve that equation for the charge.

$$E = k\frac{Q}{r^2} \ \rightarrow\ Q = \frac{Er^2}{k} = \frac{\left(3\times10^6\,\mathrm{N/C}\right)\left(3.75\times10^{-3}\,\mathrm{m}\right)^2}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}} = \boxed{5\times10^{-9}\,\mathrm{C}}$$

This corresponds to about 3 billion electrons.

82. There will be a rightward force on Q_1 due to Q_2, given by Coulomb's law. There will be a leftward force on Q_1 due to the electric field created by the parallel plates. Let right be the positive direction.

$$\sum F = k\frac{|Q_1Q_2|}{x^2} - |Q_1|E$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.7\times10^{-6}\,\mathrm{C}\right)\left(1.8\times10^{-6}\,\mathrm{C}\right)}{\left(0.34\,\mathrm{m}\right)^2} - \left(6.7\times10^{-6}\,\mathrm{C}\right)\left(7.3\times10^4\,\mathrm{N/C}\right)$$

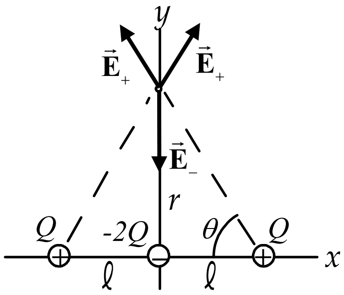
$$= \boxed{0.45\,\mathrm{N},\ \text{right}}$$

83. The weight of the sphere is the density times the volume. The electric force is given by Eq. 21-1, with both spheres having the same charge, and the separation distance equal to their diameter.

$$mg = k\frac{Q^2}{(d)^2} \rightarrow \rho\tfrac{4}{3}\pi r^3 g = \frac{kQ^2}{(2r)^2} \rightarrow$$

$$Q=\sqrt{\frac{16\rho\pi g r^5}{3k}} = \sqrt{\frac{16\left(35\,\mathrm{kg/m^3}\right)\pi\left(9.80\,\mathrm{m/s^2}\right)\left(1.0\times10^{-2}\,\mathrm{m}\right)^5}{3\left(8.99\times10^9\,\mathrm{N\bullet m^2/C^2}\right)}} = \boxed{8.0\times10^{-9}\,\mathrm{C}}$$

84. From the symmetry, we see that the resultant field will be in the y direction. So we take the vertical component of each field.



$$E_{\text{net}} = 2E_+\sin\theta - E_- = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}} - \frac{2Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{2Q}{4\pi\varepsilon_0}\left[\frac{r}{\left(r^2+\ell^2\right)^{3/2}} - \frac{1}{r^2}\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2}r^2}\left[r^3-\left(r^2+\ell^2\right)^{3/2}\right]$$

$$= \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}}$$

Use the binomial expansion, assuming $r \gg \ell$.

$$E_{\text{net}} = \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}} \approx \frac{2Qr^3\left[1-\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)} = \frac{2Qr^3\left(-\frac{3}{2}\frac{\ell^2}{r^2}\right)}{4\pi\varepsilon_0 r^5(1)} = \boxed{-\frac{3Q\ell^2}{4\pi\varepsilon_0 r^4}}$$

Notice that the field points toward the negative charges.

85. This is a constant acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be ℓ, the vertical gap between the plates be h, and the initial velocity be v_0. Notice that the vertical motion has a maximum displacement of $h/2$. Let upwards be the positive vertical direction. We calculate the vertical acceleration produced by the electric field and the time t for the electron to cross the region of the field. We then use constant acceleration equations to solve for the angle.

$$F_y = ma_y = qE = -eE \rightarrow a_y = -\frac{eE}{m} \;;\; \ell = v_0\cos\theta_0(t) \rightarrow t = \frac{\ell}{v_0\cos\theta_0}$$

$$v_{y_{\text{top}}} = v_{0y} + a_y t_{\text{top}} \rightarrow 0 = v_0\sin\theta_0 - \frac{eE}{m}\left(\frac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) \rightarrow v_0^2 = \frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)$$

$$y_{\text{top}} = y_0 + v_{0y}t_{\text{top}} + \tfrac{1}{2}a_y t^2 \;\rightarrow\; \tfrac{1}{2}h = v_0\sin\theta_0\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) - \tfrac{1}{2}\frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right)^2 \;\rightarrow$$

$$h = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1\;\;^{2}}{v_0^2} = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)} = \ell\tan\theta_0 - \tfrac{1}{2}\ell\tan\theta_0$$

$$h = \tfrac{1}{2}\ell\tan\theta_0 \;\rightarrow\; \theta_0 = \tan^{-1}\frac{2h}{\ell} = \tan^{-1}\frac{2(1.0\,\text{cm})}{6.0\,\text{cm}} = \boxed{18^\circ}$$

86. (*a*) The electric field from the long wire is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}, \text{radially away from the wire}$$

(*b*) The force on the electron will point radially in, producing a centripetal acceleration.

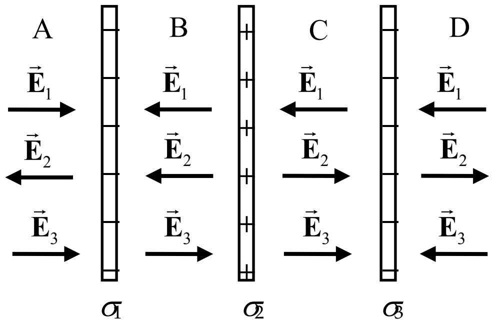
$$|F| = |qE| = \frac{e}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{mv^2}{r} \;\rightarrow$$

$$v = \sqrt{2\frac{1}{4\pi\varepsilon_0}\frac{e\lambda}{m}} = \sqrt{2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\text{C}\right)\left(0.14\times10^{-6}\,\text{C/m}\right)}{9.11\times10^{-31}\text{kg}}}$$

$$= \boxed{2.1\times10^7\,\text{m/s}}$$

Note that this speed is independent of *r*.

87. We treat each of the plates as if it were infinite, and then use Eq. 21-7. The fields due to the first and third plates point towards their respective plates, and the fields due to the second plate point away from it. See the diagram. The directions of the fields are given by the arrows, so we calculate the magnitude of the fields from Eq. 21-7. Let the positive direction be to the right.



$$E_A = E_1 - E_2 + E_3 = \frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{3.4\times10^4\,\text{N/C, to the right}}$$

$$E_B = -E_1 - E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(-0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = -2.3\times10^4\,\text{N/C} = \boxed{2.3\times10^4\,\text{N/C to the left}}$$

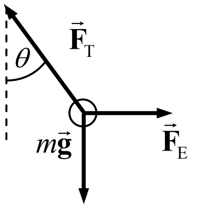
$$E_C = -E_1 + E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

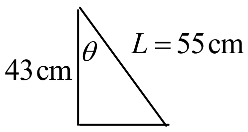
$$= \frac{(-0.50 + 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{5.6\times10^3\,\text{N/C to the right}}$$

$$E_{\mathrm{D}} = -E_1 + E_2 - E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(-0.50 + 0.25 - 0.35)\times 10^{-6}\,\mathrm{C/m^2}}{2\left(8.85\times 10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)} = -3.4\times 10^{4}\,\mathrm{N/C} = \boxed{3.4\times 10^{3}\,\mathrm{N/C}\text{ to the left}}$$

88. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is $\boxed{\text{positive}}$. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.





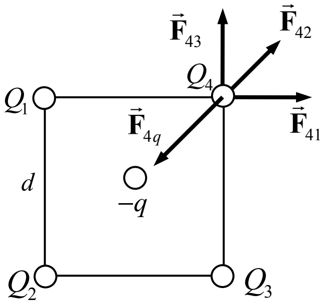
$$\theta = \cos^{-1}\frac{43\,\mathrm{cm}}{55\,\mathrm{cm}} = 38.6^{\circ}$$

$$\sum F_x = F_{\mathrm{E}} - F_{\mathrm{T}}\sin\theta = 0 \;\rightarrow\; F_{\mathrm{E}} = F_{\mathrm{T}}\sin\theta = QE$$

$$\sum F_y = F_{\mathrm{T}}\cos\theta - mg = 0 \;\rightarrow\; F_{\mathrm{T}} = \frac{mg}{\cos\theta} \;\rightarrow\; QE = mg\tan\theta$$

$$Q = \frac{mg\tan\theta}{E} = \frac{\left(1.0\times 10^{-3}\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan 38.6^{\circ}}{\left(1.5\times 10^{4}\,\mathrm{N/C}\right)} = \boxed{5.2\times 10^{-7}\mathrm{C}}$$

89. A negative charge must be placed at the center of the square. Let $Q = 8.0\,\mu\mathrm{C}$ be the charge at each corner, let $-q$ be the magnitude of negative charge in the center, and let $d = 9.2\,\mathrm{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.



$$F_{41} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{41x} = k\frac{Q^2}{d^2}\,,\; F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \;\rightarrow\; F_{42x} = k\frac{Q^2}{2d^2}\cos 45^{\circ} = k\frac{\sqrt{2}Q^2}{4d^2}\,,\; F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \;\rightarrow\; F_{43x} = 0\,,\; F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \;\rightarrow\; F_{4qx} = -k\frac{2qQ}{d^2}\cos 45^{\circ} = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \;\rightarrow$$

$$q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = \left(8.0\times 10^{-6}\mathrm{C}\right)\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times 10^{-6}\mathrm{C}$$

So the charge to be placed is $-q = \boxed{-7.7\times 10^{-6}\mathrm{C}}$.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.

90. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_{\mathrm{AB}} = \boxed{\frac{kQ^2}{R^2}\text{, away from B}}$$

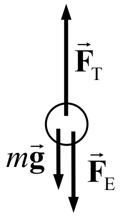
(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_{\mathrm{AB}} = k\frac{QQ/2}{R^2} = \boxed{\frac{kQ^2}{2R^2}\text{, away from B}}$$

(*c*) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

$$F_{\mathrm{AB}} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}\text{, away from B}}$$

$\boxed{91.}$ (*a*) The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed $\boxed{\text{down}}$. Use the free-body diagram to write an expression for the magnitude of the electric field.



$$\sum F = F_{\mathrm{T}} - mg - F_{\mathrm{E}} = 0 \;\rightarrow\; F_{\mathrm{E}} = QE = F_{\mathrm{T}} - mg \;\rightarrow$$

$$E = \frac{F_{\mathrm{T}} - mg}{Q} = \frac{5.18\,\mathrm{N} - (0.210\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{3.40\times10^{-7}\,\mathrm{C}} = \boxed{9.18\times10^{6}\,\mathrm{N/C}}$$

(*b*) Use Eq. 21-7.

$$E = \frac{\sigma}{2\varepsilon_0} \;\rightarrow\; \sigma = 2E\varepsilon_0 = 2(9.18\times10^{6}\,\mathrm{N/C})(8.854\times10^{-12}) = \boxed{1.63\times10^{-4}\,\mathrm{C/m^2}}$$

92. (*a*) The force will be attractive. Each successive charge is another distance *d* farther than the previous charge. The magnitude of the charge on the electron is *e*.

$$F = k\frac{eQ}{(d)^2} + k\frac{eQ}{(2d)^2} + k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)$$

$$= k\frac{eQ}{d^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{4\pi\varepsilon_0}\frac{eQ}{d^2}\frac{\pi^2}{6} = \boxed{\frac{\pi eQ}{24\varepsilon_0 d^2}}$$

(*b*) Now the closest *Q* is a distance of 3*d* from the electron.

$$F = k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + k\frac{eQ}{(5d)^2} + k\frac{eQ}{(6d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots\right)$$

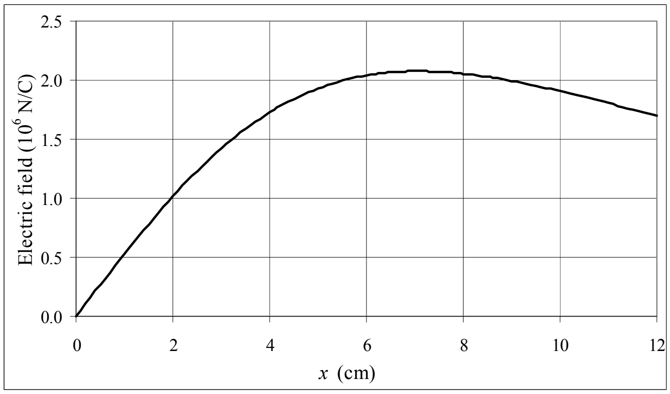
$$= k\frac{eQ}{d^2}\sum_{n=3}^{\infty}\frac{1}{n^2} = k\frac{eQ}{d^2}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right) - \frac{1}{1^2} - \frac{1}{2^2}\right] = k\frac{eQ}{d^2}\left[\frac{\pi^2}{6} - \frac{5}{4}\right] = \boxed{\frac{eQ}{4\pi\varepsilon_0 d^2}\left[\frac{\pi^2}{6} - \frac{5}{4}\right]}$$

93. (*a*) Take $\dfrac{dE}{dx}$, set it equal to 0, and solve for the location of the maximum.

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$$

$$\frac{dE}{dx}=\frac{Q}{4\pi\varepsilon_0}\left[\frac{\left(x^2+a^2\right)^{3/2}-x\frac{3}{2}\left(x^2+a^2\right)^{1/2}2x}{\left(x^2+a^2\right)^3}\right]=\frac{Q}{4\pi\varepsilon_0}\frac{\left(a^2-2x^2\right)}{\left(x^2+a^2\right)^{5/2}}=0\ \rightarrow\ a^2-2x^2=0\ \rightarrow$$

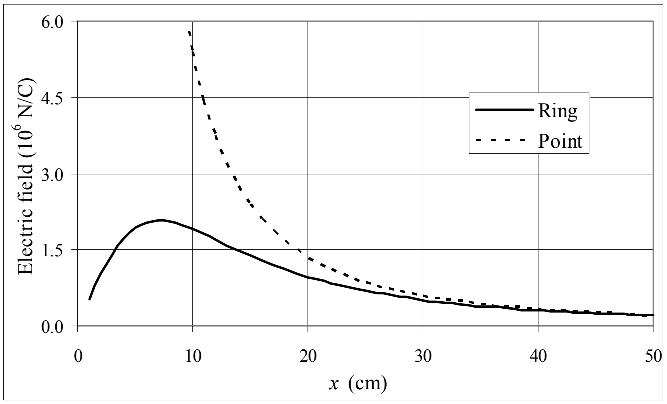
$$x=\frac{a}{\sqrt{2}}=\frac{10.0\,\text{cm}}{\sqrt{2}}=\boxed{7.07\,\text{cm}}$$



(*b*) $\boxed{\text{Yes}}$, the maximum of the graph does coincide with the analytic maximum. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93b."

(*c*) The field due to the ring is $E_{\text{ring}}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{\left(x^2+a^2\right)^{3/2}}$.

(*d*) The field due to the point charge is $E_{\text{ring}}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x^2}$. Both are plotted on the graph. The graph shows that the two fields converge at large distances from the origin. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93cd."

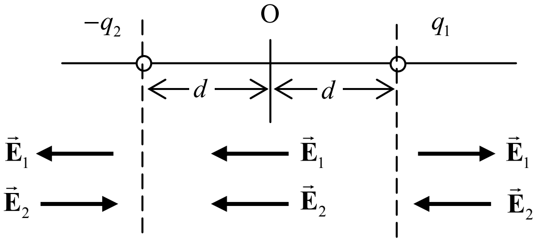


(*e*) According to the spreadsheet, $E_{\text{ring}}=0.9E_{\text{point}}$ at about $\boxed{37\text{ cm}}$. An analytic calculation gives the same result.

$$E_{\text{ring}}=0.9E_{\text{point}}\ \rightarrow\ \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}=0.9\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}\ \rightarrow$$

$$x^3=0.9\left(x^2+a^2\right)^{3/2}=0.9x^3\left(1+\frac{a^2}{x^2}\right)^{3/2}\ \rightarrow\ x=\frac{a}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}}=\frac{10.0\,\text{cm}}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}}=37.07\,\text{cm}$$

94. (*a*) Let $q_1 = 8.00\mu\text{C}$, $q_2 = 2.00\mu\text{C}$, and $d = 0.0500\,\text{m}$. The field directions due to the charges are shown in the diagram. We take care with the signs of the *x* coordinate used to calculate the magnitude of the field.

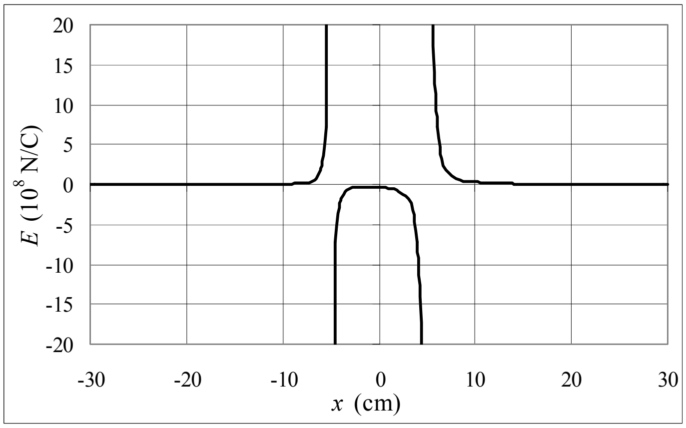


$$E_{x<-d} = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(|x|-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(-x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

$$E_{-d<x<0} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d-|x|\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

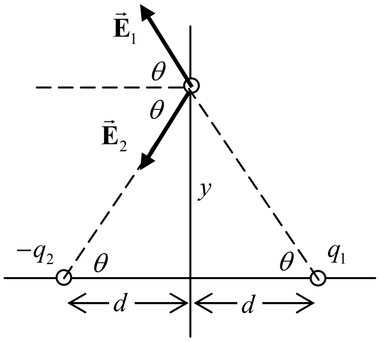
$$E_{0<x<d} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(d-x\right)^2}$$

$$E_{d<x} = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(x+d\right)^2}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94a."

(*b*) Now for points on the y axis. See the diagram for this case.



$$E_x = -E_1\cos\theta - E_2\cos\theta$$

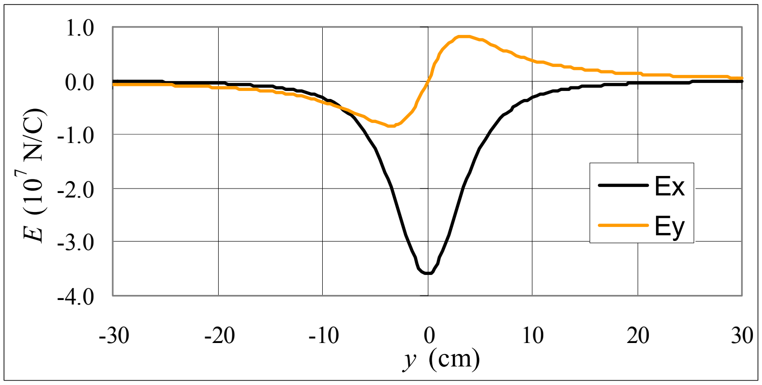
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1\cos\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\cos\theta}{\left(d^2+y^2\right)}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\frac{d}{\sqrt{d^2+y^2}}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)d}{\left(d^2+y^2\right)^{3/2}}$$

$$E_y = E_1\sin\theta - E_2\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q_1\sin\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\sin\theta}{\left(d^2+y^2\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\frac{y}{\sqrt{d^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)y}{\left(d^2+y^2\right)^{3/2}}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94b."

CHAPTER 22: Gauss's Law

Responses to Questions

$\boxed{1.}$ No. If the net electric flux through a surface is zero, then the net charge contained in the surface is zero. However, there may be charges both inside and outside the surface that affect the electric field at the surface. The electric field could point outward from the surface at some points and inward at others. Yes. If the electric field is zero for all points on the surface, then the net flux through the surface must be zero and no net charge is contained within the surface.

2. No. The electric field in the expression for Gauss's law refers to the *total* electric field, not just the electric field due to any enclosed charge. Notice, though, that if the electric field is due to a charge outside the Gaussian surface, then the net flux through the surface due to this charge will be zero.

3. The electric flux will be the same. The flux is equal to the net charge enclosed by the surface divided by ε_0. If the same charge is enclosed, then the flux is the same, regardless of the shape of the surface.

4. The net flux will be zero. An electric dipole consists of two charges that are equal in magnitude but opposite in sign, so the net charge of an electric dipole is zero. If the closed surface encloses a zero net charge, than the net flux through it will be zero.

5. Yes. If the electric field is zero for all points on the surface, then the integral of $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}$ over the surface will be zero, the flux through the surface will be zero, and no net charge will be contained in the surface. No. If a surface encloses no net charge, then the net electric flux through the surface will be zero, but the electric field is not necessarily zero for all points on the surface. The integral of $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}$ over the surface must be zero, but the electric field itself is not required to be zero. There may be charges outside the surface that will affect the values of the electric field at the surface.

6. The electric flux through a surface is the scalar (dot) product of the electric field vector and the area vector of the surface. Thus, in magnitude, $\Phi_{\mathrm{E}} = EA\cos\theta$. By analogy, the gravitational flux through a surface would be the product of the gravitational field (or force per unit mass) and the area, or $\Phi_{\mathrm{g}} = gA\cos\theta$. Any mass, such as a planet, would be a "sink" for gravitational field. Since there is not "anti-gravity" there would be no sources.

$\boxed{7.}$ No. Gauss's law is most useful in cases of high symmetry, where a surface can be defined over which the electric field has a constant value and a constant relationship to the direction of the outward normal to the surface. Such a surface cannot be defined for an electric dipole.

8. When the ball is inflated and charge is distributed uniformly over its surface, the field inside is zero. When the ball is collapsed, there is no symmetry to the charge distribution, and the calculation of the electric field strength and direction inside the ball is difficult (and will most likely give a non-zero result).

9. For an infinitely long wire, the electric field is radially outward from the wire, resulting from contributions from all parts of the wire. This allows us to set up a Gaussian surface that is cylindrical, with the cylinder axis parallel to the wire. This surface will have zero flux through the top and bottom of the cylinder, since the net electric field and the outward surface normal are perpendicular at all points over the top and bottom. In the case of a short wire, the electric field is not radially outward from the wire near the ends; it curves and points directly outward along the axis of

the wire at both ends. We cannot define a useful Gaussian surface for this case, and the electric field must be computed directly.

10. In Example 22-6, there is no flux through the flat ends of the cylindrical Gaussian surface because the field is directed radially outward from the wire. If instead the wire extended only a short distance past the ends of the cylinder, there would be a component of the field through the ends of the cylinder. The result of the example would be altered because the value of the field at a given point would now depend not only on the radial distance from the wire but also on the distance from the ends.

11. The electric flux through the sphere remains the same, since the same charge is enclosed. The electric field at the surface of the sphere is changed, because different parts of the sphere are now at different distances from the charge. The electric field will not have the same magnitude for all parts of the sphere, and the direction of the electric field will not be parallel to the outward normal for all points on the surface of the sphere. The electric field will be stronger on the side closer to the charge and weaker on the side further from the charge.

12. (*a*) A charge of ($Q - q$) will be on the outer surface of the conductor. The total charge Q is placed on the conductor but since $+q$ will reside on the inner surface, the leftover, ($Q - q$), will reside on the outer surface.
    (*b*) A charge of $+q$ will reside on the inner surface of the conductor since that amount is attracted by the charge $-q$ in the cavity. (Note that E must be zero inside the conductor.)

$\boxed{13.}$ Yes. The charge q will induce a charge $-q$ on the inside surface of the thin metal shell, leaving the outside surface with a charge $+q$. The charge Q outside the sphere will feel the same electric force as it would if the metal shell were not present.

14. The total flux through the balloon's surface will not change because the enclosed charge does not change. The flux per unit surface area will decrease, since the surface area increases while the total flux does not change.

Solutions to Problems

$\boxed{1.}$ The electric flux of a uniform field is given by Eq. 22-1b.

(*a*) $\Phi_E = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = EA\cos\theta = (580\,\text{N/C})\pi(0.13\,\text{m})^2\cos 0 = \boxed{31\,\text{N}\bullet\text{m}^2/\text{C}}$

(*b*) $\Phi_E = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = EA\cos\theta = (580\,\text{N/C})\pi(0.13\,\text{m})^2\cos 45^\circ = \boxed{22\,\text{N}\bullet\text{m}^2/\text{C}}$

(*c*) $\Phi_E = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = EA\cos\theta = (580\,\text{N/C})\pi(0.13\,\text{m})^2\cos 90^\circ = \boxed{0}$

2. Use Eq. 22-1b for the electric flux of a uniform field. Note that the surface area vector points radially outward, and the electric field vector points radially inward. Thus the angle between the two is 180°.

$$\Phi_E = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = EA\cos\theta = (150\,\text{N/C})4\pi R_E^2\cos 180^\circ = -4\pi(150\,\text{N/C})\left(6.38\times10^6\,\text{m}\right)^2$$
$$= \boxed{-7.7\times10^{16}\,\text{N}\bullet\text{m}^2/\text{C}}$$

3. (*a*) Since the field is uniform, no lines originate or terminate inside the cube, and so the net flux is $\Phi_{\text{net}} = \boxed{0}$.

   (*b*) There are two opposite faces with field lines perpendicular to the faces. The other four faces have field lines parallel to those faces. For the faces parallel to the field lines, no field lines enter or exit the faces. Thus $\Phi_{\text{parallel}} = \boxed{0}$.

   Of the two faces that are perpendicular to the field lines, one will have field lines entering the cube, and so the angle between the field lines and the face area vector is 180°. The other will have field lines exiting the cube, and so the angle between the field lines and the face area vector is 0°. Thus we have $\Phi_{\text{entering}} = \vec{E}\bullet\vec{A} = E_0 A\cos 180° = \boxed{-E_0\ell^2}$ and

   $\Phi_{\text{leaving}} = \vec{E}\bullet\vec{A} = E_0 A\cos 0° = \boxed{E_0\ell^2}$.

4. (*a*) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base, $\vec{\mathbf{E}} \parallel \vec{\mathbf{A}}$. Thus $\Phi_{\text{E}} = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = \boxed{\pi r^2 E}$.

   (b) $\vec{\mathbf{E}}$ is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_{\text{E}} = \boxed{0}$.

5. Use Gauss's law to determine the enclosed charge.

$$\Phi_{\text{E}} = \frac{Q_{\text{encl}}}{\varepsilon_o} \rightarrow Q_{\text{encl}} = \Phi_{\text{E}}\varepsilon_o = \left(1840\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.63\times10^{-8}\,\text{C}}$$

6. The net flux through each closed surface is determined by the net charge inside. Refer to the picture in the textbook.

$$\Phi_1 = \left(+Q-3Q\right)/\varepsilon_0 = \boxed{-2Q/\varepsilon_0} \;;\; \Phi_2 = \left(+Q+2Q-3Q\right)/\varepsilon_0 = \boxed{0} \;;$$

$$\Phi_3 = \left(+2Q-3Q\right)/\varepsilon_0 = \boxed{-Q/\varepsilon_0} \;;\; \Phi_4 = \boxed{0} \;;\; \Phi_5 = \boxed{+2Q/\varepsilon_0}$$

$\boxed{7.}$ (*a*) Use Gauss's law to determine the electric flux.

$$\Phi_{\text{E}} = \frac{Q_{\text{encl}}}{\varepsilon_o} = \frac{-1.0\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{-1.1\times10^5\,\text{N}\cdot\text{m}^2/\text{C}}$$

   (*b*) Since there is no charge enclosed by surface A_2, $\Phi_{\text{E}} = \boxed{0}$.

8. The net flux is only dependent on the charge enclosed by the surface. Since both surfaces enclose the same amount of charge, the flux through both surfaces is the same. Thus the ratio is $\boxed{1:1}$.

9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}$ on each of these two surfaces.

$$\Phi_{\text{E}} = \left(\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}\right)_{\text{right}} + \left(\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}\right)_{\text{left}} = E_{\text{right}}\ell^2 - E_{\text{left}}\ell^2 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$Q_{\text{encl}} = \left(E_{\text{right}} - E_{\text{left}}\right)\ell^2\varepsilon_0 = \left(410\,\text{N/C} - 560\,\text{N/C}\right)\left(25\,\text{m}\right)^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{-8.3\times10^{-7}\,\text{C}}$$

10. Because of the symmetry of the problem one sixth of the total flux will pass through each face.

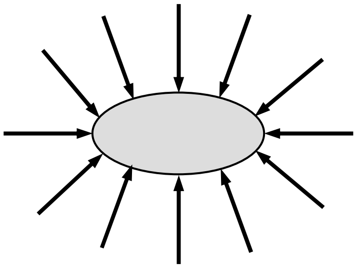
$$\Phi_{\text{face}} = \tfrac{1}{6}\Phi_{\text{total}} = \tfrac{1}{6}\frac{Q_{\text{encl}}}{\varepsilon_0} = \boxed{\frac{Q_{\text{encl}}}{6\varepsilon_0}}$$

Notice that the side length of the cube did not enter into the calculation.

11. The charge density can be found from Eq. 22-4, Gauss's law. The charge is the charge density times the length of the rod.

$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0} = \quad\rightarrow\quad \lambda = \frac{\Phi\varepsilon_0}{\ell} = \frac{\left(7.3\times10^5\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}{0.15\,\text{m}} = \boxed{4.3\times10^{-5}\,\text{C/m}}$$

12.



$\boxed{13.}$ The electric field can be calculated by Eq. 21-4a, and that can be solved for the magnitude of the charge.

$$E = k\frac{Q}{r^2} \quad\rightarrow\quad Q = \frac{Er^2}{k} = \frac{\left(6.25\times10^2\,\text{N/C}\right)\left(3.50\times10^{-2}\,\text{m}\right)^2}{8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 8.52\times10^{-11}\text{C}$$

This corresponds to about 5×10^8 electrons. Since the field points toward the ball, the charge must be negative. Thus $Q = \boxed{-8.52\times10^{-11}\text{C}}$.

14. The charge on the spherical conductor is uniformly distributed over the surface area of the sphere, so $\sigma = \dfrac{Q}{4\pi R^2}$. The field at the surface of the sphere is evaluated at $r = R$.

$$E\left(r = R\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R^2} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

15. The electric field due to a long thin wire is given in Example 22-6 as $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R}$.

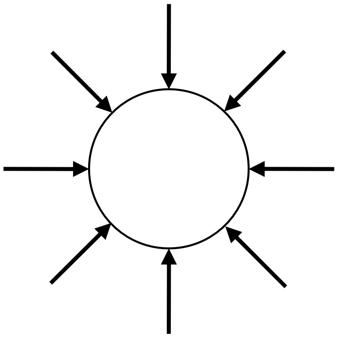
(*a*) $$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(-7.2\times10^{-6}\,\text{C/m}\right)}{\left(5.0\,\text{m}\right)} = \boxed{-2.6\times10^4\,\text{N/C}}$$

The negative sign indicates the electric field is pointed $\boxed{\text{towards the wire}}$.

(*b*) $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{R} = \left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\dfrac{2\left(-7.2\times10^{-6}\,\mathrm{C/m}\right)}{\left(1.5\,\mathrm{m}\right)} = \boxed{-8.6\times10^{4}\,\mathrm{N/C}}$

The negative sign indicates the electric field is pointed $\boxed{\text{towards the wire}}$.

16. Because the globe is a conductor, the net charge of -1.50 mC will be arranged symmetrically around the sphere.



17. Due to the spherical symmetry of the problem, the electric field can be evaluated using Gauss's law and the charge enclosed by a spherical Gaussian surface of radius *r*.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\mathrm{encl}}}{\varepsilon_0} \quad\rightarrow\quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2}$$

Since the charge densities are constant, the charge enclosed is found by multiplying the appropriate charge density times the volume of charge enclosed by the Gaussian sphere. Let $r_1 = 6.0\,\mathrm{cm}$ and $r_2 = 12.0\,\mathrm{cm}$.

(*a*) Negative charge is enclosed for $r < r_1$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{\rho_{(-)}r}{3\varepsilon_0} = \frac{\left(-5.0\,\mathrm{C/m^3}\right)r}{3\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}$$

$$= \boxed{\left(-1.9\times10^{11}\,\mathrm{N/C\bullet m}\right)r}$$

(*b*) In the region $r_1 < r < r_2$, all of the negative charge and part of the positive charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)}\left[\frac{4}{3}\pi\left(r^3 - r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)} - \rho_{(+)}\right)\left(r_1^3\right)}{3\varepsilon_0 r^2} + \frac{\rho_{(+)}r}{3\varepsilon_0}$$

$$= \frac{\left[\left(-5.0\,\mathrm{C/m^3}\right) - \left(8.0\,\mathrm{C/m^3}\right)\right]\left(0.060\,\mathrm{m}\right)^3}{3\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)r^2} + \frac{\left(8.0\,\mathrm{C/m^3}\right)r}{3\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}$$

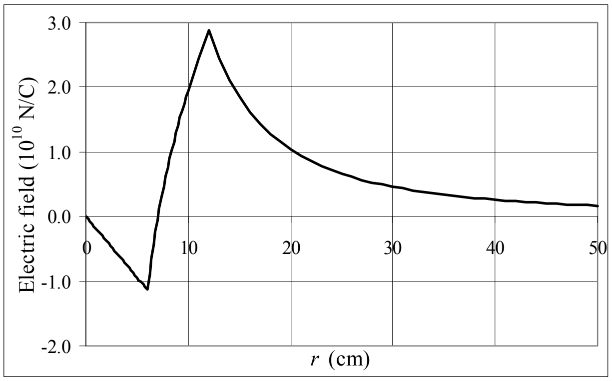
$$= \boxed{\frac{\left(-1.1\times10^{8}\,\mathrm{N\bullet m^2/C}\right)}{r^2} + \left(3.0\times10^{11}\,\mathrm{N/C\bullet m}\right)r}$$

(*c*) In the region $r_2 < r$, all of the charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)}\left[\frac{4}{3}\pi\left(r_2^3 - r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)} - \rho_{(+)}\right)\left(r_1^3\right) + \rho_{(+)}\left(r_2^3\right)}{3\varepsilon_0 r^2} =$$

$$= \frac{\left[\left(-5.0\,\mathrm{C/m^3}\right) - \left(8.0\,\mathrm{C/m^3}\right)\right]\left(0.060\,\mathrm{m}\right)^3 + \left(8.0\,\mathrm{C/m^3}\right)\left(0.120\,\mathrm{m}\right)^3}{3\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)r^2} = \boxed{\frac{\left(4.1\times10^{8}\,\mathrm{N\bullet m^2/C}\right)}{r^2}}$$

(*d*) See the adjacent plot. The field is continuous at the edges of the layers. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.17d."



18. See Example 22-3 for a detailed discussion related to this problem.

(*a*) Inside a solid metal sphere the electric field is $\boxed{0}$.

(*b*) Inside a solid metal sphere the electric field is $\boxed{0}$.

(*c*) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.50\times10^{-6}\,\text{C}\right)}{\left(3.10\,\text{m}\right)^2} = \boxed{5140\,\text{N/C}}$$

The field would point towards the center of the sphere.

(*d*) Same reasoning as in part (*c*).

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.50\times10^{-6}\,\text{C}\right)}{\left(8.00\,\text{m}\right)^2} = \boxed{772\,\text{N/C}}$$

The field would point towards the center of the sphere.

(*e*) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.

(*f*) The solid sphere of charge is dealt with in Example 22-4. We see from that Example that the field inside the sphere is given by $|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r_0^3}r.$ Outside the sphere the field is no different.

So we have these results for the solid sphere.

$$\left|E\left(r=0.250\,\text{m}\right)\right| = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(3.00\,\text{m}\right)^3}\left(0.250\,\text{m}\right) = \boxed{458\,\text{N/C}}$$

$$\left|E\left(r=2.90\,\text{m}\right)\right| = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(3.00\,\text{m}\right)^3}\left(2.90\,\text{m}\right) = \boxed{5310\,\text{N/C}}$$

$$\left|E\left(r=3.10\,\text{m}\right)\right| = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(3.10\,\text{m}\right)^2} = \boxed{5140\,\text{N/C}}$$

$$\left|E\left(r=8.00\,\text{m}\right)\right| = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(3.10\,\text{m}\right)^2} = \boxed{772\,\text{N/C}}$$

All point towards the center of the sphere.

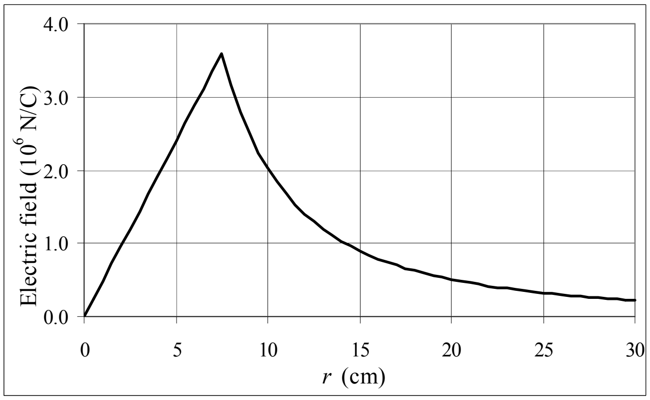
$\boxed{19.}$ For points inside the nonconducting spheres, the electric field will be determined by the charge inside the spherical surface of radius *r*.

$$Q_{\text{encl}} = Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3}\right) = Q\left(\frac{r}{r_0}\right)^3$$

The electric field for $r \le r_0$ can be calculated from Gauss's law.

$$E(r \le r_0) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

$$= Q\left(\frac{r}{r_0}\right)^3 \frac{1}{4\pi\varepsilon_0 r^2} = \left(\frac{Q}{4\pi\varepsilon_0 r_0^3}\right) r$$



The electric field outside the sphere is calculated from Gauss's law with $Q_{\text{encl}} = Q$.

$$E\left(r \ge r_0\right) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.19."

20. (*a*) When close to the sheet, we approximate it as an infinite sheet, and use the result of Example 22-7. We assume the charge is over both surfaces of the aluminum.

$$E = \frac{\sigma}{2\varepsilon_o} = \frac{\dfrac{275\times10^{-9}\,\text{C}}{\left(0.25\,\text{m}\right)^2}}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.5\times10^5\,\text{N/C, away from the sheet}}$$

(*b*) When far from the sheet, we approximate it as a point charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{275\times10^{-9}\,\text{C}}{\left(15\,\text{m}\right)^2} = \boxed{11\,\text{N/C, away from the sheet}}$$

21. (*a*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \quad \rightarrow$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(3.00\times10^{-2}\,\text{m}\right)^2} = \boxed{5.49\times10^7\,\text{N/C, radially outward}}$$

(*b*) A radius of 6.00 cm is inside the conducting material, and so the field must be $\boxed{0}$. Note that there must be an induced charge of $-5.50\times10^{-6}\,\text{C}$ on the surface at $r = 4.50$ cm, and then an induced charge of $5.50\times10^{-6}\,\text{C}$ on the outer surface of the sphere.

(*c*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \quad \rightarrow$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.50\times10^{-6}\,\text{C}}{\left(30.0\times10^{-2}\,\text{m}\right)^2} = \boxed{5.49\times10^5\,\text{N/C, radially outward}}$$

22. (*a*) Inside the shell, the field is that of the point charge, $\boxed{E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}}$.

    (*b*) There is no field inside the conducting material: $\boxed{E=0}$.

    (*c*) Outside the shell, the field is that of the point charge, $\boxed{E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}}$.

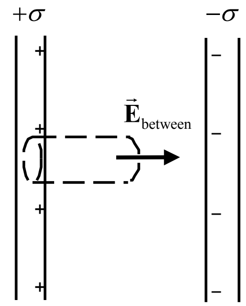
    (*d*) The shell does not affect the field due to Q alone, except in the shell material, where the field is 0. The charge Q does affect the shell – it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell, and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.

23. (*a*) There can be no field inside the conductor, and so there must be an induced charge of $\boxed{-8.00\mu\text{C}}$ on the surface of the spherical cavity.

    (*b*) Any charge on the conducting material must reside on its boundaries. If the net charge of the cube is $-6.10\mu\text{C}$, and there is a charge of $-8.00\mu\text{C}$ on its inner surface, there must be a charge of $\boxed{+1.90\mu\text{C}}$ on the outer surface.

24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

    (*a*) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

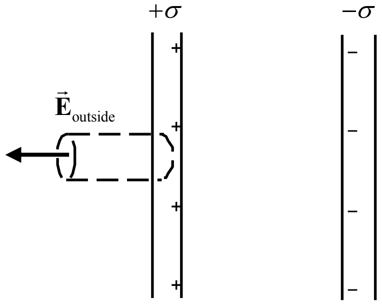


$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{right}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{between}}A = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E_{\text{between}} = \frac{\sigma}{\varepsilon_0}}$$

The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

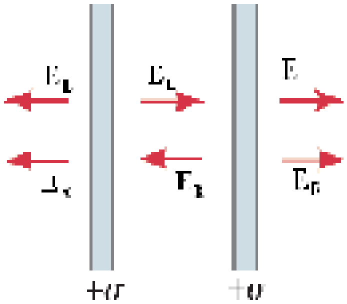
    (*b*) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.



$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}}A = \frac{0}{\varepsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{0}{\varepsilon_0}}$$

(*c*) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.



25. Example 22-7 gives the electric field from a positively charged plate as $E = \sigma/2\varepsilon_0$ with the field pointing away from the plate. The fields from the two plates will add, as shown in the figure.

(*a*) Between the plates the fields are equal in magnitude, but point in opposite directions.

$$E_{\text{between}} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$$

(*b*) Outside the two plates the fields are equal in magnitude and point in the same direction.

$$E_{\text{outside}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

(*c*) When the plates are conducting the charge lies on the surface of the plates. For nonconducting plates the same charge will be spread across the plate. This will not affect the electric field between or outside the two plates. It will, however, allow for a non-zero field inside each plate.

26. Because 3.0 cm << 1.0 m, we can consider the plates to be infinite in size, and ignore any edge effects. We use the result from Problem 24(*a*).

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} \rightarrow Q = EA\varepsilon_0 = (160\,\text{N/C})(1.0\,\text{m})^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.4\times10^{-9}\,\text{C}}$$

27. (*a*) In the region $0 < r < r_1$, a gaussian surface would enclose no charge. Thus, due to the spherical symmetry, we have the following.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = 0 \rightarrow E = \boxed{0}$$

(*b*) In the region $r_1 < r < r_2$, only the charge on the inner shell will be enclosed.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2}{\varepsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2}{\varepsilon_0 r^2}}$$

(*c*) In the region $r_2 < r$, the charge on both shells will be enclosed.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2 + \sigma_2 4\pi r_2^2}{\varepsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2 + \sigma_2 r_2^2}{\varepsilon_0 r^2}}$$

(*d*) To make $E = 0$ for $r_2 < r$, we must have $\boxed{\sigma_1 r_1^2 + \sigma_2 r_2^2 = 0}$. This implies that the shells are of opposite charge.

(*e*) To make $E = 0$ for $r_1 < r < r_2$, we must have $\boxed{\sigma_1 = 0}$. Or, if a charge $Q = -4\pi\sigma_1 r_1^2$ were placed at the center of the shells, that would also make $E = 0$.

28. If the radius is to increase from r_0 to $2r_0$ linearly during an elapsed time of *T*, then the rate of increase must be r_0/T. The radius as a function of time is then $r = r_0 + \frac{r_0}{T}t = r_0\left(1+\frac{t}{T}\right)$. Since the balloon is spherical, the field outside the balloon will have the same form as the field due to a point charge.

(*a*) Here is the field just outside the balloon surface.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2\left(1+\frac{t}{T}\right)^2}}$$

(*b*) Since the balloon radius is always smaller than $3.2r_0$, the total charge enclosed in a gaussian surface at $r = 3.2r_0$ does not change in time.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(3.2r_0\right)^2}}$$

29. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius *r* leads to the following.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(*a*) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \boxed{0}$$

(*b*) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho = \frac{Q}{\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_1^3}$ $= \frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}$.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho V_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \frac{\frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}\frac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)}}$$

(*c*) For the region $r > r_0$, the enclosed charge is the total charge, *Q*.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

30. By the superposition principle for electric fields (Section 21-6), we find the field for this problem by adding the field due to the point charge at the center to the field found in Problem 29. At any location $r > 0$, the field due to the point charge is $E = \frac{q}{4\pi\varepsilon_0 r^2}$.

(*a*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + 0 = \boxed{\dfrac{q}{4\pi\varepsilon_0 r^2}}$

(*b*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + \dfrac{Q}{4\pi\varepsilon_0 r^2}\dfrac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)} = \boxed{\dfrac{1}{4\pi\varepsilon_0 r^2}\left[q + Q\dfrac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)}\right]}$

(*c*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + \dfrac{Q}{4\pi\varepsilon_0 r^2} = \boxed{\dfrac{q+Q}{4\pi\varepsilon_0 r^2}}$

31. (*a*) Create a gaussian surface that just encloses the inner surface of the spherical shell. Since the electric field inside a conductor must be zero, Gauss's law requires that the enclosed charge be zero. The enclosed charge is the sum of the charge at the center and charge on the inner surface of the conductor.

$$Q_{\text{enc}} = q + Q_{\text{inner}} = 0$$

Therefore $Q_{\text{inner}} = \boxed{-q}$.

(*b*) The total charge on the conductor is the sum of the charges on the inner and outer surfaces.

$$Q = Q_{\text{outer}} + Q_{\text{inner}} \quad \to \quad Q_{\text{outer}} = Q - Q_{\text{inner}} = \boxed{Q + q}$$

(*c*) A gaussian surface of radius $r < r_1$ only encloses the center charge, *q*. The electric field will therefore be the field of the single charge.

$$\boxed{E(r < r_1) = \frac{q}{4\pi\varepsilon_0 r^2}}$$

(*d*) A gaussian surface of radius $r_1 < r < r_0$ is inside the conductor so $\boxed{E = 0}$.

(*e*) A gaussian surface of radius $r > r_0$ encloses the total charge $q + Q$. The electric field will then be the field from the sum of the two charges.

$$\boxed{E(r > r_0) = \frac{q+Q}{4\pi\varepsilon_0 r^2}}$$

32. (*a*) For points inside the shell, the field will be due to the point charge only.

$$E\left(r < r_0\right) = \boxed{\frac{q}{4\pi\varepsilon_0 r^2}}$$

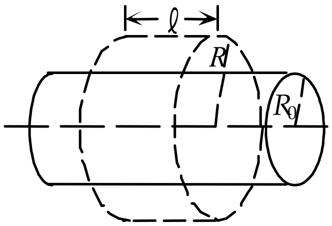
(*b*) For points outside the shell, the field will be that of a point charge, equal to the total charge.

$$E\left(r > r_0\right) = \boxed{\frac{q+Q}{4\pi\varepsilon_0 r^2}}$$

(*c*) If $q = Q$, we have $E\left(r < r_0\right) = \boxed{\dfrac{Q}{4\pi\varepsilon_0 r^2}}$ and $E\left(r > r_0\right) = \boxed{\dfrac{2Q}{4\pi\varepsilon_0 r^2}}$.

(*d*) If $q = -Q$, we have $E\left(r < r_0\right) = \boxed{\dfrac{-Q}{4\pi\varepsilon_0 r^2}}$ and $E\left(r > r_0\right) = \boxed{0}$.

33. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, R, from the symmetry axis of the shell. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the shell. The gaussian surface is of radius r and length ℓ. $\vec{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since $\vec{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma A_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\sigma A_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

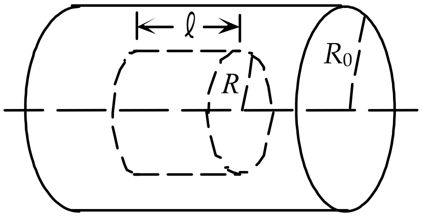
(*a*) For $R > R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 2\pi R_0 \ell$.

$$E = \frac{\sigma A_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\sigma 2\pi R_0 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\sigma R_0}{\varepsilon_0 R}\text{, radially outward}}$$

(*b*) For $R < R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 0$, and so $E = \boxed{0}$.

(*c*) The field for $R > R_0$ due to the shell is the same as the field due to the long line of charge, if we substitute $\boxed{\lambda = 2\pi R_0 \sigma}$.

34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho_{\text{E}} V_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $R > R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R_0^2 \ell$.

$$E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_{\text{E}} R_0^2}{2\varepsilon_0 R}\text{, radially outward}}$$

(*b*) For $R < R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R^2 \ell$.

$$E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_{\text{E}} R}{2\varepsilon_0}\text{, radially outward}}$$

35. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $0 < R < R_1$, no charge is enclosed, and so $E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*b*) For $R_1 < R < R_2$, charge $+Q$ is enclosed, and so $E = \boxed{\dfrac{Q}{2\pi\varepsilon_0 R\ell}\text{, radially outward}}$.

(*c*) For $R > R_2$, both charges of $+Q$ and $-Q$ are enclosed, and so $E = \dfrac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*d*) The force on an electron between the cylinders points in the direction opposite to the electric field, and so the force is inward. The electric force produces the centripetal acceleration for the electron to move in the circular orbit.

$$F_{\text{centrip}} = eE = \frac{eQ}{2\pi\varepsilon_0 R\ell} = m\frac{v^2}{R} \rightarrow K = \tfrac{1}{2}mv^2 = \boxed{\frac{eQ}{4\pi\varepsilon_0\ell}}$$

Note that this is independent of the actual value of the radius, as long as $R_1 < R < R_2$.

36. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(2\pi R\ell\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) At a distance of $R = 3.0\,\text{cm}$, no charge is enclosed, and so $E = \dfrac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*b*) At a distance of $R = 7.0\,\text{cm}$, the charge on the inner cylinder is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{2}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{R\ell} = 2\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(-0.88\times10^{-6}\text{C}\right)}{\left(0.070\,\text{m}\right)\left(5.0\,\text{m}\right)} = \boxed{-4.5\times10^{4}\,\text{N/C}}$$

The negative sign indicates that the field points radially inward.

(*c*) At a distance of $R = 12.0\,\text{cm}$, the charge on both cylinders is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{2}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{R\ell} = 2\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.56-0.88\right)\times10^{-6}\text{C}}{\left(0.120\,\text{m}\right)\left(5.0\,\text{m}\right)} = \boxed{2.0\times10^{4}\,\text{N/C}}$$

The field points radially outward.

$\boxed{37.}$ (*a*) The final speed can be calculated from the work-energy theorem, where the work is the integral of the force on the electron between the two shells.

$$W = \int \vec{F}\bullet d\vec{r} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

Setting the force equal to the electric field times the charge on the electron, and inserting the electric field from Problem 36 gives the work done on the electron.

$$W = \int_{R_1}^{R_2}\frac{qQ}{2\pi\varepsilon_0 \ell R}dR = \frac{qQ}{2\pi\varepsilon_0\ell}\ln\left(\frac{R_2}{R_1}\right)$$

$$= \frac{\left(-1.60\times10^{-19}\text{C}\right)\left(-0.88\mu\text{C}\right)}{2\pi\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(5.0\,\text{m}\right)}\ln\left(\frac{9.0\,\text{cm}}{6.5\,\text{cm}}\right) = 1.65\times10^{-16}\text{ J}$$

Solve for the velocity from the work-energy theorem.

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\left(1.65\times10^{-16}\text{ J}\right)}{9.1\times10^{-31}\text{ kg}}} = \boxed{1.9\times10^{7}\,\text{m/s}}$$

(*b*) The electric force on the proton provides its centripetal acceleration.

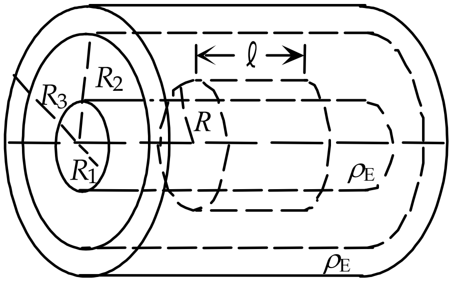
$$F_c = \frac{mv^2}{R} = qE = \frac{|qQ|}{2\pi\varepsilon_0 \ell R}$$

The velocity can be solved for from the centripetal acceleration.

$$v = \sqrt{\frac{(1.60\times10^{-19}\,\mathrm{C})(0.88\,\mu\mathrm{C})}{2\pi(8.85\times10^{-12}\,\mathrm{C^2/Nm^2})(1.67\times10^{-27}\,\mathrm{kg})(5.0\,\mathrm{m})}} = \boxed{5.5\times10^{5}\,\mathrm{m/s}}$$

Note that as long as the proton is between the two cylinders, the velocity is independent of the radius.

38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $0 < R < R_1$, the enclosed charge is the volume of charge enclosed, times the charge density.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}}$$

(*b*) For $R_1 < R < R_2$, the enclosed charge is all of the charge on the inner cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R_1^2}{2\varepsilon_0 R}}$$

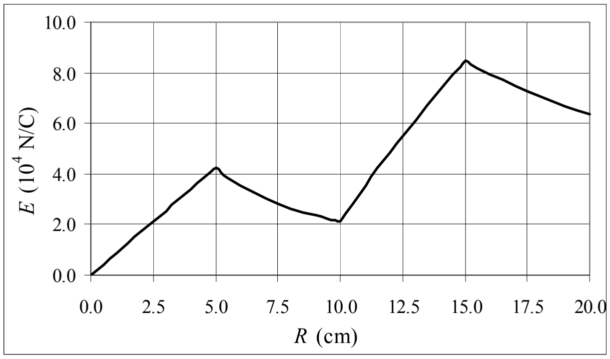
(*c*) For $R_2 < R < R_3$, the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E(\pi R^2 \ell - \pi R_2^2 \ell)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E(R_1^2 + R^2 - R_2^2)}{2\varepsilon_0 R}}$$

(*d*) For $R > R_3$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E(\pi R_3^2 \ell - \pi R_2^2 \ell)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E(R_1^2 + R_3^2 - R_2^2)}{2\varepsilon_0 R}}$$

(*e*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."



39. Due to the spherical symmetry of the geometry, we have the following to find the electric field at any radius *r*. The field will point either radially out or radially in.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(*a*) For $0 < r < r_0$, the enclosed charge is due to the part of the charged sphere that has a radius smaller than *r*.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_{\text{E}}\left(\frac{4}{3}\pi r^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_{\text{E}} r}{3\varepsilon_0}}$$

(*b*) For $r_0 < r < r_1$, the enclosed charge is due to the entire charged sphere of radius r_0.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_{\text{E}}\left(\frac{4}{3}\pi r_0^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_{\text{E}} r_0^3}{3\varepsilon_0 r^2}}$$

(*c*) For $r_1 < r < r_2$, *r* is in the interior of the conducting spherical shell, and so $E = \boxed{0}$. This implies that $Q_{\text{encl}} = 0$, and so there must be an induced charge of magnitude $-\frac{4}{3}\rho_{\text{E}}\pi r_0^3$ on the inner surface of the conducting shell, at r_1.

(*d*) For $r > r_2$, the enclosed charge is the total charge of both the sphere and the shell.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{Q + \rho_{\text{E}}\left(\frac{4}{3}\pi r_0^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\left(\frac{Q}{4\pi\varepsilon_0} + \frac{\rho_{\text{E}} r_0^3}{3\varepsilon_0}\right)\frac{1}{r^2}}$$

40. The conducting outer tube is uncharged, and the electric field is 0 everywhere within the conducting material. Because there will be no electric field inside the conducting material of the outer cylinder tube, the charge on the inner nonconducting cylinder will induce an oppositely signed, equal magnitude charge on the inner surface of the conducting tube. This charge will NOT be uniformly distributed, because the inner cylinder is not in the center of the tube. Since the conducting tube has no net charge, there will be an induced charge on the OUTER surface of the conducting tube, equal in magnitude to the charge on the inner cylinder, and of the same sign. This charge will be uniformly distributed. Since there is no electric field in the conducting material of the tube, there is no way for the charges in the region interior to the tube to influence the charge distribution on the outer surface. Thus the field outside the tube is due to a cylindrically symmetric distribution of charge. Application of Gauss's law as in Example 22-6, for a Gaussian cylinder with a radius larger than the conducting tube, and a length ℓ leads to $E\left(2\pi R\ell\right) = \frac{Q_{\text{encl}}}{\varepsilon_0}$. The enclosed charge is the amount of charge on the inner cylinder.

$$Q_{\text{encl}} = \rho_{\text{E}}\pi R_1^2 \ell \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{\varepsilon_0\left(2\pi R\ell\right)} = \boxed{\frac{\rho_{\text{E}} R_1^2}{2\varepsilon_0 R}}$$

41. We treat the source charge as a disk of positive charge of radius concentric with a disk of negative charge of radius R_0. In order for the net charge of the inner space to be 0, the charge per unit area of the source disks must both have the same magnitude but opposite sign. The field due to the annulus is then the sum of the fields due to both the positive and negative rings.

(*a*) At a distance of $0.25R_0$ from the center of the ring, we can approximate both of the disks as infinite planes, each producing a uniform field. Since those two uniform fields will be of the same magnitude and opposite sign, the net field is $\boxed{0}$.

(*b*) At a distance of $75R_0$ from the center of the ring, it appears to be approximately a point charge, and so the field will approximate that of a point charge, $E = \boxed{\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\left(75R_0\right)^2}}$

42. The conducting sphere is uncharged, and the electric field is 0 everywhere within its interior, except for in the cavities. When charge Q_1 is placed in the first cavity, a charge $-Q_1$ will be drawn from the conducting material to the inner surface of the cavity, and the electric field will remain 0 in the conductor. That charge $-Q_1$ will NOT be distributed symmetrically on the cavity surface. Since the conductor is neutral, a compensating charge Q_1 will appear on the outer surface of the conductor (charge can only be on the surfaces of conductors in electrostatics). Likewise, when charge Q_2 is placed in the second cavity, a charge $-Q_2$ will be drawn from the conducting material, and a compensating charge Q_2 will appear on the outer surface. Since there is no electric field in the conducting material, there is no way for the charges in the cavities to influence the charge distribution on the outer surface. So the distribution of charge on the outer surface is uniform, just as it would be if there were no inner charges, and instead a charge $Q_1 + Q_2$ were simply placed on the conductor. Thus the field outside the conductor is due to a spherically symmetric distribution of $Q_1 + Q_2$. Application of Gauss's law leads to $E = \boxed{\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1+Q_2}{r^2}}$. If $Q_1 + Q_2 > 0$, the field will point radially outward. If $Q_1 + Q_2 < 0$, the field will point radially inward.

$\boxed{43.}$ (*a*) Choose a cylindrical gaussian surface with the flat ends parallel to and equidistant from the slab. By symmetry the electric field must point perpendicularly away from the slab, resulting in no flux passing through the curved part of the gaussian cylinder. By symmetry the flux through each end of the cylinder must be equal with the electric field constant across the surface. $\oint \vec{E}\cdot d\vec{A} = 2EA$

The charge enclosed by the surface is the charge density of the slab multiplied by the volume of the slab enclosed by the surface.

$$q_{enc} = \rho_E\left(Ad\right)$$

Gauss's law can then be solved for the electric field.

$$\oint \vec{E}\cdot d\vec{A} = 2EA = \frac{\rho_E Ad}{\varepsilon_0} \quad \rightarrow \quad E = \boxed{\frac{\rho_E d}{2\varepsilon_0}}$$

Note that this electric field is independent of the distance from the slab.

(*b*) When the coordinate system of this problem is changed to axes parallel $\left(\hat{\mathbf{z}}\right)$ and perpendicular $\left(\hat{\mathbf{r}}\right)$ to the slab, it can easily be seen that the particle will hit the slab if the initial perpendicular velocity is sufficient for the particle to reach the slab before the acceleration decreases its velocity to zero. In the new coordinate system the axes are rotated by 45°.

$$\vec{r}_0 = y_0\cos 45°\hat{\mathbf{r}} + y_0\sin 45°\hat{\mathbf{z}} = \frac{y_0}{\sqrt{2}}\hat{\mathbf{r}} + \frac{y_0}{\sqrt{2}}\hat{\mathbf{z}}$$

$$\vec{v}_0 = -v_0 \sin 45°\hat{\mathbf{r}} + v_0 \cos 45°\hat{\mathbf{z}} = -\frac{v_0}{\sqrt{2}}\hat{\mathbf{r}} + \frac{v_0}{\sqrt{2}}\hat{\mathbf{z}}$$

$$\vec{a} = qE/m\hat{\mathbf{r}}$$

The perpendicular components are then inserted into Eq. 2-12c, with the final velocity equal to zero.

$$0 = v_{r0}^2 - 2a(r - r_0) = \frac{v_0^2}{2} - 2\frac{q}{m}\left(\frac{\rho_E d}{2\varepsilon_0}\right)\left(\frac{y_0}{\sqrt{2}} - 0\right)$$

Solving for the velocity gives the minimum speed that the particle can have to reach the slab.

$$v_0 \geq \boxed{\sqrt{\frac{\sqrt{2}q\rho_E d y_0}{m\varepsilon_0}}}$$

44. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius *r* leads to the following.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(*a*) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \boxed{0}$$

(*b*) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho = \rho_0 \frac{r_1}{r}$. We must integrate to find the total charge. We follow the procedure given in Example 22-5. We divide the sphere up into concentric thin shells of thickness *dr*, as shown in Fig. 22-14. We then integrate to find the charge.

$$Q_{\text{encl}} = \int \rho_{\text{E}} dV = \int_{r_1}^{r} \rho_0 \frac{r_1}{r'} 4\pi \left(r'\right)^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^{r} r' dr' = 2\pi r_1 \rho_0 \left(r^2 - r_1^2\right)$$

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{2\pi r_1 \rho_0 \left(r^2 - r_1^2\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 \left(r^2 - r_1^2\right)}{2\varepsilon_0 r^2}}$$

(*c*) For the region $r > r_0$, the enclosed charge is the total charge, found by integration in a similar fashion to part (*b*).

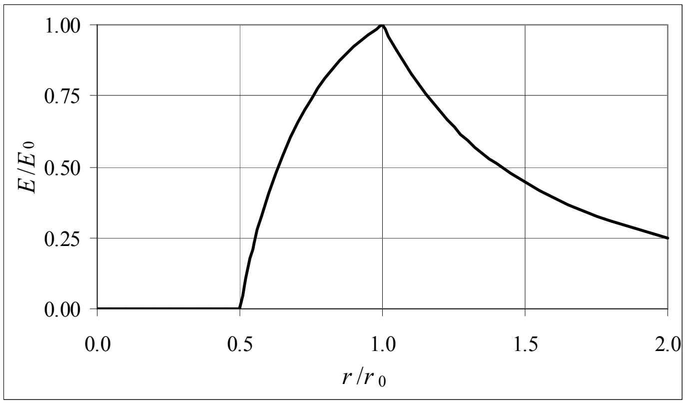
$$Q_{\text{encl}} = \int \rho_{\text{E}} dV = \int_{r_1}^{r_0} \rho_0 \frac{r_1}{r'} 4\pi \left(r'\right)^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^{r_0} r' dr' = 2\pi r_1 \rho_0 \left(r_0^2 - r_1^2\right)$$

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{2\pi r_1 \rho_0 \left(r_0^2 - r_1^2\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 \left(r_0^2 - r_1^2\right)}{2\varepsilon_0 r^2}}$$

(*d*) See the attached graph. We have chosen $r_1 = \frac{1}{2}r_0$. Let

$$E_0 = E(r = r_0) = \frac{\rho_0 r_1 (r_0^2 - r_1^2)}{2\varepsilon_0 r_0^2}.$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.44d."



45. (*a*) The force felt by one plate will be the charge on that plate multiplied by the electric field caused by the other plate. The field due to one plate is found in Example 22-7. Let the positive plate be on the left, and the negative plate on the right. We find the force on the negative plate due to the positive plate.

$$F_{\substack{\text{on}\\\text{plate}\\\text{"b"}}} = q_{\substack{\text{on}\\\text{plate}\\\text{"b"}}} E_{\substack{\text{due to}\\\text{plate}\\\text{"a"}}} = (\sigma_b A_b) E_a = (\sigma_b A_b)\frac{\sigma_a}{2\varepsilon_0}$$

$$= \frac{(-15\times10^{-6}\,\text{C/m}^2)(1.0\,\text{m}^2)(-15\times10^{-6}\,\text{C/m}^2)}{2(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)} = -12.71\,\text{N}$$

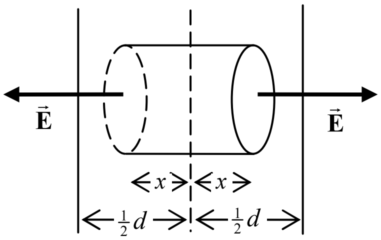
$$\approx \boxed{13\,\text{N, towards the other plate}}$$

(*b*) Since the field due to either plate is constant, the force on the other plate is constant, and then the work is just the force times the distance. Since the plates are oppositely charged, they will attract, and so a force equal to and opposite the force above will be required to separate them. The force will be in the same direction as the displacement of the plates.

$$W = \vec{\mathbf{F}}\bullet\Delta\vec{\mathbf{x}} = (12.71\,\text{N})(\cos 0°)(5.0\times10^{-3}\text{m}) = \boxed{0.064\,\text{J}}$$

46. Because the slab is very large, and we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in the field being perpendicular to the slab, with a constant magnitude for a constant distance from the center. We assume that $\rho_E > 0$ and so the electric field points away from the center of the slab.

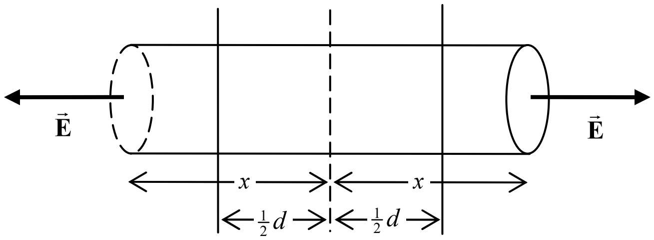
(*a*) To determine the field inside the slab, choose a cylindrical gaussian surface, of length $2x < d$ and cross-sectional area A. Place it so that it is centered in the slab. There will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on both ends, and is the same magnitude on both ends. Apply Gauss's law to find the electric field at a distance $x < \frac{1}{2}d$ from the center of the slab. See the first diagram.



$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow 2EA = \frac{\rho(2xA)}{\varepsilon_0} \rightarrow$$

$$\boxed{E_{\text{inside}} = \frac{\rho x}{\varepsilon_0};\ |x| < \tfrac{1}{2}d}$$

(*b*) Use a similar arrangement to determine the field outside the slab. Now let $2x > d$. See the second diagram.



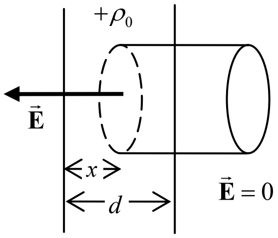
$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$2EA = \frac{\rho(dA)}{\varepsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{\rho d}{2\varepsilon_0};\ |x| > \tfrac{1}{2}d}$$

Notice that electric field is continuous at the boundary of the slab.

47. (*a*) In Problem 46, it is shown that the field outside a flat slab of nonconducting material with a uniform charge density is given by $E = \dfrac{\rho d}{2\varepsilon_0}$. If the charge density is positive, the field points away from the slab, and if the charge density is negative, the field points towards the slab. So for this problem's configuration, the field outside of both half-slabs is the vector sum of the fields from each half-slab. Since those fields are equal in magnitude and opposite in direction, the field outside the slab is $\boxed{0}$.

(*b*) To find the field in the positively charged half-slab, we use a cylindrical gaussian surface of cross sectional area *A*. Place it so that its left end is in the positively charged half-slab, a distance $x > 0$ from the center of the slab. Its right end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the left. Apply Gauss's law to find the electric field a distance $0 < x < d$ from the center of the slab. See the diagram.

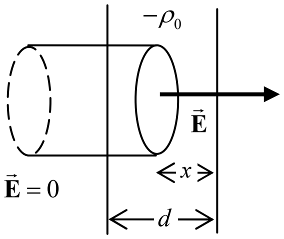


$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$EA = \frac{\rho_0(d-x)A}{\varepsilon_0} \rightarrow E_{x>0} = \frac{\rho_0(d-x)}{\varepsilon_0}$$

Since the field is pointing to the left, we can express this as $\boxed{E_{x>0} = -\dfrac{\rho_0(d-x)}{\varepsilon_0}\hat{\mathbf{i}}}$.

(*c*) To find the field in the negatively charged half-slab, we use a cylindrical gaussian surface of cross sectional area *A*. Place it so that its right end is in the negatively charged half-slab, a distance $x < 0$ from the center of the slab. Its left end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the right. Apply Gauss's law to find the electric field at a distance $-d < x < 0$ from the center of the slab. See the diagram.



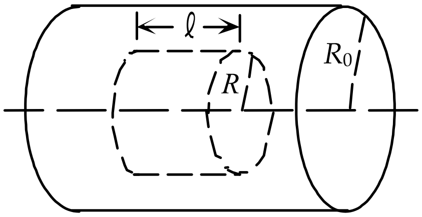
$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{right}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$EA = \frac{-\rho_0\left(d+x\right)A}{\varepsilon_0} \quad \rightarrow \quad E_{x<0} = \frac{-\rho_0\left(d+x\right)}{\varepsilon_0}$$

Since the field is pointing to the left, we can express this as $\boxed{E_{x<0} = -\frac{\rho_0\left(d+x\right)}{\varepsilon_0}\hat{\mathbf{i}}}$.

Notice that the field is continuous at all boundaries. At the left edge $\left(x=-d\right)$, $E_{x<0} = E_{\text{outside}}$. At the center $\left(x=0\right)$, $E_{x<0} = E_{>0}$. And at the right edge $\left(x=d\right)$, $E_{x>0} = E_{\text{outside}}$.

48. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, *R*, from the symmetry axis of the cylinder. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the cylinder.



The gaussian surface is of radius *r* and length *ℓ*. $\vec{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since $\vec{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(2\pi R\ell\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

To find the field inside the cylinder, we must find the charge enclosed in the gaussian cylinder. We divide the gaussian cylinder up into coaxial thin cylindrical shells of length *ℓ* and thickness *dR*. That shell has volume $dV = 2\pi R\ell dR$. The total charge in the gaussian cylinder is found by integration.

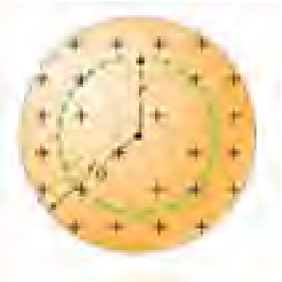
$$R < R_0: \;\; Q_{\text{encl}} = \int_0^R \rho_{\text{E}} dV = \int_0^R \rho_0\left(\frac{R}{R_0}\right)^2 2\pi R\ell\, dR = \frac{2\pi\rho_0\ell}{R_0^2}\int_0^R R^3\, dR = \frac{\pi\rho_0\ell R^4}{2R_0^2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\frac{\pi\rho_0\ell R^4}{2R_0^2}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_0 R^3}{4\varepsilon_0 R_0^2}\text{, radially out}}$$

$$R < R_0: \;\; Q_{\text{encl}} = \int_0^{R_0} \rho_{\text{E}} dV = \frac{2\pi\rho_0\ell}{R_0^2}\int_0^{R_0} R^3\, dR = \frac{\pi\rho_0\ell R_0^2}{2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\frac{\pi\rho_0\ell R_0^2}{2}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_0 R_0^2}{4\varepsilon_0 R}\text{, radially out}}$$

$\boxed{49.}$ The symmetry of the charge distribution allows the electric field inside the sphere to be calculated using Gauss's law with a concentric gaussian sphere of radius $r \le r_0$. The enclosed charge will be found by integrating the charge density over the enclosed volume.



$$Q_{\text{encl}} = \int \rho_{\text{E}} dV = \int_0^r \rho_0\left(\frac{r'}{r_0}\right)4\pi r'^2 dr' = \frac{\rho_0\pi r^4}{r_0}$$

The enclosed charge can be written in terms of the total charge by setting

$r=r_0$ and solving for the charge density in terms of the total charge.

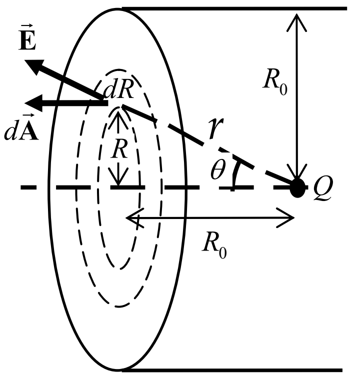
$$Q=\frac{\rho_0\pi r_0^4}{r_0}=\rho_0\pi r_0^3 \;\rightarrow\; \rho_0=\frac{Q}{\pi r_0^3} \;\rightarrow\; Q_{\text{encl}}(r)=\frac{\rho_0\pi r^4}{r_0}=Q\left(\frac{r}{r_0}\right)^4$$

The electric field is then found from Gauss's law

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\frac{Q_{\text{encl}}}{\varepsilon_0} \;\rightarrow\; E\left(4\pi r^2\right)=\frac{Q}{\varepsilon_0}\left(\frac{r}{r_0}\right)^4 \;\rightarrow\; E=\boxed{\frac{Q}{4\pi\varepsilon_0}\frac{r^2}{r_0^4}}$$

The electric field points $\boxed{\text{radially outward}}$ since the charge distribution is positive.

50. By Gauss's law, the total flux through the cylinder is Q/ε_0. We find the flux through the ends of the cylinder, and then subtract that from the total flux to find the flux through the curved sides. The electric field is that of a point charge. On the ends of the cylinder, that field will vary in both magnitude and direction. Thus we must do a detailed integration to find the flux through the ends of the cylinder. Divide the ends into a series of concentric circular rings, of radius *R* and thickness *dR*. Each ring will have an area of $2\pi R dR$. The angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ is θ, where $\tan\theta=R/R_0$. See the diagram of the left half of the cylinder.



$$\Phi_{\substack{\text{left}\\\text{end}}}=\int\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\int_0^{R_0}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\cos\theta\left(2\pi R\right)dR$$

The flux integral has three variables: *r*, *R*, and θ. We express *r* and θ in terms of *R* in order to integrate. The anti-derivative is found in Appendix B-4.

$$r=\sqrt{R^2+R_0^2}\;;\;\cos\theta=\frac{R_0}{r}=\frac{R_0}{\sqrt{R^2+R_0^2}}$$

$$\Phi_{\substack{\text{left}\\\text{end}}}=\int_0^{R_0}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2+R_0^2\right)}\frac{R_0}{\sqrt{R^2+R_0^2}}\left(2\pi R\right)dR=\frac{2\pi QR_0}{4\pi\varepsilon_0}\int_0^{R_0}\frac{RdR}{\left(R^2+R_0^2\right)^{3/2}}=\frac{QR_0}{2\varepsilon_0}\left[-\frac{1}{\sqrt{R^2+R_0^2}}\right]_0^{R_0}$$

$$=\frac{Q}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{2}}\right]\;;\;\Phi_{\substack{\text{both}\\\text{ends}}}=2\Phi_{\substack{\text{left}\\\text{end}}}=\frac{Q}{\varepsilon_0}\left[1-\frac{1}{\sqrt{2}}\right]$$

$$\Phi_{\text{total}}=\frac{Q}{\varepsilon_0}=\Phi_{\text{sides}}+\Phi_{\substack{\text{both}\\\text{ends}}} \;\rightarrow\; \Phi_{\text{sides}}=\frac{Q}{\varepsilon_0}-\Phi_{\substack{\text{both}\\\text{ends}}}=\frac{Q}{\varepsilon_0}-\frac{Q}{\varepsilon_0}\left[1-\frac{1}{\sqrt{2}}\right]=\boxed{\frac{Q}{\sqrt{2}\varepsilon_0}}$$

51. The gravitational field a distance *r* from a point mass *M* is given by Eq. 6-8, $\vec{\mathbf{g}}=-\frac{GM}{r^2}\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from mass *M*. Compare this to the electric field of a point charge, $\vec{\mathbf{E}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$. To change the electric field to the gravitational field, we would make these changes: $\vec{\mathbf{E}}\rightarrow\vec{\mathbf{g}}$; $Q/\varepsilon_0\rightarrow-4\pi GM$. Make these substitutions in Gauss's law.

$$\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\frac{Q_{\text{encl}}}{\varepsilon_0} \;\rightarrow\; \boxed{\oint\vec{\mathbf{g}}\cdot d\vec{\mathbf{A}}=-4\pi GM_{\text{encl}}}$$

52. (*a*) We use Gauss's law for a spherically symmetric charge distribution, and assume that all the charge is on the surface of the Earth. Note that the field is pointing radially inward, and so the dot product introduces a negative sign.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = -E\left(4\pi r^2\right) = Q_{\text{encl}}/\varepsilon_0 \quad \rightarrow$$

$$Q_{\text{encl}} = -4\pi\varepsilon_0 E R_{\text{Earth}}^2 = \frac{-\left(150\,\text{N/C}\right)\left(6.38\times10^6\,\text{m}\right)^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = -6.793\times10^5\,\text{C} \approx \boxed{-6.8\times10^5\,\text{C}}$$

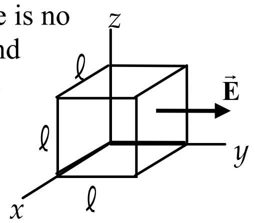
(*b*) Find the surface density of electrons. Let *n* be the total number of electrons.

$$\sigma = \frac{Q}{A} = -\frac{ne}{A} \quad \rightarrow$$

$$\frac{n}{A} = -\frac{Q}{eA} = -\frac{-4\pi\varepsilon_0 E R_{\text{Earth}}^2}{e\left(4\pi R_{\text{Earth}}^2\right)} = \frac{\varepsilon_0 E}{e} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(150\,\text{N/C}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)}$$

$$= \boxed{8.3\times10^9\,\text{electrons/m}^2}$$

53. The electric field is strictly in the *y* direction. So, referencing the diagram, there is no flux through the top, bottom, front, or back faces of the cube. Only the "left" and "right" faces will have flux through them. And since the flux is only dependent on the *y* coordinate, the flux through each of those two faces is particularly simple. Calculate the flux and use Gauss's law to find the enclosed charge.



$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{face}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\substack{\text{right}\\\text{face}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$= \int_{\substack{\text{left}\\\text{face}}} b\hat{\mathbf{j}} \cdot \left(-\hat{\mathbf{j}}dA\right) + \int_{\substack{\text{right}\\\text{face}}} \left(a\ell + b\right)\hat{\mathbf{j}} \cdot \left(\hat{\mathbf{j}}dA\right) = -b\ell^2 + a\ell^3 + b\ell^2$$

$$= a\ell^3 = Q_{\text{encl}}/\varepsilon_0 \quad \rightarrow \quad Q_{\text{encl}} = \boxed{\varepsilon_0 a\ell^3}$$

54. (*a*) Find the value of *b* by integrating the charge density over the entire sphere. Follow the development given in Example 22-5.

$$Q = \int \rho_{\text{E}} dV = \int_0^{r_0} br\left(4\pi r^2 dr\right) = 4\pi b\left(\tfrac{1}{4}r_0^4\right) \quad \rightarrow \quad b = \boxed{\frac{Q}{\pi r_0^4}}$$

(*b*) To find the electric field inside the sphere, we apply Gauss's law to an imaginary sphere of radius *r*, calculating the charge enclosed by that sphere. The spherical symmetry allows us to evaluate the flux integral simply.

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \ ; \ Q = \int \rho_{\text{E}} dV = \int_0^{r} \frac{Q}{\pi r_0^4} r\left(4\pi r^2 dr\right) = \frac{Qr^4}{r_0^4} \quad \rightarrow$$

$$E = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^4},\ r < r_0}$$

(*c*) As discussed in Example 22-4, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},\ r > r_0}$$

$\boxed{55.}$ The flux through a gaussian surface depends only on the charge enclosed by the surface. For both of these spheres the two point charges are enclosed within the sphere. Therefore the flux is the same for both spheres.

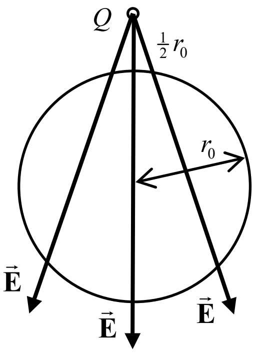
$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\left(9.20\times10^{-9}\,\text{C}\right)+\left(-5.00\times10^{-9}\,\text{C}\right)}{8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2} = \boxed{475\,\text{N}\bullet\text{m}^2/\text{C}}$$

56. (*a*) The flux through any closed surface containing the total charge must be the same, so the flux through the larger sphere is the same as the flux through the smaller sphere, $\boxed{+235\,\text{N}\bullet\text{m}^2/\text{C}}$.

(*b*) Use Gauss's law to determine the enclosed charge.

$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow Q_{\text{encl}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(+235\,\text{N}\bullet\text{m}^2/\text{C}\right) = \boxed{2.08\times10^{-9}\,\text{C}}$$

57. (*a*) There is no charge enclosed within the sphere, and so no flux lines can originate or terminate inside the sphere. All field lines enter and leave the sphere. Thus the net flux is $\boxed{0}$.



(*b*) The maximum electric field will be at the point on the sphere closest to *Q*, which is the top of the sphere. The minimum electric field will be at the point on the sphere farthest from Q, which is the bottom of the sphere.

$$E_{\text{max}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{closest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{1}{2}r_0\right)^2} = \boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

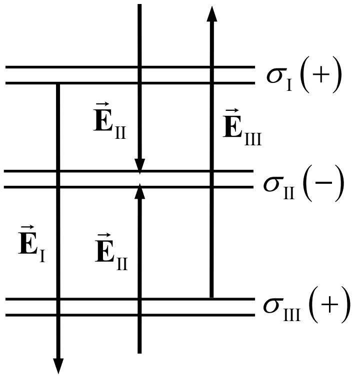
$$E_{\text{min}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{farthest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{5}{2}r_0\right)^2} = \boxed{\frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

Thus the range of values is $\boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2} \le E_{\substack{\text{sphere}\\\text{surface}}} \le \frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$.

(*c*) $\vec{\mathbf{E}}$ is $\boxed{\text{not perpendicular}}$ at all points. It is only perpendicular at the two points already discussed: the point on the sphere closest to the point charge, and the point on the sphere farthest from the point charge.

(*d*) The electric field is not perpendicular or constant over the surface of the sphere. Therefore Gauss's law is $\boxed{\text{not useful}}$ for obtaining E at the surface of the sphere because a gaussian surface cannot be chosen that simplifies the flux integral.

58. The force on a sheet is the charge on the sheet times the average electric field due to the other sheets: But the fields due to the "other" sheets is uniform, so the field is the same over the entire sheet. The force per unit area is then the charge per unit area, times the field due to the other sheets.



$$F_{\substack{\text{on}\\\text{sheet}}} = q_{\substack{\text{on}\\\text{sheet}}}\bar{E}_{\substack{\text{other}\\\text{sheets}}} = q_{\substack{\text{on}\\\text{sheet}}}E_{\substack{\text{other}\\\text{sheets}}} \rightarrow$$

$$\left(\frac{F}{A}\right)_{\substack{\text{on}\\\text{sheet}}} = \left(\frac{q}{A}\right)_{\substack{\text{on}\\\text{sheet}}}E_{\substack{\text{other}\\\text{sheets}}} = \sigma_{\substack{\text{on}\\\text{sheet}}}E_{\substack{\text{other}\\\text{sheets}}}$$

The uniform fields from each of the three sheets are indicated on the diagram. Take the positive direction as upwards. We take the direction from the diagram, and so use the absolute value of each charge density. The electric field magnitude due to each sheet is given by $E = \sigma/2\varepsilon_0$.

$$\left(\frac{F}{A}\right)_{\mathrm{I}} = \sigma_{\mathrm{I}}\left(E_{\mathrm{III}} - E_{\mathrm{II}}\right) = \frac{\sigma_{\mathrm{I}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{III}}| - |\sigma_{\mathrm{II}}|\right) = \frac{6.5\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(5.0-2.0)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{1.1\times10^{-6}\ \mathrm{N/m^2}\ (\mathrm{up})}$$

$$\left(\frac{F}{A}\right)_{\mathrm{II}} = \sigma_{\mathrm{II}}\left(E_{\mathrm{III}} - E_{\mathrm{I}}\right) = \frac{\sigma_{\mathrm{II}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{III}}| - |\sigma_{\mathrm{I}}|\right) = \frac{-2.0\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(5.0-6.5)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{1.7\times10^{-7}\ \mathrm{N/m^2}\ (\mathrm{up})}$$

$$\left(\frac{F}{A}\right)_{\mathrm{III}} = \sigma_{\mathrm{III}}\left(E_{\mathrm{III}} - E_{\mathrm{I}}\right) = \frac{\sigma_{\mathrm{III}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{II}}| - |\sigma_{\mathrm{I}}|\right) = \frac{5.0\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(2.0-6.5)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{-1.3\times10^{-6}\ \mathrm{N/m^2}\ (\mathrm{down})}$$

59. (*a*) The net charge inside a sphere of radius a_0 will be made of two parts – the positive point charge at the center of the sphere, and some fraction of the total negative charge, since the negative charge is distributed over all space, as described by the charge density. To evaluate the portion of the negative charge inside the sphere, we must determine the coefficient *A*. We do that by integrating the charge density over all space, in the manner of Example 22-5. Use an integral from Appendix B-5.

$$-e = \int \rho_E dV = \int_0^\infty \left(-Ae^{-2r/a_0}\right)\left(4\pi r^2 dr\right) = -4\pi A\int_0^\infty e^{-2r/a_0} r^2 dr = -4\pi A\frac{2!}{\left(2/a_0\right)^3} = -\pi A a_0^3 \ \rightarrow$$

$$A = \frac{e}{\pi a_0^3}$$

Now we find the negative charge inside the sphere of radius a_0, using an integral from Appendix B-4. We are indicating the elementary charge by (e), so as to not confuse it with the base of the natural logarithms.

$$Q_{\mathrm{neg}} = \int_0^{a_0}\left(-Ae^{-2r/a_0}\right)\left(4\pi r^2 dr\right) = -\frac{4\pi(e)}{\pi a_0^3}\int_0^{a_0} e^{-2r/a_0} r^2 dr$$

$$= -\frac{4\pi(e)}{\pi a_0^3}\left\{\left[-\frac{e^{-2r/a_0}}{\left(2/a_0\right)^3}\right]\left[\left(2/a_0\right)^2 r^2 + 2\left(2/a_0\right)r + 2\right]\right\}_0^{a_0} = (e)\left[5e^{-2} - 1\right]$$

$$Q_{\mathrm{net}} = Q_{\mathrm{neg}} + Q_{\mathrm{pos}} = (e)\left[5e^{-2}-1\right] + (e) = (e)5e^{-2} = \left(1.6\times10^{-19}\mathrm{C}\right)5e^{-2} = 1.083\times10^{-19}\mathrm{C}$$

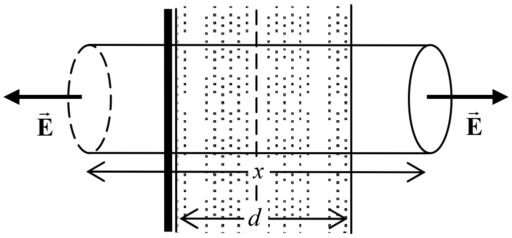
$$\approx \boxed{1.1\times10^{-19}\mathrm{C}}$$

(*b*) The field at a distance $r = a_0$ is that of a point charge of magnitude Q_{net} at the origin, because of the spherical symmetry and Gauss's law.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{net}}}{a_0^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.083\times10^{-19}\mathrm{C}\right)}{\left(0.53\times10^{-10}\mathrm{m}\right)^2} = \boxed{3.5\times10^{11}\ \mathrm{N/C}}$$

60. The field due to the plane is $E_{\text{plane}} = \dfrac{\sigma}{2\varepsilon_0}$, as discussed in Example 22-7. Because the slab is very large, and we assume that we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in its field being perpendicular to the slab, with a constant magnitude for a constant distance from its center. We also assume that $\rho_E > 0$ and so the electric field of the slab points away from the center of the slab.

(*a*) To determine the field to the left of the plane, we choose a cylindrical gaussian surface, of length $x > d$ and cross-sectional area A. Place it so that the plane is centered inside the cylinder. See the diagram. There will be no flux through the curved wall of the cylinder. From the symmetry, the electric field is parallel to the surface area vector on both ends. We already know that the field due to the plane is the same on both ends, and by the symmetry of the problem, the field due to the slab must also be the same on both ends. Thus the total field is the same magnitude on both ends.



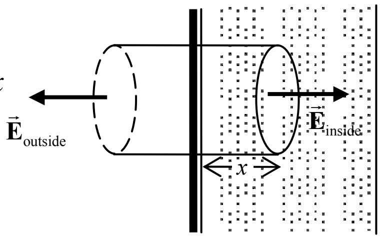
$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow 2E_{\text{outside}}A = \frac{\sigma A + \rho_E dA}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}} = E_{\substack{\text{left}\\ \text{of plane}}} = \boxed{\frac{\sigma + \rho_E d}{2\varepsilon_0}}$$

(*b*) As argued above, the field is symmetric on the outside of the charged matter.

$$E_{\substack{\text{right}\\ \text{of plane}}} = \boxed{\frac{\sigma + \rho_E d}{2\varepsilon_0}}$$

(*c*) To determine the field inside the slab, we choose a cylindrical gaussian surface of cross-sectional area A with one face to the left of the plane, and the other face inside the slab, a distance x from the plane. Due to symmetry, the field again is parallel to the surface area vector on both ends, has a constant value on each end, and no flux pierces the curved walls. Apply Gauss's law.



$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\ \text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\substack{\text{right}\\ \text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E_{\text{outside}}A + E_{\text{inside}}A + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

$$Q_{\text{encl}} = \sigma A + \rho_E xA \rightarrow \left(\frac{\sigma + \rho_E d}{2\varepsilon_0}\right)A + E_{\text{inside}}A = \frac{\sigma A + \rho_E xA}{\varepsilon_0} \rightarrow$$

$$E_{\text{inside}} = \boxed{\frac{\sigma + \rho_E\left(2x - d\right)}{2\varepsilon_0}},\ 0 < x < d$$

Notice that the field is continuous from "inside" to "outside" at the right edge of the slab, but not at the left edge of the slab. That discontinuity is due to the surface charge density.

61. Consider this sphere as a combination of two spheres. Sphere 1 is a solid sphere of radius r_0 and charge density ρ_E centered at A and sphere 2 is a second sphere of radius $r_0/2$ and density $-\rho_E$ centered at C.

(*a*) The electric field at A will have zero contribution from sphere 1 due to its symmetry about point A. The electric field is then calculated by creating a gaussian surface centered at point C with radius $r_0/2$.

$$\oint \mathbf{E}\cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E\cdot 4\pi\left(\tfrac{1}{2}r_0\right)^2 = \frac{\left(-\rho_E\right)\frac{4}{3}\pi\left(\frac{1}{2}r_0\right)^3}{\varepsilon_0} \rightarrow \boxed{E = -\frac{\rho_E r_0}{6\varepsilon_0}}$$

Since the electric field points into the gaussian surface (negative) the electric field at point A $\boxed{\text{points to the right}}$.

(*b*) At point B the electric field will be the sum of the electric fields from each sphere. The electric field from sphere 1 is calculated using a gaussian surface of radius r_0 centered at A.

$$\oint \vec{\mathbf{E}}_1\cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E_1\cdot 4\pi r_0^2 = \frac{\frac{4}{3}\pi r_0^3\left(\rho_E\right)}{\varepsilon_0} \rightarrow E_1 = \frac{\rho_E r_0}{3\varepsilon_0}$$

At point B the field from sphere 1 points toward the left. The electric field from sphere 2 is calculated using a gaussian surface centered at C of radius $3r_0/2$.

$$\oint \vec{\mathbf{E}}_2\cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E_2\cdot 4\pi\left(\tfrac{3}{2}r_0\right)^2 = \frac{\left(-\rho_E\right)\frac{4}{3}\pi\left(\frac{1}{2}r_0\right)^3}{\varepsilon_0} \rightarrow E_2 = -\frac{\rho_E r_0}{54\varepsilon_0}$$

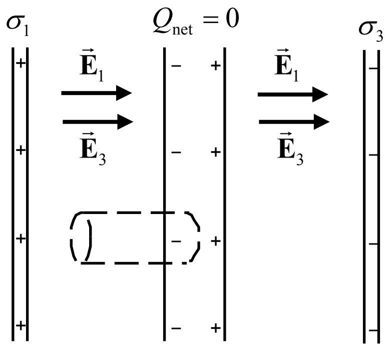
At point B, the electric field from sphere 2 points toward the right. The net electric field is the sum of these two fields. The net field $\boxed{\text{points to the left}}$.

$$E = E_1 + E_2 = \frac{\rho_E r_0}{3\varepsilon_0} + \frac{-\rho_E r_0}{54\varepsilon_0} = \boxed{\frac{17\rho_E r_0}{54\varepsilon_0}}.$$

62. We assume the charge is uniformly distributed, and so the field of the pea is that of a point charge.

$$E\left(r=R\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \rightarrow$$

$$Q = E4\pi\varepsilon_0 R^2 = \left(3\times10^6\ \text{N/C}\right)4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.00375\,\text{m}\right)^2 = \boxed{5\times10^{-9}\,\text{C}}$$



63. (*a*) In an electrostatic situation, there is no electric field inside a conductor. Thus $E = \boxed{0}$ inside the conductor.

(*b*) The positive sheet produces an electric field, external to itself, directed away from the plate with a magnitude as given in Example 22-7, of $E_1 = \frac{\left|\sigma_1\right|}{2\varepsilon_0}$. The negative sheet produces an electric field, external to itself, directed towards the plate with a magnitude of $E_2 = \frac{\left|\sigma_2\right|}{2\varepsilon_0}$. Between the left and middle sheets, those two fields are parallel and so add to each other.

$$E_{\substack{\text{left}\\ \text{middle}}} = E_1 + E_2 = \frac{\left|\sigma_1\right| + \left|\sigma_2\right|}{2\varepsilon_0} = \frac{2\left(5.00\times10^{-6}\ \text{C/m}^2\right)}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{5.65\times10^5\ \text{N/C}}, \text{ to the right}$$

(*c*) The same field is between the middle and right sheets. See the diagram.

$$E_{\substack{\text{middle}\\\text{right}}} = \boxed{5.65\times10^{5}\ \text{N/C}}\text{, to the right}$$

(*d*) To find the charge density on the surface of the left side of the middle sheet, choose a gaussian cylinder with ends of area *A*. Let one end be inside the conducting sheet, where there is no electric field, and the other end be in the area between the left and middle sheets. Apply Gauss's law in the manner of Example 22-16. Note that there is no flux through the curved sides of the cylinder, and there is no flux through the right end since it is in conducting material. Also note that the field through the left end is in the opposite direction as the area vector of the left end.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{end}}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + \int_{\substack{\text{right}\\\text{end}}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = -E_{\substack{\text{left}\\\text{middle}}} A + 0 + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_{\text{left}} A}{\varepsilon_0} \rightarrow$$

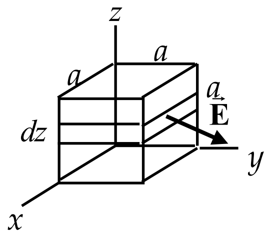
$$\sigma_{\text{left}} = -\varepsilon_0 E_{\substack{\text{left}\\\text{middle}}} = -\varepsilon_0\left(\frac{|\sigma_1| + |\sigma_2|}{2\varepsilon_0}\right) = \boxed{-5.00\times10^{-6}\ \text{C/m}^2}$$

(*e*) Because the middle conducting sheet has no net charge, the charge density on the right side must be the opposite of the charge density on the left side.

$$\sigma_{\text{right}} = -\sigma_{\text{left}} = \boxed{5.00\times10^{-6}\ \text{C/m}^2}$$

Alternatively, we could have applied Gauss's law on the right side in the same manner that we did on the left side. The same answer would result.

64. Because the electric field has only *x* and *y* components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a horizontal strip of height *dz* and width *a* for a differential element and integrate to find the flux. The total flux is used to determine the enclosed charge.



$$\Phi_{\substack{\text{front}\\(x=a)}} = \int \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\bullet\left(adz\hat{\mathbf{i}}\right)$$

$$= E_0 a\int_0^a\left(1+\frac{z}{a}\right)dz = E_0 a\left(z+\frac{z^2}{2a}\right)_0^a = \tfrac{3}{2}E_0a^2$$

$$\Phi_{\substack{\text{back}\\(x=0)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\bullet\left(-adz\hat{\mathbf{i}}\right) = -\tfrac{3}{2}E_0a^2$$

$$\Phi_{\substack{\text{right}\\(y=a)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\bullet\left(adz\hat{\mathbf{j}}\right) = E_0 a\int_0^a\left(\frac{z}{a}\right)dz = E_0 a\left(\frac{z^2}{2a}\right)_0^a = \tfrac{1}{2}E_0a^2$$

$$\Phi_{\substack{\text{left}\\(y=a)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\bullet\left(-adz\hat{\mathbf{j}}\right) = -\tfrac{1}{2}E_0a^2$$

$$\Phi_{\text{total}} = \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}} = \tfrac{3}{2}E_0a^2 - \tfrac{3}{2}E_0a^2 + \tfrac{1}{2}E_0a^2 - \tfrac{1}{2}E_0a^2 = 0 + 0$$

$$= 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow \boxed{Q_{\text{encl}} = 0}$$

65. (*a*) Because the shell is a conductor, there is no electric field in the conducting material, and all charge must reside on its surfaces. All of the field lines that originate from the point charge at the center must terminate on the inner surface of the shell. Therefore the inner surface must have an equal but opposite charge to the point charge at the center. Since the conductor has the same magnitude of charge as the point charge at the center, all of the charge on the conductor is $\boxed{\text{on the inner surface of the shell}}$, in a spherically symmetric distribution.

(*b*) By Gauss's law and the spherical symmetry of the problem, the electric field can be calculated by $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$.

$$r < 0.10\,\text{m}:\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.0\times10^{-6}\,\text{C}\right)}{r^2} = \boxed{\frac{2.7\times10^4\,\text{N}\cdot\text{m}^2/\text{C}}{r^2}}$$

$$r > 0.15\,\text{m}:\ E = \boxed{0}$$

And since there is no electric field in the shell, we could express the second answer as $r > 0.10\,\text{m}:\ E = \boxed{0}$.

66. (*a*) At a strip such as is marked in the textbook diagram, $d\vec{\mathbf{A}}$ is perpendicular to the surface, and $\vec{\mathbf{E}}$ is inclined at an angle θ relative to $d\vec{\mathbf{A}}$.

$$\Phi_{\text{hemisphere}} = \int \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_0^{\pi/2} E\cos\theta\left(2\pi R^2\sin\theta d\theta\right)$$

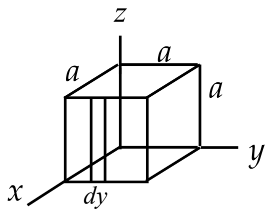
$$= 2\pi R^2 E\int_0^{\pi/2}\cos\theta\sin\theta d\theta = 2\pi R^2 E\left(\tfrac{1}{2}\sin^2\theta\right)_0^{\pi/2} = \pi R^2 E$$

(*b*) Choose a closed gaussian surface consisting of the hemisphere and the circle of radius *R* at the base of the hemisphere. There is no charge inside that closed gaussian surface, and so the total flux through the two surfaces (hemisphere and base) must be zero. The field lines are all perpendicular to the circle, and all of the same magnitude, and so that flux is very easy to calculate.

$$\Phi_{\text{circle}} = \int \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int E\left(\cos180^\circ\right)d\vec{\mathbf{A}} = -EA = -E\pi R^2$$

$$\Phi_{\text{total}} = 0 = \Phi_{\text{circle}} + \Phi_{\text{hemisphere}} = -E\pi R^2 + \Phi_{\text{hemisphere}} \ \rightarrow\ \Phi_{\text{hemisphere}} = \boxed{\pi R^2 E}$$

67. The flux is the sum of six integrals, each of the form $\iint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}$. Because the electric field has only *x* and *y* components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a vertical strip of height *a* and width *dy* (for the front and back faces) or *dx* (for the left and right faces). See the diagram for an illustration of a strip on the front face. The total flux is then calculated, and used to determine the enclosed charge.



$$\Phi_{\substack{\text{front}\\(x=a)}} = \int_0^a\left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}}\right)\bullet ady\,\hat{\mathbf{i}} = aE_{x0}\int_0^a e^{-\left(\frac{a+y}{a}\right)^2}dy$$

This integral does not have an analytic anti-derivative, and so must be integrated numerically. We approximate the integral by a sum: $\int_0^a e^{-\left(\frac{a+y}{a}\right)^2}dy \approx \sum_{i=1}^{n} e^{-\left(\frac{a+y_i}{a}\right)^2}\Delta y$. The region of integration is divided into *n* elements, and so $\Delta y = \dfrac{a-0}{n}$ and $y_i = i\Delta y$. We initially evaluate the sum for *n* = 10. Then we evaluate it for *n* = 20. If the two sums differ by no more than 2%, we take that as the value of the integral. If they differ by more than 2%, we choose a larger *n*, compute the sum, and compare that to the result for *n* = 20. We continue until a difference of 2% or less is reached. This integral, for *n* = 100 and *a* = 1.0 m, is 0.1335 m. So we have this intermediate result.

$$\underset{(x=a)}{\Phi_{\text{front}}} = aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{a+y_i}{a}\right)^2}\Delta y = (1.0\,\text{m})(50\,\text{N/C})(0.1335\,\text{m}) = 6.675\,\text{N}\bullet\text{m}^2/\text{C}$$

Now do the integral over the back face.

$$\underset{(x=0)}{\Phi_{\text{back}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}} \right)\bullet\left(-a\,dy\,\hat{\mathbf{i}}\right) = -aE_{x0}\int_0^a e^{-\left(\frac{y}{a}\right)^2}dy$$

We again get an integral that cannot be evaluated analytically. A similar process to that used for the front face is applied again, and so we make this approximation: $-aE_{x0}\int_0^a e^{-\left(\frac{y}{a}\right)^2}dy \approx -aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{y_i}{a}\right)^2}\Delta y$.

The numeric integration gives a value of 0.7405 m.

$$\underset{(x=0)}{\Phi_{\text{back}}} = -aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{y_i}{a}\right)^2}\Delta y = -(1.0\,\text{m})(50\,\text{N/C})(0.7405\,\text{m}) = -37.025\,\text{N}\bullet\text{m}^2/\text{C}.$$

Now consider the right side.

$$\underset{(y=a)}{\Phi_{\text{right}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}} \right)\bullet a\,dx\,\hat{\mathbf{j}} = aE_{y0}\int_0^a e^{-\left(\frac{x+a}{a}\right)^2}dx$$

Notice that the same integral needs to be evaluated as for the front side. All that has changed is the variable name. Thus we have the following.

$$\underset{(y=a)}{\Phi_{\text{right}}} = aE_{y0}\int_0^a e^{-\left(\frac{x+a}{a}\right)^2}dx \approx (1.0\,\text{m})(25\,\text{N/C})(0.1335\,\text{m}) = 3.3375\,\text{N}\bullet\text{m}^2/\text{C}$$

Finally, do the left side, following the same process. The same integral arises as for the back face.

$$\underset{(y=0)}{\Phi_{\text{left}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}} \right)\bullet\left(-a\,dx\,\hat{\mathbf{j}}\right) = -aE_{y0}\int_0^a e^{-\left(\frac{x}{a}\right)^2}dx$$

$$\approx -(1.0\,\text{m})(25\,\text{N/C})(0.7405\,\text{m}) = -18.5125\,\text{N}\bullet\text{m}^2/\text{C}$$

Sum to find the total flux, and multiply by ε_0 to find the enclosed charge.

$$\Phi_{\text{total}} = \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}}$$

$$= (6.675 - 37.025 + 3.3375 - 18.5125)\,\text{N}\bullet\text{m}^2/\text{C} = -45.525\,\text{N}\bullet\text{m}^2/\text{C} \approx \boxed{-46\,\text{N}\bullet\text{m}^2/\text{C}}$$

$$Q_{\text{encl}} = \varepsilon_0\Phi_{\text{total}} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(-45.525\,\text{N}\bullet\text{m}^2/\text{C}\right) = \boxed{-4.0\times10^{-10}\,\text{C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.67."

CHAPTER 23: Electric Potential

Responses to Questions

1. Not necessarily. If two points are at the same potential, then no *net* work is done in moving a charge from one point to the other, but work (both positive and negative) could be done at different parts of the path. No. It is possible that positive work was done over one part of the path, and negative work done over another part of the path, so that these two contributions to the net work sum to zero. In this case, a non-zero force would have to be exerted over both parts of the path.

2. The negative charge will move toward a region of higher potential and the positive charge will move toward a region of lower potential. In both cases, the potential energy of the charge will decrease.

3. (*a*) The electric potential is the electric potential energy per unit charge. The electric potential is a scalar. The electric field is the electric force per unit charge, and is a vector.
   (*b*) Electric potential is the electric potential energy per unit charge.

4. Assuming the electron starts from rest in both cases, the final speed will be twice as great. If the electron is accelerated through a potential difference that is four times as great, then its increase in kinetic energy will also be greater by a factor of four. Kinetic energy is proportional to the square of the speed, so the final speed will be greater by a factor of two.

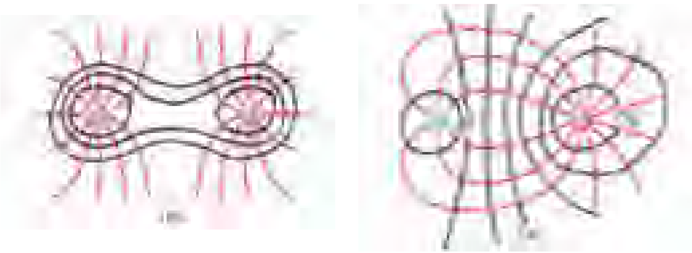
5. Yes. If the charge on the particle is negative and it moves from a region of low electric potential to a region of high electric potential, its electric potential energy will decrease.

6. No. Electric potential is the *potential energy* per unit charge at a point in space and electric field is the *electric force* per unit charge at a point in space. If one of these quantities is zero, the other is not necessarily zero. For example, the point exactly between two charges with equal magnitudes and opposite signs will have a zero electric potential because the contributions from the two charges will be equal in magnitude and opposite in sign. (Net electric potential is a *scalar* sum.) This point will not have a zero electric field, however, because the electric field contributions will be in the same direction (towards the negative and away from the positive) and so will add. (Net electric field is a *vector* sum.) As another example, consider the point exactly between two equal positive point charges. The electric potential will be positive since it is the sum of two positive numbers, but the electric field will be zero since the field contributions from the two charges will be equal in magnitude but opposite in direction.

7. (*a*) V at other points would be lower by 10 V. E would be unaffected, since E is the negative gradient of V, and a change in V by a constant value will not change the value of the gradient.
   (*b*) If V represents an absolute potential, then yes, the fact that the Earth carries a net charge would affect the value of V at the surface. If V represents a potential difference, then no, the net charge on the Earth would not affect the choice of V.

8. No. An equipotential line is a line connecting points of equal electric potential. If two equipotential lines crossed, it would indicate that their intersection point has two different values of electric potential simultaneously, which is impossible. As an analogy, imagine contour lines on a topographic map. They also never cross because one point on the surface of the Earth cannot have two different values for elevation above sea level.

9. The equipotential lines (in black) are perpendicular to the electric field lines (in red).



10. The electric field is zero in a region of space where the electric potential is constant. The electric field is the gradient of the potential; if the potential is constant, the gradient is zero.

11. The Earth's gravitational equipotential lines are roughly circular, so the orbit of the satellite would have to be roughly circular.

12. The potential at point P would be unchanged. Each bit of positive charge will contribute an amount to the potential based on its charge and its distance from point P. Moving charges to different locations on the ring does not change their distance from P, and hence does not change their contributions to the potential at P.

    The value of the electric field will change. The electric field is the vector sum of all the contributions to the field from the individual charges. When the charge Q is distributed uniformly about the ring, the y-components of the field contributions cancel, leaving a net field in the x-direction. When the charge is not distributed uniformly, the y-components will not cancel, and the net field will have both x- and y-components, and will be larger than for the case of the uniform charge distribution. There is no discrepancy here, because electric potential is a scalar and electric field is a vector.

13. The charge density and the electric field strength will be greatest at the pointed ends of the football because the surface there has a smaller radius of curvature than the middle.

14. No. You cannot calculate electric potential knowing only electric field at a point and you cannot calculate electric field knowing only electric potential at a point. As an example, consider the uniform field between two charged, conducting plates. If the potential difference between the plates is known, then the distance between the plates must also be known in order to calculate the field. If the field between the plates is known, then the distance to a point of interest between the plates must also be known in order to calculate the potential there. In general, to find V, you must know E and be able to integrate it. To find E, you must know V and be able to take its derivative. Thus you need E or V in the region around the point, not just at the point, in order to be able to find the other variable.

15. (*a*) Once the two spheres are placed in contact with each other, they effectively become one larger conductor. They will have the same potential because the potential everywhere on a conducting surface is constant.
    (*b*) Because the spheres are identical in size, an amount of charge $Q/2$ will flow from the initially charged sphere to the initially neutral sphere so that they will have equal charges.
    (*c*) Even if the spheres do not have the same radius, they will still be at the same potential once they are brought into contact because they still create one larger conductor. However, the amount of charge that flows will not be exactly equal to half the total charge. The larger sphere will end up with the larger charge.

16. If the electric field points due north, the change in the potential will be (*a*) greatest in the direction opposite the field, south; (*b*) least in the direction of the field, north; and (*c*) zero in a direction perpendicular to the field, east and west.

17. Yes. In regions of space where the equipotential lines are closely spaced, the electric field is stronger than in regions of space where the equipotential lines are farther apart.

18. If the electric field in a region of space is uniform, then you can infer that the electric potential is increasing or decreasing uniformly in that region. For example, if the electric field is 10 V/m in a region of space then you can infer that the potential difference between two points 1 meter apart (measured parallel to the direction of the field) is 10 V. If the electric potential in a region of space is uniform, then you can infer that the electric field there is zero.

19. The electric potential energy of two unlike charges is negative. The electric potential energy of two like charges is positive. In the case of unlike charges, work must be done to separate the charges. In the case of like charges, work must be done to move the charges together.

Solutions to Problems

1. Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.

$$\Delta U = q\Delta V = -\Delta K \quad \rightarrow$$

$$\Delta V = \frac{-\Delta K}{q} = \frac{K_{\text{initial}} - K_{\text{final}}}{q} = \frac{mv^2}{2q} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(5.0\times10^{5}\,\text{m/s}\right)^2}{2\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{-0.71\,\text{V}}$$

The final potential should be lower than the initial potential in order to stop the electron.

2. The work done by the electric field can be found from Eq. 23-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \quad \rightarrow \quad W_{\text{ba}} = -qV_{\text{ba}} = -\left(1.60\times10^{-19}\,\text{C}\right)\left[-55\,\text{V} - 185\,\text{V}\right] = \boxed{3.84\times10^{-17}\,\text{J}}$$

3. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 23-2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{5.25\times10^{-16}\,\text{J}}{\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{3280\,\text{V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

4. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 23-2b.

$$W_{\text{external}} + W_{\text{electric}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \quad \rightarrow \quad W_{\text{external}} - q\left(V_{\text{b}} - V_{\text{a}}\right) = \text{KE}_{\text{final}} \quad \rightarrow$$

$$\left(V_{\text{b}} - V_{\text{a}}\right) = \frac{W_{\text{external}} - \text{KE}_{\text{final}}}{q} = \frac{7.00\times10^{-4}\,\text{J} - 2.10\times10^{-4}\,\text{J}}{-9.10\times10^{-6}\text{C}} = \boxed{-53.8\,\text{V}}$$

Since the potential difference is negative, we see that $V_{\text{a}} > V_{\text{b}}$.

5. As an estimate, the length of the bolt would be the voltage difference of the bolt divided by the breakdown electric field of air.

$$\frac{1\times10^{8}\,\text{V}}{3\times10^{6}\,\text{V/m}}=33\,\text{m}\approx\boxed{30\,\text{m}}$$

6. The distance between the plates is found from Eq. 23-4b, using the magnitude of the electric field.

$$|E|=\frac{V_{\text{ba}}}{d}\rightarrow d=\frac{V_{\text{ba}}}{|E|}=\frac{45\,\text{V}}{1300\,\text{V/m}}=\boxed{3.5\times10^{-2}\,\text{m}}$$

$\boxed{7.}$ The maximum charge will produce an electric field that causes breakdown in the air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\text{ and }V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\rightarrow$$

$$Q=4\pi\varepsilon_0r_0^2E_{\text{breakdown}}=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(0.065\,\text{m})^2\left(3\times10^{6}\,\text{V/m}\right)=\boxed{1.4\times10^{-6}\,\text{C}}$$

8. We assume that the electric field is uniform, and so use Eq. 23-4b, using the magnitude of the electric field.

$$E=\frac{V_{\text{ba}}}{d}=\frac{110\,\text{V}}{4.0\times10^{-3}\,\text{m}}=\boxed{2.8\times10^{4}\,\text{V/m}}$$

9. To find the limiting value, we assume that the E-field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\rightarrow r_0=\frac{V_{\text{surface}}}{E_{\text{breakdown}}}=\frac{35{,}000\,\text{V}}{3\times10^{6}\,\text{V/m}}=0.0117\,\text{m}\approx\boxed{0.012\,\text{m}}$$

$$V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\rightarrow Q=4\pi\varepsilon_0V_{\text{surface}}r_0=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(35{,}000\,\text{V})(0.0117\,\text{m})$$

$$=\boxed{4.6\times10^{-8}\,\text{C}}$$

10. If we assume the electric field is uniform, then we can use Eq. 23-4b to estimate the magnitude of the electric field. From Problem 22-24 we have an expression for the electric field due to a pair of oppositely charged planes. We approximate the area of a shoe as 30 cm x 8 cm.

$$E=\frac{V}{d}=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0A}\rightarrow$$

$$Q=\frac{\varepsilon_0AV}{d}=\frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2\right)\left(0.024\text{m}^2\right)\left(5.0\times10^{3}\,\text{V}\right)}{1.0\times10^{-3}\,\text{m}}=\boxed{1.1\times10^{-6}\,\text{C}}$$

11. Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.

(*a*) $\boxed{V_{\text{BA}}=0}$. The distance between the two points is exactly perpendicular to the field lines.

(*b*) $V_{\text{CB}}=V_{\text{C}}-V_{\text{B}}=(-4.20\,\text{N/C})(7.00\,\text{m})=\boxed{-29.4\,\text{V}}$

(*c*) $V_{\text{CA}}=V_{\text{C}}-V_{\text{A}}=V_{\text{C}}-V_{\text{B}}+V_{\text{B}}-V_{\text{A}}=V_{\text{CB}}+V_{\text{BA}}=-29.4\,\text{V}+0=\boxed{-29.4\,\text{V}}$

12. From Example 22-7, the electric field produced by a large plate is uniform with magnitude $E=\dfrac{\sigma}{2\varepsilon_0}$.

The field points away from the plate, assuming that the charge is positive. Apply Eq. 23-41.

$$V(x)-V(0)=V(x)-V_0=-\int_0^x\vec{\mathbf{E}}\cdot\left(d\vec{\boldsymbol{\ell}}\right)=-\int_0^x\left(\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{i}}\right)\cdot\left(dx\hat{\mathbf{i}}\right)=-\frac{\sigma x}{2\varepsilon_0}\quad\rightarrow\quad\boxed{V(x)=V_0-\frac{\sigma x}{2\varepsilon_0}}$$

$\boxed{13.}$ (*a*) The electric field at the surface of the Earth is the same as that of a point charge, $E=\dfrac{Q}{4\pi\varepsilon_0 r_0^2}$.

The electric potential at the surface, relative to $V(\infty)=0$ is given by Eq. 23-5. Writing this in terms of the electric field and radius of the earth gives the electric potential.

$$V=\frac{Q}{4\pi\varepsilon_0 r_0}=Er_0=\left(-150\,\text{V/m}\right)\left(6.38\times10^6\,\text{m}\right)=\boxed{-0.96\text{ GV}}$$

(*b*) Part (*a*) demonstrated that the potential at the surface of the earth is 0.96 GV lower than the potential at infinity. Therefore if the potential at the surface of the Earth is taken to be zero, the potential at infinity must be $V(\infty)=\boxed{0.96\text{ GV}}$. If the charge of the ionosphere is included in the calculation, the electric field outside the ionosphere is basically zero. The electric field between the earth and the ionosphere would remain the same. The electric potential, which would be the integral of the electric field from infinity to the surface of the earth, would reduce to the integral of the electric field from the ionosphere to the earth. This would result in a negative potential, but of a smaller magnitude.

14. (*a*) The potential at the surface of a charged sphere is derived in Example 23-4.

$$V_0=\frac{Q}{4\pi\varepsilon_0 r_0}\quad\rightarrow\quad Q=4\pi\varepsilon_0 r_0 V_0\quad\rightarrow$$

$$\sigma=\frac{Q}{\text{Area}}=\frac{Q}{4\pi r_0^2}=\frac{4\pi\varepsilon_0 r_0 V_0}{4\pi r_0^2}=\frac{V_0\varepsilon_0}{r_0}=\frac{\left(680\,\text{V}\right)\left(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2\right)}{\left(0.16\,\text{m}\right)}=3.761\times10^{-8}\,\text{C/m}^2$$

$$\approx\boxed{3.8\times10^{-8}\,\text{C/m}^2}$$

(*b*) The potential away from the surface of a charged sphere is also derived in Example 23-4.

$$V=\frac{Q}{4\pi\varepsilon_0 r}=\frac{4\pi\varepsilon_0 r_0 V_0}{4\pi\varepsilon_0 r}=\frac{r_0 V_0}{r}\quad\rightarrow\quad r=\frac{r_0 V_0}{V}=\frac{\left(0.16\,\text{m}\right)\left(680\,\text{V}\right)}{\left(25\,\text{V}\right)}=4.352\,\text{m}\approx\boxed{4.4\,\text{m}}$$

15. (*a*) After the connection, the two spheres are at the $\boxed{\text{same potential}}$. If they were at different potentials, then there would be a flow of charge in the wire until the potentials were equalized.

(*b*) We assume the spheres are so far apart that the charge on one sphere does not influence the charge on the other sphere. Another way to express this would be to say that the potential due to either of the spheres is zero at the location of the other sphere. The charge splits between the spheres so that their potentials (due to the charge on them only) are equal. The initial charge on sphere 1 is *Q*, and the final charge on sphere 1 is Q_1.

$$V_1=\frac{Q_1}{4\pi\varepsilon_0 r_1}\ ;\ V_2=\frac{Q-Q_1}{4\pi\varepsilon_0 r_2}\ ;\ V_1=V_2\quad\rightarrow\quad\frac{Q_1}{4\pi\varepsilon_0 r_1}=\frac{Q-Q_1}{4\pi\varepsilon_0 r_2}\quad\rightarrow\quad Q_1=Q\frac{r_1}{\left(r_1+r_2\right)}$$

$$\text{Charge transferred }Q-Q_1=Q-Q\frac{r_1}{\left(r_1+r_2\right)}=\boxed{Q\frac{r_2}{\left(r_1+r_2\right)}}$$

16. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_{\text{b}} - V_{\text{a}} = -\int_{R_{\text{a}}}^{R_{\text{b}}} \vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_{\text{a}}}^{R_{\text{b}}} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}\,dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_{\text{b}} - R_{\text{a}}\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_{\text{a}}}{\text{R}_{\text{b}}}}$$

17. (*a*) The width of the end of a finger is about 1 cm, and so consider the fingertip to be a part of a sphere of diameter 1 cm. We assume that the electric field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}} = r_0 E_{\text{breakdown}} = \left(0.005\,\text{m}\right)\left(3\times10^{6}\ \text{V/m}\right) = \boxed{15{,}000\,\text{V}}$$

Since this is just an estimate, we might expect anywhere from 10,000 V to 20,000 V.

(*b*) $V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi r_0^2\sigma}{r_0} \quad \rightarrow$

$$\sigma = V_{\text{surface}}\frac{\varepsilon_0}{r_0} = \left(15{,}000\,\text{V}\right)\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)}{0.005\,\text{m}} = \boxed{2.7\times10^{-5}\ \text{C/m}^2}$$

Since this is an estimate, we might say the charge density is on the order of $30\,\mu\text{C/m}^2$.

18. We assume the field is uniform, and so Eq. 23-4b applies.

$$E = \frac{V}{d} = \frac{0.10\,\text{V}}{10\times10^{-9}\,\text{m}} = \boxed{1\times10^{7}\ \text{V/m}}$$

19. (*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest will give the potential at that radius.

$$E\left(r \ge r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2}\ ;\ V\left(r \ge r_0\right) = -\int_{\infty}^{r}\frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{\infty}^{r} = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius *r*.

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} \quad \rightarrow \quad E\left(r < r_0\right) = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$$

Integrating the electric field from the surface to $r < r_0$ gives the electric potential inside the sphere.

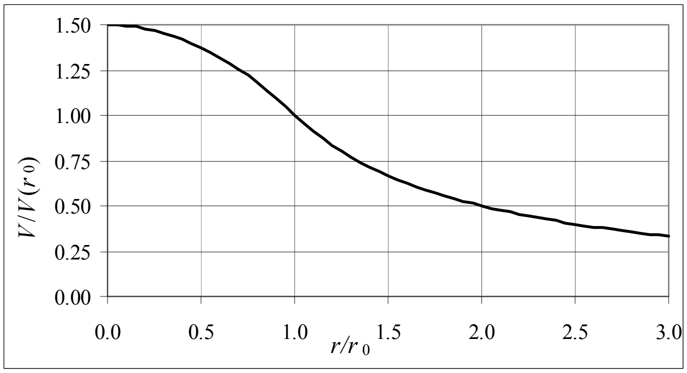
$$V\left(r < r_0\right) = V\left(r_0\right) - \int_{r_0}^{r}\frac{Qr}{4\pi\varepsilon_0 r_0^3}\,dr = \frac{Q}{4\pi\varepsilon_0 r_0} - \left.\frac{Qr^2}{8\pi\varepsilon_0 r_0^3}\right|_{r_0}^{r} = \boxed{\frac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \frac{r^2}{r_0^2}\right)}$$

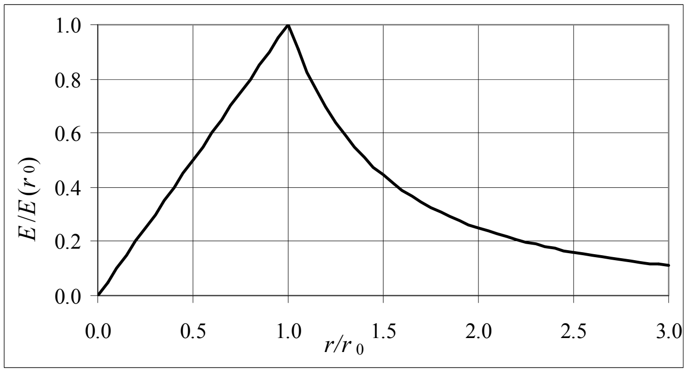
(*c*) To plot, we first calculate $V_0 = V\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0 = E\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r < r_0$: $$V/V_0 = \frac{\dfrac{Q}{8\pi\varepsilon_0 r_0}\left(3-\dfrac{r^2}{r_0^2}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \tfrac{1}{2}\left(3-\frac{r^2}{r_0^2}\right) \;;\; E/E_0 = \frac{\dfrac{Qr}{4\pi\varepsilon_0 r_0^3}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \frac{r}{r_0}$$

For $r > r_0$: $$V/V_0 = \frac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \frac{r_0}{r} = \left(r/r_0\right)^{-1} \;;\; E/E_0 = \frac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \frac{r_0^2}{r^2} = \left(r/r_0\right)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.19c."





20. We assume the total charge is still Q, and let $\rho_E = kr^2$. We evaluate the constant k by calculating the total charge, in the manner of Example 22-5.

$$Q = \int \rho_E dV = \int_0^{r_0} kr^2\left(4\pi r^2 dr\right) = \tfrac{4}{5}k\pi r_0^5 \;\rightarrow\; k = \frac{5Q}{4\pi r_0^5}$$

(*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest gives the potential at that radius.

$$E\left(r \ge r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \;;\; V\left(r \ge r_0\right) = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_\infty^r = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius *r*.

$$4\pi r^2 E = \frac{Q_{\text{encl}}}{\varepsilon_0} \;;\; Q_{\text{encl}} = \int \rho_E dV = \frac{5Q}{4\pi r_0^5}\int_0^r r^2\left(4\pi r^2 dr\right) = \frac{5Q}{4\pi r_0^5}\tfrac{4}{5}\pi r^5 = \frac{Qr^5}{r_0^5} \;\rightarrow$$

$$E\left(r<r_0\right)=\frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}=\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}$$

Integrating the electric field from the surface to $r<r_0$ gives the electric potential inside the sphere.

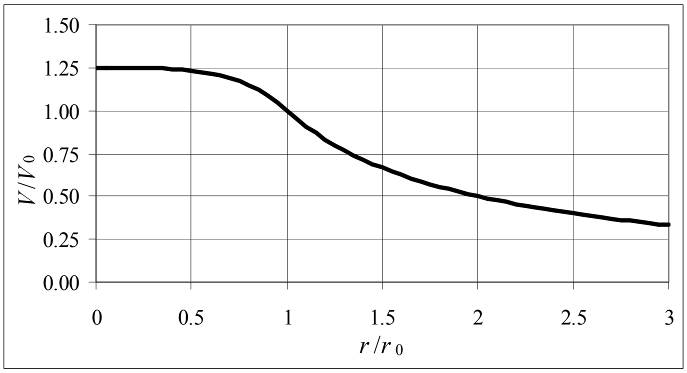
$$V\left(r<r_0\right)=V\left(r_0\right)-\int_{r_0}^{r}\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}dr=\frac{Q}{4\pi\varepsilon_0 r_0}-\left.\frac{Qr^4}{16\pi\varepsilon_0 r_0^5}\right|_{r_0}^{r}=\frac{Q}{16\pi\varepsilon_0 r_0}\left(5-\frac{r^4}{r_0^4}\right)$$

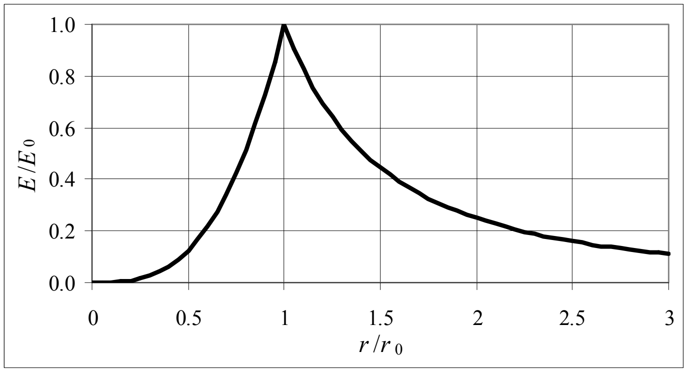
(*c*) To plot, we first calculate $V_0=V\left(r=r_0\right)=\dfrac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0=E\left(r=r_0\right)=\dfrac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r<r_0$: $$V/V_0=\frac{\dfrac{Q}{16\pi\varepsilon_0 r_0}\left(5-\dfrac{r^4}{r_0^4}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\tfrac{1}{4}\left(5-\frac{r^4}{r_0^4}\right)\ ;\ E/E_0=\frac{\dfrac{Qr^3}{4\pi\varepsilon_0 r_0^5}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\frac{r^3}{r_0^3}$$

For $r>r_0$: $$V/V_0=\frac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\frac{r_0}{r}=\left(r/r_0\right)^{-1}\ ;\ E/E_0=\frac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\frac{r_0^2}{r^2}=\left(r/r_0\right)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.20c."





21. We first need to find the electric field. Since the charge distribution is spherically symmetric, Gauss's law tells us the electric field everywhere.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$$

If $r < r_0$, calculate the charge enclosed in the manner of Example 22-5.

$$Q_{\text{encl}} = \int \rho_E dV = \int_0^r \rho_0\left[1-\frac{r^2}{r_0^2}\right]4\pi r^2 dr = 4\pi\rho_0\int_0^r\left[r^2-\frac{r^4}{r_0^2}\right]dr = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5r_0^2}\right]$$

The total charge in the sphere is the above expression evaluated at $r = r_0$.

$$Q_{\text{total}} = 4\pi\rho_0\left[\frac{r_0^3}{3}-\frac{r_0^5}{5r_0^2}\right] = \frac{8\pi\rho_0 r_0^3}{15}$$

Outside the sphere, we may treat it as a point charge, and so the potential at the surface of the sphere is given by Eq. 23-5, evaluated at the surface of the sphere.

$$V\left(r=r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{total}}}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{8\pi\rho_0 r_0^3}{15}}{r_0} = \frac{2\rho_0 r_0^2}{15\varepsilon_0}$$

The potential inside is found from Eq. 23-4a. We need the field inside the sphere to use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = E dr$.

$$E\left(r<r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5r_0^2}\right]}{r^2} = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^3}{5r_0^2}\right]$$

$$V_r - V_{r_0} = -\int_{r_0}^{r}\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = -\int_{r_0}^{r} E\,dr = -\int_{r_0}^{r}\frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^3}{5r_0^2}\right]dr = -\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right]_{r_0}^{r}$$

$$V_r = V_{r_0} + \left(-\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right]_{r_0}^{r}\right) = \frac{2\rho_0 r_0^2}{15\varepsilon_0} - \frac{\rho_0}{\varepsilon_0}\left[\left(\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right)-\left(\frac{r_0^2}{6}-\frac{r_0^4}{20r_0^2}\right)\right]$$

$$= \boxed{\frac{\rho_0}{\varepsilon_0}\left(\frac{r_0^2}{4}-\frac{r^2}{6}+\frac{r^4}{20r_0^2}\right)}$$

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.

(*a*) For $r > r_2$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r^2} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

For $r_1 < r < r_2$: $E = \boxed{0}$, because the electric field is 0 inside of conducting material.

For $0 < r < r_1$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{2}Q}{r^2} = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

(*b*) For $r > r_2$, the potential is that of a point charge at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}},\ r > r_2$$

(*c*) For $r_1 < r < r_2$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$V = V\left(r = r_2\right) = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}},\quad r_1 < r < r_2$$

(*d*) For $0 < r < r_1$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$V_r - V_{r_1} = -\int_{r_1}^{r} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{r_1}^{r} E\,dr = -\int_{r_1}^{r} \frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$V_r = V_{r_1} + \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) = \boxed{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)},\quad 0 < r < r_1$$

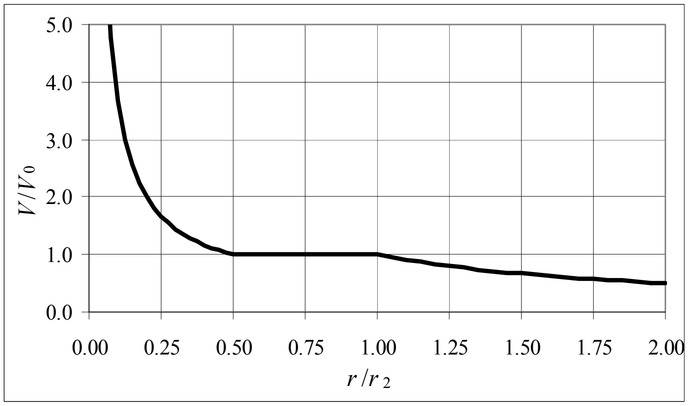
(*e*) To plot, we first calculate $V_0 = V\left(r = r_2\right) = \dfrac{3Q}{8\pi\varepsilon_0 r_2}$ and $E_0 = E\left(r = r_2\right) = \dfrac{3Q}{8\pi\varepsilon_0 r_2^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_2.

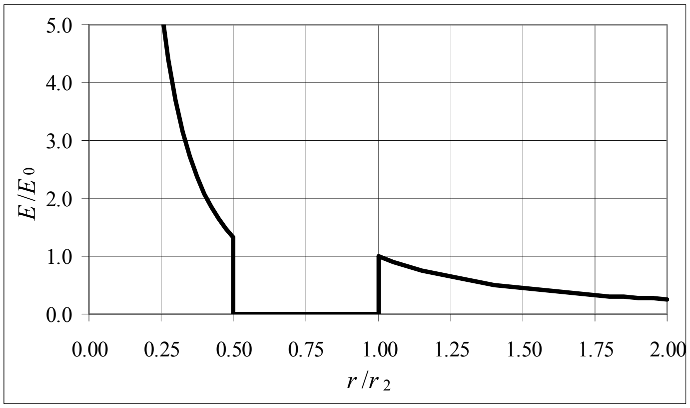
For $0 < r < r_1$: $$\frac{V}{V_0} = \frac{\dfrac{Q}{8\pi\varepsilon_0}\left(\dfrac{1}{r_2} + \dfrac{1}{r}\right)}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \tfrac{1}{3}\left[1 + \left(r/r_2\right)^{-1}\right]\;;\; \frac{E}{E_0} = \frac{\dfrac{1}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \tfrac{1}{3}\frac{r_2^2}{r^2} = \tfrac{1}{3}\left(r/r_2\right)^{-2}$$

For $r_1 < r < r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r_2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = 1\;;\; \frac{E}{E_0} = \frac{0}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = 0$$

For $r > r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \frac{r_2}{r} = \left(r/r_2\right)^{-1}\;;\; \frac{E}{E_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{r_2^2}{r^2} = \left(r/r_2\right)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.22e."





23. The field is found in Problem 22-33. The field inside the cylinder is 0, and the field outside the cylinder is $\dfrac{\sigma R_0}{\varepsilon_0 R}$.

(*a*) Use Eq. 23-4a to find the potential. Integrate along a radial line, so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = E dR$.

$$V_R - V_{R_0} = -\int_{R_0}^{R} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{R_0}^{R} E\,dR = -\int_{R_0}^{R} \frac{\sigma R_0}{\varepsilon_0 R}dR = -\frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0} \rightarrow$$

$$V_R = \boxed{V_0 - \frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0}},\ R > R_0$$

(*b*) The electric field inside the cylinder is 0, so the potential inside is constant and equal to the potential on the surface, $\boxed{V_0}$.

(*c*) $\boxed{\text{No}}$, we are not able to assume that $V = 0$ at $R = \infty$. $V \neq 0$ because there would be charge at infinity for an infinite cylinder. And from the formula derived in (*a*), if $R = \infty$, $V_R = -\infty$.

24. Use Eq. 23-5 to find the charge.

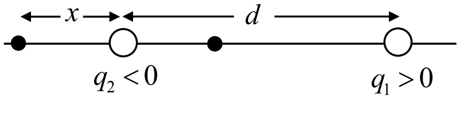
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \rightarrow Q = \left(4\pi\varepsilon_0\right)rV = \left(\frac{1}{8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2}\right)\left(0.15\ \text{m}\right)\left(185\,\text{V}\right) = \boxed{3.1\times10^{-9}\text{C}}$$

$\boxed{25.}$ (*a*) The electric potential is given by Eq. 23-5.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{1.60\times10^{-19}\text{C}}{0.50\times10^{-10}\text{m}} = 28.77\,\text{V} \approx \boxed{29\,\text{V}}$$

(*b*) The potential energy of the electron is the charge of the electron times the electric potential due to the proton.

$$U = QV = \left(-1.60\times10^{-19}\text{C}\right)\left(28.77\,\text{V}\right) = \boxed{-4.6\times10^{-18}\,\text{J}}$$



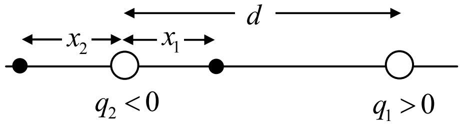
26. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge $\left(q_2\right)$. Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge,

but not between them. In the diagram, this is the point to the left of q_2. Take rightward as the positive direction.

$$E=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{q_1}{(d+x)^2}=0 \;\rightarrow\; |q_2|(d+x)^2=q_1x^2 \;\rightarrow$$

$$x=\frac{\sqrt{|q_2|}}{\sqrt{q_1}-\sqrt{|q_2|}}d=\frac{\sqrt{2.0\times10^{-6}\,\mathrm{C}}}{\sqrt{3.4\times10^{-6}\,\mathrm{C}}-\sqrt{2.0\times10^{-6}\,\mathrm{C}}}(5.0\,\mathrm{cm})=\boxed{16\,\mathrm{cm\ left\ of\ }q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.



$$V_{\text{location 1}}=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d-x_1)}+\frac{q_2}{x_1}\right]=0 \;\rightarrow\; x_1=\frac{q_2d}{(q_2-q_1)}=\frac{(-2.0\times10^{-6}\mathrm{C})(5.0\,\mathrm{cm})}{(-5.4\times10^{-6}\mathrm{C})}=1.852\,\mathrm{cm}$$

$$V_{\text{location 2}}=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d+x_2)}+\frac{q_2}{x_2}\right]=0 \;\rightarrow$$

$$x_2=-\frac{q_2d}{(q_1+q_2)}=-\frac{(-2.0\times10^{-6}\mathrm{C})(5.0\,\mathrm{cm})}{(1.4\times10^{-6}\mathrm{C})}=7.143\,\mathrm{cm}$$

So the two locations where the potential is zero are 1.9 cm from the negative charge towards the positive charge, and 7.1 cm from the negative charge away from the positive charge.

27. The work required is the difference in the potential energy of the charges, calculated with the test charge at the two different locations. The potential energy of a pair of charges is given in Eq. 23-10 as $U=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$. So to find the work, calculate the difference in potential energy with the test charge at the two locations. Let *Q* represent the $25\mu\mathrm{C}$ charge, let *q* represent the $0.18\mu\mathrm{C}$ test charge, *D* represent the 6.0 cm distance, and let *d* represent the 1.0 cm distance. Since the potential energy of the two $25\mu\mathrm{C}$ charges doesn't change, we don't include it in the calculation.

$$U_{\text{initial}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2} \qquad U_{\text{final}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]}$$

$$\begin{aligned}\text{Work}_{\substack{\text{external}\\\text{force}}}&=U_{\text{final}}-U_{\text{initial}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]}-2\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}\right)\\&=\frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{[D-2d]}+\frac{1}{[D+2d]}-\frac{1}{D/2}\right]\\&=2(8.99\times10^9\,\mathrm{N\bullet m^2/C^2})(25\times10^{-6}\mathrm{C})(0.18\times10^{-6}\mathrm{C})\left[\frac{1}{0.040\,\mathrm{m}}+\frac{1}{0.080\,\mathrm{m}}-\frac{1}{0.030\,\mathrm{m}}\right]\\&=\boxed{0.34\,\mathrm{J}}\end{aligned}$$

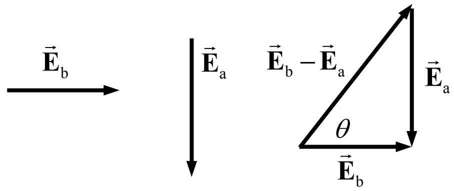
An external force needs to do positive work to move the charge.

28. (*a*) The potential due to a point charge is given by Eq. 23-5.

$$V_{\text{ba}} = V_{\text{b}} - V_{\text{a}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{\text{b}}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{\text{a}}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_{\text{b}}} - \frac{1}{r_{\text{a}}}\right)$$

$$= \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-3.8\times10^{-6}\text{C}\right)\left(\frac{1}{0.36\,\text{m}} - \frac{1}{0.26\,\text{m}}\right) = \boxed{3.6\times10^{4}\,\text{V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 21-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.



$$\vec{\mathbf{E}}_{\text{b}} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_{\text{b}}^2}\hat{\mathbf{i}} = \frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.36\,\text{m}\right)^2}\hat{\mathbf{i}} = 2.636\times10^{5}\ \text{V/m}\,\hat{\mathbf{i}}$$

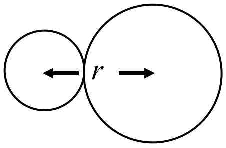
$$\vec{\mathbf{E}}_{\text{a}} = -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_{\text{a}}^2}\hat{\mathbf{j}} = -\frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.26\,\text{m}\right)^2}\hat{\mathbf{j}} = -5.054\times10^{5}\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_{\text{b}} - \vec{\mathbf{E}}_{\text{a}} = 2.636\times10^{5}\ \text{V/m}\,\hat{\mathbf{i}} + 5.054\times10^{5}\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{E}}_{\text{b}} - \vec{\mathbf{E}}_{\text{a}}\right| = \sqrt{\left(2.636\times10^{5}\ \text{V/m}\right)^2 + \left(5.054\times10^{5}\ \text{V/m}\right)^2} = \boxed{5.7\times10^{5}\ \text{V/m}}$$

$$\theta = \tan^{-1}\frac{-E_{\text{a}}}{E_{\text{b}}} = \tan^{-1}\frac{5.054\times10^{5}}{2.636\times10^{5}} = \boxed{62^\circ}$$

29. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.



$$U_{\text{initial}} = U_{\text{final}} \rightarrow eV_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{e(14e)}{r} \rightarrow$$

$$V_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{14e}{r} = \left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(14)\left(1.60\times10^{-19}\text{C}\right)}{(1.2+3.6)\times10^{-15}\text{m}} = \boxed{4.2\times10^{6}\text{V}}$$

30. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r} = 2\left(\tfrac{1}{2}mv^2\right) \rightarrow$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{mr}} = \sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(5.5\times10^{-6}\text{C}\right)^2}{\left(1.0\times10^{-6}\text{kg}\right)\left(0.065\,\text{m}\right)}} = \boxed{2.0\times10^{3}\ \text{m/s}}$$

$\boxed{31.}$ By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed, and so has no kinetic energy. When the electron is far away, there is no potential energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} \rightarrow \frac{(-e)(Q)}{4\pi\varepsilon_0 r} = \tfrac{1}{2}mv^2 \rightarrow$$

$$v = \sqrt{\frac{2(-e)(Q)}{(4\pi\varepsilon_0)mr}} = \sqrt{\frac{2\left(8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\,\text{C}\right)\left(-1.25\times10^{-10}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.425\,\text{m}\right)}}$$

$$= \boxed{9.64\times10^{5}\,\text{m/s}}$$

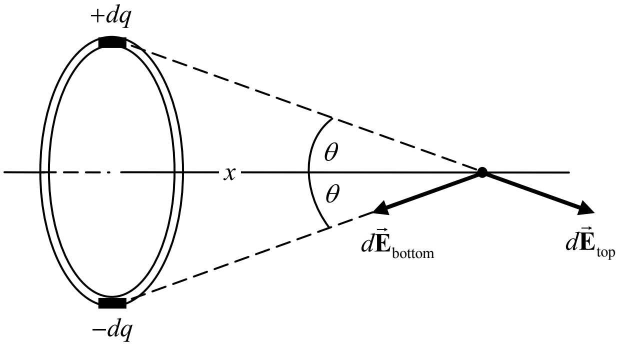
32. Use Eq. 23-2b and Eq. 23-5.

$$V_{\text{BA}} = V_{\text{B}} - V_{\text{A}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d-b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{b}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{d-b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right) = 2\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2q(2b-d)}{4\pi\varepsilon_0 b(d-b)}}$$

33. (*a*) For every element *dq* as labeled in Figure 23-14 on the top half of the ring, there will be a diametrically opposite element of charge –*dq*. The potential due to those two infinitesimal elements will cancel each other, and so the potential due to the entire ring is $\boxed{0}$.

(*b*) We follow Example 21-9 from the textbook. But because the upper and lower halves of the ring are oppositely charged, the parallel components of the fields from diametrically opposite infinitesimal segments of the ring will cancel each other, and the perpendicular components add, in the negative *y* direction. We know then that $\boxed{E_x = 0}$.



$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{2\pi R}d\ell}{\left(x^2+R^2\right)}\frac{R}{\left(x^2+R^2\right)^{1/2}} = -\frac{Q}{8\pi^2\varepsilon_0}\frac{d\ell}{\left(x^2+R^2\right)^{3/2}}$$

$$E_y = \int_0^{2\pi R} dE_y = -\frac{Q}{8\pi^2\varepsilon_0}\frac{1}{\left(x^2+R^2\right)^{3/2}}\int_0^{2\pi R} d\ell = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}} \rightarrow$$

$$\vec{\mathbf{E}} = \boxed{-\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}}\hat{\mathbf{j}}}$$

Note that for $x \gg R$, this reduces to $\vec{\mathbf{E}} = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{x^3}\hat{\mathbf{j}}$, which has the typical distance dependence for the field of a dipole, along the axis of the dipole.

34. The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 23-5.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{(3Q)}{\ell}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell}+\frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\ell}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}\left(1+\frac{1}{\sqrt{2}}\right)=\boxed{\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}Q}{2\ell}\left(\sqrt{2}+1\right)}$$

35. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius R and thickness dR is $dq=\sigma dA=\sigma\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{\sigma\left(2\pi R dR\right)}{\sqrt{x^2+R^2}}=\frac{\sigma}{2\varepsilon_0}\int_{R_1}^{R_2}\frac{R}{\sqrt{x^2+R^2}}dR=\frac{\sigma}{2\varepsilon_0}\left(x^2+R^2\right)^{1/2}\Big|_{R_1}^{R_2}$$

$$=\boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R_2^2}-\sqrt{x^2+R_1^2}\right)}$$

36. All of the charge is the same distance from the center of the semicircle – the radius of the semicircle. Use Eq 23-6b to calculate the potential.

$$\ell=\pi r_0\ \rightarrow\ r_0=\frac{\ell}{\pi}\ ;\ V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0 r_0}\int dq=\frac{Q}{4\pi\varepsilon_0\frac{\ell}{\pi}}=\boxed{\frac{Q}{4\varepsilon_0\ell}}$$

37. The electric potential energy is the product of the point charge and the electric potential at the location of the charge. Since all points on the ring are equidistant from any point on the axis, the electric potential integral is simple.

$$U=qV=q\int\frac{dq}{4\pi\varepsilon_0\sqrt{r^2+x^2}}=\frac{q}{4\pi\varepsilon_0\sqrt{r^2+x^2}}\int dq=\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

Energy conservation is used to obtain a relationship between the potential and kinetic energies at the center of the loop and at a point 2.0 m along the axis from the center.

$$K_0+U_0=K+U$$

$$0+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2}}=\tfrac{1}{2}mv^2+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

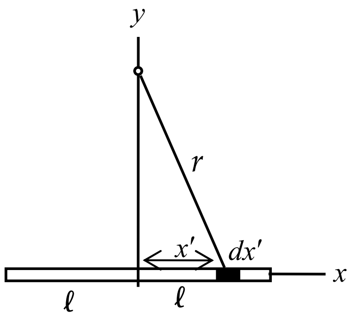
This is equation is solved to obtain the velocity at $x = 2.0$ m.

$$v=\sqrt{\frac{qQ}{2\pi\varepsilon_0 m}\left(\frac{1}{r}-\frac{1}{\sqrt{r^2+x^2}}\right)}$$

$$=\sqrt{\frac{(3.0\,\mu\text{C})(15.0\,\mu\text{C})}{2\pi\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(7.5\times10^{-3}\text{kg}\right)}\left(\frac{1}{0.12\,\text{m}}-\frac{1}{\sqrt{(0.12\,\text{m})^2+(2.0\,\text{m})^2}}\right)}$$

$$=\boxed{29\text{ m/s}}$$

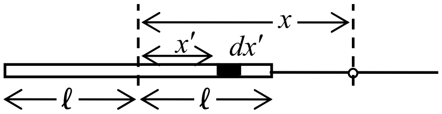
38. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the y axis. Use an indefinite integral from Appendix B-4, page A-7.



$$V_{y\,\text{axis}} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{\frac{Q}{2\ell}dx'}{\sqrt{x'^2+y^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[\ln\left(\sqrt{x'^2+y^2}+x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2+y^2}+\ell}{\sqrt{\ell^2+y^2}-\ell}\right)\right]}$$

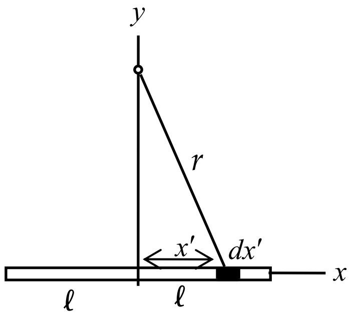
39. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $x - x'$ from a point outside the rod on the x axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{\frac{Q}{2\ell}dx'}{x-x'} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[-\ln\left(x-x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x+\ell}{x-\ell}\right)\right]},\ x > \ell$$

40. For both parts of the problem, use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \lambda dx' = ax'dx'$.

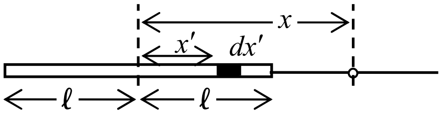
(*a*) The element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the y axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{ax'dx'}{\sqrt{x'^2+y^2}} = 0$$

The integral is equal to 0 because the region of integration is "even" with respect to the origin, while the integrand is "odd." Alternatively, the antiderivative can be found, and the integral can be shown to be 0. This is to be expected since the potential from points symmetric about the origin would cancel on the y axis.

(*b*) The element is a distance $x - x'$ from a point outside the rod on the x axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{ax'dx'}{x-x'} = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{x'dx'}{x-x'}$$

A substitution of $z = x - x'$ can be used to do the integration.

$$V = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{x'dx'}{x-x'} = \frac{a}{4\pi\varepsilon_0}\int_{x+\ell}^{x-\ell}\frac{(x-z)(-dz)}{z} = \frac{a}{4\pi\varepsilon_0}\int_{x-\ell}^{x+\ell}\left(\frac{x}{z}-1\right)dz$$

$$= \frac{a}{4\pi\varepsilon_0}\left(x\ln z - z\right)_{x-\ell}^{x+\ell} = \boxed{\frac{a}{4\pi\varepsilon_0}\left[x\ln\left(\frac{x+\ell}{x-\ell}\right)-2\ell\right]}, x > \ell$$

41. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius R and thickness dR will now be $dq = \sigma dA = \left(aR^2\right)\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_0^{R_0}\frac{\left(aR^2\right)\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3 dR}{\sqrt{x^2+R^2}}$$

A substitution of $x^2 + R^2 = u^2$ can be used to do the integration.

$$x^2 + R^2 = u^2 \quad \to \quad R^2 = u^2 - x^2 \ ; \ 2RdR = 2udu$$

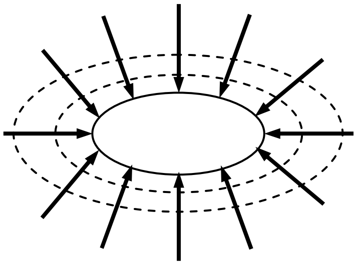
$$V = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3 dR}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_{R=0}^{R=R_0}\frac{\left(u^2-x^2\right)udu}{u} = \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}u^3 - ux^2\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}\left(x^2+R^2\right)^{3/2} - x^2\left(x^2+R^2\right)^{1/2}\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\left\{\tfrac{1}{3}\left(x^2+R_0^2\right)^{3/2} - x^2\left(x^2+R_0^2\right)^{1/2}\right\} + \tfrac{2}{3}x^3\right]$$

$$= \boxed{\frac{a}{6\varepsilon_0}\left[\left(R_0^2-2x^2\right)\left(x^2+R_0^2\right)^{1/2}+2x^3\right]}, x > 0$$

42.



$\boxed{43.}$ The electric field from a large plate is uniform with magnitude $E = \sigma/2\varepsilon_0$, with the field pointing away from the plate on both sides. Equation 23-4(a) can be integrated between two arbitrary points to calculate the potential difference between those points.

$$\Delta V = -\int_{x_0}^{x_1}\frac{\sigma}{2\varepsilon_0}dx = \frac{\sigma(x_0-x_1)}{2\varepsilon_0}$$

Setting the change in voltage equal to 100 V and solving for $x_0 - x_1$ gives the distance between field lines.

$$x_0 - x_1 = \frac{2\varepsilon_0\Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/Nm^2}\right)\left(100\ \mathrm{V}\right)}{0.75\times10^{-6}\mathrm{C/m^2}} = 2.36\times10^{-3}\mathrm{m} \approx \boxed{2\ \mathrm{mm}}$$

44. The potential at the surface of the sphere is $V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$. The potential outside the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = V_0\frac{r_0}{r}$, and decreases as you move away from the surface. The difference in potential between a given location and the surface is to be a multiple of 100 V.

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(\frac{0.50\times10^{-6}\mathrm{C}}{0.44\,\mathrm{m}}\right) = 10{,}216\,\mathrm{V}$$

$$V_0 - V = V_0 - V_0\frac{r_0}{r} = (100\,\mathrm{V})n \;\rightarrow\; r = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})n\right]}r_0$$

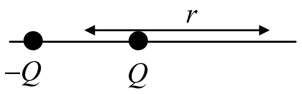
(*a*) $r_1 = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})1\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{10{,}116\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.444\,\mathrm{m}}$

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a 100-V difference.

(*b*) $r_{10} = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})10\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{9{,}216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.49\,\mathrm{m}}$

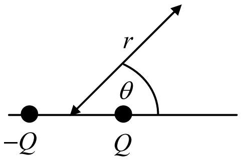
(*c*) $r_{100} = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})100\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{21\,\mathrm{m}}$

45. The potential due to the dipole is given by Eq. 23-7.



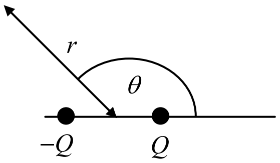
(*a*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 0}{\left(4.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{2.6\times10^{-3}\mathrm{V}}$



(*b*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 45^\circ}{\left(4.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{1.8\times10^{-3}\mathrm{V}}$



(*c*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 135^\circ}{\left(1.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{-1.8\times10^{-3}\mathrm{V}}$

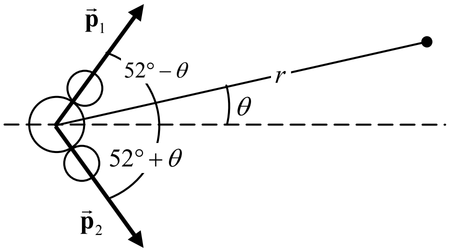
46. (*a*) We assume that $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ are equal in magnitude, and that each makes a 52° angle with $\vec{\mathbf{p}}$. The magnitude of $\vec{\mathbf{p}}_1$ is also given by $p_1 = qd$, where q is the net charge on the hydrogen atom, and d is the distance between the H and the O.

$$p = 2p_1\cos 52^\circ \;\rightarrow\; p_1 = \frac{p}{2\cos 52^\circ} = qd \;\rightarrow$$

$$q = \frac{p}{2d\cos 52^\circ} = \frac{6.1\times10^{-30}\mathrm{C{\bullet}m}}{2\left(0.96\times10^{-10}\mathrm{m}\right)\cos 52^\circ} = \boxed{5.2\times10^{-20}\mathrm{C}}$$

This is about 0.32 times the charge on an electron.

(*b*) Since we are considering the potential far from the dipoles, we will take the potential of each dipole to be given by Eq. 23-7. See the diagram for the angles involved. From part (a), $p_1 = p_2 = \dfrac{p}{2\cos 52^\circ}$.



$$
\begin{aligned}
V &= V_{p_1} + V_{p_2} \\
&= \frac{1}{4\pi\varepsilon_0}\frac{p_1\cos(52^\circ - \theta)}{r} + \frac{1}{4\pi\varepsilon_0}\frac{p_2\cos(52^\circ + \theta)}{r} \\
&= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left[\cos(52^\circ - \theta) + \cos(52^\circ + \theta)\right] \\
&= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(\cos 52^\circ\cos\theta + \sin 52^\circ\cos\theta + \cos 52^\circ\cos\theta - \sin 52^\circ\cos\theta\right) \\
&= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(2\cos 52^\circ\cos\theta\right) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}}
\end{aligned}
$$

47. $$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = -\frac{q}{4\pi\varepsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right) = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r^2}\right) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

48. The potential gradient is the negative of the electric field. Outside of a spherically symmetric charge distribution, the field is that of a point charge at the center of the distribution.

$$\frac{dV}{dr} = -E = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = -\left(8.99\times10^9\ \text{N•m}^2/\text{C}^2\right)\frac{(92)\left(1.60\times10^{-19}\text{C}\right)}{\left(7.5\times10^{-15}\text{m}\right)^2} = \boxed{-2.4\times10^{21}\ \text{V/m}}$$

$\boxed{49.}$ The electric field between the plates is obtained from the negative derivative of the potential.

$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left[(8.0\ \text{V/m})\,x\ +\ 5.0\ \text{V}\right] = -8.0\ \text{V/m}$$

The charge density on the plates (assumed to be conductors) is then calculated from the electric field between two large plates, $E = \sigma/\varepsilon_0$.

$$\sigma = E\varepsilon_0 = (8.0\,\text{V/m})\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right) = \boxed{7.1\times10^{-11}\text{C/m}^2}$$

The plate at the origin has the charge $-7.1\times10^{-11}\text{C/m}^2$ and the other plate, at a positive *x*, has charge $+7.1\times10^{-11}\text{C/m}^2$ so that the electric field points in the negative direction.

50. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = 0\ ;\ E_z = -\frac{\partial V}{\partial z} = 0$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{by}{\left(a^2+y^2\right)}\right] = -\frac{\left(a^2+y^2\right)b - by(2y)}{\left(a^2+y^2\right)^2} = \frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}$$

$$\vec{\mathbf{E}} = \boxed{\frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}\hat{\mathbf{j}}}$$

51. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = -2.5y + 3.5yz \ ; \ E_y = -\frac{\partial V}{\partial y} = -2y - 2.5x + 3.5xz \ ; E_z = -\frac{\partial V}{\partial z} = 3.5xy$$

$$\vec{\mathbf{E}} = \boxed{\left(-2.5y + 3.5yz\right)\hat{\mathbf{i}} + \left(-2y - 2.5x + 3.5xz\right)\hat{\mathbf{j}} + \left(3.5xy\right)\hat{\mathbf{k}}}$$

52. We use the potential to find the electric field, the electric field to find the force, and the force to find the acceleration.

$$E_x = -\frac{\partial V}{\partial x} \ ; \ F_x = qE_x \ ; \ a_x = \frac{F_x}{m} = \frac{qE_x}{m} = -\frac{q}{m}\frac{\partial V}{\partial x} = -\frac{q}{m}\frac{\partial V}{\partial x}$$

$$a_x\left(x = 2.0\,\text{m}\right) = -\frac{2.0\times10^{-6}\,\text{C}}{5.0\times10^{-5}\,\text{kg}}\left[2\left(2.0\,\text{V/m}^2\right)\left(2.0\,\text{m}\right) - 3\left(3.0\,\text{V/m}^3\right)\left(2.0\,\text{m}\right)^2\right] = \boxed{1.1\,\text{m/s}^2}$$

53. (*a*) The potential along the *y* axis was derived in Problem 38.

$$V_{y\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2 + y^2} + \ell}{\sqrt{\ell^2 + y^2} - \ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\sqrt{\ell^2 + y^2} + \ell\right) - \ln\left(\sqrt{\ell^2 + y^2} - \ell\right)\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{\frac{1}{2}\left(\ell^2 + y^2\right)^{-1/2} 2y}{\sqrt{\ell^2 + y^2} + \ell} - \frac{\frac{1}{2}\left(\ell^2 + y^2\right)^{-1/2} 2y}{\sqrt{\ell^2 + y^2} - \ell}\right] = \frac{Q}{4\pi\varepsilon_0 y\sqrt{\ell^2 + y^2}}$$

From the symmetry of the problem, this field will point along the *y* axis.

$$\vec{\mathbf{E}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{y\sqrt{\ell^2 + y^2}}\hat{\mathbf{j}}}$$

Note that for $y \gg \ell,$ this reduces to the field of a point charge at the origin.

(*b*) The potential along the *x* axis was derived in Problem 39.

$$V_{x\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x + \ell}{x - \ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(x + \ell\right) - \ln\left(x - \ell\right)\right]$$

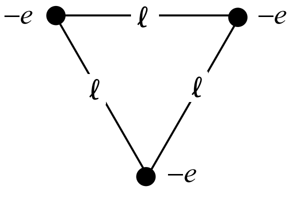
$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{1}{x + \ell} - \frac{1}{x - \ell}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2 - \ell^2}\right)$$

From the symmetry of the problem, this field will point along the *x* axis.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2 - \ell^2}\right)\hat{\mathbf{i}}$$

Note that for $x \gg \ell,$ this reduces to the field of a point charge at the origin.

54. Let the side length of the equilateral triangle be ℓ. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.



Thus $W_2 = \left(-e\right)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}$. The work done in bringing the third electron to its final location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell} - \frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell}$. The total work done is the sum $W_1 + W_2 + W_3$.

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\ell} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{\ell} = \frac{3\left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(1.0\times10^{-10}\,\text{m}\right)}$$

$$= \boxed{6.9\times10^{-18}\,\text{J}} = 6.9\times10^{-18}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{43\,\text{eV}}$$

$\boxed{55.}$ The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge. The helium nucleus has a charge of 2*e*.

$$\Delta V = \frac{\Delta U}{q} = -\frac{\Delta K}{q} = -\frac{125\times10^3\,\text{eV}}{2e} = \boxed{-62.5\,\text{kV}}$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.

56. The kinetic energy of the particle is given in each case. Use the kinetic energy to find the speed.

(*a*) $\frac{1}{2}mv^2 = K \;\rightarrow\; v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = \boxed{2.3\times10^7\,\text{m/s}}$

(*b*) $\frac{1}{2}mv^2 = K \;\rightarrow\; v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{1.67\times10^{-27}\,\text{kg}}} = \boxed{5.4\times10^5\,\text{m/s}}$

57. The potential energy of the two-charge configuration (assuming they are both point charges) is given by Eq. 23-10.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_{\text{initial}}} - \frac{1}{r_{\text{final}}}\right)$$

$$= \left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2\left(\frac{1}{0.110\times10^{-9}\,\text{m}} - \frac{1}{0.100\times10^{-9}\,\text{m}}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$$

$$= -1.31\,\text{eV}$$

Thus $\boxed{1.3\text{ eV}}$ of potential energy was lost.

58. The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$\tfrac{1}{2}mv^2 = K \;\rightarrow\; v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.53\times10^6\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{6.64\times10^{-27}\,\text{kg}}} = \boxed{1.63\times10^7\,\text{m/s}}$$

59. Following the same method as presented in Section 23-8, we get the following results.

(*a*) 1 charge: No work is required to move a single charge into a position, so $U_1 = 0$.

2 charges: This represents the interaction between Q_1 and Q_2.

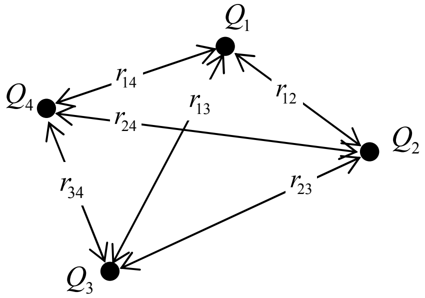
$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$

3 charges: This now adds the interactions between $Q_1\,\&\,Q_3$ and $Q_2\,\&\,Q_3$.

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_2Q_3}{r_{23}}\right)$$

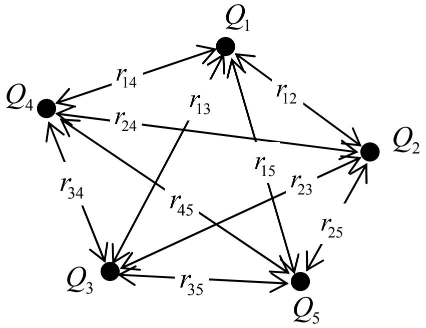
4 charges: This now adds the interaction between $Q_1\,\&\,Q_4$, $Q_2\,\&\,Q_4$, and $Q_3\,\&\,Q_4$.

$$U_4 = \boxed{\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_3Q_4}{r_{34}}\right)}$$



(*b*) 5 charges: This now adds the interaction between $Q_1\,\&\,Q_5$, $Q_2\,\&\,Q_5$, $Q_3\,\&\,Q_5$, and $Q_4\,\&\,Q_5$.

$$U_5 = \boxed{\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_1Q_5}{r_{15}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_2Q_5}{r_{25}}+\frac{Q_3Q_4}{r_{34}}+\frac{Q_3Q_5}{r_{35}}+\frac{Q_4Q_5}{r_{45}}\right)}$$



60. (*a*) The potential energy of the four-charge configuration was derived in Problem 59. Number the charges clockwise, starting in the upper right hand corner of the square.

$$U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{b}+\frac{1}{\sqrt{2}b}+\frac{1}{b}+\frac{1}{b}+\frac{1}{\sqrt{2}b}+\frac{1}{b}\right) = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4+\sqrt{2}\right)}$$

(*b*) The potential energy of the fifth charge is due to the interaction between the fifth charge and each of the other four charges. Each of those interaction terms is of the same magnitude since the fifth charge is the same distance from each of the other four charges.

$$U_{\substack{\text{5th}\\ \text{charge}}} = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4\sqrt{2}\right)}$$

(*c*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the charges are all of the same sign, by moving closer, the center charge would be repelled back towards its original position. Thus it is in a place of $\boxed{\text{stable}}$ equilibrium.

(*d*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the corner charges are of the opposite sign as the center charge, the center charge would be attracted towards those closer charges, making the center charge move even farther from the center. So it is in a place of $\boxed{\text{unstable}}$ equilibrium.

$\boxed{61.}$ (*a*) The electron was accelerated through a potential difference of 1.33 kV (moving from low potential to high potential) in gaining 1.33 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same kinetic energy, $\boxed{1.33\text{ keV}}$.

(*b*) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$\tfrac{1}{2}m_\text{p}v_\text{p}^2 = \tfrac{1}{2}m_\text{e}v_\text{e}^2 \rightarrow \frac{v_\text{e}}{v_\text{p}} = \sqrt{\frac{m_\text{p}}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{42.8}$$

The lighter electron is moving about 43 times faster than the heavier proton.

62. We find the energy by bringing in a small amount of charge at a time, similar to the method given in Section 23-8. Consider the sphere partially charged, with charge $q < Q$. The potential at the surface of the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}$, and the work to add a charge dq to that sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire charge to find the total potential energy.

$$U = \int dU = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}dq = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q^2}{r_0}}$$

63. The two fragments can be treated as point charges for purposes of calculating their potential energy. Use Eq. 23-10 to calculate the potential energy. Using energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$E_\text{initial} = E_\text{final} \rightarrow U_\text{initial} = K_\text{final} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}V$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60\times10^{-19}\text{C}\right)^2}{\left(5.5\times10^{-15}\text{m}\right)+\left(6.2\times10^{-15}\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 250\times10^6\,\text{eV}$$

$$= \boxed{250\,\text{MeV}}$$

This is about 25% greater than the observed kinetic energy of 200 MeV.

64. We find the energy by bringing in a small amount of spherically symmetric charge at a time, similar to the method given in Section 23-8. Consider that the sphere has been partially constructed, and so has a charge $q < Q$, contained in a radius $r < r_0$. Since the sphere is made of uniformly charged material, the charge density of the sphere must be $\rho_E = \frac{Q}{\frac{4}{3}\pi r_0^3}$. Thus the partially constructed sphere also satisfies $\rho_E = \frac{q}{\frac{4}{3}\pi r^3}$, and so $\frac{q}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi r_0^3} \rightarrow q = \frac{Qr^3}{r_0^3}$. The potential at the surface of that sphere can now found.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Qr^3}{r_0^3}}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}$$

We now add another infinitesimally thin shell to the partially constructed sphere. The charge of that shell is $dq = \rho_E 4\pi r^2 dr$. The work to add charge dq to the sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire sphere to find the total potential energy.

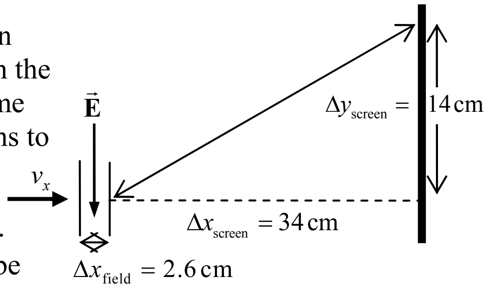
$$U = \int dU = \int Vdq = \int_0^{r_0} \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}\rho_E 4\pi r^2 dr = \frac{\rho_E}{\varepsilon_0}\frac{Q}{r_0^3}\int_0^{r_0} r^4 dr = \boxed{\frac{3Q^2}{20\pi\varepsilon_0 r_0}}$$

65. The ideal gas model, from Eq. 18-4, says that $K = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$.

$$K = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT \rightarrow v_{\substack{\text{rms}\\ 273\,\text{K}}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(273\,\text{K}\right)}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.11\times10^{5}\,\text{m/s}}$$

$$v_{\substack{\text{rms}\\ 2700\,\text{K}}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(2700\,\text{K}\right)}{9.11\times10^{-31}\,\text{kg}}} = \boxed{3.5\times10^{5}\,\text{m/s}}$$

66. If there were no deflecting field, the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons towards one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the electron's motion, and see the diagram, which is a top view. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, v_y. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.



Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \rightarrow eV = \frac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \rightarrow t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = eE = ma_y \rightarrow a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \rightarrow t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\frac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \frac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}} \Delta x_{\text{field}}} = \frac{2\left(6.0\times 10^3\,\text{V}\right)\left(0.14\,\text{m}\right)}{\left(0.34\text{ m}\right)\left(0.026\,\text{m}\right)}$$

$$= 1.90\times 10^5\ \text{V/m} \approx 1.9\times 10^5\ \text{V/m}$$

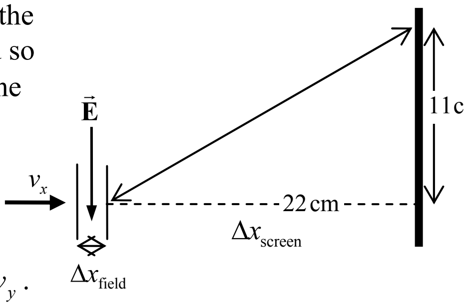
As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE\left(\Delta x_{\text{field}}\right)^2}{2m\left(\frac{2eV}{m}\right)} = \frac{E\left(\Delta x_{\text{field}}\right)^2}{4V}$$

$$= \frac{\left(1.90\times 10^5\ \text{V/m}\right)\left(0.026\,\text{m}\right)^2}{4\left(6000\,\text{V}\right)} = 5.4\times 10^{-3}\,\text{m}$$

This is about 4% of the total 15 cm vertical deflection, and so for an estimation, our approximation is acceptable. And so the field must vary from $\boxed{+1.9\times 10^5\ \text{V/m to } -1.9\times 10^5\ \text{V/m}}$

$\boxed{67.}$ Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential, V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron an upward velocity, v_y.



We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen.

Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \rightarrow eV = \tfrac{1}{2} m v_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \dfrac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2(7200\,\text{V})(0.11\,\text{m})}{(0.22\,\text{m})(0.028\,\text{m})}$$

$$= 2.57\times10^5\ \text{V/m} \approx \boxed{2.6\times10^5\ \text{V/m}}$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE(\Delta x_{\text{field}})^2}{2m\left(\dfrac{2eV}{m}\right)} = \frac{E(\Delta x_{\text{field}})^2}{4V}$$

$$= \frac{(2.97\times10^5\ \text{V/m})(0.028\,\text{m})^2}{4(7200\,\text{V})} = 8.1\times10^{-3}\,\text{m}$$

This is about 7% of the total 11 cm vertical deflection, and so for an estimation, our approximation is acceptable.

68. The potential of the earth will increase because the "neutral" Earth will now be charged by the removing of the electrons. The excess charge will be the elementary charge times the number of electrons removed. We approximate this change in potential by using a spherical Earth with all the excess charge at the surface.

$$Q = \left(\frac{1.602\times10^{-19}\,\text{C}}{e^-}\right)\left(\frac{10\ e^-}{\text{H}_2\text{O molecule}}\right)\left(\frac{6.02\times10^{23}\,\text{molecules}}{0.018\,\text{kg}}\right)\left(\frac{1000\,\text{kg}}{\text{m}^3}\right)\tfrac{4}{3}\pi(0.00175\,\text{m})^3$$

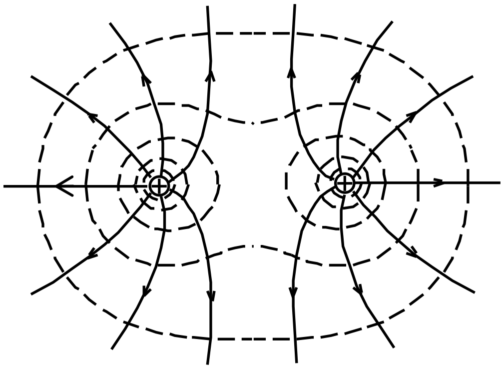
$$= 1203\,\text{C}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_{\text{Earth}}} = (8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)\frac{1203\,\text{C}}{6.38\times10^6\,\text{m}} = \boxed{1.7\times10^6\,\text{V}}$$

69. The potential at the surface of a charged sphere is that of a point charge of the same magnitude, located at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)\frac{(1\times10^{-8}\,\text{C})}{(0.15\,\text{m})} = 599.3\,\text{V} \approx \boxed{600\,\text{V}}$$

70.



71. Let d_1 represent the distance from the left charge to point b, and let d_2 represent the distance from the right charge to point b. Let *Q* represent the positive charges, and let *q* represent the negative charge that moves. The change in potential energy is given by Eq. 23-2b.

$$d_1 = \sqrt{12^2+14^2}\,\text{cm} = 18.44\text{ cm} \qquad d_2 = \sqrt{14^2+24^2}\,\text{cm} = 27.78\text{ cm}$$

$$U_b - U_a = q\left(V_b - V_a\right) = q\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Q}{0.1844\,\text{m}}+\frac{Q}{0.2778\,\text{m}}\right)-\left(\frac{Q}{0.12\,\text{m}}+\frac{Q}{0.24\,\text{m}}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}Qq\left[\left(\frac{1}{0.1844\,\text{m}}+\frac{1}{0.2778\,\text{m}}\right)-\left(\frac{1}{0.12\,\text{m}}+\frac{1}{0.24\,\text{m}}\right)\right]$$

$$= \left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.5\times10^{-6}\text{C}\right)\left(33\times10^{-6}\text{C}\right)\left(-3.477\,\text{m}^{-1}\right) = 1.547\,\text{J} \approx \boxed{1.5\,\text{J}}$$

72. (*a*) All eight charges are the same distance from the center of the cube. Use Eq. 23-5 for the potential of a point charge.

$$V_{\text{center}} = 8\frac{1}{4\pi\varepsilon_0}\frac{Q}{\frac{\sqrt{3}}{2}\ell} = \boxed{\frac{16}{\sqrt{3}}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 9.24\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*b*) For the seven charges that produce the potential at a corner, three are a distance ℓ away from that corner, three are a distance $\sqrt{2}\ell$ away from that corner, and one is a distance $\sqrt{3}\ell$ away from that corner.

$$V_{\text{corner}} = 3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}+3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{3}\ell} = \boxed{\left(3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 5.70\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*c*) The total potential energy of the system is half the energy found by multiplying each charge times the potential at a corner. The factor of half comes from the fact that if you took each charge times the potential at a corner, you would be counting each pair of charges twice.

$$U = \tfrac{1}{2}8\left(QV_{\text{corner}}\right) = \boxed{4\left(3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}} \approx 22.8\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}$$

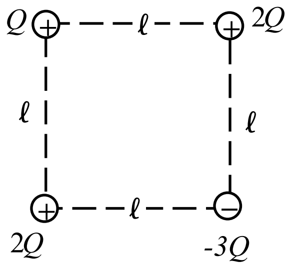
73. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

$$F_{\mathrm{E}} = mg \;;\; F_{\mathrm{E}} = |q|E = eV/d \;\rightarrow\; eV/d = mg \;\rightarrow$$

$$V = \frac{mgd}{e} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(0.035\,\mathrm{m}\right)}{1.60\times10^{-19}\,\mathrm{C}} = \boxed{2.0\times10^{-12}\,\mathrm{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

74. From Problem 59, the potential energy of a configuration of four charges is $U = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q_1Q_2}{r_{12}} + \dfrac{Q_1Q_3}{r_{13}} + \dfrac{Q_1Q_4}{r_{14}} + \dfrac{Q_2Q_3}{r_{23}} + \dfrac{Q_2Q_4}{r_{24}} + \dfrac{Q_3Q_4}{r_{34}}\right)$.



Let a side of the square be ℓ, and number the charges clockwise starting with the upper left corner.

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q(2Q)}{\ell} + \frac{Q(-3Q)}{\sqrt{2}\ell} + \frac{Q(2Q)}{\ell} + \frac{(2Q)(-3Q)}{\ell} + \frac{(2Q)(2Q)}{\sqrt{2}\ell} + \frac{(-3Q)(2Q)}{\ell}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0\ell}\left(\frac{1}{\sqrt{2}} - 8\right) = \left(8.99\times10^{9}\,\mathrm{N\bullet m^2/C^2}\right)\frac{\left(3.1\times10^{-6}\,\mathrm{C}\right)^2}{0.080\,\mathrm{m}}\left(\frac{1}{\sqrt{2}} - 8\right) = \boxed{-7.9\,\mathrm{J}}$$

75. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\mathrm{KE}_{\mathrm{initial}} = \mathrm{PE}_{\mathrm{final}} \;\rightarrow\; \tfrac{1}{2}mv^2 = qV \;\rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\left(-1.60\times10^{-19}\,\mathrm{C}\right)\left(-3.02\,\mathrm{V}\right)}{9.11\times10^{-31}\,\mathrm{kg}}} = \boxed{1.03\times10^{6}\,\mathrm{m/s}}$$

76. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

Horizontal:

$$\mathrm{PE}_{\mathrm{inital}} = \mathrm{KE}_{\mathrm{final}} \;\rightarrow\; qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Vertical:

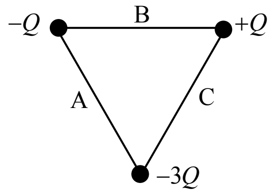
$$F_{\mathrm{E}} = qE_y = ma = m\frac{\left(v_y - v_{y0}\right)}{t} \;\rightarrow\; v_y = \frac{qE_yt}{m} = \frac{qE_y\Delta x}{mv_x}$$

Combined:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\frac{250\,\text{V}}{0.013\,\text{m}}\right)(0.065\,\text{m})}{2(5500\,\text{V})} = 0.1136$$

$$\theta = \tan^{-1} 0.1136 = \boxed{6.5^\circ}$$

77. Use Eq. 23-5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3}\ell/2$.



$$V_{\text{A}} = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\sqrt{3}\ell/2} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(-4+\frac{1}{\sqrt{3}}\right)$$

$$= \boxed{\frac{Q}{\pi\varepsilon_0\ell}\left(\frac{\sqrt{3}}{6}-2\right)}$$

$$V_{\text{B}} = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{6Q}{\sqrt{3}\ell} = \boxed{-\frac{\sqrt{3}Q}{2\pi\varepsilon_0\ell}}$$

$$V_{\text{C}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(2+\frac{1}{\sqrt{3}}\right) = \boxed{-\frac{Q}{\pi\varepsilon_0\ell}\left(1+\frac{\sqrt{3}}{6}\right)}$$

78. Since the E-field points downward, the surface of the Earth is a lower potential than points above the surface. Call the surface of the Earth 0 volts. Then a height of 2.00 m has a potential of 300 V. We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0) and at ground level (where their electrical and gravitational potential energies are 0).

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad mgh + qV = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{2\left(gh + \frac{qV}{m}\right)}$$

$$v_+ = \sqrt{2\left[\left(9.80\,\text{m/s}^2\right)(2.00\,\text{m}) + \frac{\left(4.5\times10^{-4}\,\text{C}\right)(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.3241\,\text{m/s}$$

$$v_- = \sqrt{2\left[\left(9.80\,\text{m/s}^2\right)(2.00\,\text{m}) + \frac{\left(-4.5\times10^{-4}\,\text{C}\right)(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.1972\,\text{m/s}$$

$$v_+ - v_- = 6.3241\,\text{m/s} - 6.1972\,\text{m/s} = \boxed{0.13\,\text{m/s}}$$

$\boxed{79.}$ (*a*) The energy is related to the charge and the potential difference by Eq. 23-3.

$$\Delta U = q\Delta V \quad\rightarrow\quad \Delta V = \frac{\Delta U}{q} = \frac{4.8\times10^6\,\text{J}}{4.0\,\text{C}} = \boxed{1.2\times10^6\,\text{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C.

$$Q = mc\Delta T + mL_{\text{f}} \quad\rightarrow$$

$$m=\frac{Q}{c\Delta T+L_{\mathrm{f}}}=\frac{4.8\times 10^{6}\,\mathrm{J}}{\left(4186\,\frac{\mathrm{J}}{\mathrm{kg}\bullet\mathrm{C}^{\circ}}\right)\left(80\,\mathrm{C}^{\circ}\right)+\left(22.6\times 10^{5}\,\frac{\mathrm{J}}{\mathrm{kg}}\right)}=\boxed{1.8\,\mathrm{kg}}$$

80. Use Eq. 23-7 for the dipole potential, and use Eq. 23-9 to determine the electric field.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}=\frac{p}{4\pi\varepsilon_0}\frac{\frac{x}{\left(x^2+y^2\right)^{1/2}}}{x^2+y^2}=\frac{p}{4\pi\varepsilon_0}\frac{x}{\left(x^2+y^2\right)^{3/2}}$$

$$E_x=-\frac{\partial V}{\partial x}=-\frac{p}{4\pi\varepsilon_0}\left[\frac{\left(x^2+y^2\right)^{3/2}-x\tfrac{3}{2}\left(x^2+y^2\right)^{1/2}2x}{\left(x^2+y^2\right)^{3}}\right]=\boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{2x^2-y^2}{\left(x^2+y^2\right)^{5/2}}\right]}$$

$$=\frac{p}{4\pi\varepsilon_0}\left[\frac{2\cos^2\theta-\sin^2\theta}{r^3}\right]$$

$$E_y=-\frac{\partial V}{\partial y}=-\frac{px}{4\pi\varepsilon_0}\left[-\tfrac{3}{2}\left(x^2+y^2\right)^{-5/2}2y\right]=\boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{3xy}{\left(x^2+y^2\right)^{5/2}}\right]}\frac{p}{4\pi\varepsilon_0}\left[\frac{3\cos\theta\sin\theta}{r^3}\right]$$

Notice the $\frac{1}{r^3}$ dependence in both components, which is indicative of a dipole field.

81. (*a*) Since the reference level is given as $V=0$ at $r=\infty$, the potential outside the shell is that of a point charge with the same total charge.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\rho_{\mathrm{E}}\left(\tfrac{4}{3}\pi r_2^3-\tfrac{4}{3}\pi r_1^3\right)}{r}=\boxed{\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\frac{\left(r_2^3-r_1^3\right)}{r}},\ r>r_2$$

Note that the potential at the surface of the shell is $V_{r_2}=\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\left(r_2^2-\frac{r_1^3}{r_2}\right)$.

(*b*) To find the potential in the region $r_1<r<r_2$, we need the electric field in that region. Since the charge distribution is spherically symmetric, Gauss's law may be used to find the electric field.

$$\oint\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}=E\left(4\pi r^2\right)=\frac{Q_{\mathrm{encl}}}{\varepsilon_0}\ \rightarrow\ E=\frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{\rho_{\mathrm{E}}\left(\tfrac{4}{3}\pi r^3-\tfrac{4}{3}\pi r_1^3\right)}{r^2}=\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\frac{\left(r^3-r_1^3\right)}{r^2}$$

The potential in that region is found from Eq. 23-4a. The electric field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}}=E\,dr$.

$$V_r-V_{r_2}=-\int_{r_2}^{r}\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}}=-\int_{r_2}^{r}E\,dr=-\int_{r_2}^{r}\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\frac{\left(r^3-r_1^3\right)}{r^2}dr=-\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\int_{r_2}^{r}\left(r-\frac{r_1^3}{r^2}\right)dr=-\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\left(\tfrac{1}{2}r^2+\frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}$$

$$V_r=V_{r_2}+\left[-\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\left(\tfrac{1}{2}r^2+\frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}\right]=\frac{\rho_{\mathrm{E}}}{3\varepsilon_0}\left(\tfrac{3}{2}r_2^2-\tfrac{1}{2}r^2-\frac{r_1^3}{r}\right)=\boxed{\frac{\rho_{\mathrm{E}}}{\varepsilon_0}\left(\tfrac{1}{2}r_2^2-\tfrac{1}{6}r^2-\tfrac{1}{3}\frac{r_1^3}{r}\right)},\ r_1<r<r_2$$

(*c*) Inside the cavity there is no electric field, so the potential is constant and has the value that it has on the cavity boundary.

$$V_{r_1} = \frac{\rho_E}{\varepsilon_0}\left(\tfrac{1}{2}r_2^2 - \tfrac{1}{6}r_1^2 - \tfrac{1}{3}\frac{r_1^3}{r_1}\right) = \boxed{\frac{\rho_E}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)},\ r < r_1$$

$\boxed{\text{The potential is continuous at both boundaries.}}$

82. We follow the development of Example 23-9, with Figure 23-15. The charge density of the ring is $\sigma = \left(\dfrac{Q}{\pi R_0^2 - \pi\left(\frac{1}{2}R_0\right)^2}\right) = \dfrac{4Q}{3\pi R_0^2}$. The charge on a thin ring of radius *R* and thickness *dR* is $dq = \sigma dA = \dfrac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{\frac{1}{2}R_0}^{R_0}\frac{\frac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{2Q}{3\varepsilon_0\pi R_0^2}\int_{\frac{1}{2}R_0}^{R_0}\frac{R}{\sqrt{x^2+R^2}}dR = \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(x^2+R^2\right)^{1/2}\Big|_{\frac{1}{2}R_0}^{R_0}$$

$$= \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(\sqrt{x^2+R_0^2} - \sqrt{x^2+\tfrac{1}{4}R_0^2}\right)$$

83. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_a - V_b = -\int_{R_b}^{R_a}\vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_b}^{R_a}\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_a - R_b\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_b}{\mathrm{R}_a}}$$

84. (*a*) We may treat the sphere as a point charge located at the center of the field. Then the electric field at the surface is $E_{\text{surface}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_0^2}$, and the potential at the surface is $V_{\text{surface}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_0}$.

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = E_{\text{surface}}r_0 = E_{\text{breakdown}}r_0 = \left(3\times10^6\ \mathrm{V/m}\right)\left(0.20\,\mathrm{m}\right) = \boxed{6\times10^5\,\mathrm{V}}$$

(*b*) $$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} \quad\rightarrow\quad Q = \left(4\pi\varepsilon_0\right)r_0V_{\text{surface}} = \frac{\left(0.20\,\mathrm{m}\right)\left(6\times10^5\,\mathrm{V}\right)}{\left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)} = 1.33\times10^{-5}\,\mathrm{C} \approx \boxed{1\times10^{-5}\,\mathrm{C}}$$

$\boxed{85.}$ (*a*) The voltage at $x = 0.20\ \mathrm{m}$ is obtained by inserting the given data directly into the voltage equation.

$$V\left(0.20\,\mathrm{m}\right) = \frac{B}{\left(x^2+R^2\right)^2} = \frac{150\,\mathrm{V{\bullet}m^4}}{\left[\left(0.20\,\mathrm{m}\right)^2 + \left(0.20\,\mathrm{m}\right)^2\right]^2} = \boxed{23\ \mathrm{kV}}$$

(*b*) The electric field is the negative derivative of the potential.

$$\vec{\mathbf{E}}(x) = -\frac{d}{dx}\left[\frac{B}{\left(x^2+R^2\right)^2}\right]\hat{\mathbf{i}} = \boxed{\frac{4Bx\,\hat{\mathbf{i}}}{\left(x^2+R^2\right)^3}}$$

Since the voltage only depends on *x* the electric field points in the positive *x* direction.

(*c*) Inserting the given values in the equation of part (*b*) gives the electric field at $x = 0.20\text{ m}$

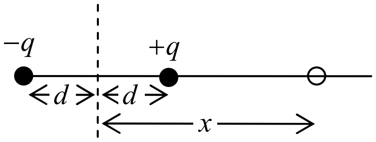
$$\vec{\mathbf{E}}(0.20\text{ m}) = \frac{4\left(150\,\text{V}\bullet\text{m}^4\right)\left(0.20\,\text{m}\right)\hat{\mathbf{i}}}{\left[\left(0.20\,\text{m}\right)^2+\left(0.20\,\text{m}\right)^2\right]^3} = \boxed{2.3\times10^5\ \text{V/m}\,\hat{\mathbf{i}}}$$

86. Use energy conservation, equating the energy of charge $-q_1$ at its initial position to its final position at infinity. Take the speed at infinity to be 0, and take the potential of the point charges to be 0 at infinity.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}} \rightarrow \tfrac{1}{2}mv_0^2 + \left(-q_1\right)V_{\substack{\text{initial}\\\text{point}}} = \tfrac{1}{2}mv_{\text{final}}^2 + \left(-q_1\right)V_{\substack{\text{final}\\\text{point}}}$$

$$\tfrac{1}{2}mv_0^2 + \left(-q_1\right)\frac{1}{4\pi\varepsilon_0}\frac{2q_2}{\sqrt{a^2+b^2}} = 0+0 \rightarrow v_0 = \boxed{\sqrt{\frac{1}{m\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{a^2+b^2}}}}$$

87. (*a*) From the diagram, the potential at *x* is the potential of two point charges.



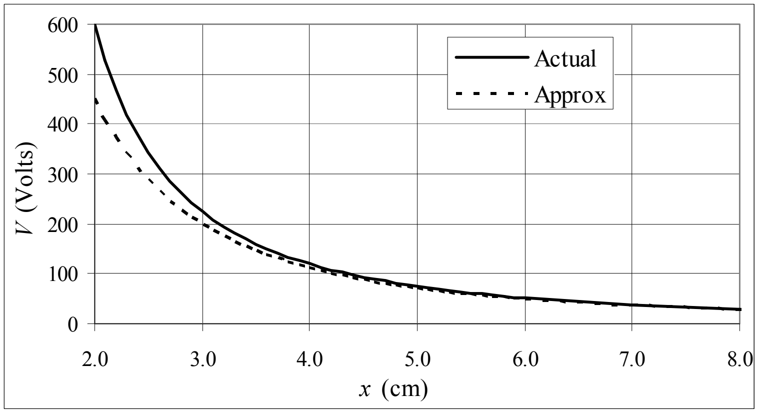
$$V_{\text{exact}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x-d}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{x+d}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2qd}{\left(x^2-d^2\right)}\right], \quad q = 1.0\times10^{-9}\text{C},\ d = 0.010\,\text{m}$$

(*b*) The approximate potential is given by Eq. 23-7, with $\theta = 0$, $p = 2qd$, and $r = x$.

$$V_{\text{approx}} = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{x^2}$$

To make the difference at small distances more apparent, we have plotted from 2.0 cm to 8.0 cm. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH23.XLS,” on tab “Problem 23.87.”



88. The electric field can be determined from the potential by using Eq. 23-8, differentiating with respect to *x*.

$$E(x)=-\frac{dV(x)}{dx}=-\frac{d}{dx}\left[\frac{Q}{2\pi\varepsilon_0R_0^2}\left[\left(x^2+R_0^2\right)^{1/2}-x\right]\right]=-\frac{Q}{2\pi\varepsilon_0R_0^2}\left[\tfrac{1}{2}\left(x^2+R_0^2\right)^{-1/2}(2x)-1\right]$$

$$=\boxed{\frac{Q}{2\pi\varepsilon_0R_0^2}\left[1-\frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right]}$$

Express *V* and *E* in terms of x/R_0. Let $X=x/R_0$.

$$V(x)=\frac{Q}{2\pi\varepsilon_0R_0^2}\left[\left(x^2+R_0^2\right)^{1/2}-x\right]=\frac{2Q}{4\pi\varepsilon_0R_0}\left(\sqrt{X^2+1}-X\right)$$

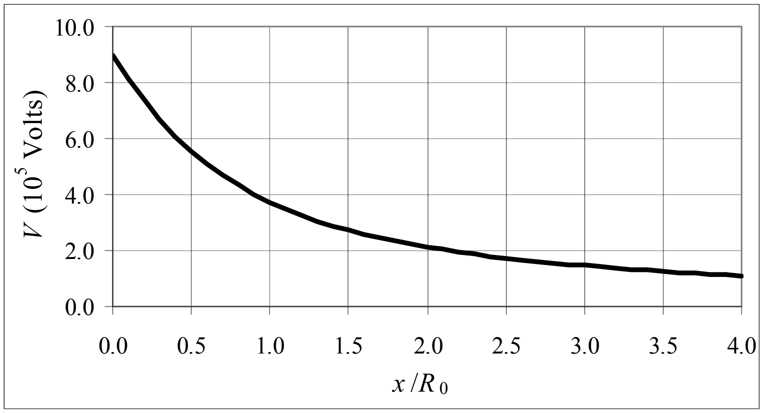
$$=\left(8.99\times10^9\ \text{N•m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\text{C}\right)}{0.10\,\text{m}}\left(\sqrt{X^2+1}-X\right)=\left(8.99\times10^5\text{V}\right)\left(\sqrt{X^2+1}-X\right)$$

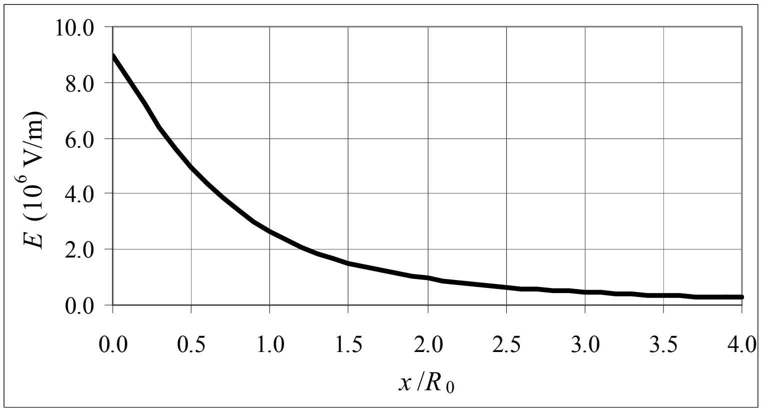
$$E(x)=\frac{Q}{2\pi\varepsilon_0R_0^2}\left[1-\frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right]=\frac{2Q}{4\pi\varepsilon_0R_0^2}\left[1-\frac{X}{\sqrt{X^2+1}}\right]$$

$$=\left(8.99\times10^9\ \text{N•m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\text{C}\right)}{(0.10\,\text{m})^2}\left[1-\frac{X}{\sqrt{X^2+1}}\right]$$

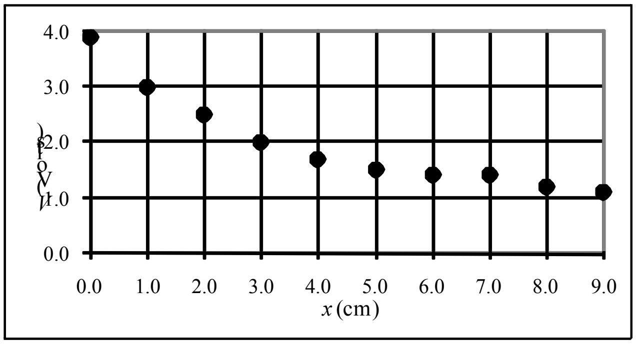
$$=\left(8.99\times10^6\ \text{V/m}\right)\left[1-\frac{X}{\sqrt{X^2+1}}\right]$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.88."





89. (*a*) If the field is caused by a point charge, the potential will have a graph that has the appearance of $1/r$ behavior, which means that the potential change per unit of distance will decrease as potential is measured farther from the charge. If the field is caused by a sheet of charge, the potential will have a linear decrease with distance. The graph indicates that the field is caused by a point charge. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89a."

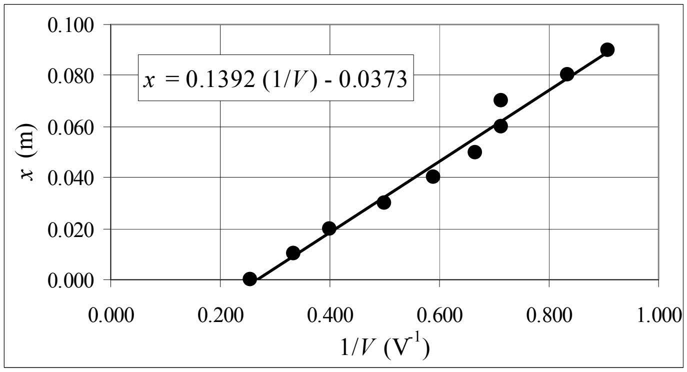


(*b*) Assuming the field is caused by a point charge, we assume the charge is at $x = d$, and then the potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d}$. This can be rearranged to the following.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d} \quad\rightarrow$$

$$x = \frac{1}{V}\frac{Q}{4\pi\varepsilon_0} + d$$

If we plot x vs. $\frac{1}{V}$, the slope is $\frac{Q}{4\pi\varepsilon_0}$, which can be used to determine the charge.



$$\text{slope} = 0.1392\,\text{m}\bullet\text{V} = \frac{Q}{4\pi\varepsilon_0} \quad\rightarrow$$

$$Q = 4\pi\varepsilon_0\left(0.1392\,\text{m}\bullet\text{V}\right) = \frac{\left(0.1392\,\text{m}\bullet\text{V}\right)}{\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)} = \boxed{1.5\times10^{-11}\text{C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89b."

(*c*) From the above equation, the *y* intercept of the graph is the location of the charge, *d*. So the charge is located at $x = d = -0.0373\,\text{m} \approx \boxed{3.7\,\text{cm from the first measured position}}$.

CHAPTER 24: Capacitance, Dielectrics, Electric Energy Storage

Responses to Questions

$\boxed{1.}$ Yes. If the conductors have different shapes, then even if they have the same charge, they will have different charge densities and therefore different electric fields near the surface. There can be a potential difference between them. The definition of capacitance $C = Q/V$ cannot be used here because it is defined for the case where the charges on the two conductors of the capacitor are equal and opposite.

2. Underestimate. If the separation between the plates is not very small compared to the plate size, then fringing cannot be ignored and the electric field (for a given charge) will actually be smaller. The capacitance is inversely proportional to potential and, for parallel plates, also inversely proportional to the field, so the capacitance will actually be larger than that given by the formula.

3. Ignoring fringing field effects, the capacitance would decrease by a factor of 2, since the area of overlap decreases by a factor of 2. (Fringing effects might actually be noticeable in this configuration.)

4. When a capacitor is first connected to a battery, charge flows to one plate. Because the plates are separated by an insulating material, charge cannot cross the gap. An equal amount of charge is therefore repelled from the opposite plate, leaving it with a charge that is equal and opposite to the charge on the first plate. The two conductors of a capacitor will have equal and opposite charges even if they have different sizes or shapes.

5. Charge a parallel-plate capacitor using a battery with a known voltage *V*. Let the capacitor discharge through a resistor with a known resistance *R* and measure the time constant. This will allow calculation of the capacitance *C*. Then use $C = \varepsilon_0 A/d$ and solve for ε_0.

6. Parallel. The equivalent capacitance of the three capacitors in parallel will be greater than that of the same three capacitors in series, and therefore they will store more energy when connected to a given potential difference if they are in parallel.

$\boxed{7.}$ If a large copper sheet of thickness ℓ is inserted between the plates of a parallel-plate capacitor, the charge on the capacitor will appear on the large flat surfaces of the copper sheet, with the negative side of the copper facing the positive side of the capacitor. This arrangement can be considered to be two capacitors in series, each with a thickness of $\frac{1}{2}(d-\ell)$. The new net capacitance will be $C' = \varepsilon_0 A/(d-\ell)$, so the capacitance of the capacitor will be reduced.

8. A force is required to increase the separation of the plates of an isolated capacitor because you are pulling a positive plate away from a negative plate. The work done in increasing the separation goes into increasing the electric potential energy stored between the plates. The capacitance decreases, and the potential between the plates increases since the charge has to remain the same.

9. (*a*) The energy stored quadruples since the potential difference across the plates doubles and the capacitance doesn't change: $U = \frac{1}{2}CV^2$.

   (*b*) The energy stored quadruples since the charge doubles and the capacitance doesn't change: $U = \frac{1}{2}\frac{Q^2}{C}$.

(*c*) If the separation between the plates doubles, the capacitance is halved. The potential difference across the plates doesn't change if the capacitor remains connected to the battery, so the energy stored is also halved: $U = \frac{1}{2}CV^2$.

10. (*c*) If the voltage across a capacitor is doubled, the amount of energy it can store is quadrupled: $U = \frac{1}{2}CV^2$.

11. The dielectric will be pulled into the capacitor by the electrostatic attractive forces between the charges on the capacitor plates and the polarized charges on the dielectric's surface. (Note that the addition of the dielectric decreases the energy of the system.)

12. If the battery remains connected to the capacitor, the energy stored in the electric field of the capacitor will increase as the dielectric is inserted. Since the energy of the system increases, work must be done and the dielectric will have to be pushed into the area between the plates. If it is released, it will be ejected.

$\boxed{13.}$ (*a*) If the capacitor is isolated, Q remains constant, and $U = \frac{1}{2}\frac{Q^2}{C}$ becomes $U' = \frac{1}{2}\frac{Q^2}{KC}$ and the stored energy decreases.

(*b*) If the capacitor remains connected to a battery so V does not change, $U = \frac{1}{2}CV^2$ becomes $U' = \frac{1}{2}KCV^2$, and the stored energy increases.

14. For dielectrics consisting of polar molecules, one would expect the dielectric constant to decrease with temperature. As the thermal energy increases, the molecular vibrations will increase in amplitude, and the polar molecules will be less likely to line up with the electric field.

15. When the dielectric is removed, the capacitance decreases. The potential difference across the plates remains the same because the capacitor is still connected to the battery. If the potential difference remains the same and the capacitance decreases, the charge on the plates and the energy stored in the capacitor must also decrease. (Charges return to the battery.) The electric field between the plates will stay the same because the potential difference across the plates and the distance between the plates remain constant.

16. For a given configuration of conductors and dielectrics, C is the proportionality constant between the voltage between the plates and the charge on the plates.

17. The dielectric constant is the ratio of the capacitance of a capacitor with the dielectric between the plates to the capacitance without the dielectric. If a conductor were inserted between the plates of a capacitor such that it filled the gap and touched both plates, the capacitance would drop to zero since charge would flow from one plate to the other. So, the dielectric constant of a good conductor would be zero.

Solutions to Problems

$\boxed{1.}$ The capacitance is found from Eq. 24-1.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{2.8\times10^{-3}\,\mathrm{C}}{930\,\mathrm{V}} = 3.0\times10^{-6}\,\mathrm{F} = \boxed{3.0\,\mu\mathrm{F}}$$

2. We assume the capacitor is fully charged, according to Eq. 24-1.

$$Q = CV = \left(12.6\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right) = \boxed{1.51\times10^{-4}\,\text{C}}$$

3. The capacitance is found from Eq. 24-1.

$$Q = CV \;\rightarrow\; C = \frac{Q}{V} = \frac{75\times10^{-12}\,\text{C}}{24.0\,\text{V}} = 3.1\times10^{-12}\,\text{F} = \boxed{3.1\,\text{pF}}$$

4. Let Q_1 and V_1 be the initial charge and voltage on the capacitor, and let Q_2 and V_2 be the final charge and voltage on the capacitor. Use Eq. 24-1 to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \quad Q_2 = CV_2 \quad Q_2 - Q_1 = CV_2 - CV_1 = C\left(V_2 - V_1\right) \;\rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{26\times10^{-6}\,\text{C}}{50\,\text{V}} = 5.2\times10^{-7}\,\text{F} = \boxed{0.52\,\mu\text{F}}$$

5. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{\underset{\text{initial}}{1}} \qquad Q_{\underset{\text{final}}{1}} = C_1 V_{\text{final}} \qquad Q_{\underset{\text{final}}{2}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{\underset{\text{final}}{1}} + Q_{\underset{\text{final}}{2}} = \left(C_1 + C_2\right)V_{\text{final}} \;\rightarrow\; C_1 V_{\underset{\text{initial}}{1}} = \left(C_1 + C_2\right)V_{\text{final}} \;\rightarrow$$

$$C_2 = C_1\left(\frac{V_{\underset{\text{initial}}{1}}}{V_{\text{final}}} - 1\right) = \left(7.7\times10^{-6}\,\text{F}\right)\left(\frac{125\,\text{V}}{15\,\text{V}} - 1\right) = 5.6\times10^{-5}\,\text{F} = \boxed{56\,\mu\text{F}}$$

6. The total charge will be conserved, and the final potential difference across the capacitors will be the same.

$$Q_0 = Q_1 + Q_2 \;\; ; \;\; V_1 = V_2 \;\rightarrow\; \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_0 - Q_1}{C_2} \;\rightarrow\; \boxed{Q_1 = Q_0\frac{C_1}{C_1 + C_2}}$$

$$Q_2 = Q_0 - Q_1 = Q_0 - Q_0\frac{C_1}{C_1 + C_2} = \boxed{Q_2 = Q_0\left(\frac{C_2}{C_1 + C_2}\right)}$$

$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_0\dfrac{C_1}{C_1 + C_2}}{C_1} = \boxed{V = \frac{Q_0}{C_1 + C_2}}$$

$\boxed{7.}$ The work to move the charge between the capacitor plates is $W = qV$, where V is the voltage difference between the plates, assuming that $q \ll Q$ so that the charge on the capacitor does not change appreciably. The charge is then found from Eq. 24-1. The assumption that $q \ll Q$ is justified.

$$W = qV = q\left(\frac{Q}{C}\right) \;\rightarrow\; Q = \frac{CW}{q} = \frac{\left(15\,\mu\text{F}\right)\left(15\,\text{J}\right)}{0.20\,\text{mC}} = \boxed{1.1\,\text{C}}$$

8. (*a*) The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge, and there is no neutralization of charge by combining positive and negative charges. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_{\substack{1\\ \text{initial}}} = C_1 V_{\substack{1\\ \text{initial}}} \qquad Q_{\substack{2\\ \text{initial}}} = C_2 V_{\substack{2\\ \text{initial}}} \qquad Q_{\substack{1\\ \text{final}}} = C_1 V_{\text{final}} \qquad Q_{\substack{2\\ \text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{\substack{1\\ \text{initial}}} + Q_{\substack{2\\ \text{initial}}} = Q_{\substack{1\\ \text{final}}} + Q_{\substack{2\\ \text{final}}} = C_1 V_{\substack{1\\ \text{initial}}} + C_2 V_{\substack{2\\ \text{initial}}} = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{C_1 V_{\substack{1\\ \text{initial}}} + C_2 V_{\substack{2\\ \text{initial}}}}{C_1 + C_2} = \frac{\left(2.70\times10^{-6}\,\text{F}\right)\left(475\,\text{V}\right) + \left(4.00\times10^{-6}\,\text{F}\right)\left(525\,\text{V}\right)}{\left(6.70\times10^{-6}\,\text{F}\right)}$$

$$= 504.85\,\text{V} \approx \boxed{505\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\ \text{final}}} = C_1 V_{\text{final}} = \left(2.70\times10^{-6}\,\text{F}\right)\left(504.85\,\text{V}\right) = \boxed{1.36\times10^{-3}\,\text{C}}$$

$$Q_{\substack{2\\ \text{final}}} = C_2 V_{\text{final}} = \left(4.00\times10^{-6}\,\text{F}\right)\left(504.85\,\text{V}\right) = \boxed{2.02\times10^{-3}\,\text{C}}$$

(*b*) By connecting plates of opposite charge, the total charge will be the difference of the charges on the two individual capacitors. Once the charges have equalized, the two capacitors will again be at the same potential.

$$Q_{\substack{1\\ \text{initial}}} = C_1 V_{\substack{1\\ \text{initial}}} \qquad Q_{\substack{2\\ \text{initial}}} = C_2 V_{\substack{2\\ \text{initial}}} \qquad Q_{\substack{1\\ \text{final}}} = C_1 V_{\text{final}} \qquad Q_{\substack{2\\ \text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = \left| Q_{\substack{1\\ \text{initial}}} - Q_{\substack{2\\ \text{initial}}} \right| = Q_{\substack{1\\ \text{final}}} + Q_{\substack{2\\ \text{final}}} \rightarrow \left| C_1 V_{\substack{1\\ \text{initial}}} - C_2 V_{\substack{2\\ \text{initial}}} \right| = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{\left| C_1 V_{\substack{1\\ \text{initial}}} - C_2 V_{\substack{2\\ \text{initial}}} \right|}{C_1 + C_2} = \frac{\left|\left(2.70\times10^{-6}\,\text{F}\right)\left(475\,\text{V}\right) - \left(4.00\times10^{-6}\,\text{F}\right)\left(525\,\text{V}\right)\right|}{\left(6.70\times10^{-6}\,\text{F}\right)}$$

$$= 122.01\,\text{V} \approx \boxed{120\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\ \text{final}}} = C_1 V_{\text{final}} = \left(2.70\times10^{-6}\,\text{F}\right)\left(122.01\,\text{V}\right) = \boxed{3.3\times10^{-4}\,\text{C}}$$

$$Q_{\substack{2\\ \text{final}}} = C_2 V_{\text{final}} = \left(4.00\times10^{-6}\,\text{F}\right)\left(122.01\,\text{V}\right) = \boxed{4.9\times10^{-4}\,\text{C}}$$

9. Use Eq. 24-1.

$$\Delta Q = C\Delta V \; ; \; t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{C\Delta V}{\Delta Q/\Delta t} = \frac{\left(1200\,\text{F}\right)\left(6.0\,\text{V}\right)}{1.0\times10^{-3}\,\text{C/s}} = 7.2\times10^{6}\,\text{s}\left(\frac{1\,\text{d}}{86{,}400\,\text{s}}\right) = \boxed{83\,\text{d}}$$

10. (*a*) The absolute value of the charge on each plate is given by Eq. 24-1. The plate with electrons has a net negative charge.

$$Q = CV \quad \rightarrow \quad N\left(-e\right) = -CV \quad \rightarrow$$

$$N=\frac{CV}{e}=\frac{\left(35\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{1.60\times10^{-19}\,\text{C}}=3.281\times10^{5}\approx\boxed{3.3\times10^{5}\,\text{electrons}}$$

(*b*) Since the charge is directly proportional to the potential difference, a 1.0% decrease in potential difference corresponds to a 1.0% decrease in charge.

$\Delta Q=0.01Q$;

$$\Delta t=\frac{\Delta Q}{\Delta Q/\Delta t}=\frac{0.01Q}{\Delta Q/\Delta t}=\frac{0.01CV}{\Delta Q/\Delta t}=\frac{0.01\left(35\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{0.30\times10^{-15}\,\text{C/s}}=1.75\,\text{s}\approx\boxed{1.8\,\text{s}}$$

11. Use Eq. 24-2.

$$C=\varepsilon_0\frac{A}{d}\quad\rightarrow\quad A=\frac{Cd}{\varepsilon_0}=\frac{\left(0.40\times10^{-6}\,\text{F}\right)\left(2.8\times10^{-3}\,\text{m}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}=126.6\,\text{m}^2\approx\boxed{130\,\text{m}^2}$$

If the capacitor plates were square, they would be about 11.2 m on a side.

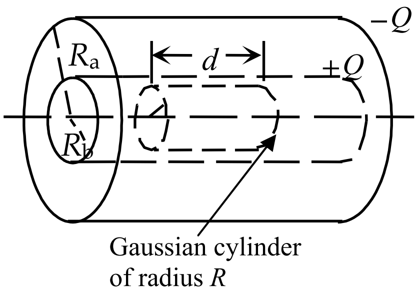
12. The capacitance per unit length of a coaxial cable is derived in Example 24-2

$$\frac{C}{\ell}=\frac{2\pi\varepsilon_0}{\ln\left(R_{\text{outside}}/R_{\text{inside}}\right)}=\frac{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}{\ln\left(5.0\,\text{mm}/1.0\,\text{mm}\right)}=\boxed{3.5\times10^{-11}\,\text{F/m}}$$

$\boxed{13.}$ Inserting the potential at the surface of a spherical conductor into Eq. 24.1 gives the capacitance of a conducting sphere. Then inserting the radius of the Earth yields the Earth's capacitance.

$$C=\frac{Q}{V}=\frac{Q}{\left(Q/4\pi\varepsilon_0 r\right)}=4\pi\varepsilon_0 r=4\pi\left(8.85\times10^{-12}\,\text{F/m}\right)\left(6.38\times10^{6}\,\text{m}\right)=\boxed{7.10\times10^{-4}\,\text{F}}$$

14. From the symmetry of the charge distribution, any electric field must be radial, away from the cylinder axis, and its magnitude must be independent of the location around the axis (for a given radial location). We assume the cylinders have charge of magnitude Q in a length ℓ. Choose a Gaussian cylinder of length d and radius R, centered on the capacitor's axis, with $d\ll\ell$ and the Gaussian cylinder far away from both ends of the capacitor. On the ends of this cylinder, $\vec{\mathbf{E}}\perp d\vec{\mathbf{A}}$ and so there is no flux through the ends. On the curved side of the cylinder, the field has a constant magnitude and $\vec{\mathbf{E}}\parallel d\vec{\mathbf{A}}$. Thus $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}=EdA$. Write Gauss's law.



$$\oiint\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}=E_{\substack{\text{curved}\\\text{walls}}}A_{\substack{\text{curved}\\\text{walls}}}=E\left(2\pi Rd\right)=\frac{Q_{\text{encl}}}{\varepsilon_0}$$

For $R<R_{\text{b}}$, $Q_{\text{encl}}=0\ \rightarrow\ E\left(2\pi Rd\right)\varepsilon_0=0\ \rightarrow\ E=0.$

For $R>R_{\text{a}}$, $Q_{\text{encl}}=\frac{Q}{\ell}d+\frac{-Q}{\ell}d=0$, and so $Q_{\text{encl}}=0\ \rightarrow\ E\left(2\pi Rd\right)\varepsilon_0=0\ \rightarrow\ E=0.$

15. We assume there is a uniform electric field between the capacitor plates, so that $V=Ed$, and then use Eqs. 24-1 and 24-2.

$$Q_{\max}=CV_{\max}=\varepsilon_0\frac{A}{d}\left(E_{\max}d\right)=\varepsilon_0 AE_{\max}=\left(8.85\times10^{-12}\,\text{F/m}\right)\left(6.8\times10^{-4}\,\text{m}^2\right)\left(3.0\times10^{6}\,\text{V/m}\right)$$

$$=\boxed{1.8\times10^{-8}\,\text{C}}$$

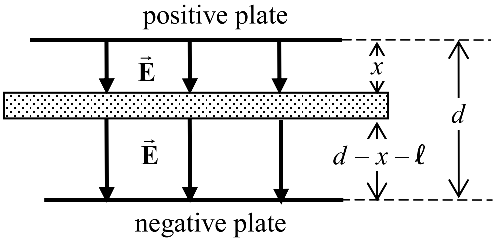
16. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0 \frac{A}{d}(Ed) = \varepsilon_0 AE = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(21.0\times10^{-4}\,\text{m}^2\right)\left(4.80\times10^{5}\,\text{V/m}\right)$$
$$= \boxed{8.92\times10^{-9}\,\text{C}}$$

17. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = CEd \;\rightarrow\; E = \frac{Q}{Cd} = \frac{92\times10^{-6}\,\text{C}}{\left(0.80\times10^{-6}\,\text{F}\right)\left(2.0\times10^{-3}\,\text{m}\right)} = \boxed{5.8\times10^{4}\,\text{V/m}}$$

18. (*a*) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let *x* be the distance from one original plate to the nearest face of the sheet, and so $d - \ell - x$ is the distance from the other original plate to the other face of the sheet.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;;\; V_1 = Ex = \frac{Qx}{A\varepsilon_0} \;;\; V_2 = E(d-\ell-x) = \frac{Q(d-\ell-x)}{A\varepsilon_0}$$

$$\Delta V = V_1 + V_2 = \frac{Qx}{A\varepsilon_0} + \frac{Q(d-\ell-x)}{A\varepsilon_0} = \frac{Q(d-\ell)}{A\varepsilon_0} = \frac{Q}{C} \;\rightarrow\; C = \boxed{\varepsilon_0 \frac{A}{(d-\ell)}}$$

(*b*) $$C_{\text{initial}} = \varepsilon_0\frac{A}{d} \;;\; C_{\text{final}} = \varepsilon_0\frac{A}{(d-\ell)} \;;\; \frac{C_{\text{final}}}{C_{\text{initial}}} = \frac{\varepsilon_0\dfrac{A}{(d-\ell)}}{\varepsilon_0\dfrac{A}{d}} = \frac{d}{d-\ell} = \frac{d}{d-0.40d} = \frac{1}{0.60} = \boxed{1.7}$$

$\boxed{19.}$ (*a*) The distance between plates is obtained from Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{x} \;\rightarrow\; x = \frac{\varepsilon_0 A}{C}$$

Inserting the maximum capacitance gives the minimum plate separation and the minimum capacitance gives the maximum plate separation.

$$x_{\min} = \frac{\varepsilon_0 A}{C_{\max}} = \frac{(8.85\,\text{pF/m})\left(25\times10^{-6}\,\text{m}^2\right)}{1000.0\times10^{-12}\,\text{F}} = 0.22\,\mu\text{m}$$

$$x_{\max} = \frac{\varepsilon_o A}{C_{\min}} = \frac{(8.85\,\text{pF/m})\left(25\times10^{-6}\,\text{m}^2\right)}{1.0\,\text{pF}} = 0.22\,\text{mm} = 220\,\mu\text{m}$$

So $\boxed{0.22\,\mu\text{m} \le x \le 220\,\mu\text{m}}$.

(*b*) Differentiating the distance equation gives the approximate uncertainty in distance.

$$\Delta x \approx \frac{dx}{dC}\Delta C = \frac{d}{dC}\left[\frac{\varepsilon_0 A}{C}\right]\Delta C = -\frac{\varepsilon_0 A}{C^2}\Delta C .$$

The minus sign indicates that the capacitance increases as the plate separation decreases. Since only the magnitude is desired, the minus sign can be dropped. The uncertainty is finally written in terms of the plate separation using Eq. 24-2.

$$\Delta x \approx \frac{\varepsilon_0 A}{\left(\dfrac{\varepsilon_0 A}{x}\right)^2}\Delta C = \boxed{\frac{x^2\Delta C}{\varepsilon_0 A}}$$

(*c*) The percent uncertainty in distance is obtained by dividing the uncertainty by the separation distance.

$$\frac{\Delta x_{\text{min}}}{x_{\text{min}}}\times 100\% = \frac{x_{\text{min}}\Delta C}{\varepsilon_{\text{o}} A}\times 100\% = \frac{(0.22\,\mu\text{m})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{0.01\%}$$

$$\frac{\Delta x_{\text{max}}}{x_{\text{max}}}\times 100\% = \frac{x_{\text{max}}\Delta C}{\varepsilon_{\text{o}} A}\times 100\% = \frac{(0.22\,\text{mm})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{10\%}$$

20. The goal is to have an electric field of strength E_{S} at a radial distance of 5.0 R_{b} from the center of the cylinder. Knowing the electric field at a specific distance allows us to calculate the linear charge density on the inner cylinder. From the linear charge density and the capacitance we can find the potential difference needed to create the field. From the cylindrically symmetric geometry and Gauss's law, the field in between the cylinders is given by $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R}$. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$E(R = 5.0R_{\text{b}}) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{(5.0R_{\text{b}})} = E_{\text{S}} \;\rightarrow\; \lambda = 2\pi\varepsilon_0(5.0R_{\text{b}})E_{\text{S}} = \frac{Q}{\ell}$$

$$Q = CV \;\rightarrow\; V = \frac{Q}{C} = \frac{Q}{\dfrac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}} = \frac{Q}{\ell}\frac{\ln(R_{\text{a}}/R_{\text{b}})}{2\pi\varepsilon_0} = \left[2\pi\varepsilon_0(5.0R_{\text{b}})E_{\text{S}}\right]\frac{\ln(R_{\text{a}}/R_{\text{b}})}{2\pi\varepsilon_0}$$

$$= (5.0R_{\text{b}})E_{\text{S}}\ln(R_{\text{a}}/R_{\text{b}}) = \left[5.0(1.0\times10^{-4}\,\text{m})\right](2.7\times10^{6}\,\text{N/C})\ln\left(\frac{0.100\,\text{m}}{1.0\times10^{-4}\,\text{m}}\right) = \boxed{9300\,\text{V}}$$

21. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; \frac{1}{C_2} = \frac{1}{C_{\text{eq}}} - \frac{1}{C_1} = \frac{C_1 - C_{\text{eq}}}{C_1 C_{\text{eq}}} \;\rightarrow$$

$$C_2 = \frac{C_1 C_{\text{eq}}}{C_1 - C_{\text{eq}}} = \frac{(2.9\times10^{-9}\,\text{F})(1.6\times10^{-9}\,\text{F})}{(2.9\times10^{-9}\,\text{F}) - (1.6\times10^{-9}\,\text{F})} = 3.57\times10^{-9}\,\text{F} \approx \boxed{3600\,\text{pF}}$$

$\boxed{\text{Yes}}$, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

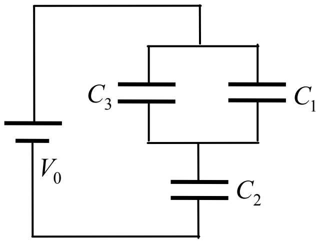
22. (*a*) Capacitors in parallel add according to Eq. 24-3.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6(3.8\times10^{-6}\,\text{F}) = \boxed{2.28\times10^{-5}\,\text{F}} = 22.8\,\mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 24-4.

$$C_{\rm eq} = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}+\frac{1}{C_5}+\frac{1}{C_6}\right)^{-1} = \left(\frac{6}{3.8\times10^{-6}\,{\rm F}}\right)^{-1} = \frac{3.8\times10^{-6}\,{\rm F}}{6} = \boxed{6.3\times10^{-7}\,{\rm F}}$$
$$= 0.63\,\mu{\rm F}$$

23. We want a small voltage drop across C_1. Since $V = Q/C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put C_1 and C_3 in parallel with each other, and then put that combination in series with C_2. See the diagram. To calculate the voltage across C_1, find the equivalent capacitance and the net charge. That charge is used to find the voltage drop across C_2, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.



$$\frac{1}{C_{\rm eq}} = \frac{1}{C_2} + \frac{1}{C_1+C_3} = \frac{C_1+C_2+C_3}{C_2\left(C_1+C_3\right)} \;\rightarrow\; C_{\rm eq} = \frac{C_2\left(C_1+C_3\right)}{C_1+C_2+C_3} \;;\; Q_{\rm eq} = C_{\rm eq}V_0 \;;\; V_2 = \frac{Q_2}{C_2} = \frac{Q_{\rm eq}}{C_2} \;;$$

$$V_1 = V_0 - V_2 = V_0 - \frac{Q_{\rm eq}}{C_2} = V_0 - \frac{C_{\rm eq}V_0}{C_2} = V_0 - \frac{\dfrac{C_2\left(C_1+C_3\right)}{C_1+C_2+C_3}V_0}{C_2} = \frac{C_2}{C_1+C_2+C_3}V_0 = \frac{1.5\,\mu{\rm F}}{6.5\,\mu{\rm F}}\left(12\,{\rm V}\right)$$
$$= \boxed{2.8\,{\rm V}}$$

24. The capacitors are in parallel, and so the potential is the same for each capacitor, and the total charge on the capacitors is the sum of the individual charges. We use Eqs. 24-1 and 24-2.

$$Q_1 = C_1V = \varepsilon_0\frac{A_1}{d_1}V \;;\; Q_2 = C_2V = \varepsilon_0\frac{A_2}{d_2}V \;;\; Q_3 = C_3V = \varepsilon_0\frac{A_3}{d_3}V$$

$$Q_{\rm total} = Q_1+Q_2+Q_3 = \varepsilon_0\frac{A_1}{d_1}V + \varepsilon_0\frac{A_2}{d_2}V + \varepsilon_0\frac{A_3}{d_3}V = \left(\varepsilon_0\frac{A_1}{d_1} + \varepsilon_0\frac{A_2}{d_2} + \varepsilon_0\frac{A_3}{d_3}\right)V$$

$$C_{\rm net} = \frac{Q_{\rm total}}{V} = \frac{\left(\varepsilon_0\dfrac{A_1}{d_1} + \varepsilon_0\dfrac{A_2}{d_2} + \varepsilon_0\dfrac{A_3}{d_3}\right)V}{V} = \left(\varepsilon_0\frac{A_1}{d_1} + \varepsilon_0\frac{A_2}{d_2} + \varepsilon_0\frac{A_3}{d_3}\right) = C_1+C_2+C_3$$

25. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\,\mu{\rm F}$ capacitor.

$$5.0\,\mu{\rm F} + C = 16\,\mu{\rm F} \;\rightarrow\; C = \boxed{11\,\mu{\rm F}\text{ connected in parallel}}$$

26. (*a*) The two capacitors are in $\boxed{\text{parallel}}$. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0A}{d_1} + \frac{\varepsilon_0A}{d_2} = \boxed{\varepsilon_0A\left(\frac{1}{d_1}+\frac{1}{d_2}\right)} = \varepsilon_0A\left(\frac{d_1+d_2}{d_1d_2}\right)$$

(*c*) Let $\ell = d_1 + d_2 =$ constant. Then $C = \dfrac{\varepsilon_0 A\ell}{d_1 d_2} = \dfrac{\varepsilon_0 A\ell}{d_1\left(\ell - d_1\right)}$. We see that $C \to \infty$ as $d_1 \to 0$ or $d_1 \to \ell$ (which is $d_2 \to 0$). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\dfrac{dC}{d\left(d_1\right)} = 0$ and solve for d_1.

$$\frac{dC}{d\left(d_1\right)} = \frac{d}{d\left(d_1\right)}\left[\frac{\varepsilon_0 A\ell}{d_1\ell - d_1^2}\right] = \varepsilon_0 A\ell \frac{\left(\ell - 2d_1\right)}{\left(d_1\ell - d_1^2\right)^2} = 0 \;\;\rightarrow\;\; d_1 = \tfrac{1}{2}\ell = d_2$$

$$C_{\min} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)_{d_1 = \frac{1}{2}\ell} = \varepsilon_0 A\left(\frac{\ell}{\left(\frac{1}{2}\ell\right)\left(\frac{1}{2}\ell\right)}\right) = \varepsilon_0 A\left(\frac{4}{\ell}\right) = \varepsilon_0 A\left(\frac{4}{d_1 + d_2}\right)$$

$$\boxed{C_{\min} = \frac{4\varepsilon_0 A}{d_1 + d_2} \;\; ; \; C_{\max} = \infty}$$

27. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max} = C_1 + C_2 + C_3 = 3.6\times10^{-9}\,\mathrm{F} + 5.8\times10^{-9}\,\mathrm{F} + 1.00\times10^{-8}\,\mathrm{F} = \boxed{1.94\times10^{-8}\,\mathrm{F\ in\ parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3.6\times10^{-9}\,\mathrm{F}} + \frac{1}{5.8\times10^{-9}\,\mathrm{F}} + \frac{1}{1.00\times10^{-8}\,\mathrm{F}}\right)^{-1} = \boxed{1.82\times10^{-9}\,\mathrm{F\ in\ series}}$$

28. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $Q_1 = Q_2 = Q_{\mathrm{eq}} = C_{\mathrm{eq}}V = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1} V = \left(\dfrac{1}{0.50\times10^{-6}\,\mathrm{F}} + \dfrac{1}{0.80\times10^{-6}\,\mathrm{F}}\right)^{-1}\left(9.0\,\mathrm{V}\right)$

$= 2.769\times10^{-6}\,\mathrm{C}$

$$V_1 = \frac{Q_1}{C_1} = \frac{2.769\times10^{-6}\,\mathrm{C}}{0.50\times10^{-6}\,\mathrm{F}} = 5.538\,\mathrm{V} \approx \boxed{5.5\,\mathrm{V}} \qquad V_2 = \frac{Q_2}{C_2} = \frac{2.769\times10^{-6}\,\mathrm{C}}{0.80\times10^{-6}\,\mathrm{F}} = 3.461\,\mathrm{V} \approx \boxed{3.5\,\mathrm{V}}$$

(*b*) $Q_1 = Q_2 = Q_{\mathrm{eq}} = 2.769\times10^{-6}\,\mathrm{C} \approx \boxed{2.8\times10^{-6}\,\mathrm{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\,\mathrm{V}}$ $\quad V_2 = \boxed{9.0\,\mathrm{V}}$ $\quad Q_1 = C_1V_1 = \left(0.50\times10^{-6}\,\mathrm{F}\right)\left(9.0\,\mathrm{V}\right) = \boxed{4.5\times10^{-6}\,\mathrm{C}}$

$Q_2 = C_2V_2 = \left(0.80\times10^{-6}\,\mathrm{F}\right)\left(9.0\,\mathrm{V}\right) = \boxed{7.2\times10^{-6}\,\mathrm{C}}$

29. (*a*) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3, and then that combination is in series with C_4. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \;\;\rightarrow\;\; C_{12} = \tfrac{1}{2}C \;\; ; \;\; C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C \;\; ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \;\;\rightarrow\;\; C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V = \frac{3}{5}CV$. This is the charge on both of the series components of C_{1234}.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \quad \rightarrow \quad V_{123} = \tfrac{2}{5}V$$

$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \quad \rightarrow \quad V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\frac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12} \ ; \ Q_{12} = C_{12}V_{12} = \left(\tfrac{1}{2}C\right)\left(\tfrac{2}{5}V\right) = \tfrac{1}{5}CV$$

$$V_{123} = \tfrac{2}{5}V = V_3 \ ; \ Q_3 = C_3V_3 = C\left(\tfrac{2}{5}V\right) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12}.

$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \quad \rightarrow \quad V_1 = \tfrac{1}{5}V \ ; \ Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \quad \rightarrow \quad V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV \ ; \ Q_3 = \tfrac{2}{5}CV \ ; \ Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V \ ; \ V_3 = \tfrac{2}{5}V \ ; \ V_4 = \tfrac{3}{5}V \end{array}}$$

30. C_1 and C_2 are in series, so they both have the same charge. We then use that charge to find the voltage across each of C_1 and C_2. Then their combined voltage is the voltage across C_3. The voltage across C_3 is used to find the charge on C_3.

$$Q_1 = Q_2 = 12.4\mu\text{C} \ ; \ V_1 = \frac{Q_1}{C_1} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V} \ ; \ V_2 = \frac{Q_2}{C_2} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V}$$

$$V_3 = V_1 + V_2 = 1.55\,\text{V} \ ; \ Q_3 = C_3V_3 = (16.0\mu\text{F})(1.55\,\text{V}) = 24.8\mu\text{C}$$

From the diagram, C_4 must have the same charge as the sum of the charges on C_1 and C_3. Then the voltage across the entire combination is the sum of the voltages across C_4 and C_3.

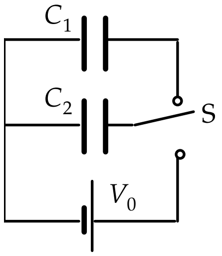
$$Q_4 = Q_1 + Q_3 = 12.4\mu\text{C} + 24.8\mu\text{C} = 37.2\mu\text{C} \ ; \ V_4 = \frac{Q_4}{C_4} = \frac{37.2\mu\text{C}}{28.5\mu\text{F}} = 1.31\,\text{V}$$

$$V_{\text{ab}} = V_4 + V_3 = 1.31\,\text{V} + 1.55\,\text{V} = 2.86\,\text{V}$$

Here is a summary of all results.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = 12.4\mu\text{C} \ ; \ Q_3 = 24.8\mu\text{C} \ ; \ Q_4 = 37.2\mu\text{C} \\ V_1 = V_2 = 0.775\,\text{V} \ ; \ V_3 = 1.55\,\text{V} \ ; \ V_4 = 1.31\,\text{V} \ ; \ V_{\text{ab}} = 2.86\,\text{V} \end{array}}$$

$\boxed{31.}$ When the switch is down the initial charge on C_2 is calculated from Eq. 24-1.



$$Q_2 = C_2V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \quad \rightarrow \quad Q_2' = Q_1'\frac{C_2}{C_1}$$

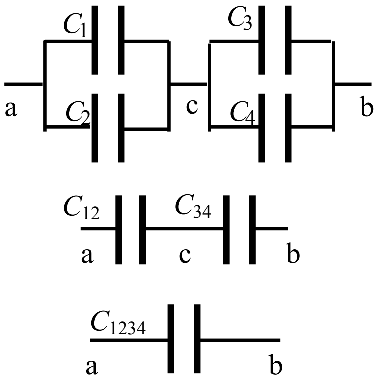
The sum of the charges on the two capacitors is equal to the initial charge on C_2.

$$Q_2 = Q_2' + Q_1' = Q_1'\frac{C_2}{C_1} + Q_1' = Q_1'\left(\frac{C_2 + C_1}{C_1}\right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1'\left(\frac{C_2 + C_1}{C_1}\right) = C_2 V_0 \quad \rightarrow \quad Q_1' = \boxed{\frac{C_1 C_2}{C_2 + C_1}V_0} \;;\; Q_2' = Q_1'\frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_2 + C}V_0}$$

32. (*a*) From the diagram, we see that C_1 and C_2 are in parallel, and C_3 and C_4 are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.



$$C_{12} = C_1 + C_2 \;;\; C_{34} = C_3 + C_4 \;;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4} \quad \rightarrow$$

$$C_{1234} = \boxed{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V$. This is the charge on both of the series components of C_{1234}. Note that $V_{12} + V_{34} = V$.

$$Q_{12} = C_{1234}V = C_{12}V_{12} \quad \rightarrow \quad V_{12} = \frac{C_{1234}}{C_{12}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_1 + C_2}V = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$Q_{34} = C_{1234}V = C_{34}V_{34} \quad \rightarrow \quad V_{34} = \frac{C_{1234}}{C_{34}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_3 + C_4}V = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

The voltage across the equivalent capacitor C_{12} is the voltage across both of its parallel components, and the voltage across the equivalent C_{34} is the voltage across both its parallel components.

$$V_{12} = \boxed{V_1 = V_2 = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \;;$$

$$C_1 V_1 = \boxed{Q_1 = \frac{C_1(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \;;\; C_2 V_2 = \boxed{Q_2 = \frac{C_2(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V}$$

$$V_{34} = \boxed{V_3 = V_2 = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \;;$$

$$C_3 V_3 = \boxed{Q_3 = \frac{C_3(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \;;\; C_4 V_4 = \boxed{Q_4 = \frac{C_4(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V}$$

33. (*a*) The voltage across C_3 and C_4 must be the same, since they are in parallel.

$$V_3 = V_4 \rightarrow \frac{Q_3}{C_3} = \frac{Q_4}{C_4} \rightarrow Q_4 = Q_3\frac{C_4}{C_3} = (23\mu\text{C})\frac{16\mu\text{F}}{8\mu\text{F}} = \boxed{46\mu\text{C}}$$

The parallel combination of C_3 and C_4 is in series with the parallel combination of C_1 and C_2, and so $Q_3 + Q_4 = Q_1 + Q_2$. That total charge then divides between C_1 and C_2 in such a way that $V_1 = V_2$.

$$Q_1 + Q_2 = Q_3 + Q_4 = 69\mu\text{C} \ ; \ V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_4}{C_4} = \frac{69\mu\text{C} - Q_1}{C_4} \rightarrow$$

$$Q_1 = \frac{C_1}{C_4 + C_1}(69\mu\text{C}) = \frac{8.0\mu\text{F}}{24.0\mu\text{F}}(69\mu\text{C}) = \boxed{23\mu\text{C}} \ ; Q_2 = 69\mu\text{C} - 23\mu\text{C} = \boxed{46\mu\text{C}}$$

Notice the symmetry in the capacitances and the charges.

(*b*) Use Eq. 24-1.

$$V_1 = \frac{Q_1}{C_1} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \ ; V_2 = V_1 = \boxed{2.9\,\text{V}}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \ ; V_4 = V_3 = \boxed{2.9\,\text{V}}$$

(*c*) $V_{\text{ba}} = V_1 + V_3 = 2.875\,\text{V} + 2.875\,\text{V} = 5.75\text{V} \approx \boxed{5.8\,\text{V}}$

34. We have $C_\text{P} = C_1 + C_2$ and $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2}$. Solve for C_1 and C_2 in terms of C_P and C_S.

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{C_\text{P} - C_1} = \frac{(C_\text{P} - C_1) + C_1}{C_1(C_\text{P} - C_1)} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow$$

$$\frac{1}{C_\text{S}} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow C_1^2 - C_\text{P}C_1 + C_\text{P}C_\text{S} = 0 \rightarrow$$

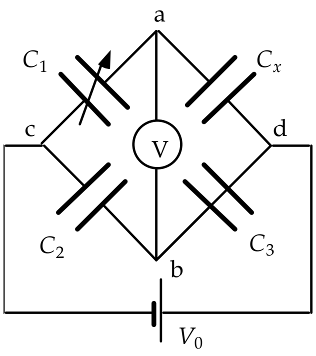
$$C_1 = \frac{C_\text{P} \pm \sqrt{C_\text{P}^2 - 4C_\text{P}C_\text{S}}}{2} = \frac{35.0\mu\text{F} \pm \sqrt{(35.0\mu\text{F})^2 - 4(35.0\mu\text{F})(5.5\mu\text{F})}}{2}$$

$$= 28.2\mu\text{F},\ 6.8\mu\text{F}$$

$$C_2 = C_\text{P} - C_1 = 35.0\mu\text{F} - 28.2\mu\text{F} = 6.8\mu\text{F} \text{ or } 35.0\mu\text{F} - 6.8\mu\text{F} = 28.2\mu\text{F}$$

So the two values are $\boxed{28.2\mu\text{F and } 6.8\mu\text{F}}$.

35. Since there is no voltage between points a and b, we can imagine there being a connecting wire between points a and b. Then capacitors C_1 and C_2 are in parallel, and so have the same voltage. Also capacitors C_3 and C_x are in parallel, and so have the same voltage.



$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \ ; \ V_3 = V_x \rightarrow \frac{Q_3}{C_3} = \frac{Q_x}{C_x}$$

Since no charge flows through the voltmeter, we could also remove it from the circuit and have no change in the circuit. In that case, capacitors C_1 and C_x are in series and so have the same charge. Likewise capacitors C_2 and C_3 are in series, and so have the same charge.

$$Q_1 = Q_x \ ; \ Q_2 = Q_3$$

Solve this system of equations for C_x.

$$\frac{Q_3}{C_3} = \frac{Q_x}{C_x} \ \rightarrow \ C_x = C_3\frac{Q_x}{Q_3} = C_3\frac{Q_1}{Q_2} = C_3\frac{C_1}{C_2} = (4.8\mu\text{F})\left(\frac{8.9\mu\text{F}}{18.0\mu\text{F}}\right) = \boxed{2.4\mu\text{F}}$$

36. The initial equivalent capacitance is the series combination of the two individual capacitances. Each individual capacitor will have the same charge as the equivalent capacitance. The sum of the two initial charges will be the sum of the two final charges, because charge is conserved. The final potential of both capacitors will be equal.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \ \rightarrow$$

$$C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2} \ ; \ Q_{\text{eq}} = C_{\text{eq}}V_0 = \frac{C_1C_2}{C_1 + C_2}V_0 = \frac{(3200\,\text{pF})(1800\,\text{pF})}{5000\,\text{pF}}(12.0\,\text{V}) = 13{,}824\,\text{pC}$$

$$Q_{1\,\text{final}} + Q_{2\,\text{final}} = 2Q_{\text{eq}} \ ; \ V_{1\,\text{final}} = V_{2\,\text{final}} \ \rightarrow \ \frac{Q_{1\,\text{final}}}{C_1} = \frac{Q_{2\,\text{final}}}{C_2} = \frac{2Q_{\text{eq}} - Q_{1\,\text{final}}}{C_2} \ \rightarrow$$

$$Q_{1\,\text{final}} = 2\frac{C_1}{C_1 + C_2}Q_{\text{eq}} = 2\frac{3200\,\text{pF}}{5000\,\text{pF}}(13{,}824\,\text{pC}) = 17{,}695\,\text{pC} \approx \boxed{1.8\times10^{-8}\,\text{C}}$$

$$Q_{2\,\text{final}} = 2Q_{\text{eq}} - Q_{1\,\text{final}} = 2(13{,}824\,\text{pC}) - 17{,}695\,\text{pC} = 9953\,\text{pC} \approx \boxed{1.0\times10^{-8}\,\text{C}}$$

$\boxed{37.}$ (*a*) The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \left(\frac{C_3}{C_2C_3} + \frac{C_2}{C_2C_3}\right)^{-1} = C_1 + \left(\frac{C_2 + C_3}{C_2C_3}\right)^{-1} = \boxed{C_1 + \frac{C_2C_3}{C_2 + C_3}}$$

(*b*) For each capacitor, the charge is found by multiplying the capacitance times the voltage. For C_1, the full 35.0 V is across the capacitance, so $Q_1 = C_1V = (24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) =$ $\boxed{8.40\times10^{-4}\,\text{C}}$. The equivalent capacitance of the series combination of C_2 and C_3 has the full 35.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C/2}\right)^{-1} = \frac{C}{3} \qquad Q_{\text{eq}} = C_{\text{eq}}V = \tfrac{1}{3}(24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) = \boxed{2.80\times10^{-4}\,\text{C}} = Q_2 = Q_3$$

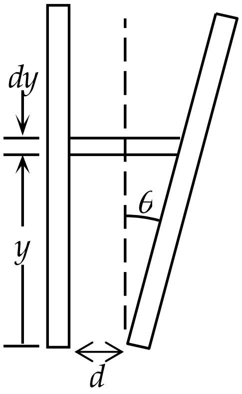
38. From the circuit diagram, we see that C_1 is in parallel with the voltage, and so $\boxed{V_1 = 24\,\text{V}}$.

Capacitors C_2 and C_3 both have the same charge, so their voltages are inversely proportional to their capacitance, and their voltages must total to 24.0 V.

$$Q_2 = Q_3 \ \rightarrow \ C_2V_2 = C_3V_3 \ ; \ V_2 + V_3 = V$$

$$V_2 + \frac{C_2}{C_3}V_2 = V \ \rightarrow \ V_2 = \frac{C_3}{C_2 + C_3}V = \frac{4.00\mu\text{F}}{7.00\mu\text{F}}(24.0\,\text{V}) = \boxed{13.7\,\text{V}}$$

$$V_3 = V - V_2 = 24.0\,\text{V} - 13.7\,\text{V} = \boxed{10.3\,\text{V}}$$



39. For an infinitesimal area element of the capacitance a distance y up from the small end, the distance between the plates is $d+x=d+y\tan\theta\approx d+y\theta.$ Since the capacitor plates are square, they are of dimension $\sqrt{A}\times\sqrt{A},$ and the area of the infinitesimal strip is $dA=\sqrt{A}\,dy.$ The infinitesimal capacitance dC of the strip is calculated, and then the total capacitance is found by adding together all of the infinitesimal capacitances, in parallel with each other.

$$C=\varepsilon_0\frac{A}{d}\quad\rightarrow\quad dC=\varepsilon_0\frac{dA}{d+y\theta}=\varepsilon_0\frac{\sqrt{A}\,dy}{d+y\theta}$$

$$C=\int dC=\int_0^{\sqrt{A}}\varepsilon_0\frac{\sqrt{A}\,dy}{d+y\theta}=\frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(d+y\theta\right)\Big|_0^{\sqrt{A}}$$

$$=\frac{\varepsilon_0\sqrt{A}}{\theta}\left[\ln\left(d+\theta\sqrt{A}\right)-\ln d\right]=\frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(\frac{d+\theta\sqrt{A}}{d}\right)=\frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right)$$

We use the approximation from page A-1 that $\ln(1+x)\approx x-\frac{1}{2}x^2.$

$$C=\frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right)=\frac{\varepsilon_0\sqrt{A}}{\theta}\left[\frac{\theta\sqrt{A}}{d}-\frac{1}{2}\left(\frac{\theta\sqrt{A}}{d}\right)^2\right]=\boxed{\frac{\varepsilon_0 A}{d}\left(1-\frac{\theta\sqrt{A}}{2d}\right)}$$

40. No two capacitors are in series or in parallel in the diagram, and so we may not simplify by that method. Instead use the hint as given in the problem. We consider point a as the higher voltage. The equivalent capacitance must satisfy $Q_{\text{tot}}=C_{\text{eq}}V.$

(*a*) The potential between a and b can be written in three ways. Alternate but equivalent expressions are shown in parentheses.

$$V=V_2+V_1\ ;\ V=V_2+V_3+V_4\ ;\ V=V_5+V_4\quad\left(V_2+V_3=V_5\ ;\ V_3+V_4=V_1\right)$$

There are also three independent charge relationships. Alternate but equivalent expressions are shown in parentheses. Convert the charge expressions to voltage – capacitance expression.

$$Q_{\text{tot}}=Q_2+Q_5\quad ;\ Q_{\text{tot}}=Q_4+Q_1\quad ;\ Q_2=Q_1+Q_3\quad\left(Q_4=Q_3+Q_5\right)$$

$$C_{\text{eq}}V=C_2V_2+C_5V_5\ ;\ C_{\text{eq}}V=C_4V_4+C_1V_1\ ;\ C_2V_2=C_1V_1+C_3V_3$$

We have a set of six equations: $V=V_2+V_1\ (1)\ ;\ V=V_2+V_3+V_4\ (2)\ ;\ V=V_5+V_4\ (3)$

$$C_{\text{eq}}V=C_2V_2+C_5V_5\ (4)\ ;\ C_{\text{eq}}V=C_4V_4+C_1V_1\ (5)\ ;\ C_2V_2=C_1V_1+C_3V_3\ (6)$$

Solve for C_{eq} as follows.

(i) From Eq. (1), $V_1=V-V_2.$ Rewrite equations (5) and (6). V_1 has been eliminated.

$$C_{\text{eq}}V=C_4V_4+C_1V-C_1V_2\ (5)\ ;\ C_2V_2=C_1V-C_1V_2+C_3V_3\ (6)$$

(ii) From Eq. (3), $V_5=V-V_4.$ Rewrite equation (4). V_5 has been eliminated.

$$C_{\text{eq}}V=C_2V_2+C_5V-C_5V_4\ (4)$$

(iii) From Eq. (2), $V_3=V-V_2-V_4.$ Rewrite equation (6). V_3 has been eliminated.

$$C_2V_2=C_1V-C_1V_2+C_3V-C_3V_2-C_3V_4\ (6)\ \rightarrow$$

$$\left(C_1+C_2+C_3\right)V_2+C_3V_4=\left(C_1+C_3\right)V\ (6)$$

Here is the current set of equations.

$$C_{\text{eq}}V = C_2V_2 + C_5V - C_5V_4 \quad (4)$$
$$C_{\text{eq}}V = C_4V_4 + C_1V - C_1V_2 \quad (5)$$
$$(C_1 + C_2 + C_3)V_2 + C_3V_4 = (C_1 + C_3)V \quad (6)$$

(iv) From Eq. (4), $V_4 = \frac{1}{C_5}\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)$. Rewrite equations (5) and (6).

$$C_5C_{\text{eq}}V = C_4\left[\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)\right] + C_5C_1V - C_5C_1V_2 \quad (5)$$
$$C_5(C_1 + C_2 + C_3)V_2 + C_3\left[\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)\right] = C_5(C_1 + C_3)V \quad (6)$$

(v) Group all terms by common voltage.

$$\left(C_5C_{\text{eq}} + C_4C_{\text{eq}} - C_4C_5 - C_5C_1\right)V = \left(C_4C_2 - C_5C_1\right)V_2 \quad (5)$$
$$\left[C_5(C_1 + C_3) + C_3C_{\text{eq}} - C_3C_5\right]V = \left[C_5(C_1 + C_2 + C_3) + C_3C_2\right]V_2 \quad (6)$$

(vi) Divide the two equations to eliminate the voltages, and solve for the equivalent capacitance.

$$\frac{\left(C_5C_{\text{eq}} + C_4C_{\text{eq}} - C_4C_5 - C_5C_1\right)}{\left[C_5(C_1 + C_3) + C_3C_{\text{eq}} - C_3C_5\right]} = \frac{\left(C_4C_2 - C_5C_1\right)}{\left[C_5(C_1 + C_2 + C_3) + C_3C_2\right]} \quad \rightarrow$$

$$C_{\text{eq}} = \boxed{\frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}}$$

(*b*) Evaluate with the given data. Since all capacitances are in μF, and the expression involves capacitance cubed terms divided by capacitance squared terms, the result will be in μF.

$$C_{\text{eq}} = \frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}$$
$$= \frac{C_1\left[C_2(C_3 + C_4 + C_5) + C_5(C_3 + C_4)\right] + C_4\left(C_2C_3 + C_2C_5 + C_3C_5\right)}{C_1(C_3 + C_4 + C_5) + C_2(C_3 + C_4 + C_5) + C_3(C_4 + C_5)}$$
$$= \frac{(4.5)\left\{(8.0)(17.0) + (4.5)(12.5)\right\} + (8.0)\left[(8.0)(4.5) + (8.0)(4.5) + (4.5)(4.5)\right]}{(4.5)(17.0) + (8.0)(17.0) + (4.5)(12.5)}\,\mu\text{F}$$
$$= \boxed{6.0\,\mu\text{F}}$$

41. The stored energy is given by Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(2.8\times10^{-9}\,\text{F}\right)\left(2200\,\text{V}\right)^2 = \boxed{6.8\times10^{-3}\,\text{J}}$$

42. The energy density is given by Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0E^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(150\,\text{V/m}\right)^2 = \boxed{1.0\times10^{-7}\,\text{J/m}^3}$$

43. The energy stored is obtained from Eq. 24-5, with the capacitance of Eq. 24-2.

$$U = \frac{Q^2}{2C} = \frac{Q^2d}{2\varepsilon_0A} = \frac{\left(4.2\times10^{-4}\,\text{C}\right)^2\left(0.0013\,\text{m}\right)}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.080\,\text{m}\right)^2} = \boxed{2.0\times10^{3}\,\text{J}}$$

44. (*a*) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3.

$$\frac{U_2}{U_1}=\frac{\dfrac{Q^2}{2C_2}}{\dfrac{Q^2}{2C_1}}=\frac{C_1}{C_2}=\frac{\varepsilon_0\dfrac{A}{d}}{\varepsilon_0\dfrac{A}{3d}}=\boxed{3}$$

(*b*) The work done is the change in energy stored in the capacitor.

$$U_2-U_1=3U_1-U_1=2U_1=2\frac{Q^2}{2C_1}=\frac{Q^2}{\varepsilon_0\dfrac{A}{d}}=\boxed{\frac{Q^2d}{\varepsilon_0A}}$$

45. The equivalent capacitance is formed by C_1 in parallel with the series combination of C_2 and C_3. Then use Eq. 24-5 to find the energy stored.

$$C_{\text{net}}=C_1+\frac{C_2C_3}{C_2+C_3}=C+\frac{C^2}{2C}=\tfrac{3}{2}C$$

$$U=\tfrac{1}{2}C_{\text{net}}V^2=\tfrac{3}{4}CV^2=\tfrac{3}{4}\left(22.6\times10^{-6}\,\text{F}\right)\left(10.0\,\text{V}\right)^2=\boxed{1.70\times10^{-3}\,\text{J}}$$

46. (*a*) Use Eqs. 24-3 and 24-5.

$$U_{\text{parallel}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(C_1+C_2\right)V^2=\tfrac{1}{2}\left(0.65\times10^{-6}\,\text{F}\right)\left(28\,\text{V}\right)^2=2.548\times10^{-4}\,\text{J}\approx\boxed{2.5\times10^{-4}\,\text{J}}$$

(*b*) Use Eqs. 24-4 and 24-5.

$$U_{\text{series}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2=\tfrac{1}{2}\left(\frac{\left(0.45\times10^{-6}\,\text{F}\right)\left(0.20\times10^{-6}\,\text{F}\right)}{0.65\times10^{-6}\,\text{F}}\right)\left(28\,\text{V}\right)^2$$

$$=5.428\times10^{-5}\,\text{J}\approx\boxed{5.4\times10^{-5}\,\text{J}}$$

(*c*) The charge can be found from Eq. 24-5.

$$U=\tfrac{1}{2}QV\ \rightarrow\ Q=\frac{2U}{V}\ \rightarrow\ Q_{\text{parallel}}=\frac{2\left(2.548\times10^{-4}\,\text{J}\right)}{28\,\text{V}}=\boxed{1.8\times10^{-5}\,\text{C}}$$

$$Q_{\text{series}}=\frac{2\left(5.428\times10^{-5}\,\text{J}\right)}{28\,\text{V}}=\boxed{3.9\times10^{-6}\,\text{C}}$$

47. The capacitance of a cylindrical capacitor is given in Example 24-2 as $C=\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}$.

(*a*) If the charge is constant, the energy can be calculated by $U=\tfrac{1}{2}\dfrac{Q^2}{C}$.

$$\frac{U_2}{U_1}=\frac{\tfrac{1}{2}\dfrac{Q^2}{C_2}}{\tfrac{1}{2}\dfrac{Q^2}{C_1}}=\frac{C_1}{C_2}=\frac{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(3R_a/R_b\right)}}=\boxed{\frac{\ln\left(3R_a/R_b\right)}{\ln\left(R_a/R_b\right)}}>1$$

The energy comes from the work required to separate the capacitor components.

(*b*) If the voltage is constant, the energy can be calculated by $U=\frac{1}{2}CV^2$.

$$\frac{U_2}{U_1}=\frac{\frac{1}{2}C_2V^2}{\frac{1}{2}C_1V^2}=\frac{C_2}{C_1}=\frac{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(3R_\text{a}/R_\text{b}\right)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_\text{a}/R_\text{b}\right)}}=\boxed{\frac{\ln\left(R_\text{a}/R_\text{b}\right)}{\ln\left(3R_\text{a}/R_\text{b}\right)}}<1$$

Since the voltage remained constant, and the capacitance decreased, the amount of charge on the capacitor components decreased. Charge flowed back into the battery that was maintaining the constant voltage.

48. (*a*) Before the capacitors are connected, the only stored energy is in the initially-charged capacitor. Use Eq. 24-5.

$$U_1=\tfrac{1}{2}C_1V_0^2=\tfrac{1}{2}\left(2.20\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right)^2=1.584\times10^{-4}\,\text{J}\approx\boxed{1.58\times10^{-4}\,\text{J}}$$

(*b*) The total charge available is the charge on the initial capacitor. The capacitance changes to the equivalent capacitance of the two capacitors in parallel.

$$Q=Q_1=C_1V_0\ ;\ C_\text{eq}=C_1+C_2\ ;\ U_2=\tfrac{1}{2}\frac{Q^2}{C_\text{eq}}=\tfrac{1}{2}\frac{C_1^2V_0^2}{C_1+C_2}=\tfrac{1}{2}\frac{\left(2.20\times10^{-6}\,\text{F}\right)^2\left(12.0\,\text{V}\right)^2}{\left(5.70\times10^{-6}\,\text{F}\right)}$$

$$=6.114\times10^{-5}\,\text{J}\approx\boxed{6.11\times10^{-5}\,\text{J}}$$

(*c*) $\Delta U=U_2-U_1=6.114\times10^{-5}\,\text{J}-1.584\times10^{-4}\,\text{J}=\boxed{-9.73\times10^{-5}\,\text{J}}$

$\boxed{49.}$ (*a*) With the plate inserted, the capacitance is that of two series capacitors of plate separations $d_1=x$ and $d_2=d-\ell-x$.

$$C_i=\left[\frac{x}{\varepsilon_0A}+\frac{d-x-\ell}{\varepsilon_0A}\right]^{-1}=\frac{\varepsilon_0A}{d-\ell}$$

With the plate removed the capacitance is obtained directly from Eq. 24-2.

$$C_f=\frac{\varepsilon_0A}{d}$$

Since the voltage remains constant the energy of the capacitor will be given by Eq. 24-5 written in terms of voltage and capacitance. The work will be the change in energy as the plate is removed.

$$W=U_f-U_i=\tfrac{1}{2}\left(C_f-C_i\right)V^2$$

$$=\tfrac{1}{2}\left(\frac{\varepsilon_0A}{d}-\frac{\varepsilon_0A}{d-\ell}\right)V^2=\boxed{-\frac{\varepsilon_0A\ell V^2}{2d\left(d-\ell\right)}}$$

The net work done is negative. Although the person pulling the plate out must do work, charge is returned to the battery, resulting in a net negative work done.

(*b*) Since the charge now remains constant, the energy of the capacitor will be given by Eq. 24-5 written in terms of capacitance and charge.

$$W=\frac{Q^2}{2}\left(\frac{1}{C_f}-\frac{1}{C_i}\right)=\frac{Q^2}{2}\left(\frac{d}{\varepsilon_0A}-\frac{d-\ell}{\varepsilon_0A}\right)=\frac{Q^2\ell}{2\varepsilon_0A}$$

The original charge is $Q = CV_0 = \dfrac{\varepsilon_0 A}{d-\ell}V_0$ and so $W = \dfrac{\left(\dfrac{\varepsilon_0 A}{d-\ell}V_0\right)^2 \ell}{2\varepsilon_0 A} = \boxed{\dfrac{\varepsilon_0 A V_0^2 \ell}{2\left(d-\ell\right)^2}}$.

50. (*a*) The charge remains constant, so we express the stored energy as $U = \frac{1}{2}\dfrac{Q^2}{C} = \frac{1}{2}\dfrac{Q^2 x}{\varepsilon_0 A}$, where *x* is the separation of the plates. The work required to increase the separation by dx is $dW = Fdx$, where *F* is the force on one plate exerted by the other plate. That work results in an increase in potential energy, dU.

$$dW = Fdx = dU = \tfrac{1}{2}\frac{Q^2 dx}{\varepsilon_0 A} \quad \rightarrow \quad \boxed{F = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}}$$

(*b*) We cannot use $F = QE = Q\dfrac{\sigma}{\varepsilon_0} = Q\dfrac{Q}{\varepsilon_0 A} = \dfrac{Q^2}{\varepsilon_0 A}$ because the electric field is due to both plates, and charge cannot put a force on itself by the field it creates. By the symmetry of the geometry, the electric field at one plate, due to just the other plate, is $\frac{1}{2}E$. See Example 24-10.

51. (*a*) The electric field outside the spherical conductor is that of an equivalent point charge at the center of the sphere, so $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}, r > R$. Consider a differential volume of radius *dr*, and volume $dV = 4\pi r^2 dr$, as used in Example 22-5. The energy in that volume is $dU = udV$. Integrate over the region outside the conductor.

$$U = \int dU = \int udV = \tfrac{1}{2}\varepsilon_0 \int E^2 dV = \tfrac{1}{2}\varepsilon_0 \int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = -\frac{Q^2}{8\pi\varepsilon_0}\frac{1}{r}\bigg|_R^\infty$$

$$= \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*b*) Use Eq. 24-5 with the capacitance of an isolated sphere, from the text immediately after Example 24-3.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*c*) When there is a charge *q* < *Q* on the sphere, the potential of the sphere is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$. The work required to add a charge *dq* to the sphere is then $dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. That work increase the potential energy by the same amount, so $dU = dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. Build up the entire charge from 0 to *Q*, calculating the energy as the charge increases.

$$U = \int dU = \int dW = \int Vdq = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{R}dq = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q qdq = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

52. In both configurations, the voltage across the combination of capacitors is the same. So use $U = \frac{1}{2}CV^2$.

$$U_{\mathrm{P}} = \tfrac{1}{2}C_{\mathrm{P}}V^2 = \tfrac{1}{2}\left(C_1 + C_2\right)V^2 \ ; \ U_{\mathrm{S}} = \tfrac{1}{2}C_{\mathrm{S}}V^2 = \tfrac{1}{2}\frac{C_1C_2}{\left(C_1 + C_2\right)}V^2$$

$$U_{\mathrm{P}} = 5\,U_{\mathrm{S}} \ \rightarrow \ \tfrac{1}{2}\left(C_1 + C_2\right)V^2 = 5\left(\tfrac{1}{2}\right)\frac{C_1C_2}{\left(C_1 + C_2\right)}V^2 \ \rightarrow \ \left(C_1 + C_2\right)^2 = 5C_1C_2 \ \rightarrow$$

$$C_1^2 - 3C_1C_2 + C_2^2 = 0 \ \rightarrow \ C_1 = \frac{3C_2 \pm \sqrt{9C_2^2 - 4C_2^2}}{2} = C_2\frac{3 \pm \sqrt{5}}{2} \ \rightarrow$$

$$\frac{C_1}{C_2} = \boxed{\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2} = 2.62, 0.382}$$

53. First find the ratio of energy requirements for a logical operation in the past to the current energy requirements for a logical operation.

$$\frac{E_{\mathrm{past}}}{E_{\mathrm{present}}} = \frac{N\left(\frac{1}{2}CV^2\right)_{\mathrm{past}}}{N\left(\frac{1}{2}CV^2\right)_{\mathrm{present}}} = \left(\frac{C_{\mathrm{past}}}{C_{\mathrm{present}}}\right)\left(\frac{V_{\mathrm{past}}}{V_{\mathrm{present}}}\right)^2 = \left(\frac{20}{1}\right)\left(\frac{5.0}{1.5}\right)^2 = 220$$

So past operations would have required 220 times more energy. Since 5 batteries in the past were required to hold the same energy as a present battery, it would have taken 1100 times as many batteries in the past. And if it takes 2 batteries for a modern PDA, it would take $\boxed{\text{2200 batteries}}$ to power the PDA in the past. It $\boxed{\text{would not fit}}$ in a pocket or purse. The volume of a present-day battery is $V = \pi r^2 \ell = \pi\left(0.5\,\mathrm{cm}\right)^2\left(4\,\mathrm{cm}\right) = 3\,\mathrm{cm}^3$. The volume of 2200 of them would be $6600\,\mathrm{cm}^3$, which would require a cube about 20 cm in side length.

54. Use Eq. 24-8 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0\frac{A}{d} = \left(2.2\right)\left(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2\right)\frac{\left(4.2\times10^{-2}\,\mathrm{m}\right)^2}{\left(1.8\times10^{-3}\,\mathrm{m}\right)} = \boxed{1.9\times10^{-11}\,\mathrm{F}}$$

$\boxed{55.}$ The change in energy of the capacitor is obtained from Eq. 24-5 in terms of the constant voltage and the capacitance.

$$\Delta U = U_{\mathrm{f}} - U_{\mathrm{i}} = \tfrac{1}{2}C_0V^2 - \tfrac{1}{2}KC_0V^2 = -\tfrac{1}{2}\left(K - 1\right)C_0V^2$$

The work done by the battery in maintaining a constant voltage is equal to the voltage multiplied by the change in charge, with the charge given by Eq. 24-1.

$$W_{\mathrm{battery}} = V\left(Q_{\mathrm{f}} - Q_{\mathrm{i}}\right) = V\left(C_0V - KC_0V\right) = -\left(K - 1\right)C_0V^2$$

The work done in pulling the dielectric out of the capacitor is equal to the difference between the change in energy of the capacitor and the energy done by the battery.

$$W = \Delta U - W_{\mathrm{battery}} = -\tfrac{1}{2}\left(K - 1\right)C_0V^2 + \left(K - 1\right)C_0V^2$$

$$= \tfrac{1}{2}\left(K - 1\right)C_0V^2 = \left(3.4 - 1\right)\left(8.8\times10^{-9}\,\mathrm{F}\right)\left(100\,\mathrm{V}\right)^2 = \boxed{1.1\times10^{-4}\,\mathrm{J}}$$

56. We assume the charge and dimensions are the same as in Problem 43. Use Eq. 24-5 with charge and capacitance.

$$U=\tfrac{1}{2}\frac{Q^2}{C}=\tfrac{1}{2}\frac{Q^2}{KC_0}=\tfrac{1}{2}\frac{Q^2d}{K\varepsilon_0 A}=\tfrac{1}{2}\frac{\left(420\times10^{-6}\,\mathrm{C}\right)^2\left(0.0013\,\mathrm{m}\right)}{\left(7\right)\left(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2\right)\left(64\times10^{-4}\,\mathrm{m}^2\right)}=289.2\,\mathrm{J}\approx\boxed{290\,\mathrm{J}}$$

57. From Problem 10, we have $C=35\times10^{-15}\,\mathrm{F}$. Use Eq. 24-8 to calculate the area.

$$C=K\varepsilon_0\frac{A}{d}\quad\to\quad A=\frac{Cd}{K\varepsilon_0}=\frac{\left(35\times10^{-15}\,\mathrm{F}\right)\left(2.0\times10^{-9}\,\mathrm{m}\right)}{\left(25\right)\left(8.85\times10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2\right)}=3.164\times10^{-13}\,\mathrm{m}^2\left(\frac{10^6\,\mu\mathrm{m}}{1\,\mathrm{m}}\right)^2$$

$$=0.3164\,\mu\mathrm{m}^2\approx\boxed{0.32\,\mu\mathrm{m}^2}$$

Half of the area of the cell is used for capacitance, so $1.5\,\mathrm{cm}^2$ is available for capacitance. Each capacitor is one "bit."

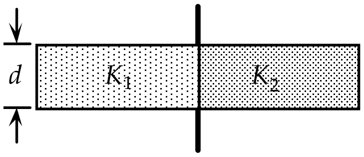
$$1.5\,\mathrm{cm}^2\left(\frac{10^6\,\mu\mathrm{m}}{10^2\,\mathrm{cm}}\right)^2\left(\frac{1\,\mathrm{bit}}{0.32\mu\mathrm{m}^2}\right)\left(\frac{1\,\mathrm{byte}}{8\,\mathrm{bits}}\right)=5.86\times10^7\,\mathrm{bytes}\approx\boxed{59\,\mathrm{Mbytes}}$$

58. The initial charge on the capacitor is $Q_{\mathrm{initial}}=C_{\mathrm{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\mathrm{final}}=KC_{\mathrm{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\mathrm{final}}=C_{\mathrm{final}}V$.

$$\Delta Q=Q_{\mathrm{final}}-Q_{\mathrm{initial}}=C_{\mathrm{final}}V-C_{\mathrm{initial}}V=\left(K-1\right)C_{\mathrm{initial}}V=\left(7-1\right)\left(3.5\times10^{-9}\,\mathrm{F}\right)\left(32\,\mathrm{V}\right)$$

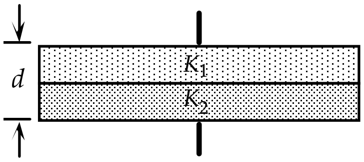
$$=\boxed{6.7\times10^{-7}\,\mathrm{C}}$$

59. The potential difference is the same on each half of the capacitor, so it can be treated as two capacitors in parallel. Each parallel capacitor has half of the total area of the original capacitor.



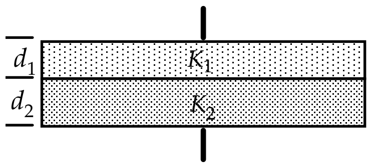
$$C=C_1+C_2=K_1\varepsilon_0\frac{\tfrac{1}{2}A}{d}+K_2\varepsilon_0\frac{\tfrac{1}{2}A}{d}=\boxed{\tfrac{1}{2}\left(K_1+K_2\right)\varepsilon_0\frac{A}{d}}$$

60. The intermediate potential at the boundary of the two dielectrics can be treated as the "low" potential plate of one half and the "high" potential plate of the other half, so we treat it as two capacitors in series. Each series capacitor has half of the inter-plate distance of the original capacitor.



$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{\tfrac{1}{2}d}{K_1\varepsilon_0 A}+\frac{\tfrac{1}{2}d}{K_2\varepsilon_0 A}=\frac{d}{2\varepsilon_0 A}\frac{K_1+K_2}{K_1K_2}\quad\to\quad C=\boxed{\frac{2\varepsilon_0 A}{d}\frac{K_1K_2}{K_1+K_2}}$$

$\boxed{61.}$ The capacitor can be treated as two series capacitors with the same areas, but different plate separations and dielectrics. Substituting Eq. 24-8 into Eq. 24-4 gives the effective capacitance.



$$C=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\left(\frac{d_1}{K_1A\varepsilon_0}+\frac{d_2}{K_2A\varepsilon_0}\right)^{-1}=\boxed{\frac{A\varepsilon_0K_1K_2}{d_1K_2+d_2K_1}}$$

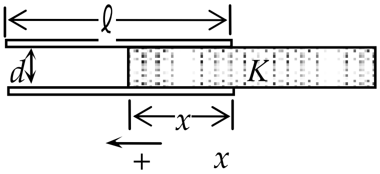
62. (*a*) Since the capacitors each have the same charge and the same voltage in the initial situation, each has the same capacitance of $C=\frac{Q_0}{V_0}$. When the dielectric is inserted, the total charge of $2Q_0$ will not change, but the charge will no longer be divided equally between the two capacitors. Some charge will move from the capacitor without the dielectric (C_1) to the capacitor with the dielectric (C_2). Since the capacitors are in parallel, their voltages will be the same.

$$V_1=V_2 \rightarrow \frac{Q_1}{C_1}=\frac{Q_2}{C_2} \rightarrow \frac{Q_1}{C}=\frac{2Q_0-Q_1}{KC} \rightarrow$$

$$Q_1=\frac{2}{(K+1)}Q_0=\frac{2}{4.2}Q_0=\boxed{0.48Q_0}\ ;\ Q_2=\boxed{1.52Q_0}$$

(*b*) $V_1=V_2=\frac{Q_1}{C_1}=\frac{0.48Q_0}{Q_0/V_0}=\boxed{0.48V_0}=\frac{Q_2}{C_2}=\frac{1.52Q_0}{3.2Q_0/V_0}$

63. (*a*) We treat this system as two capacitors, one with a dielectric, and one without a dielectric. Both capacitors have their high voltage plates in contact and their low voltage plates in contact, so they are in parallel. Use Eq. 24-2 and 24-8 for the capacitance. Note that *x* is measured from the right edge of the capacitor, and is positive to the left in the diagram.



$$C=C_1+C_2=\varepsilon_0\frac{\ell(\ell-x)}{d}+K\varepsilon_0\frac{\ell x}{d}=\boxed{\varepsilon_0\frac{\ell^2}{d}\left[1+(K-1)\frac{x}{\ell}\right]}$$

(*b*) Both "capacitors" have the same potential difference, so use $U=\frac{1}{2}CV^2$.

$$U=\tfrac{1}{2}(C_1+C_2)V_0^2=\boxed{\varepsilon_0\frac{\ell^2}{2d}\left[1+(K-1)\frac{x}{\ell}\right]V_0^2}$$

(*c*) We must be careful here. When the voltage across a capacitor is constant and a dielectric is inserted, charge flows from the battery to the capacitor. So the battery will lose energy and the capacitor gain energy as the dielectric is inserted. As in Example 24-10, we assume that work is done by an external agent (W_{nc}) in such a way that the dielectric has no kinetic energy. Then the work-energy principle (Chapter 8) can be expressed as $W_{\text{nc}}=\Delta U$ or $dW_{\text{nc}}=dU$. This is analogous to moving an object vertically at constant speed. To increase (decrease) the gravitational potential energy, positive (negative) work must be done by an outside, non-gravitational source.

In this problem, the potential energy of the voltage source and the potential energy of the capacitor both change as *x* changes. Also note that the change in charge stored on the capacitor is the opposite of the change in charge stored in the voltage supply.

$$dW_{\text{nc}}=dU=dU_{\text{cap}}+dU_{\text{battery}} \rightarrow F_{\text{nc}}dx=d\left(\tfrac{1}{2}CV_0^2\right)+d\left(Q_{\text{battery}}V_0\right) \rightarrow$$

$$F_{\text{nc}}=\tfrac{1}{2}V_0^2\frac{dC}{dx}+V_0\frac{dQ_{\text{battery}}}{dx}=\tfrac{1}{2}V_0^2\frac{dC}{dx}-V_0\frac{dQ_{\text{cap}}}{dx}=\tfrac{1}{2}V_0^2\frac{dC}{dx}-V_0^2\frac{dC}{dx}=-\tfrac{1}{2}V_0^2\frac{dC}{dx}$$

$$=-\tfrac{1}{2}V_0^2\varepsilon_0\frac{\ell^2}{d}\left[\frac{(K-1)}{\ell}\right]=-\frac{V_0^2\varepsilon_0\ell}{2d}(K-1)$$

Note that this force is in the opposite direction of dx, and so is to the right. Since this force is being applied to keep the dielectric from accelerating, there must be a force of equal magnitude to the left pulling on the dielectric. This force is due to the attraction of the charged plates and the induced charge on the dielectric. The magnitude and direction of this attractive force are $\boxed{\frac{V_0^2\varepsilon_0\ell}{2d}(K-1),\text{ left}}$.

64. (*a*) We consider the cylinder as two cylindrical capacitors in parallel. The two "negative plates" are the (connected) halves of the inner cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). The two "positive plates" are the (connected) halves of the outer cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). Schematically, it is like Figure 24-30 in Problem 59. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$C = C_{\text{liq}} + C_{\text{V}} = \frac{2\pi\varepsilon_0 K_{\text{liq}} h}{\ln(R_{\text{a}}/R_{\text{b}})} + \frac{2\pi\varepsilon_0 K_{\text{V}}(\ell - h)}{\ln(R_{\text{a}}/R_{\text{b}})} = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = C \rightarrow$$

$$\frac{h}{\ell} = \boxed{\frac{1}{(K_{\text{liq}} - K_{\text{V}})}\left[\frac{C\ln(R_{\text{a}}/R_{\text{b}})}{2\pi\varepsilon_0\ell} - K_{\text{V}}\right]}$$

(*b*) For the full tank, $\frac{h}{\ell} = 1$, and for the empty tank, $\frac{h}{\ell} = 0$.

Full: $$C = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0\ell K_{\text{liq}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$
$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.4)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.5\times10^{-9}\,\text{F}}$$

Empty: $$C = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0\ell K_{\text{V}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$
$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.0)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.1\times10^{-9}\,\text{F}}$$

65. Consider the dielectric as having a layer of equal and opposite charges at each side of the dielectric. Then the geometry is like three capacitors in series. One air gap is taken to be d_1, and then the other air gap is $d - d_1 - \ell$.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\varepsilon_0 A} + \frac{\ell}{K\varepsilon_0 A} + \frac{d - d_1 - \ell}{\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left(\left[\frac{\ell}{K} + (d - \ell)\right]\right) \rightarrow$$

$$C = \frac{\varepsilon_0 A}{\left[\frac{\ell}{K} + (d - \ell)\right]} = \frac{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.50\times10^{-2}\,\text{m}^2)}{\left[\frac{1.00\times10^{-3}\,\text{m}}{3.50} + (1.00\times10^{-3}\,\text{m})\right]} = \boxed{1.72\times10^{-10}\,\text{F}}$$

66. By leaving the battery connected, the voltage will not change when the dielectric is inserted, but the amount of charge will change. That will also change the electric field.

(*a*) Use Eq. 24-2 to find the capacitance.

$$C_0 = \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(\frac{2.50\times10^{-2}\,\mathrm{m^2}}{2.00\times10^{-3}\,\mathrm{m}}\right) = 1.106\times10^{-10}\,\mathrm{F} \approx \boxed{1.11\times10^{-10}\,\mathrm{F}}$$

(*b*) Use Eq. 24-1 to find the initial charge on each plate.

$$Q_0 = C_0V = \left(1.106\times10^{-10}\,\mathrm{F}\right)\left(150\,\mathrm{V}\right) = 1.659\times10^{-8}\,\mathrm{C} \approx \boxed{1.66\times10^{-8}\,\mathrm{C}}$$

In Example 24-12, the charge was constant, so it was simple to calculate the induced charge and then the electric fields from those charges. But now the voltage is constant, and so we calculate the fields first, and then calculate the charges. So we are solving the problem parts in a different order.

(*d*) We follow the same process as in part (*f*) of Example 24-12.

$$V = E_0\left(d-\ell\right) + E_{\mathrm{D}}\ell = E_0\left(d-\ell\right) + \frac{E_0}{K}\ell \;\rightarrow$$

$$E_0 = \frac{V}{d-\ell+\dfrac{\ell}{K}} = \frac{\left(150\,\mathrm{V}\right)}{\left(2.00\times10^{-3}\,\mathrm{m}\right)-\left(1.00\times10^{-3}\,\mathrm{m}\right)+\dfrac{\left(1.00\times10^{-3}\,\mathrm{m}\right)}{\left(3.50\right)}} = 1.167\times10^{5}\,\mathrm{V/m}$$

$$\approx \boxed{1.17\times10^{5}\,\mathrm{V/m}}$$

(*e*) $$E_{\mathrm{D}} = \frac{E_0}{K} = \frac{1.167\times10^{5}\,\mathrm{V/m}}{3.50} = 3.333\times10^{4}\,\mathrm{V/m} \approx \boxed{3.33\times10^{4}\,\mathrm{V/m}}$$

(*h*) $$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;\rightarrow$$

$$Q = EA\varepsilon_0 = \left(1.167\times10^{5}\,\mathrm{V/m}\right)\left(0.0250\,\mathrm{m^2}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right) = 2.582\times10^{-8}\,\mathrm{C}$$

$$\approx \boxed{2.58\times10^{-8}\,\mathrm{C}}$$

(*c*) $$Q_{\mathrm{ind}} = Q\left(1-\frac{1}{K}\right) = \left(2.582\times10^{-8}\,\mathrm{C}\right)\left(1-\frac{1}{3.50}\right) = \boxed{1.84\times10^{-8}\,\mathrm{C}}$$

(*f*) Because the battery voltage does not change, the potential difference between the plates is unchanged when the dielectric is inserted, and so is $V = \boxed{150\,\mathrm{V}}$.

(*g*) $$C = \frac{Q}{V} = \frac{2.582\times10^{-8}\,\mathrm{C}}{150\,\mathrm{V}} = \boxed{1.72\times10^{-10}\,\mathrm{pF}}$$

Notice that the capacitance is the same as in Example 24-12. Since the capacitance is a constant (function of geometry and material, not charge and voltage), it should be the same value.

$\boxed{67.}$ The capacitance will be given by $C = Q/V$. When a charge Q is placed on one plate and a charge $-Q$ is placed on the other plate, an electric field will be set up between the two plates. The electric field in the air-filled region is just the electric field between two charged plates, $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$. The electric field in the dielectric is equal to the electric field in the air, divided by the dielectric constant: $E_D = \dfrac{E_0}{K} = \dfrac{Q}{KA\varepsilon_0}$.

The voltage drop between the two plates is obtained by integrating the electric field between the two plates. One plate is set at the origin with the dielectric touching this plate. The dielectric ends at $x = \ell$. The rest of the distance to $x = d$ is then air filled.

$$V = -\int_0^d \vec{\mathbf{E}} \cdot d\vec{\mathbf{x}} = \int_0^\ell \frac{Qdx}{KA\varepsilon_0} + \int_\ell^d \frac{Qdx}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0}\left(\frac{\ell}{K} + (d-\ell)\right)$$

The capacitance is the ratio of the voltage to the charge.

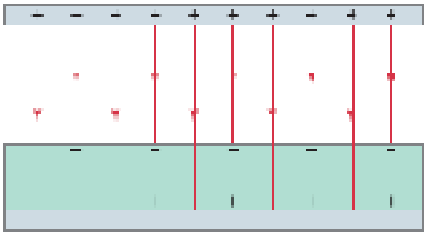
$$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{A\varepsilon_0}\left(\dfrac{\ell}{K} + (d-\ell)\right)} = \boxed{\frac{\varepsilon_0 A}{d - \ell + \dfrac{\ell}{K}}}$$

68. Find the energy in each region from the energy density and the volume. The energy density in the "gap" is given by $u_{\text{gap}} = \tfrac{1}{2}\varepsilon_0 E_{\text{gap}}^2$, and the energy density in the dielectric is given by $u_{\text{D}} = \tfrac{1}{2}\varepsilon_D E_{\text{D}}^2$ $= \tfrac{1}{2}K\varepsilon_0\left(\dfrac{E_{\text{gap}}}{K}\right)^2 = \tfrac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}$, where Eq. 24-10 is used.

$$\frac{U_{\text{D}}}{U_{\text{total}}} = \frac{U_{\text{D}}}{U_{\text{gap}} + U_{\text{D}}} = \frac{u_{\text{D}}\text{Vol}_{\text{D}}}{u_{\text{gap}}\text{Vol}_{\text{gap}} + u_{\text{D}}\text{Vol}_{\text{D}}} = \frac{\tfrac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}A\ell}{\tfrac{1}{2}\varepsilon_0 E_{\text{gap}}^2 A(d-\ell) + \tfrac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}A\ell}$$

$$= \frac{\dfrac{\ell}{K}}{(d-\ell) + \dfrac{\ell}{K}} = \frac{\ell}{(d-\ell)K + \ell} = \frac{(1.00\,\text{mm})}{(1.00\,\text{mm})(3.50) + (1.00\,\text{mm})} = \boxed{0.222}$$

69. There are two uniform electric fields – one in the air, and one in the gap. They are related by Eq. 24-10. In each region, the potential difference is the field times the distance in the direction of the field over which the field exists.



$$V = E_{\text{air}}d_{\text{air}} + E_{\text{glass}}d_{\text{glass}} = E_{\text{air}}d_{\text{air}} + \frac{E_{\text{air}}}{K_{\text{glass}}}d_{\text{glass}} \rightarrow$$

$$E_{\text{air}} = V\frac{K_{\text{glass}}}{d_{\text{air}}K_{\text{glass}} + d_{\text{glass}}}$$

$$= (90.0\,\text{V})\frac{5.80}{(3.00\times10^{-3}\,\text{m})(5.80) + (2.00\times10^{-3}\,\text{m})}$$

$$= \boxed{2.69\times10^4\ \text{V/m}}$$

$$E_{\text{glass}} = \frac{E_{\text{air}}}{K_{\text{glass}}} = \frac{2.69\times10^4\ \text{V/m}}{5.80} = \boxed{4.64\times10^3\ \text{V/m}}$$

The charge on the plates can be calculated from the field at the plate, using Eq. 22-5. Use Eq. 24-11b to calculate the charge on the dielectric.

$$E_{\text{air}} = \frac{\sigma_{\text{plate}}}{\varepsilon_0} = \frac{Q_{\text{plate}}}{\varepsilon_0 A} \quad \rightarrow$$

$$Q_{\text{plate}} = E_{\text{air}}\varepsilon_0 A = \left(2.69\times10^4\,\text{V/m}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.45\,\text{m}^2\right) = \boxed{3.45\times10^{-7}\,\text{C}}$$

$$Q_{\text{ind}} = Q\left(1-\frac{1}{K}\right) = \left(3.45\times10^{-7}\,\text{C}\right)\left(1-\frac{1}{5.80}\right) = \boxed{2.86\times10^{-7}\,\text{C}}$$

70. (*a*) The capacitance of a single isolated conducting sphere is given after example 24-3.

$$C = 4\pi\varepsilon_0 r \quad \rightarrow$$

$$\frac{C}{r} = 4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \left(1.11\times10^{-10}\,\frac{\text{F}}{\text{m}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)\left(\frac{10^{12}\,\text{pF}}{1\,\text{F}}\right) = 1.11\,\text{pF/cm}$$

And so $C = \left(1.11\,\text{pF/cm}\right)r \quad \rightarrow \quad \boxed{C(\text{pF}) \approx r(\text{cm})}$.

(*b*) We assume that the human body is a sphere of radius 100 cm. Thus the rule $C(\text{pF}) \approx r(\text{cm})$ says that the capacitance of the human body is about $\boxed{100\,\text{pF}}$.

(*c*) A 0.5-cm spark would require a potential difference of about 15,000 V. Use Eq. 24-1.

$$Q = CV = (100\,\text{pF})(15{,}000\,\text{V}) = \boxed{1.5\mu\text{C}}$$

71. Use Eq. 24-5 to find the capacitance.

$$U = \tfrac{1}{2}CV^2 \quad \rightarrow \quad C = \frac{2U}{V^2} = \frac{2(1200\,\text{J})}{(7500\,\text{V})^2} = \boxed{4.3\times10^{-5}\,\text{F}}$$

72. (*a*) We approximate the configuration as a parallel-plate capacitor, and so use Eq. 24-2 to calculate the capacitance.

$$C = \varepsilon_0\frac{A}{d} = \varepsilon_0\frac{\pi r^2}{d} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{\pi\left[(4.5\,\text{in})(0.0254\,\text{m/in})\right]^2}{0.050\,\text{m}} = 7.265\times10^{-12}\,\text{F}$$

$$\approx \boxed{7\times10^{-12}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.265\times10^{-12}\,\text{F}\right)(9\,\text{V}) = 6.539\times10^{-11}\,\text{C} \approx \boxed{7\times10^{-11}\,\text{C}}$$

(*c*) The electric field is uniform, and is the voltage divided by the plate separation.

$$E = \frac{V}{d} = \frac{9\,\text{V}}{0.050\,\text{m}} = 180\,\text{V/m} \approx \boxed{200\,\text{V/m}}$$

(*d*) The work done by the battery to charge the plates is equal to the energy stored by the capacitor. Use Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(7.265\times10^{-12}\,\text{F}\right)(9\,\text{V})^2 = 2.942\times10^{-10}\,\text{J} \approx \boxed{3\times10^{-10}\,\text{J}}$$

(*e*) The electric field will stay the same, because the voltage will stay the same (since the capacitor is still connected to the battery) and the plate separation will stay the same. The $\boxed{\text{capacitance}}$ changes, and so the $\boxed{\text{charge}}$ changes (by Eq. 24-1), and so the $\boxed{\text{work}}$ done by the battery changes (by Eq. 24-5).

$\boxed{73.}$ Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of *K*, the dielectric constant.

$$Q = C_{\text{initial}}V_{\text{initial}} = C_{\text{final}}V_{\text{final}} \rightarrow V_{\text{final}} = V_{\text{initial}}\frac{C_{\text{initial}}}{C_{\text{final}}} = V_{\text{initial}}\frac{C_{\text{initial}}}{KC_{\text{initial}}} = (34.0\,\text{V})\frac{1}{2.2} = \boxed{15\,\text{V}}$$

74. The energy is given by Eq. 24-5. Calculate the energy difference for the two different amounts of charge, and then solve for the difference.

$$U = \tfrac{1}{2}\frac{Q^2}{C} \rightarrow \Delta U = \tfrac{1}{2}\frac{(Q+\Delta Q)^2}{C} - \tfrac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left[(Q+\Delta Q)^2 - Q^2\right] = \frac{\Delta Q}{2C}\left[2Q+\Delta Q\right] \rightarrow$$

$$Q = \frac{C\Delta U}{\Delta Q} - \tfrac{1}{2}\Delta Q = \frac{(17.0\times10^{-6}\,\text{F})(18.5\,\text{J})}{(13.0\times10^{-3}\,\text{C})} - \tfrac{1}{2}(13.0\times10^{-3}\,\text{C}) = \boxed{17.7\times10^{-3}\,\text{C}} = 17.7\,\text{mC}$$

75. The energy in the capacitor, given by Eq. 24-5, is the heat energy absorbed by the water, given by Eq. 19-2.

$$U = Q_{\text{heat}} \rightarrow \tfrac{1}{2}CV^2 = mc\Delta T \rightarrow$$

$$V = \sqrt{\frac{2mc\Delta T}{C}} = \sqrt{\frac{2(3.5\,\text{kg})\left(4186\frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(95^\circ\text{C} - 22^\circ\text{C})}{3.0\,\text{F}}} = 844\,\text{V} \approx \boxed{840\,\text{V}}$$

76. (*a*) The capacitance per unit length of a cylindrical capacitor with no dielectric is derived in Example 24-2, as $\frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln(R_{\text{outside}}/R_{\text{inside}})}$. The addition of a dielectric increases the capacitance by a factor of *K*.

$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})}}$$

(*b*) $$\frac{C}{\ell} = \frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})} = \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)2.6}{\ln(9.0\,\text{mm}/2.5\,\text{mm})} = \boxed{1.1\times10^{-10}\,\text{F/m}}$$

77. The potential can be found from the field and the plate separation. Then the capacitance is found from Eq. 24-1, and the area from Eq. 24-8.

$$E = \frac{V}{d}\ ;\ Q = CV = CEd \rightarrow$$

$$C = \frac{Q}{Ed} = \frac{(0.675\times10^{-6}\,\text{C})}{(9.21\times10^{4}\,\text{V/m})(1.95\times10^{-3}\,\text{m})} = 3.758\times10^{-9}\,\text{F} \approx \boxed{3.76\times10^{-9}\,\text{F}}$$

$$C = K\varepsilon_0\frac{A}{d} \rightarrow A = \frac{Cd}{K\varepsilon_0} = \frac{(3.758\times10^{-9}\,\text{F})(1.95\times10^{-3}\,\text{m})}{(3.75)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{0.221\,\text{m}^2}$$

78. (*a*) If *N* electrons flow onto the plate, the charge on the top plate is $-Ne$, and the positive charge associated with the capacitor is $Q = Ne$. Since $Q = CV$, we have $Ne = CV \rightarrow \boxed{V = Ne/C}$, showing that *V* is proportional to *N*.

(*b*) Given $\Delta V = 1\,\text{mV}$ and we want $\Delta N = 1$, solve for the capacitance.

$$V = \frac{Ne}{C} \rightarrow \Delta V = \frac{e\Delta N}{C} \rightarrow$$

$$C = e\frac{\Delta N}{\Delta V} = \left(1.60\times10^{-19}\,\text{C}\right)\frac{1}{1\times10^{-3}\,\text{V}} = 1.60\times10^{-16}\,\text{F} \approx \boxed{2\times10^{-16}\,\text{F}}$$

(*c*) Use Eq. 24-8.

$$C = \varepsilon_0 K\frac{A}{d} = \varepsilon_0 K\frac{\ell^2}{d} \rightarrow$$

$$\ell = \sqrt{\frac{Cd}{\varepsilon_0 K}} = \sqrt{\frac{\left(1.60\times10^{-16}\,\text{F}\right)\left(100\times10^{-9}\,\text{m}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(3)}} = 7.76\times10^{-7}\,\text{m}\left(\frac{10^6\,\mu\text{m}}{1\,\text{m}}\right) = \boxed{0.8\,\mu\text{m}}$$

$\boxed{79.}$ The relative change in energy can be obtained by inserting Eq. 24-8 into Eq. 24-5.

$$\frac{U}{U_0} = \frac{\dfrac{Q^2}{2C}}{\dfrac{Q^2}{2C_0}} = \frac{C_0}{C} = \frac{\dfrac{A\varepsilon_0}{d}}{\dfrac{KA\varepsilon_0}{\left(\frac{1}{2}d\right)}} = \boxed{\frac{1}{2K}}$$

The dielectric is attracted to the capacitor. As such, the dielectric will gain kinetic energy as it enters the capacitor. An external force is necessary to stop the dielectric. The negative work done by this force results in the decrease in energy within the capacitor.

Since the charge remains constant, and the magnitude of the electric field depends on the charge, and not the separation distance, the electric field will not be affected by the change in distance between the plates. The electric field between the plates will be reduced by the dielectric constant, as given in Eq. 24-10.

$$\frac{E}{E_0} = \frac{E_0/K}{E_0} = \boxed{\frac{1}{K}}$$

80. (*a*) Use Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(120\times10^6\,\text{m}^2\right)}{(1500\,\text{m})} = 7.08\times10^{-7}\,\text{F} \approx \boxed{7.1\times10^{-7}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.08\times10^{-7}\,\text{F}\right)\left(3.5\times10^7\,\text{V}\right) = 24.78\,\text{C} \approx \boxed{25\,\text{C}}$$

(*c*) Use Eq. 24-5.

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}(24.78\,\text{C})\left(3.5\times10^7\,\text{V}\right) = 4.337\times10^8\,\text{J} \approx \boxed{4.3\times10^8\,\text{J}}$$

81. We treat this as *N* capacitors in parallel, so that the total capacitance is *N* times the capacitance of a single capacitor. The maximum voltage and dielectric strength are used to find the plate separation of a single capacitor.

$$d = \frac{V}{E_\text{S}} = \frac{100\,\text{V}}{30\times10^6\,\text{V/m}} = 3.33\times10^{-6}\,\text{m} \;;\; N = \frac{\ell}{d} = \frac{6.0\times10^{-3}\,\text{m}}{3.33\times10^{-6}\,\text{m}} = 1800$$

$$C_\text{eq} = NC = N\varepsilon_0 K\frac{A}{d} \rightarrow$$

$$K=\frac{C_{\text{eq}}d}{N\varepsilon_0 A}=\frac{\left(1.0\times10^{-6}\,\text{F}\right)\left(3.33\times10^{-6}\,\text{m}\right)}{1800\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(12.0\times10^{-3}\,\text{m}\right)\left(14.0\times10^{-3}\,\text{m}\right)}=1.244\approx\boxed{1.2}$$

82. The total charge doesn't change when the second capacitor is connected, since the two-capacitor combination is not connected to a source of charge. The final voltage across the two capacitors must be the same. Use Eq. 24-1.

$$Q_0=C_1V_0=Q_1+Q_2=C_1V_1+C_2V_2=C_1V_1+C_2V_1$$

$$C_2=C_1\frac{\left(V_0-V_1\right)}{V_1}=\left(3.5\mu\text{F}\right)\left(\frac{12.4\,\text{V}-5.9\,\text{V}}{5.9\,\text{V}}\right)=3.856\mu\text{F}\approx\boxed{3.9\mu\text{F}}$$

83. (*a*) Use Eq. 24-5 to calculate the stored energy.

$$U=\tfrac{1}{2}CV^2=\tfrac{1}{2}\left(8.0\times10^{-8}\,\text{F}\right)\left(2.5\times10^4\,\text{V}\right)^2=\boxed{25\,\text{J}}$$

(*b*) The power is the energy converted per unit time.

$$P=\frac{\text{Energy}}{\text{time}}=\frac{0.15\left(25\,\text{J}\right)}{4.0\times10^{-6}\,\text{s}}=9.38\times10^5\,\text{W}\approx\boxed{940\,\text{kW}}$$

84. The pressure is the force per unit area on a face of the dielectric. The force is related to the potential energy stored in the capacitor by Eq. 8-7, $F=-\frac{dU}{dx}$, where x is the separation of the capacitor plates.

$$U=\tfrac{1}{2}CV^2=\tfrac{1}{2}\left(K\varepsilon_0\frac{A}{x}\right)V^2\ \rightarrow\ F=-\frac{dU}{dx}=\frac{K\varepsilon_0AV^2}{2x^2}\ ;P=\frac{F}{A}=\frac{K\varepsilon_0V^2}{2x^2}\ \rightarrow$$

$$V=\sqrt{\frac{2x^2P}{K\varepsilon_0}}=\sqrt{\frac{2\left(1.0\times10^{-4}\,\text{m}\right)^2\left(40.0\,\text{Pa}\right)}{\left(3.1\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}}=\boxed{170\,\text{V}}$$

$\boxed{85.}$ (*a*) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{\text{eq}}=7C=7\varepsilon_0\frac{A}{d}$$

$$C_{\min}=7\varepsilon_0\frac{A_{\min}}{d}=7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(2.0\times10^{-4}\,\text{m}^2\right)}{\left(1.6\times10^{-3}\,\text{m}\right)}=7.7\times10^{-12}\,\text{F}$$

$$C_{\max}=7\varepsilon_0\frac{A_{\max}}{d}=7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(9.0\times10^{-4}\,\text{m}^2\right)}{\left(1.6\times10^{-3}\,\text{m}\right)}=3.5\times10^{-11}\,\text{F}$$

And so the range is $\boxed{\text{from }7.7\,\text{pF to }35\,\text{pF}}$.

86. (*a*) Since the capacitor is charged and then disconnected from the power supply, the charge is constant. Use Eq. 24-1 to find the new voltage.

$$Q = CV = \text{constant} \rightarrow C_1V_1 = C_2V_2 \rightarrow V_2 = V_1\frac{C_1}{C_2} = (7500\,\text{V})\frac{8.0\,\text{pF}}{1.0\,\text{pF}} = \boxed{6.0\times10^4\,\text{V}}$$

(*b*) In using this as a high voltage power supply, once it discharges, the voltage drops, and it needs to be recharged. So it is not a constant source of high voltage. You would also have to be sure it was designed to not have breakdown of the capacitor material when the voltage gets so high. Another disadvantage is that it has only a small amount of energy stored: $U = \frac{1}{2}CV^2$ $= \frac{1}{2}(1.0\times10^{-12}\,\text{C})(6.0\times10^4\,\text{V})^2 = 1.8\times10^{-3}\,\text{J}$, and so could actually only supply a small amount of power unless the discharge time was extremely short.

87. Since the two capacitors are in series, they will both have the same charge on them.

$$Q_1 = Q_2 = Q_{\text{series}} \;;\; \frac{1}{C_{\text{series}}} = \frac{V}{Q_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow$$

$$C_2 = \frac{Q_{\text{series}}C_1}{C_1V - Q_{\text{series}}} = \frac{(125\times10^{-12}\,\text{C})(175\times10^{-12}\,\text{F})}{(175\times10^{-12}\,\text{F})(25.0\,\text{V}) - (125\times10^{-12}\,\text{C})} = \boxed{5.15\times10^{-12}\,\text{F}}$$

88. (*a*) The charge can be determined from Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0\frac{A}{d}V = (8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)\frac{(2.0\times10^{-4}\,\text{m}^2)}{(5.0\times10^{-4}\,\text{m})}(12\,\text{V}) = 4.248\times10^{-11}\,\text{C}$$

$$\approx \boxed{4.2\times10^{-11}\,\text{C}}$$

(*b*) Since the battery is disconnected, no charge can flow to or from the plates. Thus the charge is constant.

$$Q = \boxed{4.2\times10^{-11}\,\text{C}}$$

(*c*) The capacitance has changed and the charge has stayed constant, and so the voltage has changed.

$$Q = CV = \text{constant} \rightarrow C_1V_1 = C_0V_0 \rightarrow \varepsilon_0\frac{A}{d_1}V_1 = \varepsilon_0\frac{A}{d_0}V_0 \rightarrow$$

$$V_1 = \frac{d_1}{d_0}V_0 = \frac{0.75\,\text{mm}}{0.50\,\text{mm}}(12\,\text{V}) = \boxed{18\,\text{V}}$$

(*d*) The work is the change in stored energy.

$$W = \Delta U = \tfrac{1}{2}QV_1 - \tfrac{1}{2}QV_0 = \tfrac{1}{2}Q(V_1 - V_0) = \tfrac{1}{2}(4.248\times10^{-11}\,\text{C})(6.0\,\text{V}) = \boxed{1.3\times10^{-10}\,\text{J}}$$

89. The first capacitor is charged, and so has a certain amount of charge on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, and so the final charge is equal to the initial charge. Initially treat capacitors C_2 and C_3 as their equivalent capacitance, $C_{23} = \frac{C_2C_3}{C_2 + C_3} = \frac{(2.0\mu\text{F})(2.4\mu\text{F})}{4.4\mu\text{F}} = 1.091\mu\text{F}$. The final voltage across C_1 and C_{23} must be the same. The charge on C_2 and C_3 must be the same. Use Eq. 24-1.

$$Q_0 = C_1V_0 = Q_1 + Q_{23} = C_1V_1 + C_{23}V_{23} = C_1V_1 + C_{23}V_1 \quad \rightarrow$$

$$V_1 = \frac{C_1}{C_1 + C_{23}}V_0 = \frac{1.0\mu\text{F}}{1.0\mu\text{F} + 1.091\mu\text{F}}\left(24\,\text{V}\right) = 11.48\,\text{V} = V_1 = V_{23}$$

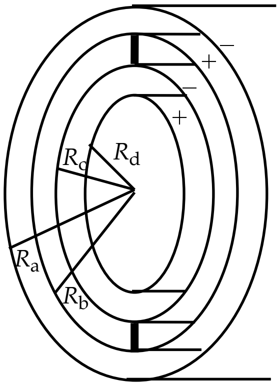
$$Q_1 = C_1V_1 = \left(1.0\mu\text{F}\right)\left(11.48\,\text{V}\right) = 11.48\mu\text{C}$$

$$Q_{23} = C_{23}V_{23} = \left(1.091\mu\text{F}\right)\left(11.48\,\text{V}\right) = 12.52\mu\text{C} = Q_2 = Q_3$$

$$V_2 = \frac{Q_2}{C_2} = \frac{12.52\mu\text{C}}{2.0\mu\text{F}} = 6.26\,\text{V} \;;\; V_3 = \frac{Q_3}{C_3} = \frac{12.52\mu\text{C}}{2.4\mu\text{F}} = 5.22\,\text{V}$$

To summarize: $\boxed{Q_1 = 11\mu\text{C}\,,\, V_1 = 11\,\text{V} \;;\; Q_2 = 13\mu\text{C}\,,\, V_2 = 6.3\,\text{V} \;;\; Q_3 = 13\mu\text{C}\,,\, V_3 = 5.2\,\text{V}}$

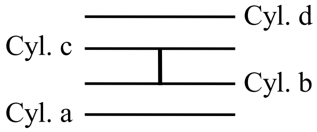
90. The metal conducting strips connecting cylinders b and c mean that b and c are at the same potential. Due to the positive charge on the inner cylinder and the negative charge on the outer cylinder, cylinders b and c will polarize according to the first diagram, with negative charge on cylinder c, and positive charge on cylinder b. This is then two capacitors in series, as illustrated in the second diagram. The capacitance per unit length of a cylindrical capacitor is derived in Example 24-2.



$$C_1 = \frac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)} \;;\; C_2 = \frac{2\pi\varepsilon_0\ell}{\ln\left(R_c/R_d\right)} \;;\; \frac{1}{C_{\text{net}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow$$

$$C_{\text{net}} = \frac{C_1C_2}{C_1 + C_2} = \frac{\left[\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}\right]\left[\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_c/R_d\right)}\right]}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)} + \dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_c/R_d\right)}}$$

$$= \frac{2\pi\varepsilon_0\ell}{\ln\left(R_c/R_d\right) + \ln\left(R_a/R_b\right)} = \frac{2\pi\varepsilon_0\ell}{\ln\left(R_aR_c/R_bR_d\right)} \quad \rightarrow$$



$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0}{\ln\left(R_aR_c/R_bR_d\right)}}$$

$\boxed{91.}$ The force acting on one plate by the other plate is equal to the electric field produced by one charged plate multiplied by the charge on the second plate.

$$F = EQ = \left(\frac{Q}{2A\varepsilon_0}\right)Q = \frac{Q^2}{2A\varepsilon_0}$$

The force is attractive since the plates are oppositely charged. Since the force is constant, the work done in pulling the two plates apart by a distance *x* is just the force times distance.

$$W = Fx = \boxed{\frac{Q^2x}{2A\varepsilon_0}}$$

The change in energy stored between the plates is obtained using Eq. 24-5.

$$W = \Delta U = \frac{Q^2}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right) = \frac{Q^2}{2}\left(\frac{2x}{\varepsilon_0A} - \frac{x}{\varepsilon_0A}\right) = \boxed{\frac{Q^2x}{2\varepsilon_0A}}$$

The work done in pulling the plates apart is equal to the increase in energy between the plates.

92. Since the other values in this problem manifestly have 2 significant figures, we assume that the capacitance also has 2 significant figures.

(*a*) The number of electrons is found from the charge on the capacitor.

$$Q = CV = Ne \ \rightarrow \ N = \frac{CV}{e} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{1.60\times10^{-19}\,\text{C}} = \boxed{2.8\times10^{5}\,e\text{'s}}$$

(*b*) The thickness is determined from the dielectric strength.

$$E_{\text{max}} = \frac{V}{d_{\text{min}}} \ \rightarrow \ d_{\text{min}} = \frac{V}{E_{\text{max}}} = \frac{1.5\,\text{V}}{1.0\times10^{9}\,\text{V/m}} = \boxed{1.5\times10^{-9}\,\text{m}}$$

(*c*) The area is found from Eq. 24-8.

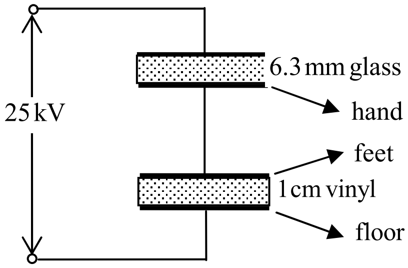
$$C = K\varepsilon_0\frac{A}{d} \ \rightarrow \ A = \frac{Cd}{K\varepsilon_0} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\times10^{-9}\,\text{m}\right)}{25\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.0\times10^{-13}\,\text{m}}$$

93. Use Eq. 24-2 for the capacitance.

$$C = \frac{\varepsilon_0 A}{d} \ \rightarrow \ d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)}{\left(1\,\text{F}\right)} = \boxed{9\times10^{-16}\,\text{m}}$$

$\boxed{\text{No}}$, this is not practically achievable. The gap would have to be smaller than the radius of a proton.

94. See the schematic diagram for the arrangement. The two "capacitors" are in series, and so have the same charge. Thus their voltages, which must total 25kV, will be inversely proportional to their capacitances. Let C_1 be the glass-filled capacitor, and C_2 be the vinyl capacitor. The area of the foot is approximately twice the area of the hand, and since there are two feet on the floor and only one hand on the screen, the area ratio is $\frac{A_{\text{foot}}}{A_{\text{hand}}} = \frac{4}{1}$.



$$Q = C_1V_1 = C_2V_2 \ \rightarrow \ V_1 = V_2\frac{C_2}{C_1}$$

$$C_1 = \frac{\varepsilon_0 K_{\text{glass}} A_{\text{hand}}}{d_{\text{glass}}} \ ; \ C_2 = \frac{\varepsilon_0 K_{\text{vinyl}} A_{\text{foot}}}{d_{\text{vinyl}}}$$

$$\frac{C_2}{C_1} = \frac{\dfrac{\varepsilon_0 K_{\text{vinyl}} A_{\text{foot}}}{d_{\text{vinyl}}}}{\dfrac{\varepsilon_0 K_{\text{glass}} A_{\text{hand}}}{d_{\text{glass}}}} = \frac{K_{\text{vinyl}} A_{\text{foot}} d_{\text{glass}}}{K_{\text{glass}} A_{\text{hand}} d_{\text{vinyl}}} = \frac{(3)(4)(0.63)}{(5)(1)(1.0)} = 1.5$$

$$V = V_1 + V_2 = V_2\frac{C_2}{C_1} + V_2 = 2.5V_2 = 25{,}000\,\text{V} \ \rightarrow \ V_2 = \boxed{10{,}000\,\text{V}}$$

95. (*a*) Use Eq. 24-2 to calculate the capacitance.

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.0\,\text{m}^2\right)}{\left(3.0\times10^{-3}\,\text{m}\right)} = \boxed{5.9\times10^{-9}\,\text{F}}$$

Use Eq. 24-1 to calculate the charge.

$$Q_0 = C_0V_0 = \left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 2.655\times10^{-7}\,\text{C} \approx \boxed{2.7\times10^{-7}\,\text{C}}$$

The electric field is the potential difference divided by the plate separation.

$$E_0 = \frac{V_0}{d} = \frac{45\,\text{V}}{3.0\times10^{-3}\,\text{m}} = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U_0 = \tfrac{1}{2}C_0V_0^2 = \tfrac{1}{2}\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{6.0\times10^{-6}\,\text{J}}$$

(*b*) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$C = KC_0 = 3.2\left(5.9\times10^{-9}\,\text{F}\right) = 1.888\times10^{-8}\,\text{F} \approx \boxed{1.9\times10^{-8}\,\text{F}}$$

The voltage doesn't change. Use Eq. 24-1 to calculate the charge.

$$Q = CV = KC_0V = 3.2\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 8.496\times10^{-7}\,\text{C} \approx \boxed{8.5\times10^{-7}\,\text{C}}$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn't change.

$$E = E_0 = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}KC_0V^2 = \tfrac{1}{2}\left(3.2\right)\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{1.9\times10^{-5}\,\text{J}}$$

96. (*a*) For a plane conducting surface, the electric field is given by Eq. 22-5.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad\rightarrow\quad Q_{\text{max}} = E_{\text{S}}\varepsilon_0 A = \left(3\times10^{6}\,\text{N/C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(150\times10^{-4}\,\text{m}^2\right)$$

$$= 3.98\times10^{-7}\,\text{C} \approx \boxed{4\times10^{-7}\,\text{C}}$$

(*b*) The capacitance of an isolated sphere is derived in the text, right after Example 24-3.

$$C = 4\pi\varepsilon_0 r = 4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1\,\text{m}\right) = 1.11\times10^{-10}\,\text{F} \approx \boxed{1\times10^{-10}\,\text{F}}$$

(*c*) Use Eq. 24-1, with the maximum charge from part (a) and the capacitance from part (b).

$$Q = CV \quad\rightarrow\quad V = \frac{Q}{C} = \frac{3.98\times10^{-7}\,\text{C}}{1.11\times10^{-10}\,\text{F}} = 3586\,\text{V} \approx \boxed{4000\,\text{V}}$$

$\boxed{97.}$ (*a*) The initial capacitance is obtained directly from Eq. 24-8.

$$C_0 = \frac{K\varepsilon_0 A}{d} = \frac{3.7\left(8.85\ \text{pF/m}\right)\left(0.21\,\text{m}\right)\left(0.14\,\text{m}\right)}{0.030\times10^{-3}\,\text{m}} = \boxed{32\,\text{nF}}$$

(*b*) Maximum charge will occur when the electric field between the plates is equal to the dielectric strength. The charge will be equal to the capacitance multiplied by the maximum voltage, where the maximum voltage is the electric field times the separation distance of the plates.

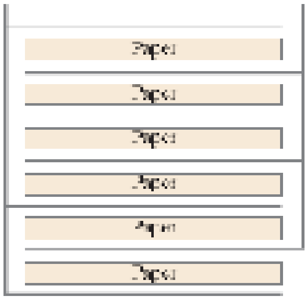
$$Q_{\text{max}} = C_0V = C_0Ed = \left(32\,\text{nF}\right)\left(15\times10^{6}\ \text{V/m}\right)\left(0.030\times10^{-3}\ \text{m}\right)$$

$$= \boxed{14\,\mu\text{C}}$$

(*c*) The sheets of foil would be separated by sheets of paper with alternating sheets connected together on each side. This capacitor would consist of 100 sheets of paper with 101 sheets of foil.

$$t = 101d_{\text{Al}} + 100d_{\text{paper}} = 101\left(0.040\ \text{mm}\right) + 100\left(0.030\ \text{mm}\right)$$

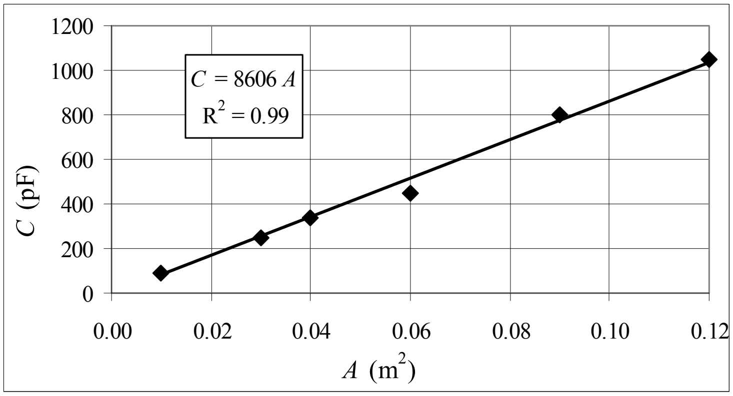
$$= \boxed{7.0\ \text{mm}}$$



(*d*) Since the capacitors are in parallel, each capacitor has the same voltage which is equal to the total voltage. Therefore breakdown will occur when the voltage across a single capacitor provides an electric field across that capacitor equal to the dielectric strength.

$$V_{\text{max}} = E_{\text{max}} d = \left(15\times10^{6}\,\text{V/m}\right)\left(0.030\times10^{-3}\,\text{m}\right) = \boxed{450\,\text{V}}$$

98. From Eq. 24-2, $C = \frac{\varepsilon_0}{d}A$. So if we plot C vs. A, we should get a straight line with a slope of $\frac{\varepsilon_0}{d}$.



$$\frac{\varepsilon_0}{d} = \text{slope} \rightarrow$$

$$d = \frac{\varepsilon_0}{\text{slope}} = \frac{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}{8606\times10^{-12}\,\text{F/m}^2} = 1.03\times10^{-3}\,\text{m} \approx \boxed{1.0\,\text{mm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH24.XLS," on tab "Problem 24.98."

CHAPTER 25: Electric Currents and Resistance

Responses to Questions

$\boxed{1.}$ A battery rating in ampere-hours gives the total amount of charge available in the battery.

2. The chemical reactions within the cell cause electrons to pile up on the negative electrode. If the terminals of the battery are connected in a circuit, then electrons flow from the negative terminal because it has an excess of electrons. Once the electrons return to the cell, the electrolyte again causes them to move to the negative terminal.

3. When a flashlight is operated, the battery energy is being used up.

4. The terminal of the car battery connected to "ground" is actually connected to the metal frame of the car. This provides a large "sink" or "source" for charge. The metal frame serves as the common ground for all electrical devices in the car, and all voltages are measured with respect to the car's frame.

5. Generally, water is already in the faucet spout, but it will not come out until the faucet valve is opened. Opening the valve provides the pressure difference needed to force water out of the spout. The same thing is essentially true when you connect a wire to the terminals of a battery. Electrons already exist in the wires. The battery provides the potential that causes them to move, producing a current.

6. Yes. They might have the same resistance if the aluminum wire is thicker. If the lengths of the wires are the same, then the ratios of resistivity to cross-sectional area must also be the same for the resistances to be the same. Aluminum has a higher resistivity than copper, so if the cross-sectional area of the aluminum is also larger by the same proportion, the two wires will have the same resistance.

$\boxed{7.}$ If the emf in a circuit remains constant and the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease. Both power equations support this result. If the current in a circuit remains constant and the resistance is increased, then the emf must increase and the power dissipated in the circuit will increase. Both equations also support this result. There is no contradiction, because the voltage, current, and resistance are related to each other by $V = IR$.

8. When a lightbulb burns out, the filament breaks, creating a gap in the circuit so that no current flows.

9. If the resistance of a small immersion heater were increased, it would slow down the heating process. The emf in the circuit made up of the heater and the wires that connect it to the wall socket is maintained at a constant rms value. If the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease, slowing the heating process.

10. Resistance is proportional to length and inversely proportional to cross-sectional area.
    (*a*) For the least resistance, you want to connect the wires to maximize area and minimize length. Therefore, connect them opposite to each other on the faces that are $2a$ by $3a$.
    (*b*) For the greatest resistance, you want to minimize area and maximize length. Therefore, connect the wires to the faces that are $1a$ by $2a$.

11. When a light is turned on, the filament is cool, and has a lower resistance than when it is hot. The current through the filament will be larger, due to the lower resistance. This momentary high current will heat the wire rapidly, possibly causing the filament to break due to thermal stress or vaporize. After the light has been on for some time, the filament is at a constant high temperature, with a higher resistance and a lower current. Since the temperature is constant, there is less thermal stress on the filament than when the light is first turned on.

12. When connected to the same potential difference, the 100-W bulb will draw more current ($P = IV$). The 75-W bulb has the higher resistance ($V = IR$ or $P = V^2/R$).

$\boxed{13.}$ The electric power transferred by the lines is $P = IV$. If the voltage across the transmission lines is large, then the current in the lines will be small. The power lost in the transmission lines is $P = I^2R$. The power dissipated in the lines will be small, because I is small.

14. If the circuit has a 15-A fuse, then it is rated to carry current of no more than 15 A. Replacing the 15-A fuse with a 25-A fuse will allow the current to increase to a level that is dangerously high for the wiring, which might result in overheating and possibly a fire.

15. The human eye and brain cannot distinguish the on-off cycle of lights when they are operated at the normal 60 Hz frequency. At much lower frequencies, such as 5 Hz, the eye and brain are able to process the on-off cycle of the lights, and they will appear to flicker.

16. The electrons are not “used up” as they pass through the lamp. Their energy is dissipated as light and heat, but with each cycle of the alternating voltage, their potential energy is raised again. As long as the electrons keep moving (converting potential energy into kinetic energy, light, and heat) the lamp will stay lit.

17. Immediately after the toaster is turned on, the Nichrome wire heats up and its resistance increases. Since the (rms) potential across the element remains constant, the current in the heating element must decrease.

18. No. Energy is dissipated in a resistor but current, the rate of flow of charge, is not “used up.”

$\boxed{19.}$ In the two wires described, the drift velocities of the electrons will be about the same, but the current density, and therefore the current, in the wire with twice as many free electrons per atom will be twice as large as in the other wire.

20. (*a*) If the length of the wire doubles, its resistance also doubles, and so the current in the wire will be reduced by a factor of two. Drift velocity is proportional to current, so the drift velocity will be halved.
    (*b*) If the wire’s radius is doubled, the drift velocity remains the same. (Although, since there are more charge carriers, the current will quadruple.)
    (*c*) If the potential difference doubles while the resistance remains constant, the drift velocity and current will also double.

21. If you turn on an electric appliance when you are outside with bare feet, and the appliance shorts out through you, the current has a direct path to ground through your feet, and you will receive a severe shock. If you are inside wearing socks and shoes with thick soles, and the appliance shorts out, the current will not have an easy path to ground through you, and will most likely find an alternate path. You might receive a mild shock, but not a severe one.

Solutions to Problems

$\boxed{1.}$ Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}\ \rightarrow\ 1.30\,\mathrm{A}=\frac{1.30\,\mathrm{C}}{\mathrm{s}}\times\frac{1\text{ electron}}{1.60\times10^{-19}\mathrm{C}}=\boxed{8.13\times10^{18}\text{ electrons/s}}$$

2. Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}\ \rightarrow\ \Delta Q=I\Delta t=(6.7\,\mathrm{A})(5.0\,\mathrm{h})(3600\,\mathrm{s/h})=\boxed{1.2\times10^{5}\,\mathrm{C}}$$

3. Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}=\frac{(1200\text{ ions})(1.60\times10^{-19}\text{ C/ion})}{3.5\times10^{-6}\mathrm{s}}=\boxed{5.5\times10^{-11}\,\mathrm{A}}$$

4. Solve Eq. 25-2a for resistance.

$$R=\frac{V}{I}=\frac{120\,\mathrm{V}}{4.2\,\mathrm{A}}=\boxed{29\,\Omega}$$

5. (*a*) Use Eq. 25-2b to find the current.

$$V=IR\ \rightarrow\ I=\frac{V}{R}=\frac{240\,\mathrm{V}}{8.6\,\Omega}=27.91\,\mathrm{A}\approx\boxed{28\,\mathrm{A}}$$

(*b*) Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}\ \rightarrow\ \Delta Q=I\Delta t=(27.91\,\mathrm{A})(50\,\mathrm{min})(60\,\mathrm{s/min})=\boxed{8.4\times10^{4}\,\mathrm{C}}$$

6. (*a*) Solve Eq. 25-2a for resistance.

$$R=\frac{V}{I}=\frac{120\,\mathrm{V}}{9.5\,\mathrm{A}}=12.63\,\Omega\approx\boxed{13\,\Omega}$$

(*b*) Use the definition of average current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}\ \rightarrow\ \Delta Q=I\Delta t=(9.5\,\mathrm{A})(15\,\mathrm{min})(60\,\mathrm{s/min})=\boxed{8600\,\mathrm{C}}$$

$\boxed{7.}$ Use Ohm's Law, Eq. 25-2a, to find the current. Then use the definition of current, Eq. 25-1a, to calculate the number of electrons per minute.

$$I=\frac{V}{R}=\frac{\Delta Q}{\Delta t}=\frac{4.5\,\mathrm{V}}{1.6\,\Omega}=\frac{2.8\,\mathrm{C}}{\mathrm{s}}\times\frac{1\text{ electron}}{1.60\times10^{-19}\mathrm{C}}\times\frac{60\,\mathrm{s}}{1\,\mathrm{min}}=\boxed{1.1\times10^{21}\,\frac{\text{electrons}}{\text{minute}}}$$

8. Find the potential difference from the resistance and the current.

$$R=(2.5\times10^{-5}\,\Omega/\mathrm{m})(4.0\times10^{-2}\mathrm{m})=1.0\times10^{-6}\Omega$$

$$V=IR=(3100\,\mathrm{A})(1.0\times10^{-6}\Omega)=\boxed{3.1\times10^{-3}\mathrm{V}}$$

9. (*a*) Use Eq. 25-2b to find the resistance.

$$R = \frac{V}{I} = \frac{12\,\text{V}}{0.60\,\text{A}} = \boxed{20\,\Omega}\ \text{(2 sig. fig.)}$$

(*b*) An amount of charge ΔQ loses a potential energy of $(\Delta Q)V$ as it passes through the resistor. The amount of charge is found from Eq. 25-1a.

$$\Delta U = (\Delta Q)V = (I\Delta t)V = (0.60\,\text{A})(60\,\text{s})(12\,\text{V}) = \boxed{430\,\text{J}}$$

10. (*a*) If the voltage drops by 15%, and the resistance stays the same, then by Eq. 25-2b, $V = IR$, the current will also drop by 15%.

$$I_{\text{final}} = 0.85 I_{\text{initial}} = 0.85(6.50\,\text{A}) = 5.525\,\text{A} \approx \boxed{5.5\,\text{A}}$$

(*b*) If the resistance drops by 15% (the same as being multiplied by 0.85), and the voltage stays the same, then by Eq. 25-2b, the current must be divided by 0.85.

$$I_{\text{final}} = \frac{I_{\text{initial}}}{0.85} = \frac{6.50\,\text{A}}{0.85} = 7.647\,\text{A} \approx \boxed{7.6\,\text{A}}$$

11. Use Eq. 25-3 to find the diameter, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} \rightarrow d = \sqrt{\frac{4\ell\rho}{\pi R}} = \sqrt{\frac{4(1.00\,\text{m})(5.6\times10^{-8}\,\Omega\text{•m})}{\pi(0.32\,\Omega)}} = \boxed{4.7\times10^{-4}\,\text{m}}$$

12. Use Eq. 25-3 to calculate the resistance, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = (1.68\times10^{-8}\,\Omega\text{•m})\frac{4(4.5\,\text{m})}{\pi(1.5\times10^{-3}\,\text{m})^2} = \boxed{4.3\times10^{-2}\,\Omega}$$

$\boxed{13.}$ Use Eq. 25-3 to calculate the resistances, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2}.$$

$$\frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{\rho_{\text{Al}}\dfrac{4\ell_{\text{Al}}}{\pi d_{\text{Al}}^2}}{\rho_{\text{Cu}}\dfrac{4\ell_{\text{Cu}}}{\pi d_{\text{Cu}}^2}} = \frac{\rho_{\text{Al}}\ell_{\text{Al}}d_{\text{Cu}}^2}{\rho_{\text{Cu}}\ell_{\text{Cu}}d_{\text{Al}}^2} = \frac{(2.65\times10^{-8}\,\Omega\text{•m})(10.0\,\text{m})(1.8\,\text{mm})^2}{(1.68\times10^{-8}\,\Omega\text{•m})(20.0\,\text{m})(2.0\,\text{mm})^2} = \boxed{0.64}$$

14. Use Eq. 25-3 to express the resistances, with the area as $A = \pi r^2 = \pi d^2/4$, and so $R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2}$.

$$R_{\text{W}} = R_{\text{Cu}} \rightarrow \rho_{\text{W}}\frac{4\ell}{\pi d_{\text{W}}^2} = \rho_{\text{Cu}}\frac{4\ell}{\pi d_{\text{Cu}}^2} \rightarrow$$

$$d_{\text{W}} = d_{\text{Cu}}\sqrt{\frac{\rho_{\text{W}}}{\rho_{\text{Cu}}}} = (2.2\,\text{mm})\sqrt{\frac{5.6\times10^{-8}\,\Omega\text{•m}}{1.68\times10^{-8}\,\Omega\text{•m}}} = \boxed{4.0\,\text{mm}}$$

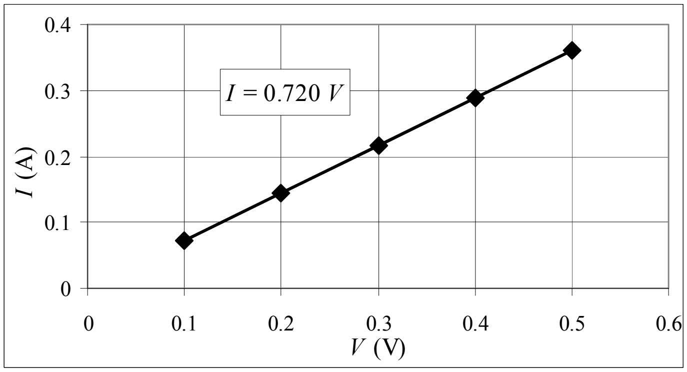
The diameter of the tungsten should be 4.0 mm.

15. (*a*) If the wire obeys Ohm's law, then $V = IR$ or $I = \frac{1}{R}V$, showing a linear relationship between I and V. A graph of I vs. V should give a straight line with a slope of $\frac{1}{R}$ and a y-intercept of 0.

(*b*) From the graph and the calculated linear fit, we see that the wire obeys Ohm's law.

$$\text{slope} = \frac{1}{R} \rightarrow$$

$$R = \frac{1}{0.720} \text{A/V} = \boxed{1.39\,\Omega}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.15b."

(*c*) Use Eq. 25-3 to find the resistivity.

$$R = \rho\frac{\ell}{A} \rightarrow \rho = \frac{AR}{\ell} = \frac{\pi d^2 R}{4\ell} = \frac{\pi\left(3.2\times10^{-4}\text{m}\right)^2\left(1.39\,\Omega\right)}{4\left(0.11\text{m}\right)} = \boxed{1.0\times10^{-6}\,\Omega\text{•m}}$$

From Table 25-1, the material is $\boxed{\text{nichrome}}$.

16. Use Eq. 25-5 multiplied by ℓ/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] = 1.15R_0 \rightarrow 1+\alpha\left(T-T_0\right) = 1.15 \rightarrow$$

$$T - T_0 = \frac{0.15}{\alpha} = \frac{0.15}{.0068\left(\text{C}°\right)^{-1}} = \boxed{22\,\text{C}°}$$

So raise the temperature by $22\,\text{C}°$ to a final temperature of $42°\text{C}$.

17. Since the resistance is directly proportional to the length, the length of the long piece must be 4.0 times the length of the short piece.

$$\ell = \ell_{\text{short}} + \ell_{\text{long}} = \ell_{\text{short}} + 4.0\ell_{\text{short}} = 5.0\ell_{\text{short}} \rightarrow \ell_{\text{short}} = 0.20\ell,\ \ell_{\text{long}} = 0.80\ell$$

Make the cut at $\boxed{\text{20\% of the length of the wire}}$.

$$\ell_{\text{short}} = 0.20\ell,\ \ell_{\text{long}} = 0.80\ell \rightarrow R_{\text{short}} = 0.2R = \boxed{2.0\,\Omega},\ R_{\text{long}} = 0.8R = \boxed{8.0\,\Omega}$$

18. Use Eq. 25-5 for the resistivity.

$$\rho_{\text{T Al}} = \rho_{0\text{ Al}}\left[1+\alpha_{\text{Al}}\left(T-T_0\right)\right] = \rho_{0\text{ W}} \rightarrow$$

$$T = T_0 + \frac{1}{\alpha_{\text{Al}}}\left(\frac{\rho_{0\text{ W}}}{\rho_{0\text{ Al}}} - 1\right) = 20°\text{C} + \frac{1}{0.00429\left(\text{C}°\right)^{-1}}\left(\frac{5.6\times10^{-8}\,\Omega\text{•m}}{2.65\times10^{-8}\,\Omega\text{•m}} - 1\right) = 279.49°\text{C} \approx \boxed{280°\text{C}}$$

$\boxed{19.}$ Use Eq. 25-5 multiplied by ℓ/A so that it expresses resistances instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \rightarrow$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \frac{1}{0.0045\left(\text{C}°\right)^{-1}}\left(\frac{140\,\Omega}{12\,\Omega} - 1\right) = 2390°\text{C} \approx \boxed{2400°\text{C}}$$

20. Calculate the voltage drop by combining Ohm's Law (Eq. 25-2b) with the expression for resistance, Eq. 25-3.

$$V = IR = I\frac{\rho\ell}{A} = I\frac{4\rho\ell}{\pi d^2} = (12\,\text{A})\frac{4\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)(26\,\text{m})}{\pi\left(1.628\times10^{-3}\,\text{m}\right)^2} = \boxed{2.5\,\text{V}}$$

21. The wires have the same resistance and the same resistivity.

$$R_{\text{long}} = R_{\text{short}} \rightarrow \frac{\rho\ell_{\text{long}}}{A_1} = \frac{\rho\ell_{\text{short}}}{A_2} \rightarrow \frac{(4)2\ell_{\text{short}}}{\pi d_{\text{long}}^2} = \frac{4\ell_{\text{short}}}{\pi d_{\text{short}}^2} \rightarrow \boxed{\frac{d_{\text{long}}}{d_{\text{short}}} = \sqrt{2}}$$

22. In each case calculate the resistance by using Eq. 25-3 for resistance.

(*a*) $$R_x = \frac{\rho\ell_x}{A_{yz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(1.0\times10^{-2}\,\text{m}\right)}{\left(2.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = 3.75\times10^{-4}\,\Omega \approx \boxed{3.8\times10^{-4}\,\Omega}$$

(*b*) $$R_y = \frac{\rho\ell_y}{A_{xz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = \boxed{1.5\times10^{-3}\,\Omega}$$

(*c*) $$R_z = \frac{\rho\ell_z}{A_{xy}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)} = \boxed{6.0\times10^{-3}\,\Omega}$$

23. The original resistance is $R_0 = V/I_0$, and the high temperature resistance is $R = V/I$, where the two voltages are the same. The two resistances are related by Eq. 25-5, multiplied by ℓ/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha(T-T_0)\right] \rightarrow T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{V/I}{V/I_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{I_0}{I}-1\right)$$

$$= 20.0°\text{C} + \frac{1}{0.00429\left(\text{C}°\right)^{-1}}\left(\frac{0.4212\,\text{A}}{0.3818\,\text{A}}-1\right) = \boxed{44.1°\text{C}}$$

24. For the cylindrical wire, its (constant) volume is given by $V = \ell_0 A_0 = \ell A$, and so $A = \frac{V}{\ell}$. Combine this relationship with Eq. 25-3. We assume that $\Delta\ell \ll \ell_0$.

$$R_0 = \rho\frac{\ell_0}{A_0} = \rho\frac{\ell_0^2}{V_0} \;;\; R = \rho\frac{\ell}{A} = \rho\frac{\ell^2}{V_0} \;;\; \frac{dR}{d\ell} = 2\rho\frac{\ell}{V_0}$$

$$\Delta R \approx \frac{dR}{d\ell}\Delta\ell = 2\rho\frac{\ell}{V_0}\Delta\ell \rightarrow \Delta\ell = \frac{V_0\Delta R}{2\rho\ell} \rightarrow \frac{\Delta\ell}{\ell} = \frac{V_0\Delta R}{2\rho\ell^2} = \frac{\Delta R}{2\frac{\rho\ell^2}{V_0}} = \frac{1}{2}\frac{\Delta R}{R}$$

This is true for any initial conditions, and so $\boxed{\frac{\Delta\ell}{\ell_0} = \frac{1}{2}\frac{\Delta R}{R_0}}$

$\boxed{25.}$ The resistance depends on the length and area as $R=\rho\ell/A$. Cutting the wire and running the wires side by side will halve the length and double the area.

$$R_2=\frac{\rho\left(\frac{1}{2}\ell\right)}{2A}=\frac{1}{4}\frac{\rho\ell}{A}=\boxed{\frac{1}{4}R_1}$$

26. The total resistance is to be 3700 ohms $\left(R_{\text{total}}\right)$ at all temperatures. Write each resistance in terms of Eq.25-5 (with $T_0=0^{\circ}\text{C}$), multiplied by ℓ/A to express resistance instead of resistivity.

$$R_{\text{total}}=R_{0\text{C}}\left[1+\alpha_{\text{C}}T\right]+R_{0\text{N}}\left[1+\alpha_{\text{N}}T\right]=R_{0\text{C}}+R_{0\text{C}}\alpha_{\text{C}}T+R_{0\text{N}}+R_{0\text{N}}\alpha_{\text{N}}T$$
$$=R_{0\text{C}}+R_{0\text{N}}+\left(R_{0\text{C}}\alpha_{\text{C}}+R_{0\text{N}}\alpha_{\text{N}}\right)T$$

For the above to be true, the terms with a temperature dependence must cancel, and the terms without a temperature dependence must add to R_{total}. Thus we have two equations in two unknowns.

$$0=\left(R_{0\text{C}}\alpha_{\text{C}}+R_{0\text{N}}\alpha_{\text{N}}\right)T\ \rightarrow\ R_{0\text{N}}=-\frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}}$$

$$R_{\text{total}}=R_{0\text{C}}+R_{0\text{N}}=R_{0\text{C}}-\frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}}=\frac{R_{0\text{C}}\left(\alpha_{\text{N}}-\alpha_{\text{C}}\right)}{\alpha_{\text{N}}}\ \rightarrow$$

$$R_{0\text{C}}=R_{\text{total}}\frac{\alpha_{\text{N}}}{\left(\alpha_{\text{N}}-\alpha_{\text{C}}\right)}=\left(3700\,\Omega\right)\frac{0.0004\left(\text{C}^{\circ}\right)^{-1}}{0.0004\left(\text{C}^{\circ}\right)^{-1}+0.0005\left(\text{C}^{\circ}\right)^{-1}}=1644\Omega\approx\boxed{1600\,\Omega}$$

$$R_{0\text{N}}=R_{\text{total}}-R_{0\text{C}}=3700\,\Omega-1644\,\Omega=2056\,\Omega\approx\boxed{2100\,\Omega}$$

27. We choose a spherical shell of radius *r* and thickness *dr* as a differential element. The area of this element is $4\pi r^2$. Use Eq. 25-3, but for an infinitesimal resistance. Then integrate over the radius of the sphere.

$$R=\rho\frac{\ell}{A}\ \rightarrow\ dR=\rho\frac{d\ell}{A}=\frac{dr}{4\pi\sigma r^2}\ \rightarrow\ R=\int dR=\int_{r_1}^{r_2}\frac{dr}{4\pi\sigma r^2}=\frac{1}{4\pi\sigma}\left(-\frac{1}{r}\right)_{r_1}^{r_2}=\boxed{\frac{1}{4\pi\sigma}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

28. (*a*) Let the values at the lower temperature be indicated by a subscript "0". Thus $R_0=\rho_0\frac{\ell_0}{A_0}$ $=\rho_0\frac{4\ell_0}{\pi d_0^2}$. The change in temperature results in new values for the resistivity, the length, and the diameter. Let α represent the temperature coefficient for the resistivity, and α_{T} represent the thermal coefficient of expansion, which will affect the length and diameter.

$$R=\rho\frac{\ell}{A}=\rho\frac{4\ell}{\pi d^2}=\rho_0\left[1+\alpha\left(T-T_0\right)\right]\frac{4\ell_0\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]}{\pi\left\{d_0\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]\right\}^2}=\rho_0\frac{4\ell_0}{\pi d_0^2}\frac{\left[1+\alpha\left(T-T_0\right)\right]}{\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]}$$
$$=R_0\frac{\left[1+\alpha\left(T-T_0\right)\right]}{\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]}\ \rightarrow\ R\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]=R_0\left[1+\alpha\left(T-T_0\right)\right]\ \rightarrow$$

$$T = T_0 + \frac{(R - R_0)}{(R_0\alpha - R\alpha_T)} = 20°\mathrm{C} + \frac{(140\,\Omega - 12\,\Omega)}{\left[(12\,\Omega)\left(0.0045\,\mathrm{C}^{\circ -1}\right) - (140\,\Omega)\left(5.5\times10^{-6}\,\mathrm{C}^{\circ -1}\right)\right]}$$

$$= 20°\mathrm{C} + 2405°\mathrm{C} = 2425°\mathrm{C} \approx \boxed{2400°\mathrm{C}}$$

(*b*) The net effect of thermal expansion is that both the length and diameter increase, which lowers the resistance.

$$\frac{R}{R_0} = \frac{\rho_0 \dfrac{4\ell}{\pi d^2}}{\rho_0 \dfrac{4\ell_0}{\pi d_0^2}} = \frac{\ell d_0^2}{\ell_0 d^2} = \frac{\ell_0\left[1 + \alpha_T (T - T_0)\right]}{\ell_0}\frac{d_0^2}{\left\{d_0\left[1 + \alpha_T (T - T_0)\right]\right\}^2} = \frac{1}{\left[1 + \alpha_T (T - T_0)\right]}$$

$$= \frac{1}{\left[1 + \left(5.5\times10^{-6}\,\mathrm{C}^{\circ -1}\right)(2405°\mathrm{C})\right]} = 0.9869$$

$$\%\,\text{change} = \left(\frac{R - R_0}{R_0}\right)100 = \left(\frac{R}{R_0} - 1\right)100 = -1.31 \approx \boxed{-1.3\%}$$

The net effect of resistivity change is that the resistance increases.

$$\frac{R}{R_0} = \frac{\rho \dfrac{4\ell_0}{\pi d_0^2}}{\rho_0 \dfrac{4\ell_0}{\pi d_0^2}} = \frac{\rho}{\rho_0} = \frac{\rho_0\left[1 + \alpha (T - T_0)\right]}{\rho_0} = \left[1 + \alpha (T - T_0)\right] = \left[1 + \left(0.0045\,\mathrm{C}^{\circ -1}\right)(2405°\mathrm{C})\right]$$

$$= 11.82$$

$$\%\,\text{change} = \left(\frac{R - R_0}{R_0}\right)100 = \left(\frac{R}{R_0} - 1\right)100 = 1082 \approx \boxed{1100\%}$$

29. (*a*) Calculate each resistance separately using Eq. 25-3, and then add the resistances together to find the total resistance.

$$R_{\mathrm{Cu}} = \frac{\rho_{\mathrm{Cu}}\ell}{A} = \frac{4\rho_{\mathrm{Cu}}\ell}{\pi d^2} = \frac{4\left(1.68\times10^{-8}\,\Omega\bullet\mathrm{m}\right)(5.0\,\mathrm{m})}{\pi\left(1.4\times10^{-3}\,\mathrm{m}\right)^2} = 0.054567\,\Omega$$

$$R_{\mathrm{Al}} = \frac{\rho_{\mathrm{Al}}\ell}{A} = \frac{4\rho_{\mathrm{Al}}\ell}{\pi d^2} = \frac{4\left(2.65\times10^{-8}\,\Omega\bullet\mathrm{m}\right)(5.0\,\mathrm{m})}{\pi\left(1.4\times10^{-3}\,\mathrm{m}\right)^2} = 0.086074\,\Omega$$

$$R_{\mathrm{total}} = R_{\mathrm{Cu}} + R_{\mathrm{Al}} = 0.054567\,\Omega + 0.086074\,\Omega = 0.140641\,\Omega \approx \boxed{0.14\,\Omega}$$

(*b*) The current through the wire is the voltage divided by the total resistance.

$$I = \frac{V}{R_{\mathrm{total}}} = \frac{85\times10^{-3}\,\mathrm{V}}{0.140641\,\Omega} = 0.60438\,\mathrm{A} \approx \boxed{0.60\,\mathrm{A}}$$

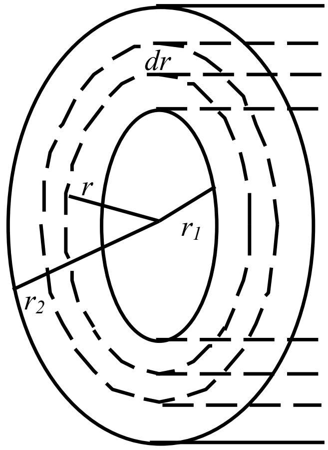
(*c*) For each segment of wire, Ohm's law is true. Both wires have the current found in (*b*) above.

$$V_{\mathrm{Cu}} = IR_{\mathrm{Cu}} = (0.60438\,\mathrm{A})(0.054567\,\Omega) \approx \boxed{0.033\,\mathrm{V}}$$

$$V_{\mathrm{Al}} = IR_{\mathrm{Al}} = (0.60438\,\mathrm{A})(0.086074\,\Omega) \approx \boxed{0.052\,\mathrm{V}}$$

Notice that the total voltage is 85 mV.

30. (*a*) Divide the cylinder up into concentric cylindrical shells of radius *r*, thickness *dr*, and length ℓ. See the diagram. The resistance of one of those shells, from Eq. 25-3, is found. Note that the "length" in Eq. 25-3 is in the direction of the current flow, so we must substitute in *dr* for the "length" in Eq. 25-3. The area is the surface area of the thin cylindrical shell. Then integrate over the range of radii to find the total resistance.



$$R = \rho \frac{\text{"}\ell\text{"}}{A} \rightarrow dR = \rho \frac{dr}{2\pi r \ell} ;$$

$$R = \int dR = \int_{r_1}^{r_2} \rho \frac{dr}{2\pi r \ell} = \boxed{\frac{\rho}{2\pi \ell} \ln \frac{r_2}{r_1}}$$

(*b*) Use the data given to calculate the resistance from the above formula.

$$R = \frac{\rho}{2\pi\ell} \ln \frac{r_2}{r_1} = \frac{15\times10^{-5}\,\Omega\text{•m}}{2\pi(0.024\,\text{m})} \ln\left(\frac{1.8\,\text{mm}}{1.0\,\text{mm}}\right) = \boxed{5.8\times10^{-4}\,\Omega}$$

(*c*) For resistance along the axis, we again use Eq. 25-3, but the current is flowing in the direction of length ℓ. The area is the cross-sectional area of the face of the hollow cylinder.

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi\left(r_2^2 - r_1^2\right)} = \frac{\left(15\times10^{-5}\,\Omega\text{•m}\right)\left(0.024\,\text{m}\right)}{\pi\left[\left(1.8\times10^{-3}\text{m}\right)^2 - \left(1.0\times10^{-3}\text{m}\right)^2\right]} = \boxed{0.51\,\Omega}$$

$\boxed{31.}$ Use Eq. 25-6 to find the power from the voltage and the current.

$$P = IV = (0.27\,\text{A})(3.0\,\text{V}) = \boxed{0.81\,\text{W}}$$

32. Use Eq. 25-7b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{3300\,\text{W}} = \boxed{17\,\Omega}$$

33. Use Eq. 25-7b to find the voltage from the power and the resistance.

$$P = \frac{V^2}{R} \rightarrow V = \sqrt{RP} = \sqrt{(3300\,\Omega)(0.25\,\text{W})} = \boxed{29\,\text{V}}$$

34. Use Eq. 25-7b to find the resistance, and Eq. 25-6 to find the current.

(*a*) $$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(110\,\text{V})^2}{75\,\text{W}} = 161.3\,\Omega \approx \boxed{160\,\Omega}$$

$$P = IV \rightarrow I = \frac{P}{V} = \frac{75\,\text{W}}{110\,\text{V}} = 0.6818\,\text{A} \approx \boxed{0.68\,\text{A}}$$

(*b*) $$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(110\,\text{V})^2}{440\,\text{W}} = 27.5\,\Omega \approx \boxed{28\,\Omega}$$

$$P = IV \rightarrow I = \frac{P}{V} = \frac{440\,\text{W}}{110\,\text{V}} = \boxed{4.0\,\text{A}}$$

35. (*a*) From Eq. 25-6, if power P is delivered to the transmission line at voltage V, there must be a current $I = P/V$. As this current is carried by the transmission line, there will be power losses of I^2R due to the resistance of the wire. This power loss can be expressed as $\Delta P = I^2R = \boxed{P^2R/V^2}$. Equivalently, there is a voltage drop across the transmission lines of $V' = IR$. Thus the voltage available to the users is $V - V'$, and so the power available to the users is $P' = (V - V')I = VI - V'I = VI - I^2R = P - I^2R$. The power loss is $\Delta P = P - P' = P - (P - I^2R) = I^2R = \boxed{P^2R/V^2}$.

(*b*) Since $\Delta P \propto \dfrac{1}{V^2}$, V should be $\boxed{\text{as large as possible}}$ to minimize ΔP.

36. (*a*) Since $P = \dfrac{V^2}{R} \rightarrow R = \dfrac{V^2}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.

(*b*) $R = \dfrac{V^2}{P} = \dfrac{(120\,\text{V})^2}{850\,\text{W}} = \boxed{17\,\Omega}$

(*c*) $R = \dfrac{V^2}{P} = \dfrac{(120\,\text{V})^2}{1250\,\text{W}} = \boxed{12\,\Omega}$

$\boxed{37.}$ (*a*) Use Eq. 25-6 to find the current.

$$P = IV \rightarrow I = \frac{P}{V} = \frac{95\,\text{W}}{115\,\text{V}} = \boxed{0.83\,\text{A}}$$

(*b*) Use Eq. 25-7b to find the resistance.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(115\,\text{V})^2}{95\,\text{W}} \approx \boxed{140\,\Omega}$$

38. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. 25-7b, $P = \dfrac{V^2}{R}$. Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120V, the power will be reduced by a factor of 4. Thus the bulb will appear only about $\boxed{\text{1/4 as bright}}$ in the United States as in Europe.

39. To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$\text{Energy} = P(\text{in kW})\,t(\text{in h}) = (550\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(6.0\,\text{min})\left(\frac{1\,\text{h}}{60\,\text{min}}\right) = \boxed{0.055\,\text{kWh}}$$

To find the cost of the energy used in a month, multiply times 4 days per week of usage, times 4 weeks per month, times the cost per kWh.

$$\text{Cost} = \left(0.055\frac{\text{kWh}}{\text{d}}\right)\left(\frac{4\,\text{d}}{1\,\text{week}}\right)\left(\frac{4\,\text{week}}{1\,\text{month}}\right)\left(\frac{9.0\,\text{cents}}{\text{kWh}}\right) = \boxed{7.9\,\text{cents/month}}$$

40. To find the cost of the energy, multiply the kilowatts of power consumption by the number of hours in operation times the cost per kWh.

$$\text{Cost} = (25\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(365\,\text{day})\left(\frac{24\,\text{h}}{1\,\text{day}}\right)\left(\frac{\$0.095}{\text{kWh}}\right) \approx \boxed{\$21}$$

41. The A•h rating is the amount of charge that the battery can deliver. The potential energy of the charge is the charge times the voltage.

$$U = QV = (75\,\text{A}\bullet\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)(12\,\text{V}) = \boxed{3.2\times10^6\,\text{J}} = 0.90\,\text{kWh}$$

42. (*a*) Calculate the resistance from Eq. 25-2b and the power from Eq. 25-6.

$$R = \frac{V}{I} = \frac{3.0\,\text{V}}{0.38\,\text{A}} = 7.895\,\Omega \approx \boxed{7.9\,\Omega} \qquad P = IV = (0.38\,\text{A})(3.0\,\text{V}) = 1.14\,\text{W} \approx \boxed{1.1\,\text{W}}$$

(*b*) If four D-cells are used, the voltage will be doubled to 6.0 V. Assuming that the resistance of the bulb stays the same (by ignoring heating effects in the filament), the power that the bulb would need to dissipate is given by Eq. 25-7b, $P = \frac{V^2}{R}$. A doubling of the voltage means the power is increased by a factor of $\boxed{4}$. This should not be tried because the bulb is probably not rated for such a high wattage. The filament in the bulb would probably burn out, and the glass bulb might even explode if the filament burns violently.

$\boxed{43.}$ Each bulb will draw an amount of current found from Eq. 25-6.

$$P = IV \quad \rightarrow \quad I_{\text{bulb}} = \frac{P}{V}$$

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

$$I_{\text{total}} = nI_{\text{bulb}} = n\frac{P}{V} \quad \rightarrow \quad n = \frac{VI_{\text{total}}}{P} = \frac{(120\,\text{V})(15\,\text{A})}{75\,\text{W}} = \boxed{24\text{ bulbs}}$$

44. Find the power dissipated in the cord by Eq. 25-7a, using Eq. 25-3 for the resistance.

$$P = I^2R = I^2\rho\frac{\ell}{A} = I^2\rho\frac{\ell}{\pi d^2/4} = I^2\rho\frac{4\ell}{\pi d^2} = (15.0\,\text{A})^2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\frac{4(5.4\,\text{m})}{\pi\left(0.129\times10^{-2}\,\text{m}\right)^2}$$

$$= 15.62\,\text{W} \approx \boxed{16\,\text{W}}$$

45. Find the current used to deliver the power in each case, and then find the power dissipated in the resistance at the given current.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} \qquad P_{\text{dissipated}} = I^2R = \frac{P^2}{V^2}R$$

$$P_{\substack{\text{dissipated}\\ 12{,}000\,\text{V}}} = \frac{\left(7.5\times10^5\,\text{W}\right)^2}{\left(1.2\times10^4\,\text{V}\right)^2}(3.0\,\Omega) = 11719\,\text{W}$$

$$P_{\substack{\text{dissipated}\\ 50{,}000\,\text{V}}} = \frac{\left(7.5\times10^5\,\text{W}\right)^2}{\left(5\times10^4\,\text{V}\right)^2}(3.0\,\Omega) = 675\,\text{W} \qquad \text{difference} = 11719\,\text{W} - 675\,\text{W} = \boxed{1.1\times10^4\,\text{W}}$$

46. (*a*) By conservation of energy and the efficiency claim, 75% of the electrical power dissipated by the heater must be the rate at which energy is absorbed by the water.

$$0.75_{\substack{\text{emitted by}\\ \text{electromagnet}}} = P_{\substack{\text{absorbed}\\ \text{by water}}} \rightarrow 0.75\left(IV\right) = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \rightarrow$$

$$I = \frac{mc\Delta T}{0.75Vt} = \frac{\left(0.120\,\text{kg}\right)\left(4186\,\text{J/kg}\right)\left(95^\circ\text{C} - 25^\circ\text{C}\right)}{\left(0.75\right)\left(12\,\text{V}\right)\left(480\,\text{s}\right)} = 8.139\,\text{A} \approx \boxed{8.1\,\text{A}}$$

(*b*) Use Ohm's law to find the resistance of the heater.

$$V = IR \rightarrow R = \frac{V}{I} = \frac{12\,\text{V}}{8.139\,\text{A}} = \boxed{1.5\,\Omega}$$

47. The water temperature rises by absorbing the heat energy that the electromagnet dissipates. Express both energies in terms of power, which is energy per unit time.

$$P_{\text{electric}} = P_{\substack{\text{to heat}\\ \text{water}}} \rightarrow IV = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \rightarrow$$

$$\frac{m}{t} = \frac{IV}{c\Delta T} = \frac{\left(17.5\,\text{A}\right)\left(240\,\text{V}\right)}{\left(4186\,\text{J/kg}\bullet{}^\circ\text{C}\right)\left(6.50\,\text{C}^\circ\right)} = 0.154\ \text{kg/s} \approx \boxed{0.15\,\text{kg/s}}$$

This is $154\,\text{mL/s}$.

48. For the wire to stay a constant temperature, the power generated in the resistor is to be dissipated by radiation. Use Eq. 25-7a and 19-18, both expressions of power (energy per unit time). We assume that the dimensions requested and dimensions given are those at the higher temperature, and do not take any thermal expansion effects into account. We also use Eq. 25-3 for resistance.

$$I^2R = \varepsilon\sigma A\left(T_{\text{high}}^4 - T_{\text{low}}^4\right) \rightarrow I^2\frac{4\rho\ell}{\pi d^2} = \varepsilon\sigma\pi d\ell\left(T_{\text{high}}^4 - T_{\text{low}}^4\right) \rightarrow$$

$$d = \left(\frac{4I^2\rho}{\pi^2\varepsilon\sigma\left(T_{\text{high}}^4 - T_{\text{low}}^4\right)}\right)^{1/3} = \left(\frac{4\left(15.0\,\text{A}\right)^2\left(5.6\times10^{-8}\,\Omega\bullet\text{m}\right)}{\pi^2\left(1.0\right)\left(5.67\times10^{-8}\,\text{W/m}^2\bullet\text{K}^4\right)\left[\left(3100\,\text{K}\right)^4 - \left(293\,\text{K}\right)^4\right]}\right)^{1/3}$$

$$= 9.92\times10^{-5}\,\text{m} \approx \boxed{0.099\,\text{mm}}$$

49. Use Ohm's law and the relationship between peak and rms values.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{R} = \sqrt{2}\frac{220\,\text{V}}{2700\,\Omega} = \boxed{0.12\,\text{A}}$$

50. Find the peak current from Ohm's law, and then find the rms current from Eq. 25-9a.

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{180\,\text{V}}{380\,\Omega} = 0.47368\,\text{A} \approx \boxed{0.47\,\text{A}} \qquad I_{\text{rms}} = I_{\text{peak}}\big/\sqrt{2} = \left(0.47368\,\text{A}\right)\big/\sqrt{2} = \boxed{0.33\,\text{A}}$$

51. (*a*) When everything electrical is turned off, no current will be flowing into the house, even though a voltage is being supplied. Since for a given voltage, the more resistance, the lower the current, a zero current corresponds to an $\boxed{\text{infinite resistance}}$.

(*b*) Use Eq. 25-7a to calculate the resistance.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{\left(120\,\text{V}\right)^2}{2\left(75\,\text{W}\right)} = \boxed{96\,\Omega}$$

52. The power and current can be used to find the peak voltage, and then the rms voltage can be found from the peak voltage.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \quad\rightarrow\quad V_{\text{rms}} = \frac{\sqrt{2}\,\overline{P}}{I_{\text{peak}}} = \frac{\sqrt{2}\left(1500\,\text{W}\right)}{5.4\,\text{A}} = \boxed{390\,\text{V}}$$

53. Use the average power and rms voltage to calculate the peak voltage and peak current.

(*a*) $V_{\text{peak}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}\left(660\,\text{V}\right) = 933.4\,\text{V} \approx \boxed{930\,\text{V}}$

(*b*) $\overline{P} = I_{\text{rms}} V_{\text{rms}} = \dfrac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \quad\rightarrow\quad I_{\text{peak}} = \dfrac{\sqrt{2}\,\overline{P}}{V_{\text{rms}}} = \dfrac{\sqrt{2}\left(1800\,\text{W}\right)}{660\,\text{V}} = \boxed{3.9\,\text{A}}$

54. (*a*) We assume that the 2.5 hp is the average power, so the maximum power is twice that, or 5.0 hp, as seen in Figure 25-22.

$$5.0\,\text{hp}\left(\frac{746\,\text{W}}{1\,\text{hp}}\right) = 3730\,\text{W} \approx \boxed{3700\,\text{W}}$$

(*b*) Use the average power and the rms voltage to find the peak current.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \quad\rightarrow\quad I_{\text{peak}} = \frac{\sqrt{2}\,\overline{P}}{V_{\text{rms}}} = \frac{\sqrt{2}\left[\frac{1}{2}\left(3730\,\text{W}\right)\right]}{240\,\text{V}} = \boxed{11\,\text{A}}$$

$\boxed{55.}$ (*a*) The average power used can be found from the resistance and the rms voltage by Eq. 25-10c.

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(240\,\text{V}\right)^2}{44\,\Omega} = 1309\,\text{W} \approx \boxed{1300\,\text{W}}$$

(*b*) The maximum power is twice the average power, and the minimum power is 0.

$$P_{\text{max}} = 2\overline{P} = 2\left(1309\,\text{W}\right) \approx \boxed{2600\,\text{W}} \qquad P_{\text{min}} = \boxed{0\,\text{W}}$$

56. (*a*) Find V_{rms}. Use an integral from Appendix B-4, page A-7.

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T \left(V_0 \sin\frac{2\pi t}{T}\right)^2 dt\right]^{1/2} = \left[\frac{V_0^2}{T}\left(\frac{t}{2} - \frac{\sin\left(\frac{4\pi t}{T}\right)}{\frac{8\pi}{T}}\right)\Bigg|_0^T\right]^{1/2} = \left(\frac{V_0^2}{2}\right)^{1/2} = \boxed{\frac{V_0}{\sqrt{2}}}$$

(*b*) Find V_{rms}.

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T V^2 dt\right]^{1/2} = \left[\frac{1}{T}\int_0^{T/2} V_0^2 dt + \frac{1}{T}\int_{T/2}^{T} \left(0\right)^2 dt\right]^{1/2} = \left[\frac{V_0^2}{T}\frac{T}{2} + 0\right]^{1/2} = \boxed{\frac{V_0}{\sqrt{2}}}$$

57. (*a*) We follow the derivation in Example 25-14. Start with Eq. 25-14, in absolute value.

$$j = nev_{\text{d}} \quad\rightarrow\quad v_{\text{d}} = \frac{j}{ne} = \frac{I}{neA} = \frac{I}{\left(\dfrac{N\left(1\text{ mole}\right)}{m\left(1\text{ mole}\right)}\rho_{\text{D}}\right)e\left[\pi\left(\frac{1}{2}d\right)^2\right]} = \frac{4Im}{N\rho_{\text{D}}e\pi d^2}$$

$$v_{\text{d}} = \frac{4\left(2.3\times10^{-6}\,\text{A}\right)\left(63.5\times10^{-3}\,\text{kg}\right)}{\left(6.02\times10^{23}\right)\left(8.9\times10^{3}\,\text{kg/m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)\pi\left(0.65\times10^{-3}\,\text{m}\right)^2} = \boxed{5.1\times10^{-10}\,\text{m/s}}$$

(*b*) Calculate the current density from Eq. 25-11.

$$j=\frac{I}{A}=\frac{I}{\pi r^2}=\frac{4I}{\pi d^2}=\frac{4\left(2.3\times10^{-6}\,\text{A}\right)}{\pi\left(6.5\times10^{-4}\,\text{m}\right)^2}=6.931\,\text{A/m}^2\approx\boxed{6.9\,\text{A/m}^2}$$

(*c*) The electric field is calculated from Eq. 25-17.

$$j=\frac{1}{\rho}E\ \rightarrow\ E=\rho j=\left(1.68\times10^{-8}\,\Omega\text{•m}\right)\left(6.931\,\text{A/m}^2\right)=\boxed{1.2\times10^{-7}\,\text{V/m}}$$

58. (*a*) Use Ohm's law to find the resistance.

$$V=IR\ \rightarrow\ R=\frac{V}{I}=\frac{0.0220\,\text{V}}{0.75\,\text{A}}=0.02933\,\Omega\approx\boxed{0.029\,\Omega}$$

(*b*) Find the resistivity from Eq. 25-3.

$$R=\frac{\rho\ell}{A}\ \rightarrow$$

$$\rho=\frac{RA}{\ell}=\frac{R\pi r^2}{\ell}=\frac{\left(0.02933\,\Omega\right)\pi\left(1.0\times10^{-3}\,\text{m}\right)^2}{\left(5.80\,\text{m}\right)}=1.589\times10^{-8}\,\Omega\text{•m}\approx\boxed{1.6\times10^{-8}\,\Omega\text{•m}}$$

(*c*) Use Eq. 25-11 to find the current density.

$$j=\frac{I}{A}=\frac{I}{\pi r^2}=\frac{0.75}{\pi\left(0.0010\,\text{m}\right)^2}=2.387\times10^{5}\,\text{A/m}^2\approx\boxed{2.4\times10^{5}\,\text{A/m}^2}$$

(*d*) Use Eq. 25-17 to find the electric field.

$$j=\frac{1}{\rho}E\ \rightarrow$$

$$E=\rho j=\left(1.589\times10^{-8}\,\Omega\text{•m}\right)\left(2.387\times10^{5}\,\text{A/m}^2\right)=3.793\times10^{-3}\,\text{V/m}\approx\boxed{3.8\times10^{-3}\,\text{V/m}}$$

(*e*) Find the number of electrons per unit volume from the absolute value of Eq. 25-14.

$$j=nev_\text{d}\ \rightarrow\ n=\frac{j}{v_\text{d}e}=\frac{2.387\times10^{5}\,\text{A/m}^2}{\left(1.7\times10^{-5}\,\text{m/s}\right)\left(1.60\times10^{-19}\,\text{C}\right)}=\boxed{8.8\times10^{28}\,\text{e}^-/\text{m}^3}$$

59. We are given a charge density and a speed (like the drift speed) for both types of ions. From that we can use Eq. 25-13 (without the negative sign) to determine the current per unit area. Both currents are in the same direction in terms of conventional current – positive charge moving north has the same effect as negative charge moving south – and so they can be added.

$$I=neAv_\text{d}\ \rightarrow$$

$$\frac{I}{A}=\left(nev_\text{d}\right)_\text{He}+\left(nev_\text{d}\right)_\text{O}=\left[\left(2.8\times10^{12}\,\text{ions/m}^3\right)2\left(1.60\times10^{-19}\,\text{C/ion}\right)\left(2.0\times10^{6}\,\text{m/s}\right)\right]+$$

$$\left[\left(7.0\times10^{11}\,\text{ions/m}^3\right)\left(1.60\times10^{-19}\,\text{C/ion}\right)\left(6.2\times10^{6}\,\text{m/s}\right)\right]$$

$$=2.486\,\text{A/m}^2\approx\boxed{2.5\,\text{A/m}^2\text{, North}}$$

60. The magnitude of the electric field is the voltage change per unit meter.

$$\left|E\right|=\frac{\Delta V}{\Delta x}=\frac{70\times10^{-3}\,\text{V}}{1.0\times10^{-8}\,\text{m}}=\boxed{7.0\times10^{6}\,\text{V/m}}$$

$\boxed{61.}$ The speed is the change in position per unit time.

$$v=\frac{\Delta x}{\Delta t}=\frac{7.20\times10^{-2}\,\mathrm{m}-3.40\times10^{-2}\,\mathrm{m}}{0.0063\,\mathrm{s}-0.0052\,\mathrm{s}}=\boxed{35\,\mathrm{m/s}}$$

Two measurements are needed because there may be a time delay from the stimulation of the nerve to the generation of the action potential.

62. The power is the work done per unit time. The work done to move a charge through a potential difference is the charge times the potential difference. The charge density must be multiplied by the surface area of the cell (the surface area of an open tube, length times circumference) to find the actual charge moved.

$$P=\frac{W}{t}=\frac{QV}{t}=\frac{Q}{t}V$$

$$=\left(3\times10^{-7}\,\frac{\mathrm{mol}}{\mathrm{m}^2\bullet\mathrm{s}}\right)\left(6.02\times10^{23}\,\frac{\mathrm{ions}}{\mathrm{mol}}\right)\left(1.6\times10^{-19}\,\frac{\mathrm{C}}{\mathrm{ion}}\right)(0.10\,\mathrm{m})\pi\left(20\times10^{-6}\,\mathrm{m}\right)(0.030\,\mathrm{V})$$

$$=\boxed{5.4\times10^{-9}\,\mathrm{W}}$$

63. The energy supplied by the battery is the energy consumed by the lights.

$$E_{\text{supplied}}=E_{\text{consumed}}\quad\rightarrow\quad Q\Delta V=Pt\quad\rightarrow$$

$$t=\frac{Q\Delta V}{P}=\frac{(85\mathrm{A}\bullet\mathrm{h})(3600\,\mathrm{s/h})(12\mathrm{V})}{92\,\mathrm{W}}=39913\,\mathrm{s}\left(\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right)=11.09\,\mathrm{h}\approx\boxed{11\,\mathrm{h}}$$

64. The ampere-hour is a unit of charge.

$$(1.00\mathrm{A}\bullet\mathrm{h})\left(\frac{1\,\mathrm{C/s}}{1\mathrm{A}}\right)\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right)=\boxed{3600\,\mathrm{C}}$$

65. Use Eqs. 25-3 and 25-7b.

$$R=\rho\frac{\ell}{A}=\rho\frac{\ell}{\pi r^2}=\frac{4\rho\ell}{\pi d^2}\ ;\ P=\frac{V^2}{R}=\frac{V^2}{\dfrac{4\rho\ell}{\pi d^2}}\quad\rightarrow$$

$$\ell=\frac{V^2\pi d^2}{4\rho P}=\frac{(1.5\,\mathrm{V})^2\,\pi\left(5.0\times10^{-4}\,\mathrm{m}\right)^2}{4\left(1.68\times10^{-8}\,\Omega\bullet\mathrm{m}\right)(15\,\mathrm{W})}=1.753\,\mathrm{m}\approx\boxed{1.8\,\mathrm{m}}$$

If the voltage increases by a factor of 6 without the resistance changing, the power will increase by a factor of 36. The blanket would theoretically be able to deliver 540 W of power, which might make the material catch on fire or burn the occupant.

66. Use Eq. 25-6 to calculate the current.

$$P=IV\quad\rightarrow\quad I=\frac{P}{V}=\frac{746\,\mathrm{W}}{120\,\mathrm{V}}=\boxed{6.22\,\mathrm{A}}$$

$\boxed{67.}$ From Eq. 25-2b, if $R=V/I$, then $G=I/V$

$$G=\frac{I}{V}=\frac{0.48\,\mathrm{A}}{3.0\,\mathrm{V}}=\boxed{0.16\,\mathrm{S}}$$

68. Use Eq. 25-7b to express the resistance in terms of the power, and Eq. 25-3 to express the resistance in terms of the wire geometry.

$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P} \qquad R=\rho\frac{\ell}{A}=\rho\frac{\ell}{\pi r^2}=4\rho\frac{\ell}{\pi d^2}$$

$$4\rho\frac{\ell}{\pi d^2}=\frac{V^2}{P} \rightarrow d=\sqrt{\frac{4\rho\ell P}{\pi V^2}}=\sqrt{\frac{4\left(9.71\times10^{-8}\Omega\bullet\text{m}\right)(3.5\,\text{m})(1500\,\text{W})}{\pi(110\,\text{V})^2}}=\boxed{2.3\times10^{-4}\,\text{m}}$$

69. (*a*) Calculate the total kWh used per day, and then multiply by the number of days and the cost per kWh.

$$(1.8\,\text{kW})(2.0\,\text{h/d})+4(0.1\,\text{kW})(6.0\,\text{h/d})+(3.0\,\text{kW})(1.0\,\text{h/d})+(2.0\,\text{kWh/d})$$

$$=11.0\,\text{kWh/d}$$

$$\text{Cost}=(11.0\,\text{kWh/d})(30\,\text{d})\left(\frac{\$0.105}{\text{kWh}}\right)=\$34.65\approx\boxed{\$35\text{ per month}}$$

(*b*) The energy required by the household is 35% of the energy that needs to be supplied by the power plant.

$$\text{Household Energy}=0.35(\text{coal mass})(\text{coal energy per mass}) \rightarrow$$

$$\text{coal mass}=\frac{\text{Household Energy}}{(0.35)(\text{coal energy per mass})}=\frac{(11.0\,\text{kWh/d})(365\,\text{d})\left(\frac{1000\,\text{W}}{\text{kW}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)}{(0.35)\left(7500\frac{\text{kcal}}{\text{kg}}\right)\left(\frac{4186\,\text{J}}{1\,\text{kcal}}\right)}$$

$$=1315\,\text{kg}\approx\boxed{1300\,\text{kg of coal}}$$

70. To deliver 15 MW of power at 120 V requires a current of $I=\frac{P}{V}=\frac{15\times10^6\,\text{W}}{120\,\text{V}}=1.25\times10^5\,\text{A}$.

Calculate the power dissipated in the resistors using the current and the resistance.

$$P=I^2R=I^2\rho\frac{L}{A}=I^2\rho\frac{L}{\pi r^2}=4I^2\rho\frac{L}{\pi d^2}=4\left(1.25\times10^5\,\text{A}\right)^2\left(1.68\times10^{-8}\Omega\bullet\text{m}\right)\frac{2(1.0\,\text{m})}{\pi\left(5.0\times10^{-3}\,\text{m}\right)^2}$$

$$=2.674\times10^7\,\text{W}$$

$$\text{Cost}=(\text{Power})(\text{time})(\text{rate per kWh})=\left(2.674\times10^7\,\text{W}\right)\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(1\,\text{h})\left(\frac{\$0.090}{\text{kWh}}\right)$$

$$=\$2407\approx\boxed{\$2,400\text{ per hour per meter}}$$

71. (*a*) Use Eq. 25-7b to relate the power to the voltage for a constant resistance.

$$P=\frac{V^2}{R} \rightarrow \frac{P_{105}}{P_{117}}=\frac{(105\,\text{V})^2/R}{(117\,\text{V})^2/R}=\frac{(105\,\text{V})^2}{(117\,\text{V})^2}=0.805\text{ or a }\boxed{19.5\%\text{ decrease}}$$

(*b*) The lower power output means that the resistor is generating less heat, and so the resistor's temperature would be lower. The lower temperature results in a lower value of the resistance, which would increase the power output at the lower voltages. Thus the $\boxed{\text{decrease would be smaller}}$ than the value given in the first part of the problem.

72. Assume that we have a meter of wire, carrying 35 A of current, and dissipating 1.5 W of heat. The power dissipated is $P_R = I^2 R$, and the resistance is $R = \frac{\rho \ell}{A}$.

$$P_R = I^2 R = I^2 \frac{\rho \ell}{A} = I^2 \frac{\rho \ell}{\pi r^2} = I^2 \frac{4\rho \ell}{\pi d^2} \quad \rightarrow$$

$$d = \sqrt{I^2 \frac{4\rho \ell}{P_R \pi}} = 2I\sqrt{\frac{\rho \ell}{P_R \pi}} = 2\left(35\,\text{A}\right)\sqrt{\frac{\left(1.68\times 10^{-8}\,\Omega\bullet\text{m}\right)\left(1.0\,\text{m}\right)}{\left(1.5\,\text{W}\right)\pi}} = \boxed{4.2\times 10^{-3}\,\text{m}}$$

73. (*a*) The resistance at the operating temperature can be calculated directly from Eq. 25-7.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{\left(120\,\text{V}\right)^2}{75\,\text{W}} = \boxed{190\,\Omega}$$

(*b*) The resistance at room temperature is found by converting Eq. 25-5 into an equation for resistances and solving for R_0.

$$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$$

$$R_0 = \frac{R}{\left[1 + \alpha\left(T - T_0\right)\right]} = \frac{192\,\Omega}{\left[1 + \left(0.0045\,\text{K}^{-1}\right)\left(3000\,\text{K} - 293\,\text{K}\right)\right]} = \boxed{15\,\Omega}$$

74. (*a*) The angular frequency is $\omega = 210\,\text{rad/s}$.

$$f = \frac{\omega}{2\pi} = \frac{210\,\text{rad/s}}{2\pi} = 33.42\,\text{Hz} \approx \boxed{33\,\text{Hz}}$$

(*b*) The maximum current is 1.80 A.

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{1.80\,\text{A}}{\sqrt{2}} = \boxed{1.27\,\text{A}}$$

(*c*) For a resistor, $V = IR$.

$$V = IR = \left(1.80\,\text{A}\right)\left(\sin 210\,t\right)\left(24.0\,\Omega\right) = \boxed{\left(43.2\sin 210\,t\right)\text{V}}$$

75. (*a*) The power delivered to the interior is 65% of the power drawn from the source.

$$P_{\text{interior}} = 0.65 P_{\text{source}} \quad \rightarrow \quad P_{\text{source}} = \frac{P_{\text{interior}}}{0.65} = \frac{950\,\text{W}}{0.65} = 1462\,\text{W} \approx \boxed{1500\,\text{W}}$$

(*b*) The current drawn is current from the source, and so the source power is used to calculate the current.

$$P_{\text{source}} = IV_{\text{source}} \quad \rightarrow \quad I = \frac{P_{\text{source}}}{V_{\text{source}}} = \frac{1462\,\text{W}}{120\,\text{V}} = 12.18\,\text{A} \approx \boxed{12\,\text{A}}$$

76. The volume of wire is unchanged by the stretching. The volume is equal to the length of the wire times its cross-sectional area, and since the length was increased by a factor of 1.20, the area was decreased by a factor of 1.20. Use Eq. 25-3.

$$R_0 = \rho\frac{\ell_0}{A_0} \qquad \ell = 1.20\ell_0 \qquad A = \frac{A_0}{1.20} \qquad R = \rho\frac{\ell}{A} = \rho\frac{1.20\ell_0}{\frac{A_0}{1.20}} = \left(1.20\right)^2 \rho\frac{\ell_0}{A_0} = 1.44R_0 = \boxed{1.44\,\Omega}$$

77. The long, thick conductor is labeled as conductor number 1, and the short, thin conductor is labeled as number 2. The power transformed by a resistor is given by Eq. 25-7b, $P = V^2/R$, and both have the same voltage applied.

$$R_1 = \rho\frac{\ell_1}{A_1} \quad R_2 = \rho\frac{\ell_2}{A_2} \quad \ell_1 = 2\ell_2 \quad A_1 = 4A_2 \ \left(\text{diameter}_1 = 2\text{diameter}_2\right)$$

$$\frac{P_1}{P_2} = \frac{V_1^2/R_1}{V_2^2/R_2} = \frac{R_2}{R_1} = \frac{\rho\ell_2/A_2}{\rho\ell_1/A_1} = \frac{\ell_2}{\ell_1}\frac{A_1}{A_2} = \frac{1}{2}\times 4 = 2 \quad \boxed{P_1 : P_2 = 2:1}$$

78. The heater must heat $108\,\text{m}^3$ of air per hour from 5°C to 20°C, and also replace the heat being lost at a rate of 850 kcal/h. Use Eq. 19-2 to calculate the energy needed to heat the air. The density of air is found in Table 13-1.

$$Q = mc\Delta T \ \rightarrow \ \frac{Q}{t} = \frac{m}{t}c\Delta T = \left(108\frac{\text{m}^3}{\text{h}}\right)\left(1.29\frac{\text{kg}}{\text{m}^3}\right)\left(0.17\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}\right)\left(15\text{C}^\circ\right) = 355\frac{\text{kcal}}{\text{h}}$$

$$\text{Power required} = 355\frac{\text{kcal}}{\text{h}} + 850\frac{\text{kcal}}{\text{h}} = 1205\frac{\text{kcal}}{\text{h}}\left(\frac{4186\,\text{J}}{\text{kcal}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 1401\,\text{W} \approx \boxed{1400\,\text{W}}$$

$\boxed{79.}$ (*a*) Use Eq. 25-7b.

$$P = \frac{V^2}{R} \ \rightarrow \ R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{2800\,\text{W}} = 20.57\,\Omega \approx \boxed{21\Omega}$$

(*b*) Only 75% of the heat from the oven is used to heat the water. Use Eq. 19-2.

$$0.75\left(P_{\text{oven}}\right)t = \text{Heat absorbed by water} = mc\Delta T \ \rightarrow$$

$$t = \frac{mc\Delta T}{0.75\left(P_{\text{oven}}\right)} = \frac{(0.120\,\text{L})\left(\frac{1\,\text{kg}}{1\,\text{L}}\right)(4186\,\text{J/kg}\bullet\text{C}^\circ)(85\,\text{C}^\circ)}{0.75(2800\,\text{W})} = 20.33\,\text{s} \approx \boxed{20\,\text{s}}\ (2 \text{ sig. fig.})$$

(*c*) $$\frac{11\,\text{cents}}{\text{kWh}}(2.8\,\text{kW})(20.33\,\text{s})\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{0.17\,\text{cents}}$$

80. (*a*) The horsepower required is the power dissipated by the frictional force, since we are neglecting the energy used for acceleration.

$$P = Fv = (240\,\text{N})(45\,\text{km/hr})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right) = 3000\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = \boxed{4.0\,\text{hp}}$$

(*b*) The charge available by each battery is $Q = 95\text{A}\bullet\text{h} = 95\,\text{C/s}\bullet 3600\,\text{s} = 3.42\times10^5\,\text{C}$, and so the total charge available is 24 times that. The potential energy of that charge is the charge times the voltage. That energy must be delivered (batteries discharged) in a certain amount of time to produce the 3000 W necessary. The speed of the car times the discharge time is the range of the car between recharges.

$$P = \frac{U}{t} = \frac{QV}{t} \ \rightarrow \ t = \frac{QV}{P} = \frac{d}{v} \ \rightarrow$$

$$d = vt = v\frac{QV}{P} = v\frac{QV}{Fv} = \frac{QV}{F} = \frac{24(3.42\times10^5\,\text{C})(12\,\text{V})}{240\,\text{N}} = \boxed{410\,\text{km}}$$

81. The mass of the wire is the density of copper times the volume of the wire, and the resistance of the wire is given by Eq. 25-3. We represent the mass density by ρ_{m} and the resistivity by ρ.

$$R=\rho\frac{\ell}{A}\ \rightarrow\ A=\frac{\rho\ell}{R}\qquad m=\rho_{\mathrm{m}}\ell A=\rho_{\mathrm{m}}\ell\frac{\rho\ell}{R}\ \rightarrow$$

$$\ell=\sqrt{\frac{mR}{\rho_{\mathrm{m}}\rho}}=\sqrt{\frac{(0.0155\,\mathrm{kg})(12.5\,\Omega)}{(8.9\times10^{3}\,\mathrm{kg/m^3})(1.68\times10^{-8}\,\Omega\bullet\mathrm{m})}}=35.997\,\mathrm{m}\approx\boxed{36.0\,\mathrm{m}}$$

$$A=\frac{\rho\ell}{R}=\pi\left(\tfrac{1}{2}d\right)^2\ \rightarrow\ d=\sqrt{\frac{4\rho\ell}{\pi R}}=\sqrt{\frac{4(1.68\times10^{-8}\,\Omega\bullet\mathrm{m})(35.997\,\mathrm{m})}{\pi(12.5\,\Omega)}}=\boxed{2.48\times10^{-4}\,\mathrm{m}}$$

82. The resistance can be calculated from the power and voltage, and then the diameter of the wire can be calculated from the resistance.

$$P=\frac{V^2}{R}\ \rightarrow\ R=\frac{V^2}{P}\qquad R=\frac{\rho L}{A}=\frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2}\ \rightarrow\ \frac{V^2}{P}=\frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2}\ \rightarrow$$

$$d=\sqrt{\frac{4\rho LP}{\pi V^2}}=\sqrt{\frac{4(100\times10^{-8}\,\Omega\bullet\mathrm{m})(3.8\,\mathrm{m})(95\,\mathrm{W})}{\pi(120\,\mathrm{V})^2}}=1.787\times10^{-4}\,\mathrm{m}\approx\boxed{1.8\times10^{-4}\,\mathrm{m}}$$

83. Use Eq. 25-7b.

(*a*) $P=\frac{V^2}{R}=\frac{(120\,\mathrm{V})^2}{12\,\Omega}=\boxed{1200\,\mathrm{W}}$

(*b*) $P=\frac{V^2}{R}=\frac{(120\,\mathrm{V})^2}{140\,\Omega}=103\,\mathrm{W}\approx\boxed{100\,\mathrm{W}}$ (2 sig. fig.)

84. Use Eq. 25-7b for the power in each case, assuming the resistance is constant.

$$\frac{P_{13.8\,\mathrm{V}}}{P_{12.0\,\mathrm{V}}}=\frac{\left(V^2/R\right)_{13.8\,\mathrm{V}}}{\left(V^2/R\right)_{12.0\,\mathrm{V}}}=\frac{13.8^2}{12.0^2}=1.3225=\boxed{32\%\text{ increase}}$$

$\boxed{85.}$ Model the protons as moving in a continuous beam of cross-sectional area *A*. Then by Eq. 25-13, $I=neAv_{\mathrm{d}}$, where we only consider the absolute value of the current. The variable *n* is the number of protons per unit volume, so $n=\frac{N}{A\ell}$, where *N* is the number of protons in the beam and ℓ is the circumference of the ring. The "drift" velocity in this case is the speed of light.

$$I=neAv_{\mathrm{d}}=\frac{N}{A\ell}eAv_{\mathrm{d}}=\frac{N}{\ell}ev_{\mathrm{d}}\ \rightarrow$$

$$N=\frac{I\ell}{ev_{\mathrm{d}}}=\frac{(11\times10^{-3})(6300\,\mathrm{m})}{(1.60\times10^{-19}\,\mathrm{C})(3.00\times10^{8}\,\mathrm{m/s})}=\boxed{1.4\times10^{12}\,\text{protons}}$$

86. (*a*) The current can be found from Eq. 25-6.

$$I=P/V\qquad I_A=P_A/V_A=40\mathrm{W}/120\,\mathrm{V}=\boxed{0.33\,\mathrm{A}}\qquad I_B=P_B/V_B=40\mathrm{W}/12\,\mathrm{V}=\boxed{3.3\,\mathrm{A}}$$

(*b*) The resistance can be found from Eq. 25-7b.

$$R=\frac{V^2}{P}\qquad R_A=\frac{V_A^2}{P_A}=\frac{(120\,\text{V})^2}{40\,\text{W}}=\boxed{360\,\Omega}\qquad R_B=\frac{V_B^2}{P_B}=\frac{(12\,\text{V})^2}{40\,\text{W}}=\boxed{3.6\,\Omega}$$

(*c*) The charge is the current times the time.

$$Q=It\qquad Q_A=I_At=(0.33\,\text{A})(3600\,\text{s})=\boxed{1200\,\text{C}}$$

$$Q_B=I_Bt=(3.3\,\text{A})(3600\,\text{s})=\boxed{12{,}000\,\text{C}}$$

(*d*) The energy is the power times the time, and the power is the same for both bulbs.

$$E=Pt\qquad E_A=E_B=(40\,\text{W})(3600\,\text{s})=\boxed{1.4\times10^5\,\text{J}}$$

(*e*) $\boxed{\text{Bulb B}}$ requires a larger current, and so should have larger diameter connecting wires to avoid overheating the connecting wires.

87. (*a*) The power is given by $P=IV$.

$$P=IV=(14\,\text{A})(220\,\text{V})=3080\text{W}\approx\boxed{3100\,\text{W}}$$

(*b*) The power dissipated is given by $P_R=I^2R$, and the resistance is $R=\frac{\rho\ell}{A}$.

$$P_R=I^2R=I^2\frac{\rho\ell}{A}=I^2\frac{\rho\ell}{\pi r^2}=I^2\frac{4\rho\ell}{\pi d^2}=(14\,\text{A})^2\frac{4(1.68\times10^{-8}\Omega\bullet\text{m})(15\,\text{m})}{\pi(1.628\times10^{-3}\text{m})^2}=23.73\,\text{W}$$

$$\approx\boxed{24\,\text{W}}$$

(*c*) $$P_R=I^2\frac{4\rho L}{\pi d^2}=(14\,\text{A})^2\frac{4(1.68\times10^{-8}\Omega\bullet\text{m})(15\,\text{m})}{\pi(2.053\times10^{-3}\text{m})^2}=14.92\,\text{W}\approx\boxed{15\,\text{W}}$$

(*d*) The savings is due to the power difference.

$$\text{Savings}=(23.73\,\text{W}-14.92\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(30\,\text{d})\left(\frac{12\,\text{h}}{1\,\text{d}}\right)\left(\frac{\$0.12}{1\,\text{kWh}}\right)$$

$$=\$0.3806/\text{month}\approx\boxed{38\,\text{cents per month}}$$

88. The wasted power is due to losses in the wire. The current in the wire can be found by $I=P/V$.

(*a*) $$P_R=I^2R=\frac{P^2}{V^2}R=\frac{P^2}{V^2}\frac{\rho L}{A}=\frac{P^2}{V^2}\frac{\rho L}{\pi r^2}=\frac{P^2}{V^2}\frac{4\rho L}{\pi d^2}$$

$$=\frac{(1750\,\text{W})^2}{(120\,\text{V})^2}\frac{4(1.68\times10^{-8}\Omega\bullet\text{m})(25.0\,\text{m})}{\pi(2.59\times10^{-3}\text{m})^2}=16.954\,\text{W}\approx\boxed{17.0\,\text{W}}$$

(*b*) $$P_R=\frac{P^2}{V^2}\frac{4\rho L}{\pi d^2}=\frac{(1750\,\text{W})^2}{(120\,\text{V})^2}\frac{4(1.68\times10^{-8}\Omega\bullet\text{m})(25.0\,\text{m})}{\pi(4.12\times10^{-3}\text{m})^2}=\boxed{6.70\,\text{W}}$$

89. (*a*) The D-cell provides 25 mA at 1.5 V for 820 h, at a cost of $1.70.

$$\text{Energy}=Pt=VIt=(1.5\,\text{V})(0.025\,\text{A})(820\,\text{h})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)=0.03075\,\text{kWh}$$

$$\text{Cost/kWh} = \frac{\$1.70}{0.03075\,\text{kWh}} = \$55.28/\text{kWh} \approx \boxed{\$55/\text{kWh}}$$

(*b*) The AA-cell provides 25 mA at 1.5 V for 120 h, at a cost of \$1.25.

$$\text{Energy} = Pt = VIt = (1.5\,\text{V})(0.025\,\text{A})(120\,\text{h})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right) = 0.0045\,\text{kWh}$$

$$\text{Cost/kWh} = \frac{\$1.25}{0.0045\,\text{kWh}} = \$277.78/\text{kWh} \approx \boxed{\$280/\text{kWh}}$$

The D-cell is $\frac{\$55.28/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{550 \times \text{as costly}}$. The AA-cell is $\frac{\$277.78/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{2800 \times \text{as costly}}$.

90. The electrons are assumed to be moving with simple harmonic motion. During one cycle, an object in simple harmonic motion will move a distance equal to the amplitude from its equilibrium point. From Eq. 14-9a, we know that $v_{\max} = A\omega$, where ω is the angular frequency of oscillation. From Eq. 25-13 in absolute value, we see that $I_{\max} = neAv_{\max}$. Finally, the maximum current can be related to the power by Eqs. 25-9 and 25-10. The charge carrier density, *n*, is calculated in Example 25-14.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}} = \tfrac{1}{\sqrt{2}} I_{\max}V_{\text{rms}}$$

$$A = \frac{v_{\max}}{\omega} = \frac{I_{\max}}{\omega neA} = \frac{\sqrt{2}\overline{P}}{\omega ne\frac{\pi d^2}{4}V_{\text{rms}}}$$

$$= \frac{4\sqrt{2}(550\,\text{W})}{2\pi(60\,\text{Hz})(8.4\times10^{28}\,\text{m}^{-3})(1.60\times10^{-19}\,\text{C})\pi(1.7\times10^{-3}\,\text{m})^2(120\,\text{V})} = \boxed{5.6\times10^{-7}\,\text{m}}$$

The electron will move this distance in both directions from its equilibrium point.

$\boxed{91.}$ Eq. 25-3 can be used. The area to be used is the cross-sectional area of the pipe.

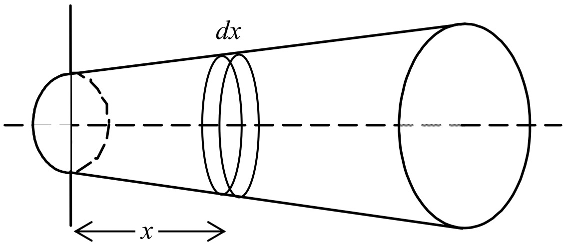
$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi\left(r_{\text{outside}}^2 - r_{\text{inside}}^2\right)} = \frac{(1.68\times10^{-8}\,\Omega\bullet\text{m})(10.0\,\text{m})}{\pi\left[(2.50\times10^{-2}\,\text{m})^2 - (1.50\times10^{-2}\,\text{m})^2\right]} = \boxed{1.34\times10^{-4}\,\Omega}$$

92. We assume that all of the current that enters at *a* leaves at *b*, so that the current is the same at each end. The current density is given by Eq. 25-11.

$$j_{\text{a}} = \frac{I}{A_{\text{a}}} = \frac{I}{\pi\left(\frac{1}{2}a\right)^2} = \frac{4I}{\pi a^2} = \frac{4(2.0\,\text{A})}{\pi(2.5\times10^{-3}\,\text{m})^2} = \boxed{4.1\times10^5\,\text{A/m}^2}$$

$$j_{\text{b}} = \frac{I}{A_{\text{b}}} = \frac{I}{\pi\left(\frac{1}{2}b\right)^2} = \frac{4I}{\pi b^2} = \frac{4(2.0\,\text{A})}{\pi(4.0\times10^{-3}\,\text{m})^2} = \boxed{1.6\times10^5\,\text{A/m}^2}$$

93. Using Eq. 25-3, we find the infinitesimal resistance first of a thin vertical slice at a horizontal distance x from the center of the left side towards the center of the right side. Let the thickness of that slice be dx. That thickness corresponds to the variable ℓ in Eq. 25-3. The diameter of this slice is $a+\frac{x}{\ell}(b-a)$. Then integrate over all the slices to find the total resistance.



$$R=\rho\frac{\ell}{A}\quad\to\quad dR=\rho\frac{dx}{\pi\frac{1}{4}\left(a+\frac{x}{\ell}(b-a)\right)^2}\quad\to$$

$$R=\int dR=\int_0^{\ell}\rho\frac{dx}{\pi\frac{1}{4}\left(a+\frac{x}{\ell}(b-a)\right)^2}=-\frac{4\rho}{\pi}\frac{\ell}{b-a}\frac{1}{\left(a+\frac{x}{\ell}(b-a)\right)}\Bigg|_0^{\ell}=\boxed{\frac{4\rho}{\pi}\frac{\ell}{ab}}$$

94. The resistance of the filament when the flashlight is on is $R=\frac{V}{I}=\frac{3.2\,\text{V}}{0.20\,\text{A}}=16\,\Omega$. That can be used with a combination of Eqs. 25-3 and 25-5 to find the temperature.

$$R=R_0\left[1+\alpha\left(T-T_0\right)\right]\quad\to$$

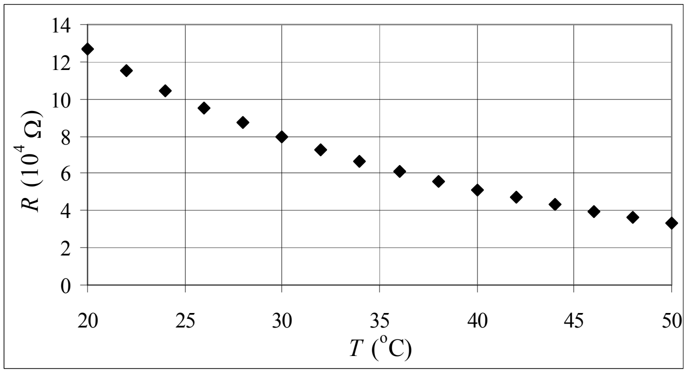
$$T=T_0+\frac{1}{\alpha}\left(\frac{R}{R_0}-1\right)=20^{\circ}\text{C}+\frac{1}{0.0045\left(\text{C}^{\circ}\right)^{-1}}\left(\frac{16\,\Omega}{1.5\,\Omega}-1\right)=2168^{\circ}\text{C}\approx\boxed{2200^{\circ}\text{C}}$$

95. When the tank is empty, the entire length of the wire is in a non-superconducting state, and so has a non-zero resistivity, which we call ρ. Then the resistance of the wire when the tank is empty is given by $R_0=\rho\frac{\ell}{A}=\frac{V_0}{I}$. When a length x of the wire is superconducting, that portion of the wire has 0 resistance. Then the resistance of the wire is only due to the length $\ell-x$, and so $R=\rho\frac{\ell-x}{A}=\rho\frac{\ell}{A}\frac{\ell-x}{\ell}=R_0\frac{\ell-x}{\ell}$. This resistance, combined with the constant current, gives $V=IR$.

$$V=IR=\left(\frac{V_0}{R_0}\right)R_0\frac{\ell-x}{\ell}=V_0\left(1-\frac{x}{\ell}\right)=V_0\left(1-f\right)\quad\to\quad\boxed{f=1-\frac{V}{V_0}}$$

Thus a measurement of the voltage can give the fraction of the tank that is filled with liquid helium.

96. We plot resistance vs. temperature. The graph is shown as follows, with no curve fitted to it. It is apparent that a linear fit will not be a good fit to this data. Both quadratic and exponential equations fit the data well, according to the R-squared coefficient as given by Excel. The equations and the predictions are given below.



$$R_{\text{exp}} = \left(30.1\times10^4 e^{-0.0442T}\right)\Omega$$

$$R_{\text{quad}} = \left[\left(7.39\times10^4\right)T^2 - 8200T + 25.9\times10^4\right]\Omega$$

Solving these expressions for $R = 57{,}641\,\Omega$ (using the spreadsheet) gives $T_{\text{exp}} = 37.402°\text{C}$ and $T_{\text{quad}} = 37.021°\text{C}$. So the temperature is probably in the range between those two values: $\boxed{37.021°\text{C} < T < 37.402°\text{C}}$. The average of those two values is $\boxed{T = 37.21°\text{C}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.96."

As an extra comment, how might you choose between the exponential and quadratic fits? While they both give almost identical predictions for this intermediate temperature, they differ significantly at temperatures near $0°\text{C}$. The exponential fit would give a resistance of about $301{,}000\,\Omega$ at $0°\text{C}$, while the quadratic fit would give a resistance of about 259,000 Ω at $0°\text{C}$. So a measurement of resistance near $0°\text{C}$ might be very useful.

CHAPTER 26: DC Circuits

Responses to Questions

$\boxed{1.}$ Even though the bird's feet are at high potential with respect to the ground, there is very little potential difference between them, because they are close together on the wire. The resistance of the bird is much greater than the resistance of the wire between the bird's feet. These two resistances are in parallel, so very little current will pass through the bird as it perches on the wire. When you put a metal ladder up against a power line, you provide a direct connection between the high potential line and ground. The ladder will have a large potential difference between its top and bottom. A person standing on the ladder will also have a large potential difference between his or her hands and feet. Even if the person's resistance is large, the potential difference will be great enough to produce a current through the person's body large enough to cause substantial damage or death.

2. Series: The main disadvantage of Christmas tree lights connected in series is that when one bulb burns out, a gap is created in the circuit and none of the bulbs remains lit. Finding the burned-out bulb requires replacing each individual bulb one at a time until the string of bulbs comes back on. As an advantage, the bulbs are slightly easier to wire in series.

   Parallel: The main advantage of connecting the bulbs in parallel is that one burned-out bulb does not affect the rest of the strand, and is easy to identify and replace. As a disadvantage, wiring the bulbs in parallel is slightly more difficult.

3. Yes. You can put 20 of the 6-V lights in series, or you can put several of the 6-V lights in series with a large resistance.

4. When the bulbs are connected in series, they have the same current through them. R_2, the bulb with the greater resistance, will be brighter in this case, since $P = I^2R$. When the bulbs are connected in parallel, they will have the same voltage across them. In this case, R_1, the bulb with the lower resistance, will have a larger current flowing through it and will be brighter: $P = V^2/R$.

5. Double outlets are connected in parallel, since each has 120 V across its terminals and they can be used independently.

6. Arrange the two batteries in series with each other and the two bulbs in parallel across the combined voltage of the batteries. This configuration maximizes the voltage gain and minimizes the equivalent resistance, yielding the maximum power.

$\boxed{7.}$ The battery has to supply less power when the two resistors are connected in series than it has to supply when only one resistor is connected. $P = IV = \dfrac{V^2}{R}$, so if V is constant and R increases, the power decreases.

8. The overall resistance decreases and more current is drawn from the source. A bulb rated at 60-W and 120-V has a resistance of 240 Ω. A bulb rated at 100-W and 120-V has a resistance of 144 Ω. When only the 60-W bulb is on, the total resistance is 240 Ω. When both bulbs are lit, the total resistance is the combination of the two resistances in parallel, which is only 90 Ω.

9. No. The sign of the battery's emf does not depend on the direction of the current through the battery. Yes, the terminal voltage of the battery does depend on the direction of the current through the

battery. Note that the sign of the battery's emf in the loop equation does depend on the direction the loop is traversed (+ in the direction of the battery's potential, – in the opposite direction), and the terminal voltage sign and magnitude depend on whether the loop is traversed with or against the current.

10. When resistors are connected in *series*, the equivalent resistance is the *sum* of the individual resistances, $R_{\text{eq,series}} = R_1 + R_2 + \ldots$. The current has to go through each additional resistance if the resistors are in series and therefore the equivalent resistance is greater than any individual resistance. In contrast, when capacitors are in *parallel* the equivalent capacitance is equal to the sum of the individual capacitors, $C_{\text{eq,parallel}} = C_1 + C_2 + \ldots$. Charge drawn from the battery can go down any one of the different branches and land on any one of the capacitors, so the overall capacitance is greater than that of each individual capacitor.

    When resistors are connected in *parallel*, the current from the battery or other source divides into the different branches and so the equivalent resistance is less than any individual resistor in the circuit. The corresponding expression is $1/R_{\text{eq,parallel}} = 1/R_1 + 1/R_2 + \ldots$. The formula for the equivalent capacitance of capacitors in *series* follows this same form, $1/C_{\text{eq,series}} = 1/C_1 + 1/C_2 + \ldots$. When capacitors are in series, the overall capacitance is less than the capacitance of any individual capacitor. Charge leaving the first capacitor lands on the second rather than going straight to the battery.

    Compare the expressions defining resistance ($R = V/I$) and capacitance ($C = Q/V$). Resistance is proportional to voltage, whereas capacitance is inversely proportional to voltage.

11. When batteries are connected in series, their emfs add together, producing a larger potential. The batteries do not need to be identical in this case. When batteries are connected in parallel, the currents they can generate add together, producing a larger current over a longer time period. Batteries in this case need to be nearly identical, or the battery with the larger emf will end up charging the battery with the smaller emf.

12. Yes. When a battery is being charged, current is forced through it "backwards" and then $V_{\text{terminal}} = \text{emf} + Ir$, so $V_{\text{terminal}} > \text{emf}$.

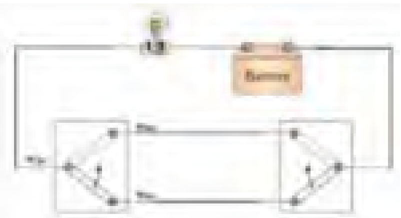
$\boxed{13.}$ Put the battery in a circuit in series with a very large resistor and measure the terminal voltage. With a large resistance, the current in the circuit will be small, and the potential across the battery will be mainly due to the emf. Next put the battery in parallel with the large resistor (or in series with a small resistor) and measure the terminal voltage and the current in the circuit. You will have enough information to use the equation $V_{\text{terminal}} = \text{emf} - Ir$ to determine the internal resistance r.

14. No. As current passes through the resistor in the *RC* circuit, energy is dissipated in the resistor. Therefore, the total energy supplied by the battery during the charging is the combination of the energy dissipated in the resistor and the energy stored in the capacitor.

15. (*a*) Stays the same; (*b*) Increases; (*c*) Decreases; (*d*) Increases; (*e*) Increases; (*f*) Decreases; (*g*) Decreases; (*h*) Increases; (*i*) Remains the same.

16. The capacitance of a parallel plate capacitor is inversely proportional to the distance between the plates: ($C = \varepsilon_0 A/d$). As the diaphragm moves in and out, the distance between the plates changes and therefore the capacitance changes with the same frequency. This changes the amount of charge that can be stored on the capacitor, creating a current as the capacitor charges or discharges. The current oscillates with the same frequency as the diaphragm, which is the same frequency as the incident sound wave, and produces an oscillating V_{output}.

17. See the adjacent figure. If both switches are connected to the same wire, the circuit is complete and the light is on. If they are connected to opposite wires, the light will remain off.



18. In an analog ammeter, the internal resistor, or shunt resistor, has a small value and is in parallel with the galvanometer, so that the overall resistance of the ammeter is very small. In an analog voltmeter, the internal resistor has a large value and is in series with the galvanometer, and the overall resistance of the voltmeter is very large.

$\boxed{19.}$ If you use an ammeter where you need to use a voltmeter, you will short the branch of the circuit. Too much current will pass through the ammeter and you will either blow the fuse on the ammeter or burn out its coil.

20. An ammeter is placed in series with a given circuit element in order to measure the current through that element. If the ammeter did not have very low (ideally, zero) resistance, its presence in the circuit would change the current it is attempting to measure by adding more resistance in series. An ideal ammeter has zero resistance and thus does not change the current it is measuring.

    A voltmeter is placed in parallel with a circuit element in order to measure the voltage difference across that element. If the voltmeter does not have a very high resistance, than its presence in parallel will lower the overall resistance and affect the circuit. An ideal voltmeter has infinite resistance so that when placed in parallel with circuit elements it will not change the value of the voltage it is reading.

21. When a voltmeter is connected across a resistor, the voltmeter is in parallel with the resistor. Even if the resistance of the voltmeter is large, the parallel combination of the resistor and the voltmeter will be slightly smaller than the resistor alone. If R_{eq} decreases, then the overall current will increase, so that the potential drop across the rest of the circuit will increase. Thus the potential drop across the parallel combination will be less than the original voltage drop across the resistor.

22. A voltmeter has a very high resistance. When it is connected to the battery very little current will flow. A small current results in a small voltage drop due to the internal resistance of the battery, and the emf and terminal voltage (measured by the voltmeter) will be very close to the same value. However, when the battery is connected to the lower-resistance flashlight bulb, the current will be higher and the voltage drop due to the internal resistance of the battery will also be higher. As a battery is used, its internal resistance increases. Therefore, the terminal voltage will be significantly lower than the emf: $V_{\text{terminal}} = \text{emf} - Ir$. A lower terminal voltage will result in a dimmer bulb, and usually indicates a "used-up" battery.

23. (*a*) With the batteries in series, a greater voltage is delivered to the lamp, and the lamp will burn brighter.
    (*b*) With the batteries in parallel, the voltage across the lamp is the same as for either battery alone. Each battery supplies only half of the current going through the lamp, so the batteries will last twice as long.

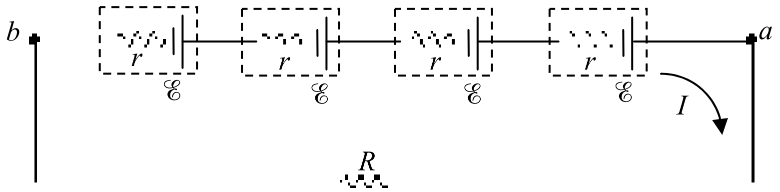
Solutions to Problems

$\boxed{1.}$ See Figure 26-2 for a circuit diagram for this problem. Using the same analysis as in Example 26-1, the current in the circuit is $I = \dfrac{\mathscr{E}}{R+r}$. Use Eq. 26-1 to calculate the terminal voltage.

$$(a)\quad V_{\text{ab}} = \mathscr{E} - Ir = \mathscr{E} - \left(\frac{\mathscr{E}}{R+r}\right)r = \frac{\mathscr{E}(R+r) - \mathscr{E}r}{R+r} = \mathscr{E}\frac{R}{R+r} = (6.00\,\text{V})\frac{81.0\,\Omega}{(81.0+0.900)\,\Omega} = \boxed{5.93\,\text{V}}$$

$$(b)\quad V_{\text{ab}} = \mathscr{E}\frac{R}{R+r} = (6.00\,\text{V})\frac{810\,\Omega}{(810+0.900)\,\Omega} = \boxed{5.99\,\text{V}}$$

2. See the circuit diagram below. The current in the circuit is *I*. The voltage V_{ab} is given by Ohm's law to be $V_{\text{ab}} = IR$. That same voltage is the terminal voltage of the series EMF.

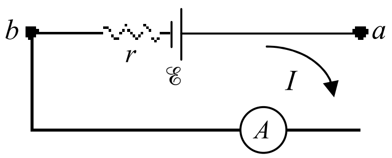


$$V_{\text{ab}} = (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) = 4(\mathscr{E} - Ir) \quad \text{and} \quad V_{\text{ab}} = IR$$

$$4(\mathscr{E} - Ir) = IR \;\rightarrow\; r = \frac{\mathscr{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = 0.333\,\Omega \approx \boxed{0.3\,\Omega}$$

3. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$V_{\text{ab}} = \mathscr{E} - Ir = 0 \;\rightarrow\; r = \frac{\mathscr{E}}{I} = \frac{1.5\,\text{V}}{25\,\text{A}} = \boxed{0.060\,\Omega}$$

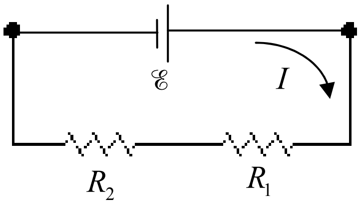


4. See Figure 26-2 for a circuit diagram for this problem. Use Eq. 26-1.

$$V_{\text{ab}} = \mathscr{E} - Ir \;\rightarrow\; r = \frac{\mathscr{E} - V_{\text{ab}}}{I} = \frac{12.0\,\text{V} - 8.4\,\text{V}}{95\,\text{A}} = \boxed{0.038\,\Omega}$$

$$V_{\text{ab}} = IR \;\rightarrow\; R = \frac{V_{\text{ab}}}{I} = \frac{8.4\,\text{V}}{95\,\text{A}} = \boxed{0.088\,\Omega}$$

5. The equivalent resistance is the sum of the two resistances: $R_{\text{eq}} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \dfrac{\mathscr{E}}{R_{\text{eq}}} = \dfrac{\mathscr{E}}{R_1 + R_2}$. The voltage across the 2200-Ω resistor is given by Ohm's law.



$$V_{2200} = IR_2 = \frac{\mathscr{E}}{R_1 + R_2}R_2 = \mathscr{E}\frac{R_2}{R_1 + R_2} = (12.0\,\text{V})\frac{2200\,\Omega}{650\,\Omega + 2200\,\Omega} = \boxed{9.3\,\text{V}}$$

6. (*a*) For the resistors in series, use Eq. 26-3, which says the resistances add linearly.

$$R_{\text{eq}} = 3(45\,\Omega) + 3(65\,\Omega) = \boxed{330\,\Omega}$$

(*b*) For the resistors in parallel, use Eq. 26-4, which says the resistances add reciprocally.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{45\,\Omega} + \frac{1}{45\,\Omega} + \frac{1}{45\,\Omega} + \frac{1}{65\,\Omega} + \frac{1}{65\,\Omega} + \frac{1}{65\,\Omega} = \frac{3}{45\,\Omega} + \frac{3}{65\,\Omega} = \frac{3(65\,\Omega) + 3(45\,\Omega)}{(65\,\Omega)(45\,\Omega)} \rightarrow$$

$$R_{\text{eq}} = \frac{(65\,\Omega)(45\,\Omega)}{3(65\,\Omega) + 3(45\,\Omega)} = \boxed{8.9\,\Omega}$$

$\boxed{7.}$ (*a*) The maximum resistance is made by combining the resistors in series.

$$R_{\text{eq}} = R_1 + R_2 + R_3 = 680\,\Omega + 720\,\Omega + 1200\,\Omega = \boxed{2.60\,\text{k}\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

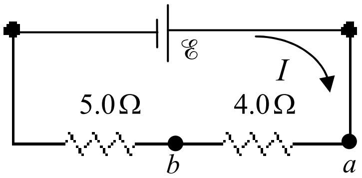
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow$$

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{720\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{270\,\Omega}$$

8. The equivalent resistance of five 100-Ω resistors in parallel is found, and then that resistance is divided by 10 Ω to find the number of 10-Ω resistors needed.

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{5}{100\,\Omega}\right)^{-1} = 20\,\Omega = n(10\,\Omega) \rightarrow n = \frac{20\,\Omega}{10\,\Omega} = \boxed{2}$$

9. Connecting nine of the resistors in series will enable you to make a voltage divider with a 4.0 V output. To get the desired output, measure the voltage across four consecutive series resistors.



$$R_{\text{eq}} = 9(1.0\,\Omega) \qquad I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{9.0\,\Omega}$$

$$V_{\text{ab}} = (4.0\,\Omega)I = (4.0\,\Omega)\frac{\mathcal{E}}{9.0\,\Omega} = (4.0\,\Omega)\frac{9.0\,\text{V}}{9.0\,\Omega} = \boxed{4.0\,\text{V}}$$

10. The resistors can all be connected in series.

$$R_{\text{eq}} = R + R + R = 3(1.70\,\text{k}\Omega) = \boxed{5.10\,\text{k}\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{\text{eq}} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{1.70\,\text{k}\Omega}{3} = \boxed{567\,\Omega}$$

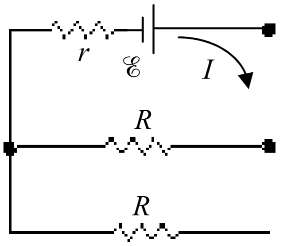
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{\text{eq}} = \frac{2R}{3} = \frac{2(1.70\,\text{k}\Omega)}{3} = \boxed{1.13\,\text{k}\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(1.70\,\text{k}\Omega) = \boxed{2.55\,\text{k}\Omega}$$

11. The resistance of each bulb can be found from its power rating.



$$P=\frac{V^2}{R} \quad \rightarrow \quad R=\frac{V^2}{P}=\frac{(12.0\,\text{V})^2}{4.0\,\text{W}}=36\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R} \quad \rightarrow \quad R_{\text{eq}}=\frac{R}{2}=\frac{36\,\Omega}{2}=18\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{\text{ab}}=IR_{\text{eq}} \quad \rightarrow \quad I=\frac{V_{\text{ab}}}{R_{\text{eq}}}=\frac{V_{\text{ab}}}{R/2}=\frac{2V_{\text{ab}}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 26-1.

$$V_{\text{ab}}=\mathscr{E}-Ir \quad \rightarrow \quad r=\frac{\mathscr{E}-V_{\text{ab}}}{I}=\frac{\mathcal{E}-V_{\text{ab}}}{\left(\frac{2V_{\text{ab}}}{R}\right)}=R\frac{\mathscr{E}-V_{\text{ab}}}{2V_{\text{ab}}}=(36\,\Omega)\frac{12.0\,\text{V}-11.8\,\text{V}}{2(11.8\,\text{V})}=0.305\,\Omega\approx\boxed{0.3\,\Omega}$$

12. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text{eq}}=8R$. The current flowing through the bulbs is then $V_{\text{tot}}=IR_{\text{eq}} \rightarrow I=\frac{V_{\text{tot}}}{R_{\text{eq}}}=\frac{V_{\text{tot}}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V=IR=\frac{V_{\text{tot}}}{8R}R=\frac{V_{\text{tot}}}{8}=\frac{110\,\text{V}}{8}=13.75\,\text{V}\approx\boxed{14\,\text{V}}$$

(*b*) $$I=\frac{V_{\text{tot}}}{8R} \quad \rightarrow \quad R=\frac{V_{\text{tot}}}{8I}=\frac{110\,\text{V}}{8(0.42\,\text{A})}=32.74\,\Omega\approx\boxed{33\,\Omega}$$

$$P=I^2R=(0.42\,\text{A})^2(32.74\,\Omega)=5.775\,\text{W}\approx\boxed{5.8\,\text{W}}$$

$\boxed{13.}$ We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I=8I_R$. The voltage drop across the long leads is $V_{\text{leads}}=Ir=8I_Rr=8(0.24\,\text{A})(1.4\,\Omega)=2.688\,\text{V}$. Thus the voltage across each of the parallel resistors is $V_R=V_{\text{tot}}-V_{\text{leads}}=110\,\text{V}-2.688\,\text{V}=107.3\,\text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_R=I_RR \quad \rightarrow \quad R=\frac{V_R}{I_R}=\frac{107.3\,\text{V}}{0.24\,\text{A}}=447.1\,\Omega=\boxed{450\,\Omega}$$

The total power delivered is $P=V_{\text{tot}}I$, and the "wasted" power is I^2r. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted}=\frac{I^2r}{IV_{\text{tot}}}=\frac{Ir}{V_{\text{tot}}}=\frac{8(0.24\,\text{A})(1.4\,\Omega)}{110\,\text{V}}=\boxed{0.024}$$

So about 2.5% of the power is wasted.

14. The power delivered to the starter is equal to the square of the current in the circuit multiplied by the resistance of the starter. Since the resistors in each circuit are in series we calculate the currents as the battery emf divided by the sum of the resistances.

$$\frac{P}{P_0}=\frac{I^2R_S}{I_0^2R_S}=\left(\frac{I}{I_0}\right)^2=\left(\frac{\mathscr{E}/R_{\text{eq}}}{\mathscr{E}/R_{0\text{eq}}}\right)^2=\left(\frac{R_{0\text{eq}}}{R_{\text{eq}}}\right)^2=\left(\frac{r+R_S}{r+R_S+R_C}\right)^2$$

$$=\left(\frac{0.02\,\Omega+0.15\,\Omega}{0.02\,\Omega+0.15\,\Omega+0.10\,\Omega}\right)^2=\boxed{0.40}$$

15. To fix this circuit, connect another resistor in parallel with the 480-Ω resistor so that the equivalent resistance is the desired 370 Ω.

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R_1}+\frac{1}{R_2}\quad\rightarrow\quad R_2=\left(\frac{1}{R_{\text{eq}}}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{370\,\Omega}-\frac{1}{480\,\Omega}\right)^{-1}=1615\,\Omega\approx\boxed{1600\,\Omega}$$

So solder a 1600-Ω resistor in parallel with the 480-Ω resistor.

16. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 960 Ω resistor in series with that parallel combination.

$$R_{\text{eq}}=\left(\frac{1}{820\,\Omega}+\frac{1}{680\,\Omega}\right)^{-1}+960\,\Omega=372\,\Omega+960\,\Omega=1332\,\Omega\approx\boxed{1330\,\Omega}$$

(*b*) The current delivered by the battery is $I=\dfrac{V}{R_{\text{eq}}}=\dfrac{12.0\,\text{V}}{1332\,\Omega}=9.009\times10^{-3}\,\text{A}$. This is the current in the 960 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470}=IR=\left(9.009\times10^{-3}\,\text{A}\right)\left(960\,\Omega\right)=8.649\,\text{V}\approx\boxed{8.6\,\text{V}}$$

Thus the voltage across the parallel combination must be $12.0\,\text{V}-8.6\,\text{V}=\boxed{3.4\,\text{V}}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

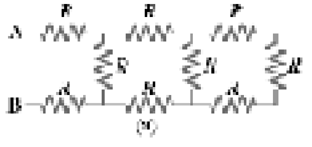
$$V_{\text{parallel}}=IR_{\text{parallel}}=\left(9.009\times10^{-3}\,\text{A}\right)\left(372\,\Omega\right)=3.351\,\text{V}\approx3.4\,\text{V}$$

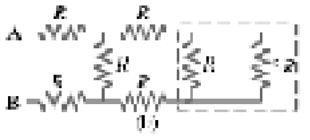
17. The resistance of each bulb can be found by using Eq. 25-7b, $P=V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P=V^2/R\quad\rightarrow\quad\frac{1}{R}=\frac{P}{V^2}$$

$$R_{\text{eq}}=\left(\frac{1}{R_{75}}+\frac{1}{R_{40}}\right)^{-1}=\left(\frac{75\,\text{W}}{\left(110\,\text{V}\right)^2}+\frac{25\,\text{W}}{\left(110\,\text{V}\right)^2}\right)^{-1}=121\,\Omega\approx\boxed{120\,\Omega}$$

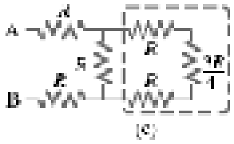
18. (*a*) The three resistors on the far right are in series, so their equivalent resistance is 3*R*. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the second figure. The equivalent resistance of the dashed box is found as follows.

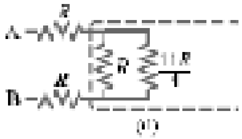




$$R_{\text{eq1}}=\left(\frac{1}{R}+\frac{1}{3R}\right)^{-1}=\tfrac{3}{4}R$$

This equivalent resistance of $\tfrac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the third figure (on the next page). The equivalent resistance of that dashed box is $R_{\text{eq2}}=2R+\tfrac{3}{4}R=\tfrac{11}{4}R.$ This $\tfrac{11}{4}R$ is in





parallel with the next resistor to the left, as shown in the fourth figure. The equivalent resistance of that dashed box is found as follows.

$$R_{\mathrm{eq2}} = \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1} = \tfrac{11}{15}R.$$

This is in series with the last two resistors, the ones connected directly to A and B. The final equivalent resistance is given below.

$$R_{\mathrm{eq}} = 2R + \tfrac{11}{15}R = \tfrac{41}{15}R = \tfrac{41}{15}(125\,\Omega) = 341.67\,\Omega \approx \boxed{342\,\Omega}$$

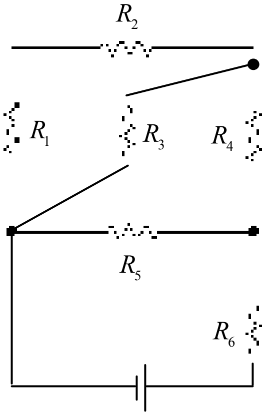
(*b*) The current flowing from the battery is found from Ohm's law.

$$I_{\mathrm{total}} = \frac{V}{R_{\mathrm{eq}}} = \frac{50.0\,\mathrm{V}}{341.67\,\Omega} = 0.1463\,\mathrm{A} \approx \boxed{0.146\,\mathrm{A}}$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\tfrac{11}{4}R$, as shown in the last figure. The voltage across R and across $\tfrac{11}{4}R$ must be the same, since they are in parallel. Use this to find the desired current.

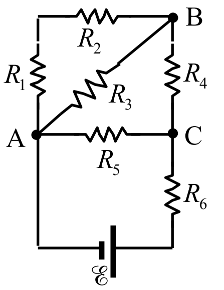
$$V_R = V_{\frac{11}{4}R} \rightarrow I_R R = I_{\frac{11}{4}R}\left(\tfrac{11}{4}R\right) = \left(I_{\mathrm{total}} - I_R\right)\left(\tfrac{11}{4}R\right) \rightarrow$$

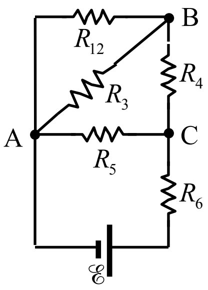
$$I_R = \tfrac{11}{15}I_{\mathrm{total}} = \tfrac{11}{15}(0.1463\,\mathrm{A})\,I_{\mathrm{total}} = \boxed{0.107\,\mathrm{A}}$$



$\boxed{19.}$ The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \tfrac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \tfrac{2}{3}R + R = \tfrac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \tfrac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.

$$R_{\mathrm{eq}} = \tfrac{5}{8}R + R = \boxed{\tfrac{13}{8}R}$$





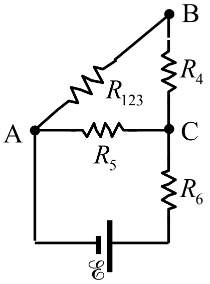
20. We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams.

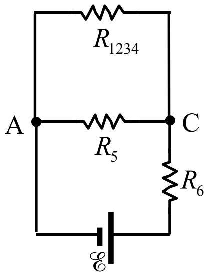
R_1 and R_2 are in series.

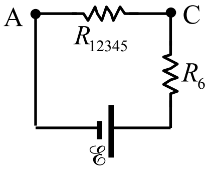
$$R_{12} = R_1 + R_2 = R + R = 2R$$

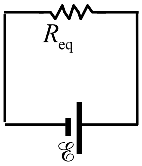
R_{12} and R_3 are in parallel.

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \tfrac{2}{3}R$$









R_{123} and R_4 are in series.

$$R_{1234} = R_{123} + R_4 = \tfrac{2}{3}R + R = \tfrac{5}{3}R$$

R_{1234} and R_5 are in parallel.

$$R_{12345} = \left(\frac{1}{R_{1234}} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{\tfrac{5}{3}R} + \frac{1}{R}\right)^{-1} = \tfrac{5}{8}R$$

R_{12345} and R_6 are in series, producing the equivalent resistance.

$$R_{\text{eq}} = R_{12345} + R_6 = \tfrac{5}{8}R + R = \tfrac{13}{8}R$$

Now work "backwards" from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the same voltage as their equivalent resistance. To avoid rounding errors, we do not use numeric values until the end of the problem.

$$I_{\text{eq}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{\mathscr{E}}{\tfrac{13}{8}R} = \boxed{\frac{8\mathscr{E}}{13R} = I_6} = I_{12345}$$

$$V_5 = V_{1234} = V_{12345} = I_{12345}R_{12345} = \left(\frac{8\mathscr{E}}{13R}\right)\left(\tfrac{5}{8}R\right) = \tfrac{5}{13}\mathscr{E} \;;\; I_5 = \frac{V_5}{R_5} = \frac{\tfrac{5}{13}\mathscr{E}}{R} = \boxed{\frac{5\mathscr{E}}{13R} = I_5}$$

$$I_{1234} = \frac{V_{1234}}{R_{1234}} = \frac{\tfrac{5}{13}\mathscr{E}}{\tfrac{5}{3}R} = \boxed{\frac{3\mathscr{E}}{13R} = I_4} = I_{123} \;;\; V_{123} = I_{123}R_{123} = \left(\frac{3\mathscr{E}}{13R}\right)\left(\tfrac{2}{3}R\right) = \tfrac{2}{13}\mathscr{E} = V_{12} = V_3$$

$$I_3 = \frac{V_3}{R_3} = \boxed{\frac{2\mathscr{E}}{13R} = I_3} \;;\; I_{12} = \frac{V_{12}}{R_{12}} = \frac{\tfrac{2}{13}\mathscr{E}}{2R} = \boxed{\frac{\mathscr{E}}{13R} = I_1 = I_2}$$

Now substitute in numeric values.

$$I_1 = I_2 = \frac{\mathscr{E}}{13R} = \frac{12.0\,\text{V}}{13(1.20\,\text{k}\Omega)} = \boxed{0.77\,\text{mA}} \;;\; I_3 = \frac{2\mathscr{E}}{13R} = \boxed{1.54\,\text{mA}} \;;\, I_4 = \frac{3\mathscr{E}}{13R} = \boxed{2.31\,\text{mA}} \;;$$

$$I_5 = \frac{5\mathscr{E}}{13R} = \boxed{3.85\,\text{mA}} \;;\; I_6 = \frac{8\mathscr{E}}{13R} = \boxed{6.15\,\text{mA}} \;;\; V_{\text{AB}} = V_3 = \tfrac{2}{13}\mathscr{E} = \boxed{1.85\,\text{V}}$$

21. The resistors *r* and *R* are in series, so the equivalent resistance of the circuit is $R + r$ and the current in the resistors is $I = \dfrac{\mathscr{E}}{R + r}$. The power delivered to load resistor is found from Eq. 25-7a. To find the value of *R* that maximizes this delivered power, set $\dfrac{dP}{dR} = 0$ and solve for *R*.

$$P = I^2R = \left(\frac{\mathscr{E}}{R + r}\right)R = \frac{\mathscr{E}^2R}{(R + r)^2} \;;\; \frac{dP}{dR} = \mathscr{E}^2\left[\frac{(R + r)^2 - R(2)(R + r)}{(R + r)^4}\right] = 0 \;\rightarrow$$

$$(R + r)^2 - R(2)(R + r) = 0 \;\rightarrow\; R^2 + 2Rr + r^2 - 2R^2 - 2Rr = 0 \;\rightarrow\; \boxed{R = r}$$

22. It is given that the power used when the resistors are in series is one-fourth the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$P_{\text{series}} = \tfrac{1}{4}P_{\text{parallel}} \rightarrow \frac{V^2}{R_{\text{series}}} = \tfrac{1}{4}\frac{V^2}{R_{\text{parallel}}} \rightarrow R_{\text{series}} = 4R_{\text{parallel}} \rightarrow \left(R_1+R_2\right) = 4\frac{R_1R_2}{\left(R_1+R_2\right)} \rightarrow$$

$$\left(R_1+R_2\right)^2 = 4R_1R_2 \rightarrow R_1^2+2R_1R_2+R_2^2-4R_1R_2 = 0 = \left(R_1-R_2\right)^2 \rightarrow R_1 = R_2$$

Thus the two resistors must be the same, and so the "other" resistor is $\boxed{3.8\,\text{k}\Omega}$.

23. We label identical resistors from left to right as R_{left}, R_{middle}, and R_{right}. When the switch is opened, the equivalent resistance of the circuit increases from $\tfrac{3}{2}R+r$ to $2R+r$. Thus the current delivered by the battery decreases, from $\dfrac{\mathscr{E}}{\tfrac{3}{2}R+r}$ to $\dfrac{\mathscr{E}}{2R+r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle}.

$\boxed{V_{\text{left}}\text{ decreases ; }V_{\text{middle}}\text{ increases ; }V_{\text{right}}\text{ goes to 0}}$.

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$\boxed{I_{\text{left}}\text{ decreases ; }I_{\text{middle}}\text{ increases ; }I_{\text{right}}\text{ goes to 0}}$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the $\boxed{\text{terminal voltage increases}}$.

(*d*) With the switch closed, the equivalent resistance is $\tfrac{3}{2}R+r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathscr{E}}{\tfrac{3}{2}R+r}$, and the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathscr{E} - I_{\text{closed}}r = \mathscr{E} - \frac{\mathscr{E}}{\tfrac{3}{2}R+r}r = \mathscr{E}\left(1-\frac{r}{\tfrac{3}{2}R+r}\right) = (9.0\,\text{V})\left(1-\frac{0.50\,\Omega}{\tfrac{3}{2}(5.50\,\Omega)+0.50\,\Omega}\right)$$

$$= 8.486\,\text{V} \approx \boxed{8.5\,\text{V}}$$

(*e*) With the switch open, the equivalent resistance is $2R+r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathscr{E}}{2R+r}$, and again the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathscr{E} - I_{\text{closed}}r = \mathscr{E} - \frac{\mathscr{E}}{2R+r}r = \mathscr{E}\left(1-\frac{r}{2R+r}\right) = (9.0\,\text{V})\left(1-\frac{0.50\,\Omega}{2(5.50\,\Omega)+0.50\,\Omega}\right)$$

$$= 8.609\,\text{V} \approx \boxed{8.6\,\text{V}}$$

24. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2R = \frac{V^2}{R} \rightarrow I = \sqrt{\frac{P}{R}}\,,\ V = \sqrt{RP}$$

$$I_{1800} = \sqrt{\frac{0.5\,\text{W}}{1.8\times10^3\,\Omega}} = 0.0167\,\text{A} \qquad V_{1800} = \sqrt{(0.5\,\text{W})(1.8\times10^3\,\Omega)} = 30.0\,\text{V}$$

$$I_{2800}=\sqrt{\frac{0.5\,\text{W}}{2.8\times10^{3}\,\Omega}}=0.0134\,\text{A}\qquad V_{2800}=\sqrt{\left(0.5\,\text{W}\right)\left(2.8\times10^{3}\,\Omega\right)}=37.4\,\text{V}$$

$$I_{3700}=\sqrt{\frac{0.5\,\text{W}}{3.7\times10^{3}\,\Omega}}=0.0116\,\text{A}\qquad V_{3700}=\sqrt{\left(0.5\,\text{W}\right)\left(3.7\times10^{3}\,\Omega\right)}=43.0\,\text{V}$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 37.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}}=\frac{V_{\text{parallel}}}{R_{\text{parallel}}}=V_{\text{parallel}}\left(\frac{1}{R_{2800}}+\frac{1}{R_{2100}}\right)=\left(37.4\,\text{V}\right)\left(\frac{1}{2800\,\Omega}+\frac{1}{3700\,\Omega}\right)=0.0235\,\text{A}$$

This is more than the maximum current that can be in R_{1800}. Thus the maximum current that R_{1800} can carry, $0.0167\,\text{A}$, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of R_{2800} and R_{3700} added to R_{1800}.

$$V_{\max}=I_{\max}R_{\text{eq}}=I_{\max}\left[R_{1800}+\left(\frac{1}{R_{2800}}+\frac{1}{R_{3700}}\right)^{-1}\right]=\left(0.0167\,\text{A}\right)\left[1800\,\Omega+\left(\frac{1}{2800\,\Omega}+\frac{1}{3700\,\Omega}\right)^{-1}\right]$$

$$=56.68\,\text{V}\approx\boxed{57\,\text{V}}$$

$\boxed{25.}$ (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with R_3 and R_4, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since R_1 is in series with the battery, its voltage will increase. Because of that increase, the voltage across R_3 and R_4 must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across R_2 until the switch was closed, its voltage will increase. To summarize:

$$\boxed{V_1\text{ and }V_2\text{ increase };\ V_3\text{ and }V_4\text{ decrease}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$$\boxed{I_1\text{ and }I_2\text{ increase };\ I_3\text{ and }I_4\text{ decrease}}$$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is R_3 and R_4 in parallel, combined with R_1 in series.

$$R_{\text{eq}}=R_1+\left(\frac{1}{R_3}+\frac{1}{R_4}\right)^{-1}=125\,\Omega+\left(\frac{2}{125\,\Omega}\right)^{-1}=187.5\,\Omega$$

The current delivered by the battery is the same as the current through R_1.

$$I_{\text{total}}=\frac{V_{\text{battery}}}{R_{\text{eq}}}=\frac{22.0\,\text{V}}{187.5\,\Omega}=0.1173\,\text{A}=I_1$$

The voltage across R_1 is found by Ohm's law.

$$V_1 = IR_1 = (0.1173\,\text{A})(125\,\Omega) = 14.66\,\text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across R_1.

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 14.66\,\text{V} = 7.34\,\text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_\text{p}}{R_2} = \frac{7.34\,\text{V}}{125\,\Omega} = 0.0587\,\text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.117\,\text{A} \quad I_3 = I_4 = 0.059\,\text{A}}$$

After the switch is closed, the equivalent resistance is R_2, R_3, and R_4 in parallel, combined with R_1 in series. Do a similar analysis.

$$R_\text{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{3}{125\,\Omega}\right)^{-1} = 166.7\,\Omega$$

$$I_\text{total} = \frac{V_\text{battery}}{R_\text{eq}} = \frac{22.0\,\text{V}}{166.7\,\Omega} = 0.1320\,\text{A} = I_1 \qquad V_1 = IR_1 = (0.1320\,\text{A})(125\,\Omega) = 16.5\,\text{V}$$

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 16.5\,\text{V} = 5.5\,\text{V} \quad I_2 = \frac{V_\text{p}}{R_2} = \frac{5.5\,\text{V}}{125\,\Omega} = 0.044\,\text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.132\,\text{A} \quad I_2 = I_3 = I_4 = 0.044\,\text{A}}$$

$\boxed{\text{Yes}}$, the predictions made in part (*b*) are all confirmed.

26. The goal is to determine *r* so that $\left.\frac{dP_R}{dR}\right|_{R=R_0} = 0$. This ensures that *R* produce very little change in P_R, since $\Delta P_R \approx \frac{dP_R}{dR}\Delta R.$ The power delivered to the heater can be found by $P_\text{heater} = V_\text{heater}^2/R,$ and so we need to determine the voltage across the heater. We do this by calculating the current drawn from the voltage source, and then subtracting the voltage drop across *r* from the source voltage.

$$R_\text{eq} = r + \frac{Rr}{R+r} = \frac{2Rr + r^2}{R+r} = \frac{r(2R+r)}{R+r} \;;\; I_\text{total} = \frac{\mathscr{E}}{R_\text{eq}} = \frac{\mathscr{E}}{\frac{r(2R+r)}{R+r}} = \frac{\mathscr{E}(R+r)}{r(2R+r)}$$

$$V_\text{heater} = \mathscr{E} - I_\text{total}r = \mathscr{E} - \frac{\mathscr{E}(R+r)}{r(2R+r)}r = \mathscr{E} - \frac{\mathscr{E}(R+r)}{(2R+r)} = \frac{\mathscr{E}R}{(2R+r)} \;;\; P_\text{heater} = \frac{V_\text{heater}^2}{R} = \frac{\mathscr{E}^2 R}{(2R+r)^2}$$

$$\left.\frac{dP_\text{heater}}{dR}\right|_{R=R_0} = \mathscr{E}^2\frac{(2R_0+r)^2 - R_0(2)(2R_0+r)(2)}{(2R_0+r)^4} = 0 \rightarrow (2R_0+r)^2 - R_0(2)(2R_0+r)(2) = 0 \rightarrow$$

$$4R_0^2 + 4R_0r + r^2 - 8R_0^2 - 4R_0r = 0 \rightarrow r^2 = 4R_0^2 \rightarrow \boxed{r = 2R_0}$$

27. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{9.0\,\text{V}}{(9.5 + 12.0 + 2.0)\,\Omega} = 0.383\,\text{A} \approx \boxed{0.38\,\text{A}}$$

$$\begin{aligned}\sum \text{voltages} &= 9.0\,\text{V} - (9.5\,\Omega)(0.383\,\text{A}) - (12.0\,\Omega)(0.383\,\text{A}) - (2.0\,\Omega)(0.383\,\text{A}) \\ &= 9.0\,\text{V} - 3.638\,\text{V} - 4.596\,\text{V} - 0.766\,\text{V} = \boxed{0.00\,\text{V}}\end{aligned}$$

28. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(2.0\Omega) + 18\,\text{V} - I(6.6\ \Omega) - 12\,\text{V} - I(1.0\Omega) = 0 \quad \rightarrow \quad I = \frac{6\,\text{V}}{9.6\ \Omega} = 0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery:}\ V_{\text{terminal}} = -I(2.0\Omega) + 18\,\text{V} = -(0.625\,\text{A})(2.0\Omega) + 18\,\text{V} = 16.75\text{V} \approx \boxed{17\,\text{V}}$$

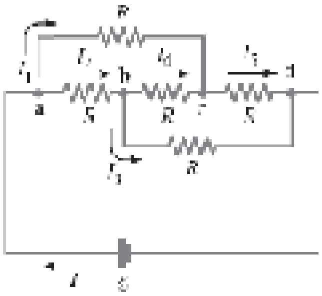
$$12\,\text{V battery:}\ V_{\text{terminal}} = I(1.0\Omega) + 12\,\text{V} = (0.625\,\text{A})(1.0\Omega) + 12\,\text{V} = 12.625\,\text{V} \approx \boxed{13\,\text{V}}$$

29. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR + \mathscr{E} - IR - IR + \mathscr{E} - IR = 0 \quad \rightarrow \quad I = \frac{\mathscr{E}}{2R}$$

$$V_{\text{ab}} = V_{\text{a}} - V_{\text{b}} = -IR + \mathscr{E} - IR = \mathscr{E} - 2IR = \mathscr{E} - 2\frac{\mathscr{E}}{2R}R = \boxed{0\,\text{V}}$$

30. (*a*) We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a-c) we write equations relating the entering and exiting currents.



$$I = I_1 + I_2 \qquad [1]$$

$$I_2 = I_3 + I_4 \qquad [2]$$

$$I_1 + I_4 = I_5 \qquad [3]$$

We use Kirchhoff's loop rule to write equations for loops abca, abcda, and bdcb.

$$0 = -I_2R - I_4R + I_1R \qquad [4]$$

$$0 = -I_2R - I_3R + \mathscr{E} \qquad [5]$$

$$0 = -I_3R + I_5R + I_4R \qquad [6]$$

We have six unknown currents and six equations. We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current I_5. Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate I_2.

$$0=-I_3R+\left(I_1+I_4\right)R+I_4R \quad \rightarrow 0=-I_3R+I_1R+2I_4R \qquad [6^*]$$

$$I=I_1+I_3+I_4 \qquad [1^*]$$

$$0=-\left(I_3+I_4\right)R-I_4R+I_1R \rightarrow 0=-I_3R-2I_4\mathrm{R}+I_1R \qquad [4^*]$$

$$0=-\left(I_3+I_4\right)R-I_3R+\mathscr{E} \rightarrow 0=-I_4R-2I_3R+\mathscr{E} \qquad [5^*]$$

Next we solve Eq. [4*] for I_4 and insert the result into Eqs. [1*], [5*], and [6*].

$$0=-I_3R-2I_4R+I_1R \rightarrow I_4=\tfrac{1}{2}I_1-\tfrac{1}{2}I_3$$

$$I=I_1+I_3+\tfrac{1}{2}I_1-\tfrac{1}{2}I_3 \rightarrow I=\tfrac{3}{2}I_1+\tfrac{1}{2}I_3 \qquad [1^{**}]$$

$$0=-I_3R+I_1R+2\left(\tfrac{1}{2}I_1-\tfrac{1}{2}I_3\right)R=-2I_3R+2I_1R \rightarrow I_1=I_3 \qquad [6^{**}]$$

$$0=-\left(\tfrac{1}{2}I_1-\tfrac{1}{2}I_3\right)R-2I_3R+\mathscr{E} \rightarrow 0=-\tfrac{1}{2}I_1R-\tfrac{3}{2}I_3R+\mathscr{E} \qquad [5^{**}]$$

Finally we substitute Eq. [6**] into Eq [5**] and solve for I_1. We insert this result into Eq. [1**] to write an equation for the current through the battery in terms of the battery emf and resistance.

$$0=-\tfrac{1}{2}I_1R-\tfrac{3}{2}I_1R+\mathscr{E} \rightarrow I_1=\frac{\mathscr{E}}{2R} \ ; \ I=\tfrac{3}{2}I_1+\tfrac{1}{2}I_1 \ =2I_1 \rightarrow \boxed{I=\frac{\mathscr{E}}{R}}$$

(*b*) We divide the battery emf by the current to determine the effective resistance.

$$R_{eq}=\frac{\mathscr{E}}{I}=\frac{\mathscr{E}}{\mathscr{E}/R}=\boxed{R}$$

31. This circuit is identical to Example 26-9 and Figure 26-13 except for the numeric values. So we may copy the same equations as developed in that Example, but using the current values.

Eq. (*a*): $I_3=I_1+I_2$; Eq. (*b*): $-34I_1+45-48I_3=0$

Eq. (*c*): $-34I_1+19I_2-75=0$ Eq. (*d*): $I_2=\dfrac{75+34I_1}{19}=3.95+1.79I_1$

Eq. (*e*): $I_3=\dfrac{45-34I_1}{48}=0.938-0.708I_1$

$$I_3=I_1+I_2 \quad \rightarrow \quad 0.938-0.708I_1=I_1+3.95+1.79I_1 \quad \rightarrow \quad I_1=-0.861\,\mathrm{A}$$

$$I_2=3.95+1.79I_1=2.41\,\mathrm{A} \ ; \ I_3=0.938-0.708I_1=1.55\,\mathrm{A}$$

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\mathrm{ad}}=V_{\mathrm{d}}-V_{\mathrm{a}}=-I_1\left(34\,\Omega\right)=-\left(-0.861\,\mathrm{A}\right)\left(34\,\Omega\right)=29.27\,\mathrm{V}\approx\boxed{29\,\mathrm{V}}$$

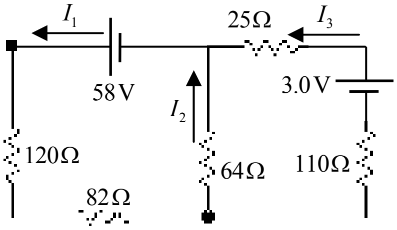
Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$V_{\mathrm{ad}}=V_{\mathrm{d}}-V_{\mathrm{a}}=\mathscr{E}_1-I_2\left(19\,\Omega\right)=75\,\mathrm{V}-\left(2.41\,\mathrm{A}\right)\left(19\,\Omega\right)=29.21\,\mathrm{V}\approx 29\,\mathrm{V}$$

(*b*) For the 75-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

75 V battery: $V_{\text{terminal}}=\mathscr{E}_1-I_2r=75\,\mathrm{V}-\left(2.41\,\mathrm{A}\right)\left(1.0\,\Omega\right)=\boxed{73\,\mathrm{V}}$

45 V battery: $V_{\text{terminal}}=\mathscr{E}_2-I_3r=45\,\mathrm{V}-\left(1.55\,\mathrm{A}\right)\left(1.0\,\Omega\right)=\boxed{43\,\mathrm{V}}$



32. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58\,\text{V} - I_1(120\Omega) - I_1(82\Omega) - I_2(64\Omega) = 0 \;\rightarrow\; 58 = 202I_1 + 64I_2$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\,\text{V} - I_3(25\Omega) + I_2(64\Omega) - I_3(110\Omega) = 0 \;\rightarrow\; 3 = -64I_2 + 135I_3$$

Substitute $I_1 = I_2 + I_3$ into the left loop equation, so that there are two equations with two unknowns.

$$58 = 202(I_2 + I_3) + 64I_2 = 266I_2 + 202I_3$$

Solve the right loop equation for I_2 and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$3 = -64I_2 + 135I_3 \;\rightarrow\; I_2 = \frac{135I_3 - 3}{64} \;;\; 58 = 266I_2 + 202I_3 = 266\left(\frac{135I_3 - 3}{64}\right) + 202I_3 \;\rightarrow$$

$$I_3 = 0.09235\,\text{A} \;;\; I_2 = \frac{135I_3 - 3}{64} = 0.1479\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.24025\,\text{A}$$

The current in each resistor is as follows:

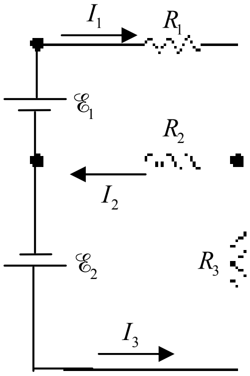
$$\boxed{120\Omega\text{: } 0.24\,\text{A} \quad 82\Omega\text{: } 0.24\,\text{A} \quad 64\Omega\text{: } 0.15\,\text{A} \quad 25\Omega\text{: } 0.092\,\text{A} \quad 110\Omega\text{: } 0.092\,\text{A}}$$

33. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

$$V_3 - I_1R_1 + V_1 = 0 \;\rightarrow\; I_1 = \frac{V_3 + V_1}{R_1} = \frac{6.0\,\text{V} + 9.0\,\text{V}}{22\Omega} = \boxed{0.68\,\text{A, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3 - I_2R_2 = 0 \;\rightarrow\; I_2 = \frac{V_3}{R_2} = \frac{6.0\,\text{V}}{18\Omega} = \boxed{0.33\,\text{A, left}}$$



34. (a) There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.

$$I_2 = I_1 + I_3 \;\rightarrow\; I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathscr{E}_1 - I_1R_1 - I_2R_2 = 0 \;\rightarrow\; 9 = 25I_1 + 48I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \rightarrow 12 = 35I_3 + 48I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 48I_2 = 25(I_2 - I_3) + 48I_2 = 73I_2 - 25I_3 \; ; \; 12 = 35I_3 + 48I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 35I_3 + 48I_2 \rightarrow I_2 = \frac{12 - 35I_3}{48}$$

$$9 = 73I_2 - 25I_3 = 73\left(\frac{12 - 35I_3}{48}\right) - 25I_3 \rightarrow 432 = 876 - 2555I_3 - 1200I_3 \rightarrow$$

$$I_3 = \frac{444}{3755} = 0.1182\,\text{A} \approx \boxed{0.12\,\text{A}\text{ , up}} \; ; \; I_2 = \frac{12 - 35I_3}{48} = 0.1638\,\text{A} \approx \boxed{0.16\,\text{A}\text{ , left}}$$

$$I_1 = I_2 - I_3 = 0.0456\,\text{A} \approx \boxed{0.046\,\text{A}\text{ , right}}$$

(*b*) We can include the internal resistances simply by adding 1.0Ω to R_1 and R_3. So let $R_1 = 26\Omega$ and let $R_3 = 36\Omega$. Now re-work the problem exactly as in part (*a*).

$$I_2 = I_1 + I_3 \rightarrow I_1 = I_2 - I_3$$

$$\mathcal{E}_1 - I_1R_1 - I_2R_2 = 0 \rightarrow 9 = 26I_1 + 48I_2$$

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \rightarrow 12 = 36I_3 + 48I_2$$

$$9 = 26I_1 + 48I_2 = 26(I_2 - I_3) + 48I_2 = 74I_2 - 26I_3 \; ; \; 12 = 36I_3 + 48I_2$$

$$12 = 36I_3 + 48I_2 \rightarrow I_2 = \frac{12 - 36I_3}{48} = \frac{1 - 3I_3}{4}$$

$$9 = 74I_2 - 26I_3 = 74\left(\frac{1 - 3I_3}{4}\right) - 26I_3 \rightarrow 36 = 74 - 222I_3 - 104I_3 \rightarrow$$

$$I_3 = \frac{38}{326} = 0.1166\,\text{A} \approx \boxed{0.12\,\text{A}\text{ , up}} \; ; \; I_2 = \frac{1 - 3I_3}{4} = 0.1626\,\text{A} \approx \boxed{0.16\,\text{A, left}}$$

$$I_1 = I_2 - I_3 = \boxed{0.046\,\text{A}\text{ , right}}$$

The currents are unchanged to 2 significant figures by the inclusion of the internal resistances.

35. We are to find the ratio of the power used when the resistors are in series, to the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$R_{\text{series}} = R_1 + R_2 + \cdots R_n = nR \; ; \; R_{\text{parallel}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)^{-1} = \left(\frac{n}{R}\right)^{-1} = \frac{R}{n}$$

$$\frac{P_{\text{series}}}{P_{\text{parallel}}} = \frac{V^2/R_{\text{series}}}{V^2/R_{\text{parallel}}} = \frac{R_{\text{parallel}}}{R_{\text{series}}} = \frac{R/n}{nR} = \boxed{\frac{1}{n^2}}$$

36. (*a*) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise. We add series resistances.

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(35\,\Omega) = 0 \;\rightarrow\; 24 = 35I_1 + 12I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 35I_1 + 12I_2 = 35(I_2 + I_3) + 12I_2 = 47I_2 + 35I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for I_3.

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 47I_2 + 35I_3 = 47\left(\frac{6 + 34I_3}{12}\right) + 35I_3 \;\rightarrow$$

$$I_3 = \boxed{2.97\,\text{mA}} \;;\; I_2 = \frac{6 + 34I_3}{12} = \boxed{0.508\,\text{A}} \;;\; I_1 = I_2 + I_3 = \boxed{0.511\,\text{A}}$$

(*b*) The terminal voltage of the 6.0-V battery is $6.0\,\text{V} - I_3 r = 6.0\,\text{V} - (2.97 \times 10^{-3}\,\text{A})(1.0\,\Omega)$

$= 5.997\,\text{V} \approx \boxed{6.0\,\text{V}}$.

$\boxed{37.}$ This problem is the same as Problem 36, except the total resistance in the top branch is now $23\,\Omega$ instead of $35\,\Omega$. We simply reproduce the adjusted equations here without the prose.

$$I_1 = I_2 + I_3$$

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(23\,\Omega) = 0 \;\rightarrow\; 24 = 23I_1 + 12I_2$$

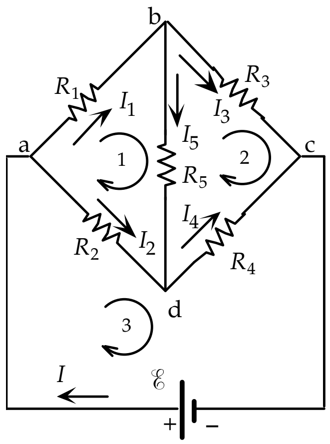
$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

$$24 = 23I_1 + 12I_2 = 23(I_2 + I_3) + 12I_2 = 35I_2 + 23I_3$$

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 35I_2 + 23I_3 = 35\left(\frac{6 + 34I_3}{12}\right) + 23I_3 \;\rightarrow$$

$$I_3 = 0.0532\,\text{A} \;;\; I_2 = \frac{6 + 34I_3}{12} = 0.6508\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.704\,\text{A} \approx \boxed{0.70\,\text{A}}$$

38. The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.



$$1)\ I = I_1 + I_2 \ ;\ 2)\ I_1 = I_3 + I_5 \ ;\ 3)\ I_3 + I_4 = I$$

$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0 \ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate *I* using equations 1) and 3).

$$1)\ I_3 + I_4 = I_1 + I_2 \ ;\ 2)\ I_1 = I_3 + I_5$$

$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0 \ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_1 using equation 2.

$$1)\ I_3 + I_4 = I_3 + I_5 + I_2 \ \to\ I_4 = I_5 + I_2$$

$$4)\ -\left(I_3 + I_5\right)R_1 - I_5R_5 + I_2R_2 = 0 \ \to\ -I_3R_1 - I_5\left(R_1 + R_5\right) + I_2R_2 = 0$$

$$5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_4 using equation 1.

$$4)\ -I_3R_1 - I_5\left(R_1 + R_5\right) + I_2R_2 = 0$$

$$5)\ -I_3R_3 + \left(I_5 + I_2\right)R_4 + I_5R_5 = 0 \ \to\ -I_3R_3 + I_5\left(R_4 + R_5\right) + I_2R_4 = 0$$

$$6)\ \mathscr{E} - I_2R_2 - \left(I_5 + I_2\right)R_4 = 0 \ \to\ \mathscr{E} - I_2\left(R_2 + R_4\right) - I_5R_4 = 0$$

Eliminate I_2 using equation 4: $I_2 = \dfrac{1}{R_2}\left[I_3R_1 + I_5\left(R_1 + R_5\right)\right]$.

$$5)\ -I_3R_3 + I_5\left(R_4 + R_5\right) + \frac{1}{R_2}\left[I_3R_1 + I_5\left(R_1 + R_5\right)\right]R_4 = 0 \ \to$$

$$I_3\left(R_1R_4 - R_2R_3\right) + I_5\left(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4\right) = 0$$

$$6)\ \mathscr{E} - \frac{1}{R_2}\left[I_3R_1 + I_5\left(R_1 + R_5\right)\right]\left(R_2 + R_4\right) - I_5R_4 = 0 \ \to$$

$$\mathscr{E}R_2 - I_3R_1\left(R_2 + R_4\right) - I_5\left(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4\right) = 0$$

Eliminate I_3 using equation 5: $I_3 = -I_5\dfrac{\left(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4\right)}{\left(R_1R_4 - R_2R_3\right)}$

$$\mathscr{E}R_2 + \left[I_5\frac{\left(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4\right)}{\left(R_1R_4 - R_2R_3\right)}\right]R_1\left(R_2 + R_4\right) - I_5\left(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4\right) = 0$$

$$\mathscr{E} = -\frac{I_5}{R_2}\left\{\left[\frac{\left(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4\right)}{\left(R_1R_4 - R_2R_3\right)}\right]R_1\left(R_2 + R_4\right) - \left(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4\right)\right\}$$

$$= -\frac{I_5}{25\Omega}\left\{\begin{matrix}\left[\dfrac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)}\right](22\Omega)(25\Omega + 14\Omega) \\ -\left[(22\Omega)(25\Omega) + (22)(14) + (15\Omega)(25\Omega) + (15\Omega)(14\Omega) + (25\Omega)(14\Omega)\right]\end{matrix}\right\}$$

$$= -I_5\left(5261\,\Omega\right) \rightarrow I_5 = -\frac{6.0\,\text{V}}{5261\,\Omega} = -1.140\,\text{mA}\,\left(\text{upwards}\right)$$

$$I_3 = -I_5\frac{\left(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4\right)}{\left(R_1R_4 - R_2R_3\right)}$$

$$= -\left(-1.140\,\text{mA}\right)\frac{\left(25\,\Omega\right)\left(14\,\Omega\right)+\left(25\,\Omega\right)\left(15\,\Omega\right)+\left(22\,\Omega\right)\left(14\,\Omega\right)+\left(15\,\Omega\right)\left(14\,\Omega\right)}{\left(22\,\Omega\right)\left(14\,\Omega\right)-\left(25\,\Omega\right)\left(12\,\Omega\right)} = 0.1771\,\text{A}$$

$$I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5\left(R_1 + R_5\right)\right] = \frac{1}{25\,\Omega}\left[\left(0.1771\,\text{A}\right)\left(22\,\Omega\right) + \left(-0.00114\,\text{A}\right)\left(37\,\Omega\right)\right] = 0.1542\,\text{A}$$

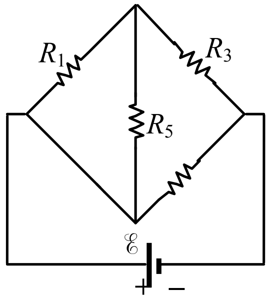
$$I_4 = I_5 + I_2 = -0.00114\,\text{A} + 0.1542\,\text{A} = 0.1531\,\text{A}$$

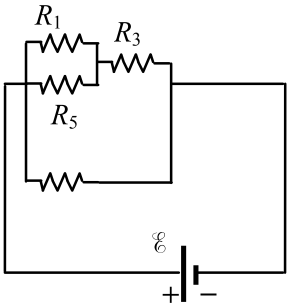
$$I_1 = I_3 + I_5 = 0.1771\,\text{A} - 0.00114\,\text{A} = 0.1760\,\text{A}$$

We keep an extra significant figure to show the slight difference in the currents.

$$\boxed{I_{22\Omega} = 0.176\text{A} \quad I_{25\Omega} = 0.154\,\text{A} \quad I_{12\Omega} = 0.177\,\text{A} \quad I_{14\Omega} = 0.153\,\text{A} \quad I_{15\Omega} = 0.001\,\text{A, upwards}}$$

39. The circuit diagram from Problem 38 is reproduced, with $R_2 = 0$. This circuit can now be simplified significantly. Resistors R_1 and R_5 are in parallel. Call that combination R_{15}. That combination is in series with R_3. Call that combination R_{153}. That combination is in parallel with R_4. See the second diagram. We calculate the equivalent resistance R_{153}, use that to find the current through the top branch in the second diagram, and then use that current to find the current through R_5.





$$R_{153} = \left(\frac{1}{R_1} + \frac{1}{R_5}\right)^{-1} + R_3 = \left(\frac{1}{22\,\Omega} + \frac{1}{15\,\Omega}\right)^{-1} + 12\,\Omega = 20.92\,\Omega$$

Use the loop rule for the outside loop to find the current in the top branch.

$$\mathcal{E} - I_{153}R_{153} = 0 \rightarrow I_{153} = \frac{\mathcal{E}}{R_{153}} = \frac{6.0\,\text{V}}{20.92\,\Omega} = 0.2868\,\text{A}$$

This current is the sum of the currents in R_1 and R_5. Since those two resistors are in parallel, the voltage across them must be the same.

$$V_1 = V_5 \rightarrow I_1R_1 = I_5R_5 \rightarrow \left(I_{153} - I_5\right)R_1 = I_5R_5 \rightarrow$$

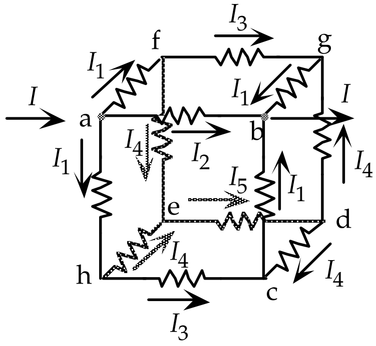
$$I_5 = I_{153}\frac{R_1}{\left(R_5 + R_1\right)} = \left(0.2868\,\text{A}\right)\frac{22\,\Omega}{37\,\Omega} = \boxed{0.17\,\text{A}}$$

40. (*a*) As shown in the diagram, we use symmetry to reduce the number of independent currents to six. Using Kirchhoff's junction rule, we write equations for junctions a, c, and d. We then use Kirchhoff's loop rule to write the loop equations for loops afgba, hedch, and aba (through the voltage source).

$I = 2I_1 + I_2$ [1] ; $I_3 + I_4 = I_1$ [2] ; $I_5 = 2I_4$ [3]

$0 = -2I_1R - I_3R + I_2R$ [4] ; $0 = -2I_4R - I_5R + I_3R$ [5]

$0 = \mathcal{E} - I_2R$ [6]



We have six equations with six unknown currents. We use the method of substitution to reduce the equations to a single equation relating the emf from the power source to the current through the power source. This resulting ratio is the effective resistance between points a and b. We insert Eqs. [2], [3], and [6] into the other three equations to eliminate I_1, I_2, and I_5.

$$I = 2(I_3 + I_4) + \frac{\mathscr{E}}{R} = 2I_3 + 2I_4 + \frac{\mathscr{E}}{R} \qquad [1^*]$$

$$0 = -2(I_3 + I_4)R - I_3R + \frac{\mathscr{E}}{R}R = -2I_4R - 3I_3R + \mathscr{E} \qquad [4^*]$$

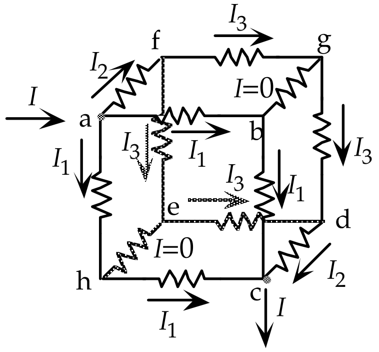
$$0 = -2I_4R - 2I_4R + I_3R = -4I_4R + I_3R \qquad [5^*]$$

We solve Eq. [5*] for I_3 and insert that into Eq. [4*]. We then insert the two results into Eq. [1*] and solve for the effective resistance.

$$I_3 = 4I_4 \ ; \ 0 = -2I_4R - 3(4I_4)R + \mathscr{E} \to I_4 = \frac{\mathscr{E}}{14R}$$

$$I = 2(4I_4) + 2I_4 + \frac{\mathscr{E}}{R} = 10I_4 + \frac{\mathscr{E}}{R} = \frac{10\mathscr{E}}{14R} + \frac{\mathscr{E}}{R} = \frac{24\mathscr{E}}{14R} = \frac{12\mathscr{E}}{7R} \to R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{7}{12}R}$$

(*b*) As shown in the diagram, we use symmetry to reduce the number of currents to four. We use Kirchhoff's junction rule at junctions a and d and the loop rule around loops abca (through the voltage source) and afgdcha. This results in four equations with four unknowns. We solve these equations for the ratio of the voltage source to current *I*, to obtain the effective resistance.



$$I = 2I_1 + I_2 \ [1] \quad ; \ 2I_3 = I_2 \ [2]$$

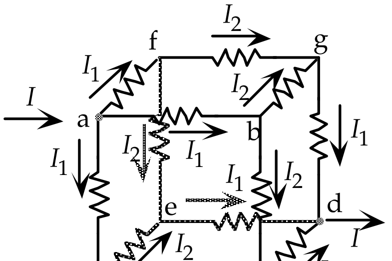
$$0 = -2I_2R + \mathscr{E} \ [3] \ ; \ 0 = -2I_2R - 2I_3R + 2I_1R \ [4]$$

We solve Eq. [3] for I_2 and Eq. [2] for I_3. These results are inserted into Eq. [4] to determine I_1. Using these results and Eq. [1] we solve for the effective resistance.

$$I_2 = \frac{\mathscr{E}}{2R} \ ; \ I_3 = \frac{I_2}{2} = \frac{\mathscr{E}}{4R} \ ; \ I_1 = I_2 + I_3 = \frac{\mathscr{E}}{2R} + \frac{\mathscr{E}}{4R} = \frac{3\mathscr{E}}{4R}$$

$$I = 2I_1 + I_2 = 2\left(\frac{3\mathscr{E}}{4R}\right) + \frac{\mathscr{E}}{2R} = \frac{2\mathscr{E}}{R} \ ; \ R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{1}{2}R}$$

(*c*) As shown in the diagram, we again use symmetry to reduce the number of currents to three. We use Kirchhoff's junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.



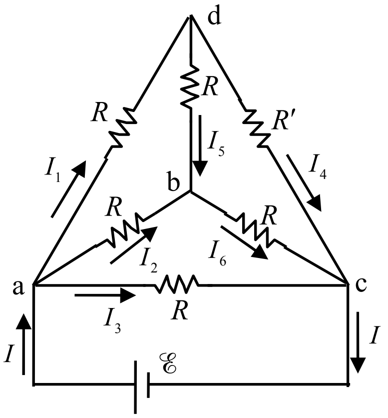
$$I = 3I_1 \ [1] \ ; \ I_1 = 2I_2 \ [2]$$

$$0 = -2I_1R - I_2R + \mathscr{E} \ [3]$$

We insert Eq. [2] into Eq. [3] and solve for I_1. Inserting I_1 into Eq. [1] enables us to solve for the effective resistance.

$$0 = -2I_1R - \tfrac{1}{2}I_1R + \mathscr{E} \to I_1 = \frac{2\mathscr{E}}{5R} \ ; \ I = 3I_1 = \frac{6\mathscr{E}}{5R} \to R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{5}{6}R}$$

41\. (*a*) To find the equivalent resistance between points a and c, apply a voltage between points a and c, find the current that flows from the voltage source, and then calculate $R_{\text{eq}} = \mathcal{E}/I$.



There is no symmetry to exploit.

$$\text{(Bottom Loop)} \quad 1)\quad \mathcal{E} - RI_3 = 0$$

$$\text{(a - d - b)} \quad 2)\quad -RI_1 - RI_5 + RI_2 = 0$$

$$\text{(a - b - c)} \quad 3)\quad -RI_2 - RI_6 + RI_3 = 0$$

$$\text{(d - b - c)} \quad 4)\quad -RI_5 - RI_6 + R'I_4 = 0$$

$$\text{(junction a)} \quad 5)\quad I = I_1 + I_2 + I_3$$

$$\text{(junction d)} \quad 6)\quad I_1 = I_4 + I_5$$

$$\text{(junction b)} \quad 7)\quad I_2 + I_5 = I_6$$

From Eq. 1, substitute $I_3 = \mathcal{E}/R$.

$$2)\quad -RI_1 - RI_5 + RI_2 = 0 \;\rightarrow\; I_1 + I_5 = I_2$$

$$3)\quad -RI_2 - RI_6 + R\frac{\mathcal{E}}{R} = 0 \;\rightarrow\; I_2 + I_6 = \frac{\mathcal{E}}{R}$$

$$4)\quad -RI_5 - RI_6 + R'I_4 = 0 \;\rightarrow\; R\left(I_5 + I_6\right) = R'I_4$$

$$5)\quad I = I_1 + I_2 + \frac{\mathcal{E}}{R} \quad ; \quad 6)\quad I_1 = I_4 + I_5 \quad ; \quad 7)\quad I_2 + I_5 = I_6$$

From Eq. 7, substitute $I_6 = I_2 + I_5$

$$2)\quad I_1 + I_5 = I_2 \quad ; \quad 3)\quad I_2 + I_2 + I_5 = \frac{\mathcal{E}}{R} \;\rightarrow\; 2I_2 + I_5 = \frac{\mathcal{E}}{R}$$

$$4)\quad R\left(2I_5 + I_2\right) = R'I_4 \quad ; \quad 5)\quad I = I_1 + I_2 + \frac{\mathcal{E}}{R} \quad ; \quad 6)\quad I_1 = I_4 + I_5$$

From Eq. 6, substitute $I_1 = I_4 + I_5 \;\rightarrow\; I_5 = I_1 - I_4$

$$2)\quad 2I_1 - I_4 = I_2 \quad ; \quad 3)\quad 2I_2 + I_1 - I_4 = \frac{\mathcal{E}}{R}$$

$$4)\quad R\left(2I_1 - 2I_4 + I_2\right) = R'I_4 \quad ; \quad 5)\quad I = I_1 + I_2 + \frac{\mathcal{E}}{R}$$

From Eq. 2, substitute $2I_1 - I_4 = I_2 \;\rightarrow\; I_4 = 2I_1 - I_2$

$$3)\quad 2I_2 + I_1 - \left(2I_1 - I_2\right) = \frac{\mathcal{E}}{R} \;\rightarrow\; 3I_2 - I_1 = \frac{\mathcal{E}}{R}$$

$$4)\quad R\left(2I_1 - 2\left(2I_1 - I_2\right) + I_2\right) = R'\left(2I_1 - I_2\right) \;\rightarrow\; R\left(3I_2 - 2I_1\right) = R'\left(2I_1 - I_2\right)$$

$$5)\quad I = I_1 + I_2 + \frac{\mathcal{E}}{R}$$

From Eq. 3, substitute $3I_2 - I_1 = \dfrac{\mathcal{E}}{R} \;\rightarrow\; I_1 = 3I_2 - \dfrac{\mathcal{E}}{R}$

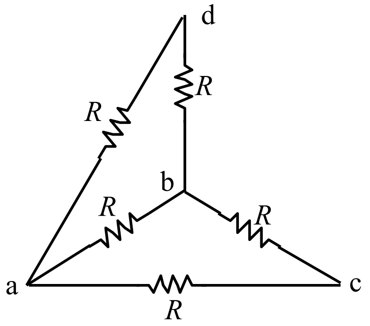
$$4)\quad R\left(3I_2 - 2\left(3I_2 - \frac{\mathcal{E}}{R}\right)\right) = R'\left(2\left(3I_2 - \frac{\mathcal{E}}{R}\right) - I_2\right) \;\rightarrow\; R\left(-3I_2 + 2\frac{\mathcal{E}}{R}\right) = R'\left(5I_2 - 2\frac{\mathcal{E}}{R}\right)$$

$$5)\quad I = 3I_2 - \frac{\mathcal{E}}{R} + I_2 + \frac{\mathcal{E}}{R} \quad \rightarrow \quad I = 4I_2$$

From Eq. 5, substitute $I_2 = \frac{1}{4}I$

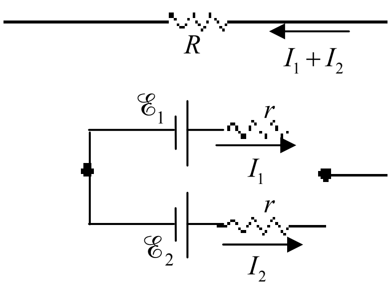
$$4)\quad R\left(-3\left(\tfrac{1}{4}I\right) + 2\frac{\mathcal{E}}{R}\right) = R'\left(5\left(\tfrac{1}{4}I\right) - 2\frac{\mathcal{E}}{R}\right) \quad \rightarrow \quad \frac{\mathcal{E}}{I} = \boxed{R_{\text{eq}} = \frac{R(5R' + 3R)}{8(R + R')}}$$

(*b*) In this case, apply a voltage between points a and b. Now there is symmetry. In this case no current would flow through resistor R', and so that branch can be eliminated from the circuit. See the adjusted diagram. Now the upper left two resistors (from a to d to b) are in series, and the lower right two resistors (from a to c to b) are in series. These two combinations are in parallel with each other, and with the resistor between a and b. The equivalent resistance is now relatively simple to calculate.



$$R_{\text{eq}} = \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}\right)^{-1} = \left(\frac{4}{2R}\right)^{-1} = \boxed{\tfrac{1}{2}R}$$

42. Define I_1 to be the current to the right through the 2.00 V battery $(\mathcal{E}_1)$, and I_2 to be the current to the right through the 3.00 V battery $(\mathcal{E}_2)$. At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.



$$\mathcal{E}_1 - I_1 r - (I_1 + I_2)R = 0 \quad \rightarrow \quad \mathcal{E}_1 - (R + r)I_1 - RI_2 = 0$$

$$\mathcal{E}_2 - I_2 r - (I_1 + I_2)R = 0 \quad \rightarrow \quad \mathcal{E}_2 - RI_1 - (R + r)I_2 = 0$$

Solve the first equation for I_2 and substitute into the second equation to solve for I_1.

$$\mathcal{E}_1 - (R + r)I_1 - RI_2 = 0 \quad \rightarrow \quad I_2 = \frac{\mathcal{E}_1 - (R + r)I_1}{R} = \frac{2.00 - 4.450I_1}{4.00} = 0.500 - 1.1125I_1$$

$$\mathcal{E}_2 - RI_1 - (R + r)I_2 = 3.00\,\text{V} - (4.00\,\Omega)I_1 - (4.45\,\Omega)(0.500 - 1.1125I_1) = 0 \quad \rightarrow$$

$$I_1 = -0.815\,\text{A} \ ; \ I_2 = 0.500 - 1.1125I_1 = 1.407\,\text{A}$$

The voltage across R is its resistance times $I = I_1 + I_2$.

$$V_R = R(I_1 + I_2) = (4.00\,\Omega)(-0.815\,\text{A} + 1.407\,\text{A}) = 2.368\,\text{V} \approx \boxed{2.37\,\text{V}}$$

Note that the top battery is being charged – the current is flowing through it from positive to negative.

$\boxed{43.}$ We estimate the time between cycles of the wipers to be from 1 second to 15 seconds. We take these times as the time constant of the *RC* combination.

$$\tau = RC \quad \rightarrow \quad R_{1\text{s}} = \frac{\tau}{C} = \frac{1\,\text{s}}{1 \times 10^{-6}\,\text{F}} = 10^6\,\Omega \ ; \ R_{1\text{s}} = \frac{\tau}{C} = \frac{15\,\text{s}}{1 \times 10^{-6}\,\text{F}} = 15 \times 10^6\,\Omega$$

So we estimate the range of resistance to be $\boxed{1\,\text{M}\Omega \ - \ 15\,\text{M}\Omega}$.

44. (*a*) From Eq. 26-7 the product *RC* is equal to the time constant.

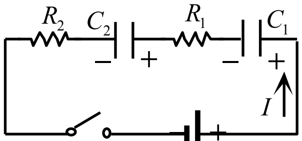
$$\tau = RC \quad \rightarrow \quad C = \frac{\tau}{R} = \frac{24.0 \times 10^{-6}\,\text{s}}{15.0 \times 10^{3}\,\Omega} = \boxed{1.60 \times 10^{-9}\,\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathscr{E}\left(1 - e^{-t/\tau}\right) \quad \rightarrow \quad e^{-t/\tau} = \left(1 - \frac{V_C}{\mathscr{E}}\right) \quad \rightarrow \quad -\frac{t}{\tau} = \ln\left(1 - \frac{V_C}{\mathscr{E}}\right) \quad \rightarrow$$

$$t = -\tau \ln\left(1 - \frac{V_C}{\mathscr{E}}\right) = -\left(24.0 \times 10^{-6}\,\text{s}\right)\ln\left(1 - \frac{8.0\,\text{V}}{24.0\,\text{V}}\right) = \boxed{9.73 \times 10^{-6}\,\text{s}}$$

45. The current for a capacitor-charging circuit is given by Eq. 26-8, with *R* the equivalent series resistance and *C* the equivalent series capacitance.



$$I = \frac{\mathscr{E}}{R_{\text{eq}}} e^{-\left(\frac{t}{R_{\text{eq}}C_{\text{eq}}}\right)} \quad \rightarrow$$

$$t = -R_{\text{eq}}C_{\text{eq}} \ln\left(\frac{IR_{\text{eq}}}{\mathscr{E}}\right) = -\left(R_1 + R_2\right)\left(\frac{C_1C_2}{C_1 + C_2}\right)\ln\left[\frac{I\left(R_1 + R_2\right)}{\mathscr{E}}\right]$$

$$= -\left(4400\,\Omega\right)\left[\frac{\left(3.8 \times 10^{-6}\,\text{F}\right)^2}{7.6 \times 10^{-6}\,\text{F}}\right]\ln\left[\frac{\left(1.50 \times 10^{-3}\,\text{A}\right)\left(4400\,\Omega\right)}{\left(12.0\text{V}\right)}\right] = \boxed{5.0 \times 10^{-3}\,\text{s}}$$

46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the capacitor is given by Eq. 26-6a.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{\left[C\mathscr{E}\left(1 - e^{-t/\tau}\right)\right]^2}{C} = \tfrac{1}{2}C\mathscr{E}^2\left(1 - e^{-t/\tau}\right)^2 = U_{\text{max}}\left(1 - e^{-t/\tau}\right)^2 \quad ;$$

$$U = 0.75U_{\text{max}} \quad \rightarrow \quad U_{\text{max}}\left(1 - e^{-t/\tau}\right)^2 = 0.75U_{\text{max}} \quad \rightarrow \quad \left(1 - e^{-t/\tau}\right)^2 = 0.75 \quad \rightarrow$$

$$t = -\tau \ln\left(1 - \sqrt{0.75}\right) = \boxed{2.01\tau}$$

47. The capacitance is given by Eq. 24-8 and the resistance by Eq. 25-3. The capacitor plate separation *d* is the same as the resistor length *ℓ*. Calculate the time constant.

$$\tau = RC = \left(\frac{\rho d}{A}\right)\left(K\varepsilon_0 \frac{A}{d}\right) = \boxed{\rho K \varepsilon_0} = \left(1.0 \times 10^{12}\,\Omega\text{•m}\right)\left(5.0\right)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N•m}^2\right) = \boxed{44\,\text{s}}$$

48. The voltage of the discharging capacitor is given by $V_{\text{C}} = V_0 e^{-t/RC}$. The capacitor voltage is to be $0.0010V_0$.

$$V_{\text{C}} = V_0 e^{-t/RC} \quad \rightarrow \quad 0.0010V_0 = V_0 e^{-t/RC} \quad \rightarrow \quad 0.0010 = e^{-t/RC} \quad \rightarrow \quad \ln\left(0.010\right) = -\frac{t}{RC} \quad \rightarrow$$

$$t = -RC\ln\left(0.010\right) = -\left(8.7 \times 10^{3}\Omega\right)\left(3.0 \times 10^{-6}\,\text{F}\right)\ln\left(0.0010\right) = \boxed{0.18\,\text{s}}$$

$\boxed{49.}$ (*a*) At $t = 0$, the capacitor is uncharged and so there is no voltage difference across it. The capacitor is a "short," and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \tfrac{3}{2}R \rightarrow I_1 = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{\mathscr{E}}{\tfrac{3}{2}R} = \boxed{\frac{2\mathscr{E}}{3R}} \; ; I_2 = I_3 = \tfrac{1}{2}I_1 = \boxed{\frac{\mathscr{E}}{3R}}$$

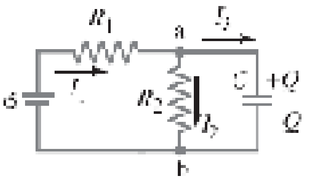
(*b*) At $t = \infty$, the capacitor will be fully charged and there will be no current in the branch containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$R_{\text{eq}} = R + R = 2R \rightarrow I_1 = I_2 = \frac{\mathscr{E}}{R_{\text{eq}}} = \boxed{\frac{\mathscr{E}}{2R}} \; ; \; I_3 = \boxed{0}$$

(*c*) At $t = \infty$, since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which I_2 flows.

$$V_C = V_{R_2} = I_2R = \left(\frac{\mathscr{E}}{2R}\right)R = \boxed{\tfrac{1}{2}\mathscr{E}}$$

50. (*a*) With the currents and junctions labeled as in the diagram, we use point a for the junction rule and the right and left loops for the loop rule. We set current I_3 equal to the derivative of the charge on the capacitor and combine the equations to obtain a single differential equation in terms of the capacitor charge. Solving this equation yields the charging time constant.



$$I_1 = I_2 + I_3 \;\; [1] \; ; \; \mathscr{E} - I_1R_1 - I_2R_2 = 0 \;\; [2] \; ; \; -\frac{Q}{C} + I_2R_2 = 0 \;\; [3]$$

We use Eq. [1] to eliminate I_1 in Eq. [2]. Then we use Eq. [3] to eliminate I_2 from Eq. [2].

$$0 = \mathscr{E} - (I_2 + I_3)R_1 - I_2R_2 \; ; \; 0 = \mathscr{E} - I_2(R_1 + R_2) - I_3R_1 \; ; \; 0 = \mathscr{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - I_3R_1$$

We set I_3 as the derivative of the charge on the capacitor and solve the differential equation by separation of variables.

$$0 = \mathscr{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - \frac{dQ}{dt}R_1 \rightarrow \int_0^Q \frac{dQ'}{Q' - \left(\dfrac{R_2C\mathscr{E}}{R_1 + R_2}\right)} = \int_0^t \frac{-(R_1 + R_2)}{R_1R_2C}dt' \rightarrow$$

$$\ln\left[Q' - \left(\frac{R_2C\mathscr{E}}{R_1 + R_2}\right)\right]_0^Q = -\frac{(R_1 + R_2)}{R_1R_2C}t'\Bigg|_0^t \rightarrow \ln\left[\frac{Q - \left(\dfrac{R_2C\mathscr{E}}{R_1 + R_2}\right)}{\left(\dfrac{R_2C\mathscr{E}}{R_1 + R_2}\right)}\right] = -\frac{(R_1 + R_2)}{R_1R_2C}t \rightarrow$$

$$Q = \frac{R_2C\mathscr{E}}{R_1 + R_2}\left(1 - e^{-\frac{(R_1+R_2)}{R_1R_2C}t}\right)$$

From the exponential term we obtain the time constant, $\tau = \boxed{\dfrac{R_1R_2C}{R_1 + R_2}}$.

(*b*) We obtain the maximum charge on the capacitor by taking the limit as time goes to infinity.

$$Q_{\max} = \lim_{t\to\infty} \frac{R_2 C\mathscr{E}}{R_1 + R_2}\left(1 - e^{-\frac{(R_1+R_2)}{R_1 R_2 C}t}\right) = \boxed{\frac{R_2 C\mathscr{E}}{R_1 + R_2}}$$

51. (*a*) With the switch open, the resistors are in series with each other, and so have the same current. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \;\rightarrow\; I = \frac{V}{R_1 + R_2} = \frac{24\,\text{V}}{8.8\,\Omega + 4.4\,\Omega} = 1.818\,\text{A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\,\text{A})(4.4\,\Omega) = \boxed{8.0\,\text{V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.48\mu\text{F})(0.36\mu\text{F})}{(0.48\mu\text{F} + 0.36\mu\text{F})} = 0.2057\mu\text{F}$$

$$Q_{\text{eq}} = VC_{\text{eq}} = (24.0\,\text{V})(0.2057\mu\text{F}) = 4.937\mu\text{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\mu\text{F}$-capacitor.

$$V_b = \frac{Q_2}{C_2} = \frac{4.937\mu\text{C}}{0.36\mu\text{F}} = 13.7\,\text{V} \approx \boxed{14\,\text{V}}$$

(*c*) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, and so point b must be at the same potential as point a, $\boxed{8.0\,\text{V}}$. This also means that the voltage across C_2 is 8.0 V, and the voltage across C_1 is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1 C_1 = (16\,\text{V})(0.48\mu\text{F}) = 7.68\mu\text{C}$$

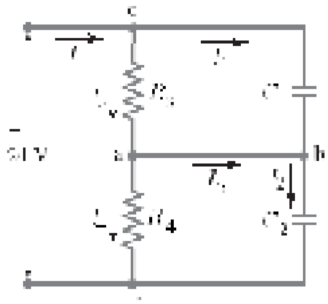
$$Q_2 = V_2 C_2 = (8.0\,\text{V})(0.36\mu\text{F}) = 2.88\mu\text{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of C_1 had the same magnitude as the charge on the positive plate of C_2. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of C_1 and the charge on the positive plate of C_2.

$$Q_{\text{b}} = -Q_1 + Q_2 = -7.68\mu\text{C} + 2.88\mu\text{C} = -4.80\mu\text{C} \approx -4.8\mu\text{C}$$

Thus $\boxed{4.8\mu\text{C}}$ of charge has passed through the switch, from right to left.

52. Because there are no simple series or parallel connections in this circuit, we use Kirchhoff's rules to write equations for the currents, as labeled in our diagram. We write junction equations for the junctions c and d. We then write loop equations for each of the three loops. We set the current through the capacitor equal to the derivative of the charge on the capacitor.



$$I = I_1 + I_3 \ \ [1] \ ; \ I = I_2 + I_4 \ \ [2] \ ; \ \mathscr{E} - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \ \ [3]$$

$$\frac{Q_1}{C_1} - I_3 R_3 = 0 \ \ [4] \ ; \ \frac{Q_2}{C_2} - I_4 R_4 = 0 \ \ [5]$$

We differentiate Eq. [3] with respect to time and set the derivative of the charge equal to the current.

$$0 = \frac{d\mathscr{E}}{dt} - \frac{dQ_1}{dt}\frac{1}{C_1} - \frac{dQ_2}{dt}\frac{1}{C_2} = 0 - \frac{I_1}{C_1} - \frac{I_2}{C_2} \rightarrow I_2 = -I_1 \frac{C_2}{C_1}$$

We then substitute Eq. [1] into Eq. [2] to eliminate I. Then using Eqs. [4] and [5] we eliminate I_3 and I_4 from the resulting equation. We eliminate I_2 using the derivative equation above.

$$I_1 + I_3 = I_2 + I_4 \ \ ; \ I_1 + \frac{Q_1}{R_3 C_1} = -I_1 \frac{C_2}{C_1} + \frac{Q_2}{R_4 C_2}$$

Finally, we eliminate Q_2 using Eq.[3].

$$I_1 + \frac{Q_1}{R_3 C_1} = -I_1 \frac{C_2}{C_1} + \frac{1}{R_4}\left(\mathscr{E} - \frac{Q_1}{C_1}\right) \ \rightarrow \ \mathscr{E} = I_1 R_4 \left(\frac{C_1 + C_2}{C_1}\right) + Q_1 \left(\frac{R_4 + R_3}{R_3 C_1}\right) \ \rightarrow$$

$$\mathscr{E} = I_1 R + \frac{Q_1}{C} \qquad \text{where} \quad R = R_4 \left(\frac{C_1 + C_2}{C_1}\right) \text{ and } C = C_1 \left(\frac{R_3}{R_4 + R_3}\right)$$

This final equation represents a simple RC circuit, with time constant $\tau = RC$.

$$\tau = RC = R_4 \left(\frac{C_1 + C_2}{C_1}\right) C_1 \left(\frac{R_3}{R_4 + R_3}\right) = \frac{R_4 R_3 (C_1 + C_2)}{R_4 + R_3}$$

$$= \frac{(8.8\,\Omega)(4.4\,\Omega)(0.48\,\mu\text{F} + 0.36\,\mu\text{F})}{8.8\,\Omega + 4.4\,\Omega} = \boxed{2.5\,\mu\text{s}}$$

53. The full-scale current is the reciprocal of the sensitivity.

$$I_{\substack{\text{full-}\\\text{scale}}} = \frac{1}{35{,}000\,\Omega/\text{V}} = \boxed{2.9 \times 10^{-5}\,\text{A}} \ \text{ or } \ 29\,\mu\text{A}$$

54. The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\substack{\text{full-}\\\text{scale}}} (\text{sensitivity}) = (250\,\text{V})(35{,}000\,\Omega/\text{V}) = 8.75 \times 10^{6}\,\Omega \approx \boxed{8.8 \times 10^{6}\,\Omega}$$

$\boxed{55.}$ (*a*) The current for full-scale deflection of the galvanometer is

$$I_{\text{G}} = \frac{1}{\text{sensitivity}} = \frac{1}{45{,}000\,\Omega/\text{V}} = 2.222 \times 10^{-5}\,\text{A}$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A. See Figure 26-28 for a circuit diagram.

$$I_G r_G = I_s R_s \rightarrow R_s = \frac{I_G}{I_s} r_G = \frac{I_G}{I_{\text{full}} - I_G} r_G = \frac{2.222\times10^{-5}\,\text{A}}{2.0\text{A} - 2.222\times10^{-5}\,\text{A}}(20.0\,\Omega)$$

$$= 2.222\times10^{-4}\,\Omega \approx \boxed{2.2\times10^{-4}\,\Omega \text{ in parallel}}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram. The total current must be the full-scale deflection current.

$$V_{\text{full}} = I_G(r_G + R) \rightarrow$$

$$R = \frac{V_{\text{full}}}{I_G} - r_G = \frac{1.00\text{V}}{2.222\times10^{-5}\,\text{A}} - 20.0\,\Omega = 44985\,\Omega \approx \boxed{45\,\text{k}\Omega \text{ in series}}$$

56. (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 26-28 for a circuit diagram.

$$V_{\text{shunt}} = V_G \rightarrow (I_{\text{full}} - I_G)R_{\text{shunt}} = I_G R_G \rightarrow$$

$$R_{\text{shunt}} = \frac{I_G R_G}{(I_{\text{full}} - I_G)} = \frac{(55\times10^{-6}\,\text{A})(32\,\Omega)}{(25\,\text{A} - 55\times10^{-6}\,\text{A})} = \boxed{7.0\times10^{-5}\,\Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram.

$$V_{\text{full scale}} = I_G(R_{\text{ser}} + R_G) \rightarrow R_{\text{ser}} = \frac{V_{\text{full scale}}}{I_G} - R_G = \frac{250\,\text{V}}{55\times10^{-6}\,\text{A}} - 30\,\Omega = \boxed{4.5\times10^6\,\Omega}$$

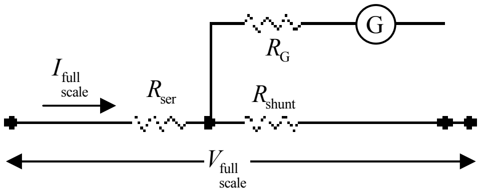
57. We divide the full-scale voltage of the electronic module by the module's internal resistance to determine the current through the module that will give full-scale deflection. Since the module and R_2 are in parallel they will have the same voltage drop across them (400 mV) and their currents will add to equal the current through R_1. We set the voltage drop across R_1 and R_2 equal to the 40 volts and solve the resulting equation for R_2.

$$I_{\text{meter}} = \frac{V_{\text{meter}}}{r} = \frac{400\text{ mV}}{100\text{ M}\Omega} = 4.00\text{ nA} \;;\; I_2 = \frac{V_{\text{meter}}}{R_2} \;;\; I_1 = I_2 + I_{\text{meter}} = \frac{V_{\text{meter}}}{R_2} + I_{\text{meter}}$$

$$V = I_1 R_1 + V_{\text{meter}} \rightarrow (V - V_{\text{meter}}) = \left(\frac{V_{\text{meter}}}{R_2} + I_{\text{meter}}\right)R_1 \rightarrow$$

$$R_2 = \frac{R_1 V_{\text{meter}}}{(V - V_{\text{meter}}) - I_{\text{meter}}R_1} = \frac{(10\times10^6\;\Omega)(0.400\text{ V})}{(40\text{ V} - 0.400\text{ V}) - (4.00\times10^{-9}\text{ A})(10\times10^6\;\Omega)} = \boxed{100\text{ k}\Omega}$$

58. To make a voltmeter, a resistor R_{ser} must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 25 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 25 V when the current through the meter is 25 mA.



$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{\text{eq}} = I_{\substack{\text{full}\\\text{scale}}} \left[R_{\text{ser}} + \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}} \right)^{-1} \right] \rightarrow$$

$$R_{\text{ser}} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}} \right)^{-1} = \frac{25\,\text{V}}{25\times10^{-3}\,\text{A}} - \left(\frac{1}{33\,\Omega} + \frac{1}{0.20\,\Omega} \right)^{-1} = 999.8\,\Omega \approx \boxed{1000\,\Omega}$$

The sensitivity is $\frac{1000\,\Omega}{25\,\text{V}} = \boxed{40\,\Omega/\text{V}}$

59. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{\text{tot}} = R_1 + R_2 = 44\,\text{k}\Omega + 27\,\text{k}\Omega = 71\,\text{k}\Omega \ ; \ I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{71\times10^3\,\Omega} = 6.338\times10^{-4}\,\text{A}$$

$$V_{44} = IR_1 = \left(6.338\times10^{-4}\,\text{A}\right)\left(44\times10^3\,\Omega\right) = 27.89\,\text{V}$$

$$V_{27} = IR_2 = \left(6.338\times10^{-4}\,\text{A}\right)\left(27\times10^3\,\Omega\right) = 17.11\,\text{V}$$

Now put the voltmeter in parallel with the $44\,\text{k}\Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{\text{eq}} = \left(\frac{1}{44\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 30.07\,\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_2 = 30.07\,\text{k}\Omega + 27\text{k}\Omega = 57.07\,\text{k}\Omega \quad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{57.07\times10^3\,\Omega} = 7.885\times10^{-4}\,\text{A}$$

$$V_{44} = V_{\text{eq}} = IR_{\text{eq}} = \left(7.885\times10^{-4}\,\text{A}\right)\left(30.07\times10^3\,\Omega\right) = 23.71\,\text{V} \approx \boxed{24\,\text{V}}$$

$$\%\ \text{error} = \frac{23.71\,\text{V} - 27.89\,\text{V}}{27.89\,\text{V}} \times 100 = \boxed{-15\%\left(\text{reading too low}\right)}$$

And now put the voltmeter in parallel with the $27\,\text{k}\Omega$ resistor, and repeat the process.

$$R_{\text{eq}} = \left(\frac{1}{27\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 21.02\,\text{k}\Omega$$

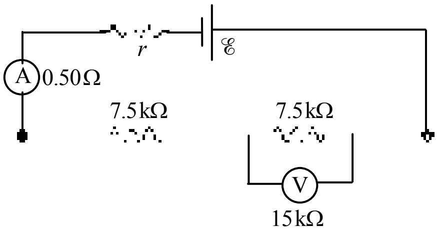
$$R_{\text{tot}} = R_{\text{eq}} + R_1 = 21.02\,\text{k}\Omega + 44\,\text{k}\Omega = 65.02\,\text{k}\Omega \quad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{65.02\times10^3\,\Omega} = 6.921\times10^{-4}\,\text{A}$$

$$V_{27} = V_{\text{eq}} = IR_{\text{eq}} = \left(6.921\times10^{-4}\,\text{A}\right)\left(21.02\times10^3\,\Omega\right) = 14.55\text{V} \approx \boxed{15\,\text{V}}$$

$$\%\ \text{error} = \frac{14.55\,\text{V} - 17.11\,\text{V}}{17.11\,\text{V}} \times 100 = \boxed{-15\%\ \left(\text{reading too low}\right)}$$

60. The total resistance with the ammeter present is $R_{\text{eq}} = 650\,\Omega + 480\,\Omega + 53\,\Omega = 1183\,\Omega$. The voltage supplied by the battery is found from Ohm’s law to be $V_{\text{battery}} = IR_{\text{eq}} = \left(5.25\times10^{-3}\,\text{A}\right)\left(1183\,\Omega\right)$ $= 6.211\,\text{V}$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{\text{eq}} = 1130\,\Omega$, and the new current is again found from Ohm’s law.

$$I=\frac{V_{\text{battery}}}{R'_{\text{eq}}}=\frac{6.211\,\text{V}}{1130\,\Omega}=\boxed{5.50\times10^{-3}\,\text{A}}$$



$\boxed{61.}$ Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.

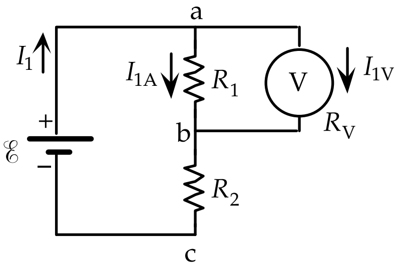
$$R_{\text{eq}}=1.0\,\Omega+0.50\,\Omega+7500\,\Omega+\frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega+15000\,\Omega)}$$

$$=12501.5\,\Omega\approx12500\,\Omega\ ;\ I_{\text{source}}=\frac{\mathscr{E}}{R_{\text{eq}}}=\frac{12.0\,\text{V}}{12500\,\Omega}=\boxed{9.60\times10^{-4}\,\text{A}}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{\text{meter}}=I_{\text{source}}R_{\text{eq}}=\left(9.60\times10^{-4}\,\text{A}\right)\frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega+15000\,\Omega)}=\boxed{4.8\,\text{V}}$$

62. From the first diagram, write the sum of the currents at junction a, and then substitute in for those currents as shown.

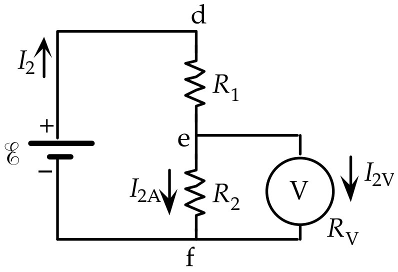


$$I_1=I_{1\text{A}}+I_{1\text{V}}$$

$$\mathscr{E}-V_{R_1}-I_1R_2=0\ \rightarrow\ I_1=\frac{\mathscr{E}-V_{R_1}}{R_2}\ ;\ I_{1A}=\frac{V_{R_1}}{R_1}\ ;\ I_{1V}=\frac{V_{1\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E}-V_{R_1}}{R_2}=\frac{V_{R_1}}{R_1}+\frac{V_{1\text{V}}}{R_{\text{V}}}$$

Then do a similar procedure for the second diagram.



$$I_2=I_{2\text{A}}+I_{2\text{V}}$$

$$\mathscr{E}-I_2R_1-V_{R_2}=0\ \rightarrow\ I_2=\frac{\mathscr{E}-V_{R_2}}{R_1}\ ;\ I_{2A}=\frac{V_{R_2}}{R_2}\ ;\ I_{2V}=\frac{V_{2\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E}-V_{R_2}}{R_1}=\frac{V_{R_2}}{R_2}+\frac{V_{2\text{V}}}{R_{\text{V}}}$$

Now there are two equations in the two unknowns of R_1 and R_2. Solve for the reciprocal values and then find the resistances. Assume that all resistances are measured in kilohms.

$$\frac{\mathscr{E}-V_{R_1}}{R_2}=\frac{V_{R_1}}{R_1}+\frac{V_{1\text{V}}}{R_{\text{V}}}\ \rightarrow\ \frac{12.0-5.5}{R_2}=\frac{5.5}{R_1}+\frac{5.5}{18.0}\ \rightarrow\ \frac{6.5}{R_2}=\frac{5.5}{R_1}+0.30556$$

$$\frac{\mathscr{E}-V_{R_2}}{R_1}=\frac{V_{R_2}}{R_2}+\frac{V_{2\text{V}}}{R_{\text{V}}}\ \rightarrow\ \frac{12.0-4.0}{R_1}=\frac{4.0}{R_2}+\frac{4.0}{18.0}\ \rightarrow\ \frac{8.0}{R_1}=\frac{4.0}{R_2}+0.22222$$

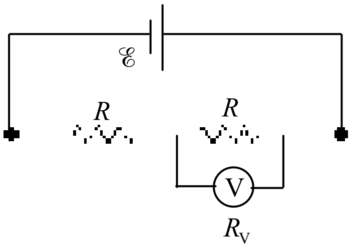
$$\frac{8.0}{R_1}=\frac{4.0}{R_2}+0.22222\ \rightarrow\ \frac{1}{R_2}=\frac{2}{R_1}-0.05556$$

$$\frac{6.5}{R_2}=\frac{5.5}{R_1}+0.30556\ \rightarrow\ 6.5\left(\frac{2}{R_1}-0.05556\right)=\frac{5.5}{R_1}+0.30556\ \rightarrow\ \frac{1}{R_1}=\frac{0.66667}{7.5}\ \rightarrow$$

$$R_1 = 11.25\,\text{k}\Omega \;\; ; \;\; \frac{1}{R_2} = \frac{2}{R_1} - 0.05556 \;\rightarrow\; R_2 = 8.18\,\text{k}\Omega$$

So the final results are $\boxed{R_1 = 11\,\text{k}\Omega \;\; ; \;\; R_2 = 8.2\,\text{k}\Omega}$

63. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.



$$R_p = \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = \frac{R_V R}{R_V + R} = \frac{(3000\,\Omega)(9400\,\Omega)}{3000\,\Omega + 9400\,\Omega} = 2274\,\Omega$$

$$R_{eq} = R + R_p = 2274\,\Omega + 9400\,\Omega = 11674\,\Omega$$

Using the meter reading of 2.3 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$I = \frac{V}{R_p} = \frac{2.3\,\text{V}}{2274\,\Omega} = 1.011 \times 10^{-3}\,\text{A}$$

$$\mathcal{E} = IR_{eq} = \left(1.011 \times 10^{-3}\,\text{A}\right)\left(11674\,\Omega\right) = 11.80\,\text{V} \approx \boxed{12\,\text{V}}$$

64. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

$$\text{Case 1: } V_{\text{meter}} = V_1 = I_1 R_1 \quad \mathcal{E} - I_1 r - I_1 R_1 = 0 \;\rightarrow\; \mathcal{E} = I_1\left(r + R_1\right) = \frac{V_1}{R_1}\left(r + R_1\right)$$

$$\text{Case 2: } V_{\text{meter}} = V_2 = I_2 R_2 \quad \mathcal{E} - I_2 r - I_2 R_2 = 0 \;\rightarrow\; \mathcal{E} = I_2\left(r + R_2\right) = \frac{V_2}{R_2}\left(r + R_2\right)$$

Solve these two equations for the two unknowns of $\mathcal{E}$ and r.

$$\mathcal{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{V_2}{R_2}\left(r + R_2\right) \;\rightarrow$$

$$r = R_1 R_2 \left(\frac{V_2 - V_1}{V_1 R_2 - V_2 R_1}\right) = (35\,\Omega)(14.0\,\Omega)\left(\frac{8.1\,\text{V} - 9.7\,\text{V}}{(9.7\,\text{V})(14.0\,\Omega) - (8.1\,\text{V})(35\,\Omega)}\right) = 5.308\,\Omega \approx \boxed{5.3\,\Omega}$$

$$\mathcal{E} = \frac{V_1}{R_1}\left(r + R_1\right) = \frac{9.7\,\text{V}}{35\,\Omega}\left(5.308\,\Omega + 35\,\Omega\right) = 11.17\,\text{V} \approx \boxed{11\,\text{V}}$$

65. We connect the battery in series with the body and a resistor. The current through this series circuit is the voltage supplied by the battery divided by the sum of the resistances. The voltage drop across the body is equal to the current multiplied by the body's resistance. We set the voltage drop across the body equal to 0.25 V and solve for the necessary resistance.

$$I = \frac{\mathcal{E}}{R + R_B}$$

$$V = IR_B = \frac{\mathcal{E}\,R_B}{R + R_B} \rightarrow R = \left(\frac{\mathcal{E}}{V} - 1\right)R_B = \left(\frac{1.5\text{ V}}{0.25\text{ V}} - 1\right)(1800\ \Omega) = 9000\ \Omega = \boxed{9.0\text{ k}\Omega}$$

66. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50 W output, use the higher-resistance filament}}$. For the $\boxed{\text{100 W output, use the lower-resistance filament}}$. For the $\boxed{\text{150 W output, use the filaments in parallel}}$.

(*b*) $P = V^2/R \;\rightarrow$

$$R = \frac{V^2}{P} \qquad R_{50\,\text{W}} = \frac{(120\,\text{V})^2}{50\,\text{W}} = 288\,\Omega \approx \boxed{290\,\Omega} \qquad R_{100\,\text{W}} = \frac{(120\,\text{V})^2}{100\,\text{W}} = 144\,\Omega \approx \boxed{140\,\Omega}$$

As a check, the parallel combination of the resistors gives the following.

$$R_\text{p} = \frac{R_1R_2}{R_1 + R_2} = \frac{(288\,\Omega)(144\,\Omega)}{288\,\Omega + 144\,\Omega} = 96\,\Omega \qquad P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{96\,\Omega} = 150\,\text{W}\,.$$

$\boxed{67.}$ The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$V_\text{wires} = IR_\text{wires} = (3.0\,\text{A})(0.0065\,\Omega/\text{m})(130\,\text{m})\,R_\text{p} = 2.535\,\text{V} \approx \boxed{2.5\,\text{V}}$$

Thus the voltage applied to the apparatus is $V = V_\text{source} - V_\text{wires} = 120\,\text{V} - 2.535\,\text{V} = 117.465\,\text{V} \approx \boxed{117\,\text{V}}$.

68. The charge on the capacitor and the current in the resistor both decrease exponentially, with a time constant of $\tau = RC$. The energy stored in the capacitor is given by $U = \frac{1}{2}\frac{Q^2}{C}$, and the power dissipated in the resistor is given by $P = I^2R$.

$$Q = Q_0e^{-t/RC} \;;\; I = I_0e^{-t/RC} = \frac{V_0}{R}e^{-t/RC} = \frac{Q_0}{RC}e^{-t/RC}$$

$$U_\text{decrease} = -\Delta U = U_{t=0} - U_{t=\tau} = \tfrac{1}{2}\left(\frac{Q^2}{C}\right)_{t=0} - \tfrac{1}{2}\left(\frac{Q^2}{C}\right)_{t=\tau} = \tfrac{1}{2}\frac{Q_0^2}{C} - \tfrac{1}{2}\left(\frac{Q_0e^{-1}}{C}\right)^2 = \tfrac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

$$U_\text{dissipated} = \int P\,dt = \int_0^\tau I^2R\,dt = \int_0^\tau\left(\frac{Q_0}{RC}e^{-t/RC}\right)^2 R\,dt = \frac{Q_0^2}{RC^2}\int_0^\tau e^{-2t/RC}dt = \frac{Q_0^2}{RC^2}\left(-\frac{RC}{2}\right)\left(e^{-2t/RC}\right)_0^\tau$$

$$= -\tfrac{1}{2}\frac{Q_0^2}{C}\left(e^{-2} - 1\right) = \tfrac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

And so we see that $\boxed{U_\text{decrease} = U_\text{dissipated}}$.

69. The capacitor will charge up to 75% of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$t_\text{beat} = \frac{1\,\text{min}}{72\,\text{beats}} \times \frac{60\,\text{s}}{1\,\text{min}} = 0.8333\,\text{s}$$

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \;\rightarrow\; 0.75V_0 = V_0\left(1 - e^{-\frac{t_\text{beat}}{RC}}\right) \;\rightarrow\; e^{-\frac{t_\text{beat}}{RC}} = 0.25 \;\rightarrow\; \left(-\frac{t_\text{beat}}{RC}\right) = \ln(0.25) \;\rightarrow$$

$$R = -\frac{t_\text{beat}}{C\ln(0.25)} = -\frac{0.8333\,\text{s}}{(6.5\times10^{-6}\,\text{F})(-1.3863)} = \boxed{9.2\times10^4\,\Omega}$$

70. (*a*) Apply Ohm's law to find the current.

$$I=\frac{V_{\text{body}}}{R_{\text{body}}}=\frac{110\,\text{V}}{950\,\Omega}=0.116\,\text{A}\approx\boxed{0.12\,\text{A}}$$

(*b*) The description of "alternative path to ground" is a statement that the 35Ω path is in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: $\boxed{0.12\,\text{A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}}=V_{\text{alternate}}\;\rightarrow\;I_{\text{body}}R_{\text{body}}=I_{\text{alternate}}R_{\text{alternate}}=\left(I_{\text{total}}-I_{\text{body}}\right)R_{\text{alternate}}\;\rightarrow$$

$$I_{\text{body}}=I_{\text{total}}\frac{R_{\text{alternate}}}{\left(R_{\text{body}}+R_{\text{alternate}}\right)}=\left(1.5\,\text{A}\right)\frac{35\Omega}{950\Omega+35\Omega}=0.0533\,\text{A}\approx\boxed{53\,\text{mA}}$$

This is still a very dangerous current.

71. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}}=V_{\text{DA}}\;\rightarrow\;I_3R_3=I_1R_1\;\rightarrow\;\frac{R_3}{R_1}=\frac{I_1}{I_3}$$

$$V_{\text{CB}}=V_{\text{CD}}\;\rightarrow\;I_3R_x=I_1R_2\;\rightarrow\;R_x=R_2\frac{I_1}{I_3}=\boxed{R_2R_3/R_1}$$

(*b*) $$R_x=R_2\frac{R_3}{R_1}=\left(972\Omega\right)\left(\frac{78.6\Omega}{630\Omega}\right)=\boxed{121\Omega}$$

72. From the solution to problem 71, the unknown resistance is given by $R_x=R_2R_3/R_1$. We use that with Eq. 25-3 to find the length of the wire.

$$R_x=R_2\frac{R_3}{R_1}=\frac{\rho L}{A}=\frac{\rho L}{\pi\left(d/2\right)^2}=\frac{4\rho L}{\pi d^2}\;\rightarrow$$

$$L=\frac{R_2R_3\pi d^2}{4R_1\rho}=\frac{\left(29.2\Omega\right)\left(3.48\Omega\right)\pi\left(1.22\times10^{-3}\text{m}\right)^2}{4\left(38.0\Omega\right)\left(10.6\times10^{-8}\Omega\text{•m}\right)}=\boxed{29.5\,\text{m}}$$

$\boxed{73.}$ Divide the power by the required voltage to determine the current drawn by the hearing aid.

$$I=\frac{P}{V}=\frac{2.5\text{ W}}{4.0\text{ V}}=0.625\text{ A}$$

Use Eq. 26-1 to calculate the terminal voltage across the three batteries for mercury and dry cells.

$$V_{\text{Hg}}=3\left(\mathscr{E}-Ir\right)=3\left[1.35\text{ V}-\left(0.625\text{ A}\right)\left(0.030\ \Omega\right)\right]=3.99\text{ V}$$

$$V_{\text{D}}=3\left(\mathscr{E}-Ir\right)=3\left[1.50\text{ V}-\left(0.625\text{ A}\right)\left(0.35\ \Omega\right)\right]=3.84\text{ V}$$

The terminal voltage of the mercury cell batteries is closer to the required 4.0 V than the voltage from the dry cell.

74. One way is to connect *N* resistors in series. If each resistor can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{\text{eq}}}{7} = \frac{3200\,\Omega}{7} = 457\,\Omega \approx 460\,\Omega$$

So connect $\boxed{\text{7 resistors of } 460\,\Omega \text{ each, rated at ½ W, in series}}$.

Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{\text{eq}}} = 7\left(\frac{1}{R}\right) \rightarrow R = 7R_{\text{eq}} = 7(3200\,\Omega) = 22.4\,\text{k}\Omega$$

So connect $\boxed{\text{7 resistors of } 22.4\,\text{k}\Omega \text{ each, rated at ½ W, in parallel}}$.

75. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\,\text{V}}{0.80\,\text{V/cell}} = 150 \text{ cells}$ are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add making a total of $\frac{1.3\,\text{A}}{350\times10^{-3}\,\text{A/bank}} = 3.71 \text{ banks} \approx 4 \text{ banks}$. So the total number of cells is $\boxed{\text{600 cells}}$. The panel area is $600 \text{ cells}\left(9.0\times10^{-4}\,\text{m}^2/\text{cell}\right) = \boxed{0.54\,\text{m}^2}$. The cells should be wired in $\boxed{\text{4 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.4 A at 120 V. To optimize the output, $\boxed{\text{always have the panel pointed directly at the sun}}$.

76. (*a*) If the terminal voltage is to be 3.0 V, then the voltage across R_1 will be 9.0 V. This can be used to find the current, which then can be used to find the value of R_2.

$$V_1 = IR_1 \rightarrow I = \frac{V_1}{R_1} \qquad V_2 = IR_2 \rightarrow$$

$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (14.5\,\Omega)\frac{3.0\,\text{V}}{9.0\,\text{V}} = 4.833\,\Omega \approx \boxed{4.8\,\Omega}$$

(*b*) If the load has a resistance of $7.0\,\Omega$, then the parallel combination of R_2 and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2R_{\text{load}}}{R_2 + R_{\text{load}}} = \frac{(4.833\,\Omega)(7.0\,\Omega)}{11.833\,\Omega} = 2.859\,\Omega \qquad R_{eq} = 2.859\,\Omega + 14.5\,\Omega = 17.359\,\Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{17.359\,\Omega} = 0.6913\,\text{A} \qquad V_{\text{T}} = IR_{2+\text{load}} = (0.6913\,\text{A})(2.859\,\Omega) = 1.976\,\text{V} \approx \boxed{2.0\,\text{V}}$$

The presence of the load has affected the terminal voltage significantly.

77. There are two answers because it is not known which direction the given current is flowing through the $4.0\,\mathrm{k\Omega}$ resistor. Assume the current is to the right. The voltage across the $4.0\,\mathrm{k\Omega}$ resistor is given by Ohm's law as $V = IR = \left(3.10\times10^{-3}\,\mathrm{A}\right)\left(4000\,\Omega\right) = 12.4\,\mathrm{V}$. The voltage drop across the $8.0\,\mathrm{k\Omega}$ must be the same, and the current through it is $I = \frac{V}{R} = \frac{12.4\,\mathrm{V}}{8000\,\Omega} = 1.55\times10^{-3}\,\mathrm{A}$. The total current in the circuit is the sum of the two currents, and so $I_{\mathrm{tot}} = 4.65\times10^{-3}\,\mathrm{A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{\mathrm{ab}} - \left(5000\,\Omega\right)I_{\mathrm{tot}} - 12.4\,\mathrm{V} - 12.0\,\mathrm{V} - \left(1.0\,\Omega\right)I_{\mathrm{tot}} \quad \rightarrow$$

$$V_{\mathrm{ab}} = 24.4\mathrm{V} + \left(5001\,\Omega\right)\left(4.65\times10^{-3}\,\mathrm{A}\right) = 47.65\,\mathrm{V} \approx \boxed{48\,\mathrm{V}}$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 12.4 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$V_{\mathrm{ab}} + \left(5000\,\Omega\right)I_{\mathrm{tot}} + 12.4\,\mathrm{V} - 12.0\,\mathrm{V} + \left(1.0\,\Omega\right)I_{\mathrm{tot}} \quad \rightarrow$$

$$V_{\mathrm{ab}} = -0.4\mathrm{V} - \left(5001\,\Omega\right)\left(4.65\times10^{-3}\,\mathrm{A}\right) = -23.65\,\mathrm{V} \approx \boxed{-24\,\mathrm{V}}$$

78. The terminal voltage and current are given for two situations. Apply Eq. 26-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$V_1 = \mathscr{E} - I_1 r \;;\; V_2 = \mathscr{E} - I_2 r \;\rightarrow\; \mathscr{E} = V_1 + I_1 r = V_2 + I_2 r \;\rightarrow$$

$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\,\mathrm{V} - 40.8\,\mathrm{V}}{7.40\,\mathrm{A} - 2.80\,\mathrm{A}} = 1.413\,\Omega \approx \boxed{1.4\,\Omega}$$

$$\mathscr{E} = V_1 + I_1 r = 40.8\,\mathrm{V} + \left(7.40\,\mathrm{A}\right)\left(1.413\,\Omega\right) = \boxed{51.3\,\mathrm{V}}$$

$\boxed{79.}$ The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2 R \;\rightarrow\; I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.80\,\mathrm{W}}{33\,\Omega}} = 0.1557\,\mathrm{A}$$

$$R_{\mathrm{eq}} = 33\,\Omega + \left(\frac{1}{68\,\Omega} + \frac{1}{75\,\Omega}\right)^{-1} = 68.66\,\Omega \qquad V = IR_{\mathrm{eq}} = \left(0.1557\,\mathrm{A}\right)\left(68.66\,\Omega\right) = 10.69\,\mathrm{V} \approx \boxed{11\,\mathrm{V}}$$

80. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\,\mathrm{V} - I\left(50\,\Omega + 20\,\Omega + 10\,\Omega\right) = 0 \;\rightarrow\; I = 6.0\,\mathrm{V}/80\,\Omega = 0.075\,\mathrm{A}$$

If the switches are both closed, the 20-Ω resistor is in parallel with *R*. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the 20-Ω resistor having the current found previously.

$$6.0\,\mathrm{V} - I\left(50\,\Omega\right) - \left(0.075\,\mathrm{A}\right)\left(20\,\Omega\right) = 0 \;\rightarrow\; I = \frac{6.0\,\mathrm{V} - \left(0.075\,\mathrm{A}\right)\left(20\,\Omega\right)}{50\,\Omega} = 0.090\,\mathrm{A}$$

This is the current in the parallel combination. Since 0.075 A is in the 20-Ω resistor, 0.015 A must be in *R*. The voltage drops across *R* and the 20-Ω resistor are the same since they are in parallel.

$$V_{20} = V_R \;\rightarrow\; I_{20}R_{20} = I_R R \;\rightarrow\; R = R_{20}\frac{I_{20}}{I_R} = \left(20\,\Omega\right)\frac{0.075\,\mathrm{A}}{0.015\,\mathrm{A}} = \boxed{100\,\Omega}$$

81. (*a*) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance R_V. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$V = IR_{\text{eq}} = I\frac{1}{\frac{1}{R}+\frac{1}{R_V}} \rightarrow V\left(\frac{1}{R}+\frac{1}{R_V}\right) = I \rightarrow \frac{1}{R}+\frac{1}{R_V} = \frac{I}{V} \rightarrow \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_V}}$$

(*b*) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance R_A. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$V = IR_{\text{eq}} = I\left(R+R_A\right) \rightarrow R+R_A = \frac{V}{I} \rightarrow \boxed{R = \frac{V}{I} - R_A}$$

82. (*a*) The 12-Ω and the 25-Ω resistors are in parallel, with a net resistance $R_{1\text{-}2}$ as follows.

$$R_{1\text{-}2} = \left(\frac{1}{12\,\Omega}+\frac{1}{25\,\Omega}\right)^{-1} = 8.108\,\Omega$$

$R_{1\text{-}2}$ is in series with the 4.5-Ω resistor, for a net resistance $R_{1\text{-}2\text{-}3}$ as follows.

$$R_{1\text{-}2\text{-}3} = 4.5\,\Omega + 8.108\,\Omega = 12.608\,\Omega$$

That net resistance is in parallel with the 18-Ω resistor, for a final equivalent resistance as follows.

$$R_{\text{eq}} = \left(\frac{1}{12.608\,\Omega}+\frac{1}{18\,\Omega}\right)^{-1} = 7.415\,\Omega \approx \boxed{7.4\,\Omega}$$

(*b*) Find the current in the 18-Ω resistor by using Kirchhoff's loop rule for the loop containing the battery and the 18-Ω resistor.

$$\mathscr{E} - I_{18}R_{18} = 0 \rightarrow I_{18} = \frac{\mathscr{E}}{R_{18}} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A}}$$

(*c*) Find the current in $R_{1\text{-}2}$ and the 4.5-Ω resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1\text{-}2}$ and the 4.5-Ω resistor.

$$\mathscr{E} - I_{1\text{-}2}R_{1\text{-}2} - I_{1\text{-}2}R_{4.5} = 0 \rightarrow I_{1\text{-}2} = \frac{\mathcal{E}}{R_{1\text{-}2}+R_{4.5}} = \frac{6.0\,\text{V}}{12.608\,\Omega} = 0.4759\,\text{A}$$

This current divides to go through the 12-Ω and 25-Ω resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the 12-Ω resistor.

$$I_{1\text{-}2} = I_{12} + I_{25} \rightarrow I_{25} = I_{1\text{-}2} - I_{12}$$

$$V_{R_{12}} = V_{R_{25}} \rightarrow I_{12}R_{12} = I_{25}R_{25} = \left(I_{1\text{-}2} - I_{12}\right)R_{25} \rightarrow$$

$$I_{12} = I_{1\text{-}2}\frac{R_{25}}{\left(R_{12}+R_{25}\right)} = \left(0.4759\,\text{A}\right)\frac{25\,\Omega}{37\,\Omega} = \boxed{0.32\,\text{A}}$$

(*d*) The current in the 4.5-Ω resistor was found above to be $I_{1\text{-}2} = 0.4759\,\text{A}$. Find the power accordingly.

$$P_{4.5} = I_{1\text{-}2}^2 R_{4.5} = \left(0.4759\,\text{A}\right)^2\left(4.5\,\Omega\right) = 1.019\,\text{W} \approx \boxed{1.0\,\text{W}}$$

83. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$P_{\text{bulb}}=\frac{V_{\text{bulb}}^2}{R_{\text{bulb}}} \rightarrow R_{\text{bulb}}=\frac{V_{\text{bulb}}^2}{P_{\text{bulb}}} \qquad P_{\text{bulb}}=I_{\text{bulb}}V_{\text{bulb}} \rightarrow I_{\text{bulb}}=\frac{P_{\text{bulb}}}{V_{\text{bulb}}}$$

$$\mathscr{E}-I_{\text{bulb}}R-I_{\text{bulb}}R_{\text{bulb}}=0 \rightarrow$$

$$R=\frac{\mathscr{E}}{I_{\text{bulb}}}-R_{\text{bulb}}=\frac{\mathscr{E}}{P_{\text{bulb}}/V_{\text{bulb}}}-\frac{V_{\text{bulb}}^2}{P_{\text{bulb}}}=\frac{V_{\text{bulb}}}{P_{\text{bulb}}}\left(\mathscr{E}-V_{\text{bulb}}\right)=\frac{3.0\,\text{V}}{2.0\,\text{W}}\left(9.0\,\text{V}-3.0\,\text{V}\right)=\boxed{9.0\,\Omega}$$

84. The equivalent resistance of the circuit is the parallel combination of the bulb and the lower portion of the potentiometer, in series with the upper portion of the potentiometer. With the slide at position *x*, the resistance of the lower portion is xR_{var}, and the resistance of the upper portion is $(1-x)R_{\text{var}}$. From that equivalent resistance, we find the current in the loop, the voltage across the bulb, and then the power expended in the bulb.

$$R_{\text{parallel}}=\left(\frac{1}{R_{\text{lower}}}+\frac{1}{R_{\text{bulb}}}\right)^{-1}=\frac{R_{\text{lower}}R_{\text{bulb}}}{R_{\text{lower}}+R_{\text{bulb}}}=\frac{xR_{\text{var}}R_{\text{bulb}}}{xR_{\text{var}}+R_{\text{bulb}}}$$

$$R_{\text{eq}}=(1-x)R_{\text{var}}+R_{\text{parallel}}\ ;\ I_{\text{loop}}=\frac{\mathscr{E}}{R_{\text{eq}}}\ ;\ V_{\text{bulb}}=I_{\text{loop}}R_{\text{parallel}}\ ;\ P_{\text{bulb}}=\frac{V_{\text{bulb}}^2}{R_{\text{bulb}}}$$

(*a*) Consider the case in which $x=1.00$. In this case, the full battery potential is across the bulb, and so it is obvious that $V_{\text{bulb}}=120\,\text{V}$. Thus $P_{\text{bulb}}=\frac{V_{\text{bulb}}^2}{R_{\text{bulb}}}=\frac{(120\,\text{V})^2}{240\,\Omega}=\boxed{60\,\text{W}}$.

(*b*) Consider the case in which $x=0.65$.

$$R_{\text{parallel}}=\frac{xR_{\text{var}}R_{\text{bulb}}}{xR_{\text{var}}+R_{\text{bulb}}}=\frac{(0.65)(150\,\Omega)(240\,\Omega)}{(0.65)(150\,\Omega)+240\,\Omega}=69.33\,\Omega$$

$$R_{\text{eq}}=(1-x)R_{\text{var}}+R_{\text{parallel}}=(0.35)(150\,\Omega)+69.33\,\Omega=121.83\,\Omega$$

$$I_{\text{loop}}=\frac{\mathscr{E}}{R_{\text{eq}}}=\frac{120\,\text{V}}{121.83\,\Omega}=0.9850\,\text{A}\ ;\ V_{\text{bulb}}=(0.9850\,\text{A})(69.33\,\Omega)=68.29\,\text{V}$$

$$P_{\text{bulb}}=\frac{(68.29\,\text{V})^2}{240\,\Omega}=19.43\,\text{W}\approx\boxed{19\,\text{W}}$$

(*c*) Consider the case in which $x=0.35$.

$$R_{\text{parallel}}=\frac{xR_{\text{var}}R_{\text{bulb}}}{xR_{\text{var}}+R_{\text{bulb}}}=\frac{(0.35)(150\,\Omega)(240\,\Omega)}{(0.35)(150\,\Omega)+240\,\Omega}=43.08\,\Omega$$

$$R_{\text{eq}}=(1-x)R_{\text{var}}+R_{\text{parallel}}=(0.65)(150\,\Omega)+69.33\,\Omega=140.58\,\Omega$$

$$I_{\text{loop}}=\frac{\mathscr{E}}{R_{\text{eq}}}=\frac{120\,\text{V}}{140.58\,\Omega}=0.8536\,\text{A}\ ;\ V_{\text{bulb}}=(0.8536\,\text{A})(43.08\,\Omega)=36.77\,\text{V}$$

$$P_{\text{bulb}}=\frac{(36.77\,\text{V})^2}{240\,\Omega}=5.63\,\text{W}\approx\boxed{5.6\,\text{W}}$$

$\boxed{85.}$ (*a*) When the galvanometer gives a null reading, no current is passing through the galvanometer or the emf that is being measured. All of the current is flowing through the slide wire resistance. Application of the loop rule to the lower loop gives $\mathscr{E} - IR = 0,$ since there is no current through the emf to cause voltage drop across any internal resistance. The amount of current flowing through the slide wire resistor will be the same no matter what emf is used since no current is flowing through the lower loop. Apply this relationship to the two emf's.

$$\mathscr{E}_x - IR_x = 0 \;\; ; \;\; \mathscr{E}_\text{s} - IR_\text{s} = 0 \;\; \rightarrow \;\; ; \;\; I = \frac{\mathscr{E}_x}{R_x} = \frac{\mathscr{E}_\text{s}}{R_\text{s}} \;\; \rightarrow \;\; \boxed{\mathscr{E}_x = \left(\frac{R_x}{R_\text{s}}\right)\mathscr{E}_\text{s}}$$

(*b*) Use the equation derived above. We use the fact that the resistance is proportional to the length of the wire, by Eq. 25-3, $R = \rho \ell / A$.

$$\mathscr{E}_x = \left(\frac{R_x}{R_\text{s}}\right)\mathscr{E}_\text{s} = \left(\frac{\rho\dfrac{\ell_x}{A}}{\rho\dfrac{\ell_\text{s}}{A}}\right)\mathscr{E}_\text{s} = \left(\frac{\ell_x}{\ell_\text{s}}\right)\mathscr{E}_\text{s} = \left(\frac{45.8\,\text{cm}}{33.6\,\text{cm}}\right)(1.0182\,\text{V}) = \boxed{1.39\,\text{V}}$$

(*c*) If there is current in the galvanometer, then the voltage between points A and C is uncertainty by the voltage drop across the galvanometer, which is $V_\text{G} = I_\text{G}R_\text{G} = \left(0.012 \times 10^{-3}\,\text{A}\right)\left(35\,\Omega\right)$ $= \boxed{4.2 \times 10^{-4}\,\text{V}}$. The uncertainty might of course be more than this, due to uncertainties compounding from having to measure distance for both the standard emf and the unknown emf. Measuring the distances also has some uncertainty associated with it.

(*d*) Using this null method means that the (unknown) internal resistance of the unknown emf does not enter into the calculation. No current passes through the unknown emf, and so there is no voltage drop across that internal resistance.

86. (*a*) In normal operation, the capacitor is fully charged by the power supply, and so the capacitor voltage is the same as the power supply voltage, and there will be no current through the resistor. If there is an interruption, the capacitor voltage will decrease exponentially – it will discharge. We want the voltage across the capacitor to be at 75% of the full voltage after 0.20 s. Use Eq. 26-9b for the discharging capacitor.

$$V = V_0 e^{-t/RC} \;\; ; \;\; 0.75V_0 = V_0 e^{-(0.20\text{s})/RC} \;\; \rightarrow \;\; 0.75 = e^{-(0.20\text{s})/RC} \;\; \rightarrow$$

$$R = \frac{-(0.20\,\text{s})}{C\ln(0.75)} = \frac{-(0.20\,\text{s})}{\left(8.5 \times 10^{-6}\,\text{F}\right)\ln(0.75)} = 81790\,\Omega \approx \boxed{82\,\text{k}\Omega}$$

(*b*) When the power supply is functioning normally, there is no voltage across the resistor, so the device should NOT be connected between terminals a and b. If the power supply is not functioning normally, there will be a larger voltage across the capacitor than across the capacitor–resistor combination, since some current might be present. This current would result in a voltage drop across the resistor. To have the highest voltage in case of a power supply failure, the device should be connected between terminals $\boxed{\text{b and c}}$.

87. Note that, based on the significant figures of the resistors, that the $1.0\text{-}\Omega$ resistor will not change the equivalent resistance of the circuit as determined by the resistors in the switch bank.

Case 1: $n = 0$ switch closed. The effective resistance of the circuit is $16.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{16.0\,\text{k}\Omega} = 1.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (1.0\,\text{mA})(1.0\,\Omega) = \boxed{1.0\,\text{mV}}$.

Case 2: $n = 1$ switch closed. The effective resistance of the circuit is $8.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{8.0\,\text{k}\Omega} = 2.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (2.0\,\text{mA})(1.0\,\Omega) = \boxed{2.0\,\text{mV}}$.

Case 3: $n = 2$ switch closed. The effective resistance of the circuit is $4.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{4.0\,\text{k}\Omega} = 4.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (4.0\,\text{mA})(1.0\,\Omega) = \boxed{4.0\,\text{mV}}$.

Case 4: $n = 3$ and $n = 1$ switches closed. The effective resistance of the circuit is found by the parallel combination of the $2.0\text{-k}\Omega$ and $8.0\text{-k}\Omega$ resistors.

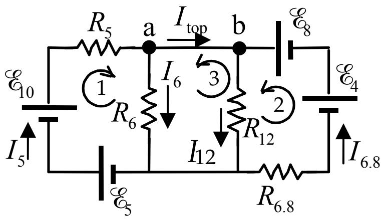
$$R_{\text{eq}} = \left(\frac{1}{2.0\,\text{k}\Omega} + \frac{1}{8.0\,\text{k}\Omega}\right)^{-1} = 1.6\,\text{k}\Omega$$

The current in the circuit is $I = \dfrac{16\,\text{V}}{1.6\,\text{k}\Omega} = 10\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is

$V = IR = (10\,\text{mA})(1.0\,\Omega) = \boxed{10\,\text{mV}}$.

So in each case, the voltage across the $1.0\text{-}\Omega$ resistor, if taken in mV, is the expected analog value corresponding to the digital number set by the switches.

88. We have labeled the resistors and the currents through the resistors with the value of the specific resistance, and the emf's with the appropriate voltage value. We apply the junction rule to points a and b, and then apply the loop rule to loops 1, 2, and 3. This enables us to solve for all of the currents.



$$I_5 = I_6 + I_{\text{top}} \;;\; I_{\text{top}} + I_{6.8} = I_{12} \;\to\; I_5 - I_6 = I_{12} - I_{6.8} \;\to$$

$$I_5 + I_{6.8} = I_{12} + I_6 \qquad [1]$$

$$\mathcal{E}_5 + \mathcal{E}_{10} - I_5R_5 - I_6R_6 = 0 \qquad [2]\ \ (\text{loop } 1)$$

$$\mathcal{E}_4 + \mathcal{E}_8 - I_{12}R_{12} - I_{6.8}R_{6.8} = 0 \qquad [3]\ \ (\text{loop } 2)$$

$$I_{12}R_{12} - I_6R_6 = 0 \qquad [4]\ \ (\text{loop } 3)$$

Use Eq. 4 to substitute $I_6R_6 = I_{12}R_{12}$ and $I_6 = I_{12}\dfrac{R_{12}}{R_6} = 2I_{12}$. Also combine the emf's by adding the voltages.

$$I_5 + I_{6.8} = 3I_{12}\ [1]\ ;\ \mathcal{E}_{15} - I_5R_5 - I_{12}R_{12} = 0\ [2]\ ;\ \mathcal{E}_{12} - I_{12}R_{12} - I_{6.8}R_{6.8} = 0\ [3]$$

Use Eq. 1 to eliminate $I_{6.8}$ by $I_{6.8} = 3I_{12} - I_5$.

$$\mathcal{E}_{15} - I_5R_5 - I_{12}R_{12} = 0 \qquad [2]$$

$$\mathcal{E}_{12} - I_{12}R_{12} - (3I_{12} - I_5)R_{6.8} = 0 \;\to\; \mathcal{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + I_5R_{6.8} = 0 \qquad [3]$$

Use Eq. 2 to eliminate I_5 by $I_5 = \dfrac{\mathcal{E}_{15} - I_{12}R_{12}}{R_5}$, and then solve for I_{12}.

$$\mathcal{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + \left[\frac{\mathcal{E}_{15} - I_{12}R_{12}}{R_5}\right]R_{6.8} = 0 \;\to$$

$$I_{12}=\frac{\mathcal{E}_{12}R_5+\mathcal{E}_{15}R_{6.8}}{R_{12}R_5+3R_{6.8}R_5+R_{12}R_{6.8}}=\frac{(12.00\,\text{V})(5.00\,\Omega)+(15.00\,\text{V})(6.800\,\Omega)}{(12.00\,\Omega)(5.00\,\Omega)+3(6.800\,\Omega)(5.00\,\Omega)+(12.00\,\Omega)(6.800\,\Omega)}$$

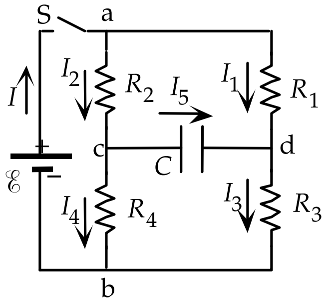
$$=0.66502\,\text{A}\approx\boxed{0.665\,\text{A}=I_{12}}$$

$$I_5=\frac{\mathcal{E}_{15}-I_{12}R_{12}}{R_5}=\frac{(15.00\,\text{V})-(0.66502\,\text{A})(12.00\,\Omega)}{(5.00\,\Omega)}=1.40395\,\text{A}\approx\boxed{1.40\,\text{A}=I_5}$$

$$I_{6.8}=3I_{12}-I_5=3(0.66502\,\text{A})-1.40395\,\text{A}=0.59111\,\text{A}\approx\boxed{0.591\,\text{A}=I_{6.8}}$$

$$I_6=2I_{12}=2(0.66502\,\text{A})\approx\boxed{1.33\,\text{A}=I_6}$$

89. (*a*) After the capacitor is fully charged, there is no current through it, and so it behaves like an "open" in the circuit. In the circuit diagram, this means that $I_5=0$, $I_1=I_3$, and $I_2=I_4$. Write loop equations for the leftmost loop and the outer loop in order to solve for the currents.



$$\mathcal{E}-I_2(R_2+R_4)=0\;\rightarrow\;I_2=\frac{\mathcal{E}}{R_2+R_4}=\frac{12.0\,\text{V}}{10.0\,\Omega}=1.20\,\text{A}$$

$$\mathcal{E}-I_1(R_1+R_3)=0\;\rightarrow\;I_1=\frac{\mathcal{E}}{R_1+R_3}=\frac{12.0\,\text{V}}{15.0\,\Omega}=0.800\,\text{A}$$

Use these currents to find the voltage at points c and d, which will give the voltage across the capacitor.

$$V_c=\mathcal{E}-I_2R_2=12.0\,\text{V}-(1.20\,\text{A})(1.0\,\Omega)=10.8\,\text{V}$$

$$V_d=\mathcal{E}-I_1R_1=12.0\,\text{V}-(0.800\,\text{A})(10.0\,\Omega)=4.00\,\text{V}$$

$$V_{cd}=10.8\,\text{V}-4.00\,\text{V}=\boxed{6.8\,\text{V}}\;;\;Q=CV=(2.2\,\mu\text{F})(6.8\,\text{V})=14.96\,\mu\text{C}\approx\boxed{15\,\mu\text{C}}$$

(*b*) When the switch is opened, the emf is taken out of the circuit. Then we have the capacitor discharging through an equivalent resistance. That equivalent resistance is the series combination of R_1 and R_2, in parallel with the series combination of R_3 and R_4. Use the expression for discharging a capacitor, Eq. 26-9a.

$$R_{\text{eq}}=\left(\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}\right)^{-1}=\left(\frac{1}{11.0\,\Omega}+\frac{1}{14.0\,\Omega}\right)^{-1}=6.16\,\Omega$$

$$Q=Q_0e^{-t/R_{\text{eq}}C}=0.030Q_0\;\rightarrow$$

$$t=-R_{\text{eq}}C\ln(0.030)=-(6.16\,\Omega)(2.2\times10^{-6}\,\text{F})\ln(0.030)=\boxed{4.8\times10^{-5}\,\text{s}}$$

90. (*a*) The time constant of the *RC* circuit is given by Eq. 26-7.

$$\tau=RC=(33.0\text{ k}\Omega)(4.00\;\mu\text{F})=132\text{ ms}$$

During the charging cycle, the charge and the voltage on the capacitor increases exponentially as in Eq. 26-6b. We solve this equation for the time it takes the circuit to reach 90.0 V.

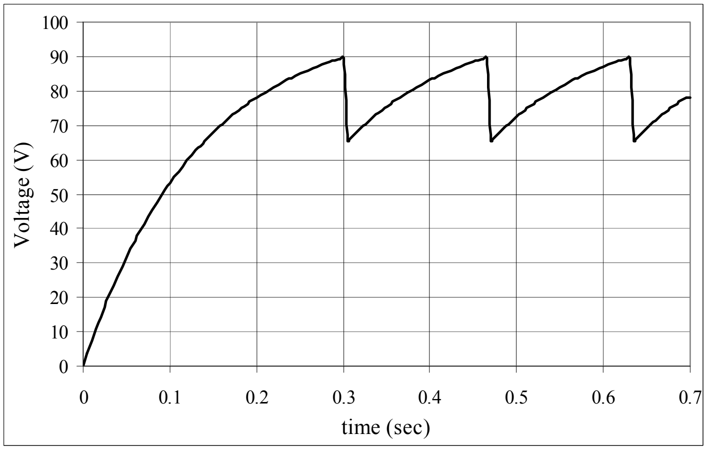
$$V=\mathcal{E}\left(1-e^{-t/\tau}\right)\rightarrow t=-\tau\ln\left(1-\frac{V}{\mathcal{E}}\right)=-(132\text{ ms})\ln\left(1-\frac{90.0\text{ V}}{100.0\text{ V}}\right)=\boxed{304\text{ ms}}$$

(*b*) When the neon bulb starts conducting, the voltage on the capacitor drops quickly to 65.0 V and then starts charging. We can find the recharging time by first finding the time for the capacitor to reach 65.0 V, and then subtract that time from the time required to reach 90.0 V.

$$t=-\tau\ln\left(1-\frac{V}{\mathscr{E}}\right)=-(132\text{ ms})\ln\left(1-\frac{65.0\text{ V}}{100.0\text{ V}}\right)=139\text{ ms}$$

$$\Delta t=304\text{ ms}-139\text{ ms}=165\text{ ms}\ ;\ t_2=304\,\text{ms}+165\,\text{ms}=\boxed{469\,\text{ms}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.90c."



$\boxed{91.}$ We represent the 10.00-MΩ resistor by R_{10}, and the resistance of the voltmeter as R_V. In the first configuration, we find the equivalent resistance R_eqA, the current in the circuit I_A, and the voltage drop across R.

$$R_\text{eqA}=R+\frac{R_{10}R_\text{V}}{R_{10}+R_\text{V}}\ ;\ I_A=\frac{\mathscr{E}}{R_\text{eqA}}\ ;\ V_\text{R}=I_\text{A}R=\mathscr{E}-V_\text{A}\ \rightarrow\ \mathscr{E}\frac{R}{R_\text{eqA}}=\mathscr{E}-V_\text{A}$$

In the second configuration, we find the equivalent resistance R_eqB, the current in the circuit I_B, and the voltage drop across R_{10}.

$$R_\text{eqB}=R_{10}+\frac{RR_\text{V}}{R+R_\text{V}}\ ;\ I_B=\frac{\mathscr{E}}{R_\text{eqB}}\ ;\ V_{\text{R}_{10}}=I_\text{B}R_{10}=\mathscr{E}-V_\text{B}\ \rightarrow\ \mathscr{E}\frac{R_{10}}{R_\text{eqB}}=\mathscr{E}-V_\text{B}$$

We now have two equations in the two unknowns of R and R_V. We solve the second equation for R_V and substitute that into the first equation. We are leaving out much of the algebra in this solution.

$$\mathscr{E}\frac{R}{R_\text{eqA}}=\mathscr{E}\frac{R}{R+\dfrac{R_{10}R_\text{V}}{R_{10}+R_\text{V}}}=\mathscr{E}-V_\text{A}\ ;$$

$$\mathscr{E}\frac{R_{10}}{R_\text{eqB}}=\mathscr{E}\frac{R_{10}}{R_{10}+\dfrac{RR_\text{V}}{R+R_\text{V}}}=\mathscr{E}-V_\text{B}\ \rightarrow\ R_\text{V}=\frac{V_\text{B}R_{10}R}{\left(\mathscr{E}R-V_\text{B}R_{10}-V_\text{B}R\right)}$$

$$\mathscr{E}-V_\text{A}=\mathscr{E}\frac{R}{R+\dfrac{R_{10}R_\text{V}}{R_{10}+R_\text{V}}}=\mathscr{E}\frac{R}{R+\dfrac{R_{10}\left[\dfrac{V_\text{B}R_{10}R}{\left(\mathscr{E}R-V_\text{B}R_{10}-V_\text{B}R\right)}\right]}{R_{10}+\left[\dfrac{V_\text{B}R_{10}R}{\left(\mathscr{E}R-V_\text{B}R_{10}-V_\text{B}R\right)}\right]}}\ \rightarrow$$

$$R=\frac{V_\text{B}}{V_\text{A}}R_{10}=\frac{7.317\,\text{V}}{0.366\,\text{V}}(10.00\,\text{M}\Omega)=199.92\,\text{M}\Omega\approx\boxed{200\,\text{M}\Omega}\ (3\text{ sig. fig.})$$

92. Let the internal resistance of the voltmeter be indicated by R_V, and let the 15-MΩ resistance be indicated by R_{15}. We calculate the current through the probe and voltmeter as the voltage across the probe divided by the equivalent resistance of the problem and the voltmeter. We then set the voltage drop across the voltmeter equal to the product of the current and the parallel combination of R_V and R_{15}. This can be solved for the unknown resistance.

$$I=\frac{V}{R+\dfrac{R_{15}R_V}{R_{15}+R_V}}\ ;\ V_V=I\frac{R_{15}R_V}{R_{15}+R_V}=\frac{V}{R+\dfrac{R_{15}R_V}{R_{15}+R_V}}\frac{R_{15}R_V}{R_{15}+R_V}=\frac{VR_{15}R_V}{R\left(R_{15}+R_V\right)+R_{15}R_V}\ \rightarrow$$

$$R=\frac{\dfrac{V}{V_V}R_{15}R_V-R_{15}R_V}{\left(R_{15}+R_V\right)}=\frac{R_{15}R_V}{\left(R_{15}+R_V\right)}\left(\frac{V}{V_V}-1\right)=\frac{(15\,\text{M}\Omega)(10\,\text{M}\Omega)}{(25\,\text{M}\Omega)}\left(\frac{50{,}000\,\text{V}}{50\,\text{V}}-1\right)$$

$$=5994\,\text{M}\Omega\approx 6000\,\text{M}\Omega=\boxed{6\,\text{G}\Omega}$$

93. The charge and current are given by Eq. 26-6a and Eq. 26-8, respectively.

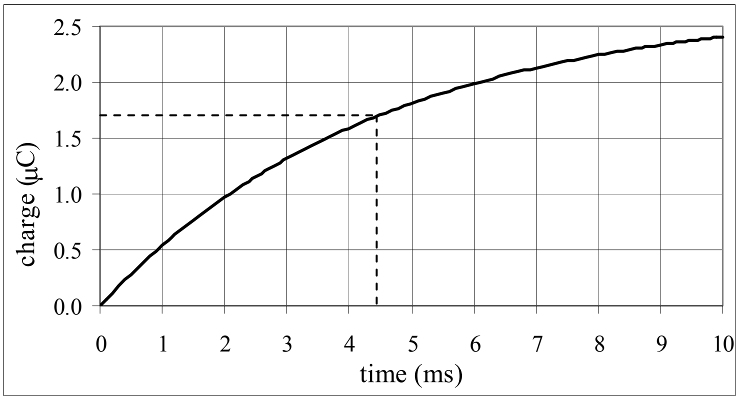
$$Q=C\mathscr{E}\left(1-e^{-t/RC}\right)\ ;\ I=\frac{\mathscr{E}}{R}e^{-t/RC}\ ;\ \tau=RC=\left(1.5\times10^{4}\,\Omega\right)\left(3.0\times10^{-7}\,\text{F}\right)=4.5\times10^{-3}\,\text{s}$$

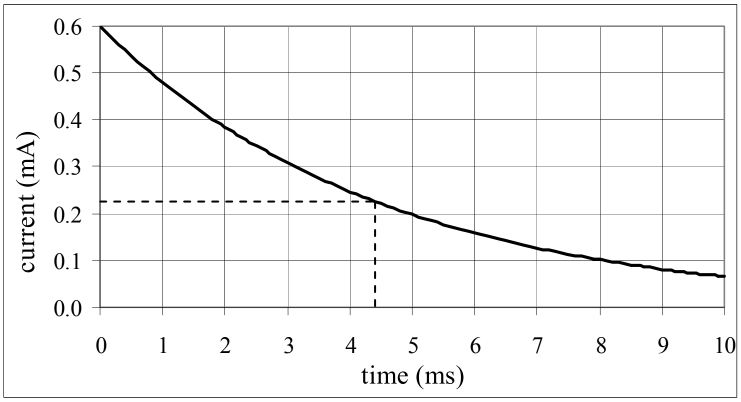
$$0.63Q_{\text{final}}=0.63C\mathscr{E}=0.63\left(3.0\times10^{-7}\,\text{F}\right)\left(9.0\,\text{V}\right)=1.70\times10^{-6}\,\text{C}$$

$$0.37I_{\text{initial}}=0.37\frac{\mathscr{E}}{R}=0.37\left(\frac{9.0\,\text{V}}{1.5\times10^{4}\,\Omega}\right)=2.22\times10^{-4}\,\text{A}$$

The graphs are shown. The times for the requested values are about 4.4 or 4.5 ms, about one time constant, within the accuracy of estimation on the graphs.

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH26.XLS,” on tab “Problem 26.93.”



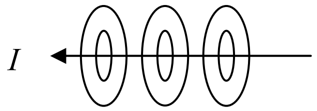


CHAPTER 27: Magnetism

Responses to Questions

1. The compass needle aligns itself with the local magnetic field of the Earth, and the Earth's magnetic field lines are not always parallel to the surface of the Earth.

2. The magnetic field lines are concentric circles around the wire. With the current running to the left, the field is directed counterclockwise when looking from the left end. So, the field goes into the page above the wire and comes out of the page below the wire.



3. The force is downward. The field lines point from the north pole to the south pole, or left to right. Use the right hand rule. Your fingers point in the direction of the current (away from you). Curl them in the direction of the field (to the right). Your thumb points in the direction of the force (downward).

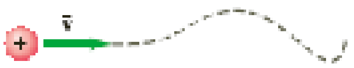
4. $\vec{\mathbf{F}}$ is always perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\ell}$. $\vec{\mathbf{B}}$ and $\vec{\ell}$ can be at any angle with respect to each other.

5. Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

6. The kinetic energy of the particle will stay the same. The magnetic force on the particle will be perpendicular to the particle's velocity vector and so will do no work on the particle. The force will change the direction of the particle's velocity but not the speed.

7. Positive particle in the upper left: force is downward toward the wire. Negative particle in the upper right: force is to the left. Positive particle in the lower right: force is to the left. Negative particle in the lower left: force is upward toward the wire.

8. In the areas where the particle's path is curving up towards the top of the page, the magnetic field is directed into the page. Where the particle's path curves downward towards the bottom of the page, the magnetic field is directed out of the page. Where the particle is moving in a straight line, the magnetic field direction is parallel or anti-parallel to the particle's velocity. The strength of the magnetic field is greatest where the radius of curvature of the path is the smallest.



9. (*a*) Near one pole of a very long bar magnet, the magnetic field is proportional to $1/r^2$.
   (*b*) Far from the magnet as a whole, the magnetic field is proportional to $1/r^3$.

10. The picture is created when moving charged particles hit the back of the screen. A strong magnet held near the screen can deflect the particles from their intended paths, and thus distort the picture. If the magnet is strong enough, it is possible to deflect the particles so much that they do not even reach the screen, and the picture "goes black."

11. The negative particle will curve down (toward the negative plate) if $v > E/B$ because the magnetic force (down) will be greater than the electric force (up). If $v < E/B$ the negative particle will curve up toward the positive plate because the electric force will be greater than the magnetic force. The motion of a positive particle would be exactly opposite that of a negative particle.

12. No, you cannot set a resting electron into motion with a static magnetic field. In order for a charged particle to experience a magnetic force, it must already have a velocity with a component perpendicular to the magnetic field: $F = qvB\sin\theta$. If $v = 0$, $F = 0$. Yes, you can set an electron into motion with an electric field. The electric force on a charged particle does not depend on velocity: $F = qE$.

13. The particle will move in an elongating helical path in the direction of the electric field (for a positive charge). The radius of the helix will remain constant.

14. Consider a positive ion. It will experience a force downward due to the applied electric field. Once it begins moving downward, it will then experience a force out (in the direction of the red arrow) because of its motion in the magnetic field. A negative ion will experience a force up due to the electric field and then, because it is a negative particle moving up in the magnetic field directed to the right, it will experience a force out. The positive and negative ions therefore each feel a force in the same direction.

15. The beam is deflected to the right. The current in the wire creates a magnetic field into the page surrounding the beam of electrons. This results in a magnetic force on the negative particles that is to the right.

16. Yes. One possible situation is that the magnetic field is parallel or anti-parallel to the velocity of the charged particle. In this case, the magnetic force would be zero, and the particle would continue moving in a straight line. Another possible situation is that there is an electric field with a magnitude and direction (perpendicular to the magnetic field) such that the electric and magnetic forces on the particle cancel each other out. The net force would be zero and the particle would continue moving in a straight line.

17. No. A charged particle may be deflected sideways by an electric field if a component of its velocity is perpendicular to the field.

18. If the direction of the velocity of the electrons is changing but their speed is not, then they are being deflected by a magnetic field only, and their path will be circular or helical. If the speed of the electrons is changing but the direction is not, then they are being accelerated by an electric field only. If both speed and direction are changing, the particles are possibly being deflected by both magnetic and electric fields, or they are being deflected by an electric field that is not parallel to the initial velocity of the particles. In the latter case, the component of the electron velocity antiparallel to the field direction will continue to increase, and the component of the electron velocity perpendicular to the field direction will remain constant. Therefore, the electron will asymptotically approach a straight path in the direction opposite the field direction. If the particles continue with a circular component to their path, there must be a magnetic field present.

19. Use a small current-carrying coil or solenoid for the compass needle.

20. Suspend the magnet in a known magnetic field so that it is aligned with the field and free to rotate. Measure the torque necessary to rotate the magnet so that it is perpendicular to the field lines. The magnetic moment will be the torque divided by the magnetic field strength. $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ so $\tau = \mu B$ when the magnetic moment and the field are perpendicular.

21. (*a*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is parallel to the field lines, the loop will be in stable equilibrium. Small displacements from this position will result in a torque that tends to return the loop to this position.
    (*b*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is anti-parallel to the field lines, the loop will be in unstable equilibrium.

22. The charge carriers are positive. Positive particles moving to the right in the figure will experience a magnetic force into the page, or toward point *a*. Therefore, the positive charge carriers will tend to move toward the side containing *a*; this side will be at a higher potential than the side with point *b*.

23. The distance 2*r* to the singly charged ions will be twice the distance to the doubly charged ions.

Solutions to Problems

$\boxed{1.}$ (*a*) Use Eq. 27-1 to calculate the force with an angle of 90° and a length of 1 meter.

$$F = I\ell B\sin\theta \;\rightarrow\; \frac{F}{\ell} = IB\sin\theta = (9.40\,\text{A})(0.90\,\text{T})\sin 90^\circ = \boxed{8.5\,\text{N/m}}$$

(*b*) $$\frac{F}{\ell} = IB\sin\theta = (9.40\,\text{A})(0.90\,\text{T})\sin 35.0^\circ = \boxed{4.9\,\text{N/m}}$$

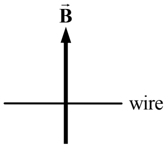
2. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (150\,\text{A})(240\,\text{m})\left(5.0\times10^{-5}\,\text{T}\right)\sin 68^\circ = \boxed{1.7\,\text{N}}$$

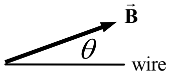
3. The dip angle is the angle between the Earth's magnetic field and the current in the wire. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (4.5\,\text{A})(1.6\,\text{m})\left(5.5\times10^{-5}\,\text{T}\right)\sin 41^\circ = \boxed{2.6\times10^{-4}\,\text{N}}$$

4. To have the maximum force, the current must be perpendicular to the magnetic field, as shown in the first diagram. Use $\frac{F}{\ell} = 0.25\frac{F_{\text{max}}}{\ell}$ to find the angle between the wire and the magnetic field, illustrated in the second diagram.



$$\frac{F}{\ell} = 0.25\frac{F_{\text{max}}}{\ell} \;\rightarrow\; IB\sin\theta = 0.25IB \;\rightarrow\; \theta = \sin^{-1}0.25 = \boxed{14^\circ}$$



5. (*a*) By the right hand rule, the magnetic field must be pointing up, and so the top pole face must be a $\boxed{\text{South pole}}$.
    (*b*) Use Eq. 27-2 to relate the maximum force to the current. The length of wire in the magnetic field is equal to the diameter of the pole faces.

$$F_{\text{max}} = I\ell B \;\to\; I = \frac{F_{\text{max}}}{\ell B} = \frac{\left(7.50\times10^{-2}\,\text{N}\right)}{\left(0.100\,\text{m}\right)\left(0.220\,\text{T}\right)} = 3.4091\,\text{A} \approx \boxed{3.41\,\text{A}}$$

(*c*) Multiply the maximum force by the sine of the angle between the wire and the magnetic field.

$$F = F_{\text{max}}\sin\theta = \left(7.50\times10^{-2}\,\text{N}\right)\sin 80.0^\circ = \boxed{7.39\times10^{-2}\,\text{N}}$$

6. The magnetic force must be equal in magnitude to the force of gravity on the wire. The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field. The mass of the wire is the density of copper times the volume of the wire.

$$F_{\text{B}} = mg \;\to\; I\ell B = \rho\pi\left(\tfrac{1}{2}d\right)^2\ell g \;\to$$

$$I = \frac{\rho\pi d^2 g}{4B} = \frac{\left(8.9\times10^3\,\text{kg/m}^3\right)\pi\left(1.00\times10^{-3}\,\text{m}\right)^2\left(9.80\,\text{m/s}^2\right)}{4\left(5.0\times10^{-5}\,\text{T}\right)} = \boxed{1400\,\text{A}}$$

This answer does not seem feasible. The current is very large, and the resistive heating in the thin copper wire would probably melt it.

$\boxed{7.}$ We find the force using Eq. 27-3, where the vector length is broken down into two parts: the portion along the z-axis and the portion along the line *y=2x*.

$$\vec{\boldsymbol{\ell}}_1 = -0.250\,\text{m}\,\hat{\mathbf{k}} \qquad \vec{\boldsymbol{\ell}}_2 = 0.250\,\text{m}\left(\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)$$

$$\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\left(\vec{\boldsymbol{\ell}}_1+\vec{\boldsymbol{\ell}}_2\right)\times\vec{\mathbf{B}} = \left(20.0\,\text{A}\right)\left(0.250\,\text{m}\right)\left(-\hat{\mathbf{k}}+\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)\times\left(0.318\hat{\mathbf{i}}\,\text{T}\right)$$

$$= \left(1.59\text{ N}\right)\left(-\hat{\mathbf{k}}\times\hat{\mathbf{i}}+\frac{2}{\sqrt{5}}\hat{\mathbf{j}}\times\hat{\mathbf{i}}\right) = -\left(1.59\hat{\mathbf{j}}+1.42\hat{\mathbf{k}}\right)\text{N}$$

$$F = \left|\vec{\mathbf{F}}\right| = \sqrt{1.59^2+1.42^2}\text{ N} = \boxed{2.13\text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{-1.42\text{ N}}{-1.59\text{ N}}\right) = \boxed{41.8^\circ\text{ below the negative y-axis}}$$

8. We find the force per unit length from Eq. 27-3. Note that while the length is not known, the direction is given, and so $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_{\text{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\ell\hat{\mathbf{i}}\times\vec{\mathbf{B}} \;\to$$

$$\frac{\vec{\mathbf{F}}_{\text{B}}}{\ell} = I\,\hat{\mathbf{i}}\times\vec{\mathbf{B}} = \left(3.0\,\text{A}\right)\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 0 & 0\\ 0.20\,\text{T} & -0.36\,\text{T} & 0.25\,\text{T}\end{vmatrix} = \left(-0.75\hat{\mathbf{j}}-1.08\hat{\mathbf{k}}\right)\text{N/m}\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)$$

$$= \boxed{-\left(7.5\hat{\mathbf{j}}+11\hat{\mathbf{k}}\right)\times10^{-3}\text{ N/cm}}$$

9. We find the net force on the loop by integrating the infinitesimal force on each infinitesimal portion of the loop within the magnetic field. The infinitesimal force is found using Eq. 27-4 with the current in an infinitesimal portion of the loop given by $Id\vec{\boldsymbol{\ell}} = I\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta$.

$$\vec{\mathbf{F}} = \int Id\vec{\boldsymbol{\ell}}\times\vec{B} = I\int_{\theta_0}^{2\pi-\theta_0}\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta\times B_0\hat{\mathbf{k}} = IB_0 r\int_{\theta_0}^{2\pi-\theta_0}\left(\cos\theta\hat{\mathbf{j}}+\sin\theta\hat{\mathbf{i}}\right)d\theta$$

$$= IB_0r\left(\sin\theta\hat{\mathbf{j}} - \cos\theta\hat{\mathbf{i}}\right)\Big|_{\theta_0}^{2\pi-\theta_0} = IB_0r\left[\sin(2\pi-\theta_0)\hat{\mathbf{j}} - \sin\theta_0\hat{\mathbf{j}} - \cos(2\pi-\theta_0)\hat{\mathbf{i}} + \cos\theta_0\hat{\mathbf{i}}\right]$$

$$= \boxed{-2IB_0r\sin\theta_0\hat{\mathbf{j}}}$$

The trigonometric identities $\sin(2\pi-\theta) = -\sin\theta$ and $\cos(2\pi-\theta) = \cos\theta$ are used to simplify the solution.

10. We apply Eq. 27-3 to each circumstance, and solve for the magnetic field. Let $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$.

For the first circumstance, $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = (8.2\,\text{A})\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2.0\,\text{m} & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix} = (-16.4\,\text{A}\bullet\text{m})B_z\hat{\mathbf{j}} + (16.4\,\text{A}\bullet\text{m})B_y\hat{\mathbf{k}} = (-2.5\hat{\mathbf{j}})\,\text{N} \rightarrow$$

$$B_y = 0\,;\ (-16.4\,\text{A}\bullet\text{m})B_z = -2.5\,\text{N} \rightarrow B_z = \frac{2.5\,\text{N}}{16.4\,\text{A}\bullet\text{m}} = 0.1524\,\text{T}\,;\ B_x \text{ unknown}$$

For the second circumstance, $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{j}}$.

$$\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = (8.2\,\text{A})\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0\,\text{m} & 0 \\ B_x & 0 & 0.1524\,\text{T} \end{vmatrix} = (2.5\,\text{N})\hat{\mathbf{i}} + (-16.4\,\text{A}\bullet\text{m})B_x\hat{\mathbf{k}} = (2.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{k}})\,\text{N} \rightarrow$$

$$(-16.4\,\text{A}\bullet\text{m})B_x = -5.0\,\text{N} \rightarrow B_x = \frac{5.0\,\text{N}}{16.4\,\text{A}\bullet\text{m}} = 0.3049\,\text{T}$$

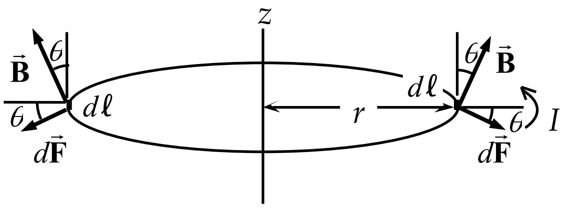
Thus $\vec{\mathbf{B}} = \boxed{(0.30\hat{\mathbf{i}} + 0.15\hat{\mathbf{k}})\,\text{T}}$.

11. We find the force along the wire by integrating the infinitesimal force from each path element (given by Eq. 27-4) along an arbitrary path between the points *a* and *b*.

$$\vec{\mathbf{F}} = \int_a^b I d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = I\int_a^b \left(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy\right) \times B_0\hat{\mathbf{k}} = IB_0\int_a^b \left(-\hat{\mathbf{j}}dx + \hat{\mathbf{i}}dy\right) = IB_0\left(-\Delta x\hat{\mathbf{j}} + \Delta y\hat{\mathbf{i}}\right)$$

The resultant magnetic force on the wire depends on the displacement between the points *a* and *b*, and not on the path taken by the wire. Therefore, the resultant force must be the same for the curved path, as for the straight line path between the points.

12. The net force on the current loop is the sum of the infinitesimal forces obtained from each current element. From the figure, we see that at each current segment, the magnetic field is perpendicular to the current. This results in a force with only radial and vertical components. By symmetry, we find that the radial force components from segments on opposite sides of the loop cancel. The net force then is purely vertical. Symmetry also shows us that each current element contributes the same magnitude of force.



$$\vec{\mathbf{F}} = \int I d\vec{\boldsymbol{\ell}} \times B = -IB_r\hat{\mathbf{k}}\int d\ell = -I(B\sin\theta)\hat{\mathbf{k}}(2\pi r) = \boxed{-2\pi IB\frac{r^2}{\sqrt{r^2+d^2}}\hat{\mathbf{k}}}$$

$\boxed{13.}$ The maximum magnetic force as given in Eq. 27-5b can be used since the velocity is perpendicular to the magnetic field.

$$F_{\max} = qvB = \left(1.60\times10^{-19}\,\text{C}\right)\left(8.75\times10^{5}\,\text{m/s}\right)\left(0.45\,\text{T}\right) = \boxed{6.3\times10^{-14}\,\text{N}}$$

By the right hand rule, the force must be directed to the $\boxed{\text{North}}$.

14. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\max} = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.70\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.480\,\text{T}\right)} = \boxed{2.02\times10^{-5}\,\text{m}}$$

15. In this scenario, the magnetic force is causing centripetal motion, and so must have the form of a centripetal force. The magnetic force is perpendicular to the velocity at all times for circular motion.

$$F_{\max} = qvB = m\frac{v^2}{r} \rightarrow B = \frac{mv}{qr} = \frac{\left(6.6\times10^{-27}\,\text{kg}\right)\left(1.6\times10^{7}\,\text{m/s}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.18\,\text{m}\right)} = \boxed{1.8\,\text{T}}$$

16. Since the charge is negative, the answer is the OPPOSITE of the result given from the right hand rule applied to the velocity and magnetic field.

(*a*) left
(*b*) left
(*c*) upward
(*d*) inward into the paper
(*e*) no force
(*f*) downward

17. The right hand rule applied to the velocity and magnetic field would give the direction of the force. Use this to determine the direction of the magnetic field given the velocity and the force.

(*a*) downward
(*b*) inward into the paper
(*c*) right

18. The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force.

$$F_{\text{E}} = F_{\text{B}} \rightarrow qE = qvB \rightarrow v = \frac{E}{B} = \frac{8.8\times10^{3}\,\text{V/m}}{7.5\times10^{-3}\,\text{T}} = 1.173\times10^{6}\,\text{m/s} \approx \boxed{1.2\times10^{6}\,\text{m/s}}$$

If the electric field is turned off, the magnetic force will cause circular motion.

$$F_{\text{B}} = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.173\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(7.5\times10^{-3}\,\text{T}\right)} = \boxed{8.9\times10^{-4}\,\text{m}}$$

$\boxed{19.}$ (*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow qV = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$F_{\max} = qvB = m\frac{v^2}{r} \quad \to$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,\text{T}}\sqrt{\frac{2\left(6.6\times10^{-27}\,\text{kg}\right)\left(2700\,\text{V}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)}} = \boxed{3.1\times10^{-2}\,\text{m}}$$

(*b*) The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above.

$$v = \frac{2\pi r}{T} \quad \to \quad T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi\left(6.6\times10^{-27}\,\text{kg}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.340\,\text{T}\right)} = \boxed{3.8\times10^{-7}\,\text{s}}$$

20. The velocity of each charged particle can be found using energy conservation. The electrical potential energy of the particle becomes kinetic energy as it is accelerated. Then, since the particle is moving perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path, and the radius can be determined in terms of the mass and charge of the particle.

$$E_{\text{initial}} = E_{\text{final}} \quad \to \quad qV = \tfrac{1}{2}mv^2 \quad \to \quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{\max} = qvB = m\frac{v^2}{r} \quad \to \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_{\text{d}}}{r_{\text{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\text{d}}V}{q_{\text{d}}}}}{\frac{1}{B}\sqrt{\frac{2m_{\text{p}}V}{q_{\text{p}}}}} = \frac{\sqrt{\frac{m_{\text{d}}}{m_{\text{p}}}}}{\sqrt{\frac{q_{\text{d}}}{q_{\text{p}}}}} = \frac{\sqrt{2}}{\sqrt{1}} = \sqrt{2} \quad \to \quad \boxed{r_{\text{d}} = \sqrt{2}r_{\text{p}}}$$

$$\frac{r_{\alpha}}{r_{\text{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\alpha}V}{q_{\alpha}}}}{\frac{1}{B}\sqrt{\frac{2m_{\text{p}}V}{q_{\text{p}}}}} = \frac{\sqrt{\frac{m_{\alpha}}{m_{\text{p}}}}}{\sqrt{\frac{q_{\alpha}}{q_{\text{p}}}}} = \frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2} \quad \to \quad \boxed{r_{\alpha} = \sqrt{2}r_{\text{p}}}$$

21. (*a*) From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. The kinetic energy is given by

$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m}$ and so we see that $\boxed{K \propto r^2}$.

(*b*) The angular momentum of a particle moving in a circular path is given by $L = mvr$. From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. Combining these relationships gives

$L = mvr = m\frac{rqB}{m}r = \boxed{qBr^2}$.

22. The force on the electron is given by Eq. 27-5a.

$$\vec{\mathbf{F}}_{\rm B} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 7.0\times10^4\ \mathrm{m/s} & -6.0\times10^4\ \mathrm{m/s} & 0 \\ -0.80\,\mathrm{T} & 0.60\,\mathrm{T} & 0 \end{vmatrix} = -e\left(4.2-4.8\right)\times10^4\ \mathrm{T{\bullet}m/s}\,\hat{\mathbf{k}}$$

$$= -\left(1.60\times10^{-19}\mathrm{C}\right)\left(-0.6\times10^4\ \mathrm{T{\bullet}m/s}\,\hat{\mathbf{k}}\right) = 9.6\times10^{-16}\mathrm{N}\,\hat{\mathbf{k}} \approx \boxed{1\times10^{-15}\mathrm{N}\,\hat{\mathbf{k}}}$$

23. The kinetic energy of the proton can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined.

$$K = \tfrac{1}{2}mv^2 \ \rightarrow\ v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \ \rightarrow\ r = \frac{mv}{qB}$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\dfrac{2K}{m}}}{qB} = \frac{\sqrt{2Km}}{qB} = \frac{\sqrt{2\left(6.0\times10^6\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)\left(1.67\times10^{-27}\mathrm{kg}\right)}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(0.20\,\mathrm{T}\right)} = \boxed{1.8\,\mathrm{m}}$$

24. The magnetic field can be found from Eq. 27-5b, and the direction is found from the right hand rule. Remember that the charge is negative.

$$F_{\rm max} = qvB \ \rightarrow\ B = \frac{F_{\rm max}}{qv} = \frac{8.2\times10^{-13}\mathrm{N}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(2.8\times10^6\ \mathrm{m/s}\right)} = \boxed{1.8\,\mathrm{T}}$$

The direction would have to be $\boxed{\text{East}}$ for the right hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

$\boxed{25.}$ The total force on the proton is given by the Lorentz equation, Eq. 27-7.

$$\vec{\mathbf{F}}_{\rm B} = q\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right) = e\left[\left(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}}\right)\times10^3\ \mathrm{V/m} + \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.0\times10^3\ \mathrm{m/s} & 3.0\times10^3\ \mathrm{m/s} & -5.0\times10^3\ \mathrm{m/s} \\ 0.45\,\mathrm{T} & 0.38\,\mathrm{T} & 0 \end{vmatrix}\right]$$

$$= \left(1.60\times10^{-19}\mathrm{C}\right)\left[\left(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}}\right)+\left(1.9\hat{\mathbf{i}}-2.25\hat{\mathbf{j}}+0.93\hat{\mathbf{k}}\right)\right]\times10^3\ \mathrm{N/C}$$

$$= \left(1.60\times10^{-19}\mathrm{C}\right)\left[\left(4.9\hat{\mathbf{i}}-6.45\hat{\mathbf{j}}+0.93\hat{\mathbf{k}}\right)\right]\times10^3\ \mathrm{N/C}$$

$$= \left(7.84\times10^{-16}\hat{\mathbf{i}}-1.03\times10^{-15}\hat{\mathbf{j}}+1.49\times10^{-16}\hat{\mathbf{k}}\right)\mathrm{N/C}$$

$$= \boxed{\left[\left(0.78\hat{\mathbf{i}}-1.0\hat{\mathbf{j}}+0.15\hat{\mathbf{k}}\right)\right]\times10^{-15}\mathrm{N}}$$

26. The force on the electron is given by Eq. 27-5a. Set the force expression components equal and solve for the velocity components.

$$\vec{\mathbf{F}}_{\rm B} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \ \rightarrow\ F_x\hat{\mathbf{i}}+F_y\hat{\mathbf{j}} = -e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ 0 & 0 & B_z \end{vmatrix} = -ev_yB_z\hat{\mathbf{i}} - e\left(-v_xB_z\right)\hat{\mathbf{j}} \ \rightarrow$$

$$F_x = -ev_yB_z \ \rightarrow\ v_y = -\frac{F_x}{eB_z} = -\frac{3.8\times10^{-13}\mathrm{N}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(0.85\mathrm{T}\right)} = -2.8\times10^6\ \mathrm{m/s}$$

$$F_y = ev_xB_z \quad \rightarrow \quad v_x = \frac{F_y}{eB_z} = \frac{-2.7\times10^{-13}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.85\,\text{T}\right)} = -2.0\times10^{6}\,\text{m/s}$$

$$\vec{\mathbf{v}} = \boxed{-\left(2.0\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}}\right)\times10^{6}\,\text{m/s}}$$

Notice that we have not been able to determine the *z* component of the electron's velocity.

27. The kinetic energy of the particle can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined. Inserting the radius and velocity into the equation for angular momentum gives the angular momentum in terms of the kinetic energy and magnetic field.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB}$$

$$L = mvr = m\sqrt{\frac{2K}{m}}\left(\frac{m\sqrt{\frac{2K}{m}}}{qB}\right) = \frac{2mK}{qB}$$

From the equation for the angular momentum, we see that doubling the magnetic field while keeping the kinetic energy constant will cut the angular momentum in half.

$$\boxed{L_{\text{final}} = \tfrac{1}{2}L_{\text{initial}}}$$

28. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad \rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(3.0\times10^{6}\,\text{m/s}\right)\sin45^\circ}{\left(1.60\times10^{-19}\text{C}\right)\left(0.28\,\text{T}\right)} = 4.314\times10^{-5}\,\text{m} \approx \boxed{4.3\times10^{-5}\,\text{m}}$$

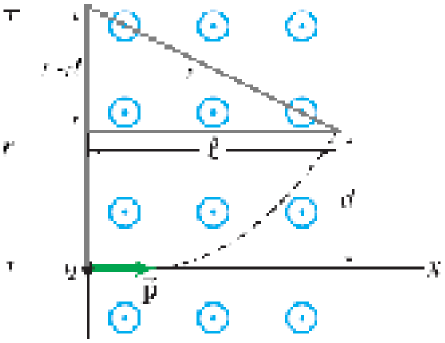
The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_{\parallel}T = v\cos45^\circ\left(\frac{2\pi m}{qB}\right) = \left(3.0\times10^{6}\,\text{m/s}\right)\cos45^\circ\frac{2\pi\left(9.11\times10^{-31}\text{kg}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.28\,\text{T}\right)} = \boxed{2.7\times10^{-4}\,\text{m}}$$

29. (*a*) For the particle to move upward the magnetic force must point upward, by the right hand rule we see that the force on a positively charged particle would be downward. Therefore, the charge on the particle must be $\boxed{\text{negative}}$.

(*b*) In the figure we have created a right triangle to relate the horizontal distance ℓ, the displacement *d*, and the radius of curvature, *r*. Using the Pythagorean theorem we can write an expression for the radius in terms of the other two distances.



$$r^2 = (r-d)^2 + \ell^2 \rightarrow r = \frac{d^2+\ell^2}{2d}$$

Since the momentum is perpendicular to the magnetic field, we can solve for the momentum by relating the maximum force (Eq. 27-5b) to the centripetal force on the particle.

$$F_{\max} = qvB_0 = \frac{mv^2}{r} \quad \rightarrow \quad p = mv = qB_0 r = \boxed{\frac{qB_0\left(d^2+\ell^2\right)}{2d}}$$

30. In order for the path to be bent by 90° within a distance *d*, the radius of curvature must be less than or equal to *d*. The kinetic energy of the protons can be used to find their velocity. The magnetic force produces centripetal acceleration, and from this, the magnetic field can be determined.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad B = \frac{mv}{qr}$$

$$B \geq \frac{mv}{ed} = \frac{m\sqrt{\frac{2K}{m}}}{ed} = \boxed{\left(\frac{2Km}{e^2d^2}\right)^{1/2}}$$

$\boxed{31.}$ The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is $6.38\times10^6\,\text{km}$, and the altitude is added to that.

$$F_B = qvB = m\frac{v^2}{r} \quad \rightarrow \quad v = \frac{qrB}{m} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(6.385\times10^6\,\text{m}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)} = \boxed{1.3\times10^8\,\text{m/s}}$$

Compare the size of the magnetic force to the force of gravity on the ion.

$$\frac{F_B}{F_g} = \frac{qvB}{mg} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.3\times10^8\,\text{m/s}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = 2.3\times10^8$$

$\boxed{\text{Yes}}$, we may ignore gravity. The magnetic force is more than 200 million times larger than gravity.

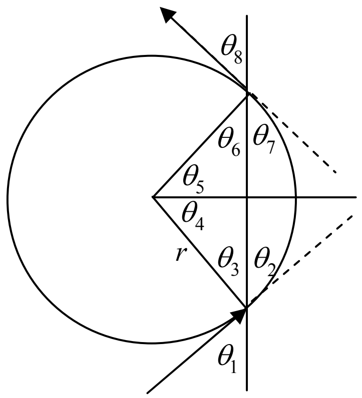
32. The magnetic force produces an acceleration that is perpendicular to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity *v* and the distance traveled ℓ. The starting speed in the perpendicular direction will be zero.

$$F_\perp = ma_\perp = qvB \quad \rightarrow \quad a_\perp = \frac{qvB}{m}$$

$$d_\perp = v_{0\perp}t + \tfrac{1}{2}a_\perp t^2 = \tfrac{1}{2}\frac{qvB}{m}\left(\frac{\ell}{v}\right)^2 = \frac{qB\ell^2}{2mv} = \frac{\left(18.5\times10^{-9}\,\text{C}\right)\left(5.00\times10^{-5}\,\text{T}\right)\left(1.00\times10^3\,\text{m}\right)^2}{2\left(3.40\times10^{-3}\,\text{kg}\right)\left(155\,\text{m/s}\right)}$$

$$= \boxed{8.8\times10^{-7}\,\text{m}}$$

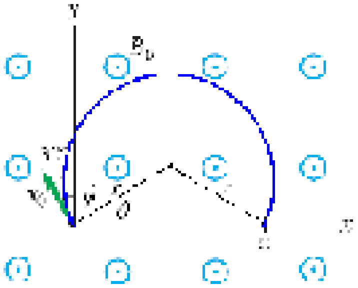
This small distance justifies the assumption of constant acceleration.



33. (*a*) In the magnetic field, the proton will move along an arc of a circle. The distance x in the diagram is a chord of that circle, and so the center of the circular path lies on the perpendicular bisector of the chord. That perpendicular bisector bisects the central angle of the circle which subtends the chord. Also recall that a radius is perpendicular to a tangent. In the diagram, $\theta_1 = \theta_2$ because they are vertical angles. Then $\theta_2 = \theta_4$, because they are both complements of θ_3, so $\theta_1 = \theta_4$. We have $\theta_4 = \theta_5$ since the central angle is bisected by the perpendicular bisector of the chord. $\theta_5 = \theta_7$ because they are both complements of θ_6, and $\theta_7 = \theta_8$ because they are vertical angles. Thus $\theta_1 = \theta_2 = \theta_4 = \theta_5 = \theta_7 = \theta_8$, and so in the textbook diagram, the angle at which the proton leaves is $\boxed{\theta = 45^\circ}$.

(*b*) The radius of curvature is given by $r = \dfrac{mv}{qB}$, and the distance x is twice the value of $r\cos\theta$.

$$x = 2r\cos\theta = 2\frac{mv}{qB}\cos\theta = 2\frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.3\times10^{5}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.850\,\text{T}\right)}\cos45^\circ = \boxed{2.3\times10^{-3}\,\text{m}}$$



34. (*a*) Since the velocity is perpendicular to the magnetic field, the particle will follow a circular trajectory in the *x*-*y* plane of radius *r*. The radius is found using the centripetal acceleration.

$$qvB = \frac{mv^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB}$$

From the figure we see that the distance α is the chord distance, which is twice the distance $r\cos\theta$. Since the velocity is perpendicular to the radial vector, the initial direction and the angle ϕ are complementary angles. The angles ϕ and θ are also complementary angles, so $\theta = 30^\circ$.

$$\alpha = 2r\cos\theta = \frac{2mv_0}{qB_0}\cos30^\circ = \boxed{\sqrt{3}\frac{mv_0}{qB_0}}$$

(*b*) From the diagram, we see that the particle travels a circular path, that is 2ϕ short of a complete circle. Since the angles ϕ and θ are complementary angles, so $\phi = 60^\circ$. The trajectory distance is equal to the circumference of the circular path times the fraction of the complete circle. Dividing the distance by the particle speed gives t_α.

$$t_\alpha = \frac{\ell}{v_0} = \frac{2\pi r}{v_0}\left(\frac{360^\circ - 2\left(60^\circ\right)}{360^\circ}\right) = \frac{2\pi}{v_0}\frac{mv_0}{qB_0}\left(\frac{2}{3}\right) = \boxed{\frac{4\pi m}{3qB_0}}$$

35. The work required by an external agent is equal to the change in potential energy. The potential energy is given by Eq. 27-12, $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}$.

(*a*) $W = \Delta U = \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} - \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} = \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} - \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$

$= NIAB\left(\cos0^\circ - \cos180^\circ\right) = \boxed{2NIAB}$

(*b*) $W = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right) = NIAB\left(\cos90^\circ - \cos\left(-90^\circ\right)\right) = \boxed{0}$

36. With the plane of the loop parallel to the magnetic field, the torque will be a maximum. We use Eq. 27-9.

$$\tau = NIAB\sin\theta \rightarrow B = \frac{\tau}{NIAB\sin\theta} = \frac{0.185\,\text{m}\bullet\text{N}}{(1)(4.20\,\text{A})\pi(0.0650\,\text{m})^2\sin 90^\circ} = \boxed{3.32\,\text{T}}$$

$\boxed{37.}$ (*a*) The torque is given by Eq. 27-9. The angle is the angle between the B-field and the perpendicular to the coil face.

$$\tau = NIAB\sin\theta = 12(7.10\,\text{A})\left[\pi\left(\frac{0.180\,\text{m}}{2}\right)^2\right](5.50\times10^{-5}\,\text{T})\sin 24^\circ = \boxed{4.85\times10^{-5}\,\text{m}\bullet\text{N}}$$

(*b*) In Example 27-11 it is stated that if the coil is free to turn, it will rotate toward the orientation so that the angle is 0. In this case, that means the $\boxed{\text{north}}$ edge of the coil will rise, so that a perpendicular to its face will be parallel with the Earth's magnetic field.

38. The magnetic dipole moment is defined in Eq. 27-10 as $\mu = NIA$. The number of turns, N, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume the electron is moving in a circular orbit of radius *r*, then the area is πr^2. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mrv$. Combine these relationships to find the magnetic moment.

$$\mu = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emrv}{2m} = \frac{e}{2m}mrv = \frac{e}{2m}L$$

39. (*a*) The magnetic moment of the coil is given by Eq. 27-10. Since the current flows in the clockwise direction, the right hand rule shows that the magnetic moment is down, or in the negative *z*-direction.

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}} = 15(7.6\,\text{A})\pi\left(\frac{0.22\,\text{m}}{2}\right)^2(-\hat{\mathbf{k}}) = -4.334\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2 \approx \boxed{-4.3\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2}$$

(*b*) We use Eq. 27-11 to find the torque on the coil.

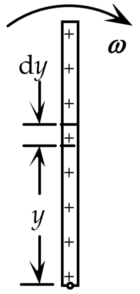
$$\vec{\tau} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} = (-4.334\,\hat{\mathbf{k}}\ \text{A}\bullet\text{m}^2)\times(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}})\text{T} = \boxed{(2.6\hat{\mathbf{i}} - 2.4\hat{\mathbf{j}})\text{m}\bullet\text{N}}$$

(*c*) We use Eq. 27-12 to find the potential energy of the coil.

$$U = -\vec{\boldsymbol{\mu}}\bullet\vec{\mathbf{B}} = -(-4.334\,\hat{\mathbf{k}}\ \text{A}\bullet\text{m}^2)(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}})\text{T} = -(4.334\text{A}\bullet\text{m}^2)(0.65\ \text{T})$$
$$= \boxed{-2.8\ \text{J}}$$

40. To find the total magnetic moment, we divide the rod into infinitesimal pieces of thickness *dy*. As the rod rotates on its axis the charge in each piece, $(Q/d)dy$, creates a current loop around the axis of rotation. The magnitude of the current is the charge times the frequency of rotation, $\omega/2\pi$. By integrating the infinitesimal magnetic moments from each piece, we find the total magnetic moment.

$$\vec{\boldsymbol{\mu}} = \int d\vec{\boldsymbol{\mu}} = \int \vec{\mathbf{A}}dI = \int_0^d (\pi y^2)\left(\frac{\omega}{2\pi}\frac{Q}{d}dy\right) = \frac{Q\omega}{2d}\int_0^d y^2 dy = \boxed{\frac{Q\omega d^2}{6}}.$$



41. From Section 27-5, we see that the torque is proportional to the current, so if the current drops by 12%, the output torque will also $\boxed{\text{drop by 12\%}}$. Thus the final torque is 0.88 times the initial torque.

42. In Section 27-6, it is shown that the deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.860 times its original value, the current must be increased by dividing the original value by 0.860 to obtain the same deflection.

$$(IB)_{\text{initial}} = (IB)_{\text{final}} \quad \to \quad I_{\text{final}} = \frac{I_{\text{initial}}B_{\text{initial}}}{B_{\text{final}}} = \frac{(63.0\,\mu\text{A})\,B_{\text{initial}}}{0.800B_{\text{initial}}} = \boxed{78.8\,\mu\text{A}}$$

$\boxed{43.}$ From the galvanometer discussion in Section 27-6, the amount of deflection is proportional to the ratio of the current and the spring constant: $\phi \propto \frac{I}{k}$. Thus if the spring constant decreases by 15%, the current can decrease by 15% to produce the same deflection. The new current will be 85% of the original current.

$$I_{\text{final}} = 0.85I_{\text{initial}} = 0.85(46\mu\text{A}) = \boxed{39\mu\text{A}}$$

44. Use Eq. 27-13.

$$\frac{q}{m} = \frac{E}{B^2r} = \frac{(260\,\text{V/m})}{(0.46\text{T})^2(0.0080\,\text{m})} = \boxed{1.5\times10^5\,\text{C/kg}}$$

45. The force from the electric field must be equal to the weight.

$$|qE| = (ne)\left(\frac{V}{d}\right) = mg \quad \to \quad n = \frac{mgd}{eV} = \frac{(3.3\times10^{-15}\text{kg})(9.80\,\text{m/s}^2)(0.010\,\text{m})}{(1.60\times10^{-19})(340\,\text{V})} = 5.94 \approx \boxed{6\text{ electrons}}$$

46. (*a*) Eq. 27-14 shows that the Hall emf is proportional to the magnetic field perpendicular to the conductor's surface. We can use this proportionality to determine the unknown resistance. Since the new magnetic field is oriented 90° to the surface, the full magnetic field will be used to create the Hall potential.

$$\frac{\mathscr{E}_H'}{\mathscr{E}_H} = \frac{B_\perp'}{B_\perp} \to B_\perp' = \frac{\mathscr{E}_H'}{\mathscr{E}_H}B_\perp = \frac{63\,\text{mV}}{12\,\text{mV}}(0.10\,\text{T}) = \boxed{0.53\text{T}}$$

(*b*) When the field is oriented at 60° to the surface, the magnetic field, $B\sin60°$, is used to create the Hall potential.

$$B_\perp'\sin60° = \frac{\mathscr{E}_H'}{\mathscr{E}_H}B_\perp \to B_\perp' = \frac{63\text{ mV}}{12\text{ mV}}\frac{(0.10\,\text{T})}{\sin60°} = \boxed{0.61\text{T}}$$

47. (*a*) We use Eq. 27-14 for the Hall Potential and Eq. 25-13 to write the current in terms of the drift velocity.

$$K_H = \frac{\mathscr{E}_H}{IB} = \frac{v_dBd}{\left[en(td)v_d\right]B} = \boxed{\frac{1}{ent}}$$

(*b*) We set the magnetic sensitivities equal and solve for the metal thickness.

$$\frac{1}{en_st_s} = \frac{1}{en_mt_m} \to t_m = \frac{n_s}{n_m}t_s = \frac{3\times10^{22}\,\text{m}^{-3}}{1\times10^{29}\text{ m}^{-3}}(0.15\times10^{-3}\text{m}) = \boxed{5\times10^{-11}\text{ m}}$$

This is less than one sixth the size of a typical metal atom.

(*c*) Use the magnetic sensitivity to calculate the Hall potential.

$$\mathcal{E}_H = K_H IB = \frac{IB}{ent} = \frac{(100\text{ mA})(0.1\text{T})}{(1.6\times10^{-19}\text{C})(3\times10^{22}\text{m}^{-3})(0.15\times10^{-3}\text{m})} = 14\,\text{mV} \approx \boxed{10\,\text{mV}}$$

48. (*a*) We find the Hall field by dividing the Hall emf by the width of the metal.

$$E_H = \frac{\mathcal{E}_H}{d} = \frac{6.5\ \mu\text{V}}{0.03\text{ m}} = 2.167\times10^{-4}\,\text{V/m} \approx \boxed{2.2\times10^{-4}\,\text{V/m}}$$

(*b*) Since the forces from the electric and magnetic fields are balanced, we can use Eq. 27-14 to calculate the drift velocity.

$$v_d = \frac{E_H}{B} = \frac{2.167\times10^{-4}\,\text{V/m}}{0.80\text{T}} = 2.709\times10^{-4}\text{ m/s} \approx \boxed{2.7\times10^{-4}\text{m/s}}$$

(*c*) We now find the density using Eq. 25-13.

$$n = \frac{I}{eAv_d} = \frac{42\,\text{A}}{(1.6\times10^{-19}\text{C})(6.80\times10^{-4}\text{m})(0.03\text{m})(2.709\times10^{-4}\text{m/s})}$$
$$= \boxed{4.7\times10^{28}\text{electrons/m}^3}$$

$\boxed{49.}$ We find the magnetic field using Eq. 27-14, with the drift velocity given by Eq. 25-13. To determine the electron density we divide the density of sodium by its atomic weight. This gives the number of moles of sodium per cubic meter. Multiplying the result by Avogadro's number gives the number of sodium atoms per cubic meter. Since there is one free electron per atom, this is also the density of free electrons.

$$B = \frac{\mathcal{E}_H}{v_d d} = \frac{\mathcal{E}_H}{\left(\dfrac{I}{ne(td)}\right)d} = \frac{\mathcal{E}_H net}{I} = \frac{\mathcal{E}_H et}{I}\left(\frac{\rho N_A}{m_A}\right)$$
$$= \frac{(1.86\times10^{-6}\text{V})(1.60\times10^{-19}\text{C})(1.30\times10^{-3}\text{m})}{12.0\text{ A}}\,\frac{(0.971)(1000\text{ kg/m}^3)(6.022\times10^{23}\text{e/mole})}{0.02299\text{ kg/mole}}$$
$$= \boxed{0.820\text{T}}$$

50. (*a*) The sign of the ions will not change the magnitude of the Hall emf, but will $\boxed{\text{determine the polarity of the emf}}$.

(*b*) The flow velocity corresponds to the drift velocity in Eq. 27-14.

$$\mathcal{E}_\text{H} = vBd \quad\to\quad \to\quad v = \frac{\mathcal{E}_\text{H}}{Bd} = \frac{(0.13\times10^{-3}\text{V})}{(0.070\text{T})(0.0033\text{m})} = \boxed{0.56\,\text{m/s}}$$

51. The magnetic force on the ions causes them to move in a circular path, so the magnetic force is a centripetal force. This results in the ion mass being proportional to the path's radius of curvature.

$$qvB = mv^2/r \quad\to\quad m = qBr/v \quad\to\quad m/r = qB/v = \text{constant} = 76\quad \text{u}/22.8\,\text{cm}$$

$$\frac{m_{21.0}}{21.0\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \quad\to\quad m_{21.0} = 70\,\text{u} \qquad \frac{m_{21.6}}{21.6\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \quad\to\quad m_{21.6} = 72\,\text{u}$$

$$\frac{m_{21.9}}{21.9\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \quad\to\quad m_{21.9} = 73\,\text{u} \qquad \frac{m_{22.2}}{22.2\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \quad\to\quad m_{22.2} = 74\,\text{u}$$

The other masses are $\boxed{\text{70 u, 72 u, 73 u, and 74 u}}$.

52. The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, since the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ions to move in a circular path.

$$qvB = \frac{mv^2}{R} \quad \rightarrow \quad v = \frac{qBR}{m} \qquad qV = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m} \quad \rightarrow \quad m = \frac{qR^2B^2}{2V}$$

53. The location of each line on the film is twice the radius of curvature of the ion. The radius of curvature can be found from the expression given in Section 27-9.

$$m = \frac{qBB'r}{E} \quad \rightarrow \quad r = \frac{mE}{qBB'} \quad \rightarrow \quad 2r = \frac{2mE}{qBB'}$$

$$2r_{12} = \frac{2(12)\left(1.67\times10^{-27}\,\text{kg}\right)\left(2.48\times10^{4}\,\text{V/m}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.58\,\text{T}\right)^2} = 1.8467\times10^{-2}\,\text{m}$$

$$2r_{13} = 2.0006\times10^{-2}\,\text{m} \qquad 2r_{14} = 2.1545\times10^{-2}\,\text{m}$$

The distances between the lines are

$$2r_{13} - 2r_{12} = 2.0006\times10^{-2}\,\text{m} - 1.8467\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 2.1545\times10^{-2}\,\text{m} - 2.0006\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

If the ions are doubly charged, the value of *q* in the denominator of the expression would double, and so the actual distances on the film would be halved. Thus the distances between the lines would also be halved.

$$2r_{13} - 2r_{12} = 1.0003\times10^{-2}\,\text{m} - 9.2335\times10^{-3}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 1.07725\times10^{-2}\,\text{m} - 1.0003\times10^{-2}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

54. The particles in the mass spectrometer follow a semicircular path as shown in Fig. 27-33. A particle has a displacement of 2*r* from the point of entering the semicircular region to where it strikes the film. So if the separation of the two molecules on the film is 0.65 mm, the difference in radii of the two molecules is 0.325 mm. The mass to radius ratio is the same for the two molecules.

$$qvB = mv^2/r \quad \rightarrow \quad m = qBr/v \quad \rightarrow \quad m/r = \text{constant}$$

$$\left(\frac{m}{r}\right)_{\text{CO}} = \left(\frac{m}{r}\right)_{\text{N}_2} \quad \rightarrow \quad \frac{28.0106\,\text{u}}{r} = \frac{28.0134\,\text{u}}{r + 3.25\times10^{-4}\,\text{m}} \quad \rightarrow \quad r = 3.251\,\text{m} \approx \boxed{3.3\,\text{m}}$$

$\boxed{55.}$ Since the particle is undeflected in the crossed fields, its speed is given by Eq. 27-8. Without the electric field, the particle will travel in a circle due to the magnetic force. Using the centripetal acceleration, we can calculate the mass of the particle. Also, the charge must be an integer multiple of the fundamental charge.

$$qvB = \frac{mv^2}{r} \quad \rightarrow$$

$$m = \frac{qBr}{v} = \frac{qBr}{(E/B)} = \frac{neB^2r}{E} = \frac{n\left(1.60\times10^{-19}\,\text{C}\right)\left(0.034\,\text{T}\right)^2\left(0.027\,\text{m}\right)}{1.5\times10^{3}\,\text{V/m}} = n\left(3.3\times10^{-27}\,\text{kg}\right) \approx n\left(2.0\,\text{u}\right)$$

The particle has an atomic mass of a multiple of 2.0 u. The simplest two cases are that it could be a hydrogen-2 nucleus (called a deuteron), or a helium-4 nucleus (called an alpha particle): $\boxed{{}^{2}_{1}\text{H}, {}^{4}_{2}\text{He}}$.

56. The radius and magnetic field values can be used to find the speed of the protons. The electric field is then found from the fact that the magnetic force must be the same magnitude as the electric force for the protons to have straight paths.

$$qvB = mv^2/r \rightarrow v = qBr/m \qquad F_{\text{E}} = F_{\text{B}} \rightarrow qE = qvB \rightarrow$$

$$E = vB = qB^2r/m = \frac{(1.60\times10^{-19}\,\text{C})(0.625\,\text{T})^2(5.10\times10^{-2}\,\text{m})}{1.67\times10^{-27}\,\text{kg}} = \boxed{1.91\times10^{6}\,\text{V/m}}$$

The direction of the electric field must be perpendicular to both the velocity and the magnetic field, and must be in the opposite direction to the magnetic force on the protons.

57. The magnetic force produces centripetal acceleration.

$$qvB = mv^2/r \rightarrow mv = p = qBr \rightarrow B = \frac{p}{qr} = \frac{3.8\times10^{-16}\,\text{kg}\bullet\text{m/s}}{(1.60\times10^{-19}\,\text{C})(1.0\times10^{3}\,\text{m})} = \boxed{2.4\,\text{T}}$$

The magnetic field must point $\boxed{\text{upward}}$ to cause an inward-pointing (centripetal) force that steers the protons clockwise.

58. The kinetic energy is used to determine the speed of the particles, and then the speed can be used to determine the radius of the circular path, since the magnetic force is causing centripetal acceleration.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{m\sqrt{\dfrac{2K}{m}}}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\frac{r_{\text{p}}}{r_{\text{e}}} = \frac{\dfrac{\sqrt{2m_pK}}{qB}}{\dfrac{\sqrt{2m_eK}}{qB}} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{42.8}$$

59. (*a*) There will be one force on the rod, due to the magnetic force on the charge carriers in the rod. That force is of magnitude $F_{\text{B}} = IdB$, and by Newton's second law is equal to the mass of the rod times its acceleration. That force is constant, so the acceleration will be constant, and constant acceleration kinematics can be used.

$$F_{\text{net}} = F_{\text{B}} = IdB = ma \rightarrow a = \frac{IdB}{m} = \frac{v - v_0}{t} = \frac{v}{t} \rightarrow v = \boxed{\frac{IdB}{m}t}$$

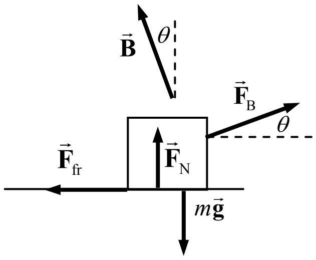
(*b*) Now the net force is the vector sum of the magnetic force and the force of kinetic friction.

$$F_{\text{net}} = F_{\text{B}} - F_{\text{fr}} = IdB - \mu_{\text{k}}F_{\text{N}} = IdB - \mu_{\text{k}}mg = ma \rightarrow$$

$$a = \frac{IdB}{m} - \mu_{\text{k}}g = \frac{v - v_0}{t} = \frac{v}{t} \rightarrow v = \boxed{\left(\frac{IdB}{m} - \mu_{\text{k}}g\right)t}$$

(*c*) Using the right hand rule, we find that the force on the rod is to the east, and the rod moves $\boxed{\text{east}}$.

60. Assume that the magnetic field makes an angle θ with respect to the vertical. The rod will begin to slide when the horizontal magnetic force $(IB\ell\cos\theta)$ is equal to the maximum static friction (μ_sF_N). Find the normal force by setting the sum of the vertical forces equal to zero. See the free body diagram.



$$F_B\sin\theta + F_N - mg = 0 \rightarrow F_N = mg - F_B\sin\theta = mg - I\ell B\sin\theta$$

$$IB\ell\cos\theta=\mu_sF_N=\mu_s\left(mg-I\ell B\sin\theta\right)\;\rightarrow\;B=\frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta+\cos\theta\right)}$$

We find the angle for the minimum magnetic field by setting the derivative of the magnetic field with respect to the angle equal to zero and solving for the angle.

$$\frac{dB}{d\theta}=0=\frac{-\mu_s mg\left(\mu_s\cos\theta-\sin\theta\right)}{I\ell\left(\mu_s\sin\theta+\cos\theta\right)^2}\;\rightarrow\theta=\tan^{-1}\mu_s=\tan^{-1}0.5=26.6^\circ$$

$$B=\frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta+\cos\theta\right)}=\frac{0.5\left(0.40\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(36\,\text{A}\right)\left(0.22\,\text{m}\right)\left(0.5\sin26.6^\circ+\cos26.6^\circ\right)}=0.22\,\text{T}$$

The minimum magnetic field that will cause the rod to move is $\boxed{\text{0.22 T at 27° from the vertical}}$.

61. The magnetic force must be equal in magnitude to the weight of the electron.

$$mg=qvB\quad\rightarrow\quad v=\frac{mg}{qB}=\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.50\times10^{-4}\text{T}\right)}=\boxed{1.1\times10^{-6}\ \text{m/s}}$$

The magnetic force must point upwards, and so by the right hand rule and the negative charge of the electron, the electron must be moving $\boxed{\text{west}}$.

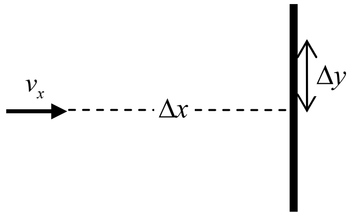
62. The airplane is a charge moving in a magnetic field. Since it is flying perpendicular to the magnetic field, Eq. 27-5b applies.

$$F_{\max}=qvB=\left(1850\times10^{-6}\text{C}\right)\left(120\,\text{m/s}\right)\left(5.0\times10^{-5}\text{T}\right)=\boxed{1.1\times10^{-5}\text{N}}$$

63. The maximum torque is found using Eq. 27-9 with $\sin\theta=1$. Set the current equal to the voltage divided by resistance and the area as the square of the side length.

$$\tau=NIAB=N\left(\frac{V}{R}\right)\ell^2B=20\left(\frac{9.0\,\text{V}}{24\,\Omega}\right)\left(0.050\,\text{m}\right)^2\left(0.020\,\text{T}\right)=\boxed{3.8\times10^{-4}\text{m}\cdot\text{N}}$$

64. The speed of the electrons is found by assuming the energy supplied by the accelerating voltage becomes kinetic energy of the electrons. We assume that those electrons are initially directed horizontally, and that the television set is oriented so that the electron velocity is perpendicular to the Earth's magnetic field, resulting in the largest possible force. Finally, we assume that the magnetic force on the electrons is small enough that the electron velocity is essentially perpendicular to the Earth's field for the entire trajectory. This results in a constant acceleration for the electrons.



(*a*) Acceleration:

$$U_{\text{initial}}=K_{\text{final}}\quad\rightarrow\quad eV=\tfrac{1}{2}mv_x^2\quad\rightarrow\quad v_x=\sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: }\Delta x_{\text{field}}=v_xt_{\text{field}}\quad\rightarrow\quad t_{\text{field}}=\frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y=qv_xB_{\text{Earth}}=ma_y\quad\rightarrow\quad a_y=\frac{qv_xB_{\text{Earth}}}{m}=\frac{e\sqrt{\dfrac{2eV}{m}}B_{\text{Earth}}}{m}=\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}$$

$$\Delta y=\tfrac{1}{2}a_yt^2=\tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2=\tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\Delta x\right)^2\frac{m}{2eV}$$

$$=\sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2=\sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.0\times10^{3}\,\text{V}\right)}}\left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$=5.37\times10^{-3}\,\text{m}\approx\boxed{5.4\,\text{mm}}$$

(*b*) $$\Delta y=\sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2=\sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(28{,}000\,\text{V}\right)}}\left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$=\boxed{1.4\times10^{-3}\,\text{m}}$$

Note that the deflection is significantly smaller than the horizontal distance traveled, and so the assumptions made above are verified.

65. From Fig. 27-22 we see that when the angle θ is positive, the torque is negative. The magnitude of the torque is given by Eq. 27-9. For small angles we use the approximation $\sin\theta\approx\theta$. Using Eq. 10-14, we can write the torque in terms of the angular acceleration, showing that it is a harmonic oscillator.

$$\tau=-NIAB\sin\theta\approx-IabB\theta=I_M\alpha\ \rightarrow\ \alpha=-\left(\frac{IabB}{I_M}\right)\theta=-\omega^2\theta$$

We obtain the period of motion from the angular frequency, using $T=2\pi/\omega$. First we determine the moment of inertia of the loop, as two wires rotating about their centers of mass and two wires rotating about an axis parallel to their lengths.

$$I_M=2\left[\frac{1}{12}\left(\frac{b}{2a+2b}m\right)b^2\right]+2\left(\frac{a}{2a+2b}m\right)\left(\frac{b}{2}\right)^2=\frac{\left(3a+b\right)mb^2}{12\left(a+b\right)}$$

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I_M}{NIabB}}=2\pi\sqrt{\frac{mb^2\left(3a+b\right)}{12\left(a+b\right)NIabB}}=\boxed{\pi\sqrt{\frac{mb\left(3a+b\right)}{3\left(a+b\right)NIaB}}}$$

66. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles, so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 27-7 as $r=\frac{mv}{qB}$. For an object moving in circular motion, the period is given by $T=\frac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T=\frac{2\pi r}{v}\ \rightarrow\ f=\frac{v}{2\pi r}=\frac{v}{2\pi\frac{mv}{qB}}=\boxed{\frac{Bq}{2\pi m}}$$

In particular we note that this frequency is independent of the radius, and so the same frequency can be used throughout the acceleration.

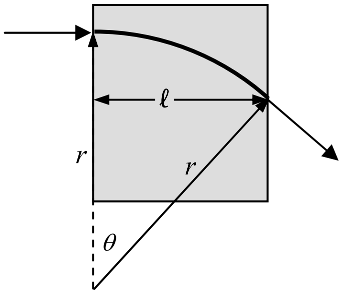
(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, the particles receive an energy increase of

$K = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of 2 gaps, so the total kinetic energy increase is $\boxed{2qV_0}$.

(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$K_{\max} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m\left(\frac{r_{\max}qB}{m}\right)^2 = \tfrac{1}{2}\frac{r_{\max}^2q^2B^2}{m} = \tfrac{1}{2}\frac{(0.50\,\text{m})^2(1.60\times10^{-19}\,\text{C})^2(0.60\,\text{T})^2}{1.67\times10^{-27}\,\text{kg}}$$

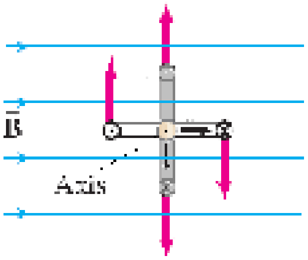
$$= 6.898\times10^{-13}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = \boxed{4.3\,\text{MeV}}$$



$\boxed{67.}$ The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 27-7 as $r = \dfrac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large and so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small and so $\boxed{\text{they will exit below the second tube}}$. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.

$$\sin\theta = \frac{\ell}{r} \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell qB}{mv} = \sin^{-1}\frac{(5.0\times10^{-2}\,\text{m})(1.60\times10^{-19}\,\text{C})(0.38\,\text{T})}{(1.67\times10^{-27}\,\text{kg})(0.85\times10^7\,\text{m/s})} = \sin^{-1}0.214 = \boxed{12^\circ}$$



68. (*a*) The force on each of the vertical wires in the loop is perpendicular to the magnetic field and is given by Eq. 27-1, with $\theta = 90^\circ$. When the face of the loop is parallel to the magnetic field, the forces point radially away from the axis. This provides a tension in the two horizontal sides. When the face of the loop is perpendicular to the magnetic field, the force on opposite vertical wires creates a shear force in the horizontal wires. From Table 12-2, we see that the tensile and shear strengths of aluminum are the same, so either can be used to determine the minimum strength. We set tensile strength multiplied by the cross-sectional area of the two wires equal the tensile strength multiplied by the safety factor and solve for the wire diameter.

$$\frac{F}{A}\pi\left(\frac{d}{2}\right)^2 = 10(I\ell B) \quad \rightarrow \quad d = 2\sqrt{\frac{10(I\ell B)}{\pi(F/A)}} = 2\sqrt{\frac{10(15.0\text{ A})(0.200\text{ m})(1.35\text{ T})}{\pi(200\times10^6\text{ N/m}^2)}}$$

$$= 5.0777\times10^{-4}\text{ m} \approx \boxed{0.508\text{ mm}}$$

(*b*) The resistance is found from the resistivity using Eq. 25-3.

$$R = \rho\frac{\ell}{A} = (2.65\times10^{-8}\,\Omega\bullet\text{m})\frac{4(0.200\,\text{m})}{\pi\left(\dfrac{5.0777\times10^{-4}}{2}\,\text{m}\right)^2} = \boxed{0.105\,\Omega}$$

69. The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used.

$$v^2 = v_0^2 + 2a\Delta x \;\rightarrow\; a = \frac{v^2 - 0}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{\text{net}} = ma = IdB \;\rightarrow\; I = \frac{ma}{dB} = \frac{m\left(\dfrac{v^2}{2\Delta x}\right)}{dB} = \frac{mv^2}{2\Delta x dB} = \frac{\left(1.5\times10^{-3}\,\text{kg}\right)\left(25\,\text{m/s}\right)^2}{2\left(1.0\,\text{m}\right)\left(0.24\,\text{m}\right)\left(1.8\,\text{T}\right)} = \boxed{1.1\,\text{A}}$$

Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in Fig. 27-53, the magnetic field must be $\boxed{\text{down}}$.

70. (*a*) For the beam of electrons to be undeflected, the magnitude of the magnetic force must equal the magnitude of the electric force. We assume that the magnetic field will be perpendicular to the velocity of the electrons so that the maximum magnetic force is obtained.

$$F_{\text{B}} = F_{\text{E}} \;\rightarrow\; qvB = qE \;\rightarrow\; B = \frac{E}{v} = \frac{8400\,\text{V/m}}{4.8\times10^{6}\,\text{m/s}} = 1.75\times10^{-3}\,\text{T} \approx \boxed{1.8\times10^{-3}\,\text{T}}$$

(*b*) Since the electric field is pointing up, the electric force is down. Thus the magnetic force must be up. Using the right hand rule with the negative electrons, the magnetic field must be $\boxed{\text{out}}$ of the plane of the plane formed by the electron velocity and the electric field.

(*c*) If the electric field is turned off, then the magnetic field will cause a centripetal force, moving the electrons in a circular path. The frequency is the reciprocal of the period of the motion.

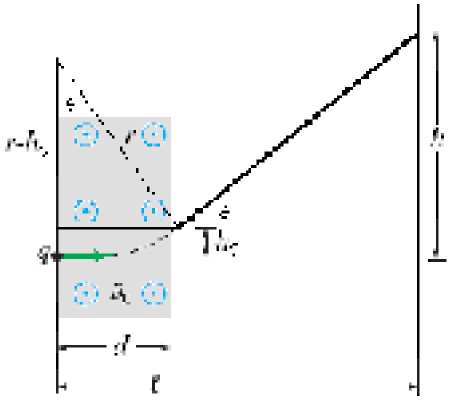
$$qvB = \frac{mv^2}{r} \;\rightarrow\; v = \frac{qBr}{m}$$

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\dfrac{qBr}{m}}{2\pi r} = \frac{qB}{2\pi m} = \frac{qE}{2\pi mv} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(8400\,\text{V/m}\right)}{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)\left(4.8\times10^{6}\,\text{m/s}\right)} = \boxed{4.9\times10^{7}\,\text{Hz}}$$

71. We find the speed of the electron using conservation of energy. The accelerating potential energy becomes the kinetic energy of the electron.

$$eV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2eV}{m}}$$

Upon entering the magnetic field the electron is traveling horizontally. The magnetic field will cause the path of the electron to rotate an angle θ from the horizontal. While in the field, the electron will travel a horizontal distance d and a vertical distance h_0. Using the Pythagorean theorem, and trigonometric relations, we can write three equations which relate the unknown parameters, r, h_0, and θ.



$$\tan\theta = \frac{h - h_0}{\ell - d} \qquad \sin\theta = \frac{d}{r} \qquad r^2 = d^2 + \left(r - h_0\right)^2 \rightarrow h_0 = r - \sqrt{r^2 - d^2}$$

These three equations can be directly solved, for the radius of curvature. However, doing so requires solving a 3rd order polynomial. Instead, we can guess at a value for h_0, such as 1.0 cm. Then we use the tangent equation to calculate an approximate value for θ. Then insert the approximate value into the sine equation to solve for r. Finally, inserting the value of r into the third equation we solve for h_0. We then use the new value of h_0 as our guess and reiterated the process a couple of times until the value of h_0 does not significantly change.

$$\theta=\tan^{-1}\left(\frac{11\,\text{cm}-1.0\,\text{cm}}{22\,\text{cm}-3.5\,\text{cm}}\right)=28.39^\circ\rightarrow r=\frac{3.5\,\text{cm}}{\sin 28.39^\circ}=7.36\,\text{cm}$$

$$\rightarrow h_0=7.36\,\text{cm}-\sqrt{(7.36\,\text{cm})^2-(3.5\,\text{cm})^2}=0.885\,\text{cm}$$

$$\theta=\tan^{-1}\left(\frac{11\text{ cm}-0.885\text{ cm}}{22\text{ cm}-3.5\text{ cm}}\right)=28.67^\circ\rightarrow r=\frac{3.5\text{ cm}}{\sin 28.67^\circ}=7.30\text{ cm}$$

$$\rightarrow h_0=7.30\text{ cm}-\sqrt{(7.30\text{ cm})^2-(3.5\text{ cm})^2}=0.894\text{ cm}$$

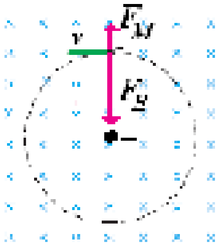
$$\theta=\tan^{-1}\left(\frac{11\text{ cm}-0.894\text{ cm}}{22\text{ cm}-3.5\text{ cm}}\right)=28.65^\circ\rightarrow r=\frac{3.5\text{ cm}}{\sin 28.67^\circ}=7.30\,\text{cm}$$

$$\rightarrow h_0=7.30\,\text{cm}-\sqrt{(7.30\,\text{cm})^2-(3.5\,\text{cm})^2}=0.894\,\text{cm}$$

The magnetic field can be determined from the trajectory's radius, as done in Example 27-7.

$$r=\frac{mv}{eB}\quad\rightarrow\quad B=\frac{mv}{er}=\frac{m}{er}\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2mV}{er^2}}=\sqrt{\frac{2(9.11\times10^{-31}\,\text{kg})(25\times10^{3}\,\text{V})}{(1.60\times10^{-19}\text{ C})(0.0730\,\text{m})^2}}=\boxed{7.3\,\text{mT}}$$

72. (*a*) As the electron orbits the nucleus in the absence of the magnetic field, its centripetal acceleration is caused solely by the electrical attraction between the electron and the nucleus. Writing the velocity of the electron as the circumference of its orbit times its frequency, enables us to obtain an equation for the frequency of the electron's orbit.



$$\frac{ke^2}{r^2}=m\frac{v^2}{r}=m\frac{(2\pi r f_0)^2}{r}\rightarrow f_0^2=\frac{ke^2}{4\pi^2 m r^3}$$

When the magnetic field is added, the magnetic force adds or subtracts from the centripetal acceleration (depending on the direction of the field) resulting in the change in frequency.

$$\frac{ke^2}{r^2}\pm q(2\pi r f)B=m\frac{(2\pi r f)^2}{r}\rightarrow f^2\mp\frac{qB}{2\pi m}f-f_0^2=0$$

We can solve for the frequency shift by setting $f=f_0+\Delta f$, and only keeping the lowest order terms, since $\Delta f\ll f_0$.

$$(f_0+\Delta f)^2\mp\frac{qB}{2\pi m}(f_0+\Delta f)-f_0^2=0$$

$$\cancel{f_0^2}+2f_0\Delta f+\cancel{\Delta f^2}\mp\frac{qB}{2\pi m}f_0\mp\cancel{\frac{qB}{2\pi m}\Delta f}-\cancel{f_0^2}=0\quad\rightarrow\quad\boxed{\Delta f=\pm\frac{qB}{4\pi m}}$$

(*b*) The "±" indicates whether the magnetic force adds to or subtracts from the centripetal acceleration. If the magnetic force adds to the centripetal acceleration, the frequency increases. If the magnetic force is opposite in direction to the acceleration, the frequency decreases.

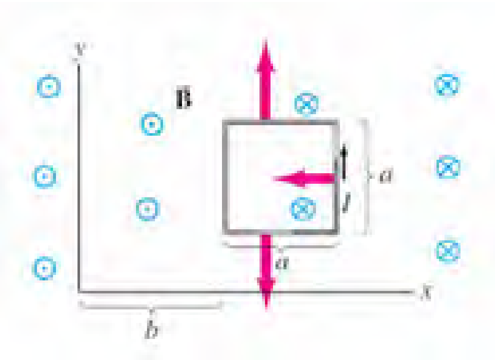
$\boxed{73.}$ The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB=\frac{mv^2}{r}\quad\rightarrow\quad v=\frac{qBr}{m}\qquad K=\tfrac{1}{2}mv^2=\tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2=\frac{q^2B^2r^2}{2m}$$

$$\Delta K=\frac{q^2B^2}{2m}\left(r_2^2-r_1^2\right)=\frac{(1.60\times10^{-19}\text{C})^2(0.018\text{T})^2}{2(1.67\times10^{-27}\text{kg})}\left[(8.5\times10^{-3}\text{m})^2-(10.0\times10^{-3}\text{m})^2\right]$$

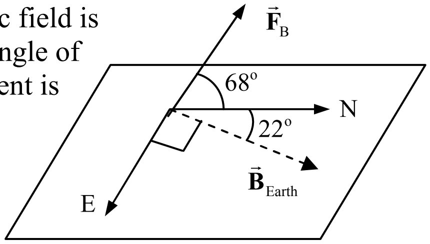
$$=\boxed{-6.9\times10^{-20}\text{J}}\text{ or }-0.43\,\text{eV}$$

74. The forces on each of the two horizontal sides of the loop have the same magnitude, but opposite directions, so these forces sum to zero. The left side of the loop is located at $x = b$, where the magnetic field is zero, and therefore the force is zero. The net force is the force acting on the right side of the loop. By the right hand rule, with the current directed toward the top of the page and the magnetic field into the page, the force will point in the negative x direction with magnitude given by Eq. 27-2.



$$\vec{\mathbf{F}} = I\ell B\left(-\hat{\mathbf{i}}\right) = IaB_0\left(1-\frac{b+a}{b}\right)\hat{\mathbf{i}} = \boxed{-\frac{Ia^2B_0}{b}\hat{\mathbf{i}}}$$

75. We assume that the horizontal component of the Earth's magnetic field is pointing due north. The Earth's magnetic field also has the dip angle of 22°. The angle between the magnetic field and the eastward current is 90°. Use Eq. 27-1 to calculate the magnitude of the force.

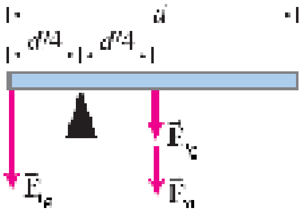


$$F = I\ell B\sin\theta = (330\,\text{A})(5.0\,\text{m})\left(5.0\times10^{-5}\,\text{T}\right)\sin 90^\circ$$

$$= \boxed{0.083\,\text{N}}$$

Using the right hand rule with the eastward current and the Earth's magnetic field, the force on the wire is $\boxed{\text{northerly and 68° above the horizontal}}$.

76. Since the magnetic and gravitational force along the entire rod is uniform, we consider the two forces acting at the center of mass of the rod. To be balanced, the net torque about the fulcrum must be zero. Using the usual sign convention for torques and Eq. 10-10a, we solve for the magnetic force on the rod.



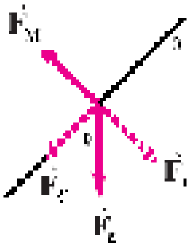
$$\sum\tau = 0 = Mg\left(\tfrac{1}{4}d\right) - mg\left(\tfrac{1}{4}d\right) - F_M\left(\tfrac{1}{4}d\right) \quad\rightarrow\quad F_M = (M-m)g$$

We solve for the current using Eq. 27-2.

$$I = \frac{F}{\ell B} = \frac{(M-m)g}{dB} = \frac{(8.0m-m)g}{dB} = \boxed{\frac{7.0mg}{dB}}$$

The right hand rule indicates that the current must flow $\boxed{\text{toward the left}}$ since the magnetic field is into the page and the magnetic force is downward.

77. (*a*) For the rod to be in equilibrium, the gravitational torque and the magnetic torque must be equal and opposite. Since the rod is uniform, the two torques can be considered to act at the same location (the center of mass). Therefore, components of the two forces perpendicular to the rod must be equal and opposite. Since the gravitational force points downward, its perpendicular component will point down and to the right. The magnetic force is perpendicular to the rod and must point towards the left to oppose the perpendicular component of the gravitational force. By the right hand rule, with a magnetic field pointing out of the page, the current must flow $\boxed{\text{downward from the pivot}}$ to produce this force.



(*b*) We set the magnitude of the magnetic force, using Eq. 27-2, equal to the magnitude of the perpendicular component of the gravitational force, $F_\perp = mg\sin\theta$, and solve for the magnetic field.

$$I\ell B = mg\sin\theta \rightarrow B = \frac{mg\sin\theta}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 13^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.028\,\text{T}}$$

(*c*) The largest magnetic field that could be measured is when $\theta = 90^\circ$.

$$B_{\text{max}} = \frac{mg\sin 90^\circ}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 90^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.12\,\text{T}}$$

CHAPTER 28: Sources of Magnetic Field

Responses to Questions

1. Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

2. The magnetic field due to a long straight current is proportional to the current strength. The electric field due to a long straight line of electric charge at rest is proportional to the charge per unit length. Both fields are inversely proportional to the distance from the wire or line of charge. The magnetic field lines form concentric circles around the wire; the electric field lines are directed radially outward if the line of charge is positive and radially inward if the line of charge is negative.

3. The magnetic forces exerted on one wire by the other try to align the wires. The net force on either wire is zero, but the net torque is not zero.

4. Yes. Assume the upper wire is fixed in position. Since the currents in the wires are in the same direction, the wires will attract each other. The lower wire will be held in equilibrium if this force of attraction (upward) is equal in magnitude to the weight of the wire (downward).

5. (*a*) The current in the lower wire is opposite in direction to the current in the upper wire.
   (*b*) The upper wire can be held in equilibrium due to the balance between the magnetic force from the lower wire and the gravitational force. The equilibrium will be stable for small vertical displacements, but not for horizontal displacements.

6. (*a*) Let $I_2 = I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left(I_1 + I_2\right) = 2\mu_0 I_1$
   (*b*) Let $I_2 = -I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left(I_1 + I_2\right) = 0$

7. Inside the cavity $\vec{\mathbf{B}} = 0$ since the geometry is cylindrical and no current is enclosed.

8. Construct a closed path similar to that shown in part (*a*) of the figure, such that sides *ab* and *cd* are perpendicular to the field lines and sides *bc* and *da* lie along the field lines. Unlike part (*a*), the path will not form a rectangle; the sides *ab* and *cd* will flare outward so that side *bc* is longer than side *da*. Since the field is stronger in the region of *da* than it is in the region of *bc*, but *da* is shorter than *bc*, the contributions to the integral in Ampère's law may cancel. Thus, $\mu_0 I_{\text{encl}} = \oint \vec{\mathbf{B}} \cdot d\vec{\ell} = 0$ is possible and the field is consistent with Ampère's law. The lines could not curve upward instead of downward, because then *bc* would be shorter than *da* and it would not be possible for the contributions to sum to zero.

9. The equation for the magnetic field strength inside a solenoid is given by $B = \mu_0 nI$.
   (*a*) The magnetic field strength is not affected if the diameter of the loops doubles.
   (*b*) If the spacing between the loops doubles, the number of loops per unit length decreases by a factor of 2, and the magnetic field strength then also decreases by a factor of 2.

(*c*) If the solenoid's length is doubled along with the doubling of the total number of loops, then the number of loops per unit length remains the same, and the magnetic field strength is not affected.

10. The Biot-Savart law states that the net field at a point in space is the vector sum of the field contributions due to each infinitesimal current element. As shown in Example 28-12, the magnetic field along the axis of a current loop is parallel to the axis because the perpendicular field contributions cancel. However, for points off the axis, the perpendicular contributions will not cancel. The net field for a point off the axis will be dominated by the current elements closest to it. For example, in Figure 28-21, the field lines inside the loop but below the axis curve downward, because these points in space are closer to the lower segment of the loop (where the current goes into the page) than they are to the upper segment (where the current comes out of the page).

11. No. The magnetic field varies in strength and direction for points in the plane of the loop. The magnetic field is strongest at the center of the loop.

12. The lead-in wires to electrical devices have currents running in opposite directions. The magnetic fields due to these currents are therefore also opposite in direction. If the wires are twisted together, then the distance from a point outside the wires to each of the individual wires is about the same, and the field contributions from the two wires will cancel. If the wires were not twisted and were separate from each other, then a point outside the wires would be a different distance from one of the wires than from the other, and there would be a net field due to the currents in the wires.

$\boxed{13.}$ The Biot-Savart law and Coulomb's law are both inverse-square in the radius and both contain a proportionality constant. Coulomb's law describes a central force; the Biot-Savart law involves a cross product of vectors and so cannot describe a central force.

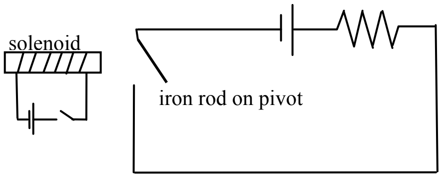
14. (*a*) The force between two identical electric charges is given by Coulomb's law: $F = \frac{kq^2}{r^2}$.

Magnetic pole strength of a bar magnet could be defined using an analogous expression for the magnetic force between the poles of two identical magnets: $F = \frac{\mu m^2}{4\pi r^2}$. Then, magnetic pole strength, *m*, would be given by $m = \sqrt{\frac{4\pi F r^2}{\mu}}$. To determine *m*, place two identical magnets with their poles facing each other a distance *r* apart and measure the force between them.

(*b*) The magnetic pole strength of a current loop could be defined the same way by using two identical current loops instead of two bar magnets.

15. Determine the magnetic field of the Earth at one of the magnetic poles (north or south), and use Equation 28-7b to calculate the magnetic moment. In this equation, *x* will be (approximately) the radius of the Earth.

16. To design a relay, place an iron rod inside a solenoid, with the solenoid oriented such that one end of it is facing a second iron rod on a pivot. The second iron rod functions as a switch for the large-current circuit and is normally held open by a spring. When current flows through the solenoid, the iron rod inside it becomes magnetized and attracts the second iron rod, closing the switch and allowing current to flow.



17. (*a*) The source of the kinetic energy is the attractive force produced by the magnetic field from the magnet acting on the magnetic moments of the atoms in the iron.

    (*b*) When the block strikes the magnet, some of the kinetic energy from the block is converted into kinetic energy in the iron atoms in the magnet, randomizing their magnetic moments and decreasing the overall field produced by the magnet. Some of the kinetic energy of the block as a whole is also converted into the kinetic energy of the individual atoms in the block, resulting in an increase in thermal energy.

18. No, a magnet with a steady field will only attract objects made of ferromagnetic materials. Aluminum is not ferromagnetic, so the magnetic field of the magnet will not cause the aluminum to become a temporary magnet and therefore there will be no attractive force. Iron is ferromagnetic, so in the presence of a magnet, the domains in a piece of iron will align such that it will be attracted to the magnet.

19. An unmagnetized nail has randomly oriented domains and will not generate an external magnetic field. Therefore, it will not attract an unmagnitized paper clip, which also has randomly oriented domains. When one end of the nail is in contact with a magnet, some of the domains in the nail align, producing an external magnetic field and turning the nail into a magnet. The magnetic nail will cause some of the domains in the paper clip to align, and it will be attracted to the nail.

20. Yes, an iron rod can attract a magnet and a magnet can attract an iron rod. Consider Newton's third law. If object A attracts object B then object B attracts object A.

21. Domains in ferromagnetic materials in molten form were aligned by the Earth's magnetic field and then fixed in place as the material cooled.

22. Yes. When a magnet is brought near an unmagnetized piece of iron, the magnet's field causes a temporary alignment of the domains of the iron. If the magnet's north pole is brought near the iron, then the domains align such that the temporary south pole of the iron is facing the magnet, and if the magnet's south pole is closest to the iron, then the alignment will be the opposite. In either case, the magnet and the iron will attract each other.

23. The two rods that have ends that repel each other will be the magnets. The unmagnetized rod will be attracted to both ends of the magnetized rods.

24. No. If they were both magnets, then they would repel one another when they were placed with like poles facing each other. However, if one is a magnet and the other isn't, they will attract each other no matter which ends are placed together. The magnet will cause an alignment of the domains of the non-magnet, causing an attraction.

25. (*a*) The magnetization curve for a paramagnetic substance is a straight line with slope slightly greater than 1. It passes through the origin; there is no hysteresis.

    (*b*) The magnetization curve for a diamagnetic substance is a straight line with slope slightly less than 1. It passes through the origin; there is no hysteresis.

    The magnetization curve for a ferromagnetic substance is a hysteresis curve (see Figure 28-29).

26. (*a*) Yes. Diamagnetism is present in all materials but in materials that are also paramagnetic or ferromagnetic, its effects will not be noticeable.

    (*b*) No. Paramagnetic materials are nonferromagnetic materials with a relative permeability greater than one.

    (*c*) No. Ferromagnetic materials are those that can be magnetized by alignment of their domains.

Solutions to Problems

1. We assume the jumper cable is a long straight wire, and use Eq. 28-1.

$$B_{\text{cable}}=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(65\,\text{A}\right)}{2\pi\left(0.035\,\text{m}\right)}=3.714\times10^{-4}\,\text{T}\approx\boxed{3.7\times10^{-4}\,\text{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\text{T}$.

$$B_{\text{cable}}/B_{\text{Earth}}=\frac{3.714\times10^{-4}\,\text{T}}{5.0\times10^{-5}\,\text{T}}=7.43\text{, so }\boxed{\text{the field of the cable is over 7 times that of the Earth.}}$$

2. We assume that the wire is long and straight, and use Eq. 28-1.

$$B_{\text{wire}}=\frac{\mu_0 I}{2\pi r}\;\rightarrow\; I=\frac{2\pi r B_{\text{wire}}}{\mu_0}=\frac{2\pi\left(0.15\,\text{m}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=37.5\,\text{A}\approx\boxed{38\,\text{A}}$$

3. Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 28-2 to calculate the magnitude of the force.

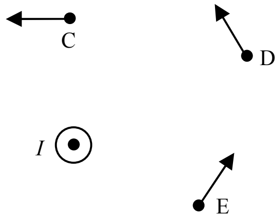
$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{\left(35\,\text{A}\right)^2}{\left(0.040\,\text{m}\right)}\left(25\,\text{m}\right)=\boxed{0.15\,\text{N, attractive}}$$

4. Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 28-2 to calculate the magnitude of the second current.

$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2\;\rightarrow$$

$$I_2=\frac{2\pi}{\mu_0}\frac{F_2}{\ell_2}\frac{d}{I_1}=\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(7.8\times10^{-4}\,\text{N/m}\right)\frac{0.070\,\text{m}}{28\,\text{A}}=9.75\,\text{A}\approx\boxed{9.8\,\text{A upward}}$$

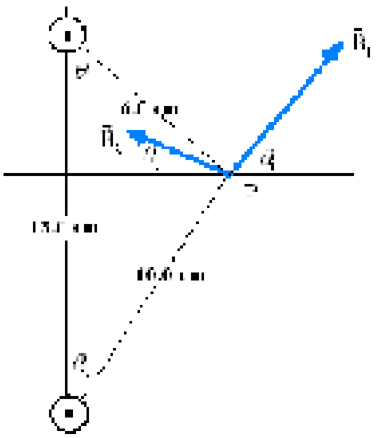
5. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle.



6. For the experiment to be accurate to ±2.0%, the magnetic field due to the current in the cable must be less than or equal to 2.0% of the Earth's magnetic field. Use Eq. 28-1 to calculate the magnetic field due to the current in the cable.

$$B_{\text{cable}}=\frac{\mu_0 I}{2\pi r}\le 0.020B_{\text{Earth}}\;\rightarrow\; I\le\frac{2\pi r\left(0.020B_{\text{Earth}}\right)}{\mu_0}=\frac{2\pi\left(1.00\,\text{m}\right)\left(0.020\right)\left(0.5\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=5.0\,\text{A}$$

Thus the maximum allowable current is $\boxed{5.0\,\text{A}}$.



7. Since the magnetic field from a current carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define $\vec{\mathbf{B}}_1$ as the field from the top wire, and $\vec{\mathbf{B}}_2$ as the field from the bottom wire. We use Eq. 28-1 to calculate the magnitude of each individual field.

$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}\right)(35\,\mathrm{A})}{2\pi(0.060\,\mathrm{m})} = 1.17 \times 10^{-4}\,\mathrm{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}\right)(35\,\mathrm{A})}{2\pi(0.100\,\mathrm{m})} = 7.00 \times 10^{-5}\,\mathrm{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal.

$$\theta_1 = \cos^{-1}\left(\frac{(0.060\text{ m})^2 + (0.130\text{ m})^2 - (0.100\text{ m})^2}{2(0.060\text{ m})(0.130\text{ m})}\right) = 47.7^\circ$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.100\text{ m})^2 + (0.130\text{ m})^2 - (0.060\text{ m})^2}{2(0.100\text{ m})(0.130\text{ m})}\right) = 26.3^\circ$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

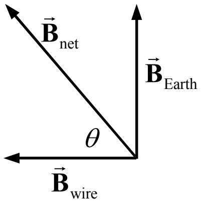
$$B_{\mathrm{net}\,x} = B_1\cos(\theta_1) - B_2\cos\theta_2 = \left(1.174\times10^{-4}\,\mathrm{T}\right)\cos 47.7^\circ - \left(7.00\times10^{-5}\,\mathrm{T}\right)\cos 26.3^\circ = 1.626\times10^{-5}\,\mathrm{T}$$

$$B_{\mathrm{net}\,y} = B_1\sin(\theta_1) + B_2\sin\theta_1 = \left(1.17\times10^{-4}\,\mathrm{T}\right)\sin 47.7^\circ + \left(7.00\times10^{-5}\,\mathrm{T}\right)\sin 26.3^\circ = 1.18\times10^{-4}\,\mathrm{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{\left(1.626\times10^{-5}\text{ T}\right)^2 + \left(1.18\times10^{-4}\text{ T}\right)^2} = 1.19\times10^{-4}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{1.18\times10^{-4}\text{ T}}{1.626\times10^{-5}\text{ T}} = 82.2^\circ$$

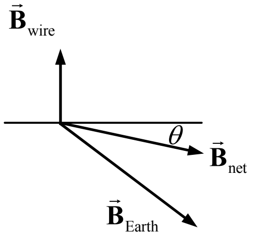
$$\vec{\mathbf{B}} = 1.19\times10^{-4}\text{ T @ }82.2^\circ \approx \boxed{1.2\times10^{-4}\,\mathrm{T}\text{ @ }82^\circ}$$



8. At the location of the compass, the magnetic field caused by the wire will point to the west, and the Earth's magnetic field points due North. The compass needle will point in the direction of the NET magnetic field.

$$B_{\mathrm{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A}\right)(43\,\mathrm{A})}{2\pi(0.18\,\mathrm{m})} = 4.78\times10^{-5}\,\mathrm{T}$$

$$\theta = \tan^{-1}\frac{B_{\mathrm{Earth}}}{B_{\mathrm{wire}}} = \tan^{-1}\frac{4.5\times10^{-5}\,\mathrm{T}}{4.78\times10^{-5}\,\mathrm{T}} = \boxed{43^\circ\mathrm{N\ of\ W}}$$



9. The magnetic field due to the long horizontal wire points straight up at the point in question, and its magnitude is given by Eq. 28-1. The two fields are oriented as shown in the diagram. The net field is the vector sum of the two fields.

$$B_{\mathrm{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A}\right)(24.0\,\mathrm{A})}{2\pi(0.200\,\mathrm{m})} = 2.40\times10^{-5}\,\mathrm{T}$$

$$B_{\text{Earth}} = 5.0\times10^{-5}\,\text{T}$$

$$B_{\text{net}\,x} = B_{\text{Earth}}\cos 44^\circ = 3.60\times10^{-5}\,\text{T} \qquad B_{\text{net}\,y} = B_{\text{wire}} - B_{\text{Earth}}\sin 44^\circ = -1.07\times10^{-5}\,\text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(3.60\times10^{-5}\,\text{T}\right)^2 + \left(-1.07\times10^{-5}\,\text{T}\right)^2} = \boxed{3.8\times10^{-5}\,\text{T}}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-1.07\times10^{-5}\,\text{T}}{3.60\times10^{-5}\,\text{T}} = \boxed{17^\circ\,\text{below the horizontal}}$$

10. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate times the charge of a proton. Then use Eq. 28-1 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(2.5\times10^{9}\,\text{protons/s}\right)\left(1.60\times10^{-19}\,\text{C/proton}\right)}{2\pi\left(2.0\,\text{m}\right)} = \boxed{4.0\times10^{-17}\,\text{T}}$$

11. (*a*) If the currents are in the same direction, the magnetic fields at the midpoint between the two currents will oppose each other, and so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I - 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I - 25\,\text{A}\right)}$$

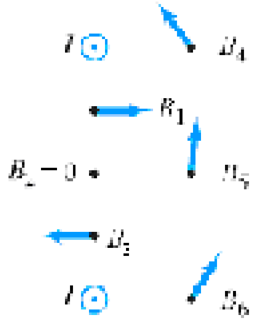
(*b*) If the currents are in the opposite direction, the magnetic fields at the midpoint between the two currents will reinforce each other, and so their magnitudes should be added.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I + 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I + 25\,\text{A}\right)}$$

12. Using the right-hand-rule we see that if the currents flow in the same direction, the magnetic fields will oppose each other between the wires, and therefore can equal zero at a given point. Set the sum of the magnetic fields from the two wires equal to zero at the point 2.2 cm from the first wire and use Eq. 28-1 to solve for the unknown current.

$$B_{net} = 0 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} \rightarrow I_2 = \left(\frac{r_2}{r_1}\right) I_1 = \left(\frac{6.0\,\text{cm} - 2.2\,\text{cm}}{2.2\,\text{cm}}\right)\left(2.0\,\text{A}\right) = \boxed{3.5\,\text{A}}$$

$\boxed{13.}$ Use the right hand rule to determine the direction of the magnetic field from each wire. Remembering that the magnetic field is inversely proportional to the distance from the wire, qualitatively add the magnetic field vectors. The magnetic field at point #2 is zero.



14. The fields created by the two wires will oppose each other, so the net field is the difference of the magnitudes of the two fields. The positive direction for the fields is taken to be into the paper, and so the closer wire creates a field in the positive direction, and the farther wire creates a field in the negative direction. Let *d* be the separation distance of the wires.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{\text{closer}}} - \frac{\mu_0 I}{2\pi r_{\text{farther}}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}} - \frac{1}{r_{\text{farther}}}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r - \frac{1}{2}d} - \frac{1}{r + \frac{1}{2}d}\right)$$

$$=\frac{\mu_0 I}{2\pi}\left(\frac{d}{\left(r-\frac{1}{2}d\right)\left(r+\frac{1}{2}d\right)}\right)$$

$$=\frac{\left(4\pi\times10^{-7}\,\mathrm{T\bullet m/A}\right)\left(28.0\,\mathrm{A}\right)}{2\pi}\left(\frac{0.0028\,\mathrm{m}}{\left(0.10\,\mathrm{m}-0.0014\,\mathrm{m}\right)\left(0.10\,\mathrm{m}+0.0014\,\mathrm{m}\right)}\right)$$

$$=1.568\times10^{-6}\,\mathrm{T}\approx\boxed{1.6\times10^{-6}\,\mathrm{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$.

$$B_{\mathrm{net}}/B_{\mathrm{Earth}}=\frac{1.568\times10^{-6}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}}=0.031$$

$\boxed{\text{The field of the wires is about 3\% that of the Earth.}}$

15. The center of the third wire is 5.6 mm from the left wire, and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. 28-2 to calculate the force per unit length.

$$F_{\mathrm{near}}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{near}}}\ell_{\mathrm{near}}\rightarrow$$

$$\frac{F_{\mathrm{near}}}{\ell_{\mathrm{near}}}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{near}}}=\frac{4\pi\times10^{-7}\,\mathrm{T\bullet m/A}}{2\pi}\frac{\left(25.0\,\mathrm{A}\right)\left(28.0\,\mathrm{A}\right)}{2.8\times10^{-3}\,\mathrm{m}}=\boxed{0.050\,\mathrm{N/m},\text{ attractive}}$$

$$F_{\mathrm{far}}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{far}}}\ell_{\mathrm{far}}\rightarrow$$

$$\frac{F_{\mathrm{far}}}{\ell_{\mathrm{far}}}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{far}}}=\frac{4\pi\times10^{-7}\,\mathrm{T\bullet m/A}}{2\pi}\frac{\left(25.0\,\mathrm{A}\right)\left(28.0\,\mathrm{A}\right)}{5.6\times10^{-3}\,\mathrm{m}}=\boxed{0.025\,\mathrm{N/m},\text{ repelling}}$$

16. (*a*) We assume that the power line is long and straight, and use Eq. 28-1.

$$B_{\mathrm{line}}=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\mathrm{T\bullet m/A}\right)\left(95\,\mathrm{A}\right)}{2\pi\left(8.5\,\mathrm{m}\right)}=2.235\times10^{-6}\,\mathrm{T}\approx\boxed{2.2\times10^{-6}\,\mathrm{T}}$$

The direction at the ground, from the right hand rule, is $\boxed{\text{south}}$. Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$, which points approximately north.

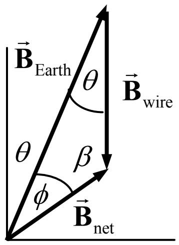
$$B_{\mathrm{line}}/B_{\mathrm{Earth}}=\frac{2.235\times10^{-6}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}}=0.0447$$

$\boxed{\text{The field of the cable is about 4\% that of the Earth.}}$

(*b*) We solve for the distance where $B_{\mathrm{line}}=B_{\mathrm{Earth}}$.

$$B_{\mathrm{line}}=\frac{\mu_0 I}{2\pi r}=B_{\mathrm{Earth}}\rightarrow r=\frac{\mu_0 I}{2\pi B_{\mathrm{Earth}}}=\frac{\left(4\pi\times10^{-7}\,\mathrm{T\bullet m/A}\right)\left(95\,\mathrm{A}\right)}{2\pi\left(0.5\times10^{-4}\,\mathrm{T}\right)}=0.38\,\mathrm{m}\approx\boxed{0.4\,\mathrm{m}}$$

So about 0.4 m below the wire, the net B-field would be 0, assuming the Earth's field points straight north at this location.



17. The Earth's magnetic field is present at both locations in the problem, and we assume it is the same at both locations. The field east of a vertical wire must be pointing either due north or due south. The compass shows the direction of the net magnetic field, and it changes from 28° E of N to 55° E of N when taken inside. That is a "southerly" change (rather than a "northerly" change), and so the field due to the wire must be pointing due south. See the diagram. For the angles, $\theta=28^\circ,\ \theta+\phi=55^\circ$, and $\beta+\theta+\phi=180^\circ$ and so $\phi=27^\circ$ and $\beta=125^\circ$. Use the law of sines to find the magnitude of $\vec{\mathbf{B}}_{\text{wire}}$, and then use Eq. 28-1 to find the magnitude of the current.

$$\frac{B_{\text{wire}}}{\sin\phi}=\frac{B_{\text{Earth}}}{\sin\beta}\rightarrow B_{\text{wire}}=B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}=\frac{\mu_0}{2\pi}\frac{I}{r}\rightarrow$$

$$I=B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}\frac{2\pi}{\mu_0}r=\left(5.0\times10^{-5}\,\text{T}\right)\frac{\sin 27^\circ}{\sin 125^\circ}\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(0.120\,\text{m}\right)=\boxed{17\,\text{A}}$$

Since the field due to the wire is due south, the current in the wire must be $\boxed{\text{down}}$.

18. The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 28-1. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 28-2.

$$F_{\text{net}}=F_{\text{near}}-F_{\text{far}}=\frac{\mu_0}{2\pi}\frac{I_1I_2}{d_{\text{near}}}\ell_{\text{near}}-\frac{\mu_0}{2\pi}\frac{I_1I_2}{d_{\text{far}}}\ell_{\text{far}}=\frac{\mu_0}{2\pi}I_1I_2\ell\left(\frac{1}{d_{\text{near}}}-\frac{1}{d_{\text{far}}}\right)$$

$$=\frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\left(3.5\,\text{A}\right)^2\left(0.100\,\text{m}\right)\left(\frac{1}{0.030\,\text{m}}-\frac{1}{0.080\,\text{m}}\right)=\boxed{5.1\times10^{-6}\,\text{N, towards wire}}$$

$\boxed{19.}$ The left wire will cause a field on the *x* axis that points in the *y* direction, and the right wire will cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d-x$.

$$\vec{\mathbf{B}}_{\text{net}}=\frac{\mu_0 I}{2\pi x}\hat{\mathbf{j}}-\frac{\mu_0 I}{2\pi\left(d-x\right)}\hat{\mathbf{j}}=\boxed{\frac{\mu_0 I}{2\pi}\left(\frac{1}{x}-\frac{1}{d-x}\right)\hat{\mathbf{j}}}=\boxed{\frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x\left(d-x\right)}\right)\hat{\mathbf{j}}}$$

20. The left wire will cause a field on the *x* axis that points in the negative *y* direction, and the right wire will also cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d-x$.

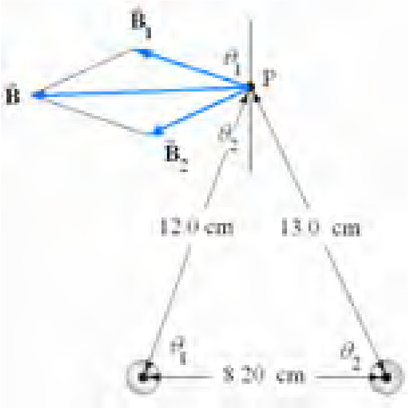
$$\vec{\mathbf{B}}_{\text{net}}=-\frac{\mu_0\left(2I\right)}{2\pi x}\hat{\mathbf{j}}-\frac{\mu_0 I}{2\pi\left(d-x\right)}\hat{\mathbf{j}}=\boxed{-\frac{\mu_0 I}{2\pi}\left(\frac{2}{x}+\frac{1}{d-x}\right)\hat{\mathbf{j}}}$$

21. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r}\sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi(0.100\,\text{m})}\sqrt{(20.0\,\text{A})^2 + (12.0\,\text{A})^2}$$

$$= \boxed{4.66\times10^{-5}\,\text{T}}$$

22. The net magnetic field is the vector sum of the magnetic fields produced by each current carrying wire. Since the individual magnetic fields encircle the wire producing it, the field is perpendicular to the radial line from the wire to point P. We let $\vec{\mathbf{B}}_1$ be the field from the left wire, and $\vec{\mathbf{B}}_2$ designate the field from the right wire. The magnitude of the magnetic field vectors is calculated from Eq. 28-1.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(16.5\,\text{A})}{2\pi(0.12\,\text{m})} = 2.7500\times10^{-5}\,\text{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(16.5\text{A})}{2\pi(0.13\,\text{m})} = 2.5385\times10^{-5}\,\text{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the horizontal. Since the magnetic fields are perpendicular to the radial lines, these angles are the same as the angles the magnetic fields make with the vertical.

$$\theta_1 = \cos^{-1}\left(\frac{(0.12\text{ m})^2 + (0.082\text{ m})^2 - (0.13\text{ m})^2}{2(0.12\text{ m})(0.082\text{ m})}\right) = 77.606°$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.13\text{ m})^2 + (0.082\text{ m})^2 - (0.12\text{ m})^2}{2(0.13\text{ m})(0.082\text{ m})}\right) = 64.364°$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net}\,x} = -B_1\sin(\theta_1) - B_2\sin\theta_2 = -(2.7500\times10^{-5}\text{T})\sin 77.606° - (2.5385\times10^{-5}\text{T})\sin 64.364°$$

$$= -49.75\times10^{-6}\text{T}$$

$$B_{\text{net}\,y} = B_1\cos(\theta_1) - B_2\cos\theta_1 = (2.7500\times10^{-5}\text{T})\cos 77.606° - (2.5385\times10^{-5}\text{T})\cos 64.364°$$

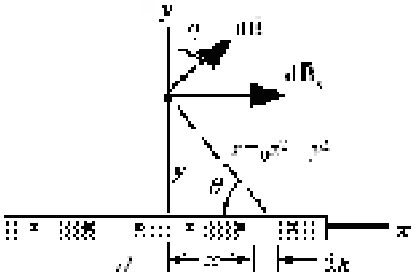
$$= -5.080\times10^{-6}\text{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(-49.75\times10^{-6}\text{ T})^2 + (-5.080\times10^{-6}\text{ T})^2} = 5.00\times10^{-5}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{-5.08\times10^{-6}\text{ T}}{-49.75\times10^{-6}\text{ T}} = 5.83°$$

$$\boxed{\vec{\mathbf{B}} = 5.00\times10^{-5}\text{ T @ } 5.83°\text{ below the negative } x\text{-axis}}$$

23. (*a*) The net magnetic field a point y above the center of the strip can be found by dividing the strip into infinitely thin wires and integrating the field contribution from each wire. Since the point is directly above the center of the strip, we see that the vertical contributions to the magnetic field from symmetric points on either side of the center cancel out. Therefore, we only need to integrate the horizontal component of the magnetic field. We use Eq. 28-1 for the magnitude of the magnetic field, with the current given by



$$dI = \frac{I}{d}dx.$$

$$B_x = \int \frac{\mu_0 \sin\theta}{2\pi r} dI = \frac{\mu_0 I}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d} \frac{dx}{\sqrt{x^2+y^2}}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{\mu_0 I y}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dx}{x^2+y^2}$$

$$= \frac{\mu_0 I y}{2\pi d}\frac{1}{y}\tan^{-1}\frac{x}{y}\bigg|_{-d/2}^{d/2} = \boxed{\frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right)}$$

(*b*) In the limit of large y, $\tan^{-1} d/2y \approx d/2y.$

$$B_x = \frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right) \approx \frac{\mu_0 I}{\pi d}\frac{d}{2y} = \boxed{\frac{\mu_0 I}{2\pi y}}$$

This is the same as the magnetic field for a long wire.

24. We break the current loop into the three branches of the triangle and add the forces from each of the three branches. The current in the parallel branch flows in the same direction as the long straight wire, so the force is attractive with magnitude given by Eq. 28-2.

$$F_1 = \frac{\mu_0 I I'}{2\pi d}a$$

By symmetry the magnetic force for the other two segments will be equal. These two wires can be broken down into infinitesimal segments, each with horizontal length *dx*. The net force is found by integrating Eq. 28-2 over the side of the triangle. We set x=0 at the left end of the left leg. The distance of a line segment to the wire is then given by $r = d + \sqrt{3}x$. Since the current in these segments flows opposite the direction of the current in the long wire, the force will be repulsive.

$$F_2 = \int_0^{a/2}\frac{\mu_0 I I'}{2\pi\left(d+\sqrt{3}x\right)}dx = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(d+\sqrt{3}x\right)\bigg|_0^{a/2} = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)\bigg|$$

We calculate the net force by summing the forces from the three segments.

$$F = F_1 - 2F_2 = \frac{\mu_0 I I'}{2\pi d}a - 2\frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right) = \boxed{\frac{\mu_0 I I'}{\pi}\left[\frac{a}{2d} - \frac{\sqrt{3}}{3}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)\right]}$$

$\boxed{25.}$ Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 I N}{\ell} \quad\to\quad I = \frac{B\ell}{\mu_0 N} = \frac{\left(0.385\times10^{-3}\,\mathrm{T}\right)\left(0.400\,\mathrm{m}\right)}{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(765\right)} = \boxed{0.160\,\mathrm{A}}$$

26. The field inside a solenoid is given by Eq. 28-4.

$$B=\frac{\mu_0 IN}{\ell}\quad\rightarrow\quad N=\frac{B\ell}{\mu_0 I}=\frac{(0.30\,\text{T})(0.32\,\text{m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(4.5\,\text{A})}=\boxed{1.7\times10^{4}\,\text{turns}}$$

27. (*a*) We use Eq. 28-1, with *r* equal to the radius of the wire.

$$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m})}=\boxed{5.3\ \text{mT}}$$

(*b*) We use the results of Example 28-6, for points inside the wire. Note that $r=(1.25-0.50)\,\text{mm}$ $=0.75\,\text{mm}$.

$$B=\frac{\mu_0 Ir}{2\pi R^2}=\frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(33\,\text{A})(0.75\times10^{-3}\,\text{m})}{2\pi(1.25\times10^{-3}\,\text{m})^2}=\boxed{3.2\,\text{mT}}$$

(*c*) We use Eq. 28-1, with *r* equal to the distance from the center of the wire.

$$B=\frac{\mu_0 I}{2\pi r}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m}+2.5\times10^{-3}\ \text{m})}=\boxed{1.8\ \text{mT}}$$

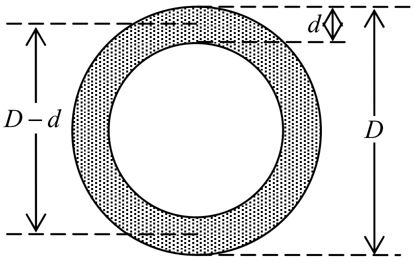
28. We use the results of Example 28-10 to find the maximum and minimum fields.

$$B_{\text{min}}=\frac{\mu_0 NI}{2\pi r_{\text{max}}}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.270\ \text{m})}=12.7\ \text{mT}$$

$$B_{\text{max}}=\frac{\mu_0 NI}{2\pi r_{\text{min}}}=\frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.250\ \text{m})}=13.7\ \text{mT}$$

$$\boxed{12.7\ \text{mT}<B<13.7\ \text{mT}}$$

29. (*a*) The copper wire is being wound about an average diameter that is approximately equal to the outside diameter of the solenoid minus the diameter of the wire, or $D-d$. See the (not to scale) end-view diagram. The length of each wrapping is $\pi(D-d)$. We divide the length of the wire L by the length of a single winding to determine the number of loops. The length of the solenoid is the number of loops multiplied by the outer diameter of the wire, *d*.



$$\ell=d\frac{L}{\pi(D-d)}=(2.00\times10^{-3}\text{m})\frac{20.0\,\text{m}}{\pi\left[2.50\times10^{-2}\text{m}-(2.00\times10^{-3}\text{m})\right]}=\boxed{0.554\,\text{m}}$$

(*b*) The field inside the solenoid is found using Eq. 28-4. Since the coils are wound closely together, the number of turns per unit length is equal to the reciprocal of the wire diameter.

$$n=\frac{\#\ \text{turns}}{\ell}=\frac{\dfrac{L}{\pi(D-d)}}{\ell}=\frac{\ell/d}{\ell}=\frac{1}{d}$$

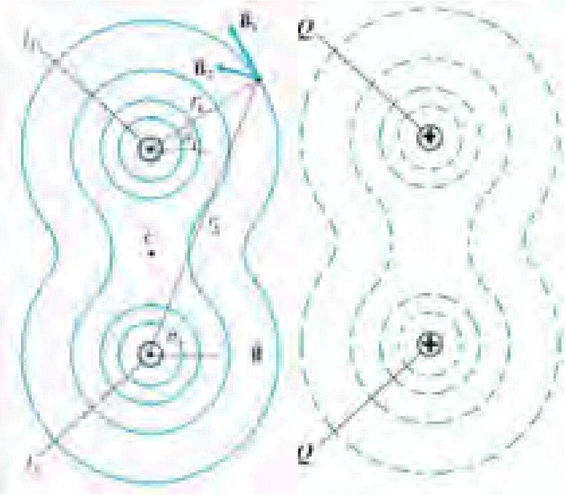
$$B=\mu_0 nI=\frac{\mu_0 I}{d}=\frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(16.7\,\text{A})}{2.00\times10^{-3}\text{m}}=\boxed{10.5\,\text{mT}}$$

30. (*a*) The magnitude of the magnetic field from each wire is found using Eq. 28-1. The direction of the magnetic field is perpendicular to the radial vector from the current to the point of interest. Since the currents are both coming out of the page, the magnetic fields will point counterclockwise from the radial line. The total magnetic field is the vector sum of the individual fields.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 I}{2\pi r_1}\left(-\sin\theta_1\hat{\mathbf{i}} + \cos\theta_1\hat{\mathbf{j}}\right) + \frac{\mu_0 I}{2\pi r_2}\left(-\sin\theta_2\hat{\mathbf{i}} + \cos\theta_2\hat{\mathbf{j}}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left[\left(-\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_1}{r_2}\right)\hat{\mathbf{i}} + \left(\frac{\cos\theta_1}{r_1} + \frac{\cos\theta_2}{r_2}\right)\hat{\mathbf{j}}\right]$$

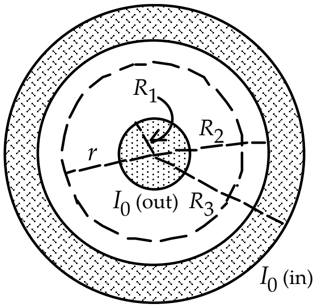
This equation for the magnetic field shows that the *x*-component of the magnetic field is symmetric and the *y*-component is anti-symmetric about $\theta = 90°$.



(*b*) See sketch.

(*c*) The two diagrams are similar in shape, as both form loops around the central axes. However, the magnetic field lines form a vector field, showing the direction, not necessarily the magnitude of the magnetic field. The equipotential lines are from a scalar field showing the points of constant magnitude. The equipotential lines do not have an associated direction.

31. Because of the cylindrical symmetry, the magnetic fields will be circular. In each case, we can determine the magnetic field using Ampere's law with concentric loops. The current densities in the wires are given by the total current divided by the cross-sectional area.



$$J_{\text{inner}} = \frac{I_0}{\pi R_1^2} \qquad J_{\text{outer}} = -\frac{I_0}{\pi\left(R_3^2 - R_2^2\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by the current density of the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left(J_{\text{inner}}\pi R^2\right)$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^2}{\pi R_1^2} \rightarrow \boxed{B = \frac{\mu_0 I_0 R}{2\pi R_1^2}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

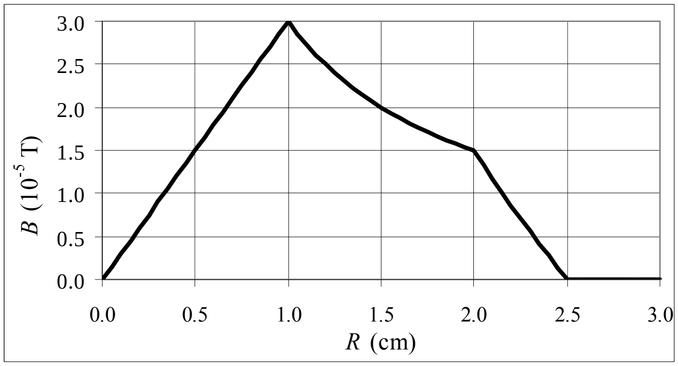
$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left[I_0 + J_{\text{outer}}\pi\left(R^2 - R_2^2\right)\right]$$

$$B(2\pi r) = \mu_0\left[I_0 - I_0\frac{\pi\left(R^2 - R_2^2\right)}{\pi\left(R_3^2 - R_2^2\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^2 - R^2\right)}{\left(R_3^2 - R_2^2\right)}}$$

(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \;\rightarrow\; B(2\pi R) = 0 \;\rightarrow\; \boxed{B=0}$$

(*e*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.31e."



32. We first find the constants C_1 and C_2 by integrating the currents over each cylinder and setting the integral equal to the total current.

$$I_0 = \int_0^{R_1} C_1 R 2\pi R dR = 2\pi C_1 \int_0^{R_1} R^2 dR = \frac{2}{3}\pi R_1^3 C_1 \;\rightarrow\; C_1 = \frac{3I_0}{2\pi R_1^3}$$

$$-I_0 = 2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2}{3}\pi\left(R_3^3 - R_2^3\right)C_2 \;\rightarrow\; C_2 = \frac{-3I_0}{2\pi\left(R_3^3 - R_2^3\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by integrating the current density inside the radius R.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \int_0^R (C_1 R')2\pi R' dR' = \frac{2}{3}\mu_0 \pi C_1 R^3 = \frac{2}{3}\mu_0 \pi\left(\frac{3I_0}{2\pi R_1^3}\right)R^3$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^3}{\pi R_1^3} \rightarrow \boxed{B = \frac{\mu_0 I_0 R^2}{2\pi R_1^3}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} \;\rightarrow\; B(2\pi R) = \mu_0 I_0 \;\rightarrow\; \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left[I_0 + \int_{R_2}^{R} (C_2 R)2\pi R dR\right] = \mu_0\left[I_0 + \int_{R_2}^{R} (C_2 R)2\pi R dR\right]$$

$$= \mu_0 I_0\left[1 - \frac{2}{3}\pi C_2\left(R^3 - R_2^3\right)\right] = \mu_0\left[I_0 - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)}\right]$$

$$B(2\pi r) = \mu_0 I_0\left[\frac{\left(R_3^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^3 - R^3\right)}{\left(R_3^3 - R_2^3\right)}}$$

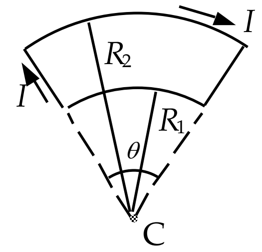
(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \;\rightarrow\; B(2\pi R) = 0 \;\rightarrow\; \boxed{B=0}$$

33. Use Eq. 28-7b to write a ratio of the magnetic fields at the surface of the earth and 13,000 km above the surface. Use the resulting ratio to determine the magnetic field above the surface.

$$\frac{B_2}{B_1}=\frac{\dfrac{\mu_0}{2\pi}\dfrac{\mu}{x_2^3}}{\dfrac{\mu_0}{2\pi}\dfrac{\mu}{x_1^3}}=\frac{x_1^3}{x_2^3}\ \rightarrow\ B_2=B_1\frac{x_1^3}{x_2^3}=\left(1.0\times10^{-4}\text{T}\right)\left(\frac{6.38\times10^3\text{km}}{19.38\times10^3\text{km}}\right)^3=\boxed{3.6\times10^{-6}\text{T}}$$

34. Since the point C is along the line of the two straight segments of the current, these segments do not contribute to the magnetic field at C. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration the line segment is perpendicular to the radial vector and the radial distance is constant.



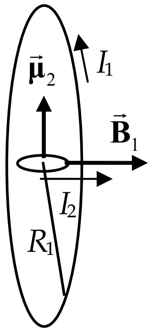
$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\int_0^{R_1\theta}\frac{d\vec{\ell}\times\hat{r}}{R_1^2}+\frac{\mu_0 I}{4\pi}\int_{R_2\theta}^{0}\frac{d\vec{\ell}\times\hat{r}}{R_2^2}=\frac{\mu_0 I}{4\pi R_1^2}\hat{\mathbf{k}}\int_0^{R_1\theta}ds+\frac{\mu_0 I}{4\pi R_2^2}\hat{\mathbf{k}}\int_{R_2\theta}^{0}ds$$

$$=\frac{\mu_0 I\theta}{4\pi R_1}\hat{\mathbf{k}}-\frac{\mu_0 I\theta}{4\pi R_2}\hat{\mathbf{k}}=\boxed{\frac{\mu_0 I\theta}{4\pi}\left(\frac{R_2-R_1}{R_1R_2}\right)\hat{\mathbf{k}}}$$

35. Since the current in the two straight segments flows radially toward and away from the center of the loop, they do not contribute to the magnetic field at the center. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration segment, the current is perpendicular to the radial vector and the radial distance is constant. By the right-hand-rule the magnetic field from the upper portion will point into the page and the magnetic field from the lower portion will point out of the page.

$$\vec{\mathbf{B}}=\frac{\mu_0 I_1}{4\pi}\int_{\text{upper}}\frac{ds}{R^2}\hat{\mathbf{k}}+\frac{\mu_0 I_2}{4\pi}\int_{\text{lower}}\frac{ds}{R^2}\left(-\hat{\mathbf{k}}\right)=\frac{\mu_0\left(\pi R\right)}{4\pi R^2}\hat{\mathbf{k}}\left(I_1-I_2\right)=\frac{\mu_0}{4R}\hat{\mathbf{k}}\left(0.35I-0.65I\right)=\boxed{-\frac{3\mu_0 I}{40R}}$$

36. We assume that the inner loop is sufficiently small that the magnetic field from the larger loop can be considered to be constant across the surface of the smaller loop. The field at the center of the larger loop is illustrated in Example 28-12. Use Eq. 27-10 to calculate the magnetic moment of the small loop, and Eq. 27-11 to calculate the torque.



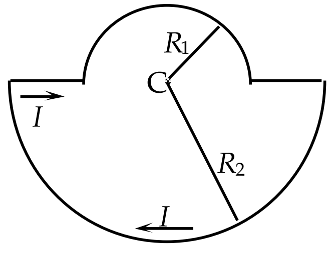
$$\vec{\mathbf{B}}=\frac{\mu_0 I}{2R}\hat{\mathbf{i}}\qquad\qquad \vec{\boldsymbol{\mu}}=I\vec{\mathbf{A}}=I\pi R_2^2\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}=I\pi R_2^2\hat{\mathbf{j}}\times\frac{\mu_0 I}{2R}\hat{\mathbf{i}}=-\frac{\mu_0\pi I^2R_2^2}{2R}\hat{\mathbf{k}}$$

$$=-\frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\pi\left(7.0\ \text{A}\right)^2\left(0.018\ \text{m}\right)^2}{2\left(0.25\ \text{m}\right)}\hat{\mathbf{k}}=\boxed{-1.3\times10^{-7}\hat{\mathbf{k}}\ \text{m}\bullet\text{N}}$$

This torque would cause the inner loop to rotate into the same plane as the outer loop with the currents flowing in the same direction.

37. (*a*) The magnetic field at point C can be obtained using the Biot-Savart law (Eq. 28-5, integrated over the current). First break the loop into four sections: 1) the upper semi-circle, 2) the lower semi-circle, 3) the right straight segment, and 4) the left straight segment. The two straight segments do not contribute to the magnetic field as the point C is in the same direction that the



current is flowing. Therefore, along these segments $\hat{r}$ and $d\hat{\ell}$ are parallel and $d\hat{\ell}\times\hat{r}=0$. For the upper segment, each infinitesimal line segment is perpendicular to the constant magnitude radial vector, so the magnetic field points downward with constant magnitude.

$$\vec{\mathbf{B}}_{\text{upper}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_1{}^2}\left(\pi R_1\right)=-\frac{\mu_0 I}{4R_1}\hat{k}\,.$$

Along the lower segment, each infinitesimal line segment is also perpendicular to the constant radial vector.

$$\vec{\mathbf{B}}_{\text{lower}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_2{}^2}\left(\pi R_2\right)=-\frac{\mu_0 I}{4R_2}\hat{k}$$

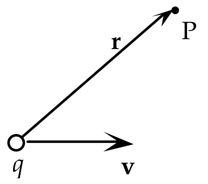
Adding the two contributions yields the total magnetic field.

$$\vec{\mathbf{B}}=\vec{\mathbf{B}}_{\text{upper}}+\vec{\mathbf{B}}_{\text{lower}}=-\frac{\mu_0 I}{4R_1}\hat{k}-\frac{\mu_0 I}{4R_2}\hat{k}=\boxed{-\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\hat{k}}$$

(*b*) The magnetic moment is the product of the area and the current. The area is the sum of the two half circles. By the right-hand-rule, curling your fingers in the direction of the current, the thumb points into the page, so the magnetic moment is in the $-\hat{k}$ direction.

$$\vec{\boldsymbol{\mu}}=-\left(\frac{\pi R_1^2}{2}+\frac{\pi R_2^2}{2}\right)I\hat{k}=\boxed{\frac{-\pi I}{2}\left(R_1^2+R_2^2\right)\hat{k}}$$

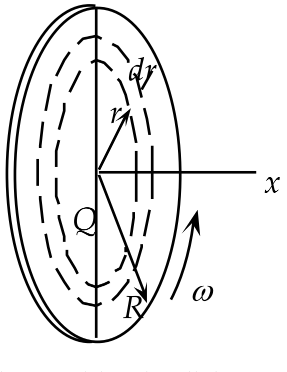
38. Treat the moving point charge as a small current segment. We can write the product of the charge and velocity as the product of a current and current segment. Inserting these into the Biot-Savart law gives us the magnetic field at point P.



$$q\vec{\mathbf{v}}=q\frac{d\vec{\boldsymbol{\ell}}}{dt}=\frac{dq}{dt}d\vec{\boldsymbol{\ell}}=Id\vec{\boldsymbol{\ell}}$$

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{r}}{r^2}=\boxed{\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}}$$

39. (*a*) The disk can be broken down into a series of infinitesimal thick rings. As the charge in each of these rings rotates it produces a current of magnitude $dI=\left(\omega/2\pi\right)dq$, where dq is the surface charge density multiplied by the area of the ring. We use Eq. 27-10 to calculate the magnetic dipole moment of each current loop and integrate the dipole moments to obtain the total magnetic dipole moment.



$$d\vec{\boldsymbol{\mu}}=dI\vec{\mathbf{A}}=\left(\frac{Q}{\pi R^2}2\pi r dr\frac{\omega}{2\pi}\right)\left(\pi r^2\right)=\frac{Q\omega}{R^2}r^3dr\hat{\mathbf{i}}$$

$$\vec{\boldsymbol{\mu}}=\int_0^R\frac{Q\omega}{R^2}r^3dr\,\hat{\mathbf{i}}=\boxed{\frac{Q\omega R^2}{4}\hat{\mathbf{i}}}$$

(*b*) To find the magnetic field a distance *x* along the axis of the disk, we again consider the disk as a series of concentric currents. We use the results of Example 28-12 to determine the magnetic field from each current loop in the disk, and then integrate to obtain the total magnetic field.

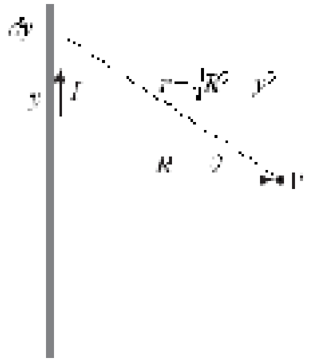
$$d\vec{\mathbf{B}}=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}dI=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}\frac{Q\omega}{\pi R^2}rdr$$

$$\vec{\mathbf{B}}=\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\int_0^R\frac{r^3}{\left(r^2+x^2\right)^{\frac{3}{2}}}dr=\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\frac{\left(r^2+2x^2\right)}{\sqrt{r^2+x^2}}\Bigg|_0^R=\boxed{\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left[\frac{\left(R^2+2x^2\right)}{\sqrt{R^2+x^2}}-2x\right]}$$

(*c*) When we take the limit $x \gg R$ our equation reduces to Eq. 28-7b.

$$\vec{\mathbf{B}}\approx\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left[2x\left(1+\frac{R^2}{2x^2}\right)\left(1-\frac{R^2}{2x^2}+\frac{3R^4}{8x^4}+\ldots\right)-2x\right]\approx\frac{\mu_0 Q\omega\hat{\mathbf{i}}}{2\pi R^2}\left(\frac{R^4}{4x^3}\right)=\boxed{\frac{\mu_0}{2\pi}\frac{\vec{\boldsymbol{\mu}}}{x^3}}$$

40. (*a*) Choose the *y* axis along the wire and the *x* axis passing from the center of the wire through the point P. With this definition we calculate the magnetic field at P by integrating Eq. 28-5 over the length of the wire. The origin is at the center of the wire.



$$\vec{\mathbf{B}}=\int\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2}=\int\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3}=\frac{\mu_0 I}{4\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy\hat{\mathbf{j}}\times\left(R\hat{\mathbf{i}}-y\hat{\mathbf{j}}\right)}{\left(R^2+y^2\right)^{3/2}}$$

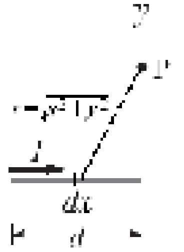
$$=-\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left(R^2+y^2\right)^{3/2}}$$

$$=-\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\frac{y}{R^2\left(R^2+y^2\right)^{1/2}}\Bigg|_{-d/2}^{d/2}=\boxed{-\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

(*b*) If we take the limit as $d\to\infty$, this equation reduces to Eq. 28-1.

$$B=\lim_{d\to\infty}\left(\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\right)\approx\frac{\mu_0 I}{2\pi R}$$

41. (*a*) The magnetic field at point Q can be obtained by integrating Eq. 28-5 over the length of the wire. In this case, each infinitesimal current segment dℓ is parallel to the *x* axis, as is each radial vector. Since the magnetic field is proportional to the cross-product of the current segment and the radial vector, each segment contributes zero field. Thus the magnetic field at point Q is $\boxed{\text{zero.}}$

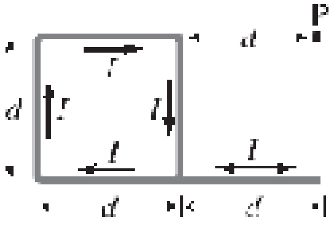


(*b*) The magnetic field at point P is found by integrating Eq. 28-5 over the length of the current segment.

$$\vec{\mathbf{B}}=\int\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2}=\int\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3}=\frac{\mu_0 I}{4\pi}\int_{-d}^{0}\frac{dx\hat{\mathbf{i}}\times\left(-x\hat{\mathbf{i}}+y\hat{\mathbf{j}}\right)}{\left(x^2+y^2\right)^{3/2}}=\frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\int_{-d}^{0}\frac{dx}{\left(x^2+y^2\right)^{3/2}}$$

$$=\frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\frac{x}{y^2\left(x^2+y^2\right)^{1/2}}\Bigg|_{-d}^{0}=\boxed{\frac{\mu_0 I}{4\pi y}\frac{d}{\left(y^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

42. We treat the loop as consisting of 5 segments, The first has length *d*, is located a distance *d* to the left of point P, and has current flowing toward the right. The second has length *d*, is located a distance 2*d* to left of point P, and has current flowing upward. The third has length *d*, is located a distance *d* to the left of point P, and has current flowing downward. The fourth has length 2*d*, is located a distance *d*



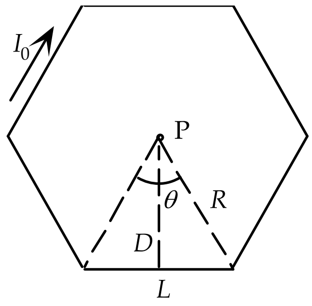
below point P, and has current flowing toward the left. Note that the fourth segment is twice as long as the actual fourth current. We therefore add a fifth line segment of length *d*, located a distance *d* below point P with current flowing to the right. This fifth current segment cancels the added portion, but allows us to use the results of Problem 41 in solving this problem. Note that the first line points radially toward point P, and therefore by Problem 41(*a*) does not contribute to the net magnetic field. We add the contributions from the other four segments, with the contribution in the positive *z*-direction if the current in the segment appears to flows counterclockwise around the point P.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 + \vec{\mathbf{B}}_5$$

$$= -\frac{\mu_0 I}{4\pi(2d)}\frac{d}{\left(4d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} - \frac{\mu_0 I}{4\pi d}\frac{2d}{\left(d^2+4d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}}$$

$$= \boxed{\frac{\mu_0 I}{4\pi d}\left(\sqrt{2} - \frac{\sqrt{5}}{2}\right)\hat{\mathbf{k}}}$$

$\boxed{43.}$ (*a*) The angle subtended by one side of a polygon, *θ*, from the center point P is 2π divided by the number of sides, *n*. The length of the side *L* and the distance from the point to the center of the side, *D*, are obtained from trigonometric relations.



$$L = 2R\sin(\theta/2) = 2R\sin(\pi/n)$$

$$D = R\cos(\theta/2) = R\cos(\pi/n)$$

The magnetic field contribution from each side can be found using the result of Problem 40.

$$B = \frac{\mu_0 I}{2\pi D}\frac{L}{\left(L^2+4D^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi\left(R\cos(\pi/n)\right)}\frac{2R\sin(\pi/n)}{\left(\left(2R\sin(\pi/n)\right)^2 + 4\left(R\cos(\pi/n)\right)^2\right)^{\frac{1}{2}}}$$

$$= \frac{\mu_0 I}{2\pi R}\tan(\pi/n)$$

The contributions from each segment add, so the total magnetic field is *n* times the field from one side.

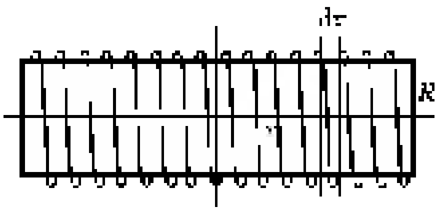
$$\boxed{B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\tan(\pi/n)}$$

(*b*) In the limit of large *n*, π/n, becomes very small, so $\tan(\pi/n) \approx \pi/n$.

$$B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\frac{\pi}{n} = \boxed{\frac{\mu_0 I}{2R}}$$

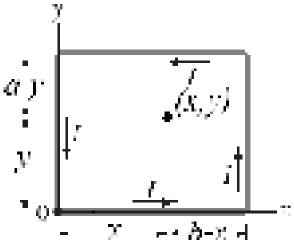
This is the magnetic field at the center of a circle.

44. The equation derived in Eq. 28-12 gives the magnetic field a distance *x* from a single loop. We expand this single loop to the field of an infinite solenoid by multiplying the field from a single loop by *n dx*, the density of loops times the infinitesimal thickness, and integrating over all values of *x*. Use the table in Appendix B-4 to evaluate the integral.



$$B = \int_{-\infty}^{\infty}\frac{\mu_0 I R^2 n\,dx}{2\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\int_{-\infty}^{\infty}\frac{dx}{\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\left.\frac{x}{R^2\left(R^2+x^2\right)^{\frac{1}{2}}}\right|_{-\infty}^{\infty} = \boxed{\mu_0 I n}$$

45. To find the magnetic field at point (*x*,*y*) we break each current segment into two segments and sum fields from each of the eight segments to determine the magnetic field at the center. We use the results of Problem 41(*b*) to calculate the magnetic field of each segment.



$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi y}\frac{x}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi y}\frac{(b-x)}{\left(y^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}}$$
$$+\frac{\mu_0 I}{4\pi(b-x)}\frac{y}{\left((b-x)^2+y^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi(b-x)}\frac{(a-y)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\mathbf{k}$$
$$+\frac{\mu_0 I}{4\pi(a-y)}\frac{(b-x)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi(a-y)}\frac{x)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$
$$+\frac{\mu_0 I}{4\pi x}\frac{(a-y)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi x}\frac{y}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$

We simplify this equation by factoring out common constants and combining terms with similar roots.

$$\vec{\mathbf{B}}=\boxed{\frac{\mu_0 I}{4\pi}\left(\frac{\sqrt{y^2+x^2}}{xy}+\frac{\sqrt{y^2+(b-x)^2}}{(b-x)y}+\frac{\sqrt{(a-y)^2+(b-x)^2}}{(a-y)(b-x)}+\frac{\sqrt{(a-y)^2+x^2}}{x(a-y)}\right)\hat{\mathbf{k}}}$$

46. (*a*) By symmetry we see that on the *x* axis the magnetic field can only have an *x* component. To justify this assertion, imagine that the magnetic field had a component off the axis. If the current loop were rotated by 90° about the *x* axis, the loop orientation would be identical to the original loop, but the off-axis magnetic field component would have changed. This is not possible, so the field only has an *x* component. The contribution to this field is the same for each loop segment, and so the total magnetic field is equal to 4 times the *x* component of the magnetic field from one segment. We integrate Eq. 28-5 to find this magnetic field.

$$\vec{\mathbf{B}}=4\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{\mu_0 I}{4\pi}\frac{dy\,\hat{\mathbf{j}}\times\left(\frac{1}{2}d\,\hat{\mathbf{k}}\right)}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}}=\frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}}$$
$$=\frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\frac{y}{\left[\left(\frac{1}{2}d\right)^2+x^2\right]\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{1/2}}\Bigg|_{-\frac{1}{2}d}^{\frac{1}{2}d}=\boxed{\frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(d^2+4x^2\right)\left(d^2+2x^2\right)^{1/2}}}$$

(*b*) Let $x\gg d$ to show that the magnetic field reduces to a dipole field of Eq. 28-7b.

$$\vec{\mathbf{B}}\approx\frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(4x^2\right)\left(2x^2\right)^{1/2}}=\frac{d^2\mu_0 I\,\hat{\mathbf{i}}}{2\pi x^3}$$

Comparing our magnetic field to Eq. 28-7b we see that it is a dipole field with the magnetic moment $\boxed{\vec{\boldsymbol{\mu}}=d^2 I\,\hat{\mathbf{i}}}$

47. (*a*) If the iron bar is completely magnetized, all of the dipoles are aligned. The total dipole moment is equal to the number of atoms times the dipole moment of a single atom.

$$\mu=N\mu_1=\frac{N_A\rho V}{M_m}\mu_1$$

$$= \frac{\left(6.022\times10^{23}\,\text{atoms/mole}\right)\left(7.80\,\text{g/cm}^3\right)\left(9.0\,\text{cm}\right)\left(1.2\,\text{cm}\right)\left(1.0\,\text{cm}\right)}{55.845\,\text{g/mole}}\left(1.8\times10^{-23}\,\frac{\text{A}\bullet\text{m}^2}{\text{atom}}\right)$$

$$= 16.35\ \text{A}\bullet\text{m}^2 \approx \boxed{16\ \text{A}\bullet\text{m}^2}$$

(*b*) We use Eq. 27-9 to find the torque.

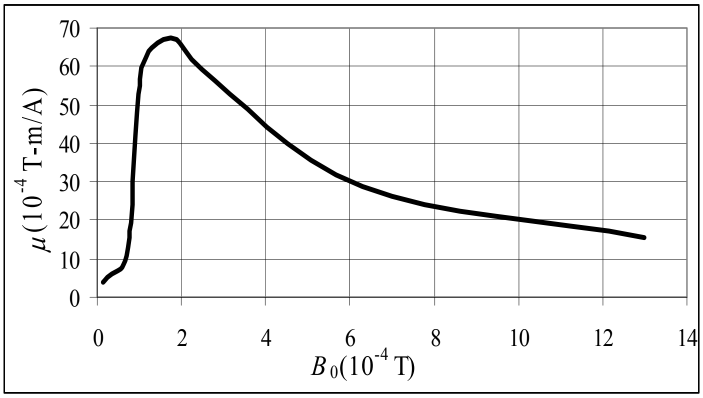
$$\tau = \mu B\sin\theta = \left(16.35\ \text{A}\bullet\text{m}^2\right)\left(0.80\ \text{T}\right)\sin 90^\circ = \boxed{13\ \text{m}\bullet\text{N}}$$

48. The magnetic permeability is found from the two fields.

$$B_0 = \mu_0 nI \ ;\ B = \mu nI \ ;$$

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} \ \rightarrow\ \mu = \mu_0\frac{B}{B_0}$$

For the graph, we have not plotted the last three data points so that the structure for low fields is seen. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.48."



49. The magnetic field of a long, thin torus is the same as the field given by a long solenoid, as in Eq. 28-9.

$$B = \mu nI = \left(2200\right)\left(4\pi\times10^{-7}\ \text{Tm/A}\right)\left(285\ \text{m}^{-1}\right)\left(3.0\ \text{A}\right) = \boxed{2.4\ \text{T}}$$

50. The field inside the solenoid is given by Eq. 28-4 with μ_0 replaced by the permeability of the iron.

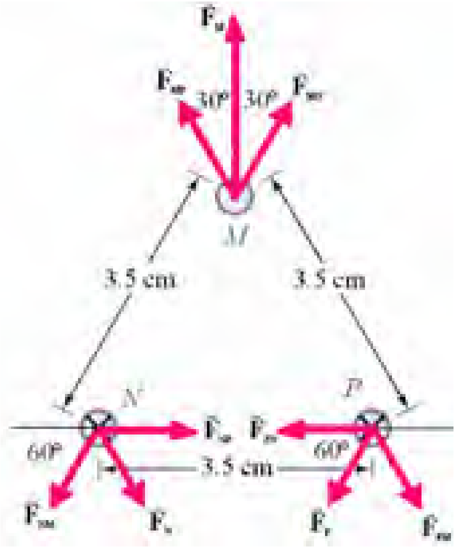
$$B = \frac{\mu NI}{\ell} \ \rightarrow\ \mu = \frac{B\ell}{NI} = \frac{\left(2.2\,\text{T}\right)\left(0.38\,\text{m}\right)}{\left(640\right)\left(48\,\text{A}\right)} = \boxed{2.7\times10^{-5}\ \text{T}\bullet\text{m/A}} \approx 22\mu_0$$

51. Since the wires all carry the same current and are equidistant from each other, the magnitude of the force per unit length between any two wires is the same and is given by Eq. 28-2.

$$\frac{F}{\ell} = \frac{\mu_0 I^2}{2\pi d} = \frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\left(8.00\ \text{A}\right)^2}{2\pi\left(0.035\ \text{m}\right)}$$

$$= 3.657\times10^{-4}\,\text{N/m}$$

The direction of the force between two wires is along the radial line and attractive for currents traveling in the same direction and repulsive for currents traveling in opposite directions. The forces acting on wire M are radially away from the other two wires. By symmetry, the horizontal components of these forces cancel and the net force is the sum of the vertical components.



$$F_M = F_{MP}\cos 30^\circ + F_{MN}\cos 30^\circ$$

$$= 2\left(3.657\times10^{-4}\,\text{N/m}\right)\cos 30^\circ = \boxed{6.3\times10^{-4}\,\text{N/m at } 90^\circ}$$

The force on wire N is found by adding the components of the forces from the other two wires. By symmetry we see that this force is directed at an angle of 300°. The force on wire P, will have the same magnitude but be directed at 240°.

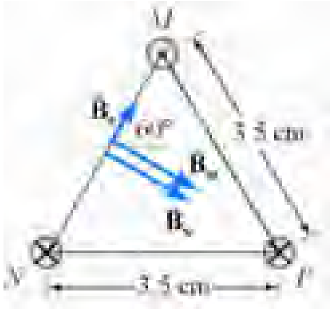
$$F_{N,x} = F_{NP} - F_{NM}\cos 60° = 3.657\times10^{-4}\,\text{N/m} - \left(3.657\times10^{-4}\,\text{N/m}\right)\cos 60° = 1.829\times10^{-4}\,\text{N/m}$$

$$F_{N,\text{y}} = -F_{NM}\sin 60° = -\left(3.657\times10^{-4}\,\text{N/m}\right)\sin 60° = -3.167\times10^{-4}\,\text{N/m}$$

$$F_N = \sqrt{\left(1.829\times10^{-4}\,\text{N/m}\right)^2 + \left(-3.167\times10^{-4}\,\text{N/m}\right)^2} = \boxed{3.7\times10^{-4}\,\text{N/m at }300°}$$

$$F_P = \boxed{3.7\times10^{-4}\,\text{N/m at }240°}$$

52. The magnetic field at the midpoint between currents M and N is the vector sum of the magnetic fields from each wire, given by Eq. 28-1. Each field points perpendicularly to the line connecting the wire to the midpoint.



$$\vec{\mathbf{B}}_{\text{net}} = \vec{\mathbf{B}}_{\text{M}} + \vec{\mathbf{B}}_{\text{N}} + \vec{\mathbf{B}}_{\text{P}}$$

$$B_{\text{M}} = B_{\text{N}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{M}}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{0.0175\,\text{m}} = 9.143\times10^{-5}\,\text{T}$$

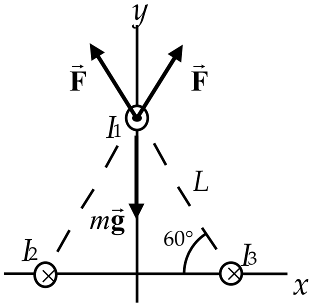
$$B_{\text{P}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{P}}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{\sqrt{3}\left(0.0175\,\text{m}\right)} = 5.279\times10^{-5}\,\text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(1.849\times10^{-4}\,\text{T}\right)^2 + \left(-4.571\times10^{-5}\,\text{T}\right)} = \boxed{4.93\times10^{-4}\,\text{T}}$$

$$\theta_{\text{net}} = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,y}} = \tan^{-1}\frac{-4.210\times10^{-5}\,\text{T}}{1.702\times10^{-4}\,\text{T}} = \boxed{-14°}$$

The net field points slightly below the horizontal direction.

53. For the wire to be suspended the net magnetic force must equal the gravitational force. Since the same current flows through the two lower wires, the net magnetic force is the sum of the vertical components of the force from each wire, given by Eq. 28-2. We solve for the unknown current by setting this force equal to the weight of the wire.



$$F_M = 2\frac{\mu_0 I_M I_{NP}}{2\pi r}\ell\cos30° = \rho g\left(\tfrac{1}{4}\pi d^2\ell\right)$$

$$I_M = \frac{\rho g\pi^2 r d^2}{4\mu_0 I_{NP}\cos30°}$$

$$= \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.035\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{4\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(40.0\,\text{A}\right)\cos30°} = \boxed{170\,\text{A}}$$

54. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b. From this force we can calculate the radius of curvature.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad\rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.3\times10^{7}\,\text{m/s}\right)\sin7°}{\left(1.60\times10^{-19}\text{C}\right)\left(3.3\times10^{-2}\text{T}\right)} = 2.734\times10^{-4}\,\text{m} \approx \boxed{0.27\text{ mm}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T=\frac{2\pi r}{v_\perp}=\frac{2\pi\frac{mv_\perp}{qB}}{v_\perp}=\frac{2\pi m}{qB}$$

$$p=v_\parallel T=v\cos 7^\circ\left(\frac{2\pi m}{qB}\right)=\left(1.3\times10^{7}\,\mathrm{m/s}\right)\cos 7^\circ\frac{2\pi\left(9.11\times10^{-31}\,\mathrm{kg}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(3.3\times10^{-2}\,\mathrm{T}\right)}=\boxed{1.4\text{ cm}}$$

$\boxed{55.}$ (*a*) Use Eq. 28-1 to calculate the field due to a long straight wire.

$$B_{\text{A at B}}=\frac{\mu_0 I_{\text{A}}}{2\pi r_{\text{A to B}}}=\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(2.0\,\mathrm{A}\right)}{2\pi\left(0.15\,\mathrm{m}\right)}=2.667\times10^{-6}\,\mathrm{T}\approx\boxed{2.7\times10^{-6}\,\mathrm{T}}$$

(*b*) $$B_{\text{B at A}}=\frac{\mu_0 I_{\text{B}}}{2\pi r_{\text{B to A}}}=\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(4.0\,\mathrm{A}\right)}{2\pi\left(0.15\,\mathrm{m}\right)}=5.333\times10^{-6}\,\mathrm{T}\approx\boxed{5.3\times10^{-6}\,\mathrm{T}}$$

(*c*) The two fields are not equal and opposite. Each individual field is due to a single wire, and has no dependence on the other wire. The magnitude of current in the second wire has nothing to do with the value of the field caused by the first wire.

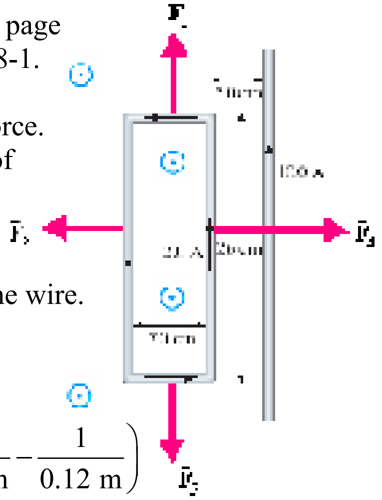
(*d*) Use Eq. 28-2 to calculate the force due to one wire on another. The forces are attractive since the currents are in the same direction.

$$\frac{F_{\text{on A due to B}}}{\ell_{\text{A}}}=\frac{F_{\text{on B due to A}}}{\ell_{\text{B}}}=\frac{\mu_0}{2\pi}\frac{I_{\text{A}}I_{\text{B}}}{d_{\text{A to B}}}=\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)}{2\pi}\frac{\left(2.0\,\mathrm{A}\right)\left(4.0\,\mathrm{A}\right)}{\left(0.15\,\mathrm{m}\right)}$$

$$=1.067\times10^{-5}\,\mathrm{N/m}\approx\boxed{1.1\times10^{-5}\,\mathrm{N/m}}$$

These two forces per unit length are equal and opposite because they are a Newton's third law pair of forces.

56. (*a*) The magnetic field from the long straight wire will be out of the page in the region of the wire loop with its magnitude given by Eq. 28-1. By symmetry, the forces from the two horizontal segments are equal and opposite, therefore they do not contribute to the net force. We use Eq. 28-2 to find the force on the two vertical segments of the loop and sum the results to determine the net force. Note that the segment with the current parallel to the straight wire will be attracted to the wire, while the segment with the current flowing in the opposite direction will be repelled from the wire.

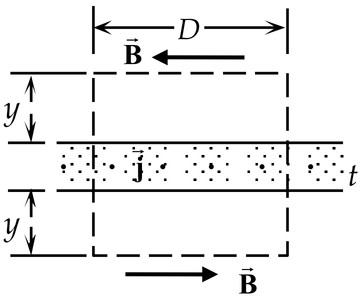


$$F_{net}=F_2+F_4=-\frac{\mu_0 I_1 I_2}{2\pi d_2}\ell+\frac{\mu_0 I_1 I_2}{2\pi d_1}\ell=\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$=\frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)\left(2.0\text{ A}\right)\left(10.0\text{ A}\right)\left(0.26\text{ m}\right)}{2\pi}\left(\frac{1}{0.05\text{ m}}-\frac{1}{0.12\text{ m}}\right)$$

$$=\boxed{1.2\times10^{-5}\text{ N toward the wire}}$$

(*b*) Since the forces on each segment lie in the same plane, the net torque on the loop is $\boxed{\text{zero}}$.



57. The sheet may be treated as an infinite number of parallel wires. The magnetic field at a location *y* above the wire will be the sum of the magnetic fields produced by each of the wires. If we consider the magnetic field from two wires placed symmetrically on either side of where we are measuring the magnetic field, we see that the vertical magnetic field components cancel each other out. Therefore, the field above the wire must be horizontal and to the left. By symmetry, the field a location *y* below the wire must have the same magnitude, but point in the opposite direction. We calculate the magnetic field using Ampere's law with a rectangular loop that extends a distance *y* above and below the current sheet, as shown in the figure.

$$\oint \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = \int_{\text{sides}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{\text{top}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} + \int_{\text{bottom}} \vec{\mathbf{B}}\cdot d\vec{\boldsymbol{\ell}} = 0 + 2B_{\parallel}D = \mu_0 I_{\text{encl}} = \mu_0\left(jtD\right)$$

$$\rightarrow B_{\parallel} = \boxed{\tfrac{1}{2}\mu_0 jt, \text{ to the left above the sheet}}$$

58. (*a*) We set the magnetic force, using Eq. 28-2, equal to the weight of the wire and solve for the necessary current. The current must flow in the same direction as the upper current, for the magnetic force to be upward.

$$F_M = \frac{\mu_0 I_1 I_2}{2\pi r}\boldsymbol{\ell} = \rho g\left(\frac{\pi d^2}{4}\ell\right) \rightarrow$$

$$I_2 = \frac{\rho g \pi^2 r d^2}{4\mu_0 I_1} = \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.050\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(48.0\,\text{A}\right)} = \boxed{360\,\text{A, right}}$$

(*b*) The lower wire is in $\boxed{\text{unstable equilibrium}}$, since if it is raised slightly from equilibrium, the magnetic force would be increased, causing the wire to move further from equilibrium.

(*c*) If the wire is suspended above the first wire at the same distance, the same current is needed, but in the opposite direction, as the wire must be repelled from the lower wire to remain in equilibrium. Therefore the current must be $\boxed{\text{360 A to the left}}$. This is a $\boxed{\text{stable equilibrium}}$ for vertical displacement since if the wire is moved slightly off the equilibrium point the magnetic force will increase or decrease to push the wire back to the equilibrium height.

59. The magnetic field at the center of the square loop is four times the magnetic field from one of the sides. It will be directed out of the page. We can use the result of Problem 40 for the magnitude of the field from one side, with $R = \tfrac{1}{2}d$. If the current is flowing counterclockwise around the square loop, the magnetic field due to each piece will point upwards.

$$\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \frac{\mu_0 I}{2\pi R}\frac{d\,\hat{\mathbf{k}}}{\left(4R^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{2\pi\left(\tfrac{1}{2}d\right)}\frac{d\,\hat{\mathbf{k}}}{\left(4\left(\tfrac{1}{2}d\right)^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{\sqrt{2}\pi d}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}}_{\text{total}} = 4\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \boxed{\frac{2\sqrt{2}\mu_0 I}{\pi d}\hat{\mathbf{k}}}$$

60. The magnetic field at the center of a circular loop was calculated in Example 28-12. To determine the radius of the loop, we set the circumferences of the loops equal.

$$2\pi R = 4d \rightarrow R = \frac{2d}{\pi} \;;\; B_{\text{circle}} = \frac{\mu_0 I}{2R} = \frac{\mu_0 I\pi}{4d} < \frac{2\sqrt{2}\mu_0 I}{\pi d} = B_{\text{square}}$$

Therefore, changing the shape to a circular loop will $\boxed{\text{decrease}}$ the magnetic field.

61. (*a*) Choose $x = 0$ at the center of one coil. The center of the other coil will then be at $x = R$. Since the currents flow in the same direction in both coils, the right-hand-rule shows that the magnetic fields from the two coils will point in the same direction along the axis. The magnetic field from a current loop was found in Example 28-12. Adding the two magnetic fields together yields the total field.

$$B(x) = \boxed{\frac{\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{3/2}}}$$

(*b*) Evaluate the derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{dB}{dx} = -\frac{3\mu_0 NIR^2 x}{2\left[R^2 + x^2\right]^{5/2}} - \frac{3\mu_0 NIR^2 (x-R)}{2\left[R^2 + (x-R)^2\right]^{5/2}} = -\frac{3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} - \frac{-3\mu_0 NIR^3}{4\left[R^2 + R^2/4\right]^{5/2}} = \boxed{0}$$

Evaluate the second derivative of the magnetic field at $x = \frac{1}{2}R$.

$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 NIR^2}{2\left[R^2 + x^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 x^2}{2\left[R^2 + x^2\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[R^2 + (x-R)^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 (x-R)^2}{2\left[R^2 + (x-R)^2\right]^{7/2}}$$

$$= -\frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}}$$

$$= \frac{\mu_0 NIR^2}{\left[5R^2/4\right]^{5/2}}\left(-\frac{3}{2} + \frac{15\cdot 4}{8\cdot 5} - \frac{3}{2} + \frac{15\cdot 4}{8\cdot 5}\right) = \boxed{0}$$

Therefore, at the midpoint $\dfrac{dB}{dx} = 0$ and $\dfrac{d^2B}{dx^2} = 0$.

(*c*) We insert the given data into the magnetic field equation to calculate the field at the midpoint.

$$B\left(\tfrac{1}{2}R\right) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}} = \frac{\mu_0 NIR^2}{\left[R^2 + \left(\frac{1}{2}R\right)^2\right]^{3/2}}$$

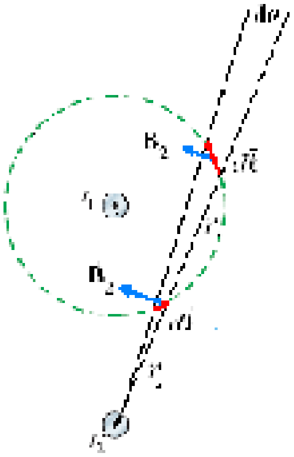
$$= \frac{\left(4\pi\times 10^{-7}\,\mathrm{T\bullet m/A}\right)(250)(2.0\,\mathrm{A})(0.10\,\mathrm{m})^2}{\left[(0.10\,\mathrm{m})^2 + (0.05\,\mathrm{m})^2\right]^{3/2}} = \boxed{4.5\,\mathrm{mT}}$$

62. The total field is the vector sum of the fields from the two currents. We can therefore write the path integral as the sum of two such integrals.

$$\oint \vec{\mathbf{B}}\bullet d\vec{\ell} = \oint \vec{\mathbf{B}}_1\bullet d\vec{\ell} + \oint \vec{\mathbf{B}}_2\bullet d\vec{\ell}$$

To evaluate the integral for current 1, we use Eq. 28-1, with the magnetic field constant and parallel to the loop at each line segment.

$$\oint \vec{\mathbf{B}}_1\bullet d\vec{\ell} = \frac{\mu_0 I_1}{2\pi r}\int_0^{2\pi} r\,d\theta = \mu_0 I_1$$



To evaluate the integral for current 2, we consider a different angle $d\theta$ centered at I_2 and crossing the path of the loop at two locations, as shown in the diagram. If we integrate clockwise around the path, the components of $d\ell$ parallel to the field will be $-r_1 d\theta$ and $r_2 d\theta$. Multiplying these components by the magnetic field at both locations gives the contribution to the integral from the sum of these segments.

$$B_1 d\ell_1 + B_2 d\ell_2 = \frac{\mu_0 I_2}{2\pi r_1}\left(-r_1 d\theta\right) + \frac{\mu_0 I_2}{2\pi r_2}\left(r_2 d\theta\right) = 0$$

The total integral will be the sum of these pairs resulting in a zero net integral.

$$\oint \vec{\mathbf{B}}\bullet d\vec{\ell} = \oint \vec{\mathbf{B}}_1\bullet d\vec{\ell} + \oint \vec{\mathbf{B}}_2\bullet d\vec{\ell} = \mu_0 I_1 + 0 = \boxed{\mu_0 I_1}$$

63. From Example 28-12, the magnetic field on the axis of a circular loop of wire of radius R carrying current I is $B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$, where x is the distance along the axis from the center of the loop.

For the loop described in this problem, we have $R = x = R_{\text{Earth}}$.

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \rightarrow I = \frac{2B\left(R^2 + x^2\right)^{3/2}}{\mu_0 R^2} = \frac{2B\left(R_{\text{Earth}}^2 + R_{\text{Earth}}^2\right)^{3/2}}{\mu_0 R_{\text{Earth}}^2} = \frac{2(2)^{3/2} B R_{\text{Earth}}}{\mu_0}$$

$$= \frac{2(2)^{3/2}\left(1\times10^{-4}\,\text{T}\right)\left(6.38\times10^{6}\,\text{m}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}} = \boxed{3\times10^{9}\,\text{A}}$$

64. The magnetic field from the wire at the location of the plane is perpendicular to the velocity of the plane since the plane is flying parallel to the wire. We calculate the force on the plane, and thus the acceleration, using Eq. 27-5b, with the magnetic field of the wire given by Eq. 28-1.

$$F = qvB = qv\frac{\mu_0 I}{2\pi r}$$

$$a = \frac{F}{m} = \frac{qv}{m}\frac{\mu_0 I}{2\pi r} = \frac{\left(18\times10^{-3}\text{ C}\right)\left(2.8\text{ m/s}\right)\left(4\pi\times10^{-7}\text{ T}\bullet\text{m/A}\right)\left(25\text{ A}\right)}{2\pi\left(0.175\text{ kg}\right)\left(0.086\text{ m}\right)}$$

$$= 1.67\times10^{-5}\text{ m/s}^2 = \boxed{1.7\times10^{-6}\,g's}$$

65. (*a*) To find the length of wire that will give the coil sufficient resistance to run at maximum power, we write the power equation (Eq. 25-7b) with the resistance given by Eq. 25-3. We divide the length by the circumference of one coil to determine the number of turns.

$$P_{\max} = \frac{V^2}{R} = \frac{V^2}{\rho\ell/\left(d^2\right)} \rightarrow \ell = \frac{V^2 d^2}{\rho P_{\max}}$$

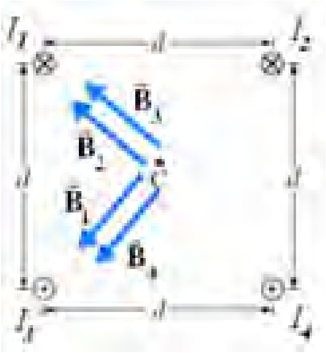
$$N = \frac{\ell}{\pi D} = \frac{V^2 d^2}{\pi D \rho P_{\max}} = \frac{\left(35\text{ V}\right)^2\left(2.0\times10^{-3}\text{ m}\right)^2}{\pi\left(2.0\text{ m}\right)\left(1.68\times10^{-8}\ \Omega\text{m}\right)\left(1.0\times10^{3}\text{ W}\right)} = \boxed{46\text{ turns}}$$

(*b*) We use the result of Example 28-12 to determine the magnetic field at the center of the coil, with the current obtained from Eq. 25-7b.

$$B = \frac{\mu_0 N I}{D} = \frac{\mu_0 N}{D}\frac{P_{\max}}{V} = \frac{\left(4\pi\times10^{-7}\text{ T}\bullet\text{m/A}\right)\left(46\right)\left(1.0\times10^{3}\text{ W}\right)}{\left(2.0\text{ m}\right)\left(35\text{ V}\right)} = \boxed{0.83\text{ mT}}$$

(*c*) Increasing the number of turns will proportionately increase the resistance and therefore decrease the current. The net result is $\boxed{\text{no change in the magnetic field}}$.

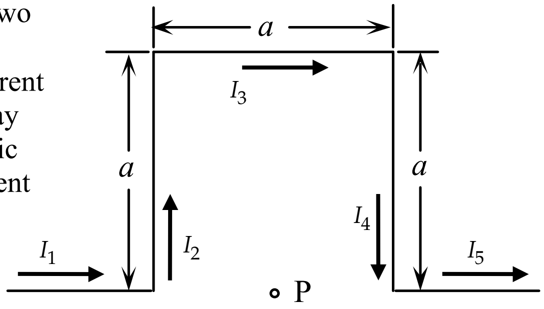
66. The magnetic field at the center of the square is the vector sum of the magnetic field created by each current. Since the magnitudes of the currents are equal and the distance from each corner to the center is the same, the magnitude of the magnetic field from each wire is the same and is given by Eq. 28-1. The direction of the magnetic field is directed by the right-hand-rule and is shown in the diagram. By symmetry, we see that the vertical components of the magnetic field cancel and the horizontal components add.



$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 = -4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°\hat{\mathbf{i}}$$

$$= -4\left(\frac{\mu_0 I}{2\pi \frac{\sqrt{2}}{2} d}\right)\frac{\sqrt{2}}{2}\hat{\mathbf{i}} = \boxed{-\frac{2\mu_0 I}{\pi d}\hat{\mathbf{i}}}$$

$\boxed{67.}$ The wire can be broken down into five segments: the two long wires, the left vertical segment, the right vertical segment, and the top horizontal segment. Since the current in the two long wires either flow radially toward or away from the point P, they will not contribute to the magnetic field. The magnetic field from the top horizontal segment points into the page and is obtained from the solution to Problem 40.



$$B_{top} = \frac{\mu_0 I}{2\pi a}\frac{a}{\left(a^2 + 4a^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi a\sqrt{5}}$$

The magnetic fields from the two vertical segments both point into the page with magnitudes obtained from the solution to Problem 41.

$$B_{vert} = \frac{\mu_0 I}{4\pi(a/2)}\frac{a}{\left(a^2 + (a/2)^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{\pi a\sqrt{5}}$$

Summing the magnetic fields from all the segments yields the net field.

$$B = B_{top} + 2B_{vert} = \frac{\mu_0 I}{2\pi a\sqrt{5}} + 2\frac{\mu_0 I}{\pi a\sqrt{5}} = \boxed{\frac{\mu_0 I\sqrt{5}}{2\pi a}\text{, into the page.}}$$

68. Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 IN}{\ell} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2.0\,\text{A})(420)}{0.12\,\text{m}} = \boxed{8.8\times10^{-3}\,\text{T}}$$

69. The field due to the solenoid is given by Eq. 28-4. Since the field due to the solenoid is perpendicular to the current in the wire, Eq. 27-2 can be used to find the force on the wire segment.

$$F = I_{\text{wire}}\ell_{\text{wire}}B_{\text{solenoid}} = I_{\text{wire}}\ell_{\text{wire}}\frac{\mu_0 I_{\text{solenoid}}N}{\ell_{\text{solenoid}}} = (22\,\text{A})(0.030\text{m})\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(33\,\text{A})(550)}{(0.15\,\text{m})}$$

$$= \boxed{0.10\,\text{N to the south}}$$

70. Since the mass of copper is fixed and the density is fixed, the volume is fixed, and we designate it as $V_{\text{Cu}} = m_{\text{Cu}}\rho_{\text{Cu}} = \ell_{\text{Cu}}A_{\text{Cu}}$. We call the fixed voltage V_0. The magnetic field in the solenoid is given by Eq. 28-4.

$$B = \frac{\mu_0 IN}{\ell_{\text{sol}}} = \mu_0 V_0 \frac{N}{R_{\text{Cu}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\frac{\ell_{\text{Cu}}}{A_{\text{Cu}}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}} \frac{A_{\text{Cu}}}{\ell_{\text{Cu}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}} \frac{m_{\text{Cu}}\rho_{\text{Cu}}}{\ell_{\text{Cu}}^2}$$

$$= \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2}$$

The number of turns of wire is the length of wire divided by the circumference of the solenoid.

$$N = \frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}} \rightarrow B = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}} \frac{\frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}}}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{2\pi\rho_{\text{R Cu}}} \frac{1}{\ell_{\text{sol}} r_{\text{sol}} \ell_{\text{Cu}}}$$

The first factor in the expression for B is made of constants, so we have $B \propto \frac{1}{\ell_{\text{sol}} r_{\text{sol}} \ell_{\text{Cu}}}$. Thus we want the wire to be $\boxed{\text{short and fat}}$. Also the radius of the solenoid should be small and the length of the solenoid small.

71. The magnetic field inside the smaller solenoid will equal the sum of the fields from both solenoids. The field outside the inner solenoid will equal the field produced by the outer solenoid only. We set the sum of the two fields given by Eq. 28-4 equal to $-\frac{1}{2}$ times the field of the outer solenoid and solve for the ratio of the turn density.

$$\mu_0(-I)n_a + \mu_0 I n_b = -\tfrac{1}{2}(\mu_0 I n_b) \rightarrow \boxed{\frac{n_b}{n_a} = \frac{2}{3}}$$

72. Take the origin of coordinates to be at the center of the semicircle.The magnetic field at the center of the semicircle is the vector sum of the magnetic fields from each of the two long wires and from the semicircle. By the right-hand-rule each of these fields point into the page, so we can sum the magnitudes of the fields. The magnetic field for each of the long segments is obtained by integrating Eq. 28-5 over the straight segment.

$$\vec{\mathbf{B}}_{\text{straight}} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{R}}{R^2} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{\mathbf{R}}}{R^3} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{0} \frac{dx\,\hat{\mathbf{i}} \times \left(-x\hat{\mathbf{i}} - r\hat{\mathbf{j}}\right)}{\left(x^2 + r^2\right)^{3/2}} = -\frac{\mu_0 Ir}{4\pi}\hat{\mathbf{k}} \int_{-\infty}^{0} \frac{dx}{\left(x^2 + r^2\right)^{3/2}}$$

$$= -\frac{\mu_0 Ir}{4\pi}\hat{\mathbf{k}} \left. \frac{x}{r^2\left(x^2 + r^2\right)^{1/2}} \right|_{-\infty}^{0} = -\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}}$$

The magnetic field for the curved segment is obtained by integrating Eq. 28-5 over the semicircle.

$$\vec{\mathbf{B}}_{\text{curve}} = \frac{\mu_0 I}{4\pi} \int_0^{\pi r} \frac{d\vec{\ell} \times \hat{r}}{r^2} = -\frac{\mu_0 I}{4\pi r^2}\hat{\mathbf{k}} \int_0^{\pi r} ds = -\frac{\mu_0 I}{4r}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = 2\vec{\mathbf{B}}_{\text{straight}} + \vec{\mathbf{B}}_{\text{curve}} = -2\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}} - \frac{\mu_0 I}{4r}\hat{\mathbf{k}} = \boxed{-\frac{\mu_0 I}{4\pi r}(2 + \pi)\hat{\mathbf{k}}}$$

73. (*a*) Set $x = 0$ at the midpoint on the axis between the two loops. Since the loops are a distance R apart, the center of one loop will be at $x = -\frac{1}{2}R$ and the center of the other at $x = \frac{1}{2}R$. The currents in the loops flow in opposite directions, so by the right-hand-rule the magnetic fields from the two wires will subtract from each other. The magnitude of each field can be obtained from Example 28-12.

$$B(x) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R - x\right)^2\right]^{3/2}} - \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R + x\right)^2\right]^{3/2}}$$

Factoring out $\frac{1}{8}R^3$ from each of the denominators yields the desired equation.

$$B(x) = \frac{4\mu_0 NI}{R\left[4 + \left(1 - 2x/R\right)^2\right]^{3/2}} - \frac{4\mu_0 NI}{R\left[4 + \left(1 + 2x/R\right)^2\right]^{3/2}}$$

$$= \boxed{\frac{4\mu_0 NI}{R}\left\{\left[4 + \left(1 - \frac{2x}{R}\right)^2\right]^{-3/2} - \left[4 + \left(1 + \frac{2x}{R}\right)^2\right]^{-3/2}\right\}}$$

(*b*) For small values of x, we can use the approximation $\left(1 \pm \frac{2x}{R}\right)^2 \approx 1 \pm \frac{4x}{R}$.

$$B(x) = \frac{4\mu_0 NI}{R}\left\{\left[4 + 1 - \frac{4x}{R}\right]^{-3/2} - \left[4 + 1 + \frac{4x}{R}\right]^{-3/2}\right\}$$

$$= \frac{4\mu_0 NI}{5R\sqrt{5}}\left\{\left[1 - \frac{4x}{5R}\right]^{-3/2} - \left[1 + \frac{4x}{5R}\right]^{-3/2}\right\}$$

Again we can use the expansion for small deviations $\left(1 \pm \frac{4x}{5R}\right)^{-3/2} \approx 1 \mp \frac{6x}{5R}$

$$B(x) = \frac{4\mu_0 NI}{5R\sqrt{5}}\left[\left(1 + \frac{6x}{5R}\right) - \left(1 - \frac{6x}{5R}\right)\right] = \boxed{\frac{48\mu_0 NIx}{25R^2\sqrt{5}}}$$

This magnetic field has the expected linear dependence on x with a coefficient of $C = 48\mu_0 NI/\left(25R^2\sqrt{5}\right)$.

(*c*) Set C equal to 0.15 T/m and solve for the current.

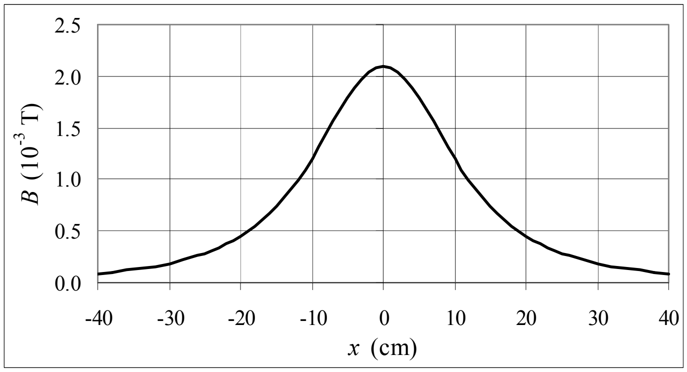
$$I = \frac{25CR^2\sqrt{5}}{48\mu_0 N} = \frac{25(0.15\text{ T/m})(0.04\text{ m})^2\sqrt{5}}{48\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(150)} = \boxed{1.5\text{ A}}$$

74. We calculate the peak current using Eqs. 25-7 and 25-9. Then we use the peak current in Eq. 28-1 to calculate the maximum magnetic field.

$$I_{\text{max}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{P_{\text{avg}}}{V_{\text{rms}}} \rightarrow B_{\text{max}} = \frac{\mu_0 I_{\text{max}}}{2\pi r} = \frac{\sqrt{2}\mu_0}{2\pi r}\frac{P_{\text{avg}}}{V_{\text{rms}}}$$

$$= \frac{\sqrt{2}\left(4\pi\times10^{-7}\text{ T}\bullet\text{m/A}\right)}{2\pi(12\text{ m})}\frac{\left(45\times10^6\text{ W}\right)}{\left(15\times10^3\text{ V}\right)} = \boxed{71\ \mu\text{T}}$$

75. We use the results of Example 28-12 to calculate the magnetic field as a function of position. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.75."

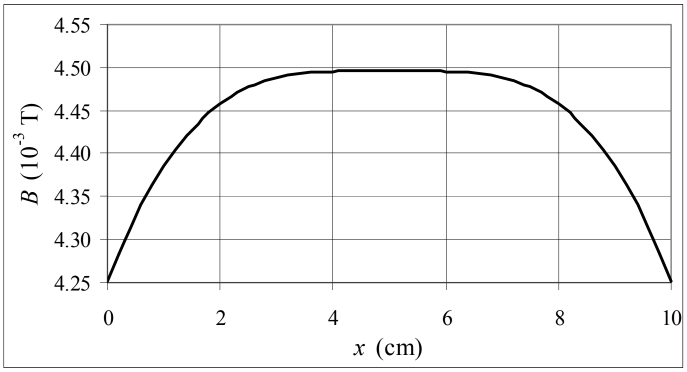


$$B=\frac{N\mu_0 IR^2}{2\left(R^2+x^2\right)^{3/2}}=\frac{(250)\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(2.0\,\mathrm{A})(0.15\,\mathrm{m})^2}{2\left[(0.15\,\mathrm{m})^2+x^2\right]^{3/2}}=\frac{7.0686\times10^{-6}\,\mathrm{T{\bullet}m^3}}{\left[(0.15\,\mathrm{m})^2+x^2\right]^{3/2}}.$$

76. (*a*) Use the results of Problem 61(a) to write the magnetic field.

$$B(x)=\frac{\mu_0 NIR^2}{2\left[R^2+x^2\right]^{3/2}}+\frac{\mu_0 NIR^2}{2\left[R^2+(x-R)^2\right]^{3/2}}$$

(*b*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.76b."



(*c*) Use the values from the spreadsheet to find the % difference.

$$\%\,\mathrm{diff}=\frac{B(x=6.0\,\mathrm{cm})-B(x=5.0\,\mathrm{cm})}{B(x=5.0\,\mathrm{cm})}(100)=\frac{4.49537\,\mathrm{mT}-4.49588\,\mathrm{mT}}{4.49588\,\mathrm{mT}}(100)$$

$$=\boxed{-1.1\times10^{-2}\%}$$

CHAPTER 29: Electromagnetic Induction and Faraday's Law

Responses to Questions

1. Using coils with many (N) turns increases the values of the quantities to be experimentally measured, because the induced emf and therefore the induced current are proportional to N.

2. Magnetic flux is a quantitative measure of the number of magnetic field lines passing through a given area. It depends not only on the field itself, but also on the area and on the angle between the field and the area.

3. Yes, the current is induced clockwise. No, there is no induced current if the magnet is steady, because there is no changing flux through the ring. Yes, the current is induced counterclockwise.

4. There is no induced current in the loop that is moving parallel to the wire because there is no change of magnetic flux through the loop. The induced current in the loop moving away from the wire is clockwise. The magnetic field through the loop due to the current is directed into the page, and the loop is moving such that its distance from the wire is increasing, resulting in a decrease in magnetic field strength and therefore a decrease in magnetic flux through the loop. By Lenz's law, a decreasing magnetic flux into the page results in a clockwise induced current.

5. Yes. The force is attractive. The induced clockwise current in the right loop will induce a counterclockwise current in the left loop which will slow the relative motion of the loops.

6. (*a*) Yes.
   (*b*) The current starts as soon as the battery is connected and current begins to flow in the first loop.
   (*c*) The induced current stops as soon as the current in the first loop has reached its steady value.
   (*d*) The induced current in the second loop will be counterclockwise, in order to oppose the change.
   (*e*) While there is an induced current, there will be a force between the two loops.
   (*f*) The force will be repulsive, since the currents are in opposite directions.

7. Yes, a current will be induced in the second coil. It will start when the battery is disconnected from the first coil and stop when the current falls to zero in the first coil. The current in the second loop will be clockwise.

8. Counterclockwise. If the area of the loop decreases, the flux through the loop (directed out of the page) decreases. By Lenz's law, the resulting induced current will be counterclockwise to oppose the change. Another way to approach this question is to use the right-hand rule. As the bar moves to the left, the negative electrons in the bar will experience a force down, which results in a counterclockwise current.

9. (*a*) The current through R_A will be to the right. The field due to the current in coil B will be to the left. As coil B is moved toward coil A, the flux through A will increase, so the induced field in coil A will be to the right, to oppose the change. This field corresponds to an induced current flowing from left to right in R_A.
   (*b*) The current through R_A will be to the left. When coil B is moved away from coil A, the flux through coil A will decrease, so the induced field will be to the left, to oppose the change. This field corresponds to an induced current flowing from right to left in R_A.
   (*c*) If R_B is increased, the current in the circuit will decrease, decreasing the flux through coil A, resulting in a current through R_A to the left.

10. The shielding prevents external fields from inducing a current which would cause a false signal in the inner signal wire.

11. The currents in the two wires will be 180º out of phase. If they are very close together, or wrapped around each other, then the magnetic fields created by the currents in the wires will very nearly cancel each other.

12. The straight wire will fall faster. Since the magnetic field is non-uniform, the flux through the loop will change as the loop falls, inducing a current which will oppose the change and therefore resist the downward motion. Eddy currents will also be induced in the straight wire, but they will be much smaller since the straight wire does not form a closed loop.

$\boxed{13.}$ (*a*) Yes. If a rapidly changing magnetic field exists outside, then currents will be induced in the metal sheet. These currents will create magnetic fields which will partially cancel the external fields.
(*b*) Yes. Since the metal sheet is permeable, it will partially shield the interior from the exterior static magnetic field; some of the magnetic field lines will travel through the metal sheet.
(*c*) The superconducting sheet will shield the interior from magnetic fields.

14. Each of the devices mentioned has a different operating current and voltage, and each needs its own transformer with its own ratio of primary to secondary turns designed to convert normal household current and voltage into the required current and voltage. If the devices were designed to operate with the same current and voltage, they could all run on identical transformers.

15. You could hook the transformer up to a known ac voltage source. The ratio of the output voltage to the input voltage will give the ratio of turns on the two coils. If you pair up the leads incorrectly (one lead from each coil, rather than both leads from the same coil), there will be no output voltage. Alternatively, you could attach an ohmmeter to two of the leads. The resistance will be infinite if you have one lead from each pair, and nearly zero if you have both leads from the same pair.

16. Higher voltages are inherently more dangerous because of the increased risk of establishing large currents and large electromagnetic fields. The large potential differences between the wires and the ground could cause arcing and short circuits, leading to accidental electrocutions. In addition, higher-voltage power lines will have higher electromagnetic fields associated with them than lower-voltage power lines. Biological effects of exposure to high electromagnetic fields are not well understood, but there is evidence of increased health risks to people who live close to high voltage power lines.

17. When the transformer is connected to the 120-V dc source no back emf is generated, as would happen with an ac source. Therefore, the current in the transformer connected to the dc source will be very large. Because transformers generally are made with fine, low resistance wires, the large current could cause the wires to overheat, melt the insulation, and burn out.

18. A motor uses electric energy to create mechanical energy. When a large electric motor is running, the current in the motor's coil creates a back emf. When the motor is first turned on, the back emf is small, allowing the motor to draw maximum current. The back emf has a maximum value when the motor is running at full speed, reducing the amount of current required to run the motor. As the current flow in the motor's coil stabilizes, the motor will operate at its lower, normal current. The lights will dim briefly when the refrigerator motor starts due to the increased current load on the house circuit. Electric heaters operate by sending a large current through a large resistance, generating heat. When an electric heater is turned on, the current will increase quickly to its

maximum value (no coil, so no back emf) and will stay at its maximum value as long as the heater is on. Therefore, the lights will stay dim as long as the heater is on.

19. At the moment shown in Figure 29-15, the armature is rotating clockwise and so the current in length *b* of the wire loop on the armature is directed outward. (Use the right-hand rule: the field is north to south and the wire is moving with a component downward, therefore force on positive charge carriers is out.) This current is increasing, because as the wire moves down, the downward component of the velocity increases. As the current increases, the flux through the loop also increases, and therefore there is an induced emf to oppose this change. The induced emf opposes the current flowing in section *b* of the wire, and therefore creates a counter-torque.

20. Eddy currents exist in any conducting material, so eddy current brakes could work with wheels made of copper or aluminum.

21. The nonferrous materials are not magnetic but they are conducting. As they pass by the permanent magnets, eddy currents will be induced in them. The eddy currents provide a “braking” mechanism which will cause the metallic materials to slide more slowly down the incline than the nonmetallic materials. The nonmetallic materials will reach the bottom with larger speeds. The nonmetallic materials can therefore be separated from the metallic, nonferrous materials by placing bins at different distances from the bottom of the incline. The closest bin will catch the metallic materials, since their projectile velocities off the end of the incline will be small. The bin for the nonmetallic materials should be placed farther away to catch the higher-velocity projectiles.

22. The slots in the metal bar prevent the formation of large eddy currents, which would slow the bar’s fall through the region of magnetic field.

23. As the aluminum sheet is moved through the magnetic field, eddy currents are created in the sheet. The magnetic force on these induced currents opposes the motion. Thus it requires some force to pull the sheet out. (See Figure 29-21.)

24. As the bar magnet falls, it sets up eddy currents in the metal tube which will interact with the magnet and slow its fall. The magnet will reach terminal velocity (due to the interactions with the magnetic dipoles set up by the eddy currents, not air resistance) when the weight of the magnet is balanced by the upward force from the eddy currents.

25. As the bar moves in the magnetic field, induced eddy currents are created in the bar. The magnetic field exerts a force on these currents that opposes the motion of the bar. (See Figure 29-21.)

26. Although in principle you could use a loudspeaker in reverse as a microphone, it would probably not work in actual practice. The membrane of the microphone is very lightweight and sensitive to the sound waves produced by your voice. The cardboard cone of a loudspeaker is much stiffer and would significantly dampen the vibrations so that the frequency of the impinging sound waves would not be translated into an induced emf with the same frequency.

Solutions to Problems

1. The average induced emf is given by Eq. 29-2b.

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = -N\frac{\Delta\Phi_B}{\Delta t} = -2\frac{38\,\text{Wb} - (-58\,\text{Wb})}{0.42\,\text{s}} = \boxed{-460\,\text{V}}$$

2. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

3. As the coil is pushed into the field, the magnetic flux through the coil increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is $\boxed{\text{counterclockwise}}$.

4. The flux changes because the loop rotates. The angle between the field and the normal to the loop changes from 0° to 90°. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathcal{E}_{\text{avg}} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{AB\Delta\cos\theta}{\Delta t} = -\frac{\pi\left(0.110\,\text{m}\right)^2 1.5\,\text{T}\left(\cos 90^\circ - \cos 0^\circ\right)}{0.20\,\text{s}}$$

$$= -\frac{\pi\left(0.110\,\text{m}\right)^2 1.5\,\text{T}\left(0-1\right)}{0.20\,\text{s}} = \boxed{0.29\,\text{V}}$$

5. Use Eq. 29-2a to calculate the emf. Setting the flux equal to the magnetic field multiplied by the area of the loop, $A = \pi r^2$, and the emf equal to zero, we can solve for the rate of change in the coil radius.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B\pi r^2\right) = -\frac{dB}{dt}\pi r^2 - 2\pi B r\frac{dr}{dt} = 0$$

$$\frac{dr}{dt} = -\frac{dB}{dt}\frac{r}{2B} = -\left(-0.010\text{ T/s}\right)\frac{0.12\text{ m}}{2\left(0.500\text{ T}\right)} = 0.0012\text{ m/s} = \boxed{1.2\text{ mm/s}}$$

6. We choose up as the positive direction. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi\left(0.054\,\text{m}\right)^2\left(-0.25\,\text{T} - 0.68\,\text{T}\right)}{0.16\,\text{s}} = \boxed{5.3\times10^{-2}\,\text{V}}$$

$\boxed{7.}$ (*a*) When the plane of the loop is perpendicular to the field lines, the flux is given by the maximum of Eq. 29-1a.

$$\Phi_B = BA = B\pi r^2 = \left(0.50\,\text{T}\right)\pi\left(0.080\,\text{m}\right)^2 = \boxed{1.0\times10^{-2}\,\text{Wb}}$$

(*b*) The angle is $\theta = \boxed{55^\circ}$

(*c*) Use Eq. 29-1a.

$$\Phi_B = BA\cos\theta = B\pi r^2 = \left(0.50\,\text{T}\right)\pi\left(0.080\,\text{m}\right)^2\cos 55^\circ = \boxed{5.8\times10^{-3}\,\text{Wb}}$$

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

9. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is $\boxed{\text{counterclockwise}}$ as viewed from the right end of the solenoid.

10. (*a*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.040\,\text{m})^2(-0.45\,\text{T} - 0.52\,\text{T})}{0.18\,\text{s}} = \boxed{2.7\times10^{-2}\,\text{V}}$$

(*b*) The positive result for the induced emf means the induced field is away from the observer, so the induced current is $\boxed{\text{clockwise}}$.

11. (*a*) The magnetic flux through the loop is into the paper and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the paper, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\left|\mathscr{E}_{\text{avg}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B|\Delta A|}{\Delta t} = \frac{(0.75\,\text{T})\pi\left[(0.100\,\text{m})^2 - (0.030\,\text{m})^2\right]}{0.50\,\text{s}}$$

$$= 4.288\times10^{-2}\,\text{V} \approx \boxed{4.3\times10^{-2}\,\text{V}}$$

(*c*) We find the average induced current from Ohm's law.

$$I = \frac{\mathscr{E}}{R} = \frac{4.288\times10^{-2}\,\text{V}}{2.5\,\Omega} = \boxed{1.7\times10^{-2}\,\text{A}}$$

12. As the loop is pulled from the field, the flux through the loop decreases, causing an induced EMF whose magnitude is given by Eq. 29-3, $\mathscr{E} = B\ell v$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I = \mathscr{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F = I\ell B$.

$$F = I\ell B = \frac{\mathscr{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R} = \frac{(0.650\,\text{T})^2(0.350\,\text{m})^2(3.40\,\text{m/s})}{0.280\,\Omega} = \boxed{0.628\,\text{N}}$$

$\boxed{13.}$ (*a*) Use Eq. 29-2a to calculate the emf induced in the ring, where the flux is the magnetic field multiplied by the area of the ring. Then using Eq. 25-7, calculate the average power dissipated in the ring as it is moved away. The thermal energy is the average power times the time.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta BA}{\Delta t} = -\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}$$

$$Q = P\Delta t = \left(\frac{\mathscr{E}^2}{R}\right)\Delta t = \left(\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}\right)^2\left(\frac{\Delta t}{R}\right) = \frac{(\Delta B)^2\pi^2 d^4}{16R\Delta t}$$

$$= \frac{(0.80\,\text{T})^2\pi^2(0.015\,\text{m})^4}{16(55\times10^{-6}\,\Omega)(45\times10^{-3}\,\text{s})} = 8.075\times10^{-3}\,\text{J} \approx \boxed{8.1\,\text{mJ}}$$

(*b*) The temperature change is calculated from the thermal energy using Eq. 19-2.

$$\Delta T = \frac{Q}{mc} = \frac{8.075\times10^{-3}\,\text{J}}{(15\times10^{-3}\,\text{kg})(129\,\text{J/kg}\bullet{}^\circ\text{C})} = \boxed{4.2\times10^{-3}\,\text{C}^\circ}$$

14. The average emf induced in the short coil is given by the "difference" version of Eq. 29-2b. N is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the field created by the solenoid. The field in a solenoid is given by Eq. 28-4, $B = \mu_0 I N_{\text{solenoid}} / \ell_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathcal{E}| = \frac{N_{\text{short}} A_{\text{short}} \Delta B}{\Delta t} = \frac{N_{\text{short}} A_{\text{short}} \Delta\left(\dfrac{\mu_0 I N_{\text{solenoid}}}{\ell_{\text{solenoid}}}\right)}{\Delta t} = \frac{\mu_0 N_{\text{short}} N_{\text{solenoid}} A_{\text{short}}}{\ell_{\text{solenoid}}} \frac{\Delta I}{\Delta t}$$

$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(15)(420)\pi(0.0125\,\text{m})^2}{(0.25\,\text{m})} \frac{(5.0\,\text{A})}{(0.60\,\text{s})} = \boxed{1.3\times10^{-4}\,\text{V}}$$

15. (*a*) There is an emf induced in the coil since the flux through the coil changes. The current in the coil is the induced emf divided by the resistance of the coil. The resistance of the coil is found from Eq. 25-3.

$$|\mathcal{E}| = N A_{\text{coil}} \frac{dB}{dt} \qquad R = \frac{\rho \ell}{A_{\text{wire}}}$$

$$I = \frac{\mathcal{E}}{R} = \frac{N A_{\text{coil}} \dfrac{dB}{dt}}{\dfrac{\rho \ell}{A_{\text{wire}}}} = \frac{N A_{\text{coil}} A_{\text{wire}}}{\rho \ell} \frac{dB}{dt}$$

$$= \frac{28\left[\pi(0.110\,\text{m})^2\right]\left[\pi\left(1.3\times10^{-3}\,\text{m}\right)^2\right]\left(8.65\times10^{-3}\,\text{T/s}\right)}{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)28(2\pi)(0.110\,\text{m})} = 0.1504\,\text{A} \approx \boxed{0.15\,\text{A}}$$

(*b*) The rate at which thermal energy is produced in the wire is the power dissipated in the wire.

$$P = I^2 R = I^2 \frac{\rho \ell}{A_{\text{wire}}} = (0.1504\,\text{A})^2 \frac{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)28(2\pi)(0.11)}{\pi\left(1.3\times10^{-3}\,\text{m}\right)^2} = \boxed{1.4\times10^{-3}\,\text{W}}$$

16. The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, given by Eq. 28-1. Using Eq. 29-1b we integrate this field over the area of the rectangle to determine the flux through it. Differentiating the flux as in Eq. 29-2b gives the emf around the rectangle. Finally, by setting the maximum emf equal to 170 V we can solve for the necessary length of the rectangle.

$$B(t) = \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t) \ ;$$

$$\Phi_B(t) = \int B\,dA = \int_{5.0\,\text{m}}^{7.0\,\text{m}} \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t)\,\ell\,dr = \frac{\mu_0 I_0}{2\pi}\ell\cos(2\pi f t)\int_{5.0\,\text{m}}^{7.0\,\text{m}}\frac{dr}{r} = \frac{\mu_0 I_0}{2\pi}\ln(1.4)\,\ell\cos(2\pi f t)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{N\mu_0 I_0}{2\pi}\ln(1.4)\,\ell\left[\frac{d}{dt}\cos(2\pi f t)\right] = N\mu_0 I_0 f\,\ln(1.4)\,\ell\sin(2\pi f t) \ ;$$

$$\mathcal{E}_0 = N\mu_0 I_0 f\,\ln(1.4)\,\ell \ \rightarrow$$

$$\ell = \frac{\mathcal{E}_0}{N\mu_0 I_0 f\,\ln(1.4)} = \frac{170\,\text{V}}{10\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(55{,}000\,\text{A})(60\,\text{Hz})\ln(1.4)} = \boxed{12\,\text{m}}$$

This is unethical because the current in the rectangle creates a back emf in the initial wire. This results in a power loss to the electric company, just as if the wire had been physically connected to the line.

17. The charge that passes a given point is the current times the elapsed time, $Q = I\Delta t$. The current will be the emf divided by the resistance, $I = \dfrac{\mathcal{E}}{R}$. The resistance is given by Eq. 25-3, $R = \dfrac{\rho\ell}{A_{\text{wire}}}$, and the emf is given by the "difference" version of Eq. 29-2a. Combine these equations to find the charge during the operation.

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}|\Delta B|}{\Delta t} \;;\; R = \frac{\rho\ell}{A_{\text{wire}}} \;;\; I = \frac{\mathcal{E}}{R} = \frac{\dfrac{A_{\text{loop}}|\Delta B|}{\Delta t}}{\dfrac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2|\Delta B|}{\rho(2\pi)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2|\Delta B|}{2\rho}$$

$$= \frac{(0.091\,\text{m})\pi\left(1.175\times10^{-3}\,\text{m}\right)^2(0.750\,\text{T})}{2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{8.81\,\text{C}}$$

18. (*a*) Use Eq. 29-2b to calculate the emf.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = (-75)\frac{d}{dt}\left[\left(8.8t - 0.51t^3\right)\times10^{-2}\,\text{T}\bullet\text{m}^2\right] = \left(-6.6 + 1.1475t^2\right)\text{V}$$

$$\approx \boxed{\left(-6.6 + 1.1t^2\right)\text{V}}$$

(*b*) Evaluate at the specific times.

$$\mathcal{E}(t = 1.0\,\text{s}) = \left(-6.6 + 1.1475(1.0)^2\right)\text{V} = \boxed{-5.5\,\text{V}}$$

$$\mathcal{E}(t = 4.0\,\text{s}) = \left(-6.6 + 1.1475(4.0)^2\right)\text{V} = \boxed{12\,\text{V}}$$

$\boxed{19.}$ The energy dissipated in the process is the power dissipated by the resistor, times the elapsed time that the current flows. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} \;;\; P = \frac{\mathcal{E}^2}{R} \;;$$

$$E = P\Delta t = \frac{\mathcal{E}^2}{R}\Delta t = \left(\frac{\Delta\Phi_B}{\Delta t}\right)^2\frac{\Delta t}{R} = \frac{A^2(\Delta B)^2}{R\Delta t} = \frac{\left[\pi(0.125\text{m})^2\right]^2(0.40\,\text{T})^2}{(150\,\Omega)(0.12\,\text{s})} = \boxed{2.1\times10^{-5}\,\text{J}}$$

20. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -(0.28\,\text{T})\left(-3.50\times10^{-2}\,\text{m}^2/\text{s}\right) = \boxed{9.8\,\text{mV}}$$

Since the area changes at a constant rate, and the area has not shrunk to 0 at $t = 2.00$ s, the emf is the same for both times.

21. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area of a circle. We calculate the initial radius from the initial area. To calculate the radius after one second we add the change in radius to the initial radius.

$$\mathscr{E}(t) = \frac{d\Phi_B}{dt} = B\frac{d\left(\pi r^2\right)}{dt} = 2\pi B r\frac{dr}{dt} \qquad A_0 = \pi r_0^2 \to r_0 = \sqrt{\frac{A_0}{\pi}}$$

$$\mathscr{E}(0) = 2\pi\left(0.28\,\text{T}\right)\sqrt{\frac{0.285\,\text{m}^2}{\pi}}\left(0.043\,\text{m/s}\right) = \boxed{23\,\text{mV}}$$

$$\mathscr{E}(1.00\,\text{s}) = 2\pi\left(0.28\,\text{T}\right)\left[\sqrt{\frac{0.285\,\text{m}^2}{\pi}} + \left(0.043\,\text{m/s}\right)\left(1.00\,\text{s}\right)\right]\left(0.043\,\text{m/s}\right) = \boxed{26\,\text{mV}}$$

22. The magnetic field inside the solenoid is given by Eq. 28-4, $B = \mu_0 nI$. Use Eq. 29-2a to calculate the induced emf. The flux causing the emf is the flux through the small loop.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -A_1\frac{dB_{\text{solenoid}}}{dt} = -A_1\mu_0 n\frac{dI}{dt} = -A_1\mu_0 n\left(-\omega I_0 \sin\omega t\right) = \boxed{A_1\mu_0 n\omega I_0 \sin\omega t}$$

23. (*a*) If the magnetic field is parallel to the plane of the loop, no magnetic flux passes through the loop at any time. Therefore, the emf and the current in the loop are $\boxed{\text{zero}}$.

(*b*) When the magnetic field is perpendicular to the plane of the loop, we differentiate Eq. 29-1a with respect to time to obtain the emf in the loop. Then we divide the emf by the resistance to calculate the current in the loop.

$$I = \frac{\mathscr{E}}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right) = -\frac{1}{R}\frac{d}{dt}\left[\left(\alpha t\right)\left(A_0 + \beta t\right)\right] = -\frac{\alpha}{R}\left[A_0 + 2\beta t\right]$$

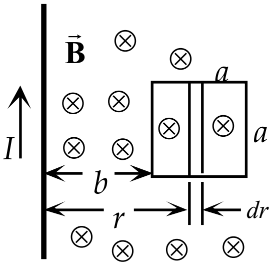
$$= -\frac{\left(0.60\text{ T/s}\right)\left[\left(0.50\text{ m}^2\right) + 2\left(0.70\text{ m}^2\text{/s}\right)\left(2.0\text{ s}\right)\right]}{2.0\ \Omega} = -0.99\text{ A}$$

Since the magnetic field is pointing down into the page, the downward flux is increasing. The current then flows in a direction to create an upward flux. The resulting current is then $\boxed{\text{0.99 A in the counterclockwise direction}}$.

24. The magnetic field across the primary coil is constant and is that of a solenoid (Eq. 28-4). We multiply this magnetic field by the area of the secondary coil to calculate the flux through the secondary coil. Then using Eq. 29-2b we differentiate the flux to calculate the induced emf.

$$\Phi_B = BA = \mu_0 n_p I_0 \sin\left(2\pi ft\right)\left(\pi d^2/4\right)$$

$$\mathscr{E}_2 = N\frac{d\Phi_B}{dt} = N\mu_0 n_p I_0\left(\pi d^2/4\right)\left[\frac{d}{dt}\sin\left(2\pi ft\right)\right] = \boxed{-\tfrac{1}{2}\pi^2 d^2 fN\mu_0 n_p I_0 \cos\left(2\pi ft\right)}$$



25. (*a*) The magnetic field a distance *r* from the wire is perpendicular to the wire and given by Eq. 28-1. Integrating this magnetic field over the area of the loop gives the flux through the loop.

$$\Phi_B = \int B\,dA = \int_b^{b+a}\frac{\mu_0 I}{2\pi r}a\,dr = \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(1 + \frac{a}{b}\right)}$$

(*b*) Since the loop is being pulled away, $v = \frac{db}{dt}$. Differentiate the magnetic flux with respect to time to calculate the emf in the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 Ia}{2\pi}\frac{d}{db}\left[\ln\left(1+\frac{a}{b}\right)\right]\frac{db}{dt} = \boxed{\frac{\mu_0 Ia^2 v}{2\pi b(b+a)}}$$

Note that this is the emf at the instant the loop is a distance *b* from the wire. The value of *b* is changing with time.

(*c*) Since the magnetic field at the loop points into the page, and the flux is decreasing, the induced current will create a downward magnetic field inside the loop. The current in the loop then flows $\boxed{\text{clockwise}}$.

(*d*) The power dissipated in the loop as it is pulled away is related to the emf and resistance by Eq. 25-7b. This power is provided by the force pulling the loop away. We calculate this force from the power using Eq. 8-21. As in part (*b*), the value of *b* is changing with time.

$$F = \frac{P}{v} = \frac{\mathcal{E}^2}{Rv} = \boxed{\frac{\mu_0^2 I^2 a^4 v}{4\pi^2 R b^2 (b+a)^2}}$$

26. From Problem 25, the flux through the loop is given by $\Phi_B = \frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)$. The emf is found from Eq. 29-2a.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{b}\right)\frac{dI}{dt}$$

$$= -\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(0.120\,\text{m})}{2\pi}\ln\left(1+\frac{12.0}{15.0}\right)(15.0\,\text{A})(2500\,\text{rad/s})\cos(2500t)$$

$$= \boxed{\left(5.3\times10^{-4}\,\text{V}\right)\cos(2500t)}$$

27. The velocity is found from Eq. 29-3.

$$\mathcal{E} = B\ell v \rightarrow v = \frac{\mathcal{E}}{B\ell} = \frac{0.12\,\text{V}}{(0.90\,\text{T})(0.132\,\text{m})} = \boxed{1.0\,\text{m/s}}$$

28. Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.800\,\text{T})(0.120\,\text{m})(0.150\,\text{m/s}) = \boxed{1.44\times10^{-2}\,\text{V}}$$

29. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.35\,\text{T})(0.250\,\text{m})(1.3\,\text{m/s}) = 0.1138\,\text{V} \approx \boxed{0.11\,\text{V}}$$

(*b*) Find the induced current from Ohm's law, using the **total** resistance.

$$I = \frac{\mathcal{E}}{R} = \frac{0.1138\,\text{V}}{25.0\,\Omega + 2.5\,\Omega} = 4.138\times10^{-3}\,\text{A} \approx \boxed{4.1\,\text{mA}}$$

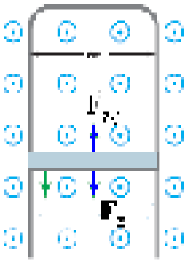
(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by Eq. 27-1.

$$F = I\ell B = \left(4.138\times10^{-3}\,\text{A}\right)(0.250\,\text{m})(0.35\,\text{T}) = 3.621\times10^{-4}\,\text{N} \approx \boxed{0.36\,\text{mN}}$$

30. The emf is given by Eq. 29-3 as $\mathcal{E} = B\ell v$. The resistance of the conductor is given by Eq. 25-3. The length in Eq. 25-3 is the length of resistive material. Since the movable rod starts at the bottom of the U at time $t = 0$, in a time t it will have moved a distance vt.

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{\frac{\rho L}{A}} = \frac{B\ell v}{\frac{\rho(2vt+\ell)}{A}} = \boxed{\frac{B\ell v A}{\rho(2vt+\ell)}}$$

31. The rod will descend at its terminal velocity when the magnitudes of the magnetic force (found in Example 29-8) and the gravitational force are equal. We set these two forces equal and solve for the terminal velocity.



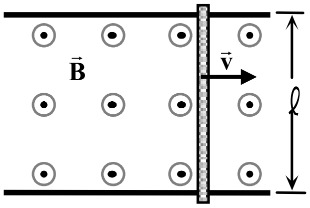
$$\frac{B^2\ell^2 v_t}{R} = mg \quad \rightarrow$$

$$v_t = \frac{mgR}{B^2\ell^2} = \frac{(3.6\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(0.0013\ \Omega)}{(0.060\ \text{T})^2(0.18\ \text{m})^2} = \boxed{0.39\ \text{m/s}}$$

32. Since the antenna is vertical, the maximum emf will occur when the car is traveling perpendicular to the horizontal component of the Earth's magnetic field. This occurs when the car is traveling in the east or west direction. We calculate the magnitude of the emf using Eq. 29-3, where B is the horizontal component of the Earth's magnetic field.

$$\mathcal{E} = B_x\ell v = (5.0\times10^{-5}\,\text{T}\cos45^\circ)(0.750\,\text{m})(30.0\,\text{m/s}) = 8.0\times10^{-4}\,\text{V} = \boxed{0.80\,\text{mV}}$$

33. (*a*) As the rod moves through the magnetic field an emf will be built up across the rod, but no current can flow. Without the current, there is no force to oppose the motion of the rod, so yes, the rod travels at constant speed.



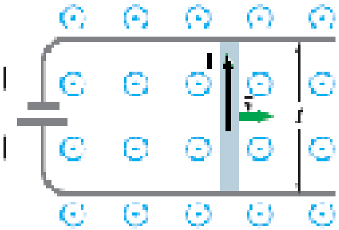
(*b*) We set the force on the moving rod, obtained in Example 29-8, equal to the mass times the acceleration of the rod. We then write the acceleration as the derivative of the velocity, and by separation of variables we integrate the velocity to obtain an equation for the velocity as a function of time.

$$F = ma = m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v \rightarrow \frac{dv}{v} = -\frac{B^2\ell^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B^2\ell^2}{mR}\int_0^t dt' \rightarrow \ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = v_0 e^{-\frac{B^2\ell^2}{mR}t}}$$

The magnetic force is proportional to the velocity of the rod and opposes the motion. This results in an exponentially decreasing velocity.

34. (*a*) For a constant current, of polarity shown in the figure, the magnetic force will be constant, given by Eq. 27-2. Using Newton's second law we can integrate the acceleration to calculate the velocity as a function of time.



$$F = m\frac{dv}{dt} = I\ell B \rightarrow \int_0^v dv = \frac{I\ell B}{m}\int_0^t dt \rightarrow \boxed{v(t) = \frac{I\ell B}{m}t}$$

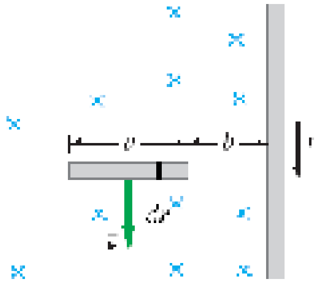
(*b*) For a constant emf, the current will vary with the speed of the rod, as motional emf opposes the motion of the rod. We again use Eq. 27-2 for the force on the rod, with the current given by Ohm's law, and the induced motional emf given by Eq 29-3. The current produced by the induced emf opposes the current produced by the battery.

$$F = m\frac{dv}{dt} = I\ell B = \left(\frac{\mathscr{E}_0 - B\ell v}{R}\right)\ell B \rightarrow \frac{dv}{\mathscr{E}_0 - B\ell v} = \frac{\ell B}{mR}dt \rightarrow \frac{dv}{v - \mathscr{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}dt \rightarrow$$

$$\int_0^v \frac{dv}{v - \mathscr{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}\int_0^t dt \rightarrow \ln\left(\frac{v - \mathscr{E}_0/B\ell}{-\mathscr{E}_0/B\ell}\right) = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = \frac{\mathscr{E}_0}{B\ell}\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right)}$$

(*c*) With constant current, the acceleration is constant and so the velocity does not reach a terminal velocity. However, with constant emf, the increasing motional emf decreases the applied force. This results in a limiting, or terminal velocity of $\boxed{v_t = \mathscr{E}_0/B\ell}$.

35. (*a*) The magnetic field is perpendicular to the rod, with the magnetic field decreasing with distance from the rod, as in Eq. 28-1. The emf, $d\mathscr{E}$, across a short segment, *dr*, of the rod is given by the differential version of Eq. 29-3. Integrating this emf across the length of the wire gives the total emf.



$$d\mathscr{E} = Bvdr \rightarrow$$

$$\mathscr{E} = \int d\mathscr{E} = \int_b^{b+a} \frac{\mu_0 I}{2\pi r}vdr = \boxed{\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b+a}{b}\right)}$$

This emf points $\boxed{\text{toward the wire}}$, as positive charges are attracted toward the current.

(*b*) The only change is the direction of the current, so the magnitude of the emf remains the same, but points $\boxed{\text{away from the wire}}$, since positive charges are repelled from the current.

36. From Eq. 29-4, the induced voltage is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathscr{E}_1}{\omega_1} = \frac{\mathscr{E}_2}{\omega_2} \rightarrow \mathscr{E}_2 = \mathscr{E}_1\frac{\omega_2}{\omega_1} = (12.4\,\text{V})\frac{1550\,\text{rpm}}{875\,\text{rpm}} = \boxed{22.0\,\text{V}}$$

$\boxed{37.}$ We find the number of turns from Eq. 29-4. The factor multiplying the sine term is the peak output voltage.

$$\mathscr{E}_{\text{peak}} = NB\omega A \rightarrow N = \frac{\mathscr{E}_{\text{peak}}}{B\omega A} = \frac{24.0\,\text{V}}{(0.420\,\text{T})(2\pi\,\text{rad/rev})(60\,\text{rev/s})(0.0515\,\text{m})^2} = \boxed{57.2\text{ loops}}$$

38. From Eq. 29-4, the peak voltage is $\mathscr{E}_{\text{peak}} = NB\omega A$. Solve this for the rotation speed.

$$\mathscr{E}_{\text{peak}} = NB\omega A \rightarrow \omega = \frac{\mathscr{E}_{\text{peak}}}{NBA} = \frac{120\,\text{V}}{480(0.550\,\text{T})(0.220\,\text{m})^2} = 9.39\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{9.39\,\text{rad/s}}{2\pi\,\text{rad/rev}} = \boxed{1.49\,\text{rev/s}}$$

39. From Eq. 29-4, the peak voltage is $\mathscr{E}_{\text{peak}} = NAB\omega$. The rms voltage is the peak voltage divided by $\sqrt{2}$, and so $V_{\text{rms}} = \mathscr{E}_{\text{peak}}/\sqrt{2} = NAB\omega/\sqrt{2}$.

40. Rms voltage is found from the peak induced emf. Peak induced emf is calculated from Eq. 29-4.

$$\mathscr{E}_{\text{peak}} = NB\omega A \rightarrow$$

$$V_{\text{rms}} = \frac{\mathcal{E}_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(250)(0.45\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.050\,\text{m})^2}{\sqrt{2}}$$

$$= 471.1\,\text{V} \approx \boxed{470\,\text{V}}$$

To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s.

41. From Eq. 29-4, the induced voltage (back emf) is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \;\rightarrow\; \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (72\,\text{V})\frac{2500\,\text{rpm}}{1200\,\text{rpm}} = \boxed{150\,\text{V}}$$

42. When the motor is running at full speed, the back emf opposes the applied emf, to give the net across the motor.

$$\mathcal{E}_{\text{applied}} - \mathcal{E}_{\text{back}} = IR \;\rightarrow\; \mathcal{E}_{\text{back}} = \mathcal{E}_{\text{applied}} - IR = 120\,\text{V} - (7.20\,\text{A})(3.05\,\Omega) = \boxed{98\,\text{V}}$$

$\boxed{43.}$ The back emf is proportional to the rotation speed (Eq. 29-4). Thus if the motor is running at half speed, the back emf is half the original value, or 54 V. Find the new current from writing a loop equation for the motor circuit, from Figure 29-20.

$$\mathcal{E} - \mathcal{E}_{\text{back}} - IR = 0 \;\rightarrow\; I = \frac{\mathcal{E} - \mathcal{E}_{\text{back}}}{R} = \frac{120\,\text{V} - 54\,\text{V}}{5.0\,\Omega} = \boxed{13\,\text{A}}$$

44. The magnitude of the back emf is proportional to both the rotation speed and the magnetic field, from Eq. 29-4. Thus $\frac{\mathcal{E}}{B\omega}$ is constant.

$$\frac{\mathcal{E}_1}{B_1\omega_1} = \frac{\mathcal{E}_2}{B_2\omega_2} \;\rightarrow\; B_2 = \frac{\mathcal{E}_2}{\omega_2}\frac{B_1\omega_1}{\mathcal{E}_1} = \frac{(75\,\text{V})}{(2300\,\text{rpm})}\frac{B_1(1100\,\text{rpm})}{(85\,\text{V})} = 0.42B_1$$

So $\boxed{\text{reduce the magnetic field to 42\% of its original value}}$.

45. (*a*) The generator voltage rating is the generator emf less the back emf. The ratio of the generator voltage rating to the generator emf is equal to the ratio of the effective resistance to the armature resistance. We solve this ratio for the generator emf, which is the same as the "no load" voltage.

$$V_{\text{nl}} = \mathcal{E} = V_{\text{load}}\frac{R_{\text{load}}}{R_{\text{nl}}} = V_{\text{load}}\frac{V_{\text{load}}/I_{\text{load}}}{R_{\text{nl}}} = 250\,\text{V}\left(\frac{250\,\text{V}/64\,\text{A}}{0.40\,\Omega}\right) = 2441\,\text{V} \approx \boxed{2.4\,\text{kV}}$$

(*b*) The generator voltage is proportional to the rotation frequency. From this proportionality we solve for the new generator voltage.

$$\frac{V_2}{V_1} = \frac{\omega_2}{\omega_1} \rightarrow V_2 = V_1\frac{\omega_2}{\omega_1} = (250\,\text{V})\frac{750\,\text{rpm}}{1000\,\text{rpm}} = \boxed{190\,\text{V}}$$

46. Because $N_{\text{S}} < N_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer. Use Eq. 29-5 to find the voltage ratio, and Eq. 29-6 to find the current ratio.

$$\frac{V_{\text{S}}}{V_{\text{P}}} = \frac{N_{\text{S}}}{N_{\text{P}}} = \frac{85\,\text{turns}}{620\,\text{turns}} = \boxed{0.14} \qquad \frac{I_{\text{S}}}{I_{\text{P}}} = \frac{N_{\text{P}}}{N_{\text{S}}} = \frac{620\,\text{turns}}{85\,\text{turns}} = \boxed{7.3}$$

47. We find the ratio of the number of turns from Eq. 21-6.

$$\frac{N_{\text{S}}}{N_{\text{P}}}=\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{12000\,\text{V}}{240\,\text{V}}=\boxed{50}$$

If the transformer is connected backward, the role of the turns will be reversed:

$$\frac{N_{\text{S}}}{N_{\text{P}}}=\frac{V_{\text{S}}}{V_{\text{P}}}\;\rightarrow\;\frac{1}{50}=\frac{V_{\text{S}}}{240\,\text{V}}\;\rightarrow\;V_{\text{S}}=\frac{1}{50}\left(240\,\text{V}\right)=\boxed{4.8\,\text{V}}$$

48. (*a*) Use Eqs. 29-5 and 29-6 to relate the voltage and current ratios.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}}\;;\;\frac{I_{\text{S}}}{I_{\text{P}}}=\frac{N_{\text{P}}}{N_{\text{S}}}\;\rightarrow\;\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{I_{\text{P}}}{I_{\text{S}}}\;\rightarrow\;V_{\text{S}}=V_{\text{P}}\frac{I_{\text{P}}}{I_{\text{S}}}=\left(120\,\text{V}\right)\frac{0.35\,\text{A}}{7.5\,\text{A}}=\boxed{5.6\,\text{V}}$$

(*b*) Because $V_{\text{S}}<V_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer.

$\boxed{49.}$ (*a*) We assume 100% efficiency, and find the input voltage from $P=IV$.

$$P=I_{\text{P}}V_{\text{P}}\;\rightarrow\;V_{\text{P}}=\frac{P}{I_{\text{P}}}=\frac{75\,\text{W}}{22\,\text{A}}=3.409\,\text{V}$$

Since $V_{\text{P}}<V_{\text{S}}$, this is a $\boxed{\text{step-up}}$ transformer.

(*b*) $\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{12\,\text{V}}{3.409\,\text{V}}=\boxed{3.5}$

50. (*a*) The current in the transmission lines can be found from Eq. 25-10a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}}=V_{\text{rms}}I_{\text{rms}}\;\rightarrow\;I_{\text{rms}}=\frac{P_{\text{town}}}{V_{\text{rms}}}=\frac{65\times10^{6}\,\text{W}}{45\times10^{3}\,\text{V}}=1444\,\text{A}$$

$$\mathcal{E}-IR-V_{\text{output}}=0\;\rightarrow$$

$$\mathcal{E}=IR+V_{\text{output}}=\frac{P_{\text{town}}}{V_{\text{rms}}}R+V_{\text{rms}}=\frac{65\times10^{6}\,\text{W}}{45\times10^{3}\,\text{V}}\left(3.0\,\Omega\right)+45\times10^{3}\,\text{V}=49333\,\text{V}=\boxed{49\,\text{kV}\left(\text{rms}\right)}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}}=I_{\text{rms}}^{2}R$.

$$\text{Fraction wasted}=\frac{P_{\text{loss}}}{P_{\text{total}}}=\frac{P_{\text{loss}}}{P_{\text{town}}+P_{\text{loss}}}=\frac{I_{\text{rms}}^{2}R}{P_{\text{town}}+I_{\text{rms}}^{2}R}=\frac{\left(1444\text{A}\right)^{2}\left(3.0\,\Omega\right)}{65\times10^{6}\,\text{W}+\left(1444\,\text{A}\right)^{2}\left(3.0\,\Omega\right)}$$

$$=\boxed{0.088}=8.8\%$$

51. (*a*) If the resistor *R* is connected between the terminals, then it has a voltage V_0 across it and current I_0 passing through it. Then by Ohm's law the equivalent resistance is equal to the resistance of the resistor.

$$R_{\text{eq}}=\frac{V_0}{I_0}=\boxed{R}$$

(*b*) We use Eqs. 29-5 and 29-6 to write the voltage drop and current through the resistor in terms of the source voltage and current to calculate the effective resistance.

$$R=\frac{V_{\text{s}}}{I_{\text{s}}}=\frac{\frac{N_{\text{S}}}{N_{\text{P}}}V_0}{\frac{N_{\text{P}}}{N_{\text{S}}}I_0}\rightarrow R_{\text{eq}}=\frac{V_0}{I_0}=\boxed{\left(\frac{N_{\text{P}}}{N_{\text{S}}}\right)^{2}R}$$

52. We set the power loss equal to 2% of the total power. Then using Eq. 25-7a we write the power loss in terms of the current (equal to the power divided by the voltage drop) and the resistance. Then, using Eq. 25-3, we calculate the cross-sectional area of each wire and the minimum wire diameter. We assume there are two lines to have a complete circuit.

$$P_{\text{loss}} = 0.020P = I^2R = \left(\frac{P}{V}\right)^2\left(\frac{\rho\ell}{A}\right) \rightarrow A = \frac{P\rho\ell}{0.020V^2} = \frac{\pi d^2}{4} \rightarrow$$

$$d = \sqrt{\frac{4}{0.020\pi}\frac{P\rho\ell}{V^2}} = \sqrt{\frac{4\left(225\times10^6\,\text{W}\right)\left(2.65\times10^{-8}\,\Omega\bullet\text{m}\right)2\left(185\times10^3\ \text{m}\right)}{0.020\pi\left(660\times10^3\,\text{V}\right)^2}} = 0.01796\,\text{m} \approx \boxed{1.8\,\text{cm}}$$

The transmission lines must have a diameter greater than or equal to 1.8 cm.

53. Without the transformers, we find the delivered current, which is the current in the transmission lines, from the delivered power, and the power lost in the transmission lines.

$$P_{\text{out}} = V_{\text{out}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{P_{\text{out}}}{V_{\text{out}}} = \frac{85000\,\text{W}}{120\,\text{V}} = 708.33\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(708.33\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 100346\,\text{W}$$

Thus there must be $85000\,\text{W} + 100346\,\text{W} = 185346\text{W} \approx 185\,\text{kW}$ of power generated at the start of the process.

With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer must still be 708.33 A. Using the step-down transformer efficiency, we calculate the current in the transmission lines, and the loss in the transmission lines.

$$P_{\text{out}} = 0.99P_{\substack{\text{line}\\\text{end}}} \rightarrow V_{\text{out}}I_{\text{out}} = 0.99V_{\text{line}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{V_{\text{out}}I_{\text{out}}}{0.99V_{\text{line}}} = \frac{\left(120\,\text{V}\right)\left(708.33\,\text{A}\right)}{\left(0.99\right)\left(1200\,\text{V}\right)} = 71.548\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(71.548\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 1024\,\text{W}$$

The power to be delivered is 85000 W. The power that must be delivered to the step-down transformer is $\frac{85000\,\text{W}}{0.99} = 85859\,\text{W}$. The power that must be present at the start of the transmission must be $85859\,\text{W} + 1024\,\text{W} = 86883\,\text{W}$ to compensate for the transmission line loss. The power that must enter the transmission lines from the 99% efficient step-up transformer is $\frac{86883\,\text{W}}{0.99} = 87761 \approx 88\,\text{kW}$. So the power saved is $185346\,\text{W} - 87761\,\text{W} = 97585\,\text{W} \approx \boxed{98\,\text{kW}}$.

54. We choose a circular path centered at the origin with radius 10 cm. By symmetry the electric field is uniform along this path and is parallel to the path. We then use Eq. 29-8 to calculate the electric field at each point on this path. From the electric field we calculate the force on the charged particle.

$$\oint\vec{E}\bullet d\vec{\ell} = E\left(2\pi r\right) = -\frac{d\Phi_B}{dt} = -\left(\pi r^2\right)\frac{dB}{dt}$$

$$F = QE = -Q\frac{r}{2}\frac{dB}{dt} = -\left(1.0\times10^{-6}\ \text{C}\right)\frac{0.10\ \text{m}}{2}\left(-0.10\ \text{T/s}\right) = \boxed{5.0\ \text{nN}}$$

Since the magnetic field points into the page and is decreasing, Lenz's law tells us that an induced circular current centered at the origin would flow in the clockwise direction. Therefore, the force on a positive charge along the positive x-axis would be down, or $\boxed{\text{in the } -\hat{\mathbf{j}} \text{ direction.}}$

55. (*a*) The increasing downward magnetic field creates a circular electric field along the electron path. This field applies an electric force to the electron causing it to accelerate.

(*b*) For the electrons to move in a circle, the magnetic force must provide a centripetal acceleration. With the magnetic field pointing downward, the right-hand-rule requires the electrons travel in a clockwise direction for the force to point inward.

(*c*) For the electrons to accelerate, the electric field must point in the counterclockwise direction. A current in this field would create an upward magnetic flux. So by Lenz's law, the downward magnetic field must be increasing.

(*d*) For the electrons to move in a circle and accelerate, the field must be pointing downward and increasing in magnitude. For a sinusoidal wave, the field is downward half of the time and upward the other half. For the half that it is downward its magnitude is decreasing half of the time and increasing the other half. Therefore, the magnetic field is pointing downward and increasing for only one fourth of every cycle.

56. In Example 29-14 we found the electric field along the electron's path from Faraday's law. Multiplying this field by the electron charge gives the force on the electron, and from the force, we calculate the change in tangential velocity.

$$\frac{dv}{dt}=\frac{F}{m}=E\frac{q}{m}=\frac{q}{m}\frac{r}{2}\frac{dB_{\text{avg}}}{dt}$$

We set the centripetal force on the electron equal to the magnetic force (using Eq. 27-5b) and solve for the velocity. Differentiating the velocity with respect to time (keeping the radius constant) yields a relation for the acceleration in terms of the changing magnetic field.

$$qvB=m\frac{v^2}{r}\quad\rightarrow\quad v=\frac{qBr}{m}\quad\rightarrow\quad\frac{dv}{dt}=\frac{q}{m}r\frac{dB_0}{dt}$$

Equating these two equations for the electron acceleration, we see that the change in magnetic field at the electron must equal $\frac{1}{2}$ of the average change in magnetic field. This relation is satisfied if at all times $B_0=\frac{1}{2}B_{\text{avg}}$.

57. (*a*) The electric field is the change in potential across the rod (obtained from Ohm's law) divided by the length of the rod.

$$E=\frac{\Delta V}{\ell}=\boxed{\frac{IR}{\ell}}$$

(*b*) Again the electric field is the change in potential across the rod divided by the length of the rod. The electric potential is the supplied potential less the motional emf found using Eq. 29-3 and the results of Problem 34(b).

$$E=\frac{\Delta V}{\ell}=\frac{\mathscr{E}_0-B\ell v}{\ell}=\frac{\mathscr{E}_0-B\ell\,\mathscr{E}_0/B\ell\left(1-e^{-\frac{B^2\ell^2}{mR}t}\right)}{\ell}=\boxed{\frac{\mathscr{E}_0}{\ell}e^{-\frac{B^2\ell^2}{mR}t}}$$

58. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field that is directed out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be clockwise.

(*b*) After a long time, the current in the left-hand loop is constant, so there will be no induced current in the right-hand coil.

(*c*) If the second loop is pulled to the right, the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{counterclockwise}}$.

59. The electrical energy is dissipated because there is current flowing in a resistor. The power dissipation by a resistor is given by $P = I^2R$, and so the energy dissipated is $E = P\Delta t = I^2R\Delta t$. The current is created by the induced emf caused by the changing B-field. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} \qquad I = \frac{\mathscr{E}}{R} = -\frac{A\Delta B}{R\Delta t}$$

$$E = P\Delta t = I^2R\Delta t = \frac{A^2(\Delta B)^2}{R^2(\Delta t)^2}R\Delta t = \frac{A^2(\Delta B)^2}{R(\Delta t)} = \frac{\left[(0.270\,\text{m})^2\right]^2\left[(0-0.755\,\text{T})\right]^2}{(7.50\,\Omega)(0.0400\,\text{s})}$$

$$= \boxed{1.01\times10^{-2}\,\text{J}}$$

60. Because there are perfect transformers, the power loss is due to resistive heating in the transmission lines. Since the town requires 65 MW, the power at the generating plant must be $\frac{65\,\text{MW}}{0.985} = 65.99$ MW. Thus the power lost in the transmission is 0.99 MW. This can be used to determine the current in the transmission lines.

$$P = I^2R \;\rightarrow\; I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.99\times10^6\,\text{W}}{2(85\,\text{km})0.10\,\Omega/\text{km}}} = 241.3\,\text{A}$$

To produce 65.99 MW of power at 241.3 A requires the following voltage.

$$V = \frac{P}{I} = \frac{65.99\times10^6\,\text{W}}{241.3\,\text{A}} = 2.73\times10^5\,\text{V} \approx \boxed{270\,\text{kV}}$$

$\boxed{61.}$ The charge on the capacitor can be written in terms of the voltage across the battery and the capacitance using Eq. 24-1. When fully charged the voltage across the capacitor will equal the emf of the loop, which we calculate using Eq. 29-2b.

$$Q = CV = C\frac{d\Phi_B}{dt} = CA\frac{dB}{dt} = \left(5.0\times10^{-12}\text{ F}\right)\left(12\text{ m}^2\right)\left(10\text{ T/s}\right) = \boxed{0.60\text{ nC}}$$

62. (*a*) From the efficiency of the transformer, we have $P_\text{S} = 0.85P_\text{P}$. Use this to calculate the current in the primary.

$$P_\text{S} = 0.85P_\text{P} = 0.85I_\text{P}V_\text{P} \;\rightarrow\; I_\text{P} = \frac{P_\text{S}}{0.85V_\text{P}} = \frac{75\,\text{W}}{0.85(110\,\text{V})} = 0.8021\,\text{A} \approx \boxed{0.80\,\text{A}}$$

(*b*) The voltage in both the primary and secondary is proportional to the number of turns in the respective coil. The secondary voltage is calculated from the secondary power and resistance since $P = V^2/R$.

$$\frac{N_\text{P}}{N_\text{S}} = \frac{V_\text{P}}{V_\text{S}} = \frac{V_\text{P}}{\sqrt{P_\text{S}R_\text{S}}} = \frac{110\,\text{V}}{\sqrt{(75\,\text{W})(2.4\,\Omega)}} = \boxed{8.2}$$

63. (*a*) The voltage drop across the lines is due to the resistance.

$$V_{\text{out}} = V_{\text{in}} - IR = 42000\,\text{V} - (740\,\text{A})(2)(0.80\,\Omega) = 40816\,\text{V} \approx \boxed{41\,\text{kV}}$$

(*b*) The power input is given by $P_{\text{in}} = IV_{\text{in}}$.

$$P_{\text{in}} = IV_{\text{in}} = (740\,\text{A})(42000\,\text{V}) = 3.108\times10^7\,\text{W} \approx \boxed{3.1\times10^7\,\text{W}}$$

(*c*) The power loss in the lines is due to the current in the resistive wires.

$$P_{\text{loss}} = I^2R = (740\,\text{A})^2(1.60\,\Omega) = 8.76\times10^5\,\text{W} \approx \boxed{8.8\times10^5\,\text{W}}$$

(*d*) The power output is given by $P_{\text{out}} = IV_{\text{out}}$.

$$P_{\text{out}} = IV_{\text{out}} = (740\,\text{A})(40816\,\text{V}) = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}.$$

This could also be found by subtracting the power lost from the input power.

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 3.108\times10^7\,\text{W} - 8.76\times10^5\,\text{W} = 3.020\times10^7\,\text{W} \approx \boxed{3.0\times10^7\,\text{W}}$$

64. We find the current in the transmission lines from the power transmitted to the user, and then find the power loss in the lines.

$$P_{\text{T}} = I_{\text{L}}V \rightarrow I_{\text{L}} = \frac{P_{\text{T}}}{V} \qquad P_{\text{L}} = I_{\text{L}}^2R_{\text{L}} = \left(\frac{P_{\text{T}}}{V}\right)^2 R_{\text{L}} = \boxed{\frac{P_{\text{T}}^2R_{\text{L}}}{V^2}}$$

65. (*a*) Because $V_{\text{S}} < V_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer.

(*b*) Assuming 100% efficiency, the power in both the primary and secondary is 35 W. Find the current in the secondary from the relationship $P = IV$.

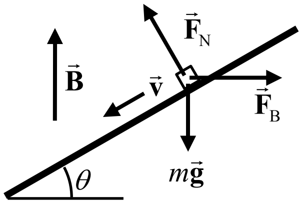
$$P_{\text{S}} = I_{\text{S}}V_{\text{S}} \rightarrow I_{\text{S}} = \frac{P_{\text{S}}}{V_{\text{S}}} = \frac{35\,\text{W}}{12\,\text{V}} = \boxed{2.9\,\text{A}}$$

(*c*) $P_{\text{P}} = I_{\text{P}}V_{\text{P}} \rightarrow I_{\text{P}} = \frac{P_{\text{P}}}{V_{\text{P}}} = \frac{35\,\text{W}}{120\,\text{V}} = \boxed{0.29\,\text{A}}$

(*d*) Find the resistance of the bulb from Ohm's law. The bulb is in the secondary circuit.

$$V_{\text{S}} = I_{\text{S}}R \rightarrow R = \frac{V_{\text{S}}}{I_{\text{S}}} = \frac{12\,\text{V}}{2.9\,\text{A}} = \boxed{4.1\,\Omega}$$

66. A side view of the rail and bar is shown in the figure. From Section 21-3, the emf in the bar is produced by the components of the magnetic field, the length of the bar, and the velocity of the bar, which are all mutually perpendicular. The magnetic field and the length of the bar are already perpendicular. The component of the velocity of the bar that is perpendicular to the magnetic field is $v\cos\theta$, and so the induced emf is given by the following.



$$\mathcal{E} = B\ell v\cos\theta$$

This produces a current in the wire, which can be found by Ohm's law. That current is pointing into the page on the diagram.

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field can be calculated from Eq. 27-2, and will be horizontal, as shown in the diagram.

$$F_B = I\ell B = \frac{B\ell v\cos\theta}{R}\ell B = \frac{B^2\ell^2 v\cos\theta}{R}$$

For the wire to slide down at a steady speed, the net force along the rail must be zero. Write Newton's second law for forces along the rail, with up the rail being positive.

$$F_{\text{net}} = F_B\cos\theta - mg\sin\theta = 0 \quad\rightarrow\quad \frac{B^2\ell^2 v\cos^2\theta}{R} = mg\sin\theta \quad\rightarrow$$

$$v = \frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta} = \frac{(0.60\,\Omega)(0.040\,\text{kg})(9.80\,\text{m/s}^2)\sin 6.0^\circ}{(0.55\,\text{T})^2(0.32\,\text{m})^2\cos^2 6.0^\circ} = \boxed{0.80\,\text{m/s}}$$

$\boxed{\text{67.}}$ The induced current in the coil is the induced emf divided by the resistance. The induced emf is found from the changing flux by Eq. 29-2a. The magnetic field of the solenoid, which causes the flux, is given by Eq. 28-4. For the area used in Eq. 29-2a, the cross-sectional area of the solenoid (not the coil) must be used, because all of the magnetic flux is inside the solenoid.

$$I = \frac{\mathcal{E}_{\text{ind}}}{R} \qquad \left|\mathcal{E}_{\text{ind}}\right| = N_{\text{coil}}\frac{d\Phi}{dt} = N_{\text{coil}}A_{\text{sol}}\frac{dB_{\text{sol}}}{dt} \qquad B_{\text{sol}} = \mu_0\frac{N_{\text{sol}}I_{\text{sol}}}{\ell_{\text{sol}}}$$

$$I = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0\dfrac{N_{\text{sol}}}{\ell_{\text{sol}}}\dfrac{dI_{\text{sol}}}{dt}}{R} = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0}{R}\frac{N_{\text{sol}}}{\ell_{\text{sol}}}\frac{dI_{\text{sol}}}{dt}$$

$$= \frac{(150\,\text{turns})\pi(0.045\,\text{m})^2(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}{12\,\Omega}\frac{(230\,\text{turns})}{(0.01\,\text{m})}\frac{2.0\,\text{A}}{0.10\,\text{s}} = \boxed{4.6\times10^{-2}\,\text{A}}$$

As the current in the solenoid increases, a magnetic field from right to left is created in the solenoid and the loop. The induced current will flow in such a direction as to oppose that field, and so must flow from $\boxed{\text{left to right}}$ through the resistor.

68. The average induced emf is given by the "difference" version of Eq. 29-2b. Because the coil orientation changes by 180°, the change in flux is the opposite of twice the initial flux. The average current is the induced emf divided by the resistance, and the charge that flows in a given time is the current times the elapsed time.

$$\mathcal{E}_{\text{avg}} = -N\frac{\Delta\Phi_B}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{\left[(-B)-(+B)\right]}{\Delta t} = \frac{2NAB}{\Delta t}$$

$$Q = I\Delta t = \frac{\mathcal{E}_{\text{avg}}}{R}\Delta t = \frac{\left(\dfrac{2NAB}{\Delta t}\right)}{R}\Delta t = \frac{2NAB}{R} \quad\rightarrow\quad \boxed{B = \frac{RQ}{2NA}}$$

69. Calculate the current in the ring from the magnitude of the emf (from Eq. 29-2a) divided by the resistance. Setting the current equal to the derivative of the charge, we integrate the charge and flux over the 90° rotation, with the flux given by Eq. 29-1a. This results in the total charge flowing past a given point in the ring. Note that the initial orientation of the ring area relative to the magnetic field is not given.

$$\mathcal{E}=\frac{d\Phi_B}{dt}\ ;\ I=\frac{\mathcal{E}}{R}=\frac{dQ}{dt}\ \rightarrow\ dQ=\frac{\mathcal{E}}{R}dt\ \rightarrow$$

$$Q=\int dQ=\int\frac{\mathcal{E}}{R}dt=\frac{1}{R}\int\frac{d\Phi}{dt}dt=\frac{1}{R}\int d\Phi=\frac{1}{R}\int_{BA\cos(\theta)}^{BA\cos(\theta+90^\circ)}d\Phi=\frac{BA\left[\cos\left(\theta+90^\circ\right)-\cos\theta\right]}{R}$$

$$=\frac{(0.23\,\mathrm{T})\pi(0.030\,\mathrm{m})^2}{0.025\,\Omega}\left[\cos\left(\theta+90^\circ\right)-\cos\theta\right]=0.02601\,\mathrm{C}\left[\cos\left(\theta+90^\circ\right)-\cos\theta\right]$$

To find the maximum charge, we set the derivative of the charge with respect to the starting angle, θ, equal to zero to find the extremes. Inserting the maximum angle into our equation, we find the maximum charge passing through the ring. Finally, we divide the maximum charge by the charge of a single electron to obtain the number of electrons passing the point in the ring.

$$\frac{dQ}{d\theta}=0.02601\ \mathrm{C}\left[-\sin\left(\theta+90^\circ\right)+\sin\theta\right]=0.02601\ \mathrm{C}\left[-\cos\theta+\sin\theta\right]=0\ \rightarrow\ \tan\theta=1\ \rightarrow$$

$$\theta=45^\circ\text{ or }225^\circ$$

$$Q_{\max}=0.02601\ \mathrm{C}\left[\cos\left(225^\circ+90^\circ\right)-\cos 225^\circ\right]=0.03678\ \mathrm{C}$$

$$N_{\max}=\frac{Q_{\max}}{q}=\frac{0.03678\ \mathrm{C}}{1.60\times10^{-19}\ \mathrm{C/e}}=\boxed{2.3\times10^{17}\,\text{electrons}}$$

70. The coil should have a diameter about equal to the diameter of a standard flashlight D-cell so that it will be simple to hold and use. This would give the coil a radius of about 1.5 cm. As the magnet passes through the coil the field changes direction, so the change in flux for each pass is twice the maximum flux. Let us assume that the magnet is shaken with a frequency of about two shakes per second, so the magnet passes through the coil four times per second. We obtain the number of turns in the coil using Eq. 29-2b.

$$N=\frac{\mathcal{E}}{\Delta\Phi/\Delta t}=\frac{\mathcal{E}\Delta t}{\Delta\Phi}=\frac{\mathcal{E}\Delta t}{2B_0A}=\frac{(3.0\,\mathrm{V})(0.25\,\mathrm{s})}{2(0.050\,\mathrm{T})\pi(0.015\,\mathrm{m})^2}\approx\boxed{11{,}000\text{ turns}}$$

71. (*a*) Since the coils are directly connected to the wheels, the torque provided by the motor (Eq. 27-9) balances the torque caused by the frictional force.

$$NIAB=Fr\rightarrow I=\frac{Fr}{NAB}=\frac{(250\ \mathrm{N})(0.29\ \mathrm{m})}{270(0.12\ \mathrm{m})(0.15\ \mathrm{m})(0.60\ \mathrm{T})}=24.86\ \mathrm{A}\approx\boxed{25\ \mathrm{A}}$$

(*b*) To maintain this speed the power loss due to the friction (Eq. 8-21) must equal the net power provided by the coils. The power provided by the coils is the current through the coils multiplied by the back emf.

$$P=Fv=I\mathcal{E}_{\text{back}}\rightarrow\mathcal{E}_{\text{back}}=\frac{Fv}{I}=\frac{(250\,\mathrm{N})(35\,\mathrm{km/h})}{24.86\,\mathrm{A}}\left(\frac{1000\,\mathrm{m/km}}{3600\ \mathrm{s/h}}\right)=97.76\,\mathrm{V}\approx\boxed{98\,\mathrm{V}}$$

(*c*) The power dissipated in the coils is the difference between the power produced by the coils and the net power provided to the wheels.

$$P_{\text{loss}}=P-P_{\text{net}}=I\mathcal{E}-I\mathcal{E}_{\text{back}}=(24.86\,\mathrm{A})(120\,\mathrm{V}-97.76\,\mathrm{V})=553\,\mathrm{W}\approx\boxed{600\,\mathrm{W}}$$

(*d*) We divide the net power by the total power to determine the percent used to drive the car.

$$\frac{P_{\text{net}}}{P}=\frac{I\mathcal{E}_{\text{back}}}{I\mathcal{E}}=\frac{97.76\,\text{V}}{120\,\text{V}}=0.8147\approx\boxed{81\%}$$

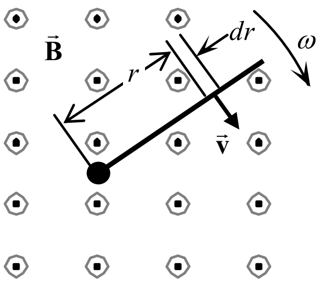
72. The energy is dissipated by the resistance. The power dissipated by the resistor is given by Eq. 25-7b, and the energy is the integral of the power over time. The induced emf is given by Eq. 29-2a.

$$\mathcal{E}=-N\frac{d\Phi_B}{dt}=-NA\frac{dB}{dt}=\frac{NAB_0}{\tau}e^{-t/\tau}\ \ ;\ P=I^2R=\frac{\mathcal{E}^2}{R}=\left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}$$

$$E=\int Pdt=\int_0^t\left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}dt=\left(\frac{N^2A^2B_0^2}{R\tau^2}\right)\left(-\frac{\tau}{2}e^{-2t/\tau}\right)_0^t=\frac{\left(NAB_0\right)^2}{2R\tau}\left(1-e^{-2t/\tau}\right)$$

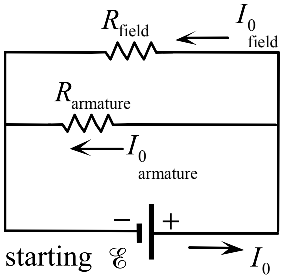
$$=\frac{\left[18\pi\left(0.100\,\text{m}\right)^2\left(0.50\,\text{T}\right)\right]^2}{2\left(2.0\,\Omega\right)\left(0.10\,\text{s}\right)}\left(1-e^{-2t/(0.10\,\text{s})}\right)=\boxed{\left(0.20\,\text{J}\right)\left(1-e^{-20t}\right)}$$

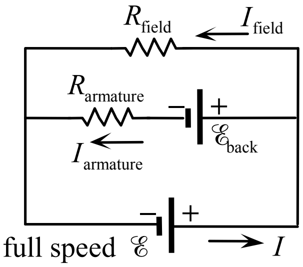
73. The total emf across the rod is the integral of the differential emf across each small segment of the rod. For each differential segment, *dr*, the differential emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. The figure is a top view of the spinning rod.



$$d\mathcal{E}=Bvd\ell=B\omega rdr\ \rightarrow\ \mathcal{E}=\int d\mathcal{E}=\int_0^\ell B\omega rdr=\boxed{\tfrac{1}{2}B\omega\ell^2}$$

74. (*a*)





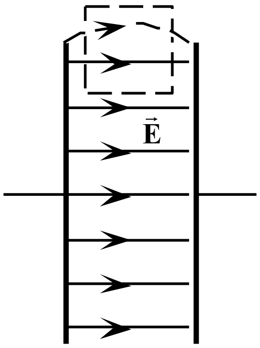
(*b*) At startup there is no back emf. We therefore treat the circuit as two parallel resistors, each with the same voltage drop. The current through the battery is the sum of the currents through each resistor.

$$I=I_{0\,\text{field}}+I_{0\,\text{armature}}=\frac{\mathcal{E}}{R_{\text{field}}}+\frac{\mathcal{E}}{R_{\text{armature}}}=\frac{115\text{ V}}{36.0\ \Omega}+\frac{115\text{ V}}{3.00\ \Omega}=\boxed{41.5\text{ A}}$$

(*c*) At full speed the back emf decreases the voltage drop across the armature resistor.

$$I=I_{\text{field}}+I_{\text{armature}}=\frac{\mathcal{E}}{R_{\text{field}}}+\frac{\mathcal{E}-\mathcal{E}_{\text{back}}}{R_{\text{armature}}}=\frac{115\,\text{V}}{36.0\,\Omega}+\frac{115\,\text{V}-105\,\text{V}}{3.00\,\Omega}=\boxed{6.53\,\text{A}}$$

75. Assume that the electric field does not fringe, but only has a horizontal component between the plates and zero field outside the plates. Apply Faraday's law (Eq. 29-8) to this situation for a rectangular loop with one horizontal leg inside the plates and the second horizontal leg outside the plates. We integrate around this path in the counterclockwise direction. Since the field only has a horizontal component between the plates, only the horizontal leg will contribute to the electric field integral. Since the field is constant in this region, the integral is the electric field times the length of the leg.

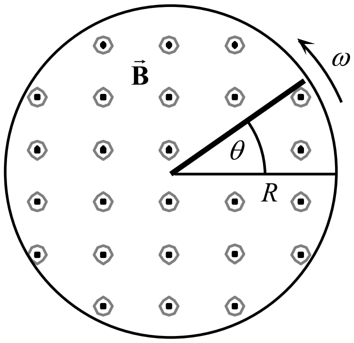


$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = \int_0^{\ell} E\,d\ell = E\ell$$

For a static electric field, the magnetic flux is unchanging. Therefore $\frac{d\Phi_B}{dt} = 0.$

Using Faraday's law, we have $\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \rightarrow E\ell = 0,$ which is not possible. Thus one of the initial assumptions must be false. We conclude that the field must have some fringing at the edges.

76. The total emf across the disk is the integral of the differential emf across each small segment of the radial line passing from the center of the disk to the edge. For each differential segment, dr, the emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. Since the disk is rotating in the counterclockwise direction, and the field is out of the page, the emf is increasing with increasing radius. Therefore the rim is at the higher potential.



$$d\mathscr{E} = Bv\,d\ell = B\omega r\,dr \rightarrow$$

$$\mathscr{E} = \int d\mathscr{E} = \int_0^R B\omega r\,dr = \boxed{\tfrac{1}{2}B\omega R^2}$$

77. We set the electric field equal to the negative gradient of the electric potential (Eq. 23-8), with the differential potential given by Eq. 29-3, as in Problem 76.

$$\vec{\mathbf{E}} = -\frac{d\mathscr{E}}{dr}\hat{\mathbf{r}} = -\frac{Br\omega dr}{dr}\hat{\mathbf{r}} = \boxed{-Br\omega\hat{\mathbf{r}}}$$

The electric field has magnitude $Br\omega$ and points radially inwards, toward the center of the disk.

78. The emf around the loop is equal to the time derivative of the flux, as in Eq. 29-2a. Since the area of the coil is constant, the time derivative of the flux is equal to the derivative of the magnetic field multiplied by the area of the loop. To calculate the emf in the loop we add the voltage drop across the capacitor to the voltage drop across the resistor. The current in the loop is the derivative of the charge on the capacitor (Eq. 24-1).

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = \frac{d}{dt}\left[CV_0\left(1-e^{-t/\tau}\right)\right] = \frac{CV_0}{\tau}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau}$$

$$\mathscr{E} = IR + V_C = \left(\frac{V_0}{R}e^{-t/\tau}\right)R + V_0\left(1-e^{-t/\tau}\right) = V_0 = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} \rightarrow \boxed{\frac{dB}{dt} = \frac{V_0}{\pi r^2}}$$

Since the charge is building up on the top plate of the capacitor, the induced current is flowing clockwise. By Lenz's law this produces a downward flux, so the external downward magnetic field must be $\boxed{\text{decreasing}}$.

$\boxed{79.}$ (*a*) As the loop falls out of the magnetic field, the flux through the loop decreases with time creating an induced emf in the loop. The current in the loop is equal to the emf divided by the resistance, which can be written in terms of the resistivity using Eq. 25-3.

$$I=\frac{\mathcal{E}}{R}=\left(\frac{\pi d^2/4}{\rho 4\ell}\right)\frac{d\Phi_B}{dt}=\left(\frac{\pi d^2}{16\rho\ell}\right)B\frac{dA}{dt}=\frac{\pi d^2}{16\rho\ell}B\ell v$$

This current induces a force on the three sides of the loop in the magnetic field. The forces on the two vertical sides are equal and opposite and therefore cancel.

$$F=I\ell B=\frac{\pi d^2}{16\rho\ell}B\ell v\ell B=\boxed{\frac{\pi d^2B^2\ell v}{16\rho}}$$

By Lenz's law this force is upward to slow the decrease in flux.

(*b*) Terminal speed will occur when the gravitational force is equal to the magnetic force.

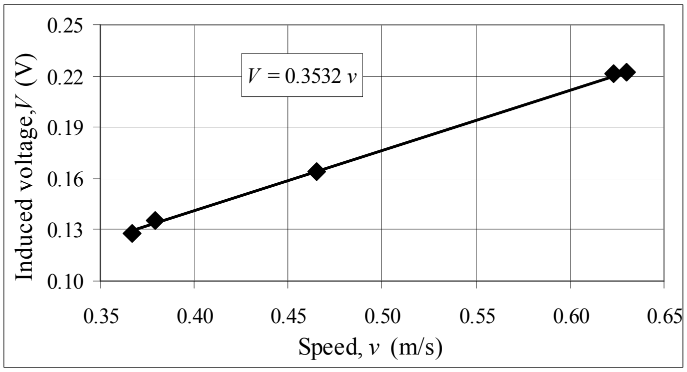
$$F_g=\rho_m\left(4\pi\ell\frac{d^2}{4}\right)g=\frac{\pi d^2B^2\ell v_T}{16\rho}\rightarrow v_T=\boxed{\frac{16\rho\rho_m g}{B^2}}$$

(*c*) We calculate the terminal velocity using the given magnetic field, the density of copper from Table 13-1, and the resistivity of copper from Table 25-1

$$v_T=\frac{16\left(8.9\times10^3\text{ kg/m}^3\right)\left(1.68\times10^{-8}\ \Omega\text{m}\right)\left(9.80\text{ m/s}^2\right)}{\left(0.80\text{ T}\right)^2}=\boxed{3.7\text{ cm/s}}$$

80. (*a*) See the graph, with best fit linear trend line (with the *y* intercept forced to be 0).

(*b*) The theoretical slope is the induced voltage divided by the velocity. Take the difference between the experimental value found in part (*a*) and the theoretical value and divide the result by the theoretical value to obtain the percent difference.



$$\%\text{ diff}=\left(\frac{m_{\text{exp}}-m_{\text{theory}}}{m_{\text{theory}}}\right)100=\left(\frac{m_{\text{exp}}}{BN\ell}-1\right)100=\left(\frac{0.3532\text{ V}\bullet\text{s/m}}{(0.126\text{T})(50)(0.0561\text{m})}-1\right)100$$

$$=\boxed{-0.065\%}$$

(*c*) Use the theoretical equation to calculate the voltage at each experimental speed. Then calculate the percent difference at each speed.

Speed (m/s)	Induced Voltage (V)	Theoretical Induced Voltage (V)	% diff.
0.367	0.128	0.130	− 1.32%
0.379	0.135	0.134	0.78%
0.465	0.164	0.164	− 0.21%
0.623	0.221	0.220	0.37%
0.630	0.222	0.223	− 0.30%

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH29.XLS," on tab "Problem 29.80."

CHAPTER 30: Inductance, Electromagnetic Oscillations, and AC Circuits

Responses to Questions

1. (*a*) For the maximum value of the mutual inductance, place the coils close together, face to face, on the same axis.
   (*b*) For the least possible mutual inductance, place the coils with their faces perpendicular to each other.

2. The magnetic field near the end of the first solenoid is less than it is in the center. Therefore the flux through the second coil would be less than that given by the formula, and the mutual inductance would be lower.

3. Yes. If two coils have mutual inductance, then they each have the capacity for self-inductance. Any coil that experiences a changing current will have a self-inductance.

4. The energy density is greater near the center of a solenoid, where the magnetic field is greater.

5. To create the greatest self-inductance, bend the wire into as many loops as possible. To create the least self-inductance, leave the wire as a straight piece of wire.

6. (*a*) No. The time needed for the *LR* circuit to reach a given fraction of its maximum possible current depends on the time constant, $\tau = L/R$, which is independent of the emf.
   (*b*) Yes. The emf determines the maximum value of the current ($I_{\max} = V_0/R$,) and therefore will affect the time it takes to reach a particular value of current.

7. A circuit with a large inductive time constant is resistant to changes in the current. When a switch is opened, the inductor continues to force the current to flow. A large charge can build up on the switch, and may be able to ionize a path for itself across a small air gap, creating a spark.

8. Although the current is zero at the instant the battery is connected, the rate at which the current is changing is a maximum and therefore the rate of change of flux through the inductor is a maximum. Since, by Faraday's law, the induced emf depends on the rate of change of flux and not the flux itself, the emf in the inductor is a maximum at this instant.

9. When the capacitor has discharged completely, energy is stored in the magnetic field of the inductor. The inductor will resist a change in the current, so current will continue to flow and will charge the capacitor again, with the opposite polarity.

10. Yes. The instantaneous voltages across the different elements in the circuit will be different, but the current through each element in the series circuit is the same.

11. The energy comes from the generator. (A generator is a device that converts mechanical energy to electrical energy, so ultimately, the energy came from some mechanical source, such as falling water.) Some of the energy is dissipated in the resistor and some is stored in the fields of the capacitor and the inductor. An increase in *R* results in an increase in energy dissipated by the circuit. *L*, *C*, *R*, and the frequency determine the current flow in the circuit, which determines the power supplied by generator.

12. $X_L = X_C$ at the resonant frequency. If the circuit is predominantly inductive, such that $X_L > X_C$, then the frequency is greater than the resonant frequency and the voltage leads the current. If the circuit is predominantly capacitive, such that $X_C > X_L$, then the frequency is lower than the resonant frequency and the current leads the voltage. Values of L and C cannot be meaningfully compared, since they are in different units. Describing the circuit as "inductive" or "capacitive" relates to the values of X_L and X_C, which are both in ohms and which both depend on frequency.

$\boxed{13.}$ Yes. When ω approaches zero, X_L approaches zero, and X_C becomes infinitely large. This is consistent with what happens in an ac circuit connected to a dc power supply. For the dc case, ω is zero and X_L will be zero because there is no changing current to cause an induced emf. X_C will be infinitely large, because steady direct current cannot flow across a capacitor once it is charged.

14. The impedance in an *LRC* circuit will be a minimum at resonance, when $X_L = X_C$. At resonance, the impedance equals the resistance, so the smallest R possible will give the smallest impedance.

15. Yes. The power output of the generator is $P = IV$. When either the instantaneous current or the instantaneous voltage in the circuit is negative, and the other variable is positive, the instantaneous power output can be negative. At this time either the inductor or the capacitor is discharging power back to the generator.

16. Yes, the power factor depends on frequency because X_L and X_C, and therefore the phase angle, depend on frequency. For example, at resonant frequency, $X_L = X_C$, the phase angle is 0º, and the power factor is one. The average power dissipated in an *LRC* circuit also depends on frequency, since it depends on the power factor: $P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\varphi$. Maximum power is dissipated at the resonant frequency. The value of the power factor decreases as the frequency gets farther from the resonant frequency.

17. (*a*) The impedance of a pure resistance is unaffected by the frequency of the source emf.
    (*b*) The impedance of a pure capacitance decreases with increasing frequency.
    (*c*) The impedance of a pure inductance increases with increasing frequency.
    (*d*) In an *LRC* circuit near resonance, small changes in the frequency will cause large changes in the impedance.
    (*e*) For frequencies far above the resonance frequency, the impedance of the *LRC* circuit is dominated by the inductive reactance and will increase with increasing frequency. For frequencies far below the resonance frequency, the impedance of the *LRC* circuit is dominated by the capacitive reactance and will decrease with increasing frequency.

18. In all three cases, the energy dissipated decreases as R approaches zero. Energy oscillates between being stored in the field of the capacitor and being stored in the field of the inductor.
    (*a*) The energy stored in the fields (and oscillating between them) is a maximum at resonant frequency and approaches an infinite value as R approaches zero.
    (*b*) When the frequency is near resonance, a large amount of energy is stored in the fields but the value is less than the maximum value.
    (*c*) Far from resonance, a much lower amount of energy is stored in the fields.

$\boxed{19.}$ In an *LRC* circuit, the current and the voltage in the circuit both oscillate. The energy stored in the circuit also oscillates and is alternately stored in the magnetic field of the inductor and the electric field of the capacitor.

20. In an *LRC* circuit, energy oscillates between being stored in the magnetic field of the inductor and being stored in the electric field of the capacitor. This is analogous to a mass on a spring, with energy alternating between kinetic energy of the mass and spring potential energy as the spring compresses and extends. The energy stored in the magnetic field is analogous to the kinetic energy of the moving mass, and *L* corresponds to the mass, *m*, on the spring. The energy stored in the electric field of the capacitor is analogous to the spring potential energy, and *C* corresponds to the reciprocal of the spring constant, 1/*k*.

Solutions to Problems

$\boxed{1.}$ (*a*) The mutual inductance is found in Example 30-1.

$$M=\frac{\mu N_1N_2A}{\ell}=\frac{1850\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(225)(115)\pi\left(0.0200\,\mathrm{m}\right)^2}{2.44\,\mathrm{m}}=\boxed{3.10\times10^{-2}\,\mathrm{H}}$$

(b) The emf induced in the second coil can be found from Eq. 30-3b.

$$\mathscr{E}_2=-M\frac{dI_1}{dt}=-M\frac{\Delta I_1}{\Delta t}=\left(-3.10\times10^{-2}\,\mathrm{H}\right)\frac{(-12.0\mathrm{A})}{0.0980\,\mathrm{ms}}=\boxed{3.79\,\mathrm{V}}$$

2. If we assume the outer solenoid is carrying current I_1, then the magnetic field inside the outer solenoid is $B=\mu_0n_1I_1$. The flux in each turn of the inner solenoid is $\Phi_{21}=B\pi r_2^2=\mu_0n_1I_1\pi r_2^2$. The mutual inductance is given by Eq. 30-1.

$$M=\frac{N_2\Phi_{21}}{I_1}=\frac{n_2\ell\mu_0n_1I_1\pi r_2^2}{I_1}\quad\rightarrow\quad\frac{M}{\ell}=\boxed{\mu_0n_1n_2\pi r_2^2}$$

3. We find the mutual inductance of the inner loop. If we assume the outer solenoid is carrying current I_1, then the magnetic field inside the outer solenoid is $B=\mu_0\frac{N_1}{\ell}I_1$. The magnetic flux through each loop of the small coil is the magnetic field times the area perpendicular to the field. The mutual inductance is given by Eq. 30-1.

$$\Phi_{21}=BA_2\sin\theta=\mu_0\frac{N_1I_1}{\ell}A_2\sin\theta\ ;\ M=\frac{N_2\Phi_{21}}{I_1}=\frac{N_2\mu_0\frac{N_1I_1}{\ell}A_2\sin\theta}{I_1}=\boxed{\frac{\mu_0N_1N_2A_2\sin\theta}{\ell}}$$

4. We find the mutual inductance of the system using Eq. 30-1, with the flux equal to the integral of the magnetic field of the wire (Eq. 28-1) over the area of the loop.

$$M=\frac{\Phi_{12}}{I_1}=\frac{1}{I_1}\int_{\ell_1}^{\ell_2}\frac{\mu_0I_1}{2\pi r}w\,dr=\boxed{\frac{\mu_0w}{2\pi}\ln\left(\frac{\ell_2}{\ell_1}\right)}$$

5. Find the induced emf from Eq. 30-5.

$$\mathscr{E}=-L\frac{dI}{dt}=-L\frac{\Delta I}{\Delta t}=-(0.28\,\mathrm{H})\frac{(10.0\,\mathrm{A}-25.0\,\mathrm{A})}{0.36\,\mathrm{s}}=\boxed{12\,\mathrm{V}}$$

6. Use the relationship for the inductance of a solenoid, as given in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.13\,\text{H})(0.300\,\text{m})}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\pi(0.021\,\text{m})^2}}\approx\boxed{4700\,\text{turns}}$$

$\boxed{7.}$ Because the current in increasing, the emf is negative. We find the self-inductance from Eq. 30-5.

$$\mathscr{E}=-L\frac{dI}{dt}=-L\frac{\Delta I}{\Delta t} \quad\rightarrow\quad L=-\mathscr{E}\frac{\Delta t}{\Delta I}=-(-2.50\,\text{V})\frac{0.0120\,\text{s}}{\left[0.0250\,\text{A}-(-0.0280\,\text{A})\right]}=\boxed{0.566\,\text{H}}$$

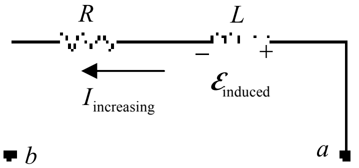
8. (*a*) The number of turns can be found from the inductance of a solenoid, which is derived in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2800)^2\,\pi(0.0125\,\text{m})^2}{(0.217\,\text{m})}=0.02229\,\text{H}\approx\boxed{0.022\,\text{H}}$$

(*b*) Apply the same equation again, solving for the number of turns.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.02229\,\text{H})(0.217\,\text{m})}{(1200)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\pi(0.0125\,\text{m})^2}}\approx\boxed{81\,\text{turns}}$$

9. We draw the coil as two elements in series, and pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 30-5. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, and so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.



$$V_{ab}=IR+L\frac{dI}{dt}=(3.00\,\text{A})(3.25\,\Omega)+(0.44\,\text{H})(3.60\,\text{A/s})=\boxed{11.3\,\text{V}}$$

10. We use the result for inductance per unit length from Example 30-5.

$$\frac{L}{\ell}=\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}\le 55\times10^{-9}\,\text{H/m} \quad\rightarrow\quad r_1\ge r_2 e^{-\frac{2\pi}{\mu_0}\left(55\times10^{-9}\,\text{H/m}\right)}=(0.0030\,\text{m})e^{-\frac{2\pi\left(55\times10^{-9}\,\text{H/m}\right)}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}}=0.00228\,\text{m}$$

$$\boxed{r_1\ge 0.0023\,\text{m}}$$

11. The self-inductance of an air-filled solenoid was determined in Example 30-3. We solve this equation for the length of the tube, using the diameter of the wire as the length per turn.

$$L=\frac{\mu_0 N^2 A}{\ell}=\mu_0 n^2 A\ell=\frac{\mu_0 A\ell}{d^2}$$

$$\ell=\frac{Ld^2}{\mu_0\pi r^2}=\frac{(1.0\text{H})\left(0.81\times10^{-3}\text{m}\right)^2}{\left(4\pi\times10^{-7}\text{T}\bullet\text{m/A}\right)\pi(0.060\,\text{m})^2}=46.16\,\text{m}\approx\boxed{46\,\text{m}}$$

The length of the wire is equal to the number of turns (the length of the solenoid divided by the diameter of the wire) multiplied by the circumference of the turn.

$$L=\frac{\ell}{d}\pi D=\frac{46.16\,\text{m}}{0.81\times10^{-3}\text{m}}\pi(0.12\,\text{m})=21{,}490\,\text{m}\approx\boxed{21\text{km}}$$

The resistance is calculated from the resistivity, area, and length of the wire.

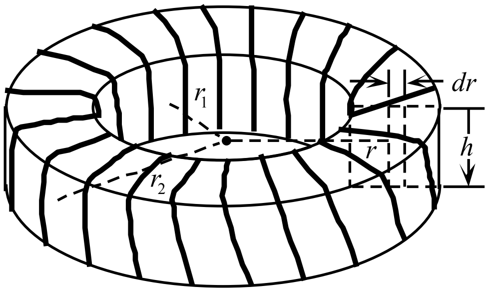
$$R=\frac{\rho\ell}{A}=\frac{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(21{,}490\,\text{m}\right)}{\pi\left(0.405\times10^{-3}\,\text{m}\right)^2}=\boxed{0.70\,\text{k}\Omega}$$

12. The inductance of the solenoid is given by $L=\dfrac{\mu_0N^2A}{\ell}=\dfrac{\mu_0N^2}{\ell}\dfrac{\pi d^2}{4}$. The (constant) length of the wire is given by $\ell_{\text{wire}}=N\pi d_{\text{sol}}$, and so since $d_{\text{sol 2}}=2.5d_{\text{sol 1}}$, we also know that $N_1=2.5N_2$. The fact that the wire is tightly wound gives $\ell_{\text{sol}}=Nd_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1}=\frac{\dfrac{\mu_0\pi}{4}\dfrac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{\mu_0\pi}{4}\dfrac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\dfrac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\dfrac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\dfrac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 2}}}}{\dfrac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 1}}}}=\frac{\ell_{\text{sol 1}}}{\ell_{\text{sol 2}}}=\frac{N_1d_{\text{wire}}}{N_2d_{\text{wire}}}=\frac{N_1}{N_2}=\boxed{2.5}$$

13. We use Eq. 30-4 to calculate the self-inductance, where the flux is the integral of the magnetic field over a cross-section of the toroid. The magnetic field inside the toroid was calculated in Example 28-10.

$$L=\frac{N}{I}\Phi_B=\frac{N}{I}\int_{r_1}^{r_2}\frac{\mu_0NI}{2\pi r}h\,dr=\boxed{\frac{\mu_0N^2h}{2\pi}\ln\left(\frac{r_2}{r_1}\right)}$$



14. (*a*) When connected in series the voltage drops across each inductor will add, while the currents in each inductor are the same.

$$\mathscr{E}=\mathscr{E}_1+\mathscr{E}_2=-L_1\frac{dI}{dt}-L_2\frac{dI}{dt}=-\left(L_1+L_2\right)\frac{dI}{dt}=-L_{\text{eq}}\frac{dI}{dt}\rightarrow\boxed{L_{\text{eq}}=L_1+L_2}$$

(*b*) When connected in parallel the currents in each inductor add to the equivalent current, while the voltage drop across each inductor is the same as the equivalent voltage drop.

$$\frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}\rightarrow\frac{\mathscr{E}}{L_{\text{eq}}}=\frac{\mathscr{E}}{L_1}+\frac{\mathscr{E}}{L_2}\rightarrow\boxed{\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}}$$

Therefore, inductors in series and parallel add the same as resistors in series and parallel.

15. The magnetic energy in the field is derived from Eq. 30-7.

$$u=\frac{\text{Energy stored}}{\text{Volume}}=\tfrac{1}{2}\frac{B^2}{\mu_0}\rightarrow$$

$$\text{Energy}=\tfrac{1}{2}\frac{B^2}{\mu_0}\left(\text{Volume}\right)=\tfrac{1}{2}\frac{B^2}{\mu_0}\pi r^2\ell=\tfrac{1}{2}\frac{\left(0.600\,\text{T}\right)^2}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\pi\left(0.0105\,\text{m}\right)^2\left(0.380\,\text{m}\right)=\boxed{18.9\,\text{J}}$$

16. (*a*) We use Eq. 24-6 to calculate the energy density in an electric field and Eq. 30-7 to calculate the energy density in the magnetic field.

$$u_E=\tfrac{1}{2}\varepsilon_0E^2=\tfrac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.0\times10^4\ \text{N/C}\right)^2=\boxed{4.4\times10^{-4}\ \text{J/m}^3}$$

$$u_B = \frac{B^2}{2\mu_0} = \frac{(2.0\text{ T})^2}{2(4\pi\times10^{-7}\text{ T•m/A})} = 1.592\times10^6\text{ J/m}^3 \approx \boxed{1.6\times10^6\text{ J/m}^3}$$

(*b*) Use Eq. 24-6 to calculate the electric field from the energy density for the magnetic field given in part (*a*).

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = u_B \quad\to\quad E = \sqrt{\frac{2u_B}{\varepsilon_0}} = \sqrt{\frac{2(1.592\times10^6\text{ J/m}^3)}{(8.85\times10^{-12}\text{ C}^2/\text{N•m}^2)}} = \boxed{6.0\times10^8\text{ N/C}}$$

17. We use Eq. 30-7 to calculate the energy density with the magnetic field calculated in Example 28-12.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2R}\right)^2 = \frac{\mu_0 I^2}{8R^2} = \frac{(4\pi\times10^{-7}\text{ T•m/A})(23.0\text{ A})^2}{8(0.280\text{ m})^2} = \boxed{1.06\times10^{-3}\text{ J/m}^3}$$

18. We use Eq. 30-7 to calculate the magnetic energy density, with the magnetic field calculated using Eq. 28-1.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi R}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 R^2} = \frac{(4\pi\times10^{-7}\text{ T•m/A})(15\text{ A})^2}{8\pi^2(1.5\times10^{-3}\text{ m})^2} = \boxed{1.6\text{ J/m}^3}$$

To calculate the electric energy density with Eq. 24-6, we must first calculate the electric field at the surface of the wire. The electric field will equal the voltage difference along the wire divided by the length of the wire. We can calculate the voltage drop using Ohm's law and the resistance from the resistivity and diameter of the wire.

$$E = \frac{V}{\ell} = \frac{IR}{\ell} = \frac{I\rho\ell}{\ell\pi r^2} = \frac{I\rho}{\pi r^2}$$

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{I\rho}{\pi r^2}\right)^2 = \tfrac{1}{2}(8.85\times10^{-12}\text{C}^2/\text{N•m}^2)\left[\frac{(15\text{A})(1.68\times10^{-8}\Omega\text{•m})}{\pi(1.5\times10^{-3}\text{m})^2}\right]^2$$

$$= \boxed{5.6\times10^{-15}\text{J/m}^3}$$

$\boxed{19.}$ We use Eq. 30-7 to calculate the energy density in the toroid, with the magnetic field calculated in Example 28-10. We integrate the energy density over the volume of the toroid to obtain the total energy stored in the toroid. Since the energy density is a function of radius only, we treat the toroid as cylindrical shells each with differential volume $dV = 2\pi rhdr$.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 NI}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}}$$

$$U = \int u_B dV = \int_{r_1}^{r_2}\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}2\pi rhdr = \frac{\mu_0 N^2 I^2 h}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 N^2 I^2 h}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

20. The magnetic field between the cables is given in Example 30-5. Since the magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate over the radius between the two cables.

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_{r_1}^{r_2}\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi rdr = \frac{\mu_0 I^2}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 I^2}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

21. We create an Amperean loop of radius *r* to calculate the magnetic field within the wire using Eq. 28-3. Since the resulting magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r \ell dr$ and integrate from zero to the radius of the wire.

$$\oint \vec{\mathbf{B}} \bullet d\vec{\ell} = \mu_0 I_{enc} \to B(2\pi r) = \mu_0 \left(\frac{I}{\pi R^2}\right)(\pi r^2) \to B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_0^R \frac{1}{2\mu_0}\left(\frac{\mu_0 I r}{2\pi R^2}\right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3 dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

22. For an *LR* circuit, we have $I = I_{\max}\left(1 - e^{-t/\tau}\right)$. Solve for t.

$$I = I_{\max}\left(1 - e^{-t/\tau}\right) \quad \to \quad e^{-t/\tau} = 1 - \frac{I}{I_{\max}} \quad \to \quad t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right)$$

(*a*) $I = 0.95 I_{\max} \quad \to \quad t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.95) = \boxed{3.0\,\tau}$

(*b*) $I = 0.990 I_{\max} \quad \to \quad t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.990) = \boxed{4.6\,\tau}$

(*c*) $I = 0.9990 I_{\max} \quad \to \quad t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.9990) = \boxed{6.9\,\tau}$

23. We set the current in Eq. 30-11 equal to $0.03I_0$ and solve for the time.

$$I = 0.03 I_0 = I_0 e^{-t/\tau} \to t = -\tau \ln(0.03) \approx \boxed{3.5\tau}$$

24. (*a*) We set *I* equal to 75% of the maximum value in Eq. 30-9 and solve for the time constant.

$$I = 0.75 I_0 = I_0\left(1 - e^{-t/\tau}\right) \to \tau = -\frac{t}{\ln(0.25)} = -\frac{(2.56\text{ ms})}{\ln(0.25)} = 1.847\text{ ms} \approx \boxed{1.85\text{ ms}}$$

(*b*) The resistance can be calculated from the time constant using Eq. 30-10.

$$R = \frac{L}{\tau} = \frac{31.0\text{ mH}}{1.847\text{ ms}} = \boxed{16.8\ \Omega}$$

$\boxed{25.}$ (*a*) We use Eq. 30-6 to determine the energy stored in the inductor, with the current given by Eq. Eq 30-9.

$$U = \tfrac{1}{2} L I^2 = \boxed{\frac{L V_0^2}{2R^2}\left(1 - e^{-t/\tau}\right)^2}$$

(*b*) Set the energy from part (*a*) equal to 99.9% of its maximum value and solve for the time.

$$U = 0.999\frac{V_0^2}{2R^2} = \frac{V_0^2}{2R^2}\left(1 - e^{-t/\tau}\right)^2 \to t = \tau \ln\left(1 - \sqrt{0.999}\right) \approx \boxed{7.6\tau}$$

26. (*a*) At the moment the switch is closed, no current will flow through the inductor. Therefore, the resistors R_1 and R_2 can be treated as in series.

$$\mathcal{E} = I(R_1 + R_2) \to \boxed{I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2},\ I_3 = 0}$$

(*b*) A long time after the switch is closed, there is no voltage drop across the inductor so resistors R_2 and R_3 can be treated as parallel resistors in series with R_1.

$$I_1 = I_2 + I_3, \quad \mathcal{E} = I_1R_1 + I_2R_2, \quad I_2R_2 = I_3R_3$$

$$\frac{\mathcal{E} - I_2R_2}{R_1} = I_2 + \frac{I_2R_2}{R_3} \rightarrow \boxed{I_2 = \frac{\mathcal{E}R_3}{R_2R_3 + R_1R_3 + R_1R_2}}$$

$$I_3 = \frac{I_2R_2}{R_3} = \boxed{\frac{\mathcal{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}} \qquad I_1 = I_2 + I_3 = \boxed{\frac{\mathcal{E}\left(R_3 + R_2\right)}{R_2R_3 + R_1R_3 + R_1R_2}}$$

(*c*) Just after the switch is opened the current through the inductor continues with the same magnitude and direction. With the open switch, no current can flow through the branch with the switch. Therefore the current through R_2 must be equal to the current through R_3, but in the opposite direction.

$$I_3 = \boxed{\frac{\mathcal{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}}, \quad I_2 = \boxed{\frac{-\mathcal{E}R_2}{R_2R_3 + R_1R_3 + R_1R_2}}, \quad I_1 = \boxed{0}$$

(*d*) After a long time, with no voltage source, the energy in the inductor will dissipate and no current will flow through any of the branches.

$$I_1 = I_2 = I_3 = \boxed{0}$$

27. (*a*) We use Eq. 30-5 to determine the emf in the inductor as a function of time. Since the exponential term decreases in time, the maximum emf occurs when $t = 0$.

$$\mathcal{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[I_0e^{-tR/L}\right] = \frac{LI_0R}{L}e^{-t/\tau} = V_0e^{-t/\tau} \rightarrow \boxed{\mathcal{E}_{\text{max}} = V_0}.$$

(*b*) The current is the same just before and just after the switch moves from A to B. We use Ohm's law for a steady state current to determine I_0 before the switch is thrown. After the switch is thrown, the same current flows through the inductor, and therefore that current will flow through the resistor *R'*. Using Kirchhoff's loop rule we calculate the emf in the inductor. This will be a maximum at $t = 0$.

$$I_0 = \frac{V_0}{R}, \qquad \mathcal{E} - IR' = 0 \rightarrow \mathcal{E} = R'\frac{V_0}{R}e^{-t/\tau'} \rightarrow \mathcal{E}_{\text{max}} = \left(\frac{R'}{R}\right)V_0 = \left(\frac{55R}{R}\right)\left(120\,\text{V}\right) = \boxed{6.6\,\text{kV}}$$

28. The steady state current is the voltage divided by the resistance while the time constant is the inductance divided by the resistance, Eq. 30-10. To cut the time constant in half, we must double the resistance. If the resistance is doubled, we must double the voltage to keep the steady state current constant.

$$R' = 2R = 2\left(2200\ \Omega\right) = \boxed{4400\ \Omega} \qquad V_0' = 2V_0 = 2\left(240\ \text{V}\right) = \boxed{480\ \text{V}}$$

29. We use Kirchhoff's loop rule in the steady state (no voltage drop across the inductor) to determine the current in the circuit just before the battery is removed. This will be the maximum current after the battery is removed. Again using Kirchhoff's loop rule, with the current given by Eq. 30-11, we calculate the emf as a function of time.

$$V - I_0R = 0 \ \rightarrow \ I_0 = \frac{V}{R}$$

$$\mathcal{E} - IR = 0 \ \rightarrow \ \mathcal{E} = I_0Re^{-t/\tau} = V\,\text{e}^{-tR/L} = \left(12\,\text{V}\right)e^{-t(2.2\,\text{k}\Omega)/(18\,\text{mH})} = \boxed{\left(12\,\text{V}\right)e^{-\left(1.22\times10^5\,\text{s}^{-1}\right)t}}$$

The emf across the inductor is greatest at $\boxed{t = 0}$ with a value of $\boxed{\mathcal{E}_{\text{max}} = 12\,\text{V}}$.

30. We use the inductance of a solenoid, as derived in Example 30-3: $L_{\text{sol}} = \dfrac{\mu_0 N^2 A}{\ell}$.

(*a*) Both solenoids have the same area and the same length. Because the wire in solenoid 1 is 1.5 times as thick as the wire in solenoid 2, solenoid 2 will have 1.5 times the number of turns as solenoid 1.

$$\frac{L_2}{L_1} = \frac{\dfrac{\mu_0 N_2^2 A}{\ell}}{\dfrac{\mu_0 N_1^2 A}{\ell}} = \frac{N_2^2}{N_1^2} = \left(\frac{N_2}{N_1}\right)^2 = 1.5^2 = 2.25 \rightarrow \boxed{\frac{L_2}{L_1} = 2.25}$$

(*b*) To find the ratio of the time constants, both the inductance and resistance ratios need to be known. Since solenoid 2 has 1.5 times the number of turns as solenoid 1, the length of wire used to make solenoid 2 is 1.5 times that used to make solenoid 1, or $\ell_{\text{wire 2}} = 1.5\ell_{\text{wire 1}}$, and the diameter of the wire in solenoid 1 is 1.5 times that in solenoid 2, or $d_{\text{wire 1}} = 1.5 d_{\text{wire 2}}$. Use this to find their relative resistances, and then the ratio of time constants.

$$\frac{R_1}{R_2} = \frac{\dfrac{\rho \ell_{\text{wire 1}}}{A_{\text{wire 1}}}}{\dfrac{\rho \ell_{\text{wire 2}}}{A_{\text{wire 2}}}} = \frac{\dfrac{\ell_{\text{wire 1}}}{\pi\left(d_{\text{wire 1}}/2\right)^2}}{\dfrac{\ell_{\text{wire 2}}}{\pi\left(d_{\text{wire 2}}/2\right)^2}} = \frac{\dfrac{\ell_{\text{wire 1}}}{d_{\text{wire 1}}^2}}{\dfrac{\ell_{\text{wire 2}}}{d_{\text{wire 2}}^2}} = \frac{\ell_{\text{wire 1}}}{\ell_{\text{wire 2}}}\left(\frac{d_{\text{wire 2}}}{d_{\text{wire 1}}}\right)^2 = \left(\frac{1}{1.5}\right)\left(\frac{1}{1.5}\right)^2 = \frac{1}{1.5^3} \rightarrow$$

$$\frac{R_1}{R_2} = \frac{1}{1.5^3} \ ; \ \frac{\tau_1}{\tau_2} = \frac{L_1/R_1}{L_2/R_2} = \frac{L_1}{L_2}\frac{R_2}{R_1} = \left(\frac{1}{2.25}\right)\left(1.5^3\right) = 1.5 \rightarrow \boxed{\frac{\tau_1}{\tau_2} = 1.5}$$

31. (*a*) The AM station received by the radio is the resonant frequency, given by Eq. 30-14. We divide the resonant frequencies to create an equation relating the frequencies and capacitances. We then solve this equation for the new capacitance.

$$\frac{f_1}{f_2} = \frac{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_1}}}{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = C_1\left(\frac{f_1}{f_2}\right)^2 = \left(1350\,\text{pF}\right)\left(\frac{550\,\text{kHz}}{1600\,\text{kHz}}\right)^2 = \boxed{0.16\,\text{nF}}$$

(*b*) The inductance is obtained from Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1}} \rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(550\times10^3\,\text{Hz}\right)^2\left(1350\times10^{-12}\,\text{F}\right)} = \boxed{62\,\mu\text{H}}$$

32. (*a*) To have maximum current and no charge at the initial time, we set $t = 0$ in Eqs. 30-13 and 30-15 to solve for the necessary phase factor ϕ.

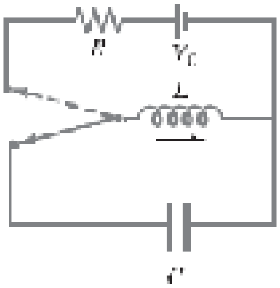
$$I_0 = I_0 \sin\phi \rightarrow \phi = \frac{\pi}{2} \rightarrow I(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos\omega t$$

$$Q(0) = Q_0 \cos\left(\frac{\pi}{2}\right) = 0 \rightarrow Q = Q_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -Q_0 \sin\left(\omega t\right)$$

Differentiating the charge with respect to time gives the negative of the current. We use this to write the charge in terms of the known maximum current.

$$I = -\frac{dQ}{dt} = -Q_0\omega\cos\left(\omega t\right) = I_0\cos\left(\omega t\right) \rightarrow Q_0 = \frac{I_0}{\omega} \rightarrow \boxed{Q(t) = \frac{I_0}{\omega}\sin\left(\omega t\right)}$$

(*b*) As in the figure, attach the inductor to a battery and resistor for an extended period so that a steady state current flows through the inductor. Then at time $t = 0$, flip the switch connecting the inductor in series to the capacitor.



33. (*a*) We write the oscillation frequency in terms of the capacitance using Eq. 30-14, with the parallel plate capacitance given by Eq. 24-2. We then solve the resulting equation for the plate separation distance.

$$2\pi f = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L(\varepsilon_0 A/x)}} \rightarrow x = \boxed{4\pi^2 A\varepsilon_0 f^2 L}$$

(*b*) For small variations we can differentiate *x* and divide the result by *x* to determine the fractional change.

$$dx = 4\pi^2 A\varepsilon_0 (2fdf)L \;\; ; \;\; \frac{dx}{x} = \frac{4\pi^2 A\varepsilon_0 (2fdf)L}{4\pi^2 A\varepsilon_0 f^2 L} = \frac{2df}{f} \rightarrow \boxed{\frac{\Delta x}{x} \approx \frac{2\Delta f}{f}}$$

(*c*) Inserting the given data, we can calculate the fractional variation on *x.*

$$\frac{\Delta x}{x} \approx \frac{2(1\text{ Hz})}{1\text{ MHz}} = 2\times 10^{-6} = \boxed{0.0002\%}$$

34. (*a*) We calculate the resonant frequency using Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.175\text{ H})(425\times 10^{-12}\text{ F})}} = 18{,}450\text{ Hz} \approx \boxed{18.5\text{ kHz}}$$

(*b*) As shown in Eq. 30-15, we set the peak current equal to the maximum charge (from Eq. 24-1) multiplied by the angular frequency.

$$I = Q_0\omega = CV(2\pi f) = (425\times 10^{-12}\text{ F})(135\text{ V})(2\pi)(18{,}450\text{ Hz})$$

$$= 6.653\times 10^{-3}\text{ A} \approx \boxed{6.65\text{ mA}}$$

(*c*) We use Eq. 30-6 to calculate the maximum energy stored in the inductor.

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(0.175\text{ H})(6.653\times 10^{-3}\text{ A})^2 = \boxed{3.87\ \mu\text{J}}$$

35. (*a*) When the energy is equally shared between the capacitor and inductor, the energy stored in the capacitor will be one half of the initial energy in the capacitor. We use Eq. 24-5 to write the energy in terms of the charge on the capacitor and solve for the charge when the energy is equally shared.

$$\frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C} \rightarrow Q = \boxed{\frac{\sqrt{2}}{2}Q_0}$$

(*b*) We insert the charge into Eq. 30-13 and solve for the time.

$$\frac{\sqrt{2}}{2}Q_0 = Q_0\cos\omega t \rightarrow t = \frac{1}{\omega}\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{T}{2\pi}\left(\frac{\pi}{4}\right) = \boxed{\frac{T}{8}}$$

36. Since the circuit loses 3.5% of its energy per cycle, it is an underdamped oscillation. We use Eq. 24-5 for the energy with the charge as a function of time given by Eq. 30-19. Setting the change in energy equal to 3.5% and using Eq. 30-18 to determine the period, we solve for the resistance.

$$\frac{\Delta E}{E}=\frac{\dfrac{Q_0^2e^{-\frac{R}{L}T}\cos^2\left(2\pi\right)}{2C}-\dfrac{Q_0^2\cos^2\left(0\right)}{2C}}{\dfrac{Q_0^2\cos^2\left(0\right)}{2C}}=e^{-\frac{R}{L}T}-1=-0.035\rightarrow\frac{RT}{L}=\ln(1-0.035)=0.03563$$

$$0.03563=\frac{R}{L}\left(\frac{2\pi}{\omega'}\right)=\frac{R}{L}\frac{2\pi}{\sqrt{1/LC-R^2/4L^2}}\rightarrow R=\sqrt{\frac{4L\left(0.03563\right)^2}{C\left[16\pi^2+\left(0.03563\right)^2\right]}}$$

$$R=\sqrt{\frac{4\left(0.065\,\text{H}\right)\left(0.03563\right)^2}{\left(1.00\times10^{-6}\,\text{F}\right)\left[16\pi^2+\left(0.03563\right)^2\right]}}=1.4457\,\Omega\approx\boxed{1.4\,\Omega}$$

$\boxed{37.}$ As in the derivation of 30-16, we set the total energy equal to the sum of the magnetic and electric energies, with the charge given by Eq. 30-19. We then solve for the time that the energy is 75% of the initial energy.

$$U=U_E+U_B=\frac{Q^2}{2C}+\frac{LI^2}{2}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\cos^2\left(\omega't+\phi\right)+\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\sin^2\left(\omega't+\phi\right)=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}$$

$$0.75\frac{Q_0^2}{2C}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\rightarrow t=-\frac{L}{R}\ln\left(0.75\right)=-\frac{L}{R}\ln\left(0.75\right)\approx\boxed{0.29\frac{L}{R}}$$

38. As shown by Eq. 30-18, adding resistance will $\boxed{\text{decrease}}$ the oscillation frequency. We use Eq. 30-14 for the pure LC circuit frequency and Eq. 30-18 for the frequency with added resistance to solve for the resistance.

$$\omega'=(1-.0025)\omega\quad\rightarrow\quad\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}=0.9975\sqrt{\frac{1}{LC}}\quad\rightarrow$$

$$R=\sqrt{\frac{4L}{C}\left(1-0.9975^2\right)}=\sqrt{\frac{4\left(0.350\,\text{H}\right)}{\left(1.800\times10^{-9}\,\text{F}\right)}\left(1-0.9975^2\right)}=\boxed{2.0\,\text{k}\Omega}$$

39. We find the frequency from Eq. 30-23b for the reactance of an inductor.

$$X_L=2\pi fL\quad\rightarrow\quad f=\frac{X_L}{2\pi L}=\frac{660\,\Omega}{2\pi\left(0.0320\,\text{H}\right)}=3283\,\text{Hz}\approx\boxed{3300\,\text{Hz}}$$

40. The reactance of a capacitor is given by Eq. 30-25b, $X_C=\dfrac{1}{2\pi fC}$.

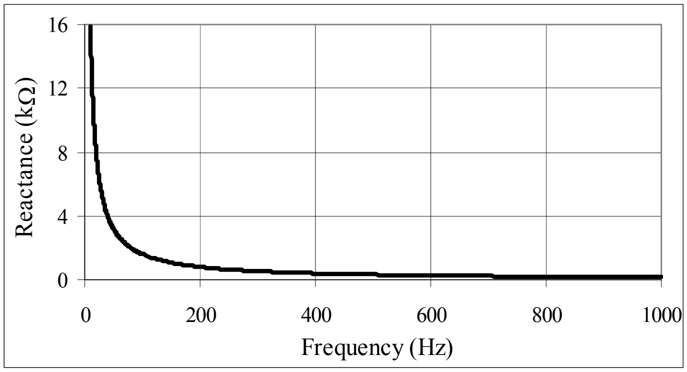
(*a*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\left(60.0\,\text{Hz}\right)\left(9.2\times10^{-6}\,\text{F}\right)}=\boxed{290\,\Omega}$$

(*b*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\left(1.00\times10^{6}\,\text{Hz}\right)\left(9.2\times10^{-6}\,\text{F}\right)}=\boxed{1.7\times10^{-2}\,\Omega}$$

41. The impedance is $X_C = \dfrac{1}{2\pi fC}$. The extreme values are as follows.

$$X_{\max} = \frac{1}{2\pi\left(10\,\text{Hz}\right)\left(1.0\times10^{-6}\,\text{F}\right)} = 16{,}000\,\Omega$$

$$X_{\min} = \frac{1}{2\pi\left(1000\,\text{Hz}\right)\left(1.0\times10^{-6}\,\text{F}\right)} = 160\,\Omega$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.41."

42. We find the reactance from Eq. 30-23b, and the current from Ohm's law.

$$X_L = 2\pi fL = 2\pi\left(33.3\times10^3\,\text{Hz}\right)\left(0.0360\,\text{H}\right) = 7532\,\Omega \approx \boxed{7530\,\Omega}$$

$$V = IX_L \quad\rightarrow\quad I = \frac{V}{X_L} = \frac{250\,\text{V}}{7532\,\Omega} = 0.03319\,\text{A} \approx \boxed{3.3\times10^{-2}\,\text{A}}$$

$\boxed{43.}$ (*a*) At $\omega = 0$, the impedance of the capacitor is infinite. Therefore the parallel combination of the resistor *R* and capacitor *C* behaves as the resistor only, and so is *R*. Thus the impedance of the entire circuit is equal to the resistance of the two series resistors.

$$Z = \boxed{R + R'}$$

(*b*) At $\omega = \infty$,, the impedance of the capacitor is zero. Therefore the parallel combination of the resistor *R* and capacitor *C* is equal to zero. Thus the impedance of the entire circuit is equal to the resistance of the series resistor only.

$$Z = \boxed{R'}$$

44. We use Eq. 30-22a to solve for the impedance.

$$V_{\text{rms}} = I_{\text{rms}}\omega L \rightarrow L = \frac{V_{\text{rms}}}{I_{\text{rms}}\omega} = \frac{110\,\text{V}}{\left(3.1\,\text{A}\right)2\pi\left(60\,\text{Hz}\right)} = \boxed{94\,\text{mH}}$$

45. (*a*) We find the reactance from Eq. 30-25b.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(660\,\text{Hz}\right)\left(8.6\times10^{-8}\,\text{F}\right)} = 2804\,\Omega \approx \boxed{2800\,\Omega}$$

(*b*) We find the peak value of the current from Ohm's law.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{X_C} = \sqrt{2}\frac{22{,}000\,\text{V}}{2804\,\Omega} = \boxed{11\,\text{A at }660\,\text{Hz}}$$

46. (*a*) Since the resistor and capacitor are in parallel, they will have the same voltage drop across them. We use Ohm's law to determine the current through the resistor and Eq. 30-25 to determine the current across the capacitor. The total current is the sum of the currents across each element.

$$I_R = \frac{V}{R} \ \ ; \ I_C = \frac{V}{X_C} = V\left(2\pi fC\right)$$

$$\frac{I_C}{I_R + I_C} = \frac{V\left(2\pi fC\right)}{V\left(2\pi fC\right) + V/R} = \frac{R\left(2\pi fC\right)}{R\left(2\pi fC\right)+1} = \frac{\left(490\ \Omega\right)2\pi\left(60\ \text{Hz}\right)\left(0.35\times10^{-6}\ \text{F}\right)}{\left(490\ \Omega\right)2\pi\left(60\ \text{Hz}\right)\left(0.35\times10^{-6}\ \text{F}\right)+1}$$

$$= 0.0607 \approx \boxed{6.1\%}$$

(*b*) We repeat part (*a*) with a frequency of 60,000 Hz.

$$\frac{I_C}{I_R + I_C} = \frac{\left(490\ \Omega\right)2\pi\left(60{,}000\ \text{Hz}\right)\left(0.35\times10^{-6}\ \text{F}\right)}{\left(490\ \Omega\right)2\pi\left(60{,}000\ \text{Hz}\right)\left(0.35\times10^{-6}\ \text{F}\right)+1} = 0.9847 \approx \boxed{98\%}$$

47. The power is only dissipated in the resistor, so we use the power dissipation equation obtained in section 25-7.

$$P_{\text{avg}} = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2}\left(1.80\,\text{A}\right)^2\left(1350\,\Omega\right) = 2187\,\text{W} \approx \boxed{2.19\,\text{kW}}$$

48. The impedance of the circuit is given by Eq. 30-28a without a capacitive reactance. The reactance of the inductor is given by Eq. 30-23b.

(*a*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{\left(10.0\times10^3\Omega\right)^2 + 4\pi^2\left(55.0\,\text{Hz}\right)^2\left(0.0260\,\text{H}\right)^2}$

$= \boxed{1.00\times10^4\Omega}$

(*b*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{\left(10.0\times10^3\Omega\right)^2 + 4\pi^2\left(5.5\times10^4\,\text{Hz}\right)^2\left(0.0260\,\text{H}\right)^2}$

$= \boxed{1.34\times10^4\Omega}$

$\boxed{49.}$ The impedance of the circuit is given by Eq. 30-28a without an inductive reactance. The reactance of the capacitor is given by Eq. 30-25b.

(*a*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{\left(75\Omega\right)^2 + \dfrac{1}{4\pi^2\left(60\,\text{Hz}\right)^2\left(6.8\times10^{-6}\text{F}\right)^2}} = 397\,\Omega$

$\approx \boxed{400\,\Omega}\left(2\ \text{sig. fig.}\right)$

(*b*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}} = \sqrt{\left(75\Omega\right)^2 + \dfrac{1}{4\pi^2\left(60000\,\text{Hz}\right)^2\left(6.8\times10^{-6}\text{F}\right)^2}} = \boxed{75\,\Omega}$

50. We find the impedance from Eq. 30-27.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{70\times10^{-3}\,\text{A}} = \boxed{1700\,\Omega}$$

51. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \left(2\pi fL\right)^2}$$

$$Z_f = 2Z_{60} \ \rightarrow \ \sqrt{R^2 + 4\pi^2 f^2 L^2} = 2\sqrt{R^2 + 4\pi^2\left(60\,\text{Hz}\right)^2 L^2} \ \rightarrow$$

$$R^2 + 4\pi^2 f^2 L^2 = 4\left[R^2 + 4\pi^2\left(60\,\text{Hz}\right)^2 L^2\right] = 4R^2 + 16\pi^2\left(60\,\text{Hz}\right)^2 L^2 \ \rightarrow$$

$$f=\sqrt{\frac{3R^2+16\pi^2(60\,\text{Hz})^2L^2}{4\pi^2L^2}}=\sqrt{\frac{3R^2}{4\pi^2L^2}+4(60\,\text{Hz})^2}=\sqrt{\frac{3(2500\,\Omega)^2}{4\pi^2(0.42\,\text{H})^2}+4(60\,\text{Hz})^2}$$

$$=1645\,\text{Hz}\approx\boxed{1.6\,\text{kHz}}$$

52. (*a*) The rms current is the rmv voltage divided by the impedance. The impedance is given by Eq. 30-28a with no inductive reactance,

$$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{(2\pi fC)^2}}\;.$$

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{V_{\text{rms}}}{\sqrt{R^2+\frac{1}{4\pi^2f^2L^2}}}=\frac{120\,\text{V}}{\sqrt{(3800\,\Omega)^2+\frac{1}{4\pi^2(60.0\,\text{Hz})^2(0.80\times10^{-6}\,\text{F})^2}}}$$

$$=\frac{120\,\text{V}}{5043\,\Omega}=2.379\times10^{-2}\,\text{A}\approx\boxed{2.4\times10^{-2}\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no inductive reactance.

$$\phi=\tan^{-1}\frac{-X_C}{R}=\tan^{-1}\frac{-\frac{1}{2\pi fC}}{R}=\tan^{-1}\frac{-\frac{1}{2\pi(60.0\,\text{Hz})(0.80\times10^{-6}\,\text{F})}}{3800\,\Omega}=\boxed{-41^\circ}$$

The current is leading the source voltage.

(*c*) The power dissipated is given by $P=I_{\text{rms}}^2R=(0.02379\text{A})^2(6.0\times10^3\Omega)=\boxed{2.2\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\text{rms}\\R}}=I_{\text{rms}}R=(2.379\times10^{-2}\text{A})(3800\,\Omega)=90.4\,\text{V}\approx\boxed{90\,\text{V}}\quad(2\text{ sig. fig.})$$

$$V_{\substack{\text{rms}\\C}}=I_{\text{rms}}X_C=\frac{I_{\text{rms}}}{2\pi fC}=\frac{(2.379\times10^{-2}\text{A})}{2\pi(60.0\,\text{Hz})(0.80\times10^{-6}\text{F})}=78.88\,\text{V}\approx\boxed{79\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

53. We use the rms voltage across the resistor to determine the rms current through the circuit. Then, using the rms current and the rms voltage across the capacitor in Eq. 30-25 we determine the frequency.

$$I_{\text{rms}}=\frac{V_{R,\text{rms}}}{R}\qquad V_{C,\text{rms}}=\frac{I_{\text{rms}}}{2\pi fC}$$

$$f=\frac{I_{\text{rms}}}{2\pi CV_{C,\text{rms}}}=\frac{V_{R,\text{rms}}}{2\pi CRV_{C,\text{rms}}}=\frac{(3.0\text{ V})}{2\pi(1.0\times10^{-6}\text{ C})(750\ \Omega)(2.7\text{ V})}=\boxed{240\text{ Hz}}$$

Since the voltages in the resistor and capacitor are not in phase, the rms voltage across the power source will not be the sum of their rms voltages.

54. The total impedance is given by Eq. 30-28a.

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

$$=\sqrt{\left(8.70\times10^3\,\Omega\right)^2+\left[2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right)-\frac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}\right]^2}$$

$$=8716.5\,\Omega\approx\boxed{8.72\,\text{k}\Omega}$$

The phase angle is given by Eq. 30-29a.

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{2\pi fL-\frac{1}{2\pi fC}}{R}$$

$$=\tan^{-1}\frac{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right)-\frac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}}{R}$$

$$=\tan^{-1}\frac{-535.9\,\Omega}{8.70\times10^3\,\Omega}=\boxed{-3.52^\circ}$$

The voltage is lagging the current, or the current is leading the voltage.
The rms current is given by Eq. 30-27.

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{725\,\text{V}}{8716.5\,\Omega}=\boxed{8.32\times10^{-2}\,\text{A}}$$

$\boxed{55.}$ (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z=\sqrt{R^2+X_L^2}=\sqrt{R^2+\left(2\pi fL\right)^2}\ .$$

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{V_{\text{rms}}}{\sqrt{R^2+4\pi^2f^2L^2}}=\frac{120\,\text{V}}{\sqrt{\left(965\,\Omega\right)^2+4\pi^2\left(60.0\,\text{Hz}\right)^2\left(0.225\,\text{H}\right)^2}}$$

$$=\frac{120\,\text{V}}{968.7\,\Omega}=\boxed{0.124\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no capacitive reactance.

$$\phi=\tan^{-1}\frac{X_L}{R}=\tan^{-1}\frac{2\pi fL}{R}=\tan^{-1}\frac{2\pi\left(60.0\,\text{Hz}\right)\left(0.225\,\text{H}\right)}{965\,\Omega}=\boxed{5.02^\circ}$$

The current is lagging the source voltage.

(*c*) The power dissipated is given by $P=I_{\text{rms}}^2R=\left(0.124\,\text{A}\right)^2\left(965\,\Omega\right)=\boxed{14.8\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\underset{R}{\text{rms}}}=I_{\text{rms}}R=\left(0.124\,\text{A}\right)\left(965\,\Omega\right)=119.7\,\text{V}\approx\boxed{120\,\text{V}}$$

$$V_{\underset{L}{\text{rms}}}=I_{\text{rms}}X_L=I_{\text{rms}}2\pi fL=\left(0.124\,\text{A}\right)2\pi\left(60.0\,\text{Hz}\right)\left(0.25\,\text{H}\right)=\boxed{10.5\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

56. (*a*) The current is found from the voltage and impedance. The impedance is given by Eq. 30-28a.

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

$$=\sqrt{\left(2.0\,\Omega\right)^2+\left[2\pi\left(60\,\text{Hz}\right)\left(0.035\,\text{H}\right)-\frac{1}{2\pi\left(60\,\text{Hz}\right)\left(26\times10^{-6}\,\text{F}\right)}\right]^2}=88.85\,\Omega$$

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{45\,\text{V}}{88.85\,\Omega}=0.5065\,\text{A}\approx\boxed{0.51\,\text{A}}$$

(*b*) Use Eq. 30-29a to find the phase angle.

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{2\pi fL-\dfrac{1}{2\pi fC}}{R}$$

$$=\tan^{-1}\frac{2\pi\left(60\,\text{Hz}\right)\left(0.035\,\text{H}\right)-\dfrac{1}{2\pi\left(60\,\text{Hz}\right)\left(26\times10^{-6}\,\text{F}\right)}}{2.0\,\Omega}=\tan^{-1}\frac{-88.83\,\Omega}{2.0\,\Omega}=\boxed{-88^\circ}$$

(*c*) The power dissipated is given by $P=I_{\text{rms}}^2R=\left(0.5065\,\text{A}\right)^2\left(2.0\,\Omega\right)=\boxed{0.51\,\text{W}}$

57. For the current and voltage to be in phase, the reactances of the capacitor and inductor must be equal. Setting the two reactances equal enables us to solve for the capacitance.

$$X_L=2\pi fL=X_C=\frac{1}{2\pi fC}\rightarrow C=\frac{1}{4\pi^2f^2L}=\frac{1}{4\pi^2\left(360\,\text{Hz}\right)^2\left(0.025\,\text{H}\right)}=\boxed{7.8\,\mu\text{F}}$$

58. The light bulb acts like a resistor in series with the inductor. Using the desired rms voltage across the resistor and the power dissipated by the light bulb we calculate the rms current in the circuit and the resistance. Then using this current and the rms voltage of the circuit we calculate the impedance of the circuit (Eq. 30-27) and the required inductance (Eq. 30-28b).

$$I_{\text{rms}}=\frac{P}{V_{R,\text{rms}}}=\frac{75\,\text{W}}{120\,\text{V}}=0.625\,\text{A}\qquad R=\frac{V_{R,\text{rms}}}{I_{\text{rms}}}=\frac{120\,\text{V}}{0.625\,\text{A}}=192\,\Omega$$

$$Z=\frac{V_{\text{rms}}}{I_{\text{rms}}}=\sqrt{R^2+\left(2\pi fL\right)^2}$$

$$L=\frac{1}{2\pi f}\sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2-R^2}=\frac{1}{2\pi\left(60\text{ Hz}\right)}\sqrt{\left(\frac{240\text{ V}}{0.625\text{ A}}\right)^2-\left(192\text{ }\Omega\right)^2}=\boxed{0.88\text{ H}}$$

59. We multiply the instantaneous current by the instantaneous voltage to calculate the instantaneous power. Then using the trigonometric identity for the summation of sine arguments (inside back cover of text) we can simplify the result. We integrate the power over a full period and divide the result by the period to calculate the average power.

$$P=IV=\left(I_0\sin\omega t\right)V_0\sin\left(\omega t+\phi\right)=I_0V_0\sin\omega t\left(\sin\omega t\cos\phi+\sin\phi\cos\omega t\right)$$

$$=I_0V_0\left(\sin^2\omega t\cos\phi+\sin\omega t\cos\omega t\sin\phi\right)$$

$$\bar{P}=\frac{1}{T}\int_0^T P\,dT=\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0V_0\left(\sin^2\omega t\cos\phi+\sin\omega t\cos\omega t\sin\phi\right)dt$$

$$=\frac{\omega}{2\pi}I_0V_0\cos\phi\int_0^{\frac{2\pi}{\omega}}\sin^2\omega t\;dt+\frac{\omega}{2\pi}I_0V_0\sin\phi\int_0^{\frac{2\pi}{\omega}}\sin\omega t\cos\omega t\;dt$$

$$=\frac{\omega}{2\pi}I_0V_0\cos\phi\left(\frac{1}{2}\frac{2\pi}{\omega}\right)+\frac{\omega}{2\pi}I_0V_0\sin\phi\left(\frac{1}{\omega}\sin^2\omega t\Big|_0^{\frac{2\pi}{\omega}}\right)=\boxed{\tfrac{1}{2}I_0V_0\cos\phi}$$

60. Given the resistance, inductance, capacitance, and frequency, we calculate the impedance of the circuit using Eq. 30-28b.

$$X_L=2\pi fL=2\pi(660\text{ Hz})(0.025\text{ H})=103.67\ \Omega$$

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(660\text{ Hz})(2.0\times10^{-6}\text{ F})}=120.57\ \Omega$$

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(150\ \Omega)^2+(103.67\ \Omega-120.57\ \Omega)^2}=150.95\ \Omega$$

(*a*) From the impedance and the peak voltage we calculate the peak current, using Eq. 30-27.

$$I_0=\frac{V_0}{Z}=\frac{340\text{ V}}{150.95\ \Omega}=2.252\text{ A}\approx\boxed{2.3\text{ A}}$$

(*b*) We calculate the phase angle of the current from the source voltage using Eq. 30-29a.

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{103.67\ \Omega-120.57\ \Omega}{150\ \Omega}=\boxed{-6.4^\circ}$$

(*c*) We multiply the peak current times the resistance to obtain the peak voltage across the resistor. The voltage across the resistor is in phase with the current, so the phase angle is the same as in part (*b*).

$$V_{0,R}=I_0R=(2.252\text{ A})(150\ \Omega)=\boxed{340\text{ V}}\ ;\ \boxed{\phi=-6.4^\circ}$$

(*d*) We multiply the peak current times the inductive reactance to calculate the peak voltage across the inductor. The voltage in the inductor is 90º ahead of the current. Subtracting the phase difference between the current and source from the 90º between the current and inductor peak voltage gives the phase angle between the source voltage and the inductive peak voltage.

$$V_{0,L}=I_0X_L=(2.252\text{ A})(103.67\ \Omega)=\boxed{230\text{ V}}$$

$$\phi_L=90.0^\circ-\phi=90.0^\circ-(-6.4)^\circ=\boxed{96.4^\circ}$$

(*e*) We multiply the peak current times the capacitive reactance to calculate the peak voltage across the capacitor. Subtracting the phase difference between the current and source from the -90º between the current and capacitor peak voltage gives the phase angle between the source voltage and the capacitor peak voltage.

$$V_{0,C}=I_0X_C=(2.252\text{ A})(120.57\ \Omega)=\boxed{270\text{ V}}$$

$$\phi_C=-90.0^\circ-\phi=-90.0^\circ-(-6.4^\circ)=\boxed{-83.6^\circ}$$

$\boxed{61.}$ Using Eq. 30-23b we calculate the impedance of the inductor. Then we set the phase shift in Eq. 30-29a equal to 25º and solve for the resistance. We calculate the output voltage by multiplying the current through the circuit, from Eq. 30-27, by the inductive reactance (Eq. 30-23b).

$$X_L=2\pi fL=2\pi(175\text{ Hz})(0.055\text{ H})=60.48\ \Omega$$

$$\tan\phi=\frac{X_L}{R}\Longrightarrow R=\frac{X_L}{\tan\phi}=\frac{60.48\ \Omega}{\tan\ 25^\circ}=129.7\Omega\approx\boxed{130\ \Omega}$$

$$\frac{V_{\text{output}}}{V_0}=\frac{V_R}{V_0}=\frac{IR}{IZ}=\frac{R}{Z}=\frac{129.70\,\Omega}{\sqrt{(129.70\,\Omega)^2+(60.48\,\Omega)^2}}=\boxed{0.91}$$

62. The resonant frequency is found from Eq. 30-32. The resistance does not influence the resonant frequency.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\frac{1}{2\pi}\sqrt{\frac{1}{(26.0\times10^{-6}\,\text{H})(3800\times10^{-12}\,\text{F})}}=\boxed{5.1\times10^5\,\text{Hz}}$$

63. We calculate the resonant frequency using Eq. 30-32 with the inductance and capacitance given in the example. We use Eq. 30-30 to calculate the power dissipation, with the impedance equal to the resistance.

$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(0.0300\text{ H})(12.0\times10^{-6}\text{ F})}}=\boxed{265\text{ Hz}}$$

$$\bar{P}=I_{\text{rms}}V_{\text{rms}}\cos\phi=\left(\frac{V_{\text{rms}}}{R}\right)V_{\text{rms}}\left(\frac{R}{R}\right)=\frac{V^2_{\text{rms}}}{R}=\frac{(90.0\,\text{V})^2}{25.0\,\Omega}=\boxed{324\,\text{W}}$$

64. (*a*) We find the capacitance from the resonant frequency, Eq. 30-32.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}\rightarrow C=\frac{1}{4\pi^2Lf_0^2}=\frac{1}{4\pi^2(4.15\times10^{-3}\,\text{H})(33.0\times10^3\,\text{Hz})^2}=\boxed{5.60\times10^{-9}\,\text{F}}$$

(*b*) At resonance the impedance is the resistance, so the current is given by Ohm's law.

$$I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{136\,\text{V}}{3800\,\Omega}=\boxed{35.8\,\text{mA}}$$

65. (*a*) The peak voltage across the capacitor is the peak current multiplied by the capacitive reactance. We calculate the current in the circuit by dividing the source voltage by the impedance, where at resonance the impedance is equal to the resistance.

$$V_{C0}=X_CI_0=\frac{1}{2\pi f_0C}\frac{V_0}{R}=\frac{V_0}{2\pi(RC)}\frac{1}{f_0}=\boxed{\frac{V_0}{2\pi\tau}T_0}$$

(*b*) We set the amplification equal to 125 and solve for the resistance.

$$\beta=\frac{T_0}{2\pi\tau}=\frac{1}{2\pi f_0RC}\rightarrow R=\frac{1}{2\pi f_0\beta C}=\frac{1}{2\pi(5000\,\text{Hz})(125)(2.0\times10^{-9}\,\text{F})}=\boxed{130\,\Omega}$$

66. (*a*) We calculate the resonance frequency from the inductance and capacitance using Eq.30-32.

$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(0.055\text{ H})(1.0\times10^{-9}\text{ F})}}=21460\text{ Hz}\approx\boxed{21\text{ kHz}}$$

(*b*) We use the result of Problem 65 to calculate the voltage across the capacitor.

$$V_{C0}=\frac{V_0}{2\pi(RC)}\frac{1}{f_0}=\frac{2.0\text{ V}}{2\pi(35\ \Omega)(1.0\times10^{-9}\text{ F})(21460\text{ Hz})}=\boxed{420\text{ V}}$$

(*c*) We divide the voltage across the capacitor by the voltage source.

$$\frac{V_{C0}}{V_0}=\frac{420\text{ V}}{2.0\text{ V}}=\boxed{210}$$

67. (*a*) We write the average power using Eq. 30-30, with the current in terms of the impedance (Eq. 30-27) and the power factor in terms of the resistance and impedance (Eq. 30-29b). Finally we write the impedance using Eq. 30-28b.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{V_{\text{rms}}}{Z}V_{\text{rms}}\frac{R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2} = \boxed{\frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]}}$$

(*b*) The power dissipation will be a maximum when the inductive reactance is equal to the capacitive reactance, which is the resonant frequency.

$$f = \boxed{\frac{1}{2\pi\sqrt{LC}}}$$

(*c*) We set the power dissipation equal to ½ of the maximum power dissipation and solve for the angular frequencies.

$$\bar{P} = \frac{1}{2}\bar{P}_{\max} = \frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]} = \frac{1}{2}\left(\frac{V_0^2 R}{2R^2}\right) \rightarrow \left(\omega L - 1/\omega C\right) = \pm R$$

$$\rightarrow\ 0 = \omega^2 LC \pm RC\omega - 1\ \rightarrow\ \omega = \frac{\pm RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

We require the angular frequencies to be positive and for a sharp peak, $R^2C^2 \ll 4LC$. The angular width will then be the difference between the two positive frequencies.

$$\omega = \frac{2\sqrt{LC} \pm RC}{2LC} = \frac{1}{\sqrt{LC}} \pm \frac{R}{2L}\ \rightarrow\ \Delta\omega = \left(\frac{1}{\sqrt{LC}} + \frac{R}{2L}\right) - \left(\frac{1}{\sqrt{LC}} - \frac{R}{2L}\right) = \boxed{\frac{R}{L}}$$

68. (*a*) We write the charge on the capacitor using Eq. 24-1, where the voltage drop across the capacitor is the inductive capacitance multiplied by the circuit current (Eq. 30-25a) and the circuit current is found using the source voltage and circuit impedance (Eqs. 30-27 and 30-28b).

$$Q_0 = CV_{C0} = CI_0X_C = C\left(\frac{V_0}{Z}\right)X_C = \frac{CV_0}{\omega C\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}} = \boxed{\frac{V_0}{\sqrt{\omega^2R^2 + \left(\omega^2 L - 1/C\right)^2}}}$$

(*b*) We set the derivative of the charge with respect to the frequency equal to zero to calculate the frequency at which the charge is a maximum.

$$\frac{dQ_0}{d\omega} = \frac{d}{d\omega}\frac{V_0}{\sqrt{\omega'^2R^2 + \left(\omega'^2L - 1/C\right)^2}} = \frac{-V_0\left(2\omega' R^2 + 4\omega'^3L^2 - 4\omega' L/C\right)}{\left[\omega'^2R^2 + \left(\omega'^2L - 1/C\right)^2\right]^{\frac{3}{2}}} = 0$$

$$\rightarrow \omega' = \boxed{\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

(*c*) The amplitude in a forced damped harmonic oscillation is given by Eq. 14-23. This is equivalent to the *LRC* circuit with $F_0 \leftrightarrow V_0,\ k \leftrightarrow 1/C,\ \ m \leftrightarrow L,$ and $b \leftrightarrow R.$

69. Since the circuit is in resonance, we use Eq. 30-32 for the resonant frequency to determine the necessary inductance. We set this inductance equal to the solenoid inductance calculated in Example 30-3, with the area equal to the area of a circle of radius *r*, the number of turns equal to the length of the wire divided by the circumference of a turn, and the length of the solenoid equal to the diameter of the wire multiplied by the number of turns. We solve the resulting equation for the number of turns.

$$f_0=\frac{1}{2\pi\sqrt{LC}}\quad\rightarrow\quad L=\frac{1}{4\pi^2 f_o^2 C}=\frac{\mu_0 N^2 A}{\ell}=\frac{\mu_0\left(\dfrac{\ell_{wire}}{2\pi r}\right)^2\pi r^2}{Nd}\quad\rightarrow$$

$$N=\frac{\pi f_o^2 C\mu_0 \ell^2{}_{wire}}{d}=\frac{\pi\left(18.0\times10^3\,\text{Hz}\right)^2\left(2.20\times10^{-7}\,\text{F}\right)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(12.0\,\text{m}\right)^2}{1.1\times10^{-3}\,\text{m}}=\boxed{37\text{ loops}}$$

70. The power on each side of the transformer must be equal. We replace the currents in the power equation with the number of turns in the two coils using Eq. 29-6. Then we solve for the turn ratio.

$$P_p=I_p^2 Z_p=P_s=I_s^2 Z_s\rightarrow\frac{Z_p}{Z_s}=\left(\frac{I_s}{I_p}\right)^2=\left(\frac{N_p}{N_s}\right)^2$$

$$\rightarrow\frac{N_p}{N_s}=\sqrt{\frac{Z_p}{Z_s}}=\sqrt{\frac{45\times10^3\ \Omega}{8.0\ \Omega}}=\boxed{75}$$

71. (*a*) We calculate the inductance from the resonance frequency.

$$f_0=\frac{1}{2\pi\sqrt{LC}}\quad\rightarrow$$

$$L=\frac{1}{4\pi^2 f_o^2 C}=\frac{1}{4\pi^2\left(17\times10^3\,\text{Hz}\right)^2\left(2.2\times10^{-9}\,\text{F}\right)}=0.03982\,\text{H}\approx\boxed{0.040\,\text{H}}$$

(*b*) We set the initial energy in the electric field, using Eq. 24-5, equal to the maximum energy in the magnetic field, Eq. 30-6, and solve for the maximum current.

$$\tfrac{1}{2}CV_0^2=\tfrac{1}{2}LI_{\max}^2\quad\rightarrow\quad I_{\max}=\sqrt{\frac{CV_0^2}{L}}=\sqrt{\frac{\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2}{\left(0.03984\,\text{H}\right)}}=\boxed{0.028\,\text{A}}$$

(*c*) The maximum energy in the inductor is equal to the initial energy in the capacitor.

$$U_{L,\max}=\tfrac{1}{2}CV_0^2=\tfrac{1}{2}\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2=\boxed{16\,\mu\text{J}}$$

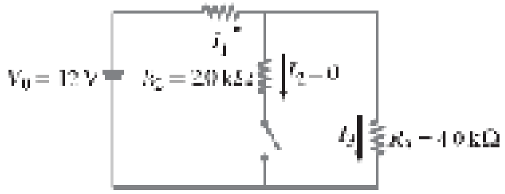
72. We use Eq. 30-6 to calculate the initial energy stored in the inductor.

$$U_0=\tfrac{1}{2}LI_0^2=\tfrac{1}{2}\left(0.0600\,\text{H}\right)\left(0.0500\,\text{A}\right)^2=\boxed{7.50\times10^{-5}\,\text{J}}$$

We set the energy in the inductor equal to five times the initial energy and solve for the current. We set the current equal to the initial current plus the rate of increase multiplied by time and solve for the time.

$$U=\tfrac{1}{2}LI^2\quad\rightarrow\quad I=\sqrt{\frac{2U}{L}}=\sqrt{\frac{2\left(5.0\times7.50\times10^{-5}\,\text{J}\right)}{0.0600\,\text{H}}}=111.8\,\text{mA}$$

$$I=I_0+\beta t\quad\rightarrow\quad t=\frac{I-I_0}{\beta}=\frac{111.8\,\text{mA}-50.0\,\text{mA}}{78.0\,\text{mA/s}}=\boxed{0.79\,\text{s}}$$



$\boxed{73.}$ When the currents have acquired their steady-state values, the capacitor will be fully charged, and so no current will flow through the capacitor. At this time, the voltage drop across the inductor will be zero, as the current flowing through the inductor is constant. Therefore, the current through R_1 is zero, and the resistors R_2 and R_3 can be treated as in series.

$$I_1 = I_3 = \frac{V_0}{R_1 + R_3} = \frac{12\,\text{V}}{5.0\,\text{k}\Omega} = \boxed{2.4\,\text{mA}} \ ; \ I_2 = \boxed{0}$$

74. (*a*) The self inductance is written in terms of the magnetic flux in the toroid using Eq. 30-4. We set the flux equal to the magnetic field of a toroid, from Example 28-10. The field is dependent upon the radius of the solenoid, but if the diameter of the solenoid loops is small compared with the radius of the solenoid, it can be treated as approximately constant.

$$L = \frac{N\Phi_B}{I} = \frac{N\left(\pi d^2/4\right)\left(\mu_0 NI/2\pi r_0\right)}{I} = \boxed{\frac{\mu_0 N^2 d^2}{8r_0}}$$

This is consistent with the inductance of a solenoid for which the length is $\ell = 2\pi r_0$.

(*b*) We calculate the value of the inductance from the given data, with r_0 equal to half of the diameter.

$$L = \frac{\mu_0 N^2 d^2}{8r_0} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\bullet\text{m/A}\right)\left(550\right)^2\left(0.020\ \text{m}\right)^2}{8\left(0.33\ \text{m}\right)} = \boxed{58\ \mu\text{H}}$$

75. We use Eq. 30-4 to calculate the self inductance between the two wires. We calculate the flux by integrating the magnetic field from the two wires, using Eq. 28-1, over the region between the two wires. Dividing the inductance by the length of the wire gives the inductance per unit length.

$$L = \frac{\Phi_B}{I} = \frac{1}{I}\int_r^{\ell-r}\left[\frac{\mu_0 I}{2\pi r'} + \frac{\mu_0 I}{2\pi\left(\ell - r'\right)}\right]h\,dr' = \frac{\mu_0 h}{2\pi}\int_r^{\ell-r}\left[\frac{1}{r'} + \frac{1}{\left(\ell - r'\right)}\right]dr'$$

$$\frac{L}{h} = \frac{\mu_0}{2\pi}\left[\ln\left(r'\right) - \ln\left(\ell - r'\right)\right]_r^{\ell-r} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{\ell - r}{r}\right) - \ln\left(\frac{r}{\ell - r}\right)\right] = \boxed{\frac{\mu_0}{\pi}\ln\left(\frac{\ell - r}{r}\right)}$$

76. The magnetic energy is the energy density (Eq. 30-7) multiplied by the volume of the spherical shell enveloping the earth.

$$U = u_B V = \frac{B^2}{2\mu_0}\left(4\pi r^2 h\right) = \frac{\left(0.50 \times 10^{-4}\text{T}\right)^2}{2\left(4\pi \times 10^{-7}\,\text{T}\bullet\text{m/A}\right)}\left[4\pi\left(6.38 \times 10^6\text{m}\right)^2\left(5.0 \times 10^3\text{m}\right)\right] = \boxed{2.5 \times 10^{15}\text{J}}$$

77. (*a*) For underdamped oscillation, the charge on the capacitor is given by Eq. 30-19, with $\phi = 0$. Differentiating the current with respect to time gives the current in the circuit.

$$Q(t) = Q_0 e^{-\frac{R}{2L}t}\cos\omega' t \ ; \ I(t) = \frac{dQ}{dt} = -Q_0 e^{-\frac{R}{2L}t}\left(\frac{R}{2L}\cos\omega' t + \omega'\sin\omega' t\right)$$

The total energy is the sum of the energies stored in the capacitor (Eq. 24-5) and the energy stored in the inductor (Eq. 30-6). Since the oscillation is underdamped ($\omega' \gg R/2L$), the cosine term in the current is much smaller than the sine term and can be ignored. The frequency of oscillation is approximately equal to the undamped frequency of Eq. 30-14.

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{LI^2}{2} = \frac{\left(Q_0 e^{-\frac{R}{2L}t}\cos\omega' t\right)^2}{2C} + \frac{L\left(Q_0 e^{-\frac{R}{2L}t}\right)^2\left(\omega'\sin\omega' t\right)^2}{2}$$

$$= \frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\left(\cos^2\omega' t + \omega'^2 LC\sin^2\omega' t\right) \approx \boxed{\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}}$$

(*b*) We differentiate the energy with respect to time to show the average power dissipation. We then set the power loss per cycle equal to the resistance multiplied by the square of the current. For a lightly damped oscillation, the exponential term does not change much in one cycle, while the sine squared term averages to ½ .

$$\frac{dU}{dt}=\frac{d}{dt}\left(\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\right)=-\frac{RQ_0^2 e^{-\frac{R}{L}t}}{2LC}$$

$$P=-I^2R=-Q_0^2 e^{-\frac{R}{L}t}\left(\omega'^2\sin^2\omega' t\right)\approx -Q_0^2 e^{-\frac{R}{L}t}\left(\frac{1}{LC}\right)\left(\frac{1}{2}\right)=\boxed{-\frac{RQ_0^2 e^{-\frac{R}{L}t}}{2LC}}$$

The change in power in the circuit is equal to the power dissipated by the resistor.

78. Putting an inductor in series with the device will protect it from sudden surges in current. The growth of current in an *LR* circuit is given is Eq. 30-9.

$$I=\frac{V}{R}\left(1-e^{-tR/L}\right)=I_{\max}\left(1-e^{-tR/L}\right)$$

The maximum current is 33 mA, and the current is to have a value of 7.5 mA after a time of 75 microseconds. Use this data to solve for the inductance.

$$I=I_{\max}\left(1-e^{-tR/L}\right)\;\rightarrow\; e^{-tR/L}=1-\frac{I}{I_{\max}}\;\rightarrow$$

$$L=-\frac{tR}{\ln\left(1-\frac{I}{I_{\max}}\right)}=-\frac{\left(75\times10^{-6}\,\text{sec}\right)\left(150\,\Omega\right)}{\ln\left(1-\frac{7.5\,\text{mA}}{33\,\text{mA}}\right)}=4.4\times10^{-2}\,\text{H}$$

$\boxed{\text{Put an inductor of value } 4.4\times10^{-2}\,\text{H in series with the device.}}$

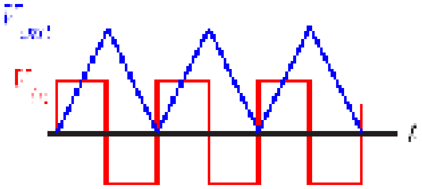
$\boxed{79.}$ We use Kirchhoff's loop rule to equate the input voltage to the voltage drops across the inductor and resistor. We then multiply both sides of the equation by the integrating factor $e^{\frac{Rt}{L}}$ and integrate the right-hand side of the equation using a *u* substitution with $u=IRe^{\frac{Rt}{L}}$ and $du=dIRe^{\frac{Rt}{L}}+Ie^{\frac{Rt}{L}}dt/L$

$$V_{in}=L\frac{dI}{dt}+IR\;\rightarrow$$

$$\int V_{in}e^{\frac{Rt}{L}}dt=\int\left(L\frac{dI}{dt}+IR\right)e^{\frac{Rt}{L}}dt=\frac{L}{R}\int du=IR\frac{L}{R}e^{\frac{Rt}{L}}=V_{out}\frac{L}{R}e^{\frac{Rt}{L}}$$

For $L/R<<t$, $e^{\frac{Rt}{L}}\approx 1$. Setting the exponential term equal to unity on both sides of the equation gives the desired results.

$$\int V_{in}dt=V_{out}\frac{L}{R}$$



80. (*a*) Since the capacitor and resistor are in series, the impedance of the circuit is given by Eq. 30-28a. Divide the source voltage by the impedance to determine the current in the circuit. Finally, multiply the current by the resistance to determine the voltage drop across the resistor.

$$V_R=IR=\frac{V_{in}}{Z}R=\frac{V_{in}R}{\sqrt{R^2+1/\left(2\pi fC\right)^2}}$$

$$=\frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2+1\Big/\left[2\pi(60\text{ Hz})\left(1.2\times10^{-6}\text{ F}\right)\right]^2}}=\boxed{31\text{ mV}}$$

(*b*) Repeat the calculation with a frequency of 6.0 kHz.

$$V_R=\frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2+1\Big/\left[2\pi(6000\text{ Hz})\left(1.2\times10^{-6}\text{ F}\right)\right]^2}}=\boxed{130\text{ mV}}$$

Thus the capacitor allows the higher frequency to pass, but attenuates the lower frequency.

81. (*a*) We integrate the power directly from the current and voltage over one cycle.

$$\bar{P}=\frac{1}{T}\int_0^T IVdt=\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}I_0\sin(\omega t)V_0\sin(\omega t+90^\circ)dt=\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}I_0\sin(\omega t)V_0\cos(\omega t)dt$$

$$=\frac{\omega}{2\pi}I_0V_0\left.\frac{\sin^2(\omega t)}{2\omega}\right|_0^{\frac{2\pi}{\omega}}=\frac{I_0V_0}{4\pi}\left[\sin^2\left(\cancel{\omega}\frac{2\pi}{\cancel{\omega}}\right)-\sin^2(0)\right]=\boxed{0}$$

(*b*) We apply Eq. 30-30, with $\phi=90^\circ$.

$$\bar{P}=I_{\rm rms}V_{\rm rms}\cos 90^\circ=\boxed{0}$$

As expected the average power is the same for both methods of calculation.

82. Since the current lags the voltage one of the circuit elements must be an $\boxed{\text{inductor}}$. Since the angle is less than 90º, the other element must be a $\boxed{\text{resistor}}$. We use 30-29a to write the resistance in terms of the impedance. Then using Eq. 30-27 to determine the impedance from the voltage and current and Eq. 30-28b, we solve for the unknown inductance and resistance.

$$\tan\phi=\frac{2\pi fL}{R}\to R=2\pi fL\cot\phi$$

$$Z=\frac{V_{\rm rms}}{I_{\rm rms}}=\sqrt{R^2+(2\pi fL)^2}=\sqrt{(2\pi fL\cot\phi)^2+(2\pi fL)^2}=2\pi fL\sqrt{1+\cot^2\phi}$$

$$L=\frac{V_{\rm rms}}{2\pi f\,I_{\rm rms}\sqrt{1+\cot^2\phi}}=\frac{120\text{V}}{2\pi(60\text{Hz})(5.6\text{A})\sqrt{1+\cot^2 65^\circ}}=51.5\text{mH}\approx\boxed{52\text{mH}}$$

$$R=2\pi f\,L\cot\phi=2\pi(60\text{Hz})(51.5\text{mH})\cot 65^\circ=\boxed{9.1\Omega}$$

83. We use Eq. 30-28b to calculate the impedance at 60 Hz. Then we double that result and solve for the required frequency.

$$Z_0=\sqrt{R^2+(2\pi f_0L)^2}=\sqrt{(3500\Omega)^2+\left[2\pi(60\text{Hz})(0.44\text{H})\right]^2}=3504\Omega$$

$$2Z_0=\sqrt{R^2+(2\pi fL)^2}\to f=\frac{\sqrt{4Z_0^2-R^2}}{2\pi L}=\frac{\sqrt{4(3504\Omega)^2-(3500\Omega)^2}}{2\pi(0.44\text{H})}=\boxed{2.2\text{kHz}}$$

84. (*a*) We calculate capacitive reactance using Eq. 30-25b. Then using the resistance and capacitive reactance we calculate the impedance. Finally, we use Eq. 30-27 to calculate the rms current.

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(60.0\text{Hz})\left(1.80\times10^{-6}\text{F}\right)}=1474\Omega$$

$$Z=\sqrt{R^2+X_C^2}=\sqrt{(5700\Omega)^2+(1474\Omega)^2}=5887\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\,\text{V}}{5887\,\Omega} = 20.38\,\text{mA} \approx \boxed{20.4\,\text{mA}}$$

(*b*) We calculate the phase angle using Eq. 30-29a.

$$\phi = \tan^{-1}\frac{-X_C}{R} = \tan^{-1}\frac{-1474\,\Omega}{5700\,\Omega} = \boxed{-14.5^\circ}$$

(*c*) The average power is calculated using Eq. 30-30.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (0.0204\,\text{A})(120\,\text{V})\cos(-14.5^\circ) = \boxed{2.37\,\text{W}}$$

(*d*) The voltmeter will read the rms voltage across each element. We calculate the rms voltage by multiplying the rms current through the element by the resistance or capacitive reactance.

$$V_R = I_{\text{rms}}R = (20.38\,\text{mA})(5.70\,\text{k}\Omega) = \boxed{116\,\text{V}}$$

$$V_C = I_{\text{rms}}X_C = (20.38\,\text{mA})(1474\,\Omega) = \boxed{30.0\,\text{V}}$$

Note that since the voltages are out of phase they do not sum to the applied voltage. However, since they are 90º out of phase their squares sum to the square of the input voltage.

$\boxed{85.}$ We find the resistance using Ohm's law with the dc voltage and current. When then calculate the impedance from the ac voltage and current, and using Eq. 30-28b.

$$R = \frac{V}{I} = \frac{45\,\text{V}}{2.5\,\text{A}} = \boxed{18\,\Omega}\ ;\ Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{3.8\,\text{A}} = 31.58\,\Omega$$

$$\sqrt{R^2 + (2\pi fL)^2} \rightarrow L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} = \frac{\sqrt{(31.58\,\Omega)^2 - (18\,\Omega)^2}}{2\pi(60\,\text{Hz})} = \boxed{69\,\text{mH}}$$

86. (*a*) From the text of the problem, the *Q* factor is the ratio of the voltage across the capacitor or inductor to the voltage across the resistor, at resonance. The resonant frequency is given by Eq. 30-32.

$$Q = \frac{V_L}{V_R} = \frac{I_{\text{res}}X_L}{I_{\text{res}}R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\frac{1}{2\pi}\sqrt{\frac{1}{LC}}L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

(*b*) Find the inductance from the resonant frequency, and the resistance from the *Q* factor.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \rightarrow$$

$$L = \frac{1}{4\pi^2 C f_0^2} = \frac{1}{4\pi^2(1.0\times10^{-8}\,\text{F})(1.0\times10^{6}\,\text{Hz})^2} = 2.533\times10^{-6}\,\text{H} \approx \boxed{2.5\times10^{-6}\,\text{H}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \rightarrow R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{350}\sqrt{\frac{2.533\times10^{-6}\,\text{H}}{1.0\times10^{-8}\,\text{F}}} = \boxed{4.5\times10^{-2}\,\Omega}$$

87. We calculate the period of oscillation as 2π divided by the angular frequency. Then set the total energy of the system at the beginning of each cycle equal to the charge on the capacitor as given by Eq. 24-5, with the charge given by Eq. 30-19, with $\cos(\omega' t + \phi) = \cos[\omega'(t+T)+\phi] = 1$. We take the difference in energies at the beginning and end of a cycle, divided by the initial energy. For small damping, the argument of the resulting exponential term is small and we replace it with the first two terms of the Taylor series expansion.

$$T=\frac{2\pi}{\omega'}\approx\frac{2\pi}{\omega}\qquad U_{\max}=\frac{Q_0^2e^{-\frac{R}{L}t}\cos^2\left(\omega't+\phi\right)}{2C}=\frac{Q_0^2e^{-\frac{R}{L}t}}{2C}$$

$$\frac{\Delta U}{U}=\frac{Q_0^2e^{-\frac{R}{L}t}-Q_0^2e^{-\frac{R}{L}\left(t+\frac{2\pi}{\omega}\right)}}{Q_0^2e^{-\frac{R}{L}t}}=1-e^{-\frac{2\pi R}{\omega L}}\approx1-\left(1-\frac{2\pi R}{\omega L}\right)=\frac{2\pi R}{\omega L}=\boxed{\frac{2\pi}{Q}}$$

88. We set the power factor equal to the resistance divided by the impedance (Eq. 30-28a) with the impedance written in terms of the angular frequency (Eq. 30-28b). We rearrange the resulting equation to form a quadratic equation in terms of the angular frequency. We divide the positive angular frequencies by 2π to determine the desired frequencies.

$$\cos\phi=\frac{R}{Z}=\frac{R}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}\quad\rightarrow\quad\omega^2LC\pm\omega C\sqrt{R^2\left(\frac{1}{\cos^2\phi}-1\right)}-1=0$$

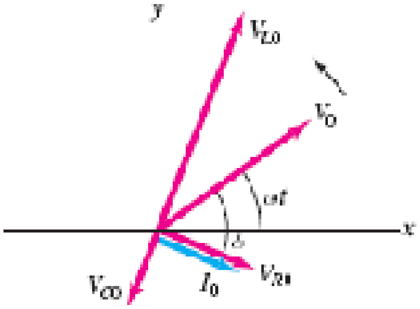
$$\omega^2\left(0.033\,\text{H}\right)\left(55\times10^{-9}\,\text{F}\right)\pm\omega\left(55\times10^{-9}\,\text{F}\right)\sqrt{\left(1500\,\Omega\right)^2\left(\frac{1}{0.17^2}-1\right)}-1=0$$

$$\left(1.815\times10^{-9}\,\text{F}\bullet\text{H}\right)\omega^2\pm\left(4.782\times10^{-4}\,\Omega\bullet\text{F}\right)\omega-1=0$$

$$\omega=\frac{\pm4.78225\times10^{-4}\,\Omega\bullet\text{F}\pm4.85756\times10^{-4}\,\Omega\bullet\text{F}}{3.63\times10^{-9}\,\text{F}\bullet\text{H}}=\pm2.65\times10^5\,\text{rad/s},\ \pm2.07\times10^3\,\text{rad/s}$$

$$f=\frac{\omega}{2\pi}=\frac{2.65\times10^5\,\text{rad/s}}{2\pi}=\boxed{42\,\text{kHz}}\quad\text{and}\quad\frac{2.07\times10^3\,\text{rad/s}}{2\pi}=\boxed{330\,\text{Hz}}$$

89. (*a*) We set $V=V_0\sin\omega t$ and assume the inductive reactance is greater than the capacitive reactance. The current will lag the voltage by an angle ϕ. The voltage across the resistor is in phase with the current and the voltage across the inductor is 90° ahead of the current. The voltage across the capacitor is smaller than the voltage in the inductor, and antiparallel to it.



(*b*) From the diagram, the current is the projection of the maximum current onto the *y* axis, with the current lagging the voltage by the angle ϕ. This is the same angle obtained in Eq. 30-29a. The magnitude of the maximum current is the voltage divided by the impedance, Eq. 30-28b.

$$I(t)=I_0\sin\left(\omega t-\phi\right)=\boxed{\frac{V_0}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}\sin\left(\omega t-\phi\right)}\ ;\ \phi=\tan^{-1}\frac{\omega L-1/\omega C}{R}$$

90. (*a*) We use Eq. 30-28b to calculate the impedance and Eq. 30-29a to calculate the phase angle.

$$X_L=\omega L=\left(754\,\text{rad/s}\right)\left(0.0220\,\text{H}\right)=16.59\,\Omega$$

$$X_C=1/\omega C=1\Big/\left(754\,\text{rad/s}\right)\left(0.42\times10^{-6}\,\text{F}\right)=3158\,\Omega$$

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{\left(23.2\times10^3\,\Omega\right)^2+\left[16.59\,\Omega-3158\,\Omega\right]^2}=\boxed{23.4\,\text{k}\Omega}$$

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{16.59\,\Omega-3158\,\Omega}{23.2\times10^3\,\Omega}=\boxed{-7.71^\circ}$$

(*b*) We use Eq. 30-30 to obtain the average power. We obtain the rms voltage by dividing the maximum voltage by $\sqrt{2}$. The rms current is the rms voltage divided by the impedance.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{V_{\text{rms}}^2}{Z}\cos\phi = \frac{V_0^2}{2Z}\cos\phi = \frac{(0.95\,\text{V})^2}{2(23.4\times10^3\,\Omega)}\cos(-7.71^\circ) = \boxed{19\,\mu\text{W}}$$

(*c*) The rms current is the peak voltage, divided by $\sqrt{2}$, and then divided by the impedance.

$$I_{\text{rms}} = \frac{V_0/\sqrt{2}}{Z} = \frac{0.95\text{V}/\sqrt{2}}{23.4\times10^3\Omega} = 2.871\times10^{-5}\,\text{A} \approx \boxed{29\,\mu\text{A}}$$

The rms voltage across each element is the rms current times the resistance or reactance of the element.

$$V_R = I_{\text{rms}}R = (2.871\times10^{-5}\text{A})(23.2\times10^3\Omega) = \boxed{0.67\text{V}}$$

$$V_C = I_{\text{rms}}X_C = (2.871\times10^{-5}\text{A})(3158\Omega) = \boxed{0.091\text{V}}$$

$$V_L = I_{\text{rms}}X_L = (2.871\times10^{-5}\text{A})(16.59\Omega) = \boxed{4.8\times10^{-4}\text{V}}$$

$\boxed{91.}$ (*a*) The impedance of the circuit is given by Eq. 30-28b with $X_L > X_C$ and $R = 0$. We divide the magnitude of the ac voltage by the impedance to get the magnitude of the ac current in the circuit. Since $X_L > X_C$, the voltage will lead the current by $\phi = \pi/2$. No dc current will flow through the capacitor.

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \omega L - 1/\omega C \qquad I_0 = \frac{V_{20}}{Z} = \frac{V_{20}}{\omega L - 1/\omega C}$$

$$I(t) = \boxed{\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)}$$

(*b*) The voltage across the capacitor at any instant is equal to the charge on the capacitor divided by the capacitance. This voltage is the sum of the ac voltage and dc voltage. There is no dc voltage drop across the inductor so the dc voltage drop across the capacitor is equal to the input dc voltage.

$$V_{\text{out,ac}} = V_{\text{out}} - V_1 = \frac{Q}{C} - V_1$$

We treat the emf as a superposition of the ac and dc components. At any instant of time the sum of the voltage across the inductor and capacitor will equal the input voltage. We use Eq. 30-5 to calculate the voltage drop across the inductor. Subtracting the voltage drop across the inductor from the input voltage gives the output voltage. Finally, we subtract off the dc voltage to obtain the ac output voltage.

$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left[\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)\right] = \frac{V_{20}L\omega}{\omega L - 1/\omega C}\cos(\omega t - \pi/2)$$

$$= \frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)$$

$$V_{\text{out}} = V_{\text{in}} - V_L = V_1 + V_{20}\sin\omega t - \left(\frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)\right)$$

$$= V_1 + V_{20}\left(1 - \frac{L\omega}{\omega L - 1/\omega C}\right)\sin(\omega t) = V_1 - V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t)$$

$$V_{\text{out,ac}} = V_{\text{out}} - V_1 = -V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t) = \boxed{\left(\frac{V_{20}}{\omega^2 LC - 1}\right)\sin(\omega t - \pi)}$$

(*c*) The attenuation of the ac voltage is greatest when the denominator is large.

$$\omega^2 LC \gg 1 \to \omega L \gg \frac{1}{\omega C} \to X_L \gg X_C$$

We divide the output ac voltage by the input ac voltage to obtain the attenuation.

$$\frac{V_{2,\text{out}}}{V_{2,\text{in}}} = \frac{\dfrac{V_{20}}{\omega^2 LC - 1}}{V_{20}} = \frac{1}{\omega^2 LC - 1} \approx \boxed{\frac{1}{\omega^2 LC}}$$

(*d*) The dc output is equal to the dc input, since there is no dc voltage drop across the inductor.

$$\boxed{V_{1,\text{out}} = V_1}$$

92. Since no dc current flows through the capacitor, there will be no dc current through the resistor. Therefore the dc voltage passes through the circuit with little attenuation. The ac current in the circuit is found by dividing the input ac voltage by the impedance (Eq. 30-28b) We obtain the output ac voltage by multiplying the ac current by the capacitive reactance. Dividing the result by the input ac voltage gives the attenuation.

$$V_{2,\text{out}} = IX_C = \frac{V_{20}X_C}{\sqrt{R^2 + X_C^2}} \quad \to \quad \frac{V_{2,\text{out}}}{V_{20}} = \frac{1}{\sqrt{R^2\omega^2C^2 + 1}} \approx \boxed{\frac{1}{R\omega C}}$$

93. (*a*) Since the three elements are connected in parallel, at any given instant in time they will all three have the same voltage drop across them. That is the voltages across each element will be in phase with the source. The current in the resistor is in phase with the voltage source with magnitude given by Ohm's law.

$$I_R(t) = \boxed{\frac{V_0}{R}\sin \omega t}$$

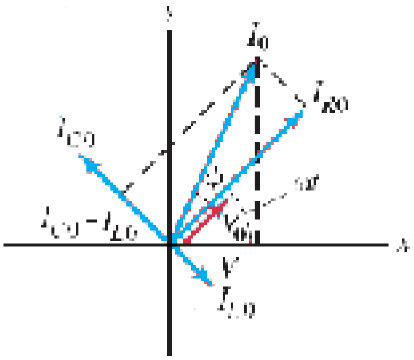
(*b*) The current through the inductor will lag behind the voltage by π/2, with magnitude equal to the voltage source divided by the inductive reactance.

$$I_L(t) = \boxed{\frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)}$$

(*c*) The current through the capacitor leads the voltage by π/2, with magnitude equal to the voltage source divided by the capacitive reactance.

$$I_C(t) = \boxed{\frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)}$$

(*d*) The total current is the sum of the currents through each element. We use a phasor diagram to add the currents, as was used in Section 30-8 to add the voltages with different phases. The net current is found by subtracting the current through the inductor from the current through the capacitor. Then using the Pythagorean theorem to add the current through the resistor. We use the tangent function to find the phase angle between the current and voltage source.



$$I_0 = \sqrt{I_{R0}^2 + (I_{C0} - I_{L0})^2} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2} = \frac{V_0}{R}\sqrt{1 + \left(R\omega C - \frac{1}{R\omega L}\right)^2}$$

$$I(t)=\boxed{\frac{V_0}{R}\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}\,\sin(\omega t+\phi)}$$

$$\tan\phi=\frac{\dfrac{V_0}{X_C}-\dfrac{V_0}{X_L}}{\dfrac{V_0}{R}}\rightarrow\phi=\tan^{-1}\left(\frac{R}{X_C}-\frac{R}{X_L}\right)=\boxed{\tan^{-1}\left(R\omega C-\frac{R}{\omega L}\right)}$$

(*e*) We divide the magnitude of the voltage source by the magnitude of the current to find the impedance.

$$Z=\frac{V_0}{I_0}=\frac{V_0}{\dfrac{V_0}{R}\sqrt{1+\left(R\omega C-\dfrac{R}{\omega L}\right)^2}}=\boxed{\frac{R}{\sqrt{1+\left(R\omega C-\dfrac{R}{\omega L}\right)^2}}}$$

(*f*) The power factor is the ratio of the power dissipated in the circuit divided by the product of the rms voltage and current.

$$\frac{I_{R,\text{rms}}^2R}{V_{\text{rms}}I_{\text{rms}}}=\frac{I_R^2R}{V_0I_0}=\frac{\left(\dfrac{V_0}{R}\right)^2R}{V_0\dfrac{V_0}{R}\sqrt{1+\left(R\omega C-\dfrac{R}{\omega L}\right)^2}}=\boxed{\frac{1}{\sqrt{1+\left(R\omega C-\dfrac{R}{\omega L}\right)^2}}}$$

94. We find the equivalent values for each type of element in series. From the equivalent values we calculate the impedance using Eq. 30-28b.

$$R_{\text{eq}}=R_1+R_2\qquad\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}\qquad L_{\text{eq}}=L_1+L_2$$

$$Z=\sqrt{R_{\text{eq}}^2+\left(\omega L_{\text{eq}}-\frac{1}{\omega C_{\text{eq}}}\right)^2}=\boxed{\sqrt{(R_1+R_2)^2+\left(\omega L_1+\omega L_2-\frac{1}{\omega C_1}-\frac{1}{\omega C_2}\right)^2}}$$

95. If there is no current in the secondary, there will be no induced emf from the mutual inductance. Therefore, we set the ratio of the voltage to current equal to the inductive reactance and solve for the inductance.

$$\frac{V_{\text{rms}}}{I_{\text{rms}}}=X_L=2\pi fL\rightarrow L=\frac{V_{\text{rms}}}{2\pi f\,I_{\text{rms}}}=\frac{220\text{ V}}{2\pi(60\text{ Hz})(4.3\text{ A})}=\boxed{0.14\text{ H}}$$

96. (*a*) We use Eq. 24-2 to calculate the capacitance, assuming a parallel plate capacitor.

$$C=\frac{K\varepsilon_0A}{d}=\frac{(5.0)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.0\times10^{-4}\text{m}^2\right)}{2.0\times10^{-3}\text{m}}=2.213\times10^{-12}\,\text{F}\approx\boxed{2.2\,\text{pF}}$$

(*b*) We use Eq. 30-25b to calculate the capacitive reactance.

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(12000\,\text{Hz})\left(2.2\times10^{-12}\,\text{F}\right)}=5.995\times10^6\,\Omega\approx\boxed{6.0\,\text{M}\Omega}$$

(*c*) Assuming that the resistance in the plasma and in the person is negligible compared with the capacitive reactance, calculate the current by dividing the voltage by the capacitive reactance.

$$I_{\text{o}}\approx\frac{V_{\text{o}}}{X_C}=\frac{2500\text{ V}}{5.995\times10^6\,\Omega}=4.17\times10^{-4}\,\text{A}\approx\boxed{0.42\,\text{mA}}$$

This is not a dangerous current level.

(*d*) We replace the frequency with 1.0 MHz and recalculate the current.

$$I_0 \approx \frac{V_0}{X_C} = 2\pi f C V_0 = 2\pi\left(1.0\times10^{6}\,\text{Hz}\right)\left(2.2\times10^{-12}\,\text{F}\right)\left(2500\,\text{V}\right) = \boxed{35\,\text{mA}}$$

This current level is dangerous.

97. We calculate the resistance from the power dissipated and the current. Then setting the ratio of the voltage to current equal to the impedance, we solve for the inductance.

$$\overline{P} = I_{\text{rms}}^2 R \rightarrow R = \frac{\overline{P}}{I_{\text{rms}}^2} = \frac{350\,\text{W}}{\left(4.0\,\text{A}\right)^2} = 21.88\,\Omega \approx \boxed{22\,\Omega}$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + \left(2\pi f L\right)^2} \quad \rightarrow$$

$$L = \frac{\sqrt{\left(V_{\text{rms}}/I_{\text{rms}}\right)^2 - R^2}}{2\pi f} = \frac{\sqrt{\left(120\,\text{V}/4.0\,\text{A}\right)^2 - \left(21.88\,\Omega\right)^2}}{2\pi\left(60\,\text{Hz}\right)} = \boxed{54\,\text{mH}}$$

98. We insert the proposed current into the differential equation and solve for the unknown peak current and phase.

$$\begin{aligned} V_0 \sin\omega t &= L\frac{d}{dt}\left[I_0 \sin\left(\omega t - \phi\right)\right] + RI_0 \sin\left(\omega t - \phi\right) \\ &= L\omega I_0 \cos\left(\omega t - \phi\right) + RI_0 \sin\left(\omega t - \phi\right) \\ &= L\omega I_0\left(\cos\omega t\cos\phi + \sin\omega t\sin\phi\right) + RI_0\left(\sin\omega t\cos\phi - \cos\omega t\sin\phi\right) \\ &= \left(L\omega I_0\cos\phi - RI_0\sin\phi\right)\cos\omega t + \left(L\omega I_0\sin\phi + RI_0\cos\phi\right)\sin\omega t \end{aligned}$$

For the given equation to be a solution for all time, the coefficients of the sine and cosine terms must independently be equal.

For the $\cos\omega t$ term:

$$0 = L\omega I_0\cos\phi - RI_0\sin\phi \rightarrow \tan\phi = \frac{\omega L}{R} \rightarrow \boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$$

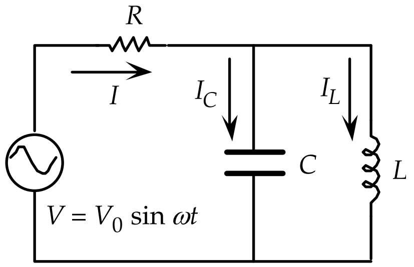
For the $\sin\omega t$ term:

$$V_0 = L\omega I_0\sin\phi + RI_0\cos\phi$$

$$I_0 = \frac{V_0}{L\omega\sin\phi + R\cos\phi} = \frac{V_0}{L\omega\dfrac{\omega L}{\sqrt{R^2+\omega^2L^2}} + R\dfrac{R}{\sqrt{R^2+\omega^2L^2}}} = \boxed{\frac{V_0}{\sqrt{R^2+\omega^2L^2}}}$$

99. The peak voltage across either element is the current through the element multiplied by the reactance. We set the voltage across the inductor equal to six times the voltage across the capacitor and solve for the frequency in terms of the resonant frequency, Eq. 30-14.

$$V_L = I_0 2\pi f C = 6V_C = \frac{6I_0}{2\pi f C} \quad \rightarrow \quad \rightarrow \ f = \frac{1}{2\pi}\sqrt{\frac{6}{LC}} = \boxed{\sqrt{6}f_0}$$



100. We use Kirchhoff's junction rule to write an equation relating the currents in each branch, and the loop rule to write two equations relating the voltage drops around each loop. We write the voltage drops across the capacitor and inductor in terms of the charge and derivative of the current.

$$I_R = I_L + I_C$$

$$V_0 \sin \omega t - I_R R - \frac{Q_C}{C} = 0 \ ; \ V_0 \sin \omega t - I_R R - L\frac{dI_L}{dt} = 0$$

We combine these equations to eliminate the charge in the capacitor and the current in the inductor to write a single differential equation in terms of the current through the resistor.

$$\frac{dI_L}{dt} = \frac{V_0}{L}\sin \omega t - \frac{I_R R}{L}$$

$$\frac{dI_C}{dt} = \frac{d^2 Q_C}{dt^2} = \frac{d^2}{dt^2}\left(CV_0 \sin \omega t - I_R RC\right) = -CV_0\omega^2 \sin \omega t - RC\frac{dI_R{}^2}{dt^2}$$

$$\frac{dI_R}{dt} = \frac{dI_L}{dt} + \frac{dI_C}{dt} = -\frac{V_0}{L}\sin \omega t + \frac{I_R R}{L} - CV_0\omega^2 \sin \omega t - RC\frac{dI_R{}^2}{dt^2}$$

We set the current in the resistor, $I_R = I_0 \sin\left(\omega t + \phi\right) = I_0\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$, equal to the current provided by the voltage source and take the necessary derivatives.

$$I_0 \frac{d}{dt}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right) = \frac{V_0}{L}\sin \omega t - \left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)\frac{I_0 R}{L} - CV_0\omega^2 \sin \omega t$$
$$- RCI_0 \frac{d^2}{dt^2}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$$

$$I_0\omega \cos \omega t \cos \phi - I_0\omega \sin \omega t \sin \phi = \frac{V_0}{L}\sin \omega t - \frac{I_0 R}{L}\sin \omega t \cos \phi + \frac{I_0 R}{L}\cos \omega t \sin \phi - CV_0\omega^2 \sin \omega t$$
$$+ RCI_0\omega^2 \sin \omega t \cos \phi + RCI_0\omega^2 \cos \omega t \sin \phi$$

Setting the coefficients of the time dependent sine and cosine terms separately equal to zero enables us to solve for the magnitude and phase of the current through the voltage source. We also use Eq. 30-23b and Eq. 30-25b to write the inductance and capacitance in terms of their respective reactances.

From the $\cos(\omega t)$ term:

$$I_0\omega \cos \phi = \frac{I_0\omega R}{X_L}\sin \phi - \frac{I_0 R\omega}{X_C}\sin \phi \rightarrow \tan \phi = \frac{X_L X_C}{R\left(X_L - X_C\right)} \rightarrow \phi = \tan^{-1}\left[\frac{X_L X_C}{R\left(X_L - X_C\right)}\right]$$

From the $\sin(\omega t)$ term:

$$-I_0\omega \sin \phi = \frac{V_0\omega}{X_L} - \frac{I_0\omega R}{X_L}\cos \phi - \frac{V_0\omega}{X_C} + \frac{RI_0\omega}{X_C}\cos \phi$$

$$I_0 = \frac{V_0\left(X_C - X_L\right)}{X_C X_L \sin \phi + R\left(X_C - X_L\right)\cos \phi}$$

$$= \frac{V_0\left(X_C - X_L\right)}{X_C X_L \frac{X_C X_L}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}} + R\left(X_C - X_L\right)\frac{R\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}}$$

$$= \frac{V_0\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}$$

This gives us the current through the power source and resistor. We insert these values back into the junction and loop equations to determine the current in each element as a function of time. We calculate the impedance of the circuit by dividing the peak voltage by the peak current through the voltage source.

$$Z=\frac{V_0}{I_0}=\boxed{\frac{\sqrt{\left(X_C X_L\right)^2+R^2\left(X_C-X_L\right)^2}}{\left(X_C-X_L\right)}}\ ;\ \phi=\boxed{\tan^{-1}\left[\frac{X_L X_C}{R\left(X_L-X_C\right)}\right]}\ ;\ I_R=\boxed{\frac{V_0}{Z}\sin\left(\omega t+\phi\right)}$$

$$I_C=\frac{dQ_C}{dt}=\frac{d}{dt}\left(CV_0\sin\omega t-I_R RC\right)=CV_0\omega\cos\omega t-RC\frac{dI_R}{dt}$$

$$=\boxed{\frac{V_0}{X_C}\left[\cos\omega t-\frac{R}{Z}\cos\left(\omega t+\phi\right)\right]}$$

$$I_L=I_R-I_C=\frac{V_0}{Z}\sin\left(\omega t+\phi\right)-\frac{V_0}{X_C}\left[\cos\omega t-\frac{R}{Z}\cos\left(\omega t+\phi\right)\right]$$

$$=\boxed{\frac{V_0}{Z}\left[\sin\left(\omega t+\phi\right)+\frac{R}{X_C}\cos\left(\omega t+\phi\right)\right]-\frac{V_0}{X_C}\cos\omega t}$$

101. (*a*) The resonant frequency is given by Eq. 30-32. At resonance, the impedance is equal to the resistance, so the rms voltage of the circuit is equal to the rms voltage across the resistor.

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{\left(0.0050\,\text{H}\right)\left(0.10\times10^{-6}\,\text{F}\right)}}=7118\,\text{Hz}\approx\boxed{7.1\,\text{kHz}}$$

$$\left(V_R\right)_{\text{rms}}=\boxed{V_{\text{rms}}}$$

(*b*) We set the inductance equal to 90% of the initial inductance and use Eq. 30-28b to calculate the new impedance. Dividing the rms voltage by the impedance gives the rms current. We multiply the rms current by the resistance to determine the voltage drop across the resistor.

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\left(7118\,\text{Hz}\right)\left(0.10\times10^{-6}\,\text{F}\right)}=223.6\,\Omega$$

$$X_L=2\pi fL=2\pi\left(7118\,\text{Hz}\right)\left(0.90\right)\left(0.0050\,\text{H}\right)=201.3\,\Omega$$

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{\left(45\,\Omega\right)^2+\left(201.3\,\Omega-223.6\,\Omega\right)^2}=50.24\,\Omega$$

$$\left(V_R\right)_{\text{rms}}=\left(\frac{R}{Z}\right)V_{\text{rms}}=\left(\frac{45\,\Omega}{50.24\,\Omega}\right)V_{\text{rms}}=\boxed{0.90V_{\text{rms}}}$$

102. With the given applied voltage, calculate the rms current through each branch as the rms voltage divided by the impedance in that branch.

$$I_{C,\text{rms}}=\frac{V_{\text{rms}}}{\sqrt{R_1^2+X_C^2}}\qquad I_{L,\text{rms}}=\frac{V_{\text{rms}}}{\sqrt{R_2^2+X_L^2}}$$

Calculate the potential difference between points a and b in two ways. First pass through the capacitor and then through R_2. Then pass through R_1 and the inductor.

$$V_{\text{ab}}=I_C X_C-I_L R_2=\frac{V_{\text{rms}}X_C}{\sqrt{R_1^2+X_C^2}}-\frac{V_{\text{rms}}R_2}{\sqrt{R_2^2+X_L^2}}$$

$$V_{\text{ab}} = -I_C R_1 + X_L I_L = -\frac{V_{\text{rms}} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{\text{rms}} X_L}{\sqrt{R_2^2 + X_L^2}}$$

Set these voltage differences equal to zero, and rearrange the equations.

$$\frac{V_{\text{rms}} X_C}{\sqrt{R_1^2 + X_C^2}} - \frac{V_{\text{rms}} R_2}{\sqrt{R_2^2 + X_L^2}} = 0 \to X_C\sqrt{R_2^2 + X_L^2} = R_2\sqrt{R_1^2 + X_C^2}$$

$$-\frac{V_{\text{rms}} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{\text{rms}} X_L}{\sqrt{R_2^2 + X_L^2}} = 0 \to R_1\sqrt{R_2^2 + X_L^2} = X_L\sqrt{R_1^2 + X_C^2}$$

Divide the resulting equations and solve for the product of the resistances. Write the reactances in terms of the capacitance and inductance to show that the result is frequency independent.

$$\frac{X_C\sqrt{R_2^2 + X_L^2}}{R_1\sqrt{R_2^2 + X_L^2}} = \frac{R_2\sqrt{R_1^2 + X_C^2}}{X_L\sqrt{R_1^2 + X_C^2}} \to R_1 R_2 = X_L X_C = \frac{\omega L}{\omega C} \to \boxed{R_1 R_2 = \frac{L}{C}}$$

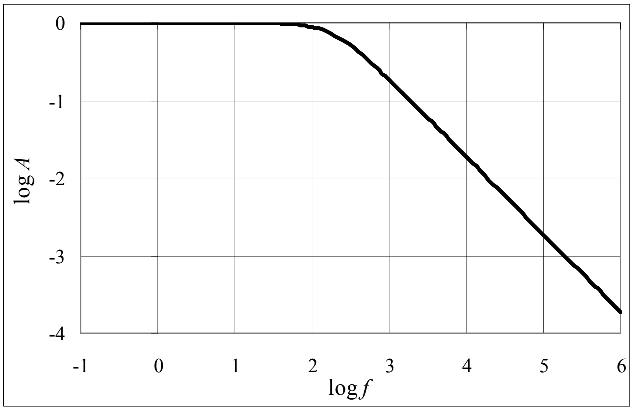
103. (*a*) The output voltage is the voltage across the capacitor, which is the current through the circuit multiplied by the capacitive reactance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{\text{out}} = IX_C = \frac{V_{\text{in}} X_C}{\sqrt{R^2 + X_C^2}} = \frac{V_{\text{in}}}{\sqrt{(R/X_C)^2 + 1}} = \frac{V_{\text{in}}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A = \frac{V_{\text{out}}}{V_{\text{in}}} = \boxed{\frac{1}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}}$$

(*b*) As the frequency goes to zero, the gain becomes one. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage is equal to the input voltage. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain goes to zero.

(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies less than about 100 Hz the gain is ~ 1. For higher frequencies the gain drops off proportionately to the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.103c."



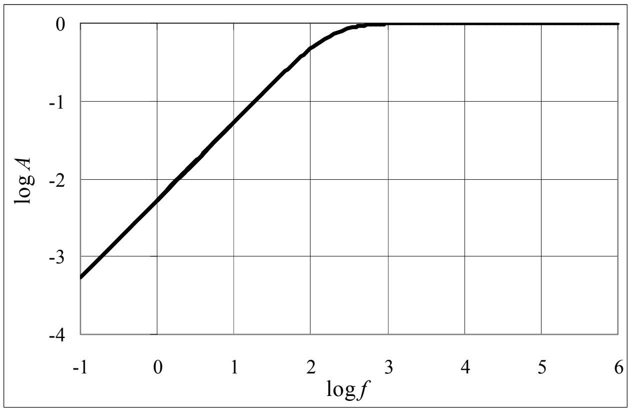
104. (*a*) The output voltage is the voltage across the resistor, which is the current through the circuit multiplied by the resistance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{\text{out}} = IR = \frac{V_{\text{in}} R}{\sqrt{R^2 + X_C^2}} = \frac{V_{\text{in}} R}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} = \frac{2\pi fCRV_{\text{in}}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A=\frac{V_{\text{out}}}{V_{\text{in}}}=\boxed{\frac{2\pi fCR}{\sqrt{4\pi^2f^2C^2R^2+1}}}$$

(*b*) As the frequency goes to zero, the gain drops to zero. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage drops to zero. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain is equal to unity.

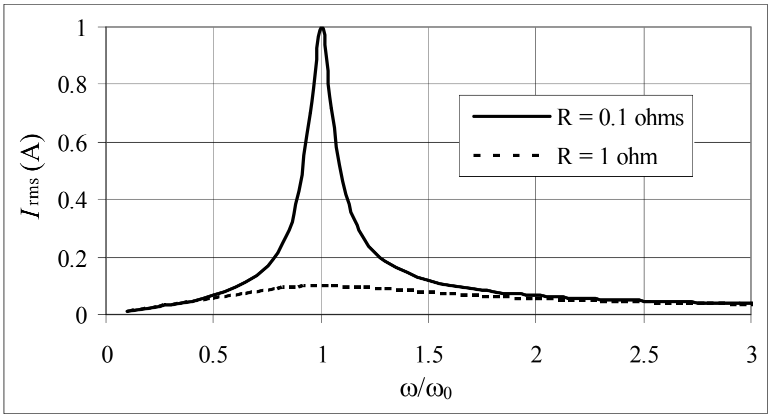
(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies greater than about 1000 Hz the gain is ~ 1. For lower frequencies the gain drops off proportionately to the inverse of the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.104c."



105. We calculate the resonant frequency using Eq. 30-32.

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{\left(50\times10^{-6}\,\text{H}\right)\left(50\times10^{-6}\,\text{F}\right)}}=20{,}000\,\text{rad/s}$$

Using a spreadsheet, we calculate the impedance as a function of frequency using Eq. 30-28b. We divide the rms voltage by the impedance to plot the rms current as a function of frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.105."



CHAPTER 31: Maxwell's Equations and Electromagnetic Waves

Responses to Questions

$\boxed{1.}$ The magnetic field will be clockwise in both cases. In the first case, the electric field is away from you and is increasing. The direction of the displacement current (proportional to $\frac{d\Phi_E}{dt}$) is therefore away from you and the corresponding magnetic field is clockwise. In the second case, the electric field is directed towards you and is decreasing; the displacement current is still away from you, and the magnetic field is still clockwise.

2. The displacement current is to the right.

3. The displacement current is spread out over a larger area than the conduction current. Thus, the displacement current produces a less intense field at any given point.

4. One possible reason the term $\varepsilon_0 \frac{d\Phi_E}{dt}$ can be called a "current" is because it has units of amperes.

5. The magnetic field vector will oscillate up and down, perpendicular to the direction of propagation and to the electric field vector.

6. No. Sound is a longitudinal mechanical wave. It requires the presence of a medium; electromagnetic waves do not require a medium.

$\boxed{7.}$ EM waves are self-propagating and can travel through a perfect vacuum. Sound waves are mechanical waves which require a medium, and therefore cannot travel through a perfect vacuum.

8. No. Electromagnetic waves travel at a very large but finite speed. When you flip on a light switch, it takes a very small amount of time for the electrical signal to travel along the wires.

9. The wavelengths of radio and television signals are longer than those of visible light.

10. The wavelength of the current is 5000 km; the house is only 200 km away. The phase of the current at the position of the house is 2π/25 radians different from the phase at the source due to the position of the house.

11. The signals travel through the wires at close to the speed of light, so the length of the wires in a normal room will have an insignificant effect.

12. 10^3 km: radio wave; 1 km: radio wave; 1 m: microwave; 1 cm: microwave; 1 mm: microwave or infrared; 1 μm: infrared.

$\boxed{13.}$ Yes, although the wavelengths for radio waves will be much longer than for sound waves, since the radio waves travel at the speed of light.

14. Both cordless phones and cellular phones are radio receivers and transmitters. When you speak, the phone converts the sound waves into electrical signals which are amplified, modulated, and transmitted. The receiver picks up the EM waves and converts them back into sound. Cordless phones and cell phones use different frequency ranges and different intensities.

15. Yes. If one signal is sent by amplitude modulation and the other signal is sent by frequency modulation, both could be carried over the same carrier frequency. There are other ways two signals can be sent on the same carrier frequency which are more complex.

16. The receiver's antenna should also be vertical for the best reception.

17. Diffraction is significant when the order of magnitude of the wavelength of the waves is the same as the size of the obstacles. AM waves have longer wavelengths than FM waves and will be more likely to diffract around hills and other landscape barriers.

18. It is amplitude modulated, or AM. The person flashing the light on and off is changing the amplitude of the light ("on" is maximum amplitude and "off" is zero). The frequency of the carrier wave is just the frequency of the visible light, approximately 10^{14} to 10^{15} Hz.

Solutions to Problems

$\boxed{1.}$ The electric field between the plates is given by $E=\frac{V}{d}$, where d is the distance between the plates.

$$E=\frac{V}{d} \quad\rightarrow\quad \frac{dE}{dt}=\frac{1}{d}\frac{dV}{dt}=\left(\frac{1}{0.0011\,\text{m}}\right)(120\,\text{V/s})=\boxed{1.1\times10^{5}\,\frac{\text{V/m}}{\text{s}}}$$

2. The displacement current is shown in section 31-1 to be $I_\text{D}=\varepsilon_0 A\frac{dE}{dt}$.

$$I_\text{D}=\varepsilon_0 A\frac{dE}{dt}=\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.058\,\text{m})^2\left(2.0\times10^{6}\,\frac{\text{V}}{\text{m•s}}\right)=\boxed{6.0\times10^{-8}\,\text{A}}$$

3. The current in the wires must also be the displacement current in the capacitor. Use the displacement current to find the rate at which the electric field is changing.

$$I_D=\varepsilon_0 A\frac{dE}{dt} \quad\rightarrow\quad \frac{dE}{dt}=\frac{I_\text{D}}{\varepsilon_0 A}=\frac{(2.8\,\text{A})}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.0160\,\text{m})^2}=\boxed{1.2\times10^{15}\,\frac{\text{V}}{\text{m•s}}}$$

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire.

$$B=\frac{\mu_0 I}{2\pi R}=\left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R}=\frac{\left(10^{-7}\,\text{T•m/A}\right)2\left(38.0\times10^{-3}\,\text{A}\right)}{(0.100\,\text{m})}=\boxed{7.60\times10^{-8}\,\text{T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero.}}$

5. The electric field between the plates is given by $E = \frac{V}{d}$, where *d* is the distance between the plates.

The displacement current is shown in section 31-1 to be $I_{\mathrm{D}} = \varepsilon_0 A \frac{dE}{dt}$.

$$I_{\mathrm{D}} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{1}{d}\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = \boxed{C\frac{dV}{dt}}$$

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\mu\mathrm{A}}$.

(*b*) The charge on the pages is given by $Q = CV = C\mathscr{E}_0 \cos\omega t$. The current is the derivative of this.

$$I = \frac{dQ}{dt} = -\omega C\mathscr{E}_0 \sin\omega t \;\;;\;\; I_{\max} = \omega C\mathscr{E}_0 \;\;\rightarrow$$

$$\mathscr{E}_0 = \frac{I_{\max}}{\omega \mathrm{C}} = \frac{I_{\max} d}{2\pi f \varepsilon_0 A} = \frac{\left(35\times10^{-6}\,\mathrm{A}\right)\left(1.6\times10^{-3}\,\mathrm{m}\right)}{2\pi\left(76.0\,\mathrm{Hz}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\pi\left(0.025\mathrm{m}\right)^2}$$

$$= 6749\,\mathrm{V} \approx \boxed{6700\,\mathrm{V}}$$

(*c*) From Eq. 31-3, $I_{\mathrm{D}} = \varepsilon_0 \frac{d\Phi_E}{dt}$.

$$I_{\mathrm{D}} = \varepsilon_0 \frac{d\Phi_E}{dt} \;\rightarrow\; \left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{\left(I_{\mathrm{D}}\right)_{\max}}{\varepsilon_0} = \frac{35\times10^{-6}\,\mathrm{A}}{8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}} = \boxed{4.0\times10^{6}\,\mathrm{V{\bullet}m/s}}$$

$\boxed{7.}$ (*a*) We follow the development and geometry given in Example 31-1, using R for the radial distance. The electric field between the plates is given by $E = \frac{V}{d}$, where *d* is the distance between the plates.

$$\oint \vec{\mathbf{B}}\bullet d\vec{\boldsymbol{\ell}} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} \;\rightarrow\; B\left(2\pi R_{\text{path}}\right) = \mu_0\varepsilon_0 \frac{d\left(\pi R_{\text{flux}}^2 E\right)}{dt}$$

The subscripts are used on the radial variable because there might not be electric field flux through the entire area bounded by the amperian path. The electric field between the plates is given by $E = \frac{V}{d} = \frac{V_0 \sin\left(2\pi f t\right)}{d}$, where *d* is the distance between the plates.

$$B\left(2\pi R_{\text{path}}\right) = \mu_0\varepsilon_0 \frac{d\left(\pi R_{\text{flux}}^2 E\right)}{dt} \;\rightarrow$$

$$B = \frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{d\left(E\right)}{dt} = \frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{2\pi f V_0}{d}\cos\left(2\pi f t\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d}\cos\left(2\pi f t\right)$$

We see that the functional form of the magnetic field is $\boxed{B = B_0\left(R\right)\cos\left(2\pi f t\right)}$.

(*b*) If $R \le R_0$, then there is electric flux throughout the area bounded by the amperian loop, and so $R_{\text{path}} = R_{\text{flux}} = R$.

$$B_0\left(R \le R_0\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0}{d}R = \frac{\pi\left(60\,\text{Hz}\right)\left(150\,\text{V}\right)}{\left(3.00\times10^8\,\text{m/s}\right)^2\left(5.0\times10^{-3}\,\text{m}\right)}R$$

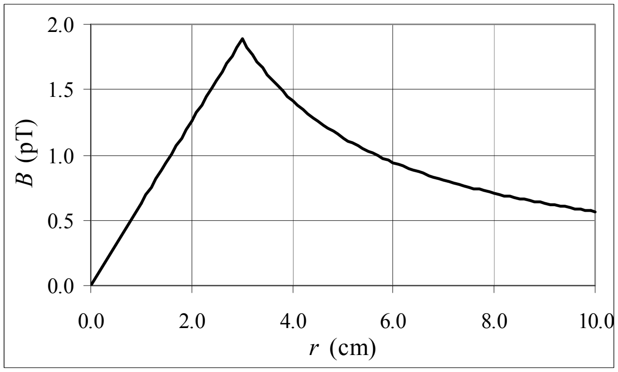
$$= \left(6.283\times10^{-11}\,\text{T/m}\right)R \approx \boxed{\left(6.3\times10^{-11}\,\text{T/m}\right)R}$$

If $R > R_0$, then there is electric flux only out a radial distance of R_0. Thus $R_{\text{path}} = R$ and $R_{\text{flux}} = R_0$.

$$B_0\left(R > R_0\right) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0 R_0^2}{d}\frac{1}{R} = \frac{\pi\left(60\,\text{Hz}\right)\left(150\,\text{V}\right)\left(0.030\,\text{m}\right)^2}{\left(3.00\times10^8\,\text{m/s}\right)^2\left(5.0\times10^{-3}\,\text{m}\right)}\frac{1}{R}$$

$$= \left(5.655\times10^{-14}\,\text{T}\bullet\text{m}\right)\frac{1}{R} \approx \boxed{\left(5.7\times10^{-14}\,\text{T}\bullet\text{m}\right)\frac{1}{R}}$$

(*c*) See the adjacent graph. Note that the magnetic field is continuous at the transition from "inside" to "outside" the capacitor radius. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH31.XLS," on tab "Problem 31.7c."



8. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \quad\rightarrow\quad B_0 = \frac{E_0}{c} = \frac{0.57\times10^{-4}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{1.9\times10^{-13}\,\text{T}}$$

9. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \quad\rightarrow\quad E_0 = B_0 c = \left(12.5\times10^{-9}\,\text{T}\right)\left(3.00\times10^8\,\text{m/s}\right) = \boxed{3.75\,\text{V/m}}$$

10. The frequency of the two fields must be the same: $\boxed{80.0\,\text{kHz}}$. The rms strength of the electric field can be found from Eq. 31-11 with $v = c$.

$$E_{\text{rms}} = cB_{\text{rms}} = \left(3.00\times10^8\,\text{m/s}\right)\left(7.75\times10^{-9}\,\text{T}\right) = \boxed{2.33\,\text{V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

11. (*a*) If we write the argument of the cosine function as $kz + \omega t = k(z + ct)$, we see that the wave is traveling in the $\boxed{-z\ \text{direction}}$, or $\boxed{-\hat{\mathbf{k}}}$.

(*b*) $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation. At the origin, the electric field is pointing in the positive x direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must point in the negative z direction, $\vec{\mathbf{B}}$ must point in the the $\boxed{-y\text{ direction}}$, or $\boxed{-\hat{\mathbf{j}}}$. The magnitude of the magnetic field is found from Eq. 31-11 as $B_0=\boxed{E_0/c}$.

12. The wave equation to be considered is $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$.

(*a*) Given $E(x,t)=Ae^{-\alpha(x-vt)^2}$.

$$\frac{\partial E}{\partial x}=Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial x^2}=Ae^{-\alpha(x-vt)^2}(-2\alpha)+Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]^2=-2\alpha Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

$$\frac{\partial E}{\partial t}=Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)(-v)\right]=Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial t^2}=Ae^{-\alpha(x-vt)^2}\left(-2\alpha v^2\right)+Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]^2=-2\alpha v^2Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

We see that $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$, and so the wave equation is satisfied.

(*b*) Given $E(x,t)=Ae^{-\left(\alpha x^2-vt\right)}$.

$$\frac{\partial E}{\partial x}=Ae^{-\left(\alpha x^2-vt\right)}(-2\alpha x)$$

$$\frac{\partial^2 E}{\partial x^2}=Ae^{-\left(\alpha x^2-vt\right)}(-2\alpha)+Ae^{-\left(\alpha x^2-vt\right)}(-2\alpha x)^2=-2\alpha Ae^{-\left(\alpha x^2-vt\right)}\left[1-2\alpha x^2\right]$$

$$\frac{\partial E}{\partial t}=Ave^{-\left(\alpha x^2-vt\right)}\quad;\quad\frac{\partial^2 E}{\partial t^2}=Av^2e^{-\left(\alpha x^2-vt\right)}$$

This does NOT satisfy $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$, since $-2\alpha v^2Ae^{-\left(\alpha x^2-vt\right)}\left[1-2\alpha x^2\right]\neq Av^2e^{-\left(\alpha x^2-vt\right)}$ in general.

$\boxed{13.}$ Use Eq. 31-14 to find the frequency of the microwave.

$$c=\lambda f\quad\rightarrow\quad f=\frac{c}{\lambda}=\frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.50\times10^{-2}\text{m}\right)}=\boxed{2.00\times10^{10}\text{ Hz}}$$

14. Use Eq. 31-14 to find the wavelength and frequency.

(*a*) $c=\lambda f\quad\rightarrow\quad\lambda=\dfrac{c}{f}=\dfrac{\left(3.000\times10^8\,\text{m/s}\right)}{\left(25.75\times10^9\text{Hz}\right)}=\boxed{1.165\times10^{-2}\text{m}}$

(*b*) $c=\lambda f\quad\rightarrow\quad f=\dfrac{c}{\lambda}=\dfrac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(0.12\times10^{-9}\text{m}\right)}=\boxed{2.5\times10^{18}\text{ Hz}}$

15. Use the relationship that $d = vt$ to find the time.

$$d = vt \quad \rightarrow \quad t = \frac{d}{v} = \frac{\left(1.50 \times 10^{11}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = 5.00 \times 10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

16. Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(8.56 \times 10^{14}\,\text{Hz}\right)} = \boxed{3.50 \times 10^{-7}\,\text{m}} = 311\,\text{nm}$$

This wavelength is just outside the violet end of the visible region, so it is $\boxed{\text{ultraviolet.}}$

17. (*a*) Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{3.00 \times 10^{5}\,\text{m}}$$

(*b*) Again use Eq. 31-14, with the speed of sound in place of the speed of light.

$$v = \lambda f \quad \rightarrow \quad \lambda = \frac{v}{f} = \frac{\left(341\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{0.341\,\text{m}}$$

(*c*) $\boxed{\text{No}}$, you cannot hear a 1000-Hz EM wave. It takes a pressure wave to excite the auditory system. However, if you applied the 1000-Hz EM wave to a speaker, you could hear the 1000-Hz pressure wave.

18. The length of the pulse is $\Delta d = c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N = \frac{\left(c\Delta t\right)}{\lambda} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(38 \times 10^{-12}\,\text{s}\right)}{\left(1062 \times 10^{-9}\,\text{m}\right)} = 10734 \approx \boxed{11{,}000\ \text{wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{c} = \frac{\left(1062 \times 10^{-9}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = \boxed{3.54 \times 10^{-15}\,\text{s}}$$

$\boxed{19.}$ (*a*) The radio waves travel at the speed of light, and so $\Delta d = v\Delta t$. The distance is found from the radii of the orbits. For Mars when nearest the Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} - 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{261\,\text{s}}$$

(*b*) For Mars when farthest from Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} + 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{1260\,\text{s}}$$

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x = E_0 \sin\left(kz - \omega t\right)$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.077\text{m}^{-1}} = 81.60\,\text{m} \approx \boxed{82\,\text{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{2.3\times10^{7}\,\text{rad/s}}{2\pi} = 3.661\times10^{6}\,\text{Hz} \approx \boxed{3.7\,\text{MHz}}$$

Note that $\lambda f = \left(81.60\,\text{m}\right)\left(3.661\times10^{6}\,\text{Hz}\right) = 2.987\times10^{8}\,\text{m/s} \approx c.$

(*b*) The magnitude of the magnetic field is given by $B_0 = E_0/c$. The wave is traveling in the $\hat{\mathbf{k}}$ direction, and so the magnetic field must be in the $\hat{\mathbf{j}}$ direction, since the direction of travel is given by the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$.

$$B_0 = \frac{E_0}{c} = \frac{225\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = 7.50\times10^{-7}\,\text{T} \quad\rightarrow$$

$$\vec{\mathbf{B}} = \boxed{\hat{\mathbf{j}}\left(7.50\times10^{-7}\text{T}\right)\sin\left[\left(0.077\text{m}^{-1}\right)z-\left(2.3\times10^{7}\,\text{rad/s}\right)t\right]}$$

21. The eight-sided mirror would have to rotate 1/8 of a revolution for the succeeding mirror to be in position to reflect the light in the proper direction. During this time the light must travel to the opposite mirror and back.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{1}{8}(2\pi\,\text{rad})}{(2\Delta x/c)} = \frac{(\pi\,\text{rad})c}{8\Delta x} = \frac{(\pi\,\text{rad})\left(3.00\times10^8\,\text{m/s}\right)}{8\left(35\times10^3\,\text{m}\right)} = \boxed{3400\,\text{rad/s}}\ \left(3.2\times10^4\,\text{rev/min}\right)$$

22. The average energy transferred across unit area per unit time is the average magnitude of the Poynting vector, and is given by Eq. 31-19a.

$$\bar{S} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(0.0265\,\text{V/m}\right) = \boxed{9.32\times10^{-7}\,\text{W/m}^2}$$

23. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time ΔT.

$$S = \frac{cB_{\text{rms}}^2}{\mu_0} = \frac{\Delta U}{A\Delta t} \quad\rightarrow$$

$$\Delta t = \frac{\mu_0\Delta U}{AcB_{\text{rms}}^2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(335\,\text{J})}{\left(1.00\times10^{-4}\,\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(22.5\times10^{-9}\,\text{T}\right)^2} = 0.194\,\text{W/m}^2$$

$$= \boxed{2.77\times10^{7}\,\text{s}} \approx 321\ \text{days}$$

24. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time Δt.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{\Delta U}{A\Delta t} \quad\rightarrow$$

$$\frac{\Delta U}{\Delta t} = c\varepsilon_0 E_{\text{rms}}^2 A$$

$$= \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.0328\,\text{V/m}\right)^2\left(1.00\times10^{-4}\text{m}^2\right)\left(3600\text{s/h}\right)$$

$$= \boxed{1.03\times10^{-6}\,\text{J/h}}$$

$\boxed{25.}$ The intensity is the power per unit area, and also is the time averaged value of the Poynting vector. The area is the surface area of a sphere, since the wave is spreading spherically.

$$\bar{S} = \frac{P}{A} = \frac{(1500\,\text{W})}{\left[4\pi(5.0\,\text{m})^2\right]} = 4.775\,\text{W/m}^2 \approx \boxed{4.8\,\text{W/m}^2}$$

$$\bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \quad\rightarrow\quad E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{4.775\,\text{W/m}^2}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{42\,\text{V/m}}$$

26. (*a*) We find E using Eq. 31-11 with $v = c$.

$$E = cB = \left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(2.5\times10^{-7}\,\mathrm{T}\right) = \boxed{75\,\mathrm{V/m}}$$

(*b*) The average power per unit area is given by the Poynting vector, from Eq. 31-19a.

$$\bar{I} = \frac{E_0B_0}{\left(2\mu_0\right)} = \frac{\left(75\,\mathrm{V/m}\right)\left(2.5\times10^{-7}\,\mathrm{T}\right)}{\left[2\left(4\pi\times10^{-7}\,\mathrm{N{\bullet}s^2/C^2}\right)\right]} = \boxed{7.5\,\mathrm{W/m^2}}$$

27. From Eq. 31-16b, the instantaneous energy density is $u = \varepsilon_0E^2$. From Eq. 31-17, we see that this instantaneous energy density is also given by S/c. The time-averaged value is therefore $\bar{S}/c$. Multiply that times the volume to get the energy.

$$U = uV = \frac{\bar{S}}{c}V = \frac{1350\,\mathrm{W/m^2}}{3.00\times10^{8}\,\mathrm{m/s}}\left(1.00\,\mathrm{m^3}\right) = \boxed{4.50\times10^{-6}\,\mathrm{J}}$$

28. The power output per unit area is the intensity, and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0E_{\mathrm{rms}}^2 \quad\rightarrow$$

$$E_{\mathrm{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158\,\mathrm{W}}{\pi\left(1.00\times10^{-3}\,\mathrm{m}\right)^2\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}}$$

$$= 1376.3\,\mathrm{V/m} \approx \boxed{1380\,\mathrm{V/m}}$$

$$B_{\mathrm{rms}} = \frac{E_{\mathrm{rms}}}{c} = \frac{1376.3\,\mathrm{V/m}}{3.00\times10^{8}\,\mathrm{m/s}} = \boxed{4.59\times10^{-6}\,\mathrm{T}}$$

29. The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The $1350\,\mathrm{W/m^2}$ is the intensity, or the magnitude of the Poynting vector.

$$S = \frac{P}{A} \quad\rightarrow\quad P = SA = 4\pi R^2S = 4\pi\left(1.496\times10^{11}\,\mathrm{m}\right)^2\left(1350\,\mathrm{W/m^2}\right) = \boxed{3.80\times10^{26}\,\mathrm{W}}$$

30. (*a*) The energy emitted in each pulse is the power output of the laser times the time duration of the pulse.

$$P = \frac{\Delta W}{\Delta t} \quad\rightarrow\quad \Delta W = P\Delta t = \left(1.8\times10^{11}\,\mathrm{W}\right)\left(1.0\times10^{-9}\,\mathrm{s}\right) = \boxed{180\,\mathrm{J}}$$

(*b*) We find the rms electric field from the intensity, which is the power per unit area. That is also the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0E_{\mathrm{rms}}^2 \quad\rightarrow$$

$$E_{\mathrm{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{\left(1.8\times10^{11}\,\mathrm{W}\right)}{\pi\left(2.2\times10^{-3}\,\mathrm{m}\right)^2\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}}$$

$$= \boxed{2.1\times10^{9}\,\mathrm{V/m}}$$

$\boxed{31.}$ In each case, the required area is the power requirement of the device divided by 10% of the intensity of the sunlight.

(*a*) $A = \dfrac{P}{I} = \dfrac{50\times10^{-3}\,\text{W}}{100\,\text{W/m}^2} = 5\times10^{-4}\,\text{m}^2 = \boxed{5\,\text{cm}^2}$

A typical calculator is about 17 cm x 8 cm, which is about $140\,\text{cm}^2$. So $\boxed{\text{yes}}$, the solar panel can be mounted directly on the calculator.

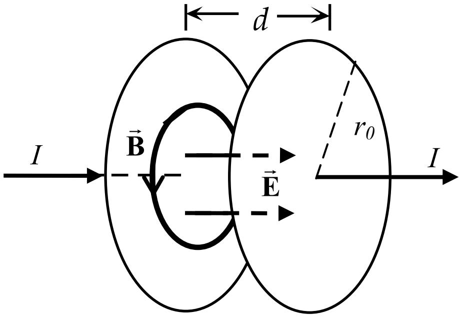
(*b*) $A = \dfrac{P}{I} = \dfrac{1500\,\text{W}}{100\,\text{W/m}^2} = 15\,\text{m}^2 \approx \boxed{20\,\text{m}^2}$ (to one sig. fig.)

A house of floor area $1000\,\text{ft}^2$ would have on the order of $100\,\text{m}^2$ of roof area. So $\boxed{\text{yes}}$, a solar panel on the roof should be able to power the hair dryer.

(*c*) $A = \dfrac{P}{I} = \dfrac{20\,\text{hp}\left(746\,\text{W/hp}\right)}{100\,\text{W/m}^2} = 149\,\text{m}^2 \approx \boxed{100\,\text{m}^2}$ (to one sig. fig.)

This would require a square panel of side length about 12 m. So $\boxed{\text{no}}$, this panel could not be mounted on a car and used for real-time power.

32. (*a*) Example 31-1 refers back to Example 21-13 and Figure 21-31. In that figure, and the figure included here, the electric field between the plates is to the right. The magnetic field is shown as counterclockwise circles. Take any point between the capacitor plates, and find the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. For instance, at the top of the circle shown in Figure 31-4, $\vec{\mathbf{E}}$ is toward the viewer, and $\vec{\mathbf{B}}$ is to the left. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points down, directly to the line connecting the center of the plates. Or take the rightmost point on the circle. $\vec{\mathbf{E}}$ is again toward the viewer, and $\vec{\mathbf{B}}$ is upwards. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points to the left, again directly to the line connecting the center of the plates. In cylindrical coordinates, $\vec{\mathbf{E}} = E\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B\hat{\boldsymbol{\varphi}}$. The cross product $\hat{\mathbf{k}}\times\hat{\boldsymbol{\varphi}} = -\hat{\mathbf{r}}$.



(*b*) We evaluate the Poynting vector, and then integrate it over the curved cylindrical surface between the capacitor plates. The magnetic field (from Example 31-1) is $B = \frac{1}{2}\mu_0\varepsilon_0 r_0\dfrac{dE}{dt}$, evaluated at $r = r_0$. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other, so $\vec{\mathbf{S}} = \dfrac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{2}\varepsilon_0 r_0 E\dfrac{dE}{dt}$, inward. In calculating $\iint\vec{\mathbf{S}}\bullet d\vec{\mathbf{A}}$ for energy flow into the capacitor volume, note that both $\vec{\mathbf{S}}$ and $d\vec{\mathbf{A}}$ point inward, and that *S* is constant over the curved surface of the cylindrical volume.

$$\iint\vec{\mathbf{S}}\bullet d\vec{\mathbf{A}} = \iint S\,dA = S\iint dA = SA = S2\pi r_0 d = \tfrac{1}{2}\varepsilon_0 r_0 E\frac{dE}{dt}2\pi r_0 d = \varepsilon_0 d\pi r_0^2 E\frac{dE}{dt}$$

The amount of energy stored in the capacitor is the energy density times the volume of the capacitor. The energy density is given by Eq. 24-6, $u = \frac{1}{2}\varepsilon_0 E^2$, and the energy stored is the energy density times the volume of the capacitor. Take the derivative of the energy stored with respect to time.

$$U = u\left(\text{Volume}\right) = \tfrac{1}{2}\varepsilon_0 E^2\pi r_0^2 d \;\rightarrow\; \frac{dU}{dt} = \varepsilon_0 E\pi r_0^2 d\frac{dE}{dt}$$

We see that $\iint\vec{\mathbf{S}}\bullet d\vec{\mathbf{A}} = \dfrac{dU}{dt}$.

33. (*a*) The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Sun}}}{I_{\text{Star}}}=\frac{r^2_{\substack{\text{Star-}\\\text{Earth}}}}{r^2_{\substack{\text{Sun-}\\\text{Earth}}}} \rightarrow r_{\substack{\text{Star-}\\\text{Earth}}}=r_{\substack{\text{Sun-}\\\text{Earth}}}\sqrt{\frac{I_{\text{Sun}}}{I_{\text{Star}}}}=\left(1.496\times10^{11}\,\text{m}\right)\sqrt{\frac{1350\,\text{W/m}^2}{1\times10^{-23}\,\text{W/m}^2}}\left(\frac{1\,\text{ly}}{9.46\times10^{15}\,\text{m}}\right)$$

$$=1.84\times10^{8}\,\text{ly}\approx\boxed{2\times10^{8}\,\text{ly}}$$

(*b*) Compare this distance to the galactic size.

$$\frac{r_{\substack{\text{Star-}\\\text{Earth}}}}{\text{galactic size}}=\frac{1.84\times10^{8}\,\text{ly}}{1\times10^{5}\,\text{ly}}=1840\approx\boxed{2000}$$

The distance to the star is about 2000 times the size of our galaxy.

34. We assume the light energy is all absorbed, and so use Eq. 31-21a.

$$P=\frac{\overline{S}}{c}=\frac{\dfrac{75\,\text{W}}{4\pi\left(8.0\times10^{-2}\,\text{m}\right)^2}}{\left(3.00\times10^{8}\,\text{m/s}\right)}=3.108\times10^{-6}\,\text{N/m}^2\approx\boxed{3.1\times10^{-6}\,\text{N/m}^2.}$$

The force is pressure times area. We approximate the area of a fingertip to be 1.0 cm^2.

$$F=PA=\left(3.108\times10^{-6}\,\text{N/m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)=\boxed{3.1\times10^{-10}\,\text{N}}$$

35. The acceleration of the cylindrical particle will be the force on it (due to radiation pressure) divided by its mass. The light is delivering electromagnetic energy to an area *A* at a rate of $\frac{dU}{dt}=1.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\overline{S}=\frac{\frac{dU}{dt}}{A}$. From Eq. 31-21a, that causes a pressure on the particle of $P=\frac{\overline{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships with Newton's second law to calculate the acceleration. The mass of the particle is its volume times the density of water.

$$F_{\text{laser}}=PA=\frac{\overline{S}}{c}A=\frac{1}{c}\frac{dU}{dt}=ma=\rho_{\text{H}_2\text{O}}\pi r^2 ra \rightarrow$$

$$a=\frac{dU/dt}{c\rho_{\text{H}_2\text{O}}\pi r^3}=\frac{(1.0\text{W})}{\left(3.00\times10^{8}\,\text{m/s}\right)\left(1000\,\text{kg/m}^2\right)\pi\left(5\times10^{-7}\,\text{m}\right)^3}=\boxed{8\times10^{6}\,\text{m/s}^2}$$

36. (*a*) The light is delivering electromagnetic energy to an area *A* of the suit at a rate of $\frac{dU}{dt}=3.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\overline{S}=\frac{dU/dt}{A}$. From Eq. 31-21b, that causes a pressure on the suit of $P=\frac{2\overline{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships to calculate the force.

$$F_{\text{laser}}=PA=\frac{2\overline{S}}{c}A=\frac{2}{c}\frac{dU}{dt}=\frac{2(3.0\,\text{W})}{3.00\times10^{8}\,\text{m/s}}=\boxed{2.0\times10^{-8}\,\text{N}}$$

(*b*) Use Newton's law of universal gravitation, Eq. 6-1, to estimate the gravitational force. We take the 20 m distance as having 2 significant figures.

$$F_{\text{grav}} = G\frac{m_{\text{shuttle}}m_{\text{astronaut}}}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.03\times10^5\,\text{kg}\right)\left(120\,\text{kg}\right)}{\left(20\,\text{m}\right)^2}$$

$$= 2.061\times10^{-6}\,\text{N} \approx \boxed{2.1\times10^{-6}\,\text{N}}$$

(*c*) $\boxed{\text{The gravity force is larger, by a factor of approximately 100.}}$

$\boxed{37.}$ The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Earth}}}{I_{\text{Jupiter}}} = \frac{r^2_{\substack{\text{Sun-}\\\text{Jupiter}}}}{r^2_{\substack{\text{Sun-}\\\text{Earth}}}} = \frac{\left(7.78\times10^{11}\,\text{m}\right)^2}{\left(1.496\times10^{11}\,\text{m}\right)^2} = 27.0$$

So it would take an area of $\boxed{27\text{m}^2}$ at Jupiter to collect the same radiation as a 1.0-m^2 solar panel at the Earth.

38. Use Eq. 31-14. Note that the higher frequencies have the shorter wavelengths.

(*a*) For FM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.08\times10^8\,\text{Hz}\right)} = \boxed{2.78\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(8.8\times10^7\,\text{Hz}\right)} = \boxed{3.41\,\text{m}}$$

(*b*) For AM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.7\times10^6\,\text{Hz}\right)} = \boxed{180\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(5.35\times10^5\,\text{Hz}\right)} = \boxed{561\,\text{m}}$$

39. Use Eq. 31-14.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.9\times10^9\,\text{Hz}\right)} = \boxed{0.16\,\text{m}}$$

40. The resonant frequency of an *LC* circuit is given by $f = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{LC}}$. We assume the inductance is constant, and form the ratio of the two frequencies.

$$\frac{f_1}{f_2} = \frac{\frac{2\pi}{\sqrt{LC_1}}}{\frac{2\pi}{\sqrt{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = \left(\frac{f_1}{f_2}\right)^2 C_1 = \left(\frac{550\,\text{kHz}}{1610\,\text{kHz}}\right)^2\left(2200\,\text{pF}\right) = \boxed{260\,\text{pF}}$$

41. The resonant frequency of an *LC* circuit is given by $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$. Solve for the inductance.

$$f = \frac{1}{2\pi\sqrt{LC}} \rightarrow L = \frac{1}{4\pi^2 f^2 C}$$

$$L_1 = \frac{1}{4\pi^2 f_1^2 C} = \frac{1}{4\pi^2\left(88\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)} = 5.3\times10^{-9}\,\text{H}$$

$$L_2=\frac{1}{4\pi^2 f_2^2 C}=\frac{1}{4\pi^2\left(108\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)}=3.5\times10^{-9}\,\text{H}$$

The range of inductances is $\boxed{3.5\times10^{-9}\,\text{H}\le L\le 5.3\times10^{-9}\,\text{H}}$

42. The rms electric field strength of the beam can be found from the Poynting vector.

$$S=\frac{P}{A}=c\varepsilon_0 E_{\text{rms}}^2 \quad\rightarrow$$

$$E_{\text{rms}}=\sqrt{\frac{P}{Ac\varepsilon_0}}=\sqrt{\frac{1.2\times10^4\,\text{W}}{\pi\left(750\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}}=\boxed{1.6\,\text{V/m}}$$

$\boxed{43.}$ The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the magnitude of the Poynting vector.

$$E_{\text{rms}}=\frac{V_{\text{rms}}}{d}=\frac{1.00\times10^{-3}\,\text{V}}{1.60\,\text{m}}=\boxed{6.25\times10^{-4}\,\text{V/m}}$$

$$S=c\varepsilon_0 E_{\text{rms}}^2=c\varepsilon_0\frac{V_{\text{rms}}^2}{d^2}=\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(1.00\times10^{-3}\,\text{V}\right)^2}{\left(1.60\,\text{m}\right)^2}$$

$$=\boxed{1.04\times10^{-9}\,\text{W/m}^2}$$

44. We ignore the time for the sound to travel to the microphone. Find the difference between the time for sound to travel to the balcony and for a radio wave to travel 3000 km.

$$\Delta t=t_{\text{radio}}-t_{\text{sound}}=\left(\frac{d_{\text{radio}}}{c}\right)-\left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right)=\left(\frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}}\right)-\left(\frac{50\,\text{m}}{343\,\text{m/s}}\right)=-0.14\,\text{s},$$

so the $\boxed{\text{person at the radio hears the voice 0.14 s sooner.}}$

45. The length is found from the speed of light and the duration of the burst.

$$d=ct=\left(3.00\times10^8\,\text{m/s}\right)\left(10^{-8}\text{s}\right)=\boxed{3\,\text{m}}$$

46. The time travel delay is the distance divided by the speed of radio waves (which is the speed of light).

$$t=\frac{d}{c}=\frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}}=\boxed{0.01\,\text{s}}$$

47. The time consists of the time for the radio signal to travel to Earth and the time for the sound to travel from the loudspeaker. We use 343 m/s for the speed of sound.

$$t=t_{\text{radio}}+t_{\text{sound}}=\left(\frac{d_{\text{radio}}}{c}\right)+\left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right)=\left(\frac{3.84\times10^8\,\text{m}}{3.00\times10^8\,\text{m/s}}\right)+\left(\frac{25\,\text{m}}{343\,\text{m/s}}\right)=\boxed{1.35\,\text{s}}$$

Note that about 5% of the time is for the sound wave.

48. (*a*) The rms value of the associated electric field is found from Eq. 24-6.

$$u=\tfrac{1}{2}\varepsilon_0E^2=\varepsilon_0E_{\text{rms}}^2 \quad\rightarrow\quad E_{\text{rms}}=\sqrt{\frac{u}{\varepsilon_0}}=\sqrt{\frac{4\times10^{-14}\,\text{J/m}^3}{8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2}}=0.0672\,\text{V/m}\approx\boxed{0.07\,\text{V/m}}$$

(*b*) A comparable value can be found using the magnitude of the Poynting vector.

$$\overline{S}=\varepsilon_0 cE_{\text{rms}}^2=\frac{P}{4\pi r^2}\quad\rightarrow$$

$$r=\frac{1}{E_{\text{rms}}}\sqrt{\frac{P}{4\pi\varepsilon_0 c}}=\frac{1}{0.0672\,\text{V/m}}\sqrt{\frac{7500\,\text{W}}{4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)}}$$

$$=7055\,\text{m}\approx\boxed{7\,\text{km}}$$

$\boxed{49.}$ The light has the same intensity in all directions, so use a spherical geometry centered on the source to find the value of the Poynting vector. Then use Eq. 31-19a to find the magnitude of the electric field, and Eq. 31-11 with $v=c$ to find the magnitude of the magnetic field.

$$S=\frac{P_0}{A}=\frac{P_0}{4\pi r^2}=\tfrac{1}{2}c\varepsilon_0 E_0^2\quad\rightarrow$$

$$E_0=\sqrt{\frac{P_0}{2\pi r^2 c\varepsilon_0}}=\sqrt{\frac{(75\,\text{W})}{2\pi(2.00\,\text{m})^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}}=33.53\,\text{V/m}$$

$$\approx\boxed{34\,\text{V/m}}$$

$$B_0=\frac{E_0}{c}=\frac{(33.53\,\text{V/m})}{\left(3.00\times10^8\,\text{m/s}\right)}=\boxed{1.1\times10^{-7}\,\text{T}}$$

50. The radiation from the Sun has the same intensity in all directions, so the rate at which energy passes through a sphere centered at the Sun is $P=S4\pi R^2$. This rate must be the same at any distance from the Sun. Use this fact to calculate the magnitude of the Poynting vector at Mars, and then use the Poynting vector to calculate the rms magnitude of the electric field at Mars.

$$S_{\text{Mars}}\left(4\pi R_{\text{Mars}}^2\right)=S_{\text{Earth}}\left(4\pi R_{\text{Earth}}^2\right)\quad\rightarrow\quad S_{\text{Mars}}=S_{\text{Earth}}\left(\frac{R_{\text{Earth}}^2}{R_{\text{Mars}}^2}\right)=c\varepsilon_0 E_{\substack{\text{rms}\\\text{Mars}}}^2\quad\rightarrow$$

$$E_{\substack{\text{rms}\\\text{Mars}}}=\sqrt{\frac{S_{\text{Earth}}}{c\varepsilon_0}}\left(\frac{R_{\text{Earth}}}{R_{\text{Mars}}}\right)=\sqrt{\frac{1350\,\text{W/m}^2}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}}\left(\frac{1}{1.52}\right)=\boxed{469\,\text{V/m}}$$

51. The direction of the wave velocity is the direction of the cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. "South" crossed into "west" gives the direction $\boxed{\text{downward}}$. The electric field is found from the Poynting vector, Eq. 31-19a, and then the magnetic field is found from Eq. 31-11 with $v=c$.

$$S=\tfrac{1}{2}c\varepsilon_0 E_0{}^2\quad\rightarrow$$

$$E_0=\sqrt{\frac{2S}{c\varepsilon_0}}=\sqrt{\frac{2\left(560\,\text{W/m}^2\right)}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{n}\bullet\text{m}^2\right)}}=649\,\text{V/m}\approx\boxed{650\,\text{V/m}}$$

$$B_0=\frac{E_0}{c}=\frac{(649\,\text{V/m})}{\left(3.00\times10^8\,\text{m/s}\right)}=\boxed{2.2\times10^{-6}\,\text{T}}$$

52. From the hint, we use Eq. 29-4, which says $\mathscr{E}=\mathscr{E}_0\sin\omega t=NBA\omega\sin\omega t$. The intensity is given, and this can be used to find the magnitude of the magnetic field.

$$\overline{S}=\frac{E_{\text{rms}}B_{\text{rms}}}{\mu_0}=\frac{cB_{\text{rms}}^2}{\mu_0}\quad\rightarrow\quad B_{\text{rms}}=\sqrt{\frac{\mu_0\overline{S}}{c}}\quad;\quad\mathscr{E}=\mathscr{E}_0\sin\omega t=NBA\omega\sin\omega t\quad\rightarrow$$

$$\mathcal{E}_{\text{rms}} = NA\omega B_{\text{rms}} = NA\omega\sqrt{\frac{\mu_0 \bar{S}}{c}}$$

$$= (320)\pi(0.011\,\text{m})^2\, 2\pi\left(810\times10^3\,\text{Hz}\right)\sqrt{\frac{\left(4\pi\times10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2\right)\left(1.0\times10^{-4}\,\text{W/m}^2\right)}{3.00\times10^8\,\text{m/s}}}$$

$$= \boxed{4.0\times10^{-4}\,\text{V}}$$

53. (*a*) Since intensity is energy per unit time per unit area, the energy is found by multiplying the intensity times the area of the antenna times the elapsed time.

$$U = IAt = \left(1.0\times10^{-13}\,\text{W/m}^2\right)\pi\left(\frac{0.33\,\text{m}}{2}\right)^2(6.0\,\text{h})(3600\,\text{s/h}) = \boxed{1.8\times10^{-10}\,\text{J}}$$

(*b*) The electric field amplitude can be found from the intensity, which is the magnitude of the Poynting vector. The magnitude of the magnetic field is then found from Eq. 31-11 with $v = c$.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{2\bar{I}}{\varepsilon_0 c}} = \sqrt{\frac{2\left(1.0\times10^{-13}\,\text{W/m}^2\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)}} = 8.679\times10^{-6}\,\text{V/m}$$

$$\approx \boxed{8.7\times10^{-6}\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{8.679\times10^{-6}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{2.9\times10^{-14}\,\text{T}}$$

54. Use the relationship between average intensity (the magnitude of the Poynting vector) and electric field strength, as given by Eq. 31-19a. Also use the fact that intensity is power per unit area. We assume that the power is spherically symmetric about source.

$$\bar{S} = \tfrac{1}{2}\varepsilon_0 c E_0{}^2 = \frac{P}{A} = \frac{P}{4\pi r^2} \quad \rightarrow$$

$$r = \sqrt{\frac{P}{2\pi\varepsilon_0 c E_0{}^2}} = \sqrt{\frac{25{,}000\,\text{W}}{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(0.020\,\text{V/m}\right)^2}} = 61{,}200\,\text{m}$$

$$\approx \boxed{61\,\text{km}}$$

Thus, to receive the transmission one should be within 61 km of the station.

$\boxed{55.}$ The light has the same intensity in all directions. Use a spherical geometry centered at the source with the definition of the Poynting vector.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0{}^2 = \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 \quad \rightarrow \quad \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 = \frac{P_0}{4\pi r^2} \quad \rightarrow \quad \boxed{E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}}$$

56. (*a*) The radio waves have the same intensity in all directions. The power crossing a given area is the intensity times the area. The intensity is the total power through the area of a sphere centered at the source.

$$P = IA = \frac{P_0}{A_{\text{total}}}A = \frac{35{,}000\,\text{W}}{4\pi\left(1.0\times10^3\,\text{m}\right)^2}\left(1.0\,\text{m}^2\right) = 2.785\times10^{-3}\,\text{W} \approx \boxed{2.8\,\text{mW}}$$

(*b*) We find the rms value of the electric field from the intensity, which is the magnitude of the Poynting vector.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{P_0}{4\pi r^2} \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(1.0\times10^3\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 1.024\,\text{V/m} \approx \boxed{1.0\,\text{V/m}}$$

(*c*) The voltage over the length of the antenna is the electric field times the length of the antenna.

$$V_{\text{rms}} = E_{\text{rms}} d = \left(1.024\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{1.0\,\text{V}}$$

(*d*) We calculate the electric field at the new distance, and then calculate the voltage.

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(5.0\times10^4\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 2.048\times10^{-2}\,\text{V/m} \;\;;\;\; V_{\text{rms}} = E_{\text{rms}} d = \left(2.048\times10^{-2}\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{2.0\times10^{-2}\,\text{V}}$$

57. The power output of the antenna would be the intensity at a given distance from the antenna, times the area of a sphere surrounding the antenna. The intensity is the magnitude of the Poynting vector.

$$S = \tfrac{1}{2} c\varepsilon_0 E_0^2$$

$$P = 4\pi r^2 S = 2\pi r^2 c\varepsilon_0 E_0^2 = 2\pi\left(0.50\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3\times10^6\,\text{V/m}\right)^2$$

$$\approx \boxed{4\times10^{10}\,\text{W}}$$

This is many orders of magnitude higher than the power output of commercial radio stations, which are no higher than the 10's of kilowatts.

58. We calculate the speed of light in water according to the relationship given.

$$v_{\text{water}} = \frac{1}{\sqrt{K\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}c = \frac{1}{\sqrt{1.77}}\left(3.00\times10^8\,\text{m/s}\right) = \boxed{2.25\times10^8\,\text{m/s}}$$

$$\frac{v_{\text{water}}}{c} = \frac{\frac{1}{\sqrt{K}}c}{c} = \frac{1}{\sqrt{K}} = 0.752 = \boxed{75.2\%}$$

59. A standing wave has a node every half-wavelength, including the endpoints. For this wave, the nodes would occur at the spacing calculated here.

$$\tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{c}{f} = \tfrac{1}{2}\frac{3.00\times10^8\,\text{m/s}}{2.45\times10^9\,\text{Hz}} = \boxed{0.0612\,\text{m}}$$

Thus there would be nodes at the following distances from a wall:

0, 6.12 cm, 12.2 cm, 18.4 cm, 24.5 cm, 30.6 cm, and 36.7 cm (approximately the other wall).

So there are $\boxed{\text{5 nodes}}$, not counting the ones at (or near) the walls.

60. (*a*) Assume that the wire is of length ℓ and cross-sectional area A. There must be a voltage across the ends of the wire to make the current flow $(V = IR)$, and there must be an electric field associated with that voltage $(E = V/\ell)$. Use these relationships with the definition of displacement current, Eq. 31-3.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d(V/\ell)}{dt} = \varepsilon_0 \frac{A}{\ell}\frac{dV}{dt} = \varepsilon_0 \rho \frac{A}{\rho\ell}\frac{d(IR)}{dt}$$

$$= \varepsilon_0 \rho \frac{1}{R} R \frac{dI}{dt} = \boxed{\varepsilon_0 \rho \frac{dI}{dt}}$$

(*b*) Calculate the displacement current found in part (*a*).

$$I_D = \varepsilon_0 \rho \frac{dI}{dt} = \left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\left(1.68\times10^{-8}\,\Omega{\bullet}\mathrm{m}\right)\left(\frac{1.0\,\mathrm{A}}{1.0\times10^{-3}\,\mathrm{s}}\right)$$

$$= 1.4868\times10^{-16}\,\mathrm{A} \approx \boxed{1.5\times10^{-16}\,\mathrm{A}}$$

(*c*) From example 28-6, Ampere's law gives the magnetic field created by a cylinder of current as $B = \dfrac{\mu_0 I}{2\pi r}$ at a distance of *r* from the axis of the cylindrical wire. This is true whether the current is displacement current or steady current.

$$B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{N{\bullet}s^2/C^2}\right)\left(1.486\times10^{-16}\,\mathrm{A}\right)}{2\pi\left(1.0\times10^{-3}\,\mathrm{m}\right)} = 2.97\times10^{-20}\,\mathrm{T} \approx \boxed{3.0\times10^{-20}\,\mathrm{T}}$$

$$\frac{B_D}{B_{\text{steady}}} = \frac{\dfrac{\mu_0 I_D}{2\pi r}}{\dfrac{\mu_0 I_{\text{steady}}}{2\pi r}} = \frac{I_D}{I_{\text{steady}}} = \frac{1.486\times10^{-16}\,\mathrm{A}}{1.0\,\mathrm{A}} = 1.486\times10^{-16} \approx \boxed{1.5\times10^{-16}}$$

61. (*a*) We note that $-\alpha x^2 - \beta^2 t^2 + 2\alpha\beta x t = -(\alpha x - \beta t)^2$ and so $E_y = E_0 e^{-(\alpha x-\beta t)^2} = E_0 e^{-\alpha^2\left(x-\frac{\beta}{\alpha}t\right)^2}$. Since the wave is of the form $f(x-vt)$, with $v=\beta/\alpha$, the wave is moving in the $\boxed{+x\text{ direction}}$.

(*b*) The speed of the wave is $v=\beta/\alpha=c$, and so $\boxed{\beta=\alpha c}$.

(*c*) The electric field is in the *y* direction, and the wave is moving in the *x* direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must be in the direction of motion, the magnetic field must be in the *z* direction. The magnitudes are related by $\left|\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{E}}\right|/c$.

$$\boxed{B_z = \frac{E_0}{c} e^{-(\alpha x-\beta t)^2}}$$

62. (*a*) Use the $\sin A \pm \sin B = 2\sin\left(\dfrac{A\pm B}{2}\right)\cos\left(\dfrac{A\mp B}{2}\right)$ from page A-4 in Appendix A.

$$E_y = E_0\left[\sin(kx-\omega t)+\sin(kx+\omega t)\right]$$

$$= 2E_0\sin\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right) = 2E_0\sin(kx)\cos(-\omega t)$$

$$= \boxed{2E_0\sin(kx)\cos(\omega t)}$$

$$B_z = B_0\left[\sin(kx-\omega t)-\sin(kx+\omega t)\right]$$

$$= 2B_0\sin\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right) = 2B_0\sin(-\omega t)\cos(kx)$$

$$= \boxed{-2B_0\cos(kx)\sin(\omega t)}$$

(*b*) The Poynting vector is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2E_0\sin(kx)\cos(\omega t) & 0 \\ 0 & 0 & -2B_0\cos(kx)\sin(\omega t) \end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-4E_0B_0\sin(kx)\cos(kx)\sin(\omega t)\cos(\omega t)\right] = \boxed{-\frac{1}{\mu_0}\hat{\mathbf{i}}\,E_0B_0\sin(2kx)\sin(2\omega t)}$$

This is 0 for all times at positions where $\sin(2kx) = 0$.

$$\sin(2kx) = 0 \;\rightarrow\; 2kx = n\pi \;\rightarrow\; \boxed{x = \frac{n\pi}{2k},\ n = 0, \pm 1, \pm 2, \ldots}$$

63. (*a*) To show that $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular, calculate their dot product.

$$\vec{\mathbf{E}}\bullet\vec{\mathbf{B}} = \left[E_0\sin(kx-\omega t)\hat{\mathbf{j}} + E_0\cos(kx-\omega t)\hat{\mathbf{k}}\right]\bullet\left[B_0\cos(kx-\omega t)\hat{\mathbf{j}} - B_0\sin(kx-\omega t)\hat{\mathbf{k}}\right]$$

$$= E_0\sin(kx-\omega t)B_0\cos(kx-\omega t) - E_0\cos(kx-\omega t)B_0\sin(kx-\omega t) = 0$$

Since $\vec{\mathbf{E}}\bullet\vec{\mathbf{B}} \equiv 0$, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other at all times.

(*b*) The wave moves in the direction of the Poynting vector, which is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & E_0\sin(kx-\omega t) & E_0\cos(kx-\omega t) \\ 0 & B_0\cos(kx-\omega t) & -B_0\sin(kx-\omega t) \end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-E_0B_0\sin^2(kx-\omega t) - E_0B_0\cos^2(kx-\omega t)\right] + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(0) = -\frac{1}{\mu_0}E_0B_0\hat{\mathbf{i}}$$

We see that the Poynting vector points in the negative $\underline{x}$ direction, and so the wave moves in the negative *x* direction, which is perpendicular to both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

(*c*) We find the magnitude of the electric field vector and the magnetic field vector.

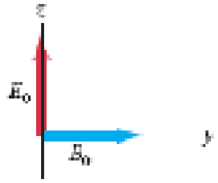
$$\left|\vec{\mathbf{E}}\right| = E = \left(\left[E_0\sin(kx-\omega t)\right]^2 + \left[E_0\cos(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[E_0^2\sin^2(kx-\omega t) + E_0^2\cos^2(kx-\omega t)\right]^{1/2} = E_0$$

$$\left|\vec{\mathbf{B}}\right| = B = \left(\left[B_0\cos(kx-\omega t)\right]^2 + \left[B_0\sin(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[B_0^2\cos^2(kx-\omega t) + B_0^2\sin^2(kx-\omega t)\right]^{1/2} = B_0$$

(*d*) At *x* = 0 and *t* = 0, $\vec{\mathbf{E}} = E_0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}$. See the figure. The *x* axis is coming out of the page toward the reader. As time increases, the component of the electric field in the *z* direction electric field begins to get smaller and the component in the negative *y* direction begins to get larger. At the same time, the component of the magnetic field in the *y* direction begins to get smaller, and the component in the *z* direction begins to get larger. The net effect is that $\boxed{\text{both vectors rotate counterclockwise}}$.



CHAPTER 32: Light: Reflection and Refraction

Responses to Questions

$\boxed{1.}$ (*a*) The Moon would look just like it does now, since the surface is rough. Reflected sunlight is scattered by the surface of the Moon in many directions, making the surface appear white.

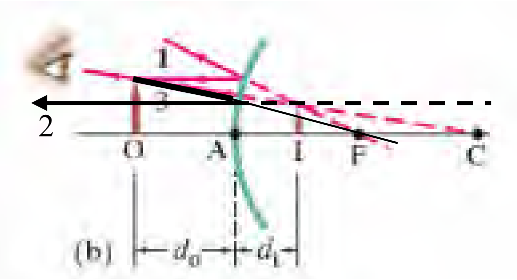
(*b*) With a polished, mirror-like surface, the Moon would reflect an image of the Sun, the stars, and the Earth. The appearance of the Moon would be different as seen from different locations on the Earth.

2. Yes, it would have been possible, although certainly difficult. Several attempts have been made to reenact the event in order to test its feasibility. Two of the successful attempts include a 1975 experiment directed by Greek scientist Dr. Ioannis Sakkas and a 2005 experiment performed by a group of engineering students at MIT. (See www.mit.edu for links to both these and other similar experiments.) In both these cases, several individual mirrors operating together simulated a large spherical mirror and were used to ignite a wooden boat. If in fact the story is true, Archimedes would have needed good weather and an enemy fleet that cooperated by staying relatively still while the focused sunlight heated the wood.

3. The focal length of a plane mirror is infinite. The magnification of a plane mirror is 1.

4. The image is real and inverted, because the magnification is negative. The mirror is concave, because convex mirrors can only form virtual images. The image is on the same side of the mirror as the object; real images are formed by converging light rays and light rays cannot actually pass through a mirror.

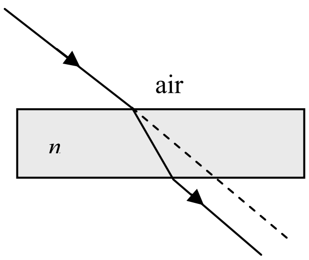
5. Ray 2 is directed as if it were going through the focal point and is reflected from the convex mirror parallel to the principal axis.



6. Yes. For a plane mirror, $d_o = -d_i$, since the object and image are equidistant from the mirror and the image is virtual, or behind the mirror. The focal length of a plane mirror is infinite, so the result of the mirror equation, Eq. 32-2, is $\frac{1}{d_o} + \frac{1}{d_i} = 0$, or $d_o = -d_i$, as expected.

$\boxed{7.}$ Yes. When a concave mirror produces a real image of a real object, both d_o and d_i are positive. The magnification equation, $m = -\frac{d_i}{d_o}$, results in a negative magnification, which indicates that the image is inverted.

8. A light ray entering the solid rectangular object will exit the other side following a path that is parallel to its original path but displaced slightly from it. The angle of refraction in the glass can be determined geometrically from this displacement and the thickness of the object. The index of refraction can then be determined using Snell's Law with this angle of refraction and the original angle of incidence. The speed of light in the material follows from the definition of the index of refraction: $n = c/v$.

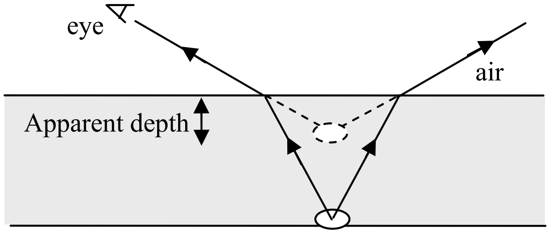


9. This effect is similar to diffuse reflection off of a rough surface. A ripply sea has multiple surfaces which are at an angle to reflect the image of the Moon into your eyes. This makes the image of the Moon appear elongated.

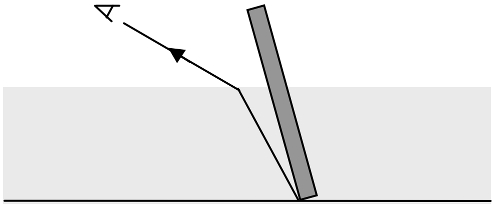
10. A negative object distance corresponds to a virtual object. This could occur if converging rays from another mirror or lens were intercepted by the mirror before actually forming an image. This image would be the object for the mirror.

11. The angle of refraction and the angle of incidence are both zero in this case.

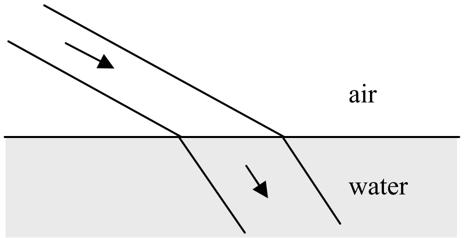
12. Underestimate. The light rays leaving the bottom of the pool bend away from the normal as they enter the air, so their source appears to be more shallow than it actually is. The greater the viewing angle, the more the bending of the light and therefore the less the apparent depth.



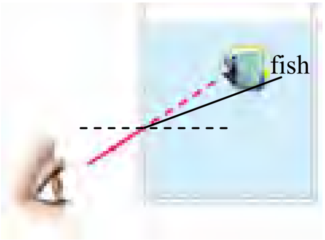
13. Your brain interprets the refracted rays as if the part of the stick that is under water is closer to the surface than it actually is, so the stick appears bent.



14. Because the broad beam hits the surface of the water at an angle, it illuminates an area of the surface that is wider than the beam width. Light from the beam bends towards the normal. The refracted beam is wider than the incident beam because one edge of the beam strikes the surface first, while the other edge travels farther in the air. (See the adjacent diagram.)



15. The light rays from the fish are bent away from the normal as they leave the tank. The fish will appear closer to the side of the tank than it really is.



16. The water drop acts like a lens, and refracts light as the light passes through it. Also, some of the light incident on the air/water boundary is reflected at the surface, so the drop can be seen in reflected light.

17. When the light ray passes from the blue material to the green material, the ray bends toward the normal. This indicates that the index of refraction of the blue material is less than that of the green material. When the light ray passes from the green material to the yellow material, the ray bends away from the normal, but not far enough to make the ray parallel to the initial ray, indicating that the index of refraction of the yellow material is less than that of the green material but larger than the index of refraction of the blue material. The ranking of the indices of refraction is, least to greatest, blue, yellow, and green.

18. No. Total internal reflection can only occur when light travels from a medium of higher index of refraction to a medium of lower index of refraction.

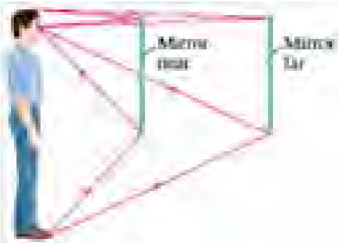
$\boxed{19.}$ No. The refraction of light as it enters the pool will make the object look smaller. See Figure 32-32 and Conceptual Example 32-11.

20. The mirror is concave, and the person is standing inside the focal point so that a virtual, upright image is formed. (A convex mirror would also form a virtual, upright image but the image would be smaller than the object.) In addition, an image is also present at the far right edge of the mirror, which is only possible if the mirror is concave.

21. (*a*) Since the light is coming from a vacuum into the atmosphere, which has a larger index of refraction, the light rays should bend toward the normal (toward the vertical direction).
(*b*) The stars are closer to the horizon than they appear to be from the surface of the Earth.

Solutions to Problems

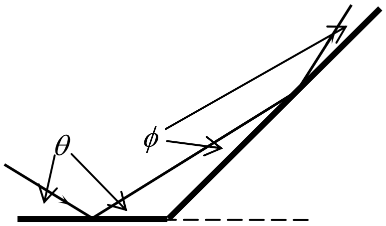
$\boxed{1.}$ Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.



2. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is twice the distance from the object to the mirror, or $\boxed{5.6\,\text{m}}$.

3. The law of reflection can be applied twice. At the first reflection, the angle is θ, and at the second reflection, the angle is ϕ. Consider the triangle formed by the mirrors and the first reflected ray.

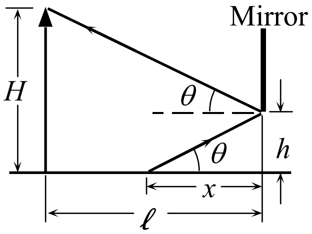
$$\theta + \alpha + \phi = 180^\circ \quad\rightarrow\quad 38^\circ + 135^\circ + \phi = 180^\circ \quad\rightarrow\quad \boxed{\phi = 7^\circ}$$



4. The angle of incidence is the angle of reflection. See the diagram for the appropriate lengths.

$$\tan\theta=\frac{(H-h)}{\ell}=\frac{h}{x}\quad\rightarrow$$

$$\frac{(1.64\,\mathrm{m}-0.38\,\mathrm{m})}{(2.30\,\mathrm{m})}=\frac{(0.38\,\mathrm{m})}{x}\quad\rightarrow\quad x=\boxed{0.69\,\mathrm{m}}$$



5. The incoming ray is represented by line segment DA. For the first reflection at *A* the angles of incidence and reflection are θ_1. For the second reflection at *B* the angles of incidence and reflection are θ_2. We relate θ_1 and θ_2 to the angle at which the mirrors meet, ϕ, by using the sum of the angles of the triangle *ABC*.

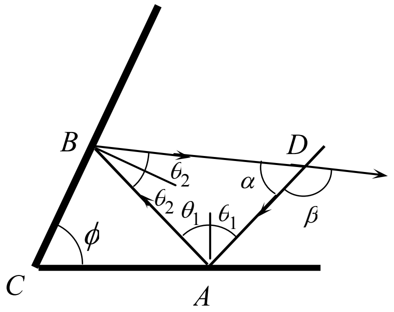
$$\phi+(90^\circ-\theta_1)+(90^\circ-\theta_2)=180^\circ\quad\rightarrow\quad\phi=\theta_1+\theta_2$$

Do the same for triangle *ABD*.

$$\alpha+2\theta_1+2\theta_2=180^\circ\quad\rightarrow\quad\alpha=180^\circ-2(\theta_1+\theta_2)=180^\circ-2\phi$$

At point *D* we see that the deflection is as follows.

$$\beta=180^\circ-\alpha=180^\circ-(180^\circ-2\phi)=\boxed{2\phi}$$

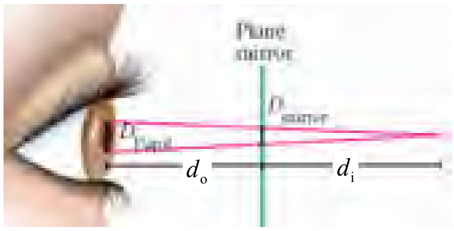


6. The rays entering your eye are diverging from the virtual image position behind the mirror. Thus the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image.

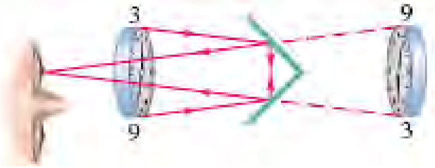
$$\frac{D_{\text{mirror}}}{d_{\text{i}}}=\frac{D_{\text{pupil}}}{(d_{\text{o}}+d_{\text{i}})}\quad\rightarrow\quad D_{\text{mirror}}=D_{\text{pupil}}\frac{d_{\text{i}}}{(d_{\text{o}}+d_{\text{i}})}=\tfrac{1}{2}D_{\text{pupil}}$$

$$A_{\text{mirror}}=\tfrac{1}{4}\pi D_{\text{mirror}}^2=\tfrac{1}{4}\pi\tfrac{1}{4}D_{\text{pupil}}^2=\frac{\pi}{16}\left(4.5\times10^{-3}\,\mathrm{m}\right)^2$$

$$=\boxed{4.0\times10^{-6}\,\mathrm{m}^2}$$



$\boxed{7.}$ See the "top view" ray diagram.



8. (*a*) The velocity of the incoming light wave is in the direction of the initial light wave. We can write this velocity in component form, where the three axes of our coordinate system are chosen to be perpendicular to the plane of each of the three mirrors. As the light reflects off any of the three mirrors, the component of the velocity perpendicular to that mirror reverses direction. The other two velocity components will remain unchanged. After the light has reflected off of each of the three mirrors, each of the three velocity components will be reversed and the light will be traveling directly back from where it came.

(*b*) If the mirrors are assumed to be large enough, the light can only reflect off two of the mirrors if the velocity component perpendicular to the third mirror is zero. Therefore, in this case the light is still reflected back directly to where it came.

9. The rays from the Sun will be parallel, so the image will be at the focal point, which is half the radius of curvature.

$$r = 2f = 2(18.8\,\text{cm}) = \boxed{37.6\,\text{cm}}$$

10. To produce an image at infinity, the object must be at the focal point, which is half the radius of curvature.

$$d_\text{o} = f = \tfrac{1}{2}r = \tfrac{1}{2}(24.0\,\text{cm}) = \boxed{12.0\,\text{cm}}$$

11. The image flips at the focal point, which is half the radius of curvature. Thus the radius is $\boxed{1.0\text{ m}}$.

12. (*a*) The focal length is half the radius of curvature, so $f = \tfrac{1}{2}r = \tfrac{1}{2}(24\,\text{cm}) = \boxed{12\,\text{cm}}$.

(*b*) Use Eq. 32-2.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad\rightarrow\quad d_\text{i} = \frac{d_\text{o}f}{d_\text{o} - f} = \frac{(35\,\text{cm})(24\,\text{cm})}{35\,\text{cm} - 24\,\text{cm}} = \boxed{76\,\text{cm}}$$

(*c*) The image is $\boxed{\text{inverted}}$, since the magnification is negative.

$\boxed{13.}$ The ball is a convex mirror with a focal length $f = \tfrac{1}{2}r = \tfrac{1}{2}(-4.6\,\text{cm}) = -2.3\,\text{cm}$. Use Eq. 32-3 to locate the image.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad\rightarrow\quad d_\text{i} = \frac{d_\text{o}f}{d_\text{o} - f} = \frac{(25.0\,\text{cm})(-2.3\,\text{cm})}{25.0\,\text{cm} - (-2.3\,\text{cm})} = -2.106\,\text{cm} \approx -2.1\,\text{cm}$$

The image is $\boxed{\text{2.1 cm behind the surface of the ball, virtual, and upright}}$. Note that the magnification is $m = -\dfrac{d_\text{i}}{d_\text{o}} = \dfrac{-(-2.106\,\text{cm})}{(25.0\,\text{cm})} = +0.084.$

14. The image distance can be found from the object distance of 1.7 m and the magnification of +3. With the image distance and object distance, the focal length and radius of curvature can be found.

$$m = \frac{-d_\text{i}}{d_\text{o}} \quad\rightarrow\quad d_\text{i} = -md_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad\rightarrow\quad f = \frac{d_\text{o}d_\text{i}}{d_\text{o} + d_\text{i}} = \frac{d_\text{o}(-md_\text{o})}{d_\text{o} - md_\text{o}} = \frac{md_\text{o}}{m-1} = \frac{3(1.7\,\text{m})}{3-1} = 2.55\,\text{m}$$

$$r = 2f = 2(2.55\,\text{m}) = \boxed{5.1\,\text{m}}$$

15. The object distance of 2.00 cm and the magnification of +4.0 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_\text{i}}{d_\text{o}} \quad\rightarrow\quad d_\text{i} = -md_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \quad\rightarrow\quad f = \frac{d_\text{o}d_\text{i}}{d_\text{o} + d_\text{i}} = \frac{d_\text{o}(-md_\text{o})}{d_\text{o} - md_\text{o}} = \frac{md_\text{o}}{m-1} = \frac{4(2.00\,\text{cm})}{4-1} = 2.677\,\text{cm}$$

$$r = 2f = 2(2.667\,\text{cm}) = \boxed{5.3\,\text{cm}}$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$.

16. The mirror must be $\boxed{\text{convex}}$. Only convex mirrors produce images that are upright and smaller than the object. The object distance of 18.0 m and the magnification of +0.33 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.33(18.0\,\text{m})}{0.33-1} = -8.866\,\text{m}$$

$$r = 2f = 2(-8.866\,\text{m}) = \boxed{-17.7\,\text{m}}$$

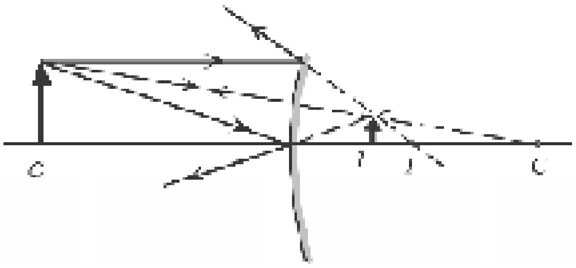
17. The object distance of 3.0 m and the magnification of +0.5 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.5(3.0\,\text{m})}{0.5-1} = -3.0\,\text{m}$$

$$r = 2f = 2(-3.0\,\text{m}) = \boxed{-6.0\,\text{m}}$$

18. (*a*) From the ray diagram it is seen that the image is virtual. We estimate the image distance as $\boxed{-6\,\text{cm.}}$



(*b*) Use a focal length of $-9.0\,\text{cm}$ with the object distance of $18.0\,\text{cm}$.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow$$

$$d_i = \frac{d_o f}{d_o - f} = \frac{(18.0\,\text{cm})(-9.0\,\text{cm})}{18.0\,\text{cm} - (-9.0\,\text{cm})} = \boxed{-6.0\,\text{cm}}$$

(*c*) We find the image size from the magnification:

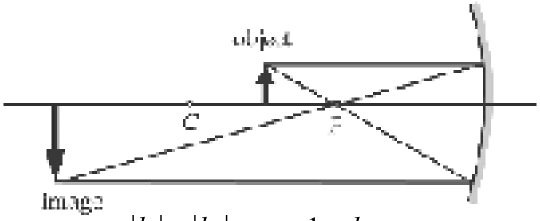
$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \rightarrow h_i = h_o\left(\frac{-d_i}{d_o}\right) = (3.0\,\text{mm})\left(\frac{6.0\,\text{cm}}{18.0\,\text{cm}}\right) = \boxed{1.0\,\text{mm}}$$

$\boxed{19.}$ Take the object distance to be ∞, and use Eq. 32-3. Note that the image distance is negative since the image is behind the mirror.

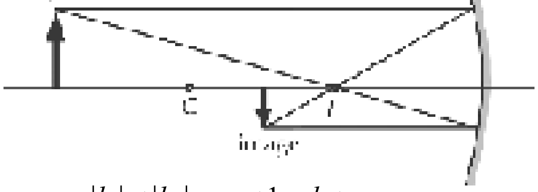
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = d_i = -16.0\,\text{cm} \rightarrow r = 2f = \boxed{-32.0\,\text{cm}}$$

Because the focal length is negative, the mirror is $\boxed{\text{convex.}}$

20. (*a*)



$|h_i| > |h_o|$; $m > 1$; $d_o < r$



$|h_i| < |h_o|$; $m < 1$; $d_o > r$

(*b*) Apply Eq. 32-3 and Eq. 32-4.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{2}{r} \rightarrow d_i=\frac{rd_o}{(2d_o-r)} \ ; \ m=\frac{-d_i}{d_o}=\frac{-r}{(2d_o-r)}$$

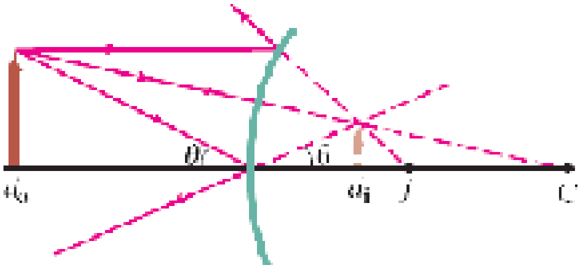
If $d_o > r$, then $(2d_o-r) > r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(>r)}<1.$

If $d_o < r$, then $(2d_o-r) < r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(<r)}>1.$

21. Consider the ray that reflects from the center of the mirror, and note that $d_i < 0$.

$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i} \rightarrow \frac{-d_i}{d_o}=\frac{h_i}{h_o}$$

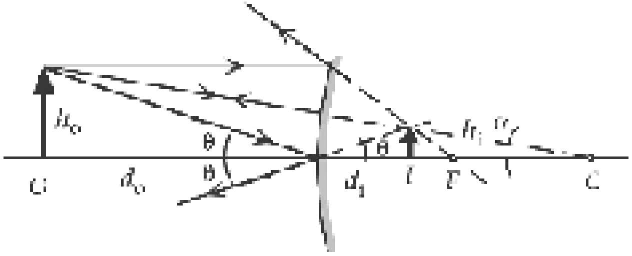
$$m=\frac{h_i}{h_o}=\boxed{-\frac{d_i}{d_o}}$$



22. From the ray diagram, we see that with a negative image distance, we have the following.

$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i}$$

$$\tan\alpha=\frac{h_o}{(d_o+r)}=\frac{h_i}{(r+d_i)}$$



When we divide the two equations, we get

$$\frac{(d_o+r)}{d_o}=-\frac{(r+d_i)}{d_i} \rightarrow 1+\frac{r}{d_o}=-1-\frac{r}{d_i} \rightarrow \frac{r}{d_o}+\frac{r}{d_i}=-2 \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=-\frac{2}{r}$$

If we define $f=\frac{r}{2}$ and consider the radius of curvature and focal length to be negative, then we have Eq. 32-2, $\boxed{\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}}$..

23. Use Eq. 32-2 and 32-3.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{(0.55)(3.2\,\text{m})}{0.55-1}=\boxed{-3.9\,\text{m}}$$

24. (*a*) We are given that $d_i=d_o$. Use Eq. 32-3.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{2}{d_o}=\frac{1}{f} \rightarrow \boxed{d_o=2f=r}$$

The object should be placed at the $\boxed{\text{center of curvature.}}$

(*b*) Because the image is in front of the mirror, $d_i>0$, it is $\boxed{\text{real.}}$

(*c*) The magnification is $m = \frac{-d_\text{i}}{d_\text{o}} = \frac{-d_\text{o}}{d_\text{o}} = -1$. Because the magnification is negative, the image is $\boxed{\text{inverted.}}$

(*d*) As found in part (*c*), $m = \boxed{-1.}$

$\boxed{25.}$ (*a*) To produce a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror.}}$

(*b*) Find the image distance from the magnification.

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}} \rightarrow d_\text{i} = -\frac{d_\text{o}h_\text{i}}{h_\text{o}} = -\frac{(26\,\text{cm})(3.5\,\text{cm})}{(4.5\,\text{cm})} = -20.2\,\text{cm} \approx \boxed{-20\,\text{cm}} \text{ (2 sig. fig.)}$$

As expected, $d_\text{i} < 0$. The image is located $\boxed{\text{20 cm behind the surface.}}$

(*c*) Find the focal length from Eq. 32.3.

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \rightarrow f = \frac{d_\text{o}d_\text{i}}{d_\text{o}+d_\text{i}} = \frac{(26\,\text{cm})(-20.2\,\text{cm})}{(26\,\text{cm})+(-20.2\,\text{cm})} = -90.55\,\text{cm} \approx \boxed{-91\,\text{cm}}$$

(*d*) The radius of curvature is twice the focal length.

$$r = 2f = 2(-90.55\,\text{cm}) = -181.1\,\text{cm} \approx \boxed{-180\,\text{cm}}$$

26. (*a*) To produce a larger upright image requires a $\boxed{\text{concave mirror.}}$

(*b*) The image will be $\boxed{\text{upright and virtual.}}$

(*c*) We find the image distance from the magnification:

$$m = \frac{h_\text{i}}{h_\text{o}} = \frac{-d_\text{i}}{d_\text{o}} \rightarrow d_\text{i} = -md_\text{o}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \rightarrow f = \frac{d_\text{o}d_\text{i}}{d_\text{o}+d_\text{i}} = \frac{d_\text{o}(-md_\text{o})}{d_\text{o}-md_\text{o}} = \frac{md_\text{o}}{m-1} \rightarrow$$

$$r = 2f = \frac{2md_\text{o}}{m-1} = \frac{2(1.35)(20.0\,\text{cm})}{1.35-1} = \boxed{154\,\text{cm}}$$

27. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length.

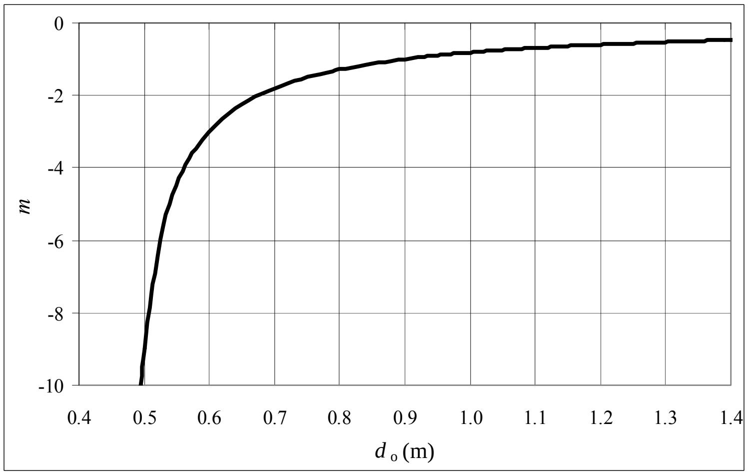
$$m = -d_\text{i}/d_\text{o} \rightarrow d_\text{i} = -md_\text{o}$$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \rightarrow$$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{-md_\text{o}} \rightarrow$$

$$\boxed{m = \frac{f}{f-d_\text{o}}}$$

(*b*) We set $f = 0.45$ m and draw a graph of the magnification as a function of the object distance. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.27b."



(*c*) The image and object will have the same lateral size when the magnification is equal to negative one. Setting the magnification equal to negative one, we solve the equation found in part (a) for the object distance.

$$m=\frac{f}{f-d_o}=-1 \rightarrow d_o=2f=\boxed{0.90\,\text{m}}$$

(*d*) From the graph we see that for the image to be much larger than the object, the object should be placed at a point $\boxed{\text{just beyond the focal point.}}$

28. We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length, with the focal length given as $f=-|f|$.

$$m=-\frac{d_i}{d_o} \rightarrow d_i=-md_o \rightarrow \frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{1}{-|f|}=\frac{1}{d_o}+\frac{1}{-md_o}\rightarrow\boxed{m=\frac{|f|}{|f|+d_o}}$$

From this relation, the closer the object is to the mirror (i.e., smaller object distance) the greater the magnification. Since a person's nose is closer to the mirror than the rest of the face, its image appears larger.

29. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the object distance in terms of the magnification and the focal length.

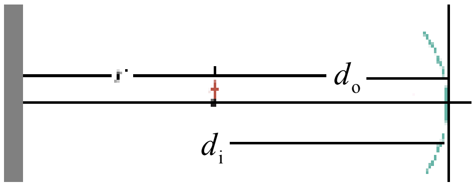
$$m=-\frac{d_i}{d_o}\rightarrow d_i=-md_o$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{1}{f}=\frac{1}{d_o}+\frac{1}{-md_o}=\frac{1}{d_o}\left(1-\frac{1}{m}\right)\rightarrow\boxed{d_o=f\left(1-\frac{1}{m}\right)}$$

(*b*) We set the object distance equal to the range of all positive numbers. Since the focal length of a convex lens is negative, the term in parentheses in the above equation must be the range of all negative numbers for the object distance to include the range of all positive numbers. We solve the resulting equation for all possible values of the magnification.

$$\left(1-\frac{1}{m}\right)\le 0\rightarrow 1\le\frac{1}{m}\rightarrow\boxed{0\le m\le 1}$$

30. The distance between the mirror and the wall is equal to the image distance, which we can calculate using Eq. 32-2. The object is located a distance *r* from the wall, so the object distance will be *r* less than the image distance. The focal length is given by Eq. 32-1. For the object distance to be real, the image distance must be greater than *r*.

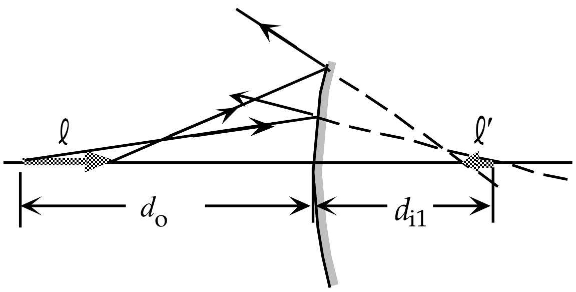


$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{2}{r}=\frac{1}{d_i-r}+\frac{1}{d_i}\rightarrow 2d_i^2-4d_ir+r^2=0$$

$$d_i=\frac{4r\pm\sqrt{16r^2-8r^2}}{4}=r\left(1\pm\frac{\sqrt{2}}{2}\right)\approx 0.292r \text{ or } \boxed{1.71r}$$

Use Eq. 32-3 to calculate the magnification: $m=-\frac{d_i}{d_o}=\frac{1.71r}{1.71r-r}=\boxed{-2.41}$

$\boxed{31.}$ The lateral magnification of an image equals the height of the image divided by the height of the object. This can be written in terms of the image distance and focal length with Eqs. 32-2 and 32-3.



$$\frac{1}{f}=\frac{1}{d_i}+\frac{1}{d_o} \rightarrow d_i=\left(\frac{fd_o}{d_o-f}\right)$$

$$m=\frac{-d_i}{d_o}=-\frac{f}{d_o-f}$$

The longitudinal magnification will be the difference in image distances of the two ends of the object divided by the length of the image. Call the far tip of the wire object 1 with object distance d_o. The close end of the wire will be object 2 with object distance $d_o-\ell$. Using Eq. 32-2 we can find the image distances for both ends.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_{i1}} \rightarrow d_{i1}=\frac{d_of}{d_o-f} \quad ; \quad \frac{1}{f}=\frac{1}{d_o-\ell}+\frac{1}{d_{i2}} \rightarrow d_{i2}=\frac{(d_o-\ell)f}{d_o-\ell-f}$$

Taking the difference in image distances and dividing by the object length gives the longitudinal magnification.

$$m_\ell=\frac{d_{i1}-d_{i2}}{\ell}=\frac{1}{\ell}\left(\frac{d_of}{d_o-f}-\frac{(d_o-\ell)f}{d_o-\ell-f}\right)=\frac{d_of(d_o-\ell-f)-(d_o-f)(d_o-\ell)f}{\ell(d_o-f)(d_o-\ell-f)}$$

$$=\frac{-f^2}{(d_o-f)(d_o-\ell-f)}$$

Set $\ell \ll d_o$, so that the ℓ drops out of the second factor of the denominator. Then rewrite the equation in terms of the lateral magnification, using the expression derived at the beginning of the problem.

$$m_\ell=\frac{-f^2}{(d_o-f)^2}=-\left[\frac{f}{(d_o-f)}\right]^2=\boxed{-m^2}$$

The negative sign indicates that the $\boxed{\text{image is reversed front to back}}$, as shown in the diagram.

32. We find the index of refraction from Eq. 32-1.

$$n=\frac{c}{v}=\frac{3.00\times10^8\ \text{m/s}}{2.29\times10^8\ \text{m/s}}=\boxed{1.31}$$

33. In each case, the speed is found from Eq. 32-1 and the index of refraction.

(*a*) Ethyl alcohol: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.36}=\boxed{2.21\times10^8\ \text{m/s}}$

(*b*) Lucite: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.51}=\boxed{1.99\times10^8\ \text{m/s}}$

(*c*) Crown glass: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.52}=\boxed{1.97\times10^8\ \text{m/s}}$

34. Find the distance traveled by light in 4.2 years.

$$d=c\Delta t=(3.00\times10^8\ \text{m/s})(4.2\ \text{yr})(3.16\times10^7\ \text{s/yr})=\boxed{4.0\times10^{16}\ \text{m}}$$

35. The time for light to travel from the Sun to the Earth is found from the distance between them and the speed of light.

$$\Delta t = \frac{d}{c} = \frac{1.50\times10^{11}\,\text{m}}{3.00\times10^{8}\,\text{m/s}} = 5.00\times10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

36. We find the index of refraction from Eq. 32-1.

$$n = \frac{c}{v} = \frac{c}{0.88v_{\text{water}}} = \frac{c}{0.88\left(\dfrac{c}{n_{\text{water}}}\right)} = \frac{n_{\text{water}}}{0.88} = \frac{1.33}{0.88} = \boxed{1.51}$$

$\boxed{37.}$ The length in space of a burst is the speed of light times the elapsed time.

$$d = ct = \left(3.00\times10^{8}\,\text{m/s}\right)\left(10^{-8}\,\text{s}\right) = \boxed{3\,\text{m}}$$

38. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 38.5^\circ\right) = \boxed{55.9^\circ}$$

39. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.56}\sin 63^\circ\right) = \boxed{35^\circ}$$

40. We find the incident angle in the air (relative to the normal) from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 33.0^\circ\right) = 46.4^\circ$$

Since this is the angle relative to the horizontal, the angle as measured from the horizon is $90.0^\circ - 46.4^\circ = \boxed{43.6^\circ}$.

41. We find the incident angle in the water from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 56.0^\circ\right) = \boxed{38.6^\circ}$$

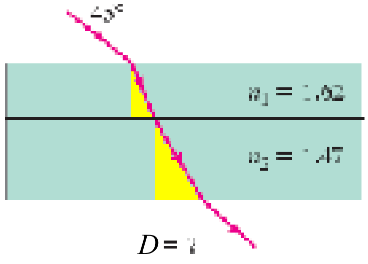
42. The angle of reflection is equal to the angle of incidence: $\theta_{\text{refl}} = \theta_1 = 2\theta_2$. Use Snell's law

$$n_{\text{air}}\sin\theta_1 = n_{\text{glass}}\sin\theta_2 \;\rightarrow\; (1.00)\sin 2\theta_2 = (1.56)\sin\theta_2$$

$$\sin 2\theta_2 = 2\sin\theta_2\cos\theta_2 = (1.56)\sin\theta_2 \;\rightarrow\; \cos\theta_2 = 0.780 \;\rightarrow\; \theta_2 = 38.74^\circ$$

$$\theta_1 = 2\theta_2 = \boxed{77.5^\circ}$$

$\boxed{43.}$ The beam forms the hypotenuse of two right triangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement *D*. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5).



$$\sin 45 = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 45}{n_1}\right) = \sin^{-1}\left(\frac{\sin 45}{1.62}\right) = 25.88°$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 45}{n_2}\right) = \sin^{-1}\left(\frac{\sin 45}{1.47}\right) = 28.75°$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan\theta_1 + h_1 \tan\theta_1 = (2.0\,\text{cm})\tan 25.88° + (3.0\,\text{cm})\tan 28.75° = \boxed{2.6\,\text{cm}}$$

44. (*a*) We use Eq. 32-5 to calculate the refracted angle as the light enters the glass (n=1.56) from the air (n=1.00).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \to \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.56}\sin\ 43.5°\right] = 26.18° \approx \boxed{26.2°}$$

(*b*) We again use Eq. 32-5 using the refracted angle in the glass and the indices of refraction of the glass and water.

$$\theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.56}{1.33}\sin\ 26.18°\right] = 31.17° \approx \boxed{31.2°}$$

(*c*) We repeat the same calculation as in part (a), but using the index of refraction of water.

$$\theta_3 = \sin^{-1}\left[\frac{n_1}{n_3}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33}\sin\ 43.5°\right] = 31.17° \approx \boxed{31.2°}$$

As expected the refracted angle in the water is the same whether the light beam first passes through the glass, or passes directly into the water.

45. We find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{\ell_1}{h_1} = \frac{(2.5\,\text{m})}{(1.3\,\text{m})} = 1.9231 \quad\to\quad \theta_1 = 62.526°$$

For the refraction from air into water, we have

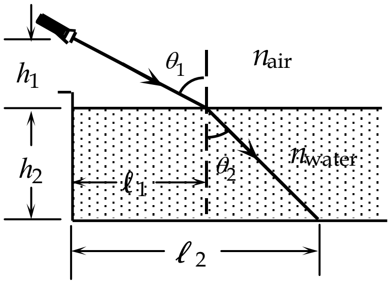
$$n_{\text{air}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

$$(1.00)\sin 62.526° = (1.33)\sin\theta_2 \quad\to\quad \theta_2 = 41.842°$$

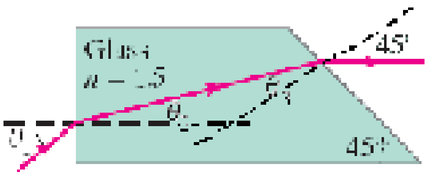
We find the horizontal distance from the edge of the pool from

$$\ell = \ell_1 + \ell_2 = \ell_1 + h_2 \tan\theta_2$$

$$= 2.5\,\text{m} + (2.1\,\text{m})\tan 41.842° = 4.38\,\text{m} \approx \boxed{4.4\,\text{m}}$$



46. Since the light ray travels parallel to the base when it exits the glass, and the back edge of the glass makes a 45° angle to the horizontal, the exiting angle of refraction is 45°. We use Snell's law, Eq. 32-5, to calculate the incident angle at the back pane.



$$\theta_3 = \sin^{-1}\left[\frac{n_4}{n_3}\sin\theta_4\right] = \sin^{-1}\left[\frac{1.0}{1.5}\sin\ 45°\right] = 28.13°$$

We calculate the refracted angle at the front edge of the glass by noting that the angles θ_2 and θ_3 in the figure form two angles of a triangle. The third angle, as determined by the perpendiculars to the surface, is 135°.

$$\theta_2 + \theta_3 + 135° = 180° \rightarrow \theta_2 = 45° - \theta_3 = 45° - 28.13° = 16.87°$$

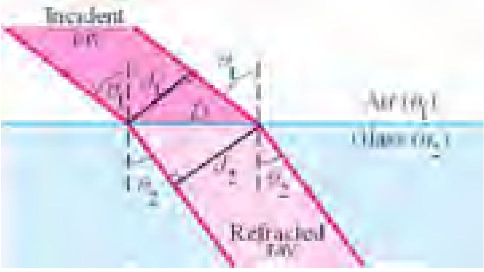
Finally, we use Snell's law at the front face of the glass to calculate the incident angle.

$$\theta_1 = \sin^{-1}\left[\frac{n_2}{n_1}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.5}{1.0}\sin\ 16.87°\right] = 25.81° \approx \boxed{26°}$$

47. As the light ray passes from air into glass with an angle of incidence of 25°, the beam will refract. Determine the angle of refraction by applying Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow$$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.5}\sin\ 25°\right] = 16.36°$$



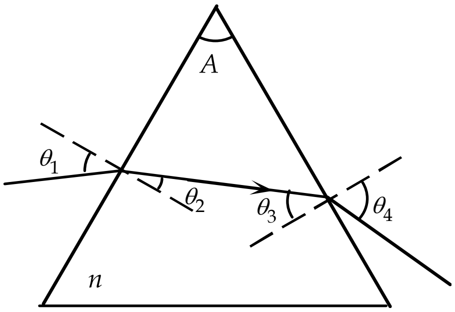
We now consider the two right triangles created by the diameters of the incident and refracted beams with the air–glass interface, as shown in the figure. The diameters form right angles with the ray direction and using complementary angles we see that the angle between the diameter and the interface is equal to the incident and refracted angles. Since the air–glass interface creates the hypotenuse for both triangles we use the definition of the cosine to solve for this length in each triangle and set the lengths equal. The resulting equation is solved for the diameter of the refracted ray.

$$D = \frac{d_1}{\cos\theta_1} = \frac{d_2}{\cos\theta_2} \quad \rightarrow \quad d_2 = d_1\frac{\cos\theta_2}{\cos\theta_1} = (3.0\,\text{mm})\frac{\cos 16.36°}{\cos 25°} = \boxed{3.2\,\text{mm}}$$

48. Find the angle θ_2 for the refraction at the first surface.

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2$$

$$(1.00)\sin 45.0° = (1.54)\sin\theta_2 \quad \rightarrow \quad \theta_2 = 27.33°$$



Find the angle of incidence at the second surface from the triangle formed by the two sides of the prism and the light path.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \rightarrow$$

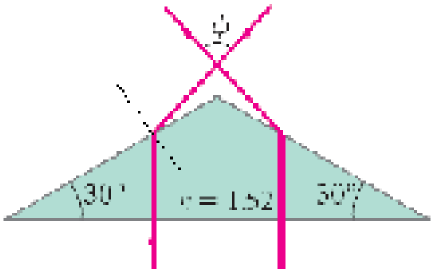
$$\theta_3 = A - \theta_2 = 60° - 27.33° = 32.67°$$

Use refraction at the second surface to find θ_4.

$$n\sin\theta_3 = n_{\text{air}} \sin\theta_4 \quad \rightarrow \quad (1.54)\sin 32.67° = (1.00)\sin\theta_4 \quad \rightarrow \quad \theta_4 = \boxed{56.2° \text{ from the normal}}$$

$\boxed{49.}$ Since the angle of incidence at the base of the prism is $0°$, the rays are undeflected there. The angle of incidence at the upper face of the prism is $30°$. Use Snell's law to calculate the angle of refraction as the light exits the prism.

$$n_1 \sin\theta_1 = \sin\theta_r \rightarrow \theta_r = \sin^{-1}(1.52\sin 30°) = 49.46°$$



From the diagram, note that a normal to either top surface makes a 30° angle from the vertical. Subtracting 30° from the refracted angle will give the angle of the beam with respect to the vertical. By symmetry, the angle ϕ is twice the angle of the refracted beam from the vertical.

$$\phi = 2(\theta_r - 30°) = 2(49.46° - 30°) = \boxed{38.9°}$$

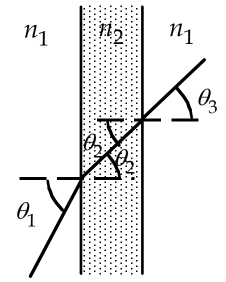
50. Because the surfaces are parallel, the angle of refraction from the first surface is the angle of incidence at the second. Thus for the two refractions, we have the following.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \;\; ; \;\; n_2 \sin\theta_2 = n_1 \sin\theta_3$$

Substitute the second equation into the first.

$$n_1 \sin\theta_1 = n_1 \sin\theta_3 \;\; \rightarrow \;\; \boxed{\theta_3 = \theta_1}$$

Because the ray emerges in the same index of refraction, it is undeviated.

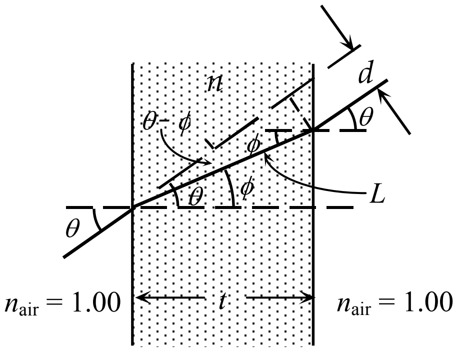


51. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam. Find the angle inside the glass from Snell's law, $n_{\text{air}} \sin\theta = n \sin\phi$. Since the angles are small, $\cos\phi \approx 1$ and $\sin\phi \approx \phi$, where ϕ is in radians.

$$(1.00)\,\theta = n\phi \;\; \rightarrow \;\; \phi = \frac{\theta}{n}$$

Find the distance along the ray in the glass from $L = \dfrac{t}{\cos\phi} \approx t$, and then find the perpendicular displacement from the original direction.

$$d = L\sin(\theta - \phi) \approx t(\theta - \phi) = t\left[\theta - \left(\frac{\theta}{n}\right)\right] = \boxed{\frac{t\theta(n-1)}{n}}$$



52. We find the speed of light from the speed of light in a vacuum divided by the index of refraction. Examining the graph we estimate that the index of refraction of 450 nm light in silicate flint glass is 1.643 and of 680 nm light is 1.613. There will be some variation in the answers due to estimation from the graph.

$$\frac{v_{\text{red}} - v_{\text{blue}}}{v_{\text{red}}} = \frac{c/n_{680} - c/n_{450}}{c/n_{680}} = \frac{1/1.613 - 1/1.643}{1/1.613} = 0.01826 \approx \boxed{1.8\%}$$

53. We find the angles of refraction in the glass from Snell's law, Eq. 32-5.

$$(1.00)\sin 60.00^\circ = (1.4831)\sin\theta_{2,\text{blue}} \;\; \rightarrow \;\; \theta_{2,\text{blue}} = 35.727^\circ$$

$$(1.00)\sin 60.00^\circ = (1.4754)\sin\theta_{2,\text{red}} \;\; \rightarrow \;\; \theta_{2,\text{red}} = 35.943^\circ \text{ which gives } \theta_{2,700} = 35.943^\circ.$$

Thus the angle between the refracted beams is

$$\theta_{2,\text{red}} - \theta_{2,\text{blue}} = 35.943^\circ - 35.727^\circ = 0.216^\circ \approx \boxed{0.22^\circ.}$$

54. The indices of refraction are estimated from Figure 32-28 as 1.642 for 465 nm and 1.619 for 652 nm. Consider the refraction at the first surface.

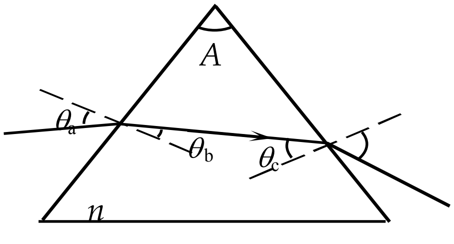
$$n_{\text{air}} \sin\theta_{\text{a}} = n \sin\theta_{\text{b}} \;\; \rightarrow$$

$$(1.00)\sin 45^\circ = (1.642)\sin\theta_{\text{b1}} \;\; \rightarrow \;\; \theta_{\text{b1}} = 25.51^\circ$$

$$(1.00)\sin 45^\circ = (1.619)\sin\theta_{\text{b2}} \;\; \rightarrow \;\; \theta_{\text{b2}} = 25.90^\circ$$

We find the angle of incidence at the second surface from the upper triangle.

$$(90^\circ - \theta_{\text{b}}) + (90^\circ - \theta_{\text{c}}) + A = 180^\circ \;\; \rightarrow$$



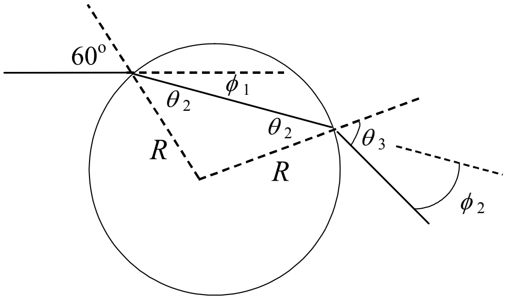
$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49°$; $\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$

Apply Snell's law at the second surface.

$$n\sin\theta_c = n_{air}\sin\theta_d$$

$$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1} \rightarrow \boxed{\theta_{d1} = 68.4°\text{ from the normal}}$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2} \rightarrow \boxed{\theta_{d2} = 65.2°\text{ from the normal}}$$



$\boxed{55.}$ At the first surface, the angle of incidence $\theta_1 = 60^{\circ}$ from air $(n_1 = 1.000)$ and the angle of refraction θ_2 into water $(n_2 = n)$ is found using Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow$$

$$(1.000)\sin 60° = (n)\sin\theta_2 \rightarrow$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 60°}{n}\right)$$

Note that at this surface the ray has been deflected from its initial direction by angle $\phi_1 = 60° - \theta_2$. From the figure we see that the triangle that is interior to the drop is an isosceles triangle, so the angle of incidence from water $(n_2 = n)$ at the second surface is θ_2 and angle of refraction is θ_3 into air $(n_3 = 1.000)$. This relationship is identical to the relationship at the first surface, showing that the refracted angle as the light exits the drop is again 60°.

$$n_2\sin\theta_2 = n_3\sin\theta_3 \rightarrow (n)\sin\theta_2 = (1.000)\sin\theta_3 \rightarrow \sin\theta_3 = n\sin\theta_2 \rightarrow$$

$$\sin\theta_3 = n\left(\frac{\sin 60^{\circ}}{n}\right) = \sin 60^{\circ} \rightarrow \theta_3 = 60^{\circ}$$

Note that at this surface the ray has been deflected from its initial direction by the angle $\phi_2 = \theta_3 - \theta_2 = 60^{\circ} - \theta_2$. The total deflection of the ray is equal to the sum of the deflections at each surface.

$$\phi = \phi_1 + \phi_2 = (60^{\circ} - \theta_2) + (60^{\circ} - \theta_2) = 120^{\circ} - 2\theta_2 = 120^{\circ} - 2\sin^{-1}\left(\frac{\sin 60^{\circ}}{n}\right)$$

Inserting the indices of refraction for the two colors and subtracting the angles gives the difference in total deflection.

$$\Delta\phi = \phi_{violet} - \phi_{red} = \left\{120^{\circ} - 2\sin^{-1}\left[\frac{\sin 60^{\circ}}{n_{violet}}\right]\right\} - \left\{120^{\circ} - 2\sin^{-1}\left[\frac{\sin 60^{\circ}}{n_{red}}\right]\right\}$$

$$= 2\left\{\sin^{-1}\left[\frac{\sin 60^{\circ}}{n_{red}}\right] - \sin^{-1}\left[\frac{\sin 60^{\circ}}{n_{violet}}\right]\right\} = 2\left\{\sin^{-1}\left[\frac{\sin 60^{\circ}}{1.330}\right] - \sin^{-1}\left[\frac{\sin 60^{\circ}}{1.341}\right]\right\} = \boxed{0.80^{\circ}}$$

56. (*a*) We solve Snell's law for the refracted angle. Then, since the index varies by only about 1%, we differentiate the angle with respect to the index of refraction to determine the spread in angle.

$$\sin\theta_1 = n\sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) \rightarrow$$

$$\frac{\Delta\theta_2}{\Delta n} \approx \frac{d\theta_2}{dn} = \frac{\sin\theta_1}{n^2\sqrt{1 - \frac{\sin^2\theta_1}{n^2}}} \rightarrow \boxed{\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}}$$

(*b*) We set $n = 1.5$ and $\theta_1 = 0° = 0\,\text{rad}$ and solve for the spread in refracted angle.

$$\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}} = (0.01)\frac{\sin\ 0}{\sqrt{1.5^2 - \sin^2 0}} = \boxed{0}$$

(*c*) We set $n = 1.5$ and $\theta_1 = 90°$ and solve for the spread in refracted angle. We must convert the spread from radians back to degrees.

$$\Delta\theta_2 = (0.01)\frac{\sin\ 90°}{\sqrt{1.5^2 - \sin^2 90°}} = 0.0089\ \text{rad}\left(\frac{180°}{\pi\ \text{rad}}\right) = \boxed{0.5°}$$

57. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

$$n_{\text{diamond}}\sin\theta_1 = n_{\text{water}}\sin\theta_2 \quad\rightarrow\quad \theta_1 = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{diamond}}}\right) = \sin^{-1}\frac{1.33}{2.42} = \boxed{33.3°}$$

Because diamond has the higher index, the light must start in $\boxed{\text{diamond.}}$

58. When the light in the liquid is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow\quad n_{\text{liquid}} = n_{\text{air}}\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\frac{1}{\sin 49.6°} = \boxed{1.31}$$

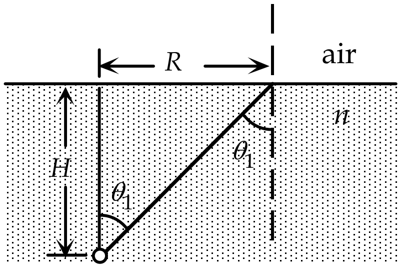
59. We find the critical angle for light leaving the water:

$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow$$

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\frac{1.00}{1.33} = 48.75°$$

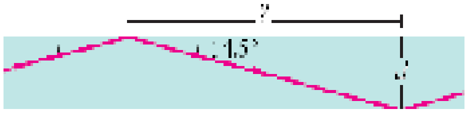
If the light is incident at a greater angle than this, it will totally reflect. Find *R* from the diagram.

$$R > H\tan\theta_1 = (72.0\,\text{cm})\tan 48.75° = \boxed{82.1\,\text{cm}}$$



60. The ray reflects at the same angle, so each segment makes a 14.5° angle with the side. We find the distance ℓ between reflections from the definition of the tangent function.

$$\tan\theta = \frac{d}{\ell} \rightarrow \ell = \frac{d}{\tan\theta} = \frac{1.40\times10^{-4}\ \text{m}}{\tan\ 14.5°} = \boxed{5.41\times10^{-4}\ \text{m}}$$



$\boxed{61.}$ We find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{\ell}{h} = \frac{(7.6\,\text{cm})}{(8.0\,\text{cm})} = 0.95 \quad\rightarrow\quad \theta_1 = 43.53°$$

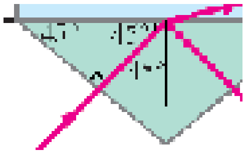
The relationship for the maximum incident angle for refraction from liquid into air gives this.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow\quad n_{\text{liquid}}\sin\theta_{1\,\text{max}} = (1.00)\sin 90° \quad\rightarrow\quad \sin\theta_{1\,\text{max}} = \frac{1}{n_{\text{liquid}}}$$

Thus we have the following.

$$\sin\theta_1 \geq \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}} \quad\rightarrow\quad \sin 43.53^\circ = 0.6887 \geq \frac{1}{n_{\text{liquid}}} \quad\rightarrow\quad \boxed{n_{\text{liquid}} \geq 1.5}$$

62. For the device to work properly, the light should experience total internal reflection at the top surface of the prism when it is a prism to air interface, but not total internal reflection when the top surface is a prism to water interface. Since the incident ray is perpendicular to the lower surface of the prism, light does not experience refraction at that surface. As shown in the diagram, the incident angle for the upper surface will be 45°. We then use Eq. 32-7 to determine the minimum index of refraction for total internal reflection with an air interface, and the maximum index of refraction for a water interface. The usable indices of refraction will lie between these two values.



$$\frac{n_2}{n_1} = \sin\theta_c \quad\rightarrow\quad n_{1,\min} = \frac{n_{\text{air}}}{\sin\theta_c} = \frac{1.00}{\sin 45^\circ} = 1.41 \quad\rightarrow\quad n_{1,\max} = \frac{n_{\text{water}}}{\sin\theta_c} = \frac{1.33}{\sin 45^\circ} = 1.88$$

The index of refraction must fall within the range $1.41 < n < 1.88$. $\boxed{\text{A Lucite prism will work.}}$

63. (*a*) We calculate the critical angle using Eq. 32-7. We calculate the time for each ray to pass through the fiber by dividing the length the ray travels by the speed of the ray in the fiber. The length for ray A is the horizontal length of the fiber. The length for ray B is equal to the length of the fiber divided by the critical angle, since ray B is always traveling along a diagonal line at the critical angle relative to the horizontal. The speed of light in the fiber is the speed of light in a vacuum divided by the index of refraction in the fiber.

$$\sin\theta_c = \frac{n_2}{n_1} \;;\; \Delta t = t_B - t_A = \frac{\ell_B}{v} - \frac{\ell_A}{v} = \frac{\ell_A}{v\sin\theta_c} - \frac{\ell_A}{v} = \frac{\ell_A}{c/n_1}\left(\frac{n_1}{n_2} - 1\right)$$

$$= \frac{(1.0\,\text{km})(1.465)}{(3.00\times10^5\,\text{km/s})}\left(\frac{1.465}{1.000} - 1\right) = \boxed{2.3\times10^{-6}\,\text{s}}$$

(*b*) We now replace the index of refraction of air (n = 1.000) with the index of refraction of the glass "cladding" (n = 1.460).

$$\Delta t = \frac{\ell_A n_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.0\,\text{km})(1.465)}{3.00\times10^5\,\text{km/s}}\left(\frac{1.465}{1.460} - 1\right) = \boxed{1.7\times10^{-8}\,\text{s}}$$

64. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad\rightarrow\quad n_1\sin 45^\circ = (1.00)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$, and so $n_1 \geq \dfrac{1}{\sin 45^\circ} = \boxed{1.41}$.

(*b*) When there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad\rightarrow\quad (1.58)\sin 45^\circ = (1.33)\sin\theta_2 \quad\rightarrow\quad \sin\theta_2 = 0.84$$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

(*c*) For total reflection when there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad\rightarrow\quad n_1\sin 45^\circ = (1.33)\sin\theta_2$$

$$n_1\sin 45^\circ = (1.33)\sin\theta_2.$$

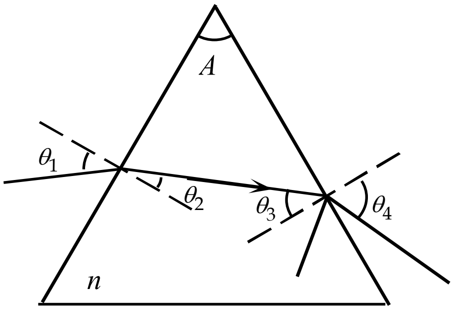
For total internal reflection to occur, $\sin\theta_2 \geq 1$.

$$n_1 \geq \frac{1.33}{\sin 45^\circ} = \boxed{1.88}$$

65. For the refraction at the first surface, we have the following.

$$n_{\text{air}}\sin\theta_1 = n\sin\theta_2 \;\rightarrow\; (1.00)\sin\theta_1 = n\sin\theta_2 \;\rightarrow$$

$$\sin\theta_2 = \sin\frac{\theta_1}{n}.$$



Find the angle of incidence at the second surface.

$$(90^\circ - \theta_2) + (90^\circ - \theta_3) + A = 180^\circ \;\rightarrow$$

$$\theta_3 = A - \theta_2 = 60.0^\circ - \theta_2$$

For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 = (1.00)\sin\theta_4$$

The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. For internal reflection to occur, we have the following.

$$n\sin\theta_3 = n\sin(A-\theta_2) \ge 1 \;\rightarrow\; n(\sin A\cos\theta_2 - \cos A\sin\theta_2) \ge 1$$

Use the result from the first surface to eliminate *n*.

$$\frac{\sin\theta_1(\sin A\cos\theta_2 - \cos A\sin\theta_2)}{(\sin\theta_2)} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2} - \cos A\right) \ge 1 \;\rightarrow$$

$$\frac{1}{\tan\theta_2} \ge \frac{\left[\left(\dfrac{1}{\sin\theta_1}\right) + \cos A\right]}{\sin A} = \frac{\left[\left(\dfrac{1}{\sin 45.0^\circ}\right) + \cos 60.0^\circ\right]}{\sin 60.0^\circ} = 2.210 \;\rightarrow \text{or}$$

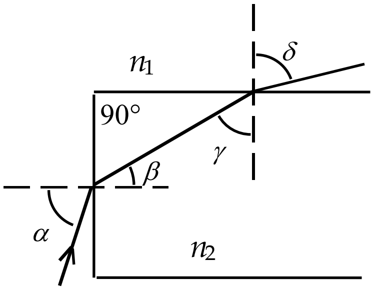
$$\tan\theta_2 \le 0.452 \;\rightarrow\; \theta_2 \le 24.3^\circ$$

Use the result from the first surface.

$$n_{\min} = \frac{\sin\theta_1}{\sin\theta_{2\max}} = \frac{\sin 45.0^\circ}{\sin 24.3^\circ} = 1.715 \;\rightarrow\; \boxed{n \ge 1.72}$$

66. For the refraction at the side of the rod, we have $n_2\sin\gamma = n_1\sin\delta$. The minimum angle for total reflection $\gamma_{\min}$ occurs when $\delta = 90^\circ$.

$$n_2\sin\gamma_{\min} = (1.00)(1) = 1 \;\rightarrow\; \sin\gamma_{\min} = \frac{1}{n_2}$$



Find the maximum angle of refraction at the end of the rod.

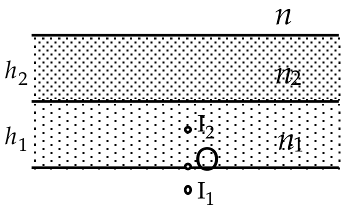
$$\beta_{\max} = 90^\circ - \gamma_{\min}$$

Because the sine function increases with angle, for the refraction at the end of the rod, we have the following.

$$n_1\sin\alpha_{\max} = n_2\sin\beta_{\max} \;\rightarrow\; (1.00)\sin\alpha_{\max} = n_2\sin(90^\circ - \gamma_{\min}) = n_2\cos\gamma_{\min}$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of α is 90°, so $n_2\cos\gamma_{\min} = 1$. When we divide this by the result for the refraction at the side, we get $\tan\gamma_{\min} = 1 \rightarrow \gamma_{\min} = 45^\circ$. Thus we have the following.

$$\boxed{n_2 \ge \frac{1}{\sin\gamma_{\min}} = \frac{1}{\sin 45^\circ} = 1.414}$$



$\boxed{67.}$ We find the location of the image of a point on the bottom from the refraction from water to glass, using Eq. 32-8, with $R = \infty$.

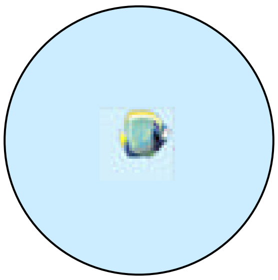
$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} = 0 \rightarrow$$

$$d_i = -\frac{n_2 d_o}{n_1} = -\frac{1.58(12.0\,\text{cm})}{1.33} = -14.26\,\text{cm}$$

Using this image distance from the top surface as the object for the refraction from glass to air gives the final image location, which is the apparent depth of the water.

$$\frac{n_2}{d_{o2}} + \frac{n_3}{d_{i2}} = \frac{n_3 - n_2}{R} = 0 \rightarrow d_{i2} = -\frac{n_3 d_{o2}}{n_2} = -\frac{1.00(13.0\,\text{cm} + 14.26\,\text{cm})}{1.58} = -17.25\,\text{cm}$$

Thus the bottom appears to be $\boxed{\text{17.3 cm below the surface of the glass}}$. In reality it is 25 cm.



68. (*a*) We use Eq. 32-8 to calculate the location of the image of the fish. We assume that the observer is outside the circle in the diagram, to the right of the diagram. The fish is located at the center of the sphere so the object distance is 28.0 cm. Since the glass is thin we use the index of refraction of the water and of the air. Index 1 refers to the water, and index 2 refers to the air. The radius of curvature of the right side of the bowl is negative.

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} \rightarrow$$

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{28.0\,\text{cm}}\right]^{-1} = \boxed{-28.0\,\text{cm}}$$

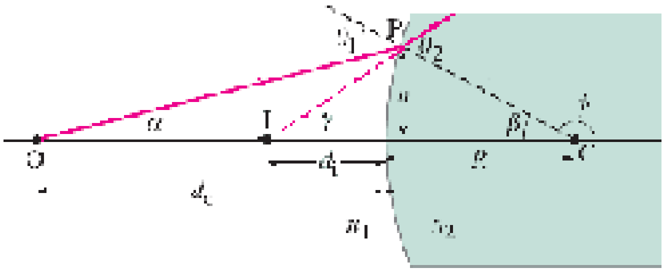
The image is also at the center of the bowl. When the fish is at the center of the bowl, all small-angle light rays traveling outward from the fish are approximately perpendicular to the surface of the bowl, and therefore do not refract at the surface. This causes the image of the fish to also be located at the center of the bowl.

(*b*) We repeat the same calculation as above with the object distance 20.0 from the right side of the bowl, so $d_o = 20.0\,\text{cm}$.

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{20.0\,\text{cm}}\right]^{-1} = \boxed{-18.3\,\text{cm}}$$

The fish appears closer to the center of the bowl than it actually is.

69. (*a*) The accompanying figure shows a light ray originating at point O and entering the convex spherical surface at point P. In this case $n_2 < n_1$. The ray bends away from the normal and creates a virtual image at point I. From the image and supplementary angles we obtain the relationships between the angles.



$$\theta_1 = \alpha + \beta \qquad \theta_2 = \gamma + \beta$$

We then use Snell's law to relate the incident and refracted angle. For this derivation we assume these are small angles.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow n_1\theta_1 = n_2\theta_2$$

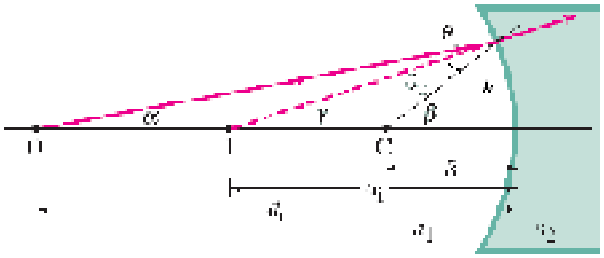
From the diagram we can create three right triangles, each with height h and lengths d_o, d_i, and R. Again, using the small angle approximation we obtain a relationship between the angles and lengths. Combining these definitions to eliminate the angles we obtain Eq. 32-8, noting that by our definition d_i is a negative value.

$$\alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{R} \;;\; \beta = \frac{h}{(-d_i)}$$

$$n_1\theta_1 = n_2\theta_2 \rightarrow n_1(\alpha+\beta) = n_2(\gamma+\beta) = n_1\alpha + n_1\beta = n_2\gamma + n_2\beta \rightarrow$$

$$n_1\frac{h}{d_o} + n_1\frac{h}{R} = n_2\frac{h}{(-d_i)} + n_2\frac{h}{R} \rightarrow \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

(*b*) This image shows a concave surface with $n_2 > n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.

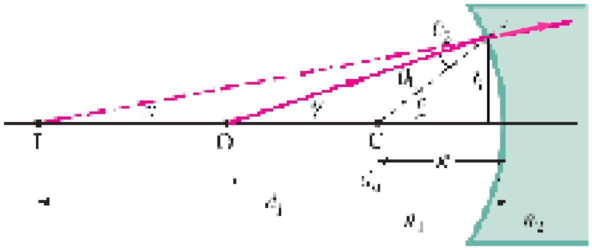


$$\theta_1 = \beta - \alpha \;;\; \theta_2 = \beta - \gamma \;;\; \alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{(-d_i)} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \rightarrow n_1(\beta-\alpha) = n_2(\beta-\gamma) = n_1\beta - n_1\alpha = n_2\beta - n_2\gamma \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \rightarrow \boxed{\frac{n_1}{d_0} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

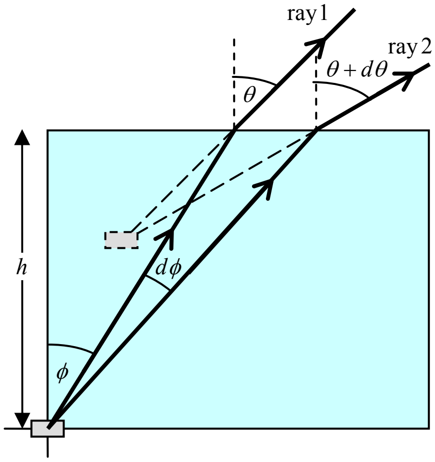
(*c*) This image shows a concave surface with $n_2 < n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.



$$\theta_1 = \beta - \gamma \;;\; \theta_2 = \beta - \alpha \;;\; \alpha = \frac{h}{(-d_i)} \;;\; \gamma = \frac{h}{d_o} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \rightarrow n_1(\beta-\gamma) = n_2(\beta-\alpha) = n_1\beta - n_1\gamma = n_2\beta - n_2\alpha \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \rightarrow \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$



70. We consider two rays leaving the coin. These rays refract upon leaving the surface and reach the observer's eye with angles of refraction all very near $\theta = 45°$. Let the origin of coordinates be at the actual location of the coin. We will write straight-line equations for each of two refracted rays, one with a refraction angle of θ and the other with a refraction angle of $\theta + d\theta$, and extrapolate them back to where they intersect to find the location of the image. We utilize the relationship $f(x+dx) = f(x) + \left(\frac{df}{dx}\right)dx.$

First, apply Snell's law to both rays.

Ray # 1, leaving the coin at angle ϕ.

$$n\sin\phi = \sin\theta$$

Ray # 2, leaving the coin at angle $\phi + d\phi$.

$$n\sin(\phi + d\phi) = \sin(\theta + d\theta)$$

Note the following relationship involving the differential angles.

$$\sin(\phi + d\phi) = \sin\phi + \frac{d(\sin\phi)}{d\phi}d\phi = \sin\phi + \cos\phi d\phi \ ; \ \sin(\theta + d\theta) = \sin\theta + \cos\theta d\theta$$

So for Ray # 2, we would have the following Snell's law relationship.

$$n[\sin\phi + \cos\phi d\phi] = [\sin\theta + \cos\theta d\theta] \ \rightarrow \ n\sin\phi + n\cos\phi d\phi = \sin\theta + \cos\theta d\theta \ \rightarrow$$

$$n\cos\phi d\phi = \cos\theta d\theta \ \rightarrow \ d\phi = \frac{\cos\theta}{n\cos\phi}d\theta$$

This relationship between $d\phi$ and $d\theta$ will be useful later in the solution.

Ray # 1 leaves the water at coordinates $x_1 = h\tan\phi,\ y_1 = h$ and has a slope after it leaves the water of $m_1 = \tan(90° - \theta) = \cot\theta.$ Thus a straight-line equation describing ray # 1 after it leaves the water is as follows.

$$y - y_1 = (x - x_1)m_1 \ \rightarrow \ y = h + (x - h\tan\phi)\cot\theta$$

Ray # 2 leaves the water at the following coordinates.

$$x_2 = h\tan(\phi + d\phi) = h\left[\tan\phi + \frac{d(\tan\phi)}{d\phi}d\phi\right] = h\left[\tan\phi + \sec^2\phi d\phi\right],\ y_2 = h$$

Ray # 2 has the following slope after it leaves the water.

$$m_2 = \tan\left[90° - (\theta + d\theta)\right] = \cot(\theta + d\theta) = \cot\theta + \frac{d(\cot\theta)}{d\theta}d\theta = \cot\theta - \csc^2\theta d\theta$$

Thus a straight-line equation describing ray # 2 after it leaves the water is as follows.

$$y - y_2 = (x - x_2)m_2 \ \rightarrow \ y = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right]$$

To find where these rays intersect, which is the image location, set the two expressions for y equal to each other.

$$h + (x - h\tan\phi)\cot\theta = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right] \ \rightarrow$$

Expanding the terms and subtracting common terms gives us the following.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta + h\sec^2\phi d\phi\csc^2\theta d\theta$$

The first three terms each have a differential factor, but the last term has two differential factors.

That means the last term is much smaller than the other terms, and so can be ignored. So we delete the last term, and use the relationship between the differentials derived earlier.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta \quad ; \quad d\phi = \frac{\cos\theta}{n\cos\phi}d\theta \quad \rightarrow$$

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi\frac{\cos\theta}{n\cos\phi}d\theta\cot\theta$$

$$x = h\left[\tan\phi - \sec^2\phi\frac{\cos\theta}{n\cos\phi}\frac{\cot\theta}{\csc^2\theta}\right] = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right]$$

Now we may substitute in values. We know that $\theta = 45°$ and $h = 0.75\,\text{m}$. We use the original relationship for ray # 1 to solve for ϕ. And once we solve for x, we use the straight-line equation for ray # 1 to solve for y.

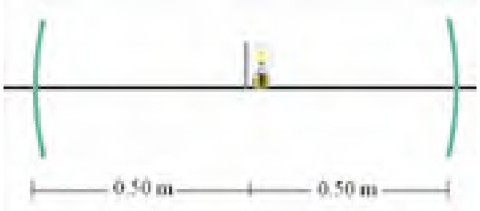
$$n\sin\phi = \sin\theta \quad \rightarrow \quad \phi = \sin^{-1}\frac{\theta}{n} = \sin^{-1}\frac{\sin 45°}{1.33} = 32.12°$$

$$x = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right] = 0.75\left[\tan 32.12 - \frac{\cos^2 45\sin 45}{1.33\cos^3 32.12}\right] = 0.1427\,\text{m}$$

$$y = h + \left(x - h\tan\phi\right)\cot\theta = 0.75 + \left(0.1427 - 0.75\tan 32.12\right)\cot 45 = 0.4264\text{m}$$

The image of the coin is located $\boxed{\text{0.14 m toward the viewer and 0.43 m above}}$ the actual coin.

71. Use Eq. 32-2 to determine the location of the image from the right mirror, in terms of the focal length. Since this distance is measured from the right mirror, we subtract that distance from the separation distance between the two mirrors to obtain the object distance for the left mirror. We then insert this object distance back into Eq. 32-2, with the known image distance and combine terms to write a quadratic equation for the focal length.



$$\frac{1}{f} = \frac{1}{d_{\text{o1}}} + \frac{1}{d_{\text{i1}}} \quad \rightarrow \quad d_{\text{i1}} = \left(\frac{1}{f} - \frac{1}{d_{\text{o1}}}\right)^{-1} = \frac{fd_{\text{o1}}}{d_{\text{o1}} - f}$$

$$d_{\text{o2}} = D - d_{\text{i1}} = D - \frac{fd_{\text{o1}}}{d_{\text{o1}} - f} = \frac{Dd_{\text{o1}} - fD - fd_{\text{o1}}}{d_{\text{o1}} - f}$$

$$\frac{1}{f} = \frac{1}{d_{\text{o2}}} + \frac{1}{d_{\text{i2}}} = \frac{d_{\text{o1}} - f}{Dd_{\text{o1}} - fD - fd_{\text{o1}}} + \frac{1}{d_{\text{i2}}} = \frac{d_{\text{i2}}d_{\text{o1}} - fd_{\text{i2}} + Dd_{\text{o1}} - fD - fd_{\text{o1}}}{d_{\text{i2}}\left(Dd_{\text{o1}} - fD - fd_{\text{o1}}\right)}$$

$$d_{\text{i2}}\left(Dd_{\text{o1}} - fD - fd_{\text{o1}}\right) = d_{\text{i2}}d_{\text{o1}}f - f^2d_{\text{i2}} + fDd_{\text{o1}} - f^2D - f^2d_{\text{o1}}$$

$$f^2\left[d_{\text{i2}} + D + d_{\text{o1}}\right] - f\left[2d_{\text{i2}}d_{\text{o1}} + Dd_{\text{o1}} + Dd_{\text{i2}}\right] + Dd_{\text{o1}}d_{\text{i2}} = 0$$

We insert the values for the initial object distance, final image distance, and mirror separation distance and then solve the quadratic equation.

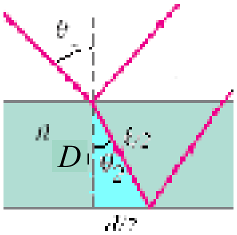
$$f^2\left[0.50\,\text{m} + 1.00\,\text{m} + 0.50\,\text{m}\right] - f\left[2\left(0.50\,\text{m}\right)^2 + 2\left(1.00\,\text{m}\right)\left(0.50\,\text{m}\right)\right] + \left(1.00\,\text{m}\right)\left(0.50\,\text{m}\right)^2 = 0$$

$$\left(2.00\,\text{m}\right)f^2 - \left(1.50\,\text{m}^2\right)f + 0.25\,\text{m}^3 = 0$$

$$f = \frac{1.50\,\text{m}^2 \pm \sqrt{\left(1.50\,\text{m}^2\right)^2 - 4\left(2.00\,\text{m}\right)\left(0.25\,\text{m}^3\right)}}{2\left(2.00\,\text{m}\right)} = \boxed{0.25\,\text{m} \text{ or } 0.50\,\text{m}}$$

If the focal length is 0.25 m, the right mirror creates an image at the location of the object. With the paper in place, this image would be blocked out. With a focal length of 0.50 m, the light from the right mirror comes out as parallel light. No image is formed from the right mirror. When this parallel light enters the second mirror it is imaged at the focal point (0.50 m) of the second mirror.

72. (*a*) We use Snell's law to calculate the refracted angle within the medium. Then using the right triangle formed by the ray within the medium, we can use the trigonometric identities to write equations for the horizontal displacement and path length.



$$\sin\theta_1 = n\sin\theta_2 \quad\rightarrow\quad \sin\theta_2 = \frac{\sin\theta_1}{n}$$

$$\cos\theta_2 = \frac{D}{\ell/2} \quad\rightarrow\quad \ell = \frac{2D}{\cos\theta_2} = \frac{2D}{\sqrt{1-\sin^2\theta_2}} = \boxed{\frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}}$$

$$\sin\theta_2 = \frac{d/2}{\ell/2} \quad\rightarrow\quad d = \ell\sin\theta_2 = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}\frac{\sin\theta_1}{n} = \boxed{\frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}}}$$

(*b*) Evaluate the above expressions for $\theta_1 = 0°$.

$$\ell = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}} = \frac{2nD}{\sqrt{n^2}} = 2D \;;\; \sin\theta_2 = \frac{d/2}{\ell/2} \quad\rightarrow\quad d = \frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}} = 0$$

These are the expected values.

$\boxed{73.}$ (*a*) The first image seen will be due to a single reflection off the front glass. This image will be equally far behind the mirror as you are in front of the mirror.

$$D_1 = 2\times 1.5 \text{ m} = \boxed{3.0 \text{ m}}$$

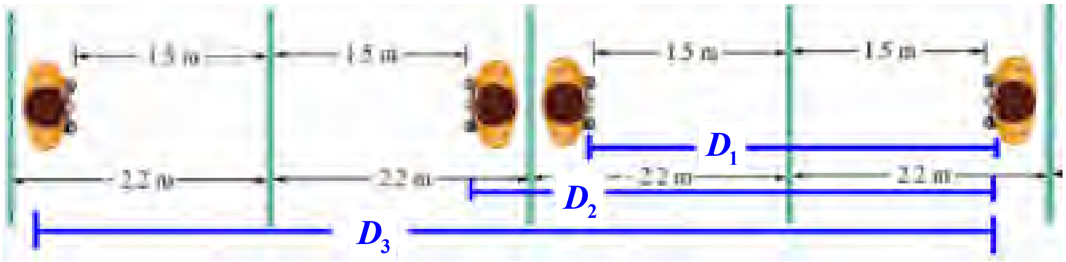
The second image seen will be the image reflected once off the front mirror and once off the back mirror. As seen in the diagram, this image will appear to be twice the distance between the mirrors.

$$D_2 = 1.5 \text{ m} + 2.2 \text{ m} + (2.2 \text{ m} - 1.5 \text{ m}) = 2\times 2.2 \text{ m} = \boxed{4.4 \text{ m}}$$

The third image seen will be the image reflected off the front mirror, the back mirror, and off the front mirror again. As seen in the diagram this image distance will be the sum of twice your distance to the mirror and twice the distance between the mirrors.

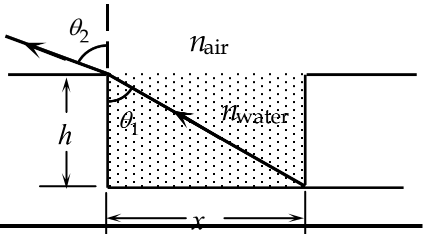
$$D_3 = 1.5 \text{ m} + 2.2 \text{ m} + 2.2 \text{ m} + 1.5 \text{ m} = 2\times 1.5 \text{ m} + 2\times 2.2 \text{ m} = \boxed{7.4 \text{ m}}$$

The actual person is to the far right in the diagram.



(*b*) We see from the diagram that the first image is facing $\boxed{\text{toward you}}$; the second image is facing $\boxed{\text{away from you}}$; and the third image is facing $\boxed{\text{toward you}}$.

74. Find the angle of incidence for refraction from water into air.



$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \quad\rightarrow$$

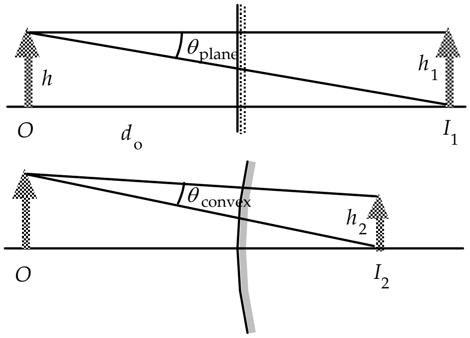
$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°) \quad\rightarrow\quad \theta_1 = 47.11°$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°),$$

We find the depth of the pool from $\tan\theta_1 = x/h$.

$$\tan 47.11^\circ = (5.50\,\mathrm{m})/h \;\rightarrow\; h = \boxed{5.11\,\mathrm{m}}$$

75. The apparent height of the image is related to the angle subtended by the image. For small angles, this angle is the height of the image divided by the distance between the image and viewer. Since both images are virtual, which gives a negative image distance, the image to viewer (object) distance will be the object distance minus the image distance. For the plane mirror the object and image heights are the same, and the image distance is the negative of the object distance.



$$h_\text{i} = h_\text{o} \;;\; d_\text{i} = -d_\text{o} \;;\; \theta_{\text{plane}} = \frac{h_\text{i}}{d_\text{o} - d_\text{i}} = \frac{h_\text{o}}{2d_\text{o}}$$

We use Eq. 32-2 and 32-3 to write the angle of the image in the convex mirror in terms of the object size and distance.

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \;\rightarrow\; d_\text{i} = \frac{d_\text{o} f}{d_\text{o} - f} \;\rightarrow\; d_\text{o} - d_\text{i} = \frac{d_\text{o}^2 - 2d_\text{o} f}{d_\text{o} - f}$$

$$\frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}} \;\rightarrow\; h_\text{i} = -\frac{h_\text{o} d_\text{i}}{d_\text{o}} = -\frac{h_\text{o} f}{d_\text{o} - f}$$

$$\theta_{\text{convex}} = \frac{h_\text{i}}{d_\text{o} - d_\text{i}} = -\left(\frac{h_\text{o} f}{d_\text{o} - f}\right)\left(\frac{d_\text{o} - f}{d_\text{o}^2 - 2d_\text{o} f}\right) = \frac{-h_\text{o} f}{d_\text{o}^2 - 2d_\text{o} f}$$

We now set the angle in the convex mirror equal to ½ of the angle in the plane mirror and solve for the focal length.

$$\theta_{\text{convex}} = \tfrac{1}{2}\theta_{\text{plane}} \;\rightarrow\; \frac{-h_\text{o} f}{d_\text{o}^2 - 2d_\text{o} f} = \frac{h_\text{o}}{4d_\text{o}} \;\rightarrow\; -4d_\text{o} f = d_\text{o}^2 - 2d_\text{o} f \;\rightarrow\; f = -\tfrac{1}{2} d_\text{o}$$

We use Eq. 32-1 to calculate the radius of the mirror.

$$r = 2f = 2\left(-\tfrac{1}{2} d_\text{o}\right) = -d_\text{o} = \boxed{-3.80\,\mathrm{m}}$$

76. For the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have the following.

$$n_{\text{plastic}} \sin\theta_{\text{plastic}} = n_{\text{air}} \sin\theta_{\text{air}} \;\rightarrow\; n_{\text{plastic}} \sin 39.3^\circ = (1.00)\sin 90^\circ \;\rightarrow\; n_{\text{plastic}} = 1.5788$$

For the refraction from plastic to water, we have the following.

$$n_{\text{plastic}} \sin\theta'_{\text{plastic}} = n_{\text{water}} \sin\theta_{\text{water}} \;\rightarrow\; (1.5788)\sin\theta'_{\text{plastic}} = (1.33)\sin 90^\circ \;\rightarrow\; \theta'_{\text{plastic}} = \boxed{57.4^\circ}$$

77. The two students $\boxed{\text{chose different signs for the magnification,}}$ i.e., one upright and one inverted. The focal length of the concave mirror is $f = \frac{1}{2}R = \frac{1}{2}(46\,\mathrm{cm}) = 23\,\mathrm{cm}$. We relate the object and image distances from the magnification.

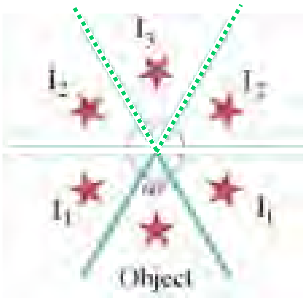
$$m = -\frac{d_\text{i}}{d_\text{o}} \;\rightarrow\; \pm 3 = -\frac{d_\text{i}}{d_\text{o}} \;\rightarrow\; d_\text{i} = \mp 3d_\text{o}$$

Use this result in the mirror equation.

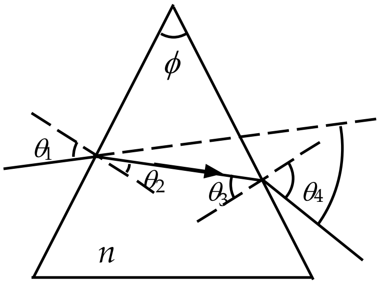
$$\left(\frac{1}{d_\text{o}}\right) + \left(\frac{1}{d_\text{i}}\right) = \frac{1}{f} \;\rightarrow\; \left(\frac{1}{d_\text{o}}\right) + \left[\frac{1}{(\mp 3d_\text{o})}\right] = \frac{1}{f} \;\rightarrow\; d_\text{o} = \frac{2f}{3}, \frac{4f}{3} = 15.3\,\mathrm{cm}, 30.7\,\mathrm{cm}$$

So the object distances are $\boxed{\text{15 cm (produces virtual image)}}$, and $\boxed{\text{+31 cm (produces real image)}}$.

78. The object "creates" the I_1 images as reflections from the actual mirrors. The I_2 images can be considered as images of the I_1 "objects," formed by the original mirrors. A specific I_2 image is the image of the I_1 "object" that is diametrically opposite it. Then the I_3 image can be considered as an image of the I_2 "objects." Each I_2 "object" would make the I_3 "image" at the same location. We can consider the extension of the actual mirrors, shown as dashed lines, to help understand the image formation.



79. The total deviation of the beam is the sum of the deviations at each surface. The deviation at the first surface is the refracted angle θ_2 subtracted from the incident angle θ_1. The deviation at the second surface is the incident angle θ_3 subtracted from the refracted angle θ_4. This gives the total deviation.



$$\delta = \delta_1 + \delta_2 = \theta_1 - \theta_2 + \theta_4 - \theta_3$$

We will express all of the angles in terms of θ_2. To minimize the deviation, we will take the derivative of the deviation with respect to θ_2, and then set that derivative equal to zero. Use Snell's law at the first surface to write the incident angle in terms of the refracted angle.

$$\sin\theta_1 = n\sin\theta_2 \quad\to\quad \theta_1 = \sin^{-1}\left(n\sin\theta_2\right)$$

The angle of incidence at the second surface is found using complementary angles, such that the sum of the refracted angle from the first surface and the incident angle at the second surface must equal the apex angle.

$$\phi = \theta_2 + \theta_3 \quad\to\quad \theta_3 = \phi - \theta_2$$

The refracted angle from the second surface is again found using Snell's law with the deviation in angle equal to the difference between the incident and refracted angles at the second surface.

$$n\sin\theta_3 = \sin\theta_4 \quad\to\quad \theta_4 = \sin^{-1}\left(n\sin\theta_3\right) = \sin^{-1}\left(n\sin\left(\phi-\theta_2\right)\right)$$

Inserting each of the angles into the deviation and setting the derivative equal to zero allows us to solve for the angle at which the deviation is a minimum.

$$\delta = \sin^{-1}\left(n\sin\theta_2\right) - \theta_2 + \sin^{-1}\left(n\sin\left(\phi-\theta_2\right)\right) - \left(\phi-\theta_2\right)$$

$$= \sin^{-1}\left(n\sin\theta_2\right) + \sin^{-1}\left(n\sin\left(\phi-\theta_2\right)\right) - \phi$$

$$\frac{d\delta}{d\theta_2} = \frac{n\cos\theta_2}{\sqrt{1-n^2\sin^2\theta_2}} - \frac{n\cos\left(\phi-\theta_2\right)}{\sqrt{1-n^2\sin^2\left(\phi-\theta_2\right)}} = 0 \quad\to\quad \theta_2 = \phi-\theta_2 \quad\to\quad \theta_2 = \theta_3 = \tfrac{1}{2}\phi$$

In order for $\theta_2 = \theta_3$, the ray must pass through the prism horizontally, which is perpendicular to the bisector of the apex angle ϕ. Set $\theta_2 = \tfrac{1}{2}\phi$ in the deviation equation (for the minimum deviation, δ_m) and solve for the index of refraction.

$$\delta_m = \sin^{-1}\left(n\sin\theta_2\right) + \sin^{-1}\left(n\sin\left(\phi-\theta_2\right)\right) - \phi$$

$$= \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) + \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi = 2\sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi$$

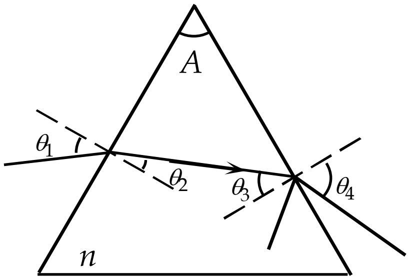
$$\to \boxed{n = \frac{\sin\left(\tfrac{1}{2}\left(\delta_m+\phi\right)\right)}{\sin\tfrac{1}{2}\phi}}$$

80. For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 \;\rightarrow\; (1.58)\sin\theta_3 = (1.00)\sin\theta_4$$

The maximum value of θ_4 before internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.58)\sin\theta_3 \le 1.00 \;\rightarrow\; \sin\theta_3 \le 0.6329 \;\rightarrow\; \theta_3 \le 39.27°$$



We find the refraction angle at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \;\rightarrow$$

$$\theta_2 = A - \theta_3 = 72° - \theta_3$$

Thus $\theta_2 \ge 72° - 39.27° = 32.73°$.

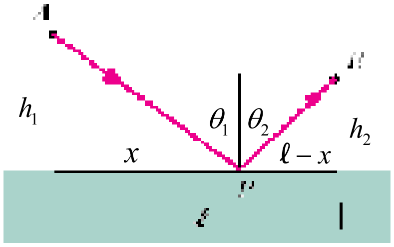
For the refraction at the first surface, we have the following.

$$n_{\text{air}}\sin\theta_1 = n\sin\theta_2 \;\rightarrow\; (1.00)\sin\theta_1 = (1.58)\sin\theta_2 \;\rightarrow\; \sin\theta_1 = (1.50)\sin\theta_2$$

Now apply the limiting condition.

$$\sin\theta_1 \ge (1.58)\sin 32.73° = 0.754 \;\rightarrow\; \boxed{\theta_1 \ge 58.69°}$$

81. (*a*) Consider the light ray shown in the figure. A ray of light starting at point A reflects off the surface at point P before arriving at point B, a horizontal distance ℓ from point A. We calculate the length of each path and divide the length by the speed of light to determine the time required for the light to travel between the two points.



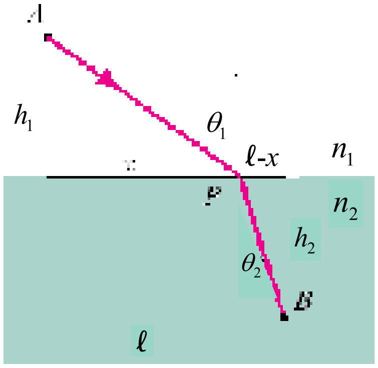
$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c}$$

To minimize the time we set the derivative of the time with respect to x equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(\ell - x)}{c\sqrt{(\ell - x)^2 + h_2^2}} \;\rightarrow$$

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^2}} \;\rightarrow\; \sin\theta_1 = \sin\theta_2 \;\rightarrow\; \boxed{\theta_1 = \theta_2}$$

(*b*) Now we consider a light ray traveling from point A to point B in media with different indices of refraction, as shown in the figure. The time to travel between the two points is the distance in each medium divided by the speed of light in that medium.



$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c/n_2}$$

To minimize the time we set the derivative of the time with respect to x equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

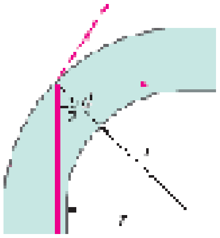
$$0=\frac{dt}{dx}=\frac{n_1x}{c\sqrt{x^2+h_1^2}}+\frac{-n_2\left(\ell-x\right)}{c\sqrt{\left(\ell-x\right)^2+h_2^2}}\rightarrow\frac{n_1x}{\sqrt{x^2+h_1^2}}=\frac{n_2\left(\ell-x\right)}{\sqrt{\left(\ell-x\right)^2+h_2^2}}\rightarrow\boxed{n_1\sin\theta_1=n_2\sin\theta_2}$$

82. We use Eq. 32-8 to calculate the location of the image and Eq. 32-3 to calculate the height of the image.

$$\frac{n_1}{d_o}+\frac{n_2}{d_i}=\frac{n_2-n_1}{R}\rightarrow d_i=n_2\left[\frac{n_2-n_1}{R}-\frac{n_1}{d_o}\right]^{-1}=1.53\left[\frac{1.53-1.33}{2.00\,\text{cm}}-\frac{1.33}{23\,\text{cm}}\right]^{-1}=\boxed{36.3\,\text{cm}}$$

$$\frac{h_i}{h_o}=-\frac{d_i}{d_o}\rightarrow h_i=-h_o\frac{d_i}{d_o}=-\left(2.0\,\text{mm}\right)\frac{36.3\,\text{cm}}{23\,\text{cm}}=\boxed{3.2\,\text{mm}}$$

83. A ray of light initially on the inside of the beam will strike the far surface at the smallest angle, as seen in the associated figure. The angle is found using the triangle shown in the figure, with side *r* and hypotenuse *r*+*d*. We set this angle equal to the critical angle, using Eq. 32-7, and solve for the minimum radius of curvature.



$$\sin\theta_C=\frac{r}{r+d}=\frac{n_2}{n_1}=\frac{1}{n}\rightarrow\boxed{r=\frac{d}{n-1}}$$

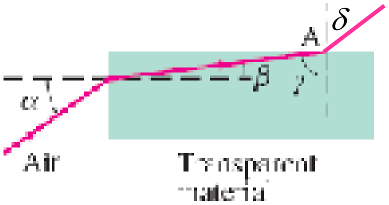
84. A relationship between the image and object distances can be obtained from the given information.

$$m=-\frac{1}{2}=-\frac{d_i}{d_o}\quad\rightarrow\quad d_i=\tfrac{1}{2}d_o=\boxed{7.5\,\text{cm}}$$

Now we find the focal length and the radius of curvature.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\quad\rightarrow\quad\frac{1}{15\,\text{cm}}+\frac{1}{7.5\,\text{cm}}=\frac{1}{f}\quad\rightarrow\quad f=5.0\,\text{cm}\quad\rightarrow\quad\boxed{r=10\,\text{cm}}$$

$\boxed{85.}$ If total internal reflection fails at all, it fails for $\alpha\approx90°$. Assume $\alpha=90°$ and use Snell's law to determine the maximum β.



$$n_2\sin\beta=n_1\sin\alpha=n_1\sin90°=n_1\quad\rightarrow\quad\sin\beta=\frac{n_1}{n_2}$$

Snell's law can again be used to determine the angle δ for which light (if not totally internally reflected) would exit the top surface, using the relationship $\beta+\gamma=90°$ since they form two angles of a right triangle.

$$n_1\sin\delta=n_2\sin\gamma=n_2\sin\left(90°-\beta\right)=n_2\cos\beta\quad\rightarrow\quad\sin\delta=\frac{n_2}{n_1}\cos\beta$$

Using the trigonometric relationship $\cos\beta=\sqrt{1-\sin^2\beta}$ we can solve for the exiting angle in terms of the indices of refraction.

$$\sin\delta=\frac{n_2}{n_1}\sqrt{1-\sin^2\beta}=\frac{n_2}{n_1}\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}$$

Insert the values for the indices ($n_1=1.00$ and $n_2=1.51$) to determine the sine of the exit angle.

$$\sin\delta=\frac{1.51}{1.00}\sqrt{1-\left(\frac{1.00}{1.51}\right)^2}=1.13$$

Since the sine function has a maximum value of 1, the light totally internally reflects at the glass–air interface for any incident angle of light.

If the glass is immersed in water, then $n_1 = 1.33$ and $n_2 = 1.51$.

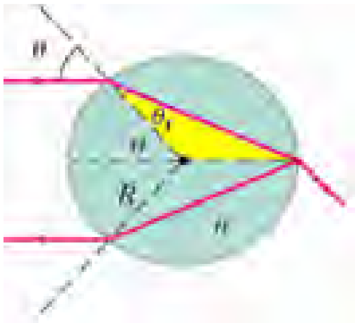
$$\sin\delta = \frac{1.51}{1.33}\sqrt{1-\left(\frac{1.33}{1.51}\right)^2} = 0.538 \quad\rightarrow\quad \delta = \sin^{-1}0.538 = 32.5^\circ$$

Light entering the glass from water at 90° can escape out the top at 32.5°, therefore total internal reflection only occurs for incident angles $\boxed{\le 32.5^\circ}$.

86. The path of the ray in the sphere forms an isosceles triangle with two radii. The two identical angles of the triangle are equal to the refracted angle. Since the incoming ray is horizontal, the third angle is the supplementary angle of the incident angle. We set the sum of these angles equal to 180° and solve for the ratio of the incident and refracted angles. Finally we use Snell's law in the small angle approximation to calculate the index of refraction.

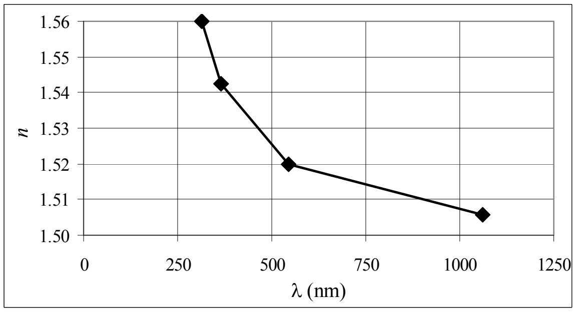
$$2\theta_r + (180^\circ - \theta) = 180^\circ \quad\rightarrow\quad \theta = 2\theta_r$$

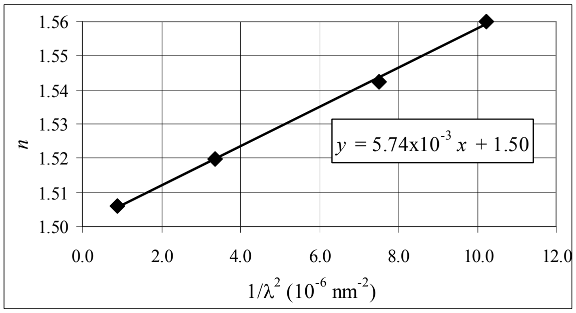
$$n_1\sin\theta = n_2\sin\theta_r \quad\rightarrow\quad \theta = n\theta_r = 2\theta_r \quad\rightarrow\quad \boxed{n = 2}$$



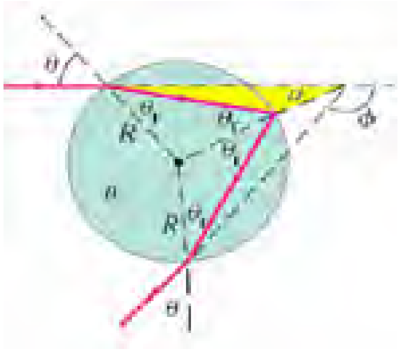
87. The first graph is a graph of *n* vs. λ. The second graph is a graph *n* vs. of $1/\lambda^2$. By fitting a line of the form $n = A + B/\lambda^2$, we have $A = 1.50$ and $B = \left(5.74\times10^{-3}\right)/10^{-6}\,\text{nm}^{-2} = \boxed{5740\,\text{nm}^2}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.87."





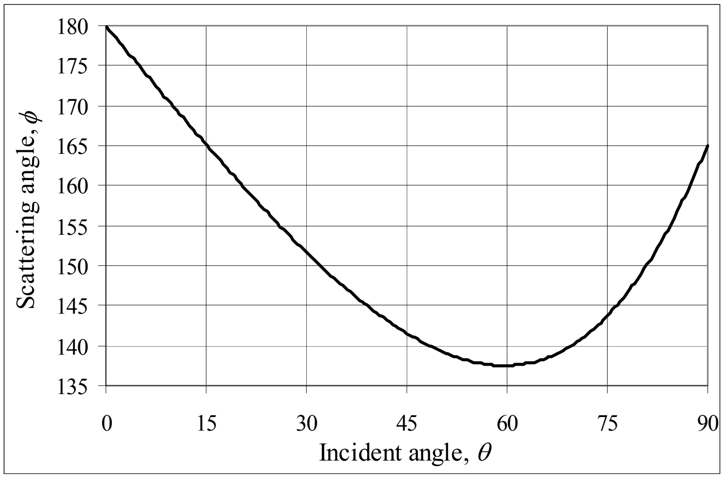
88. (*a*) As the light ray enters the water drop, its path changes by the difference between the incident and refracted angles. We use Snell's law to calculate the refracted angle. The light ray then reflects off the back surface of the droplet. At this surface its path changes by $180^\circ - 2\theta_r$, as seen in the diagram. As the light exits the droplet it refracts again, changing its path by the difference between the incident and refracted angles. Summing these three angles gives the total path change.



$$\sin\theta = n\sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin\theta}{n}\right)$$

$$\phi = (\theta - \theta_r) + (180^\circ - 2\theta_r) + (\theta - \theta_r) = 180^\circ + 2\theta - 4\theta_r = \boxed{180^\circ + 2\theta - 4\sin^{-1}\left(\frac{\sin\theta}{n}\right)}$$

(*b*) Here is the graph of ϕ vs θ. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.88."



(*c*) On the spreadsheet, the incident angles that give scattering angles from 138° to 140° are approximately $48.5° \le \theta \le 54.5°$ and $64.5° \le \theta \le 69.5°$. This is 11/90 of the possible incident angles, or about $\boxed{12\%}$.

CHAPTER 33: Lenses and Optical Instruments

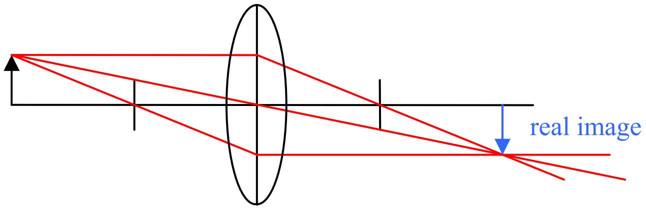
Responses to Questions

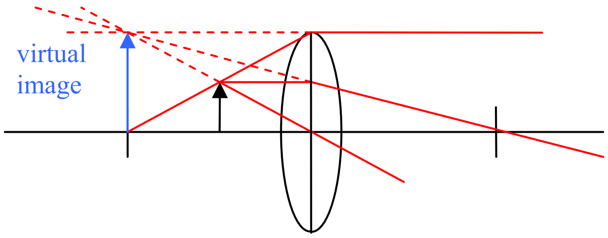
1. The film must be placed behind the lens at the focal length of the lens.

2. The lens moves farther away from the film. When the photographer moves closer to his subject, the object distance decreases. The focal length of the lens does not change, so the image distance must increase, by Eq. 33-2, $\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}$.

3. Yes. Diverging lenses, by definition, cause light rays to diverge and will not bring rays from a real object to a focal point as required to form a real image. However, if another optical element (for example, a converging lens) forms a virtual object for the diverging lens, it is possible for the diverging lens to form a real image.

4. A real image formed by a thin lens is on the opposite side of the lens as the object, and will always be inverted as shown in the top diagram. A virtual image is formed on the same side of the lens as the real object, and will be upright, as shown in the bottom diagram.





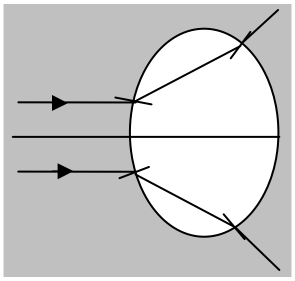
5. Yes. In the thin-lens equation, the variables for object distance and image distance can be interchanged and the formula remains the same.

6. Yes, real images can be projected on a screen. No, virtual images cannot, because they are formed by diverging rays, which do not come to a focus on the screen. Both kinds of images can be photographed. The lenses in a camera are designed to focus either converging or diverging light rays down onto the film.

7. (*a*) Yes. The image moves farther from the lens.
   (*b*) Yes. The image also gets larger.

8. The mirror equation and the lens equation are identical. According to the sign conventions, $d > 0$ indicates a real object or image and $d < 0$ indicates a virtual object or image, for both mirrors and lenses. But the positions of the objects and images are different for a mirror and a lens. For a mirror, a real object or image will be in front of the mirror and a virtual object or image will be behind the mirror. For a lens, a real image will be on the opposite side of the lens from a real object, and a virtual image will be on the same side of the lens as the real object.

9. No. The lens will be a diverging lens when placed in water because the index of refraction of the lens is less than the index of refraction of the medium surrounding it. Rays going from water to lens material will bend away from the normal instead of toward the normal, and rays going from the lens back to the water will bend towards the normal.



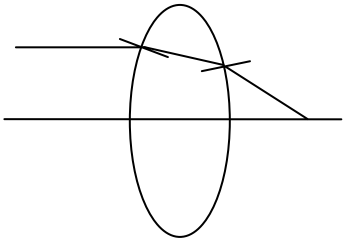
10. A virtual image created by a previous lens can serve as a virtual object for a second lens. If the previous lens creates an image behind the position of the second lens, that image will also serve as a virtual object for the second lens.

11. Assuming that the lens remains fixed and the screen is moved, the dog's head will have the greater magnification. The object distance for the head is less than the object distance for the tail, because the dog is facing the mirror. The image distance for the head will therefore be greater than the image distance for the tail. Magnification is the ratio of the image distance to the object distance, so will be greater for the head.

12. If the cat's nose is closer to the lens than the focal point and the tail is farther from the lens than the focal point, the image of the nose will be virtual and the image of the tail will be real. The virtual image of the front part of the cat will be spread out from the image of the nose to infinity on the same side of the lens as the cat. The real image of the back part of the cat will be spread out from the image of the tail to infinity on the opposite side of the lens.

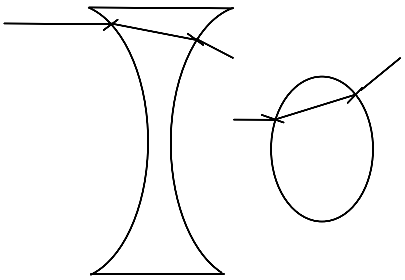
$\boxed{13.}$ The technique for determining the focal length of the diverging lens in Example 33-6 requires the combination of the two lenses together to project a real image of the sun onto a screen. The focal length of the lens combination can be measured. If the focal length of the converging lens is longer than the focal length of the diverging lens (the converging lens is weaker than the diverging lens), then the lens combination will be diverging, and will not form a real image of the sun. In this case the focal length of the combination of lenses cannot be measured, and the focal length of the diverging lens alone cannot be determined.

14. A double convex lens causes light rays to converge because the light bends towards the normal as it enters the lens and away from the normal as it exits the lens. The result, due to the curvature of the sides of the lens, is that the light bends towards the principal axis at both surfaces. The more strongly the sides of the lens are curved, the greater the bending, and the shorter the focal length.



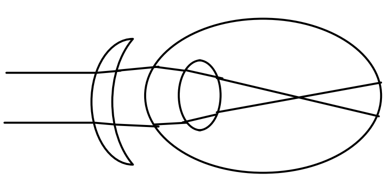
15. Yes. The relative values of the index of refraction of the fluid and the index of refraction of the lens will determine the refraction of light as it passes from the fluid through the lens and back into the fluid. The amount of refraction of light determines the focal length of the lens, so the focal length will change if the lens is immersed in a fluid. No, the image formation of the spherical mirror is determined by reflection, not refraction, and is independent of the medium in which the mirror is immersed.

16. The lens material is air and the medium in which the lens is placed is water. Air has a lower index of refraction than water, so the light rays will bend away from the normal when entering the lens and towards the normal when leaving the lens.
    (*a*) A converging lens can be made by a shape that is thinner in the middle than it is at the edges.
    (*b*) A diverging lens will be thicker in the middle than it is at the edges.



17. If the object of the second lens (the image from the first lens) is exactly at the focal point, then a virtual image will be formed at infinity and can be viewed with a relaxed eye.

18. The corrective lenses will not work the same underwater as in air, and so the nearsighted person will probably not be able to see clearly underwater. The difference in the index of refraction of water and glass is much smaller than the difference in the indices for air and glass, so the lenses will not cause the incoming rays to diverge sufficiently.



19. Nearsighted. Diverging lenses are used to correct nearsightedness and converging lenses are used to correct farsightedness. If the person's face appears narrower through the glasses, then the image of the face produced by the lenses is smaller than the face, virtual, and upright. Thus, the lenses must be diverging, and therefore the person is nearsighted.

20. All light entering the camera lens while the shutter is open contributes to a single picture. If the camera is moved while the shutter is open, the position of the image on the film moves. The new image position overlaps the previous image position, causing a blurry final image. With the eye, new images are continuously being formed by the nervous system, so images do not "build up" on the retina and overlap with each other.

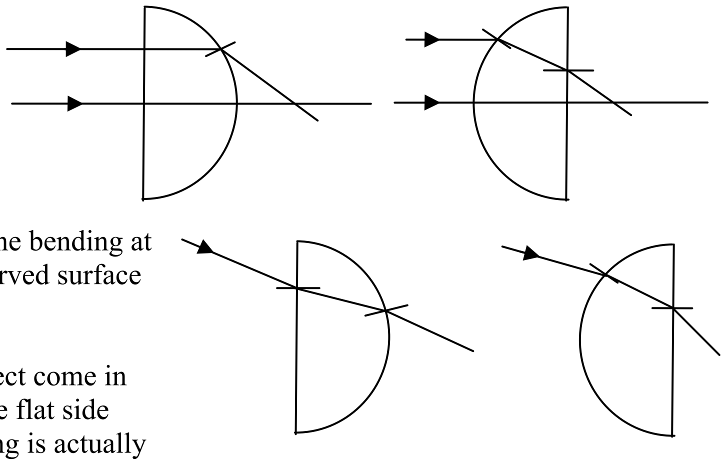
21. Squinting limits the off-axis rays that enter the eye and results in an image that is formed primarily by the center part of the lens, reducing spherical aberration and spreading of the image.

22. The image formed on the retina is inverted. The human brain then processes the image so that we interpret the world we see correctly.

23. Both reading glasses and magnifiers are converging lenses used to produce magnified images. A magnifier, generally a short focal length lens, is typically used by adjusting the distance between the lens and the object so that the object is exactly at or just inside the focal point. An object exactly at the focal point results in an image that is at infinity and can be viewed with a relaxed eye. If the lens is adjusted so that it focuses the image at the eye's near point, the magnification is slightly greater. The lenses in reading glasses typically are a fixed distance from the eye. These lenses cause the rays from a nearby object to converge somewhat before they reach the eye, allowing the eye to focus on an object that is inside the near point. The focal length of the lens needed for reading glasses will depend on the individual eye. The object does not have to be inside the focal point of the lens. For both reading glasses and magnifiers, the lenses allow the eye to focus on an object closer than the near point.

24. The relationship between d_{i} and d_{o} for a given lens of focal length f is given by Eq. 33-2, $\frac{1}{d_{\text{o}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}$. The focal length is fixed for a camera lens, so if the lens focuses on a closer object, d_{o} decreases and therefore d_{i} must increase. An increase in d_{i} means that the lens must be farther from the film.

$\boxed{25.}$ The curved surface should face the object. If the flat surface faces the object and the rays come in parallel to the optical axis, then no bending will occur at the first surface and all the bending will occur at the second surface. Bending at the two surfaces will clearly not be equal in this case. The bending at the two surfaces may be equal if the curved surface faces the object.



If the parallel rays from the distant object come in above or below the optical axis with the flat side towards the object, then the first bending is actually away from the axis. In this case also, bending at both surfaces can be equal if the curved side of the lens faces the object.

26. For both converging and diverging lenses, the focal point for violet light is closer to the lens than the focal point for red light. The index of refraction for violet light is slightly greater than for red light for glass, so the violet light bends more, resulting in a smaller magnitude focal length.

Solutions to Problems

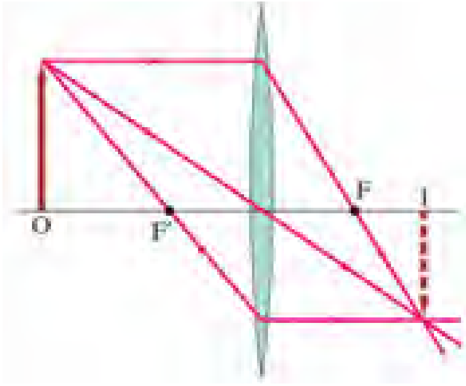
$\boxed{1.}$ (*a*) From the ray diagram, the object distance is about $\boxed{480\text{ cm}}$.

(*b*) We find the object distance from Eq. 33-2.

$$\frac{1}{d_{\text{o}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f} \quad\rightarrow$$

$$d_{\text{o}}=\frac{fd_{\text{i}}}{d_{\text{i}}-f}=\frac{(215\,\text{mm})(373\,\text{mm})}{373\,\text{mm}-215\,\text{mm}}=\boxed{508\,\text{mm}}$$

NOTE: In the first printing of the textbook, a different set of values was given: $f=75.0\,\text{mm}$ and $d_{\text{i}}=88.0\,\text{mm}$. Using that set of values gives the same object distance as above. But the ray diagram would be much more elongated, with the object distance almost 7 times the focal length.



2. (*a*) To form a real image from parallel rays requires a $\boxed{\text{converging lens.}}$

(*b*) We find the power of the lens from Eqs. 33-1 and 33-2.

$$\frac{1}{d_{\text{o}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}=P=\frac{1}{\infty}+\frac{1}{0.185\,\text{m}}=\boxed{5.41\,\text{D}}$$

3. (*a*) The power of the lens is given by Eq. 33.1

$$P = \frac{1}{f} = \frac{1}{0.235\,\text{m}} = \boxed{4.26\,\text{D}}$$

This lens is $\boxed{\text{converging}}$.

(*b*) We find the focal length of the lens from Eq. 33.1

$$P = \frac{1}{f} \rightarrow f = \frac{1}{D} = -\frac{1}{6.75\,\text{D}} = \boxed{-0.148\,\text{m}}$$

This lens is $\boxed{\text{diverging}}$.

4. To form a real image from a real object requires a $\boxed{\text{converging lens}}$. We find the focal length of the lens from Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{(1.85\,\text{m})(0.483\,\text{m})}{1.85\,\text{m} + 0.483\,\text{m}} = \boxed{0.383\,\text{m}}$$

Because $d_i > 0$, the image is $\boxed{\text{real.}}$

5. (*a*) We find the image distance from Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(10.0\,\text{m})(0.105\,\text{m})}{10.0\,\text{m} - 0.105\,\text{m}} = 0.106\,\text{m} = \boxed{106\,\text{mm}}$$

(*b*) Use the same general calculation.

$$d_i = \frac{d_o f}{d_o - f} = \frac{(3.0\,\text{m})(0.105\,\text{m})}{3.0\,\text{m} - 0.105\,\text{m}} = 0.109\,\text{m} = \boxed{109\,\text{mm}}$$

(*c*) Use the same general calculation.

$$d_i = \frac{d_o f}{d_o - f} = \frac{(1.0\,\text{m})(0.105\,\text{m})}{1.0\,\text{m} - 0.105\,\text{m}} = 0.117\,\text{m} = \boxed{117\,\text{mm}}$$

(*d*) We find the smallest object distance from the maximum image distance.

$$\frac{1}{d_{o_{\text{min}}}} + \frac{1}{d_{i_{\text{max}}}} = \frac{1}{f} \rightarrow d_{o_{\text{min}}} = \frac{d_{i_{\text{max}}} - f}{d_{i_{\text{max}}} \; f} = \frac{(132\,\text{mm})(105\,\text{mm})}{132\,\text{mm} - 105\,\text{mm}} = 513\,\text{mm} = \boxed{0.513\,\text{m}}$$

6. (*a*) We locate the image using Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(18\,\text{cm})(28\,\text{cm})}{18\,\text{cm} - 28\,\text{cm}} = -50.4\,\text{cm} \approx -50\,\text{cm}$$

The negative sign means the image is $\boxed{\text{50 cm behind the lens (virtual).}}$

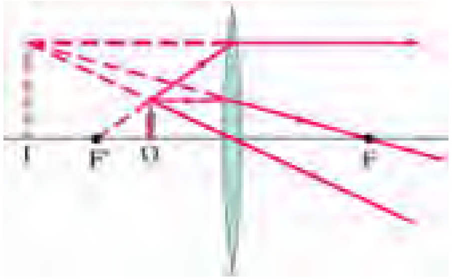
(*b*) We find the magnification from Eq. 33-3.

$$m = -\frac{d_i}{d_o} = -\frac{(-50.4\,\text{cm})}{(18\,\text{cm})} = \boxed{+2.8}$$

$\boxed{7.}$ (*a*) The image should be upright for reading. The image will be $\boxed{\text{virtual, upright, and magnified}}$.

(*b*) To form a virtual, upright magnified image requires a $\boxed{\text{converging lens}}$.

(*c*) We find the image distance, then the focal length, and then the power of the lens. The object distance is given.



$$m=-\frac{d_\text{i}}{d_\text{o}} \;\rightarrow\; d_\text{i}=-md_\text{o}$$

$$P=\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{d_\text{i}+d_\text{o}}{d_\text{o}d_\text{i}}=\frac{-md_\text{o}+d_\text{o}}{d_\text{o}\left(-md_\text{o}\right)}=\frac{m-1}{md_\text{o}}=\frac{2.5-1}{(2.5)(0.090\,\text{m})}=\boxed{6.7\,\text{D}}$$

8. Use Eqs. 33-1 and 33-2 to find the image distance, and Eq. 33-3 to find the image height.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}=P \;\rightarrow\; d_\text{i}=\frac{d_\text{o}}{Pd_\text{o}-1}=\frac{(0.125\,\text{m})}{(-8.00\,\text{D})(0.125\,\text{m})-1}=-0.0625\,\text{m}=\boxed{-6.25\,\text{cm}}$$

Since the image distance is negative, the image is $\boxed{\text{virtual and behind the lens}}$.

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}} \;\rightarrow\; h_\text{i}=-\frac{d_\text{i}}{d_\text{o}}h_\text{o}=-\frac{(-6.25\,\text{cm})}{12.5\,\text{cm}}(1.00\,\text{mm})=\boxed{0.500\,\text{mm}\,(\text{upright})}$$

9. First, find the original image distance from Eqs. 33-1 and 33-2.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}=P \;\rightarrow\; d_\text{i}=\frac{d_\text{o}}{Pd_\text{o}-1}=\frac{(1.50\,\text{m})}{(8.00\,\text{D})(1.50\,\text{m})-1}=0.1364\,\text{m}$$

(*a*) With $d_\text{o}=0.60\,\text{m}$, find the new image distance.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}=P \;\rightarrow\; d_\text{i}=\frac{d_\text{o}}{Pd_\text{o}-1}=\frac{(0.60\,\text{m})}{(8.00\,\text{D})(0.60\,\text{m})-1}=0.1579\,\text{m}$$

Thus the image has moved $0.1579\,\text{m}-0.1364\,\text{m}=0.0215\,\text{m}\approx\boxed{0.02\,\text{m}}$ away from the lens.

(*b*) With $d_\text{o}=2.40\,\text{m}$, find the new image distance.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}=P \;\rightarrow\; d_\text{i}=\frac{d_\text{o}}{Pd_\text{o}-1}=\frac{(2.40\,\text{m})}{(8.00\,\text{D})(2.40\,\text{m})-1}=0.1319\,\text{m}$$

The image has moved $0.1319\,\text{m}-0.1364\,\text{m}=-0.0045\,\text{m}\approx\boxed{0.004\,\text{m}}$ toward the lens.

10. (*a*) If the image is real, the focal length must be positive, the image distance must be positive, and the magnification is negative. Thus $d_\text{i}=2.50d_\text{o}$. Use Eq. 33-2.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}+\frac{1}{2.50d_\text{o}}=\frac{1}{f} \;\rightarrow\; d_\text{o}=\left(\frac{3.50}{2.50}\right)f=\left(\frac{3.50}{2.50}\right)(50.0\,\text{mm})=\boxed{70.0\,\text{mm}}$$

(*b*) If the image is magnified, the lens must have a positive focal length, because negative lenses always form reduced images. Since the image is virtual the magnification is positive. Thus $d_\text{i}=-2.50d_\text{o}$. Again use Eq. 33-2.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}-\frac{1}{2.5d_\text{o}}=\frac{1}{f} \;\rightarrow\; d_\text{o}=\left(\frac{1.50}{2.50}\right)f=\left(\frac{1.50}{2.50}\right)(50.0\,\text{mm})=\boxed{30.0\,\text{mm}}$$

11. From Eq. 33-3, $|h_\text{i}|=|h_\text{o}|$ when $d_\text{i}=d_\text{o}$. So find d_o from Eq. 33-2.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{o}}=\frac{1}{f} \;\rightarrow\; d_\text{o}=2f=\boxed{50\,\text{cm}}$$

12. (*a*) Use Eqs. 33-2 and 33-3.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f} \;\rightarrow\; d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}=\frac{(1.30\,\text{m})(0.135\,\text{m})}{1.30\,\text{m}-0.135\,\text{m}}=0.1506\,\text{m}\approx\boxed{151\,\text{mm}}$$

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}}\rightarrow h_\text{i}=-\frac{d_\text{i}}{d_\text{o}}h_\text{o}=-\frac{0.1506\,\text{m}}{1.30\,\text{m}}(2.80\,\text{cm})=\boxed{-0.324\,\text{m}}$$

The image is behind the lens a distance of 151 mm, is real, and is inverted.

(*b*) Again use Eqs. 33-2 and 33-3.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}\rightarrow d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}=\frac{(1.30\,\text{m})(-0.135\,\text{m})}{1.30\,\text{m}-(-0.135\,\text{m})}=-0.1223\,\text{m}\approx\boxed{-122\,\text{mm}}$$

$$m=\frac{h_\text{i}}{h_\text{o}}=-\frac{d_\text{i}}{d_\text{o}}\rightarrow h_\text{i}=-\frac{d_\text{i}}{d_\text{o}}h_\text{o}=-\frac{(-0.1223\,\text{m})}{1.30\,\text{m}}(2.80\,\text{cm})=\boxed{0.263\,\text{m}}$$

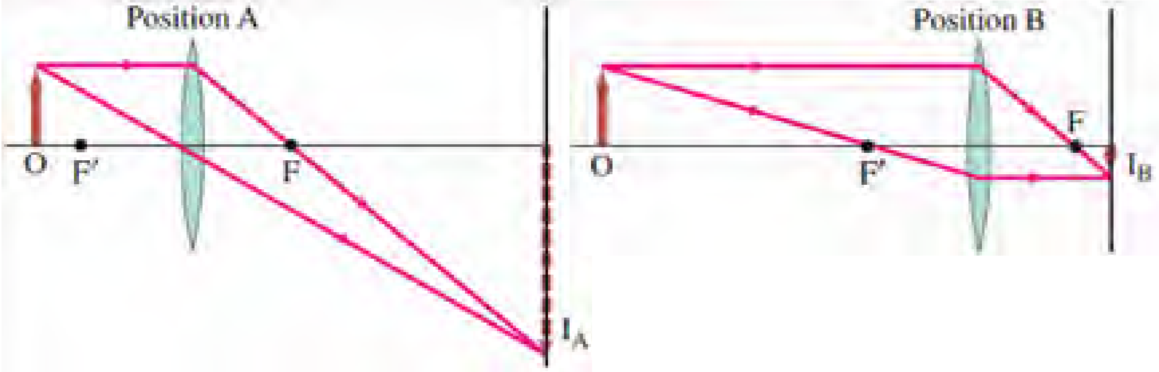
The image is in front of the lens a distance of 122 mm, is virtual, and is upright.

$\boxed{13.}$ The sum of the object and image distances must be the distance between object and screen, which we label as d_T. We solve this relationship for the image distance, and use that expression in Eq. 33-2 in order to find the object distance.

$$d_\text{o}+d_\text{i}=d_\text{T}\rightarrow d_\text{i}=d_\text{T}-d_\text{o}\ ;\ \frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}+\frac{1}{(d_\text{T}-d_\text{o})}=\frac{1}{f}\rightarrow d_\text{o}^2-d_\text{T}d_\text{o}+fd_\text{T}=0\rightarrow$$

$$d_\text{o}=\frac{d_\text{T}\pm\sqrt{d_\text{T}^2-4fd_\text{T}}}{2}=\frac{(86.0\,\text{cm})\pm\sqrt{(86.0\,\text{cm})^2-4(16.0\,\text{cm})(86.0\,\text{cm})}}{2}=\boxed{21.3\,\text{cm},\ 64.7\,\text{cm}}$$

Note that to have real values for d_o, we must in general have $d_\text{T}^2-4fd_\text{T}>0\rightarrow d_\text{T}>4f$.



14. For a real image both the object distance and image distances are positive, and so the magnification is negative. Use Eqs. 33-2 and 33-3 to find the object and image distances. Since they are on opposite sides of the lens, the distance between them is their sum.

$$m=\frac{-d_\text{i}}{d_\text{o}}\rightarrow d_\text{i}=-md_\text{o}=2.95d_\text{o}$$

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}+\frac{1}{2.95d_\text{o}}=\frac{1}{f}\rightarrow d_\text{o}=\left(\frac{3.95}{2.95}\right)f=\left(\frac{3.95}{2.95}\right)(85\,\text{cm})=113.8\,\text{cm}$$

$$d_\text{i}=2.95d_\text{o}=2.95(113.8\,\text{cm})=335.7\,\text{cm}$$

$$d_\text{o}+d_\text{i}=113.8\,\text{cm}+335.7\,\text{cm}=449.5\,\text{cm}\approx\boxed{450\,\text{cm}}$$

15. (*a*) Use Eq. 33-2 to write an expression for the image distance in terms of the object distance and focal length. We then use Eq. 33-3 to write an expression for the magnification.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}\rightarrow d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}\ ;\ m=-\frac{d_\text{i}}{d_\text{o}}=-\frac{f}{d_\text{o}-f}$$

These expressions show that when $d_o > f$, the image distance is positive, producing a real image, and the magnification is negative, which gives an inverted image.

(*b*) From the above equations, when $d_o < f$, the image distance is negative, producing a virtual image, and the magnification is positive, which gives an upright image.

(*c*) We set $-d_o = f$ and calculate the limiting image distance and magnification.

$$d_i = \frac{(-f)f}{-f-f} = \frac{f}{2} \qquad m = -\frac{d_i}{d_o} = -\frac{f}{-f-f} = \frac{1}{2}$$

We also take the limit of large negative object distance.

$$d_i = \frac{(-\infty)f}{-\infty - f} = f \qquad m = -\frac{d_i}{d_o} = -\frac{f}{-\infty - f} = 0$$

From these limiting cases, we see that when $-d_o > f$, the image is $\boxed{\text{real and upright}}$ with $\frac{1}{2}f < d_i < f$ and $0 < m < \frac{1}{2}$.

(*d*) We take the limiting condition $d_o \to 0$, and determine the resulting image distance and magnification.

$$d_i = \frac{(0)f}{0-f} = 0 \qquad m = -\frac{d_i}{d_o} = -\frac{f}{0-f} = 1$$

From this limit and that found in part (*c*), we see that when $0 < -d_o < f$, the image is $\boxed{\text{real and upright}}$, with $0 < d_i < \frac{1}{2}f$ and $\frac{1}{2} < m < 1$.

16. (*a*) We use the magnification equation, Eq. 33-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length.

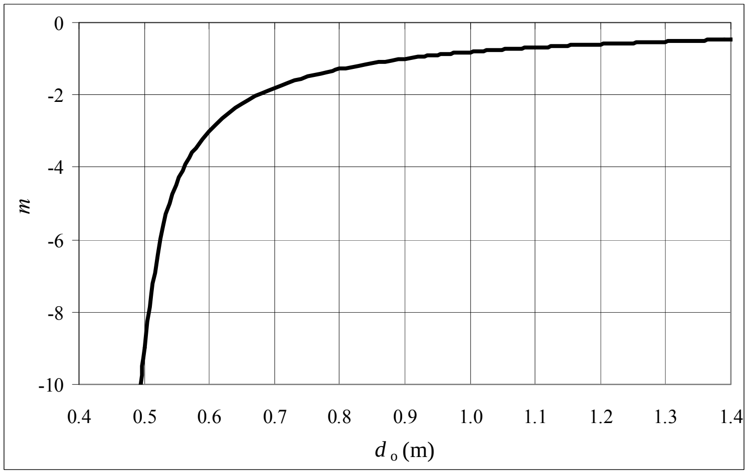
$$m = -d_i/d_o \quad \to \quad d_i = -md_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \to$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{-md_o} \quad \to$$

$$\boxed{m = \frac{f}{f - d_o}}$$

(*b*) We set $f = 0.45$ m and draw a graph of the magnification as a function of the object distance. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH33.XLS," on tab "Problem 33.16b."



(*c*) The image and object will have the same lateral size when the magnification is equal to negative one. Setting the magnification equal to negative one, we solve the equation found in part (a) for the object distance.

$$m = \frac{f}{f - d_o} = -1 \quad \to \quad d_o = 2f = \boxed{0.90\,\text{m}}$$

(*d*) From the graph we see that for the image to be much larger than the object, the object should be placed at a point $\boxed{\text{just beyond the focal point.}}$

17. Find the object distance from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{fd_i}{d_i-f}=\frac{(0.105\,\text{m})(6.50\,\text{m})}{6.50\,\text{m}-0.105\,\text{m}}=\boxed{0.107\,\text{m}}$$

Find the size of the image from Eq. 33-3.

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow |h_i|=\frac{d_i}{d_o}h_o=\frac{6.50\,\text{m}}{0.107\,\text{m}}(36\,\text{mm})=2187\,\text{mm}\approx\boxed{2.2\,\text{m}}$$

18. (*a*) Use Eq. 33-2 with $d_o+d_i=d_T \rightarrow d_i=d_T-d_o$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(d_T-d_o)}=\frac{1}{f} \rightarrow d_o^2-d_Td_o+fd_T=0 \rightarrow d_o=\frac{d_T\pm\sqrt{d_T^2-4fd_T}}{2}$$

There are only real solutions for d_o if $d_T^2-4fd_T>0 \rightarrow d_T>4f$. If that condition is met, then there will be two locations for the lens, at distances $d_o=\frac{1}{2}\left(d_T\pm\sqrt{d_T^2-4fd_T}\right)$ from the object, that will form sharp images on the screen.

(*b*) If $d_T<4f$, then Eq. 33-2 cannot be solved for real values of d_o or d_i.

(*c*) If $d_T>4f$, the lens locations relative to the object are given by $d_{o1}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)$ and $d_{o2}=\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)$.

$$\Delta d=d_{o1}-d_{o2}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)=\boxed{\sqrt{d_T^2-4fd_T}}$$

Find the ratio of image sizes using Eq. 33-3.

$$\frac{h_{i2}}{h_{i1}}=\frac{-h_o\frac{d_{i2}}{d_{o2}}}{-h_o\frac{d_{i1}}{d_{o1}}}=\frac{d_{i2}}{d_{o2}}\frac{d_{o1}}{d_{i1}}=\frac{d_T-d_{o2}}{d_{o2}}\frac{d_{o1}}{d_T-d_{o1}}$$

$$=\left[\frac{d_T-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}{\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}\right]\left[\frac{\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}{d_T-\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}\right]=\boxed{\left(\frac{d_T+\sqrt{d_T^2-4fd_T}}{d_T-\sqrt{d_T^2-4fd_T}}\right)^2}$$

$\boxed{19.}$ (*a*) With the definitions as given in the problem, $x=d_o-f \rightarrow d_o=x+f$ and $x'=d_i-f \rightarrow d_i=x'+f$. Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{x+f}+\frac{1}{x'+f}=\frac{1}{f} \rightarrow \frac{(x'+f)+(x+f)}{(x+f)(x'+f)}=\frac{1}{f} \rightarrow$$

$$(2f+x+x')f=(x+f)(x'+f) \rightarrow 2f^2+xf+x'f=x'x+xf+fx'+f^2 \rightarrow \boxed{f^2=x'x}$$

(*b*) Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_of}{d_o-f}=\frac{(48.0\,\text{cm})(38.0\,\text{cm})}{48.0\,\text{cm}-38.0\,\text{cm}}=\boxed{182\,\text{cm}}$$

(*c*) Use the Newtonian form.

$$xx'=f^2 \rightarrow x'=\frac{f^2}{x}=\frac{(38.0\,\text{cm})^2}{(48.0\,\text{cm}-38.0\,\text{cm})}=144.2\,\text{cm}$$

$$d_i=x'+f=144.2\,\text{cm}+38.0\,\text{cm}=\boxed{182\,\text{cm}}$$

20. The first lens is the converging lens. An object at infinity will form an image at the focal point of the converging lens, by Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1}=\frac{1}{\infty}+\frac{1}{d_{i1}} \quad\rightarrow\quad d_{i1}=f_1=20.0\,\text{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2}=-6.0\,\text{cm}$. Again use Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \quad\rightarrow\quad d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-6.0\,\text{cm})(-33.5\,\text{cm})}{(-6.0\,\text{cm})-(-33.5\,\text{cm})}=7.3\,\text{cm}$$

Thus the final image is real, $\boxed{\text{7.3 cm beyond the second lens.}}$

21. Find the image formed by the first lens, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \quad\rightarrow\quad d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(35.0\,\text{cm})(25.0\,\text{cm})}{(35.0\,\text{cm})-(25.0\,\text{cm})}=87.5\,\text{cm}$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance.

$$d_{o2}=16.5\,\text{cm}-87.5\,\text{cm}=-71.0\,\text{cm}$$

Find the image formed by the second lens, again using Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \quad\rightarrow\quad d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-71.0\,\text{cm})(25.0\,\text{cm})}{(-71.0\,\text{cm})-(25.0\,\text{cm})}=18.5\,\text{cm}$$

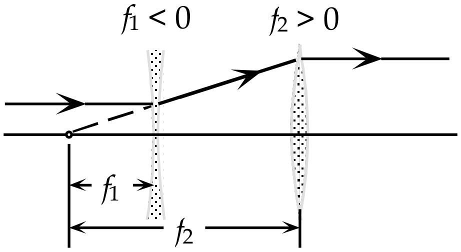
Thus the final image is real, $\boxed{\text{18.5 cm beyond second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(+87.5\,\text{cm})(+18.5\,\text{cm})}{(+35.0\,\text{cm})(-71.0\,\text{cm})}=\boxed{-0.651\times(\text{inverted})}$$

22. From the ray diagram, the image from the first lens is a virtual image at the focal point of the first lens. This is a real object for the second lens. Since the light is parallel after leaving the second lens, the object for the second lens must be at its focal point. Let the separation of the lenses be ℓ. Note that the focal length of the diverging lens is negative.



$$|f_1|+\ell=f_2 \quad\rightarrow$$

$$|f_1|=f_2-\ell=34.0\,\text{cm}-24.0\,\text{cm}=10.0\,\text{cm} \quad\rightarrow$$

$$f_1=\boxed{-10.0\,\text{cm}}$$

23. (*a*) The first image is formed as in Example 33-5, and so $d_{iA}=30.0\,\text{cm}$. This image becomes the object for the lens B, at a distance $d_{oB}=20.0\,\text{cm}-30.0\,\text{cm}=-10.0\,\text{cm}$. This is a virtual object since it is behind lens N. Use Eq. 33-2 to find the image formed by lens B, which is the final image.

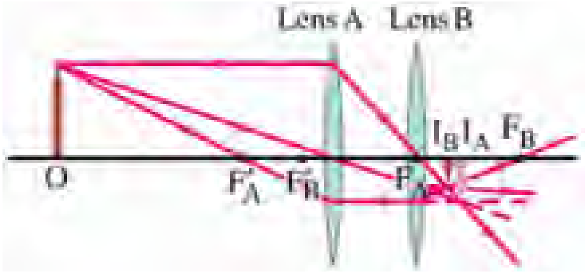
$$\frac{1}{d_{oB}}+\frac{1}{d_{iB}}=\frac{1}{f_B} \quad\rightarrow\quad d_{iB}=\frac{d_{oB}f_B}{d_{oB}-f_B}=\frac{(-10.0\,\text{cm})(25.0\,\text{cm})}{-10.0\,\text{cm}-25.0\,\text{cm}}=7.14\,\text{cm}$$

So the final image is $\boxed{\text{7.14 cm beyond lens B}}$.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{\mathrm{iA}}}{d_{\mathrm{oA}}}\right)\left(-\frac{d_{\mathrm{iB}}}{d_{\mathrm{oB}}}\right) = \frac{d_{\mathrm{iA}} d_{\mathrm{iB}}}{d_{\mathrm{oA}} d_{\mathrm{oB}}} = \frac{(30.0\,\mathrm{cm})(7.14\,\mathrm{cm})}{(60.0\,\mathrm{cm})(-10.0\,\mathrm{cm})} = \boxed{-0.357}$$

(*c*) See the ray diagram here.



24. (*a*) Find the image formed by the converging lens, using Eq. 33-2.

$$\frac{1}{d_{\mathrm{o1}}} + \frac{1}{d_{\mathrm{i1}}} = \frac{1}{f_1} \quad \rightarrow \quad d_{\mathrm{i1}} = \frac{d_{\mathrm{o1}} f_1}{d_{\mathrm{o1}} - f_1} = \frac{(33\,\mathrm{cm})(18\,\mathrm{cm})}{(33\,\mathrm{cm}) - (18\,\mathrm{cm})} = 39.6\,\mathrm{cm}$$

This image is the object for the second lens. The image is to the right of the second lens, and so is virtual. Use that image to find the final image.

$$d_{\mathrm{o2}} = 12\,\mathrm{cm} - 39.6\,\mathrm{cm} = -27.6\,\mathrm{cm} \ ; \quad \frac{1}{d_{\mathrm{o2}}} + \frac{1}{d_{\mathrm{i2}}} = \frac{1}{f_2} \quad \rightarrow$$

$$d_{\mathrm{i2}} = \frac{d_{\mathrm{o2}} f_2}{d_{\mathrm{o2}} - f_2} = \frac{(-27.6\,\mathrm{cm})(-14\,\mathrm{cm})}{(-27.6\,\mathrm{cm}) - (-14\,\mathrm{cm})} = -28.4\,\mathrm{cm}$$

So the final image is 28 cm to the left of the diverging lens, or $\boxed{\text{16 cm to the left of the converging lens}}$.

(*b*) The initial image is unchanged. With the change in the distance between the lenses, the image distance for the second lens has changed.

$$d_{\mathrm{o2}} = 38\,\mathrm{cm} - 39.6\,\mathrm{cm} = -1.6\,\mathrm{cm} \ ; \quad \frac{1}{d_{\mathrm{o2}}} + \frac{1}{d_{\mathrm{i2}}} = \frac{1}{f_2} \quad \rightarrow$$

$$d_{\mathrm{i2}} = \frac{d_{\mathrm{o2}} f_2}{d_{\mathrm{o2}} - f_2} = \frac{(-1.6\,\mathrm{cm})(-14\,\mathrm{cm})}{(-1.6\,\mathrm{cm}) - (-14\,\mathrm{cm})} = 1.8\,\mathrm{cm}$$

Now the final image is $\boxed{\text{1.8 cm to the right of the diverging lens.}}$

$\boxed{25.}$ (*a*) The first lens is the converging lens. Find the image formed by the first lens.

$$\frac{1}{d_{\mathrm{o1}}} + \frac{1}{d_{\mathrm{i1}}} = \frac{1}{f_1} \quad \rightarrow \quad d_{\mathrm{i1}} = \frac{d_{\mathrm{o1}} f_1}{d_{\mathrm{o1}} - f_1} = \frac{(60.0\,\mathrm{cm})(20.0\,\mathrm{cm})}{(60.0\,\mathrm{cm}) - (20.0\,\mathrm{cm})} = 30.0\,\mathrm{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{\mathrm{o2}} = 25.0\,\mathrm{cm} - 30.0\,\mathrm{cm} = -5.0\,\mathrm{cm}$. Use Eq. 33-2.

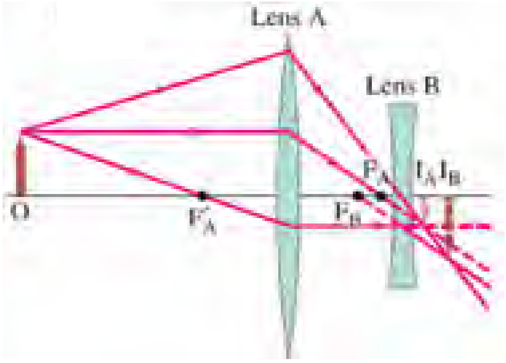
$$\frac{1}{d_{\mathrm{o2}}} + \frac{1}{d_{\mathrm{i2}}} = \frac{1}{f_2} \quad \rightarrow \quad d_{\mathrm{i2}} = \frac{d_{\mathrm{o2}} f_2}{d_{\mathrm{o2}} - f_2} = \frac{(-5.0\,\mathrm{cm})(-10.0\,\mathrm{cm})}{(-5.0\,\mathrm{cm}) - (-10.0\,\mathrm{cm})} = 10\,\mathrm{cm}$$

Thus the final image is real, $\boxed{\text{10 cm beyond the second lens}}$. The distance has two significant figures.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{\mathrm{i1}}}{d_{\mathrm{o1}}}\right)\left(-\frac{d_{\mathrm{i2}}}{d_{\mathrm{o2}}}\right) = \frac{d_{\mathrm{i1}} d_{\mathrm{i2}}}{d_{\mathrm{o1}} d_{\mathrm{o2}}} = \frac{(30.0\,\mathrm{cm})(10.0\,\mathrm{cm})}{(60.0\,\mathrm{cm})(-5.0\,\mathrm{cm})} = \boxed{-1.0\times}$$

(*c*) See the diagram here.



26. We find the focal length of the combination by finding the image distance for an object very far away. For the converging lens, we have the following from Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_C}=\frac{1}{\infty}+\frac{1}{d_{i1}} \rightarrow d_{i1}=f_C$$

The first image is the object for the second lens. Since the first image is real, the second object distance is negative. We also assume that the lenses are thin, and so $d_{o2}=-d_{i1}=-f_C$.

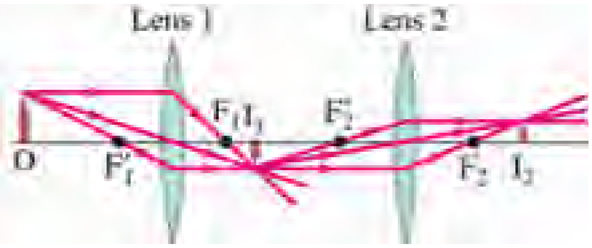
For the second diverging lens, we have the following from Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_D}=-\frac{1}{f_C}+\frac{1}{d_{i2}}$$

Since the original object was at infinity, the second image must be at the focal point of the combination, and so $d_{i2}=f_T$.

$$\frac{1}{f_D}=-\frac{1}{f_C}+\frac{1}{d_{i2}}=-\frac{1}{f_C}+\frac{1}{f_T}$$

27. (*a*) We see that the image is real and upright. We estimate that it is 30 cm beyond the second lens, and that the final image height is half the original object height.



(*b*) Find the image formed by the first lens, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow$$

$$d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(36\,\text{cm})(13\,\text{cm})}{(36\,\text{cm})-(13\,\text{cm})}=20.35\,\text{cm}$$

This image is the object for the second lens. Because it is between the lenses, it has a positive object distance.

$$d_{o2}=56\,\text{cm}-20.35\,\text{cm}=35.65\,\text{cm}$$

Find the image formed by the second lens, again using Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(35.65\,\text{cm})(16\,\text{cm})}{(35.65\,\text{cm})-(16\,\text{cm})}=29.25\,\text{cm}$$

Thus the final image is real, $\boxed{\text{29 cm beyond the second lens.}}$

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{(20.35\,\text{cm})(29.25\,\text{cm})}{(36\,\text{cm})(35.65\,\text{cm})}=\boxed{0.46\times}$$

28. Use Eq. 33-4, the lensmaker's equation.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.58-1)}\left(\frac{(-33.4\,\text{cm})(-28.8\,\text{cm})}{(-33.4\,\text{cm})+(-28.8\,\text{cm})}\right)=-26.66\,\text{cm}\approx\boxed{-27\,\text{cm}}$$

29. Find the index from Eq. 33-4, the lensmaker's equation.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow n=1+\frac{1}{f}\left(\frac{R_1R_2}{R_1+R_2}\right)=1+\left(\frac{1}{28.9\,\text{cm}}\right)\left(\tfrac{1}{2}(31.4\,\text{cm})\right)=\boxed{1.54}$$

30. With the surfaces reversed, we have $R_1=-46.2\,\text{cm}$ and $R_2=+22.4\,\text{cm}$. Use Eq. 33-4 to find the focal length.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.50-1)}\left(\frac{(-46.2\,\text{cm})(+22.4\,\text{cm})}{(-46.2\,\text{cm})+(+22.4\,\text{cm})}\right)=\boxed{87.0\,\text{cm}}$$

$\boxed{31.}$ The plane surface has an infinite radius of curvature. Let the plane surface be surface 2, so $R_2=\infty$. The index of refraction is found in Table 32-1.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{\infty}\right)=\frac{(n-1)}{R_1} \rightarrow$$

$$R_1=(n-1)f=(1.46-1)(18.7\,\text{cm})=\boxed{8.6\,\text{cm}}$$

32. First we find the focal length from Eq. 33-3, the lensmaker's equation. Then we use Eq. 33-2 to find the image distance, and Eq. 33-3 to find the magnification.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.52-1)}\left(\frac{(-22.0\,\text{cm})(+18.5\,\text{cm})}{(-22.0\,\text{cm})+(+18.5\,\text{cm})}\right)=223.6\,\text{cm}$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_of}{d_o-f}=\frac{(90.0\,\text{cm})(223.6\,\text{m})}{90.0\,\text{cm}-223.6\,\text{cm}}=-150.6\,\text{cm}\approx\boxed{-151\,\text{cm}}$$

$$m=-\frac{d_i}{d_o}=-\frac{-150.6\,\text{cm}}{90.0\,\text{cm}}=\boxed{+1.67}$$

The image is virtual, in front of the lens, and upright.

33. Find the radius from the lensmaker's equation, Eq. 33-4.:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow P=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$R_2=\frac{(n-1)R_1}{PR_1-(n-1)}=\frac{(1.56-1)(0.300\,\text{m})}{(3.50\,\text{D})(0.300\,\text{m})-(1.56-1)}=\boxed{0.34\,\text{m}}$$

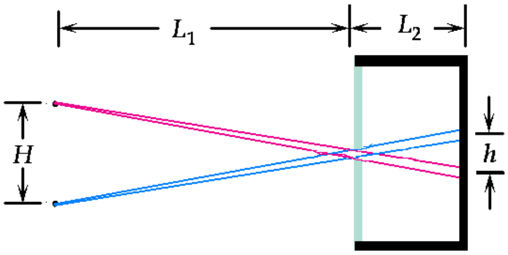
34. The exposure is proportional to the product of the lens opening area and the exposure time, with the square of the *f*-stop number inversely proportional to the lens opening area. Setting the exposures equal for both exposure times we solve for the needed *f*-stop number.

$$t_1\left(f\text{-stop}_1\right)^{-2}=t_2\left(f\text{-stop}_2\right)^{-2}\ \rightarrow\ f\text{-stop}_2=f\text{-stop}_1\sqrt{\frac{t_2}{t_1}}=16\sqrt{\frac{1/1000\text{ s}}{1/120\,\text{s}}}=5.54\text{ or }\boxed{\frac{f}{5.6}}$$

35. We find the *f*-number from $f\text{-stop}=f/D$.

$$f\text{-stop}=\frac{f}{D}=\frac{(17\,\text{cm})}{(6.0\,\text{cm})}=\boxed{\frac{f}{2.8}}$$

36. We use similar triangles, created from the distances between the centers of the two objects (*H*) and their ray traces to the hole (L_1) and the distance between the centers of the two images (*h*) and the distance of the screen to the hole (L_2) to determine the distance between the center of the two image circles. We then create similar triangles from the two ray traces for a single source with the base of one triangle equal to the diameter of the hole (*d*) , and the base of the second triangle equal to the diameter of the image circle (*D*). The heights for these two triangles are the distance from object to hole (L_1) and the distance from object to image ($L_1 + L_2$).



$$\frac{H}{L_1}=\frac{h}{L_2}\ \rightarrow h=H\frac{L_2}{L_1}=(2.0\text{ cm})\frac{7.0\text{ cm}}{100\text{ cm}}=0.14\text{ cm}=1.4\text{ mm}$$

$$\frac{d}{L_1}=\frac{D}{L_1+L_2}\ \rightarrow D=d\frac{L_1+L_2}{L_1}=(1.0\,\text{mm})\frac{100\,\text{cm}+7.0\,\text{cm}}{100\,\text{cm}}=1.07\,\text{mm}$$

Since the separation distance of the two images is greater than their diameters, the two circles do not overlap.

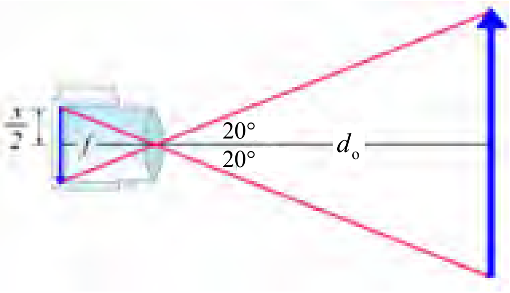
$\boxed{37.}$ We calculate the effective *f*-number for the pinhole camera by dividing the focal length by the diameter of the pinhole. The focal length is equal to the image distance. Setting the exposures equal for both cameras, where the exposure is proportional to the product of the exposure time and the area of the lens opening (which is inversely proportional to the square of the *f*-stop number), we solve for the exposure time.

$$f\text{-stop}_2=\frac{f}{D}=\frac{(70\,\text{mm})}{(1.0\,\text{mm})}=\frac{f}{70}.$$

$$t_1\left(f\text{-stop}_1\right)^{-2}=t_2\left(f\text{-stop}_2\right)^{-2}\ \rightarrow\ t_2=t_1\left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2=\frac{1}{250\text{ s}}\left(\frac{70}{11}\right)^2=0.16\text{ s}\approx\boxed{\tfrac{1}{6}\text{s}}$$

38. Consider an object located a distance d_o from a converging lens of focal length f and its real image formed at distance d_i. If the distance d_o is much greater than the focal length, the lens equation tells us that the focal length and image distance are equal.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}\ \rightarrow\ d_\text{i}=\frac{fd_\text{o}}{d_\text{o}-f}\approx\frac{fd_\text{o}}{d_\text{o}}=f$$



Thus, in a camera, the recording medium of spatial extent x is placed a distance equal to f behind the lens to form a focused image of a distant object. Assume the distant object subtends an angle of 40° at the position of the lens, so that the half-angle subtended is 20°, as shown in the figure. We then use the tangent of this angle to determine the relationship between the focal length and half the image height.

$$\tan 20^\circ = \frac{\frac{1}{2}x}{f} \rightarrow f = \frac{x}{2\tan 20^\circ}$$

(*a*) For a 35-mm camera, we set $x = 36\text{ mm}$ to calculate the focal length.

$$f = \frac{36\text{ mm}}{2\tan 20^\circ} = \boxed{49\text{ mm}}$$

(*b*) For a digital camera, we set $x = 1.0\text{ cm} = 10\text{ mm}$.

$$f = \frac{10\text{ mm}}{2\tan 20^\circ} = \boxed{14\text{ mm}}$$

39. The image distance is found from Eq. 33-3, and then the focal length from Eq. 33-2. The image is inverted.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \rightarrow d_i = -d_o\frac{h_i}{h_o} = -(65\text{m})\frac{(-24\text{mm})}{(38\text{m})} = 41\text{mm}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{(65\text{m})(0.041\text{m})}{65\text{m} - 0.041\text{m}} = 0.041\text{m} = \boxed{41\text{mm}}$$

The object is essentially at infinity, so the image distance is equal to the focal length.

40. The length of the eyeball is the image distance for a far object, i.e., the focal length of the lens. We find the *f*-number from $f\text{-stop} = f/D$.

$$f\text{-stop} = \frac{f}{D} = \frac{(20\text{mm})}{(8.0\text{mm})} = \boxed{2.5 \text{ or } \frac{f}{2.5}}.$$

41. The actual near point of the person is 55 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 55 cm from the eye, or 53 cm from the lens. We find the power of the lens from Eqs. 33-1 and 33-3.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.23\text{m}} + \frac{1}{-0.53\text{m}} = \boxed{2.5\text{D}}$$

42. The screen placed 55 cm from the eye, or 53.2 cm from the lens, is to produce a virtual image 105 cm from the eye, or 103.2 cm from the lens. Find the power of the lens from Eqs. 33-1 and 33-2.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.532\text{m}} + \frac{1}{-1.032\text{m}} = \boxed{0.91\text{D}}$$

$\boxed{43.}$ With the contact lens, an object at infinity should form a virtual image at the far point of the eye, 17 cm from the contact lens. Use that with Eq. 33-2 to find the focal length of the contact lens. We find the power of the lens from

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = d_i = -17\text{cm}$$

Find the new near point as the object location that forms a virtual image at the actual near point of 12 cm from the contact lens. Again use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{fd_i}{d_i-f}=\frac{(-17\,\text{cm})(-12\,\text{cm})}{(-12\,\text{cm})-(-17\,\text{cm})}=\boxed{41\,\text{cm}}$$

So the person would have to hold the object 41 cm from their eye to see it clearly. With glasses, they only had to hold the object 32 cm from the eye. So $\boxed{\text{glasses would be better.}}$

44. (*a*) Since the lens power is negative, the lens is diverging, so it produces images closer than the object. Thus the person is $\boxed{\text{nearsighted.}}$

(*b*) We find the far point by finding the image distance for an object at infinity. Since the lens is 2.0 cm in front of the eye, the far point is 2.0 cm farther than the absolute value of the image distance.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow \frac{1}{\infty}+\frac{1}{d_i}=-4.50\,\text{D} \rightarrow d_i=-\frac{1}{4.50\text{ D}}=-0.222\text{ m}=-22.2\text{ cm}$$

$$\text{FP}=\left|-22.2\text{ cm}\right|+2.0\text{ cm}=\boxed{24.2\text{ cm}}\text{ from eye}$$

45. (*a*) The lens should put the image of an object at infinity at the person's far point of 78 cm. Note that the image is still in front of the eye, so the image distance is negative. Use Eqs. 33-2 and 33-1.

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}+\frac{1}{(-0.78\,\text{m})}=-1.282\,\text{D}\approx\boxed{-1.3\,\text{D}}$$

(*b*) To find the near point with the lens in place, we find the object distance to form an image 25 cm in front of the eye.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_o=\frac{d_i}{Pd_i-1}=\frac{(-0.25\,\text{m})}{(-1.282\,\text{D})(-0.25\,\text{m})-1}=0.37\,\text{m}=\boxed{37\,\text{cm}}$$

46. The image of an object at infinity is to be formed 14 cm in front of the eye. So for glasses, the image distance is to be $d_i=-12\,\text{cm}$, and for contact lenses, the image distance is to be $d_i=-14\,\text{cm}$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{1}{\infty}+\frac{1}{d_i} \rightarrow f=d_i \rightarrow P=\frac{1}{f}=\frac{1}{d_i}$$

$$P_{\text{glasses}}=\frac{1}{-0.12\,\text{m}}=\boxed{-8.3\,\text{D}}\ ;\ P_{\text{contacts}}=\frac{1}{-0.14\,\text{m}}=\boxed{-7.1\,\text{D}}$$

47. Find the far point of the eye by finding the image distance FROM THE LENS for an object at infinity, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=f_1=-23.0\,\text{cm}$$

Since the image is 23.0 in front of the lens, the image is 24.8 cm in front of the eye. The contact lens must put the image of an object at infinity at this same location. Use Eq. 33-2 for the contact lens with an image distance of -24.8 cm and an object distance of infinity.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i2}}=\frac{1}{f_1} \rightarrow f_1=d_{i2}=\boxed{-24.8\,\text{cm}}$$

48. (*a*) We find the focal length of the lens for an object at infinity and the image on the retina. The image distance is thus 2.0 cm. Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{\infty}+\frac{1}{2.0\,\text{cm}}=\frac{1}{f} \rightarrow f=\boxed{2.0\,\text{cm}}$$

(*b*) We find the focal length of the lens for an object distance of 38 cm and an image distance of 2.0 cm. Again use Eq. 33.2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{(38\,\text{cm})(2.0\,\text{cm})}{(38\,\text{cm})+(2.0\,\text{cm})}=\boxed{1.9\,\text{cm}}$$

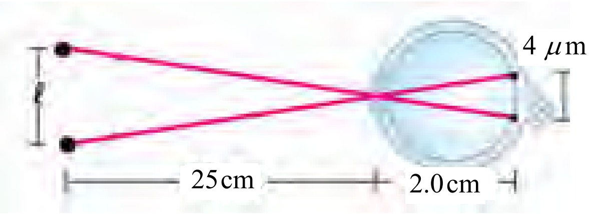
$\boxed{49.}$ Find the object distance for the contact lens to form an image at the eye's near point, using Eqs. 33-2 and 33-1.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_o=\frac{d_i}{Pd_i-1}=\frac{-0.106\,\text{m}}{(-4.00\,\text{D})(-0.106\,\text{m})-1}=0.184\,\text{m}=\boxed{18.4\,\text{cm}}$$

Likewise find the object distance for the contact lens to form an image at the eye's far point.

$$d_o=\frac{d_i}{Pd_i-1}=\frac{-0.200\,\text{m}}{(-4.0\,\text{D})(-0.200\,\text{m})-1}=1.00\,\text{m}=\boxed{100\,\text{cm}}\ \text{(3 sig. fig.)}$$

50. In the image we show the principal rays from each of the two points as they pass directly through the cornea and onto the lens. These two rays and the distance between the two objects, ℓ, and the distance between the two images (4 μm) create similar triangles. We set the ratio of the bases and heights of these two triangles equal to solve for ℓ.



$$\frac{\ell}{25\text{ cm}}=\frac{4\ \mu\text{m}}{2.0\text{ cm}} \rightarrow \ell=25\text{ cm}\frac{4\ \mu\text{m}}{2.0\text{ cm}}=\boxed{50\mu\text{m}}$$

51. We find the focal length from Eq. 33-6

$$M=\frac{N}{f} \rightarrow f=\frac{N}{M}=\frac{25\,\text{cm}}{3.8}=\boxed{6.6\,\text{cm}}$$

52. Find the magnification from Eq. 33-6.

$$M=\frac{N}{f}=\frac{(25\,\text{cm})}{(13\,\text{cm})}=\boxed{1.9\times}$$

53. (*a*) We find the focal length with the image at the near point from Eq. 33-6b.

$$M=1+\frac{N}{f} \rightarrow f=\frac{N}{M-1}=\frac{25\,\text{cm}}{3.0-1}=12.5\,\text{cm}\approx\boxed{13\,\text{cm}}$$

$$3.0=1+\frac{(25\,\text{cm})}{f_1},\ \text{which gives } f_1=12.5\,\text{cm}\approx\boxed{13\,\text{cm.}}$$

(*b*) If the eye is relaxed, the image is at infinity, and so use Eq. 33-6a.

$$M=\frac{N}{f} \rightarrow f=\frac{N}{M}=\frac{25\,\text{cm}}{3.0}=\boxed{8.3\,\text{cm}}$$

54. Maximum magnification is obtained with the image at the near point (which is negative). We find the object distance from Eq. 33-2, and the magnification from Eq. 33-6b.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{d_if}{d_i-f}=\frac{(-25.0\,\text{cm})(8.80\,\text{cm})}{(-25.0\,\text{cm})-(8.80\,\text{cm})}=\boxed{6.51\,\text{cm}}$$

$$M = 1 + \frac{N}{f} = 1 + \frac{25.0\,\text{cm}}{8.80\,\text{cm}} = \boxed{3.84\times}$$

55. (*a*) We find the image distance from Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow\; d_i = \frac{fd_o}{d_o - f} = \frac{(6.00\,\text{cm})(5.85\,\text{cm})}{5.85\,\text{cm} - 6.00\,\text{cm}} = \boxed{-234\,\text{cm}}$$

(*b*) The angular magnification is given by Eq. 33-6a, since the eye will have to focus over 2 m away.

$$M = \frac{N}{f} = \frac{25.0\,\text{cm}}{6.00\,\text{cm}} = \boxed{4.17\times}$$

56. (*a*) We use Eq. 33-6b to calculate the angular magnification.

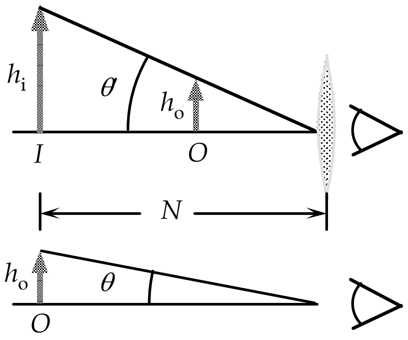
$$M = 1 + \frac{N}{f} = 1 + \frac{(25.0\,\text{cm})}{(9.60\,\text{cm})} = \boxed{3.60\times}$$

(*b*) Because the object without the lens and the image with the lens are at the near point, the angular magnification is also the ratio of widths. Using this relationship we calculate the image width.

$$M = \frac{h_i}{h_o} \rightarrow h_i = Mh_o = 3.60(3.40\,\text{mm}) = \boxed{12.3\,\text{mm}}$$

(*c*) We use Eq. 33-2 to calculate the object distance, with the image distance at -25.0 cm.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow\; d_o = \frac{fd_i}{d_i - f} = \frac{(9.60\,\text{cm})(-25.0\,\text{cm})}{-25.0\,\text{cm} - 9.60\,\text{cm}} = \boxed{6.94\,\text{cm}}$$



57. (*a*) We find the image distance using Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow\; d_i = \frac{fd_o}{d_o - f} = \frac{(9.5\,\text{cm})(8.3\,\text{cm})}{8.3\,\text{cm} - 9.5\,\text{cm}} = \boxed{-66\,\text{cm}}$$

(*b*) The angular magnification is found using Eq. 33-5, with the angles given as defined in Figure 33-33.

$$M = \frac{\theta'}{\theta} = \frac{(h_o/d_o)}{(h_o/N)} = \frac{N}{d_o} = \frac{25\text{ cm}}{8.3\text{ cm}} = \boxed{3.0\times}$$

58. First, find the focal length of the magnifying glass from Eq. 33-6a, for a relaxed eye (focused at infinity).

$$M = \frac{N}{f} \;\rightarrow\; f = \frac{N}{M} = \frac{25.0\,\text{cm}}{3.0} = 8.33\,\text{cm}$$

(*a*) Again use Eq. 33-6a for a different near point.

$$M_1 = \frac{N_1}{f} = \frac{(65\,\text{cm})}{(8.33\,\text{cm})} = \boxed{7.8\times}$$

(*b*) Again use Eq. 33-6a for a different near point.

$$M_2 = \frac{N_2}{f} = \frac{(17\,\text{cm})}{(8.33\,\text{cm})} = \boxed{2.0\times}$$

Without the lens, the closest an object can be placed is the near point. A farther near point means a smaller angle subtended by the object without the lens, and thus greater magnification.

59. The focal length is 10 cm. First, find the object distance for an image at infinity. Then, find the object distance for an image 25 cm in front of the eye.

$$\text{Initial: } \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{\infty}=\frac{1}{f} \rightarrow d_o = f = 12\,\text{cm}$$

$$\text{Final: } \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{d_i f}{d_i - f}=\frac{(-25\,\text{cm})(12\,\text{cm})}{(-25\,\text{cm})-(12\,\text{cm})}=8.1\,\text{cm}$$

The lens was moved $12\,\text{cm}-8.1\,\text{cm}=3.9\,\text{cm}\approx$ $\boxed{\text{4 cm toward the fine print}}$.

60. The magnification of the telescope is given by Eq. 33-7.

$$M=-\frac{f_o}{f_e}=-\frac{(78\,\text{cm})}{(2.8\,\text{cm})}=\boxed{-28\times}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o + f_e = 78\,\text{cm} + 2.8\,\text{cm} = \boxed{81\,\text{cm}}$$

$\boxed{61.}$ We find the focal length of the eyepiece from the magnification by Eq. 33-7.

$$M=-\frac{f_o}{f_e} \rightarrow f_e=-\frac{f_o}{M}=-\frac{88\,\text{cm}}{35\times}=\boxed{2.5\,\text{cm}}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o + f_e = 88\,\text{cm} + 2.5\,\text{cm} = \boxed{91\,\text{cm}}$$

62. We find the focal length of the objective from Eq. 33-7.

$$M = f_o/f_e \rightarrow f_o = Mf_e = (7.0)(3.0\,\text{cm}) = \boxed{21\,\text{cm}}$$

63. The magnification is given by Eq. 33-7.

$$M=-f_o/f_e == -f_oP_e = -(0.75\,\text{m})(35\,\text{D}) = \boxed{-26\times}$$

64. For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 33-7 to find the magnification.

$$\ell = f_o + f_e \;;\; M=-\frac{f_o}{f_e}=-\frac{f_o}{\ell - f_o}=-\frac{75.5\,\text{cm}}{78.0\,\text{cm}-75.5\,\text{cm}}=\boxed{-30\times}$$

65. For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 33-7 to find the magnification.

$$\ell = f_o + f_e \;;\; M=-\frac{f_o}{f_e}=-\frac{f_o}{\ell - f_o}=-\frac{36.0\,\text{cm}}{33.8\,\text{cm}-36.0\,\text{cm}}=\boxed{+16\times}$$

66. The focal length of the objective is just half the radius of curvature. Use Eq. 33-7 for the magnification.

$$M=-\frac{f_o}{f_e}=-\frac{\frac{1}{2}r}{f_e}=-\frac{3.2\,\text{m}}{0.028\text{m}}=-114\times\approx\boxed{-110\times}$$

$\boxed{67.}$ The focal length of the mirror is found from Eq. 33-7. The radius of curvature is twice the focal length.

$$M=-\frac{f_\text{o}}{f_\text{e}} \rightarrow f_\text{o}=-Mf_\text{e}=-(120)(0.031\,\text{m})=3.72\,\text{m}\approx\boxed{3.7\,\text{m}}\ ;\ r=2f=\boxed{7.4\,\text{m}}$$

68. The relaxed eye means that the image is at infinity, and so the distance between the two lenses is 1.25 m. Use that relationship with Eq. 33-7 to solve for the focal lengths. Note that the magnification for an astronomical telescope is negative.

$$\ell=f_\text{o}+f_\text{e}\ ;\ M=-\frac{f_\text{o}}{f_\text{e}}=-\frac{f_\text{o}}{\ell-f_\text{o}} \rightarrow f_\text{o}=\frac{M\ell}{M-1}=\frac{-120(1.25\,\text{m})}{-120-1}=\boxed{1.24\,\text{m}}$$

$$f_\text{e}=\ell-f_\text{o}=1.25\,\text{m}-1.24\,\text{m}=0.01\,\text{m}=\boxed{1\text{ cm}}$$

69. We use Eq. 33-6a and the magnification of the eyepiece to calculate the focal length of the eyepiece. We set the sum of the focal lengths equal to the length of the telescope to calculate the focal length of the objective. Then using both focal lengths in Eq. 33-7 we calculate the maximum magnification.

$$f_\text{e}=\frac{N}{M}=\frac{25\,\text{cm}}{5}=5\,\text{cm}\ ;\ \ell=f_\text{e}+f_\text{o} \rightarrow f_\text{o}=\ell-f_\text{e}=50\,\text{cm}-5\,\text{cm}=45\,\text{cm}$$

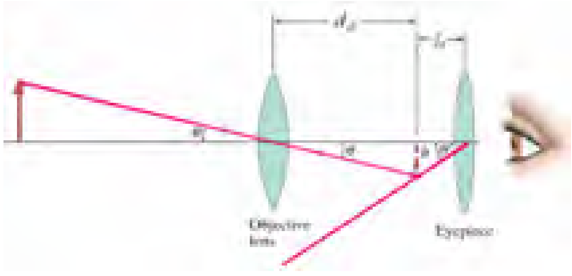
$$M=-\frac{f_\text{o}}{f_\text{e}}=-\frac{45\,\text{cm}}{5\,\text{cm}}=\boxed{-9\times}$$

70. Since the star is very far away, the image of the star from the objective mirror will be at the focal length of the objective, which is equal to one-half its radius of curvature (Eq. 32-1). We subtract this distance from the separation distance to determine the object distance for the second mirror. Then, using Eq. 33-2, we calculate the final image distance, which is where the sensor should be placed.

$$d_\text{i1}=f_\text{o}=\frac{R_\text{o}}{2}=\frac{3.00\,\text{m}}{2}=1.50\,\text{m}\ ;\ d_\text{o2}=\ell-d_\text{i1}=0.90\,\text{m}-1.50\,\text{m}=-0.60\,\text{m}$$

$$\frac{1}{d_\text{o2}}+\frac{1}{d_\text{i2}}=\frac{1}{f_\text{e}}=\frac{2}{R_\text{e}} \rightarrow d_\text{i}=\frac{R_\text{e}d_\text{o2}}{2d_\text{o2}-R_\text{e}}=\frac{(-1.50\,\text{m})(-0.60\,\text{m})}{2(-0.60\,\text{m})-(-1.50\,\text{m})}=\boxed{3.0\,\text{m}}$$

71. We assume a prism binocular so the magnification is positive, but simplify the diagram by ignoring the prisms. We find the focal length of the eyepiece using Eq. 33-7, with the design magnification.



$$f_\text{e}=\frac{f_\text{o}}{M}=\frac{26\,\text{cm}}{7.5}=3.47\,\text{cm}$$

Using Eq. 33-2 and the objective focal length, we calculate the intermediate image distance. With the final image at infinity (relaxed eye), the secondary object distance is equal to the focal length of the eyepiece. We calculate the angular magnification using Eq. 33-5, with the angles shown in the diagram.

$$\frac{1}{d_\text{o1}}+\frac{1}{d_\text{i1}}=\frac{1}{f_\text{o}} \rightarrow d_\text{i1}=\frac{f_\text{o}d_{\text{o`}}}{d_\text{o1}-f_\text{o}}=\frac{(26\,\text{cm})(400\,\text{cm})}{400\,\text{cm}-26\,\text{cm}}=27.81\,\text{cm}$$

$$M=\frac{\theta'}{\theta}=\frac{h/f_\text{e}}{h/d_\text{i1}}=\frac{d_\text{i1}}{f_\text{e}}=\frac{27.81\,\text{cm}}{3.47\,\text{cm}}=\boxed{8.0\times}$$

72. The magnification of the microscope is given by Eq. 33-10b.

$$M=\frac{N\ell}{f_{\mathrm{o}}f_{\mathrm{e}}}=\frac{(25\,\mathrm{cm})(17.5\,\mathrm{cm})}{(0.65\,\mathrm{cm})(1.50\,\mathrm{cm})}=448.7\times\approx\boxed{450\times}$$

$\boxed{73.}$ We find the focal length of the eyepiece from the magnification of the microscope, using the approximate results of Eq. 33-10b. We already know that $f_{\mathrm{o}}\ll\ell$.

$$M\approx\frac{N\ell}{f_{\mathrm{o}}f_{\mathrm{e}}}\;\rightarrow\;f_{\mathrm{e}}=\frac{N\ell}{Mf_{\mathrm{o}}}=\frac{(25\,\mathrm{cm})(17.5\,\mathrm{cm})}{(680)(0.40\,\mathrm{cm})}=\boxed{1.6\,\mathrm{cm}}$$

Note that this also satisfies the assumption that $f_{\mathrm{e}}\ll\ell$.

74. We use Eq. 33-10b.

$$M\approx\frac{N\ell}{f_{\mathrm{e}}f_{\mathrm{o}}}=\frac{(25\,\mathrm{cm})(17\,\mathrm{cm})}{(2.5\,\mathrm{cm})(0.28\,\mathrm{cm})}=607.1\times\approx\boxed{610\times}$$

75. (*a*) The total magnification is found from Eq. 33-10a.

$$M=M_{\mathrm{o}}M_{\mathrm{e}}=(58.0)(13.0)=\boxed{754\times}$$

(*b*) With the final image at infinity, we find the focal length of the eyepiece using Eq. 33-9.

$$M_{\mathrm{e}}=\frac{N}{f_{\mathrm{e}}}\;\rightarrow\;f_{\mathrm{e}}=\frac{N}{M_{\mathrm{e}}}=\frac{25.0\,\mathrm{cm}}{13.0}=1.923\,\mathrm{cm}\approx\boxed{1.92\,\mathrm{cm}}$$

Since the image from the objective is at the focal point of the eyepiece, we set the image distance from the objective as the distance between the lenses less the focal length of the eyepiece. Using the image distance and magnification in Eq. 33-3, we calculate the initial object distance. Then using the image and object distance in Eq. 33-2 we calculate the objective focal length.

$$d_{\mathrm{i}}=\ell-f_{\mathrm{e}}=20.0\,\mathrm{cm}-1.92\,\mathrm{cm}=18.08\,\mathrm{cm}$$

$$m=\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}}\rightarrow d_{\mathrm{o}}=\frac{d_{\mathrm{i}}}{m}=\frac{18.08\,\mathrm{cm}}{58.0}=0.312\,\mathrm{cm}$$

$$\frac{1}{f_{\mathrm{o}}}=\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}}\rightarrow f_{\mathrm{o}}=\frac{d_{\mathrm{o}}d_{\mathrm{i}}}{d_{\mathrm{o}}+d_{\mathrm{i}}}=\frac{(0.312\,\mathrm{cm})(18.08\,\mathrm{cm})}{0.312\,\mathrm{cm}+18.08\,\mathrm{cm}}=\boxed{0.307\,\mathrm{cm}}$$

(*c*) We found the object distance, in part (*b*), $d_{\mathrm{o}}=\boxed{0.312\,\mathrm{cm}}$.

76. (*a*) The total magnification is the product of the magnification of each lens, with the magnification of the eyepiece increased by one, as in Eq. 33-6b.

$$M=M_{\mathrm{o}}(M_{\mathrm{e}}+1)=(58.0)(13.0+1.0)=\boxed{812\times}$$

(*b*) We find the focal length of the eyepiece using Eq. 33-6b.

$$(M_{\mathrm{e}}+1)=\frac{N}{f_{\mathrm{e}}}+1\;\rightarrow f_{\mathrm{e}}=\frac{N}{M_{\mathrm{e}}}=\frac{25\,\mathrm{cm}}{13.0}=\boxed{1.92\,\mathrm{cm}}$$

Since the image from the eyepiece is at the near point, we use Eq. 33-2 to calculate the location of the object. This object distance is the location of the image from the objective. Subtracting this object distance from the distance between the lenses gives us the image distance from the objective. Using the image distance and magnification in Eq. 33-3, we calculate the initial object distance. Then using the image and object distance in Eq. 33-2 we calculate the objective focal length.

$$\frac{1}{f_e}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}} \rightarrow d_{o2}=\frac{f_e d_{i2}}{d_{i2}-f_e}=\frac{(1.92\,\text{cm})(-25.0\,\text{cm})}{-25.0\,\text{cm}-1.92\,\text{cm}}=1.78\,\text{cm}$$

$$d_{i1}=\ell-d_{o2}=20.0\,\text{cm}-1.78\,\text{cm}=18.22\,\text{cm}$$

$$m=\frac{d_i}{d_o} \rightarrow d_o=\frac{d_i}{m}=\frac{18.22\,\text{cm}}{58.0}=0.314\,\text{cm}$$

$$\frac{1}{f_o}=\frac{1}{d_o}+\frac{1}{d_i} \rightarrow f_o=\frac{d_o d_i}{d_o+d_i}=\frac{(0.314\,\text{cm})(18.22\,\text{cm})}{0.314\,\text{cm}+18.22\,\text{cm}}=\boxed{0.308\,\text{cm}}$$

(*c*) We found the object distance, in part (*b*), $d_o=\boxed{0.314\,\text{cm}}$.

77. (*a*) Since the final image is at infinity (relaxed eye) the image from the objective is at the focal point of the eyepiece. We subtract this distance from the distance between the lenses to calculate the objective image distance. Then using Eq. 33-2, we calculate the object distance.

$$d_{i1}=\ell-f_e=16.8\,\text{cm}-1.8\,\text{cm}=15.0\,\text{cm}$$

$$\frac{1}{f_o}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}} \rightarrow d_{o1}=\frac{f_o d_{i1}}{d_{i1}-f_o}=\frac{(0.80\,\text{cm})(15.0\,\text{cm})}{15.0\,\text{cm}-0.80\,\text{cm}}=\boxed{0.85\,\text{cm}}$$

(*b*) With the final image at infinity, the magnification of the eyepiece is given by Eq. 33-10a.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25.0\,\text{cm})}{(1.8\,\text{cm})}\left(\frac{16.8\,\text{cm}-1.8\,\text{cm}}{0.85\,\text{cm}}\right)=247\times\approx\boxed{250\times}$$

78. (*a*) We find the image distance from the objective using Eq. 33-2. For the final image to be at infinity (viewed with a relaxed eye), the objective image distance must be at the focal distance of the eyepiece. We calculate the distance between the lenses as the sum of the objective image distance and the eyepiece focal length.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_o} \rightarrow d_{i1}=\frac{f_o d_{o1}}{d_{o1}-f_o}=\frac{(0.740\,\text{cm})(0.790\,\text{cm})}{0.790\,\text{cm}-0.740\,\text{cm}}=11.7\,\text{cm}$$

$$\ell=d_{i1}+f_e=11.7\,\text{cm}+2.80\,\text{cm}=\boxed{14.5\,\text{cm}}$$

(*b*) We use Eq. 33-10a to calculate the total magnification.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25.0\,\text{cm})}{(2.80\,\text{cm})}\left(\frac{14.5\,\text{cm}-2.80\,\text{cm}}{0.790\,\text{cm}}\right)=\boxed{132\times}$$

$\boxed{79.}$ For each objective lens we set the image distance equal to the sum of the focal length and 160 mm. Then, using Eq. 33-2 we write a relation for the object distance in terms of the focal length. Using this relation in Eq. 33-3 we write an equation for the magnification in terms of the objective focal length. The total magnification is the product of the magnification of the objective and focal length.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f_o} \rightarrow \frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{d_i} \rightarrow \frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{f_o+160\text{ mm}} \rightarrow d_o=\frac{f_o(f_o+160\text{ mm})}{160\text{ mm}}$$

$$m_o=\frac{d_i}{d_o}=\frac{f_o+160\,\text{mm}}{\left[\dfrac{f_o(f_o+160\,\text{mm})}{160\text{ mm}}\right]}=\frac{160\text{ mm}}{f_o}$$

Since the objective magnification is inversely proportional to the focal length, the objective with the smallest focal length $(f_o=3.9\text{ mm})$ combined with the largest eyepiece magnification $(M_e=10)$ yields the largest overall magnification. The objective with the largest focal length $(f_o=32\text{ mm})$

coupled with the smallest eyepiece magnification $(M_e = 5)$ yields the smallest overall magnification.

$$M_{\text{largest}} = \frac{160\,\text{mm}}{3.9\,\text{mm}}(10\times) = \boxed{410\times} \;;\; M_{\text{smallest}} = \frac{160\,\text{mm}}{32\,\text{mm}}(5\times) = \boxed{25\times}$$

80. (*a*) For this microscope both the objective and eyepiece have focal lengths of 12 cm. Since the final image is at infinity (relaxed eye) the image from the objective must be at the focal length of the eyepiece. The objective image distance must therefore be equal to the distance between the lenses less the focal length of the objective. We calculate the object distance by inserting the objective focal length and image distance into Eq. 33-2.

$$d_{i1} = \ell - f_e = 55\,\text{cm} - 12\,\text{cm} = 43\,\text{cm}$$

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_{i1}} \rightarrow d_o = \frac{f_o d_{i1}}{d_{i1} - f_o} = \frac{(12\,\text{cm})(43\,\text{cm})}{43\,\text{cm} - 12\,\text{cm}} = 16.65\,\text{cm} \approx \boxed{17\,\text{cm}}$$

(*b*) We calculate the magnification using Eq. 33-10a.

$$M = \frac{N}{f_e}\left(\frac{\ell - f_e}{d_o}\right) = \frac{(25\,\text{cm})}{(12\,\text{cm})}\left(\frac{55\,\text{cm} - 12\,\text{cm}}{16.65\,\text{cm}}\right) = 5.38\times \approx \boxed{5.4\times}$$

(*c*) We calculate the magnification using Eq. 33-10b, and divide the result by the answer to part (*b*) to determine the percent difference.

$$M_{\text{approx}} \approx \frac{N\ell}{f_e f_o} = \frac{(25\,\text{cm})(55\,\text{cm})}{(12\,\text{cm})(\,12\,\text{cm})} = 9.55\times \;;\; \frac{M_{\text{approx}} - M}{M} = \frac{9.55 - 5.38}{5.38} = 0.775 \approx \boxed{78\%}$$

81. We use Eq. 33-4 to find the focal length for each color, and then Eq. 33-2 to find the image distance. For the plano-convex lens, $R_1 > 0$ and $R_2 = \infty$.

$$\frac{1}{f_{\text{red}}} = (n_{\text{red}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5106 - 1)\left[\left(\frac{1}{18.4\,\text{cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{red}} = 36.036\,\text{cm}$$

$$\frac{1}{f_{\text{yellow}}} = (n_{\text{yellow}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5226 - 1)\left[\left(\frac{1}{18.4\,\text{cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{orange}} = 35.209\,\text{cm}$$

We find the image distances from

$$\frac{1}{d_o} + \frac{1}{d_{i_{\text{red}}}} = \frac{1}{f_{\text{red}}} \rightarrow d_{i_{\text{red}}} = \frac{d_o f_{\text{red}}}{d_o - f_{\text{red}}} = \frac{(66.0\,\text{cm})(36.036\,\text{cm})}{(66.0\,\text{cm}) - (36.036\,\text{cm})} = 79.374\,\text{cm} \approx \boxed{79.4\,\text{cm}}$$

$$\frac{1}{d_o} + \frac{1}{d_{i_{\text{yellow}}}} = \frac{1}{f_{\text{yellow}}} \rightarrow d_{i_{\text{yellow}}} = \frac{d_o f_{\text{yellow}}}{d_o - f_{\text{yellow}}} = \frac{(66.0\,\text{cm})(35.209\,\text{cm})}{(66.0\,\text{cm}) - (35.209\,\text{cm})} = 75.469\,\text{cm} \approx \boxed{75.5\,\text{cm}}$$

The images are 3.9 cm apart, an example of chromatic aberration.

82. From Problem 26 we have a relationship between the individual focal lengths and the focal length of the combination.

$$\frac{1}{f_D} = -\frac{1}{f_C} + \frac{1}{f_T} \rightarrow \frac{1}{f_T} = \frac{1}{f_D} + \frac{1}{f_C} \rightarrow f_T = \frac{f_C f_D}{f_C + f_D} = \frac{(25\,\text{cm})(-28\,\text{cm})}{(25\,\text{cm}) + (-28\,\text{cm})} = 233\,\text{cm}$$

(*a*) The combination is $\boxed{\text{converging}}$, since the focal length is positive. Also, the converging lens is "stronger" than the diverging lens since it has a smaller absolute focal length (or higher absolute power).

(*b*) From above, $f_T \approx \boxed{230\,\text{cm}}$.

83. We calculate the range object distances from Eq. 33-2 using the given focal length and maximum and minimum image distances.

$$\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}} \rightarrow d_{\text{o,min}}=\frac{fd_{\text{i,max}}}{d_{\text{i,max}}-f}=\frac{(200.0\,\text{mm})(206.4\,\text{mm})}{206.4\,\text{mm}-200.0\,\text{mm}}=6450\,\text{mm}=6.45\,\text{m}$$

$$d_{\text{o,max}}=\frac{fd_{\text{i,min}}}{d_{\text{i,min}}-f}=\frac{(200.0\,\text{mm})(200.0\,\text{mm})}{200.0\,\text{mm}-200.0\,\text{mm}}=\infty$$

Thus the range of object distances is $\boxed{6.45\,\text{m}\le d_\text{o}<\infty}$.

84. We calculate the maximum and minimum image distances from Eq. 33-2, using the given focal length and maximum and minimum object distances. Subtracting these two distances gives the distance over which the lens must move relative to the plane of the sensor or film.

$$\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}} \rightarrow d_{\text{i,max}}=\frac{fd_{\text{o,min}}}{d_{\text{o,min}}-f}=\frac{(135\,\text{mm})(1.30\,\text{m})}{1300\,\text{mm}-135\,\text{mm}}=0.151\,\text{m}=151\,\text{mm}$$

$$d_{\text{i,min}}=\frac{fd_{\text{o,max}}}{d_{\text{o,max}}-f}=\frac{(135\,\text{mm})(\infty)}{\infty-135\,\text{mm}}=135\,\text{mm}$$

$$\Delta d=d_{\text{i,max}}-d_{\text{i,min}}=151\,\text{mm}-135\,\text{mm}=\boxed{16\,\text{mm}}$$

$\boxed{85.}$ Since the object height is equal to the image height, the magnification is –1. We use Eq. 33-3 to obtain the image distance in terms of the object distance. Then we use this relationship with Eq. 33-2 to solve for the object distance.

$$m=-1=-\frac{d_\text{i}}{d_\text{o}} \rightarrow d_\text{i}=d_\text{o}$$

$$\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{o}}=\frac{2}{d_\text{o}} \rightarrow d_o=2f=2(58\,\text{mm})=\boxed{116\,\text{mm}}$$

The distance between the object and the film is the sum of the object and image distances.

$$d=d_\text{o}+d_\text{i}=d_\text{o}+d_\text{o}=2d_\text{o}=2(116\text{ mm})=\boxed{232\,\text{mm}}$$

86. When an object is very far away, the image will be at the focal point. We set the image distance in Eq. 33-3 equal to the focal length to show that the magnification is proportional to the focal length.

$$m=-\frac{d_\text{i}}{d_\text{o}}=-\frac{f}{d_\text{o}}=\left(-\frac{1}{d_\text{o}}\right)f=(\text{constant})f \rightarrow \boxed{m\propto f}$$

87. We use Eq. 33-2 with the final image distance and focal length of the converging lens to determine the location of the object for the second lens. Subtracting this distance from the separation distance between the lenses gives us the image distance from the first lens. Inserting this image distance and object distance into Eq. 33-2, we calculate the focal length of the diverging lens.

$$\frac{1}{d_{\text{o}2}}+\frac{1}{d_{\text{i}2}}=\frac{1}{f_2} \rightarrow d_{\text{o}2}=\frac{d_{\text{i}2}f_2}{d_{\text{i}2}-f_2}=\frac{(17.0\,\text{cm})(12.0\,\text{cm})}{17.0\,\text{cm}-12.0\,\text{cm}}=40.8\,\text{cm}$$

$$d_{\text{i}1}=\ell-d_{\text{o}2}=30.0\,\text{cm}-40.8\,\text{cm}=-10.8\,\text{cm}$$

$$\frac{1}{d_{\text{o}1}}+\frac{1}{d_{\text{i}1}}=\frac{1}{f_1} \rightarrow f_1=\frac{d_{i1}d_{\text{o}1}}{d_{\text{i}1}+d_{\text{o}1}}=\frac{(-10.8\,\text{cm})(25.0\,\text{cm})}{-10.8\,\text{cm}+25.0\,\text{cm}}=\boxed{-19.0\,\text{cm}}$$

88. The relationship between two lenses in contact was found in Problem 26. We use this resulting equation to solve for the combination focal length.

$$\frac{1}{f_T}=\frac{1}{f_D}+\frac{1}{f_C} \rightarrow f_T=\frac{f_D f_C}{f_D+f_C}=\frac{(-20.0\,\text{cm})(13.0\,\text{cm})}{-20.0\,\text{cm}+13.0\,\text{cm}}=\boxed{37.1\,\text{cm}}$$

Since the focal length is positive, the combination is a $\boxed{\text{converging lens}}$.

89. We use Eq. 33-7, which relates the magnification to the focal lengths, to write the focal length of the objective lens in terms of the magnification and focal length of the eyepiece. Then setting the sum of the focal lengths equal to the length of the telescope we solve for the focal length of the eyepiece and the focal length of the objective.

$$M=-\frac{f_o}{f_e} \rightarrow f_o=-Mf_e \;;\; \ell=f_e+f_o=f_e(1-M) \rightarrow f_e=\frac{\ell}{1-M}=\frac{28\,\text{cm}}{1-(-8.0)}=\boxed{3.1\,\text{cm}}$$

$$f_o=\ell-f_e=28\,\text{cm}-3.1\,\text{cm}=\boxed{25\,\text{cm}}$$

90. (*a*) When two lenses are placed in contact, the negative of the image of the first lens is the object distance of the second. Using Eq. 33-2, we solve for the image distance of the first lens. Inserting the negative of this image distance into the lens equation for the second lens we obtain a relationship between the initial object distance and final image distance. Again using the lens equation with this relationship, we obtain the focal length of the lens combination.

$$\frac{1}{f_1}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}}=\frac{1}{f_1}-\frac{1}{d_{o1}}=-\frac{1}{d_{o2}}$$

$$\frac{1}{f_2}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{d_{o2}}-\left(\frac{1}{f_1}-\frac{1}{d_{o1}}\right)\Rightarrow\frac{1}{f_2}+\frac{1}{f_1}=\frac{1}{d_{o2}}+\frac{1}{d_{o1}}=\frac{1}{f_T}$$

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2} \rightarrow \boxed{f_T=\frac{f_1 f_2}{f_1+f_2}}$$

(*b*) Setting the power equal to the inverse of the focal length gives the relationship between powers of adjacent lenses.

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow\boxed{P_T=P_1+P_2}$$

$\boxed{91.}$ (*a*) Because the Sun is very far away, the image will be at the focal point, or $d_i=f$. We find the magnitude of the size of the image using Eq. 33-3, with the image distance equal to 28 mm.

$$\frac{h_i}{h_o}=\frac{-d_i}{d_o} \rightarrow |h_i|=\frac{h_o d_i}{d_o}=\frac{(1.4\times10^6\,\text{km})(28\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{0.26\,\text{mm}}$$

(*b*) We repeat the same calculation with a 50 mm image distance.

$$|h_i|=\frac{(1.4\times10^6\,\text{km})(50\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{0.47\,\text{mm}}$$

(*c*) Again, with a 135 mm image distance.

$$|h_i|=\frac{(1.4\times10^6\,\text{km})(135\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{1.3\text{ mm}}$$

(*d*) The equations show that image height is directly proportional to focal length. Therefore the relative magnifications will be the ratio of focal lengths.

$$\frac{28\,\text{mm}}{50\,\text{mm}}=\boxed{0.56\times}\text{ for the 28 mm lens ; }\frac{135\,\text{mm}}{50\,\text{mm}}=\boxed{2.7\times}\text{ for the 135 mm lens.}$$

92. We solve this problem by working through the lenses "backwards." We use the image distances and focal lengths to calculate the object distances. Since the final image from the right lens is halfway between the lenses, we set the image distance of the second lens equal to the negative of half the distance between the lenses. Using Eq. 33-2, we solve for the object distance of this lens. By subtracting this object distance from the distance between the two lenses, we find the image distance from the first lens. Then using Eq. 33-2 again, we solve for the initial object distance.

$$d_{i2} = -\tfrac{1}{2}\ell = -\tfrac{1}{2}(30.0\,\text{cm}) = -15.0\,\text{cm}$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \rightarrow d_{o2} = \frac{d_{i2}f_2}{d_{i2}-f_2} = \frac{(-15.0\,\text{cm})(20.0\,\text{cm})}{-15.0\,\text{cm}-20.0\,\text{cm}} = 8.57\,\text{cm}$$

$$d_{i1} = \ell - d_{o2} = 30.0\text{ cm} - 8.57\text{ cm} = 21.4\text{ cm}$$

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \rightarrow d_{o1} = \frac{d_{i1}f_1}{d_{i1}-f_1} = \frac{(21.4\,\text{cm})(15.0\,\text{cm})}{21.4\,\text{cm}-15.0\,\text{cm}} = \boxed{50.0\,\text{cm}}$$

93. We set d_i as the original image distance and $d_i + 10.0\,\text{cm}$ as the new image distance. Then using Eq. 33-2 for both cases, we eliminate the focal length and solve for the image distance. We insert the real image distance into the initial lens equation and solve for the focal length.

$$\frac{1}{d_{o1}} + \frac{1}{d_i} = \frac{1}{f} = \frac{1}{d_{o2}} + \frac{1}{d_i + 10.0\,\text{cm}} \rightarrow \frac{1}{d_{o1}} - \frac{1}{d_{o2}} = \frac{1}{d_i + 10.0\,\text{cm}} - \frac{1}{d_i} = \frac{-10.0\,\text{cm}}{d_i(d_i + 10.0\,\text{cm})}$$

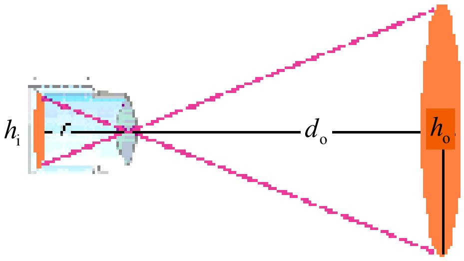
$$\frac{1}{60.0\,\text{cm}} - \frac{1}{40.0\,\text{cm}} = \frac{-10.0\,\text{cm}}{d_i(d_i + 10.0\,\text{cm})} \rightarrow d_i^2 + (10.0\,\text{cm})d_i - 1200\,\text{cm}^2 = 0$$

$$d_i = -40.0\,\text{cm} \text{ or } 30.0\,\text{cm}$$

Only the positive image distance will produce the real image.

$$\frac{1}{f} = \frac{1}{d_{o1}} + \frac{1}{d_i} \Rightarrow f = \frac{d_i d_{o1}}{d_i + d_{o1}} = \frac{(30.0\,\text{cm})(60.0\,\text{cm})}{30.0\,\text{cm} + 60.0\,\text{cm}} = \boxed{20.0\,\text{cm}}$$

94. Since the distance to the sun is much larger than the telescope's focal length, the image distance is about equal to the focal length. Rays from the top and bottom edges of the sun pass through the lens unrefracted. These rays with the object and image heights form similar triangles. We calculate the focal length of the telescope by setting the ratio of height to base for each triangle equal.



$$\frac{f}{h_i} = \frac{d_o}{h_o} \rightarrow$$

$$f = h_i\frac{d_o}{h_o} = (15\text{ mm})\frac{1.5\times10^8\text{ km}}{1.4\times10^6\text{ km}} = 1607\text{ mm} \approx \boxed{1.6\text{ m}}$$

95. We use Eq. 33-3 to write the image distance in terms of the object distance, image height, and object height. Then using Eq. 33-2 we solve for the object distance, which is the distance between the photographer and the subject.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \rightarrow \frac{1}{d_i} = -\frac{h_o}{h_i}\frac{1}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \left(-\frac{h_o}{h_i}\frac{1}{d_o}\right) = \left(1 - \frac{h_o}{h_i}\right)\frac{1}{d_o} \rightarrow$$

$$d_o=\left(1-\frac{h_o}{h_i}\right)f=\left(1-\frac{1750\,\text{mm}}{-8.25\,\text{mm}}\right)(220\,\text{mm})=46{,}900\,\text{mm}\approx\boxed{47\,\text{m}}$$

96. The exposure is proportional to the intensity of light, the area of the shutter, and the time. The area of the shutter is proportional to the square of the diameter or inversely proportional to the square of the *f*-stop. Setting the two proportionalities equal, with constant time, we solve for the change in intensity.

$$\frac{I_1t}{(f\text{-stop}_1)^2}=\frac{I_2t}{(f\text{-stop}_2)^2}\quad\rightarrow\quad\frac{I_2}{I_1}=\left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2=\left(\frac{16}{5.6}\right)^2=\boxed{8.2}$$

$\boxed{97.}$ The maximum magnification is achieved with the image at the near point, using Eq. 33-6b.

$$M_1=1+\frac{N_1}{f}=1+\frac{(15.0\,\text{cm})}{(8.5\,\text{cm})}=\boxed{2.8\times}$$

For an adult we set the near point equal to 25.0 cm.

$$M_2=1+\frac{N_2}{f}=1+\frac{(25.0\,\text{cm})}{(8.5\,\text{cm})}=\boxed{3.9\times}$$

The $\boxed{\text{person with the normal eye}}$ (adult) sees more detail.

98. The actual far point of the person is 155 cm. With the lens, an object far away is to produce a virtual image 155 cm from the eye, or 153 cm from the lens. We calculate the power of the upper part of the bifocals using Eq. 33-2 with the power equal to the inverse of the focal length in meter.

$$P_1=\frac{1}{f_1}=\left(\frac{1}{d_{o1}}\right)+\left(\frac{1}{d_{i1}}\right)=\left(\frac{1}{\infty}\right)+\left(\frac{1}{-1.53\text{ m}}\right)=\boxed{-0.65\text{ D (upper part)}}$$

The actual near point of the person is 45 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens. We again calculate the power using Eq. 33-2.

$$P_2=\frac{1}{f_2}=\left(\frac{1}{d_{o2}}\right)+\left(\frac{1}{d_{i2}}\right)=\left(\frac{1}{0.23\text{ m}}\right)+\left(\frac{1}{-0.43\text{ m}}\right)=\boxed{+2.0\text{ D (lower part)}}$$

99. The magnification for a relaxed eye is given by Eq. 33-6a.

$$M=N/f=NP=(0.25\,\text{m})(+4.0\,\text{D})=\boxed{1.0\times.}$$

100. (*a*) The magnification of the telescope is given by Eq. 33-7. The focal lengths are expressed in terms of their powers.

$$M=-\frac{f_o}{f_e}=-\frac{P_e}{P_o}=-\frac{(4.5\,\text{D})}{(2.0\,\text{D})}=-2.25\times\approx\boxed{-2.3\times}$$

(*b*) To get a magnification greater than 1, for the eyepiece we use the lens with the smaller focal length, or greater power: $\boxed{4.5\text{ D.}}$

101. We calculate the man's near point (d_i) using Eq. 33-2, with the initial object at 0.32 m with a 2.5 D lens. To give him a normal near point, we set the final object distance as 0.25 m and calculate the power necessary to have the image at his actual near point.

$$P_1 = \frac{1}{d_\mathrm{i}} + \frac{1}{d_\mathrm{o1}} \rightarrow \frac{1}{d_\mathrm{i}} = P_1 - \frac{1}{d_\mathrm{o1}} \rightarrow d_\mathrm{i} = \frac{d_\mathrm{o1}}{P_1 d_\mathrm{o1} - 1} = \frac{0.32\,\mathrm{m}}{(2.5\,\mathrm{D})(0.32\,\mathrm{m}) - 1} = -1.6\,\mathrm{m}$$

$$P_2 = \frac{1}{d_\mathrm{i}} + \frac{1}{d_\mathrm{o2}} = \left(P_1 - \frac{1}{d_\mathrm{o1}}\right) + \frac{1}{d_\mathrm{o2}} = \left(+2.5\ \mathrm{D} - \frac{1}{0.32\ \mathrm{m}}\right) + \frac{1}{0.25\ \mathrm{m}} = \boxed{+3.4\ \mathrm{D}}$$

102. (*a*) We solve Eq. 33-2 for the image distance. Then taking the time derivative of the image distance gives the image velocity. If the velocity of the object is taken to be positive, then the image distance is decreasing, and so $v_\mathrm{o} = -\frac{d}{dt}(d_\mathrm{o})$.

$$\frac{1}{f} = \frac{1}{d_\mathrm{i}} + \frac{1}{d_\mathrm{o}} \rightarrow d_\mathrm{i} = \frac{fd_\mathrm{o}}{d_\mathrm{o} - f}$$

$$v_\mathrm{i} = \frac{d}{dt}(d_\mathrm{i}) = \frac{d}{dt}\left(\frac{fd_\mathrm{o}}{d_\mathrm{o} - f}\right) = \frac{f}{d_\mathrm{o} - f}(-v_\mathrm{o}) - \frac{fd_\mathrm{o}}{(d_\mathrm{o} - f)^2}(-v_\mathrm{o}) = \frac{f(d_\mathrm{o} - f) - fd_\mathrm{o}}{(d_\mathrm{o} - f)^2}(-v_\mathrm{o})$$

$$= \boxed{\frac{f^2 v_\mathrm{o}}{(d_\mathrm{o} - f)^2}}$$

(*b*) The velocity of the image is positive, which means the image is moving the same direction as the object. But since the image is on the opposite side of the lens as the object, the image must be moving $\boxed{\text{away}}$ from the lens.

(*c*) We set the image and object velocities equal and solve for the image distance.

$$v_\mathrm{i} = v_\mathrm{o} \rightarrow \frac{f^2 v_\mathrm{o}}{(d_\mathrm{o} - f)^2} = v_\mathrm{o} \rightarrow (d_\mathrm{o} - f)^2 = f^2 \rightarrow d_\mathrm{o} - f = f \rightarrow \boxed{d_\mathrm{o} = 2f}$$

$\boxed{103.}$ The focal length of the eyepiece is found using Eq. 33-1.

$$f_\mathrm{e} = \frac{1}{P_\mathrm{e}} = \frac{1}{23\ \mathrm{D}} = 4.3 \times 10^{-2}\ \mathrm{m} = 4.3\,\mathrm{cm}.$$

For both object and image far away, we find the focal length of the objective from the separation of the lenses.

$$\ell = f_\mathrm{o} + f_\mathrm{e} \rightarrow f_\mathrm{o} = \ell - f_\mathrm{e} = 85\,\mathrm{cm} - 4.3\,\mathrm{cm} = 80.7\,\mathrm{cm}$$

The magnification of the telescope is given by Eq. 33-7.

$$M = -\frac{f_\mathrm{o}}{f_\mathrm{e}} = -\frac{(80.7\,\mathrm{cm})}{(4.3\,\mathrm{cm})} = \boxed{-19\times}$$

104. (*a*) The length of the telescope is the sum of the focal lengths. The magnification is the ratio of the focal lengths (Eq. 33-7). For a magnification greater than one, the lens with the smaller focal length should be the eyepiece. Therefore the $\boxed{\text{4.0 cm lens should be the eyepiece}}$.

$$\ell = f_\mathrm{o} + f_\mathrm{e} = 4.0\,\mathrm{cm} + 44\,\mathrm{cm} = \boxed{48\,\mathrm{cm}}$$

$$M = -\frac{f_\mathrm{o}}{f_\mathrm{e}} = -\frac{(44\,\mathrm{cm})}{(4.0\,\mathrm{cm})} = \boxed{-11\times}$$

(*b*) We use Eq. 33-10b to solve for the length, ℓ, of the microscope.

$$M = -\frac{N\ell}{f_\mathrm{e} f_\mathrm{o}} \Rightarrow \ell = \frac{-Mf_e f_o}{N} = \frac{-(25)(4.0\ \mathrm{cm})(44\ \mathrm{cm})}{25\ \mathrm{cm}} = 180\ \mathrm{cm} = \boxed{1.8\ \mathrm{m}}$$

This is far too long to be practical.

105. (*a*) The focal length of the lens is the inverse of the power.

$$f=\frac{1}{P}=\frac{1}{3.50\,\mathrm{D}}=0.286\,\mathrm{m}=\boxed{28.6\,\mathrm{cm.}}$$

(*b*) The lens produces a virtual image at his near point. We set the object distance at 23 cm from the glass (25 cm from the eyes) and solve for the image distance. We add the two centimeters between the glass and eyes to determine the near point.

$$P=\frac{1}{f}=\frac{1}{d_\mathrm{o}}+\frac{1}{d_\mathrm{i}}\ \rightarrow\ d_\mathrm{i}=\left(P-\frac{1}{d_\mathrm{o}}\right)^{-1}=\left(3.50\,\mathrm{D}-\frac{1}{0.23\,\mathrm{m}}\right)^{-1}=-1.18\,\mathrm{m}$$

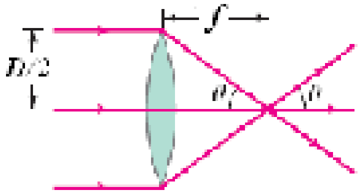
$$N=|d_i|+0.02\,\mathrm{m}=1.18\,\mathrm{m}+0.02\,\mathrm{m}=\boxed{1.20\,\mathrm{m}}$$

(*c*) For Pam, find the object distance that has an image at her near point, –0.23 m from the lens.

$$P=\frac{1}{f}=\frac{1}{d_\mathrm{o}}+\frac{1}{d_\mathrm{i}}\ \rightarrow\ d_\mathrm{o}=\left(P-\frac{1}{d_\mathrm{i}}\right)^{-1}=\left(3.50\ \mathrm{D}-\frac{1}{-0.23\ \mathrm{m}}\right)^{-1}=0.13\,\mathrm{m}$$

Pam's near point with the glasses is 13 cm from the glasses or $\boxed{15\ \mathrm{cm}}$ from her eyes.

106. As shown in the image, the parallel rays will pass through a single point located at the focal distance from the lens. The ray passing through the edge of the lens (a distance D/2 from the principal axis) makes an angle θ with the principal axis. We set the tangent of this angle equal to the ratio of the opposite side ($D/2$) to the adjacent side (f) and solve for the focal length.



$$\tan\theta=\frac{D/2}{f}\rightarrow f=\frac{D}{2\tan\theta}=\frac{5.0\ \mathrm{cm}}{2\tan 3.5^\circ}=\boxed{41\ \mathrm{cm}}$$

107. We use Eq. 33-6b to calculate the necessary focal length for a magnifying glass held at the near point ($N=25\ \mathrm{cm}$) to have a magnification of $M=3.0$.

$$M=\frac{N}{f}+1\ \rightarrow\ f=\frac{N}{M-1}=\frac{25\ \mathrm{cm}}{3.0-1}=12.5\ \mathrm{cm}$$

In the text, the lensmaker's equation (Eq. 33-4) is derived assuming the lens is composed of material with index of refraction n and is surrounded by air, whose index of refraction is $n_a=1$. We now modify this derivation, with the lens composed of air with index of refraction $n_a=1$ surrounded by water, whose index of refraction is $n_\mathrm{w}=1.33$. In the proof of the lensmaker's equation, Snell's law at small angles is first applied at both surfaces of the lens.

$$n_\mathrm{w}\sin\theta_1=n\sin\theta_2\ \rightarrow\ n_\mathrm{w}\theta_1\approx\theta_2\ \rightarrow\ \theta_1\approx\frac{1}{n_\mathrm{w}}\theta_2$$

$$n\sin\theta_3=n_\mathrm{w}\sin\theta_4\ \rightarrow\ \theta_3\approx n_\mathrm{w}\theta_4\ \rightarrow\ \frac{1}{n_\mathrm{w}}\theta_3\approx\theta_4$$

These equations are the same as those following Fig. 33-16, but with n replaced by $1/n_w$. The rest of the derivation is the same, so we can rewrite the lensmaker's equation with this single modification. We assume the radii are equal, insert the necessary focal length, and solve for the radius of curvature

$$\frac{1}{f}=\left(\frac{1}{n_\mathrm{w}}-1\right)\left[\frac{1}{R_1}+\frac{1}{R_2}\right]=\left(\frac{1}{n_\mathrm{w}}-1\right)\left[\frac{1}{R}+\frac{1}{R}\right]=\left(\frac{1}{n_\mathrm{w}}-1\right)\left[\frac{2}{R}\right]$$

$$R=2f\left(\frac{1}{n_\mathrm{w}}-1\right)=2\left(12.5\ \mathrm{cm}\right)\left(\frac{1}{1.33}-1\right)=-6.20\,\mathrm{cm}\approx\boxed{-6.2\,\mathrm{cm}}$$

The lens is therefore a $\boxed{\text{concave lens}}$ with radii of curvature –6.2 cm.

108. (*a*) We use Eq. 32-2 to calculate the image distance and then use the object and image distances in Eq. 32-3 to calculate the magnification. We finally make the approximation that the object distance is much larger than the focal length.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f_1} \rightarrow d_i=\left(\frac{1}{f_1}-\frac{1}{d_o}\right)^{-1}=\frac{f_1d_o}{d_o-f_1}$$

$$m_1=-\frac{d_i}{d_o}=-\frac{1}{d_o}\left(\frac{f_1d_o}{d_o-f_1}\right)=-\frac{f_1}{d_o-f_1}\approx\boxed{-\frac{f_1}{d_o}}$$

This real image, located near the focal distance from lens 1, becomes the object for the second lens. We subtract the focal length from the separation distance to determine the object distance for lens 2. Using Eq. 32-2, we calculate the second image distance and Eq. 32-3 to calculate the second magnification. Multiplying the two magnifications gives the total magnification.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{f_2d_{o2}}{d_{o2}-f_2}$$

$$m_2=-\frac{d_{i2}}{d_{o2}}=-\frac{1}{d_{o2}}\left(\frac{f_2d_{o2}}{d_{o2}-f_2}\right)=-\frac{f_2}{d_{o2}-f_2}=-\frac{\left(-\frac{1}{2}f_1\right)}{\left(\frac{3}{4}f_1-f_1\right)-\left(-\frac{1}{2}f_1\right)}=2$$

$$m_1m_2=\left(-\frac{f_1}{d_o}\right)(2)=\boxed{-\frac{2f_1}{d_o}}$$

(*b*) If the object is at infinity, the image from the first lens will form a focal length behind that lens. Subtracting this distance from the separation distance gives the object distance for the second lens. We use Eq. 32-2 to calculate the image distance from the second lens. Adding this distance to the separation distance between the lenses gives the distance the image is from the first lens.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{f_2d_{o2}}{d_{o2}-f_2}=\frac{\left(-\frac{1}{2}f_1\right)\left(\frac{3}{4}f_1-f_1\right)}{\left(\frac{3}{4}f_1-f_1\right)-\left(-\frac{1}{2}f_1\right)}=\frac{1}{2}f_1$$

$$d=\ell+d_{i2}=\frac{3}{4}f_1+\frac{1}{2}f_1=\boxed{\frac{5}{4}f_1}$$

(*c*) We set the magnification equal to the total magnification found in part (*a*) and solve for the focal length.

$$m=-\frac{250\,\text{mm}}{d_o}=-\frac{2f_1}{d_o}\Rightarrow f_1=\frac{250\,\text{mm}}{2}=\boxed{125\,\text{mm}}$$

We use the results of part (*b*) to determine the distance of the lens to the film. We subtract this distance from 250 mm to determine how much closer the lens can be to the film in the two lens system.

$$d=\frac{5}{4}f_1=\frac{5}{4}(125\,\text{mm})=\boxed{156\,\text{mm}}\ ;\ \Delta d=250\,\text{mm}-156\,\text{mm}=\boxed{94\,\text{mm}}$$

109. (*a*) We use Eqs. 33-2 and 33-3.

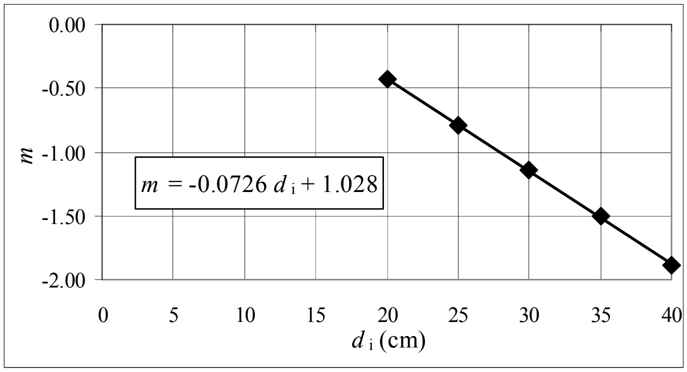
$$m=-\frac{d_i}{d_o} \rightarrow d_o=-\frac{d_i}{m}\ ;\ \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=-\frac{m}{d_i}+\frac{1}{d_i} \rightarrow m=-\frac{d_i}{f}+1$$

This is a straight line with $\boxed{\text{slope}=-\frac{1}{f}\ \text{and}\ y\text{-intercept}=1}$.

(*b*) A plot of m vs. d_i is shown here.

$$f=-\frac{1}{\text{slope}}=-\frac{1}{-.0726\,\text{cm}^{-1}}$$

$$=13.8\,\text{cm}\approx\boxed{14\,\text{cm}}$$



The y-intercept is 1.028. $\boxed{\text{Yes}}$, it is close to the expected value of 1. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH33.XLS," on tab "Problem 33.109b."

(*c*) Use the relationship derived above.

$$m=-\frac{d_\text{i}}{f}+1=-\frac{d'+\ell_\text{i}}{f}+1=-\frac{d'_\text{i}}{f}+\left(1-\frac{\ell_\text{i}}{f}\right)$$

A plot of m vs. d'_i would still have a slope of $-\frac{1}{f}$, so $f=\boxed{-\frac{1}{\text{slope}}}$ as before. The y-intercept will have changed, to $1-\frac{\ell_\text{i}}{f}$.

CHAPTER 34: The Wave Nature of Light; Interference

Responses to Questions

1. Yes, Huygens’ principle applies to all waves, including sound and water waves.

2. Light from the Sun can be focused by a converging lens on a piece of paper and burn a hole in the paper. This provides evidence that light is energy. Also, you can feel the heat from the Sun beating down on you on a hot summer day. When you move into the shade you may still feel hot, but you don’t feel the Sun’s energy directly.

3. A ray shows the direction of propagation of a wave front. If this information is enough for the situation under discussion, then light can be discussed as rays. Sometimes, however, the wave nature of light is essential to the discussion. For instance, the double slit interference pattern depends on the interference of the waves, and could not be explained by examining light as only rays.

4. The bending of waves around corners or obstacles is called diffraction. Diffraction is most prominent when the size of the obstacle is on the order of the size of the wavelength. Sound waves have much longer wavelengths than do light waves. As a result, the diffraction of sound waves around a corner is noticeable and we can hear the sound in the “shadow region,” but the diffraction of light waves around a corner is not noticeable.

5. The wavelength of light cannot be determined from reflection measurements alone, because the law of reflection is the same for all wavelengths. However, thin film interference, which involves interference of the rays reflecting from the front and back surfaces of the film, can be used to determine wavelength. Refraction can also be used to determine wavelength because the index of refraction for a given medium is different for different wavelengths.

6. For destructive interference, the path lengths must differ by an odd number of half wavelengths, such as $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc. In general, the path lengths must differ by $\lambda(m + \frac{1}{2})$, where m is an integer.

7. Blue light has a shorter wavelength than red light. The angles to each of the bright fringes for the blue light would be smaller than for the corresponding orders for the red light, so the bright fringes would be closer together for the blue light.

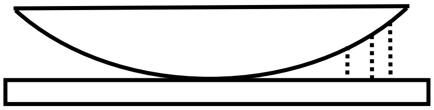
8. The fringes would be closer together because the wavelength of the light underwater is less than the wavelength in air.

9. The two experiments are the same in principle. Each requires coherent sources and works best with a single frequency source. Each produces a pattern of alternating high and low intensity. Sound waves have much longer wavelengths than light waves, so the appropriate source separation for the sound experiment would be larger. Also, sound waves are mechanical waves which require a medium through which to travel, so the sound experiment could not be done in a vacuum and the light experiment could.

10. The red light and the blue light coming from the two different slits will have different wavelengths (and different frequencies) and will not have a constant phase relationship. In order for a double-slit pattern to be produced, the light coming from the slits must be coherent. No distinct double-slit interference pattern will appear. However, each slit will individually produce a “single-slit diffraction” pattern, as will be discussed in Chapter 35.

11. Light from the two headlights would not be coherent, so would not maintain a consistent phase relationship and therefore no stable interference pattern would be produced.

12. As the thickness of the film increases, the number of different wavelengths in the visible range that meet the constructive interference criteria increases. For a thick piece of glass, many different wavelengths will undergo constructive interference and these will all combine to produce white light.

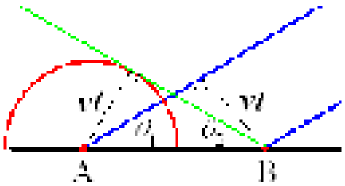


$\boxed{13.}$ Bright colored rings will occur when the path difference between the two interfering rays is λ/2, 3λ/2, 5λ/2, and so forth. A given ring, therefore, has a path difference that is exactly one wavelength longer than the path difference of its neighboring ring to the inside and one wavelength shorter than the path difference of its neighboring ring to the outside. Newton's rings are created by the thin film of air between a glass lens and the flat glass surface on which it is placed. Because the glass of the lens is curved, the thickness of this air film does not increase linearly. The farther a point is from the center, the less the horizontal distance that corresponds to an increase in vertical thickness of one wavelength. The horizontal distance between two neighboring rings therefore decreases with increasing distance from the center.

14. These lenses probably are designed to eliminate wavelengths at both the red and the blue ends of the spectrum. The thickness of the coating is designed to cause destructive interference for reflected red and blue light. The reflected light then appears yellow-green.

15. The index of refraction of the oil must be less than the index of refraction of the water. If the oil film appears bright at the edge, then the interference between the light reflected from the top of the oil film and from the bottom of the oil film at that point must be constructive. The light reflecting from the top surface (the air/oil interface) undergoes a 180º phase shift since the index of refraction of the oil is greater than that of air. The thickness of the oil film at the edge is negligible, so for there to be constructive interference, the light reflecting from the bottom of the oil film (the oil/water interface) must also undergo a 180º phase shift. This will occur only if the index of refraction of the oil is less than that of the water.

Solutions to Problems



$\boxed{1.}$ Consider a wave front traveling at an angle θ_1 relative to a surface. At time $t = 0$, the wave front touches the surface at point A, as shown in the figure. After a time *t*, the wave front, moving at speed *v*, has moved forward such that the contact position has moved to point B. The distance between the two contact points is calculated using simple geometry: $\text{AB} = \dfrac{vt}{\sin\theta_1}$.

By Huygens' principle, at each point the wave front touches the surface, it creates a new wavelet. These wavelets expand out in all directions at speed *v*. The line passing through the surface of each of these wavelets is the reflected wave front. Using the radius of the wavelet created at $t = 0$, the center of the wavelet created at time *t*, and the distance between the two contact points (AB) we create a right triangle. Dividing the radius of the wavelet centered at AB (*vt*) by distance between the contact points gives the sine of the angle between the contact surface and the reflected wave, θ_2.

$$\sin\theta_2 = \frac{vt}{\text{AB}} = \frac{vt}{\dfrac{vt}{\sin\theta_1}} = \sin\theta_1 \rightarrow \boxed{\theta_2 = \theta_1}$$

Since these two angles are equal, their complementary angles (the incident and reflected angles) are also equal.

2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the fifth order.

$$d\sin\theta = m\lambda \rightarrow \lambda = \frac{d\sin\theta}{m} = \frac{\left(1.8\times10^{-5}\,\text{m}\right)\sin 9.8^\circ}{5} = \boxed{6.1\times10^{-7}\,\text{m}}$$

3. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the third order.

$$d\sin\theta = m\lambda \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{3\left(610\times10^{-9}\,\text{m}\right)}{\sin 28^\circ} = \boxed{3.9\times10^{-6}\,\text{m}}$$

4. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d}$$

$$x_1 = \frac{\lambda m_1\ell}{d} \;;\; x_2 = \frac{\lambda(m+1)\ell}{d} \rightarrow \Delta x = x_2 - x_1 = \frac{\lambda(m+1)\ell}{d} - \frac{\lambda m\ell}{d} = \frac{\lambda\ell}{d}$$

$$\lambda = \frac{d\Delta x}{\ell} = \frac{\left(4.8\times10^{-5}\,\text{m}\right)\left(0.085\,\text{m}\right)}{6.00\,\text{m}} = \boxed{6.8\times10^{-7}\,\text{m}} \;;\; f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,\text{m/s}}{6.8\times10^{-7}\,\text{m}} = \boxed{4.4\times10^{14}\,\text{Hz}}$$

5. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1 m\ell}{d} \;;\; x_2 = \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{(\lambda_2 - \lambda_1)m\ell}{d} = \frac{\left[(720-660)\times10^{-9}\,\text{m}\right](2)(1.0\,\text{m})}{\left(6.8\times10^{-4}\,\text{m}\right)} = 1.76\times10^{-4}\,\text{m} \approx \boxed{0.2\,\text{mm}}$$

This justifies using the small angle approximation, since $x \ll \ell$.

6. The slit spacing and the distance from the slits to the screen is the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe (m = 1) relative to the central fringe. We indicate the lab laser with subscript 1, and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1\ell}{d} \;;\; x_2 = \frac{\lambda_2\ell}{d} \rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = \left(632.8\,\text{nm}\right)\frac{5.14\,\text{mm}}{5.00\,\text{mm}} = 650.52\,\text{nm} \approx \boxed{651\,\text{nm}}$$

$\boxed{7.}$ Using a ruler on Fig. 35-9a, the distance from the $m = 0$ fringe to the $m = 10$ fringe is found to be about 13.5 mm. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; \lambda = \frac{dx}{m\ell} = \frac{dx}{m\ell} = \frac{\left(1.7\times10^{-4}\,\text{m}\right)\left(0.0135\,\text{m}\right)}{\left(10\right)\left(0.35\,\text{m}\right)} = \boxed{6.6\times10^{-7}\,\text{m}}$$

8. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; d = \frac{\lambda m\ell}{x} = \frac{\left(680\times10^{-9}\,\text{m}\right)\left(3\right)\left(2.6\,\text{m}\right)}{38\times10^{-3}\,\text{m}} = \boxed{1.4\times10^{-4}\,\text{m}}$$

9. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;\rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = \left(1\right)\frac{\left(633\times10^{-9}\,\text{m}\right)\left(3.8\,\text{m}\right)}{\left(6.8\times10^{-5}\,\text{m}\right)} = 0.035\,\text{m} = \boxed{3.5\,\text{cm}}$$

10. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; d = \frac{\lambda m\ell}{x} = \frac{\left(633\times10^{-9}\,\text{m}\right)\left(1\right)\left(5.0\,\text{m}\right)}{\left(0.25\,\text{m}\right)} = \boxed{1.3\times10^{-5}\,\text{m}}$$

11. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{max}} = m\lambda,\; m = 0, 1, 2, \cdots \;\rightarrow\; d\sin\theta_{\text{max}} = \left(m - \tfrac{1}{2}\right)\lambda,\; m = 1, 2, \cdots$$

We could express the result as $d\sin\theta_{\text{max}} = \left(m + \tfrac{1}{2}\right)\lambda,\; m = 0, 1, 2, \cdots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{min}} = \left(m + \tfrac{1}{2}\right)\lambda,\; m = 0, 1, 2, \cdots \;\rightarrow\; d\sin\theta_{\text{min}} = m\lambda,\; m = 0, 1, 2, \cdots$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern}}$. There will be a dark central line. Every place there was a bright fringe will now have a dark line, and vice versa.

12. We equate the expression from Eq. 34-2a for the second order blue light to Eq. 34-2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$d\sin\theta = m\lambda_b = \left(2\right)\left(480\,\text{nm}\right) = 960\,\text{nm} \;\; ; \;\; d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda,\; m' = 0, 1, 2, \cdots$$

$$\left(m' + \tfrac{1}{2}\right)\lambda = 960\,\text{nm} \quad m' = 0 \;\rightarrow\; \lambda = 1920\,\text{nm} \;\; ; \;\; m' = 1 \;\rightarrow\; \lambda = 640\,\text{nm}$$

$$m' = 2 \;\rightarrow\; \lambda = 384\,\text{nm}$$

The only one visible is $\boxed{640\text{ nm}}$. 384 nm is near the low-wavelength limit for visible light.

13. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;\rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{\left(544\times10^{-9}\,\text{m}\right)(5.0\,\text{m})}{\left(1.0\times10^{-3}\,\text{m}\right)} = \boxed{2.7\times10^{-3}\,\text{m}}$$

14. An expression is derived for the slit separation from the data for the 500 nm light. That expression is then used to find the location of the maxima for the 650 nm light. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; d = \frac{\lambda m\ell}{x} = \frac{\lambda_1 m_1 \ell}{x_1} \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;\rightarrow$$

$$x_2 = \frac{\lambda_2 m_2 \ell}{\frac{\lambda_1 m_1 \ell}{x_1}} = x_1\frac{\lambda_2 m_2}{\lambda_1 m_1} = (12\,\text{mm})\frac{(650\,\text{nm})(2)}{(500\,\text{nm})(3)} = 10.4\,\text{mm} \approx \boxed{10\,\text{mm}}\;\;(2\text{ sig. fig.})$$

15. The presence of the water changes the wavelength according to Eq. 34-1, and so we must change λ to $\lambda_n = \lambda/n$. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda_n \;\rightarrow\; d\frac{x}{\ell} = m\lambda_n \;\rightarrow\; x = \frac{\lambda_n m\ell}{d}\;;\; x_1 = \frac{\lambda m_1\ell}{d}\;;\; x_2 = \frac{\lambda(m+1)\ell}{d} \;\rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n\ell}{d} = \frac{\lambda\ell}{nd} = \frac{\left(470\times10^{-9}\,\text{m}\right)(0.500\,\text{m})}{(1.33)\left(6.00\times10^{-5}\,\text{m}\right)} = \boxed{2.94\times10^{-3}\,\text{m}}$$

16. To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be equivalent to half a wavelength. The wavelength of the light is shorter in the plastic than in the air, so the number of wavelengths in the plastic must be ½ greater than the number in the same thickness of air. The number of wavelengths in the distance equal to the thickness of the plate is the thickness of the plate divided by the appropriate wavelength.

$$N_{\text{plastic}} - N_{\text{air}} = \frac{t}{\lambda_{\text{plastic}}} - \frac{t}{\lambda} = \frac{tn_{\text{plastic}}}{\lambda} - \frac{t}{\lambda} = \frac{t}{\lambda}\left(n_{\text{plastic}} - 1\right) = \tfrac{1}{2} \;\rightarrow$$

$$t = \frac{\lambda}{2\left(n_{\text{plastic}} - 1\right)} = \frac{680\,\text{nm}}{2(1.60 - 1)} = \boxed{570\,\text{nm}}$$

17. The intensity is proportional to the square of the amplitude. Let the amplitude at the center due to one slit be E_0. The amplitude at the center with both slits uncovered is $2E_0$.

$$\frac{I_{1\text{ slit}}}{I_{2\text{ slits}}} = \left(\frac{E_0}{2E_0}\right)^2 = \boxed{\frac{1}{4}}$$

Thus the amplitude due to a single slit is one-fourth the amplitude when both slits are open.

18. The intensity as a function of angle from the central maximum is given by Eq. 34-6.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}} \;\rightarrow\; \frac{\pi d \sin\theta}{\lambda} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 45^\circ \pm n\left(90^\circ\right) = \frac{\pi}{4} \pm n\frac{\pi}{2} \;\rightarrow$$

$$2d\sin\theta = \left(\tfrac{1}{2} \pm n\right)\lambda$$

To only consider $\theta \ge 0,$ we take just the plus sign.

$$\boxed{2d\sin\theta = \left(n + \tfrac{1}{2}\right)\lambda,\; n = 0, 1, 2, \cdots}$$

$\boxed{19.}$ The intensity of the pattern is given by Eq. 34-6. We find the angle where the intensity is half its maximum value.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \frac{1}{\sqrt{2}} \;\rightarrow$$

$$\frac{\pi d \sin\theta_{1/2}}{\lambda} = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4} \;\rightarrow\; \sin\theta_{1/2} = \frac{\lambda}{4d}$$

If $\lambda \ll d$, then $\sin\theta = \dfrac{\lambda}{4d} \ll 1$ and so $\sin\theta \approx \theta.$ This is the angle from the central maximum to the location of half intensity. The angular displacement from the half-intensity position on one side of the central maximum to the half-intensity position on the other side would be twice this.

$$\Delta\theta = 2\theta_{1/2} = 2\frac{\lambda}{4d} = \boxed{\frac{\lambda}{2d}}$$

20. (*a*) The phase difference is given in Eq. 34-4. We are given the path length difference, $d\sin\theta.$

$$\frac{\delta}{2\pi} = \frac{d\sin\theta}{\lambda} \;\rightarrow\; \delta = 2\pi\frac{1.25\lambda}{\lambda} = \boxed{2.50\pi}$$

(*b*) The intensity is given by Eq. 34-6.

$$I = I_0\cos^2\left(\frac{\delta}{2}\right) = I_0\cos^2\left(1.25\pi\right) = \boxed{0.500 I_0}$$

21. A doubling of the intensity means that the electric field amplitude has increased by a factor of $\sqrt{2}.$ We set the amplitude of the electric field of one slit equal to E_0 and of the other equal to $\sqrt{2}E_0$. We use Eq. 34-3 to write each of the electric fields, where the phase difference, δ, is given by Eq. 34-4. Summing these two electric fields gives the total electric field.

$$E_\theta = E_0\sin\omega t + \sqrt{2}E_0\sin\left(\omega t + \delta\right) = E_0\sin\omega t + \sqrt{2}E_0\sin\omega t\cos\delta + \sqrt{2}E_0\cos\omega t\sin\delta$$

$$= E_0\left(1 + \sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta$$

We square the total electric field intensity and integrate over the period to determine the average intensity.

$$\overline{E}_\theta^2 = \frac{1}{T}\int_0^T E_\theta^2 dt = \frac{1}{T}\int_0^T \left[E_0\left(1 + \sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta\right]^2 dt$$

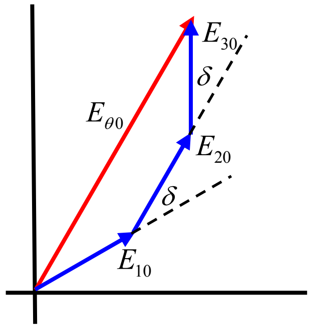
$$= \frac{E_0^2}{T}\int_0^T \left[\left(1 + \sqrt{2}\cos\delta\right)^2\sin^2\omega t + 2\cos^2\omega t\sin^2\delta + 2\sqrt{2}\left(1 + \sqrt{2}\cos\delta\right)\sin\delta\sin\omega t\cos\omega t\right]dt$$

$$= \frac{E_0^2}{2}\left[\left(1+\sqrt{2}\cos\delta\right)^2 + 2\sin^2\delta\right] = \frac{E_0^2}{2}\left[3+2\sqrt{2}\cos\delta\right]$$

Since the intensity is proportional to this average square of the electric field, and the intensity is maximum when $\delta = 0$, we obtain the relative intensity by dividing the square of the electric field by the maximum square of the electric field.

$$\frac{I_\theta}{I_0} = \frac{\overline{E}_\theta^2}{E_{\delta=0}^2} = \boxed{\frac{3+2\sqrt{2}\cos\delta}{3+2\sqrt{2}}, \text{ with } \delta = \frac{2\pi}{\lambda} d\sin\theta}$$

22. (*a*) If the sources have equal intensities, their electric fields will have the same magnitudes. We show a phasor diagram with each of the electric fields shifted by an angle δ. As shown in the sketch, the three electric fields and their sum form a symmetric trapezoid. Since E_{20} and $E_{\theta 0}$ are parallel, and E_{20} is rotated from E_{10} and E_{30} by the angle δ, the magnitude of $E_{\theta 0}$ is the sum of the components of E_{10}, E_{20}, and E_{30} that are parallel to E_{20}.



$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta = E_{10}\left(1+2\cos\delta\right)$$

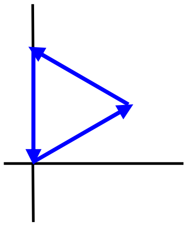
We set the intensity proportional to the square of the electric field magnitude and divide by the maximum intensity (at $\delta = 0$) to determine the relative intensity.

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{\delta=0}^2} = \frac{\left[E_{10}\left(1+2\cos\delta\right)\right]^2}{\left[E_{10}\left(1+2\cos 0\right)\right]^2} = \boxed{\frac{\left(1+2\cos\delta\right)^2}{9}, \ \delta = \frac{2\pi}{\lambda} d\sin\theta}$$

(*b*) The intensity will be at its maximum when $\cos\delta = 1$. In this case the three phasors are all in line.

$$\cos\delta_{\max} = 1 \rightarrow \delta_{\max} = 2m\pi = \frac{2\pi}{\lambda} d\sin\theta_{\max} \rightarrow \boxed{\sin\theta_{\max} = \frac{m\lambda}{d}, \quad m = 0, 1, 2, \cdots}$$

The intensity will be a minimum when $1+2\cos\delta = 0$. In this case the three phasors add to 0 and form an equilateral triangle as shown in the second diagram, for the case of $k = 1$, where k is defined below.



$$1+2\cos\delta_{\min} = 0 \rightarrow$$

$$\delta_{\min} = \cos^{-1}\left(-\tfrac{1}{2}\right) = \begin{cases} \tfrac{2}{3}\pi + 2m\pi = 2\pi\left(m+\tfrac{1}{3}\right) \\ \tfrac{4}{3}\pi + 2m\pi = 2\pi\left(m+\tfrac{2}{3}\right) \end{cases}, \ m = 0, 1, 2, \cdots$$

This can be written as one expression with two parameters.

$$\delta_{\min} = 2\pi\left(m+\tfrac{1}{3}k\right) = \frac{2\pi}{\lambda} d\sin\theta_{\min}, \ k = 1, 2; \ m = 0, 1, 2, \cdots \rightarrow$$

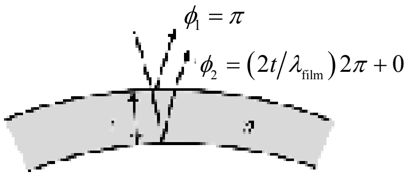
$$\boxed{\sin\theta_{\min} = \frac{\lambda}{d}\left(m+\tfrac{1}{3}k\right) = , \ k = 1, 2; \ m = 0, 1, 2, \cdots}$$

23. From Example 34-7, we see that the thickness is related to the bright color wavelength by $t = \lambda/4n$.

$$t = \lambda/4n \rightarrow \lambda = 4nt = 4(1.32)(120\,\text{nm}) = \boxed{634\,\text{nm}}$$

24. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals over the length of the plates.

$$\frac{28.5\,\text{cm}}{24.5\,\text{intervals}} = \boxed{1.16\,\text{cm}}$$



25. (*a*) An incident wave that reflects from the outer surface of the bubble has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the inner surface of the bubble has a phase change due to the additional path length, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi$. For destructive interference with a minimum non-zero thickness of bubble, the net phase change must be π.

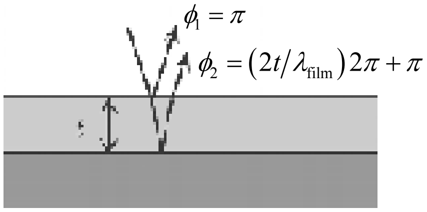
$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = \pi \quad\rightarrow\quad t = \tfrac{1}{2}\lambda_{\text{film}} = \frac{\lambda}{2n} = \frac{480\,\text{nm}}{2(1.33)} = \boxed{180\,\text{nm}}$$

(*b*) For the next two larger thicknesses, the net phase change would be 3π and 5π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 3\pi \quad\rightarrow\quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \frac{480\,\text{nm}}{(1.33)} = \boxed{361\,\text{nm}}$$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 5\pi \quad\rightarrow\quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \tfrac{3}{2}\frac{480\,\text{nm}}{(1.33)} = \boxed{541\,\text{nm}}$$

(*c*) If the thickness were much less than one wavelength, then there would be very little phase change introduced by additional path length, and so the two reflected waves would have a phase difference of about $\phi_1 = \pi$. This would produce destructive interference.



26. An incident wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference with a minimum non-zero thickness of coating, the net phase change must be 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = 2\pi \quad\rightarrow\quad t = \tfrac{1}{2}\lambda_{\text{film}} = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right).$$

The lens reflects the most for $\lambda = 570\,\text{nm}$. The minimum non-zero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\,\text{nm})}{2(1.25)} = \boxed{228\,\text{nm}}$$

Since the middle of the spectrum is being selectively reflected, the transmitted light will be stronger in the red and blue portions of the visible spectrum.

27. (*a*) When illuminated from above at A, a light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift. If the oil thickness at A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path distance traveled by a ray in the oil. Thus the light reflected from the two surfaces will destructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(*b*) From the discussion in part (*a*), the ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray that reflects from the oil-water interface has no phase change due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference, the net phase change must be a multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for 580 nm, and so we use *m* = 1. (The first location that yields constructive interference would be for *m* = 0.)

$$t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}} = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\frac{580\,\text{nm}}{1.50} = \boxed{290\,\text{nm}}$$

28. When illuminated from above, the light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference to occur, the net phase change must be a multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

For $\lambda = 650\,\text{nm}$, the possible thicknesses are as follows.

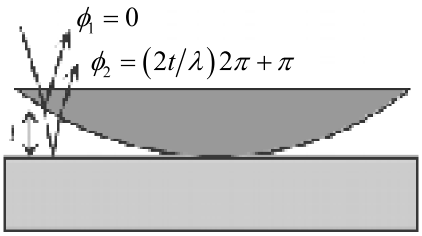
$$t_{650} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{650\,\text{nm}}{1.50} = 108\,\text{nm},\ 325\,\text{nm},\ 542\,\text{nm},\ \cdots$$

For $\lambda = 390\,\text{nm}$, the possible thicknesses are as follows.

$$t_{390} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{390\,\text{nm}}{1.50} = 65\,\text{nm},\ 195\,\text{nm},\ 325\,\text{nm},\ 455\,\text{nm},\ \cdots$$

The minimum thickness of the oil slick must be $\boxed{325\,\text{nm}}$.

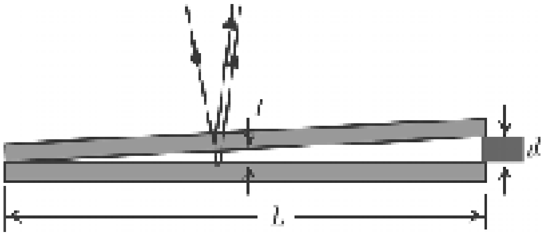
29. An incident wave that reflects from the convex surface of the lens has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the glass underneath the lens has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark rings), the net phase change must be an odd-integer multiple of π, so $\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the dark center, *m* represents the number of the ring.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi,\ m = 0, 1, 2, \cdots \rightarrow$$

$$t = \tfrac{1}{2}m\lambda_{\text{air}} = \tfrac{1}{2}(31)(560\,\text{nm}) = 8680\,\text{nm} = \boxed{8.68\,\mu\text{m}}$$

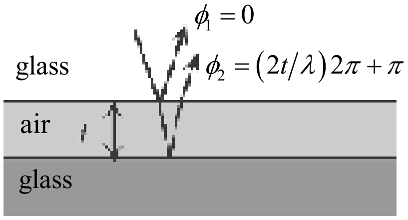
The thickness of the lens is the thickness of the air at the edge of the lens:



30. An incident wave that reflects from the second surface of the upper piece of glass has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the first surface of the second piece of glass has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark lines), the net phase change must be an odd-integer multiple of π, so $\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the left edge of the diagram, the 28th dark line corresponds to *m* = 27. The 28th dark line also has a gap thickness of *d*.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda \quad \rightarrow$$

$$d = \tfrac{1}{2}(27)(670\,\text{nm}) = 9045\,\text{nm} \approx \boxed{9.0\,\mu\text{m}}$$



$\boxed{31.}$ With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of $\phi_1 = 0$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = 2m\pi \quad \rightarrow \quad t = \tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \cdots.$$

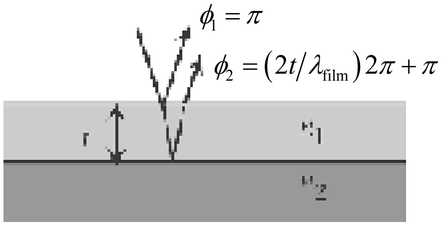
The minimum thickness is with $m = 1$.

$$t_{\min} = \tfrac{1}{2}(450\,\text{nm})\left(1 - \tfrac{1}{2}\right) = \boxed{113\,\text{nm}}$$

For destructive interference, the net phase change must be an odd-integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda,\ = 0, 1, 2, \cdots$$

The minimum non-zero thickness is $t_{\min} = \tfrac{1}{2}(450\,\text{nm})(1) = \boxed{225\,\text{nm}}$.



32. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{1\text{film}}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{1\text{film}}m_1 = \tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1,\ m_1 = 1, 2, 3, \ldots .$$

For destructive interference, the net phase change must be an odd-integer multiple π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{2\,\text{film}}}\right)2\pi + \pi\right] - \pi = (2m_2+1)\pi \quad\rightarrow\quad t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1),\ m_2 = 0, 1, 2, \ldots .$$

Set the two expressions for the thickness equal to each other.

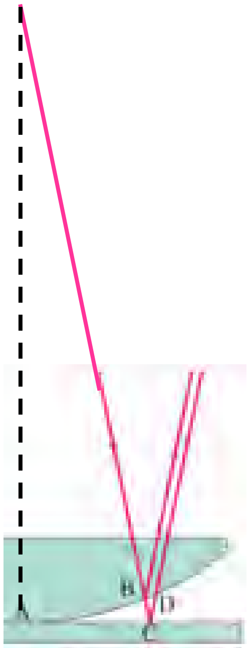
$$\tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) \quad\rightarrow\quad \frac{2m_2+1}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(635\,\text{nm})}{(512\,\text{nm})} = 1.24 \approx 1.25 = \frac{5}{4}$$

Thus we see that $m_1 = m_2 = 2$, and the thickness of the film is

$$t = \tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{2}\left(\frac{635\,\text{nm}}{1.36}\right)(2) = \boxed{467\,\text{nm}} \ \text{ or } \ t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) = \tfrac{1}{4}\left(\frac{512\,\text{nm}}{1.36}\right)(5) = \boxed{471\,\text{nm}}$$

With 2 sig.fig., the thickness is 470 nm. The range of answers is due to rounding λ_1/λ_2.

33. With respect to the incident wave, the wave that reflects from point B in the first diagram will not undergo a phase change, and so $\phi_B = 0$. With respect to the incident wave, the wave that reflects from point C in the first diagram has a phase change due to both the additional path length in air, and a phase change of π on reflection, and so we say that $\phi_D = \frac{2y}{\lambda}(2\pi) + \pi$, where y is the thickness of the air gap from B to C (or C to D). For dark rings, the net phase difference of the waves that recombine as they leave the glass moving upwards must be an odd-integer multiple of π.

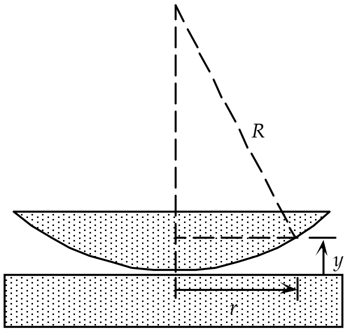


$$\phi_{\text{net}} = \phi_D - \phi_B = \frac{2y}{\lambda}(2\pi) + \pi = (2m+1)\pi \quad\rightarrow$$

$$y_{\text{dark}} = \tfrac{1}{2}m\lambda,\ m = 0, 1, 2, \cdots$$

Because $m = 0$ corresponds to the dark center, m represents the number of the dark ring.

Let the air gap of y be located a horizontal distance r from the center of the lens, as seen in the second diagram. Consider the dashed right triangle in the second diagram.



$$R^2 = r^2 + (R-y)^2 \quad\rightarrow$$

$$R^2 = r^2 + R^2 - 2Ry + y^2 \quad\rightarrow$$

$$r^2 = 2Ry - y^2$$

If we assume that $y \ll R$, then $r^2 \approx 2Ry$.

$$r^2 = 2Ry \quad\rightarrow\quad r_{\text{dark}}^2 = 2Ry_{\text{dark}} = 2R\left(\tfrac{1}{2}m\lambda\right) \quad\rightarrow\quad \boxed{r_{\text{dark}} = \sqrt{m\lambda R},\ m = 0, 1, 2, \cdots}$$

34. From Problem 33, we have $r = \sqrt{m\lambda R} = (m\lambda R)^{1/2}$. To find the distance between adjacent rings, we assume $m >> 1 \ \rightarrow\ \Delta m = 1 \ll m$. Since $\Delta m \ll m$, $\Delta r \approx \frac{dr}{dm}\Delta m$.

$$r = (m\lambda R)^{1/2} \quad ; \quad \frac{dr}{dm} = \tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R$$

$$\Delta r \approx \frac{dr}{dm}\Delta m = \left[\tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R\right](1) = \left[\frac{\lambda^2 R^2}{4m\lambda R}\right]^{1/2} = \boxed{\sqrt{\frac{\lambda R}{4m}}}$$

35. The radius of the m-th ring in terms of the wavelength of light and the radius of curvature is derived in Problem 33 as $r=\sqrt{m\lambda R}$. Using this equation, with the wavelength of light in the liquid given by Eq. 34-1, we divide the two radii and solve for the index of refraction.

$$\frac{r_{\text{air}}}{r_{\text{liquid}}}=\frac{\sqrt{m\lambda R}}{\sqrt{m(\lambda/n)R}}=\sqrt{n}\;\rightarrow\; n=\left(\frac{r_{\text{air}}}{r_{\text{liquid}}}\right)^2=\left(\frac{2.92\text{ cm}}{2.54\text{ cm}}\right)^2=\boxed{1.32}$$

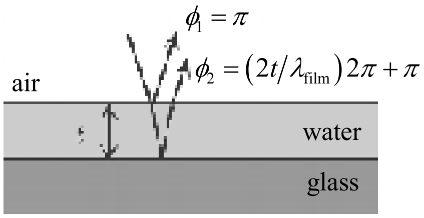
36. We use the equation derived in Problem 33, where r is the radius of the lens (1.7 cm) to solve for the radius of curvature. Since the outer edge is the 44th bright ring, which would be halfway between the 44th and 45th dark fringes, we set m=44.5

$$r=\sqrt{m\lambda R}\;\rightarrow\; R=\frac{r^2}{m\lambda}=\frac{(0.017\,\text{m})^2}{(44.5)(580\times10^{-9}\,\text{m})}=11.20\,\text{m}\approx\boxed{11\,\text{m}}$$

We calculate the focal length of the lens using Eq. 33-4 (the lensmaker's equation) with the index of refraction of lucite taken from Table 32-1.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=(1.51-1)\left(\frac{1}{11.2\,\text{m}}+\frac{1}{\infty}\right)=0.0455\,\text{m}^{-1}\;\rightarrow\; f=\frac{1}{0.0455\,\text{m}^{-1}}=\boxed{22\,\text{m}}$$

$\boxed{37.}$ (*a*) Assume the indices of refraction for air, water, and glass are 1.00, 1.33, and 1.50, respectively. When illuminated from above, a ray reflected from the air-water interface undergoes a phase shift of $\phi_1=\pi$, and a ray reflected at the water-glass interface also undergoes a phase shift of π. Thus, the two rays are unshifted in phase relative to each other due to reflection. For constructive interference, the path difference $2t$ must equal an integer number of wavelengths in water.



$$2t=m\lambda_{\text{water}}=m\frac{\lambda}{n_{\text{water}}},\; m=0,1,2,\cdots\;\rightarrow\;\boxed{\lambda=\frac{2n_{\text{water}}t}{m}}$$

(*b*) The above relation can be solved for the m-value associated with the reflected color. If this m-value is an integer the wavelength undergoes constructive interference upon reflection.

$$\lambda=\frac{2n_{\text{water}}t}{m}\;\rightarrow\; m=\frac{2n_{\text{water}}t}{\lambda}$$

For a thickness $t=200\mu\text{m}=2\times10^5\,\text{nm}$ the m-values for the two wavelengths are calculated.

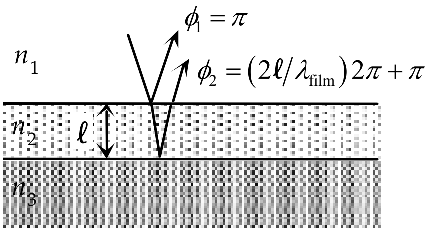
$$m_{700\,\text{nm}}=\frac{2n_{\text{water}}t}{\lambda}=\frac{2(1.33)(2\times10^5\,\text{nm})}{700\,\text{nm}}=760$$

$$m_{400\,\text{nm}}=\frac{2n_{\text{water}}t}{\lambda}=\frac{2(1.33)(2\times10^5\,\text{nm})}{400\,\text{nm}}=1330$$

Since both wavelengths yield integers for m, they are both reflected.

(*c*) All m-values between m = 760 and m = 1330 will produce reflected visible colors. There are 1330 – (760 – 1) = $\boxed{571}$ such values.

(*d*) This mix of a large number of wavelengths from throughout the visible spectrum will give the thick layer a white or grey appearance.



38. We assume $n_1 < n_2 < n_3$ and that most of the incident light is transmitted. If the amplitude of an incident ray is taken to be E_0, then the amplitude of a reflected ray is rE_0, with $r << 1$. The light reflected from the top surface of the film therefore has an amplitude of rE_0 and is phase shifted by $\phi_1 = \pi$ from the incident wave, due to the higher index of refraction. The light transmitted at that top surface has an amplitude of $(1-r)E_0$. That light is then reflected off the bottom surface of the film, and we assume that it has the same reflection coefficient. Thus the amplitude of that second reflected ray is $r(1-r)E_0 = (r-r^2)E_0 \approx rE_0$, the same amplitude as the first reflected ray. Due to traveling through the film and reflecting from the glass, the second ray has a phase shift of $\phi_2 = \pi + 2\pi(2\ell/\lambda_{\text{film}}) = \pi + 4\pi\ell n_2/\lambda$, where ℓ is the thickness of the film. Summing the two reflected rays gives the net reflected wave.

$$E = rE_0\cos(\omega t+\pi) + rE_0\cos(\omega t+\pi+4\pi\ell/\lambda_n)$$
$$= rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]$$

As with the double slit experiment, we set the intensity proportional to the square of the wave amplitude and integrate over one period to calculate the average intensity.

$$I \propto \frac{1}{T}\int_0^T E^2 dt = \frac{1}{T}\int_0^T \left[rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]\right]^2 dt$$
$$= \frac{r^2E_0^2}{T}\int_0^T \begin{bmatrix} (1+\cos 4\pi\ell n_2/\lambda)^2\cos^2(\omega t+\pi) + \sin^2(4\pi\ell n_2/\lambda)\sin^2(\omega t+\pi) \\ -2(1+\cos 4\pi\ell n_2/\lambda)\sin(4\pi\ell n_2/\lambda)\cos(\omega t+\pi)\sin(\omega t+\pi) \end{bmatrix} dt$$
$$= \frac{r^2E_0^2}{2}\left[(1+\cos 4\pi\ell n_2/\lambda)^2 + \sin^2(4\pi\ell n_2/\lambda)\right] = r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)$$

The reflected intensity without the film is proportional to the square of the intensity of the single reflected electric field.

$$I_0 \propto \frac{1}{T}\int_0^T E_{\text{no film}}^2 dt = \frac{1}{T}\int_0^T \left[rE_0\cos(\omega t+\pi)\right]^2 dt = \frac{r^2E_0^2}{T}\int_0^T\left[\cos^2(\omega t+\pi)\right]dt = \frac{r^2E_0^2}{2}$$

Dividing the intensity with the film to that without the film gives the factor by which the intensity is reduced.

$$\frac{I}{I_0} = \frac{r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)}{\frac{1}{2}r^2E_0^2} = 2(1+\cos 4\pi\ell n_2/\lambda)$$

To determine the thickness of the film, the phase difference between the two reflected waves with $\lambda = 550\,\text{nm}$ must be an odd integer multiple of π so that there is destructive interference. The minimum thickness will be for $m = 0$.

$$\phi_2 - \phi_1 = [\pi + 4\pi\ell n_2/\lambda] - \pi = (2m+1)\pi \quad\rightarrow\quad \ell = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{550\,\text{nm}}{4n}$$

It is interesting to see that the same result is obtained if we set the reflected intensity equal to zero for a wavelength of 550 nm.

$$\frac{I}{I_0} = 2(1+\cos 4\pi\ell n_2/\lambda) = 0 \;\rightarrow\; \cos 4\pi\ell n_2/\lambda = -1 \;\rightarrow\; 4\pi\ell n_2/\lambda = \pi \;\rightarrow\; \ell = \frac{550\,\text{nm}}{4n}$$

Finally, we insert the two given wavelengths (430 nm and 670 nm) into the intensity equation to determine the reduction in intensities.

For $\lambda = 430\text{ nm},\ \dfrac{I}{I_0} = 2\left(1+\cos 4\pi\dfrac{550\text{ nm}/4n}{430\text{ nm}/n}\right) = 2\left(1+\cos 4\pi\dfrac{550\text{ nm}}{4(430\text{ nm})}\right) = 0.721 \approx \boxed{72\%}$

For $\lambda = 670\text{ nm},\ \dfrac{I}{I_0} = 2\left(1+\cos 4\pi\dfrac{550\text{ nm}}{4(670\text{ nm})}\right) = 0.308 \approx \boxed{31\%}$

39. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of Δx.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(650)\left(589\times10^{-9}\text{m}\right) = \boxed{1.91\times10^{-4}\text{m}}$$

40. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of Δx.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \lambda = \frac{2\Delta x}{N} = \frac{2\left(1.25\times10^{-4}\text{m}\right)}{384} = 6.51\times10^{-7}\text{m} = \boxed{651\text{nm}}$$

41. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of Δx. The thickness of the foil is the distance that the mirror moves during the 272 fringe shifts.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(272)\left(589\times10^{-9}\text{m}\right) = \boxed{8.01\times10^{-5}\text{m}}$$

42. One fringe shift corresponds to an effective change in path length of λ. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a length *d*, the number of wavelengths in vacuum is $\dfrac{d}{\lambda}$, and the (greater) number with the gas present is $\dfrac{d}{\lambda_{\text{gas}}} = \dfrac{n_{\text{gas}}d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is twice the difference in the number of wavelengths in the two media.

$$N = 2\left(\frac{n_{\text{gas}}d}{\lambda} - \frac{d}{\lambda}\right) = 2\frac{d}{\lambda}\left(n_{\text{gas}}-1\right) \rightarrow n_{\text{gas}} = \frac{N\lambda}{2d}+1 = \frac{(176)\left(632.8\times10^{-9}\text{m}\right)}{2\left(1.155\times10^{-2}\text{m}\right)}+1 = \boxed{1.00482}$$

$\boxed{43.}$ There are two interference patterns formed, one by each of the two wavelengths. The fringe patterns overlap but do not interfere with each other. Accordingly, when the bright fringes of one pattern occurs at the same locations as the dark fringes of the other patterns, there will be no fringes seen, since there will be no dark bands to distinguish one fringe from the adjacent fringes.

To shift from one "no fringes" occurrence to the next, the mirror motion must produce an integer number of fringe shifts for each wavelength, and the number of shifts for the shorter wavelength must be one more than the number for the longer wavelength. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of Δx.

$$N_1 = 2\frac{\Delta x}{\lambda_1} \;;\; N_2 = 2\frac{\Delta x}{\lambda_2} \;;\; N_2 = N_1 + 1 \;\rightarrow\; 2\frac{\Delta x}{\lambda_2} = 2\frac{\Delta x}{\lambda_1} + 1 \;\rightarrow$$

$$\Delta x = \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{(589.6\,\text{nm})(589.0\,\text{nm})}{2(0.6\,\text{nm})} = 2.89\times10^5\,\text{nm} \approx \boxed{2.9\times10^{-4}\,\text{m}}$$

44. We assume the luminous flux is uniform, and so is the same in all directions.

$$F_\ell = E_\ell A = E_\ell 4\pi r^2 = \left(10^5\,\text{lm/m}^2\right)4\pi\left(1.496\times10^{11}\,\text{m}\right)^2 = 2.81\times10^{28}\,\text{lm} \approx \boxed{3\times10^{28}\,\text{lm}}$$

$$I_\ell = \frac{F_\ell}{4\pi\,\text{sr}} = \frac{2.81\times10^{28}\,\text{lm}}{4\pi\,\text{sr}} = 2.24\times10^{27}\,\text{cd} \approx \boxed{2\times10^{27}\,\text{cd}}$$

45. (*a*) The wattage of the bulb is the electric power input to the bulb.

$$\text{luminous efficiency} = \frac{F_\ell}{P} = \frac{1700\,\text{lm}}{100\,\text{W}} = \boxed{17\,\text{lm/W}}$$

(*b*) The illuminance is the luminous flux incident on a surface, divided by the area of the surface. Let *N* represent the number of lamps, each contributing an identical amount of luminous flux.

$$E_\ell = \frac{F_\ell}{A} = \frac{N\left[\frac{1}{2}(\text{luminous efficiency})P\right]}{A} \rightarrow$$

$$N = \frac{2E_\ell A}{(\text{luminous efficiency})P} = \frac{2\left(250\,\text{lm/m}^2\right)(25\,\text{m})(30\,\text{m})}{(60\,\text{lm/W})(40\,\text{W})} = 156\,\text{lamps} \approx \boxed{160\,\text{lamps}}$$

46. (*a*) For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

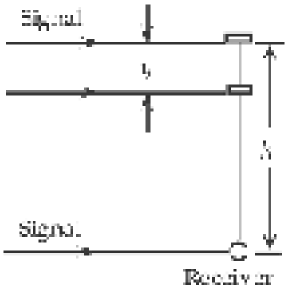
$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;\rightarrow\; \Delta x = \Delta m\frac{\lambda\ell}{d} \;\rightarrow$$

$$d = \frac{\lambda\ell\Delta m}{\Delta x} = \frac{\left(5.0\times10^{-7}\,\text{m}\right)(4.0\,\text{m})(1)}{\left(2.0\times10^{-2}\,\text{m}\right)} = \boxed{1.0\times10^{-4}\,\text{m}}$$

(*b*) For minima, we use Eq. 34-2b. The fourth-order minimum corresponds to *m* = 3, and the fifth-order minimum corresponds to *m* = 4. The slit separation, screen distance, and location on the screen are the same for the two wavelengths.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; d\frac{x}{\ell} = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; \left(m_\text{A} + \tfrac{1}{2}\right)\lambda_\text{A} = \left(m_\text{B} + \tfrac{1}{2}\right)\lambda_\text{B} \;\rightarrow$$

$$\lambda_\text{B} = \lambda_\text{A}\frac{\left(m_\text{A} + \frac{1}{2}\right)}{\left(m_\text{B} + \frac{1}{2}\right)} = \left(5.0\times10^{-7}\,\text{m}\right)\frac{3.5}{4.5} = \boxed{3.9\times10^{-7}\,\text{m}}$$



47. The wavelength of the signal is $\lambda = \dfrac{v}{f} = \dfrac{\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(75\times10^{6}\,\text{Hz}\right)} = 4.00\,\text{m}.$

(*a*) There is a phase difference between the direct and reflected signals from both the path difference, $\left(\dfrac{h}{\lambda}\right)2\pi$, and the reflection, π.

The total phase difference is the sum of the two.

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \frac{(122\,\text{m})}{(4.00\,\text{m})}2\pi + \pi = 62\pi = 31(2\pi)$$

Since the phase difference is an integer multiple of 2π, the interference is $\boxed{\text{constructive.}}$

(*b*) When the plane is 22 m closer to the receiver, the phase difference is as follows.

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi = \left[\frac{(122\,\text{m}-22\,\text{m})}{(4.00\,\text{m})}\right]2\pi + \pi = 51\pi = \frac{51}{2}(2\pi)$$

Since the phase difference is an odd-half-integer multiple of 2π, the interference is $\boxed{\text{destructive.}}$

48. Because the measurements are made far from the antennas, we can use the analysis for the double slit. Use Eq. 34-2a for constructive interference, and 34-2b for destructive interference. The wavelength of the signal is $\lambda = \dfrac{v}{f} = \dfrac{\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(88.5\times10^{6}\,\text{Hz}\right)} = 3.39\,\text{m}.$

For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots;\quad \rightarrow\quad \theta = \sin^{-1}\frac{m\lambda}{d}$$

$$\underset{\max}{\theta_1} = \sin^{-1}\frac{(1)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{22^\circ}\ ;\ \underset{\max}{\theta_2} = \sin^{-1}\frac{(2)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{49^\circ}\ ;$$

$$\underset{\max}{\theta_3} = \sin^{-1}\frac{(3)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

For destructive interference, the path difference is an odd multiple of half a wavelength:

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots;\quad \rightarrow\quad \theta = \sin^{-1}\frac{\left(m+\frac{1}{2}\right)\lambda}{d}$$

$$\underset{\max}{\theta_0} = \sin^{-1}\frac{\left(\frac{1}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{11^\circ}\ ;\ \underset{\max}{\theta_1} = \sin^{-1}\frac{\left(\frac{3}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{34^\circ}\ ;$$

$$\underset{\max}{\theta_2} = \sin^{-1}\frac{\left(\frac{5}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{70^\circ}\ ;\ \underset{\max}{\theta_3} = \sin^{-1}\frac{\left(\frac{7}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

These angles are applicable both above and below the midline, and both to the left and the right of the antennas.

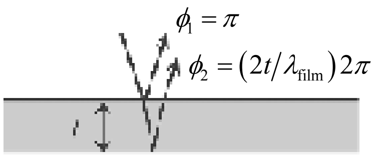
$\boxed{49.}$ For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda\ \rightarrow\ d\frac{x}{\ell} = m\lambda\ \rightarrow\ x = \frac{\lambda m\ell}{d}\ ;\ x_1 = \frac{\lambda_1 m\ell}{d}\ ;\ x_2 = \frac{\lambda_2 m\ell}{d}\ \rightarrow$$

$$\Delta x = x_1 - x_2 = \frac{\lambda_1 m\ell}{d} - \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\lambda_2 = \lambda_1 - \frac{d\Delta x}{m\ell} = 690\times10^{-9}\,\text{m} - \frac{\left(6.6\times10^{-4}\,\text{m}\right)\left(1.23\times10^{-3}\,\text{m}\right)}{2\left(1.60\,\text{m}\right)} = 4.36\times10^{-7}\,\text{m} \approx \boxed{440\,\text{nm}}$$

50. PLEASE NOTE: In early versions of the textbook, in which the third line of this problem states that "… light is a minimum only for …," the resulting answer does not work out properly. It yields values of $m = 6$ and $m = 4$ for the integers in the interference relationship. Accordingly, the problem was changed to read "… light is a maximum only for … ." The solution here reflects that change.



With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the bottom surface of the film has a phase change due to the additional path length and no phase change due to reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0$. For constructive interference, the net phase change must be an integer multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = 2\pi m \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{film}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, \ldots .$$

Evaluate the thickness for the two wavelengths.

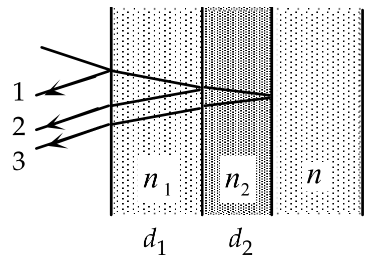
$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} \rightarrow \frac{\left(m_2+\frac{1}{2}\right)}{\left(m_1+\frac{1}{2}\right)} = \frac{\lambda_1}{\lambda_2} = \frac{688.0\,\text{nm}}{491.4\,\text{nm}} = 1.40 = \frac{7}{5}$$

Thus $m_2 = 3$ and $m_1 = 2$. Evaluate the thickness with either value and the corresponding wavelength.

$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{5}{2}\right)\frac{688.0\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}\ ;\ t = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{7}{2}\right)\frac{491.4\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}$$

51. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. The phase shift is 2π for every wavelength of path length change. The intensity as a function of phase shift is given by Eq. 34-6.

$$\frac{\delta}{2\pi} = \frac{\text{path change}}{\lambda} = \frac{2x}{\lambda} \rightarrow \delta = \frac{4\pi x}{\lambda}\ ;\ I = I_0\cos^2\frac{\delta}{2} = \boxed{I_0\cos^2\left(\frac{2\pi x}{\lambda}\right)}$$



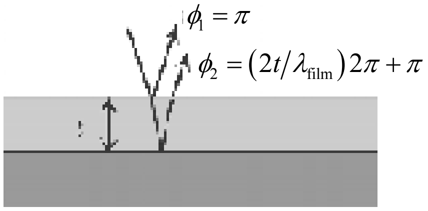
52. To maximize reflection, the three rays shown in the figure should be in phase. We first compare rays 2 and 3. Ray 2 reflects from $n_2 > n_1$, and so has a phase shift of $\phi_2 = \pi$. Ray 3 will have a phase change due to the additional path length in material 2, and a phase shift of π because of reflecting from $n > n_2$. Thus $\phi_3 = \left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi$. For constructive interference the net phase change for rays 2 and 3 must be a non-zero integer multiple of 2π.

$$\Delta\phi_{2-3} = \phi_3 - \phi_2 = \left[\left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi\right] - \pi = 2m\pi \rightarrow d_2 = \tfrac{1}{2}m\lambda_2,\ m = 1, 2, 3\cdots$$

The minimum thickness is for $m = 1$, and so $d_2 = \frac{1}{2}m\lambda_2 = \boxed{\dfrac{\lambda}{2n_2}}$.

Now consider rays 1 and 2. The exact same analysis applies, because the same relationship exists between the indices of refraction: $n_1 > n$ and $n_2 > n_1$. Thus $d_1 = \boxed{\dfrac{\lambda}{2n_1}}$.

53. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For destructive interference, the net phase change must be an odd-integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = (2m+1)\pi \quad \rightarrow$$

$$t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}} = \tfrac{1}{4}(2m+1)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, \ldots$$

The minimum thickness has $m = 0$, and so $t_{\min} = \frac{1}{4}\dfrac{\lambda}{n_{\text{film}}}$.

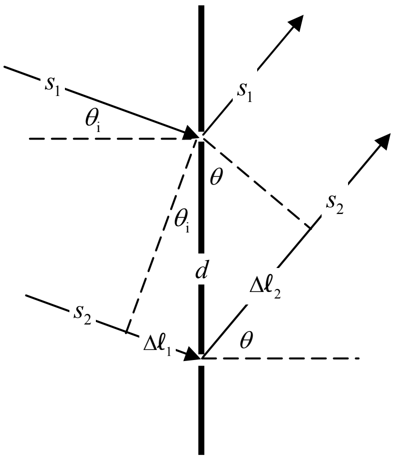
(*a*) For the blue light: $t_{\min} = \frac{1}{4}\dfrac{(450\,\text{nm})}{(1.38)} = 81.52\,\text{nm} \approx \boxed{82\,\text{nm}}$.

(*b*) For the red light: $t_{\min} = \frac{1}{4}\dfrac{(700\,\text{nm})}{(1.38)} = 126.8\,\text{nm} \approx \boxed{130\,\text{nm}}$.

54. The phase difference caused by the path difference back and forth through the coating must correspond to half a wavelength in order to produce destructive interference.

$$2t = \tfrac{1}{2}\lambda \quad \rightarrow \quad t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2\,\text{cm}) = \boxed{0.5\,\text{cm}}$$

$\boxed{55.}$ We consider a figure similar to Figure 34-12, but with the incoming rays at an angle of θ_i to the normal. Ray s_2 will travel an extra distance $\Delta\ell_1 = d\sin\theta_i$ before reaching the slits, and an extra distance $\Delta\ell_2 = d\sin\theta$ after leaving the slits. There will be a phase difference between the waves due to the path difference $\Delta\ell_1 + \Delta\ell_2$. When this total path difference is a multiple of the wavelength, constructive interference will occur.



$$\Delta\ell_1 + \Delta\ell_2 = d\sin\theta_i + d\sin\theta = m\lambda \quad \rightarrow$$

$$\sin\theta = \frac{m\lambda}{d} - \sin\theta_i,\quad m = 0, 1, 2, \cdots$$

Since the rays leave the slits at all angles in the forward direction, we could have drawn the leaving rays with a downward tilt instead of an upward tilt. This would make the ray s_2 traveling a longer distance from the slits to the screen. In this case the path difference would be $\Delta\ell_2 - \Delta\ell_1$, and would result in the following expression.

$$\Delta\ell_2 - \Delta\ell_1 = d\sin\theta - d\sin\theta_i = m\lambda \ \rightarrow \ \sin\theta = \frac{m\lambda}{d} + \sin\theta_i, \ m = 0, 1, 2, \cdots$$

$$\Delta\ell_1 - \Delta\ell_2 = d\sin\theta_i - d\sin\theta = m\lambda \ \rightarrow \ \sin\theta = -\frac{m\lambda}{d} + \sin\theta_i, \ m = 0, 1, 2, \cdots$$

We combine the statements as follows.

$$\boxed{\sin\theta = \frac{m\lambda}{d} \pm \sin\theta_i, \ m = 0, 1, 2, \cdots}$$

Because of an arbitrary choice of taking $\Delta\ell_2 - \Delta\ell_1$, we could also have formulated the problem so that the result would be expressed as $\boxed{\sin\theta = \sin\theta_i \pm \frac{m\lambda}{d}, \ m = 0, 1, 2, \cdots}$.

56. The signals will be out of phase when the path difference equals an odd number of half-wavelengths. Let the 175-m distance be represented by *d*.

$$\sqrt{y^2 + d^2} - y = \left(m + \tfrac{1}{2}\right)\lambda, \ m = 0, 1, 2, 3, ... \ \rightarrow \ \sqrt{y^2 + d^2} = y + \left(m + \tfrac{1}{2}\right)\lambda \ \rightarrow$$

$$y^2 + d^2 = y^2 + 2y\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \ \rightarrow \ y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda}$$

We evaluate this for the first three values of *m*. The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.0 \times 10^6 \text{Hz}} = 50 \text{m}$.

$$y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{(175\text{m})^2 - \left(m + \tfrac{1}{2}\right)^2(50\text{m})^2}{2\left(m + \tfrac{1}{2}\right)(50\text{m})} = 600\text{m}, 167\text{m}, 60\text{m}, 0\text{m}$$

The first three points on the *y* axis where the signals are out of phase are at $y = \boxed{0, 60\text{m, and } 167\text{m}}$.

57. As explained in Example 34-6 the $\tfrac{1}{2}$-cycle phase change at the lower surface means that destructive interference occurs when the thickness *t* is such that $2t = m\lambda, \ m = 0, 1, 2, ...$. Set $m = 1$ to find the smallest nonzero value of *t*.

$$t = \tfrac{1}{2}\lambda = \tfrac{1}{2}(680\text{nm}) = \boxed{340\text{nm}}$$

As also explained in Example 34-6, constructive interference will occur when $2t = \left(m + \tfrac{1}{2}\right)\lambda$, $m = 0, 1, 2, ...$. We set $m = 0$ to find the smallest value of *t*:

$$t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(680\text{nm}) = \boxed{170\text{nm}}$$

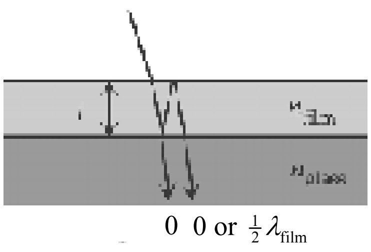
58. The reflected wave appears to be coming from the virtual image, so this corresponds to a double slit, with the separation being *d* = 2S. The reflection from the mirror produces a π phase shift, however, so the maxima and minima are interchanged, as described in Problem 11.

$$\sin\theta_{\text{max}} = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2\text{S}}, \ m = 0, 1, 2, \cdots \ ; \ \sin\theta_{\text{min}} = m\frac{\lambda}{2\text{S}}, \ m = 0, 1, 2, \cdots$$

59. Since the two sources are 180° out of phase, destructive interference will occur when the path length difference between the two sources and the receiver is 0, or an integer number of wavelengths. Since the antennae are separated by a distance of $d = \lambda/2$, the path length difference can never be greater than $\lambda/2$, so the only points of destructive interference occur when the receiver is equidistant from each antenna, that is, at $\boxed{\theta_{\text{destructive}} = 0° \text{ and } 180°}$. Constructive interference occurs when the path difference is a half integer wavelength. Again, since the separation distance between the two

antennas is $d = \lambda/2$, the maximum path length difference is $\lambda/2$, which occurs along the line through the antennae, therefore the constructive interference only occurs at $\boxed{\theta_{\text{constructive}} = 90° \text{ and } 270°}$. As expected, these angles are reversed from those in phase, found in Example 34-5c.

60. If we consider the two rays shown in the diagram, we see that the first ray passes through with no reflection, while the second ray has reflected twice. If $n_{\text{film}} < n_{\text{glass}}$, the first reflection from the glass produces a phase shift equivalent to $\frac{1}{2}\lambda_{\text{film}}$, while the second reflection from the air produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift in phase, due to its longer path length (2*t*) and reflection $\left(\frac{1}{2}\lambda_{\text{film}}\right)$. We set this path difference equal to an integer number of wavelengths for maximum intensity and equal to a half-integer number of wavelengths for minimum intensity.



$$\text{max:}\quad 2t + \tfrac{1}{2}\lambda_{\text{film}} = m\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$$

$$\text{min:}\quad 2t + \tfrac{1}{2}\lambda_{\text{film}} = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a minimum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

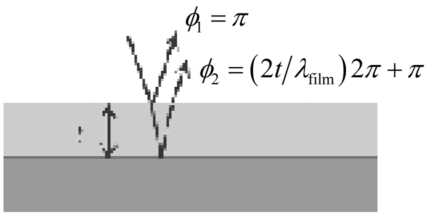
If $n_{\text{film}} > n_{\text{glass}}$, the first reflection from the glass produces no shift, while the second reflection from the air also produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift due solely to the difference in path lengths, 2*t*.
For maxima, we have

$$\text{max:}\quad 2t = m\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \frac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$$

$$\text{min:}\quad 2t = \left(m - \tfrac{1}{2}\right)\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \frac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a maximum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

$\boxed{61.}$ With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = m2\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda_{\text{film}} = \tfrac{1}{2}m\frac{\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \dots .$$

The minimum non-zero thickness occurs for $m = 1$.

$$t_{\min} = \frac{\lambda}{2n_{\text{film}}} = \frac{643\,\text{nm}}{2(1.34)} = \boxed{240\,\text{nm}}$$

62. The path difference to a point on the *x* axis from the two sources is $\Delta d = d_2 - d_1 = \sqrt{x^2 + d^2} - x$. For the two signals to be out of phase, this path difference must be an odd number of half-wavelengths, so $\Delta d = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots$. Also, the maximum path difference is $d = 3\lambda$. Thus the path difference must be $\tfrac{1}{2}\lambda,\ \tfrac{3}{2}\lambda$, or $\tfrac{5}{2}\lambda$ for the signals to be out of phase (*m* = 0, 1, or 2). We solve for *x* for the three path differences.

$$\Delta d = \sqrt{x^2 + d^2} - x = \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow \quad \sqrt{x^2 + d^2} = x + \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow$$

$$x^2 + d^2 = x^2 + 2x\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \quad \rightarrow$$

$$x = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{9\lambda^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{9 - \left(m + \tfrac{1}{2}\right)^2}{2\left(m + \tfrac{1}{2}\right)}\lambda$$

$$m = 0 : x = \frac{9 - \left(0 + \tfrac{1}{2}\right)^2}{2\left(0 + \tfrac{1}{2}\right)}\lambda = \boxed{8.75\lambda}\ ;\ m = 1 : x = \frac{9 - \left(1 + \tfrac{1}{2}\right)^2}{2\left(1 + \tfrac{1}{2}\right)}\lambda = \boxed{2.25\lambda}$$

$$m = 2 : x = \frac{9 - \left(2 + \tfrac{1}{2}\right)^2}{2\left(2 + \tfrac{1}{2}\right)}\lambda = \boxed{0.55\lambda}$$

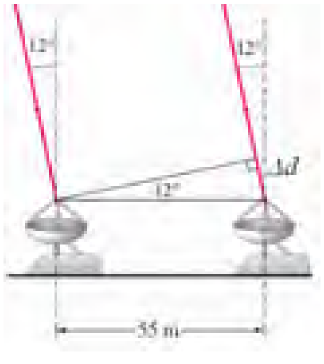
63. For both configurations, we have $d\sin\theta = m\lambda$. The angles and the orders are to be the same. The slit separations and wavelengths will be different. Use the fact that frequency and wavelength are related by $v = f\lambda$. The speed of sound in room-temperature air is given in Chapter 16 as 343 m/s.

$$d\sin\theta = m\lambda \quad \rightarrow \quad \frac{\sin\theta}{m} = \frac{\lambda}{d} = \frac{\lambda_{\text{L}}}{d_{\text{L}}} = \frac{\lambda_{\text{S}}}{d_{\text{S}}} \quad \rightarrow$$

$$d_{\text{S}} = d_{\text{L}}\frac{\lambda_{\text{S}}}{\lambda_{\text{L}}} = d_{\text{L}}\frac{\frac{v_{\text{S}}}{f_{\text{S}}}}{\frac{v_{\text{L}}}{f_{\text{L}}}} = d_{\text{L}}\frac{v_{\text{S}}f_{\text{L}}}{v_{\text{L}}f_{\text{S}}} = \left(1.0\times10^{-4}\,\text{m}\right)\left(\frac{343\,\text{m/s}}{3.00\times10^{8}\,\text{m/s}}\right)\left(\frac{4.6\times10^{14}\,\text{Hz}}{262\,\text{Hz}}\right) = \boxed{200\,\text{m}}$$

The answer has 2 significant figures.

64. Light traveling from a region 12° from the vertical would have to travel a slightly longer distance to reach the far antenna. Using trigonometry we calculate this distance, as was done in Young's double slit experiment. Dividing this additional distance by the speed of light gives us the necessary time shift.



$$\Delta t = \frac{\Delta d}{c} = \frac{\ell\sin\theta}{c} = \frac{(55\ \text{m})\sin 12^\circ}{3.00\times10^{8}\ \text{m/s}} = 3.81\times10^{-8}\ \text{s} = \boxed{38.1\ \text{ns}}$$

65. In order for the two reflected halves of the beam to be 180° out of phase with each other, the minimum path difference (2*t*) should be $\frac{1}{2}\lambda$ in the plastic. Notice that there is no net phase difference between the two halves of the beam due to reflection, because both halves reflect from the same material.

$$2t = \tfrac{1}{2}\frac{\lambda}{n} \quad \rightarrow \quad t = \frac{\lambda}{4n} = \frac{780\,\text{nm}}{4(1.55)} = \boxed{126\,\text{nm}}$$

66. We determine *n* for each angle using a spreadsheet. The results are shown below.

N	25	50	75	100	125	150
$\theta(\text{degree})$	5.5	6.9	8.6	10.0	11.3	12.5
n	1.75	2.19	2.10	2.07	2.02	1.98

The average value is $n_{\text{avg}} = \boxed{2.02}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH34.XLS," on tab "Problem 34.66."

CHAPTER 35: Diffraction and Polarization

Responses to Questions

1. Radio waves have a much longer wavelength than visible light and will diffract around normal-sized objects (like hills). The wavelengths of visible light are very small and will not diffract around normal-sized objects.

2. You see a pattern of dark and bright lines parallel to your fingertips in the narrow opening between your fingers.

3. Light from all points of an extended source produces diffraction patterns, and these many different diffraction patterns overlap and wash out each other so that no distinct pattern can be easily seen. When using white light, the diffraction patterns of the different wavelengths will overlap because the locations of the fringes depend on wavelength. Monochromatic light will produce a more distinct diffraction pattern.

4. (*a*) If the slit width is increased, the diffraction pattern will become more compact.
   (*b*) If the wavelength of the light is increased, the diffraction pattern will spread out.

5. (*a*) A slit width of 50 nm would produce a central maximum so spread out that it would cover the entire width of the screen. No minimum (and therefore no diffraction pattern) will be seen. The different wavelengths will all overlap, so the light on the screen will be white. It will also be dim, compared to the source, because it is spread out.
   (*b*) For the 50,000 nm slit, the central maximum will be very narrow, about a degree in width for the blue end of the spectrum and about a degree and a half for the red. The diffraction pattern will not be distinct, because most of the intensity will be in the small central maximum and the fringes for the different wavelengths of white light will not coincide.

6. (*a*) If the apparatus is immersed in water, the wavelength of the light will decrease $\left(\lambda' = \frac{\lambda}{n}\right)$ and the diffraction pattern will become more compact.
   (*b*) If the apparatus is placed in a vacuum, the wavelength of the light will increase slightly, and the diffraction pattern will spread out very slightly.

7. The intensity pattern is actually a function of the form $\left(\frac{\sin x}{x}\right)^2$ (see equations 35-7 and 35-8). The maxima of this function do not coincide exactly with the maxima of $\sin^2 x$. You can think of the intensity pattern as the combination of a $\sin^2 x$ function and a $1/x^2$ function, which forces the intensity function to zero and shifts the maxima slightly.

8. Similarities: Both have a regular pattern of light and dark fringes. The angular separation of the fringes is proportional to the wavelength of the light, and inversely proportional to the slit size or slit separation. Differences: The single slit diffraction maxima decrease in brightness from the center. Maxima for the double slit interference pattern would be equally bright (ignoring single slit effects) and are equally spaced.

9. No. D represents the slit width and d the distance between the centers of the slits. It is possible for the distance between the slit centers to be greater than the width of the slits; it is not possible for the distance between the slit centers to be less than the width of the slits.

10. (*a*) Increasing the wavelength, λ, will spread out the diffraction pattern, since the locations of the minima are given by $\sin\theta = m\lambda/D$. The interference pattern will also spread out; the interference maxima are given by $\sin\theta = m\lambda/d$. The number of interference fringes in the central diffraction maximum will not change.
    (*b*) Increasing the slit separation, *d*, will decrease the spacing between the interference fringes without changing the diffraction, so more interference maxima will fit in the central maximum of the diffraction envelope.
    (*c*) Increasing the slit width, *D*, will decrease the angular width of the diffraction central maximum without changing the interference fringes, so fewer bright fringes will fit in the central maximum.

11. Yes. As stated in Section 35-5, "It is not possible to resolve detail of objects smaller than the wavelength of the radiation being used."

12. Yes. Diffraction effects will occur for both real and virtual images.

$\boxed{13.}$ A large mirror has better resolution and gathers more light than a small mirror.

14. No. The resolving power of a lens is on the order of the wavelength of the light being used, so it is not possible to resolve details smaller than the wavelength of the light. Atoms have diameters of about 10^{-8} cm and the wavelength of visible light is on the order of 10^{-5} cm.

15. Violet light would give the best resolution in a microscope, because the wavelengths are shortest.

16. Yes. (See the introduction to Section 35-7.) The analysis for a diffraction grating of many slits is essentially the same as for Young's double slit interference. However, the bright maxima of a multiple-slit grating are much sharper and narrower than those in a double-slit pattern.

17. The answer depends on the slit spacing of the grating being used. If the spacing is small enough, only the first order will appear so there will not be any overlap. For wider slit spacing there can be overlap. If there is overlap, it will be the higher orders of the shorter wavelength light overlapping with lower orders of the longer wavelength light. See, for instance, Example 35-9, which shows the overlap of the third order blue light with the second order red light.

18. The bright lines will coincide, but those for the grating will be much narrower with wider dark spaces in between. The grating will produce a much sharper pattern.

$\boxed{19.}$ (*a*) Violet light will be at the top of the rainbow created by the diffraction grating. Principal maxima for a diffraction grating are at positions given by $\sin\theta = \frac{m\lambda}{d}$. Violet light has a shorter wavelength than red light and so will appear at a smaller angle away from the direction of the horizontal incident beam.
    (*b*) Red light will appear at the top of the rainbow created by the prism. The index of refraction for violet light in a given medium is slightly greater than for red light in the same medium, and so the violet light will bend more and will appear farther from the direction of the horizontal incident beam.

20. The tiny peaks are produced when light from some but not all of the slits interferes constructively. The peaks are tiny because light from only some of the slits interferes constructively.

21. Polarization demonstrates the transverse wave nature of light, and cannot be explained if light is considered only as particles.

22. Take the sunglasses outside and look up at the sky through them. Rotate the sunglasses (about an axis perpendicular to the lens) through at least 180°. If the sky seems to lighten and darken as you rotate the sunglasses, then they are polarizing. You could also look at a liquid crystal display or reflections from the floor while rotating the glasses, or put one pair of glasses on top of the other and rotate them. If what you see through the glasses changes as you rotate them, then the glasses are polarizing.

23. Black. If there were no atmosphere, there would be no scattering of the sunlight coming to Earth.

Solutions to Problems

$\boxed{1.}$ We use Eq. 35-1 to calculate the angular distance from the middle of the central peak to the first minimum. The width of the central peak is twice this angular distance.

$$\sin\theta_1 = \frac{\lambda}{D} \to \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{680\times10^{-9}\,\text{m}}{0.0365\times10^{-3}\,\text{m}}\right) = 1.067°$$

$$\Delta\theta = 2\theta_1 = 2(1.067°) = \boxed{2.13°}$$

2. The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using this angle and Eq. 35-1, we calculate the wavelength used.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(32°) = 16°$$

$$\sin\theta_1 = \frac{\lambda}{D} \to \lambda = D\sin\theta_1 = \left(2.60\times10^{-3}\text{ mm}\right)\sin(16°) = 7.17\times10^{-4}\text{ mm} = \boxed{717\text{ nm}}$$

3. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2 to calculate the angular distance to the first and second minima. Then we average these to values to determine the approximate location of the first maximum. Finally, using trigonometry, we set the linear distance equal to the distance to the screen multiplied by the tangent of the angle.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 8.678° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 17.774°$$

$$\theta = \frac{\theta_1+\theta_2}{2} = \frac{8.678°+17.774°}{2} = 13.23°$$

$$y = \ell\tan\theta_1 = (10.0\text{ m})\tan(13.23°) = \boxed{2.35\text{ m}}$$

4. (*a*) We use Eq. 35-2, using m=1,2,3,… to calculate the possible diffraction minima, when the wavelength is 0.50 cm.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times0.50\text{ cm}}{1.6\text{ cm}}\right) = 18.2° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times0.50\text{ cm}}{1.6\text{ cm}}\right) = 38.7°$$

$$\theta_3 = \sin^{-1}\left(\frac{3\times 0.50\text{ cm}}{1.6\text{ cm}}\right) = 69.6^\circ \qquad \theta_4 = \sin^{-1}\left(\frac{4\times 0.50\text{ cm}}{1.6\text{ cm}}\right) \to \text{ no solution}$$

There are three diffraction minima: $\boxed{18^\circ, 39^\circ, \text{ and } 70^\circ.}$

(*b*) We repeat the process from part (*a*) using a wavelength of 1.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = 38.7^\circ \qquad \theta_2 = \sin^{-1}\left(\frac{2\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

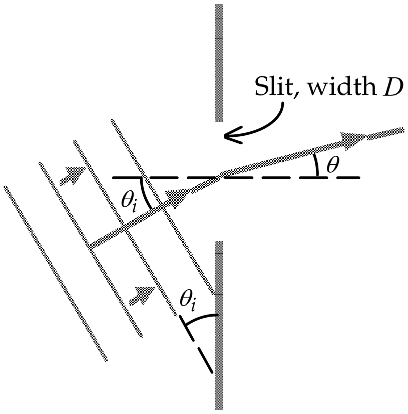
The only diffraction minima is at $\boxed{39^\circ}$

(*c*) We repeat the process from part (*a*) using a wavelength of 3.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 3.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

There are $\boxed{\text{no diffraction minima}}$.

5. The path-length difference between the top and bottom of the slit for the incident wave is $D \sin \theta_i$. The path-length difference between the top and bottom of the slit for the diffracted wave is $D \sin \theta$. When the net path-length difference is equal to a multiple of the wavelength, there will be an even number of segments of the wave having a path-length difference of $\lambda/2$. We set the path-length difference equal to m (an integer) times the wavelength and solve for the angle of the diffraction minimum.



$$D\sin\theta_i - D\sin\theta = m\lambda \quad \to$$

$$\boxed{\sin\theta = \sin\theta_i - \frac{m\lambda}{D}, \quad m = \pm 1, \pm 2, ...}$$

From this equation we see that when $\theta = 23.0^\circ$, the minima will be symmetrically distributed around a central maximum at 23.0°

6. The angle from the central maximum to the first bright maximum is half the angle between the first bright maxima on either side of the central maximum. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2, setting $m = 3/2$, to calculate the slit width, D.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(35^\circ) = 17.5^\circ$$

$$D\sin\theta_m = m\lambda \to D = \frac{m\lambda}{\sin\theta_1} = \frac{(3/2)(633\text{ nm})}{\sin 17.5^\circ} = 3157.6\text{ nm} \approx \boxed{3.2\,\mu\text{m}}$$

$\boxed{7.}$ We use the distance to the screen and half the width of the diffraction maximum to calculate the angular distance to the first minimum. Then using this angle and Eq. 35-1 we calculate the slit width. Then using the slit width and the new wavelength we calculate the angle to the first minimum and the width of the diffraction maximum.

$$\tan\theta_1 = \frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} \quad \to \quad \theta_1 = \tan^{-1}\frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} = \tan^{-1}\frac{\left(\frac{1}{2}\times 0.06\,\text{m}\right)}{2.20\,\text{m}} = 0.781^\circ$$

$$\sin\theta_1 = \frac{\lambda_1}{D} \quad \to \quad D = \frac{\lambda_1}{\sin\theta_1} = \frac{580\,\text{nm}}{\sin 0.781^\circ} = 42{,}537\,\text{nm}$$

$$\sin\theta_2 = \frac{\lambda_2}{D} \rightarrow \theta_2 = \sin^{-1}\left(\frac{\lambda_2}{D}\right) = \sin^{-1}\left(\frac{460\,\text{nm}}{42{,}537\,\text{nm}}\right) = 0.620°$$

$$\Delta y_2 = 2\ell\tan\theta_2 = 2(2.20\,\text{m})\tan(0.620°) = 0.0476\,\text{m} \approx \boxed{4.8\,\text{cm}}$$

8. (*a*) There will be no diffraction minima if the angle for the first minimum is greater than 90°. We set the angle in Eq. 35-1 equal to 90° and solve for the slit width.

$$\sin\theta = \frac{\lambda}{D} \rightarrow D = \frac{\lambda}{\sin 90°} = \boxed{\lambda}$$

(*b*) For no visible light to exhibit a diffraction minimum, the slit width must be equal to the shortest visible wavelength.

$$D = \lambda_{\text{min}} = \boxed{400\text{ nm}}.$$

9. We set the angle to the first minimum equal to half of the separation angle between the dark bands. We insert this angle into Eq. 35-1 to solve for the slit width.

$$\theta = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(55.0°) = 27.5°$$

$$\sin\theta = \frac{\lambda}{D} \rightarrow D = \frac{\lambda}{\sin\theta} = \frac{440\,\text{nm}}{\sin 27.5°} = \boxed{953\,\text{nm}}$$

10. We find the angle to the first minimum using Eq. 35-1. The distance on the screen from the central maximum is found using the distance to the screen and the tangent of the angle. The width of the central maximum is twice the distance from the central maximum to the first minimum.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{450\times10^{-9}\,\text{m}}{1.0\times10^{-3}\,\text{m}}\right) = \underline{\underline{0.02578°}}$$

$$y_1 = \ell\tan\theta_1 = (5.0\text{ m})\tan 0.02578° = 0.00225\text{ m}$$

$$\Delta y = 2y_1 = 2(0.00225\text{ m}) = 0.0045\text{ m} = \boxed{0.45\text{ cm}}$$

11. (*a*) For vertical diffraction we use the height of the slit (1.5 μm) as the slit width in Eq. 35-1 to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is equal to twice this angle.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{1.5\times10^{-6}\,\text{m}} = 31.3°$$

$$\Delta\theta = 2\theta_1 = 2(31.3°) \approx \boxed{63°}$$

(*b*) To find the horizontal diffraction we use the width of the slit (3.0 μm) in Eq. 35-1.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{3.0\times10^{-6}\,\text{m}} = 15.07°$$

$$\Delta\theta = 2\theta_1 = 2(15.07°) \approx \boxed{30°}$$

12. (*a*) If we consider the slit made up of *N* wavelets each of amplitude E_0, the total amplitude at the central maximum, where they are all in phase, is NE_0. Doubling the size of the slit doubles the number of wavelets and thus the total amplitude of the electric field. Because the intensity is proportional to the square of the electric field amplitude, the intensity at the central maximum is increased by a factor of 4.

$$I \propto E^2 = (2E_0)^2 = 4E_0^2 \propto \boxed{4I_0}$$

(*b*) From Eq. 35-1 we see that, for small angles, the width of the central maximum is inversely proportional to the slit width. Therefore doubling the slit width will cut the area of the central peak in half. Since the intensity is spread over only half the area, where the intensity is four times the initial intensity, the average intensity (or energy) over the central maximum has doubled. This is true for all fringes, so when the slit width is doubled, allowing twice the energy to pass through the slit, the average energy within each slit will also double, in accord with the conservation of energy.

13. We use Eq. 35-8 to calculate the intensity, where the angle θ is found from the displacement from the central maximum (15 cm) and the distance to the screen.

$$\tan\theta = \frac{y}{\ell} \rightarrow \theta = \tan^{-1}\left(\frac{15\,\text{cm}}{25\,\text{cm}}\right) = 31.0^\circ$$

$$\beta = \frac{2\pi}{\lambda}D\sin\theta = \frac{2\pi}{\left(750\times10^{-9}\,\text{m}\right)}\left(1.0\times10^{-6}\,\text{m}\right)\sin 31.0^\circ = 4.31\,\text{rad}$$

$$\frac{I}{I_0} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \left(\frac{\sin\left(4.31\,\text{rad}/2\right)}{4.31\,\text{rad}/2}\right)^2 = 0.1498 \approx \boxed{0.15}$$

So the light intensity at 15 cm is about 15% of the maximum intensity.

14. (*a*) The secondary maxima do not occur precisely where $\sin(\beta/2)$ is a maximum, that is at $\beta/2 = (m+\frac{1}{2})\pi$ where $m = 1,2,3,...,$ because the diffraction intensity (Eq. 35-7) is the ratio of the sine function and $\beta/2$. Near the maximum of the sine function, the denominator of the intensity function causes the intensity to decrease more rapidly than the sine function causes it to increase. This results in the intensity reaching a maximum slightly before the sine function reaches its maximum.

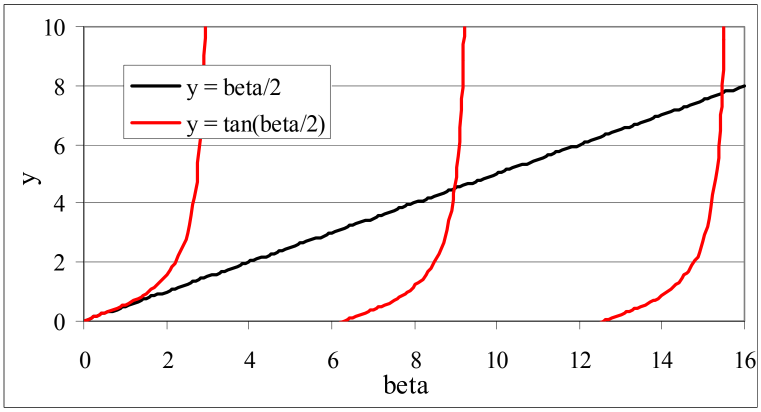
(*b*) We set the derivative of Eq. 35-7 with respect to β equal to zero to determine the intensity extrema.

$$0 = \frac{dI}{d\beta} = \frac{d}{d\beta}I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = 2I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]\left[\frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2}\right]$$

When the first term in brackets is zero, the intensity is a minimum, so the intensity is a maximum when the second term in brackets is zero.

$$0 = \frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2} \rightarrow \boxed{\beta/2 = \tan(\beta/2)}$$

(*c*) The first and secondary maxima are found where these two curves intersect, or $\boxed{\beta_1 = 8.987}$ and $\boxed{\beta_2 = 15.451.}$ We calculate the percent difference between these and the maxima of the sine curve, $\beta_1' = 3\pi$ and $\beta_2' = 5\pi$.



$$\left.\frac{\Delta\beta}{\beta}\right|_1=\frac{\beta_1-\beta_1'}{\beta_1'}=\frac{8.987-3\pi}{3\pi}=-0.0464=\boxed{-4.64\%}$$

$$\left.\frac{\Delta\beta}{\beta}\right|_2=\frac{15.451-5\pi}{5\pi}=-0.0164=\boxed{-1.64\%}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.14."

15. If the central diffraction peak contains nine fringes, there will be four fringes on each side of the central peak. Thus the fifth maximum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2a with m = 5 to find the angle of the fifth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta=m\lambda \rightarrow \sin\theta=\frac{5\lambda}{d}\ ;\ \sin\theta=\frac{\lambda}{D}=\frac{5\lambda}{d} \rightarrow \boxed{d=5D}$$

16. (*a*) If the central diffraction peak is to contain seventeen fringes, there will be eight fringes on each side of the central peak. Thus, the ninth minimum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2b with m = 8 to find the angle of the ninth interference minimum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta=\left(m+\tfrac{1}{2}\right)\lambda \rightarrow \sin\theta=\frac{\left(8+\frac{1}{2}\right)\lambda}{d}=\frac{8.5\lambda}{d}$$

$$\sin\theta=\frac{\lambda}{D}=\frac{8.5\lambda}{d} \rightarrow \boxed{d=8.5D}$$

Therefore, for the first diffraction minimum to be at the ninth interference minimum, the separation of slits should be 8.5 times the slit width.

(*b*) If the first diffraction minimum is to occur at the ninth interference maximum, we use Eq. 34-2a with m = 9 to find the angle of the ninth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta=m\lambda \rightarrow \sin\theta=\frac{9\lambda}{d}=\frac{9\lambda}{d}\ ;\ \sin\theta=\frac{\lambda}{D}=\frac{9\lambda}{d} \rightarrow \boxed{d=9D}$$

Therefore, for the first diffraction minimum to be at the ninth interference maximum, the separation of slits should be 9 times the slit width.

17. Given light with $\lambda=605\text{ nm}$ passing through double slits with separation $d=0.120\text{ mm}$, we use Eq. 34-2a to find the highest integer m value for the interference fringe that occurs before the angle $\theta=90^\circ$.

$$d\sin\theta=m\lambda \rightarrow m=\frac{\left(0.120\times10^{-3}\text{m}\right)\sin 90^{\circ\circ}}{605\times10^{-9}\text{m}}=198$$

So, including the $m=0$ fringe, and the symmetric pattern of interference fringes on each side of $\theta=0$, there are potentially a total of $198+198+1=397$ fringes. However, since slits have width $a=0.040$ mm, the potential interference fringes that coincide with the slits' diffraction minima will be absent. Let the diffraction minima be indexed by $m'=1,2,3,$ etc. We then set the diffraction angles in Eq. 34-2a and Eq. 35-2 equal to solve for the m values of the absent fringes.

$$\sin\theta = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \quad \to \quad \frac{m}{m'} = \frac{d}{D} = \frac{0.120\text{ mm}}{0.040\text{ mm}} = 3 \quad \to \quad m = 3m'$$

Using $m' = 1, 2, 3,$ etc., the 66 interference fringes on each side of $\theta = 0$ with $m = 3, 6, 9, ..., 198$ will be absent. Thus the number of fringes on the screen is $397 - 2(66) = \boxed{265}$.

18. In a double-slit experiment, if the central diffraction peak contains 13 interference fringes, there is the $m = 0$ fringe, along with fringes up to $m = 6$ on each side of $\theta = 0$. Then, at angle θ, the $m = 7$ interference fringe coincides with the first diffraction minima. We set this angle in Eq. 34-2a and 35-2 equal to solve for the relationship between the slit width and separation.

$$\sin\theta_1 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \quad \to \quad \frac{d}{D} = \frac{m}{m'} = \frac{7}{1} = 7 \quad \to \quad d = 7D$$

Now, we use these equations again to find the *m* value at the second diffraction minimum, $m' = 0$.

$$\sin\theta_2 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \quad \to \quad m = m'\frac{d}{D} = 2\frac{7D}{D} = 14$$

Thus, the $\boxed{\text{six fringes}}$ corresponding to $m = 8$ to $m = 13$ will occur within the first and second diffraction minima.

$\boxed{19.}$ (*a*) The angle to each of the maxima of the double slit are given by Eq. 34-2a. The distance of a fringe on the screen from the center of the pattern is equal to the distance between the slit and screen multiplied by the tangent of the angle. For small angles, we can set the tangent equal to the sine of the angle. The slit spacing is found by subtracting the distance between two adjacent fringes.

$$\sin\theta_m = \frac{m\lambda}{d} \qquad y_m = \ell\tan\theta_m \approx \ell\sin\theta_m = \ell\frac{m\lambda}{d}$$

$$\Delta y = y_{m+1} - y_m = \ell\frac{(m+1)\lambda}{d} - \ell\frac{m\lambda}{d} = \frac{\ell\lambda}{d} = \frac{(1.0\,\text{m})(580\times10^{-9}\,\text{m})}{0.030\times10^{-3}\text{m}} = 0.019\,\text{m} = \boxed{1.9\,\text{cm}}$$

(*b*) We use Eq. 35-1 to determine the angle between the center and the first minimum. Then by multiplying the distance to the screen by the tangent of the angle we find the distance from the center to the first minima. The distance between the two first order diffraction minima is twice the distance from the center to one of the minima.

$$\sin\theta_1 = \frac{\lambda}{D} \quad \to \quad \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{580\times10^{-9}\,\text{m}}{0.010\times10^{-3}\text{m}} = 3.325^\circ$$

$$y_1 = \ell\tan\theta_1 = (1.0\,\text{m})\tan 3.325^\circ = 0.0581\text{ m}$$

$$\Delta y = 2y_1 = 2(0.0581\,\text{m}) = 0.116\,\text{m} \approx \boxed{12\,\text{cm}}$$

20. We set $d = D$ in Eqs. 34-4 and 35-6 to show $\beta = \delta$. Replacing δ with β in Eq. 35-9, and using the double angle formula we show that Eq. 35-9 reduces to Eq. 35-7, with $\beta' = 2\beta$. Finally using Eq. 35-6 again, we show that $\beta' = 2\beta$ implies that the new slit width D' is simply double the initial slit width.

$$\delta = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{\lambda}D\sin\theta = \beta$$

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2\cos^2(\delta/2) = I_0\frac{\sin^2(\beta/2)\cos^2(\beta/2)}{(\beta/2)^2} = I_0\frac{\frac{1}{4}\sin^2[2(\beta/2)]}{(\beta/2)^2}$$

$$= I_0 \frac{\sin^2\beta}{\beta^2} = \boxed{I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2}}, \text{ where } \beta' = 2\beta.$$

$$\beta' = \frac{2\pi}{\lambda} D' \sin\theta = 2\left(\frac{2\pi}{\lambda} D\sin\theta\right) \rightarrow \boxed{D' = 2D}$$

21. Using Eq. 34-2a we determine the angle at which the third-order interference maximum occurs. Then we use Eq. 35-9 to determine the ratio of the intensity of the third-order maximum, where β is given by Eq. 35-6 and δ is given by Eq. 34-4.

$$d\sin\theta = m\lambda \;\Rightarrow\; \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\lambda}{40.0\lambda}\right) = 4.301°$$

$$\frac{\beta}{2} = \frac{2\pi}{2\lambda} D\sin\theta = \frac{\pi(40.0\lambda/5)}{\lambda}\sin(4.301°) = 1.885 \text{ rad}$$

$$\frac{\delta}{2} = \frac{2\pi d}{2\lambda}\sin\theta = \frac{\pi(40.0\lambda)}{\lambda}\sin(4.301°) = 9.424 \text{ rad}$$

$$I = I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2\left[\cos\left(\frac{\delta}{2}\right)\right]^2 = I_o\left[\frac{\sin(1.885 \text{ rad})}{1.885 \text{ rad}}\right]^2\left[\cos(9.424)\right]^2 = \boxed{0.255 I_o}$$

22. We use Eq. 34-2a to determine the order of the double slit maximum that corresponds to the same angle as the first order single slit minimum, from Eq. 35-1. Since this double slit maximum is darkened, inside the central diffraction peak, there will be the zeroth order fringe and on either side of the central peak a number of maximum equal to one less than the double slit order. Therefore, there will be 2(*m* – 1*)*+1, or 2*m* – 1 fringes.

$$d\sin\theta = m\lambda \;\rightarrow\; m = \frac{d\sin\theta}{\lambda} = \frac{d}{\lambda}\left(\frac{\lambda}{D}\right) = \frac{d}{D} \;\;;\; N = 2m - 1 = 2\frac{d}{D} - 1$$

(*a*) We first set the slit separation equal to twice the slit width, *d* = 2.00 *D*.

$$N = 2\frac{2.00D}{D} - 1 = \boxed{3}$$

(*b*) Next we set *d* = 12.0 *D*.

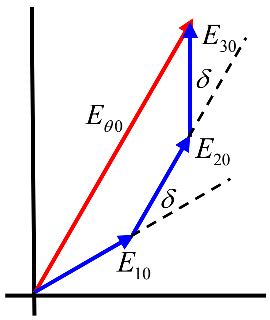
$$N = 2\frac{12.00D}{D} - 1 = \boxed{23}$$

(*c*) For the previous two parts, the ratio of slits had been an integer value. This corresponded to the single slit minimum overlapping the double slit maximum. Now that *d* = 4.50 *D*, the single slit minimum overlaps a double slit minimum. Therefore, the last order maximum, *m* = 4, is not darkened and *N* = 2*m* + 1.

$$N = 2m + 1 = 2(4) + 1 = \boxed{9}$$

(*d*) In this case the ratio of the slit separation to slit width is not an integer, nor a half-integer value. The first order single-slit minimum falls between the seventh order maximum and the seventh order minimum. Therefore, the seventh order maximum will partially be seen as a fringe.

$$N = 2m + 1 = 2(7) + 1 = \boxed{15}$$



23. (*a*) If $D \approx \lambda$, the central maximum of the diffraction pattern will be very wide. Thus we need consider only the interference between slits. We construct a phasor diagram for the interference, with $\delta = \frac{2\pi}{\lambda} d\sin\theta$ as the phase difference between adjacent slits. The magnitude of the electric fields of the slits will have the same magnitude, $E_{10} = E_{20} = E_{30} = E_0$. From the symmetry of the phasor diagram we see that $\phi = \delta$. Adding the three electric field vectors yields the net electric field.

$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta = E_0\left(1+2\cos\delta\right)$$

The central peak intensity occurs when $\delta = 0$. We set the intensity proportional to the square of the electric field and calculate the ratio of the intensities.

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{00}^2} = \frac{E_0^2\left(1+2\cos\delta\right)^2}{E_0^2\left(1+2\cos 0\right)^2} = \boxed{\frac{\left(1+2\cos\delta\right)^2}{9}}$$

(*b*) We find the locations of the maxima and minima by setting the first derivative of the intensity equal to zero.

$$\frac{dI_\theta}{d\delta} = \frac{d}{d\delta}\frac{I_0}{9}\left(1+2\cos\delta\right)^2 = \frac{2I_0}{9}\left(1+2\cos\delta\right)\left(-2\sin\delta\right) = 0$$

This equation is satisfied when either of the terms in parentheses is equal to zero. When $1+2\cos\delta = 0$, the intensity equals zero and is a minimum.

$$1+2\cos\delta = 0 \rightarrow \delta = \cos^{-1}\left(-\tfrac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots$$

Maxima occur for $\sin\delta = 0$, which also says $\cos\delta = \pm 1$.

$$\sin\delta = 0 \rightarrow \delta = \sin^{-1} 0 = 0, \pi, 2\pi, 3\pi, \ldots$$

When $\cos\delta = 1$, the intensity is a principal maximum. When $\cos\delta = -1$, the intensity is a secondary maximum.

$$I_\theta(0) = I_0\frac{\left(1+2\cos\delta\right)^2}{9} = I_0\frac{\left(1+2\cos 0\right)^2}{9} = I_0$$

$$I_\theta(\pi) = I_0\frac{\left(1+2\cos\pi\right)^2}{9} = I_0\frac{\left(1+2(-1)\right)^2}{9} = \frac{I_0}{9}$$

$$I_\theta(2\pi) = I_0\frac{\left(1+2\cos 2\pi\right)^2}{9} = I_0\frac{\left(1+2\right)^2}{9} = I_0$$

Thus we see that, since $\cos\delta$ alternates between +1 and –1, there is only a single secondary maximum between each principal maximum.

24. The angular resolution is given by Eq. 35-10.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times10^{-9}\text{ m}}{254\times10^{-2}\text{ m}} = 2.69\times10^{-7}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{0.055''}$$

$\boxed{25.}$ The angular resolution is given by Eq. 35-10. The distance between the stars is the angular resolution times the distance to the stars from the Earth.

$$\theta = 1.22\frac{\lambda}{D}\ ;\ \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{\left(16\text{ly}\right)\left(\dfrac{9.46\times10^{15}\text{m}}{1\text{ly}}\right)\left(550\times10^{-9}\text{m}\right)}{\left(0.66\text{m}\right)} = \boxed{1.5\times10^{11}\text{m}}$$

26. We find the angle θ subtended by the planet by dividing the orbital radius by the distance of the star to the earth. Then using Eq. 35-10 we calculate the minimum diameter aperture needed to resolve this angle.

$$\theta = \frac{r}{d} = \frac{1.22\lambda}{D} \quad \to$$

$$D = \frac{1.22\lambda d}{r} = \frac{1.22\left(550\times 10^{-9}\text{m}\right)\left(4\,\text{ly}\right)\left(9.461\times 10^{15}\,\text{m/ly}\right)}{\left(1\,\text{AU}\right)\left(1.496\times 10^{11}\,\text{m/AU}\right)} = 0.17\,\text{m} \approx \boxed{20\,\text{cm}}$$

27. We find the angular half-width of the flashlight beam using Eq. 35-10 with $D = 5$ cm and $\lambda = 550$ nm. We set the diameter of the beam equal to twice the radius, where the radius of the beam is equal to the angular half-width multiplied by the distance traveled, $3.84\times 10^{8}\,\text{m}$.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22\left(550\times 10^{-9}\,\text{m}\right)}{0.050\ \text{m}} = 1.3\times 10^{-5}\,\text{rad}$$

$$d = 2\left(r\theta\right) = 2\left(3.84\times 10^{8}\,\text{m}\right)\left(1.3\times 10^{-5}\,\text{rad}\right) = \boxed{1.0\times 10^{4}\,\text{m}}$$

28. To find the focal length of the eyepiece we use Eq. 33-7, where the objective focal length is 2.00 m, *θ'* is the ratio of the minimum resolved distance and 25 cm, and θ is the ratio of the object on the moon and the distance to the moon. We ignore the inversion of the image.

$$\frac{f_o}{f_e} = \frac{\theta'}{\theta} \to f_e = f_o\frac{\theta}{\theta'} = f_o\frac{\left(d_o/\ell\right)}{\left(d/N\right)} = \left(2.0\ \text{m}\right)\frac{\left(7.5\ \text{km}/384{,}000\ \text{km}\right)}{\left(0.10\ \text{mm}/250\ \text{mm}\right)} = 0.098\ \text{m} = \boxed{9.8\ \text{cm}}$$

We use Eq. 35-10 to determine the resolution limit.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times 10^{-9}\ \text{m}}{0.11\ \text{m}} = \boxed{6.2\times 10^{-6}\ \text{rad}}$$

This corresponds to a minimum resolution distance, $r = \left(384{,}000\,\text{km}\right)\left(6.2\times 10^{-6}\,\text{rad}\right) = \boxed{2.4\,\text{km}}$, which is smaller than the 7.5 km object we wish to observe.

29. We set the resolving power as the focal length of the lens multiplied by the angular resolution, as in Eq. 35-11. The resolution is the inverse of the resolving power.

$$\frac{1}{RP(f/2)} = \left[\frac{1.22\lambda f}{D}\right]^{-1} = \frac{D}{1.22\lambda f} = \frac{25\ \text{mm}}{1.22\left(560\times 10^{-6}\text{mm}\right)\left(50.0\ \text{mm}\right)} = \boxed{730\ \text{lines/mm}}$$

$$\frac{1}{RP(f/16)} = \frac{3.0\ \text{mm}}{1.22\left(560\times 10^{-6}\text{mm}\right)\left(50.0\ \text{mm}\right)} = \boxed{88\ \text{lines/mm}}$$

30. We use Eq. 35-13 to calculate the angle for the second order maximum.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2\left(480\times 10^{-9}\ \text{m}\right)}{1.35\times 10^{-5}\ \text{m}}\right) = \boxed{4.1^\circ}$$

31. We use Eq. 35-13 to calculate the wavelengths from the given angles. The slit separation, *d*, is the inverse of the number of lines per cm, *N*. We assume that 12,000 is good to 3 significant figures.

$$d\sin\theta = m\lambda \to \lambda = \frac{\sin\theta}{Nm}$$

$$\lambda_1 = \frac{\sin 28.8^\circ}{12{,}000\ /\text{cm}} = 4.01\times10^{-5}\ \text{cm} = \boxed{401\ \text{nm}} \qquad \lambda_2 = \frac{\sin 36.7^\circ}{12{,}000\ /\text{cm}} = 4.98\times10^{-5}\ \text{cm} = \boxed{498\ \text{nm}}$$

$$\lambda_3 = \frac{\sin 38.6^\circ}{12{,}000\ /\text{cm}} = 5.201\times10^{-5}\ \text{cm} = \boxed{520\ \text{nm}} \qquad \lambda_4 = \frac{\sin 47.9^\circ}{12{,}000\ /\text{cm}} = 6.18\times10^{-5}\ \text{cm} = \boxed{618\ \text{nm}}$$

32. We use Eq. 35-13 to find the wavelength, where the number of lines, *N,* is the inverse of the slit separation, or $d=1/N$.

$$d\sin\theta = m\lambda \to \lambda = \frac{\sin\theta}{mN} = \frac{\sin 26.0^\circ}{3(3500\ /\text{cm})} = 4.17\times10^{-5}\text{cm} \approx \boxed{420\,\text{nm}}$$

33. Because the angle increases with wavelength, to have a complete order we use the largest wavelength. We set the maximum angle is 90° to determine the largest integer *m* in Eq. 35-13.

$$d\sin\theta = m\lambda \to m = \frac{\sin\theta}{\lambda N} = \frac{\sin 90^\circ}{(700\times10^{-9}\ \text{m})(6800\ /\text{cm})(100\ \text{cm/m})} = 2.1$$

Thus, $\boxed{\text{two}}$ full spectral orders can be seen on each side of the central maximum, and a portion of the third order.

34. We find the slit separation from Eq. 35-13. Then set the number of lines per centimeter equal to the inverse of the slit separation, $N=1/d$.

$$d\sin\theta = m\lambda \to N = \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 15.0^\circ}{3(650\times10^{-7}\text{cm})} = \boxed{1300\,\text{lines/cm}}$$

35. Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. We solve Eq. 35-13 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

$$d\sin\theta = m\lambda \quad\Rightarrow\quad d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \quad\Rightarrow\quad \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6^\circ}{\sin 53.2^\circ}(632.8\,\text{nm}) = \boxed{556\,\text{nm}}$$

36. We find the first order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Then we set the distance from the central maximum of the maximum and minimum wavelength equal to the distance to the screen multiplied by the tangent of the first order angle. The width of the spectrum is the difference in these distances.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[(410\times10^{-7}\text{cm})(7800\ \text{lines/cm})\right] = 18.65^\circ$$

$$\theta_2 = \sin^{-1}\left[(750\times10^{-7}\text{cm})(7800\ \text{lines/cm})\right] = 35.80^\circ$$

$$\Delta y = y_2 - y_1 = \ell(\tan\theta_2 - \tan\theta_1) = (2.80\ \text{m})(\tan 35.80^\circ - \tan 18.65^\circ) = \boxed{1.1\text{m}}$$

$\boxed{37.}$ We find the second order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Subtracting these two angles gives the angular width.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[2\left(4.5\times10^{-7}\text{ m}\right)\left(6.0\times10^5\text{ /m}\right)\right] = 32.7^\circ$$
$$\theta_2 = \sin^{-1}\left[2\left(7.0\times10^{-7}\text{ m}\right)\left(6.0\times10^5\text{ /m}\right)\right] = 57.1^\circ$$
$$\Delta\theta = \theta_2 - \theta_1 = 57.1^\circ - 32.7^\circ = \boxed{24^\circ}$$

38. The $m=1$ brightness maximum for the wavelength of 1200 nm occurs at angle θ. At this same angle $m=2$, $m=3$, etc. brightness maximum will form for other wavelengths. To find these wavelengths, we use Eq. 35-13, where the right hand side of the equation remains constant, and solve for the wavelengths of higher order.

$$d\sin\theta = m_1\lambda_1 = m\lambda_m \Rightarrow \lambda_m = \frac{m_1\lambda_1}{m} = \frac{\lambda_1}{m}$$
$$\lambda_2 = \frac{1200\text{ nm}}{2} = 600\text{ nm} \qquad \lambda_3 = \frac{1200\text{ nm}}{3} = 400\text{ nm} \qquad \lambda_4 = \frac{1200\text{ nm}}{4} = 300\text{ nm}$$

Higher order maxima will have shorter wavelengths. Therefore in the range 360 nm to 2000 nm, the only wavelengths that have a maxima at the angle θ are $\boxed{\text{600 nm and 400 nm}}$ besides the 1200 nm.

39. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order, using Eq. 35-13, by calculating the ratio of the sines for each angle. Since this ratio is greater than one, the maximum angle for the second order is larger than the minimum angle for the first order and the spectra overlap.

$$d\sin\theta = m\lambda \quad\to\quad \sin\theta = \left(\frac{m\lambda}{d}\right) \;;\; \frac{\sin\theta_2}{\sin\theta_3} = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} = \frac{2\left(700\text{ nm}\right)}{3\left(400\text{ nm}\right)} = 1.2$$

To determine which wavelengths overlap, we set this ratio of sines equal to one and solve for the second order wavelength that overlaps with the shortest wavelength of the third order. We then repeat this process to find the wavelength of the third order that overlaps with the longest wavelength of the second order.

$$\frac{\sin\theta_2}{\sin\theta_3} = 1 = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} \quad\to\quad \lambda_3 = \frac{2}{3}\lambda_{2,\text{max}} = \frac{2}{3}\left(700\text{ nm}\right) = 467\text{ nm}$$
$$\to\quad \lambda_2 = \frac{3}{2}\lambda_{3,\text{min}} = \frac{3}{2}\left(400\text{ nm}\right) = 600\text{ nm}$$

Therefore, the wavelengths $\boxed{\text{600 nm – 700 nm of the second order}}$ overlap with the wavelengths $\boxed{\text{400 nm – 467 nm of the third order.}}$ Note that these wavelengths are independent of the slit spacing.

40. We set the diffraction angles as one half the difference between the angles on opposite sides of the center. Then we solve Eq. 35-13 for the wavelength, with *d* equal to the inverse of the number of lines per centimeter.

$$\theta_1 = \frac{\theta_r - \theta_\ell}{2} = \frac{26^\circ38' - \left(-26^\circ18'\right)}{2} = 26^\circ28' = 26 + 28/60 = 26.47^\circ$$
$$\lambda_1 = d\sin\theta = \frac{\sin\theta}{N} = \frac{\sin 26.47^\circ}{9650\text{ line/cm}} = 4.618\times10^{-5}\text{ cm} = \boxed{462\text{ nm}}$$
$$\theta_2 = \frac{\theta_{2r} - \theta_{2\ell}}{2} = \frac{41^\circ02' - \left(-40^\circ27'\right)}{2} = 40^\circ44.5' = 40 + 44.5/60 = 40.742^\circ$$
$$\lambda_2 = \frac{\sin 40.742^\circ}{9650\text{ line/cm}} = 6.763\times10^{-5}\text{ cm} = \boxed{676\text{ nm}}$$

41. If the spectrometer were immersed in water, the wavelengths calculated in Problem 40 would be wavelengths in water. To change those wavelengths into wavelengths in air, we must multiply by the index of refraction.

$$\lambda_{\underset{\text{air}}{1}} = \left(4.618\times10^{-5}\,\text{cm}\right)(1.33) = \boxed{614\,\text{nm}} \;;\; \lambda_{\underset{\text{air}}{2}} = \left(6.763\times10^{-5}\,\text{cm}\right)(1.33) = \boxed{899\,\text{nm}}$$

Note that the second wavelength is not in the visible range.

42. We solve Eq. 35-13 for the slit separation width, *d,* using the given information. Then setting *m*=3, we solve for the angle of the third order maximum.

$$\sin\theta = \frac{m\lambda}{d} \;\to\; d = \frac{m\lambda}{\sin\theta} = \frac{1(589\,\text{nm})}{\sin 16.5^\circ} = 2074\,\text{nm} = \boxed{2.07\,\mu\text{m}}$$

$$\theta_3 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times589\,\text{nm}}{2074\,\text{nm}}\right) = \boxed{58.4^\circ}$$

$\boxed{43.}$ We find the angle for each "boundary" color from Eq. 35-13, and then use the fact that the displacement on the screen is given by $\tan\theta = \frac{y}{L}$, where *y* is the displacement on the screen from the central maximum, and *L* is the distance from the grating to the screen.

$$\sin\theta = \frac{m\lambda}{d} \;;\; d = \frac{1}{610\,\text{lines/mm}}\left(\frac{1\text{m}}{10^3\text{mm}}\right) = \left(1/6.1\times10^5\right)\text{m} \;;\; y = L\tan\theta = L\tan\left[\sin^{-1}\frac{m\lambda}{d}\right]$$

$$\ell_1 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= (0.32\,\text{m})\left\{\tan\left[\sin^{-1}\frac{(1)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(1)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.0706\,\text{m} \approx \boxed{7\,\text{cm}}$$

$$\ell_2 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= (0.32\,\text{m})\left\{\tan\left[\sin^{-1}\frac{(2)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(2)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.3464\,\text{m} \approx \boxed{35\,\text{cm}}$$

The $\boxed{\text{second order}}$ rainbow is dispersed over a larger distance.

44. (*a*) Missing orders occur when the angle to the interference maxima (Eq. 34-2a) is equal to the angle of a diffraction minimum (Eq. 35-2). We set $d = 2D$ and show that the even interference orders are missing.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \;\to\; \frac{m_1}{m_2} = \frac{d}{D} = \frac{2D}{D} = 2 \;\to\; m_1 = 2m_2$$

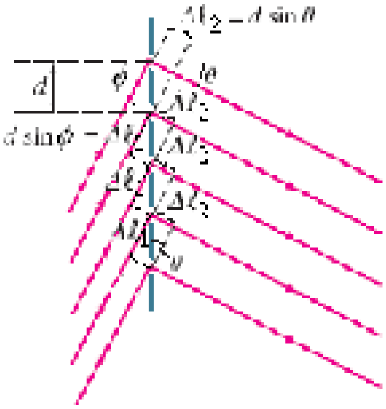
Since $m_2 = 1,2,3,4,\ldots,$ all even orders of m_1 correspond to the diffraction minima and will be missing from the interference pattern.

(*b*) Setting the angle of interference maxima equal to the angle of diffraction minimum, with the orders equal to integers we determine the relationship between the slit size and separation that will produce missing orders.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \quad \rightarrow \quad \boxed{\frac{d}{D} = \frac{m_1}{m_2}}$$

(*c*) When $d = D$, all interference maxima will overlap with diffraction minima so that no fringes will exist. This is expected because if the slit width and separation distance are the same, the slits will merge into one single opening.

45. (*a*) Diffraction maxima occur at angles for which the incident light constructive interferes. That is, when the path length difference between two rays is equal to an integer number of wavelengths. Since the light is incident at an angle ϕ relative to the grating, each succeeding higher ray, as shown in the diagram, travels a distance $\Delta\ell_1 = d\sin\phi$ farther to reach the grating. After passing through the grating the higher rays travel a distance to the screen that is again longer by $\Delta\ell_2 = d\sin\theta$. By setting the total path length difference equal to an integer number of wavelengths, we are able to determine the location of the bright fringes.



$$\Delta\ell = \Delta\ell_1 + \Delta\ell_1 = d\left(\sin\phi + \sin\theta\right) = \pm m\lambda, \qquad m = 0,1,2,....$$

(*b*) The ± allows for the incident angle and the diffracted angle to have positive and negative values.

(*c*) We insert the given data, with *m*=1, to solve for the angles θ.

$$\theta = \sin^{-1}\left(-\sin\phi \pm \frac{m\lambda}{d}\right) = \sin^{-1}\left(-\sin 15^\circ \pm \frac{550\times10^{-9}\,\text{m}}{0.01\,\text{m}/5000\,\text{lines}}\right) = \boxed{0.93^\circ \text{ and } -32^\circ}$$

46. Using Eq. 35-13 we calculate the maximum order possible for this diffraction grating, by setting the angle equal to 90°. Then we set the resolving power equal to the product of the number of grating lines and the order, where the resolving power is the wavelength divided by the minimum separation in wavelengths (Eq. 35-19) and solve for the separation.

$$\sin\theta = \frac{m\lambda}{d} \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda} = \frac{\left(0.01\,\text{m}/6500\,\text{lines}\right)\sin 90^\circ}{624\times10^{-9}\text{m}} = 2.47 \approx 2$$

$$\frac{\lambda}{\Delta\lambda} = Nm \;\Rightarrow\; \Delta\lambda = \frac{\lambda}{Nm} = \frac{624\text{ nm}}{\left(6500\text{ lines/cm}\right)\left(3.18\text{ cm}\right)\left(2\right)} = \boxed{0.015\text{ nm}}$$

The resolution is best for the $\boxed{\text{second order}}$, since it is more spread out than the first order.

47. (*a*) The resolving power is given by Eq. 35-19.

$$\text{R} = Nm \;\rightarrow\; \text{R}_1 = \left(16{,}000\right)\left(1\right) = \boxed{16{,}000} \;;\; \text{R}_1 = \left(16{,}000\right)\left(2\right) = \boxed{32{,}000}$$

(*b*) The wavelength resolution is also given by Eq. 35-19.

$$\text{R} = \frac{\lambda}{\Delta\lambda} = Nm \;\rightarrow\; \Delta\lambda = \frac{\lambda}{Nm}$$

$$\Delta\lambda_1 = \frac{410\,\text{nm}}{\left(16{,}000\right)\left(1\right)} = 2.6\times10^{-2}\,\text{nm} = \boxed{26\,\text{pm}} \;;\; \Delta\lambda_1 = \frac{410\,\text{nm}}{\left(32{,}000\right)\left(1\right)} = 1.3\times10^{-2}\,\text{nm} = \boxed{13\,\text{pm}}$$

48. (*a*) We use Eq. 35-13, with the angle equal to 90° to determine the maximum order.

$$\sin\theta = \frac{m\lambda}{d} \quad\rightarrow\quad m = \frac{d\sin\theta}{\lambda} = \frac{(1050\,\text{nm})\sin 90^\circ}{580\,\text{nm}} = 1.81$$

Since the order must be an integer number there will only be one principal maximum on either side of the central maximum. Counting the central maximum and the two other principal maxima there will be a total of $\boxed{\text{three principal maxima.}}$

(*b*) We use Eq. 35-17 to calculate the peak width, where the full peak width is double the half-peak width and the angle to the peak is given by Eq. 35-13.

$$\theta_0 = 0$$

$$\Delta\theta_0 = 2\frac{\lambda}{Nd\cos\theta_0} = \frac{2\lambda}{\ell\cos\theta_0} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos 0^\circ} = 6.4\times10^{-5}\,\text{rad} = \boxed{0.0037^\circ}$$

$$\theta_{\pm1} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{\pm1\times580\,\text{nm}}{1050\,\text{nm}}\right) = \pm33.5^\circ$$

$$\Delta\theta_{\pm1} = \frac{2\lambda}{\ell\cos\theta_{\pm1}} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos(\pm33.5^\circ)} = 7.7\times10^{-5}\,\text{rad} = \boxed{0.0044^\circ}$$

$\boxed{49.}$ We use Eq. 35-20, with $m = 1$.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \phi = \sin^{-1}\frac{m\lambda}{2d} = \sin^{-1}\frac{(1)(0.138\,\text{nm})}{2(0.285\,\text{nm})} = \boxed{14.0^\circ}$$

50. We use Eq. 35-20 for X-ray diffraction.

(*a*) Apply Eq. 35-20 to both orders of diffraction.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \frac{m_1}{m_2} = \frac{\sin\phi_1}{\sin\phi_2} \quad\rightarrow\quad \phi_2 = \sin^{-1}\left(\frac{m_2}{m_1}\sin\phi_1\right) = \sin^{-1}\left(\frac{2}{1}\sin 26.8^\circ\right) = \boxed{64.4^\circ}$$

(*b*) Use the first order data.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \lambda = \frac{2d\sin\phi}{m} = \frac{2(0.24\,\text{nm})\sin 26.8^\circ}{1} = \boxed{0.22\,\text{nm}}$$

51. For each diffraction peak, we can measure the angle and count the order. Consider Eq. 35-20.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \lambda = 2d\sin\phi_1 \;;\; 2\lambda = 2d\sin\phi_2 \;;\; 3\lambda = 2d\sin\phi_3$$

From each equation, all we can find is the ratio $\frac{\lambda}{d} = 2\sin\phi = \sin\phi_2 = \frac{2}{3}\sin\phi_3$. $\boxed{\text{No}}$, we cannot separately determine the wavelength or the spacing.

52. Use Eq. 35-21. Since the initial light is unpolarized, the intensity after the first polarizer will be half the initial intensity. Let the initial intensity be I_0.

$$I_1 = \tfrac{1}{2}I_0 \;;\; I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \quad\rightarrow\quad \frac{I_2}{I_0} = \frac{\cos^2 65^\circ}{2} = \boxed{0.089}$$

53. If I_0 is the intensity passed by the first Polaroid, the intensity passed by the second will be I_0 when the two axes are parallel. To calculate a reduction to half intensity, we use Eq. 35-21.

$$I = I_0\cos^2\theta = \tfrac{1}{2}I_0 \quad\rightarrow\quad \cos^2\theta = \tfrac{1}{2} \quad\rightarrow\quad \theta = \boxed{45^\circ}$$

54. We assume that the light is coming from air to glass, and use Eq. 35-22b.

$$\tan\theta_\mathrm{p} = n_\mathrm{glass} = 1.58 \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}1.58 = \boxed{57.7^\circ}$$

$\boxed{55.}$ The light is traveling from water to diamond. We use Eq. 35-22a.

$$\tan\theta_\mathrm{p} = \frac{n_\mathrm{diamond}}{n_\mathrm{water}} = \frac{2.42}{1.33} = 1.82 \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}1.82 = \boxed{61.2^\circ}$$

56. The critical angle exists when light passes from a material with a higher index of refraction (n_1) into a material with a lower index of refraction (n_2). Use Eq. 32-7.

$$\frac{n_2}{n_1} = \sin\theta_\mathrm{C} = \sin 55^\circ$$

To find the Brewster angle, use Eq. 35-22a. If light is passing from high index to low index, we have the following.

$$\frac{n_2}{n_1} = \tan\theta_\mathrm{p} = \sin 55^\circ \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}\left(\sin 55^\circ\right) = \boxed{39^\circ}$$

If light is passing from low index to high index, we have the following.

$$\frac{n_1}{n_2} = \tan\theta_\mathrm{p} = \frac{1}{\sin 55^\circ} \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}\left(\frac{1}{\sin 55^\circ}\right) = \boxed{51^\circ}$$

57. Let the initial intensity of the unpolarized light be I_0. The intensity after passing through the first Polaroid will be $I_1 = \frac{1}{2}I_0$. Then use Eq. 35-21.

$$I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \quad\rightarrow\quad \theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}}$$

(*a*) $\theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}} = \cos^{-1}\sqrt{\frac{2}{3}} = \boxed{35.3^\circ}$

(*b*) $\theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}} = \cos^{-1}\sqrt{\frac{2}{10}} = \boxed{63.4^\circ}$

58. For the first transmission, the angle between the light and the polarizer is 18.0°. For the second transmission, the angle between the light and the polarizer is 36.0°. Use Eq. 35-21 twice.

$$I_1 = I_0\cos^2 18.0^\circ \ ;\ I_2 = I_1\cos^2 36.0^\circ = I_0\cos^2 18.0^\circ\cos^2 36.0^\circ = 0.592 I_0$$

Thus the transmitted intensity is $\boxed{59.2\%}$ of the incoming intensity.

59. First case: the light is coming from water to air. Use Eq. 35-22a.

$$\tan\theta_\mathrm{p} = \frac{n_\mathrm{air}}{n_\mathrm{water}} \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}\frac{n_\mathrm{air}}{n_\mathrm{water}} = \tan^{-1}\frac{1.00}{1.33} = \boxed{36.9^\circ}$$

Second case: for total internal reflection, the light must also be coming from water into air. Use Eq. 32-7.

$$\sin\theta_\mathrm{C} = \frac{n_\mathrm{air}}{n_\mathrm{water}} \quad\rightarrow\quad \theta_\mathrm{p} = \sin^{-1}\frac{n_\mathrm{air}}{n_\mathrm{water}} = \sin^{-1}\frac{1.00}{1.33} = \boxed{48.8^\circ}$$

Third case: the light is coming from air to water. Use Eq. 35-22b.

$$\tan\theta_\mathrm{p} = n_\mathrm{water} \quad\rightarrow\quad \theta_\mathrm{p} = \tan^{-1}n_\mathrm{water} = \tan^{-1}1.33 = \boxed{53.1^\circ}$$

Note that the two Brewster's angles add to give 90.0°.

60. When plane-polarized light passes through a sheet oriented at an angle θ, the intensity decreases according to Eq. 35-21, $I = I_0\cos^2\theta$. For $\theta = 45°$, $\cos^2\theta = \frac{1}{2}$. Thus sheets 2 through 6 will each reduce the intensity by a factor of $\frac{1}{2}$. The first sheet reduces the intensity of the unpolarized incident light by $\frac{1}{2}$ as well. Thus we have the following.

$$I = I_0\left(\tfrac{1}{2}\right)^6 = \boxed{0.016 I_0}$$

$\boxed{61.}$ We assume vertically polarized light of intensity I_0 is incident upon the first polarizer. The angle between the polarization direction and the polarizer is θ. After the light passes that first polarizer, the angle between that light and the next polarizer will be $90° - \theta$. Apply Eq. 35-21.

$$I_1 = I_0\cos^2\theta \ \ ; \ I = I_1\cos^2(90° - \theta) = I_0\cos^2\theta\cos^2(90° - \theta) = \boxed{I_0\cos^2\theta\sin^2\theta}$$

We can also use the trigonometric identity $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ to write the final intensity as

$$I = I_0\cos^2\theta\sin^2\theta = \boxed{\tfrac{1}{4}I_0\sin^2 2\theta}.$$

$$\frac{dI}{d\theta} = \frac{d}{d\theta}\left(\tfrac{1}{4}I_0\sin^2 2\theta\right) = \tfrac{1}{4}I_0(2\sin 2\theta)(\cos 2\theta)2 = I_0\sin 2\theta\cos 2\theta = \boxed{\tfrac{1}{2}I_0\sin 4\theta}$$

$$\tfrac{1}{2}I_0\sin 4\theta = 0 \ \rightarrow \ 4\theta = 0, 180°, 360° \ \rightarrow \ \theta = 0, 45°, 90°$$

Substituting the three angles back into the intensity equation, we see that the angles $0°$ and $90°$ both give minimum intensity. The angle $\boxed{45° \text{ gives the maximum intensity}}$ of $\frac{1}{4}I_0$.

62. We set the intensity of the beam as the sum of the maximum and minimum intensities. Using Eq. 35-21, we determine the intensity of the beam after it has passed through the polarizer. Since I_{min} is polarized perpendicular to I_{max} and the polarizer is rotated at an angle ϕ from the polarization of I_{max}, the polarizer is oriented at an angle of $(90° - \phi)$ from I_{min}.

$$I_0 = I_{\text{max}} + I_{\text{min}}$$

$$I = I_0\cos^2\phi = I_{\text{max}}\cos^2\phi + I_{\text{min}}\cos^2(90 - \phi) = I_{\text{max}}\cos^2\phi + I_{\text{min}}\sin^2\phi$$

We solve the percent polarization equation for I_{min} and insert the result into our intensity equation.

$$p = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \ \rightarrow \ I_{\text{min}} = \frac{1-p}{1+p}I_{\text{max}}$$

$$I = I_{\text{max}}\cos^2\phi + \left(\frac{1-p}{1+p}I_{\text{max}}\right)\sin^2\phi = I_{\text{max}}\left[\frac{(1+p)\cos^2\phi + (1-p)\sin^2\phi}{1+p}\right]$$

$$= I_{\text{max}}\left[\frac{(\cos^2\phi + \sin^2\phi) + p(\cos^2\phi - \sin^2\phi)}{1+p}\right] = \boxed{I_{\text{max}}\left[\frac{1 + p\cos 2\phi}{1+p}\right]}$$

63. Because the width of the pattern is much smaller than the distance to the screen, the angles from the diffraction pattern for this first order will be small. Thus we may make the approximation that $\sin\theta = \tan\theta$. We find the angle to the first minimum from the distances, using half the width of the full first order pattern. Then we use Eq. 35-2 to find the slit width.

$$\tan\theta_{1\,\text{min}} = \tfrac{1}{2}\frac{(8.20\,\text{cm})}{(285\,\text{cm})} = 0.01439 = \sin\theta_{1\,\text{min}}$$

$$D\sin\theta = m\lambda \ \rightarrow \ D = \frac{m\lambda}{\sin\theta} = \frac{(1)(415\,\text{nm})}{0.01439} = 2.88\times10^4\,\text{nm} = \boxed{2.88\times10^{-5}\,\text{m}}$$

64. If the original intensity is I_0, the first polarizers will reduce the intensity to one half the initial intensity, or $I_1 = \frac{1}{2}I_0$. Each subsequent polarizer oriented at an angle θ to the preceding one will reduce the intensity by $\cos^2\theta$, as given by Eq. 35-21. We set the final intensity equal to one quarter of the initial intensity, with $\theta = 10°$ for each polarizer and solve for the minimum number of polarizers.

$$I = \tfrac{1}{2}I_0\left(\cos^2\theta\right)^{n-1} \Rightarrow n = 1 + \frac{\ln\left(2I/I_0\right)}{\ln\left(\cos^2\theta\right)} = 1 + \frac{\ln\left(2\times 0.25\right)}{\ln\left(\cos^2 10°\right)} = 23.6 \approx \boxed{24\text{ polarizers}}$$

We round the number of lenses up to the integer number of polarizers, so that the intensity will be less than 25% of the initial intensity.

65. The lines act like a grating. We assume that we see the first diffractive order, so $m = 1$. Use Eq. 35-13.

$$d\sin\theta = m\lambda \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{(1)(480\,\text{nm})}{\sin 56°} = \boxed{580\,\text{nm}}$$

66. We assume the sound is diffracted when it passes through the doorway, and find the angles of the minima from Eq. 35-2.

$$\lambda = \frac{v}{f}\ ;\ D\sin\theta = m\lambda = \frac{mv}{f} \rightarrow \theta = \sin^{-1}\frac{mv}{Df},\ m = 1, 2, 3, ...$$

$$m = 1:\ \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(1)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{27°}$$

$$m = 2:\ \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(2)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{65°}$$

$$m = 3:\ \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(3)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \sin^{-1}1.36 = \text{impossible}$$

Thus the whistle would not be heard clearly at angles of $\boxed{27° \text{ and } 65° \text{ on either side of the normal.}}$

$\boxed{67.}$ We find the angles for the first order from Eq. 35-13.

$$\theta_1 = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)\left(4.4\times10^{-7}\text{ m}\right)}{0.01\text{ m}/7600} = 19.5°$$

$$\theta_2 = \sin^{-1}\frac{(1)\left(6.8\times10^{-7}\text{ m}\right)}{0.01\text{ m}/7600} = 31.1°$$

The distances from the central white line on the screen are found using the tangent of the angle and the distance to the screen.

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 19.5° = 0.89\,\text{m}$$

$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 31.1° = 1.51\,\text{m}$$

Subtracting these two distances gives the linear separation of the two lines.

$$y_2 - y_1 = 1.51\,\text{m} - 0.89\,\text{m} = \boxed{0.6\,\text{m}}$$

68. Because the angle increases with wavelength, to miss a complete order we use the smallest visible wavelength, 400 nm. The maximum angle is 90°. With these parameters we use Eq. 35-13 to find the slit separation, *d*. The inverse of the slit separation gives the number of lines per unit length.

$$d\sin\theta = m\lambda \;\rightarrow\; d = \frac{m\lambda}{\sin\theta} = \frac{2(400\,\text{nm})}{\sin 90°} = \boxed{800\,\text{nm}}$$

$$\frac{1}{d} = \frac{1}{800\times 10^{-7}\,\text{cm}} = \boxed{12{,}500\,\text{lines/cm}}$$

69. We find the angles for the two first-order peaks from the distance to the screen and the distances along the screen to the maxima from the central peak.

$$\tan\theta_1 = \frac{y_1}{\ell} \;\rightarrow\; \theta_1 = \tan^{-1}\frac{y_1}{\ell} = \tan^{-1}\frac{(3.32\,\text{cm})}{(66.0\,\text{cm})} = 2.88°$$

$$\tan\theta_2 = \frac{y_2}{\ell} \;\rightarrow\; \theta_2 = \tan^{-1}\frac{y_2}{\ell} = \tan^{-1}\frac{(3.71\,\text{cm})}{(66.0\,\text{cm})} = 3.22°$$

Inserting the wavelength of yellow sodium light and the first order angle into Eq. 35-13, we calculate the separation of lines. Then, using the separation of lines and the second angle, we calculate the wavelength of the second source. Finally, we take the inverse of the line separation to determine the number of lines per centimeter on the grating.

$$d\sin\theta_1 = m\lambda_1 \;\rightarrow\; d = \frac{m\lambda_1}{\sin\theta_1} = \frac{1(589\text{ nm})}{\sin 2.88°} = 11{,}720\,\text{nm}$$

$$\lambda_2 = \frac{d\sin\theta_1}{m} = (11{,}720\,\text{nm})\sin 3.22° = \boxed{658\,\text{nm}}$$

$$\frac{1}{d} = \frac{1\text{ line}}{11{,}720\times 10^{-7}\,\text{cm}} = \boxed{853\,\text{lines/cm}}$$

70. We find the angles for the first order from Eq. 35-13, with $m = 1$. The slit spacing is the inverse of the lines/cm of the grating.

$$d = \frac{1}{8100\,\text{lines/cm}}\times\frac{1\,\text{m}}{100\,\text{cm}} = \frac{1}{8.1\times 10^{5}}\,\text{m} \;;\; d\sin\theta = m\lambda \;\rightarrow\; \theta = \sin^{-1}\frac{m\lambda}{d} \;\rightarrow$$

$$\Delta\theta = \sin^{-1}\frac{\lambda_1}{d} - \sin^{-1}\frac{\lambda_2}{d} = \sin^{-1}\frac{656\times 10^{-9}\,\text{m}}{\left(\dfrac{1}{8.1\times 10^{5}}\,\text{m}\right)} - \sin^{-1}\frac{410\times 10^{-9}\,\text{m}}{\left(\dfrac{1}{8.1\times 10^{5}}\,\text{m}\right)} = \boxed{13°}$$

71. (*a*) This is very similar to Example 35-6. We use the same notation as in that Example, and solve for the distance ℓ.

$$s = \ell\theta = \ell\frac{1.22\lambda}{D} \;\rightarrow\; \ell = \frac{Ds}{1.22\lambda} = \frac{(6.0\times 10^{-3}\,\text{m})(2.0\,\text{m})}{1.22(560\times 10^{-9}\,\text{m})} = \boxed{1.8\times 10^{4}\,\text{m}} = 18\,\text{km}$$

(*b*) We use the same data for the eye and the wavelength.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(560\times 10^{-9}\,\text{m})}{(6.0\times 10^{-3}\,\text{m})} = 1.139\times 10^{-4}\,\text{rad}\left(\frac{180°}{\pi\,\text{rad}}\right)\left(\frac{3600''}{1°}\right) = \boxed{23''}$$

Our answer is less than the real resolution, because of atmospheric effects and aberrations in the eye.

72. We first find the angular half-width for the first order, using Eq. 35-1, $\sin\theta = \dfrac{\lambda}{D}$. Since this angle is small, we may use the approximation that $\sin\theta \approx \tan\theta$. The width from the central maximum to the first minimum is given by $y = L\tan\theta$. That width is then doubled to find the width of the beam, from the first diffraction minimum on one side to the first diffraction minimum on the other side.

$$y = L\tan\theta = L\sin\theta$$

$$\Delta y = 2y = 2L\sin\theta = 2L\frac{\lambda}{D} = \frac{2\left(3.8\times10^{8}\,\text{m}\right)\left(633\times10^{-9}\,\text{m}\right)}{0.010\,\text{m}} = \boxed{4.8\times10^{4}\,\text{m}}$$

$\boxed{73.}$ The distance between lines on the diffraction grating is found by solving Eq. 35-13 for *d*, the grating spacing. The number of lines per meter is the reciprocal of *d*.

$$d = \frac{m\lambda}{\sin\theta} \quad\rightarrow\quad \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 21.5^\circ}{(1)6.328\times10^{-7}\,\text{m}} = \boxed{5.79\times10^{5}\,\text{lines/m}}$$

74. (*a*) We calculate the wavelength of the mother's sound by dividing the speed of sound by the frequency of her voice. We use Eq. 34-2b to determine the double slit interference minima with $d = 3.0$ m.

$$\lambda = v/f = (340\text{ m/s})/(400\text{ Hz}) = 0.85\text{ m}$$

$$\theta = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)\lambda}{d}\right] = \sin^{-1}\left[\frac{\left(m+\frac{1}{2}\right)(0.85\text{ m})}{(3.0\text{ m})}\right] = \sin^{-1}\left[0.2833\left(m+\tfrac{1}{2}\right)\right],\ m = 0,1,2,...$$

$$= \boxed{8.1^\circ,\ 25^\circ,\ 45^\circ,\ \text{and } 83^\circ}$$

We use Eq. 35-2 to determine the angles for destructive interference from single slit diffraction, with $D = 1.0$ m.

$$\theta = \sin^{-1}\left[\frac{m\lambda}{D}\right] = \sin^{-1}\left[\frac{m(0.85\text{ m})}{(1.0\text{ m})}\right] = \sin^{-1}\left[0.85m\right],\ m = 1,2,...$$

$$\theta = \boxed{58^\circ}$$

(*b*) We use the depth and length of the room to determine the angle the sound would need to travel to reach the son.

$$\theta = \tan^{-1}\left(\frac{8.0\text{ m}}{5.0\text{ m}}\right) = 58^\circ$$

This angle is close to the single slit diffraction minimum, so the son has a good explanation for not hearing her.

75. We use the Brewster angle, Eq. 35-22b, for light coming from air to water.

$$\tan\theta_{\text{p}} = n \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1} n = \tan^{-1} 1.33 = 53.1^\circ$$

This is the angle from the normal, as seen in Fig. 35-41, so the angle above the horizontal is the complement of $90.0^\circ - 53.1^\circ = \boxed{36.9^\circ}$.

76. (*a*) Let the initial unpolarized intensity be I_0. The intensity of the polarized light after passing the first polarizer is $I_1 = \frac{1}{2}I_0$. Apply Eq. 35-21 to find the final intensity.

$$I_2 = I_1\cos^2\theta = I_1\cos^2 90^\circ = \boxed{0}.$$

(*b*) Now the third polarizer is inserted. The angle between the first and second polarizers is 66°, so the angle between the second and third polarizers is 24°. It is still true that $I_1 = \frac{1}{2}I_0$.

$$I_2 = I_1\cos^2 66^\circ = \tfrac{1}{2}I_0\cos^2 66^\circ \ ; \ I_3 = I_2\cos^2 24^\circ = \tfrac{1}{2}I_0\cos^2 66^\circ\cos^2 24^\circ = 0.069 \ \rightarrow$$

$$\frac{I_3}{I_1} = \boxed{0.069}$$

(*c*) The two crossed polarizers, which are now numbers 2 and 3, will still not allow any light to pass through them if they are consecutive to each other. Thus $\frac{I_3}{I_1} = \boxed{0}$.

77. The reduction being investigated is that which occurs when the polarized light passes through the second Polaroid. Let I_1 be the intensity of the light that emerges from the first Polaroid, and I_2 be the intensity of the light after it emerges from the second Polaroid. Use Eq. 35-21.

(*a*) $I_2 = I_1\cos^2\theta = 0.25I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.25} = \boxed{60^\circ}$

(*b*) $I_2 = I_1\cos^2\theta = 0.10I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.10} = \boxed{72^\circ}$

(*c*) $I_2 = I_1\cos^2\theta = 0.010I_1 \ \rightarrow \ \theta = \cos^{-1}\sqrt{0.010} = \boxed{84^\circ}$

78. (*a*) We apply Eq. 35-21 through the successive polarizers. The initial light is unpolarized. Each polarizer is then rotated 30° from the previous one.

$$I_1 = \tfrac{1}{2}I_0 \ ; \ I_2 = I_1\cos^2\theta_2 = \tfrac{1}{2}I_0\cos^2\theta_2 \ ; \ I_3 = I_2\cos^2\theta_3 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3 \ ;$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_2\cos^2\theta_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 30^\circ\cos^2 30^\circ\cos^2 30^\circ = \boxed{0.21I_0}$$

(*b*) If we remove the second polarizer, then the angle between polarizers # 1 and # 3 is now 60°.

$$I_1 = \tfrac{1}{2}I_0 \ ; \ I_3 = I_1\cos^2\theta_3 = \tfrac{1}{2}I_0\cos^2\theta_3 \ ;$$

$$I_4 = I_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2\theta_3\cos^2\theta_4 = \tfrac{1}{2}I_0\cos^2 60^\circ\cos^2 30^\circ = 0.094\,I_0$$

The same value would result by removing the third polarizer, because then the angle between polarizers # 2 and # 4 would be 60°. Thus we can decrease the intensity by removing either the $\boxed{\text{second or third polarizer.}}$

(*c*) If we remove both the $\boxed{\text{second and third polarizers,}}$ we will have two polarizers with their axes perpendicular, so no light will be transmitted.

$\boxed{79.}$ For the minimum aperture the angle subtended at the lens by the smallest feature is the angular resolution, given by Eq. 35-10. We let ℓ represent the spatial separation, and *r* represent the altitude of the camera above the ground.

$$\theta = \frac{1.22\lambda}{D} = \frac{\ell}{r} \ \rightarrow \ D = \frac{1.22\lambda r}{\ell} = \frac{1.22\left(580\times10^{-9}\,\text{m}\right)\left(25000\,\text{m}\right)}{\left(0.05\,\text{m}\right)} = 0.3538\,\text{m} \approx \boxed{0.4\,\text{m}}$$

80. Let I_0 be the initial intensity. Use Eq. 35-21 for both transmissions of the light.

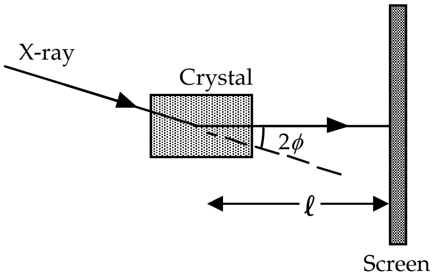
$$I_1 = I_0\cos^2\theta_1 \ ; \ I_2 = I_1\cos^2\theta_2 = I_0\cos^2\theta_1\cos^2\theta_2 = 0.25I_0 \ \rightarrow$$

$$\theta_1 = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos\theta_2}\right) = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos 48^\circ}\right) = \boxed{42^\circ}$$

81. We find the spacing from Eq. 35-20.

$$m\lambda = 2d\sin\phi \ \rightarrow \ d = \frac{m\lambda}{2\sin\phi} = \frac{(2)\left(9.73\times10^{-11}\,\text{m}\right)}{2\sin 23.4^\circ} = \boxed{2.45\times10^{-10}\,\text{m}}$$

82. The angles for Bragg scattering are found from Eq. 35-20, for $m = 1$ and $m = 2$. If the distance from the crystal to the screen is ℓ, the radius of the diffraction ring is given by $r = \ell \tan 2\phi$.



$$2d\sin\phi = m\lambda \ ; \ r = \ell\tan 2\phi = \ell\tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$r_1 = \ell\tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$= (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(1)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.059\,\text{m}}$$

$$r_2 = \ell\tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right] = (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(2)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.17\,\text{m}}$$

83. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between the Earth and Moon to obtain the minimum distance between two objects on the Moon that the Hubble can resolve.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\,\text{m})}{2.4\,\text{m}} = 2.796\times10^{-7}\,\text{rad}$$

$$\ell = s\theta = (3.84\times10^{8}\,\text{m})(2.796\times10^{-7}\,\text{rad}) = \boxed{110\,\text{m}}$$

84. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between Mars and Earth to obtain the minimum distance between two objects that can be resolved by a person on Mars

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\ \text{m})}{0.005\ \text{m}} = 1.34\times10^{-4}\ \text{rad}$$

$$\ell = s\theta = (8\times10^{10}\ \text{m})(1.34\times10^{-4}\ \text{rad}) = \boxed{1.07\times10^{7}\ \text{m}}$$

Since the minimum resolvable distance is much less than the Earth-Moon distance, a person standing on Mars could resolve the Earth and Moon as two separate objects without a telescope.

85. The distance x is twice the distance to the first minima. We can write x in terms of the slit width D using Eq. 35-2, with $m = 1$. The ratio $\frac{\lambda}{D}$ is small, so we may approximate $\sin\theta \approx \tan\theta \approx \theta$.

$$\sin\theta = \frac{\lambda}{D} \approx \theta \ ; \ x = 2y = 2\ell\tan\theta = 2\ell\theta = 2\ell\frac{\lambda}{D}$$

When the plate is heated up the slit width increases due to thermal expansion. Eq. 17-1b is used to determine the new slit width, with the coefficient of thermal expansion, α, given in Table 17-1. Each slit width is used to determine a value for x. Subtracting the two values for x gives the change Δx. We use the binomial expansion to simplify the evaluation.

$$\Delta x = x - x_0 = 2\ell\left(\frac{\lambda}{D_0(1+\alpha\Delta T)}\right) - 2\ell\left(\frac{\lambda}{D_0}\right) = \frac{2\ell\lambda}{D_0}\left(\frac{1}{(1+\alpha\Delta T)} - 1\right) = \frac{2\ell\lambda}{D_0}\left((1+\alpha\Delta T)^{-1} - 1\right)$$

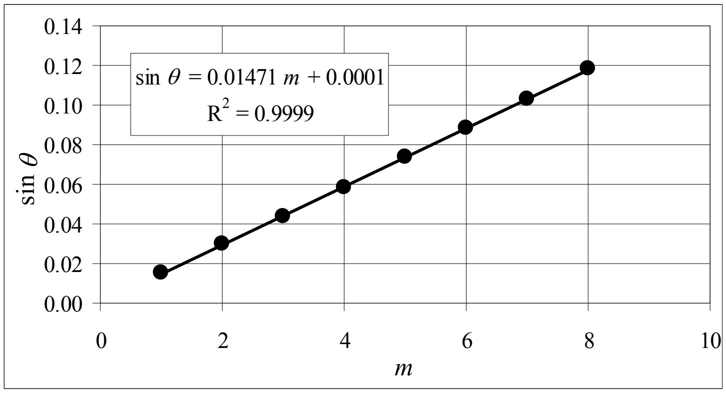
$$= \frac{2\ell\lambda}{D_0}(1-\alpha\Delta T - 1) = -\frac{2\ell\lambda}{D_0}\alpha\Delta T = -\frac{2(2.0\,\text{m})(650\times10^{-9}\,\text{m})}{(22\times10^{-6}\,\text{m})}\left[25\times10^{-6}\,(\text{C}^\circ)^{-1}\right](55\text{C}^\circ)$$

$$= \boxed{-1.7\times10^{-4}\,\text{m}}$$

86. The tangent of the angle for each order is the distance in the table divided by the distance to the screen. If we call the distance in the table y and the distance to the screen ℓ, then we have this relationship.

$$\tan\theta = \frac{y}{\ell} \quad\rightarrow\quad \theta = \tan^{-1}\frac{y}{\ell}$$

The relationship between the angle and the wavelength is given by Eq. 35-2, $D\sin\theta = m\lambda$, which can be written as $\sin\theta = \frac{\lambda}{D}m$. A plot of $\sin\theta$ vs. m should have a slope of $\frac{\lambda}{D}$, and so the wavelength can be determined from the slope and the slit width. The graph is shown, and the slope used to calculate the wavelength.



$$\frac{\lambda}{D} = \text{slope} \quad\rightarrow\quad \lambda = (\text{slope})D = (0.01471)(4.000\times10^{-5}\,\text{m}) = \boxed{588.4\,\text{nm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.86."

87. We have N polarizers providing a rotation of 90°. Thus, each polarizer must rotate the light by an angle of $\theta_N = (90/N)°$. As the light passes through each polarizer, the intensity will be reduced by a factor of $\cos^2\theta_N$. Let the original intensity be I_0.

$$I_1 = I_0\cos^2\theta_N \;;\; I_2 = I_1\cos^2\theta_N = I_0\cos^4\theta_N \;;\; I_3 = I_2\cos^2\theta_N = I_0\cos^6\theta_N$$

$$I_N = I_0(\cos\theta_N)^{2N} = 0.90I_0 \quad\rightarrow\quad [\cos(90°/N)]^{2N} = 0.90$$

We evaluate $[\cos(90°/N)]^{2N}$ for various values of N. A table for a few values of N is shown here. We see that N = 24 satisfies the criteria, and so $\theta_N = (90/24N)° = (90/24N)° = 3.75°$. So we need to $\boxed{\text{put 24 polarizers in the path of the original polarized light, each rotated } 3.75° \text{ from the previous one.}}$ The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.87."

N	$[\cos(90/N)]^{2N}$
21	0.8890
22	0.8938
23	0.8982
24	0.9022
25	0.9060

88. (*a*) The intensity of the diffraction pattern is given by Eqs. 35-6 and 35-7. We want to find the angle where $I = \frac{1}{2}I_0$. Doubling this angle will give the desired $\Delta\theta$.

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \tfrac{1}{2}I_0 \quad\rightarrow\quad \sin\beta/2 = \frac{\beta/2}{\sqrt{2}} \text{ or } \sin\alpha = \frac{\alpha}{\sqrt{2}}, \text{ with } \alpha = \tfrac{1}{2}\beta$$

This equation must be solved numerically. A spreadsheet was developed to find the non-zero values of α that satisfy $\sin\alpha - \frac{\alpha}{\sqrt{2}} = 0$. It is apparent from this expression that there will be no solutions for $\alpha > \sqrt{2}$. The only non-zero value is $\alpha = 1.392$. Now use Eq. 35-6 to find θ.

$$\beta = \frac{2\pi}{\lambda}D\sin\theta \quad\rightarrow\quad \theta = \sin^{-1}\frac{\lambda\beta}{2\pi D} = \sin^{-1}\frac{2\lambda\alpha}{2\pi D} = \sin^{-1}\frac{\lambda(1.392)}{\pi D} \;;$$

$$\Delta\theta = 2\theta = \boxed{2\sin^{-1}\frac{\lambda(1.392)}{\pi D}}$$

(*b*) For $D=\lambda$: $\Delta\theta = 2\sin^{-1}\dfrac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\dfrac{(1.392)}{\pi} = \boxed{52.6^\circ}$

For $D=100\lambda$: $\Delta\theta = 2\sin^{-1}\dfrac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\dfrac{(1.392)}{100\pi} = \boxed{0.508^\circ}$

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH35.XLS,” on tab “Problem 35.88.”

CHAPTER 36: The Special Theory of Relativity

Responses to Questions

1. No. The train is an inertial reference frame, and the laws of physics are the same in all inertial reference frames, so there is no experiment you can perform inside the train car to determine if you are moving.

2. The fact that you instinctively think you are moving is consistent with the relativity principle applied to mechanics. Even though you are at rest relative to the ground, when the car next to you creeps forward, you are moving backward relative to that car.

3. As long as the railroad car is traveling with a constant velocity, the ball will land back in his hand.

4. The relativity principle refers only to inertial reference frames. Neither the reference frame of the Earth nor the reference frame of the Sun is inertial. Either reference frame is valid, but the laws of physics will not be the same in each of the frames.

5. The starlight would pass at c, regardless of your spaceship's speed. This is consistent with the second postulate of relativity which states that the speed of light through empty space is independent of the speed of the source or the observer.

6. It deals with space-time (sometimes called "the fabric of space-time") and the actual passage of time in the reference frame, not with the mechanical workings of clocks. Any measurement of time (heartbeats or decay rates, for instance) would be measured as slower than normal when viewed by an observer outside the moving reference frame.

7. Time *actually* passes more slowly in the moving reference frames, according to observers outside the moving frames.

8. This situation is an example of the "twin paradox" applied to parent-child instead of to twins. This might be possible if the woman was traveling at high enough speeds during her trip. Time would have passed more slowly for her and she could have aged less than her son, who stayed on Earth. (Note that the situations of the woman and son are not symmetric; she must undergo acceleration during her journey.)

9. No, you would not notice any change in your heartbeat, mass, height, or waistline, because you are in the inertial frame of the spaceship. Observers on Earth, however, would report that your heartbeat is slower and your mass greater than if you were at rest with respect to them. Your height and waistline will depend on your orientation with respect to the motion. If you are "standing up" in the spaceship such that your height is perpendicular to the direction of travel, then your height would not change but your waistline would shrink. If you happened to be "lying down" so that your body is parallel to the direction of motion when the Earth observers peer through the telescope, then you would appear shorter but your waistline would not change.

10. Yes. However, at a speed of only 90 km/hr, v/c is very small, and therefore γ is very close to one, so the effects would not be noticeable.

11. Length contraction and time dilation would not occur. If the speed of light were infinite, *v/c* would be zero for all finite values of *v*, and therefore γ would always be one, resulting in $\Delta t = \Delta t_0$ and $\ell = \ell_0$.

12. The effects of special relativity, such as time dilation and length contraction, would be noticeable in our everyday activities because everyday speeds would no longer be so small compared to the speed of light. There would be no "absolute time" on which we would all agree, so it would be more difficult, for instance, to plan to meet friends for lunch at a certain time! In addition, 25 m/s would be the limiting speed and nothing in the universe would move faster than that.

$\boxed{13.}$ Both the length contraction and time dilation formulas include the term $\sqrt{1-v^2/c^2}$. If *c* were not the limiting speed in the universe, then it would be possible to have a situation with $v > c$. However, this would result in a negative number under the square root, which gives an imaginary number as a result, indicating that *c* must be the limiting speed.

14. Mr. Tompkins appears shrunk in the horizontal direction, since that is the direction of his motion, and normal size in the vertical direction, perpendicular to his direction of motion. This length contraction is a result of the fact that, to the people on the sidewalk, Mr. Tompkins is in a moving frame of reference. If the speed of light were only 20 mi/h, then the amount of contraction, which depends on γ, would be enough to be noticeable. Therefore, Mr. Tompkins and his bicycle appear very skinny. (Compare to the chapter-opening figure, which is shown from Mr. Tompkin's viewpoint. In this case, Mr. Tompkins sees himself as "normal" but all the objects moving with respect to him are contracted.)

15. No. The relativistic momentum of the electron is given by $p = \gamma mv = \dfrac{mv}{\sqrt{1-v^2/c^2}}$. At low speeds (compared to *c*) this reduces to the classical momentum, $p = mv$. As *v* approaches *c*, γ approaches infinity so there is no upper limit to the electron's momentum.

16. No. To accelerate a particle with nonzero rest mass up to the speed of light would require an infinite amount of energy, and so is not possible.

17. No. $E = mc^2$ does not conflict with the principle of conservation of energy as long as it is understood that mass is a form of energy.

18. Yes, mass is a form of energy so technically it is correct to say that a spring has more mass when compressed. However, the change in mass of the spring is very small and essentially negligible.

$\boxed{19.}$ "Energy can be neither created nor destroyed." Mass is a form of energy, and mass can be "destroyed" when it is converted to other forms of energy. The total amount of energy remains constant.

20. Technically yes, the notion that velocities simply add is wrong. However, at everyday speeds, the relativistic equations reduce to classical ones, so our ideas about velocity addition are essentially true for velocities that are low compared to the speed of light.

Solutions to Problems

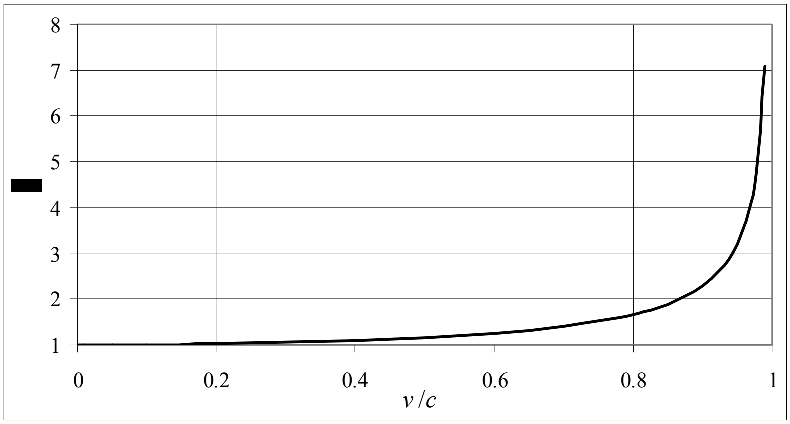
1. You measure the contracted length. Find the rest length from Eq. 36-3a.

$$\ell_0=\frac{\ell}{\sqrt{1-v^2/c^2}}=\frac{38.2\,\mathrm{m}}{\sqrt{1-(0.850)^2}}=\boxed{72.5\,\mathrm{m}}$$

2. We find the lifetime at rest from Eq. 36-1a.

$$\Delta t_0=\Delta t\sqrt{1-v^2/c^2}=\left(4.76\times10^{-6}\mathrm{s}\right)\sqrt{1-\left(\frac{2.70\times10^{8}\,\mathrm{m/s}}{3.00\times10^{8}\,\mathrm{m/s}}\right)^2}=\boxed{2.07\times10^{-6}\mathrm{s}}$$

3. The numerical values and graph were generated in a spreadsheet. The graph is shown also. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH36.XLS," on tab "Problem 36.3."



v/c	0.00	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
γ	1.000	1.000	1.001	1.005	1.021	1.048	1.091	1.155	1.250	1.400	1.667	2.294	7.089

4. The measured distance is the contracted length. Use Eq. 36-3a.

$$\ell=\ell_0\sqrt{1-v^2/c^2}=\left(135\text{ ly}\right)\sqrt{1-\left(\frac{2.80\times10^{8}\,\mathrm{m/s}}{3.00\times10^{8}\,\mathrm{m/s}}\right)^2}=\boxed{48.5\text{ ly}}$$

5. The speed is determined from the time dilation relationship, Eq. 36-1a.

$$\Delta t_0=\Delta t\sqrt{1-v^2/c^2}\quad\rightarrow$$

$$v=c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}=c\sqrt{1-\left(\frac{2.60\times10^{-8}\mathrm{s}}{4.40\times10^{-8}\mathrm{s}}\right)^2}=\boxed{0.807c}=2.42\times10^{8}\,\mathrm{m/s}$$

6. The speed is determined from the length contraction relationship, Eq. 36-3a.

$$\ell=\ell_0\sqrt{1-v^2/c^2}\quad\rightarrow\quad v=c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}=c\sqrt{1-\left(\frac{35\,\mathrm{ly}}{56\,\mathrm{ly}}\right)^2}=\boxed{0.78c}=2.3\times10^{8}\,\mathrm{m/s}$$

7. The speed is determined from the length contraction relationship, Eq. 36-3a. Then the time is found from the speed and the contracted distance.

$$\ell=\ell_0\sqrt{1-v^2/c^2}\quad\rightarrow$$

$$v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} \quad ; \quad t = \frac{\ell}{v} = \frac{\ell}{c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}} = \frac{25\,\text{ly}}{c\sqrt{1-\left(\frac{25\,\text{ly}}{65\,\text{ly}}\right)^2}} = \frac{(25\,\text{y})c}{c(0.923)} = \boxed{27\,\text{y}}$$

8. The speed is determined from the length contraction relationship, Eq. 36-3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-(0.900)^2} = \boxed{0.436c}$$

9. The change in length is determined from the length contraction relationship, Eq. 36-3a. The speed is very small compared to the speed of light.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow$$

$$\frac{\ell}{\ell_0} = \sqrt{1-v^2/c^2} = \left(1-\frac{v^2}{c^2}\right)^{1/2} \approx 1-\tfrac{1}{2}\frac{v^2}{c^2} = 1-\tfrac{1}{2}\left(\frac{11.2\times10^3\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2 = 1-6.97\times10^{-10}$$

So the percent decrease is $\boxed{\left(6.97\times10^{-8}\right)\%}$.

10. (*a*) The measured length is the contracted length. We find the rest length from Eq. 36-3a.

$$\ell_0 = \frac{\ell}{\sqrt{1-v^2/c^2}} = \frac{4.80\,\text{m}}{\sqrt{1-(0.760)^2}} = \boxed{7.39\,\text{m}}$$

Distances perpendicular to the motion do not change, so the rest height is $\boxed{1.35\,\text{m}}$.

(*b*) The time in the spacecraft is the rest time, found from Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (20.0\,\text{s})\sqrt{1-(0.760)^2} = \boxed{13.0\,\text{s}}$$

(*c*) To your friend, you moved at the same relative speed: $\boxed{0.760c}$.

(*d*) She would measure the same time dilation: $\boxed{13.0\,\text{s}}$.

11. (*a*) We use Eq. 36-3a for length contraction with the contracted length 99.0% of the rest length.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-(0.990)^2} = \boxed{0.141c}$$

(*b*) We use Eq. 36-1a for time dilation with the time as measured from a relative moving frame 1.00% greater than the rest time.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{1}{1.0100}\right)^2} = \boxed{0.140c}$$

We see that a speed of 0.14 c results in about a 1% relativistic effect.

12. (*a*) To an observer on Earth, 18.6 ly is the rest length, so the time will be the distance divided by the speed.

$$t_{\text{Earth}} = \frac{\ell_0}{v} = \frac{(18.6\text{ ly})}{0.950c} = 19.58\,\text{yr} \approx \boxed{19.6\,\text{yr}}$$

(*b*) The time as observed on the spacecraft is shorter. Use Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (19.58\,\text{yr})\sqrt{1-(0.950)^2} = 6.114\,\text{yr} \approx \boxed{6.11\,\text{yr}}$$

(*c*) To the spacecraft observer, the distance to the star is contracted. Use Eq. 36-3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} = (18.6\text{ ly})\sqrt{1-(0.950)^2} = 5.808\text{ ly} \approx \boxed{5.81\text{ ly}}$$

(*d*) To the spacecraft observer, the speed of the spacecraft is their observed distance divided by their observed time.

$$v = \frac{\ell}{\Delta t_0} = \frac{(5.808\text{ ly})}{6.114\text{ yr}} = \boxed{0.950c}$$

$\boxed{13.}$ (*a*) In the Earth frame, the clock on the *Enterprise* will run slower. Use Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (5.0\text{ yr})\sqrt{1-(0.74)^2} = \boxed{3.4\text{ yr}}$$

(*b*) Now we assume the 5.0 years is the time as measured on the *Enterprise*. Again use Eq. 36-1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad\rightarrow\quad \Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{(5.0\text{ yr})}{\sqrt{1-(0.74)^2}} = \boxed{7.4\text{ yr}}$$

14. We find the speed of the particle in the lab frame, and use that to find the rest frame lifetime and distance.

$$v = \frac{\Delta x_{\text{lab}}}{\Delta t_{\text{lab}}} = \frac{1.00\text{ m}}{3.40\times10^{-9}\text{ s}} = 2.941\times10^8\text{ m/s} = 0.9803c$$

(*a*) Find the rest frame lifetime from Eq. 36-1a.

$$\Delta t_0 = \Delta t_{\text{lab}}\sqrt{1-v^2/c^2} = (3.40\times10^{-9}\text{ s})\sqrt{1-(0.9803)^2} = \boxed{6.72\times10^{-10}\text{ s}}$$

(*b*) In its rest frame, the particle will travel the distance given by its speed and the rest lifetime.

$$\Delta x_0 = v\Delta t_0 = (2.941\times10^8\text{ m/s})(6.72\times10^{-10}\text{ s}) = \boxed{0.198\text{ m}}$$

This could also be found from the length contraction relationship: $\Delta x_0 = \dfrac{\Delta x_{\text{lab}}}{\sqrt{1-v^2/c^2}}$.

15. Since the number of particles passing per second is reduced from *N* to *N* / 2, a time T_0 must have elapsed in the particles' rest frame. The time *T* elapsed in the lab frame will be greater, according to Eq. 36-1a. The particles moved a distance of $2cT_0$ in the lab frame during that time.

$$T_0 = T\sqrt{1-v^2/c^2} \quad\rightarrow\quad T = \frac{T_0}{\sqrt{1-v^2/c^2}} \;;\; v = \frac{x}{T} = \frac{2cT_0}{\dfrac{T_0}{\sqrt{1-v^2/c^2}}} \quad\rightarrow\quad v = \sqrt{\tfrac{4}{5}}c = \boxed{0.894c}$$

16. The dimension along the direction of motion is contracted, and the other two dimensions are unchanged. Use Eq. 36-3a to find the contracted length.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \;;\; V = \ell(\ell_0)^2 = (\ell_0)^3\sqrt{1-v^2/c^2} = (2.0\text{ m})^3\sqrt{1-(0.80)^2} = \boxed{4.8\text{ m}^3}$$

17. The vertical dimensions of the ship will not change, but the horizontal dimensions will be contracted according to Eq. 36-3a. The base will be contracted as follows.

$$\ell_{\text{base}} = \ell\sqrt{1-v^2/c^2} = \ell\sqrt{1-(0.95)^2} = \boxed{0.31\ell}$$

When at rest, the angle of the sides with respect to the base is given by $\theta = \cos^{-1}\dfrac{0.50\ell}{2.0\ell} = 75.52°$.

The vertical component of $\ell_{\text{vert}} = 2\ell\sin\theta = 2\ell\sin 75.52° = 1.936\ell$ is unchanged. The horizontal component, which is $2\ell\cos\theta = 2\ell\left(\frac{1}{4}\right) = 0.50\ell$ at rest, will be contracted in the same way as the base.

$$\ell_{\text{horizontal}} = 0.50\ell\sqrt{1-v^2/c^2} = 0.50\ell\sqrt{1-(0.95)^2} = 0.156\ell$$

Use the Pythagorean theorem to find the length of the leg.

$$\ell_{\text{leg}} = \sqrt{\ell^2_{\text{horizontal}} + \ell^2_{\text{vert}}} = \sqrt{(0.156\ell)^2 + (1.936\ell)^2} = 1.942\ell \approx \boxed{1.94\ell}$$

18. In the Earth frame, the average lifetime of the pion will be dilated according to Eq. 36-1a. The speed of the pion will be the distance moved in the Earth frame times the dilated time.

$$v = \frac{d}{\Delta t} = \frac{d}{\Delta t_0}\sqrt{1-v^2/c^2} \quad \rightarrow$$

$$v = c\sqrt{\frac{1}{1+\left(\frac{c\Delta t_0}{d}\right)^2}} = c\sqrt{\frac{1}{1+\left(\frac{(3.00\times10^8\,\text{m/s})(2.6\times10^{-8}\,\text{s})}{(25\,\text{m})}\right)^2}} = \boxed{0.95c}$$

$\boxed{19.}$ We take the positive direction in the direction of the *Enterprise*. Consider the alien vessel as reference frame S, and the Earth as reference frame S′. The velocity of the Earth relative to the alien vessel is $v = -0.60c$. The velocity of the *Enterprise* relative to the Earth is $u'_x = 0.90c$. Solve for the velocity of the *Enterprise* relative to the alien vessel, u_x, using Eq. 36-7a.

$$u_x = \frac{(u'_x + v)}{\left(1+\frac{vu'_x}{c^2}\right)} = \frac{(0.90c - 0.60c)}{\left[1+(-0.60)(0.90)\right]} = \boxed{0.65c}$$

We could also have made the *Enterprise* as reference frame S, with $v = -0.90c$, and the velocity of the alien vessel relative to the Earth as $u'_x = 0.60c$. The same answer would result.

Choosing the two spacecraft as the two reference frames would also work. Let the alien vessel be reference frame S, and the *Enterprise* be reference frame S′. Then we have the velocity of the Earth relative to the alien vessel as $u_x = -0.60c$, and the velocity of the Earth relative to the *Enterprise* as $u'_x = -0.90c$. We solve for v, the velocity of the *Enterprise* relative to the alien vessel.

$$u_x = \frac{(u'_x + v)}{\left(1+\frac{vu'_x}{c^2}\right)} \quad \rightarrow \quad v = \frac{u_x - u'_x}{\left(1-\frac{u'_x u_x}{c^2}\right)} = \frac{(-.60c)-(-0.90c)}{\left(1-\frac{(-0.90c)(-.60c)}{c^2}\right)} = \boxed{0.65c}$$

20. The Galilean transformation is given in Eq. 36-4.

(*a*) $(x,y,z) = (x'+vt', y', z') = (25\,\text{m} + (30\,\text{m/s})(3.5\,\text{s}), 20\,\text{m}, 0) = \boxed{(130\,\text{m}, 20\,\text{m}, 0)}$

(*b*) $(x,y,z) = (x'+vt', y', z') = (25\,\text{m} + (30\,\text{m/s})(10.0\,\text{s}), 20\,\text{m}, 0) = \boxed{(325\,\text{m}, 20\,\text{m}, 0)}$

21. (*a*) The person's coordinates in S are found using Eq. 36-6, with $x' = 25$ m, $y' = 20$ m, $z' = 0$, and $t' = 3.5\,\mu\text{s}$. We set $v = 1.80\times10^8\,\text{m/s}$.

$$x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}} = \frac{25\,\text{m} + (1.8\times10^8\,\text{m/s})(3.5\,\mu\text{s})}{\sqrt{1-(1.8\times10^8\,\text{m/s})^2/(3.0\times10^8\,\text{m/s})^2}} = \boxed{820\,\text{m}}$$

$$y = y' = \boxed{20\,\text{m}} \;;\; z = z' = \boxed{0}$$

(*b*) We repeat part (*a*) using the time $t' = 10.0\,\mu\text{s}$.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} = \frac{25\,\text{m} + \left(1.8 \times 10^8\,\text{m/s}\right)\left(10.0\,\mu\text{s}\right)}{\sqrt{1 - \left(1.8 \times 10^8\,\text{m/s}\right)^2 \big/ \left(3.0 \times 10^8\,\text{m/s}\right)^2}} = \boxed{2280\,\text{m}}$$

$$y = y' = \boxed{20\,\text{m}} \; ; \; z = z' = \boxed{0}$$

22. We determine the components of her velocity in the S frame using Eq. 36-7, where $u'_x = u'_y = 1.10 \times 10^8\,\text{m/s}$ and $v = 1.80 \times 10^8\,\text{m/s}$. Then using trigonometry we combine the components to determine the magnitude and direction.

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2} = \frac{1.10 \times 10^8\,\text{m/s} + 1.80 \times 10^8\,\text{m/s}}{1 + \left(1.80 \times 10^8\,\text{m/s}\right)\left(1.10 \times 10^8\,\text{m/s}\right)/\left(3.00 \times 10^8\,\text{m/s}\right)^2} = 2.38 \times 10^8\,\text{m/s}$$

$$u_y = \frac{u'_y\sqrt{1 - v^2/c^2}}{1 + vu'_x/c^2} = \frac{\left(1.10 \times 10^8\,\text{m/s}\right)\sqrt{1 - \left(1.8 \times 10^8\,\text{m/s}\right)^2 \big/ \left(3.0 \times 10^8\,\text{m/s}\right)^2}}{1 + \left(1.80 \times 10^8\,\text{m/s}\right)\left(1.10 \times 10^8\,\text{m/s}\right)/\left(3.00 \times 10^8\,\text{m/s}\right)^2} = 7.21 \times 10^7\,\text{m/s}$$

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{\left(2.38 \times 10^8\,\text{m/s}\right)^2 + \left(7.21 \times 10^7\,\text{m/s}\right)^2} = \boxed{2.49 \times 10^8\,\text{m/s}}$$

$$\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\frac{7.21 \times 10^7\,\text{m/s}}{2.38 \times 10^8\,\text{m/s}} = \boxed{16.9^\circ}$$

23. (*a*) We take the positive direction to be the direction of motion of spaceship 1. Consider spaceship 2 as reference frame S, and the Earth reference frame S′. The velocity of the Earth relative to spaceship 2 is $v = 0.60c$. The velocity of spaceship 1 relative to the Earth is $u'_x = 0.60c$. Solve for the velocity of spaceship 1 relative to spaceship 2, u_x, using Eq. 36-7a.

$$u_x = \frac{\left(u'_x + v\right)}{\left(1 + \dfrac{vu'_x}{c^2}\right)} = \frac{\left(0.60c + 0.60c\right)}{\left[1 + \left(0.60\right)\left(0.60\right)\right]} = \boxed{0.88c}$$

(*b*) Now consider spaceship 1 as reference frame S. The velocity of the Earth relative to spaceship 1 is $v = -0.60c$. The velocity of spaceship 2 relative to the Earth is $u'_x = -0.60c$. Solve for the velocity of spaceship 2 relative to spaceship 1, u_x, using Eq. 36-7a.

$$u_x = \frac{\left(u'_x + v\right)}{\left(1 + \dfrac{vu'_x}{c^2}\right)} = \frac{\left(-0.60c - 0.60c\right)}{\left[1 + \left(-0.60\right)\left(-0.60\right)\right]} = \boxed{-0.88c}$$

As expected, the two relative velocities are the opposite of each other.

24. (*a*) The Galilean transformation is given in Eq. 36-4.

$$x = x' + vt' = x' + vt = 100\,\text{m} + \left(0.92\right)\left(3.00 \times 10^8\,\text{m/s}\right)\left(1.00 \times 10^{-6}\,\text{s}\right) = \boxed{376\,\text{m}}$$

(*b*) The Lorentz transformation is given in Eq. 36-6. Note that we are given *t*, the clock reading in frame S.

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \;\rightarrow\; t' = \frac{t}{\gamma} - \frac{vx'}{c^2}$$

$$x = \gamma\left(x' + vt'\right) = \gamma\left[x' + v\left(\frac{t}{\gamma} - \frac{vx'}{c^2}\right)\right] = \gamma\left[x' + \frac{v}{c}\left(\frac{ct}{\gamma} - \frac{vx'}{c}\right)\right]$$

$$= \frac{1}{\sqrt{1-0.92^2}}\left[(100\,\text{m})+(0.92)\left(\sqrt{1-0.92^2}\left(3.00\times10^8\,\text{m/s}\right)\left(1.00\times10^{-6}\,\text{s}\right)-0.92(100\,\text{m})\right)\right]$$

$$= \boxed{316\,\text{m}}$$

$\boxed{25.}$ (*a*) We take the positive direction in the direction of the first spaceship. We choose reference frame S as the Earth, and reference frame S' as the first spaceship. So $v = 0.61c$. The speed of the second spaceship relative to the first spaceship is $u'_x = 0.87c$. We use Eq. 36-7a to solve for the speed of the second spaceship relative to the Earth, *u*.

$$u_x = \frac{(u'_x + v)}{\left(1+\frac{vu'_x}{c^2}\right)} = \frac{(0.87c + 0.61c)}{\left[1+(0.61)(0.87)\right]} = \boxed{0.97c}$$

(*b*) The only difference is now that $u'_x = -0.87c$.

$$u_x = \frac{(u'_x + v)}{\left(1+\frac{vu'_x}{c^2}\right)} = \frac{(-0.87c + 0.61c)}{\left[1+(0.61)(-0.87)\right]} = -0.55c$$

The problem asks for the speed, which would be $\boxed{0.55c}$

26. We assume that the given speed of 0.90*c* is relative to the planet that you are approaching. We take the positive direction in the direction that you are traveling. Consider your spaceship as reference frame S, and the planet as reference frame S'. The velocity of the planet relative to you is $v = -0.90c$. The velocity of the probe relative to the planet is $u'_x = 0.95c$. Solve for the velocity of the probe relative to your spaceship, u_x, using Eq. 36-7a.

$$u_x = \frac{(u'_x + v)}{\left(1+\frac{vu'_x}{c^2}\right)} = \frac{(0.95c - 0.90c)}{\left[1+(-0.90)(0.95)\right]} = \boxed{0.34c}$$

27. We set frame S' as the frame at rest with the spaceship. In this frame the module has speed $u' = u'_y = 0.82c$. Frame S is the frame that is stationary with respect to the Earth. The spaceship, and therefore frame S' moves in the *x*-direction with speed 0.76c in this frame, or $v = 0.76c$. We use Eq. 36-7a and 36-7b to determine the components of the module velocity in frame S. Then using trigonometry we combine the components to determine the speed and direction of travel.

$$u_x = \frac{u'_x + v}{1+vu'_x/c^2} = \frac{0+0.76c}{1+0} = 0.76c \;\; ; \;\; u_y = \frac{u'_y\sqrt{1-v^2/c^2}}{1+vu'_x/c^2} = \frac{0.82c\sqrt{1-0.76^2}}{1+0} = 0.533c$$

$$u = \sqrt{u_x^2+u_y^2} = \sqrt{(0.76c)^2+(0.533c)^2} = \boxed{0.93c} \;\; ; \;\; \theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\frac{0.533c}{0.76c} = \boxed{35^\circ}$$

28. The velocity components of the particle in the S frame are $u_x = u\cos\theta$ and $u_y = u\sin\theta$. We find the components of the particle in the S' frame from the velocity transformations given in Eqs. 36-7a and 36-7b. Those transformations are for the S' frame moving with speed *v* relative to the S frame. We can find the transformations from the S frame to the S' frame by simply changing *v* to –*v* and primed to unprimed variables.

$$u_x = \frac{(u'_x + v)}{\left(1+vu'_x/c^2\right)} \rightarrow u'_x = \frac{(u_x - v)}{\left(1-vu_x/c^2\right)} \;\; ; \;\; u_y = \frac{u'_y\sqrt{1-v^2/c^2}}{\left(1+vu_x/c^2\right)} \rightarrow u'_y = \frac{u_y\sqrt{1-v^2/c^2}}{\left(1-vu_x/c^2\right)}$$

$$\tan\theta' = \frac{u'_y}{u'_x} = \frac{\dfrac{u_y\sqrt{1-v^2/c^2}}{\left(1-vu_x/c^2\right)}}{\dfrac{\left(u_x-v\right)}{\left(1-vu_x/c^2\right)}} = \frac{u_y\sqrt{1-v^2/c^2}}{\left(u_x-v\right)} = \frac{u\sin\theta\sqrt{1-v^2/c^2}}{\left(u\cos\theta-v\right)} = \boxed{\frac{\sin\theta\sqrt{1-v^2/c^2}}{\left(\cos\theta-v/u\right)}}$$

29. (*a*) In frame S' the horizontal component of the stick length will be contracted, while the vertical component remains the same. We use the trigonometric relations to determine the *x*- and *y*-components of the length of the stick. Then using Eq. 36-3a we determine the contracted length of the *x*-component. Finally, we use the Pythagorean theorem to determine stick length in frame S'.

$$\ell_x = \ell_0\cos\theta \;\; ; \;\; \ell_y = \ell_0\sin\theta = \ell'_y \;\; ; \;\; \ell'_x = \ell_x\sqrt{1-v^2/c^2} = \ell_0\cos\theta\sqrt{1-v^2/c^2}$$

$$\ell = \sqrt{\ell'^2_x + \ell'^2_y} = \sqrt{\ell_0^2\cos^2\theta\left(1-v^2/c^2\right) + \ell_0^2\sin^2\theta} = \boxed{\ell_0\sqrt{1-\left(v\cos\theta/c\right)^2}}$$

(*b*) We calculate the angle from the length components in the moving frame.

$$\theta' = \tan^{-1}\frac{\ell'_y}{\ell'_x} = \tan^{-1}\left(\frac{\ell_0\sin\theta}{\ell_0\cos\theta\sqrt{1-v^2/c^2}}\right) = \tan^{-1}\left(\frac{\tan\theta}{\sqrt{1-v^2/c^2}}\right) = \boxed{\tan^{-1}\left(\gamma\tan\theta\right)}$$

30. (*a*) We choose the train as frame S' and the Earth as frame S. Since the guns fire simultaneously in S', we set these times equal to zero, that is $t'_A = t'_B = 0$. To simplify the problem we also set the location of gunman A equal to zero in frame S' when the guns were fired, $x'_A = 0$. This places gunman B at $x'_B = 55.0\,\text{m}$. Use Eq. 36-6 to determine the time that each gunman fired his weapon in frame S.

$$t_A = \gamma\left(t'_A + \frac{vx'_A}{c^2}\right) = \gamma\left(0 + \frac{v\times 0}{c^2}\right) = 0$$

$$t_B = \gamma\left(t'_B + \frac{vx'_B}{c^2}\right) = \frac{1}{\sqrt{1-\left(35.0\,\text{m/s}/3.00\times10^8\,\text{m/s}\right)^2}}\left(0 + \frac{(35\,\text{m/s})(55.0\,\text{m})}{\left(3.00\times10^8\,\text{m/s}\right)^2}\right) = 2.14\times10^{-14}\,\text{s}$$

Therefore, in Frame S, $\boxed{\text{A fired first}}$.

(*b*) As found in part (*a*), the difference in time is $\boxed{2.14\times10^{-14}\,\text{s}}$.

(*c*) In the Earth frame of reference, since A fired first, $\boxed{\text{B was struck first}}$. In the train frame, A is moving away from the bullet fired toward him, and B is moving toward the bullet fired toward him. Thus B will be struck first in this frame as well.

$\boxed{31.}$ We set frame S' as the frame moving with the observer. Frame S is the frame in which the two light bulbs are at rest. Frame S is moving with velocity *v* with respect to frame S'. We solve Eq. 36-6 for the time t' in terms of *t, x,* and *v*. Using the resulting equation we determine the time in frame S' that each bulb is turned on, given that in frame S the bulbs are turned on simultaneously at $t_A = t_B = 0$. Taking the difference in these times gives the time interval as measured by the observing moving with velocity *v*.

$$x = \gamma\left(x' + vt'\right) \rightarrow x' = \frac{x}{\gamma} - vt'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) = \gamma\left[t' + \frac{v}{c^2}\left(\frac{x}{\gamma} - vt'\right)\right] = \gamma t'\left(1 - \frac{v^2}{c^2}\right) + \frac{vx}{c^2} = \frac{t'}{\gamma} + \frac{vx}{c^2} \;\;\rightarrow\;\; t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$t'_A = \gamma\left(t_A - \frac{vx_A}{c^2}\right) = \gamma\left(0 - \frac{v\times 0}{c^2}\right) = 0 \ ; \ t'_B = \gamma\left(t_B - \frac{vx_B}{c^2}\right) = \gamma\left(0 - \frac{v\ell}{c^2}\right) = -\gamma\frac{v\ell}{c^2}$$

$$\Delta t' = t'_B - t'_A = \boxed{-\gamma\frac{v\ell}{c^2}}$$

According to the observer, $\boxed{\text{bulb B turned on first}}$.

32. We set up the two frames such that in frame S, the first object is located at the origin and the second object is located 220 meters from the origin, so $x_\mathrm{A} = 0$ and $x_\mathrm{B} = 220$ m. We set the time when event A occurred equal to zero, so $t_\mathrm{A} = 0$ and $t_\mathrm{B} = 0.80\,\mu$s. We then set the location of the two events in frame S′ equal, and using Eq. 36-6 we solve for the velocity.

$$x'_\mathrm{A} = x'_\mathrm{B} \ \rightarrow \ \gamma\left(x_\mathrm{A} - vt_A\right) = \gamma\left(x_\mathrm{B} - vt_\mathrm{B}\right) \ ; \ v = \frac{x_\mathrm{A} - x_\mathrm{B}}{t_\mathrm{A} - t_\mathrm{B}} = \frac{0 - 220\,\mathrm{m}}{0 - 0.88\ \mu\mathrm{s}} = \boxed{2.5\times 10^8\,\mathrm{m/s}}$$

33. From the boy's frame of reference, the pole remains at rest with respect to him. As such, the pole will always remain 12.0 m long. As the boy runs toward the barn, relativity requires that the (relatively moving) barn contract in size, making the barn even shorter than its rest length of 10.0 m. Thus it is impossible, in the boy's frame of reference, for the barn to be longer than the pole. So according to the boy, the pole will never completely fit within the barn.

In the frame of reference at rest with respect to the barn, it is possible for the pole to be shorter than the barn. We use Eq. 36-3a to calculate the speed that the boy would have to run for the contracted length of the pole, ℓ, to equal the length of the barn.

$$\ell = \ell_0\sqrt{1 - v^2/c^2} \ \rightarrow \ v = c\sqrt{1 - \ell^2/\ell_0^2} = c\sqrt{1 - (10.0\ \mathrm{m})^2/(12.0\ \mathrm{m})^2} = 0.5528c$$

If persons standing at the front and back door of the barn were to close both doors exactly when the pole was completely inside the barn, we would have two simultaneous events in the barn's rest frame S with the pole completely inside the barn. Let us set the time for these two events as $t_\mathrm{A} = t_\mathrm{B} = 0$. In frame S these two events occur at the front and far side of the barn, or at $x_\mathrm{A} = 0$ and $x_\mathrm{B} = 10.0\,\mathrm{m}$. Using Eq. 36-6, we calculate the times at which the barn doors close in the boy's frame of reference.

$$t'_A = \gamma\left(t_A - \frac{vx_A}{c^2}\right) = \gamma\left(0 - \frac{v\times 0}{c^2}\right) = 0$$

$$t'_B = \gamma\left(t_B - \frac{vx_B}{c^2}\right) = \frac{1}{\sqrt{1 - 0.5528^2}}\left[0 - \frac{0.5528(10.0\,\mathrm{m})}{3.00\times 10^8\,\mathrm{m/s}}\right] = -2.211\times 10^{-8}\,\mathrm{s}$$

Therefore, in the boy's frame of reference the far door of the barn closed 22.1 ns before the front door. If we multiply the speed of the boy by this time difference, we calculate the distance the boy traveled between the closing of the two doors.

$$\Delta x = vt = 0.5528\left(3.00\times 10^8\ \mathrm{m/s}\right)\left(2.211\times 10^{-8}\ \mathrm{s}\right) = 3.67\ \mathrm{m}.$$

We use Eq. 36-3a to determine the length of the barn in the boy's frame of reference.

$$\ell = \ell_0\sqrt{1 - v^2/c^2} = (10.0\ \mathrm{m})\sqrt{1 - 0.5528^2} = 8.33\ \mathrm{m}$$

Subtracting the distance traveled between closing the doors from the length of the pole, we find the length of the barn in the boy's frame of reference.

$$\ell_{0,\mathrm{pole}} - \Delta x = 12.0\ \mathrm{m} - 3.67\ \mathrm{m} = 8.33\ \mathrm{m} = \ell_\mathrm{barn}$$

Therefore, in the boy's frame of reference, when the front of the pole reached the far door it was closed. Then 22.1 ns later, when the back of the pole reached the front door, that door was closed. In the boy's frame of reference these two events are not simultaneous.

34. The momentum of the proton is given by Eq. 36-8.

$$p=\gamma mv=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{\left(1.67\times10^{-27}\,\text{kg}\right)(0.75)\left(3.00\times10^{8}\,\text{m/s}\right)}{\sqrt{1-(0.75)^2}}=\boxed{5.7\times10^{-19}\,\text{kg}\cdot\text{m/s}}$$

35. (*a*) We compare the classical momentum to the relativistic momentum.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}}=\frac{mv}{\dfrac{mv}{\sqrt{1-v^2/c^2}}}=\sqrt{1-v^2/c^2}=\sqrt{1-(0.10)^2}=0.995$$

The classical momentum is about $\boxed{-0.5\%}$ in error.

(*b*) We again compare the two momenta.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}}=\frac{mv}{\dfrac{mv}{\sqrt{1-v^2/c^2}}}=\sqrt{1-v^2/c^2}=\sqrt{1-(0.60)^2}=0.8$$

The classical momentum is $\boxed{-20\%}$ in error.

36. The momentum at the higher speed is to be twice the initial momentum. We designate the initial state with a subscript "0", and the final state with a subscript "f".

$$\frac{p_{\text{f}}}{p_0}=\frac{\dfrac{mv_{\text{f}}}{\sqrt{1-v_{\text{f}}^2/c^2}}}{\dfrac{mv_0}{\sqrt{1-v_0^2/c^2}}}=2\;\rightarrow\;\frac{\dfrac{v_{\text{f}}^2}{1-v_{\text{f}}^2/c^2}}{\dfrac{v_0^2}{1-v_0^2/c^2}}=4\;\rightarrow\;\left(\frac{v_{\text{f}}^2}{1-v_{\text{f}}^2/c^2}\right)=4\left[\frac{(0.26c)^2}{1-(0.26)^2}\right]=0.29c^2\;\rightarrow$$

$$v_{\text{f}}^2=\left(\frac{0.29}{1.29}\right)c^2\;\rightarrow\;v_{\text{f}}=\boxed{0.47c}$$

$\boxed{37.}$ The two momenta, as measured in the frame in which the particle was initially at rest, will be equal to each other in magnitude. The lighter particle is designated with a subscript "1", and the heavier particle with a subscript "2".

$$p_1=p_2\;\rightarrow\;\frac{m_1v_1}{\sqrt{1-v_1^2/c^2}}=\frac{m_2v_2}{\sqrt{1-v_2^2/c^2}}\;\rightarrow$$

$$\frac{v_1^2}{\left(1-v_1^2/c^2\right)}=\left(\frac{m_2}{m_1}\right)^2\frac{v_2^2}{\left(1-v_2^2/c^2\right)}=\left(\frac{6.68\times10^{-27}\,\text{kg}}{1.67\times10^{-27}\,\text{kg}}\right)^2\left[\frac{(0.60c)^2}{1-(0.60)^2}\right]=9.0c^2\;\rightarrow$$

$$v_1=\sqrt{0.90}\,c=\boxed{0.95c}$$

38. We find the proton's momenta using Eq. 36-8.

$$p_{0.45}=\frac{m_{\text{p}}v_1}{\sqrt{1-\dfrac{v_1^2}{c^2}}}=\frac{m_{\text{p}}(0.45c)}{\sqrt{1-(0.45)^2}}=0.5039m_{\text{p}}c\;;\;p_{0.80}=\frac{m_{\text{p}}v_2}{\sqrt{1-\dfrac{v_2^2}{c^2}}}=\frac{m_{\text{p}}(0.80c)}{\sqrt{1-(0.80)^2}}=1.3333m_{\text{p}}c$$

$$p_{0.98}=\frac{m_{\text{p}}v_2}{\sqrt{1-\dfrac{v_2^2}{c^2}}}=\frac{m_{\text{p}}(0.98c)}{\sqrt{1-(0.98)^2}}=4.9247m_{\text{p}}c$$

(*a*) $$\left(\frac{p_2-p_1}{p_1}\right)100=\left(\frac{1.3333m_{\mathrm{p}}c-0.5039m_{\mathrm{p}}c}{0.5039m_{\mathrm{p}}c}\right)100=164.6\approx\boxed{160\%}$$

(*b*) $$\left(\frac{p_2-p_1}{p_1}\right)100=\left(\frac{4.9247m_{\mathrm{p}}c-1.3333m_{\mathrm{p}}c}{1.3333m_{\mathrm{p}}c}\right)100=269.4\approx\boxed{270\%}$$

39. The rest energy of the electron is given by Eq. 36-12.

$$E=mc^2=\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(3.00\times10^8\,\mathrm{m/s}\right)^2=\boxed{8.20\times10^{-14}\,\mathrm{J}}$$

$$=\frac{\left(8.20\times10^{-14}\,\mathrm{J}\right)}{\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}=\boxed{0.511\,\mathrm{MeV}}$$

40. We find the loss in mass from Eq. 36-12.

$$m=\frac{E}{c^2}=\frac{\left(200\,\mathrm{MeV}\right)\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}{\left(3.00\times10^8\,\mathrm{m/s}\right)^2}=3.56\times10^{-28}\,\mathrm{kg}\approx\boxed{4\times10^{-28}\,\mathrm{kg}}$$

41. We find the mass conversion from Eq. 36-12.

$$m=\frac{E}{c^2}=\frac{\left(8\times10^{19}\,\mathrm{J}\right)}{\left(3.00\times10^8\,\mathrm{m/s}\right)^2}=\boxed{900\,\mathrm{kg}}$$

42. We calculate the mass from Eq. 36-12.

$$m=\frac{E}{c^2}=\frac{1}{c^2}\left(mc^2\right)=\frac{1}{c^2}\frac{\left(1.6726\times10^{-27}\,\mathrm{kg}\right)\left(2.9979\times10^8\,\mathrm{m/s}\right)^2}{\left(1.6022\times10^{-13}\,\mathrm{J/MeV}\right)}=\boxed{938.2\,\mathrm{MeV}/c^2}$$

$\boxed{43.}$ Each photon has momentum 0.50 MeV/*c*. Thus each photon has mass 0.50 MeV. Assuming the photons have opposite initial directions, then the total momentum is 0, and so the product mass will not be moving. Thus all of the photon energy can be converted into the mass of the particle. Accordingly, the heaviest particle would have a mass of $\boxed{1.00\,\mathrm{MeV}/c^2}$, which is $1.78\times10^{-30}\,\mathrm{kg}$. .

44. (*a*) The work is the change in kinetic energy. Use Eq. 36-10b. The initial kinetic energy is 0.

$$W=\Delta K=K_{\text{final}}=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-0.998^2}}-1\right)\left(938.3\,\mathrm{MeV}\right)=1.39\times10^4\,\mathrm{MeV}$$

$$=\boxed{13.9\,\mathrm{GeV}}$$

(*b*) The momentum of the proton is given by Eq. 36-8.

$$p=\gamma mv=\frac{1}{\sqrt{1-0.998^2}}\left(938.3\,\mathrm{MeV}/c^2\right)\left(0.998c\right)=1.48\times10^4\,\mathrm{MeV}/c=\boxed{14.8\,\mathrm{GeV}/c}$$

45. We find the energy equivalent of the mass from Eq. 36-12.

$$E=mc^2=\left(1.0\times10^{-3}\mathrm{kg}\right)\left(3.00\times10^8\,\mathrm{m/s}\right)^2=\boxed{9.0\times10^{13}\,\mathrm{J}}$$

We assume that this energy is used to increase the gravitational potential energy.

$$E=mgh\quad\rightarrow\quad m=\frac{E}{hg}=\frac{9.0\times10^{13}\,\mathrm{J}}{\left(1.0\times10^3\mathrm{m}\right)\left(9.80\,\mathrm{m/s^2}\right)}=\boxed{9.2\times10^9\,\mathrm{kg}}$$

46. The work is the change in kinetic energy. Use Eq. 36-10b. The initial kinetic energy is 0.

$$W_1 = (\gamma_{0.90} - 1)mc^2 \;\; ; \;\; W_2 = K_{0.99c} - K_{0.90c} = (\gamma_{0.99} - 1)mc^2 - (\gamma_{0.90} - 1)mc^2$$

$$\frac{W_2}{W_1} = \frac{(\gamma_{0.99} - 1)mc^2 - (\gamma_{0.90} - 1)mc^2}{(\gamma_{0.90} - 1)mc^2} = \frac{\gamma_{0.99} - \gamma_{0.90}}{\gamma_{0.90} - 1} = \frac{\dfrac{1}{\sqrt{1-0.99^2}} - \dfrac{1}{\sqrt{1-0.90^2}}}{\dfrac{1}{\sqrt{1-0.90^2}} - 1} = \boxed{3.7}$$

47. The kinetic energy is given by Eq. 36-10.

$$K = (\gamma - 1)mc^2 = mc^2 \;\; \to \;\; \gamma = 2 = \frac{1}{\sqrt{1 - v^2/c^2}} \;\; \to \;\; v = \sqrt{\frac{3}{4}}c = \boxed{0.866c}$$

48. The total energy of the proton is the kinetic energy plus the mass energy. Use Eq. 36-13 to find the momentum.

$$E = K + mc^2 \;\; ;$$

$$(pc)^2 = E^2 - (mc^2)^2 = (K + mc^2)^2 - (mc^2)^2 = K^2 + 2K(mc^2)$$

$$pc = \sqrt{K^2 + 2K(mc^2)} = K\sqrt{1 + 2\frac{mc^2}{K}} = (950\,\text{MeV})\sqrt{1 + 2\frac{938.3\,\text{MeV}}{950\,\text{MeV}}} = 1638\,\text{MeV}$$

$$p = 1638\,\text{MeV}/c \approx \boxed{1.6\,\text{GeV}/c}$$

$\boxed{49.}$ We find the speed in terms of *c*. The kinetic energy is given by Eq. 36-10 and the momentum by Eq. 36-8.

$$v = \frac{(2.80 \times 10^8\,\text{m/s})}{(3.00 \times 10^8\,\text{m/s})} = 0.9333c$$

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - 0.9333^2}} - 1\right)(938.3\,\text{MeV}) = 1674.6\,\text{MeV} \approx \boxed{1.67\,\text{GeV}}$$

$$p = \gamma mv = \frac{1}{\sqrt{1 - 0.9333^2}}(938.3\,\text{MeV}/c^2)(0.9333c) = 2439\,\text{MeV}/c \approx \boxed{2.44\,\text{GeV}/c}$$

50. We use Eq. 36-10 to find the speed from the kinetic energy.

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 \;\; \to$$

$$v = c\sqrt{1 - \frac{1}{\left(\dfrac{K}{mc^2} + 1\right)^2}} = c\sqrt{1 - \frac{1}{\left(\dfrac{1.25\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^2}} = \boxed{0.957c}$$

51. Since the proton was accelerated by a potential difference of 125 MV, its potential energy decreased by 125 MeV, and so its kinetic energy increased from 0 to 125 MeV. Use Eq. 36-10 to find the speed from the kinetic energy.

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 \;\; \to$$

$$v = c\sqrt{1-\frac{1}{\left(\frac{K}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{125\,\text{MeV}}{938.3\,\text{MeV}}+1\right)^2}} = \boxed{0.470c}$$

52. We let M represent the rest mass of the new particle. The initial energy is due to both incoming particles, and the final energy is the rest energy of the new particle. Use Eq. 36-11 for the initial energies.

$$E = 2\left(\gamma mc^2\right) = Mc^2 \quad \rightarrow \quad M = 2\gamma m = \boxed{\frac{2m}{\sqrt{1-v^2/c^2}}}$$

We assumed that energy is conserved, and so there was $\boxed{\text{no loss of energy}}$ in the collision. The final kinetic energy is 0, so all of the kinetic energy was lost.

$$K_{\text{lost}} = K_{\text{initial}} = 2(\gamma-1)mc^2 = \boxed{\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)2mc^2}$$

53. Since the electron was accelerated by a potential difference of 28 kV, its potential energy decreased by 28 keV, and so its kinetic energy increased from 0 to 28 MeV. Use Eq. 36-10 to find the speed from the kinetic energy.

$$K = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 \quad \rightarrow$$

$$v = c\sqrt{1-\frac{1}{\left(\frac{K}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{0.028\,\text{MeV}}{0.511\,\text{MeV}}+1\right)^2}} = \boxed{0.32c}$$

54. We use Eqs. 36-11 and 36-13 in order to find the mass.

$$E^2 = p^2c^2 + m^2c^4 = \left(K+mc^2\right)^2 = K^2 + 2Kmc^2 + m^2c^4 \quad \rightarrow$$

$$m = \frac{p^2c^2 - K^2}{2Kc^2} = \frac{\left(121\,\text{MeV}/c\right)^2c^2 - \left(45\,\text{MeV}\right)^2}{2\left(45\,\text{MeV}\right)c^2} = \boxed{140\,\text{MeV}/c^2} \approx 2.5\times10^{-28}\,\text{kg}$$

The particle is most likely a probably a π^0 meson.

$\boxed{55.}$ (*a*) Since the kinetic energy is half the total energy, and the total energy is the kinetic energy plus the rest energy, the kinetic energy must be equal to the rest energy. We also use Eq. 36-10.

$$K = \tfrac{1}{2}E = \tfrac{1}{2}\left(K+mc^2\right) \quad \rightarrow \quad K = mc^2$$

$$K = (\gamma-1)mc^2 = mc^2 \quad \rightarrow \quad \gamma = 2 = \frac{1}{\sqrt{1-v^2/c^2}} \quad \rightarrow \quad v = \sqrt{\tfrac{3}{4}}c = \boxed{0.866c}$$

(*b*) In this case, the kinetic energy is half the rest energy.

$$K = (\gamma-1)mc^2 = \tfrac{1}{2}mc^2 \quad \rightarrow \quad \gamma = \tfrac{3}{2} = \frac{1}{\sqrt{1-v^2/c^2}} \quad \rightarrow \quad v = \sqrt{\tfrac{5}{9}}c = \boxed{0.745c}$$

56. We use Eq. 36-10 for the kinetic energy and Eq. 36-8 for the momentum.

$$K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=\left(\frac{1}{\sqrt{1-\left(\frac{8.15\times10^7\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)^2}}-1\right)(938.3\,\mathrm{MeV})$$

$$=\boxed{36.7\,\mathrm{MeV}}$$

$$p=\gamma mv=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{mc^2(v/c)}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{(938.3\,\mathrm{MeV})\left(\frac{8.15\times10^7\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)}{\sqrt{1-\left(\frac{8.15\times10^7\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)^2}}=\boxed{265\,\mathrm{MeV}/c}$$

Evaluate with the classical expressions.

$$K_\mathrm{c}=\tfrac{1}{2}mv^2=\tfrac{1}{2}mc^2\left(\frac{v}{c}\right)^2=\tfrac{1}{2}(938.3\,\mathrm{MeV})\left(\frac{8.15\times10^7\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)^2=34.6\,\mathrm{MeV}$$

$$p_\mathrm{c}=mv=\frac{1}{c}mc^2\left(\frac{v}{c}\right)=(938.3\,\mathrm{MeV})\left(\frac{8.15\times10^7\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)=255\,\mathrm{MeV}/c$$

Calculate the percent error.

$$\mathrm{error}_K=\frac{K_\mathrm{c}-K}{K}\times100=\frac{34.6-36.7}{36.7}\times100=\boxed{-5.7\%}$$

$$\mathrm{error}_p=\frac{p_\mathrm{c}-p}{p}\times100=\frac{255-265}{265}\times100=\boxed{-3.8\%}$$

57. (*a*) The kinetic energy is found from Eq. 36-10.

$$K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=\left(\frac{1}{\sqrt{1-0.18^2}}-1\right)(1.7\times10^4\,\mathrm{kg})(3.00\times10^8\,\mathrm{m/s})^2$$

$$=2.541\times10^{19}\,\mathrm{J}\approx\boxed{2.5\times10^{19}\,\mathrm{J}}$$

(*b*) Use the classical expression and compare the two results.

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}(1.7\times10^4\,\mathrm{kg})\left[(0.18)(3.00\times10^8\,\mathrm{m/s})\right]^2=2.479\times10^{19}\,\mathrm{J}$$

$$\%\,\mathrm{error}=\frac{(2.479\times10^{19}\,\mathrm{J})-(2.541\times10^{19}\,\mathrm{J})}{(2.541\times10^{19}\,\mathrm{J})}\times100=\boxed{-2.4\%}$$

The classical value is 2.4% too low.

58. The kinetic energy of 998 GeV is used to find the speed of the protons. Since the energy is 1000 times the rest mass, we expect the speed to be very close to *c*. Use Eq. 36-10.

$$K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2\quad\rightarrow$$

$$v=c\sqrt{1-\frac{1}{\left(\frac{K}{mc^2}+1\right)^2}}=c\sqrt{1-\frac{1}{\left(\frac{998\,\mathrm{GeV}}{0.938\,\mathrm{GeV}}+1\right)^2}}=c(\text{to 7 sig. fig.})$$

$$B=\frac{\gamma mv^2}{rqv}=\frac{\gamma mv}{rq}\approx\frac{\left(\frac{K}{mc^2}-1\right)mc}{rq}=\frac{\left(\frac{998\,\text{GeV}}{0.938\,\text{GeV}}-1\right)\left(1.673\times10^{-27}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.0\times10^{3}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)}=\boxed{3.3\,\text{T}}$$

59. By conservation of energy, the rest energy of the americium nucleus is equal to the rest energies of the other particles plus the kinetic energy of the alpha particle.

$$m_{\text{Am}}c^2=\left(m_{\text{Np}}+m_\alpha\right)c^2+K_\alpha \quad\rightarrow$$

$$m_{\text{Np}}=m_{\text{Am}}-m_\alpha-\frac{K_\alpha}{c^2}=241.05682\,\text{u}-4.00260\,\text{u}-\frac{5.5\,\text{MeV}}{c^2}\left(\frac{1\,\text{u}}{931.49\,\text{MeV}/c^2}\right)=\boxed{237.04832\,\text{u}}$$

60. (*a*) For a particle of non-zero mass, we derive the following relationship between kinetic energy and momentum.

$$E=K+mc^2\;;\;(pc)^2=E^2-\left(mc^2\right)^2=\left(K+mc^2\right)^2-\left(mc^2\right)^2=K^2+2K\left(mc^2\right)$$

$$K^2+2K\left(mc^2\right)-(pc)^2=0\;\rightarrow\;K=\frac{-2mc^2\pm\sqrt{4\left(mc^2\right)^2+4(pc)^2}}{2}$$

For the kinetic energy to be positive, we take the positive root.

$$K=\frac{-2mc^2+\sqrt{4\left(mc^2\right)^2+4(pc)^2}}{2}=-mc^2+\sqrt{\left(mc^2\right)^2+(pc)^2}$$

If the momentum is large, we have the following relationship.

$$K=-mc^2+\sqrt{\left(mc^2\right)^2+(pc)^2}\approx pc-mc^2$$

Thus there should be a linear relationship between kinetic energy and momentum for large values of momentum.

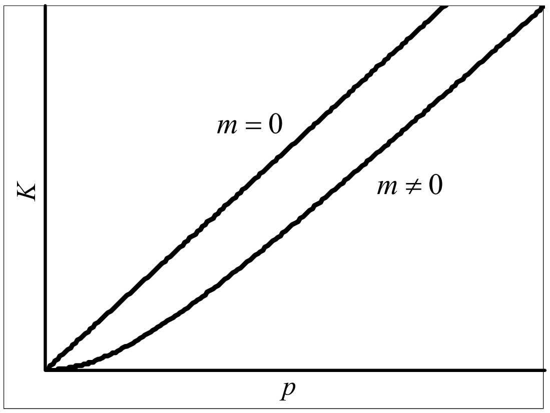
If the momentum is small, we use the binomial expansion to derive the classical relationship.

$$K=-mc^2+\sqrt{\left(mc^2\right)^2+(pc)^2}=-mc^2+mc^2\sqrt{1+\left(\frac{pc}{mc^2}\right)^2}$$

$$\approx-mc^2+mc^2\left(1+\tfrac{1}{2}\left(\frac{pc}{mc^2}\right)^2\right)=\frac{p^2}{2m}$$

Thus we expect a quadratic relationship for small values of momentum. The adjacent graph verifies these approximations.

(*b*) For a particle of zero mass, the relationship is simply $K=pc$. See the included graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH36.XLS," on tab "Problem 36.60."



$\boxed{61.}$ All of the energy, both rest energy and kinetic energy, becomes electromagnetic energy. We use Eq. 36-11. Both masses are the same.

$$E_{\text{total}} = E_1 + E_2 = \gamma_1 mc^2 + \gamma_2 mc^2 = \left(\gamma_1 + \gamma_2\right) mc^2 = \left(\frac{1}{\sqrt{1-0.43^2}} + \frac{1}{\sqrt{1-0.55^2}}\right)(105.7\,\text{MeV})$$

$$= 243.6\,\text{MeV} \approx \boxed{240\,\text{MeV}}$$

62. We use Eqs. 36-11 and 36-13.

$$E = K + mc^2 \;\; ; \;\; (pc)^2 = E^2 - \left(mc^2\right)^2 = \left(K + mc^2\right)^2 - \left(mc^2\right)^2 = K^2 + 2K\left(mc^2\right) \;\; \rightarrow$$

$$\boxed{p = \frac{\sqrt{K^2 + 2K\left(mc^2\right)}}{c}}$$

63. (*a*) We assume the mass of the particle is *m*, and we are given that the velocity only has an *x*-component, u_x. We write the momentum in each frame using Eq. 36-8, and we use the velocity transformation given in Eq. 36-7. Note that there are three relevant velocities: u_x, the velocity in reference frame S; u'_x, the velocity in reference frame S′; and *v*, the velocity of one frame relative to the other frame. There is no velocity in the *y* or *z* directions, in either frame. We reserve the symbol γ for $\frac{1}{\sqrt{1-v^2/c^2}}$, and also use Eq. 36-11 for energy.

$$p_x = \frac{mu_x}{\sqrt{1-u_x^2/c^2}} \;\; ; \;\; p_y = 0 \;\; ; \;\; p_z = 0$$

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2} \;\; \rightarrow \;\; u'_x = \frac{u_x - v}{1 - vu_x/c^2} \;\; ; \;\; u'_y = u_y \frac{1 + vu'_x/c^2}{\sqrt{1-v^2/c^2}} = 0 \;\; ; \;\; u'_z = u_z \frac{1 + vu'_x/c^2}{\sqrt{1-v^2/c^2}} = 0$$

$$p'_x = \frac{mu'_x}{\sqrt{1-u'^2_x/c^2}} \;\; ; \;\; p'_y = 0\left(\text{since } u'_y = 0\right) \;\; ; \;\; p'_z = 0\left(\text{since } u'_z = 0\right)$$

Substitute the expression for u'_x into the expression for p'_x.

$$p'_x = \frac{mu'_x}{\sqrt{1-u'^2_x/c^2}} = \frac{m\dfrac{\left(u_x - v\right)}{\left(1 - vu_x/c^2\right)}}{\sqrt{1 - \dfrac{1}{c^2}\dfrac{\left(u_x - v\right)^2}{\left(1 - vu_x/c^2\right)^2}}} = m\frac{\left(u_x - v\right)}{\left(1 - vu_x/c^2\right)} \frac{1}{\sqrt{\dfrac{\left(1 - vu_x/c^2\right)^2}{\left(1 - vu_x/c^2\right)^2} - \dfrac{\dfrac{\left(u_x - v\right)^2}{c^2}}{\left(1 - vu_x/c^2\right)^2}}}$$

$$= m\frac{\left(u_x - v\right)}{\left(1 - vu_x/c^2\right)} \frac{1}{\dfrac{1}{\left(1 - vu_x/c^2\right)}\sqrt{\left(1 - vu_x/c^2\right)^2 - \dfrac{\left(u_x - v\right)^2}{c^2}}} = \frac{m\left(u_x - v\right)}{\sqrt{\left(1 - vu_x/c^2\right)^2 - \dfrac{\left(u_x - v\right)^2}{c^2}}}$$

$$= \frac{m\left(u_x - v\right)}{\sqrt{1 - 2\dfrac{vu_x}{c^2} + \left(\dfrac{vu_x}{c^2}\right)^2 - \dfrac{u_x^2}{c^2} + \dfrac{2u_x v}{c^2} - \dfrac{v^2}{c^2}}} = \frac{m\left(u_x - v\right)}{\sqrt{1 + \left(\dfrac{vu_x}{c^2}\right)^2 - \dfrac{u_x^2}{c^2} - \dfrac{v^2}{c^2}}}$$

$$=\frac{m\left(u_x-v\right)}{\sqrt{\left(1-v^2/c^2\right)\left(1-u_x^2/c^2\right)}}=\frac{\dfrac{mu_x}{\sqrt{\left(1-u_x^2/c^2\right)}}-\dfrac{mv}{\sqrt{\left(1-u_x^2/c^2\right)}}}{\sqrt{\left(1-v^2/c^2\right)}}$$

$$=\frac{\dfrac{mu_x}{\sqrt{\left(1-u_x^2/c^2\right)}}-\dfrac{mc^2}{\sqrt{\left(1-u_x^2/c^2\right)}}\dfrac{v}{c^2}}{\sqrt{\left(1-v^2/c^2\right)}}=\frac{p_x-\dfrac{mc^2}{\sqrt{\left(1-u_x^2/c^2\right)}}\dfrac{v}{c^2}}{\sqrt{\left(1-v^2/c^2\right)}}=\boxed{\frac{p_x-vE/c^2}{\sqrt{\left(1-v^2/c^2\right)}}}$$

It is obvious from the first few equations of the problem that $\boxed{p_y'=p_y}(=0)$ and $\boxed{p_z'=p_z}(=0)$.

$$E'=\frac{mc^2}{\sqrt{1-u_x'^2/c^2}}=\frac{mc^2}{\sqrt{1-\dfrac{1}{c^2}\dfrac{\left(u_x-v\right)^2}{\left(1-vu_x/c^2\right)^2}}}=\frac{mc^2}{\sqrt{\dfrac{\left(1-vu_x/c^2\right)^2}{\left(1-vu_x/c^2\right)^2}-\dfrac{\dfrac{\left(u_x-v\right)^2}{c^2}}{\left(1-vu_x/c^2\right)^2}}}$$

$$=\frac{mc^2\left(1-vu_x/c^2\right)}{\sqrt{\left(1-vu_x/c^2\right)^2-\dfrac{\left(u_x-v\right)^2}{c^2}}}=\frac{\left(mc^2-mvu_x\right)}{\sqrt{\left(1-v^2/c^2\right)\left(1-u_x^2/c^2\right)}}=\frac{\dfrac{mc^2}{\sqrt{\left(1-u_x^2/c^2\right)}}-\dfrac{mvu_x}{\sqrt{\left(1-u_x^2/c^2\right)}}}{\sqrt{\left(1-v^2/c^2\right)}}$$

$$=\boxed{\frac{E-p_xv}{\sqrt{\left(1-v^2/c^2\right)}}}$$

(*b*) We summarize these results, and write the Lorentz transformation from Eq. 36-6, but solved in terms of the primed variables. That can be easily done by interchanged primed and unprimed quantities, and changing v to $-v$.

$$p_x'=\frac{p_x-vE/c^2}{\sqrt{\left(1-v^2/c^2\right)}}\ ;\ p_y'=p_y\ ;\ p_y'=p_y\ ;\ E'=\frac{E-p_xv}{\sqrt{\left(1-v^2/c^2\right)}}$$

$$x'=\frac{x-vt}{\sqrt{\left(1-v^2/c^2\right)}}\ ;\ y'=y\ ;\ z'=z\ ;\ t'=\frac{t-vx/c^2}{\sqrt{\left(1-v^2/c^2\right)}}$$

These transformations are identical if we exchange p_x with *x*, p_y with *y*, p_z with *z*, and E/c^2 with *t* (or E/c with *ct*).

64. The galaxy is moving away from the Earth, and so we use Eq. 36-15b.

$$f_0-f=0.0987f_0\ \rightarrow f=0.9013f_0$$

$$f=f_0\sqrt{\frac{c-v}{c+v}}\quad\rightarrow\quad v=\frac{\left[1-\left(f/f_0\right)^2\right]}{\left[1+\left(f/f_0\right)^2\right]}c=\left(\frac{1-0.9013^2}{1+0.9013^2}\right)c=\boxed{0.1035c}$$

65. For source and observer moving towards each other, use Eq. 36-14b.

$$f=f_0\sqrt{\frac{c+v}{c-v}}=f_0\sqrt{\frac{1+v/c}{1-v/c}}=\left(95.0\,\text{MHz}\right)\sqrt{\frac{1+0.70}{1-0.70}}=226\,\text{MHz}\approx\boxed{230\,\text{MHz}}$$

66. We use Eq. 36-15a, and assume that $v \ll c$.

$$\lambda = \lambda_0\sqrt{\frac{c+v}{c-v}} = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} = \lambda_0\sqrt{\frac{(1+v/c)(1+v/c)}{(1-v/c)(1+v/c)}} = \lambda_0(1+v/c)\sqrt{\frac{1}{(1-v^2/c^2)}}$$

$$= \lambda_0(1+v/c)(1-v^2/c^2)^{-1/2} \approx \lambda_0(1+v/c) = \lambda_0 + \lambda_0 v/c \rightarrow \frac{\lambda-\lambda_0}{\lambda_0} = \boxed{\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}}$$

$\boxed{67.}$ (*a*) We apply Eq. 36-14b to determine the received/reflected frequency *f*. Then we apply this same equation a second time using the frequency *f* as the source frequency to determine the Doppler-shifted frequency f'. We subtract the initial frequency from this Doppler-shifted frequency to obtain the beat frequency. The beat frequency will be much smaller than the emitted frequency when the speed is much smaller than the speed of light. We then set $c - v \approx c$ and solve for *v*.

$$f = f_0\sqrt{\frac{c+v}{c-v}} \qquad f' = f\sqrt{\frac{c+v}{c-v}} = f_0\sqrt{\frac{c+v}{c-v}}\sqrt{\frac{c+v}{c-v}} = f_0\left(\frac{c+v}{c-v}\right)$$

$$f_{\text{beat}} = f' - f_0 = f_0\left(\frac{c+v}{c-v}\right) - f_0\left(\frac{c-v}{c-v}\right) = f_0\frac{2v}{c-v} \approx f_0\frac{2v}{c} \rightarrow \boxed{v \approx \frac{cf_{\text{beat}}}{2f_0}}$$

$$v \approx \frac{(3.00\times10^8\,\text{m/s})(6670\,\text{Hz})}{2(36.0\times10^9\,\text{Hz})} = \boxed{27.8\,\text{m/s}}$$

(*b*) We find the change in velocity and solve for the resulting change in beat frequency. Setting the change in the velocity equal to 1 km/h we solve for the change in beat frequency.

$$v = \frac{cf_{\text{beat}}}{2f_0} \rightarrow \Delta v = \frac{c\Delta f_{\text{beat}}}{2f_0} \rightarrow \boxed{\Delta f_{\text{beat}} = \frac{2f_0\Delta v}{c}}$$

$$\Delta f_{\text{beat}} = \frac{2(36.0\times10^9\,\text{Hz})(1\,\text{km/h})}{(3.00\times10^8\,\text{m/s})}\left(\frac{1\,\text{m/s}}{3.600\,\text{km/h}}\right) = \boxed{70\,\text{Hz}}$$

68. We consider the difference between Doppler-shifted frequencies for atoms moving directly towards the observer and atoms moving directly away. Use Eqs. 36-14b and 36-15b.

$$\Delta f = f_0\sqrt{\frac{c+v}{c-v}} - f_0\sqrt{\frac{c-v}{c+v}} = f_0\left(\sqrt{\frac{c+v}{c-v}} - \sqrt{\frac{c-v}{c+v}}\right) = f_0\left(\frac{2v}{\sqrt{c^2-v^2}}\right) = f_0\left(\frac{2v/c}{\sqrt{1-v^2/c^2}}\right)$$

We take the speed to be the rms speed of thermal motion, given by Eq. 18-5. We also assume that the thermal energy is much less than the rest energy, and so $3kT \ll mc^2$.

$$v = v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \rightarrow \frac{v}{c} = \sqrt{\frac{3kT}{mc^2}} \rightarrow \frac{\Delta f}{f_0} = 2\sqrt{\frac{3kT}{mc^2}}\left(1-\frac{3kT}{mc^2}\right)^{-1/2} \approx \boxed{2\sqrt{\frac{3kT}{mc^2}}}$$

We evaluate for a gas of H atoms (not H_2 molecules) at 550 K. Use Appendix F to find the mass.

$$\frac{\Delta f}{f_0} = 2\sqrt{\frac{3kT}{mc^2}} = 2\sqrt{\frac{3(1.38\times10^{-23}\,\text{J/K})(550\,\text{K})}{(1.008\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(3.00\times10^8\,\text{m/s})^2}} = \boxed{2.5\times10^{-5}}$$

69. At the North Pole the clock is at rest, while the clock on the equator travels the circumference of the Earth each day. We divide the circumference of the Earth by the length of the day to determine the speed of the equatorial clock. We set the dilated time equal to 2.0 years and solve for the change in rest times for the two clocks.

$$v = \frac{2\pi R}{T} = \frac{2\pi\left(6.38\times10^6\,\text{m}\right)}{\left(24\,\text{hr}\right)\left(3600\,\text{s/hr}\right)} = 464\,\text{m/s}$$

$$\Delta t = \frac{\Delta t_{0,\text{eq}}}{\sqrt{1-v^2/c^2}} \;\rightarrow\; \Delta t_{0,\text{eq}} = \Delta t\sqrt{1-v^2/c^2} \approx \Delta t\left(1+\frac{v^2}{2c^2}\right)$$

$$\Delta t = \frac{\Delta t_{0,\text{pole}}}{\sqrt{1-0}} \;\rightarrow\; \Delta t_{0,\text{pole}} = \Delta t$$

$$\left|\Delta t_{0,\text{eq}} - \Delta t_{0,\text{pole}}\right| = \Delta t\left(1+\frac{v^2}{2c^2}\right) - \Delta t$$

$$= \Delta t\frac{v^2}{2c^2} = \frac{\left(2.0\,\text{yr}\right)\left(464\,\text{m/s}\right)^2\left(3.156\times10^7\,\text{s/yr}\right)}{2\left(3.00\times10^8\,\text{m/s}\right)^2} = \boxed{75\,\mu\text{s}}$$

70. We take the positive direction in the direction of the motion of the second pod. Consider the first pod as reference frame S, and the spacecraft as reference frame S′. The velocity of the spacecraft relative to the first pod is $v = 0.60c$. The velocity of the first pod relative to the spacecraft is $u_x' = 0.50c$. Solve for the velocity of the second pod relative to the first pod, u_x, using Eq. 36-7a.

$$u_x = \frac{\left(u_x' + v\right)}{\left(1+\dfrac{vu_x'}{c^2}\right)} = \frac{\left(0.50c + 0.60c\right)}{\left[1 + \left(0.60\right)\left(0.50\right)\right]} = \boxed{0.846c}$$

71. We treat the Earth as the stationary frame, and the airplane as the moving frame. The elapsed time in the airplane will be dilated to the observers on the Earth. Use Eq. 36-1a.

$$t_{\text{Earth}} = \frac{2\pi r_{\text{Earth}}}{v} \;;\; t_{\text{plane}} = t_{\text{Earth}}\sqrt{1-v^2/c^2} = \frac{2\pi r_{\text{Earth}}}{v}\sqrt{1-v^2/c^2}$$

$$\Delta t = t_{\text{Earth}} - t_{\text{plane}} = \frac{2\pi r_{\text{Earth}}}{v}\left(1-\sqrt{1-v^2/c^2}\right) \approx \frac{2\pi r_{\text{Earth}}}{v}\left[1-\left(1-\tfrac{1}{2}\frac{v^2}{c^2}\right)\right] = \frac{\pi r_{\text{Earth}} v}{c^2}$$

$$= \frac{\pi\left(6.38\times10^6\,\text{m}\right)\left[1300\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]}{\left(3.00\times10^8\,\text{m/s}\right)^2} = \boxed{8.0\times10^{-8}\,\text{s}}$$

72. (*a*) To travelers on the spacecraft, the distance to the star is contracted, according to Eq. 36-3a. This contracted distance is to be traveled in 4.6 years. Use that time with the contracted distance to find the speed of the spacecraft.

$$v = \frac{\Delta x_{\text{spacecraft}}}{\Delta t_{\text{spacecraft}}} = \frac{\Delta x_{\text{Earth}}\sqrt{1-v^2/c^2}}{\Delta t_{\text{spacecraft}}} \;\rightarrow$$

$$v = c\frac{1}{\sqrt{1+\left(\dfrac{c\Delta t_{\text{spacecraft}}}{\Delta x_{\text{Earth}}}\right)^2}} = c\frac{1}{\sqrt{1+\left(\dfrac{4.6\,\text{ly}}{4.3\,\text{ly}}\right)^2}} = 0.6829c \approx \boxed{0.68c}$$

(*b*) Find the elapsed time according to observers on Earth, using Eq. 36-1a.

$$\Delta t_{\text{Earth}} = \frac{\Delta t_{\text{spaceship}}}{\sqrt{1-v^2/c^2}} = \frac{4.6\,\text{y}}{\sqrt{1-0.6829^2}} = \boxed{6.3\,\text{y}}$$

Note that this agrees with the time found from distance and speed.

$$t_{\text{Earth}} = \frac{\Delta x_{\text{Earth}}}{v} = \frac{4.31\text{y}}{0.6829\,c} = 6.3\,\text{yr}$$

$\boxed{73.}$ (*a*) We use Eq. 36-15a. To get a longer wavelength than usual means that the object is moving away from the Earth.

$$\lambda = \lambda_0\sqrt{\frac{c+v}{c-v}} = 1.070\lambda_0 \rightarrow \frac{\left(1.070^2-1\right)}{\left(1.070^2+1\right)}c = v = 0.067c$$

(*b*) We assume that the quasar is moving and the Earth is stationary. Then we use Eq. 16-9b.

$$f = \frac{f_0}{1+v/c} \rightarrow \frac{c}{\lambda} = \frac{c}{\lambda_0}\left(\frac{1}{1+v/c}\right) \rightarrow \lambda = \lambda_0\left(1+v/c\right) = 1.070\lambda_0 \rightarrow v = \boxed{0.070\,c}$$

74. We assume that some kind of a light signal is being transmitted from the astronaut to Earth, with a frequency of the heartbeat. That frequency will then be Doppler shifted, according to Eq. 36-15b. We express the frequencies in beats per minute.

$$f = f_0\sqrt{\frac{c-v}{c+v}} \rightarrow v = c\frac{\left(f_0^2-f^2\right)}{\left(f^2+f_0^2\right)} = c\frac{\left(60^2-30^2\right)}{\left(60^2+30^2\right)} = \boxed{0.60\,c}$$

75. (*a*) The velocity components of the light in the S′ frame are $u_x' = 0$ and $u_y' = c$. We transform those velocities to the S frame according to Eq. 36-7.

$$u_x = \frac{u_x'+v}{1+vu_x'/c^2} = \frac{0+v}{1+0} = v \;;\; u_y = \frac{u_y'\sqrt{1-v^2/c^2}}{1+vu_x'/c^2} = \frac{c\sqrt{1-v^2/c^2}}{1+0} = c\sqrt{1-v^2/c^2}$$

$$\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\frac{c\sqrt{1-v^2/c^2}}{v} = \boxed{\tan^{-1}\sqrt{\frac{c^2}{v^2}-1}}$$

(*b*) $u = \sqrt{u_x^2+u_y^2} = \sqrt{v^2+c^2\left(1-v^2/c^2\right)} = \sqrt{v^2+c^2-v^2} = \boxed{c}$

(*c*) In a Galilean transformation, we would have the following.

$$u_x = u_x'+v = v \;;\; u_y = u_y' = c \;;\; \boxed{u = \sqrt{v^2+c^2}}\,(>c) \;;\; \boxed{\theta = \tan^{-1}\frac{c}{v}}$$

76. We take the positive direction as the direction of motion of rocket A. Consider rocket A as reference frame S, and the Earth as reference frame S′. The velocity of the Earth relative to rocket A is $v = -0.65c$. The velocity of rocket B relative to the Earth is $u_x' = 0.85c$. Solve for the velocity of rocket B relative to rocket A, u_x, using Eq. 36-7a.

$$u_x = \frac{\left(u_x'+v\right)}{\left(1+\frac{vu_x'}{c^2}\right)} = \frac{\left(0.85c-0.65c\right)}{\left[1+\left(-0.65\right)\left(0.85\right)\right]} = \boxed{0.45c}$$

Note that a Galilean analysis would have resulted in $u_x = 0.20c$.

77. (*a*) We find the speed from Eq. 36-10.

$$K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=14{,}000mc^2 \quad\rightarrow$$

$$v=c\sqrt{1-\left(\frac{1}{14{,}001}\right)^2}\approx c-\frac{c}{2}\left(\frac{1}{14{,}001}\right)^2 \quad\rightarrow$$

$$c-v=\frac{c}{2}\left(\frac{1}{14{,}001}\right)^2=\frac{\left(3.00\times10^8\,\text{m/s}\right)}{2}\left(\frac{1}{14{,}001}\right)^2=\boxed{0.77\,\text{m/s}}$$

(*b*) The tube will be contracted in the rest frame of the electron, according to Eq. 36-3a.

$$\ell_0=\ell\sqrt{1-v^2/c^2}=\left(3.0\times10^3\text{m}\right)\sqrt{1-\left[1-\left(\frac{1}{14{,}001}\right)^2\right]}=\boxed{0.21\,\text{m}}$$

78. The electrostatic force provides the radial acceleration. We solve that relationship for the speed of the electron.

$$\left|F_{\text{electrostatic}}\right|=\left|F_{\text{centripetal}}\right| \quad\rightarrow\quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}=\frac{m_{\text{electron}}v^2}{r} \quad\rightarrow$$

$$v=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_{\text{electron}}r}}=\sqrt{\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{9.11\times10^{-31}\text{kg}}\frac{\left(1.60\times10^{-19}\text{C}\right)^2}{\left(0.53\times10^{-10}\text{m}\right)}}=2.18\times10^6\,\text{m/s}=0.0073c$$

Because this is much less than $0.1c$, the electron is $\boxed{\text{not relativistic.}}$

$\boxed{79.}$ The minimum energy required would be the energy to produce the pair with no kinetic energy, so the total energy is their rest energy. They both have the same mass. Use Eq. 36-12.

$$E=2mc^2=2\left(0.511\,\text{MeV}\right)=\boxed{1.022\,\text{MeV}\ \left(1.64\times10^{-13}\,\text{J}\right)}$$

80. The wattage times the time is the energy required. We use Eq. 36-12 to calculate the mass.

$$E=Pt=mc^2 \quad\rightarrow\quad m=\frac{Pt}{c^2}=\frac{(75\,\text{W})\left(3.16\times10^7\text{s}\right)}{\left(3.00\times10^8\,\text{m/s}\right)^2}\left(\frac{1000\,\text{g}}{1\,\text{kg}}\right)=\boxed{2.6\times10^{-5}\,\text{g}}$$

81. Use Eqs. 36-13, 36-8, and 36-11.

$$E^2=p^2c^2+m^2c^4 \quad\rightarrow\quad E=\left(p^2c^2+m^2c^4\right)^{1/2} \quad\rightarrow$$

$$\frac{dE}{dp}=\tfrac{1}{2}\left(p^2c^2+m^2c^4\right)^{-1/2}2pc^2=\frac{pc^2}{E}=\frac{pc^2}{E}=\frac{\gamma mvc^2}{\gamma mc^2}=\boxed{v}$$

82. The kinetic energy available comes from the decrease in rest energy.

$$K=m_{\text{n}}c^2-\left(m_{\text{p}}c^2+m_ec^2+m_\nu c^2\right)=939.57\,\text{MeV}-\left(938.27\,\text{MeV}+0.511\,\text{MeV}+0\right)=\boxed{0.79\,\text{MeV}}$$

83. (*a*) We find the rate of mass loss from Eq. 36-12.

$$E=mc^2 \quad\rightarrow\quad \Delta E=(\Delta m)c^2 \quad\rightarrow$$

$$\frac{\Delta m}{\Delta t}=\frac{1}{c^2}\left(\frac{\Delta E}{\Delta t}\right)=\frac{4\times10^{26}\,\text{J/s}}{\left(3.00\times10^8\,\text{m/s}\right)^2}=4.44\times10^9\,\text{kg/s}\approx\boxed{4\times10^9\,\text{kg/s}}$$

(*b*) Find the time from the mass of the Sun and the rate determined in part (*a*).

$$\Delta t=\frac{m_{\text{Earth}}}{\Delta m/\Delta t}=\frac{\left(5.98\times10^{24}\text{ kg}\right)}{\left(4.44\times10^{9}\text{ kg/s}\right)\left(3.156\times10^{7}\text{ s/y}\right)}=4.27\times10^{7}\text{ y}\approx\boxed{4\times10^{7}\text{ y}}$$

(*c*) We find the time for the Sun to lose all of its mass at this same rate.

$$\Delta t=\frac{m_{\text{Sun}}}{\Delta m/\Delta t}=\frac{\left(1.99\times10^{30}\text{ kg}\right)}{\left(4.44\times10^{9}\text{ kg/s}\right)\left(3.156\times10^{7}\text{ s/y}\right)}=1.42\times10^{13}\text{ y}\approx\boxed{1\times10^{13}\text{ y}}$$

84. Use Eq. 36-8 for the momentum to find the mass.

$$p=\gamma mv=\frac{mv}{\sqrt{1-v^2/c^2}}\quad\rightarrow$$

$$m=\frac{p\sqrt{1-v^2/c^2}}{v}=\frac{\left(3.07\times10^{-22}\text{ kg}\bullet\text{m/s}\right)\sqrt{1-\left(\dfrac{2.24\times10^{8}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2}}{2.24\times10^{8}\text{ m/s}}=9.12\times10^{-31}\text{kg}$$

This particle has the mass of an electron, and a negative charge, so it must be an $\boxed{\text{electron}}$.

$\boxed{85.}$ The total binding energy is the energy required to provide the increase in rest energy.

$$E=\left[\left(2m_{\text{p+e}}+2m_{\text{n}}\right)-m_{\text{He}}\right]c^2$$

$$=\left[2\left(1.00783\text{u}\right)+2\left(1.00867\text{u}\right)-4.00260\text{u}\right]c^2\left(\frac{931.5\text{MeV}/c^2}{\text{u}}\right)=\boxed{28.32\text{ MeV}}$$

86. The momentum is given by Eq. 36-8, and the energy is given by Eq. 36-11 and Eq. 36-13.

$$P=\gamma mv=\frac{\gamma mc^2v}{c^2}=\frac{Ev}{c^2}\quad\rightarrow\quad v=\boxed{\frac{pc^2}{E}}=\frac{pc^2}{\sqrt{m^2c^4+p^2c^2}}=\boxed{\frac{pc}{\sqrt{m^2c^2+p^2}}}$$

87. (*a*) The magnitudes of the momenta are equal. We use Eq. 36-8.

$$p=\gamma mv=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{mc^2\left(v/c\right)}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{\left(938.3\text{MeV}\right)\left(0.985\right)}{\sqrt{1-0.985^2}}=5356\text{ MeV}/c$$

$$\approx\boxed{5.36\text{GeV}/c}=\left(5.36\text{GeV}/c\right)\left(\frac{1c}{3.00\times10^{8}\text{ m/s}}\right)\left(\frac{1.602\times10^{-10}\text{ J/GeV}}{1\text{GeV}}\right)$$

$$=\boxed{2.86\times10^{-18}\text{ kg}\bullet\text{m/s}}$$

(*b*) Because the protons are moving in opposite directions, the vector sum of the momenta is $\boxed{0.}$

(*c*) In the reference frame of one proton, the laboratory is moving at 0.985*c*. The other proton is moving at $+0.985c$ relative to the laboratory. We find the speed of one proton relative to the other, and then find the momentum of the moving proton in the rest frame of the other proton by using that relative velocity.

$$u_x=\frac{\left(v+u_x'\right)}{\left(1+\dfrac{vu_x'}{c^2}\right)}=\frac{\left[0.985c+\left(0.985c\right)\right]}{\left[1+\left(0.985\right)\left(0.985\right)\right]}\approx0.9999c$$

$$p=\gamma mu_x=\frac{mu_x}{\sqrt{1-u_x^2/c^2}}=\frac{1}{c}\frac{mc^2\left(u_x/c\right)}{\sqrt{1-u_x^2/c^2}}=\frac{1}{c}\frac{\left(938.3\,\text{MeV}\right)\left(\dfrac{2(0.985)}{1+0.985^2}\right)}{\sqrt{1-\left(\dfrac{2(0.985)}{1+0.985^2}\right)^2}}=62081\,\text{MeV}/c$$

$$\approx\boxed{62.1\,\text{GeV}/c}=\left(62.1\,\text{GeV}/c\right)\left(\frac{1c}{3.00\times10^8\,\text{m/s}}\right)\left(\frac{1.602\times10^{-10}\,\text{J/GeV}}{1\,\text{GeV}}\right)$$

$$=\boxed{3.31\times10^{-17}\,\text{kg}\bullet\text{m/s}}$$

88. We find the loss in mass from Eq. 36-12.

$$\Delta m=\frac{E}{c^2}=\frac{484\times10^3\,\text{J}}{\left(3.00\times10^8\,\text{m/s}\right)^2}=\boxed{5.38\times10^{-12}\,\text{kg}}$$

Two moles of water has a mass of 36×10^{-3}kg. Find the percentage of mass lost.

$$\frac{5.38\times10^{-12}\,\text{kg}}{36\times10^{-3}\,\text{kg}}=1.49\times10^{-10}=\boxed{1.5\times10^{-8}\,\%}$$

89. Use Eq. 36-10 for kinetic energy, and Eq. 36-12 for rest energy.

$$K=(\gamma-1)m_{\text{Enterprise}}c^2=m_{\text{converted}}c^2\quad\rightarrow$$

$$m_{\text{converted}}=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)m_{\text{Enterprise}}=\left(\frac{1}{\sqrt{1-0.10^2}}-1\right)\left(6\times10^9\,\text{kg}\right)=\boxed{3\times10^7\,\text{kg}}$$

90. We set the kinetic energy of the spacecraft equal to the rest energy of an unknown mass. Use Eqs. 36-10 and 36-12.

$$K=(\gamma-1)m_{\text{ship}}c^2=mc^2\quad\rightarrow$$

$$m=(\gamma-1)m_{\text{ship}}=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)m_{\text{ship}}=\left(\frac{1}{\sqrt{1-0.70^2}}-1\right)\left(1.8\times10^5\,\text{kg}\right)=\boxed{7.2\times10^4\,\text{kg}}$$

From the Earth's point of view, the distance is 35 ly and the speed is 0.70*c*. That data is used to calculate the time from the Earth frame, and then Eq. 36-1a is used to calculate the time in the spaceship frame.

$$\Delta t=\frac{d}{v}=\frac{(35\,\text{y})c}{0.70c}=50\,\text{y}\ ;\ \Delta t_0=\Delta t\sqrt{1-v^2/c^2}=(50\,\text{y})\sqrt{1-0.70^2}=\boxed{36\,\text{y}}$$

$\boxed{91.}$ We assume one particle is moving in the negative direction in the laboratory frame, and the other particle is moving in the positive direction. We consider the particle moving in the negative direction as reference frame S, and the laboratory as reference frame S′. The velocity of the laboratory relative to the negative-moving particle is $v=0.85c$, and the velocity of the positive-moving particle relative to the laboratory frame is $u_x'=0.85c$. Solve for the velocity of the positive-moving particle relative to the negative-moving particle, u_x.

$$u_x=\frac{\left(u_x'+v\right)}{\left(1+\dfrac{vu_x'}{c^2}\right)}=\frac{\left(0.85c+0.85c\right)}{\left[1+(0.85)(0.85)\right]}=\boxed{0.987c}$$

92. We consider the motion from the reference frame of the spaceship. The passengers will see the trip distance contracted, as given by Eq. 36-3a. They will measure their speed to be that contracted distance divided by the year of travel time (as measured on the ship). Use that speed to find the work done (the kinetic energy of the ship).

$$v=\frac{\ell}{\Delta t_0}=\frac{\ell_0\sqrt{1-v^2/c^2}}{\Delta t_0}\quad\rightarrow\quad\frac{v}{c}=\frac{1}{\left(\sqrt{1+\left(\dfrac{c\Delta t_0}{\ell_0}\right)^2}\right)}=\frac{1}{\left(\sqrt{1+\left(\dfrac{1.0\,\text{ly}}{6.6\,\text{ly}}\right)^2}\right)}=0.9887c$$

$$W=\Delta K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2$$

$$=\left(\frac{1}{\sqrt{1-0.9887^2}}-1\right)\left(3.2\times10^4\,\text{kg}\right)\left(3.00\times10^8\,\text{m/s}\right)^2=\boxed{1.6\times10^{22}\,\text{J}}$$

93. The kinetic energy is given by Eq. 36-10.

$$K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=\left(\frac{1}{\sqrt{1-(0.98)^2}}-1\right)(14{,}500\,\text{kg})\left(3.00\times10^8\,\text{m/s}\right)^2$$

$$=\boxed{5.3\times10^{21}\,\text{J}}$$

We compare this with annual U.S. energy consumption: $\dfrac{5.3\times10^{21}\,\text{J}}{10^{20}\,\text{J}}=53.$

The spaceship's kinetic energy is over 50 times as great.

94. The pi meson decays at rest, and so the momentum of the muon and the neutrino must each have the same magnitude (and opposite directions). The neutrino has no rest mass, and the total energy must be conserved. We combine these relationships using Eq. 36-13.

$$E_v=\left(p_v{}^2c^2+m_v{}^2c^4\right)^{1/2}=p_vc\ \ ;\ p_\mu=p_v=p$$

$$E_\pi=E_\mu+E_v\ \rightarrow\ m_\pi c^2=\left(p_\mu{}^2c^2+m_\mu{}^2c^4\right)^{1/2}+p_vc=\left(p^2c^2+m_\mu{}^2c^4\right)^{1/2}+pc\ \rightarrow$$

$$m_\pi c^2-pc=\left(p^2c^2+m_\mu{}^2c^4\right)^{1/2}\ \rightarrow\ \left(m_\pi c^2-pc\right)^2=\left(p^2c^2+m_\mu{}^2c^4\right)$$

Solve for the momentum.

$$m_\pi{}^2c^4-2m_\pi c^2pc+p^2c^2=p^2c^2+m_\mu{}^2c^4\ \rightarrow\ pc=\frac{m_\pi{}^2c^2-m_\mu{}^2c^2}{2m_\pi}$$

Write the kinetic energy of the muon using Eqs. 36-11 and 36-13.

$$K_\mu=E_\mu-m_\mu c^2\ \ ;\ E_\mu=E_\pi-E_v=m_\pi c^2-pc\ \rightarrow$$

$$K_\mu=\left(m_\pi c^2-pc\right)-m_\mu c^2=m_\pi c^2-m_\mu c^2-\frac{\left(m_\pi{}^2c^2-m_\mu{}^2c^2\right)}{2m_\pi}$$

$$=\frac{2m_\pi\left(m_\pi c^2-m_\mu c^2\right)}{2m_\pi}-\frac{\left(m_\pi{}^2c^2-m_\mu{}^2c^2\right)}{2m_\pi}$$

$$=\frac{\left(2m_\pi{}^2-2m_\mu m_\pi-m_\pi{}^2+m_\mu{}^2\right)c^2}{2m_\pi}=\frac{\left(m_\pi{}^2-2m_\mu m_\pi+m_\mu{}^2\right)c^2}{2m_\pi}=\boxed{\frac{\left(m_\pi-m_\mu\right)^2c^2}{2m_\pi}}$$

95. (*a*) The relative speed can be calculated in either frame, and will be the same value in both frames. The time as measured on the Earth will be longer than the time measured on the spaceship, as given by Eq. 36-1a.

$$v=\frac{\Delta x_{\text{Earth}}}{\Delta t_{\text{Earth}}}\ ;\ \Delta t_{\text{Earth}}=\frac{\Delta t_{\text{spaceship}}}{\sqrt{1-v^2/c^2}}=\frac{\Delta t_{\text{spaceship}}}{\sqrt{1-\left(\dfrac{\Delta x_{\text{Earth}}}{c\Delta t_{\text{Earth}}}\right)^2}}\ \rightarrow$$

$$\left(\Delta t_{\text{Earth}}\right)^2-\left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2=\left(\Delta t_{\text{spaceship}}\right)^2\ \rightarrow\ \left(\Delta t_{\text{Earth}}\right)^2-\left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2=\left(\Delta t_{\text{spaceship}}\right)^2\ \rightarrow$$

$$\Delta t_{\text{Earth}}=\sqrt{\left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2+\left(\Delta t_{\text{spaceship}}\right)^2}=\sqrt{(6.0\,\text{y})^2+(2.50\,\text{y})^2}=\boxed{6.5\,\text{y}}$$

(b) The distance as measured by the spaceship will be contracted.

$$v=\frac{\Delta x_{\text{Earth}}}{\Delta t_{\text{Earth}}}=\frac{\Delta x_{\text{spaceship}}}{\Delta t_{\text{spaceship}}}\ \rightarrow\ \Delta x_{\text{spaceship}}=\frac{\Delta t_{\text{spaceship}}}{\Delta t_{\text{Earth}}}\Delta x_{\text{Earth}}=\frac{2.50\,\text{y}}{6.5\,\text{y}}(6.0\,\text{ly})=\boxed{2.3\,\text{ly}}$$

This is the same distance as found using the length contraction relationship.

96. (*a*) To observers on the ship, the period is non-relativistic. Use Eq. 14-7b.

$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{(1.88\,\text{kg})}{(84.2\,\text{N/m})}}=\boxed{0.939\,\text{s}}$$

(*b*) The oscillating mass is a clock. According to observers on Earth, clocks on the spacecraft run slow.

$$T_{\text{Earth}}=\frac{T}{\sqrt{1-v^2/c^2}}=\frac{(0.939\,\text{s})}{\sqrt{1-(0.900)^2}}=\boxed{2.15\,\text{s}}$$

$\boxed{97.}$ We use the Lorentz transformations to derive the result.

$$x=\gamma\left(x'+vt'\right)\ \rightarrow\ \Delta x=\gamma\left(\Delta x'+v\Delta t'\right)\ ;\ t=\gamma\left(t'+\frac{vx'}{c^2}\right)\ \rightarrow\ \Delta t=\gamma\left(\Delta t'+\frac{v\Delta x'}{c^2}\right)$$

$$\left(c\Delta t\right)^2-\left(\Delta x\right)^2=\left[c\gamma\left(\Delta t'+\frac{v\Delta x'}{c^2}\right)\right]^2-\left[\gamma\left(\Delta x'+v\Delta t'\right)\right]^2=\gamma^2\left\{\left[\left(c\Delta t'+\frac{v\Delta x'}{c}\right)\right]^2-\left(\Delta x'+v\Delta t'\right)\right\}$$

$$=\gamma^2\left\{c^2\left(\Delta t'\right)^2+2c\Delta t'\frac{v\Delta x'}{c}+\left(\frac{v\Delta x'}{c}\right)^2-\left(\Delta x'\right)^2-2\Delta x'v\Delta t'-\left(v\Delta t'\right)^2\right\}$$

$$=\frac{1}{\left(1-v^2/c^2\right)}\left\{\left(c^2-v^2\right)\left(\Delta t'\right)^2+\left[\left(\frac{v}{c}\right)^2-1\right]\left(\Delta x'\right)^2\right\}$$

$$=\frac{1}{1-v^2/c^2}\left\{c^2\left(1-v^2/c^2\right)\left(\Delta t'\right)^2-\left(1-v^2/c^2\right)\left(\Delta x'\right)^2\right\}$$

$$=\frac{\left(1-v^2/c^2\right)}{\left(1-v^2/c^2\right)}\left\{\left(c\Delta t'\right)^2-\left(\Delta x'\right)^2\right\}=\left(c\Delta t'\right)^2-\left(\Delta x'\right)^2$$

98. We assume that the left edge of the glass is even with point A when the flash of light is emitted. There is no loss of generality with that assumption. We do the calculations in the frame of reference in which points A and B are at rest, and the glass is then moving to the right with speed *v*.

If the glass is not moving, we would have this "no motion" result.

$$t_{v=0} = t_{\text{glass}} + t_{\text{vacuum}} = \frac{\text{distance in glass}}{\text{speed in glass}} + \frac{\text{distance in vacuum}}{\text{speed in vacuum}} = \frac{d}{v_{\text{glass}}} + \frac{\ell - d}{c}$$

$$= \frac{d}{c/n} + \frac{\ell - d}{c} = \frac{nd}{c} + \frac{\ell - d}{c} = \frac{nd + \ell - d}{c} = \frac{\ell + (n-1)d}{c}$$

If the index of refraction is $n = 1$, then the glass will have no effect on the light, and the time would simply be the distance divided by the speed of light.

$$t_{n=1} = t_{\text{glass}} + t_{\text{vacuum}} = \frac{\text{distance in glass}}{\text{speed in glass}} + \frac{\text{distance in vacuum}}{\text{speed in vacuum}} = \frac{d}{c} + \frac{\ell - d}{c} = \frac{d + \ell - d}{c} = \frac{\ell}{c}$$

Now, let us consider the problem from a relativistic point of view. The speed of light in the glass will be the relativistic sum of the speed of light in stationary glass, c/n, and the speed of the glass, *v*, by Eq. 36-7a. We define β to simplify further expressions.

$$v_{\substack{\text{light}\\ \text{in glass}}} = \frac{\frac{c}{n} + v}{1 + \frac{cv}{nc^2}} = \frac{\frac{c}{n} + v}{1 + \frac{v}{nc}} = \frac{c}{n}\frac{\left(1 + \frac{vn}{c}\right)}{\left(1 + \frac{v}{nc}\right)} = \beta\frac{c}{n} \qquad \left[\beta = \frac{\left(1 + \frac{vn}{c}\right)}{\left(1 + \frac{v}{nc}\right)}\right]$$

The contracted width of the glass, from the Earth frame of reference, is given by Eq. 36-3a.

$$d_{\substack{\text{moving}\\ \text{glass}}} = d\sqrt{1 - v^2/c^2} = \frac{d}{\gamma}$$

We assume the light enters the block when the left edge of the block is at point A, and write simple equations for the displacement of the leading edge of the light, and the leading edge of the block. Set them equal and solve for the time when the light exits the right edge of the block.

$$x_{\text{light}} = v_{\substack{\text{light}\\ \text{in glass}}}\, t = \beta\frac{c}{n}t \ ; \ x_{\substack{\text{right}\\ \text{edge}}} = \frac{d}{\gamma} + vt \ ;$$

$$x_{\text{light}} = x_{\substack{\text{right}\\ \text{edge}}} \ \rightarrow \ \beta\frac{c}{n}t_{\text{glass}} = \frac{d}{\gamma} + vt_{\text{glass}} \ \rightarrow \ t_{\text{glass}} = \frac{d}{\gamma}\frac{n}{(\beta c - nv)}$$

Where is the front edge of the block when the light emerges? Use $t_{\text{glass}} = \frac{d}{\gamma}\frac{n}{(\beta c - nv)}$ with either expression – for the leading edge of the light, or the leading edge of the block.

$$x_{\text{light}} = v_{\substack{\text{light}\\ \text{in glass}}}\, t_{\text{glass}} = \beta\frac{c}{n}\frac{d}{\gamma}\frac{n}{(\beta c - nv)} = \frac{\beta cd}{\gamma(\beta c - nv)}$$

$$x_{\substack{\text{right}\\ \text{edge}}} = \frac{d}{\gamma} + vt_{\text{glass}} = \frac{d}{\gamma} + v\frac{d}{\gamma}\frac{n}{(\beta c - nv)} = \frac{d(\beta c - nv) + vdn}{\gamma(\beta c - nv)} = \frac{\beta cd}{\gamma(\beta c - nv)}$$

The part of the path that is left, $\ell - \frac{\beta cd}{\gamma(\beta c - nv)}$, will be traveled at speed *c* by the light. We express that time, and then find the total time.

$$t_{\text{vacuum}} = \frac{\ell - \frac{\beta cd}{\gamma(\beta c - nv)}}{c}$$

$$t_{\text{total}}=t_{\text{glass}}+t_{\text{vacuum}}=t_{\text{glass}}=\frac{d}{\gamma}\frac{n}{\left(\beta c-nv\right)}+\frac{\ell-\dfrac{\beta cd}{\gamma\left(\beta c-nv\right)}}{c}=\frac{\ell}{c}+\frac{d}{\gamma}\frac{n-\beta}{\left(\beta c-nv\right)}$$

$$=\boxed{\frac{\ell}{c}+\frac{\left(n-1\right)d}{c}\sqrt{\frac{c-v}{c+v}}}$$

We check this for the appropriate limiting cases.

Case 1: $\underset{v=c}{t_{\text{total}}}=\dfrac{\ell}{c}+\dfrac{\left(n-1\right)d}{c}\sqrt{\dfrac{c-v}{c+v}}=\dfrac{\ell}{c}+\dfrac{\left(n-1\right)d}{c}\sqrt{\dfrac{c-c}{c+c}}=\dfrac{\ell}{c}$

This result was expected, because the speed of the light would always be *c*.

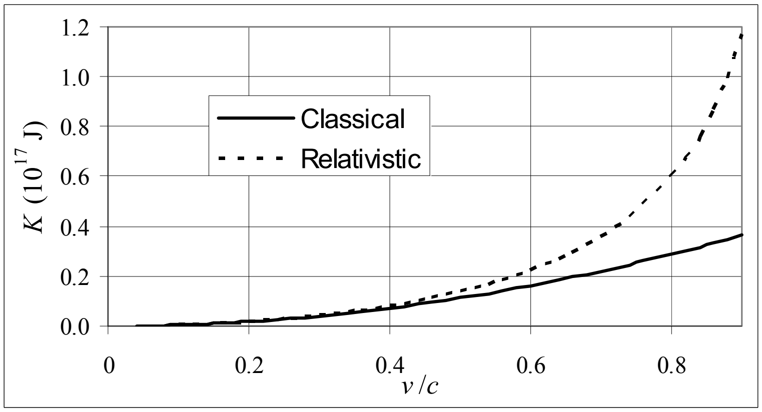
Case 2: $\underset{v=0}{t_{\text{total}}}=\dfrac{\ell}{c}+\dfrac{\left(n-1\right)d}{c}\sqrt{\dfrac{c-v}{c+v}}=\dfrac{\ell}{c}+\dfrac{\left(n-1\right)d}{c}\left(1\right)=\dfrac{\ell+\left(n-1\right)d}{c}$

This result was obtained earlier in the solution.

Case 3: $\underset{n=1}{t_{\text{total}}}=\dfrac{\ell}{c}+\dfrac{\left(n-1\right)d}{c}\sqrt{\dfrac{c-v}{c+v}}=\dfrac{\ell}{c}$

This result was expected, because then there is no speed change in the glass.

99. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH36.XLS," on tab "Problem 36.99."



100. (*a*) We use Eq. 36-98. Since there is motion in two dimensions, we have $\gamma=\dfrac{1}{\sqrt{1-\dfrac{v_x^2}{c^2}-\dfrac{v_y^2}{c^2}}}$.

$$\vec{\mathbf{F}}=-F\hat{\mathbf{j}}=\frac{d\vec{\mathbf{p}}}{dt}\ ;\ \frac{dp_x}{dt}=0\ \rightarrow\ p_x=\gamma mv_x=p_0\ ;\ \frac{dp_y}{dt}=-F\ \rightarrow\ p_y=-Ft=\gamma mv_y$$

Use the component equations to obtain expressions for v_x^2 and v_y^2.

$$\gamma mv_x=p_0\ \rightarrow\ v_x=\frac{p_0}{\gamma m}\ \rightarrow\ v_x^2=\frac{p_0^2}{\gamma^2m^2}=\frac{p_0^2}{m^2}\left(1-\frac{v_x^2}{c^2}-\frac{v_y^2}{c^2}\right)\ \rightarrow\ v_x^2=p_0^2\frac{\left(c^2-v_y^2\right)}{\left(m^2c^2+p_0^2\right)}$$

$$\gamma mv_y=-Ft\ \rightarrow\ v_y=\frac{-Ft}{\gamma m}\ \rightarrow\ v_y^2=\frac{F^2t^2}{\gamma^2m^2}=\frac{F^2t^2}{m^2}\left(1-\frac{v_x^2}{c^2}-\frac{v_y^2}{c^2}\right)\ \rightarrow$$

$$v_y^2=F^2t^2\frac{\left(c^2-v_x^2\right)}{\left(m^2c^2+F^2t^2\right)}$$

Substitute the expression for v_y^2 into the expression for v_x^2.

$$v_x^2 = p_0^2 \frac{\left(c^2 - v_y^2\right)}{\left(m^2c^2 + p_0^2\right)} = p_0^2 \frac{\left(c^2 - F^2t^2 \dfrac{\left(c^2 - v_x^2\right)}{\left(m^2c^2 + F^2t^2\right)}\right)}{\left(m^2c^2 + p_0^2\right)} = p_0^2 \frac{\left(m^2c^4 + F^2t^2v_x^2\right)}{\left(m^2c^2 + p_0^2\right)\left(m^2c^2 + F^2t^2\right)} \rightarrow$$

$$v_x^2\left(m^2c^2 + p_0^2\right)\left(m^2c^2 + F^2t^2\right) = p_0^2\left(m^2c^4 + F^2t^2v_x^2\right) \rightarrow$$

$$v_x^2m^4c^4 + v_x^2m^2c^2p_0^2 + v_x^2F^2t^2m^2c^2 + v_x^2F^2t^2p_0^2 = p_0^2m^2c^4 + p_0^2F^2t^2v_x^2 \rightarrow$$

$$v_x^2m^2c^2 + v_x^2p_0^2 + v_x^2F^2t^2 = p_0^2c^2 \rightarrow \boxed{v_x = \frac{p_0c}{\left(m^2c^2 + p_0^2 + F^2t^2\right)^{1/2}}}$$

Use the expression for v_x to solve for v_y.

$$v_y^2 = F^2t^2 \frac{\left(c^2 - v_x^2\right)}{\left(m^2c^2 + F^2t^2\right)} = F^2t^2 \frac{\left(c^2 - \dfrac{p_0^2c^2}{\left(m^2c^2 + p_0^2 + F^2t^2\right)}\right)}{\left(m^2c^2 + F^2t^2\right)}$$

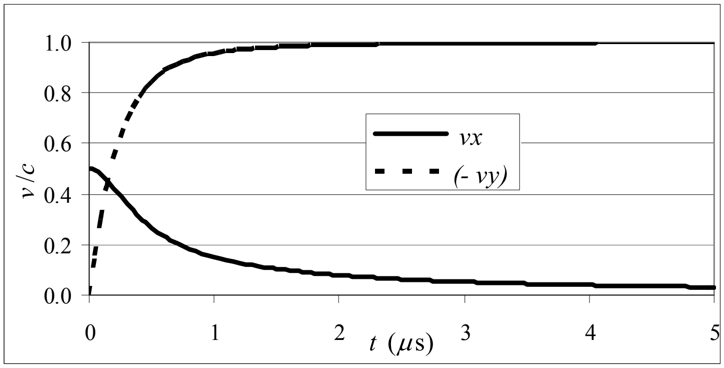
$$= F^2t^2 \frac{\left(c^2\left(m^2c^2 + p_0^2 + F^2t^2\right) - p_0^2c^2\right)}{\left(m^2c^2 + F^2t^2\right)\left(m^2c^2 + p_0^2 + F^2t^2\right)} = F^2t^2 \frac{c^2\left(m^2c^2 + F^2t^2\right)}{\left(m^2c^2 + F^2t^2\right)\left(m^2c^2 + p_0^2 + F^2t^2\right)} \rightarrow$$

$$\boxed{v_y = \frac{-Ftc}{\left(m^2c^2 + p_0^2 + F^2t^2\right)^{1/2}}}$$

The negative sign comes from taking the negative square root of the previous equation. We know that the particle is moving down.

(*b*) See the graph. We are plotting v_x/c and $-v_y/c$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH36.XLS," on tab "Problem 36.100."



(*c*) The path is not parabolic, because the v_x is not constant. Even though there is no force in the *x*-direction, as the net speed of the particle increases, γ increases. Thus v_x must decrease as time elapses in order for p_x to stay constant.

CHAPTER 37: Early Quantum Theory and Models of the Atom

Responses to Questions

1. A reddish star is the coolest, followed by a whitish-yellow star. Bluish stars have the highest temperatures. The temperature of the star is related to the frequency of the emitted light. Since red light has a lower frequency than blue light, red stars have a lower temperature than blue stars.

2. The energy radiated by an object may not be in the visible part of the electromagnetic spectrum. The spectrum of a blackbody with a temperature of 1000 K peaks in the IR and the object appears red, since it includes some radiation at the red end of the visible spectrum. Cooler objects will radiate less overall energy and peak at even longer wavelengths. Objects that are cool enough will not radiate any energy at visible wavelengths.

3. The lightbulb will not produce light as white as the Sun, since the peak of its emitted light is in the infrared. The lightbulb will appear more yellowish than the Sun, which has a spectrum that peaks in the visible range.

4. A bulb which appears red would emit very little radiant energy at higher visible frequencies and therefore would not expose black and white photographic paper. This strategy would not work in a darkroom for developing color photographs since the photographic paper would be sensitive to light at all visible frequencies, including red.

5. If the threshold wavelength increases for the second metal, then it has a smaller work function than the first metal. Longer wavelength corresponds to lower energy. It will take less energy for the electron to escape the surface of the second metal.

6. According to the wave theory, light of any frequency can cause electrons to be ejected as long as the light is intense enough. A higher intensity corresponds to a greater electric field magnitude and more energy. Therefore, there should be no frequency below which the photoelectric effect does not occur. According to the particle theory, however, each photon carries an amount of energy which depends upon its frequency. Increasing the intensity of the light increases the number of photons but does not increase the energy of the individual photons. The cutoff frequency is that frequency at which the energy of the photon equals the work function. If the frequency of the incoming light is below the cutoff, the electrons will not be ejected because no individual photon has enough energy to impart to an electron.

7. Individual photons of ultraviolet light are more energetic than photons of visible light and will deliver more energy to the skin, causing burns. UV photons also can penetrate farther into the skin, and, once at the deeper level, can deposit a large amount of energy that can cause damage to cells.

8. Cesium will give a higher maximum kinetic energy for the electrons. Cesium has a lower work function, so more energy is available for the kinetic energy of the electrons.

9. (*a*) No. The energy of a beam of photons depends not only on the energy of each individual photon but also on the total number of photons. If there are enough infrared photons, the infrared beam may have more energy than the ultraviolet beam.
   (*b*) Yes. The energy of a single photon depends on its frequency: $E = hf$. Since infrared light has a lower frequency than ultraviolet light, a single IR photon will always have less energy than a single UV photon.

10. Fewer electrons are emitted from the surface struck by the 400 nm photons. Each 400 nm photon has a higher energy than each 450 nm photon, so it will take fewer 400 nm photons to produce the same intensity (energy per unit area per unit time) as the 450 nm photon beam. The maximum kinetic energy of the electrons emitted from the surface struck by the 400 nm photons will be greater than the maximum kinetic energy of the electrons emitted from the surface struck by the 450 nm photons, again because each 400 nm photon has a higher energy.

11. (*a*) In a burglar alarm, when the light beam is interrupted (by an intruder, or a door or window opening), the current stops flowing in the circuit. An alarm could be set to go off when the current stops.
    (*b*) In a smoke detector, when the light beam is obscured by smoke, the current in the circuit would decrease or stop. An alarm could be set to go off when the current decreased below a certain level.
    (*c*) The amount of current in the circuit depends on the intensity of the light, as long as the frequency of the light is above the threshold frequency. The ammeter in the circuit could be calibrated to reflect the light intensity.

12. Yes, the wavelength increases. In the scattering process, some of the energy of the incident photon is transferred to the electron, so the scattered photon has less energy, and therefore a lower frequency and longer wavelength, than the incident photon. ($E = hf = hc/\lambda$.)

13. In the photoelectric effect the photon energy is completely absorbed by the electron. In the Compton effect, the photon is scattered from the electron and travels off at a lower energy.

14. According to both the wave theory and the particle theory the intensity of a point source of light decreases as the inverse square of the distance from the source. In the wave theory, the intensity of the waves obeys the inverse square law. In the particle theory, the surface area of a sphere increases with the square of the radius, and therefore the density of particles decreases with distance, obeying the inverse square law. The variation of intensity with distance cannot be used to help distinguish between the two theories.

15. The proton will have the shorter wavelength, since it has a larger mass than the electron and therefore a larger momentum $\left(\lambda = h/p\right)$.

16. Light demonstrates characteristics of both waves and particles. Diffraction and interference are wave characteristics, and are demonstrated, for example, in Young's double-slit experiment. The photoelectric effect and Compton scattering are examples of experiments in which light demonstrates particle characteristics. We can't say that light IS a wave or a particle, but it has properties of each.

17. Electrons demonstrate characteristics of both waves and particles. Electrons act like waves in electron diffraction and like particles in the Compton effect and other collisions.

18. Both a photon and an electron have properties of waves and properties of particles. They can both be associated with a wavelength and they can both undergo scattering. An electron has a negative charge and a rest mass, obeys the Pauli exclusion principle, and travels at less than the speed of light. A photon is not charged, has no rest mass, does not obey the Pauli exclusion principle, and travels at the speed of light.

19. Opposite charges attract, so the attractive Coulomb force between the positive nucleus and the negative electrons keeps the electrons from flying off into space.

20. Look at a solar absorption spectrum, measured above the Earth's atmosphere. If there are dark (absorption) lines at the wavelengths corresponding to oxygen transitions, then there is oxygen near the surface of the Sun.

21. At room temperature, nearly all the atoms in hydrogen gas will be in the ground state. When light passes through the gas, photons are absorbed, causing electrons to make transitions to higher states and creating absorption lines. These lines correspond to the Lyman series since that is the series of transitions involving the ground state or n = 1 level. Since there are virtually no atoms in higher energy states, photons corresponding to transitions from $n \geq 2$ to higher states will not be absorbed.

22. The closeness of the spacing between energy levels near the top of Figure 37-26 indicates that the energy differences between these levels are small. Small energy differences correspond to small wavelength differences, leading to the closely spaced spectral lines in Figure 37-21.

23. There is no direct connection between the size of a particle and its de Broglie wavelength. It is possible for the wavelength to be smaller or larger than the particle.

24. On average the electrons of helium are closer to the nucleus than the electrons of hydrogen. The nucleus of helium contains two protons (positive charges), and so attracts each electron more strongly than the single proton in the nucleus of hydrogen. (There is some shielding of the nuclear charge by the second electron, but each electron still feels the attractive force of more than one proton's worth of charge.)

25. The lines in the spectrum of hydrogen correspond to all the possible transitions that the electron can make. The Balmer lines, for example, correspond to an electron moving from all higher energy levels to the n = 2 level. Although an individual hydrogen atom only contains one electron, a sample of hydrogen gas contains many atoms and all the different atoms will be undergoing different transitions.

26. The Balmer series spectral lines are in the visible light range and could be seen by early experimenters without special detection equipment.

27. The photon carries momentum, so according to conservation of momentum, the hydrogen atom will recoil as the photon is ejected. Some of the energy emitted in the transition of the atom to a lower energy state will be the kinetic energy of the recoiling atom, so the photon will have slightly less energy than predicted by the simple difference in energy levels.

28. No. At room temperature, virtually all the atoms in a sample of hydrogen gas will be in the ground state. Thus, the absorption spectrum will contain primarily just the Lyman lines, as photons corresponding to transitions from the n = 1 level to higher levels are absorbed. Hydrogen at very high temperatures will have atoms in excited states. The electrons in the higher energy levels will fall to all lower energy levels, not just the n = 1 level. Therefore, emission lines corresponding to transitions to levels higher than n = 1 will be present as well as the Lyman lines. In general, you would expect to see only Lyman lines in the absorption spectrum of room temperature hydrogen, but you would find Lyman, Balmer, Paschen, and other lines in the emission spectrum of high-temperature hydrogen.

Solutions to Problems

In several problems, the value of *hc* is needed. We often use the result of Problem 96, $hc = 1240\,\mathrm{eV{\bullet}nm}$.

1. We use Wien's law, Eq. 37-1.

(*a*) $$\lambda_\mathrm{P} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(273\,\mathrm{K}\right)} = 1.06\times10^{-5}\,\mathrm{m} = \boxed{10.6\,\mu\mathrm{m}}$$

This wavelength is in the $\boxed{\text{far infrared.}}$

(*b*) $$\lambda_\mathrm{P} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(3500\,\mathrm{K}\right)} = 8.29\times10^{-7}\,\mathrm{m} = \boxed{829\,\mathrm{nm}}$$

This wavelength is in the $\boxed{\text{infrared.}}$

(*c*) $$\lambda_\mathrm{P} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(4.2\,\mathrm{K}\right)} = 6.90\times10^{-4}\,\mathrm{m} = \boxed{0.69\,\mathrm{mm}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

(*d*) $$\lambda_\mathrm{P} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(2.725\,\mathrm{K}\right)} = 1.06\times10^{-3}\,\mathrm{m} = \boxed{1.06\,\mathrm{mm}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

2. We use Wien's law to find the temperature for a peak wavelength of 460 nm.

$$T = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\lambda_\mathrm{P}} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(460\times10^{-9}\,\mathrm{m}\right)} = \boxed{6300\,\mathrm{K}}$$

3. Because the energy is quantized according to Eq. 37-2, the difference in energy between adjacent levels is simply $E = nhf$.

$$\Delta E = hf = \left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(8.1\times10^{13}\,\mathrm{Hz}\right) = \boxed{5.4\times10^{-20}\,\mathrm{J} = 0.34\,\mathrm{eV}}$$

4. We use Eq. 37-1 with a temperature of $98^\circ\mathrm{F} = 37^\circ\mathrm{C} = 310\,\mathrm{K}$.

$$\lambda_\mathrm{P} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{T} = \frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)}{\left(310\,\mathrm{K}\right)} = 9.4\times10^{-6}\,\mathrm{m} = \boxed{9.4\,\mu\mathrm{m}}$$

5. (*a*) Wien's displacement law says that $\lambda_\mathrm{P}T =$ constant. We must find the wavelength at which $I(\lambda,T)$ is a maximum for a given temperature. This can be found by setting $\partial I/\partial\lambda = 0$.

$$\frac{\partial I}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right) = 2\pi hc^2\frac{\partial}{\partial\lambda}\left(\frac{\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)$$

$$= 2\pi hc^2\left[\frac{\left(e^{hc/\lambda kT}-1\right)\left(-5\lambda^{-6}\right) - \lambda^{-5}e^{hc/\lambda kT}\left(-\dfrac{hc}{kT\lambda^2}\right)}{\left(e^{hc/\lambda kT}-1\right)^2}\right]$$

$$= \frac{2\pi hc^2}{\lambda^6\left(e^{hc/\lambda kT}-1\right)^2}\left[5 + e^{hc/\lambda kT}\left(\frac{hc}{kT\lambda}-5\right)\right] = 0 \;\rightarrow\; 5 = e^{hc/\lambda kT}\left(5 - \frac{hc}{kT\lambda}\right) \;\rightarrow$$

$$e^{x}\left(5-x\right)=5\ ;\ x=\frac{hc}{\lambda_{\mathrm{P}}kT}$$

This transcendental equation will have some solution x = constant, and so $\frac{hc}{\lambda_{\mathrm{P}}kT}$ = constant, and so $\boxed{\lambda_{\mathrm{P}}T=\text{constant}}$. The constant could be evaluated from solving the transcendental equation,

(*b*) To find the value of the constant, we solve $e^{x}\left(5-x\right)=5$, or $5-x=5e^{-x}$. This can be done graphically, by graphing both $y=5-x$ and $y=5e^{-x}$ on the same set of axes and finding the intersection point. Or, the quantity $5-x-5e^{-x}$ could be calculated, and find for what value of x that expression is 0. The answer is x = 4.966. We use this value to solve for h. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.5."

$$\frac{hc}{\lambda_{\mathrm{P}}kT}=4.966\ \rightarrow$$

$$h=4.966\frac{\lambda_{\mathrm{P}}Tk}{c}=4.966\frac{\left(2.90\times10^{-3}\,\mathrm{m{\bullet}K}\right)\left(1.38\times10^{-23}\,\mathrm{J/K}\right)}{3.00\times10^{8}\,\mathrm{m/s}}=\boxed{6.62\times10^{-34}\,\mathrm{J{\bullet}s}}$$

(*c*) We integrate Planck's radiation formula over all wavelengths.

$$\int_0^{\infty}I\left(\lambda,T\right)d\lambda=\int_0^{\infty}\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda\ ;\ \text{let}\ \frac{hc}{\lambda kT}=x\ ;\ \lambda=\frac{hc}{xkT}\ ;\ d\lambda=-\frac{hc}{x^2kT}dx$$

$$\int_0^{\infty}I\left(\lambda,T\right)d\lambda=\int_0^{\infty}\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda=\int_{\infty}^{0}\left(\frac{2\pi hc^2\left(\frac{hc}{xkT}\right)^{-5}}{e^{x}-1}\right)\left(-\frac{hc}{x^2kT}dx\right)=\frac{2\pi k^4T^4}{h^3c^2}\int_0^{\infty}\left(\frac{x^3}{e^{x}-1}\right)dx$$

$$=\frac{2\pi k^4}{h^3c^2}\left[\int_0^{\infty}\left(\frac{x^3}{e^{x}-1}\right)dx\right]T^4\propto T^4$$

Thus the total radiated power per unit area is proportional to T^4. Everything else in the expression is constant with respect to temperature.

6. We use Eq. 37-3.

$$E=hf=\left(6.626\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(104.1\times10^{6}\,\mathrm{Hz}\right)=\boxed{6.898\times10^{-26}\,\mathrm{J}}$$

$\boxed{7.}$ We use Eq. 37-3 along with the fact that $f=c/\lambda$ for light. The longest wavelength will have the lowest energy.

$$E_1=hf_1=\frac{hc}{\lambda_1}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(410\times10^{-9}\,\mathrm{m}\right)}=4.85\times10^{-19}\,\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right)=3.03\,\mathrm{eV}$$

$$E_2=hf_2=\frac{hc}{\lambda_2}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{\left(750\times10^{-9}\,\mathrm{m}\right)}=2.65\times10^{-19}\,\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right)=1.66\,\mathrm{eV}$$

Thus the range of energies is $\boxed{2.7\times10^{-19}\,\mathrm{J}<E<4.9\times10^{-19}\,\mathrm{J}}$ or $\boxed{1.7\,\mathrm{eV}<E<3.0\,\mathrm{eV}}$.

8. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(380\times10^{3}\,\text{eV})} = 3.27\times10^{-12}\,\text{m} \approx \boxed{3.3\times10^{-3}\,\text{nm}}$$

Significant diffraction occurs when the opening is on the order of the wavelength. Thus there would be $\boxed{\text{insignificant diffraction}}$ through the doorway.

9. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$E_{\text{min}} = hf_{\text{min}} \rightarrow f_{\text{min}} = \frac{E_{\text{min}}}{h} = \frac{(0.1\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})} = 2.41\times10^{13}\,\text{Hz} \approx \boxed{2\times10^{13}\,\text{Hz}}$$

$$\lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{(3.00\times10^{8}\,\text{m/s})}{(2.41\times10^{13}\,\text{Hz})} = 1.24\times10^{-5}\,\text{m} \approx \boxed{1\times10^{-5}\,\text{m}}$$

10. We use Eq. 37-5.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(6.20\times10^{-7}\,\text{m})} = \boxed{1.07\times10^{-27}\,\text{kg}\bullet\text{m/s}}$$

11. At the minimum frequency, the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\text{min}} - W_0 = 0 \rightarrow f_{\text{min}} = \frac{W_0}{h} = \frac{4.8\times10^{-19}\,\text{J}}{6.63\times10^{-34}\,\text{J}\bullet\text{s}} = \boxed{7.2\times10^{14}\,\text{Hz}}$$

12. The longest wavelength corresponds to the minimum frequency. That occurs when the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\text{min}} - W_0 = 0 \rightarrow f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{W_0}{h} \rightarrow$$

$$\lambda_{\text{max}} = \frac{ch}{W_0} = \frac{(3.00\times10^{8}\,\text{m/s})(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(3.70\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{3.36\times10^{-7}\,\text{m}} = 336\,\text{nm}$$

$\boxed{13.}$ The energy of the photon will equal the kinetic energy of the baseball. We use Eq. 37-3.

$$K = hf \rightarrow \tfrac{1}{2}mv^2 = h\frac{c}{\lambda} \rightarrow \lambda = \frac{2hc}{mv^2} = \frac{2(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(0.145\,\text{kg})(30.0\,\text{m/s})^2} = \boxed{3.05\times10^{-27}\,\text{m}}$$

14. We divide the minimum energy by the photon energy at 550 nm to find the number of photons.

$$E = nhf = E_{\text{min}} \rightarrow n = \frac{E_{\text{min}}}{hf} = \frac{E_{\text{min}}\lambda}{hc} = \frac{(10^{-18}\,\text{J})(550\times10^{-9}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})} = 2.77 \approx \boxed{3\text{ photons}}$$

15. The photon of visible light with the maximum energy has the least wavelength. We use 410 nm as the lowest wavelength of visible light.

$$hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(410\times10^{-9}\,\text{m})} = 3.03\,\text{eV}$$

Electrons will not be emitted if this energy is less than the work function.
The metals with work functions greater than 3.03 eV are $\boxed{\text{copper and iron.}}$

16. (*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so the work function is equal to the energy of the photon.

$$W_0 = hf - K_{\max} = hf = \frac{hc}{\lambda} = \frac{1240\,\text{eV•nm}}{520\,\text{nm}} = \boxed{2.4\,\text{eV}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{\max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV•nm}}{470\,\text{nm}} - 2.38\,\text{eV} = 0.25\,\text{eV}$$

$$V_0 = \frac{K_{\max}}{e} = \frac{0.25\,\text{eV}}{e} = \boxed{0.25\,\text{V}}$$

17. The photon of visible light with the maximum energy has the minimum wavelength. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{\max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV•nm}}{410\text{ nm}} - 2.48\text{ eV} = \boxed{0.54\text{ eV}}$$

18. We use Eq. 37-4b to calculate the maximum kinetic energy. Since the kinetic energy is much less than the rest energy, we use the classical definition of kinetic energy to calculate the speed.

$$K_{\max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV•nm}}{365\,\text{nm}} - 2.48\,\text{eV} = \boxed{0.92\,\text{eV}}$$

$$K_{\max} = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K_{\max}}{m}} = \sqrt{\frac{2(0.92\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\text{kg}}} = \boxed{5.7\times10^5\,\text{m/s}}$$

$\boxed{19.}$ We use Eq. 37-4b to calculate the work function.

$$W_0 = hf - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{1240\text{ eV•nm}}{285\text{ nm}} - 1.70\text{ eV} = \boxed{2.65\text{ eV}}$$

20. Electrons emitted from photons at the threshold wavelength have no kinetic energy. We use Eq. 37-4b with the threshold wavelength to determine the work function.

$$W_0 = \frac{hc}{\lambda} - K_{\max} = \frac{hc}{\lambda_{\max}} = \frac{1240\text{ eV•nm}}{320\text{ nm}} = 3.88\text{ eV}.$$

(*a*) We now use Eq. 36-4b with the work function determined above to calculate the kinetic energy of the photoelectrons emitted by 280 nm light.

$$K_{\max} = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV•nm}}{280\text{ nm}} - 3.88\text{ eV} = \boxed{0.55\text{ eV}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons.}}$

21. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy of the photoelectrons. We use Eq. 37-4b to calculate the work function where the maximum kinetic energy is the product of the stopping voltage and electron charge.

$$W_0 = \frac{hc}{\lambda} - K_{\max} = \frac{hc}{\lambda} - eV_0 = \frac{1240\text{ eV•nm}}{230\text{ nm}} - (1.84\text{ V})e = \boxed{3.55\text{ eV}}$$

22. The energy required for the chemical reaction is provided by the photon. We use Eq. 37-3 for the energy of the photon, where $f = c/\lambda$.

$$E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\bullet\text{nm}}{630 \text{ nm}} = \boxed{2.0 \text{ eV}}$$

Each reaction takes place in a molecule, so we use the appropriate conversions to convert eV/molecule to kcal/mol.

$$E = \left(\frac{2.0 \text{ eV}}{\text{molecule}}\right)\left(\frac{1.60\times10^{-19} \text{ J}}{\text{eV}}\right)\left(\frac{6.02\times10^{23} \text{ molecules}}{\text{mol}}\right)\left(\frac{\text{kcal}}{4186 \text{ J}}\right) = \boxed{45 \text{ kcal/mole}}$$

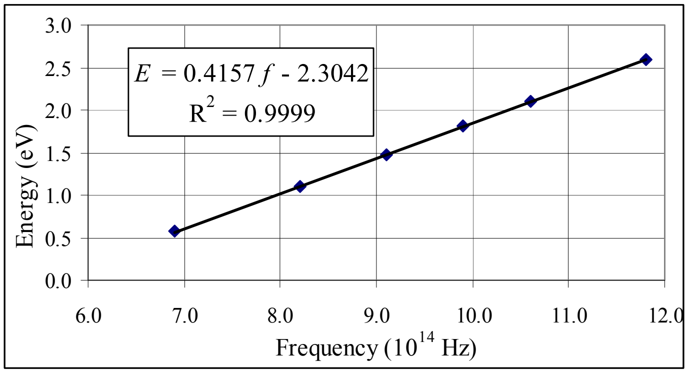
23. (*a*) Since $f = c/\lambda$, the photon energy given by Eq. 37-3 can be written in terms of the wavelength as $E = hc/\lambda$. This shows that the photon with the largest wavelength has the smallest energy. The 750-nm photon then delivers the minimum energy that will excite the retina.

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\text{J}}\right) = \boxed{1.66\,\text{eV}}$$

(*b*) The eye cannot see light with wavelengths less than 410 nm. Obviously, these wavelength photons have more energy than the minimum required to initiate vision, so they must not arrive at the retina. That is, wavelength less than 410 nm are absorbed near the front portion of the eye. The threshold photon energy is that of a 410-nm photon.

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\text{J}}\right) = \boxed{3.03\,\text{eV}}$$

24. We plot the maximum (kinetic) energy of the emitted electrons vs. the frequency of the incident radiation. Eq. 37-4b says $K_{\text{max}} = hf - W_0$. The best-fit straight line is determined by linear regression in Excel. The slope of the best-fit straight line to the data should give Planck's constant, the *x*-intercept is the cutoff frequency, and the *y*-intercept is the opposite of the work function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.24."



(*a*) $h = \left(0.4157\,\text{eV}/10^{14}\text{Hz}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \boxed{6.7\times10^{-34}\,\text{J}\bullet\text{s}}$

(*b*) $hf_{\text{cutoff}} = W_0 \;\rightarrow\; f_{\text{cutoff}} = \dfrac{W_0}{h} = \dfrac{2.3042\,\text{eV}}{\left(0.4157\,\text{eV}/10^{14}\text{Hz}\right)} = \boxed{5.5\times10^{14}\,\text{Hz}}$

(*c*) $W_0 = \boxed{2.3\,\text{eV}}$

$\boxed{25.}$ (*a*) Since $f = c/\lambda$, the photon energy is $E = hc/\lambda$ and the largest wavelength has the smallest energy. In order to eject electrons for all possible incident visible light, the metal's work function must be less than or equal to the energy of a 750-nm photon. Thus the maximum value for the metal's work function W_o is found by setting the work function equal to the energy of the 750-nm photon.

$$W_\text{o}=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=\boxed{1.66\,\text{eV}}$$

(*b*) If the photomultiplier is to function only for incident wavelengths less than 410-nm, then we set the work function equal to the energy of the 410-nm photon.

$$W_\text{o}=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=\boxed{3.03\,\text{eV}}$$

26. Since $f=c/\lambda$, the energy of each emitted photon is $E=hc/\lambda$. We multiply the energy of each photon by $1.0\times10^{6}/\text{s}$ to determine the average power output of each atom. At distance of $r=25\,\text{cm}$, the light sensor measures an intensity of $I=1.6\,\text{nW}/1.0\,\text{cm}^2$. Since light energy emitted from atoms radiates equally in all directions, the intensity varies with distance as a spherical wave. Thus, from Section 15–3 in the text, the average power emitted is $\bar{P}=4\pi r^2 I$. Dividing the total average power by the power from each atom gives the number of trapped atoms.

$$N=\frac{\bar{P}}{\bar{P}_\text{atom}}=\frac{4\pi r^2 I}{nhc/\lambda}=\frac{4\pi\left(25\text{cm}\right)^2\left(1.6\times10^{-9}\,\text{W/cm}^2\right)}{\left(1.0\times10^{6}/\text{s}\right)\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{m/s}\right)/\left(780\times10^{-9}\,\text{m}\right)}$$
$$=\boxed{4.9\times10^{7}\,\text{atoms}}$$

27. We set the kinetic energy in Eq. 37-4b equal to the stopping voltage, eV_0, and write the frequency of the incident light in terms of the wavelength, $f=c/\lambda$. We differentiate the resulting equation and solve for the fractional change in wavelength, and we take the absolute value of the final expression.

$$eV_0=\frac{hc}{\lambda}-W_0\ \rightarrow\ e\,dV_0=-\frac{hc}{\lambda^2}d\lambda\ \rightarrow\ \frac{d\lambda}{\lambda}=-\frac{e\,dV_0\lambda}{hc}\ \approx\ \boxed{\frac{\Delta\lambda}{\lambda}=\frac{e\lambda}{hc}\Delta V_0}$$

$$\frac{\Delta\lambda}{\lambda}=\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(550\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}\left(0.01\text{V}\right)=\boxed{0.004}$$

28. We use Eq. 37-6b. Note that the answer is correct to two significant figures.

$$\Delta\lambda=\frac{h}{m_\text{e}c}\left(1-\cos\phi\right)\ \rightarrow$$

$$\phi=\cos^{-1}\left(1-\frac{m_\text{e}c\Delta\lambda}{h}\right)=\cos^{-1}\left(1-\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(1.5\times10^{-13}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}\right)=\boxed{20^\circ}$$

29. The Compton wavelength for a particle of mass *m* is h/mc.

(*a*) $$\frac{h}{m_\text{e}c}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{2.43\times10^{-12}\,\text{m}}$$

(*b*) $$\frac{h}{m_\text{p}c}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{1.32\times10^{-15}\,\text{m}}$$

(*c*) The energy of the photon is given by Eq. 37-3.

$$E_\text{photon}=hf=\frac{hc}{\lambda}=\frac{hc}{\left(h/mc\right)}=mc^2=\text{rest energy}$$

30. We find the Compton wavelength shift for a photon scattered from an electron, using Eq. 37-6b. The Compton wavelength of a free electron is given in the text right after Eq. 37-6b.

$$\lambda' - \lambda = \left(\frac{h}{m_\text{e}c}\right)(1-\cos\theta) = \lambda_\text{C}(1-\cos\theta) = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos\theta)$$

(*a*) $\lambda'_a - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos60^\circ) = \boxed{1.22\times10^{-3}\,\text{nm}}$

(*b*) $\lambda'_b - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos90^\circ) = \boxed{2.43\times10^{-3}\,\text{nm}}$

(*c*) $\lambda'_c - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos180^\circ) = \boxed{4.86\times10^{-3}\,\text{nm}}$

$\boxed{31.}$ (*a*) In the Compton effect, the maximum change in the photon's wavelength is when scattering angle $\phi = 180^\circ$. We use Eq. 37-6b to determine the maximum change in wavelength. Dividing the maximum change by the initial wavelength gives the maximum fractional change.

$$\Delta\lambda = \frac{h}{m_\text{e}c}(1-\cos\theta) \quad\rightarrow$$

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_\text{e}c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(1-\cos180^\circ)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(550\times10^{-9}\text{m}\right)} = \boxed{8.8\times10^{-6}}$$

(*b*) We replace the initial wavelength with $\lambda = 0.10$ nm.

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_\text{e}c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(1-\cos180^\circ)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(0.10\times10^{-9}\text{m}\right)} = \boxed{0.049}$$

32. We find the change in wavelength for each scattering event using Eq. 37-6b, with a scattering angle of $\phi = 0.50^\circ$. To calculate the total change in wavelength, we subtract the initial wavelength, obtained from the initial energy, from the final wavelength. We divide the change in wavelength by the wavelength change from each event to determine the number of scattering events.

$$\Delta\lambda = \frac{h}{m_\text{e}c}\left(1-\cos0.5^\circ\right) = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)(1-\cos0.5^\circ)}{\left(9.11\times10^{-31}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)} = 9.24\times10^{-17}\,\text{m} = 9.24\times10^{-8}\,\text{nm}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.0\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 1.24\times10^{-12}\,\text{m} = 0.00124\,\text{nm}\,.$$

$$n = \frac{\lambda - \lambda_0}{\Delta\lambda} = \frac{(555\text{ nm})-(0.00124\text{ nm})}{9.24\times10^{-8}\text{ nm}} = \boxed{6\times10^{9}\,\text{events}}$$

33. (*a*) We use conservation of momentum to set the initial momentum of the photon equal to the sum of the final momentum of the photon and electron, where the momentum of the photon is given by Eq. 37-5 and the momentum of the electron is written in terms of the total energy (Eq. 36-13). We multiply this equation by the speed of light to simplify.

$$\frac{h}{\lambda} + 0 = -\left(\frac{h}{\lambda'}\right) + p_e \quad\rightarrow\quad \frac{hc}{\lambda} = -\left(\frac{hc}{\lambda'}\right) + \sqrt{E^2 - E_0^2}$$

Using conservation of energy we set the initial energy of the photon and rest energy of the electron equal to the sum of the final energy of the photon and the total energy of the electron.

$$\left(\frac{hc}{\lambda}\right) + E_0 = \left(\frac{hc}{\lambda'}\right) + E$$

By summing these two equations, we eliminate the final wavelength of the photon. We then solve the resulting equation for the kinetic energy of the electron, which is the total energy less the rest energy.

$$2\left(\frac{hc}{\lambda}\right)+E_0=\sqrt{E^2-E_0^{\,2}}+E \quad\rightarrow\quad \left[2\left(\frac{hc}{\lambda}\right)+E_0-E\right]^2=E^2-E_0^{\,2}$$

$$\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2-2E\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]+E^2=E^2-E_0^{\,2} \quad\rightarrow\quad E=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$K=E-E_0=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}-\frac{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]E_0}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}=\frac{2\left(\frac{hc}{\lambda}\right)^2}{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$=\frac{2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)^2}{\left[2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)+5.11\times10^5\text{eV}\right]}=\boxed{228\,\text{eV}}$$

(*b*) We solve the energy equation for the final wavelength.

$$\left(\frac{hc}{\lambda}\right)+E_0=\left(\frac{hc}{\lambda'}\right)+E$$

$$\lambda'=\frac{hc}{\left(\frac{hc}{\lambda}\right)+E_0-E}=\left[\frac{1}{\lambda}-\frac{K}{hc}\right]^{-1}=\left[\frac{1}{0.160\,\text{nm}}-\frac{228\,\text{eV}}{1240\,\text{eV}\bullet\text{nm}}\right]^{-1}=\boxed{0.165\,\text{nm}}$$

34. First we use conservation of energy, where the energy of the photon is written in terms of the wavelength, to relate the initial and final energies. Solve this equation for the electron's final energy.

$$\left(\frac{hc}{\lambda}\right)+mc^2=\left(\frac{hc}{\lambda'}\right)+E \;\Rightarrow\; E=\left(\frac{hc}{\lambda}\right)-\left(\frac{hc}{\lambda'}\right)+mc^2$$

Next, we define the *x*-direction as the direction of the initial motion of the photon. We write equations for the conservation of momentum in the horizontal and vertical directions, where θ is the angle the photon makes with the initial direction of the photon and ϕ is the angle the electron makes.

$$p_x:\quad \frac{h}{\lambda}=p_e\cos\phi+\frac{h}{\lambda'}\cos\theta \qquad\qquad p_y:\quad 0=p_e\sin\phi+\frac{h}{\lambda'}\sin\theta$$

To eliminate the variable ϕ we solve the momentum equations for the electron's momentum, square the resulting equations and add the two equations together using the identity $\cos^2\theta+\sin^2\theta=1$.

$$\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2=\left(p_e\cos\phi\right)^2 \qquad\qquad \left(\frac{h}{\lambda'}\sin\theta\right)^2=\left(p_e\sin\phi\right)^2$$

$$\left(p_e\cos\phi\right)^2+\left(p_e\sin\phi\right)^2=\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2+\left(\frac{h}{\lambda'}\sin\theta\right)^2$$

$$p_e^2=\left(\frac{h}{\lambda}\right)^2-\frac{2h^2}{\lambda\lambda'}\cos\theta+\left(\frac{h}{\lambda'}\right)^2$$

We now apply the relativistic invariant equation, Eq. 36-13, to write the electron momentum in terms of the electron energy. Then using the electron energy obtained from the conservation of energy equation, we eliminate the electron energy and solve for the change in wavelength.

$$\left(\frac{h}{\lambda}\right)^2-\frac{2h^2}{\lambda\lambda'}\cos\theta+\left(\frac{h}{\lambda'}\right)^2=\frac{E^2-m^2c^4}{c^2}=\left[\left(\frac{h}{\lambda}\right)-\left(\frac{h}{\lambda'}\right)+mc\right]^2-m^2c^2$$

$$=\left(\frac{h}{\lambda}\right)^2+\left(\frac{h}{\lambda'}\right)^2+m^2c^2+2hmc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)-\frac{h^2}{\lambda\lambda'}-m^2c^2$$

$$-\frac{2h^2}{\lambda\lambda'}\cos\theta=2hmc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)-\frac{h^2}{\lambda\lambda'}$$

$$-h\cos\theta=mc(\lambda'-\lambda)-h \;\rightarrow\; \boxed{\lambda'-\lambda=\frac{h}{mc}(1-\cos\theta)}$$

35. The photon energy must be equal to the kinetic energy of the products plus the mass energy of the products. The mass of the positron is equal to the mass of the electron.

$$E_{\text{photon}}=K_{\text{products}}+m_{\text{products}}c^2 \;\rightarrow$$

$$K_{\text{products}}=E_{\text{photon}}-m_{\text{products}}c^2=E_{\text{photon}}-2m_{\text{electron}}c^2=2.67\,\text{MeV}-2(0.511\,\text{MeV})=\boxed{1.65\,\text{MeV}}$$

36. The photon with the longest wavelength has the minimum energy in order to create the masses with no additional kinetic energy. Use Eq. 37-5.

$$\lambda_{\text{max}}=\frac{hc}{E_{\text{min}}}=\frac{hc}{2mc^2}=\frac{h}{2mc}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{2(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\,\text{m/s})}=\boxed{6.62\times10^{-16}\,\text{m}}$$

This must take place in the presence of some other object in order for momentum to be conserved.

$\boxed{37.}$ The minimum energy necessary is equal to the rest energy of the two muons.

$$E_{\text{min}}=2mc^2=2(207)(0.511\,\text{MeV})=\boxed{212\,\text{MeV}}$$

The wavelength is given by Eq. 37-5.

$$\lambda=\frac{hc}{E}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(212\times10^6\,\text{eV})}=\boxed{5.86\times10^{-15}\,\text{m}}$$

38. Since $v<0.001c$, the total energy of the particles is essentially equal to their rest energy. Both particles have the same rest energy of 0.511 MeV. Since the total momentum is 0, each photon must have half the available energy and equal momenta.

$$E_{\text{photon}}=m_{\text{electron}}c^2=\boxed{0.511\,\text{MeV}}\quad;\quad p_{\text{photon}}=\frac{E_{\text{photon}}}{c}=\boxed{0.511\,\text{MeV}/c}$$

39. The energy of the photon is equal to the total energy of the two particles produced. Both particles have the same kinetic energy and the same mass.

$$E_{\text{photon}}=2(K+mc^2)=2(0.375\,\text{MeV}+0.511\,\text{MeV})=\boxed{1.772\,\text{MeV}}$$

The wavelength is found from Eq. 37-5.

$$\lambda=\frac{hc}{E}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(1.772\times10^6\,\text{eV})}=\boxed{7.02\times10^{-13}\,\text{m}}$$

40. We find the wavelength from Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.23\,\text{kg})(0.10\,\text{m/s})}=\boxed{2.9\times10^{-32}\,\text{m}}$$

41. The neutron is not relativistic, so we can use $p=mv$. We also use Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(8.5\times10^{4}\,\text{m/s}\right)}=\boxed{4.7\times10^{-12}\,\text{m}}$$

42. We assume the electron is non-relativistic, and check that with the final answer. We use Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{mv}\quad\rightarrow\quad v=\frac{h}{m\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.21\times10^{-9}\,\text{m}\right)}=3.466\times10^{6}\,\text{m/s}=0.01155c$$

Our use of classical expressions is justified. The kinetic energy is equal to the potential energy change.

$$eV=K=\tfrac{1}{2}mv^2=\frac{\tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.466\times10^{6}\,\text{m/s}\right)^2}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=34.2\,\text{eV}$$

Thus the required potential difference is $\boxed{34\text{ V.}}$

$\boxed{43.}$ The theoretical resolution limit is the wavelength of the electron. We find the wavelength from the momentum, and find the momentum from the kinetic energy and rest energy. We use the result from Problem 94. The kinetic energy of the electron is 85 keV.

$$\lambda=\frac{hc}{\sqrt{K^2+2mc^2K}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\sqrt{\left(85\times10^{3}\text{eV}\right)^2+2\left(0.511\times10^{6}\text{eV}\right)\left(85\times10^{3}\text{eV}\right)}}$$

$$=\boxed{4.1\times10^{-12}\,\text{m}}$$

44. We use the relativistic expression for momentum, Eq. 36-8.

$$p=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{h}{\lambda}\quad\rightarrow$$

$$\lambda=\frac{h\sqrt{1-v^2/c^2}}{mv}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\sqrt{1-(0.98)^2}}{\left(9.11\times10^{-31}\,\text{kg}\right)(0.98)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{4.9\times10^{-13}\,\text{m}}$$

45. Since the particles are not relativistic, we may use $K=p^2/2m$. We then form the ratio of the kinetic energies, using Eq. 37-7.

$$K=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}\quad;\quad\frac{\lambda_{\text{e}}}{\lambda_{\text{p}}}=\frac{\dfrac{h^2}{2m_{\text{e}}\lambda^2}}{\dfrac{h^2}{2m_{\text{p}}\lambda^2}}=\frac{m_{\text{p}}}{m_{\text{e}}}=\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}=\boxed{1840}$$

46. We assume the neutron is not relativistic. If the resulting velocity is small, our assumption will be valid. We use Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{mv}\quad\rightarrow\quad v=\frac{h}{m\lambda}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)}{\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(0.3\times10^{-9}\,\mathrm{m}\right)}=1300\,\mathrm{m/s}\approx\boxed{1000\,\mathrm{m/s}}$$

This is not relativistic, so our assumption was valid.

47. (*a*) We find the momentum from Eq. 37-7.

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{6.0\times10^{-10}\,\mathrm{m}}=\boxed{1.1\times10^{-24}\,\mathrm{kg\bullet m/s}}$$

(*b*) We assume the speed is non-relativistic.

$$\lambda=\frac{h}{p}=\frac{h}{mv}\quad\rightarrow\quad v=\frac{h}{m\lambda}==\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(6.0\times10^{-10}\,\mathrm{m}\right)}=\boxed{1.2\times10^{6}\,\mathrm{m/s}}$$

Since $v/c=4.04\times10^{-3}$, our assumption is valid.

(*c*) We calculate the kinetic energy classically.

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}\left(mc^2\right)\left(v/c\right)^2=\tfrac{1}{2}\left(0.511\,\mathrm{MeV}\right)\left(4.04\times10^{-3}\right)^2=4.17\times10^{-6}\,\mathrm{MeV}=4.17\,\mathrm{eV}$$

This is the energy gained by an electron if accelerated through a potential difference of $\boxed{4.2\ \mathrm{V}}$.

48. Because all of the energies to be considered are much less than the rest energy of an electron, we can use non-relativistic relationships. We use Eq. 37-7 to calculate the wavelength.

$$K=\frac{p^2}{2m}\quad\rightarrow\quad p=\sqrt{2mK}\ \ ;\ \ \lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}$$

(*a*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\sqrt{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(20\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}=2.7\times10^{-10}\,\mathrm{m}\approx\boxed{3\times10^{-10}\,\mathrm{m}}$$

(*b*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\sqrt{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(200\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}=8.7\times10^{-11}\,\mathrm{m}\approx\boxed{9\times10^{-11}\,\mathrm{m}}$$

(*c*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\mathrm{J\bullet s}}{\sqrt{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2.0\times10^{3}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}=\boxed{2.7\times10^{-11}\,\mathrm{m}}$$

$\boxed{49.}$ Since the particles are not relativistic, we may use $K=p^2/2m$. We then form the ratio of the wavelengths, using Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}\ \ ;\ \ \frac{\lambda_{\mathrm{p}}}{\lambda_{\mathrm{e}}}=\frac{\dfrac{h}{\sqrt{2m_{\mathrm{p}}K}}}{\dfrac{h}{\sqrt{2m_{\mathrm{e}}K}}}=\sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{p}}}}<1$$

Thus we see the proton has the shorter wavelength, since $m_{\mathrm{e}}<m_{\mathrm{p}}$.

50. The final kinetic energy of the electron is equal to the negative change in potential energy of the electron as it passes through the potential difference. We compare this energy to the rest energy of the electron to determine if the electron is relativistic.

$$K=-q\Delta V=\left(1\mathrm{e}\right)\left(33\times10^{3}\,\mathrm{V}\right)=33\times10^{3}\mathrm{eV}$$

Because this is greater than 1% of the electron rest energy, $\boxed{\text{the electron is relativistic}}$. We use Eq. 36-13 to determine the electron momentum and then Eq. 37-5 to determine the wavelength.

$$E^2 = \left[K + mc^2\right]^2 = p^2c^2 + m^2c^4 \;\Rightarrow p = \frac{\sqrt{K^2 + 2Kmc^2}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} = \frac{1240\,\text{eV}\bullet\text{nm}}{\sqrt{\left(33\times10^3\,\text{eV}\right)^2 + 2\left(33\times10^3\,\text{eV}\right)\left(511\times10^3\,\text{eV}\right)}} = 0.0066\,\text{nm}$$

Because $\lambda \ll 5$ cm, $\boxed{\text{diffraction effects are negligible.}}$

51. We will assume that the electrons are non-relativistic, and then examine the result in light of that assumption. The wavelength of the electron can be found from Eq. 34-2a. The speed can then be found from Eq. 37-7.

$$d\sin\theta = m_{\text{order}}\lambda \;\rightarrow\; \lambda = \frac{d\sin\theta}{m_{\text{order}}} \;;\; \lambda = \frac{h}{p} = \frac{h}{m_e v} \;\rightarrow$$

$$v = \frac{hm_{\text{order}}}{m_e d\sin\theta} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(2)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.0\times10^{-6}\text{m}\right)\left(\sin55^\circ\right)} = \boxed{590\,\text{m/s}}$$

This is far from being relativistic, so our original assumption was fine.

52. We relate the kinetic energy to the momentum with a classical relationship, since the electrons are non-relativistic. We also use Eq. 37-7. We then assume that the kinetic energy was acquired by electrostatic potential energy.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = eV \;\rightarrow$$

$$V = \frac{h^2}{2me\lambda^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.60\times10^{-19}\text{C}\right)\left(0.28\times10^{-9}\text{m}\right)^2} = \boxed{19\,\text{V}}$$

53. The kinetic energy is 3450 eV. That is small enough compared to the rest energy of the electron for the electron to be non-relativistic. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{\left(2mK\right)^{1/2}} = \frac{hc}{\left(2mc^2K\right)^{1/2}} = \frac{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\text{m/s}\right)}{\left(1.60\times10^{-19}\text{J/eV}\right)\left[2\left(0.511\times10^{6}\,\text{eV}\right)\left(3450\text{ eV}\right)\right]^{1/2}}$$

$$= 2.09\times10^{-11}\text{m} = \boxed{20.9\,\text{pm}}$$

54. The energy of a level is $E_n = -\dfrac{\left(13.6\text{ eV}\right)}{n^2}$.

(*a*) The transition from $n = 1$ to $n' = 3$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 3$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -\left(13.6\text{ eV}\right)\left[\left(\frac{1}{3^2}\right) - \left(\frac{1}{1^2}\right)\right] = 12.1\text{ eV}$$

(*b*) The transition from $n = 6$ to $n' = 2$ is an $\boxed{\text{emission,}}$ because the $\boxed{\text{initial state,}}$ $n' = 2$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = -\left(E_{n'} - E_n\right) = \left(13.6\text{ eV}\right)\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{6^2}\right)\right] = 3.0\text{ eV}$$

(*c*) The transition from $n = 4$ to $n' = 5$ is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n' = 5$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{5^2}\right)-\left(\frac{1}{4^2}\right)\right] = 0.31\text{ eV}$$

The photon for the transition from $\boxed{n = 1 \text{ to } n' = 3}$ has the largest energy.

55. To ionize the atom means removing the electron, or raising it to zero energy.

$$E_{\text{ionization}} = 0 - E_n = 0 - \frac{(-13.6\,\text{eV})}{n^2} = \frac{(13.6\,\text{eV})}{3^2} = \boxed{1.51\,\text{eV}}$$

56. We use the equation that appears above Eq. 37-15 in the text.

(*a*) The second Balmer line is the transition from $n = 4$ to $n = 2$.

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-0.85\,\text{eV} - (-3.4\,\text{eV})]} = \boxed{490\,\text{nm}}$$

(*b*) The third Lyman line is the transition from $n = 4$ to $n = 1$.

$$\lambda = \frac{hc}{(E_4 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-0.85\text{ eV} - (-13.6\text{ eV})]} = \boxed{97.3\,\text{nm}}$$

(*c*) The first Balmer line is the transition from $n = 3$ to $n = 2$.
For the jump from $n = 5$ to $n = 2$, we have

$$\lambda = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

57. Doubly ionized lithium is similar to hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\,\text{eV})}{n^2} = -\frac{3^2(13.6\,\text{eV})}{n^2} = -\frac{(122\,\text{eV})}{n^2}$$

$$E_{\text{ionization}} = 0 - E_1 = 0 - \left[-\frac{(122\,\text{eV})}{(1)^2}\right] = \boxed{122\,\text{eV}}$$

58. We evaluate the Rydberg constant using Eq. 37-8 and 37-15. We use hydrogen so $Z = 1$.

$$\frac{1}{\lambda} = R\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c}\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) \rightarrow$$

$$R = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c} = \frac{(1)^2(1.602176\times10^{-19}\,\text{C})^4(9.109382\times10^{-31}\,\text{kg})}{8(8.854188\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)^2(6.626069\times10^{-34}\,\text{J}\bullet\text{s})^3(2.997925\times10^8\text{ m/s})}$$

$$= 1.0974\times10^7\,\frac{\text{C}^4\bullet\text{kg}}{\dfrac{\text{C}^4}{\text{N}^2\bullet\text{m}^4}\,\text{J}^3\text{s}^3\text{ m/s}} = \boxed{1.0974\times10^7\,\text{m}^{-1}}$$

59. The longest wavelength corresponds to the minimum energy, which is the ionization energy:

$$\lambda = \frac{hc}{E_{\text{ion}}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\text{ J/eV})(13.6\,\text{eV})} = 9.14\times10^{-8}\,\text{m} = \boxed{91.4\,\text{nm}}$$

60. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2.

$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}$$

We find the energy of the photon from the $n = 5$ to $n = 2$ transition in singly-ionized helium.

$$\Delta E = E_5 - E_2 = -(54.4\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{2^2}\right)\right] = 11.4\text{ eV}$$

Because this is NOT the energy difference between any two specific energy levels for hydrogen, the photon CANNOT be absorbed by hydrogen.

61. The energy of the photon is the sum of the ionization energy of 13.6 eV and the kinetic energy of 20.0eV. The wavelength is found from Eq. 37-3.

$$hf = \frac{hc}{\lambda} = E_{\text{total}} \rightarrow \lambda = \frac{hc}{E_{\text{total}}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(33.6\text{ eV})} = 3.70\times10^{-8}\,\text{m} = \boxed{37.0\,\text{nm}}$$

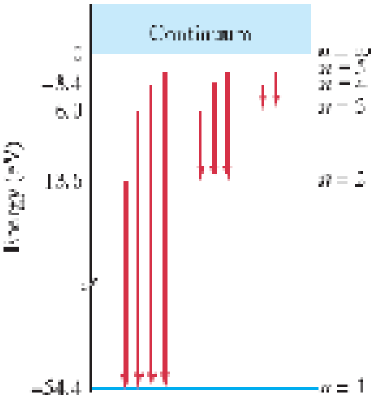
62. A collision is elastic if the kinetic energy before the collision is equal to the kinetic energy after the collision. If the hydrogen atom is in the ground state, then the smallest amount of energy it can absorb is the difference in the $n = 1$ and $n = 2$ levels. So as long as the kinetic energy of the incoming electron is less than that difference, the collision must be elastic.

$$K < E_2 - E_1 = \left(-\frac{13.6\text{eV}}{4}\right) - (-13.6\text{eV}) = \boxed{10.2\,\text{eV}}$$

63. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{2^2(13.6\text{eV})}{n^2} = -\frac{(54.4\text{eV})}{n^2}$$

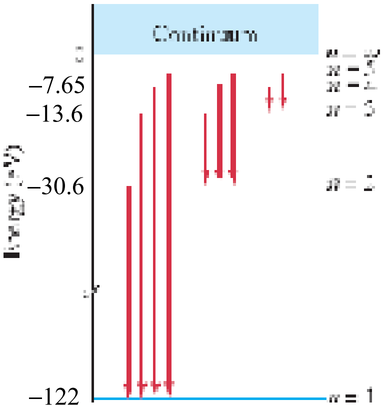
$$E_1 = -54.5\text{eV},\ E_2 = -13.6\text{eV},\ E_3 = -6.0\text{eV},\ E_4 = -3.4\text{eV}$$



64. Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 by Ze^2:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{3^2(13.6\text{eV})}{n^2} = -\frac{(122.4\text{eV})}{n^2}$$

$$E_1 = -122\,\text{eV},\ E_2 = -30.6\text{eV},\ E_3 = -13.6\text{eV},$$
$$E_4 = -7.65\text{eV}$$



65. The potential energy for the ground state is given by the charge of the electron times the electric potential caused by the proton.

$$U=(-e)V_{\text{proton}}=(-e)\frac{1}{4\pi\varepsilon_0}\frac{e}{r_1}=-\frac{\left(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2\left(1\text{eV}/1.60\times10^{-19}\,\text{J}\right)}{\left(0.529\times10^{-10}\,\text{m}\right)}$$

$$=\boxed{-27.2\,\text{eV}}$$

The kinetic energy is the total energy minus the potential energy.

$$K=E_1-U=-13.6\,\text{eV}-(-27.2\,\text{eV})=\boxed{+13.6\,\text{eV}}$$

66. The value of *n* is found from $r_n=n^2r_1$, and then find the energy from Eq. 37-14b.

$$r_n=n^2r_1\quad\rightarrow\quad n=\sqrt{\frac{r_n}{r_1}}=\sqrt{\frac{\frac{1}{2}\left(0.10\times10^{-3}\,\text{m}\right)}{0.529\times10^{-10}\,\text{m}}}=\boxed{972}$$

$$E=-\frac{(13.6\text{ eV})}{n^2}=-\frac{(13.6\text{ eV})}{972^2}=-\frac{(13.6\text{ eV})}{1375^2}=\boxed{-1.4\times10^{-5}\text{eV}}$$

$\boxed{67.}$ The velocity is found from Eq. 37-10 evaluated for *n* = 1.

$$mvr_n=\frac{nh}{2\pi}\quad\rightarrow$$

$$v=\frac{h}{2\pi r_1m_e}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{2\pi\left(0.529\times10^{-10}\,\text{m}\right)\left(9.11\times10^{-31}\,\text{kg}\right)}=2.190\times10^6\,\text{m/s}=\boxed{7.30\times10^{-3}c}$$

We see that $v\ll c$, and so $\boxed{\text{yes}}$, non-relativistic formulas are justified.
The relativistic factor is as follows.

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}\approx1-\tfrac{1}{2}\left(\frac{v}{c}\right)^2=1-\tfrac{1}{2}\left(\frac{2.190\times10^6\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2=\boxed{1-2.66\times10^{-5}}\approx0.99997$$

We see that $\sqrt{1-v^2/c^2}$ is essentially 1, and so again the answer is $\boxed{\text{yes}}$, non-relativistic formulas are justified.

68. The angular momentum can be used to find the quantum number for the orbit, and then the energy can be found from the quantum number. Use Eqs. 37-10 and 37-14b.

$$L=n\frac{h}{2\pi}\quad\rightarrow\quad n=\frac{2\pi L}{h}=\frac{2\pi\left(5.273\times10^{-34}\,\text{kg}\bullet\text{m}^2/\text{s}\right)}{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)}=5.000\approx5$$

$$E_n=-(13.6\text{eV})\frac{Z^2}{n^2}=-\frac{13.6\text{eV}}{25}=\boxed{0.544\,\text{eV}}$$

69. Hydrogen atoms start in the $n=1$ orbit ("ground state"). Using Eq. 37-9 and Eq. 37-14b, we determine the orbit to which the atom is excited when it absorbs a photon of 12.75 Ev via collision with an electron. Then, using Eq. 37-15, we calculate all possible wavelengths that can be emitted as the electron cascades back to the ground state.

$$\Delta E=E_U-E_L\quad\rightarrow\quad E_U=-\frac{13.6\text{ eV}}{n^2}=E_L+\Delta E\quad\rightarrow$$

$$n=\sqrt{\frac{-13.6\text{ eV}}{E_L+\Delta E}}=\sqrt{\frac{-13.6\text{ eV}}{-13.6\text{ eV}+12.75\text{ eV}}}=4$$

Starting with the electron in the $n=4$ orbit, the following transitions are possible: $n=4$ to $n=3$; $n=4$ to $n=2$; $n=4$ to $n=1$; $n=3$ to $n=2$; $n=3$ to $n=1$; $n=2$ to $n=1$.

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=5.333\times10^{5}\text{ m}^{-1}\Rightarrow\lambda=\boxed{1875\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=2.057\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{486.2\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{4^2}\right)=1.028\times10^{7}\text{ m}^{-1}\Rightarrow\lambda=\boxed{97.23\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.524\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{656.3\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=9.751\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{102.6\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=8.228\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{121.5\text{ nm}}$$

70. When we compare the gravitational and electric forces we see that we can use the same expression for the Bohr orbits, Eq. 37-11 and 37-14a, if we replace $Ze^2/4\pi\varepsilon_0$ with Gm_em_p.

$$r_1=\frac{h^2\varepsilon_0}{\pi m_eZe^2}=\frac{h^2}{4\pi^2m_e}\frac{4\pi\varepsilon_0}{Ze^2}\quad\rightarrow$$

$$r_1=\frac{h^2}{4\pi^2Gm^2{}_em_p}=\frac{\left(6.626\times10^{-34}\text{J•s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\text{N•m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\text{kg}\right)^2\left(1.67\times10^{-27}\text{kg}\right)}$$

$$=\boxed{1.20\times10^{29}\text{m}}$$

$$E_1=-\frac{Z^2e^4m_e}{8\varepsilon_0^2h^2}=-\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2m_e}{h^2}\quad\rightarrow\quad E_1=-\frac{2\pi^2G^2m_e{}^3m_p{}^2}{h^2}$$

$$=-\frac{2\pi^2\left(6.67\times10^{-11}\text{ N•m}^2/\text{kg}^2\right)^2\left(9.11\times10^{-31}\text{kg}\right)^3\left(1.67\times10^{-27}\text{kg}\right)^2}{\left(6.626\times10^{-34}\text{J•s}\right)^2}=\boxed{-4.22\times10^{-97}\text{J}}$$

71. We know that the radii of the orbits are given by $r_n=n^2r_1$. Find the difference in radius for adjacent orbits.

$$\Delta r=r_n-r_{n-1}=n^2r_1-\left(n-1\right)^2r_1=n^2r_1-\left(n^2-2n+1\right)r_1=\left(2n-1\right)r_1$$

If $n\gg1$, we have $\Delta r\approx2nr_1=2n\frac{r_n}{n^2}=\frac{2r_n}{n}$.

In the classical limit, the separation of radii (and energies) should be very small. We see that letting $n\rightarrow\infty$ accomplishes this. If we substitute the expression for r_1 from Eq. 37-11, we have this.

$$\Delta r\approx2nr_1=\frac{2nh^2\varepsilon_0}{\pi me^2}$$

We see that $\Delta r\propto h^2$, and so letting $h\rightarrow0$ is equivalent to considering $n\rightarrow\infty$.

72. We calculate the energy from the light bulb that enters the eye by calculating the intensity of the light at a distance of 250 m by dividing the power in the visible spectrum by the area of a sphere of radius 250 m. We multiply the intensity of the light by the area of the pupil to determine the energy entering the eye per second. We divide this energy by the energy of a photon (Eq. 37-3) to calculate the number of photons entering the eye per second.

$$I=\frac{P}{4\pi\ell^2}\qquad P_e=I\left(\pi D^2/4\right)=\frac{P}{16}\left(\frac{D}{\ell}\right)^2$$

$$n=\frac{P_e}{hc/\lambda}=\frac{P\lambda}{16hc}\left(\frac{D}{\ell}\right)^2=\frac{0.030(75\,\text{W})\left(550\times10^{-9}\,\text{m}\right)}{16\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}\left(\frac{4.0\times10^{-3}\,\text{m}}{250\,\text{m}}\right)^2$$

$$=\boxed{1.0\times10^{8}\,\text{photons/sec}}$$

73. To produce a photoelectron, the hydrogen atom must be ionized, so the minimum energy of the photon is 13.6 eV. We find the minimum frequency of the photon from Eq. 37-3.

$$E=hf\;\rightarrow\;f=\frac{E}{h}\;\rightarrow\;f_{\min}=\frac{E_{\min}}{h}=\frac{(13.6\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}=\boxed{3.28\times10^{15}\,\text{Hz}}$$

74. From Section 35-10, the spacing between planes, d, for the first-order peaks is given by Eq. 35-20, $\lambda=2d\sin\theta$. The wavelength of the electrons can be found from their kinetic energy. The electrons are not relativistic at the energy given.

$$K=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}\;\rightarrow\;\lambda=\frac{h}{\sqrt{2mK}}=2d\sin\theta\;\rightarrow$$

$$d=\frac{h}{2\sin\theta\sqrt{2mK}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{2\left(\sin38^\circ\right)\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)(125\,\text{eV})\left(1.60\times10^{-19}\,\text{J/eV}\right)}}=\boxed{8.9\times10^{-11}\,\text{m}}$$

75. The power rating is the amount of energy produced per second. If this is divided by the energy per photon, then the result is the number of photons produced per second.

$$E_{\text{photon}}=hf=\frac{hc}{\lambda}\;;\;\frac{P}{E_{\text{photon}}}=\frac{P\lambda}{hc}=\frac{(860\,\text{W})\left(12.2\times10^{-2}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{5.3\times10^{26}\text{ photons/s}}$$

76. The intensity is the amount of energy per second per unit area reaching the Earth. If that intensity is divided by the energy per photon, the result will be the photons per second per unit area reaching the Earth. We use Eq. 37-3.

$$E_{\text{photon}}=hf=\frac{hc}{\lambda}$$

$$I_{\text{photons}}=\frac{I_{\text{sunlight}}}{E_{\text{photon}}}=\frac{I_{\text{sunlight}}\lambda}{hc}=\frac{\left(1350\,\text{W/m}^2\right)\left(550\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}=\boxed{3.7\times10^{21}\text{ photons/s}\bullet\text{m}^2}$$

77. The impulse on the wall is due to the change in momentum of the photons. Each photon is absorbed, and so its entire momentum is transferred to the wall.

$$F_{\text{on wall}}\Delta t = \Delta p_{\text{wall}} = -\Delta p_{\text{photons}} = -\left(0 - np_{\text{photon}}\right) = np_{\text{photon}} = \frac{nh}{\lambda} \rightarrow$$

$$\frac{n}{\Delta t} = \frac{F\lambda}{h} = \frac{\left(6.5 \times 10^{-9}\,\text{N}\right)\left(633 \times 10^{-9}\,\text{m}\right)}{\left(6.63 \times 10^{-34}\,\text{J•s}\right)} = \boxed{6.2 \times 10^{18}\ \text{photons/s}}$$

78. We find the peak wavelength from Wien's law, Eq. 37-1.

$$\lambda_{\text{P}} = \frac{\left(2.90 \times 10^{-3}\,\text{m•K}\right)}{T} = \frac{\left(2.90 \times 10^{-3}\,\text{m•K}\right)}{\left(2.7\,\text{K}\right)} = 1.1 \times 10^{-3}\,\text{m} = \boxed{1.1\,\text{mm}}$$

$\boxed{79.}$ The total energy of the two photons must equal the total energy (kinetic energy plus mass energy) of the two particles. The total momentum of the photons is 0, so the momentum of the particles must have been equal and opposite. Since both particles have the same mass and the same momentum, they each have the same kinetic energy.

$$E_{\text{photons}} = E_{\text{particles}} = 2\left(m_{\text{e}}c^2 + K\right) \rightarrow$$

$$K = \tfrac{1}{2}E_{\text{photons}} - m_{\text{e}}c^2 = 0.755\,\text{MeV} - 0.511\,\text{MeV} = \boxed{0.244\,\text{MeV}}$$

80. We calculate the required momentum from de Broglie's relation, Eq. 37-7.

$$p = \frac{h}{\lambda} = \frac{\left(6.63 \times 10^{-34}\,\text{J•s}\right)}{\left(6.0 \times 10^{-12}\,\text{m}\right)} = 1.11 \times 10^{22}\ \text{kg•m/s}$$

(*a*) For the proton, we use the classical definition of momentum to determine the speed of the electron, and then the kinetic energy. We divide the kinetic energy by the charge of the proton to determine the required potential difference.

$$v = \frac{p}{m} = \frac{1.11 \times 10^{-22}\,\text{kg•m/s}}{1.67 \times 10^{-27}\,\text{kg}} = 6.65 \times 10^{4}\,\text{m/s} \ll c$$

$$V = \frac{K}{e} = \frac{mv^2}{2e} = \frac{\left(1.67 \times 10^{-27}\,\text{kg}\right)\left(6.65 \times 10^{4}\,\text{m/s}\right)^2}{2\left(1.60 \times 10^{-19}\,\text{C}\right)} = \boxed{23\,\text{V}}$$

(*b*) For the electron, if we divide the momentum by the electron mass we obtain a speed greater than 10% of the speed of light. Therefore, we must use the relativistic invariant equation to determine the energy of the electron. We then subtract the rest energy from the total energy to determine the kinetic energy of the electron. Finally, we divide the kinetic energy by the electron charge to calculate the potential difference.

$$E = \left[\left(pc\right)^2 + \left(m_0c^2\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\left(1.11 \times 10^{-22}\,\text{kg•m/s}\right)^2\left(3.00 \times 10^{8}\,\text{m/s}\right)^2 + \left(9.11 \times 10^{-31}\,\text{kg}\right)^2\left(3.00 \times 10^{8}\,\text{m/s}\right)^4\right]^{\frac{1}{2}}$$

$$= 8.85 \times 10^{-14}\,\text{J}$$

$$K = E - m_0c^2 = 8.85 \times 10^{-14}\,\text{J} - \left(9.11 \times 10^{-31}\,\text{kg}\right)\left(3.00 \times 10^{8}\,\text{m/s}\right)^2 = 6.50 \times 10^{-15}\,\text{J}$$

$$V = \frac{K}{e} = \frac{6.50 \times 10^{-15}\,\text{J}}{1.60 \times 10^{-19}\,\text{C}} = \boxed{41\,\text{kV}}$$

81. If we ignore the recoil motion, at the closest approach the kinetic energy of both particles is zero. The potential energy of the two charges must equal the initial kinetic energy of the α particle:

$$K_\alpha = U = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}}e)}{r_{\min}} \quad \rightarrow$$

$$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}}e)}{K_\alpha} = \frac{(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2)(2)(47)(1.60\times10^{-19}\,\text{C})^2}{(4.8\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})} = \boxed{2.8\times10^{-14}\,\text{m}}$$

82. The electrostatic potential energy is given by Eq. 23-5. The kinetic energy is given by the total energy, Eq. 37-14a, minus the potential energy. The Bohr radius is given by Eq. 37-11.

$$U = -eV = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\pi m Ze^2}{n^2h^2\varepsilon_0} = -\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}$$

$$K = E - U = -\frac{Z^2e^4m}{8\varepsilon_0^2h^2n^2} - \left(-\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\right) = \frac{Z^2e^4m}{8n^2h^2\varepsilon_0^2} \quad ; \quad \frac{|U|}{K} = \frac{\dfrac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}}{\dfrac{Z^2e^4m}{8n^2h^2\varepsilon_0^2}} = \frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\frac{8n^2h^2\varepsilon_0^2}{Z^2e^4m} = \boxed{2}$$

83. We calculate the ratio of the forces.

$$\frac{F_{\text{gravitational}}}{F_{\text{electric}}} = \frac{\left(\dfrac{Gm_em_p}{r^2}\right)}{\left(\dfrac{ke^2}{r^2}\right)} = \frac{Gm_em_p}{ke^2} = \frac{(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\,\text{kg})^2(1.67\times10^{-27}\,\text{kg})}{(9.00\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}$$

$$= \boxed{4.4\times10^{-40}}$$

$\boxed{\text{Yes,}}$ the gravitational force may be safely ignored.

84. The potential difference gives the electrons a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is $-13.6\text{ eV} + 12.3\text{ eV} = -1.3\text{ eV}$. From the energy level diagram, Figure 37-26, we see that this means the atom could be excited to the $n = 3$ state, so the possible transitions when the atom returns to the ground state are $n = 3$ to $n = 2$, $n = 3$ to $n = 1$, and $n = 2$ to $n = 1$. We calculate the wavelengths from the equation above Eq. 37-15.

$$\lambda_{3\rightarrow2} = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

$$\lambda_{3\rightarrow1} = \frac{hc}{(E_3 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-13.6\text{ eV})]} = \boxed{102\,\text{nm}}$$

$$\lambda_{2\rightarrow1} = \frac{hc}{(E_2 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-3.4\text{ eV} - (-13.6\text{ eV})]} = \boxed{122\,\text{nm}}$$

$\boxed{85.}$ The stopping potential is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to first find the work function, and then find the stopping potential for the higher wavelength.

$$K_{\max} = eV_0 = \frac{hc}{\lambda} - W_0 \quad \rightarrow \quad W_0 = \frac{hc}{\lambda_0} - eV_0$$

$$eV_1 = \frac{hc}{\lambda_1} - W_0 = \frac{hc}{\lambda_1} - \left(\frac{hc}{\lambda_0} - eV_0\right) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right) + eV_0$$

$$= \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}\left(\frac{1}{440\times10^{-9}\,\text{m}} - \frac{1}{380\times10^{-9}\,\text{m}}\right) + 2.70\,\text{eV} = 2.25\,\text{eV}$$

The potential difference needed to cancel an electron kinetic energy of 2.25 eV is $\boxed{2.25\text{ V}}$.

86. (*a*) The electron has a charge *e*, so the potential difference produces a kinetic energy of *eV*. The shortest wavelength photon is produced when all the kinetic energy is lost and a photon is emitted.

$$hf_{\max} = \frac{hc}{\lambda_0} = eV \quad\rightarrow\quad \lambda_0 = \boxed{\frac{hc}{eV}} \text{ which gives } \lambda_0 = \frac{hc}{eV}.$$

(*b*) $\lambda_0 = \dfrac{hc}{eV} = \dfrac{1240\,\text{eV}\bullet\text{nm}}{33\times10^{3}\,\text{eV}} = \boxed{0.038\,\text{nm}}$

87. The average force on the sail is equal to the impulse on the sail divided by the time (Eq. 9-2). Since the photons bounce off the mirror the impulse is equal to twice the incident momentum. We use Eq. 37-5 to write the momentum of the photon in terms of the photon energy. The total photon energy is the intensity of the sunlight multiplied by the area of the sail

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2(E/c)}{\Delta t} = \frac{2(E/\Delta t)}{c} = \frac{2IA}{c} = \frac{2\left(1350\text{ W/m}^2\right)\left(1000\text{ m}\right)^2}{3.00\times10^{8}\text{ m/s}} = \boxed{9.0\text{ N}}$$

88. We first find the work function from the given data. A photon energy of 9.0 eV corresponds with a stopping potential of 4.0 V.

$$eV_0 = hf - W_0 \quad\rightarrow\quad W_0 = hf - eV_0 = 9.0\,\text{eV} - 4.0\,\text{eV} = 5.0\,\text{eV}$$

If the photons' wavelength is doubled, the energy is halved, from 9.0 eV to 4.5 eV. This is smaller than the work function, and so no current flows. Thus the maximum kinetic energy is $\boxed{0}$. Likewise, if the photon's wavelength is tripled, the energy is only 3.0 eV, which is still less than the work function, and so $\boxed{\text{no current flows}}$.

89. The electrons will be non-relativistic at that low energy. The maximum kinetic energy of the photoelectrons is given by Eq. 37-4b. The kinetic energy determines the momentum, and the momentum determines the wavelength of the emitted electrons. The shortest electron wavelength corresponds to the maximum kinetic energy.

$$K_{\text{electron}} = \frac{hc}{\lambda} - W_0 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_{\text{electron}}^2} \quad\rightarrow\quad \lambda_{\text{electron}} = \frac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda} - W_0\right)}}$$

$$= \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(\dfrac{1240\,\text{eV}\bullet\text{nm}}{360\,\text{nm}} - 2.4\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = \boxed{1.2\times10^{-9}\,\text{m}}$$

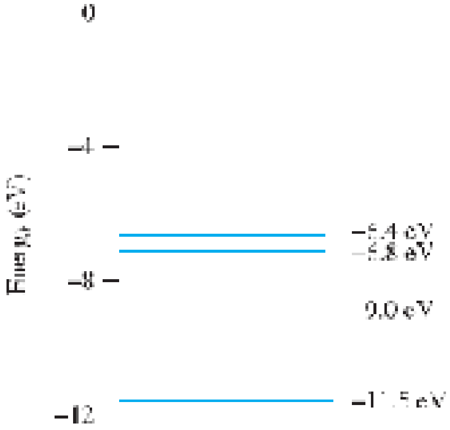
90. The wavelength is found from Eq. 35-13. The velocity of electrons with the same wavelength (and thus the same diffraction pattern) is found from their momentum, assuming they are not relativistic. We use Eq. 37-7 to relate the wavelength and momentum.

$$d\sin\theta = n\lambda \quad\rightarrow\quad \lambda = \frac{d\sin\theta}{n} = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow$$

$$v = \frac{hn}{md\sin\theta} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)(1)}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(0.012\times10^{-3}\,\mathrm{m}\right)\left(\sin 3.5^\circ\right)} = \boxed{990\,\mathrm{m/s}}$$

91. (*a*) See the adjacent figure.

(*b*) Absorption of a 5.1 eV photon represents a transition from the $\boxed{\text{ground state}}$ to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state.



$$-6.4\,\mathrm{eV} - (-6.8\,\mathrm{eV}) = \boxed{0.4\,\mathrm{eV}}$$

$$-6.4\,\mathrm{eV} - (-9.0\,\mathrm{eV}) = \boxed{2.6\,\mathrm{eV}}$$

$$-6.4\,\mathrm{eV} - (-11.5\,\mathrm{eV}) = \boxed{5.1\,\mathrm{eV}}$$

$$-6.8\,\mathrm{eV} - (-9.0\,\mathrm{eV}) = \boxed{2.2\,\mathrm{eV}}$$

$$-6.8\,\mathrm{eV} - (-11.5\,\mathrm{eV}) = \boxed{4.7\,\mathrm{eV}}$$

$$-9.0\,\mathrm{eV} - (-11.5\,\mathrm{eV}) = \boxed{2.5\,\mathrm{eV}}$$

92. (*a*) We use Eq. 37-4b to calculate the maximum kinetic energy of the electron and set this equal to the product of the stopping voltage and the electron charge.

$$K_{\mathrm{max}} = hf - W_0 = eV_0 \quad\rightarrow\quad V_0 = \frac{hf - W_0}{e} = \frac{hc/\lambda - W_0}{e}$$

$$V_0 = \frac{(1240\,\mathrm{eV{\bullet}nm})/(424\,\mathrm{nm}) - 2.28\,\mathrm{eV}}{\mathrm{e}} = \boxed{0.65\,\mathrm{V}}$$

(*b*) We calculate the speed from the non-relativistic kinetic energy equation and the maximum kinetic energy found in part (*a*).

$$K_{\mathrm{max}} = \tfrac{1}{2}mv_{\mathrm{max}}^2 \quad\rightarrow\quad v_{\mathrm{max}} = \sqrt{\frac{2K_{\mathrm{max}}}{m}} = \sqrt{\frac{2(0.65\,\mathrm{eV})\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{9.11\times10^{-31}\,\mathrm{kg}}} = \boxed{4.8\times10^5\,\mathrm{m/s}}$$

(*c*) We use Eq. 37-7 to calculate the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(4.8\times10^5\,\mathrm{m/s}\right)} = 1.52\times10^{-9}\,\mathrm{m} = \boxed{1.5\,\mathrm{nm}}$$

93. (*a*) We use Bohr's analysis of the hydrogen atom, where we replace the proton mass with Earth's mass, the electron mass with the Moon's mass, and the electrostatic force $F_e = \dfrac{ke^2}{r^2}$ with the gravitational force $F_g = \dfrac{Gm_E m_M}{r^2}$. To account for the change in force, we replace ke^2 with $Gm_E m_M$. With these replacements, we write expressions similar to Eq. 37-11 and Eq. 37-14a for the Bohr radius and energy.

$$r_n = \frac{h^2 n^2}{4\pi^2 mke^2} \quad \rightarrow$$

$$r_n = \frac{h^2 n^2}{4\pi^2 G m_M^2 m_E} = \frac{\left(6.626\times10^{-34}\,\text{J•s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)^2\left(5.98\times10^{24}\,\text{kg}\right)} n^2$$

$$= \boxed{n^2\left(5.16\times10^{-129}\,\text{m}\right)}$$

$$E_n = -\frac{2\pi^2 e^4 mk^2}{n^2 h^2} \quad \rightarrow$$

$$E_n = -\frac{2\pi^2 G^2 m_E^2 m_M^3}{n^2 h^2} = -\frac{2\pi^2\left(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2\right)^2\left(5.98\times10^{24}\,\text{kg}\right)^2\left(7.35\times10^{22}\,\text{kg}\right)^3}{n^2\left(6.626\times10^{-34}\,\text{J•s}\right)^2}$$

$$= \boxed{-\frac{2.84\times10^{165}\,\text{J}}{n^2}}$$

(*b*) We insert the known masses and Earth–Moon distance into the Bohr radius equation to determine the Bohr state.

$$n = \sqrt{\frac{4\pi^2 G m_M^2 m_E r_n}{h^2}}$$

$$= \sqrt{\frac{4\pi^2\left(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)^2\left(5.98\times10^{24}\ \text{kg}\right)\left(3.84\times10^{8}\ \text{m}\right)}{\left(6.626\times10^{-34}\ \text{Js}\right)^2}}$$

$$= 2.73\times10^{68}$$

Since $n \approx 10^{68}$, a value of $\Delta n = 1$ is negligible compared to *n*. Hence the quantization of energy and radius is $\boxed{\text{not apparent.}}$

94. We use Eqs. 36-13, 36-11, and 37-7 to derive the expression.

$$p^2c^2 + m^2c^4 = E^2\ ;\ E = K + mc^2 \ \rightarrow\ p^2c^2 + m^2c^4 = \left(K + mc^2\right)^2 = K^2 + 2mc^2K + m^2c^4 \ \rightarrow$$

$$K^2 + 2mc^2K = p^2c^2 = \frac{h^2c^2}{\lambda^2} \ \rightarrow\ \lambda^2 = \frac{h^2c^2}{\left(K^2 + 2mc^2K\right)} \ \rightarrow\ \boxed{\lambda = \frac{hc}{\sqrt{K^2 + 2mc^2K}}}$$

95. As light leaves the flashlight it gains momentum. This change in momentum is given by Eq. 31-20. Dividing the change in momentum by the elapsed time gives the force the flashlight must apply to the light to produce this momentum. This is equal to the reaction force that light applies to the flashlight.

$$\frac{\Delta p}{\Delta t} = \frac{\Delta U}{c\Delta t} = \frac{P}{c} = \frac{3.0\,\text{W}}{3.00\times10^8\,\text{m/s}} = \boxed{1.0\times10^{-8}\,\text{N}}$$

96. (*a*) Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We insert the values for *h* and *c* and convert the resulting units to eV•nm.

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\,\text{J•s}\right)\left(2.998\times10^8\,\text{m/s}\right)}{\lambda}\frac{\left(1\,\text{eV}/1.602\times10^{-19}\,\text{J}\right)}{\left(10^{-9}\text{m}/1\,\text{nm}\right)} = \boxed{\frac{1240\ \text{eV•nm}}{\lambda\left(\text{in nm}\right)}}$$

(*b*) Insert 650 nm into the above equation.

$$E = \frac{1240\ \text{eV}\bullet\text{nm}}{650\ \text{nm}} = \boxed{1.9\ \text{eV}}$$

97. (*a*) We write the Planck time as $t_\text{P} = G^\alpha h^\beta c^\gamma$, and the units of t_P must be $[T]$.

$$t_\text{P} = G^\alpha h^\beta c^\gamma \rightarrow [T] = \left[\frac{L^3}{MT^2}\right]^\alpha \left[\frac{ML^2}{T}\right]^\beta \left[\frac{L}{T}\right]^\gamma = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[T]$, and so $\beta = \alpha$, and $[T] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no length units in $[T]$, and so $\gamma = -5\alpha$ and $[T] = [T]^{-3\alpha+5\alpha} = [T]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{5}{2}$.

$$\boxed{t_\text{P} = G^{1/2}h^{1/2}c^{-5/2} = \sqrt{\frac{Gh}{c^5}}}$$

(*b*) $$t_\text{P} = \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^5}} = \boxed{1.35\times10^{-43}\text{s}}$$

(*c*) We write the Planck length as $\lambda_\text{P} = G^\alpha h^\beta c^\gamma$, and the units of λ_P must be $[L]$.

$$\lambda_\text{P} = G^\alpha h^\beta c^\gamma \rightarrow [L] = \left[\frac{L^3}{MT^2}\right]^\alpha \left[\frac{ML^2}{T}\right]^\beta \left[\frac{L}{T}\right]^\gamma = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[L]$, and so $\beta = \alpha$, and $[L] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no time units in $[L]$, and so $\gamma = -3\alpha$ and $[L] = [L]^{5\alpha-3\alpha} = [L]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{3}{2}$.

$$\boxed{t_\text{P} = G^{1/2}h^{1/2}c^{-3/2} = \sqrt{\frac{Gh}{c^3}}}$$

(*d*) $$\lambda_\text{P} = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^5}} = \boxed{4.05\times10^{-35}\text{m}}$$

98. For standing matter waves, there are nodes at the two walls. For the ground state (first harmonic), the wavelength is twice the distance between the walls, or $\ell = \frac{1}{2}\lambda$ (see Figure 15-26b). We use Eq. 37-7 to find the velocity and then the kinetic energy.

$$\ell = \tfrac{1}{2}\lambda \rightarrow \lambda = 2\ell \ ; \ p = \frac{h}{\lambda} = \frac{h}{2\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\ell}\right)^2 = \boxed{\frac{h^2}{8m\ell^2}}$$

For the second harmonic, the distance between the walls is a full wavelength, and so $\ell = \lambda$.

$$\ell = \lambda \rightarrow p = \frac{h}{\lambda} = \frac{h}{\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\ell}\right)^2 = \boxed{\frac{h^2}{2m\ell^2}}$$

99. (*a*) Apply conservation of momentum before and after the emission of the photon to determine the recoil speed of the atom, where the momentum of the photon is given by Eq. 37-7.

$$0=\frac{h}{\lambda}-mv \quad\rightarrow\quad v=\frac{h}{m\lambda}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)}{85\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(780\times10^{-9}\,\mathrm{m}\right)}=\boxed{6.0\times10^{-3}\,\mathrm{m/s}}$$

(*b*) We solve Eq. 18-5 for the lowest achievable temperature, where the recoil speed is the rms speed of the rubidium gas.

$$v=\sqrt{\frac{3kT}{m}} \quad\rightarrow\quad T=\frac{mv^2}{3k}=\frac{85\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(6.0\times10^{-3}\,\mathrm{m/s}\right)^2}{3\left(1.38\times10^{-23}\,\mathrm{J/K}\right)}=1.2\times10^{-7}\,\mathrm{K}=\boxed{0.12\,\mu\mathrm{K}}$$

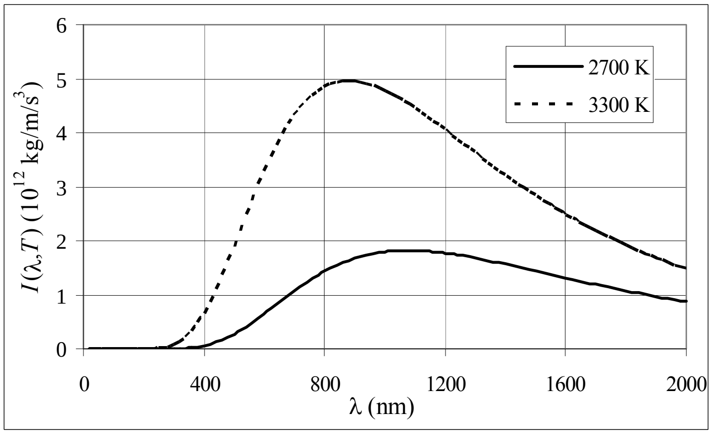
100. Each time the rubidium atom absorbs a photon its momentum decreases by the momentum of the photon. Dividing the initial momentum of the rubidium atom by the momentum of the photon, Eq. 37-7, gives the number of collisions necessary to stop the atom. Multiplying the number of collisions by the absorption time, 25 ns per absorption, provides the time to completely stop the atom.

$$n=\frac{mv}{h/\lambda}=\frac{mv\lambda}{h}=\frac{\left(8\,\mathrm{u}\right)\left(1.66\times10^{-27}\,\mathrm{kg/u}\right)\left(290\,\mathrm{m/s}\right)\left(780\times10^{-9}\,\mathrm{m}\right)}{6.63\times10^{-34}\,\mathrm{J\bullet s}}=48{,}140$$

$$T=48{,}140\left(25\,\mathrm{ns}\right)=\boxed{1.2\,\mathrm{ms}}$$

101. (*a*) See the adjacent graphs.

(*b*) To compare the intensities, the two graphs are numerically integrated from 400 nm to 760 nm, which is approximately the range of wavelengths for visible light. The result of those integrations is that the higher temperature bulb is about $\boxed{\text{4.8 times more intense}}$ than the lower temperature bulb.



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.101."

102. Planck's radiation formula $I\left(\lambda,T\right)$ was calculated for a temperature of 6000 K, for wavelengths from 20 nm to 2000 nm. A plot of those calculations is in the spreadsheet for this problem. To estimate the % of emitted sunlight that is in the visible, this ratio was calculated by numeric integration. The details are in the spreadsheet.

$$\%\,\text{visible}=\frac{\displaystyle\int_{400\,\mathrm{nm}}^{700\,\mathrm{nm}} I\left(\lambda,T\right)d\lambda}{\displaystyle\int_{20\,\mathrm{nm}}^{2000\,\mathrm{nm}} I\left(\lambda,T\right)d\lambda}=\boxed{0.42}$$

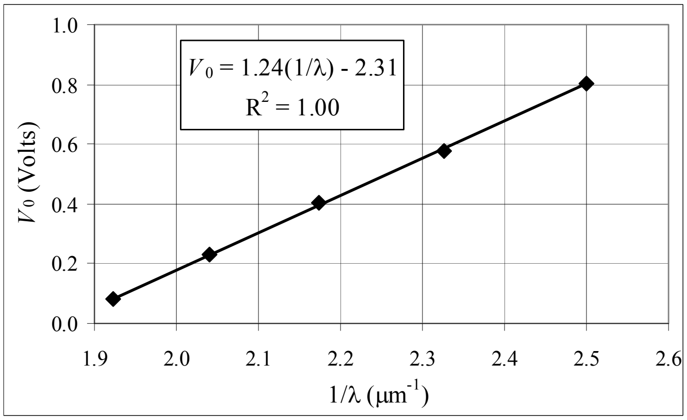
So our estimate is that 42% of emitted sunlight is in the visible wavelengths. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.102."

$\boxed{103.}$ (*a*) For the photoelectric effect experiment, Eq. 37-4b can be expressed as $K_{\max} = hf - W_0$. The maximum kinetic energy is equal to the potential energy associated with the stopping voltage, so $K_{\max} = eV_0$. We also have $f = c/\lambda$. Combine those relationships as follows.

$$K_{\max} = hf - W_0 \rightarrow eV_0 = \frac{hc}{\lambda} - W_0 \rightarrow V_0 = \frac{hc}{e}\frac{1}{\lambda} - \frac{W_0}{e}$$

A plot of V_0 vs. $\frac{1}{\lambda}$ should yield a straight line with a slope of $\boxed{\frac{hc}{e}}$ and a y-intercept of $\boxed{-\frac{W_0}{e}}$.

(*b*) The graph is shown, with a linear regression fit as given by Excel.



(*c*) The slope is $a = \frac{hc}{e} = 1.24\,\text{V}\bullet\mu\text{m}$, and the y-intercept is $b = -2.31\,\text{V}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.103."

(*d*) $b = -\frac{W_0}{e} = -2.31\,\text{V} \rightarrow W_0 = \boxed{2.31\,\text{eV}}$

(*e*) $h = \frac{ea}{c} = \frac{(1.60\times10^{-19}\,\text{C})(1.24\times10^{-6}\,\text{V}\bullet\text{m})}{3.00\times10^{8}\,\text{m/s}} = \boxed{6.61\times10^{-34}\,\text{J}\bullet\text{s}}$

CHAPTER 38: Quantum Mechanics

Responses to Questions

$\boxed{1.}$ (*a*) A matter wave ψ does not need a medium as a wave on a string does. The square of the wave function for a matter wave ψ describes the probability of finding a particle within a certain spatial range, whereas the equation for a wave on a string describes the displacement of a piece of string from its equilibrium position.

(*b*) An EM wave also does not need a medium. The equation for the EM wave describes the way in which the amplitudes of the electric and magnetic fields change as the wave passes a point in space. An EM wave represents a vector field and can be polarized. A matter wave is a scalar and cannot be polarized.

2. According to Bohr's theory, each electron in an atom travels in a circular orbit and has a precise position and momentum at any point in time. This view is inconsistent with the postulates of quantum mechanics and the uncertainty principle, which does not allow both the position and momentum to be known precisely. According to quantum mechanics, the "orbitals" of electrons do not have precise radii, but describe the probability of finding an electron in a given spatial range.

3. As mass increases, the uncertainty in the momentum of the object increases, and, from the Heisenberg uncertainty principle, the uncertainty in the position of the object decreases, making the future position of the object easier to predict.

4. Planck's constant is so small that on the scale of a baseball the uncertainties in position and momentum are negligible compared with the values of the position and momentum. If visible light is being used to observe the baseball, then the uncertainty in the baseball's position will be on the order of the wavelength of visible light. (See Section 38-3.) A baseball is very large compared to the wavelength of light, so any uncertainty in the position of the baseball will be much smaller than the extent of the object itself.

5. No. According to the uncertainty principle, if the needle were balanced the position of the center of mass would be known exactly, and there would have to be some uncertainty in its momentum. The center of mass of the needle could not have a zero momentum, and therefore would fall over. If the initial momentum of the center of mass of the needle were exactly zero, then there would be uncertainty in its position, and the needle could not be perfectly balanced (with the center of mass over the tip).

6. Yes, some of the air escapes the tire in the act of measuring the pressure and it is impossible to avoid this escape. The act of measuring the air pressure in a tire therefore actually changes the pressure, although not by much since very little air escapes compared to the total amount of air in the tire. This is similar to the uncertainty principle, in which one of the two factors limiting the precision of measurement is the interaction between the object begin observed, or measured, and the observing instrument.

$\boxed{7.}$ Yes. In energy form, the uncertainty principle is $\Delta E \Delta t \geq h/2\pi$. For the ground state, Δt is very large, since electrons remain in that state for a very long time, so ΔE is very small and the energy of the state can be precisely known. For excited states, which can decay to the ground state, Δt is much smaller, and ΔE is corresponding larger. Therefore the energy of the state is less well known.

8. If Planck's constant were much larger than it is, then the consequences of the uncertainty principle would be noticeable with macroscopic objects. For instance, attempts to determine a baseball's speed would mean that you could not find its position very accurately. Using a radar gun to find the speed of a pitcher's fastball would significantly change the actual course of the ball.

9. According to Newtonian mechanics, all objects have an exact position and momentum at a point in time. This information can be used to predict the future motion of an object. According to quantum mechanics, there is unavoidable uncertainty in the position and momentum of all objects. It is impossible to exactly determine both position and momentum at the same time, which introduces uncertainly into the prediction of the future motion of the object.

10. If you knew the position precisely, then you would know nothing about the momentum.

11. No. Some of the energy of the soup would be used to heat up the thermometer, so the temperature registered on the thermometer would be slightly less than the original temperature of the soup.

12. No. However, the greater the precision of the measurement of position, the greater the uncertainty in the measurement of the momentum of the object will be.

13. A particle in a box is confined to a region of space. Since the uncertainty in position is limited by the box, there must be some uncertainty in the particle's momentum, and the momentum cannot be zero. The zero point energy reflects the uncertainty in momentum.

14. Yes, the probability of finding the particle at these points is zero. It is possible for the particle to pass by these points. Since the particle is acting like a wave, these points correspond to the nodes in a standing wave pattern in the box.

15. For large values of n, the probability density varies rapidly between zero and the maximum value. It can be averaged easily to the classical result as n becomes large.

16. As n increases, the energy of the corresponding state increases, but $\Delta E/E$ approaches zero. For large n, the probability density varies rapidly between zero and the maximum value and is easily averaged to the classical result, which is a uniform probability density for all points in the well.

17. As the potential decreases, the wave function extends into the forbidden region as an exponential decay function. When the potential drops below the particle energy, the wave function outside the well changes from an exponential decay function to an oscillating function with a longer wavelength than the function within the well. When the potential is zero, the wavelengths of the wave function will be the same everywhere. The ground state energy of the particle in a well becomes the energy of the free particle.

18. The hydrogen atom will have a greater probability of tunneling through the barrier because it has a smaller mass and therefore a larger transmission coefficient. (See Equations 38-17a and 38-17b.)

Solutions to Problems

1. We find the wavelength of the neutron from Eq. 37-7. The peaks of the interference pattern are given by Eq. 34-2a and Figure 34-10. For small angles, we have $\sin\theta = \tan\theta$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0K}} \quad ; \; d\sin\theta = m\lambda,\ m = 1,2,\ldots \quad ; \; y = \ell\tan\theta$$

$$\sin\theta = \tan\theta \;\rightarrow\; \frac{m\lambda}{d} = \frac{y}{\ell} \;\rightarrow\; y = \frac{m\lambda\ell}{d},\ m = 1,2,\ldots \;\rightarrow$$

$$\Delta y = \frac{\lambda\ell}{d} = \frac{h\ell}{d\sqrt{2m_0K}} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(1.0\,\mathrm{m}\right)}{\left(6.0\times10^{-4}\,\mathrm{m}\right)\sqrt{2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(0.030\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}$$

$$= \boxed{2.8\times10^{-7}\,\mathrm{m}}$$

2. We find the wavelength of a pellet from Eq. 37-7. The half-angle for the central circle of the diffraction pattern is given in Section 35-4 as $\sin\theta = \frac{1.22\lambda}{D}$, where D is the diameter of the opening. Assuming the angle is small, the diameter of the spread of the bullet beam is $d = 2\ell\tan\theta = 2\ell\sin\theta$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad ; \; d = 2\ell\tan\theta = 2\ell\sin\theta = 2\ell\frac{1.22\lambda}{D} = 2\ell\frac{1.22h}{Dmv} \;\rightarrow$$

$$\ell = \frac{Dmvd}{2.44h} = \frac{\left(3.0\times10^{-3}\,\mathrm{m}\right)\left(3.0\times10^{-3}\,\mathrm{kg}\right)\left(150\,\mathrm{m/s}\right)\left(0.010\,\mathrm{m}\right)}{2.44\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)} = \boxed{8.3\times10^{27}\,\mathrm{m}}$$

This is almost 10^{12} light years.

3. The uncertainty in the velocity is given. Use Eq. 38-1 to find the uncertainty in the position.

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(1200\,\mathrm{m/s}\right)} = \boxed{5.3\times10^{-11}\,\mathrm{m}}$$

4. The minimum uncertainty in the energy is found from Eq. 38-2.

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left(1\times10^{-8}\,\mathrm{s}\right)} = 1.055\times10^{-26}\,\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right) = 6.59\times10^{-8}\,\mathrm{eV} \approx \boxed{10^{-7}\,\mathrm{eV}}$$

5. The uncertainty in position is given. Use Eq. 38-1 to find the uncertainty in the momentum.

$$\Delta p = m\Delta v \geq \frac{\hbar}{\Delta x} \;\rightarrow\; \Delta v \geq \frac{\hbar}{m\Delta x} = \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2.6\times10^{-8}\,\mathrm{m}\right)} = 4454\,\mathrm{m/s} \approx \boxed{4500\,\mathrm{m/s}}$$

6. The uncertainty in the energy is found from the lifetime and the uncertainty principle.

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{h}{2\pi\Delta t} \; ; \; E = hv = \frac{hc}{\lambda}$$

$$\frac{\Delta E}{E} = \frac{\dfrac{h}{2\pi\Delta t}}{\dfrac{hc}{\lambda}} = \frac{\lambda}{2\pi c\Delta t} = \frac{500\times10^{-9}\,\mathrm{m}}{2\pi\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(10\times10^{-9}\,\mathrm{s}\right)} = 2.65\times10^{-8} \approx \boxed{3\times10^{-8}}$$

$$E=\frac{hc}{\lambda} \rightarrow dE=-\frac{hc}{\lambda^2}d\lambda \rightarrow \Delta E\approx-\frac{hc}{\lambda^2}\Delta\lambda=-\frac{E}{\lambda}\Delta\lambda \rightarrow \frac{\Delta E}{E}=-\frac{\Delta\lambda}{\lambda}$$

The wavelength uncertainty is the absolute value of this expression, and so $\frac{\Delta\lambda}{\lambda}=\boxed{3\times10^{-8}}$

$\boxed{7.}$ The uncertainty in the energy is found from the lifetime and the uncertainty principle.

$$\Delta E=\frac{\hbar}{\Delta t}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(12\times10^{-6}\text{s}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=5.49\times10^{-11}\,\text{eV}$$

$$\frac{\Delta E}{E}=\frac{5.49\times10^{-11}\,\text{eV}}{5500\,\text{eV}}=\boxed{1.0\times10^{-14}}$$

8. (*a*) We find the wavelength from Eq. 37-7.

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.012\,\text{kg})(180\,\text{m/s})}=\boxed{3.1\times10^{-34}\,\text{m}}$$

(*b*) Use Eq. 38-1 to find the uncertainty in momentum

$$\Delta p_y\geq\frac{\hbar}{\Delta y}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.0065\,\text{m})}=\boxed{1.6\times10^{-32}\,\text{kg}\bullet\text{m/s}}$$

9. The uncertainty in the position is found from the uncertainty in the velocity and Eq. 38-1.

$$\Delta x_{\text{electron}}\geq\frac{\hbar}{\Delta p}=\frac{\hbar}{m\Delta v}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\text{kg}\right)(95\,\text{m/s})\left(8.5\times10^{-4}\right)}=\boxed{1.4\times10^{-3}\,\text{m}}$$

$$\Delta x_{\text{baseball}}\geq\frac{\hbar}{\Delta p}=\frac{\hbar}{m\Delta v}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.14\,\text{kg})(95\,\text{m/s})\left(8.5\times10^{-4}\right)}=\boxed{9.3\times10^{-33}\,\text{m}}$$

$$\frac{\Delta x_{\text{electron}}}{\Delta x_{\text{baseball}}}=\frac{m_{\text{baseball}}}{m_{\text{electron}}}=\frac{(0.14\,\text{kg})}{\left(9.11\times10^{-31}\,\text{kg}\right)}=1.5\times10^{29}$$

The uncertainty for the electron is greater by a factor of 1.5×10^{29}.

10. We find the uncertainty in the energy of the muon from Eq. 38-2, and then find the uncertainty in the mass.

$$\Delta E\geq\frac{\hbar}{\Delta t}\ ;\ \Delta E=(\Delta m)c^2 \rightarrow$$

$$\Delta m\geq\frac{\hbar}{c^2\Delta t}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{c^2\left(2.20\times10^{-6}\,\text{s}\right)}=\left(4.7955\times10^{-29}\,\frac{\text{J}}{c^2}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=\boxed{3.00\times10^{-10}\,\text{eV}/c^2}$$

11. We find the uncertainty in the energy of the free neutron from Eq. 38-2, and then the mass uncertainty from Eq. 36-12. We assume the lifetime of the neutron is good to two significant figures. The current experimental lifetime of the neutron is 886 seconds, so the 900 second value is certainly good to at least 2 significant figures.

$$\Delta E\geq\frac{\hbar}{\Delta t}\ ;\ \Delta E=(\Delta m)c^2 \rightarrow \Delta m\geq\frac{\hbar}{c^2\Delta t}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(3.00\times10^{8}\,\text{m/s}\right)^2(900\,\text{s})}=\boxed{1.3\times10^{-54}\,\text{kg}}$$

12. Use the radius as the uncertainty in position for the electron. We find the uncertainty in the momentum from Eq. 38-1, and then find the energy associated with that momentum from Eq. 36-13.

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{\left(1.055 \times 10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.0 \times 10^{-15}\,\text{m}\right)} = 1.055 \times 10^{-19}\,\text{kg}\bullet\text{m/s}.$$

If we assume that the lowest value for the momentum is the least uncertainty, we can estimate the lowest possible energy.

$$E = \left(p^2c^2 + m_0{}^2c^4\right)^{1/2} = \left[\left(\Delta p\right)^2 c^2 + m_0{}^2c^4\right]^{1/2}$$

$$= \left[\left(1.055 \times 10^{-19}\,\text{kg}\bullet\text{m/s}\right)^2\left(3.00 \times 10^8\,\text{m/s}\right)^2 + \left(9.11 \times 10^{-31}\,\text{kg}\right)^2\left(3.00 \times 10^8\,\text{m/s}\right)^4\right]^{1/2}$$

$$= 3.175 \times 10^{-11}\,\text{J}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right) \approx \boxed{200\,\text{MeV}}$$

$\boxed{13.}$ (*a*) The minimum uncertainty in the energy is found from Eq. 38-2.

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055 \times 10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1\times10^{-8}\,\text{s}\right)} = 1.055\times10^{-26}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 6.59\times10^{-8}\,\text{eV} \approx \boxed{10^{-7}\,\text{eV}}$$

(*b*) The transition energy can be found from Eq. 37-14b. $Z = 1$ for hydrogen.

$$E_n = -\left(13.6\,\text{eV}\right)\frac{Z^2}{n^2} \rightarrow E_2 - E_1 = \left[-\left(13.6\,\text{eV}\right)\frac{1^2}{2^2}\right] - \left[-\left(13.6\,\text{eV}\right)\frac{1^2}{1^2}\right] = 10.2\,\text{eV}$$

$$\frac{\Delta E}{E_2 - E_1} = \frac{6.59\times10^{-8}\,\text{eV}}{10.2\,\text{eV}} = 6.46\times10^{-9} \approx \boxed{10^{-8}}$$

(*c*) The wavelength is given by Eq. 37-3.

$$E = hv = \frac{hc}{\lambda} \rightarrow$$

$$\lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{\left(10.2\,\text{eV}\right)\left(\frac{1.60\times10^{-19}\,\text{J}}{\text{eV}}\right)} = 1.22\times10^{-7}\,\text{m} = 122\,\text{nm} \approx \boxed{100\,\text{nm}}$$

Take the derivative of the above relationship to find $\Delta\lambda$.

$$\lambda = \frac{hc}{E} \rightarrow d\lambda = -\frac{hc}{E^2}dE \rightarrow \Delta\lambda = -\frac{hc}{E^2}\Delta E = -\lambda\frac{\Delta E}{E} \rightarrow$$

$$\left|\Delta\lambda\right| = \lambda\frac{\Delta E}{E} = \left(122\,\text{nm}\right)\left(6.46\times10^{-9}\right) = 7.88\times10^{-7}\,\text{nm} \approx \boxed{10^{-6}\,\text{nm}}$$

14. We assume the electron is non-relativistic. The momentum is calculated from the kinetic energy, and the position uncertainty from the momentum uncertainty, Eq. 38-1. Since the kinetic energy is known to 1.00%, we have $\Delta K/K = 1.00\times10^{-2}$.

$$p = \sqrt{2mK}\ ;\ \frac{dp}{dK} = \sqrt{2m}\left(\frac{1}{2\sqrt{K}}\right) = \frac{\sqrt{2mK}}{2K} \rightarrow \Delta p \approx \frac{\sqrt{2mK}}{2K}\Delta K = \tfrac{1}{2}\sqrt{2mK}\frac{\Delta K}{K}$$

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{\tfrac{1}{2}\sqrt{2mK}\frac{\Delta K}{K}} = \frac{\left(1.055 \times 10^{-34}\,\text{J}\bullet\text{s}\right)}{\tfrac{1}{2}\sqrt{2\left(9.11 \times 10^{-31}\,\text{kg}\right)\left(3.50\,\text{keV}\right)\left(1.60 \times 10^{-16}\,\text{J/keV}\right)}\left(1.00\times10^{-2}\right)}$$

$$= \boxed{6.61 \times 10^{-10}\,\text{m}}$$

15. Let us assume that the electron has an initial *x* momentum p_x, so that it has a wavelength of $\lambda = h/p_x$. The maxima of the double-slit interference pattern occur at locations satisfying Eq. 34-2a, $d\sin\theta = m\lambda,\ m = 0,1,2,\cdots$. If the angles are small, then we replace $\sin\theta$ by θ, and so the maxima are given by $\theta = m\lambda/d$. The angular separation of the maxima is then $\Delta\theta = \lambda/d$, and the angular separation between a maximum and the adjacent minimum is $\Delta\theta = \lambda/2d$. The separation of a maximum and the adjacent minimum on the screen is then $\Delta y_{\text{screen}} = \lambda\ell/2d$, where ℓ is the distance from the slits to the detection screen. This means that many electrons hit the screen at a maximum position, and very few electrons hit the screen a distance $\lambda\ell/2d$ to either side of that maximum position.

 If the particular slit that an electron passes through is known, then Δy for the electrons at the location of the slits is $d/2$. The uncertainty principle says $\Delta p_y \geq \dfrac{\hbar}{\Delta y_{\text{slits}}} = \dfrac{\hbar}{\frac{1}{2}d} = \dfrac{h}{\pi d}$. We assume that p_y for the electron must be at least that big. Because of this uncertainty in *y* momentum, the electron has an uncertainty in its location on the screen, as $\dfrac{\Delta y_{\text{screen}}}{\ell} = \dfrac{\Delta p_y}{p_x} \rightarrow \Delta y_{\text{screen}} = \ell\dfrac{\frac{h}{\pi d}}{\frac{h}{\lambda}} = \dfrac{\lambda\ell}{\pi d}$.

 Since this is about the same size as the separation between maxima and minima, the interference pattern will be “destroyed.” The electrons will not be grouped near the maxima locations. They will instead be “spread out” on the screen, and no interference pattern will be visible.

16. We are given that $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to the Schrödinger equation. Substitute the function $A\Psi_1(x,t) + B\Psi_2(x,t)$ into the Schrödinger equation.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[A\Psi_1 + B\Psi_2\right] + U(x)\left[A\Psi_1 + B\Psi_2\right] = -\frac{\hbar^2}{2m}\frac{\partial^2(A\Psi_1)}{\partial x^2} - \frac{\hbar^2}{2m}\frac{\partial^2(B\Psi_2)}{\partial x^2} + U(x)A\Psi_1 + U(x)B\Psi_2$$

$$= -A\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1}{\partial x^2} - B\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2}{\partial x^2} + AU(x)\Psi_1 + BU(x)\Psi_2$$

$$= A\left(-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1}{\partial x^2} + U(x)\Psi_1\right) + B\left(-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2}{\partial x^2} + U(x)\Psi_2\right)$$

$$= A\left(i\hbar\frac{\partial\Psi_1}{\partial t}\right) + B\left(i\hbar\frac{\partial\Psi_2}{\partial t}\right) = i\hbar\frac{\partial}{\partial t}\left[A\Psi_1 + B\Psi_2\right]$$

 So, since $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}\left[A\Psi_1 + B\Psi_2\right] + U(x)\left[A\Psi_1 + B\Psi_2\right] = i\hbar\dfrac{\partial}{\partial t}\left[A\Psi_1 + B\Psi_2\right]$, the combination $A\Psi_1(x,t) + B\Psi_2(x,t)$ is also a solution to the time-dependent Schrödinger equation.

17. (*a*) Substitute $\Psi(x,t) = Ae^{i(kx-\omega t)}$ into both sides of the time-dependent Schrödinger equation, Eq. 38-7, and compare the functional form of the results.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U_0\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2 Ae^{i(kx-\omega t)}}{\partial x^2} + U_0 Ae^{i(kx-\omega t)} = \left(\frac{\hbar^2k^2}{2m} + U_0\right)Ae^{i(kx-\omega t)}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = i\hbar\frac{\partial Ae^{i(kx-\omega t)}}{\partial t} = \hbar\omega Ae^{i(kx-\omega t)}$$

Both sides of the equation give a result of $(\text{constant})Ae^{i(kx-\omega t)}$, and so t $\Psi(x,t)=Ae^{i(kx-\omega t)}$ is a valid solution, if the constants are equal.

Now repeat the process for $\Psi(x,t)=A\cos(kx-\omega t)$.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+U_0\Psi=-\frac{\hbar^2}{2m}\frac{\partial^2 A\cos(kx-\omega t)}{\partial x^2}+U_0A\cos(kx-\omega t)$$
$$=\left(\frac{\hbar^2k^2}{2m}+U_0\right)A\cos(kx-\omega t)$$

$$i\hbar\frac{\partial\Psi}{\partial t}=i\hbar\frac{\partial A\cos(kx-\omega t)}{\partial t}=i\hbar\omega A\sin(kx-\omega t)$$

Because $\cos(kx-\omega t)\neq\sin(kx-\omega t)$ for arbitrary values of *x* and *t*, $\Psi(x,t)=A\cos(kx-\omega t)$ is NOT a valid solution.

Now repeat the process for $\Psi(x,t)=A\sin(kx-\omega t)$.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+U_0\Psi=-\frac{\hbar^2}{2m}\frac{\partial^2 A\sin(kx-\omega t)}{\partial x^2}+U_0A\sin(kx-\omega t)$$
$$=\left(\frac{\hbar^2k^2}{2m}+U_0\right)A\sin(kx-\omega t)$$

$$i\hbar\frac{\partial\Psi}{\partial t}=i\hbar\frac{\partial A\sin(kx-\omega t)}{\partial t}=-i\hbar\omega A\cos(kx-\omega t)$$

Because $\cos(kx-\omega t)\neq\sin(kx-\omega t)$ for arbitrary values of *x* and *t*, $\Psi(x,t)=A\sin(kx-\omega t)$ is NOT a valid solution.

(*b*) Conservation of energy gives the following result.

$$E=K+U=\frac{p^2}{2m}+U_0\ ;\ p=\frac{h}{\lambda}=\frac{hk}{2\pi}=\hbar k\ \rightarrow\ \hbar\omega=\frac{\hbar^2k^2}{2m}+U_0$$

We equate the two results from the valid solution.

$$\left(\frac{\hbar^2k^2}{2m}+U_0\right)Ae^{i(kx-\omega t)}=\hbar\omega Ae^{i(kx-\omega t)}\ \rightarrow\ \frac{\hbar^2k^2}{2m}+U_0=\hbar\omega$$

The expressions are the same.

18. The wave function is given in the form $\psi(x)=A\sin kx$.

(*a*) $\lambda=\dfrac{2\pi}{k}=\dfrac{2\pi}{2.0\times10^{10}\,\text{m}^{-1}}=3.142\times10^{-10}\,\text{m}\approx\boxed{3.1\times10^{-10}\,\text{m}}$

(*b*) $p=\dfrac{h}{\lambda}=\dfrac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{3.142\times10^{-10}\,\text{m}}=2.110\times10^{-24}\,\text{kg}\bullet\text{m/s}\approx\boxed{2.1\times10^{-24}\,\text{kg}\bullet\text{m/s}}$

(*c*) $v=\dfrac{p}{m}=\dfrac{2.110\times10^{-24}\,\text{kg}\bullet\text{m/s}}{9.11\times10^{-31}\,\text{kg}}=\boxed{2.3\times10^{6}\,\text{m/s}}$

(*d*) $K=\dfrac{p^2}{2m}=\dfrac{\left(2.110\times10^{-24}\,\text{kg}\bullet\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\dfrac{1}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=\boxed{15\,\text{eV}}$

$\boxed{19.}$ The general expression for the wave function of a free particle is given by Eq. 38-3a. The particles are not relativistic.

(*a*) $$k=\frac{2\pi}{\lambda}=\frac{2\pi p}{h}=\frac{mv}{\hbar}=\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.0\times10^{5}\,\text{m/s}\right)}{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}=2.6\times10^{9}\,\text{m}^{-1}$$

$$\psi=\boxed{A\sin\left[\left(2.6\times10^{9}\,\text{m}^{-1}\right)x\right]+B\cos\left[\left(2.6\times10^{9}\,\text{m}^{-1}\right)x\right]}$$

(*b*) $$k=\frac{2\pi}{\lambda}=\frac{2\pi p}{h}=\frac{mv}{\hbar}=\frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(3.0\times10^{5}\,\text{m/s}\right)}{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}=4.7\times10^{12}\,\text{m}^{-1}$$

$$\psi=\boxed{A\sin\left[\left(4.7\times10^{12}\,\text{m}^{-1}\right)x\right]+B\cos\left[\left(4.7\times10^{12}\,\text{m}^{-1}\right)x\right]}$$

20. This is similar to the analysis done in Chapter 16 Section 6 for beats. Referring to Figure 16-17, we see the distance from one node to the next can be considered a wave packet. We add the two wave functions, employ the trigonometric identity for the sine of a sum of two angles, and then find the distance between nodes. The wave numbers are related to the wavelengths by $k=\frac{2\pi}{\lambda}$. Since $\lambda_1\approx\lambda_2$, it is also true that $k_1\approx k_2$ and so $\frac{1}{2}(k_1+k_2)\approx k_{\text{avg}}$. We define $\Delta k=k_1-k_2$.

$$\psi=\psi_1+\psi_2=A\sin k_1x+A\sin k_2x=A\left(\sin k_1x+\sin k_2x\right)=2A\sin\left[\tfrac{1}{2}(k_1+k_2)x\right]\cos\left[\tfrac{1}{2}(k_1-k_2)x\right]$$
$$=2A\sin k_{\text{avg}}x\cos\left[\tfrac{1}{2}(\Delta k)x\right]$$

The sum function will take on a value of 0 if $\frac{1}{2}(\Delta k)x=\left(n+\frac{1}{2}\right)\pi,\ n=0,1,2\ldots$. The distance between these nodal locations is found as follows.

$$x=\frac{2\left(n+\frac{1}{2}\right)\pi}{\Delta k}=\frac{(2n+1)\pi}{\Delta k},\ n=0,1,2\ldots\ \rightarrow\ \Delta x=\frac{\left(2(n+1)+1\right)\pi}{\Delta k}-\frac{(2n+1)\pi}{\Delta k}=\frac{2\pi}{\Delta k}$$

Now use the de Broglie relationship between wavelength and momentum.

$$p=\frac{h}{\lambda}=\hbar k\ \rightarrow\ \Delta k=\frac{\Delta p}{\hbar}\ ;\ \Delta x=\frac{2\pi}{\Delta k}=\frac{2\pi\hbar}{\Delta p}\ \rightarrow\ \boxed{\Delta x\Delta p=h}$$

21. The minimum speed corresponds to the lowest energy state. The energy is given by Eq. 38-13.

$$E_{\text{min}}=\frac{h^2}{8m\ell^2}=\tfrac{1}{2}mv_{\text{min}}^2\ \rightarrow\ v_{\text{min}}=\frac{h}{2m\ell}=\frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.20\times10^{-9}\,\text{m}\right)}=\boxed{1.8\times10^{6}\,\text{m/s}}$$

22. We assume the particle is not relativistic. The energy levels are given by Eq. 38-13, and the wave functions are given by Eq. 38-14.

$$E_n=\frac{h^2n^2}{8m\ell^2}=\frac{p^2}{2m}\ \rightarrow\ p_n=\frac{hn}{2\ell}\ ;\ \psi_n=\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi x}{\ell}\right)\ \rightarrow\ \frac{n\pi}{\ell}=k_n=\frac{2\pi}{\lambda_n}\ \rightarrow$$

$\lambda_n=\frac{2\ell}{n}=\frac{h}{p_n}$, which is the de Broglie wavelength

23. (*a*) The longest wavelength photon will be the photon with the lowest frequency, and thus the lowest energy. The difference between energy levels increases with high states, so the lowest energy transition is from *n* = 2 to *n* = 1. The energy levels are given by Eq. 38-13.

$$E_n=n^2\frac{h^2}{8m\ell^2}=n^2E_1$$

$$\lambda=\frac{c}{\nu}=\frac{hc}{\Delta E}=\frac{hc}{E_2-E_1}=\frac{hc}{4E_1-E_1}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}{3\left(9.0\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{4.6\times10^{-8}\,\mathrm{m}}$$

(*b*) We use the ground state energy and Eq. 38-13.

$$E_1=\frac{h^2}{8m\ell^2}\quad\rightarrow$$

$$\ell=\frac{h}{\sqrt{8mE_1}}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)}{\sqrt{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(9.0\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}=\boxed{2.0\times10^{-10}\,\mathrm{m}}$$

24. The energy levels for a particle in a rigid box are given by Eq. 38-13. Use that equation, evaluated for $n = 4$ and $n = 1$, to calculate the width of the box. We also use Eq. 37-3.

$$\Delta E=h\nu=\frac{hc}{\lambda}=E_4-E_1=\frac{h^2}{8m\ell^2}\left(4^2-1^2\right)\quad\rightarrow$$

$$\ell=\sqrt{\frac{15h\lambda}{8mc}}=\sqrt{\frac{15\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(340\times10^{-9}\,\mathrm{m}\right)}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}}=\boxed{1.2\times10^{-9}\,\mathrm{m}}$$

$\boxed{25.}$ We assume the particle is not relativistic. The energy levels give the kinetic energy of the particles in the box.

$$E_1=\frac{h^2}{8m\ell^2}=\frac{p_1^2}{2m}\quad\rightarrow\quad p_1^2=\frac{h^2}{4\ell^2}\quad\rightarrow\quad\left|p_1\right|=\frac{h}{2\ell}\quad\rightarrow\quad\Delta p\approx2\left|p_1\right|=\frac{h}{\ell}$$

$$\Delta x\Delta p\approx\ell\frac{h}{\ell}=h$$

This is consistent with the uncertainty principle.

26. The longest wavelength photon will be the photon with the lowest frequency, and thus the lowest energy. The difference between energy levels increases with high states, so the lowest energy transition is from $n = 2$ to $n = 1$. The energy levels are given by Eq. 38-13.

$$\Delta E=h\nu=\frac{hc}{\lambda}=E_2-E_1=\frac{h^2}{8m\ell^2}\left(2^2-1^2\right)\quad\rightarrow$$

$$\ell=\sqrt{\frac{3h\lambda}{8mc}}=\sqrt{\frac{3\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)\left(610\times10^{-9}\,\mathrm{m}\right)}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}}=\boxed{7.4\times10^{-10}\,\mathrm{m}}$$

27. The energy levels for a particle in an infinite potential well are given by Eq. 38-13. The wave functions are given by Eq. 38-14 with $A=\sqrt{\frac{2}{\ell}}$.

$$E_1=\frac{h^2}{8m\ell^2}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)^2}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2.0\times10^{-9}\,\mathrm{m}\right)^2\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=9.424\times10^{-2}\,\mathrm{eV}\approx\boxed{0.094\,\mathrm{eV}}$$

$$E_2=2^2E_1=4\left(9.424\times10^{-2}\,\mathrm{eV}\right)=\boxed{0.38\,\mathrm{eV}}$$

$$E_3=3^2E_1=9\left(9.424\times10^{-2}\,\mathrm{eV}\right)=\boxed{0.85\,\mathrm{eV}}$$

$$E_4=4^2E_1=16\left(9.424\times10^{-2}\,\mathrm{eV}\right)=\boxed{1.5\,\mathrm{eV}}$$

$$\psi_n = \sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi}{\ell}x\right) \;;\; \psi_1 = \sqrt{\frac{2}{2.0\,\text{nm}}}\sin\left(\frac{\pi}{2.0\,\text{nm}}x\right) = \boxed{\left(1.0\,\text{nm}^{-1/2}\right)\sin\left[\left(1.6\,\text{nm}^{-1}\right)x\right]}$$

$$\psi_2 = \boxed{\left(1.0\,\text{nm}^{-1/2}\right)\sin\left[\left(3.1\,\text{nm}^{-1}\right)x\right]} \;;\; \psi_3 = \boxed{\left(1.0\,\text{nm}^{-1/2}\right)\sin\left[\left(4.7\,\text{nm}^{-1}\right)x\right]} \;;$$

$$\psi_4 = \boxed{\left(1.0\,\text{nm}^{-1/2}\right)\sin\left[\left(6.3\,\text{nm}^{-1}\right)x\right]}$$

28. The wave functions for an infinite square well are given by Eq. 38-14.

$$\psi_n = A\sin\left(\frac{n\pi}{\ell}x\right) \;;\; \left|\psi_n\right|^2 = A^2\sin^2\left(\frac{n\pi}{\ell}x\right)$$

(*a*) The maxima occur at locations where $\left|\psi_n\right|^2 = A^2$.

$$\sin^2\left(\frac{n\pi}{\ell}x\right) = 1 \;\rightarrow\; \frac{n\pi}{\ell}x = \left(m+\tfrac{1}{2}\right)\pi,\; m = 0,1,2,\cdots n-1 \;\rightarrow$$

$$\boxed{x_{\max} = \left(\frac{2m+1}{2n}\right)\ell,\; m = 0,1,2,\cdots n-1}$$

The values of *m* are limited because $x \le \ell$.

(*b*) The minima occur at locations where $\left|\psi_n\right|^2 = 0$.

$$\sin^2\left(\frac{n\pi}{\ell}x\right) = 0 \;\rightarrow\; \frac{n\pi}{\ell}x = m\pi,\; m = 0,1,2,\cdots n \;\rightarrow\; \boxed{x_{\min} = \frac{m}{n}\ell,\; m = 0,1,2,\cdots n}$$

29. The energy levels for a particle in a rigid box are given by Eq. 38-13. We substitute the appropriate mass in for each part of the problem.

(*a*) For an electron we have the following:

$$E = \frac{h^2n^2}{8m\ell^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.0\times10^{-14}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)} = \boxed{940\,\text{MeV}}$$

(*b*) For a neutron we have the following:

$$E = \frac{h^2n^2}{8m\ell^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(1.675\times10^{-27}\,\text{kg}\right)\left(2.0\times10^{-14}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)} = \boxed{0.51\,\text{MeV}}$$

(*c*) For a proton we have the following:

$$E = \frac{h^2n^2}{8m\ell^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(1.673\times10^{-27}\,\text{kg}\right)\left(2.0\times10^{-14}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)} = \boxed{0.51\,\text{MeV}}$$

30. The energy released is calculated by Eq. 38-13, with *n* = 2 for the initial state and *n* = 1 for the final state.

$$\Delta E = E_2 = E_1 = \left(2^2-1^1\right)\frac{h^2}{8m\ell^2} = \frac{3\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.0\times10^{-14}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)}$$

$$= \boxed{6.17\,\text{MeV}}$$

$\boxed{31.}$ (*a*) The ground state energy is given by Eq. 38-13 with $n = 1$.

$$E_1 = \left.\frac{h^2n^2}{8m\ell^2}\right|_{n=1} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2(1)}{8(32\,\mathrm{u})\left(1.66\times10^{-27}\,\mathrm{kg/u}\right)\left(4.0\times10^{-3}\,\mathrm{m}\right)^2\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}$$

$$= 4.041\times10^{-19}\,\mathrm{eV} \approx \boxed{4.0\times10^{-19}\,\mathrm{eV}}$$

(*b*) We equate the thermal energy expression to Eq. 38-13 in order to find the quantum number.

$$\tfrac{1}{2}kT = \frac{h^2n^2}{8m\ell^2} \quad \rightarrow$$

$$n = 2\sqrt{kTm}\,\frac{\ell}{h} = 2\sqrt{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(300\,\mathrm{K})(32\,\mathrm{u})\left(1.66\times10^{-27}\,\mathrm{kg/u}\right)}\,\frac{\left(4.0\times10^{-3}\,\mathrm{m}\right)}{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)}$$

$$= 1.789\times10^{8} \approx \boxed{2\times10^{8}}$$

(*c*) Use Eq. 38-13 with a large-*n* approximation.

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8m\ell^2}\left[(n+1)^2 - n^2\right] = \frac{h^2}{8m\ell^2}(2n+1) \approx 2n\frac{h^2}{8m\ell^2} = 2nE_1$$

$$= 2\left(1.789\times10^{8}\right)\left(4.041\times10^{-19}\,\mathrm{eV}\right) = \boxed{1.4\times10^{-10}\,\mathrm{eV}}$$

32. Because the wave function is normalized, the probability is found as in Example 38-8. Change the variable to $\theta = \frac{n\pi x}{\ell}$, and then $d\theta = \frac{n\pi}{\ell}dx$.

$$P = \int_{x_1}^{x_2}|\psi_n|^2\,dx = \frac{2}{\ell}\int_{x_1}^{x_2}\sin^2\frac{n\pi x}{\ell}\,dx = \frac{2}{\ell}\frac{\ell}{n\pi}\int_{n\pi x_1/\ell}^{n\pi x_2/\ell}\sin^2\theta\,d\theta = \frac{2}{n\pi}\int_{0.35n\pi}^{0.65n\pi}\sin^2\theta\,d\theta$$

$$= \frac{2}{n\pi}\left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_{0.35n\pi}^{0.65n\pi}$$

(*a*) For the $n = 1$ state we have the following:

$$P = \frac{2}{\pi}\left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_{0.35\pi}^{0.65\pi} = \frac{2}{\pi}\left[\tfrac{1}{2}(0.30\pi) - \tfrac{1}{4}(\sin 1.3\pi - \sin 0.70\pi)\right] = 0.5575 \approx 0.56$$

(*b*) For the $n = 5$ state we have the following:

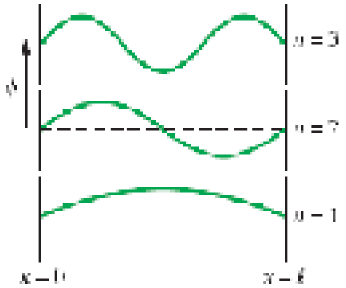
$$P = \frac{2}{5\pi}\left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_{1.75n\pi}^{3.25n\pi} = \frac{2}{5\pi}\left[\tfrac{1}{2}(1.5\pi) - \tfrac{1}{4}(\sin 6.5\pi - \sin 3.5\pi)\right] = 0.2363 \approx 0.24$$

(*c*) For the $n = 20$ state we have the following:

$$P = \frac{2}{20\pi}\left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_{7\pi}^{13\pi} = \frac{1}{10\pi}\left[\tfrac{1}{2}6\pi - \tfrac{1}{4}(\sin 26\pi - \sin 14\pi)\right] = 0.30$$

(*d*) The classical prediction would be that the particle has an equal probability of being at any location, so the probability of being in the given range is $P = \frac{0.65\,\mathrm{nm} - 0.35\,\mathrm{nm}}{1.00\,\mathrm{nm}} = 0.30$. We see that $\boxed{\text{the probabilities approach the classical value for large } n.}$

33. Consider Figure 38-9, copied here. To consider the problem with the boundaries shifted, we would not expect any kind of physics to change. So we expect the same wave functions in terms of their actual shape, and we expect the same energies if all that is done is to change the labeling of the walls to $x = -\frac{1}{2}\ell$ and $x = \frac{1}{2}\ell$. The mathematical descriptions of the wave functions would change because of the change of coordinates. All we should have to do is



shift the origin of coordinates to the right by $\frac{1}{2}\ell$. Thus we might expect the following wave functions and energies.

$$\psi_n=\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi}{\ell}x\right)\quad\rightarrow$$

$$\psi_n=\sqrt{\frac{2}{\ell}}\sin\left[\frac{n\pi}{\ell}\left(x+\tfrac{1}{2}\ell\right)\right]=\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi}{\ell}x+\tfrac{1}{2}n\pi\right)$$

$$n=1:\ \psi_1=\sqrt{\frac{2}{\ell}}\sin\left(\frac{\pi}{\ell}x+\tfrac{1}{2}\pi\right)=\boxed{\sqrt{\frac{2}{\ell}}\cos\left(\frac{\pi}{\ell}x\right)}\ ;\ E_1=\boxed{\frac{h^2}{8m\ell^2}}$$

$$n=2:\ \psi_2=\sqrt{\frac{2}{\ell}}\sin\left(\frac{2\pi}{\ell}x+\pi\right)=\boxed{-\sqrt{\frac{2}{\ell}}\sin\left(\frac{2\pi}{\ell}x\right)}\ ;\ E_2=\boxed{\frac{4h^2}{8m\ell^2}}$$

$$n=3:\ \psi_3=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{3}{2}\pi\right)=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{1}{2}\pi+\pi\right)=-\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{1}{2}\pi\right)$$

$$=\boxed{-\sqrt{\frac{2}{\ell}}\cos\left(\frac{3\pi}{\ell}x\right)}\ ;\ E_3=\boxed{\frac{9h^2}{8m\ell^2}}$$

$$n=4:\ \psi_4=\sqrt{\frac{2}{\ell}}\sin\left(\frac{4\pi}{\ell}x+2\pi\right)=\boxed{\sqrt{\frac{2}{\ell}}\sin\left(\frac{4\pi}{\ell}\right)}\ ;\ E_4=\boxed{\frac{16h^2}{8m\ell^2}}$$

For any higher orders, we simply add another 2π of phase to the arguments of the above functions. They can be summarized as follows.

$$n\text{ odd: }\psi=\left[(-1)^{(n-1)/2}\right]\sqrt{\frac{2}{\ell}}\cos\left(\frac{n\pi x}{\ell}\right),\ E_n=\frac{n^2h^2}{8m\ell^2}$$

$$n\text{ even: }\psi=\left[(-1)^{n/2}\right]\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi x}{\ell}\right),\ E_n=\frac{n^2h^2}{8m\ell^2}$$

Of course, this is not a "solution" in the sense that we have not derived these solutions from the Schrödinger equation. We now show a solution that arises from solving the Schrödinger equation. We follow the development as given in Section 38-8.

As suggested, let $\psi(x)=A\sin(kx+\phi)$. For a region where $U(x)=0,\ k=\sqrt{\frac{2mE}{\hbar^2}}$ (Eq. 38-11a).

The boundary conditions are $\psi\left(-\frac{1}{2}\ell\right)=A\sin\left(-\frac{1}{2}k\ell+\phi\right)=0$ and $\psi\left(\frac{1}{2}\ell\right)=A\sin\left(\frac{1}{2}k\ell+\phi\right)=0$. To guarantee the boundary conditions, we must have the following:

$$A\sin\left(-\tfrac{1}{2}k\ell+\phi\right)=0\ \rightarrow\ -\tfrac{1}{2}k\ell+\phi=m\pi\ ;\ A\sin\left(\tfrac{1}{2}k\ell+\phi\right)=0\ \rightarrow\ \tfrac{1}{2}k\ell+\phi=n\pi$$

Both n and m are integers. Add these two results, and subtract the two results, to get two new expressions.

$$\begin{aligned}\tfrac{1}{2}k\ell+\phi&=n\pi\\-\tfrac{1}{2}k\ell+\phi&=m\pi\end{aligned}\quad\rightarrow\quad\phi=\tfrac{1}{2}(n+m)\pi\ ;\ k=\frac{1}{\ell}(n-m)\pi$$

So again we have an energy quantization, with $E=\frac{\hbar^2k^2}{2m}=\frac{\hbar^2\pi^2(n-m)^2}{2m\ell^2}=\frac{h^2(n-m)^2}{8m\ell^2}$. Note that $m=n$ is not allowed, because this leads to $k=0,\ \phi=n\pi,$ and $\psi(x)\equiv0$.

Next we normalize the wave functions. We use an indefinite integral from Appendix B-4.

$$\psi(x)=A\sin(kx+\phi)=A\sin\left[\frac{1}{\ell}(n-m)\pi x+\tfrac{1}{2}(n+m)\pi\right]$$

$$\int_{-\infty}^{\infty}|\psi|^2\,dx=\int_{-\frac{1}{2}\ell}^{\frac{1}{2}\ell}A^2\sin^2\left[\frac{1}{\ell}(n-m)\pi x+\tfrac{1}{2}(n+m)\pi\right]dx=1$$

$$\text{let } \theta=\frac{1}{\ell}(n-m)\pi x+\tfrac{1}{2}(n+m)\pi \;\rightarrow\; dx=\frac{\ell}{(n-m)\pi}d\theta$$

$$x=\tfrac{1}{2}\ell \;\rightarrow\; \theta=n\pi \;;\; x=-\tfrac{1}{2}\ell \;\rightarrow\; \theta=m\pi$$

$$1=\int_{-\infty}^{\infty}|\psi|^2\,dx=\frac{A^2\ell}{(n-m)\pi}\int_{m\pi}^{n\pi}\sin^2\theta\,d\theta=\frac{A^2\ell}{(n-m)\pi}\left(\tfrac{1}{2}\theta-\tfrac{1}{4}\sin 2\theta\right)_{m\pi}^{n\pi}\frac{A^2\ell}{(n-m)\pi}\frac{(n-m)\pi}{2}=\frac{A^2\ell}{2}$$

$$A^2=\frac{2}{\ell} \;\rightarrow\; A=\sqrt{\frac{2}{\ell}}$$

This is the same as in Section 38-8. Finally, let us examine a few allowed cases.

$$n=1,\, m=0:\; k=\frac{\pi}{\ell},\; \phi=\tfrac{1}{2}\pi \;\rightarrow\; \psi_{1,0}=\sqrt{\frac{2}{\ell}}\sin\left(\frac{\pi}{\ell}x+\tfrac{1}{2}\pi\right)=\boxed{\sqrt{\frac{2}{\ell}}\cos\left(\frac{\pi}{\ell}x\right)}\;;\; E_{1,0}=\boxed{\frac{h^2}{8m\ell^2}}$$

$$n=2,\; m=0:\; k=\frac{2\pi}{\ell},\; \phi=\pi \;\rightarrow\; \psi_{2,0}=\sqrt{\frac{2}{\ell}}\sin\left(\frac{2\pi}{\ell}x+\pi\right)=\boxed{-\sqrt{\frac{2}{\ell}}\sin\left(\frac{2\pi}{\ell}x\right)}\;;\; E_{2,0}=\boxed{\frac{4h^2}{8m\ell^2}}$$

$$n=3,\; m=0:\; k=\frac{3\pi}{\ell},\; \phi=\tfrac{3}{2}\pi \;\rightarrow\; \psi_{3,0}=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{3}{2}\pi\right)=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{1}{2}\pi+\pi\right)$$

$$=-\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{1}{2}\pi\right)=\boxed{-\sqrt{\frac{2}{\ell}}\cos\left(\frac{3\pi}{\ell}x\right)}\;;\; E_{3,0}=\boxed{\frac{9h^2}{8m\ell^2}}$$

$$n=4,\; m=0:\; k=\frac{4\pi}{\ell},\; \phi=2\pi \;\rightarrow\; \psi_{4,0}=\sqrt{\frac{2}{\ell}}\sin\left(\frac{4\pi}{\ell}x+2\pi\right)=\boxed{\sqrt{\frac{2}{\ell}}\sin\left(\frac{4\pi}{\ell}\right)}\;;\; E_{4,0}=\boxed{\frac{16h^2}{8m\ell^2}}$$

These are the same results as those obtained in the less formal method. Other combinations of m and n would give essentially these same results for the lowest four energies and the associated wave functions. For example, consider $n=4,\; m=1$.

$$n=4,\; m=1:\; k=\frac{3\pi}{\ell},\; \phi=\tfrac{5}{2}\pi \;\rightarrow\; \psi_{4,1}=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{5}{2}\pi\right)=\sqrt{\frac{2}{\ell}}\sin\left(\frac{3\pi}{\ell}x+\tfrac{1}{2}\pi\right)$$

$$=\boxed{\sqrt{\frac{2}{\ell}}\cos\left(\frac{4\pi}{\ell}\right)}\;;\; E_{4,1}=\boxed{\frac{9h^2}{8m\ell^2}}$$

We see that $\psi_{4,1}=-\psi_{3,0}$ and that both states have the same energy. Since the only difference in the wave functions is the algebraic sign, any physical measurement predictions, which depend on the absolute square of the wave function, would be the same.

34. We choose the zero of potential energy to be at the bottom of the well. Thus in free space, outside the well, the potential is $U_0=56\text{eV}$. Thus the total energy of the electron is $E=K+U_0=236$ eV.

(*a*) In free space, the kinetic energy of the particle is 180 eV. Use that to find the momentum and then the wavelength.

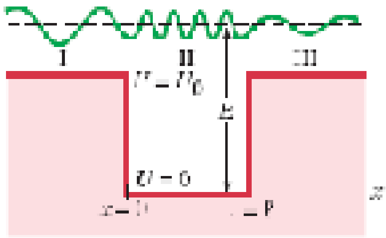
$$K=\frac{p^2}{2m} \;\rightarrow\; p=\sqrt{2mK}=\frac{h}{\lambda} \;\rightarrow$$

$$\lambda=\frac{h}{\sqrt{2mK}}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)}{\left[2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(180\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)\right]^{1/2}}=\boxed{9.15\times10^{-11}\,\mathrm{m}}$$

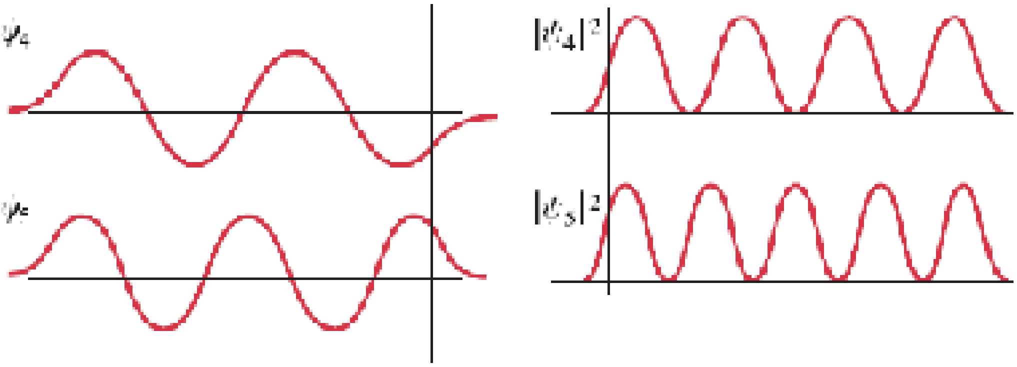
(*b*) Over the well, the kinetic energy is 236 eV.

$$\lambda=\frac{h}{\sqrt{2mK}}=\frac{\left(6.63\times10^{-34}\,\mathrm{J\bullet s}\right)}{\left[2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(236\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)\right]^{1/2}}=\boxed{7.99\times10^{-11}\,\mathrm{m}}$$

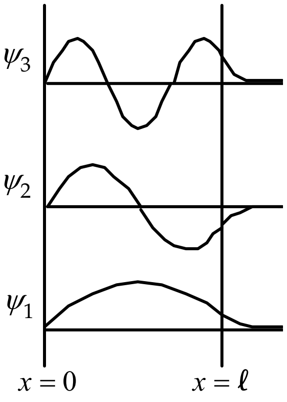
(*c*) The diagram is qualitatively the same as Figure 38-14, reproduced here. Notice that the wavelength is longer when the particle is not over the well, and shorter when the particle is over the well.



35. We pattern our answer after Figure 38-13.



36. (*a*) We assume that the lowest three states are bound in the well, so that $E<U_0$. See the diagrams for the proposed wave functions. Note that, in the well, the wave functions are similar to those for the infinite well. Outside the well, for $x>\ell$, the wave functions are drawn with an exponential decay, similar to the right side of Figure 38-13.



(*b*) In the region $x<0$, $\boxed{\psi=0}$.

In the well, with $0<x<\ell$, the wave function is similar to that of a free particle or a particle in an infinite potential well, since $U=0$.

Thus $\boxed{\psi=A\sin kx+B\sin kx}$, where $k=\dfrac{\sqrt{2mE}}{\hbar}$.

In the region $x>\ell$, $\boxed{\psi=De^{-Gx}}$, where $G=\dfrac{\sqrt{2m\left(U_0-E\right)}}{\hbar}$.

$\boxed{37.}$ We will consider the "left" wall of the square well, using Figure 38-12,l and assume that our answer is applicable at either wall due to the symmetry of the potential well. As in Section 38-9, let $\psi=Ce^{Gx}$ for $x<0$, with G given in Eq. 38-16. Since the wave function must be continuous, $\psi\left(x=0\right)=C$. The energy of the electron is to be its ground state energy, approximated by Eq. 38-

13 for the infinite well. If that energy is much less than the depth of the well, our approximation will be reasonable. We want to find the distance *x* for which $\psi(x) = 0.010\psi(0)$.

$$E = \frac{h^2}{8m\ell^2} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(0.16\times10^{-9}\,\mathrm{m}\right)^2\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = 14.73\,\mathrm{eV} \ll U_0$$

$$\frac{\psi(x)}{\psi(0)} = \frac{Ce^{Gx}}{C} = e^{Gx} = 0.010 \;\rightarrow\; x = \frac{\ln(0.010)}{G} = \frac{\hbar\ln(0.010)}{\sqrt{2m(U_0-E)}}$$

$$x = \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)\ln(0.010)}{\sqrt{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(2000\,\mathrm{eV}-14.73\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}} = -2.0\times10^{-11}\,\mathrm{m}$$

The wave function will be 1.0% of its value at the walls at a distance of $\boxed{2.0\times10^{-11}\,\mathrm{m} = 0.020\,\mathrm{nm}}$ from the walls.

38. We use Eqs. 38-17a and 38-17b.

$$T = e^{-2G\ell} \;\rightarrow\; G = -\frac{\ln T}{2\ell} \;;\; G = \sqrt{\frac{2m(U_0-E)}{\hbar^2}} = -\frac{\ln T}{2\ell} \;\rightarrow$$

$$E = U_0 - \frac{\hbar^2}{2m}\left(-\frac{\ln T}{2\ell}\right)^2 = 14\,\mathrm{eV} - \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2}{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)}\left(-\frac{\ln(0.00050)}{2\left(0.85\times10^{-9}\,\mathrm{m}\right)}\right)^2\left(\frac{1}{1.60\times10^{-19}\,\mathrm{J/eV}}\right)$$

$$= 14\,\mathrm{eV} - 0.76\,\mathrm{eV} = 13.24\,\mathrm{eV} \approx \boxed{13\,\mathrm{eV}}$$

39. We use Eqs. 38-17a and 38-17b to solve for the particle's energy.

$$T = e^{-2G\ell} \;\rightarrow\; G = -\frac{\ln T}{2\ell} \;;\; G = \sqrt{\frac{2m(U_0-E)}{\hbar^2}} = -\frac{\ln T}{2\ell} \;\rightarrow$$

$$E = U_0 - \frac{\hbar^2}{2m}\left(-\frac{\ln T}{2\ell}\right)^2 = 18\,\mathrm{eV} - \frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2}{2\left(9.11\times10^{-31}\,\mathrm{kg}\right)}\left(-\frac{\ln(0.010)}{2\left(0.55\times10^{-9}\,\mathrm{m}\right)}\right)^2\left(\frac{1}{1.60\times10^{-19}\,\mathrm{J/eV}}\right)$$

$$= 17.33\,\mathrm{eV} \approx \boxed{17\,\mathrm{eV}}$$

40. We use Eqs. 38-17a and 38-17b to solve for the transmission coefficient, which can be interpreted in terms of probability. For the mass of the helium nucleus, we take the mass of 2 protons and 2 neutrons, ignoring the (small) binding energy.
Proton:

$$G = \frac{\sqrt{2m(U_0-E)}}{\hbar} = \frac{\sqrt{2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(20.0\,\mathrm{MeV}\right)\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}}{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)} = 9.799\times10^{14}\,\mathrm{m^{-1}}$$

$$2G\ell = 2\left(9.799\times10^{14}\,\mathrm{m^{-1}}\right)\left(3.6\times10^{-15}\,\mathrm{m}\right) = 7.056 \;;\; T_{\mathrm{proton}} = e^{-2G\ell} = e^{-7.056} = \boxed{8.6\times10^{-4}}$$

Helium: Mass = $2m_{\mathrm{proton}} + 2m_{\mathrm{neutron}} = 2\left(1.673\times10^{-27}\,\mathrm{kg}\right) + 2\left(1.675\times10^{-27}\,\mathrm{kg}\right) = 6.70\times10^{-27}\,\mathrm{kg}$.

$$G = \frac{\sqrt{2m(U_0-E)}}{\hbar} = \frac{\sqrt{2\left(6.70\times10^{-27}\,\mathrm{kg}\right)\left(20.0\,\mathrm{MeV}\right)\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}}{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)} = 1.963\times10^{15}\,\mathrm{m^{-1}}$$

$$2G\ell = 2\left(1.963\times10^{15}\,\mathrm{m^{-1}}\right)\left(3.6\times10^{-15}\,\mathrm{m}\right) = 14.112 \;;\; T_{\mathrm{He}} = e^{-2G\ell} = e^{-14.112} = \boxed{7.4\times10^{-7}}$$

41. (*a*) The probability of the electron passing through the barrier is given by Eqs. 38-17a and 38-17b.

$$T = e^{-2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar}}$$

$$2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar} = 2\left(0.25\times10^{-9}\,\text{m}\right)\frac{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.2\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}{\left(1.055\times10^{-34}\,\text{J•s}\right)} = 2.803$$

$$T = e^{-2.803} = 6.063\times10^{-2} \approx \boxed{6.1\%}$$

(*b*) The probability of reflecting is the probability of NOT tunneling, and so is $\boxed{93.9\%}$.

42. The transmitted current is caused by protons that tunnel through the barrier. Since current is directly proportional to the number of charges moving, the transmitted current is the incident current times the transmission coefficient. We use Eqs. 38-17a and 38-17b.

$$T = e^{-2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar}}$$

$$2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar} = 2\left(2.8\times10^{-13}\,\text{m}\right)\frac{\sqrt{2\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.0\,\text{MeV}\right)\left(1.60\times10^{-13}\,\text{J/eV}\right)}}{\left(1.055\times10^{-34}\,\text{J•s}\right)} = 122.86$$

$$T = e^{-122.86} \;\rightarrow\; \log T = \left(-122.86\right)\log e = -53.357 \;\rightarrow\; T = 10^{-53.357} = 4.4\times10^{-54}$$

$$I = I_0 T = \left(1.0\,\text{mA}\right)\left(4.4\times10^{-54}\right) = \boxed{4.4\times10^{-54}\,\text{mA}}$$

$\boxed{43.}$ The transmission coefficient is given by Eqs. 38-17a and 38-17b.

(*a*) The barrier height is now $1.02\left(70\,\text{eV}\right) = 71.4\,\text{eV}$.

$$2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar} = 2\left(0.10\times10^{-9}\,\text{m}\right)\frac{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(21.4\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}{\left(1.055\times10^{-34}\,\text{J•s}\right)}$$

$$= 4.735$$

$$T = e^{-2\ell\frac{\sqrt{2m(U\ - E)}}{\hbar}} = e^{-4.735} = 8.782\times10^{-3} \;;\; \frac{T}{T_0} = \frac{8.782\times10^{-3}}{0.010} = 88 \;\rightarrow\; \boxed{12\%\ \text{decrease}}$$

(*b*) The barrier width is now $1.02\left(0.10\,\text{nm}\right) = 0.102\,\text{nm}$.

$$2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar} = 2\left(0.102\times10^{-9}\,\text{m}\right)\frac{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(20\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}{\left(1.055\times10^{-34}\,\text{J•s}\right)}$$

$$= 4.669$$

$$T = e^{-2\ell\frac{\sqrt{2m(U_0 - E)}}{\hbar}} = e^{-4.669} = 9.382\times10^{-3} \;;\; \frac{T}{T_0} = \frac{9.382\times10^{-3}}{0.010} = 93.8 \;\rightarrow\; \boxed{6.2\%\ \text{decrease}}$$

44. We assume that the wave function inside the barrier is given by a decaying exponential, so $\psi\left(x\right) = Ae^{-Gx}$.

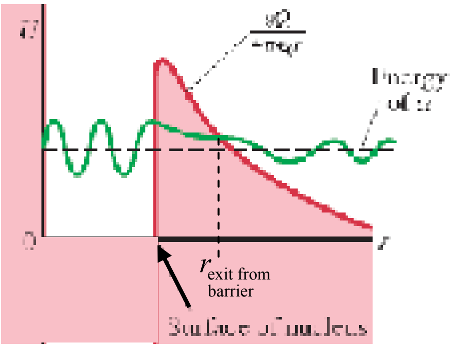
$$T = \frac{\left|\psi\left(x=\ell\right)\right|^2}{\left|\psi\left(x=0\right)\right|^2} = \frac{\left(Ae^{-G\ell}\right)^2}{A^2} = \boxed{e^{-2G\ell}}$$

45. (*a*) We assume that the alpha particle is at the outer edge of the nucleus. The potential energy is electrostatic potential energy, and is found from Eq. 23-10.

$$U_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{\substack{\text{surface of}\\ \text{nucleus}}}} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(90)\left(1.60\times10^{-19}\,\text{C}\right)^2}{8\times10^{-15}\,\text{m}}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)$$

$$= 32.36\,\text{MeV} \approx \boxed{32\,\text{MeV}}$$

(*b*) The kinetic energy of the free alpha particle is also its total energy. Since the free alpha has 4 MeV, by conservation of energy the alpha particle had 4 MeV of potential energy at the exit from the barrier. See the diagram, a copy of Figure 38-17, modified to show U = 0 inside the barrier, and stated in part (*c*).



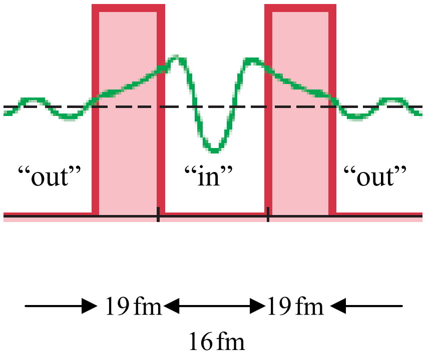
$$U_{\text{exit}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{\substack{\text{exit from}\\ \text{barrier}}}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{\substack{\text{surface of}\\ \text{nucleus}}}}\frac{r_{\substack{\text{surface of}\\ \text{nucleus}}}}{r_{\substack{\text{exit from}\\ \text{barrier}}}}$$

$$= U_{\text{surface}}\frac{r_{\substack{\text{surface of}\\ \text{nucleus}}}}{r_{\substack{\text{exit from}\\ \text{barrier}}}} \quad \rightarrow$$

$$r_{\substack{\text{exit from}\\ \text{barrier}}} = \frac{U_{\text{surface}}}{U_{\text{exit}}}r_{\substack{\text{surface of}\\ \text{nucleus}}} = \frac{32.36\,\text{MeV}}{4\,\text{MeV}}(8\,\text{fm}) = 64.72\,\text{fm}$$

$$\Delta r = r_{\substack{\text{exit from}\\ \text{barrier}}} - r_{\substack{\text{surface of}\\ \text{nucleus}}} = 64.72\,\text{fm} - 8\,\text{fm} = 56.72\,\text{fm} \approx \boxed{57\,\text{fm}}$$

(*c*) We now model the barrier as being rectangular, with a width of $r_{\text{barrier}} = \frac{1}{3}(56.72\,\text{fm}) = 18.9\,\text{fm}$. The barrier exists at both boundaries of the nucleus, if we imagine the nucleus as 1-dimensional. See the diagram (not to scale). We calculate the speed of the alpha particle and use that to find the frequency of collision with the barrier.



$$E = K = \tfrac{1}{2}mv^2 \quad \rightarrow$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(4\,\text{MeV})\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{4\left(1.67\times10^{-27}\,\text{kg}\right)}}$$

$$= 1.38\times10^7\,\text{m/s} \approx \boxed{1.4\times10^7\,\text{m/s}}$$

Note that the speed of the alpha is less than 5% of the speed of light, so we can treat the alpha without using relativistic concepts. The time between collisions is the diameter of the nucleus (16 fm) divided by the speed of the alpha particles. The frequency of collision is the reciprocal of the time between collisions.

$$f = \frac{v}{d} = \frac{1.38\times10^7\,\text{m/s}}{16\times10^{-15}\,\text{m}} = 8.625\times10^{20}\,\text{collisions/s} \approx \boxed{8.6\times10^{20}\,\text{collisions/s}}$$

If we multiply this collision frequency times the probability of tunneling, T, then we will have an estimate of "effective" collisions/s, or in other words, the decays/s. The reciprocal of this effective frequency is an estimate of the time the alpha spends inside the nucleus – the life of the uranium nucleus.

$$G=\frac{\sqrt{2m\left(U_0-E\right)}}{\hbar}=\frac{\sqrt{2\left(4\right)\left(1.67\times10^{-27}\,\text{kg}\right)\left(32.36\,\text{MeV}-4\,\text{MeV}\right)\left(1.60\times10^{-13}\,\text{J/MeV}\right)}}{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}$$

$$=2.33\times10^{15}\,\text{m}^{-1}$$

$$T=e^{-2G\ell}\;;\;\text{Lifetime}=\frac{1}{fT}=\frac{1}{fe^{-2G\ell}}=\frac{1}{\left(8.625\times10^{20}\,\text{collisions/s}\right)e^{-2\left(2.33\times10^{15}\,\text{m}^{-1}\right)\left(18.9\times10^{-15}\,\text{m}\right)}}$$

$$=2.06\times10^{17}\,\text{s}\left(\frac{1\,\text{yr}}{3.156\times10^{7}\,\text{s}}\right)\approx\boxed{7\times10^{9}\,\text{yr}}$$

46. We find the lifetime of the particle from Eq. 38-2.

$$\Delta t\geq\frac{\hbar}{\Delta E}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(2.5\,\text{GeV}\right)\left(1.60\times10^{-10}\,\text{J/GeV}\right)}=\boxed{2.6\times10^{-25}\,\text{s.}}$$

47. We use the radius as the uncertainty in position for the neutron. We find the uncertainty in the momentum from Eq. 38-1. If we assume that the lowest value for the momentum is the least uncertainty, we estimate the lowest possible kinetic energy (non-relativistic) as

$$E=\frac{\left(\Delta p\right)^2}{2m}=\frac{\left(\frac{\hbar}{\Delta x}\right)^2}{2m}=\frac{\left(1.055\times10^{-34}\,\text{kg}\bullet\text{m/s}\right)^2}{2\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.2\times10^{-15}\,\text{m}\right)^2}=2.314\times10^{-12}\,\text{J}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)$$

$$=14.46\,\text{MeV}\approx\boxed{14\text{ MeV}}$$

48. The energy levels for a particle in an infinite potential well are given by Eq. 38-13. The wave functions are given by Eq. 38-14. with $A=\sqrt{\frac{2}{\ell}}$.

(*a*) $$E_1=\frac{h^2}{8m\ell^2}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(1.67\times10^{-27}\,\text{kg}\right)\left(2.5\times10^{-15}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)}=32.90\,\text{MeV}\approx\boxed{33\,\text{MeV}}$$

$$E_2=2^2E_1=4\left(32.90\,\text{MeV}\right)=\boxed{130\,\text{MeV}}\;;\;E_3=3^2E_1=9\left(32.90\,\text{MeV}\right)=\boxed{300\,\text{MeV}}\;\left(2\text{ sig. fig.}\right)$$

$$E_4=4^2E_1=16\left(32.90\,\text{MeV}\right)=\boxed{530\,\text{MeV}}$$

(*b*) $$\psi_n=\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi}{\ell}x\right)$$

$$\psi_1=\sqrt{\frac{2}{2.5\times10^{-15}\,\text{m}}}\sin\left(\frac{\pi}{2.5\times10^{-15}\,\text{m}}x\right)=\boxed{\left(2.8\times10^{7}\,\text{m}^{-1/2}\right)\sin\left[\left(1.3\times10^{15}\,\text{m}^{-1}\right)x\right]}$$

$$\psi_2=\sqrt{\frac{2}{2.5\times10^{-15}\,\text{m}}}\sin\left(\frac{2\pi}{2.5\times10^{-15}\,\text{m}}x\right)=\boxed{\left(2.8\times10^{7}\,\text{m}^{-1/2}\right)\sin\left[\left(2.5\times10^{15}\,\text{m}^{-1}\right)x\right]}$$

$$\psi_3=\sqrt{\frac{2}{2.5\times10^{-15}\,\text{m}}}\sin\left(\frac{3\pi}{2.5\times10^{-15}\,\text{m}}x\right)=\boxed{\left(2.8\times10^{7}\,\text{m}^{-1/2}\right)\sin\left[\left(3.8\times10^{15}\,\text{m}^{-1}\right)x\right]}$$

$$\psi_4=\sqrt{\frac{2}{2.5\times10^{-15}\,\text{m}}}\sin\left(\frac{4\pi}{2.5\times10^{-15}\,\text{m}}x\right)=\boxed{\left(2.8\times10^{7}\,\text{m}^{-1/2}\right)\sin\left[\left(5.0\times10^{15}\,\text{m}^{-1}\right)x\right]}$$

(*c*) $\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 = 3(32.90\,\text{MeV}) = 98.7\,\text{MeV} \approx \boxed{99\,\text{MeV}}$

$$\Delta E = \frac{hc}{\lambda} \;\rightarrow\; \lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(98.7\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})} = \boxed{1.3\times10^{-14}\,\text{m}} = 13\,\text{fm}$$

This is in the gamma-ray region of the EM spectrum, as seen in Fig. 31-12.

$\boxed{49.}$ We find the wavelength of the protons from their kinetic energy, and then use the two-slit interference formulas from Chapter 34, with a small angle approximation. If the protons were accelerated by a 650-volt potential difference, then they will have 650 eV of kinetic energy.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0K}} \;;\; d\sin\theta = m\lambda,\ m = 1,2,\ldots \;;\; y = \ell\tan\theta$$

$$\sin\theta = \tan\theta \;\rightarrow\; \frac{m\lambda}{d} = \frac{y}{\ell} \;\rightarrow\; y = \frac{m\lambda\ell}{d},\ m = 1,2,\ldots \;\rightarrow$$

$$\Delta y = \frac{\lambda\ell}{d} = \frac{h\ell}{d\sqrt{2m_0K}} = \frac{(6.63\times10^{-34}\,\text{J•s})(18\,\text{m})}{(8.0\times10^{-4}\,\text{m})\sqrt{2(1.67\times10^{-27}\,\text{kg})(650\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}}$$

$$= \boxed{2.5\times10^{-8}\,\text{m}}$$

50. We assume that the particles are not relativistic. Conservation of energy is used to find the speed of each particle. That speed then can be used to find the momentum and finally the de Broglie wavelength. We let the magnitude of the accelerating potential difference be *V*.

$$U_{\text{initial}} = K_{\text{final}} \;\rightarrow\; eV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2eV}{m}} \;;\; \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} = \Delta x$$

$$\Delta x\Delta p = \frac{h}{2\pi} \;\rightarrow\; \Delta p = \frac{2\pi h}{\Delta x}$$

$$\frac{\Delta p_{\text{proton}}}{\Delta p_{\text{electron}}} = \frac{\dfrac{2\pi h}{\Delta x_{\text{proton}}}}{\dfrac{2\pi h}{\Delta x_{\text{electron}}}} = \frac{\Delta x_{\text{electron}}}{\Delta x_{\text{proton}}} = \frac{\dfrac{h}{\sqrt{2m_{\text{electron}}eV}}}{\dfrac{h}{\sqrt{2m_{\text{proton}}eV}}} = \sqrt{\frac{m_{\text{proton}}}{m_{\text{electron}}}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{43}$$

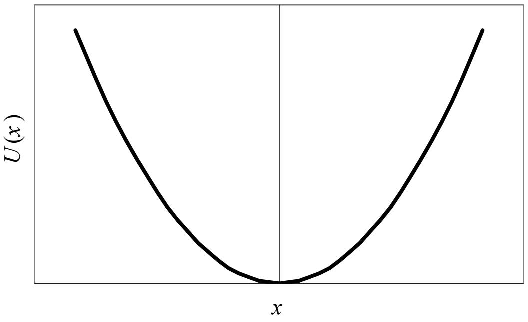
51. We use Eq. 37-10, Bohr's quantum condition.

$$mvr_n = n\frac{h}{2\pi} \;\rightarrow\; mvr_1 = \hbar \;\rightarrow\; mv = p = \frac{\hbar}{r_1} = \Delta p$$

$$\Delta x\Delta p \approx \hbar \;\rightarrow\; \Delta x \approx \frac{\hbar}{\Delta p} = \frac{\hbar}{\hbar/r_1} = r_1$$

The uncertainty in position is comparable to the Bohr radius.

52. (*a*) See the diagram.



(*b*) We use the solution $\psi(x) = Ae^{-Bx^2}$ in the Schrödinger equation.

$$\psi = Ae^{-Bx^2} \; ; \; \frac{d\psi}{dx} = -2ABxe^{-Bx^2}$$

$$\frac{d^2\psi}{dx^2} = -2ABe^{-Bx^2} - 2ABx\left(-2Bxe^{-Bx^2}\right)$$

$$= 2\left(2Bx^2 - 1\right)ABe^{-Bx^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\Psi = -\frac{\hbar^2}{2m}\left[2\left(2Bx^2-1\right)ABe^{-Bx^2}\right] + \tfrac{1}{2}Cx^2Ae^{-Bx^2} = EAe^{-Bx^2} \rightarrow$$

$$\left(\frac{\hbar^2 B}{m} - E\right) + \left(\tfrac{1}{2}C - \frac{2\hbar^2B^2}{m}\right)x^2 = 0$$

This is a solution if $\frac{\hbar^2 B}{m} = E$ and $\tfrac{1}{2}C = \frac{2\hbar^2 B^2}{m}$. Solve these two equations for *E* in terms of *C*, and let $\omega \equiv \sqrt{C/m}$.

$$\tfrac{1}{2}C = \frac{2\hbar^2B^2}{m} \rightarrow B = \frac{\sqrt{mC}}{2\hbar} \; ; \; E = \frac{\hbar^2 B}{m} = \frac{\hbar^2}{m}\frac{\sqrt{mC}}{2\hbar} = \tfrac{1}{2}\hbar\sqrt{C/m} = \tfrac{1}{2}\hbar\omega$$

$$B = \frac{\sqrt{mC}}{2\hbar} = \frac{m}{2\hbar}\sqrt{C/m} = \frac{\omega m}{2\hbar}$$

53. We assume the alpha particle is in the ground state. The energy is given by Eq. 38-13.

$$E_1 = \frac{h^2}{8m\ell^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8(4)\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.5\times10^{-14}\,\text{m}\right)^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)} = 0.2285\text{MeV}$$

$$\approx \boxed{0.23\,\text{MeV}}$$

The speed can be found from the kinetic energy. The alpha is non-relativistic.

$$E_1 = \frac{h^2}{8m\ell^2} = \tfrac{1}{2}mv^2 \rightarrow v = \frac{h}{2m\ell} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{2(4)\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.5\times10^{-14}\,\text{m}\right)} = \boxed{3.3\times10^6\,\text{m/s}}$$

54. From energy conservation, the speed of a ball after falling a height *H*, or the speed needed to rise to a height *H*, is $v = \sqrt{2gH}$. We say that the starting height is H_0, and so the speed just before the ball hits the ground before the first bounce is $v_0 = \sqrt{2gH_0}$. After that bounce, the ball rebounded to $H_1 = 0.65H_0$, and so the speed right after the first bounce, and right before the second bounce, is $v_1 = \sqrt{2gH_1} = \sqrt{2g(0.65)H_0}$. Repeated application of this idea gives the maximum height after *n* bounces as $H_n = (0.65)^n H_0$, and the maximum speed after *n* bounces as $v_n = \sqrt{2g(0.65)^n H_0}$. The uncertainty principle will come into play in the problem when the maximum speed after a bounce is of the same order as the uncertainty in the speed. We take the maximum height as the uncertainty in the position.

$$m\Delta v_y = \Delta p_y \; ; \; \Delta y \Delta p_y \approx \hbar \rightarrow m\Delta v_y \approx \frac{\hbar}{\Delta y} \rightarrow mv_n \approx \frac{\hbar}{H_n} \rightarrow$$

$$m\sqrt{2g(0.65)^n H_0} \approx \frac{\hbar}{(0.65)^n H_0} \quad \rightarrow \quad 2m^2 g(0.65)^n H_0 \approx \frac{\hbar^2}{(0.65)^{2n} H_0^2} \quad \rightarrow$$

$$(0.65)^{3n} \approx \frac{\hbar^2}{2m^2 g H_0^3} \quad \rightarrow \quad n \approx \frac{\ln\left(\dfrac{\hbar^2}{2m^2 g H_0^3}\right)}{3\ln(0.65)} = \frac{\ln\left[\dfrac{(1.055\times10^{-34}\,\text{J}\bullet\text{s})^2}{2(3.0\times10^{-6}\,\text{kg})^2(9.80\,\text{m/s}^2)(2.0\,\text{m})^3}\right]}{3\ln(0.65)} = \boxed{105}$$

After about 105 bounces, the uncertainty principle will be important to consider.

$\boxed{55.}$ We model the electrons as being restricted from leaving the surface of the sodium by an energy barrier, similar to Figure 38-15a. The difference between the barrier's height and the energy of the electrons is the work function, and so $U_0 - E = W_0 = 2.28\,\text{eV}$. But quantum mechanically, some electrons will "tunnel" through that barrier without ever being given the work function energy, and thus get outside the barrier, as shown in Figure 38-15b. This is the tunneling current as indicated in Figure 38-18. The distance from the sodium surface to the tip of the microscope is the width of the barrier, ℓ. We calculate the transmission probability as a function of barrier width by Eqs. 38-17a and 38-17b. The barrier is then increased to $\ell + \Delta\ell$, which will lower the transmission probability.

$$2G\Delta\ell = 2\Delta\ell\frac{\sqrt{2m(U_0-E)}}{\hbar} = 2(0.02\times10^{-9}\,\text{m})\frac{\sqrt{2(9.11\times10^{-31}\,\text{kg})(2.28\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}}{(1.055\times10^{-34}\,\text{J}\bullet\text{s})}$$

$$= 0.3091$$

$$T_0 = e^{-2G\ell} \ ; \ T = e^{-2G(\ell+\Delta\ell)} \ ; \ \frac{T}{T_0} = \frac{e^{-2G(\ell+\Delta\ell)}}{e^{-2G\ell}} = e^{-2G\Delta\ell} = e^{-0.3091} = 0.734$$

The tunneling current is caused by electrons that tunnel through the barrier. Since current is directly proportional to the number of electrons making it through the barrier, any change in the transmission probability is reflected as a proportional change in current. So we see that the change in the transmission probability, which will be reflected as a change in current, is $\boxed{\text{a decrease of 27\%}}$. Note that this change is only a fraction of the size of an atom.

56. The time independent Schrödinger equation with $U = 0$ is $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} = E\psi$.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(Ae^{ikx}\right) = -\frac{\hbar^2}{2m}\left(-k^2\right)Ae^{ikx} = \frac{\hbar^2k^2}{2m}\psi = \frac{\hbar^2\left(\sqrt{\dfrac{2mE}{\hbar}}\right)^2}{2m}\psi = E\psi$$

We see that the function solves the Schrödinger equation.

57. The wave functions for the particle in the infinite well are $\psi_n = \sqrt{\dfrac{2}{\ell}}\sin\left(\dfrac{n\pi}{\ell}x\right)$, as derived in Section 38-8. A table of integrals was consulted to find $\int x^2\left(\sin^2 ax\right)dx$.

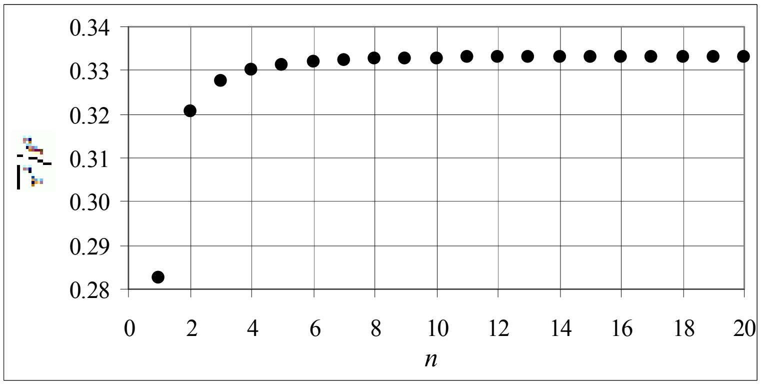
$$\overline{x^2} = \int x^2|\psi_n|^2\,dx = \int_0^\ell x^2\left[\sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi}{\ell}x\right)\right]^2 dx = \frac{2}{\ell}\int_0^\ell x^2\sin^2\left(\frac{n\pi}{\ell}x\right)dx$$

Note that $\int x^2\left(\sin^2 ax\right)dx = \dfrac{x^3}{6} - \left(\dfrac{x^2}{4a} - \dfrac{1}{8a^3}\right)\sin 2ax - \dfrac{x\cos 2ax}{4a^2}$.

$$\overline{x^2} = \frac{2}{\ell}\int_0^\ell x^2 \sin^2\left(\frac{n\pi}{\ell}x\right)dx = \frac{2}{\ell}\left\{\frac{x^3}{6} - \left(\frac{x^2}{4\frac{n\pi}{\ell}} - \frac{1}{8\left(\frac{n\pi}{\ell}\right)^3}\right)\sin\left(\frac{2n\pi}{\ell}x\right) - \frac{x\cos\left(\frac{2n\pi}{\ell}x\right)}{4\left(\frac{n\pi}{\ell}\right)^2}\right\}_0^\ell$$

$$= \boxed{\ell^2\left[\tfrac{1}{3} - \tfrac{1}{2}(n\pi)^{-2}\right]}$$

See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH38.XLS," on tab "Problem 38.57."



58. (*a*) To check that the wave function is normalized, we calculate $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx$.

$$\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = \int_{-\infty}^{\infty}\frac{1}{b}\left|\frac{x}{b}\right|e^{-\frac{x^2}{b^2}}dx = \frac{2}{b^2}\int_0^{\infty}xe^{-\frac{x^2}{b^2}}dx = \frac{2}{b^2}\left[-\frac{b^2}{2}e^{-\frac{x^2}{b^2}}\right]_0^{\infty} = -(0-1) = 1$$

We see that the function is normalized.

(*b*) The most probable position is that for which $|\psi(x)|^2$ is maximized. That point can be found by solving $\frac{d|\psi(x)|^2}{dx} = 0$ for *x*. Since we are only considering $x > 0$, we need not use the absolute value signs in the function.

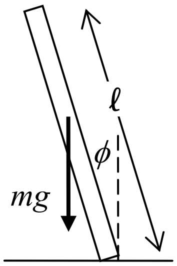
$$\frac{d|\psi(x)|^2}{dx} = \frac{d\left[\frac{x}{b^2}e^{-\frac{x^2}{b^2}}\right]}{dx} = \frac{1}{b^2}e^{-\frac{x^2}{b^2}} + \frac{x}{b^2}\left(-\frac{2x}{b^2}\right)e^{-\frac{x^2}{b^2}} = 0 \rightarrow 1 = \frac{2x^2}{b^2} \rightarrow$$

$$x = \frac{b}{\sqrt{2}} = \frac{1.0\,\text{nm}}{\sqrt{2}} = \boxed{0.71\,\text{nm}}$$

This value for *x* maximizes the function, because the function must be positive, and the function is 0 at $x = 0$ and $x = \infty$. Thus this single local extreme point must be a maximum.

(*c*) To find the probability, we integrate the probability density function between the given limits.

$$P = \int_0^{0.50\,\text{nm}}|\psi(x)|^2\,dx = \int_0^{0.50\,\text{nm}}\frac{x}{b^2}e^{-\frac{x^2}{b^2}}dx = \left[-\tfrac{1}{2}e^{-\frac{x^2}{b^2}}\right]_0^{0.50\,\text{nm}} = \left(-\tfrac{1}{2}e^{-0.25}\right) - \left(-\tfrac{1}{2}\right) = \boxed{0.11}$$



59. (*a*) We assume the pencil is a uniform rod, and that it makes an angle of ϕ with the vertical. If the bottom point is fixed, then the torque due to its weight about the bottom point will cause an angular acceleration. See the diagram.

$$\tau = I\alpha \ \rightarrow \ mg\left(\tfrac{1}{2}\ell\right)\sin\phi = \tfrac{1}{3}m\ell^2\frac{d^2\phi}{dt^2}$$

From the equation, if the pencil is exactly upright, so that $\phi = 0,$ then the angular acceleration will be exactly 0 and the pencil will remain stationary. But according to the uncertainty principle (as expressed at the bottom of page 1023), the angle cannot be known with 0 uncertainty. Let the *z* axis be coming out of the page.

$$\Delta L_z\Delta\phi \ge \hbar \ \rightarrow \ \Delta\phi \ge \frac{\hbar}{\Delta L_z} > 0$$

Thus the pencil cannot have exactly $\phi = 0,$ and so there will be a torque and hence rotation.

(*b*) For the initial part of the motion, the angle will be very small, and so the differential equation can be expressed as $\dfrac{3g}{2\ell}\phi = \dfrac{d^2\phi}{dt^2}.$ The solutions to this differential equation are of the form $\phi = Ae^{kt} + Be^{-kt},$ where $k = \sqrt{\dfrac{3g}{2\ell}} = \sqrt{\dfrac{3\left(9.80\,\text{m/s}^2\right)}{2\left(0.18\,\text{m}\right)}} = 9.037\,\text{s}^{-1}$ Since the angle will be increasing in time, we ignore the second term, which decreases in time. Thus $\phi \approx Ae^{kt},$ with $\phi(t=0) = \phi_0 \approx A.$ The angular velocity of the pencil is approximated as $\omega = \dfrac{d\phi}{dt} = kAe^{kt},$ and the initial angular velocity is $\omega_0 = kA.$ We take the initial position and the initial angular velocity as their smallest possible values, which are their uncertainties – the magnitude of a quantity must be at least as big as its uncertainties. Apply the uncertainty principle in angular form.

$$\Delta L_z\Delta\phi \ge \hbar \ \rightarrow \ \left(I\Delta\omega\right)\Delta\phi = \tfrac{1}{3}m\ell^2\left(kA\right)\left(A\right) = \tfrac{1}{3}m\ell^2kA^2 \ge \hbar \ \rightarrow$$

$$A \ge \sqrt{\frac{3\hbar}{m\ell^2k}} = \sqrt{\frac{3\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(7.0\times10^{-3}\,\text{kg}\right)\left(0.18\,\text{m}\right)^2\left(9.037\,\text{s}^{-1}\right)}} = 3.930\times10^{-16}\,\text{rad} = \phi_0$$

$$\omega_0 = kA = \left(9.037\,\text{s}^{-1}\right)\left(3.930\times10^{-16}\,\text{rad}\right) = 3.552\times10^{-15}\,\text{rad/s}$$

With this initial position and initial angular velocity, we can then do a numeric integration to find the time when the angle is $\phi = \pi/2\,\text{rad}.$ For a step size of 0.01 s, the time of fall is about 4.07 s. For a step size of 0.001 s, the time of fall is about 3.99 s. This is only about a 2% change in the final result, so the time is pretty stable around 4 s. Even changing the starting angle to a value 100 times bigger than that above (so $\phi_0 = 3.930\times10^{-14}\,\text{rad}$) still gives a time of fall of 3.48 s. So within a factor of 2, we estimate the time of fall as $\boxed{\text{4 seconds}}$.

Note that if the solution of the approximate differential equation is used, $\phi \approx Ae^{kt},$ we get the following time of fall.

$$t_{\text{max}} = \frac{1}{k}\ln\left(\frac{\phi_{\text{max}}}{A}\right) = \frac{1}{9.037\text{s}^{-1}}\ln\left(\frac{\pi/2}{3.930\times10^{-16}}\right) = 3.98\,\text{s}.$$

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH38.XLS,” on tab “Problem 38.59.”

60. The ground state wave function for the particle in the infinite well is $\psi = \sqrt{\frac{2}{\ell}}\sin\left(\frac{\pi}{\ell}x\right)$. Let $x_{\text{center}} = \frac{1}{2}\ell$, so the region of interest extends from $x_{\text{min}} = \frac{1}{2}\ell - \frac{1}{2}\Delta x$ to $x_{\text{max}} = \frac{1}{2}\ell + \frac{1}{2}\Delta x$. We are to find the largest value of Δx so that the approximate probability of $\left|\psi\left(x = \frac{1}{2}\ell\right)\right|^2 \Delta x = \frac{2\Delta x}{\ell}$ (from Example 38-7) is no more than 10% different than $\int_{x_{\text{min}}}^{x_{\text{max}}} \left[\sqrt{\frac{2}{\ell}}\sin\left(\frac{\pi}{\ell}x\right)\right]^2 dx$, the exact probability. We calculate the value of the integral using numeric integration (as described in Section 2-9), first finding the number of steps needed between x_{min} and x_{max} that gives a stable value. Then we compare the integral to the approximation. (Note that the integral could be evaluated exactly.) To aid in the evaluation of the integral, we make the substitution that $u = x/\ell$. Then the integral becomes as follows.

$$\int_{u_{\text{min}}=\frac{1}{2}\left(1-\frac{\Delta x}{\ell}\right)}^{u_{\text{max}}=\frac{1}{2}\left(1-\frac{\Delta x}{\ell}\right)} \left[2\sin^2(\pi u)\right]du$$

In doing the numeric integrations, we found that for any value of Δx up to ℓ, breaking the numeric integration up into 50 steps gave the same answer to 3 significant digits as breaking it up into 100 steps. So we did all numeric integrations with 50 steps. We then numerically calculated the integral for values of $\Delta x/\ell$, starting at 0.01, and increasing by steps of 0.01, until we found a 10% difference between the approximation and the numeric integration. This happens at $\Delta x/\ell = 0.34$, and so the approximation is good within 10% up to $\Delta x = 0.34\ell = 0.34(0.10\,\text{nm}) = \boxed{0.034\,\text{nm}}$. This is much broader than we might have guessed initially, indicating that the wave function is varying rather slowly over the central region of the potential well. The spreadsheet used for this problem can be found on the Media Manager, with filename"PSE4_ISM_CH38.XLS," on tab "Problem 38.60."

$\boxed{61.}$ (*a*) See the graph.

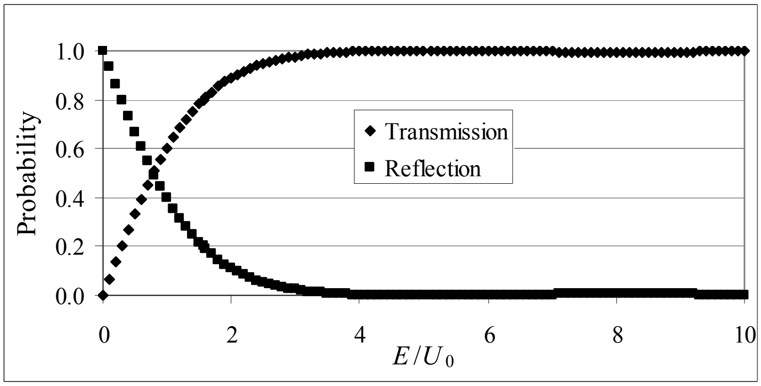
(*b*) From the graph and the spreadsheet, we find these results.

$T = 10\%$ at $E/U_0 = 0.146$

$T = 20\%$ at $E/U_0 = 0.294$

$T = 50\%$ at $E/U_0 = 0.787$

$T = 80\%$ at $E/U_0 = 1.56$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH38.XLS," on tab "Problem 38.61."

CHAPTER 39: Quantum Mechanics of Atoms

Responses to Questions

$\boxed{1.}$ The Bohr model placed electrons in definite circular orbits described by a single quantum number (*n*). The Bohr model could not explain the spectra of atoms more complex than hydrogen and could not explain fine structure in the spectra. The quantum-mechanical model uses the concept of electron "probability clouds," with the probability of finding the electron at a given position determined by the wave function. The quantum model uses four quantum numbers to describe the electron (n, ℓ, m_ℓ, m_s) and can explain the spectra of more complex atoms and fine structure.

2. The quantity $|\psi|^2$ is maximum at $r = 0$ because of its dependence on the factor e^{-r/r_0}. In the ground state, the electron is expected to be found near the nucleus. The radial probability density $4\pi r^2 |\psi|^2$ gives the probability of finding the electron in a thin spherical shell located at *r*. Since $r = 0$ is at the center of the nucleus, the radial probability density is zero here.

3. The quantum-mechanical model predicts that the electron spends more time near the nucleus. In the Bohr model, the electron in the ground state is in a fixed orbit of definite radius. The electron cannot come any closer to the nucleus than that distance. In the quantum-mechanical model, the electron is most often found at the Bohr radius, but it can also be found closer to the nucleus (and farther away).

4. As the number of electrons goes up, the number of protons in the nucleus increases, which increases the attraction of the electrons to the center of the atom. Even though the outer electrons are partially screened from the increased nuclear charge by the inner electrons, they are all pulled closer to the more positive nucleus. Also, more states are available in the upper shells to accommodate many more electrons at approximately the same radius.

5. Because the nuclei of hydrogen and helium are different, the energy levels of the atoms are different. The presence of the second electron in helium will also affect its energy levels. If the energy levels are different, then the energy difference between the levels will be different and the spectra will be different.

6. The two levels have different orbital quantum numbers. The orbital quantum number for the upper level is $\ell = 2$. This results in five different possible values of m_ℓ (–2, –1, 0, 1, and 2) so the energy level is split into five separate levels in the presence of a magnetic field. The lower level shown has an orbital quantum number of $\ell = 1$, so only three different values of m_ℓ (–1, 0 and 1) are possible, and therefore the energy level is split into only three separate levels.

$\boxed{7.}$ In the time-independent Schrödinger equation, the wave function and the potential depend on the three spatial variables. The three quantum numbers result from application of boundary conditions to the wave function.

8. The Zeeman effect is the splitting of an energy level in the presence of a magnetic field. In the reference frame of the electron, the nucleus orbits the electron. The "internal" Zeeman effect, as seen in sodium, is caused by the magnetic field produced by the "orbiting" nucleus.

9. (*a*) and (*c*) are allowed for atoms in an excited state. (*b*) is not allowed. Only six electrons are allowed in the 2*p* state.

10. The complete electron configuration for a uranium atom is as follows.
$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^3 6s^2 6p^6 6d^1 7s^2$

11. (*a*) Group II; (*b*) Group VIII; (*c*) Group I; (*d*) Group VII.

12. The periodicity of the periodic table depends on the number and arrangements of the electrons in the atom. It therefore depends on all the factors which determine this arrangement. One of these, the Pauli exclusion principle, states that no two electrons can occupy the same quantum state. The number of electrons that can be in any principle state depends on how many different substates are available, which is determined by the number of possible orbital quantum numbers for each principle state, the number of possible magnetic quantum numbers for each orbital quantum number, and finally, the number of spin orientations for each electron. Therefore, quantization of angular momentum, direction of angular momentum, and spin all play a role in the periodicity of the periodic table. (See Table 39-1 for a summary of the quantum numbers.)

13. If there were no electron spin, then, according to the Pauli exclusion principle, *s*-subshells would be filled with one electron, *p*-subshells with three electrons, and *d*-subshells with five electrons. The first 20 elements of the periodic table would look like the following:

H 1 $1s^1$								
He 2 $2s^1$						Li 3 $2p^1$	Be 4 $2p^2$	B 5 $2p^3$
C 6 $3s^1$						N 7 $3p^1$	O 8 $3p^2$	F 9 $3p^3$
Ne 10 $4s^1$	Na 11 $3d^1$	Mg 12 $3d^2$	Al 13 $3d^3$	Si 14 $3d^4$	P 15 $3d^5$	S 16 $4p^1$	Cl 17 $4p^2$	Ar 18 $4p^3$
K 19 $5s^1$	Ca 20 $4d^1$							

14. Neon is a noble gas and does not react readily with other elements. Neon has its outermost subshell completely filled, and so the electron distribution is spherically symmetric, making it harder to remove an electron. Sodium is in the first column of the periodic table and is an alkali metal. Sodium has a single outer *s* electron, which is outside the inner closed shells and shielded from the nuclear charge by the inner electrons, making it easier to remove. Therefore, neon has a higher ionization energy than sodium, even though they differ in number of protons by only one.

15. Chlorine and iodine are in the same column of the periodic table. They are each one electron away from having a complete outermost shell and will react readily with atoms having only one electron in the outermost shell. Interactions with other atoms depend largely on the outermost electrons; therefore these two elements will have similar properties because their outermost electrons are in similar configurations.

16. Potassium and sodium are in the same column of the periodic table. They each have only one electron in the outermost shell, and their inner shells are completely filled. Sodium has only one electron in the $n = 3$ shell, or principle energy level, and potassium has only one electron in the $n = 4$ level. Interactions with other atoms depend largely on the outermost electrons, and therefore these two elements will have similar properties because their outermost electrons are in similar configurations.

17. Rare earth elements have similar chemical properties because the electrons in the filled 6*s* or 7*s*-suborbitals serve as the valence electrons for all these elements. They all have partially filled inner *f*-suborbitals, which are very close together in energy. The different numbers of electrons in the *f*-suborbitals have very little effect on the chemical properties of these elements.

18. When we use the Bohr theory to calculate the X-ray line wavelengths, we estimate the nuclear charge seen by the transitioning electron as $Z - 1$, assuming that the second electron in the ground state is partially shielding the nuclear charge. This is only an estimate, so we do not expect the calculated wavelengths to agree exactly with the measured values.

$\boxed{19.}$ In helium and other complex atoms, electrons interact with other electrons in addition to their interactions with the nucleus. The Bohr theory only works well for atoms that have a single outer electron in an *s* state. X-ray emissions generally involve transitions to the 1*s* or 2*s* states. In these cases the Bohr theory can be modified to correct for screening from a second electron by using the factor $Z - 1$ for the nuclear charge and can yield good estimates of the transition energies. Transitions involving outer electrons in more complex atoms will be affected by additional complex screening effects and cannot be adequately described by the Bohr theory.

20. The continuous portion of the X-ray spectrum is due to the "bremsstrahlung" radiation. An incoming electron gives up energy in the collision and emits light. Electrons can give up all or part of their kinetic energy. The maximum amount of energy an electron can give up is its total amount of kinetic energy. In the photon description of light, the maximum electron kinetic energy will correspond to the energy of the shortest wavelength (highest energy) photons that can be produced in the collisions. The result is the existence of "cut-off" wavelength in the X-ray spectrum. An increase in the number of electrons will not change the cut-off wavelength. According to wave theory, an increase in the number of electrons could result in the production of shorter-wavelength photons, which is not observed experimentally.

21. To figure out which lines in an X-ray spectrum correspond to which transitions, you would use the Bohr model to estimate the energies of the transitions between levels and match these to the energies of the observed lines. The energies of transitions to the $n = 1$ level (K) will be the greatest, followed by the transitions to the $n = 2$ level (L). Within a level, the α line will have the lowest energy (because it corresponds to a transition between adjacent levels), followed by the β line, and so on.

22. The characteristic X-ray spectra occur when inner electrons are knocked out of their shells. X-rays are the high energy photons emitted when other electrons fall to replace the knocked-out electrons. Because the shells involved are close to the nucleus, Z will have a direct influence on the energies. The visible spectral lines due to transitions between upper levels have energies less influenced by Z because the inner electrons shield the outer electrons from the nuclear charge.

23. The difference in energy between adjacent energy levels in an atom decreases with increasing *n*. Therefore, transitions of electrons between inner energy levels will produce higher energy (shorter wavelength) photons than transitions between outer energy levels.

24. The electron has a negative charge.

$\boxed{25.}$ Consider a silver atom in its ground state for which the entire magnetic moment is due to the spin of only one of its electrons. In a uniform magnetic field, the dipole will experience a torque that would tend to align it with the field. In a non-uniform field, each pole of the dipole will experience a force of different magnitude. Consequently, the dipole will experience a net force that varies with the spatial orientation of the dipole. The Stern-Gerlach experiment provided the first evidence of space quantization, since it clearly indicated that there are two opposite spin orientations for the outermost electron in the silver atom.

26. Spontaneous emission occurs randomly when an electron in an excited state falls to a lower energy level and emits a photon. Stimulated emission also results when an electron falls to a lower energy level, but it occurs when a photon of the same energy as the transition stimulates the electron to fall sooner than it would have naturally.

27. No. The intensity of a spherical wave, which spreads out in all directions, follows the inverse-square law. A laser produces light that is very nearly a plane wave; its intensity is nearly constant with distance.

28. Laser light is monochromatic, coherent, and in a narrow beam that spreads very little if at all. Ordinary light is usually made up of many different wavelengths, incoherent, and spreads out in all directions. Both types of light can be created when electrons fall to lower energy levels and emit photons.

29. Since laser light is a plane wave, its intensity remains approximately constant with distance. The light produced by a street lamp spreads out with an intensity that decreases as 1/*r5*. Thus, at a sufficient distance, the laser light will be more intense than the light from a street lamp.

Solutions to Problems

$\boxed{1.}$ The value of ℓ can range from 0 to $n-1$. Thus for $n = 7$, $\boxed{\ell = 0, 1, 2, 3, 4, 5, 6}$.

2. The value of m_ℓ can range from $-\ell$ to $+\ell$. Thus for $\ell = 3$, $\boxed{m_\ell = -3, -2, -1, 0, 1, 2, 3}$.

   The possible values of m_s are $\boxed{-\frac{1}{2}, +\frac{1}{2}}$.

3. The value of ℓ ranges from 0 to $n-1$. Thus for $n = 3$, $\ell = 0,1,2$. For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell + 1$ values. For each m_ℓ there are 2 values of m_s. Thus the number of states for each ℓ is $2(2\ell+1)$. The number of states is $N = 2(0+1) + 2(2+1) + 2(4+1)$ $= \boxed{18\text{states}}$. We start with $\ell = 0$, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{\begin{array}{l} (3,0,0,-\tfrac{1}{2}), (3,0,0,+\tfrac{1}{2}), (3,1,-1,-\tfrac{1}{2}), (3,1,-1,+\tfrac{1}{2}), (3,1,0,-\tfrac{1}{2}), (3,1,0,+\tfrac{1}{2}), \\ (3,1,1,-\tfrac{1}{2}), (3,1,1,+\tfrac{1}{2}), (3,2,-2,-\tfrac{1}{2}), (3,2,-2,+\tfrac{1}{2}), (3,2,-1,-\tfrac{1}{2}), (3,2,-1,+\tfrac{1}{2}), \\ (3,2,0,-\tfrac{1}{2}), (3,2,0,+\tfrac{1}{2}), (3,2,1,-\tfrac{1}{2}), (3,2,1,+\tfrac{1}{2}), (3,2,2,-\tfrac{1}{2}), (3,2,2,+\tfrac{1}{2}) \end{array}}$$

4. The value of m_ℓ can range from $-\ell$ to $+\ell$, so we have $\boxed{\ell \geq 4}$.

   The value of ℓ can range from 0 to $n-1$. Thus we have $\boxed{n \geq \ell + 1 (\text{minimum} 5)}$.

   There are two values of m_s: $\boxed{m_s = -\frac{1}{2}, +\frac{1}{2}}$.

5. The value of ℓ can range from 0 to $n-1$. Thus for $\ell = 5$, we have $\boxed{n \geq 6}$.
For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}$.
There are two values of m_s: $\boxed{m_s = -\tfrac{1}{2}, +\tfrac{1}{2}.}$

6. The magnitude of the angular momentum depends only on ℓ.
$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = \sqrt{12}\left(1.055\times10^{-34}\text{ J}\bullet\text{s}\right) = \boxed{3.65\times10^{-34}\text{ J}\bullet\text{s}}$$

$\boxed{7.}$ (*a*) The principal quantum number is $n = \boxed{7}$.
(*b*) The energy of the state is
$$E_7 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{7^2} = \boxed{-0.278\,\text{eV}}.$$
(*c*) The "*g*" subshell has $\ell = \boxed{4}$. The magnitude of the angular momentum depends on ℓ only:
$$L = \hbar\sqrt{\ell(\ell+1)} = \boxed{\sqrt{20}\hbar} = \sqrt{20}\left(1.055\times10^{-34}\text{ J}\bullet\text{s}\right) = \boxed{4.72\times10^{-34}\text{ J}\bullet\text{s}}$$
(*d*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$: $\boxed{m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4}$.

8. (*a*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these there are two values of m_s. Thus the total number of states in a subshell is $N = \boxed{2(2\ell+1)}$.
(*b*) For $\ell = 0, 1, 2, 3, 4, 5$, and 6, $N = \boxed{2, 6, 10, 14, 18, 22, \text{and } 26}$, respectively.

9. For a given *n*, $0 \leq \ell \leq n-1$. Since for each ℓ the number of possible states is $2(2\ell+1)$, the number of possible states for a given *n* is as follows.
$$\sum_{\ell=0}^{n-1} 2(2\ell+1) = 4\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1} 2 = 4\left(\frac{n(n-1)}{2}\right) + 2n = \boxed{2n^2}$$

10. Photon emission means a jump to a lower state, so for the final state, $n = 1, 2, 3$, or 4. For a *d* subshell, $\ell = 2$, and because $\Delta\ell = \pm1$, the new value of ℓ must be 1 or 3.
(*a*) $\ell = 1$ corresponds to a *p* subshell, and $\ell = 3$ corresponds to an *f* subshell. Keeping in mind that $0 \leq \ell \leq n-1$, we find the following possible destination states: $2p, 3p, 4p, 4f$.
(*b*) In a hydrogen atom, ℓ has no appreciable effect on energy, and so for energy purposes there are four possible destination states, corresponding to *n* = 2, 3, and 4. Thus there are $\boxed{\text{three different photon wavelengths}}$ corresponding to three possible changes in energy.

11. We use Eq. 39-3 to find ℓ and Eq. 39-4 to find m_ℓ.
$$L = \sqrt{\ell(\ell+1)}\hbar \;\rightarrow\; \ell(\ell+1) = \frac{L^2}{\hbar^2} = \frac{\left(6.84\times10^{-34}\text{J}\bullet\text{s}\right)^2}{\left(1.055\times10^{-34}\text{ J}\bullet\text{s}\right)^2} = 42 \;\rightarrow\; \boxed{\ell = 6}$$
$$L_z = m_\ell\hbar \;\rightarrow\; m_\ell = \frac{L_z}{\hbar} = \frac{2.11\times10^{-34}\text{J}\bullet\text{s}}{1.055\times10^{-34}\text{ J}\bullet\text{s}} = \boxed{2}$$
Since $\ell = 6$, we must have $\boxed{n \geq 7}$.

12. To show that the ground-state wave function is normalized, we integrate $|\psi_{100}|^2$ over all space. Use substitution of variables and an integral from Appendix B-5.

$$\int_{\substack{\text{all}\\ \text{space}}} |\psi_{100}|^2\, dV = \int_0^\infty \frac{1}{\pi r_0^3} e^{-\frac{2r}{r_0}} 4\pi r^2 dr \ ; \quad \text{let } x = \frac{2r}{r_0} \ \rightarrow \ r = \tfrac{1}{2} r_0 x \ , \ dr = \tfrac{1}{2} r_0 dx$$

Note that if $r = 0, x = 0$ and if $r = \infty, x = \infty$.

$$\int_{\substack{\text{all}\\ \text{space}}} |\psi_{100}|^2\, dV = \int_0^\infty \frac{1}{\pi r_0^3} e^{-\frac{2r}{r_0}} 4\pi r^2 dr = \frac{4\pi}{\pi r_0^3}\int_0^\infty e^{-x}\left(\tfrac{1}{4} r_0^2 x^2\right)\left(\tfrac{1}{2} r_0 dx\right) = \tfrac{1}{2}\int_0^\infty e^{-x} x^2 dx = \tfrac{1}{2}(2!) = 1$$

And so we see that the ground-state wave function is normalized.

13. The ground state wave function is $\psi_{100} = \frac{1}{\sqrt{\pi r_0^3}} e^{-r/r_0}$.

(*a*) $\left(\psi_{100}\right)_{r=1.5r_0} = \boxed{\frac{1}{\sqrt{\pi r_0^3}} e^{-1.5}}$

(*b*) $\left(|\psi_{100}|^2\right)_{r=1.5r_0} = \boxed{\frac{1}{\pi r_0^3} e^{-3}}$

(*c*) $P_{\text{r}} = \left(4\pi r^2 |\psi_{100}|^2\right)_{r=1.5r_0} = 4\pi r_0^2\left(\frac{1}{\pi r_0^3} e^{-2}\right) = \boxed{\frac{4}{r_0} e^{-3}}$

14. The state $n = 2, \ell = 0$ must have $m_\ell = 0$ and so the wave function is $\psi_{200} = \frac{1}{\sqrt{32\pi r_0^3}}\left(2 - \frac{r}{r_0}\right) e^{-\frac{r}{2r_0}}$.

(*a*) $\left(\psi_{200}\right)_{r=4r_0} = \frac{1}{\sqrt{32\pi r_0^3}}\left(2 - \frac{4r_0}{r_0}\right) e^{-\frac{4r_0}{2r_0}} = \boxed{-\frac{1}{\sqrt{8\pi r_0^3}} e^{-2}}$

(*b*) $\left(|\psi_{200}|^2\right)_{r=4r_0} = \frac{1}{32\pi r_0^3}\left(2 - \frac{4r_0}{r_0}\right)^2 e^{-2\frac{4r_0}{2r_0}} = \boxed{\frac{1}{8\pi r_0^3} e^{-4}}$

(*c*) $P_{\text{r}} = \left(4\pi r^2 |\psi_{200}|^2\right)_{r=4r_0} = 4\pi (4r_0)^2\left(\frac{1}{8\pi r_0^3} e^{-4}\right) = \boxed{\frac{8}{r_0} e^{-4}}$

15. The factor is found from the ratio of the radial probability densities for ψ_{100}. Use Eq. 39-7.

$$\frac{P_{\text{r}}(r = r_0)}{P_{\text{r}}(r = 2r_0)} = \frac{\left(4\frac{r^2}{r_0^2} e^{-\frac{2r}{r_0}}\right)_{r=r_0}}{\left(4\frac{r^2}{r_0^2} e^{-\frac{2r}{r_0}}\right)_{r=2r_0}} = \frac{\left(4\frac{r_0^2}{r_0^2} e^{-\frac{2r_0}{r_0}}\right)_{r=r_0}}{\left(4\frac{(2r_0)^2}{r_0^2} e^{-\frac{2(2r_0)}{r_0}}\right)_{r=2r_0}} = \frac{e^{-2}}{4e^{-4}} = \boxed{\frac{e^2}{4}} \approx 1.85$$

16. (*a*) To find the probability, integrate the radial probability distribution for the ground state. We follow Example 39-4, and use the last integral in Appendix B-4.

$$\text{P} = \int_0^{r_0} 4\frac{r^2}{r_0^3} e^{-\frac{2r}{r_0}} dr \ ; \ \text{let } x = 2\frac{r}{r_0} \ \rightarrow$$

$$\mathrm{P}=\tfrac{1}{2}\int_0^2 x^2e^{-x}dx=\tfrac{1}{2}\left[-e^{-x}\left(x^2+2x+2\right)\right]_0^2=1-5e^{-2}=0.32=\boxed{32\%}$$

(*b*) We follow the same process here.

$$\mathrm{P}=\int_{r_0}^{2r_0}4\frac{r^2}{r_0^3}e^{-\frac{2r}{r_0}}dr\ ;\ \text{let } x=2\frac{r}{r_0}\ \rightarrow$$

$$\mathrm{P}=\tfrac{1}{2}\int_2^4 x^2e^{-x}dx=\tfrac{1}{2}\left[-e^{-x}\left(x^2+2x+2\right)\right]_2^4=5e^{-2}-13e^{-4}=0.44=\boxed{44\%}$$

17. To find the probability for the electron to be within a sphere of radius *r*, we must integrate the radial probability density for the ground state from 0 to *r*. The density is given in Eq. 39-7.

$$\mathrm{P}=\int_0^{r_{\text{sphere}}}4\frac{r^2}{r_0^3}e^{-\frac{2r}{r_0}}dr\ ;\ \text{let } x'=2\frac{r}{r_0}\ ;\ \text{let } 2\frac{r_{\text{sphere}}}{r_0}=x\ \rightarrow$$

$$\mathrm{P}=\tfrac{1}{2}\int_0^x x'^2e^{-x'}dx'=\tfrac{1}{2}\left[-e^{-x'}\left(x'^2+2x'+2\right)\right]_0^x=\tfrac{1}{2}\left\{\left[-e^{-x}\left(x^2+2x+2\right)\right]+2\right\}=1-e^{-x}\left(\tfrac{1}{2}x^2+x+1\right)$$

We solve this equation numerically for values of *x* that give P = 0.50, 0.90, and 0.99.

(*a*) The equation for P = 0.50 is solved by $x=2.674$, and so $r_{\text{sphere}}=\tfrac{1}{2}(2.674)r_0\approx\boxed{1.3r_0}$.

(*b*) The equation for P = 0.90 is solved by $x=5.322$, and so $r_{\text{sphere}}=\tfrac{1}{2}(5.322)r_0\approx\boxed{2.7r_0}$.

(*c*) The equation for P = 0.99 is solved by $x=8.406$, and so $r_{\text{sphere}}=\tfrac{1}{2}(8.406)r_0\approx\boxed{4.2r_0}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH39.XLS," on tab "Problem 39.17."

18. (*a*) To find the probability for the electron to be within a sphere of radius *r*, we must integrate the radial probability density for the ground state from 0 to *r*. The density is given in Eq. 39-7. Since $r\ll r_0$, we approximate $e^{-2r/r_0}\approx 1$.

$$\mathrm{P}=\int_0^{1.1\,\text{fm}}4\frac{r^2}{r_0^3}e^{-\frac{2r}{r_0}}dr\approx\int_0^{1.1\,\text{fm}}4\frac{r^2}{r_0^3}dr=\frac{4}{3}\frac{r^3}{r_0^3}=\frac{4}{3}\frac{\left(1.1\times10^{-15}\,\text{m}\right)^3}{\left(0.529\times10^{-10}\,\text{m}\right)^3}=\boxed{1.2\times10^{-14}}$$

(*b*) The Bohr radius, r_0, is inversely proportional to the mass of the particle. So now the Bohr radius is smaller by a factor of 207.

$$\mathrm{P}=\int_0^{1.1\,\text{fm}}4\frac{r^2}{r_0^3}e^{-\frac{2r}{r_0}}dr\approx\int_0^{1.1\,\text{fm}}4\frac{r^2}{r_0^3}dr=\frac{4}{3}\frac{r^3}{r_0^3}=\frac{4}{3}\frac{\left(1.1\times10^{-15}\,\text{m}\right)^3}{\left(0.529\times10^{-10}\,\text{m}/207\right)^3}=\boxed{1.1\times10^{-7}}$$

$\boxed{19.}$ We follow the directions as given in the problem. We use the first integral listed in Appendix B-5.

$$\overline{r}=\int_0^\infty r\left|\psi_{100}\right|^2 4\pi r^2dr=\int_0^\infty r\frac{1}{\pi r_0^3}e^{-2\frac{r}{r_0}}4\pi r^2dr=4\int_0^\infty\frac{r^3}{r_0^3}e^{-2\frac{r}{r_0}}dr\ ;\ \text{let } x=2\frac{r}{r_0}\ \rightarrow$$

$$\overline{r}=\tfrac{1}{4}r_0\int_0^\infty x^3e^{-x}dx=\tfrac{1}{4}r_0\left(3!\right)=\boxed{\tfrac{3}{2}r_0}$$

20. To show that ψ_{200} is normalized, we integrate $|\psi_{200}|^2$ over all space. Use substitution of variables and an integral from Appendix B-5.

$$\int_{\substack{\text{all}\\\text{space}}} |\psi_{200}|^2\, dV = \int_0^\infty \frac{1}{32\pi r_0^3}\left(2-\frac{r}{r_0}\right) e^{-\frac{r}{r_0}} 4\pi r^2 dr\ ; \qquad \text{let } x = \frac{r}{r_0} \ \rightarrow\ r = r_0 x\ ,\ dr = r_0 dx$$

Note that if $r = 0, x = 0$ and if $r = \infty, x = \infty$.

$$\int_{\substack{\text{all}\\\text{space}}} |\psi_{200}|^2\, dV = \int_0^\infty \frac{1}{32\pi r_0^3}\left(2-\frac{r}{r_0}\right)^2 e^{-\frac{r}{r_0}} 4\pi r^2 dr = \frac{4\pi}{32\pi r_0^3}\int_0^\infty (2-x)^2 e^{-x} r_0^2 x^2 r_0 dx$$

$$= \frac{1}{8}\int_0^\infty (2-x)^2 e^{-x} x^2 dx = \frac{1}{8}\int_0^\infty \left(4x^2 - 4x^3 + x^4\right) e^{-x} dx = \frac{1}{2}\int_0^\infty x^2 e^{-x} dx - \frac{1}{2}\int_0^\infty x^3 e^{-x} dx + \frac{1}{8}\int_0^\infty x^4 e^{-x} dx$$

$$= \frac{1}{2}(2!) - \frac{1}{2}(3!) + \frac{1}{8}(4!) = 1 - 3 + 3 = 1$$

And so we see that ψ_{200} is normalized.

21. We follow the directions as given in the problem. The three wave functions are given in Eq. 39-9. We explicitly show the expressions involving the complex conjugate.

$$P_{\mathrm{r}} = 4\pi r^2\left[\tfrac{1}{3}|\psi_{210}|^2 + \tfrac{1}{3}|\psi_{211}|^2 + \tfrac{1}{3}|\psi_{21-1}|^2\right]$$

$$= 4\pi r^2\left[\tfrac{1}{3}\left(\frac{z^2}{32\pi r_0^5} e^{-\frac{r}{r_0}}\right) + \tfrac{1}{3}\left(\frac{(x+iy)(x-iy)}{64\pi r_0^5} e^{-\frac{r}{r_0}}\right) + \tfrac{1}{3}\left(\frac{(x+iy)(x-iy)}{64\pi r_0^5} e^{-\frac{r}{r_0}}\right)\right]$$

$$= 4\pi r^2\left[\tfrac{1}{3}\frac{z^2}{32\pi r_0^5} e^{-\frac{r}{r_0}} + \tfrac{1}{3}\frac{\left(x^2+y^2\right)}{64\pi r_0^5} e^{-\frac{r}{r_0}} + \tfrac{1}{3}\frac{\left(x^2+y^2\right)}{64\pi r_0^5} e^{-\frac{r}{r_0}}\right] = \frac{r^2}{24 r_0^5}\left(z^2 + x^2 + y^2\right) e^{-\frac{r}{r_0}}$$

$$= \boxed{\frac{r^4}{24 r_0^5} e^{-\frac{r}{r_0}}}$$

22. From Problem 21, we have that $P_{\mathrm{r}} = \frac{r^4}{24 r_0^5} e^{-\frac{r}{r_0}}$ for the 2*p* state. We find the most probable distance by setting $\frac{dP_{\mathrm{r}}}{dr} = 0$ and solving for *r*. This is very similar to Example 39-3.

$$P_{\mathrm{r}} = \frac{r^4}{24 r_0^5} e^{-\frac{r}{r_0}}\ ;\ \ \frac{dP_{\mathrm{r}}}{dr} = \frac{4r^3}{24 r_0^5} e^{-\frac{r}{r_0}} - \frac{1}{r_0}\frac{r^4}{24 r_0^5} e^{-\frac{r}{r_0}} = (4r_0 - r)\frac{r^3 e^{-\frac{r}{r_0}}}{r_0^6 24} = 0 \ \rightarrow\ \boxed{r = 4r_0}$$

23. The probability is found by integrating the radial probability density over the range of radii given.

$$\mathrm{P} = \int_{0.99 r_0}^{1.01 r_0} 4\frac{r^2}{r_0^3} e^{-\frac{2r}{r_0}} dr\ ;\ \text{let } x = 2\frac{r}{r_0} \ \rightarrow$$

$$\mathrm{P} = \tfrac{1}{2}\int_{1.98}^{2.02} x^2 e^{-x} dx = \tfrac{1}{2}\left[-e^{-x}\left(x^2 + 2x + 2\right)\right]_{1.98}^{2.02} = e^{-1.98}(4.9402) - e^{-2.02}(5.0602) = 0.0108 \approx \boxed{1.1\%}$$

Because the range of radii is small and the radial probability density is relatively constant over that range (see Figure 39-7), we can approximate the probability as follows.

$$\text{P} = \int_{0.99r_0}^{1.01r_0} 4\frac{r^2}{r_0^3} e^{-\frac{2r}{r_0}} dr \approx P(r = r_0)\Delta r = 4\frac{r_0^2}{r_0^3} e^{-\frac{2r_0}{r_0}} (0.02r_0) = 0.08e^{-2} = 0.0108 \approx \boxed{1.1\%}$$

24. The probability is found by integrating the radial probability density over the range of radii given. The radial probability density is given after Eq. 39-8.

$$\text{P} = \int_{4.00r_0}^{5.00r_0} \frac{r^2}{8r_0^3}\left(2 - \frac{r}{r_0}\right)^2 e^{-\frac{r}{r_0}} dr \text{ ; let } x = \frac{r}{r_0} \quad \rightarrow$$

$$\text{P} = \tfrac{1}{8}\int_{4.00}^{5.00} x^2 (2 - x)^2 e^{-x} dx = \tfrac{1}{8}\int_{4.00}^{5.00} \left(x^4 - 4x^3 + 4x^2\right) e^{-x} dx$$

There are some difficult integrals to evaluate. We use integration by parts.

$$\int x^2 e^{-x} dx: \quad u = x^2;\ dv = e^{-x};\ du = 2x dx;\ v = -e^{-x}$$

$$\int x^2 e^{-x} dx = -x^2 e^{-x} + \int 2x e^{-x} dx = -x^2 e^{-x} + 2\left(-e^{-x}\right)(x+1) = -e^{-x}\left(x^2 + 2x + 2\right)$$

$$\int x^3 e^{-x} dx: \quad u = x^3;\ dv = e^{-x};\ du = 3x^2 dx;\ v = -e^{-x}$$

$$\int x^3 e^{-x} dx = -x^3 e^{-x} + 3\int x^2 e^{-x} dx = -e^{-x}\left(x^3 + 3x^2 + 6x + 6\right)$$

$$\int x^4 e^{-x} dx: \quad u = x^4;\ dv = e^{-x};\ du = 4x^3 dx;\ v = -e^{-x}$$

$$\int x^4 e^{-x} dx = -x^4 e^{-x} + 4\int x^3 e^{-x} dx = -e^{-x}\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right)$$

We substitute the integrals above into the expression for the probability. We are not showing the algebra.

$$\text{P} = -\tfrac{1}{8}\left[e^{-x}\left(x^4 + 4x^2 + 8x + 8\right)\right]_{4.00}^{5.00} = -\tfrac{1}{8}\left(773e^{-5} - 360e^{-4}\right) = 0.173 = \boxed{17.3\%}$$

$\boxed{25.}$ The wave function is given in Eq. 39-5a. Note that $r = \left(x^2 + y^2 + z^2\right)^{1/2}$. We will need the derivative relationship derived in the first line below.

$$\frac{\partial r}{\partial x} = \tfrac{1}{2}\left(x^2 + y^2 + z^2\right)2x = \frac{x}{r} \ ; \ \psi_{100} = \frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}} \ ; \ \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial r}\frac{\partial r}{\partial x} = -\frac{1}{r_0}\frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}} \frac{x}{r}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{1}{r_0}\frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}} \frac{x}{r}\right) = \left(-\frac{1}{r_0}\frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}} \frac{1}{r}\right) + \frac{\partial}{\partial r}\left(-\frac{1}{r_0}\frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}} \frac{x}{r}\right)\frac{\partial r}{\partial x}$$

$$= \frac{-1}{rr_0}\frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}}\left[1 - x^2\left(\frac{1}{rr_0} + \frac{1}{r^2}\right)\right] = -\psi\frac{1}{rr_0}\left[1 - x^2\left(\frac{1}{rr_0} + \frac{1}{r^2}\right)\right]$$

Similarly, we would have $\dfrac{\partial r}{\partial y} = \dfrac{y}{r}$; $\dfrac{\partial^2 \psi}{\partial y^2} = -\psi\dfrac{1}{rr_0}\left[1 - y^2\left(\dfrac{1}{rr_0} + \dfrac{1}{r^2}\right)\right]$; $\dfrac{\partial r}{\partial z} = \dfrac{z}{r}$; and

$\dfrac{\partial^2 \psi}{\partial z^2} = -\psi\dfrac{1}{rr_0}\left[1 - z^2\left(\dfrac{1}{rr_0} + \dfrac{1}{r^2}\right)\right]$. Substitute into the time-independent Schrödinger equation.

$$E\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x^2}\right) - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\psi$$

$$= \left(-\frac{\hbar^2}{2m}\right)\left(-\psi\frac{1}{rr_0}\right)\left\{\left[1 - x^2\left(\frac{1}{rr_0} + \frac{1}{r^2}\right)\right] + \left[1 - y^2\left(\frac{1}{rr_0} + \frac{1}{r^2}\right)\right] + \left[1 - z^2\left(\frac{1}{rr_0} + \frac{1}{r^2}\right)\right]\right\} - \frac{e^2}{4\pi\varepsilon_0 r}\psi$$

$$=\psi\left[\frac{\hbar^2}{2m}\frac{1}{rr_0}\left\{3-\left(x^2+y^2+z^2\right)\left(\frac{1}{rr_0}+\frac{1}{r^2}\right)\right\}-\frac{e^2}{4\pi\varepsilon_0 r}\right]$$

$$=\psi\left[\frac{\hbar^2}{2m}\frac{1}{rr_0}\left\{3-r^2\left(\frac{1}{rr_0}+\frac{1}{r^2}\right)\right\}-\frac{e^2}{4\pi\varepsilon_0 r}\right]=\psi\left[\frac{\hbar^2}{2m}\frac{1}{rr_0}\left(2-\frac{r^2}{rr_0}\right)-\frac{e^2}{4\pi\varepsilon_0 r}\right]$$

$$=\psi\left[\frac{\hbar^2}{mrr_0}-\frac{\hbar^2}{2mr_0^2}-\frac{e^2}{4\pi\varepsilon_0 r}\right]$$

Since the factor in square brackets must be a constant, the terms with the *r* dependence must cancel.

$$\frac{\hbar^2}{mrr_0}-\frac{e^2}{4\pi\varepsilon_0 r}=0 \ \rightarrow\ r_0=\frac{4\pi\varepsilon_0\hbar^2}{me^2}=\frac{\varepsilon_0 h^2}{\pi me^2}$$

Note from Equation 37-11 that this expression for r_0 is the same as the Bohr radius. Since those two terms cancel, we are left with the following.

$$E\psi=-\frac{\hbar^2}{2mr_0^2}\psi \ \rightarrow\ E=-\frac{\hbar^2}{2mr_0^2}=-\frac{\hbar^2}{2m\left(\frac{\varepsilon_0 h^2}{\pi me^2}\right)^2}=\boxed{-\frac{me^4}{8h^2\varepsilon_0^2}}$$

26. (*a*) The probability is found by integrating the radial probability density over the range of radii given. The radial probability density is given after Eq. 39-8.

$$\mathrm{P}=\int_0^{r_0}\frac{r^2}{8r_0^3}\left(2-\frac{r}{r_0}\right)^2 e^{-\frac{r}{r_0}}dr \ ;\ \text{let } x=\frac{r}{r_0} \ \rightarrow$$

$$\mathrm{P}=\tfrac{1}{8}\int_0^1 x^2\left(2-x\right)^2 e^{-x}dx=\tfrac{1}{8}\int_0^1\left(x^4-4x^3+4x^2\right)e^{-x}dx$$

The following integrals are derived in the solution to Problem 24.

$$\int x^2e^{-x}dx=-e^{-x}\left(x^2+2x+2\right) \ ;\ \int x^3e^{-x}dx=-e^{-x}\left(x^3+3x^2+6x+6\right)$$

$$\int x^4e^{-x}dx=-e^{-x}\left(x^4+4x^3+12x^2+24x+24\right)$$

$$\mathrm{P}=\tfrac{1}{8}\int_0^1\left(x^4-4x^3+4x^2\right)e^{-x}dx$$

$$=\tfrac{1}{8}\left\{-e^{-x}\left[\left(x^4+4x^3+12x^2+24x+24\right)-4\left(x^3+3x^2+6x+6\right)+4\left(x^2+2x+2\right)\right]\right\}_0^1$$

$$=\tfrac{1}{8}\left[e^{-x}\left(x^4+4x^2+8x+8\right)\right]_1^0=\tfrac{1}{8}\left(8-21e^{-1}\right)=0.0343\approx\boxed{3.4\%}$$

(*b*) From Problem 21, we have that the radial probability density for this state is $P_{\mathrm{r}}=\frac{r^4}{24r_0^5}e^{-\frac{r}{r_0}}$. We proceed as in part (*a*).

$$\mathrm{P}=\int_0^{r_0}\frac{r^4}{24r_0^5}e^{-\frac{r}{r_0}}dr \ ;\ \text{let } x=\frac{r}{r_0} \ \rightarrow$$

$$\mathrm{P}=\tfrac{1}{24}\int_0^1 x^4e^{-x}dx=\tfrac{1}{24}\left[-e^{-x}\left(x^4+4x^3+12x^2+24x+24\right)\right]_0^1=\tfrac{1}{24}\left(24-65e^{-1}\right)=3.66\times10^{-3}$$

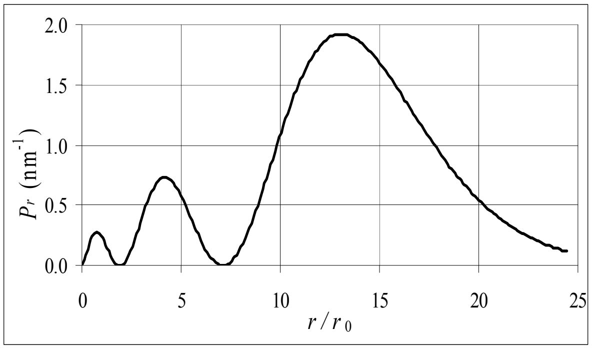
$$\approx\boxed{0.37\%}$$

27. (*a*) The radial probability distribution is given by Eq. 39-6. Use the wave function given.

$$P_{\mathrm{r}} = 4\pi r^2 \left|\psi_{300}\right|^2 = 4\pi r^2 \frac{1}{27\pi r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)^2 e^{-\frac{2r}{3r_0}} = \boxed{\frac{4r^2}{27r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)^2 e^{-\frac{2r}{3r_0}}}$$

(*b*) See the graph.

(*c*) The most probable distance is the radius for which the radial probability distribution has a global maximum. We find that location by setting $\frac{dP_{\mathrm{r}}}{dr}=0$ and solving for *r*. We see from the graph that the global maximum is approximately at $r = 13r_0$.



$$\frac{dP_{\mathrm{r}}}{dr} = \frac{8r}{27r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)^2 e^{-\frac{2r}{3r_0}} + \frac{8r^2}{27r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)\left(-\frac{2}{3r_0}+\frac{4r}{27r_0^2}\right)e^{-\frac{2r}{3r_0}}$$

$$+\frac{4r^2}{27r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)^2\left(-\frac{2}{3r_0}\right)e^{-\frac{2r}{3r_0}}$$

$$= \frac{8r}{27r_0^3}\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)\left(1-\frac{5r}{3r_0}+\frac{12r^2}{27r_0^2}-\frac{2r^3}{81r_0^3}\right)e^{-\frac{2r}{3r_0}} = 0$$

The above system has 6 non-infinite solutions. One solution is *r* = 0, which leads to $P_{\mathrm{r}} = 0$, which is not a maximum for the radial distribution. The second-order polynomial, $\left(1-\frac{2r}{3r_0}+\frac{2r^2}{27r_0^2}\right)$, is a factor of the radial probability distribution, and so its zeros also give locations where $P_{\mathrm{r}} = 0$. So the maxima must be found from the roots of the third-order polynomial. A spreadsheet was used to find the roots of $1-\frac{5}{3}x+\frac{12}{27}x^2-\frac{2}{81}x^3=0$. Those roots are $x = 0.74$, 4.19, and 13.07. So the most probable distance is $r = \boxed{13.1r_0}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH39.XLS," on tab "Problem 39.27."

28. For oxygen, *Z* = 8. We start with the $n = 1$ shell, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s).

$$\boxed{\begin{array}{l}\left(1,0,0,-\tfrac{1}{2}\right),\left(1,0,0,+\tfrac{1}{2}\right),\left(2,0,0,-\tfrac{1}{2}\right),\left(2,0,0,+\tfrac{1}{2}\right),\\ \left(2,1,-1,-\tfrac{1}{2}\right),\left(2,1,-1,+\tfrac{1}{2}\right),\left(2,1,0,-\tfrac{1}{2}\right),\left(2,1,0,+\tfrac{1}{2}\right)\end{array}}$$

Note that, without additional information, there are two other possibilities that could substitute for any of the last four electrons.

29. (*a*) For carbon, *Z* = 6. We start with the $n = 1$ shell, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s).

$$\boxed{\left(1,0,0,-\tfrac{1}{2}\right),\left(1,0,0,+\tfrac{1}{2}\right),\left(2,0,0,-\tfrac{1}{2}\right),\left(2,0,0,+\tfrac{1}{2}\right),\left(2,1,-1,-\tfrac{1}{2}\right),\left(2,1,-1,+\tfrac{1}{2}\right)}$$

Note that, without additional information, there are other possibilities for the last two electrons.

(*b*) For aluminum, $Z = 13$. We start with the $n=1$ shell, and list the quantum numbers in the order (n,ℓ,m_ℓ,m_s).

$$\boxed{\begin{array}{l}(1,0,0,-\tfrac{1}{2}),(1,0,0,+\tfrac{1}{2}),(2,0,0,-\tfrac{1}{2}),(2,0,0,+\tfrac{1}{2}),(2,1,-1,-\tfrac{1}{2}),(2,1,-1,+\tfrac{1}{2}),\\(2,1,0,-\tfrac{1}{2}),(2,1,0,+\tfrac{1}{2}),(2,1,1,-\tfrac{1}{2}),(2,1,1,+\tfrac{1}{2}),(3,0,0,-\tfrac{1}{2}),(3,0,0,+\tfrac{1}{2}),(3,1,-1,-\tfrac{1}{2})\end{array}}$$

Note that, without additional information, there are other possibilities for the last electron.

30. The number of electrons in the subshell is determined by the value of ℓ. For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$, which is $2\ell+1$ values. For each m_ℓ value there are two values of m_s. Thus the total number of states for a given ℓ is $N = 2(2\ell+1)$.

$$N = 2(2\ell+1) = 2\left[2(4)+1\right] = \boxed{18\text{ electrons}}$$

$\boxed{31.}$ Since the electron is in its lowest energy state, we must have the lowest possible value of *n*. Since $m_\ell = 2$, the smallest possible value of ℓ is $\boxed{\ell = 2}$, and the smallest possible value of *n* is $\boxed{n=3}$.

32. Limiting the number of electron shells to six would mean that the periodic table stops with radon (Rn), since the next element, francium (Fr), begins filling the seventh shell. Including all elements up through radon means $\boxed{86}$ elements.

33. (*a*) Nickel has $Z = 28$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2}$$

(*b*) Silver has $Z = 47$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^1}$$

(*c*) Uranium has $Z = 92$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6 5f^3 6d^1 7s^2}$$

34. The third electron in lithium is in the 2*s* subshell, which is outside the more tightly bound filled 1*s* shell. This makes it appear as if there is a "nucleus" with a net charge of $+1\text{e}$. Thus we use the energy of the hydrogen atom.

$$E_2 = -\frac{(13.6\,\text{eV})}{n^2} = -\frac{(13.6\,\text{eV})}{2^2} = -3.4\,\text{eV}$$

We predict the binding energy to be $\boxed{3.4\text{ eV}}$. Our assumption of complete shielding of the nucleus by the 2*s* electrons is probably not correct. The partial shielding means the net charge of the "nucleus" is higher than +1e, and so it holds the outer electron more tightly, requiring more energy to remove it.

35. We use Eq. 37-13, which says that the radius of a Bohr orbit is inversely proportional to the atomic number. We also use Eq. 37-14b, which says that the energy of Bohr orbit is proportional to the square of the atomic number. The energy to remove the electron is the opposite of the total energy.

$$r_n = \frac{n^2}{Z}\left(0.529\times10^{-10}\,\text{m}\right) = \frac{1}{92}\left(0.529\times10^{-10}\,\text{m}\right) = \boxed{5.75\times10^{-13}\,\text{m}}$$

$$|E_n| = (13.6\,\text{eV})\frac{Z^2}{n^2} = (13.6\,\text{eV})\frac{92^2}{1^2} = \boxed{1.15\times10^5\,\text{eV}}$$

36. The energy levels of the infinite square well are given in Eq. 38-13. Each energy level can have a maximum of two electrons, since the only quantum numbers are n and m_s. Thus the lowest energy level will have two electrons in the $n = 1$ state, two electrons in the $n = 2$ state, and 1 electron in the $n = 3$ state.

$$E = 2E_1 + 2E_2 + E_3 = \left[2(1)^2 + 2(2^2) + 1(3)^2\right]\frac{h^2}{8m\ell^2} = \boxed{19\frac{h^2}{8m\ell^2}}$$

$\boxed{37.}$ In a filled subshell, there are an even number of electrons. All of the possible quantum number combinations for electrons in that subshell represent an electron that is present. Thus for every m_ℓ value, both values of m_s are filled, representing a spin "up" state and a spin "down" state. The total angular momentum of that pair is zero, and since all of the electrons are paired, the total angular momentum is $\boxed{\text{zero.}}$

38. The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(0.027\times10^{-9}\,\text{m})} = 4.6\times10^{4}\,\text{eV} = 46\,\text{keV}$$

Thus the operating voltage of the tube is $\boxed{46\,\text{kV}}$.

39. The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(32.5\times10^{3}\,\text{eV})} = 3.825\times10^{-11}\,\text{m} = \boxed{0.0383\,\text{nm}}$$

The longest wavelength of the continuous spectrum would be at the limit of the X-ray region of the electromagnetic spectrum, generally on the order of $\boxed{\text{1 nm.}}$

40. The energy of the photon with the shortest wavelength must equal the maximum kinetic energy of an electron. We assume V is in volts.

$$E = hf_0 = \frac{hc}{\lambda_0} = eV \quad \rightarrow$$

$$\lambda_0 = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})(10^{9}\,\text{nm/m})}{(1.60\times10^{-19}\,\text{C})(V\,\text{V})} = \frac{1243\,\text{nm}}{V} \approx \boxed{\frac{1240\,\text{nm}}{V}}$$

41. With the shielding provided by the remaining $n = 1$ electron, we use the energies of the hydrogen atom with Z replaced by $Z - 1$. The energy of the photon is found, and then the wavelength.

$$hf = \Delta E = -(13.6\,\text{eV})(26-1)^2\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{1^2}\right)\right] = 6.40\times10^{3}\,\text{eV}.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(6.40\times10^{3}\,\text{eV})} = 1.94\times10^{-10}\,\text{m} = \boxed{0.194\,\text{nm}}$$

42. We follow the procedure of Example 39-6, of using the Bohr formula, Eq. 37-15, with Z replaced by $Z-1$.

$$\frac{1}{\lambda}=\left(\frac{e^4m}{8\varepsilon_0^2h^3c}\right)(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)=\left(1.097\times10^7\,\text{m}^{-1}\right)(27-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=5.562\times10^9\,\text{m}^{-1}\rightarrow$$

$$\lambda=\frac{1}{5.562\times10^9\,\text{m}^{-1}}=\boxed{1.798\times10^{-10}\,\text{m}}$$

$\boxed{43.}$ The wavelength of the K_α line is calculated for molybdenum in Example 39-6. We use that same procedure. Note that the wavelength is inversely proportional to $(Z-1)^2$.

$$\frac{\lambda_{\text{unknown}}}{\lambda_{\text{Fe}}}=\frac{(Z_{\text{Fe}}-1)^2}{(Z_{\text{unknown}}-1)^2}\rightarrow Z_{\text{unknown}}=\left[(26-1)\sqrt{\frac{194\,\text{pm}}{229\,\text{pm}}}\right]+1=24$$

The unknown material has Z = 24, and so is $\boxed{\text{chromium}}$.

44. We assume that there is "shielding" provided by the 1*s* electron that is already at that level. Thus the effective charge "seen" by the transitioning electron is 42 – 1 = 41. We use Eqs. 37-9 and 37-14b.

$$hf=\Delta E=(13.6\,\text{eV})(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)$$

$$\lambda=\frac{hc}{\Delta E}=\frac{hc}{(13.6\,\text{eV})(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{(13.6\,\text{eV})\left(41^2\right)\left(\frac{1}{1^2}-\frac{1}{3^2}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}$$

$$=6.12\times10^{-11}\,\text{m}=\boxed{0.0612\,\text{nm}}$$

We do not expect perfect agreement because there is some $\boxed{\text{partial shielding provided by the } n=2 \text{ shell,}}$ which was ignored when we replaced Z by $Z-1$. That would make the effective atomic number a little smaller, which would lead to a larger wavelength. The amount of shielding could be estimated by using the actual wavelength and solving for the effective atomic number.

45. Momentum and energy will be conserved in any inertial reference frame. Consider the frame of reference that is moving with the same velocity as the electron's initial velocity. In that frame of reference, the initial momentum of the electron is 0, and its initial total energy is mc^2. Let the emitted photon have frequency f, and let the direction of motion of that photon be considered the positive direction. The momentum of the photon is then $p_\gamma=\frac{h}{\lambda}=\frac{hf}{c}$, and so the momentum of the electron must be $p_e=-\frac{hf}{c}$. The final energy of the photon is $E_\gamma=hf=p_\gamma c$, and the final energy of the electron is, from Eq. 36-13, $E_{\substack{\text{electron}\\ \text{final}}}=\sqrt{p_\text{e}^2c^2+m^2c^4}$. We write the conservation conditions, and then solve for the frequency of the emitted photon.

Momentum: $0=\frac{hf}{c}+p_\text{e}\rightarrow p_\text{e}=-\frac{hf}{c}$

Energy:
$$mc^2=hf+\sqrt{p_\text{e}^2c^2+m^2c^4}$$
$$mc^2-hf=\sqrt{\left(\frac{hf}{c}\right)^2c^2+m^2c^2}=\sqrt{h^2f^2+m^2c^4}$$
$$m^2c^4-2mc^2hf+h^2f^2=h^2f^2+m^2c^4\rightarrow 2mc^2hf=0\rightarrow f=0$$

Since the photon must have $f = 0$, no photon can be emitted and still satisfy the conservation laws.

Another way to consider this situation is that if an electron at rest emits a photon, the energy of the electron must decrease for energy to be conserved. But the energy of a stationary electron cannot decrease, unless its mass were to change. Then it would no longer be an electron.

So we conclude that a third object (with mass) must be present in order for both energy and momentum to be conserved.

46. The Bohr magneton is given by Eq. 39-12.

$$\mu_B = \frac{e\hbar}{2m} = \frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(1.054\times10^{-34}\,\text{J}\bullet\text{s}\right)}{2\left(9.109\times10^{-31}\text{kg}\right)} = \boxed{9.27\times10^{-24}\,\text{J/T}}$$

47. We use Eq. 39-14 for the magnetic moment, since the question concerns spin angular momentum. The energy difference is the difference in the potential energies of the two spin states.

$$\Delta U = \left(\mu_z B\right)_{\text{spin up}}^{\text{spin down}} = -g\mu_B B\Delta m_s = -\left(2.0023\right)\frac{\left(9.27\times10^{-24}\,\text{J/T}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}\left(2.5\text{T}\right)\left(-\tfrac{1}{2}-\tfrac{1}{2}\right) = \boxed{2.9\times10^{-4}\text{eV}}$$

48. (*a*) The energy difference is the difference in the potential energies of the two spin states. Use Eq. 39-14 for the magnetic moment.

$$\Delta U = \left(\mu_z B\right)_{\text{spin up}}^{\text{spin down}} = -g\mu_B B\Delta m_s = -\left(2.0023\right)\frac{\left(9.27\times10^{-24}\,\text{J/T}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}\left(1.0\text{T}\right)\left(-\tfrac{1}{2}-\tfrac{1}{2}\right)$$

$$= 1.160\times10^{-4}\text{eV} \approx \boxed{1.2\times10^{-4}\text{eV}}$$

(*b*) Calculate the wavelength associated with this energy change.

$$\Delta U = E = h\frac{c}{\lambda} \quad \rightarrow$$

$$\lambda = \frac{hc}{\Delta U} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.160\times10^{-4}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.072\times10^{-2}\text{m} \approx \boxed{1.1\text{cm}}$$

(*c*) The answer would be no different for hydrogen. The splitting for both atoms is due to an *s*-state electron: 1*s* for hydrogen, 5*s* for silver. See the discussion on page 1058 concerning the Stern-Gerlach experiment.

$\boxed{49.}$ (*a*) Refer to Figure 39-14 and the equation following it. A constant magnetic field gradient will produce a constant force on the silver atoms. Atoms with the valence electron in one of the spin states will experience an upward force, and atoms with the valence electron in the opposite spin state will experience a downward force. That constant force will produce a constant acceleration, leading to the deflection from the original direction of the atoms as they leave the oven. We assume the initial direction of the atoms is the *x* direction, and the magnetic field gradient is in the *z* direction. If undeflected, that atoms would hit the screen at $z = 0$.

$$z = \tfrac{1}{2}at^2 = \tfrac{1}{2}\frac{F}{m_{\text{Ag}}}\left(\frac{\Delta x}{v}\right)^2 = \tfrac{1}{2}\frac{\mu_z\frac{dB_z}{dz}}{m_{\text{Ag}}}\left(\frac{\Delta x}{v}\right)^2 = \tfrac{1}{2}\frac{\left(-g\mu_B m_s\right)\frac{dB_z}{dz}}{m_{\text{Ag}}}\left(\frac{\Delta x}{v}\right)^2$$

One beam is deflected up, and the other down. There separation is the difference in the two deflections due to the two spin states.

$$\Delta z = z_{m_s=-\frac{1}{2}} - z_{m_s=\frac{1}{2}} = \tfrac{1}{2}\frac{\left[-g\mu_B\left(-\frac{1}{2}-\frac{1}{2}\right)\right]\dfrac{dB_z}{dz}}{m_{\text{Ag}}}\left(\frac{\Delta x}{v}\right)^2$$

$$= \tfrac{1}{2}\frac{(2.0023)\left(9.27\times10^{-24}\,\text{J/T}\right)(1800\,\text{T/m})}{(107.87\,\text{u})\left(1.66\times10^{-27}\,\text{kg/u}\right)}\left(\frac{0.050\,\text{m}}{780\,\text{m/s}}\right)^2 = 3.833\times10^{-4}\,\text{m} \approx \boxed{0.38\,\text{mm}}$$

(*b*) The separation is seen in the above equation to be proportional to the *g*-factor. So to find the new deflection, divide the answer to part (a) by the original *g*-factor.

$$\Delta z_{g=1} = \frac{3.833\times10^{-4}\,\text{m}}{2.0023} \approx \boxed{0.19\,\text{mm}}$$

50. For the 5*g* state, $\ell = 4$ and $s = \frac{1}{2}$. Thus the possible values of *j* are $j = \ell \pm s = 4 \pm \frac{1}{2} = \boxed{\frac{7}{2}, \frac{9}{2}}$.

Let $j = \frac{7}{2}$. Then we have the following.

$$\boxed{\begin{array}{l} m_j = -\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \qquad J = \sqrt{\frac{7}{2}\left(\frac{7}{2}+1\right)}\hbar = \dfrac{3\sqrt{7}}{2}\hbar \\ J_z = m_j\hbar = -\frac{7}{2}\hbar, -\frac{5}{2}\hbar, -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \frac{7}{2}\hbar \end{array}}$$

Let $j = \frac{9}{2}$. Then we have the following.

$$\boxed{\begin{array}{l} m_j = -\frac{9}{2}, -\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2} \qquad J = \sqrt{\frac{9}{2}\left(\frac{9}{2}+1\right)}\hbar = \dfrac{3\sqrt{11}}{2}\hbar \\ J_z = m_j\hbar = -\frac{9}{2}\hbar, -\frac{7}{2}\hbar, -\frac{5}{2}\hbar, -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \frac{7}{2}\hbar, \frac{9}{2}\hbar \end{array}}$$

51. (*a*) For the 4*p* state, $\ell = 1$. Since $s = \frac{1}{2}$, the possible values for *j* are $j = \ell + s = \boxed{\frac{3}{2}}$ and $j = \ell - s = \boxed{\frac{1}{2}}$.

(*b*) For the 4*f* state, $\ell = 3$. Since $s = \frac{1}{2}$, the possible values for *j* are $j = \ell + s = \boxed{\frac{7}{2}}$ and $j = \ell - s = \boxed{\frac{5}{2}}$.

(*c*) For the 3*d* state, $\ell = 2$. Since $s = \frac{1}{2}$, the possible values for *j* are $j = \ell + s = \boxed{\frac{5}{2}}$ and $j = \ell - s = \boxed{\frac{3}{2}}$.

(*d*) The values of *J* are found from Eq. 39-15.

$$4p:\ J = \sqrt{j(j+1)}\hbar = \boxed{\frac{\sqrt{15}}{2}\hbar \text{ and } \frac{\sqrt{3}}{2}\hbar}$$

$$4f:\ J = \sqrt{j(j+1)}\hbar = \boxed{\frac{\sqrt{63}}{2}\hbar \text{ and } \frac{\sqrt{35}}{2}\hbar}$$

$$3d:\ J = \sqrt{j(j+1)}\hbar = \boxed{\frac{\sqrt{35}}{2}\hbar \text{ and } \frac{\sqrt{15}}{2}\hbar}$$

52. (*a*) Gallium has *Z* = 31. We list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{\begin{array}{l} \left(1,0,0,-\frac{1}{2}\right), \left(1,0,0,+\frac{1}{2}\right), \left(2,0,0,-\frac{1}{2}\right), \left(2,0,0,+\frac{1}{2}\right), \left(2,1,-1,-\frac{1}{2}\right), \left(2,1,-1,+\frac{1}{2}\right), \\ \left(2,1,0,-\frac{1}{2}\right), \left(2,1,0,+\frac{1}{2}\right), \left(2,1,1,-\frac{1}{2}\right), \left(2,1,1,+\frac{1}{2}\right), \left(3,0,0,-\frac{1}{2}\right), \left(3,0,0,+\frac{1}{2}\right), \end{array}}$$

$$\boxed{\begin{array}{l}\left(3,1,-1,-\tfrac{1}{2}\right),\left(3,1,-1,+\tfrac{1}{2}\right),\left(3,1,0,-\tfrac{1}{2}\right),\left(3,1,0,+\tfrac{1}{2}\right),\left(3,1,1,-\tfrac{1}{2}\right),\left(3,1,1,+\tfrac{1}{2}\right),\\ \left(3,2,-2,-\tfrac{1}{2}\right),\left(3,2,-2,+\tfrac{1}{2}\right),\left(3,2,-1,-\tfrac{1}{2}\right),\left(3,2,-1,+\tfrac{1}{2}\right),\left(3,2,0,-\tfrac{1}{2}\right),\left(3,2,0,+\tfrac{1}{2}\right),\\ \left(3,2,1,-\tfrac{1}{2}\right),\left(3,2,1,+\tfrac{1}{2}\right),\left(3,2,2,-\tfrac{1}{2}\right),\left(3,2,-2,+\tfrac{1}{2}\right),\left(4,0,0,-\tfrac{1}{2}\right),\left(4,0,0,+\tfrac{1}{2}\right),\\ \left(4,1,0,+\tfrac{1}{2}\right)\end{array}}$$

The last electron listed could have other quantum numbers for m_ℓ and m_s.

(*b*) The $\boxed{1s,\ 2s,\ 2p,\ 3s,\ 3p,\ 3d,\ \text{and } 4s}$ subshells are filled.

(*c*) For a 4*p* state, $\ell = 1$. Since $s = \tfrac{1}{2}$, the possible values for *j* are $j = \ell + s = \boxed{\tfrac{3}{2}}$ and $j = \ell - s = \boxed{\tfrac{1}{2}}$.

(*d*) The 4*p* electron is the only electron not in a filled subshell. The angular momentum of a filled subshell is zero, so the total angular momentum of the atom is the angular momentum of the 4*p* electron.

(*e*) When the beam passes through the magnetic field gradient, the deflecting force will be proportional to m_j. If $j = \tfrac{1}{2}$, the values of m_j are $\pm\tfrac{1}{2}$, and there will be two lines. If $j = \tfrac{3}{2}$, the values of m_j are $\pm\tfrac{1}{2}, \pm\tfrac{3}{2}$, and there will be four lines. $\boxed{\text{The number of lines indicates the value of } j.}$

53. (*a*) The additional term for the spin-orbit interaction is given in the text as $U_{\substack{\text{spin}\\ \text{orbit}}} = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}_{\text{n}} = \mu_z B_{\text{n}}$.

The separation of the energy levels due to the two different electron spins is twice this.

$$\Delta U_{\substack{\text{spin}\\ \text{orbit}}} = \left(\mu_z B_{\text{n}}\right)_{\text{spin up}}^{\text{spin down}} = -g\mu_{\text{B}} B_{\text{n}} \Delta m_s \quad \rightarrow$$

$$B_{\text{n}} = \frac{\Delta U_{\substack{\text{spin}\\ \text{orbit}}}}{-g\mu_{\text{B}}\Delta m_s} = \frac{\left(5\times10^{-5}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(2.0023\right)\left(9.27\times10^{-24}\,\text{J/T}\right)\left(-\tfrac{1}{2}-\tfrac{1}{2}\right)} = 0.431\,\text{T} \approx \boxed{0.4\,\text{T}}$$

(*b*) If we consider the nucleus to be a loop of current with radius *r*, then the magnetic field due to the nucleus at the center of the loop (the location of the electron) is given in Example 28-12 as $B_{\text{n}} = \dfrac{\mu_0 I}{2r}$. Model the current as the charge of the nucleus moving in a circle, with a period as given by circular motion.

$$I = \frac{\Delta q}{\Delta t} = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r} = \frac{e\left(m_e r\right)v}{2\pi r\left(m_e r\right)}$$

Note that classically, $m_e r v = L_e$, the angular momentum of the electron, and so $m_e r v = \sqrt{\ell\left(\ell+1\right)}\hbar$ with $\ell = 2$. Thus we have the following:

$$B_{\text{n}} = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\frac{e\sqrt{\ell\left(\ell+1\right)}\hbar}{2\pi m_e r^2} = \frac{e\hbar}{2m_e}\frac{\mu_0\sqrt{\ell\left(\ell+1\right)}}{2\pi r^3} = \frac{\mu_{\text{B}}\mu_0\sqrt{\ell\left(\ell+1\right)}}{2\pi r^3}$$

From Figure 39-9 (b), we see that the most probable radius for the $n = 2$, $\ell = 1$ state is approximately $r = 4r_0$. We can now calculate the approximate magnetic field.

$$B_{\text{n}} = \frac{\mu_{\text{B}}\mu_0\sqrt{\ell\left(\ell+1\right)}}{2\pi\left(4r_0\right)^3} = \frac{\left(9.27\times10^{-24}\,\text{J/T}\right)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\sqrt{6}}{2\pi\left[4\left(0.529\times10^{-10}\,\text{m}\right)\right]^3} = 0.479\,\text{T} \approx \boxed{0.5\,\text{T}}$$

The two values are about $\dfrac{0.479\,\text{T} - 0.431\,\text{T}}{\frac{1}{2}\left(0.479\,\text{T} + 0.431\,\text{T}\right)} \times 100 \approx 11\%$ different, and so are consistent.

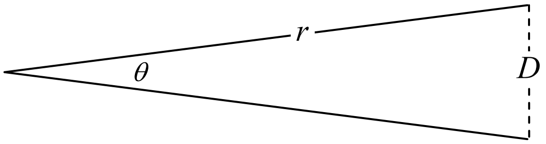
54. The energy of a pulse is the power of the pulse times the duration in time.

$$E = P\Delta t = \left(0.63\,\text{W}\right)\left(23\times10^{-3}\,\text{s}\right) = 0.01449\,\text{J} \approx \boxed{0.014\,\text{J}}$$

The number of photons in a pulse is the energy of a pulse, divided by the energy of a photon as given in Eq. 37-3.

$$N = \frac{E}{hf} = \frac{E\lambda}{hc} = \frac{\left(0.01449\,\text{J}\right)\left(640\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{4.7\times10^{16}\text{ photons}}$$

55. The angular half-width of the beam can be found in Section 35-4, and is given by $\theta_{1/2} = \dfrac{1.22\lambda}{d}$, where d is the diameter of the diffracting circle. The angular width of the beam is twice this. The linear diameter of the beam is then the angular width times the distance from the source of the light to the observation point, $D = r\theta$. See the diagram.



(*a*) $$D = r\theta = r\frac{2.44\lambda}{d} = \left(380\times10^{3}\,\text{m}\right)\frac{2.44\left(694\times10^{-9}\,\text{m}\right)}{3.6\times10^{-3}\,\text{m}} = \boxed{180\,\text{m}}$$

(*b*) $$D = r\theta = r\frac{2.44\lambda}{d} = \left(384\times10^{6}\,\text{m}\right)\frac{2.44\left(694\times10^{-9}\,\text{m}\right)}{3.6\times10^{-3}\,\text{m}} = \boxed{1.8\times10^{5}\,\text{m}}$$

56. Intensity equals power per unit area. The area of the light from the laser is assumed to be in a circular area, while the area intercepted by the light from a light bulb is the surface area of a sphere.

(*a*) $$I = \frac{P}{S} = \frac{P}{\pi r^2} = \frac{0.50\times10^{-3}\,\text{W}}{\pi\left(1.5\times10^{-3}\,\text{m}\right)^2} = 70.74\,\text{W/m}^2 \approx \boxed{71\,\text{W/m}^2}$$

(*b*) $$I = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{15\,\text{W}}{4\pi\left(2.0\,\text{m}\right)^2} = 0.2984\,\text{W/m}^2 \approx 0.30\,\text{W/m}^2$$

The laser beam is more intense by a factor of $\dfrac{70.74\,\text{W/m}^2}{0.2984\,\text{W/m}^2} = 237 \approx \boxed{240}$.

57. Transition from the $E_3{}'$ state to the $E_2{}'$ state releases photons with energy 1.96 eV, as shown in Figure 39-21. The wavelength is determined from the energy.

$$\lambda = \frac{hc}{\Delta E} = \frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(1.96\,\text{eV}\right)} = 6.34\times10^{-7}\,\text{m} = \boxed{634\,\text{nm}}$$

58. We use Eq. 39-16b.

$$\frac{N_2}{N_0} = e^{-\left(\frac{E_2-E_0}{kT}\right)} = e^{-\left[\frac{\left(2.2\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}\right]} = e^{-85.0} = \boxed{1.2\times10^{-37}}$$

$$\frac{N_1}{N_0} = e^{-\left(\frac{E_2-E_0}{kT}\right)} = e^{-\left[\frac{\left(1.8\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}\right]} = e^{-69.6} = \boxed{6.1\times10^{-31}}$$

59. The relative numbers of atoms in two energy states at a given temperature is given by Eq. 39-16b. From Figure 39-20, the energy difference between the two states is 2.2 eV.

$$\frac{N_2}{N_0}=e^{-\left(\frac{E_2-E_0}{kT}\right)} \rightarrow T=-\left(\frac{E_2-E_0}{k\ln\dfrac{N_2}{N_0}}\right)=-\left[\frac{(2.2\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln 0.5}\right]=\boxed{3.7\times10^{4}\,\text{K}}$$

60. Consider Eq. 39-16b, with $E_n > E_{n'}$. To have a population inversion means that $N_n > N_{n'}$.

$$\frac{N_n}{N_{n'}}=e^{-\left(\frac{E_n-E_{n'}}{kT}\right)}>1 \rightarrow \ln\frac{N_n}{N_{n'}}=-\left(\frac{E_n-E_{n'}}{kT}\right)>0 \rightarrow \frac{E_n-E_{n'}}{kT}<0$$

Since $E_n > E_{n'}$, to satisfy the last condition we must have $T < 0$, a negative temperature.

This negative "temperature" is not a contradiction. The Boltzmann distribution assumes that a system is in thermal equilibrium, and the inverted system is not in thermal equilibrium. The inversion cannot be maintained without adding energy to the system. If left to itself, the excited states will decay and the inversion will not be maintained.

61. (*a*) Boron has $Z = 4$, so the outermost electron has $n = 2$. We use the Bohr result with an effective *Z*. We might naively expect to get $Z_{\text{eff}} = 1$, indicating that the other three electrons shield the outer electron from the nucleus, or $Z_{\text{eff}} = 2$, indicating that only the inner two electrons accomplish the shielding.

$$E_2=-\frac{(13.6\,\text{eV})(Z_{\text{eff}})^2}{n^2} \rightarrow -8.26\,\text{eV}=-\frac{(13.6\,\text{eV})(Z_{\text{eff}})^2}{2^2} \rightarrow Z_{\text{eff}}=\boxed{1.56}$$

This indicates that the second electron in the $n = 2$ shell does partially shield the electron that is to be removed.

(*b*) We find the average radius from the expression below.

$$r=\frac{n^2 r_1}{Z_{\text{eff}}}=\frac{2^2\left(0.529\times10^{-10}\,\text{m}\right)}{(1.56)}=\boxed{1.36\times10^{-10}\,\text{m}}$$

62. An *h* subshell has $\ell = 5$. For a given ℓ value, m_ℓ ranges from $-\ell$ to $+\ell$, taking on $2\ell + 1$ different values. For each m_ℓ there are 2 values of m_s. Thus the number of states for a given ℓ value is $2(2\ell + 1)$. Thus there are $2(2\ell + 1) = 2(11) = \boxed{22}$ possible electron states.

63. (*a*) $Z = 25$ is manganese.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^2}$$

(*b*) $Z = 34$ is selenium.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4}$$

(*c*) $Z = 39$ is yttrium.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^1 5s^2}$$

64. The value of ℓ can range from 0 to $n-1$. Thus for $n=6$, we have $0 \le \ell \le 5$. The magnitude of $\vec{\mathbf{L}}$ is given by Eq. 39-3, $L=\sqrt{\ell(\ell+1)}\hbar$.

$$\boxed{L_{\text{min}}=0\ ;\ L_{\text{max}}=\sqrt{30}\hbar}$$

65. (*a*) We treat the Earth as a particle in rotation about the Sun. The angular momentum of a particle is given in Example 11-7 as $L=mvr$, where *r* is the orbit radius. We equate this to the quantum mechanical expression in Eq. 39-3. We anticipate that the quantum number will be very large, and so approximate $\left[\ell(\ell+1)\right]^{\frac{1}{2}}$ as ℓ.

$$L=M_{\text{Earth}}vr_{\substack{\text{Sun–}\\ \text{Earth}}}=\frac{M2\pi r^2}{T}=\hbar\left[\ell(\ell+1)\right]^{\frac{1}{2}}=\hbar\ell \quad\rightarrow$$

$$\ell=\frac{M_{\text{Earth}}2\pi r^2}{\hbar T}=\frac{\left(5.98\times10^{24}\,\text{kg}\right)2\pi\left(1.496\times10^{11}\,\text{m}\right)^2}{\left(1.055\times10^{-34}\,\text{J•s}\right)\left(3.156\times10^{7}\,\text{s}\right)}=2.5255\times10^{74}\approx\boxed{2.5\times10^{74}}$$

(*b*) There are $2\ell+1$ values of m_ℓ for a value of ℓ, so the number of orientations is as follows.

$$N=2\ell+1=2\left(2.5255\times10^{74}\right)+1=5.051\times10^{74}\approx\boxed{5.1\times10^{74}}.$$

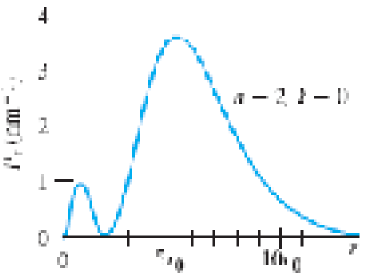
66. Eq. 37-15 gives the Bohr-theory result for the wavelength of a spectral line. For the Mosley plot, the wavelengths are for the K_α line, which has $n=2$ and $n'=1$. We assume that the shielding of the other *n* = 1 electron present reduces the effective atomic number to *Z* – 1. We use the value of the Rydberg constant from Section 37-11.

$$\frac{1}{\lambda}=\left(\frac{Z^2e^4m}{8\varepsilon_0^2h^3c}\right)\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)\ \rightarrow\ \frac{1}{\lambda}=\left(\frac{e^4m}{8\varepsilon_0^2h^3c}\right)Z^2\left(\frac{1}{1}-\frac{1}{4}\right)\ \rightarrow\ \frac{1}{\lambda}=\left(\frac{3e^4m}{32\varepsilon_0^2h^3c}\right)Z^2\ \rightarrow$$

$$\frac{1}{\lambda}=\left(\frac{3e^4m}{32\varepsilon_0^2h^3c}\right)(Z-1)^2\ \rightarrow\ \boxed{\sqrt{\frac{1}{\lambda}}=a(Z-b)}\ ,\ a=\left(\frac{3e^4m}{32\varepsilon_0^2h^3c}\right)^{1/2}\ ,\ b=1$$

$$a=\left(\frac{3e^4m}{32\varepsilon_0^2h^3c}\right)^{1/2}=\left(\tfrac{3}{4}R\right)^{1/2}=\left[\tfrac{3}{4}\left(1.0974\times10^{7}\,\text{m}^{-1}\right)\right]^{1/2}=\boxed{2868.9\,\text{m}^{-1/2}}$$

$\boxed{67.}$ This is very similar to Example 39-3. We find the radial probability distribution for the *n* = 2, ℓ = 0 wave function, and find the position at which that distribution has a maximum. We see from Figure 39-8 that there will be two local maxima in the probability distribution function, and the global maximum is at approximately $5r_0$. The wave function is given in Eq. 39-8.



$$\psi_{200}=\frac{1}{\sqrt{32\pi r_0^3}}\left(2-\frac{r}{r_0}\right)e^{-\frac{r}{2r_0}}$$

$$P_{\text{r}}=4\pi r^2\left|\psi_{200}\right|^2=\frac{r^2}{8r_0^3}\left(2-\frac{r}{r_0}\right)^2e^{-\frac{r}{r_0}}$$

$$\frac{dP_{\text{r}}}{dr}=\frac{2r}{8r_0^3}\left(2-\frac{r}{r_0}\right)^2e^{-\frac{r}{r_0}}+\frac{r^2}{8r_0^3}2\left(2-\frac{r}{r_0}\right)\left(-\frac{1}{r_0}\right)e^{-\frac{r}{r_0}}+\frac{r^2}{8r_0^3}\left(2-\frac{r}{r_0}\right)^2\left(-\frac{1}{r_0}\right)e^{-\frac{r}{r_0}}$$

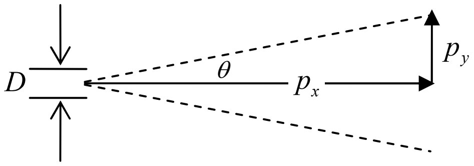
$$=\frac{(2r_0-r)re^{-\frac{r}{r_0}}}{8r_0^6}\left(r^2-6r_0r+4r_0^2\right)=0 \;\rightarrow\; r=0,\; r=2r_0,\; r^2-6r_0r+4r_0^2=0$$

$$r^2-6r_0r+4r_0^2=0 \;\rightarrow\; r=\frac{6r_0\pm\sqrt{36r_0^2-16r_0^2}}{2}=\left(3\pm\sqrt{5}\right)r\approx 0.764r_0,\; 5.24r_0$$

So there are four extrema: $r=0,\ 0.76r_0,\ 2r_0,\ 5.2r_0$. From Figure 39-8 we see that the most probable distance is $r=\boxed{5.2r_0}$.

68. The "location" of the beam is uncertain in the transverse direction by an amount equal to the aperture opening, *D*. This gives a value for the uncertainty in the transverse momentum. The momentum must be at least as big as its uncertainty, and so we obtain a value for the transverse momentum.

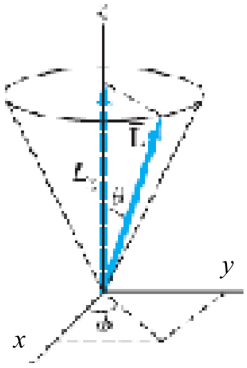
$$\Delta p_y\Delta y\ge\hbar \;\rightarrow\; \Delta p_y\ge\frac{\hbar}{\Delta y}=\frac{\hbar}{D} \;\rightarrow\; p_y\approx\frac{\hbar}{D}$$



The momentum in the forward direction is related to the wavelength of the light by $p_x=\frac{h}{\lambda}$. See the diagram to relate the momentum to the angle.

$$\theta\approx\frac{p_y}{p_x}\approx\frac{\hbar/D}{h/\lambda}=\frac{\lambda}{2\pi D}\quad;\ \text{“spread”}=2\theta=\frac{\lambda}{\pi D}\approx\boxed{\frac{\lambda}{D}}$$

69. The magnitude of the angular momentum is given by Eq. 39-3, and L_z is given by Eq. 39-4. The cosine of the angle between $\vec{\mathbf{L}}$ and the *z* axis is found from L and L_z.



$$L=\sqrt{\ell(\ell+1)}\hbar\ ;\ L_z=m_\ell\hbar\ ;\ \theta=\cos^{-1}\frac{L_z}{L}=\cos^{-1}\frac{m_\ell}{\sqrt{\ell(\ell+1)}}$$

(*a*) For $\ell=1,\ m_\ell=-1,\ 0,\ 1.$

$$\theta_{1,1}=\cos^{-1}\frac{1}{\sqrt{2}}=\boxed{45^\circ}\ ;\ \theta_{1,0}=\cos^{-1}\frac{0}{\sqrt{2}}=\boxed{90^\circ}\ ;$$

$$\theta_{1,-1}=\cos^{-1}\frac{-1}{\sqrt{2}}=\boxed{135^\circ}$$

(*b*) For $\ell=2,\ m_\ell=-2,-1,\ 0,\ 1,\ 2.$

$$\theta_{2,2}=\cos^{-1}\frac{2}{\sqrt{6}}=\boxed{35.3^\circ}\ ;\ \theta_{2,1}=\cos^{-1}\frac{1}{\sqrt{6}}=\boxed{65.9^\circ}\ ;\ \theta_{2,0}=\cos^{-1}\frac{0}{\sqrt{2}}=\boxed{90^\circ}$$

$$\theta_{2,-1}=\cos^{-1}\frac{-1}{\sqrt{6}}=\boxed{114.1^\circ}\ ;\ \theta_{2,2}=\cos^{-1}\frac{-2}{\sqrt{6}}=\boxed{144.7^\circ}$$

(*c*) For $\ell=3,\ m_\ell=-3,-2,-1,\ 0,\ 1,\ 2,\ 3.$

$$\theta_{3,3}=\cos^{-1}\frac{3}{\sqrt{12}}=\boxed{30^\circ}\ ;\ \theta_{3,2}=\cos^{-1}\frac{2}{\sqrt{12}}=\boxed{54.7^\circ}\ ;\ \theta_{3,1}=\cos^{-1}\frac{1}{\sqrt{12}}=\boxed{73.2^\circ}$$

$$\theta_{3,0}=\cos^{-1}\frac{0}{\sqrt{12}}=\boxed{90^\circ}\ ;\ \theta_{3,-1}=\cos^{-1}\frac{-1}{\sqrt{12}}=\boxed{106.8^\circ}\ ;\ \theta_{3,-2}=\cos^{-1}\frac{-2}{\sqrt{12}}=\boxed{125.3^\circ}\ ;$$

$$\theta_{3,-3}=\cos^{-1}\frac{-3}{\sqrt{12}}=\boxed{150^\circ}$$

(*d*) We see from the previous parts that the smallest angle occurs for $m_\ell = \ell$.

$$\theta_{100,100} = \cos^{-1}\frac{100}{\sqrt{(100)(101)}} = \boxed{5.71°}$$

$$\theta_{10^6,10^6} = \cos^{-1}\frac{10^6}{\sqrt{(10^6)(10^6+1)}} = \boxed{0.0573°}$$

This is consistent with the correspondence principle, which would say that the angle between $\vec{\mathbf{L}}$ and the *z* axis could be any value classically, which is represented by letting $\ell \to \infty$ (which also means $n \to \infty$).

70. (*a*) Since $\Delta L_z = 0,\ \Delta\phi \to \infty$, and so $\boxed{\phi \text{ is unknown}}$. We can say nothing about the value of ϕ.

(*b*) Since ϕ is completely unknown, we have no knowledge of the component of the angular momentum perpendicular to the *z*-axis. Thus $\boxed{L_x \text{ and } L_y \text{ are unknown}}$.

(*c*) The square of the total angular momentum is given by $L^2 = L_x^2 + L_y^2 + L_z^2$. Use this with the quantization conditions for L and L_z given in Eqs. 39-3 and 39-4.

$$L^2 = L_x^2 + L_y^2 + L_z^2 \ \rightarrow\ \ell(\ell+1)\hbar^2 = L_x^2 + L_y^2 + m_\ell^2\hbar^2 \ \rightarrow\ L_x^2 + L_y^2 = \left[\ell(\ell+1) - m_\ell^2\right]\hbar^2$$

$$\sqrt{L_x^2 + L_y^2} = \boxed{\left[\ell(\ell+1) - m_\ell^2\right]^{1/2}\hbar}$$

71. (*a*) The mean value can be found as described in Problem 19. We use the first definite integral given in Appendix B-5, with *n* = 1 and *a* = 1.

$$\overline{\left(\frac{1}{r}\right)} = \int_0^\infty \frac{1}{r}\left|\psi_{100}\right|^2 4\pi r^2 dr = \int_0^\infty \frac{1}{r}\frac{1}{\pi r_0^3}e^{-2\frac{r}{r_0}}4\pi r^2 dr = \frac{4}{r_0^2}\int_0^\infty \frac{r}{r_0}e^{-2\frac{r}{r_0}}dr \text{ ; let } x = 2\frac{r}{r_0} \ \rightarrow$$

$$= \frac{1}{r_0}\int_0^\infty xe^{-x}dx = \frac{1}{r_0}(1) = \frac{1}{r_0}$$

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \ \rightarrow\ \bar{U} = -\frac{e^2}{4\pi\varepsilon_0}\int_0^\infty \frac{1}{r}\left|\psi_{100}\right|^2 4\pi r^2 dr = \boxed{-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_0}}$$

(*b*) For the ground state of hydrogen, Eq. 37-14a gives the energy, and Eq. 37-11 gives the Bohr radius. Substitute those expressions into $E = \bar{U} + \bar{K}$.

$$E = -\frac{e^4 m}{8\varepsilon_0^2 h^2} \ ;\ \bar{U} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_0}$$

$$\bar{K} = E - \bar{U} = -\frac{e^4 m}{8\varepsilon_0^2 h^2} - \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_0}\right) = \frac{e^2}{4\pi\varepsilon_0 r_0} - \frac{e^4 m}{8\varepsilon_0^2 h^2} = \frac{e^2}{4\pi\varepsilon_0 r_0} - \frac{e^4 m\left(\dfrac{h^2\varepsilon_0}{\pi m e^2}\right)}{8\varepsilon_0^2 h^2 r_0}$$

$$= \frac{e^2}{4\pi\varepsilon_0 r_0} - \frac{e^2}{8\pi\varepsilon_0 r_0} = \frac{e^2}{8\pi\varepsilon_0 r_0} = \boxed{-\tfrac{1}{2}\bar{U}}$$

72. In the Bohr model, $L_{\text{Bohr}} = n\dfrac{h}{2\pi} = \boxed{2\hbar}$. In quantum mechanics, $L_{\text{QM}} = \sqrt{\ell(\ell+1)}\,\hbar$. For $n = 2,\ \ell = 0$ or $\ell = 1$, so that $L_{\text{QM}} = \sqrt{0(0+1)}\,\hbar = \boxed{0}$ or $L_{\text{QM}} = \sqrt{1(1+1)}\,\hbar = \boxed{\sqrt{2}\hbar}$.

$\boxed{73.}$ (*a*) The $4p \to 3p$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = 0 \neq \pm 1$.

(*b*) The $3p \to 1s$ transition is $\boxed{\text{allowed,}}$ because $\Delta\ell = -1$.

(*c*) The $4d \to 3d$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = 0 \neq \pm 1$.

(*d*) The $4d \to 3s$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = -2 \neq \pm 1$.

(*e*) The $4s \to 2p$ transition is $\boxed{\text{allowed,}}$ because $\Delta\ell = +1$.

74. The binding energy is given by the opposite of Eq. 37-14b.

$$-E_n = (13.6\,\text{eV})\frac{Z^2}{n^2} = \frac{13.6\,\text{eV}}{45^2} = \boxed{6.72\times10^{-3}\,\text{eV}}$$

The radius is given by Eq. 37-13.

$$r_n = \frac{n^2}{Z}\left(0.529\times10^{-10}\,\text{m}\right) = 45^2\left(0.529\times10^{-10}\,\text{m}\right) = \boxed{1.07\times10^{-7}\,\text{m}}$$

The effective cross-sectional area is as follows.

$$\sigma = \pi r^2 = \pi\left(1.07\times10^{-7}\,\text{m}\right)^2 = \boxed{3.60\times10^{-14}\,\text{m}^2}$$

75. The wavelengths of emitted lines from one-electron atoms are given by Eq. 37-15. We can simplify the equation by using the Rydberg constant, so $\dfrac{1}{\lambda} = Z^2R\left(\dfrac{1}{n'^2} - \dfrac{1}{n^2}\right)$. The Lyman series with hydrogen has $Z = 1$ and $n' = 1$. Every fourth line from the unknown element match the wavelengths of the first three Lyman lines. This gives three equations in three unknowns.

$$\frac{1}{\lambda_{\substack{\text{Lyman}\\2\to1}}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Z^2R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad\to\quad \frac{3}{4} = Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\frac{1}{\lambda_{\substack{\text{Lyman}\\3\to1}}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = Z^2R\left(\frac{1}{n^2} - \frac{1}{(m+4)^2}\right) \quad\to\quad \frac{8}{9} = Z^2\left(\frac{1}{n^2} - \frac{1}{(m+4)^2}\right)$$

$$\frac{1}{\lambda_{\substack{\text{Lyman}\\4\to1}}} = R\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = Z^2R\left(\frac{1}{n^2} - \frac{1}{(m+8)^2}\right) \quad\to\quad \frac{15}{16} = Z^2\left(\frac{1}{n^2} - \frac{1}{(m+8)^2}\right)$$

Subtract the first equation from each of the other two equations.

$$\frac{8}{9} - \frac{3}{4} = Z^2\left(\frac{1}{n^2} - \frac{1}{(m+4)^2}\right) - Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad\to\quad \frac{5}{36} = Z^2\left(\frac{1}{m^2} - \frac{1}{(m+4)^2}\right)$$

$$\frac{15}{16} - \frac{3}{4} = Z^2\left(\frac{1}{n^2} - \frac{1}{(m+8)^2}\right) - Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad\to\quad \frac{3}{16} = Z^2\left(\frac{1}{m^2} - \frac{1}{(m+8)^2}\right)$$

Divide the resulting equations to eliminate *Z*, solve for *m*, and then substitute to find *Z*.

$$\frac{\dfrac{5}{36}}{\dfrac{3}{16}} = \frac{Z^2\left(\dfrac{1}{m^2} - \dfrac{1}{(m+4)^2}\right)}{Z^2\left(\dfrac{1}{m^2} - \dfrac{1}{(m+8)^2}\right)} \quad\to\quad \frac{20}{27} = \frac{\dfrac{(m+4)^2 - m^2}{m^2(m+4)^2}}{\dfrac{(m+8)^2 - m^2}{m^2(m+8)^2}} = \frac{(m+2)(m+8)^2}{2(m+4)^3} \quad\to$$

$$40(m+4)^3 = 27(m+2)(m+8)^2 \quad\to\quad 13m^3 + 514m^2 + 608m - 896 = 0$$

This is a cubic equation, which can be solved by numerical techniques. We first drew a graph, and saw that two of the 0's of the function were negative, and one was near $m = 8$. The only acceptable results are for $m > 0$, and substitution verifies that $m = 8$ solves the equation. We use that result to find Z.

$$\frac{3}{16} = Z^2\left(\frac{1}{m^2} - \frac{1}{(m+8)^2}\right) \rightarrow Z = \sqrt{\left(\frac{3}{16}\right)\left(\frac{1}{m^2} - \frac{1}{(m+8)^2}\right)^{-1}} = \sqrt{\left(\frac{3}{16}\right)\left(\frac{1}{8^2} - \frac{1}{(8+8)^2}\right)^{-1}} = 4$$

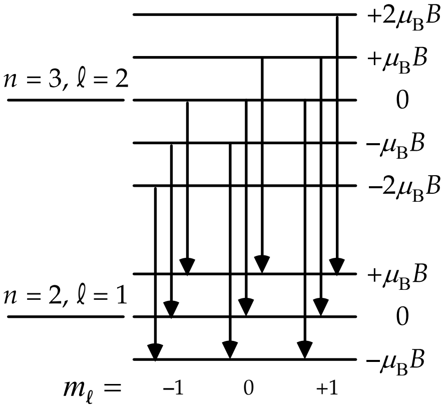
Thus the element is $\boxed{\text{beryllium}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH39.XLS," on tab "Problem 39.75."

76. (*a*) The additional energy due to the presence of a magnetic field is derived in Section 39-7, as $U = \mu_B m_\ell B_z$. We use this to calculate the energy spacing between adjacent m_ℓ values.

$$U = \mu_B m_\ell B_z \rightarrow$$

$$U = \mu_B B_z \Delta m_\ell = \frac{(9.27\times10^{-24}\,\text{J/T})}{(1.60\times10^{-19}\,\text{J/eV})}(1.6\,\text{T}) = 9.27\times10^{-5}\,\text{eV} \approx \boxed{9.3\times10^{-5}\,\text{eV}}$$

(*b*) As seen in Figure 39-4, the $n = 3$, $\ell = 2$ level will split into 5 levels, and the $n = 2$, $\ell = 1$ level will split into 3 levels. With no restrictions, there would be 15 different transitions possible. All transitions would have the same $\Delta n = 1$. Thus there are only $\boxed{\text{three unique wavelengths}}$ possible: the one corresponding to a transition with $\Delta m_\ell = +1$ (a slightly larger energy change than in the $B = 0$ case), the one corresponding to a transition with $\Delta m_\ell = 0$ (the same energy change as in the $B = 0$ case), and the one corresponding to a transition with $\Delta m_\ell = -1$ (a slightly smaller energy change than in the $B = 0$ case). See the diagram, showing 9 possible transitions grouped into 3 actual energy changes. The value along the right side is the change in energy level due to the magnetic field interaction.



(*c*) Eq. 37-15 gives the wavelength for hydrogen, considering only a change in principal quantum number. The energies for those transitions is on the order of eV. The energy change due to the magnetic field interaction is much smaller than that, so we can use an approximation, knowing ΔE from part (*a*). We obtain E from Eq. 37-14b.

$$\lambda_{n=3\to2} = \left[R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\right]^{-1} = \left[1.0974\times10^7\,\text{m}^{-1}\left(\tfrac{1}{4} - \tfrac{1}{9}\right)\right]^{-1} = 6.56096\times10^{-7}\,\text{m} \approx 656.10\,\text{nm}$$

$$\lambda = h\frac{c}{E} \rightarrow \Delta\lambda = -\frac{hc}{E^2}\Delta E = -\lambda\frac{\Delta E}{E} \;;\; E_{n=3\to2} = 13.6\,\text{eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.889\,\text{eV}$$

$$\Delta\lambda = -\lambda\frac{\Delta E}{E} = -(656.10\,\text{nm})\left(\frac{9.27\times10^{-5}\,\text{eV}}{1.889\,\text{eV}}\right) = 0.032\,\text{nm}$$

$$\lambda_{\Delta m_\ell=+1} = \lambda_{n=3\to2} + \Delta\lambda = 656.10\,\text{nm} - 0.032\text{nm} = \boxed{656.07\,\text{nm}}$$

$$\lambda_{\Delta m_\ell=+1} = \lambda_{n=3\to2} + 0 = \boxed{656.10\,\text{nm}}$$

$$\lambda_{\Delta m_\ell=-1} = \lambda_{n=3\to2} + \Delta\lambda = 656.10\,\text{nm} + 0.032\text{nm} = \boxed{656.13\,\text{nm}}$$

77. (*a*) We use Eq. 39-16b, with the "note" as explained in the problem, multiplying the initial expression times $\frac{8}{2}$.

$$\frac{N_2}{N_1}=\tfrac{8}{2}e^{-\left(\frac{E_2-E_1}{kT}\right)}=\tfrac{8}{2}e^{-\left[\frac{\left(\frac{-13.6\,\text{eV}}{4}-\frac{-13.6\,\text{eV}}{1}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}\right]}=4e^{-394.2}$$

Many calculators will not directly evaluate $e^{-394.2}$, so we do the following.

$$x=e^{-394.2}\ ;\ \log x=-394.2\log e=-394.2\left(0.4343\right)=-171.2\ ;$$

$$x=10^{-171.2}=10^{-0.2}10^{-171}=0.631\times10^{-171}$$

$$\frac{N_2}{N_1}=4e^{-394.2}=4\left(0.631\times10^{-171}\right)=2.52\times10^{-171}\approx\boxed{3\times10^{-171}}$$

There are 18 states with n = 3, so we multiply by $\frac{18}{2}$.

$$\frac{N_3}{N_1}=\tfrac{18}{2}e^{-\left(\frac{E_3-E_1}{kT}\right)}=\tfrac{18}{2}e^{-\left[\frac{\left(\frac{-13.6\,\text{eV}}{9}-\frac{-13.6\,\text{eV}}{1}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}\right]}=9e^{-467.2}=1.13\times10^{-202}\approx\boxed{1\times10^{-202}}$$

(*b*) We repeat the evaluations for the higher temperature.

$$\frac{N_2}{N_1}=\tfrac{8}{2}e^{-\left(\frac{E_2-E_1}{kT}\right)}=\tfrac{8}{2}e^{-\left[\frac{\left(\frac{-13.6\,\text{eV}}{4}-\frac{-13.6\,\text{eV}}{1}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(6000\,\text{K}\right)}\right]}=4e^{-19.71}=1.10\times10^{-8}\approx\boxed{1\times10^{-8}}$$

$$\frac{N_3}{N_1}=\tfrac{18}{2}e^{-\left(\frac{E_3-E_1}{kT}\right)}=\tfrac{18}{2}e^{-\left[\frac{\left(\frac{-13.6\,\text{eV}}{9}-\frac{-13.6\,\text{eV}}{1}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(6000\,\text{K}\right)}\right]}=9e^{-23.36}=6.44\times10^{-10}\approx\boxed{6\times10^{-10}}$$

(*c*) Since the fraction of atoms in each excited state is very small, we assume that N_1 is the number of hydrogen atoms given. 1.0 g of H atoms contains 6.02×10^{23} atoms.

$$N_2=N_1\left(1.10\times10^{-8}\right)=\left(6.02\times10^{23}\right)\left(1.10\times10^{-8}\right)=6.62\times10^{15}\approx\boxed{7\times10^{15}}$$

$$N_3=N_1\left(6.44\times10^{-10}\right)=\left(6.02\times10^{23}\right)\left(6.44\times10^{-10}\right)=3.88\times10^{14}\approx\boxed{4\times10^{14}}$$

(*d*) We assume the lifetime of an excited state atom is 10^{-8}s. Each atom would emit one photon as its electron goes to the ground state. The number of photons emitted per second can be estimated by the number of atoms, divided by the lifetime.

$$n_3=\frac{N_3}{\tau}=\frac{4\times10^{14}}{10^{-8}\,\text{s}}\boxed{4\times10^{22}\,\text{photons/s}}\ ;\ n_2=\frac{N_2}{\tau}=\frac{7\times10^{15}}{10^{-8}\,\text{s}}\approx\boxed{7\times10^{23}\,\text{photons/s}}$$

CHAPTER 40: Molecules and Solids

Responses to Questions

1. (*a*) Covalent; (*b*) ionic; (*c*) metallic.

2. A neutral calcium atom has 20 electrons. Its outermost electrons are in the $4s^2$ state. The inner 18 electrons form a spherically symmetric distribution and partially shield the outer two electrons from the nuclear charge. A neutral chlorine atom has 17 electrons; it lacks just one electron to have its outer shell filled. A $CaCl_2$ molecule could be formed when the outer two electrons of the calcium atom are “shared” with two chlorine atoms. These electrons will be attracted by both the Ca and the Cl nuclei and will spend part of their time between the Ca and Cl nuclei. The nuclei will be attracted to this negatively charged area, forming a covalent bond. As is the case with other asymmetric covalent bonds, this bond will have a partial ionic character as well. The two electrons will partly orbit the Ca nucleus and partly orbit each of the two Cl nuclei. Since each Cl nucleus will now have an extra electron part of the time, it will have a net negative charge. The Ca nucleus will “lose” two electrons for part of the time, giving it a net positive charge.

3. No, neither the H_2 nor the O_2 molecule has a permanent dipole moment. The outer electrons are shared equally between the two atoms in each molecule, so there are no polar ends that are more positively or negatively charged. The H_2O molecule does have a permanent dipole moment. The electrons associated with the hydrogen atoms are pulled toward the oxygen atom, leaving each hydrogen with a small net positive charge and the oxygen with a small net negative charge. Because of the shape of the H_2O molecule (see Figure 40-6), one end of the molecule will be positive and the other end will be negative, resulting in a permanent dipole moment.

4. The molecule H_3 has three electrons. According to the Pauli exclusion principle, no two of these electrons can be in the same quantum state. Two of them will be $1s^2$ electrons and will form a “closed shell” and a spherically symmetric distribution, and the third one will be outside this distribution and unpaired. This third electron will be partially shielded from the nucleus and will thus be easily “lost,” resulting in an unstable molecule. The ion H_3^+ only has two electrons. These $1s^2$ electrons will form a closed shell and a spherically symmetric distribution, resulting in a stable configuration.

5. The energy of a molecule can be divided into four categories: translational kinetic energy, rotational kinetic energy, vibrational kinetic energy, and electrostatic potential energy.

6. Yes. The electron will spend most of its time between the two nuclei. Both positive nuclei will be attracted to this negative charge, forming a bond.

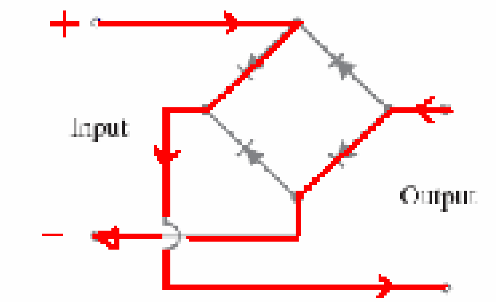
7. The carbon atom ($Z = 6$) usually forms four bonds because carbon requires four electrons to form a closed $2p$ shell, and each hydrogen-like atom contributes one electron.

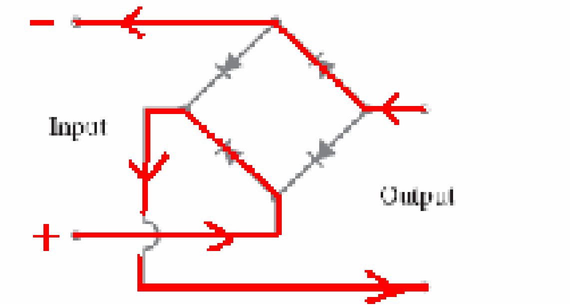
8. The last valence electron of a sodium atom is shielded from most of the sodium nuclear charge and experiences a net nuclear charge of $+1e$. The outer shell of a chlorine atom is the $3p$ shell, which contains five electrons. Due to shielding effects, the $3p$ electrons of chlorine experience a net nuclear charge of $+5e$. In NaCl, the last valence electron of sodium is strongly bound to a chlorine nucleus. This strong ionic bonding produces a large energy gap between the valence band and the conduction band in NaCl, characteristic of a good insulator.

9. The conduction electrons are not strongly bound to particular nuclei, so a metal can be viewed as a collection of positive ions and a negative electron "gas." (The positive ions are just the metal atoms without their outermost electrons, since these "free" electrons make up the gas.) The electrostatic attraction between the freely-roaming electrons and the positive ions keeps the electrons from leaving the metal.

10. As temperature increases, the thermal motion of ions in a metal lattice will increase. More electrons will collide with the ions, increasing the resistivity of the metal. When the temperature of a semiconductor increases, more electrons are able to move into the conduction band, making more charge carriers available and therefore decreasing the resistivity. (Note: the thermal motion increases in semiconductors as well, but the increase in the number of charge carriers is a larger effect.)

11. When the top branch of the input circuit is at the high voltage (current is flowing in this direction for half the cycle), then the bottom branch of the output is at the high voltage. The current follows the path through the bridge in the diagram on the left. When the bottom branch of the input circuit is at the high voltage (current is flowing in this direction during the other half of the cycle), then the bottom branch of the output is still at high voltage. The current follows the path through the bridge in the diagram on the right.





12. In an ideal gas, it is possible for all of the gas particles to have the same energy. (The velocity distribution of the particles in an ideal gas usually follows the Maxwell velocity distribution.) As the temperature of the gas increases, the kinetic energy of the gas increases. In a Fermi electron gas, only two electrons can have the same energy (Pauli exclusion principle). The electrons fill up the energy states up to the Fermi level. As the temperature of the Fermi gas increases, only the electrons in the top few levels can move to higher energy levels. The result is that the energy of the Fermi gas is not strongly temperature dependent.

13. For an ideal *pn* junction diode connected in reverse bias, the holes and electrons that would normally be near the junction are pulled apart by the reverse voltage, preventing current flow across the junction. The resistance is essentially infinite. A real diode does allow a small amount of reverse current to flow if the voltage is high enough, so the resistance in this case is very high but not infinite. A *pn* junction diode connected in forward bias has a low resistance (the holes and electrons are close together at the junction) and current flows easily.

14. The general shape of Figure 40-28 is the same for most metals. The scale of the graph (especially the x-axis scale and the Fermi energy) is peculiar to copper and will change from metal to metal.

15. The base current (between the base and the emitter) controls the collector current (between the collector and the emitter). If there is no base current, then no collector current flows. Thus, controlling the relatively small base current allows the transistor to act as a switch, turning the larger collector current on and off.

16. The main difference between *n*-type and *p*-type semiconductors is the sign on the charge carriers. In an *n*-type semiconductor the charge carriers are negative electrons. In a *p*-type semiconductor, the charge carriers are positive holes.

17. A transistor can be used as an amplifier because small changes in the base current can make much larger changes in the collector current. (See Figure 40-43.) For a *pnp* transistor, both the collector and the base voltages are negative, and holes move from the emitter to the collector. The diagram for a *pnp* amplifier looks just like Figure 40-43, with the polarity of $\mathscr{E}_B$ and $\mathscr{E}_C$ reversed, I_B and I_C flowing in opposite directions, and the emitter arrow pointing toward the base.

18. In Figure 40-43, the base–collector junction is reverse-biased and the base–emitter junction is forward-biased.

$\boxed{19.}$ The energy comes from the power supplied by the collector/emitter voltage source. The input signal to the base just regulates how much current, and therefore power, can be drawn from the collector's voltage source.

20. The phosphorus atoms will be donor atoms. Phosphorus has five valence electrons. It will form four covalent bonds with the silicon atoms around it, and will have one "extra" electron which is weakly bound to the atom and can be easily excited up to the conduction band. This process results in extra electrons in the conduction band. Silicon doped with phosphorus is therefore an *n*-type semiconductor.

21. No. Ohmic devices (those that obey Ohm's law) have a constant resistance and therefore a linear relationship between voltage and current. The voltage-current relationship for diodes is not linear. The resistance of a diode operated in reverse-bias is very large. The same diode operated in a forward-bias mode has a much smaller resistance. Since a transistor can be thought of as made up of diodes, it is also non-ohmic.

22. No. Single diodes can be used to rectify signals, but cannot amplify signals. The diode will allow the signal to pass, if forward-biased, or not allow the signal to pass, if reverse-biased. Combinations of diodes with additional power sources, as in a transistor, are able to amplify a signal.

23. Reversing the collector voltage would reverse the roles of the collector and emitter of the transistor. Unless the base-emitter voltage is also reversed, the transistor cannot act as an amplifier.

Solutions to Problems

Note: A factor that appears in the analysis of electron energies is

$$\frac{e^2}{4\pi\varepsilon_0} = \left(9.00\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2 = 2.30\times10^{-28}\,\text{J}\bullet\text{m}.$$

$\boxed{1.}$ We calculate the binding energy as the opposite of the electrostatic potential energy. We use Eq. 23-10 for the potential energy.

$$\text{Binding energy} = -U = -\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} = -\frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{0.28\times10^{-9}\,\text{m}}\right) = -\frac{2.30\times10^{-28}\,\text{J}\bullet\text{m}}{0.28\times10^{-9}\,\text{m}}$$

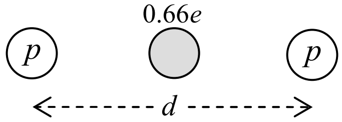
$$= 8.214\times10^{-19}\,\text{J} \approx \boxed{8.2\times10^{-19}\,\text{J}}$$

$$= 8.214\times10^{-19}\,\text{J}\left(\frac{1\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 5.134\,\text{eV} \approx \boxed{5.1\,\text{eV}}$$

2. From Problem 1, the "point electron" binding energy is 5.134 eV. With the repulsion of the electron clouds included, the actual binding energy is 4.43 eV. Use these values to calculate the contribution of the electron clouds.

$$5.134\,\text{eV} - 4.43\,\text{eV} = 0.704\,\text{eV} \approx \boxed{0.70\,\text{eV}}$$

3. We calculate the binding energy as the difference between the energy of two isolated hydrogen atoms and the energy of the bonded combination of particles. We estimate the energy of the bonded combination as the negative potential energy of the two electron-proton combinations, plus the positive potential energy of the proton-proton combination. We approximate the electrons as a single object with a charge of 0.33 of the normal charge of two electrons, since the electrons only spend that fraction of time between the nuclei. A simple picture illustrating our bonded model is shown.



$$U_{\text{isolated}} = 2(-13.6\,\text{eV}) = -27.2\,\text{eV}$$

$$U_{\text{bonded}} = U_{\text{p-p}} + 2U_{\text{e-p}} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{d}\right) - 2\left[\frac{e(0.66)e}{4\pi\varepsilon_0}\left(\frac{1}{\frac{1}{2}d}\right)\right] = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{d}(1-2.64)$$

$$= (-1.64)\frac{\left(2.30\times10^{-28}\,\text{J}\bullet\text{m}\right)}{\left(0.074\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = -31.9\,\text{eV}$$

$$U_{\text{binding}} = U_{\text{isolated}} - U_{\text{bonded}} = -27.2\,\text{eV} - (-31.9\,\text{eV}) = \boxed{4.7\,\text{eV}}$$

This is reasonably close to the actual value of 4.5 eV quoted in the text.

4. We follow the procedure outlined in the statement of the problem.

HN: $\frac{1}{2}\left(d_{\text{H}_2} + d_{\text{N}_2}\right) = \frac{1}{2}(74\,\text{pm} + 145\,\text{pm}) = \boxed{110\,\text{pm}}$

CN: $\frac{1}{2}\left(d_{\text{C}_2} + d_{\text{N}_2}\right) = \frac{1}{2}(154\,\text{pm} + 145\,\text{pm}) = \boxed{150\,\text{pm}}$

NO: $\frac{1}{2}\left(d_{\text{N}_2} + d_{\text{O}_2}\right) = \frac{1}{2}(145\,\text{pm} + 121\,\text{pm}) = \boxed{133\,\text{pm}}$

5. According to the problem statement, 5.39 eV of energy is required to make an Li^+ ion from neutral Li, and 3.41 eV of energy is released when an F atom becomes an F^- ion. That means that a net energy input of 5.39 eV – 3.41 eV = 1.98 eV is needed to form the ions. We calculate the negative potential energy of the attraction between the two ions.

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{\left(2.30\times10^{-28}\,\text{J}\cdot\text{m}\right)}{\left(0.156\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = -9.21\,\text{eV}$$

The binding energy should therefore be 9.21 eV – 1.98 eV = 7.23 eV. But the actual binding energy is only 5.95 eV. Thus the energy associated with the repulsion of the electron clouds is 7.23 eV – 5.95 eV = $\boxed{1.28\text{ eV}}$.

6. We convert the units from kcal/mole to eV/molecule.

$$1\frac{\text{kcal}}{\text{mole}}\times\frac{4186\,\text{J}}{1\,\text{kcal}}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\,\text{J}}\times\frac{1\,\text{mole}}{6.022\times10^{23}\,\text{molecules}} = \boxed{4.339\times10^{-2}\,\frac{\text{eV}}{\text{molecule}}}$$

Now convert 4.43 eV per molecule into kcal per mole.

$$4.43\frac{\text{eV}}{\text{molecule}}\times\frac{1\,\text{kcal/mol}}{4.339\times10^{-2}\,\text{eV/molecule}} = \boxed{102\,\text{kcal/mol}}$$

7. (*a*) The neutral He atom has two electrons in the ground state, $n = 1,\ \ell = 0,\ m_\ell = 0$. Thus the two electrons have opposite spins, $m_s = \pm\frac{1}{2}$. If we try to form a covalent bond, we see that an electron from one of the atoms will have the same quantum numbers as one of the electrons on the other atom. From the exclusion principle, this is not allowed, so the electrons cannot be shared.

(*b*) We consider the $\mathrm{He_2}^+$ molecular ion to be formed from a neutral He atom and an He^+ ion. It will have three electrons. If the electron on the ion has a certain spin value, it will have the opposite spin as one of the electrons on the neutral atom. Thus those two electrons can be in the same spatial region, and so a bond can be formed.

8. The units of $\dfrac{\hbar^2}{I}$ are $\dfrac{(\mathrm{J{\bullet}s})^2}{(\mathrm{kg{\bullet}m^2})} = \dfrac{\mathrm{J}^2}{(\mathrm{kg{\bullet}m/s^2})\mathrm{m}} = \dfrac{\mathrm{J}^2}{(\mathrm{N{\bullet}m})} = \dfrac{\mathrm{J}^2}{\mathrm{J}} = \mathrm{J}.$

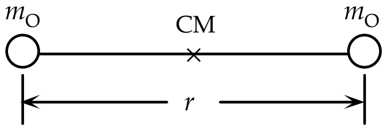
9. The reduced mass is given in Section 40-4 as $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$. We calculate in atomic mass units.

(*a*) KCl: $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(39.10\,\mathrm{u})(35.45\,\mathrm{u})}{(39.10\,\mathrm{u}) + (35.45\,\mathrm{u})} = \boxed{18.59\,\mathrm{u}}$$

(*b*) O_2: $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(16.00\,\mathrm{u})(16.00\,\mathrm{u})}{(16.00\,\mathrm{u}) + (16.00\,\mathrm{u})} = \boxed{8.00\,\mathrm{u}}$$

(*c*) HCl: $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(1.008\,\mathrm{u})(35.45\,\mathrm{u})}{(1.008\,\mathrm{u}) + (35.45\,\mathrm{u})} = \boxed{0.9801\,\mathrm{u}}$$

10. (*a*) The moment of inertia of O_2 about its CM is given by



$$I = 2m_O\left(\frac{r}{2}\right)^2 = \frac{m_O r^2}{2}.$$

$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_O r^2} = \frac{(1.055\times10^{-34}\,\mathrm{J{\bullet}s})^2}{(16)(1.66\times10^{-27}\,\mathrm{kg})(0.121\times10^{-9}\,\mathrm{m})^2(1.60\times10^{-19}\,\mathrm{J/eV})} = 1.789\times10^{-4}\,\mathrm{eV}$$

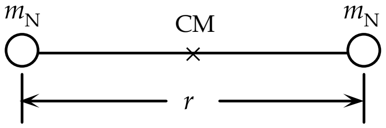
$$\approx \boxed{1.79\times10^{-4}\,\mathrm{eV}}$$

(*b*) From Figure 40-17, we see that the energy involved in the $\ell = 2$ to $\ell = 1$ transition is $\dfrac{2\hbar^2}{I}$.

$$\Delta E = \frac{2\hbar^2}{I} = 4\frac{\hbar^2}{2I} = 4(1.789\times10^{-4}\,\mathrm{eV}) = 7.156\times10^{-4}\,\mathrm{eV} \approx \boxed{7.16\times10^{-4}\,\mathrm{eV}}$$

$$\Delta E = h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\,\mathrm{J{\bullet}s})(3.00\times10^{8}\,\mathrm{m/s})}{(7.156\times10^{-4}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})} = \boxed{1.74\times10^{-3}\,\mathrm{m}}$$

11. Use the rotational energy and the moment of inertia of N_2 about its CM to find the bond length.



$$I = 2m_N\left(\frac{r}{2}\right)^2 = \frac{m_N r^2}{2};\ E_{\mathrm{rot}} = \frac{\hbar^2}{2I} = \frac{\hbar^2}{m_N r^2} \rightarrow$$

$$r=\frac{\hbar}{\sqrt{E_{\text{rot}}m_{\text{N}}}}=\frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\sqrt{\left(2.48\times10^{-4}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(14.01\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)}}=\boxed{1.10\times10^{-10}\text{m}}$$

12. The longest wavelength emitted will be due to the smallest energy change. From Figure 40-17, the smallest rotational energy change is $\Delta E=\hbar^2/I$. We find the rotational inertia from Eq. 40-4.

$$\Delta E=\frac{\hbar^2}{I}=\frac{hc}{\lambda}\quad\rightarrow$$

$$\lambda=\frac{hc}{\hbar^2}I=\frac{4\pi^2c}{h}\left(\frac{m_1m_2}{m_1+m_2}\right)r^2$$

$$=\frac{4\pi^2\left(3.00\times10^8\,\text{m/s}\right)}{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)}\left[\frac{(6.941\,\text{u})(1.008\,\text{u})}{6.941\,\text{u}+1.008\,\text{u}}\right]\left(1.66\times10^{-27}\,\text{kg/u}\right)\left(0.16\times10^{-9}\,\text{m}\right)^2$$

$$=\boxed{6.7\times10^{-4}\text{m}}$$

$\boxed{13.}$ The energies involved in the transitions are given in Figure 40-17. We find the rotational inertial from Eq. 40-4. The basic amount of rotational energy is $\hbar^2/I$.

$$\frac{\hbar^2}{I}=\frac{\hbar^2}{\left(\frac{m_1m_2}{m_1+m_2}r^2\right)}=\frac{\hbar^2}{\left(\frac{1}{2}m_{\text{H}}r^2\right)}=\frac{2\hbar^2}{m_{\text{H}}r^2}=\frac{2\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{(1.008\,\text{u})\left(1.66\times10^{-27}\,\text{kg/u}\right)\left(0.074\times10^{-9}\,\text{m}\right)^2}$$

$$=2.429\times10^{-21}\text{J}$$

(*a*) For ℓ = 1 to ℓ = 0:

$$\Delta E=\frac{\hbar^2}{I}=2.429\times10^{-21}\text{J}\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right)=\boxed{1.5\times10^{-2}\text{eV}}$$

$$\lambda=\frac{hc}{\Delta E}=\frac{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{2.429\times10^{-21}\text{J}}=\boxed{8.2\times10^{-5}\text{m}}$$

(*b*) For ℓ = 2 to ℓ = 1:

$$\Delta E=2\frac{\hbar^2}{I}=2\left(2.429\times10^{-21}\text{J}\right)\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right)=\boxed{3.0\times10^{-2}\text{eV}}$$

$$\lambda=\frac{hc}{\Delta E}=\frac{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{2\left(2.429\times10^{-21}\text{J}\right)}=\boxed{4.1\times10^{-5}\text{m}}$$

(*c*) For ℓ = 3 to ℓ = 2:

$$\Delta E=3\frac{\hbar^2}{I}=3\left(2.429\times10^{-21}\text{J}\right)\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right)=\boxed{4.6\times10^{-2}\text{eV}}$$

$$\lambda=\frac{hc}{\Delta E}=\frac{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{3\left(2.429\times10^{-21}\text{J}\right)}=\boxed{2.7\times10^{-5}\text{m}}$$

14. The energy change for transitions between combined rotational and vibrational states is given above Eq. 40-8a.

$$\Delta E = \Delta E_{\text{vib}} + \Delta E_{\text{rot}} = hf + \Delta E_{\text{rot}}$$

If $\Delta E = hf$ is to be in the spectrum, then $\Delta E_{\text{rot}} = 0$. But the selection rules state that $\Delta \ell = \pm 1$ for a transition. It is not possible to have $\Delta \ell = 0$ for a transition. The only way to have $\Delta E_{\text{rot}} = 0$ is for $\Delta \ell = 0$, which is forbidden. Thus $\Delta E = hf$ is not possible. Here is a mathematical statement as well.

$$\Delta E_{\text{rot}} = E_{(\ell+\Delta\ell)} - E_{\ell} = \left(\ell + \Delta\ell\right)\left(\ell + \Delta\ell + 1\right)\frac{\hbar^2}{2I} - \ell\left(\ell+1\right)\frac{\hbar^2}{2I} = \Delta\ell\left(2\ell + \Delta\ell + 1\right)\frac{\hbar^2}{2I}$$

For $\Delta E_{\text{rot}} = 0$, mathematically we must have $\Delta\ell = 0$, which is forbidden.

15. (*a*) The reduced mass is defined in Eq. 40-4.

$$\mu = \frac{m_{\text{C}} m_{\text{O}}}{m_{\text{C}} + m_{\text{O}}} = \frac{\left(12.01\,\text{u}\right)\left(16.00\,\text{u}\right)}{\left(12.01\,\text{u}\right) + \left(16.00\,\text{u}\right)} = \boxed{6.86\,\text{u}}$$

(*b*) We find the effective spring constant from Eq. 40-5.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \quad \rightarrow$$

$$k = 4\pi f^2 \mu = 4\pi^2\left(6.42\times10^{13}\,\text{Hz}\right)^2\left(6.86\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right) = \boxed{1850\,\text{N/m}}$$

The spring constant for H_2 is estimated in Example 40-6 as 550 N/m.

$$\frac{k_{\text{CO}}}{k_{\text{H}_2}} = \frac{1850\,\text{N/m}}{550\,\text{N/m}} = \boxed{3.4}$$

16. The effective spring constant can be found from Eq. 40-5, using the vibrational frequency and the reduced mass.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \quad \rightarrow \quad k = 4\pi f^2 \mu = 4\pi^2 f^2 \frac{m_1 m_2}{m_1 + m_2}$$

$$= 4\pi^2\left(1.7\times10^{13}\,\text{Hz}\right)^2 \frac{\left(6.941\,\text{u}\right)\left(79.904\,\text{u}\right)}{\left(6.941\,\text{u} + 79.904\,\text{u}\right)}\left(1.66\times10^{-27}\,\text{kg/u}\right) = \boxed{120\,\text{N/m}}$$

17. We first find the energies of the transitions represented by the wavelengths.

$$\Delta E_1 = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(23.1\times10^{-3}\,\text{m}\right)} = 5.38\times10^{-5}\,\text{eV}$$

$$\Delta E_2 = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(11.6\times10^{-3}\,\text{m}\right)} = 10.72\times10^{-5}\,\text{eV}$$

$$\Delta E_3 = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(7.71\times10^{-3}\,\text{m}\right)} = 16.12\times10^{-5}\,\text{eV}$$

Since $\frac{\Delta E_2}{\Delta E_1} = \frac{10.72}{5.38} \approx 2$ and $\frac{\Delta E_3}{\Delta E_1} = \frac{16.12}{5.38} \approx 3$, from the energy levels indicated in Figure 40-17, and from the selection rule that $\Delta\ell = \pm1$, we see that these three transitions must represent the $\ell = 1$ to $\ell = 0$ transition, the $\ell = 2$ to $\ell = 1$ transition, and the $\ell = 3$ to $\ell = 2$ transition. Thus $\Delta E_1 = \hbar^2/I$. We use that relationship along with Eq. 40-4 to find the bond length.

$$\Delta E_1 = \frac{\hbar^2}{I} = \frac{\hbar^2}{\mu r^2} \rightarrow$$

$$r = \frac{\hbar}{\sqrt{\mu \Delta E_1}} = \frac{\left(1.055 \times 10^{-34}\,\text{J}\bullet\text{s}\right)}{\sqrt{\left[\frac{(22.990\,\text{u})(35.453\,\text{u})}{(22.990\,\text{u} + 35.453\,\text{u})}\right]\left(1.66\times10^{-27}\,\text{kg/u}\right)\left(5.38\times10^{-5}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}$$

$$= \boxed{2.36\times10^{-10}\text{m}}$$

18. (*a*) The curve for $U = \frac{1}{2}k\left(x - r_0\right)^2 - 4.5\,\text{eV}$ is shown in Figure 40–18 as a dotted line. Measuring on the graph with a ruler gives the distance from the origin to the 0.1 nm mark as 38 mm. The measured distance from the origin to the largest x-intercept of the parabola is 45 mm. Taking a ratio gives the distance from the origin to the largest x-intercept as 0.118 nm. We fit a parabolic curve to data.

$$U = \tfrac{1}{2}k\left(x - r_0\right)^2 - 4.5\,\text{eV} \;;\; U_{x=r_0} = \tfrac{1}{2}k\left(0\right)^2 - 4.5\,\text{eV} = -4.5\,\text{eV} \;\left(\text{check}\right)$$

$$U_{x=0.118\,\text{nm}} = \tfrac{1}{2}k\left(0.118\,\text{nm} - r_0\right)^2 - 4.5\,\text{eV} = 0 \;\rightarrow$$

$$k = \frac{2\left(4.5\,\text{eV}\right)}{\left(0.118\,\text{nm} - 0.074\,\text{nm}\right)^2} = 4649\frac{\text{eV}}{\text{nm}^2}\left(\frac{1.60\times10^{-19}\,\text{J}}{\text{eV}}\right)\left(\frac{10^9\,\text{nm}}{1\,\text{m}}\right)^2 = 743.8\,\text{N/m}$$

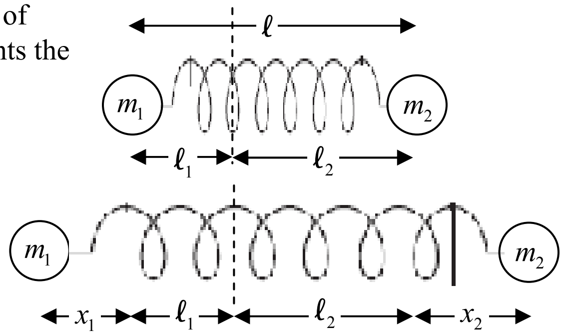
$$\approx \boxed{740\,\text{N/m}}$$

(*b*) The frequency of vibration is given by Eq. 14-7a, using the reduced mass. Use this relationship to find the wavelength.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{c}{\lambda} \;\rightarrow$$

$$\lambda = 2\pi c\sqrt{\frac{\mu}{k}} = 2\pi\left(3.00\times10^8\,\text{m/s}\right)\sqrt{\frac{0.5\left(1.00794\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)}{743.8\,\text{N/m}}} = \boxed{2.0\times10^{-6}\,\text{m}}$$

$\boxed{19.}$ Consider the system in equilibrium, to find the center of mass. See the first diagram. The dashed line represents the location of the center of mass.



$$\ell = \ell_1 + \ell_2 \;;\; m_1\ell_1 = m_2\ell_2 = m_2\left(\ell - \ell_1\right) \;\rightarrow$$

$$\ell_1 = \frac{m_2}{m_1 + m_2}\ell \;;\; \ell_2 = \frac{m_1}{m_1 + m_2}\ell$$

Now let the spring be stretched to the left and right, but let the center of mass be unmoved.

$$x = x_1 + x_2 \;;\; m_1\left(\ell_1 + x_1\right) = m_2\left(\ell_2 + x_2\right) \;\rightarrow$$

$$m_1\ell_1 + m_1x_1 = m_2\ell_2 + m_2x_2 \;\rightarrow\; \boxed{m_1x_1 = m_2x_2}$$

This is the second relationship requested in the problem. Now use the differential relationships.

$$m_1\frac{d^2x_1}{dt^2} = -kx \;;\; m_2\frac{d^2x_2}{dt^2} = -kx \;\rightarrow\; \frac{d^2x_1}{dt^2} = -\frac{k}{m_1}x \;;\; \frac{d^2x_2}{dt^2} = -\frac{k}{m_2}x$$

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = -kx\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \;\rightarrow\; \frac{d^2\left(x_1 + x_2\right)}{dt^2} = -kx\frac{m_1 + m_2}{m_1m_2} = -\frac{k}{\mu}x \;\rightarrow\; \mu\frac{d^2x}{dt^2} = -kx$$

This last equation is the differential equation for simple harmonic motion, as in Eq. 14-3, with m replaced by μ. The frequency is given by Eq. 14-7a, $f=\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$, which is the same as Eq. 40-5.

20. The ionic cohesive energy is given right after Eq. 40-9, and derived in the solution to Problem 25. The Madelung constant is 1.75.

$$U_0=-\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0}\left(1-\frac{1}{m}\right)=-(1.75)\frac{\left(2.30\times10^{-28}\,\mathrm{J\bullet m}\right)}{\left(0.28\times10^{-9}\,\mathrm{m}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}\left(1-\tfrac{1}{8}\right)=\boxed{-7.9\,\mathrm{eV}}$$

21. Because each ion occupies a cell of side s, a molecule occupies two cells. Use the value of the density to solve for the desired distance.

$$\rho=\frac{\text{mass}}{\text{volume}}=\frac{m_{\mathrm{NaCl}}}{2s^3}\quad\rightarrow$$

$$s=\left(\frac{m_{\mathrm{NaCl}}}{2\rho}\right)^{1/3}=\left[\frac{(58.44\,\mathrm{u})\left(1.661\times10^{-27}\,\mathrm{kg/u}\right)}{2\left(2.165\frac{\mathrm{g}}{\mathrm{cm}^3}\right)\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(\frac{10^6\,\mathrm{cm}^3}{1\,\mathrm{m}^3}\right)}\right]^{1/3}=\boxed{2.826\times10^{-10}\,\mathrm{m}}$$

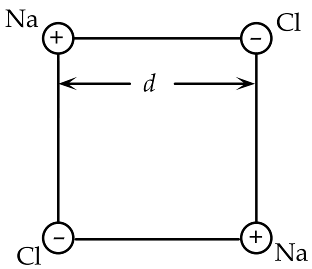
22. Because each ion occupies a cell of side s, a molecule occupies two cells. Use the value of the density to solve for the desired distance.

$$\rho=\frac{\text{mass}}{\text{volume}}=\frac{m_{\mathrm{KCl}}}{2s^3}\quad\rightarrow$$

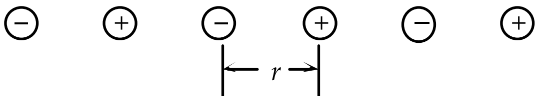
$$s=\left(\frac{m_{\mathrm{KCl}}}{2\rho}\right)^{1/3}=\left[\frac{(39.10\,\mathrm{u}+35.45\,\mathrm{u})\left(1.661\times10^{-27}\,\mathrm{kg/u}\right)}{2\left(1.99\frac{\mathrm{g}}{\mathrm{cm}^3}\right)\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(\frac{10^6\,\mathrm{cm}^3}{1\,\mathrm{m}^3}\right)}\right]^{1/3}=\boxed{3.15\times10^{-10}\,\mathrm{m}}$$

23. According to Section 40-5, the NaCl crystal is face-centered cubic. It is illustrated in Figure 40-24. We consider four of the labeled ions from Figure 40-24. See the adjacent diagram. The distance from an Na ion to a Cl ion is labeled as d, and the distance from an Na ion to the nearest neighbor Na ion is called D.

$$D=d\sqrt{2}=(0.24\,\mathrm{nm})\sqrt{2}=\boxed{0.34\,\mathrm{nm}}$$



24. See the diagram. Select a charge in the middle of the chain. There will be two charges of opposite sign a distance r away, two charges of the same sign a distance $2r$ away, etc. Calculate the potential energy of the chosen charge.



$$U=\frac{1}{4\pi\varepsilon_0}\left(\frac{-2e^2}{r}\right)+\frac{1}{4\pi\varepsilon_0}\left(\frac{+2e^2}{2r}\right)+\frac{1}{4\pi\varepsilon_0}\left(\frac{-2e^2}{3r}\right)+\frac{1}{4\pi\varepsilon_0}\left(\frac{+2e^2}{4r}\right)+\cdots$$

$$=-\frac{2e^2}{4\pi\varepsilon_0 r}\left(1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\cdots\right)$$

From Appendix A, $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^2 - \frac{1}{4}x^4 + \cdots$. Evaluate this expansion at $x = 1$.

$$\ln(1+1) = 1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots = \ln 2 \;\rightarrow\; U = -\frac{2e^2}{4\pi\varepsilon_0 r}\left(1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots\right) = -\frac{2e^2}{4\pi\varepsilon_0 r}\ln 2$$

From Section 40-5, the potential energy is given as $U = -\dfrac{\alpha e^2}{4\pi\varepsilon_0 r}$. Equate the two expressions for the potential energy to evaluate the Madelung constant.

$$U = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} = -\frac{2e^2}{4\pi\varepsilon_0 r}\ln 2 \;\rightarrow\; \boxed{\alpha = 2\ln 2}$$

$\boxed{25.}$ (*a*) Start with Eq. 40-9 and find the equilibrium distance, which minimizes the potential energy. Call that equilibrium distance r_0.

$$U = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r} + \frac{B}{r^m} \;;\; \left.\frac{dU}{dr}\right|_{r=r_0} = \left(\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r^2} - m\frac{B}{r^{m+1}}\right)_{r=r_0} = \frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0^2} - m\frac{B}{r_0^{m+1}} = 0 \;\rightarrow$$

$$B = \frac{\alpha}{4\pi\varepsilon_0}\frac{e^2 r_0^{m-1}}{m}$$

$$U_0 = U(r = r_0) = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0} + \frac{B}{r_0^m} = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0} + \frac{\dfrac{\alpha}{4\pi\varepsilon_0}\dfrac{e^2 r_0^{m-1}}{m}}{r_0^m} = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0}\left(1 - \frac{1}{m}\right)$$

(*b*) For NaI, we evaluate U_0 with $m = 10$, $\alpha = 1.75$, and $r_0 = 0.33\,\text{nm}$.

$$U_0 = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0}\left(1 - \frac{1}{m}\right) = -(1.75)\frac{2.30\times10^{-28}\,\text{J}\bullet\text{m}}{(0.33\times10^{-9}\,\text{m})(1.60\times10^{-19}\,\text{J/eV})}\left(1 - \tfrac{1}{10}\right) = -6.861\,\text{eV}$$

$$\approx \boxed{-6.9\,\text{eV}}$$

(*c*) For MgO, we evaluate U_0 with $m = 10$, $\alpha = 1.75$, and $r_0 = 0.21\,\text{nm}$.

$$U_0 = -\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0}\left(1 - \frac{1}{m}\right) = -(1.75)\frac{2.30\times10^{-28}\,\text{J}\bullet\text{m}}{(0.21\times10^{-9}\,\text{m})(1.60\times10^{-19}\,\text{J/eV})}\left(1 - \tfrac{1}{10}\right) = -10.78\,\text{eV}$$

$$\approx \boxed{-11\,\text{eV}}$$

(*d*) Calculate the % difference using $m = 8$ instead of $m = 10$.

$$\frac{U_{0\,m=8} - U_{0\,m=10}}{U_{0\,m=10}} = \frac{-\dfrac{\alpha}{4\pi\varepsilon_0}\dfrac{e^2}{r_0}\left(1 - \tfrac{1}{8}\right) - -\dfrac{\alpha}{4\pi\varepsilon_0}\dfrac{e^2}{r_0}\left(1 - \tfrac{1}{10}\right)}{-\dfrac{\alpha}{4\pi\varepsilon_0}\dfrac{e^2}{r_0}\left(1 - \tfrac{1}{10}\right)} = \frac{\left(1 - \tfrac{1}{8}\right) - \left(1 - \tfrac{1}{10}\right)}{\left(1 - \tfrac{1}{10}\right)} = \frac{\tfrac{1}{10} - \tfrac{1}{8}}{\left(1 - \tfrac{1}{10}\right)} = -0.0278$$

$$\approx \boxed{-2.8\%}$$

26. We follow Example 40-9. The density of occupied states (number of states per unit volume in an infinitesimal energy range) is given by Eq. 40-15. Because we are using a small energy range, we estimate the calculation with a difference expression. We let *N* represent the number of states, and *V* represent the volume under consideration.

$$N \approx g(E)V\Delta E = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}V\Delta E$$

$$E = \tfrac{1}{2}(E_\text{F} + 0.985E_\text{F}) = 0.9925E_\text{F} \;;\; \Delta E = E_\text{F} - 0.985E_\text{F} = 0.015E_\text{F} = 0.0822\,\text{eV}$$

$$N=\frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\,\text{kg}\right)^{3/2}}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^{3}}\sqrt{0.9925\left(5.48\,\text{eV}\right)}\left(1.0\times10^{-6}\,\text{m}^{3}\right)\left(0.0822\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)^{3/2}$$

$$=\boxed{1.3\times10^{21}\,\text{states}}$$

27. We follow Example 40-9. The density of occupied states (number of states per unit volume in an infinitesimal energy range) is given by Eq. 40-15. Because we are using a small energy range, we estimate the calculation with a difference expression. We let N represent the number of states, and V represent the volume under consideration.

$$N\approx g\left(E\right)V\Delta E=\frac{8\sqrt{2}\pi m^{3/2}}{h^{3}}E^{1/2}V\Delta E$$

$$=\frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\,\text{kg}\right)^{3/2}}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^{3}}\sqrt{\left(7.025\,\text{eV}\right)}\left(1.0\times10^{-6}\,\text{m}^{3}\right)\left(0.05\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)^{3/2}$$

$$=\boxed{9.0\times10^{20}\,\text{states}}$$

28. The density of molecules in an ideal gas can be found from the ideal gas law.

$$PV=NkT\quad\rightarrow\quad\left(\frac{N}{V}\right)_{\text{gas}}=\frac{P}{kT}=\frac{1.013\times10^{5}\,\text{Pa}}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(285\,\text{K}\right)}=2.576\times10^{25}\,\text{m}^{-3}$$

We assume that each copper atom contributes one free electron, and use the density of copper as given in Table 13-1.

$$\left(\frac{N}{V}\right)_{\text{e's}}=\left(\frac{1\text{e}}{\text{Cu atom}}\right)\left(\frac{6.02\times10^{23}\,\text{Cu atoms}}{63.546\times10^{-3}\,\text{kg}}\right)\left(\frac{8.9\times10^{3}\,\text{kg}}{\text{m}^{3}}\right)=8.431\times10^{28}\,\text{m}^{-3}$$

$$\frac{\left(\frac{N}{V}\right)_{\text{gas}}}{\left(\frac{N}{V}\right)_{\text{e's}}}=\frac{2.576\times10^{25}\,\text{m}^{-3}}{8.431\times10^{28}\,\text{m}^{-3}}=\boxed{3.1\times10^{-4}}$$

29. We use Eq. 40-14 for the occupancy probability, and solve for the energy. The Fermi energy is 7.0 eV.

(*a*) Evaluate for T = 295 K.

$$f\left(E\right)=\frac{1}{e^{\left(E-E_{\text{F}}\right)/kT}+1}\quad\rightarrow$$

$$E=kT\ln\left[\frac{1}{f\left(E\right)}-1\right]+E_{\text{F}}=\frac{\left(1.38\times10^{-23}\,\text{J/K}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}\left(295\,\text{K}\right)\ln\left(\frac{1}{0.850}-1\right)+7.0\,\text{eV}$$

$$=\boxed{6.96\,\text{eV}}$$

(*b*) Evaluate for T = 750 K.

$$f\left(E\right)=\frac{1}{e^{\left(E-E_{\text{F}}\right)/kT}+1}\quad\rightarrow$$

$$E=kT\ln\left[\frac{1}{f\left(E\right)}-1\right]+E_{\text{F}}=\frac{\left(1.38\times10^{-23}\,\text{J/K}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}\left(750\,\text{K}\right)\ln\left(\frac{1}{0.850}-1\right)+7.0\,\text{eV}$$

$$=\boxed{6.89\,\text{eV}}$$

30. We use Eq. 40-14 for the occupancy probability, and solve for the energy. The Fermi energy is 7.0 eV.

(*a*) Evaluate for *T* = 295 K.

$$f(E)=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1} \quad\rightarrow$$

$$E=kT\ln\left[\frac{1}{f(E)}-1\right]+E_\mathrm{F}=\frac{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}(295\,\mathrm{K})\ln\left(\frac{1}{0.150}-1\right)+7.0\,\mathrm{eV}$$

$$=\boxed{7.04\,\mathrm{eV}}$$

(*b*) Evaluate for *T* = 950 K.

$$f(E)=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1} \quad\rightarrow$$

$$E=kT\ln\left[\frac{1}{f(E)}-1\right]+E_\mathrm{F}=\frac{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}(950\,\mathrm{K})\ln\left(\frac{1}{0.150}-1\right)+7.0\,\mathrm{eV}$$

$$=\boxed{7.14\,\mathrm{eV}}$$

$\boxed{31.}$ The occupancy probability is given by Eq. 40-14. The Fermi level for copper is 7.0 eV.

$$f=\frac{1}{e^{(E-E_\mathrm{F})/kT}+1}=\frac{1}{e^{(1.015E_\mathrm{F}-E_\mathrm{F})/kT}+1}=\frac{1}{e^{\left[0.015(7.0\,\mathrm{eV})\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)\right]/\left[\left(1.38\times10^{-23}\,\mathrm{J/K}\right)(295\,\mathrm{K})\right]}+1}=0.0159$$

$$\approx\boxed{1.6\%}$$

32. We follow Example 40-10.

(*a*) Because each zinc atom contributes two free electrons, the density of free electrons is twice the density of atoms.

$$\frac{N}{V}=\left(7100\,\mathrm{kg/m^3}\right)\left(\frac{6.02\times10^{23}\,\mathrm{atoms/mol}}{65.409\times10^{-3}\,\mathrm{kg/mol}}\right)(2\,\text{free electrons/atom})=1.307\times10^{29}\,\mathrm{m^{-3}}$$

$$\approx\boxed{1.3\times10^{29}\,\mathrm{m^{-3}}}$$

(*b*) The Fermi energy is given by Eq. 40-12.

$$E_\mathrm{F}=\frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2}{8\left(9.11\times10^{-31}\,\mathrm{kg}\right)}\left[\frac{3}{\pi}\left(1.307\times10^{29}\,\mathrm{m^{-3}}\right)\right]^{2/3}\left(\frac{1}{1.60\times10^{-19}\,\mathrm{J/eV}}\right)$$

$$=9.414\,\mathrm{eV}\approx\boxed{9.4\,\mathrm{eV}}$$

(*c*) The Fermi speed is the speed of electrons with the Fermi energy.

$$v_\mathrm{F}=\sqrt{\frac{2E_\mathrm{F}}{m}}=\sqrt{\frac{2(9.414\,\mathrm{eV})\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)}}=\boxed{1.8\times10^{6}\,\mathrm{m/s}}$$

33. We follow Example 40-10. We need the number of conduction electrons per unit volume of sodium.

$$\frac{N}{V}=\left(970\,\mathrm{kg/m^3}\right)\left(\frac{6.02\times10^{23}\,\mathrm{atoms/mol}}{22.99\times10^{-3}\,\mathrm{kg/mol}}\right)(1\,\text{free electrons/atom})=2.540\times10^{28}\,\mathrm{m^{-3}}$$

The Fermi energy is given by Eq. 40-12.

$$E_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(9.11\times10^{-31}\,\text{kg}\right)}\left[\frac{3}{\pi}\left(2.540\times10^{28}\,\text{m}^{-3}\right)\right]^{2/3}\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right) = 3.159\,\text{eV}$$

$$\approx \boxed{3.2\,\text{eV}}$$

The Fermi speed is the speed of electrons with the Fermi energy.

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2\left(3.159\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}} = \boxed{1.1\times10^6\,\text{m/s}}$$

34. (*a*) Find the density of free electrons from Eq. 40-12.

$$E_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} \rightarrow$$

$$\frac{N}{V} = \frac{\pi}{3}\left(\frac{8mE_F}{h^2}\right)^{3/2} = \frac{\pi}{3}\left[\frac{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(11.63\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}\right]^{3/2}$$

$$= 1.7945\times10^{29}\,\text{m}^{-3} \approx \boxed{1.79\times10^{29}\,\text{m}^{-3}}$$

(*b*) Let *n* represent the valence number, so there are *n* free electrons per atom.

$$\frac{N}{V} = \left(270\,\text{kg/m}^3\right)\left(\frac{6.02\times10^{23}\,\text{atoms/mol}}{27.0\times10^{-3}\,\text{kg/mol}}\right)\left(n\,\text{free electrons/atom}\right) \rightarrow$$

$$n = \left(\frac{27.0\times10^{-3}\,\text{kg/mol}}{6.02\times10^{23}\,\text{atoms/mol}}\right)\left(\frac{1}{270\,\text{kg/m}^3}\right)\left(1.7945\times10^{29}\,\text{m}^{-3}\right) = 2.981 \approx \boxed{3}$$

This agrees nicely with aluminum's position in the periodic table, and its electron configuration of $1s^2 2s^2 2p^6 3s^2 3p^1$. The level 3 electrons are the valence electrons.

35. We calculate the given expression, with $T = 0$, so that the maximum energy is E_F. The value of $f(E)$ at $T = 0$ is given below Eq. 40-14.

$$\bar{E} = \frac{\int_0^{E_F} E n_0(E)\,dE}{\int_0^{E_F} n_0(E)\,dE} = \frac{\int_0^{E_F} E g(E) f(E)\,dE}{\int_0^{E_F} g(E) f(E)\,dE} = \frac{\int_0^{E_F} E\left(\frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}\right)(1)\,dE}{\int_0^{E_F}\left(\frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}\right)(1)\,dE} = \frac{\int_0^{E_F} E^{3/2}\,dE}{\int_0^{E_F} E^{1/2}\,dE}$$

$$= \frac{\frac{2}{5}E_F^{5/2}}{\frac{2}{3}E_F^{3/2}} = \frac{3}{5}E_F$$

36. We first find the density of neutrons, and then use Eq. 40-12.

$$\frac{N}{V} = \left(\frac{1\text{ neutron}}{1.675\times10^{-27}\,\text{kg}}\right)\left[2.5\left(1.99\times10^{30}\,\text{kg}\right)\right]\left[\frac{1}{\frac{4}{3}\pi\left(12{,}000\,\text{m}\right)^3}\right] = 4.103\times10^{44}\,\text{m}^{-3}$$

$$E_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{8\left(1.675\times10^{-27}\,\text{kg}\right)}\left[\frac{3}{\pi}\left(4.103\times10^{44}\,\text{m}^{-3}\right)\right]^{2/3}\left(\frac{1}{1.60\times10^{-13}\,\text{J/MeV}}\right)$$

$$= 109.8\,\text{MeV} \approx \boxed{110\,\text{MeV}}$$

$\boxed{37.}$ We start with Eq. 38-13 for the energy level as a function of *n*. If we solve for *n*, we have the number of levels with energies between 0 and *E*. Taking the differential of that expression will give the number of levels with energies between E and dE. Finally, we multiply by 2 since there can be 2 electrons (with opposite spins) in each energy level.

$$E = n^2 \frac{h^2}{8m\ell^2} \rightarrow n = \sqrt{\frac{8m\ell^2}{h^2}}\sqrt{E} \rightarrow dn = \sqrt{\frac{8m\ell^2}{h^2}}\frac{1}{2}\frac{dE}{\sqrt{E}} \rightarrow$$

$$g_\ell = 2\frac{dn}{dE} = 2\frac{\sqrt{\frac{8m\ell^2}{h^2}}\frac{1}{2}\frac{dE}{\sqrt{E}}}{dE} = \boxed{\sqrt{\frac{8m\ell^2}{h^2 E}}}$$

38. We use Eq. 40-14, with $E = E_F$.

$$f = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^0+1} = \frac{1}{1+1} = \frac{1}{2}$$

The result is independent of the value of *T*.

39. (*a*) Eq. 38-13 gives the energy levels as a function of *n*, the number of levels. Since there are 2 electrons in every energy level, *n* = *N* / 2. The Fermi energy will be the highest energy level occupied with *N* electrons.

$$E_n = n^2 \frac{h^2}{8m\ell^2} = \left(\tfrac{1}{2}N\right)^2 \frac{h^2}{8m\ell^2} = \frac{h^2N^2}{32m\ell^2} \rightarrow \boxed{E_F = \frac{h^2N^2}{32m\ell^2}}$$

(*b*) The smallest amount of energy that this metal can absorb is the spacing between energy levels.

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8m\ell^2}\left[(n+1)^2 - n^2\right] = \frac{h^2}{8m\ell^2}(2n+1) = \boxed{\frac{h^2}{8m\ell^2}(N+1)}$$

(*c*) We calculate the limit requested.

$$\frac{\Delta E}{E_F} = \frac{\frac{h^2}{8m\ell^2}(N+1)}{\frac{h^2N^2}{32m\ell^2}} = \boxed{\frac{4}{N}}$$

For large *N*, this is a very small change in energy. Thus a very small change in energy will allow an electron to change energy levels, and so the metal conducts very easily.

40. (*a*) We use Eq. 40-14, with the data as given in the problem.

$$\frac{E-E_F}{kT} = \frac{(0.12\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{k(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})} = 4.74848$$

$$f = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{4.74848}+1} = \frac{1}{116.409} = 8.590\times10^{-3} \approx \boxed{8.6\times10^{-3}}$$

This is reasonable. Very few states this far above the Fermi energy are occupied at this relatively low temperature.

(*b*) Use a similar calculation to part (*a*).

$$\frac{E-E_F}{kT} = \frac{(-0.12\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{k(1.38\times10^{-23}\,\text{J/K})(293\,\text{K})} = -4.74848$$

$$f = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{-4.74848}+1} = \frac{1}{1.008665} = 0.991409 \approx \boxed{0.99}$$

(*c*) Since the probability that the state is occupied is 0.991409, the probability that the state is unoccupied is $1-0.991409=8.591\times10^{-3}\approx\boxed{8.6\times10^{-3}}$. This is the same as part (*a*).

41. We consider the cube to be a three-dimensional infinite well, with a width of ℓ in each dimension. We apply the boundary conditions as in Section 38-8 separately to each dimension. Each dimension gives a quantum number which we label as $n_1,\ n_2,$ and n_3. We then have a contribution to the energy of the bound particle from each quantum number, as in Eq. 38-13.

$$E=E_1+E_2+E_3=n_1^2\frac{h^2}{8m\ell^2}+n_2^2\frac{h^2}{8m\ell^2}+n_3^2\frac{h^2}{8m\ell^2}=\frac{h^2}{8m\ell^2}\left(n_1^2+n_2^2+n_3^2\right),\ n_1,n_2,n_3=1,2,3,\cdots$$

Specifying the three quantum numbers gives a state and the corresponding energy.

Choosing axes as specified in the problem, the equation of a sphere of radius *R* in that coordinate system is $R^2=n_1^2+n_2^2+n_3^2$. Each state "contained" in that sphere could be indicated by a cube of side length 1, and each state can have two electrons (two spin states). The "volume" of that sphere is $\frac{1}{8}$ of a full sphere. From that we calculate the number of states in one octant, and then *g*(*E*).

$$N=2\left(\tfrac{1}{8}\right)\tfrac{4}{3}\pi R^3=\frac{\pi}{3}\left(n_1^2+n_2^2+n_3^2\right)^{3/2}=\frac{\pi}{3}\left(\frac{8m\ell^2}{h^2}E\right)^{3/2}$$

$$g(E)=\frac{1}{V}\frac{dN}{dE}=\frac{1}{\ell^3}\frac{\pi}{3}\left(\frac{8m\ell^2}{h^2}\right)^{3/2}\frac{3}{2}E^{1/2}=\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2}E^{1/2}=\boxed{\frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}}$$

42. The photon with the minimum frequency for conduction must have an energy equal to the energy gap.

$$E_g=hf=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(580\times10^{-9}\,\text{m}\right)}=\boxed{2.14\,\text{eV}}$$

43. The photon with the longest wavelength or minimum frequency for conduction must have an energy equal to the energy gap:

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{hc}{E_g}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(1.14\,\text{eV}\right)}=1.09\times10^{-6}\,\text{m}=\boxed{1.09\,\mu\text{m.}}$$

44. The energy of the photon must be greater than or equal to the energy gap. Thus the longest wavelength that will excite an electron is

$$\lambda=\frac{c}{f}=\frac{hc}{hf}=\frac{hc}{E_g}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(0.72\,\text{eV}\right)}=1.7\times10^{-6}\,\text{m}=1.7\,\mu\text{m}$$

Thus the wavelength range is $\boxed{\lambda\le1.7\,\mu\text{m}}$.

45. (*a*) In the 2*s* shell of an atom, $\ell=0$, so there are two states: $m_s=\pm\frac{1}{2}$. When *N* atoms form bands, each atom provides 2 states, so the total number of states in the band is $\boxed{2N.}$

(*b*) In the 2*p* shell of an atom, $\ell=1$, so there are three states from the m_ℓ values: $m_\ell=0,\pm1$; each of which has two states from the m_s values: $m_s=\pm\frac{1}{2}$, for a total of 6 states. When *N* atoms form bands, each atom provides 6 states, so the total number of states in the band is $\boxed{6N.}$

(*c*) In the 3*p* shell of an atom, $\ell = 1$, so there are three states from the m_ℓ values: $m_\ell = 0, \pm 1$; each of which has two states from the m_s values: $m_s = \pm\frac{1}{2}$, for a total of 6 states. When *N* atoms form bands, each atom provides 6 states, so the total number of states in the band is $\boxed{6N.}$

(*d*) In general, for a value of ℓ, there are $2\ell + 1$ states from the m_ℓ values: $m_\ell = 0, \pm 1, \dots, \pm\ell$. For each of these there are two states from the m_s values: $m_s = \pm\frac{1}{2}$, for a total of $2(2\ell + 1)$ states. When *N* atoms form bands, each atom provides $2(2\ell + 1)$ states, so the total number of states in the band is $\boxed{2N(2\ell + 1).}$

46. The minimum energy provided to an electron must be equal to the energy gap. Divide the total available energy by the energy gap to estimate the maximum number of electrons that can be made to jump.

$$N = \frac{hf}{E_g} = \frac{(760 \times 10^3 \,\text{eV})}{(0.72\,\text{eV})} = \boxed{1.1 \times 10^6}$$

47. Calculate the number of conduction electrons in a mole of pure silicon. Also calculate the additional conduction electrons provided by the doping, and then take the ratio of those two numbers of conduction electrons.

$$N_{\text{Si}} = \left[\frac{(28.09 \times 10^{-3}\,\text{kg/mol})}{(2330\,\text{kg/m}^3)}\right](10^{16}\,\text{electrons/m}^3) = 1.206 \times 10^{11}\,\text{electrons/mole}$$

$$N_{\text{doping}} = \frac{(6.02 \times 10^{23}\,\text{atoms})}{1.2 \times 10^6} = 5.017 \times 10^{17} \text{ added conduction electrons.}$$

$$\frac{N_{\text{doping}}}{N_{\text{Si}}} = \frac{(5.017 \times 10^{17})}{(1.206 \times 10^{11})} = 4.16 \times 10^6 \approx \boxed{4 \times 10^6}$$

48. The wavelength is found from the energy gap.

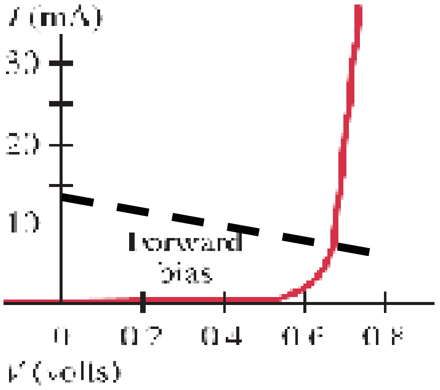
$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_g} = \frac{(6.63 \times 10^{-34}\,\text{J}\bullet\text{s})(3.00 \times 10^8\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{J/eV})(1.6\,\text{eV})} = 7.8 \times 10^{-7}\,\text{m} = \boxed{0.78\,\mu\text{m}}$$

$\boxed{49.}$ The photon will have an energy equal to the energy gap:

$$E_g = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\,\text{J}\bullet\text{s})(3.00 \times 10^8\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{J/eV})(680 \times 10^{-9}\,\text{m})} = \boxed{1.8\,\text{eV}}$$

50. From the current-voltage characteristic graph in Figure 40-38, we see that a current of 12 mA means a voltage of about 0.68 V across the diode. The battery voltage is the sum of the voltages across the diode and the resistor.

$$V_{\text{battery}} = V_{\text{diode}} + V_{\text{R}} = 0.68\,\text{V} + (0.012\,\text{A})(860\,\Omega) = \boxed{11\,\text{V}}$$



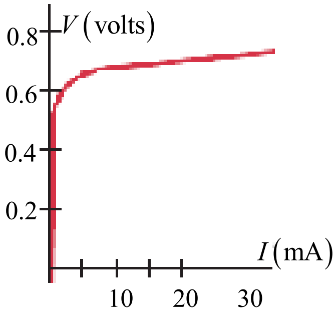
51. The battery voltage is the sum of the voltages across the diode and the resistor.

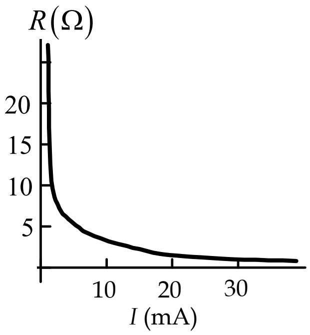
$$V_{\text{battery}} = V_{\text{diode}} + V_{\text{R}};$$

$$2.0\,\text{V} = V_{\text{diode}} + I\left(150\,\Omega\right) \rightarrow I = -\frac{V_{\text{diode}}}{150\,\Omega} + \frac{2.0\,\text{V}}{150\,\Omega}$$

This is the equation for a straight line which passes through the points (0 V, 13.3 mA) and (0.8 V, 8 mA). The line has a y-intercept of 13.3 mA and a slope of 6.67 mA/V. If we assume the operating voltage of the diode is about 0.7 V, then the current is about $\boxed{8.6\text{ mA}}$. There is some approximation involved in the answer.

52. We have copied the graph for $V > 0$ and rotated it so that it shows V as a function of I. This is the first diagram below. The resistance is the slope of that first graph. The slope, and thus the resistance, is very high for low currents, and decreases for larger currents, approaching 0. As an approximate value, we see that the voltage changes from about 0.55 V to 0.65 V as the current goes from 0 to 10 mA. That makes the resistance about 10 ohms when the current is about 5 mA. The second diagram is a sketch of the resistance.





53. (*a*) For a half-wave rectifier without a capacitor, the current is zero for half the time. We approximate the average current as half of the full rms current.

$$I_{\text{av}} = \tfrac{1}{2}\frac{V_{\text{rms}}}{R} = \tfrac{1}{2}\frac{\left(120\,\text{V}\right)}{\left(35\,\text{k}\Omega\right)} = \boxed{1.7\,\text{mA}}$$

(*b*) For a full-wave rectifier without a capacitor, the current is positive all the time. We approximate the average current as equal to the full rms current.

$$I_{\text{av}} = \frac{V_{\text{rms}}}{R} = \frac{\left(120\,\text{V}\right)}{\left(35\,\text{k}\Omega\right)} = \boxed{3.4\,\text{mA}}$$

54. The band gap is the energy corresponding to the emitted wavelength.

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.3\times10^{-6}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{0.96\,\text{eV}}$$

$\boxed{55.}$ There will be a current in the resistor while the ac voltage varies from 0.6 V to 9.0 V rms. Because the 0.6 V is small, the voltage across the resistor will be almost sinusoidal, so the rms voltage across the resistor will be close to $9.0\,\text{V} - 0.6\,\text{V} = 8.4\,\text{V}$.

(*a*) For a half-wave rectifier without a capacitor, the current is zero for half the time. We ignore the short time it takes for the voltage to increase from 0 to 0.6 V, and so current is flowing in the resistor for about half the time. We approximate the average current as half of the full rms current.

$$I_{\text{av}} = \tfrac{1}{2}\frac{V_{\text{rms}}}{R} = \tfrac{1}{2}\frac{8.4\,\text{V}}{0.120\,\text{k}\Omega} = \boxed{35\,\text{mA}}$$

(*b*) For a full-wave rectifier without a capacitor, the current is positive all the time. We ignore the short times it takes for the voltage to increase from 0 to 0.6 V, and so current is flowing in the resistor all the time. We approximate the average current as the full rms current.

$$I_{\text{av}} = \frac{V_{\text{rms}}}{R} = \frac{8.4\,\text{V}}{0.120\,\text{k}\Omega} = \boxed{70\,\text{mA}}$$

56. (*a*) The time constant for the circuit is $\tau_1 = RC_1 = \left(28\times10^3\,\Omega\right)\left(35\times10^{-6}\,\text{F}\right) = 0.98\,\text{s}$. As seen in Figure 40-40(c), there are two peaks per cycle. The period of the rectified voltage is $T = \frac{1}{120}\,\text{s} = 0.0083\,\text{s}$. Because $\tau_1 \gg T$, the voltage across the capacitor will be essentially constant during a cycle, so the average voltage is the same as the peak voltage. The average current is basically constant.

$$I_{\text{avg}} = \frac{V_{\text{avg}}}{R} = \frac{V_{\text{peak}}}{R} = \frac{\sqrt{2}V_{\text{rms}}}{R} = \frac{\sqrt{2}\left(120\,\text{V}\right)}{\left(28\times10^3\,\Omega\right)} = \boxed{6.1\,\text{mA}}$$

(*b*) With a different capacitor, the time constant for the circuit changes.

$$\tau_2 = RC_2 = \left(28\times10^3\,\Omega\right)\left(0.10\times10^{-6}\,\text{F}\right) = 0.0028\,\text{s}$$

Now the period of the rectified voltage is about 3 time constants, and so the voltage will decrease to about 5% $\left(e^{-3}\right)$ of the peak value during each half-cycle. We approximate the voltage as dropping linearly from its peak value to 0 over each half-cycle, and so take the average voltage as half the peak voltage.

$$I_{\text{avg}} = \frac{V_{\text{avg}}}{R} = \tfrac{1}{2}\frac{\sqrt{2}V_{\text{rms}}}{R} = \tfrac{1}{2}\frac{\sqrt{2}\left(120\,\text{V}\right)}{\left(28\times10^3\,\Omega\right)} = \boxed{3.0\,\text{mA}}$$

57. By Ohm's law, the output (collector) current times the output resistor will be the output voltage.

$$V_{\text{out}} = i_{\text{C}}R_{\text{C}} \rightarrow R_{\text{C}} = \frac{V_{\text{out}}}{i_{\text{C}}} = \frac{V_{\text{out}}}{\beta i_{\text{B}}} = \frac{0.35\,\text{V}}{95\left(1.0\times10^{-6}\,\text{A}\right)} = 3684\,\Omega \approx \boxed{3700\,\Omega}$$

58. By Ohm's law, the output (collector) current times the output resistor will be the output voltage.

$$V_{\text{out}} = i_{\text{C}}R_{\text{C}} = \beta i_{\text{B}}R_{\text{C}} = \left(85\right)\left(2.0\times10^{-6}\,\text{A}\right)\left(4300\,\Omega\right) = \boxed{0.73\,\text{V}}$$

59. By Ohm's law, the output (collector) current times the output resistor will be the output voltage.

$$V_{\text{out}} = i_{\text{C}}R \rightarrow i_{\text{C}} = \frac{V_{\text{out}}}{R} = \frac{\beta_V V_{\text{input}}}{R} = \frac{65\left(0.080\,\text{V}\right)}{25{,}000\,\Omega} = 2.08\times10^{-4}\,\text{A} \approx \boxed{0.21\,\text{mA}}$$

60. (*a*) The voltage gain is the collector ac voltage divided by the base ac voltage.

$$\beta_V = \frac{V_{\text{C}}}{V_{\text{B}}} = \frac{i_{\text{C}}R_{\text{C}}}{i_{\text{B}}R_{\text{B}}} = \beta_I\frac{R_{\text{C}}}{R_{\text{B}}} = 75\left(\frac{7.8\,\text{k}\Omega}{3.8\,\text{k}\Omega}\right) = 153.9 \approx \boxed{150}$$

(*b*) The power amplification is the output power divided by the input power.

$$\beta_{\text{P}} = \frac{i_{\text{C}}V_{\text{C}}}{i_{\text{B}}V_{\text{B}}} = \beta_I\beta_V = \left(75\right)\left(153.9\right) = 11{,}543 \approx \boxed{12{,}000}$$

$\boxed{61.}$ The arrow at the emitter terminal, E, indicates the direction of current I_E. The current into the transistor must equal the current out of the transistor.

$$\boxed{I_B + I_C = I_E}$$

62. For an electron confined in 1 dimension, we find the uncertainty in the momentum from Eq. 38-1, $\Delta p \approx \frac{\hbar}{\Delta x}$. The momentum of the electron must be at least as big as the uncertainty in the momentum, so we approximate $p \approx \frac{\hbar}{\Delta x}$. Finally, we calculate the kinetic energy by $K = \frac{p^2}{2m}$. Find the difference in the two kinetic energies based on the two position uncertainties.

$$K = \frac{p^2}{2m} = \frac{\hbar^2}{2m(\Delta x)^2}$$

$$\Delta K = K_{\text{in atoms}} - K_{\text{molecule}} = \frac{\hbar^2}{2m}\left[\frac{1}{(\Delta x)^2_{\text{in atoms}}} - \frac{1}{(\Delta x)^2_{\text{molecule}}}\right]$$

$$= \frac{\left(1.055\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left[\frac{1}{\left(0.053\times10^{-9}\,\text{m}\right)^2_{\text{in atoms}}} - \frac{1}{\left(0.074\times10^{-9}\,\text{m}\right)^2_{\text{molecule}}}\right]\left(\frac{1}{1.60\times10^{-19}\,\text{J/eV}}\right)$$

$$= 6.62\,\text{eV}$$

There are two electrons, and each one has this kinetic energy difference, so the total kinetic energy difference is $2(6.62\,\text{eV}) = 13.2\,\text{eV} \approx \boxed{13\,\text{eV}}$.

63. We find the temperature from the given relationship.

(*a*) $$K = \tfrac{3}{2}kT \;\rightarrow\; T = \frac{2K}{3k} = \frac{2(4.0\,\text{eV})\left(1.60\times10^{-19}\,\text{J/eV}\right)}{3\left(1.38\times10^{-23}\,\text{J/K}\right)} = \boxed{3.1\times10^4\,\text{K}}$$

(*b*) $$K = \tfrac{3}{2}kT \;\rightarrow\; T = \frac{2K}{3k} = \frac{2(0.12\,\text{eV})\left(1.60\times10^{-19}\,\text{J/eV}\right)}{3\left(1.38\times10^{-23}\,\text{J/K}\right)} = \boxed{930\,\text{K}}$$

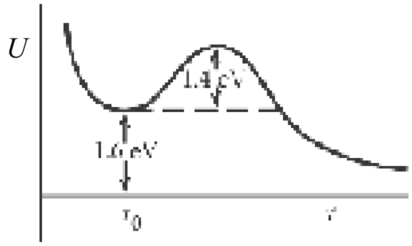
64. (*a*) The potential energy for the point charges is found as from Eq. 23-10.

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{\left(2.30\times10^{-28}\,\text{J}\bullet\text{m}\right)}{\left(0.27\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = -5.32\,\text{eV} \approx \boxed{-5.3\,\text{eV}}$$

(*b*) Because the potential energy of the ions is negative, 5.32 eV is released when the ions are brought together. The other energies quoted involve the transfer of the electron from the K atom to the F atom. 3.41 eV is released and 4.34 eV is absorbed in the individual electron transfer processes. Thus the total binding energy is as follows.

$$\text{Binding energy} = 5.32\,\text{eV} + 3.41\,\text{eV} - 4.34\,\text{eV} = 4.39\,\text{eV} \approx \boxed{4.4\,\text{eV}}$$

65. The diagram here is similar to Figure 40-9 and Figure 40-11. The activation energy is the energy needed to get the (initially) stable system over the barrier in the potential energy. The activation energy is 1.4 eV for this molecule. The dissociation energy is the energy that is released when the bond is broken. The dissociation energy is 1.6 eV for this molecule.



66. (*a*) The equilibrium position is the location where the potential energy is a minimum. We find that location by setting the derivative of the potential energy equal to 0.

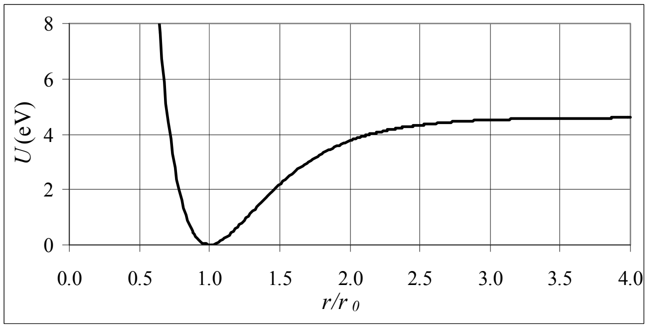
$$U = U_0\left[1-e^{-a(r-r_0)}\right]^2 \ ; \ \frac{dU}{dr} = 2U_0\left[1-e^{-a(r-r_0)}\right]\left(ae^{-a(r-r_0)}\right) = 0 \ \rightarrow \ 1-e^{-a(r-r_0)} = 0$$

$$e^{-a(r-r_0)} = 1 \ \rightarrow \ a(r-r_0) = 0 \ \rightarrow \ \boxed{r = r_0}$$

The dissociation energy is the energy difference between the two states of equilibrium separation and infinite separation.

$$\Delta U = U_{r=\infty} - U_{r=r_0} = U_0\left[1-e^{-a(\infty-r_0)}\right]^2 - U_0\left[1-e^{-a(r_0-r_0)}\right]^2 = U_0[1-0]^2 - U_0[1-1]^2 = \boxed{U_0}$$

(*b*) See the included graph. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH40.XLS,” on tab “Problem 40.66b.”



$\boxed{67.}$ (*a*) The reduced mass is defined in Eq. 40-4.

$$\mu = \frac{m_{\mathrm{H}}m_{\mathrm{Cl}}}{m_{\mathrm{H}}+m_{\mathrm{Cl}}} = \frac{(1.008\,\mathrm{u})(35.453\,\mathrm{u})}{(1.008\,\mathrm{u})+(35.453\,\mathrm{u})} = \boxed{0.9801\,\mathrm{u}}$$

(*b*) We find the effective spring constant from Eq. 40-5.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \ \rightarrow$$

$$k = 4\pi f^2\mu = 4\pi^2\left(8.66\times10^{13}\,\mathrm{Hz}\right)^2(0.9801\,\mathrm{u})\left(1.6605\times10^{-27}\,\mathrm{kg/u}\right) = \boxed{482\,\mathrm{N/m}}$$

The spring constant for H_2 is estimated in Example 40-6 as 550 N/m.

$$\frac{k_{\mathrm{CO}}}{k_{\mathrm{H_2}}} = \frac{482\,\mathrm{N/m}}{550\,\mathrm{N/m}} = \boxed{0.88}$$

68. Vibrational states have a constant energy difference of $\Delta E_{\mathrm{vib}} = 0.54\,\mathrm{eV}$, as found in Example 40-7. Rotational states have a varying energy difference, depending on the ℓ value, of $\Delta E_{\mathrm{rot}} = \frac{\hbar^2\ell}{I}$, where ℓ represents the upper energy state, as given in Eq. 40-3.

$$\Delta E_{\mathrm{rot}} = \frac{\hbar^2\ell}{I} = \frac{\hbar^2\ell}{\mu r_0^2} = \ell\frac{\left(1.055\times10^{-34}\,\mathrm{J{\bullet}s}\right)^2}{(0.5)(1.008\,\mathrm{u})\left(1.66\times10^{-27}\,\mathrm{kg/u}\right)\left(0.074\times10^{-9}\,\mathrm{m}\right)^2\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}$$

$$= (0.0152\,\mathrm{eV})\ell$$

Each gap, as represented in Figure 40-17, is larger. We add those gaps until we reach $0.54\,\mathrm{eV}$.

$$\ell = 1 \rightarrow \ell = 0:\ \Delta E_{\mathrm{rot}} = 0.0152\,\mathrm{eV} \qquad \sum\Delta E_{\mathrm{rot}} = 0.0152\,\mathrm{eV}$$

$$\ell = 2 \rightarrow \ell = 1:\ \Delta E_{\mathrm{rot}} = 2(0.0152\,\mathrm{eV}) \qquad \sum\Delta E_{\mathrm{rot}} = 3(0.0152\,\mathrm{eV}) = 0.0456\,\mathrm{eV}$$

$$\ell = 3 \rightarrow \ell = 2:\ \Delta E_{\mathrm{rot}} = 3(0.0152\,\mathrm{eV}) \qquad \sum\Delta E_{\mathrm{rot}} = 6(0.0152\,\mathrm{eV}) = 0.0912\,\mathrm{eV}$$

$$\ell=4\ \rightarrow\ \ell=3:\ \Delta E_{\text{rot}}=4(0.0152\,\text{eV})\qquad \sum\Delta E_{\text{rot}}=10(0.0152\,\text{eV})=0.152\,\text{eV}$$
$$\ell=5\ \rightarrow\ \ell=4:\ \Delta E_{\text{rot}}=5(0.0152\,\text{eV})\qquad \sum\Delta E_{\text{rot}}=15(0.0152\,\text{eV})=0.228\,\text{eV}$$
$$\ell=6\ \rightarrow\ \ell=5:\ \Delta E_{\text{rot}}=6(0.0152\,\text{eV})\qquad \sum\Delta E_{\text{rot}}=21(0.0152\,\text{eV})=0.3192\,\text{eV}$$
$$\ell=7\ \rightarrow\ \ell=8:\ \Delta E_{\text{rot}}=7(0.0152\,\text{eV})\qquad \sum\Delta E_{\text{rot}}=28(0.0152\,\text{eV})=0.4256\,\text{eV}$$
$$\ell=8\ \rightarrow\ \ell=7:\ \Delta E_{\text{rot}}=8(0.0152\,\text{eV})\qquad \sum\Delta E_{\text{rot}}=36(0.0152\,\text{eV})=0.5472\,\text{eV}$$

So we see that rotational states from $\ell=0$ to $\ell=7$ can be “between” vibrational states, or a total of $\boxed{\text{8 rotational states}}$.

69. The Boltzmann indicates that the population of a state decreases as the energy of the state increases, according to $N_n \propto e^{-E_n/kT}$. The rotation energy of states increases with higher ℓ values, according to Eq. 40-2. Thus states with higher values of ℓ have higher energies, and so there are fewer molecules in those states. Since the higher states are less likely to be populated, they are less likely to absorb a photon. As an example, the probability of absorption between $\ell=1$ and $\ell=2$ is more likely than between $\ell=2$ and $\ell=3$, and so the peak representing the transition between $\ell=1$ and $\ell=2$ is higher than the peak representing the transition between $\ell=2$ and $\ell=3$.

    The molecule is not rigid, and so the distance between the two ions is not constant. The moment of inertia depends on the bond length, and the energy levels depend on the moment of inertia. Thus the energy levels are not exactly equally spaced.

70. From Figure 40-17, a rotational absorption spectrum would show peaks at energies of $\hbar^2/I$, $2\hbar^2/I$, $3\hbar^2/I$, etc. Adjacent peaks are separated by an energy of $\hbar^2/I$. We use the photon frequency at that energy to determine the rotational inertia.

$$\Delta E=\frac{\hbar^2}{I}\quad\rightarrow\quad I=\frac{\hbar^2}{\Delta E}=\frac{\hbar^2}{hf}=\frac{h}{4\pi^2 f}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{4\pi^2(8.4\times10^{11}\,\text{Hz})}=\boxed{2.0\times10^{-47}\,\text{kg}\bullet\text{m}^2}$$

71. To use silicon to filter the wavelengths, wavelengths below the IR should cause the electron to be raised to the conduction band, so the photon is absorbed in the silicon. Let us find the shortest wavelength that will cause the electron to jump.

$$\lambda=\frac{c}{f}=\frac{hc}{hf}>\frac{hc}{E_g}=\frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(1.14\,\text{eV})}=1.09\times10^{-6}\,\text{m}=\boxed{1.09\,\mu\text{m}}$$

    Because this is in the IR region of the spectrum, the shorter wavelengths of visible light will excite the electron and the photon would be absorbed. So silicon $\boxed{\text{could be used}}$ as a window.

72. The kinetic energy of the baton is $\frac{1}{2}I\omega^2$, and the quantum number can be found from Eq. 40-2. Let the length of the baton be *d*. We assume the quantum number will be very large.

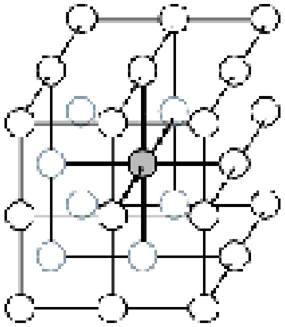
$$\tfrac{1}{2}I\omega^2=\frac{\ell(\ell+1)\hbar^2}{2I}\approx\frac{\ell^2\hbar^2}{2I}\quad\rightarrow$$
$$\ell=\frac{I\omega}{\hbar}=\left[2m_{\text{end}}\left(\tfrac{1}{2}d\right)^2+\tfrac{1}{12}m_{\text{bar}}d^2\right]\frac{2\pi f}{\hbar}=$$
$$=\left[2(0.38\,\text{kg})(0.16\,\text{m})^2+\tfrac{1}{12}(0.26\,\text{kg})(0.32\,\text{m})^2\right]\frac{2\pi(1.6\,\text{s}^{-1})}{(1.055\times10^{-34}\,\text{J}\bullet\text{s})}=2.07\times10^{33}$$

The spacing between rotational energy levels is given by Eq. 40-3. We compare that value to the rotational kinetic energy.

$$\frac{\Delta E}{E}=\frac{\frac{\ell\hbar^2}{I}}{\frac{\ell^2\hbar^2}{2I}}=\frac{1}{2\ell}=\frac{1}{2\left(2.07\times10^{33}\right)}=2.4\times10^{-34}$$

This is such a small difference that it would not be detectable, so $\boxed{\text{no}}$, we do not need to consider quantum effects.

$\boxed{73.}$ From the diagram of the cubic lattice, we see that an atom inside the cube is bonded to the six nearest neighbors. Because each bond is shared by two atoms, the number of bonds per atom is 3 (as long as the sample is large enough that most atoms are in the interior, and not on the boundary surface). We find the heat of fusion from the energy required to break the bonds:



$$L_{\text{fusion}}=\left(\frac{\text{number of bonds}}{\text{atom}}\right)\left(\frac{\text{number of atoms}}{\text{mol}}\right)E_{\text{bond}}$$
$$=(3)\left(6.02\times10^{23}\,\text{atoms/mol}\right)\left(3.9\times10^{-3}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)$$
$$=1127\,\text{J/mol}\approx\boxed{1100\,\text{J/mol}}$$

74. The longest wavelength will be the photon with the minimum energy.

$$\Delta E_{\min}=\frac{hc}{\lambda_{\max}}\ \rightarrow\ \lambda_{\max}=\frac{hc}{\Delta E_{\min}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(3.6\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}=3.5\times10^{-7}\,\text{m}$$

So the photon must have $\boxed{\lambda\le3.5\times10^{-7}\,\text{m}}$.

75. The photon with the minimum frequency for conduction must have an energy equal to the energy gap.

$$E_g=hf=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(226\times10^{-9}\,\text{m}\right)}=\boxed{5.50\,\text{eV}}$$

76. (*a*) We calculate the Fermi temperature, for a Fermi energy of 7.0 eV.

$$T_{\text{F}}=\frac{E_{\text{F}}}{k}=\frac{\left(7.0\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{1.38\times10^{-23}\,\text{J/K}}=\boxed{8.1\times10^{4}\,\text{K}}$$

(b) We are given that $T\gg T_{\text{F}}$, and we assume that $e^{E/kT}>>1$.

$$f(E)=\frac{1}{e^{(E-E_{\text{F}})/kT}+1}=\frac{1}{e^{\left(\frac{E}{kT}-\frac{kT_{\text{F}}}{kT}\right)}+1}\approx\frac{1}{e^{\left(\frac{E}{kT}\right)}+1}\approx\frac{1}{e^{\left(\frac{E}{kT}\right)}}=e^{-E/kT}$$

This is not useful for conductors like copper, because the Fermi temperature is higher than the melting point, and we would no longer have a solid conductor.

77. We use Eq. 40-11 with the limits given in order to determine the number of states.

$$N=V\int_{E_1}^{E_2}g(E)\,dE=\frac{8\sqrt{2}\pi m^{3/2}}{h^3}V\int_{E_1}^{E_2}E^{1/2}dE=\frac{8\sqrt{2}\pi m^{3/2}}{h^3}V\frac{2}{3}\left(E_2^{3/2}-E_1^{3/2}\right)$$

$$=\frac{16\sqrt{2}\pi\left(9.11\times10^{-31}\,\text{kg}\right)^{3/2}}{3\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^{3}}\left(0.1\,\text{m}\right)^{3}\left[\left(6.2\,\text{eV}\right)^{3/2}-\left(4.0\,\text{eV}\right)^{3/2}\right]\left(1.60\times10^{-19}\,\text{J/eV}\right)^{3/2}$$

$$=3.365\times10^{25}\approx\boxed{3\times10^{25}}$$

78. (*a*) For the glass to be transparent to the photon, the photon must have less energy than 1.14 eV, and so the wavelength of the photon must be longer than the wavelength corresponding to 1.14 eV.

$$E_{\text{band gap}}=\frac{hc}{\lambda_{\min}}\quad\rightarrow$$

$$\lambda_{\min}=\frac{hc}{E_{\text{band gap}}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.14\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}=1.09\times10^{-6}\,\text{m}\quad\rightarrow\quad\boxed{\lambda>1.09\times10^{-6}\,\text{m}}$$

The minimum wavelength for transparency is in the $\boxed{\text{infrared}}$ region of the spectrum. Since IR has longer wavelengths than visible light, the silicon would not be transparent for visible light. The silicon would be opaque, as in Example 40-14.

(*b*) The minimum possible band gap energy for light to be transparent would mean that the band gap energy would have to be larger than the most energetic visible photon. The most energetic photon corresponds to the shortest wavelength, which is 450 nm in this problem.

$$E_{\text{band gap}}>E_{\lambda_{\min}}=\frac{hc}{\lambda_{\min}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(450\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}=2.7625\,\text{eV}\approx\boxed{2.8\,\text{eV}}$$

$\boxed{79.}$ The photon with the maximum wavelength for absorption must have an energy equal to the energy gap.

$$E_g=hf=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(1.92\times10^{-3}\,\text{m}\right)}=\boxed{6.47\times10^{-4}\,\text{eV}}$$

80. (*a*) The electrons will not be moving fast enough at this low temperature to use relativistic expressions, so the momentum is just the mass times the speed. The kinetic energy of the electrons can be found from the temperature, by Eq. 18-4. The kinetic energy is used to calculate the momentum, and the momentum is used to calculate the wavelength.

$$K=\tfrac{3}{2}kT=\frac{p^2}{2m}\quad\rightarrow\quad p=\sqrt{3mkT}$$

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{3mkT}}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\sqrt{3\left(9\times10^{-31}\,\text{kg}\right)\left(1.38\times10^{-23}\,\text{J/K}\right)\left(300\,\text{K}\right)}}=6.27\times10^{-9}\,\text{m}\approx\boxed{6\,\text{nm}}$$

(*b*) The wavelength is much longer than the opening, and so electrons at this temperature would experience $\boxed{\text{diffraction}}$ when passing through the lattice.

81. The photon with the longest wavelength has the minimum energy.

$$E_g=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(1100\times10^{-9}\,\text{m}\right)}=1.130\,\text{eV}\approx\boxed{1.1\,\text{eV}}$$

If the energy gap is any larger than this, some solar photons will not have enough energy to cause an electron to jump levels, and so will not be absorbed.

82. The energy gap is related to photon wavelength by $E_g = hf = hc/\lambda$. Use this for both colors of LED.

Green: $$E_g = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(525\times10^{-9}\,\text{m})} = \boxed{2.37\,\text{eV}}$$

Blue: $$E_g = \frac{(6.63\times10^{-34}\,\text{J•s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(465\times10^{-9}\,\text{m})} = \boxed{2.67\,\text{eV}}$$

83. The arsenic ion has a charge of +1, since we consider the ion as having been formed by removing one electron from the arsenic atom. Thus the effective Z will be 1, and we can use the Bohr theory results for hydrogen. We also substitute $K\epsilon_0$ in place of ϵ_0.

(*a*) The Bohr energy is given in Eq. 37-14a and b. The binding energy is the opposite of the ground state energy, so we use $n = 1$.

$$E_{\text{binding}} = \frac{Z^2e^4m}{8(K\epsilon_0)^2h^2}\frac{1}{n^2} = \frac{1}{K^2}\left(\frac{Z^2e^4m}{8\epsilon_0^2h^2}\right) = \frac{1}{12^2}(13.6\,\text{eV}) = \boxed{0.094\,\text{eV}}$$

(*b*) The Bohr radius is given in Eq. 37-11.

$$r_1 = \frac{h^2(K\epsilon_0)}{\pi mZe^2} = K\frac{h^2\epsilon_0}{\pi me^2} = 12(0.529\times10^{-10}\,\text{m}) = \boxed{6.3\times10^{-10}\,\text{m}}$$

84. From Eq. 25-13, the number of charge carriers per unit volume in a current is given by $n = \frac{I}{ev_{\text{drift}}A}$, where v_{drift} is the drift velocity of the charge carriers, and *A* is the cross-sectional area through which the carriers move. From Eq. 27-14, the drift velocity is given by $v_{\text{drift}} = \frac{\mathcal{E}_{\text{H}}}{Bd}$, where $\mathcal{E}_{\text{H}}$ is the Hall-effect voltage and *d* is the width of the strip carrying the current (see Figure 27-32). The distance *d* is the shorter dimension on the "top" of Figure 40-47. We combine these equations to find the density of charge carriers. We define the thickness of the current-carrying strip by $t = A/d$.

$$n = \frac{I}{ev_{\text{drift}}A} = \frac{IBd}{e\mathcal{E}_{\text{H}}A} = \frac{IB}{e\mathcal{E}_{\text{H}}t} = \frac{(0.28\times10^{-3}\,\text{A})(1.3\,\text{T})}{(1.60\times10^{-19}\,\text{C})(0.018\,\text{V})(1.0\times10^{-3}\,\text{m})} = 1.264\times10^{20}\,\text{electrons/m}^3$$

The actual density of atoms per unit volume in the silicon is found from the density and the atomic weight. We let that be represented by *N*.

$$N = (2330\,\text{kg/m}^3)\left(\frac{1\,\text{mole}}{28.0855\times10^{-3}\,\text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{1\,\text{mole}}\right) = 4.994\times10^{28}\,\text{atoms/m}^3$$

$$\frac{n}{N} = \frac{1.264\times10^{20}\,\text{electrons/m}^3}{4.994\times10^{28}\,\text{atoms/m}^3} = \boxed{2.5\times10^{-9}\,\text{electrons/atom}}$$

$\boxed{85.}$ We assume the 130 V value is given to the nearest volt.

(*a*) The current through the load resistor must be maintained at a constant value.

$$I_{\text{load}} = \frac{V_{\text{output}}}{R_{\text{load}}} = \frac{(130\,\text{V})}{(18.0\,\text{k}\Omega)} = 7.22\,\text{mA}$$

At the minimum supply voltage, there will be no current through the diode, so the current through *R* is also 7.22 mA. The supply voltage is equal to the voltage across *R* plus the output voltage.

$$V_R = I_{\text{load}}R = (7.22\,\text{mA})(2.80\,\text{k}\Omega) = 20.2\,\text{V} \;;\; V_{\substack{\text{supply}\\ \text{min}}} = V_R + V_{\text{output}} = \boxed{150\,\text{V}}$$

At the maximum supply voltage, the current through the diode will be 120 mA, and so the current through *R* is 127 mA.

$$V_R = I_{\text{load}}R = (127.22\,\text{mA})(2.80\,\text{k}\Omega) = 356\,\text{V} \;;\; V_{\substack{\text{supply}\\ \text{max}}} = V_R + V_{\text{output}} = \boxed{486\,\text{V}}$$

(*b*) The supply voltage is fixed at 245 V, and the output voltage is still to be 130 V. The voltage across *R* is fixed at $245\,\text{V} - 130\,\text{V} = 115\,\text{V}$. We calculate the current through *R*.

$$I_R = \frac{V_R}{R} = \frac{(115\,\text{V})}{(2.80\,\text{k}\Omega)} = 41.1\,\text{mA}$$

If there is no current through the diode, this current will be in the load resistor.

$$R_{\text{load}} = \frac{V_{\text{load}}}{I_{\text{load}}} = \frac{130\,\text{V}}{41.1\,\text{mA}} = 3.16\,\text{k}\Omega$$

If R_{load} is less than this, there will be a greater current through *R*, meaning a greater voltage drop across *R*, and a smaller voltage across the load. Thus regulation would be lost, so $3.16\,\text{k}\Omega$ is the minimum load resistance for regulation.

If R_{load} is greater than $3.16\,\text{k}\Omega$, the current through R_{load} will have to decrease in order for the voltage to be regulated, which means there must be current through the diode. The current through the diode is 41.1 mA when R_{load} is infinite, which is less than the diode maximum of 120 mA. Thus the range for load resistance is $\boxed{3.16\,\text{k}\Omega \le R_{\text{load}} < \infty.}$

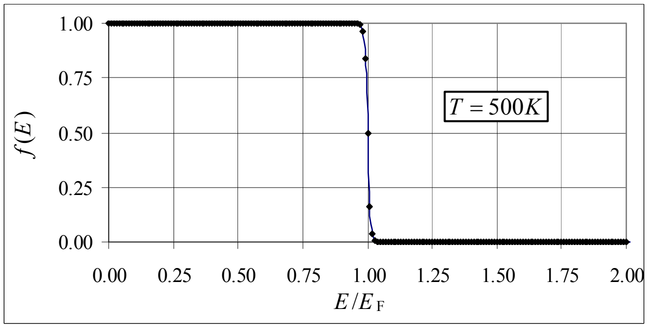
86. The voltage as graphed in Figure 40-40c decays exponentially, according to Eq. 26-9b. As suggested in the problem, we use a linear approximation for the decay, using an expansion from Appendix A-3. From Figure 40-40c, we see that the decay lasts for approximately one-half of a cycle, before it increases back to the peak value.

$$V_{\text{min}} = V_{\text{peak}}e^{-t/\tau} \quad\to\quad \frac{V_{\text{min}}}{V_{\text{peak}}} = e^{-t/\tau} \approx 1 - \frac{t}{\tau} = 1 - \frac{t}{RC} = 1 - \frac{\frac{1}{2}\left(\frac{1}{60}\,\text{s}\right)}{(7.8\times10^3\,\Omega)(36\times10^{-6}\,\text{F})} = 0.97$$

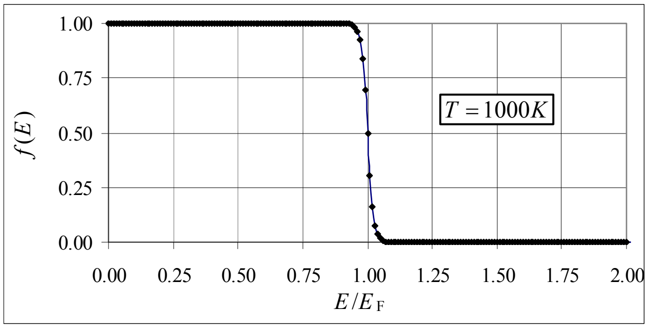
$\boxed{\text{The voltage will decrease 3\% from its maximum, or 1.5\% above and below its average.}}$

87. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH40.XLS," on tab "Problem 40.87."

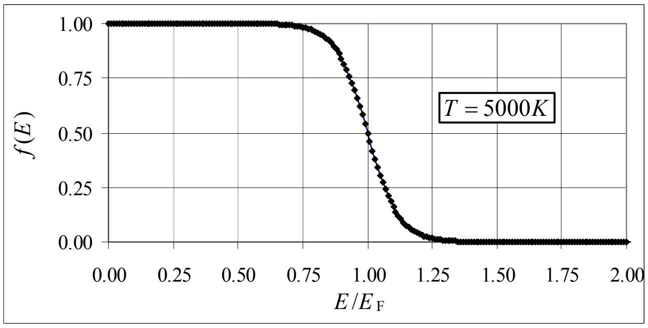
(*a*)



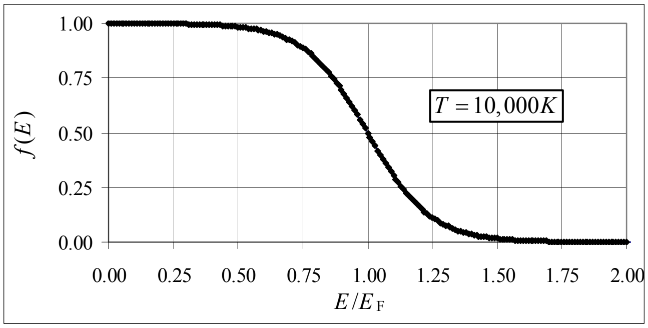
(*b*)



(*c*)



(*d*)



We see that for temperatures even as high as 1000 K, there is very little probability of electrons being above the Fermi level. Only when the temperature gets quite high do we see a significant increase in the probability of electrons to have an energy higher than the Fermi energy.

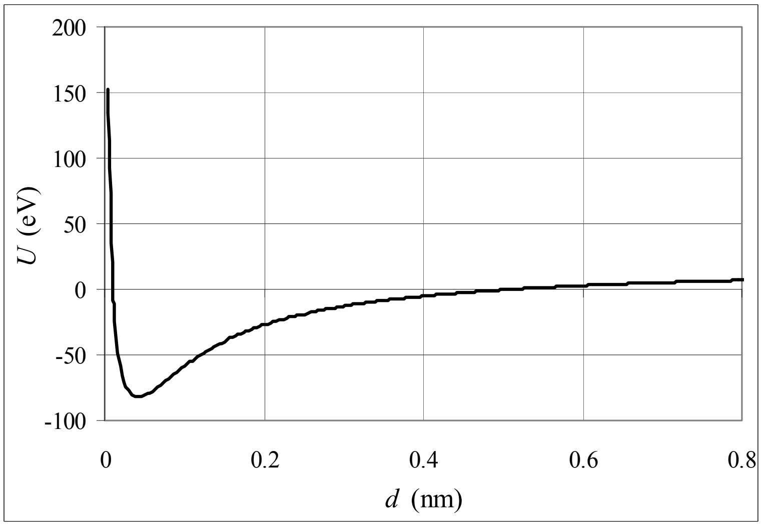
88. (*a*) The total potential energy is due to the electron-electron interaction, the proton-proton interaction, and 4 electron-proton interactions.

$$U = U_{\text{e-e}} + U_{\text{p-p}} + 4U_{\text{p-e}} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{d} + \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_0} + 4\left(-\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{\sqrt{\left(\tfrac{1}{2}d\right)^2 + \left(\tfrac{1}{2}r_0\right)^2}}$$

$$= \boxed{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{d} + \frac{1}{r_0} - \frac{8}{\sqrt{r_0^2 + d^2}}\right)}$$

(*b*) *U* has a minimum at $d \approx \boxed{0.043\,\text{nm}}$. $U < 0$ for the approximate range $0.011\,\text{nm} \le d \le 0.51\,\text{nm}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH40.XLS," on tab "Problem 40.88b."



(*c*) To find the point of greatest stability, set the derivative of *U* with respect to *d* (indicated by U') equal to 0 and solve for *d*.

$$U = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{d} + \frac{1}{r_0} - \frac{8}{\sqrt{r_0^2 + d^2}}\right) \rightarrow$$

$$U' = \frac{e^2}{4\pi\varepsilon_0}\left(-\frac{1}{d^2} - \left(-\frac{1}{2}\right)\frac{8(2d)}{\left(r_0^2 + d^2\right)^{3/2}}\right) = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{8d}{\left(r_0^2 + d^2\right)^{3/2}} - \frac{1}{d^2}\right) = 0 \rightarrow$$

$$\frac{8d}{\left(r_0^2 + d^2\right)^{3/2}} = \frac{1}{d^2} \rightarrow 8d^3 = \left(r_0^2 + d^2\right)^{3/2} \rightarrow 2d = \sqrt{\left(r_0^2 + d^2\right)} \rightarrow$$

$$4d^2 = r_0^2 + d^2 \rightarrow d = \frac{r_0}{\sqrt{3}} = \frac{0.074}{\sqrt{3}} = \boxed{0.0427\,\text{nm}}$$

89. We first find the wavelength for a photon that has 1.14 eV of energy. This is the maximum wavelength that will be able to make electron-hole pairs.

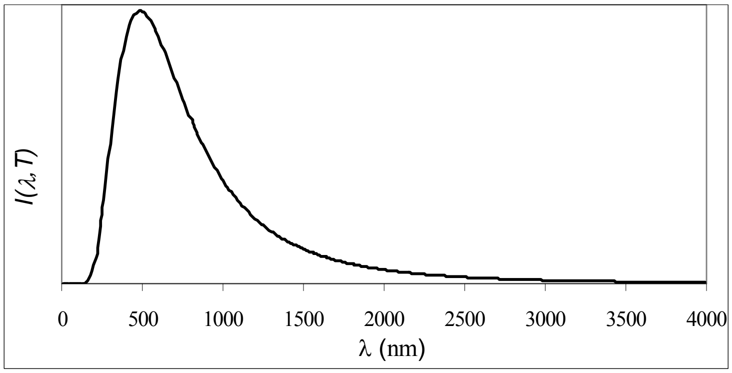
$$E_{\text{gap}} = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E_{\text{gap}}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.14\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.09\times10^{-6}\,\text{m} = 1090\,\text{nm}$$

Any wavelength shorter than 1100 nm will be effective. The $1000\,\text{W/m}^2$ value includes all wavelengths of solar photons reaching the Earth, so we need to find what fraction of solar photons have wavelengths below 1100 nm. We do this using the Planck formula, from Section 37-1. We find the following using numeric integration, for a temperature of 6000 K.

$$\text{fraction of effective photons} = \frac{\displaystyle\int_0^{1100\,\text{nm}} \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}\,d\lambda}{\displaystyle\int_0^{\infty} \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}\,d\lambda}$$

The Planck function is shown in the figure for $T = 6000$ K. We approximated the upper limit for the full integration as 4000 nm, and obtained a ratio of 0.79. Thus we use an effective solar energy input of $790\,\text{W/m}^2$. To estimate the number of incoming photons, we use an average photon wavelength of 500 nm, estimated simply by looking at the Planck function graph. We also assume that each photon produces only one electron-hole pair, even though many of them would have enough energy to create more than one.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH40.XLS," on tab "Problem 40.89."



$$\frac{I}{A}=\left(\frac{\text{Solar energy}}{\text{s}\bullet\text{m}^2}\right)\left(\frac{1\text{ "average" photon}}{\text{Energy for 500 nm photon}}\right)\left(\frac{1\text{ electron produced}}{1\text{ solar photon}}\right)\left(\frac{\text{Charge}}{\text{electron}}\right)$$

$$=\left(790\,\text{W/m}^2\right)\left(\frac{\lambda}{hc}\right)\left(1.60\times10^{-19}\,\text{C}\right)=\left(790\,\text{W/m}^2\right)\frac{\left(500\times10^{-9}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)}{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}$$

$$=318\,\text{C/s}\bullet\text{m}^2=\boxed{32\,\text{mA/cm}^2}$$

CHAPTER 41: Nuclear Physics and Radioactivity

Responses to Questions

1. All isotopes of the same element have the same number of protons in their nuclei (and electrons in the atom) and will have very similar chemical properties. Isotopes of the same element have different numbers of neutrons in their nuclei and therefore different atomic masses.

2. (*a*) uranium; (*b*) nitrogen; (*c*) hydrogen; (*d*) strontium; (*e*) berkelium. The element is determined by the atomic number.

3. Z is the number of protons and N is the number of neutrons.
(*a*) $Z = 92$, $N = 140$; (*b*) $Z = 7$, $N = 11$; (*c*) $Z = 1$, $N = 0$; (*d*) $Z = 38$, $N = 44$; (*e*) $Z = 97$, $N = 150$.

4. If there are 88 nucleons and 50 neutrons, then there are 38 protons. Strontium is the element with 38 protons.

5. The atomic masses given in the periodic table are averaged over the isotopes of the element in the percentages in which they occur in nature. For instance, the most common form of hydrogen has one proton and no neutrons, but other naturally occurring isotopes include deuterium (one proton and one neutron) and tritium (one proton and 2 neutrons). These latter two are much less common, and the resulting “weighted average” for the mass of hydrogen is 1.0079.

6. A force other than the gravitational force or the electromagnetic force is necessary to explain the stability of nuclei. In most nuclei, several protons and neutrons are confined to a very small space. The gravitational attractive force between the nucleons is very small compared with electromagnetic repulsion between the protons. The strong nuclear force overcomes the electromagnetic repulsion and holds the nucleus together.

7. The strong force and the electromagnetic force are two of the four fundamental forces in nature. They are both involved in holding atoms together: the strong force binds quarks into nucleons and nucleons together in the nucleus; the electromagnetic force is responsible for binding negatively-charged electrons to positively-charged nuclei and atoms into molecules. The strong force is the strongest of the four fundamental forces; the electromagnetic force is about 100 times weaker at distances on the order of 10^{-17} m. The strong force operates at short range and is negligible for distances greater than about the size of the nucleus. The electromagnetic force is a long range force that decreases as the inverse square of the distance between the two interacting charged particles. The electromagnetic force operates only between charged particles. The strong force is always attractive; the electromagnetic force can be attractive or repulsive. Both these forces have mediating field particles associated with them. The gluon is the particle for the strong force and the photon is the particle for the electromagnetic force.

8. Chemical processes are the result of interactions between electrons. Radioactivity is not affected by the external conditions that normally affect chemical processes, such as temperature, pressure, or strong chemical reagents. Therefore, radioactivity is not a chemical process, but a nuclear one. In addition, the energies associated with radioactivity are generally larger than energies corresponding to electron orbital transitions, indicating that radioactivity is a nuclear process.

9. The resulting nuclide for gamma decay is the same isotope in a lower energy state:

$$ {}^{64}_{29}\text{Cu}^* \rightarrow {}^{64}_{29}\text{Cu} + \gamma . $$

The resulting nuclide for beta-minus decay is an isotope of zinc, ${}^{64}_{30}\text{Zn}$:

$${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{30}\text{Zn} + \text{e}^- + \bar{\nu} .$$

The resulting nuclide for beta-plus decay is an isotope of nickel, ${}^{64}_{28}\text{Ni}$:

$${}^{64}_{29}\text{Cu} \rightarrow {}^{64}_{28}\text{Ni} + \text{e}^+ + \nu .$$

10. ${}^{238}_{92}\text{U}$ decays by alpha emission into ${}^{234}_{90}\text{Th}$, which has 144 neutrons.

11. Alpha (*α*) particles are helium nuclei. Each α particle consists of 2 protons and 2 neutrons, and therefore has a charge of +2*e* and an atomic mass value of 4 u. Beta (*β*) particles are electrons (beta-minus) or positrons (beta-plus). Electrons have a charge of –*e* and positrons have a charge of +*e*. In terms of mass, beta particles are much lighter than protons or neutrons, by a factor of about 2000, so are lighter than alpha particles by a factor of about 8000. Gamma (*γ*) particle are photons. They have no rest mass and no charge.

12. (*a*) Magnesium is formed: ${}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + \text{e}^- + \bar{\nu}$.

    (*b*) Neon is formed: ${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + \text{e}^+ + \nu$.

    (c) Lead is formed: ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$.

13. (*a*) Sulfur is formed: ${}^{32}_{15}\text{Pb} \rightarrow {}^{32}_{16}\text{S} + \text{e}^- + \bar{\nu}$.

    (*b*) Chlorine is formed: ${}^{35}_{16}\text{S} \rightarrow {}^{35}_{17}\text{Cl} + \text{e}^- + \bar{\nu}$.

    (c) Thallium is formed: ${}^{211}_{83}\text{Bi} \rightarrow {}^{207}_{81}\text{Tl} + {}^{4}_{2}\text{He}$.

14. (*a*) ${}^{45}_{21}\text{Sc}$; (*b*) ${}^{58}_{29}\text{Cu}$; (*c*) $\text{e}^+ + \nu$; (*d*) ${}^{230}_{92}\text{U}$; (*e*) $\text{e}^- + \bar{\nu}$

15. The two extra electrons held by the newly formed thorium will be very loosely held, as the number of protons in the nucleus will have been reduced from 92 to 90, reducing the nuclear charge. It will be easy for these extra two electrons to escape from the thorium atom through a variety of mechanisms.

16. When a nucleus undergoes either β^- or β^+ decay it becomes a different element, since it has either converted a neutron to a proton or a proton to a neutron and therefore its atomic number has changed. The energy levels of the atomic electrons will adjust to become the energy levels of the new element. Photons are likely to be emitted from the atom as electrons change energies to occupy the new levels.

17. Alpha particles from an alpha-emitting nuclide are part of a two-body decay. The energy carried off by the decay fragments is determined by the principles of conservation of energy and of momentum. With only two decay fragments, these two constraints require the alpha particles to be monoenergetic. Beta particles from a beta-emitting nucleus are part of a three-body decay. Again, the energy carried off by all of the decay fragments is determined by the principles of conservation of energy and of momentum. However, with three decay fragments, the energy distribution between the fragments is not determined by these two constraints. The beta particles will therefore have a range of energies.

18. Below. During electron capture, a proton in the nucleus becomes a neutron. Therefore, the isotopes that undergo electron capture will most likely be those that have too few neutrons in the nucleus (compared to their number of protons) and will lie below the line of stability in Figure 41-2.

19. No. Hydrogen has only one proton. Deuterium has one proton and one neutron. Neither has the two protons and two neutrons required to form an alpha particle.

20. Many artificially produced radioactive isotopes have very short half-lives, and so are rare in nature because they do not last long if they are produced naturally. Many of these isotopes also have a very high energy of formation, which is generally not available in nature.

21. No. At the end of one month, ½ the sample will remain. At the end of two months, ¼ of the original sample will remain.

22. For $Z > 92$, the short range of the attractive strong nuclear force means that no number of neutrons is able to overcome the electrostatic repulsion of the large concentration of protons.

23. Helium-3, ${}^{3}_{2}\mathrm{He}$, will be the other particle released. There are a total of four protons and three neutrons in the reactant particles. The alpha particle carries off two protons and two neutrons, leaving two protons and one neutron.

24. No. Carbon-14 dating can only be used to date objects that were once living. The stone used to build walls was never alive.

25. In β decay, a neutrino and a β particle (electron or positron) will be emitted from the nucleus, and the number of protons in the nucleus changes. Because there are three decay products (the neutrino, the β particle, and the nucleus), the momentum of the β particle can have a range of values. In internal conversion, only an electron is emitted from the atom, and the number of protons in the nucleus stays the same. Because there are only two decay products (the electron and the nucleus), the electron will have a unique momentum and, therefore, a unique energy.

26. Figure 41-6 shows the potential energy curve for an alpha particle and daughter nucleus for the case of radioactive nuclei. The alpha particle tunnels through the barrier from point A to point B in the figure. In the case of stable nuclei, the probability of this happening must be essentially zero. The maximum height of the Coulomb potential energy curve must be larger and/or the Q-value of the reaction must be smaller so that the probability of tunneling is extremely low.

27. The decay series of Figure 41-12 begins with a nucleus that has many more neutrons than protons and lies far above the line of stability in Figure 41-2. In a β^{+} decay, a proton is converted to a neutron, which would take the nuclei in this decay series farther from the line of stability and is not energetically preferred.

28. There are four alpha particles and four β^{-} particles (electrons) emitted, no matter which decay path is chosen. The nucleon number drops by 16 as ${}^{222}_{86}\mathrm{Rn}$ decays into ${}^{206}_{82}\mathrm{Pb}$ ^{206}Pb, indicating the presence of four alpha decays. The proton number only drops by four, from $Z = 86$ to $Z = 82$, but four alpha decays would result in a decrease of eight protons. Four β^{-} decays will convert four neutrons into protons, making the decrease in the number of protons only four, as required. (See Figure 41-12.)

Solutions to Problems

$\boxed{1.}$ Convert the units from MeV/c^2 to atomic mass units.

$$m=\left(139\,\mathrm{MeV}/c^2\right)\left(\frac{1\,\mathrm{u}}{931.49\,\mathrm{MeV}/c^2}\right)=\boxed{0.149\,\mathrm{u}}$$

2. The α particle is a helium nucleus and has $A = 4$. Use Eq. 41-1.

$$r=\left(1.2\times10^{-15}\,\mathrm{m}\right)A^{\frac{1}{3}}=\left(1.2\times10^{-15}\,\mathrm{m}\right)\left(4\right)^{\frac{1}{3}}=\boxed{1.9\times10^{-15}\,\mathrm{m}}=1.9\,\mathrm{fm}$$

3. The radii of the two nuclei can be calculated with Eq. 41-1. Take the ratio of the two radii.

$$\frac{r_{238}}{r_{232}}=\frac{\left(1.2\times10^{-15}\,\mathrm{m}\right)\left(238\right)^{1/3}}{\left(1.2\times10^{-15}\,\mathrm{m}\right)\left(232\right)^{1/3}}=\left(\frac{238}{232}\right)^{1/3}=1.0085$$

So the radius of ${}^{238}_{92}\mathrm{U}$ is $\boxed{0.85\%}$ larger than the radius of ${}^{232}_{92}\mathrm{U}$.

4. Use Eq. 41-1 for both parts.

(*a*) $r=\left(1.2\times10^{-15}\,\mathrm{m}\right)A^{1/3}=\left(1.2\times10^{-15}\,\mathrm{m}\right)\left(112\right)^{1/3}=\boxed{5.8\times10^{-15}\,\mathrm{m}}=5.8\,\mathrm{fm}$

(*b*) $r=\left(1.2\times10^{-15}\,\mathrm{m}\right)A^{1/3}\;\rightarrow\;A=\left(\frac{r}{1.2\times10^{-15}\,\mathrm{m}}\right)^3=\left(\frac{3.7\times10^{-15}\,\mathrm{m}}{1.2\times10^{-15}\,\mathrm{m}}\right)^3=29.3\approx\boxed{29}$

5. To find the rest mass of an α particle, we subtract the rest mass of the two electrons from the rest mass of a helium atom:

$$\begin{aligned}m_\alpha&=m_{\mathrm{He}}-2m_{\mathrm{e}}\\&=\left(4.002603\,\mathrm{u}\right)\left(931.5\,\mathrm{MeV}/\mathrm{u}c^2\right)-2\left(0.511\,\mathrm{MeV}/c^2\right)=\boxed{3727\,\mathrm{MeV}/c^2}\end{aligned}$$

This is less than the sum of the masses of two protons and two neutrons because of the binding energy.

6. Each particle would exert a force on the other through the Coulomb electrostatic force. The distance between the particles is twice the raiius of one of the particles. The Coulomb force is given by Eq. 21-2.

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\alpha}{\left(2r_\alpha\right)^2}=\frac{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\left(2\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)\right]^2}{\left[\left(2\right)\left(4^{1/3}\right)\left(1.2\times10^{-15}\,\mathrm{m}\right)\right]^2}=63.41\,\mathrm{N}\approx\boxed{63\,\mathrm{N}}$$

The acceleration is found from Newton's second law. We use the mass of a "bare" alpha calculated in Problem 5.

$$F=ma\;\rightarrow\;a=\frac{F}{m}=\frac{63.41\,\mathrm{N}}{3727\,\mathrm{MeV}/c^2\left(\frac{1.6605\times10^{-27}\,\mathrm{kg}}{931.49\,\mathrm{MeV}/c^2}\right)}=\boxed{9.5\times10^{27}\,\mathrm{m/s^2}}$$

$\boxed{7.}$ (*a*) The mass of a nucleus with mass number A is approximately (A u) and its radius is $r=\left(1.2\times10^{-15}\,\text{m}\right)A^{1/3}$. Calculate the density.

$$\rho=\frac{m}{V}=\frac{A\left(1.66\times10^{-27}\,\text{kg/u}\right)}{\frac{4}{3}\pi r^3}=\frac{A\left(1.66\times10^{-27}\,\text{kg/u}\right)}{\frac{4}{3}\pi\left(1.2\times10^{-15}\,\text{m}\right)^3A}=2.293\times10^{17}\,\text{kg/m}^3\approx$$

$$\approx\boxed{2.3\times10^{17}\,\text{kg/m}^3}$$

We see that this is independent of A.

(*b*) We find the radius from the mass and the density.

$$M=\rho\tfrac{4}{3}\pi R^3\ \rightarrow\ R=\left(\frac{3M}{4\pi\rho}\right)^{1/3}=\left[\frac{3\left(5.98\times10^{24}\,\text{kg}\right)}{4\pi\left(2.293\times10^{17}\,\text{kg/m}^3\right)}\right]^{1/3}=184\,\text{m}\approx\boxed{180\,\text{m}}$$

(*c*) We set the density of the Earth equal to the density of the uranium nucleus. We approximate the mass of the uranium nucleus as 238 u.

$$\rho_{\text{Earth}}=\rho_U\ \rightarrow\ \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}^3}=\frac{m_{\text{U}}}{\frac{4}{3}\pi r_{\text{U}}^3}\ \rightarrow$$

$$r_{\text{U}}=R_{\text{Earth}}\left(\frac{m_{\text{U}}}{M_{\text{Earth}}}\right)^{1/3}=\left(6.38\times10^6\,\text{m}\right)\left[\frac{238\left(1.66\times10^{-27}\,\text{kg}\right)}{5.98\times10^{24}\,\text{kg}}\right]^{1/3}=\boxed{2.58\times10^{-10}\,\text{m}}$$

8. Use Eq. 41-1 to find the value for A. We use uranium-238 since it is the most common isotope.

$$\frac{r_{\text{unknown}}}{r_{\text{U}}}=\frac{\left(1.2\times10^{-15}\,\text{m}\right)A^{1/3}}{r=\left(1.2\times10^{-15}\,\text{m}\right)\left(238\right)^{1/3}}=0.5\ \rightarrow\ A=238\left(0.5\right)^3=29.75\approx30$$

From Appendix F, a stable nucleus with $A\approx30$ is $\boxed{{}^{31}_{15}\text{P}}$.

9. The basic principle to use is that of conservation of energy. We assume that the centers of the two particles are located a distance from each other equal to the sum of their radii. That distance is used to calculate the initial electrical potential energy. Then we also assume that, since the nucleus is much heavier than the alpha, that the alpha has all of the final kinetic energy when the particles are far apart from each other (and so have no potential energy).

$$K_{\text{i}}+U_{\text{i}}=K_{\text{f}}+U_{\text{f}}\ \rightarrow\ 0+\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Fm}}}{\left(r_\alpha+r_{\text{Fm}}\right)}=K_\alpha+0\ \rightarrow$$

$$K_\alpha=\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(2\right)\left(100\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(4^{1/3}+257^{1/3}\right)\left(1.2\times10^{-15}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}=3.017\times10^7\,\text{eV}$$

$$\approx\boxed{30\,\text{MeV}}$$

10. (*a*) The hydrogen atom is made of a proton and an electron.

$$\frac{m_{\text{p}}}{\left(m_{\text{p}}+m_{\text{e}}\right)}=\frac{\left(1.67\times10^{-27}\,\text{kg}\right)}{\left(1.67\times10^{-27}\,\text{kg}+9.11\times10^{-31}\,\text{kg}\right)}=\boxed{0.99945}$$

(*b*) Compare the volume of the nucleus to the volume of the atom. The nuclear radius is given by Eq. 41-1. For the atomic radius we use the Bohr radius, given in Eq. 37-12.

$$\left(\frac{r_{\text{nucleus}}}{r_{\text{atom}}}\right)^3=\left[\frac{\left(1.2\times10^{-15}\,\text{m}\right)}{\left(0.53\times10^{-10}\,\text{m}\right)}\right]^3=\boxed{1.2\times10^{-14}}$$

11. Electron mass is negligible compared to nucleon mass, and one nucleon weighs about 1.0 atomic mass unit. Therefore, in a 1.0-kg object,

$$N = \frac{(1.0\,\text{kg})\left(6.02\times10^{26}\ \text{u/kg}\right)}{1.0\,\text{u/nucleon}} \approx \boxed{6\times10^{26}\,\text{nucleons.}}$$

$\boxed{\text{No}}$, it does not matter what the element is, because $\boxed{\text{the mass of one nucleon is essentially the same for all elements.}}$

12. The initial kinetic energy of the alpha must be equal to the electrical potential energy when the alpha just touches the uranium. The distance between the two particles is the sum of their radii.

$$K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f} \ \rightarrow\ K_\alpha + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{U}}{(r_\alpha + r_\text{U})} \ \rightarrow$$

$$K_\alpha = \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(92)\left(1.60\times10^{-19}\text{C}\right)^2}{\left(4^{1/3}+238^{1/3}\right)\left(1.2\times10^{-15}\text{m}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 2.832\times10^{7}\,\text{eV}$$

$$\approx \boxed{28\,\text{MeV}}$$

$\boxed{13.}$ From Figure 41–1, we see that the average binding energy per nucleon at A = 63 is about 8.7 MeV. Multiply this by the number of nucleons in the nucleus.

$$(63)(8.7\,\text{MeV}) = 548.1\,\text{MeV} \approx \boxed{550\,\text{MeV}}$$

14. Deuterium consists of one proton, one neutron, and one electron. Ordinary hydrogen consists of one proton and one electron. We use the atomic masses from Appendix F, and the electron masses cancel.

$$\begin{aligned}\text{Binding energy} &= \left[m\left({}^1_1\text{H}\right) + m\left({}^1_0\text{n}\right) - m\left({}^2_1\text{H}\right)\right]c^2\\ &= \left[(1.007825\,\text{u}) + (1.008665\,\text{u}) - (2.014082\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)\\ &= \boxed{2.243\,\text{MeV}}\end{aligned}$$

15. We find the binding energy of the last neutron from the masses of the isotopes.

$$\begin{aligned}\text{Binding energy} &= \left[m\left({}^{31}_{15}\text{P}\right) + m\left({}^1_0\text{n}\right) - m\left({}^{32}_{15}\text{P}\right)\right]c^2\\ &= \left[(30.973762\,\text{u}) + (1.008665\,\text{u}) - (31.973907\,\text{u})\right]c^2\left(931.5\,\text{MeV}/c^2\right)\\ &= \boxed{7.94\,\text{MeV}}\end{aligned}$$

16. (*a*) ${}^7_3\text{Li}$ consists of three protons and three neutrons. We find the binding energy from the masses, using hydrogen atoms in place of protons so that we account for the mass of the electrons.

$$\begin{aligned}\text{Binding energy} &= \left[3m\left({}^1_1\text{H}\right) + 4m\left({}^1_0\text{n}\right) - m\left({}^7_3\text{Li}\right)\right]c^2\\ &= \left[3(1.007825\,\text{u}) + 4(1.008665\,\text{u}) - (7.016005\,\text{u})\right]c^2\left(931.5\,\text{MeV}/c^2\right)\\ &= 39.24\,\text{MeV}\end{aligned}$$

$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{39.24\,\text{MeV}}{7\,\text{nucleons}} = \boxed{5.61\,\text{MeV/nucleon}}$$

(*b*) $^{197}_{79}\text{Au}$ consists of 79 protons and 118 neutrons. We find the binding energy as in part (*a*).

$$\begin{aligned}\text{Binding energy} &= \left[79m\left({}^{1}_{1}\text{H}\right)+118m\left({}^{1}_{0}\text{n}\right)-m\left({}^{197}_{79}\text{Au}\right)\right]c^2\\ &= \left[79\left(1.007825\,\text{u}\right)+118\left(1.008665\,\text{u}\right)-\left(196.966569\,\text{u}\right)\right]c^2\left(931.5\,\text{MeV}/c^2\right)\\ &= 1559\,\text{MeV}\end{aligned}$$

$$\frac{\text{Binding energy}}{\text{nucleon}}=\frac{1559\,\text{MeV}}{197\,\text{nucleons}}=\boxed{7.91\,\text{MeV/nucleon}}$$

17. $^{23}_{11}\text{Na}$ consists of 11 protons and 12 neutrons. We find the binding energy from the masses:

$$\begin{aligned}\text{Binding energy} &= \left[11m\left({}^{1}_{1}\text{H}\right)+12m\left({}^{1}_{0}\text{n}\right)-m\left({}^{23}_{11}\text{Na}\right)\right]c^2\\ &= \left[11\left(1.007825\,\text{u}\right)+12\left(1.008665\,\text{u}\right)-\left(22.989769\,\text{u}\right)\right]c^2\left(931.5\,\text{MeV/u}c^2\right)\\ &= 186.6\,\text{MeV}\end{aligned}$$

$$\frac{\text{Binding energy}}{\text{nucleon}}=\frac{186.6\,\text{MeV}}{23}=\boxed{8.113\,\text{MeV/nucleon}}$$

We do a similar calculation for $^{24}_{11}\text{Na}$, consisting of 11 protons and 13 neutrons.

$$\begin{aligned}\text{Binding energy} &= \left[11m\left({}^{1}_{1}\text{H}\right)+13m\left({}^{1}_{0}\text{n}\right)-m\left({}^{24}_{11}\text{Na}\right)\right]c^2\\ &= \left[11\left(1.007825\,\text{u}\right)+13\left(1.008665\,\text{u}\right)-\left(23.990963\,\text{u}\right)\right]c^2\left(931.5\,\text{MeV/u}c^2\right)\\ &= 193.5\,\text{MeV}\end{aligned}$$

$$\frac{\text{Binding energy}}{\text{nucleon}}=\frac{193.5\,\text{MeV}}{24}=\boxed{8.063\,\text{MeV/nucleon}}$$

By this measure, the nucleons in $^{23}_{11}\text{Na}$ are more tightly bound than those in $^{24}_{11}\text{Na}$.

18. We find the required energy by calculating the difference in the masses.

(*a*) Removal of a proton creates an isotope of carbon. To balance electrons, the proton is included as a hydrogen atom: $^{15}_{7}\text{N}\rightarrow{}^{1}_{1}\text{H}+{}^{14}_{6}\text{C}$.

$$\begin{aligned}\text{Energy needed} &= \left[m\left({}^{14}_{6}\text{C}\right)+m\left({}^{1}_{1}\text{H}\right)-m\left({}^{15}_{7}\text{N}\right)\right]c^2\\ &= \left[\left(14.003242\,\text{u}\right)+\left(1.007825\,\text{u}\right)-\left(15.000109\,\text{u}\right)\right]\left(931.5\,\text{MeV/u}c^2\right)\\ &= \boxed{10.21\,\text{MeV}}\end{aligned}$$

(*b*) Removal of a neutron creates another isotope of nitrogen: $^{15}_{7}\text{N}\rightarrow{}^{1}_{0}\text{n}+{}^{14}_{7}\text{N}$.

$$\begin{aligned}\text{Energy needed} &= \left[m\left({}^{14}_{7}\text{N}\right)+m\left({}^{1}_{0}\text{n}\right)-m\left({}^{15}_{7}\text{N}\right)\right]c^2\\ &= \left[\left(14.003074\,\text{u}\right)+\left(1.008665\,\text{u}\right)-\left(15.000109\,\text{u}\right)\right]\left(931.5\,\text{MeV/u}c^2\right)\\ &= \boxed{10.83\,\text{MeV}}\end{aligned}$$

The nucleons are held by the attractive strong nuclear force. It takes less energy to remove the proton because there is also the $\boxed{\text{repulsive electric force}}$ from the other protons.

$\boxed{19.}$ (*a*) We find the binding energy from the masses.

$$\begin{aligned}\text{Binding Energy} &= \left[2m\left({}^{4}_{2}\text{He}\right)-m\left({}^{8}_{4}\text{Be}\right)\right]c^2\\ &= \left[2\left(4.002603\,\text{u}\right)-\left(8.005305\,\text{u}\right)\right]c^2\left(931.5\,\text{MeV/u}c^2\right)=-0.092\,\text{MeV}\end{aligned}$$

Because the binding energy is negative, the nucleus is unstable. It will be in a lower energy state as two alphas instead of a beryllium.

(*b*) We find the binding energy from the masses.

$$\text{Binding Energy} = \left[3m\left({}^{4}_{2}\text{He}\right) - m\left({}^{12}_{6}\text{C}\right)\right]c^2$$

$$= \left[3(4.002603\,\text{u}) - (12.000000\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = +7.3\,\text{MeV}$$

Because the binding energy is positive, the nucleus is $\boxed{\text{stable.}}$

20. The decay is ${}^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$. When we add one electron to both sides to use atomic masses, we see that the mass of the emitted β particle is included in the atomic mass of ${}^{3}_{2}\text{He}$. The energy released is the difference in the masses.

$$\text{Energy released} = \left[m\left({}^{3}_{1}\text{H}\right) - m\left({}^{3}_{2}\text{He}\right)\right]c^2$$

$$= \left[(3.016049\,\text{u}) - (3.016029\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right) = \boxed{0.019\,\text{MeV}}$$

21. The decay is ${}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \bar{\nu}$. The electron mass is accounted for if we use the atomic mass of ${}^{1}_{1}\text{H}$. If we ignore the recoil of the proton and the neutrino, and any possible mass of the neutrino, we get the maximum kinetic energy.

$$K_{\text{max}} = \left[m\left({}^{1}_{0}\text{n}\right) - m\left({}^{1}_{1}\text{H}\right)\right]c^2 = \left[(1.008665\,\text{u}) - (1.007825\,\text{u})\right]c^2\left(931.5\,\text{MeV}/\text{u}c^2\right)$$

$$= \boxed{0.782\,\text{MeV}}$$

22. For the decay ${}^{11}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + {}^{1}_{1}\text{p}$, we find the difference of the initial and the final masses:

$$\Delta m = m\left({}^{11}_{6}\text{C}\right) - m\left({}^{10}_{5}\text{B}\right) - m\left({}^{1}_{1}\text{H}\right)$$

$$= (11.011434\,\text{u}) - (10.012937\,\text{u}) - (1.007825\,\text{u}) = -0.009328\,\text{u}$$

$$= (11.011433\,\text{u}) - (10.012936\,\text{u}) - (1.007825\,\text{u}) = -0.0099318\,\text{u}.$$

Since the final masses are more than the original mass, energy would not be conserved.

23. The wavelength is determined from the energy change between the states.

$$\Delta E = hf = h\frac{c}{\lambda} \quad \rightarrow \quad \lambda = \frac{hc}{\Delta E} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(0.48\,\text{MeV}\right)\left(1.60\times10^{-13}\,\text{J/MeV}\right)} = \boxed{2.6\times10^{-12}\,\text{m}}$$

24. For each decay, we find the difference of the initial and the final masses. If the final mass is more than the initial mass, then the decay is not possible.

(*a*) $\Delta m = m\left({}^{232}_{92}\text{U}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{233}_{92}\text{U}\right) = 232.037156\,\text{u} + 1.008665\,\text{u} - 233.039635\,\text{u} = 0.006816\,\text{u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*b*) $\Delta m = m\left({}^{13}_{7}\text{N}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{14}_{7}\text{N}\right) = 13.005739\,\text{u} + 1.008665\,\text{u} - 14.003074\,\text{u} = 0.011330\,\text{u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

(*c*) $\Delta m = m\left({}^{39}_{19}\text{K}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{40}_{19}\text{K}\right) = 38.963707\,\text{u} + 1.008665\,\text{u} - 39.963998\,\text{u} = 0.008374\,\text{u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible.}}$

$\boxed{25.}$ (*a*) From Appendix F, ${}^{24}_{11}\text{Na}$ is a $\boxed{\beta^- \text{ emitter}}$.

(*b*) The decay reaction is $\boxed{{}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + \beta^- + \bar{\nu}}$. We add 11 electrons to both sides in order to use atomic masses. Then the mass of the beta is accounted for in the mass of the magnesium. The maximum kinetic energy of the β^- corresponds to the neutrino having no kinetic energy (a limiting case). We also ignore the recoil of the magnesium.

$$K_{\beta^-} = \left[m\left({}^{24}_{11}\text{Na}\right) - m\left({}^{24}_{12}\text{Mg}\right)\right]c^2$$
$$= \left[(23.990963\,\text{u}) - (23.985042\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{5.52\,\text{MeV}}$$

26. The kinetic energy of the electron will be maximum if the (essentially) massless neutrino has no kinetic energy. We also ignore the recoil energy of the sodium. The maximum kinetic energy of the reaction is then the *Q*-value of the reaction. Note that the "new" electron mass is accounted for by using atomic masses.

$$K = Q = \left[m\left({}^{23}_{10}\text{Ne}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2 == \left[(22.9945\,\text{u}) - (22.9898\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)$$
$$= \boxed{4.4\,\text{MeV}}$$

If the neutrino were to have all of the kinetic energy, then the minimum kinetic energy of the electron is $\boxed{0.}$ The sum of the kinetic energy of the electron and the energy of the neutrino must be the *Q*-value, and so the neutrino energies are $\boxed{0}$ and $\boxed{4.4\text{ MeV}}$, respectively.

27. (*a*) We find the final nucleus by balancing the mass and charge numbers.

$$Z(X) = Z(\text{U}) - Z(\text{He}) = 92 - 2 = 90$$
$$A(X) = A(\text{U}) - A(\text{He}) = 238 - 4 = 234$$

Thus the final nucleus is $\boxed{{}^{234}_{90}\text{Th}}$.

(*b*) If we ignore the recoil of the thorium, the kinetic energy of the α particle is equal to the *Q*-value of the reaction. The electrons are balanced.

$$K = Q = \left[m\left({}^{238}_{92}\text{U}\right) - m\left({}^{234}_{90}\text{Th}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2 \quad \rightarrow$$
$$m\left({}^{234}_{90}\text{Th}\right) = m\left({}^{238}_{92}\text{U}\right) - m\left({}^{4}_{2}\text{He}\right) - \frac{K}{c^2}$$
$$= \left[238.050788\,\text{u} - 4.002603\,\text{u} - \frac{4.20\,\text{MeV}}{c^2}\left(\frac{1\,\text{u}}{931.5\,\text{MeV}/c^2}\right)\right]$$
$$= \boxed{234.04368\,\text{u}}$$

This answer assumes that the 4.20 MeV value does not limit the sig. fig. of the answer.

28. The reaction is ${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + \beta^- + \bar{\nu}$. The kinetic energy of the β^- will be maximum if the (essentially) massless neutrino has no kinetic energy. We also ignore the recoil of the nickel.

$$K_\beta = \left[m\left({}^{60}\text{Co}\right) - m\left({}^{60}\text{Ni}\right)\right]c^2$$
$$= \left[(59.933822\,\text{u}) - (59.930791\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{2.82\,\text{MeV.}}$$

29. We use conservation of momentum – the momenta of the two particles must be equal and opposite if there are only two products. The energies are small enough that we may use non-relativistic relationships.

$$p_\alpha = p_X \;\rightarrow\; \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_X K_X} \;\rightarrow\; K_X = \frac{m_\alpha}{m_X}K_\alpha = \frac{4}{256}(5.0\,\text{MeV}) = \boxed{0.078\,\text{MeV}}$$

30. For alpha decay we have ${}^{218}_{84}\text{Po} \rightarrow {}^{214}_{82}\text{Pb} + {}^{4}_{2}\text{He}$. We find the Q value.

$$\begin{aligned} Q &= \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{214}_{82}\text{Pb}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2 \\ &= [218.008965\,\text{u} - 213.999805\,\text{u} - 4.002603\,\text{u}]c^2\left(931.5\,\text{MeV/u}c^2\right) \\ &= \boxed{6.108\,\text{MeV}} \end{aligned}$$

For beta decay we have ${}^{218}_{84}\text{Po} \rightarrow {}^{218}_{85}\text{At} + {}^{0}_{-1}\text{e} + \bar{\nu}$. We assume the neutrino is massless, and find the Q value.

$$\begin{aligned} Q &= \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{218}_{85}\text{At}\right)\right]c^2 \\ &= [218.008965\,\text{u} - 218.008694\,\text{u}]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{0.252\,\text{MeV}} \end{aligned}$$

$\boxed{31.}$ (*a*) We find the final nucleus by balancing the mass and charge numbers.

$$Z(X) = Z(\text{P}) - Z(\text{e}) = 15 - (-1) = 16$$
$$A(X) = A(\text{P}) - A(\text{e}) = 32 - 0 = 32$$

Thus the final nucleus is $\boxed{{}^{32}_{16}\text{S}}$.

(*b*) If we ignore the recoil of the sulfur and the energy of the neutrino, the maximum kinetic energy of the electron is the Q-value of the reaction. The reaction is ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + \beta^- + \bar{\nu}$. We add 15 electrons to each side of the reaction, and then we may use atomic masses. The mass of the emitted beta is accounted for in the mass of the sulfur.

$$K = Q = \left[m\left({}^{32}_{15}\text{P}\right) - m\left({}^{32}_{16}\text{S}\right)\right]c^2 \;\rightarrow$$

$$m\left({}^{32}_{16}\text{S}\right) = m\left({}^{32}_{15}\text{P}\right) - \frac{K}{c^2} = \left[31.973907\,\text{u} - \frac{1.71\,\text{MeV}}{c^2}\left(\frac{1\,\text{u}}{931.5\,\text{MeV}/c^2}\right)\right] = \boxed{31.972071\,\text{u}}$$

32. We find the energy from the wavelength.

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.00\times10^{-13}\,\text{m}\right)\left(1.602\times10^{-19}\,\text{J/eV}\right)} = \boxed{12.4\,\text{MeV}}$$

This has to be a $\boxed{\gamma\text{ ray from the nucleus}}$ rather than a photon from the atom. Electron transitions do not involve this much energy. Electron transitions involve energies on the order of a few eV.

33. We add three electron masses to each side of the reaction ${}^{7}_{4}\text{Be} + {}^{0}_{-1}\text{e} \rightarrow {}^{7}_{3}\text{Li} + \nu$. Then for the mass of the product side, we may use the atomic mass of ${}^{7}_{3}\text{Li}$. For the reactant side, including the three electron masses and the mass of the emitted electron, we may use the atomic mass of ${}^{7}_{4}\text{Be}$. The energy released is the Q-value.

$$\begin{aligned} Q &= \left[m\left({}^{7}_{4}\text{Be}\right) - m\left({}^{7}_{3}\text{Li}\right)\right]c^2 \\ &= \left[(7.016930\,\text{u}) - (7.016005\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right) = \boxed{0.862\,\text{MeV}} \end{aligned}$$

34. The emitted photon and the recoiling nucleus have the same magnitude of momentum. We find the recoil energy from the momentum. We assume the energy is small enough that we can use classical relationships.

$$p_\gamma = \frac{E_\gamma}{c} = p_\mathrm{K} = \sqrt{2m_\mathrm{K}K_\mathrm{K}} \quad \rightarrow$$

$$K_\mathrm{K} = \frac{E_\gamma^2}{2m_\mathrm{K}c^2} = \frac{(1.46\,\mathrm{MeV})^2}{2(39.96\,\mathrm{u})\left(931.5\,\mathrm{MeV/u}c^2\right)c^2} = 2.86\times10^{-5}\,\mathrm{MeV} = \boxed{28.6\,\mathrm{eV}}$$

35. The kinetic energy of the β^+ particle will be maximum if the (almost massless) neutrino has no kinetic energy. We ignore the recoil of the boron. Note that if the mass of one electron is added to the mass of the boron, then we may use atomic masses. We also must include the mass of the β^+. (See Problem 38 for details.)

$${}^{11}_{6}\mathrm{C} \rightarrow \left({}^{11}_{5}\mathrm{B} + {}^{0}_{-1}\mathrm{e}\right) + {}^{0}_{1}\beta^+ + v$$

$$K = \left[m\left({}^{11}_{6}\mathrm{C}\right) - m\left({}^{11}_{5}\mathrm{B}\right) - m\left({}^{0}_{-1}\mathrm{e}\right) - m\left({}^{0}_{1}\beta^+\right)\right]c^2 = \left[m\left({}^{11}_{6}\mathrm{C}\right) - m\left({}^{11}_{5}\mathrm{B}\right) - 2m\left({}^{0}_{-1}\mathrm{e}\right)\right]c^2$$

$$= \left[(11.011434\,\mathrm{u}) - (11.009305) - 2(0.00054858\,\mathrm{u})\right]c^2\left(931.5\,\mathrm{MeV/u}c^2\right) = \boxed{0.9612\,\mathrm{MeV}}$$

If the β^+ has no kinetic energy, then the maximum kinetic energy of the neutrino is also $\boxed{0.9612\,\mathrm{MeV}}$. The minimum energy of each is $\boxed{0}$, when the other has the maximum.

36. We assume that the energies are low enough that we may use classical kinematics. In particular, we will use $p = \sqrt{2mK}$. The decay is ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$. If the uranium nucleus is at rest when it decays, the magnitude of the momentum of the two daughter particles must be the same.

$$p_\alpha = p_\mathrm{Th} \;\; ; \;\; K_\mathrm{Th} = \frac{p_\mathrm{Th}{}^2}{2m_\mathrm{Th}} = \frac{p_\alpha{}^2}{2m_\mathrm{Th}} = \frac{2m_\alpha K_\alpha}{2m_\mathrm{Th}} = \frac{m_\alpha}{m_\mathrm{Th}}K_\alpha = \left(\frac{4\,\mathrm{u}}{234\,\mathrm{u}}\right)(4.20\,\mathrm{MeV}) = \boxed{0.0718\,\mathrm{MeV}}$$

The Q-value is the total kinetic energy produced.

$$Q = K_\alpha + K_\mathrm{Th} = 4.20\,\mathrm{MeV} + 0.0718\,\mathrm{MeV} = \boxed{4.27\,\mathrm{MeV}}$$

$\boxed{37.}$ Both energy and momentum are conserved. Therefore, the momenta of the product particles are equal in magnitude. We assume that the energies involved are low enough that we may use classical kinematics; in particular, $p = \sqrt{2mK}$.

$$p_\alpha = p_\mathrm{Pb} \;\; ; \;\; K_\mathrm{Pb} = \frac{p_\mathrm{Pb}^2}{2m_\mathrm{Pb}} = \frac{p_\alpha^2}{2m_\mathrm{Pb}} = \frac{2m_\alpha K_\alpha}{2m_\mathrm{Pb}} = \left(\frac{m_\alpha}{m_\mathrm{Pb}}\right)K_\alpha = \frac{4.0026}{205.97}K_\alpha$$

The sum of the kinetic energies of the product particles must be equal to the Q-value for the reaction.

$$K_\mathrm{Pb} + K_\alpha = \left[m\left({}^{210}_{84}\mathrm{Po}\right) - m\left({}^{206}_{82}\mathrm{Pb}\right) - m\left({}^{4}_{2}\mathrm{He}\right)\right]c^2 = \frac{4.0026}{205.97}K_\alpha + K_\alpha \quad \rightarrow$$

$$K_\alpha = \frac{\left[m\left({}^{210}_{84}\mathrm{Po}\right) - m\left({}^{206}_{82}\mathrm{Pb}\right) - m\left({}^{4}_{2}\mathrm{He}\right)\right]c^2}{\left(\frac{4.0026}{205.97}+1\right)}$$

$$= \frac{\left[(209.982874\,\mathrm{u}) - (205.974465\,\mathrm{u}) - (4.002603\,\mathrm{u})\right]c^2}{\left(\frac{4.0026}{205.97}+1\right)}\left(931.5\,\mathrm{MeV/u}c^2\right) = \boxed{5.31\,\mathrm{MeV}}$$

38. For the positron-emission process, ${}^{A}_{Z}\mathrm{N} \rightarrow {}^{A}_{Z-1}\mathrm{N}' + \mathrm{e}^{+} + \nu$. We must add Z electrons to the nuclear mass of N to be able to use the atomic mass, and so we must also add Z electrons to the reactant side. On the reactant side, we use $Z-1$ electrons to be able to use the atomic mass of N′. Thus we have 1 "extra" electron mass and the β-particle mass, which means that we must include 2 electron masses on the right-hand side. We find the Q-value given this constraint.

$$Q = \left[M_{\mathrm{P}} - \left(M_{\mathrm{D}} + 2m_{\mathrm{e}}\right)\right]c^2 = \left(M_{\mathrm{P}} - M_{\mathrm{D}} - 2m_{\mathrm{e}}\right)c^2.$$

39. (*a*) The decay constant can be found from the half-life, using Eq. 41-8.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.5\times 10^{9}\,\mathrm{yr}} = \boxed{1.5\times 10^{-10}\,\mathrm{yr}^{-1}} = 4.9\times 10^{-18}\,\mathrm{s}^{-1}$$

(*b*) The half-life can be found from the decay constant, using Eq. 41-8.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3.2\times 10^{-5}\,\mathrm{s}^{-1}} = 21660\,\mathrm{s} = \boxed{6.0\,\mathrm{h}}$$

40. We find the half-life from Eq. 41-7d and Eq. 41-8.

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{\ln 2}{T_{1/2}}t} \rightarrow T_{1/2} = -\frac{\ln 2}{\ln\dfrac{R}{R_0}}t = -\frac{\ln 2}{\ln\dfrac{320}{1280}}(3.6\,\mathrm{h}) = \boxed{1.8\,\mathrm{h}}$$

We can see this also from the fact that the rate dropped by a factor of 4, which takes 2 half-lives.

41. We use Eq. 41.6 to find the fraction remaining.

$$N = N_0 e^{-\lambda t} \rightarrow \frac{N}{N_0} = e^{-\lambda t} = e^{-\left[\frac{(\ln 2)(2.0\,\mathrm{yr})(12\,\mathrm{mo/yr})}{9\,\mathrm{mo}}\right]} = 0.158 \approx \boxed{0.16}$$

42. The activity at a given time is given by Eq. 41-7b. The half-life is found in Appendix F.

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{(5730\,\mathrm{yr})(3.16\times 10^{7}\,\mathrm{s/yr})}\left(8.1\times 10^{20}\,\mathrm{nuclei}\right) = \boxed{3.1\times 10^{9}\,\mathrm{decays/s}}$$

$\boxed{\text{43.}}$ Every half-life, the sample is multiplied by one-half.

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^n = \left(\tfrac{1}{2}\right)^6 = \boxed{0.015625}$$

44. The activity of a sample is given by Eq. 41-7a. There are two different decay constants involved. Note that Appendix F gives half-lives, not activities.

$$\lambda_{\mathrm{I}}N_{\mathrm{I}} = \lambda_{\mathrm{Co}}N_{\mathrm{Co}} \rightarrow \lambda_{\mathrm{I}}N_0 e^{-\lambda_{\mathrm{I}}t} = \lambda_{\mathrm{Co}}N_0 e^{-\lambda_{\mathrm{Co}}t} \rightarrow \frac{\lambda_{\mathrm{I}}}{\lambda_{\mathrm{Co}}} = e^{(\lambda_{\mathrm{I}}-\lambda_{\mathrm{Co}})t} \rightarrow$$

$$t = \frac{\ln\left(\lambda_{\mathrm{I}}/\lambda_{\mathrm{Co}}\right)}{\lambda_{\mathrm{I}} - \lambda_{\mathrm{Co}}} = \frac{\ln\left(\dfrac{T_{\underset{1/2}{\mathrm{Co}}}}{T_{\underset{1/2}{\mathrm{I}}}}\right)}{\dfrac{\ln 2}{T_{\underset{1/2}{\mathrm{I}}}} - \dfrac{\ln 2}{T_{\underset{1/2}{\mathrm{Co}}}}} = \frac{\ln\left[\dfrac{(5.2710\,\mathrm{y})(365.25\,\mathrm{d/y})}{(8.0233\,\mathrm{d})}\right]}{\ln 2\left[\dfrac{1}{(8.0233\,\mathrm{d})} - \dfrac{1}{(5.2710\,\mathrm{y})(365.25\,\mathrm{d/y})}\right]} = \boxed{63.703\,\mathrm{d}}$$

45. We find the number of nuclei from the activity of the sample and the half-life.

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N \quad \to$$

$$N = \frac{T_{1/2}}{\ln 2}\left|\frac{dN}{dt}\right| = \frac{\left(4.468\times10^{9}\,\text{yr}\right)\left(3.156\times10^{7}\,\text{s/yr}\right)}{\ln 2}\left(340\,\text{decays/s}\right) = \boxed{6.9\times10^{19}\,\text{nuclei}}$$

46. Each α emission decreases the mass number by 4 and the atomic number by 2. The mass number changes from 235 to 207, which is a change of 28. Thus there must be $\boxed{7\,\alpha\,\text{particles}}$ emitted. With the 7 α emissions, the atomic number would have changed from 92 to 78. Each β^{-} emission increases the atomic number by 1, so to have a final atomic number of 82, there must be $\boxed{4\,\beta^{-}\,\text{particles}}$ emitted.

47. We need both the decay constant and the initial number of nuclei.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(8.0233\,\text{days}\right)\left(24\,\text{h/day}\right)\left(3600\,\text{s/h}\right)} = 9.99905\times10^{-7}\,\text{s}^{-1}$$

$$N_0 = \left[\frac{\left(782\times10^{-6}\,\text{g}\right)}{\left(130.906\,\text{g/mol}\right)}\right]\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 3.596\times10^{18}\,\text{nuclei.}$$

(*a*) We Eq. 41-7b to evaluate the initial activity.

$$\left|\frac{dN}{dt}\right|_0 = \left(9.99905\times10^{-7}\text{s}^{-1}\right)\left(3.596\times10^{18}\right) = 3.5957\times10^{12}\,\text{decays/s} \approx \boxed{3.60\times10^{12}\,\text{decays/s}}$$

(*b*) We evaluate Eq. 41-7c at $t = 1.0\,\text{h}$.

$$\left|\frac{dN}{dt}\right| = \left|\frac{dN}{dt}\right|_0 e^{-\lambda t} = \left(3.5957\times10^{12}\,\text{decays/s}\right)e^{-\left(9.99905\times10^{-7}\,\text{s}^{-1}\right)\left(3600\,\text{s}\right)}$$

$$\approx \boxed{3.58\times10^{12}\,\text{decays/s}}$$

(*c*) We evaluate Eq. 41-7c at t = 4 months. We use a time of 1/3 year for the 4 months.

$$\left|\frac{dN}{dt}\right| = \left|\frac{dN}{dt}\right|_0 e^{-\lambda t} = \left(3.5957\times10^{12}\,\text{decays/s}\right)e^{-\left(9.99905\times10^{-7}\,\text{s}^{-1}\right)\left(0.333\text{yr}\right)\left(3.156\times10^{7}\,\text{s/yr}\right)}$$

$$\approx \boxed{9.72\times10^{7}\,\text{decays/s}}$$

48. We will use the decay constant frequently, so we calculate it here.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{30.8\,\text{s}} = 0.022505\,\text{s}^{-1}$$

(*a*) We find the initial number of nuclei from the atomic mass.

$$N_0 = \frac{\left(7.8\times10^{-6}\,\text{g}\right)}{\left(124\,\text{g/mol}\right)}\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 3.787\times10^{16} \approx \boxed{3.8\times10^{16}\,\text{nuclei.}}$$

(*b*) Evaluate Eq. 41-6 at $t = 2.6\,\text{min}$.

$$N = N_0 e^{-\lambda t} = \left(3.787\times10^{16}\right)e^{-\left(0.022505\text{s}^{-1}\right)\left(2.6\,\text{min}\right)\left(60\text{s/min}\right)} = 1.131\times10^{15} \approx \boxed{1.1\times10^{15}\,\text{nuclei}}$$

(*c*) The activity is found by Eq. 41-7a.

$$\lambda N = \left(0.022505\,\text{s}^{-1}\right)\left(1.131\times10^{15}\right) = 2.545\times10^{13} \approx \boxed{2.5\times10^{13}\,\text{decays/s}}$$

(*d*) We find the time from Eq. 41-7a.

$$\lambda N = \lambda N_0 e^{-\lambda t} \quad \rightarrow$$

$$t = -\frac{\ln\left(\dfrac{\lambda N}{\lambda N_0}\right)}{\lambda} = -\frac{\ln\left[\dfrac{1\,\text{decay/s}}{\left(0.022505\,\text{s}^{-1}\right)\left(3.787\times 10^{16}\right)\text{decay/s}}\right]}{0.022505\,\text{s}^{-1}} = 1528\,\text{s} = 25.46\,\text{min} \approx \boxed{25\,\text{min}}$$

$\boxed{49.}$ We find the mass from the initial decay rate and Eq. 41-7b.

$$\left|\frac{dN}{dt}\right|_0 = \lambda N_0 = \lambda m \frac{6.02\times 10^{23}\,\text{nuclei/mole}}{(\text{atomic weight})\,\text{g/mole}} \quad \rightarrow$$

$$m = \left|\frac{dN}{dt}\right|_0 \frac{1}{\lambda}\frac{(\text{atomic weight})}{\left(6.02\times 10^{23}\right)} = \left|\frac{dN}{dt}\right|_0 \frac{T_{1/2}}{\ln 2}\frac{(\text{atomic weight})}{\left(6.02\times 10^{23}\right)}$$

$$= \left(2.0\times 10^5\,\text{s}^{-1}\right)\frac{\left(1.265\times 10^9\,\text{yr}\right)\left(3.156\times 10^7\,\text{s/yr}\right)}{(\ln 2)}\frac{(39.963998\,\text{g})}{\left(6.02\times 10^{23}\right)} = \boxed{0.76\,\text{g}}$$

50. The number of nuclei is found from the mass and the atomic weight. The activity is then found from number of nuclei and the half-life, using Eq. 41-7b.

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{1.23\times 10^6\,\text{s}}\left[\frac{\left(8.7\times 10^{-6}\,\text{g}\right)}{(31.974\,\text{g/mol})}\right]\left(6.02\times 10^{23}\,\text{atoms/mol}\right) = 9.2\times 10^{10}\,\text{decays/s}$$

51. We find the mass from the initial decay rate and Eq. 41-7b.

$$\left|\frac{dN}{dt}\right|_0 = \lambda N_0 = \lambda m \frac{6.02\times 10^{23}\,\text{nuclei/mole}}{(\text{atomic weight})\,\text{g/mole}} \quad \rightarrow$$

$$m = \left|\frac{dN}{dt}\right|_0 \frac{1}{\lambda}\frac{(\text{atomic weight})}{\left(6.02\times 10^{23}\right)} = \left|\frac{dN}{dt}\right|_0 \frac{T_{1/2}}{\ln 2}\frac{(\text{atomic weight})}{\left(6.02\times 10^{23}\right)}$$

$$= \left(3.65\times 10^4\,\text{s}^{-1}\right)\frac{(87.32\,\text{d})(86{,}400\,\text{s/d})}{(\ln 2)}\frac{(34.969032\,\text{g})}{\left(6.02\times 10^{23}\right)} = \boxed{2.31\times 10^{-11}\,\text{g}}$$

52. (*a*) The decay constant is found from Eq. 41-8.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(1.59\times 10^5\,\text{yr}\right)\left(3.156\times 10^7\,\text{s/yr}\right)} = \boxed{1.38\times 10^{-13}\,\text{s}^{-1}}$$

(*b*) The activity is the decay constant times the number of nuclei.

$$\lambda N = \left(1.38\times 10^{-13}\,\text{s}^{-1}\right)\left(5.50\times 10^{18}\right) = 7.59\times 10^5\,\text{decays/s}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{4.55\times 10^7\,\text{decays/min.}}$$

53. We use Eq. 41-7c.

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{\ln 2}{T_{1/2}}t} \quad \rightarrow \quad T_{1/2} = -\frac{\ln 2}{\ln\dfrac{R}{R_0}}t = -\frac{\ln 2}{\ln\frac{1}{4}}(8.6\,\text{min}) = \boxed{4.3\,\text{min}}$$

54. Because the fraction of atoms that are ${}^{14}_{6}\text{C}$ is so small, we use the atomic weight of ${}^{12}_{6}\text{C}$ to find the number of carbon atoms in the sample. The activity is found from Eq. 41-7a.

$$N=\left[\frac{(385\,\text{g})}{(12\,\text{g/mol})}\right]\left(6.02\times10^{23}\,\text{atoms/mol}\right)=1.93\times10^{25}\,\text{atoms}$$

$$N_{14}=\left(\frac{1.3}{10^{12}}\right)\left(1.93\times10^{25}\right)=2.51\times10^{13}\,\text{nuclei}$$

$$\lambda N=\left[\frac{\ln 2}{(5730\,\text{yr})\left(3.156\times10^{7}\,\text{s/yr}\right)}\right]\left(2.51\times10^{13}\right)=\boxed{96\,\text{decays/s}}$$

$\boxed{55.}$ We find the mass from the activity. Note that N_{A} is used to represent Avogadro's number.

$$R=\lambda N=\frac{\ln 2}{T_{1/2}}\frac{mN_{\text{A}}}{A}\quad\rightarrow$$

$$m=\frac{RT_{1/2}A}{N_{\text{A}}\ln 2}=\frac{(370\,\text{decays/s})\left(4.468\times10^{9}\,\text{yr}\right)\left(3.156\times10^{7}\,\text{s/yr}\right)(238.05\,\text{g/mole})}{\left(6.02\times10^{23}\,\text{nuclei/mole}\right)\ln 2}=\boxed{2.98\times10^{-2}\,\text{g}}$$

56. We assume that the elapsed time is much smaller than the half-life, so that can approixmate the decay rate as being constant. We also assume that the ${}^{87}_{38}\text{Sr}$ is stable, and there was none present when the rocks were formed. Thus every atom of ${}^{87}_{37}\text{Rb}$ that decayed is now an atom of ${}^{87}_{38}\text{Sr}$.

$$N_{\text{Sr}}=-\Delta N_{\text{Rb}}=\lambda N_{\text{Rb}}\,\Delta t\quad\rightarrow\quad\Delta t=\frac{N_{\text{Sr}}}{N_{\text{Rb}}}\frac{T_{1/2}}{\ln 2}=(0.0260)\frac{4.75\times10^{10}\,\text{yr}}{\ln 2}=\boxed{1.78\times10^{9}\,\text{yr}}$$

This is $\approx 4\%$ of the half-life, so our original assumption is valid.

57. The activity is given by Eq. 41-7a.

$$0.975\lambda N_0=\lambda N_0e^{-\lambda t}\quad\rightarrow\quad\ln 0.975=-\lambda t=-\frac{\ln 2}{T_{1/2}}t\quad\rightarrow$$

$$T_{1/2}=-\frac{\ln 2}{\ln 0.975}(31.0\,\text{h})=848.71\,\text{h}\left(\frac{1\,\text{d}}{24\,\text{h}}\right)=\boxed{35.4\,\text{d}}$$

58. The activity is given by Eq. 41-7a.

(*a*) We use Eq. 41-7c.We find the number of half-lives from

$$R=R_0e^{-\lambda t}\quad\rightarrow\quad t=-\frac{1}{\lambda}\ln\frac{R}{R_0}=-\frac{T_{1/2}}{\ln 2}\ln\frac{R}{R_0}=-\frac{53\,\text{d}}{\ln 2}\left(\ln\frac{15\,\text{decays/s}}{350\,\text{decays/s}}\right)=240.85\,\text{d}\approx\boxed{240\,\text{d}}$$

(*b*) We find the mass from the activity. Note that N_{A} is used to represent Avogadro's number.

$$R_0=\lambda N_0=\frac{\ln 2}{T_{1/2}}\frac{m_0N_{\text{A}}}{A}\quad\rightarrow$$

$$m=\frac{R_0T_{1/2}A}{N_{\text{A}}\ln 2}=\frac{(350\,\text{decays/s})(53\,\text{d})(86{,}400\,\text{s/d})(7.017\,\text{g/mole})}{\left(6.02\times10^{23}\,\text{nuclei/mole}\right)\ln 2}=\boxed{2.7\times10^{-14}\,\text{g}}$$

59. ${}^{232}_{90}\text{Th}\rightarrow\boxed{{}^{228}_{88}\text{Ra}}+{}^{4}_{2}\alpha$; ${}^{228}_{88}\text{Ra}\rightarrow\boxed{{}^{228}_{89}\text{Ac}}+\beta^{-}$; ${}^{228}_{89}\text{Ac}\rightarrow\boxed{{}^{228}_{90}\text{Th}}+\beta^{-}$; ${}^{228}_{90}\text{Th}\rightarrow\boxed{{}^{224}_{88}\text{Ra}}+{}^{4}_{2}\alpha$; ${}^{224}_{88}\text{Ra}\rightarrow\boxed{{}^{220}_{86}\text{Rn}}+{}^{4}_{2}\alpha$

$${}^{235}_{92}\mathrm{U} \to \boxed{{}^{231}_{90}\mathrm{Th}} + {}^{4}_{2}\alpha \;;\; {}^{231}_{90}\mathrm{Th} \to \boxed{{}^{231}_{91}\mathrm{Pa}} + \beta^- \;;\; {}^{231}_{91}\mathrm{Pa} \to \boxed{{}^{227}_{89}\mathrm{Ac}} + {}^{4}_{2}\alpha \;;\; {}^{227}_{89}\mathrm{Ac} \to \boxed{{}^{227}_{90}\mathrm{Th}} + \beta^- \;;$$

$${}^{227}_{90}\mathrm{Th} \to \boxed{{}^{223}_{88}\mathrm{Ra}} + {}^{4}_{2}\alpha$$

60. Because the fraction of atoms that are ${}^{14}_{6}\mathrm{C}$ is so small, we use the atomic weight of ${}^{12}_{6}\mathrm{C}$ to find the number of carbon atoms in 85 g. We then use the ratio to find the number of ${}^{14}_{6}\mathrm{C}$ atoms present when the club was made. Finally, we use the activity as given in Eq. 41-7c to find the age of the club.

$$N_{{}^{12}_{6}\mathrm{C}} = \left[\frac{(85\,\mathrm{g})}{(12\,\mathrm{g/mol})}\right]\left(6.02\times10^{23}\ \mathrm{atoms/mol}\right) = 4.264\times10^{24}\ \mathrm{atoms}$$

$$N_{{}^{14}_{6}\mathrm{C}} = \left(1.3\times10^{-12}\right)\left(4.264\times10^{24}\right) = 5.543\times10^{12}\ \mathrm{nuclei}$$

$$\left(\lambda N_{{}^{14}_{6}\mathrm{C}}\right)_{\mathrm{today}} = \left(\lambda N_{{}^{14}_{6}\mathrm{C}}\right)_0 e^{-\lambda t} \;\to$$

$$t = -\frac{1}{\lambda}\ln\frac{\left(\lambda N_{{}^{14}_{6}\mathrm{C}}\right)_{\mathrm{today}}}{\left(\lambda N_{{}^{14}_{6}\mathrm{C}}\right)_0} = -\frac{T_{1/2}}{\ln 2}\ln\frac{\left(\lambda N_{{}^{14}_{6}\mathrm{C}}\right)_{\mathrm{today}}}{\left(\dfrac{\ln 2}{T_{1/2}}N_{{}^{14}_{6}\mathrm{C}}\right)_0}$$

$$= -\frac{5730\,\mathrm{yr}}{\ln 2}\ln\frac{(7.0\,\mathrm{decays/s})}{\left[\dfrac{\ln 2}{(5730\,\mathrm{yr})(3.156\times10^{7}\ \mathrm{s/yr})}\left(5.543\times10^{12}\ \mathrm{nuclei}\right)\right]_0} = 9178\,\mathrm{yr} \approx \boxed{9200\,\mathrm{yr}}$$

61. The number of radioactive nuclei decreases exponentially, and every radioactive nucleus that decays becomes a daughter nucleus.

$$N = N_0 e^{-\lambda t}$$

$$N_{\mathrm{D}} = N_0 - N = \boxed{N_0\left(1 - e^{-\lambda t}\right)}$$

62. The activity is given by Eq. 41-7d.

$$R = R_0 e^{-\lambda t} \;\to\; \lambda = -\frac{\ln\dfrac{R}{R_0}}{t} = \frac{\ln 2}{T_{1/2}} \;\to\; T_{1/2} = -\frac{t\ln 2}{\ln\dfrac{R}{R_0}} = -\frac{(4.00\,\mathrm{h})\ln 2}{\ln 0.01050} = 0.6085\,\mathrm{h} = 36.5\,\mathrm{min}$$

From Appendix F we see that the isotope is $\boxed{{}^{211}_{82}\mathrm{Pb}}$.

63. Because the carbon is being replenished in living trees, we assume that the amount of ${}^{14}_{6}\mathrm{C}$ is constant until the wood is cut, and then it decays. We use Eq. 41-6.

$$N = N_0 e^{-\lambda t} \;\to\; \lambda = -\frac{\ln\dfrac{N}{N_0}}{t} = \frac{\ln 2}{T_{1/2}} \;\to\; t = -\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0} = -\frac{(5730\,\mathrm{yr})\ln 0.060}{\ln 2} = \boxed{23{,}000\,\mathrm{yr}}$$

64. (*a*) The mass number is found from the radius, using Eq. 41-1.

$$r=\left(1.2\times10^{-15}\,\text{m}\right)A^{1/3} \;\rightarrow\; A=\left(\frac{r}{1.2\times10^{-15}\,\text{m}}\right)^3=\left(\frac{5000\,\text{m}}{1.2\times10^{-15}\,\text{m}}\right)^3=7.23\times10^{55}\approx\boxed{7\times10^{55}}$$

(*b*) The mass of the neutron star is the mass number times the atomic mass unit conversion in kg.

$$m=A\left(1.66\times10^{-27}\,\text{kg/u}\right)=\left(7.23\times10^{55}\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)=1.20\times10^{29}\,\text{kg}\approx\boxed{1\times10^{29}\,\text{kg}}$$

Note that this is about 6% of the mass of the Sun.

(*c*) The acceleration of gravity on the surface of the neutron star is found from Eq. 6-4 applied to the neutron star.

$$g=\frac{Gm}{r^2}=\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.20\times10^{29}\,\text{kg}\right)}{\left(5000\,\text{m}\right)^2}=3.20\times10^{11}\,\text{m/s}^2\approx\boxed{3\times10^{11}\,\text{m/s}^2}$$

65. Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We use Eq. 41-6.

$$N=N_0e^{-\lambda t} \;\rightarrow\; \lambda=-\frac{\ln\frac{N}{N_0}}{t}=\frac{\ln 2}{T_{1/2}} \;\rightarrow\; t=-\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0}=-\frac{(12.3\,\text{yr})\ln 0.10}{\ln 2}=\boxed{41\,\text{yr}}$$

66. We assume a mass of 70 kg of water, and find the number of protons, given that there are 10 protons in a water molecule.

$$N_{\text{protons}}=\left[\frac{\left(70\times10^3\,\text{g water}\right)}{\left(18\,\text{g water/mol water}\right)}\right]\left(6.02\times10^{23}\,\text{molecules water/mol water}\right)\left(\frac{10\,\text{protons}}{\text{water molecule}}\right)$$

$$=2.34\times10^{28}\,\text{protons}$$

We assume that the time is much less than the half-life so that the rate of decay is constant.

$$\frac{\Delta N}{\Delta t}=\lambda N=\left(\frac{\ln 2}{T_{1/2}}\right)N \;\rightarrow\; \Delta t=\frac{\Delta N}{N}\left(\frac{T_{1/2}}{\ln 2}\right)=\frac{1\,\text{proton}}{2.34\times10^{28}\,\text{protons}}\left(\frac{10^{33}\,\text{yr}}{\ln 2}\right)=\boxed{60{,}000\,\text{yr}}$$

This is almost 1000 times a normal life expectancy.

$\boxed{67.}$ Consider the reaction $\text{n}\rightarrow\text{p}+\text{e}^-+\bar{\nu}$. The neutron, proton, and electron are all spin $\frac{1}{2}$ particles. If the proton and neutron spins are aligned (both are $\frac{1}{2}$, for example), then the electron and neutrino spins must cancel. Since the electron is spin $\frac{1}{2}$, the neutrino must also be spin $\frac{1}{2}$ in this case.

The other possibility is if the proton and neutron spins are opposite of each other. Consider the case of the neutron having spin $\frac{1}{2}$ and the proton having spin $-\frac{1}{2}$. If the electron has spin $\frac{1}{2}$, then the spins of the electron and proton cancel, and the neutrino must have spin $\frac{1}{2}$ for angular momentum to be conserved. If the electron has spin $-\frac{1}{2}$, then the spin of the neutrino must be $\frac{3}{2}$ for angular momentum to be conserved.

A similar argument could be made for positron emission, with $\text{p}\rightarrow\text{n}+\text{e}^++\nu$.

68. We assume that all of the kinetic energy of the alpha particle becomes electrostatic potential energy at the distance of closest approach. Note that the distance found is the distance from the center of the alpha to the center of the gold nucleus.

$$K_{\mathrm{i}} + U_{\mathrm{i}} = K_{\mathrm{f}} + U_{\mathrm{f}} \rightarrow K_\alpha + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\mathrm{Au}}}{r} \rightarrow$$

$$r = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\mathrm{Au}}}{K_\alpha} = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{(2)(79)\left(1.60\times10^{-19}\,\mathrm{C}\right)^2}{(7.7\,\mathrm{MeV})\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)} = 2.951\times10^{-14}\,\mathrm{m}$$

$$\approx \boxed{3.0\times10^{-14}\,\mathrm{m}}$$

We use Eq. 41-1 to compare to the size of the gold nucleus.

$$\frac{r_{\mathrm{approach}}}{r_{\mathrm{Au}}} = \frac{2.951\times10^{-14}\,\mathrm{m}}{197^{1/3}\left(1.2\times10^{-15}\,\mathrm{m}\right)} = 4.2$$

So the distance of approach is about $\boxed{4.2\times}$ the radius of the gold nucleus.

69. We find the number of half-lives from the change in activity.

$$\frac{\left|\frac{dN}{dt}\right|}{\left|\frac{dN}{dt}\right|_0} = \left(\tfrac{1}{2}\right)^n = 0.0100 \rightarrow n = \frac{\ln 0.0100}{\ln\frac{1}{2}} = \boxed{6.64\text{ half-lives}}$$

70. We find the mass from the activity. Note that N_{A} is used to represent Avogadro's number.

$$R = \lambda N = \frac{\ln 2}{T_{1/2}}\frac{m_{40}N_{\mathrm{A}}}{A} \rightarrow$$

$$m_{40} = \frac{RT_{1/2}A}{N_{\mathrm{A}}\ln 2} = \frac{(45\,\mathrm{decays/s})\left(1.265\times10^9\,\mathrm{yr}\right)\left(3.156\times10^7\,\mathrm{s/yr}\right)(39.964\,\mathrm{g/mole})}{\left(6.02\times10^{23}\,\mathrm{nuclei/mole}\right)\ln 2} = 1.721\times10^{-4}\,\mathrm{g}$$

$$\approx \boxed{1.7\times10^{-4}\,\mathrm{g}}$$

We find the number of $^{39}_{19}\mathrm{K}$ atoms from the number of $^{40}_{19}\mathrm{K}$ atoms and the abundance given in Appendix F. That is then used to find the mass of $^{39}_{19}\mathrm{K}$.

$$R = \lambda N_{40} \rightarrow N_{40} = \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} \; ; \; N_{40} = 0.000117N_K \; ; \; N_{39} = 0.93258N_K = \frac{0.93258}{0.000117}N_{40}$$

$$m_{39} = N_{39}\frac{A}{N_{\mathrm{A}}} = \frac{0.93258}{0.000117}N_{40}\frac{A}{N_{\mathrm{A}}} = \frac{0.93258}{0.000117}\frac{RT_{1/2}}{\ln 2}\frac{A}{N_{\mathrm{A}}}$$

$$= \left(\frac{0.93258}{0.000117}\right)\frac{(45\,\mathrm{decays/s})\left(1.265\times10^9\,\mathrm{yr}\right)\left(3.156\times10^7\,\mathrm{s/yr}\right)}{\ln 2}\frac{(38.964\,\mathrm{g/mole})}{\left(6.02\times10^{23}\,\mathrm{nuclei/mole}\right)}$$

$$= \boxed{1.3\,\mathrm{g}}$$

71. (*a*) If the initial nucleus is at rest when it decays, momentum conservation says that the magnitude of the momentum of the alpha particle will be equal to the magnitude of the momentum of the daughter particle. We use that to calculate the (non-relativistic) kinetic energy of the daughter particle. The mass of each particle is essentially equal to its atomic mass number, in atomic mass units.

$$p_\alpha = p_{\mathrm{D}} \;;\; K_{\mathrm{D}} = \frac{p_{\mathrm{D}}^2}{2m_{\mathrm{D}}} = \frac{p_\alpha^2}{2m_{\mathrm{D}}} = \frac{2m_\alpha K_\alpha}{2m_{\mathrm{D}}} = \frac{m_\alpha}{m_{\mathrm{D}}}K_\alpha = \frac{A_\alpha}{A_{\mathrm{D}}}K_\alpha = \frac{4}{A_{\mathrm{D}}}K_\alpha$$

$$\frac{K_{\mathrm{D}}}{K_\alpha + K_{\mathrm{D}}} = \frac{\frac{4}{A_{\mathrm{D}}}K_\alpha}{\left[K_\alpha + \frac{4}{A_{\mathrm{D}}}K_\alpha\right]} = \frac{K_\alpha}{\left[\frac{A_{\mathrm{D}}}{4}K_\alpha + K_\alpha\right]} = \boxed{\frac{1}{1+\frac{1}{4}A_{\mathrm{D}}}}$$

(*b*) We specifically consider the decay of ${}^{226}_{88}\mathrm{Ra}$. The daughter has $A_{\mathrm{D}} = 222$.

$$\frac{K_{\mathrm{D}}}{K_\alpha + K_{\mathrm{D}}} = \frac{1}{1+\frac{1}{4}A_{\mathrm{D}}} = \frac{1}{1+\frac{1}{4}(222)} = 0.017699 \approx 1.8\%$$

Thus the alpha particle carries away $1 - 0.018 = 0.982 = \boxed{98.2\%}$.

72. We see from the periodic chart that Sr is in the same column as $\boxed{\text{calcium.}}$ If strontium is ingested, the body may treat it chemically as if it were calcium, which means it might be $\boxed{\text{stored by the body in bones.}}$ We use Eq. 41-6 to find the time to reach a 1% level.

$$N = N_0 e^{-\lambda t} \;\rightarrow\; \lambda = -\frac{\ln\frac{N}{N_0}}{t} = \frac{\ln 2}{T_{1/2}} \;\rightarrow$$

$$t = -\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0} = -\frac{(29\,\mathrm{yr})\ln 0.010}{\ln 2} = 192.67\,\mathrm{yr} \approx \boxed{190\,\mathrm{yr}}$$

The decay reactions are as follows. We assume the daughter undergoes beta decay.

$$\boxed{{}^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y} + {}^{0}_{-1}\mathrm{e} + \bar{\nu} \;;\; {}^{90}_{39}\mathrm{Y} \rightarrow {}^{90}_{40}\mathrm{Zr} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}}$$

$\boxed{73.}$ We take the momentum of the nucleon to be equal to the uncertainty in the momentum of the nucleon, as given by the uncertainty principle. The uncertainty in position is estimate as the radius of the nucleus. With that momentum, we calculate the kinetic energy, using a classical formula.

$$\Delta p \Delta x \approx \hbar \;\rightarrow\; p \approx \Delta p \approx \frac{\hbar}{\Delta x} = \frac{\hbar}{r}$$

$$K = \frac{p^2}{2m} = \frac{\hbar^2}{2mr^2} = \frac{\left((1.055\times10^{-34}\,\mathrm{J{\bullet}s})\right)^2}{2(1.67\times10^{-27}\,\mathrm{kg})\left[(56^{1/3})(1.2\times10^{-15}\,\mathrm{m})\right]^2(1.60\times10^{-13}\,\mathrm{J/MeV})}$$

$$= 0.988\,\mathrm{MeV} \approx \boxed{1\,\mathrm{MeV}}$$

74. (*a*) The reaction is ${}^{236}_{92}\mathrm{U} \rightarrow {}^{232}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$. If we assume the uranium nucleus is initially at rest, then the magnitude of the momenta of the two products must be the same. The kinetic energy available to the products is the *Q*-value of the reaction. We use the non-relativistic relationship that $p^2 = 2mK$.

$$p_{\mathrm{He}} = p_{\mathrm{Th}} \;;\; K_{\mathrm{Th}} = \frac{p_{\mathrm{Th}}^2}{2m_{\mathrm{Th}}} = \frac{p_{\mathrm{He}}^2}{2m_{\mathrm{Th}}} = \frac{2m_{\mathrm{He}}K_{\mathrm{He}}}{2m_{\mathrm{Th}}} = \frac{m_{\mathrm{He}}}{m_{\mathrm{Th}}}K_{\mathrm{He}}$$

$$Q = K_{\mathrm{Th}} + K_{\mathrm{He}} = \left(\frac{m_{\mathrm{He}}}{m_{\mathrm{Th}}} + 1\right)K_{\mathrm{He}} \;\rightarrow$$

$$K_{\text{He}}=\left(\frac{m_{\text{Th}}}{m_{\text{He}}+m_{\text{Th}}}\right)Q=\left(\frac{m_{\text{Th}}}{m_{\text{He}}+m_{\text{Th}}}\right)\left[m_{\text{U}}-m_{\text{Th}}-m_{\text{He}}\right]c^2$$
$$=\left(\frac{232.038055\,\text{u}}{4.002603\,\text{u}+232.038055\,\text{u}}\right)\left[236.045568\,\text{u}-232.038055\,\text{u}-4.002603\,\text{u}\right]c^2$$
$$=0.004827\text{u}\left(931.5\,\text{MeV}/\text{u}c^2\right)=\boxed{4.496\,\text{MeV}}$$

(*b*) We use Eq. 41-1 to estimate the radii.

$$r_{\text{He}}=\left(1.2\times10^{-15}\,\text{m}\right)\left(4\right)^{1/3}=1.905\times10^{-15}\,\text{m}\approx\boxed{1.9\times10^{-15}\,\text{m}}$$
$$r_{\text{Th}}=\left(1.2\times10^{-15}\,\text{m}\right)\left(232\right)^{1/3}=7.374\times10^{-15}\,\text{m}\approx\boxed{7.4\times10^{-15}\,\text{m}}$$

(*c*) The maximum height of the Coulomb barrier will correspond to the alpha particle and the thorium nucleus being separated by the sum of their radii. We use Eq. 23-10.

$$U=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{He}}q_{\text{Th}}}{r_{\text{A}}}=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{He}}q_{\text{Th}}}{\left(r_{\text{He}}+r_{\text{Th}}\right)}$$
$$=\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(90)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(4^{1/3}+232^{1/3}\right)\left(1.2\times10^{-15}\,\text{m}\right)\left(1.60\times10^{-13}\,\text{J/MeV}\right)}$$
$$=27.898\,\text{MeV}\approx\boxed{28\,\text{MeV}}$$

(*d*) At position “A”, the product particles are separated by the sum of their radii, about 9.3 fm. At position “B”, the alpha particle will have a potential energy equal to its final kinetic energy, 4.496 MeV. Use Eq. 23-10 to solve for the separation distance at position “B”.

$$U_{\text{B}}=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{He}}q_{\text{Th}}}{r_{\text{B}}}\quad\rightarrow$$
$$r_{\text{B}}=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{He}}q_{\text{Th}}}{U_{\text{B}}}=\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2)(90)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(4.496\,\text{MeV}\right)\left(1.60\times10^{-13}\,\text{J/MeV}\right)}$$
$$=57.57\times10^{-15}\,\text{m}$$
$$r_{\text{B}}-r_{\text{A}}=57.6\,\text{fm}-9.3\,\text{fm}=\boxed{48.3\,\text{fm}}$$

Note that this is a center-to-center distance.

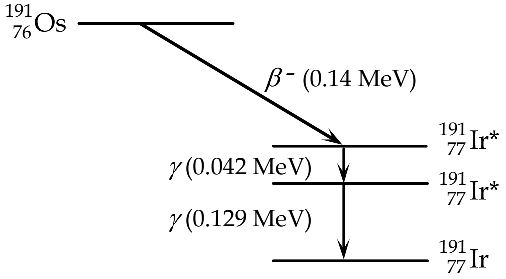
75. (*a*) We find the daughter nucleus by balancing the mass and charge numbers:

$$Z(X)=Z(\text{Os})-Z\left(\text{e}^-\right)=76-(-1)=77$$
$$A(X)=A(\text{Os})-A\left(\text{e}^-\right)=191-0=191$$

The daughter nucleus is $\boxed{{}^{191}_{77}\text{Ir}}$.

(*b*) See the included diagram.

(*c*) Because there is only one β energy, the β decay must be to the higher excited state.



76. The activity is the decay constant times the number of nuclei, as given by Eq. 41-7a.

(*a*) We calculate the activity for ${}^{131}_{53}\text{I}$.

$$R=\lambda N=\frac{\ln 2}{T_{1/2}}N=\frac{\ln 2}{\left(8.02\,\text{d}\right)\left(86{,}400\,\text{s/d}\right)}\left(\frac{1.0\,\text{g}}{130.906\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{nuclei/mol}\right)$$

$$= \boxed{4.6\times10^{15}\text{ decays/s}}$$

(*b*) We calculate the activity for ${}^{238}_{92}\text{U}$.

$$R = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{\left(4.47\times10^{9}\text{ yr}\right)\left(3.156\times10^{7}\text{ s/yr}\right)}\left(\frac{1.0\text{g}}{238.051\text{g/mol}}\right)\left(6.02\times10^{23}\text{ nuclei/mol}\right)$$
$$= \boxed{1.2\times10^{4}\text{ decays/s}}$$

77. From Figure 30–1, we see that the average binding energy per nucleon at $A = 63$ is ~8.6 MeV. We use the mass average atomic weight as the average number of nucleons for the two stable isotopes of copper. That gives a binding energy of $(63.546)(8.6\text{MeV}) = 546.5\text{MeV} \approx \boxed{550\text{MeV}}$.

The number of atoms in a penny is found from the atomic weight.

$$N = \frac{(3.0\text{g})}{(63.546\text{g/mol})}\left(6.02\times10^{23}\text{ atoms/mol}\right) = 2.842\times10^{22}\text{ atoms}$$

Thus the total energy needed is the product of the number of atoms times the binding energy.

$$\left(2.842\times10^{22}\text{atoms}\right)\left(546.5\text{MeV/atom}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right) = \boxed{2.5\times10^{12}\text{ J}}$$

78. (*a*) $\Delta\left({}^{4}_{2}\text{He}\right) = m\left({}^{4}_{2}\text{He}\right) - A\left({}^{4}_{2}\text{He}\right) = 4.002603\text{u} - 4 = \boxed{0.002603\text{u}}$

$$= (0.002603\text{u})\left(931.5\text{MeV/u}c^2\right) = \boxed{2.425\text{Mev}/c^2}$$

(*b*) $\Delta\left({}^{12}_{6}\text{C}\right) = m\left({}^{12}_{6}\text{C}\right) - A\left({}^{12}_{6}\text{C}\right) = 12.000000\text{u} - 12 = \boxed{0}$

(*c*) $\Delta\left({}^{86}_{38}\text{Sr}\right) = m\left({}^{86}_{38}\text{Sr}\right) - A\left({}^{86}_{38}\text{Sr}\right) = 85.909260\text{u} - 86 = \boxed{-0.090740\text{u}}$

$$= (-0.090740\text{u})\left(931.5\text{MeV/u}c^2\right) = \boxed{-84.52\text{MeV}/c^2}$$

(*d*) $\Delta\left({}^{235}_{92}\text{U}\right) = m\left({}^{235}_{92}\text{U}\right) - A\left({}^{235}_{92}\text{U}\right) = 235.043930\text{u} - 235 = \boxed{0.043930\text{u}}$

$$= (0.043930\text{u})\left(931.5\text{MeV/u}c^2\right) = \boxed{40.92\text{MeV}/c^2}$$

(*e*) From the Appendix we see that

$$\boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le Z \le 8 \text{ and } Z \ge 85;\\ \Delta < 0 \text{ for } 9 \le Z \le 84.\end{array}} \qquad \boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le A \le 15 \text{ and } A > 214;\\ \Delta < 0 \text{ for } 16 \le A \le 214.\end{array}}$$

$\boxed{79.}$ The reaction is ${}^{1}_{1}\text{H} + {}^{1}_{0}\text{n} \to {}^{2}_{1}\text{H}$. If we assume the initial kinetic energies are small, then the energy of the gamma is the *Q*-value of the reaction.

$$Q = \left[m\left({}^{1}_{1}\text{He}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{2}_{1}\text{He}\right)\right]c^2$$
$$= \left[(1.007825\text{u}) + (1.008665\text{u}) - (2.014082\text{u})\right]c^2\left(931.5\text{MeV/u}c^2\right) = \boxed{2.243\text{MeV}}$$

80. (*a*) We use the definition of the mean life given in the problem. We use a definite integral formula from Appendix B-5.

$$\tau = \frac{\int_0^\infty tN(t)\,dt}{\int_0^\infty N(t)\,dt} = \frac{\int_0^\infty tN_0e^{-\lambda t}\,dt}{\int_0^\infty N_0e^{-\lambda t}\,dt} = \frac{\int_0^\infty te^{-\lambda t}\,dt}{\int_0^\infty e^{-\lambda t}\,dt} = \frac{\frac{1}{\lambda^2}}{-\frac{1}{\lambda}e^{-\lambda t}\Big|_0^\infty} = \frac{\frac{1}{\lambda^2}}{\frac{1}{\lambda}} = \boxed{\frac{1}{\lambda}}$$

(*b*) We evaluate at time $t=\tau=\frac{1}{\lambda}$.

$$\frac{N(t=\tau)}{N(t=0)}=\frac{N_0e^{-\lambda\tau}}{N_0e^{-0}}=e^{-\frac{\lambda}{\lambda}}=e^{-1}=\boxed{0.368}$$

81. (*a*) The usual fraction of ${}^{14}_{6}\text{C}$ is 1.3×10^{-12}. Because the fraction of atoms that are ${}^{14}_{6}\text{C}$ is so small, we use the atomic weight of ${}^{12}_{6}\text{C}$ to find the number of carbon atoms in 72 g. We use Eq. 41-6 to find the time.

$$N_{12}=\left(\frac{72\,\text{g}}{12\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right)=3.612\times10^{24}\,\text{atoms}$$

$$N_{14}=\left(3.612\times10^{24}\,\text{atoms}\right)\left(1.3\times10^{-12}\right)=4.6956\times10^{12}\,\text{atoms}$$

$$N=N_0e^{-\lambda t}\ \rightarrow\ t=-\frac{1}{\lambda}\ln\frac{N}{N_0}=-\frac{T_{1/2}}{\ln2}\ln\frac{N}{N_0}=-\frac{(5730\,\text{yr})}{\ln2}\ln\frac{1}{4.6956\times10^{12}}=\boxed{2.4\times10^5\,\text{yr}}$$

(*b*) We do a similar calculation for an initial mass of 270 grams.

$$N_{14}=\left(\frac{270\,\text{g}}{12\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right)\left(1.3\times10^{-12}\right)=1.761\times10^{13}\,\text{atoms}$$

$$N=N_0e^{-\lambda t}\ \rightarrow\ t=-\frac{1}{\lambda}\ln\frac{N}{N_0}=-\frac{T_{1/2}}{\ln2}\ln\frac{N}{N_0}=-\frac{(5730\,\text{yr})}{\ln2}\ln\frac{1}{1.761\times10^{13}}=\boxed{2.5\times10^5\,\text{yr}}$$

This shows that, for times on the order of 10^5 yr, the sample amount has fairly little effect on the age determined. Thus, times of this magnitude are not accurately measured by carbon dating.

82. (*a*) This reaction would turn the protons and electrons in atoms into neutrons. This would eliminate chemical reactions, and thus eliminate life as we know it.

(*b*) We assume that there is no kinetic energy brought into the reaction, and solve for the increase of mass necessary to make the reaction energetically possible. For calculating energies, we write the reaction as ${}^{1}_{1}\text{H}\ \rightarrow\ {}^{1}_{0}\text{n}+v$, and we assume the neutrino has no mass or kinetic energy.

$$Q=\left[m\left({}^{1}_{1}\text{H}\right)-m\left({}^{1}_{0}\text{n}\right)\right]c^2=\left[(1.007825\,\text{u})-(1.008665\,\text{u})\right]c^2\left(931.5\,\text{MeV/u}c^2\right)$$
$$=-0.782\,\text{MeV}$$

This is the amount that the proton would have to increase in order to make this energetically possible. We find the percentage change.

$$\left(\frac{\Delta m}{m}\right)(100)=\left[\frac{\left(0.782\,\text{MeV}/c^2\right)}{\left(938.27\,\text{MeV}/c^2\right)}\right](100)=\boxed{0.083\%}$$

83. We assume the particles are not relativistic, so that $p=\sqrt{2mK}$. The radius is given in Example 27-7 as $r=\frac{mv}{qB}$. Set the radii of the two particles equal. Note that the charge of the alpha particle is twice that of the electron (in absolute value). We also use the "bare" alpha particle mass, subtracting the two electrons from the helium atomic mass.

$$\frac{m_\alpha v_\alpha}{2eB}=\frac{m_\beta v_\beta}{eB}\ \rightarrow\ m_\alpha v_\alpha=2m_\beta v_\beta\ \rightarrow\ p_\alpha=2p_\beta$$

$$\frac{K_\alpha}{K_\beta}=\frac{\dfrac{p_\alpha^2}{2m_\alpha}}{\dfrac{p_\beta^2}{2m_\beta}}=\frac{\dfrac{4p_\beta^2}{2m_\alpha}}{\dfrac{p_\beta^2}{2m_\beta}}=\frac{4m_\beta}{m_\alpha}=\frac{4(0.000549\,\text{u})}{4.002603\,\text{u}-2(0.000549\,\text{u})}=\boxed{5.48\times10^{-4}}$$

84. Natural samarium has an atomic mass of 150.36 grams per mole. We find the number of nuclei in the natural sample, and then take 15% of that to find the number of ${}^{147}_{62}\text{Sm}$ nuclei. We first find the number of ${}^{147}\text{Sm}$ nuclei from the mass and proportion information.

$$N_{{}^{147}_{62}\text{Sm}}=(0.15)N_{\text{natural}}=\frac{(0.15)(1.00\,\text{g})\left(6.02\times10^{23}\,\text{nuclei / mol}\right)}{150.36\,\text{g/mol}}=6.006\times10^{20}\,\text{nuclei of }{}^{147}_{62}\text{Sm}$$

The activity level is used to calculate the half-life.

$$\text{Activity}=R=\lambda N=\frac{\ln 2}{T_{1/2}}N\quad\rightarrow$$

$$T_{1/2}=\frac{\ln 2}{R}N=\frac{\ln 2}{120\,\text{decays/s}}\left(6.006\times10^{20}\right)=3.469\times10^{18}\,\text{s}\left(\frac{1\,\text{yr}}{3.156\times10^{7}\,\text{s}}\right)=\boxed{1.1\times10^{11}\,\text{yr}}$$

$\boxed{85.}$ Since amounts are not specified, we will assume that "today" there is 0.720 g of ${}^{235}_{92}\text{U}$ and $100.000-0.720=99.280\,\text{g}$ of ${}^{238}_{92}\text{U}$. We use Eq. 41-6.

(*a*) Relate the amounts today to the amounts 1.0×10^{9} years ago.

$$N=N_0e^{-\lambda t}\quad\rightarrow\quad N_0=Ne^{\lambda t}=Ne^{\frac{t}{T_{1/2}}\ln 2}$$

$$\left(N_0\right)_{235}=\left(N_{235}\right)e^{\frac{t}{T_{1/2}}\ln 2}=(0.720\,\text{g})e^{\frac{\left(1.0\times10^{9}\,\text{yr}\right)}{\left(7.04\times10^{8}\,\text{yr}\right)}\ln 2}=1.927\,\text{g}$$

$$\left(N_0\right)_{238}=\left(N_{238}\right)e^{\frac{t}{T_{1/2}}\ln 2}=(99.280\,\text{g})e^{\frac{\left(1.0\times10^{9}\right)}{\left(4.468\times10^{9}\right)}\ln 2}=115.94\,\text{g}$$

$$N_{0,238}=N_{238}e^{\frac{0.693t}{T_{1/2}}}=(99.28\,\text{g})e^{\frac{0.693\left(1.0\times10^{9}\right)}{\left(4.468\times10^{9}\right)}}=115.937\,\text{g}.$$

The percentage of ${}^{235}_{92}\text{U}$ was $\dfrac{1.927}{1.927+115.94}\times100\%=\boxed{1.63\%}$

(*b*) Relate the amounts today to the amounts 100×10^{6} years from now.

$$N=N_0e^{-\lambda t}\quad\rightarrow\quad\left(N_{235}\right)=\left(N_0\right)_{235}e^{-\frac{t}{T_{1/2}}\ln 2}=(0.720\,\text{g})e^{-\frac{\left(100\times10^{6}\,\text{yr}\right)}{\left(7.04\times10^{8}\,\text{yr}\right)}\ln 2}=0.6525\,\text{g}$$

$$\left(N_{238}\right)=\left(N_0\right)_{238}e^{-\frac{t}{T_{1/2}}\ln 2}=(99.280\,\text{g})e^{-\frac{\left(100\times10^{6}\,\text{yr}\right)}{\left(4.468\times10^{9}\,\text{yr}\right)}\ln 2}=97.752\,\text{g}$$

The percentage of ${}^{235}_{92}\text{U}$ will be $\dfrac{0.6525}{0.6525+97.752}\times100\%=\boxed{0.663\%}$

86. We determine the number of ${}^{40}_{19}\mathrm{K}$ nuclei in the sample, and then use the half-life to determine the activity.

$$N_{{}^{40}_{19}\mathrm{K}} = (0.000117)N_{{}^{39}_{19}\mathrm{K}} = (0.000117)\frac{(400\times10^{-3}\,\mathrm{g})(6.02\times10^{23}\,\mathrm{atoms/mol})}{38.9637\,\mathrm{g/mol}} = 7.231\times10^{17}$$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{1.265\times10^{9}\,\mathrm{yr}}(7.231\times10^{17})\left(\frac{1\,\mathrm{yr}}{3.156\times10^{7}\,\mathrm{s}}\right) = 12.55\,\mathrm{decay/s} \approx \boxed{13\,\mathrm{decay/s}}$$

87. We use Eq. 41-7a to relate the activity to the half-life.

$$\left|\frac{dN}{dt}\right| = R = \lambda N = \frac{\ln 2}{T_{1/2}}N \quad\rightarrow$$

$$T_{1/2} = \frac{\ln 2}{R}N = \frac{\ln 2}{1\,\mathrm{decay/s}}(1.5\times10^{7}\,\mathrm{g})\left(\frac{6.02\times10^{23}\,\mathrm{nuclei}}{152\,\mathrm{g}}\right)\left(\frac{1\,\mathrm{yr}}{3.156\times10^{7}\,\mathrm{s}}\right) = \boxed{1.3\times10^{21}\,\mathrm{yr}}$$

88. The mass number changes only with α decay, and changes by -4. If the mass number is $4n$, then the new number is $4n-4=4(n-1)=4n'$. There is a similar result for each family, as shown here.

$$4n \;\rightarrow\; 4n-4=4(n-1)=4n'$$
$$4n+1 \;\rightarrow\; 4n-4+1=4(n-1)+1=4n'+1$$
$$4n+2 \;\rightarrow\; 4n-4+2=4(n-1)+2=4n'+2$$
$$4n+3 \;\rightarrow\; 4n-4+3=4(n-1)+3=4n'+3$$

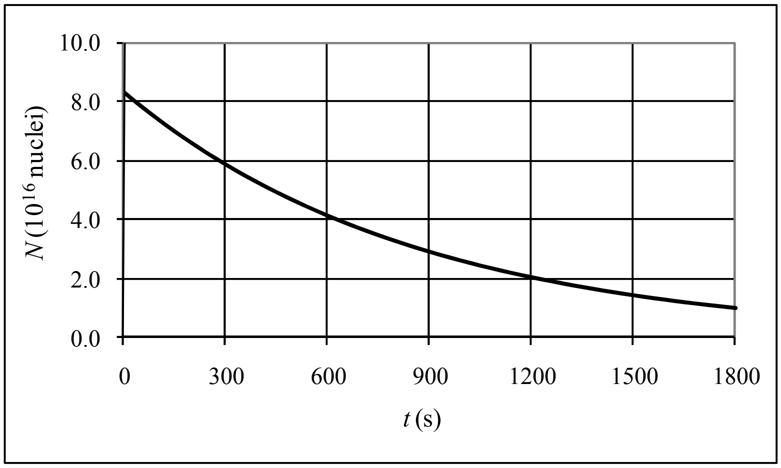
Thus the daughter nuclides are always in the same family.

89. We calculate the initial number of nuclei from the initial mass and the atomic mass.

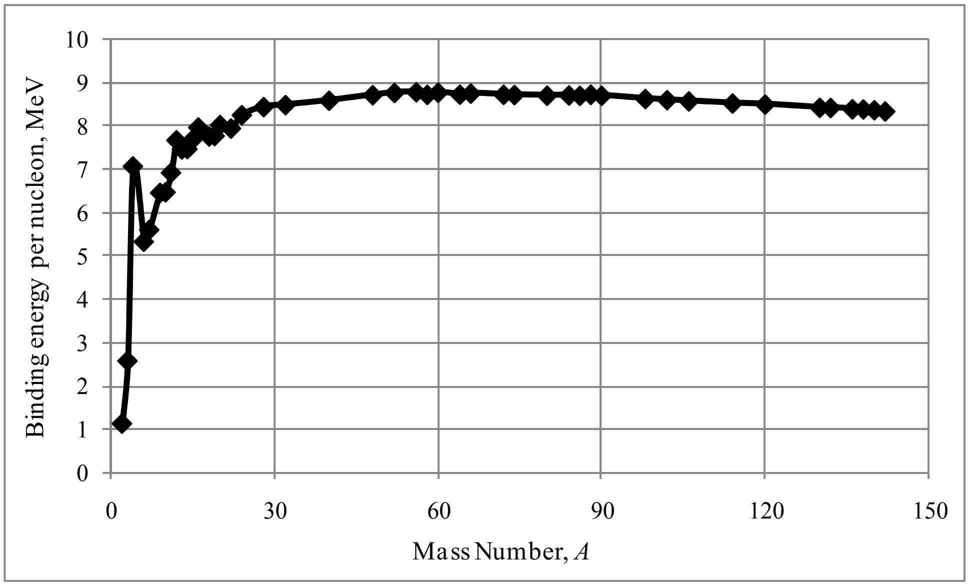
$$N = (1.80\times10^{-9}\,\mathrm{kg})\frac{1\ \mathrm{atom}}{(13.005739\,\mathrm{u})(1.6605\times10^{-27}\,\mathrm{kg})} = 8.3349\times10^{16}\,\mathrm{nuclei} \approx \boxed{8.33\times10^{16}\,\mathrm{nuclei}}$$

See the adjacent graph. From the graph, the half-life is approximately $\boxed{\text{600 seconds}}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH41.XLS," on tab "Problem 41.89."



90. See the following graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH41.XLS," on tab "Problem 41.90."



CHAPTER 42: Nuclear Energy: Effects and Uses of Radiation

Responses to Questions

$\boxed{1.}$ (*a*) $^{138}_{56}\text{Ba}$; (*b*) p or $^{1}_{1}\text{H}$; (*c*) γ ; (*d*) $^{199}_{80}\text{Hg}$

2. $^{32}_{16}\text{S}$

3. $^{20}_{10}\text{Ne}$

4. Neutrons have no net charge, and therefore do not have to overcome the Coulomb barrier (Coulomb repulsion) to get into the nucleus. Neutrons are also massive and can carry more energy than a lighter particle.

5. $^{17}_{9}\text{F}$ is the residual nucleus. The reaction equation is: $^{20}_{10}\text{Ne}+\text{p} \rightarrow \alpha + {}^{17}_{9}\text{F}$.

6. Fission fragments have more neutrons than are required for nuclear stability and will decay by β^- emission in order to convert a neutron to a proton.

$\boxed{7.}$ The energy from nuclear fission appears in the thermal (kinetic) energy of the fission fragments and the neutrons that are emitted, and in the thermal energy of nearby atoms with which they collide.

8. $^{239}_{94}\text{Pu}$ has a smaller critical mass than $^{235}_{92}\text{U}$. Since there are more neutrons released per decay in Pu-239, fewer nuclei are needed to release sufficient neutrons to create and sustain a chain reaction.

9. Yes, a chain reaction would be possible, since the multiplication factor f is greater than 1. However, the chain reaction would progress slowly, and care would have to be taken to prevent neutron loss.

10. When uranium is enriched, the percentage of U-235 nuclei in a given mass of uranium is increased. This process involves the nuclei of the atoms. Chemical processes typically involve the electrons in atoms, and not the nuclei. The chemical behavior of all the isotopes of uranium is nearly identical, therefore chemical means could not be used to separate isotopes and enrich uranium.

11. Neutrons are neutral; they are not repelled by the electrons surrounding the atom or by the positively charged protons in the nucleus. They are able to penetrate easily into the nucleus, and, once there, are held in place by the strong nuclear force. A neutron brings to the nucleus its kinetic energy and the binding energy given up as the neutron is bound to the rest of the nucleus. This binding energy can be very large, and is enough to move the nucleus into an excited state, from which it will fission.

12. The hydrogen atoms in water act as a moderator to slow down the neutrons released during fission reactions. The slower neutrons are more likely to be absorbed by other uranium nuclei to produce further fission reactions, creating a chain reaction that could lead to an explosion. A porous block of uranium in air would be less likely to undergo a chain reaction due to the absence of an effective moderator.

$\boxed{13.}$ Ordinary water does not moderate, or slow down, neutrons as well as heavy water; more neutrons will also be lost to absorption in ordinary water. However, if the uranium in a reactor is highly enriched, there will be many fissionable nuclei available in the fuel rods. It will be likely that the few moderated neutrons will be absorbed by a fissionable nucleus, and it will be possible for a chain reaction to occur.

14. A useful fission reaction is one that is self-sustaining. The neutrons released from an initial fission process can go on to initiate further fission reactions, creating a self-sustaining reaction. If no neutrons were released, then the process would end after a single reaction and not be very useful.

15. Heavy nuclei decay because they are neutron-rich, especially after neutron capture. After fission, the smaller daughter nuclei will still be neutron-rich and relatively unstable, and will emit neutrons in order to move to a more stable configuration. Lighter nuclei are generally more stable with approximately equal numbers of protons and neutrons; heavier nuclei need additional neutrons in order to overcome Coulomb repulsion between the protons.

16. The water in the primary system flows through the core of the reactor and therefore could contain radioactive materials, including deuterium, tritium, and radioactive oxygen isotopes. The use of a secondary system provides for isolation of these potentially hazardous materials from the external environment.

17. Fission is the process in which a larger nucleus splits into two or more fragments, roughly equal in size. Fusion is the process in which smaller nuclei combine to form larger nuclei.

18. Fossil fuel power plants are less expensive to construct and the technology is well known. However, the mining of coal is dangerous and can be environmentally destructive, the transportation of oil can be damaging to the environment through spills, the production of power from both coal and oil contributes to air pollution and the release of greenhouse gases into the environment, and there is a limited supply of both coal and oil. Fission power plants produce no greenhouse gases and virtually no air pollution, and the technology is well known. However, they are expensive to build, produce thermal pollution and radioactive waste, and when accidents occur they tend to be very destructive. Uranium is also dangerous to mine. Fusion power plants produce very little radioactive waste and virtually no air pollution or greenhouse gases. Unfortunately, the technology for large-scale sustainable power production is not yet known, and the pilot plants are very expensive to build.

$\boxed{19.}$ To ignite a fusion reaction, the two nuclei must have enough kinetic energy to overcome electrostatic repulsion and approach each other very closely in a collision. Electrostatic repulsion is proportional to charge and inversely proportional to the square of the distance between the centers of the charge distributions. Both deuterium and tritium have one positive charge, so the charge effect is the same for d-d and d-t ignition. Tritium has one more neutron than deuterium and thus has a larger nucleus. In the d-t ignition, the distance between the centers of the nuclei will be greater than in d-d ignition, reducing the electrostatic repulsion and requiring a lower temperature for fusion ignition.

20. The interiors of stars contain ionized atoms (a plasma) at very high temperature and with a high density of nuclei. The nuclei have high enough kinetic energy and a great enough likelihood of colliding with other nuclei to allow fusion to occur.

21. Stars maintain fusion confinement with gravity. The large amount of mass in a star creates a tremendous gravitational attraction on the gas particles which is able to overcome the repulsive Coulomb force and radiation pressure.

22. Younger women may suffer damage to reproductive cells as well as somatic damage. Genetic damage caused by radiation can cause mutations and be passed on to future generations. A fetus is particularly susceptible to radiation damage due to its small mass and rapid cell development. Since it is quite possible, especially early in a pregnancy, for a woman to be pregnant and not know it, it is

reasonable to have lower recommended dose levels for women of childbearing years. Beyond the reproductive years, the acceptable exposure dosage for women can be increased.

23. Alpha particles are relatively large and are generally emitted with relatively low kinetic energies. They are not able to penetrate the skin, and so are not very destructive or dangerous as long as they stay outside the body. If alpha emitters are ingested or breathed, however, the protective layer of skin is bypassed, and the alpha particles, which are charged, can do tremendous amounts of damage to lung or other delicate internal tissue due to ionizing effects. Thus, there are strong rules against eating and drinking around alpha-emitters, and the machining of such materials, which would produce fine dust particles that could be inhaled, can be done only in sealed conditions.

24. The absorbed dose measures the amount of energy deposited per unit mass of absorbing material and is measured in Grays (SI) (1 Gy = 1J/kg) or rads. The effective dose takes into account the type of radiation depositing the energy and is used to determine the biological damage done by the radiation. The effective dose is the absorbed dose multiplied by a quality factor, QF. The effective dose is measured in rem or siverts (SI). 1 Sv of any type of radiation does approximately the same amount of biological damage.

$\boxed{25.}$ Appropriate levels of radiation can kill possibly harmful bacteria and viruses on medical supplies or in food.

26. Allow a radioactive tracer to be introduced into the liquid that flows through the pipe. Then check the pipe with a Geiger counter. When you find the tracer on the outside of the pipe (radiation levels will be higher at that point), you will have found the leak.

Solutions to Problems

$\boxed{1.}$ By absorbing a neutron, the mass number increases by one and the atomic number is unchanged. The product nucleus is $\boxed{{}^{28}_{13}\text{Al}}$. Since the nucleus now has an "extra" neutron, it will decay by $\boxed{\beta^-}$, according to this reaction: ${}^{28}_{13}\text{Al} \rightarrow {}^{28}_{14}\text{Si} + \beta^- + \bar{\nu}_e$. Thus the product is $\boxed{{}^{28}_{14}\text{Si}}$.

2. If the Q-value is positive, then no threshold energy is needed.

$$Q = 2m_{{}^2_1\text{H}}c^2 - m_{{}^3_2\text{He}}c^2 - m_{\text{n}}c^2 = \left[2\left(2.014082\,\text{u}\right) - 3.016029\,\text{u} - 1.008665\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= 3.232\,\text{MeV}$$

Thus $\boxed{\text{no threshold energy is required}}$.

3. A "slow" neutron means that it has negligible kinetic energy. If the Q-value is positive, then the reaction is possible.

$$Q = m_{{}^{238}_{92}\text{U}}c^2 + m_{\text{n}}c^2 - m_{{}^{239}_{92}\text{U}}c^2 = \left[238.050788\,\text{u} + 1.008665\,\text{u} - 239.054293\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= 4.807\,\text{MeV}$$

Thus $\boxed{\text{the reaction is possible}}$.

4. The Q-value tells whether the reaction requires or releases energy.

$$Q = m_{\text{p}}c^2 + m_{{}^7_3\text{Li}}c^2 - m_{{}^4_2\text{He}}c^2 - m_\alpha c^2 = \left[1.007825\,\text{u} + 7.016005\,\text{u} - 2\left(4.002603\right)\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= 17.35\,\text{MeV}$$

The reaction $\boxed{\text{releases } 17.35\,\text{MeV}}$.

5. The Q-value tells whether the reaction requires or releases energy.

$$Q = m_\alpha c^2 + m_{{}^9_4\text{Be}}c^2 - m_{{}^{12}_6\text{C}}c^2 - m_{\text{n}}c^2$$

$$= \left[4.002603\,\text{u} + 9.012182\,\text{u} - 12.000000\,\text{u} - 1.008665\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 5.701\,\text{MeV}$$

The reaction $\boxed{\text{releases } 5.701\,\text{MeV}}$.

6. (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_{\text{n}}c^2 + m_{{}^{24}_{12}\text{Mg}}c^2 - m_{{}^{23}_{11}\text{Na}}c^2 - m_{\text{d}}c^2$$

$$= \left[1.008665\,\text{u} + 23.985042\,\text{u} - 22.989769\,\text{u} - 2.014082\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = -9.449\,\text{MeV}$$

Thus more energy is required if this reaction is to occur. The 16.00 MeV of kinetic energy is more than sufficient, and so $\boxed{\text{the reaction can occur}}$.

(*b*) $16.00\,\text{MeV} - 9.449\,\text{MeV} = \boxed{6.55\,\text{MeV of energy is released}}$

$\boxed{7.}$ (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_{\text{p}}c^2 + m_{{}^7_3\text{Li}}c^2 - m_{{}^4_2\text{He}}c^2 - m_\alpha c^2$$

$$= \left[1.007825\,\text{u} + 7.016005\,\text{u} - 2\left(4.002603\,\text{u}\right)\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 17.348\,\text{MeV}$$

Since the Q-value is positive, $\boxed{\text{the reaction can occur}}$.

(*b*) The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$K_{\text{total}} = K_{\text{reactants}} + Q = 3.5\,\text{MeV} + 17.348\,\text{MeV} = \boxed{20.8\,\text{MeV}}$$

8. (*a*) If the Q-value is positive, then no threshold energy is needed.

$$Q = m_\alpha c^2 + m_{{}^{14}_7\text{N}}c^2 - m_{{}^{17}_8\text{O}}c^2 - m_{\text{p}}c^2$$

$$= \left[4.002603\,\text{u} + 14.003074\,\text{u} - 16.999132\,\text{u} - 1.007825\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = -1.192\,\text{MeV}$$

Thus more energy is required if this reaction is to occur. The 9.68 MeV of kinetic energy is more than sufficient, and so $\boxed{\text{the reaction can occur}}$.

(*b*) The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$K_{\text{total}} = K_{\text{reactants}} + Q = 9.68\,\text{MeV} - 1.192\,\text{MeV} = \boxed{8.49\,\text{MeV}}$$

9. $Q = m_\alpha c^2 + m_{{}^{16}_{8}\text{O}} c^2 - m_{{}^{20}_{10}\text{Ne}}$

$$= \left[4.002603\,\text{u} + 15.994915\,\text{u} - 19.992440\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{4.730\,\text{MeV}}$$

10. The Q-value tells whether the reaction requires or releases energy.

$$Q = m_\text{d}c^2 + m_{{}^{13}_{6}\text{C}}c^2 - m_{{}^{14}_{7}\text{N}}c^2 - m_\text{n}c^2$$

$$= \left[2.014082\,\text{u} + 13.003355\,\text{u} - 14.003074\,\text{u} - 1.008665\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 5.308\,\text{MeV}$$

The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$K_\text{total} = K_\text{reactants} + Q = 44.4\,\text{MeV} + 5.308\,\text{MeV} = \boxed{49.7\,\text{MeV}}$$

11. The nitrogen-14 absorbs a neutron. Carbon-12 is a product. Thus the reaction is $\text{n} + {}^{14}_{7}\text{N} \rightarrow {}^{14}_{6}\text{C} + ?$. The reactants have 7 protons and 15 nucleons, which means 8 neutrons. Thus the products have 7 protons and 15 nucleons. The unknown product must be a proton. Thus the reaction is $\boxed{\text{n} + {}^{14}_{7}\text{N} \rightarrow {}^{14}_{6}\text{C} + \text{p}}$.

$$Q = m_\text{n}c^2 + m_{{}^{14}_{7}\text{N}}c^2 - m_{{}^{14}_{6}\text{C}}c^2 - m_\text{p}c^2$$

$$= \left[1.008665\,\text{u} + 14.003074\,\text{u} - 14.003242\,\text{u} - 1.007825\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{0.626\,\text{MeV}}$$

12. (*a*) The deuteron is ${}^{2}_{1}\text{H}$, and so the reactants have 4 protons and 8 nucleons. Therefore the reactants have 4 neutrons. Thus the products must have 4 protons and 4 neutrons. That means that X must have 3 protons and 4 neutrons, and so X is $\boxed{{}^{7}_{3}\text{Li}}$.

(*b*) This is called a "stripping" reaction because the lithium nucleus has "stripped" a neutron from the deuteron.

(*c*) The Q-value tells whether the reaction requires or releases energy.

$$Q = m_\text{d}c^2 + m_{{}^{6}_{3}\text{Li}}c^2 - m_{{}^{7}_{3}\text{Li}}c^2 - m_\text{p}c^2$$

$$= \left[2.014082\,\text{u} + 6.015123\,\text{u} - 7.016005\,\text{u} - 1.007825\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{5.007\,\text{MeV}}$$

Since the Q-value is positive, the reaction is $\boxed{\text{exothermic}}$.

$\boxed{13.}$ (*a*) This is called a "pickup" reaction because the helium has "picked up" a neutron from the carbon nucleus.

(*b*) The alpha is ${}^{4}_{2}\text{He}$. The reactants have 8 protons and 15 nucleons, and so have 7 neutrons. Thus the products must also have 8 protons and 7 neutrons. The alpha has 2 protons and 2 neutrons, and so X must have 6 protons and 5 neutrons. Thus X is $\boxed{{}^{11}_{6}\text{C}}$.

(*c*) The *Q*-value tells whether the reaction requires or releases energy.

$$Q = m_{^3_2\mathrm{He}}c^2 + m_{^{12}_6\mathrm{C}}c^2 - m_{^{11}_6\mathrm{C}}c^2 - m_\alpha c^2$$

$$= [3.016029\,\mathrm{u} + 12.000000\,\mathrm{u} - 11.011434\,\mathrm{u} - 4.002603\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.856\,\mathrm{MeV}}$$

Since the *Q*-value is positive, the reaction is $\boxed{\text{exothermic}}$.

14. (*a*) The product has 16 protons and 16 neutrons. Thus the reactants must have 16 protons and 16 neutrons. Thus the missing nucleus has 15 protons and 16 neutrons, and so is $\boxed{^{31}_{15}\mathrm{P}}$.

(*b*) The *Q*-value tells whether the reaction requires or releases energy.

$$Q = m_\mathrm{p}c^2 + m_{^{31}_{15}\mathrm{P}}c^2 - m_{^{32}_{16}\mathrm{S}}c^2$$

$$= [1.007825\,\mathrm{u} + 30.973762\,\mathrm{u} - 31.972071\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{8.864\,\mathrm{MeV}}$$

15. We assume that all of the particles are essentially at rest, and so ignore conservation of momentum. To just make the fluorine nucleus, the *Q*-value plus the incoming kinetic energy should add to 0.

$$K + Q = K + m_\mathrm{p}c^2 + m_{^{18}_8\mathrm{O}}c^2 - m_{^{18}_9\mathrm{F}}c^2 - m_\mathrm{n}c^2 = 0 \quad \rightarrow$$

$$m_{^{18}_9\mathrm{F}}c^2 = K + m_\mathrm{p}c^2 + m_{^{18}_8\mathrm{O}}c^2 - m_\mathrm{n}c^2 =$$

$$= 2.438\,\mathrm{MeV} + [1.007825\,\mathrm{u} + 17.999161\,\mathrm{u} - 1.008665\,\mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$

$$= 1.6767874\times10^4\,\mathrm{MeV}$$

$$m_{^{18}_9\mathrm{F}} = (1.6767874\times10^4\,\mathrm{MeV})c^2\left(\frac{1\,\mathrm{u}}{931.5\,\mathrm{MeV}/c^2}\right) = \boxed{18.000938\,\mathrm{u}}$$

16. We assume that the energies are small enough that classical mechanics is applicable, particularly $p = \sqrt{2mK}$ and $K = \frac{1}{2}mv^2$. The least proton kinetic energy is required when the product particles move together and so have the same speed. We write equations for 1-D momentum conservation and for energy conservation, and then combine those to find the required proton energy.

$$p_\mathrm{p} = p_\mathrm{n} + p_\mathrm{N} = (m_\mathrm{n} + m_\mathrm{N})v$$

$$K_\mathrm{p} = \frac{p_\mathrm{p}^2}{2m_\mathrm{p}} = \frac{[(m_\mathrm{n} + m_\mathrm{N})v]^2}{2m_\mathrm{p}} = \frac{(m_\mathrm{n} + m_\mathrm{N})}{m_\mathrm{p}}\tfrac{1}{2}(m_\mathrm{n} + m_\mathrm{N})v^2 = \frac{(m_\mathrm{n} + m_\mathrm{N})}{m_\mathrm{p}}(K_\mathrm{n} + K_\mathrm{N}) \quad \rightarrow$$

$$K_\mathrm{n} + K_\mathrm{N} = \frac{m_\mathrm{p}}{(m_\mathrm{n} + m_\mathrm{N})}K_\mathrm{p}$$

$$\text{Energy conservation}: K_\mathrm{p} + Q = K_\mathrm{n} + K_\mathrm{N} = \frac{m_\mathrm{p}}{(m_\mathrm{n} + m_\mathrm{N})}K_\mathrm{p} \quad \rightarrow \quad K_\mathrm{p} = -Q\frac{m_\mathrm{n} + m_\mathrm{N}}{m_\mathrm{n} + m_\mathrm{N} - m_\mathrm{p}}$$

Note that this result is also derived in Problem 87. We now substitute in the values for this specific problem.

$$K_\mathrm{p} = -Q\frac{m_\mathrm{n} + m_\mathrm{N}}{m_\mathrm{n} + m_\mathrm{N} - m_\mathrm{p}} = -\left[m_{^{13}_6\mathrm{C}}c^2 + m_\mathrm{p}c^2 - m_{^{13}_7\mathrm{N}}c^2 - m_\mathrm{n}c^2\right]Q\frac{m_\mathrm{n} + m_\mathrm{N}}{m_\mathrm{n} + m_\mathrm{N} - m_\mathrm{p}}$$

$$= -\left\{ \begin{array}{l} \left[\left[13.003355\,\text{u} + 1.007825\,\text{u} - 13.005739\,\text{u} - 1.008665\,\text{u} \right] \left(931.5 \dfrac{\text{MeV}/c^2}{\text{u}} \right) c^2 \right] \\ \times \left(\dfrac{1.008665 + 13.005739}{1.008665 + 13.005739 - 1.007825} \right) \end{array} \right\}$$

$$= \boxed{3.24\,\text{MeV}}$$

If some intermediate values are used, such as $Q = -3.003\,\text{MeV}$, and $\dfrac{m_\text{n} + m_\text{N}}{m_\text{n} + m_\text{N} - m_\text{p}} = 1.077$, the mass factor as 1.077, then the value of 3.23 MeV will be obtained.

17. We use the same derivation algebra as in Problem 16, and so jump to the final expression.

$$K_\text{p} = -Q \frac{m_\text{n} + m_\text{N}}{m_\text{n} + m_\text{N} - m_\text{p}} = -\left[m_{{}^{14}_{6}\text{C}} c^2 + m_\text{p} c^2 - m_{{}^{14}_{7}\text{N}} c^2 - m_\text{n} c^2 \right] Q \frac{m_\text{n} + m_\text{N}}{m_\text{n} + m_\text{N} - m_\text{p}}$$

$$= -\left\{ \begin{array}{l} \left[\left[14.003242\,\text{u} + 1.007825\,\text{u} - 14.003074\,\text{u} - 1.008665\,\text{u} \right] \left(931.5 \dfrac{\text{MeV}/c^2}{\text{u}} \right) c^2 \right] \\ \times \left(\dfrac{1.008665 + 14.003074}{1.008665 + 14.003074 - 1.007825} \right) \end{array} \right\}$$

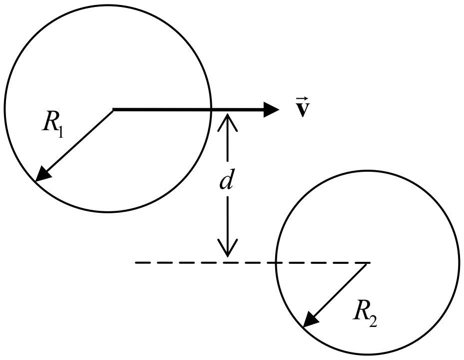
$$= \boxed{0.671\,\text{MeV}}$$

18. We use Eq, 42-3.

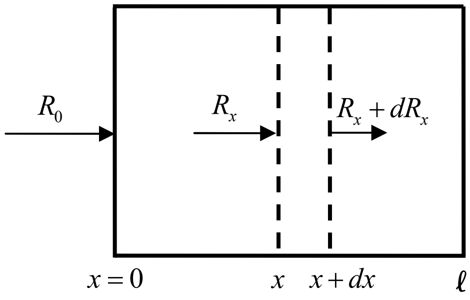
$$\frac{R}{R_0} = n\ell\sigma = \left(1.7 \times 10^{21}\ \text{nuclei}/\text{m}^3\right)\left(0.120\,\text{m}\right)\left(40 \times 10^{-28}\,\text{m}^2\right) \approx \boxed{8 \times 10^{-7}}$$

$\boxed{19.}$ From the figure we see that a collision will occur if $d \le R_1 + R_2$. We calculate the area of the effective circle presented by R_2 to the center of R_1.

$$\sigma = \pi d^2 = \pi \left(R_1 + R_2\right)^2$$



20. At a distance x into the target material, particles are arriving at a rate R_x. Due to interactions between the particles and the target material, which remove particles from the stream, particles are arriving at a distance of $x + dx$ at a lower rate of $R_x + dR_x$, where $dR_x < 0$. Thus the collision rate is $-dR_x$. The cross section, given in Eq. 42-3, gives the relationship between the two rates. Also see the diagram.



$$\sigma = \frac{-dR_x}{R_x n dx} \quad \rightarrow \quad \boxed{-dR_x = R_x n\sigma dx}$$

Integrate the above differential relationship.

$$dR_x = -R_x n\sigma dx \quad \rightarrow \quad \int_{R_0}^{R_x} \frac{1}{R_x} dR_x = -n\sigma \int_0^x dx \quad \rightarrow \quad \ln\left(\frac{R_x}{R_0}\right) = -n\sigma x \quad \rightarrow \quad \boxed{R_x = R_0 e^{-n\sigma x}}$$

$R_0 e^{-n\sigma\ell}$ represents the rate at which particles leave the target material, unaffected by the target material.

21. We use the result from Problem 20, where $R_\ell = R_0 e^{-n\sigma\ell}$. We use the data for the 1.0-cm-thick target to get an expression for $n\sigma$.

$$R_\ell = R_0 e^{-n\sigma\ell} \quad \rightarrow \quad n\sigma = -\frac{1}{\ell}\ln\frac{R_\ell}{R_0} = -\frac{1}{0.010\,\text{m}}\ln 0.25 = 138.63\,\text{m}^{-1}$$

$$R_x = R_0 e^{-n\sigma x} \quad \rightarrow \quad x = -\frac{1}{n\sigma}\ln\frac{R_x}{R_0} = -\left(\frac{1}{138.63\,\text{m}^{-1}}\right)\ln\frac{1}{10^6} = 9.966\times10^{-2}\,\text{m} \approx \boxed{10\,\text{cm}}$$

Note that the answer is correct to 2 significant figures.

22. We assume a 2.0% reaction rate allows us to treat the target as thin. We use Eq. 42-3. We need the volume density of the cadmium atoms.

$$n = \left(8650\,\text{kg/m}^3\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{0.1139\,\text{kg}}\right) = 4.572\times10^{28}\,\text{nuclei/m}^3$$

(*a*) The cross section for 0.1-eV neutrons is ~ 3000 bn.

$$\frac{R}{R_0} = n\ell\sigma \quad \rightarrow$$

$$\ell = \frac{R}{R_0}\frac{1}{n\sigma} = (0.020)\frac{1}{\left(4.572\times10^{28}\,\text{nuclei/m}^3\right)\left(3000\times10^{-28}\,\text{m}^2\right)} = 1.458\times10^{-6}\,\text{m}$$

$$\approx \boxed{1.5\,\mu\text{m}}$$

(*b*) The cross section for 5.0-eV neutrons is ~ 2 bn.

$$\ell = \frac{R}{R_0}\frac{1}{n\sigma} = (0.020)\frac{1}{\left(4.572\times10^{28}\,\text{nuclei/m}^3\right)\left(2\times10^{-28}\,\text{m}^2\right)} = 2.187\times10^{-3}\,\text{m}$$

$$\approx \boxed{2.2\,\text{mm}}$$

23. The *Q*-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q = m_\text{n}c^2 + m_{{}^{235}_{92}\text{U}}c^2 - m_{{}^{141}_{56}\text{Ba}}c^2 - m_{{}^{92}_{36}\text{Kr}}c^2 - 3m_\text{n}c^2$$

$$= \left[1.008665\,\text{u} + 235.043930\,\text{u} - 140.914411\,\text{u} - 91.926156\,\text{u} - 3(1.008665\,\text{u})\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= \boxed{173.3\,\text{MeV}}$$

24. The Q-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q = m_{\mathrm{n}}c^2 + m_{{}^{235}_{92}\mathrm{U}}c^2 - m_{{}^{88}_{38}\mathrm{Sr}}c^2 - m_{{}^{136}_{54}\mathrm{Xe}}c^2 - 12m_{\mathrm{n}}c^2$$

$$= \left[1.008665\,\mathrm{u} + 235.043930\,\mathrm{u} - 87.905612\,\mathrm{u} - 135.907219\,\mathrm{u} - 12(1.008665\,\mathrm{u})\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$

$$= \boxed{126.5\,\mathrm{MeV}}$$

$\boxed{25.}$ The power released is the energy released per reaction times the number of reactions per second.

$$P = \frac{\text{energy}}{\text{reaction}} \times \frac{\#\,\text{reactions}}{\text{s}} \;\rightarrow$$

$$\frac{\#\,\text{reactions}}{\text{s}} = \frac{P}{\dfrac{\text{energy}}{\text{reaction}}} = \frac{200\times10^6\,\mathrm{W}}{(200\times10^6\,\mathrm{eV/reaction})(1.60\times10^{-19}\,\mathrm{J/eV})} = \boxed{6\times10^{18}\,\text{reactions/s}}$$

26. Compare the energy per fission with the rest mass energy.

$$\frac{\text{energy per fission}}{\text{rest mass energy } mc^2} = \frac{200\,\mathrm{MeV}}{(235\,\mathrm{u})(931.5\,\mathrm{MeV}/c^2)c^2} = 9.1\times10^{-4} \approx \boxed{\frac{1}{1100}}$$

27. We convert the 880 watts over a year's time to a mass of uranium.

$$\left(\frac{880}{1}\frac{\mathrm{J}}{\mathrm{s}}\right)\left(\frac{3.156\times10^7\,\mathrm{s}}{1\,\mathrm{y}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{1\,\text{fission}}{200\,\mathrm{MeV}}\right)\left(\frac{0.235\,\mathrm{kg}\;{}^{235}_{92}\mathrm{U}}{6.02\times10^{23}\,\text{atoms}}\right) = 3.388\times10^{-4}\,\mathrm{kg}\;{}^{235}_{92}\mathrm{U}$$

$$\approx \boxed{0.34\,\mathrm{g}\;{}^{235}_{92}\mathrm{U}}$$

28. (*a*) The total number of nucleons for the reactants is 236, and so the total number of nucleons for the products must also be 236. The two daughter nuclei have a total of 231 nucleons, so $\boxed{\text{5 neutrons}}$ must be produced in the reaction: ${}^{235}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{133}_{51}\mathrm{Sb} + {}^{98}_{41}\mathrm{Nb} + 5\mathrm{n}$.

(*b*)
$$Q = m_{{}^{235}_{92}\mathrm{U}}c^2 + m_{\mathrm{n}}c^2 - m_{{}^{133}_{51}\mathrm{Sb}}c^2 - m_{{}^{98}_{41}\mathrm{Nb}}c^2 - 5m_{\mathrm{n}}c^2$$

$$= \left[235.043930\,\mathrm{u} + 1.008665\,\mathrm{u} - 132.915250\,\mathrm{u} - 97.910328\,\mathrm{u} - 5(1.008665\,\mathrm{u})\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$

$$= \boxed{171.1\,\mathrm{MeV}}$$

29. We assume as stated in problems 26 and 27 that an average of 200 MeV is released per fission of a uranium nucleus. Also, note that the problem asks for the mass of ${}^{238}_{92}\mathrm{U}$, but it is the ${}^{235}_{92}\mathrm{U}$ nucleus that undergoes the fission. Since ${}^{238}_{92}\mathrm{U}$ is almost 100% of the natural abundance, we can use the abundance of ${}^{235}_{92}\mathrm{U}$ from Appendix F as a ratio of ${}^{235}_{92}\mathrm{U}$ to ${}^{238}_{92}\mathrm{U}$.

$$(3\times10^7\,\mathrm{J})\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{1\,\text{nucleus}\;{}^{235}_{92}\mathrm{U}}{200\,\mathrm{MeV}}\right)\left(\frac{1\,\text{atom}\;{}^{238}_{92}\mathrm{U}}{0.0072\,\text{atoms}\;{}^{235}_{92}\mathrm{U}}\right)\left(\frac{0.238\,\mathrm{kg}}{6.02\times10^{23}\,\text{nuclei}\;{}^{238}_{92}\mathrm{U}}\right)$$

$$= 5.15\times10^{-5}\,\mathrm{kg}\;{}^{238}_{92}\mathrm{U} \approx \boxed{5\times10^{-5}\,\mathrm{kg}\;{}^{238}_{92}\mathrm{U}}$$

30. Since the reaction is 38% efficient, the fission needs to generate $(950/0.38)\,\text{MW}$ of power. Convert the power rating to a mass of uranium using the factor-label method. We assume 200 MeV is released per fission, as in other problems.

$$\frac{950\times10^{6}}{0.38}\frac{\text{J}}{\text{s}}\times\frac{1\text{ atom}}{200\times10^{6}\text{eV}}\times\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\times\frac{0.235\,\text{kg U}}{6.02\times10^{23}\,\text{atoms}}\times\frac{3.156\times10^{7}\,\text{s}}{1\,\text{y}}=962\,\text{kg}\ {}^{235}_{92}\text{U}$$

$$\approx\boxed{960\,\text{kg}\ {}^{235}_{92}\text{U}}$$

$\boxed{31.}$ We find the number of collisions from the relationship $E_n=E_0\left(\tfrac{1}{2}\right)^n$, where n is the number of collisions.

$$E_n=E_0\left(\tfrac{1}{2}\right)^n\ \rightarrow\ n=\frac{\ln\frac{E_n}{E_0}}{\ln\frac{1}{2}}=\frac{\ln\frac{0.040\,\text{eV}}{1.0\times10^{6}\text{eV}}}{\ln\frac{1}{2}}=24.58\approx\boxed{25\,\text{collisions}}$$

32. If the uranium splits into equal fragments, each will have an atomic mass number of half of 236, or 118. Each will have a nuclear charge of half of 92, or 46. Calculate the electrical potential energy using Eq. 23-10. The distance between the nuclei will be twice the radius of a nucleus, and the radius is given in Eq. 41-1.

$$U=\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}=\left(8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60\times10^{-19}\text{C}\right)^2}{2\left(1.2\times10^{-15}\text{m}\right)(118)^{1/3}}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)=\boxed{260\,\text{MeV}}$$

This is about $\boxed{\text{30\% larger}}$ than the nuclear fission energy released.

33. The height of the Coulomb barrier is given by the electrostatic potential energy, Eq. 23-10. The distance to use is the sum of the radii of the two particles involved. For the alpha decay, the daughter nucleus is ${}^{232}_{90}\text{Th}$. We assume the fission results in two equal fragments, each with $Z=46$ and $A=128$. These are palladium nuclei.

$$U_\alpha=\frac{1}{4\pi\varepsilon_0}\frac{Q_\alpha Q_{\text{Th}}}{r_\alpha+r_{\text{Th}}}\ ;\ U_{\text{fission}}=\frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{Pd}}Q_{\text{Pd}}}{r_{\text{Pd}}+r_{\text{Pd}}}$$

$$\frac{U_\alpha}{U_{\text{fission}}}=\frac{\frac{1}{4\pi\varepsilon_0}\frac{Q_\alpha Q_{\text{Th}}}{r_\alpha+r_{\text{Th}}}}{\frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{Pd}}Q_{\text{Pd}}}{r_{\text{Pd}}+r_{\text{Pd}}}}=\frac{\dfrac{(2)(90)}{\left(1.2\times10^{-15}\text{m}\right)\left(4^{1/3}+232^{1/3}\right)}}{\dfrac{(46)^2}{\left(1.2\times10^{-15}\text{m}\right)2\left(128^{1/3}\right)}}=\frac{(2)(90)2\left(128^{1/3}\right)}{(46)^2\left(4^{1/3}+232^{1/3}\right)}=\boxed{0.11}$$

34. The reaction rate is proportional to the number of neutrons causing the reactions. For each fission the number of neutrons will increase by a factor of 1.0004, so in 1000 milliseconds the number of neutrons will increase by a factor of $(1.0004)^{1000}=\boxed{1.5}$.

35. $K=\tfrac{3}{2}kT=\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)\left(2\times10^{7}\,\text{K}\right)=\boxed{4\times10^{-16}\,\text{J}}$

$$=\tfrac{3}{2}\left(1.38\times10^{-23}\,\text{J/K}\right)\left(2\times10^{7}\,\text{K}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=2588\,\text{eV}\approx\boxed{3000\,\text{eV}}$$

36. The Q-value gives the energy released in the reaction.

$$Q = m_{^2_1\text{H}}c^2 + m_{^3_1\text{H}}c^2 - m_{^4_2\text{He}}c^2 - m_{\text{n}}c^2$$

$$= [2.014082\,\text{u} + 3.016049\,\text{u} - 4.002603\,\text{u} - 1.008665\,\text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{17.57\,\text{MeV}}$$

$\boxed{37.}$ Calculate the Q-value for the reaction $^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \text{n}$

$$Q = 2m_{^2_1\text{H}}c^2 - m_{^3_2\text{He}}c^2 - m_{\text{n}}c^2$$

$$= [2(2.014082\,\text{u}) - 3.016029\,\text{u} - 1.008665\,\text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{3.23\,\text{MeV}}$$

38. For the reaction in Eq. 42-7a, if atomic masses are to be used, then one more electron needs to be added to the products side of the equation. Notice that charge is not balanced in the equation as written. The balanced reaction is $^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + \text{e}^+ + \nu + \text{e}^-$

$$Q = 2m_{^1_1\text{H}}c^2 - m_{^2_1\text{H}}c^2 - m_{\text{e}^+}c^2 - m_{\text{e}^-}c^2$$

$$= [2(1.007825\,\text{u}) - 2.014082\,\text{u} - 2(0.000549\,\text{u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 0.4378\,\text{MeV} \approx \boxed{0.44\,\text{MeV}}$$

For the reaction in Eq. 42-7b, use atomic masses since there would be two electrons on each side.

$$Q = m_{^1_1\text{H}}c^2 + m_{^2_1\text{H}}c^2 - m_{^3_2\text{He}}c^2$$

$$= [1.007825\,\text{u} + 2.014082\,\text{u} - 3.016029\,\text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 5.4753\,\text{MeV} \approx \boxed{5.48\,\text{MeV}}$$

For the reaction in Eq. 42-7c, use atomic masses since there would be two electrons on each side.

$$Q = 2m_{^3_2\text{He}}c^2 - m_{^4_2\text{He}}c^2 - 2m_{^1_1\text{H}}c^2$$

$$= [2(3.016029\,\text{u}) - 4.002603 - 2(1.007825\,\text{u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{12.86\,\text{MeV}}$$

39. (*a*) Reaction 42-9a: $\dfrac{4.00\,\text{MeV}}{2(2.014082\,\text{u})} \times \dfrac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}} \times \dfrac{1\,\text{kg}}{1000\,\text{g}} = \boxed{5.98\times10^{23}\,\text{MeV/g}}$

Reaction 42-9b: $\dfrac{3.23\,\text{MeV}}{2(2.014082\,\text{u})} \times \dfrac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}} \times \dfrac{1\,\text{kg}}{1000\,\text{g}} = \boxed{4.83\times10^{23}\,\text{MeV/g}}$

Reaction 42-9c: $\dfrac{17.57\,\text{MeV}}{(2.014082\,\text{u} + 3.016049\,\text{u})} \times \dfrac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}} \times \dfrac{1\,\text{kg}}{1000\,\text{g}} = \boxed{2.10\times10^{24}\,\text{MeV/g}}$

(*b*) Uranium fission (200 MeV per nucleus):

$$\frac{200\,\text{MeV}}{(235\,\text{u})} \times \frac{1\,\text{u}}{1.66\times10^{-27}\,\text{kg}} \times \frac{1\,\text{kg}}{1000\,\text{g}} = \boxed{5.13\times10^{23}\,\text{MeV/g}}$$

Reaction 42-9a gives about 17% more energy per gram than uranium fission. Reaction 42-9b gives about 6% less energy per gram than uranium fission. Reaction 42-9c gives about 4 times as much energy per gram than uranium fission.

40. Calculate the Q-value for the reaction ${}^{238}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U}$

$$Q = m_{{}^{238}_{92}\mathrm{U}}c^2 + m_{\mathrm{n}}c^2 - m_{{}^{239}_{92}\mathrm{U}}c^2$$

$$= \left[238.050788\,\mathrm{u} + 1.008665\,\mathrm{u} - 239.054293\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.807\,\mathrm{MeV}}$$

41. The reaction of Eq. 42-9b consumes 2 deuterons and releases 3.23 MeV of energy. The amount of energy needed is the power times the elapsed time, and the energy can be related to the mass of deuterium by the reaction.

$$\left(850\frac{\mathrm{J}}{\mathrm{s}}\right)(1\ \mathrm{yr})\left(3.156\times10^{7}\,\frac{\mathrm{s}}{\mathrm{yr}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\frac{2\,\mathrm{d}}{3.23\,\mathrm{MeV}}\right)\left(\frac{2.014\times10^{-3}\mathrm{kg}}{6.02\times10^{23}\,\mathrm{d}}\right)$$

$$= 3.473\times10^{-4}\,\mathrm{kg} = \boxed{0.35\,\mathrm{g}}$$

42. (*a*) The reactants have a total of 3 protons and 7 neutrons, and so the products should have the same. After accounting for the helium, there are 3 neutrons and 1 proton in the other product, and so it must be tritium, ${}^{3}_{1}\mathrm{He}$. The reaction is ${}^{6}_{3}\mathrm{Li} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H}$.

(*b*) The Q-value gives the energy released.

$$Q = m_{{}^{6}_{3}\mathrm{Li}}c^2 + m_{{}^{1}_{0}\mathrm{n}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - m_{{}^{3}_{1}\mathrm{He}}c^2$$

$$= \left[6.015123\,\mathrm{u} + 1.008665\,\mathrm{u} - 4.002603\,\mathrm{u} - 3.016049\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.784\,\mathrm{MeV}}$$

$\boxed{43.}$ Assume that the two reactions take place at equal rates, so they are both equally likely. Then from the reaction of 4 deuterons, there would be a total of 7.228 MeV of energy released, or 1.807 MeV per deuteron on the average. A total power of $\dfrac{1250\,\mathrm{MW}}{0.33} = 3788\,\mathrm{MW}$ must be obtained from the fusion reactions to provide the required 1250 MW output, because of the 330% efficiency. We convert the power to a number of deuterons based on the energy released per reacting deuteron, and then convert that to an amount of water using the natural abundance of deuterium.

$$3788\,\mathrm{MW} \rightarrow \left[\begin{array}{l}\left(3788\times10^{6}\,\dfrac{\mathrm{J}}{\mathrm{s}}\right)\left(\dfrac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right)\left(\dfrac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)\left(\dfrac{1\ \mathrm{d}}{1.807\,\mathrm{MeV}}\right)\left(\dfrac{1\ \mathrm{H\ atom}}{0.000115\ \mathrm{d's}}\right)\times \\ \left(\dfrac{1\,\mathrm{H_2O\,molecule}}{2\ \mathrm{H\ atoms}}\right)\left(\dfrac{0.018\ \mathrm{kg\,H_2O}}{6.02\times10^{23}\,\mathrm{molecules}}\right)\end{array}\right]$$

$$= 6131\,\mathrm{kg/h} \approx \boxed{6100\,\mathrm{kg/h}}$$

44. We assume that the reactants are at rest when they react, and so the total momentum of the system is 0. As a result, the momenta of the two products are equal in magnitude. The available energy of 17.57 MeV is much smaller than the masses involved, and so we use the non-relativistic relationship between momentum and kinetic energy, $K = \dfrac{p^2}{2m} \rightarrow p = \sqrt{2mK}$.

$$K_{{}^{4}_{2}\mathrm{He}} + K_{\mathrm{n}} = K_{\mathrm{total}} = 17.57\,\mathrm{MeV} \qquad p_{{}^{4}_{2}\mathrm{He}} = p_{\mathrm{n}} \rightarrow \sqrt{2m_{{}^{4}_{2}\mathrm{He}}K_{{}^{4}_{2}\mathrm{He}}} = \sqrt{2m_{\mathrm{n}}K_{\mathrm{n}}} \rightarrow$$

$$m_{{}^{4}_{2}\mathrm{He}}K_{{}^{4}_{2}\mathrm{He}} = m_{\mathrm{n}}K_{\mathrm{n}} \rightarrow m_{{}^{4}_{2}\mathrm{He}}K_{{}^{4}_{2}\mathrm{He}} = m_{\mathrm{n}}\left(K_{\mathrm{total}} - K_{{}^{4}_{2}\mathrm{He}}\right) \rightarrow$$

$$K_{{}^4_2\mathrm{He}} = \frac{m_\mathrm{n}}{m_{{}^4_2\mathrm{He}} + m_\mathrm{n}} K_\mathrm{total} = \left(\frac{1.008665}{4.002603 + 1.008665}\right) 17.57\,\mathrm{MeV} = 3.536\,\mathrm{MeV} \approx \boxed{3.5\,\mathrm{MeV}}$$

$$K_\mathrm{n} = K_\mathrm{total} - K_{{}^4_2\mathrm{He}} = 17.57\,\mathrm{MeV} - 3.54\,\mathrm{MeV} = 14.03\,\mathrm{MeV} \approx \boxed{14\,\mathrm{MeV}}$$

If the plasma temperature were significantly higher, then the approximation of 0 kinetic energy being brought into the reaction would not be reasonable. Thus the results would depend on plasma temperature. A higher plasma temperature would result in higher values for the energies.

45. In Eq. 42-9a, 4.00 MeV of energy is released for every 2 deuterium atoms. The mass of water can be converted to a number of deuterium atoms.

$$\left(1.00\,\mathrm{kg\,H_2O}\right)\left(\frac{6.02\times10^{23}\mathrm{H_2O}}{0.018\,\mathrm{kg\,H_2O}}\right)\left(\frac{2\,\mathrm{H}}{1\,\mathrm{H_2O}}\right)\left(\frac{1.15\times10^{-4}\,\mathrm{d}}{1\,\mathrm{H}}\right) = 7.692\times10^{21}\,\mathrm{d\ nuclei} \rightarrow$$

$$\left(7.692\times10^{21}\,\mathrm{d\ nuclei}\right)\left(\frac{4.00\times10^{6}\,\mathrm{eV}}{2\,\mathrm{d\,atoms}}\right)\left(\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right) = \boxed{2.46\times10^{9}\,\mathrm{J}}$$

As compared to gasoline: $\frac{2.46\times10^{9}\,\mathrm{J}}{5\times10^{7}\,\mathrm{J}} \approx \boxed{\text{50 times more than gasoline}}$

46. (*a*) We follow the method of Example 42-10. The reaction is ${}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$. We calculate the potential energy of the particles when they are separated by the sum of their radii. The radii are calculated from Eq. 41-1.

$$K_\mathrm{total} = \frac{1}{4\pi\varepsilon_0}\frac{q_\mathrm{C}q_\mathrm{H}}{r_\mathrm{C} + r_\mathrm{H}} = \left(8.99\times10^{9}\,\mathrm{N{\bullet}m^2/C^2}\right)\frac{(6)(1)\left(1.60\times10^{-19}\,\mathrm{C}\right)^2}{\left(1.2\times10^{-15}\mathrm{m}\right)\left(1^{1/3}+12^{1/3}\right)}\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right)$$

$$= 2.19\,\mathrm{MeV}$$

For the d-t reaction, Example 42-10 shows $K_\mathrm{total} = 0.45\,\mathrm{MeV}$. Find the ratio of the two energies.

$$\frac{K_\mathrm{C}}{K_\mathrm{d-t}} = \frac{2.19\,\mathrm{MeV}}{0.45\,\mathrm{MeV}} = 4.9$$

$\boxed{\text{The carbon reaction requires about 5 times more energy than the d-t reaction.}}$

(*b*) Since the kinetic energy is proportional to the temperature by $\overline{K} = \frac{3}{2}kT$, since the kinetic energy has to increase by a factor of 5, so does the temperature.

$$T = 4.9\left(3\times10^{8}\,\mathrm{K}\right) \approx \boxed{1.5\times10^{9}\,\mathrm{K}}.$$

47. (*a*) No carbon is consumed in this cycle because one ${}^{12}_{6}\mathrm{C}$ nucleus is required in the first step of the cycle, and one ${}^{12}_{6}\mathrm{C}$ nucleus is produced in the last step of the cycle. The net effect of the cycle can be found by adding all the reactants and all the products together, and canceling what appears on both sides of the reaction.

$${}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

$${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + \mathrm{e}^{+} + \nu$$

$${}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$$

$${}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma$$

$$^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \mathrm{e}^{+} + \nu$$

$$^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$$

$$^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} + {}^{13}_{7}\mathrm{N} + {}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} + {}^{15}_{8}\mathrm{O} + {}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H}$$

$$\rightarrow \quad {}^{13}_{7}\mathrm{N} + \gamma + {}^{13}_{6}\mathrm{C} + \mathrm{e}^{+} + \nu + {}^{14}_{7}\mathrm{N} + \gamma + {}^{15}_{8}\mathrm{O} + \gamma + {}^{15}_{7}\mathrm{N} + \mathrm{e}^{+} + \nu + {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$$

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,\mathrm{e}^{+} + 2\nu + 3\gamma$$

There is a difference of one gamma ray in the process, as mentioned in the text.

(*b*) To use the values from Appendix F, we must be sure that number of electrons is balanced as well as the number of protons and neutrons. The above "net" equation does not consider the electrons that neutral nuclei would have, because it does not conserve charge. What the above reaction really represents (ignoring the gammas and neutrinos) is the following.

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,\mathrm{e}^{+} \quad \Rightarrow \quad 4\,{}^{1}_{1}\mathrm{p} \rightarrow 2\,{}^{1}_{1}\mathrm{p} + 2\,{}^{1}_{0}\mathrm{n} + 2\mathrm{e}^{+}$$

To use the values from Appendix F, we must add 4 electrons to each side of the reaction.

$$\left(4\,{}^{1}_{1}\mathrm{p} + 4\mathrm{e}^{-}\right) \rightarrow \left(2\,{}^{1}_{1}\mathrm{p} + 2\mathrm{e}^{-} + 2\,{}^{1}_{0}\mathrm{n}\right) + 2\mathrm{e}^{+} + 2\mathrm{e}^{-} \quad \Rightarrow \quad 4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\mathrm{e}^{+} + 2\mathrm{e}^{-}$$

The energy produced in the reaction is the *Q*-value.

$$Q = 4m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - 4m_{\mathrm{e}}$$

$$= \left[4\left(1.007825\,\mathrm{u}\right) - 4.002603\,\mathrm{u} - 4\left(0.000549\,\mathrm{u}\right)\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 24.69\,\mathrm{MeV}$$

As mentioned at the top of page 1143, the positrons and the electrons annihilate to produce another 2.04 MeV, so the total energy released is $24.69\,\mathrm{MeV} + 2\left(1.02\,\mathrm{MeV}\right) = \boxed{26.73\,\mathrm{MeV}}$.

(*c*) In some reactions extra electrons must be added in order to use the values from Appendix F.

The <u>first equation</u> is electron-balanced, and so Appendix F can be used.

$$Q = m_{{}^{12}_{6}\mathrm{C}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{13}_{7}\mathrm{N}}c^2$$

$$= \left[12.000000\,\mathrm{u} + 1.007825\,\mathrm{u} - 13.005739\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.943\,\mathrm{MeV}}$$

The <u>second equation</u> needs to have another electron, so that ${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + \mathrm{e}^{-} + \mathrm{e}^{+} + \nu$.

$$Q = m_{{}^{13}_{7}\mathrm{N}}c^2 - m_{{}^{13}_{6}\mathrm{C}}c^2 - 2m_{\mathrm{e}}c^2$$

$$= \left[13.005739\,\mathrm{u} - 13.003355\,\mathrm{u} - 2\left(0.000549\,\mathrm{u}\right)\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 1.198\,\mathrm{MeV}$$

We must include an electron-positron annihilation in this reaction.

$$1.198\,\mathrm{MeV} + 1.02\,\mathrm{MeV} = \boxed{2.218\,\mathrm{MeV}}$$

The <u>third equation</u> is electron-balanced.

$$Q = m_{{}^{13}_{6}\mathrm{C}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{14}_{7}\mathrm{N}}c^2$$

$$= \left[13.003355\,\mathrm{u} + 1.007825\,\mathrm{u} - 14.003074\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.551\,\mathrm{MeV}}$$

The fourth equation is electron-balanced.

$$Q = m_{^{14}_{7}\mathrm{N}}c^2 + m_{^{1}_{1}\mathrm{H}}c^2 - m_{^{15}_{8}\mathrm{O}}c^2$$

$$= \left[14.003074\,\mathrm{u} + 1.007825\,\mathrm{u} - 15.003066\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.296\,\mathrm{MeV}}$$

The fifth equation needs to have another electron, so that $^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \mathrm{e}^- + \mathrm{e}^+ + \nu$.

$$Q = m_{^{15}_{8}\mathrm{O}}c^2 - m_{^{15}_{7}\mathrm{N}}c^2 - 2m_{\mathrm{e}}c^2$$

$$= \left[15.003066\,\mathrm{u} - 15.000109\,\mathrm{u} - 2\left(0.000549\,\mathrm{u}\right)\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 1.732\,\mathrm{MeV}$$

We must include an electron-positron annihilation in this reaction.

$$1.732\,\mathrm{MeV} + 1.02\,\mathrm{MeV} = \boxed{2.752\,\mathrm{MeV}}$$

The sixth equation is electron-balanced.

$$Q = m_{^{15}_{7}\mathrm{N}}c^2 + m_{^{1}_{1}\mathrm{H}}c^2 - m_{^{12}_{6}\mathrm{C}}c^2 - m_{^{4}_{2}\mathrm{He}}c^2$$

$$= \left[15.000109\,\mathrm{u} + 1.007825\,\mathrm{u} - 12.000000\,\mathrm{u} - 4.002603\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$

$$= \boxed{4.966\,\mathrm{MeV}}$$

The total is found as follows.

$$1.943\,\mathrm{MeV} + 2.218\,\mathrm{MeV} + 7.551\,\mathrm{MeV} + 7.296\,\mathrm{MeV} + 2.752\,\mathrm{MeV} + 4.966\,\mathrm{MeV}$$

$$= 26.73\,\mathrm{MeV}$$

(*d*) It takes a higher temperature for this reaction than for a proton-proton reaction because the reactants have to have more initial kinetic energy to overcome the Coulomb repulsion of one nucleus to another. In particular, the carbon and nitrogen nuclei have higher Z values leading to the requirement of a high temperature in order for the protons to get close enough to fuse with them.

48. Because the quality factor of alpha particles is 20 and the quality factor of X-rays is 1, it takes 20 times as many rads of X-rays to cause the same biological damage as compared to alpha particles. Thus the 250 rads of alpha particles is equivalent to $250\,\mathrm{rad} \times 20 = \boxed{5000\,\mathrm{rad}}$ of X-rays.

$\boxed{49.}$ Use Eq. 42-11b to relate Sv to Gy. From Table 42.1, the quality factor of gamma rays is 1, and so the number of Sv is equal to the number of Gy. Thus $4.0\,\mathrm{Sv} = \boxed{4.0\,\mathrm{Gy}}$.

50. A gray is 1 Joule per kg, according to Eq. 42-10.

$$3.0\frac{\mathrm{J}}{\mathrm{kg}} \times 65\,\mathrm{kg} = 195\,\mathrm{J} \approx \boxed{200\,\mathrm{J}} \text{ (2 sig. fig.)}$$

51. The biological damage is measured by the effective dose, Eq. 42-11b.

$$65\,\mathrm{rad\ fast\ neutrons} \times 10 = x\,\mathrm{rad\ slow\ neutrons} \times 3 \quad \rightarrow$$

$$x = \frac{65\,\mathrm{rad} \times 10}{3} = \boxed{220\,\mathrm{rad\ slow\ neutrons}}$$

52. (*a*) Since the quality factor for protons is 1, the effective dose (in rem) is the same as the absorbed dose (in rad). Thus the absorbed dose is $\boxed{1.0\text{ rad or }0.010\text{ Gy}}$.

(*b*) A Gy is a J per kg.

$$(0.010\,\text{Gy})\left(\frac{1\,\text{J/kg}}{1\,\text{Gy}}\right)(0.25\,\text{kg})\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{p}}{1.2\times10^{6}\,\text{eV}}\right)=\boxed{1.3\times10^{10}\,\text{p}}$$

53. The counting rate will be 85% of 25% of the activity.

$$(0.035\times10^{-6}\,\text{Ci})\left(\frac{3.7\times10^{10}\,\text{decays/s}}{1\,\text{Ci}}\right)\left(\frac{1\beta}{1\,\text{decay}}\right)(0.25)(0.85)=275.2\,\text{counts/s}\approx\boxed{280\,\text{counts/s}}$$

54. The two definitions of roentgen are 1.6×10^{12} ion pairs/g produced by the radiation, and the newer definition of 0.878×10^{-2} J/kg deposited by the radiation. Start with the current definition, and relate them by the value of 35 eV per ion pair.

$$1\,\text{R}=(0.878\times10^{-2}\,\text{J/kg})(1\,\text{kg}/1000\,\text{g})(1\,\text{eV}/1.60\times10^{-19}\,\text{J})(1\,\text{ion pair}/35\,\text{eV})$$

$$=1.567\times10^{12}\,\text{ion pairs/g}$$

The two values of ion pairs per gram are within about 2% of each other.

$\boxed{55.}$ We approximate the decay rate as constant, and find the time to administer 36 Gy. If that calculated time is significantly shorter than the half-life of the isotope, then the approximation is reasonable. If 1.0 mCi delivers about 10 mGy/min, then 1.6 mCi would deliver 16 mGy/min.

$$\text{dose}=\text{rate}\times\text{time}\;\rightarrow\;\text{time}=\frac{\text{dose}}{\text{rate}}=\frac{36\,\text{Gy}}{16\times10^{-3}\,\text{Gy/min}}\left(\frac{1\,\text{day}}{1440\,\text{min}}\right)=1.56\,\text{day}\approx\boxed{1.6\,\text{day}}$$

This is only about 11% of a half life, so our approximation is reasonable.

56. Since the half-life is long (5730 yr) we will consider the activity as constant over a short period of time. Use the definition of the curie from Section 42-6.

$$(2.00\times10^{-6}\,\text{Ci})\left(\frac{3.70\times10^{10}\,\text{decays/s}}{1\,\text{Ci}}\right)=7.40\times10^{4}\,\text{decays/s}=\left|\frac{dN}{dt}\right|=\lambda N=\frac{\ln2}{T_{1/2}}N\;\rightarrow$$

$$N=\left|\frac{dN}{dt}\right|\frac{T_{1/2}}{\ln2}=(7.4\times10^{4}\,\text{decays/s})\left(\frac{5730\,\text{y}}{\ln2}\right)(3.156\times10^{7}\,\text{s/y})=1.931\times10^{16}\,\text{nuclei}$$

$$1.931\times10^{16}\,\text{nuclei}\left(\frac{0.014\,\text{kg}}{6.02\times10^{23}\,\text{nuclei}}\right)=\boxed{4.49\times10^{-10}\,\text{kg}}$$

57. (*a*) According to Appendix F, ${}^{131}_{53}\text{I}$ decays by beta decay.

$$\boxed{{}^{131}_{53}\text{I}\rightarrow{}^{131}_{54}\text{Xe}+\beta^{-}+\overline{\nu}}$$

(*b*) The number of nuclei present is given by Eq. 41-6.

$$N=N_0e^{-\lambda t}\;\rightarrow\;t=-\frac{\ln\frac{N}{N_0}}{\lambda}=-\frac{T_{1/2}\ln\frac{N}{N_0}}{\ln2}=-\frac{(8.0\,\text{d})\ln0.070}{\ln2}=30.69\,\text{d}\approx\boxed{31\,\text{d}}$$

(*c*) The activity is given by $dN/dt = -\lambda N$. This can be used to find the number of nuclei, and then the mass can be found. Note that the numeric value of dN/dt is negative since the number of undecayed nuclei is decreasing.

$$\frac{dN}{dt} = -\lambda N \quad \rightarrow$$

$$N = -\frac{dN/dt}{\lambda} = \frac{(T_{1/2})(-dN/dt)}{\ln 2} = \frac{(8.0\,\mathrm{d})(86400\,\mathrm{s/d})(1\times10^{-3}\,\mathrm{Ci})(3.70\times10^{10}\,\mathrm{decays/s})}{\ln 2}$$

$$= 3.69\times10^{13}\,\text{nuclei}$$

$$3.69\times10^{13}\,\text{nuclei}\left(\frac{0.131\,\mathrm{kg}}{6.02\times10^{23}\,\text{nuclei}}\right) \approx \boxed{8\times10^{-12}\,\mathrm{kg}} = 8\,\mathrm{ng}$$

58. The activity is converted to decays per day, then to energy per year, and finally to a dose per year. The potassium decays by gammas and betas, according to Appendix F. Gammas and betas have a quality factor of 1, so the number of Sv is the same as the number of Gy, and the number of rem is the same as the number of rad.

$$\left(2000\times10^{-12}\,\frac{\mathrm{Ci}}{\mathrm{L}}\right)\left(3.70\times10^{10}\,\frac{\mathrm{decays/s}}{1\,\mathrm{Ci}}\right)(12\,\mathrm{hr})\left(\frac{3600\,\mathrm{s}}{\mathrm{hr}}\right)\left(0.5\frac{\mathrm{L}}{\mathrm{day}}\right) = 1.598\times10^{6}\,\frac{\mathrm{decays}}{\mathrm{day}}$$

$$\left(1.598\times10^{6}\,\frac{\mathrm{decays}}{\mathrm{day}}\right)\left(365\frac{\mathrm{day}}{\mathrm{yr}}\right)(0.10)\left(1.5\frac{\mathrm{MeV}}{\mathrm{decay}}\right)\left(\frac{1.60\times10^{-13}\,\mathrm{J}}{\mathrm{MeV}}\right) = 1.40\times10^{-5}\,\frac{\mathrm{J}}{\mathrm{yr}}$$

(*a*) For the adult, use a mass of 60 kg.

$$\text{Effective dose} = \left(1.40\times10^{-5}\,\frac{\mathrm{J}}{\mathrm{yr}}\right)\left(\frac{1}{60\,\mathrm{kg}}\right)\left(\frac{1\,\mathrm{Gy}}{1\,\mathrm{J/kg}}\right)\left(\frac{1\,\mathrm{Sv}}{1\,\mathrm{Gy}}\right)$$

$$= 2.33\times10^{-7}\,\mathrm{Sv/yr}\left(\frac{10^{5}\,\mathrm{mrem}}{\mathrm{Sv}}\right) = 2.33\times10^{-2}\,\mathrm{mrem/yr}$$

$$\approx \boxed{2\times10^{-7}\,\mathrm{Sv/yr}} \text{ or } \boxed{2\times10^{-2}\,\mathrm{mrem/yr}}$$

$$\text{fraction of allowed dose} = \frac{2.33\times10^{-2}\,\frac{\mathrm{mrem}}{\mathrm{year}}}{100\frac{\mathrm{mrem}}{\mathrm{year}}} \approx \boxed{2\times10^{-4}\,\text{times the allowed dose}}$$

(*b*) For the baby, the only difference is that the mass is 10 times smaller, so the effective dose is 10 times bigger. The results are as follows.

$$\approx \boxed{2\times10^{-6}\,\mathrm{Sv/yr}},\ \boxed{0.2\,\mathrm{mrem/yr}},\ \text{and } \boxed{2\times10^{-3}\,\text{times the allowed dose}}$$

59. Each decay releases one gamma ray of energy 122 keV. Half of that energy is deposited in the body. The activity tells at what rate the gamma rays are released into the body. We assume the activity is constant.

$$\left(1.55\times10^{-6}\,\mathrm{Ci}\right)\left(3.70\times10^{10}\,\frac{\gamma/\mathrm{s}}{\mathrm{Ci}}\right)(86400\,\mathrm{s/day})(0.50)(122\,\mathrm{keV}/\gamma)\left(1.60\times10^{-16}\,\mathrm{J/keV}\right)\left(\frac{1}{58\,\mathrm{kg}}\right)$$

$$= 8.338\times10^{-7}\,\frac{\mathrm{J/kg}}{\mathrm{day}} \approx \boxed{8.3\times10^{-7}\,\frac{\mathrm{Gy}}{\mathrm{day}}}$$

60. We use the dose, the mass of the beef, and the energy per electron to find the number of electrons.

$$4.5\times10^{3}\,\text{Gy}\left(\frac{1\,\text{J/kg}}{1\,\text{Gy}}\right)(5\,\text{kg})\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)\left(\frac{1\,\text{e}^-}{1.2\,\text{MeV}}\right)=\boxed{1\times10^{17}\,\text{e}^-}$$

$\boxed{61.}$ (*a*) The reaction has $Z = 86$ and $A = 222$ for the parent nucleus. The alpha has $Z = 2$ and $A = 4$, so the daughter nucleus must have $Z = 84$ and $A = 218$. That makes the daughter nucleus $\boxed{{}^{218}_{84}\text{Po}}$.

(*b*) From Figure 41-12, polonium-218 is $\boxed{\text{radioactive}}$. It decays via both $\boxed{\text{alpha and beta decay}}$, each with a half-life of $\boxed{3.1\text{ minutes}}$.

(*c*) The daughter nucleus is not a noble gas, so it is $\boxed{\text{chemically reacting}}$. It is in the same group as oxygen, so it might react with many other elements chemically.

(*d*) The activity is given by Eq. 41-7a, $R=\lambda N=\frac{\ln 2}{T_{1/2}}N$.

$$R=\frac{\ln 2}{T_{1/2}}N=\frac{\ln 2}{(3.8235\,\text{d})(86400\,\text{s/d})}(1.6\times10^{-9}\,\text{g})\frac{6.02\times10^{23}\,\text{nuclei}}{222\text{ g}}$$

$$=9.104\times10^{6}\text{ decays/s}\approx\boxed{9.1\times10^{6}\,\text{Bq}}=0.25\,\text{mCi}$$

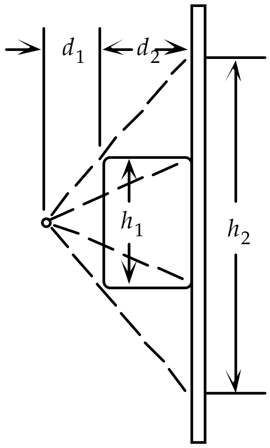
To find the activity after 1 month, use Eq. 41-7d.

$$R=R_0e^{-\frac{\ln 2}{T_{1/2}}t}=(9.104\times10^{6}\text{ decays/s})e^{-\frac{\ln 2}{(3.8235\text{d})}(30\text{d})}=3.956\times10^{4}\text{ decays/s}$$

$$\approx\boxed{4.0\times10^{4}\,\text{Bq}}=1.1\,\mu\text{Ci}$$

62. (*a*) For parallel rays, the object and the image will be the same size, and so the magnification is $\boxed{1}$.

(*b*) When the film is pressed against the back, the image of the back on the film will be the same size as the back, since there is no appreciable spreading of the rays from the back to the film. So $m_{\text{back}} = 1$. But, from the diagram, we see that the rays which define the boundary of the area on the chest will have a much larger image with the film at the back. The height of the image is proportional to the distance from the point source, since the rays travel in straight lines.



$$m_{\text{front}}=\frac{h_2}{h_1}=\frac{d_1+d_2}{d_1}=1+\frac{d_2}{d_1}=1+\frac{25}{15}=2.67$$

So the range of magnifications is $1\le m\le 2.67$, depending on which part of the body is being imaged.

63. The frequency is given in Example 42-14 to be 42.58 MHz. Use that to find the wavelength.

$$c=f\lambda\;\rightarrow\;\lambda=\frac{c}{f}=\frac{2.998\times10^{8}\,\text{m/s}}{42.58\times10^{6}\,\text{Hz}}=\boxed{7.041\,\text{m}}$$

This lies in the $\boxed{\text{radio wave}}$ portion of the spectrum.

64. We use Eq. 42-12, but with the neutron's magnetic moment.

$$hf = 2\mu_n B_T \quad \rightarrow$$

$$B_T = \frac{hf}{2\mu_n} = \frac{hf}{2(0.7023)\mu_N} = \frac{hf}{2(0.7023)\dfrac{eh}{4\pi m_p}} = \frac{2\pi f m_p}{(0.7023)e}$$

$$= \frac{2\pi(42.58\times10^6\,\text{Hz})(1.673\times10^{-27}\,\text{kg})}{(0.7023)(1.602\times10^{-19}\,\text{C})} = \boxed{3.978\,\text{T}}$$

65. (*a*) The reaction is ${}^9_4\text{Be} + {}^4_2\text{He} \rightarrow \text{n} + ?$. There are 6 protons and 13 nucleons in the reactants, and so there must be 6 protons and 13 nucleons in the products. The neutron is 1 nucleon, so the other product must have 6 protons ad 12 nucleons. Thus it is $\boxed{{}^{12}_6\text{C}}$.

(*b*) $Q = m_{{}^9_4\text{Be}}c^2 + m_{{}^4_2\text{He}}c^2 - m_\text{n}c^2 - m_{{}^{12}_6\text{C}}c^2$

$$= [9.012182\,\text{u} + 4.002603\,\text{u} - 1.008665\,\text{u} - 12.000000\,\text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{5.701\,\text{MeV}}$$

66. The energy and temperature are related by the Boltzmann constant, which has units of energy/temperature.

$$k = 1.381\times10^{-23}\,\frac{\text{J}}{\text{K}}\times\frac{1\,\text{eV}}{1.602\times10^{-19}\,\text{J}}\times\frac{1\,\text{keV}}{1000\,\text{eV}} = \boxed{8.620\times10^{-8}\,\text{keV/K}}$$

$\boxed{67.}$ From Eq. 18-5, the average speed of a gas molecule (root mean square speed) is inversely proportional to the square root of the mass of the molecule, if the temperature is constant. We assume that the two gases are in the same environment and so at the same temperature. We use UF_6 molecules for the calculations.

$$\frac{v_{{}^{235}_{92}\text{UF}_6}}{v_{{}^{238}_{92}\text{UF}_6}} = \sqrt{\frac{m_{{}^{238}_{92}\text{UF}_6}}{m_{{}^{235}_{92}\text{UF}_6}}} = \sqrt{\frac{238+6(19)}{235+6(19)}} = \boxed{1.0043:1}$$

68. (*a*) We assume that the energy produced by the fission was 200 MeV per fission, as in Eq. 42-6.

$$(20\,\text{kilotons TNT})\left(\frac{5\times10^{12}\,\text{J}}{1\,\text{kiloton}}\right)\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\text{MeV}}\right)\left(\frac{0.235\,\text{kg}}{6.02\times10^{23}\,\text{atom}}\right)$$

$$= 1.220\,\text{kg} \approx \boxed{1\,\text{kg}}$$

(*b*) Use $E = mc^2$.

$$E = mc^2 \quad \rightarrow \quad m = \frac{E}{c^2} = \frac{(20\,\text{kilotons TNT})\left(\dfrac{5\times10^{12}\,\text{J}}{1\,\text{kiloton}}\right)}{(3.0\times10^8\,\text{m/s})^2} = 1.11\times10^{-3}\,\text{kg} \approx \boxed{1\,\text{g}}$$

This is consistent with the result found in Problem 26.

69. The effective dose (in rem) is equal to the actual dose (in rad) times the quality factor, from Eq. 42-11a.

$$\text{dose}(\text{rem}) = (29\,\text{mrad/yr X-ray, } \gamma\text{-ray})(1) + (3.6\,\text{mrad/yr})(10) = \boxed{65\,\text{mrem/yr}}$$

70. The oceans cover about 70% of the Earth, to an average depth of approximately 4 km. The density of the water is approximately $1000\,\text{kg/m}^3$. Find the volume of water using the surface area of the Earth. Then convert that volume of water to mass, to the number of water molecules, to the number of hydrogen atoms, and then finally to the number of deuterium atoms using the natural abundance of deuterium from Appendix F.

$$\text{Mass of water} = (\text{surface area})(\text{depth})(\text{density}) = 4\pi(6.38\times10^6\,\text{m})^2(4000\,\text{m})(1000\,\text{kg/m}^3)$$

$$= 2.05\times10^{21}\,\text{kg water}$$

$$(2.05\times10^{21}\,\text{kg water})\left(\frac{6.02\times10^{23}\,\text{molecules}}{0.018\text{ kg water}}\right)\left(\frac{2\text{ H atoms}}{1\text{ molecule}}\right)\left(\frac{0.000115\text{ d atoms}}{1\text{ H atom}}\right)$$

$$= 1.58\times10^{43}\,\text{d} \approx \boxed{2\times10^{43}\,\text{d atoms}}$$

$$= 1.58\times10^{43}\,\text{d}\left(\frac{2\times10^{-3}\,\text{kg}}{6.02\times10^{23}\,\text{d atoms}}\right) \approx \boxed{5\times10^{16}\,\text{kg d}}$$

From Eqs. 42-9a and 42-9b, if the two reactions are carried out at the same rate then 4 deuterons would produce 7.23 MeV of energy. Use that relationship to convert the number of deuterons in the oceans to energy.

$$(1.58\times10^{43}\,\text{d})\times\frac{7.23\,\text{MeV}}{4\text{ d}}\times\frac{1.60\times10^{-13}\,\text{J}}{1\,\text{MeV}} = 4.57\times10^{30}\,\text{J} \approx \boxed{5\times10^{30}\,\text{J}}$$

71. Because the quality factor for gamma rays is 1, the dose in rem is equal in number to the dose in rad. Since the intensity falls off as r^2, the square of the distance, the exposure rate times r^2 is constant.

$$\text{Allowed dose} = \frac{5\,\text{rem}}{\text{year}}\left(\frac{1\,\text{rad}}{1\,\text{rem}}\right)\left(\frac{1\text{ year}}{52\text{ weeks}}\right)\left(\frac{1\,\text{week}}{35\,\text{hours}}\right) = 2.747\times10^{-3}\,\frac{\text{rad}}{\text{hour}}$$

$$\left(2.747\times10^{-3}\,\frac{\text{rad}}{\text{hour}}\right)r^2 = \left(5.2\times10^{-2}\,\frac{\text{rad}}{\text{hour}}\right)(1\,\text{m})^2 \quad\to$$

$$r = \sqrt{\frac{\left(5.2\times10^{-2}\,\frac{\text{rad}}{\text{hour}}\right)(1\,\text{m})^2}{\left(2.747\times10^{-3}\,\frac{\text{rad}}{\text{hour}}\right)}} = 4.351\,\text{m} \approx \boxed{4.4\,\text{m}}$$

72. (*a*) The reaction is of the form ? $\to {}^4_2\text{He} + {}^{222}_{86}\text{Rn}$. There are 88 protons and 226 nucleons as products, so there must be 88 protons and 226 nucleons as reactants. Thus the parent nucleus is ${}^{226}_{88}\text{Ra}$.

$$\boxed{{}^{226}_{88}\text{Ra} \to {}^4_2\text{He} + {}^{222}_{86}\text{Rn}}$$

(*b*) If we ignore the kinetic energy of the daughter nucleus, then the kinetic energy of the alpha particle is the *Q*-value of the reaction.

$$K_\alpha = m_{{}^{226}_{88}\text{Ra}}c^2 - m_{{}^{4}_{2}\text{He}}c^2 - m_{{}^{222}_{86}\text{Rn}}c^2$$

$$= \left[226.025410\,\text{u} - 4.002603\,\text{u} - 222.017578\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{4.871\,\text{MeV}}$$

(*c*) From momentum conservation, the momentum of the alpha particle will be equal in magnitude to the momentum of the daughter particle. At the energy above, the alpha particle is not relativistic, and so $K_\alpha = \frac{p_\alpha^2}{2m_\alpha} \rightarrow p_\alpha = \sqrt{2m_\alpha K_\alpha}$.

$$p_\alpha = \sqrt{2m_\alpha K_\alpha} = \sqrt{2\left(4.002603\,\text{u}\right)\left(\frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\right)\left(4.871\,\text{MeV}\right)} = \boxed{191\,\text{MeV}/c}$$

(*d*) Since $p_\alpha = p_{\text{daughter}}$, $K_{\text{daughter}} = \frac{p_{\text{daughter}}^2}{2m_{\text{daughter}}} = \frac{p_\alpha^2}{2m_{\text{daughter}}}$.

$$K_{\text{daughter}} = \frac{p_\alpha^2}{2m_{\text{daughter}}} = \frac{\left(191\,\text{MeV}/c\right)^2}{2\left(222\,\text{u}\right)\left(\frac{931.5\,\text{MeV}/c^2}{1\,\text{u}}\right)} = \boxed{8.82\times10^{-2}\,\text{MeV}}$$

Thus we see that our original assumption of ignoring the kinetic energy of the daughter nucleus is valid. The kinetic energy of the daughter is less than 2% of the *Q*-value.

$\boxed{73.}$ (*a*) The mass of fuel can be found by converting the power to energy to number of nuclei to mass.

$$\left(2400\times10^6\,\text{J/s}\right)\left(1\text{ y}\right)\left(\frac{3.156\times10^7\text{s}}{1\,\text{y}}\right)\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\text{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\text{MeV}}\right)\left(\frac{0.235\,\text{kg}}{6.02\times10^{23}\text{atom}}\right)$$

$$= 9.240\times10^4\text{kg} \approx \boxed{920\,\text{kg}}$$

(*b*) The product of the first 5 factors above gives the number of U atoms that fission.

$$\#\text{Sr atoms} = 0.06\left(2400\times10^6\text{ J/s}\right)\left(1\text{ y}\right)\left(\frac{3.156\times10^7\text{s}}{1\,\text{y}}\right)\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\text{J}}\right)\left(\frac{1\,\text{fission atom}}{200\,\text{MeV}}\right)$$

$$= 1.42\times10^{26}\,\text{Sr atoms}$$

The activity is given by Eq. 41-7a.

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{\left(29\,\text{yr}\right)\left(3.156\times10^7\text{ s/yr}\right)}\left(1.42\times10^{26}\right) = 1.076\times10^{17}\text{ decays/s}$$

$$= \left(1.076\times10^{17}\text{ decays/s}\right)\left(\frac{1\,\text{Ci}}{3.70\times10^{10}\text{ decays/s}}\right) = 2.91\times10^6\,\text{Ci} \approx \boxed{3\times10^6\,\text{Ci}}$$

74. This “heat of combustion” is 26.2 MeV / 4 hydrogen atoms.

$$\frac{26.2\,\text{MeV}}{4\text{ H atoms}}\left(\frac{1.60\times10^{-13}\text{J}}{1\,\text{MeV}}\right)\left(\frac{1\,\text{H atom}}{1.0078\,\text{u}}\right)\left(\frac{1\,\text{u}}{1.66\times10^{-27}\text{kg}}\right) = \boxed{6.26\times10^{14}\text{ J/kg}}$$

This is about $\boxed{2\times10^7}$ times the heat of combustion of coal.

75. (*a*) The energy is radiated uniformly over a sphere with a radius equal to the orbit radius of the Earth.

$$\left(1300\,\mathrm{W/m^2}\right)4\pi\left(1.496\times10^{11}\,\mathrm{m}\right)^2 = 3.656\times10^{26}\,\mathrm{W} \approx \boxed{3.7\times10^{26}\,\mathrm{W}}$$

(*b*) The reaction of Eq. 42-8 releases 26.2 MeV for every 4 protons consumed, assuming we ignore the energy carried away with the neutrinos.

$$\left(3.656\times10^{26}\,\frac{\mathrm{J}}{\mathrm{s}}\right)\left(\frac{4\text{ protons}}{26.2\,\mathrm{MeV}}\right)\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\,\mathrm{J}}\right) = 3.489\times10^{38}\text{ protons/s}$$

$$\approx \boxed{3.5\times10^{38}\text{ protons/s}}$$

(*c*) Convert the Sun's mass to a number of protons, and then use the above result to estimate the Sun's lifetime.

$$2.0\times10^{30}\,\mathrm{kg}\left(\frac{1\,\text{proton}}{1.673\times10^{-27}\,\mathrm{kg}}\right)\left(\frac{1\,\mathrm{s}}{3.489\times10^{38}\,\text{protons}}\right)\left(\frac{1\,\mathrm{yr}}{3.156\times10^{7}}\right) \approx \boxed{1.1\times10^{11}\,\mathrm{yr}}$$

76. For the net proton cycle, Eq. 42-8, we see that there are two neutrinos produced for every four protons consumed. Thus the net number of neutrinos generated per second from the sun is just half the value of protons consumed per second. That proton consumption rate is calculated in Problem 75b.

$$\left(3.489\times10^{38}\text{ protons/s}\right)\left(\frac{2\nu}{4\mathrm{p}}\right) = 1.745\times10^{38}\;\nu/\mathrm{s}$$

We assume the neutrinos are spread out uniformly over a sphere centered at the Sun. So the fraction that would pass through the area of the ceiling can be found by a ratio of areas, assuming the ceiling is perpendicular to the neutrino flux. But since the window is not perpendicular, a cosine factor is included to account for the angle difference, as discussed in Eq. 22-1a. Finally, we adjust for the one-hour duration, assuming the relative angle is constant over that hour.

$$\left(1.745\times10^{38}\;\nu/\mathrm{s}\right)\frac{180\,\mathrm{m}^2}{4\pi\left(1.496\times10^{11}\,\mathrm{m}\right)^2}\left(\cos 38^\circ\right)\left(3600\,\mathrm{s}\right) = \boxed{3.2\times10^{20}\,\nu}$$

77. We use the common value of 200 MeV of energy released per fission. We then multiply that by the number of fissions, which we take as 5.0% of the number of U-238 atoms.

$$\text{Total energy} = \left[\begin{array}{l}\dfrac{200\,\mathrm{MeV}}{1\text{ nucleus of }{}^{235}_{92}\mathrm{U}}\left(\dfrac{1.602\times10^{-13}\,\mathrm{J}}{1\,\mathrm{MeV}}\right)\left(0.05\dfrac{{}^{235}_{92}\mathrm{U}\text{ nuclei}}{{}^{238}_{92}\mathrm{U}\text{ nuclei}}\right)\left(2.0\,\mathrm{kg}\,{}^{238}_{92}\mathrm{U}\right)\\ \times\left(\dfrac{6.022\times10^{23}\text{ nuclei of }{}^{238}_{92}\mathrm{U}\text{ nuclei}}{0.238\,\mathrm{kg}\,{}^{238}_{92}\mathrm{U}}\right)\end{array}\right]$$

$$= 8.107\times10^{12}\,\mathrm{J} \approx \boxed{8\times10^{12}\,\mathrm{J}}$$

78. (*a*) The energy released is given by the *Q*-value.

$$Q = 2m_{{}^{12}_{6}\mathrm{C}}c^2 - m_{{}^{24}_{12}\mathrm{Mg}}c^2 = \left[2\left(12.000000\,\mathrm{u}\right) - 23.985042\,\mathrm{u}\right]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{13.93\,\mathrm{MeV}}$$

(*b*) The total kinetic energy of the two nuclei must equal their potential energy when separated by 6.0 fm.

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} \quad \rightarrow$$

$$K = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \tfrac{1}{2}\left(8.99\times10^{9}\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left[(6)\left(1.60\times10^{-19}\,\text{C}\right)\right]^2}{6.0\times10^{-15}\text{m}}\left(\frac{1\,\text{MeV}}{1.60\times10^{-13}\,\text{J}}\right)$$
$$= 4.315\,\text{MeV} \approx \boxed{4.3\,\text{MeV}}$$

(*c*) The kinetic energy and temperature are related by Eq. 18-4.

$$K = \tfrac{3}{2}kT \ \rightarrow\ T = \tfrac{2}{3}\frac{K}{k} = \tfrac{2}{3}\frac{\left(4.315\,\text{MeV}\right)\left(\frac{1.60\times10^{-13}\,\text{J}}{1\,\text{MeV}}\right)}{1.38\times10^{-23}\,\text{J/K}} = \boxed{3.3\times10^{10}\,\text{K}}$$

$\boxed{79.}$ (*a*) A Curie is 3.7×10^{10} decays/s.

$$\left(0.10\times10^{-6}\text{Ci}\right)\left(3.7\times10^{10}\ \text{decays/s}\right) = \boxed{3700\,\text{decays/s}}$$

(*b*) The beta particles have a quality factor of 1. We calculate the dose in gray and then convert to sieverts. The half life is over a billion years, so we assume the activity is constant.

$$\left(3700\,\text{decays/s}\right)\left(1.4\,\text{MeV/decay}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)\left(3.156\times10^{7}\ \text{s/y}\right)\left(\frac{1}{55\,\text{kg}}\right)$$
$$= 4.756\times10^{-4}\ \text{J/kg/y} = 4.756\times10^{-4}\ \text{Gy/y} = \boxed{4.8\times10^{-4}\ \text{Sv/y}}$$

This is about $\frac{4.756\times10^{-4}\ \text{Sv/y}}{3.6\times10^{-3}\ \text{Sv/y}} = 0.13$ or $\boxed{\text{13\% of the background rate}}$.

80. The surface area of a sphere is $4\pi r^2$.

$$\frac{\text{Activity}}{\text{m}^2} = \frac{2.0\times10^{7}\text{Ci}}{4\pi r_{\text{Earth}}^2} = \frac{\left(2.0\times10^{7}\text{Ci}\right)\left(3.7\times10^{10}\ \text{decays/s}\right)}{4\pi\left(6.38\times10^{6}\text{m}\right)^2} = \boxed{1400\frac{\text{decays/s}}{\text{m}^2}}$$

81. $Q = 3m_{^4_2\text{He}}c^2 - m_{^{12}_{6}\text{C}}c^2 = \left[3\left(4.002603\,\text{u}\right) - 12.000000\,\text{u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{7.274\,\text{MeV}}$

82. Since the half-life is 30 years, we assume that the activity does not change during the 2.0 hours of exposure. We calculate the total energy absorbed, and then calculate the effective dose. The two energies can be added directly since the quality factor for both gammas and betas is about 1.

$$\text{Energy} = \left[\begin{array}{l}\left(1.2\times10^{-6}\text{Ci}\right)\left(3.7\times10^{10}\frac{\text{decays}}{\text{s}}\right)\left(1.6\,\text{hr}\right)\left(3600\frac{\text{s}}{1\,\text{hr}}\right)\times \\ \left(850\times10^{3}\frac{\text{eV}}{\text{decay}}\right)\left(1.60\times10^{-19}\frac{\text{J}}{\text{eV}}\right)\end{array}\right] = 3.478\times10^{-5}\,\text{J}$$

$$\text{dose} = \frac{3.478\times10^{-5}\,\text{J}}{65\,\text{kg}}\times\frac{100\text{rad}}{1\,\text{J/kg}} = 5.351\times10^{-5}\,\text{rad} \approx \boxed{5.4\times10^{-5}\text{rem}}$$

83. The half life of the strontium isotope is 28.79 years. Use that with Eq. 41-7c to find the time for the activity to be reduced to 15% of its initial value.

$$R = R_0e^{-\lambda t} \ \rightarrow\ t = -\frac{1}{\lambda}\ln\frac{R}{R_0} = -\frac{T_{1/2}}{\ln 2}\ln\frac{R}{R_0} = -\frac{\left(28.79\,\text{y}\right)\ln\left(0.15\right)}{\ln 2} \approx \boxed{79\,\text{y}}$$

84. Source B is more dangerous than source A because of its higher energy. Since both sources have the same activity, they both emit the same number of gammas. Source B can deposit twice as much energy per gamma and therefore cause more biological damage.

Source C is more dangerous than source B because the alphas have a quality factor up to 20 times larger than the gammas. Thus a number of alphas may have an effective dose up to 20 times higher than the effective dose of the same number of like-energy gammas.

So from most dangerous to least dangerous, the ranking of the sources is $\boxed{C > B > A}$.

We might say that source B is twice as dangerous as source A, and source C is 20 times more dangerous than source B.

$\boxed{85.}$ The whole-body dose can be converted into a number of decays, which would be the maximum number of nuclei that could be in the Tc sample. The quality factor of gammas is 1.

$$50\,\text{mrem} = 50\,\text{mrad} \;\rightarrow\; \left(50\times10^{-3}\,\text{rad}\right)\left(\frac{1\,\text{J/kg}}{100\,\text{rad}}\right)\left(60\,\text{kg}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 1.875\times10^{17}\,\text{eV}$$

$$\left(1.875\times10^{17}\,\text{eV}\right)\left(\frac{1\text{ effective}\,\gamma}{140\times10^{3}\text{eV}}\right)\left(\frac{2\,\gamma\text{ decays}}{1\text{ effective }\gamma}\right)\left(\frac{1\text{ nucleus}}{1\,\gamma\text{ decay}}\right) = 2.679\times10^{12}\,\text{nuclei}$$

This is the total number of decays that will occur. The activity for this number of nuclei can be calculated from Eq. 41-7a.

$$R = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2\left(2.67910^{12}\,\text{decays}\right)}{\left(6\,\text{h}\right)\left(3600\,\text{s/h}\right)}\left(\frac{1\,\text{Ci}}{3.70\times10^{10}\,\text{decays/s}}\right) \approx 2.32\times10^{-3}\,\text{Ci} \approx \boxed{2\,\text{mCi}}$$

86. The number of ${}^{60}_{27}\text{Co}$ nuclei $\left(N_{60}\right)$ can be calculated from the activity and the half-life of ${}^{60}_{27}\text{Co}$. The number of ${}^{60}_{27}\text{Co}$ nuclei can also be calculated from the cross section and the number of ${}^{59}_{27}\text{Co}$ nuclei $\left(N_{59}\right)$ present in the paint. By combining these two calculations, we can find the number of ${}^{59}_{27}\text{Co}$ in the paint. We assume that, since the half life is relatively long, that all ${}^{60}_{27}\text{Co}$ are made initially, without any of them decaying.

$$\left|\frac{dN_{60}}{dt}\right| = \lambda N_{60} = \frac{\ln 2}{T_{1/2}}N_{60} \;\rightarrow\; N_{60} = \left|\frac{dN_{60}}{dt}\right|\frac{T_{1/2}}{\ln 2}$$

We assume the paint is thin and so use Eq. 42-3 for the cross section. Let *R* represent the rate at which ${}^{60}_{27}\text{Co}$ nuclei are made (i.e., the collision rate), and *t* the elapsed time of neutron bombardment. R_0 is the rate at which the neutrons hit the painting, and so is the given neutron flux times the area of the painting.

$$N_{60} = Rt = R_0 n\ell\sigma t = R_0\left(\frac{N_{59}}{\text{volume of paint}}\right)\left(\frac{\text{volume of paint}}{\text{surface area of paint}}\right)\sigma t = \frac{R_0 N_{59}\sigma t}{A}$$

$$= \frac{\left(\text{neutron flux}\right)AN_{59}\sigma t}{A} = \left(\text{neutron flux}\right)N_{59}\sigma t \;\rightarrow$$

$$N_{59} = \frac{N_{60}}{\left(\text{neutron flux}\right)\sigma t} = \left|\frac{dN_{60}}{dt}\right|\frac{T_{1/2}}{\ln 2\left(\text{neutron flux}\right)\sigma t}$$

$$=\left(55\,\text{decays/s}\right)\frac{\left(5.27\,\text{yr}\right)\left(3.156\times10^{7}\,\text{s/yr}\right)}{\left(\ln 2\right)\left(\dfrac{5.0\times10^{12}}{\text{cm}^{2}\bullet\text{s}}\right)\left(10^{4}\,\dfrac{\text{cm}^{2}}{\text{m}^{2}}\right)\left(19\times10^{-28}\,\text{m}^{2}\right)\left(300\,\text{s}\right)}$$

$$=4.631\times10^{17}\,\text{atoms}\;\rightarrow\;m_{59}=\left(4.631\times10^{17}\,\text{atoms}\right)\left(\frac{58.933\,\text{g}}{6.022\times10^{23}\,\text{atoms}}\right)=\boxed{4.5\times10^{-5}\,\text{g}}$$

87. Since all speeds are relativistic, we may use $p=\sqrt{2mK}$ to relate momentum and kinetic energy. If we assume the target is at rest, then the total momentum of the products must equal the momentum of the bombarding particle. The total (kinetic) energy of the products comes from the kinetic energy of the bombarding particle and the Q-value of the reaction.

$$p_{\text{pr}}=p_{\text{b}}\;\rightarrow\;\sqrt{2m_{\text{pr}}K_{\text{pr}}}=\sqrt{2m_{\text{b}}K_{\text{br}}}\;\rightarrow\;K_{\text{pr}}=\frac{m_{\text{b}}}{m_{\text{pr}}}K_{\text{b}}=Q+K_{\text{b}}\;\rightarrow\;\boxed{K_{\text{b}}=-\frac{Qm_{\text{pr}}}{m_{\text{pr}}-m_{\text{b}}}}$$

88. (*a*) We assume a thin target, and use Eq. 42-3.

$$\sigma=\frac{R}{R_0 n\ell}=\frac{R}{R_0}\frac{1}{n\ell}=\left(1.6\times10^{-5}\right)\frac{1}{\left(5.9\times10^{28}\,\text{m}^{-3}\right)\left(4.0\times10^{-7}\,\text{m}\right)}\left(\frac{1\,\text{bn}}{10^{-28}\,\text{m}^{2}}\right)=\boxed{6.8\,\text{bn}}$$

(*b*) We assume that the cross section is the area presented by the gold nucleus.

$$\sigma=\pi r^{2}=\tfrac{1}{4}\pi d^{2}\;\rightarrow\;d=\sqrt{\frac{4\left(6.8\,\text{bn}\right)}{\pi}}=\left(\frac{1\,\text{bn}}{10^{-28}\,\text{m}^{2}}\right)=\boxed{2.9\times10^{-14}\,\text{m}}$$

CHAPTER 43: Elementary Particles

Responses to Questions

1. $p + n \rightarrow p + p + \pi^-$.

2. No. In the rest frame of the proton, this decay is energetically impossible, and so it is impossible in every other frame as well. In a frame in which the proton is moving very fast, the decay products must be moving very fast as well to conserve momentum. With this constraint, there will not be enough energy to make the decay possible.

3. “Antiatoms” would be made up of antiprotons, antineutrons, and positrons. If antimatter came into contact with matter, the corresponding pairs of particles would annihilate, producing energy in the form of gamma rays and other particles.

4. The photon signals the electromagnetic interaction.

5. (*a*) Yes, if a neutrino is produced during a decay, the weak interaction is responsible.
   (*b*) No, for example, a weak interaction decay could produce a Z^0 instead of a neutrino.

6. The neutron decay process also produces an electron and an antineutrino; these two particles only interact via the weak force. In addition, the strong force dominates at extremely small distances and at other distances the weak force dominates.

7. An electron takes part in electromagnetic, weak, and gravitational interactions. A neutrino takes part in weak and gravitational interactions. A proton takes part in all four interactions: strong, electromagnetic, weak, and gravitational.

8. All of the gauge bosons, leptons, and mesons have baryon number equal to zero and there are no baryons produced in the decays, so baryon number is conserved for these groups. The chart below shows charge conservation for a few examples of decays in these groups.

Particle name	Decay	Charge conservation check
W	$W^+ \rightarrow e^+ + \nu_e$	+1 = +1 + 0
muon	$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$	–1 = –1 + 0 + 0
tau	$\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$ $\tau^- \rightarrow e^- + \overline{\nu}_e + \nu_\tau$	–1 = –1 + 0 + 0 –1 = –1 + 0 + 0
pion	$\pi^+ \rightarrow \mu^+ + \nu_\mu$	+1 = +1 + 0
eta	$\eta^0 \rightarrow 3\pi^o$ $\eta^0 \rightarrow \pi^+ + \pi^- + \pi^0$	0 = 0 + 0 + 0 0 = +1 –1 + 0

The next chart shows charge conservation and baryon number conservation for a sample of the baryon decays shown in Table 43-2.

Particle name	Decay	Charge conservation check	Baryon number (*B*) conservation check
neutron	$n \rightarrow p + e^- + \overline{\nu}_e$	0 = +1 –1 + 0	+1 = +1 + 0 + 0

sigma	$\Sigma^+ \rightarrow p + \pi^o$ $\Sigma^+ \rightarrow n + \pi^+$	+1 = +1 + 0 +1 = 0 + 1	+1 = +1 + 0 +1 = +1 + 0

9. Decays via the electromagnetic interaction are indicated by the production of photons. In Table 43-2, the decays of π^0, Σ^0, and η^0 proceed via the electromagnetic interaction.

10. All of the decays listed in Table 43-2 in which a neutrino or antineutrino is one of the decay products occur via the weak interaction. These include the W, muon, tau, pion, kaon, and neutron. In addition, the Z particle decays via the weak interaction.

11. The Δ baryon has a baryon number of one and is therefore made of three quarks. The u, c, and t quarks have a charge of +2/3 *e* each and the d, s, and b quarks have a charge of -1/3 *e* each. There is no way to combine three quarks for a total charge of -2*e*.

12. As evidenced by their shorter lifetimes, the J/ψ and Y particles decay via the electromagnetic interaction.

13. Based on the lifetimes listed, all of the particles in Table 43-4, except the J/ψ and the Y, decay via the weak interaction.

14. Baryons are formed from three quarks or antiquarks, each of spin $\frac{1}{2}$ or $-\frac{1}{2}$, respectively. Any combination of quarks and antiquarks will yield a spin magnitude of either $\frac{1}{2}$ or $\frac{3}{2}$. Mesons are formed from two quarks or antiquarks. Any combination of two quarks or antiquarks will yield a spin magnitude of either 0 or 1.

15. The "neutrinolet" would not interact via the gravitational force (no mass), the strong force (no color charge), or the electromagnetic force (no electrical charge). In addition, it does not feel the weak force. However, it could possibly exist. The photon, for example, also has no rest mass, color charge, or electric charge and does not feel the weak force.

16. (*a*) No. Leptons are fundamental particles with no known internal structure. Baryons are made up of three quarks.
    (*b*) Yes. All baryons are hadrons.
    (*c*) No. A meson is a quark–antiquark pair.
    (*d*) No. Hadrons are made up of quarks and leptons are fundamental particles.

17. No. A particle made up of two quarks would have a particular color. Three quarks or a quark–antiquark pair are necessary for the particle to be white or colorless. A combination of two quarks and two antiquarks is possible, as the resulting particle could be white or colorless.

18. Inside the nucleus, a neutron will not decay because the dominant interaction is the strong interaction with the other nucleons. A free neutron will decay through the weak interaction.

19. The reaction is not possible, because it does not conserve lepton number. L = 1 on the left-hand side of the reaction equation, and L = –1 on the right-hand side of the reaction equation.

20. The reaction proceeds by the weak force. We know this because an electron anti-neutrino is produced in the reaction, which only happens in reactions governed by the weak interaction.

Solutions to Problems

$\boxed{1.}$ The total energy is given by Eq. 36-11a.

$$E = m_0c^2 + K = 0.938\,\text{GeV} + 4.65\text{GeV} = \boxed{5.59\,\text{GeV}}$$

2. Because the energy of the electrons is much greater than their rest mass, we have $K = E = pc$. Combine that with Eq. 43-1 for the de Broglie wavelength.

$$E = pc\ ;\ p = \frac{h}{\lambda} \ \rightarrow\ E = \frac{hc}{\lambda} \ \rightarrow\ \lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(28\times10^{9}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{4.4\times10^{-17}\,\text{m}}$$

3. The frequency is related to the magnetic field in Eq. 43-2.

$$f = \frac{qB}{2\pi m} \ \rightarrow\ B = \frac{2\pi mf}{q} = \frac{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)\left(3.1\times10^{7}\,\text{Hz}\right)}{1.60\times10^{-19}\,\text{C}} = \boxed{2.0\,\text{T}}$$

4. The time for one revolution is the period of revolution, which is the circumference of the orbit divided by the speed of the protons. Since the protons have very high energy, their speed is essentially the speed of light.

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.0\times10^{3}\,\text{m}\right)}{3.0\times10^{8}\,\text{m/s}} = \boxed{2.1\times10^{-5}\,\text{s}}$$

5. Use Eq. 43-2 to calculate the frequency. The alpha particle has a charge of +2*e* and a mass of 4 times the proton mass.

$$f = \frac{qB}{2\pi m} = \frac{2\left(1.60\times10^{-19}\,\text{C}\right)\left(1.7\,\text{T}\right)}{2\pi\left[4\left(1.67\times10^{-27}\,\text{kg}\right)\right]} = \boxed{1.3\times10^{7}\,\text{Hz}} = 13\,\text{MHz}$$

6. (*a*) The maximum kinetic energy is $K = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2$. Compared to Example 43-2, the charge has been doubled and the mass has been multiplied by 4. These two effects cancel each other in the equation, and so the maximum kinetic energy is unchanged. The kinetic energy from that example was 8.653 MeV.

$$K = \boxed{8.7\,\text{MeV}}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(8.653\,\text{MeV}\right)\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{4\left(1.66\times10^{-27}\,\text{kg}\right)}} = 2.042\times10^{7}\,\text{m/s} \approx \boxed{2.0\times10^{7}\,\text{m/s}}$$

(*b*) The maximum kinetic energy is $K = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2$. Compared to Example 43-2, the charge is unchanged and the mass has been multiplied by 2. Thus the kinetic energy will be half of what it was in Example 43-2 (8.653 MeV).

$$K = \boxed{4.3\,\text{MeV}}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left[\frac{1}{2}\left(8.653\,\text{MeV}\right)\right]\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{2\left(1.66\times10^{-27}\,\text{kg}\right)}} = 2.042\times10^{7}\,\text{m/s} \approx \boxed{2.0\times10^{7}\,\text{m/s}}$$

The alpha and the deuteron have the same charge to mass ratio, and so move at the same speed.

(*c*) The frequency is given by $f=\dfrac{qB}{2\pi m}$. Since the charge to mass ratio of both the alpha and the deuteron is half that of the proton, the frequency for the alpha and the deuteron will both be half the frequency found in Example 43-2 for the proton.

$$\boxed{f=13\,\mathrm{MHz}}$$

7. From Eq. 41-1, the diameter of a nucleon is about $d_{\text{nucleon}}=2.4\times10^{-15}\,\mathrm{m}$. The 25-MeV alpha particles and protons are not relativistic, so their momentum is given by $p=mv=\sqrt{2mK}$. The wavelength is given by Eq. 43-1, $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2mK}}$.

$$\lambda_\alpha=\frac{h}{\sqrt{2m_\alpha \mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{\sqrt{2(4)\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(25\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=2.88\times10^{-15}\,\mathrm{m}$$

$$\lambda_{\mathrm{p}}=\frac{h}{\sqrt{2m_{\mathrm{p}} \mathrm{KE}}}=\frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{\sqrt{2\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(25\times10^{6}\,\mathrm{eV}\right)\left(1.6\times10^{-19}\,\mathrm{J/eV}\right)}}=5.75\times10^{-15}$$

We see that $\boxed{\lambda_\alpha\approx d_{\text{nucleon}}}$ and $\boxed{\lambda_{\mathrm{p}}\approx 2d_{\text{nucleon}}}$. Thus the $\boxed{\text{alpha particle will be better}}$ for picking out details in the nucleus.

8. Because the energy of the protons is much greater than their rest mass, we have $K=E=pc$. Combine this with the expression (given above Example 43-2) relating the momentum and radius of curvature for a particle in a magnetic field.

$$v=\frac{qBr}{m}\;\rightarrow\; mv=qBr\;\rightarrow\; p=qBr\;\rightarrow\;\frac{E}{c}=qBr\;\rightarrow$$

$$B=\frac{E}{qrc}=\frac{\left(1.0\times10^{12}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(1.0\times10^{3}\,\mathrm{m}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}=\boxed{3.3\,\mathrm{T}}$$

9. Because the energy of the protons is much greater than their rest mass, we have $K=E=pc$. A relationship for the magnetic field is given right before Eq. 43-2.

$$v=\frac{qBr}{m}\;\rightarrow\; mv=qBr\;\rightarrow\; p=qBr\;\rightarrow\;\frac{E}{c}=qBr\;\rightarrow$$

$$B=\frac{E}{qrc}=\frac{\left(7.0\times10^{15}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(4.25\times10^{3}\,\mathrm{m}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)}=\boxed{5.5\,\mathrm{T}}$$

10. (*a*) The magnetic field is found from the maximum kinetic energy as derived in Example 43-2.

$$K=\frac{q^2B^2R^2}{2m}\;\rightarrow\; B=\frac{\sqrt{2mK}}{qR}\;\rightarrow$$

$$B=\frac{\sqrt{2(2.014)\left(1.66\times10^{-27}\,\mathrm{kg}\right)\left(12\times10^{6}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(1.0\,\mathrm{m}\right)}=0.7082\,\mathrm{T}\approx\boxed{0.71\,\mathrm{T}}$$

(*b*) The cyclotron frequency is given by Eq. 43-2.

$$f=\frac{qB}{2\pi m}=\frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(0.7082\,\mathrm{T}\right)}{2\pi\left(2.014\right)\left(1.66\times10^{-27}\,\mathrm{kg}\right)}=5.394\times10^{6}\,\mathrm{Hz}\approx\boxed{5.4\,\mathrm{MHz}}$$

(*c*) The deuteron will be accelerated twice per revolution, and so will gain energy equal to twice its charge times the voltage on each revolution.

$$\text{number of revolutions}=n=\frac{12\times10^{6}\,\mathrm{eV}}{2\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(22\times10^{3}\,\mathrm{V}\right)}\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)$$

$$=273\text{ revolutions}\approx\boxed{270\text{ revolutions}}$$

(*d*) The time is the number of revolutions divided by the frequency (which is revolutions per second).

$$\Delta t=\frac{n}{f}=\frac{273\,\text{revolutions}}{5.394\times10^{6}\,\mathrm{rev/s}}=\boxed{5.1\times10^{-6}\,\mathrm{s}}=51\,\mu\mathrm{s}$$

(*e*) If we use an average radius of half the radius of the cyclotron, then the distance traveled is the average circumference times the number of revolutions.

$$\text{distance}=\tfrac{1}{2}2\pi rn=\pi\left(1.0\,\mathrm{m}\right)\left(273\right)=\boxed{860\,\mathrm{m}}$$

11. Because the energy of the protons is much greater than their rest mass, we have $K=E=pc$. Combine that with Eq. 43-1 for the de Broglie wavelength. That is the minimum size that protons of that energy could resolve.

$$E=pc\ ;\ p=\frac{h}{\lambda}\ \rightarrow\ E=\frac{hc}{\lambda}\ \rightarrow\ \lambda=\frac{hc}{E}=\frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{\left(7.0\times10^{12}\,\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}=\boxed{1.8\times10^{-19}\,\mathrm{m}}$$

12. If the speed of the protons is *c*, then the time for one revolution is found from uniform circular motion. The number of revolutions is the total time divided by the time for one revolution. The energy per revolution is the total energy gained divided by the number of revolutions.

$$v=\frac{2\pi r}{T}\ \rightarrow\ T=\frac{2\pi r}{v}=\frac{2\pi r}{c}\qquad n=\frac{t}{T}=\frac{ct}{2\pi r}$$

$$\text{Energy/revolution}=\frac{\Delta E}{n}=\frac{\left(\Delta E\right)2\pi r}{ct}=\frac{\left(1.0\times10^{6}\,\mathrm{MeV}-150\times10^{3}\,\mathrm{MeV}\right)2\pi\left(1.0\times10^{3}\,\mathrm{m}\right)}{\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(20\,\mathrm{s}\right)}$$

$$=0.89\,\mathrm{MeV/rev}\approx\boxed{0.9\,\mathrm{MeV/rev}}$$

$\boxed{13.}$ Start with an expression from Section 42-1, relating the momentum and radius of curvature for a particle in a magnetic field, with *q* replaced by *e*.

$$v=\frac{eBr}{m}\ \rightarrow\ mv=eBr\ \rightarrow\ p=eBr$$

In the relativistic limit, $p=E/c$ and so $\frac{E}{c}=eBr$. To put the energy in electron volts, divide the energy by the charge of the object.

$$\frac{E}{c}=eBr\ \rightarrow\ \boxed{\frac{E}{e}=Brc}$$

14. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\Lambda^0}c^2 - m_n c^2 - m_{\pi^0}c^2 = 1115.7\,\text{MeV} - 939.6\,\text{MeV} - 135.0\,\text{MeV} = \boxed{41.1\,\text{MeV}}$$

15. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\pi^+}c^2 - m_{\mu^+}c^2 - m_{\nu_\mu}c^2 = 139.6\,\text{MeV} - 105.7\,\text{MeV} - 0 = \boxed{33.9\,\text{MeV}}$$

16. Use Eq. 43-3 to estimate the range of the force based on the mass of the mediating particle.

$$mc^2 \approx \frac{hc}{2\pi d} \quad \to \quad d \approx \frac{hc}{2\pi mc^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{2\pi\left(497.7\times10^{6}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} \approx \boxed{3.98\times10^{-16}\text{m}}$$

17. The energy required is the mass energy of the two particles.

$$E = 2m_n c^2 = 2\left(939.6\,\text{MeV}\right) = \boxed{1879.2\,\text{MeV}}$$

18. The reaction is multi-step, and can be written as shown here: $\Sigma^0 \to \Lambda^0 + \gamma$
The energy released is the initial rest energy minus the $\qquad \downarrow\!\to p + \pi^-$
final rest energy of the proton and pion, using Table 43-2.

$$\Delta E = \left(m_{\Sigma^0} - m_{\text{p}} - m_{\pi^-}\right)c^2 = 1192.6\,\text{Mev} - 938.3\,\text{MeV} - 139.6\,\text{MeV} = \boxed{114.7\,\text{MeV}}$$

$\boxed{19.}$ Because the two protons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all three particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the π^0. Each proton will have half of that kinetic energy. From Table 43-2, the mass of the π^0 is $135.0\,\text{MeV}/c^2$.

$$2K_{\text{proton}} = m_{\pi^0}c^2 = 135.0\,\text{MeV} \quad \to \quad K_{\text{proton}} = \boxed{67.5\,\text{MeV}}$$

20. Because the two neutrons are heading towards each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all four particles at rest, and so the minimum kinetic energy of the collision must be equal to the mass energy of the K^+K^- pair. Each neutron will have half of that kinetic energy. From Table 43-2, the mass of each of the K^+ and the K^- is $493.7\,\text{MeV}/c^2$.

$$2K_{\text{neutron}} = 2m_{\text{K}}c^2 \quad \to \quad K_{\text{neutron}} = m_{\text{K}}c^2 = \boxed{493.7\,\text{MeV}}$$

21. We treat the neutrino as massless, but it still has momentum and energy. We use conservation of momentum and conservation of energy, along with Eqs. 36-11 and 36-13.

(*a*) To find the maximum kinetic energy of the positron, we assume that the pion has no kinetic energy, and so the magnitude of the momenta of the positron and the neutrino are the same.

$$p_{\text{e}} = p_\nu \ ;$$

$$m_{\text{K}}c^2 = m_\pi c^2 + E_{\text{e}} + E_\nu \quad \to \quad \left(m_{\text{K}}c^2 - m_\pi c^2 - E_{\text{e}}\right)^2 = E_\nu^2 = p_\nu^2 c^2 = p_{\text{e}}^2 c^2 = E_{\text{e}}^2 - m_{\text{e}}^2 c^4 \quad \to$$

$$E_{\text{e}} = \frac{\left(m_{\text{K}}c^2 - m_\pi c^2\right)^2 + m_{\text{e}}^2 c^4}{2\left(m_{\text{K}}c^2 - m_\pi c^2\right)} = K_{\text{e}} + m_{\text{e}}c^2 \quad \to$$

$$K_e = \frac{\left(m_K c^2 - m_\pi c^2\right)^2 + m_e^2 c^4}{2\left(m_K c^2 - m_\pi c^2\right)} - m_e c^2$$

$$= \frac{\left(497.7\,\text{MeV} - 139.6\,\text{MeV}\right)^2 + \left(0.511\,\text{MeV}\right)^2}{2\left(497.7\,\text{MeV} - 139.6\,\text{MeV}\right)} - \left(0.511\,\text{MeV}\right) = \boxed{178.5\,\text{MeV}}$$

(*b*) To find the maximum kinetic energy of the pion, we assume that the positron has no kinetic energy, and so the magnitude of the momenta of the positron and the neutrino are the same. The result is found by just interchanging the pion and positron.

$$K_e = \frac{\left(m_K c^2 - m_e c^2\right)^2 + m_\pi^2 c^4}{2\left(m_K c^2 - m_e c^2\right)} - m_\pi c^2$$

$$= \frac{\left(497.7\,\text{MeV} - 0.511\,\text{MeV}\right)^2 + \left(139.6\,\text{MeV}\right)^2}{2\left(497.7\,\text{MeV} - 0.511\,\text{MeV}\right)} - \left(139.6\,\text{MeV}\right) = \boxed{128.6\,\text{MeV}}$$

22. The energy of the two photons (assumed to be equal so that momentum is conserved) must be the combined rest mass energy of the proton and antiproton.

$$2m_0c^2 = 2hf = 2h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{m_0c^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{\left(938.3\times10^6\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{1.32\times10^{-15}\,\text{m}}$$

23. (*a*) $\Lambda^0 \rightarrow \text{n} + \pi^-$ Charge conservation is violated, since $0 \neq 0 - 1$
Strangeness is violated, since $-1 \neq 0 + 0$

(*b*) $\Lambda^0 \rightarrow \text{p} + \text{K}^-$ Energy conservation is violated, since
$1115.7\,\text{MeV}/c^2 < 938.3\,\text{MeV}/c^2 + 493.7\,\text{MeV}/c^2 = 1432.0\,\text{MeV}/c^2$

(*c*) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number conservation is violated, since $1 \neq 0 + 0$
Strangeness is violated, since $-1 \neq 0 + 0$
Spin is violated, since $\frac{1}{2} \neq 0 + 0$

24. (*a*) The *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\Lambda^0}c^2 - \left(m_p c^2 + m_{\pi^-}c^2\right) = 1115.7\,\text{MeV} - \left(938.3\,\text{MeV} + 139.6\,\text{MeV}\right) = \boxed{37.8\,\text{MeV}}$$

(*b*) Energy conservation for the decay gives the following.

$$m_{\Lambda^0}c^2 = E_p + E_{\pi^-} \rightarrow E_{\pi^-} = m_{\Lambda^0}c^2 - E_p$$

Momentum conservation says that the magnitudes of the momenta of the two products are equal. Then convert that relationship to energy using $E^2 = p^2c^2 + m_0^2c^4$, with energy conservation.

$$p_p = p_{\pi^-} \rightarrow \left(p_p c\right)^2 = \left(p_{\pi^-}c\right)^2 \rightarrow$$

$$E_p^2 - m_p^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Lambda^0}c^2 - E_p\right)^2 - m_{\pi^-}^2c^4$$

$$E_p^2 - m_p^2c^4 = \left(m_{\Lambda^0}^2c^4 - 2E_p m_{\Lambda^0}c^2 + E_p^2\right) - m_{\pi^-}^2c^4 \rightarrow$$

$$E_p = \frac{m_{\Lambda^0}^2c^4 + m_p^2c^4 - m_{\pi^-}^2c^4}{2m_{\Lambda^0}c^2} = \frac{\left(1115.7\,\text{MeV}\right)^2 + \left(938.3\,\text{MeV}\right)^2 - \left(139.6\,\text{MeV}\right)^2}{2\left(1115.7\,\text{MeV}\right)} = 943.7\,\text{MeV}$$

$$E_{\pi^-} = m_{\Lambda^0}c^2 - E_{\rm p} = 1115.7\,\text{MeV} - 943.7\,\text{MeV} = 172.0\,\text{MeV}$$

$$K_{\rm p} = E_{\rm p} - m_{\rm p}c^2 = 943.7\,\text{MeV} - 938.3\,\text{MeV} = \boxed{5.4\,\text{MeV}}$$

$$K_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 172.0\,\text{MeV} - 139.6\,\text{MeV} = \boxed{32.4\,\text{MeV}}$$

$\boxed{25.}$ (*a*) We work in the rest frame of the isolated electron, so that it is initially at rest. Energy conservation gives the following.

$$m_{\rm e}c^2 = K_{\rm e} + m_{\rm e}c^2 + E_\gamma \;\rightarrow\; K_{\rm e} = -E_\gamma \;\rightarrow\; K_{\rm e} = E_\gamma = 0$$

Since the photon has no energy, it does not exist, and so has not been emitted.

(*b*) For the photon exchange in Figure 43-8, the photon exists for such a short time that the uncertainty principle allows energy to not be conserved during the exchange.

26. The total momentum of the electron and positron is 0, and so the total momentum of the two photons must be 0. Thus each photon has the same momentum, and so each photon also has the same energy. The total energy of the photons must be the total energy of the electron / positron pair.

$$E_{\rm e^+/e^-\,pair} = E_{\rm photons} \;\rightarrow\; 2\left(m_0c^2 + \text{KE}\right) = 2hf = 2h\frac{c}{\lambda} \;\rightarrow$$

$$\lambda = \frac{hc}{m_0c^2 + \text{KE}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(0.511\times10^{6}\,\text{eV} + 420\times10^{3}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.335\times10^{-12}\,\text{m} \approx \boxed{1.3\times10^{-12}\,\text{m}}$$

27. Since the pion decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the magnitudes of the momenta of the positron and the neutrino are equal. We also treat the neutrino as massless. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\pi^+}c^2 = E_{\rm e^+} + E_\nu \;;\; p_{\rm e^+} = p_\nu \;\rightarrow\; \left(p_{\rm e^+}^2c^2\right) = \left(p_\nu^2c^2\right) \;\rightarrow\; E_{\rm e^+}^2 - m_{\rm e^+}^2c^4 = E_\nu^2$$

$$E_{\rm e^+}^2 - m_{\rm e^+}^2c^4 = \left(m_{\pi^+}c^2 - E_{\rm e^+}\right)^2 = m_{\pi^+}^2c^4 - 2E_{\rm e^+}m_{\pi^+}c^2 + E_{\rm e^+}^2 \;\rightarrow\; 2E_{\rm e^+}m_{\pi^+}c^2 = m_{\pi^+}^2c^4 + m_{\rm e^+}^2c^4$$

$$E_{\rm e^+} = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{\rm e^+}^2c^2}{2m_{\pi^+}} \;\rightarrow\; K_{\rm e^+} + m_{\rm e^+}c^2 = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{\rm e^+}^2c^2}{2m_{\pi^+}} \;\rightarrow$$

$$K_{\rm e^+} = \tfrac{1}{2}m_{\pi^+}c^2 - m_{\rm e^+}c^2 + \frac{m_{\rm e^+}^2c^2}{2m_{\pi^+}} = \tfrac{1}{2}\left(139.6\,\text{MeV}\right) - 0.511\,\text{MeV} + \frac{\left(0.511\,\text{MeV}/c^2\right)\left(0.511\,\text{MeV}\right)}{2\left(139.6\,\text{MeV}/c^2\right)}$$

$$= \boxed{69.3\,\text{MeV}}$$

28. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{n} + \eta^0$, the conservation laws are as follows.

Charge: $-1+1=0+0$	Charge is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=0+0$	Strangeness is conserved.

$\boxed{\text{The reaction is possible.}}$

(*b*) For the reaction $\pi^+ + \text{p} \rightarrow \text{n} + \pi^0$, the conservation laws are as follows.

Charge: $1+1\neq0+0$	Charge is NOT conserved.

$\boxed{\text{The reaction is forbidden, because charge is not conserved.}}$

(*c*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{e}^+$, the conservation laws are as follows.

Charge: $1+1=1+1$ Charge is conserved.
Baryon number: $0+1=1+0$ Baryon number is conserved.
Lepton number: $0+0\neq 0+1$ Lepton number is NOT conserved.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*d*) For the reaction $\mathrm{p} \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Charge: $1=1+0$ Charge is conserved.
Baryon number: $1\neq 0+0$ Baryon number NOT conserved.
Mass energy is fine, because $m_\mathrm{p} > m_\mathrm{e} + m_\nu$.

$\boxed{\text{The reaction is forbidden, because baryon number is not conserved.}}$

(*e*) For the reaction $\mu^+ \rightarrow \mathrm{e}^+ + \bar{\nu}_\mu$, the conservation laws are as follows.

Charge: $1=1+0$ Charge is conserved.
Baryon number: $0=0+0$ Baryon number is conserved.
Electron lepton number: $0\neq -1+0$ Lepton number is NOT conserved.
Mass energy is fine, because $m_\mu > m_\mathrm{e} + m_\nu$.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*f*) For the reaction $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Mass energy: $938.3\,\mathrm{MeV}/c^2 < 939.6\,\mathrm{MeV}/c^2 + 0.511\,\mathrm{MeV}/c^2$
Mass energy is NOT conserved.

$\boxed{\text{The reaction is forbidden, because energy is not conserved.}}$

29. Since the Ξ^- decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, and so the momenta of the Λ^0 and π^- are equal in magnitude. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\Xi^-}c^2 = E_{\Lambda^0} + E_{\pi^-} \rightarrow E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0}$$

$$p_{\Lambda^0} = p_{\pi^-} \rightarrow \left(p_{\Lambda^0}c\right)^2 = \left(p_{\pi^-}c\right)^2 \rightarrow$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(m_{\Xi^-}c^2 - E_{\Lambda^0}\right)^2 - m_{\pi^-}^2c^4$$

$$E_{\Lambda^0}^2 - m_{\Lambda^0}^2c^4 = \left(m_{\Xi^-}^2c^4 - 2E_{\Lambda^0}m_{\Xi^-}c^2 + E_{\Lambda^0}^2\right) - m_{\pi^-}^2c^4 \rightarrow$$

$$E_{\Lambda^0} = \frac{m_{\Xi^-}^2c^4 + m_{\Lambda^0}^2c^4 - m_{\pi^-}^2c^4}{2m_{\Xi^-}c^2} = \frac{(1321.3\,\mathrm{MeV})^2 + (1115.7\,\mathrm{MeV})^2 - (139.6\,\mathrm{MeV})^2}{2(1321.3\,\mathrm{MeV})} = 1124.3\,\mathrm{MeV}$$

$$E_{\pi^-} = m_{\Xi^-}c^2 - E_{\Lambda^0} = 1321.3\,\mathrm{MeV} - 1124.3\,\mathrm{MeV} = 197.0\,\mathrm{MeV}$$

$$K_{\Lambda^0} = E_{\Lambda^0} - m_{\Lambda^0}c^2 = 1124.3\,\mathrm{MeV} - 1115.7\,\mathrm{MeV} = \boxed{8.6\,\mathrm{MeV}}$$

$$K_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 197.0\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{57.4\,\mathrm{MeV}}$$

30. $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \overline{\mathrm{p}}$: This reaction will not happen because charge is not conserved $(2\neq 0)$, and baryon number is not conserved $(2\neq 0)$.

$p + p \rightarrow p + p + \overline{p}$: This reaction will not happen because charge is not conserved $(2 \neq 1)$, and baryon number is not conserved $(2 \neq 1)$.

$p + p \rightarrow p + p + p + \overline{p}$: This reaction is possible. All conservation laws are satisfied.

$p + p \rightarrow p + e^+ + e^+ + \overline{p}$: This reaction will not happen. Baryon number is not conserved $(2 \neq 0)$, and lepton number is not conserved $(0 \neq -2)$.

31. The two neutrinos must move together, in the opposite direction of the electron, in order for the electron to have the maximum kinetic energy, and thus the total momentum of the neutrinos will be equal in magnitude to the momentum of the electron. Since a neutrino is (essentially) massless, we have $E_\nu = p_\nu c$. We assume that the muon is at rest when it decays. Use conservation of energy and momentum, along with their relativistic relationship.

$$p_{e^-} = p_{\overline{\nu}_e} + p_{\nu_\mu}$$

$$m_{\mu^-}c^2 = E_{e^-} + E_{\overline{\nu}_e} + E_{\nu_\mu} = E_{e^-} + p_{\overline{\nu}_e}c + p_{\nu_\mu}c = E_{e^-} + \left(p_{\overline{\nu}_e} + p_{\nu_\mu}\right)c = E_{e^-} + p_{e^-}c \rightarrow$$

$$m_{\mu^-}c^2 - E_{e^-} = p_{e^-}c \rightarrow \left(m_{\mu^-}c^2 - E_{e^-}\right)^2 = \left(p_{e^-}c\right)^2 = E_{e^-}^2 - m_{e^-}^2c^4 \rightarrow$$

$$m_{\mu^-}^2c^4 - 2m_{\mu^-}c^2E_{e^-} + E_{e^-}^2 = E_{e^-}^2 - m_{e^-}^2c^4 \rightarrow E_{e^-} = \frac{m_{\mu^-}^2c^4 + m_{e^-}^2c^4}{2m_{\mu^-}c^2} = K_{e^-} + m_{e^-}c^2 \rightarrow$$

$$K_{e^-} = \frac{m_{\mu^-}^2c^4 + m_{e^-}^2c^4}{2m_{\mu^-}c^2} - m_{e^-}c^2 = \frac{(105.7\,\text{MeV})^2 + (0.511\,\text{MeV})^2}{2(105.7\,\text{MeV})} - (0.511\,\text{MeV}) = \boxed{52.3\,\text{MeV}}$$

32. A π^+ could NOT be produced by $p + p \rightarrow p + n + \pi^+$. The pion has a mass energy of 139.6 MeV, and so the extra 100 MeV of energy could not create it. The *Q*-value for the reaction is $Q = 2m_pc^2 - \left(2m_pc^2 + m_{\pi^+}c^2\right) = -139.6\,\text{MeV}$, and so more than 139.6 MeV of kinetic energy is needed. The minimum initial kinetic energy would produce the particles all moving together at the same speed, having the same total momentum as the incoming proton. We consider the products to be one mass $M = m_p + m_n + m_{\pi^+}$ since they all move together with the velocity. We use energy and momentum conservation, along with their relativistic relationship, $E^2 = p^2c^2 + m_0^2c^4$.

$$E_p + m_pc^2 = E_M \quad ; \quad p_p = p_M \rightarrow \left(p_pc\right)^2 = \left(p_Mc\right)^2 \rightarrow E_p^2 - m_{\pi^-}^2c^4 = E_M^2 - M^2c^4 \rightarrow$$

$$E_p^2 - m_p^2c^4 = \left(E_p + m_pc^2\right)^2 - M^2c^4 = E_p^2 + 2E_pm_pc^2 + m_p^2c^4 - M^2c^4 \rightarrow$$

$$E_p = \frac{M^2c^4 - 2m_p^2c^4}{2m_pc^2} = K_p + m_pc^2 \rightarrow$$

$$K_p = \frac{M^2c^4 - 2m_p^2c^4}{2m_pc^2} - m_pc^2 = \frac{M^2c^4}{2m_pc^2} - 2m_pc^2$$

$$= \frac{(938.3\,\text{MeV} + 939.6\,\text{MeV} + 139.6\,\text{MeV})^2}{2(938.3\,\text{MeV})} - 2(938.3\,\text{MeV}) = \boxed{292.4\,\text{MeV}}$$

33. We use the uncertainty principle to estimate the uncertainty in rest energy.

$$\Delta E \approx \frac{h}{2\pi\Delta t} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)}{2\pi\left(7\times10^{-20}\,\mathrm{s}\right)\left(1.60\times10^{-16}\,\mathrm{J/eV}\right)} = 9420\,\mathrm{eV} \approx \boxed{9\,\mathrm{keV}}$$

34. We estimate the lifetime from the energy width and the uncertainty principle.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \quad\rightarrow\quad \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{2\pi\left(300\times10^{3}\,\mathrm{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{2\times10^{-21}\,\mathrm{s}}$$

Apply the uncertainty principle, which says that $\Delta E \approx \dfrac{h}{2\pi\Delta t}$.

$$\Delta E \approx \frac{h}{2\pi\Delta t} \quad\rightarrow\quad \Delta t \approx \frac{h}{2\pi\Delta E} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{2\pi\left(88\times10^{3}\,\mathrm{eV}\right)\left(\dfrac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)} = \boxed{7.5\times10^{-21}\,\mathrm{s}}$$

36. (*a*) For $\mathrm{B}^{-} = \mathrm{b}\,\overline{\mathrm{u}}$, we have

Charge:	$-1 = -\frac{1}{3} - \frac{2}{3}$	Spin:	$0 = \frac{1}{2} - \frac{1}{2}$
Baryon number:	$0 = \frac{1}{3} - \frac{1}{3}$	Strangeness:	$0 = 0 + 0$
Charm:	$0 = 0 + 0$	Bottomness:	$-1 = -1 + 0$
Topness:	$0 = 0 + 0$		

(*b*) Because B^{+} is the antiparticle of B^{-}, $\boxed{\mathrm{B}^{+} = \overline{\mathrm{b}}\,\mathrm{u}}$. The B^{0} still must have a bottom quark, but must be neutral. Therefore $\boxed{\mathrm{B}^{0} = \mathrm{b}\,\overline{\mathrm{d}}}$. Because $\overline{\mathrm{B}}^{0}$ is the antiparticle to B^{0}, we must have $\boxed{\overline{\mathrm{B}}^{0} = \overline{\mathrm{b}}\,\mathrm{d}}$.

37. We find the energy width from the lifetime in Table 42-2 and the uncertainty principle.

(*a*) $\Delta t = 10^{-18}\,\mathrm{s}$ $$\Delta E \approx \frac{h}{2\pi\Delta t} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{2\pi\left(10^{-18}\,\mathrm{s}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = 659\,\mathrm{eV} \approx \boxed{700\,\mathrm{eV}}$$

(*b*) $\Delta t = 10^{-23}\,\mathrm{s}$ $$\Delta E \approx \frac{h}{2\pi\Delta t} = \frac{6.63\times10^{-34}\,\mathrm{J{\bullet}s}}{2\pi\left(10^{-23}\,\mathrm{s}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = 6.59\times10^{7}\,\mathrm{eV} \approx \boxed{70\,\mathrm{MeV}}$$

38. (*a*)

Charge:	$(0) = (+1) + (-1)$	Charge is conserved.
Baryon number:	$(+1) = (+1) + (0)$	Baryon number is conserved.
Lepton number:	$(0) = (0) + (0)$	Lepton number is conserved.
Strangeness:	$(-2) \neq (-1) + (0)$	Strangeness is NOT conserved.
Energy:	$1314.9\,\mathrm{Mev}/c^{2} > 1189.4\,\mathrm{Mev}/c^{2} + 139.6\,\mathrm{Mev}/c^{2} = 1329\,\mathrm{Mev}/c^{2}$	Energy is NOT conserved.

$\boxed{\text{The decay is not possible, because energy is not conserved.}}$

(*b*) Charge: $(-1)=(0)+(-1)+(0)$ Charge is conserved.

Baryon number: $(+1)=(+1)+(0)+(0)$ Baryon number is conserved.

Lepton number: $(0)=(0)+(0)+(1)$ Lepton number is NOT conserved.

Strangeness: $(-3)\neq(-1)+(0)+(0)$ Strangeness is NOT conserved.

Energy: $1672.5\,\text{Mev}/c^2 > 1192.6\,\text{Mev}/c^2 + 139.6\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1332.2\,\text{Mev}/c^2$

Energy is conserved.

$\boxed{\text{The decay is not possible, because lepton number is not conserved.}}$

(*c*) Charge: $(0)=(0)+(0)+(0)$ Charge is conserved.

Baryon number: $(0)=(0)+(0)+(0)$ Baryon number is conserved.

Lepton number: $(0)=(0)+(0)+(0)$ Lepton number is conserved.

Strangeness: $(-1)=(-1)+(0)+(0)$ Strangeness is conserved.

Energy: $1192.6\,\text{Mev}/c^2 > 1115.7\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 + 0\,\text{Mev}/c^2 = 1115.7\,\text{Mev}/c^2$

Energy is conserved.

$\boxed{\text{The decay is possible.}}$

39. (*a*) The Ξ^0 has a strangeness of –2, and so must contain two strange quarks. In order to make a neutral particle, the third quark must be an up quark. So $\boxed{\Xi^0 = \text{u s s}}$.

(*b*) The Ξ^- has a strangeness of –2, and so must contain two strange quarks. In order to make a particle with a total charge of –1, the third quark must be a down quark. So $\boxed{\Xi^- = \text{d s s}}$.

40. (*a*) The neutron has a baryon number of 1, so there must be three quarks. The charge must be 0, as must be the strangeness, the charm, the bottomness, and the topness. Thus $\boxed{n = \text{u d d}}$.

(*b*) The antineutron is the anti particle of the neutron, and so $\boxed{\overline{n} = \overline{\text{u}}\,\overline{\text{d}}\,\overline{\text{d}}}$.

(*c*) The Λ^0 has a strangeness of -1, so it must contain an "s" quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\Lambda^0 = \text{u d s}}$.

(*d*) The $\overline{\Sigma}^0$ has a strangeness of +1, so it must contain an $\overline{\text{s}}$ quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\overline{\Sigma}^0 = \overline{\text{u}}\,\overline{\text{d}}\,\overline{\text{s}}}$.

41. (*a*) The combination u u d has charge = +1, baryon number = +1, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\text{u u d} = \text{p}}$.

(*b*) The combination $\overline{\text{u}}\,\overline{\text{u}}\,\overline{\text{s}}$ has charge = -1, baryon number = -1, strangeness = +1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\text{u}}\,\overline{\text{u}}\,\overline{\text{s}} = \overline{\Sigma}^-}$.

(*c*) The combination $\overline{\text{u}}\,\text{s}$ has charge = -1, baryon number = 0, strangeness = -1, and charm, bottomness, and topness all equal to 0. Thus $\boxed{\overline{\text{u}}\,\text{s} = \text{K}^-}$.

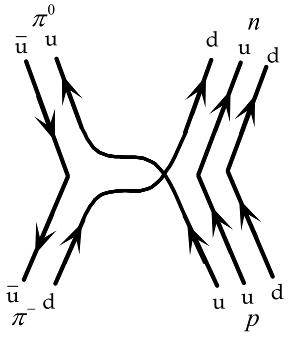
(*d*) The combination $\mathrm{d\,\overline{u}}$ has charge = -1, baryon number = 0, and strangeness, charm, bottomness, and topness all equal to 0. Thus $\boxed{\mathrm{d\,\overline{u}} = \pi^-}$

(*e*) The combination $\mathrm{\overline{c}\,s}$ has charge = -1, baryon number = 0, strangeness = -1, charm = -1, and bottomness and topness of 0. Thus $\boxed{\mathrm{\overline{c}\,s} = \mathrm{D}_S^-}$

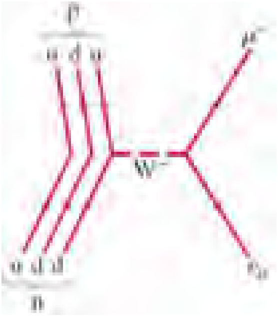
42. To form the D^0 meson, we must have a total charge of 0, a baryon number of 0, a strangeness of 0, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a “c” quark, with a charge of $+\frac{2}{3}e$. To have a neutral meson, there must be another quark with a charge of $-\frac{2}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. The only candidate with those properties is an anti-up quark. Thus $\boxed{D^0 = \mathrm{c\,\overline{u}}}$.

$\boxed{43.}$ To form the D_S^+ meson, we must have a total charge of +1, a baryon number of 0, a strangeness of +1, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a “c” quark, with a charge of $+\frac{2}{3}e$. To have a total charge of +1, there must be another quark with a charge of $+\frac{1}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. To have a strangeness of +1, the other quark must be an anti-strange. Thus $\boxed{D_S^+ = \mathrm{c\,\overline{s}}}$.

44. Here is a Feynman diagram for the reaction $\pi^- + \mathrm{p} \rightarrow \pi^0 + \mathrm{n}$.



45. Since leptons are involved, the reaction $\mathrm{n} + \nu_\mu \rightarrow \mathrm{p} + \mu^-$ is a weak interaction. Since there is a charge change in the lepton, a W boson must be involved in the interaction. If we consider the neutron as having emitted the boson, then it is a W^-, which interacts with the neutrino. If we consider the neutrino as having emitted the boson, then it is a W^+, which interacts with the neutron.



46. To find the length in the lab, we need to know the speed of the particle which is moving relativistically. Start with Eq. 36-10a.

$$K = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \rightarrow v = c\sqrt{1-\frac{1}{\left(\frac{K}{m_0c^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{950\,\mathrm{MeV}}{1777\,\mathrm{MeV}}+1\right)^2}} = 0.7585c$$

$$\Delta t_{\text{lab}}=\frac{\Delta t_0}{\sqrt{1-v^2/c^2}}=\frac{2.91\times10^{-13}\,\text{s}}{\sqrt{1-(0.7585)^2}}=4.465\times10^{-13}\,\text{s}$$

$$\Delta x_{\text{lab}}=v\Delta t_{\text{lab}}=(0.7585)\left(3.00\times10^{8}\,\text{m/s}\right)\left(4.465\times10^{-13}\,\text{s}\right)=\boxed{1.02\times10^{-4}\,\text{m}}$$

47. (*a*) At an energy of 1.0 TeV, the protons are moving at practically the speed of light. From uniform circular motion we find the time for the protons to complete one revolution of the ring. Then the total charge that passes any point in the ring during that time is the charge of the entire group of stored protons. The current is then the total charge divided by the period.

$$v=\frac{2\pi R}{T}\ \rightarrow\ T=\frac{2\pi R}{v}=\frac{2\pi R}{c}$$

$$I=\frac{Ne}{T}=\frac{Nec}{2\pi R}=\frac{\left(5.0\times10^{13}\,\text{protons}\right)\left(1.60\times10^{-19}\,\text{C/proton}\right)\left(3.0\times10^{8}\,\text{m/s}\right)}{2\pi\left(1.0\times10^{3}\,\text{m}\right)}=\boxed{0.38\,\text{A}}$$

(*b*) The 1.0 TeV is equal to the kinetic energy of the proton beam.

$$K_{\text{beam}}=K_{\text{car}}\ \rightarrow\ K_{\text{beam}}=\tfrac{1}{2}mv^2\ \rightarrow$$

$$v=\sqrt{\frac{2K_{\text{beam}}}{m}}=\sqrt{\frac{2\left(1.0\times10^{12}\,\text{eV/proton}\right)\left(5.0\times10^{13}\,\text{protons}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{1500\,\text{kg}}}=103\,\text{m/s}$$

$$\approx\boxed{1.0\times10^{2}\,\text{m/s}}$$

48. By assuming that the kinetic energy is approximately 0, the total energy released is the rest mass energy of the annihilating pair of particles.

(*a*) $E_{\text{total}}=2m_0c^2=2(0.511\,\text{MeV})=\boxed{1.022\,\text{MeV}}$

(*b*) $E_{\text{total}}=2m_0c^2=2(938.3\,\text{MeV})=\boxed{1876.6\,\text{MeV}}$

$\boxed{49.}$ These protons will be moving at essentially the speed of light for the entire time of acceleration. The number of revolutions is the total gain in energy divided by the energy gain per revolution. Then the distance is the number of revolutions times the circumference of the ring, and the time is the distance of travel divided by the speed of the protons.

$$N=\frac{\Delta E}{\Delta E/\text{rev}}=\frac{\left(1.0\times10^{12}\,\text{eV}-150\times10^{9}\,\text{eV}\right)}{2.5\times10^{6}\,\text{eV/rev}}=3.4\times10^{5}\,\text{rev}$$

$$d=N(2\pi R)=\left(3.4\times10^{5}\right)2\pi\left(1.0\times10^{3}\,\text{m}\right)=2.136\times10^{9}\,\text{m}\approx\boxed{2.1\times10^{9}\,\text{m}}$$

$$t=\frac{d}{c}=\frac{2.136\times10^{9}\,\text{m}}{3.00\times10^{8}\,\text{m/s}}=\boxed{7.1\,\text{s}}$$

50. (*a*) For the reaction $\pi^-+\text{p}\rightarrow\text{K}^0+\text{p}+\pi^0$, the conservation laws are as follows.

Charge: $-1+1\neq0+1+0$ Charge is NOT conserved.

$\boxed{\text{The reaction is not possible, because charge is not conserved.}}$

Also we note that the reactants would have to have significant kinetic energy to be able to "create" the K^0.

(*b*) For the reaction $\mathrm{K}^- + \mathrm{p} \rightarrow \Lambda^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1=0+0$	Charge is conserved.
Spin: $0+\frac{1}{2}=\frac{1}{2}+0$	Spin is conserved.
Baryon number: $0+1=1+0$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $-1+0=-1+0$	Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\mathrm{K}^+ + \mathrm{n} \rightarrow \Sigma^+ + \pi^0 + \gamma$, the conservation laws are as follows.

Charge: $1+0=1+0+0$	Charge is conserved.
Spin: $0+\frac{1}{2}=-\frac{1}{2}+0+1$	Spin is conserved.
Baryon number: $0+1=1+0+0$	Baryon number is conserved.
Lepton number: $0+0=0+0+0$	Lepton number is conserved.
Strangeness: $1+0\neq-1+0+0$	Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*d*) For the reaction $\mathrm{K}^+ \rightarrow \pi^0 + \pi^0 + \pi^+$, the conservation laws are as follows.

Charge: $1=0+0+1$	Charge is conserved.
Spin: $0=0+0+0$	Spin is conserved.
Baryon number: $0=0+0+0$	Baryon number is conserved.
Lepton number: $0=0+0+0$	Lepton number is conserved.
Strangeness: $1\neq0+0+0$	Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*e*) For the reaction $\pi^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows.

Charge: $1=1+0$	Charge is conserved.
Spin: $0=-\frac{1}{2}+\frac{1}{2}$	Spin is conserved.
Baryon number: $0=0+0$	Baryon number is conserved.
Lepton number: $0=-1+1$	Lepton number is conserved.
Strangeness: $0+0=0+0+0$	Strangeness is conserved.

The reaction is possible, via the weak interaction.

51. (*a*) For the reaction $\pi^- + \mathrm{p} \rightarrow \mathrm{K}^+ + \Sigma^-$, the conservation laws are as follows.

Charge: $-1+1=1-1$	Charge is conserved.
Baryon number: $0+1=0+1$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=1-1$	Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*b*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \mathrm{K}^+ + \Sigma^+$, the conservation laws are as follows.

Charge: $1+1=1+1$	Charge is conserved.
Baryon number: $0+1=0+1$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=1-1$	Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $\pi^- + \mathrm{p} \rightarrow \Lambda^0 + \mathrm{K}^0 + \pi^0$, the conservation laws are as follows.

Charge: $-1+1=0+0+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+0+0$ Lepton number is conserved.
Strangeness: $0+0=-1+1+0$ Strangeness is conserved.

$\boxed{\text{The reaction is possible, via the strong interaction.}}$

(*d*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \Sigma^0 + \pi^0$, the conservation laws are as follows.

Charge: $1+1 \neq 0+0$ Charge is NOT conserved.

$\boxed{\text{The reaction is not possible, because charge is not conserved.}}$

(*e*) For the reaction $\pi^- + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_{\mathrm{e}}$, the conservation laws are as follows.

Charge: $-1+1=1-1+0$ Charge is conserved.
Baryon number: $0+1=1+0+0$ Baryon number is conserved.
Lepton number: $0+0=0+1-1$ Lepton number is conserved.
Strangeness: $0+0=0+0+0$ Strangeness is conserved.

$\boxed{\text{The reaction is possible, via the weak interaction.}}$

Note that we did not check mass conservation, because in a collision, there is always some kinetic energy brought into the reaction. Thus the products can be heavier than the reactants.

52. The π^- is the anti-particle of the π^+, so the reaction is $\boxed{\pi^- \rightarrow \mu^- + \overline{\nu}_\mu}$. The conservation rules are as follows.

Charge: $-1=-1+0$ Charge is conserved.
Baryon number: $0=0+0$ Baryon number is conserved.
Lepton number: $0=1-1$ Lepton number is conserved.
Strangeness: $0=0+0$ Strangeness is conserved.
Spin: $0=\frac{1}{2}-\frac{1}{2}$ Spin is conserved

53. Use Eq. 43-3 to estimate the mass of the particle based on the given distance.

$$mc^2 \approx \frac{hc}{2\pi d} = \frac{\left(6.63\times10^{-34}\,\mathrm{J{\bullet}s}\right)\left(3.0\times10^{8}\,\mathrm{m/s}\right)}{2\pi\left(10^{-18}\,\mathrm{m}\right)}\left(\frac{1}{1.60\times10^{-19}\,\mathrm{J/eV}}\right) = 1.98\times10^{11}\,\mathrm{eV} \approx \boxed{200\,\mathrm{GeV}}$$

This value is of the same order of magnitude as the mass of the $\mathrm{W}^{\pm}$.

54. The *Q*-value is the mass energy of the reactants minus the mass energy of the products.
For the first reaction, $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+\pi^0$:

$$Q = 2m_{\mathrm{p}}c^2 - \left(2m_{\mathrm{p}}c^2 + m_{\pi^0}c^2\right) = -m_{\pi^0}c^2 = \boxed{-135.0\,\mathrm{MeV}}$$

For the second reaction, $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{n}+\pi^+$:

$$Q = 2m_{\mathrm{p}}c^2 - \left(m_{\mathrm{p}}c^2 + m_{\mathrm{n}}c^2 + m_{\pi^+}c^2\right) = m_{\mathrm{p}}c^2 - m_{\mathrm{n}}c^2 - m_{\pi^+}c^2a$$

$$= 938.3\,\mathrm{MeV} - 939.6\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{-140.9\,\mathrm{MeV}}$$

$\boxed{55.}$ The fundamental fermions are the quarks and electrons. In a water molecule there are 2 hydrogen atoms consisting of one electron and one proton each, and 1 oxygen atom, consisting of 8 electrons, 8 protons, and 8 neutrons. Thus there are 18 nucleons, consisting of 3 quarks each, and 10 electrons. The total number of fermions is thus $18\times 3+10=\boxed{64\text{ fermions}}$.

56. We assume that the interaction happens essentially at rest, so that there is no initial kinetic energy or momentum. Thus the momentum of the neutron and the momentum of the π^0 will have the same magnitude. From energy conservation we find the total energy of the π^0.

$$m_{\pi^-}c^2+m_{\text{p}}c^2=E_{\pi^0}+m_{\text{n}}c^2+K_{\text{n}}\quad\rightarrow$$

$$E_{\pi^0}=m_{\pi^-}c^2+m_{\text{p}}c^2-\left(m_{\text{n}}c^2+K_{\text{n}}\right)=139.6\,\text{MeV}+938.3\,\text{MeV}-\left(939.6\,\text{MeV}+0.60\,\text{MeV}\right)$$
$$=137.7\,\text{MeV}$$

From momentum conservation, we can find the mass energy of the π^0. We utilize Eq. 36-13 to relate momentum and energy.

$$p_{\text{n}}=p_{\pi^0}\ \rightarrow\ \left(p_{\text{n}}c\right)^2=\left(p_{\pi^0}c\right)^2\ \rightarrow E_{\text{n}}^2-m_{\text{n}}^2c^4=E_{\pi^0}^2-m_{\pi^0}^2c^4\ \rightarrow\ m_{\pi^0}^2c^4=E_{\pi^0}^2-E_{\text{n}}^2+m_{\text{n}}^2c^4\ \rightarrow$$

$$m_{\pi^0}c^2=\sqrt{E_{\pi^0}^2-E_{\text{n}}^2+m_{\text{n}}^2c^4}=\left[\left(137.7\,\text{MeV}\right)^2-\left(939.6\,\text{MeV}+0.60\,\text{MeV}\right)^2+\left(939.6\,\text{MeV}\right)^2\right]^{1/2}$$
$$=133.5\,\text{MeV}\ \rightarrow\ m_{\pi^0}=\boxed{133.5\,\text{MeV}/c^2}$$

The value from Table 43-2 is $135.0\,\text{MeV}$.

57. (*a*) First we use the uncertainty principle, Eq. 38-1. The energy is so high that we assume $E = pc$, and so $\Delta p=\dfrac{\Delta E}{c}$.

$$\Delta x\Delta p\approx\frac{h}{2\pi}\quad\rightarrow\quad\Delta x\frac{\Delta E}{c}\approx\frac{h}{2\pi}\quad\rightarrow$$

$$\Delta E\approx\frac{hc}{2\pi\Delta x}=\frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{2\pi\left(10^{-32}\,\text{m}\right)}\frac{\left(1\,\text{GeV}/10^{9}\,\text{eV}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=\boxed{2\times10^{16}\,\text{GeV}}$$

Next, we use de Broglie's wavelength formula. We take the de Broglie wavelength as the unification distance.

$$\lambda=\frac{h}{p}=\frac{h}{Ec}\quad\rightarrow$$

$$E=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(10^{-32}\,\text{m}\right)}\frac{\left(1\,\text{GeV}/10^{9}\,\text{eV}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)}=\boxed{1\times10^{17}\,\text{GeV}}$$

Both energies are reasonably close to $10^{16}\,\text{GeV}$. This energy is the amount that could be violated in conservation of energy if the universe were the size of the unification distance.

(*b*) From Eq. 18-4, we have $E=\frac{3}{2}kT$.

$$E=\tfrac{3}{2}kT\quad\rightarrow\quad T=\frac{2E}{3k}=\frac{2\left(10^{25}\,\text{eV}\right)\left(1.6\times10^{-19}\,\text{J/eV}\right)}{3\left(1.38\times10^{-23}\,\text{J/K}\right)}=7.7\times10^{28}\,\text{K}\approx\boxed{10^{29}\,\text{K}}$$

58. The Q-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\pi^-}c^2 + m_{\mathrm{p}}c^2 - \left(m_{\Lambda^0}c^2 + m_{\mathrm{K}^0}c^2\right) = 139.6\,\mathrm{MeV} + 938.3\,\mathrm{MeV} - \left(1115.7\,\mathrm{MeV} + 497.7\,\mathrm{MeV}\right)$$
$$= \boxed{-535.5\,\mathrm{MeV}}$$

We consider the products to be one mass $M = m_{\Lambda^0} + m_{\mathrm{K}^0} = 1613.4\,\mathrm{MeV}/c^2$ since they both have the same velocity. Energy conservation gives the following: $E_{\pi^-} + m_{\mathrm{p}}c^2 = E_M$. Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m^2c^4$.

$$p_{\pi^-} = p_{\mathrm{M}} \rightarrow \left(p_{\pi^-}c\right)^2 = \left(p_{\mathrm{M}}c\right)^2 \rightarrow E_{\pi^-}^2 - m_{\pi^-}^2c^4 = E_M^2 - M^2c^4 \rightarrow$$
$$E_{\pi^-}^2 - m_{\pi^-}^2c^4 = \left(E_{\pi^-} + m_{\mathrm{p}}c^2\right)^2 - M^2c^4 = E_{\pi^-}^2 + 2E_{\pi^-}m_{\mathrm{p}}c^2 + m_{\mathrm{p}}^2c^4 - M^2c^4 \rightarrow$$
$$E_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\mathrm{p}}^2c^4}{2m_{\mathrm{p}}c^2} = K_{\pi^-} + m_{\pi^-}c^2 \rightarrow$$
$$K_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\mathrm{p}}^2c^4}{2m_{\mathrm{p}}c^2} - m_{\pi^-}c^2$$
$$= \frac{\left(1613.4\,\mathrm{MeV}\right)^2 - \left(139.6\,\mathrm{MeV}\right)^2 - \left(938.3\,\mathrm{MeV}\right)^2}{2\left(938.3\,\mathrm{MeV}\right)} - \left(139.6\,\mathrm{MeV}\right) = \boxed{768.0\,\mathrm{MeV}}$$

59. Since there is no initial momentum, the final momentum must add to zero. Thus each of the pions must have the same magnitude of momentum, and therefore the same kinetic energy. Use energy conservation to find the kinetic energy of each pion.

$$2m_{\mathrm{p}}c^2 = 2K_\pi + 2m_\pi c^2 \rightarrow K_\pi = m_{\mathrm{p}}c^2 - m_\pi c^2 = 938.3\,\mathrm{MeV} - 139.6\,\mathrm{MeV} = \boxed{798.7\,\mathrm{MeV}}$$

60. The Q-value is the energy of the reactants minus the energy of the products. We assume that one of the initial protons is at rest, and that all four final particles have the same velocity and therefore the same kinetic energy, since they all have the same mass. We consider the products to be one mass $M = 4m_{\mathrm{p}}$ since they all have the same velocity.

$$Q = 2m_{\mathrm{p}}c^2 - 4m_{\mathrm{p}}c^2 = 2m_{\mathrm{p}}c^2 - Mc^2 = -2m_{\mathrm{p}}c^2$$

Energy conservation gives the following, where K_{th} is the threshold energy.

$$\left(K_{\mathrm{th}} + m_{\mathrm{p}}c^2\right) + m_{\mathrm{p}}c^2 = E_M = K_M + Mc^2$$

Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m_0^2c^4$.

$$p_{\mathrm{p}} = p_{\mathrm{M}} \rightarrow \left(p_{\mathrm{p}}c\right)^2 = \left(p_{\mathrm{M}}c\right)^2 \rightarrow \left(K_{\mathrm{th}} + m_{\mathrm{p}}c^2\right)^2 - m_{\mathrm{p}}^2c^4 = \left(K_M + Mc^2\right)^2 - M^2c^4 \rightarrow$$
$$K_{\mathrm{th}}^2 + 2K_{\mathrm{th}}m_{\mathrm{p}}c^2 + m_{\mathrm{p}}^2c^4 - m_{\mathrm{p}}^2c^4 = K_{\mathrm{th}}^2 + 4K_{\mathrm{th}}m_{\mathrm{p}}c^2 + 4m_{\mathrm{p}}^2c^4 - \left(4m_{\mathrm{p}}\right)^2c^4 \rightarrow$$
$$2K_{\mathrm{th}}m_{\mathrm{p}}c^2 = 4K_{\mathrm{th}}m_{\mathrm{p}}c^2 + 4m_{\mathrm{p}}^2c^4 - 16m_{\mathrm{p}}^2c^4 \rightarrow 2K_{\mathrm{th}}m_{\mathrm{p}}c^2 = 12m_{\mathrm{p}}^2c^4 \rightarrow$$
$$K_{\mathrm{th}} = 6m_{\mathrm{p}}c^2 = 3\left|Q\right|$$

$\boxed{61.}$ The total energy is the sum of the kinetic energy and the mass energy. The wavelength is found from the relativistic momentum.

$$E = K + mc^2 = 15\times10^9\,\text{eV} + 938\times10^6\,\text{eV} = 1.594\times10^{10}\,\text{eV} \approx \boxed{16\,\text{GeV}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\dfrac{\sqrt{E^2-\left(mc^2\right)^2}}{c}} = \frac{hc}{\sqrt{E^2-\left(mc^2\right)^2}}$$

$$= \frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{\sqrt{\left(1.594\times10^{10}\,\text{eV}\right)^2-\left(938\times10^6\,\text{eV}\right)^2}}\frac{1}{\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{7.8\times10^{-17}\,\text{m}}$$

62. We use λ_0 to represent the actual wavelength, and λ to define the approximate wavelength. The approximation is to ignore the rest mass in the expression for the total energy, $E = K + mc^2$. We also use Eqs. 36-10, 36-11, 36-13, and 43-1.

$$p^2c^2 = E^2 - \left(mc^2\right)^2 = \left(K+mc^2\right)^2 - \left(mc^2\right)^2 = K^2\left(1+2\frac{mc^2}{K}\right)$$

$$\lambda_0 = \frac{h}{p} = \frac{hc}{K\left(1+\dfrac{2mc^2}{K}\right)^{1/2}} \ ; \ \lambda = \frac{hc}{K} > \lambda_0 \ ; \ \lambda = 1.01\lambda_0 \ \rightarrow$$

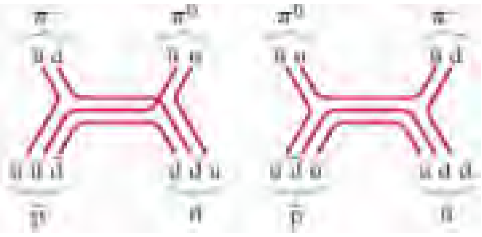
$$\frac{hc}{K} = 1.01\frac{hc}{K\left(1+\dfrac{2mc^2}{K}\right)^{1/2}} \ \rightarrow \ K = \frac{2mc^2}{\left(1.01\right)^2-1} = \frac{2\left(9.38\times10^8\,\text{eV}\right)}{0.0201} = 9.333\times10^{10}\,\text{eV} \approx \boxed{9.3\times10^{10}\,\text{eV}}$$

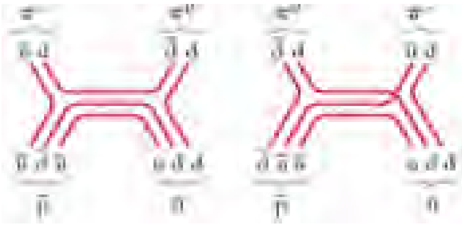
$$\lambda = \frac{hc}{K} = \frac{\left(6.63\times10^{-34}\,\text{J•s}\right)\left(3.00\times10^8\,\text{m/s}\right)}{\left(9.333\times10^{10}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.332\times10^{-17}\,\text{m} \approx \boxed{1.3\times10^{-17}\,\text{m}}$$

$$K = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 \ \rightarrow$$

$$v = c\sqrt{1-\left(\frac{K}{mc^2}+1\right)^{-2}} = c\sqrt{1-\left(\frac{9.333\times10^{10}\,\text{eV}}{9.38\times10^8\,\text{eV}}+1\right)^{-2}} = \boxed{0.99995c}$$

63. As mentioned in Example 43-9, the π^0 can be considered as either $\text{u}\,\overline{\text{u}}$ or $\text{d}\,\overline{\text{d}}$. There are various models to describe this reaction. Four are shown here.





64. (*a*) To conserve charge, the missing particle must be neutral. To conserve baryon number, the missing particle must be a meson. To conserve strangeness, charm, topness, and bottomness, the missing particle must be made of up and down quarks and antiquarks only. With all this information, the missing particle is $\boxed{\pi^0}$.

    (*b*) This is a weak interaction since one product is a lepton. To conserve charge, the missing particle must be neutral. To conserve the muon lepton number, the missing particle must be an antiparticle in the muon family. With this information, the missing particle is $\boxed{\bar{\nu}_\mu}$.

65. A relationship between total energy and speed is given by Eq. 36-11b.

$$E=\frac{mc^2}{\sqrt{1-v^2/c^2}} \rightarrow$$

$$\frac{v}{c}=\sqrt{1-\left(\frac{mc^2}{E}\right)^2}=\sqrt{1-\left(\frac{9.38\times10^8\,\text{eV}}{7.0\times10^{12}\text{eV}}\right)^2}\approx1-\tfrac{1}{2}\left(\frac{9.38\times10^8\,\text{eV}}{7.0\times10^{12}\text{eV}}\right)^2=\boxed{1-9.0\times10^9}$$

66. We write equations for both conservation of energy and conservation of momentum. The magnitudes of the momenta of the products are equal. We also use Eqs. 36-11 and 36-13.

$$p_1=p_2\ ;\ E_0=mc^2=E_1+E_2 \rightarrow$$

$$\left(mc^2-E_1\right)^2=E_2^2=p_2^2c^2+m_2^2c^4=p_1^2c^2+m_2^2c^4=E_1^2-m_1^2c^4+m_2^2c^4 \rightarrow$$

$$m^2c^4-2mc^2E_1+E_1^2=E_1^2-m_1^2c^4+m_2^2c^4 \rightarrow E_1=\frac{m^2c^4+m_1^2c^4-m_2^2c^4}{2mc^2}$$

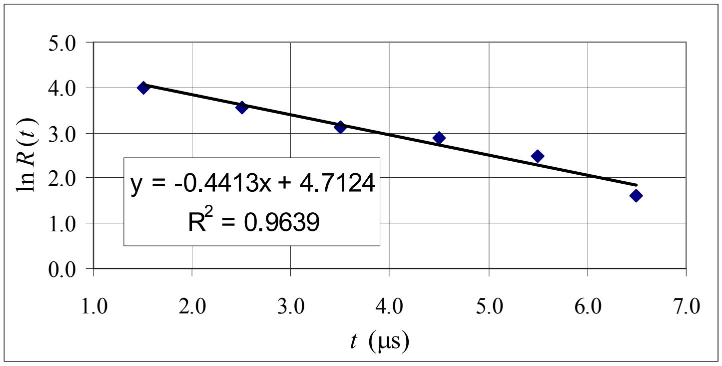
$$K_1=E_1-m_1c^2=\frac{m^2c^4+m_1^2c^4-m_2^2c^4}{2mc^2}-m_1c^2=\frac{m^2c^4+m_1^2c^4-m_2^2c^4-2mc^2m_1c^2}{2mc^2}$$

$$=\boxed{\frac{\left(mc^2-m_1c^2\right)^2-m_2^2c^4}{2mc^2}}$$

$\boxed{67.}$ The value of R_0 is not known until we draw the graph. We note the following:

$$R=R_0e^{-t/\tau} \rightarrow R/R_0=e^{-t/\tau} \rightarrow \ln R-\ln R_0=-t/\tau \rightarrow \ln R=-t/\tau+\ln R_0$$

A graph of $\ln R$ vs. t should give a straight line with a slope of $-1/\tau$ and a y-intercept of $\ln R_0$. The determination of the mean life does not depend on R_0, and so to find the mean life, we may simply plot $\ln R$ vs. t. That graph is shown, along with the slope and y-intercept. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH43.XLS," on tab "Problem 43.67."



$$\tau=\frac{1}{0.4413}=2.266\mu\text{s}\approx\boxed{2.3\mu\text{s}} \qquad \%\,\text{diff}=\frac{2.266-2.197}{2.197}\times100=\boxed{3.1\%}$$

CHAPTER 44: Astrophysics and Cosmology

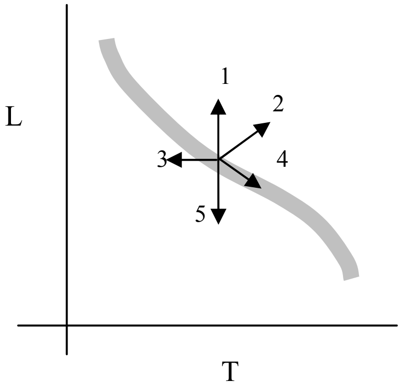
Responses to Questions

1. The Milky Way appears "murky" or "milky" to the naked eye, and so before telescopes were used it was thought to be cloud-like. When viewed with a telescope, much of the "murkiness" is resolved into stars and star clusters, so we no longer consider the Milky Way to be milky.

2. If a star generates more energy in its interior than it radiates away, its temperature will increase. Consequently, there will be greater outward pressure opposing the gravitational force directed inward. To regain equilibrium, the star will expand. If a star generates less energy than it radiates away, then its temperature will decrease. There will be a smaller outward pressure opposing the gravitational force directed inward, and, in order to regain equilibrium, the star will contract.

3. Red giants are extremely large stars with relatively cool surface temperatures, resulting in their reddish colors. These stars are very luminous because they are so large. When the Sun becomes a red giant, for instance, its radius will be on the order of the distance from the Earth to the Sun. A red giant has run out of hydrogen in its inner core and is fusing hydrogen to helium in a shell surrounding the core. Red giants have left their main sequence positions on the H–R diagram and moved up (more luminous) and to the right (cooler).

4. A star moving along arrow #1 would increase in luminosity while maintaining the same surface temperature. It would therefore also have to increase in size, since each square meter of its surface would have the same color and therefore same energy output as before. A star moving along arrow #2 would increase in luminosity and decrease in temperature. It would also increase in size, since it would need to produce a greater luminosity even though each unit area of its surface would now be producing less energy. A star moving along arrow #3 maintains the same luminosity while increasing its surface temperature. It will become smaller, since a unit area of this star will increase its energy and therefore a smaller overall area will be needed to maintain the same luminosity. A star moving along arrow #4 decreases in both surface temperature and luminosity. Finally, a star moving along arrow #5 will decrease in luminosity while maintaining the same surface temperature and decreasing in size. Note that these arrows do not necessarily represent "natural" paths for stars on the H–R diagram.



5. The H–R diagram is a plot of luminosity versus surface temperature of a star and therefore does not directly tell us anything about the core of a star. However, when considered in conjunction with theories of stellar evolution, the H–R diagram does relate to the interior of a star. For instance, all main sequence stars are fusing hydrogen to helium in their cores, so the location of a particular star on the main sequence does give us that information.

6. The fate of a star depends on the mass of the star remaining after the red giant phase. If the mass is less than about 1.4 solar masses, the star will become a white dwarf. If the mass is greater than this limit, than the exclusion principle applied to electrons is not enough to hold the star up against its own gravity and it continues to contract, eventually becoming a neutron star or, if its mass at this stage is more than two or three solar masses, a black hole.

7. Yes. Hotter stars are found on the main sequence above and to the left of cooler stars. If H–R diagrams of clusters of stars are compared, it is found that older clusters are missing the upper left portions of their main sequences. All the stars in a given cluster are formed at about the same time, and the absence of the hotter main sequence stars in a cluster indicates that they have shorter lives and have already used up their core hydrogen and become red giants. In fact, the "turn-off" point, or point at which the upper end of the main sequence stops, can be used to determine the ages of clusters.

8. The baseline used in measuring parallaxes from the Earth is the distance from the Earth to the Sun. (See Figure 44-11.) If you were measuring parallaxes from the Moon instead, you would need to make a slight correction based on the position of the Moon with respect to the Earth–Sun line at the time of the measurement. If you were measuring parallaxes from Mars, you would need to use the distance from Mars to the Sun as the baseline. In addition, you would need to wait half a Martian year between measurements instead of half an Earth year.

9. Watch the star over a period of several days and determine its period through observation. Use the known relationship between period and luminosity to find its absolute luminosity. Compare its absolute luminosity to its apparent luminosity (observed) to determine the distance to the galaxy in which it is located.

10. A geodesic is the shortest distance between two points. For instance, on a flat plane the shortest distance between two points is a straight line, and on the surface of a sphere the shortest distance is an arc of a great circle. According to general relativity, space–time is curved. Determining the nature of a geodesic, for instance by observing the motion of a body or light near a large mass, will help determine the nature of the curvature of space–time there.

11. If the redshift of spectral lines of galaxies were discovered to be due to something other than expansion of the universe, then the Big Bang theory and the idea that the universe is expanding would be called into question. However, the evidence of the cosmic background microwave radiation would conflict with this view, unless it too was determined to result from some cause other than expansion.

12. No. In an expanding universe, all galaxies are moving away from all other galaxies on a large scale. (On a small scale, neighboring galaxies may be gravitationally bound to each other.) Therefore, the view from any galaxy would be the same. Our observations do not indicate that we are at the center. (See Figure 44-23.)

13. They would appear to be receding. In an expanding universe, the distances between galaxies are increasing, and so the view from any galaxy is that all other galaxies are moving away.

14. An explosion on Earth would be affected by the Earth's gravity and air resistance. Each piece of debris would act like a projectile, with its individual initial velocity. More distant particles would not spread at a higher speed. This corresponds somewhat to a closed universe, in which the galaxies eventually stop and then all come back together again. In the case of the explosion on Earth, most of the particles would eventually stop. Most would land on the ground. Some might escape into space. The particles would not all reassemble, as in the "big crunch."

15. Black holes have tremendous gravity, so we can detect them by the gravitational deflection of other objects in their vicinity. Also, matter accelerating toward a black hole gives off x-rays, which can be detected. In addition, gravitational lensing, the bending of light coming from stars and galaxies located behind the black hole, can indicate that the black hole is present.

16. $R = 2GM/c^2$, so $M = Rc^2/2G$.

    $M = (5.29 \times 10^{-11})(3.00 \times 10^{8})^2/[2(6.67 \times 10^{-11})] = 3.57 \times 10^{16}$ kg

17. Both the formation of the Earth and the time during which people have lived on Earth are on the far right edge of Figure 44-30, in the era of dark energy.

18. The 2.7 K cosmic microwave background radiation is the remnant radiation of the Big Bang. As the universe expanded, the wavelengths of the Big Bang radiation lengthened and became redshifted. The 2.7 K blackbody curve peaks at a wavelength of about 7.35 cm, in the microwave region. The temperature of this radiation is low because the energy spread out over an increasingly large volume as the universe expanded.

19. The early universe was too hot for atoms to exist. The average kinetic energies of particles were high and frequent collisions prevented electrons from remaining with nuclei.

20. (*a*) Type Ia supernovae have a range of luminosities that can be extracted from their observable characteristics and can be derived from the rate at which they brighten and fade away.
    (*b*) The distance to a supernova can be determined by comparing the relative intensity to the luminosity.

21. The initial Big Bang was not perfectly symmetric. Deviations in the symmetry enabled the development of galaxies and other structures.

22. If the average mass density of the universe is above the critical density, then the universe will eventually stop its expansion and contract, collapsing on itself and ending finally in a “big crunch.” This scenario corresponds to a closed universe, or one with positive curvature.

23. If there were 7 protons for every neutron, and it takes two protons and two neutrons to create a single helium nucleus, then for every helium nucleus there would be 12 hydrogen nuclei. Since the mass of helium is four times the mass of hydrogen, the ratio of the total mass of hydrogen to the total mass of helium should be 12:4, or 3:1.

24. (*a*) Gravity between galaxies should be pulling the galaxies back together, slowing the expansion of the universe.
    (*b*) Astronomers could measure the redshift of light from distant supernovae and deduce the recession velocities of the galaxies in which they lie. By obtaining data from a large number of supernovae, they could establish a history of the recessional velocity of the universe, and perhaps tell whether the expansion of the universe is slowing down.

Solutions to Problems

1. Convert the angle to seconds of arc, reciprocate to find the distance in parsecs, and then convert to light years.

$$\phi = \left(2.9 \times 10^{-4}\right)^{\circ}\left(\frac{3600''}{1^{\circ}}\right) = 1.044''$$

$$d\left(\text{pc}\right) = \frac{1}{\phi''} = \frac{1}{1.044''} = 0.958\,\text{pc}\left(\frac{3.26\,\text{ly}}{1\,\text{pc}}\right) = \boxed{3.1\,\text{ly}}$$

2. Use the angle to calculate the distance in parsecs, and then convert to light years.

$$d(\text{pc}) = \frac{1}{\phi''} = \frac{1}{0.27''} = 3.704\,\text{pc} \rightarrow 3.704\,\text{pc}\left(\frac{3.26\,\text{ly}}{1\,\text{pc}}\right) = \boxed{12\,\text{ly}}$$

3. Convert the light years to parsecs, and then take the reciprocal of the number of parsecs to find the parallax angle in seconds of arc.

$$65\,\text{ly}\left(\frac{1\,\text{pc}}{3.26\,\text{ly}}\right) = 19.94\,\text{pc} \approx \boxed{20\,\text{pc}}\ (2\text{ sig. fig.}) \qquad \phi = \frac{1}{19.94\,\text{pc}} = \boxed{0.050''}$$

4. The reciprocal of the distance in parsecs is the angle in seconds of arc.

(*a*) $\phi'' = \dfrac{1}{d(\text{pc})} = \dfrac{1}{56\,\text{pc}} = 0.01786 \approx \boxed{0.018''}$

(*b*) $0.01786''\left(\dfrac{1^\circ}{3600''}\right) = \left(4.961\times10^{-6}\right)^\circ \approx \boxed{\left(5.0\times10^{-6}\right)^\circ}$

5. The parallax angle is $\boxed{\text{smaller}}$ for the further star. Since $\tan\phi = d/D$, as the distance D to the star increases, the tangent decreases, so the angle decreases. And since for small angles, $\tan\phi \approx \phi$, we have that $\phi \approx d/D$. Thus if the distance D is doubled, the angle ϕ will be $\boxed{\text{smaller by a factor of 2}}$.

6. Find the distance in light years. That value is also the time for light to reach us.

$$85\,\text{pc}\left(\frac{3.26\,\text{ly}}{1\,\text{pc}}\right) = 277\,\text{ly} \approx 280\,\text{ly} \rightarrow \text{It takes light } \boxed{280\text{ years}} \text{ to reach us.}$$

$\boxed{7.}$ The apparent brightness of an object is inversely proportional to the square of the observer's distance from the object, given by Eq. 44-1. To find the relative brightness at one location as compared to another, take a ratio of the apparent brightness at each location.

$$\frac{b_{\text{Jupiter}}}{b_{\text{Earth}}} = \frac{\dfrac{L}{4\pi d^2_{\text{Jupiter}}}}{\dfrac{L}{4\pi d^2_{\text{Earth}}}} = \frac{d^2_{\text{Earth}}}{d^2_{\text{Jupiter}}} = \left(\frac{d_{\text{Earth}}}{d_{\text{Jupiter}}}\right)^2 = \left(\frac{1}{5.2}\right)^2 = \boxed{0.037}$$

8. (*a*) The apparent brightness is the solar constant, $\boxed{1.3\times10^3\ \text{W/m}^2}$.

(*b*) Use Eq. 44-1 to find the intrinsic luminosity.

$$b = \frac{L}{4\pi d^2} \rightarrow L = 4\pi d^2 b = 4\pi\left(1.496\times10^{11}\,\text{m}^2\right)^2\left(1.3\times10^3\ \text{W/m}^2\right) = \boxed{3.7\times10^{26}\,\text{W}}$$

9. The density is the mass divided by the volume.

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{1.99\times10^{30}\,\text{kg}}{\frac{4}{3}\pi\left(6\times10^{10}\,\text{m}\right)^3} = \boxed{2\times10^{-3}\ \text{kg/m}^3}$$

10. The angular width is the inverse tangent of the diameter of our Galaxy divided by the distance to the nearest galaxy. According to Figure 44-2, our Galaxy is about 100,000 ly in diameter.

$$\phi = \tan^{-1}\frac{\text{Galaxy diameter}}{\text{Distance to nearest galaxy}} = \tan^{-1}\frac{1.0\times10^{5}\,\text{ly}}{2.4\times10^{6}\,\text{ly}} = \boxed{0.042\,\text{rad}} \approx 2.4^{\circ}$$

$$\phi_{\text{Moon}} = \tan^{-1}\frac{\text{Moon diameter}}{\text{Distance to Moon}} = \tan^{-1}\frac{3.48\times10^{6}\,\text{m}}{3.84\times10^{8}\,\text{m}} = \boxed{9.1\times10^{-3}\,\text{rad}} \approx 0.52^{\circ}$$

$\boxed{\text{The Galaxy width is about 4.5 times the Moon width.}}$

11. The *Q*-value is the mass energy of the reactants minus the mass energy of the products. The masses are found in Appendix F.

$${}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} \rightarrow {}^{8}_{4}\text{Be}$$

$$Q = 2m_{\text{He}}c^{2} - m_{\text{Be}}c^{2} = \left[2\left(4.002603\,\text{u}\right) - 8.005305\,\text{u}\right]c^{2}\left(931.5\,\text{Mev}/c^{2}\right) = \boxed{-0.092\,\text{MeV}}$$

$${}^{4}_{2}\text{He} + {}^{8}_{4}\text{Be} \rightarrow {}^{12}_{6}\text{C}$$

$$Q = m_{\text{Be}}c^{2} + m_{\text{He}}c^{2} - m_{\text{C}}c^{2} = \left[4.002603\,\text{u} + 8.005305\,\text{u} - 12.000000\right]c^{2}\left(931.5\,\text{Mev}/c^{2}\right)$$

$$= \boxed{7.366\,\text{MeV}}$$

12. The angular width is the inverse tangent of the diameter of the Moon divided by the distance to the Sun.

$$\phi = \tan^{-1}\frac{\text{Moon diameter}}{\text{Distance to Sun}} = \tan^{-1}\frac{3.48\times10^{6}\,\text{m}}{1.496\times10^{11}\,\text{m}} = \boxed{2.33\times10^{-5}\,\text{rad}} \approx \left(1.33\times10^{-3}\right)^{\circ} \approx 4.79''$$

$\boxed{13.}$ The density is the mass divided by the volume.

$$\rho = \frac{M}{V} = \frac{M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Earth}}^{3}} = \frac{1.99\times10^{30}\,\text{kg}}{\frac{4}{3}\pi\left(6.38\times10^{6}\,\text{m}\right)^{3}} = \boxed{1.83\times10^{9}\,\text{kg}/\text{m}^{3}}$$

Since the volumes are the same, the ratio of the densities is the same as the ratio of the masses.

$$\frac{\rho}{\rho_{\text{Earth}}} = \frac{M}{M_{\text{Earth}}} = \frac{1.99\times10^{30}\,\text{kg}}{5.98\times10^{24}\,\text{kg}} = \boxed{3.33\times10^{5}\text{ times larger}}$$

14. The density of the neutron star is its mass divided by its volume. Use the proton to calculate the density of nuclear matter. The radius of the proton is taken from Eq. 41-1.

$$\rho_{\substack{\text{neutron}\\\text{star}}} = \frac{M}{V} = \frac{1.5\left(1.99\times10^{30}\,\text{kg}\right)}{\frac{4}{3}\pi\left(11\times10^{3}\,\text{m}\right)^{3}} = 5.354\times10^{17}\,\text{kg}/\text{m}^{3} \approx \boxed{5.4\times10^{17}\,\text{kg}/\text{m}^{3}}$$

$$\frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{white}\\\text{dwarf}}}} = \frac{5.354\times10^{17}\,\text{kg}/\text{m}^{3}}{1.83\times10^{9}\,\text{kg}/\text{m}^{3}} = \boxed{2.9\times10^{8}} \qquad \frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{nuclear}\\\text{matter}}}} = \frac{5.354\times10^{17}\,\text{kg}/\text{m}^{3}}{\dfrac{1.673\times10^{-27}\,\text{kg}}{\frac{4}{3}\pi\left(1.2\times10^{-15}\,\text{m}\right)^{3}}} = \boxed{2.3}$$

15. Wien's law (Eq. 37-1) says that the $\lambda_{\text{P}}T = \alpha$, where α is a constant, and so $\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2$. The Stefan–Boltzmann equation (Eq. 19-17) says that the power output of a star is given by $P = \beta A T^4$, where β is a constant, and *A* is the radiating area. The *P* in the Stefan–Boltzmann equation is the same as the luminosity *L* in this chapter. The luminosity *L* is related to the apparent brightness *b* by Eq. 44-1. It is given that $b_1/b_2 = 0.091$, $d_1 = d_2$, $\lambda_{\text{P1}} = 470\,\text{nm}$, and $\lambda_{\text{P2}} = 720\,\text{nm}$.

$$\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2 \;\rightarrow\; \frac{T_2}{T_1} = \frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}} \;;\; b_1 = 0.091 b_2 \;\rightarrow\; \frac{L_1}{4\pi d_1^2} = 0.091\frac{L_2}{4\pi d_2^2} \;\rightarrow$$

$$1 = \frac{d_2^2}{d_1^2} = \frac{0.091 L_2}{L_1} = \frac{0.091 P_2}{P_1} = \frac{0.091 A_2 T_2^4}{A_1 T_1^4} = \frac{(0.091)4\pi r_2^2 T_2^4}{4\pi r_1^2 T_1^4} = 0.091\frac{T_2^4}{T_1^4}\frac{r_2^2}{r_1^2} \;\rightarrow$$

$$\frac{r_1}{r_2} = \sqrt{0.091}\left(\frac{T_2}{T_1}\right)^2 = \sqrt{0.091}\left(\frac{\lambda_{\text{P2}}}{\lambda_{\text{P1}}}\right)^2 = \sqrt{0.091}\left(\frac{470\,\text{nm}}{720\,\text{nm}}\right)^2 = 0.1285$$

The ratio of the diameters is the same as the ratio of radii, so $\frac{D_1}{D_2} = \boxed{0.13}$.

16. Wien's law (Eq. 37-1) says that the $\lambda_{\text{P}}T = \alpha$, where α is a constant, and so $\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2$. The Stefan–Boltzmann equation (Eq. 19-17) says that the power output of a star is given by $P = \beta A T^4$, where β is a constant, and *A* is the radiating area. The *P* in the Stefan–Boltzmann equation is the same as the luminosity *L* in this chapter. The luminosity *L* is related to the apparent brightness *b* by Eq. 44-1. It is given that $b_1 = b_2$, $r_1 = r_2$, $\lambda_{\text{P1}} = 750\,\text{nm}$, and $\lambda_{\text{P2}} = 450\,\text{nm}$.

$$\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2 \;\rightarrow\; \frac{T_2}{T_1} = \frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}$$

$$b_1 = b_2 \;\rightarrow\; \frac{L_1}{4\pi d_1^2} = \frac{L_2}{4\pi d_2^2} \;\rightarrow\; \frac{d_2^2}{d_1^2} = \frac{L_2}{L_1} = \frac{P_2}{P_1} = \frac{\beta A_2 T_2^4}{\beta A_1 T_1^4} = \frac{4\pi r_2^2 T_2^4}{4\pi r_1^2 T_1^4} = \frac{T_2^4}{T_1^4} = \left(\frac{T_2}{T_1}\right)^4 \;\rightarrow$$

$$\frac{d_2}{d_1} = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}\right)^2 = \left(\frac{750}{450}\right)^2 = 2.8$$

$\boxed{\text{The star with the peak at 450 nm is 2.8 times further away than the star with the peak at 750 nm.}}$

17. The Schwarzschild radius is $\frac{2GM}{c^2}$.

$$R_{\text{Earth}} = \frac{2GM_{\text{Earth}}}{c^2} = \frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(3.00\times10^{8}\,\text{m/s}\right)^2} = 8.86\times10^{-3}\,\text{m} \approx \boxed{8.9\,\text{mm}}$$

18. The Schwarzschild radius is given by $R = \frac{2GM}{c^2}$. An approximate mass for our Galaxy is calculated in Example 44-1.

$$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(2\times10^{41}\,\text{kg}\right)}{\left(3.00\times10^{8}\,\text{m/s}\right)^2} = \boxed{3\times10^{14}\,\text{m}}$$

$\boxed{19.}$ The limiting value for the angles in a triangle on a sphere is $\boxed{540^\circ}$. Imagine drawing an equilateral triangle near the north pole, enclosing the north pole. If that triangle were small, the surface would be approximately flat, and each angle in the triangle would be 60°. Then imagine "stretching" each side of that triangle down towards the equator, while keeping sure that the north pole stayed inside the triangle. The angle at each vertex of the triangle would expand, with a limiting value of 180°. The three 180° angles in the triangle would sum to 540°.

20. To just escape from an object, the kinetic energy of the body at the surface of the body must be equal to the magnitude of the gravitational potential energy at the surface. Use Eq. 8-19.

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R_{\text{Schwarzchild}}}} = \sqrt{\frac{2GM}{2GM/c^2}} = \boxed{c}$$

21. We find the time for the light to cross the elevator, and then find how far the elevator moves during that time due to its acceleration.

$$\Delta t = \frac{\Delta x}{c}\ ;\ \Delta y = \tfrac{1}{2}g(\Delta t)^2 = \frac{g(\Delta x)^2}{2c^2} = \frac{(9.80\,\text{m/s}^2)(2.4\,\text{m})}{2(3.00\times10^8\,\text{m/s})^2} = \boxed{1.3\times10^{-16}\,\text{m}}$$

Note that this is smaller than the size of a proton.

22. Use Eq. 44-4, Hubble's law.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{1850\,\text{km/s}}{22\,\text{km/s/Mly}} = \boxed{84\,\text{Mly}} = 8.4\times10^7\,\text{ly}$$

23. Use Eq. 44-4, Hubble's law.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{(0.015)(3.00\times10^8\,\text{m/s})}{2.2\times10^4\,\text{m/s/Mly}} = 204.5\,\text{Mly} \approx \boxed{2.0\times10^2\,\text{Mly}} = 2.0\times10^8\,\text{ly}$$

24. (*a*) Use Eq. 44-6 to solve for the speed of the galaxy.

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} \approx \frac{v}{c} \rightarrow v = c\left(\frac{455\,\text{nm} - 434\,\text{nm}}{434\,\text{nm}}\right) = 0.04839c \approx \boxed{0.048c}$$

(*b*) Use Hubble's law, Eq. 44-4, to solve for the distance.

$$v = Hd \rightarrow d = \frac{v}{H} = \frac{0.04839(3.00\times10^8\,\text{m/s})}{22000\,\text{m/s/Mly}} = 660\,\text{Mly} \approx \boxed{6.6\times10^8\,\text{ly}}$$

$\boxed{25.}$ We find the velocity from Hubble's law, Eq. 44-4, and the observed wavelength from the Doppler shift, Eq. 44-3.

(*a*) $$\frac{v}{c} = \frac{Hd}{c} = \frac{(22000\,\text{m/s/Mly})(7.0\,\text{Mly})}{3.00\times10^8\,\text{m/s}} = 5.133\times10^{-4}$$

$$\lambda = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} = (656\,\text{nm})\sqrt{\frac{1+5.133\times10^{-4}}{1-5.133\times10^{-4}}} = 656.34\,\text{nm} \approx \boxed{656\,\text{nm}}$$

(*b*) $$\frac{v}{c}=\frac{Hd}{c}=\frac{\left(22000\,\text{m/s/Mly}\right)\left(70\,\text{Mly}\right)}{3.00\times10^{8}\,\text{m/s}}=5.133\times10^{-3}$$

$$\lambda=\lambda_0\sqrt{\frac{1+v/c}{1-v/c}}=\left(656\,\text{nm}\right)\sqrt{\frac{1+5.133\times10^{-3}}{1-5.133\times10^{-3}}}=659.38\,\text{nm}\approx\boxed{659\,\text{nm}}$$

26. Use Eqs. 44-3 and 44-4 to solve for the distance to the galaxy.

$$\lambda_{\text{obs}}=\lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}}\quad\rightarrow\quad v=c\frac{\left(\lambda_{\text{obs}}^2-\lambda_{\text{rest}}^2\right)}{\left(\lambda_{\text{obs}}^2+\lambda_{\text{rest}}^2\right)}$$

$$d=\frac{v}{H}=\frac{c}{H}\frac{\left(\lambda_{\text{obs}}^2-\lambda_{\text{rest}}^2\right)}{\left(\lambda_{\text{obs}}^2+\lambda_{\text{rest}}^2\right)}=\frac{\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(2.2\times10^{4}\,\text{m/s/Mly}\right)}\frac{\left[\left(423.4\,\text{nm}\right)^2-\left(393.4\,\text{nm}\right)^2\right]}{\left[\left(423.4\,\text{nm}\right)^2+\left(393.4\,\text{nm}\right)^2\right]}$$

$$=1.0\times10^{3}\,\text{Mly}=\boxed{1.0\times10^{9}\,\text{ly}}$$

27. Use Eqs. 44-3 and 44-5a to solve for the speed of the galaxy.

$$z=\frac{\lambda_{\text{obs}}-\lambda_{\text{rest}}}{\lambda_{\text{rest}}}=\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}-1=\sqrt{\frac{1+v/c}{1-v/c}}-1\quad\rightarrow$$

$$\frac{v}{c}=\frac{\left(z+1\right)^2-1}{\left(z+1\right)^2+1}=\frac{1.060^2-1}{1.060^2+1}=0.05820\quad\rightarrow\quad v=\boxed{0.058\,c}$$

The approximation of Eq. 44-6 gives $v=zc=\boxed{0.060\,c}$.

28. Use Eqs. 44-3 and 44-5a to solve for the redshift parameter.

$$z=\frac{\lambda_{\text{obs}}-\lambda_{\text{rest}}}{\lambda_{\text{rest}}}=\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}-1=\sqrt{\frac{1+v/c}{1-v/c}}-1=\sqrt{\frac{1+0.075}{1-0.075}}-1=\boxed{0.078}$$

Or, we use the approximation given in Eq. 44-6.

$$z\approx\frac{v}{c}=\boxed{0.075}$$

29. Eq. 44-3 states $\lambda=\lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}}$.

$$\lambda=\lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}}=\lambda_{\text{rest}}\left(1+\frac{v}{c}\right)^{1/2}\left(1-\frac{v}{c}\right)^{-1/2}\approx\lambda_{\text{rest}}\left(1+\tfrac{1}{2}\frac{v}{c}\right)\left(1-\left(-\tfrac{1}{2}\right)\frac{v}{c}\right)=\lambda_{\text{rest}}\left(1+\tfrac{1}{2}\frac{v}{c}\right)^2$$

$$\lambda\approx\lambda_{\text{rest}}\left(1+2\left(\tfrac{1}{2}\frac{v}{c}\right)\right)=\lambda_{\text{rest}}\left(1+\frac{v}{c}\right)=\lambda_{\text{rest}}+\lambda_{\text{rest}}\frac{v}{c}\quad\rightarrow\quad\lambda-\lambda_{\text{rest}}=\Delta\lambda=\lambda_{\text{rest}}\frac{v}{c}\quad\rightarrow\quad\boxed{\frac{\Delta\lambda}{\lambda_{\text{rest}}}=\frac{v}{c}}$$

30. For small relative wavelength shifts, we may use Eq. 44-6 to find the speed. We use Eq. 44-4 to find the distance.

$$\frac{v}{c}=\frac{\Delta\lambda}{\lambda_{\text{rest}}} \quad\rightarrow\quad v=c\frac{\Delta\lambda}{\lambda_{\text{rest}}}\ ;\ v=Hd \quad\rightarrow$$

$$d=\frac{v}{H}=\frac{c}{H}\frac{\Delta\lambda}{\lambda_{\text{rest}}}=\left(\frac{3.00\times10^{8}\,\text{m/s}}{22{,}000\,\text{m/s/Mly}}\right)\left(\frac{0.10\,\text{cm}}{21\,\text{cm}}\right)=\boxed{65\,\text{Mly}}$$

$\boxed{31.}$ Wien's law is given in Eq. 37-1.

$$\lambda_{\text{P}}T=2.90\times10^{-3}\,\text{m}\bullet\text{K} \quad\rightarrow\quad \lambda_{\text{P}}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{T}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{2.7\,\text{K}}=\boxed{1.1\times10^{-3}\,\text{m}}$$

32. We use Wien's law, Eq. 37-1. From Figure 44-30, the temperature is about 10^{10} K.

$$\lambda_{\text{P}}T=2.90\times10^{-3}\,\text{m}\bullet\text{K} \quad\rightarrow\quad \lambda_{\text{P}}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{T}=\frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{10^{10}\,\text{K}}\succ=\boxed{3\times10^{-13}\,\text{m}}$$

From Figure 31-12, that wavelength is in the $\boxed{\text{gamma ray}}$ region of the EM spectrum.

33. We use the proton as typical nuclear matter.

$$\left(10^{-26}\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1\,\text{nucleon}}{1.67\times10^{-27}\,\text{kg}}\right)=\boxed{6\,\text{nucleons/m}^3}$$

34. If the universe's scale is inversely proportional to the temperature, the scale times the temperature should be constant. If we call the current scale "1," and knowing the current temperature to be about 3 K, then the product of scale and temperature should be about 3. Use Figure 44-30 to estimate the temperature at various times. For purposes of illustration, we assume the universe has a current size of about 10^{10} ly. There will be some variation in the answer due to reading the figure.

(*a*) At $t=10^{6}\,\text{yr}$, the temperature is about 1000 K. Thus the scale is found as follows.

$$(\text{Scale})(\text{Temperature})=3 \quad\rightarrow\quad \text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{1000}=\boxed{3\times10^{-3}} \quad\rightarrow$$

$$\text{Size}\approx(3\times10^{-3})(10^{10}\,\text{ly})=\boxed{3\times10^{7}\,\text{ly}}$$

(*b*) At $t=1$ s, the temperature is about $10^{10}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{10}}=\boxed{3\times10^{-10}} \quad\rightarrow\quad \text{Size}\approx(3\times10^{-10})(10^{10}\,\text{ly})=\boxed{3\,\text{ly}}$$

(*c*) At $t=10^{-6}\,\text{s}$, the temperature is about $10^{13}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{13}}=\boxed{3\times10^{-13}} \quad\rightarrow$$

$$\text{Size}\approx(3\times10^{-13})(10^{10}\,\text{ly})=\boxed{3\times10^{-3}\,\text{ly}}\approx3\times10^{13}\,\text{m}$$

(*d*) At $t=10^{-35}\,\text{s}$, the temperature is about $10^{27}\,\text{K}$.

$$\text{Scale}=\frac{3}{\text{Temperature}}=\frac{3}{10^{27}}=\boxed{3\times10^{-27}}$$

$$\text{Size}\approx(3\times10^{-27})(10^{10}\,\text{ly})=3\times10^{-17}\,\text{ly}\approx\boxed{0.3\,\text{m}}$$

35. We approximate the temperature–energy relationship by $kT = E = mc^2$ as suggested on page 1217.

$$kT = mc^2 \quad \to \quad T = \frac{mc^2}{k}.$$

(*a*) $$T = \frac{mc^2}{k} = \frac{\left(500\,\text{MeV}/c^2\right)c^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{1.38\times10^{-23}\,\text{J/K}} = 6\times10^{12}\,\text{K}$$

From Figure 44-30, this corresponds to a time of $\boxed{\sim 10^{-5}\,\text{s}}$.

(*b*) $$T = \frac{mc^2}{k} = \frac{\left(9500\,\text{MeV}/c^2\right)c^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{1.38\times10^{-23}\,\text{J/K}} = 1\times10^{14}\,\text{K}$$

From Figure 44-30, this corresponds to a time of $\boxed{\sim 10^{-7}\,\text{s}}$.

(*c*) $$T = \frac{mc^2}{k} = \frac{\left(100\,\text{MeV}/c^2\right)c^2\left(1.60\times10^{-13}\,\text{J/MeV}\right)}{1.38\times10^{-23}\,\text{J/K}} = 1\times10^{12}\,\text{K}$$

From Figure 44-30, this corresponds to a time of $\boxed{\sim 10^{-4}\,\text{s}}$.

There will be some variation in the answers due to reading the figure.

36. (*a*) According to the text, near Figure 44-33, the visible matter makes up about one-tenth of the total baryonic matter. The average baryonic density is therefore 10 times the density of visible matter.

$$\begin{aligned}\rho_{\text{baryon}} &= 10\rho_{\text{visible}} = 10\frac{M_{\text{visible}}}{\frac{4}{3}\pi R^3} \\ &= 10\frac{\left(10^{11}\,\text{galaxies}\right)\left(10^{11}\,\text{stars/galaxy}\right)\left(2.0\times10^{30}\,\text{kg/star}\right)}{\frac{4}{3}\pi\left[\left(14\times10^{9}\,\text{ly}\right)\left(9.46\times10^{15}\,\text{m/ly}\right)\right]^3} \\ &= 2.055\times10^{-26}\,\text{kg/m}^3 \approx \boxed{2.1\times10^{-26}\,\text{kg/m}^3}\end{aligned}$$

(*b*) Again, according to the text, dark matter is about 4 times more plentiful than normal matter.

$$\rho_{\text{dark}} = 4\rho_{\text{baryon}} = 4\left(2.055\times10^{-26}\,\text{kg/m}^3\right) \approx \boxed{8.2\times10^{-26}\,\text{kg/m}^3}$$

$\boxed{37.}$ (*a*) From page 1201, a white dwarf with a mass equal to that of the Sun has a radius about the size of the Earth's radius, $\boxed{6380\,\text{km}}$. From page 1202, a neutron star with a mass equal to 1.5 solar masses has a radius of about $\boxed{20\,\text{km}}$. For the black hole, we use the Schwarzschild radius formula.

$$R = \frac{2GM}{c^2} = \frac{2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)3\left(1.99\times10^{30}\,\text{kg}\right)}{\left(3.00\times10^{8}\,\text{m/s}\right)^2} = 8849\text{m} \approx \boxed{8.85\,\text{km}}$$

(*b*) The ratio is $6380 : 20 : 8.85 = 721 : 2.26 : 1 \approx \boxed{700 : 2 : 1}$.

38. The angular momentum is the product of the rotational inertia and the angular velocity.

$$(I\omega)_{\text{initial}} = (I\omega)_{\text{final}} \quad \rightarrow$$

$$\omega_{\text{final}} = \omega_{\text{initial}}\left(\frac{I_{\text{initial}}}{I_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{\frac{2}{5}MR^2_{\text{initial}}}{\frac{2}{5}MR^2_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{R_{\text{initial}}}{R_{\text{final}}}\right)^2 = \left(1\,\text{rev/month}\right)\left(\frac{7\times10^8\,\text{m}}{8\times10^3\,\text{m}}\right)^2$$

$$= 7.66\times10^9\,\text{rev/month} = 7.66\times10^9\,\frac{\text{rev}}{\text{month}}\times\frac{1\,\text{month}}{30\,\text{d}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}} = 2953\,\text{rev/s}$$

$$\approx \boxed{3000\,\text{rev/s}}$$

39. The rotational kinetic energy is given by $\frac{1}{2}I\omega^2$. The final angular velocity, from problem 43, is $7.66\times10^9\,\text{rev/month}$.

$$\frac{K_{\text{final}}}{K_{\text{initial}}} = \frac{\frac{1}{2}I_{\text{final}}\omega^2_{\text{final}}}{\frac{1}{2}I_{\text{initial}}\omega^2_{\text{initial}}} = \frac{\frac{2}{5}MR^2_{\text{final}}\omega^2_{\text{final}}}{\frac{2}{5}MR^2_{\text{initial}}\omega^2_{\text{initial}}} = \left(\frac{R_{\text{final}}\omega_{\text{final}}}{R_{\text{initial}}\omega_{\text{initial}}}\right)^2$$

$$= \left(\frac{\left(8\times10^3\,\text{m}\right)\left(7.66\times10^9\,\text{rev/month}\right)}{\left(7\times10^8\,\text{m}\right)\left(1\,\text{rev/month}\right)}\right)^2 = \boxed{8\times10^9}$$

40. The apparent luminosity is given by Eq. 44-1. Use that relationship to derive an expression for the absolute luminosity, and equate that for two stars.

$$b = \frac{L}{4\pi d^2} \quad \rightarrow \quad L = 4\pi d^2 b$$

$$L_{\substack{\text{distant}\\\text{star}}} = L_{\text{Sun}} \quad \rightarrow \quad 4\pi d^2_{\substack{\text{distant}\\\text{star}}} b_{\substack{\text{distant}\\\text{star}}} = 4\pi d^2_{\text{Sun}} b_{\text{Sun}} \quad \rightarrow$$

$$d_{\substack{\text{distant}\\\text{star}}} = d_{\text{Sun}}\sqrt{\frac{l_{\text{Sun}}}{l_{\substack{\text{distant}\\\text{star}}}}} = \left(1.5\times10^{11}\,\text{m}\right)\sqrt{\frac{1}{10^{-11}}}\left(\frac{1\,\text{ly}}{9.461\times10^{15}\,\text{m}}\right) = \boxed{5\,\text{ly}}$$

41. A: The temperature increases, the luminosity stays the same, and the size decreases.
B: The temperature stays the same, the luminosity decreases, and the size decreases.
C: The temperature decreases, the luminosity increases, and the size increases.

42. The power output is the energy loss divided by the elapsed time.

$$P = \frac{\Delta K}{\Delta t} = \frac{K_{\text{initial}}\left(\text{fraction lost}\right)}{\Delta t} = \frac{\frac{1}{2}I\omega^2\left(\text{fraction lost}\right)}{\Delta t} = \frac{\frac{1}{2}\frac{2}{5}MR^2\omega^2\left(\text{fraction lost}\right)}{\Delta t}$$

$$= \frac{1}{5}\frac{(1.5)\left(1.99\times10^{30}\,\text{kg}\right)\left(8.0\times10^3\,\text{m}\right)^2\left(2\pi\,\text{rad/s}\right)^2\left(1\times10^{-9}\right)}{(1\,\text{d})(24\,\text{h/d})(3600\,\text{s/h})} = 1.74610^{25}\,\text{W} \approx \boxed{1.7\times10^{25}\,\text{W}}$$

43. Use Newton's law of universal gravitation.

$$F = G\frac{m_1 m_2}{r^2} = \left(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2\right)\frac{\left(3\times10^{41}\,\text{kg}\right)^2}{\left[\left(2\times10^6\,\text{ly}\right)\left(9.46\times10^{15}\,\text{m/ly}\right)\right]^2} = 1.68\times10^{28}\,\text{N}$$

$$\approx \boxed{2\times10^{28}\,\text{N}}$$

44. (*a*) Assume that the nucleons make up only 2% of the critical mass density.

$$\text{nucleon mass density} = 0.02\left(10^{-26}\ \text{kg/m}^3\right)$$

$$\text{nucleon number density} = \frac{0.02\left(10^{-26}\ \text{kg/m}^3\right)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 0.12\,\text{nucleon/m}^3$$

$$\text{neutrino number density} = 10^9\left(\text{nucleon number density}\right) = 1.2\times10^8\ \text{neutrino/m}^3$$

$$\frac{0.98\left(10^{-26}\ \text{kg/m}^3\right)}{1.2\times10^8\ \text{neutrino/m}^3} = 8.17\times10^{-35}\,\frac{\text{kg}}{\text{neutrino}}\times\frac{9.315\times10^8\ \text{eV}/c^2}{1.66\times10^{-27}\,\text{kg}} = \boxed{46\,\text{eV}/c^2}$$

(*b*) Assume that the nucleons make up only 5% of the critical mass density.

$$\text{nucleon mass density} = 0.05\left(10^{-26}\ \text{kg/m}^3\right)$$

$$\text{nucleon number density} = \frac{0.05\left(10^{-26}\ \text{kg/m}^3\right)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 0.30\,\text{nucleon/m}^3$$

$$\text{neutrino number density} = 10^9\left(\text{nucleon number density}\right) = 3.0\times10^8\ \text{neutrino/m}^3$$

$$\frac{0.95\left(10^{-26}\ \text{kg/m}^3\right)}{3.0\times10^8\ \text{neutrino/m}^3} = 3.17\times10^{-35}\,\frac{\text{kg}}{\text{neutrino}}\times\frac{9.315\times10^8\ \text{eV}/c^2}{1.66\times10^{-27}\,\text{kg}} = \boxed{18\,\text{eV}/c^2}$$

45. The temperature of each star can be found from Wien's law.

$$\lambda_{\text{P}}T = 2.90\times10^{-3}\,\text{m}\bullet\text{K} \ \rightarrow$$

$$T_{660} = \frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{660\times10^{-9}\,\text{m}} = 4390\,\text{K} \qquad T_{480} = \frac{2.90\times10^{-3}\,\text{m}\bullet\text{K}}{480\times10^{-9}\,\text{m}} = 6040\,\text{K}$$

The luminosity of each star can be found from the H–R diagram.

$$L_{660} \approx 3\times10^{25}\,\text{W} \qquad L_{480} \approx 3\times10^{26}\,\text{W}$$

The Stefan–Boltzmann equation says that the power output of a star is given by $P = \beta AT^4$, where β is a constant, and A is the radiating area. The P in the Stefan–Boltzmann equation is the same as the luminosity L given in Eq. 44-1. Form the ratio of the two luminosities.

$$\frac{L_{480}}{L_{660}} = \frac{\beta A_{480}T_{480}^4}{\beta A_{660}T_{660}^4} = \frac{4\pi r_{480}^2 T_{480}^4}{4\pi r_{660}^2 T_{660}^4} \ \rightarrow \ \frac{r_{480}}{r_{660}} = \sqrt{\frac{L_{480}}{L_{660}}\frac{T_{660}^2}{T_{480}^2}} = \sqrt{\frac{3\times10^{26}\,\text{W}}{3\times10^{25}\,\text{W}}\frac{\left(4390\,\text{K}\right)^2}{\left(6040\,\text{K}\right)^2}} = 1.67$$

The diameters are in the same ratio as the radii.

$$\frac{d_{480}}{d_{660}} = 1.67 \approx \boxed{1.7}$$

The luminosities are fairly subjective, since they are read from the H–R diagram. Different answers may arise from different readings of the H–R diagram.

46. (*a*) The number of parsecs is the reciprocal of the angular resolution in seconds of arc.

$$100\ \text{parsec} = \frac{1}{\phi''} \ \rightarrow \ \phi = \left(0.01''\right)\left(\frac{1'}{60''}\right)\left(\frac{1^\circ}{60'}\right) = \left(2.78\times10^{-6}\right)^\circ \approx \boxed{\left(3\times10^{-6}\right)^\circ}$$

(*b*) We use the Rayleigh criterion, Eq. 35-10, which relates the angular resolution to the diameter of the optical element. We choose a wavelength of 550 nm, in the middle of the visible range.

$$\theta = \frac{1.22\lambda}{D} \rightarrow D = \frac{1.22\lambda}{\theta} = \frac{1.22\left(550\times10^{-9}\,\mathrm{m}\right)}{\left[\left(2.78\times10^{-6}\right)^{\circ}\right]\left(\pi\mathrm{rad}/180^{\circ}\right)} = 13.83\mathrm{m} \approx \boxed{14\,\mathrm{m}}$$

The largest optical telescopes currently in use are about 10 m in diameter.

47. We approximate the temperature–kinetic energy relationship by $kT = K$ as given on page 1217.

$$kT = K \rightarrow T = \frac{K}{k} = \frac{\left(1.96\times10^{12}\mathrm{eV}\right)\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)}{1.38\times10^{-23}\,\mathrm{J/K}} = \boxed{2\times10^{16}\,\mathrm{K}}$$

From Figure 44-30, this is in the $\boxed{\text{hadron era}}$.

48. We assume that gravity causes a centripetal force on the gas. Solve for the speed of the rotating gas, and use Eq. 44-6.

$$F_{\text{gravity}} = F_{\text{centripetal}} \rightarrow G\frac{m_{\text{gas}}m_{\substack{\text{black}\\\text{hole}}}}{r^2} = \frac{m_{\text{gas}}v_{\text{gas}}^2}{r} \rightarrow$$

$$v_{\text{gas}} = \sqrt{G\frac{m_{\substack{\text{black}\\\text{hole}}}}{r}} = \sqrt{\left(6.67\times10^{-11}\,\mathrm{N{\bullet}m^2/kg^2}\right)\frac{\left(2\times10^{9}\right)\left(1.99\times10^{30}\mathrm{kg}\right)}{\left(68\,\mathrm{ly}\right)\left(\frac{9.46\times10^{15}\mathrm{m}}{1\,\mathrm{ly}}\right)}} = 6.42\times10^{5}\,\mathrm{m/s}$$

$$z \approx \frac{v}{c} = \frac{6.42\times10^{5}\,\mathrm{m/s}}{3.00\times10^{8}\,\mathrm{m/s}} = 2.14\times10^{-3} \approx \boxed{2\times10^{-3}}$$

$\boxed{49.}$ (*a*) To find the energy released in the reaction, we calculate the *Q*-value for this reaction. From Eq. 42-2a, the *Q*-value is the mass energy of the reactants minus the mass energy of the products. The masses are found in Appendix F.

$$Q = 2m_{\mathrm{C}}c^2 - m_{\mathrm{Mg}}c^2 = \left[2\left(12.000000\,\mathrm{u}\right) - 23.985042\,\mathrm{u}\right]c^2\left(931.5\,\mathrm{Mev}/c^2\right) = \boxed{13.93\,\mathrm{MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 41-1. Each nucleus will have half the total kinetic energy.

$$r = \left(1.2\times10^{-15}\mathrm{m}\right)\left(A\right)^{1/3} = \left(1.2\times10^{-15}\mathrm{m}\right)\left(12\right)^{1/3} \qquad U = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{nucleus}}^2}{2r}$$

$$K = \tfrac{1}{2}U = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{nucleus}}^2}{2r}$$

$$= \tfrac{1}{2}\left(8.988\times10^{9}\,\mathrm{N{\bullet}n^2/C^2}\right)\frac{\left(6\right)^2\left(1.60\times10^{-19}\mathrm{C}\right)^2}{2\left(1.2\times10^{-15}\mathrm{m}\right)\left(12\right)^{1/3}}\left(\frac{1\,\mathrm{MeV}}{1.60\times10^{-13}\mathrm{J}}\right) = 4.711\,\mathrm{MeV}$$

$$\approx \boxed{4.7\,\mathrm{MeV}}$$

(*c*) We approximate the temperature–kinetic energy relationship by $kT = K$ as given on page 1217.

$$kT = K \rightarrow T = \frac{K}{k} = \frac{\left(4.711\,\mathrm{MeV}\right)\left(1.60\times10^{-13}\,\mathrm{J/MeV}\right)}{1.38\times10^{-23}\,\mathrm{J/K}} = \boxed{5.5\times10^{10}\,\mathrm{K}}$$

50. (*a*) Find the *Q*-value for this reaction. From Eq. 42-2a, the *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} \rightarrow {}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He}$$

$$Q = 2m_{\mathrm{O}}c^2 - m_{\mathrm{Si}}c^2 - m_{\mathrm{He}}c^2 = \left[2(15.994915\,\mathrm{u}) - 27.976927\,\mathrm{u} - 4.002603\right]c^2\left(931.5\,\mathrm{Mev}/c^2\right)$$

$$= \boxed{9.594\,\mathrm{MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 41-1. Each nucleus will have half the total kinetic energy.

$$r = \left(1.2\times10^{-15}\,\mathrm{m}\right)(A)^{1/3} = \left(1.2\times10^{-15}\,\mathrm{m}\right)(16)^{1/3} \qquad U = \frac{1}{4\pi\varepsilon_0}\frac{q^2_{\mathrm{nucleus}}}{2r}$$

$$K_{\mathrm{nucleus}} = \tfrac{1}{2}U = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2_{\mathrm{nucleus}}}{2r}$$

$$= \tfrac{1}{2}\left(8.988\times10^9\,\mathrm{N{\bullet}n^2/C^2}\right)\frac{(8)^2\left(1.60\times10^{-19}\,\mathrm{C}\right)^2}{2\left(1.2\times10^{-15}\,\mathrm{m}\right)(16)^{1/3}}\times\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}} = 7.609\,\mathrm{MeV}$$

$$\approx \boxed{7.6\,\mathrm{MeV}}$$

(*c*) We approximate the temperature–kinetic energy relationship by $kT = K$ as given on page 1217.

$$kT = K \rightarrow T = \frac{K}{k} = \frac{\left(7.609\times10^6\mathrm{eV}\right)\left(\frac{1.60\times10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right)}{1.38\times10^{-23}\,\mathrm{J/K}} = \boxed{8.8\times10^{10}\,\mathrm{K}}$$

51. We treat the energy of the photon as a "rest mass," and so $m_{\mathrm{photon}}\,"="\,E_{\mathrm{photon}}/c^2$. To just escape from a spherical mass *M* of radius *R*, the energy of the photon must be equal to the magnitude of the gravitational potential energy at the surface.

$$E_{\mathrm{photon}} = \frac{GMm_{\mathrm{photon}}}{R} \rightarrow R = \frac{GMm_{\mathrm{photon}}}{E_{\mathrm{photon}}} = \frac{GM\left(E_{\mathrm{photon}}/c^2\right)}{E_{\mathrm{photon}}} = \boxed{\frac{GM}{c^2}}$$

52. We use the Sun's mass and given density to calculate the size of the Sun.

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3_{\mathrm{Sun}}} \rightarrow$$

$$r_{\mathrm{Sun}} = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left[\frac{3\left(1.99\times10^{30}\mathrm{kg}\right)}{4\pi\left(10^{-26}\,\mathrm{kg/m^3}\right)}\right]^{1/3} = 3.62\times10^{18}\,\mathrm{m}\left(\frac{1\,\mathrm{ly}}{9.46\times10^{15}\,\mathrm{m}}\right) = 382\,\mathrm{ly} \approx \boxed{400\,\mathrm{ly}}$$

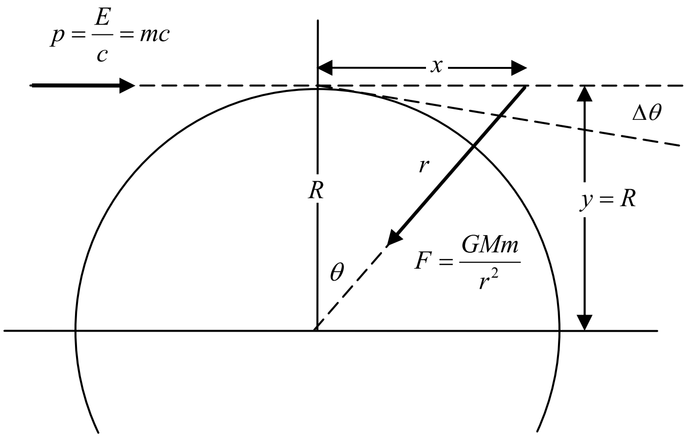
$$\frac{r_{\mathrm{Sun}}}{d_{\mathrm{Earth\text{-}Sun}}} = \frac{3.62\times10^{18}\mathrm{m}}{1.50\times10^{11}\mathrm{m}} \approx \boxed{2\times10^7} \;;\; \frac{r_{\mathrm{Sun}}}{d_{\mathrm{galaxy}}} = \frac{382\,\mathrm{ly}}{100{,}000\,\mathrm{ly}} \approx \boxed{4\times10^{-3}}$$

53. We approximate the temperature–kinetic energy relationship by $kT = K$ as given on page 1217.

$$kT = K \quad \rightarrow \quad T = \frac{K}{k} = \frac{\left(14\times10^{12}\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.38\times10^{-23}\text{ J/K}} = 1.6\times10^{17}\text{ K}$$

From Figure 44-30, this might correspond to a time around $\boxed{10^{-15}\text{s}}$. Note that this is just a very rough estimate due to the qualitative nature of Figure 44-30.

54. (a) We consider the photon as entering from the left, grazing the Sun, and moving off in a new direction. The deflection is assumed to be very small. In particular, we consider a small part of the motion in which the photon moves a horizontal distance $dx = cdt$ while located at (x,y) relative to the center of the Sun. Note that $y \approx R$ and $r^2 = x^2 + y^2$. If the photon has energy E, it will have a "mass" of $m = E/c^2$, and a momentum of magnitude $p = E/c = mc$. To find the change of momentum in the y-direction, we use the impulse produced by the y-component of the gravitational force.



$$dp_y = F_y dt = -\frac{GMm}{r^2}\cos\theta\frac{dx}{c} = -\frac{GMm}{r^2}\frac{R}{r}\frac{dx}{c} = -\frac{GMmR}{c}\frac{dx}{\left(x^2+R^2\right)^{3/2}}$$

To find the total change in the y-momentum, we integrate over all x (the entire path of the photon). We use an integral from Appendix B-4.

$$\Delta p_y = \int_{-\infty}^{\infty} -\frac{GMmR}{c}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = -\frac{GMmR}{c}\frac{x}{R^2\left(x^2+R^2\right)^{1/2}}\Bigg|_{-\infty}^{\infty} = -\frac{2GMm}{cR} = -\frac{2GMp}{c^2R}$$

The total magnitude of deflection is the change in momentum divided by the original momentum.

$$\Delta\theta = \frac{\left|\Delta p_y\right|}{p} = \frac{\frac{2GMp}{c^2R}}{p} = \boxed{\frac{2GM}{c^2R}}$$

(*b*) We use data for the Sun.

$$\Delta\theta = \frac{2GM}{c^2R} = \frac{2\left(6.67\times10^{-11}\frac{\text{N•m}^2}{\text{kg}^2}\right)\left(1.99\times10^{30}\text{kg}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2\left(6.96\times10^8\text{m}\right)}$$

$$= 4.238\times10^{-6}\text{rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{0.87^\circ}$$

$\boxed{55.}$ Because Venus has a more negative apparent magnitude, $\boxed{\text{Venus is brighter}}$. We write the logarithmic relationship as follows, letting *m* represent the magnitude and *b* the brightness.

$$m = k\log b \;\; ; \;\; m_2 - m_1 = k\left(\log b_2 - \log b_1\right) = k\log\left(b_2/b_1\right) \;\;\rightarrow$$

$$k = \frac{m_2 - m_1}{\log\left(b_2/b_1\right)} = \frac{+5}{\log\left(0.01\right)} = -2.5$$

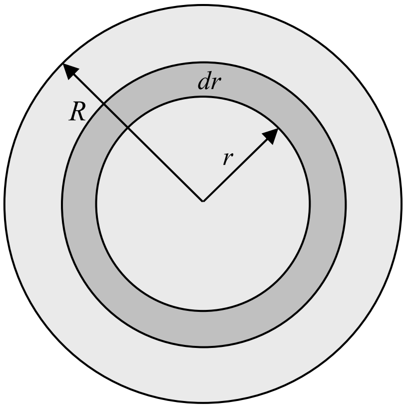
$$m_2 - m_1 = k\log\left(b_2/b_1\right) \;\rightarrow\; \frac{b_2}{b_1} = 10^{\frac{m_2 - m_1}{k}} \;\rightarrow\; \frac{b_{\text{Venus}}}{b_{\text{Sirius}}} = 10^{\frac{m_{\text{Venus}} - m_{\text{Sirius}}}{-2.5}} = 10^{\frac{-4.4+1.4}{-2.5}} = \boxed{16}$$

56. If there are *N* nucleons, we assume that there are approximately $\frac{1}{2}N$ neutrons and $\frac{1}{2}N$ protons. Thus, for the star to be neutral, there would also be $\frac{1}{2}N$ electrons.

(*a*) From Eq. 40-12 and 40-13, we find that if all electron levels are filled up to the Fermi energy E_{F}, the average electron energy is $\frac{3}{5}E_{\text{F}}$.

$$E_{\text{e}} = N_{\text{e}}\left(\tfrac{3}{5}E_{\text{F}}\right) = \left(\tfrac{1}{2}N\right)\tfrac{3}{5}\frac{h^2}{8m_{\text{e}}}\left(\frac{3}{\pi}\frac{N_{\text{e}}}{V}\right)^{2/3} = \boxed{\frac{3}{5}\left(\frac{1}{2}N\right)\frac{h^2}{8m_{\text{e}}}\left(\frac{3}{\pi}\frac{N}{2V}\right)^{2/3}}$$

(*b*) The Fermi energy for nucleons would be a similar expression, but the mass would be the mass of a nucleon instead of the mass of the electron. Nucleons are about 2000 times heavier than electrons, so the Fermi energy for the nucleons would be on the order of 1/1000 the Fermi energy for the electrons. We will ignore that small correction.



To calculate the potential energy of the star, think about the mass in terms of shells. Consider the inner portion of the star with radius *r* < *R* and mass *m*, surrounded by a shell of thickness *dr* and mass *dM*. See the diagram. From Gauss's law applied to gravity, the gravitational effects of the inner portion of the star on the shell are the same as if all of its mass were at the geometric center. Likewise, the spherically-symmetric outer portion of the star has no gravitational effect on the shell. Thus the gravitational energy of the inner portion–shel) combination is given by a form of Eq. 8-17, $dU = -G\dfrac{mdM}{r}$. The density of the star is given by $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$. We use that density to calculate the masses, and then integrate over the full radius of the star to find the total gravitational energy of the star.

$$m = \rho\left(\tfrac{4}{3}\pi r^3\right) = \frac{M}{\frac{4}{3}\pi R^3}\left(\tfrac{4}{3}\pi r^3\right) = M\frac{r^3}{R^3}$$

$$dM = \rho\left(4\pi r^2 dr\right) = \frac{M}{\frac{4}{3}\pi R^3}\left(4\pi r^2 dr\right) = \frac{3Mr^2}{R^3}dr$$

$$dU = -G\frac{mdM}{r} = -G\frac{M\dfrac{r^3}{R^3}\dfrac{3Mr^2}{R^3}dr}{r} = -\frac{3GM^2}{R^6}r^4 dr$$

$$U=\int_0^R\left(-\frac{3GM^2}{R^6}r^4\right)dr=-\frac{3GM^2}{R^6}\int_0^R\left(r^4\right)dr=-\frac{3GM^2}{R^6}\frac{R^5}{5}=\boxed{-\frac{3}{5}\frac{GM^2}{R}}$$

(*c*) The total energy is the sum of the two terms calculated above. The mass of the star is primarily due to the nucleons, and so $M=Nm_{\text{nucleon}}$.

$$E_{\text{total}}=E_{\text{e}}+U=\frac{3}{5}\left(\frac{1}{2}N\right)\frac{h^2}{8m_{\text{e}}}\left(\frac{3}{\pi}\frac{N}{2V}\right)^{2/3}-\frac{3}{5}\frac{GM^2}{R}$$

$$=\frac{3h^2}{80m_{\text{e}}}\left(\frac{M}{m_{\text{nucleon}}}\right)\left(\frac{3}{\pi}\frac{M}{2m_{\text{nucleon}}\,\frac{4}{3}\pi R^3}\right)^{2/3}-\frac{3}{5}\frac{GM^2}{R}=\boxed{\frac{9^{2/3}3h^2M^{5/3}}{320\pi^{4/3}m_{\text{e}}m_{\text{nucleon}}^{5/3}R^2}-\frac{3}{5}\frac{GM^2}{R}}$$

Let $a=\dfrac{9^{2/3}3h^2M^{5/3}}{320\pi^{4/3}m_{\text{e}}m_{\text{nucleon}}^{5/3}}$ and $b=\dfrac{3}{5}GM^2$, so $E_{\text{total}}=\dfrac{a}{R^2}-\dfrac{b}{R}$. We set $dE_{\text{total}}/dR=0$ to find the equilibrium radius.

$$\frac{dE_{\text{total}}}{dR}=-\frac{2a}{R^3}+\frac{b}{R^2}=0\ \rightarrow\ R_{\text{eq}}=\frac{2a}{b}=\frac{2\dfrac{9^{2/3}3h^2M^{5/3}}{320\pi^{4/3}m_{\text{e}}m_{\text{nucleon}}^{5/3}}}{\dfrac{3}{5}GM^2}=\boxed{\frac{9^{2/3}h^2}{32\pi^{4/3}GM^{1/3}m_{\text{e}}m_{\text{nucleon}}^{5/3}}}$$

We evaluate the equilibrium radius using the Sun's mass.

$$R_{\text{eq}}=\frac{9^{2/3}h^2}{32\pi^{4/3}GM^{1/3}m_{\text{e}}m_{\text{nucleon}}^{5/3}}$$

$$=\frac{9^{2/3}\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{32\pi^{4/3}\left(6.67\times10^{-11}\,\dfrac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(2.0\times10^{30}\,\text{kg}\right)^{1/3}\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.67\times10^{-27}\,\text{kg}\right)^{5/3}}$$

$$=7.178\times10^6\,\text{m}\approx\boxed{7.2\times10^3\,\text{km}}$$

57. There are *N* neutrons. The mass of the star is due only to neutrons, and so $M=Nm_{\text{n}}$. From Eqs. 40-12 and 40-13, we find that if all energy levels are filled up to the Fermi energy E_{F}, the average energy is $\frac{3}{5}E_{\text{F}}$. We follow the same procedure as in Problem 56. The expression for the gravitational energy does not change.

$$E_{\text{n}}=N_{\text{n}}\left(\tfrac{3}{5}E_{\text{F}}\right)=\left(N\right)\tfrac{3}{5}\frac{h^2}{8m_{\text{n}}}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}=\left(\frac{M}{m_{\text{n}}}\right)\frac{3h^2}{40m_{\text{n}}}\left(\frac{3}{\pi}\frac{M}{\frac{4}{3}\pi R^3m_{\text{n}}}\right)^{2/3}=\frac{3\left(18\right)^{2/3}h^2M^{5/3}}{160\pi^{4/3}m_{\text{n}}^{8/3}R^2}$$

$$E_{\text{total}}=E_{\text{n}}+U=\frac{3\left(18\right)^{2/3}h^2M^{5/3}}{160\pi^{4/3}m_{\text{n}}^{8/3}R^2}-\frac{3}{5}\frac{GM^2}{R}$$

Let $a=\dfrac{3\left(18\right)^{2/3}h^2M^{5/3}}{160\pi^{4/3}m_{\text{n}}^{8/3}}$ and $b=\dfrac{3}{5}GM^2$, so $E_{\text{total}}=\dfrac{a}{R^2}-\dfrac{b}{R}$. We set $dE_{\text{total}}/dR=0$ to find the equilibrium radius.

$$\frac{dE_{\text{total}}}{dR}=-\frac{2a}{R^3}+\frac{b}{R^2}=0\ \rightarrow\ R_{\text{eq}}=\frac{2a}{b}=\frac{2\dfrac{3(18)^{2/3}h^2M^{5/3}}{160\pi^{4/3}m_{\text{n}}^{8/3}}}{\dfrac{3}{5}GM^2}=\frac{(18)^{2/3}h^2}{16\pi^{4/3}GM^{1/3}m_{\text{n}}^{8/3}}$$

We evaluate the equilibrium radius for a mass of 1.5 solar masses.

$$R_{\text{eq}}=\frac{(18)^{2/3}h^2}{16\pi^{4/3}GM^{1/3}m_{\text{n}}^{8/3}}$$

$$=\frac{18^{2/3}\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{16\pi^{4/3}\left(6.67\times10^{-11}\,\dfrac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left[1.5\left(2.0\times10^{30}\text{kg}\right)\right]^{1/3}\left(1.67\times10^{-27}\,\text{kg}\right)^{8/3}}$$

$$=1.086\times10^4\,\text{m}\approx\boxed{11\,\text{km}}$$

58. We must find a combination of c, G, and $\hbar$ that has the dimensions of time. The dimensions of c are $\left[\frac{L}{T}\right]$, the dimensions of G are $\left[\frac{L^3}{MT^2}\right]$, and the dimensions of $\hbar$ are $\left[\frac{ML^2}{T}\right]$.

$$t_P=c^\alpha G^\beta\hbar^\gamma\ \rightarrow\ [T]=\left[\frac{L}{T}\right]^\alpha\left[\frac{L^3}{MT^2}\right]^\beta\left[\frac{ML^2}{T}\right]^\gamma=[L]^{\alpha+3\beta+2\gamma}[M]^{\gamma-\beta}[T]^{-\alpha-2\beta-\gamma}$$

$$\alpha+3\beta+2\gamma=0\ ;\ \gamma-\beta=0\ ;\ -\alpha-2\beta-\gamma=1\ \rightarrow\ \alpha+5\beta=0\ ;\ \alpha=-1-3\beta\ \rightarrow$$
$$-5\beta=-1-3\beta\ \rightarrow\ \beta=\tfrac{1}{2}\ ;\ \gamma=\tfrac{1}{2}\ ;\ \alpha=-\tfrac{5}{2}$$

$$t_P=c^{-5/2}G^{1/2}\hbar^{1/2}=\sqrt{\frac{G\hbar}{c^5}}=\sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\dfrac{1}{2\pi}\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(3.00\times10^8\,\text{m/s}\right)^5}}=\boxed{5.38\times10^{-44}\text{s}}$$