Instructor's Manual/Test Bank
for

THE LOGIC BOOK

Fourth Edition

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OVERVIEW

In *The Logic Book* we have tried to develop elementary symbolic logic in a way that is at once fully rigorous and readily accessible to the beginning student. Important concepts are formally defined and informally explicated. These key concepts are illustrated both with multiple examples and in numerous exercises. Solutions to unstarred exercises are given in the solutions manual; solutions to starred exercises are given in this instructor's manual. All of these features combine to make *The Logic Book* clear, comprehensive, and self-contained.

There are several different "paths" through *The Logic Book* that an instructor can follow. Which is most appropriate will depend upon the aims of the course, the instructor's interests, and the amount of time available. If an instructor has two quarters or two semesters (or one semester with very good students), it is possible to work through the entire book. The instructor who does not want to emphasize metatheory can simply omit Chapters 6 and 11. The chapters on truth-trees and the chapters on derivations are independent, so that it is possible to do truth-trees but not derivations and the reverse. The chapters on truth-trees do depend on the chapters on semantics; that is, Chapter 4 depends on Chapter 3, and Chapter 9 depends on Chapter 8. And, although the standard practice is to do semantics before doing the corresponding derivations, the opposite order is possible in both sentential logic and predicate logic. The sections at the ends of Chapters 3, 5, 8, and 10 can also be omitted.

Support software can be found on The Bertie/Twootie Home Page <http://www.ucc.uconn.edu/~wwwphil/software.html>. Bertie3 helps students with derivations and Twootie helps with truth trees. These programs were developed by Professor Austen Clark, Department of Philosophy U-54, University of Connecticut, Storrs, CT 06269–2054 USA. His email is austen.clark@uconn.edu. This site also contains information about developing self-paced courses and using test-generating software.

THE OPTION OF A SELF-PACED COURSE

Because *The Logic Book* contains a comprehensive development of elementary symbolic logic and includes a great many exercises (with solutions), it is ideal for use in a self-paced course. The self-paced approach to teaching is sometimes called "the Keller Plan" or "Personalized System of Instruction." In this system students are allowed to learn at their own pace. The course is divided into units that may be entire chapters or sections of chapters. After a student has studied the material in the first unit, a test on the material in that unit is taken. If the test is passed, the student proceeds to the next unit. If the test is not passed, the student continues to work on the unit until a test on that unit is passed. The more units a student completes, the higher his or her grade is for the course. Because a student can take more than one test on a given unit, the standard for passing is usually set rather high. That is, the student is expected to show a thorough mastery of a subject before proceeding to the next unit. Obviously, this approach can be easily modified to fit local needs and practices. For example, a self-paced course can consist of many small units or a few large units. Tests can be graded either "pass/fail" or "A," "B", . . . , "F". The former option would be appropriate if students are allowed to take an unlimited number of tests on a given unit, the latter if this number is limited. And lectures can be given to supplement the text and to help students who are having difficulties.

Instructors of self-paced courses commonly report that students in such courses work harder and learn more than do students in traditional courses. The explanation given is that in such courses tests are a teaching tool, as well as a measurement tool, and faster students are not held back by slower students while slower students are not intimidated by faster students.

Teaching logic according to a self-paced format usually requires the use of student assistants (to grade tests and discuss test performance with students) and a large supply of different tests for each unit. As an aid to those who wish to develop such courses, we include in this manual four tests for each chapter, except those on metatheory, of *The Logic Book*. Instructors are free to duplicate these tests, as well as to modify them to suit their own needs. (They may also duplicate the answers for starred exercises found in this manual.) Further tests, as well as advice on how to set up a self-paced course, can be obtained from James Moor, Philosophy Department, Dartmouth College, Hanover, New Hampshire 03755.

TESTS AND ANSWERS

CHAPTER ONE

LOGIC TEST 1

1. Define the following:
a. Logical truth
b. Deductive validity
c. Logical equivalence

2. For each of the following, indicate whether it has a truth-value (i.e., is either true or false). If it does not, explain why not.
a. Mars is the planet closest to the sun.
b. May you inherit a hotel and die in every room.
c. Police cars use left lane.
d. Ask not for whom the bell tolls.

3. Which of the following passages are best understood as arguments? For those that are, recast the passage in standard form and determine whether the resulting argument is deductively valid. Evaluate the inductive strength of those that are not deductively valid.
a. That boxer cannot be hurt too badly because he is still moving well and throwing hard punches.
b. One should not open the bidding with a five-point bridge hand.

c. Either the members of the electoral college do their job or they don't. Thus the members of the electoral college are either useless or dangerous, for if they do their job, they are useless, and if they don't, they are dangerous.

4. For each of the following, circle **T** if the sentence is true or **F** if the sentence is false.

T F a. A set all of whose members are true is logically consistent.
T F b. All valid arguments have true conclusions.
T F c. Every sound argument is deductively valid.
T F d. Every argument that has inductive strength is also deductively valid.
T F e. A sound argument is a valid argument with a true conclusion.

5. Give an example of each of the following where one exists. If there can be no such example, explain why.

a. A valid argument that has true premises and a true conclusion.
b. A sound argument with a false conclusion.
c. A consistent set all of whose members are false.

ANSWERS

1. a. A sentence is *logically true* if and only if it is not possible for the sentence to be false.

b. An argument is *deductively valid* if and only if it is not possible for the premises to be true and the conclusion false. An argument is *deductively invalid* if and only if it is not deductively valid.

c. The members of a pair of sentences are *logically equivalent* if and only if it is not possible for one of the sentences to be true while the other sentence is false.

2. a. This sentence does have a truth-value (as it happens, the value 'false').

b. This sentence is a curse. It has no truth-value.

c. This marginally grammatical sentence might be an instruction to police officers, in which case it does not have a truth-value. It might also be a warning to motorists, in which case it does have a truth-value.

d. This is a bit of advice. It has no truth-value.

3. a. There is an argument here:

That boxer is moving well and throwing hard punches.
___
That boxer cannot be hurt too badly.

The argument is deductively invalid. (The boxer may have injuries that are not apparent.) It does have considerable inductive strength.

b. This is not an argument. It is a claim, probably a true one, that certain behavior in bridge is inadvisable.

c. This is an argument:

Either the members of the electoral college do their job or they don't.

If the members of the electoral college do their job, then they are useless.

If the members of the electoral college don't do their job, then they are dangerous.

The members of the electoral college are either useless or dangerous.

This argument is deductively valid. It is not possible for the premises to be true and the conclusion false.

4. a. **T**

b. **F** A valid argument can have a false conclusion so long as at least one of its premises is also false. Here is an example:

The largest city in each state is the capital of that state.

Los Angeles is the largest city in California.

Los Angeles is the capital of California.

The first premise and the conclusion of this argument are both false.

c. **T**

d. **F** Many arguments with some degree of inductive strength are deductively invalid. See, for example, the above argument concerning a boxer.

e. **F** Sound arguments do have true conclusions, but not all valid arguments with true conclusions are sound, for not all such arguments have true premises. An example is

The largest city in each state is the capital of that state.

Denver is the largest city in Colorado.

Denver is the capital of Colorado.

This argument is valid and it has a true conclusion, but it is not sound.

5. a. A valid argument that has true premises and a true conclusion:

Honolulu is the largest city in Hawaii and the capital of Hawaii.

Honolulu is the capital of Hawaii.

b. A sound argument with a false conclusion:

There can be no such argument, for a sound argument is by definition a valid argument with true premises, and a valid argument is one where it is impossible for the premises to be true and the conclusion false.

c. A consistent set all of whose members are false:

{Los Angeles is the capital of California, Miami is the capital of Florida}

LOGIC TEST 2

1. Define the following:
a. Logical indeterminacy
b. Deductive soundness
c. Logical consistency

2. For each of the following, indicate whether it has a truth-value (i.e., is either true or false). If it does not, explain why not.
a. Count no man lucky until he is dead.
b. Whenever a general runs for President, he or she gets elected.
c. I promise to be on time for class.
d. There once was a barber of Seville who shaved all and only those citizens of Seville who shaved but did not shave themselves.

3. Which of the following passages are best understood as arguments? For those that are, recast the passage in standard form and determine whether the resulting argument is deductively valid. Evaluate the inductive strength of those that are not deductively valid.
a. This ship is going to sink. She has been holed in three places and the pumps aren't working. No ship with that kind of damage can stay afloat.
b. Black clouds are moving in and the barometer is dropping rapidly, so there is a storm coming.
c. Truman was elected because he had experience, Eisenhower because he was a war hero, Kennedy because he was young, and Nixon for no reason at all.

4. For each of the following, circle **T** if the sentence is true or **F** if the sentence is false.
T F a. Every argument whose conclusion is logically true is valid.
T F b. Every argument with true premises and a true conclusion is valid.
T F c. Every sound argument has a true conclusion.
T F d. As explicated in Chapter 1, the term 'sound' applies to both arguments and sentences.
T F e. Every pair of true sentences is a pair of logically equivalent sentences.

5. Give an example of each of the following where one exists. If there can be no such example, explain why.
a. A valid argument with at least one false premise and a true conclusion.
b. An invalid argument with true premises and a true conclusion.
c. A consistent set containing at least one logically false sentence.

ANSWERS

1. a. A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

b. An argument is *deductively sound* if and only if it is deductively valid and all of its premises are true. An argument is *deductively unsound* if and only if it is not deductively sound.

c. A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true at the same time. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

2. a. This sentence does not have a truth-value. It is an ancient Greek adage that, grammatically, is an instruction or command.

b. This sentence does have a truth-value. (It is false. George B. McClellan ran for President in 1864 and lost to Lincoln.)

c. Whenever this sentence is said or written, in a normal context, it thereby becomes true. So it does have a truth-value. (Sentences of this sort, which cannot be said falsely, are called *performatives.*)

d. This sentence does have a truth-value: It may be either true or false. That is, we do not know whether there had been such a barber, but we do know that if such a barber existed, he or she was a person who did not shave.

3. a. There is an argument here:

This ship has been holed in three places and the pumps aren't working.

No ship that has been holed in three places and whose pumps are not working can stay afloat.

This ship is going to sink.

This argument is deductively valid. It is impossible for the premises to be true and the conclusion false.

b. There is an argument here:

Black clouds are moving in and the barometer is dropping rapidly.

There is a storm coming.

This argument is not deductively valid. It is possible that the premise is true but that a storm will not materialize. However, the argument does have inductive strength. Black clouds and a falling barometer are indicators of a coming storm.

c. There is no argument here. The passage is best construed as a commentary on the American political system.

4. a. **T**

b. **F** Many such arguments are invalid. Here is an example:

Kennedy was President and a Democrat.

Roosevelt was President and a Democrat.

Carter was President and a Democrat.

Clinton is President.

Clinton is a Democrat.

It is possible for the premises of this argument to be true and the conclusion false. This would be the case were Clinton to change his party affiliation and become a Republican. Note that the premises do not negate this possibility.

c. **T**

d. **F** The term *sound* applies only to arguments.

e. **F** The sentences 'Clinton defeated Bush' and 'Reagan defeated Carter' are both true, but this alone does not make them logically equivalent. It is possible for one of them to be true and the other false. For example, had Carter succeeded in rescuing the hostages in Iran he might have defeated Reagan, in which case 'Clinton defeated Bush' would be true and 'Reagan defeated Carter' false.

5. a. Anchorage is the capital of Alaska and Anchorage is a seaport.

The capital of Alaska is a seaport.

The premise is false: Juneau, not Anchorage, is the capital of Alaska. But Juneau is a seaport, so the conclusion is true. Therefore the argument is valid, for it is impossible for the premise to be true and the conclusion false.

b. Some women are lawyers.

Some lawyers are dishonest.

Some women are dishonest.

The premises and the conclusion are all true, but the argument is deductively invalid. It is possible for the premises to be true but the conclusion false: This would be the case if all women were honest (and accordingly the only lawyers who are dishonest would be males).

c. There can be no such set. If a sentence is logically false, then it cannot be true. Therefore not *all* the members of a set that has that sentence as a member can be true, so such a set must be inconsistent.

LOGIC TEST 3

1. Define the following:

a. Argument

b. Logical falsity

c. Deductive validity

2. For each of the following, indicate whether it has a truth-value (i.e., is either true or false). If it does not, explain why not.

a. Who's in charge here anyway?

b. No pain, no gain.

c. Never look a gift horse in the mouth.

d. Sentence **2.**c. is true.

3. Which of the following passages are best understood as arguments? For those that are, recast the passage in standard form and determine whether the resulting argument is deductively valid. Evaluate the inductive strength of those that are not deductively valid.

a. The stock market has risen dramatically, but wages have not. Whenever the stock market rises, the upper class benefits. Whenever wages are flat, the lower and middle classes are hurt. So if the present trend continues, the social fabric of the country will suffer.

b. FDR was a Democrat and didn't balance the budget. Kennedy was a Democrat and didn't balance the budget. Carter was a Democrat and didn't balance the budget. Clinton is a Democrat. So the budget won't be balanced during Clinton's administration.

c. War is too important to leave to the generals and peace is too important to leave to the politicians.

4. For each of the following, circle **T** if the sentence is true of **F** if the sentence is false.

T F a. Every pair of logically true sentences is a pair of logically equivalent sentences.

T F b. Every argument has two premises and one conclusion.

T F c. Every set that contains at least one false sentence is logically inconsistent.

T F d. Every valid argument has a true conclusion.

T F e. If a set is consistent then every member of the set is true.

5. Give an example of each of the following where one exists. If there can be no such example, explain why.

a. A valid argument with a logically false conclusion.

b. An inconsistent set at least one member of which is logically true.

c. A sentence that is neither logically true, nor logically false, nor logically indeterminate.

ANSWERS

1. a. An *argument* is a set of sentences one of which (the conclusion) is taken to be supported by the remaining sentences (the premises).

b. A sentence is *logically false* if and only if it is not possible for the sentence to be true.

c. An argument is *deductively valid* if and only if it is not possible for the premises to be true and the conclusion false.

2. a. This sentence is a question, hence it is neither true nor false.

b. This sentence is short for something like 'If there is no pain involved, then there is no gain to be had' and as such does have a truth-value.

c. This sentence is an imperative or command. It gives instructions, not information. Hence it is neither true nor false.

d. This sentence says, falsely, that the preceding sentence is true. Therefore it does have a truth-value (false).

3. a. There is an argument here:

The stock market has risen dramatically, but wages have not

Whenever the stock market rises, the upper class benefits.

Whenever wages are flat, the lower and middle classes are hurt.

If the present trend continues, the social fabric of the country will suffer.

The argument is deductively invalid, for no explicit connection is given between the upper classes benefiting, the lower and middle classes being hurt, and the social fabric of the country suffering. However, the argument does have considerable inductive strength. It is probable that the social fabric of the country will suffer if the lower and middle classes are hurt while the upper class benefits.

b. There is an argument here:

FDR was a Democrat and didn't balance the budget.

Kennedy was a Democrat and didn't balance the budget.

Carter was a Democrat and didn't balance the budget.

Clinton is a Democrat.

The budget won't be balanced during Clinton's administration.

The argument is deductively invalid. Even though Democratic Presidents of the recent past have not balanced the budget, it is possible that Clinton will be different, that he will balance the budget. The argument does have some inductive strength. That recent Democratic Presidents have not balanced the budget, together with the fact that Clinton is a Democrat, gives some reason to believe that Clinton will not balance the budget.

c. There is no argument here. This sentence makes two claims: one about war and generals, the other about peace and politicians.

4. a. **T**

b. **F** All arguments do have one conclusion, but they can have any number of premises.

c. **F** The set whose only members are 'Ford succeeded Nixon' and 'Bush defeated Clinton' contains two sentences, one of which is false. But the set is not inconsistent. It is possible, had history gone other than it did, that Bush defeated Clinton. So it is possible that all the members of this set are true, and hence the set is consistent.

d. **F** In a valid argument, the conclusion must be true *if* the premises are all true, but if one or more premises is false, the conclusion may also be false. For example:

Every president elected in the 20th century has been a Democrat.

Reagan was elected in the 20th century.

Reagan was a Democrat

This is a valid argument with one false premise and a false conclusion.

e. **F** The set given in the answer to **4.**c above is consistent, but has one false member.

5. a. Everyone in the class will pass and someone in the class will not pass.

The final will, and will not, be given on Friday.

The conclusion of this argument is logically false, but so is the premise. Therefore it is impossible for the premise to be true, and thus impossible for the premise to be true *and* the conclusion false. So the argument is deductively valid.

b. Consider the set whose three members are 'Samantha will graduate', 'Samantha will not graduate', and 'Either Samantha will graduate or she will not graduate'. The last listed member is logically true, but the set is inconsistent, for it is impossible for both the first and second listed members to be true at the same time.

c. This text is devoted to the study of the relations among sentences that are either true or false, and each such sentence is either logically true, logically false, or logically indeterminate. But there are sentences falling outside the purview of this text—for example questions and commands—that are neither true nor false and hence neither logically true, nor logically false, nor logically indeterminate. One such sentence is 'Who thought up this exercise anyway?'

LOGIC TEST 4

1. Define the following:

a. Inductive strength
b. Logical consistency
c. Logical indeterminacy

2. For each of the following, indicate whether it has a truth-value (i.e., is either true or false). If it does not, explain why not.

a. Abandon hope all ye who enter here.
b. This sentence is false.
c. 2 is the smallest prime number.
d. Who made up this test anyway?

3. Which of the following passages are best understood as arguments? For those that are, recast the passage in standard form and determine whether the resulting argument is deductively valid. Evaluate the inductive strength of those that are not deductively valid.

a. The demand for Ph.D.s in engineering is down. The production of Ph.D.s in engineering is up. So there are more Ph.D.s in engineering than there is demand.

b. If it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic.

c. When all is said and done, there will be no more to say or do.

4. For each of the following, circle **T** if the sentence is true or **F** if the sentence is false.

T F a. If the members of a pair of sentences are logically equivalent, then either both are logically true, or both are logically false, or both are logically indeterminate.

T F b. An argument whose conclusion is logically equivalent to one of its premises is logically valid.

T F c. Every logically inconsistent set contains at least one logically false sentence.

T F d. An argument can be deductively sound without being deductively valid.

T F e. Every argument has two premises and one conclusion.

5. Give an example of each of the following where one exists. If there can be no such example, explain why.

a. A pair of logically indeterminate sentences that are logically equivalent.

b. A pair of sentences both of which are logically indeterminate but that are not logically equivalent.

c. A valid argument whose conclusion is logically false and no premise of which is logically false.

ANSWERS

1. a. An argument has *inductive strength* to the extent that the conclusion is probable given the premises.

b. A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true at the same time. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

c. A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

2. a. This sentence does not have a truth-value. It is an instruction, or bit of advice. (It is the sentence written over the gate to Hell in Dante's *Inferno.*)

b. This sentence might appear to have a truth-value, but it does not.

Were sentence b true, then what it says, that sentence b is false, would be false. That is, were sentence b true it would also have to be false. And were sentence b false, then what sentence b says (that sentence b is false) would be true. That is, were sentence b false it would also have to be true. No sentence can be both true and false, so sentence b must be neither true nor false. The apparent paradox embodied by sentence b is brought about by the reference of that sentence to itself. To avoid paradox, logicians generally disallow this sort of self-reference.

c. This sentence is true. (In mathematics, by definition 1 is not a prime number.)

d. This sentence is a question and as such is neither true nor false.

3. a. There is an argument here:

The demand for Ph.D.s in engineering is down.

The production of Ph.D.s in engineering is up.

There are more Ph.D.s in engineering than there is demand for.

The argument is not deductively valid. To see this, note that in the past the demand for Ph.D.s in engineering may have been enormously greater than the supply. If so, then even if the demand is down and the supply up, the demand may still be greater than the supply. In the absence of information about the past relation of supply to demand, the argument also has little inductive strength.

b. This passage, from Lewis Carroll's *Through the Looking Glass,* can be construed as the following argument plus the comment 'That's logic'.

If it was so, it might be.

If it were so, it would be.

It isn't

It ain't.

The conclusion, 'It ain't', is colloquial for 'It is not', and does follow from the third premise alone. So the argument is deductively valid.

c. There is no argument here. The sentence 'When all is said and done, there will be no more to say or do' is, however, logically true.

4. a. **T**

b. **T** In such an argument, it is impossible for all the premises to be true and the conclusion false, for if the conclusion is false, so is the premise to which it is logically equivalent.

c. **F** The set whose only two members are the sentences 'The President is a Democrat' and 'The President is not a Democrat' is logically inconsistent (both members cannot be true at the same time), but no member is logically false (both are logically indeterminate).

d. **F** A deductively sound argument is, by definition, a deductively valid argument with true premises.

e. **F** Every Aristotelean syllogism has two premises and one conclusion, but not every argument is an Aristotelean syllogism. In fact, while every argument has one and only one conclusion, an argument may have any (finite) number of premises.

5. a. One such pair consists of 'Alice and Irene both got As' and 'Irene and Alice both got A's'.

b. One such pair consists of 'Alice got an A' and 'Irene got an A'.

c. One such argument is

The President is a Democrat.

The President is not a Democrat.

Whatever will be won't be.

LOGIC TEST 1

1. Give a truth-functional paraphrase of each of the following. Symbolize each paraphrase in *SL*, being sure to indicate which sentences your sentence letters abbreviate.

a. I won't go to Boston unless it rains, although Bill will go if and only if his car is running.

b. Mares eat oats and bears eat oats and little lambs eat ivy.

c. William fancied that leprechauns or poltergeists were persecuting him.

d. Michael Crichton is one of the most creative people alive, and, if he tries his hand at television, he'll be a success.

e. Two heads aren't better than one.

2. Give a truth-functional paraphrase of the following argument, and put it into standard form. Symbolize the paraphrased argument in *SL*, indicating which sentences your sentence letters abbreviate.

> Unless the local bookstore carries novels by Mishima, I won't be able to complete my bibliography. None of Mishima's novels is in the library, and, if there aren't any there, there won't be any in the bookstore either. If I don't complete my bibliography, I can't go home this weekend or next. So I can't go home next weekend.

3. Give the chracteristic truth-tables for the following connectives:

P	**~ P**
T	
F	

P	**Q**	**P & Q**	**P ∨ Q**	**P ⊃ Q**	**P ≡ Q**
T	**T**				
T	**F**				
F	**T**				
F	**F**				

4. Assuming that 'A' has the truth-value **T** and 'B' has the truth-value **F**, what is the truth-value of each of the following sentences?

a. A ≡ B

b. B ∨ B

c. B ⊃ A

d. ~ A

e. B & A

5. Use the following abbreviations to give clear, idiomatic English readings for a–c.

R: The race track is muddy.

B: Bright Horse is the favorite.

G: The odds on Bright Horse are good.

C: The odds on Cinch Bet are good.

a. $(C \equiv B) \,\&\, \sim G$
b. $[(\sim B \vee \sim G) \supset R] \,\&\, C$
c. $\sim (G \vee C)$

6. Which of the following are sentences (well-formed formulas) of *SL*? For those that are not, state why not.
a. $B \supset (K \sim \vee D)$
b. $(D \equiv (J \supset B)) \equiv J)$
c. $\sim J \equiv \sim (J \vee \sim K)$
d. $(W \vee \sim W) \supset \sim W \vee W$

ANSWERS

1. a. Both either it is not the case that I will go to Boston or it rains and (Bill will go to Boston if and only if Bill's car is running).

$(\sim W \vee R) \,\&\, (B \equiv C)$

b. Both both mares eat oats and bears eat oats and little lambs eat ivy.

$(M \,\&\, B) \,\&\, L$

c. Not a truth-functional compound. It is its own paraphrase and can be symbolized as 'W'.

d. Both Michael Crichton is one of the most creative people alive and if Michael Crichton tries his hand at television, then Michael Crichton will be a success.

$C \,\&\, (T \supset S)$

e. It is not the case that two heads are better than one.

$\sim T$

2. Either the local bookstore carries novels by Mishima or it is not the case that I will be able to complete my bibliography.

Both it is not the case that some of Mishima's novels are in the library and if it is not the case that some of Mishima's novels are in the library then it is not the case that the local bookstore carries novels by Mishima.

If it is not the case that I will be able to complete my bibliography then it is not the case that either I can go home this weekend or I can go home next weekend.

It is not the case that I can go home next weekend.

$B \vee \sim C$

$\sim S \;\&\; (\sim S \supset \sim B)$

$\sim C \supset \sim (H \vee N)$

$\sim N$

3.

P	**~ P**
T	**F**
F	**T**

P	**Q**	**P & Q**	**P ∨ Q**	**P ⊃ Q**	**P ≡ Q**
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**T**

4. a. **F**
b. **F**
c. **T**
d. **F**
e. **F**

5. a. The odds on Cinch Bet are good if and only if Bright Horse is the favorite, but the odds on him aren't good.

b. If Bright Horse isn't the favorite or the odds on him aren't good, then the track is muddy; and the odds on Cinch Bet are good.

c. Neither the odds on Bright Horse nor the odds on Cinch Bet are good.

6. a. Not a sentence: '~' cannot immediately precede a binary connective
b. Not a sentence: too many right parentheses
c. Sentence
d. Not a sentence: too few parentheses

LOGIC TEST 2

1. Give a truth-functional paraphrase of each of the following. Symbolize each paraphrase in *SL*, being sure to indicate which sentences your sentence letters abbreviate.

a. The track is muddy, and, although Bright Horse is favored to win, I'll bet he won't.

b. We'll have a flood in the basement if that pipe doesn't stop leaking.

c. Jazz is better than rock, but neither beats country music!

d. This novel will sell if and only if the author gets the right publisher.
e. Miracles probably never happen.

2. Give a truth-functional paraphrase of the following argument and put it into standard form. Symbolize the paraphrased argument in *SL*, indicating which sentences your sentence letters abbreviate.

> You may have another drink only if Jeff or Stanley can drive you home. Neither Herbert nor Jeff is sober. If either Jeff or Stanley is not sober, then neither is sober. Furthermore, if neither is sober, neither can drive you home. Hence you may not have another drink.

3. Give the characteristic truth-tables for the following connectives:

P	**~ P**
T	
F	

P	**Q**	**P & Q**	**P ∨ Q**	**P ⊃ Q**	**P ≡ Q**
T	**T**				
T	**F**				
F	**T**				
F	**F**				

4. Assuming that 'A' has the truth-value **T** and 'B' has the truth-value **F**, what is the truth-value of each of the following sentences?
a. $\sim A$
b. $B \vee A$
c. $B \supset A$
d. $A \equiv B$
e. $B \,\&\, B$

5. Use the following abbreviations to give clear, idiomatic English readings for a–c.
G: Granola is a healthy food.
H: Perky Dog Chow is a healthy food.
C: Granola looks appetizing.
D: Perky Dog Chow looks appetizing.

a. $\sim C \,\&\, \sim (D \vee H)$
b. $C \equiv (\sim G \vee \sim H)$
c. $G \supset C$

6. Which of the following are sentences (well-formed formulas) of *SL*? For those that are not, state why not.
a. $A \equiv (C \vee C) \vee D$
b. $B \supset (A \equiv \sim B)$
c. $\sim \sim A$
d. $A(\& B \& A)$
e. $(A \supset B) \,\&\, \sim (A \supset B)$

ANSWERS

1. a. Both the track is muddy and both Bright Horse is favored to win and I'll bet Bright Horse will not win.

$T \& (B \& I)$

b. If it is not the case that that pipe does stop leaking then we'll have a flood in the basement.

$\sim P \supset F$

c. Both jazz is better than rock and it is not the case that either jazz beats country music or rock beats country music.

$B \& \sim (J \vee R)$

d. This novel will sell if and only if the author gets the right publisher.

$N \equiv A$

e. This cannot be paraphrased as a truth-functional compound. Hence it serves as its own paraphrase. It can be symbolized as an atomic sentence 'M'.

2. If you may have another drink then either Jeff can drive you home or Stanley can drive you home.

It is not the case that either Herbert is sober or Jeff is sober.

If either it is not the case that Jeff is sober or it is not the case that Stanley is sober then both it is not the case that Jeff is sober and it is not the case that Stanley is sober.

If both it is not the case that Jeff is sober and it is not the case that Stanley is sober then it is not the case that either Jeff can drive you home or Stanley can drive you home.

It is not the case that you may have another drink.

$Y \supset (J \vee S)$

$\sim (H \vee B)$

$(\sim B \vee \sim O) \supset (\sim B \& \sim O)$

$(\sim B \& \sim O) \supset \sim (J \vee S)$

$\sim Y$

3.

P	~ P
T	F
F	T

P	Q	P & Q	P ∨ Q	P ⊃ Q	P ≡ Q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

4. a. **F**
b. **T**
c. **T**
d. **F**
e. **F**

5. a. Granola doesn't look appetizing, and Perky Dog Chow neither looks appetizing nor is a healthy food.

b. Granola looks appetizing if and only if either granola or Perky Dog Chow isn't a healthy food.

c. If Granola is a healthy food, then it looks appetizing.

6. a. Not a sentence: needs more parentheses.
b. Sentence
c. Sentence
d. Not a sentence: can't have '(&'
e. Sentence

LOGIC TEST 3

1. Give a truth-functional paraphrase of each of the following. Symbolize each paraphrase in *SL*, being sure to indicate which sentences your sentence letters abbreviate.

a. If Mary is here she'll tell us how to set the lenses.

b. At most one of the two brothers, Bill and Rob, will be able to come today.

c. Food prices will decline only if wholesale costs decline as well, and wholesale costs won't decline unless inflation is halted.

d. The doctor said that measles are nothing to worry about.

e. The museum is open on Sundays if and only if it received a new federal grant.

2. Give a truth-functional paraphrase of the following argument and put it into standard form. Symbolize the paraphrased argument in *SL*, indicating which sentences your sentence letters abbreviate.

> This painting is either by Kandinsky or Picasso. If Kandinsky painted it, then it's unusual. But the painting is neither unusual nor inexpensive. Therefore Picasso painted it.

3. Give the characteristic truth-tables for the following connectives:

P	**~ P**
T	
F	

P	**Q**	**P & Q**	**P ∨ Q**	**P ⊃ Q**	**P ≡ Q**
T	**T**				
T	**F**				
F	**T**				
F	**F**				

4. Assuming that 'A' has the truth-value **F** and 'B' has the truth-value **F**, what is the truth-value of each of the following sentences?

a. ~ A
b. B ⊃ A
c. B ≡ A
d. A ∨ A
e. A & B

5. Use the following abbreviations to give clear, idiomatic English readings for a–c.

J: Judy Garland was a childhood star.
S: Shirley Temple was a childhood star.
M: My memory fails me.
H: Hollywood is falling apart.

a. M ≡ (S & ~ J)
b. ((M & S) & J) ⊃ H
c. S ⊃ J

6. Which of the following are sentences (well-formed formulas) of *SL*? For those that are not, state why not.

a. ((B & D) ≡ B) ≡ D
b. ~ ((B & ~ D)B)
c. (A ~ B) ⊃ (D ∨ G)
d. (~ P ⊃ (~ C ∨ ~ H)
e. A & B & D

ANSWERS

1. a. <u>If</u> Mary is here <u>then</u> Mary will tell us how to set the lenses.

M ⊃ T

b. <u>It is not the case that</u> <u>both</u> Bill will be able to come today <u>and</u> Rob will be able to come today.

~ (B & R)

c. Both if food prices decline then wholesale costs decline as well, and if it is not the case that inflation is halted, then it is not the case that wholesale costs decline.

$(F \supset W)\ \&\ (\sim W \vee I)$

d. This is not a truth-functional compound. It is its own paraphrase and can be symbolized as 'D'.

e. The museum is open on Sundays if and only if the museum received a new federal grant.

$M \equiv B$

2. Either this painting is by Kandinsky or this painting is by Picasso.

If this painting is by Kandinsky then this painting is unusual.

It is not the case that either this painting is unusual or the painting is inexpensive.

This painting is by Picasso.

$K \vee P$

$K \supset U$

$\sim (U \vee I)$

P

3.

P	~ P
T	F
F	T

P	Q	P & Q	P ∨ Q	P ⊃ Q	P ≡ Q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

4. a. **T**
b. **T**
c. **T**
d. **F**
e. **F**

5. a. My memory fails me just in case Shirley Temple but not Judy Garland was a childhood star.

b. If my memory fails me and Shirley Temple and Judy Garland were childhood stars, then Hollywood is falling apart.

c. If Shirley Temple was a childhood star, so was Judy Garland.

6. a. Sentence
b. Not a sentence: missing a binary connective
c. Not a sentence: '~' is not a binary connective
d. Not a sentence: missing right parenthesis
e. Not a sentence: missing parentheses

LOGIC TEST 4

1. Give a truth-functional paraphrase of each of the following. Symbolize each paraphrase in *SL*, being sure to indicate which sentences your sentence letters abbreviate.

a. Neither the football team nor the soccer team won yesterday.

b. Probably ice or butter will soothe your burn.

c. If we're in Hartford, then we must be close to New Haven, and, if that is the Prudential Building, then we're in Hartford.

d. Our efforts are not in vain if and only if at least one student learns how to symbolize sentences.

e. This novel isn't so bad; I've either read or imagined worse.

2. Give a truth-functional paraphrase of the following argument and put it into standard form. Symbolize the paraphrased argument in *SL*, indicating which sentences your sentence letters abbreviate.

> The duplicating machine hasn't worked for days, and the local scribe will be looking for work. If I don't have fifteen copies of this manuscript by five o'clock, I'll lose my job and will be looking for work. Neither the local scribe nor I will be looking for work if I don't lose my job. So the local scribe will be looking for work if and only if I don't have fifteen copies of this manuscript by five o'clock.

3. Give the characteristic truth-tables for the following connectives:

P	~ P
T	
F	

P	Q	P & Q	P ∨ Q	P ⊃ Q	P ≡ Q
T	T				
T	F				
F	T				
F	F				

4. Assuming that 'A' has the truth-value **T** and 'B' has the truth-value **F**, what is the truth-value of each of the following sentences?

a. B ⊃ B

b. A & B

c. B ≡ B

d. ~ B

e. B ∨ A

5. Use the following abbreviations to give clear, idiomatic English readings for a–c.

Q: The quality of Scandinavian rugs has declined.
R: This rug cleaner works well.
D: Dry cleaning is good for quality rugs.
O: Oriental rugs are as good as ever.

a. $(O \And \sim Q) \equiv \sim D$
b. $D \And (O \supset Q)$
c. $\sim R \vee \sim O$

6. Which of the following are sentences (well-formed formulas) of *SL*? For those that are not, state why not.

a. $(A, B) \equiv (B, A)$
b. $\sim \sim \sim \sim K$
c. $\sim (A \vee B) \vee A)$
d. $G \supset (G \supset (G \supset (G \supset G)))$
e. $C \vee \sim B \vee (A \supset B)$

ANSWERS

1. a. It is not the case that either the football team won yesterday or the soccer team won yesterday.

$$\sim (F \vee S)$$

b. Not a truth-functional compound. The sentence is its own paraphrase and can be symbolized as ‘P’.

c. Both if we are in Hartford then we must be close to New Haven and if that is the Prudential Building then we are in Hartford.

$$(W \supset M) \And (P \supset W)$$

d. (It is not the case that our efforts are in vain) if and only if at least one student learns how to symbolize sentences.

$$\sim O \equiv L$$

e. Both it is not the case that this novel is so bad and either I’ve read worse or I’ve imagined worse.

$$\sim N \And (R \vee I)$$

2. Both it is not the case that the duplicating machine has worked for days and the local scribe will be looking for work.

If it is not the case that I have fifteen copies of this manuscript by five o’clock then both I’ll lose my job and I’ll be looking for work.

If it is not the case that I lose my job then it is not the case that either the local scribe will be looking for work or I’ll be looking for work.

The local scribe will be looking for work if and only if it is not the case that I have fifteen copies of this manuscript by five o’clock.

$\sim D \;\&\; S$

$\sim F \supset (J \;\&\; L)$

$\sim J \supset \sim (S \vee L)$

$S \equiv \sim F$

3.

P	**~ P**
T	**F**
F	**T**

P	**Q**	**P & Q**	**P ∨ Q**	**P ⊃ Q**	**P ≡ Q**
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**T**

4. a. **T**
b. **F**
c. **T**
d. **T**
e. **T**

5. a. Oriental rugs and Scandinavian rugs are as good as ever just in case dry cleaning isn't good for quality rugs.

b. Dry cleaning is good for quality rugs and, if Oriental rugs are as good as ever, then the quality of Scandinavian rugs has declined.

c. Either this rug cleaner doesn't work well or Oriental rugs aren't as good as ever.

6. a. Not a sentence: ',' is not a symbol of *SL*
b. Sentence
c. Not a sentence: left parenthesis missing
d. Sentence
e. Not a sentence: parentheses missing

LOGIC TEST 1

1. Define
a. Truth-functional consistency
b. Truth-functional truth

2. Circle the main connective of each of the following sentences and underline the immediate component(s).
a. $[A \supset [B \supset (C \equiv D)]] \supset \sim A$
b. $\sim [(B \supset \sim \sim M) \;\&\; \sim (B \supset \sim M)]$
c. $\sim [\sim (A \vee B) \;\&\; B] \supset A$
d. $[(A \;\&\; B) \supset C] \equiv [A \supset (B \supset C)]$

3. Which of the sentences in question 2 are of the form $\mathbf{P} \supset \mathbf{Q}$?

4. Using appropriate sentences letters, symbolize the following in *SL.*
a. George will arrive safely only if his uncle and brother are good drivers.
b. Nine out of ten doctors recommend either aspirin or Bufferin in case of a headache.
c. If food prices rise beyond current levels, then widespread famine will result if government policies are not changed. Government policies will not change and food prices will rise. So widespread famine will result.
d. We won't buy a painting by Picasso unless we have sufficient funds.

5. Use the truth-table method to answer the following questions. In each case construct a full truth-table. Be sure to state your results.
a. Is the following sentence truth-functionally true, truth-functionally false, or truth-functionally indeterminate?

$$\sim [F \;\&\; (H \equiv J)] \supset (\sim H \vee J)$$

b. Is the following argument truth-functionally valid?

$\sim A \supset (C \equiv A)$
$(H \vee A) \equiv C$

$\sim C \supset (A \supset H)$

6. Construct a shortened truth-table that shows that the following set is truth-functionally consistent.

$$\{(A \;\&\; B) \vee \sim C, \sim B \equiv \sim C, \sim A \equiv \sim C\}$$

Why is it necessary to construct a *full* truth-table to show a set is truth-functionally inconsistent, whereas a shortened truth-table suffices to establish truth-functional consistency?

7. Assume that the argument

P

Q

is truth-functionally valid, no matter what sentence **P** is. What kind of sentence must **Q** be? Explain.

ANSWERS

1. a. A set of sentences of *SL* is *truth-functionally consistent* if and only if there is at least one truth-value assignment on which all of the members of the set are true.

b. A sentence **P** of *SL* is *truth-functionally true* if and only if **P** is true on every truth-value assignment.

2. a. [A ⊃ [B ⊃ (C ≡ D)]] ⊃ ~ A
b. ~ [(B ⊃ ~ ~ M) & ~ (B ⊃ ~ M)]
c. ~ [~ (A ∨ B) & B] ⊃ A
d. [(A & B) ⊃ C] ≡ [A ⊃ (B ⊃ C)]

3. a, c

4. a. G ⊃ (U & B)
b. N
c. F ⊃ (~ G ⊃ W)

~ G & F

W

d. ~ B ∨ S

5. a. Truth-functionally indeterminate

F	H	J	~	[F	&	(H	≡	J)]	↓ ⊃	(~	H	∨	J)
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**

b. Truth-functionally valid

					↓								↓				↓			
A	C	H	~	A	⊃	(C	≡	A)	(H	∨	A)	≡	C	~	C	⊃	(A	⊃	H)
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

6.

						↓					↓					↓		
A	B	C	(A	&	B)	∨	~	C	~	B	≡	~	C	~	A	≡	~	C
F	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**

A full table is necessary to establish truth-functional inconsistency, since we must show that there is *no* assignment on which all the set members are true. A shortened truth-table suffices to establish truth-functional consistency because here we need only show that there is *at least one* truth-value assignment on which all the set members are true.

7. **Q** must be a truth-functionally true sentence, for **Q** must be true on every truth-value assignment on which **P** is true, whatever sentence **P** may be. In particular, if **P** is truth-functionally true, then, given the validity of the argument, **Q** must be true on every assignment on which **P** is true—**Q** must be true on every truth-value assignment.

LOGIC TEST 2

1. Define
a. Truth-functional equivalence
b. Truth-functional falsity

2. Circle the main connective of each of the following sentences and underline the immediate component(s).
a. B ∨ [H ≡ (A ∨ [B ≡ (H ∨ A)])]
b. ~ [(B ∨ C) ⊃ ~ (C & ~ A)]
c. [A ∨ (B ∨ C)] ∨ ~ (A ∨ B)
d. [(B ∨ A) ⊃ (B ∨ A)] ∨ B

3. Which of the sentences is question 2 are of the form **P** ∨ **Q**?

4. Using appropriate sentence letters, symbolize the following in *SL*.

a. The dictator will be assassinated only if his countrymen rebel or the CIA intervenes.

b. It is a common police view that capital punishment will both deter crime and curb population growth, and they'll defend it to the death.

c. As we all know, everyone has a price; but Milo's price is too high to pay.

d. Although neither of the two candidates, Smith and Jones, is worthy of holding public office, one of them will be elected.

5. Use the truth-table method to answer the following questions. In each case construct a full truth-table. Be sure to state your results.

a. Is the following sentence truth-functionally true, truth-functionally false, or truth-functionally indeterminate?

$$[(M \vee \sim M) \equiv (\sim N \supset N)] \vee (N \supset K)$$

b. Is the following set of sentences truth-functionally consistent?

$$\{\sim B,\ D \equiv (B \ \& \sim D),\ D \supset \sim B\}$$

6. Construct a shortened truth-table that shows that the following argument is truth-functionally invalid.

$$\begin{array}{l} A \vee (\sim B \ \& \ C) \\ \underline{B \supset \sim A \qquad\quad} \\ \sim \sim B \end{array}$$

Why is it necessary to construct a *full* truth-table to show that an argument is truth-functionally valid, whereas a shortened truth-table suffices to establish truth-functional invalidity?

7. What is a corresponding material conditional? What important relationship holds between arguments and their corresponding material conditionals?

ANSWERS

1. a. Sentences **P** and **Q** of *SL* are *truth-functionally equivalent* if and only if there is no truth-value assignment on which **P** and **Q** have different truth-values.

b. A sentence **P** of *SL* is *truth-functionally false* if and only if **P** is false on every truth-value assignment.

2. a. $\underline{B} \vee \underline{[H \equiv (A \vee [B \equiv (H \vee A)])]}$

b. $\sim \underline{[(B \vee C) \supset \sim (C \ \& \sim A)]}$

c. $\underline{[A \vee (B \vee C)]} \vee \underline{\sim (A \vee B)}$

d. $\underline{[(B \vee A) \supset (B \vee A)]} \vee \underline{B}$

3. a, c, d

4. a. A ⊃ (R ∨ I)
b. C & D
c. E & M
d. ~ (S ∨ J) & (M ∨ D)

5. a. Truth-functionally true

K	M	N	[(M	∨	~	M)	≡	(~	N	⊃	N)]	↓ ∨	(N	⊃	K)
T	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**
F	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**
F	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
F	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**

b. Truth-functionally consistent

B	D	↓ ~	B	D	↓ ≡	(B	&	~	D)	D	↓ ⊃	~	B
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**

6.

A	B	C	A	↓ ∨	(~	B	&	C)	B	↓ ⊃	~	A	↓ ~	~	B
T	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

A full table is necessary to establish truth-functional validity, for we must show that there is *no* truth-value assignment on which all the premises are true and the conclusion is false. A shortened truth-table suffices to establish truth-functional invalidity because here we need only show that there is *at least one* truth-value assignment on which all premises are true and the conclusion is false.

7. The corresponding material conditional for an argument

$$\begin{array}{l} \mathbf{P}_1 \\ \vdots \\ \underline{\mathbf{P}_n} \\ \mathbf{Q} \end{array}$$

is the sentence

$(\ldots(\mathbf{P}_1 \ \& \ \mathbf{P}_2) \ \& \ \ldots \mathbf{P}_n) \supset \mathbf{Q}$

An argument with a finite number of premises is truth-functionally valid if and only if its corresponding material conditional is truth-functionally true.

LOGIC TEST 3

1. Define

a. Truth-functional equivalence

b. Truth-functional validity

2. Circle the main connective of each of the following sentences and underline the immediate component(s).

a. $([A \vee (B \vee D)] \equiv K) \equiv (B \,\&\, D)$

b. $\sim (\sim A \vee \sim\sim B) \vee \sim\sim C$

c. $\sim [\sim (B \vee C) \supset (\sim B \,\&\, \sim C)] \supset D$

d. $\sim [\sim (A \,\&\, B) \vee \sim (D \,\&\, E)]$

3. Which of the sentences in question 2 are of the form $\sim$ **P**?

4. Using appropriate sentence letters, symbolize the following in *SL*.

a. Milo can make money on his egg transactions if but only if he can sell them for three times what he pays for them and he doesn't have to bribe more than three generals for his use of army aircraft.

b. If we're only two days behind schedule, we may be able to catch up if we are lucky.

c. Milo will agree to bomb his own airfield only if the Germans pay in advance and in cash, but if the Germans pay in cash they will pay in counterfeit bills.

d. William doesn't know that Bill loves either Joan or Maude.

5. Use the truth-table method to answer the following questions. In each case construct a full truth-table. Be sure to state your results.

a. Is the following argument truth-functionally valid?

$H \equiv (K \vee \sim J)$

$(K \,\&\, H) \supset \sim J)$

$\sim J$

b. Is the following sentence truth-functionally true, truth-functionally false, or truth-functionally indeterminate?

$\sim [(A \equiv B) \supset [(B \supset A) \vee \sim A]]$

6. Construct a shortened truth-table that shows that the following set is truth-functionally consistent.

$\{A \,\&\, \sim D,\ D \equiv M,\ \sim M\}$

Why is it necessary to construct a *full* truth-table to show that a set is truth-functionally inconsistent, whereas a shortened truth-table suffices to establish that a set is truth-functionally consistent?

7. Assume that the argument

$$\frac{\mathbf{P}}{\mathbf{Q}}$$

is truth-functionally valid. Given this assumption, is it possible that

$$\frac{\mathbf{P}}{\sim \mathbf{Q}}$$

is also truth-functionally valid? Explain.

ANSWERS

1. a. Sentences **P** and **Q** of *SL* are *truth-functionally equivalent* if and only if there is no truth-value assignment on which **P** and **Q** have different truth-values.

b. An argument of *SL* is *truth-functionally valid* if and only if there is no truth-value assignment on which all the premises are true and the conclusion is false.

2. a. $\underline{([A \vee (B \vee D)] \equiv K)}$ $\equiv$ $\underline{(B \,\&\, D)}$

b. $\underline{\sim (\sim A \vee \sim\sim B)}$ $\vee$ $\underline{\sim\sim C}$

c. $\underline{\sim [\sim (B \vee C) \supset (\sim B \,\&\, \sim C)]}$ $\supset$ $\underline{D}$

d. $\sim$ $\underline{[\sim (A \,\&\, B) \vee \sim (D \,\&\, E)]}$

3. d

4. a. $M \equiv (S \,\&\, \sim B)$

b. $T \supset (L \supset C)$

c. $[B \supset (G \,\&\, C)] \,\&\, (C \supset F)$

d. $\sim W$

5. a. Truth-functionally invalid

H	J	K	H	↓ ≡	(K	∨	~ J)	(K	&	H)	↓ ⊃	~ J	↓ ~ J
T	**T**	**T**	**T**	**T**	**T**	**T**	**F T**	**T**	**T**	**T**	**F**	**F T**	**F T**
T	**T**	**F**	**T**	**F**	**F**	**F**	**F T**	**F**	**F**	**T**	**T**	**F T**	**F T**
T	**F**	**T**	**T**	**T**	**T**	**T**	**T F**	**T**	**T**	**T**	**T**	**T F**	**T F**
T	**F**	**F**	**T**	**T**	**F**	**T**	**T F**	**F**	**F**	**T**	**T**	**T F**	**T F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**F T**	**T**	**F**	**F**	**T**	**F T**	**F T**
F	**T**	**F**	**F**	**T**	**F**	**F**	**F T**	**F**	**F**	**F**	**T**	**F T**	**F T**
F	**F**	**T**	**F**	**F**	**T**	**T**	**T F**	**T**	**F**	**F**	**T**	**T F**	**T F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**T F**	**F**	**F**	**F**	**T**	**T F**	**T F**

b. Truth-functionally false

		↓										
A	B	~	[(A	≡	B)	⊃	[(B	⊃	A)	∨	~	A]]
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**
T	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**
F	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**

6.

				↓				↓		↓	
A	D	M	A	&	~	D	D	≡	M	~	M
T	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**

A full table is necessary to establish truth-functional inconsistency, for we must show that there is *no* truth-value assignment on which all the set members are true. A shortened truth-table suffices to establish truth-functional consistency because here we need only show that there is *at least one* truth-value assignment on which all the set members are true.

7. Yes, if **P** is truth-functionally false. For then, as there is no truth-value assignment on which **P** is true, it follows that there is no truth-value assignment on which **P** is true and ~ **Q** is false, as well as no truth-value assignment on which **P** is true and **Q** is false.

LOGIC TEST 4

1. Define
a. Truth-functional equivalence
b. Truth-functional indeterminancy

2. Circle the main connective of each of the following sentences and underline the immediate component(s).
a. (B & ~ D) ≡ [A ∨ (B & C)]
b. ~ [~ (B & ~ D) ∨ [A ∨ (B & C)]]
c. ~ [A ≡ (B & ~ C)]
d. ~ (~ ~ B ⊃ ~ ~ A) ≡ ~ ~ B

3. Which of the sentences in question 2 are of the form **P** ≡ **Q**?

4. Using appropriate sentence letters, symbolize the following in *SL.*

a. Russian astronauts will land on the moon only if Russian scientists perfect a soft-landing technique, and Russian scientists will perfect a soft-landing technique if and only if their space program is well financed.

b. Neither Israel nor Egypt presently has a fondness for the Russians, but Libya does.

c. The United Nations will not survive unless it is significantly strengthened and becomes a world government.

d. Henry doesn't believe a word Charles says.

5. Use the truth-table method to answer the following questions. In each case construct a full truth-table. Be sure to state your results.

a. Is the following argument truth-functionally valid?

$$(G \,\&\, {\sim} B) \supset [(E \vee B) \equiv {\sim} G]$$
$$(B \vee E) \vee {\sim} G$$
$$\overline{}$$
$${\sim} E \equiv B$$

b. Is the following set truth-functionally consistent?

$$\{B \supset D,\ {\sim} (B \vee {\sim} D),\ (B \,\&\, {\sim} D) \supset {\sim} B\}$$

6. Construct a shortened truth-table that shows that the following sentence is not truth-functionally false.

$$([(A \equiv B) \equiv A] \,\&\, B) \equiv A$$

Why is it necessary to construct a *full* truth-table to show that a sentence is truth-functionally false, whereas a shortened truth-table suffices to establish that a sentence is not truth-functionally false?

7. Let Γ be a truth-functionally consistent set of sentences and **P** a truth-functionally indeterminate sentence. Is it possible that $\Gamma \cup \{\mathbf{P}\}$ is truth-functionally inconsistent? Explain.

ANSWERS

1. a. Sentences **P** and **Q** of *SL* are *truth-functionally equivalent* if and only if there is no truth-value assignment on which **P** and **Q** have different truth-values.

b. A sentence **P** of *SL* is truth-functionally indeterminate if and only if **P** is neither truth-functionally true nor truth-functionally false, that is, if and only if **P** is true on at least one truth-value assignment and false on at least one truth-value assignment.

2. a. $\underline{(B \,\&\, {\sim} D)} \equiv \underline{[A \vee (B \,\&\, C)]}$

b. ${\sim} \underline{[{\sim} (B \,\&\, {\sim} D) \vee [A \vee (B \,\&\, C)]]}$

c. ${\sim} \underline{[A \equiv (B \,\&\, {\sim} C)]}$

d. $\underline{{\sim} ({\sim} {\sim} B \supset {\sim} {\sim} A)} \equiv \underline{{\sim} {\sim} B}$

3. a, d

4. a. $(L \supset P) \,\&\, (P \equiv S)$

b. ${\sim} (I \vee E) \,\&\, L$

c. ${\sim} U \vee (S \,\&\, W)$

d. ${\sim} H$

5. a. Truth-functionally invalid

B	E	G	(G	&	~	B)	↓ ⊃	[(E	∨	B)	≡	~	G]	(B	∨	E)	↓ ∨	~	G	~	E	↓ ≡	B
T	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**
T	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**
F	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**

b. Truth-functionally consistent

B	D	B	↓ ⊃	D	↓ ~	(B	∨	~	D)	(B	&	~	D)	↓ ⊃	~	B
T	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**
T	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**

6.

A	B	([(A	≡	B)	≡	A]	&	B)	↓ ≡	A
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**

A full table is necessary to establish truth-functional falsity, for we must show that the sentence is false on *every* truth-value assignment. A shortened truth-table suffices to establish that a sentence is *not* truth-functionally false because in that case we only have to show that the sentence is true on *at least one* truth-value assignment.

7. Yes. For example, let Γ be {A & B, C} and let **P** be '~ B'. Then, although Γ is truth-functionally consistent and **P** is truth-functionally indeterminate, Γ ∪ {**P**}—which is {A & B, C, ~ B}—is truth-functionally inconsistent.

LOGIC TEST 1

1. Explicate in terms of open and/or closed truth-trees:

a. Truth-functional validity*

b. Truth-functional indeterminacy

*Note: we are here considering only arguments with a finite number of premises.

2. Use the truth-tree method to answer a and b below. In each case state your result. Where appropriate, recover a fragment of a truth-value assignment that supports your answer.

a. Is the following argument truth-functionally valid?

$$\sim C \supset (D \vee E)$$
$$(D \equiv \sim E) \;\&\; C$$
$$\sim E \supset \sim C$$
$$\overline{}$$
$$\sim C$$

b. Is the following set truth-functionally consistent?

$$\{(A \vee \sim B) \supset \sim (B \;\&\; C),\ \sim C,\ B \supset \sim A\}$$

3. Suppose a truth-tree for a set of sentences closes. What do we know about *any* argument that has all of the sentences in the set among its premises? Explain.

ANSWERS

1. a. An argument of *SL* is *truth-functionally valid* if and only if the set consisting of the premises and the negation of the conclusion has a closed truth-tree.

b. A sentence **P** of *SL* is *truth-functionally indeterminate* if and only if neither the set {**P**} nor the set {~ **P**} has a closed truth-tree.

2. a. Truth-functionally invalid

C	D	E
T	**F**	**T**

1.	$\sim C \supset (D \vee E)$ ✓	SM
2.	$(D \equiv \sim E)\ \&\ C$ ✓	SM
3.	$\sim E \supset \sim C$ ✓	SM
4.	$\sim \sim C$ ✓	SM
5.	C	$4 \sim\sim D$
6.	$D \equiv \sim E$ ✓	2 &D
7.	C	2 &D
8.	$\sim\sim E$ ✓ $\sim C$ ×	$3 \supset D$
9.	E	$8 \sim\sim D$
10.	D $\sim D$	$6 \equiv D$
11.	$\sim E$ × $\sim\sim E$ ✓	$6 \equiv D$
12.	E	$11 \sim\sim D$
13.	$\sim\sim C$ ✓ $D \vee E$ ✓	$1 \supset D$
14.	C	$13 \sim\sim D$
15.	D × E	$13 \vee D$

b. Truth-functionally consistent

A	B	C
F	**T**	**F**
T	**F**	**F**
F	**F**	**F**

1.	$(A \vee \sim B) \supset \sim (B \,\&\, C)$✓					SM
2.	$\sim C$					SM
3.	$B \supset \sim A$✓					SM
4.	$\sim (A \vee \sim B)$✓		$\sim (B \,\&\, C)$✓			$1 \supset D$
5.	$\sim A$					$4 \sim \vee D$
6.	$\sim \sim B$✓					$4 \sim \vee D$
7.	B					$6 \sim \sim D$
8.	$\sim B$ ×	$\sim A$	$\sim B$		$\sim A$	$3 \supset D$
9.			$\sim B$	$\sim C$	$\sim B$ $\quad$ $\sim C$	$4 \sim \& D$

3. Any such argument is truth-functionally valid. For, if the truth-tree for the set closes, the set is truth-functionally inconsistent; there is no truth-value assignment on which every member of the set is true. So, for any argument that has all those set members among its premises, there is no assignment on which all of its premises are true and its conclusion is false.

LOGIC TEST 2

1. Explicate in terms of open and/or closed truth-trees:
a. Truth-functional entailment
b. Truth-functional equivalence

2. Use the truth-tree method to answer a and b below. In each case state your result. Where appropriate, recover a fragment truth-value assignment that supports your answer.

a. Is the following argument truth-functionally valid?

$[C \supset (A \,\&\, U)] \equiv (C \vee A)$

$\sim (A \,\&\, \sim B)$

$(B \,\&\, A) \supset (\sim C \vee \sim U)$

$C \vee B$

b. Are the following sentences truth-functionally equivalent?

A ⊃ (B ⊃ C)   (A ⊃ B) ⊃ C

3. Suppose that the truth-tree for a sentence **P** has at least one open branch and that the truth-tree for a sentence **Q** has at least one open branch. Does it follow that the tree for **P** & **Q** must have at least one open branch? Explain, using an example.

ANSWERS

1. a. A finite set Γ of sentences of *SL* truth-functionally entails a sentence **P** of *SL* if and only if the set Γ ∪ {~ **P**} has a closed truth-tree.

b. Sentences **P** and **Q** of *SL* are truth-functionally equivalent if and only if the set {~ (**P** ≡ **Q**} has a closed truth-tree.

2. a. Truth-functionally valid

1.	[C ⊃ (A & U)] ≡ (C ∨ A)✓			SM
2.	~ (A & ~ B)✓			SM
3.	(B & A) ⊃ (~ C ∨ ~ U)			SM
4.	~ (C ∨ B)✓			SM
5.	~ C			4 ~ ∨D
6.	~ B			4 ~ ∨D
7.	~ A		~ ~ B✓	2 ~ &D
8.			B ×	7 ~ ~ D
9.	C ⊃ (A & U)	~ (C ⊃ (A & U))✓		1 ≡D
10.	C ∨ A✓	~ (C ∨ A)		1 ≡D
11.	C × A ×			10 ∨D
12.		C		9 ~ ⊃D
13.		~ (A & U) ×		9 ~ ⊃D

b. Not truth-functionally equivalent

A	B	C
F	**T**	**F**
F	**F**	**F**

1.	~ ([A ⊃ (B ⊃ C)]) ≡ [(A ⊃ B) ⊃ C])✓							SM
2.	[(A ⊃ (B ⊃ C)]✓						~ [A ⊃ (B ⊃ C)]✓	1 ~ ≡D
3.	~ [(A ⊃ B) ⊃ C]✓						(A ⊃ B) ⊃ C✓	1 ~ ≡D
4.	A ⊃ B✓							3 ~ ⊃D
5.	~ C							3 ~ ⊃D
6.	~ A		B ⊃ C✓					2 ⊃D
7.	~ A	B	~ A		B			4 ⊃D
8.			~ B	C ✕	~ B ✕	C ✕		6 ⊃D
9.							A	2 ~ ⊃D
10.							~ (B ⊃ C)✓	2 ~ ⊃D
11.							B	10 ~ ⊃D
12.							~ C	10 ~ ⊃D
13.							~ (A ⊃ B)✓ / C ✕	3 ⊃D
14.							A	13 ~ ⊃D
15.							~ B ✕	13 ~ ⊃D

3. No. For example, the truth-tree for 'A & B' has an open branch, and the truth-tree for '~ A & ~ B' has an open branch, but the truth-tree for '(A & B) & (~ A & ~ B)' is closed:

1.	(A & B) & (~ A & ~ B)✓	SM
2.	A & B✓	1 &D
3.	~ A & ~ B✓	1 &D
4.	A	2 &D
5.	B	2 &D
6.	~ A	3 &D
7.	~ B	3 &D
	✕	

LOGIC TEST 3

1. Explicate in terms of open and/or closed truth-trees:
a. Truth-functional truth
b. Truth-functional validity

2. Use the truth-tree method to answer a and b below. In each case state your result. Where appropriate, recover a fragment of a truth-value assignment that supports your answer.

a. Is the following argument truth-functionally valid?

$$[E \supset (G \vee \sim T)] \supset \sim A$$
$$A \equiv E$$
$$\sim (G \,\&\, T)$$
$$\overline{\sim A}$$

b. Is the following sentence truth-functionally false?

$$\sim (A \equiv [(A \,\&\, B) \vee (A \,\&\, \sim B)])$$

3. Suppose that **P** is truth-functionally true and that we construct a truth-tree for **P** (*not* for the negation of **P**). Must all the branches of this truth-tree remain open? Explain, using an example.

ANSWERS

1. a. A sentence **P** of *SL* is *truth-functionally true* if and only if the set {~ **P**} has a closed truth-tree.

b. An argument of *SL* is *truth-functionally valid* if and only if the set consisting of the premises and the negation of the conclusion has a closed truth-tree.

2. a. Truth-functionally invalid

A	E	G	T
T	**T**	**F**	**T**

1.	[E ⊃ (G ∨ ~ T)] ⊃ ~ A ✓				SM
2.	A ≡ E ✓				SM
3.	~ (G & T) ✓				SM
4.	~ ~ A ✓				SM
5.	A				4 ~ ~ D
6.	A			~ A	2 ≡D
7.	E			~ E	2 ≡D
				×	
8.	~ [E ⊃ (G ∨ ~ T)] ✓		~ A		1 ⊃D
			×		
9.	E				8 ~ ⊃D
10.	~ (G ∨ ~ T) ✓				8 ~ ⊃D
11.	~ G				10 ~ ∨D
12.	~ ~ T ✓				10 ~ ∨D
13.	T				12 ~ ~ D
14.	~ G	~ T			3 ~ &D
		×			

b. Truth-functionally false

1.	~ (A ≡ [(A & B) ∨ (A & ~ B)]) ✓					SM
2.	A			~ A		1 ~ ≡D
3.	~ [(A & B) ∨ (A & ~ B)] ✓			(A & B) ∨ (A & ~ B) ✓		1 ~ ≡D
4.	~ (A & B) ✓					3 ~ ∨D
5.	~ (A & ~ B) ✓					3 ~ ∨D
6.	~ A	~ B				4 ~ &D
	×					
7.		~ A	~ ~ B ✓			5 ~ &D
		×				
8.			B			7 ~ ~ D
			×			
9.				A & B ✓	A & ~ B ✓	3 ∨D
10.				A	A	9 &D
11.				B	~B	9 &D
				×	×	

3. No. For example, the sentence '(A ∨ ~ A) ∨ (B & ~ B)' is truth-functionally true, yet the tree for the sentence has a closed branch:

1.	(A ∨ ~ A) ∨ (B & ~ B) ✓			SM
2.	A ∨ ~ A ✓		B & ~ B ✓	1 ∨D
3.			B	2 &D
4.			~ B	2 &D
			×	
5.	A	~ A		2 ∨D

LOGIC TEST 4

1. Explicate in terms of open and/or closed truth-trees:

a. Truth-functional falsity

b. Truth-functional equivalence

2. Use the truth-tree method to answer a and b below. In each case state your result. Where appropriate, recover a fragment of a truth-value assignment that supports your answer.

a. Is the following argument truth-functionally valid?

J ⊃ (K ⊃ L)

K ⊃ (~ L ⊃ M)

(L ∨ M) ⊃ N

J ⊃ N

b. Is the following sentence truth-functionally true?

A ∨ [(~ A ≡ B) & (~ A ⊃ ~ B)]

3. Suppose that all the branches of a tree for a sentence **P** remain open. Does it follow that **P** is truth-functionally true? Explain, using an example.

ANSWERS

1. a. A sentence **P** of *SL* is *truth-functionally false* if and only if the set {**P**} has a closed truth-tree.

b. Sentences **P** and **Q** of *SL* are truth-functionally equivalent if and only if the set {~ (**P** ≡ **Q**} has a closed truth-tree.

2. a. Truth-functionally invalid

J	K	L	M	N
T	**F**	**F**	**F**	**F**

1.	J ⊃ (K ⊃ L) ✓	SM
2.	K ⊃ (~ L ⊃ M) ✓	SM
3.	(L ∨ M) ⊃ N ✓	SM
4.	~ (J ⊃ N) ✓	SM
5.	J	4 ~ ⊃D
6.	~ N	4 ~ ⊃D
7.	~ J × K ⊃ L ✓	1 ⊃D
8.	~ (L ∨ M) ✓ N ×	3 ⊃D
9.	~ L	8 ~ ∨D
10.	~ M	8 ~ ∨D
11.	~ K L ×	7 ⊃D
12.	~ K ~ L ⊃ M ✓	2 ⊃D
13.	~ ~ L ✓ M ×	12 ⊃D
14.	L ×	13 ~ ~ D

b. Not truth-functionally true

A	B
F	**T**
F	**F**

1.	$\sim (A \vee [(\sim A \equiv B)\ \&\ (\sim A \supset \sim B)])$ ✓			SM
2.	$\sim A$			$1 \sim \vee D$
3.	$\sim [(\sim A \equiv B)\ \&\ (\sim A \supset \sim B)]$ ✓			$1 \sim \vee D$
4.	$\sim (\sim A \equiv B)$ ✓		$\sim (\sim A \supset \sim B)$ ✓	$3 \sim \& D$
5.			$\sim A$	$4 \sim \supset D$
6.			$\sim \sim B$ ✓	$4 \sim \supset D$
7.			B	$6 \sim \sim D$
8.	$\sim A$	$\sim \sim A$ ✓		$4 \sim \equiv D$
9.	$\sim B$	B		$4 \sim \equiv D$
10.		A		$8 \sim \sim D$
		×		

3. No. For example, the sentence ‘A ∨ B’ is not truth-functionally true, yet no branch on its truth-tree closes:

1.	$A \vee B$ ✓		SM
2.	A	B	$1 \vee D$

LOGIC TEST 1

1. Define
a. Validity in *SD*
b. Inconsistency in *SD*

2. Construct derivations in *SD* that establish the following:
a. $\{(A \vee \sim C) \supset B, \sim B \equiv (Q \,\&\, \sim Q), \sim C \vee A\}$ is inconsistent in *SD*.
b. '$[A \supset (B \supset C)] \supset [(A \,\&\, B) \supset C]$' is a theorem in *SD*.

3. Sherlock Holmes considered the following evidence and then announced correctly who the murderer was. What did Holmes conclude? Justify your answer by symbolizing the following argument in *SL* (using the suggested sentence letters) and constructing a derivation that shows that the symbolic argument is valid in *SD+*.

> The maid did it or the cook did it unless the butler did it or the gardener did it. If the maid did it, then it was done silently; but, if the cook did it, then it was done quickly. However, it was done neither silently nor quickly. The gardener did it if and only if it was done with a rope. If it was done with a rope, then it was not done quickly. On the other hand, if the cook did it or the butler did it, then a pistol was the weapon. But, if a pistol was the weapon, then it was not done silently. It was not done with a rope. (M, C, B, G, S, Q, R, P)
>
> Therefore the ________ did it.

4. Show that the following argument is valid in *SD+*:

$$\begin{array}{l} [B \,\&\, (E \vee G)] \,\&\, (\sim F \vee G) \\ (B \,\&\, G) \equiv H \\ (\sim F \supset \sim H) \,\&\, \sim E \\ \hline C \supset F \end{array}$$

5. Explain why any argument of *SL* whose premises form a set that is inconsistent in *SD* is valid in *SD*.

ANSWERS

1. a. An argument of *SL* is *valid in SD* if and only if the conclusion of the argument is derivable in *SD* from the set consisting of the premises.

b. A set Γ of sentences of *SL* is *inconsistent in SD* if and only if a sentence **P** of *SL* and its negation ~ **P** are derivable in *SD* from Γ.

2. a.

1	$(A \vee \sim C) \supset \sim B$	Assumption
2	$\sim B \equiv (Q \,\&\, \sim Q)$	Assumption
3	$\sim C \vee A$	Assumption
4	$\sim C$	Assumption
5	$A \vee \sim C$	4 $\vee$I
6	$\sim B$	1, 5 $\supset$E
7	$Q \,\&\, \sim Q$	2, 6 $\equiv$E
8	A	Assumption
9	$A \vee \sim C$	8 $\vee$I
10	$\sim B$	1, 9 $\supset$E
11	$Q \,\&\, \sim Q$	2, 10 $\equiv$E
12	$Q \,\&\, \sim Q$	3, 4–7, 8–11 $\vee$E
13	Q	12 &E
14	$\sim Q$	12 &E

b. Derive: $[A \supset (B \supset C)] \supset [(A \,\&\, B) \supset C]$

1	$A \supset (B \supset C)$	Assumption
2	$A \,\&\, B$	Assumption
3	A	2 &E
4	$B \supset C$	1, 3 $\supset$E
5	B	2 &E
6	C	4, 5 $\supset$E
7	$(A \,\&\, B) \supset C$	2–6 $\supset$I
8	$[A \supset (B \supset C)] \supset [(A \,\&\, B) \supset C]$	1–7 $\supset$I

3. The butler did it.
Derive: B

1	$(M \vee C) \vee (B \vee G)$	Assumption
2	$(M \supset S)\ \&\ (C \supset Q)$	Assumption
3	$\sim S\ \&\ \sim Q$	Assumption
4	$G \equiv R$	Assumption
5	$R \supset \sim Q$	Assumption
6	$(C \vee B) \supset P$	Assumption
7	$P \supset \sim S$	Assumption
8	$\sim R$	Assumption
9	$M \supset S$	2 &E
10	$\sim S$	3 &E
11	$\sim M$	9, 10 MT
12	$C \supset Q$	2 &E
13	$\sim Q$	3 &E
14	$\sim C$	12, 13 MT
15	$\sim M\ \&\ \sim C$	11, 14 &I
16	$\sim (M \vee C)$	15 DeM
17	$B \vee G$	1, 16 DS
18	$(G \supset R)\ \&\ (R \supset G)$	4 Equiv
19	$G \supset R$	18 &E
20	$\sim G$	8, 19 MT
21	B	17, 20 DS

4. Derive: $C \supset F$

1	$[B\ \&\ (E \vee G)]\ \&\ (\sim F \vee G)$	Assumption
2	$(B\ \&\ G) \equiv H$	Assumption
3	$(\sim F \supset \sim H)\ \&\ \sim E$	Assumption
4	C	Assumption
5	$B\ \&\ (E \vee G)$	1 &E
6	B	5 &E
7	$E \vee G$	5 &E
8	$\sim E$	3 &E
9	G	7, 8 DS
10	$B\ \&\ G$	6, 9 &I
11	H	2, 10 $\equiv$E
12	$\sim F \supset \sim H$	3 &E
13	$H \supset F$	12 Trans
14	F	11, 13 $\supset$E
15	$C \supset F$	4–14 $\supset$I

5. Consider an argument whose premises form a set that is inconsistent in *SD*. Since the set is inconsistent in *SD*, there is a derivation of some sentence **P** and a sentence ~ **P** from that set. We can continue the derivation as follows, where **Q** is the conclusion of the argument in question:

i	**P**	
n	~ **P**	
n + 1	| ~ **Q**	Assumption
n + 2	| **P**	**i** R
n + 3	| ~ **P**	**n** R
n + 4	**Q**	**n** + 1 − **n** + 3 ~ E

Hence there is a derivation of **Q** from the set consisting of the premises; so the argument is valid in *SD*.

LOGIC TEST 2

1. Define
a. Theorem in *SD*
b. Equivalence in *SD*

2. Construct derivations in *SD* that establish the following:
a. 'A ⊃ ~ B' and 'B ⊃ ~ A' are equivalent in *SD*
b. The following argument is valid in *SD*

A ⊃ (B & ~ C)
C ∨ ~ B
―――――――
C ∨ ~ A

3. Sherlock Holmes considered the following evidence and then announced correctly who the murderer was. What did Holmes conclude? Justify your answer by symbolizing the following argument in *SL* (using the suggested sentence letters) and constructing a derivation that shows that the symbolic argument is valid in *SD*+.

The butler did it only if the maid did it; and the maid did it if and only if it was done with a revolver. Now either the butler did it or the maid did it unless, of course, the gardener or the cook did it. If the cook did it, then it was not done with a revolver. Furthermore, if it was not done with an axe, then the gardener didn't do it. If it was done with an axe or a revolver, then it was done both swiftly and with premeditation. Now, the facts indicate it was not done swiftly. (B, M, R, G, C, A, S, P)
Therefore the ______ did it.

4. Show that the following argument is valid in *SD*+:

B ∨ M
B ⊃ C
(M ⊃ G) & (G ⊃ Z)
(E ∨ R) ≡ ~ (C ∨ Z)
―――――――
E ⊃ ~ K

5. Explain why any argument of *SL* whose conclusion is a theorem in *SD* is valid in *SD*.

ANSWERS

1. a. A sentence **P** of *SL* is a *theorem* in *SD* if and only if **P** is derivable in *SD* from the empty set.

b. Sentences **P** and **Q** of *SL* are *equivalent in SD* if and only if **Q** is derivable in *SD* from {**P**} and **P** is derivable in *SD* from {**Q**}.

2. a. Derive: B ⊃ ~ A

```
1  | A ⊃ ~ B              Assumption
   |---------
2  | | B                  Assumption
   | |-------
3  | | | A                Assumption
   | | |-----
4  | | | ~ B              1, 3 ⊃E
5  | | | B                2 R
6  | | ~ A                3–5 ~ I
7  | B ⊃ ~ A              2–6 ⊃I
```

Derive: A ⊃ ~ B

```
1  | B ⊃ ~ A              Assumption
   |---------
2  | | A                  Assumption
   | |-------
3  | | | B                Assumption
   | | |-----
4  | | | ~ A              1, 3 ⊃E
5  | | | A                2 R
6  | | ~ B                3–5 ~ I
7  | A ⊃ ~ B              2–6 ⊃I
```

b. Derive: C ∨ ~ A

```
1  | A ⊃ (B & ~ C)        Assumption
2  | C ∨ ~ B              Assumption
   |---------------
3  | | C                  Assumption
   | |-------------
4  | | C ∨ ~ A            3 ∨I
5  | | ~ B                Assumption
   | |-------------
6  | | | A                Assumption
   | | |-----------
7  | | | B & ~ C          1, 6 ⊃E
8  | | | B                7 &E
9  | | | ~ B              5 R
10 | | ~ A                6–9 ~ I
11 | | C ∨ ~ A            10 ∨I
12 | C ∨ ~ A              2, 3–4, 5–11 ∨E
```

3. The cook did it.

Derive: C

1	(B ⊃ M) & (M ≡ R)	Assumption
2	(B ∨ M) ∨ (G ∨ C)	Assumption
3	C ⊃ ~ R	Assumption
4	~ A ⊃ ~ G	Assumption
5	(A ∨ R) ⊃ (S & P)	Assumption
6	~ S	Assumption
7	~ S ∨ ~ P	6 ∨I
8	~ (S & P)	7 DeM
9	~ (A ∨ R)	5, 8 MT
10	~ A & ~ R	9 DeM
11	~ A	10 &E
12	~ G	4, 11 ⊃E
13	M ≡ R	1 &E
14	(M ⊃ R) & (R ⊃ M)	13 Equiv
15	M ⊃ R	14 &E
16	~ R	10 &E
17	~ M	15, 16 MT
18	B ⊃ M	1 &E
19	~ B	17, 18 MT
20	~ B & ~ M	19, 17 &I
21	~ (B ∨ M)	20 DeM
22	G ∨ C	2, 21 DS
23	C	12, 22 DS

4. Derive: E ⊃ ~ K

1	B ∨ M	Assumption
2	B ⊃ C	Assumption
3	(M ⊃ G) & (G ⊃ Z)	Assumption
4	(E ∨ R) ≡ ~ (C ∨ Z)	Assumption
5	│ E	Assumption
6	│ E ∨ R	5 ∨I
7	│ ~ (C ∨ Z)	4, 6 ≡E
8	│ ~ C & ~ Z	7 DeM
9	│ ~ C	8 &E
10	│ ~ B	2, 9 MT
11	│ M	1, 10 DS
12	│ M ⊃ G	3 &E
13	│ G	11, 12 ⊃E
14	│ G ⊃ Z	3 &E
15	│ │ K	Assumption
16	│ │ Z	13, 14 ⊃E
17	│ │ ~ Z	8 &E
18	│ ~ K	15–17 ~ I
19	E ⊃ ~ K	5–18 ⊃I

5. Consider an argument whose conclusion is a theorem in *SD*. Since the conclusion is a theorem, there is a derivation in *SD* of that conclusion from the empty set. We may add the premises of the argument in question as primary assumptions at the top of this derivation, renumbering other lines appropriately, and thus we obtain a derivation of the conclusion of the argument from the set consisting of the premises. Therefore the argument is valid in *SD*.

LOGIC TEST 3

1. Define
a. Inconsistency in *SD*
b. Equivalence in *SD*

2. Construct derivations in *SD* that establish parts a and b below.
a. The following sentence is a theorem in *SD*:

$(\sim A \equiv B) \supset (A \equiv \sim B)$

b. The following claim holds:

$\vdash [(\sim A \;\&\; \sim B) \vee (A \;\&\; B)] \supset (A \equiv B)$

3. Sherlock Holmes considered the following evidence and then announced correctly who the murderer was. What did Holmes conclude? Justify your answer by symbolizing the following argument in *SL* (using the suggested sentence letters) and constructing a derivation that shows that the symbolic argument is valid in *SD+*.

> The cook did it or the gardener did it unless, of course, the maid or butler did it. If the crime was done in the study, then the revolver was the murder weapon. Furthermore, if the revolver was the murder weapon, then the butler did it. The cook did it if and only if it was done in the dining room. But it was done neither in the dining room nor in the study. The gardener did it only if he was in love with the maid; and, if he was in love with the maid, then poison was the cause. On the other hand, if poison was not the cause, then either the cook did it or the maid didn't do it. It is not the case that either the revolver was the murder weapon or poison was the cause. (C, G, M, B, S, R, D, L, P)
>
> Therefore, the ______ did it.

4. Show that the following argument is valid in *SD+*:

$J \supset (\sim Z \supset H)$

$[\sim J \supset (L \,\&\, M)] \,\&\, \sim H$

$[\sim (K \supset \sim K) \supset \sim (L \,\&\, M)] \,\&\, \sim Z$

$\sim K$

5. Explain the difference between the rules of inference and the rules of replacement in *SD+*.

ANSWERS

1. a. A set Γ of sentences of *SL* is inconsistent in *SD* if and only if a sentence **P** and its negation ~ **P** are derivable in *SD* from Γ.

b. Sentences **P** and **Q** of *SL* are equivalent in *SD* if and only if **Q** is derivable in *SD* from {**P**} and **P** is derivable in *SD* from {**Q**}.

2. a. Derive: $(\sim A \equiv B) \supset (A \equiv \sim B)$

1	$\sim A \equiv B$	Assumption
2	A	Assumption
3	B	Assumption
4	$\sim A$	1, 3 ≡E
5	A	2 R
6	$\sim B$	3–5 ~ I
7	$\sim B$	Assumption
8	$\sim A$	Assumption
9	B	1, 8 ≡E
10	$\sim B$	7 R
11	A	8–10 ~ E
12	$A \equiv \sim B$	2–6, 7–11 ≡I
13	$(\sim A \equiv B) \supset (A \equiv \sim B)$	1–12 ⊃I

b. Derive: $[(\sim A \& \sim B) \vee (A \& B)] \supset (A \equiv B)$

1	$(\sim A \& \sim B) \vee (A \& B)$	Assumption
2	A	Assumption
3	$\sim A \& \sim B$	Assumption
4	$\sim B$	Assumption
5	A	2 R
6	$\sim A$	3 &E
7	B	4–6 ~ E
8	$A \& B$	Assumption
9	B	8 &E
10	B	1, 3–7, 8–9 ∨E
11	B	Assumption
12	$\sim A \& \sim B$	Assumption
13	$\sim A$	Assumption
14	B	11 R
15	$\sim B$	12 &E
16	A	13–15 ~ E
17	$A \& B$	Assumption
18	A	17 &E
19	A	1, 12–16, 17–18 ∨E
20	$A \equiv B$	2–10, 11–19 ≡I
21	$[(\sim A \& \sim B) \vee (A \& B)] \supset (A \equiv B)$	1–20 ⊃I

3. The butler did it.

Derive: B

1	$(C \vee G) \vee (M \vee B)$	Assumption
2	$S \supset R$	Assumption
3	$R \supset B$	Assumption
4	$C \equiv D$	Assumption
5	$\sim D \;\&\; \sim S$	Assumption
6	$(G \supset L) \;\&\; (L \supset P)$	Assumption
7	$\sim P \supset (C \vee \sim M)$	Assumption
8	$\sim (R \vee P)$	Assumption
9	$\sim R \;\&\; \sim P$	8 DeM
10	$\sim P$	9 &E
11	$C \vee \sim M$	7, 10 ⊃E
12	$\sim D$	5 &E
13	$(C \supset D) \;\&\; (D \supset C)$	4 Equiv
14	$C \supset D$	13 &E
15	$\sim C$	12, 14 MT
16	$\sim M$	11, 15 DS
17	$G \supset L$	6 &E
18	$L \supset P$	6 &E
19	$G \supset P$	17, 18 HS
20	$\sim G$	10, 19 MT
21	$\sim C \;\&\; \sim G$	15, 20 &I
22	$\sim (C \vee G)$	21 DeM
23	$M \vee B$	1, 22 DS
24	B	16, 23 DS

4. Derive: ~ K

1	$J \supset (\sim Z \supset H)$	Assumption
2	$[\sim J \supset (L \;\&\; M)] \;\&\; \sim H$	Assumption
3	$[\sim (K \supset \sim K) \supset \sim (L \;\&\; M)] \;\&\; \sim Z$	Assumption
4	$\quad K$	Assumption
5	$\quad \sim (K \supset \sim K) \supset \sim (L \;\&\; M)$	3 &E
6	$\quad \sim (\sim K \vee \sim K) \supset \sim (L \;\&\; M)$	5 Impl
7	$\quad \sim \sim K \supset \sim (L \;\&\; M)$	6 Idem
8	$\quad K \supset \sim (L \;\&\; M)$	7 DN
9	$\quad \sim (L \;\&\; M)$	4, 8 ⊃E
10	$\quad \sim J \supset (L \;\&\; M)$	2 &E
11	$\quad \sim \sim J$	9, 10 MT
12	$\quad J$	11 DN
13	$\quad \sim Z \supset H$	1, 12 ⊃E
14	$\quad \sim H$	2 &E
15	$\quad \sim \sim Z$	13, 14 MT
16	$\quad \sim Z$	3 &E
17	$\sim K$	4–16 ~ I

5. The rules of inference must be applied to entire sentences on a line and are one-way rules. Rules of replacement may be applied to sentential components within a sentence and are two-way rules.

LOGIC TEST 4

1. Define

a. Validity in *SD*

b. Theorem in *SD*

2. Construct derivations in *SD* that establish parts a and b below.

a. The following set is inconsistent in *SD*:

$\{A \ \& \ (\sim B \supset C), \sim A \equiv (C \vee B), \sim C \vee B\}$

b. The following argument is valid in *SD*:

$(A \vee C) \vee B$

$(A \vee B) \vee (B \vee C)$

3. Sherlock Holmes considered the following evidence and then announced correctly who the murderer was. What did Holmes conclude? Justify your answer by symbolizing the following argument in *SL* (using the suggested sentence letters) and constructing a derivation that shows that the symbolic argument is valid in *SD+*.

> If the maid did it, then it was neatly done; and, if it was neatly done, then it was done with the revolver. The butler did it if and only if his affair with the maid needed to be hidden. If the butler didn't do it, then either the maid did it, the gardener did it, or the cook did it. Neither the gardener nor the cook did it, provided that it was done with the revolver. If it was done with a knife, then the cook did it. However, it was not done with a knife. Although it was done neatly, the butler's affair with the maid did not need to be hidden. (M, N, R, B, A, G, C, K)
>
> Therefore the ______ did it.

4. Show that the following argument is valid in *SD+*:

$(\sim A \supset C) \ \& \ (\sim G \vee W)$

$(A \supset C) \ \& \ (A \supset \sim H)$

$[(\sim C \ \& \sim H) \supset K] \supset W$

W

5. Give a routine using only the derivation rules of *SD* that could be used in place of Modus Tollens in *SD+*.

ANSWERS

1. a. An argument of *SL* is *valid in SD* if and only if the conclusion of the argument is derivable in *SD* from the set consisting of the premises.

b. A sentence **P** of *SL* is a *theorem in SD* if and only if **P** is derivable in *SD* from the empty set.

2. a.

```
 1 | A & (~ B ⊃ C)                 Assumption
 2 | ~ A ≡ (C ∨ B)                 Assumption
 3 | ~ C ∨ B                       Assumption
   |----------------
 4 | | ~ C                         Assumption
   | |--------------
 5 | | ~ B ⊃ C                     1 &E
 6 | | | ~ B                       Assumption
   | | |------------
 7 | | | C                         5, 6 ⊃E
 8 | | | ~ C                       4 R
 9 | | B                           6–8 ~ E
10 | | C ∨ B                       9 ∨ I
11 | | ~ A                         2, 10 ≡E
12 | | B                           Assumption
   | |--------------
13 | | C ∨ B                       12 ∨I
14 | | ~ A                         2, 13 ≡E
15 | ~ A                           3, 4–11, 12–15 ∨E
16 | A                             1 &E
```

b. Derive: (A ∨ B) ∨ (B ∨ C)

```
 1 | (A ∨ C) ∨ B                   Assumption
   |----------------
 2 | | A ∨ C                       Assumption
   | |--------------
 3 | | | A                         Assumption
   | | |------------
 4 | | | A ∨ B                     3 ∨I
 5 | | | (A ∨ B) ∨ (B ∨ C)         4 ∨I
 6 | | | C                         Assumption
   | | |------------
 7 | | | B ∨ C                     6 ∨I
 8 | | | (A ∨ B) ∨ (B ∨ C)         7 ∨I
 9 | | (A ∨ B) ∨ (B ∨ C)           2, 3–5, 6–8 ∨E
10 | | B                           Assumption
   | |--------------
11 | | B ∨ C                       10 ∨I
12 | | (A ∨ B) ∨ (B ∨ C)           11 ∨I
13 | (A ∨ B) ∨ (B ∨ C)             1, 2–9, 10–12 ∨E
```

3. The maid did it.

Derive: M

1	$(M \supset N) \,\&\, (N \supset R)$	Assumption
2	$B \equiv A$	Assumption
3	$\sim B \supset [M \vee (G \vee C)]$	Assumption
4	$R \supset (\sim G \,\&\, \sim C)$	Assumption
5	$K \supset C$	Assumption
6	$\sim K$	Assumption
7	$N \,\&\, \sim A$	Assumption
8	$(B \supset A) \,\&\, (A \supset B)$	2 Equiv
9	$B \supset A$	8 &E
10	$\sim A$	7 &E
11	$\sim B$	9, 10 MT
12	$M \vee (G \vee C)$	3, 11 ⊃E
13	N	7 &E
14	$N \supset R$	1 &E
15	R	13, 14 ⊃E
16	$\sim G \,\&\, \sim C$	4, 15 ⊃E
17	$\sim (G \vee C)$	16 DeM
18	M	12, 17 DS

4. Derive: W

1	$(\sim A \supset C) \,\&\, (\sim G \vee W)$	Assumption
2	$(A \supset C) \,\&\, (A \supset \sim H)$	Assumption
3	$[(\sim C \,\&\, \sim H) \supset K] \supset W$	Assumption
4	$\quad \sim W$	Assumption
5	$\quad \sim [(\sim C \,\&\, \sim H) \supset K]$	3, 4 MT
6	$\quad \sim [\sim (\sim C \,\&\, \sim H) \vee K]$	5 Impl
7	$\quad \sim \sim (\sim C \,\&\, \sim H) \,\&\, \sim K$	6 DeM
8	$\quad (\sim C \,\&\, \sim H) \,\&\, \sim K$	7 DN
9	$\quad \sim C \,\&\, \sim H$	8 &E
10	$\quad \sim C$	9 &E
11	$\quad \sim A \supset C$	1 &E
12	$\quad \sim \sim A$	10, 11 MT
13	$\quad A \supset C$	2 &E
14	$\quad \sim A$	10, 13 MT
15	W	4–14 ~ E

5.

$\mathbf{n}$	$\mathbf{P} \supset \mathbf{Q}$	
$\mathbf{n} + 1$	$\sim \mathbf{Q}$	
$\mathbf{n} + 2$	$\quad \mathbf{P}$	Assumption
$\mathbf{n} + 3$	$\quad \mathbf{Q}$	$\mathbf{n}, \mathbf{n} + 2$ ⊃E
$\mathbf{n} + 4$	$\quad \sim \mathbf{Q}$	$\mathbf{n} + 1$ R
$\mathbf{n} + 5$	$\sim \mathbf{P}$	$\mathbf{n} + 2 - \mathbf{n} + 4$ ~ I

CHAPTER SIX

No tests.

CHAPTER SEVEN

LOGIC TEST 1

1. List the logical operators of *PL*.

2. Give the recursive definition of 'formula of *PL*'. (You may assume that the atomic formulas have been specified.)

3. Indicate which of the following are formulas of *PL*, and which of those are sentences of *PL*. For each formula, underline the main logical operator and indicate whether the formula is atomic, truth-functionally compound, or quantified. For those that are not formulas, explain why not, and for those that are formulas but not sentences, explain why they are not sentences.

a. $\sim \sim \sim (\forall y)Fya$
b. $(\forall z)(Faz \supset Gz)$
c. $(\exists x)Fxa \supset (\exists y)Gay$
d. $(\forall z)(Fza \supset Gwy)$
e. $\sim (\exists a)Fa$

4. List all the subformulas of each of the following:

$\sim Ga \supset (\forall y)(\exists x)Fxy$ $\qquad$ $\sim (\exists y)(\exists x)(Fxy \equiv Fyx)$

5. Symbolize English sentences a–d in *PL*, and give English readings for e–h, using the following symbolization key:

UD: living things
Lxy: x likes y
Axy: x is afraid of y
Fx: x is friendly
Nx: x is a nut-eater
Bx: x is a black squirrel
Gx: x is a gray squirrel
Rx: x is a red squirrel
j: Julia
a: Aaron

a. All gray squirrels are nut-eaters.
b. No gray squirrels are nut-eaters.
c. Some gray squirrels are nut-eaters.
d. Some gray squirrels are not nut-eaters.
e. $(\exists x)(Gx \ \& \ Fx) \ \& \sim (\forall y)(Gy \supset Fy)$
f. $(\forall y)[(Gy \vee Ry) \supset Ny]$
g. $(\exists x)(\exists y)[(Gx \ \& \ Ry) \ \& \ Lxy]$
h. $\sim Ra \ \& \ (\forall y)(Ry \supset Lay)$

6. Symbolize a–d in *PL* and give English readings of e–h, using the symbolization key given in Exercise 5.

a. Julia likes all gray squirrels and all red squirrels, but no black squirrels.
b. All black squirrels are afraid of Aaron and Aaron is afraid of all gray squirrels.
c. Every gray squirrel likes at least one red squirrel.
d. No gray squirrel is afraid of any red squirrel.
e. $(\exists x)[Rx \ \& \ (Axa \ \& \sim Axj)]$
f. $(\forall y)[(Ry \ \& \ Fy) \supset Ljy]$
g. $(\exists z)[Rz \ \& \ (\forall y)(Gy \supset Lyz)]$
h. $(\forall y)[(By \ \& \ Ljy) \supset (\forall z)(Rz \supset Lzy)]$

7. Indicate which of the listed expressions are substitution instances of '$(\forall x)(\exists y)(Bxy \supset Byx)$'.

a. $Bab \supset Bba$
b. $(\exists y)(Bay \supset Bya)$
c. $(\exists y)Bay \supset Bya$
d. $(\exists y)(Bcy \supset Byc)$

8. Using the symbolization key given below, symbolize English sentences a–c in *PLE*, and give English readings of d–f.

UD:	Positive integers
a:	1
b:	3
Gxy:	x is greater than y
Ox:	x is odd
Ex:	x is even
$f(x)$:	the successor of x

a. The successor of 3 is greater than the successor of 1.
b. The successor of an odd number is even.
c. The successor of the successor of an even number is even.
d. $\sim (\exists x)x = f(x)$
e. $\sim (\forall x)(\exists y)x = f(y)$
f. $(\forall x)(\exists y)y = f(x)$.

ANSWERS

1. ~, &, ∨, ⊃, ≡, (∀**x**), (∃**x**)

2. 1. Every atomic formula of *PL is* a formula of *PL*.
   2. If **P** is a formula of *PL*, *so* is ~ **P**.
   3. If **P** and **Q** are formulas of *PL*, so are (**P** & **Q**), (**P** ∨ **Q**), (**P** ⊃ **Q**), and (**P** ≡ **Q**).
   4. If **P** is a formula of *PL* that contains at least one occurrence of **x** and no **x**-quantifier, then (∀**x**)**P** and (∃**x**)**P** are both formulas of *PL*.
   5. Nothing is a formula of *PL* unless it can be formed by repeated applications of clauses 1 to 4.

3.

a. ~ ~ ~ (∀y)Fya	formula and sentence	truth-functional
b. (∀z)(Faz ⊃ Gz)	formula and sentence	quantified
c. (∃x) Fxa ⊃ (∃y)Gay	formula and sentence	truth-functional
d. (∀z)(Fza ⊃ Gwy)	formula, not a sentence (free 'w')	quantified
e. ~ (∃a)Fa	not a formula: 'a' is not a variable	

4.

~ Ga ⊃ (∀y)(∃x)Fxy	~ (∃y)(∃x)(Fxy ≡ Fyx)
~ Ga	(∃y)(∃x)(Fxy ≡ Fyx)
(∀y)(∃x)Fxy	(∃x)(Fxy ≡ Fyx)
Ga	Fxy ≡ Fyx
(∃x)Fxy	Fxy
Fxy	Fyx

5. a. (∀x)(Gx ⊃ Nx)
   b. (∀x)(Gx ⊃ ~ Nx)
   c. (∃x)(Gx & Nx)
   d. (∃x)(Gx & ~ Nx)
   e. Some, but not all, gray squirrels are friendly.
   f. Gray squirrels and red squirrels are nut-eaters.
   g. At least one gray squirrel likes at least one red squirrel.
   h. Aaron is not a red squirrel, and he likes red squirrels.

6. a. (∀y)[(Gy ∨ Ry) ⊃ Ljy] & ~ (∃y)(By & Ljy)
   b. (∀x)(Bx ⊃ Axa) & (∀y)(Gy ⊃ Aay)
   c. (∀z)[Gz ⊃ (∃x)(Rx & Lzx)]
   d. ~ (∃x)(∃y)[(Gx & Ry) & Axy]
   e. There is a red squirrel who is afraid of Aaron and who is not afraid of Julia.
   f. Julia likes all friendly red squirrels.
   g. There is a red squirrel whom every gray squirrel likes.
   h. Every black squirrel that Julia likes is liked by every red squirrel.

7. b and d are substitution instances.

8. a. $Gf(b)f(a)$
b. $(\forall x)(Ox \supset Ef(x))$
c. $(\forall x)(Ex \supset Ef(f(x)))$
d. No positive integer is identical to its successor.
e. Not every positive integer is the successor of a positive integer.
f. Every positive integer has a successor.

LOGIC TEST 2

1. Specify the atomic formulas of *PL*.

2. Give the recursive definition of 'formula of *PL*'.

3. Indicate which of the following are formulas of *PL*, and which of those are sentences of *PL*. For each formula, underline the main logical operator and indicate whether the formula is atomic, truth-functionally compound, or quantified. For those that are not formulas, explain why not, and for those that are formulas but not sentences, explain why they are not sentences.

a. $(\forall x)[Fxa \supset (\forall x)Gax]$
b. $(\forall z)Fza \supset \sim (\exists z)Gaz$
c. $\sim (\forall y)Gyy$
d. $Faz \supset (\forall x)Fxa$
e. $\sim (\exists x)Fab$

4. List all the subformulas of each of the following:

$(\forall x)[(\exists y)Fxy \supset Gax]$ $\qquad \sim Fab \equiv (\forall x) \sim Fxb$

5. Symbolize English sentences a–d in *PL*, and give English readings for e–h, using the following symbolization key:

UD: living things
Lxy: x likes y
Tx: x is a toad
Ux: x is ugly
Bx: x is brown
s: Sarah

a. All toads are ugly.
b. No toads are ugly.
c. Some toads are not ugly.
d. Some toads are ugly.
e. $(\forall x)[(Tx \& Bx) \supset Lsx]$
f. $(\exists y)[(By \& Ty) \& Lys] \& \sim (\forall y)[(Ty \& By) \supset Lys]$
g. $(\forall y)[Ty \supset (Lsy \equiv By)]$
h. $\sim (\exists z)(Tz \& Lsz) \& \sim (\exists w)(Tw \& Lws)$

6. Symbolize a–d in *PL* and give English readings of e–h, using the symbolization key given in Exercise 5 plus:

Fx: x is a frog
Gx: x is a green
Cxy: x is chasing y
c: Charles
s: Sarah

a. Charles is chasing Sarah and Sarah is chasing a green frog.
b. No frog likes every toad.
c. No frog likes any toad.
d. Every frog Sarah likes is liked by every toad.
e. $\sim (\exists x)(\exists y)[(Fx \ \& \ Ty) \ \& \ Lxy]$
f. $(\exists x)[Fx \ \& \ (\forall y)(Ty \supset Lyx)]$
g. $(\forall x)(\forall y)([(Fx \ \& \ Gx) \ \& \ (Ty \ \& \ By)] \supset Lxy)$
h. $(\forall x)([Tx \ \& \ (\exists y)(Fy \ \& \ Lyx)] \supset Lsx)$

7. Indicate which of the listed expressions are substitution instances of '$(\exists x) \sim (\forall w) \sim Mwx$'.

a. $\sim (\forall w) \sim Mwc$
b. $(\exists w)Mwc$
c. $\sim (\forall w) \sim Maa$
d. $\sim (\forall w) \sim Mwb$

8. Using the symbolization key given below, symbolize English sentences a–c in *PLE*, and give English readings of d–f.

UD: Positive integers
a: 2
Gxy: x is greater than y
Px: x is prime
f(x): the successor of x

a. Two is prime and its successor is prime.
b. The successor of an integer is greater than that integer.
c. No integer other than 2 is prime and has a successor that is prime.
d. $\sim (\exists x)x = f(x)$
e. $(\forall x)[Px \supset (\exists y)x = f(y)]$
f. $\sim (\exists x)(\exists y)[\sim x = y \ \& \ f(x) = f(y)]$

ANSWERS

1. Every sentence letter of *SL* is an atomic formula of *PL*.

Every *n*-place predicate of *PL* followed by *n* individual terms is an atomic formula of *PL*.

2. 1. Every atomic formula of *PL is* a formula of *PL*.
2. If **P** is a formula of *PL*, *so* is ~ **P**.
3. If **P** and **Q** are formulas of *PL*, so are (**P** & **Q**), (**P** $\vee$ **Q**), (**P** $\supset$ **Q**), and (**P** $\equiv$ **Q**).
4. If **P** is a formula of *PL* that contains at least one occurrence of **x** and no **x**-quantifier, then ($\forall$**x**)**P** and ($\exists$**x**)**P** are both formulas of *PL*.
5. Nothing is a formula of *PL* unless it can be formed by repeated applications of clauses 1 to 4.

3.

a. ($\forall$x)[Fxa $\supset$ ($\forall$x)Gax]	not a formula, '($\forall$x)' occurs in '[Fxa $\supset$ ($\forall$x)Gax]'	
b. ($\forall$z)Fza $\supseteq$ ~ (uQz)Gaz	formula and sentence	truth-functional
c. $\simeq$ ($\forall$y)Gyy	formula and sentence	truth-functional
d. Faz $\supseteq$ ($\forall$x)Fxa	formula, not a sentence ('z' is free)	truth-functional
e. ~ ($\exists$x)Fab	not a formula, 'x' does not occur in 'Fab'	

4.

($\forall$x)[($\exists$y)Fxy $\supset$ Gax]	~ Fab $\equiv$ ($\forall$x) ~ Fxb
($\exists$y)Fxy $\supset$ Gax	~ Fab
($\exists$y)Fxy	($\forall$x) ~ Fxb
Gax	Fab
Fxy	~ Fxb
	Fxb

5. a. ($\forall$x)(Tx $\supset$ Ux)
b. ~ ($\exists$z)(Ty & Uy)
c. ($\exists$z)(Tz & ~ Uz)
d. ($\exists$z)(Tz & Uz)
e. Sarah likes all brown toads.
f. At least one brown toad likes Sarah, but not all brown toads do.
g. Among toads, Sarah likes all and only brown ones.
h. Sarah likes no toad and no toad likes Sarah.

6. a. Ccs & ($\exists$x)[(Fx & Gx) & Csx]
b. ~ ($\exists$x)[Fx & ($\forall$y)(Ty $\supset$ Lxy)]
c. ~ ($\exists$x)($\exists$y)[(Fx & Ty) & Lxy]
d. ($\forall$y)[(Fy & Lsy) $\supset$ ($\forall$z)(Tz $\supset$ Lzy)]
e. No frog likes any toad.
f. There is a frog whom every toad likes.
g. Green frogs like brown toads.
h. Sarah likes every toad who is liked by at least one frog.

7. a and d are substitution instances.

8. a. Pa & Pf(a)
b. ($\forall$x)Gf(x)x
c. ~ ($\exists$x)[(~ x = 2 & Px) & Pf(x)]
d. No integer is its own successor.
e. Every prime is the successor of an integer.
f. No two integers have the same successor.

LOGIC TEST 3

1. Give the recursive definition of 'formula of *PL*'. (You may assume that the atomic formulas have been specified.)

2. Define 'sentence of *PL*'.

3. Indicate which of the following are formulas, and which of those are sentences, of *PL*. For each formula, underline the main logical operator and indicate whether the formula is atomic, truth-functionally compound, or quantified. For those that are not formulas, explain why not, and for those that are formulas but not sentences, explain why they are not sentences.

a. $\sim (\forall x)[Fxa \supset (\exists y)Gay]$
b. $\sim (\forall y)Gyx$
c. $(\forall x)(\forall y)(\sim Fxy \supset \sim Fyx)$
d. $(\forall z)Faz \supset (\forall x)Fxa$
e. $\sim (\exists x)Fabx$

4. List all the subformulas of each of the following:

$(\forall x)[(\exists y)Fxy \supset Gax]$ $\qquad \sim Fab \equiv (\forall x) \sim Fxb$

5. Symbolize English sentences a–d in *PL*, and give English readings for e–h, using the following symbolization key:

UD:	living things	Lx:	x is a lily pad
Oxy:	x is on y	Gx:	x is green
Rxy:	x respects y	Bx:	x is brown
Fx:	x is a frog	Hx:	x is happy
Tx:	x is a toad	k:	Kermit

a. All happy frogs are green.
b. No green toads are happy.
c. Some brown toads are happy.
d. Some green frogs are not happy.
e. $(\exists x)(Gx \,\&\, Fx) \,\&\, \sim (\forall y)(Gy \supset Fy)$
f. $(\forall y)[(Fy \vee Ty) \supset Hy]$
g. $(\exists x)(\exists y)[(Fx \,\&\, Ty) \,\&\, Rxy]$
h. $\sim Tk \,\&\, (\forall y)(Ty \supset Ryk)$

6. Symbolize a–d in *PL* and give English readings of e–h, using the symbolization key given in Exercise 5.

a. There is a frog on every lily pad.
b. There are no toads on lily pads.
c. Every green frog respects Kermit and Kermit respects every green frog.
d. Every self-respecting frog is on a lily pad.
e. $(\exists x)[(Bx \,\&\, Tx) \,\&\, Rkx]$

f. $(\forall y)[(Fy \; \& \; Ryy) \supset Rky]$
g. $(\exists z)[Tz \; \& \; (\forall y)(Fy \supset Ryz)]$
h. $(\exists y)[(Ly \; \& \; Oky) \; \& \; (\forall z)((Fz \; \& \; Rkz) \supset Ozy)]$

7. Indicate which of the listed expressions are substitution instances of '$(\exists y)(\exists z)(\exists w)[(My \; \& \; Nzy) \; \& \; Owy]$'.

a. $(Ma \; \& \; Nab) \; \& \; Oab$
b. $(\exists z)(\exists w)[(Ma \; \& \; Nzb) \; \& \; Owc]$
c. $(\exists z)(\exists w)[(Ma \; \& \; Nza) \; \& \; Owa]$
d. $(\exists z)(\exists w)[(Mc \; \& \; Nzc) \; \& \; Owc]$

8. Using the symbolization key given below, symbolize English sentences a–c in *PLE*, and give English readings of d–f.

UD:	Positive integers
a:	2
b:	3
Gxy:	x is greater than y
Ex:	x is even
$f(x)$:	the successor of x
$g(x,y)$:	the product of x and y
$h(x,y)$:	the sum of x and y

a. The product of two and three is greater than the sum of two and three.
b. There is a pair of integers such that the product of those integers equals the sum of those integers.
c. Every integer is such that the product of it and its successor is even.
d. $\sim (\forall x)Gg(x,x)h(x,x)$
e. $(\exists x)f(x) = h(x,x)$
f. $\sim (\forall x)(\forall y)Gg(x,y)h(x,y)$

ANSWERS

1. 1. Every atomic formula of *PL is* a formula of *PL*.
2. If **P** is a formula of *PL*, *so* is $\sim$ **P**.
3. If **P** and **Q** are formulas of *PL*, so are (**P** & **Q**), (**P** $\vee$ **Q**), (**P** $\supset$ **Q**), and (**P** $\equiv$ **Q**).
4. If **P** is a formula of *PL* that contains at least one occurrence of **x** and no **x**-quantifier, then ($\forall$**x**)**P** and ($\exists$**x**)**P** are both formulas of *PL*.
5. Nothing is a formula of *PL* unless it can be formed by repeated applications of clauses 1 to 4.

2. A sentence of *PL* is a formula of *PL* in which there are no free variables.

3.

a.	$\sim (\forall x)[Fxa \supset (\exists y)Gay]$	formula and sentence	truth-functional
b.	$\underline{\sim} (\forall y)Gyx$	formula, not a sentence 'x' is a free variable	truth-functional
c.	$\underline{(\forall x)}(\forall y)(\sim Fxy \supset \sim Fyx)$	formula and sentence	quantified
d.	$(\forall z)Faz \underline{\supset} (\forall x)Fxa$	formula and sentence	truth-functional
e.	$\underline{\sim} (\exists x)Fabx$	formula and sentence	truth-functional

4.

$(\forall x)[(\exists y)Fxy \supset Gax]$	$\sim Fab \equiv (\forall x) \sim Fxb$
$(\exists y)Fxy \supset Gax$	$\sim Fab$
$(\exists y)Fxy$	$(\forall x) \sim Fxb$
Gax	Fab
Fxy	$\sim Fxb$
	Fxb

5. a. $(\forall x)[(Hx \ \& \ Fx) \supset Gx]$
b. $\sim (\exists x)[(Gx \ \& \ Tx) \ \& \ Hx]$
c. $(\exists y)[(By \ \& \ Ty) \ \& \ Hy]$
d. $(\exists z)[(Gz \ \& \ Fz) \ \& \sim Hz]$
e. Some green things are frogs, but not all are.
f. All frogs and toads are happy.
g. At least one frog respects at least one toad.
h. Kermit is not a toad, and all toads respect Kermit.

6. a. $(\forall x)[Lx \supset (\exists y)(Fy \ \& \ Oyx)]$
b. $\sim (\exists x)(\exists y)[(Tx \ \& \ Ly) \ \& \ Oxy]$
c. $(\forall x)[(Gx \ \& \ Fx) \supset (Rxk \ \& \ Rkx)]$
d. $(\forall x)[(Fx \ \& \ Rxx) \supset (\exists y)(Ly \ \& \ Oxy)]$
e. There is a brown toad whom Kermit respects.
f. Kermit respects every self-respecting frog.
g. There is a toad whom every frog respects.
h. Kermit is on a lily pad and every frog whom Kermit respects is also on that lily pad.

7. c and d are substitution instances.

8. a. $Gg(a,b)h(a,b)$
b. $(\exists x)(\exists y)g(x,y) = h(x,y)$
c. $(\forall x)Eg(x,f(x))$
d. The product of an integer and itself is not always greater than the sum of that integer and itself.
e. There is an integer whose successor is identical to the sum of that integer and itself.
f. The product of a pair of integers is not always greater than the sum of those integers.

LOGIC TEST 4

1. Specify the atomic formulas of *PL*.

2. Give the recursive definition of 'formula of *PL*'.

3. Indicate which of the following are formulas, and which of those are sentences, of *PL*. For each formula, underline the main logical operator and indicate whether the formula is atomic, truth-functionally compound, or quantified. For those that are not formulas, explain why not, and for those that are formulas but not sentences, explain why they are not sentences.

a. $(\forall a)Fax$
b. $\sim (\forall z)Faz \supset Gza$
c. $(\exists x)[Fxa \,\&\, (\exists y)Gxy]$
d. $\sim (\exists w)(\exists x)Fwax$
e. $(\forall x)(\forall y)[Fxy \supset (\exists w)Gwyx]$

4. List all of the subformulas of each of the following

$\sim (\exists y)(\exists x)(Fxy \equiv \sim Fyx)$ $\qquad$ $(\forall y)(\exists z) \sim Lyz \supset (\forall x)(\forall y) \sim Lxy$

5. Symbolize English sentences a–d in *PL*, and give English readings for e–h, using the following symbolization key:

UD:	living things		
Lxy:	x is larger than y	Bx:	x is a black squirrel
Gx:	x is a gray squirrel	Fx:	x is friendly
Rx:	x is a red squirrel	Tx:	x is timid
Rxy:	x respects y	a:	Aaron

a. All red squirrels are timid.
b. Some black squirrels are timid and some are not.
c. No gray squirrels are timid.
d. Aaron is a timid red squirrel whom all gray squirrels respect.
e. $\sim (\exists x)(Gx \,\&\, Tx)$
f. $(\forall y)[(Fy \,\&\, Gy) \supset \sim Ty]$
g. $(\forall z)[(Gz \vee Bz) \supset \sim Fz] \,\&\, (\forall x)(Rx \supset Fx)$
h. $\sim (\forall w)[Gw \supset (Fw \,\&\, Tw)]$

6. Symbolize a–d in *PL* and give English readings of e–h, using the symbolization key given in Exercise 5.

a. Every black squirrel is larger than at least one gray squirrel.

b. Aaron is a timid black squirrel and he is larger than every red squirrel.

c. Every gray squirrel is larger than every red squirrel.

d. There is a timid, friendly, black squirrel who is respected by every gray squirrel and every red squirrel.

e. $(\forall x)[Gx \vee Rx) \supset Lax]$

f. $(\exists x)[(Bx \ \& \ Lxa) \ \& \sim (\forall y)(By \supset Lya)]$
g. $(\forall x)[(Bx \ \& \ Lxa) \supset (\exists y)(Gy \supset Lxy)]$
h. $(\forall x)(\forall y)([Rx \ \& \ By) \ \& \ Lxy] \supset Ryx)$

7. Indicate which of the listed expressions are substitution instances of '$(\forall z)[Bz \supset (\exists x)Czxz]$'

a. $Ba \supset (\exists x)Cbxc$
b. $Ba \supset (\exists x)Caxa$
c. $(\forall z)Bz \supset Czaz$
d. $Bc \supset (\exists x)Ccxc$

8. Using the symbolization key given below, symbolize English sentences a–c in *PLE*, and give English readings of d–f.

UD:	Positive integers
a:	1
b:	2
Gxy:	x is greater than y
Ex:	x is even
f(x):	the successor of x
g(x,y):	the product of x and y
h(x,y):	the sum of x and y

a. The sum of 1 and itself is greater than the product of 1 and itself.
b. For every integer greater than 2, the product of that integer and itself is greater than the sum of that integer and itself.
c. The sum of 1 and an integer is always greater than the product of 1 and that integer.
d. $(\forall x)g(a,x) = x$
e. $(\exists x)h(x,x) = g(x,x)$
f. $(\forall x)(\forall y)[(Gxb \ \& \ Gyb) \supset Gg(x,y)h(x,y)]$

ANSWER

1. Every sentence letter of *SL* is an atomic formula of *PL*.
Every *n*-place predicate of *PL* followed by *n* individual terms is an atomic formula of *PL*.

2. 1. Every atomic formula of *PL* *is* a formula of *PL*.
2. If **P** is a formula of *PL*, *so* is ~ **P**.
3. If **P** and **Q** are formulas of *PL*, so are (**P** & **Q**), (**P** ∨ **Q**), (**P** ⊃ **Q**), and (**P** ≡ **Q**).
4. If **P** is a formula of *PL* that contains at least one occurrence of **x** and no **x**-quantifier, then (∀**x**)**P** and (∃**x**)**P** are both formulas of *PL*.
5. Nothing is a formula of *PL* unless it can be formed by repeated applications of clauses 1 to 4.

3. a.	(∀a)Fax	not a formula (‘(∀a)’ is not a quantifier)	
b.	~ (∀z)Faz ⊇ Gza	formula, not a sentence (‘z’ in ‘Gza’ is free)	truth-functional
c.	(∃x)[Fxa & (∃y)Gxy]	formula and sentence	quantified
d.	~ (∃w)(∃x)Fwax	formula and sentence	truth-functional
e.	(∀x)(∀y)[Fxy ⊃ (∃w)Gwyx]	formula and sentence	quantified

4. ~ (∃y)(∃x)(Fxy ≡ ~ Fyx)	(∀y)(∃z) ~ Lyz ⊃ (∀x)(∀y) ~ Lxy
(∃y)(∃x)(Fxy ≡ ~ Fyx)	(∀y)(∃z) ~ Lyz
(∃x)(Fxy ≡ ~ Fyx)	(∀x)(∀y) ~ Lxy
Fxy ≡ ~ Fyx	(∃z) ~ Lyz
Fxy	~ Lyz
~ Fyx	Lyz
Fyx	(∀y) ~ Lxy
	~ Lxy
	Lxy

5. a. (∀x)(Rx ⊃ Tx)
b. (∃z)(Bz & Tz) & (∃z)(Bz & ~ Tz)
c. (∀y)(Gy ⊃ ~ Ty)
d. (Ta & Ra) & (∀x)(Gx ⊃ Rxa)
e. There are no timid gray squirrels.
f. Friendly gray squirrels are not timid
g. Gray squirrels and black squirrels are not friendly, but red squirrels are.
h. Not all gray squirrels are both friendly and timid.

6. a. (∀y)[By ⊃ (∃w)(Gw & Lyw)]
b. (Ta & Ba) & (∀x)(Rx ⊃ Lax)
c. (∀w)(∀z)] (Gw & Rz) ⊃ Lwz]
d. (∃x)([Tx & (Fx & Bx)] & (∀y[(Gy ∨ Ry) ⊃ Ryx])
e. Aaron is larger than all gray squirrels as well as all red squirrels.
f. At least one black squirrel is larger than Aaron but not all black squirrels are.
g. Every black squirrel that is larger than Aaron is also larger than every gray squirrel.
h. Every black squirrel respects every red squirrel that is larger than it.

7. b and d are substitution instances.

8. a. G*h*(a,a)*g*(a,a)
b. (∀x)[Gxb ⊃ G*g*(x,x)*h*(x,x)]
c. (∀x)G*h*(a,x)*g*(a,x)
d. The product of 1 and any integer is that integer.
e. There is an integer such that the sum of that integer and itself equals the product of that integer and itself.
f. Every pair of integers such that both integers are greater than 2 is such that their product is greater than their sum.

LOGIC TEST 1

1. Define

a. Quantificational validity
b. Quantificational consistency

2. Symbolize the following sentences, using the symbolization key given.

UD: Everything
Px: x is a person
Tx: x is a time
Fxyz: x fools y at z

a. No one can fool all of the people all of the time.
b. Someone is never fooled.

3. Determine the truth-values of a–d on the following interpretation.

UD: Set of people
Ox: x is one year old
Sx: x is sixteen years old
Txy: x is taller than y
Dxy: x is older than y

a. $(\forall x)[Sx \supset (\forall y)(Oy \supset Dxy)]$
b. $(\exists x)(\exists y)Dyx$
c. $(\forall x)(\forall y)(\sim Txy \supset Tyx)$
d. $(\exists x)(\forall y)(\sim y = x \supset Txy)$

4. Determine the truth-values of a–d on the following interpretation.

UD: Set of U.S. cities
Bxyz: x is between y and z
n: New York City
c: Chicago
s: San Francisco

a. $Bnsc \vee Bsnc$
b. $(\forall x)Bxcs \supset (\exists x) \sim Bxcs$
c. $(\forall x)(\sim Bxsc \equiv Bscn)$
d. $(\forall x)(\forall y)(\forall z)(Bxyz \supset \sim y = z)$

5. Construct an expansion of the following sentence for the set of constants {‘a’, ‘b’, ‘s’}.

$$(\forall w)(\sim Dsw \equiv (\exists x)Nx)$$

6. Show that the following argument is not quantificationally valid, both by the interpretation method and by the truth-functional expansion method.

$(\exists w)(Nw \;\&\; Sw)$
$(\forall w)(Hw \supset Sw)$
$\overline{}$
$(\exists w)Hw$

7. Suppose some sentence of *PL* is true on every interpretation with a one-member UD. Does it follow that the sentence is quantificationally true? Explain.

ANSWERS

1. a. An argument of *PL* is *quantificationally valid* if and only if there is no interpretation on which all the premises are true and the conclusion is false.

b. A set of sentences of *PL* is *quantificationally consistent* if and only if there is at least one interpretation on which all the members of the set are true.

2. a. $(\forall x)[Px \supset \sim (\forall y)(\forall z)((Py \;\&\; Tz) \supset Fxyz)]$
b. $(\exists x)[Px \;\&\; \sim (\exists y)(\exists z)((Py \;\&\; Tz) \;\&\; Fyxz)]$

3. a. True
b. True
c. False
d. True

4. a. False
b. True
c. False
d. True

5. $[(\sim Dsa \equiv [(Na \vee Nb) \vee Ns)] \;\&\; (\sim Dsb \equiv [(Na \vee Nb) \vee Ns])] \;\&\; (\sim Dss \equiv [(Na \vee Nb) \vee Ns])$

6. Interpretation method:

UD: Set of positive integers
Hx: x is negative
Nx: x is prime
Sx: x is odd

Truth-functional expansion method:

Ha	Na	Sa	Na	↓ &	Sa	Ha	↓ ⊃	Sa	↓ Ha
F	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**

7. No. For example, the following sentence is true on every interpretation with a one-member UD:

$$\sim [(\exists x)Fx \ \& \ (\exists x) \sim Fx]$$

since the one member of the UD is either F or not F but not both. But the sentence is not quantificationally true, since it is false on some interpretations with larger UDs, for example,

UD: Set of positive integers
Fx: x is even

LOGIC TEST 2

1. Define

a. Quantificational truth
b. Quantificational equivalence

2. Symbolize the following sentences, using the symbolization key given.

UD: Everything
Ixy: x is identical to y
Mx: x is a math problem
Lx: x is a logic problem
Sxy: x is easier to solve than is y

a. Math problems are easier to solve than are logic problems.
b. Some logic problems are easier to solve than are others.

3. Determine the truth-values of a–d on the following interpretation.

UD: Set of people and planets
Hx: x is a human being
Lxy: x lives on y
Px: x is a planet
Sx: x is in the solar system
e: earth

a. $(\exists x)(\forall y)[Hx \ \& \ (Py \supset Lxy)]$
b. $(Pe \ \& \ Se) \ \& \ (\exists w)(Hw \ \& \ Lwe)$
c. $(\exists x)(Hx \ \& \ Lxe) \equiv Se$
d. $(\forall x)(\forall y)[(Hx \ \& \ Py) \supset \sim x = y]$

4. Determine the truth-values of a–d on the following interpretation.

UD: Set of people
Mx: x is a male
Sx: x is a scientist
Oxy: x is older than y
a: Albert Einstein

a. $(\forall x)(Mx \ \& \ Sx) \supset \sim Sa$
b. $(\forall x)[(Mx \ \& \ Sx) \supset \sim Sa]$
c. $(\forall x)(\forall y)(Oxy \supset \sim Oyx)$
d. $(\forall x)(\sim y = a \ \& \ (My \ \& \ Sy))$

5. Construct an expansion of each of the following sentences for the set of constants {'a', 's'}.

a. $(\forall x)(\exists y)Sxy \ \& \ D$
b. $(\exists x)Fx \equiv \sim (\forall y)Gsy$

6. Show that the following argument is not quantificationally valid, both by the interpretation method and by the truth-functional expansion method.

$(\forall y)(\exists x)(Py \supset Cyx)$
$(\exists x)\ Cxx$

$(\exists x) \sim Cxx$

7. Are the sentences

'$(\forall y)Fy$' and '$\sim (\exists y) \sim Fy$'

quantificationally equivalent?

ANSWERS

1. a. A sentence **P** of *PL* is *quantificationally true* if and only if **P** is true on every interpretation.

b. Sentences **P** and **Q** of *PL* are *quantificationally equivalent* if and only if there is no interpretation on which they have different truth-values.

2. a. $(\forall x)[Mx \supset (\forall y)(Ly \supset Sxy)]$
b. $(\exists x)(\exists y)[(Lx \ \& \ Ly) \ \& \ (\sim Ixy \ \& \ Sxy)]$

3. a. False
b. True
c. True
d. True

4. a. True
b. False
c. True
d. True

5. a. $[(Saa \vee Sas)\ \&\ (Ssa \vee Sss)]\ \&\ D$
b. $(Fa \vee Fs) \equiv \sim (Gsa\ \&\ Gss)$

6. Interpretation method:

UD: Set of positive integers
Py: y is prime
Cxy: x equals y

Truth-functional expansion method:

Caa	Pa	Pa	↓ ⊃	Caa	↓ Caa	↓ ~	Caa
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**

7. Yes. First assume that '(∀y)Fy' is true on the interpretation. Then everything in the UD is F. So nothing in the UD is *not* F; hence '~ (∃y) ~ Fy' is true. Now assume that '(∀y)Fy' is false on an interpretation. Then something in the UD is not F. So '(∃y) ~ Fy' is true, and '~ (∃y) ~ Fy' is false.

LOGIC TEST 3

1. Define
a. Quantificational entailment
b. Quantificational falsehood

2. Symbolize the following sentences, using the symbolization key given.

UD: Everything
Px: x is a person
Bx: x is a New York banker
Lxy: x lives in y
Rx: x is rich
m: Manhattan

a. Every New York banker lives in Manhattan.
b. No rich Manhattan dweller is a New York banker.

3. Determine the truth-values of a–d on the following interpretation.

UD: Set of people
Cx: x is a child
Pxy: x is a parent of y
Mx: x is male

a. $(\forall x)[Mx \supset (\exists y)(My \,\&\, Pyx)]$
b. $\sim (\exists x)(Cx \,\&\, Mx) \equiv (\forall x)[(\exists y)Pxy \supset Mx]$
c. $(\forall x)(\forall y)(Pxy \supset \sim Pyx)$
d. $(\forall x)(\forall y)(Pxy \supset \sim x = y)$

4. Determine the truth-values of a–d on the following interpretation.

UD: Set of positive integers
Nx: x is greater than 16
Dxyz: x plus y equals z
Ex: x is even
t: 10
s: 7

a. $(\exists x)(\exists y)[(Ex \,\&\, Ey) \,\&\, Dtxy]$
b. $(\exists x)(Ex \equiv \sim Dtss)$
c. $(\forall x)Nx \supset Es$
d. $(\forall x)(Nx \equiv \sim x = t)$

5. Construct an expansion of the following sentences for the set of constants {'a', 'b'}.

a. $\sim D \supset \sim (\forall x)Mx$
b. $(\forall x)(Mx \equiv \sim D)$

6. Show that the following sentences are not quantificationally equivalent, both by the interpretation method and by the truth-functional expansion method.

$(\exists x)(\forall y)Fxy \qquad (\forall y)(\exists x)Fxy$

7. Is the following argument quantificationally valid? Explain.

$(\forall x)(Bx \supset G)$
$(\exists x)Bx$
———
G

ANSWERS

1. a. A set Γ of sentences of *PL quantificationally entails* a sentence **P** of *PL* if and only if there is no interpretation on which every member of Γ is true and **P** is false.

b. A sentence **P** of *PL* is *quantificationally false* if and only if **P** is false on every interpretation.

2. a. $(\forall x)(Bx \supset Lxm)$
b. $(\forall x)[((Px \& Rx) \& Lxm) \supset \sim Bx]$

3. a. True
b. True
c. True
d. True

4. a. True
b. True
c. True
d. False

5. a. $\sim D \supset \sim (Ma \& Mb)$
b. $(Ma \equiv \sim D) \& (Mb \equiv \sim D)$

6. Interpretation method:

UD: Set of positive integers
Fxy: x is greater than y

Truth-functional expansion method:

Faa	Fab	Fba	Fbb	(Faa	&	Fab)	↓ ∨	(Fba	&	Fbb)
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**

(Faa	∨	Fba)	↓ &	(Fab	∨	Fbb)
T	**T**	**F**	**T**	**F**	**T**	**T**

7. Yes. The first premise says that everything is such that, if it is B, then G. The second premise says that at least one thing is B. Then, since that thing satisfies the condition in the first premise, G must be true.

LOGIC TEST 4

1. Define
a. Quantificational validity
b. Quantificational indeterminacy

2. Symbolize the following sentences, using the symbolization key given.

UD:	Everything
Px:	x is a person
Gx:	x is made of glass
Hx:	x is a house
Txy:	x throws y
Sx:	x is a stone
Lxy:	x lives in y

a. People who live in glass houses don't throw stones.
b. If anyone throws stones, then everyone does.

3. Determine the truth-values of a–d on the following interpretation.

UD:	Set of natural numbers
Gxy:	x is greater than y
Exyz:	x plus y equals z
o:	0
t:	2
f:	4

a. $(\forall x)(\forall y)(Gxy \supset \sim Exoy)$
b. $Ettf \supset Eooo$
c. $(\exists x)(\exists y)(\exists z) \sim (Exyz \vee Exyx)$
d. $(\forall x)(\sim x = o \supset Gxt)$

4. Determine the truth-values of a–d on the following interpretation.

UD:	Set of people and automobiles
Px:	x is a person
Ax:	x is an automobile
Rx:	x is rich
Oxy:	x owns y

a. $(\forall x)[(Px \ \& \ (\exists y)(Oxy \ \& \ Ay)) \supset Rx]$
b. $\sim (\exists x)Rx$
c. $(\forall x)(Px \equiv \sim Ax)$
d. $(\exists x)(Rx \ \& \ (\forall y)(Ry \supset y = x))$

5. Construct an expansion of the following sentences for the set of constants {'n', 'p'}.
a. $(Snn \ \& \ Snp) \supset (\forall x)Sxx$
b. $(\exists x)(\forall y)Bxy$

6. Show that the following set is quantificationally consistent, both by the interpretation method and by the truth-functional expansion method.

$\{(\forall x)(Fx \supset Gx), (\forall x)(Hx \supset Gx), (\exists x) \sim (Fx \vee Hx)\}$

7. Is the sentence '$(\forall x)Mx \supset Ma$' quantificationally true? Explain.

ANSWERS

1. a. An argument of *PL* is *quantificationally valid* if and only if there is no interpretation on which all the premises are true and the conclusion is false.

b. A sentence **P** of *PL* is *quantificationally indeterminate* if and only if **P** is neither quantificationally true nor quantificationally false—that is, if and only if **P** is true on at least one interpretation and false on at least one interpretation.

2. a. $(\forall x)[[Px \ \& \ (\exists y)((Gy \ \& \ Hy) \ \& \ Lxy)] \supset \sim (\exists z)(Sz \ \& \ Txz)]$
b. $(\exists x)(\exists y)[(Px \ \& \ Sy) \ \& \ Txy] \supset (\forall x)[Px \supset (\exists y)(Sy \ \& \ Txy)]$

3. a. True
b. True
c. True
d. False

4. a. False
b. False
c. True
d. False

5. a. $(Snn \ \& \ Snp) \supset (Snn \ \& \ Spp)$
b. $(Bnn \ \& \ Bnp) \vee (Bpn \ \& \ Bpp)$

6. Interpretation method:

UD: Set of positive integers
Fx: x is odd
Gx: x is positive
Hx: x is prime

Truth-functional expansion method:

Fa	Ga	Ha	Fa	↓ $\supset$	Ga	Ha	↓ $\supset$	Ga	↓ $\sim$	(Fa	$\vee$	Ha)
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**

7. Yes. If every member of the UD is M, then any member that ‘a’ designates is M.

LOGIC TEST 1

1. Explicate in terms of open and/or closed truth-trees:
a. Quantificational entailment
b. Quantificational equivalence

2. Use the tree method to determine whether the arguments a and b are quantificationally valid, whether sentence c is quantificationally true, and whether set d is quantificationally consistent. In each case state your result; if no result is obtainable, explain why. Where a result is obtained way what it is about your tree that shows that this is the result. (If a result is unobtainable, make sure the tree you start is a systematic tree.)

a. $(\exists x)[(\forall y)Gyx \supset Fx]$
$\sim (\exists x)Fx$
$\overline{\sim (\forall x)(\forall y)Gxy}$

b. $\sim [(\exists x)Fx \vee (\forall y)Hy]$
$\overline{(\forall y) \sim Hy}$

c. $(\exists x)(\forall y)[(Gxy \;\&\; Gyx) \supset Gxx]$

d. $\{(\forall x)(\exists y)y = f(x),\ (\exists x) \sim (\exists y)y = f(x)\}$

3. Why does The System require, at stage 2, that, for each universally quantified sentence being decomposed, every constant occurring on the branch must be used as an instantiating constant?

ANSWERS

1. a. A finite set Γ of sentences of *PL/PLE quantificationally entails* a sentence **P** of *PL/PLE* if and only if Γ ∪ {~ **P**} has a closed truth-tree.
b. Sentences **P** and **Q** of PL/PLE are *quantificationally equivalent* if and only if the set {~ **P** ≡ **Q**} has a closed truth-tree.

2. a. The tree is closed, so the argument is quantificationally valid.

1.	$(\exists x)[(\forall y)Gyx \supset Fx]$ ✓		SM
2.	$\sim (\exists x)Fx$ ✓		SM
3.	$\sim \sim (\forall x)(\forall y)Gxy$ ✓		SM
4.	$(\forall x)(\forall y)Gxy$		$3 \sim \sim D$
5.	$(\forall x) \sim Fx$		$2 \sim \exists D$
6.	$(\forall y)Gya \supset Fa$ ✓		$1\ \exists D$
7.	$\sim Fa$		$5\ \forall D$
8.	$\sim (\forall y)Gya$ ✓	Fa	$6 \supset D$
		×	
9.	$(\exists y) \sim Gya$ ✓		$8 \sim \forall D$
10.	$\sim Gba$		$9\ \exists D$
11.	$(\forall y)Gby$		$4\ \forall D$
12.	Gba		$11\ \forall D$
	×		

b. The tree has a completed open branch, so the argument is quantificationally invalid.

1.	$\sim [(\exists x)Fx \vee (\forall y)Hy]$ ✓	SM
2.	$\sim (\forall y) \sim Hy$ ✓	SM
3.	$\sim (\exists x)Fx$ ✓	$1 \sim \vee D$
4.	$\sim (\forall y)Hy$ ✓	$1 \sim \vee D$
5.	$(\exists y) \sim \sim Hy$ ✓	$2 \sim \forall D$
6.	$\sim \sim Ha$ ✓	$5\ \exists D$
7.	Ha	$6 \sim \sim D$
8.	$(\exists y) \sim Hy$ ✓	$4 \sim \forall D$
9.	$\sim Hb$	$8\ \exists D$
10.	$(\forall x) \sim Fx$	$3 \sim \exists D$
11.	$\sim Fa$	$10\ \forall D$
12.	$\sim Fb$	$10\ \forall D$

c. This systematic tree has a completed open branch (left-most branch). Therefore the sentence being tested is not quantificationally true.

1.		~ (∃x)(∀y)[(Gxy & Gyx) ⊃ Gxx]✓		SM
2.		(∀x) ~ (∀y)[(Gxy & Gyx) ⊃ Gxx]		1 ~ ∃D
3.		~ (∀y)[(Gay & Gya) ⊃ Gaa]✓		3 ∀D
4.		(∃y) ~ [(Gay & Gya ⊃ Gaa]✓		3 ~ ∀D
5.	~ [(Gaa & Gaa) ⊃ Gaa]✓	~ [(Gab & Gba) ⊃ Gaa]✓		4 ∃D2
6.	Gaa & Gaa✓	Gab & Gba✓		5 ~ ⊃D
7.	~ Gaa	~ Gaa		5 ~ ⊃D
8.	Gaa	Gab		6 &D
9.	Gaa	Gba		6 &D
10.	×	~ (∀y)[(Gby & Gyb) ⊃ Gbb]✓		2 ∀D
11.		(∃y) ~ [(Gby & Gyb) ⊃ Gbb]✓		10 ~ ∀D
12.	~ [(Gba & Gab) ⊃ Gbb]✓	~ [(Gbb & Gbb) ⊃ Gbb]✓	~ [(Gbc & Gcb) ⊃ Gbb]✓	11 ∃D2
13.	Gba & Gab✓	Gbb & Gbb✓	Gbc & Gcb✓	12 ~ ⊃D
14.	~ Gbb	~ Gbb	~ Gbb	12 ~ ⊃D
15.	Gba	Gbb	Gbc	13 &D
16.	Gab	Gbb	Gcb	13 &D
		×	⋮	

d. The tree is closed. Therefore, the set is quantificationally inconsistent.

1.	(∀x)(∃y)y = *f*(x)		SM
2.	(∃x) ~ (∃y)y = *f*(x)✓		SM
3.	~ (∃y)y = *f*(a)✓		2 ∃D2
4.	(∀y) ~ y = *f*(a)		3 ~ ∃D
5.	(∃y)y = *f*(a)		1 ∀D
6.	~ a = *f*(a)		4 ∀D
7.	a = *f*(a)	b = *f*(a)	5 ∃D2
8.	×	(∃y)y = *f*(b)	1 ∀D
9.		~ b = *f*(a)	4 ∀D
		×	

3. The System is designed to find closed trees for quantificationally inconsistent sets in a finete number of steps. If we do not follow the requirement at stage 2, we will sometimes fail to find a closed tree when testing a set that *is* quantificationally inconsistent. For Example,

1.	$(\forall x)Fx$	SM
2.	$(\exists x) \sim Fx \checkmark$	SM
3.	$\sim Fa$	2 $\exists$D
4.	Fb	1 $\forall$D
5.	Fc	1 $\forall$D
6.	Fd	1 $\forall$D
	$\vdots$	

Here the branch will go on indefinitely *unless* the sentence on line 1 is instantiated with the constant 'a', which occurs on the tree on line 3. The System ensures that we shall do this.

LOGIC TEST 2

1. Explicate, in terms of open and/or closed truth-trees:
a. Quantificational validity
b. Quantificational equivalence

2. Use the tree method to determine whether sentence a is quantificationally true, whether argument b is quantificationally valid, whether the sentences in c are quantificationally equivalent, and whether the alleged entailment d holds. In each case state your result. If no result is obtainable, explain why. Where a result is obtained, say what it about your tree that shows that this is the result. (If a result is unobtainable make sure the tree you start is a systematic tree.)

a. $[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$

b. $(\forall x)[Nx \supset (\exists y)Rxy]$
$\sim (\exists x)Rxx \ \& \ Na$

$(\exists y)Ray$

c. $(\forall x)Fx \supset Ga \qquad (\exists x)(Fx \supset Ga)$

d. $\{(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x), Ha, a = g(a,b)\} \vDash (\exists y)Bg(y,y)$

3. Why does the rule Existential Decomposition require that the instantiating constant **a** be foreign to all preceding lines of the branch?

ANSWERS

1. a. An argument of PL/PLE is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion has a closed truth-tree.

b. Sentences **P** and **Q** of PL/PLE are quantificationally equivalent if and only if the set {~ **P** ≡ **Q**} has a closed truth-tree.

2. a. The tree has a completed open branch. Therefore, the given sentence is not quantificationally true.

1.	$\sim([Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx])$ ✓		SM
2.	$Fa \supset (\forall x)Fx$ ✓		$1 \sim\supset D$
3.	$\sim[(\exists x)Fx \supset (\forall x)Fx]$ ✓		$1 \sim\supset D$
4.	$(\exists x)Fx$ ✓		$3 \sim\supset D$
5.	$\sim(\forall x)Fx$ ✓		$3 \sim\supset D$
6.	$(\exists x) \sim Fx$ ✓		$5 \sim\forall D$
7.	Fb		$4\ \exists D$
8.	$\sim Fc$		$6\ \exists D$
9.	$\sim Fa$	$(\forall x)Fx$	$2 \supset D$
10.		Fc	$9\ \forall D$
		×	

b. The tree is closed, so the argument is quantificationally valid.

1.	$(\forall x)[Nx \supset (\exists y)Rxy]$		SM
2.	$\sim(\exists x)Rxx\ \&\ Na$ ✓		SM
3.	$\sim(\exists y)Ray$ ✓		SM
4.	$\sim(\exists x)Rxx$		2 &D
5.	Na		2 &D
6.	$Na \supset (\exists y)Ray$ ✓		$1\ \forall D$
7.	$\sim Na$	$(\exists y)Ray$ ✓	$6 \supset D$
	×		
8.		Rab	$7\ \exists D$
9.		$(\forall y) \sim Ray$	$3 \sim\exists D$
10.		$\sim Rab$	$9\ \forall D$
		×	

c. The tree is closed, so the sentences are quantificationally equivalent.

1.	~ ([(∀x)Fx ⊃ Ga] ≡ (∃x)(Fx ⊃ Ga))✓				SM
2.	(∀x)Fx ⊃ Ga✓		~ [(∀x)Fx ⊃ Ga]✓		1 ~ ≡D
3.	~ (∃x)(Fx ⊃ Ga)✓		(∃x)(Fx ⊃ Ga)✓		1 ~ ≡D
4.	(∀x) ~ (Fx ⊃ Ga)				3 ~ ∃D
5.	~ (∀x)Fx✓	Ga			2 ⊃D
6.	(∃x) ~ Fx✓				5 ~ ∀D
7.	~ Fb				6 ∃D
8.	~ (Fb ⊃ Ga)✓	~ (Fb ⊃ Ga)✓			4 ∀D
9.	Fb	Fb			8 ~ ⊃D
10.	~ Ga	~ Ga			8 ~ ⊃D
	×	×			
11.			Fc ⊃ Ga✓		3 ∃D
12.			(∀x)Fx		2 ~ ⊃D
13.			~ Ga		2 ~ ⊃D
14.			~ Fc	Ga	11 ⊃D
				×	
15.			Fc		12 ∀D
			×		

d. The tree is closed. Therefore, the alleged entailment does hold.

1.		$(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)$		SM
2.		Ha		SM
3.		$a = g(a,b)$		SM
4.		$\sim (\exists y)Bg(y,y)$ ✓		SM
5.		$(\forall y) \sim Bg(y,y)$		$4 \sim \exists D$
6.		$(\exists y)Hg(a,y) \supset Bg(a,a)$ ✓		$1\ \forall D$
7.		$(\exists y)Hg(b,y) \supset Bg(b,b)$		$1\ \forall D$
8.		$\sim Bg(a,a)$		$5\ \forall D$
9.		$\sim Bg(b,b)$		$5\ \forall D$
10.		$\sim (\exists y)Hg(a,y)$ ✓	$Bg(a,a)$	$6 \supset D$
11.		$(\forall y) \sim Hg(a,y)$	×	$10 \sim\exists D$
12.	$\sim (\exists y)Hg(b,y)$ ✓	$Bg(b,b)$		$7 \supset D$
13.	$(\forall y) \sim Hg(b,y)$	×		$12 \sim \exists D$
14.	$\sim Hg(a,a)$			$11\ \forall D$
15.	$\sim Hg(a,b)$			$11\ \forall D$
16.	$\sim Hg(b,a)$			$13\ \forall D$
17.	$\sim Hg(b,b)$			$13\ \forall D$
18.	$\sim Ha$			$3,15 = D$
	×			

3. Because otherwise the instantiating constant we choose may not be arbitrary. In violating the restriction we may incorrectly show that a quantificationally consistent set is inconsistent. Here is an example:

1.	$Fa \ \& \ (\exists x) \sim Fx$ ✓	SM	
2.	Fa	1 &D	
3.	$(\exists x) \sim Fx$ ✓	1 &D	
4.	$\sim Fa$	$3\ \exists D$	**MISTAKE!**
	×		

The tree is closed, yet the set {Fa & (∃x) ~ Fx} is obviously quantificationally consistent.

LOGIC TEST 3

1. Explicate in terms of open and/or closed truth-trees:
a. Quantificational validity
b. Quantificational truth

2. Use the tree method to determine whether arguments a and b are quantificationally valid, whether sentence c is quantificationally false, and whether set d is quantificationally consistent. In each case state your result. If no result is obtainable, explain why. Where a result is obtained, say what it about your tree that shows that this is the result. (If a result is unobtainable make sure the tree you start is a systematic tree.)

a. $(\exists x) \sim Fxx$

$\overline{(\exists x) \sim (\forall y)(\sim Fyx \supset Fxy)}$

b. $(\exists x)(Gx \,\&\, \sim Fx)$

$\overline{(\exists x)Fx \,\&\, (\forall x) \sim Gx}$

c. $(\exists x)(\forall y)[(Fxy \,\&\, Ga) \supset \sim Fay]$

d. $\{(\forall x)\ Lxf(x),\ (\exists y) \sim Lf(y)y\}$

3. Can a sentence that contains *no* universal quantifiers have a systematic tree with a nonterminating branch? Explain.

ANSWERS

1. a. An argument of PL/PLE is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion has a closed truth-tree.

b. A sentence **P** of PL/PLE is quantificationally true if and only if the set {~ **P**} has a closed truth-tree.

2. a. The tree is closed. Therefore, the argument is quantificationally valid.

1.	(∃x) ~ Fxx ✓		SM
2.	~ (∃x) ~ (∀y) (~ Fyx ⊃ Fxy) ✓		SM
3.	~ Faa		1 ∃D
4.	(∀x) ~ ~ (∀y) (~ Fyx ⊃ Fxy)		2 ~ ∃D
5.	~ ~ (∀y) (~ Fya ⊃ Fay) ✓		4 ∀D
6.	(∀y) (~ Fya ⊃ Fay)		5 ~ ~ D
7.	~ Faa ⊃ Faa ✓		6 ∀D
8.	~ ~ Faa ✓	Faa ×	7 ⊃D
9.	Faa ×		8 ~ ~ D

b. The tree has a completed open branch, so the argument is quantificationally invalid.

1.	(∃x) (Gx & ~ Fx) ✓		SM
2.	~ [(∃x)Fx & (∀x) ~ Gx] ✓		SM
3.	Ga & ~ Fa ✓		1 ∃D
4.	Ga		3 &D
5.	~ Fa		3 &D
6.	~ (∃x)Fx ✓	~ (∀x) ~ Gx ✓	2 ~ &D
7.		(∃x) ~ ~ Gx ✓	6 ~ ∀D
8.		~ ~ Gb ✓	7 ∃D
9.		Gb	8 ~ ~ D
10.	(∀x) ~ Fx		6 ~ ∃D
11.	~ Fa		10 ∀D

c. The following systematic tree contains a completed open branch (the second branch from the left). Therefore the sentence being tested is not quantificationally false.

1.	(∃x)(∀y)[(Fxy & Ga) ⊃ ~ Fay]✓				SM
2.	(∀y)[(Fay & Ga) ⊃ ~ Fay]		(∀y)[(Fby & Ga) ⊃ ~ Fay]		1 ∃D2
3.	(Faa & Ga) ⊃ ~ Faa✓		(Fba & Ga) ⊃ ~ Faa✓		2 ∀D
4.			(Fbb & Ga) ⊃ ~ Fab		2 ∀D
5.	~ (Faa & Ga)	~ Faa	~ (Fba & Ga)	~ Faa	3 ⊃D
	⋮		⋮	⋮	

d. The tree has a completed open branch. Therefore, the set is quantificationally consistent.

1.	(∀x) Lx*f*(x)		SM
2.	(∃y) ~ L*f*(y)y✓		SM
3.	~ L*f*(a)a		2 ∃D2
4.	La*f*(a)		1 ∀D
5.	a = *f*(a)	b = *f*(a)	4 CTD
6.	~ Laa	~ Lba	5,3 =D
7.	Laa	Lab	5,4 ID
	×	|	
8.		b = b	5,5 =D

3. Yes, for example,

1.	~ (∃x) ~ (∃y)Fxy✓		SM
2.	(∀x) ~ ~ (∃y)Fxy		1 ~∃D
3.	~~ (∃y)Fay✓		2 ∀D
4.	(∃y)Fay✓		3 ~~D
5.	Faa	Fab	4 ∃D2
		⋮	

While this tree has a completed open branch (the left branch), the right branch will generate at least one nonterminating branch.

LOGIC TEST 4

1. Explicate in terms of open and/or closed truth-trees:
a. Quantificational entailment
b. Quantificational falsity

2. Use the tree method to determine whether sentence a is quantificationally true, whether argument b is quantificationally valid, whether the sentences in c are quantificationally equivalent, and whether alleged entailment d holds. In each case state your result. If no result is obtainable, explain why. Where a result is obtained, say what it about your tree that shows that this is the result. (If a result is unobtainable make sure the tree you start is a systematic tree.)

a. $(\exists x)(\exists y)Hxy \vee (\forall x) \sim (\exists y)Hxy$

b. $(\forall x)[Fx \supset (\exists y)Hy]$
$(\forall x) \sim Hx$
———————————
$\sim Fa$

c. $(\forall x)(\exists y)Gxy \qquad (\exists x)(\forall y)Gxy$

d. $\{(\forall x)(Gx \supset Gh(x)\} \vDash (\exists x) \sim (\sim Gx \,\&\, \sim Gh(x))$

3. Can a nonsystematic tree for a quantificationally valid argument have an infinite branch? Explain, using an example.

ANSWERS

1. a. A finite set Γ of sentences of PL/PLE quantificationally entails a sentence **P** of PL/PLE if and only if $\Gamma \cup \{\sim \mathbf{P}\}$ has a closed truth-tree.

b. A sentence **P** of PL/PLE is quantificationally false if and only if the set $\{\mathbf{P}\}$ has a closed truth-tree.

2. a. The tree for the negation of the given sentence is closed. Therefore, the given sentence is quantificationally true.

1.	$\sim [(\exists x)(\exists y)Hxy \vee (\forall x) \sim (\exists y)Hxy]$ ✓	SM
2.	$\sim (\exists x)(\exists y)Hxy$ ✓	$1 \sim \vee D$
3.	$\sim (\forall x) \sim (\exists y)Hxy$ ✓	$1 \sim \vee D$
4.	$(\exists x) \sim \sim (\exists y)Hxy$ ✓	$3 \sim \forall D$
5.	$\sim \sim (\exists y)Hay$ ✓	$4\ \exists D$
6.	$(\exists y)Hay$ ✓	$5 \sim \sim D$
7.	Hab	$6\ \exists D$
8.	$(\forall x) \sim (\exists y)Hxy$	$2 \sim \exists D$
9.	$\sim (\exists y)Hay$ ✓	$8\ \forall D$
10.	$(\forall y) \sim Hay$	$9 \sim \exists D$
11.	$\sim Hab$	$10\ \forall D$
	×	

b. The tree is closed, so the argument is quantificationally valid.

1.	$(\forall x)[Fx \supset (\exists y)Hy]$		SM
2.	$(\forall x) \sim Hx$		SM
3.	$\sim \sim Fa$ ✓		SM
4.	Fa		$3 \sim \sim D$
5.	$Fa \supset (\exists y)Hy$ ✓		$1\ \forall D$
6.	$\sim Fa$	$(\exists y)Hy$ ✓	$5 \supset D$
	×		
7.		Hb	$6\ \exists D$
8.		$\sim Hb$	$2\ \forall D$
		×	

c. The tree has a completed open branch (the rightmost branch). Therefore, the sentences are not quantificationally equivalent. The left-hand branch will generate at least one nonterminating branch.

1.	$\sim [(\forall x)(\exists y)Gxy \equiv (\exists x)(\forall y)Gxy]$ ✓			SM
2.	$(\forall x)(\exists y)Gxy$	$\sim (\forall x)(\exists y)Gxy$ ✓		$1 \sim \equiv D$
3.	$\sim (\exists x)(\forall y)Gxy$ ✓	$(\exists x)(\forall y)Gxy$ ✓		$1 \sim \equiv D$
4.	$(\forall x) \sim (\forall y)Gxy$			$3 \sim \forall D$
	⋮			
5.		$(\exists x) \sim (\exists y)Gxy$ ✓		$2 \sim \forall D$
6.		$\sim (\exists y)Gay$ ✓		$5\ \exists D2$
7.		$(\forall y) \sim Gay$		$6 \sim \forall D$
8.		$(\forall y)Gay$	$(\forall y)Gby$	$3\ \exists D2$
9.		$\sim Gaa$	$\sim Gaa$	$7\ \forall D$
10.		Gaa	Gba	$8\ \forall D$
11.		×	Gbb	$8\ \forall D$
			$\sim Gab$	$7\ \forall D$

d. The tree has a completed open branch (the left branch). Therefore, the alleged entailment does not hold.

1.	$(\forall x)(Gx \supset Gh(x))$			SM
2.	$\sim (\exists x) \sim (\sim Gx \;\&\; \sim Gh(x))$ ✓			SM
3.	$(\forall x) \sim\sim (Gx \;\&\; \sim Gh(x))$			$2 \sim \exists D$
4.	$Ga \supset Gh(a)$ ✓			$1\ \forall D$
5.	$\sim\sim (\sim Ga \;\&\; \sim Gh(a))$ ✓			$3\ \forall D$
6.	$(\sim Ga \;\&\; \sim Gh(a))$			$5 \sim\sim D$
7.	$\sim Ga$			6 &D
8.	$\sim Gh(a)$			6 &D
9.	$\sim Ga$		$Gh(a)$ ×	$4 \supset D$
10.	$a = h(a)$	$b = h(a)$		8 CTD
11.	$\sim Ga$	$\sim Gb$		10,8 =D
12.	$a = a$	$b = b$		10,10 =D

3. Yes. For example, the argument

$$\frac{(\forall x)Fxx}{Fcc}$$

is quantificationally valid. Yet, if we always use constants other than 'c' as the instantiating constant in applications of universal decomposition, the tree will not close and will not be completed in a finite number of steps.

1.	$(\forall x)Fxx$	SM
2.	$\sim Fcc$	SM
3.	Fdd	$1\ \forall D$
4.	Fee	$1\ \forall D$
5.	Fff	$1\ \forall D$
	⋮	

LOGIC TEST 1

1. Define
a. Validity in *PD*
b. Inconsistency in *PD*

2. Construct derivations that show each of the following:
a. '$(\forall x)(Fx \vee (\exists y)Gxy) \supset (\forall x)(\exists y)(Fx \vee Gxy)$' is a theorem in *PD*
b. $[(\exists x)(Fx \supset Ga)] \vdash (\forall x)Fx \supset Ga$

3. Casino Slim, world-renowned riverboat gambler, reasons as follows:

> Sneaky Sally's hand is a flush. Every full house beats Sneaky Sally's hand. Sneaky Sally's hand beats any pair. The relation of one hand beating another is transitive (that is, for any poker hand, if the first beats the second and the second beats the third, then the first beats the third). Thus any full house beats any pair.

Use the following symbolization key to symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

U.D.:	set of poker hands
Hx:	x is a full house
Fx:	x is a flush
Px:	x is a pair
Bxy:	x beats y
s:	Sneaky Sally's hand

4. Show that the following argument is valid in *PD+*:

$(\exists x)[Sx \;\&\; (\forall y)(Ty \supset Lxy)]$

$(\exists x)(Tx \;\&\; Fx)$

$(\forall x)[(\exists y)(Fy \;\&\; Lxy) \supset Fx]$

$(\exists x)(Sx \;\&\; Fx)$

5. Universal Elimination, Existential Elimination, Universal Introduction, and Existential Introduction are rules of inference, not rules of replacement. Give an example that illustrates the misuse of one of these rules as a rule of replacement, and give an interpretation that shows that such a misuse could lead from true sentences to a false one.

6. Show in *PDE* that {$\sim$ a = b} $\vdash$ $\sim$ b = a

7. Complete the following derivation in *PDE*:

Derive: $(\exists x)Kf(x)f(x)$

1	$(\forall x)(\forall y)(Hx \supset Kxy)$	Assumption
2	$(\exists x)Hf(x)$	Assumption

ANSWERS

1. a. An argument of *PL* is *valid in PD* if and only if the conclusion of the argument is derivable in *PD* from the set consisting of the premises.

b. A set Γ of sentences of *PL* is *inconsistent in PD* if and only if a sentence **P** and its negation $\sim$ **P** are derivable in *PD* from Γ.

2. a. Derive: $(\forall x)(Fx \vee (\exists y)Gxy) \supset (\forall x)(\exists y)(Fx \vee Gxy)$

1	$(\forall x)(Fx \vee (\exists y)Gxy)$	Assumption
2	$Fa \vee (\exists y)Gay$	1 $\forall$E
3	Fa	Assumption
4	$Fa \vee Gab$	3 $\vee$I
5	$(\exists y)(Fa \vee Gay)$	4 $\exists$I
6	$(\exists y)Gay$	Assumption
7	Gac	Assumption
8	$Fa \vee Gac$	7 $\vee$I
9	$(\exists y)(Fa \vee Gay)$	8 $\exists$I
10	$(\exists y)(Fa \vee Gay)$	6, 7–9 $\exists$E
11	$(\exists y)(Fa \vee Gay)$	2, 3–5, 6–10 $\vee$E
12	$(\forall x)(\exists y)(Fx \vee Gxy)$	11 $\forall$I
13	$(\forall x)(Fx \vee (\exists y)Gxy) \supset (\forall x)(\exists y)(Fx \vee Gxy)$	1–12 $\supset$I

b. Derive: $(\forall x)Fx \supset Ga$

1	$(\exists x)(Fx \supset Ga)$	Assumption
2	$Fb \supset Ga$	Assumption
3	$(\forall x)Fx$	Assumption
4	Fb	3 $\forall$E
5	Ga	2, 4 $\supset$E
6	$(\forall x)Fx \supset Ga$	3–5 $\supset$I
7	$(\forall x)Fx \supset Ga$	1, 2–6 $\exists$E

3. Derive: (∀x)(∀y)[(Hx & Py) ⊃ Bxy]

1	Fs	Assumption
2	(∀x)(Hx ⊃ Bxs)	Assumption
3	(∀x)(Px ⊃ Bsx)	Assumption
4	(∀x)(∀y)(∀z)[(Bxy & Byz) ⊃ Bxz]	Assumption
5	Ha & Pb	Assumption
6	Ha ⊃ Bas	2 ∀E
7	Ha	5 &E
8	Bas	6, 7 ⊃E
9	Pb ⊃ Bsb	3 ∀E
10	Pb	5 &E
11	Bsb	9, 10 ⊃E
12	Bas & Bsb	8, 11 &I
13	(∀y)(∀z)[(Bay & Byz) ⊃ Baz]	4 ∀E
14	(∀z)[(Bas & Bsz) ⊃ Baz]	13 ∀E
15	(Bas & Bsb) ⊃ Bab	14 ∀E
16	Bab	12, 15 ⊃E
17	(Ha & Pb) ⊃ Bab	5–16 ⊃I
18	(∀y)[(Ha & Py) ⊃ Bay]	17 ∀I
19	(∀x)(∀y)[(Hx & Py) ⊃ Bxy]	18 ∀I

4. Derive: (∃x)(Sx & Fx)

1	(∃x)[Sx & (∀y)(Ty ⊃ Lxy)]	Assumption
2	(∃x)(Tx & Fx)	Assumption
3	(∀x)[(∃y)(Fy & Lxy) ⊃ Fx]	Assumption
4	Sa & (∀y)(Ty ⊃ Lay)	Assumption
5	Tb & Fb	Assumption
6	(∃y)(Fy & Lay) ⊃ Fa	3 ∀E
7	(∀y)(Ty ⊃ Lay)	4 &E
8	Tb ⊃ Lab	7 ∀E
9	Tb	5 &E
10	Lab	8, 9 ⊃E
11	Fb	5 &E
12	Fb & Lab	11, 10 &I
13	(∃y)(Fy & Lay)	12 ∃I
14	Fa	6, 13 ⊃E
15	Sa	4 &E
16	Sa & Fa	15, 14 &I
17	(∃x)(Sx & Fx)	16 ∃I
18	(∃x)(Sx & Fx)	2, 5–17 ∃E
19	(∃x)(Sx & Fx)	1, 4–18 ∃E

5.

1	$(\forall x)Vxs \supset Es$	Assumption	
2	$Vss \supset Es$	1 $\forall$E	**MISTAKE!**

On the following interpretation, the first sentence is true and the second false:

U.D.: set of positive integers
Vxy: x is equal to y
Ex: x is even
s: seven

6. Derive: $\sim b = a$

1	$\sim a = b$	Assumption
2	$\quad b = a$	Assumption
3	$\quad a = a$	2, 2 =E
4	$\quad a = b$	2, 3 =E
5	$\quad \sim a = b$	1 R
6	$\sim b = a$	2–5 ~I

7. Derive: $(\exists x)Kf(x)f(x)$

1	$(\forall x)(\forall y)(Hx \supset Kxy)$	Assumption
2	$(\exists x)Hf(x)$	Assumption
3	$\quad Hf(a)$	Assumption
4	$\quad (\forall y)(Hf(a) \supset Kf(a)y)$	1 $\forall$E
5	$\quad Hf(a) \supset Kf(a)f(a)$	4 $\forall$E
6	$\quad Kf(a)f(a)$	3, 5 $\supset$E
7	$\quad (\exists x)Kf(x)f(x)$	6 $\exists$I
8	$(\exists x)Kf(x)f(x)$	2, 3–7 $\exists$E

LOGIC TEST 2

1. Define
a. Theorem in *PD*
b. Equivalence in *PD*

2. Construct derivations that show each of the following:
a. $\{\sim (\forall x)(\exists y)Lxy,\ (\exists y)(\forall x)\ Lxy\}$ is inconsistent in *PD*
b. ‘$((\exists x)Fx \vee (\exists x)Gx) \supset (\exists x)(Fx \vee Gx)$’ is a theorem in *PD*

3. Casino Slim, world-renowned riverboat gambler, reasons as follows:

In poker, a flush beats a straight. Either Wild-Eyed Harry’s hand or Fast Fingers Flora’s hand is a flush, but Slippery Sam’s hand is a straight. Hence either Wild-Eyed Harry’s hand or Fast Fingers Flora’s hand beats some hand.

Use the following symbolization key to symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

U.D.:	set of poker hands
Fx:	x is a flush
Sx:	x is a straight
Bxy:	x beats y
h:	Wild-Eyed Harry's hand
f:	Fast Fingers Flora's hand
s:	Slippery Sam's hand

4. Show that the following argument is valid in *PD+*:

$(\forall x)\,[(\exists x)\,(Byb\ \&\ Lxyb) \supset Fx]$

$(\exists x)\,(Cxb\ \&\ Lxab)$

$(\forall x)\,(Cxb \supset \sim Fx) \supset \sim Bab$

5. Suppose that a set is inconsistent in *PD*. Is an argument that has the sentences in the sets as premises valid in *PD*? Prove that you are right.

6. Show in *PDE* that $\{a = b,\ b = c\} \vdash c = a$

7. Complete the following derivation in *PDE*:

Derive: $(\forall x)\,(Rf(x)g(x) \equiv Rg(x)f(x))$

1	$(\forall x)\,(\forall y)\,(Ryx \supset Rxy)$	Assumption

ANSWERS

1. a. A sentence **P** of *PL* is a *theorem in PD* if and only if **P** is derivable in *PD* from the empty set.

b. Sentences **P** and **Q** of *PL* are *equivalent in PD* if and only if **Q** is derivable in *PD* from {**P**} and **P** is derivable in *PD* from {**Q**}.

2. a.

1	$\sim (\forall x)\,(\exists y)\,Lxy$	Assumption
2	$(\exists y)\,(\forall x)\,Lxy$	Assumption
3	$\quad (\forall x)\,Lxa$	Assumption
4	$\quad Lba$	3 ∀E
5	$\quad (\exists y)\,Lby$	4 ∃I
6	$\quad (\forall x)\,(\exists y)\,Lxy$	5 ∀I
7	$(\forall x)\,(\exists y)\,Lxy$	2, 3–6 ∃E
8	$(\forall x)\,(\exists y)\,Lxy\ \&\ \sim (\forall x)\,(\exists y)\,Lxy$	7, 1 &I

b. Derive: $((\exists x)Fx \vee (\exists x)Gx) \supset (\exists x)(Fx \vee Gx)$

1	$\quad (\exists x)Fx \vee (\exists x)Gx$	Assumption
2	$\quad\quad (\exists x)Fx$	Assumption
3	$\quad\quad\quad Fa$	Assumption
4	$\quad\quad\quad Fa \vee Ga$	3 $\vee$I
5	$\quad\quad\quad (\exists x)(Fx \vee Gx)$	4 $\exists$I
6	$\quad\quad (\exists x)(Fx \vee Gx)$	2, 3–5 $\exists$E
7	$\quad\quad (\exists x)Gx$	Assumption
8	$\quad\quad\quad Gb$	Assumption
9	$\quad\quad\quad Fb \vee Gb$	8 $\vee$I
10	$\quad\quad\quad (\exists x)(Fx \vee Gx)$	9 $\exists$I
11	$\quad\quad (\exists x)(Fx \vee Gx)$	7, 8–10 $\exists$E
12	$\quad (\exists x)(Fx \vee Gx)$	1, 2–6, 7–11 $\vee$E
13	$((\exists x)Fx \vee (\exists x)Gx) \supset (\exists x)(Fx \vee Gx)$	1–12 $\supset$I

3. Derive: $(\exists x)(Bhx \vee Bfx)$

1	$(\forall x)(\forall y)[(Fx \,\&\, Sy) \supset Bxy]$	Assumption
2	$(Fh \vee Ff) \,\&\, Ss$	Assumption
3	$Fh \vee Ff$	2 &E
4	$\quad Fh$	Assumption
5	$\quad (\forall y)[(Fh \,\&\, Sy) \supset Bhy]$	1 $\forall$E
6	$\quad (Fh \,\&\, Ss) \supset Bhs$	5 $\forall$E
7	$\quad Ss$	2 &E
8	$\quad Fh \,\&\, Ss$	4, 7 &I
9	$\quad Bhs$	6, 8 $\supset$E
10	$\quad Bhs \vee Bfs$	9 $\vee$I
11	$\quad Ff$	Assumption
12	$\quad (\forall y)[(Ff \,\&\, Sy) \supset Bfy]$	1 $\forall$E
13	$\quad (Ff \,\&\, Ss) \supset Bfs$	12 $\forall$E
14	$\quad Ss$	2 &E
15	$\quad Ff \,\&\, Ss$	11, 14 &I
16	$\quad Bfs$	13, 15 $\supset$E
17	$\quad Bhs \vee Bfs$	16 $\vee$I
18	$Bhs \vee Bfs$	3, 4–10, 11–17 $\vee$E
19	$(\exists x)(Bhx \vee Bfx)$	18 $\exists$I

4. Derive: (∀x)(Cxb ⊃ ~ Fx) ⊃ ~ Bab

1	(∀x)[(∃y)(Byb & Lxyb) ⊃ Fx]	Assumption
2	(∃x)(Cxb & Lxab)	Assumption
3	(∀x)(Cxb ⊃ ~ Fx)	Assumption
4	Cib & Liab	Assumption
5	Cib	4 &E
6	Cib ⊃ ~ Fi	3 ∀E
7	~ Fi	5, 6 ⊃D
8	(∃y)(Byb & Liyb) ⊃ Fi	1 ∀E
9	~ (∃y)(Byb & Liyb)	7, 8 MT
10	(∀y) ~ (Byb & Liyb)	9 QN
11	~ (Bab & Liab)	10 ∀E
12	~ Bab ∨ ~ Liab	11 DeM
13	Liab	4 &E
14	~ ~ Liab	13 DN
15	~ Bab	12, 14 DS
16	~ Bab	2, 4–15 ∃E
17	(∀x)(Cxb ⊃ ~ Fx) ⊃ ~ Bab	3–16 ⊃I

5. Yes. If the premises of a given argument form a set that is inconsistent in *PD*, then there is a derivation of a sentence **P** and its negation ~ **P** from the set. We can continue that derivation as follows, where **Q** is the conclusion of the argument in question:

i	**P**	
n	~ **P**	
n + 1	~ **Q**	Assumption
n + 2	**P**	**i** R
n + 3	~ **P**	**n** R
n + 4	**Q**	**n** + 1 − **n** + 3 ~ E

Thus we obtain a derivation of the conclusion from the set consisting of the premises, which establishes the validity of the argument in *PD*.

6. Derive: c = a

1	a = b	Assumption
2	b = c	Assumption
3	c = c	2, 2 =E
4	c = b	2, 3 =E
5	c = a	1, 4 =E

7. Derive: $(\forall x)(Rf(x)g(x) \equiv Rg(x)f(x))$

1	$(\forall x)(\forall y)(Ryx \supset Rxy)$	Assumption
2	$Rf(a)g(a)$	Assumption
3	$(\forall y)(Ryg(a) \supset Rg(a)y)$	1 ∀E
4	$Rf(a)g(a) \supset Rg(a)f(a)$	4 ∀E
5	$Rg(a)f(a)$	2, 6 ⊃E
6	$Rg(a)f(a)$	Assumption
7	$(\forall y)(Ryf(a) \supset Rf(a)y)$	1 ∀E
8	$Rg(a)f(a) \supset Rf(a)g(a)$	7 ∀E
9	$Rf(a)g(a)$	6, 8 ⊃E
10	$Rf(a)g(a) \equiv Rg(a)f(a)$	2–6, 6–9 ≡I
11	$(\forall x)(Rf(x)g(x) \equiv Rg(x)f(x))$	10 ∀I

LOGIC TEST 3

1. Define
a. Inconsistency in *PD*
b. Equivalence in *PD*

2. Construct derivations that show each of the following:
a. '$(\exists x)(Fx \vee Gx) \supset ((\forall x) \sim Fx \supset (\exists x)Gx)$' is a theorem in *PD*
b. '$(\forall x)(Fx \supset Ga)$' and '$(\exists x)Fx \supset Ga$' are equivalent in *PD*

3. Casino Slim, world-renowned riverboat gambler, reasons as follows:

In poker, any flush is a good hand, but there is a hand that beats every flush. One hand beats another if and only if the latter hand loses to the former. So every flush loses to some hand.

Use the following symbolization key to symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

U.D.: set of poker hands
Fx: x is a flush
Gx: x is a good hand
Bxy: x beats y
Lxy: x loses to y

4. Show that the following argument is valid in *PD+*:

$\sim (\forall x)(\sim Fx \vee \sim Gx) \supset (\forall x)[Kx \ \& \ (\forall y)(My \supset Nxy)]$
$(\exists x)[Gx \ \& \ (\forall y)(My \supset Nxy)] \supset (\forall x)(Rx \ \& \ (\forall y)Sxy)$

$\sim (\forall x)(\forall y)Sxy \supset (\forall x)(\sim Fx \vee \sim Gx)$

5. One of the restrictions on the use of Existential Elimination is that the instantiating constant must not occur in an undischarged assumption. Explain, using an example, why this restriction is necessary.

6. Show in *PDE* that ⊢ ~ a = b ⊃ ~ b = a

7. Complete the following derivation in *PDE*:

Derive: ~ J*f*(*f*(a,b))

1	(∀x)(Jx ⊃ Kx)	Assumption
2	(∀x)~ Kx	Assumption

ANSWERS

1. a. A set Γ on sentences of *PL* is *inconsistent in PD* if and only if a sentence **P** and its negation ~ **P** are derivable in *PD* from Γ.

b. Sentences **P** and **Q** of *PL* are *equivalent in PD* if and only if **Q** is derivable in *PD* from {**P**} and **P** is derivable in *PD* from {**Q**}.

2. a. Derive: (∃x)(Fx ∨ Gx) ⊃ ((∀x) ~ Fx ⊃ (∃x)Gx)

1	│ │ (∃x)(Fx ∨ Gx)	Assumption
2	│ │ │ (∀x) ~ Fx	Assumption
3	│ │ │ │ Fa ∨ Ga	Assumption
4	│ │ │ │ │ Fa	Assumption
5	│ │ │ │ │ │ ~ Ga	Assumption
6	│ │ │ │ │ │ Fa	4 R
7	│ │ │ │ │ │ ~ Fa	2 ∀E
8	│ │ │ │ │ Ga	5–7 ~ E
9	│ │ │ │ │ Ga	Assumption
10	│ │ │ │ │ Ga	9 R
11	│ │ │ │ Ga	3, 4–8, 9–10 ∨E
12	│ │ │ │ (∃x)Gx	11 ∃I
13	│ │ │ (∃x)Gx	1, 3–12 ∃E
14	│ │ (∀x) ~ Fx ⊃ (∃x)Gx	2–13 ⊃I
15	│ (∃x)(Fx ∨ Gx) ⊃ ((∀x) ~ Fx ⊃ (∃x)Gx)	1–14 ⊃I

b. Derive: (∃x)Fx ⊃ Ga

1	│ (∀x)(Fx ⊃ Ga)	Assumption
2	│ │ (∃x)Fx	Assumption
3	│ │ │ Fb	Assumption
4	│ │ │ Fb ⊃ Ga	1 ∀E
5	│ │ │ Ga	3, 4 ⊃E
6	│ │ Ga	2, 3–5 ∃E
7	│ (∃x)Fx ⊃ Ga	2–6 ⊃I

Derive: (∀x)(Fx ⊃ Ga)

1	(∃x)Fx ⊃ Ga	Assumption
2	Fb	Assumption
3	(∃x)Fx	2 ∃I
4	Ga	1, 3 ⊃E
5	Fb ⊃ Ga	2–4 ⊃I
6	(∀x)(Fx ⊃ Ga)	5 ∀I

3. Derive: (∀x)(Fx ⊃ (∃y)Lxy)

1	(∀x)(Fx ⊃ Gx) & (∃y)(∀x)(Fx ⊃ Byx)	Assumption
2	(∀x)(∀y)(Bxy ≡ Lyx)	Assumption
3	Fa	Assumption
4	(∃y)(∀x)(Fx ⊃ Byx)	1 &E
5	(∀x)(Fx ⊃ Bbx)	Assumption
6	Fa ⊃ Bba	5 ∀E
7	Bba	3, 6 ⊃E
8	(∀y)(Bby ≡ Lyb)	2 ∀E
9	Bba ≡ Lab	8 ∀E
10	Lab	7, 9 ≡E
11	(∃y)Lay	10 ∃I
12	(∃y)Lay	4, 5–11 ∃E
13	Fa ⊃ (∃y)Lay	3–12 ⊃I
14	(∀x)(Fx ⊃ (∃y)Lxy)	13 ∀I

4. Derive: ∼ (∀x)(∀y)Sxy ⊃ (∀x)(∼ Fx ∨ ∼ Gx)

1	∼ (∀x)(∼ Fx ∨ ∼ Gx) ⊃ (∀x)[Kx & (∀y)(My ⊃ Nxy)]	Assumption
2	(∃x)[Gx & (∀y)(My ⊃ Nxy)] ⊃ (∀x)(Rx & (∀y)Sxy)	Assumption
3	∼ (∀x)(∼ Fx ∨ ∼ Gx)	Assumption
4	(∀x)[Kx & (∀y)(My ⊃ Nxy)]	1, 3 ⊃E
5	(∃x) ∼ (∼ Fx ∨ ∼ Gx)	3 QN
6	∼ (∼ Fa ∨ ∼ Ga)	Assumption
7	∼ ∼ Fa & ∼ ∼ Ga	6 DeM
8	∼ ∼ Ga	7 &E
9	Ga	8 DN
10	Ka & (∀y)(My ⊃ Nay)	4 ∀E
11	(∀y)(My ⊃ Nay)	10 &E
12	Ga & (∀y)(My ⊃ Nay)	9, 11 &E
13	(∃x)[Gx & (∀y)(My ⊃ Nxy)]	12 ∃I
14	(∃x)[Gx & (∀y)(My ⊃ Nxy)]	5, 6–13 ∃E
15	(∀x)(Rx & (∀y)Sxy)	2, 14 ⊃E
16	Rb & (∀y)Sby	15 ∀E
17	(∀y)Sby	16 &E
18	(∀x)(∀y)Sxy	17 ∀I
19	∼ (∀x)(∼ Fx ∨ ∼ Gx) ⊃ (∀x)(∀y)Sxy	3–18 ⊃I
20	∼ (∀x)(∀y)Sxy ⊃ ∼ ∼ (∀x)(∼ Fx ∨ ∼ Gx)	19 Trans
21	∼ (∀x)(∀y)Sxy ⊃ (∀x)(∼ Fx ∨ ∼ Gx)	20 DN

5. Without the restriction our derivation might not be truth-preserving; for example:

1	Et	Assumption	
2	(∃x)Ox	Assumption	
3	Ot	Assumption	
4	Et & Ot	1, 3 &I	
5	(∃x)(Ex & Ox)	4 ∃I	
6	(∃x)(Ex & Ox)	2, 3–5 ∃E	**MISTAKE!**

The first two sentences in the derivation are true and the last one is false on the following interpretation:

U.D.: set of positive integers
Ex: x is even
Ox: x is odd
t: 2

6. Derive: ~ a = b ⊃ ~ b = a

1	~ a = b	Assumption
2	b = a	Assumption
3	~ a = a	1, 2 =E
4	(∀x)x = x	=I
5	a = a	4 ∀E
6	~ b = a	2–5 ~ I
7	~ a = b ⊃ ~ b = a	1–6 ⊃I

7. Derive: ~ J*f*(*f*(a,b))

1	(∀x)(Jx ⊃ Kx)	Assumption
2	(∀x) ~ Kx	Assumption
3	J*f*(*f*(a,b))	Assumption
4	J*f*(*f*(a,b)) ⊃ K*f*(*f*(a,b))	1 ∀E
5	K*f*(*f*(a,b))	3, 4 ⊃E
6	~ K*f*(*f*(a,b))	2 ∀E
7	~ J*f*(*f*(a,b))	3–6 ~ I

LOGIC TEST 4

1. Define
a. Validity in *PD*
b. Theorem in *PD*

2. Construct derivations showing that
a. '$(\forall x)Gx$' and '$(\forall x)(\sim Gx \supset Gx)$' are equivalent in *PD*
b. '$(\forall x) \sim Px \supset \sim (\exists x)Px$' is a theorem in *PD*

3. Casino Slim, world-renowned riverboat gambler, reasons as follows:

In poker, three of a kind is not a royal flush. Any royal flush beats every hand that is not a royal flush. One hand beats another if and only if the latter loses to the former. Sneaky Sally's hand is a royal flush, and Hairy Harry's hand is three of a kind. Hence some hand loses to Sneaky Sally's hand.

Use the following symbolization key to symbolize Casino Slim's reasoning, and construct a derivation in *PD+* that shows that the symbolized argument is valid in *PD+*.

U.D.:	set of poker hands
Rx:	x is a royal flush
Tx:	x is three of a kind
Bxy:	x beats y
Lxy:	x loses to y
s:	Sneaky Sally's hand
h:	Hairy Harry's hand

4. Show that the following argument is valid in *PD+*:

$(\forall x)(\forall y)(\forall z)[(Lxy \,\&\, Lyz) \supset Lxz]$

$\sim (\forall x)(\forall y) \sim Lxy$

$(\forall x)(\forall y)(Lxy \supset Lyx)$

$(\exists x)Lxx$

5. Give a routine using the rules of *PD+* that shows that Universal. Elimination can be eliminated, that is, that, given the other rules of *PD+*, the rule $\forall$E need never be used. (Hint: It will be useful to appeal to $\exists$I and QN.)

6. Show in *PDE* that $\{a = b, (\forall x)(x = a \equiv Gx)\} \vdash Gb$

7. Complete the following derivation in *PDE*

Derive: $(\forall x)Bf(g(x))$

1	$(\forall x)(Zx \vee Gx)$	Assumption
2	$(\forall x)(Zx \supset Bx)$	Assumption
3	$(\forall x)(Gx \supset Bx)$	Assumption

ANSWERS

1. a. An argument of *PL* is *valid in PD* if and only if the conclusion of the argument is derivable in *PD* from the set consisting of the premises.

b. A sentence **P** of *PL* is a *theorem in PD* if and only if **P** is derivable in *PD* from the empty set.

2. a. Derive: $(\forall x)(\sim Gx \supset Gx)$

1	$(\forall x)Gx$	Assumption
2	$\quad \sim Ga$	Assumption
3	$\quad Ga$	1 $\forall$E
4	$\sim Ga \supset Ga$	2–3 $\supset$I
5	$(\forall x)(\sim Gx \supset Gx)$	4 $\forall$I

Derive: $(\forall x)Gx$

1	$(\forall x)(\sim Gx \supset Gx)$	Assumption
2	$\quad \sim Ga$	Assumption
3	$\quad \sim Ga \supset Ga$	1 $\forall$E
4	$\quad Ga$	2, 3 $\supset$E
5	$\quad \sim Ga$	2 R
6	Ga	2–5 $\sim$ E
7	$(\forall x)Gx$	6 $\forall$I

b. Derive: $(\forall x) \sim Px \supset \sim (\exists x)Px$

1	$\quad (\forall x) \sim Px$	Assumption
2	$\quad\quad (\exists x)Px$	Assumption
3	$\quad\quad\quad Pa$	Assumption
4	$\quad\quad\quad\quad (\exists x)Px$	Assumption
5	$\quad\quad\quad\quad \sim Pa$	1 $\forall$E
6	$\quad\quad\quad\quad Pa$	3 R
7	$\quad\quad\quad \sim (\exists x)Px$	4–6 $\sim$ I
8	$\quad\quad \sim (\exists x)Px$	2, 3–7 $\exists$E
9	$\quad\quad (\exists x)Px$	2 R
10	$\quad \sim (\exists x)Px$	2–9 $\sim$ I
11	$(\forall x) \sim Px \supset \sim (\exists x)Px$	1–10 $\supset$I

3. Derive: $(\exists x)Lxs$

1	$(\forall x)(Tx \supset \sim Rx)$	Assumption
2	$(\forall x)[Rx \supset (\forall y)(\sim Ry \supset Bxy)]$	Assumption
3	$(\forall x)(\forall y)(Bxy \equiv Lyx)$	Assumption
4	$Rs \;\&\; Th$	Assumption
5	$Rs \supset (\forall y)(\sim Ry \supset Bsy)$	2 $\forall$E
6	Rs	4 &E
7	$(\forall y)(\sim Ry \supset Bsy)$	5, 6 $\supset$E
8	$\sim Rh \supset Bsh$	7 $\forall$E
9	$Th \supset \sim Rh$	1 $\forall$E
10	Th	4 &E
11	$\sim Rh$	9, 10 $\supset$E
12	Bsh	p8, 11 $\supset$E
13	$(\forall y)(Bsy \equiv Lys)$	3 $\forall$E
14	$Bsh \equiv Lhs$	13 $\forall$E
15	Lhs	12, 14 $\equiv$ E
16	$(\exists x)Lxs$	15 $\exists$I

4. Derive: $(\exists x)Lxx$

1	$(\forall x)(\forall y)(\forall z)[(Lxy \;\&\; Lyz) \supset Lxz]$	Assumption
2	$\sim (\forall x)(\forall y) \sim Lxy$	Assumption
3	$(\forall x)(\forall y)(Lxy \supset Lyx)$	Assumption
4	$(\exists x) \sim (\forall y) \sim Lxy$	2 QN
5	$\quad \sim (\forall y) \sim Lay$	Assumption
6	$\quad (\exists y) \sim \sim Lay$	5 QN
7	$\quad\quad \sim \sim Lab$	Assumption
8	$\quad\quad (\forall y)(Lay \supset Lya)$	3 $\forall$E
9	$\quad\quad Lab \supset Lba$	8 $\forall$E
10	$\quad\quad (\forall y)(\forall z)[(Lay \;\&\; Lyz) \supset Laz]$	1 $\forall$E
11	$\quad\quad (\forall z)[(Lab \;\&\; Lbz) \supset Laz]$	10 $\forall$E
12	$\quad\quad (Lab \;\&\; Lba) \supset Laa$	11 $\forall$E
13	$\quad\quad Lab$	7 DN
14	$\quad\quad Lba$	9, 13 $\forall$E
15	$\quad\quad Lab \;\&\; Lba$	13, 14 &I
16	$\quad\quad Laa$	12, 15 $\supset$E
17	$\quad\quad (\exists x)Lxx$	16 $\exists$I
18	$\quad (\exists x)Lxx$	6, 7–17 $\exists$E
19	$(\exists x)Lxx$	4, 5–18 $\exists$E

5.

$\mathbf{i}$	$(\forall \mathbf{x})\mathbf{P}$	
$\mathbf{n}$	$\quad \sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	Assumption
$\mathbf{n} + 1$	$\quad (\exists \mathbf{x}) \sim \mathbf{P}$	$\mathbf{n}$ ∃I
$\mathbf{n} + 2$	$\quad \sim (\forall \mathbf{x})\mathbf{P}$	$\mathbf{n}$ + 1 QN
$\mathbf{n} + 3$	$\quad (\forall \mathbf{x})\mathbf{P}$	$\mathbf{i}$ R
$\mathbf{n} + 4$	$\mathbf{P}(\mathbf{a}/\mathbf{x})$	$\mathbf{n} - \mathbf{n} + 3 \sim$ E

6. Derive: Gb

1	a = b	Assumption
2	(∀x) (x = a ≡ Gx)	Assumption
3	b = a ≡ Gb	2 ∀E
4	b = b	1, 1 =E
5	b = a	1, 4 =E
6	Gb	3, 5 ≡E

7. Derive: (∀x)B*f*(*g*(x))

1	(∀x) (Zx ∨ Gx)	Assumption
2	(∀x) (Zx ⊃ Bx)	Assumption
3	(∀x) (Gx ⊃ Bx)	Assumption
4	Z*f*(*g*(a)) ∨ G*f*(*g*(a))	1 ∀E
5	Z*f*(*g*(a))	Assumption
6	Z*f*(*g*(a)) ⊃ B*f*(*g*(a))	2 ∀E
7	B*f*(*g*(a))	5, 6 ⊃E
8	G*f*(*g*(a))	Assumption
9	G*f*(*g*(a)) ⊃ B*f*(*g*(a))	3 ∀E
10	B*f*(*g*(a))	8, 9 ⊃E
11	B*f*(*g*(a))	4, 5–7, 8–10 ∨E
12	(∀x)B*f*(*g*(x))	11 ∀I

SOLUTIONS TO EXERCISES

CHAPTER ONE

Section 1.3E

1. b. This sentence is either true or false, though we do not know which.

d. This is a lament or expression of regret, it is neither true nor false, and therefore does not fall within the scope of this text.

f. This sentence does have a truth-value (it is false), and does fall within the scope of this text. (Two, not one, is the smallest prime number.)

h. This sentence is either true or false, though we do not know which. On the date and time in question there either was, or was not, a snowstorm. It is true the concept of Eastern Standard Time had not been developed in 1134, but we can apply that concept to the date in question. If, for example, we knew that the sun was shining and no snow was falling on January 15, 1134, when the sun was so and so many degrees above the western horizon, we would know the sentence in question is false.

j. This sentences, which expresses a wish, is (if we are to believe StarTrek) used as a greeting or salutation. It is neither true nor false and hence does not fall within the scope of this text.

l. This sentence is a question. Since questions are neither true nor false, it does not fall within the scope of this text. The answer, by the way, is 'Jack Nelson'.

n. This sentence gives advice and is neither true nor false. Hence it does not fall within the scope of this text. Note that the advice this sentence gives is the opposite of that given by sentence k.

2. b. This passage is best construed as an explanation for a fact (that the press releases are always clear and upbeat), rather than an argument intended to establish that point. However, it may be construed as an argument and, if so, it would have the following form:

Jack doesn't like sound bytes.

Mike does.

Mike is the press officer.

Our press releases are always crisp and upbeat.

d. Shelby and Noreen are wonderful in dealing with irate students and faculty.

Stephanie is wonderful at managing the Chancellor's very demanding schedule, and Tina keeps everything moving and cheers everyone up.

This is a great office to work in.

f. The galvanized nails, both common and finishing, are in the first drawer.

The plain nails are in the second drawer.

The third drawer contains sheet rock screws of various sizes, and the fourth drawer contains wood screws.

The bottom drawer contains miscellaneous hardware.

We should have everything we need to repair the broken deck chair.

h. The new kitchen cabinets are done and the installers are scheduled to come Monday.

The old cabinets haven't been removed.

The carpenter who is to do the removal is off for a week of duck hunting in North Dakota.

There will probably be a delay of at least a week.

j. Sarah, John, Rita, and Bob have all worked hard and all deserve promotion.

The Company is having a cash flow problem and is offering those over 55 a $50,000 bonus if they retire at the end of this year.

Sarah, John, and Bob are all over 55 and will take early retirement.

Rita will be promoted.

l. I didn’t die yesterday.

I didn’t die the day before yesterday.

I didn’t die the day before the day before yesterday.

I didn’t die any day in the past fifty some years.

I am not going to die today.

n. The given passage can be read as expressing the following argument:

The perceived need for the military security offered by the Soviet Union disappeared with the end of the cold war.

Over 70 years of union produced few economic benefits.

The Soviet Union never successfully addressed the problem how to inspire loyalty to a single state by peoples with vastly different cultures and history.

The Soviet Union disintegrated.

p. This passage is probably not an argument. Rather it is the recitation of a fanciful series of events. The ‘So’ of the last sentence has the force of ‘And then’ or ‘After giving up their efforts to reassemble Humpty Dumpty . . .”

Section 1.4E

1. b. False. Many invalid arguments have true premises. Here is an example:

The fourth of July is a national holiday in the United States.

Memorial Day is a national holiday in the United States.

Labor Day is a national holiday in the United States.

Bastille Day is a holiday in the United States.

d. False. A valid argument is sound if all of its premises are true, but not otherwise. Valid arguments can have one or more false premises and, therefore, be unsound. Here is an example:

All Doberman pinschers are dogs and all Doberman pinschers are friendly.

All Doberman pinschers are dogs.

f. False. For an argument to be valid it must be impossible for the premises to be true and the conclusion false. That a conclusion is true does not show that it is impossible for the premises in question to be true and the conclusion false. Consider:

Temple University became part of the Commonwealth of Pennsylvania System of Higher Education in 1965.

Temple University is today part of the Commonwealth of Pennsylvania System of Higher Education.

Both the premises and the conclusion of this argument are true. But the premise could be true and the conclusion false. This would be so, for example, if Temple had disaffiliated itself from the Commonwealth system in 1982 and remained disaffiliated.

h. True. If an argument does have true premises and a false conclusion, then it is possible for the premises to be true and the conclusion false, and the argument is therefore invalid.

j. False. A sound argument is, by definition, a valid argument with true premises, and an argument is valid if and only if it is impossible for the premises to be true and the conclusion false. So a sound argument, being a valid argument with true premises, cannot have a false conclusion.

2. b. If collies are reptiles, then collies are warm blooded.
Collies are reptiles.

Collies are warm blooded.

d. Temple University is in Pennsylvania.
Smith College is in Massachusetts.
Dartmouth College is in New Hampshire.

UCLA is in California.

f. Temple University is in Pennsylvania.
Smith College is in Nevada.

UCLA is in New Hampshire.

Section 1.5E

1. b. This passage is perhaps best construed as an inductive argument. Even if some unstated assumptions of the argument are added, that sound bytes tend to be crisp and upbeat and that the press officer tends to get his way with press releases, the argument is deductively invalid.

d. This is a reasonably strong inductive argument. The conclusion, that this is a great office to work in, doesn't follow from the premises, but the premises do make it probable.

f. This is a strong inductive argument. Because there is no guarantee we won't need things other than those specified, it is not, construed as a deductive argument, deductively valid.

h. This is a strong inductive argument. It is not, construed as a deductive argument, deductively valid because the premises do not rule out the possibility that the carpenter scheduled to remove the old cabinets won't return early, or the possibility that someone else will remove the old cabinets, or even the unlikely possibility that the new cabinets can be installed before the old ones are removed.

j. This passage about Sarah, John, Rita and Bob is best construed as an inductive argument. The plausibility of the conclusion rests on the assumptions, not stated, that those who take early retirement will not be promoted, and that if all four of the named individuals deserve promotion at least one will be promoted. Unless these assumptions are made explicit (the first is reasonable, the second less clearly so), the argument is inductively weak.

l. This old argument is best construed as an inductive argument. It is a reasonably strong argument, but it is also possible, of course, that the conclusion is false. And we know that one day the conclusion will be false.

n. This passage is best construed as a purported explanation of why the Soviet Union disintegrated. It is best to construe this passage as an inductive argument (it is clearly invalid when construed as a deductive argument). How strong the argument is we leave to historians and political scientists to decide.

p. As noted in the answers to Exercise Set 1.3E, this passage is probably best construed not as an argument but as the recitation of a fanciful series of events. The 'So' of the last sentence has the force of 'And then' or 'After giving up their efforts to reassemble Humpty Dumpty . . ." Construed as an argument it is deductively invalid and inductively weak.

Section 1.6E

1. b. {Kansas City is in Missouri. St. Paul is in Wisconsin. San Francisco is in California.} The claims about, respectively, Kansas City and San Francisco are true. The claim about St. Paul is false.

d. {The Soviet Union placed an astronaut on the moon in 1972. The Mariners won the World Series in 2001.} Both members of this set are, as a matter of history, false. But things could have gone otherwise—both could be true. So the set is consistent.

2. b. This set is inconsistent. It is not possible that both Henry likes real ice cream, which is a dairy product, and that Henry doesn't like any dairy product.

d. This set is consistent. The claims about Washington, D.C. and Paris are both true. The claim about Toronto would be true if Canada made that city its capital, which is possible.

f. This set is inconsistent. If Sue is taller than Tom and Tom is taller than Henry, then Sue is clearly taller than Henry. And if this is so Henry cannot be as tall as Sue. So not all the members of the set can be true.

h. This set is inconsistent. If it is true that the United States supported Iraq in the 1980s (it is), and true that Iraq has been a dictatorship since 1979 (it is), then it is clearly not true that the United States does not support dictatorships.

j. This set is inconsistent. It cannot be true that Jones and his relatives own all the land in Gaylord, and that Smith owns land in Gaylord, and that Smith is not a relative of Jones.

l. This set is inconsistent. If it is true that Sarah likes film classics, and true that everyone who likes film classics likes *Casablanca,* then it cannot also be true that Sarah can't stand *Casablanca.*

3. b. 'Everyone's a scoundrel, but I'm not' is logically false. If everyone is a scoundrel then I, being someone, am a scoundrel. Similarly, if I am not a scoundrel, then at least one person, namely me, is not a scoundrel, so not everyone is a scoundrel.

4. b. Logically indeterminate. Some, but not all, doctors are M.D.s. (Consider veterinarians, doctors of osteopathy, dentists, Ph.D.s . . .). So Helen may well be a doctor but not an M.D.

d. Logically indeterminate. Most of us, finding ourselves in London, would have our hearts somewhere other than Texas. But Bob may be an exception.

f. Logically indeterminate. Robin might not be enrolled in the class. If so, then she might not make it to the class by starting time but not be late—only those who are in the class, or have promised to attend the class session in question, can be late.

h. Logically false. Ocean fish are a kind of fish, so if Sarah likes all kinds of fish she likes ocean fish, contrary to what the last part of this sentence says.

j. Logically indeterminate. Those committed to raising emus would like us to believe this sentence, and indeed the sentence may be true, but if so it is because emu tastes like beef, or looks like beef, or whatever, not because of any principle of logic.

l. True, but not logically true. If a disaster strikes and all but five people in the world are killed, then there could be someone who likes everyone but only likes five people.

5. b. Seattle is the largest city in Washington State.
George W. Bush succeeded Bill Clinton as President of the United States.

d. Gore succeeded Clinton as President.
Clinton was succeeded by Gore as President.

f. Not possible. Sentences that are logically equivalent are such that is impossible for one of them to be true and the other false. A pair of sentences, one of which is logically true and the other of which is logically false is such that one is true and the other false. So they are not equivalent.

6. b. These sentences are logically equivalent. Person A cannot marry person B without B also marrying A.

d. These sentences are logically equivalent. If both Bill and Mary were admitted, then each of them was admitted, and vice-versa.

f. These sentences are not logically equivalent. Having a judge pronounce a couple husband and wife is not the only way for a couple to be married. In most states cohabiting for a set period of time constitutes being married. There are also individuals other than judges who can marry people.

h. These sentences are not logically equivalent. The first sentence is true. The second is almost certainly false, no matter who is speaking. It is certainly false when I say it, for there are many large cities in America where I do not know anyone.

j. These sentences are logically equivalent. If a bad day sailing is *better than* a good day at work, then of course a good day at work isn't *as good as* a bad day sailing, and vice-versa.

l. These sentences are equivalent. If the first is true, they won't both be elected, then of course the second is true, either Sara or Anna will not be elected. And if the second is true, then of course they will not both be elected, that is, the first is also true.

n. These sentences are not logically equivalent. If the total group of people were small enough, then everyone could dislike someone, but there still be a person whom everyone likes.

p. These sentences are logically equivalent. If not everyone likes someone, then there is a person who doesn't like anyone, and vice-versa.

Section 1.7E

1. b. False. 'Stevenson defeated Eisenhower in 1952' and 'Eisenhower was defeated by Stevenson in 1952' are logically equivalent and are both false.

d. True. These two sentences cannot both be true. Therefore, any argument that has both of them as premises cannot have all true premises, and, hence, cannot have all true premises and a false conclusion. Such an argument is, therefore, deductively valid.

f. False. The sentence in question is logically false, and any argument that has that sentence as a conclusion will, therefore, always have a false conclusion. But such an argument will be deductively invalid only if it is possible for its premises to be true. If the premises are logically inconsistent, this is not possible. So not every such argument is deductively invalid.

2. b. No. Arguments, in our sense, are of use when one is interested in finding out how well founded a claim is or what the consequences of accepting it are. These interests do not arise only in the context of a dispute or disagreement.

d. If an argument is valid but has a false conclusion, then at least one of the premises must be false. Otherwise the argument would be invalid, because it would have true premises and a false conclusion.

f. Yes. If an argument has a premise that is logically equivalent to a logical falsehood, then that premise is itself a logical falsehood. An argument with a logical falsehood among its premises cannot have all true premises and a false conclusion, hence such an argument is deductively valid.

h. If the premises of an argument form an inconsistent set, then not all the premises can be true. If this is so, obviously the argument cannot have true premises and a false conclusion. Such an argument is, therefore, deductively valid, no matter what the conclusion is. However, no such argument can be sound, for a sound argument has all true premises, and premises that form an inconsistent set cannot all be true.

Section 2.1E

1. b. Both it is not the case that Bob jogs regularly and Carol jogs regularly.

~ B & C

d. Both Albert jogs regularly and Carol jogs regularly.

A & C

f. Both Bob jogs regularly and it is not the case that Albert jogs regularly.

B & ~ A

h. Both (both it is not the case that Albert jogs regularly and it is not the case that Bob jogs regularly) and it is not the case that Carol jogs regularly.

(~ A & ~ B) & ~ C

j. Both it is not the case that Carol jogs regularly and (either Bob jogs regularly or Albert jogs regularly).

~ C & (B ∨ A)

l. It is not the case that either (either Albert jogs regularly or Bob jogs regularly) or Carol jogs regularly.

~ ((A ∨ B) ∨ C)

n. It is not the case that either (either Albert jogs regularly or Carol jogs regularly) or Bob jogs regularly.

~ [(A ∨ C) ∨ B]

(or)

Both (both it is not the case that Albert jogs regularly and it is not the case that Carol jogs regularly) and it is not the case that Bob jogs regularly.

(~ A & ~ C) & ~ B

2. b. Either Albert jogs regularly or he doesn't.
d. Neither Albert nor Carol jogs regularly.
f. It isn't the case that Bob doesn't jog regularly.
h. Either Albert or Carol, but not both, jogs regularly.
j. Either Albert or Bob or Carol does not jog regularly.

3. b and j are true; d, f, and h are false.

4. Paraphrases:

b. It is not the case that Bob is a marathon runner.

d. It is not the case that all joggers are healthy.

Symbolizations:

b. Using ‘B’ for ‘Bob is a marathon runner’:

$\sim B$

d. Using ‘H’ for ‘All joggers are healthy’:

$\sim H$

5. b. If it is not the case that Bob is lazy then Bob jogs regularly.

$\sim L \supset B$

d. If Carol is a marathon runner then Carol jogs regularly.

$M \supset C$

f. If Carol jogs regularly then (if it is not the case that Bob is lazy then Bob jogs regularly).

$C \supset (\sim L \supset B)$

h. If (either Carol jogs regularly or Bob jogs regularly) then Albert jogs regularly.

$(C \vee B) \supset A$

j. If it is not the case that (either Carol jogs regularly or Bob jogs regularly) then it is not the case that Albert jogs regularly.

$\sim (C \vee B) \supset \sim A$

l. If Albert is healthy then (Albert jogs regularly if and only if Bob jogs regularly).

$H \supset (A \equiv B)$

n. Both (both Albert is healthy and it is not the case that Albert jogs regularly) and (if Bob jogs regularly then Carol jogs regularly).

$(H \;\&\; \sim A) \;\&\; (B \supset C)$

p. If Albert jogs regularly then (if Bob jogs regularly then Carol jogs regularly).

$A \supset (B \supset C)$

r. If (if Albert jogs regularly then Albert is healthy) then (if Bob is lazy then it is not the case that Bob jogs regularly).

$(A \supset H) \supset (L \supset \sim B)$

t. If it is not the case that Albert is healthy, then it is not the case that both Bob jogs regularly and Albert jogs regularly.

$\sim H \supset \sim (B \& A)$

6. b. If Carol is a marathon runner, then she jogs regularly.
d. Carol jogs regularly but Bob does not.
f. If Albert, Bob, or Carol jogs regularly, then they all do.
h. If neither Albert nor Carol is a regular jogger, then Bob is.
j. Bob jogs regularly if and only if both Bob is not lazy and Albert jogs regularly.
l. Albert is a regular jogger, but Carol is a regular jogger if and only if Bob is.
n. It isn't the case that Albert is not healthy, but he isn't a regular jogger either.
p. If Carol jogs regularly, then so does Albert, but if Albert does, so does Bob.
r. If Albert is healthy only if he jogs regularly, then Bob is not lazy only if he jogs regularly.

7. b. Both it is not the case that this dog will hunt and it is not the case that this dog is even a good pet.

$\sim H \& \sim P$

d. Either it is not the case that the tea will taste robust or the tea steeps for awhile.

$\sim R \vee S$

(or another possibility)

If it is not the case that the tea steeps for awhile then it is not the case that the tea will taste robust.

$\sim S \supset \sim R$

f. Both both it is not the case that wind will stop the mail and it is not the case that rain will stop the mail and it is not the case that night will stop the mail.

$(\sim W \& \sim R) \& \sim N$

h. Either snow storms arrive or both skiing will be impossible and snowboarding will be impossible.

$A \vee (S \& B)$

8.	P	Q	(P ∨ Q) & ~ (P & Q)	(P ≡ ~ Q)
	T	T	F	F
	T	F	T	T
	F	T	T	T
	F	F	F	F

Section 2.2E

1. b. Either it is not the case that either the French team will win at least one gold medal or the German team will win at least one gold medal or either it is not the case that either the French team will win at least one gold medal or the Danish team will win at least one gold medal or it is not the case that either the German team will win at least one gold medal or the Danish team will win at least one gold medal.

$\sim (F \vee G) \vee [\sim (F \vee D) \vee \sim (G \vee D)]$

d. It is not the case that either the French team will win at least one gold medal or either it is not the case that the German team will win at least one gold medal or it is not the case that the Danish team will win at least one gold medal.

$\sim F \vee (\sim G \vee \sim D)$

f. Either it is not the case that the French team will win at least one gold medal or either it is not the case that the German team will win at least one gold medal or it is not the case that the Danish team will win at least one gold medal.

$\sim F \vee (\sim G \vee \sim D)$

h. Both the French team will win at least one gold medal and both the German team will win at least one gold medal and the Danish team will win at least one gold medal.

$F \& (G \& D)$

2. b. Not all of them will win a gold medal.
d. At most one of them will win a gold medal.
f. At least two of them will win gold medals.
h. The French team will win a gold medal and at least one of the other two teams will win a gold medal.

3. b. If the French team will win at least one gold medal then it is not the case that the French team is plagued with injuries.

$F \supset \sim P$

d. If both it is not the case that it rains during most of the competition and it is not the case that the star German runner is disqualified then if either the French team will win at least one gold medal or the Danish team will win at least one gold medal then the German team will win at least one gold medal.

$(\sim R \,\&\, \sim S) \supset [(F \vee D) \supset G]$

f. If the German team will win at least one gold medal then both it is not the case that it rains during most of the competition and it is not the case that the star German runner is disqualified.

$G \supset (\sim R \,\&\, \sim S)$

h. Both if it is not the case that it rains during most of the competition, then the Danish team will win at least one gold medal and if it rains during most of the competition then both it is not the case that the Danish team will win at least one gold medal and both the French team will win at least one gold medal and the German team will win at least one gold medal.

$(\sim R \supset D) \,\&\, (R \supset [\sim D \,\&\, (F \,\&\, G)])$

4. b. If neither the French nor the German team will win a gold medal, the Danish team will.

d. If the French team is plagued with injuries and the star German runner is disqualified, then the Danish team will win a gold medal.

f. If it rains during most of the competition, then none of the three teams will win a gold medal.

h. The Danish team will win a gold medal unless the French team, not plagued with injuries, will win or the German team, with its star runner not disqualified, will win.

5. b. Either it is not the case that George does have a high cholesterol level or cholesterol is trapped in the walls of his arteries.

Both if cholesterol is trapped in the walls of his arteries then plaque will build up and block his arteries and if plaque will build up and block his arteries then George is a candidate for a heart attack.

George is a candidate for a heart attack.

L: George does have a high cholesterol level.

W: Cholesterol is trapped in the walls of his arteries.

P: Plaque will build up and block his arteries.

H: George is a candidate for a heart attack.

$\sim L \vee W$

$(W \supset P) \,\&\, (P \supset H)$

H

d. Both [if it is not the case that (either Henry will play the part of the lawyer or Fred will play the part of the lawyer) then it is not the case that Morris will be upset] and (if it is not the case that Morris will be upset then the drama will be a success).

Both (both it is not the case that Henry will play the part of the lawyer and it is not the case that Fred will play the part of the lawyer) and (the drama will get good reviews if and only if the drama will be a success).

The drama will get good reviews.

D: The drama will be a success.

H: Henry will play the part of the lawyer.

F: Fred will play the part of the lawyer.

U: Morris will be upset.

R: The drama will get good reviews.

$[\sim (H \vee F) \supset \sim U] \ \& \ (\sim U \supset D)$

$(\sim H \ \& \ F) \ \& \ (R \equiv D)$

R

f. If Betty is the judge then it is not the case that Peter will get a suspended sentence.

Both (if it is not the case that the district attorney is brief, then the trial will be long) and it is not the case that the district attorney is brief.

Fred is the defense lawyer.

Both (if Fred is the defense lawyer then Peter will be found guilty) and (if Peter will be found guilty then Peter will be given a sentence).

Both the trial will be long and both Peter will be given a sentence and it is not the case that Peter will get a suspended sentence.

J: Betty is the judge.

G: Peter will get a suspended sentence.

L: The trial will be long.

B: The district attorney is brief.

D: Fred is the defense lawyer.

F: Peter will be found guilty.

S: Peter will be given a sentence.

$J \supset \sim G$

$(\sim B \supset L) \ \& \sim B$

D

$(D \supset F) \ \& \ (F \supset S)$

$L \ \& \ (S \ \& \sim G)$

Section 2.3E

1. Since we do not know how these sentences are being used (e.g., as premises, conclusions, or as isolated claims), it is best to symbolize those which are non-truth-functional compounds as atomic sentences of *SL*.

b. This sentence is a disjunctive and can be paraphrased truth-functionally.

> Either Rocky knows who will arrive on the train or George knows who will arrive on the train.

A symbolization is '$R \vee G$'.

d. 'Because' has no truth-functional sense. Hence the entire sentence should be abbreviated by a capital letter such as 'L'.

f. This is a subjunctive conditional that is not a truth-functional compound. Abbreviate the entire sentence by one letter such as 'L'.

h. 'And' is the main connective and it is here used as a truth-functional connective. However, notice that the two conjuncts of this conjunction are not truth-functional compounds and must be treated as single sentences. The paraphrase is

> Both John believes that our manuscript has been stolen and Howard believes that our manuscript has been lost.

which can be symbolized as '$J \ \& \ H$'.

2. b. The second and third sentences in the argument have component sentences embedded in belief contexts. The overall belief sentences must be treated as non-truth-functional.

> If this piece of metal is gold then this piece of metal has atomic number 79.
>
> Nordvik believes this piece of metal is gold.
>
> ---
>
> Nordvik believes this piece of metal has atomic number 79.

G: This piece of metal is gold.
A: This piece of metal has atomic number 79.
N: Nordvik believes this piece of metal is gold.
M: Nordvik believes this piece of metal has atomic number 79.

$$\begin{array}{l} G \supset A \\ N \\ \hline M \end{array}$$

Section 2.4E

1. b. True.
d. False. The name of copper is not copper.
f. True.

2. b. In its first occurrence 'Deutschland' is being used, and refers to Germany. In its second occurrence 'Deutschland' occurs within single quotation marks and is thus being mentioned, not used. Note that the expression 'the German name of Germany' also refers to the word 'Deutschland'.

d. Here the word 'Deutschland' is being mentioned and is claimed to be the same as itself.

f. Here the word 'Deutschland' is being used, and hence we have a false claim, namely the claim that Deutschland, a country (Germany), is the name of Germany. It is not. Deutschland is Germany.

3. b. Not a sentence—the left conjunct is missing.
d. Not a sentence—'~' cannot be used to join two sentences.
f. Not a sentence of *SL*—the boldfaced letters are metavariables in the metalanguage.
h. Not a sentence—parentheses are missing. It could be transformed into a sentence by writing it as '(U & (C & ~ L))' or '((U & C) & ~ L)'.
j. Not a sentence—a right parenthesis is missing. It should be '[(G ∨ E) ⊃ (~ H & (K ∨ B))]'.

4. b. The main connective is '~'. The immediate sentential component is '(A & H)'. '~ (A & H)' is a component of itself. The remaining sentential components, which are also the atomic components, are 'A' and 'H'.

d. The main connective is the first occurrence of the '⊃'. The immediate sentential components are 'K' and '(~ K ⊃ K)'. The sentential components are these immediate sentential components, the sentence itself, and '~ K' and 'K'. The atomic component is, of course, 'K'.

f. The main connective is the first occurrence of the '⊃'. The immediate sentential components are 'M' and '[~ N ⊃ ((B & C) ≡ ~ [(L ⊃ J) ∨ X])]'.

Additional sentential components are the sentence itself, '~ N', '((B & C) ≡ ~ [(L ⊃ J) ∨ X])', '(B & C)', '~ [(L ⊃ J) ∨ X]', '[(L ⊃ J) ∨ X]', '(L ⊃ J)', 'N', 'B', 'C', 'L', 'J', and 'X'. The last six sentential components listed, along with 'M', are atomic components.

5. b. Yes. Here **P** is the sentence 'A' and **Q** is the sentence 'B'.
d. Yes. Here **P** is '~ A' and **Q** is 'B'.
f. Yes. Here **P** is '~ A' and **Q** is '~ B'.
h. Yes. Here **P** is '~ (A ⊃ B)' and **Q** is 'C ⊃ D'.
j. No. This sentence is a conjunction, not a conditional.

6. b. '&' may occur immediately to the left of '~' as well as immediately to the right of 'A'. An example in which '&' occurs in both ways is '(A & ~ B)'.

d. ')' may occur immediately to the right of 'A', as in '(B & ~ A)', but not immediately to the left of '~'. ')' marks the end of a sentence, and a sentence can be followed by a binary but not by a unary connective.

f. '~' may not occur immediately to the right of 'A' but may occur immediately to the left of another '~', as in '(B & ~ ~ A)'. '~' is a unary connective and as such can precede but not follow sentences of *SL*.

Section 3.1E

1. b. $2^5 = 32$.

2. b.

					↓		
A	B	(A	&	B)	≡	~	B
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**F**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**

d.

								↓					
A	B	C	[A	⊃	(B	⊃	C)]	&	[(A	⊃	B)	⊃	C]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**

f.

								↓				
A	B	(~	~	A	&	~	B)	⊃	(~	A	≡	B)
T	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**
F	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**

h.

					↓						
D	E	H	~	D	&	[~	H	∨	(D	&	E)]
T	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**
T	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**

j.

										↓					
A	B	D	~	(D	≡	(~	A	&	B))	∨	(~	D	∨	~	B)
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
T	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

l.

														↓		
E	F	J	(J	&	[(E	∨	F)	&	(~	E	&	~	F)])	⊃	~	J
T	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**
T	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**
F	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

3. b.

			↓							
A	B	C	~	[A	∨	(~	C	&	~	B)]
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**

d.

						↓			
A	B	C	(A	⊃	B)	⊃	(B	⊃	C)
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**

f.

					↓			
A	B	C	~	A	⊃	(B	≡	C)
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**

h.

			↓												
A	B	C	~	[~	A	≡	~	(B	≡	~	[A	≡	(B	&	C)])]
F	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**

j.

								↓					
A	B	C	~	(B	⊃	~	A)	&	[C	≡	(A	&	B)]
F	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**

4. b.

								↓									
D	F	G	~	(F	∨	G)	∨	[~	(F	∨	D)	∨	~	(G	∨	D)]
T	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**

d.

					↓					
D	F	G	~	F	∨	(~	G	∨	~	D)
T	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

f.

					↓					
D	F	G	~	F	∨	(~	G	∨	~	D)
T	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

h.

				↓			
D	F	G	F	&	(G	&	D)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**	**F**	**F**	**T**
F	**T**	**T**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**

5. b.

F	P	(F	∨	P)	↓ &	(P	⊃	~	F)
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**
F	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**

d.

D	F	G	R	S	(~	R	&	~	S)	↓ ⊃	[(F	∨	D)	⊃	G]
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
T	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**
F	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

f.

G	R	S	G	⊃ ↓	(~ R	&	~ S)
T	**T**	**T**	**T**	**F**	**F T**	**F**	**F T**
T	**T**	**F**	**T**	**F**	**F T**	**F**	**T F**
T	**F**	**T**	**T**	**F**	**T F**	**F**	**F T**
T	**F**	**F**	**T**	**T**	**T F**	**T**	**T F**
F	**T**	**T**	**F**	**T**	**F T**	**F**	**F T**
F	**T**	**F**	**F**	**T**	**F T**	**F**	**T F**
F	**F**	**T**	**F**	**T**	**T F**	**F**	**F T**
F	**F**	**F**	**F**	**T**	**T F**	**T**	**T F**

h.

D	F	G	R	(D	∨	R)	& ↓	(R	⊃	[~ D	&	(F	&	G)])
T	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F T**	**F**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F T**	**F**	**T**	**F**	**F**
T	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F T**	**F**	**F**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F T**	**F**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F T**	**F**	**F**	**F**	**F**
T	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F T**	**F**	**F**	**F**	**F**
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T F**	**F**	**F**	**F**	**T**
F	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T F**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T F**	**F**	**F**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T F**	**F**	**F**	**F**	**F**

Section 3.2E

1. b. Truth-functionally true

J	K	J	⊃ ↓	(K	⊃	J)
T	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**

d. Truth-functionally true

					↓					
E	H	(E	≡	H)	⊃	(~	E	⊃	~	H)
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**

f. Truth-functionally false

												↓		
C	D	E	([(C	⊃	D)	&	(D	⊃	E)]	&	C)	&	~	E
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**
T	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**

h. Truth-functionally true

		↓													
A	B	~	[[(A	∨	B)	&	(B	∨	B)]	&	(~	A	&	~	B)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
T	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**
F	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**

j. Truth-functionally true

				↓					
B	D	~	B	⊃	[(B	∨	D)	⊃	D]
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**
F	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
F	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

l. Truth-functionally false

M	N	(M	≡	~	N)	↓ &	(M	≡	N)
T	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**
F	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

2. b. Truth-functionally true

F	H	(F	∨	H)	↓ ∨	~	(~	F	⊃	H)
T	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**

d. Not truth-functionally true

A	B	A	↓ ≡	(B	≡	A)
T	**F**	**T**	**F**	**F**	**F**	**T**

f. Not truth-functionally true

C	D	[C	⊃	(C	∨	~	D)]	↓ ⊃	(C	∨	D)
F	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**

3. b. Not truth-functionally false

B	H	(B	⊃	H)	↓ &	(B	⊃	~	H)
F	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**

d. Not truth-functionally false

C	F	G	[(F	&	G)	⊃	(C	&	~	C)]	↓ &	F
T	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**

f. Not truth-functionally false

											↓								
A	B	F	[~	(A	&	F)	⊃	(B	∨	A)]	&	~	[~	B	⊃	~	(F	∨	A)]
T	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**

4. b. True. If one disjunct of **P** ∨ **Q** is true on every truth-value assignment, then the disjunction itself must be true on every truth-value assignment.

d. True. If one conjunct is truth-functionally false, then on every truth-value assignment the conjunction will have at least one false conjunct.

f. False. For example, although '(A & ~ A)' is truth-functionally false, 'A ⊃ (A & ~ A)' is not.

h. True. **P** is true on a truth-value assignment if and only if ~ **P** is false on that assignment, and **P** is false on a truth-value assignment if and only if ~ **P** is true on that assignment. So **P** is true on at least one truth-value assignment and false on at least one truth-value assignment if and only if ~ **P** is false on at least one truth-value assignment and true on at least one truth-value assignment.

j. False. In fact, a material conditional with a truth-functionally false antecedent must itself be truth-functionally true. If there is no truth-value assignment on which the antecedent is true, then there is no truth-value assignment on which the antecedent is true and the consequent false.

5. b. No. For example, 'A' and '~ A' are each truth-functionally indeterminate, but 'A & ~ A' is truth-functionally false.

d. It is truth-functionally indeterminate. Because **Q** is truth-functionally indeterminate, there are some truth-value assignments on which it is true, and **P** ⊃ **Q** will be true on these assignments. There are also some truth-value assignments on which **Q** is false, and, because **P** is true on every truth-value assignment, **P** ⊃ **Q** will be false on these assignments.

Section 3.3E

1. b. Truth-functionally equivalent

				↓								↓			
A	B	C	A	⊃	(B	⊃	A)	(C	&	~	C)	∨	(A	⊃	A)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

d. Not truth-functionally equivalent

				↓							↓	
A	B	C	C	&	(B	∨	A)	(C	&	B)	∨	A
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**
T	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**
F	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**

f. Truth-functionally equivalent

				↓				↓	
B	C	~	C	⊃	~	B	B	⊃	C
T	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**

h. Not truth-functionally equivalent

							↓					↓			
B	C	D	~	(D	∨	B)	⊃	(C	⊃	B)	C	⊃	(D	&	B)
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**

j. Truth-functionally equivalent

			↓							↓					
A	B	A	⊃	[B	⊃	(A	⊃	B)]	B	⊃	[A	⊃	(B	⊃	A)]
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

2. b. Not truth-functionally equivalent

		↓						↓	
A	B	~	(B	&	~	A)	A	∨	B
F	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**

d. Not truth-functionally equivalent

				↓							↓	
F	H	J	F	&	(J	∨	H)	(F	&	J)	∨	H
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**

f. Not truth-functionally equivalent

			↓													↓		
B	C	D	~	(~	B	∨	(~	C	∨	~	D))	(D	∨	C)	&	~	B
T	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**

3. b. Not truth-functionally equivalent

F: The new play at the Roxy is a flop.
C: Critics will ignore the new play at the Roxy.
A: The new play at the Roxy is canceled.

				↓						↓			
A	C	F	F	&	(~	C	∨	A)	F	&	(A	⊃	C)
T	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
T	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**
F	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**

d. Not truth-functionally equivalent

A: 1972 was a good vintage year.
B: 1973 was a good vintage year.
C: 1974 was a good vintage year.
D: 1975 was a good vintage year.

A	B	C	D	(~	A	&	B)	↓ &	~	(C	∨	D)	~	(C	∨	A)	↓ &	~	(B	&	D)
T	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
T	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**
T	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
T	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**
F	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
F	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
F	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**
F	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**

f. Truth-functionally equivalent

B: The blue team will win the tournament.
R: The red team will win the tournament.

B	R	(B	∨	R)	↓ &	~	(B	&	R)	R	↓ ≡	~	B
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**
F	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**

4. b. **P** and **Q** have the same truth-value on every truth-value assignment. On every truth-value assignment on which **P** is true so is **Q**; hence so is **P** & **Q**. On every truth-value assignment on which **P** is false so is **P** & **Q**. So **P** and **P** & **Q** are truth-functionally equivalent.

Section 3.4E

1. b. Truth-functionally inconsistent

				↓				↓				↓	
B	J	K	B	≡	(J	&	K)	~	J	~	B	⊃	B
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**
F	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**

d. Truth-functionally consistent

						↓			↓					↓			
A	B	C	(A	&	B)	&	C	C	∨	(B	∨	A)	A	≡	(B	⊃	C)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**
F	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**

f. Truth-functionally consistent

				↓					↓					↓		
H	U	W	U	∨	(W	&	H)	W	≡	(U	∨	H)	H	∨	~	H
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**
T	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**
T	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**
F	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**

h. Truth-functionally consistent

			↓				↓				↓					↓			
A	B	C	~	(A	&	B)	~	(B	&	C)	~	(A	&	C)	A	∨	(B	&	C)
T	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
T	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**

j. Truth-functionally consistent

				↓							↓		
A	B	C	A	⊃	(B	⊃	(C	⊃	A))	B	⊃	~	A
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**

2. b. Truth-functionally consistent

		↓				
H	H	≡	(~	H	⊃	H)
T	**T**	**T**	**F**	**T**	**T**	**T**

d. Truth-functionally inconsistent

									↓			↓		
A	B	C	~	(~	C	∨	~	B)	&	A	A	≡	~	C
T	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**
T	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**
F	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**
F	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**
F	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**

f. Truth-functionally consistent

				↓			↓			↓	
H	J	K	H	⊃	J	J	⊃	K	K	⊃	~ H
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F T**
T	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F T**
T	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F T**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T F**
F	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T F**
F	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T F**

3. b. Truth-functionally consistent

N: Newtonian mechanics is right.
E: Einsteinian mechanics is right.
S: Space is non-Euclidean.

				↓			↓			↓	
E	N	S	E	⊃	~ N	E	≡	S	S	∨	N
T	**T**	**T**	**T**	**F**	**F T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**F T**	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T F**	**T**	**T**	**T**	**T**	**T**	**F**
T	**F**	**F**	**T**	**T**	**T F**	**T**	**F**	**F**	**F**	**F**	**F**
F	**T**	**T**	**F**	**T**	**F T**	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**F**	**F**	**T**	**F T**	**F**	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**F**	**T**	**T F**	**F**	**F**	**T**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**T F**	**F**	**T**	**F**	**F**	**F**	**F**

d. Truth-functionally consistent

U: Sugar is desirable.
A: Saccharin is desirable.
G: Sugar is lethal.
C: Saccharin is lethal.

								↓					↓				↓		
A	C	G	U	~	(U	∨	A)	≡	(G	&	C)	G	≡	A	~	U	≡	~	C
T	T	T	T	F	T	T	T	F	T	T	T	T	T	T	F	T	T	F	T
T	T	T	F	F	F	T	T	F	T	T	T	T	T	T	T	F	F	F	T
T	T	F	T	F	T	T	T	T	F	F	T	F	F	T	F	T	T	F	T
T	T	F	F	F	F	T	T	T	F	F	T	F	F	T	T	F	F	F	T
T	F	T	T	F	T	T	T	T	T	F	F	T	T	T	F	T	F	T	F
T	F	T	F	F	F	T	T	T	T	F	F	T	T	T	T	F	T	T	F
T	F	F	T	F	T	T	T	T	F	F	F	F	F	T	F	T	F	T	F
T	F	F	F	F	F	T	T	T	F	F	F	F	F	T	T	F	T	T	F
F	T	T	T	F	T	T	F	F	T	T	T	T	F	F	F	T	T	F	T
F	T	T	F	T	F	F	F	T	T	T	T	T	F	F	T	F	F	F	T
F	T	F	T	F	T	T	F	T	F	F	T	F	T	F	F	T	T	F	T
F	T	F	F	T	F	F	F	F	F	F	T	F	T	F	T	F	F	F	T
F	F	T	T	F	T	T	F	T	T	F	F	T	F	F	F	T	F	T	F
F	F	T	F	T	F	F	F	F	T	F	F	T	F	F	T	F	T	T	F
F	F	F	T	F	T	T	F	T	F	F	F	F	T	F	F	T	F	T	F
F	F	F	F	T	F	F	F	F	F	F	F	F	T	F	T	F	T	T	F

f. Truth-functionally consistent

J: Johnson pleaded guilty.
H: Hartshorne pleaded guilty.
N: The newspaper story is correct.

						↓						↓				↓	
H	J	N	(J	∨	H)	∨	~	(J	∨	H)	J	⊃	~	N	N	&	H
T	T	T	T	T	T	T	F	T	T	T	T	F	F	T	T	T	T
T	T	F	T	T	T	T	F	T	T	T	T	T	T	F	F	F	T
T	F	T	F	T	T	T	F	F	T	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	F	F	T	T	F	T	T	F	F	F	T
F	T	T	T	T	F	T	F	T	T	F	T	F	F	T	T	F	F
F	T	F	T	T	F	T	F	T	T	F	T	T	T	F	F	F	F
F	F	T	F	F	F	T	T	F	F	F	F	T	F	T	T	F	F
F	F	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F

4. b. Not necessarily. For example, {A ⊃ A} is truth-functionally consistent, but {~ (A ⊃ A)} is truth-functionally inconsistent. Do note, however, that if **P** is truth-functionally indeterminate, both {**P**} and {~ **P**} are truth-functionally consistent.

d. Assume that **P** ≡ **Q** is truth-functionally true. Then on any truth-value assignment, either both **P** and **Q** are true or both are false. {**P**, ~ **Q**} is truth-functionally consistent if and only if there is at least one truth-value assignment on which **P** is true and **Q** is false (thus making ~ **Q** true). But, on our assumption, this is impossible. Therefore the set is truth-functionally *in*consistent.

Section 3.5E

1. b. Truth-functionally invalid

				↓								↓				↓		↓			
A	B	C	B	∨	(A	&	~	C)	(C	⊃	A)	≡	B	~	B	∨	A	~	(A	∨	C)
T	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**
T	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**
F	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**
F	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**

d. Truth-functionally valid

			↓				↓		↓			↓		
A	W	Y	~	(Y	≡	A)	~	Y	~	A	W	&	~	W
T	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**
T	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**F**
T	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**
F	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**F**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**

f. Truth-functionally invalid

				↓											↓											↓
B	C	D	B	∨	B	[~	B	⊃	(~	D	∨	~	C)]	&	[(~	D	∨	C)	∨	(~	B	∨	C)]	C
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**
T	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**
F	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
F	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**

h. Truth-functionally valid

								↓							↓			↓			↓	
J	T	Y	[(J	&	T)	&	Y]	∨	(~	J	⊃	~	Y)	J	⊃	T	T	⊃	Y	Y	≡	T
T	T	T	T	T	T	T	T	T	F	T	T	F	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F	T	F	T	T	T	F	T	T	T	T	F	F	F	F	T
T	F	T	T	F	F	F	T	T	F	T	T	F	T	T	F	F	F	T	T	T	F	F
T	F	F	T	F	F	F	F	T	F	T	T	T	F	T	F	F	F	T	F	F	T	F
F	T	T	F	F	T	F	T	F	T	F	F	F	T	F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T	T	F	T	T	F	F	T	T	T	F	F	F	F	T
F	F	T	F	F	F	F	T	F	T	F	F	F	T	F	T	F	F	T	T	T	F	F
F	F	F	F	F	F	F	F	T	T	F	T	T	F	F	T	F	F	T	F	F	T	F

j. Truth-functionally invalid

								↓					↓				↓	
A	B	C	[A	&	(B	∨	C)]	≡	(A	∨	B)	B	⊃	~	B	C	∨	A
T	T	T	T	T	T	T	T	T	T	T	T	T	F	F	T	T	T	T
T	T	F	T	T	T	T	F	T	T	T	T	T	F	F	T	F	T	T
T	F	T	T	T	F	T	T	T	T	T	F	F	T	T	F	T	T	T
T	F	F	T	F	F	F	F	F	T	T	F	F	T	T	F	F	T	T
F	T	T	F	F	T	T	T	F	F	T	T	T	F	F	T	T	T	F
F	T	F	F	F	T	T	F	F	F	T	T	T	F	F	T	F	F	F
F	F	T	F	F	F	T	T	T	F	F	F	F	T	T	F	T	T	F
F	F	F	F	F	F	F	F	T	F	F	F	F	T	T	F	F	F	F

2. b. Truth-functionally invalid

				↓		↓				↓
B	F	G	B	&	F	~	(B	&	G)	G
T	T	F	T	T	T	T	T	F	F	F

d. Truth-functionally invalid

				↓									↓					↓	
J	M	T	J	∨	[M	⊃	(T	≡	J)]	(M	⊃	J)	&	(T	⊃	M)	T	&	~	M
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	F	F	T

3. b. Truth-functionally valid

J	K	L	([(K	≡	L)	&	(L	⊃	J)]	&	~	J)	↓ ⊃	(~	K	∨	L)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**
F	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**
F	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**
F	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**

d. Truth-functionally invalid

A	C	H	[(A	∨	C)	&	~	H]	↓ ⊃	~	C
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**T**

f. Truth-functionally invalid

A	B	C	[~	[A	∨	~	(B	∨	~	C)]	&	(B	⊃	(A	⊃	C))]	↓ ⊃	(~	A	≡	~	B)
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**

4. b. Truth-functionally invalid

M: Many people believe that war is inevitable.
W: War is inevitable.
N: Our planet's natural resources are nonrenewable.
P: Many people believe that our planet's natural resources are nonrenewable.

M	N	P	W	↓ M	W	↓ ≡	N	↓ P
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**
T	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**F**
T	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**
T	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**F**
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**T**
F	**F**	**F**	**T**	**F**	**T**	**F**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**

d. Truth-functionally valid

T: The town hall is now a grocery store.
I: I'm mistaken.
L: The little red schoolhouse is a movie theater.
O: The old school bus is a boutique.
E: The old theater is an elementary school.

						↓				↓			↓				↓
E	I	L	O	T	T	&	(I	∨	L)	~	I	O	&	(L	⊃	E)	L
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**
T	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
T	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
T	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**
T	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**
T	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**
F	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
F	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**
F	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**
F	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**

f. Truth-functionally invalid

B: The butler murdered Devon.
M: The maid is lying.
G: The gardener murdered Devon.
W: The weapon was a slingshot.

							↓								↓					↓
B	G	M	W	(B	⊃	M)	&	(G	⊃	W)	(M	≡	~	W)	&	(~	W	⊃	B)	B
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**T**
F	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**F**	**F**

5. b. If {**P**} ⊨ **Q**, then there is no truth-value assignment on which **P** is true and **Q** is false. If {**Q**} ⊨ **P**, then there is no truth-value assignment on which **Q** is true and **P** is false. So there is no truth-value assignment on which **P** and **Q** have different truth-values; they are truth-functionally equivalent. Conversely, assume that **P** and **Q** are truth-functionally equivalent. Then **Q** is true on every truth-value assignment on which **P** is true, so {**P**} ⊨ **Q**; and **P** is true on every truth-value assignment on which **Q** is true, so {**Q**} ⊨ **P**.

d. Assume that {**P**} ⊨ **Q** and {**Q**} ⊨ **R**. Then, by the first entailment, if **P** is true on a truth-value assignment, **Q** is also true. By the second entailment, **R** must be true as well. Therefore {**P**} ⊨ **R**.

Section 3.6E

1. b. If **P** ≡ **Q** is true on every truth-value assignment, then **P** and **Q** have the same truth-value on every truth-value assignment. Hence **P** and **Q** are truth-functionally equivalent.

2. b. Assume Γ ⊨ **P** ⊃ **Q**. Then there is no truth-value assignment on which every member of Γ is true and **P** ⊃ **Q** is false. But **P** ⊃ **Q** is false on a truth-value assignment if and only if **P** is true and **Q** false on that assignment. Hence it follows from our assumption that there is no truth-value assignment on which every member of Γ is true, **P** is true, and **Q** is false; that is, Γ ∪ {**P**} ⊨ **Q**.

Assume Γ ∪ {**P**} ⊨ **Q**. Then there is no truth-value assignment on which every member of Γ ∪ {**P**} is true and **Q** is false. This means that there is no truth-value assignment on which every member of Γ is true and **P** ⊃ **Q** is false; that is, Γ ⊨ **P** ⊃ **Q**.

d. Let Γ be an arbitrary set of sentences of *SL*, and let **P** be a truth-functionally false sentence. There is no truth-value assignment on which **P** is true. Hence there is no truth-value assignment on which every member of $\Gamma \cup \{\mathbf{P}\}$ is true (since **P** is false on every assignment). So $\Gamma \cup \{\mathbf{P}\}$ is truth-functionally inconsistent.

3. b. Assume that $\Gamma \vDash \mathbf{P}$ and that $\Gamma \vDash \sim \mathbf{P}$. Also assume, contrary to what we wish to show, that Γ is truth-functionally consistent. Then there is at least one truth-value assignment on which every member of Γ is true. Since $\Gamma \vDash \mathbf{P}$, **P** is true on this truth-value assignment; and since $\Gamma \vDash \sim \mathbf{P}$, $\sim \mathbf{P}$ is also true on this truth-value assignment. But this is impossible; a sentence and its negation cannot both be true on a single truth-value assignment. So our supposition that Γ is truth-functionally consistent is false—Γ is truth-functionally inconsistent.

4. b. Assume that **P** and **Q** are truth-functionally equivalent. If $\{\mathbf{P}\} \vDash \mathbf{R}$ then, for every assignment on which **P** is true, so is **R**. Because **Q** is truth-functional equivalent to **P**, **P** is true on every truth-value assignment on which **Q** is true. So whenever **Q** is true, so is **P** and hence so is **R**. Therefore $\{\mathbf{Q}\} \vDash \mathbf{R}$ as well. Similar reasoning shows that if $\{\mathbf{Q}\} \vDash \mathbf{R}$, then $\{\mathbf{P}\} \vDash \mathbf{R}$.

Section 4.2.E

1. b.

1.	A & ~ (B & A) ✓		SM
2.	A		1 &D
3.	~ (B & A) ✓		1 &D
4.	~ B	~ A	3 ~ &D
		×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
T	**F**

d.

1.	~ A & ~ (A & B) ✓		SM
2.	B		SM
3.	~ A		1 &D
4.	~ (A & B) ✓		1 &D
5.	~ A	~ B	4 ~ &D
		×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
F	**T**

f.

1.	(J & ~ K) & I ✓		SM
2.	~ I ∨ K ✓		SM
3.	J & ~ K ✓		1 &D
4.	I		1 &D
5.	J		3 &D
6.	~ K		3 &D
7.	~ I	K	2 ∨D
	×	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

h. 1.	(H & ~ I) & (I ∨ H) ✓		SM
2.	H & ~ I ✓		1 &D
3.	I ∨ H ✓		1 &D
4.	H		2 &D
5.	~ I		2 &D
6.	I	H	3 ∨D
	×	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

j. 1.	(A & B) ∨ (A & C) ✓				SM
2.	~ (A & B) ✓				SM
3.	A & B ✓		A & C ✓		1 ∨D
4.	A		A		3 &D
5.	B		C		3 &D
6.	~ A	~ B	~ A	~ B	2 ~ &D
	×	×	×		

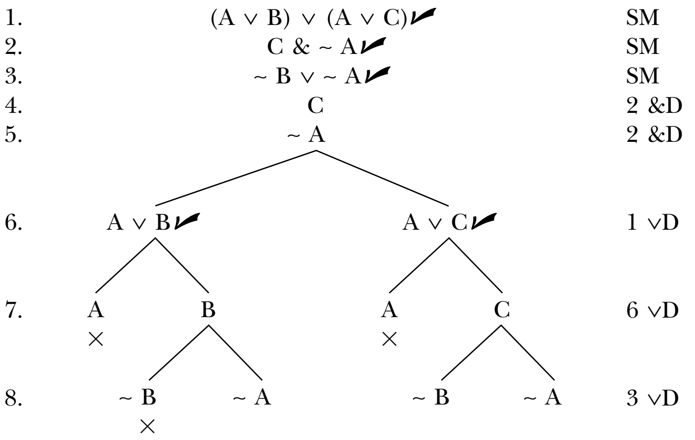
Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B	C
T	**F**	**T**

l. 1.	(A & B) ∨ (A & C) ✓		SM
2.	~ (A ∨ B) ✓		SM
3.	~ A		2 ~ ∨D
4.	~ B		2 ~ ∨D
5.	A & B ✓	A & C ✓	1 ∨D
6.	A	A	5 &D
7.	B	C	5 &D
	×	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

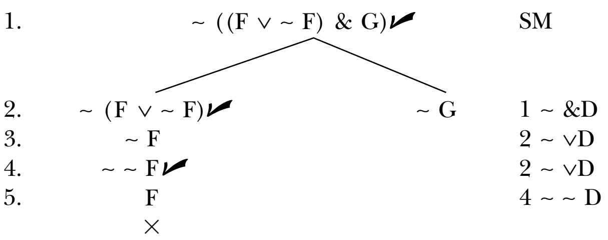
n.



Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

A	B	C
F	**T**	**T**
F	**F**	**T**

2. b.



Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

F	G
T	**F**
F	**F**

d. 1.	$\sim [(A \vee B) \vee C]$ ✓		SM
2.	$\sim D \vee C$ ✓		SM
3.	D		SM
4.	$\sim (A \vee B)$ ✓		$1 \sim \vee D$
5.	$\sim C$		$1 \sim \vee D$
6.	$\sim A$		$4 \sim \vee D$
7.	$\sim B$		$4 \sim \vee D$
8.	$\sim D$	C	$2 \vee D$
	×	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

f. 1.	$A \& (B \& \sim C)$ ✓			SM
2.	$\sim (A \& (B \& C))$ ✓			SM
3.	A			$1 \& D$
4.	$B \& \sim C$ ✓			$1 \& D$
5.	B			$4 \& D$
6.	$\sim C$			$4 \& D$
7.	$\sim A$	$\sim (B \& C)$ ✓		$2 \sim \& D$
	×			
8.		$\sim B$	$\sim C$	$7 \sim \& D$
		×		

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B	C
T	**T**	**F**

h. 1.		$\sim(\sim A \vee B)$ ✓			SM
2.		$A \vee \sim B$ ✓			SM
3.		$\sim(\sim B \ \& \sim A)$ ✓			SM
4.		$\sim\sim A$ ✓			$1 \sim\vee D$
5.		$\sim B$			$1 \sim\vee D$
6.		A			$4 \sim\sim D$
7.	A		$\sim B$		$2 \vee D$
8.	$\sim\sim B$ ✓	$\sim\sim A$ ✓	$\sim\sim B$ ✓	$\sim\sim A$ ✓	$3 \sim\& D$
9.	B	A	B	A	$8 \sim\sim D$
	×		×		

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
T	**F**

j. 1.		$\sim(\sim A \vee \sim B)$ ✓		SM
2.		$\sim(A \ \& \sim(B \ \& \ C))$ ✓		SM
3.		$A \vee (B \vee C)$ ✓		SM
4.		$\sim\sim A$ ✓		$1 \sim\vee D$
5.		$\sim\sim B$ ✓		$1 \sim\vee D$
6.		A		$4 \sim\sim D$
7.		B		$5 \sim\sim D$
8.	$\sim A$	$\sim\sim(B \ \& \ C)$ ✓		$2 \sim\& D$
9.	×	$B \ \& \ C$ ✓		$8 \sim\sim D$
10.		B		$9 \ \& D$
11.		C		$9 \ \& D$
12.	A	$B \vee C$ ✓		$3 \vee D$
13.		B	C	$12 \vee D$

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B	C
T	**T**	**T**

1.

1.			~ (A & (~ B & ~ C)) ✓				SM
2.			~ A ∨ ~ C ✓				SM
3.			~ (~ B ∨ ~ ~ C) ✓				SM
4.			~ ~ B ✓				3 ~ ∨D
5.			~ ~ ~ C ✓				3 ~ ∨D
6.			~ C				5 ~ ~ D
7.			B				4 ~ ~ D
8.	~ A			~ (~ B & ~ C) ✓			1 ~ &D
9.	~ A	~ C	~ A		~ C		2 ∨D
10.			~ ~ B ✓	~ ~ C ✓	~ ~ B ✓	~ ~ C ✓	8 ~ &D
11.			B	C	B	C	10 ~ ~ D
				×		×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

A	B	C
F	**T**	**F**
T	**T**	**F**

n.	1.	(H ∨ ~ I) & (I ∨ ~ G) ✓	SM
	2.	~ (H & G) ✓	SM
	3.	H ∨ (~ I & ~ G) ✓	SM
	4.	H ∨ ~ I ✓	1 &D
	5.	I ∨ ~ G ✓	1 &D
	6.	~ H ~ G	2 ~ &D
	7.	H ~ I & ~ G ✓ H ~ I & ~ G ✓	3 ∨D
	8.	× ~ I ~ I	7 &D
	9.	~ G ~ G	7 &D
	10.	H × ~ I H ~ I H ~ I	4 ∨D
	11.	I × ~ G I ~ G I × ~ G I × ~ G I × ~ G	5 ∨D

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

G	H	I
F	**F**	**F**
F	**T**	**T**
F	**T**	**F**

Section 4.3.E

1. b.	1.	~ [~ A ⊃ (B ⊃ C)] ✓	SM
	2.	A ⊃ C	SM
	3.	~ A ✓	1 ~⊃D
	4.	~ (B ⊃ C) ✓	1 ~⊃D
	5.	B	4 ~⊃D
	6.	~ C	4 ~⊃D
	7.	~ A C ×	2 ⊃D

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B	C
F	**T**	**F**

d. 1.	$H \equiv G$ ✓		SM
2.	$\sim H$		SM
3.	G		SM
4.	H	$\sim H$	$1 \equiv D$
5.	G	$\sim G$	$1 \equiv D$
	×	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

f. 1.	$(H \equiv G) \equiv \sim H$ ✓				SM
2.	$H \equiv G$ ✓		$\sim (H \equiv G)$ ✓		$1 \equiv D$
3.	$\sim H$		$\sim \sim H$ ✓		$1 \equiv D$
4.	H	$\sim H$			$2 \equiv D$
5.	G	$\sim G$			$2 \equiv D$
6.	×				
			H		$3 \sim \sim D$
7.			H	$\sim H$	$2 \sim \equiv D$
8.			$\sim G$	G	$2 \sim \equiv D$
				×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

G	H
F	**F**
F	**T**

h. 1.		$H \equiv \sim G$ ✓			SM
2.		$H \supset G$ ✓			SM
3.	H		$\sim H$		$1 \equiv D$
4.	$\sim G$		$\sim \sim G$ ✓		$1 \equiv D$
5.			G		$4 \sim \sim D$
6.	$\sim H$	G	$\sim H$	G	$2 \supset D$
	×	×			

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

G	H
T	**F**

j. 1.		$A \supset \sim (A \equiv B)$ ✓		SM
2.		$\sim (A \supset B)$ ✓		SM
3.		A		$2 \sim \supset D$
4.		$\sim B$		$2 \sim \supset D$
5.	$\sim A$	$\sim (A \equiv B)$ ✓		$1 \supset D$
	×			
6.		A	$\sim A$	$5 \sim \equiv D$
7.		$\sim B$	B	$5 \sim \equiv D$
			×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
T	**F**

l. 1.	B ≡ D ✓				SM
2.	B ≡ ~ D ✓				SM
3.	B		~ B		1 ≡D
4.	D		~ D		1 ≡D
5.	B	~ B	B	~ B	2 ≡D
6.	~ D	~ ~ D	~ D	~ ~ D ✓	2 ≡D
7.	×	×	×	D	6 ~ ~ D
				×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

n. 1.	[B ⊃ (A ∨ C)] & ~ ~ B ✓				SM
2.	A ≡ ~ B ✓				SM
3.	B ⊃ (A ∨ C) ✓				1 &D
4.	~ ~ B ✓				1 &D
5.	B				4 ~ ~ D
6.	~ B	A ∨ C ✓			3 ⊃D
	×				
7.		A	~ A		2 ≡D
8.		~ B	~ ~ B ✓		2 ≡D
9.		×	B		8 ~ ~ D
10.			A	C	6 ∨D
			×		

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B	C
F	**T**	**T**

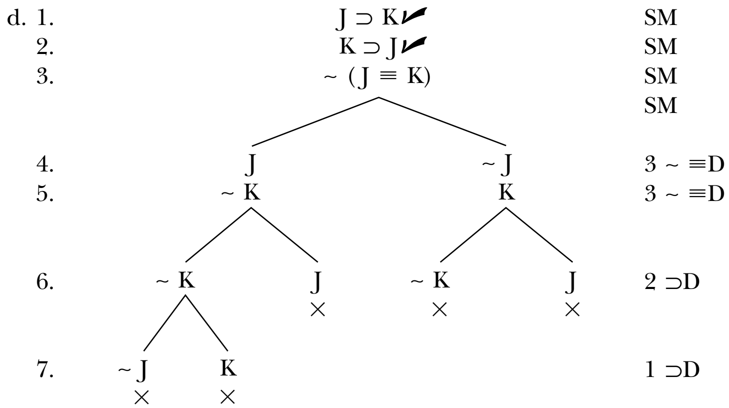
p. 1. $A \supset [B \supset (C \supset A)]$ ✓ SM
2. $\sim (B \supset \sim A)$ ✓ SM
3. B $2 \sim \supset D$
4. $\sim \sim A$ ✓ $2 \sim \supset D$
5. A $4 \sim \sim D$
6. $\sim A$ $B \supset (C \supset A)$ ✓ $1 \supset D$
$\times$
7. $\sim B$ $C \supset A$ ✓ $6 \supset D$
$\times$
8. $\sim C$ A $7 \supset D$

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

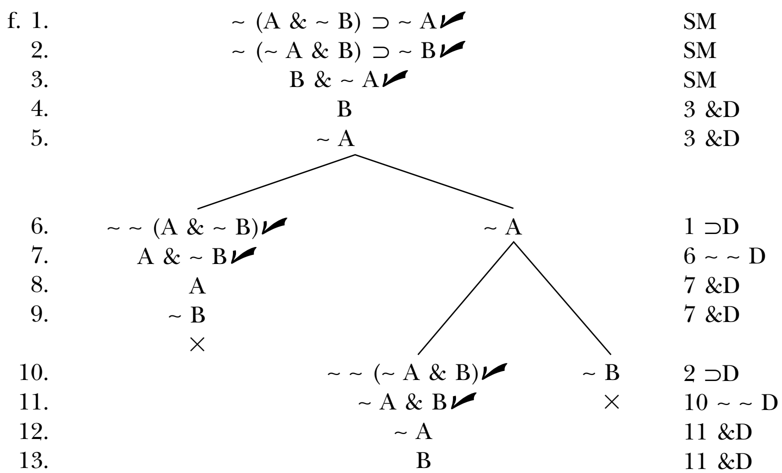
A	B	C
T	**T**	**F**
T	**T**	**T**

2. b. 1. $\sim [(A \supset \sim B) \supset (B \supset A)]$ ✓ SM
2. $\sim (\sim A \supset B)$ ✓ SM
3. $\sim A$ $2 \sim \supset D$
4. $\sim B$ $2 \sim \supset D$
5. $A \supset \sim B$ $1 \sim \supset D$
6. $\sim (B \supset A)$ ✓ $1 \sim \supset D$
7. B $6 \sim \supset D$
8. $\sim A$ $6 \sim \supset D$
$\times$

Since the truth-tree is closed, the given set is truth-functionally inconsistent.



Since the truth-tree is closed, the given set is truth-functionally inconsistent.



Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
F	**T**

h. 1.	~ (A ∨ B) ⊃ A✓				SM
2.	~ (A ∨ B) ⊃ B✓				SM
3.	~ B				SM
4.	~ ~ (A ∨ B)✓		A		1 ⊃D
5.	A ∨ B✓				4 ~ ~ D
6.	~ ~ (A ∨ B)✓	B ×	~ ~ (A ∨ B)✓	B ×	2 ⊃D
7.	A ∨ B✓		A ∨ B✓		6 ~ ~ D
8.	A B ×		A B ×		7 ∨D

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragment is

A	B
T	**F**

j. 1.	A ≡ (B & ~ C)✓					SM
2.	~ A ∨ ~ B✓					SM
3.	B ⊃ ~ C✓					SM
4.	A		~ A			1 ≡D
5.	B & ~ C✓		~ (B & ~ C)✓			1 ≡D
6.	B					5 &D
7.	~ C					5 &D
8.	~ A ×	~ B ×	~ A		~ B	2 ∨ D
9.			~ B ~ C		~ B ~ C	5 ~ &D

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

A	B	C
F	**F**	**T**
F	**F**	**F**
F	**T**	**F**

1.

1.	$\sim [A \supset (\sim B \equiv C)]$ ✓				SM
2.	$A \equiv \sim C$ ✓				SM
3.	$A \equiv B$ ✓				SM
4.	A				$1 \supset D$
5.	$\sim (\sim B \equiv C)$ ✓				$1 \supset D$
6.	A			$\sim A$	$2 \equiv D$
7.	$\sim C$			$\sim \sim C$	$2 \equiv D$
				×	
8.	A		$\sim A$		$3 \equiv D$
9.	B		$\sim B$		$3 \equiv D$
			×		
10.	$\sim B$	$\sim \sim B$			$5 \sim \equiv D$
11.	$\sim C$	C			$5 \sim \equiv D$
	×	×			

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

n.

1.					(A & ~ B) ≡ (C & ~ E)✓					SM
2.					~ A ≡ E✓					SM
3.					B ≡ C✓					SM
4.	A & ~ B✓						~ (A & ~ B)✓			1 ≡D
5.	C & ~ E✓						~ (C & ~ E)✓			1 ≡D
6.	A									4 &D
7.	~ B									4 &D
8.	C									5 &D
9.	~ E									5 &D
10.	~ A	~ ~ A		~ A				~ ~ A✓		2 ≡D
11.	E	~ E		E				~ E		2 ≡D
12.	×	A						A		10 ~ ~ D
		×								
13.		~ C			~ ~ E		~ C		~ ~ E	5 ~ &D
14.					E				E	13 ~ ~ D
									×	
15.	B	~ B			B	~ B	B		~ B	3 ≡D
16.	C	~ C			C	~ C	C		~ C	3 ≡D
	×						×			
17.	~ A	~ ~ B	~ A	~ ~ B		~ A	~ ~ B	~ A	~ ~ B	4 ~ &D
18.		B		B			B	×	B	17 ~ ~ D
		×					×		×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

A	B	C	E
F	**F**	**F**	**T**
F	**T**	**T**	**T**

Section 4.4E

1. b.

1.	K ∨ (M & ~ M) ✓		SM
2.	J & ~ C ✓		SM
3.	J		2 &D
4.	~ C		2 &D
5.	K	M & ~ M ✓	1 ∨D
6.		M	5 &D
7.		~ M	5 &D
		×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

C	J	K	M
F	**T**	**T**	**T**
F	**T**	**T**	**F**

d.

1.	~ (M & ~ N)	SM
2.	~ (K ∨ M) & ~ ~ M ✓	SM
3.	~ ~ ~ K ✓	SM
4.	~ K	3 ~ ~ D
5.	~ (K ∨ M) ✓	2 &D
6.	~ ~ M ✓	2 &D
7.	M	6 ~ ~ D
8.	~ K	5 ~ ∨D
9.	~ M	5 ~ ∨D
	×	

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

f.

1.	~ [~ (L ∨ ~ L) & (N ∨ ~ N)]			SM
2.	~ ~ (L ∨ ~ L) ✓		~ (N ∨ ~ N) ✓	1 ~ &D
3.	L ∨ ~ L ✓			2 ~ ~ D
4.	L	~ L		3 ∨D
5.			~ N	2 ~ ∨D
6.			~ ~ N ✓	2 ~ ∨D
7.			N	6 ~ ~ D
			×	

Since the truth-tree has at least one completed open branch, the set is truth-functionally consistent. The recoverable fragments are

L	N
T	**T**
T	**F**
F	**T**
F	**F**

h.

1.	$\sim [\sim (J \supset M) \equiv (J \,\&\, \sim M)]$ ✓				SM
2.	$\sim (J \supset M)$ ✓		$\sim \sim (J \supset M)$ ✓		$1 \sim \equiv D$
3.	$\sim (J \,\&\, \sim M)$ ✓		$J \,\&\, \sim M$ ✓		$1 \sim \equiv D$
4.	J				$2 \sim \supset D$
5.	$\sim M$				$2 \sim \supset D$
6.	$\sim J$ ×	$\sim \sim M$ ✓			$3 \sim \& D$
7.		M ×			$6 \sim \sim D$
8.			J		$3 \; \& D$
9.			$\sim M$		$3 \; \& D$
10.			$J \supset M$ ✓		$2 \sim \sim D$
11.			$\sim J$ ×	M ×	$10 \supset D$

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

j. 1.	$H \,\&\, (\sim K \supset M)$ ✓		SM
2.	$\sim K$		SM
3.	$\sim (H \supset M)$ ✓		SM
4.	H		1 &D
5.	$\sim K \supset M$ ✓		1 &D
6.	H		3 $\sim \supset$D
7.	$\sim M$		3 $\sim \supset$D
8.	$\sim \sim K$ ✓	M	5 $\supset$D
		×	
9.	K		8 $\sim \sim$ D
	×		

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

l. 1.	$\sim \sim \sim[(K \vee M) \supset \sim G]$ ✓		SM
2.	$G \equiv (J \,\&\, U)$ ✓		SM
3.	$U \supset (\sim G \,\&\, K)$		SM
4.	$K \,\&\, \sim U$ ✓		SM
5.	K		4 &D
6.	$\sim U$		4 &D
7.	$\sim[(K \vee M) \supset \sim G]$ ✓		1 $\sim \sim$D
8.	$K \vee M$		7 $\sim \supset$D
9.	$\sim \sim G$ ✓		7 $\sim \supset$D
10.	G		9 $\sim \sim$D
11.	G	$\sim G$	2 $\equiv$D
12.	$J \,\&\, U$ ✓	$\sim (J \,\&\, U)$	2 $\equiv$D
		×	
13.	J		12 &D
14.	U		12 &D
	×		

Since the truth-tree is closed, the given set is truth-functionally inconsistent.

2. b. False. Sometimes fragments of more than one truth-value assignment can be recovered from a single open branch. This occurs when the open branch contains neither **P** nor ~ **P** for some atomic component, **P**, of one of the members of the set being tested. Here is an example:

1.	B		SM
2.	$K \vee J$ ✓		SM
3.	K	J	2 $\vee$D

In this example neither 'J' nor '~ J' occurs on the left-hand open branch. That branch tells us that 'K' and 'B' must both be assigned the truth-value **T** and allows us to assign either the truth-value **T** or the truth-value **F** to 'J'. Thus from this one branch we can recover the following two fragments:

B	J	K
T	**T**	**T**
T	**F**	**T**

d. True. A closed truth-tree has, by definition, no open branches.

f. False. It can happen that all the branches of a truth-tree close before all the nonliteral sentences on the tree have been decomposed. Here is an example:

1.	$H \supset M$	SM
2.	$(B \equiv Z) \vee \sim U$	SM
3.	$A \ \& \sim A$ ✓	SM
4.	A	3 &D
5.	$\sim A$	3 ~ D
	×	

h. True. Every tree that is not closed is an open tree, and a tree with a completed open branch is not a closed tree.

j. True. If a set is consistent, so is every subset of that set, for we cannot generate an inconsistent set by removing members of a consistent set. The tree for a subset of a consistent set will be a part of the tree for that original set; hence it will not close.

3. b. 1.	$I \vee \sim I$ ✓	SM
2.	$\sim I \supset [(C \supset B) \vee (G \supset R)]$ ✓	SM
3.	$\sim B \;\&\; R$ ✓	SM
4.	$(E \vee \sim I) \;\&\; \sim E$ ✓	SM
5.	$\sim B$	3 &D
6.	R	3 &D
7.	$E \vee \sim I$ ✓	4 &D
8.	$\sim E$	4 &D
9.	E × $\sim I$	7 ∨D
10.	I × $\sim I$	1 ∨D
11.	$\sim \sim I$ ✓ $(C \supset B) \vee (G \supset R)$ ✓	2 ⊃D
12.	I ×	11 ~ ~ D
13.	$C \supset B$ ✓ $G \supset R$ ✓	11 ∨D
14.	$\sim C$ B × $\sim G$ R	13 ⊃D

Since the truth-tree is open, the set is truth-functionally consistent. The recoverable fragments are

B	C	E	G	I	R
F	**F**	**F**	**T**	**F**	**T**
F	**F**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**T**	**F**	**T**

d. Some care is necessary in symbolizing this passage. Acquiring work is described in various ways: finding employment, getting a job, being hired, and finding work. And we have to extract from the strong claim that Christine will certainly be hired by a good law firm the weaker claim that she will find work. We use the following abbreviations in symbolizing this passage:

A: Albert will find work when he graduates from law school.
B: Betty will find work when she graduates from law school.
C: Christine will find work when she graduates from law school.
L: Christine is a first-rate lawyer.

Here is the truth-tree for the symbolized version of this passage:

```
1.   ([(A & B) & ~ C] ∨ [(A & C) & ~ B]) ∨ [(B & C) & ~ A]✓   SM
2.   A ⊃ (B & C)✓                                              SM
3.   ~ B ∨ A✓                                                  SM
4.   L & C✓                                                    SM
5.   L                                                         4 &D
6.   C                                                         4 &D

7.   [(A & B) & ~ C] ∨ [(A & C) & ~ B]✓    (B & C) & ~ A✓      1 ∨D
8.                                          B & C✓              7 &D
9.                                          ~ A                 7 &D
10.                                         B                   8 &D
11.                                         C                   8 &D
12.  (A & B) & ~ C✓   (A & C) & ~ B✓                            7 ∨D
13.  A & B✓           A & C✓                                    12 &D
14.  ~ C              ~ B                                       12 &D
     ×

15.                   A                                         13 &D
16.                   C                                         13 &D

17.       ~ B                 A              ~ B       A        3 ∨D
                                             ×         ×

18.  ~ A   B & C✓       ~ A   B & C✓                            2 ⊃D
     ×                  ×

19.        B                  B                                 18 &D
20.        C                  C                                 18 &D
           ×                  ×
```

The truth-tree closes so the given set is truth-functionally inconsistent.

Section 4.5E

```
1. b. 1.   ~ (M ∨ ~ M)✓    SM
      2.   ~ M             1 ~ ∨D
      3.   ~ ~ M✓          1 ~ ∨D
      4.   M               3 ~ ~ D
           ×
```

Since the truth-tree for '~ (M ∨ ~ M)' is closed, 'M ∨ ~ M' is truth-functionally true.

d. 1.	~ [(C ⊃ R) ⊃ [~ R ⊃ ~ (C & J)]] ✔		SM
2.	C ⊃ R ✔		1 ~ ⊃D
3.	~ [~ R ⊃ ~ (C & J)] ✔		1 ~ ⊃D
4.	~ R		3 ~ ⊃D
5.	~ ~ (C & J) ✔		3 ~ ⊃D
6.	C & J ✔		5 ~ ~ D
7.	C		6 &D
8.	J		6 &D
9.	~ C	R	2 ⊃D
	×	×	

Since the truth-tree for '~ [(C ⊃ R) ⊃ [~ R ⊃ ~ (C & J)]]' is closed, '(C ⊃ R) ⊃ [~ R ⊃ ~ (C & J)]' is truth-functionally true.

f. 1.	(K ≡ W) ∨ (A & W) ✔			SM
2.	K ≡ W ✔		A & W ✔	1 ∨D
3.	K	~ K		2 ≡D
4.	W	~ W		2 ≡D
5.			A	2 &D
6.			W	2 &D

1.	~ [(K ≡ W) ∨ (A & W)] ✔				SM
2.	~ (K ≡ W) ✔				1 ~ ∨D
3.	~ (A & W) ✔				1 ~ ∨D
4.	K		~ K		2 ~ ≡D
5.	~ W		W		2 ~ ≡D
6.	~ A	~ W	~ A	~ W	3 ~ &D
				×	

Since neither of these two trees is closed, the sentence we are testing is truth-functionally indeterminate.

h. 1.	[L ∨ (J ∨ ~ K)] & (K & [(J ∨ L) ⊃ ~ K])✓			SM
2.	L ∨ (J ∨ ~ K)✓			1 &D
3.	K & [(J ∨ L) ⊃ ~ K]✓			1 &D
4.	K			3 &D
5.	(J ∨ L) ⊃ ~ K✓			3 &D
6.	~ (J ∨ L)✓		~ K	5 ⊃D
			×	
7.	~ J			6 ~ ∨D
8.	~ L			6 ~ ∨D
9.	L	J ∨ ~ K✓		2 ∨D
	×			
10.	J	~ K		9 ∨D
	×	×		

Since the truth-tree is closed, the sentence we are testing is truth-functionally false.

j. 1.	(A ∨ B) ⊃ ~ (A ∨ B)✓		SM
2.	~ (A ∨ B)✓	~ (A ∨ B)✓	1 ⊃D
3.	~ A	~ A	2 ~ ∨D
4.	~ B	~ B	2 ~ ∨D

1.	~ [(A ∨ B) ⊃ ~ (A ∨ B)]✓				SM
2.	A ∨ B✓				1 ~ ⊃D
3.	~ ~ (A ∨ B)✓				1 ~ ⊃D
4.	A ∨ B✓				3 ~ ~ D
5.	A		B		2 ∨D
6.	A	B	A	B	4 ∨D

Neither the tree for the sentence nor the tree for its negation is closed. Therefore the sentence is truth-functionally indeterminate.

1. 1.	~ (D ∨ F) ≡ ~ (D & F) ✓				SM
2.	~ (D ∨ F) ✓		~ ~ (D ∨ F) ✓		1 ≡D
3.	~ (D & F) ✓		~ ~ (D & F) ✓		1 ≡D
4.			D ∨ F ✓		2 ~ ~ D
5.			D & F ✓		3 ~ ~ D
6.			D		5 &D
7.			F		5 &D
8.			D	F	4 ∨D
9.	~ D				2 ~ ∨D
10.	~ F				2 ~ ∨D
11.	~ D	~ F			3 ~ &D

1.	~ [~ (D ∨ F) ≡ ~ (D & F)] ✓					SM
2.	~ (D ∨ F) ✓	~ ~ (D ∨ F) ✓				1 ~ ≡D
3.	~ ~ (D & F) ✓	~ (D & F) ✓				1 ~ ≡D
4.	~ D					2 ~ ∨D
5.	~ F					2 ~ ∨D
6.	D & F ✓					3 ~ ~ D
7.	D					6 &D
8.	F					6 &D
	×					
9.		D ∨ F ✓				2 ~ ~ D
10.		~ D		~ F		3 ~ &D
11.		D	F	D	F	9 ∨D
		×			×	

Neither the tree for the sentence nor the tree for its negation is closed. Therefore the sentence is truth-functionally indeterminate.

n. 1.	~ [~ (D ∨ F) ≡ (~ D & ~ F)] ✓				SM
2.	~ (D ∨ F) ✓		~ ~ (D ∨ F) ✓		1 ~ ≡D
3.	~ (~ D & ~ F) ✓		~ D & ~ F ✓		1 ~ ≡D
4.			~ D		3 &D
5.			~ F		3 &D
6.			D ∨ F ✓		2 ~ ~ D
7.			D	F	6 ∨D
			×	×	
8.	~ D				2 ~ ∨D
9.	~ F				2 ~ ∨D
10.	~ ~ D ✓	~ ~ F ✓			3 ~ &D
11.	D	F			10 ~ ~ D
	×	×			

The tree for the negation of the sentence is closed. Therefore the sentence is truth-functionally true.

2. b. 1.	~ [(B ⊃ L) & (L ⊃ B)] ✓		SM
2.	~ (B ⊃ L) ✓	~ (L ⊃ B) ✓	1 ~ &D
3.	B	L	2 ~ ⊃D
4.	~ L	~ B	2 ~ ⊃D

Since the truth-tree for the negation of the given sentence is not closed, the given sentence is not truth-functionally true. The recoverable fragments are

B	L
T	**F**
F	**T**

d. 1.	$\sim [(A \equiv K) \supset (\sim A \vee K)]$ ✓		SM
2.	$A \equiv K$ ✓		$1 \sim \supset D$
3.	$\sim (\sim A \vee K)$ ✓		$1 \sim \supset D$
4.	$\sim \sim A$ ✓		$3 \sim \vee D$
5.	$\sim K$		$3 \sim \vee D$
6.	A		$4 \sim \sim D$
7.	A	$\sim A$	$2 \equiv D$
8.	K	$\sim K$	$2 \equiv D$
	×	×	

Since the truth-tree for the negation of the given sentence is closed, the given sentence is truth-functionally true.

f. 1.	$\sim [[(J \supset Z)\ \&\ \sim J] \supset \sim Z]$ ✓		SM
2.	$(J \supset Z)\ \&\ \sim J$ ✓		$1 \supset D$
3.	$\sim \sim Z$ ✓		$1 \sim \supset D$
4.	Z		$3 \sim \sim D$
5.	$J \supset Z$ ✓		2 &D
6.	$\sim J$		2 &D
7.	$\sim J$	Z	$5 \supset D$

Since the truth-tree for the negation of the given sentence is not closed, the given sentence is not truth-functionally true. The recoverable fragment is

J	Z
F	**T**

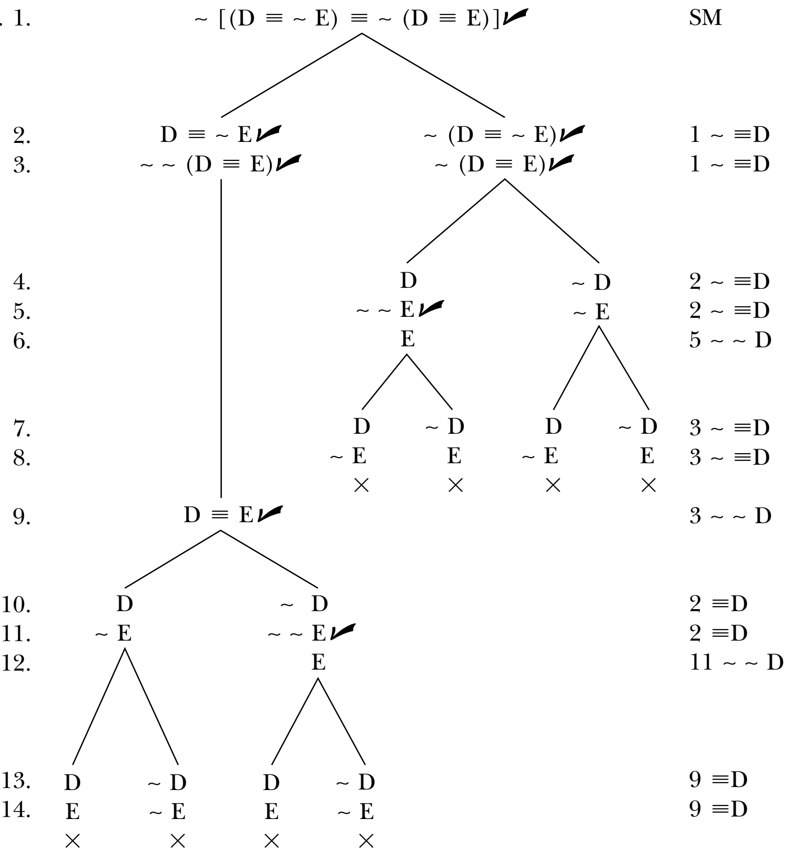
h. 1.	~ [M ⊃ [L ≡ (~ M ≡ ~ L)]] ✓				SM
2.	M				1 ~ ⊃D
3.	~ [L ≡ (~ M ≡ ~ L)] ✓				1 ~ ⊃D
4.	L		~ L		3 ~ ≡D
5.	~ (~ M ≡ ~ L) ✓		~ M ≡ ~ L ✓		3 ~ ≡D
6.	~ M	~ ~ M			5 ~ ≡D
7.	~ ~ L	~ L			5 ~ ≡D
	×	×			
8.			~ M	~ ~ M	5 ≡D
9.			~ L	~ ~ L ✓	5 ≡D
			×		
10.				L	9 ~ ~ D
				×	

Since the truth-tree for the negation of the given sentence is closed, the given sentence is truth-functionally true.

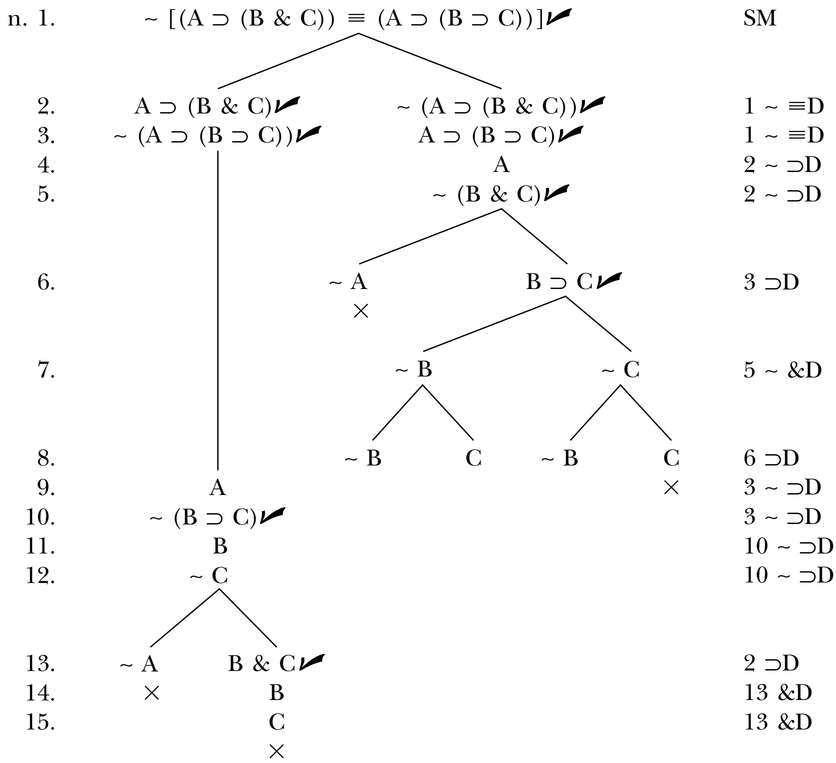
j. 1.	~ [(A & ~ B) ⊃ ~ (A & B)] ✓	SM
2.	A & ~ B ✓	1 ~ ⊃D
3.	~ ~ (A & B) ✓	1 ~ ⊃D
4.	A	2 &D
5.	~ B	2 &D
6.	A & B ✓	3 ~ ~ D
7.	A	6 &D
8.	B	6 &D
	×	

The tree for the negation of the sentence is closed. Therefore the given sentence is truth-functionally true.

1.

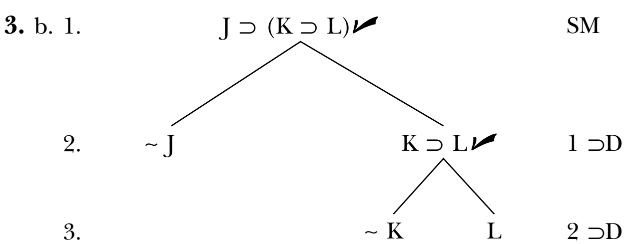


The tree for the negation of the sentence is closed. Therefore the given sentence is truth-functionally true.



The tree for the negation of the sentence is not closed. Therefore the given sentence is not truth-functionally true. The recoverable fragments are

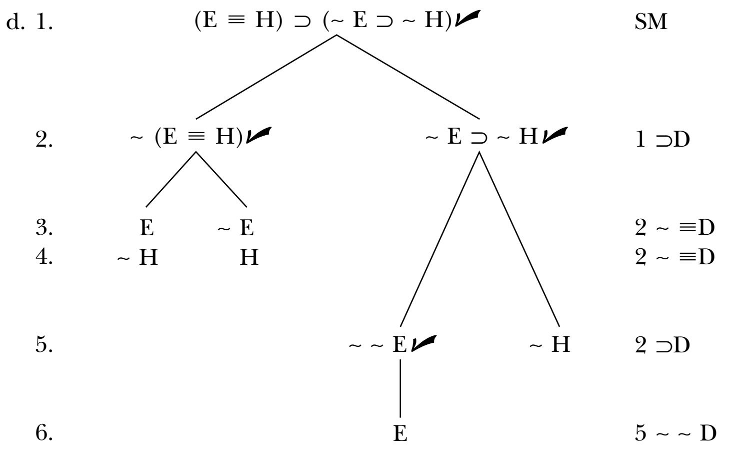
A	B	C
T	**F**	**T**
T	**F**	**F**



The tree for the sentence does not close. Therefore the sentence is not truth-functionally false. The recoverable fragments are

J	K	L
F	**T**	**T**
F	**T**	**F**
F	**F**	**T**
F	**F**	**F**
T	**F**	**T**
T	**F**	**F**
T	**T**	**T**

Since only seven of the eight relevant fragments are recoverable, the sentence is not truth-functionally true. Therefore it is truth-functionally indeterminate.



The tree for the sentence does not close. Therefore the sentence is not truth-functionally false. The recoverable fragments are

E	H
T	**F**
F	**T**
T	**T**
F	**F**

Since all four of the four relevant fragments are recoverable, the sentence is truth-functionally true.

f. 1.	([(C ⊃ D) & (D ⊃ E)] & C) & ~ E ✓				SM
2.	[(C ⊃ D) & (D ⊃ E)] & C ✓				1 &D
3.	~ E				2 &D
4.	(C ⊃ D) & (D ⊃ E) ✓				2 &D
5.	C				2 &D
6.	C ⊃ D ✓				4 &D
7.	D ⊃ E ✓				4 &D
8.	~ C	D			6 ⊃D
	×				
9.		~ D	E		7 ⊃D
		×	×		

The tree for the sentence is closed. Therefore the sentence is truth-functionally false.

h. 1.	~ ([A ∨ B) & (B ∨ C)] & (~ A & ~ B)) ✓				SM
2.	~ [(A ∨ B) & (B ∨ C)] ✓		~ (~ A & ~ B) ✓		1 ~ &D
3.	~ (A ∨ B) ✓	~ (B ∨ C) ✓	~ ~ A ✓	~ ~ B ✓	2 ~ &D
4.	~ A	~ B			3 ~ ∨D
5.	~ B	~ C			3 ~ ∨D
6.			A	B	3 ~ ~ D

The tree for the sentence does not close. Therefore the sentence is not truth-functionally false. The recoverable fragments are

A	B	C
F	**F**	**T**
F	**F**	**F**
T	**F**	**F**
T	**T**	**T**
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**T**	**F**

Since all eight of the eight relevant fragments are recoverable, the sentence is truth-functionally true.

j. 1.		~ B ⊃ ((B ∨ D) ⊃ D)✓		SM
2.	~ ~ B✓	(B ∨ D) ⊃ D✓		1 ⊃D
3.	B			2 ~ ~ D
4.		~ (B ∨ D)✓	D	2 ⊃D
5.		~ B		4 ~ ∨D
6.		~ D		4 ~ ∨D

The tree for the sentence does not close. Therefore the sentence is not truth-functionally false. The recoverable fragments are

B	D
T	**T**
T	**F**
F	**F**
F	**T**

Since all four of the four relevant fragments are recoverable, the sentence is truth-functionally true.

4. b. False. There is no relation between the number of atomic components of a truth-functionally true sentence and the number of open branches on a tree for that sentence. For example, '(A ∨ ~ A) ∨ [(B & C) & D]' is truth-functionally true and has four atomic components. But the tree for the unit set of this sentence has only three open branches:

1.		(A ∨ ~ A) ∨ [(B & C) & D]✓		SM
2.		A ∨ ~ A✓	(B & C) & D✓	1 ∨D
3.			B & C✓	2 &D
4.			D	2 &D
5.			B	3 &D
6.			C	3 &D
7.	A	~ A		2 ∨D

d. False. Some such unit sets will have closed trees; for example, {**P** & **Q**} will, but not all such unit sets will have closed trees. For example, {**P** ⊃ **Q**} will not have a closed tree. If **P** is 'A & ~ A' and **Q** is 'B & ~ B', the following tree will result:

```
1.              (A & ~ A) ⊃ (B & ~ B)✓            SM
                    /              \
2.      ~ (A & ~ A)✓              B & ~ B✓        1 ⊃D
3.         /    \                    B            2 &D
4.        /      \                  ~ B           2 &D
         /        \                  ×
5.    ~ A       ~ ~ A✓                            2 ~ &D
6.                A                               5 ~ ~ D
```

f. True. Since the tree for **P** is not closed, **P** is not truth-functionally false. The completed tree has only one open branch. That branch must contain at least one literal (if it did not, it could not be a completed open branch and the tree could not be a completed tree). If that literal is an atomic sentence, then every truth-value assignment recoverable from that branch will assign that literal **T**. If that literal is a negation of an atomic sentence, then every truth-value assignment recoverable from that branch will assign that atomic sentence **F**. In either case, there will be truth-value assignments that are not recoverable from the branch. So the sentence is not truth-functionally true. Therefore, **P** is truth-functionally indeterminate. Since the negation of a truth-functionally indeterminate sentence is also truth-functionally indeterminate, it follows that ~ **P** is truth-functionally indeterminate.

h. True. If **P** and **Q** are both truth-functionally true, then **P** & **Q**, **P** ∨ **Q**, **P** ⊃ **Q**, and **P** ≡ **Q** will also be truth-functionally true. Since they are truth-functionally true, each will have a completed open tree from which all the distinct fragments of truth-value assignments for the atomic components of **P** and **Q** can be recovered. There will have to be at least two completed open branches on each tree since each open branch has to have at least one literal on it, and such a branch will yield at most half of the relevant fragments (the half on which the literal is true).

j. True. Since **P** and **Q** are truth-functionally false, **P** & **Q** and **P** ∨ **Q** will also be truth-functionally false, and therefore have closed trees (every truth-functionally false sentence has a closed tree), and every closed tree has at least one closed branch. **P** ⊃ **Q** will be truth-functionally true, because on every truth-value assignment **P** is false and **Q** is false and so on every assignment **P** ⊃ **Q** is true. The first step in the tree for **P** ⊃ **Q** will yield ~ **P** on the left branch and **Q** on the right branch. Subsequent decomposition of **Q** will yield only, and therefore at least one, closed branches, because **Q** is truth-functionally false. **P** ≡ **Q** will also be truth-functionally true, because there can be no truth-value assignment on which **P** and **Q** have different truth values (both are false on every assignment). The tree for this sentence will yield, at lines 2 and 3, **P** and **Q** on the left and ~ **P** and ~ **Q** on the right branch. Since **P** and **Q** are both truth-functionally false, decomposition of each will yield only closed branches.

Section 4.6E

1. b.

1.	$\sim [\sim (Z \vee K) \equiv (\sim Z \vee \sim K)]$ ✓		SM
2.	$\sim (Z \vee K)$ ✓	$\sim \sim (Z \vee K)$ ✓	$1 \sim \equiv D$
3.	$\sim (\sim Z \vee \sim K)$ ✓	$\sim Z \vee \sim K$ ✓	$1 \sim \equiv D$
4.	$\sim Z$		$2 \sim \vee D$
5.	$\sim K$		$2 \sim \vee D$
6.	$\sim \sim Z$		$3 \sim \vee D$
7.	$\sim \sim K$ ✓		$3 \sim \vee D$
8.	K		$7 \sim \sim D$
	×		
9.		$Z \vee K$ ✓	$2 \sim \sim D$
10.		Z K	$9 \vee D$
11.		$\sim Z$ $\sim K$ $\sim Z$ $\sim K$	$3 \vee D$
		× ×	

Our truth-tree for the negation of the biconditional of the sentences we are testing is open. Therefore that negation is not truth-functionally false, and the biconditional of our sentences is not truth-functionally true. Hence the sentences we are testing are not truth-functionally equivalent. The recoverable fragments are

K	Z
F	**T**
T	**F**

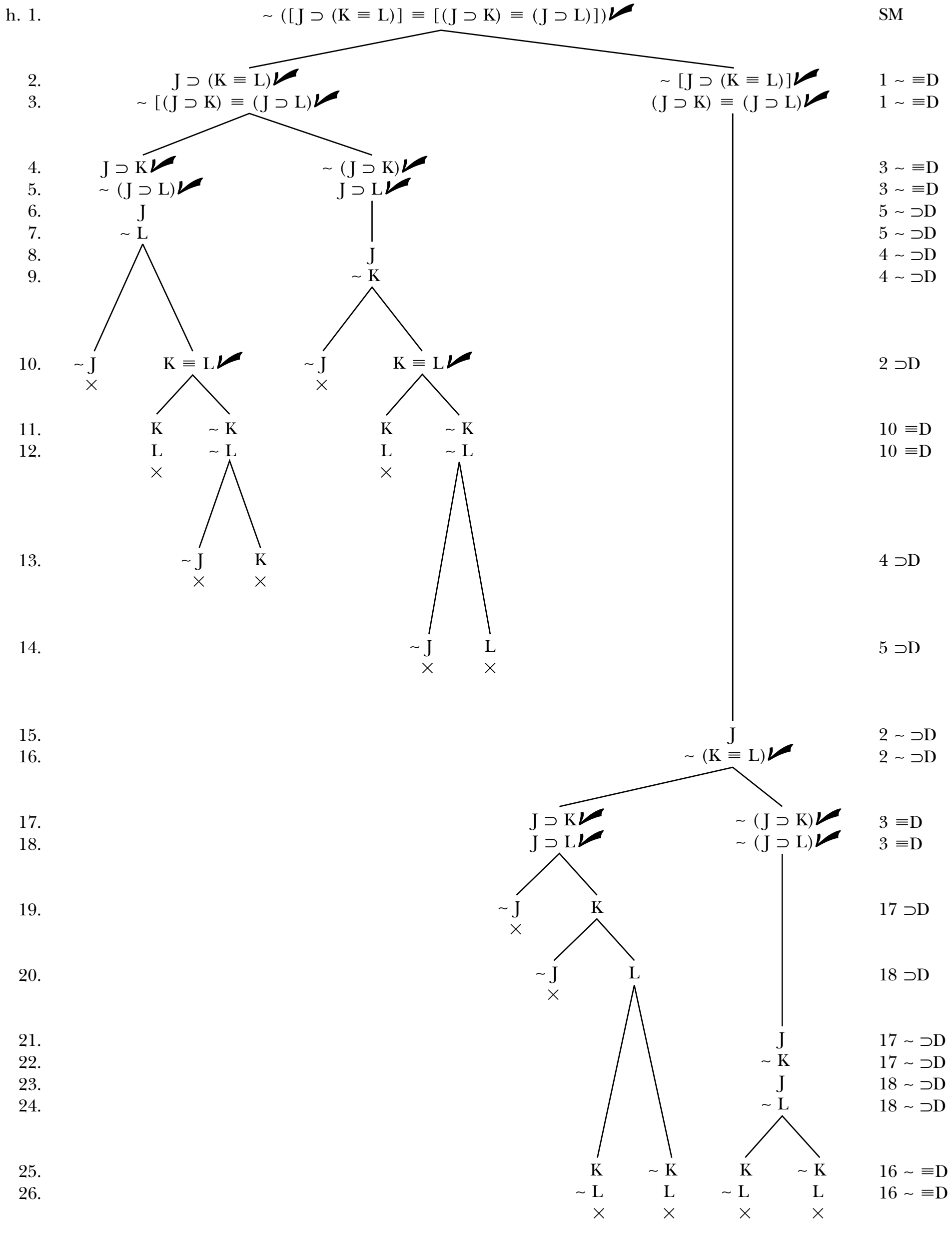
d. 1. $\sim [[(B \vee C) \supset R] \equiv [(B \supset R) \;\&\; (C \supset R)]]\checkmark$ SM

2.	$(B \vee C) \supset R\checkmark$				$\sim [(B \vee C) \supset R]\checkmark$		$1 \sim \equiv D$
3.	$\sim[(B \supset R) \;\&\; (C \supset R)]\checkmark$				$(B \supset R) \;\&\; (C \supset R)\checkmark$		$1 \sim \equiv D$
4.	$\sim(B \supset R)\checkmark$		$\sim (C \supset R)\checkmark$				$3 \sim \&D$
5.	B		C				$4 \sim \supset D$
6.	$\sim R$		$\sim R$				$4 \sim \supset D$
7.	$\sim (B \vee C)\checkmark$	R ×	$\sim (B \vee C)\checkmark$	R ×			$2 \supset D$
8.	$\sim B$		$\sim B$				$7 \sim \vee D$
9.	$\sim C$ ×		$\sim C$ ×				$7 \sim \vee D$
10.					$B \supset R\checkmark$		$3 \;\&D$
11.					$C \supset R\checkmark$		$3 \;\&D$
12.					$B \vee C\checkmark$		$2 \sim \supset D$
13.					$\sim R$		$2 \sim \supset D$
14.					$\sim B$	R ×	$10 \supset D$
15.					$\sim C$ / R ×		$11 \supset D$
16.					B × / C ×		$12 \vee D$

Since our truth-tree for the negation of the biconditional of the sentences we are testing is closed, those sentences are truth-functionally equivalent.

f. 1.	~ ([A ∨ (B & C)] ≡ [(A ∨ B) & (A ∨ C)])✓						SM
2.	A ∨ (B & C)✓				~ [A ∨ (B & C)]✓		1 ~ ≡D
3.	~[(A ∨ B) & (A ∨ C)]✓				(A ∨ B) & (A ∨ C)✓		1 ~ ≡D
4.	~ (A ∨ B)✓		~ (A ∨ C)✓				3 ~ &D
5.	~ A		~ A				4 ~ ∨D
6.	~ B		~ C				4 ~ ∨D
7.	A ×	B & C✓	A ×	B & C✓			2 ∨D
8.		B		B			7 &D
9.		C ×		C ×			7 &D
10.					A ∨ B✓		3 &D
11.					A ∨ C✓		3 &D
12.					~ A		2 ~ ∨D
13.					~ (B & C)✓		2 ~ ∨D
14.					A ×	B	10 ∨D
15.					A ×	C	11 ∨D
16.					~ B ×	~ C ×	13 ~ &D

Since our truth-tree for the negation of the biconditional of the sentences we are testing is closed, those sentences are truth-functionally equivalent.



Since our truth-tree for the negation of the biconditional of the sentences we are testing is closed, those sentences are truth-functionally equivalent.

j. 1.	~ [(A & ~ A) ≡ (B & ~ B)]✓		SM
2.	A & ~ A✓	~ (A & ~ A)	1 ~ ≡D
3.	~ (B & ~ B)	B & ~ B✓	1 ~ ≡D
4.	A		2 &D
5.	~ A		2 &D
6.	×	B	3 &D
7.		~ B	3 &D
		×	

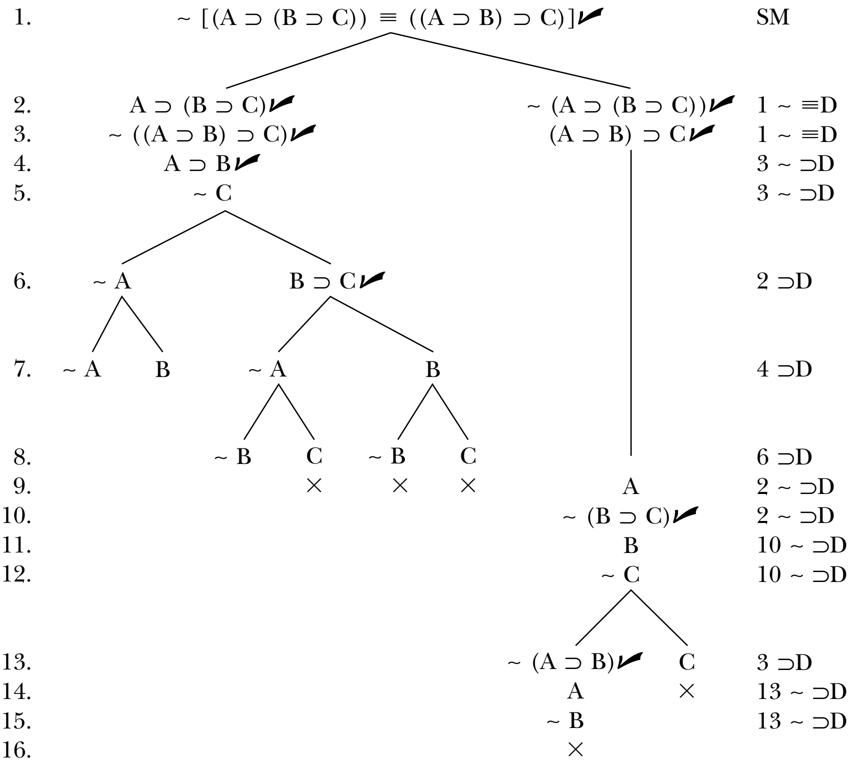
Since the tree is closed, the sentences being tested are truth-functionally equivalent.

l. 1.	~ [~ (A ∨ B) ≡ ~ (A & B)]✓				SM
2.	~ (A ∨ B)✓		~ ~ (A ∨ B)✓		1 ~ ≡D
3.	~ ~ (A & B)✓		~ (A & B)✓		1 ~ ≡D
4.	A & B✓				3 ~ ~ D
5.	A				4 &D
6.	B				4 &D
7.	~ A				2 ~ ∨D
8.	~ B				2 ~ ∨D
9.	×		A ∨ B✓		2 ~ ~ D
10.	A		B		9 ∨D
11.	~ A	~ B	~ A	~ B	3 ~ &D
	×			×	

Since the completed tree is open, the sentences being tested are not truth-functionally equivalent. The recoverable fragments are

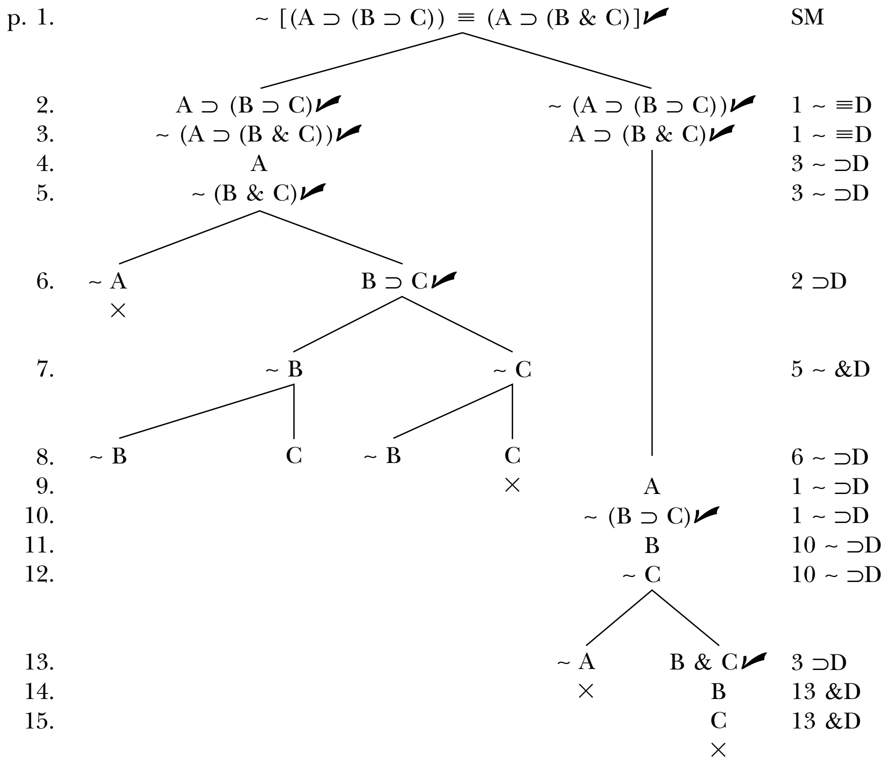
A	B
T	**F**
F	**T**

n.



Since the completed tree is open, the sentences being tested are not truth-functionally equivalent. The recoverable fragments are

A	B	C
F	**T**	**F**
F	**F**	**F**
F	**F**	**F**



Since the completed tree is open, the sentences being tested are not truth-functionally equivalent. The recoverable fragments are

A	B	C
T	**F**	**T**
T	**F**	**F**

2. b. False. There are three cases to consider: **P** and **Q** are both truth-functionally true, **P** and **Q** are both truth-functionally false, and **P** and **Q** are both truth-functionally indeterminate. In the first two cases **P** ≡ ~ **Q** will be truth-functionally false (either **P** will be truth-functionally true and ~ **Q** truth-functionally false, or vice-versa). Hence, in these cases the tree for **P** ≡ ~ **Q** will close. In the third case, where **P** and **Q** are truth-functionally indeterminate (and truth-functionally equivalent) **P** ≡ ~ **Q** is again truth-functionally false. If it were not, there would be a truth-value assignment on which **P** and ~ **Q** have the same truth-value, and hence on which **P** and **Q** have different truth-values. But there is no such assignment because **P** and **Q** are known to be truth-functionally equivalent. So, again, the tree for **P** ≡ ~ **Q** will close.

d. The completed tree for {~ **P** ≡ ~ **Q**} will be open. If were closed, there would be no truth-value assignment on which ~ **P** ≡ ~ **Q** is true, and thus no truth-value assignment on which ~ **P** and ~ **Q** have the same truth value. That is, on any truth-value assignment ~ **P** would be true and ~ **Q** false (and hence **P** false and **Q** true) or viceversa. But this cannot be since we are assuming that **P** and **Q** are truth-functionally equivalent, that is, that they have the same truth-value on every truth-value assignment.

Section 4.7E

1. b.

1.			K ⊃ H ✓			SM
2.			H ⊃ L ✓			SM
3.			L ⊃ M ✓			SM
4.			~ (K ⊃ M) ✓			SM
5.			K			4 ~ ⊃D
6.			~ M			4 ~ ⊃D
7.	~ K ×			H		1 ⊃D
8.			~ L		M ×	3 ⊃D
9.		~ H ×		L ×		2 ⊃D

Our truth-tree is closed, so the set {K ⊃ H, H ⊃ L, L ⊃ M} does truth-functionally entail ‘K ⊃ M’.

d.

1.		~ (~ A & ~ B) ✓			SM
2.		~ (A & B) ✓			SM
3.	~ ~ A ✓		~ ~ B ✓		1 ~ &D
4.	A		B		3 ~ ~D
5.	~ A ×	~ B	~ A	~ B ×	2 ~ &D

Our truth-tree is open, so the set {~ (~ A & ~ B)} does not truth-functionally entail 'A & B'. The relevant fragments of the recoverable truth-value assignments are

A	B
T	**F**
F	**T**

f. 1.	A & (B ⊃ C) ✓				SM
2.	~ [(A & C) ∨ (A & B)] ✓				SM
3.	A				1 &D
4.	B ⊃ C ✓				1 &D
5.	~ (A & C) ✓				2 ~ ∨D
6.	~ (A & B) ✓				2 ~ ∨D
7.	~ B		C		4 ⊃D
8.	~ A ×	~ C	~ A ×	~ C ×	5 ~ &D
9.	~ A ×	~ B ✓			6 ~ &D

Our tree is open, so the set {A & (B ⊃ C)} does not truth-functionally entail '(A & C) ∨ (A & B)'. The relevant fragment of the recoverable truth-value assignment is

A	B	C
T	**F**	**F**

h. 1. (G ≡ H) ∨ (~ G ≡ H)✓ SM
2. ~ [(~ G ≡ ~ H) ∨ ~ (G ≡ H)]✓ SM
3. ~ (~ G ≡ ~ H)✓ 2 ~ ∨D
4. ~ ~ (G ≡ H)✓ 2 ~ ∨D
5. G ≡ H✓ 4 ~ ~ D

6. G ≡ H | ~ G ≡ H — 1 ∨D

7. ~ G | ~ ~ G✓ | ~ G | ~ ~ G✓ — 3 ~ ≡D
8. ~ ~ H✓ | ~ H | ~ ~ H✓ | ~ H — 3 ~ ≡D
9. H | | H | — 8 ~ ~ D
10. | G | | G — 7 ~ ~ D

11. G | ~ G | G | ~ G | G | ~ G | G | ~ G — 5 ≡D
12. H | ~ H | H | ~ H | H | ~ H | H | ~ H — 5 ≡D
× | × | × | × | × | × | × | ×

Our tree is closed, so the set {(G ≡ H) ∨ (~ G ≡ H)} does truth-functionally entail '(~ G ≡ H ~ H) ∨ ~ (G ≡ H)'.

j. 1. ~ [[A ∨ ((K ⊃ ~ H) & ~ A)] ∨ ~ A]✓ SM
2. ~ [A ∨ ((K ⊃ ~ H) & ~ A)]✓ 1 ~ ∨D
3. ~ ~ A✓ 1 ~ ∨D
4. A 3 ~ ~ D
5. ~ A 2 ~ ∨D
6. ~ ((K ⊃ ~ H) & ~ A) 2 ~ ∨D
×

Our truth-tree is closed, so the entailment does hold; that is, the empty set, ∅, does entail '[A ∨ ((K ⊃ ~ H) & ~ A)] ∨ ~ A]'. Notice that in using the tree method to determine whether a sentence is entailed by the empty set, ∅, we proceed just as we would if we were determining whether that sentence is truth-functionally true.

1\. 1. ~ [~ (C ≡ C) ≡ (C ∨ ~ C)] ✔ SM

2.	~ (C ≡ C)	~ ~ (C ≡ C) ✔	1 ~ ≡D
3.	~ (C ∨ ~ C) ✔	C ∨ ~ C ✔	1 ~ ≡D
4.	~ C		3 ~ ∨D
5.	~ ~ C ✔		3 ~ ∨D
6.	C		5 ~ ~ D
	×		
7.		C ≡ C ✔	2 ~ ~ D
8.		C ~ C	7 ≡D
9.		C ~ C	7 ≡D
10.		C ~ C C ~ C	3 ∨D
		× ×	

Our truth-tree is open, so the empty set does not truth-functionally entail '~ (C ≡ C) ≡ (C ∨ ~ C)'. The relevant fragment of the recoverable truth-value assignment is

C
T
F

2. b.

1.	(~ J ∨ K) ⊃ (L & M) ✔		SM
2.	~(~ J ∨ K) ✔		SM
3.	~ ~ (L & M) ✔		SM
4.	L & M ✔		3 ~ ~ D
5.	L		4 &D
6.	M		4 &D
7.	~ ~ J ✔		2 ~ ∨D
8.	~ K		2 ~ ∨D
9.	J		7 ~ ~ D
10.	~(~ J ∨ K) ✔	L & M ✔	1 ⊃D
11.	~ ~ J ✔		10 ~ ∨D
12.	~ K		10 ~ ∨D
13.	J		11 ~ ~ D
14.		L	10 &D
15.		M	10 &D

Our truth-tree for the premises and the negation of the conclusion of the argumen we are testing is open. Therefore, there is a truth-value assignment on which those premises and the negation of the conclusion are true. This is also an assignment on which the premises are true and the conclusion false. The recoverable fragment is

J	K	L	M
T	**F**	**T**	**T**

d. 1.	$(D \equiv \sim G)\ \&\ G \checkmark$		SM
2.	$(G \vee ((A \supset D)\ \&\ A)) \supset \sim D \checkmark$		SM
3.	$\sim (G \supset \sim D) \checkmark$		SM
4.	$D \equiv \sim G$		1 &D
5.	G		1 &D
6.	G		$3 \sim \supset$D
7.	$\sim \sim D \checkmark$		$3 \sim \supset$D
8.	D		$7 \sim \sim$ D
9.	$\sim (G \vee ((A \supset D)\ \&\ A)) \checkmark$	$\sim D$	$2 \supset$D
10.	$\sim G$	$\times$	$9 \sim \vee$D
11.	$\sim ((A \supset D)\ \&\ A)$		$9 \sim \vee$D
	$\times$		

Our truth-tree for the premises and the negation of the conclusion is closed. Therefore the argument is truth-functionally valid.

f. 1.	$(J \supset T) \supset J \checkmark$			SM
2.	$(T \supset J) \supset T \checkmark$			SM
3.	$\sim (\sim J \vee \sim T) \checkmark$			SM
4.	$\sim \sim J \checkmark$			$3 \sim \vee$D
5.	$\sim \sim T \checkmark$			$3 \sim \vee$
6.	J			$4 \sim \sim$ D
7.	T			$5 \sim \sim$ D
8.	$\sim (J \supset T) \checkmark$	J		$1 \supset$D
9.	J			$8 \sim \supset$D
10.	$\sim T$			$8 \sim \supset$D
11.	$\times$	$\sim (T \supset J) \checkmark$	T	$2 \supset$D
12.		T		$11 \sim \supset$D
13.		$\sim J$		$11 \sim \supset$D
		$\times$		

Our truth-tree for the premises and the negation of the conclusion of the argument we are testing is open. Therefore that argument is truth-functionally invalid. The relevant fragment of the recoverable truth-value assignments is

J	T
T	**T**

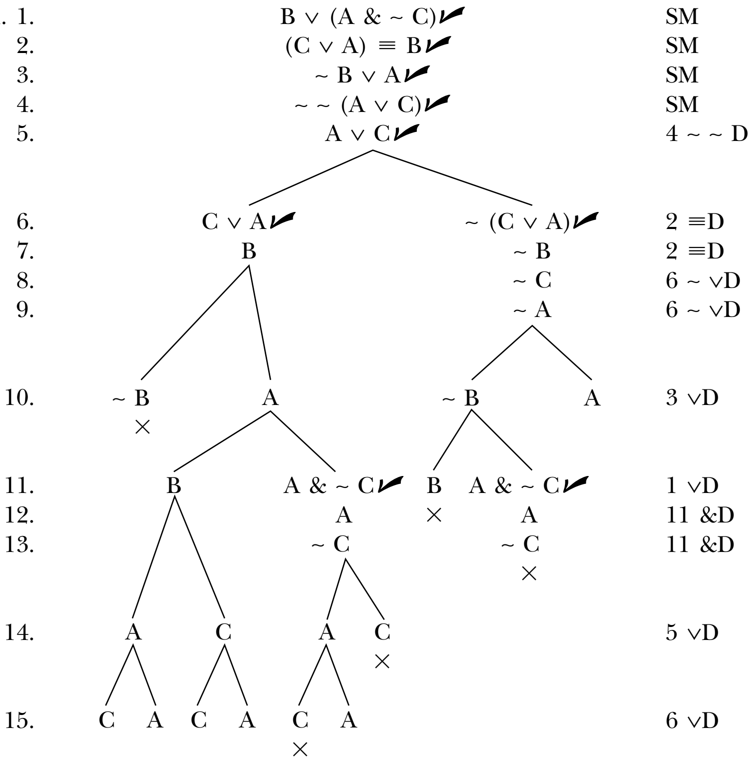
h.	1.	A ∨ ~ (B & C)✓	SM
	2.	~ B	SM
	3.	~ (A ∨ C)✓	SM
	4.	~ A	SM
	5.	~ A	3 ~ ∨D
	6.	~ C	3 ~ ∨D
	7.	A ~ (B & C)✓	1 ∨D
		×	
	8.	~ B ~ C	7 ~ &D

Our truth-tree for the premises and the negation of the conclusion of the argument we are testing is open. Therefore that argument is truth-functionally invalid. The relevant fragment of the recoverable truth-value assignments is

A	B	C
F	**F**	**F**

j.	1.	(G ≡ H) ∨ (~ G ≡ H)✓	SM
	2.	~ [(~ G ≡ ~ H) ∨ ~ (G ≡ H)]✓	SM
	3.	~ (~ G ≡ ~ H)✓	2 ~ ∨D
	4.	~ ~ (G ≡ H)✓	2 ~ ∨D
	5.	G ≡ H✓	4 ~ ~ D
	6.	G ≡ H ~ G ≡ H	1 ∨D
	7.	~ G ~ ~ G✓ ~ G ~ ~ G✓	3 ~ ≡D
	8.	~ ~ H✓ ~ H ~ ~ H✓ ~ H	3 ~ ≡D
	9.	H H	8 ~ ~ D
	10.	G G	7 ~ ~ D
	11.	G ~ G G ~ G G ~ G G ~ G	5 ≡D
	12.	H ~ H H ~ H H ~ H H ~ H	5 ≡D
		× × × × × × × ×	

Our truth-tree for the premises and the negation of the conclusion is closed. Therefore that argument is truth-functionally valid.

1.


Our truth-tree for the premises and the negation of the conclusion of the argument we are testing is open. Therefore that argument is truth-functionally invalid. The relevant fragments of the recoverable truth-value assignments are

A	B	C
T	**T**	**T**
T	**T**	**F**

3. b. In symbolizing the argument we use the following abbreviations:

M: A majority of the people support the legislation.
R: The representatives support the legislation.
S: The senators support the legislation.
P: The President supports the legislation.

Here is our tree for the premises and the negation of the conclusion:

```
1.        (P & S) ∨ (P & R)✓          SM
2.             M ⊃ R✓                 SM
3.              ~ M                   SM
4.              ~ S                   SM
               /    \
5.       P & S✓        P & R✓         1 ∨D
6.         P             P            5 &D
7.         S             R            5 &D
           ×            / \
8.                   ~ M   R          2 ⊃D
```

Since our truth-tree is open, the argument is truth-functionally invalid. The recoverable fragment is

M	P	R	S
F	**T**	**T**	**F**

d. In symbolizing the argument we use the following abbreviations:

C: Congress passes the bill.
D: A majority of Democrats vote for the bill.
H: The House of Representatives passes the bill.
S: The Senate passes the bill.
V: The President will veto the bill.

Here is our tree for the premises and the negation of the conclusion.

1.	$(C \supset \sim V) \;\&\; [C \equiv (S \;\&\; H)]$ ✓				SM
2.	$(H \supset D) \;\&\; D$ ✓				SM
3.	$\sim \sim V$ ✓				SM
4.	V				$3 \sim \sim D$
5.	$H \supset D$ ✓				2 &D
6.	D				2 &D
7.	$C \supset \sim V$ ✓				1 &D
8.	$C \equiv (S \;\&\; H)$ ✓				1 &D
9.	$\sim C$			$\sim V$ $\times$	$7 \supset D$
10.	C	$\sim C$			$8 \equiv D$
11.	$S \;\&\; H$ $\times$	$\sim (S \;\&\; H)$ ✓			$8 \equiv D$
12.		$\sim S$	$\sim H$		$11 \sim \&D$
13.		$\sim H$ D	$\sim H$ D		$5 \supset D$

Since the truth-tree is open, the argument is truth-functionally invalid. The recoverable fragments are

C	D	H	S	V
F	**T**	**F**	**F**	**T**
F	**T**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**

4. b. The first of the following arguments is truth-functionally valid, the second is truth-functionally invalid.

$$\frac{H \;\&\; \sim H}{G} \qquad\qquad \frac{H}{G \;\&\; \sim G}$$

The first argument is truth-functionally valid because there is no assignment on which the one premises is true, and hence no assignment on which it is true and the conclusion false. The second is truth-functionally invalid because there is an assignment on which the premises true and the conclusion false. In fact every assignment that assigns **T** to 'H' is of this sort, as the conclusion is truth-functionally false and thus false on every assignment.

When we do trees for the premise and the conclusion we generate, in each case, closed trees.

1.	H & ~ H✓	SM
2.	G	SM
3.	H	1 &D
4.	~ H	1 &D
	×	

1.	H	SM
2.	G & ~ G✓	SM
3.	G	2 &D
4.	~ G	2 &D
	×	

Since one of the arguments is truth-functionally valid and the other not, and both trees close, doing a tree for the premises of an argument and the conclusion of that argument and obtaining a closed tree clearly neither shows that the argument is valid nor that it is invalid.

Section 5.1.1E

b. Derive: (~ ~ S & ~ ~ S) & ~ ~ S

1	~ ~ S	Assumption
2	~ ~ S & ~ ~ S	1, 1 &I
3	(~ ~ S & ~ ~ S) & ~ ~ S	1, 2 &I

d. Derive: A & (B & C)

1	(C & B) & A	Assumption
2	A	1 &E
3	C & B	1 &E
4	B	3 &E
5	C	3 &E
6	B & C	4, 5 &I
7	A & (B & C)	2, 6 &I

Section 5.1.2E

b. Derive E ∨ K

1	(Q & M) ⊃ (E ∨ K)	Assumption
2	M & (E ∨ C)	Assumption
3	Q & ~ N	Assumption
4	Q	3 &E
5	M	2 &E
6	Q & M	4, 5 &I
7	E ∨ K	1, 6 ⊃E

d. Derive: (Z & K) ⊃ (A & E)

1	(K & Z) ⊃ (E & A)		Assumption
2		Z & K	Assumption
3		Z	2 &E
4		K	2 &E
5		K & Z	3, 4 &I
6		E & A	1, 5 ⊃E
7		E	6 &E
8		A	6 &E
9		A & E	7, 8 &I
10	(Z & K) ⊃ (A & E)		2–9 ⊃I

Section 5.1.3E

b. Derive: K

1	$M \& \sim M$	Assumption
2	$\quad \sim K$	Assumption
3	$\quad M$	1 &E
4	$\quad \sim M$	1 &E
5	K	2–4 $\sim$ E

d. Derive: R & M

1	$\sim (R \& M) \supset (L \& \sim N)$	Assumption
2	N	Assumption
3	$\quad \sim (R \& M)$	Assumption
4	$\quad L \& \sim N$	1, 3 $\supset$E
5	$\quad \sim N$	4 &E
6	$\quad N$	2 R
7	$R \& M$	3–6 $\sim$ E

Section 5.1.4E

b. Derive: Y

1	$P \vee C$	Assumption
2	$P \supset Y$	Assumption
3	$C \supset Y$	Assumption
4	$\quad P$	Assumption
5	$\quad Y$	2, 4 $\supset$E
6	$\quad C$	Assumption
7	$\quad Y$	3, 6 $\supset$E
8	Y	1, 4–5, 6–7 $\vee$E

d. Derive: ~ H

1	$(K \vee P) \supset \sim H$	Assumption
2	P	Assumption
3	$K \vee P$	2 $\vee$I
4	$\sim H$	1, 3 $\supset$E

Section 5.1.5E

b. Derive: ~ R ≡ E

```
1 | (~ R ⊃ E) & (E ⊃ ~ R)       Assumption
  |-----------------------
2 |   | ~ R                     Assumption
  |   |---------------------
3 |   | ~ R ⊃ E                 1 &E
4 |   | E                       2, 3 ⊃E
5 |   | E                       Assumption
  |   |---------------------
6 |   | E ⊃ ~ R                 1 &E
7 |   | ~ R                     5, 6 ⊃E
8 | ~ R ≡ E                     2–4, 5–7 ≡I
```

d. Derive: N

```
1 | A ∨ L                       Assumption
2 | A ≡ N                       Assumption
3 | L ⊃ N                       Assumption
  |-------
4 |   | A                       Assumption
  |   |---
5 |   | N                       2, 4 ≡E
6 |   | L                       Assumption
  |   |---
7 |   | N                       3, 6 ⊃E
8 | N                           1, 4–5, 6–7 ∨E
```

Section 5.2E

1. b. Derive: A ⊃ (B ⊃ C)

```
1 | (A & B) ⊃ C                 Assumption
  |-------------
2 |   | A                       Assumption
  |   |---------
3 |   |   | B                   Assumption
  |   |   |-----
4 |   |   | A & B               2, 3 &I
5 |   |   | C                   1, 4 ⊃E
6 |   | B ⊃ C                   3–5 ⊃I
7 | A ⊃ (B ⊃ C)                 2–6 ⊃I
```

d. Derive: A ⊃ B

1	(A & ~ B) ⊃ (~ B & C)	Assumption
2	C ⊃ ~ A	Assumption
3	│ A	Assumption
4	│ │ ~ B	Assumption
5	│ │ A & ~ B	3, 4 &I
6	│ │ ~ B & C	1, 5 ⊃E
7	│ │ C	6 &E
8	│ │ ~ A	2, 7 ⊃E
9	│ │ A	5 &E
10	│ B	4–9 ~ E
11	A ⊃ B	3–10 ⊃I

f. Derive: (A & ~ B) ∨ A

1	C ⊃ B	Assumption
2	(~ C ⊃ A) ∨ E	Assumption
3	F & ~ E	Assumption
4	B ⊃ (A & ~ B)	Assumption
5	│ ~ C ⊃ A	Assumption
6	│ │ C	Assumption
7	│ │ B	6, 1 ⊃E
8	│ │ A & ~ B	7, 4 ⊃E
9	│ │ ~ B	8 &E
10	│ ~ C	6–9 ~ I
11	│ A	5, 10 ⊃E
12	│ E	Assumption
13	│ │ ~ A	Assumption
14	│ │ E	12 R
15	│ │ ~ E	3 &E
16	│ A	13–15 ~ E
17	A	2, 5–11, 12–16 ∨E
18	(A & ~ B) ∨ A	17 ∨I

h. Derive: A ≡ (B ∨ C)

1	(A ≡ B) & (A ≡ C)	Assumption
2	A	Assumption
3	A ≡ B	1 &E
4	B	2, 3 ≡E
5	B ∨ C	4 ∨I
6	B ∨ C	Assumption
7	B	Assumption
8	A ≡ B	1 &E
9	A	7, 8 ≡E
10	C	Assumption
11	A ≡ C	1 &E
12	A	10, 11 ≡E
13	A	6, 7–9, 10–12 ∨E
14	A ≡ (B ∨ C)	2–5, 6–13 ≡I

2. b. Derive: B

1	P ⊃ (S ≡ (A & B))	Assumption	
2	P & S	Assumption	
3	S	2 &E	
4	A & B	1, 3 ≡E	**ERROR!**
5	B	4 &E	

Biconditional Elimination is a rule of inference and thus must be applied to a whole sentence which has a biconditional as its main connective.

Here is a corrective derivation:

Derive: B

1	P ⊃ (S ≡ (A & B))	Assumption
2	P & S	Assumption
3	S	2 &E
4	P	2 &E
5	S ≡ (A & B)	1, 4 ⊃E
6	A & B	3, 5 ≡E
7	B	6 &E

d. Derive: M

1	$(G \supset \sim\sim M) \& G$	Assumption	
2	G	1 &E	
3	$G \supset \sim\sim M$	1 &E	
4	$\sim\sim M$	2, 3 $\supset$E	
5	M	4 $\sim$ E	**ERROR!**

The rule of Negation Elimination requires a subderivation.

Here is a correct derivation:

Derive: M

1	$(G \supset \sim\sim M) \& G$	Assumption
2	G	1 &E
3	$G \supset \sim\sim M$	1 &E
4	$\sim\sim M$	2, 3 $\supset$E
5	$\quad \sim M$	Assumption
6	$\quad \sim\sim M$	4 R
7	$\quad \sim M$	5 R
8	M	5–7 $\sim$ E

f. Derive: K

1	$S \vee J$	Assumption	
2	$\quad K$	Assumption	
3	$\quad\quad S$	Assumption	
4	$\quad\quad K$	2 R	
5	$\quad\quad J$	Assumption	
6	$\quad\quad K$	2 R	
7	K	1, 3–4, 5–6 $\vee$E	**ERROR!**

The inner subderivations 3–4 and 5–6 are not accessible from the main derivation line. ‘K’ is derivable on line 7 within the subderivation beginning on line 2.

1	S ∨ J	Assumption
2	K	Assumption
3	S	Assumption
4	K	2 R
5	J	Assumption
6	K	2 R
7	K	1, 3–4, 5–6 ∨E ???

Now line 7 shows a correct use of the rule, but 'K' is no longer derived from the first assumption alone for it appears not immediately beside the main derivation line but within the subderivation. 'K' cannot be derived from 'S ∨ J'.

Section 5.4E

1. Goal Analysis
First Part: Indicating goals and subgoals

b. Derive: A ∨ Q

	1	R ⊃ A	Assumption
	2	R	Assumption
Subgoal →		A	
Goal →		A ∨ Q	– ∨I

d. Derive: L ≡ K

	1	(K ⊃ L) & (L ⊃ K)	Assumption
		L	Assumption
Subgoal →		K	
		K	Assumption
Subgoal →		L	
Goal →		L ≡ K	__–__, __–__ ≡I

f. Derive: Z

	1	B & (~ E ⊃ Z)	Assumption
	2	~ E	Assumption
Subgoal →		~ E ⊃ Z	
Goal →		Z	2, ___ ⊃E

h. Derive: W

	1	~ T ⊃ W	Assumption
	2	I ≡ W	Assumption
	3	~ T ∨ I	Assumption
		| ~ T	Assumption
Subgoal →		| W	
		| I	Assumption
Subgoal →		| W	
Goal →		W	3, ___–___, ___–___ ∨E

j. Derive: (Y & ~ H) & L

	1	(C ∨ A) ⊃ (Y & ~ H)	Assumption
	2	L & P	Assumption
	3	A	Assumption
Subgoal →		C ∨ A	
Goal →		Y & ~ H	1, ___ ⊃E
		L	2 &E
		(Y & ~ H) & L	___, ___ &I

l. Derive: (~ F & H) ≡ K

	1	K ⊃ (~ F & H)	Assumption
	2	(~ F & H) ⊃ J	Assumption
	3	J ⊃ K	Assumption
		| ~ F & H	Assumption
Subgoal →		| J	
Goal →		| K	3, ___ ⊃E
		| K	Assumption
		| ~ F & H	1, ___ ⊃E
		(~ F & H) ≡ K	___–___, ___–___ ≡I

Second Part: Completing derivations

b. Derive: $A \vee Q$

1	$R \supset A$	Assumption
2	R	Assumption
3	A	1, 2 $\supset$E
4	$A \vee Q$	3 $\vee$I

d. Derive: $L \equiv K$

1	$(K \supset L)\ \&\ (L \supset K)$	Assumption
2	$\quad L$	Assumption
3	$\quad L \supset K$	1 &E
4	$\quad K$	2, 3 $\supset$E
5	$\quad K$	Assumption
6	$\quad K \supset L$	1 &E
7	$\quad L$	5, 6 $\supset$E
8	$L \equiv K$	2–4, 5–7 $\equiv$I

f. Derive: Z

1	$B\ \&\ (\sim E \supset Z)$	Assumption
2	$\sim E$	Assumption
3	$\sim E \supset Z$	1 &E
4	Z	2, 3 $\supset$E

h. Derive: W

1	$\sim T \supset W$	Assumption
2	$I \equiv W$	Assumption
3	$\sim T \vee I$	Assumption
4	$\quad \sim T$	Assumption
5	$\quad W$	1, 4 $\supset$E
6	$\quad I$	Assumption
7	$\quad W$	2, 6 $\equiv$E
8	W	3, 4–5, 6–7 $\vee$E

j. Derive: $(Y\ \&\ \sim H)\ \&\ L$

1	$(C \vee A) \supset (Y\ \&\ \sim H)$	Assumption
2	$L\ \&\ P$	Assumption
3	A	Assumption
4	$C \vee A$	3 $\vee$I
5	$Y\ \&\ \sim H$	1, 4 $\supset$E
6	L	2 &E
7	$(Y\ \&\ \sim H)\ \&\ L$	5, 6 &I

l. Derive: (~ F & H) ≡ K

1	K ⊃ (~ F & H)	Assumption
2	(~ F & H) ⊃ J	Assumption
3	J ⊃ K	Assumption
4	| ~ F & H	Assumption
5	| J	2, 4 ⊃E
6	| K	3, 5 ⊃E
7	| K	Assumption
8	| ~ F & H	1, 7 ⊃E
9	(~ F & H) ≡ K	4–6, 7–8 ≡I

2. Derivability

b. Derive: R ∨ T

1	K ⊃ R	Assumption
2	B & K	Assumption
3	K	2 &E
4	R	1, 3 ⊃E
5	R ∨ T	4 ∨I

d. Derive: B & D

1	(A ∨ B) ⊃ (B ≡ D)	Assumption
2	B	Assumption
3	A ∨ B	2 ∨I
4	B ≡ D	1, 3 ⊃E
5	D	2, 4 ≡E
6	B & D	2, 5 &I

f. Derive: B

1	(~ B ∨ ~ H) ⊃ M	Assumption
2	K & ~ M	Assumption
3	| ~ B	Assumption
4	| ~ B ∨ ~ H	3 ∨I
5	| M	1, 4 ⊃E
6	| ~ M	2 &E
7	B	3–6 ~ E

3. Validity

b. Derive: ~ W

1	R & (C & ~ F)	Assumption
2	(R ∨ S) ⊃ ~ W	Assumption
3	R	1 &E
4	R ∨ S	3 ∨I
5	~ W	2, 4 ⊃E

d. Derive: D

1	(A ⊃ F) & (F ⊃ D)	Assumption
2	[(M ∨ H) ∨ C] ⊃ A	Assumption
3	~ (M ∨ H) & C	Assumption
4	C	3 &E
5	(M ∨ H) ∨ C	4 ∨I
6	A	2, 5 ⊃E
7	A ⊃ F	1 &E
8	F	6, 7 ⊃E
9	F ⊃ D	1 &E
10	D	8, 9 ⊃E

f. Derive: C ⊃ [A ⊃ (S ⊃ H)]

1	H	Assumption
2	C	Assumption
3	A	Assumption
4	S	Assumption
5	H	1 R
6	S ⊃ H	4–5 ⊃I
7	A ⊃ (S ⊃ H)	3–6 ⊃I
8	C ⊃ [A ⊃ (S ⊃ H)]	2–7 ⊃I

h. Derive: A ⊃ (D ⊃ C)

1	A ⊃ (B ⊃ C)	Assumption
2	D ⊃ B	Assumption
3	A	Assumption
4	D	Assumption
5	B ⊃ C	1, 3 ⊃E
6	B	4, 2 ⊃E
7	C	6, 5 ⊃E
8	D ⊃ C	4–7 ⊃I
9	A ⊃ (D ⊃ C)	3–8 ⊃I

j. Derive: ~ H

1	~ B ≡ Z	Assumption
2	N ⊃ B	Assumption
3	Z & N	Assumption
4	│ H	Assumption
5	│ Z	3 &E
6	│ ~ B	1, 5 ≡E
7	│ N	3 &E
8	│ B	2, 7 ⊃E
9	~ H	4–8 ~ I

4. Theorems

b. Derive: A ⊃ (B ⊃ A)

1	│ A	Assumption
2	│ │ B	Assumption
3	│ │ A	1 R
4	│ B ⊃ A	2–3 ⊃I
5	A ⊃ (B ⊃ A)	1–4 ⊃I

d. Derive: (A & ~ A) ⊃ (B & ~ B)

1	│ A & ~ A	Assumption
2	│ │ ~ (B & ~ B)	Assumption
3	│ │ A	1 &E
4	│ │ ~ A	1 &E
5	│ B & ~ B	2–4 ~ E
6	(A & ~ A) ⊃ (B & ~ B)	1–5 ⊃I

f. Derive: A ∨ ~ A

1	│ ~ (A ∨ ~ A)	Assumption
2	│ │ A	Assumption
3	│ │ A ∨ ~ A	2 ∨I
4	│ │ ~ (A ∨ ~ A)	1 R
5	│ ~ A	2–4 ~ I
6	│ A ∨ ~ A	5 ∨I
7	│ ~ (A ∨ ~ A)	1 R
8	A ∨ ~ A	1–7 ~ E

h. Derive: (A & A) ≡ A

1	A & A	Assumption
2	A	1 &E
3	A	Assumption
4	A	3 R
5	A & A	3, 4 &I
6	(A & A) ≡ A	1–2, 3–5 ≡I

j. Derive: ~ A ⊃ [(B & A) ⊃ C]

1	~ A	Assumption
2	B & A	Assumption
3	~ C	Assumption
4	A	2 &E
5	~ A	1 R
6	C	3–5 ~ E
7	(B & A) ⊃ C	2–6 ⊃I
8	~ A ⊃ [(B & A) ⊃ C]	1–7 ⊃I

5. Equivalence

b. Derive: A & (B & C)

1	(A & B) & C	Assumption
2	A & B	1 &E
3	A	2 &E
4	B	2 &E
5	C	1 &E
6	B & C	4, 5 &I
7	A & (B & C)	3, 6 &I

Derive: (A & B) & C

1	A & (B & C)	Assumption
2	A	1 &E
3	B & C	1 &E
4	B	3 &E
5	A & B	2, 4 &I
6	C	3 &E
7	(A & B) & C	5, 6 &I

d. Derive: A ∨ A

1	A & A	Assumption
2	A	1 &E
3	A ∨ A	2 ∨I

Derive: A & A

1	A ∨ A	Assumption
2	A	Assumption
3	A	2 R
4	A	Assumption
5	A	4 R
6	A	1, 2–3, 4–5 ∨E
7	A	6 R
8	A & A	6, 7 &I

f. Derive: B ≡ A

1	A ≡ B	Assumption
2	B	Assumption
3	A	1, 2 ≡E
4	A	Assumption
5	B	1, 4 ≡E
6	B ≡ A	2–3, 4–5 ≡I

Derive: A ≡ B

1	B ≡ A	Assumption
2	B	Assumption
3	A	1, 2 ≡E
4	A	Assumption
5	B	1, 4 ≡E
6	A ≡ B	2–3, 4–5 ≡I

6. Inconsistency

b.

1	P ⊃ ~ P	Assumption
2	~ P ⊃ P	Assumption
3	P	Assumption
4	~ P	1, 3 ⊃E
5	P	3 R
6	~ P	3–5 ~ I
7	~ P	Assumption
8	P	2, 7 ⊃E
9	~ P	7 R
10	P	7–9 ~ E

d.	1	(E ∨ F) ⊃ (G & ~ I)	Assumption
	2	(G ∨ F) ⊃ I	Assumption
	3	~ F ⊃ E	Assumption
	4	│ ~ F	Assumption
	5	│ E	3, 4 ⊃E
	6	│ E ∨ F	5 ∨I
	7	│ G & ~ I	1, 6 ⊃E
	8	│ G	7 &E
	9	│ G ∨ F	8 ∨I
	10	│ I	2, 9 ⊃E
	11	│ ~ I	7 &E
	12	F	4–11 ~ E
	13	E ∨ F	12 ∨I
	14	G & ~ I	1, 13 ⊃E
	15	G	14 &E
	16	G ∨ F	15 ∨I
	17	I	16, 2 ⊃E
	18	~ I	14 &E

f.	1	F ⊃ ~ G	Assumption
	2	~ F ⊃ ~ H	Assumption
	3	(~ F ∨ G) & H	Assumption
	4	H	3 &E
	5	│ ~ F	Assumption
	6	│ ~ H	5, 2 ⊃E
	7	│ H	4 R
	8	F	5–7 ~ E
	9	~ G	1, 8 ⊃E
	10	~ F ∨ G	3 &E
	11	│ ~ F	Assumption
	12	│ │ ~ G	Assumption
	13	│ │ ~ F	11 R
	14	│ │ F	8 R
	15	│ G	12–14 ~ E
	16	│ G	Assumption
	17	│ G	16 R
	18	G	10, 11–15, 16–17 ∨E

7. Derivability

b. Derive: $E \vee H$

1	$C \supset (\sim D \supset H)$	Assumption
2	$C \,\&\, \sim D$	Assumption
3	C	2 &E
4	$\sim D \supset H$	1, 3 $\supset$E
5	$\sim D$	2 &E
6	H	3 &E
7	$E \vee H$	6 $\vee$I

d. Derive: $\sim (A \,\&\, I)$

1	$A \supset \sim \sim B$	Assumption
2	$I \supset \sim B$	Assumption
3	$\mid\; A \,\&\, I$	Assumption
4	$\mid\; A$	3 &E
5	$\mid\; \sim \sim B$	1, 4 $\supset$E
6	$\mid\; I$	3 &E
7	$\mid\; \sim B$	2, 6 $\supset$E
8	$\sim (A \,\&\, I)$	4–7 $\sim$ I

8. Validity

b. Derive: A

1	$B \vee Q$	Assumption
2	$A \equiv B$	Assumption
3	$Q \supset A$	Assumption
4	$\mid\; B$	Assumption
5	$\mid\; A$	2, 4 $\equiv$E
6	$\mid\; Q$	Assumption
7	$\mid\; A$	3, 6 $\supset$E
8	A	1, 4–5, 6–7 $\vee$E

d. Derive: $G \equiv N$

1	$[\sim (M \,\&\, F) \vee N] \supset G$	Assumption
2	$\sim N \supset \sim G$	Assumption
3	$\mid\; G$	Assumption
4	$\mid\;\mid\; \sim N$	Assumption
5	$\mid\;\mid\; \sim G$	2, 4 $\supset$E
6	$\mid\;\mid\; G$	3 R
7	$\mid\; N$	4–6 $\sim$ E
8	$\mid\; N$	Assumption
9	$\mid\; \sim (M \,\&\, F) \vee N$	8 $\vee$I
10	$\mid\; G$	1, 9 $\supset$E
11	$G \equiv N$	3–7, 8–10 $\equiv$I

9. Theorems

b. Derive: $(A \vee B) \supset (B \vee A)$

1	$\mid\ \mid A \vee B$	Assumption
2	$\mid\ \mid\ \mid A$	Assumption
3	$\mid\ \mid\ \mid B \vee A$	2 $\vee$I
4	$\mid\ \mid\ \mid B$	Assumption
5	$\mid\ \mid\ \mid B \vee A$	4 $\vee$I
6	$\mid\ \mid B \vee A$	1, 2–3, 4–5 $\vee$E
7	$\mid (A \vee B) \supset (B \vee A)$	1–5 $\supset$I

d. Derive: $(A \equiv B) \supset (B \supset A)$

1	$\mid\ \mid A \equiv B$	Assumption
2	$\mid\ \mid\ \mid B$	Assumption
3	$\mid\ \mid\ \mid A$	1, 2 $\equiv$E
4	$\mid\ \mid B \supset A$	2–3 $\supset$I
5	$\mid (A \equiv B) \supset (B \supset A)$	1–4 $\supset$I

10. Equivalence

b. Derive: $A \supset (A \supset A)$

1	$\mid A \equiv A$	Assumption
2	$\mid\ \mid A$	Assumption
3	$\mid\ \mid\ \mid A$	Assumption
4	$\mid\ \mid\ \mid A$	3 R
5	$\mid\ \mid A \supset A$	3–4 $\supset$I
6	$\mid A \supset (A \supset A)$	2–5 $\supset$I

Derive: $A \equiv A$

1	$\mid A \supset (A \supset A)$	Assumption
2	$\mid\ \mid A$	Assumption
3	$\mid\ \mid A$	2 R
4	$\mid A \equiv A$	2–3, 2–3, $\equiv$I

d. Derive: $(A \& C) \vee (B \& C)$

1	$(A \vee B) \& C$	Assumption
2	$(A \vee B)$	1 &E
3	A	Assumption
4	C	1 &E
5	$A \& C$	3, 4 &I
6	$(A \& C) \vee (B \& C)$	5 $\vee$I
7	B	Assumption
8	C	1 &E
9	$B \& C$	7 &E
10	$(A \& C) \vee (B \& C)$	9 $\vee$I
11	$(A \& C) \vee (B \& C)$	2, 3–6, 7–10 $\vee$E

Derive: $(A \vee B) \& C$

1	$(A \& C) \vee (B \& C)$	Assumption
2	$A \& C$	Assumption
3	A	2 &E
4	$A \vee B$	3 $\vee$I
5	C	2 &E
6	$(A \vee B) \& C$	4, 5 &I
7	$B \& C$	Assumption
8	B	7 &E
9	$A \vee B$	8 $\vee$I
10	C	7 &E
11	$(A \vee B) \& C$	9, 10 &I
12	$(A \vee B) \& C$	1, 2–6, 7–11 $\vee$E

11. Inconsistency

b. $\{\sim G \& Y, Y \supset G\}$

1	$\sim G \& Y$	Assumption
2	$Y \supset G$	Assumption
3	Y	1 &E
4	$\sim G$	1 &E
5	G	3, 2 $\supset$I

d. {P ⊃ (A & Y), Y ⊃ (~ A & P), P ∨ Y}

```
 1  | P ⊃ (A & Y)                Assumption
 2  | Y ⊃ (~ A & P)              Assumption
 3  | P ∨ Y                      Assumption
    |----------------
 4  |  | P                       Assumption
    |  |-------------
 5  |  | A & Y                   1, 4 ⊃E
 6  |  | A                       5 &E
 7  |  | Y                       5 &E
 8  |  | ~ A & P                 2, 7 ⊃E
 9  |  | ~ A                     8 &E
10  |  | A & ~ A                 6, 9 &I
    |
11  |  | Y                       Assumption
    |  |-------------
12  |  | ~ A & P                 2, 11 ⊃E
13  |  | ~ A                     12 &E
14  |  | P                       12 &E
15  |  | A & Y                   1, 14 ⊃E
16  |  | A                       15 &E
17  |  | A & ~ A                 13, 16 &I
18  | A & ~ A                    3, 4–10, 11–17 ∨E
19  | A                          18 &E
20  | ~ A                        18 &E
```

12. Derivability

b. Derive: K

```
 1  | K ∨ (K ∨ K)                Assumption
    |----------------
 2  |  | K                       Assumption
    |  |-------------
 3  |  | K                       2 R
    |
 4  |  | K ∨ K                   Assumption
    |  |-------------
 5  |  | K                       4, 2–3, 2–3 ∨E
 6  | K                          1, 2–3, 4–5 ∨E
```

d. Derive: A

```
 1  | A ∨ B                      Assumption
 2  | ~ B                        Assumption
    |----------------
 3  |  | A                       Assumption
    |  |-------------
 4  |  | A                       3 R
    |
 5  |  | B                       Assumption
    |  |-------------
 6  |  |  | ~ A                  Assumption
    |  |  |----------
 7  |  |  | ~ B                  2 R
 8  |  |  | B                    5 R
 9  |  | A                       6–8 ~ E
10  | A                          1, 3–4, 5–9 ∨E
```

f. Derive: A

1	$A \equiv (\sim B \vee C)$	Assumption
2	$B \supset C$	Assumption
3	$\quad \sim (\sim B \vee C)$	Assumption
4	$\quad\quad \sim B$	Assumption
5	$\quad\quad \sim B \vee C$	4 $\vee$I
6	$\quad\quad \sim (\sim B \vee C)$	3 R
7	$\quad B$	4–6 $\sim$ E
8	$\quad C$	2, 7 $\supset$E
9	$\quad \sim B \vee C$	8 $\vee$I
10	$\quad \sim (\sim B \vee C)$	3 R
11	$\sim B \vee C$	3–10 $\sim$ E
12	A	1, 11 $\equiv$E

h. Derive: $P \vee (Q \vee R)$

1	$(P \vee Q) \vee R$	Assumption
2	$\quad P \vee Q$	Assumption
3	$\quad\quad P$	Assumption
4	$\quad\quad P \vee (Q \vee R)$	3 $\vee$I
5	$\quad\quad Q$	Assumption
6	$\quad\quad Q \vee R$	5 $\vee$I
7	$\quad\quad P \vee (Q \vee R)$	6 $\vee$I
8	$\quad P \vee (Q \vee R)$	2, 3–4, 5–7 $\vee$E
9	$\quad R$	Assumption
10	$\quad Q \vee R$	9 $\vee$I
11	$\quad P \vee (Q \vee R)$	10 $\vee$I
12	$P \vee (Q \vee R)$	1, 2–8, 9–11 $\vee$E

j. Derive: $\sim H$

1	$R \vee \sim H$	Assumption
2	$\sim R \vee \sim H$	Assumption
3	$\sim H$	Assumption
4	$\sim H$	3 R
5	$\sim R$	Assumption
6	$\sim H$	Assumption
7	$\sim H$	6 R
8	R	Assumption
9	H	Assumption
10	R	8 R
11	$\sim R$	5 R
12	$\sim H$	9–11 $\sim$ I
13	$\sim H$	1, 6–7, 8–12 $\vee$E
14	$\sim H$	2, 3–4, 5–13 $\vee$E

l. Derive: $\sim W \equiv \sim (S \supset L)$

1	$(S \supset L) \supset W$	Assumption
2	$(S \supset L) \vee \sim W$	Assumption
3	$\sim (S \supset L)$	Assumption
4	$(S \supset L)$	Assumption
5	W	Assumption
6	$(S \supset L)$	4 R
7	$\sim (S \supset L)$	3 R
8	$\sim W$	5–7 $\sim$ I
9	$\sim W$	Assumption
10	$\sim W$	9 R
11	$\sim W$	2, 4–8, 9–10 $\vee$E
12	$\sim W$	Assumption
13	$(S \supset L)$	Assumption
14	W	1, 13 $\supset$E
15	$\sim W$	12 R
16	$\sim (S \supset L)$	13–15 $\sim$ I
17	$\sim W \equiv \sim (S \supset L)$	3–11, 12–16 $\equiv$I

n. Derive: $E \vee M$

1	$[E \vee (L \vee M)] \;\&\; (E \equiv F)$	Assumption
2	$L \supset D$	Assumption
3	$D \supset \sim L$	Assumption
4	$E \vee (L \vee M)$	1 &E
5	$\quad E$	Assumption
6	$\quad E$	5 R
7	$\quad E \vee M$	6 ∨I
8	$\quad L \vee M$	Assumption
9	$\quad\quad L$	Assumption
10	$\quad\quad\quad \sim E$	Assumption
11	$\quad\quad\quad L$	9 R
12	$\quad\quad\quad D$	11, 2 ⊃E
13	$\quad\quad\quad \sim L$	12, 3 ⊃E
14	$\quad\quad E$	10–13 ~ E
15	$\quad\quad E \vee M$	14 ∨I
16	$\quad\quad M$	Assumption
17	$\quad\quad E \vee M$	16 ∨I
18	$\quad E \vee M$	8, 9–15, 16–17 ∨E
19	$E \vee M$	4, 5–7, 8–18 ∨E

p. Derive: ~ A ≡ B

```
 1  | ~ (A ≡ B)                      Assumption
    |----------
 2  | | ~ A                          Assumption
    | |------
 3  | | | ~ B                        Assumption
    | | |------
 4  | | | | A                        Assumption
    | | | |----
 5  | | | | | ~ B                    Assumption
    | | | | |------
 6  | | | | | ~ A                    2 R
 7  | | | | | A                      4 R
 8  | | | | B                        5–7 ~ E
    | | | |
 9  | | | | B                        Assumption
    | | | |----
10  | | | | | ~ A                    Assumption
    | | | | |------
11  | | | | | B                      9 R
12  | | | | | ~ B                    3 R
13  | | | | A                        10–12 ~ E
14  | | | A ≡ B                      4–8, 9–13 ≡I
15  | | | ~ (A ≡ B)                  1 R
16  | | B                            3–15 ~ E
    | |
17  | | B                            Assumption
    | |----
18  | | | A                          Assumption
    | | |----
19  | | | | A                        Assumption
    | | | |----
20  | | | | B                        17 R
    | | |
21  | | | | B                        Assumption
    | | | |----
22  | | | | A                        18 R
23  | | | A ≡ B                      19–20, 21–22 ≡I
24  | | | ~ (A ≡ B)                  1 R
25  | | ~ A                          18–24 ~ I
26  | ~ A ≡ B                        2–16, 17–25 ≡I
```

13. Validity

b. Derive: Q

```
 1  | K ⊃ ~ ~ Q                      Assumption
 2  | ~ ~ K                          Assumption
    |----------
 3  | | ~ Q                          Assumption
    | |------
 4  | | | ~ K                        Assumption
    | | |------
 5  | | | ~ ~ K                      2 R
 6  | | | ~ K                        4 R
 7  | | K                            4–6 ~ E
 8  | | ~ ~ Q                        1, 7 ⊃E
 9  | | ~ Q                          3 R
10  | Q                              3–9 ~ E
```

d. Derive: ~ (A ≡ C)

```
 1 | A ≡ B              Assumption
 2 | B ≡ ~ C            Assumption
   |-------
 3 | | A ≡ C            Assumption
   | |------
 4 | | | ~ C            Assumption
   | | |----
 5 | | | B              2, 4 ≡E
 6 | | | A              1, 5 ≡E
 7 | | | C              3, 6 ≡E
 8 | | | ~ C            4 R
 9 | | C                4–8 ~ E
10 | | A                3, 9 ≡E
11 | | B                1, 10 ≡E
12 | | ~ C              2, 11 ≡E
13 | ~ (A ≡ C)          3–12 ~ I
```

f. Derive: Q

```
 1 | Q ∨ (J ≡ D)        Assumption
 2 | ~ D                Assumption
 3 | J                  Assumption
   |-------
 4 | | (J ≡ D)          Assumption
   | |------
 5 | | | ~ Q            Assumption
   | | |----
 6 | | | J              3 R
 7 | | | D              4, 6 ≡E
 8 | | | ~ D            2 R
 9 | | Q                5–8 ~ E
   |
10 | | Q                Assumption
   | |------
11 | | Q                10 R
12 | Q                  1, 4–9, 10–11 ∨E
```

h. Derive: ~ A ≡ ~ (~ K ∨ C)

```
 1 │ A ∨ ~ (~ K ∨ C)                 Assumption
 2 │ ~ (~ K ∨ C) ⊃ ~ A               Assumption
   ├────────────────
 3 │ │ ~ A                           Assumption
   │ ├──────────────
 4 │ │ │ A                           Assumption
   │ │ ├────────────
 5 │ │ │ │ ~ K ∨ C                   Assumption
   │ │ │ ├──────────
 6 │ │ │ │ A                         4 R
 7 │ │ │ │ ~ A                       3 R
 8 │ │ │ ~ (~ K ∨ C)                 5–7 ~ I
   │ │
 9 │ │ │ ~ (~ K ∨ C)                 Assumption
   │ │ ├────────────
10 │ │ │ ~ (~ K ∨ C)                 9 R
11 │ │ ~ (~ K ∨ C)                   1, 4–8, 9–10 ∨E
   │
12 │ │ ~ (~ K ∨ C)                   Assumption
   │ ├──────────────
13 │ │ ~ A                           2, 12 ⊃E
14 │ ~ A ≡ ~ (~ K ∨ C)               3–11, 12–13 ≡I
```

j. Derive: A

```
 1 │ A ∨ B                           Assumption
 2 │ ~ B ∨ C                         Assumption
 3 │ ~ C                             Assumption
   ├──────────
 4 │ │ ~ B                           Assumption
   │ ├────────
 5 │ │ │ A                           Assumption
   │ │ ├──────
 6 │ │ │ A                           5 R
   │ │
 7 │ │ │ B                           Assumption
   │ │ ├──────
 8 │ │ │ │ ~ A                       Assumption
   │ │ │ ├────
 9 │ │ │ │ B                         7 R
10 │ │ │ │ ~ B                       4 R
11 │ │ │ A                           8–10 ~ E
12 │ │ A                             1, 5–6, 7–11 ∨E
   │
13 │ │ C                             Assumption
   │ ├────────
14 │ │ │ ~ A                         Assumption
   │ │ ├──────
15 │ │ │ C                           13 R
16 │ │ │ ~ C                         3 R
17 │ │ A                             14–16 ~ E
18 │ A                               2, 4–12, 13–17 ∨E
```

1. Derive: (P & [G ≡ (A & B)]) ∨ (~ C ∨ ~ E)

1	B ⊃ (E ⊃ F)	Assumption
2	A ⊃ (C ⊃ D)	Assumption
3	A ∨ B	Assumption
4	~ D & ~ F	Assumption
5		A	Assumption
6		C ⊃ D	5, 2 ⊃E
7			C	Assumption
8			D	6, 7 ⊃E
9			~ D	4 &E
10		~ C	7–9 ~ I
11		~ C ∨ ~ E	10 ∨I
12		B	Assumption
13		E ⊃ F	1, 12 ⊃E
14			E	Assumption
15			F	13, 14 ⊃E
16			~ F	4 &E
17		~ E	14–16 ~ I
18		~ C ∨ ~ E	17 ∨I
19	~ C ∨ ~ E	3, 5–11, 12–18 ∨E
20	(P & [G ≡ (A & B)]) ∨ (~ C ∨ ~ E)	19 ∨I

n. Derive: ~ K

1	(A ∨ B) ∨ (C & D)	Assumption
2	(A ≡ E) & (B ⊃ F)	Assumption
3	K ≡ ~ (E ∨ F)	Assumption
4	C ⊃ B	Assumption
5	│ K	Assumption
6	│ │ A ∨ B	Assumption
7	│ │ │ A	Assumption
8	│ │ │ A ≡ E	2 &E
9	│ │ │ E	7, 8 ≡E
10	│ │ │ E ∨ F	9 ∨I
11	│ │ │ B	Assumption
12	│ │ │ B ⊃ F	2 &E
13	│ │ │ F	11, 12 ⊃E
14	│ │ │ E ∨ F	13 ∨I
15	│ │ E ∨ F	6, 7–10, 11–14 ∨E
16	│ │ C & D	Assumption
17	│ │ C	16 &E
18	│ │ B	4, 17 ⊃E
19	│ │ B ⊃ F	2 &E
20	│ │ F	18, 19 ⊃E
21	│ │ E ∨ F	20 ∨I
22	│ E ∨ F	1, 6–15, 16–21 ∨E
23	│ ~ (E ∨ F)	3, 5 ≡E
24	~ K	5–23 ~ I

14. Theorems

b. Derive: A ≡ ~ ~ A

1	│ A	Assumption
2	│ │ ~ A	Assumption
3	│ │ A	1 R
4	│ │ ~ A	2 R
5	│ ~ ~ A	2–4 ~ I
6	│ ~ ~ A	Assumption
7	│ │ ~ A	Assumption
8	│ │ ~ ~ A	6 R
9	│ │ ~ A	7 R
10	│ A	7–9 ~ E
11	A ≡ ~ ~ A	1–5, 6–10 ≡I

d. Derive: $[(A \supset B) \supset A] \supset A$

1	$(A \supset B) \supset A$	Assumption
2	$\sim A$	Assumption
3	A	Assumption
4	$\sim B$	Assumption
5	A	3 R
6	$\sim A$	2 R
7	B	4–6 ~ E
8	$A \supset B$	3–7 ⊃I
9	A	1, 8 ⊃E
10	$\sim A$	2 R
11	A	2–10 ~ E
12	$[(A \supset B) \supset A] \supset A$	1–11 ⊃I

f. Derive: $(A \equiv B) \equiv [(A \supset B) \& (B \supset A)]$

1	$A \equiv B$	Assumption
2	A	Assumption
3	B	1, 2 ≡E
4	$A \supset B$	2–3 ⊃I
5	B	Assumption
6	A	1, 5 ≡E
7	$B \supset A$	5–6 ⊃I
8	$(A \supset B) \& (B \supset A)$	4, 7 &I
9	$(A \supset B) \& (B \supset A)$	Assumption
10	A	Assumption
11	$A \supset B$	9 &E
12	B	10, 11 ⊃E
13	B	Assumption
14	$B \supset A$	9 &E
15	A	13, 14 ⊃E
16	$A \equiv B$	10–12, 13–15 ≡I
17	$(A \equiv B) \equiv [(A \supset B) \& (B \supset A)]$	1–8, 9–16 ≡I

h. Derive: $[(A \vee B) \supset C] \equiv [(A \supset C) \,\&\, (B \supset C)]$

1	$\quad (A \vee B) \supset C$	Assumption
2	$\quad\quad A$	Assumption
3	$\quad\quad A \vee B$	2 $\vee$I
4	$\quad\quad C$	1, 3 $\supset$E
5	$\quad A \supset C$	2–4 $\supset$I
6	$\quad\quad B$	Assumption
7	$\quad\quad A \vee B$	6 $\vee$I
8	$\quad\quad C$	1, 7 $\supset$E
9	$\quad B \supset C$	6–8 $\supset$I
10	$\quad (A \supset C) \,\&\, (B \supset C)$	5, 9 &I
11	$\quad (A \supset C) \,\&\, (B \supset C)$	Assumption
12	$\quad\quad A \vee B$	Assumption
13	$\quad\quad\quad A$	Assumption
14	$\quad\quad\quad A \supset C$	11 &E
15	$\quad\quad\quad C$	13, 14 $\supset$E
16	$\quad\quad\quad B$	Assumption
17	$\quad\quad\quad B \supset C$	11 &E
18	$\quad\quad\quad C$	16, 17 $\supset$E
19	$\quad\quad C$	12, 13–15, 16–18 $\vee$E
20	$\quad (A \vee B) \supset C$	12–19 $\supset$I
21	$[(A \vee B) \supset C] \equiv [(A \supset C) \,\&\, (B \supset C)]$	1–10, 11–20 $\equiv$I

15. Equivalence

b. Derive: $B \,\&\, {\sim} B$

1	$A \,\&\, {\sim} A$	Assumption
2	$\quad {\sim} (B \,\&\, {\sim} B)$	Assumption
3	$\quad A$	1 &E
4	$\quad {\sim} A$	1 &E
5	$B \,\&\, {\sim} B$	2–4 ~ E

Derive: $A \,\&\, {\sim} A$

1	$B \,\&\, {\sim} B$	Assumption
2	$\quad {\sim} (A \,\&\, {\sim} A)$	Assumption
3	$\quad B$	1 &E
4	$\quad {\sim} B$	1 &E
5	$A \,\&\, {\sim} A$	2–4 ~ E

d. Derive: ~ A ≡ ~ B

```
 1  | A ≡ B                  Assumption
    |------
 2  | | ~ A                  Assumption
    | |------
 3  | | | B                  Assumption
    | | |------
 4  | | | A                  1, 3 ≡E
 5  | | | ~ A                2 R
 6  | | ~ B                  3–5 ~ I
 7  | | ~ B                  Assumption
    | |------
 8  | | | A                  Assumption
    | | |------
 9  | | | B                  1, 8 ≡E
10  | | | ~ B                7 R
11  | | ~ A                  8–10 ~ I
12  | ~ A ≡ ~ B              2–6, 7–11 ≡I
```

Derive: A ≡ B

```
 1  | ~ A ≡ ~ B              Assumption
    |------
 2  | | A                    Assumption
    | |------
 3  | | | ~ B                Assumption
    | | |------
 4  | | | ~ A                1, 3 ≡E
 5  | | | A                  2 R
 6  | | B                    3–5 ~ E
 7  | | B                    Assumption
    | |------
 8  | | | ~ A                Assumption
    | | |------
 9  | | | ~ B                1, 8 ≡E
10  | | | B                  7 R
11  | | A                    8–10 ~ E
12  | A ≡ B                  2–6, 7–11 ≡I
```

f. Derive: (A & ~ B) ∨ (~ A & B)

1	~ (A ≡ B)	Assumption
2	~ [(A & ~ B) ∨ (~ A & B)]	Assumption
3	A	Assumption
4	~ B	Assumption
5	A & ~ B	3, 4 &I
6	(A & ~ B) ∨ (~ A & B)	5 ∨I
7	~ [(A & ~ B) ∨ (~ A & B)]	2 R
8	B	4–7 ~ E
9	B	Assumption
10	~ A	Assumption
11	~ A & B	10, 9 &I
12	(A & ~ B) ∨ (~ A & B)	11 ∨I
13	~ [(A & ~ B) ∨ (~ A & B)]	2 R
14	A	10–13 ~ E
15	A ≡ B	3–8, 9–14 ≡I
16	~ (A ≡ B)	1 R
17	(A & ~ B) ∨ (~ A & B)	2–16 ~ E

Derive: ~ (A ≡ B)

1	(A & ~ B) ∨ (~ A & B)	Assumption
2	A & ~ B	Assumption
3	A ≡ B	Assumption
4	A	2 &E
5	B	3, 4 ≡E
6	~ B	2 &E
7	~ (A ≡ B)	3–6 ~ I
8	~ A & B	Assumption
9	A ≡ B	Assumption
10	B	8 &E
11	A	9, 10 ≡E
12	~ A	8 &E
13	~ (A ≡ B)	9–12 ~ I
14	~ (A ≡ B)	1, 2–7, 8–13 ∨E

16. Inconsistency

b.

b.	1	$B \equiv (A \,\&\, \sim A)$	Assumption
	2	$\sim B \supset (A \,\&\, \sim A)$	Assumption
	3	$\quad B$	Assumption
	4	$\quad A \,\&\, \sim A$	1, 3 ≡E
	5	$\quad \sim A$	4 &E
	6	$\quad A$	4 &E
	7	$\sim B$	3–6 ~ I
	8	$A \,\&\, \sim A$	2, 7 ⊃E
	9	A	8 &E
	10	$\sim A$	8 &E

d.	1	$A \,\&\, (B \vee C)$	Assumption
	2	$(\sim C \vee H) \,\&\, (H \supset \sim H)$	Assumption
	3	$\sim B$	Assumption
	4	$B \vee C$	1 &E
	5	$\quad B$	Assumption
	6	$\quad\quad \sim C$	Assumption
	7	$\quad\quad B$	5 R
	8	$\quad\quad \sim B$	3 R
	9	$\quad C$	6–8 ~ E
	10	$\quad C$	Assumption
	11	$\quad C$	10 R
	12	C	4, 5–9, 10–11 ∨E
	13	$\sim C \vee H$	2 &E
	14	$\quad \sim C$	Assumption
	15	$\quad\quad \sim H$	Assumption
	16	$\quad\quad C$	12 R
	17	$\quad\quad \sim C$	14 R
	18	$\quad H$	15–17 ~ E
	19	$\quad H$	Assumption
	20	$\quad H$	19 R
	21	H	13, 14–18, 19–20 ∨E
	22	$H \supset \sim H$	2 &E
	23	$\sim H$	21, 22 ⊃E

17. Validity

b. Derive: $\sim L \vee F$

1	$L \supset \sim F$	Assumption
2	$L \supset (C \equiv B)$	Assumption
3	$\sim B \,\&\, C$	Assumption
4	$\quad L$	Assumption
5	$\quad C \equiv B$	2, 4 $\supset$E
6	$\quad C$	3 &E
7	$\quad B$	5, 6 $\equiv$E
8	$\quad \sim B$	3 &E
9	$\sim L$	4–8 $\sim$ I
10	$\sim L \vee F$	9 $\vee$I

d. Derive: $(O \,\&\, G) \supset L$

1	$(O \supset L) \vee (\sim L \supset \sim G)$	Assumption
2	$\quad O \,\&\, G$	Assumption
3	$\quad\quad O \supset L$	Assumption
4	$\quad\quad O$	2 &E
5	$\quad\quad L$	3, 4 $\supset$E
6	$\quad\quad \sim L \supset \sim G$	Assumption
7	$\quad\quad\quad \sim L$	Assumption
8	$\quad\quad\quad \sim G$	6, 7 $\supset$E
9	$\quad\quad\quad G$	2 &E
10	$\quad\quad L$	7–9 $\sim$ E
11	$\quad L$	1, 3–5, 6–10 $\vee$E
12	$(O \,\&\, G) \supset L$	2–11 $\supset$I

f. Derive: $L \supset B$

1	$(T \supset A) \,\&\, (\sim T \supset B)$	Assumption
2	$A \supset \sim L$	Assumption
3	$\quad L$	Assumption
4	$\quad\quad T$	Assumption
5	$\quad\quad T \supset A$	1 &E
6	$\quad\quad A$	4, 5 $\supset$E
7	$\quad\quad \sim L$	2, 6 $\supset$E
8	$\quad\quad L$	3 R
9	$\quad \sim T$	4–8 $\sim$ I
10	$\quad \sim T \supset B$	1 &E
11	$\quad B$	9, 10 $\supset$E
12	$L \supset B$	3–11 $\supset$I

h. Derive: J ⊃ R

1	J ⊃ (H ∨ R)	Assumption
2	H ⊃ (B & A)	Assumption
3	B ⊃ (A ⊃ F)	Assumption
4	~ (F & J)	Assumption
5	J	Assumption
6	H ∨ R	1, 5 ⊃E
7	H	Assumption
8	B & A	2, 7 ⊃E
9	B	8 &E
10	A ⊃ F	3, 9 ⊃E
11	A	8 &E
12	F	10, 11 ⊃E
13	~ R	Assumption
14	F & J	12, 5 &E
15	~ (F & J)	4 R
16	R	13–15 ~ E
17	R	Assumption
18	R	17 R
19	R	6, 7–16, 17–18 ∨E
20	J ⊃ R	5–19 ⊃I

18. Inconsistency

b.

1	(~ F ∨ ~ E) ⊃ ~ T	Assumption
2	E ⊃ (T & ~ F)	Assumption
3	E	Assumption
4	T & ~ F	2, 3 ⊃E
5	~ F	4 &E
6	~ F ∨ ~ E	5 ∨I
7	~ T	1, 6 ⊃E
8	T	4 &E

d.	1	(H ≡ ~ T) ≡ D	Assumption
	2	(H ∨ P) ⊃ (D & ~ T)	Assumption
	3	H ≡ ~ P	Assumption
	4	D ⊃ ~ H	Assumption
	5	│ P	Assumption
	6	│ H ∨ P	5 ∨I
	7	│ D & ~ T	2, 6 ⊃E
	8	│ D	7 &E
	9	│ ~ H	4, 8 ⊃E
	10	│ H ≡ ~ T	1, 8 ≡E
	11	│ ~ T	7 &E
	12	│ H	10, 11 ≡E
	13	~ P	5–12 ~ I
	14	H	13, 3 ≡E
	15	H ∨ P	14 ∨I
	16	D & ~ T	15, 2 ⊃E
	17	D	16 &E
	18	~ H	4, 17 ⊃E

19. b. Schematically a use of a rule Negation Introduction can be expressed as

i	│ **P**	Assumption
j	│ **Q**	
k	│ ~ **Q**	
n	~ **P**	**i**–**k** ~ I

We can construct an alternative derivation by inserting four new lines, beginning at line **i** and renumbering the derivation in the obvious way.

i	│ ~ ~ **P**	Assumption
i + 1	│ │ ~ **P**	Assumption
i + 2	│ │ ~ **P**	**i** + 1 R
i + 3	│ │ ~ ~ **P**	**i** R
i + 4	│ **P**	**i** + 1 − **i** + 3 ~ E
j + 4	│ **Q**	
k + 4	│ ~ **Q**	
n + 4	~ **P**	**i** − **k** + 4 ~ E

Notice that ~ ~ **P** has been taken as an assumption at line **i** and **P** has been derived at line **i** + 4. Hence Negation Elimination can be used to derive ~ **P** at line **n** + 4 (where Negation Introduction had been used to derive ~ **P** on line **n** of the former derivation). This routine could be used to replace any use of Negation Introduction in favor of Negation Elimination. Thus Negation Introduction is eliminable.

d. To show that a set is inconsistent in *SD*, both a sentence and its negation must be derived with only members of the set serving as undischarged assumptions. In this case the auxiliary assumption 'B' is not discharged and is not a member of the set {A, B ⊃ ~ A}. In fact, this set is not inconsistent in *SD*.

f. The point here is that even a set that contains only one member can be inconsistent in *SD*. An obvious case is {A & ~ A}.

1	A & ~ A	Assumption
2	A	1 &E
3	~ A	1 &E

20. b. Assume that a sentence **P** of *SL* is a theorem in *SD*. Then, by definition, **P** is derivable in *SD* from the empty set. By *, **P** is truth-functionally entailed by the empty set. By Exercise 2.a of Section 3.6E, then, **P** is truth-functionally true. Assume that a sentence **P** of *SL* is truth-functionally true. By Exercise 2.a of Section 3.6E, **P** is truth-functionally entailed by ∅. By *, it follows that ∅ ⊢ **P** in *SD*. So **P** is a theorem in *SD*.

Section 5.5E

1. Derivability

b. Derive: K ∨ L

1	(H & G) ⊃ (L ∨ K)	Assumption
2	G & H	Assumption
3	H & G	2 Com
4	L ∨ K	1, 3 ⊃E
5	K ∨ L	4 Com

d. Derive: F ∨ N

1	[(K & J) ∨ I] ∨ ~ Y	Assumption
2	Y & [(I ∨ K) ⊃ F]	Assumption
3	Y	2 &E
4	~ ~ Y	3 DN
5	(K & J) ∨ I	1, 4 DS
6	I ∨ (K & J)	5 Com
7	(I ∨ K) & (I ∨ J)	6 Dist
8	I ∨ K	7 &E
9	(I ∨ K) ⊃ F	2 &E
10	F	8, 9 ⊃E
11	F ∨ N	10 ∨I

f. Derive: L ⊃ H

1	~ L ∨ (~ Z ∨ ~ U)	Assumption
2	(U & G) ∨ H	Assumption
3	Z	Assumption
4	L	Assumption
5	L ⊃ (~ Z ∨ ~ U)	1 Impl
6	~ Z ∨ ~ U	4, 5 ⊃E
7	Z ⊃ ~ U	6 Impl
8	~ U	3, 7 ⊃E
9	~ U ∨ ~ G	8 ∨I
10	~ (U & G)	9 DeM
11	H	2, 10 DS
12	L ⊃ H	4–11 ⊃I

2. Validity

b. Derive: ~ E ∨ ~ D

1	(~ A & ~ B) ∨ (~ A & ~ C)	Assumption
2	(E & D) ⊃ A	Assumption
3	~ A & (~ B ∨ ~ C)	1 Dist
4	~ A	3 &E
5	~ (E & D)	2,4 MT
6	~ E ∨ ~ D	5 DeM

d. Derive: F ⊃ J

1	F ⊃ (~ G ∨ H)	Assumption
2	F ⊃ G	Assumption
3	~ (H ∨ I)	Assumption
4	F	Assumption
5	G	2, 4 ⊃E
6	~ G ∨ H	1, 4 ⊃E
7	G ⊃ H	6 Impl
8	H	5, 7 ⊃E
9	~ H & ~ I	3 DeM
10	~ H	9 &E
11	H ∨ J	8 ∨I
12	J	10, 11 DS
13	F ⊃ J	4–12 ⊃I

f. Derive: $\sim G$

1	$G \supset (H \,\&\, \sim K)$	Assumption
2	$H \equiv (L \,\&\, I)$	Assumption
3	$\sim I \vee K$	Assumption
4	$\quad G$	Assumption
5	$\quad H \,\&\, \sim K$	1, 4 $\supset$E
6	$\quad H$	5 &E
7	$\quad L \,\&\, I$	2, 6 $\equiv$E
8	$\quad I$	7 &E
9	$\quad \sim K$	5 &E
10	$\quad \sim \sim I$	8 DN
11	$\quad \sim \sim I \,\&\, \sim K$	10, 9 &I
12	$\quad \sim (\sim I \vee K)$	11 DeM
13	$\quad \sim I \vee K$	3 R
14	$\sim G$	4–13 ~ I

3. Theorems

b. Derive: $\sim \sim \sim \sim \sim (A \,\&\, \sim A)$

1	$\quad A \,\&\, \sim A$	Assumption
2	$\quad A$	1 &E
3	$\quad \sim A$	1 &E
4	$\sim (A \,\&\, \sim A)$	1–3 ~ I
5	$\sim \sim \sim (A \,\&\, \sim A)$	4 DN
6	$\sim \sim \sim \sim \sim (A \,\&\, \sim A)$	5 DN

d. Derive: $[(A \,\&\, B) \supset (B \,\&\, A)] \,\&\, [\sim (A \,\&\, B) \supset \sim (B \,\&\, A)]$

1	$\quad A \,\&\, B$	Assumption
2	$\quad B \,\&\, A$	1 Com
3	$(A \,\&\, B) \supset (B \,\&\, A)$	1–2 $\supset$I
4	$\quad \sim (A \,\&\, B)$	Assumption
5	$\quad \sim (B \,\&\, A)$	4 Com
6	$\sim (A \,\&\, B) \supset \sim (B \,\&\, A)$	4–5 $\supset$I
7	$[(A \,\&\, B) \supset (B \,\&\, A)] \,\&\, [\sim (A \,\&\, B) \supset \sim (B \,\&\, A)]$	3, 6 Conj

f. Derive: $[A \vee (B \vee C)] \equiv [C \vee (B \vee A)]$

1	$\quad A \vee (B \vee C)$	Assumption
2	$\quad (A \vee B) \vee C$	1 Assoc
3	$\quad C \vee (A \vee B)$	2 Com
4	$\quad C \vee (B \vee A)$	3 Com
5	$\quad C \vee (B \vee A)$	Assumption
6	$\quad (B \vee A) \vee C$	5 Com
7	$\quad (A \vee B) \vee C$	6 Com
8	$\quad A \vee (B \vee C)$	7 Assoc
9	$[A \vee (B \vee C)] \equiv [C \vee (B \vee A)]$	1–4, 5–8 $\equiv$I

h. Derive: $(A \vee [B \supset (A \supset B)]) \equiv (A \vee [(\sim A \vee \sim B) \vee B])$

1	$A \vee [B \supset (A \supset B)]$	Assumption
2	$A \vee [(B \,\&\, A) \supset B]$	1 Exp
3	$A \vee [\sim (B \,\&\, A) \vee B]$	2 Impl
4	$A \vee [(\sim B \vee \sim A) \vee B]$	3 DeM
5	$A \vee [(\sim A \vee \sim B) \vee B]$	4 Com
6	$A \vee [(\sim A \vee \sim B) \vee B]$	Assumption
7	$A \vee [(\sim B \vee \sim A) \vee B]$	6 Com
8	$A \vee [\sim (B \,\&\, A) \vee B]$	7 DeM
9	$A \vee [(B \,\&\, A) \supset B]$	8 Impl
10	$A \vee [B \supset (A \supset B)]$	9 Exp
11	$(A \vee [B \supset (A \supset B)]) \equiv (A \vee [(\sim B \vee \sim A) \vee B])$	1–5, 6–10 $\equiv$I

j. Derive: $(\sim A \equiv \sim A) \equiv [\sim (\sim A \supset A) \equiv (A \supset \sim A)]$

1	$\sim A \equiv \sim A$	Assumption
2	$\sim (A \vee A) \equiv \sim A$	1 Idem
3	$\sim (\sim \sim A \vee A) \equiv \sim A$	2 DN
4	$\sim (\sim A \supset A) \equiv \sim A$	3 Impl
5	$\sim (\sim A \supset A) \equiv (\sim A \vee \sim A)$	4 Idem
6	$\sim (\sim A \supset A) \equiv (A \supset \sim A)$	5 Impl
7	$\sim (\sim A \supset A) \equiv (A \supset \sim A)$	Assumption
8	$\sim (\sim \sim A \vee A) \equiv (A \supset \sim A)$	7 Impl
9	$\sim (A \vee A) \equiv (A \supset \sim A)$	8 DN
10	$\sim A \equiv (A \supset \sim A)$	9 Idem
11	$\sim A \equiv (\sim A \vee \sim A)$	10 Impl
12	$\sim A \equiv \sim A$	11 Idem
13	$(\sim A \equiv \sim A) \equiv [\sim (\sim A \supset A) \equiv (A \supset \sim A)]$	1–6, 7–12 $\equiv$I

4. Equivalence

b. Derive: $(B \,\&\, A) \vee (C \,\&\, A)$

1	$A \,\&\, (B \vee C)$	Assumption
2	$(A \,\&\, B) \vee (A \,\&\, C)$	1 Dist
3	$(B \,\&\, A) \vee (A \,\&\, C)$	2 Com
4	$(B \,\&\, A) \vee (C \,\&\, A)$	3 Com

Derive: $A \,\&\, (B \vee C)$

1	$(B \,\&\, A) \vee (C \,\&\, A)$	Assumption
2	$(A \,\&\, B) \vee (C \,\&\, A)$	1 Com
3	$(A \,\&\, B) \vee (A \,\&\, C)$	2 Com
4	$A \,\&\, (B \vee C)$	3 Dist

d. Derive: $\sim A \supset (\sim B \supset C)$

1	$(A \vee B) \vee C$	Assumption
2	$\sim \sim (A \vee B) \vee C$	1 DN
3	$\sim (A \vee B) \supset C$	2 Impl
4	$(\sim A \;\&\; \sim B) \supset C$	3 DeM
5	$\sim A \supset (\sim B \supset C)$	4 Exp

Derive: $(A \vee B) \vee C$

1	$\sim A \supset (\sim B \supset C)$	Assumption
2	$(\sim A \;\&\; \sim B) \supset C$	1 Exp
3	$\sim (A \vee B) \supset C$	2 DeM
4	$\sim \sim (A \vee B) \vee C$	3 Impl
5	$(A \vee B) \vee C$	4 DN

f. Derive: $([(C \vee A) \;\&\; (C \vee B)] \;\&\; [(D \vee A) \;\&\; (D \vee B)]) \vee A$

1	$(A \;\&\; B) \vee [(C \;\&\; D) \vee A]$	Assumption
2	$[(A \;\&\; B) \vee (C \;\&\; D)] \vee A$	1 Assoc
3	$([(A \;\&\; B) \vee C] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	2 Dist
4	$([C \vee (A \;\&\; B)] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	3 Com
5	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	4 Dist
6	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [D \vee (A \;\&\; B)] \vee A$	5 Com
7	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [(D \vee A) \;\&\; (D \vee B)]) \vee A$	6 Dist

Derive: $(A \;\&\; B) \vee [(C \;\&\; D) \vee A]$

1	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [(D \vee A) \;\&\; (D \vee B)]) \vee A$	Assumption
2	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [D \vee (A \;\&\; B)]) \vee A$	1 Dist
3	$([(C \vee A) \;\&\; (C \vee B)] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	2 Com
4	$([C \vee (A \;\&\; B)] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	3 Dist
5	$([(A \;\&\; B) \vee C] \;\&\; [(A \;\&\; B) \vee D]) \vee A$	4 Com
6	$[(A \;\&\; B) \vee (C \;\&\; D)] \vee A$	5 Dist
7	$(A \;\&\; B) \vee [(C \;\&\; D) \vee A]$	6 Assoc

5. Inconsistency

b.

1	$\sim [(\sim C \vee \sim \sim C) \vee \sim \sim C]$	Assumption
2	$\sim (\sim C \vee \sim \sim C) \;\&\; \sim \sim \sim C$	1 DeM
3	$(\sim \sim C \;\&\; \sim \sim \sim C) \;\&\; \sim \sim \sim C$	2 DeM
4	$\sim \sim \sim C$	3 &E
5	$\sim \sim C \;\&\; \sim \sim \sim C$	3 &E
6	$\sim \sim C$	5 &E

d.	1	$B \,\&\, (H \vee Z)$	Assumption
	2	$\sim Z \supset K$	Assumption
	3	$(B \equiv Z) \supset \sim Z$	Assumption
	4	$\sim K$	Assumption
	5	$\sim\sim Z$	2, 4 MT
	6	$\quad B$	Assumption
	7	$\quad Z$	5 DN
	8	$\quad Z$	Assumption
	9	$\quad B$	1 &E
	10	$B \equiv Z$	6–7, 8–9 $\equiv$I
	11	$\sim Z$	3, 10 $\supset$E

f.	1	$[(F \supset G) \vee (\sim F \supset G)] \supset H$	Assumption
	2	$(A \,\&\, H) \supset \sim A$	Assumption
	3	$A \vee \sim H$	Assumption
	4	$\quad \sim H$	Assumption
	5	$\quad \sim [(F \supset G) \vee (\sim F \supset G)]$	1, 4 MT
	6	$\quad \sim (F \supset G) \,\&\, \sim (\sim F \supset G)$	5 DeM
	7	$\quad \sim (F \supset G)$	6 &E
	8	$\quad \sim (\sim F \vee G)$	7 Impl
	9	$\quad \sim\sim F \,\&\, \sim G$	8 DeM
	10	$\quad \sim\sim F$	9 &E
	11	$\quad \sim (\sim F \supset G)$	6 &E
	12	$\quad \sim (\sim\sim F \vee G)$	11 Impl
	13	$\quad \sim\sim\sim F \,\&\, \sim G$	12 DeM
	14	$\quad \sim\sim\sim F$	13 &E
	15	H	4–14 ~ E
	16	$\quad A$	Assumption
	17	$\quad A \,\&\, H$	16, 15 &I
	18	$\quad \sim A$	2, 17 $\supset$E
	19	$\quad A$	16 R
	20	$\sim A$	16–19 ~ I
	21	$\sim H$	20, 3 DS

6. Validity

b. Derive: L

1	M ⊃ (~ B ⊃ L)	Assumption
2	~ B	Assumption
3	M ∨ (~ B ⊃ L)	Assumption
4	(~ B ⊃ L) ∨ M	3 Com
5	~ ~ (~ B ⊃ L) ∨ M	4 DN
6	~ (~ B ⊃ L) ⊃ M	5 Impl
7	~ (~ B ⊃ L) ⊃ (~ B ⊃ L)	6, 1 HS
8	~ ~ (~ B ⊃ L) ∨ (~ B ⊃ L)	7 Impl
9	(~ B ⊃ L) ∨ (~ B ⊃ L)	8 DN
10	~ B ⊃ L	9 Idem
11	L	2, 10 ⊃E

d. Derive: D ⊃ ~ W

1	[D ∨ (S ≡ ~ R)] ⊃ W	Assumption
2	(W ⊃ S) ∨ (W ⊃ ~ D)	Assumption
3	R & ~ S	Assumption
4	(~ W ∨ S) ∨ (W ⊃ ~ D)	2 Impl
5	(~ W ∨ S) ∨ (~ W ∨ ~ D)	4 Impl
6	(S ∨ ~ W) ∨ (~ W ∨ ~ D)	5 Com
7	S ∨ [~ W ∨ (~ W ∨ ~ D)]	6 Assoc
8	~ S	3 &E
9	~ W ∨ (~ W ∨ ~ D)	7, 8 DS
10	(~ W ∨ ~ W) ∨ ~ D	9 Assoc
11	~ W ∨ ~ D	10 Idem
12	~ D ∨ ~ W	11 Com
13	D ⊃ ~ W	12 Impl

f. Derive: (~ I & ~ D) ⊃ ~ A

1	~ P ⊃ ~ (A ∨ G)	Assumption
2	(P & ~ I) ⊃ (O & D)	Assumption
3	~ I & ~ D	Assumption
4	~ D	3 &E
5	~ O ∨ ~ D	4 ∨I
6	~ (O & D)	5 DeM
7	~ (P & ~ I)	2, 6 MT
8	~ P ∨ ~ ~ I	7 DeM
9	~ P ∨ I	8 DN
10	~ I	3 &E
11	~ P	9, 10 DS
12	~ (A ∨ G)	1, 11 ⊃E
13	~ A & ~ G	12 DeM
14	~ A	13 &E
15	(~ I & ~ D) ⊃ ~ A	3–14 ⊃I

h. Derive: [(W & C) & ~ S] ∨ [(S & ~ C) & ~ W]

1	(W ≡ ~ S) & (~ S ≡ C)	Assumption
2	W ≡ ~ S	1 &E
3	~ S ≡ C	1 &E
4	~ [(W & C) & ~ S]	Assumption
5	~ (W & C) ∨ ~ ~ S	4 DeM
6	~ (W & C)	Assumption
7	~ S	Assumption
8	W	2, 7 ≡E
9	C	3, 7 ≡E
10	W & C	8, 9 &I
11	~ (W & C)	6 R
12	S	7–11 ~ E
13	~ ~ S	Assumption
14	S	13 DN
15	S	5, 6–12, 13–14 ∨E
16	W	Assumption
17	~ S	2, 16 ≡E
18	S	15 R
19	~ W	16–18 ~ I
20	C	Assumption
21	~ S	3, 20 ≡E
22	S	15 R
23	~ C	20–22 ~ I
24	S & ~ C	15, 23 &I
25	(S & ~ C) & ~ W	24, 19 &I
26	~ [(W & C) & ~ S] ⊃ [(S & ~ C) & ~ W]	4–25 ⊃I
27	~ ~ [(W & C) & ~ S] ∨ [(S & ~ C) & ~ W]	26 Impl
28	[(W & C) & ~ S] ∨ [(S & ~ C) & ~ W]	27 DN

7. Inconsistency

b.	1	$[\sim C \vee (E \ \& \ P)] \equiv B$	Assumption
	2	$\sim E \supset \sim C$	Assumption
	3	$\sim (P \ \& \ B) \ \& \sim (\sim P \ \& \sim B)$	Assumption
	4	$B \supset C$	Assumption
	5	$\quad B$	Assumption
	6	$\quad \sim C \vee (E \ \& \ P)$	1, 5 ≡E
	7	$\quad C$	4, 5 ⊃E
	8	$\quad \sim \sim C$	7 DN
	9	$\quad E \ \& \ P$	6, 8 DS
	10	$\quad \sim (P \ \& \ B)$	3 &E
	11	$\quad \sim P \vee \sim B$	10 DeM
	12	$\quad \sim \sim B$	5 DN
	13	$\quad \sim P$	11, 12 DS
	14	$\quad P$	9 &E
	15	$\sim B$	5–14 ~ I
	16	$\quad \sim C \vee (E \ \& \ P)$	Assumption
	17	$\quad B$	1, 16 ≡E
	18	$\quad \sim B$	15 R
	19	$\sim [\sim C \vee (E \ \& \ P)]$	16–18 ~ I
	20	$\sim \sim C \ \& \sim (E \ \& \ P)$	19 DeM
	21	$\sim (E \ \& \ P)$	20 &E
	22	$\sim E \vee \sim P$	21 DeM
	23	$\sim (\sim P \ \& \sim B)$	3 &E
	24	$\sim \sim P \vee \sim \sim B$	23 DeM
	25	$\sim \sim P \vee B$	24 DN
	26	$\sim \sim P$	15, 25 DS
	27	$\sim E$	22, 26 DS
	28	$\sim C$	2, 27 ⊃E
	29	$\sim \sim C$	20 &E

8. b. If an argument is valid in *SD,* then the conclusion of the argument is derivable in *SD* from the set of premises. But a derivation in *SD* is also a derivation in *SD+* since all the derivation rules of *SD* are derivation rules of *SD+*. Hence the conclusion is derivable in *SD+* from the set consisting of the premises, and the argument is valid in *SD+*.

Section 6.2E

2. c. (A & ~ B) ∨ (~ A & B)

6. In view of Metatheorem 6.2.1, to show that {'|'} is truth-functionally complete, we need only show that for every sentence containing '&', '∨', or '~' there is a truth-functionally equivalent sentence with the same atomic components containing only the connective '|'. To do this, it suffices to note that $\mathbf{P}|\mathbf{P}$ is truth-functionally equivalent to $\sim \mathbf{P}$, $(\mathbf{P}|\mathbf{P})|(\mathbf{Q}|\mathbf{Q})$ is truth-functionally equivalent to $\mathbf{P} \vee \mathbf{Q}$, and $(\mathbf{P}|\mathbf{Q})|(\mathbf{P}|\mathbf{Q})$ is truth-functionally equivalent to $\mathbf{P}$ & $\mathbf{Q}$.

Since we have shown in exercise 5 that {'↓'} is truth-functionally complete, an alternative approach is to show that for every sentence containing only '↓' there is a truth-functionally equivalent sentence with the same atomic components containing only '|'. This is simple—every sentence of the form $\mathbf{P} \downarrow \mathbf{Q}$ is truth-functionally equivalent to $[(\mathbf{P}|\mathbf{P})|(\mathbf{Q}|\mathbf{Q})]|[(\mathbf{P}|\mathbf{P})|(\mathbf{Q}|\mathbf{Q})]$. Both are true when **P** and **Q** are both false and false otherwise.

Section 6.3E

3. If the sentence at position $\mathbf{k} + 1$, $\mathbf{Q_{k+1}}$, is justified by Negation Elimination, the relevant structure of the derivation is

$$
\begin{array}{r|l|ll}
\mathbf{h} & & \sim \mathbf{Q_{k+1}} & \\ \cline{3-3}
\mathbf{m} & & \mathbf{S} & \\
\mathbf{n} & & \sim \mathbf{S} & \\
\mathbf{k}+1 & \mathbf{Q_{k+1}} & & \mathbf{h}\text{–}\mathbf{n} \sim \mathrm{E}
\end{array}
$$

By the inductive hypothesis $\Gamma_{\mathbf{m}} \vDash \mathbf{S}$ and $\Gamma_{\mathbf{n}} \vDash \sim \mathbf{S}$. Every member of $\Gamma_{\mathbf{n}}$, except possibly $\sim \mathbf{Q_{k+1}}$, is a member of $\Gamma_{\mathbf{k+1}}$ and every member of $\Gamma_{\mathbf{m}}$, except possibly $\sim \mathbf{Q_{k+1}}$, is a member of $\Gamma_{\mathbf{k+1}}$. If follows that every member of $\Gamma_{\mathbf{m}}$ is a member of $\Gamma_{\mathbf{k+1}} \cup \{\sim \mathbf{Q_{k+1}}\}$, as is every member of $\Gamma_{\mathbf{n}}$. By 6.3.2, then, $\Gamma_{\mathbf{k+1}} \cup \{\sim \mathbf{Q_{k+1}}\} \vDash \mathbf{S}$ and $\Gamma_{\mathbf{k+1}} \cup \{\sim \mathbf{Q_{k+1}}\} \vDash \sim \mathbf{S}$. By 6.3.4, $\Gamma_{\mathbf{k+1}} \cup \{\sim \mathbf{Q_{k+1}}\}$ is truth-functionally inconsistent. It follows by 6.3.5 that $\Gamma_{\mathbf{k+1}} \vDash \mathbf{Q_{k+1}}$.

4. b. We need to add a clause for the new rule to the induction in the proof of Metatheorem 6.3.1:

13. If $\mathbf{Q}_{\mathbf{k}+1}$ at position $\mathbf{k} + 1$ is justified by B ⊃ I, then $\mathbf{Q}_{\mathbf{k}+1}$ is a conditional **P** ⊃ **Q** derived as follows:

h			~ **Q**	
j			~ **P**	
k + 1		**P** ⊃ **Q**	**h–j B ⊃I**

By the inductive hypothesis, $\Gamma_{\mathbf{j}} \vDash$ ~ **P**. $\Gamma_{\mathbf{j}}$ is a subset of $\Gamma_{\mathbf{k}+1} \cup$ {~ **Q**} and so, by 6.3.2, $\Gamma_{\mathbf{k}+1} \cup$ {~ **Q**} $\vDash$ ~ **P**. By 6.3.3, $\Gamma_{\mathbf{k}+1} \vDash$ ~ **Q** ⊃ ~ **P**. Because ~ **Q** ⊃ ~ **P** is truth-functionally equivalent to **P** ⊃ **Q**, it follows that $\Gamma_{\mathbf{k}+1} \vDash$ **P** ⊃ **Q**.

d. It will suffice to give an example of a derivation in *SD** that is not truth-preserving:

1		P	Assumption
2			~ P	Assumption
3				~ Q	Assumption
4				P	1 R
5				~ P	2 R
6			Q	3–5 ~ E
7		P ⊃ Q	2–6 C ⊃I

It is straightforward to verify that {P} does not truth-functionally entail ‘P ⊃ Q’.

Section 6.4E

3. The empty set is consistent in *SD* if and only if there is at least one interpretation on which every member of the empty set is true. Because the empty set has no member, it is trivially true that every member of the empty set is true on every interpretation.

5. Proof of c: Assume that $\mathbf{P} \vee \mathbf{Q} \in \Gamma^*$ and that $\mathbf{P} \notin \Gamma^*$. Then, by a, $\sim \mathbf{P} \in \Gamma^*$. But $\mathbf{Q}$ is derivable from $\{\mathbf{P} \vee \mathbf{Q}, \sim \mathbf{P}\}$ as follows:

1	$\mathbf{P} \vee \mathbf{Q}$	Assumption
2	$\sim \mathbf{P}$	Assumption
3	$\quad \mathbf{P}$	Assumption
4	$\quad\quad \sim \mathbf{Q}$	Assumption
5	$\quad\quad \mathbf{P}$	3 R
6	$\quad\quad \sim \mathbf{P}$	2 R
7	$\quad \mathbf{Q}$	4–6 ~ E
8	$\quad \mathbf{Q}$	Assumption
9	$\quad \mathbf{Q}$	8 R
10	$\mathbf{Q}$	1, 3–7, 8–9 ∨E

So, by 6.4.9, $\mathbf{Q} \in \Gamma^*$. Hence, if $\mathbf{P} \vee \mathbf{Q} \in \Gamma^*$, then either $\mathbf{P} \in \Gamma^*$ or $\mathbf{Q} \in \Gamma^*$. Now assume that $\mathbf{P} \in \Gamma^*$ or $\mathbf{Q} \in \Gamma^*$. Since both $\{\mathbf{P}\} \vdash \mathbf{P} \vee \mathbf{Q}$ and $\{\mathbf{Q}\} \vdash \mathbf{P} \vee \mathbf{Q}$ (by ∨ I), it follows from 6.4.9 that $\mathbf{P} \vee \mathbf{Q} \in \Gamma^*$.

Proof of e: Assume that $\mathbf{P} \equiv \mathbf{Q} \in \Gamma^*$. If $\mathbf{P} \in \Gamma^*$, then, since $\{\mathbf{P} \equiv \mathbf{Q}, \mathbf{P}\} \vdash \mathbf{Q}$ (by ≡ E), it follows from 6.4.9 that $\mathbf{Q} \in \Gamma^*$. If $\mathbf{P} \notin \Gamma^*$, then $\sim\mathbf{P} \in \Gamma^*$ by a. But $\{\mathbf{P} \equiv \mathbf{Q}, \sim \mathbf{P}\} \vdash \sim \mathbf{Q}$, as follows:

1	$\mathbf{P} \equiv \mathbf{Q}$	Assumption
2	$\sim \mathbf{P}$	Assumption
3	$\quad \mathbf{Q}$	Assumption
4	$\quad \mathbf{P}$	1, 3 ≡E
5	$\quad \sim \mathbf{P}$	2 R
6	$\sim \mathbf{Q}$	3–5 ~ I

So, by 6.4.9, $\sim \mathbf{Q} \in \Gamma^*$; hence, by a, $\mathbf{Q} \notin \Gamma^*$. If $\mathbf{P} \equiv \mathbf{Q} \in \Gamma^*$, then either $\mathbf{P} \in \Gamma^*$ and $\mathbf{Q} \in \Gamma^*$ or $\mathbf{P} \notin \Gamma^*$ and $\mathbf{Q} \notin \Gamma^*$. Now assume that either $\mathbf{P} \in \Gamma^*$ and $\mathbf{Q} \in \Gamma^*$ or $\mathbf{P} \notin \Gamma^*$ (so that $\sim\mathbf{P} \in \Gamma^*$, by a) and $\mathbf{Q} \notin \Gamma^*$ (so that $\sim \mathbf{Q} \in \Gamma^*$). In either case $\mathbf{P} \equiv \mathbf{Q} \in \Gamma^*$ by 6.4.9, since $\{\mathbf{P}, \mathbf{Q}\} \vdash \mathbf{P} \equiv \mathbf{Q}$, and $\{\sim\mathbf{P}, \sim\mathbf{Q}\} \vdash \mathbf{P} \equiv \mathbf{Q}$, as the following shows:

1	$\mathbf{P}$	Assumption
2	$\underline{\mathbf{Q}}$	Assumption
3	$\mid \underline{\mathbf{P}}$	Assumption
4	$\mid \mathbf{Q}$	2 R
5	$\mid \underline{\mathbf{Q}}$	Assumption
6	$\mid \mathbf{P}$	1 R
7	$\mathbf{P} \equiv \mathbf{Q}$	3–4, 5–6 $\equiv$I

1	$\sim \mathbf{P}$	Assumption
2	$\underline{\sim \mathbf{Q}}$	Assumption
3	$\mid \underline{\mathbf{P}}$	Assumption
4	$\mid \mid \underline{\sim \mathbf{Q}}$	Assumption
5	$\mid \mid \mathbf{P}$	3 R
6	$\mid \mid \sim \mathbf{P}$	1 R
7	$\mid \mathbf{Q}$	4–6 $\sim$ E
8	$\mid \underline{\mathbf{Q}}$	Assumption
9	$\mid \mid \underline{\sim \mathbf{P}}$	Assumption
10	$\mid \mid \mathbf{Q}$	8 R
11	$\mid \mid \sim \mathbf{Q}$	2 R
12	$\mid \mathbf{P}$	9–11 $\sim$ E
13	$\mathbf{P} \equiv \mathbf{Q}$	3–7, 8–12 $\equiv$I

6. Case 3: **P** has the form $\mathbf{Q} \vee \mathbf{R}$. If $\mathbf{Q} \vee \mathbf{R}$ is true on A*, then either **Q** is true on A* or **R** is true on A*. Because **Q** and **R** each contain fewer than $\mathbf{k} + 1$ occurrences of connectives, it follows that either $\mathbf{Q} \in \Gamma^*$ or $\mathbf{R} \in \Gamma^*$. Therefore, by 6.4.11(c), $\mathbf{Q} \vee \mathbf{R} \in \Gamma^*$. If $\mathbf{Q} \vee \mathbf{R}$ is false on A*, then both **Q** and **R** are false on A*. Therefore, by the inductive hypothesis, $\mathbf{Q} \notin \Gamma^*$ and $\mathbf{R} \notin \Gamma^*$. Therefore, by 6.4.11(c), $\mathbf{Q} \vee \mathbf{R} \notin \Gamma^*$.

Case 5: **P** has the form $\mathbf{Q} \equiv \mathbf{R}$. If $\mathbf{Q} \equiv \mathbf{R}$ is true on A*, then either both **Q** and **R** are true on A* or both **Q** and **R** are false on A*. Because **Q** and **R** each contain fewer than $\mathbf{k} + 1$ occurrences of connectives, it follows that either both **Q** and **R** are members of Γ* or neither is. Therefore, by 6.4.11(e), $\mathbf{Q} \equiv \mathbf{R} \in \Gamma^*$. If $\mathbf{Q} \equiv \mathbf{R}$ is false on A*, then either **Q** is true and **R** is false on A*, or **Q** is false and **R** is true on A*. Therefore, by the inductive hypothesis, either **Q** is a member of Γ* and **R** is not, or **R** is a member of Γ* and **Q** is not. Therefore, by 6.4.11(e), $\mathbf{Q} \equiv \mathbf{R} \notin \Gamma^*$.

7. b. By reasoning similar to that given for a, this follows immediately from Exercise 13.b of Section 5.4E.

Section 7.2E

1. b. 'The Speaker of the House' is a singular term; 'Republican' is not.

x is a Republican

d. 'Bob' and 'U.Mass' are the singular terms.

y flunked out of U.Mass.
Bob flunked out of y.
y flunked out of z.

f. The singular terms are 'Oregon', 'Washington', and 'California'.

x is south of Washington and north of California.
Oregon is south of x and north of California.
Oregon is south of Washington and north of x.
x is south of y and north of California.
x is south of Washington and north of y.
Oregon is south of x and north of y.
x is south of y and north of z.

h. The singular terms are 'three', 'four', 'two', and 'six'.

x times 4 equals 2 times 6.
3 times x equals 2 times 6.
3 times 4 equals x times 6.
3 times 4 equals 2 times x.
x times y equals 2 times 6.
x times 4 equals y times 6.
x times 4 equals 2 times y.
3 times x equals y times 6.
3 times x equals 2 times y.
3 times 4 equals x times y.

x times y equals z times 6.
x times y equals 2 times z.
x times 4 equals y times z.
3 times x equals y times z.
x times y equals z times w.

Section 7.3E

2. b. Bct
d. Bds
f. (~ Bak & ~ Bbk) & [(~ Bck & ~ Bdk) & ~ Bek]
h. ~ (Atp ∨ Ath)
j. Lbp ⊃ Ldp
l. (Tbc & Tca) & ~ (Tbe ∨ Tce)
n. ~ [(Tab & Tac) & (Tad & Tae)]
p. Tcb ⊃ Aca

3. b. (Rc & Bc) & ~ Ic
d. ~ [(Rb ∨ Bb) ∨ Ib]
f. [(Ba & Ia) & (Ib & ~ Bb)] & ~ (Ra ∨ Rb)
h. (Laa & Lac) & (Dab & Dad)
j. ~ (Adc ∨ Abc) & (Acd & Acb)
l. (Aab & Aad) & ~ (Lab ∨ Lad)
n. (Abd & Adb) ⊃ (Lbd & Ldb)
p. ~ [(Rc ∨ Bc) ∨ Ic] ⊃ ~ [(Lac ∨ Lbc) ∨ (Lcc ∨ Ldc)]
r. (Rd & Id) & ~ [((Ra & Ia) ∨ (Rb & Ib)) ∨ (Rc & Ic)]

4. b. One appropriate symbolization key is

UD: Charles, Linda, Stan
Sx: x is a sailor
Tx: x is a tennis player
Yx: x is a yuppie
Wx: x wants to be a yuppie
Qx: x is a squash player
Lxy: x likes y
Mxyz: x likes y more than x likes z
c: Charles
l: Linda
s: Stan

(Sc & ~ Tc) & (Sl & Tl)
Yl & (Wc & ~ Yc)
[(Lcc & Llc) & Lsc] & ([(Mclc ∨ Mcsc) & (Mllc ∨ Mlsc)] &
(Mslc ∨ Mssc))
Ys & (Llc & Mlsc)
[(Ss & Ts) & Qs] & (Mssc & Mssl)

Note that we need different predicates for ‘x likes y’ and for ‘x likes y more than x likes z’, the first being a two-place predicate, the second a three-place predicate. Being a yuppie and wanting to be a yuppie are clearly different things (one can be either without being the other), so again we need different predicates. Since there are only three persons in this universe of discourse, for everyone to like someone more than they like Charles is just for each of the three to like either Linda or Stan more than he or she likes Charles.

d. One appropriate symbolization key is

UD:	Joan, Mark, Alice, Randy
Ly:	y is in law school
Ty:	y is studying tax law
My:	y is studying medical malpractice law
Byz:	y gets better grades than z
Fy:	y will finish law school in three years
By:	y will pass the bar exam
Jy:	y will get a job as an attorney
j:	Joan
m:	Mark
a:	Alice
r:	Randy

$(Lj \,\&\, Lm) \,\&\, (La \,\&\, Lr)$
$(Tj \,\&\, Tr) \,\&\, (Mm \,\&\, Ma)$
$Bar \,\&\, Bmj$
$[(Fj \,\&\, Fm) \,\&\, (Fa \,\&\, Fr)] \,\&\, [(Bj \,\&\, Ba) \,\&\, (Br \,\&\, \sim Bm)]$
$[(Jj \,\&\, Ja) \vee (Jj \,\&\, Jr)] \vee (Ja \,\&\, Jr)$

Note that we take 'Everyone but Mark will pass the bar exam' to mean that Mark will not pass that examination. That this is the intent is shown by the last sentence of the passage, 'At least two of the three that pass the bar exam will get jobs as attorneys.'

Section 7.4E

1. b. $(\exists x)Bx$
d. $(\exists z) \sim Bz$
f. $(\forall z)Bz \supset \sim (\exists z)Rz$
h. $\sim (\exists x)Bx \supset (\forall x)Rx$
j. $(\forall y)By \vee (\forall y)Ry$

2. b. $(\exists x)(Ox \,\&\, Ex)$
d. $\sim (\exists y)Ldy$
f. $(\forall x)(Ox \supset Lxd)$
h. $(\exists z)(Pz \,\&\, \sim Ez)$
j. $\sim Pa \,\&\, \sim (\exists x)Lxa$

3. b. $\sim (\exists x)Px \vee Pj$
d. $\sim (\forall z)Pz \,\&\, Pr$
f. $(\exists y)Py \,\&\, \sim (\forall z)Pz$
h. $(\sim Pj \supset \sim (\exists x)Px) \,\&\, (Pj \supset (\forall x)Px)$
j. $(\forall x)Sx \supset (\forall x)Px$

Section 7.5

1. b. Not a formula—'x' cannot occur alone between parentheses in a formula.

d. Not a formula—'$(\exists x)$' is a quantifier, but '(Ex)' is not.

f. Not a formula—the result of prefixing a formula with a quantifier is a formula only if the original formula includes at least one free occurrence of the variable from which the quantifier is formed.

h. Not a formula—the result of prefixing a formula with a quantifier formed from the variable x is a formula only if the original formula does not already contain a quantifier formed from the variable x.

j. Formula and sentence.

l. Formula and sentence.

n. Not a formula—'$(\exists a)$' is not a quantifier.

p. Not a formula—a z-quantifier occurs in '$(Hza \supset (\exists z)Gaz)$'.

2. b. A sentence. The subformulas are

$(\exists x) \sim (\forall y)Byx$	$(\exists x)$
$\sim (\forall y)Byx$	$\sim$
$(\forall y)Byx$	$(\forall y)$
Byx	none

d. A sentence. The subformulas are

$(\forall y)[(\forall z) \sim Byz \vee Byy]$	$(\forall y)$
$(\forall z) \sim Byz \vee Byy$	$\vee$
$(\forall z) \sim Byz$	$(\forall z)$
Byy	none
$\sim Byz$	$\sim$
Byz	none

f. Not a sentence. Both occurrences of 'x' are free. The subformulas are

$Rax \supset \sim (\forall y)Ryx$	$\supset$
Rax	none
$\sim (\forall y)Ryx$	$\sim$
$(\forall y)Ryx$	$(\forall y)$
Ryx	none

h. A sentence. The subformulas are

$(\forall x)(\forall y)(\forall z)Myzz \ \& \ (\forall z)(\forall x)(\forall y)Myzx$	&
$(\forall x)(\forall y)(\forall z)Mxyz$	$(\forall x)$
$(\forall z)(\forall x)(\forall y)Myzx$	$(\forall z)$
$(\forall y)(\forall z)Mxyz$	$(\forall y)$
$(\forall z)Mxyz$	$(\forall z)$
$Mxyz$	none
$(\forall x)(\forall y)Myzx$	$(\forall x)$
$(\forall y)Myzx$	$(\forall y)$
$Myzx$	none

j. A sentence. The subformulas are

$(\forall z)[Fz \supset (\exists w)(\sim Fw \ \& \ Gwaz)]$	$(\forall z)$
$Fz \supset (\exists w)(\sim Fw \ \& \ Gwaz)$	$\supset$
Fz	none
$(\exists w)(\sim Fw \ \& \ Gwaz)$	$(\exists w)$
$\sim Fw \ \& \ Gwaz$	&
$\sim Fw$	~
Fw	none
$Gwaz$	none

l. A sentence. The subformulas are

$\sim [(\forall x)Fx \vee (\forall x) \sim Fx]$	~
$(\forall x)Fx \vee (\forall x) \sim Fx$	$\vee$
$(\forall x)Fx$	$(\forall x)$
Fx	none
$(\forall x) \sim Fx$	$(\forall x)$
$\sim Fx$	~

n. A sentence. The subformulas are

$(\exists w)(Fw \ \& \sim Fw) \equiv (Hc \ \& \sim Hc)$	$\equiv$
$(\exists w)(Fw \ \& \sim Fw)$	$(\exists w)$
$Hc \ \& \sim Hc$	&
$Fw \ \& \sim Fw$	&
Fw	none
$\sim Fw$	~
Hc	none
$\sim Hc$	~

3. b. Quantified sentence
d. Truth-functional compound
f. Atomic sentence
h. Truth-functional compound
j. Truth-functional compound
l. Atomic sentence
n. Truth-functional compound
p. Quantified sentence

4. b. $Mba \supset Maa$
d. $(Laa \ \& \ Lab) \supset Laa$
f. $Fa \ \& \ (\forall y)(Cya \supset Caa)$
h. $(Fa \ \& \ Ha) \supset [(\exists z)(Fz \ \& \ Gz) \supset Ga]$
j. $(\forall y)[(\exists z)Hza \supset (\exists z)Hzy]$
l. $Fa \supset (\exists w)(\sim Fw \ \& \ Gwaa)$
n. $(\exists w)(\exists y)[(Fawy \equiv Fway) \equiv Fyaw]$

5. b. A substitution instance.

d. A substitution instance.

f. Not a substitution instance—the *form* of the open sentence that is the result of dropping the initial quantifier '$(\exists w)$' must not be changed in any of its substitution instances; here '$(\forall y)$' has been changed to '$(\exists y)$'.

h. Not a substitution instance—only the variable 'w' may be replaced.

6. b. Not a substitution instance—'Raa' should be 'Ray'.

d. A substitution instance.

f. Not a substitution instance—'Pya' should be 'Paa'.

h. A substitution instance.

Section 7.6E

1. b.	I-sentence	$(\exists w)(Dw \ \& \ Sw)$
d.	E-sentence	$(\forall x)(Qx \supset \sim Sx)$
f.	A-sentence	$(\forall z)(Nz \supset Kz)$
h.	O-sentence	$(\exists y)(Ny \ \& \sim By)$
j.	O-sentence	$(\exists x)(Qx \ \& \sim Mx)$
l.	E-sentence	$(\forall x)(Px \supset \sim Sx)$
n.	A-sentence	$(\forall w)(\sim Mw \supset Zy)$
p.	O-sentence	$(\exists y)(Py \ \& \sim Iy)$

2. b. $(\forall x)(Rx \supset Sx)$

d. $(\exists w)(Rw \ \& \ Cw)$

f. $(\exists y)Oy \ \& \ (\exists y) \sim Oy$

h. $(\forall w)(Rw \supset Sw) \ \& \ (\forall x)(Gx \supset Ox)$

j. $\sim (\exists z)(Rz \ \& \ Lz)$

l. $(\forall z)Rx \ \& \ [(\exists y)Cy \ \& \ (\exists y) \sim Cy]$

n. $\sim (\forall x)Lx \ \& \ (\forall x)(Lx \supset Bx)$

p. $[(\exists w)Sw \ \& \ (\exists w)Ow] \ \& \sim (\exists w)(Sw \ \& \ Ow)$

3. b. An A-sentence and the corresponding E-sentence of *PL* can both be false. Consider the English sentences 'All positive integers are even' and 'All positive integers are noneven'. Where the UD is positive integers and 'Ex' is interpreted as 'x is even' these become, in *PL*, '$(\forall x)Ex$' and '$(\forall x) \sim Ex$'. Both are false. Some, but not all, positive integers are even.

An A-sentence and the corresponding E-sentence of *PL* can both be true. Consider 'All tiggers are fast' and 'No tiggers are fast'. Where the universe of discourse is mammals, 'Tx' is interpreted as 'x is a tigger', and 'Fx' as 'x is fast', these can be symbolized, respectively, as '$(\forall x)(Tx \supset Fx)$' and '$(\forall x)(Tx \supset \sim Fx)$'. Both are vacuously true, since there are no tiggers. (For the first to be false, there would have to be at least one tigger which is not fast; for the second to be false, there would have to be at least one tigger which is fast. That is, in either case there would have to be at least one tigger. And, again, there are no tiggers.)

Section 7.7E

1. b. $\sim (\exists x)(Px \& Hx)$
d. $(\exists x)(Px \& \sim Hx)$
f. $(\exists y)[(Py \& Hy) \& Dy]$
h. $\sim (\exists z)(Iz \& Hz)$
j. $(\exists z)(Rz \& Hz) \& \sim (\forall z)(Rz \supset Hz)$
l. $(\forall x)[Ix \supset (Px \vee Rx)]$
n. $\sim Ih \& (Ph \& Hh)$
p. $(\forall w)Pw \& \sim (\forall y)Hy$
r. $(\exists w)[(Pw \& Iw) \& Hw] \& \sim (\exists w)[(Pw \& Iw) \& Lw]$

2. b. $(\forall z)(Lz \supset Fz)$
d. $(\exists x)(Lx \& Cxd)$
f. $(\forall x)[(Lx \& Fx) \supset Bx]$
h. $(\exists x)(Lx \& Cxd) \& (\exists x)(Tx \& Cxd)$
j. $(\forall z)([(Lz \vee Tz) \& Fz] \supset Bz)$
l. $Bd \& (\forall x)([(Lx \vee Tx) \& Fx] \supset Bx)$
n. $(\forall y)[Ly \supset (Fy \equiv Cdy)]$
p. $Fd \supset (\forall z)(Lz \supset Tz)$

3. b. $\sim (\exists x)(Ex \& Yx)$
d. $(\exists y)(Ey \& Yy) \& (\exists y)(Ey \& \sim Yy)$
f. $(\forall y)[(Ey \& \sim Yy) \supset \sim Iy]$
h. $(\exists x)(Ex \& Sx) \supset Sf$
j. $\sim (\exists y)(Yy \& \sim Iy)$
l. $(Yf \supset \sim Pf) \& (Pf \supset \sim If)$
n. $\sim (\exists z)[(Pz \& Rzz) \& Yz]$
p. $(\forall x)([Ex \vee Lx) \& Ix] \supset Rxx)$
r. $(\forall z)([Yz \& (Ez \vee Lz)] \supset Rzz)$
t. $(\exists x)[(Yx \& Lx) \& (Ex \& Nx)]$

4. b. $(\forall y)[(Py \& Oy) \supset Uy]$
d. $(\forall z)[Az \supset \sim (Oz \vee Uz)] \& (\forall x)[Px \supset (Ux \& Ox)]$
f. $[(\exists x)(Ax \& Ux) \supset (\forall x)(Px \supset Ux)] \&$
$[(\exists y)(Py \& Uy) \supset (\forall w)(Sw \supset Uw)]$
h. $((\forall w)[Aw \supset (Ow \& \sim Uw)] \& (\forall y)[Sy \supset (Uy \& \sim Oy)]) \&$
$(\forall z)[Pz \supset (Oz \& Uz)]$
j. $((\exists x)[Px \& (Ux \& Sx)] \& (\exists x)[Ax \& (Ox \& Px)]) \&$
$\sim (\exists x)(Ax \& Sx)$

5. b. Neither one nor four is prime.
d. No integer is both even and odd.
f. One is the smallest positive integer.
h. Every prime is larger than one.
j. No integer is both odd and evenly divisible by two.
l. Not every integer is evenly divisible by two.

n. Each integer is such that if it is evenly divisible by two then it is not evenly divisible by three.

p. There is an integer that is both prime and evenly divisible by two.

r. Every prime is larger than one.

Section 7.8E

1. b. $(\exists y)[Sy \,\&\, (Cy \,\&\, \sim Ly)]$
d. $(\forall x)(([Sx \,\&\, Cx] \,\&\, \sim Lx) \supset Yx)$
f. $\sim (\forall z)[(\exists x)(Sx \,\&\, Dzx) \supset Sz]$
h. $(\forall y)[(Sy \,\&\, (\sim Ly \,\&\, Cy)) \supset \sim (\exists x)(Dxy \vee Sxy)]$
j. $(\forall w)[(Sw \,\&\, [(\exists x)Dxy \,\&\, (\exists x)Sxy]) \supset Lw]$
l. $(\forall x)[Wx \supset (Sx \vee (\exists y)[(Dxy \vee Sxy) \,\&\, Sy])]$

2. b. $(\exists x)[Ax \,\&\, (\forall y)(Fy \supset Exy)] \supset (\forall y)(Fy \supset Ejy)$
d. $(\forall y)[(Ay \,\&\, (\forall x)(Fx \supset Eyx)) \supset Ry]$
f. $\sim (\forall w)(Fw \supset Uw) \,\&\, (\forall w)(Uw \supset Fw)$
h. $(\forall x)[(Fx \,\&\, Ax) \supset (\exists y)[(Fy \,\&\, \sim Ay) \,\&\, Exy]]$
j. $(\forall y)((Fy \,\&\, My) \supset (\forall x)[(Fx \,\&\, \sim Mx) \supset Eyx])$
l. $(\exists x)[(Ax \,\&\, Fx) \,\&\, (\forall y)[(Ay \,\&\, \sim Fy) \supset Dxy]]$
n. $(\forall x)(\exists y)Dxy \,\&\, (\forall x)[(\forall y)Dxy \supset (Ax \,\&\, \sim Fx)]$

3. b. $(\forall y)[(Py \,\&\, (\forall z)(Szy \supset Byz)) \supset Dy]$
d. $(\forall y)(Py \supset (\forall z)[Szy \supset (Byz \equiv Bzy)])$
f. $(\forall x)(\forall y)([(Py \,\&\, Sxy) \,\&\, Byx] \supset (Wy \,\&\, Wx))$
h. $\sim (\exists x)[Px \,\&\, (\forall y)Nxy]$
j. $(\forall x)[(Px \,\&\, Ux) \supset (\forall y)[Syx \supset (Bxy \vee Gxy)]]$
l. $(\forall x)(\forall y)([(Py \,\&\, Sxy) \,\&\, \sim Lyx] \supset Wx)$
n. $(\exists z)(Pz \,\&\, \sim Nzt)$
p. $\sim (\exists w)[(Pw \,\&\, Nwt) \,\&\, (\exists y)(Syw \,\&\, Bwy)]$

4. b. Hildegard loves Manfred whenever Manfred loves Hildegard.
d. Siegfried always loves Hildegard.
f. Everyone is unloved at some time.
h. At every time someone is unloved.
j. At every time someone loves someone.
l. Everybody loves somebody sometime.
n. Someone always loves everyone.

5. b. The product of odd integers is odd.
d. Any prime that is larger than a prime is odd.
f. The product of primes, both of which are larger than two, is odd.
h. There is an even prime.
j. No integer is larger than every integer and for every integer there is an integer larger than it.

l. The product of a pair of integers is even if and only if at least one of them is even.

n. Every pair of positive integers is such that if the first is larger than the second, then the second is not larger than the first.

p. The product of any pair of positive integers such that both are prime and three is larger than the first is even.

r. There is an even prime and every prime larger than that prime is odd.

Section 7.9E

1. b. $(\forall x)[(Wx \ \& \sim x = d) \supset (\exists y)[Sy \ \& \ (Dxy \vee Sxy)]]$

d. $Sdj \ \& \ (\forall y)(Syj \supset y = d)$

f. $(\forall y)[(Wy \ \& \sim y = d) \supset Sy]$

h. $(\exists x)[(Sxr \ \& \ Sx) \ \& \ (\forall y)[(Syr \ \& \ Sy) \supset y = x]]$

j. $(\exists x)(\exists y)(([(Sx \ \& \ Sy) \ \& \ (Wx \ \& \ Wy)] \ \& \sim xzy) \ \&$
$(\forall z)[(Wz \ \& \ Sz) \supset (z = x \vee z = y)])$

2. b. There is a smallest positive integer.

d. There is no even positive integer less than 2.

f. There are no prime positive integers such that their product is also prime.

h. The product of any even positive integers is itself even.

j. Every even positive integer is greater than some odd positive integer, and it is not the case that every odd positive integer is greater than some even positive integer.

3. b. Symmetric only

$(\forall x)(\forall y)(Mxy \supset Myx)$

d. Transitive only

$(\forall x)(\forall y)(\forall z)[(Nxy \ \& \ Nyz) \supset Nxz]$

f. Symmetric, transitive, and reflexive (U.D.: physical objects)

$(\forall x)Sxx$
$(\forall x)(\forall y)(Sxy \supset Syx)$
$(\forall x)(\forall y)(\forall z)[(Sxy \ \& \ Syz) \supset Sxz]$

h. Symmetric only (because author A may coauthor one book with author B, another with author C)

$(\forall x)(\forall y)(Cxy \supset Cyx)$

j. Symmetric only

$(\forall x)(\forall y)(Fxy \supset Fyx)$

k. Symmetric, transitive, reflexive (U.D.: physical objects)

$(\forall x)(\forall y)(Wxy \supset Wyx)$
$(\forall x)(\forall y)(\forall z)[(Wxy \& Wyz) \supset Wxz]$
$(\forall x)Wxx$

l. Symmetric only

$(\forall x)(\forall y)(Cxy \supset Cyx)$

n. Symmetric only

$(\forall x)(\forall y)(Cxy \supset Cyx)$

p. Neither reflexive, nor symmetric, nor transitive

4. b. $Sjc \& (\forall x)(Sxc \supset x = j)$
d. $(\exists x)[(Dxd \& (\forall y)(Dyd \supset y = x)) \& Px]$
f. $Pd \& (\exists x)[(Dxd \& (\forall y)[(Dyd \& \sim y = x) \supset Oxy]) \& Px]$
h. $(\exists x)([Sxh \& (\forall y)(Syh \supset y = x)] \& Mcx)$
j. $(\exists x)[(Dxd \& (\forall y)[(Dyd \& \sim y = x) \supset Oxy]) \&$
$(\exists w)[Swh \& ((\forall z)(Szh \supset z = w) \& Mxw)]]$
l. $(\exists x)[(Bx \& (\forall y)(By \supset y = x)) \& (\exists w)(Mxw \& Dwj)]$

5. b. $\sim Pa \& Pf(a)$
d. $(\exists x)[(Px \& Ex) \& (\forall y)[(Py \& Ey) \supset y = x]]$
f. $(\forall y)(Py \supset \sim Pq(y))$
h. $(\forall x)(Ex \supset Of(x))$
j. $(\forall x)(\forall y)[Et(x,y) \supset (Ex \vee Ey)]$
l. $(\forall x)(\forall y)[Es(x,y) \supset [(Ex \& Ey) \vee (Ox \& Oy)]]$
n. $\sim (\exists y)(\exists z)[(Py \& Pz) \& Pt(x,y)]$
p. $(\forall x)[Ex \supset Of(q(x))]$
r. $Pb \& (\forall x)[(Px \& Ex) \supset x = b]$
t. $(\exists x)[(Px \& Pf(x)) \& (\forall y)[(Py \& Pf(y)) \supset y = x]]$

Section 8.1E

1. b. **T**
d. **F**
f. **T**
h. **F**

2. b. **T**
d. **F**
f. **T**
h. **T**

3. b. One interpretation is

UD: Set of positive integers
Dx: x is odd
Fx: x is even
Gx: x is prime
a: 1
b: 3
c: 4

d. One interpretation is

UD: Set of positive integers
Wxy: x is greater than y
Ex: x is even
a: 2
b: 1

f. One interpretation is

UD: Set of positive integers
Fx: x is even
Nx: x is greater than 1
a: 1
b: 2
c: 3

4. b. One interpretation is

UD: Set of museums in New York City
Kx: x is in New York City
Mx: x is an art museum
Gx: x is a natural history museum
a: The Metropolitan Museum of Art
h: The Museum of Natural History

d. One interpretation is

UD: Set of planets
Ixy: x is different from y
a: Venus
p: Pluto

f. One interpretation is

UD: Set of positive integers
Hx: x is prime
Fxy: x equals y
a: 1
b: 2

5. b. One interpretation is

UD: Set of prime numbers
Dx: x is even
Cxy: x is smaller than y
a: 5
b: 3

On this interpretation, '(Caa & Cab) ∨ Da' is false, and '~ Da ≡ ~ (Caa & Cab)' is true.

d. One interpretation is

UD: Set of positive integers
Kxy: x is greater than y
a: 2
c: 1
d: 3

On this interpretation, the first sentence is true and the second is false.

f. One interpretation is

UD: Set of positive integers
Fxy: x plus one equals y
a: 1
b: 3
c: 2

On this interpretation, the first sentence is true and the second is false.

6. b. No. For 'Eab & ~ Eba' to be true on such an interpretation, 'Eab' must be true, and 'Eba' must be false. But if the UD contains only one member, both 'a' and 'b' must be interpreted as designating that one member. If 'Eab' is true on the interpretation, then that member stands in the relation E to itself, so 'Eba' must be true as well—contrary to the requirement that 'Eba' be false.

7. b. True. There are at least two people who are brothers, one of whom is over 40 years old.

d. False. The sentence says that for any persons x and y, x and y are sisters if and only if they are brothers. This is false. For example, the Smith Brothers are brothers, but they are not sisters.

f. True. At least one person w is neither a child nor over forty years old, so '$(\forall w)(Cw \vee Bw)$' is false.

h. False. Not every person who has a sister (and is herself a female) or a brother (and is himself a male) is a child.

j. False. Not every person w who is either a child or over 40 years old is such that there is a person y such that w and y are brothers.

8. b. True. Each U.S. President x is such that for every U.S. President y, <u>if</u> x is female and y is the first U.S. President (which is false since no U.S. President is female), <u>then</u> y held office after x's first term of office.

d. True. The first conjunct, '$(\exists x)Ax$ & ~ $(\exists z)Bz$', is true, for at least one U.S. President was the first U.S. President, and no U.S. President is female. The second conjunct, '$Ag \supset (\forall y)Uy$', is true because the antecedent and the consequent are both true.

f. True. Since no U.S. Presidents are female and all U.S. Presidents are U.S. citizens, each U.S. President w is such that w is female if and only if w is not a U.S. citizen.

h. False. '$(\exists x)(Ax \ \& \ Bx)$' is false, because the (only) first U.S. President was not female; but '$(\forall y)(Ay \supset Uy)$' is true, because the (only) first U.S. President was a U.S. citizen.

j. True. Because the first U.S. President was not female, every U.S. President y trivially satisfies the conditional that <u>if</u> y is the first U.S. President and is female, <u>then</u> George Washington held office after y's first term of office.

9. b. False. The number 2 is not equal to 3, but it is not greater than 3 either. So there are at least one positive integer x and one positive integer z such that it is *not* the case that x is equal to z if and only if x is greater than z.

d. True. For every positive integer x, there are a positive integer w and a positive integer z such that w is greater than x and z is the sum of x and w.

f. True. Every positive integer is either equal to or greater than 1.

h. False. Not every positive integer y is such that <u>if</u> 2 is even and some positive integer is equal to 2 (the conjunction is true), <u>then</u> y equals $3 - 2$.

j. False. There is no even positive integer x that is greater than 3 and that is such that no positive integer equals $x - 3$.

Section 8.2E

1. b. The sentence is false on the following interpretation:

UD: Set of positive integers
Fx: x is odd
Gx: x is positive

At least one positive integer is odd or positive, and at least one positive integer is odd, but there is no positive integer that is not positive.

d. The sentence is false on the following interpretation:

UD: Set of positive integers
Fxy: x is greater than y
Gx: x is odd
b: 1

Every positive integer either is greater than 1 or is odd, but it is not the case that either every positive integer is greater than 1 or every positive integer is odd.

f. The sentence is false on the following interpretation:

UD: Set of positive integers
Ax: x is negative
Bx: x is odd

The antecedent, '$(\forall x)(Ax \supset (\forall y)By)$', is trivially true, for no positive integer is negative. But the consequent is false. Since some positive integers are odd but no positive integer is negative, no positive integer y is such that <u>if</u> y is odd <u>then</u> every positive integer is negative.

2. b. The sentence is true on the following interpretation:

UD: Set of positive integers
Fx: x is odd

At least one positive integer is odd, and at least one positive integer is not odd.

d. The sentence is true on the following interpretation:

UD: Set of positive integers
Fx: x is negative

Every positive integer x is such that <u>if</u> some positive integer is negative (which is false), <u>then</u> x is not negative.

f. The sentence is true on the following interpretation:

UD: Set of positive integers
Dxy: x is greater than y

There is at least one positive integer x such that x is not greater than any positive integer that is greater than x (in fact, every integer satisfies this condition).

h. The sentence is true on the following interpretation:

UD: Set of positive integers
Dxy: x is less than y

3. b. The sentence is true on the following interpretation:

UD: Set of positive integers
Fx: x is positive
Cx: x is odd

Some positive integer is positive, and every odd positive integer is positive.
The sentence is false on the following interpretation:

UD: Set of positive integers
Fx: x is odd
Cx: x is positive

Some positive integer is odd, but not every positive integer that is positive is odd.

d. The sentence is true on the following interpretation:

UD: Set of positive integers
Fx: x is positive
Gx: x is less than 1

The antecedent, '$(\exists x)(Fx \supset Gx)$', is false on this interpretation. Since every positive integer is positive but no positive integer is less than 1, no positive integer is such that if x is positive, then x is less than 1.
The sentence is false on the following interpretation:

UD: Set of positive integers
Fx: x is less than 1
Gx: x is odd

The antecedent, '$(\exists x)(Fx \supset Gx)$', is trivially true, as no positive integer is less than 1. That is, each positive integer x is such that if x is less than 1 (which x is not), then x is odd. And the consequent is false, for no positive integer is both less than 1 and odd.

f. The sentence is true on the following interpretation:

UD: Set of positive integers
Mx: x is even
a: 2
b: 4

2 and 4 are both even, and at least one positive integer is not even.
The sentence is false on the following interpretation:

UD: Set of positive integers
Mx: x is even
a: 3
b: 4

The conjunct 'Ma' is false, since 3 is not even.

h. The sentence is true on the following interpretation:

UD: Set of positive integers
Gx: x is negative
Hx: x is prime

The first disjunct, for example, is true—no positive integer is negative.
The sentence is false on the following interpretation:

UD: Set of positive integers
Gx: x is even
Hx: x is prime

At least one positive integer is prime, and at least one is even, and not all positive integers are both prime and even.

4. b. If the antecedent is true on an interpretation, then either every object in the UD is in the extension of 'F' or every object in the UD is in the extension of 'G'. Either way, every object in the UD is either F or G, so the consequent is also true. If the antecedent is false on an interpretation, then the conditional is trivially true. So the sentence is true on every interpretation.

d. Assume that the antecedent is true on some interpretation. Then every member of the UD stands in the relation M to at least one member of the UD. Because a UD must be nonempty, it follows that at least one pair of members of the UD is in the extension of 'M'. So on every interpretation on which the antecedent is true, the consequent is true as well.

5. b. For the sentence to be true, all three conjuncts must be true. But if '$(\exists w)Fw$' and '$(\forall w)(Fw \supset \sim Gw)$' are both true on an interpretation, then some member w of the UD is in the extension of 'F' and not in the extension of 'G' (since no F is G). But then '$(\forall w)(Fw \supset Gw)$' is false, for at least one member of the UD is F but not G. So the three conjuncts can't all be true on the same interpretation. Hence the sentence is false on every interpretation.

d. If the first conjunct is true on an interpretation, then at least one member of the UD is in the extension of 'F' but not in the extension of 'G'. This member thus fails to satisfy '$Fx \supset Gx$', so the second conjunct must be false. Therefore there is no interpretation on which both conjuncts are true.

6. b. The sentence is quantificationally indeterminate. It is true on the interpretation

UD: Set of positive integers
Gx: x is odd
Hx: x is even

because at least one positive integer is odd and at least one is even, but no positive integer is both odd and even.

The sentence is false on the following interpretation:

UD: Set of positive integers
Gx: x is less than zero
Hx: x is even

The first conjunct is false on this interpretation.

d. The sentence is quantificationally true. If the antecedent is true, then every member of the UD is F, and so every member of the UD is either F or G, so there is no member of the UD that fails to be either F or G. Therefore it is impossible for the antecedent to be true and the consequent false.

f. The sentence is quantificationally true. If there is a member of the UD to which everything that is D bears the relation H, then everything that is D is such that there is a member of the UD to which it bears the relation H.

Section 8.3E

1. b. The first sentence is true and the second false on the following interpretation:

UD: Set of positive integers
Fx: x is a multiple of 2
Gx: x is an odd number

Some positive integer is a multiple of 2 and some positive integer is odd, but no positive integer is both a multiple of 2 and odd.

d. The first sentence is true and the second false on the following interpretation:

UD: Set of positive integers
Fx: x is a negative number
Gx: x is an odd number
a: 3
b: 2

Every positive integer x is such that either x is negative or 3 is odd, since the latter is true; but no positive integer x is such that either x is negative (which it is not) or 2 is odd (which it is not).

f. The first sentence is false and the second true on the following interpretation:

UD: Set of positive integers
Fx: x is even
Gx: x is prime

Some even positive integers are not prime, so '$(\forall x)(Fx \supset Gx)$' is false. But every positive integer y is trivially such that if *every positive integer* is even (which is false), then y is prime.

h. The first sentence is true and the second false on the following interpretation:

UD: Set of positive integers
My: x is odd
Ny: x is negative

The integer 2 is odd if and only if it is negative; but it is not the case that at least one positive integer is odd if and only if at least one positive integer is negative.

2. b. Suppose that '$(\forall x)(Fx \supset Gx)$' is true on some interpretation. Then every member of the UD which is F is also G. So no member is both F and not G; hence '$\sim (\exists x)(Fx \,\&\, \sim Gx)$' is true.

Suppose that '$(\forall x)(Fx \supset Gx)$' is false on some interpretation. Then some member of the UD is F but is not G. So '$(\exists x)(Fx \,\&\, \sim Gx)$' is true and '$\sim (\exists x)(Fx \,\&\, \sim Gx)$' is false.

d. Suppose that '$(\forall x)(\forall y)(Mxy \,\&\, Myx)$' is true on an interpretation. Then every pair x and y of members of the UD satisfies 'Mxy & Myx'—so every such pair is in the extension of 'M'. Therefore, no pair x and y satisfies '$\sim Mxy \vee \sim Myx$', and so '$\sim (\exists x)(\exists y)(\sim Mxy \vee \sim Myx)$' is true as well.

If the first sentence is false on an interpretation, then some pair x and y of members of the UD fails to satisfy 'Mxy & Myx'. So either the pair x,y (in that order) is not in the extension of ‘M’ or the pair y,x is not in the extension of ‘M’. Therefore x and y satisfy ‘~ Mxy ∨ ~ Myx’, so the second sentence is false as well.

f. ‘$(\forall x)(\forall y)(Fxy \supset Hyx)$’ tells us that every x which Fs a y is H-ed by that y. This makes it true that $\sim (\exists x)(\exists y)(Fxy \& \sim Hyx)$, for this statement says that there is no pair x and y such that x Fs y but y doesn’t Hx. The same reasoning runs in the reverse.

3. b. The sentences are not quantificationally equivalent. The first sentence is false and the second true on the following interpretation:

UD: Set of positive integers
Fx: x is odd
Gx: x is even

It is false that there is a positive integer that is both even and odd, but it is true that not every positive integer fails to be even or odd (i.e., it is true that at least one positive integer *is* even or odd).

d. The sentences are not quantificationally equivalent. The first sentence is false and the second true on the following interpretation:

UD: Set of positive integers
Hxy: x is greater than y

It is false that every positive integer is such that if some positive integer is greater than y then y is greater than itself; but it is true that for every positive integer y there is at least one positive integer z such that if z is greater than itself (which is never true) then z is greater than y.

4. b. All the set members are true on the following interpretation:

UD: Set of positive integers
Fx: x is prime
Gx: x is even

At least one positive integer is prime or at least one positive integer is even, at least one positive integer is not prime, and at least one positive integer is not even.

d. All the set members are true on the following interpretation:

UD: Set of positive integers
Dxy: x is evenly divisible by y
Bxy: x is equals y
a: 1
b: 2

The first sentence is true, for every positive integer x is such that 1 is evenly divisible by x if and only if 1 equals x. The second sentence is true, for 1 is not evenly divisible by 2, and the third sentence is true, for 2 does not equal 1.

f. All the set members are true on the following interpretation:

UD: Set of positive integers
Fx: x is odd
Bxy: x is greater than y

At least one positive integer is odd, every odd positive integer w is such that at least one positive integer is greater than w, and no positive integer is greater than itself.

h. All the set members are true on the following interpretation:

UD: Set of positive integers
Bx: x is negative
Cxy: x is greater than y

Every positive integer is negative if and only if it is greater than all positive integers, no positive integer is negative, and at least one positive integer is greater than some positive integer.

5. b. If the set is quantificationally consistent, then there is an interpretation on which both of the set members are true. But, if '$(\exists x)(\exists y)(Bxy \vee Byx)$' is true on an interpretation, then at least one member x of the UD is such that either x stands in the relation B to at least one member of the UD or at least one member of the UD stands in the relation B to x. Either way, it follows that there is at least one member of the UD that stands in the relation B to some member of the UD. Hence '$(\exists x)(\exists y)Bxy$' is true, and '$\sim (\exists x)(\exists y)Bxy$' is false. There is no interpretation on which both set members are true.

d. If the first set member is true on an interpretation, then the individual denoted by 'a' is in the extension of 'B'. If the second is true as well, then the individual denoted by 'a' stands in the relation D to at least one member of the UD. In this case, the individual denoted by 'a' fails to satisfy '$Bx \supset (\forall y) \sim Dxy$', because it satisfies the antecedent but not the consequent. Therefore on such an interpretation the third set member must be false. We conclude that there is no interpretation on which all three set members are true.

f. If the first sentence is true on an interpretation, then either everything in the UD is F or nothing in the UD is F. But then there cannot be members y and x of the UD such that y is F and x isn't, so the second sentence will be false. Thus there can be no interpretation on which both members of the set are true.

6. b. The set is quantificationally consistent, as the following interpretation shows:

UD: Set of positive integers
Gxy: x is greater than y

There is at least one pair of positive integers such that one is greater than the other, and no positive integer is greater than itself. Thus there is at least one interpretation on which both members of the set are true.

d. The set is quantificationally consistent, as the following interpretation shows:

UD: Set of positive integers
Px: x is even
Hxy: x is greater than y
a: the number 1

At least one positive integer is even, every even positive integer is greater than 1, and at least one positive integer is greater than 1, so it is true that not all positive integers fail to be greater than 1. Thus there is at least one interpretation on which both set members are true.

Section 8.4E

1. b. The set members are true and '~ Fb' false on the following interpretation:

UD: Set of positive integers
Fy: y is greater than −4
a: 3
b: 4

Since every positive integer is greater than −4 and 'Fa' is true, every positive integer satisfies the condition specified by '(∀y)(Fy ≡ Fa)'. The sentence 'Fa' is true and '~ Fb' is false, for both 3 and 4 are greater than −4.

d. The set members are true and '(∀x)Cx' false on the following interpretation:

UD: Set of positive integers
Bx: x is even
Cx: x is greater than 1

Every even positive integer is greater than 1, and there is at least one positive integer that is even, but not all positive integers are greater than 1.

f. The set members are true and ‘$(\forall x)(Hx \supset Gx)$’ false on the following interpretation:

UD: Set of positive integers
Fx: x is even
Gx: x is divisible by 2
Hx: x is odd

Every even positive integer is divisible by 2, no odd positive integer is even, and no odd positive integer is divisible by 2.

h. The set members are true and ‘$(\exists x)(\forall y)Jxy$’ false on the following interpretation:

UD: Set of positive integers
Hxy: x times y equals y
Jxy: x is greater than y

The number 1 multiplied by any number equals that number, so 1 satisfies ‘$(\forall y)(Hxy \vee Jxy)$’ and the first sentence is true. All positive integers other than 1 satisfy ‘$(\forall y) \sim Hxy$’, so the second sentence is true. But no positive integer is greater than every positive integer.

2. b. The premises are true and the conclusion false on the following interpretation:

UD: Set of positive integers
Fx: x is negative
a: 1

The second disjunct of the premises is true, and, since no positive integer is negative, ‘$(\exists z)Fz$’ is false.

d. The premises are true and the conclusion false on the following interpretation:

UD: Set of positive integers
Fx: x is odd
Gx: x is positive
a: 2

Every odd positive integer is positive and 2 is positive, but 2 is not odd.

f. The premises are true and the conclusion false on the following interpretation:

UD: Set of positive integers
Mxy: x squared equals y
Nxy: x is less than or equal to y

Every pair of positive integers x and y is such that if x squared equals y, then x is less than or equal to y, but no pair of positive integers other than the pair consisting of 1 and itself is such that if x squared equals y then x and y are each less than or equal to the other.

h. The premises are true and the conclusion false on the following interpretation:

UD:	set of positive integers
Fx:	x is odd
Gxy:	x is less than y
a:	1
b:	3

Because 1 and 3 are both odd, the first disjunct of each premise is true. But the conclusion, which says that some positive integer is less than 1, is false.

3. b. A symbolization of the first argument is

$$\frac{\sim (\forall x)Dx}{(\exists x) \sim Dx}$$

To see that this argument is quantificationally valid, assume that '$\sim (\forall x)Dx$' is true on some interpretation. Then '$(\forall x)Dx$' is false. But then some member of the UD doesn't satisfy the condition specified therein; that is, some member of the UD can't dance. Then '$(\exists x) \sim Dx$' is true as well.

A symbolization of the second argument is

$$\frac{(\forall x)(Rx \supset \sim Dx)}{(\exists x)(Rx \,\&\, \sim Dx)}$$

The following interpretation makes the premise true and the conclusion false:

UD:	Set of positive integers
Rx:	x is negative
Dx:	x is prime

d. A symbolization of the first argument is

$$\frac{(\exists x)Ex \,\&\, (\exists x)Dx}{(\exists x)Ex \equiv (\exists x)Dx}$$

To see that this argument is quantificationally valid, assume that the premise is true on some interpretation. Then both conjuncts are true. But the conjuncts are the immediate components of the biconditional conclusion, so the conclusion is true as well.

A symbolization of the second argument is

$$\frac{(\forall x)(Ex \equiv Dx)}{(\exists x)Ex}$$

The premise is true and the conclusion false on the following interpretation:

UD: Set of positive integers
Ex: x is a negative number
Dx: x equals 0

Since no positive integer is either a negative number or equal to 0, the condition specified in the premise is satisfied by every positive integer. The conclusion is obviously false.

f. A symbolization of the first argument is

$$\begin{array}{l} (\forall x)[(Sx \ \& \sim Rx) \supset Dx] \\ (\exists x)(Sx \ \& \sim Rx) \\ \hline (\exists x)(Sx \ \& \ Dx) \end{array}$$

To see that this argument is quantificationally valid, assume that the premises are true on some interpretation. By the second premise, some member of the UD is in the extension of 'S' but not 'R'. By the first premise, this member must also be in the extension of 'D'. Therefore this member satisfies both conjuncts of '(Sx & Dx)', so the conclusion is true as well.

A symbolization of the second argument is

$$\frac{(\exists x)(Sx \ \& \sim Rx)}{(\exists x)(Sx \ \& \ Dx)}$$

The premise is true and the conclusion false on the following interpretation:

UD: Set of positive integers
Sx: x is prime
Rx: x is odd
Dx: x is negative

The number 2 is both prime and not odd. But no positive integer is negative, so the conclusion is false.

4. b. The argument is quantificationally valid. If the first premise is true on an interpretation, then every member of the UD that is in the extension of 'S' is either in the extension of 'G' or in the extension of 'B'. If the second premise is also true, then at least one member of the UD is in the extension of 'S' but not in the extension of 'B'. From the first premise, it follows that

that member must therefore be in the extension of 'G'. So the conclusion is true, since at least one member of the UD is in the extension of 'G'.

d. The argument is quantificationally valid. If the first premise is true on an interpretation, then there is a pair of members of the UD such that one is in the extension of 'R', the other is in the extension of 'S', and the pair is in the extension of 'P'. If the second is true, then the member of the previous pair that is in the extension of 'R' is also in the extension of 'T'. So there is a pair of members of the UD such that the first is in the extension of 'T' and the pair is in the extension of 'P'.

Section 8.5E

1. b. Faa & Ca
d. Daa ≡ Faa
f. ~ (Ca ∨ Ba)
h. (Daa ∨ Faa) ⊃ Ba

2. Remember that, in expanding a sentence containing the individual constant 'g', we must use that constant.

b. [((Ba & Aa) ⊃ Daa) & ((Ba & Ab) ⊃ Dba)] & [((Bb & Aa) ⊃ Dab) & ((Bb & Ab) ⊃ Dbb)]

d. [(Aa ∨ Ag) & ~ (Ba ∨ Bg)] & [Ag ⊃ (Ua & Ug)]

f. (Ba ≡ ~ Ua) & (Bb ≡ ~ Ub)

h. [(Aa & Ba) ∨ (Ab & Bb)] ≡ [(Aa ⊃ Ua) & (Ab ⊃ Ub)]

3. b. [(~ Eaa ≡ Gaa) & (~ Eab ≡ Gab)] & [(~ Eba ≡ Gba) & (~ Ebb ≡ Gbb)]

d. [(Gaa & (Maaa ∨ Mbaa)) ∨ (Gba & (Maab ∨ Mbab))] & [(Gab & (Maba ∨ Mbba)) ∨ (Gbb & (Mabb ∨ Mbbb))]

f. (Eaa ∨ Gaa) & (Eba ∨ Gba)

h. [(Bb & (Ebb ∨ Ecb)) ⊃ Mcbb] & [(Bb & (Ebb ∨ Ecb)) ⊃ Mcbc]

4. b. Na ∨ ((Ba ∨ Bb) ∨ Bc)
d. ((Ba & Bb) & Bc) ∨ ~ ((Ba ∨ Bb) ∨ Bc)

6. b. One assignment to its atomic components for which the expansion

(Fa ∨ Fn) ≡ (Fa & Fn)

is true is

Fa	Fn	(Fa	∨	Fn)	↓ ≡	(Fa	&	Fn)
F	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**

Using this information, we shall construct an interpretation with a two-member UD such that neither member is in the extension of ‘F’ (since ‘Fa’ and ‘Fn’ are both false):

UD: Set consisting of Germany and Italy
Fx: x is in North America

Both ‘(∃x)Fx’ and ‘(∀x)Fx’ are false on this interpretation, so ‘(∃x)Fx ≡ (∀x)Fx’ is true.

7. b.

					↓				
Fa	Ga	(Fa	∨	Ga)	⊃	(Fa	⊃	~	Ga)
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**

d.

											↓							
Fab	Fbb	Ga	Gb	[(Fab	∨	Ga)	&	(Fbb	∨	Gb)]	⊃	([Fab	&	Fbb)	∨	(Ga	&	Gb)]
T	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**T**

f.

Aa	Ab	Ba	Bb	[(Aa	⊃	(Ba	&	Bb))	&	(Ab	⊃	(Ba	&	Bb))]
F	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**

↓											
⊃	[(Ba	⊃	(Aa	&	Ab))	&	(Bb	⊃	(Aa	&	Ab))]
F	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**F**	**F**	**F**	**F**

h.

					↓			
Ba	Ha	(Ba	≡	Ha)	⊃	(Ba	&	Ha)
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**

8. b.

					↓					
Fa	Fb	(Fa	∨	Fb)	&	(~	Fa	∨	~	Fb)
T	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**

d.

		↓		
Fa	Fa	⊃	~	Fa
F	**F**	**T**	**T**	**F**

f.

		↓		
Daa	Daa	⊃	~	Daa
F	**F**	**T**	**T**	**F**

h.

		↓		
Daa	Daa	∨	~	Daa
T	**T**	**T**	**F**	**T**

9. b. Expansion showing that the sentence is true on at least one interpretation:

			↓			
Ca	Fa	Fa	⊃	(Ca	⊃	Fa)
T	**T**	**T**	**T**	**T**	**T**	**T**

Expansion showing that the sentence is false on at least one interpretation:

							↓							
Ca	Cb	Fa	Fb	(Fa	∨	Fb)	⊃	[(Ca	⊃	Fa)	&	(Cb	⊃	Fb)]
T	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**F**

d. Expansion showing that the sentence is true on at least one interpretation:

					↓			
Fa	Ga	(Fa	⊃	Ga)	⊃	(Fa	&	Ga)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**

Expansion showing that the sentence is false on at least one interpretation:

					↓			
Fa	Ga	(Fa	⊃	Ga)	⊃	(Fa	&	Ga)
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**

f. Expansion showing that the sentence is true on at least one interpretation:

						↓								
Ma	Mb	Mc	(Ma	&	Mb)	&	[(~	Ma	∨	~	Mb)	∨	~	Mc]
T	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**F**

Expansion showing that the sentence is false on at least one interpretation:

					↓					
Ma	Mb	(Ma	&	Mb)	&	(~	Ma	∨	~	Mb)
T	**T**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**F**	**T**

h. Expansion showing that the sentence is true on at least one interpretation:

Ga	Ha	[~	Ha	∨	~	Ga]	↓ ∨	(Ha	&	Ga)
T	**T**	**F**	**T**	**F**	**F**	**T**	**T**	**T**	**T**	**T**

Expansion showing that the sentence is false on at least one interpretation:

Ga	Gb	Ha	Hb	[~	(Ha	∨	Hb)	∨	~	(Ga	∨	Gb)]
F	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**

↓ ∨	((Ha	&	Ga)	&	(Hb	&	Gb))
F	**T**	**F**	**F**	**F**	**F**	**F**	**T**

10. No. The sentence ‘Fa ⊃ (Gb ⊃ Fa)’ is a truth-functional compound, containing two atomic components. There are only two truth-values that each of these may have on any interpretation. So we may construct a truth-table for the sentence to show that on every truth-value assignment the sentence is true. This will show that the sentence is true on every interpretation and is therefore quantificationally true. And, since a sentence that contains no quantifiers is its own truth-functional expansion, we shall thus have shown by the method of truth-functional expansions that the sentence is quantificationally true.

12. b.

Fa	Fb	Ga	Gb	(Fa	∨	Fb)	↓ &	(Ga	∨	Gb)	(Fa	&	Ga)	↓ ∨	(Fb	&	Gb)
T	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**

d.

Fa	Fb	Ga	Gb	(Fa	∨	Ga)	↓ ∨	(Fb	∨	Ga)	(Fa	∨	Gb)	↓ ∨	(Fb	∨	Gb)
F	**F**	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**T**

f.

Fa	Fb	Ga	Gb	(Fa	⊃	Ga)	↓ &	(Fb	⊃	Gb)	((Fa	&	Fb)	⊃	Ga)	↓ &	((Fa	&	Fb)	⊃	Gb)
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**	**F**	**T**	**T**	**F**	**F**	**T**	**T**

h.

Ma	Mb	Na	Nb	(Ma	≡	Na)	↓ ∨	(Mb	≡	Nb)	(Ma	∨	Mb)	↓ ≡	(Na	∨	Nb)
T	**F**	**F**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**

13. b.

							↓						↓					↓		
Fa	Fb	Ga	Gb	(Fa	∨	Fb)	∨	(Ga	∨	Gb)	~	Fa	∨	~	Fb	~	Ga	∨	~	Gb
F	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**	**T**	**F**	**T**

d.

								↓				↓		↓	
Baa	Bab	Bba	Daa	Dab	(Daa	≡	Baa)	&	(Dab	≡	Bab)	~	Dab	~	Bba
T	**F**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

f.

							↓						
Baa	Bab	Bba	Bbb	Fa	Fb	Fa	∨	Fb	[Fa	⊃	(Baa	∨	Bba)]
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**

↓								↓		
&	[Fb	⊃	(Bab	∨	Bbb)]	~	Baa	&	~	Bbb
T	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**F**

h.

											↓					
Ba	Bb	Caa	Cab	Cba	Cbb	[Ba	≡	(Caa	&	Cab)]	&	[Bb	≡	(Cba	&	Cbb)]
F	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**	**T**

		↓						↓			
~	Ba	∨	~	Bb	(Caa	∨	Cab)	∨	(Cba	∨	Cbb)
T	**F**	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**

14. a. Yes. For example, all the set members are true on the following interpretation:

UD: set of positive integers
Bx: x is odd
a: 1
b: 3
c: 5
d: 7
e: 9
f: 11
g: 13

b. The minimum size is eight. We must expand the sentence for a set containing at least seven constants, for there are seven distinct individual constants occurring in the sentences in the set. But if we expand the sentences for exactly those seven constants, on every truth-value assignment at least one of the expanded sentences will be false. For suppose that ‘Ba’, ‘Bb’, ‘Bc’, ‘Bd’, ‘Be’, ‘Bf’, and ‘Bg’ are all true on some truth-value assignment. Then the iterated conjunction of these sentences will be true, so the expansion of ‘~ (∀x)Bx’

~ ((((((Ba & Bb) & Bc) & Bd) & Be) & Bf) & Bg)

will be false.

But, if we expand the sentences for an eight-constant set, using the additional individual constant ‘h’, then the expansions of all the set members are true for the following partial assignment:

Ba	Bb	Bc	Bd	Be	Bf	Bg	Bh	Ba	Bb	Bc	Bd	Be	Bf	Bg
T	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**

↓														
~	((((((Ba	&	Bb)	&	Bc)	&	Bd)	&	Be)	&	Bf)	&	Bg)	&	Bh)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**

c. Yes. The (nonexpanded) set members may all be true on an interpretation with a smaller UD if we let more than one constant designate the same individual. Here is an example of an interpretation with a two-member UD on which all the set members are true:

UD: The set {1, 2}
Bx: x is odd
a: 1
b: 1
c: 1
d: 1
e: 1
f: 1
g: 1

Note that the UD must contain more than one member. For if the UD contains only one member, then ‘Ba’ and ‘~ (∀x)Bx’, for instance, cannot both be true. If the one set member is in the extension of the predicate ‘B’, then every member of the UD is in the extension of ‘B’ and ‘~ (∀x)Bx’ is false. If the one set member is not in the extension of the predicate ‘B’, then ‘Ba’, ‘Bb’, ‘Bc’, and so on are false—for ‘a’, ‘b’, ‘c’, and so on must all designate that one member.

15. b.

					↓			↓
Fa	(~	Fa	⊃	Fa)	∨	~	Fa	Fa
F	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**

d.

			↓		↓	↓
Fa	Ga	Fa	⊃	Ga	Ga	Fa
F	**T**	**F**	**T**	**T**	**T**	**F**

f.

Maa	Mab	Mba	Mbb	Naa	Nab	Nba	Nbb
T	**F**	**T**	**T**	**T**	**F**	**T**	**T**

							↓							
[(Maa	⊃	Naa)	&	(Mab	⊃	Nab)]	&	[(Mba	⊃	Nba)	&	(Mbb	⊃	Nbb)]
T	**T**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**

[(Maa	⊃	(Naa	&	Naa))	&	(Mab	⊃	(Nab	&	Nba))]
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**F**	**F**	**T**

↓											
&	[(Mba	⊃	(Nba	&	Nab))	&	(Mbb	⊃	(Nbb	&	Nbb))]
F	**T**	**F**	**T**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**T**

h.

						↓			
Fa	Fb	Gaa	Gba	Gbb	Fa	∨	(Gaa	∨	Gba)
T	**T**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**F**

	↓							↓	
Fb	∨	(~	Gaa	∨	~	Gbb)	Gaa	∨	Gba
T	**T**	**T**	**F**	**T**	**T**	**F**	**F**	**F**	**F**

Section 8.6E

1. b. **F**
d. **F**
f. **T**
h. **T**

2. b. The sentence is false on the following interpretation:

UD: Set of positive integers
b: 1
Fx: x is even

Not every positive integer identical to 1 (there is only one such integer—1) is even.

d. The sentence is false on the following interpretation:

UD: Set of positive integers
Gx: x is even
Hxy: x is greater than y

There is no even positive integer w that is identical to every positive integer that is not greater than w.

f. The sentence is false on the following interpretation:

UD: The set {1}

It is not true that for every pair of members of the UD that are identical, there is a member of the UD that is not identical to the first of the pair.

3. b. For any members x and y of the UD for any interpretation, there is a member z that is identical to one or the other—let z be x or y.

4. b. The first sentence is true and the second false on the following interpretation:

UD: Set of positive integers
Fx: x is even

Every positive integer x is such that every positive integer y that is identical to x (only x meets this condition) is even if and only if x is even, so the first sentence is true. However, not every pair x and y of positive integers satisfies the condition that if x is even just in case y is even, then x is identical to y. The numbers 2 and 4 are both even, so 2 is even just in case 4 is even. But 2 is not identical to 4.

d. The first sentence is false and the second true on the following interpretation:

UD: Set of positive integers
Gx: x is odd

No positive integer is such that every other positive integer is odd, but at least one positive integer (indeed, every even positive integer) is such that every odd positive integer is not identical to it.

5. b. Every member of the set is true on the following interpretation:

UD: The set {1}
Fx: x is odd
Gx: x is positive

Every member of the (single-member) UD is identical to every member, at least one member of the UD is odd, and every member of the UD is positive.

d. Every member of the set is true on the following interpretation:

UD: Set of positive integers
Gx: x is positive
Hx: x is even

Every member of the UD that is positive (i.e., every member) is such that every other member is positive, every even member of the UD is positive, and at least one member of the UD is even.

6. b. The following interpretation shows that the entailment does not hold:

UD: The set {2, 3}
Hx: x is prime

The first sentence in the set, which amounts to saying that there are at least two members in the UD, is true. The second sentence, which says that at least one member of the UD is prime, is true. But the conclusion, which says that at least one member of the UD is not prime, is false.

d. The following interpretation shows that the entailment does not hold:

UD: The set {1}
Fx: x is odd

Every pair of members x and y of the UD is such that one is odd if and only if the other is, if and only if x and y are identical; at least one member of the UD is odd; but, because there is no pair x and y of nonidentical members of the UD, the last sentence is false.

7. b. One symbolization of the argument is

$$\frac{(\forall x)(\forall y)(Pyx \supset Lxy)\ \&\ (\forall x)(\exists y)(\exists z)[(Pyx\ \&\ Pzx)\ \&\ {\sim}\, y = z]}{(\forall x)(\exists y)(\exists z)[(Lxy\ \&\ Lxz)\ \&\ {\sim}\, y = z]}$$

The argument is quantificationally valid. To see this, consider any interpretation on which the premise is true. By the second conjunct, every member x of the UD is such that at least two members of the UD stand in the relation P to x, and, by the first conjunct, these two members are such that x stands in the relation L to both of them. Therefore every member x is such that it stands in the relation L to at least two members of the UD, so the conclusion is true as well.

d. One symbolization of the argument is

$$\frac{\begin{array}{l}(\forall x)(\exists y)(\exists z)[(Pyx \,\&\, Pzx) \\ \&\ ({\sim}\, y = z \,\&\, (\forall w)(Pwx \supset (w = y \vee w = z)))]\end{array}}{\begin{array}{l}(\forall x)(\exists y)(\exists z)(\exists y_1)(\exists z_1)[(((\exists w)(Pyw \,\&\, Pwx) \,\&\, (\exists w)(Pzw \,\&\, Pwx)) \\ \&\ ((\exists w)(Py_1w \,\&\, Pwx) \,\&\, (\exists w)(Pz_1w \,\&\, Pwx))) \\ \&\ ((({\sim}\, y = z \,\&\, {\sim}\, y = y_1) \,\&\, {\sim}\, y = z_1) \\ \&\ (({\sim}\, z = y_1 \,\&\, {\sim}\, z = z_1) \,\&\, {\sim}\, y_1 = z_1))) \\ \&\ (\forall w)((\exists w_1)(Pww_1 \,\&\, Pw_1x) \supset ((w = y \vee w = z) \vee \\ (w = y_1 \vee w = z_1)))]\end{array}}$$

The following interpretation shows that the argument is quantificationally invalid:

UD: The set {2, 5, 9}
Pxy: x and y are relatively prime

Every integer in the UD is such that there are exactly two integers (in the UD) to which it is relatively prime, but, because there are only three members of the UD, no integer in the UD is such that there are exactly four members of the UD to which it is relatively prime.

8. b.

						↓	
$a = a$	Fa	$(Fa$	$\supset$	$\sim$	$a = a)$	$\&$	Fa
T	**T**	**T**	**F**	**F**	**T**	**F**	**T**

$a = a$	$a = b$	$b = a$	$b = b$	Fa	Fb	$[(Fa$	$\supset$	$({\sim}$	$a = a$	$\vee$	$\sim$	$b = a))$
T	**F**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**T**	**F**

								↓			
$\&$	$(Fb$	$\supset$	$({\sim}$	$a = b$	$\vee$	$\sim$	$b = b))]$	$\&$	$(Fa$	$\vee$	$Fb)$
T	**F**	**T**	**T**	**F**	**T**	**F**	**T**	**T**	**T**	**T**	**F**

d.

					↓		↓
$a = a$	Gaa	$[(Gaa$	$\vee$	$Gaa)$	$\vee$	$a = a]$	Gaa
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**

f.

			↓			↓	
$a = a$	Gaa	$(Gaa$	$\supset$	$a = a)$	$(a = a$	$\supset$	$Gaa)$
T	**F**	**F**	**T**	**T**	**T**	**F**	**F**

9. b. False. Not all integers, when squared, produce even integers.
d. True. Every positive integer that is twice a positive integer must be even.

f. True. If the sum of x, y, and z is even then at least one of x, y, and z must be even.

h. True. The sum of two positive integers' successors is greater than the sum of the two positive integers.

10. b. The sentence is false on the following interpretation:

UD:	Set of positive integers
g(x):	the successor of x

It's not true that given any two positive integers, one must be the successor of the other.

d. The sentence is false on the following interpretation:

UD:	Set of positive integers
f(x):	the successor of x

It's not true that if x is the successor of y and y is the successor of z then x is the successor of z.

f. The sentence is false on the following interpretation:

UD:	Set of positive integers
Dx:	x is greater than 8
h(x, y):	x raised to the power y

The integers 3 and 2 are *not* such that if 3^2 is greater than 8 (which is true), then 2^3 is greater than 8 (this is false).

11. b. Assume that x, y, and z satisfy the antecent 'y = f(x) & z = f(x)'. Because f is a function, it yields exactly one value for x and so y and z must be identical. But that is what the consequent says, so it will be satisfied as well. Therefore the sentence is true on every interpretation.

12. b. The first sentence is true and the second false on the following interpretation:

UD:	Set of positive integers
Bxy:	x is greater than y
h(x):	the successor of x

Every positive integer is smaller than its successor, and no positive integer is larger than its successor.

d. The first sentence is true and the second false on the following interpretation:

UD:	Set of positive integers
f(x):	x squared

The integers 16, 4, and 2 are such that $16 = 4^2$ and $4 = 2^2$; but not every positive integer is the square of a square.

13. b. The set members are both true on the following interpretation:

UD: Set of positive integers
Lxy: x is less than y
f(x): the successor of x

Every positive integer is less than its successor, and there is at least one positive integer that is not greater than or equal to its successor.

d. The set members are both true on the following interpretation:

UD: Set of positive integers
Gx: x is odd
h(x): 2 times x

Every odd positive integer is such that its double is not odd, and there exists a positive integer that is not odd and whose double is not odd.

14. b. The argument is quantificationally valid. If the premise is true on an interpretation, then every member x of the UD is such that either x or the value of x for the function g is F. It follows that every member that is the value of the function g for some x is such that either it or the value that g yields when applied to it is F, so the conclusion must also be true on any interpretation on which the premise is true.

d. The argument is quantificationally valid. If the premise is true on an interpretation then each member x of the UD bears the relation L to f(x), but not vice versa. If members x and y of the UD satisfy the condition '$y = f(x)$' in the conclusion then it follows from the premise that x bears the relation L to y, so x and y satisfy 'Lxy ∨ Lyx' as well. Hence the conclusion of this argument will be true on every interpretation on which the premise is true.

15. b.

a = g(a,a)	a = g(a,b)	a = g(b,a)	a = g(b,b)	b = g(a,a)	b = g(a,b)	b = g(b,a)
F	**T**	**F**	**F**	**T**	**F**	**T**

b = g(b,b)	Hg(a,a)a	Hg(a,b)a	Hg(a,b)b	Hg(b,a)a	Hg(b,a)b	Hg(b,b)b
T	**F**	**T**	**F**	**F**	**F**	**F**

(Hg(a,a)a	∨	Hg(a,b)a)	↓ ∨	(Hg(b,a)b	∨	Hg(b,b)b)
F	**T**	**T**	**T**	**F**	**F**	**F**

			↓					↓	
(H*g*(a,a)a	∨	H*g*(b,a)a	∨	(H*g*(a,b)b	∨	H*g*(b,b)b)	a = *g*(a,a)	∨	b = *g*(a,a)
F	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**

	↓			↓			↓	
a = *g*(a,b)	∨	b = *g*(a,b)	a = *g*(b,a)	∨	b = *g*(b,a)	a = *g*(b,b)	∨	b = *g*(b,b)
T	**T**	**F**	**F**	**T**	**T**	**F**	**T**	**T**

d.

a = a	a = b	b = a	b = b	a = *f*(a)	a = *f*(b)	b = *f*(a)	b = *f*(b)
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**

	↓					↓			
a = *f*(b)	&	b = *f*(a)	(~ a = a	∨	~ a = b)	∨	(~ b = a	∨	~ b = b)
T	**T**	**T**	**F T**	**F**	**F T**	**F**	**F T**	**F**	**F T**

	↓			↓	
a = *f*(a)	∨	b = *f*(a)	a = *f*(b)	∨	b = *f*(b)
T	**T**	**T**	**T**	**T**	**T**

Section 8.7E

1. b. Let **d** be a variable assignment for this interpretation. **d** satisfies the first conjunct, '~ Loo', just in case **d** fails to satisfy 'Loo'. **d** fails to satisfy 'Loo' just in case ⟨**I**(o), **I**(o)⟩, which is ⟨1, 1⟩, is not a member of **I**(L). And ⟨1, 1⟩ is not a member of **I**(L) because 1 is not less than 1. So **d** satisfies '~ Loo'.

d satisfies the second conjunct, '~ (∀y) ~ Loy', just in case **d** does not satisfy '(∀y) ~ Loy'. **d** does not satisfy '(∀y) ~ Loy' just in case there is a member **u** of the UD such that **d**[**u**/y] does not satisfy '~ Loy'. **d**[**u**/y] will fail to satisfy '~ Loy' just in case **d**[**u**/y] satisfies 'Loy'. There is such a member of the UD—take 2 as an example. **d**[2/y] satisfies 'Loy' because ⟨**I**(o), **d**[2/y](y)⟩, which is ⟨1, 2⟩, is a member of **I**(L)—1 is less than 2. So **d** satisfies the second conjunct and consequently **d** satisfies the conjunction '~ Loo & ~ (∀y) ~ Loy'. The sentence is true on this interpretation.

d. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀x)(Lox ⊃ (∀y)Lxy)' just in case every member **u** of the UD is such that **d**[**u**/x] satisfies 'Lox ⊃ (∀y)Lxy'. However, this is not true of all members of the UD—take 3 as an example. **d**[3/x] satisfies 'Lox', because ⟨**I**(o), **d**[3/x](x)⟩, which is ⟨1, 3⟩, is in **I**(L)—1 is less than 3. But **d**[3/x] does not satisfy '(∀y)Lxy', because there is at least one member **u** of the UD such that **d**[3/x, **u**/y] does not satisfy 'Lxy'. Take 2 as an example—**d**[3/x, 2/y] does not satisfy 'Lxy' because ⟨**d**[3/x, 2/y](x), **d**[3/x, 2/y](y)⟩, which is ⟨3, 2⟩, is not a member of

I(L)—3 is not less than 2. Because **d**[3/x] satisfies 'Lox' but does not satisfy '(∀y)Lxy', **d**[3/x] does not satisfy 'Lox ⊃ (∀y)Lxy'. So **d** does not satisfy '(∀x)(Lox ⊃ (∀y)Lxy)'. The sentence is false on this interpretation.

f. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀x) [Ex ⊃ (∃y)(Lxy ∨ Lyo)]' if and only if every member **u** of the UD is such that **d**[**u**/x] satisfies 'Ex ⊃ (∃y)(Lyx ∨ Lyo)', and this is so if and only if either **d**[**u**/x] fails to satisfy 'Ex' or **d**[**u**/x] does satisfy '(∃y)(Lyx ∨ Lyo)'. If **u** is an odd number, then **d**[**u**/x] does not satisfy 'Ex', because **u** is not a member of **I**(E). If **u** is an even number, then **d**[**u**/x] satisfies '(∃y)(Lyx ∨ Lyo)'. This is because **d**[**u**/x, 1/y] satisfies 'Lyx ∨ Lyo'—1 is less than any even integer and so ⟨**d**[**u**/x, 1/y](y), **d**[**u**/x, 1/y](x)⟩, which is ⟨1, **u**⟩, is a member of **I**(L). Therefore the sentence is true on this interpretation.

2. b. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀x)(∀y)(Gxy ∨ Gyx)' just in case every member **u** of the UD is such that **d**[**u**/x] satisfies '(∀y)(Gxy ∨ Gyx)'. For any **u**, **d**[**u**/x] will satisfy '(∀y)(Gxy ∨ Gyx)' just in case every member $\mathbf{u}_1$ of the UD is such that **d**[**u**/x, $\mathbf{u}_1$/y] satisfies 'Gxy ∨ Gyx'. So **d** satisfies '(∀x)(∀y)(Gxy ∨ Gyx)' just in case, for every pair **u** and $\mathbf{u}_1$ of members of the UD, **d**[**u**/x, $\mathbf{u}_1$/y] satisfies 'Gxy ∨ Gyx'—that is, just in case **d**[1/x, 1/y], **d**[1/x, 3/y], **d**[3/x, 1/y], and **d**[3/x, 3/y] all satisfy 'Gxy ∨ Gyx'. The first assignment does not satisfy the disjunction (nor does the fourth) because it satisfies neither disjunct. **d**[1/x, 1/y] does not satisfy 'Gxy' because ⟨**d**[1/x, 1/y](x), **d**[1/x, 1/y](y)⟩, which is ⟨1, 1⟩, is not a member of **I**(G)—1 is not greater than itself. **d**[1/x, 1/y] does not satisfy 'Gyx' because ⟨**d**[1/x, 1/y](y), **d**[1/x, 1/y](x)⟩, which is also ⟨1, 1⟩, is not a member of **I**(G). So **d**[1/x] does not satisfy '(∀y)(Gxy ∨ Gyx)' and therefore **d** does not satisfy '(∀x)(∀y)(Gxy ∨ Gyx)'. The sentence is false on this interpretation.

d. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀x)(Et ⊃ Ex)' just in case every member **u** of the UD is such that **d**[**u**/x] satisfies 'Et ⊃ Ex', that is, just in case **d**[1/x] and **d**[3/x] both satisfy 'Et ⊃ Ex'. Both do. **d**[1/x] satisfies 'Et ⊃ Ex' because **d**[1/x] does not satisfy 'Et'; **d**[1/x] does not satisfy 'Et' because ⟨**I**(t)⟩, which is ⟨3⟩, is not a member of **I**(E)—3 is not even. **d**[3/x] satisfies 'Et ⊃ Ex' because **d**[3/x] does not satisfy 'Et'; **d**[3/x] does not satisfy 'Et' because ⟨**I**(t)⟩, which again is ⟨3⟩, is not a member of **I**(E). So **d** satisfies '(∀x)(Et ⊃ Ex)'. The sentence is true on this interpretation.

f. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀y)[Ty ∨ (∀x)(Ex ⊃ Gxy)]' if and only if for every member **u** of the UD, **d**[**u**/y] satisfies 'Ty ∨ (∀x)(Ex ⊃ Gxy)'. **d**[1/x] and **d**[3/x] both satisfy the open sentence, because both satisfy the second disjunct. **d**[1/y] satisfies the second disjunct because for every member $\mathbf{u}_1$ of the UD, **d**[1/y, $\mathbf{u}_1$/x] satisfies 'Ex ⊃ Gxy' since it does not satisfy 'Ex'; no member of the UD is in the extension of '**E**'. Similar reasoning shows that **d**[3/y] also satisfies the second disjunct. Therefore the sentence is true on this interpretation.

3. b. Let **d** be a variable assignment for this interpretation. **d** satisfies '$(\forall x)(\forall y)(Mxyo \equiv Pyox)$' just in case for every member **u** of the UD, **d**[**u**/x] satisfies '$(\forall y)(Mxyo \equiv Pyox)$'. **d**[**u**/x] will satisfy '$(\forall y)(Mxyo \equiv Pyox)$' just in case every member $\mathbf{u}_1$ of the UD is such that **d**[**u**/x, $\mathbf{u}_1$/y] satisfies '$Mxyo \equiv Pyox$'. So **d** satisfies '$(\forall x)(\forall y)(Mxyo \equiv Pyox)$' just in case every pair **u** and $\mathbf{u}_1$ of members of the UD is such that **d**[**u**/x, $\mathbf{u}_1$/y] satisfies '$Mxyo \equiv Pyox$'. **d**[**u**/x, $\mathbf{u}_1$/y] will satisfy '$Mxyo \equiv Pyox$' just in case either it satisfies both immediate components or it satisfies neither.

d[**u**/x, $\mathbf{u}_1$,/y] will satisfy 'Mxyo' just in case ⟨**d**[**u**/x, $\mathbf{u}_1$/y](x), **d**[**u**/x, $\mathbf{u}_1$/y](y), **I**(o)⟩, which is ⟨**u**, $\mathbf{u}_1$, 1⟩, is a member of **I**(M)—that is, just in case $\mathbf{u} - \mathbf{u}_1 = 1$. **d**[**u**/x, $\mathbf{u}_1$/y] will satisfy 'Pyox' just in case ⟨**d**[**u**/x, $\mathbf{u}_1$/y](y), **I**(o), **d**[**u**/x, $\mathbf{u}_1$/y](x)⟩, which is ⟨$\mathbf{u}_1$, 1, **u**⟩, is a member of **I**(P)—that is, just in case $\mathbf{u}_1 + 1 = \mathbf{u}$. Because $\mathbf{u} - \mathbf{u}_1 = 1$ if and only if $\mathbf{u}_1 + 1 = \mathbf{u}$, we may conclude that **d**[**u**/x, $\mathbf{u}_1$/y] satisfies 'Mxyo' if and only if it satisfies 'Pyox'. Therefore **d**[**u**/x, $\mathbf{u}_1$/y], whatever positive integers **u** and $\mathbf{u}_1$ may be, will satisfy '$Mxyo \equiv Pyox$', and so **d** satisfies '$(\forall x)(\forall y)(Mxyo \equiv Pyox)$'. The sentence is true on this interpretation.

d. Let **d** be a variable assignment for this interpretation. **d** satisfies '$(\exists x)(\forall y)(\forall z)(Mxyz \vee Pzyx)$' if and only if for some member **u** of the UD, **d**[**u**/x] satisfies '$(\forall y)(\forall z)(Mxyz \vee Pzyx)$', and this is so if and only if for every pair of members $\mathbf{u}_1$ and $\mathbf{u}_2$ of the UD, **d**[**u**/x, $\mathbf{u}_1$/y, $\mathbf{u}_2$/z] satisfies '$Mxyz \vee Pzyx$'. This is the case if and only if either ⟨**u**, $\mathbf{u}_1$, $\mathbf{u}_2$⟩ is a member of **I**(M) or ⟨$\mathbf{u}_2$, $\mathbf{u}_1$, **u**⟩ is a member of I(P). But there is no member **u** of the UD such that for every $\mathbf{u}_1$ and $\mathbf{u}_2$ of the UD, this holds. Therefore the sentence is false on this interpretation.

4. The sentence '$(\forall x)Fx$' is true on an interpretation **I** just in case every variable assignment **d** (for **I**) satisfies '$(\forall x)Fx$'. A variable assignment **d** satisfies '$(\forall x)Fx$' if and only if every member **u** of the UD is such that **d**[**u**/x] satisfies 'Fx', that is, if and only every member **u** of the UD is such that ⟨**u**⟩ (which is ⟨**d**[**u**/x](x)⟩) is a member of **I**(F). So '$(\forall x)Fx$' is true on **I** just in case for every member **u** of the UD, ⟨**u**⟩ is a member of **I**(F).

The sentence '$\sim (\exists x) \sim Fx$' is true on an interpretation **I** just in case every variable assignment **d** satisfies '$\sim (\exists x) \sim Fx$'. A variable assignment **d** will satisfy '$\sim (\exists x) \sim Fx$' if and only if it does not satisfy '$(\exists x) \sim Fx$', that is, if and only if there is no member **u** of the UD such that **d**[**u**/x] satisfies '$\sim Fx$'. No member **u** of the UD is such that **d**[**u**/x] satisfies $\sim Fx$', if and only if every member **u** is such that **d**[**u**/x] satisfies 'Fx'—that is, if and only if every member **u** is such that ⟨**u**⟩ is a member of **I**(F). So '$\sim (\exists x)$ '$\sim Fx$' is true on **I** if and only if every member **u** of the UD is such that ⟨**u**⟩ is a member of **I**(F).

Therefore the truth-conditions for the two sentences are identical; they are quantificationally equivalent.

6. Assume that '$(\forall x)Fx$' is true on an interpretation **I**. Then every variable assignment **d** satisfies the sentence, and every member **u** of the UD is such that **d**[**u**/x] satisfies 'Fx'. From this it follows that every member **u** of the UD is such that ⟨**u**⟩ (which is ⟨**d**[**u**/x](x)⟩) is a member of **I**(F). Now consider an arbitrary substitution instance of '(∀x)Fx'—say, 'Fa'. 'Fa' is true on **I** if and only if every variable assignment **d** satisfies 'Fa', that is, if and only if ⟨**I**(a)⟩ is a member of **I**(F). But we have just seen that every member **u** of the UD is such that ⟨**u**⟩ is a member of **I**(F), so ⟨**I**(a)⟩ must be a member of **I**(F). Therefore 'Fa' is true on **I** as well. The same reasoning applies for every other substitution instance of '(∀x)Fx'—so the set consisting of this sentence quantificationally entails every one of its substitution instances.

8. Let **I** be any interpretation. A variable assignment **d** for **I** will satisfy this sentence if and only if it satisfies both conjuncts. **d** will satisfy the first conjunct if and only if for some member **u** of the UD, **d**[**u**/x] satisfies 'Fx'. But then **d**[**u**/x] does not satisfy '~ Fx', and from this it follows that **d** does not satisfy '(∀x) ~ Fx'. Therefore no variable assignment will satisfy the sentence; it is quantificationally false.

9. b. Let **d** be a variable assignment for this interpretation. **d** satisfies '(∀x)(∀y)(x = y ∨ ~ Ey)' if and only if for every pair of positive integers **u** and $\mathbf{u}_1$, **d**[**u**/x, $\mathbf{u}_1$/y] satisfies 'x = y ∨ ~ Ey'. But the variant **d**[1/x, 2/y], for example, does not satisfy this open sentence. **d**[1/x, 2/y] does not satisfy 'x = y' because 1 and 2 are distinct members of the UD. **d**[1/x, 2/y] does not satisfy '~ Ey' because it does satisfy 'Ey'—⟨**d**[1/x, 2/y](y)⟩, which is ⟨2⟩, is a member of **I**(E). Since neither disjunct is satisfied by **d**[1/x, 2/y], neither is 'x = y ∨ ~ Ey'. We conclude that the sentence is therefore false on this interpretation.

10. b. Let **I** be any interpretation. A variable assignment **d** for **I** will satisfy this sentence if and only if either it fails to satisfy the antecedent or it does satisfy the consequent. We show that if **d** does satisfy the antecedent it will satisfy the consequent as well. **d** satisfies the antecedent if and only if for every pair **u** and $\mathbf{u}_1$ of members of the UD, **d**[**u**/x, $\mathbf{u}_1$/y] satisfies 'x = y ⊃ Fxy'. **d**[**u**/x, $\mathbf{u}_1$/y] does so if and only if either it fails to satisfy 'x = y' or it does satisfy 'Fxy'—that is, if and only if **u** and $\mathbf{u}_1$ are not identical or ⟨**u**, $\mathbf{u}_1$⟩ is a member of **I**(F). Thus, if **u** = $\mathbf{u}_1$, then ⟨**u**, **u**⟩ is a member of **I**(F). And in this case, every member of the UD is such that **d**[**u**/x] satisfies 'Fxx', so **d** satisfies '(∀x)Fxx'. Therefore the sentence is true on every interpretation.

11. b. Let **d** be a variable assignment for this interpretation. **d** satisfies the sentence just in case for each pair **u** and **u**′ of members of the UD, **d**[**u**/x, **u**′/y] satisfies 'O*g*(x,y) ⊃ (Ox ∨ Oy)'. **d**[**u**/x, **u**′/y] will satisfy the antecedent just in case the member **u**″ of the UD such that ⟨**u**, **u**′, **u**″⟩ is a member of **I**(*g*) is also a member of **I**(O), i.e., just in case the sum of **u** and **u**′ is an odd number. And this will hold if and only if either **u** is odd and **u**′ is even, or **u** is even and **u**′ is odd. Either way, **d**[**u**/x, **u**′/y] will also satisfy the consequence 'Ox ∨ Oy' since one of **u**, **u**′ will be a member of **I**(O). So every pair **u** and **u**′ of members of the UD is such that **d**[**u**/x, **u**′/y] satisfies 'O*g*(x,y) ⊃ (Ox ∨ Oy)', so **d** satisfies the universally quantified sentence and there sentence is therefore true on this interpretation.

12. b. This sentence is quantificationally true if and only if every variable assignment **d** on every interpretation **I** satisfies the sentence. A variable assignment **d** will satisfy the sentence if and only if either **d** fails to satisfy the antecedent or **d** does satisfy the consequent. Consider, then, a variable assignment **d** that satisfies the antecedent. We must show that **d** also satisfies the consequent. If **d** satisfies the antecedent, then for every member **u** of the UD, **d**[**u**/x] satisfies 'P*f*(x)', i.e., for every member **u** of the UD, the member $\mathbf{u}'$ such that $\langle \mathbf{u},\mathbf{u}' \rangle$ is a member of **I**(*f*) must be such that $\langle \mathbf{u}' \rangle$ is a member of **I**(P). But then the member $\mathbf{u}''$ such that $\langle \mathbf{u}', \mathbf{u}'' \rangle$ is a member of **I**(*f*) must also be such that $\langle \mathbf{u}'' \rangle$ is a member of **I**(P). It follows that for every member u of the UD, **d**[**u**/x] satisfies 'P*f*(*f*(x))', and so **d** satisfies the consequent. We conclude that the sentence is true on every interpretation.

Section 9.1E

b.	1.	(∃x)Fx✓	SM
	2.	(∀x) ~ Fx	SM
	3.	Fa	1 ∃D
	4.	~ Fa	2 ∀D
		×	

The tree is closed.

d.	1.	(∃x)(Fx & ~ Gx)✓		SM
	2.	(∀x)Fx ⊃ (∀x)Gx✓		SM
	3.	Fa & ~ Ga✓		1 ∃D
	4.	Fa		3 &D
	5.	~ Ga		3 &D
	6.	~ (∀x)Fx✓	(∀x)Gx	2 ⊃D
	7.	(∃x) ~ Fx✓		6 ~ ∀D
	8.	~ Fb		7 ∃D
	9.		Ga	6 ∀D
			×	

The left-hand branch is a completed open branch by the account of Chapter Four. The tree is open.

f.	1.	~ (∀x)(Fx & Gx)✓		SM
	2.	(∃y)(Fy & Gy)✓		SM
	3.	(∃x) ~ (Fx & Gx)✓		1 ~ ∀D
	4.	Fa & Ga✓		2 ∃D
	5.	Fa		4 &D
	6.	Ga		4 &D
	7.	~ (Fb & Gb)✓		3 ∃D
	8.	~ Fb	~ Gb	7 ~ &D

The tree is open.

h. 1.	$(\forall x)(Fx \supset Gx)$			SM
2.	$(\forall x)(Gx \supset Hx)$			SM
3.	$(\exists x)(Fx \,\&\, \sim Hx)$ ✓			SM
4.	$Fa \,\&\, \sim Ha$ ✓			3 $\exists$D
5.	Fa			4 &D
6.	$\sim Ha$			4 &D
7.	$Fa \supset Ga$ ✓			1 $\forall$D
8.	$\sim Fa$	Ga		7 $\supset$D
9.	×	$Ga \supset Ha$ ✓		2 $\forall$D
10.		$\sim Ga$	Ha	9 $\supset$D
		×	×	

The tree is closed.

j. 1.	$(\forall x)(\exists y)Lxy$	SM
2.	$Lta \,\&\, \sim Lat$ ✓	SM
3.	$\sim (\exists y)Lay$ ✓	SM
4.	Lta	2 &D
5.	$\sim Lat$	2 &D
6.	$(\forall y) \sim Lay$	3 $\sim \exists$D
7.	$(\exists y)Lay$ ✓	1 $\forall$D
8.	Lab	7 $\exists$D
9.	$\sim Lab$	6 $\forall$D
	×	

The tree is closed.

l. 1.	$(\forall x)(Fx \supset Gx)$		SM
2.	$\sim (\forall x) \sim Fx$ ✓		SM
3.	$(\forall x) \sim Gx$		SM
4.	$(\exists x) \sim \sim Fx$ ✓		2 $\sim \forall$D
5.	$\sim \sim Fa$ ✓		4 $\exists$D
6.	Fa		5 $\sim \sim$ D
7.	$Fa \supset Ga$ ✓		1 $\forall$D
8.	$\sim Fa$	Ga	7 $\supset$D
9.	×	$\sim Ga$	3 $\forall$D
		×	

The tree is closed.

n.

1.	(∃x)Gx ⊃ (∀x)Gx ✓		SM
2.	(∃z)Gz & (∃y) ~ Gy ✓		SM
3.	(∃z)Gz ✓		2 &D
4.	(∃y) ~ Gy ✓		2 &D
5.	Ga		3 ∃D
6.	~ Gb		4 ∃D
7.	~ (∃x)Gx ✓	(∀x)Gx	1 ⊃D
8.	(∀x) ~ Gx		7 ~ ∃D
9.	~ Ga		8 ∀D
10.	×	Gb	7 ∀D
		×	

The tree is closed.

p.

1.	(∃y)(Fy ∨ Gy) ✓				SM
2.	~ (∀y)Fy & ~ (∀y)Gy ✓				SM
3.	~ (∀x)(Fx & Gx) ✓				SM
4.	~ (∀y)Fy ✓				2 &D
5.	~ (∀y)Gy ✓				2 &D
6.	(∃x) ~ (Fx & Gx) ✓				3 ~ ∀D
7.	(∃y) ~ Fy ✓				4 ~ ∀D
8.	(∃y) ~ Gy ✓				5 ~ ∀D
9.	Fa ∨ Ga ✓				1 ∃D
10.	~ (Fb & Gb) ✓				6 ∃D
11.	~ Fc				7 ∃D
12.	~ Gd				8 ∃D
13.	Fa		Ga		9 ∨D
14.	~ Fb	~ Gb	~ Fb	~ Gb	10 ~ &D

The tree is open.

Section 9.2E

Note: In this section and the next, whenever a tree is open we give a complete tree. This is because the strategems we have suggested do not uniquely determine the order of decomposition, so the first open branch to be completed on your tree may not be the first such branch completed on our tree. In accordance with strategem 5, you should stop when your tree has one completed open branch.

b.

1.	$(\forall x)Fx \vee (\exists y)Gy$ ✓		SM
2.	$(\exists x)(\sim Fx \;\&\; Gx)$ ✓		SM
3.	$\sim Fa \;\&\; Ga$ ✓		2 ∃D
4.	$\sim Fa$		3 &D
5.	Ga		3 &D
6.	$(\forall x)Fx$	$(\exists y)Gy$ ✓	1 ∨D
7.	Fa		6 ∀D
	×		
8.		Gb	6 ∃D

The tree has one completed open branch. The set is quantificationally consistent.

d.

1.	$(\forall x)(Fx \supset Gxa)$			SM
2.	$(\exists x)Fx$ ✓			SM
3.	Fb			2 ∃D
4.	$Fb \supset Gba$ ✓			1 ∀D
5.	$\sim Fb$	Gba		4 ⊃D
6.	×	$Fa \supset Gaa$ ✓		1 ∀D
7.		$\sim Fa$	Gaa	6 ⊃D

The tree has two completed open branches. The set is quantificationally consistent.

f. 1.	(∀x)(Fx ⊃ Gxa)			SM
2.	(∃x)Fx ✓			SM
3.	(∀x)(∀y)Gxy			SM
4.	Fb			2 ∃D
5.	Fb ⊃ Gba ✓			1 ∀D
6.	~ Fb	Gba		5 ⊃D
7.	×	Fa ⊃ Gaa ✓		1 ∀D
8.		(∀y)Gay		3 ∀D
9.		(∀y)Gby		3 ∀D
10.		Gaa		8 ∀D
11.		Gab		8 ∀D
12.		Gba		9 ∀D
13.		Gbb		9 ∀D
14.		~ Fa	Gaa	7 ⊃D

The tree has two completed open branches. Therefore, the set is quantificationally consistent.

h. 1.	(∀x)(Fx ∨ Gx)		SM
2.	~ (∃y)(Fy ∨ Gy) ✓		SM
3.	Fa & ~ Gb ✓		SM
4.	(∀y) ~ (Fy ∨ Gy)		2 ∃D
5.	Fa		3 &D
6.	~ Gb		3 &D
7.	Fb ∨ Gb ✓		1 ∀D
8.	Fb	Gb	7 ∨D
9.	~ (Fb ∨ Gb) ✓	×	4 ∀D
10.	~ Fb		9 ~ ∨D
11.	~ Gb		9 ~ ∨D
	×		

The tree is closed. The set is quantificationally inconsistent.

j.	1.	(∀z) ~ Hzb		SM
	2.	(∃y)Fy ⊃ (∃x)Hxc ✓		SM
	3.	~ Hbb		1 ∀D
	4.	~ Hcb		1 ∀D
	5.	~ (∃y)Fy ✓	(∃x)Hxc ✓	2 ⊃D
	6.	(∀y) ~ Fy		5 &∃
	7.		Hac	5 ∃D
	8.		~ Hab	1 ∀D
	9.	~ Fb		6 ∀D
	10.	~ Fc		6 ∀D

The tree has two completed open branches. Therefore, the set is quantificationally consistent.

l.	1.	(∀x)(∀y)Lxy		SM
	2.	(∃z) ~ Lza ⊃ (∀z) ~ Lzb ✓		SM
	3.	(∀y)Lay		1 ∀D
	4.	(∀y)Lby		1 ∀D
	5.	Laa		3 ∀D
	6.	Lab		3 ∀D
	7.	Lba		4 ∀D
	8.	Lbb		4 ∀D
	9.	~ (∃z) ~ Lza ✓	(∀z) ~ Lzb	2 ⊃D
	10.		~ Lab	9 ∀D
			×	
	11.	(∀z) ~ ~ Lza		9 ~ ∃D
	12.	~ ~ Laa ✓		11 ∀D
	13.	Laa		12 ~ ~ D
	14.	~ ~ Lba ✓		11 ∀D
	15.	Lba		14 ~ ~ D

The tree has a completed open branch. Therefore, the set is quantificationally consistent.

n. 1.	$(\forall x)Fxa \equiv \sim (\forall x)Gxb$ ✓		SM
2.	$(\exists x)(Fxa \,\&\, \sim Gxb)$ ✓		SM
3.	$Fca \,\&\, \sim Gcb$ ✓		2 $\exists$D
4.	Fca		3 &D
5.	$\sim Gcb$		3 &D
6.	$(\forall x)Fxa$	$\sim (\forall x)Fxa$	1 $\equiv$D
7.	$\sim (\forall x)Gxb$ ✓	$\sim \sim (\forall x)Gxb$	1 $\equiv$D
8.		$(\forall x)Gxb$	7 $\sim \sim$ D
9.		Gcb	8 $\forall$D
10.	$(\exists x) \sim Gxb$ ✓	×	7 $\sim \forall$D
11.	$\sim Gdb$		10 $\exists$D
12.	Faa		6 $\forall$D
13.	Fba		6 $\forall$D
14.	Fca		6 $\forall$D
15.	Fda		6 $\forall$D

The tree has one completed open branch. The set is quantificationally consistent.

Section 9.3E

1. b. 1.	$\sim ((\exists x)Fx \vee (\exists x) \sim Fx)$ ✓	SM
2.	$\sim (\exists x)Fx$ ✓	1 $\sim \vee$D
3.	$\sim (\exists x) \sim Fx$ ✓	1 $\sim \vee$D
4.	$(\forall x) \sim Fx$	2 $\sim \exists$D
5.	$(\forall x) \sim \sim Fx$	3 $\sim \exists$D
6.	$\sim \sim Fa$ ✓	5 $\forall$D
7.	Fa	6 $\sim \sim$ D
8.	$\sim Fa$	4 $\forall$D
	×	

The tree is closed. The sentence '$(\exists x)Fx \vee (\exists x) \sim Fx$' is quantificationally true.

d. 1.	$\sim ((\forall x)Fx \vee \sim (\forall x)Fx)$ ✓	SM
2.	$\sim (\forall x)Fx$ ✓	1 $\sim \vee$D
3.	$\sim \sim (\forall x)Fx$ ✓	1 $\sim \vee$D
4.	$(\exists x) \sim Fx$ ✓	2 $\sim \forall$D
5.	$(\forall x)Fx$	3 $\sim \sim$ D
6.	$\sim Fa$	4 $\exists$D
7.	Fa	5 $\forall$D
	×	

The tree is closed. The sentence '$(\forall x)Fx \vee \sim (\forall x)Fx$' is quantificationally true.

f.	1.	$\sim((\forall x)(Fx \vee Gx) \supset [(\exists x)Fx \vee (\exists x)Gx])$ ✓		SM
	2.	$(\forall x)(Fx \vee Gx)$		$1 \sim\supset D$
	3.	$\sim[(\exists x)Fx \vee (\exists x)Gx]$ ✓		$1 \sim\supset D$
	4.	$\sim(\exists x)Fx$ ✓		$3 \sim\vee D$
	5.	$\sim(\exists x)Gx$ ✓		$3 \sim\vee D$
	6.	$(\forall x) \sim Fx$		$4 \sim\exists D$
	7.	$(\forall x) \sim Gx$		$5 \sim\exists D$
	8.	$\sim Fa$		$6\ \forall D$
	9.	$\sim Ga$		$7\ \forall D$
	10.	$Fa \vee Ga$ ✓		$2\ \forall D$
	11.	Fa	Ga	$10 \vee D$
		×	×	

The tree is closed. The sentence '$(\forall x)(Fx \vee Gx) \supset [(\exists x)Fx \vee (\exists x)Gx]$' is quantificationally true.

h.	1.	$\sim((\forall x)(Fx \vee Gx) \supset ((\exists x)Fx \vee (\forall x)Gx))$ ✓		SM
	2.	$(\forall x)(Fx \vee Gx)$		$1 \sim\supset D$
	3.	$\sim((\exists x)Fx \vee (\forall x)Gx)$ ✓		$1 \sim\supset D$
	4.	$\sim(\exists x)Fx$ ✓		$3 \sim\vee D$
	5.	$\sim(\forall x)Gx)$ ✓		$3 \sim\vee D$
	6.	$(\forall x) \sim Fx$		$4 \sim\exists D$
	7.	$(\exists x) \sim Gx$ ✓		$5 \sim\forall D$
	8.	$\sim Ga$		$7\ \exists D$
	9.	$Fa \vee Ga$ ✓		$2\ \forall D$
	10.	Fa	Ga	$9 \vee D$
	11.	$\sim Fa$	×	$6\ \forall D$
		×		

The tree is closed. The sentence '$(\forall x)(Fx \vee Gx) \supset ((\exists x)Fx \vee (\forall x)Gx)$' is quantificationally true.

j. 1.	$\sim ((\forall x)(Fx \vee Gx) \supset ((\forall x)Fx \vee (\forall x)Gx))$ ✓		SM
2.	$(\forall x)(Fx \vee Gx)$		$1 \sim \supset D$
3.	$\sim ((\forall x)Fx \vee (\forall x)Gx)$ ✓		$1 \sim \supset D$
4.	$\sim (\forall x)Fx$ ✓		$3 \sim \vee D$
5.	$\sim (\forall x)Gx$ ✓		$3 \sim \vee D$
6.	$(\exists x) \sim Fx$ ✓		$4 \sim \forall D$
7.	$(\exists x) \sim Gx$ ✓		$5 \sim \forall D$
8.	$\sim Fa$		$6\ \exists D$
9.	$\sim Gb$		$7\ \exists D$
10.	$Fa \vee Ga$ ✓		$2\ \forall D$
11.	Fa	Ga	$10\ \vee D$
12.	×	$Fb \vee Gb$ ✓	$2\ \forall D$
13.		Fb Gb	$12\ \vee D$
		×	

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

l. 1.	$\sim (((\exists x)Fx \& (\exists)Gx) \supset (\exists x)(Fx \& Gx))$ ✓		SM
2.	$(\exists x)Fx \& (\exists x)Gx$ ✓		$1 \sim \supset D$
3.	$\sim (\exists x)(Fx \& Gx)$ ✓		$1 \sim \supset D$
4.	$(\exists x)Fx$ ✓		$2\ \&D$
5.	$(\exists x)Gx$ ✓		$2\ \&D$
6.	$(\forall x) \sim (Fx \& Gx)$		$3 \sim \exists D$
7.	Fa		$4\ \exists D$
8.	Gb		$5\ \exists D$
9.	$\sim (Fa \& Ga)$ ✓		$6\ \forall D$
10.	$\sim Fa$	$\sim Ga$	$9 \sim \&D$
11.	×	$\sim (Fb \& Gb)$ ✓	$6\ \forall D$
12.		$\sim Fb$ $\sim Gb$	$11 \sim \&D$
		×	

The tree has a completed open branch, therefore the given sentence is not quantificationally true.

n.	1.	~ ((∀x)[Fx ⊃ (Gx & Hx)] ⊃ (∀x)[(Fx & Gx) ⊃ Hx])✓		SM
	2.	(∀x)(Fx ⊃ (Gx & Hx))		1 ~ ⊃D
	3.	~ (∀x)((Fx & Gx) ⊃ Hx)✓		1 ~ ⊃D
	4.	(∃x) ~ ((Fx & Gx) ⊃ Hx)✓		3 ~ ∀D
	5.	~ ((Fa & Ga) ⊃ Ha)✓		4 ∃D
	6.	Fa & Ga✓		5 ~ ⊃D
	7.	~ Ha		5 ~ ⊃D
	8.	Fa		6 &D
	9.	Ga		6 &D
	10.	Fa ⊃ (Ga & Ha)✓		2 ∀D
	11.	~ Fa	Ga & Ha✓	10 ⊃D
	12.	×	Ga	11 &D
	13.		Ha	11 &D
			×	

The tree is closed. The sentence '(∀x)[Fx ⊃ (Gx & Hx)] ⊃ (∀x)[(Fx & Gx) ⊃ Hx]' is quantificationally true.

p.	1.	~ [(∀x)(Fx & ~ Gx) ∨ (∃x)(~ Fx ∨ Gx)]✓		SM
	2.	~ (∀x)(Fx & ~ Gx)✓		1 ~ ∨D
	3.	~ (∃x)(~ Fx ∨ Gx)✓		1 ~ ∨D
	4.	(∃x) ~ (Fx & ~ Gx)✓		2 ~ ∀D
	5.	(∀x) ~ (~ Fx ∨ Gx)		3 ~ ∃D
	6.	~ (Fa & ~ Ga)✓		4 ∃D
	7.	~ (~ Fa ∨ Ga)✓		5 ∀D
	8.	~ ~ Fa✓		7 ~ ∨D
	9.	~ Ga		7 ~ ∨D
	10.	Fa		8 ~ ~ D
	11.	~ Fa	~ ~ Ga✓	6 ~ &D
	12.	×	Ga	11 ~ ~ D
			×	

The tree is closed. The sentence '(∀x)(Fx & ~ Gx) ∨ (∃x)(~ Fx ∨ Gx)' is quantificationally true.

r.	1.	$\sim ((\forall x)(\forall y)Gxy \supset (\forall x)Gxx)$ ✓	SM
	2.	$(\forall x)(\forall y)Gxy$	$1 \sim \supset D$
	3.	$\sim (\forall x)Gxx$ ✓	$1 \sim \supset D$
	4.	$(\exists x) \sim Gxx$ ✓	$3 \sim \forall D$
	5.	$\sim Gaa$	$4\ \exists D$
	6.	$(\forall y)Gay$	$2\ \forall D$
	7.	Gaa	$6\ \forall D$
		×	

The tree is closed. The sentence '$(\forall x)(\forall y)Gxy \supset (\forall x)Gxx$' is quantificationally true.

t.	1.	$\sim ((\forall x)Fxx \supset (\forall x)(\exists y)Fxy)$ ✓	SM
	2.	$(\forall x)Fxx$	$1 \sim \supset D$
	3.	$\sim (\forall x)(\exists y)Fxy$ ✓	$1 \sim \supset D$
	4.	$(\exists x) \sim (\exists y)Fxy$ ✓	$3 \sim \forall D$
	5.	$\sim (\exists y)Fay$ ✓	$4\ \exists D$
	6.	$(\forall y) \sim Fay$	$5 \sim \exists D$
	7.	Faa	$2\ \forall D$
	8.	$\sim Faa$	$6\ \forall D$
		×	

The tree is closed. The sentence '$(\forall x)Fxx \supset (\forall x)(\exists y)Fxy$' is quantificationally true.

v.	1.	$\sim (\exists x)(\exists y)(Lxy \equiv Lyx)$ ✓		SM
	2.	$(\forall x) \sim (\exists y)(Lxy \equiv Lyx)$		$1 \sim \exists D$
	3.	$\sim (\exists y)(Lay \equiv Lya)$ ✓		$2\ \forall D$
	4.	$(\forall y) \sim (Lay \equiv Lya)$		$3 \sim \exists D$
	5.	$\sim (Laa \equiv Laa)$ ✓		$4\ \forall D$
	6.	Laa	$\sim Laa$	$5 \sim \equiv D$
	7.	$\sim Laa$	Laa	$5 \sim \equiv D$
		×	×	

The tree is closed. The sentence '$(\exists x)(\exists y)(Lxy \equiv Lyx)$' is quantificationally true.

2. b.	1.	$(\forall x)Fx\ \&\sim (\exists x)Fx$ ✓	SM
	2.	$(\forall x)Fx$	$1\ \&D$
	3.	$\sim (\exists x)Fx$ ✓	$1\ \&D$
	4.	$(\forall x) \sim Fx$	$3 \sim \exists D$
	5.	Fa	$2\ \forall D$
	6.	$\sim Fa$	$4\ \forall D$
		×	

The tree is closed. Therefore the sentence is quantificationally false.

d. 1.	$(\exists x)Fx \,\&\, \sim (\forall x)Fx$ ✓	SM
2.	$(\exists x)Fx$ ✓	1 &D
3.	$\sim (\forall x)Fx$ ✓	1 &D
4.	$(\exists x) \sim Fx$ ✓	3 $\sim \forall$D
5.	Fa	2 $\exists$D
6.	$\sim Fb$	4 $\exists$D

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

f. 1.	$(\forall x)(Fx \supset \sim Fx)$		SM
2.	$Fa \supset \sim Fa$ ✓		1 $\forall$D
3.	$\sim Fa$	$\sim Fa$	2 $\supset$ D

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

h. 1.	$(\exists x)Fx \supset (\forall x) \sim Fx$ ✓		SM
2.	$\sim (\exists x)Fx$ ✓	$(\forall x) \sim Fx$	1 $\supset$D
3.	$(\forall x) \sim Fx$		2 $\sim \exists$D
4.	$\sim Fa$		3 $\forall$D
5.		$\sim Fa$	9 $\forall$D

The tree has at least one completed open branch. Therefore, the given sentence is not quantificationally false.

j. 1.	$(\exists x)Fx \,\&\, \sim (\exists y)Fy$ ✓	SM
2.	$(\exists x)Fx$ ✓	1 &D
3.	$\sim (\exists y)Fy$ ✓	1 &D
4.	$(\forall y) \sim Fy$	3 $\sim \exists$D
5.	Fa	2 $\exists$D
6.	$\sim Fa$	4 $\forall$D
	×	

The tree is closed. Therefore the sentence is quantificationally false.

l. 1.	$(\forall x)(Gx \equiv \sim Fx) \;\&\; \sim (\forall x) \sim (Gx \equiv Fx)$ ✓					SM
2.	$(\forall x)(Gx \equiv \sim Fx)$					1 &D
3.	$\sim (\forall x) \sim (Gx \equiv Fx)$ ✓					1 &D
4.	$(\exists x) \sim\sim (Gx \equiv Fx)$ ✓					3 $\sim \forall$D
5.	$\sim\sim (Ga \equiv Fa)$ ✓					4 $\exists$D
6.	$Ga \equiv Fa$ ✓					5 $\sim\sim$ D
7.	Ga		$\sim Ga$			6 $\equiv$D
8.	Fa		$\sim Fa$			6 $\equiv$D
9.	$Ga \equiv \sim Fa$ ✓		$Ga \equiv \sim Fa$ ✓			2 $\forall$D
10.	Ga	$\sim Ga$	Ga	$\sim Ga$		9 $\equiv$D
11.	$\sim Fa$	$\sim\sim Fa$	$\sim Fa$	$\sim\sim Fa$ ✓		9 $\equiv$D
12.	×	×	×	Fa		11 $\sim\sim$ D
				×		

The tree is closed. Therefore the sentence is quantificationally false.

3. b. 1.	$(\exists x)(\exists y)Fxy \supset (\exists x)Fxx$ ✓		SM
2.	$\sim (\exists x)(\exists y)Fxy$ ✓	$(\exists x)Fxx$ ✓	1 $\supset$D
3.	|	Faa	2 $\exists$D
4.	$(\forall x) \sim (\exists y)Fxy$		2 $\sim \exists$D
5.	$\sim (\exists y)Fay$ ✓		4 $\forall$D
6.	$(\forall y) \sim Fay$		5 $\sim \exists$D
7.	$\sim Faa$		6 $\forall$D

1.	$\sim [(\exists x)(\exists y)Fxy \supset (\exists x)Fxx]$ ✓	SM
2.	$(\exists x)(\exists y)Fxy$ ✓	1 $\sim \supset$D
3.	$\sim (\exists x)Fxx$ ✓	1 $\sim \supset$D
4.	$(\forall x) \sim Fxx$	3 $\sim \exists$D
5.	$(\exists y)Fay$ ✓	2 $\exists$D
6.	Fab	5 $\exists$D
7.	$\sim Faa$	4 $\forall$D
8.	$\sim Fbb$	4 $\forall$D

Both the tree for the given sentence and the tree for its negation have at least one completed open branch. Therefore, the given sentence is quantificationally indeterminate.

d. 1.	$\sim [(\forall x)(Fx \supset (\exists y)Gyx) \supset ((\exists x)Fx \supset (\exists x)(\exists y)Gxy)]$ ✓		SM
2.	$(\forall x)(Fx \supset (\exists y)Gyx)$		$1 \sim \supset D$
3.	$\sim ((\exists x)Fx \supset (\exists x)(\exists y)Gxy)$ ✓		$1 \sim \supset D$
4.	$(\exists x)Fx$ ✓		$3 \sim \supset D$
5.	$\sim (\exists x)(\exists y)Gxy$ ✓		$3 \sim \supset D$
6.	$(\forall x) \sim (\exists y)Gxy$		$5 \sim \exists D$
7.	Fa		$4\ \exists D$
8.	$Fa \supset (\exists y)Gya$ ✓		$2\ \forall D$
9.	$\sim Fa$	$(\exists y)Gya$ ✓	$8 \supset D$
10.	×	Gba	$9\ \exists D$
11.		$\sim (\exists y)Gby$ ✓	$6\ \forall D$
12.		$(\forall y) \sim Gby$	$11 \sim \exists D$
13.		$\sim Gba$	$12\ \forall D$
		×	

The tree for the negation of '$(\forall x)(Fx \supset (\exists y)Gyx) \supset ((\exists x)Fx \supset (\exists x)(\exists y)Gxy)$' is closed. Therefore the latter sentence is quantificationally true.

f. 1.	$\sim[((\exists x)Lxx \supset (\forall y)Lyy) \supset (Laa \supset Lgg)]$ ✓		SM
2.	$(\exists x)Lxx \supset (\forall y)Lyy$ ✓		$1 \sim \supset D$
3.	$\sim (Laa \supset Lgg)$ ✓		$1 \sim \supset D$
4.	Laa		$3 \sim \supset D$
5.	$\sim Lgg$		$3 \sim \supset D$
6.	$\sim (\exists x)Lxx$ ✓	$(\forall y)Lyy$	$2 \supset D$
7.	$(\forall x) \sim Lxx$		$6 \sim \exists D$
8.	$\sim Laa$		$7\ \forall D$
9.	×	Lgg	$6\ \forall D$
		×	

The tree for the negation of '$((\exists x)Lxx \supset (\forall y)Lyy) \supset (Laa \supset Lgg)$' is closed. Therefore the latter sentence is quantificationally true.

4. b. 1.	$\sim [(\exists x)(Fx \supset Ga) \equiv ((\exists x)Fx \supset Ga)] \checkmark$				SM
2.	$(\exists x)(Fx \supset Ga) \checkmark$		$\sim (\exists x)(Fx \supset Ga) \checkmark$		$1 \sim {\equiv}D$
3.	$\sim ((\exists x)Fx \supset Ga) \checkmark$		$(\exists x)Fx \supset Ga \checkmark$		$1 \sim {\equiv}D$
4.	$(\exists x)Fx \checkmark$				$3 \sim {\supset}D$
5.	$\sim Ga$				$3 \sim {\supset}D$
6.	$Fb \supset Ga \checkmark$				$2\ \exists D$
7.	Fc				$4\ \exists D$
8.	$\sim Fb$	Ga			$6 \sim {\supset}D$
9.		$\times$	$(\forall x) \sim (Fx \supset Ga)$		$2 \sim \exists D$
10.			$\sim (Fa \supset Ga) \checkmark$		$9\ \forall D$
11.			Fa		$10 \sim {\supset}D$
12.			$\sim Ga$		$10 \sim {\supset}D$
13.			$\sim (\exists x)Fx \checkmark$	Ga	$2\ {\supset}D$
14.			$(\forall x) \sim Fx$	$\times$	$13 \sim \exists D$
15.			$\sim Fa$		$14\ \forall D$
			$\times$		

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

d.	1.	~ [(Ls ≡ (∀x)Lx) ≡ (∃x)Lx] ✓				SM
	2.	Ls ≡ (∀x)Lx ✓		~ (Ls ≡ (∀x)Lx) ✓		1 ~ ≡D
	3.	~ (∃x)Lx ✓		(∃x)Lx ✓		1 ~ ≡D
	4.	(∀x) ~ Lx				3 ~ ∃D
	5.	Ls	~ Ls			2 ≡D
	6.	(∀x)Lx	~ (∀x)Lx ✓			2 ≡D
	7.	~ Ls				4 ∀D
	8.	×	(∃x) ~ Lx ✓			6 ~ ∀D
	9.		~ La			8 ∃D
	10.		~ Ls			4 ∀D
	11.			La		3 ∃D
	12.			Ls	~ Ls	2 ~ ≡
	13.			~ (∀x)Lx ✓	(∀x)Lx	2 ~ ≡
	14.			(∃x) ~ Lx ✓		13 ~ ∀D
	15.			~ Lb		14 ∃D
	16.				Ls	13 ∀D
					×	

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

f. 1.	~ [(∀x) (Fx ∨ Gx) ≡ ((∀x)Fx ∨ (∀x)Gx)]✓						SM
2.	(∀x)(Fx ∨ Gx)			~ (∀x)(Fx ∨ Gx)✓			1 ~ ≡D
3.	~ ((∀x)Fx ∨ (∀x)Gx)✓			(∀x)Fx ∨ (∀x)Gx✓			1 ~ ≡D
4.	~ (∀x)Fx✓						3 ~ ∨D
5.	~ (∀x)Gx✓						3 ~ ∨D
6.	(∃x) ~Fx✓						4 ~ ∀D
7.	(∃x) ~ Gx✓						5 ~ ∀D
8.	~ Fa						6 ∃D
9.	~ Gb						7 ∃D
10.	Fa ∨ Ga✓						2 ∀D
11.	Fa	Ga					10 ∨D
12.	×	Fb ∨ Gb✓					2 ∀D
13.		Fb	Gb				12 ∨D
14.			×	(∃x) ~ (Fx ∨ Gx)✓			2 ~ ∀D
15.				~ (Fa ∨ Ga)✓			14 ∃D
16.				~ Fa			15 ~ ∨D
17.				~ Ga			15 ~ ∨D
18.					(∀x)Fx	(∀x)Gx	3 ∨D
19.					Fa	Ga	18 ∀D
					×	×	

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

h. 1. ~ [(∃x)(Ax & Bx) ≡ ((∃x)Ax & (∃x)Bx)] ✓ SM

2.	(∃x)(Ax & Bx) ✓		~ (∃x)(Ax & Bx) ✓			1 ~ ≡D
3.	~ ((∃x)Ax & (∃x)Bx) ✓		(∃x)Ax & (∃x)Bx ✓			1 ~ ≡D
4.	Aa & Ba ✓					2 ∃D
5.	Aa					4 &D
6.	Ba					4 &D
7.	~ (∃x)Ax ✓	~ (∃x)Bx ✓				3 ~ &D
8.	(∀x) ~ Ax	(∀x) ~ Bx				7 ~ ∃D
9.	~ Aa	~ Ba				8 ∀D
10.	×	×	(∀x) ~ (Ax & Bx)			2 ~ ∃D
11.			(∃x)Ax ✓			3 &D
12.			(∃x)Bx ✓			3 &D
13.			Ab			11 ∃D
14.			Bc			12 ∃D
15.			~ (Ab & Bb) ✓			10 ∀D
16.			~ Ab	~ Bb		15 ~ &D
17.			×	~ (Ac & Bc) ✓		10 ∀D
18.				~ Ac	~ Bc	17 ~ &D
					×	

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

j. 1.	~ [(∀x)(Fx ≡ ~ Gx) ≡ (∀x) ~ (Fx ≡ Gx)]✓		SM
2.	(∀x)(Fx ≡ ~ Gx)	~ (∀x)(Fx ≡ ~ Gx)✓	1 ~ ≡D
3.	~ (∀x) ~ (Fx ≡ Gx)✓	(∀x) ~ (Fx ≡ Gx)	1 ~ ≡D
4.	(∃x) ~ ~ (Fx ≡ Gx)✓		3 ~ ∀D
5.	~ ~ (Fa ≡ Ga)✓		4 ∃D
6.	Fa ≡ Ga✓		5 ~ ~ D
7.	Fa	~ Fa		6 ≡D
8.	Ga	~ Ga		6 ≡D
9.	Fa ≡ ~ Ga✓	Fa ≡ ~ Ga✓		2 ∀D
10.	Fa	~ Fa	Fa	~ Fa		9 ≡D
11.	~ Ga	~ ~ Ga	~ Ga	~ ~ Ga✓		9 ≡D
12.	×	×	×	Ga		11 ~ ~ D
13.	× (fourth branch)	(∃x) ~ (Fx ≡ ~ Gx)✓	2 ~ ∀D
14.		~ (Fb ≡ ~ Gb)✓	13 ∃D
15.		Fb	~ Fb	14 ~ ≡D
16.		~ ~ Gb✓	~ Gb	14 ~ ≡D
17.		Gb		16 ~ ~ D
18.		~ (Fb ≡ Gb)✓	~ (Fb ≡ Gb)✓	3 ∀D
19.		Fb	~ Fb	Fb	~ Fb	18 ~ ≡D
20.		~ Gb	Gb	~ Gb	Gb	18 ~ ≡D
		×	×	×	×	

The tree is closed. Therefore the sentences '(∀x)(Fx ≡ ~ Gx)' and '(∀x) ~ (Fx ≡ Gx)' are quantificationally equivalent.

1. 1.	$\sim [(\forall x)(Fx \vee (\exists y)Gy) \equiv (\forall x)(\exists y)(Fx \vee Gy)]\checkmark$				SM
2.	$(\forall x)(Fx \vee (\exists y)Gy)$		$\sim (\forall x)(Fx \vee (\exists y)Gy)\checkmark$		$1 \sim \equiv D$
3.	$\sim (\forall x)(\exists y)(Fx \vee Gy)\checkmark$		$(\forall x)(\exists y)(Fx \vee Gy)$		$1 \sim \equiv D$
4.	$(\exists x) \sim (\exists y)(Fx \vee Gy)\checkmark$				$3 \sim \forall D$
5.	$\sim (\exists y)(Fa \vee Gy)\checkmark$				$4\ \exists D$
6.	$(\forall y) \sim (Fa \vee Gy)$				$5 \sim \exists D$
7.	$Fa \vee (\exists y)Gy\checkmark$				$2\ \forall D$
8.	Fa	$(\exists y)Gy\checkmark$			$7 \vee D$
9.	$\sim (Fa \vee Ga)\checkmark$				$6\ \forall D$
10.	$\sim Fa$				$9 \sim \vee D$
11.	$\sim Ga$				$9 \sim \vee D$
12.	$\times$	Gb			$8\ \exists D$
13.		$\sim (Fa \vee Gb)\checkmark$			$6\ \forall D$
14.		$\sim Fa$			$13 \sim \vee D$
15.		$\sim Gb$			$13 \sim \vee D$
16.		$\times$	$(\exists x) \sim (Fx \vee (\exists y)Gy)\checkmark$		$2 \sim \forall D$
17.			$\sim (Fc \vee (\exists y)Gy)\checkmark$		$16\ \exists D$
18.			$\sim Fc$		$17 \sim \vee D$
19.			$\sim (\exists y)Gy\checkmark$		$17 \sim \vee D$
20.			$(\forall y) \sim Gy$		$19 \sim \exists D$
21.			$(\exists y)(Fc \vee Gy)\checkmark$		$3\ \forall D$
22.			$Fc \vee Gd$		$21\ \exists D$
23.			Fc	Gd	$22 \vee D$
24.			$\times$	$\sim Gd$	$20\ \forall D$
				$\times$	

The tree is closed. Therefore the sentences '$(\forall x)(Fx \vee (\exists y)Gy)$' and '$(\forall x)(\exists y)(Fx \vee Gy)$' are quantificationally equivalent.

5. b. 1.	$(\forall x)(Tx \supset Lx)$		SM
2.	$\sim Lb$		SM
3.	$\sim \sim Tb\checkmark$		SM
4.	Tb		$3 \sim \sim D$
5.	$Tb \supset Lb\checkmark$		$1\ \forall D$
6.	$\sim Tb$	Lb	$5 \supset D$
	$\times$	$\times$	

The tree is closed. Therefore the argument is quantificationally valid.

d.	1.		$(\forall x)(Fx \supset Gx)$			SM
	2.		$(\forall x)(Hx \supset Gx)$			SM
	3.		$\sim (\forall x)((Fx \vee Hx) \supset Gx) \checkmark$			SM
	4.		$(\exists x) \sim ((Fx \vee Hx) \supset Gx) \checkmark$			$3 \sim \forall D$
	5.		$\sim ((Fa \vee Ha) \supset Ga) \checkmark$			$4\ \exists D$
	6.		$Fa \vee Ha \checkmark$			$5 \sim \supset D$
	7.		$\sim Ga$			$5 \sim \supset D$
	8.	Fa		Ha		$6 \vee D$
	9.	$Fa \supset Ga \checkmark$				$1\ \forall D$
	10.	$\sim Fa$	Ga			$9 \supset D$
	11.	$\times$	$\times$	$Ha \supset Ga \checkmark$		$2\ \forall D$
	12.			$\sim Ha$	Ga	$11 \supset D$
				$\times$	$\times$	

The tree is closed. Therefore the argument is quantificationally valid.

f.	1.		$(\sim (\exists y)Fy \supset (\exists y)Fy) \vee \sim Fa \checkmark$		SM
	2.		$\sim (\exists z)Fz \checkmark$		SM
	3.		$(\forall z) \sim Fz$		$2 \sim \exists D$
	4.		$\sim Fa$		$3\ \forall D$
	5.	$\sim (\exists y)Fy \supset (\exists y)Fy \checkmark$		$\sim Fa$	$1 \vee D$
	6.	$\sim \sim (\exists y)Fy \checkmark$	$(\exists y)Fy \checkmark$		$5 \supset D$
	7.	$(\exists y)Fy \checkmark$			$6 \sim \sim D$
	8.	Fb			$7\ \exists D$
	9.		Fb		$6\ \exists D$
	10.	$\sim Fb$	$\sim Fb$		$3\ \forall D$
		$\times$	$\times$		

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

h.	1.	(∀y)(Hy & (Jyy & My))		SM
	2.	~ ((∃x)Jxb & (∀x)Mx) ✓		SM
	3.	~ (∃x)Jxb ✓	~ (∀x)Mx ✓	2 ~ &D
	4.	(∀x) ~ Jxb		3 ~ ∃D
	5.		(∃x) ~ Mx ✓	3 ~ ∀D
	6.		~ Ma	5 ∃D
	7.	Hb & (Jbb & Mb) ✓	Ha & (Jaa & Ma) ✓	1 ∀D
	8.	Hb	Ha	7 &D
	9.	Jbb & Mb ✓	Jaa & Ma ✓	7 &D
	10.	Jbb	Jaa	9 &D
	11.	Mb	Ma	9 &D
	12.	~ Jbb	×	4 ∀D
		×		

The tree is closed. Therefore the argument is quantificationally valid.

j.	1.	(∃x)(Fx & Gx) ✓			SM
	2.	(∃x)(Fx & Hx) ✓			SM
	3.	~ (∃x)(Gx & Hx) ✓			SM
	4.	Fa & Ga ✓			1 ∃D
	5.	Fb & Hb ✓			2 ∃D
	6.	Fa			4 &D
	7.	Ga			4 &D
	8.	Fb			5 &D
	9.	Hb			5 &D
	10.	(∀x) ~ (Gx & Hx)			3 ~ ∃D
	11.	~ (Ga & Ha) ✓			10 ∀D
	12.	~ (Gb & Hb) ✓			10 ∀D
	13.	~ Ga	~ Ha		11 ~ &D
		×			
	14.		~ Gb	~ Hb	12 ~ &D
				×	

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

Note: '(∃z)Bzz' is misprinted as '(∃x)Bzz' in the text.

l.	1.	(∃z)Bzz ✓			SM
	2.	(∀x)(Sx ⊃ Bxx)			SM
	3.	~ ~ Sg ✓			SM
	4.	Sg			3 ~ ~ D
	5.	Baa			1 ∃D
	6.	Sg ⊃ Bgg ✓			2 ∀D
	7.	Sa ⊃ Baa ✓			2 ∀D
	8.	~ Sg ×	Bgg		6 ⊃D
	9.		~ Sa	Baa	7 ⊃D

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

n.	1.	Fa ∨ (∃y)Gya ✓				SM
	2.	Fb ∨ (∃y) ~ Gyb ✓				SM
	3.	~ (∃y)Gya ✓				SM
	4.	(∀y) ~ Gya				3 ~ ∃D
	5.	~ Gaa				4 ∀D
	6.	~ Gba				4 ∀D
	7.	Fa		(∃y)Gya ✓		1 ∨D
	8.	Fb	(∃y) ~ Gyb ✓	Fb	(∃y) ~ Gyb ✓	2 ∨D
	9.		~ Gcb		~ Gcb	8 ∃D
	10.		~ Gca		~ Gca	4 ∀D
	11.			Gda	Gda	7 ∃D
	12.			~ Gda ×	~ Gda ×	4 ∀D

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

6. b.

1.	$(\forall x)(\forall y)(Mxy \supset Nxy)$	SM
2.	$\sim (\forall x)(\forall y)(Mxy \supset (Nxy \,\&\, Nyx))$ ✓	SM
3.	$(\exists x) \sim (\forall y)(Mxy \supset (Nxy \,\&\, Nyx))$ ✓	$2 \sim \forall D$
4.	$\sim (\forall y)(May \supset (Nay \,\&\, Nya))$ ✓	$3\ \exists D$
5.	$(\exists y) \sim (May \supset (Nay \,\&\, Nya))$ ✓	$4 \sim \forall D$
6.	$\sim (Mab \supset (Nab \,\&\, Nba))$ ✓	$5\ \exists D$
7.	Mab	$6 \sim \supset D$
8.	$\sim (Nab \,\&\, Nba)$ ✓	$6 \sim \supset D$
9.	$(\forall y)(May \supset Nay)$	$1\ \forall D$
10.	$(\forall y)(Mby \supset Nby)$	$1\ \forall D$
11.	$Mab \supset Nab$ ✓	$9\ \forall D$
12.	$Maa \supset Naa$ ✓	$9\ \forall D$
13.	$Mba \supset Nba$ ✓	$10\ \forall D$
14.	$Mbb \supset Nbb$ ✓	$10\ \forall D$
15.	$\sim Mab$ × Nab	$11 \supset D$
16.	(under Nab:) $\sim Nab$ × $\sim Nba$	$8 \sim \& D$
17.	(under $\sim Nba$:) $\sim Mba$ Nba ×	$13 \supset D$
18.	(under $\sim Mba$:) $\sim Maa$ Naa	$12 \supset D$
19.	$\sim Mbb$ Nbb $\sim Mbb$ Nbb	$14 \supset D$

The tree has at least one completed open branch. Therefore, the alleged entailment does not hold.

d. 1.	(∀x) (Fx ⊃ Gx)				SM
2.	(∀x) (Hx ⊃ Gx)				SM
3.	~ (∀x) (Fx ∨ Hx)✓				SM
4.	(∃x) ~ (Fx ∨ Hx)✓				3 ~ ∀D
5.	~ (Fa ∨ Ha)✓				4 ∃D
6.	~ Fa				5 ~ ∨D
7.	~ Ha				5 ~ ∨D
8.	Fa ⊃ Ga✓				1 ∀D
9.	Ha ⊃ Ga✓				2 ∀D
10.	~ Fa		Ga		8 ⊃D
11.	~ Ha	Ga	~ Ha	Ga	9 ⊃D

The tree has at least one completed open branch. Therefore, the alleged entailment does not hold.

Section 9.4E

Note: In this section and the next we sometime stop when we have one or more completed open branches, leaving one or more branch that is neither closed nor a completed open branch.

1. b. 1.	(∀x) (Fxc ⊃ x = a)				SM
2.	~ c = a				SM
3.	(∃x) Fxc✓				SM
4.	Fbc				3 ∃D
5.	Fbc ⊃ b = a✓				1 ∀D
6.	~ Fbc	b = a			5 ⊃D
7.	×	Fac ⊃ a = a✓			1 ∀D
8.		~ Fac	a = a		7 ⊃D
9.		~ Fbc			6, 8 =D
10.		×	Fcc ⊃ c = a✓		1 ∀D
11.			~ Fcc	c = a	10 ⊃D
12.			~ c = b	×	2, 6 =D
13.			a = b		6, 8 =D
14.			Fac		4, 13 =D
15.			~ c = a		13, 12 =D
16.			Fbc		8, 14 =D

The tree has one completed open branch. The set is quantificationally consistent.

d. 1.			$(\exists x)(\exists y) \sim x = y$ ✓			SM
2.			$(\forall x)(Gxx \supset x = b)$			SM
3.			Gaa			SM
4.			$(\exists y) \sim c = y$ ✓			$1\ \exists D$
5.			$\sim c = d$			$4\ \exists D$
6.			$Gaa \supset a = b$ ✓			$2\ \forall D$
7.			$Gbb \supset b = b$ ✓			$2\ \forall D$
8.			$Gcc \supset c = b$ ✓			$2\ \forall D$
9.			$Gdd \supset d = b$ ✓			$2\ \forall D$
10.	$\sim Gaa$			$a = b$		$6 \supset D$
	×					
11.	$\sim Gbb$			$b = b$		$7 \supset D$
12.	$\sim Gaa$					$10, 11 = D$
13.	×	$\sim Gcc$		$c = b$		$8 \supset D$
14.		$\sim Gdd$	$d = b$	$\sim Gdd$	$d = b$	$9 \supset D$
15.					$c = d$	$13, 14 = D$
16.		$b = a$	$b = a$	$b = a$	×	$10, 11 = D$
17.		Gbb	Gbb	Gbb		$3, 16 = D$
18.		Gba	Gba	Gba		$3, 16 = D$
19.		Gab	Gab	Gab		$3, 16 = D$
20.			$a = d$			$14, 10 = D$
21.			$b = d$			$14, 11 = D$
22.			Gdd			$14, 17 = D$
23.			Gbd			$14, 17 = D$
24.			Gdb			$14, 17 = D$
25.			Gda			$14, 18 = D$
26.			Gad			$14, 19 = D$
27.			$\sim c = a$			$20, 5 = D$
28.			$\sim c = b$			$21, 5 = D$
29.			$a = b$			$20, 21 = D$
30.			$b = a$			$20, 21 = D$
31.			$d = a$			$10, 14 = D$
32.			Gaa			$29, 18 = D$
33.				$a = c$		$10, 11 = D$
34.				$c = a$		$10, 13 = D$
35.				$b = c$		$16, 33 = D$
36.				$\sim a = d$		$33, 5 = D$
37.				$\sim b = d$		$35, 5 = D$
38.				Gcc		$34, 3 = D$
39.				Gca		$34, 3 = D$
40.				Gac		$34, 3 = D$
41.				Gbc		$35, 38 = D$
42.				Gcb		$35, 38 = D$

The tree has three completed open branches (the leftmost three). The set is quantificationally consistent.

f. 1.	$(\exists y)(\forall x)Fxy$ ✓		SM
2.	$\sim (\forall x)(\forall y)x = y$ ✓		SM
3.	$Fab \,\&\, \sim Fba$ ✓		SM
4.	$(\exists x) \sim (\forall y)x = y$ ✓		2 $\sim \forall$D
5.	$\sim (\forall y)c = y$ ✓		4 $\exists$D
6.	$(\exists y) \sim c = y$ ✓		5 $\sim \forall$D
7.	$\sim c = d$		6 $\exists$D
8.	$(\forall x)Fxe$		1 $\exists$D
9.	Fab		3 &D
10.	$\sim Fba$		3 &D
11.	Fae		8 $\forall$D
12.	Fbe		8 $\forall$D
13.	Fce		8 $\forall$D
14.	Fde		8 $\forall$D
15.	Fee		8 $\forall$D

The tree has one completed open branch. Therefore, the set is quantificationally consistent.

h. 1.	$(\forall x)(Gxx \supset x = f(x,b)$		SM
2.	Gaa		SM
3.	$(\forall x) \sim f(a,x) = a$		SM
4.	$Gaa \supset a = f(a,b)$ ✓		1 $\forall$D
5.	$\sim Gaa$	$a = f(a,b)$	4 $\supset$D
6.	×		
7.		$\sim f(a,b) = a$	3 $\forall$D
8.		$\sim a = a$	5, 7 =D
		×	

The tree is closed. Therefore, the set is quantificationally inconsistent.

j. 1.	$(\exists x)(\exists y)f(x,y) = f(y,x)$ ✓		SM
2.	$(\forall x)[f(x,a) = f(a,x) \supset \sim a = x]$		SM
3.	$(\exists y)f(b,y) = f(y,b)$ ✓		1 $\exists$D
4.	$f(b,c) = f(c,b)$		3 $\exists$D
5.	$f(a,a) = f(a,a) \supset \sim a = a$ ✓		2 $\forall$D
6.	$f(b,a) = f(a,b) \supset \sim a = b$		2 $\forall$D
7.	$f(c,a) = f(c,x) \supset \sim a = c$		2 $\forall$D
8.	$\sim f(a,a) = f(a,a)$	$\sim a = a$	5 $\supset$D
	×	×	

The tree is closed. Therefore, the set is quantificationally inconsistent.

l.

1.	$Hf(a,b)$				SM
2.	$(\forall x)(Hx \supset \sim Gx)$				SM
3.	$(\exists y)Gy$ ✓				SM
4.	Gc				3 ∃D
5.	$Ha \supset \sim Ga$ ✓				2 ∀D
6.	$Hc \supset \sim Gc$ ✓				2 ∀D
7.	$Hb \supset \sim Gb$ ✓				2 ∀D
8.	$Hf(a,b) \supset \sim Gf(a,b)$ ✓				2 ∀D
9.	$\sim Hf(a,b)$ ×	$\sim Gf(a,b)$			8 ⊃D
10.		$\sim Hc$		$\sim Gc$ ×	6 ⊃D
11.		$\sim Hb$	Gb		7 ⊃D
12.		$\sim Ha$ $\sim Ga$	$\sim Ha$ $\sim Ga$		5 ⊃D

The tree has four completed open branches. Therefore, the set is quantificationally consistent.

n.

1.	$\sim (\exists x)[x = f(s) \ \& \ (\forall y)(y = f(s) \supset y = x)]$ ✓		SM
2.	$(\forall x) \sim [x = f(s) \ \& \ (\forall y)(y = f(s) \supset y = x)]$		1 ~ ∃D
3.	$\sim [s = f(s) \ \& \ (\forall y)(y = f(s) \supset y = s)]$		2 ∀D
4.	$\sim [f(s) = f(s) \ \& \ (\forall y)(y = f(s) \supset y = f(s))]$ ✓		2 ∀D
5.			
6.	$\sim f(s) = f(s)$	$\sim (\forall y)(y = f(s) \supset y = f(s))$ ✓	4 ~ &D
7.	×	$(\exists y) \sim (y = f(s) \supset y = f(s))$ ✓	6 ~ ∀D
8.		$\sim (a = f(s) \supset a = f(s))$ ✓	7 ∃D
9.		$a = f(s)$	8 ~ ⊃D
10.		$\sim a = f(s)$	8 ~ ⊃D
		×	

The tree is closed. Therefore, the set is quantificationally inconsistent.

2. b.

1.	~ ((~ a = b & ~ b = c) ⊃ ~ a =c)✓	SM
2.	~ a = b & ~ b = c✓	1 ~ ⊃D
3.	~ ~ a = c✓	1 ~ ⊃D
4.	a = c	3 ~ ~ D
5.	~ a = b	2 &D
6.	~ b = c	2 &D
7.	~ b = a	4, 6 =D

1.	(~ a = b & ~ b = c) ⊃ ~ a = c✓			SM
2.	~ (~ a = b & ~ b = c)✓		~ a = c	1 ⊃D
3.	~ ~ a = b✓	~ ~ b = c✓		2 ~ &D
4.	a = b	b = c		3 ~ ~ D

Both trees are open. Therefore the sentence '(~ a = b & ~ b = c) ⊃ ~ a = c' is quantificationally indeterminate.

d.

1.	~ (∀x)(∃y)x = y✓	SM
2.	(∃x) ~ (∃y)x = y✓	1 ~ ∀D
3.	~ (∃y)a = y✓	2 ∃D
4.	(∀y) ~ a = y	3 ~ ∃D
5.	~ a = a	4 ∀D
	×	

The tree is closed. Therefore, the given sentence is quantificationally true.

f.

1.	~ (∃x)x = a✓	SM
2.	(∀x) ~ x = a	1 ~ ∃D
3.	~ a = a	2 ∀D
	×	

The tree is closed. Therefore the sentence '~ (∃x)x = a' is quantificationally false.

h.

1.	~ (∀x)(∀y)x = y✓	SM
2.	(∃x) ~ (∀y)x = y✓	1 ~ ∀D
3.	~ (∀y)a = y✓	2 ∃D
4.	(∃y) ~ a = y✓	3 ~ ∀D
5.	~ a = b	4 ∃D

1.	(∀x)(∀y)x = y✓	SM
2.	(∀y)a = y	1 ∀D
3.	a = a	2 ∀D

Both trees are open. Therefore the sentence '(∀x)(∀y)x = y' is quantificationally indeterminate.

j.	1.	$\sim (\exists x)(\exists y)x = y \checkmark$		SM
	2.	$(\forall x) \sim (\exists y)x = y$		$1 \sim \exists D$
	3.	$\sim (\exists y)a = y \checkmark$		$2\ \forall D$
	4.	$(\forall y) \sim a = y$		$3 \sim \exists D$
	5.	$\sim a = a$		$4\ \forall D$
		$\times$		

The tree is closed. Therefore the sentence '$(\exists x)(\exists y)x = y$' is quantificationally true.

l.	1.	$\sim (\forall x)(\forall y)[x = y \supset (Fx \equiv Fy)] \checkmark$		SM
	2.	$(\exists x) \sim (\forall y)[x = y \supset (Fx \equiv Fy)] \checkmark$		$1 \sim \forall D$
	3.	$\sim(\forall y)[a = y \supset (Fa \equiv Fy)] \checkmark$		$2\ \exists D$
	4.	$(\exists y) \sim [a = y \supset (Fa \equiv Fy)] \checkmark$		$3 \sim \forall D$
	5.	$\sim [a = b \supset (Fa \equiv Fb)] \checkmark$		$4\ \exists D$
	6.	$a = b$		$5 \sim \supset D$
	7.	$\sim (Fa \equiv Fb) \checkmark$		$5 \sim \supset D$
	8.	Fa	$\sim Fa$	$7 \sim \equiv D$
	9.	$\sim Fb$	Fb	$7 \sim \equiv D$
	10.	$\sim Fa$	Fa	$6, 9 = D$
		$\times$	$\times$	

The tree is closed. Therefore the sentence '$(\forall x)(\forall y)\ [x = y \supset (Fx \equiv Fy)]$' is quantificationally true.

n.	1.	$\sim (\forall x)(\forall y)(x = y \supset (\forall z)(Fxz \equiv Fyz)) \checkmark$		SM
	2.	$(\exists x) \sim (\forall y)(x = y \supset (\forall z)(Fxz \equiv Fyz)) \checkmark$		$1 \sim \forall D$
	3.	$\sim (\forall y)(a = y \supset (\forall z)(Faz \equiv Fyz)) \checkmark$		$2\ \exists D$
	4.	$(\exists y) \sim (a = y \supset (\forall z)(Faz \equiv Fyz)) \checkmark$		$3 \sim \forall D$
	5.	$\sim (a = b \supset (\forall z)(Faz \equiv Fbz)) \checkmark$		$4\ \exists D$
	6.	$a = b$		$5 \sim \supset D$
	7.	$\sim (\forall z)(Faz \equiv Fbz) \checkmark$		$5 \sim \supset D$
	8.	$(\exists z) \sim (Faz \equiv Fbz) \checkmark$		$7 \sim \forall D$
	9.	$\sim (Fac \equiv Fbc) \checkmark$		$8\ \exists D$
	10.	Fac	$\sim Fac$	$9 \sim \equiv D$
	11.	$\sim Fbc$	Fbc	$8 \sim \equiv D$
	12.	$\sim Fac$	Fac	$6, 11 = D$
		$\times$	$\times$	

The tree is closed. Therefore the sentence '$(\forall x)(\forall y)(x = y \supset (\forall z)(Fxz \equiv Fyz))$' is quantificationally true.

3. b.

1.	$\sim (\forall x)(\exists y)y = f(x)$	SM
2.	$(\exists x) \sim (\exists y)y = f(x)$	$1 \sim \forall D$
3.	$\sim (\exists y)y = f(a)$	$2\ \exists D$
4.	$(\forall y) \sim y = f(a)$	$3 \sim \exists D$
5.	$\sim a = f(a)$	$4\ \forall D$
6.	$\sim f(a) = f(a)$	$4\ \forall D$
	$\times$	

The tree for the negation of the given sentence is closed. Therefore, the given sentence is quantificationally true.

d.

1.	$\sim (\exists x)(\exists y)x = f(y)$	SM
2.	$(\forall x) \sim (\exists y)x = f(y)$	$1 \sim \exists D$
3.	$\sim (\exists y) f(a) = f(y)$	$2\ \forall D$
4.	$(\forall y) \sim f(a) = f(y)$	$3 \sim \exists D$
5.	$\sim f(a) = f(a)$	$4\ \forall D$
	$\times$	

The tree for the negation of the given sentence is closed. Therefore, the given sentence is quantificationally true.

f.

1.	$\sim (\forall x)(\forall y)[x = y \supset f(x) = f(y)]$	SM
2.	$(\exists x) \sim (\forall y)[x = y \supset f(x) = f(y)]$	$1 \sim \forall D$
3.	$\sim (\forall y)[a = y \supset f(a) = f(y)]$	$2\ \exists D$
4.	$(\exists y) \sim [a = y \supset f(a) = f(y)]$	$3 \sim \forall D$
5.	$\sim [a = b \supset f(a) = f(b)]$	$4\ \exists D$
6.	$a = b$	$5 \sim \supset D$
7.	$\sim f(a) = f(b)$	$5 \sim \supset D$
8.	$\sim f(a) = f(a)$	$6, 7 = D$
	$\times$	

The tree for the negation of the given sentence is closed. Therefore, the given sentence is quantificationally true.

h.	1.	$\sim (\forall x)(\exists y)[y = f(x)\ \&\ (\forall z)(z = f(x) \supset z = y)]$		SM
	2.	$(\exists x) \sim (\exists y)[y = f(x)\ \&\ (\forall)(z = f(x) \supset z = y)]$ ✓		$1 \sim \forall D$
	3.	$\sim (\exists y)[y = f(a)\ \&\ (\forall)(z = f(a) \supset z = y)]$ ✓		$2\ \exists D$
	4.	$(\forall y) \sim [y = f(a)\ \&\ (\forall)(z = f(a) \supset z = y)]$		$3 \sim \exists D$
	5.	$\sim [a = f(a)\ \&\ (\forall)(z = f(a) \supset z = a)]$		$4\ \forall D$
	6.	$\sim [f(a) = f(a)\ \&\ (\forall)(z = f(a) \supset z = f(a)]$ ✓		$4\ \forall D$
	7.	$\sim f(a) = f(a)$	$\sim (\forall z)(z = f(a) \supset z = f(a))$	$5 \sim \&D$
	8.	×	$(\exists z) \sim (z = f(a) \supset z = f(a)$	$7 \sim \forall D$
	9.		$\sim (b = f(a) \supset b = f(a)$	$8\ \exists D$
	10.		$b = f(a)$	$9 \sim \supset D$
	11.		$\sim b = f(a)$	$9 \sim \supset D$
			×	

The tree for the negation of the given sentence is closed. Therefore, the given sentence is quantificationally true.

4. b.	1.	$\sim ((\exists x) \sim x = a \equiv (\exists x) \sim x = b)$		SM
	2.	$(\exists x) \sim x = a$ ✓	$\sim (\exists x) \sim x = a$ ✓	$1 \sim \equiv D$
	3.	$\sim (\exists x) \sim x = b$ ✓	$(\exists x) \sim x = b$ ✓	$1 \sim \equiv D$
	4.	$(\forall x) \sim\sim x = b$		$3 \sim \exists D$
	5.	$\sim c = a$		$2\ \exists D$
	6.	$\sim\sim a = b$ ✓		$4\ \forall D$
	7.	$\sim\sim b = b$		$4\ \forall D$
	8.	$\sim\sim c = b$ ✓		$4\ \forall D$
	9.	$a = b$		$6 \sim\sim D$
	10.	$c = b$		$8 \sim\sim D$
	11.	$c = a$		$9, 10 = D$
	12.	×	$(\forall x) \sim\sim x = a$	$2 \sim \exists D$
	13.		$\sim c = b$	$4\ \exists D$
	14.		$\sim\sim a = a$	$12\ \forall D$
	15.		$\sim\sim b = a$ ✓	$12\ \forall D$
	16.		$\sim\sim c = a$ ✓	$12\ \forall D$
	17.		$b = a$	$15 \sim\sim D$
	18.		$c = a$	$16 \sim\sim D$
	19.		$c = b$	$17, 18 = D$
			×	

The tree is closed. Therefore the given sentences are quantificationally equivalent.

d.

1.	~ [(a = b & b = c) ≡ (a = c & b = c)] ✓		SM
2.	a = b & b = c ✓	~ (a = b & b = c) ✓	1 ~ ≡D
3.	~ (a = c & b = c) ✓	a = c & b = c ✓	1 ~ ≡D
4.	a = b		2 &D
5.	b = c		2 &D
6.	~ a = c ~ b = c		3 ~ &D
7.	a = c ×		4, 5 =D
8.	×	a = c	3 &D
9.		b = c	3 &D
10.		~ a = b ~ b = c	2 ~ &D
11.		a = b ×	8, 9 =D
		×	

The tree is closed. Therefore the sentences '(a = b & b = c)' and '(a = c & b = c)' are quantificationally equivalent.

f.

1.	~ ((∀x)(∃y)x = y ≡ (∀y)(∃x)x = y) ✓		SM
2.	(∀x)(∃y)x = y	~ (∀x)(∃y)x = y ✓	1 ~ ≡D
3.	~ (∀y)(∃x)x = y ✓	(∀y)(∃x)x = y	1 ~ ≡D
4.	(∃y) ~ (∃x)x = y ✓		3 ~ ∀D
5.	~ (∃x)x = a ✓		4 ∃D
6.	(∀x) ~ x = a		5 ~ ∃D
7.	~ a = a		6 ∀D
8.	×	(∃x) ~ (∃y)x = y ✓	2 ~ ∀D
9.		~ (∃y)a = y ✓	8 ∃D
10.		(∀y) ~ a = y	9 ~ ∃D
11.		~ a = a	10 ∀D
		×	

The tree is closed. Therefore the sentences '(∀x)(∃y)x = y' and '(∀y)(∃x)x = y' are quantificationally equivalent.

h.

1.	$\sim [(\forall x)(x = a \vee x = b) \equiv ((\forall x)x = a \vee (\forall x)x = b)]$ ✓		SM
2.	$(\forall x)(x = a \vee x = b)$	$\sim (\forall x)(x = a \vee x = b)$ ✓	$1 \sim \equiv D$
3.	$\sim ((\forall x)x = a \vee (\forall x)x = b)$ ✓	$(\forall x)x = a \vee (\forall x)x = b$ ✓	$1 \sim \equiv D$
4.		$(\exists x) \sim (x = a \vee x = b)$ ✓	$2 \sim \forall D$
5.		$\sim (c = a \vee c = b)$ ✓	$4\ \exists D$
6.		$\sim c = a$	$5 \sim \vee D$
7.		$\sim c = b$	$5 \sim \vee D$
8.		$(\forall x)x = a$; $(\forall x)x = b$	$3 \vee D$
9.		$c = a$; $c = b$	$8\ \forall D$
		× ; ×	
10.	$\sim (\forall x)x = a$ ✓		$3 \sim \vee D$
11.	$\sim (\forall x)x = b$ ✓		$3 \sim \vee D$
12.	$(\exists x) \sim x = a$ ✓		$10 \sim \forall D$
13.	$(\exists x) \sim x = b$ ✓		$11 \sim \forall D$
14.	$\sim c = a$		$12\ \exists D$
15.	$\sim d = b$		$13\ \exists D$
16.	$a = a \vee a = b$ ✓		$2\ \forall D$
17.	$b = a \vee b = b$ ✓		$2\ \forall D$
18.	$c = a \vee c = b$ ✓		$2\ \forall D$
19.	$d = a \vee d = b$ ✓		$2\ \forall D$
20.	$c = a$ × ; $c = b$		$18 \vee D$
21.	(under $c = b$) $d = a$; $d = b$ ×		$19 \vee D$
22.	(under $d = a$) $a = a$; $a = b$		$16 \vee D$
23.	(under $a = b$) $\sim d = a$		$15, 22 = D$
24.	(under $a = a$) $b = a$; $b = b$; (under $a = b$) ×		$17 \vee D$
25.	(under $b = a$) $d = b$		$21, 24 = D$
26.	(under $b = a$) × ; (under $b = b$) $\sim d = c$		$15, 20 = D$
27.	$b = c$		$20, 24 = D$
28.	$\sim c = d$		$21, 14 = D$
29.	$a = d$		$21, 22 = D$
30.	$\sim b = d$		$27, 28 = D$
31.	$\sim b = a$		$14, 27 = D$
32.	$\sim a = b$		$15, 29 = D$
33.	$\sim c = a$		$31, 20 = D$
34.	$\sim a = c$		$20, 32 = D$

The tree has at least one completed open branch, therefore the given sentences are not quantificationally equivalent.

j. 1.	~ (a = b ≡ (∀y) (y = a ⊃ y = b))✓			SM
2.	a = b	~ a = b		1 ~ ≡D
3.	~ (∀y) (y = a ⊃ y = b)✓	(∀y) (y = a ⊃ y = b)		1 ~ ≡D
4.	(∃y) ~ (y = a ⊃ y = b)✓			3 ~ ∀D
5.	~ (c = a ⊃ c = b)✓			4 ∃D
6.	c = a			5 ~ ⊃D
7.	~ c = b			5 ~ ⊃D
8.	c = b			2, 6 =D
9.	×	a = a ⊃ a = b✓		3 ∀D
10.		b = a ⊃ b = b		3 ∀D
11.		~ a = a	a = b	9 ⊃ D
		×	×	

The tree is closed. Therefore the sentences ‘a = b’ and ‘(∀y) (y = a ⊃ y = b)’ are quantificationally equivalent.

5. b. 1.	Ge ⊃ d = e✓			SM
2.	Ge ⊃ He✓			SM
3.	~ (Ge ⊃ Hd)✓			SM
4.	Ge			3 ~ ⊃D
5.	~ Hd			3 ~ ⊃D
6.	~ Ge	d = e		1 ⊃D
	×			
7.		~ Ge	He	2 ⊃D
		×	Hd	6, 7 =D
			×	

The tree is closed. Therefore the argument is quantificationally valid.

d.

1.	(∃x)(Hx & Mx) ✓			SM
2.	Ms & ~ Hs ✓			SM
3.	~ (∃x)((Hx & Mx) & ~ x = s) ✓			SM
4.	Ms			2 &D
5.	~ Hs			2 &D
6.	(∀x) ~ ((Hx & Mx) & ~ x = s)			3 ~ ∃D
7.	Ha & Ma ✓			1 ∃D
8.	Ha			7 &D
9.	Ma			7 &D
10.	~ ((Ha & Ma) & ~ a = s) ✓			6 ∀D
11.	~ (Ha & Ma) ✓		~ ~ a = s ✓	10 ~ &D
12.	~ Ha	~ Ma		11 ~ &D
13.	×	×	a = s	11 ~ ~ D
14.			~ Ha	5, 13 =D
			×	

The tree is closed. Therefore the argument is quantificationally valid.

f.

1.	(∃x) ~ Pxx ⊃ ~ a = a ✓		SM
2.	a = c		SM
3.	~ Pac		SM
4.	~ (∃x) ~ Pxx ✓	~ a = a	1 ⊃D
5.	(∀x) ~ ~ Pxx	×	4 ~ ∃D
6.	~ ~ Pcc ✓		5 ∀D
7.	Pcc		6 ~ ~ D
8.	Pac		2, 7 =D
	×		

The tree is closed. Therefore the argument is quantificationally valid.

h.

1.	$(\exists x)Fxa$ ✓				SM
2.	$(\forall y)(y = a \supset y = b)$				SM
3.	$\sim (\exists y)Fyy$ ✓				SM
4.	$(\forall y) \sim Fyy$				$3 \sim \exists D$
5.	Fca				$1\ \exists D$
6.	$a = a \supset a = b$ ✓				$2\ \forall D$
7.	$b = a \supset b = b$ ✓				$2\ \forall D$
8.	$c = a \supset c = b$ ✓				$2\ \forall D$
9.	$\sim Faa$				$4\ \forall D$
10.	$\sim Fbb$				$4\ \forall D$
11.	$\sim Fcc$				$4\ \forall D$
12.	$\sim a = a$			$a = b$	$6 \supset D$
	×				
13.		$\sim b = a$		$b = b$	$7 \supset D$
14.		$\sim a = a$			$12, 13 = D$
15.		×	$\sim c = a$	$c = b$	$8 \supset D$
16.				$\sim Fcb$	$10, 15 = D$
17.				$\sim Fca$	$12, 16 = D$
18.			$\sim Fab$	×	$12, 10 = D$
19.			$\sim Fba$		$12, 10 = D$
20.			$b = a$		$12, 13 = D$
21.			Fcb		$20, 5 = D$
22.			$\sim c = b$		$20, 15 = D$

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

j.	1.	$(\exists x)Fxa \mathbin{\&} (\exists x)Fxb \checkmark$	SM
	2.	$\sim a = b$	SM
	3.	$\sim (\forall x)(\forall y)((Fxa \mathbin{\&} Fyb) \supset \sim x = y) \checkmark$	SM
	4.	$(\exists x)Fxa \checkmark$	1 &D
	5.	$(\exists x)Fxb \checkmark$	1 &D
	6.	$(\exists x) \sim (\forall y)((Fxa \mathbin{\&} Fyb) \supset \sim x = y) \checkmark$	3 $\sim \forall$D
	7.	Fca	4 $\exists$D
	8.	Fdb	5 $\exists$D
	9.	$\sim (\forall y)((Fea \mathbin{\&} Fyb) \supset \sim e = y) \checkmark$	6 $\exists$D
	10.	$(\exists y) \sim ((Fea \mathbin{\&} Fyb) \supset \sim e = y) \checkmark$	9 $\sim \forall$D
	11.	$\sim ((Fea \mathbin{\&} Ffb) \supset \sim e = f) \checkmark$	10 $\exists$D
	12.	$Fea \mathbin{\&} Ffb \checkmark$	11 $\sim \supset$D
	13.	$\sim \sim e = f \checkmark$	11 $\sim \supset$D
	14.	Fea	12 &D
	15.	Ffb	12 &D
	16.	$e = f$	13 $\sim \sim$ D
	17.	Feb	15, 16 =D

The tree has at least one completed open branch. Therefore the argument is quantificationally invalid.

l.	1.	$\sim (\exists x)Fxx \checkmark$	SM
	2.	$\sim (\forall x)(\forall y)(Fxy \supset \sim x = y) \checkmark$	SM
	3.	$(\forall x) \sim Fxx$	1 $\sim \exists$D
	4.	$(\exists x) \sim (\forall y)(Fxy \supset \sim x = y) \checkmark$	2 $\sim \forall$D
	5.	$\sim (\forall y)(Fay \supset \sim a = y) \checkmark$	4 $\exists$D
	6.	$(\exists y) \sim (Fay \supset \sim a = y) \checkmark$	5 $\sim \forall$D
	7.	$\sim (Fab \supset \sim a = b) \checkmark$	6 $\exists$D
	8.	Fab	7 $\sim \supset$D
	9.	$\sim \sim a = b \checkmark$	7 $\sim \supset$D
	10.	$a = b$	9 $\sim \sim$ D
	11.	$\sim Fbb$	3 $\forall$D
	12.	$\sim Fab$	10, 11 =D
		$\times$	

The tree is closed. Therefore the argument is quantificationally valid.

n.

1.	$(\forall x)(\sim x = a \equiv (\exists y)Gyx)$		SM
2.	Gbc		SM
3.	$\sim\sim c = a \checkmark$		SM
4.	$c = a$		$3 \sim\sim D$
5.	$\sim a = a \equiv (\exists y)Gya \checkmark$		$1\ \forall D$
6.	$\sim a = a$	$\sim\sim a = a \checkmark$	$5 \equiv D$
7.	$(\exists y)Gya$	$\sim (\exists y)Gya \checkmark$	$5 \equiv D$
8.	$\times$	$a = a$	$6 \sim\sim D$
9.		$(\forall y) \sim Gya$	$7 \sim \exists D$
10.		$\sim Gba$	$9\ \forall D$
11.		$\sim Gbc$	$4, 10 = D$
		$\times$	

The tree is closed. Therefore the argument is quantificationally valid.

p.

1.	$(\forall y)(Hy \supset g(y) = y)$			SM
2.	$(\exists x) \sim g(x) = x \checkmark$			SM
3.	$\sim (\exists x) \sim Hx \checkmark$			SM
4.	$\sim g(a) = a$			$2\ \exists D$
5.	$(\forall x) \sim Hx$			$3 \sim \exists D$
6.	$Ha \supset g(a) = a \checkmark$			$1\ \forall D$
7.	$\sim Ha$			$5\ \forall D$
8.	$\sim Ha$		$g(a) = a$	$6 \supset D$
9.	$Hg(a) \supset g(g(a)) = g(a) \checkmark$		$\times$	$1\ \forall D$
10.	$\sim Hg(a)$			$5\ \forall D$
11.	$\sim Hg(a)$	$g(g(a)) = g(a)$		$9 \supset D$

As of line 11, the tree has one completed open branch (the leftmost). The argument is therefore quantificationally invalid. The right hand branch is closed, the middle branch is not closed but is also not a completed open branch.

r.

1.	$(\exists x)h(x) = x \checkmark$		SM
2.	$(\forall x)(Fx \supset \sim Fh(x))$		SM
3.	$\sim (\exists x) \sim Fx \checkmark$		SM
4.	$h(a) = a$		$1\ \exists D$
5.	$(\forall x) \sim\sim Fx$		$4 \sim \exists D$
6.	$\sim\sim Fa$		$5\ \forall D$
7.	Fa		$6 \sim\sim D$
8.	$Fa \supset \sim Fh(a)$		$2\ \forall D$
9.	$\sim Fa$	$\sim Fh(a)$	$8 \supset D$
10.	$\times$	$Fh(a)$	$3, 7 = D$
		$\times$	

The tree is closed. Therefore, the argument is quantificationally valid.

t.

1.	$(\forall x)(\forall y)(Hxy \supset f(x) = y)$		SM
2.	$(\exists x)Hxx$		SM
3.	$\sim (\exists x)f(x) = x$		SM
4.	Haa		2 $\exists$D
5.	$(\forall x) \sim f(x) = x$		3 $\sim \exists$D
6.	$(\forall y)(Hay \supset f(a) = y)$		1 $\forall$D
7.	$Haa \supset f(a) = a$		6 $\forall$D
8.	$\sim Haa$	$f(a) = a$	7 $\supset$D
9.	$\times$	$\sim f(a) = a$	5 $\forall$D
		$\times$	

The tree is closed. Therefore, the argument is quantificationally valid.

6. b.

1.	$\sim (\exists x)(Fxa \vee Fxb)\checkmark$					SM
2.	$(\forall x)(\forall y)(Fxy \supset \sim x = y)$					SM
3.	$\sim \sim a = b\checkmark$					SM
4.	$a = b$					3 $\sim \sim$ D
5.	$(\forall x) \sim (Fxa \vee Fxb)$					1 $\sim \exists$D
6.	$(\forall y)(Fay \supset \sim a = y)$					2 $\forall$D
7.	$(\forall y)(Fby \supset \sim b = y)$					2 $\forall$D
8.	$Faa \supset \sim a = a\checkmark$					6 $\forall$D
9.	$Fab \supset \sim a = b\checkmark$					6 $\forall$D
10.	$Fba \supset \sim b = a\checkmark$					7 $\forall$D
11.	$Fbb \supset \sim b = b\checkmark$					7 $\forall$D
12.	$\sim (Faa \vee Fab)\checkmark$					5 $\forall$D
13.	$\sim (Fba \vee Fbb)\checkmark$					5 $\forall$D
14.	$\sim Faa$				$\sim a = a$	8 $\supset$D
					$\times$	
15.	$\sim Fab$			$\sim a = b$		9 $\supset$D
				$\times$		
16.	$\sim Fba$		$\sim b = a$			10 $\supset$D
17.	$\vert$		$\sim a = a$			4, 16 =D
18.	$\sim Faa$		$\times$			12 $\sim \vee$D
19.	$\sim Fab$					12 $\sim \vee$D
20.	$\sim Fbb$	$\sim b = b$				11 $\supset$D
21.	$\sim Fba$	$\times$				13 $\sim \vee$D
22.	$\sim Fbb$					13 $\sim \vee$D

The tree has at least one completed open branch. Therefore, the alleged entailment does not hold.

d.

1.	$(\forall x)(\exists y)(Fxy \,\&\, \sim x = y)$		SM
2.	$a = b$		SM
3.	Fab		SM
4.	$\sim (\exists y)(Fay \,\&\, y = b)$ ✓		SM
5.	$(\forall y) \sim (Fay \,\&\, y = b)$		$4 \sim \exists D$
6.	$(\exists y)(Fay \,\&\, \sim a = y)$		$1\ \forall D$
7.	$(\exists y)(Fby \,\&\, \sim b = y)$		$1\ \forall D$
8.	$\sim (Faa \,\&\, a = b)$		$5\ \forall D$
9.	$\sim (Fab \,\&\, b = b)$ ✓		$5\ \forall D$
10.	$\sim Fab$	$\sim b = b$	$9 \sim \& D$
	×	×	

The tree is closed. Therefore the entailment does hold.

f.

1.	$(\exists w)(\forall y)Gwy$ ✓				SM
2.	$(\exists w)(\forall y)(\sim w = y \supset \sim Gwy)$ ✓				SM
3.	$\sim (\exists z) \sim Gzz$ ✓				SM
4.	$(\forall z) \sim\sim Gzz$				$3 \sim \exists D$
5.	$(\forall y)\ Gay$				$1\ \exists D$
6.	$(\forall y)(\sim b = y \supset \sim Gby)$				$2\ \exists D$
7.	$\sim\sim Gaa$ ✓				$4\ \forall D$
8.	$\sim\sim Gbb$ ✓				$4\ \forall D$
9.	Gaa				$5\ \forall D$
10.	Gab				$5\ \forall D$
11.	$\sim b = a \supset \sim Gba$ ✓				$6\ \forall D$
12.	$\sim b = b \supset \sim Gbb$ ✓				$6\ \forall D$
13.	Gaa				$7 \sim\sim D$
14.	Gbb				$8 \sim\sim D$
15.	$\sim\sim b = a$ ✓		$\sim Gba$		$11 \supset D$
16.	$b = a$				$15 \sim\sim D$
17.	$\sim\sim b = b$ ✓	$\sim Gbb$	$\sim\sim b = b$ ✓	$\sim Gbb$	$12 \supset D$
18.	$b = b$	×	$b = b$	×	$17 \sim\sim D$
19.	Gba				$9, 16 = D$

The tree has at least one completed open branch. Therefore, the alleged entailment does not hold.

h. 1.	$(\forall x)(\exists y)y = f(x)$	SM
2.	$\sim (\exists z)z = f(a)$ ✓	SM
3.	$(\forall z) \sim z = f(a)$	$2 \sim \exists D$
4.	$\sim a = f(a)$	$3\ \forall D$
5.	$(\exists y)y = f(a)$ ✓	$1\ \forall D$
6.	$b = f(a)$	$5\ \exists D$
7.	$\sim b = f(a)$	$3\ \forall D$
	×	

The tree is closed. Therefore, the entailment does hold.

Section 9.5E

Note: Branches that are open but not completed are so indicated by a series of dots below the branch.

1. b. 1.	$(\forall x)(Fx \supset Cx)$				SM
2.	$\sim (\forall x)(Fx \,\&\, Cx)$ ✓				SM
3.	$(\exists x) \sim (Fx \,\&\, Cx)$ ✓				$2 \sim \forall D$
4.	$\sim (Fa \,\&\, Ca)$ ✓				$3\ \exists D2$
5.	$\sim Fa$		$\sim Ca$		$4 \sim \&D$
6.	$Fa \supset Ca$ ✓		$Fa \supset Ca$ ✓		$1\ \forall D$
7.	$\sim Fa$	Ca	$\sim Fa$	Ca	$6 \supset D$
				×	

The tree has at least one completed open branch. Therefore, the set is quantificationally consistent.

d.	1.	$\sim (\forall x) \sim Hx$ ✓	SM
	2.	$(\forall x)(Hx \supset Kx)$	SM
	3.	$\sim (\exists x)(Kx \;\&\; Hx)$ ✓	SM
	4.	$(\exists x) \sim\sim Hx$ ✓	$1 \sim \forall D$
	5.	$(\forall x) \sim (Kx \;\&\; Hx)$	$3 \sim \exists D$
	6.	$\sim\sim Ha$ ✓	$4\ \exists D2$
	7.	Ha	$6 \sim\sim D$
	8.	$Ha \supset Ka$ ✓	$2\ \forall D$
	9.	$\sim (Ka \;\&\; Ha)$ ✓	$5\ \forall D$
	10.	$\sim Ha$ × Ka	$8 \supset D$
	11.	(under Ka:) $\sim Ka$ × $\sim Ha$ ×	$9 \sim \&D$

The tree is closed. Therefore, the set is quantificationally inconsistent.

f.	1.	$(\exists x)Fx \;\&\; (\exists x) \sim Fx$ ✓	SM
	2.	$(\forall x)Fx \supset (\forall x) \sim Fx$ ✓	SM
	3.	$(\exists x)Fx$ ✓	$1\ \&D$
	4.	$(\exists x) \sim Fx$ ✓	$1\ \&D$
	5.	Fa	$3\ \exists D2$
	6.	$\sim Fa$ × $\sim Fb$	$4\ \exists D2$
	7.	(under $\sim Fb$:) $\sim (\forall x)Fx$ ✓ $(\forall x) \sim Fx$ ⋮	$2 \supset D$
	8.	$(\exists x) \sim Fx$ ✓	$7 \sim \forall D$
	9.	$\sim Fa$ × $\sim Fb$ $\sim Fc$ ⋮	$8\ \exists D2$

The tree has at least one completed open branch. Therefore, the set is quantificationally consistent.

h. 1.				(∀x)(∼ Gx ⊃ Fx)				SM
2.				(∃x)(Fx & ∼ Gx) ✓				SM
3.				Fa ⊃ ∼ Ga ✓				SM
4.		Fa & ∼ Ga ✓			Fb & ∼ Gb ✓			2 ∃D2
5.		Fa			Fb			4 &D
6.		∼ Ga			∼ Gb			4 &D
7.	∼ Fa	∼ Ga		∼ Fa		∼ Ga		3 ⊃D
8.	×	∼ Ga ⊃ Fa ✓		∼ Ga ⊃ Fa ✓		∼ Ga ⊃ Fa ✓		1 ∀D
9.				∼ Gb ⊃ Fb		∼ Gb ⊃ Fb		1 ∀D
10.		∼ ∼ Ga	Fa	∼ ∼ Ga	Fa	∼ ∼ Ga	Fa	8 ⊃D
		⋮		⋮	×	⋮	⋮	

The tree has at least one completed open branch. Therefore the set is quantificationally consistent.

j. 1.	(∃x)(∀y)Lxy ✓		SM
2.	(∃x)(∀y) ∼ Lxy ✓		SM
3.	(∀y)Lay		1 ∃D2
4.	(∀y) ∼ Lay	(∀y) ∼ Lby	2 ∃D2
5.	Laa	Laa	3 ∀D
6.	|	Lab	3 ∀D
7.	∼ Laa	∼ Lba	4 ∀D
8.	×	∼ Lbb	4 ∀D

The tree has at least one completed open branch. Therefore the set is quantificationally consistent.

l. 1.	(∀x) ~ (∃y)Lxy				SM
2.	(∀w) (∀y)(Swy ∨ ~ Lwy)				SM
3.	~ (∃x) ~ (∃z)Sxz ✓				SM
4.	(∀x) ~ ~ (∃z)Sxz				3 ~ ∃D
5.	~ (∃y)Lay ✓				1 ∀D
6.	(∀y)(Say ∨ ~ Lay)				2 ∀D
7.	~ ~ (∃z)Saz ✓				4 ∀D
8.	Saa ∨ ~ Laa ✓				6 ∀D
9.	(∀y) ~ Lay				5 ~ ∃D
10.	(∃z)Saz ✓				7 ~ ~ D
11.	Saa		~ Laa		8 ∨D
12.	Saa	Sab	Saa	Sab	10 ∃D2
13.	~ Laa	~ Laa	~ Laa	~ Laa	9 ∀D
14.		~ Lab		~ Lab	9 ∀D
		⋮		⋮	

The tree has two completed open branches. Therefore, the set is quantificationally consistent.

n. 1.	(∀x)(∀y)(∀z)((Hxy & Hyz) ⊃ Hxz)				SM
2.	(∀x)(∀y)(Hxy ⊃ Hyx)				SM
3.	(∃x) ~ Hxx ✓				SM
4.	~ Haa				3 ∃D2
5.	(∀y)(∀z)((Hay & Hyz) ⊃ Haz)				1 ∀D
6.	(∀y)(Hay ⊃ Hya)				2 ∀D
7.	(∀z)((Haa & Haz) ⊃ Haz)				5 ∀D
8.	Haa ⊃ Haa ✓				6 ∀D
9.	(Haa & Haa) ⊃ Haa ✓				7 ∀D
10.	~ (Haa & Haa) ✓			Haa ×	9 ⊃D
11.	~ Haa		Haa ×		8 ⊃D
12.	~ Haa	~ Haa			10 ~ &D

The tree has at least one completed open branch. Therefore, the set is quantificationally consistent.

p. 1.		$(\exists x)f(x) = f(a)$ ✓		SM
2.		$(\forall x) \sim f(x) = f(b)$		SM
3.				
4.	$f(a) = f(a)$	$f(b) = f(a)$	$f(c) = f(a)$	1 $\exists$D2
5.	$\sim f(a) = f(b)$	$\sim f(a) = f(b)$	$\sim f(a) = f(b)$	2 $\forall$D
6.	$\sim f(b) = f(b)$	$\sim f(b) = f(b)$	$\sim f(b) = f(b)$	2 $\forall$D
7.	×	×	$\sim f(c) = f(b)$	2 $\forall$D
8.			$\sim f(c) = f(b)$	4, 5 =D

The systematic tree has a completed branch. Therefore the set being tested is quantificationally consistent.

r. 1.	$(\forall x)f(x) = f(a)$		SM
2.	$\sim (\exists x)x = f(a)$ ✓		SM
3.	$(\forall x) \sim x = f(a)$		2 $\sim\exists$D
4.	$f(a) = f(a)$		1 $\forall$D
5.	$\sim a = f(a)$		3 $\forall$D
6.	$a = f(a)$	$b = f(a)$	5 CTD
7.	$a = a$	$b = b$	6, 6 =D
8.	$\sim a = a$	$\sim a = b$	6, 5 =D
9.	×	$f(a) = b$	6, 4 =D

The systematic tree has a completed open branch. Therefore the set being tested is quantificationally consistent.

t. 1.	$(\exists x)(\forall y)x = f(y)$ ✓		SM
2.	$(\exists x)(\forall y) \sim x = f(y)$ ✓		SM
3.	$(\forall y)a = f(y)$		1 $\exists$D2
4.	$(\forall y) \sim a = f(y)$	$(\forall y) \sim b = f(y)$	2 $\exists$D2
5.	$a = f(a)$	$a = f(a)$	3 $\forall$D
6.		$a = f(b)$	3 $\forall$D
7.	$\sim a = f(a)$	$\sim a = f(a)$	4 $\forall$D
	×	×	

This systematic tree is closed. Therefore the set being tested is quantificationally inconsistent.

v. 1.	$(\forall x)(Gx \supset Gh(x))$			SM
2.	$(\exists x)(Gx \;\&\; \sim Gh(x))$ ✓			SM
3.	$Ga \;\&\; \sim Gh(a))$ ✓			2 ∃D2
4.	Ga			4 &D
5.	$\sim Gh(a)$			4 &D
6.	$a = h(a)$	$b = h(a)$		5 CTD
7.	$a = a$	$b = b$		6, 6 =D
8.	$\sim Ga$	$\sim Gb$		6, 5 =D
9.	×	$Ga \supset Gh(a)$ ✓		1 ∀D
10.		$Gb \supset Gh(b)$		1 ∀D
11.		$\sim Ga$	$Gh(a)$	9 ⊃D
		×	×	

The tree is closed. Therefore, the set being tested is quantificationally inconsistent.

x. 1.	$(\forall x) \sim x = f(x)$				SM
2.	$(\exists x)x = f(f(x))$ ✓				SM
3.	$a = f(f(a))$				2 ∃D2
4.	$\sim a = f(a)$				1 ∀D
5.	$a = f(a)$	$b = f(a)$			4 CTD
6.	×	$b = b$			5, 5 =D
7.		$\sim a = b$			5, 4 =D
8.		$a = f(b)$			5, 3 =D
9.		$b = f(b)$			1 ∀D
10.	$a = f(b)$	$b = f(b)$	$c = f(b)$		9 CTD
11.	$a = a$		$c = c$		10, 10 =D
12.	$\sim b = a$	$\sim b = b$	$\sim c = c$		10, 9 =D
		×			
13.	$a = f(c)$	$b = f(c)$	$c = f(c)$	$d = f(c)$	12 CTD
14.	$a = a$			$d = d$	13, 13 =D
15.	$\sim c = a$	$\sim c = b$	$\sim c = c$	$\sim c = d$	
			×		

The tree has at least one completed open branch. Therefore, the set being tested is quantificationally consistent.

2. b.

1.	$\sim (\forall x)(\forall y)(x = g(y) \vee y = g(x))$ ✓							SM
2.	$(\exists x) \sim (\forall y)(x = g(y) \vee y = g(x))$ ✓							1 $\sim \forall$D
3.	$\sim (\forall y)(a = g(y) \vee y = g(a))$							2 $\exists$D2
4.	$(\exists y) \sim (a = g(y) \vee y = g(a))$							3 $\sim \forall$D
5.	$\sim [a = g(a) \vee a = g(a)]$ ✓		$\sim [a = g(b) \vee b = g(a)]$ ✓					4 $\exists$D2
6.	$\sim a = g(a)$		$\sim a = g(b)$					5 $\sim \vee$D
7.	$\sim a = g(a)$		$\sim b = g(a)$					5 $\sim \vee$D
8.	$a = g(a)$ ×	$b = g(a)$	$a = g(a)$		$b = g(a)$		$c = g(c)$	7 CTD
							incomplete	
9.			$a = g(b)$, $b = g(b)$, $c = g(b)$		$a = g(b)$, $b = g(b)$, $c = g(b)$			6 CTD
10.		$b = b$						8, 8 =D
11.		$\sim a = b$						8, 7 =D

The application of CTD at line 9 is incomplete (we need four branches under '$c = g(c)$,' with '$a = g(b)$,' '$b = g(b)$,' '$c = g(b)$,' and '$d = g(b)$' each entered on one of those branches). Were we to complete this work, the tree would be a systematic tree and have a completed open branch, the leftmost one. Therefore, the sentence being tested is not quantificationally false, and the sentence of which it is the negation, '$(\forall x)(\forall y)(x = g(y) \vee y = g(x))$' is not quantificationally true.

d.

1.	$\sim (\forall x) \sim x = f(x)$		SM
2.	$(\exists x) \sim \sim x = f(x)$ ✓		1 $\sim \forall$D
3.	$\sim \sim a = f(a)$ ✓		2 $\exists$D2
4.	$a = f(a)$		3 $\sim\sim$ D
5.	$a = f(a)$	$b = f(a)$	4 CTD
6.	$a = a$	$b = b$	5, 5 =D
7.		$a = b$	5, 4 =D

The systematic tree has two completed open branches. Therefore, the sentence being tested is not quantificationally false and the sentence of which it is a negation, '$(\forall x) \sim x = f(x)$' is not quantificationally true.

3. b.

1.	~ (∀x)(∀y)(∀z)((y = *f*(x) & z = *f*(x)) ⊃ y = z)					SM
2.	(∃x) ~ (∀y)(∀z)((y = *f*(x) & z = *f*(x)) ⊃ y = z)					1 ~ ∀D
3.	~ (∀y)(∀z)((y = *f*(a) & z = *f*(a)) ⊃ y = z)					2 ∃D2
4.	(∃y) ~ (∀z) ((y = *f*(a) & z = *f*(a)) ⊃ y = z)					3 ~ ∀D
5.	~ (∀z)((a = *f*(a) & z = *f*(a)) ⊃ a = z)		~ (∀z)((b = *f*(a) & z = *f*(a)) ⊃ b = z)			4 ∃D2
6.	(∃z) ~ ((a = *f*(a) & z = *f*(a)) ⊃ a = z)		(∃z) ~ ((b = *f*(a) & z = *f*(a)) ⊃ b = z)			5 ~ ∀D
7.	~ ((a = *f*(a) & a = *f*(a)) ⊃ a = a)	~ ((a = *f*(a) & b = *f*(a)) ⊃ a = b)	~ ((b = *f*(a) & a = *f*(a)) ⊃ b = a)	~ ((b = *f*(a) & b = *f*(a)) ⊃ b = b)	~ ((b = *f*(a) & c = *f*(a)) ⊃ b = c)	6 ∃D2
8.	a = *f*(a) & a = *f*(a)	a = *f*(a) & b = *f*(a)	b = *f*(a) & a = *f*(a)	b = *f*(a) & b = *f*(a)	b = *f*(a) & c = *f*(a)	6 ~ ⊃D
9.	~ a = a	~ a = b	~ b = a	~ b = b	~ b = c	6 ~ ⊃D
10.	×	a = *f*(a)	b = *f*(a)	×	b = *f*(a)	8 &D
11.		b = *f*(a)	a = *f*(a)		c = *f*(a)	8 &D
12.		a = b	b = a		b = c	11, 10 =D
		×	×		×	

The tree is closed. Therefore, the sentence '~ (∀x)(∀y)(∀z)((y = *f*(x) & z = *f*(x)) ⊃ y = z)' is quantificationally false and '(∀x)(∀y)(∀z)((y = *f*(x) & z = *f*(x)) ⊃ y = z)' is quantificationally true.

4. b.

1.	$\sim ((\exists x) \sim Fx \supset (Fa \supset \sim Fb))$ ✓	SM
2.	$(\exists x) \sim Fx$ ✓	$1 \sim \supset D$
3.	$\sim (Fa \supset \sim Fb)$ ✓	$1 \sim \supset D$
4.	Fa	$3 \sim \supset D$
5.	$\sim \sim Fb$ ✓	$3 \sim \supset D$
6.	Fb	$5 \sim \sim D$
7.	$\sim Fa \quad \sim Fb \quad \sim Fc$	$2\ \exists D2$
	$\times \quad\quad \times$	

1.	$(\exists x) \sim Fx \supset (Fa \supset \sim Fb)$ ✓	SM
2.	$\sim (\exists x) \sim Fx$ ✓ $\qquad Fa \supset \sim Fb$	$1 \supset D$
3.	$(\forall x) \sim \sim Fx$	$2 \sim \exists D$
4.	$\vdots \qquad\qquad \sim Fa \quad \sim Fb$	$2 \supset D$

Neither the tree for the sentence nor the tree for its negation is closed. Therefore the sentence is quantificationally indeterminate.

d.

1.	$\sim [(\exists y)(\forall x)Fxy \supset (\forall x)(\exists y)Fxy]$ ✓	SM
2.	$(\exists y)(\forall x)Fxy$ ✓	$1 \sim \supset D$
3.	$\sim (\forall x)(\exists y)Fxy$ ✓	$1 \sim \supset D$
4.	$(\exists x) \sim (\exists y)Fxy$ ✓	$3 \sim \forall D$
5.	$(\forall x)Fxa$	$2\ \exists D2$
6.	$\sim (\exists y)Fay$ ✓ $\qquad \sim (\exists y)Fby$ ✓	$4\ \exists D2$
7.	$(\forall y) \sim Fay \qquad (\forall y) \sim Fby$	$6 \sim \exists D$
8.	$Faa \qquad Fba$	$5\ \forall D$
9.	$\sim Faa \qquad \sim Fba$	$7\ \forall D$
	$\times \qquad \times$	

The tree for the negation of the sentence is closed. Therefore the sentence is quantificationally true.

f. 1.	$\sim [(\forall x)(Fx \equiv ((\exists y)Gyx \supset H)) \supset (\forall x)(Fx \supset (\exists y)(Gyx \supset H))]$ ✓			SM
2.	$(\forall x)(Fx \equiv ((\exists y)Gyx \supset H))$			$1 \sim \supset D$
3.	$\sim (\forall x)(Fx \supset (\exists y)(Gyx \supset H))$ ✓			$1 \sim \supset D$
4.	$(\exists x) \sim (Fx \supset (\exists y)(Gyx \supset H))$ ✓			$3 \sim \forall D$
5.	$\sim (Fa \supset (\exists y)(Gya \supset H))$ ✓			$4\ \exists D2$
6.	Fa			$5 \sim \supset D$
7.	$\sim (\exists y)(Gya \supset H)$ ✓			$5 \sim \supset D$
8.	$(\forall y) \sim (Gya \supset H)$			$7 \sim \exists D$
9.	$Fa \equiv ((\exists y)Gya \supset H)$ ✓			$2\ \forall D$
10.	$\sim (Gaa \supset H)$ ✓			$8\ \forall D$
11.	Gaa			$10 \sim \supset D$
12.	$\sim H$			$10 \sim \supset D$
13.	Fa		$\sim Fa$	$9 \equiv D$
14.	$(\exists y)Gya \supset H$ ✓		$\sim ((\exists y)Gya \supset H)$	$9 \equiv D$
			×	
15.	$\sim (\exists y)Gya$ ✓	H		$14 \supset D$
16.	$(\forall y) \sim Gya$	×		$15 \sim \exists D$
17.	$\sim Gaa$			$16\ \forall D$
	×			

The tree for the negation of the sentence is closed. Therefore the sentence is quantificationally true.

h. 1.	$(\forall x)Gf(x)x$		SM
2.	$Gf(a)a$		$1\ \forall D$
3.	$a = f(a)$	$b = f(a)$	2 CTD
4.	$a = a$	$b = b$	3, 3 =D
5.	Gaa	Gba	3, 2 =D

1.	$\sim (\forall x)Gf(x)x$ ✓		SM
2.	$(\exists x) \sim Gf(x)x$ ✓		$1 \sim \forall D$
3.	$\sim Gf(a)a$		$2\ \exists D2$
4.	$a = f(a)$	$b = f(b)$	3 CTD
5.	$a = a$	$b = b$	4, 4 =D
6.	$\sim Gaa$	$\sim Gba$	4, 3 =D

The tree for the given sentence has at least one completed open branch, as does the tree for its negation. Therefore, the given sentence is quantificationally indeterminate.

j. 1.	~ (∀x)(Fx ∨ ~ F*g*(x)) ✓		SM
2.	(∃x) ~ (Fx ∨ ~ F*g*(x)) ✓		1 ~ ∀D
3.	~ (Fa ∨ ~ F*g*(a)) ✓		2 ∃D2
4.	~ Fa		3 ~ ∨D
5.	~ ~ F*g*(a) ✓		3 ~ ∨D
6.	F*g*(a)		5 ~ ~ D
7.	a = *g*(a)	b = *g*(a)	6 CTD
8.	a = a	b = b	7, 7 =D
9.	Fa	Fb	7, 6 =D
	×		

1.	(∀x)(Fx ∨ ~ F*g*(x))			SM
2.	Fa ∨ ~ F*g*(a) ✓			1 ∀D
3.				
4.	Fa	~ F*g*(a)		3 ∨D
5.		a = *g*(a)	b = *g*(a)	4 CTD
6.		a = a	b = b	5, 5 =D
7.		~ Fa	~ Fb	5, 4 =D

The tree for the given sentence has at least one completed open branch, as does the tree for its negation. Therefore, the given sentence is quantificationally indeterminate.

5. b. 1.	(∀x)(Jx ∨ Ixb) ∨ (∀x)(∃y)(Hxy ⊃ Mx) ✓				SM
2.	~ Iab				SM
3.	(∀x)(Jx ∨ Ixb)			(∀x)(∃y)(Hxy ⊃ Mx)	1 ∨D
4.	Ja ∨ Iab ✓			(∃y)(Hay ⊃ Ma)	3 ∀D
5.	Jb ∨ Ibb ✓			(∃y)(Hby ⊃ Mb)	3 ∀D
				⋮	
6.		Ja	Iab		4 ∨D
			×		
7.	Jb	Ibb			5 ∨D

The tree for the premises and the negation of the conclusion has at least one completed branch. Therefore the argument is quantificationally invalid.

d. 1.	~ (∀y)Kyy ∨ (∀x)Hxx ✓		SM
2.	~ (∃x)(~ Hxx ⊃ ~ Kxx) ✓		SM
3.	(∀x) ~ (~ Hxx ⊃ ~ Kxx)		2 ~ ∃D
4.	~ (∀y)Kyy ✓	(∀x)Hxx	1 ∨D
5.	(∃y) ~ Kyy ✓		2 ~ ∀D
6.	~ Kaa		5 ∃D2
7.	~ (~ Haa ⊃ ~ Kaa) ✓	~ (~ Haa ⊃ ~ Kaa) ✓	3 ∀D
8.		Haa	4 ∀D
9.	~ Haa	~ Haa	7 ~ ⊃D
10.	~ ~ Kaa ✓	~ Kaa	7 ~ ⊃D
11.	Kaa	×	10 ~ ~ D
	×		

The tree for the premises and the negation of the conclusion is closed. Therefore the argument is quantificationally valid.

f. 1.	(∀x)(∀y)(Fx ∨ Gxy)		SM
2.	(∃x)Fx ✓		SM
3.	~ (∃x)(∃y)Gxy ✓		SM
4.	Fa		2 ∃D2
5.	(∀x) ~ (∃y)Gxy		3 ~ ∃D
6.	(∀y)(Fa ∨ Gay)		1 ∀D
7.	~ (∃y)Gay ✓		5 ∀D
8.	Fa ∨ Gaa ✓		6 ∀D
9.	(∀y) ~ Gay		7 ~ ∃D
10.	Fa	Gaa	8 ∨D
11.	~ Gaa	~ Gaa	9 ∀D
		×	

The tree for the premises and the negation of the conclusion has at least one completed open branch. Therefore the argument is quantificationally invalid.

h.	1.	$(\exists x)((Lx \vee Sx) \vee Kx)$ ✓						SM
	2.	$(\forall y) \sim (Ly \vee Ky)$						SM
	3.	$\sim (\forall x)Sx$ ✓						SM
	4.	$(La \vee Sa) \vee Ka$ ✓						1 $\exists$D2
	5.	$(\exists x) \sim Sx$ ✓						3 $\sim \forall$D
	6.	$La \vee Sa$ ✓				Ka		4 $\vee$D
	7.	La		Sa				6 $\vee$D
	8.	$\sim Sa$	$\sim Sb$	$\sim Sa$	$\sim Sb$	$\sim Sa$	$\sim Sb$	5 $\exists$D2
	9.	$\sim (La \vee Ka)$ ✓	$\sim (La \vee Ka)$ ✓	×	$\sim (La \vee Ka)$ ✓	$\sim (La \vee Ka)$ ✓	$\sim (La \vee Ka)$ ✓	2 $\forall$D
	10.	|	$\sim (Lb \vee Kb)$ ✓		$\sim (Lb \vee Kb)$ ✓	|	$\sim (Lb \vee Kb)$ ✓	2 $\forall$D
	11.	$\sim La$	$\sim La$		$\sim La$	$\sim La$	$\sim La$	9 $\sim \vee$D
	12.	$\sim Ka$	$\sim Ka$		$\sim Ka$	$\sim Ka$	$\sim Ka$	9 $\sim \vee$D
	13.	×	×		$\sim Lb$	×	×	10 $\sim \vee$D
	14.				$\sim Kb$			10 $\sim \vee$D

The tree for the premises and the negation of the conclusion has at least one completed open branch. Therefore, the argument is quantificationally invalid.

j.	1.	$(\forall x)\ Pf(f(x))$		SM
	2.	$\sim Pf(a)$		SM
	3.	$Pf(f(a))$		1 $\forall$D
	4.	$a = f(a)$	$b = f(a)$	2 CTD
	5.	$a = a$	$b = b$	4, 4 =D
	6.	$\sim Pa$	$\sim Pb$	4, 2 =D
	7.	$Pf(a)$	$Pf(b)$	4, 3 =D
	8.	Pa	Pb	4, 7 =D
		×	×	

The tree is closed. Therefore, the argument being tested is quantificationally valid.

l.

1.	(∀x)(Fx ∨ Fg(x))						SM
2.	~ (∀x)(Fx(x) ∨ Fg(g(x)))✓						SM
3.	(∃x) ~ (Fx ∨ Fg(g(x)))✓						2 ~∀D
4.	~ (Fa ∨ Fg(g(a)))✓						3 ∃D2
5.	~ Fa						4 ~ ∨D
6.	~ Fg(g(a))						4 ~ ∨D
7.	a = g(a)		b = g(a)				6 CTD
8.	a = a		b = b				7, 7 =D
9.	~ Fg(a)		~ Fg(b)				7, 6 =D
10.	~ Fa						7, 9 =D
11.	Fa ∨ Fg(a)✓		Fa ∨ Fg(a)✓				1 ∀D
12.			Fb ∨ Fg(b)				1 ∀D
13.	Fa	Fg(a)	Fa	Fg(a)			12 ∨D
14.	×	×	×				
15.					Fb	Fg(b)	13 ∨D
						×	

The tree has a completed open branch. Therefore, the argument is quantificationally valid.

n.

1.	(∀x) [Fx ⊃ ~ (∃y)Gxy]					SM
2.	(∃y)Fg(y)✓					SM
3.	~ (∃y)Gyy✓					SM
4.	(∀y) ~ Gyy					3 ~ ∃D
5.	Fg(a)					2 ∃D2
6.	Fa ⊃ ~ (∃y)Gay✓					1 ∀D
7.	~ Gaa					4 ∀D
8.	a = g(a)		b = g(a)			5 CTD
9.	a = a		b = b			8, 8 =D
10.	Fa		Fb			8, 5 =D
11.						
12.	~ Fa	~ (∃y)Gay✓	~ Fa	~ (∃y)Gay✓		6 ⊃D
13.	×	(∀y) ~ Gay	.	(∀y) ~ Gay		8, 12 =D
14.		~ Gaa	.	~ Gaa		13 ∀D
15.			.	~ Gab		13 ∀D
				.		
				.		
				.		

The tree has a completed open branch (the one ending in '~ Gaa'. Therefore, the argument being tested is quantificationally invalid.

6. b.

1.	$\sim [(\forall x)(\exists y)Lxy \equiv (\exists y)(\forall x)Lyx]$ ✓			SM
2.	$(\forall x)(\exists y)Lxy$	$\sim (\forall x)(\exists y)Lxy$ ✓		$1 \sim {\equiv}D$
3.	$\sim (\exists y)(\forall x)Lyx$ ✓	$(\exists y)(\forall x)Lyx$ ✓		$1 \sim {\equiv}D$
4.	$(\forall y) \sim (\forall x)Lyx$			$3 \sim \exists D$
5.		$(\exists x) \sim (\exists y)Lxy$ ✓		$2 \sim \forall D$
6.		$(\forall x)Lax$		$3\ \exists D2$
7.		$\sim (\exists y)Lay$ ✓	$\sim (\exists y)Lby$ ✓	$5\ \exists D2$
8.		$(\forall y) \sim Lay$	$(\forall y) \sim Lby$	$7 \sim \exists D$
9.	$(\exists y)Lay$			$2\ \forall D$
10.	$\sim (\forall x)Lax$			$4\ \forall D$
11.	⋮	Laa	Laa	$6\ \forall D$
12.			Lab	$6\ \forall D$
13.		$\sim Laa$	$\sim Lba$	$8\ \forall D$
14.		×	$\sim Lbb$	$8\ \forall D$

The tree for the negation of the corresponding biconditional has at least one completed open branch. Therefore the sentences are not quantificationally equivalent.

d.

1.	$\sim [(\forall x)(Ax \supset B) \equiv ((\forall x)Ax \supset B)]$ ✓				SM
2.	$(\forall x)(Ax \supset B)$		$\sim (\forall x)(Ax \supset B)$ ✓		$1 \sim {\equiv}D$
3.	$\sim ((\forall x)Ax \supset B)$ ✓		$(\forall x)Ax \supset B$ ✓		$1 \sim {\equiv}D$
4.	$(\forall x)Ax$				$3 \sim {\supset}D$
5.	$\sim B$				$3 \sim {\supset}D$
6.	⋮		$(\exists x) \sim (Ax \supset B)$ ✓		$2 \sim \forall D$
7.			$\sim (Aa \supset B)$ ✓		$6\ \exists D2$
8.			Aa		$7 \sim {\supset}D$
9.			$\sim B$		$7 \sim {\supset}D$
10.		$\sim (\forall x)Ax$ ✓		B	$3 \supset D$
11.		$(\exists x) \sim Ax$ ✓		×	$10 \sim \forall D$
12.		$\sim Aa$	$\sim Ab$		$11\ \exists D2$
		×			

The tree for the negation of the corresponding biconditional has at least one completed open branch. Therefore, the sentences are not quantificationally equivalent.

f.

1.	$\sim [(\exists x)(Ax \supset B) \equiv ((\exists x)Ax \supset B)]$ ✓				SM
2.	$(\exists x)(Ax \supset B)$ ✓		$\sim (\exists x)(Ax \supset B)$		$1 \sim \equiv D$
3.	$\sim ((\exists x)Ax \supset B)$ ✓		$(\exists x)Ax \supset B$		$1 \sim \equiv D$
4.	$(\exists x)Ax$ ✓		⋮		$3 \sim \supset D$
5.	$\sim B$				$3 \sim \supset D$
6.	$Aa \supset B$ ✓				$2\ \exists D2$
7.	Aa		Ab		$4\ \exists D2$
8.	$\sim Aa$	B	$\sim Aa$	B	$6 \supset D$
	×	×		×	

The tree for the negation of the corresponding biconditional has at least one completed open branch. Therefore, the sentences are not quantificationally equivalent.

h.

1.	$\sim (Labf(b) \equiv Laf(b)b)$				SM
2.	$Labf(b)$		$\sim Labf(b)$		$1 \sim \equiv D$
3.	$\sim Laf(b)b$		$Laf(b)b$		$1 \sim \equiv D$
4.	$a = f(b)$	$b = f(b)$	$a = f(b)$	$b = f(b)$	2 CTD
5.	$a = a$	$b = b$	$a = a$	$b = b$	4, 4 =D
6.	$Laba$	$Labb$	$\sim Laba$	$\sim Labb$	4, 2 =D
7.	$\sim Laab$	$\sim Labb$	$Laab$	$Labb$	4, 3 =D
		×		×	

The tree has two completed open branches, therefore the given sentences are not quantificationally equivalent.

j.

1.	~ [(∀x)B*h*(x)x ≡ (∀x)Bx*h*(x)] ✓				SM
2.	(∀x)B*h*(x)x		~ (∀x)B*h*(x)x ✓		1 ~ ≡D
3.	~ (∀x)Bx*h*(x) ✓		(∀x) Bx*h*(x)		1 ~ ≡D
4.	(∃x) ~ Bx*h*(x) ✓				3 ~ ∀D
5.			(∃x) ~ B*h*(x)x ✓		2 ~ ∀D
6.	~ Ba*h*(a)				4 ∃D2
7.			~ B*h*(a)a		5 ∃D2
8.	B*h*(a)a				2 ∀D
9.			Ba*h*(a)		3 ∀D
11.	a = *h*(a)	b = *h*(a)			8 CTD
12.	a = a	b = b			11, 11 =D
13.	~ Baa	~ Bab			11, 6 =D
14.	Baa	Bba			11, 8 =D
15.	×		a = *h*(a)	b = *h*(a)	9 CTD
16.			a = a	b = b	15, 15 =D
17.			~ Baa	~ Bba	15, 7 =D
18.			Baa	Bab	15, 9 =D
			×		

The tree has two completed open branches. Therefore, the given sentences are not quantificationally equivalent.

7. b.

1.	(∀x)(∀y)(Fx ∨ Gxy)		SM
2.	(∃x)Fx ✓		SM
3.	~ (∃x)(∃y)Gxy ✓		SM
4.	(∀x) ~ (∃y)Gxy		3 ~ ∃D
5.	Fa		2 ∃D2
6.	(∀y)(Fa ∨ Gay)		1 ∀D
7.	~ (∃y)Gay ✓		4 ∀D
8.	Fa ∨ Gaa ✓		6 ∀D
9.	(∀y) ~ Gay		7 ~ ∃D
10.	Fa	Gaa	8 ∨D
11.	~ Gaa	~ Gaa	9 ∀D
		×	

The tree has at least one completed open branch. Therefore, the given set does not quantificationally entail the given sentence.

d. 1.	(∃x) (∀y) Gxy ✓		SM
2.	~ (∀y) (∃x) Gxy ✓		SM
3.	(∀y) Gay		1 ∃D2
4.	(∃y) ~ (∃x) Gxy ✓		2 ~ ∀D
5.	~ (∃x) Gxa ✓	~ (∃x) Gxb ✓	4 ∃D2
6.	(∀x) ~ Gxa	(∀x) ~ Gxb	5 ~ ∃D
7.	Gaa	Gaa	3 ∀D
8.	|	Gab	3 ∀D
9.	~ Gaa	~ Gab	6 ∀D
	×	×	

The tree is closed. Therefore the given set does quantificationally entail the given sentence.

f. 1.	(∀y) (∃x) Gxy							SM
2.	~ (∃x) (∀y) Gxy ✓							SM
3.	(∀x) ~ (∀y) Gxy							2 ~ ∃D
4.	(∃x) Gxa ✓							1 ∀D
5.	~ (∀y) Gay ✓							3 ∀D
6.	(∃y) ~ Gay ✓							5 ~ ∀D
7.	Gaa		Gba					4 ∃D2
8.	~ Gaa	~ Gab	~ Gaa	~ Gab	~ Gac			6 ∃D2
9.	×	(∃x) Gxb ✓	(∃x) Gxb	(∃x) Gxb	(∃x) Gxb			1 ∀D
10.		|	|	|	(∃x) Gxc			1 ∀D
11.		~ (∀y) Gby ✓	~ (∀y) Gby ✓	~ (∀y) Gby ✓	~ (∀y) Gby ✓			3 ∀D
12.		|			~ (∀y) Gcy			3 ∀D
13.		(∃y) ~ Gby ✓	(∃y) ~ Gby	(∃y) ~ Gby	(∃y) ~ Gby			11 ~ ∀D
			⋮	⋮	⋮			
14.	Gab	Gbb	Gcb					9 ∃D2
	×							
15.	~ Gba	~ Gbb	~ Gbc	~ Gbb	~ Gbb	~ Gbc	~ Gbd	13 ∃D2
		×	⋮	⋮	⋮	⋮	⋮	

The tree has at least one completed open branch (the leftmost). Therefore, the set does not quantificationally entail the given sentence.

i. 1.			$(\exists x)(\forall y)x = g(y)$ ✓			SM
2.			$\sim h(a) = g(a)$			SM
3.			$(\forall y)a = g(y)$			1 ∃D2
4.			$a = g(a)$			3 ∀D
5.	$a = g(a)$			$b = g(a)$		4 CTD
6.	$a = h(a)$	$b = h(a)$	$a = h(a)$	$b = h(a)$	$c = h(a)$	2 CTD
7.	$a = a$	$a = a$		$b = b$	$b = b$	5, 5 =D
8.	|	$b = b$	$a = a$	|	$c = c$	6, 6 =D
9.	$\sim h(a) = a$	$\sim h(a) = a$	$\sim h(a) = b$	$\sim h(a) = b$	$\sim h(a) = b$	5, 2 =D
10.	$\sim a = a$	$\sim b = a$	$\sim a = b$	$\sim b = b$	$\sim c = b$	6, 9 =D
11.	×	$\sim b = g(a)$	$\sim a = g(a)$	×	$\sim c = g(a)$	6, 2 =D
12.		$a = g(b)$	$a = g(b)$		$a = g(b)$	3 ∀D
					$a = g(c)$	3 ∀D
13.	$a = g(b)$	$b = g(b)$	$c = g(b)$			12 CTD

The leftmost branch is a completed open branch. Therefore, the set $\{(\exists x)(\forall y)x = g(y)\}$ does not quantificationally entail '$\sim h(a) = g(a)$'.

8. The members of Γ contain no binary connectives. Hence, each member of Γ must either be a literal or be one of the following forms, where **P** contains no binary connectives: ~ ~ **P**, (∀**x**)**P**, ~ (∀**x**)**P**, (∃**x**)**P**, ~ (∃**x**)P. Hence, the only decomposition rules that are applicable to the members of Γ are Negated Negation Decomposition, Universal Decomposition, Negated Universal Decomposition, Existential Decomposition, and Negated Existential Decomposition. These are, as long as Existential Decomposition, rather than Existential Decomposition-2, is used, all nonbranching rules. Furthermore, applying these rules to the members of Γ can yield only literals and sentences of one of the aforementioned forms, and so on. Hence, in decomposing the members of Γ, there will never be a chance to use a branching rule, and every tree for Γ will have only one branch.

However, Existential Decomposition-2 is a branching rule, so if it is used Γ may have a tree with more than one branch.

10. The envisioned change is not a wise one. For example, though the first rule would indeed eliminate the universal quantifier from a sentence (∀**x**)**P** it would do so only at the cost of producing a sentence of the form ~ (∃**x** ~ **P**, which would then have to be decomposed by Negated Existential Decomposition, yielding a sentence (∀**x**) ~ ~ **P** that is, reintroducing a universally quantified sentence. So too, the rule for decomposing existentially quantified sentences would introduce negated universally quantified sentences that, when themselves decomposed, would produce more existentially quantified sentences.

12. Yes. For to decompose $(\forall \mathbf{x})\mathbf{P}$ we must enter at least one substitution instance of that sentence on every branch passing through $(\forall \mathbf{x})\mathbf{P}$. And since, by assumption, all substitution instances of $(\forall \mathbf{x})\mathbf{P}$ have closed trees, all those branches will close. Therefore, the tree for $(\forall \mathbf{x})\mathbf{P}$ will be closed.

Section 10.1.1E

b. Derive: Kg

1	$(\forall x)(\forall y)Hxy$	Assumption
2	$Hcf \supset Kg$	Assumption
3	$(\forall y)Hcy$	1 $\forall$E
4	Hcf	3 $\forall$E
5	Kg	2, 4 $\supset$E

d. Derive: Pi ⊃ Ai

1	$(\forall x)(Px \equiv Tx)$	Assumption
2	$(\forall z)(Tz \equiv Az)$	Assumption
3	$\quad Pi$	Assumption
4	$\quad Pi \equiv Ti$	1 $\forall$E
5	$\quad Ti$	3, 4 $\equiv$E
6	$\quad Ti \equiv Ai$	2 $\forall$E
7	$\quad Ai$	5, 6 $\equiv$E
8	$Pi \supset Ai$	3–7 $\supset$I

Section 10.1.2E

b. Derive: (∃x)Fxax

1	$Faaa$	Assumption
2	$(\exists x)Fxax$	1 $\exists$I

d. Derive: Wf

1	$(\forall x)Sx$	Assumption
2	$(\exists z)Sz \supset (\forall z)Wz$	Assumption
3	Sj	1 $\forall$E
4	$(\exists z)Sz$	3 $\exists$I
5	$(\forall z)Wz$	2, 4 $\supset$E
6	Wf	5 $\forall$E

Section 10.1.3E

b. Derive: (∀y)(Hyy & By)

1	$(\forall y)Hyy$	Assumption
2	$(\forall z)Bz$	Assumption
3	Htt	1 $\forall$E
4	Bt	2 $\forall$E
5	$Htt \& Bt$	3, 4 &I
6	$(\forall y)(Hyy \& By)$	5 $\forall$I

d. Derive: (∀w) ~ Bw

1	(∀z) (∀y) Lzy	Assumption
2	(∀x) (∀y) (Lxy ⊃ ~ Bx)	Assumption
3	(∀y) Lpy	1 ∀E
4	Lpq	3 ∀E
5	(∀y) (Lpy ⊃ ~ Bp)	2 ∀E
6	Lpq ⊃ ~ Bp	5 ∀E
7	~ Bp	4, 6 ⊃E
8	(∀w) ~ Bw	7 ∀I

Section 10.1.4E

1. b. Derive: (∃x) Lx

1	(∀x) (Fx ⊃ Lx)	Assumption
2	(∃x) Fx	Assumption
3	Fa	Assumption
4	Fa ⊃ La	1 ∀E
5	La	3, 4 ⊃E
6	(∃x) Lx	5 ∃I
7	(∃x) Lx	2, 3–6 ∃E

d. Derive: (∃y) (∃x) Gxy

1	(∃x) (∃y) Gxy	Assumption
2	(∃y) Gay	Assumption
3	Gab	Assumption
4	(∃x) Gxb	3 ∃I
5	(∃y) (∃x) Gxy	4 ∃I
6	(∃y) (∃x) Gxy	2, 3–5 ∃E
7	(∃y) (∃x) Gxy	1, 2–6 ∃E

2. b. This sentence can be derived by ∀E applied to the sentence on line 1.

d. This sentence can be derived by ∃I applied to the sentence on line 2. Note that ‘Saabb’ is a substitution instance of ‘(∃x) Saxbb’.

f. This sentence can be derived by ∃I applied to the sentence on line 2. Note that we may pick the variable of our choice in using the quantifier introduction rules.

h. This sentence cannot be derived. Note that ‘Saabb’ is *not* a substitution instance of ‘(∃x) Sxaxb’. To generate ‘Saabb’, both ‘a’ and ‘b’ would have to replace an occurrence of the free variable ‘x’ in ‘Sxaxb’.

j. This sentence can be derived by ∀I applied to the sentence on line 2.

l. This sentence cannot be derived. Note that ∀I cannot be used to obtain this sentence, for ‘b’ occurs in ‘(∀z)Saabz’, which violates the second restriction on using ∀I.

Section 10.2E

1. b. Derive: ~ (Jb & Qb)

1	(∀z) (Jz ⊃ Lz)	Assumption
2	(∀w) (~ Qw ≡ Lw)	Assumption
3	Jb & Qb	Assumption
4	Jb	3 &E
5	Jb ⊃ Lb	1 ∀E
6	Lb	4, 5 ⊃E
7	~ Qb ≡ Lb	2 ∀E
8	~ Qb	6, 7 ≡E
9	Qb	3 &E
10	~ (Jb & Qb)	3–9 ~ I

d. Derive: Jaa

1	Kmm & ~ Cmr	Assumption
2	(∃y) ~ (Kmy ⊃ Cyr) ⊃ (∀x)Jxx	Assumption
3	Kmm ⊃ Cmr	Assumption
4	Kmm	1 &E
5	Cmr	3, 4 ⊃E
6	~ Cmr	1 &E
7	~ (Kmm ⊃ Cmr)	3–6 ~ I
8	(∃y) ~ (Kmy ⊃ Cyr)	7 ∃I
9	(∀x) Jxx	2, 8 ⊃E
10	Jaa	9 ∀E

f. Derive: $(\forall w)(Lwg \vee Jw)$

1	$(\forall x)(Dxx \vee Px)$	Assumption
2	$(\forall y) \sim Dyy$	Assumption
3	$(\forall z)(Pz \supset Jz)$	Assumption
4	$\sim Daa$	2 $\forall$E
5	$Daa \vee Pa$	1 $\forall$E
6	$Pa \supset Ja$	3 $\forall$E
7	Daa	Assumption
8	$\sim Ja$	Assumption
9	Daa	7 R
10	$\sim Daa$	4 R
11	Ja	8–10 $\sim$ E
12	Pa	Assumption
13	Ja	6, 12 MP
14	Ja	5, 7–11, 12–13 $\vee$E
15	$Lag \vee Ja$	14 $\vee$I
16	$(\forall w)(Lwg \vee Jw)$	15 $\forall$I

h. Derive: $(\exists w) \sim Lwn$

1	$(\exists y)(My \,\&\, Ry)$	Assumption
2	$(\forall y)[Lyn \supset \sim (Ry \vee Cy)]$	Assumption
3	$Ma \,\&\, Ra$	Assumption
4	$Lan \supset \sim (Ra \vee Ca)$	2 $\forall$E
5	Lan	Assumption
6	$\sim (Ra \vee Ca)$	4, 5 $\supset$E
7	Ra	3 &E
8	$Ra \vee Ca$	7 $\vee$I
9	$\sim Lan$	5–8 $\sim$ I
10	$(\exists w) \sim Lwn$	9 $\exists$I
11	$(\exists w) \sim Lwn$	1, 3–10 $\exists$E

j. Derive: $(\exists y)(\forall w)(\exists x)Hxxwyxx$

1	$(\forall x)(\forall y)(\exists w)(\forall z)Hwwxyzz$	Assumption
2	$(\forall y)(\exists w)(\forall z)Hwwayzz$	1 $\forall$E
3	$(\exists w)(\forall z)Hwwabzz$	2 $\forall$E
4	$(\forall z)Hccabzz$	Assumption
5	$Hccabcc$	4 $\forall$E
6	$(\exists x)Hxxabxx$	5 $\exists$I
7	$(\exists x)Hxxabxx$	3, 4–6 $\exists$E
8	$(\forall w)(\exists x)Hxxwbxx$	7 $\forall$I
9	$(\exists y)(\forall w)(\exists x)Hxxwyxx$	8 $\exists$I

2. b. Derive: (∀x) (Bx & Mx)

1	Bk	Assumption	
2	(∀x) Mx	Assumption	
3	Mk	2 ∀E	
4	Bk & Mk	1, 3 &I	
5	(∀x) (Bx & Mx)	4 ∀I	**ERROR!**

'k' occurs in the undischarged assumption on line 1, and hence line 5 violates the second condition for Universal Introduction.

d. Derive: (∃x) Gx

1	(∀x) (Fx ⊃ Gx)	Assumption	
2	~ (∃x) Fx	Assumption	
3	Fi	Assumption	
4	Fi ⊃ Gi	1 ∀E	
5	Gi	3, 4 ⊃E	
6	(∃x) Gx	5 ∃I	
7	(∃x) Gx	1, 3–6 ∃E	**ERROR!**

This not a correct application of Existential Elimination because the assumption on line 3 is not a substitution instance of an existentially quantified sentence. The sentence on line 2 is the negation of an existentially quantified sentence.

f. Derive: (∃y) Dy

1	(∀x) (∃y) (Dx & Sy)	Assumption	
2	(∃y) (Da & Sy)	1 ∀E	
3	Da & Sz	Assumption	**ERROR!**
4	Sz	3 &E	**ERROR!**
5	(∃y) Sy	4 ∃I	**ERROR!**
6	(∃y) Sy	2, 3–5 ∃E	**ERROR!**

The error on line 3 is the occurrence of a free variable 'z'. 'Da & Sz' is not a sentence in *PL* and hence not a possible assumption. This error then contaminates the next three lines of the derivation. Only sentences can occur in derivations—not formulas with free variables in them.

Here is a correct derivation:

1	(∀x)(∃y)(Dx & Sy)	Assumption
2	(∃y)(Da & Sy)	1 ∀E
3	Da & Sb	Assumption
4	Sb	3 &E
5	(∃y)Sy	4 ∃I
6	(∃y)Sy	2, 3–5 ∃E

h. Derive: ~ Sg

1	(∀x)Rxx	Assumption	
2	(∀x)(∀y)Rxy ⊃ ~ Sg	Assumption	
3	Raa	1 ∀E	
4	(∀y)Ray	3 ∀I	**ERROR!**
5	(∀x)(∀y)Rxy	4 ∀I	
6	~ Sg	2, 5 ⊃E	

This violates the second condition for the rule Universal Instantiation. 'a' occurs in the sentence on line 4. A correct use of the rule Universal Instantiation on line 4 would yield '(∀y)Ryy', but in fact '~ Sg' is not derivable from the set of primary assumptions.

Section 10.4E

1. Goal Analysis
First Part: Indicating goals and subgoals

b. Derive: (∃z)Az ⊃ (∃z)Zz

	1	(∀x)(Ax ⊃ Zx)	Assumption
	2	(∃z)Az	Assumption
	3	Aj	Assumption
Subgoal →		(∃z)Zz	
Goal →		(∃z)Zz	2, 3–__ ∃E
		(∃z)Az ⊃ (∃z)Zz	2–__ ⊃I

Given the occurrence of the existential assumption on line 2, it would be best to set up a subderivation to apply the rule Existential Elimination. Hence, the subderivation beginning at line 3 has a substitution instance of the existential sentence on line 2.

d. Derive: $(\forall x)(\forall y)((Bx \,\&\, Ny) \supset Txy)$

	1	$(\forall x)(\forall y)(Bx \supset Txy)$	Assumption
Subgoal →		$(\forall y)((Ba \,\&\, Ny) \supset Tay)$	
Goal →		$(\forall x)(\forall y)((Bx \,\&\, Ny) \supset Txy)$	___ $\forall$I

The goal sentence is a universally quantified sentence, and hence the appropriate subgoal is a substitution instance of it.

f. Derive: $(\exists x)(Hx \equiv Mx)$

	1	Ma	Assumption
	2	$(\forall x)(Mx \supset Hx)$	Assumption
Subgoal →		$Ha \equiv Ma$	
Goal →		$(\exists x)(Hx \equiv Mx)$	___ $\exists$I

When the goal sentence is an existentially quantified sentence, a good subgoal sentence to consider is its substitution instance. If a substitution instance can be derived, then the existentially quantified sentence can be derived in one step by Existential Introduction.

h. Derive: $(\exists y)Lyy$

	1	$(\exists x)(Nx \vee Lxx)$	Assumption
	2	$(\forall x) \sim Nx$	Assumption
	3	$\quad Nh \vee Lhh$	Assumption
Subgoal →		$\quad Lhh$	
Goal →		$\quad (\exists y)Lyy$	___ $\exists$I
		$(\exists y)Lyy$	1, 3–___ $\exists$E

The goal sentence in the subderivation is an existentially quantified sentence, and hence taking a substitution instance as a subgoal is appropriate. ‘Lhh’ is the substitution instance to pick because the constant ‘h’ already occurs in the assumption for the subderivation in the subformula ‘Lhh’ on line 3.

j. Derive: (∀y)(∀x)(Cxy ⊃ Px)

	1	(∀y)(∀x)(Cxy ⊃ Qx)	Assumption
	2	(∀y)(Qy ≡ Py)	Assumption
	3	Cmb	Assumption
Subgoal →		Cmb ⊃ Qm	
Goal →		Qm	3, __ ⊃E
		Qm ≡ Pm	2 ∀E
		Pm	__, __ ≡E
		Cmb ⊃ Pm	3–__ ⊃ I
		(∀x)(Cxb ⊃ Px)	__ ∀I
		(∀y)(∀x)(Cxy ⊃ Px)	__ ∀I

The conditional 'Cmb ⊃ Qm' is a reasonable subgoal because it is a substitution instance of a substitution instance of the assumption on line 1. Note that the choice of constants used in the conditional sentence is suggested by other sentences already occurring in the derivation.

Second Part: Completing the derivations

b. Derive: (∃z)Az ⊃ (∃z)Zz

1	(∀x)(Ax ⊃ Zx)	Assumption
2	(∃z)Az	Assumption
3	Aj	Assumption
4	Aj ⊃ Zj	1 ∀E
5	Zj	3, 4 ⊃E
6	(∃z)Zz	5 ∃I
7	(∃z)Zz	2, 3–6 ∃E
8	(∃z)Az ⊃ (∃z)Zz	2–7 ⊃I

d. Derive: (∀x)(∀y)((Bx & Ny) ⊃ Txy)

1	(∀x)(∀y)(Bx ⊃ Txy)	Assumption
2	Ba & Nb	Assumption
3	(∀y)(Ba ⊃ Tay)	1 ∀E
4	Ba ⊃ Tab	3 ∀E
5	Ba	2 &E
6	Tab	4, 5 ⊃E
7	(Ba & Nb) ⊃ Tab	2–6 ⊃I
8	(∀y)((Ba & Ny) ⊃ Tay)	7 ∀I
9	(∀x)(∀y)((Bx & Ny) ⊃ Txy)	8 ∀I

f. Derive: $(\exists x)(Hx \equiv Mx)$

1	Ma	Assumption
2	$(\forall x)(Mx \supset Hx)$	Assumption
3	$\quad$ Ha	Assumption
4	$\quad$ Ma	1 R
5	$\quad$ Ma	Assumption
6	$\quad$ $Ma \supset Ha$	2 $\forall$E
7	$\quad$ Ha	5, 6 $\supset$E
8	$Ha \equiv Ma$	3–4, 5–7 $\equiv$I
9	$(\exists x)(Hx \equiv Mx)$	8 $\exists$I

h. Derive: $(\exists y)Lyy$

1	$(\exists x)(Nx \vee Lxx)$	Assumption
2	$(\forall x) \sim Nx$	Assumption
3	$\quad$ $Nh \vee Lhh$	Assumption
4	$\quad\quad$ Nh	Assumption
5	$\quad\quad\quad$ $\sim$ Lhh	Assumption
6	$\quad\quad\quad$ Nh	4 R
7	$\quad\quad\quad$ $\sim$ Nh	2 $\forall$E
8	$\quad\quad$ Lhh	5–7 $\sim$ E
9	$\quad\quad$ Lhh	Assumption
10	$\quad\quad$ Lhh	9 R
11	$\quad$ Lhh	3, 4–8, 9–10 $\vee$E
12	$\quad$ $(\exists y)Lyy$	11 $\exists$I
13	$(\exists y)Lyy$	1, 3–12 $\exists$E

j. Derive: $(\forall y)(\forall x)(Cxy \supset Px)$

1	$(\forall x)(\forall y)(Cxy \supset Qx)$	Assumption
2	$(\forall y)(Qy \equiv Py)$	Assumption
3	$\quad$ Cmb	Assumption
4	$\quad$ $(\forall y)(Cmy \supset Qm)$	1 $\forall$E
5	$\quad$ $Cmb \supset Qm$	4 $\forall$E
6	$\quad$ Qm	3, 5 $\supset$E
7	$\quad$ $Qm \equiv Pm$	2 $\forall$E
8	$\quad$ Pm	6, 7 $\equiv$E
9	$Cmb \supset Pm$	3–8 $\supset$I
10	$(\forall x)(Cxb \supset Px)$	9 $\forall$I
11	$(\forall y)(\forall x)(Cxy \supset Px)$	10 $\forall$I

Section 10.4E

2. Derivability

b. Derive: (∃z)Kzz

1	(∃x)Kxx	Assumption
2	Kcc	Assumption
3	(∃z)Kzz	2 ∃I
4	(∃z)Kzz	1, 2–3 ∃E

d. Derive: (∀x)(~ Mx ⊃ Mx)

1	(∀x)Mx	Assumption
2	~ Ma	Assumption
3	Ma	1 ∀E
4	~ Ma ⊃ Ma	2–3 ⊃I
5	(∀x)(~ Mx ⊃ Mx)	4 ∀I

f. Derive: (∃y)(Iy & Gy)

1	(∀x)[(Hx & ~ Kx) ⊃ Ix]	Assumption
2	(∃y)(Hy & Gy)	Assumption
3	(∀x)(Gx & ~ Kx)	Assumption
4	Ha & Ga	Assumption
5	(Ha & ~ Ka) ⊃ Ia	1 ∀E
6	Ga & ~ Ka	3 ∀E
7	Ha	4 &E
8	~ Ka	6 &E
9	Ha & ~ Ka	7, 8 &I
10	Ia	5, 9 ⊃E
11	Ga	6 &E
12	Ia & Ga	10, 11 &I
13	(∃y)(Iy & Gy)	12 ∃I
14	(∃y)(Iy & Gy)	2, 4–13 ∃E

3. Validity

b. Derive: ~ Tb

1	(∀x)(Tx ⊃ Lx)	Assumption
2	~ Lb	Assumption
3	Tb	Assumption
4	Tb ⊃ Lb	1 ∀E
5	Lb	3, 4 ⊃E
6	~ Lb	2 R
7	~ Tb	3–6 ~ I

d. Derive: $(\exists x)Cx \equiv Ch$

1	$(\exists x)Cx \supset Ch$	Assumption
2	$\quad (\exists x)Cx$	Assumption
3	$\quad Ch$	1, 2 $\supset$E
4	$\quad Ch$	Assumption
5	$\quad (\exists x)Cx$	4 $\exists$I
6	$(\exists x)Cx \equiv Ch$	2–3, 4–5 $\equiv$I

f. Derive: $(\exists x)Jxb \;\&\; (\forall x)Mx$

1	$(\forall y)[Hy \;\&\; (Jyy \;\&\; My)]$	Assumption
2	$Hb \;\&\; (Jbb \;\&\; Mb)$	1 $\forall$E
3	$Jbb \;\&\; Mb$	2 &E
4	Jbb	3 &E
5	$(\exists x)Jxb$	4 $\exists$I
6	Mb	3 &E
7	$(\forall x)Mx$	6 $\forall$I
8	$(\exists x)Jxb \;\&\; (\forall x)Mx$	5, 7 &I

4. Theorems

b. Derive: $(\forall x)(Ax \supset \sim\sim Ax)$

1	$\quad Aa$	Assumption
2	$\quad\quad \sim Aa$	Assumption
3	$\quad\quad Aa$	1 R
4	$\quad\quad \sim Aa$	2 R
5	$\quad \sim\sim Aa$	2–4 $\sim$I
6	$Aa \supset \sim\sim Aa$	1–5 $\supset$I
7	$(\forall x)(Ax \supset \sim\sim Ax)$	6 $\forall$I

d. Derive: $(\exists x)(Ax \;\&\; Bx) \supset ((\exists x)Ax \;\&\; (\exists x)Bx)$

1	$\quad (\exists x)(Ax \;\&\; Bx)$	Assumption
2	$\quad\quad Ac \;\&\; Bc$	Assumption
3	$\quad\quad Ac$	2 &E
4	$\quad\quad (\exists x)Ax$	3 $\exists$I
5	$\quad\quad Bc$	2 &I
6	$\quad\quad (\exists x)Bx$	5 $\exists$I
7	$\quad\quad (\exists x)Ax \;\&\; (\exists x)Bx$	4, 6 &I
8	$\quad (\exists x)Ax \;\&\; (\exists x)Bx$	1, 2–7 $\exists$E
9	$(\exists x)(Ax \;\&\; Bx) \supset ((\exists x)Ax \;\&\; (\exists x)Bx)$	1–8 $\supset$I

f. Derive: $(\forall x)(Ax \supset B) \equiv ((\exists x)Ax \supset B)$

1	│ │ $(\forall x)(Ax \supset B)$	Assumption
2	│ │ │ $(\exists x)Ax$	Assumption
3	│ │ │ │ Ac	Assumption
4	│ │ │ │ $Ac \supset B$	1 $\forall$E
5	│ │ │ │ B	3, 4 $\supset$E
6	│ │ │ B	2, 3–5 $\exists$E
7	│ │ $(\exists x)Ax \supset B$	2–6 $\supset$I
8	│ │ $(\exists x)Ax \supset B$	Assumption
9	│ │ │ Ac	Assumption
10	│ │ │ $(\exists x)Ax$	9 $\exists$I
11	│ │ │ B	8, 10 $\supset$E
12	│ │ $Ac \supset B$	9–11 $\supset$I
13	│ │ $(\forall x)(Ax \supset B)$	12 $\forall$I
14	│ $(\forall x)(Ax \supset B) \equiv ((\exists x)Ax \supset B)$	1–7, 8–13 $\equiv$I

5. Equivalence

b. Derive: $(\forall x)Ax \ \& \ (\forall y)By$

1	│ $(\forall x)(\forall y)(Ax \ \& \ By)$	Assumption
2	│ $(\forall y)(Aa \ \& \ By)$	1 $\forall$E
3	│ $Aa \ \& \ Bb$	2 $\forall$E
4	│ Aa	3 &E
5	│ $(\forall x)Ax$	4 $\forall$I
6	│ Bb	3 &E
7	│ $(\forall y)By$	6 $\forall$I
8	│ $(\forall x)Ax \ \& \ (\forall y)By$	5, 7 &I

Derive: $(\forall x)(\forall y)(Ax \ \& \ By)$

1	│ $(\forall x)Ax \ \& \ (\forall y)By$	Assumption
2	│ $(\forall x)Ax$	1 &E
3	│ Aa	2 $\forall$E
4	│ $(\forall y)By$	1 &E
5	│ Bb	4 $\forall$E
6	│ $Aa \ \& \ Bb$	3, 5 &I
7	│ $(\forall y)(Aa \ \& \ By)$	6 $\forall$I
8	│ $(\forall x)(\forall y)(Ax \ \& \ By)$	7 $\forall$I

d. Derive: ~ (∃x)Ax

```
1  | (∀x) ~ Ax                Assumption
   |---------
2  | | (∃x)Ax                 Assumption
   | |-------
3  | | | Ac                   Assumption
   | | |---
4  | | | | (∃x)Ax             Assumption
   | | | |-------
5  | | | | Ac                 3 R
6  | | | | ~ Ac               1 ∀E
7  | | | ~ (∃x)Ax             4–6 ~ I
8  | | ~ (∃x)Ax               2, 3–7 ∃E
9  | | (∃x)Ax                 2 R
10 | ~ (∃x)Ax                 2–9 ~ I
```

Derive: (∀x) ~ Ax

```
1  | ~ (∃x)Ax                 Assumption
   |---------
2  | | Ac                     Assumption
   | |---
3  | | (∃x)Ax                 2 ∃I
4  | | ~ (∃x)Ax               1 R
5  | ~ Ac                     2–4 ~ I
6  | (∀x) ~ Ax                5 ∀I
```

f. Derive: (∀x) [(∃y) (Axy & Ayx) ⊃ Axx]

```
1  | (∀x) (∀y) [(Axy & Ayx) ⊃ Axx]          Assumption
   |------------------------------
2  | | (∃y) (Aay & Aya)                     Assumption
   | |-----------------
3  | | | Aab & Aba                          Assumption
   | | |----------
4  | | | (∀y) [(Aay & Aya) ⊃ Aaa]          1 ∀E
5  | | | (Aab & Aba) ⊃ Aaa                  4 ∀E
6  | | | Aaa                                3, 5 ⊃E
7  | | Aaa                                  2, 3–6 ∃E
8  | (∃y) (Aay & Aya) ⊃ Aaa                 2–7 ⊃I
9  | (∀x) [(∃y) (Axy & Ayx) ⊃ Axx]          8 ∀I
```

Derive: (∀x) (∀y) [(Axy & Ayx) ⊃ Axx]

```
1  | (∀x) [(∃y) (Axy & Ayx) ⊃ Axx]          Assumption
   |------------------------------
2  | | Aab & Aba                            Assumption
   | |----------
3  | | (∃y) (Aay & Aya)                     2 ∃I
4  | | (∃y) (Aay & Aya) ⊃ Aaa               1 ∀E
5  | | Aaa                                  3, 4 ⊃E
6  | (Aab & Aba) ⊃ Aaa                      2–5 ⊃I
7  | (∀y) [(Aay & Aya) ⊃ Aaa]               6 ∀I
8  | (∀x) (∀y) [(Axy & Ayx) ⊃ Axx]          7 ∀I
```

6. Inconsistency

```
b.  1 | (∀x)(Rx ≡ ~ Rx)            Assumption
      |----------------
    2 | Ra ≡ ~ Ra                  1 ∀E
    3 | | Ra                       Assumption
      | |-----------
    4 | | ~ Ra                     2, 3 ≡E
    5 | | Ra                       3 R
    6 | ~ Ra                       3–5 ~ I
    7 | Ra                         2, 6 ≡E

d.  1 | (∃x)(∀y)Fxy                Assumption
    2 | ~ (∀y)(∃x)Fxy              Assumption
      |----------------
    3 | | (∀y)Fay                  Assumption
      | |-----------
    4 | | Fab                      3 ∀E
    5 | | (∃x)Fxb                  4 ∃I
    6 | | (∀y)(∃x)Fxy              5 ∀I
    7 | (∀y)(∃x)Fxy                1, 3–6 ∃E
    8 | ~ (∀y)(∃x)Fxy              2 R

f.   1 | ~ (∃y)Jy                  Assumption
     2 | ~ (∃x) ~ Hx               Assumption
     3 | (∀z)(Jz ∨ ~ Hz)           Assumption
       |----------------
     4 | Jk ∨ ~ Hk                 3 ∀E
     5 | | Jk                      Assumption
       | |-----------
     6 | | | ~ (S & ~ S)           Assumption
       | | |-----------
     7 | | | (∃y)Jy                5 ∃I
     8 | | | ~ (∃y)Jy              1 R
     9 | | S & ~ S                 6–8 ~ E
    10 | | ~ Hk                    Assumption
       | |-----------
    11 | | | ~ (S & ~ S)           Assumption
       | | |-----------
    12 | | | (∃x) ~ Hx             10 ∃I
    13 | | | ~ (∃x) ~ Hx           2 R
    14 | | S & ~ S                 11–13 ~ E
    15 | S & ~ S                   4, 5–9, 10–14 ∨E
    16 | S                         15 &E
    17 | ~ S                       15 &E
```

7. Derivability

b. Derive: (∀x)(∀y)(∀z)(Gxyz ⊃ Gzyx)

1	(∀x)(∀y)(∀z)Gxyz	Assumption
2	| Gabc	Assumption
3	| (∀y)(∀z)Gcyz	1 ∀E
4	| (∀z)Gcbz	3 ∀E
5	| Gcba	4 ∀E
6	Gabc ⊃ Gcba	2–5 ⊃I
7	(∀z)(Gabz ⊃ Gzba)	6 ∀I
8	(∀y)(∀z)(Gayz ⊃ Gzya)	7 ∀I
9	(∀x)(∀y)(∀z)(Gxyz ⊃ Gzyx)	8 ∀I

d. Derive: ~ (∀x)Kx

1	~ (∀x)(Fx & Aix) ≡ ~ (∀x)Kx	Assumption
2	(∀y)[(∃x) ~ (Fx & Aix) & Ryy]	Assumption
3	(∃x) ~ (Fx & Aix) & Rmm	2 ∀E
4	(∃x) ~ (Fx & Aix)	3 &E
5	| ~ (Fa & Aia)	Assumption
6	| | (∀x)(Fx & Aix)	Assumption
7	| | Fa & Aia	6 ∀E
8	| | ~ (Fa & Aia)	5 R
9	| ~ (∀x)(Fx & Aix)	6–8 ~ I
10	~ (∀x)(Fx & Aix)	4, 5–9 ∃E
11	~ (∀x)Kx	1, 10 ≡E

f. Derive: (∃z)(~ Az & Hzz)

1	(∃x)(Jxa & Ck)	Assumption
2	(∃x)(Sx & Hxx)	Assumption
3	(∀x)[(Cx & Sx) ⊃ ~ Ax]	Assumption
4	| Jba & Ck	Assumption
5	| | Sc & Hcc	Assumption
6	| | (Ck & Sc) ⊃ ~ Ac	3 ∀E
7	| | Ck	4 &E
8	| | Sc	5 &E
9	| | Ck & Sc	7, 8 &I
10	| | ~ Ac	6, 9 ⊃E
11	| | Hcc	5 &E
12	| | ~ Ac & Hcc	10, 11 &I
13	| | (∃z)(~ Az & Hzz)	12 ∃I
14	| (∃z)(~ Az & Hzz)	2, 5–13 ∃E
15	(∃z)(~ Az & Hzz)	1, 4–14 ∃E

h. Derive: (∃x)(∃y)(Cxy & Cyx)

1	(∀x)(∀y)(Dxy ⊃ Cxy)	Assumption
2	(∀x)(∃y)Dxy	Assumption
3	(∀x)(∀y)(Cyx ⊃ Dxy)	Assumption
4	(∃y)Day	2 ∀E
5	Dab	Assumption
6	(∀y)(Day ⊃ Cay)	1 ∀E
7	Dab ⊃ Cab	6 ∀E
8	Cab	5, 7 ⊃E
9	(∀y)(Cyb ⊃ Dby)	3 ∀E
10	Cab ⊃ Dba	9 ∀E
11	Dba	8, 10 ⊃E
12	(∀y)(Dby ⊃ Cby)	1 ∀E
13	Dba ⊃ Cba	12 ∀E
14	Cba	11, 13 ⊃E
15	Cab & Cba	8, 14 &I
16	(∃y)(Cay & Cya)	15 ∃I
17	(∃x)(∃y)(Cxy & Cyx)	16 ∃I
18	(∃x)(∃y)(Cxy & Cyx)	4, 5–17 ∃E

8. Validity

b. Derive: ~ (∀x)(Px ⊃ ~ Jx)

1	(∃x) ~ (∃y)(Rxy & ~ Uy) ⊃ (∃x)(Px & Jx)	Assumption
2	(∃x) ~ (∃y)(Rxy & ~ Uy)	Assumption
3	(∃x)(Px & Jx)	1, 2 ⊃E
4	Pa & Ja	Assumption
5	(∀x)(Px ⊃ ~ Jx)	Assumption
6	Pa ⊃ ~ Ja	5 ∀E
7	Ja	4 &E
8	Pa	4 &E
9	~ Ja	6, 8 ⊃E
10	~ (∀x)(Px ⊃ ~ Jx)	5–9 ~ I
11	~ (∀x)(Px ⊃ ~ Jx)	3, 4–10 ∃E

d. Derive: $(\exists x)(Fx \mathbin{\&} Gx)$

1	$(\forall x)((Fx \mathbin{\&} Gx) \equiv (\exists y)(Axy \mathbin{\&} Py))$	Assumption
2	$(\exists x)(\exists y)(Fx \mathbin{\&} (Axy \mathbin{\&} Py))$	Assumption
3	$(\exists y)(Fa \mathbin{\&} (Aay \mathbin{\&} Py))$	Assumption
4	$Fa \mathbin{\&} (Aab \mathbin{\&} Pb)$	Assumption
5	$(Fa \mathbin{\&} Ga) \equiv (\exists y)(Aay \mathbin{\&} Py)$	1 $\forall$E
6	$Aab \mathbin{\&} Pb$	4 &E
7	$(\exists y)(Aay \mathbin{\&} Py)$	6 $\exists$I
8	$Fa \mathbin{\&} Ga$	5, 7 $\equiv$E
9	$(\exists x)(Fx \mathbin{\&} Gx)$	8 $\exists$I
10	$(\exists x)(Fx \mathbin{\&} Gx)$	3, 4–9 $\exists$E
11	$(\exists x)(Fx \mathbin{\&} Gx)$	2, 3–10 $\exists$E

f. Derive: $((\exists x)Px \mathbin{\&} (\exists x)Qx) \equiv (\exists x)(Px \mathbin{\&} Qx)$

1	$(\forall x)(Px \supset Qx)$	Assumption
2	$(\exists x)(Px \mathbin{\&} Qx)$	Assumption
3	$Pa \mathbin{\&} Qa$	Assumption
4	Pa	3 &E
5	$(\exists x)Px$	4 $\exists$I
6	Qa	3 &E
7	$(\exists x)Qx$	6 $\exists$I
8	$(\exists x)Px \mathbin{\&} (\exists x)Qx$	5, 7 &I
9	$(\exists x)Px \mathbin{\&} (\exists x)Qx$	2, 3–8 $\exists$E
10	$(\exists x)Px \mathbin{\&} (\exists x)Qx$	Assumption
11	$(\exists x)Px$	10 &E
12	Pa	Assumption
13	$Pa \supset Qa$	1 $\forall$E
14	Qa	12, 13 $\supset$E
15	$Pa \mathbin{\&} Qa$	12, 14 &I
16	$(\exists x)(Px \mathbin{\&} Qx)$	15 $\exists$I
17	$(\exists x)(Px \mathbin{\&} Qx)$	11, 12–16 $\exists$E
18	$((\exists x)Px \mathbin{\&} (\exists x)Qx) \equiv (\exists x)(Px \mathbin{\&} Qx)$	2–9, 10–17 $\equiv$I

h. Derive: (∃x) (Rx & Ax)

1	(∀y) (My ⊃ Ay)	Assumption
2	(∃x) (∃y) [(Bx & Mx) & (Ry & Syx)]	Assumption
3	(∃x) Ax ⊃ (∀y) (∀z) (Syz ⊃ Ay)	Assumption
4	(∃y) [(Ba & Ma) & (Ry & Sya)]	Assumption
5	(Ba & Ma) & (Rb & Sba)	Assumption
6	Ba & Ma	5 &E
7	Ma	6 &E
8	Ma ⊃ Aa	1 ∀E
9	Aa	7, 8 ⊃E
10	(∃x) Ax	9 ∃I
11	(∀y) (∀z) (Syz ⊃ Ay)	3, 10 ⊃E
12	(∀z) (Sbz ⊃ Ab)	11 ∀E
13	Sba ⊃ Ab	12 ∀E
14	Rb & Sba	5 &E
15	Sba	14 &E
16	Ab	13, 15 ⊃E
17	Rb	14 &E
18	Rb & Ab	17, 16 &I
19	(∃x) (Rx & Ax)	18 ∃I
20	(∃x) (Rx & Ax)	4, 5–19 ∃E
21	(∃x) (Rx & Ax)	2, 4–20 ∃E

j. Derive: Mp

1	$(\forall x)((Fx \,\&\, {\sim} Kx) \supset (\exists y)[(Fy \,\&\, Hyx) \,\&\, {\sim} Ky])$	Assumption
2	$(\forall x)[(Fx \,\&\, (\forall y)[(Fy \,\&\, Hyx) \supset Ky]) \supset Kx] \supset Mp$	Assumption
3	| $Fa \,\&\, (\forall y)[(Fy \,\&\, Hya) \supset Ky]$	Assumption
4	| | ${\sim} Ka$	Assumption
5	| | $(Fa \,\&\, {\sim} Ka) \supset (\exists y)[(Fy \,\&\, Hya) \,\&\, {\sim} Ky]$	1 ∀E
6	| | Fa	3 &E
7	| | $Fa \,\&\, {\sim} Ka$	6, 4 &I
8	| | $(\exists y)[(Fy \,\&\, Hya) \,\&\, {\sim} Ky]$	5, 7 ⊃E
9	| | | $(Fb \,\&\, Hba) \,\&\, {\sim} Kb$	Assumption
10	| | | $(\forall y)[(Fy \,\&\, Hya) \supset Ky]$	3 &E
11	| | | $(Fb \,\&\, Hba) \supset Kb$	10 ∀E
12	| | | $Fb \,\&\, Hba$	9 &E
13	| | | | ${\sim} Ka$	Assumption
14	| | | | Kb	11, 12 ⊃E
15	| | | | ${\sim} Kb$	9 &E
16	| | | Ka	13–15 ~ E
17	| | Ka	8, 9–16 ∃E
18	| | ${\sim} Ka$	4 R
19	| Ka	4–18 ~ E
20	$(Fa \,\&\, (\forall y)[(Fy \,\&\, Hya) \supset Ky]) \supset Ka$	3–19 ⊃I
21	$(\forall x)[(Fx \,\&\, (\forall y)[(Fy \,\&\, Hyx) \supset Ky]) \supset Kx]$	20 ∀I
22	Mp	2, 21 ⊃E

l. Derive: $(\forall z)(\forall w)[(Az \,\&\, Hw) \supset Czw]$

1	$(\forall x)(\forall y)[(Ax \,\&\, By) \supset Cxy]$	Assumption
2	$(\exists y)[Ey \,\&\, (\forall w)(Hw \supset Cyw)]$	Assumption
3	$(\forall x)(\forall y)(\forall z)[(Cxy \,\&\, Cyz) \supset Cxz]$	Assumption
4	$(\forall w)(Ew \supset Bw)$	Assumption
5	$\mid\ Aa \,\&\, Hb$	Assumption
6	$\mid\ \mid\ Ec \,\&\, (\forall w)(Hw \supset Ccw)$	Assumption
7	$\mid\ \mid\ Hb$	5 &E
8	$\mid\ \mid\ (\forall w)(Hw \supset Ccw)$	6 &E
9	$\mid\ \mid\ Hb \supset Ccb$	8 ∀E
10	$\mid\ \mid\ Ccb$	7, 9 ⊃E
11	$\mid\ \mid\ Ec$	6 &E
12	$\mid\ \mid\ Ec \supset Bc$	4 ∀E
13	$\mid\ \mid\ Bc$	11, 12 ⊃E
14	$\mid\ \mid\ (\forall y)[(Aa \,\&\, By) \supset Cay]$	1 ∀E
15	$\mid\ \mid\ (Aa \,\&\, Bc) \supset Cac$	14 ∀E
16	$\mid\ \mid\ Aa$	5 &E
17	$\mid\ \mid\ Aa \,\&\, Bc$	16, 13 &I
18	$\mid\ \mid\ Cac$	17, 15 ⊃E
19	$\mid\ \mid\ (\forall y)(\forall z)[(Cay \,\&\, Cyz) \supset Caz]$	3 ∀E
20	$\mid\ \mid\ (\forall z)[(Cac \,\&\, Ccz) \supset Caz]$	19 ∀E
21	$\mid\ \mid\ (Cac \,\&\, Ccb) \supset Cab$	20 ∀E
22	$\mid\ \mid\ Cac \,\&\, Ccb$	18, 10 &I
23	$\mid\ \mid\ Cab$	22, 21 ⊃E
24	$\mid\ Cab$	2, 6–23 ∃E
25	$(Aa \,\&\, Hb) \supset Cab$	5–24 ⊃I
26	$(\forall w)[(Aa \,\&\, Hw) \supset Caw]$	25 ∀I
27	$(\forall z)(\forall w)[(Az \,\&\, Hw) \supset Czw]$	26 ∀I

9. Theorems

b. Derive: $(\forall x)(\exists y)(Ax \supset By) \supset ((\forall x)Ax \supset (\exists y)By)$

1	$\mid\ (\forall x)(\exists y)(Ax \supset By)$	Assumption
2	$\mid\ \mid\ (\forall x)Ax$	Assumption
3	$\mid\ \mid\ (\exists y)(Ac \supset By)$	1 ∀E
4	$\mid\ \mid\ \mid\ Ac \supset Bd$	Assumption
5	$\mid\ \mid\ \mid\ Ac$	2 ∀E
6	$\mid\ \mid\ \mid\ Bd$	4, 5 ⊃E
7	$\mid\ \mid\ \mid\ (\exists y)By$	6 ∃I
8	$\mid\ (\exists y)By$	3, 4–7 ∃E
9	$(\forall x)Ax \supset (\exists y)By$	2, 8 ⊃I
10	$(\forall x)(\exists y)(Ax \supset By) \supset ((\forall x)Ax \supset (\exists y)By)$	1, 9 ⊃I

d. Derive: $(\forall x)(Ax \vee Bi) \equiv ((\forall x)Ax \vee Bi)$

1	$(\forall x)(Ax \vee Bi)$	Assumption
2	$\sim ((\forall x)Ax \vee Bi)$	Assumption
3	$Ac \vee Bi$	1 $\forall$E
4	Ac	Assumption
5	Ac	4 R
6	Bi	Assumption
7	$\sim Ac$	Assumption
8	$(\forall x)Ax \vee Bi$	6 $\vee$I
9	$\sim ((\forall x)Ax \vee Bi)$	2 R
10	Ac	7–9 $\sim$ E
11	Ac	3, 4–5, 6–10 $\vee$E
12	$(\forall x)Ax$	11 $\forall$I
13	$(\forall x)Ax \vee Bi$	12 $\vee$I
14	$\sim ((\forall x)Ax \vee Bi)$	2 R
15	$(\forall x)Ax \vee Bi$	2–14 $\sim$ E
16	$(\forall x)Ax \vee Bi$	Assumption
17	$(\forall x)Ax$	Assumption
18	Ac	17 $\forall$E
19	$Ac \vee Bi$	18 $\vee$I
20	Bi	Assumption
21	$Ac \vee Bi$	20 $\vee$I
22	$Ac \vee Bi$	16, 17–19, 20–21 $\vee$E
23	$(\forall x)(Ax \vee Bi)$	22 $\forall$I
24	$(\forall x)(Ax \vee Bi) \equiv ((\forall x)Ax \vee Bi)$	1–15, 16–23 $\equiv$I

f. Derive: $(\exists x)(Ax \supset B) \equiv ((\forall x)Ax \supset B)$

1	$\mid\ \mid\ (\exists x)(Ax \supset B)$	Assumption
2	$\mid\ \mid\ \mid\ (\forall x)Ax$	Assumption
3	$\mid\ \mid\ \mid\ \mid\ Ac \supset B$	Assumption
4	$\mid\ \mid\ \mid\ \mid\ Ac$	2 $\forall$E
5	$\mid\ \mid\ \mid\ \mid\ B$	3, 4 $\supset$E
6	$\mid\ \mid\ \mid\ B$	1, 3–5 $\exists$E
7	$\mid\ \mid\ (\forall x)Ax \supset B$	2–6 $\supset$I
8	$\mid\ \mid\ (\forall x)Ax \supset B$	Assumption
9	$\mid\ \mid\ \mid\ \sim (\exists x)(Ax \supset B)$	Assumption
10	$\mid\ \mid\ \mid\ \mid\ \sim Ac$	Assumption
11	$\mid\ \mid\ \mid\ \mid\ \mid\ Ac$	Assumption
12	$\mid\ \mid\ \mid\ \mid\ \mid\ \mid\ \sim B$	Assumption
13	$\mid\ \mid\ \mid\ \mid\ \mid\ \mid\ \sim Ac$	10 R
14	$\mid\ \mid\ \mid\ \mid\ \mid\ \mid\ Ac$	11 R
15	$\mid\ \mid\ \mid\ \mid\ \mid\ B$	12–14 ~ E
16	$\mid\ \mid\ \mid\ \mid\ Ac \supset B$	11–15 $\supset$I
17	$\mid\ \mid\ \mid\ \mid\ (\exists x)(Ax \supset B)$	16 $\exists$I
18	$\mid\ \mid\ \mid\ \mid\ \sim (\exists x)(Ax \supset B)$	9 R
19	$\mid\ \mid\ \mid\ Ac$	10–18 ~ E
20	$\mid\ \mid\ \mid\ (\forall x)Ax$	19 $\forall$I
21	$\mid\ \mid\ \mid\ \mid\ Ac$	Assumption
22	$\mid\ \mid\ \mid\ \mid\ B$	8, 20 $\supset$E
23	$\mid\ \mid\ \mid\ Ac \supset B$	21–22 $\supset$I
24	$\mid\ \mid\ \mid\ (\exists x)(Ax \supset B)$	23 $\exists$I
25	$\mid\ \mid\ \mid\ \sim (\exists x)(Ax \supset B)$	9 R
26	$\mid\ \mid\ (\exists x)(Ax \supset B)$	9–25 ~ E
27	$\mid\ (\exists x)(Ax \supset B) \equiv ((\forall x)Ax \supset B)$	1–7, 8–26 $\equiv$I

10. Equivalence

b. Derive: $(\forall x)Ax$

1	$\mid\ (\forall x)(Ax \vee (Ax)$	Assumption
2	$\mid\ Ac \vee Ac$	1 $\forall$E
3	$\mid\ \mid\ Ac$	Assumption
4	$\mid\ \mid\ Ac$	3 R
5	$\mid\ Ac$	2, 3–4, 3–4 $\vee$E
6	$\mid\ (\forall x)Ax$	5 $\forall$I

Derive: (∀x) (Ax ∨ Ax)

```
 1 | (∀x)Ax                                      Assumption
   |-------
 2 | Ac                                          1 ∀E
 3 | Ac ∨ Ac                                     2 ∨I
 4 | (∀x) (Ax ∨ Ax)                              3 ∀I
```

d. Derive: (∀x) (∀y) (∀z) [(Ax & By) ⊃ Cz]

```
 1 | (∃x)Ax ⊃ ((∃y)By ⊃ (∀z)Cz)                  Assumption
   |----------------------------
 2 | | Aa & Bb                                   Assumption
   | |---------
 3 | | Aa                                        2 &E
 4 | | (∃x)Ax                                    3 ∃I
 5 | | (∃y)By ⊃ (∀z)Cz                           1, 4 ⊃E
 6 | | Bb                                        2 &E
 7 | | (∃y)By                                    6 ∃I
 8 | | (∀z)Cz                                    5, 7 ⊃E
 9 | | Cc                                        8 ∀E
10 | (Aa & Bb) ⊃ Cc                              2–9 ⊃I
11 | (∀z) [(Aa & Bb) ⊃ Cz]                       10 ∀I
12 | (∀y) (∀z) [(Aa & By) ⊃ Cz]                  11 ∀I
13 | (∀x) (∀y) (∀z) [(Ax & By) ⊃ Cz]             12 ∀I
```

Derive: (∃x)Ax ⊃ ((∃y)By ⊃ (∀z)Cz)

```
 1 | (∀x) (∀y) (∀z) [(Ax & By) ⊃ Cz]             Assumption
   |--------------------------------
 2 | | (∃x)Ax                                    Assumption
   | |--------
 3 | | | (∃y)By                                  Assumption
   | | |--------
 4 | | | | Aa                                    Assumption
   | | | |----
 5 | | | | | Bb                                  Assumption
   | | | | |----
 6 | | | | | Aa & Bb                             4, 5 &I
 7 | | | | | (∀y) (∀z) [(Aa & By) ⊃ Cz]          1 ∀E
 8 | | | | | (∀z) [(Aa & Bb) ⊃ Cz]               7 ∀E
 9 | | | | | (Aa & Bb) ⊃ Cc                      8 ∀E
10 | | | | | Cc                                  6, 9 ⊃E
11 | | | | | (∀z)Cz                              10 ∀I
12 | | | | (∀z)Cz                                3, 5–11 ∃E
13 | | | (∀z)Cz                                  2, 4–12 ∀E
14 | | (∃y)By ⊃ (∀z)Cz                           3–13 ⊃I
15 | (∃x)Ax ⊃ ((∃y)By ⊃ (∀z)Cz)                  2–14 ⊃I
```

f. Derive: (∀x)(~ (∀y)Azy ∨ ~ (∀y)Bxy)

1	(∀x)(∃y) ~ (Axy & Bxy)	Assumption
2	(∃y) ~ (Aay & Bay)	1 ∀E
3	~ (Aab & Bab)	Assumption
4	~ (~ (∀y)Aay ∨ ~ (∀y)Bay)	Assumption
5	(∀y)Aay	Assumption
6	Aab	5 ∀E
7	(∀y)Bay	Assumption
8	Bab	7 ∀E
9	Aab & Bab	6, 8 &I
10	~ (Aab & Bab)	3 R
11	~ (∀y)Bay	7–10 ~ I
12	~ (∀y)Aay ∨ ~ (∀y)Bay	11 ∨I
13	~ (~ (∀y)Aay ∨ ~ (∀y)Bay)	4 R
14	~ (∀y)Aay	5–13 ~ I
15	~ (∀y)Aay ∨ ~ (∀y)Bay	14 ∨I
16	~ (~ (∀y)Aay ∨ ~ (∀y)Bay)	4 R
17	~ (∀y)Aay ∨ ~ (∀y)Bay	4–16 ~ E
18	~ (∀y)Aay ∨ ~ (∀y)Bay	2, 3–17 ∃E
19	(∀x)(~ (∀y)Axy ∨ ~ (∀y)Bxy)	18 ∀I

Derive: (∀x)(∃y) ~ (Axy & Bxy)

1	(∀x)(~ (∀y)Axy ∨ ~ (∀y)Bxy)	Assumption
2	~ (∀y)Aay ∨ ~ (∀y)Bay	1 ∀E
3	~ (∀y)Aay	Assumption
4	~ (∃y) ~ (Aay & Bay)	Assumption
5	~ Aab	Assumption
6	Aab & Bab	Assumption
7	Aab	6 &E
8	~ Aab	5 R
9	~ (Aab & Bab)	6–8 ~ I
10	(∃y) ~ (Aay & Bay)	9 ∃I
11	~ (∃y) ~ (Aay & Bay)	4 R
12	Aab	5–11 ~ E
13	(∀y)Aay	12 ∀I
14	~ (∀y)Aay	3 R
15	(∃y) ~ (Aay & Bay)	4–14 ~ E
16	~ (∀y)Bay	Assumption
17	~ (∃y) ~ (Aay & Bay)	Assumption
18	~ Bab	Assumption
19	Aab & Bab	Assumption
20	Bab	19 &E
21	~ Bab	18 R
22	~ (Aab & Bab)	19–21 ~ I
23	(∃y) ~ (Aay & Bay)	22 ∃I
24	~ (∃y) ~ (Aay & Bay)	17 R
25	Bab	18–24 ~ E
26	(∀y)Bay	25 ∀I
27	~ (∀y)Bay	16 R
28	(∃y) ~ (Aay & Bay)	17–27 ~ E
29	(∃y) ~ (Aay & Bay)	2, 3–15, 16–28 ∨E
30	(∀x)(∃y) ~ (Axy & Bxy)	29 ∀I

11. Inconsistency

b.	1	$(\exists x)(Hx \,\&\, Mxc)$	Assumption
	2	$(\forall x)(Lx \supset \sim Hx)$	Assumption
	3	$(\exists y)(Ly \,\&\, Hy)$	Assumption
	4	$\quad$ $La \,\&\, Ha$	Assumption
	5	$\quad$ $La \supset \sim Ha$	2 ∀E
	6	$\quad\quad$ $\sim (Ri \,\&\, \sim Ri)$	Assumption
	7	$\quad\quad$ La	4 &E
	8	$\quad\quad$ $\sim Ha$	5, 7 ⊃E
	9	$\quad\quad$ Ha	4 &E
	10	$\quad$ $Ri \,\&\, \sim Ri$	6–9 ~ E
	11	$Ri \,\&\, \sim Ri$	3, 4–10 ∃E
	12	Ri	11 &E
	13	$\sim Ri$	11 &E

d.	1	$(\forall x)(\forall y)[(Jx \,\&\, Gy) \supset Hxy]$	Assumption
	2	$(\exists x)(\exists y)[(Jx \,\&\, \sim Jy) \,\&\, \sim Hxy]$	Assumption
	3	$(\forall w)Gw$	Assumption
	4	$\quad$ $(\exists y)[(Ja \,\&\, \sim Jy) \,\&\, \sim Hay]$	Assumption
	5	$\quad\quad$ $(Ja \,\&\, \sim Jb) \,\&\, \sim Hab$	Assumption
	6	$\quad\quad$ $Ja \,\&\, \sim Jb$	5 &E
	7	$\quad\quad$ Ja	6 &E
	8	$\quad\quad$ Gb	3 ∀E
	9	$\quad\quad$ $Ja \,\&\, Gb$	7, 8 &I
	10	$\quad\quad$ $(\forall y)[(Ja \,\&\, Gy) \supset Hay]$	I ∀E
	11	$\quad\quad$ $(Ja \,\&\, Gb) \supset Hab$	10 ∀E
	12	$\quad\quad\quad$ $(\forall w)Gw$	Assumption
	13	$\quad\quad\quad$ Hab	9, 11 ⊃E
	14	$\quad\quad\quad$ $\sim Hab$	5 &E
	15	$\quad\quad$ $\sim (\forall w)Gw$	12–14 ~ I
	16	$\quad$ $\sim (\forall w)Gw$	4, 5–15 ∃E
	17	$\sim (\forall w)Gw$	2, 4–16 ∃E
	18	$(\forall w)Gw$	3 R

f.

1	$(\forall x)(\exists y)(Hx \supset By)$	Assumption
2	$\sim (\exists y)(\forall x)(Hx \supset By)$	Assumption
3	Ha	Assumption
4	$(\exists y)(Ha \supset By)$	I $\forall$E
5	$Ha \supset Bb$	Assumption
6	$\sim (\forall x)(Hx \supset Bb)$	Assumption
7	Hc	Assumption
8	Bb	3, 5 $\supset$E
9	$Hc \supset Bb$	7–8 $\supset$I
10	$(\forall x)(Hx \supset Bb)$	9 $\forall$I
11	$\sim (\forall x)(Hx \supset Bb)$	6 R
12	$(\forall x)(Hx \supset Bb)$	6–11 $\sim$ E
13	$(\exists y)(\forall x)(Hx \supset By)$	12 $\exists$I
14	$(\exists y)(\forall x)(Hx \supset By)$	4, 5–13 $\exists$E
15	$\sim Bb$	Assumption
16	$\sim (\exists y)(\forall x)(Hx \supset By)$	2 R
17	$(\exists y)(\forall x)(Hx \supset By)$	14 R
18	Bb	15–17 $\sim$ E
19	$Ha \supset Bb$	3–18 $\supset$I
20	$(\forall x)(Hx \supset Bb)$	19 $\forall$I
21	$(\exists y)(\forall x)(Hx \supset By)$	20 $\exists$I
22	$\sim (\exists y)(\forall x)(Hx \supset By)$	2 R

12. Validity

b. Derive: $(\forall y)(Hy \supset \sim Sgy)$

1	$(\forall y)[(Hy \,\&\, Wyg) \supset \sim Sgy]$	Assumption
2	$(\forall y)[(Hy \,\&\, \sim Wyg) \supset \sim Sgy]$	Assumption
3	Hi	Assumption
4	Sgi	Assumption
5	Wig	Assumption
6	$(Hi \,\&\, Wig) \supset \sim Sgi$	1 $\forall$E
7	$Hi \,\&\, Wig$	3, 5 &I
8	$\sim Sgi$	6, 7 $\supset$E
9	Sgi	4 R
10	$\sim Wig$	5–9 $\sim$ I
11	$(Hi \,\&\, \sim Wig) \supset \sim Sgi$	2 $\forall$E
12	$Hi \,\&\, \sim Wig$	3, 10 &I
13	$\sim Sgi$	11, 12 $\supset$E
14	Sgi	4 R
15	$\sim Sgi$	4–14 $\sim$ I
16	$Hi \supset \sim Sgi$	3–15 $\supset$I
17	$(\forall y)(Hy \supset \sim Sgy)$	16 $\sim$ I

d. Derive: $(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)$

1	$(\forall x)((Px \,\&\, Kx) \supset (\forall z)[Rz \supset (\forall w)([Ww \,\&\, (Ewz \vee Lwz)] \supset Pxzw)])$	Assumption
2	$Rg \,\&\, \sim (\exists x)(\exists y)(Px \,\&\, Pxgy)$	Assumption
3	$\sim [(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)]$	Assumption
4	$(\exists y)(Py \,\&\, Ky)$	Assumption
5	$Pa \,\&\, Ka$	Assumption
6	$(Pa \,\&\, Ka) \supset (\forall z)[Rz \supset (\forall w)([Ww \,\&\, (Ewz \vee Lwz)] \supset Pazw)]$	1 ∀E
7	$(\forall z)[Rz \supset (\forall w)([Ww \,\&\, (Ewz \vee Lwz)] \vee Pazw)]$	5, 6 ⊃E
8	$Rg \supset (\forall w)([Ww \,\&\, (Ewg \vee Lwg)] \supset Pagw)$	7 ∀E
9	Rg	2 &E
10	$(\forall w)([Ww \,\&\, (Ewg \vee Lwg)] \supset Pagw)$	8, 9 ⊃E
11	Wb	Assumption
12	$Ebg \vee Lbg$	Assumption
13	$Wb \,\&\, (Ebg \vee Lbg)$	11, 12 &I
14	$[Wb \,\&\, (Ebg \vee Lbg)] \supset Pagb$	10 ∀E
15	$Pagb$	13, 14 ⊃E
16	Pa	5 &E
17	$Pa \,\&\, Pagb$	16, 15 &I
18	$(\exists y)(Pa \,\&\, Pagy)$	17 ∃I
19	$(\exists x)(\exists y)(Px \,\&\, Pxgy)$	18 ∃I
20	$\sim (\exists x)(\exists y)(Px \,\&\, Pxgy)$	2 &E
21	$\sim (Ebg \vee Lbg)$	12–20 ~ I
22	$Wb \supset \sim (Ebg \vee Lbg)$	11–21 ⊃I
23	$(\forall z)[Wz \supset \sim (Ezg \vee Lzg)]$	22 ∀I
24	$(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)$	23 ∨I
25	$(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)$	4, 5–24 ∃E
26	$\sim [(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)]$	3 R
27	$\sim (\exists y)(Py \,\&\, Ky)$	4–26 ~ I
28	$(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)$	27 ∨I
29	$\sim [(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)]$	3 R
30	$(\forall z)[Wz \supset \sim (Ezg \vee Lzg)] \vee \sim (\exists y)(Py \,\&\, Ky)$	3–29 ~ E

f. Derive: (∃x)(∃y)[(By & Tx) & (∃z)(Szy & Czyx)]

1	(∀x)(Bx ⊃ (∀z)[Tz ⊃ (∃y)(Cyxz & [(Fyx ∨ Myx) ∨ Syx])])	Assumption
2	(∀x)(∀y)[(Pxy & By) ≡ (Fxy ∨ Mxy)]	Assumption
3	(∃x)(∃y)[(By & Tx) & (∀z)(Pzy ⊃ ~ Czyx)]	Assumption
4	(∃y)[(By & Ta) & (∀z)(Pzy ⊃ ~ Czya)]	Assumption
5	(Bb & Ta) & (∀z)(Pzb ⊃ ~ Czba)	Assumption
6	Bb & Ta	5 &E
7	Bb	6 &E
8	Bb ⊃ (∀z)[Tz ⊃ (∃y)(Cybz & [(Fyb ∨ Myb) ∨ Syb])]	1 ∀E
9	(∀z)[Tz ⊃ (∃y)(Cybz & [(Fyb ∨ Myb) ∨ Syb])]	7, 8 ⊃E
10	Ta ⊃ (∃y)(Cyba & [(Fyb ∨ Myb) ∨ Syb])	9 ∀E
11	Ta	6 &E
12	(∃y)(Cyba & [(Fyb ∨ Myb) ∨ Syb])	10, 11 ⊃E
13	Ccba & [(Fcb ∨ Mcb) ∨ Scb]	Assumption
14	(∀y)[(Pcy & By) ≡ (Fcy ∨ Mcy)]	2 ∀E
15	(Pcb & Bb) ≡ (Fcb ∨ Mcb)	14 ∀E
16	(Fcb ∨ Mcb) ∨ Scb	13 &E
17	Fcb ∨ Mcb	Assumption
18	Pcb & Bb	15, 17 ≡E
19	(∀z)(Pzb ⊃ ~ Czba)	5 &I
20	Pcb ⊃ ~ Ccba	19 ∀E
21	Pcb	18 &E
22	~ Scb	Assumption
23	~ Ccba	20, 21 ⊃E
24	Ccba	13 &E
25	Scb	22–24 ~ E
26	Scb	Assumption
27	Scb	26 R
28	Scb	16, 17–25, 26–27 ∨E
29	Ccba	13 &E
30	Scb & Ccba	28, 29 &I
31	(∃z)(Szb & Czba)	30 ∃I
32	(Bb & Ta) & (∃z)(Szb & Czba)	6, 31 &I
33	(∃y)[(By & Ta) & (∃z)(Szy & Czya)]	32 ∃I
34	(∃x)(∃y)[(By & Tx) & (∃z)(Szy & Czyx)]	33 ∃I
35	(∃x)(∃y)[(By & Tx) & (∃z)(Szy & Czyx)]	12, 13–34 ∃E
36	(∃x)(∃y)[(By & Tx) & (∃z)(Szy, & Czyx)]	4, 5–35 ∃E
37	(∃x)(∃y)[(By & Tx) & (∃z)(Szy & Czyx)]	3, 4–36 ∃E

13. Inconsistency

b.

1	$(\forall y)(\sim Py \vee Gy)$	Assumption
2	$(\forall z)(Gz \supset Dz)$	Assumption
3	$(\exists x)(Px \,\&\, \sim Dx)$	Assumption
4	$\quad Pi \,\&\, \sim Di$	Assumption
5	$\quad \sim Pi \vee Gi$	1 ∀E
6	$\quad\quad \sim Pi$	Assumption
7	$\quad\quad\quad \sim Gi$	Assumption
8	$\quad\quad\quad Pi$	4 &E
9	$\quad\quad\quad \sim Pi$	6 R
10	$\quad\quad Gi$	7–9 ~ E
11	$\quad\quad Gi$	Assumption
12	$\quad\quad Gi$	11 R
13	$\quad Gi$	5, 6–10, 11–12 ∨E
14	$\quad Gi \supset Di$	2 ∀E
15	$\quad\quad (\exists x)(Px \,\&\, \sim Dx)$	Assumption
16	$\quad\quad Di$	13, 14 ⊃E
17	$\quad\quad \sim Di$	4 &E
18	$\quad \sim (\exists x)(Px \,\&\, \sim Dx)$	15–17 ~ I
19	$\sim (\exists x)(Px \,\&\, \sim Dx)$	3, 4–18 ∃E
20	$(\exists x)(Px \,\&\, \sim Dx)$	3 R

d.

1	$(\forall x)([Px \& (\exists y)(Wy \& Lxy)] \supset (\exists z)[(Rz \& Wz) \& Lxz])$	Assumption
2	$[Pm \& (\exists y)(Wy \& Lmy)] \& (\forall x)[(Wx \& Lmx)] \supset Sx])$	Assumption
3	$(\forall z)[Sz \supset \sim (Rz \& Wz)]$	Assumption
4	$[Pm \& (\exists y)(Wy \& Lmy)] \supset (\exists z)[(Rz \& Wz) \& Lmz]$	1 ∀E
5	$Pm \& (\exists y)(Wy \& Lmy)$	2 &E
6	$(\exists z)[(Rz \& Wz) \& Lmz]$	4, 5 ⊃E
7	$(Ra \& Wa) \& Lma$	Assumption
8	$(\forall x)[(Wx \& Lmx) \supset Sx]$	2 &E
9	$(Wa \& Lma) \supset Sa$	8 ∀E
10	$Ra \& Wa$	7 &E
11	Wa	10 &E
12	Lma	7 &E
13	$Wa \& Lma$	11, 12 &I
14	Sa	9, 13 ⊃E
15	$Sa \supset \sim (Ra \& Wa)$	3 ∀E
16	Pm	Assumption
17	$Ra \& Wa$	7 &E
18	$\sim (Ra \& Wa)$	14, 15 ⊃E
19	$\sim Pm$	16–18 ~ I
20	$\sim Pm$	6, 7–19 ∃E
21	Pm	5 &E

Section 10.5E

1. Derivability

b. Derive: $(\forall w)(Lw \supset Nw)$

1	$(\forall w)(Lw \supset Mw)$	Assumption
2	$(\forall y)(My \supset Ny)$	Assumption
3	$Lk \supset Mk$	1 ∀E
4	$Mk \supset Nk$	2 ∀E
5	$Lk \supset Nk$	3, 4 HS
6	$(\forall w)(Lw \supset Nw)$	5 ∀I

d. Derive: Rj

1	$\sim (\exists x)(\sim Rx \& Sxx)$	Assumption
2	Sjj	Assumption
3	$(\forall x) \sim (\sim Rx \& Sxx)$	1 QN
4	$\sim (\sim Rj \& Sjj)$	3 ∀E
5	$\sim \sim Rj \vee \sim Sjj$	4 DeM
6	$\sim \sim Sjj$	2 DN
7	$\sim \sim Rj$	5, 6 DS
8	Rj	7 DN

f. Derive: $\sim (\exists y)(\sim Fy \vee \sim Hy)$

1	$(\forall x)Fx$	Assumption
2	$(\forall z)Hz$	Assumption
3	Fj	1 $\forall$E
4	Hj	2 $\forall$E
5	$Fj \,\&\, Hj$	3, 4 &I
6	$\sim\sim (Fj \,\&\, Hj)$	5 DN
7	$\sim (\sim Fj \vee \sim Hj)$	6 DeM
8	$(\forall y) \sim (\sim Fy \vee \sim Hy)$	7 $\forall$I
9	$\sim (\exists y)(\sim Fy \vee \sim Hy)$	8 QN

2. Validity

b. Derive: $(\forall x)(\exists y)(Pxy \supset Qxy)$

1	$\sim (\exists x)(\forall y)(Pxy \,\&\, \sim Qxy)$	Assumption
2	$(\forall x) \sim (\forall y)(Pxy \,\&\, \sim Qxy)$	1 QN
3	$(\forall x)(\exists y) \sim (Pxy \,\&\, \sim Qxy)$	2 QN
4	$(\forall x)(\exists y)(\sim Pxy \vee \sim\sim Qxy)$	3 DeM
5	$(\forall x)(\exists y)(Pxy \supset \sim\sim Qxy)$	4 Impl
6	$(\forall x)(\exists y)(Pxy \supset Qxy)$	5 DN

d. Derive: $\sim Lb$

1	$(\forall z)(Lx \equiv Hz)$	Assumption
2	$(\forall x) \sim (Hx \vee \sim Bx)$	Assumption
3	$Lb \equiv Hb$	1 $\forall$E
4	$(Lb \supset Hb) \,\&\, (Hb \supset Lb)$	3 Equiv
5	$Lb \supset Hb$	4 &E
6	$\sim (Hb \vee \sim Bb)$	2 $\forall$E
7	$\sim Hb \,\&\, \sim\sim Bb$	6 DeM
8	$\sim Hb$	7 &E
9	$\sim Lb$	5, 8 MT

f. Derive: $(\forall x)(Cx \supset \sim Wx)$

1	$(\exists x)[\sim Bxm \,\&\, (\forall y)(Cy \supset \sim Gxy)]$	Assumption
2	$(\forall z)[\sim (\forall y)(Wy \supset Gzy) \supset Bzm]$	Assumption
3	$\quad \sim Bam \,\&\, (\forall y)(Cy \supset \sim Gay)$	Assumption
4	$\quad (\forall y)(Wy \supset Gay) \supset Bam$	2 $\forall$E
5	$\quad \sim Bam$	3 &E
6	$\quad \sim \sim (\forall y)(Wy \supset Gay)$	4, 5 MT
7	$\quad (\forall y)(Wy \supset Gay)$	6 DN
8	$\quad Wp \supset Gap$	7 $\forall$E
9	$\quad (\forall y)(Cy \supset \sim Gay)$	3 &E
10	$\quad Cp \supset \sim Gap$	9 $\forall$E
11	$\quad \sim Gap \supset \sim Wp$	8 Trans
12	$\quad Cp \supset \sim Wp$	10, 11 HS
13	$\quad (\forall x)(Cx \supset \sim Wx)$	12 $\forall$I
14	$(\forall x)(Cx \supset \sim Wx)$	1, 3–13 $\exists$E

h. Derive: $(\forall x)[(\exists y)(Kby \,\&\, Qxy) \supset (\exists z)(\sim Hz \,\&\, Qxz)]$

1	$(\forall y)(Kby \supset \sim Hy)$	Assumption
2	$\quad \sim (\exists z)(\sim Hz \,\&\, Qiz)$	Assumption
3	$\quad (\forall z) \sim (\sim Hz \,\&\, Qiz)$	2 QN
4	$\quad \sim (\sim Hj \,\&\, Qij)$	3 $\forall$E
5	$\quad \sim \sim Hj \vee \sim Qij$	4 DeM
6	$\quad \sim Hj \supset \sim Qij$	5 Impl
7	$\quad Kbj \supset \sim Hj$	1 $\forall$E
8	$\quad Kbj \supset \sim Qij$	7, 6 HS
9	$\quad \sim Kbj \vee \sim Qij$	8 Impl
10	$\quad \sim (Kbj \,\&\, Qij)$	9 DeM
11	$\quad (\forall y) \sim (Kby \,\&\, Qiy)$	10 $\forall$I
12	$\quad \sim (\exists y)(Kby \,\&\, Qiy)$	11 QN
13	$\sim (\exists z)(\sim Hz \,\&\, Qiz) \supset \sim (\exists y)(Kby \,\&\, Qiy)$	2–12 $\supset$I
14	$(\exists y)(Kby \,\&\, Qiy) \supset (\exists z)(\sim Hz \,\&\, Qiz)$	13 Trans
15	$(\forall x)[(\exists y)(Kby \,\&\, Qxy) \supset (\exists z)(\sim Hz \,\&\, Qxz)]$	14 $\forall$I

3. Theorems

b. Derive: $(\forall x)(Ax \supset (Ax \supset Bx)) \supset (\forall x)(Ax \supset Bx)$

1	$(\forall x)(Ax \supset (Ax \supset Bx))$	Assumption
2	$Ag \supset (Ag \supset Bg)$	1 $\forall$E
3	Ag	Assumption
4	$Ag \supset Bg$	2, 3 $\supset$E
5	Bg	3, 4 $\supset$E
6	$Ag \supset Bg$	3–5 $\supset$I
7	$(\forall x)(Ax \supset Bx)$	6 $\forall$I
8	$(\forall x)(Ax \supset (Ax \supset Bx)) \supset (\forall x)(Ax \supset Bx)$	1–7 $\supset$I

d. Derive: $(\forall x)(Ax \supset Bx) \vee (\exists x)Ax$

1	$\sim (\exists x)Ax$	Assumption
2	$(\forall x) \sim Ax$	1 QN
3	$\sim Ac$	2 $\forall$E
4	$\sim Ac \vee Bc$	3 $\vee$I
5	$Ac \supset Bc$	4 Impl
6	$(\forall x)(Ax \supset Bx)$	5 $\forall$I
7	$\sim (\exists x)Ax \supset (\forall x)(Ax \supset Bx)$	1–6 $\supset$I
8	$\sim \sim (\exists x)Ax \vee (\forall x)(Ax \supset Bx)$	7 Impl
9	$(\exists x)Ax \vee (\forall x)(Ax \supset Bx)$	8 DN
10	$(\forall x)(Ax \supset Bx) \vee (\exists x)Ax$	9 Com

f. Derive: (∀x) (∃y) (Ax ∨ By) ≡ (∃y) (∀x) (Ax ∨ By)

1	(∀x) (∃y) (Ax ∨ By)	Assumption
2	~ (∃y) (∀x) (Ax ∨ By)	Assumption
3	(∀y) ~ (∀x) (Ax ∨ By)	2 QN
4	~ (∀x) (Ax ∨ Bd)	3 ∀E
5	(∃x) ~ (Ax ∨ Bd)	4 QN
6	~ (Ae ∨ Bd)	Assumption
7	(∃y) (Ae ∨ By)	1 ∀E
8	Ae ∨ Bf	Assumption
9	~ (∀x) (Ax ∨ Bf)	3 ∀E
10	(∃x) ~ (Ax ∨ Bf)	9 QN
11	~ (Ag ∨ Bf)	Assumption
12	~ (∃y) (∀x) (Ax ∨ By)	Assumption
13	~ Ag & ~ Bf	11 DeM
14	~ Bf	13 &E
15	Ae	8, 14 DS
16	~ Ae & ~ Bd	6 DeM
17	~ Ae	16 &E
18	(∃y) (∀x) (Ax ∨ By)	12–17 ~ E
19	(∃y) (∀x) (Ax ∨ By)	10, 11–18 ∃E
20	(∃y) (∀x) (Ax ∨ By)	7, 8–19 ∃E
21	(∃y) (∀x) (Ax ∨ By)	5, 6–20 ∃E
22	~ (∃y) (∀x) (Ax ∨ By)	2 R
23	(∃y) (∀x) (Ax ∨ By)	2–22 ~ E
24	(∃y) (∀x) (Ax ∨ By)	Assumption
25	(∀x) (Ax ∨ Bh)	Assumption
26	Ai ∨ Bh	25 ∀E
27	(∃y) (Ai ∨ By)	26 ∃I
28	(∀x) (∃y) (Ax ∨ By)	27 ∀I
29	(∀x) (∃y) (Ax ∨ By)	24, 25–28 ∃E
30	(∀x) (∃y) (Ax ∨ By) ≡ (∃y) (∀x) (Ax ∨ By)	1–23, 24–29 ≡I

4. Equivalence

b. Derive: $(\exists x)(\exists y)Axy \equiv Aab$

1	$(\exists x)(\exists y)Axy \supset Aab$	Assumption
2	$\quad$ Aab	Assumption
3	$\quad$ $(\exists y)Aay$	2 $\exists$I
4	$\quad$ $(\exists x)(\exists y)Axy$	3 $\exists$I
5	$Aab \supset (\exists x)(\exists y)Axy$	2, 4 $\supset$I
6	$((\exists x)(\exists y)Axy \supset Aab)\ \&\ (Aab \supset (\exists x)(\exists y)Axy)$	1, 5 &I
7	$(\exists x)(\exists y)Axy \equiv Aab$	6 Equiv

Derive: $(\exists x)(\exists y)Axy \supset Aab$

1	$(\exists x)(\exists y)Axy \equiv Aab$	Assumption
2	$((\exists x)(\exists y)Axy \supset Aab)\ \&\ (Aab \supset (\exists x)(\exists y)Axy)$	1 Equiv
3	$(\exists x)(\exists y)Axy \supset Aab$	2 &E

d. Derive: $(\exists x)(\forall y)[\sim (Cy \vee Ax) \vee \sim (Cy \vee Bx)]$

1	$\sim (\forall x)(\exists y)[(Ax\ \&\ Bx) \vee Cy]$	Assumption
2	$(\exists x) \sim (\exists y)[(Ax\ \&\ Bx) \vee Cy]$	1 QN
3	$(\exists x)(\forall y) \sim [(Ax\ \&\ Bx) \vee Cy]$	2 QN
4	$(\exists x)(\forall y) \sim [Cy \vee (Ax\ \&\ Bx)]$	3 Com
5	$(\exists x)(\forall y) \sim [(Cy \vee Ax)\ \&\ (Cy \vee Bx)]$	4 Dist
6	$(\exists x)(\forall y)[\sim (Cy \vee Ax) \vee \sim (Cy \vee Bx)]$	5 DeM

Derive: $\sim (\forall x)(\exists y)[(Ax\ \&\ Bx) \vee Cy]$

1	$(\exists x)(\forall y)[\sim (Cy \vee Ax) \vee \sim (Cy \vee Bx)]$	Assumption
2	$(\exists x)(\forall y) \sim [(Cy \vee Ax)\ \&\ (Cy \vee Bx)]$	1 DeM
3	$(\exists x)(\forall y) \sim [Cy \vee (Ax\ \&\ Bx)]$	2 Dist
4	$(\exists x)(\forall y) \sim [(Ax\ \&\ Bx) \vee Cy]$	3 Com
5	$(\exists x) \sim (\exists y)[(Ax\ \&\ Bx) \vee Cy]$	4 QN
6	$\sim (\forall x)(\exists y)[(Ax\ \&\ Bx) \vee Cy]$	5 QN

f. Derive: $\sim (\exists x)[\sim Ax \vee (\forall y)(Bxy\ \&\ Bxy)]$

1	$(\forall x)(Ax\ \&\ (\exists y) \sim Bxy)$	Assumption
2	$\sim \sim (\forall x)(Ax\ \&\ (\exists y) \sim Bxy)$	1 DN
3	$\sim (\exists x) \sim (Ax\ \&\ (\exists y) \sim Bxy)$	2 QN
4	$\sim (\exists x)(\sim Ax \vee \sim (\exists y) \sim Bxy)$	3 DeM
5	$\sim (\exists x)(\sim Ax \vee (\forall y) \sim \sim Bxy)$	4 QN
6	$\sim (\exists x)(\sim Ax \vee (\forall y)Bxy)$	5 DN
7	$\sim (\exists x)[\sim Ax \vee (\forall y)(Bxy\ \&\ Bxy)]$	6 Idem

Derive: $(\forall x)(Ax \,\&\, (\exists y) \sim Bxy)$

1	$\sim (\exists x)[\sim Ax \vee (\forall y)(Bxy \,\&\, Bxy)]$	Assumption
2	$\sim (\exists x)(\sim Ax \vee (\forall y)Bxy)$	1 Idem
3	$(\forall x) \sim (\sim Ax \vee (\forall y)Bxy)$	2 QN
4	$(\forall x)(\sim \sim Ax \,\&\, \sim (\forall y)Bxy)$	3 DeM
5	$(\forall x)(Ax \,\&\, \sim (\forall y)Bxy)$	4 DN
6	$(\forall x)(Ax \,\&\, (\exists y) \sim Bxy)$	5 QN

5. Inconsistency

b.	1	$\sim Fa$	Assumption
	2	$\sim (\exists x)(\sim Fx \vee \sim Fx)$	Assumption
	3	$\sim (\exists x) \sim Fx$	2 Idem
	4	$(\forall x) \sim \sim Fx$	3 QN
	5	$\sim \sim Fa$	4 $\forall$E
	6	$\sim Fa$	1 R

d.	1	$(\exists x)(\forall y)(Hxy \supset (\forall w)Jww)$	Assumption
	2	$(\exists x) \sim Jxx \,\&\, \sim (\exists x) \sim Hxm$	Assumption
	3	$\quad (\forall y)(Hay \supset (\forall w)Jww)$	Assumption
	4	$\quad Ham \supset (\forall w)Jww$	3 $\forall$E
	5	$\quad \sim (\exists x) \sim Hxm$	2 &E
	6	$\quad (\forall x) \sim \sim Hxm$	5 QN
	7	$\quad \sim \sim Ham$	6 $\forall$E
	8	$\quad \sim Ham \vee (\forall w)Jww$	4 Impl
	9	$\quad (\forall w)Jww$	7, 8 DS
	10	$(\forall w)Jww$	1, 3–9 $\exists$E
	11	Jaa	10 $\forall$E
	12	$(\forall x)Jxx$	11 $\forall$E
	13	$(\exists x) \sim Jxx$	2 &E
	14	$\sim (\forall x)Jxx$	13 QN

f.	1	$(\forall x)[(Sx \,\&\, Bxx) \supset Kax]$	Assumption
	2	$(\forall x)(Hx \supset Bxx)$	Assumption
	3	$(\exists x)(Sx \,\&\, Hx)$	Assumption
	4	$(\forall x) \sim (Kax \,\&\, Hx)$	Assumption
	5	$\quad Sc \,\&\, Hc$	Assumption
	6	$\quad Hc \supset Bcc$	2 ∀E
	7	$\quad Hc$	5 &E
	8	$\quad Bcc$	6, 7 ⊃E
	9	$\quad Sc$	5 &E
	10	$\quad Sc \,\&\, Bcc$	9, 8 &I
	11	$\quad (Sc \,\&\, Bcc) \supset Kac$	1 ∀E
	12	$\quad Kac$	10, 11 ⊃E
	13	$\quad \sim (Kac \,\&\, Hc)$	4 ∀E
	14	$\quad \sim Kac \vee \sim Hc$	13 DeM
	15	$\quad Kac \supset \sim Hc$	14 Impl
	16	$\quad \sim Hc$	12, 15 ⊃E
	17	$\quad Hc \vee \sim (\exists x)(Sx \,\&\, Hx)$	7 ∨I
	18	$\quad \sim (\exists x)(Sx \,\&\, Hx)$	16, 17 DS
	19	$\sim (\exists x)(Sx \,\&\, Hx)$	3, 5–18 ∃E
	20	$(\exists x)(Sx \,\&\, Hx)$	3 R

6. b. Suppose there is a sentence on an accessible line **i** of a derivation to which Existential Introduction can be properly applied at line **n**. The sentence that would be derived by Existential Introduction can also be derived by using the routine beginning at line **n**:

$\mathbf{i}$	$\mathbf{P(a/x)}$	
$\mathbf{n}$	$\quad \sim (\exists \mathbf{x})\mathbf{P}$	Assumption
$\mathbf{n} + 1$	$\quad (\forall \mathbf{x}) \sim \mathbf{P}$	**n** QN
$\mathbf{n} + 2$	$\quad \sim \mathbf{P(a/x)}$	**n** + 1 ∀E
$\mathbf{n} + 3$	$\quad \mathbf{P(a/x)}$	**i** R
$\mathbf{n} + 4$	$(\exists \mathbf{x})\mathbf{P}$	**n** − **n** + 3 ~ E

Suppose there is a sentence on an accessible line **i** of a derivation and an accessible subderivation from line **j** to line **k** of a derivation to which Existential Elimination can be properly applied at line **n**. The sentence that would be derived by Existential Elimination can also be derived by using the routine beginning at line **n**:

i	$(\exists \mathbf{x})\mathbf{P}$	
j	$\mathbf{P}(\mathbf{a}/\mathbf{x})$	
k	$\mathbf{Q}$	
n	$\sim \mathbf{Q}$	Assumption
n + 1	$\mathbf{P}(\mathbf{a}/\mathbf{x})$	Assumption
n + 2	$\mathbf{P}(\mathbf{a}/\mathbf{x}) \supset \mathbf{Q}$	**j**–**k** ⊃I
n + 3	$\mathbf{Q}$	**n** + 1, **n** + 2 ⊃E
n + 4	$\sim \mathbf{Q}$	**n** R
n + 5	$\sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	**n** + 1 − **n** + 4 ~ I
n + 6	$(\forall \mathbf{x}) \sim \mathbf{P}$	**n** + 5 ∀I
n + 7	$\sim (\exists \mathbf{x})\mathbf{P}$	**n** + 6 QN
n + 8	$(\exists \mathbf{x})\mathbf{P}$	**i** R
n + 9	$\mathbf{Q}$	**n** − **n** + 8 ~ E

No restriction on the use of Universal Introduction was violated at line **n** + 6. We assumed that we could have applied Existential Elimination at line **n**. So **a** does not occur in any undischarged assumption prior to line **n**, and **a** does not occur in either (∃**x**)**P** or **Q**. So **a** does not occur in **P**. Hence

(i) **a** does not occur in any undischarged assumption prior to line **n** + 6. Note that the assumption on line **n** + 1 has been discharged and the assumption on line **n** does not contain **a**, for **a** does not occur in **Q**.

(ii) **a** does not occur in (∀**x**) ~ **P**, for **a** does not occur in **P**.

Section 10.6E

1. b. Derive: (a = b & b = c) ⊃ a = c

1	a = b & b = c	Assumption
2	a = b	1 &E
3	b = c	1 &E
4	a = c	2, 3 =E
5	(a = b & b = c) ⊃ a = c	1–4 ⊃I

d. Derive: ~ a = b ≡ ~ b = a

1	~ a = b	Assumption
2	b = a	Assumption
3	b = b	2, 2 =E
4	~ b = b	1, 2 =E
5	~ b = a	2–4 ~ I
6	~ b = a	Assumption
7	a = b	Assumption
8	a = a	7, 7 =E
9	~ a = a	6, 7 =E
10	~ a = b	7–9 ~ I
11	~ a = b ≡ ~ b = a	1–5, 6–10 ≡I

2. b. Derive: Ge ⊃ Hd

1	Ge ⊃ d = e	Assumption
2	Ge ⊃ He	Assumption
3	Ge	Assumption
4	He	2, 3 ⊃E
5	d = e	1, 3 ⊃E
6	Hd	4, 5 =E
7	Ge ⊃ Hd	3–6 ⊃I

d. Derive: (∃x) [(Hx & Mx) & ~ x = s]

1	(∃x) (Hx & Mx)	Assumption
2	Ms & ~ Hs	Assumption
3	Ha & Ma	Assumption
4	a = s	Assumption
5	~ Hs	2 &E
6	Ha	3 &E
7	Hs	4, 6 =E
8	~ a = s	4–7 ~ I
9	(Ha & Ma) & ~ a = s	3, 8 &I
10	(∃x) [(Hx & Mx) & ~ x = s]	9 ∃I
11	(∃x) [(Hx & Mx) & ~ x = s]	1, 3–10 ∃E

3. b. Derive: $(\forall x)(\forall y)(x = x \,\&\, y = y)$

1	$(\forall x)x = x$	$=$I
2	$a = a$	1 $\forall$E
3	$b = b$	1 $\forall$E
4	$a = a \,\&\, b = b$	2, 3 &I
5	$(\forall y)(a = a \,\&\, y = y)$	4 $\forall$I
6	$(\forall x)(\forall y)(x = x \,\&\, y = y)$	5 $\forall$I

d. Derive: $(\forall x)(\forall y)(\forall z)[(x = y \,\&\, y = z) \supset x = z]$

1	$\quad a = b \,\&\, b = c$	Assumption
2	$\quad a = b$	1 &E
3	$\quad b = c$	1 &E
4	$\quad a = c$	2, 3 $=$E
5	$(a = b \,\&\, b = c) \supset a = c$	1–4 $\supset$I
6	$(\forall z)[(a = b \,\&\, b = z) \supset a = z]$	5 $\forall$I
7	$(\forall y)(\forall z)[a = y \,\&\, y = z) \supset a = z]$	6 $\forall$I
8	$(\forall x)(\forall y)(\forall z)[(x = y \,\&\, y = z) \supset x = z]$	7 $\forall$I

4. b. Derive: $(\exists x)(Dx \,\&\, (\exists y)[(Py \,\&\, Iy) \,\&\, Kxy])$

1	$(\exists x)[(Px \,\&\, Ix) \,\&\, Khx]$	Assumption
2	$j = h$	Assumption
3	Dj	Assumption
4	$\quad (Pa \,\&\, Ia) \,\&\, Kha$	Assumption
5	$\quad (Pa \,\&\, Ia) \,\&\, Kja$	2, 4 $=$E
6	$\quad (\exists y)[(Py \,\&\, Iy) \,\&\, Kjy]$	5 $\exists$I
7	$(\exists y)[(Py \,\&\, Iy) \,\&\, Kjy]$	1, 4–6 $\exists$E
8	$Dj \,\&\, (\exists y)[(Py \,\&\, Iy) \,\&\, Kjy]$	3, 7 &I
9	$(\exists x)(Dx \,\&\, (\exists y)[(Py \,\&\, Iy) \,\&\, Kxy])$	8 $\exists$I

d. Derive: $Lsr \,\&\, Lrs$

1	$(\forall x)(Lrx \equiv Lxr)$	Assumption
2	$(\exists x)[(Bxc \,\&\, (\forall y)(Byc \supset x = y)) \,\&\, Lxr]$	Assumption
3	Bsc	Assumption
4	$\quad (Bac \,\&\, (\forall y)(Byc \supset a = y)) \,\&\, Lar$	Assumption
5	$\quad Bac \,\&\, (\forall y)(Byc \supset a = y)$	4 &E
6	$\quad (\forall y)(Byc \supset a = y)$	5 &E
7	$\quad Bsc \supset a = s$	6 $\forall$E
8	$\quad a = s$	3, 7 $\supset$E
9	$\quad Lar$	4 &E
10	$\quad Lsr$	8, 9 $=$E
11	$\quad Lrs \equiv Lsr$	1 $\forall$E
12	$\quad Lrs$	10, 11 $\equiv$E
13	$\quad Lsr \,\&\, Lrs$	10, 12 &I
14	$Lsr \,\&\, Lrs$	2, 4–13 $\exists$E

5. b.

1	$(\exists x)Sg(x,x)$	Assumption
2	$\quad Sg(i,i)$	Assumption
3	$\quad (\exists x)Sg(i,x)$	2 $\exists$I
4	$(\exists x)Sg(i,x)$	1, 2–3 $\exists$E

Line 4 is a mistake as the instantiating individual constant, ‘i’, used in the assumption on line 2 must be absent from the sentence derived from the subderivation.

d. Correctly done.
f. Correctly done.
h. Correctly done.

j.

1	$(\forall x)Lx$	Assumption
2	$Lf(a,a)$	1 $\forall$E
3	$(\forall x)Lf(a,x)$	2 $\forall$I

Line 3 is a mistake. Universal Introduction must be applied so that the instantiating constant, in this case ‘a’, does not occur in the derived sentence.

6. Theorems in *PDE*

b. Derive: $(\forall x)(\forall y)(\forall z)[(f(x) = g(x,y) \mathbin{\&} g(x,y) = h(x,y,z)) \supset f(x) = h(x,y,z)]$

1	$\quad f(a) = g(a,b) \mathbin{\&} g(a,b) = h(a,b,c)$	Assumption
2	$\quad f(a) = g(a,b)$	1 &E
3	$\quad g(a,b) = h(a,b,c)$	1 &E
4	$\quad f(a) = h(a,b,c)$	2, 3 =E
5	$(f(a) = g(a,b) \mathbin{\&} g(a,b) = h(a,b,c)) \supset f(a) = h(a,b,c)$	1–4 $\supset$I
6	$(\forall z)[(f(a) = g(a,b) \mathbin{\&} g(a,b) = h(a,b,z)) \supset f(a) = h(a,b,z)]$	5 $\forall$I
7	$(\forall y)(\forall z)[(f(a) = g(a,y) \mathbin{\&} g(a,y) = h(a,y,z)) \supset f(a) = h(a,y,z)]$	6 $\forall$I
8	$(\forall z)(\forall y)(\forall z)[(f(x) = g(x,y) \mathbin{\&} g(x,y) = h(x,y,z)) \supset f(x) = h(x,y,z)]$	7 $\forall$I

d. Derive: $(\forall x)[\sim f(x) = x \supset (\forall y)(f(x) = y \supset \sim x = y)]$

1	$\sim f(a) = a$	Assumption
2	$f(a) = b$	Assumption
3	$a = b$	Assumption
4	$f(a) = a$	2, 3 =E
5	$\sim f(a) = a$	1 R
6	$\sim a = b$	3–5 ~ I
7	$f(a) = b \supset \sim a = b$	2–6 ⊃I
8	$(\forall y)(f(a) = y \supset \sim a = y)$	7 ∀I
9	$\sim f(a) = a \supset (\forall y)(f(x) = y \supset \sim a = y)$	1–8 ⊃I
10	$(\forall x)[\sim f(x) = x \supset (\forall y)(f(x) = y \supset \sim x = y)]$	9 ∀I

f. Derive: $(\forall x)(\forall y)(\forall z)[(f(g(x)) = y \ \& \ f(y) = z) \supset f(f(g(x))) = z]$

1	$f(g(a)) = b \ \& \ f(b) = c$	Assumption
2	$f(g(a)) = b$	1 &E
3	$f(b) = c$	1 &E
4	$f(f(g(a))) = c$	2, 3 =E
5	$(f(g(a)) = b \ \& \ f(b) = c) \supset f(f(g(a))) = c$	1–4 ⊃I
6	$(\forall z)[(f(g(a)) = b \ \& \ f(b) = z) \supset f(f(g(a))) = z]$	5 ∀I
7	$(\forall y)(\forall z)[(f(g(a)) = y \ \& \ f(y) = z) \supset f(f(g(a))) = z]$	6 ∀I
8	$(\forall x)(\forall y)(\forall z)[(f(g(x)) = y \ \& \ f(y) = z) \supset$ $f(f(g(x))) = z]$	7 ∀I

7. Validity in *PDE*

b. Derive: $(\forall x)(Kg(x) \vee Hg(g(x)))$

1	$(\forall x)(Kx \vee Hg(x))$	Assumption
2	$Kg(a) \vee Hg(g(a))$	1 ∀E
3	$(\forall x)(Kg(x) \vee Hg(g(x)))$	1 ∀I

d. Derive: $(\forall x) \sim Rf(x,g(x))$

1	$\sim (\exists x)Rx$	Assumption
2	$Rf(a,g(a))$	Assumption
3	$(\exists x) Rx$	2 ∃I
4	$\sim (\exists x)Rx$	1 R
5	$\sim Rf(a,g(a))$	2–4 ⊃E
6	$(\forall x) \sim Rf(x,g(x))$	5 ∀I

f. Derive: $(\exists x)Lf(x)f(f(x)) \supset (\exists y)Ng(y)$

1	$(\forall x)[\sim Lxf(x) \vee (\exists y)Ng(y)$	Assumption
2	$\quad (\exists x)Lf(x)f(f(x))$	Assumption
3	$\quad\quad Lf(a)f(f(a))$	Assumption
4	$\quad\quad \sim Lf(a)f(f(a)) \vee (\exists y)Ng(y)$	1 $\forall$E
5	$\quad\quad\quad \sim Lf(a)f(f(a))$	Assumption
6	$\quad\quad\quad\quad \sim (\exists y)Ng(y)$	Assumption
7	$\quad\quad\quad\quad Lf(a)f(f(a))$	3 R
8	$\quad\quad\quad\quad \sim Lf(a)f(f(a))$	5 R
9	$\quad\quad\quad (\exists y)Ng(y)$	6–8 $\sim$ E
10	$\quad\quad\quad (\exists y)Ng(y)$	Assumption
11	$\quad\quad\quad (\exists y)Ng(y)$	10 R
12	$\quad\quad (\exists y)Ng(y)$	4, 5–9, 10–11 $\vee$E
13	$\quad (\exists y)Ng(y)$	2, 3–12 $\exists$E
14	$(\exists x)Lf(x)f(f(x)) \supset (\exists y)Ng(y)$	2–13 $\supset$I

h. Derive: $(\exists x)(\exists y) \sim (\forall z)Mzg(y)f(g(x))$

1	$(\forall x)(\forall y)(\exists z)Sf(x)yz$	Assumption
2	$(\forall x)(\forall y)(\forall z)(Sxyz \supset \sim (Cxyz \vee Mzyx))$	Assumption
3	$(\forall y)(\exists z)Sf(g(a))yz$	1 $\forall$E
4	$(\exists z)Sf(g(a))g(b)z$	3 $\forall$E
5	$\quad Sf(g(a))g(b)c$	Assumption
6	$\quad (\forall y)(\forall z)(Sf(g(a))yz \supset \sim (Cf(g(a))yz \vee Mzyf(g(a))))$	2 $\forall$E
7	$\quad (\forall z)(Sf(g(a))g(b)z \supset \sim (Cf(g(a))g(b)z \vee Mzg(b)f(g(a))))$	6 $\forall$E
8	$\quad Sf(g(a))g(b)c \supset \sim (Cf(g(a))g(b)c \vee Mcg(b)f(g(a)))$	7 $\forall$E
9	$\quad\quad (\forall z)Mzg(b)f(g(a))$	Assumption
10	$\quad\quad Mcg(b)f(g(a))$	9 $\forall$E
11	$\quad\quad Cf(g(a))g(b)c \vee Mcg(b)f(g(a))$	10 $\vee$I
12	$\quad\quad \sim (Cf(g(a))g(b)c \vee Mcg(b)f(g(a)))$	5, 8 $\supset$E
13	$\quad \sim (\forall z)Mzg(b)f(g(a))$	9–12 $\sim$ I
14	$\quad (\exists y)\sim (\forall z)Mzg(y)f(g(x))$	13 $\exists$I
15	$\quad (\exists x)(\exists y) \sim (\forall z)Mzg(y)f(g(x))$	14 $\exists$I
16	$(\exists x)(\exists y) \sim (\forall z)Mzg(y)f(g(x))$	4, 5–15 $\exists$E

Section 11.1E

1. Case 3. P has the form $\mathbf{Q} \vee \mathbf{R}$. Then $\mathbf{P(a/x)}$ is $\mathbf{Q(a/x)} \vee \mathbf{R(a/x)}$. By the definition of satisfaction, **d** satisfies $\mathbf{Q(a/x)} \vee \mathbf{R(a/x)}$ if and only if either **d** satisfies $\mathbf{Q(a/x)}$ or **d** satisfies $\mathbf{R(a/x)}$. Both immediate components contain fewer than $\mathbf{k} + 1$ occurrences of logical operators, so, by the inductive hypothesis, **d** satisfies $\mathbf{Q(a/x)}$ if and only if $\mathbf{d[I(a)/x]}$ satisfies **Q**, and **d** satisfies $\mathbf{R(a/x)}$ if and only if $\mathbf{d[I(a)/x]}$ satisfies **R**. Therefore **d** satisfies $\mathbf{Q(a/x)} \vee \mathbf{R(a/x)}$ if and only if either $\mathbf{d[I(a)/x]}$ satisfies **Q** or $\mathbf{d[I(a)/x]}$ satisfies **R**. Again by the definition of satisfaction, this is the case if and only if $\mathbf{d[I(a)/x]}$ satisfies $\mathbf{Q} \vee \mathbf{R}$.

Case 4. P has the form $\mathbf{Q} \supset \mathbf{R}$. Then $\mathbf{P(a/x)}$ is $\mathbf{Q(a/x)} \supset \mathbf{R(a/x)}$. By the definition of satisfaction, **d** satisfies $\mathbf{Q(a/x)} \supset \mathbf{R(a/x)}$ if and only if either **d** does not satisfy $\mathbf{Q(a/x)}$ or **d** does satisfy $\mathbf{R(a/x)}$. Both immediate components contain fewer than $\mathbf{k} + 1$ occurrences of logical operators so, by the inductive hypothesis, **d** does not satisfy $\mathbf{Q(a/x)}$ if and only if $\mathbf{d[I(a)/x]}$ does not satisfy **Q**, and **d** does satisfy $\mathbf{R(a/x)}$ if and only if $\mathbf{d[I(a)/x]}$ satisfies **R**. By the definition of satisfaction, $\mathbf{d[I(a)/x]}$ does not satisfy **Q** or does satisfy **R** if and only if $\mathbf{d[I(a)/x]}$ satisfies $\mathbf{Q} \supset \mathbf{R}$.

Case 5. P has the form $\mathbf{Q} \equiv \mathbf{R}$. Then $\mathbf{P(a/x)}$ is $\mathbf{Q(a/x)} \equiv \mathbf{R(a/x)}$. By the definition of satisfaction, **d** satisfies $\mathbf{Q(a/x)} \equiv \mathbf{R(a/x)}$ if and only if either **d** satisfies both $\mathbf{Q(a/x)}$ and $\mathbf{R(a/x)}$ or **d** satisfies neither. By the inductive hypothesis, this is the case if and only if either $\mathbf{d[I(a)/x]}$ satisfies both **Q** and **R** or $\mathbf{d[I(a)/x]}$ satisfies neither. And by the definition of satisfaction, this is the case if and only if $\mathbf{d[I(a)/x]}$ satisfies $\mathbf{Q} \equiv \mathbf{R}$.

2. Cases 3–5. Like case 2, except that satisfaction clauses for the connectives '&', '⊃', and '≡' are used.

Case 7. P has the form $(\exists \mathbf{x})\mathbf{Q}$. By the definition of satisfaction, **d** satisfies $(\exists \mathbf{x})\mathbf{Q}$ if and only if for some member **u** of the UD, $\mathbf{d[u/x]}$ satisfies **Q**. Because **Q** contains fewer than $\mathbf{k} + 1$ occurrences of logical operators, it follows from the inductive hypothesis that $\mathbf{d[u/x]}$ satisfies **Q** if and only if every variable assignment that assigns the same values to the free variables in **Q** as $\mathbf{d[u/x]}$ satisfies **Q**. Because the variables other than **x** that are free in **Q** are also free in $(\exists \mathbf{x})\mathbf{Q}$, every variable assignment that assigns the same values to the free variables in **Q** as $\mathbf{d[u/x]}$ is a variant $\mathbf{d'[u/x]}$ of a variable assignment $\mathbf{d'}$ that assigns the same values to the free variables in $(\exists \mathbf{x})\mathbf{Q}$ as **d**, and vice versa. So $\mathbf{d[u/x]}$ satisfies **Q** if and only if every variable assignment $\mathbf{d'}$ that assigns the same values to the free variables in $(\exists \mathbf{x})\mathbf{Q}$ as **d** is such that $\mathbf{d'[u/x]}$ satisfies **Q**. By the definition of satisfaction, this is the case if and only if every variable assignment that assigns the same values to the free variables in $(\exists \mathbf{x})\mathbf{Q}$ as **d** satisfies $(\exists \mathbf{x})\mathbf{Q}$.

3. Let $(\exists \mathbf{x})\mathbf{P}$ be any existentially quantified sentence, let $\mathbf{P}(\mathbf{a}/\mathbf{x})$ be any one of its substitution instances, and let $\mathbf{I}$ be any interpretation on which $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is true. By 11.1.3, every variable assignment satisfies $\mathbf{P}(\mathbf{a}/\mathbf{x})$ on $\mathbf{I}$. By 11.1.1, every variable assignment $\mathbf{d}$ is therefore such that its variant $\mathbf{d}[\mathbf{I}(\mathbf{a})/\mathbf{x}]$ satisfies $\mathbf{P}$ on $\mathbf{I}$. And by the definition of satisfaction, it follows that every variable assignment $\mathbf{d}$ satisfies $(\exists \mathbf{x})\mathbf{P}$ on $\mathbf{I}$, so $(\exists \mathbf{x})\mathbf{P}$ is also true on $\mathbf{I}$.

4. Let $\mathbf{a}$ be a constant that does not occur in the sentences $(\exists \mathbf{x})\mathbf{P}$ and $\mathbf{Q}$ and that does not occur in any member of the set Γ, and assume that $\Gamma \vDash (\exists \mathbf{x})\mathbf{P}$ and $\Gamma \cup \{\mathbf{P}(\mathbf{a}/\mathbf{x})\} \vDash \mathbf{Q}$. We shall assume, contrary to what we want to show, that Γ does not quantificationally entail $\mathbf{Q}$—that there is at least one interpretation, call it $\mathbf{I}$, on which every member of Γ is true and $\mathbf{Q}$ is false. Because $\Gamma \vDash (\exists \mathbf{x})\mathbf{P}$, $(\exists \mathbf{x})\mathbf{P}$ is also true on $\mathbf{I}$. Therefore, for every variable assignment $\mathbf{d}$ on $\mathbf{I}$, there is a member $\mathbf{u}$ of the UD such that $\mathbf{d}[\mathbf{u}/\mathbf{x}]$ satisfies $\mathbf{P}$. Choose one of these variants $\mathbf{d}[\mathbf{u}/\mathbf{x}]$. Let $\mathbf{I}'$ be the interpretation that is just like $\mathbf{I}$ except that it assigns $\mathbf{u}$ to $\mathbf{a}$.

We shall now show that every member of Γ and $\mathbf{P}(\mathbf{a}/\mathbf{x})$ are true on $\mathbf{I}'$, and that $\mathbf{Q}$ is false on $\mathbf{I}'$, which contradicts our second assumption, that $\Gamma \cup \{\mathbf{P}(\mathbf{a}/\mathbf{x})\} \vDash \mathbf{Q}$. That every member of Γ is true and $\mathbf{Q}$ is false on $\mathbf{I}'$ follows from 11.1.7 because, by stipulation, $\mathbf{a}$ does not occur in any of these sentences. By 11.1.6, $\mathbf{d}[\mathbf{u}/\mathbf{x}]$ satisfies $\mathbf{P}$ on $\mathbf{I}'$ ($\mathbf{a}$ was also stipulated not to occur in $(\exists \mathbf{x})\mathbf{P}$). By 11.1.1, since $\mathbf{I}(\mathbf{a}) = \mathbf{u}$, $\mathbf{d}$ satisfies $\mathbf{P}(\mathbf{a}/\mathbf{x})$ on $\mathbf{I}'$. By 11.1.3, every variable assignment satisfies $\mathbf{P}(\mathbf{a}/\mathbf{x})$ on $\mathbf{I}'$, and so it is true.

Having contradicted one of our assumptions, we conclude that $\Gamma \vDash \mathbf{Q}$ as well.

7. Assume that $\mathbf{I}$ is an interpretation on which each member of the UD is assigned to at least one individual constant, and that every substitution instance of $(\exists \mathbf{x})\mathbf{P}$ is false on $\mathbf{I}$. Assume, contrary to what we wish to prove, that $(\exists \mathbf{x})\mathbf{P}$ is true on $\mathbf{I}$. Letting $\mathbf{d}$ be a variable assignment for $\mathbf{I}$, there is at least one member $\mathbf{u}$ of the U.D. such that $\mathbf{d}[\mathbf{u}/\mathbf{x}]$ satisfies $\mathbf{P}$. Because every member of the UD is assigned to some individual constant, $\mathbf{u}$ is $\mathbf{I}(\mathbf{a})$ for some individual constant $\mathbf{a}$. By 11.1.1, then, $\mathbf{d}$ satisfies $\mathbf{P}(\mathbf{a}/\mathbf{x})$ on $\mathbf{I}$, and, by 11.1.3, $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is true on $\mathbf{I}$, contradicting our assumption that every substitution instance of $(\exists \mathbf{x})\mathbf{P}$ is false on $\mathbf{I}$. We conclude that $(\exists \mathbf{x})\mathbf{P}$ must also be false on $\mathbf{I}$.

8. Let $\mathbf{P}$ be a sentence of *PL* and $\mathbf{b}$ an individual constant that does not occur in $\mathbf{P}$, and let $\mathbf{I}$ be an interpretation on which $\mathbf{P}$ is true. Let $\mathbf{d}$ be a variable assignment for $\mathbf{I}$; $\mathbf{d}$ must satisfy $\mathbf{P}$. Let $\mathbf{P}(\mathbf{x}/\mathbf{a})$ be the result of replacing every occurrence of $\mathbf{a}$ in $\mathbf{P}$ with a variable $\mathbf{x}$ that does not occur in $\mathbf{P}$; By 11.1.1, $\mathbf{d}[\mathbf{I}(\mathbf{a})/\mathbf{x}]$ satisfies $\mathbf{P}(\mathbf{x}/\mathbf{a})$ on $\mathbf{I}$. Let $\mathbf{I}'$ be the interpretation that is just like $\mathbf{I}$ except that it assigns $\mathbf{I}(\mathbf{a})$ to $\mathbf{b}$; by 11.1.6 ($\mathbf{b}$ does not occur in $\mathbf{P}(\mathbf{a}/\mathbf{x})$), it follows that $\mathbf{d}[\mathbf{I}(\mathbf{a})/\mathbf{x}]$ satisfies $\mathbf{P}(\mathbf{x}/\mathbf{a})$ on $\mathbf{I}'$. Because $\mathbf{I}'(\mathbf{b}) = \mathbf{I}(\mathbf{a})$, $\mathbf{d}[\mathbf{I}'(\mathbf{b})/\mathbf{x}]$ satisfies $\mathbf{P}(\mathbf{x}/\mathbf{a})$ on $\mathbf{I}'$. By 11.1.1, $\mathbf{d}$ satisfies $\mathbf{P}(\mathbf{x}/\mathbf{a})(\mathbf{b}/\mathbf{x})$ on $\mathbf{I}'$. $\mathbf{P}(\mathbf{x}/\mathbf{a})(\mathbf{b}/\mathbf{x})$ is just $\mathbf{P}(\mathbf{b}/\mathbf{a})$, and so $\mathbf{d}$ satisfies $\mathbf{P}(\mathbf{b}/\mathbf{a})$ on $\mathbf{I}'$. Therefore, by 11.1.3, every variable assignment satisfies $\mathbf{P}(\mathbf{b}/\mathbf{a})$ on $\mathbf{I}'$; and so the sentence is true on $\mathbf{I}'$.

Section 11.2E

1. a. If $\mathbf{P}$ has the form $\mathbf{t}_1 = \mathbf{t}_2$, then $\mathbf{P}(\mathbf{t}/\mathbf{x})$ is $\mathbf{t}_1' = \mathbf{t}_2'$, where $\mathbf{t}_\mathbf{i}'$, $1 \le \mathbf{i} \le 2$, is $\mathbf{t}$ if $\mathbf{t}_\mathbf{i}$ is $\mathbf{x}$ and $\mathbf{t}_\mathbf{i}'$ is just $\mathbf{t}_\mathbf{i}$ otherwise. By the definition of satisfaction, $\mathbf{d}$ satisfies $\mathbf{t}_1' = \mathbf{t}_2'$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1') = \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2')$, and $\mathbf{d}'$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2)$. But now we note that $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{i}') = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{i})$, $1 \le \mathbf{i} \le 2$, by exactly the same reasoning that we used after clause c in the proof of the basis clause for the *PLE* version of 11.1.1, and so $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1') = \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2')$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2)$. We conclude that $\mathbf{d}$ satisfies $\mathbf{t}_1' = \mathbf{t}_2'$ if and only if $\mathbf{d}'$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$.

b. The proof in the case where $\mathbf{P}$ is a sentence letter remains the same as it was in Section 11.1.

If $\mathbf{P}$ has the form $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$, then by the definition of satisfaction,

a. $\mathbf{d}$ satisfies $\mathbf{P}$ if and only if $\langle \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1), \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{n})\rangle$ is a member of $\mathbf{I}(\mathbf{A})$;

and where $\mathbf{d}'$ is a variable assignment that assigns the same values to the free variables in $\mathbf{P}$ as does $\mathbf{d}$,

b. $\mathbf{d}'$ satisfies $\mathbf{P}$ if and only if $\langle \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1), \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{n})\rangle$ is a member of $\mathbf{I}(\mathbf{A})$.

But now we note that

c. $\langle \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1), \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{n})\rangle =$
$\langle \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1), \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{n})\rangle$.

For if $\mathbf{t}_\mathbf{i}$ is a constant, then $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{i}) = \mathbf{I}(\mathbf{t}_\mathbf{i})$ and $\text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{i}) = \mathbf{I}(\mathbf{t}_\mathbf{i})$. If $\mathbf{t}_\mathbf{i}$ is a variable, then $\mathbf{t}_\mathbf{i}$ is free in $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$ and is therefore by stipulation assigned the same value by $\mathbf{d}'$ as it is assigned by $\mathbf{d}$. So $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{i}) = \mathbf{d}(\mathbf{t}_\mathbf{i}) = \mathbf{d}'(\mathbf{t}_\mathbf{i}) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{i})$. If $\mathbf{t}_\mathbf{i}$ is a complex term, it follows from 11.2.2 that $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{i}) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{i})$. Hence by (a) to (c), we conclude that $\mathbf{d}$ satisfies $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$ if and only if every variable assignment $\mathbf{d}'$ that assigns the same values to the free variables in $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$ as $\mathbf{d}$ satisfies $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$.

If $\mathbf{P}$ has the form $\mathbf{t}_1 = \mathbf{t}_2$, then $\mathbf{d}$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2)$, and a variable assignment $\mathbf{d}'$ that assigns the same values to the free variables in $\mathbf{t}_1 = \mathbf{t}_2$ as does $\mathbf{d}$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2)$. But now we note that $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{i}) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_\mathbf{i})$, $1 \le \mathbf{i} \le 2$, by exactly the same reasoning that we used in the case where $\mathbf{P}$ has the form $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$ and so $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2)$ if and only if $\text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_1) = \text{den}_{\mathbf{I},\mathbf{d}'}(\mathbf{t}_2)$. We conclude that $\mathbf{d}$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$ if and only if every variable assignment $\mathbf{d}'$ that assigns the same values to the free variables in $\mathbf{t}_1 = \mathbf{t}_2$ as does $\mathbf{d}$ satisfies $\mathbf{t}_1 = \mathbf{t}_2$.

c. The proof in the case where $\mathbf{P}$ is a sentence letter remains the same as it was in Section 11.1.

If $\mathbf{P}$ is an atomic formula $\mathbf{At}_1 \ldots \mathbf{t}_\mathbf{n}$ then by the definition of satisfaction,

a. $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}$ if and only if $\langle \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1), \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_\mathbf{n})\rangle$ is a member of $\mathbf{I}(\mathbf{A})$.

b. $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}'$ if and only if
$\langle \text{den}_{\mathbf{I}',\mathbf{d}}(\mathbf{t}_1), \text{den}_{\mathbf{I}',\mathbf{d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I}',\mathbf{d}}(\mathbf{t}_\mathbf{n})\rangle$ is a member of $\mathbf{I}'(\mathbf{A})$.

We note that

c. $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle =$
$\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n})\rangle$

because if $\mathbf{t_i}$ is a constant, then $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \mathbf{I}(\mathbf{t_i})$ and $\text{den}_{\mathbf{I',d}}(\mathbf{t_i}) = \mathbf{I}'(\mathbf{t_i})$, and $\mathbf{I}'(\mathbf{t_i}) = \mathbf{I}(\mathbf{t_i})$ since **I** and **I′** assign the same values to the nonlogical symbols in P; if $\mathbf{t_i}$ is a variable, then $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \mathbf{d}(\mathbf{t_i}) = \text{den}_{\mathbf{I',d}}(\mathbf{t_i})$; and if $\mathbf{t_i}$ is a complex term, then $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \text{den}_{\mathbf{I',d}}(\mathbf{t_i})$ by 11.2.3. Moreover,

d. $\mathbf{I}(\mathbf{A}) = \mathbf{I}'(\mathbf{A})$, by our assumption about I and I′.

So by (c) and (d),

e. $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle$ is a member of $\mathbf{I}(\mathbf{A})$ if and only if $\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n})\rangle$ is a member of $\mathbf{I}'(\mathbf{A})$,

and by (a), (b), and (e) it follows that **d** satisfies $\mathbf{At}_1 \ldots \mathbf{t_n}$ on **I** if and only if it does so on **I′**.

If **P** has the form $\mathbf{t}_1 = \mathbf{t}_2$, then **d** satisfies $\mathbf{t}_1 = \mathbf{t}_2$ on **I** if and only if $\text{den}_{\mathbf{I,d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I,d}}(\mathbf{t}_2)$, and **d** satisfies $\mathbf{t}_1 = \mathbf{t}_2$ on **I′** if and only if $\text{den}_{\mathbf{I',d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I',d}}(\mathbf{t}_2)$. But now we note that $\text{den}_{\mathbf{I',d}}(\mathbf{t_i}) = \text{den}_{\mathbf{I,d}}(\mathbf{t_i})$, $1 \leq \mathbf{i} \leq 2$, by exactly the same reasoning that we used in the case where **P** has the form $\mathbf{At}_1 \ldots \mathbf{t_n}$ and so $\text{den}_{\mathbf{I,d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I,d}}(\mathbf{t}_2)$ if and only if $\text{den}_{\mathbf{I',d}}(\mathbf{t}_1) = \text{den}_{\mathbf{I',d}}(\mathbf{t}_2)$. We conclude that **d** satisfies $\mathbf{t}_1 = \mathbf{t}_2$ on **I** if and only if it does so on **I′**.

2. We shall prove 11.2.2 by mathematical induction on the number of occurrences of functors in the term.

> *Basis clause:* If a complex term **t** contains 1 functor, then for any variable assignment **d′** that assigns the same values to the variables in **t** as does **d**, $\text{den}_{\mathbf{I,d'}}(\mathbf{t}) = \text{den}_{\mathbf{I,d}}(\mathbf{t})$.
>
> **Proof of basis clause:** If a complex term **t** contains 1 functor, then **t** is $f(\mathbf{t}_1, \ldots, \mathbf{t_n})$ where f is a functor and each $\mathbf{t_i}$ is either a variable or constant. If $\mathbf{t_i}$ is a variable, then since **d′** assigns the same values to the variables in **t** as does **d**, $\text{den}_{\mathbf{I,d'}}(\mathbf{t_i}) = \text{den}_{\mathbf{I,d}}(\mathbf{t_i})$. If $\mathbf{t_i}$ is a constant, then $\text{den}_{\mathbf{I,d'}}(\mathbf{t_i}) = \mathbf{I}(\mathbf{t_i}) = \text{den}_{\mathbf{I,d}}(\mathbf{t_i})$. So we know that $\langle \text{den}_{\mathbf{I,d'}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d'}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d'}}(\mathbf{t_n})\rangle =$ $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle$. Therefore, the $\mathbf{n} + 1$-tuple $\langle \text{den}_{\mathbf{I,d'}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d'}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d'}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ if and only if $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ since these are the same **n**-tuple, so $\text{den}_{\mathbf{I,d'}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n})) = \text{den}_{\mathbf{I,d}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n}))$.
>
> *Inductive step:* If every complex term **t** that contains **k** or fewer functors is such that for any variable assignment **d′** that assigns the same values to the variables in **t** as does **d**, $\text{den}_{\mathbf{I,d'}}(\mathbf{t}) = \text{den}_{\mathbf{I,d}}(\mathbf{t})$, then every complex term that contains $\mathbf{k} + 1$ functors is such that for any variable assignment **d′** that assigns the same values to the variables in **t** as does **d**, $\text{den}_{\mathbf{I,d'}}(\mathbf{t}) = \text{den}_{\mathbf{I,d}}(\mathbf{t})$.
>
> **Proof of inductive step:** Letting **k** be an arbitrary positive integer, we assume that the inductive hypothesis holds—that our claim is true of

every complex term that contains **k** or fewer functors. We must show that the claim is also true of every complex term that contains $\mathbf{k} + 1$ functors. If **t** contains $\mathbf{k} + 1$ functors, then **t** is $f(\mathbf{t}_1, \ldots, \mathbf{t_n})$ where each $\mathbf{t_i}$ has **k** or fewer functors. So each $\mathbf{t_i}$ falls under the inductive hypothesis, i.e., $\text{den}_{\mathbf{I,d'}}(\mathbf{t_i}) = \text{den}_{\mathbf{I,d}}(\mathbf{t_i})$ and hence $\langle \text{den}_{\mathbf{I,d'}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d'}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d'}}(\mathbf{t_n})\rangle =$ $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle$. Therefore, the $\mathbf{n} + 1$-tuple $\langle \text{den}_{\mathbf{I,d'}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d'}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d'}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ if and only if $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$, so $\text{den}_{\mathbf{I,d'}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n})) = \text{den}_{\mathbf{I,d}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n}))$.

3. We shall prove 11.2.3 by mathematical induction on the number of occurrences of functors in the term.

Basis clause: If a complex term **t** contains 1 functor, then, $\text{den}_{\mathbf{I,d}}(\mathbf{t}) = \text{den}_{\mathbf{I',d}}(\mathbf{t})$.

Proof of basis clause: If a complex term **t** contains 1 functor, then **t** is $f(\mathbf{t}_1, \ldots, \mathbf{t_n})$ where f is a functor and each $\mathbf{t_i}$ is either a variable or constant. If $\mathbf{t_i}$ is a variable, then $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \text{d}(\mathbf{t_i}) = \text{den}_{\mathbf{I',d}}(\mathbf{t_i})$. If $\mathbf{t_i}$ is a constant, then $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \mathbf{I}(\mathbf{t_i}) = \mathbf{I}'(\mathbf{t_i})$ (because **I** and **I′** agree on the values assigned to constants) $= \text{den}_{\mathbf{I',d}}(\mathbf{t_i})$. So we know that $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle =$ $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n})\rangle$. Therefore, the $\mathbf{n} + 1$-tuple $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ if and only if $\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ since these are the same **n**-tuple, and $\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ if and only if $\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}'(f)$ since **I** and **I′** agree on the values assigned to functors, so $\text{den}_{\mathbf{I,d'}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n})) = \text{den}_{\mathbf{I,d}}(f(\mathbf{t}_1, \ldots, \mathbf{t_n}))$.

Inductive step: If every complex term **t** that contains **k** or fewer functors is such that $\text{den}_{\mathbf{I,d}}(\mathbf{t}) = \text{den}_{\mathbf{I',d}}(\mathbf{t})$, then every complex term that contains $\mathbf{k} + 1$ functors is such that $\text{den}_{\mathbf{I,d}}(\mathbf{t}) = \text{den}_{\mathbf{I',d}}(\mathbf{t})$.

Proof of inductive step: Letting **k** be an arbitrary positive integer, we assume that the inductive hypothesis holds—that our claim is true of every complex term that contains **k** or fewer functors. We must show that the claim is also true of every complex term that contains $\mathbf{k} + 1$ functors. If **t** contains $\mathbf{k} + 1$ functors, then **t** is $f(\mathbf{t}_1, \ldots, \mathbf{t_n})$ where each $\mathbf{t_i}$ has **k** or fewer functors. So each $\mathbf{t_i}$ falls under the inductive hypothesis, i.e., $\text{den}_{\mathbf{I,d}}(\mathbf{t_i}) = \text{den}_{\mathbf{I',d}}(\mathbf{t_i})$ and hence $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n})\rangle =$ $\langle \text{den}_{\mathbf{I',d}}(\mathbf{t}_1), \text{den}_{\mathbf{I',d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I',d}}(\mathbf{t_n})\rangle$. Therefore, the $\mathbf{n} + 1$-tuple $\langle \text{den}_{\mathbf{I,d}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$ if and only if $\langle \text{den}_{\mathbf{I,d'}}(\mathbf{t}_1), \text{den}_{\mathbf{I,d'}}(\mathbf{t}_2), \ldots, \text{den}_{\mathbf{I,d'}}(\mathbf{t_n}), \mathbf{u}\rangle$ is a member of $\mathbf{I}(f)$, and $\langle den_{I',d}(t_1), den_{I',d}(t_2), \ldots, den_{I',d}(t_n), u\rangle$ is a member of $I(f)$ if and only if $\langle den_{I',d}(t_1), den_{I',d}(t_2), \ldots, den_{I',d}(t_n), u\rangle$ is a member of $I'(f)$ since **I** and **I′** agree on the values assigned to functors, so $den_{I,d'}(f(t_1, \ldots, t_n)) = den_{I,d}(f(t_1, \ldots, t_n))$.

5. Assume that Γ is a quantificationally consistent set that contains a sentence with a complex term $f(a_1, \ldots, a_n)$, where $a_1, \ldots, a_n$ are constants, and that the constant **b** does not occur in Γ. Let **I** be an interpretation on which every member of Γ is true. We construct an interpretation **I′** that is just like **I** except that $I'(b) = den_{I,d}(f(a_1, \ldots, a_n))$ (where **d** is any variable assignment). Since **I** and **I′** agree on the assignments made to each individual constant, functor, predicate, and sentence letter that occurs in Γ (**b** does not occur in Γ), it follows from 11.1.6 for *PLE* that every member of Γ is true on **I′**. Moreover, by the way we have defined **I′**, $den_{I',d}(b) = den_{I,d}(f(a_1, \ldots, a_n))$, and $den_{I,d}(f(a_1, \ldots, a_n)) = den_{I',d}(f(a_1, \ldots, a_n))$ by 11.2.3 since **b** does not occur in $f(a_1, \ldots, a_n)$, so $den_{I',d}(b) = den_{I',d}(f(a_1, \ldots, a_n))$ and hence $b = f(a_1, \ldots, a_n)$ is true on **I′**. Therefore every member of $\Gamma \cup \{b = f(a_1, \ldots, a_n)\}$ is true on **I′**, so the set is quantificationally consistent.

Section 11.3E

1. c. Assume that sentences **P** and **Q** are equivalent in *PD*. Then $\{P\} \vdash Q$ and $\{Q\} \vdash P$. By Metatheorem 11.3.1, it follows that $\{P\} \vDash Q$ and $\{Q\} \vDash P$. But then, on every interpretation on which **P** is true, so is **Q** and the reverse. Hence **P** and **Q** are quantificationally equivalent.

4. a. We may show this by producing a derivation in which the restriction is dropped, that is not truth-preserving:

1	Rb	Assumption
2	(∀x)Rx	1 ∀I

It is straightforward to verify that {Rb} does not quantificationally entail '(∀x)Rx'.

b. We may show this by producing a derivation in which the restriction is dropped, that is not truth-preserving:

1	(∃x)Bx	Assumption
2	Bc	Assumption
3	Bc	2 R
4	Bc	1, 2–3 ∃E

It is straightforward to verify that {(∃x)Bx} does not quantificationally entail 'Bc'.

Section 11.4E

1. Assume that $\Gamma \vDash P$. Then, on any interpretation **I** on which every member of Γ is true, $I(P) = T$. Hence on such an interpretation ~ **P** is false.

Therefore there is no interpretation on which every member of $\Gamma \cup \{\sim \mathbf{P}\}$ is true; that is, $\Gamma \cup \{\sim \mathbf{P}\}$ is quantificationally inconsistent.

3. c. Assume that sentences **P** and **Q** are quantificationally equivalent. Then, on every truth-value assignment on which **P** is true, **Q** is true and the reverse. Hence $\{\mathbf{P}\} \vDash \mathbf{Q}$ and $\{\mathbf{Q}\} \vDash \mathbf{P}$. By Metatheorem 11.4.1, it follows that $\{\mathbf{P}\} \vdash \mathbf{Q}$ and $\{\mathbf{Q}\} \vdash \mathbf{P}$. So **P** and **Q** are equivalent in *PD*.

8. Let Γ and Γ_e be as described, and assume that Γ_e is quantificationally consistent. Let **I** be an interpretation on which every member of Γ_e is true, and let **I**′ be an interpretation that is like **I** except that for each constant **a** occurring in Γ, $\mathbf{I}'(\mathbf{a}) = \mathbf{I}(\mathbf{a}')$, where **a**′ is the constant obtained by doubling the subscript in **a**. We claim that every member of Γ is true on **I**′. Consider any sentence **P** in Γ. If **P** contains no individual constants then, by 11.1.6, **P** is true on **I**′ because it is true on **I**. If **P** contains individual constants, then repeated applications of 11.1.13 along with 11.1.6 show that **P** is true on **I**′ because its counterpart in Γ_e is true on **I**. Therefore Γ is also quantificationally consistent.

9. Assume that $\Gamma \vdash \mathbf{P}$ and that every member of Γ is a member of a set Γ^* that is maximally consistent in *PD*. Assume that $\mathbf{P} \notin \Gamma^*$. Then $\Gamma^* \cup \{\mathbf{P}\}$ is inconsistent in *PD*. So there is some finite subset Γ' of Γ^* such that $\Gamma' \cup \{\mathbf{P}\}$ is inconsistent in *PD*. Hence $\Gamma' \vdash \sim \mathbf{P}$ (by a derivation similar to that in Exercise 2). But then (from our initial assumption) $\Gamma \cup \Gamma' \vdash \mathbf{P}$ and $\Gamma \cup \Gamma' \vdash \sim \mathbf{P}$. It follows that Γ^* is inconsistent in *PD*. But this contradicts one of our assumptions, so $\mathbf{P} \in \Gamma^*$.

12. As in Exercise 11, it will suffice to show that with $\exists$E* and $\exists$I* instead of $\exists$E and $\exists$I, every set Γ^* of sentences of *PD** that is both maximally consistent in *PD** and $\exists$-complete has properties f and g. Here is the proof:

f. Assume that $(\forall \mathbf{x})\mathbf{P} \in \Gamma^*$. The reasoning in the text establishes that every substitution instance of $(\forall \mathbf{x})\mathbf{P}$ is also in Γ^*. Now assume that $(\forall \mathrm{x})\mathbf{P} \notin \Gamma^*$. Then $\sim (\forall \mathbf{x})\mathbf{P} \in \Gamma^*$, by property a. The following derivation shows that $\{\sim (\forall \mathbf{x})\mathbf{P}\} \vdash (\exists \mathbf{x}) \sim \mathbf{P}$:

1	$\sim (\forall \mathbf{x})\mathbf{P}$	Assumption
2	$(\forall \mathbf{x}) \sim \sim \mathbf{P}$	Assumption
3	$\sim \sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	2 $\forall$E (where **a** is foreign to **P**)
4	$\sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	Assumption
5	$\sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	4 R
6	$\sim \sim \mathbf{P}(\mathbf{a}/\mathbf{x})$	3 R
7	$\mathbf{P}(\mathbf{a}/\mathbf{x})$	4–6 $\sim$ E
8	$(\forall \mathbf{x})\mathbf{P}$	7 $\forall$I
9	$\sim (\forall \mathbf{x})\mathbf{P}$	1 R
10	$\sim (\forall \mathbf{x}) \sim \sim \mathbf{P}$	2–9 $\sim$ I
11	$(\exists \mathbf{x}) \sim \mathbf{P}$	10 $\exists$I*

The rest of property f is proved as in the book.

g. Assume that $(\exists\mathbf{x})\mathbf{P} \in \Gamma^*$. Then because Γ^* is $\exists$-complete, at least one substitution instance of $(\exists\mathbf{x})\mathbf{P}$ is also a member of Γ^*. Now assume that $(\exists\mathbf{x})\mathbf{P} \notin \Gamma^*$. Then $\sim (\forall\mathbf{x}) \sim \mathbf{P} \notin \Gamma^*$ (for if it were, then, because $\{\sim (\forall\mathbf{x}) \sim \mathbf{P}\} \vdash (\exists x)\mathbf{P}$ by $\exists$I*, $(\exists\mathbf{x})\mathbf{P}$ would be a member of Γ^*). By property a, $(\forall\mathbf{x}) \sim \mathbf{P} \in \Gamma^*$, and, by property f, every substitution instance $\sim \mathbf{P}(\mathbf{a}/\mathbf{x}) \in \Gamma^*$. By property a again, for every constant $\mathbf{a}$, $\mathbf{P}(\mathbf{a}/\mathbf{x}) \notin \Gamma^*$.

14. Let Γ be a set of sentences of *PL* that is quantificationally consistent. Then Γ is consistent in *PD* (by Metatheorem 11.3.1), and we may use the method in the proof of 11.4.3 and 11.4.4 to extend Γ to a set that is both maximally consistent in *PD* and $\exists$-complete. And, by the reasoning of Exercise 13, it follows that there is an interpretation with the set of positive integers as *UD* on which every member of Γ is true.

15. Because $\forall$I and $\exists$E are unchanged in PDE, the proof of 11.4.3 does not require any change.

Inspection shows that for 11.4.9, changes need to be made in the proofs of results 11.1.1 and 11.1.6. These were done in Section 11.2.

Section 11.5E

1. Lemma 11.5.2 is established by mathematical induction on the levels of trees. The proof of the basis clause, that Lemma 11.5.2 holds for the first level of any tree for a quantificationally consistent set of sentences, is trivial. There is just one path to the first level of a tree for a set Γ; that path consists of some sentence $\mathbf{P}$ that is a member of Γ and that has been entered on the tree with the justification SM. And $\{\mathbf{P}\}$ must be quantificationally consistent if Γ, of which $\{\mathbf{P}\}$ is a subset, is. The inductive step is

If Lemma 11.5.2 holds for every level prior to level $\mathbf{k} + 1$ of any tree, then Lemma 11.5.2 holds for level $\mathbf{k} + 1$ of any tree.

To prove the inductive step, we assume the inductive hypothesis: that for each level $\mathbf{i}$ of a tree for a quantificationally consistent set, where $\mathbf{i} < \mathbf{k} + 1$, either (a) at least one completed path to a level prior to $\mathbf{i}$ is quantificationally consistent or (b) at least one path to level $\mathbf{i}$ is quantificationally consistent. We want to establish that either (a) or (b) holds for level $\mathbf{k} + 1$ as well. Consider an arbitrary tree for a consistent set of sentences with at least $\mathbf{k} + 1$ levels. The level prior to $\mathbf{k} + 1$—$\mathbf{k}$—falls under the inductive hypothesis. That is, either some completed path to a level earlier than level $\mathbf{k}$ is quantificationally consistent or at least one path to level $\mathbf{k}$ on this tree is quantificationally consistent. In the former case, since a level prior to $\mathbf{k}$ is also prior to $\mathbf{k} + 1$, Lemma 11.5.2 follows immediately for level $\mathbf{k} + 1$—since in that case (a) holds for level $\mathbf{k} + 1$.

So assume that the latter holds—that at least one path to level $\mathbf{k}$ on the tree is quantificationally consistent. Call that path Γ^*. We have two cases to consider. First, if Γ^* is a completed path to level $\mathbf{k}$, then Lemma 11.5.2 holds for level $\mathbf{k} + 1$ because there is a completed path to a level earlier than $\mathbf{k} + 1$ that is quantificationally consistent.

Our second case is that where a rule is applied at level $\mathbf{k} + 1$ to some sentence on Γ^*. Here we must ensure that at level $\mathbf{k} + 1$ there will still be at least one quantificationally consistent path to that level. That is, if some rule is applied at $\mathbf{k} + 1$ to some sentence on Γ^*, then the consistent path to level $\mathbf{k}$ is being extended to a path to level $\mathbf{k} + 1$ (possibly into two paths if a branching rule is used) to obtain level $\mathbf{k} + 1$; we shall now show that this extension of the path Γ^* yields at least one quantificationally consistent path to level $\mathbf{k} + 1$. We therefore consider four various possible results of application of a decomposition rule at level $\mathbf{k} + 1$ to a sentence on Γ^* and show that in each case—whichever rule is applied—we are left with at least one quantificationally consistent path to level $\mathbf{k} + 1$.

(i) A sentence $\mathbf{Q}$ is entered on level $\mathbf{k} + 1$ as the result of applying one of the nonbranching rules $\sim \sim$ D, & D, $\sim \vee$ D, $\sim \supset$ D, or $\forall$D to a sentence $\mathbf{P}$ on Γ^*. Then $\{\mathbf{P}\}$, the unit set of the sentence to which the rule was applied, quantificationally entails $\mathbf{Q}$. Hence the set consisting of $\mathbf{Q}$ and all the sentences on Γ^* is quantificationally consistent. So there is a path to level $\mathbf{k} + 1$ that ends with $\mathbf{Q}$ and is quantificationally consistent.

(ii) Sentences $\mathbf{Q}$ and $\mathbf{R}$ are entered on level $\mathbf{k} + 1$ as the result of applying one of the branching rules $\sim$ & D, $\vee$ D, or $\supset$ D to a sentence $\mathbf{P}$ on Γ^*. Then on any interpretation on which $\mathbf{P}$ is true so is either $\mathbf{Q}$ or $\mathbf{R}$. Hence either the set consisting of $\mathbf{Q}$ and all the sentences on Γ^* is quantificationally consistent or the set consisting of $\mathbf{R}$ and all the sentences on Γ^* is quantificationally consistent. If the former, then the path to $\mathbf{k} + 1$ ending with $\mathbf{Q}$ is quantificationally consistent; if the latter, then the path to $\mathbf{k} + 1$ ending with $\mathbf{R}$ is quantificationally consistent.

(iii) Sentences are entered on level $\mathbf{k} + 1$ as a result of applying either $\equiv$ D or $\sim \equiv$ D to a sentence on Γ^*. Similar to (ii).

(iv) A sentence $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is entered on level $\mathbf{k} + 1$ as the result of applying $\exists$D to a sentence $(\exists\mathbf{x})\mathbf{P}$ on Γ^*. Then $\mathbf{a}$ does not occur in any sentence on Γ^*, so the set consisting of $\mathbf{P}(\mathbf{a}/\mathbf{x})$ and all the sentences on Γ^* is quantificationally consistent by 11.1.10. Thus the path to $\mathbf{k} + 1$ ending with $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is quantificationally consistent.

Lemma 11.5.2 therefore holds for every level of a tree for a quantificationally consistent set of sentences of *PL.*

2. c. Assume that $\{\sim (\mathbf{P} \equiv \mathbf{Q})\}$ has a closed systematic truth-tree. Then, by Metatheorem 11.5.1, $\{\sim (\mathbf{P} \equiv \mathbf{Q})\}$ is quantificationally inconsistent. Hence there is no interpretation on which $\sim (\mathbf{P} \equiv \mathbf{Q})$ is true. So $\mathbf{P} \equiv \mathbf{Q}$ is true on every interpretation. On each interpretation, then, $\mathbf{P}$ and $\mathbf{Q}$ have the same truth-value. So $\mathbf{P}$ and $\mathbf{Q}$ are quantificationally equivalent.

e. Assume that the set consisting of the premises and the negation of the conclusion of an argument has a closed systematic truth-tree. Then, by Metatheorem 11.5.1, that set is quantificationally inconsistent. Thus there is no interpretation on which all of the premises are true and the negation of the conclusion is true; that is, there is no interpretation on which the premises are true and the conclusion is false. Therefore the argument is quantificationally valid.

3. b. **P** or **Q** is obtained from **P** & **Q** by & D. It is straightforward that {**P** & **Q**} $\vDash$ **P** and {**P** & **Q**} $\vDash$ **Q**.

c. ~ **P** or ~ **Q** is obtained from ~ (**P** $\vee$ **Q**) by ~ $\vee$ D. On any interpretation on which ~ (**P** $\vee$ **Q**) is true, **P** $\vee$ **Q** is false—hence both **P** and **Q** are false. But then, on such an interpretation, both ~ **P** and ~ **Q** are true; that is, {~ (**P** $\vee$ **Q**)} $\vDash$ ~ **P** and {~ (**P** $\vee$ **Q**)} $\vDash$ ~ **Q**.

f. ($\exists$**x**) ~ **P** is obtained from ~ ($\forall$**x**)**P** by ~ $\forall$D. It is straightforward that {~ ($\forall$**x**)**P**} $\vDash$ ($\exists$**x**) ~ **P**.

g. ($\forall$**x**) ~ **P** is obtained from ~ ($\exists$**x**)**P** by ~ $\exists$D. It is straightforward that {~ ($\exists$**x**)**P**} $\vDash$ ($\forall$**x**) ~ **P**.

4. b. **P** and **Q** are obtained from **P** $\vee$ **Q** by $\vee$ D. It is trivial that on any interpretation on which **P** $\vee$ **Q** is true, either **P** is true or **Q** is true.

c. ~ **P** and **Q** are obtained from **P** $\supset$ **Q** by $\supset$ **D**. On any interpretation on which **P** $\supset$ **Q** is true, either **P** is false or **Q** is true. Thus, on such an interpretation, either ~ **P** is true or **Q** is true.

8. The proof of 11.5.4 must be modified as follows. When showing that an open path of a systematic tree that contains only a finite number of individual constants must be finitely long, we must consider what happens with the additional decomposition rules =D and CTD.

=D applies to a literal $\mathbf{t}_1 = \mathbf{t}_2$ and a literal **P** to produce $\mathbf{P}(\mathbf{t}_1//\mathbf{t}_2)$. Given a finite number of constants and a finite number of literals, =D can apply at most finitely many times. This is because $\mathbf{t}_1$ in the new literals $\mathbf{P}(\mathbf{t}_1//\mathbf{t}_2)$ that are produced must be a constant. Given a finite number of constants there are only finitely many literals that are like **P** except that one or more terms have been replaced by constants.

CTD adds exactly one sentence to each path, a literal, and the only way that this literal can be used to generate further sentences is via =D, which we have already discussed.

Section 11.6E

1. c. Assume that **P** and **Q** are quantificationally equivalent. Then **P** $\equiv$ **Q** is quantificationally true. So ~ (**P** $\equiv$ **Q**) is quantificationally false, and {~ (**P** $\equiv$ **Q**)} is quantificationally inconsistent. It follows from Metatheorem 11.6.1 that every systematic tree for {~ (**P** $\equiv$ **Q**)} closes.

e. If an argument of *PL* is quantificationally valid, then, on every interpretation on which all of the premises are true, the conclusion is true as well; hence the negation of the conclusion is false. So the set consisting of the premises and negation of the conclusion is quantificationally inconsistent. It follows from Metatheorem 11.6.1 that every systematic tree for that set closes.

2. c. Assume that the length of a sentence $\mathbf{Q} \equiv \mathbf{R}$ is $\mathbf{k}$. As $\mathbf{Q} \equiv \mathbf{R}$ contains at least one sentence letter or predicate that $\mathbf{R}$ does not contain and $\mathbf{Q} \equiv \mathbf{R}$ contains at least one occurrence of the triple bar that $\mathbf{R}$ does not contain, the length of $\mathbf{R}$ is $\mathbf{k} - 2$ or less. Hence the length of $\sim \mathbf{R}$ is $\mathbf{k} - 1$ or less. Similar reasoning shows that the length of $\sim \mathbf{Q}$ is $\mathbf{k} - 1$ or less.

3. b. $\mathbf{P}$ is of the form $\sim (\mathbf{Q} \vee \mathbf{R})$. Assume that $\mathbf{P} \in \Gamma$. Then by property f, both $\sim \mathbf{Q}$ and $\sim \mathbf{R}$ are members of Γ. By the inductive hypothesis, then, $\mathbf{I}(\sim \mathbf{Q}) = \mathbf{T}$ and $\mathbf{I}(\sim \mathbf{R}) = \mathbf{T}$. So $\mathbf{I}(\mathbf{Q}) = \mathbf{F}$ and $\mathbf{I}(\mathbf{R}) = \mathbf{F}$. Consequently, $\mathbf{I}(\mathbf{Q} \vee \mathbf{R}) = \mathbf{F}$ and $\mathbf{I}(\sim (\mathbf{Q} \vee \mathbf{R})) = \mathbf{T}$.

d. $\mathbf{P}$ is of the form $\sim (\mathbf{Q} \supset \mathbf{R})$. Assume that $\mathbf{P} \in \Gamma$. Then, by property h, $\mathbf{Q} \in \Gamma$ and $\sim \mathbf{R} \in \Gamma$. By the inductive hypothesis, $\mathbf{I}(\mathbf{Q}) = \mathbf{T}$ and $\mathbf{I}(\sim \mathbf{R}) = \mathbf{T}$. So $\mathbf{I}(\mathbf{R}) = \mathbf{F}$, and $\mathbf{I}(\mathbf{Q} \supset \mathbf{R}) = \mathbf{F}$.

e. $\mathbf{P}$ is of the form $\mathbf{Q} \equiv \mathbf{R}$. Assume that $\mathbf{P} \in \Gamma$. Then, by property i of Hintikka sets, either both $\mathbf{Q} \in \Gamma$ and $\mathbf{R} \in \Gamma$ or both $\sim \mathbf{Q} \in \Gamma$ and $\sim \mathbf{R} \in \Gamma$. In the former case, $\mathbf{I}(\mathbf{Q}) = \mathbf{T}$ and $\mathbf{I}(\mathbf{R}) = \mathbf{T}$, by the inductive hypothesis, and $\mathbf{I}(\mathbf{Q} \equiv \mathbf{R}) = \mathbf{T}$. In the latter case $\mathbf{I}(\mathbf{Q}) = \mathbf{F}$ and $\mathbf{I}(\mathbf{R}) = \mathbf{F}$, by the inductive hypothesis, so $\mathbf{I}(\mathbf{Q} \equiv \mathbf{R}) = \mathbf{T}$. Therefore $\mathbf{I}(\mathbf{Q} \equiv \mathbf{R}) = \mathbf{T}$.

h. $\mathbf{P}$ is of the form $\sim (\exists \mathbf{x})\mathbf{Q}$. Assume that $\mathbf{P} \in \Gamma$. Then, by property n, $(\forall \mathbf{x}) \sim \mathbf{Q} \in \Gamma$. And, by property $\mathbf{k}$, for every constant $\mathbf{a}$ that occurs in Γ, $\sim \mathbf{Q}(\mathbf{a}/\mathbf{x}) \in \Gamma$. By the inductive hypothesis, it follows that for each constant $\mathbf{a}$ that occurs in some member of Γ, $\mathbf{I}(\sim \mathbf{Q}(\mathbf{a}/\mathbf{x})) = \mathbf{T}$. By property 1 of the construction of $\mathbf{I}$, each member of the UD is $\mathbf{I}(\mathbf{a})$ for some constant $\mathbf{a}$ that occurs in Γ. By 11.1.11, $\mathbf{I}((\forall \mathbf{x}) \sim \mathbf{Q}) = \mathbf{T}$. Therefore every variable assignment satisfies $\sim \mathbf{Q}$, so no variable assignment satisfies $\mathbf{Q}$. So $(\exists \mathbf{x})\mathbf{Q}$ is false on $\mathbf{I}$, and $\mathbf{I}(\sim (\exists \mathbf{x})\mathbf{Q}) = \mathbf{T}$.

4. Let $\mathbf{I}$ be an interpretation on which for each member $\mathbf{u}$ of the U.D. there is at least one constant $\mathbf{a}$ such that $\mathbf{I}(\mathbf{a}) = \mathbf{u}$ and $\mathbf{I}(\mathbf{P}(\mathbf{a}/\mathbf{x})) = \mathbf{T}$. Assume, contrary to what we wish to prove, that $\mathbf{I}((\forall \mathbf{x})\mathbf{P}) = \mathbf{F}$. Then every variable assignment $\mathbf{d}$ fails to satisfy $(\forall \mathbf{x})\mathbf{P}$ on $\mathbf{I}$; for every one there is at least one variant $\mathbf{d}[\mathbf{u}'/\mathbf{x}]$ that does not satisfy $\mathbf{P}$. Because $\mathbf{u}' = \mathbf{I}(\mathbf{a}')$ for some constant $\mathbf{a}'$, it follows from 11.1.1 that $\mathbf{d}$ does not satisfy $\mathbf{P}(\mathbf{a}'/\mathbf{x})$. This being the case, $\mathbf{I}(\mathbf{P}(\mathbf{a}'/\mathbf{x})) = \mathbf{F}$, and the same is true of any substitution instance using a constant that denotes $\mathbf{u}'$. This contradicts our original assumption, and we conclude that $(\forall \mathbf{x})\mathbf{P}$ must be true on $\mathbf{I}$ as well.