Instructor's Manual to Accompany

FOURTH EDITION

Fundamentals of Fluid Mechanics

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TABLE OF CONTENTS

INTRODUCTION

This manual contains solutions to the problems presented at the end of the chapters in the *Fourth Edition* of FUNDAMENTALS OF FLUID MECHANICS. It is our intention that the material in this manual be used as an aid in the teaching of the course. We feel quite strongly that problem solving is an essential ingredient in the process of understanding the variety of interesting concepts involved in fluid mechanics. This solutions manual is structured to enhance the learning process.

Approximately 1220 problems are solved in a complete, detailed fashion with (in most cases) one problem per page. The problem statements and figures are included with the problem solutions to provide an easier and clearer understanding of the solution procedure. Except where a greater accuracy is warranted, all intermediate calculations and answers are given to three significant figures.

Unless otherwise indicated in the problem statement, values of fluid properties used in the solutions are those given in the tables on the inside of the front cover of the text. Other fluid properties and necessary conversion factors are found in the tables of Chapter 1 or in the appendices.

Some of the problems [those designed with an (*)] are intended to be solved with the aid of a programmable calculator or a computer. The solutions for each of these problems are presented in essentially the same format as for the non-computer problems. Where appropriate a graph of the results is also included. Further details concerning the computer and their solutions can be found in the following section entitled Computer Problems.

In most chapters there are several problems [those designated with a (†)] that are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide necessary data. There is not a unique answer to these problems. Since there are various ways that one may approach many of these problems and since specific values of data need to be assumed, looked up, or approximated, we have not included solutions to these problems in the manual. Providing solutions, we feel, would be counter to the rational for having these problems—we want students to realize that in the real world problems are not necessarily uniquely formulated to a have a specific answer.

One of the new features of the *Fourth Edition* of FUNDAMENTALS OF FLUID MECHANICS is the inclusion of new problems which refer to the fluid video segments contained in the E-book CD. These problems are clearly identified in the problem statement. Although it is not necessary to use the CD to solve these "video-related" problems, it is hoped that the use of the CD will help students relate the analysis and solution of the problem to actual fluid mechanics phenomena.

Another new feature of the *Fourth Edition* is the inclusion of laboratory-related problems. In most chapters the last few problems are based on actual data from simple laboratory experiments. These problems are clearly identified by the "click here" words in the problem statement. This allows the user of the E-book CD to link to the complete problem statement and the EXCEL data for the problem. Copies of the problem statement, the original data, the EXCEL spread sheet calculations, and the resulting graphs are given in this solution manual.

Considerable effort has been put forth to develop appropriate problems and to present their solutions in a manner that we feel is helpful to both instructors and students. Any comments or suggestions as to how we can improve this material are most welcome.

COMPUTER PROBLEMS

As noted, problems designated with an (*) in the text are intended to be solved with the aid of a programmable calculator or computer. These problems typically involve solutions requiring repetitive calculations, iterative procedures, curve fitting, numerical integration, etc. Knowledge of advanced numerical techniques is not required. Solutions to all computer problems are included in the solutions manual. Although programs for many of these problems are written in the BASIC programming language, there are obviously several other math-solver or spreadsheet programs that can be used.

A number of the solutions require the use of the same program, such as a program for curve fitting, or a numerical integration program, and these "standard" programs are included. For those requiring use of one of the standard programs, there is a statement in the problem solution which simply indicates the standard program used to solve the problem. A list of these standard programs, with their file names, follow. The actual programs are given in the appendix. Most of the standard programs are, of course, readily available in other math-solver or spreadsheet programs, and the student can simply use the programs with which they are most familiar.

Standard Programs—File Names and Use

Curve Fitting

EXPFIT.BAS	Determines the least squares fit for a function of the form $y=ae^{bx}$
LINREG1.BAS	Determines the least squares fit for a function of the form $y=bx$
LINREG2.BAS	Determines the least squares fit for a function of the form $y=a+bx$
POLREG.BAS	Determines the least squares fit for a function of the form $y=d_0+d_1x+d_2x^2+d_3x^3+\cdots$
POWER1.BAS	Determines the least squares fit for a function of the form $y=ax^b$

Numerical Integration

SIMPSON.BAS	Calculates the value of a definite integral over an odd number of equally spaced points using Simpson's rule
TRAPEZOI.BAS	Calculates the value of a definite integral using the Trapezoidal Rule

Miscellaneous

COLEBROO.BAS	Determines the friction factor for laminar or turbulent pipe flow with the Reynolds number and relative roughness specified (for turbulent flow the Colebrook formula, Eq. 8.35, is used)
CUBIC.BAS	Determines the real roots of a cubic equation
FAN_RAY.BAS	Calculates Fanno or Rayleigh flow parameters for an ideal gas with constant specific heat ratio ($k>1$) for entered Mach number
ISENTROP.BAS	Calculates one-dimensional isentropic flow parameters for an ideal gas with constant specific heat ration ($k>1$) for entered Mach number
SHOCK.BAS	Calculates normal-shock flow parameters for an ideal gas with constant specific heat ratio ($k>1$) for entered upstream Mach number (Ma_x)

1.1

1.1 Determine the dimensions, in both the *FLT* system and the *MLT* system, for **(a)** the product of mass times velocity, **(b)** the product of force times volume, and **(c)** kinetic energy divided by area.

(a) mass x velocity $\doteq (M)(LT^{-1}) \doteq \underline{\underline{MLT^{-1}}}$

Since $F \doteq MLT^{-2}$

mass x velocity $\doteq (FL^{-1}T^{2})(LT^{-1}) \doteq \underline{\underline{FT}}$

(b) force x volume $\doteq \underline{\underline{FL^3}}$

$\doteq (MLT^{-2})(L^3) \doteq \underline{\underline{ML^4T^{-2}}}$

(c) $\dfrac{\text{kinetic energy}}{\text{area}} \doteq \dfrac{FL}{L^2} \doteq \underline{\underline{FL^{-1}}}$

$\doteq \dfrac{(MLT^{-2})L}{L^2} \doteq \underline{\underline{MT^{-2}}}$

1.2

1.2 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

(a) angular velocity $= \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

(b) energy ~ capacity of body to do work

Since work = force x distance,

$$\text{energy} \doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

$$\text{energy} \doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$

(c) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(d) power = rate of doing work $\doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}}$

$$\doteq (MLT^{-2})(L)(T^{-1}) \doteq \underline{\underline{ML^2T^{-3}}}$$

(e) pressure $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

$$\doteq (MLT^{-2})(L^{-2}) \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

1.3

1.3 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

(a) acceleration $= \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

(b) stress $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} = \underline{\underline{ML^{-1}T^{-2}}}$

(c) moment of a force = force × distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

(d) volume $= (\text{length})^3 \doteq \underline{\underline{L^3}}$

(e) work = force × distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

1.4

1.4 If P is a force and x a length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

(a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \underline{\underline{FL^{-1}}}$

(b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

(c) $\int P\,dx \doteq \underline{\underline{FL}}$

1.5

1.5 If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of (a) p/ρ, (b) $pV\rho$, and (c) $p/\rho V^2$?

(a) $\dfrac{p}{\rho} \doteq \dfrac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$

(b) $pV\rho \doteq (ML^{-1}T^{-2})(LT^{-1})(ML^{-3}) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$

(c) $\dfrac{p}{\rho V^2} \doteq \dfrac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0$ (dimensionless)

1.6

1.6 If V is a velocity, ℓ a length, and ν a fluid property having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, **(d)** $V/\ell\nu$?

(a) $V\ell\nu \doteq (LT^{-1})(L)(L^2T^{-1}) \doteq L^4T^{-2}$ (not dimensionless)

(b) $\dfrac{V\ell}{\nu} \doteq \dfrac{(LT^{-1})(L)}{(L^2T^{-1})} \doteq L^0T^0$ (dimensionless)

(c) $V^2\nu \doteq (LT^{-1})^2(L^2T^{-1}) \doteq L^4T^{-3}$ (not dimensionless)

(d) $\dfrac{V}{\ell\nu} \doteq \dfrac{(LT^{-1})}{(L)(L^2T^{-1})} \doteq L^{-2}$ (not dimensionless)

1.7

1.7 Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table 1.1.

Some possible examples:

$$\frac{\text{acceleration} \times \text{time}}{\text{velocity}} \doteq \frac{(LT^{-2})(T)}{(LT^{-1})} \doteq L^0T^0$$

$$\text{frequency} \times \text{time} \doteq (T^{-1})(T) \doteq T^0$$

$$\frac{(\text{velocity})^2}{\text{length} \times \text{acceleration}} \doteq \frac{(LT^{-1})^2}{(L)(LT^{-2})} \doteq L^0T^0$$

$$\frac{\text{force} \times \text{time}}{\text{momentum}} \doteq \frac{(F)(T)}{(MLT^{-1})} \doteq \frac{(F)(T)}{(FT^2L^{-1})(LT^{-1})} \doteq F^0L^0T^0$$

$$\frac{\text{density} \times \text{velocity} \times \text{length}}{\text{dynamic viscosity}} \doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq M^0L^0T^0$$

1.8

1.8 The force, P, that is exerted on a spherical particle moving slowly through a liquid is given by the equation

$$P = 3\pi\mu DV$$

where μ is a fluid property (viscosity) having dimensions of $FL^{-2}T$, D is the particle diameter, and V is the particle velocity. What are the dimensions of the constant, 3π? Would you classify this equation as a general homogeneous equation?

$$P = 3\pi\mu D V$$

$$[F] \doteq [3\pi][FL^{-2}T][L][LT^{-1}]$$

$$[F] \doteq [3\pi][F]$$

$\therefore$ 3π is dimensionless, and the equation is a general homogeneous equation. Yes.

1.9

According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq \left[0.04 \text{ to } 0.09\right]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.10

1.10 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity ($FL^{-2}T$), ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$[FL^{-2}] \doteq [K_v]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + [K_u]\left[\frac{(L^2)}{(L^2)} - 1\right]^2 \left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$[FL^{-2}] \doteq [K_v][FL^{-2}] + [K_u][FL^{-2}]$$

Since each term must have the same dimensions, K_v and K_u are dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. Yes.

1.11

1.11 Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a(\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See Eq. 1.19.)

$$c = (E_v)^a (\rho)^b$$

Since $c \doteq LT^{-1}$ $\quad E_v \doteq FL^{-2}$ $\quad \rho = FL^{-4}T^2$

$$\left[\frac{L}{T}\right] \doteq \left[\frac{F^a}{L^{-2a}}\right]\left[\frac{F^b T^{2b}}{L^{-4b}}\right] \qquad (1)$$

For a dimensionally homogeneous equation each term in the equation must have the same dimensions. Thus, the right hand side of Eq. (1) must have the dimensions of LT^{-1}. Therefore,

$$a + b = 0 \quad \text{(to eliminate } F\text{)}$$
$$2b = -1 \quad \text{(to satisfy condition on } T\text{)}$$
$$2a + 4b = -1 \quad \text{(to satisfy condition on } L\text{)}$$

It follows that $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

So that

$$c = \sqrt{\frac{E_v}{\rho}}$$

This result is consistent with the standard formula for the speed of sound. Yes.

1.12

1.12 A formula for estimating the volume rate of flow, Q, over the spillway of a dam is

$$Q = C\sqrt{2g}\,B\,(H + V^2/2g)^{3/2}$$

where C is a constant, g the acceleration of gravity, B the spillway width, H the depth of water passing over the spillway, and V the velocity of water just upstream of the dam. Would this equation be valid in any system of units? Explain.

$$Q = C\sqrt{2g}\,B\left(H + \frac{V^2}{2g}\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C][\sqrt{2}][LT^{-2}]^{1/2}[L]\left([L] + \left[\frac{L^2}{2T^2}\cdot\frac{T^2}{L}\right]\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C][\sqrt{2}][L^{3/2}T^{-1}]\left([L] + \left[\frac{L}{2}\right]\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C\sqrt{2}][L^3T^{-1}]$$

Since each term in the equation must have the same dimensions the constant $C\sqrt{2}$ must be dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent set of units. Yes.

1.14

1.14 Make use of Table 1.3 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s², **(e)** 0.0234 lb·s/ft².

(a) $$10.2\ \frac{in.}{min} = \left(10.2\ \frac{in.}{min}\right)\left(2.540\times10^{-2}\ \frac{m}{in.}\right)\left(\frac{1\ min}{60\ s}\right)$$

$$= 4.32\times10^{-3}\ \frac{m}{s} = \underline{\underline{4.32\ \frac{mm}{s}}}$$

(b) $$4.81\ slugs = \left(4.81\ slugs\right)\left(1.459\times10\ \frac{kg}{slug}\right) = \underline{\underline{70.2\ kg}}$$

(c) $$3.02\ lb = \left(3.02\ lb\right)\left(4.448\ \frac{N}{lb}\right) = \underline{\underline{13.4\ N}}$$

(d) $$73.1\ \frac{ft}{s^2} = \left(73.1\ \frac{ft}{s^2}\right)\left(3.048\times10^{-1}\ \frac{\frac{m}{s^2}}{\frac{ft}{s^2}}\right) = \underline{\underline{22.3\ \frac{m}{s^2}}}$$

(e) $$0.0234\ \frac{lb\cdot s}{ft^2} = \left(0.0234\ \frac{lb\cdot s}{ft^2}\right)\left(4.788\times10\ \frac{\frac{N\cdot s}{m^2}}{\frac{lb\cdot s}{ft^2}}\right)$$

$$= \underline{\underline{1.12\ \frac{N\cdot s}{m^2}}}$$

1.15

1.15 Make use of Table 1.4 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, **(c)** 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) $14.2\ km = (14.2 \times 10^3\ m)\left(3.281\ \frac{ft}{m}\right) = \underline{\underline{4.66 \times 10^4\ ft}}$

(b) $8.14\ \frac{N}{m^3} = \left(8.14\ \frac{N}{m^3}\right)\left(6.366 \times 10^{-3}\ \frac{\frac{lb}{ft^3}}{\frac{N}{m^3}}\right) = \underline{\underline{5.18 \times 10^{-2}\ \frac{lb}{ft^3}}}$

(c) $1.61\ \frac{kg}{m^3} = \left(1.61\ \frac{kg}{m^3}\right)\left(1.940 \times 10^{-3}\ \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}}\right) = \underline{\underline{3.12 \times 10^{-3}\ \frac{slugs}{ft^3}}}$

(d) $0.0320\ \frac{N \cdot m}{s} = \left(0.0320\ \frac{N \cdot m}{s}\right)\left(7.376 \times 10^{-1}\ \frac{\frac{ft \cdot lb}{s}}{\frac{N \cdot m}{s}}\right)$

$= \underline{\underline{2.36 \times 10^{-2}\ \frac{ft \cdot lb}{s}}}$

(e) $5.67\ \frac{mm}{hr} = \left(5.67 \times 10^{-3}\ \frac{m}{hr}\right)\left(3.281\ \frac{ft}{m}\right)\left(\frac{1\ hr}{3600\ s}\right)$

$= \underline{\underline{5.17 \times 10^{-6}\ \frac{ft}{s}}}$

1.16

1.16 Make use of Appendix A to express the following quantities in SI units: **(a)** 160 acre, **(b)** 742 Btu, **(c)** 240 miles, **(d)** 79.1 hp, **(e)** 60.3 °F.

(a) $160 \text{ acre} = (160 \text{ acre})\left(4.356\times10^{4}\ \frac{ft^2}{acre}\right)\left(9.290\times10^{-2}\ \frac{m^2}{ft^2}\right)$

$= \underline{\underline{6.47\times10^{5}\ m^{2}}}$

(b) $742 \text{ BTU} = (742 \text{ BTU})\left(1.055\times10^{3}\ \frac{J}{BTU}\right) = \underline{\underline{7.83\times10^{5}\ J}}$

(c) $240 \text{ mi} = (240 \text{ mi})\left(1.609\times10^{3}\ \frac{m}{mi}\right) = \underline{\underline{3.86\times10^{5}\ m}}$

(d) $79.1 \text{ hp} = (79.1 \text{ hp})\left(7.457\times10^{2}\ \frac{W}{hp}\right) = \underline{\underline{5.90\times10^{4}\ W}}$

(e) $T_c = \frac{5}{9}(60.3-32) = 15.7\ °C$

$K = 15.7\ °C + 273 = \underline{\underline{289\ K}}$

1.17

1.17 Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter (g/m^3). Assume that a cumulus cloud occupies a volume of one cubic kilometer, and its liquid water content is 0.2 g/m^3. (a) What is the volume of this cloud in cubic miles? (b) How much does the water in the cloud weigh in pounds?

(a) $\mathcal{V}\text{olume} = 1\,(km)^3 = 10^9\,m^3$

Since $1\,m = 3.281\,ft$

$$\mathcal{V}\text{olume} = \frac{(10^9\,m^3)\left(3.281\,\frac{ft}{m}\right)^3}{\left(5.280\times10^3\,\frac{ft}{mi}\right)^3}$$

$$= \underline{\underline{0.240\,mi^3}}$$

(b) $\mathcal{W} = \gamma \times \mathcal{V}\text{olume}$

$$\gamma = \rho g = \left(0.2\,\frac{g}{m^3}\right)\left(10^{-3}\,\frac{kg}{g}\right)\left(9.81\,\frac{m}{s^2}\right) = 1.962\times10^{-3}\,\frac{N}{m^3}$$

$$\mathcal{W} = \left(1.962\times10^{-3}\,\frac{N}{m^3}\right)\left(10^9\,m^3\right) = 1.962\times10^6\,N$$

$$= \left(1.962\times10^6\,N\right)\left(2.248\times10^{-1}\,\frac{lb}{N}\right) = \underline{\underline{4.41\times10^5\,lb}}$$

1.18

1.18 For Table 1.3 verify the conversion relationships for: **(a)** area, **(b)** density, **(c)** velocity, and **(d)** specific weight. Use the basic conversion relationships: 1 ft = 0.3048 m; 1 lb = 4.4482 N; and 1 slug = 14.594 kg.

(a) $$1\ ft^2 = (1\ ft^2)\left[(0.3048)^2 \frac{m^2}{ft^2}\right] = 0.09290\ m^2$$

Thus, multiply ft^2 by 9.290 E-2 to convert to m^2.

(b) $$1\ \frac{slug}{ft^3} = \left(1\ \frac{slug}{ft^3}\right)\left(14.594\ \frac{kg}{slug}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 515.4\ \frac{kg}{m^3}$$

Thus, multiply $slugs/ft^3$ by 5.154 E+2 to convert to kg/m^3.

(c) $$1\ \frac{ft}{s} = \left(1\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 0.3048\ \frac{m}{s}$$

Thus, multiply ft/s by 3.048 E-1 to convert to m/s.

(d) $$1\ \frac{lb}{ft^3} = \left(1\ \frac{lb}{ft^3}\right)\left(4.4482\ \frac{N}{lb}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 157.1\ \frac{N}{m^3}$$

Thus, multiply lb/ft^3 by 1.571 E+2 to convert to N/m^3.

1.19

1.19 For Table 1.4 verify the conversion relationships for: **(a)** acceleration, **(b)** density, **(c)** pressure, and **(d)** volume flowrate. Use the basic conversion relationships: 1 m = 3.2808 ft; 1 N = 0.22481 lb; and 1 kg = 0.068521 slug.

(a) $$1\ \frac{m}{s^2} = \left(1\ \frac{m}{s^2}\right)\left(3.2808\ \frac{ft}{m}\right) = 3.281\ \frac{ft}{s^2}$$

Thus, multiply m/s^2 by $\underline{\underline{3.281}}$ to convert to ft/s^2.

(b) $$1\ \frac{kg}{m^3} = \left(1\ \frac{kg}{m^3}\right)\left(0.068521\ \frac{slugs}{kg}\right)\left[\frac{1\ m^3}{(3.2808)^3\ ft^3}\right]$$

$$= 1.940\times10^{-3}\ \frac{slugs}{ft^3}$$

Thus, multiply kg/m^3 by $\underline{\underline{1.940\ E-3}}$ to convert to $slugs/ft^3$.

(c) $$1\ \frac{N}{m^2} = \left(1\ \frac{N}{m^2}\right)\left(0.22481\ \frac{lb}{N}\right)\left[\frac{1\ m^2}{(3.2808)^2\ ft^2}\right]$$

$$= 2.089\times10^{-2}\ \frac{lb}{ft^2}$$

Thus, multiply N/m^2 by $\underline{\underline{2.089\ E-2}}$ to convert to lb/ft^2.

(d) $$1\ \frac{m^3}{s} = \left(1\ \frac{m^3}{s}\right)\left[(3.2808)^3\ \frac{ft^3}{m^3}\right] = 35.31\ \frac{ft^3}{s}$$

Thus, multiply m^3/s by $\underline{\underline{3.531\ E+1}}$ to convert to ft^3/s.

1.20

1.20 Water flows from a large drainage pipe at a rate of 1200 gal/min. What is this volume rate of flow in (a) m^3/s, (b) liters/min, and (c) ft^3/s?

(a)

$$\text{flowrate} = \left(1200\ \frac{gal}{min}\right)\left(6.309\times10^{-5}\ \frac{\frac{m^3}{s}}{\frac{gal}{min}}\right)$$

$$= \underline{\underline{7.57\times10^{-2}\ \frac{m^3}{s}}}$$

(b) Since $1\ \text{liter} = 10^{-3}\ m^3$,

$$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(\frac{10^3\ \text{liters}}{m^3}\right)\left(\frac{60\,s}{min}\right)$$

$$= \underline{\underline{4540\ \frac{\text{liters}}{min}}}$$

(c)

$$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(3.531\times10\ \frac{\frac{ft^3}{s}}{\frac{m^3}{s}}\right)$$

$$= \underline{\underline{2.67\ \frac{ft^3}{s}}}$$

1.21

1.21 A tank of oil has a mass of 30 slugs. **(a)** Determine its weight in pounds and in newtons at the earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface where the gravitational attraction is approximately one-sixth that at the earth's surface?

(a) $\text{weight} = \text{mass} \times g$

$$= (30\ \text{slugs})\left(32.2\ \frac{ft}{s^2}\right) = \underline{\underline{966\ lb}}$$

$$= (30\ \text{slugs})\left(14.59\ \frac{kg}{slug}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{4290\ N}}$$

(b) $\text{mass} = \underline{\underline{30\ \text{slugs}}}$ (mass does not depend on gravitational attraction)

$$\text{weight} = (30\ \text{slugs})\left(\frac{32.2\ \frac{ft}{s^2}}{6}\right) = \underline{\underline{161\ lb}}$$

1.22

1.22 A certain object weighs 300 N at the earth's surface. Determine the mass of the object (in kilograms) and its weight (in newtons) when located on a planet with an acceleration of gravity equal to 4.0 ft/s^2.

$$\text{mass} = \frac{\text{weight}}{g}$$

$$= \frac{300\ N}{9.81\ \frac{m}{s^2}} = \underline{\underline{30.6\ kg}}$$

For $g = 4.0\ ft/s^2$,

$$\text{weight} = (30.6\ kg)\left(4.0\ \frac{ft}{s^2}\right)\left(0.3048\ \frac{m}{ft}\right)$$

$$= \underline{\underline{37.3\ N}}$$

1.23

1.23 An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is a velocity, g the acceleration of gravity, and ℓ a length. Determine the value of the Froude number for $V = 10$ ft/s, $g = 32.2$ ft/s², and $\ell = 2$ ft. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10\ \frac{ft}{s}}{\sqrt{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft)}} = \underline{\underline{1.25}}$$

In SI units:

$$V = \left(10\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 3.05\ \frac{m}{s}$$

$$g = 9.81\ \frac{m}{s^2}$$

$$\ell = (2\ ft)\left(0.3048\ \frac{m}{ft}\right) = 0.610\ m$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.610\ m)}} = \underline{\underline{1.25}}$$

The value of a dimensionless parameter is independent of the unit system.

1.24

1.24 The specific weight of a certain liquid is 85.3 lb/ft³. Determine its density and specific gravity.

$$\rho = \frac{\gamma}{g} = \frac{85.3\ \frac{lb}{ft^3}}{32.2\ \frac{ft}{s^2}} = \underline{\underline{2.65\ \frac{slugs}{ft^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O}@\,4^\circ C} = \frac{2.65\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}} = \underline{\underline{1.37}}$$

1.25

1.25 A *hydrometer* is used to measure the specific gravity of liquids. (See **Video V2.6.**) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

$$SG = \frac{\rho}{\rho_{H_2O}@\,4^\circ C}$$

$$1.15 = \frac{\rho}{1000\ \frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000\ \frac{kg}{m^3}\right) = \underline{\underline{1150\ \frac{kg}{m^3}}}$$

$$\gamma = \rho g = \left(1150\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{11.3\ \frac{kN}{m^3}}}$$

1.26

1.26 An open, rigid-walled, cylindrical tank contains 4 ft^3 of water at 40 °F. Over a 24-hour period of time the water temperature varies from 40 °F to 90 °F. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

mass of water $= \forall \times \rho$

where $\forall$ is the volume and ρ the density. Since the mass must remain constant as the temperature changes

$$\forall_{40^\circ} \times \rho_{40^\circ} = \forall_{90^\circ} \times \rho_{90^\circ} \qquad (1)$$

From Table B.1 $\rho_{H_2O\,@\,40°F} = 1.940 \ \frac{slugs}{ft^3}$

$\rho_{H_2O\,@\,90°F} = 1.931 \ \frac{slugs}{ft^3}$

Therefore, from Eq. (1)

$$\forall_{90^\circ} = \frac{(4\ ft^3)\left(1.940\ \frac{slugs}{ft^3}\right)}{1.931\ \frac{slugs}{ft^3}} = 4.0186\ ft^3$$

Thus, the increase in volume is

$$4.0186 - 4.000 = \underline{\underline{0.0186\ ft^3}}$$

The change in water depth, $\Delta\ell$, is equal to

$$\Delta\ell = \frac{\Delta\forall}{area} = \frac{0.0186\ ft^3}{\frac{\pi}{4}(2ft)^2} = 5.92\times10^{-3}\ ft = 0.0710\ in.$$

This small change in depth would not be very noticeable. $\underline{\underline{No}}$.

Note: A slightly different value for $\Delta\ell$ will be obtained if specific weight of water is used rather than density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.

1.28

1.28 A liquid when poured into a graduated cylinder is found to weigh 8 N when occupying a volume of 500 ml (milliliters). Determine its specific weight, density, and specific gravity.

$$\gamma = \frac{\text{weight}}{\text{volume}} = \frac{8\ N}{(0.500\ \ell)\left(10^{-3}\ \frac{m^3}{\ell}\right)} = \underline{\underline{16.0\ \frac{kN}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{16 \times 10^3\ \frac{N}{m^3}}{9.81\ \frac{m}{s^2}} = \underline{\underline{1.63 \times 10^3\ \frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O}\ @\ 4°C} = \frac{1.63 \times 10^3\ \frac{kg}{m^3}}{10^3\ \frac{kg}{m^3}} = \underline{\underline{1.63}}$$

1.29

1.29 The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}} \qquad (1)$$

total weight = mass x g = $(0.369\ kg)(9.81\ \frac{m}{s^2}) = 3.62\ N$

weight of can = $0.153\ N$

volume of fluid = $(355 \times 10^{-3}\ L)(10^{-3}\ \frac{m^3}{L}) = 355 \times 10^{-6}\ m^3$

Thus, from Eq. (1)

$$\gamma = \frac{3.62\ N - 0.153\ N}{355 \times 10^{-6}\ m^3} = \underline{\underline{9770\ \frac{N}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\ \frac{N}{m^3}}{9.81\ \frac{m}{s^2}} = 996\ \frac{N \cdot s^2}{m^4} = \underline{\underline{996\ \frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O@4°C}} = \frac{996\ \frac{kg}{m^3}}{1000\ \frac{kg}{m^3}} = \underline{\underline{0.996}}$$

For water at 20°C (see Table B.2 in Appendix B)

$$\gamma_{H_2O} = 9789\ \frac{N}{m^3}\ ;\ \rho_{H_2O} = 998.2\ \frac{kg}{m^3}\ ;\ SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

1.30*

1.30* The variation in the density of water, ρ, with temperature, T, in the range 20 °C $\leq T \leq$ 60 °C, is given in the following table.

Density (kg/m^3)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2T + c_3T^2$ which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

To solve this problem use POLREG.

```
****************************************************
** This program determines the least squares fit **
** for any order polynomial of the form:         **
**       y = d0 + d1*x + d2*x^2 + d3*x^3 + ...   **
****************************************************

Enter number of terms in the polynomial: 3
Enter number of data points: 7


Enter data points (X , Y)
? 20,998.2
? 25,997.1
? 30,995.7
? 35,994.1
? 40,992.2
? 45,990.2
? 50,988.1

The coefficients of the polynomial are:
d2 = -4.0953E-03
d1 = -5.3332E-02
d0 = +1.0009E+03

     X              Y            Y(predicted)
+2.0000E+01   +9.9820E+02   +9.9825E+02
+2.5000E+01   +9.9710E+02   +9.9706E+02
+3.0000E+01   +9.9570E+02   +9.9566E+02
+3.5000E+01   +9.9410E+02   +9.9407E+02
+4.0000E+01   +9.9220E+02   +9.9226E+02
+4.5000E+01   +9.9020E+02   +9.9026E+02
+5.0000E+01   +9.8810E+02   +9.8805E+02
```

Thus,

$$\underline{\underline{\rho = 1001 - 0.05333\,T - 0.004095\,T^2}}$$

Note that ρ (predicted) is in good agreement with ρ (given).

At $T = 42.1$ °C,

$$\rho = 1001 - 0.05333\,(42.1\,°C) - 0.004095\,(42.1\,°C)^2 = \underline{\underline{991.5\ \frac{kg}{m^3}}}$$

1.32

1.32 The density of oxygen contained in a tank is 2.0 kg/m^3 when the temperature is 25 °C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

$$p = \rho R T = \left(2.0\ \frac{kg}{m^3}\right)\left(259.8\ \frac{J}{kg\cdot K}\right)\left[(25°C + 273)K\right]$$

$$= 155\ kPa\ (abs)$$

$$p\ (gage) = p_{abs} - p_{atm} = 155\ kPa - 97\ kPa = \underline{\underline{58\ kPa}}$$

1.33

1.33 Some experiments are being conducted in a laboratory in which the air temperature is 27 °C, and the atmospheric pressure is 14.3 psia. Determine the density of the air. Express your answers in slugs/ft^3 and in kg/m^3.

$$p = \rho R T$$

$$\text{Temperature} = 27°C = [1.8(27) + 32°]\ °F = 80.6\ °F$$

$$\rho = \frac{p}{RT} = \frac{\left(14.3\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1716\ \frac{ft\cdot lb}{slug\cdot °R}\right)\left[(80.6°F + 460)°R\right]}$$

$$= \underline{\underline{0.00222\ \frac{slugs}{ft^3}}}$$

$$\rho = \left(0.00222\ \frac{slugs}{ft^3}\right)\left(5.154 \times 10^2\ \frac{\frac{kg}{m^3}}{\frac{slugs}{ft^3}}\right) = \underline{\underline{1.14\ \frac{kg}{m^3}}}$$

1.34

1.34 A closed tank having a volume of 2 ft³ is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Density of gas in tank $\rho = \frac{\text{weight}}{g \times \text{volume}} = \frac{0.30 \text{ lb}}{(32.2 \frac{\text{ft}}{\text{s}^2})(2 \text{ ft}^3)}$

$$= 4.66 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}$$

Since $\rho = \frac{p}{RT}$ with $p = (12 + 14.7)$ psia

(atmospheric pressure assumed to be $\approx$ 14.7 psia)

and with $T = (80°F + 460)°R$ it follows that

$$\rho = \frac{(26.7 \frac{\text{lb}}{\text{in.}^2})(144 \frac{\text{in.}^2}{\text{ft}^2})}{R\,(540°R)} = \frac{7.12}{R} \frac{\text{slugs}}{\text{ft}^3} \qquad (1)$$

From Table 1.7 $R = 1.554 \times 10^3$ for oxygen

and $R = 1.242 \times 10^4 \frac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °R}$ for helium.

Thus, from Eq. (1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3} \frac{\text{slugs}}{\text{ft}^3} = 4.58 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4} = 5.73 \times 10^{-4} \frac{\text{slugs}}{\text{ft}^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be <u>oxygen</u>.

1.36

1.36 A tire having a volume of 3 ft³ contains air at a gage pressure of 26 psi and a temperature of 70 °F. Determine the density of the air and the weight of the air contained in the tire.

$$\rho = \frac{p}{RT} = \frac{\left(26\,\frac{lb}{in.^2} + 14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\left[(70^\circ F + 460)^\circ R\right]} = \underline{\underline{6.44 \times 10^{-3}\,\frac{slugs}{ft^3}}}$$

$$\text{weight} = \rho g \times \text{volume} = \left(6.44 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)\left(3\,ft^3\right)$$

$$= \underline{\underline{0.622\ lb}}$$

1.37

1.37 A rigid tank contains air at a pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

$$p = \rho R T \qquad \text{(Eq. 1.8)}$$

For a rigid closed tank the air mass and volume are constant so ρ = constant. Thus, from Eq. 1.8 (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1)$$

where $p_1 = 90\ \text{psia}$, $T_1 = 60°F + 460 = 520°R$, and $T_2 = 110°F + 460 = 570°R$. From Eq. (1)

$$p_2 = \frac{T_2}{T_1}\, p_1 = \left(\frac{570°R}{520°R}\right)(90\ \text{psia}) = \underline{\underline{98.7\ \text{psia}}}$$

1.38*

*1.38 Develop a computer program for calculating the density of an ideal gas when the gas pressure in pascals (abs), the temperature in degrees Celsius, and the gas constant in J/kg · K are specified.

For an ideal gas

$$p = \rho R T$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, R the gas constant, and T is absolute temperature. Thus, if the temperature is in °C then

$$T = °C + 273.15$$

A spreadsheet (EXCEL) program for calculating ρ follows.

This program calculates the density of an ideal gas
when the absolute pressure in Pascals, the temperature
in degrees C, and the gas constant in J/kg·K are specified.
To use, replace current values with desired values of
temperature, pressure, and gas constant.

A	B	C	D	
Pressure,	Temperature,	Gas constant,	Density,	
Pa	°C	J/kg·K	kg/m^3	
1.01E+05	15	286.9	1.23	Row 10

Formula:
=A10/((B10+273.15)*C10)

Example: Calculate ρ for $p = 200$ kPa, temperature = 20°C, and $R = 287$ J/kg · K.

A	B	C	D	
Pressure,	Temperature,	Gas constant,	Density,	
Pa	°C	J/kg·K	kg/m^3	
2.00E+05	20	287	2.38	Row 10

1.39*

*1.39 Repeat Problem 1.38 for the case in which the pressure is given in psi (gage), the temperature in degrees Fahrenheit, and the gas constant in ft·lb/slug·°R.

For an ideal gas

$$p = \rho R T$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, and T is absolute temperature. Thus, if temperature in °F, and pressure in psi, then

$$T = {}^\circ F + 459.67 \quad \text{and} \quad p = \left[p(\text{psi}) + p_{atm}(\text{psia}) \right] \times 144 \frac{\text{in.}^2}{\text{ft}^2}$$

A spreadsheet (EXCEL) program for calculating ρ follows.

This program calculates the density of an ideal gas						
when the gage pressure in psi, the atmospheric						
pressure in psia, the temperature in degrees F, and						
the gas constant in ft·lb/slug·deg R are specified.						
To use, replace current values with desired values of						
gage pressure, atmospheric pressure, temperature,						
and gas constant.						
A	B	C	D	E		
Pressure,	Temperature,	Gas constant,	Atm. Pressure,	Density,		
psi	°F	ft lb/slug·°F	psia	slugs/ft³		
0	59	1716	14.7	0.00238	Row 12	

Formula:
=((A12+D12)*144)/((C12)*(B12+459.67))

Example: Calculate ρ for $p = 40$ psi, temperature = 100°F, $p_{atm} = 14.7$ psia, and $R = 1716$ ft·lb/slug·°R.

A	B	C	D	E		
Pressure,	Temperature,	Gas constant,	Atm. Pressure,	Density,		
psi	°F	ft lb/slug °F	psia	slugs/ft³		
40	100	1716	14.7	0.00820	Row 12	

1.40

1.40 Make use of the data in Appendix B to determine the dynamic viscosity of mercury at 75 °F. Express your answer in BG units.

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(75\,°F - 32) = 23.9\,°C$$

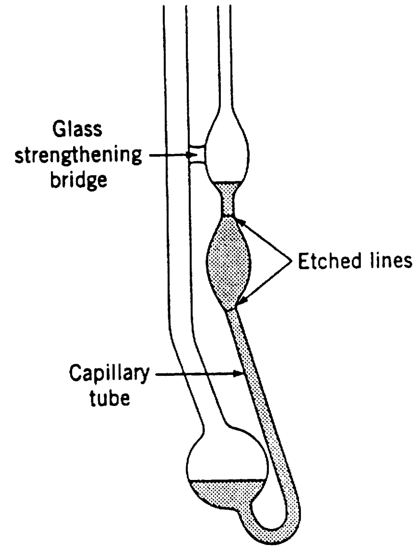
From Fig. B.1 in Appendix B:

$$\mu\,(\text{mercury at } 75\,°F\ (23.9\,°C)) \approx 1.5 \times 10^{-3}\ \frac{N \cdot s}{m^2}$$

$$\mu \approx \left(1.5 \times 10^{-3}\ \frac{N \cdot s}{m^2}\right)\left(2.089 \times 10^{-2}\ \frac{\frac{lb \cdot s}{ft^2}}{\frac{N \cdot s}{m^2}}\right) \approx \underline{\underline{3.1 \times 10^{-5}\ \frac{lb \cdot s}{ft^2}}}$$

1.41

1.41 One type of *capillary-tube viscometer* is shown in **Video V1.3** and in Fig. P1.41. For this device the liquid to be tested is drawn into the tube to a level above the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4t$ where K is a constant, R is the radius of the capillary tube in mm, and t is the drain time in seconds. When glycerin at 20°C is used as a calibration fluid in a particular viscometer the drain time is 1,430 s. When a liquid having a density of 970 kg/m^3 is tested in the same viscometer the drain time is 900 s. What is the dynamic viscosity of this liquid?



■ FIGURE P1.41

$$\nu = KR^4t$$

For glycerin @ 20°C $\nu = 1.19\times10^{-3}\ m^2/s$

$$\therefore\ 1.19\times10^{-3}\ m^2/s = (KR^4)(1{,}430\ s)$$

$$KR^4 = 8.32\times10^{-7}\ m^2/s^2$$

For unknown liquid with $t = 900s$

$$\nu = (8.32\times10^{-7}\ m^2/s^2)(900\ s)$$

$$= 7.49\times10^{-4}\ m^2/s$$

Since $\mu = \rho\nu$

$$= (970\ kg/m^3)(7.49\times10^{-4}\ m^2/s)$$

$$= 0.727\ \frac{kg}{m\cdot s} = \underline{\underline{0.727\ \frac{N\cdot s}{m^2}}}$$

1.42

1.42 The viscosity of a soft drink was determined by using a capillary tube viscometer similar to that shown in Fig. P1.41 and Video V1.3. For this device the kinematic viscosity, ν, is directly proportional to the time, t, that it takes for a given amount of liquid to flow through a small capillary tube. That is, $\nu = Kt$. The following data were obtained from regular pop and diet pop. The corresponding measured specific gravities are also given. Based on these data, by what percent is the absolute viscosity, μ, of regular pop greater than that of diet pop?

	Regular pop	Diet pop
t(s)	377.8	300.3
SG	1.044	1.003

$$\% \text{ greater} = \left[\frac{\mu_{reg} - \mu_{diet}}{\mu_{diet}}\right] \times 100 = \left[\frac{\mu_{reg}}{\mu_{diet}} - 1\right] \times 100$$

Since $\nu = \mu/\rho$, $\nu = kt$, and $\rho = (SG)\rho_{H_2O\,@\,4^\circ C}$
it follows that

$$\% \text{ greater} = \left[\frac{(\nu\rho)_{reg}}{(\nu\rho)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(t \times SG)_{reg}}{(t \times SG)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(377.8\text{ s})(1.044)}{(300.3\text{ s})(1.003)} - 1\right] \times 100$$

$$= \underline{\underline{31.0\%}}$$

1.43 The time, t, it takes to pour a liquid from a container depends on several factors, including the kinematic viscosity, ν, of the liquid. (See Video V1.1.) In some laboratory tests various oils having the same density but different viscosities were poured at a fixed tipping rate from small 150 ml beakers. The time required to pour 100 ml of the oil was measured, and it was found that an approximate equation for the pouring time in seconds was $t = 1 + 9 \times 10^2\nu + 8 \times 10^3\nu^2$ with ν in m²/s. **(a)** Is this a general homogeneous equation? Explain. **(b)** Compare the time it would take to pour 100 ml of SAE 30 oil from a 150 ml beaker at 0 °C to the corresponding time at a temperature of 60 °C. Make use of Fig. B.2 in Appendix B for viscosity data.

(a) $$t = 1 + 9\times10^2\,\nu + 8\times10^3\,\nu^2 \qquad (1)$$

$$[T] \doteq [1] + [9\times10^2]\left[\frac{L^2}{T}\right] + [8\times10^3]\left[\frac{L^4}{T^2}\right]$$

Since each term in the equation must have the same dimensions the constants appearing in the equation must have dimensions, i.e.,

$$[1] \doteq [T] \qquad [9\times10^2] \doteq \left[\frac{T^2}{L^2}\right] \qquad [8\times10^3] \doteq \left[\frac{T^3}{L^4}\right]$$

Thus, with a change in units the value of the constants would change and this is not a general homogeneous equation. No.

(b) From Table B.2 in Appendix B:

(for SAE30 oil @ 0°C) $\nu = 2.3\times10^{-3}\ m^2/s$

(for SAE30 oil @ 60°C) $\nu = 4.0\times10^{-5}\ m^2/s$

Thus, from Eq.(1)

@ 0°C $$t = 1 + 9\times10^2(2.3\times10^{-3}) + 8\times10^3(2.3\times10^{-3})^2 = \underline{\underline{3.11\ s}}$$

@ 60°C $$t = 1 + 9\times10^2(4.0\times10^{-5}) + 8\times10^3(4.0\times10^{-5})^2 = \underline{\underline{1.04\ s}}$$

1.44

1.44 The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI and BG units.

From Appendix A $10^{-1} \frac{N \cdot s}{m^2} = 1$ poise. Thus,

$$\mu = (5 \times 10^{-4} \text{ poise})\left(10^{-1} \frac{\frac{N \cdot s}{m^2}}{\text{poise}}\right) = \underline{\underline{5 \times 10^{-5} \frac{N \cdot s}{m^2}}}$$

and From Table 1.4

$$\mu = \left(5 \times 10^{-5} \frac{N \cdot s}{m^2}\right)\left(2.089 \times 10^{-2} \frac{\frac{lb \cdot s}{ft^2}}{\frac{N \cdot s}{m^2}}\right) = \underline{\underline{10.4 \times 10^{-7} \frac{lb \cdot s}{ft^2}}}$$

1.45

1.45 The kinematic viscosity of oxygen at 20 °C and a pressure of 150 kPa (abs) is 0.104 stokes. Determine the dynamic viscosity of oxygen at this temperature and pressure.

$$\mu = \nu \rho$$

$$\rho = \frac{p}{RT} = \frac{150 \times 10^3 \frac{N}{m^2}}{\left(259.8 \frac{J}{kg \cdot K}\right)\left[(20°C + 273)K\right]} = 1.97 \frac{kg}{m^3}$$

$$\nu = 0.104 \text{ stokes} = 0.104 \frac{cm^2}{s}$$

$$\mu = \left(0.104 \frac{cm^2}{s}\right)\left(10^{-4} \frac{m^2}{cm^2}\right)\left(1.97 \frac{kg}{m^3}\right)$$

$$= 2.05 \times 10^{-5} \frac{kg}{m \cdot s} = \underline{\underline{2.05 \times 10^{-5} \frac{N \cdot s}{m^2}}}$$

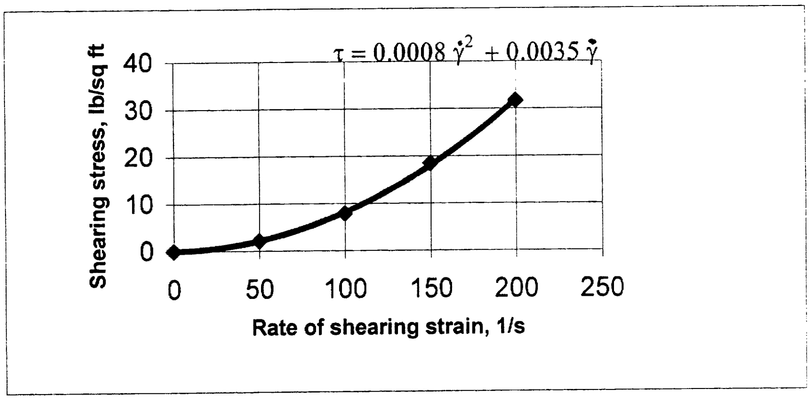
1.46*

*1.46 Fluids for which the shearing stress, τ, is not linearly related to the rate of shearing strain, $\dot{\gamma}$, are designated as non-Newtonian fluids. Such fluids are commonplace and can exhibit unusual behavior as shown in Video V1.4. Some experimental data obtained for a particular non-Newtonian fluid at 80 °F are shown below.

τ (lb/ft^2)	0	2.11	7.82	18.5	31.7
$\dot{\gamma}$ (s^{-1})	0	50	100	150	200

Plot these data and fit a second-order polynomial to the data using a suitable graphing program. What is the apparent viscosity of this fluid when the rate of shearing strain is 70 s^{-1}? Is this apparent viscosity larger or smaller than that for water at the same temperature?

Rate of shearing strain, 1/s	Shearing stress, lb/sq ft
0	0
50	2.11
100	7.82
150	18.5
200	31.7



From the graph $\tau = 0.0008\,\dot{\gamma}^2 + 0.0035\,\dot{\gamma}$ where τ is the shearing stress in lb/ft^2 and $\dot{\gamma}$ is the rate of shearing strain in s^{-1}.

$$\mu_{apparent} = \frac{d\tau}{d\dot{\gamma}} = (2)(0.0008)\dot{\gamma} + 0.0035$$

At $\dot{\gamma} = 70\ s^{-1}$

$$\mu_{apparent} = (2)\left(0.0008\ \frac{lb\cdot s^2}{ft^2}\right)\left(70\ s^{-1}\right) + 0.0035\ \frac{lb\cdot s}{ft^2}$$

$$= \underline{\underline{0.116\ \frac{lb\cdot s}{ft^2}}}$$

From Table B.1 in Appendix B, $\mu_{H_2O\,@\,80°F} = 1.791 \times 10^{-5}\ \frac{lb\cdot s}{ft^2}$, and since water is a Newtonian fluid this value is independent of $\dot{\gamma}$. Thus, the unknown non-Newtonian fluid has a much <u>larger</u> value.

1.47

1.47 Water flows near a flat surface and some measurements of the water velocity, u, parallel to the surface, at different heights, y, above the surface are obtained. At the surface $y = 0$. After an analysis of the data, the lab technician reports that the velocity distribution in the range $0 < y < 0.1$ ft is given by the equation

$$u = 0.81 + 9.2y + 4.1 \times 10^3 y^3$$

with u in ft/s when y is in ft. (**a**) Do you think that this equation would be valid in any system of units? Explain. (**b**) Do you think this equation is correct? Explain. You may want to look at Video 1.2 to help you arrive at your answer.

(a) $$u = 0.81 + 9.2\,y + 4.1 \times 10^3\, y^3$$

$$[LT^{-1}] \doteq [0.81] + [9.2][L] + [4.1 \times 10^3][L^3]$$

Each term in the equation must have the same dimensions. Thus, the constant 0.81 must have dimensions of LT^{-1}, 9.2 dimensions of T^{-1}, and 4.1×10^3 dimensions of $L^{-2}T^{-1}$. Since the constants in the equation have dimensions their values will change with a change in units. No.

(b) Equation cannot be correct since at $y=0$ $u = 0.81$ ft/s, a non-zero value which would violate the "no-slip" condition. Not correct.

1.48

1.48 Calculate the Reynolds numbers for the flow of water and for air through a 4-mm-diameter tube, if the mean velocity is 3 m/s and the temperature is 30 °C in both cases (see Example 1.4). Assume the air is at standard atmospheric pressure.

For water at 30°C (from Table B.2 in Appendix B):

$$\rho = 995.7 \ \frac{kg}{m^3} \qquad \mu = 7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(995.7 \ \frac{kg}{m^3})(3 \ \frac{m}{s})(0.004 \ m)}{7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}} = \underline{\underline{15{,}000}}$$

For air at 30°C (from Table B.4 in Appendix B):

$$\rho = 1.165 \ \frac{kg}{m^3} \qquad \mu = 1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(1.165 \ \frac{kg}{m^3})(3 \ \frac{m}{s})(0.004 \ m)}{1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}} = \underline{\underline{752}}$$

1.49

1.49 For air at standard atmospheric pressure the values of the constants that appear in the Sutherland equation (Eq. 1.10) are $C = 1.458 \times 10^{-6}$ kg/(m·s·$K^{1/2}$) and $S = 110.4$ K. Use these values to predict the viscosity of air at 10 °C and 90 °C and compare with values given in Table B.4 in Appendix B.

$$\mu = \frac{C T^{\frac{3}{2}}}{T + S} = \frac{\left(1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}}\right) T^{\frac{3}{2}}}{T + 110.4\ K}$$

For $T = 10°C = 10°C + 273.15 = 283.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(283.15\ K)^{3/2}}{283.15\ K + 110.4} = \underline{\underline{1.765 \times 10^{-5} \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 1.76 \times 10^{-5} \frac{N \cdot s}{m^2}$

For $T = 90°C = 90°C + 273.15 = 363.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(363.15\ K)^{3/2}}{363.15\ K + 110.4} = \underline{\underline{2.13 \times 10^{-5} \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 2.14 \times 10^{-5} \frac{N \cdot s}{m^2}$

1.50*

1.50* Use the values of viscosity of air given in Table B.4 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants C and S which appear in the Sutherland equation (Eq. 1.10). Compare your results with the values given in Problem 1.49. (*Hint*: Rewrite the equation in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C}$$

and plot $T^{3/2}/\mu$ versus T. From the slope and intercept of this curve C and S can be obtained.)

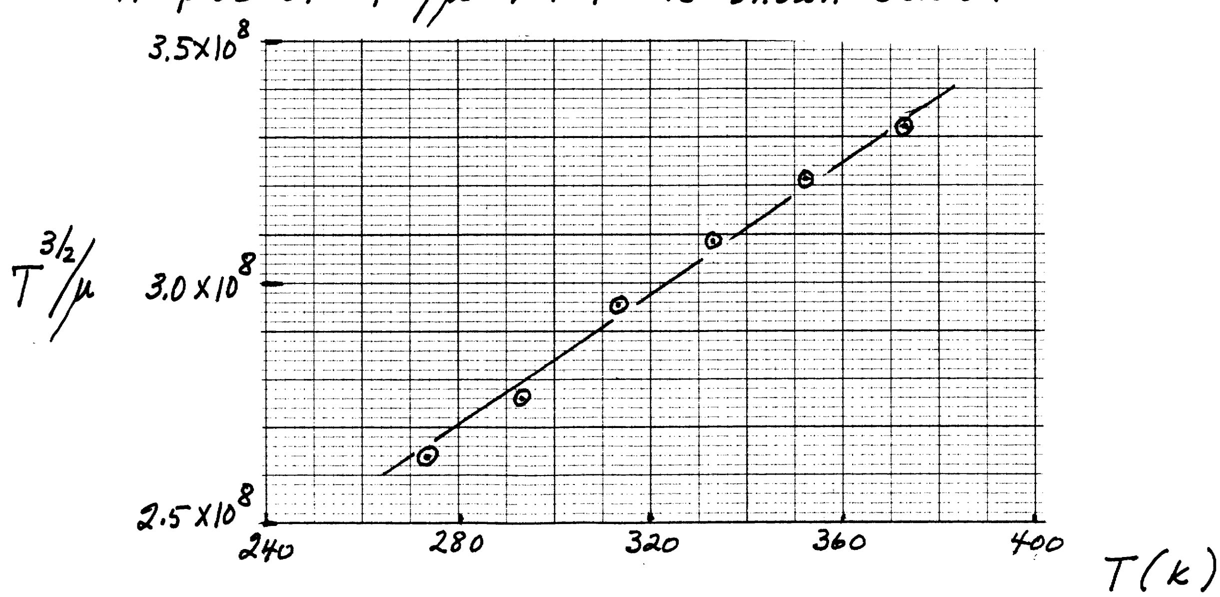
Equation 1.10 can be written in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C} \qquad (1)$$

and with the data from Table B.4:

$T(°C)$	$T(K)$	$\mu\ (N \cdot s/m^2)$	$T^{3/2}/\mu\ [K^{3/2}/(kg/m \cdot s)]$
0	273.15	1.71×10^{-5}	2.640×10^{8}
20	293.15	1.82×10^{-5}	2.758×10^{8}
40	313.15	1.87×10^{-5}	2.963×10^{8}
60	333.15	1.97×10^{-5}	3.087×10^{8}
80	353.15	2.07×10^{-5}	3.206×10^{8}
100	373.15	2.17×10^{-5}	3.322×10^{8}

A plot of $T^{3/2}/\mu$ vs. T is shown below:



(con't)

1.50* (con't)

Since the data plot as an approximate straight line, Eq. (1) can be represented by an equation of the form

$$y = bx + a$$

where $y \sim T^{3/2}/\mu$, $x \sim T$, $b \sim 1/C$, and $a \sim S/C$.

To obtain a and b use LINREG2.

```
*************************************************
** This program determines the least squares fit **
** for a function of the form  y = a + b * x     **
*************************************************

Number of points: 6
Input X, Y
? 273.15,2.640E8
? 293.15,2.758E8
? 313.15,2.963E8
? 333.15,3.087E8
? 353.15,3.206E8
? 373.15,3.322E8

a = +7.441E+07
b = +6.969E+05

      X              Y           Y(predicted)
+2.7315E+02   +2.6400E+08   +2.6476E+08
+2.9315E+02   +2.7580E+08   +2.7869E+08
+3.1315E+02   +2.9630E+08   +2.9263E+08
+3.3315E+02   +3.0870E+08   +3.0657E+08
+3.5315E+02   +3.2060E+08   +3.2051E+08
+3.7315E+02   +3.3220E+08   +3.3444E+08
```

Thus,

$$\frac{1}{C} = b = 6.969 \times 10^{5}$$

so that

$$\underline{\underline{C = 1.43 \times 10^{-6}\ kg/(m \cdot s \cdot K^{1/2})}}$$

and

$$\frac{S}{C} = a = 7.441 \times 10^{7}$$

and therefore

$$\underline{\underline{S = 107\ K}}$$

These values for C and S are in good agreement with values given in Problem 1.49.

1.51

1.51 The viscosity of a fluid plays a very important role in determining how a fluid flows. (See Video V1.1.) The value of the viscosity depends not only on the specific fluid but also on the fluid temperature. Some experiments show that when a liquid, under the action of a constant driving pressure, is forced with a low velocity, V, through a small horizontal tube, the velocity is given by the equation $V = K/\mu$. In this equation K is a constant for a given tube and pressure, and μ is the dynamic viscosity. For a particular liquid of interest, the viscosity is given by Andrade's equation (Eq. 1.11) with $D = 5 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2$ and $B = 4000\ °R$. By what percentage will the velocity increase as the liquid temperature is increased from 40 °F to 100 °F? Assume all other factors remain constant.

$$V_{40^\circ} = \frac{K}{\mu_{40^\circ}} \qquad (1)$$

$$V_{100^\circ} = \frac{K}{\mu_{100^\circ}} \qquad (2)$$

$$\%\ \text{increase in } V = \left[\frac{V_{100^\circ} - V_{40^\circ}}{V_{40^\circ}}\right] \times 100 = \left[\frac{V_{100^\circ}}{V_{40^\circ}} - 1\right] \times 100$$

and from Eq. (1) & (2)

$$\%\ \text{increase in } V = \left[\frac{K/\mu_{100^\circ}}{K/\mu_{40^\circ}} - 1\right] \times 100 = \left[\frac{\mu_{40^\circ}}{\mu_{100^\circ}} - 1\right] \times 100 \qquad (3)$$

From Andrade's equation

$$\mu_{40^\circ} = 5 \times 10^{-7} e^{\frac{4000}{(40^\circ F + 460)}}$$

and

$$\mu_{100^\circ} = 5 \times 10^{-7} e^{\frac{4000}{(100^\circ F + 460)}}$$

Thus, from Eq. (3)

$$\%\ \text{increase in } V = \left[\frac{5 \times 10^{-7} e^{\frac{4000}{500}}}{5 \times 10^{-7} e^{\frac{4000}{560}}} - 1\right] \times 100$$

$$= \underline{\underline{136\%}}$$

1.52*

1.52* Use the value of the viscosity of water given in Table B.2 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants D and B which appear in Andrade's equation (Eq. 1.11). Calculate the value of the viscosity at 50 °C and compare with the value given in Table B.2. (*Hint*: Rewrite the equation in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D$$

and plot $\ln \mu$ versus $1/T$. From the slope and intercept of this curve B and D can be obtained. If a nonlinear curve fitting program is available the constants can be obtained directly from Eq. 1.11 without rewriting the equation.)

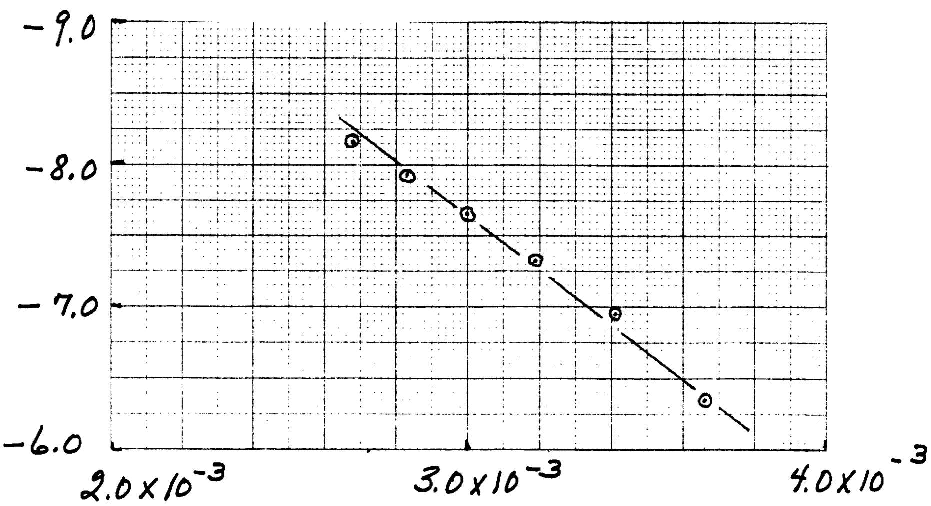
Equation 1.11 can be written in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D \qquad (1)$$

and with the data from Table B.2:

T (°C)	T (K)	$1/T$ (K)	μ ($N \cdot s/m^2$)	$\ln \mu$
0	273.15	3.661×10^{-3}	1.787×10^{-3}	-6.327
20	293.15	3.411×10^{-3}	1.002×10^{-3}	-6.906
40	313.15	3.193×10^{-3}	6.529×10^{-4}	-7.334
60	333.15	3.002×10^{-3}	4.665×10^{-4}	-7.670
80	353.15	2.832×10^{-3}	3.547×10^{-4}	-7.944
100	373.15	2.680×10^{-3}	2.818×10^{-4}	-8.174

A plot of $\ln \mu$ vs. $1/T$ is shown below:



(con't)

1.52* (con't)

Since the data plot as an approximate straight line, Eq. (1) can be used to represent these data.

To obtain B and D use EXPFIT.

```
**************************************************
** This program determines the least squares fit **
** for a function of the form  y = a * e ^ b*x   **
**************************************************

Number of points: 6
Input X, Y
? 3.661E-3,1.787E-3
? 3.411E-3,1.002E-3
? 3.193E-3,6.529E-4
? 3.002E-3,4.665E-4
? 2.832E-3,3.547E-4
? 2.680E-3,2.818E-4

a = +1.767E-06
b = +1.870E+03

      X              Y           Y(predicted)
+3.6610E-03    +1.7870E-03    +1.6629E-03
+3.4110E-03    +1.0020E-03    +1.0418E-03
+3.1930E-03    +6.5290E-04    +6.9298E-04
+3.0020E-03    +4.6650E-04    +4.8482E-04
+2.8320E-03    +3.5470E-04    +3.5277E-04
+2.6800E-03    +2.8180E-04    +2.6548E-04
```

Thus,

$$\underline{\underline{D = a = 1.767 \times 10^{-6}\ N\cdot s/m^2}}$$

and

$$\underline{\underline{B = b = 1.870 \times 10^{3}\ K}}$$

so that

$$\mu = 1.767 \times 10^{-6}\, e^{\frac{1870}{T}}$$

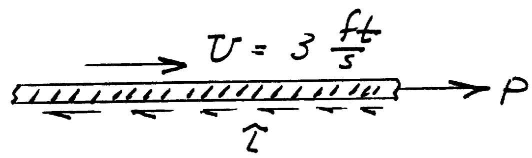
At 50°C (323.15 K),

$$\mu = 1.767 \times 10^{-6}\, e^{\frac{1870}{323.15}} = \underline{\underline{5.76 \times 10^{-4}\ N\cdot s/m^2}}$$

From Table B.2, $\mu = 5.468 \times 10^{-4}\ N\cdot s/m^2$.

1.53

1.53 Crude oil having a viscosity of 9.52×10^{-4} lb·s/ft^2 is contained between parallel plates. The bottom plate is fixed and upper plate moves when a force P is applied (see Fig. 1.3). If the distance between the two plates is 0.1 in., what value of P is required to translate the plate with a velocity of 3 ft/s? The effective area of the upper plate is 200 in.^{2}



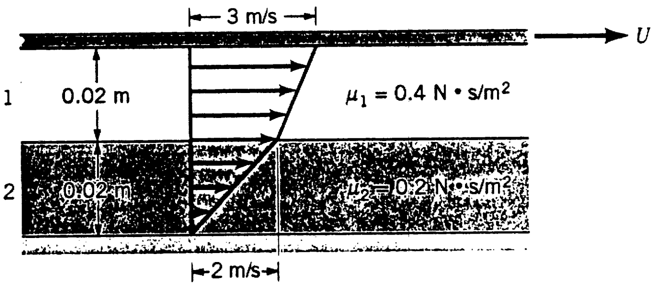
$$P = \tau \times \text{plate area}$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b}$$

$$P = \left(9.52 \times 10^{-4} \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(\frac{3 \frac{\text{ft}}{\text{s}}}{\frac{0.1}{12}\,\text{ft}}\right)(200\ \text{in.}^2)\left(\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right) = \underline{\underline{0.476\ \text{lb}}}$$

1.54

1.54 As shown in **Video V1.2**, the "no slip" condition means that a fluid "sticks" to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Fig. P1.54. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the botton plate. Determine the ratio of these two shear stresses.



■ **FIGURE P1.54**

For fluid 1

$$\tau_1 = \mu_1 \left(\frac{du}{dy}\right)_{\text{top surface}} = \left(0.4\ \frac{N\cdot s}{m^2}\right)\left(\frac{3\frac{m}{s} - 2\frac{m}{s}}{0.02\,m}\right) = 20\ \frac{N}{m^2}$$

For fluid 2

$$\tau_2 = \mu_2 \left(\frac{du}{dy}\right)_{\text{bottom surface}} = \left(0.2\ \frac{N\cdot s}{m^2}\right)\left(\frac{2\frac{m}{s}}{0.02\ m}\right) = 20\ \frac{N}{m^2}$$

Thus,

$$\frac{\tau_{\text{top surface}}}{\tau_{\text{bottom surface}}} = \frac{20\ \frac{N}{m^2}}{20\ \frac{N}{m^2}} = \underline{\underline{1}}$$

1.55

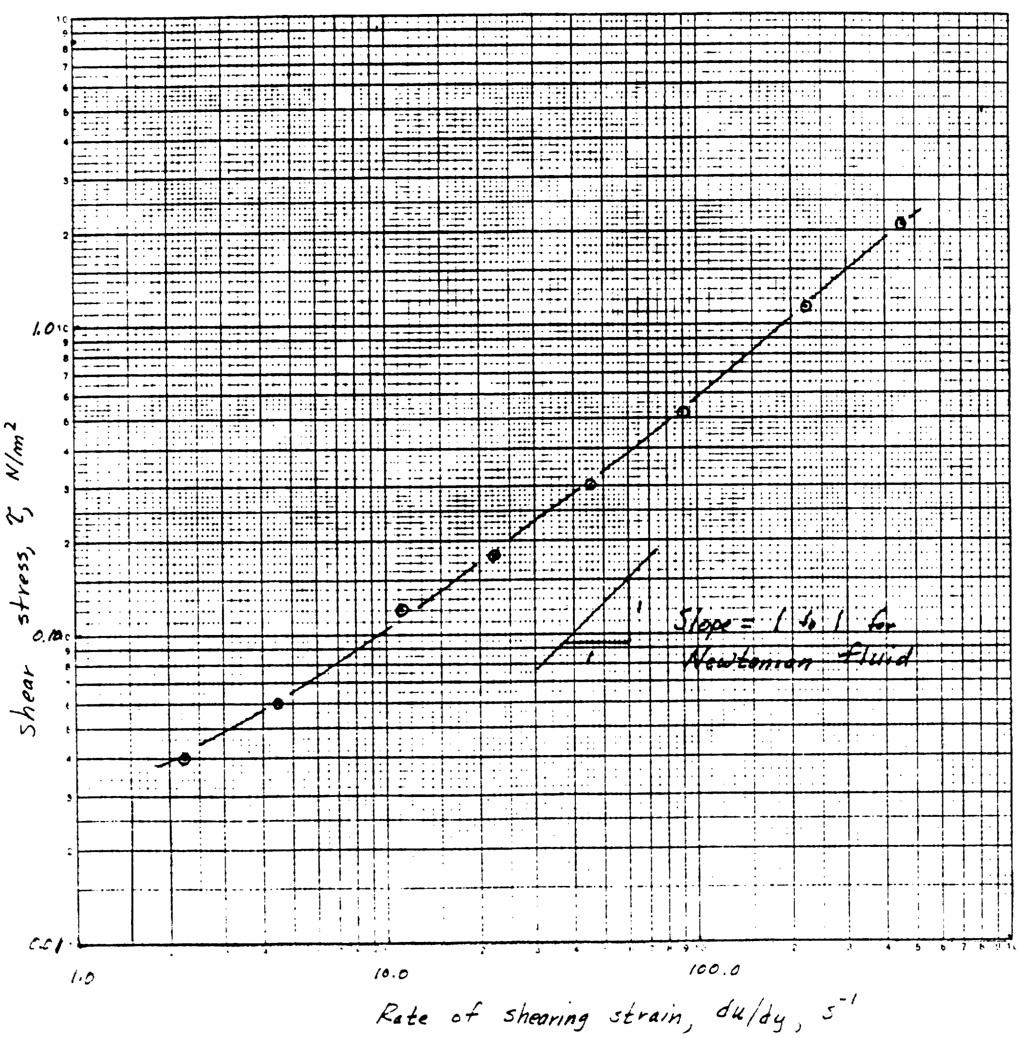
1.55 There are many fluids that exhibit non-Newtonian behavior (see for example **Video V1.4**). For a given fluid the distinction between Newtonian and non-Newtonian behavior is usually based on measurements of shear stress and rate of shearing strain. Assume that the viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, du/dy, obtained from a small blood sample tested in a suitable viscometer. Based on the data given below determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

τ (N/m^2)	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
du/dy (s^{-1})	2.25	4.50	11.25	22.5	45.0	90.0	225	450

For a Newtonian fluid the ratio of τ to du/dy is a constant. For the data given:

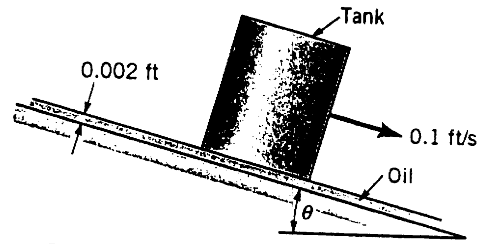
$\frac{\tau}{du/dy}$ $(N\cdot s/m^2)$	0.0178	0.0133	0.0107	0.0080	0.0067	0.0058	0.0050	0.0047

The ratio is not a constant but decreases as the rate of shearing strain increases. Thus, this fluid (blood) is a <u>non-Newtonian</u> fluid. A plot of the data is shown below. For a Newtonian fluid the curve would be a straight line with a slope of 1 to 1.

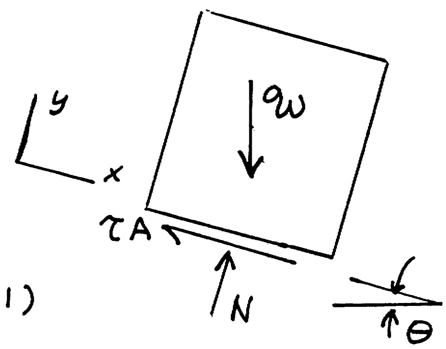


1.56

1.56 A 40-lb, 0.8-ft-diameter, 1-ft-tall cylindrical tank slides slowly down a ramp with a constant speed of 0.1 ft/s as shown in Fig. P1.56. The uniform-thickness oil layer on the ramp has a viscosity of 0.2 lb · s/ft^2. Determine the angle, θ, of the ramp.



■ FIGURE P1.56



$\Sigma F_x = 0$

Thus,

$$\mathcal{W} \sin\theta = \tau A \qquad (1)$$

Since

$$\tau = \mu \frac{V}{b}$$, where V is the velocity of tank and b is thickness of oil layer

$$\tau = \left(0.2 \frac{lb \cdot s}{ft^2}\right)\left(\frac{0.1 \frac{ft}{s}}{0.002\, ft}\right) = 10 \frac{lb}{ft^2}$$

From Eq. (1)

$$(40\, lb) \sin\theta = \left(10 \frac{lb}{ft^2}\right)\left(\frac{\pi}{4}\right)(0.8\, ft)^2$$

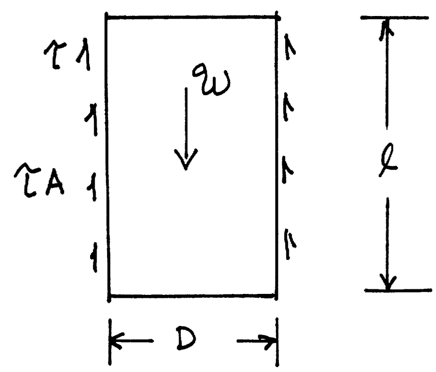
and

$$\sin\theta = 0.1257$$

so that

$$\theta = \underline{\underline{7.22^\circ}}$$

1.57 A piston having a diameter of 5.48 in. and a length of 9.50 in. slides downward with a velocity V through a vertical pipe. The downward motion is resisted by an oil film between the piston and the pipe wall. The film thickness is 0.002 in., and the cylinder weighs 0.5 lb. Estimate V if the oil viscosity is 0.016 lb·s/ft^2. Assume the velocity distribution in the gap is linear.



$$\sum F_{vertical} = 0$$

Thus, $\mathcal{W} = \tau A$

where $A = \pi D \ell$

and $$\tau = \mu \frac{(\text{velocity})}{(\text{film thickness})} = \mu \frac{V}{\delta}$$

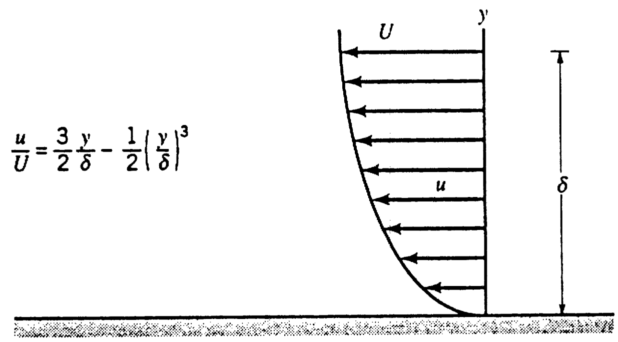
so that
$$\mathcal{W} = \left(\mu \frac{V}{\delta}\right)(\pi D \ell)$$

It follows that
$$V = \frac{\mathcal{W}\delta}{\pi D \ell \mu} = \frac{(0.5\,\text{lb})\left(\frac{0.002}{12}\,\text{ft}\right)}{\pi\left(\frac{5.48}{12}\,\text{ft}\right)\left(\frac{9.50}{12}\,\text{ft}\right)\left(0.016\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)}$$

$$= \underline{\underline{0.00459\,\frac{\text{ft}}{\text{s}}}}$$

1.58

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4}\ m^2/s$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



■ FIGURE P1.58

$$\tau_{\substack{surface \\ (y=0)}} = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\frac{du}{dy} = U\left(\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)$$

$$@\ y=0, \quad \frac{du}{dy} = \frac{3}{2}\frac{U}{\delta}$$

Since, $\mu = \nu\rho$

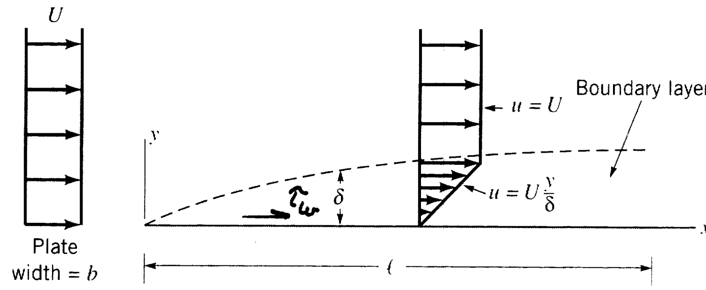
$$\tau_{surface} = \nu\rho\left(\frac{3}{2}\frac{U}{\delta}\right)$$

$$= \left(4\times10^{-4}\frac{m^2}{s}\right)\left(0.92\times10^{3}\frac{kg}{m^3}\right)\left(\frac{3}{2}\right)\frac{U}{\delta}$$

$$= \underline{\underline{0.552\frac{U}{\delta}\ N/m^2 \text{ acting to left on plate}}}$$

1.59

1.59 When a viscous fluid flows past a thin sharp-edged plate, a thin layer adjacent to the plate surface develops in which the velocity, u, changes rapidly from zero to the approach velocity, U, in a small distance, δ. This layer is called a *boundary layer*. The thickness of this layer increases with the distance x along the plate as shown in Fig. P1.59. Assume that $u = U\,y/\delta$ and $\delta = 3.5\sqrt{\nu x/U}$ where ν is the kinematic viscosity of the fluid. Determine an expression for the force (drag) that would be developed on one side of the plate of length l and width b. Express your answer in terms of l, b, ν, and ρ, where ρ is the fluid density.



Drag, $\mathcal{D} = \int_0^A \tau_w \, dA$ where $dA = b\,dx$

so that

$$\mathcal{D} = \int_0^l \tau_w \, b \, dx \qquad (1)$$

Since $\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0}$ and $\frac{du}{dy} = \frac{U}{\delta}$

with $\delta = 3.5\sqrt{\frac{\nu x}{U}}$, it follows from Eq. (1)

$$\mathcal{D} = \int_0^l \frac{\mu U^{3/2} x^{-1/2}}{3.5\,\nu^{1/2}} \, b\,dx = \frac{\mu U^{3/2} b}{3.5\,\nu^{1/2}} \int_0^l x^{-1/2} dx$$

Thus,

$$\mathcal{D} = \frac{\mu U^{3/2} b}{3.5\,\nu^{1/2}} \left(2\,l^{1/2}\right)$$

and with $\nu = \frac{\mu}{\rho}$

$$\mathcal{D} = \underline{\underline{0.571\, b\rho\sqrt{\nu l U^3}}}$$

1.60*

1.60* Standard air flows past a flat surface and velocity measurements near the surface indicate the following distribution:

y (ft)	0.005	0.01	0.02	0.04	0.06	0.08
u (ft/s)	0.74	1.51	3.03	6.37	10.21	14.43

The coordinate y is measured normal to the surface and u is the velocity parallel to the surface. **(a)** Assume the velocity distribution is of the form

$$u = C_1 y + C_2 y^3$$

and use a standard curve-fitting technique to determine the constants C_1 and C_2. **(b)** Make use of the results of part (a) to determine the magnitude of the shearing stress at the wall ($y = 0$) and at $y = 0.05$ ft.

(a) Use nonlinear regression program, such as SAS-NLIN, to obtain coefficients C_1 and C_2. This program produces least squares estimates of the parameters of a nonlinear model. For the data given,

$$\underline{\underline{C_1 = 153\ s^{-1}}} \quad \text{and} \quad \underline{\underline{C_2 = 4350\ ft^{-2}s^{-1}}}$$

(b) Since,

$$\tau = \mu \frac{du}{dy}$$

it follows that

$$\tau = \mu\left(C_1 + 3C_2 y^2\right)$$

Thus, at the wall $(y=0)$

$$\tau = \mu C_1 = \left(3.74 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\right)\left(153\ \frac{1}{s}\right) = \underline{\underline{5.72 \times 10^{-5}\ \frac{lb}{ft^2}}}$$

At $y = 0.05$ ft

$$\tau = \left(3.74 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\right)\left[153\ \frac{1}{s} + 3\left(4350\ \frac{1}{ft^2\cdot s}\right)(0.05\ ft)^2\right]$$

$$= \underline{\underline{6.94 \times 10^{-5}\ \frac{lb}{ft^2}}}$$

1.61

1.61 The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in Fig. P1.61. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque $\mathcal{T}$ required to develop ω is measured and the viscosity is calculated from these two measurements. Develop an equation relating μ, ω, $\mathcal{T}$, ℓ, R_o and R_i. Neglect end effects and assume the velocity distribution in the gap is linear.

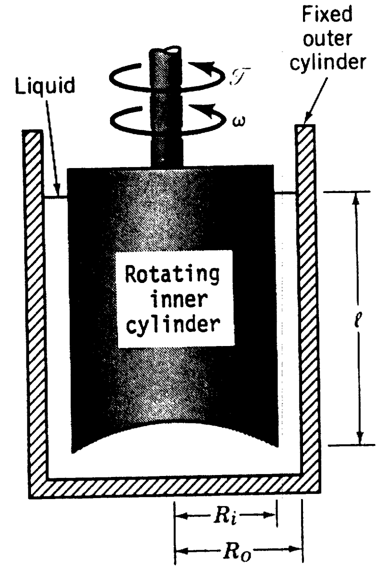


FIGURE P1.61

Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

$$d\mathcal{T} = R_i \, \tau \, dA$$

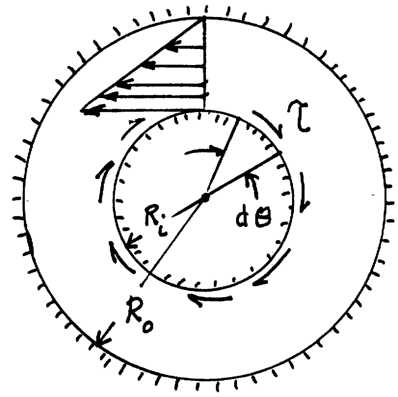
where $dA = (R_i \, d\theta)\ell$. Thus,

$$d\mathcal{T} = R_i^2 \ell \, \tau \, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2 \ell \, \tau \int_0^{2\pi} d\theta$$

$$= 2\pi R_i^2 \ell \, \tau$$



top view

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_o - R_i}$$

so that

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i}$$

1.62

1.62 The space between two 6-in. long concentric cylinders is filled with glycerin (viscosity $= 8.5 \times 10^{-3}$ lb·s/ft^2). The inner cylinder has a radius of 3 in. and the gap width between cylinders is 0.1 in. Determine the torque and the power required to rotate the inner cylinder at 180 rev/min. The outer cylinder is fixed. Assume the velocity distribution in the gap to be linear.

From Problem 1.66,

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i}$$

and with $\omega = \left(180 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = 6\pi \frac{rad}{s}$

then

$$\mathcal{T} = \frac{2\pi \left(\frac{3}{12} ft\right)^3 \left(\frac{6}{12} ft\right)\left(8.5 \times 10^{-3} \frac{lb \cdot s}{ft^2}\right)\left(6\pi \frac{rad}{s}\right)}{\left(\frac{0.1}{12} ft\right)} = \underline{\underline{0.944\ ft \cdot lb}}$$

Since $\quad$ power $= \mathcal{T} \times \omega$ it follows that

$$\text{power} = (0.944\ ft \cdot lb)\left(6\pi \frac{rad}{s}\right) = \underline{\underline{17.8 \frac{ft \cdot lb}{s}}}$$

1.63

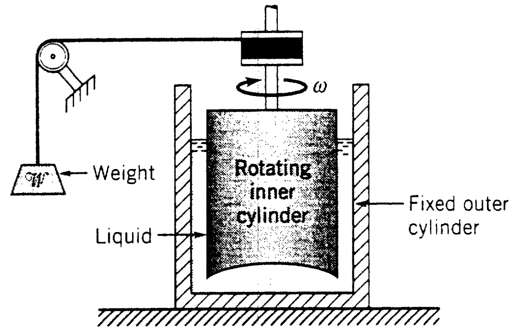
1.63 One type of rotating cylinder viscometer, called a *Stormer* viscometer, uses a falling weight, $\mathcal{W}$, to cause the cylinder to rotate with an angular velocity, ω, as illustrated in Fig. P1.63. For this device the viscosity, μ, of the liquid is related to $\mathcal{W}$ and ω through the equation $\mathcal{W} = K\mu\omega$, where K is a constant that depends only on the geometry (including the liquid depth) of the viscometer. The value of K is usually determined by using a calibration liquid (a liquid of known viscosity).

(a) Some data for a particular Stormer viscometer, obtained using glycerin at 20 °C as a calibration liquid, are given below. Plot values of the weight as ordinates and values of the angular velocity as abscissae. Draw the best curve through the plotted points and determine K for the viscometer.

$\mathcal{W}$ (lb)	0.22	0.66	1.10	1.54	2.20
ω (rev/s)	0.53	1.59	2.79	3.83	5.49

(b) A liquid of unknown viscosity is placed in the same viscometer used in part (a), and the data given below are obtained. Determine the viscosity of this liquid.

$\mathcal{W}$ (lb)	0.04	0.11	0.22	0.33	0.44
ω (rev/s)	0.72	1.89	3.73	5.44	7.42



■ FIGURE P1.63

(a) Since $\mathcal{W} = K\mu\omega$ the slope of the $\mathcal{W}$ vs. ω curve is

$$\text{slope} = K\mu = \frac{\mathcal{W}\,(\text{lb})}{\omega\,(\frac{\text{rev}}{\text{s}})}$$

so that

$$K = \frac{\text{slope}\,(\frac{\text{lb}\cdot\text{s}}{\text{rev}})}{\mu\,(\frac{\text{lb}\cdot\text{s}}{\text{ft}^2})} \qquad (1)$$

For the glycerin data (see plot on next page) the slope (based on a least squares fit of the data) is

$$\text{slope (glycerin)} = 0.398\ \frac{\text{lb}\cdot\text{s}}{\text{rev}}$$

Since $\mu\ (\text{glycerin}) = 3.13 \times 10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}$ then

$$K = \frac{0.398\ \frac{\text{lb}\cdot\text{s}}{\text{rev}}}{3.13\times10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = \underline{\underline{12.7\ \frac{\text{ft}^2}{\text{rev}}}}$$

(b) For the unknown fluid data (see plot on next page) the slope (based on a least squares fit of the data) is

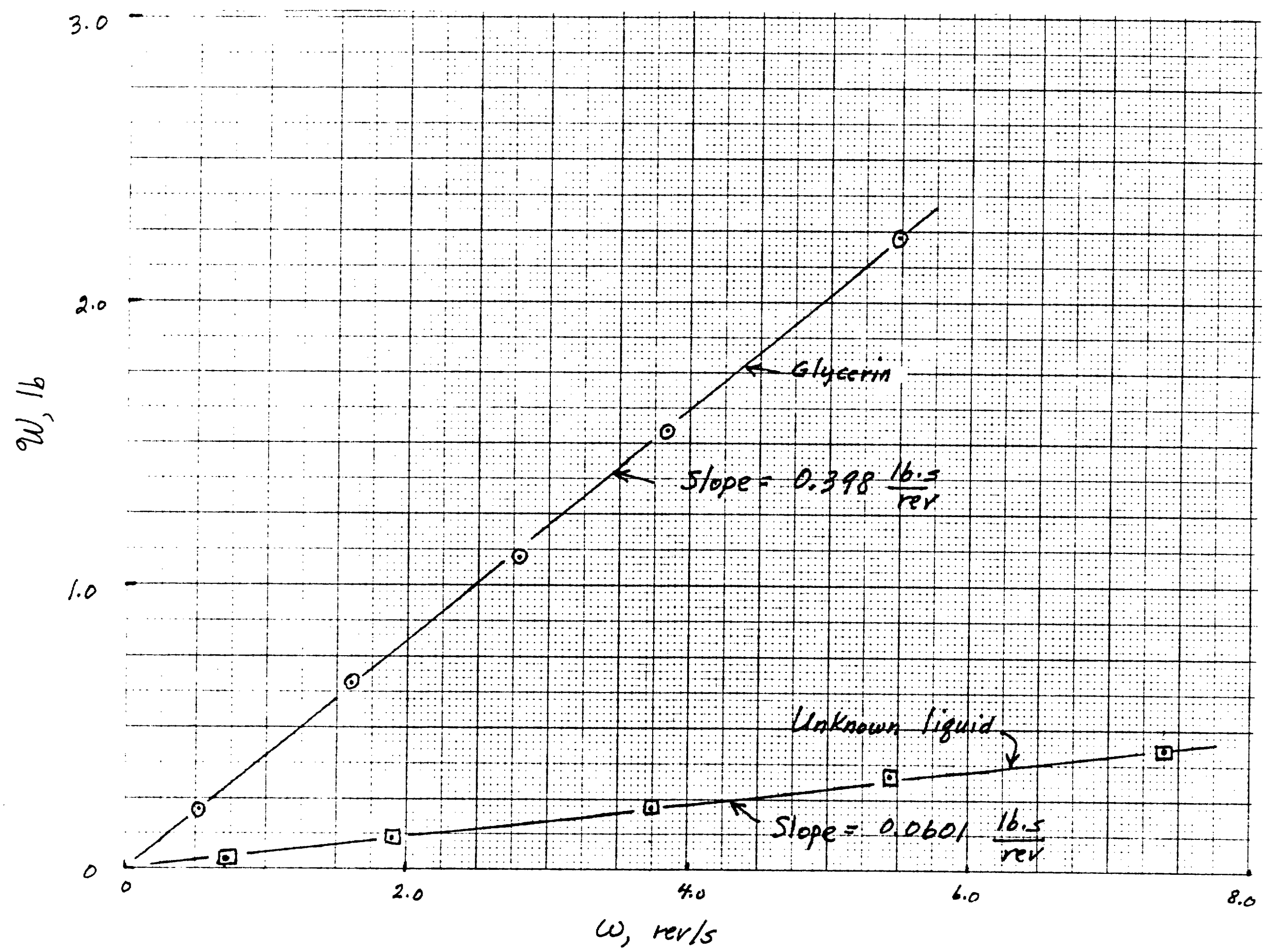
$$\text{slope (unknown fluid)} = 0.0601\ \frac{\text{lb}\cdot\text{s}}{\text{rev}}$$

(con't)

1.63 (con't)

Thus, from Eq. (1)

$$\mu(\text{unknown fluid}) = \frac{\text{slope}}{K} = \frac{0.0601 \frac{lb\cdot s}{rev}}{12.7 \frac{ft^2}{rev}} = \underline{\underline{4.73 \times 10^{-3} \frac{lb\cdot s}{ft^2}}}$$



1.64*

1.64* The following torque–angular velocity data were obtained with a rotating cylinder viscometer of the type described in Problem 1.61.

Torque (ft·lb)	13.1	26.0	39.5	52.7	64.9	78.6
Angular velocity (rad/s)	1.0	2.0	3.0	4.0	5.0	6.0

For this viscometer $R_o = 2.50$ in., $R_i = 2.45$ in., and $\ell = 5.00$ in. Make use of these data and a standard curve-fitting program to determine the viscosity of the liquid contained in the viscometer.

The torque, $\mathcal{T}$, is related to the angular velocity, ω, through the equation,

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i}\,\omega \qquad (1)$$

(see solution to Problem 1.66). Thus, for a fixed geometry and a given viscosity, Eq. (1) is of the form

$$y = bx \qquad (y \sim \mathcal{T} \text{ and } x \sim \omega)$$

where b is a constant equal to

$$b = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i} \qquad (2)$$

To obtain b use the data given with LINREG 1.

```
***************************************************
** This program determines the least squares fit **
** for a function of the form   y = b * x        **
***************************************************

Number of points: 6
Input X, Y
? 1.0,13.1
? 2.0,26.0
? 3.0,39.5
? 4.0,52.7
? 5.0,64.9
? 6.0,78.6

b = +1.308E+01  ft·lb·s

     X              Y            Y(predicted)
+1.0000E+00   +1.3100E+01   +1.3082E+01
+2.0000E+00   +2.6000E+01   +2.6165E+01
+3.0000E+00   +3.9500E+01   +3.9247E+01
+4.0000E+00   +5.2700E+01   +5.2330E+01
+5.0000E+00   +6.4900E+01   +6.5412E+01
+6.0000E+00   +7.8600E+01   +7.8495E+01
```

(con't)

1.64* (Con't)

Thus, from Eq. (2)

$$\mu = \frac{(b)(R_o - R_i)}{2\pi R_i^3 \ell}$$

and with the data given,

$$\mu = \frac{(13.08\ \text{ft}\cdot\text{lb}\cdot\text{s})\left(\frac{2.50 - 2.45}{12}\ \text{ft}\right)}{2\pi\left(\frac{2.45}{12}\ \text{ft}\right)^3\left(\frac{5.00}{12}\ \text{ft}\right)} = \underline{\underline{2.45\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}}$$

1.65

1.65 A 12-in.-diameter circular plate is placed over a fixed bottom plate with a 0.1-in. gap between the two plates filled with glycerin as shown in Fig. P1.65. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.

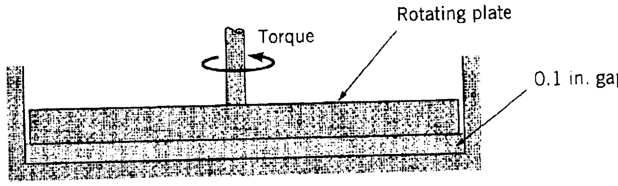


FIGURE P1.65

Torque, $d\mathcal{T}$, due to shearing stresses on plate is equal to

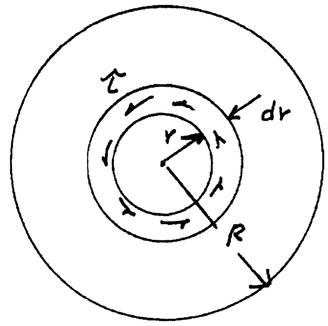
$$d\mathcal{T} = r\,\tau\, dA$$

where $dA = 2\pi r\, dr$. Thus,

$$d\mathcal{T} = r\,\tau\, 2\pi r\, dr$$

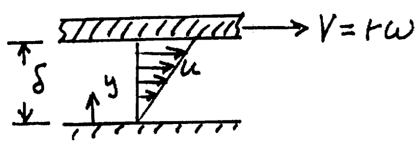
and

$$\mathcal{T} = 2\pi \int_0^R r^2 \tau\, dr$$



stresses acting on bottom of plate

Since $\tau = \mu \dfrac{du}{dy}$, and for a linear velocity distribution (see figure)



$$\frac{du}{dy} = \frac{V}{\delta} = \frac{r\omega}{\delta}$$

velocity distribution

$$\tau = \frac{\mu r \omega}{\delta}$$

Thus,

$$\mathcal{T} = \frac{2\pi\mu\omega}{\delta}\int_0^R r^3 dr = \frac{2\pi\mu\omega}{\delta}\left(\frac{R^4}{4}\right)$$

and with the data given

$$\mathcal{T} = \frac{2\pi\left(0.0313\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(2\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{6}{12}\ \text{ft}\right)^4}{\left(\frac{0.1}{12}\ \text{ft}\right)(4)}$$

$$= \underline{\underline{0.0772\ \text{ft}\cdot\text{lb}}}$$

1.67

1.67 A rigid-walled cubical container is completely filled with water at 40 °F and sealed. The water is then heated to 100 °F. Determine the pressure that develops in the container when the water reaches this higher temperature. Assume that the volume of the container remains constant and the value of the bulk modulus of the water remains constant and equal to 300,000 psi.

Since the water mass remains constant,

$$\rho_{40^\circ} \forall = \rho_{100^\circ} (\forall + \Delta\forall)$$

where $\forall$ is volume and $\Delta\forall$ is change in volume if water were unconstrained during heating. Thus,

$$\frac{\Delta\forall}{\forall} = \frac{\rho_{40^\circ}}{\rho_{100^\circ}} - 1$$

From Table B.1 in Appendix B, $\rho_{40^\circ} = 1.940 \frac{slugs}{ft^3}$ and $\rho_{100^\circ} = 1.927 \frac{slugs}{ft^3}$

So that

$$\frac{\Delta\forall}{\forall} = \frac{1.940 \frac{slugs}{ft^3}}{1.927 \frac{slugs}{ft^3}} - 1 = 0.00675$$

From Eq. 1.12

$$E_v = -\frac{dp}{\frac{d\forall}{\forall}}$$

it follows with $d\forall \simeq \Delta\forall$ and $dp \simeq \Delta p$ that the change in pressure required to compress the water back to its original volume is

$$\Delta p = -(300{,}000\ psi)(-0.00675)$$

$$= \underline{\underline{2.03 \times 10^3\ psi}}$$

1.68

1.68 In a test to determine the bulk modulus of a liquid it was found that as the absolute pressure was changed from 15 to 3000 psi the volume decreased from 10.240 to 10.138 in.³ Determine the bulk modulus for this liquid.

$$E_v = -\frac{dp}{d\forall/\forall} \qquad (\text{Eq. } 1.12)$$

Since

$$dp \approx \Delta p = 3000 - 15 = 2985 \text{ psi}$$

and

$$d\forall \approx \Delta\forall = 10.240 - 10.138 = 0.102 \text{ in.}^3$$

$$E_v \approx -\frac{2985 \frac{\text{lb}}{\text{in.}^2}}{\left(\frac{0.102 \text{ in.}^3}{10.240 \text{ in.}^3}\right)} = \underline{\underline{3.00 \times 10^5 \text{ psi}}}$$

1.69

1.69 Calculate the speed of sound in m/s for **(a)** gasoline, **(b)** mercury, and **(c)** seawater.

$$c = \sqrt{\frac{E_v}{\rho}} \qquad (\text{Eq. } 1.19)$$

(a) For gasoline: $c = \sqrt{\dfrac{1.3 \times 10^9 \frac{N}{m^2}}{680 \frac{kg}{m^3}}} = \underline{\underline{1.38 \frac{km}{s}}}$

(b) For mercury: $c = \sqrt{\dfrac{2.85 \times 10^{10} \frac{N}{m^2}}{1.36 \times 10^4 \frac{kg}{m^3}}} = \underline{\underline{1.45 \frac{km}{s}}}$

(c) For seawater: $c = \sqrt{\dfrac{2.34 \times 10^9 \frac{N}{m^2}}{1.03 \times 10^3 \frac{kg}{m^3}}} = \underline{\underline{1.51 \frac{km}{s}}}$

1.70

1.70 Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 25 psi. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 14.7 psi.

For isothermal compression, $\frac{p}{\rho}$ = constant so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus, $p_f = \frac{\rho_f}{\rho_i} p_i$

Since $\rho = \frac{\text{mass}}{\text{volume}}$, $\frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3$ (for constant mass)

and therefore

$$p_f = (3)\left[(25 + 14.7)\ \text{psi (abs)}\right] = 119 \quad \text{psi (abs)}$$

or

$$p_f\ (\text{gage}) = (119 - 14.7)\text{psi} = \underline{\underline{104\ \text{psi (gage)}}}$$

1.71

1.71 Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2%. If air is flowing through a tube such that the air pressure at one section is 9.0 psi and at a downstream section it is 8.6 psi at the same temperature, do you think that this flow could be considered an imcompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.

For isothermal change in density

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$$

so that

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$$

The percent change in air densities between sections (1) & (2) is

$$\%\ \text{change} = \frac{\rho_1 - \rho_2}{\rho_1} \times 100$$

$$= \left(1 - \frac{\rho_2}{\rho_1}\right) \times 100 = \left(1 - \frac{p_2}{p_1}\right) \times 100$$

Thus,

$$\%\ \text{change} = \left[1 - \frac{(8.6 + 14.7)\,\text{psia}}{(9.0 + 14.7)\,\text{psia}}\right] \times 100$$

$$= 1.69\%$$

Since $1.69\% < 2\%$ the flow could be considered incompressible. Yes.

1.72

1.72 Oxygen at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 120 kPa. Determine the final density of the gas.

For isothermal expansion, $\frac{p}{\rho}$ = constant so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f = \frac{p_f}{p_i}\rho_i$$

Also,

$$\rho_i = \frac{p_i}{RT_i} = \frac{300 \times 10^3 \frac{N}{m^2}}{\left(259.8 \frac{J}{kg \cdot K}\right)\left[(30°C + 273)K\right]} = 3.81 \frac{kg}{m^3}$$

so that

$$\rho_f = \left(\frac{120 \text{ kPa}}{300 \text{ kPa}}\right)\left(3.81 \frac{kg}{m^3}\right) = \underline{\underline{1.52 \frac{kg}{m^3}}}$$

1.73

1.73 Natural gas at 70 °F and standard atmospheric pressure of 14.7 psi is compressed isentropically to a new absolute pressure of 70 psi. Determine the final density and temperature of the gas.

For isentropic compression, $\frac{p}{\rho^k} = \text{constant}$ so that

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f^k = \frac{p_f}{p_i}\rho_i^k$$

or

$$\rho_f = \left(\frac{p_f}{p_i}\right)^{\frac{1}{k}}\rho_i$$

Also,

$$\rho_i = \frac{p_i}{RT_i} = \frac{\left(14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(3.099\times10^3\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)\left[(70^\circ F+460)^\circ R\right]} = 1.29\times10^{-3}\,\frac{slugs}{ft^3}$$

so that

$$\rho_f = \left[\frac{70\text{ psi (abs)}}{14.7\text{ psi (abs)}}\right]^{\frac{1}{1.31}}\left(1.29\times10^{-3}\,\frac{slugs}{ft^3}\right) = \underline{\underline{4.25\times10^{-3}\,\frac{slugs}{ft^3}}}$$

and

$$T_f = \frac{p_f}{\rho_f R} = \frac{\left(70\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(4.25\times10^{-3}\,\frac{slugs}{ft^3}\right)\left(3.099\times10^3\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)}$$

$$= 765\ {}^\circ R$$

or

$$T_f = 765\ {}^\circ R - 460 = \underline{\underline{305\ {}^\circ F}}$$

1.74

1.74 Compare the isentropic bulk modulus of air at 101 kPa (abs) with that of water at the same pressure.

For air (Eq. 1.17),

$$E_v = kp = (1.40)(101 \times 10^3 \, Pa) = 1.41 \times 10^5 \, Pa$$

For water (Table 1.6)

$$E_v = 2.15 \times 10^9 \, Pa$$

Thus,

$$\frac{E_v \text{ (water)}}{E_v \text{ (air)}} = \frac{2.15 \times 10^9 \, Pa}{1.41 \times 10^5 \, Pa} = \underline{\underline{1.52 \times 10^4}}$$

1.75 *

1.75* Develop a computer program for calculating the final gage pressure of gas when the initial gage pressure, initial and final volumes, atmospheric pressure, and the type of process (isothermal or isentropic) are specified. Use BG units. Check your program against the results obtained for Problem 1.70.

For compression or expansion,

$$\frac{p}{\rho^k} = \text{constant}$$

where $k=1$ for isothermal process, and k = specific heat ratio for isentropic process. Thus,

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$

where $i \sim$ initial state, $f \sim$ final state, so that

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i \qquad (1)$$

Since

$$\rho = \frac{\text{mass}}{\text{volume}}$$

then

$$\frac{\rho_f}{\rho_i} = \frac{V_i}{V_f}$$

where V_i, V_f, are the initial and final volumes, respectively. Thus, from Eq. (1)

$$p_{fg} + p_{atm} = \left(\frac{V_i}{V_f}\right)^k (p_{ig} + p_{atm}) \qquad (2)$$

where the subscript g refers to gage pressure. Equation (2) can be written as

$$p_{fg} = \left(\frac{V_i}{V_f}\right)^k (p_{ig} + p_{atm}) - p_{atm} \qquad (3)$$

A computer program for calculating p_{fg} follows.

(con't)

1.75* (Con't)

```
100 cls
110 print "************************************************************"
120 print "** This program calculates the final gage pressure of    **"
130 print "** an ideal gas when the initial gage pressure in psi,   **"
140 print "** the initial volume, the final volume, the             **"
150 print "** atmospheric pressure in psi, and the type of          **"
160 print "** process (isothermal or isentropic) are specified      **"
170 print "************************************************************"
180 print
190 input "Enter initial gage pressure in psi, Pi = ",p
200 input "Enter initial volume, Vi = ",vi
210 input "Enter final volume, Vf = ",vf
220 input "Enter atmospheric pressure in psi, Patm = ",patm
230 pabsi=p+patm
240 print:print "Enter type of process"
250 print "0 : Isothermal"
260 print "1 : Isentropic"
270 input pt
280 print
290 k=1
300 if pt=1 then input "Enter specific heat ratio, k = ",k
310 pabsf=pabsi*(vi/vf)^k
320 pf=pabsf-patm
330 print
340 print using "The final gage pressure of the gas

                  is Pf = +#.####^^^^ psi";pf
```

Run program using data from Problem 1.70.

```
**********************************************************
** This program calculates the final gage pressure of   **
** an ideal gas when the initial gage pressure in psi,  **
** the initial volume, the final volume, the            **
** atmospheric pressure in psi, and the type of         **
** process (isothermal or isentropic) are specified     **
**********************************************************

Enter initial gage pressure in psi, Pi =25
Enter initial volume, Vi = 1
Enter final volume, Vf = 0.3333
Enter atmospheric pressure in psi, Patm = 14.7

Enter type of process
0 : Isothermal
1 : Isentropic
? 0


The final gage pressure of the gas is Pf = +1.0441E+02 psi
```

1.76

1.76 An important dimensionless parameter concerned with very high speed flow is the *Mach number*, defined as V/c, where V is the speed of the object such as an airplane or projectile, and c is the speed of sound in the fluid surrounding the object. For a projectile traveling at 800 mph through air at 50 °F and standard atmospheric pressure, what is the value of the Mach number?

$$\text{Mach number} = \frac{V}{c}$$

From Table B.3 in Appendix B

$$c_{\text{air @ } 50^\circ F} = 1106 \ \frac{ft}{s}$$

Thus

$$\text{Mach number} = \frac{(800 \text{ mph})\left(5280 \frac{ft}{mi}\right)\left(\frac{1 \text{ hr}}{3600 s}\right)}{1106 \ \frac{ft}{s}}$$

$$= \underline{\underline{1.06}}$$

1.77 Jet airliners typically fly at altitudes between approximately 0 to 40,000 ft. Make use of the data in Appendix C to show on a graph how the speed of sound varies over this range.

$$c = \sqrt{kRT} \qquad \text{(Eq. 1.20)}$$

For $k = 1.40$ and $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}$

$$c = 49.0 \sqrt{T({}^\circ R)}$$

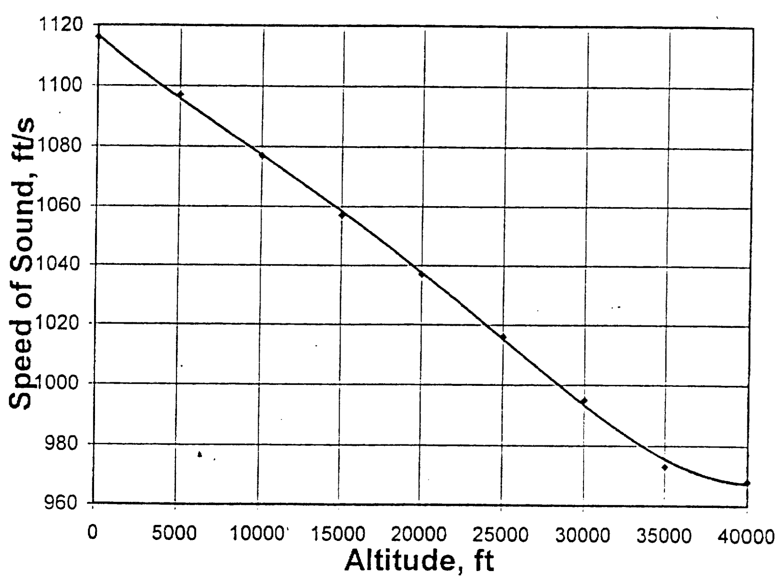
From Table C.1 in Appendix C at an altitude of 0 ft

$T = 59.00 + 460 = 519\,{}^\circ R$ so that

$$c = 49.0\sqrt{519\,{}^\circ R} = 1116 \frac{ft}{s}$$

Similar calculations can be made for other altitudes and the resulting graph is shown below.

Altitude, ft	Temp., °F	Temp., °R	c, ft/s
0	59	519	1116
5000	41.17	501.17	1097
10000	23.36	483.36	1077
15000	5.55	465.55	1057
20000	-12.26	447.74	1037
25000	-30.05	429.95	1016
30000	-47.83	412.17	995
35000	-65.61	394.39	973
40000	-69.7	390.3	968



1.78

1.78 When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

Cavitation may occur when the local pressure equals the vapor pressure. For water at 160 °F (from Table B.1 in Appendix B)

$$p_v = 4.74 \text{ psi (abs)}$$

Thus, minimum pressure $= \underline{\underline{4.74 \text{ psi (abs)}}}$

1.79

1.79 Estimate the minimum absolute pressure (in pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

Cavitation may occur when the suction pressure at the pump inlet equals the vapor pressure.
For carbon tetrachloride at 20°C $p_v = 13$ kPa (abs).
Thus, minimum pressure $= \underline{\underline{13 \text{ kPa (abs)}}}$

1.80

1.80 When water at 90 °C flows through a converging section of pipe, the pressure is reduced in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

Cavitation may occur in the converging section of pipe when the pressure equals the vapor pressure. From Table B.2 in Appendix B for water at 90°C, $p_v = 70.1$ kPa (abs). Thus,

$$\text{minimum pressure} = \underline{\underline{70.1 \text{ kPa (abs)}}} \text{ in SI units.}$$

In BG units

$$\text{minimum pressure} = \left(70.1 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{psi}{\frac{N}{m^2}}\right)$$

$$= \underline{\underline{10.2 \text{ psia}}}$$

1.81

1.81 A partially filled closed tank contains ethyl alcohol at 68 °F. If the air above the alcohol is evacuated what is the minimum absolute pressure that develops in the evacuated space?

Minimum pressure = vapor pressure = $\underline{\underline{0.85 \text{ psi (abs)}}}$

1.82

1.82 Estimate the excess pressure inside a rain drop having a diameter of 3 mm.

$$p = \frac{2\sigma}{R} \qquad \text{(Eq. 1.21)}$$

$$= \frac{2\left(7.34 \times 10^{-2} \frac{N}{m}\right)}{0.0015 \text{ m}} = \underline{\underline{97.9 \text{ Pa}}}$$

1.83

1.83 A 12-mm diameter jet of water discharges vertically into the atmosphere. Due to surface tension the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

For equilibrium (see figure),

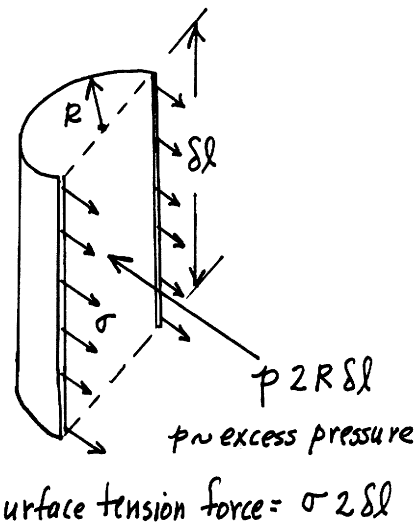
$$p(2R\delta\ell) = \sigma(2\delta\ell)$$

so that

$$p = \frac{\sigma}{R}$$

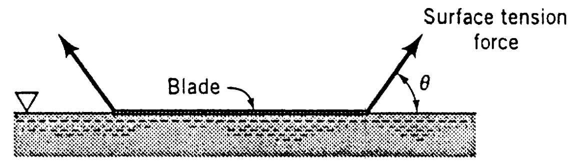
$$= \frac{7.34 \times 10^{-2}\ \frac{N}{m}}{\frac{12}{2} \times 10^{-3}\ m}$$

$$= \underline{\underline{12.2\ Pa}}$$

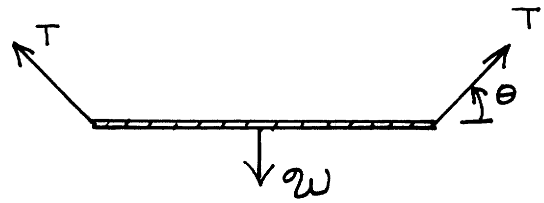


1.84

1.84 As shown in Video V1.5, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in Fig. P1.84. (a) The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. (b) The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



■ FIGURE P1.84



(a) $\sum F_{vertical} = 0$

$$\mathcal{W} = T\sin\theta$$

where $\mathcal{W} = m_{blade} \times g$ and $T = \sigma \times$ length of sides.

$$\therefore \left(0.64 \times 10^{-3}\,kg\right)\left(9.81\,m/s^2\right) = \left(7.34 \times 10^{-2}\,\frac{N}{m}\right)\left(0.206\,m\right)\sin\theta$$

$$\sin\theta = 0.415$$

$$\underline{\theta = 24.5^\circ}$$

(b) For single-edge blade

$$\mathcal{W} = m_{blade} \times g = \left(2.61 \times 10^{-3}\,kg\right)\left(9.81\,m/s^2\right)$$
$$= 0.0256\,N$$

and

$$T\sin\theta = \left(\sigma \times \text{length of blade}\right)\sin\theta$$
$$= \left(7.34 \times 10^{-2}\,N/m\right)\left(0.154\,m\right)\sin\theta$$
$$= 0.0113\sin\theta$$

In order for blade to "float" $\mathcal{W} < T\sin\theta$. Since maximum value for $\sin\theta$ is 1, it follows that $\mathcal{W} > T\sin\theta$ and single-edge blade will sink.

1.85

1.85 To measure the water depth in a large open tank with opaque walls, an open vertical glass tube is attached to the side of the tank. The height of the water column in the tube is then used as a measure of the depth of water in the tank. (a) For a true water depth in the tank of 3 ft, make use of Eq. 1.22 (with $\theta \simeq 0°$) to determine the percent error due to capillarity as the diameter of the glass tube is changed. Assume a water temperature of 80 °F. Show your results on a graph of percent error versus tube diameter, D, in the range 0.1 in. $< D <$ 1.0 in. (b) If you want the error to be less than 1%, what is the smallest tube diameter allowed?

(a) The excess height, h, caused be the surface tension is

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For $\theta \cong 0°$ with $D = 2R$

$$h = \frac{4\sigma}{\gamma D} \qquad (1)$$

From Table B.1 in Appendix B for water at 80°F
$\sigma = 4.91 \times 10^{-3}$ lb/ft and $\gamma = 62.22$ lb/ft³.

Thus, from Eq. (1)

$$h(\text{ft}) = \frac{4\left(4.91\times10^{-3}\ \frac{\text{lb}}{\text{ft}}\right)}{\left(62.22\ \frac{\text{lb}}{\text{ft}^3}\right)\frac{D(\text{in.})}{12\ \text{in./ft}}} = \frac{3.79\times10^{-3}}{D(\text{in.})} \qquad (2)$$

Since

$$\%\ \text{error} = \frac{h(\text{ft})}{3\ \text{ft}} \times 100 \quad \text{(with the true depth} = 3\ \text{ft)}$$

it follows from Eq. (2) that

$$\%\ \text{error} = \frac{3.79\times10^{-3}}{3\ D(\text{in.})} \times 100$$

$$= \frac{0.126}{D(\text{in.})} \qquad (3)$$

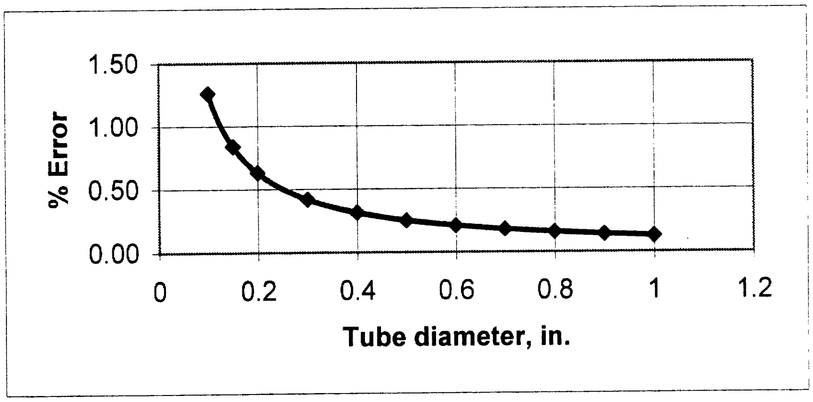
A plot of % error versus tube diameter is shown on the next page.

(con't)

1.85 (con't)

Diameter of tube, in.	% Error
0.1	1.26
0.15	0.84
0.2	0.63
0.3	0.42
0.4	0.32
0.5	0.25
0.6	0.21
0.7	0.18
0.8	0.16
0.9	0.14
1	0.13

Values obtained from Eq. (3)



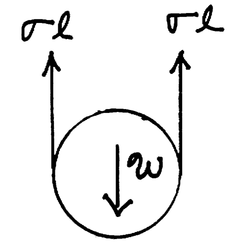
(b) For 1% error from Eq. (3)

$$1 = \frac{0.126}{D(\text{in.})}$$

$$D = \underline{\underline{0.126\ \text{in.}}}$$

1.86

1.86 Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. (See Video V1.5.) Consider placing a short length of a small diameter steel (sp. wt. = 490 lb/ft^3) rod on a surface of water. What is the maximum diameter that the rod can have before it will sink? Assume that the surface tension forces act vertically upward. *Note:* A standard paper clip has a diameter of 0.036 in. Partially unfold a paper clip and see if you can get it to float on water. Do the results of this experiment support your analysis?



l = rod length

In order for rod to float (see figure) it follows that

$$2\sigma l \geq \mathcal{W} \geq \left(\frac{\pi}{4}\right)(D^2) l \, \gamma_{steel}$$

Thus, for the limiting case

$$D_{max}^2 = \frac{2\sigma l}{\left(\frac{\pi}{4}\right) l \, \gamma_{steel}} = \frac{8\sigma}{\pi \gamma_{steel}}$$

so that

$$D_{max} = \left[\frac{8\left(5.03 \times 10^{-3} \frac{lb}{ft}\right)}{\pi\left(490 \frac{lb}{ft^3}\right)}\right]^{1/2} = 5.11 \times 10^{-3} \text{ ft}$$

$$= \underline{\underline{0.0614 \text{ in.}}}$$

Since a standard steel paper clip has a diameter of 0.036 in., which is less than 0.0614 in., it should float. A simple experiment will verify this. <u>Yes</u>.

1.87

1.87 An open, clean glass tube, having a diameter of 3 mm, is inserted vertically into a dish of mercury at 20 °C. How far will the column of mercury in the tube be depressed?

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad (Eq.\ 1.22)$$

For $\theta = 130°$,

$$h = \frac{2\left(4.66 \times 10^{-1}\ \frac{N}{m}\right)\cos 130°}{\left(133 \times 10^{3}\ \frac{N}{m^3}\right)(0.0015\ m)} = -3.00 \times 10^{-3}\ m$$

Thus, column will be depressed $\underline{\underline{3.00\ mm}}$

1.88

1.88 An open 2-mm-diameter tube is inserted into a pan of ethyl alcohol and a similar 4-mm-diameter tube is inserted into a pan of water. In which tube will the height of the rise of the fluid column due to capillary action be the greatest? Assume the angle of contact is the same for both tubes.

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad (Eq.\ 1.22)$$

Thus,

$$\frac{h\,(alcohol)}{h\,(water)} = \frac{\sigma\,(alcohol)\ \gamma\,(water)}{\sigma\,(water)\ \gamma\,(alcohol)}\left(\frac{4\ mm}{2\ mm}\right)$$

$$= \frac{\left(2.28 \times 10^{-2}\ \frac{N}{m}\right)\left(9.80 \times 10^{3}\ \frac{N}{m^3}\right)(4\ mm)}{\left(7.34 \times 10^{-2}\ \frac{N}{m}\right)\left(7.74 \times 10^{3}\ \frac{N}{m^3}\right)(2\ mm)}$$

$$= 0.787$$

Height of rise of $\underline{\underline{\text{water column}}}$ is greatest.

1.89 *

1.89* The capillary rise in a tube depends on the cleanliness of both the fluid and the tube. Typically, values of h are less than those predicted by Eq. 1.22 using values of σ and θ for clean fluids and tubes. Some measurements of the height, h, a water column rises in a vertical open tube of diameter, d, are given below. The water was tap water at a temperature of 60 °F and no particular effort was made to clean the glass tube. Fit a curve to these data and estimate the value of the product $\sigma \cos \theta$. If it is assumed that σ has the value given in Table 1.5 what is the value of θ? If it is assumed that θ is equal to 0° what is the value of σ?

d (in.)	0.3	0.25	0.20	0.15	0.10	0.05
h (in.)	0.133	0.165	0.198	0.273	0.421	0.796

From Eq. 1.22

$$h = \frac{2\sigma \cos\theta}{\gamma}\left(\frac{1}{R}\right) = \frac{4\sigma \cos\theta}{\gamma}\left(\frac{1}{d}\right) \qquad (1)$$

with $d = 2R$. Thus, Eq. (1) is of the form

$$h = b\, d' \qquad (2)$$

where:

$$b = \frac{4\sigma\cos\theta}{\gamma} \quad \text{and} \quad d' = \frac{1}{d}$$

The constant, b, can be obtained by a linear least squares fit of the given data (h and $1/d$).

$1/d$ (ft^{-1})	h (ft)
40	0.01108
48	0.01375
60	0.01650
80	0.02275
120	0.03508
240	0.06633

(con't)

1.89*

(con't)

To obtain b use LINREG 1.

```
************************************************
** This program determines the least squares fit **
** for a function of the form   y = b * x        **
************************************************

Number of points: 6
Input X, Y
? 40,0.01108
? 48,0.01375
? 60,0.01650
? 80,0.02275
? 120,0.03508
? 240,0.06633

b = +2.799E-04  ft^2

     X              Y             Y(predicted)
+4.0000E+01    +1.1080E-02    +1.1195E-02
+4.8000E+01    +1.3750E-02    +1.3434E-02
+6.0000E+01    +1.6500E-02    +1.6792E-02
+8.0000E+01    +2.2750E-02    +2.2390E-02
+1.2000E+02    +3.5080E-02    +3.3584E-02
+2.4000E+02    +6.6330E-02    +6.7169E-02
```

Thus,

$$\sigma \cos\theta = \frac{b\gamma}{4}$$

$$= \frac{(2.799 \times 10^{-4}\ ft^2)(62.4\ \frac{lb}{ft^3})}{4} = \underline{\underline{4.37 \times 10^{-3}\ \frac{lb}{ft}}}$$

If $\sigma = 5.03 \times 10^{-3}$ lb/ft, then

$$\cos\theta = \frac{4.37 \times 10^{-3}\ \frac{lb}{ft}}{5.03 \times 10^{-3}\ \frac{lb}{ft}} = 0.869$$

and

$$\underline{\underline{\theta = 29.7^\circ}}$$

If $\theta = 0^\circ$ Then $\cos\theta = 1.0$ and

$$\sigma = \frac{4.37 \times 10^{3}\ \frac{lb}{ft}}{1.0} = \underline{\underline{4.37 \times 10^{3}\ \frac{lb}{ft}}}$$

1.90

1.90 Fluid Characterization by Use of a Stormer Viscometer

Objective: As discussed in Section 1.6, some fluids can be classified as Newtonian fluids; others are non-Newtonian. The purpose of this experiment is to determine the shearing stress versus rate of strain characteristics of various liquids and, thus, to classify them as Newtonian or non-Newtonian fluids.

Equipment: Stormer viscometer containing a stationary outer cylinder and a rotating, concentric inner cylinder (see Fig. P1.90); stop watch; drive weights for the viscometer; three different liquids (silicone oil, Latex paint, and corn syrup).

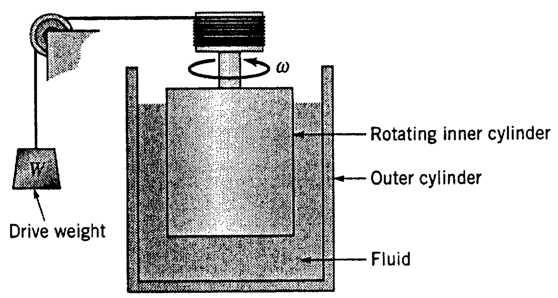
Experimental Procedure: Fill the gap between the inner and outer cylinders with one of the three fluids to be tested. Select an appropriate drive weight (of mass m) and attach it to the end of the cord that wraps around the drum to which the inner cylinder is fastened. Release the brake mechanism to allow the inner cylinder to start to rotate. (The outer cylinder remains stationary.) After the cylinder has reached its steady-state angular velocity, measure the amount of time, t, that it takes the inner cylinder to rotate N revolutions. Repeat the measurements using various drive weights. Repeat the entire procedure for the other fluids to be tested.

Calculations: For each of the three fluids tested, convert the mass, m, of the drive weight to its weight, $W = mg$, where g is the acceleration of gravity. Also determine the angular velocity of the inner cylinder, $\omega = N/t$.

Graph: For each fluid tested, plot the drive weight, W, as ordinates and angular velocity, ω, as abscissas. Draw a best fit curve through the data.

Results: Note that for the flow geometry of this experiment, the weight, W, is proportional to the shearing stress, τ, on the inner cylinder. This is true because with constant angular velocity, the torque produced by the viscous shear stress on the cylinder is equal to the torque produced by the weight (weight times the appropriate moment arm). Also, the angular velocity, ω, is proportional to the rate of strain, du/dy. This is true because the velocity gradient in the fluid is proportional to the inner cylinder surface speed (which is proportional to its angular velocity) divided by the width of the gap between the cylinders. Based on your graphs, classify each of the three fluids as to whether they are Newtonian, shear thickening, or shear thinning (see Fig. 1.5).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P1.90

(con't)

1.90 (con't)

Solution for Problem 1.90: Fluid Characterization by Use of a Stormer Viscometer

m, kg	N, revs	t, s	ω, rev/s	W, N
Silicone Oil Data				
0.02	4	59.3	0.07	0.20
0.05	12	66.0	0.18	0.49
0.10	24	64.2	0.37	0.98
0.15	20	35.0	0.57	1.47
0.20	24	31.7	0.76	1.96
0.25	30	31.0	0.97	2.45
0.30	20	17.4	1.15	2.94
0.35	25	18.8	1.33	3.43
0.40	40	26.0	1.54	3.92
Corn Syrup Data				
0.05	1	28.2	0.04	0.49
0.10	2	27.5	0.07	0.98
0.20	4	27.2	0.15	1.96
0.40	8	25.7	0.31	3.92
Latex Paint Data				
0.02	2	32.7	0.06	0.20
0.03	2	20.2	0.10	0.29
0.04	5	32.2	0.16	0.39
0.05	10	47.3	0.21	0.49
0.06	10	37.2	0.27	0.59
0.07	10	29.8	0.34	0.69
0.08	10	24.6	0.41	0.78
0.09	10	20.1	0.50	0.88
0.10	20	34.0	0.59	0.98

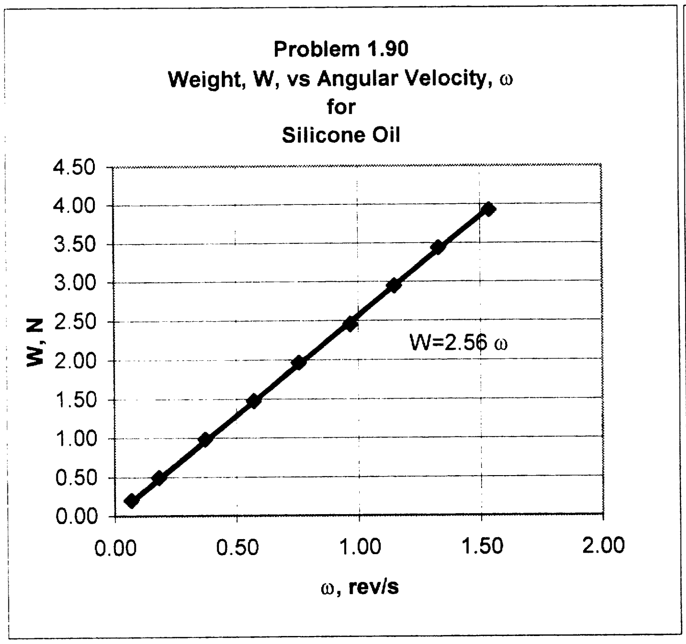
From the graphs:
Silicone oil is Newtonian
Corn Syrup is Newtonian
Latex paint is shear thinning

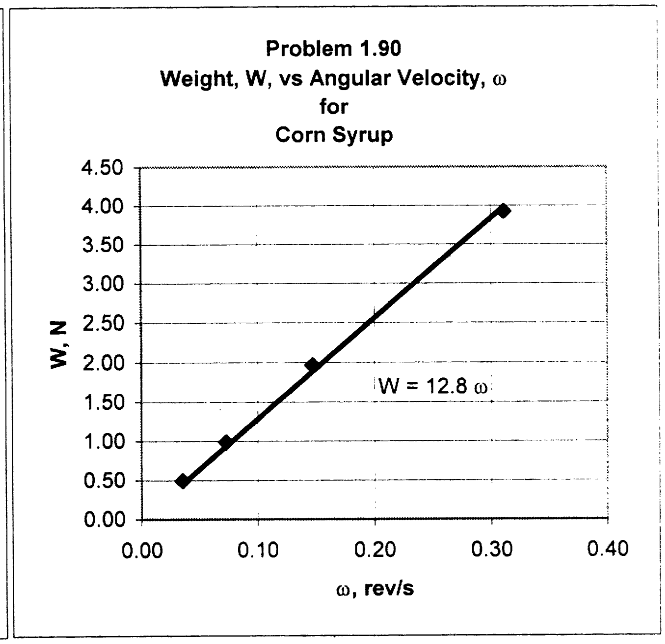
$\omega = N/t$

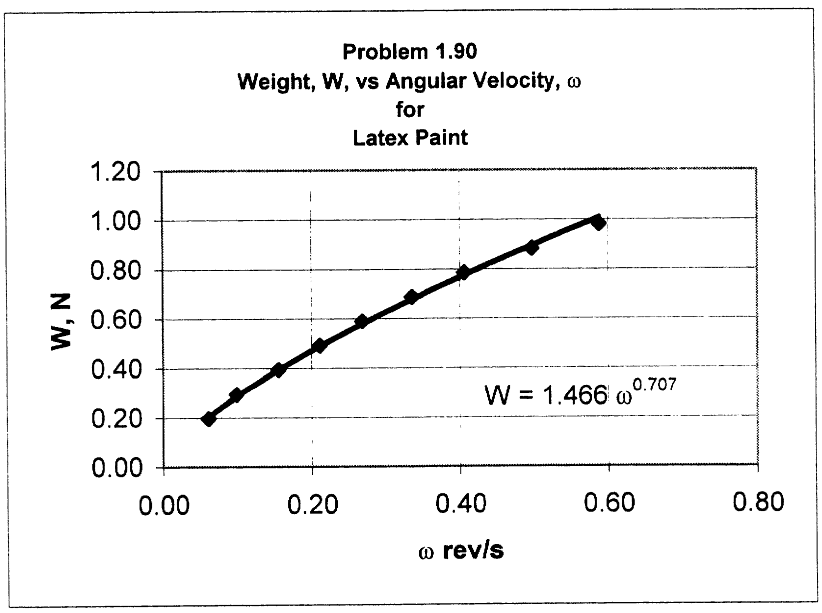
$W = mg$

(con't)

1.90 (Con't)


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Silicone Oil
W=2.56 ω
W, N
ω, rev/s
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
0.50
1.00
1.50
2.00


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Corn Syrup
W = 12.8 ω
W, N
ω, rev/s
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
0.10
0.20
0.30
0.40


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Latex Paint
W = 1.466 ω^0.707
W, N
ω rev/s
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.20
0.40
0.60
0.80

1.91

1.91 Capillary Tube Viscometer

Objective: The flowrate of a viscous fluid through a small diameter (capillary) tube is a function of the viscosity of the fluid. For the flow geometry shown in Fig. P1.91, the kinematic viscosity, ν, is inversely proportional to the flowrate, Q. That is, $\nu = K/Q$, where K is the calibration constant for the particular device. The purpose of this experiment is to determine the value of K and to use it to determine the kinematic viscosity of water as a function of temperature.

Equipment: Constant temperature water tank, capillary tube, thermometer, stop watch, graduated cylinder.

Experimental Procedure: Adjust the water temperature to 15.6°C and determine the flowrate through the capillary tube by measuring the time, t, it takes to collect a volume, V, of water in a small graduated cylinder. Repeat the measurements for various water temperatures, T. Be sure that the water depth, h, in the tank is the same for each trial. Since the flowrate is a function of the depth (as well as viscosity), the value of K obtained will be valid for only that value of h.

Calculations: For each temperature tested, determine the flowrate, $Q = V/t$. Use the data for the 15.6°C water to determine the calibration constant, K, for this device. That is, $K = \nu Q$, where the kinematic viscosity for 15.6°C water is given in Table 1.5 and Q is the measured flowrate at this temperature. Use this value of K and your other data to determine the viscosity of water as a function of temperature.

Graph: Plot the experimentally determined kinematic viscosity, ν, as ordinates and temperature, T, as abscissas.

Results: On the same graph, plot the standard viscosity-temperature data obtained from Table B.2.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

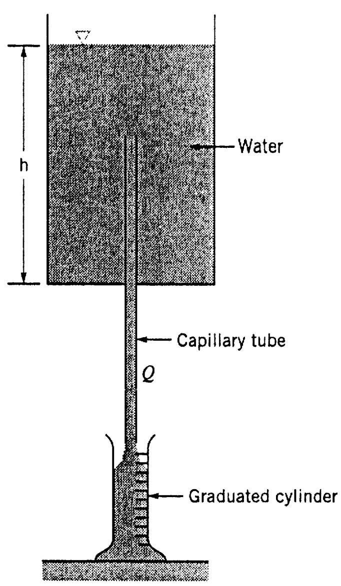


FIGURE P1.91

(con't)

1.91 (Con't)

Solution for Problem 1.91: Capillary Tube Viscometer

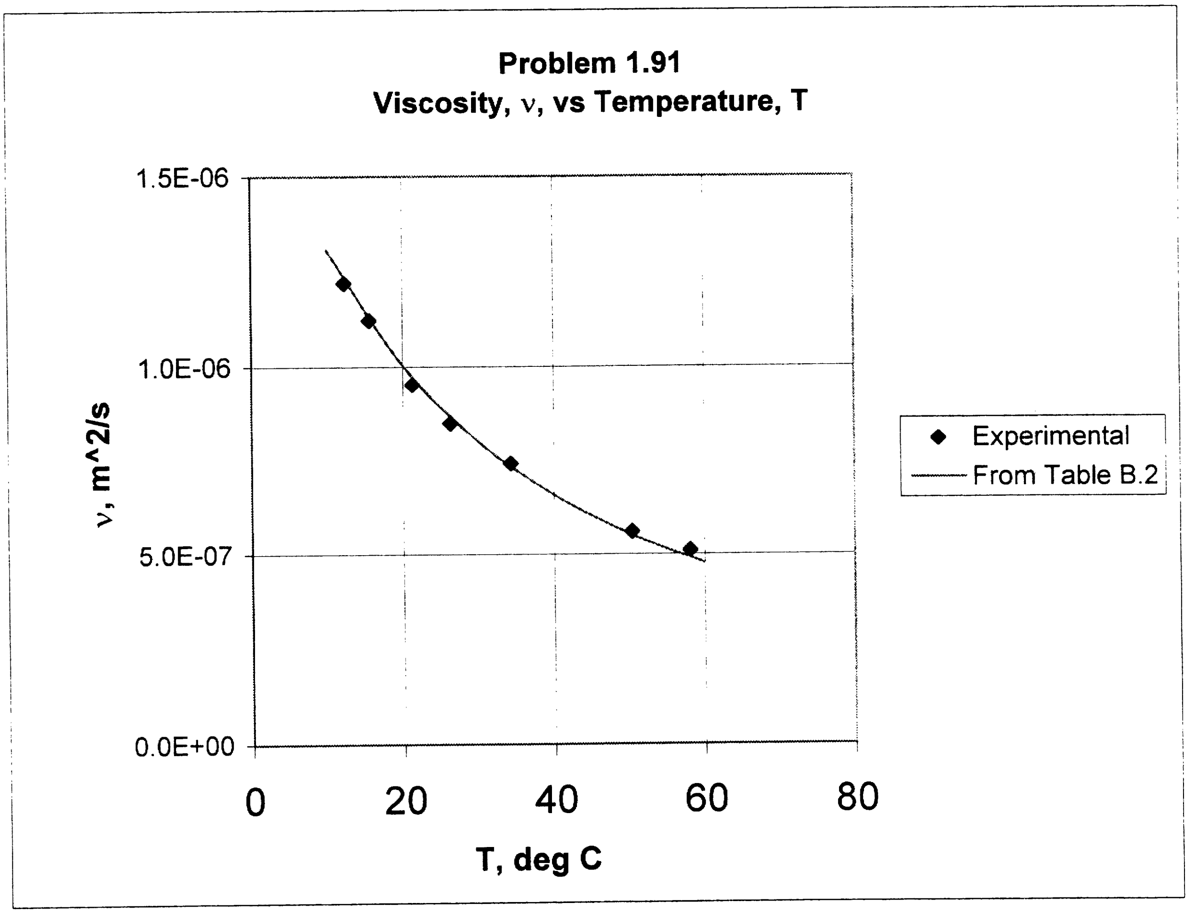
V, ml	t, s	T, deg C	Q, ml/s	ν, m^2/s
9.2	19.8	15.6	0.465	1.12E-06
9.7	15.8	26.3	0.614	8.49E-07
9.2	16.8	21.3	0.548	9.51E-07
9.1	21.3	12.3	0.427	1.22E-06
9.2	13.1	34.3	0.702	7.42E-07
9.4	10.1	50.4	0.931	5.60E-07
9.1	8.9	58.1	1.022	5.10E-07

From Table B.2

T, deg C	ν, m^2/s
10	1.31E-06
20	1.00E-06
30	8.01E-07
40	6.58E-07
50	5.53E-07
60	4.75E-07

ν = K/Q　　K, m^2 ml/s^2　5.21E-07　　ν (at 15.6 deg C), m^2/s　1.12E-06

K = ν Q = 1.12E-6 m^2/s * 0.465 ml/s = 5.21E-7 m^2 ml/s^2



2.1

2.1 The water level in an open standpipe is 80 ft above the ground. What is the static pressure at a fire hydrant that is connected to the standpipe and located at ground level? Express your answer in psi.

$$p = \gamma h + p_o$$

Since the standpipe is open $p_o = 0$, and therefore

$$p = \left(62.4 \frac{lb}{ft^3}\right)(80\ ft)\left(\frac{1\ ft^2}{144\ in.^2}\right) = \underline{\underline{34.7\ psi}}$$

2.2

2.2 Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). As shown in **Video V2.1**, such pressures are commonly measured with a mercury manometer. A typical value for this ratio for a human would be 120/70, where the pressures are in mm Hg. **(a)** What would these pressures be in pascals? **(b)** If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

$$p = \gamma h$$

(a) For 120 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.120\ m) = \underline{\underline{16.0\ kPa}}$

For 70 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.070\ m) = \underline{\underline{9.31\ kPa}}$

(b) For 120 mm Hg: $p = \left(16.0 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{lb/in.^2}{N/m^2}\right)$

$$= 2.32\ psi$$

Since a typical tire pressure is 30-35 psi, 120 mm Hg is <u>not sufficient</u> for normal driving.

2.3

2.3 What pressure, expressed in pascals, will a skin diver be subjected to at a depth of 40 m in seawater?

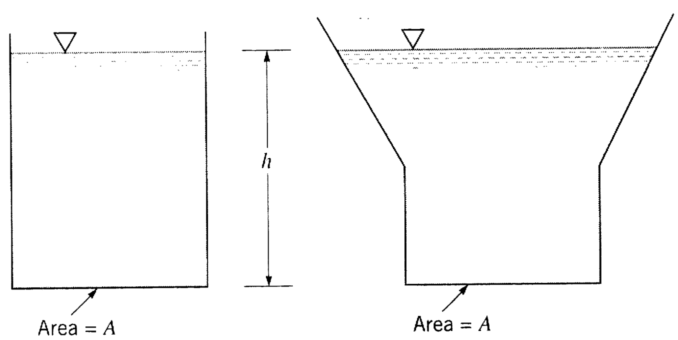
$$p = \gamma h + p_o$$

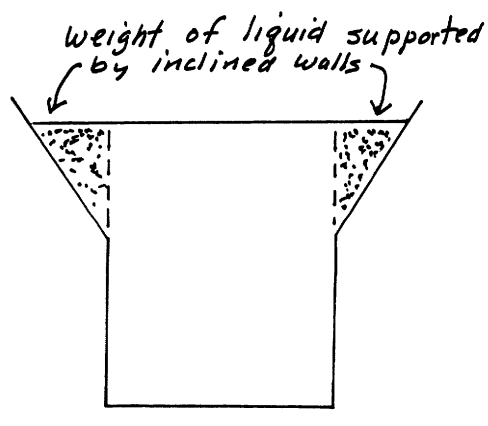
At the surface $p_o = 0$ so that

$$p = \left(10.1 \times 10^3 \frac{N}{m^3}\right)(40\,m) = 404 \times 10^3 \frac{N}{m^2} = \underline{\underline{404\ kPa}}$$

2.4

2.4 The two open tanks shown in Fig. P2.4 have the same bottom area, A, but different shapes. When the depth, h, of a liquid in the two tanks is the same, the pressure on the bottom of the two tanks will be the same in accordance with Eq. 2.7. However, the weight of the liquid in each of the tanks is different. How do you account for this apparent paradox?

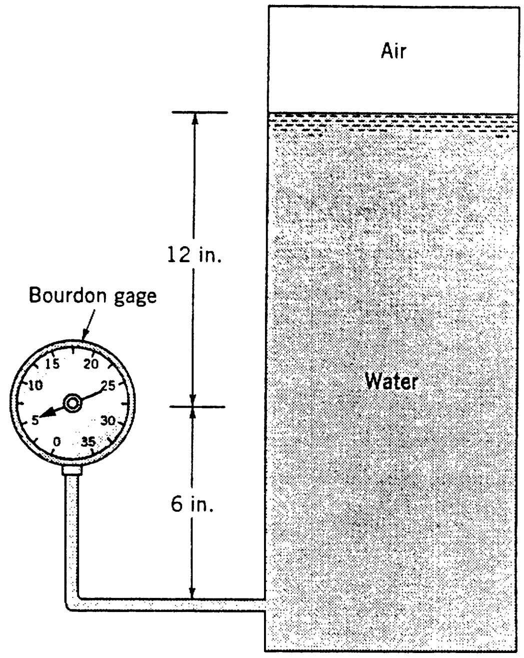




For the tank with the inclined walls, the pressure on the bottom is due to the weight of the liquid in the column directly above the bottom as shown by the dashed lines in the figure. This is the same weight as that for the tank with the straight sides. Thus, the pressure on the bottom of the two tanks is the same. The additional weight in the tank with the inclined walls is supported by the inclined walls, as illustrated in the figure.

2.5

2.5 Bourdon gages (see Video V2.2 and Fig. 2.13) are commonly used to measure pressure. When such a gage is attached to the closed water tank of Fig. P2.5 the gage reads 5 psi. What is the absolute air pressure in the tank? Assume standard atmospheric pressure of 14.7 psi.



■ FIGURE P2.5

$$p = \gamma h + p_o$$

$$p_{gage} - \left(\frac{12}{12}\ ft\right)\gamma_{H_2O} = p_{air}$$

$$p_{air} = \left(5\ \frac{lb}{in.^2} + 14.7\ \frac{lb}{in.^2}\right) - \frac{(1\ ft)(62.4\ \frac{lb}{ft^3})}{144\ \frac{in.^2}{ft^2}}$$

$$p_{air} = \underline{\underline{19.3\ psia}}$$

2.6

2.6 Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of 10.1 kN/m^3? Express your answer in pascals and psi.

$$p = \gamma h + p_o$$

At the surface $p_o = 0$ so that

$$p = (10.1 \times 10^3\ \frac{N}{m^3})(5 \times 10^3\ m) = 50.5 \times 10^6\ \frac{N}{m^2} = \underline{\underline{50.5\ MPa}}$$

Also,

$$p = \left(50.5 \times 10^6\ \frac{N}{m^2}\right)\left(1.450 \times 10^{-4}\ \frac{\frac{lb}{in.^2}}{\frac{N}{m^2}}\right) = \underline{\underline{7320\ psi}}$$

2.7

2.7 For the great depths that may be encountered in the ocean the compressibility of seawater may become an important consideration. **(a)** Assume that the bulk modulus for seawater is constant and derive a relationship between pressure and depth which takes into account the change in fluid density with depth. **(b)** Make use of part (a) to determine the pressure at a depth of 6 km assuming seawater has a bulk modulus of 2.3×10^9 Pa, and a density of 1030 kg/m^3 at the surface. Compare this result with that obtained by assuming a constant density of 1030 kg/m^3.

(a)

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad (\text{Eq. } 2.4)$$

Thus,

$$\frac{dp}{\rho} = -g\,dz \qquad (1)$$

If ρ is a function of p, we must determine $\rho = f(p)$ before integrating Eq. (1). Since,

$$E_v = \frac{dp}{d\rho/\rho} \qquad (\text{Eq. } 1.13)$$

then

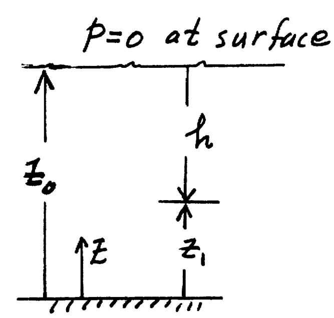
$$\int_0^p dp = E_v \int_{\rho_0}^{\rho} \frac{d\rho}{\rho}$$

so that

$$p = E_v \ln \frac{\rho}{\rho_0}$$

Thus,

$$\rho = \rho_0 e^{\frac{p}{E_v}} \qquad \text{where } \rho = \rho_0 \text{ at } p = 0$$



From Eq. (1)

$$\int_{p_1}^{0} \frac{dp}{\rho_0 e^{\frac{p}{E_v}}} = -g \int_{z_1}^{z_0} dz$$

or

$$\int_{p_1}^{0} e^{-\frac{p}{E_v}}\, dp = -\rho_0 g \int_{z_1}^{z_0} dz$$

so that

$$p = -E_v \ln\left(1 - \frac{\rho_0 g h}{E_v}\right)$$

where, $h = z_0 - z_1$, the depth below surface

(con't)

2.7 (con't)

(b) From part (a),

$$p = -E_v \ln\left(1 - \frac{\rho_0 g h}{E_v}\right)$$

so that at $h = 6\ km$

$$p = -\left(2.3\times10^9\ \frac{N}{m^2}\right)\ln\left[1 - \frac{\left(1.03\times10^3\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(6\times10^3\ m\right)}{2.3\times10^9\ \frac{N}{m^2}}\right]$$

$$= 6.14\times10^7\ \frac{N}{m^2} = \underline{\underline{61.4\ MPa}}$$

(c) For constant density

$$p = \gamma h = \rho g h = \left(1.03\times10^3\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(6\times10^3\ m\right)$$

$$= \underline{\underline{60.6\ MPa}}$$

2.8

2.8 Blood pressure is commonly measured with a cuff placed around the arm, with the cuff pressure (which is a measure of the arterial blood pressure) indicated with a mercury manometer (see Video 2.1). A typical value for the maximum value of blood pressure (systolic pressure) is 120 mm Hg. Why wouldn't it be simpler, and cheaper, to use water in the manometer rather than mercury? Explain and support your answer with the necessary calculations.

$$p = \gamma h$$

For 120 mm Hg:

$$p = \gamma h$$
$$= \left(133\times10^3\ \frac{N}{m^3}\right)(0.120\ m)$$
$$= 16.0\ kPa$$

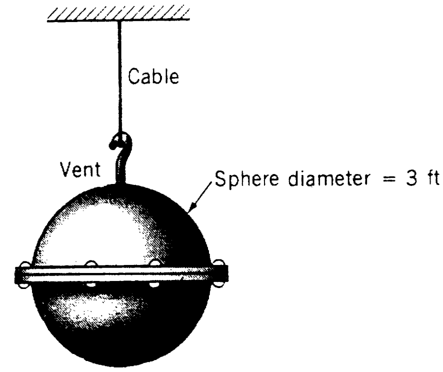
To obtain this pressure with a water column

$$h_{H_2O} = \frac{16.0\times10^3\ \frac{N}{m^2}}{9.80\times10^3\ \frac{N}{m^3}} = 1.63\ m \quad (\text{or } 5.35\ ft)$$

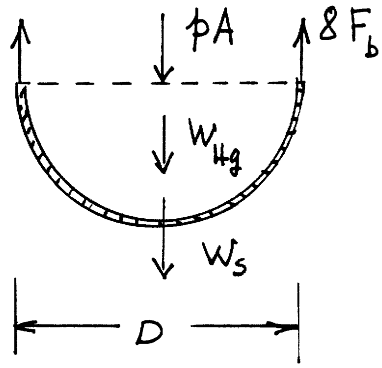
Thus, if water were used in the manometer the required column heights would be too high and impractical. $\underline{\underline{No}}$.

2.9

2.9 Two hemispherical shells are bolted together as shown in Fig. P2.9 . The resulting spherical container, which weighs 400 lb, is filled with mercury and supported by a cable as shown. The container is vented at the top. If eight bolts are symmetrically located around the circumference, what is the vertical force that each bolt must carry?



■ FIGURE P2.9



F_b ~ force in one bolt

p ~ pressure at mid-plane

A ~ area at mid-plane

W_{Hg} ~ weight of mercury in bottom half of shell

W_s ~ weight of bottom half of shell

For equilibrium,

$$\sum F_{vertical} = 0$$

Thus,

$$8F_b = pA + W_{Hg} + W_s$$

$$= \gamma_{Hg}\left(\frac{D}{2}\right)\left(\frac{\pi}{4}D^2\right) + \gamma_{Hg}\left(\frac{1}{2}\right)\left(\frac{\pi}{6}D^3\right) + \frac{1}{2}\,(400\ lb)$$

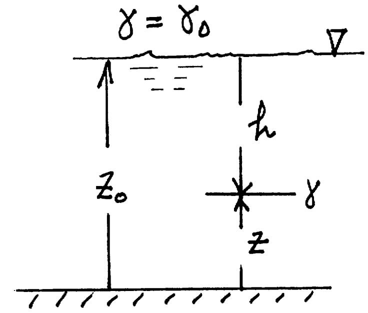
$$= \left(847\ \frac{lb}{ft^3}\right)\left(\frac{3\ ft}{2}\right)\left(\frac{\pi}{4}\right)(3\ ft)^2 + \left(847\ \frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right)(3\ ft)^3 + 200\ lb$$

and

$$F_b = \underline{\underline{1910\ lb}}$$

2.10

2.10 Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth, h, as $\gamma = Kh + \gamma_0$, where K is a constant and γ_0 is the specific weight at the free surface.



$$\frac{dp}{dz} = -\gamma \qquad \text{(Eq. 2.4)}$$

Let $h = z_0 - z$

so that $dh = -dz$

Thus,

$$dp = \gamma\, dh$$

and

$$\int_0^p dp = \int_0^h \gamma\, dh$$

For $\gamma = Kh + \gamma_0$,

$$\int_0^p dp = \int_0^h (Kh + \gamma_0)\, dh$$

and

$$\underline{\underline{p = \frac{Kh^2}{2} + \gamma_0 h}}$$

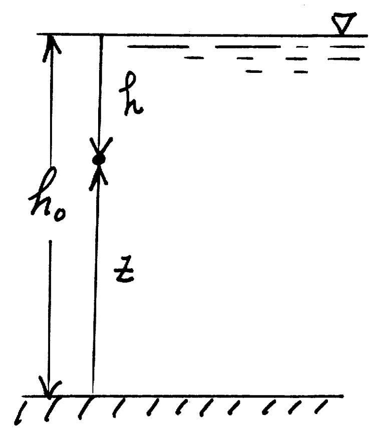
2.11 *

2.11* In a certain liquid at rest, measurements of the specific weight at various depths show the following variation:

h (ft)	γ (lb/ft³)
0	70
10	76
20	84
30	91
40	97
50	102
60	107
70	110
80	112
90	114
100	115

The depth, $h = 0$, corresponds to a free surface at atmospheric pressure. Determine, through numerical integration of Eq. 2.4, the corresponding variation in pressure, and show the results on a plot of pressure (in psf) versus depth (in feet).

$$\frac{dp}{dz} = -\gamma \qquad \text{(Eq. 2.4)}$$



Let $z = h_o - h$ (see figure) so that $dz = -dh$ and therefore

$$dp = -\gamma\, dz = \gamma\, dh$$

Thus,

$$\int_0^{p_i} dp = \int_0^{h_i} \gamma\, dh$$

or

$$p_i = \int_0^{h_i} \gamma\, dh \qquad (1)$$

where p_i is the pressure at depth h_i. Equation (1) can be integrated numerically using the following program. (Note: The numerical integration can also be accomplished through repeated use of the program TRAPEZO1).

(con't)

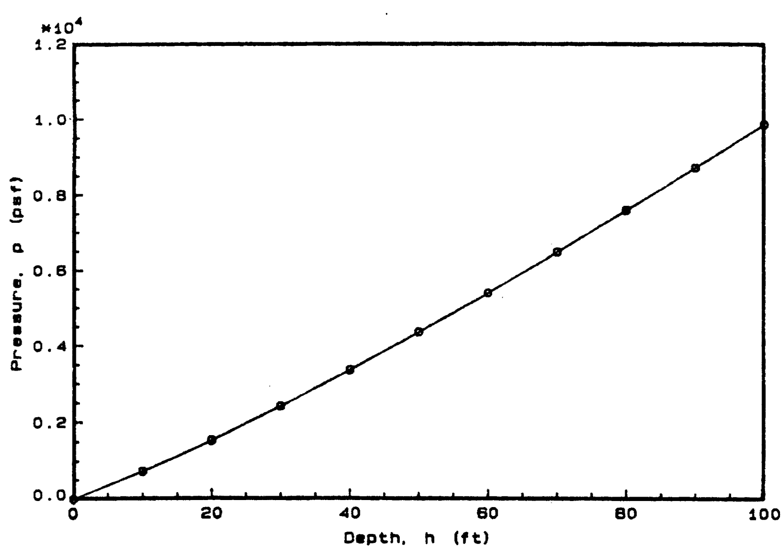
2.11 * (con't)

```
100 cls
110 print "**********************************************"
120 print "** This program integrates Eq. 2.4 numerically **"
130 print "** using the trapezoidal rule to obtain the    **"
140 print "** pressure at different depths                **"
150 print "**********************************************"
160 print
170 dim p(11),gamma(11)
180 n=11
190 dh=10
200 p(1)=0
210 for i=1 to n
220 read gamma(i)
230 next i
240 data 70,76,84,91,97,102,107,110,112,114,115
250 for i=2 to n
260 s=(gamma(1)+gamma(i))/2
270 im1=i-1
280 for j=2 to im1
290 s=s+gamma(j)
300 next j
310 p(i)=dh*s
320 next i
330 '
340 'Print the results
350 print
360 print " h (ft)   Pressure (psf)"
370 for i=1 to n
380 print using "###.#       #####.#";(i-1)*dh,p(i)
390 next i
```

The tabulated results are given below, along with the corresponding plot of pressure vs. depth.

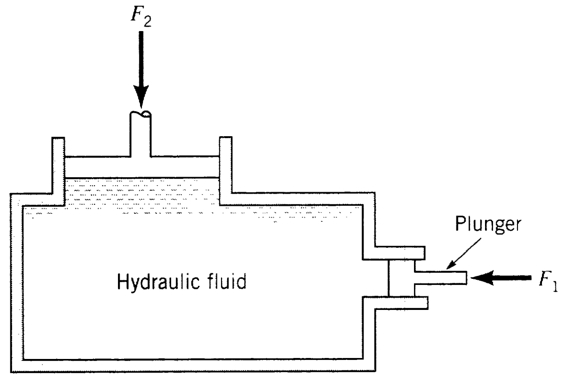
```
************************************************
** This program integrates Eq. 2.4 numerically **
** using the trapezoidal rule to obtain the    **
** pressure at different depths                **
************************************************
```

h (ft)	Pressure (psf)
0.0	0.0
10.0	730.0
20.0	1530.0
30.0	2405.0
40.0	3345.0
50.0	4340.0
60.0	5385.0
70.0	6470.0
80.0	7580.0
90.0	8710.0
100.0	9855.0



2.12

2.12 The basic elements of a hydraulic press are shown in Fig. P2.12. The plunger has an area of 1 in.^{2}, and a force, F_1, can be applied to the plunger through a lever mechanism having a mechanical advantage of 8 to 1. If the large piston has an area of 150 in.^{2}, what load, F_2, can be raised by a force of 30 lb applied to the lever? Neglect the hydrostatic pressure variation.



A force of 30 lb applied to the lever results in a plunger force, F_1, of $F_1 = (8)(30) = 240\ lb$.
Since $F_1 = pA_1$ and $F_2 = pA_2$ where p is the pressure and A_1 and A_2 are the areas of the plunger and piston, respectively. Since p is constant throughout the chamber,

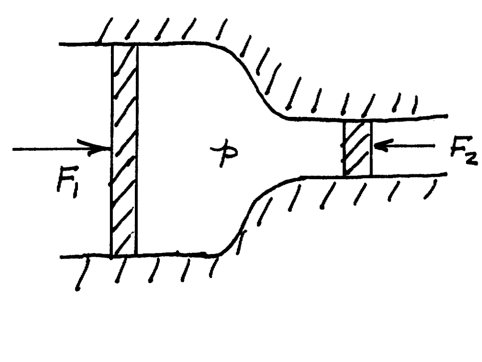
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

so that

$$F_2 = \frac{A_2}{A_1} F_1 = \left(\frac{150\ in.^2}{1\ in.^2}\right)(240\ lb) = \underline{\underline{36{,}000\ lb}}$$

2.13

2.13 A 0.3-m-diameter pipe is connected to a 0.02-m-diameter pipe and both are rigidly held in place. Both pipes are horizontal with pistons at each end. If the space between the pistons is filled with water, what force will have to be applied to the larger piston to balance a force of 80 N applied to the smaller piston? Neglect friction.



$F_1 = pA_1$

$F_2 = pA_2$

Thus,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

or

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{(0.3\ m)^2}{(0.02\ m)^2}(80\ N) = \underline{\underline{18{,}000\ N}}$$

2.15

2.15 What would be the barometric pressure reading, in mm Hg, at an elevation of 4 km in the U.S. standard atmosphere? (Refer to Table C.2 in Appendix C.)

At an elevation of 4 km, $p = 6.166 \times 10^4 \frac{N}{m^2}$ (from Table C.2 in Appendix C). Since

$$p = \gamma h$$

$$h = \frac{p}{\gamma} = \frac{6.166 \times 10^4 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}} = 0.464\,m = \underline{\underline{464\,mm}}$$

2.16

2.16 An absolute pressure of 7 psia corresponds to what gage pressure for standard atmospheric pressure of 14.7 psia?

$$p\,(abs) = p\,(gage) + p\,(atm)$$

Thus,

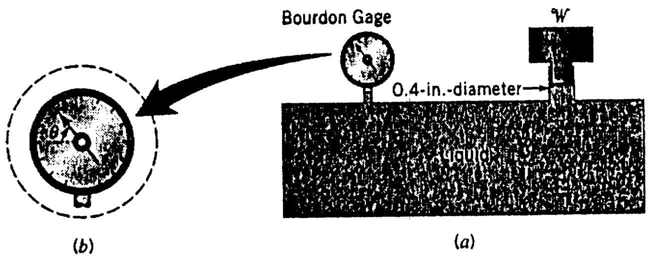
$$p\,(gage) = p\,(abs) - p\,(atm)$$

$$= 7\,psia - 14.7\,psi = \underline{\underline{-7.7\ psi}}$$

2.17*

*2.17 A Bourdon gage (see Fig. 2.13 and Video V2.2) is often used to measure pressure. One way to calibrate this type of gage is to use the arangement shown in Fig. P2.17*a*. The container is filled with a liquid and a weight, $\mathcal{W}$, placed on one side with the gage on the other side. The weight acting on the liquid through a 0.4-in.-diameter opening creates a pressure that is transmitted to the gage. This arrangement, with a series of weights, can be used to determine what a change in the dial movement, θ, in Fig. P2.17*b*, corresponds to in terms of a change in pressure. For a particular gage, some data are given below. Based on a plot of these data, determine the relationship between θ and the pressure, p, where p is measured in psi?

$\mathcal{W}$ (lb)	0	1.04	2.00	3.23	4.05	5.24	6.31
θ (deg.)	0	20	40	60	80	100	120



■ FIGURE P2.17

$$p = \frac{\mathcal{W}}{Area} = \frac{\mathcal{W}\,(lb)}{\frac{\pi}{4}(0.4\text{ in.})^2} = 7.96\,\mathcal{W}(lb) \qquad (1)$$

(where p is in psi)

From graph

$$\mathcal{W} = 0.0522\,\theta$$

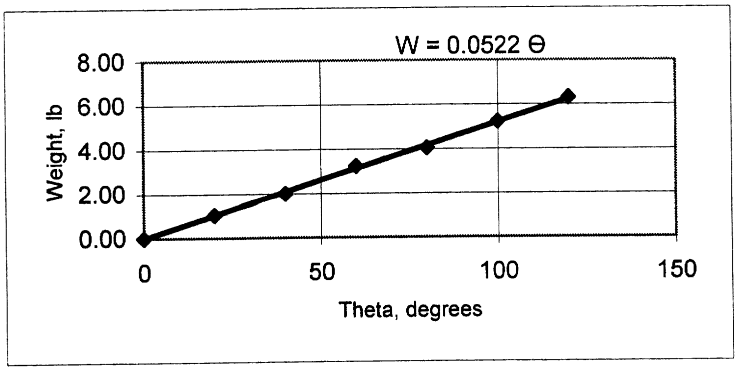
so that from Eq. (1)

$$\frac{p\,(psi)}{7.96} = 0.0522\,\theta$$

and

$$\underline{\underline{p\,(psi) = 0.416\,\theta}}$$

Theta, deg.	*W*, lb
0	0.00
20	1.04
40	2.00
60	3.23
80	4.05
100	5.24
120	6.31



2.18

2.18 For an atmospheric pressure of 101 kPa (abs) determine the heights of the fluid columns in barometers containing one of the following liquids: **(a)** mercury, **(b)** water, and **(c)** ethyl alcohol. Calculate the heights including the effect of vapor pressure, and compare the results with those obtained neglecting vapor pressure. Do these results support the widespread use of mercury for barometers? Why?

(Including vapor pressure)

$$p(atm) = \gamma h + p_v$$

where $p_v \sim$ vapor pressure

Thus, $$h = \frac{p(atm) - p_v}{\gamma}$$

(a) For mercury: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.6 \times 10^{-1} \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759 \ m}}$$

(b) For water: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.77 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.1 \ m}}$$

(c) For ethyl alcohol: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 5.9 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{12.3 \ m}}$$

(Without vapor pressure)

$$p(atm) = \gamma h$$

$$h = \frac{p(atm)}{\gamma}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759 \ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.3 \ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{13.0 \ m}}$$

<u>Yes</u>. For mercury barometers the effect of vapor pressure is negligible, and the required height of the mercury column is reasonable.

2.19

2.19 Aneroid barometers can be used to measure changes in altitude. If a barometer reads 30.1 in. Hg at one elevation, what has been the change in altitude in meters when the barometer reading is 28.3 in. Hg? Assume a standard atmosphere, and that Eq. 2.12 is applicable over the range of altitudes of interest.

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad \text{(Eq. 2.12)}$$

At $z = z_1$, $\quad p = p_1 = p_a \left(1 - \frac{\beta z_1}{T_a}\right)^{\frac{g}{R\beta}}$

or

$$\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} = 1 - \frac{\beta z_1}{T_a} \qquad (1)$$

Similarly, for $z = z_2$,

$$\left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}} = 1 - \frac{\beta z_2}{T_a} \qquad (2)$$

Subtract Eq. (2) from Eq. (1) to obtain,

$$z_2 - z_1 = \frac{T_a}{\beta}\left[\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} - \left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}}\right] \qquad (3)$$

For $T_a = 288\ K$, $\beta = 0.00650\ \frac{K}{m}$, $p_a = 101\ kPa$, $g = 9.81\ \frac{m}{s^2}$, $R = 287\ \frac{J}{kg \cdot K}$, and

$$\frac{R\beta}{g} = \frac{\left(287\ \frac{J}{kg\cdot K}\right)\left(0.00650\ \frac{K}{m}\right)}{9.81\ \frac{m}{s^2}} = 0.190$$

with $\quad p_1 = \gamma_{Hg} h_1 = \left(133 \times 10^3\ \frac{N}{m^3}\right)(30.1\ in.)\left(2.540 \times 10^{-2}\ \frac{m}{in.}\right) = 102\ kPa$

and $\quad p_2 = \gamma_{Hg} h_2 = \left(133 \times 10^3\ \frac{N}{m^3}\right)(28.3\ in.)\left(2.540 \times 10^{-2}\ \frac{m}{in.}\right) = 95.6\ kPa$

then from Eq. (3)

$$z_2 - z_1 = \frac{288\ K}{0.00650\ \frac{K}{m}}\left[\left(\frac{102\ kPa}{101\ kPa}\right)^{0.190} - \left(\frac{95.6\ kPa}{101\ kPa}\right)^{0.190}\right]$$

$$= \underline{\underline{543\ m}}$$

2.20

2.20 Pikes Peak near Denver, Colorado has an elevation of 14,110 ft. **(a)** Determine the pressure at this elevation, based on Eq. 2.12. **(b)** If the air is assumed to have a constant specific weight of 0.07647 lb/ft³, what would the pressure be at this altitude? **(c)** If the air is assumed to have a constant temperature of 59 °F what would the pressure be at this elevation? For all three cases assume standard atmospheric conditions at sea level (see Table 2.1).

(a)
$$p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad \text{(Eq. 2.12)}$$

For $p_a = 2116.2\ \frac{lb}{ft^2}$, $\beta = 0.00357\ \frac{°R}{ft}$, $g = 32.174\ \frac{ft}{s^2}$, $T_a = 518.67\ °R$, $R = 1716\ \frac{ft \cdot lb}{slug \cdot °R}$, and

$$\frac{g}{R\beta} = \frac{32.174\ \frac{ft}{s^2}}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)\left(0.00357\ \frac{°R}{ft}\right)} = 5.252$$

then

$$p = \left(2116.2\ \frac{lb}{ft^2}\right)\left[1 - \frac{\left(0.00357\ \frac{°R}{ft}\right)(14{,}110\ ft)}{518.67\ °R}\right]^{5.252}$$

$$= \underline{\underline{1240\ \frac{lb}{ft^2}\ (abs)}}$$

(b)
$$p = p_a - \gamma h$$
$$= 2116.2\ \frac{lb}{ft^2} - \left(0.07647\ \frac{lb}{ft^3}\right)(14{,}110\ ft)$$
$$= \underline{\underline{1040\ \frac{lb}{ft^2}\ (abs)}}$$

(c)
$$p = p_a\, e^{-\frac{gh}{RT_a}} \qquad \text{(Eq. 2.10)}$$
$$= \left(2116.2\ \frac{lb}{ft^2}\right) e^{-\left[\frac{\left(32.174\ \frac{ft}{s^2}\right)(14{,}110\ ft)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(518.67\ °R)}\right]}$$

$$= \underline{\underline{1270\ \frac{lb}{ft^2}\ (abs)}}$$

2.21

2.21 Equation 2.12 provides the relationship between pressure and elevation in the atmosphere for those regions in which the temperature varies linearly with elevation. Derive this equation and verify the value of the pressure given in Table C.2 in Appendix C for an elevation of 5 km.

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} \qquad (\text{Eq. } 2.9)$$

Let $p_1 \sim p_a$ for $z_1 = 0$, $p_2 \sim p$ for $z_2 = z$, and $T = T_a - \beta z$.

Thus,

$$\int_{p_a}^{p} \frac{dp}{p} = -\frac{g}{R}\int_0^z \frac{dz}{T_a - \beta z}$$

or

$$\ln\frac{p}{p_a} = -\frac{g}{R}\left[-\frac{1}{\beta}\ln(T_a - \beta z)\right]_0^z = \frac{g}{R\beta}\left[\ln(T_a - \beta z) - \ln T_a\right]$$

$$= \frac{g}{R\beta}\ln\left(1 - \frac{\beta z}{T_a}\right)$$

and taking logarithm of both sides of equation yields

$$p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad (\text{Eq. } 2.12)$$

For $z = 5\,km$ with $p_a = 101.33\,kPa$, $T_a = 288.15\,K$, $g = 9.807\,\frac{m}{s^2}$, $\beta = 0.00650\,\frac{K}{m}$, $R = 287\,\frac{J}{kg \cdot K}$,

$$p = (101.33\,kPa)\left[1 - \frac{(0.0065\frac{K}{m})(5 \times 10^3\,m)}{288.15\,K}\right]^{\frac{9.807\frac{m}{s^2}}{(287\frac{J}{kg\cdot K})(0.0065\frac{K}{m})}}$$

$$= 5.40 \times 10^4\,\frac{N}{m^2}$$

(From Table C.2 in Appendix C, $p = 5.405 \times 10^4\,\frac{N}{m^2}$.)

2.22

2.22 As shown in Fig. 2.6 for the U.S. standard atmosphere, the troposphere extends to an altitude of 11 km where the pressure is 22.6 kPa (abs). In the next layer, called the stratosphere, the temperature remains constant at −56.5 °C. Determine the pressure and density in this layer at an altitude of 15 km. Assume $g = 9.77 \text{ m/s}^2$ in your calculations. Compare your results with those given in Table C.2 in Appendix C.

For isothermal conditions,

$$p_2 = p_1 e^{-\frac{g(z_2 - z_1)}{R T_0}} \qquad \text{(Eq. 2.10)}$$

Let $z_1 = 11 \text{ km}$, $p_1 = 22.6 \text{ kPa}$, $R = 287 \frac{J}{kg \cdot K}$, $g = 9.77 \frac{m}{s^2}$,

and $T_0 = -56.5°C + 273.15 = 216.65 \text{ K}$.

Thus,

$$p_2 = (22.6 \text{ kPa})\, e^{-\left[\frac{\left(9.77 \frac{m}{s^2}\right)\left(15 \times 10^3 m - 11 \times 10^3 m\right)}{\left(287 \frac{J}{kg \cdot K}\right)(216.65 K)}\right]}$$

$$= \underline{\underline{12.1 \text{ kPa}}}$$

Also,

$$\rho_2 = \frac{p}{RT} = \frac{12.1 \times 10^3 \frac{N}{m^2}}{\left(287 \frac{J}{kg \cdot K}\right)(216.65 K)} = \underline{\underline{0.195 \frac{kg}{m^3}}}$$

(From Table C.2 in Appendix C, $p_2 = 12.11 \text{ kPa}$ and $\rho_2 = 0.1948 \frac{kg}{m^3}$.)

2.23*

2.23* Under normal conditions the temperature of the atmosphere decreases with increasing elevation. In some situations, however, a temperature inversion may exist so that the air temperature increases with elevation. A series of temperature probes on a mountain give the elevation–temperature data shown in the table below. If the barometric pressure at the base of the mountain is 12.1 psia, determine by means of numerical integration the pressure at the top of the mountain.

Elevation (ft)	Temperature (°F)
5000	50.1 (base)
5500	55.2
6000	60.3
6400	62.6
7100	67.0
7400	68.4
8200	70.0
8600	69.5
9200	68.0
9900	67.1 (top)

From Eq. 2.9,

$$\ln \frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

With the temperature data given the integral in Eq. 2.9 can be evaluated numerically using TRAPEZOI.

```
*****************************************************
** This program performs numerical integration     **
** over a set of points using the Trapezoidal Rule **
*****************************************************

Enter number of data points: 10
Enter data points (X , Y)
? 5000,1.962E-3
? 5500,1.942E-3
? 6000,1.923E-3
? 6400,1.915E-3
? 7100,1.899E-3
? 7400,1.894E-3
? 8200,1.888E-3
? 8600,1.890E-3
? 9200,1.895E-3
? 9900,1.898E-3

The approximate value of the integral is: +9.3452E+00
```

Note: $Y \sim \frac{1}{T(°R)}$

Thus,

$$\int_{5000\,ft}^{9900\,ft} \left(\frac{1}{T}\right) dz = 9.35 \frac{ft}{°R}$$

so that (with $g = 32.2\ ft/s^2$ and $R = 1716\ ft\cdot lb/slug\cdot °R$)

$$\ln \frac{p_2}{p_1} = -\frac{(32.2 \frac{ft}{s^2})(9.35 \frac{ft}{°R})}{1716\ ft\cdot lb/slug\cdot °R} = -0.1754 \qquad (1)$$

(con't)

2.23* (con't)

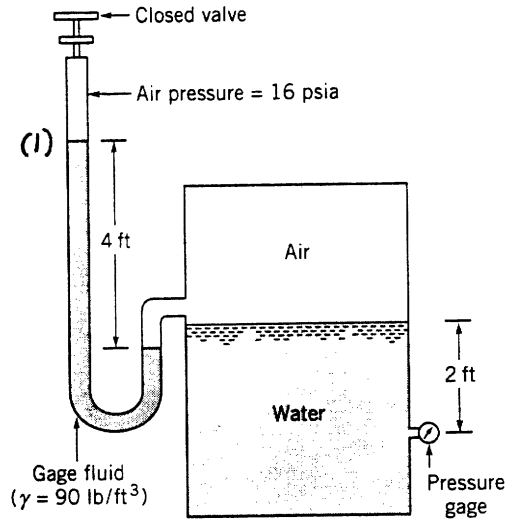
It follows from Eq.(1) with $p_1 = 12.1$ psia that

$$p_2 = (12.1 \text{ psia})\, e^{-0.1754} = \underline{\underline{10.2 \text{ psia}}}$$

(Note: Since the temperature variation is not very large, it would be expected that the assumption of a constant temperature would give good results. If the temperature is assumed to be constant at the base temperature (50.1 °F), $p_2 = 10.1$ psia, which is only slightly different from the result given above.)

2.24

2.24 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.24. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.



■ FIGURE P2.24

$$p_1 + \gamma_{gf}\,(4\text{ ft}) + \gamma_{H_2O}\,(2\text{ ft}) = p_{gage}$$

Thus,

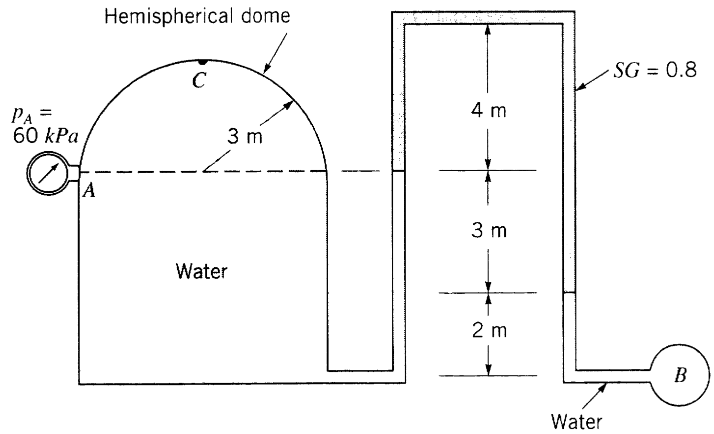
$$p_{gage} = \left(16\,\frac{\text{lb}}{\text{in.}^2} - 14.7\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(90\,\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ ft})$$

$$+ \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft})$$

$$= 672\,\frac{\text{lb}}{\text{ft}^2} = \left(672\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}^2}{144\text{ in.}^2}\right) = \underline{\underline{4.67\text{ psi}}}$$

2.25

2.25 A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.25. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at *A* is 60 kPa, determine: **(a)** the pressure in pipe *B*, and **(b)** the pressure head, in millimeters of mercury, at the top of the dome (point *C*).



(a) $$p_A + (SG)(\gamma_{H_2O})(3\,m) + \gamma_{H_2O}(2\,m) = p_B$$

$$p_B = 60\,kPa + (0.8)(9.81 \times 10^3 \tfrac{N}{m^3})(3\,m) + (9.80 \times 10^3 \tfrac{N}{m^3})(2\,m)$$

$$= \underline{\underline{103\,kPa}}$$

(b) $$p_C = p_A - \gamma_{H_2O}(3\,m)$$

$$= 60\,kPa - (9.80 \times 10^3 \tfrac{N}{m^3})(3\,m)$$

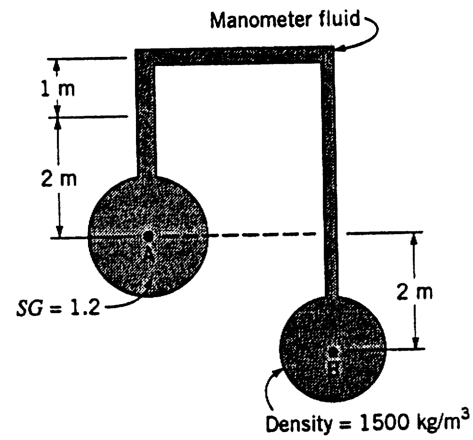
$$= 30.6 \times 10^3 \tfrac{N}{m^2}$$

$$h = \frac{p_C}{\gamma_{Hg}} = \frac{30.6 \times 10^3 \tfrac{N}{m^2}}{133 \times 10^3 \tfrac{N}{m^3}} = 0.230\,m$$

$$= 0.230\,m \left(\frac{10^3\,mm}{m}\right) = \underline{\underline{230\,mm}}$$

2.26

2.26 For the stationary fluid shown in Fig. P2.26, the pressure at point B is 20 kPa greater than at point A. Determine the specific weight of the manometer fluid.



■ FIGURE P2.26

Let: γ_m = specific weight of manometer fluid

$$\gamma_A = (SG)(\rho_{H_2O @ 4^\circ C})(g) = (1.2)(1000\,\tfrac{kg}{m^3})(9.81\,\tfrac{m}{s^2})$$

$$= 11,800\,\frac{N}{m^3}$$

$$\gamma_B = \rho_G\, g = (1500\,\tfrac{kg}{m^3})(9.81\,\tfrac{m}{s^2}) = 14,700\,\frac{N}{m^3}$$

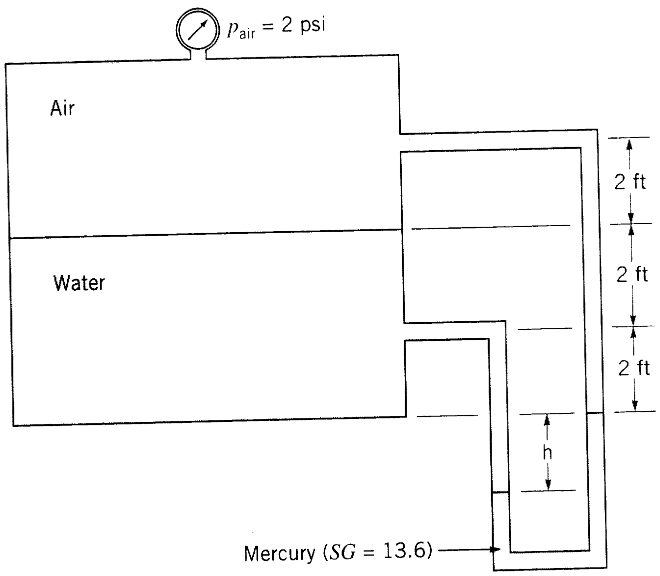
$$p_B - p_A = -\gamma_A\,(2m) + \gamma_m\,(2m) + \gamma_B\,(2m)$$

$$20\times10^3\,\frac{N}{m^2} = -(11,800\,\tfrac{N}{m^3})(2m) + \gamma_m\,(2m) + (14,700\,\tfrac{N}{m^3})(2m)$$

$$\gamma_m = \underline{\underline{7,100\,\frac{N}{m^3}}}$$

2.27

2.27 A U-tube mercury manometer is connected to a closed pressurized tank as illustrated in Fig. P2.27. If the air pressure is 2 psi, determine the differential reading, h. The specific weight of the air is negligible.

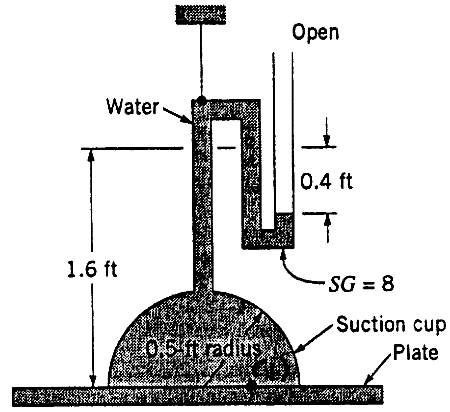


$$p_{air} + \gamma_{Hg} h - \gamma_{H_2O}(h + 4\,ft) = p_{air}$$

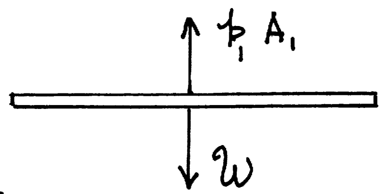
$$h = \frac{\gamma_{H_2O}(4\,ft)}{\gamma_{Hg} - \gamma_{H_2O}} = \frac{(62.4\,\frac{lb}{ft^3})(4\,ft)}{(13.6)(62.4\,\frac{lb}{ft^3}) - 62.4\,\frac{lb}{ft^3}} = \underline{\underline{0.317\,ft}}$$

2.28

2.28 A suction cup is used to support a plate of weight $\mathcal{W}$ as shown in Fig. P2.28. For the conditions shown, determine $\mathcal{W}$.



■ FIGURE P2.28



For equilibrium of forces on plate

$$\mathcal{W} = p_1 A_1 \tag{1}$$

where A_1 is area of cup and p_1 is a negative pressure.

From manometer equation:

$$p_1 - \gamma_{H_2O}(1.6\,ft) + (SG)(\gamma_{H_2O})(0.4\,ft) = 0$$

$$p_1 = \gamma_{H_2O}\left[1.6\,ft - (8)(0.4\,ft)\right]$$

$$= 62.4\,\frac{lb}{ft^3}\left[1.6\,ft - (8)(0.4\,ft)\right]$$

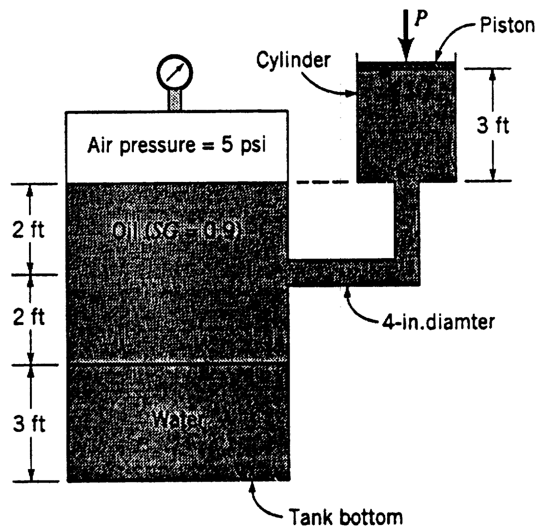
$$= -99.8\,\frac{lb}{ft^2}$$

Thus, from Eq. (1)

$$\mathcal{W} = \left(99.8\,\frac{lb}{ft^2}\right)(\pi)(0.5\,ft)^2 = \underline{\underline{78.4\,lb}}$$

2.29

2.29 A piston having a cross-sectional area of 3 ft^2 and negligible weight is located in a cylinder containing oil ($SG = 0.9$) as shown in Fig. P2.29. The cylinder is connected to a pressurized tank containing water and oil. A force, P, holds the piston in place. **(a)** Determine the required value of the force, P. **(b)** Determine the pressure head, expressed in feet of water, acting on the tank bottom.



■ FIGURE P2.29

(a) For equilibrium

$$p_1 A_1 = P \qquad (1)$$

where p_1 is pressure acting on piston. A manometer equation gives

$$p_1 + \gamma_{oil}(5\,ft) - \gamma_{oil}(2\,ft) = p_{air}$$

so that

$$p_1 = p_{air} - \gamma_{oil}(5\,ft) + \gamma_{oil}(2\,ft)$$

$$= \left(5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) - (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(5\,ft) + (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(2\,ft)$$

$$= 552\,\frac{lb}{ft^2}$$

Thus, from Eq. (1)

$$P = \left(552\,\frac{lb}{ft^2}\right)(3\,ft^2) = \underline{\underline{1660\,lb}}$$

(b) $$p_{bottom} = p_{air} + \gamma_{H_2O}(3\,ft) + \gamma_{oil}(4\,ft)$$

$$= \left(5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + \left(62.4\,\frac{lb}{ft^3}\right)(3\,ft) + (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(4\,ft)$$

$$= 1130\,\frac{lb}{ft^2}$$

$$\text{Pressure head} = \frac{p_{bottom}}{\gamma_{H_2O}} = \frac{1130\,\frac{lb}{ft^2}}{62.4\,\frac{lb}{ft^3}} = \underline{\underline{18.1\,ft}}$$

2.31

2.31 The mercury manometer of Fig. P2.3 indicates a differential reading of 0.30 m when the pressure in pipe A is 30 mm Hg vacuum. Determine the pressure in pipe B.

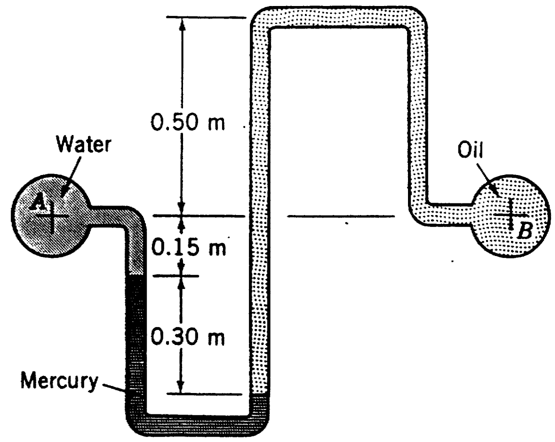


FIGURE P2.31

$$p_B + \gamma_{oil}(0.15\,m + 0.30\,m) - \gamma_{Hg}(0.3\,m) - \gamma_{H_2O}(0.15\,m) = p_A$$

where $p_A = -\gamma_{Hg}(0.030\,m)$

Thus,

$$p_B = -\gamma_{Hg}(0.030\,m) - \gamma_{oil}(0.45\,m) + \gamma_{Hg}(0.3\,m) + \gamma_{H_2O}(0.15\,m)$$

$$= -\left(133\,\frac{kN}{m^3}\right)(0.030\,m) - \left(8.95\,\frac{kN}{m^3}\right)(0.45\,m) + \left(133\,\frac{kN}{m^3}\right)(0.3\,m) + \left(9.80\,\frac{kN}{m^3}\right)(0.15\,m)$$

$$= \underline{\underline{33.4\,kPa}}$$

2.32

2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe *A* is 0.6 psi. The fluid in both pipes *A* and *B* is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe *B* corresponding to the differential reading shown?

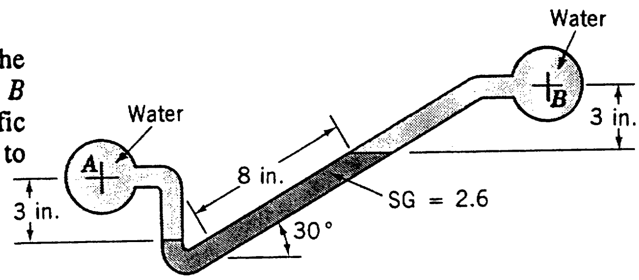


FIGURE P2.32

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30° - \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

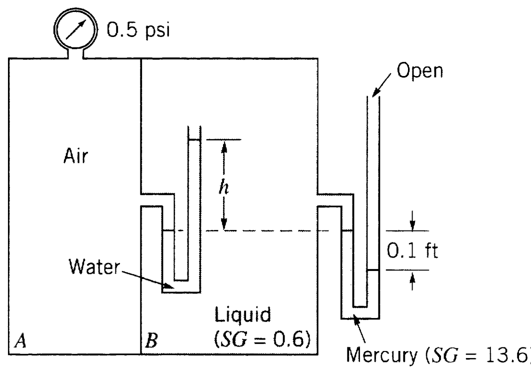
$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30°$$

$$= \left(0.6\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) - (2.6)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ ft}\right)(0.5) = 32.3\,\frac{\text{lb}}{\text{ft}^2}$$

$$= 32.3\text{ lb/ft}^2 / 144\text{ in.}^2/\text{ft}^2 = \underline{\underline{0.224\text{ psi}}}$$

2.33

2.33 Compartments A and B of the tank shown in Fig. P2.33 are closed and filled with air and a liquid with a specific gravity equal to 0.6. Determine the manometer reading, *h*, if the barometric pressure is 14.7 psia and the pressure gage reads 0.5 psi. The effect of the weight of the air is negligible.



$$p_{air} - \gamma_{H_2O}(h) + \gamma_{oil}(h) + \gamma_{Hg}(0.1\text{ ft}) = 0$$

$$h = \frac{p_{air} + \gamma_{Hg}(0.1\text{ ft})}{\gamma_{H_2O} - \gamma_{oil}}$$

$$= \frac{\left(0.5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) + (13.6)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(0.1\text{ ft})}{62.4\,\frac{\text{lb}}{\text{ft}^3} - (0.6)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)}$$

$$= \underline{\underline{6.28\text{ ft}}}$$

2.34

2.34 Small differences in gas pressures are commonly measured with a *micromanometer* of the type illustrated in Fig. P2.34. This device consists of two large reservoirs each having a cross-sectional area, A_r, which are filled with a liquid having a specific weight, γ_1, and connected by a U-tube of cross-sectional area, A_t, containing a liquid of specific weight, γ_2. When a differential gas pressure, $p_1 - p_2$, is applied a differential reading, h, develops. It is desired to have this reading sufficiently large (so that it can be easily read) for small pressure differentials. Determine the relationship between h and $p_1 - p_2$ when the area ratio A_t/A_r is small, and show that the differential reading, h, can be magnified by making the difference in specific weights, $\gamma_2 - \gamma_1$, small. Assume that initially (with $p_1 = p_2$) the fluid levels in the two reservoirs are equal.

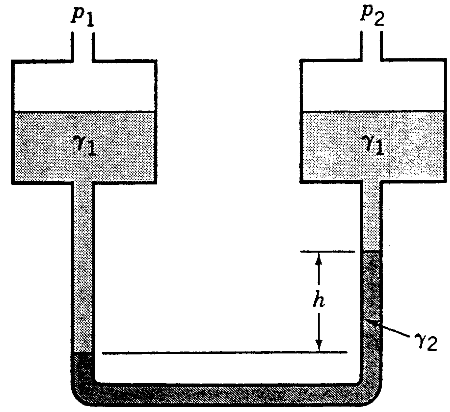
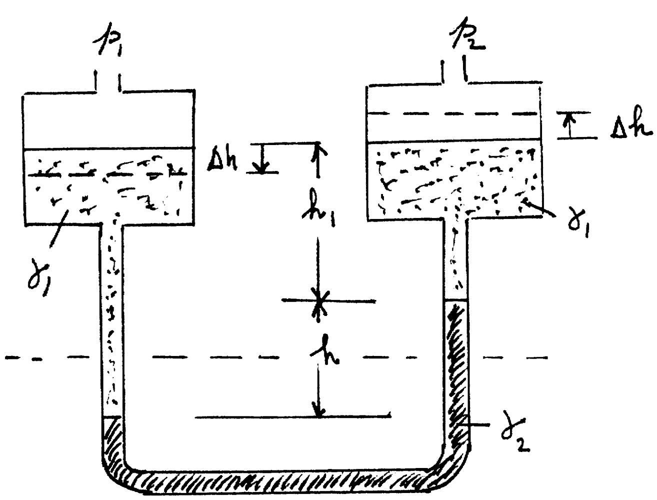


FIGURE P2.34



When a differential pressure, $p_1 - p_2$, is applied we assume that level in left reservoir drops by a distance, Δh, and right level rises by Δh. Thus, the manometer equation becomes

$$p_1 + \gamma_1(h_1 + h - \Delta h) - \gamma_2 h - \gamma_1(h_1 + \Delta h) = p_2$$

or

$$p_1 - p_2 = \gamma_2 h - \gamma_1 h + \gamma_1(2\Delta h) \qquad (1)$$

Since the liquids in the manometer are incompressible,

$$\Delta h\, A_r = \frac{h}{2} A_t \qquad \text{or} \qquad \frac{2\Delta h}{h} = \frac{A_t}{A_r}$$

and if $\frac{A_t}{A_r}$ is small then $2\Delta h << h$ and last term in Eq.(1) can be neglected. Thus,

$$p_1 - p_2 = (\gamma_2 - \gamma_1) h$$

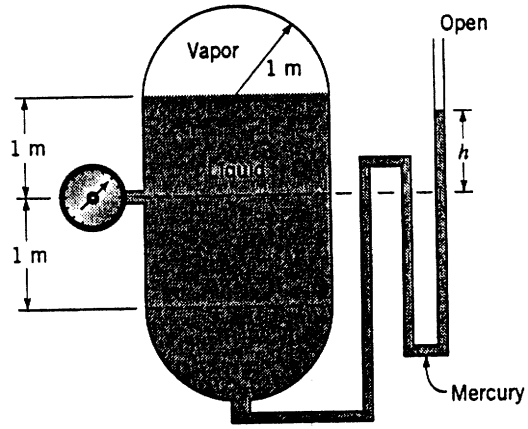
or

$$\underline{\underline{h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}}}$$

and large values of h can be obtained for small pressure differentials if $\gamma_2 - \gamma_1$ is small.

2.35

2.35 The cyclindrical tank with hemispherical ends shown in Fig. P2.35 contains a volatile liquid and its vapor. The liquid density is 800 kg/m^3, and its vapor density is negligible. The pressure in the vapor is 120 kPa (abs), and the atmospheric pressure is 101 kPa (abs). Determine: **(a)** the gage pressure reading on the pressure gage; and **(b)** the height, h, of the mercury manometer.



■ FIGURE P2.35

(a) Let γ_ℓ = sp. wt. of liquid $= \left(800 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = 7850 \frac{N}{m^3}$

and

$$p_{vapor}\ (gage) = 120\ kPa\ (abs) - 101\ kPa\ (abs) = 19\ kPa$$

Thus,

$$p_{gage} = p_{vapor} + \gamma_\ell\ (1m)$$

$$= 19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1m)$$

$$= \underline{\underline{26.9\ kPa}}$$

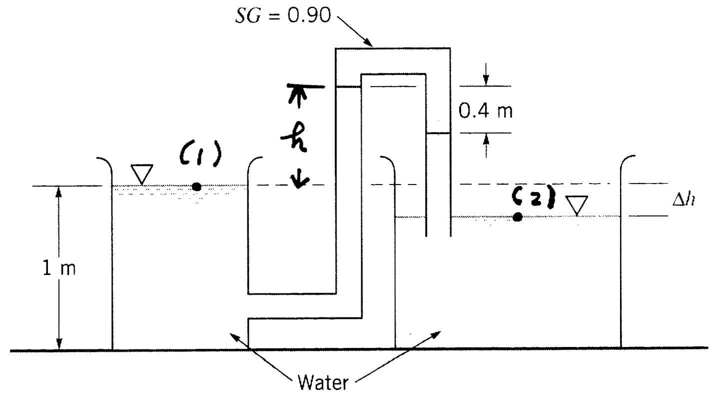
(b) $p_{vapor}\ (gage) + \gamma_\ell\ (1m) - \gamma_{Hg}\ (h) = 0$

$$19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1m) - \left(133 \times 10^3 \frac{N}{m^3}\right)(h) = 0$$

$$h = \underline{\underline{0.202\ m}}$$

2.36

2.36 Determine the elevation difference, Δh, between the water levels in the two open tanks shown in Fig. P2.36.



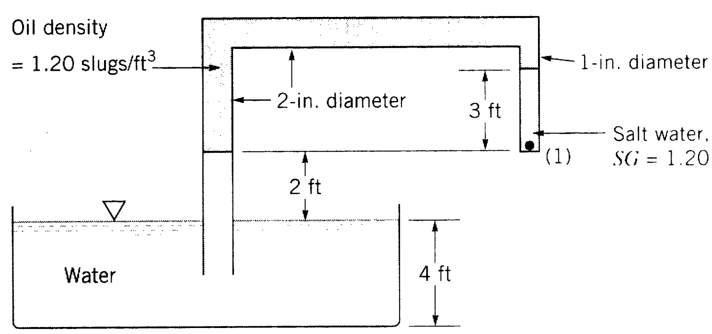
$$p_1 - \gamma_{H_2O}\, h + (SG)\gamma_{H_2O}(0.4\,m) + \gamma_{H_2O}(h - 0.4\,m) + \gamma_{H_2O}(\Delta h) = p_2$$

Since $p_1 = p_2 = 0$

$$\Delta h = 0.4\,m - (0.9)(0.4\,m) = \underline{\underline{0.040\,m}}$$

2.37

2.37 Water, oil, and salt water fill a tube as shown in Fig. P2.37. Determine the pressure at point 1 (inside the closed tube).



$$p_1 - (SG)_{\substack{salt\\water}}\gamma_{H_2O}(3\,ft) + \gamma_{oil}(3\,ft) + \gamma_{H_2O}(2\,ft) = 0$$

$$p_1 = (1.20)\left(62.4\,\frac{lb}{ft^3}\right)(3\,ft) - \left(1.20\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)(3\,ft) - \left(62.4\,\frac{lb}{ft^3}\right)(2\,ft)$$

$$= \underline{\underline{-16.1\,\frac{lb}{ft^2}}}$$

2.38

2.38 An air-filled, hemispherical shell is attached to the ocean floor at a depth of 10 m as shown in Fig. P2.38. A mercury barometer located inside the shell reads 765 mm Hg, and a mercury U-tube manometer designed to give the outside water pressure indicates a differential reading of 735 mm Hg as illustrated. Based on these data what is the atmospheric pressure at the ocean surface?

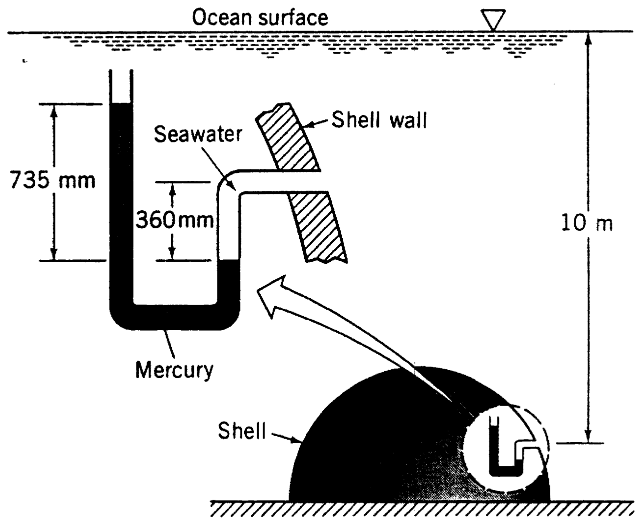


FIGURE P2.38

Let: $p_a \sim$ absolute air pressure inside shell $= \gamma_{Hg}\,(0.765\,m)$

$p_{atm} \sim$ surface atmospheric pressure

$\gamma_{SW} \sim$ specific weight of seawater

Thus, manometer equation can be written as

$$p_{atm} + \gamma_{SW}\,(10\,m) + \gamma_{SW}\,(0.360\,m) - \gamma_{Hg}\,(0.735\,m) = p_a$$

so that

$$p_{atm} = p_a - \gamma_{SW}\,(10.36\,m) + \gamma_{Hg}\,(0.735\,m)$$

$$= \left(133\,\frac{kN}{m^3}\right)(0.765\,m) - \left(10.1\,\frac{kN}{m^3}\right)(10.36\,m) + \left(133\,\frac{kN}{m^3}\right)(0.735\,m)$$

$$= \underline{\underline{94.9\ kPa}}$$

2.39 *

2.39* Both ends of the U-tube mercury manometer of Fig. P2.39 are initially open to the atmosphere and under standard atmospheric pressure. When the valve at the top of the right leg is open the level of mercury below the valve is h_i. After the valve is closed, air pressure is applied to the left leg. Determine the relationship between the differential reading on the manometer and the applied gage pressure, p_g. Show on a plot how the differential reading varies with p_g for h_i = 25, 50, 75, and 100 mm over the range $0 \le p_g \le 300$ kPa. Assume that the temperature of the trapped air remains constant.

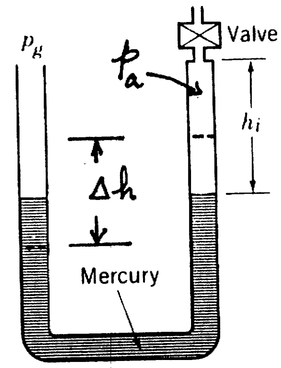


FIGURE P2.39

With the valve closed and a pressure, p_g, applied,

$$p_g - \gamma_{Hg}\,\Delta h = p_a$$

or

$$\Delta h = \frac{p_g - p_a}{\gamma_{Hg}} \qquad (1)$$

where p_g and p_a are gage pressures. For isothermal compression of trapped air

$$\frac{p}{\rho} = \text{Constant}$$

so that for constant air mass

$$p_i \forall_i = p_f \forall_f$$

where $\forall$ is air volume, p is absolute pressure, and i and f refer to initial and final states, respectively. Thus,

$$p_{atm} \forall_i = (p_a + p_{atm})\forall_f \qquad (2)$$

For air trapped in right leg, $\forall_i = h_i$(Area of tube) so that Eq.(2) can be written as

$$p_a = p_{atm}\left[\frac{h_i}{h_i - \frac{\Delta h}{2}} - 1\right] \qquad (3)$$

Substitute Eq.(3) into Eq.(1) to obtain

$$\Delta h = \frac{1}{\gamma_{Hg}}\left[p_g + p_{atm}\left(1 - \frac{h_i}{h_i - \frac{\Delta h}{2}}\right)\right] \qquad (4)$$

(con't)

2.39* (con't)

Equation (4) can be expressed in the form

$$(\Delta h)^2 - \left(2h_i + \frac{p_g + p_{atm}}{\gamma_{Hg}}\right)\Delta h + \frac{2 p_g h_i}{\gamma_{Hg}} = 0$$

and the roots of this quadratic equation are

$$\Delta h = \left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right) \pm \sqrt{\left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right)^2 - \frac{2 p_g h_i}{\gamma_{Hg}}} \qquad (5)$$

To evaluate Δh the negative sign is used since $\Delta h = 0$ for $p_g = 0$. A program for computing Δh as a function of p_g for various h_i follows (with $p_{atm} = 101\ kPa$ and $\gamma_{Hg} = 133\ kN/m^3$).

```
100 cls
110 print "*****************************************************"
120 print "** This program calculates the lower root of a    **"
130 print "** quadratic equation to give Dh (in m) for a     **"
140 print "** range of gage pressure, Pg (in kPa), and for   **"
145 print "** a set of different initial heights, hi (in m) **"
150 print "*****************************************************"
160 print
162 dim dh(5)
164 patm=101
166 ghg=133
170 print       "  Pg    Dh(hi=0.000)  Dh(hi=0.025)  Dh(hi=0.050)  Dh(hi=0.075)
 Dh(hi=0.100)
180 for pg=0 to 300 step 30
190 for i=0 to 5
195 hi=(i-1)*0.025
200 a=hi+(pg+patm)/(2*ghg)
210 dh(i)=a-(a^2-2*pg*hi/ghg)^.5
220 next i
230 print using "####.#   ###.#####     ###.#####     ###.#####     ###.#####
  ###.#####";pg,dh(1),dh(2),dh(3),dh(4),dh(5)
240 next pg
```

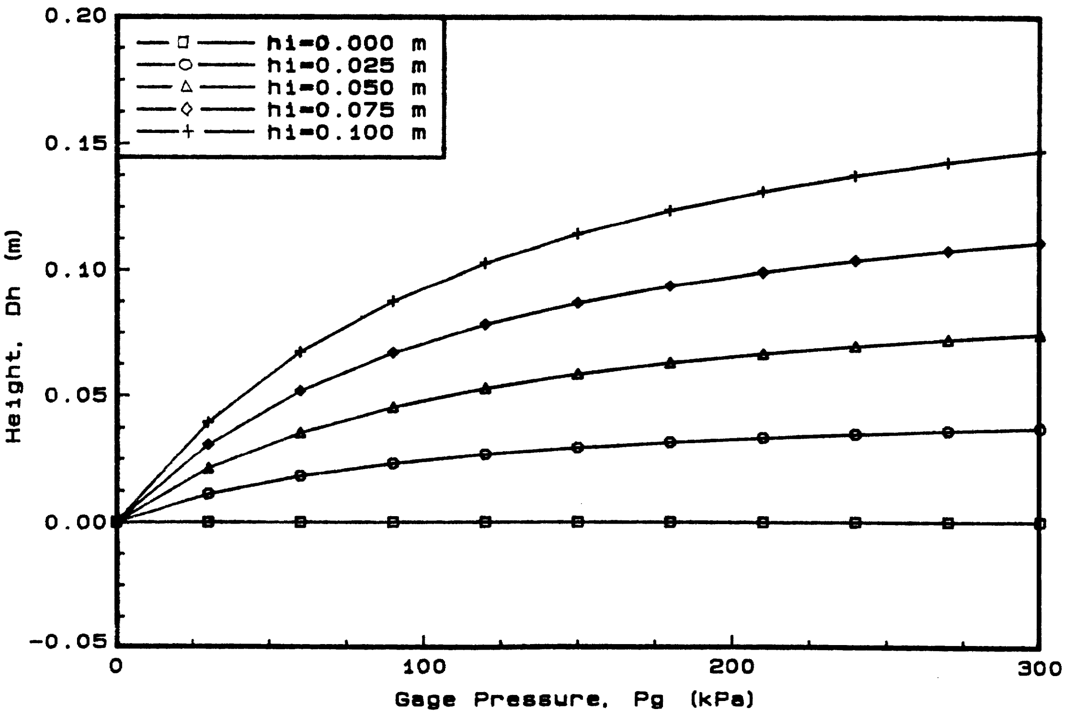
(con't)

2.39* (con't)

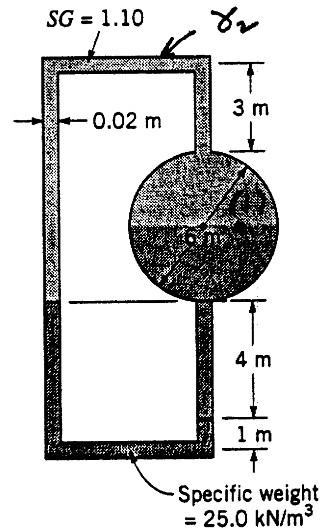
Tabulated data and a plot of the data are shown below.

```
***************************************************
** This program calculates the lower root of a   **
** quadratic equation to give Dh (in m) for a    **
** range of gage pressure, Pg (in kPa), and for  **
** a set of different initial heights, hi (in m) **
***************************************************
```

Pg	Dh(hi=0.000)	Dh(hi=0.025)	Dh(hi=0.050)	Dh(hi=0.075)	Dh(hi=0.100)
0.0	0.00000	0.00000	-0.00000	0.00000	0.00000
30.0	0.00000	0.01101	0.02120	0.03064	0.03938
60.0	0.00000	0.01816	0.03538	0.05170	0.06716
90.0	0.00000	0.02313	0.04539	0.06681	0.08739
120.0	0.00000	0.02678	0.05280	0.07807	0.10258
150.0	0.00000	0.02956	0.05847	0.08673	0.11433
180.0	0.00000	0.03175	0.06295	0.09359	0.12365
210.0	0.00000	0.03353	0.06657	0.09913	0.13119
240.0	0.00000	0.03499	0.06956	0.10370	0.13741
270.0	0.00000	0.03621	0.07205	0.10753	0.14262
300.0	0.00000	0.03725	0.07418	0.11078	0.14704



2.40



■ FIGURE P2.40

2.40 A 0.02-m-diameter manometer tube is connected to a 6-m-diameter full tank as shown in Fig. P2.40. Determine the density of the unknown liquid in the tank.

Let γ = sp. wt. of unknown fluid and

$$\gamma_2 = (1.10)(9.80 \times 10^3) = 10.8 \times 10^3 \ N/m^3.$$

Thus,

$$p_1 + \gamma(7m) - \left(25 \times 10^3 \frac{N}{m^3}\right)(4m) - \gamma_2 (3m) = p_1$$

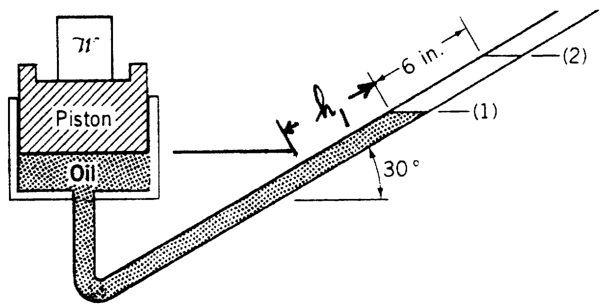
$$\gamma = 18.9 \times 10^3 \frac{N}{m^3}$$

and

$$\rho = \frac{\gamma}{g} = \frac{18.9 \times 10^3 \frac{N}{m^3}}{9.81 \frac{m}{s^2}} = \underline{\underline{1930 \frac{kg}{m^3}}}$$

2.41

2.41 A 6-in.-diameter piston is located within a cylinder which is connected to a $\frac{1}{2}$-in.-diameter inclined-tube manometer as shown in Fig. P2.41. The fluid in the cylinder and the manometer is oil (specific weight = 59 lb/ft^3). When a weight $\mathcal{W}$ is placed on the top of the cylinder the fluid level in the manometer tube rises from point (1) to (2). How heavy is the weight? Assume that the change in position of the piston is negligible.



■ FIGURE P2.41

With piston alone let pressure on face of piston = p_p, and manometer equation becomes

$$p_p - \gamma_{oil}\, h_1 \sin 30° = 0 \qquad (1)$$

With weight added pressure p_p increases to p_p' where

$$p_p' = p_p + \frac{W}{A_p} \qquad (A_p \sim \text{area of piston})$$

and manometer equation becomes

$$p_p' - \gamma_{oil}\left(h_1 + \tfrac{6}{12}\,ft\right)\sin 30° = 0 \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_p' - p_p - \gamma_{oil}\left(\tfrac{6}{12}\,ft\right)\sin 30° = 0$$

or

$$\frac{W}{A_p} = \gamma_{oil}\left(\tfrac{6}{12}\,ft\right)\sin 30°$$

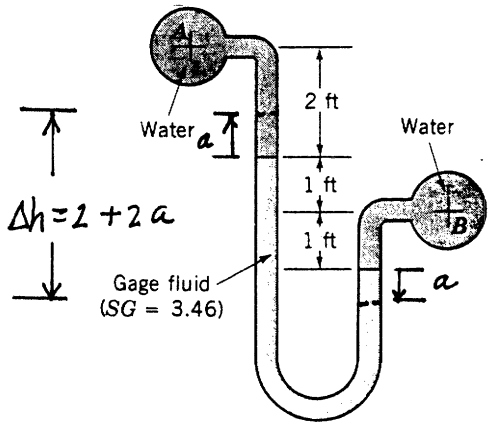
so that

$$\frac{W}{\frac{\pi}{4}\left(\frac{6}{12}\,ft\right)^2} = \left(59\,\frac{lb}{ft^3}\right)\left(\tfrac{6}{12}\,ft\right)(0.5)$$

and

$$W = \underline{\underline{2.90\ lb}}$$

2.42 The manometer fluid in the manometer of Fig. P2.42 has a specific gravity of 3.46. Pipes A and B both contain water. If the pressure in pipe A is decreased by 1.3 psi and the pressure in pipe B increases by 0.9 psi, determine the new differential reading of the manometer.



■ FIGURE P2.42

For the initial configuration:

$$p_A + \gamma_{H_2O}(2) + \gamma_{gf}(2) - \gamma_{H_2O}(1) = p_B \qquad (1)$$

where all lengths are in ft. When p_A decreases to p_A' and p_B increases to p_B' the heights of the fluid columns change as shown on figure. For the final configuration:

$$p_A' + \gamma_{H_2O}(2-a) + \gamma_{gf}(2+2a) - \gamma_{H_2O}(1+a) = p_B' \qquad (2)$$

Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_{H_2O}(a) - \gamma_{gf}(2a) + \gamma_{H_2O}(a) = p_B - p_B'$$

or

$$a = \frac{(p_B - p_B') - (p_A - p_A')}{2(\gamma_{H_2O} - \gamma_{gf})}$$

Since, $p_A - p_A' = 1.3\ psi$, $p_B - p_B' = -0.9\ psi$, and $\gamma_{gf} = 3.46\,\gamma_{H_2O}$

$$a = \frac{\left(-0.9\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) - \left(1.3\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{2\left(62.4\,\frac{lb}{ft^3}\right)(1 - 3.46)} = 1.03\ ft$$

and therefore

$$\Delta h = 2\,ft + 2a = 2\,ft + 2(1.03\,ft) = \underline{\underline{4.06\,ft}}$$

2.43 Determine the ratio of areas, A_1/A_2, of the two manometer legs of Fig. P2.43 if a change in pressure in pipe B of 0.5 psi gives a corresponding change of 1 in. in the level of the mercury in the right leg. The pressure in pipe A does not change.

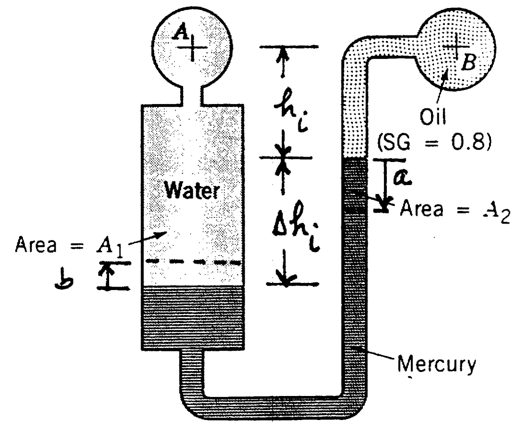


FIGURE P2.43

For the initial configuration (see figure):

$$p_A + \gamma_{H_2O}(h_i + \Delta h_i) - \gamma_{Hg}(\Delta h_i) - \gamma_{oil}(h_i) = p_B \qquad (1)$$

When p_B increases the right column falls a distance, a, and the left column rises a distance, b. Since the volume of the liquid must remain constant, $A_1 b = A_2 a$ or $\frac{A_1}{A_2} = \frac{a}{b}$.

For the final configuration, with pressure in B equal to p_B':

$$p_A + \gamma_{H_2O}(h_i + \Delta h_i - b) - \gamma_{Hg}(\Delta h_i - a - b) - \gamma_{oil}(h_i + a) = p_B' \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$-\gamma_{H_2O}(b) + \gamma_{Hg}(a+b) - \gamma_{oil}(a) = p_B' - p_B$$

or

$$b = \frac{(p_B' - p_B) - \gamma_{Hg}(a) + \gamma_{oil}(a)}{\gamma_{Hg} - \gamma_{H_2O}}$$

Since $p_B' - p_B = 0.5\ \text{psi}$ and $a = 1\ \text{in.}$, it follows that

$$b = \frac{(0.5\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2}) - (847\ \frac{lb}{ft^3})(\frac{1}{12}\ ft) + (0.8)(62.4\ \frac{lb}{ft^3})(\frac{1}{12}\ ft)}{847\ \frac{lb}{ft^3} - 62.4\ \frac{lb}{ft^3}}$$

$$= 0.00711\ ft$$

Thus,

$$\frac{A_1}{A_2} = \frac{a}{b} = \frac{\frac{1}{12}\ ft}{0.00711\ ft} = \underline{\underline{11.7}}$$

2.44

2.44 The inclined differential manometer of Fig. P2.44 contains carbon tetrachloride. Initially the pressure differential between pipes A and B, which contain a brine ($SG = 1.1$), is zero as illustrated in the figure. It is desired that the manometer give a differential reading of 12 in. (measured along the inclined tube) for a pressure differential of 0.1 psi. Determine the required angle of inclination, θ.

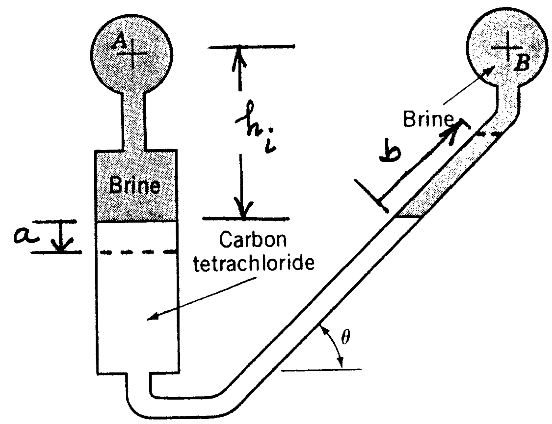


FIGURE P2.44

When $p_A - p_B$ is increased to $p_A' - p_B'$ the left column falls a distance, a, and the right column rises a distance b along the inclined tube as shown in figure. For this final configuration:

$$p_A' + \gamma_{br}(h_i + a) - \gamma_{CCl_4}(a + b\sin\theta) - \gamma_{br}(h_i - b\sin\theta) = p_B'$$

or

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(a + b\sin\theta) = 0 \qquad (1)$$

The differential reading, Δh, along the tube is

$$\Delta h = \frac{a}{\sin\theta} + b$$

Thus, from Eq.(1)

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(\Delta h \sin\theta) = 0$$

or

$$\sin\theta = \frac{-(p_A' - p_B')}{(\gamma_{br} - \gamma_{CCl_4})(\Delta h)}$$

and with $p_A' - p_B' = 0.1\,psi$

$$\sin\theta = \frac{-\left(0.1\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left[(1.1)\left(62.4\,\frac{lb}{ft^3}\right) - 99.5\,\frac{lb}{ft^3}\right]\left(\frac{12}{12}\,ft\right)} = 0.466$$

for $\Delta h = 12$ in.

Thus,

$$\underline{\underline{\theta = 27.8^\circ}}$$

2.45

2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

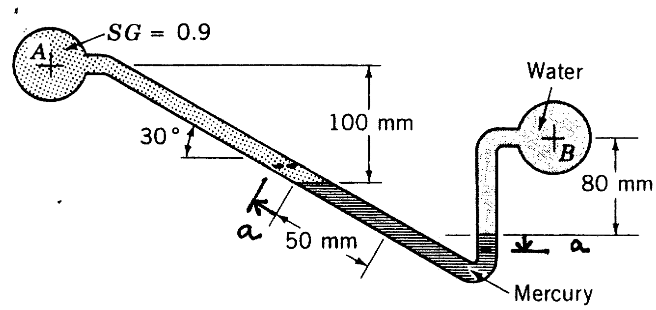


FIGURE P2.45

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05 \sin 30°) - \gamma_{H_2O}(0.08) = p_B \qquad (1)$$

where all lengths are in m. When p_A decreases left column moves up a distance, a, and right column moves down a distance, a, as shown in figure. For the final configuration:

$$p_A' + \gamma_A(0.1 - a \sin 30°) + \gamma_{Hg}(a \sin 30° + 0.05 \sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \qquad (2)$$

where p_A' is the new pressure in pipe A.
Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_A(a \sin 30°) - \gamma_{Hg}\, a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10\ kPa$

$$a = \frac{-10\ \frac{kN}{m^2}}{(0.9)(9.81\ \frac{kN}{m^3})(0.5) - (133\ \frac{kN}{m^3})(0.5+1) + 9.80\ \frac{kN}{m^3}}$$

$$= 0.0540\ m$$

New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540\ m}{0.5} + 0.05\,m + 0.0540\ m = \underline{\underline{0.212\ m}}$$

2.46

2.46 Determine the change in the elevation of the mercury in the left leg of the manometer of Fig. P2.46 as a result of an increase in pressure of 5 psi in pipe A while the pressure in pipe B remains constant.

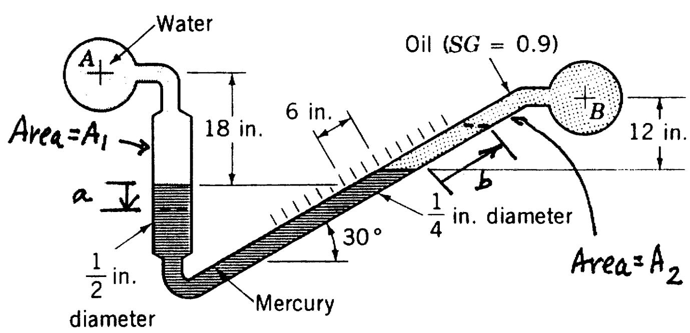


FIGURE P2.46

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30^\circ\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

where all lengths are in ft. When p_A increases to p_A' the left column falls by the distance, a, and the right column moves up the distance, b, as shown in the figure. For the final configuration:

$$p_A' + \gamma_{H_2O}\left(\frac{18}{12}+a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30^\circ + b\sin 30^\circ\right) -$$

$$\gamma_{oil}\left(\frac{12}{12} - b\sin 30^\circ\right) = p_B \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30^\circ) + \gamma_{oil}(b\sin 30^\circ) = 0 \qquad (3)$$

Since the volume of liquid must be constant $A_1 a = A_2 b$,

or
$$\left(\tfrac{1}{2}\text{ in.}\right)^2 a = \left(\tfrac{1}{4}\text{ in.}\right)^2 b$$

so that
$$b = 4a$$

Thus, Eq. (3) can be written as

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30^\circ) + \gamma_{oil}(4a\sin 30^\circ) = 0$$

and

$$a = \frac{-(p_A' - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{62.4\,\frac{lb}{ft^3} - \left(847\,\frac{lb}{ft^3}\right)(3) + (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(2)}$$

$$= \underline{\underline{0.304\text{ ft (down)}}}$$

2.47 *

2.47* Water initially fills the funnel and its connecting tube as shown in Fig. P2.47. Oil ($SG = 0.85$) is poured into the funnel until it reaches a level $h > H/2$ as indicated. Determine and plot the value of the rise in the water level in the tube, ℓ, as a function of h for $H/2 \le h \le H$, with $H = D = 2$ ft and $d = 0.1$ ft.

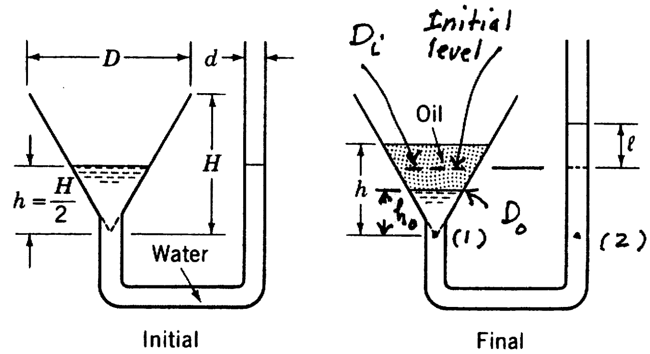


FIGURE P2.47

Since $p_1 = p_2$ (see figure), it follows that

$$\gamma_{oil}(h - h_o) + \gamma_{H_2O}\, h_o = \gamma_{H_2O}\left(\frac{H}{2} + \ell\right)$$

or

$$h = \frac{\gamma_{H_2O}}{\gamma_{oil}}\left(\frac{H}{2} + \ell - h_o\right) + h_o \qquad (1)$$

The volume of water must be conserved, and therefore

$$\frac{\pi}{4} d^2 \ell = \frac{\pi}{3}\left(\frac{D_i}{2}\right)^2 \frac{H}{2} - \frac{\pi}{3}\left(\frac{D_o}{2}\right)^2 h_o \qquad (2)$$

Also,

$$\frac{D}{H} = \frac{D_i}{H/2} = \frac{D_o}{h_o} \qquad \text{and} \qquad D_i = \frac{D}{2}$$

and Eq. (2) can be written as

$$3 d^2 \ell = \frac{D^2 H}{8} - \left(\frac{D}{H}\right)^2 h_o^3 \qquad (3)$$

For $H = 2$ ft, $D = 2$ ft, $d = 0.1$ ft, and $\frac{\gamma_{H_2O}}{\gamma_{oil}} = \frac{1}{0.85}$,

Eq. (1) becomes

$$h = \frac{1}{0.85}\left(\frac{2\text{ft}}{2} + \ell - h_o\right) + h_o$$

or

$$\ell = 0.85h + 0.15h_o - 1 \qquad (4)$$

(con't)

2.47* (con't)

Similarly, Eq. (3) becomes

$$3\,(0.1\,ft)^2\,\ell = \frac{(2\,ft)^2(2\,ft)}{8} - \left(\frac{2\,ft}{2\,ft}\right)^2 h_o^3$$

or

$$h_o = (1 - 0.03\,\ell)^{1/3} \qquad (5)$$

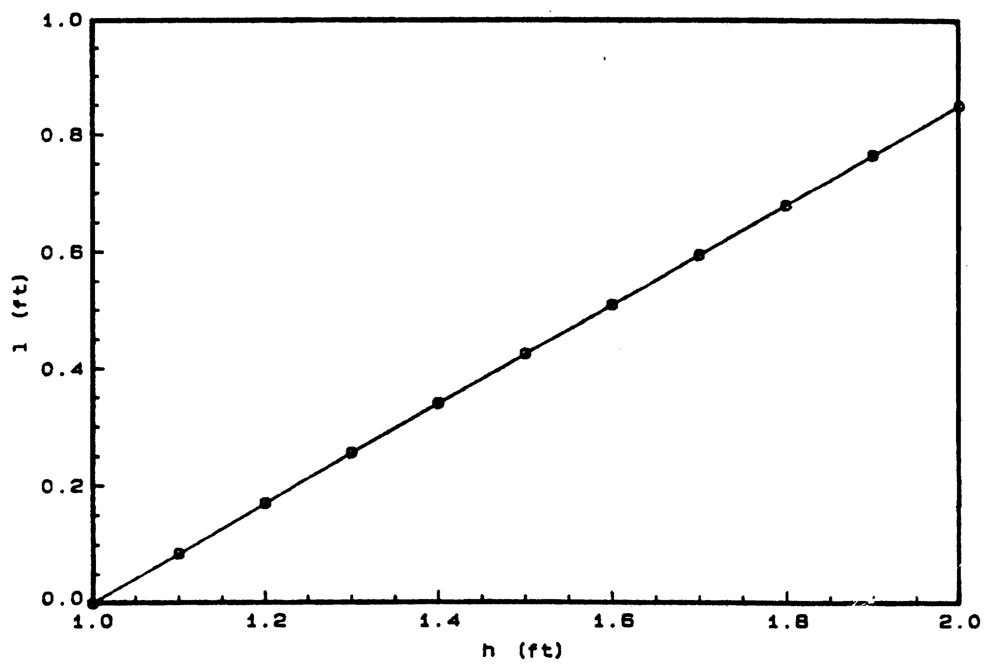
A program for computing ℓ as a function of h follows.

```
100 cls
110 print "**************************************************"
120 print "** This program solves iteratively a system of    **"
130 print "** equations to calculate the elevation l (in ft) **"
140 print "** range of heights h (in ft)                     **"
150 print "**************************************************"
160 print
165 print "     h (ft)      l (ft)"
166 ' for h=1 l=0
167 print using "    ###.###     ###.###";1.,0.
170 for h=1.10 to 2.01 step 0.10
180 l=0.0
190 las=l
200 h0=(1-0.03*las)^(1/3)
210 l=0.85*h+0.15*h0-1
220 if abs(1-las/l)>0.001 then goto 190
230 print using "    ###.###     ###.###";h,l
240 next h
```

Tabulated data and a plot of the data are shown below.

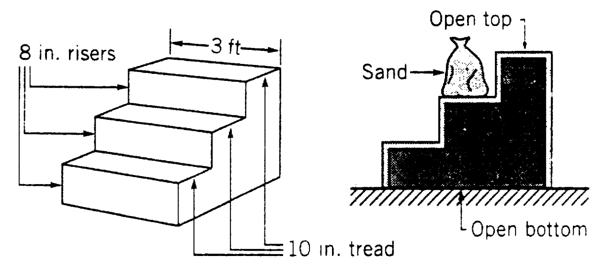
```
**************************************************
** This program solves iteratively a system of    **
** equations to calculate the elevation l (in ft) **
** range of heights h (in ft)                     **
**************************************************
```

h (ft)	l (ft)
1.000	0.000
1.100	0.085
1.200	0.170
1.300	0.255
1.400	0.339
1.500	0.424
1.600	0.509
1.700	0.594
1.800	0.679
1.900	0.764
2.000	0.849



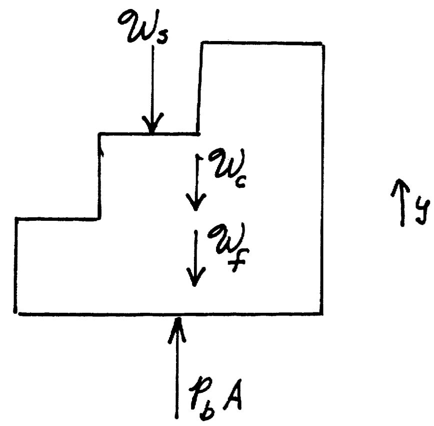
2.48

2.48 Concrete is poured into the forms as shown in Fig. P2.48 to produce a set of steps. Determine the weight of the sandbag needed to keep the bottomless forms from lifting off the ground. The weight of the forms is 85 lb, and the specific weight of the concrete is 150 lb/ft³.



■ FIGURE P2.48

From the free-body-diagram



$$(\downarrow +)\quad \Sigma F_y = 0$$

$$W_s + W_c + W_f - p_b A = 0 \qquad (1)$$

Where:

W_s = weight of sandbag

W_c = weight of concrete

W_f = weight of forms

p_b = pressure along bottom surface due to concrete

A = area of bottom surface

From the data given:

$$W_c = \left(150 \tfrac{lb}{ft^3}\right)(\text{Vol. concrete})$$

$$= \left(150 \tfrac{lb}{ft^3}\right)(3\,ft)\ \frac{[(10\,in.)(24\,in.) + (10\,in.)(16\,in.) + (10\,in.)(8\,in.)]}{144\ \frac{in.^2}{ft^2}}$$

$$= 1500\ lb$$

$$W_f = 85\ lb$$

$$p_A = \left(150 \tfrac{lb}{ft^3}\right)\left(\tfrac{24}{12} ft\right) = 300 \tfrac{lb}{ft^2}$$

$$A = \left(\tfrac{30}{12} ft\right)(3\,ft) = 7.5\,ft^2$$

Thus, from Eq. (1)

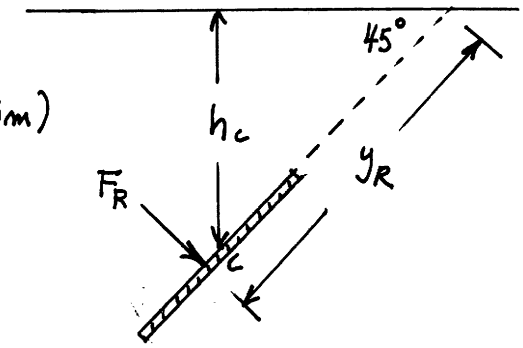
$$W_s = \left(300 \tfrac{lb}{ft^2}\right)(7.5\,ft^2) - 1500\,lb - 85\,lb$$

$$= \underline{\underline{665\ lb}}$$

2.49

2.49 A square 3 m × 3 m gate is located in the 45° sloping side of a dam. Some measurements indicate that the resultant force of the water on the gate is 500 kN. (a) Determine the pressure at the bottom of the gate. (b) Show on a sketch where this force acts.

(a) $F_R = \gamma h_c A$



$$500\,kN = \left(9.80\,\frac{kN}{m^3}\right)(h_c)(3m \times 3m)$$

$$h_c = 5.67\,m$$

$$p_{bottom} = \gamma\,(h_c + 1.5m \times \sin 45°)$$

$$= \left(9.80\,\frac{kN}{m^3}\right)(5.67\,m + 1.5\,m\,\sin 45°)$$

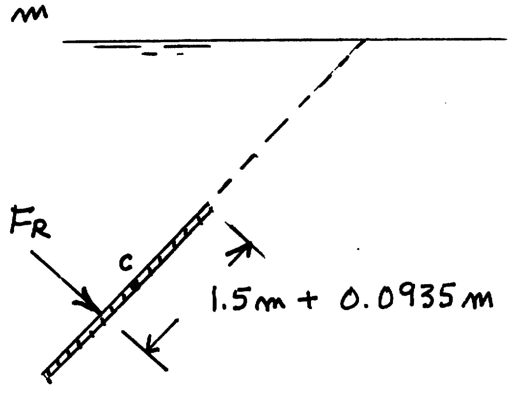
$$= 66.0\,\frac{kN}{m^2} = \underline{\underline{66.0\,kPa}}$$

(b) $$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$y_c = \frac{h_c}{\sin 45°} = \frac{5.67\,m}{\sin 45°} = 8.02\,m$$

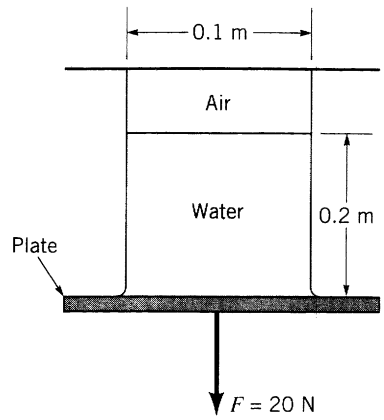
$$y_R = \frac{\frac{1}{12}(3m)(3m)^3}{(8.02m)(3m \times 3m)} + 8.02\,m = 0.0935m + 8.02m$$

$$= 8.11\,m$$



$1.5m + 0.0935\,m = \underline{\underline{1.59\,m \text{ from top of gate}}}$

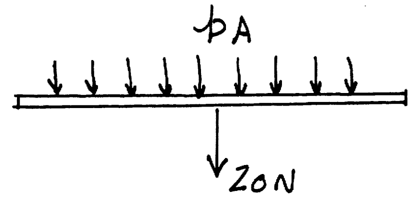
2.50 An inverted 0.1-m-diameter circular cylinder is partially filled with water and held in place as shown in Fig. P2.50. A force of 20 N is needed to pull the flat plate from the cylinder. Determine the air pressure within the cylinder. The plate is not fastened to the cylinder and has negligible mass.



For equilibrium

$$\Sigma F_{vertical} = 0$$

$$pA + 20\ N = 0$$



$$p = -\frac{20\ N}{\frac{\pi}{4}(0.1m)^2}$$

(Note that pressure must be a "suction" pressure.)

Also,

$$p_{air} + \gamma_{H_2O}(0.2m) = p$$

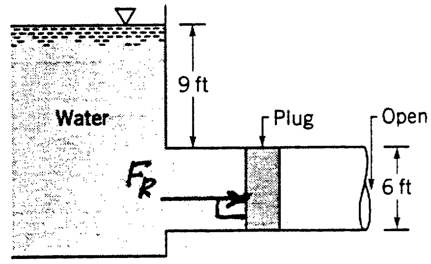
Thus,

$$p_{air} = -\frac{20\ N}{\frac{\pi}{4}(0.1m)^2} - \left(9.80 \times 10^3 \frac{N}{m^3}\right)(0.2m)$$

$$= -4510\ \frac{N}{m^2} = \underline{\underline{-4.51\ kPa}}$$

2.51

2.51 A large, open tank contains water and is connected to a 6-ft diameter conduit as shown in Fig. P2.51. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.



■ FIGURE P2.51

$$F_R = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(12\,ft)\left(\frac{\pi}{4}\right)(6\,ft)^2 = \underline{\underline{21,200\,lb}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad I_{xc} = \frac{\pi\,(3ft)^4}{4} = 63.6\,ft^4$$

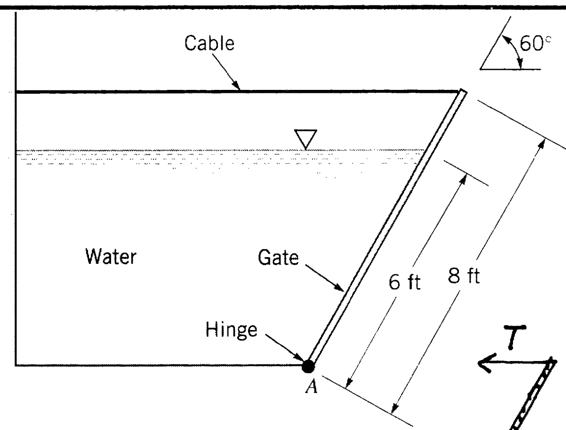
Thus,

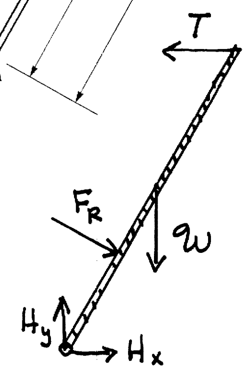
$$y_R = \frac{\frac{\pi}{4}(3ft)^4}{(12\,ft)\,\pi\,(3ft)^2} + 12\,ft = \underline{\underline{12.19\,ft}}$$

The force of 21,200 lb acts 12.19 ft below the water surface and is perpendicular to the plug surface as shown.

2.52

2.52 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.52. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.





$$F_R = \gamma h_c A \quad \text{where } h_c = \left(\frac{6\text{ ft}}{2}\right)\sin 60°$$

Thus,

$$F_R = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{6\text{ ft}}{2}\right)(\sin 60°)(6\text{ ft} \times 4\text{ ft})$$

$$= 3890\text{ lb}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where } y_c = 3\text{ ft}$$

so that

$$y_R = \frac{\frac{1}{12}(4\text{ ft})(6\text{ ft})^3}{(3\text{ ft})(6\text{ ft} \times 4\text{ ft})} + 3\text{ ft} = 4.0\text{ ft}$$

For equilibrium,

$$\Sigma M_H = 0$$

and

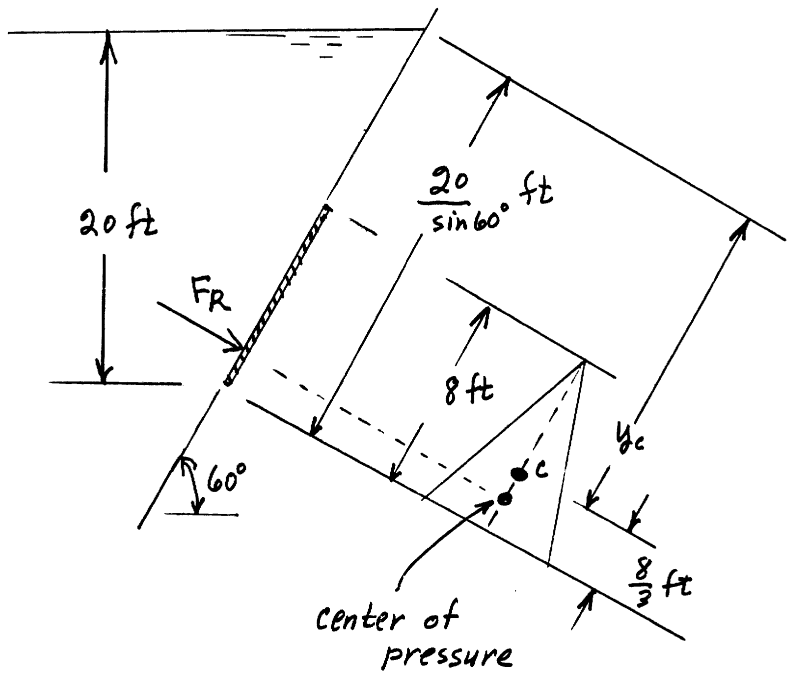
$$T\,(8\text{ ft})(\sin 60°) = \mathcal{W}\,(4\text{ ft})(\cos 60°) + F_R\,(2\text{ ft})$$

$$T = \frac{(800\text{ lb})(4\text{ ft})(\cos 60°) + (3890\text{ lb})(2\text{ ft})}{(8\text{ ft})(\sin 60°)}$$

$$= \underline{\underline{1350\text{ lb}}}$$

2.54

2.54 An area in the form of an isosceles triangle with a base width of 6 ft and an altitude of 8 ft lies in the plane forming one wall of a tank which contains a liquid having a specific weight of 79.8 lb/ft³. The side slopes upward making an angle of 60° with the horizontal. The base of the triangle is horizontal and the vertex is above the base. Determine the resultant force the fluid exerts on the area when the fluid depth is 20 ft above the base of the triangular area. Show, with the aid of a sketch, where the center of pressure is located.



$$y_c = \left(\frac{20}{\sin 60°}\right) ft - \left(\frac{8}{3}\right) ft$$

$$= 20.43 \text{ ft}$$

$$h_c = y_c \sin 60°$$

$$F_R = \gamma h_c A = \left(79.8 \frac{lb}{ft^3}\right)\left[(20.43 \text{ ft}) \sin 60°\right]\left(\frac{1}{2}\right)(6 \text{ ft} \times 8 \text{ ft})$$

$$= \underline{\underline{33,900 \text{ lb}}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where } I_{xc} = \frac{1}{36}(6 ft)(8 ft)^3$$

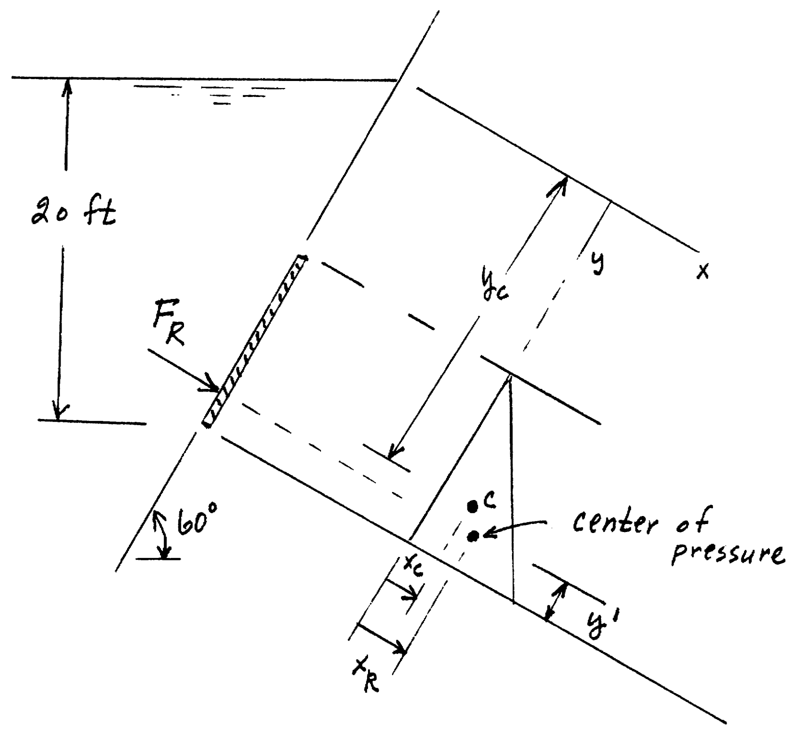
Thus,

$$y_R = \frac{\frac{1}{36}(6 ft)(8 ft)^3}{(20.43 ft)(\frac{1}{2})(6 ft \times 8 ft)} + 20.43 \text{ ft} = 20.6 \text{ ft}$$

The force, F_R, acts through the center of pressure which is located a distance of $\frac{20}{\sin 60°}$ ft $-$ 20.6 ft $=$ 2.49 ft above the base of the triangle as shown in sketch.

2.55

2.55 Solve Problem 2.54 if the isosceles triangle is replaced with a right triangle having the same base width and altitude.



$F_R = 33,900\ lb$

$y' = 2.49\ ft$

(see solution to Problem 2.54)

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad \text{(Eq. 2.20)}$$

where

$$I_{xyc} = \frac{(6\ ft)^2(8\ ft)^2}{72} = 32\ ft^4 \qquad \text{(see Fig. 2.18 d)}$$

and $y_c = 20.43\ ft$ (see solution to Problem 2.54)

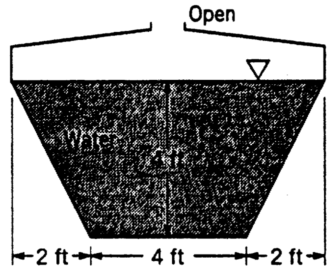
Thus,

$$x_R = \frac{32\ ft^4}{(20.43\ ft)(\frac{1}{2})(6\ ft \times 8\ ft)} + \frac{6}{3}\ ft = \underline{\underline{2.07\ ft}}$$

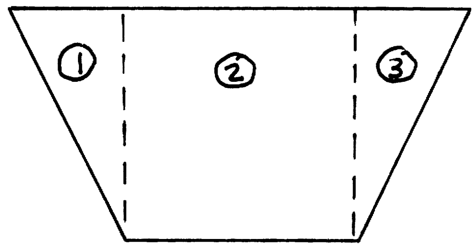
The force, F_R, acts through the center of pressure with coordinates $x_R = 2.07\ ft$ and $y' = 2.49\ ft$ (see sketch).

2.56

2.56 A tanker truck carries water, and the cross section of the truck's tank is shown in Fig. P2.56. Determine the magnitude of the force of the water against the vertical front end of the tank.



■ FIGURE P2.56



$F_R = \gamma h_c A$

Break area into 3 parts as shown. For area 1:

$F_{R_1} = \gamma h_{c_1} A_1$

$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{1}{3}\right)(4ft)\left(\frac{1}{2}\right)(2ft \times 2ft)$

$= 333\ lb$

Since $F_{R_1} = F_{R_3}$ then $F_{R_3} = 333\ lb$

For area 2:

$F_{R_2} = \gamma h_{c_2} A_2$

$= \left(62.4 \frac{lb}{ft^3}\right)(2\ ft)(4ft \times 4ft)$

and

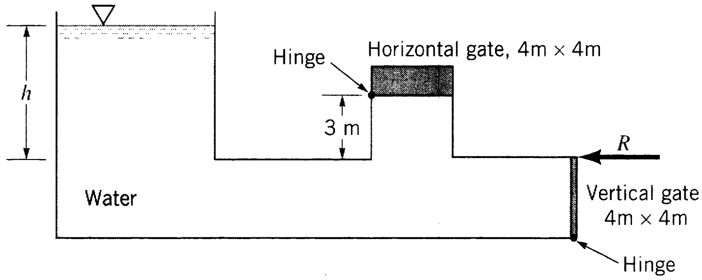
$F_R = 2F_{R_1} + F_{R_2}$

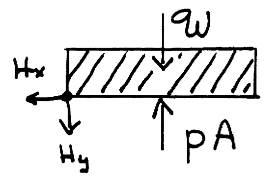
$= 2\,(333 lb) + \left(62.4 \frac{lb}{ft^3}\right)(2ft)(4ft \times 4ft)$

$= \underline{\underline{2660\ lb}}$

2.57

2.57 Two square gates close two openings in a conduit connected to an open tank of water as shown in Fig. P2.57. When the water depth, h, reaches 5 m it is desired that both gates open at the same time. Determine the weight of the homogeneous horizontal gate and the horizontal force, R, acting on the vertical gate that is required to keep the gates closed until this depth is reached. The weight of the vertical gate is negligible, and both gates are hinged at one end as shown. Friction in the hinges is negligible.





For horizontal gate,

$$\Sigma M_H = 0$$

so that

$$\mathcal{W} = pA$$ where p is the water pressure on the bottom surface.

Thus, $p = \gamma_{H_2O}(2\,m)$

so that

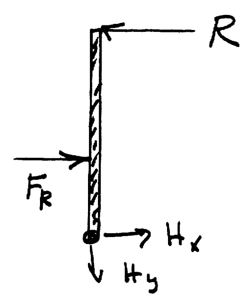
$$\mathcal{W} = \left(9800\,\frac{N}{m^3}\right)(2\,m)(4\,m \times 4\,m) = \underline{\underline{314\,kN}}$$

For vertical gate,

$F_R = \gamma h_c A$ where $h_c = 7\,m$

so that

$$F_R = \left(9800\,\frac{N}{m^3}\right)(7\,m)(4\,m \times 4\,m)$$
$$= 1100\,kN$$



To locate F_R

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,m)(4\,m)^3}{(7\,m)(4\,m \times 4\,m)} + 7\,m = 7.191\,m$$

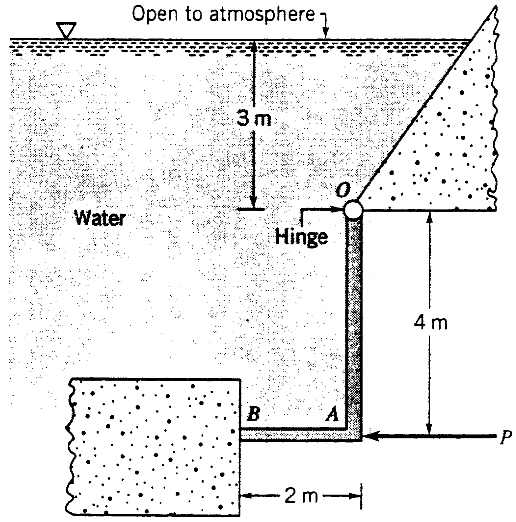
For equilibrium

$\Sigma M_H = 0$ so that

$$R = \frac{(1100\,kN)(9\,m - 7.191\,m)}{4\,m} = \underline{\underline{497\,kN}}$$

2.58

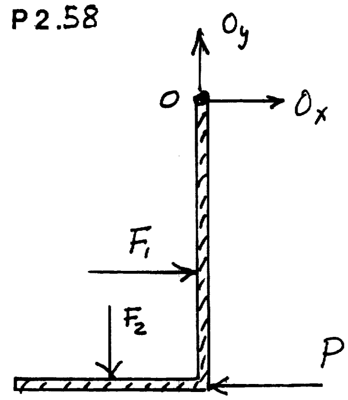
2.58 The rigid gate, *OAB*, of Fig. P2.58 is hinged at *O* and rests against a rigid support at *B*. What minimum horizontal force, *P*, is required to hold the gate closed if its width is 3 m? Neglect the weight of the gate and friction in the hinge. The back of the gate is exposed to the atmosphere.



■ FIGURE P2.58

$$F_1 = \gamma h_{c_1} A_1 \quad \text{where } h_{c_1} = 5\,m$$

Thus,
$$F_1 = \left(9800\,\frac{N}{m^3}\right)(5\,m)(4\,m \times 3\,m)$$
$$= 5.88 \times 10^5\,N$$



$$F_2 = \gamma h_{c_2} A_2 \quad \text{where } h_{c_2} = 7\,m$$

so that
$$F_2 = \left(9800\,\frac{N}{m^3}\right)(7\,m)(2\,m \times 3\,m)$$
$$= 4.12 \times 10^5\,N$$

To locate F_1,
$$y_{R_1} = \frac{I_{xc}}{y_{c_1} A_1} + y_{c_1} = \frac{\frac{1}{12}(3\,m)(4\,m)^3}{(5\,m)(4\,m \times 3\,m)} + 5\,m = 5.267\,m$$

The force F_2 acts at the center of the AB section. Thus,
$$\Sigma M_O = 0$$

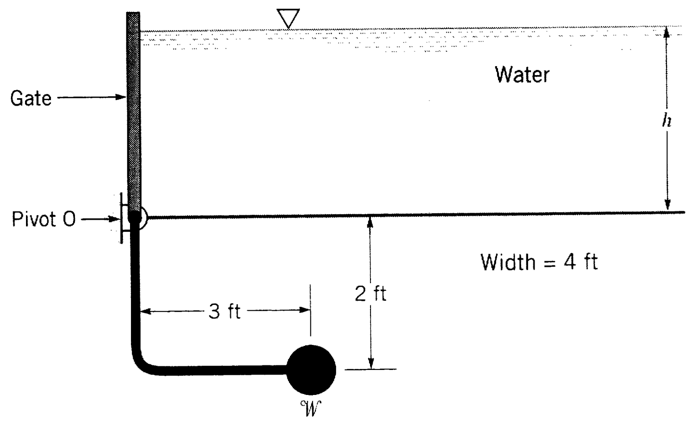
and
$$F_1(5.267\,m - 3\,m) + F_2(1\,m) = P(4\,m)$$

so that
$$P = \frac{(5.88 \times 10^5\,N)(2.267\,m) + (4.12 \times 10^5\,N)(1\,m)}{4\,m}$$

$$= \underline{\underline{436\,kN}}$$

2.59

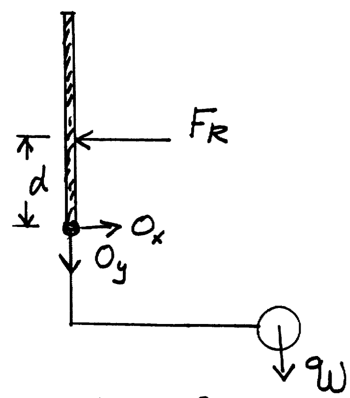
2.59 The massless, 4-ft-wide gate shown in Fig. P2.59 pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, h.



$F_R = \gamma h_c A$ where $h_c = \frac{h}{2}$

Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2}(h \times b)$$
$$= \gamma_{H_2O} \frac{h^2}{2}(4\,ft)$$



$b = 4\,ft$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,ft)(h^3)}{\frac{h}{2}(4\,ft \times h)} + \frac{h}{2}$$
$$= \frac{2}{3}h$$

For equilibrium,

$$\Sigma M_o = 0$$
$$F_R\, d = W(3\,ft) \quad \text{where} \quad d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000\,lb)(3\,ft)}{(\gamma_{H_2O})(\frac{h^2}{2})(4\,ft)}$$

Thus,

$$h^3 = \frac{(3)(2000\,lb)(3\,ft)}{(62.4\,\frac{lb}{ft^3})(\frac{1}{2})(4\,ft)}$$

$$h = \underline{\underline{5.24\,ft}}$$

2.60 *

2.60* A 200-lb homogeneous gate of 10-ft width and 5-ft length is hinged at point A and held in place by a 12-ft-long brace as shown in Fig. P2.60. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. **(a)** Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, θ, for $0 \leq \theta \leq 90°$. **(b)** Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the results as $\theta \rightarrow 0$.

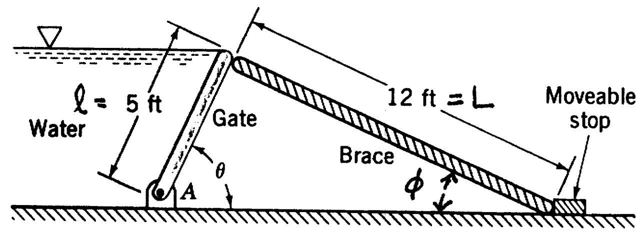
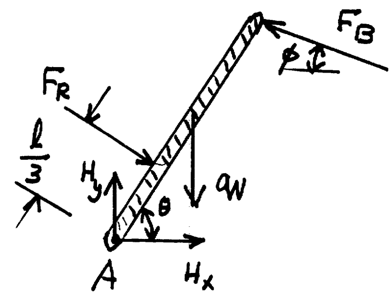


FIGURE P2.60



(a) For the free-body-diagram of the gate (see figure),

$$\sum M_A = 0$$

so that

$$F_R\left(\frac{\ell}{3}\right) + W\left(\frac{\ell}{2}\cos\theta\right) = (F_B\cos\phi)(\ell\sin\theta) + (F_B\sin\phi)(\ell\cos\theta) \qquad (1)$$

Also,

$$\ell\sin\theta = L\sin\phi$$ (assuming hinge and end of brace at same elevation)

or

$$\sin\phi = \frac{\ell}{L}\sin\theta$$

and

$$F_R = \gamma h_c A = \gamma\left(\frac{\ell\sin\theta}{2}\right)(\ell w)$$

where w is the gate width. Thus, Eq. (1) can be written as

$$\gamma\left(\frac{\ell^3}{6}\right)(\sin\theta)\,w + \frac{W\ell}{2}\cos\theta = F_B\,\ell\left(\cos\phi\sin\theta + \sin\phi\cos\theta\right)$$

so that

$$F_B = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\sin\theta + \frac{W}{2}\cos\theta}{\cos\phi\sin\theta + \sin\phi\cos\theta} = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\tan\theta + \frac{W}{2}}{\cos\phi\tan\theta + \sin\phi} \qquad (2)$$

For $\gamma = 62.4\ \text{lb/ft}^3$, $\ell = 5\text{ ft}$, $w = 10\text{ ft}$, and $W = 200\text{ lb}$,

$$F_B = \frac{\frac{(62.4\frac{\text{lb}}{\text{ft}^3})(5\text{ ft})^2(10\text{ ft})}{6}\tan\theta + \frac{200\text{ lb}}{2}}{\cos\phi\tan\theta + \sin\phi} = \frac{2600\tan\theta + 100}{\cos\phi\tan\theta + \sin\phi} \qquad (3)$$

(con't)

2.60* (con't)

Since $\sin\phi = \frac{l}{L}\sin\theta$ and $l = 5\,ft$, $L = 12\,ft$

$$\sin\phi = \frac{5}{12}\sin\theta$$

and for a given θ, ϕ can be determined. Thus, Eq.(3) can be used to determine F_B for a given θ. A computer program for calculating F_B as a function of θ follows.

```
100 cls
110 print "*********************************************************"
120 print "** Variation of the resultant Fb as a function of theta **"
130 print "*********************************************************"
140 print
150 print " Theta (deg)   Fb (lbs) (w=100 lbs)   Fb (lbs) (w=0 lbs)"
160 pi=4.0*atn(1.0)
170 for theta=pi/2 to pi/36 step -pi/36
180 sph=5/12*sin(theta)
190 phi=atn(sph/(1-sph^2)^(0.5))
200 fb1=(2600*tan(theta)+100)/(cos(phi)*tan(theta)+sin(phi))
210 fb2=2600*tan(theta)/(cos(phi)*tan(theta)+sin(phi))
220 print using "    ###.#              ####.#                   ####.#";theta*180/p
i,fb1,fb2
230 next theta
```

Tabulated data and a plot of the data are given on the following page.

(b) For $\mathcal{W} = 0$, Eq.(3) reduces to

$$F_B = \frac{2600\tan\theta}{\cos\phi\tan\theta + \sin\phi} \qquad (4)$$

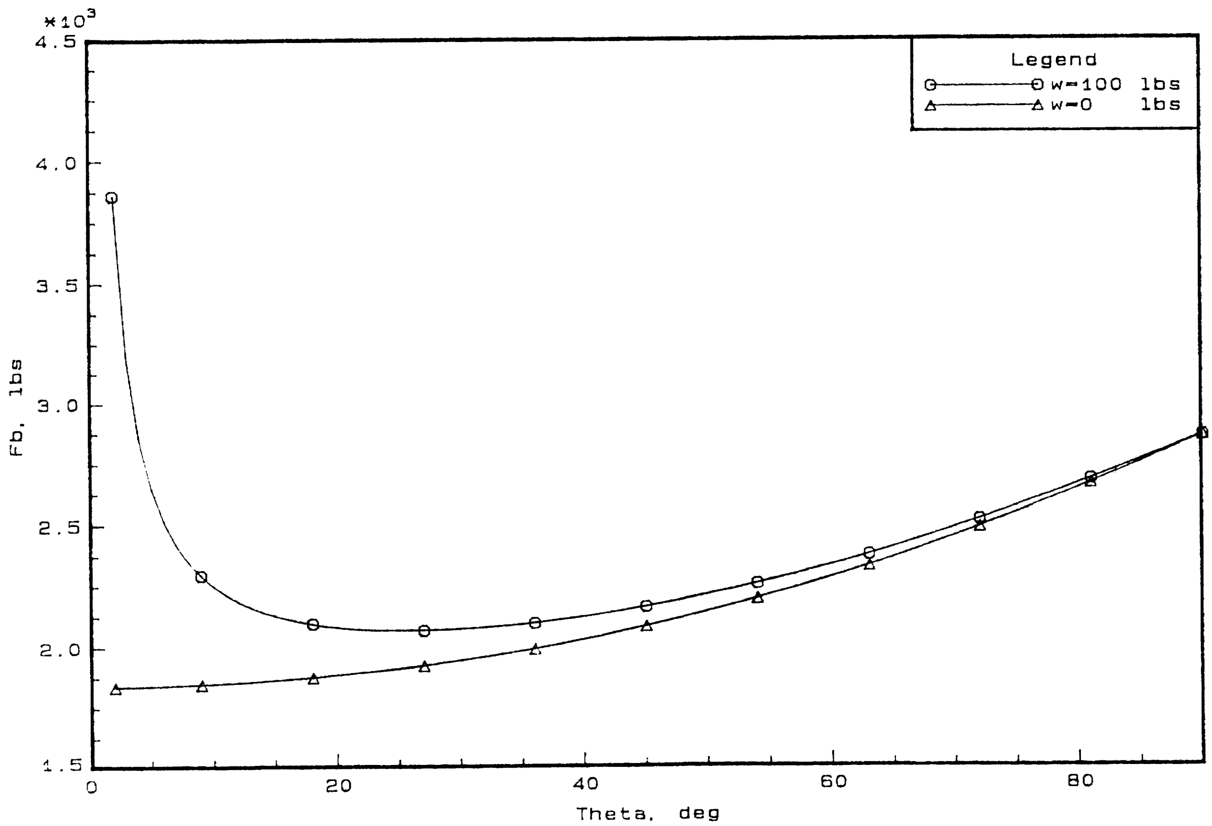
and the same program as was used in part (a) (with $\mathcal{W}$ set equal to zero) can be used to obtain F_B as a function of θ. Tabulated data and a plot of the data are given on the following page.

(con't)

2.60* (con't)

```
*************************************************************
** Variation of the resultant Fb as a function of theta **
*************************************************************
```

Theta (deg)	Fb (lbs) (w=100 lbs)	Fb (lbs) (w=0 lbs)
90.0	2860.1	2860.1
85.0	2757.4	2748.1
80.0	2659.4	2641.5
75.0	2567.0	2540.9
70.0	2480.9	2446.7
65.0	2401.6	2359.2
60.0	2329.4	2278.8
55.0	2264.8	2205.4
50.0	2208.0	2139.0
45.0	2159.6	2079.6
40.0	2120.0	2027.1
35.0	2090.0	1981.2
30.0	2071.3	1941.9
25.0	2066.4	1909.0
20.0	2081.1	1882.2
15.0	2128.8	1861.6
10.0	2249.8	1847.0
5.0	2646.3	1838.2



(con't)

2.60* (con't)

As $\theta \to 0$ the value of F_B can be determined from Eq.(*),

$$F_B = \frac{2600 \tan\theta}{\cos\phi \tan\theta + \sin\phi}$$

Since

$$\sin\phi = \frac{5}{12}\sin\theta$$

it follows that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}$$

and therefore

$$F_B = \frac{2600\tan\theta}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}\,\tan\theta + \frac{5}{12}\sin\theta} = \frac{2600}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta} + \frac{5}{12}\cos\theta}$$

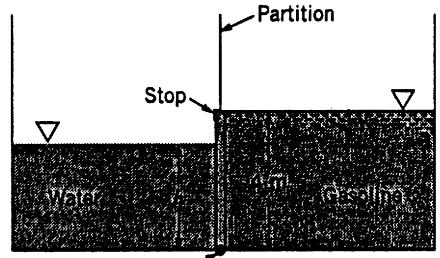
Thus, as $\theta \to 0$

$$F_B \to \frac{2600}{1+\frac{5}{12}} = 1840 \text{ lb}$$

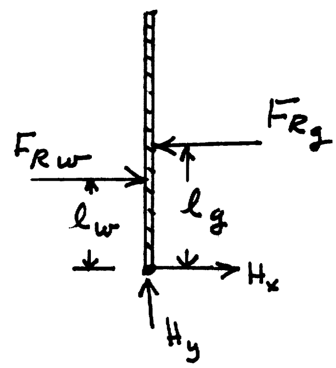
Physically this result means that for $\theta \equiv 0$, the value of F_B is indeterminate, but for any "very small" value of θ, F_B will approach 1840 lb.

2.61

2.61 An open tank has a vertical partition and on one side contains gasoline with a density ρ = 700 kg/m^3 at a depth of 4 m, as shown in Fig. P2.61. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?



■ FIGURE P2.61



$F_{R_g} = \gamma_g h_{c_g} A_g$

where g refers to gasoline.

$F_{R_g} = \left(700 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(2m)(4m \times 2m)$

$= 110 \times 10^3 \ N = 110 \ kN$

$F_{R_w} = \gamma_w h_{c_w} A_w$

where w refers to water.

$F_{R_w} = \left(9.80 \times 10^3 \frac{N}{m^3}\right)\left(\frac{h}{2}\right)(2\,m \times h)$

where h is depth of water.

$F_{R_w} = (9.80 \times 10^3) h^2$

For equilibrium,

$\sum M_H = 0$

so that

$F_{R_w} \ell_w = F_{R_g} \ell_g$ with $\ell_w = \frac{h}{3}$ and $\ell_g = \frac{4}{3}$ m

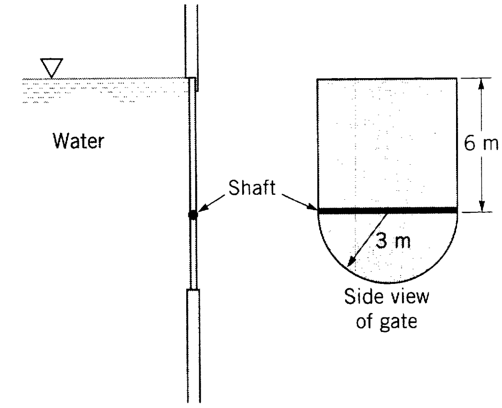
Thus, $(9.80 \times 10^3)(h^2)\left(\frac{h}{3}\right) = (110 \times 10^3 N)\left(\frac{4}{3} m\right)$

and $h = \underline{\underline{3.55 \ m}}$

which is the limiting value for h.

2.62

2.62 A gate having the shape shown in Fig. P2.62 is located in the vertical side of an open tank containing water. The gate is mounted on a horizontal shaft. **(a)** When the water level is at the top of the gate, determine the magnitude of the fluid force on the rectangular portion of the gate above the shaft and the magnitude of the fluid force on the semicircular portion of the gate below the shaft. **(b)** For this same fluid depth determine the moment of the force acting on the semicircular portion of the gate with respect to an axis which coincides with the shaft.



(a) For rectangular portion,

$$(F_R)_r = \gamma h_c A \quad \text{where} \quad h_c = 3\,m$$

So that

$$(F_R)_r = \left(9800\,\frac{N}{m^3}\right)(3\,m)(6\,m \times 6\,m) = \underline{\underline{1060\,kN}}$$

For semi-circular portion,

$$(F_R)_{sc} = \gamma h_c A \quad \text{where} \quad h_c = 6\,m + \frac{4R}{3\pi} \quad (\text{See Fig. 2.13})$$

$$= 6\,m + \frac{4(3\,m)}{3\pi} = 7.27\,m$$

so that

$$(F_R)_{sc} = \left(9800\,\frac{N}{m^3}\right)(7.27\,m)\left(\frac{\pi}{2}(3\,m)^2\right) = \underline{\underline{1010\,kN}}$$

(b) For semi-circular portion

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{0.1098 R^4}{(7.27\,m)\left(\frac{\pi}{2}\right) R^2} + 7.27\,m$$

$$= \frac{0.1098\,(3\,m)^4}{(7.27\,m)\left(\frac{\pi}{2}\right)(3\,m)^2} + 7.27\,m = 7.36\,m$$

Thus, moment with respect to shaft, M,

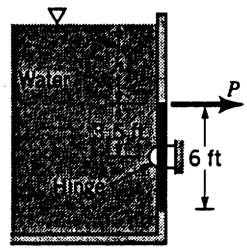
$$M = (F_R)_{sc} \times (7.36\,m - 6.00\,m)$$

$$= (1010 \times 10^3\,N)(1.36\,m)$$

$$= \underline{\underline{1.37 \times 10^6\,N \cdot m}}$$

2.63

2.63 A 6 ft × 6 ft square gate is free to pivot about the frictionless hinge shown in Fig. P2.63. In general, a force, P, is needed to keep the gate from rotating. Determine the depth, h, for the situation when $P = 0$.



■ FIGURE P2.63

For equilibrium

$$\Sigma M_H = 0$$

Thus, for $P=0$ F_R would have to pass through the hinge, i.e., $y_R = 3.5\text{ft} + h$

Since

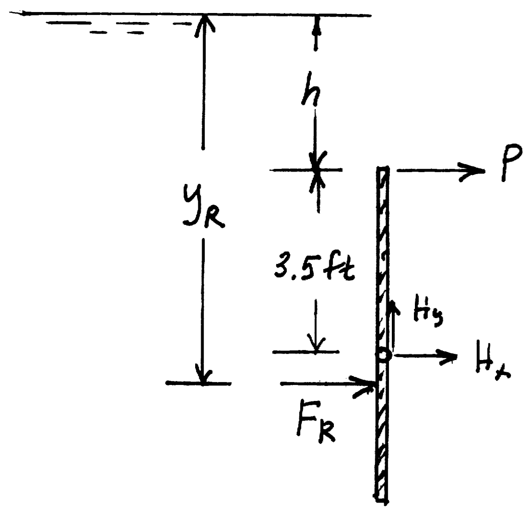
$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

and with $y_c = h + 3\text{ft}$

$$3.5\text{ft} + h = \frac{\frac{1}{12}(6\text{ft})(6\text{ft})^3}{(h+3\text{ft})(6\text{ft}\times 6\text{ft})} + h + 3\text{ft}$$

$$0.5\text{ft} = \frac{3\text{ ft}^2}{h+3\text{ft}}$$

$$h = \underline{\underline{3.00\text{ ft}}}$$



2.64

2.64 A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in Fig. P2.64. The horizontal portion of the gate covers a 1-ft-diameter drain pipe which contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.

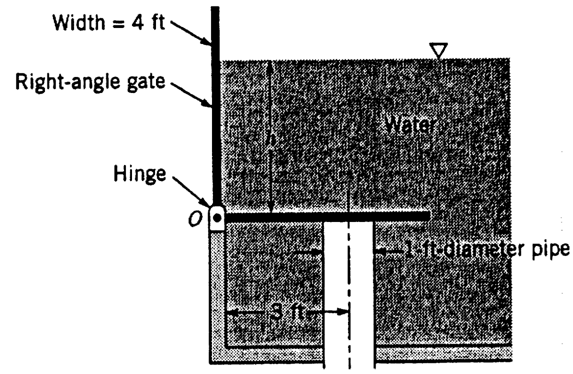
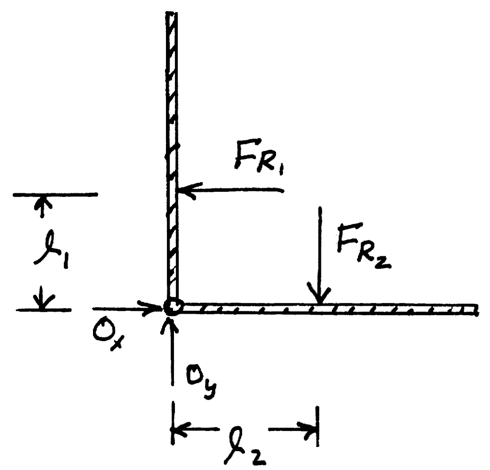


FIGURE P2.64

For equilibrium

$$\sum M_O = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$



$$F_{R_1} = \gamma h_{c_1} A_1$$
$$= \left(62.4 \tfrac{lb}{ft^3}\right)\left(\tfrac{h}{2}\right)\left(4 ft \times h\right)$$
$$= 125\, h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h \left(\tfrac{\pi}{4}\right)(1 ft)^2 = \left(62.4 \tfrac{lb}{ft^3}\right)(h)\left(\tfrac{\pi}{4}\right)(1 ft)^2$$
$$= 49.0\, h$$

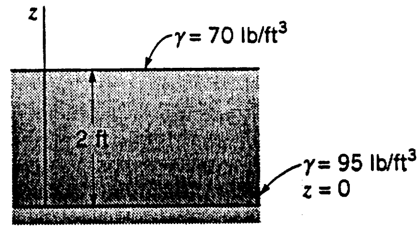
Thus, from Eq. (1) with $\ell_1 = \frac{h}{3}$ and $\ell_2 = 3 ft$

$$(125 h^2)\left(\tfrac{h}{3}\right) = (49.0\, h)(3 ft)$$

$$h = \underline{\underline{1.88 ft}}$$

2.65

2.65 The specific weight, γ, of the static liquid layer shown in Fig. P2.65 increases *linearly* with depth. At the free surface $\gamma = 70\ \text{lb/ft}^3$, and at the bottom of the layer $\gamma = 95\ \text{lb/ft}^3$. Make use of Eq. 2.4 to determine the pressure at the bottom of the layer.



■ FIGURE P2.65

$$\frac{dp}{dz} = -\gamma \qquad \text{(Eq. 2.4)}$$

For linear variation in γ

$$\gamma = 95 - 12.5\,z$$

so that

$$\int_{p_{bottom}}^{0} dp = -\int_{0}^{z=2} (95 - 12.5\,z)\,dz$$

$$-p_{bottom} = -\left[95z - \frac{12.5}{2}z^2\right]_0^2$$

$$= -\left[95(2) - 6.25(2)^2\right]$$

$$p_{bottom} = \underline{\underline{165\ \frac{\text{lb}}{\text{ft}^2}}}$$

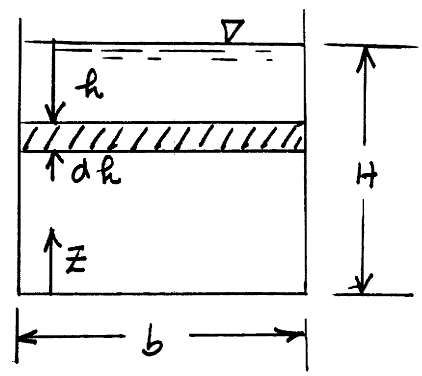
2.66*

2.66* An open rectangular settling tank contains a liquid suspension that at a given time has a specific weight that varies approximately with depth according to the following data:

h (m)	γ (kN/m³)
0	10.0
0.4	10.1
0.8	10.2
1.2	10.6
1.6	11.3
2.0	12.3
2.4	12.7
2.8	12.9
3.2	13.0
3.6	13.1

(con't)

The depth $h = 0$ corresponds to the free surface. Determine, by means of numerical integration, the magnitude and location of the resultant force that the liquid suspension exerts on a vertical wall of the tank that is 6 m wide. The depth of fluid in the tank is 3.6 m.



The magnitude of the fluid force, F_R, can be found by summing the differential forces acting on the horizontal strip shown in the figure. Thus,

$$F_R = \int_0^H dF_R = b\int_0^H p\,dh \qquad (1)$$

where p is the pressure at depth h. To find p we use Eq. 2.4

$$\frac{dp}{dz} = -\gamma$$

and with $dz = -dh$

$$p(h) = \int_0^h \gamma\,dh \qquad (2)$$

Equation (2) can be integrated numerically with the following program and using the variation in γ with h given.

```
100 cls
110 print "***********************************************"
120 print "** This program integrates Eq. 2.4 numerically **"
130 print "** using the trapezoidal rule to obtain the    **"
140 print "** pressure at different depths                **"
150 print "***********************************************"
160 print
170 dim p(10),gamma(10)
180 n=10
190 dh=0.4
200 p(1)=0
210 for i=1 to n
220 read gamma(i)
230 next i
240 data 10.0,10.1,10.2,10.6,11.3,12.3,12.7,12.9,13.0,13.1
250 for i=2 to n
260 s=(gamma(1)+gamma(i))/2
270 im1=i-1
```

(con't)

```
280 for j=2 to im1
290 s=s+gamma(j)
300 next j
310 p(i)=dh*s
320 next i
330 '
340 'Print the results
350 print
360 print " h (m)    Pressure (kPa)"
370 for i=1 to n
380 print using "###.#        ###.##";(i-1)*dh,p(i)
390 next i
```

The pressure distribution is given below.

```
*************************************************
** This program integrates Eq. 2.4 numerically **
** using the trapezoidal rule to obtain the    **
** pressure at different depths                **
*************************************************


h (m)    Pressure (kPa)
 0.0         0.00
 0.4         4.02
 0.8         8.08
 1.2        12.24
 1.6        16.62
 2.0        21.34
 2.4        26.34
 2.8        31.46
 3.2        36.64
 3.6        41.86
```

Equation (1) can now be integrated numerically using TRAPEZOI

```
*****************************************************
** This program performs numerical integration     **
** over a set of points using the Trapezoidal Rule **
*****************************************************

Enter number of data points: 10
Enter data points (X , Y)
? 0.0,0.00
? 0.4,4.02
? 0.8,8.08
? 1.2,12.24
? 1.6,16.62
? 2.0,21.34
? 2.4,26.34
? 2.8,31.46
? 3.2,36.64
? 3.6,41.86

The approximate value of the integral is: +7.1068E+01
```

(con't)

2.66* (con't)

Thus, with

$$\int_0^H p\,dh = 71.07\ \frac{kN}{m}$$

$$F_R = (6\,m)\left(71.07\ \frac{kN}{m}\right) = \underline{\underline{426\ kN}}$$

To locate F_R sum moments about axis formed by intersection of vertical wall and fluid surface. Thus,

$$F_R h_R = b\int_0^H h\,p\,dh \qquad (3)$$

The integrand hp can be determined and Eq. (3) integrated numerically using TRAPEZ01. Tabulated results are given below.

```
****************************************************
** This program performs numerical integration    **
** over a set of points using the Trapezoidal Rule **
****************************************************

Enter number of data points: 10
Enter data points (X , Y)
? 0.0,0.000
? 0.4,1.608
? 0.8,6.464
? 1.2,14.688
? 1.6,26.592
? 2.0,42.680
? 2.4,63.216
? 2.8,88.088
? 3.2,117.248
? 3.6,150.696

The approximate value of the integral is: +1.7437E+02
```

Note: $Y \sim hp$

Thus, with

$$\int_0^H h\,p\,dh = 174.4\ kN$$

it follows from Eq. (3) that

$$h_R = \frac{b\int_0^H hp\,dh}{F_R} = \frac{(6\,m)(174.4\ kN)}{426\ kN} = 2.46\,m$$

The resultant force acts $\underline{\underline{\text{2.46 m below fluid surface}}}$.

2.67

2.67 The inclined face AD of the tank of Fig. P2.67 is a plane surface containing a gate ABC, which is hinged along line BC. The shape of the gate is shown in the plan view. If the tank contains water, determine the magnitude of the force that the water exerts on the gate.

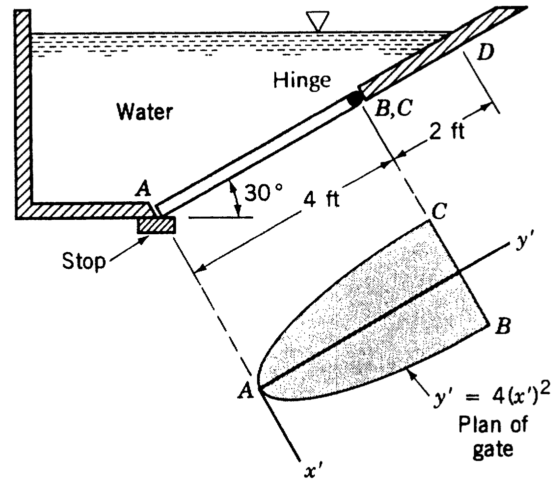
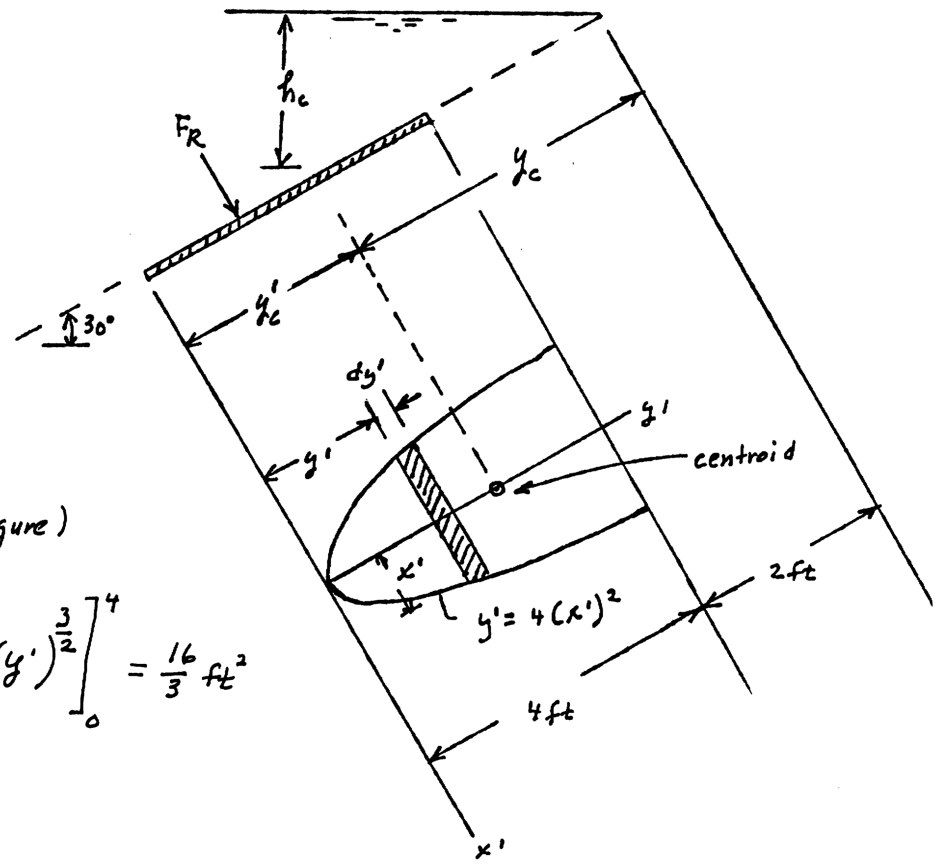


FIGURE P2.67



$$F_R = \gamma h_c A$$

where

$$A = \int_0^4 2x'\,dy' \quad \text{(see figure)}$$

$$= \int_0^4 2\left(\tfrac{1}{2}\right)\sqrt{y'}\,dy' = \tfrac{2}{3}(y')^{\frac{3}{2}}\Big]_0^4 = \tfrac{16}{3}\,ft^2$$

To locate centroid:

$$y'_c A = \int_0^4 y'\,dA = \int_0^4 2y'x'\,dy' = \int_0^4 (y')^{3/2}\,dy' = \tfrac{2}{5}(y')^{\frac{5}{2}}\Big]_0^4 = \tfrac{64}{5}\,ft^3$$

Thus,

$$y'_c = \frac{\frac{64}{5}\,ft^3}{\frac{16}{3}\,ft^2} = 2.4\,ft$$

and

$$y_c = 6\,ft - 2.4\,ft = 3.6\,ft$$

Since $h_c = y_c \sin 30°$,

$$F_R = \left(62.4\,\tfrac{lb}{ft^3}\right)(3.6\,ft)(\sin 30°)\left(\tfrac{16}{3}\,ft^2\right) = \underline{\underline{599\,lb}}$$

2.68

2.68 Dams can vary from very large structures with curved faces holding back water to great depths, as shown in Video V2.3, to relatively small structures with plane faces as shown in Fig. P2.68. Assume that the concrete dam shown in Fig. P2.68 weighs 23.6 kN/m^3 and rests on a solid foundation. Determine the minimum coefficient of friction between the dam and the foundation required to keep the dam from sliding at the water depth shown. You do not need to consider possible uplift along the base. Base your analysis on a unit length of the dam.

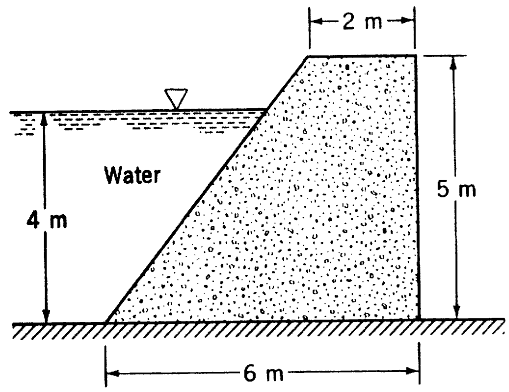
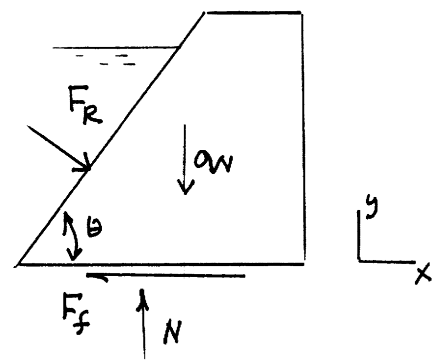


FIGURE P2.68

$$F_R = \gamma h_c A$$

where $A = \left(\frac{4\text{ m}}{\sin 51.3°}\right)(1\quad)$



so that

$$F_R = \left(9.80\,\frac{\text{kN}}{\text{m}^3}\right)\left(\frac{4\text{ m}}{2}\right)\left(\frac{4\text{ m}}{\sin 51.3°}\right)(1\text{ m})$$
$$= 100\text{ kN}$$

$$\tan\theta = \frac{5\text{ m}}{4\text{ m}}$$
$$\theta = 51.3°$$

For equilibrium,

$$\Sigma F_x = 0$$

or

$$F_R \sin 51.3° = F_f = \eta N$$ where $\eta \sim$ coefficient of friction.

Also, $\Sigma F_y = 0$

so that

$$N = \mathcal{W} + F_R \cos 51.3°$$ where

$$\mathcal{W} = (\gamma_{\text{concrete}})(\text{volume of concrete})$$

Thus,

$$N = \left(23.6\,\frac{\text{kN}}{\text{m}^3}\right)(20\text{ m}^3) + (100\text{ kN})\cos 51.3° = 534\text{ kN}$$

and

$$\eta = \frac{F_R \sin 51.3°}{N} = \frac{(100\text{ kN})\sin 51.3°}{534\text{ kN}} = \underline{\underline{0.146}}$$

2.69*

2.69* Water backs up behind a concrete dam as shown in Fig. P2.69. Leakage under the foundation gives a pressure distribution under the dam as indicated. If the water depth, h, is too great, the dam will topple over about its toe (point A). For the dimensions given, determine the maximum water depth for the following widths of the dam: ℓ = 20, 30, 40, 50, and 60 ft. Base your analysis on a unit length of the dam. The specific weight of the concrete is 150 lb/ft³.

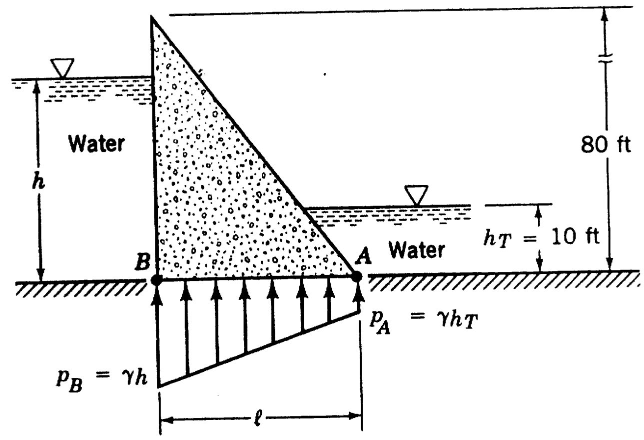
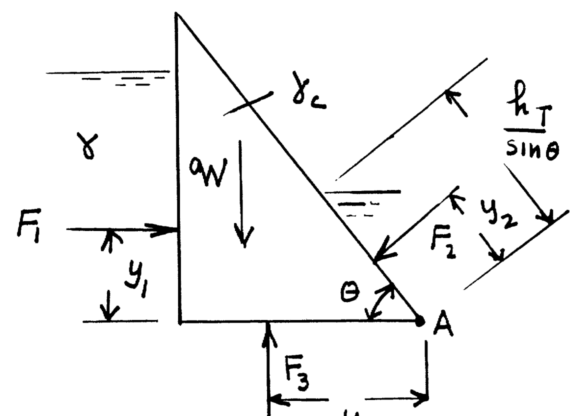


FIGURE P2.69

A free-body-diagram of the dam is shown in the figure at the right, where:



$$F_1 = \frac{\gamma h^2}{2} \quad \text{(for unit length)}$$

$$\mathcal{W} = \gamma_c\left(\tfrac{1}{2}\right)(\ell)(80) = 40\gamma_c \ell$$

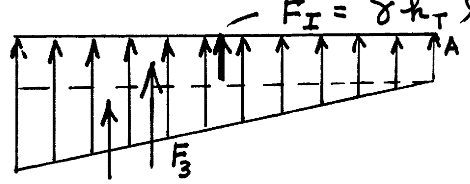
$$F_3 = \left(\frac{\gamma h + \gamma h_T}{2}\right)\ell$$

$$F_2 = \gamma\left(\frac{h_T}{2}\right)\left(\frac{h_T}{\sin\theta}\right) = \frac{\gamma h_T^2}{2\sin\theta}$$

$$\tan\theta = \frac{80}{\ell}$$

$$y_1 = \frac{h}{3} \qquad y_2 = \frac{1}{3}\left(\frac{h_T}{\sin\theta}\right)$$

To determine y_3 consider the pressure distribution on the bottom:



$$F_{II} = \frac{\gamma}{2}(h - h_T)\ell$$

Summing moments about A,

$$F_3 y_3 = F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)$$

(con't)

2.69* (con't)

so that

$$y_3 = \frac{F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)}{F_3}$$

where $F_3 = F_I + F_{II}$. Substitution of expressions for F_I and F_{II} yields,

$$y_3 = \frac{\ell\left(\frac{h_T}{3} + \frac{2}{3}h\right)}{h + h_T}$$

For equilibrium of the dam, $\Sigma M_A = 0$, so that

$$F_1\, y_1 - \mathcal{W}\left(\tfrac{2}{3}\ell\right) - F_2\, y_2 + F_3\, y_3 = 0 \qquad (1)$$

and with $\gamma = 62.4\ \text{lb/ft}^3$, $\gamma_c = 150\ \text{lb/ft}^3$, and $h_T = 10\ \text{ft}$, then:

$F_1 = 31.2\, h^2$ $\quad \mathcal{W} = 6000\,\ell$ $\quad F_2 = \dfrac{3120}{\sin\theta}$ $\quad y_2 = \dfrac{10/3}{\sin\theta}$

$F_3 = 31.2\,(h+10)\,\ell$ $\quad y_3 = \dfrac{\ell\left(\frac{10}{3} + \frac{2}{3}h\right)}{h + h_T} = \dfrac{(2h+10)\,\ell}{3(h+10)}$

Substitution of these expressions into Eq. (1) yields,

$$(31.2\,h^2)\left(\frac{h}{3}\right) - (6000\,\ell)\left(\frac{2}{3}\ell\right) - \left(\frac{3120}{\sin\theta}\right)\left(\frac{10/3}{\sin\theta}\right) + \left[31.2\,(h+10)\,\ell\right]\left[\frac{(2h+10)\,\ell}{3(h+10)}\right] = 0$$

which can be simplified to

$$\frac{31.2}{3}h^3 + 20.8\,\ell^2 h - 3896\,\ell^2 - \frac{10{,}400}{\sin^2\theta} = 0 \qquad (2)$$

Thus, for a given ℓ, θ can be determined from the condition $\tan\theta = 80/\ell$, and Eq. (2) solved for h.

A computer program for determining h for a given ℓ follows.

(con't)

2.69* (Con't)

```
100 cls
110 print "*****************************************************"
120 print "** This program solves a cubic equation to determine **"
130 print "** the maximum water, h, depth for a series of dam   **"
140 print "** widths, l                                         **"
150 print "*****************************************************"
160 print
170 print " Dam width, l (ft)     Maximum depth, h (ft)"
180 for l=20 to 60 step 10
190 theta=atn(80/l)
200 h=0
210 hp=h
220 h=(3/31.2*(3896*l^2+10400/(sin(theta)^2)-20.8*l^2*hp))^(1/3)
230 if abs(1-hp/h)>0.001 goto 210
240 print using "       ##.#                    ##.#";l,h
250 next l
```

For the dam widths specified, the maximum water depths are given below. Note that for the two largest dam widths the water would overflow the dam before it would topple.

```
*****************************************************
** This program solves a cubic equation to determine **
** the maximum water, h, depth for a series of dam   **
** widths, l                                         **
*****************************************************

 Dam width, l (ft)     Maximum depth, h (ft)
       20.0                    48.2
       30.0                    61.1
       40.0                    71.8
       50.0                    81.1
       60.0                    89.2
```

2.70

2.70 A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.70. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point *A*? Explain.

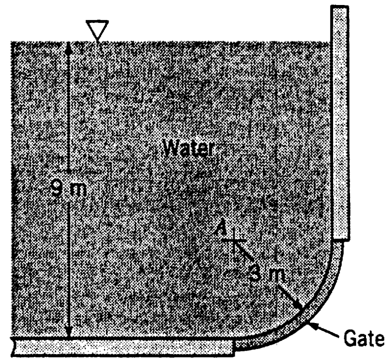
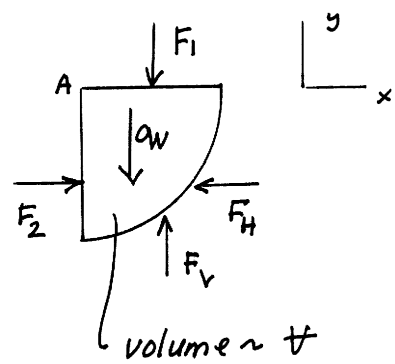


FIGURE P2.70



and
$$\forall = \frac{\pi (3\,m)^2}{4} \times 4\,m = 9\pi\ m^3$$

For equilibrium,

$$\sum F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma (6m + 1.5m)(3\,m \times 4\,m)$$

so that

$$F_H = \left(9.80 \frac{kN}{m^3}\right)(7.5\,m)(12\,m^2) = \underline{\underline{882\ kN}}$$

Similarly,

$$\sum F_y = 0$$

$$F_V = F_1 + W \qquad \text{where :}$$

$$F_1 = [\gamma (6m)](3\,m \times 4\,m) = \left(9.80 \frac{kN}{m^3}\right)(6\,m)(12\,m^2)$$

$$W = \gamma \forall = \left(9.80 \frac{kN}{m^3}\right)(9\pi\,m^3)$$

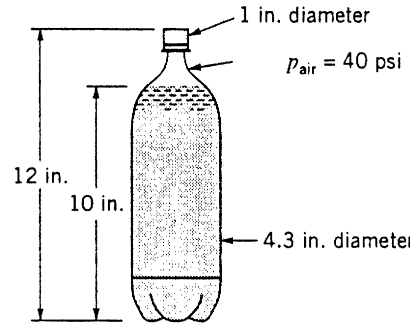
Thus, $$F_V = \left(9.80 \frac{kN}{m^3}\right)\left[72\,m^3 + 9\pi\,m^3\right] = \underline{\underline{983\ kN}}$$

(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. <u>Yes.</u>

2.71

2.71 The air pressure in the top of the two liter pop bottle shown in Video V2.4 and Fig. P2.71 is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. (a) If the bottle cap has a diameter of 1 in. what is magnitude of the axial force required to hold the cap in place? (b) Determine the force needed to secure the bottom 2 inches of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. (c) By how much does the weight of the pop increase the pressure 2 inches above the bottom? Assume the pop has the same specific weight as that of water.



■ FIGURE P2.71

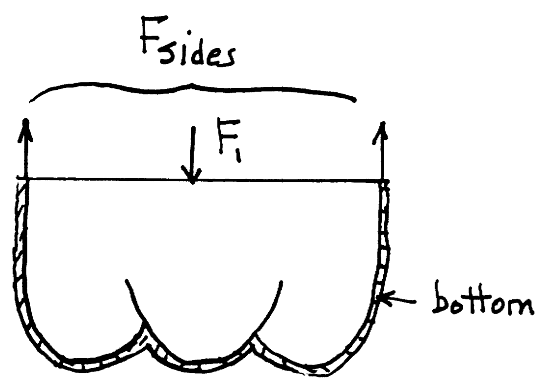
(a) $F_{cap} = p_{air} \times Area_{cap} = \left(40\,\frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(1\text{ in.})^2 = \underline{\underline{31.4\text{ lb}}}$

(b) $\sum F_{vertical} = 0$

$F_{sides} = F_1 = $ (pressure @ 2 in. above bottom) $\times$ (Area)

$= \left(40\,\frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3\text{ in.})^2$

$= \underline{\underline{581\text{ lb}}}$



(c) $p = p_{air} + \gamma h$

$= 40\,\frac{lb}{in.^2} + \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{8}{12}\text{ ft}\right)\left(\frac{1}{144\text{ in.}^2/\text{ft}^2}\right)$

$= 40\,\frac{lb}{in.^2} + 0.289\,\frac{lb}{in.^2}$

Thus, the increase in pressure due to weight = $\underline{\underline{0.289\text{ psi}}}$ (which is less than 1% of air pressure).

2-72

2.72 Hoover Dam (see Video 2.3) is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in Fig. P2.72(a). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in Figure P2.72(b) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show here this force acts.

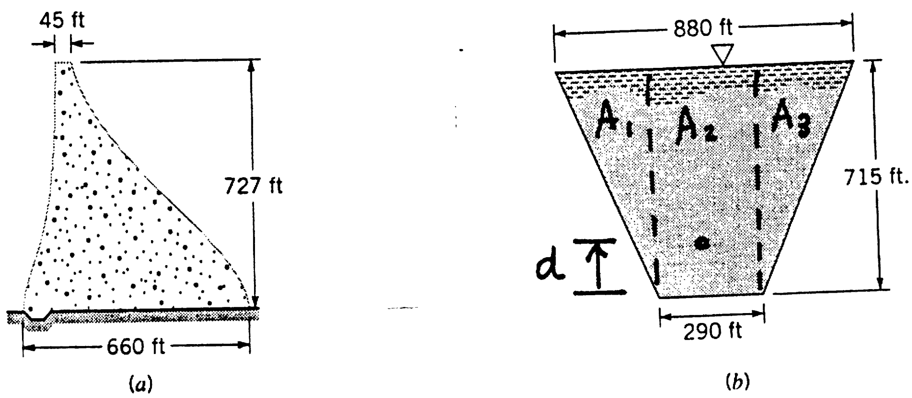


FIGURE P2.72

Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{3}\right)(715\ ft)\left(\frac{1}{2}\right)(295\ ft)(715\ ft)$$

$$= 1.57 \times 10^9\ lb$$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9\ lb$

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)(715\ ft)(290\ ft)(715\ ft)$$

$$= 4.63 \times 10^9\ lb$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ lb + 4.63 \times 10^9\ lb + 1.57 \times 10^9\ lb$$

$$= 7.77 \times 10^9\ lb$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

(con't)

2.72 (con't)

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\,ft) + F_{R_2}\left(\frac{1}{2}\right)(715\,ft) + F_{R_3}\left(\frac{2}{3}\right)(715\,ft)$$

and

$$d = \frac{(1.57\times10^9\,lb)\left(\frac{2}{3}\right)(715\,ft) + (4.63\times10^9\,lb)\left(\frac{1}{2}\right)(715\,ft) + (1.57\times10^9\,lb)\left(\frac{2}{3}\right)(715\,ft)}{7.77\times10^9\,lb}$$

$$= 406\,ft$$

Thus, the resultant horizontal force on the dam is $7.77\times10^9\,lb$ acting 406 ft up from the base of the dam along the axis of symmetry of the area.

2.73

2.73 A plug in the bottom of a pressurized tank is conical in shape as shown in Fig. P2.73. The air pressure is 50 kPa and the liquid in the tank has a specific weight of 27 kN/m³. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the 50-kPa pressure and the liquid.

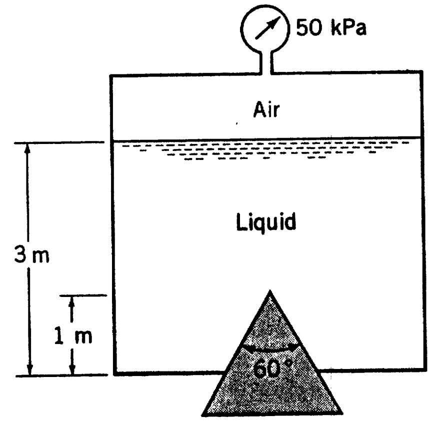
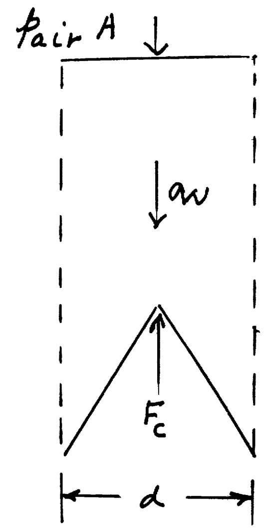


FIGURE P2.73



For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F_c = p_{air} A + \mathcal{W}$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air} A = (50\ kPa)\left(\frac{\pi}{4}\right)(d^2)$$
$$= (50\ kPa)\left(\frac{\pi}{4}\right)(1.155\,m)^2 = 52.4\ kN$$

$$\tan 30° = \frac{\frac{d}{2}}{1}$$

$$d = 2 \tan 30° = 1.155\,m$$

volume of cone $= \frac{\pi}{3}\left(\frac{d}{2}\right)^2(1)$

and

$$\mathcal{W} = \gamma\left[\frac{\pi}{4} d^2 (3\,m) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1\,m)\right]$$
$$= \gamma \pi d^2 \left[\frac{3\,m}{4} - \frac{1\,m}{12}\right]$$
$$= \left(27\ \frac{kN}{m^3}\right)(\pi)(1.155\,m)^2\left(\frac{2}{3}\,m\right) = 75.4\ kN$$

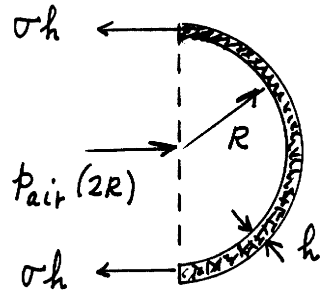
Thus,

$$F_c = 52.4\ kN + 75.4\ kN = 128\ kN$$

and the force on the cone has a magnitude of 128 kN and is directed vertically downward along the cone axis.

2.74

2.74 A 12-in.-diameter pipe contains a gas under a pressure of 140 psi. If the pipe wall thickness is $\frac{1}{4}$-in., what is the average circumferential stress developed in the pipe wall?



$\sigma \sim$ circumferential stress

For equilibrium (for a unit length of the pipe),

$$2\sigma h = p_{air}\,(2R)$$

or

$$\sigma = \frac{p_{air}\,R}{h}$$

$$= \frac{\left(140\ \frac{lb}{in.^2}\right)(6\ in.)}{\left(\frac{1}{4}\ in.\right)}$$

$$= \underline{\underline{3360\ psi}}$$

2.75

2.75 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P2.75 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

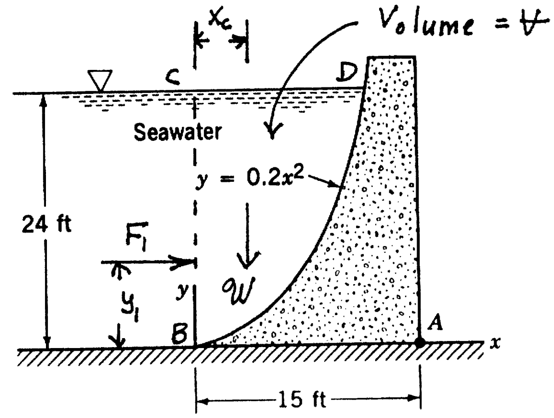


FIGURE P2.75

The components of the fluid force acting on the wall are F_1 and $\mathcal{W}$ as shown on the figure where

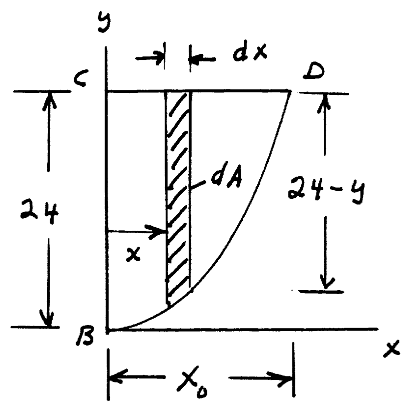
$$F_1 = \gamma h_c A = \left(64.0 \frac{lb}{ft^3}\right)\left(\frac{24\,ft}{2}\right)(24\,ft \times 1\,ft)$$

$$= 18{,}400\ lb \qquad \text{and} \qquad y_1 = \frac{24\,ft}{3} = 8\,ft$$

Also,

$$\mathcal{W} = \gamma \forall$$

To determine $\forall$ find area BCD. Thus, (see figure to right)



$x_0 = \sqrt{120}$

(Note: All lengths in ft)

$$A = \int_0^{x_0} (24 - y)\,dx = \int_0^{x_0} (24 - 0.2x^2)\,dx$$

$$= \left[24x - \frac{0.2x^3}{3}\right]_0^{x_0}$$

and with $x_0 = \sqrt{120}$, $A = 175\ ft^2$ so that

$$\forall = A \times 1\,ft = 175\ ft^3$$

Thus, $\mathcal{W} = \left(64.0 \frac{lb}{ft^3}\right)(175\ ft^3) = 11{,}200\ lb$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\,dA = \int_0^{x_0} (24 - y)\,x\,dx = \int_0^{x_0} (24x - 0.2x^3)\,dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

$$x_c = \frac{12(\sqrt{120})^2 - \frac{0.2(\sqrt{120})^4}{4}}{175} = 4.11\ ft$$

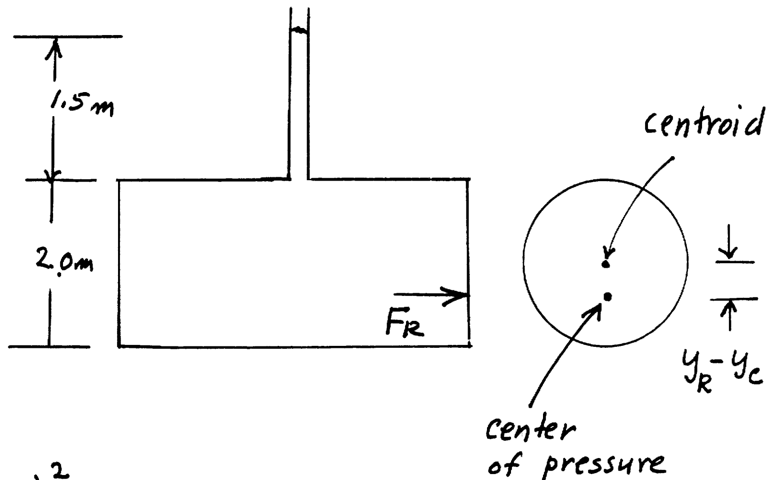
Thus,

$$M_A = F_1 y_1 - \mathcal{W}(15 - x_c)$$

$$= (18{,}400\ lb)(8\,ft) - (11{,}200\ lb)(15\,ft - 4.11\,ft) = \underline{\underline{25{,}200\ ft \cdot lb \,\circlearrowright}}$$

2.76

2.76 A cylindrical tank with its axis horizontal has a diameter of 2.0 m and a length of 4.0 m. The ends of the tank are vertical planes. A vertical, 0.1-m-diameter pipe is connected to the top of the tank. The tank and the pipe are filled with ethyl alcohol to a level of 1.5 m above the top of the tank. Determine the resultant force of the alcohol on one end of the tank and show where it acts.



$F_R = \gamma h_c A$

where $h_c = 1.5m + 1.0m = 2.5m$

so that

$$F_R = \left(7.74\,\frac{kN}{m^3}\right)(2.5\,m)\left(\frac{\pi}{4}\right)(2.0\,m)^2 = 60.8\,kN$$

Also,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

where $y_c = h_c$ so that

$$y_R = \frac{\frac{\pi (1m)^4}{4}}{(2.5m)\left(\frac{\pi}{4}\right)(2m)^2} + 2.5\,m = 2.60\,m$$

Thus, the resultant force has a magnitude of $\underline{60.8\,kN}$ and acts at a distance of $y_R - y_c = 2.60m - 2.50m = \underline{0.100m}$ below center of tank end wall.

2.77

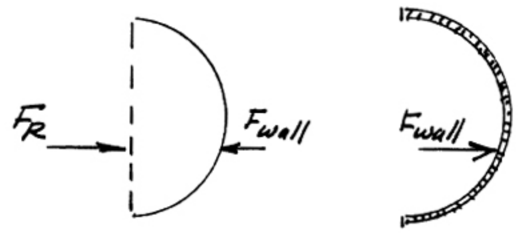
2.77 If the tank ends in Problem 2.76 are hemispherical, what is the magnitude of the resultant horizontal force of the alcohol on one of the curved ends?

For equilibrium,

$F_R = F_{wall}$ (see figure)

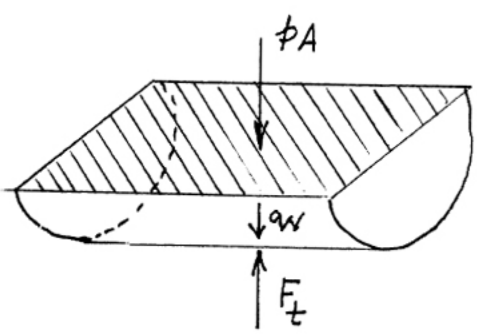
$= \underline{\underline{60.8\ kN}}$

since solution for horizontal force the same as for Problem 2.80.



2.78

2.78 Imagine the tank of Problem 2.76 split by a horizontal plane. Determine the magnitude of the resultant force of the alcohol on the bottom half of the tank.



Consider a free-body-diagram of bottom half of tank (see figure) where:

p ~ pressure of fluid on horizontal plane

A ~ area of horizontal plane

$\mathcal{W}$ ~ weight of volume of fluid in lower half of tank

F_t ~ resultant force exerted by tank on fluid

For equilibrium (refer to Problem 2.80 for tank dimensions),

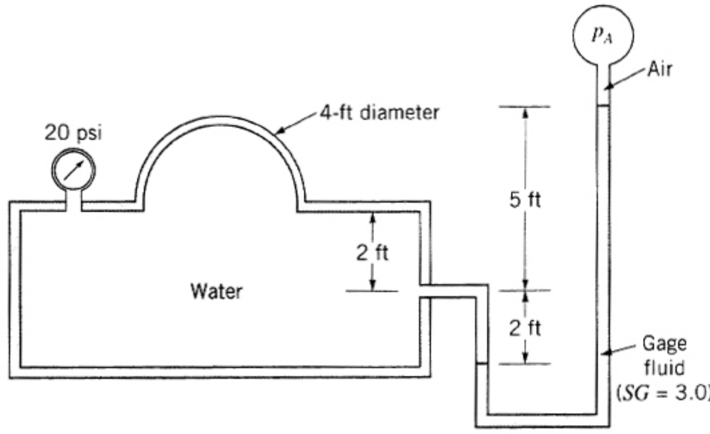
$$F_t = pA + W = \gamma\,(1.5\,m + 1\,m)(2\,m \times 4\,m) + \gamma\left[\tfrac{1}{2}\left(\tfrac{\pi}{4}\right)(2\,m)^2(4\,m)\right]$$

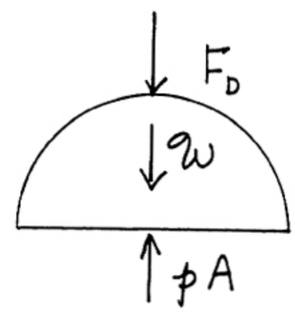
$$= \left(7.74\ \frac{kN}{m^3}\right)\left(20\,m^3 + 2\pi\,m^3\right) = 203\ kN$$

Thus, force of alcohol on tank $=$ __203 kN directed vertically downward.__

2.79

2.79 A closed tank is filled with water and has a 4-ft-diameter hemispherical dome as shown in Fig. P2.79. A U-tube manometer is connected to the tank. Determine the vertical force of the water on the dome if the differential manometer reading is 7 ft and the air pressure at the upper end of the manometer is 12.6 psi.





For equilibrium,

$$\Sigma F_{vertical} = 0$$

so that

$$F_D = pA - \mathcal{W} \qquad (1)$$

where F_D is the force the dome exerts on the fluid and p is the water pressure at the base of the dome.

From the manometer,

$$p_A + \gamma_{gf}(7\,ft) - \gamma_{H_2O}(4\,ft) = p$$

so that

$$p = \left(12.6\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + (3.0)\left(62.4\,\frac{lb}{ft^3}\right)(7\,ft) - \left(62.4\,\frac{lb}{ft^3}\right)(4\,ft)$$

$$= 2880\,\frac{lb}{ft^2}$$

Thus, from Eq. (1) with volume of sphere $= \frac{\pi}{6}(\text{diameter})^3$

$$F_D = \left(2880\,\frac{lb}{ft^2}\right)\left(\frac{\pi}{4}\right)(4\,ft)^2 - \frac{1}{2}\left[\frac{\pi}{6}(4\,ft)^3\right]\left(62.4\,\frac{lb}{ft^3}\right)$$

$$= 35{,}100\ lb$$

The force that the vertical force that the water exerts on the dome is 35,100 lb ↑.

2.80

2.80 If the bottom of a pop bottle similar to that shown in Fig. P2.71 and in Video V2.4 were changed so that it was hemispherical, as in Fig. P2.80, what would be the magnitude, line of action, and direction of the resultant force acting on the hemispherical bottom? The air pressure in the top of the bottle is 40 psi, and the pop has approximately the same specific gravity as that of water. Assume that the volume of pop remains at 2 liters.

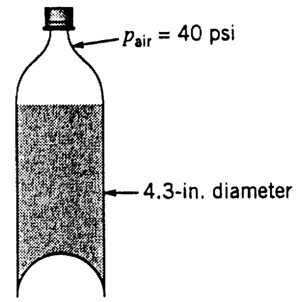


FIGURE P2.80

Force = weight of pop supported by bottom + force due to air pressure

$$\text{Weight of pop} = \gamma_{pop} \times \text{volume of pop} \qquad (1)$$

$$\text{Volume} = 2 \text{ liters} = (2 \times 10^{-3}\,m^3) \times \left(35.31 \times 10\,\frac{ft^3}{m^3}\right) = 0.0706\,ft^3$$

Thus, from Eq. (1)

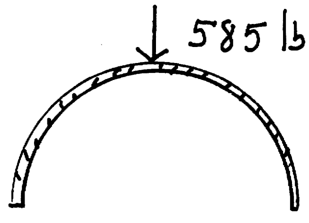
$$\text{Weight of pop} = \left(62.4\,\frac{lb}{ft^3}\right)(0.0706\,ft^3) = 4.41\,lb$$

$$\text{Force due to air pressure} = p_{air} \times \text{projected area of hemispherical bottom}$$

$$= \left(40\,\frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3\,in.)^2$$

$$= 581\,lb$$

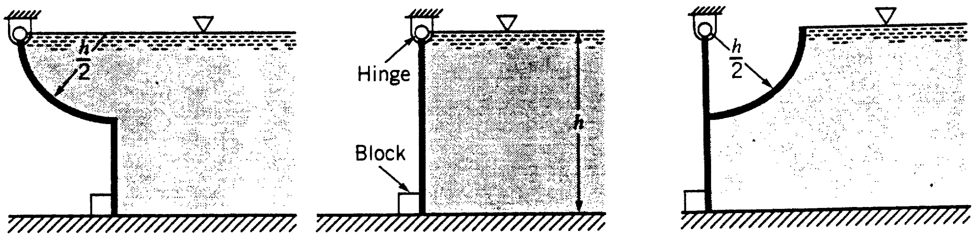
$$\text{Resultant force} = 4.41\,lb + 581\,lb = \underline{\underline{585\,lb}}$$



The resultant force is directed vertically downward, and due to symmetry, it acts on the hemispherical bottom along the vertical axis of the bottle.

2.81

2.81 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.81. The force of the gate against the block for gate **(b)** is R. Determine (in terms of R) the force against the blocks for the other two gates.



(a) (b) (c)

■ **FIGURE P2.81**

For case (b)

$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3}h$

Thus,

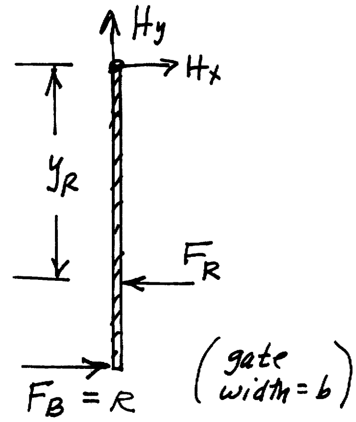
$$\Sigma M_H = 0$$

so that

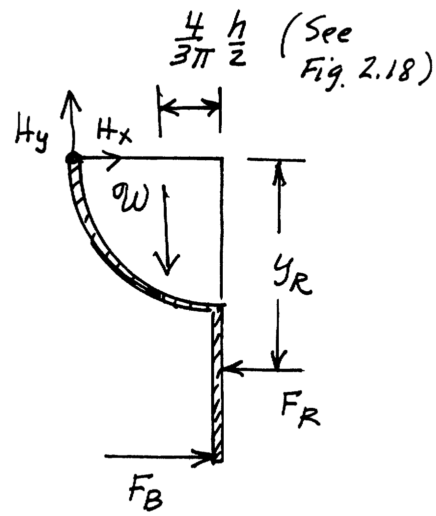
$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$



For case (a) on free-body-diagram shown



$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3}h$

and

$$\mathcal{W} = \gamma \times \text{Vol}$$

$$= \gamma\left[\frac{\pi\left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi \gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B\, h$$

and

$$\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B\, h$$

(con't)

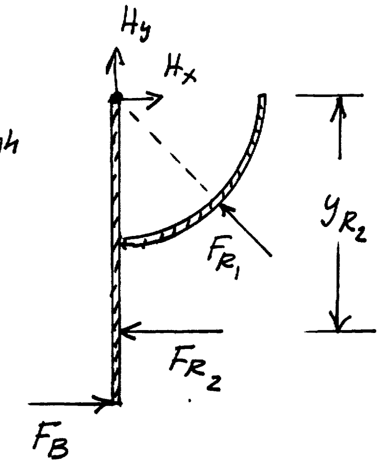
2.81 (Con't)

It follows that

$$F_B = \gamma h^2 b \,(0.390)$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$



For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus, $\Sigma M_H = 0$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\, h$$

or

$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.82

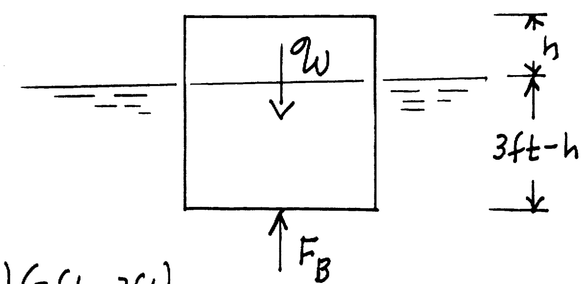
2.82 A 3 ft × 3 ft × 3 ft wooden cube (specific weight = 37 lb/ft³) floats in a tank of water. How much of the cube extends above the water surface? If the tank were pressurized so that the air pressure at the water surface was increased to 1.0 psi, how much of the cube would extend above the water surface? Explain how you arrived at your answer.

For equilibrium,

$$\Sigma F_{vertical} = 0$$

so that

$$\mathcal{W} = F_B$$



Thus,

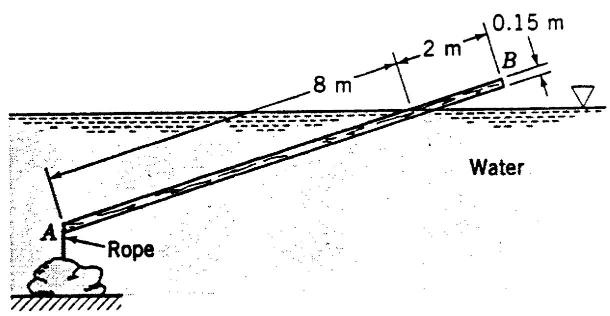
$$\left(37\,\frac{lb}{ft^3}\right)(3\,ft)^3 = \left(62.4\,\frac{lb}{ft^3}\right)(3ft-h)(3ft \times 3ft)$$

$$h = \underline{\underline{1.22\,ft}}$$

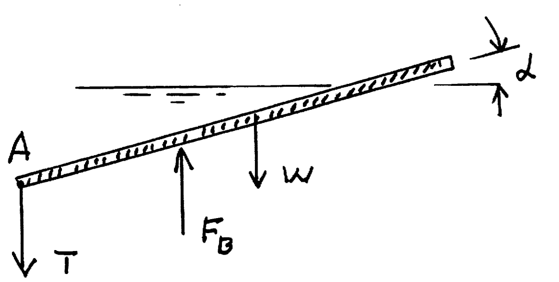
If the air pressure at the water surface increases by 1 psi there would be <u>no change</u> in the height of the cube above the water surface. The increased pressure force on the top of the cube is balanced by an equal force on the bottom of the cube since the surface pressure is transmitted throughout the fluid.

2.83

2.83 The homogeneous timber AB of Fig. P2.83 is 0.15 m by 0.35 m in cross section. Determine the specific weight of the timber and the tension in the rope.



■ FIGURE P2.83



$W = \gamma \forall$ where γ is the specific weight of the timber and $\forall$ is its volume. Thus,

$$W = \gamma\,(0.15\,m \times 0.35\,m \times 10\,m)$$
$$= 0.525\,\gamma$$

$$F_B = \gamma_{H_2O}\,\forall_{submerged} = \gamma_{H_2O}\,(0.15\,m \times 0.35\,m \times 8\,m) = 0.420\,\gamma_{H_2O}$$

For equilibrium,

$$\sum M_A = 0$$

so that

$$W\left(\frac{10\,m}{2}\right)\cos\alpha = F_B\left(\frac{8\,m}{2}\right)\cos\alpha$$

or

$$(0.525\,\gamma)(5\,m) = (0.420\,\gamma_{H_2O})(4\,m)$$

and

$$\gamma = \frac{(0.420)(9.80\,\frac{kN}{m^3})(4\,m)}{(0.525)(5\,m)} = \underline{\underline{6.27\,\frac{kN}{m^3}}}$$

Also,

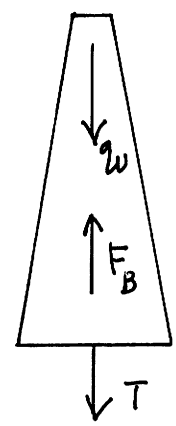
$$\sum F_{vertical} = 0$$

so that

$$T = F_B - W = (0.420\,m^3)(9.80\,\frac{kN}{m^3}) - (0.525\,m^3)(6.27\,\frac{kN}{m^3}) = \underline{\underline{824\,N}}$$

2.84

2.84 When the Tucurui dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.



$\mathcal{W} \sim$ weight
$F_B \sim$ buoyant force
$T \sim$ tension in ropes

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$T = F_B - \mathcal{W} \qquad (1)$$

For a truncated cone,

$$Volume = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

where: r_1 = base radius
r_2 = top radius
h = height

Thus,

$$\forall_{tree} = \frac{(\pi)(100\,ft)}{3}\left[(4\,ft)^2 + (4\,ft \times 1\,ft) + (1\,ft)^2\right]$$
$$= 2200\ ft^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \forall_{tree} = \left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 137{,}000\ lb$$

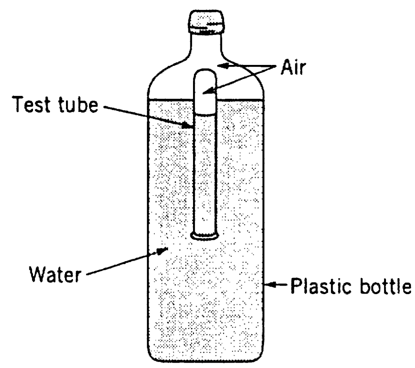
For weight,

$$\mathcal{W} = \gamma_{tree} \times \forall_{tree} = (0.6)\left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 82{,}400\,lb$$

From Eq. (1)

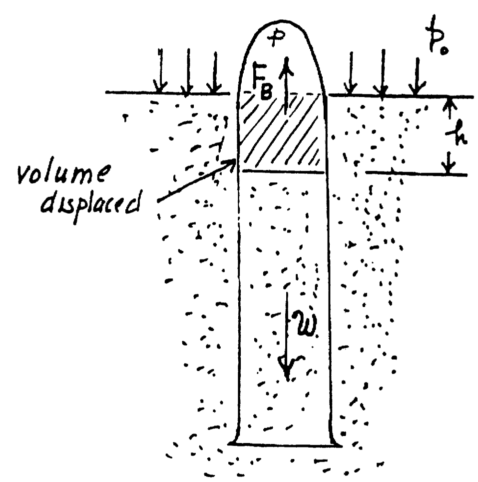
$$T = 137{,}000\ lb - 82{,}400\ lb = \underline{\underline{54{,}600\ lb}}$$

2.86 An inverted test tube partially filled with air floats in a plastic water-filled soft drink bottle as shown in Video V2.5 and Fig. P2.86. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.



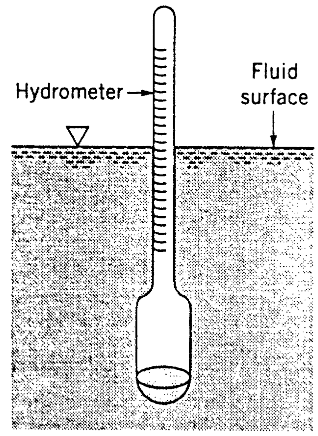
■ FIGURE P2.86

When the test tube is floating the weight of the tube, $\mathcal{W}$, is balanced by the buoyant force, F_B, as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p, trapped in the tube where $p = p_0 + \gamma_{H_2O} h$. When the bottle is squeezed, the air pressure in the bottle, p_0, is increased slightly and this in turn increases p, the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B. Since $\mathcal{W}$ is constant, a decrease in F_B will cause the test tube to sink.



2.87

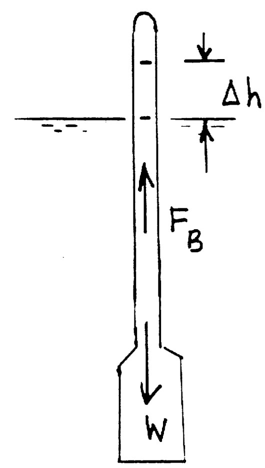
2.87 The hydrometer shown in Video V2.6 and Fig. P2.87 has a mass of 0.045 kg and the cross-sectional area of its stem is 290 mm^2. Determine the distance between graduations (on the stem) for specific gravities of 1.00 and 0.90.



■ FIGURE P2.87

When the hydrometer is floating its weight, W, and the buoyant force are equal since

$$\sum F_{vertical} = 0$$



For fluid with $SG_1 = 0.9$,

$$F_{B_1} = W$$

or

$$(SG_1)(\gamma_{H_2O})\forall_1 = W \qquad (\text{where } \gamma_{H_2O} = \gamma_{H_2O} @ 4°C)$$

Similarly, for fluid with $SG_2 = 1.0$,

$$(SG_2)(\gamma_{H_2O})\forall_2 = W$$

Let $A_s \sim$ stem area and $\forall \sim$ submerged volume

and subtracting the equations yields

$$\forall_1 - \forall_2 = \frac{W}{(SG_1)(\gamma_{H_2O})} - \frac{W}{(SG_2)(\gamma_{H_2O})}$$

Since $\forall_1 - \forall_2 = \Delta h\, A_s$

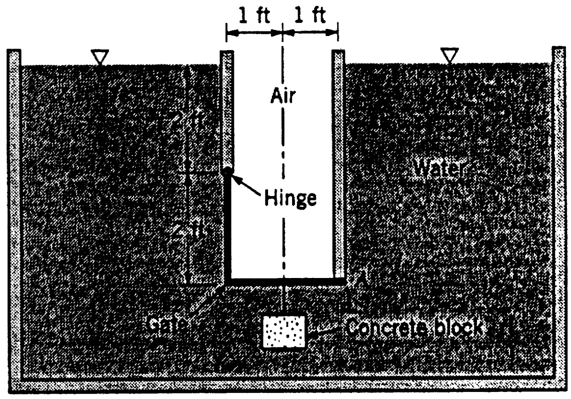
$$\Delta h = \frac{W}{A_s(\gamma_{H_2O})}\left[\frac{1}{SG_1} - \frac{1}{SG_2}\right]$$

$$= \frac{(0.045\,kg)(9.81\,\frac{m}{s^2})}{(290 \times 10^{-6}\,m^2)(9.81 \times 10^3\,\frac{N}{m^3})}\left[\frac{1}{0.9} - \frac{1}{1.0}\right]$$

$$= 1.72 \times 10^{-2}\,m = \underline{\underline{17.2\,mm}}$$

2.88

2.88 An L-shaped rigid gate is hinged at one end and is located between partitions in an open tank containing water as shown in Fig. P2.88. A block of concrete ($\gamma = 150\ \text{lb/ft}^3$) is to be hung from the horizontal portion of the gate. Determine the required volume of the block so that the reaction of the gate on the partition at A is zero when the water depth is 2 ft above the hinge. The gate is 2 ft wide with a negligible weight, and the hinge is smooth.



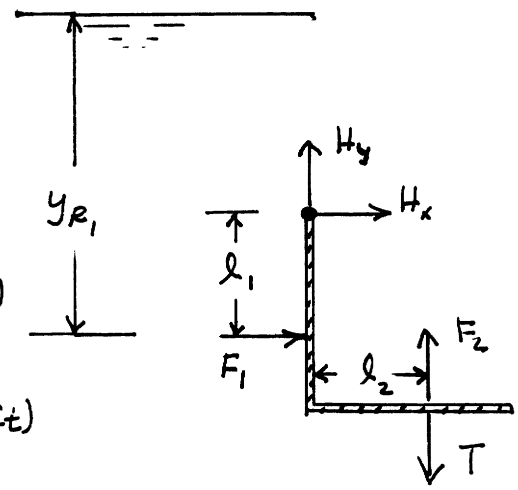
■ FIGURE P2.88

For equilibrium,

$$\sum M_H = 0$$

so that

$$F_1 \ell_1 + F_2 \ell_2 = T \ell_2 \qquad (1)$$



where:

$$F_1 = \gamma h_{c_1} A_1 = \left(62.4\ \tfrac{\text{lb}}{\text{ft}^3}\right)(3\ \text{ft})(2\ \text{ft} \times 2\ \text{ft})$$
$$= 749\ \text{lb}$$

$$F_2 = \gamma h_{c_2} A_2 = \left(62.4\ \tfrac{\text{lb}}{\text{ft}^3}\right)(4\ \text{ft})(2\ \text{ft} \times 2\ \text{ft})$$
$$= 998\ \text{lb}$$

$$y_{R_1} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(2\ \text{ft})(2\ \text{ft})^3}{(3\ \text{ft})(2\ \text{ft} \times 2\ \text{ft})} + 3\ \text{ft} = 3.11\ \text{ft}$$

$$\ell_1 = y_{R_1} - 2\ \text{ft} = 3.11\ \text{ft} - 2\ \text{ft} = 1.11\ \text{ft}$$

$$\ell_2 = 1\ \text{ft}$$

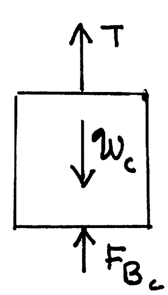
Thus, from Eq. (1)

$$(749\ \text{lb})(1.11\ \text{ft}) + (998\ \text{lb})(1\ \text{ft}) = T\ (1\ \text{ft})$$

$$T = 1830\ \text{lb}$$

For concrete block,

$$\sum F_{\text{vertical}} = 0 \quad \text{or} \quad \mathcal{W}_c = T + F_{B_c}$$



so that

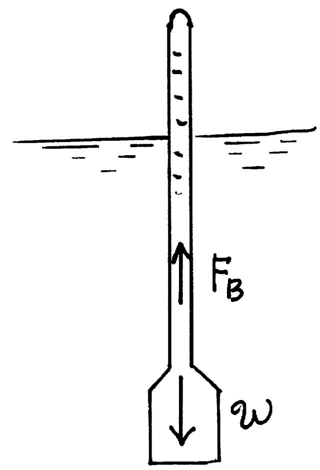
$$\gamma_c \forall_c = 1830\ \text{lb} + \gamma_{H_2O} \forall_c$$

$$\forall_c = \frac{1830\ \text{lb}}{150\ \frac{\text{lb}}{\text{ft}^3} - 62.4\ \frac{\text{lb}}{\text{ft}^3}} = \underline{\underline{20.9\ \text{ft}^3}}$$

2.89

2.89 When a hydrometer (see Fig. P2.87 and Video V2.6) having a stem diameter of 0.30 in. is placed in water, the stem protrudes 3.15 in. above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 0.042 lb.

When the hydrometer is floating its weight, $\mathcal{W}$, is balanced by the buoyant force, F_B. For equilibrium,



$$\sum F_{vertical} = 0$$

Thus, for water

$$F_B = \mathcal{W}$$

$$(\gamma_{H_2O}) \forall_1 = \mathcal{W} \qquad (1)$$

where $\forall_1$ is the submerged volume. With the new liquid

$$(SG)(\gamma_{H_2O}) \forall_2 = \mathcal{W} \qquad (2)$$

Combining Eqs. (1) and (2) with $\mathcal{W}$ constant

$$(\gamma_{H_2O}) \forall_1 = (SG)(\gamma_{H_2O}) \forall_2$$

and

$$\forall_2 = \frac{\forall_1}{SG} \qquad (3)$$

From Eq. (1)

$$\forall_1 = \frac{\mathcal{W}}{\gamma_{H_2O}} = \frac{0.042\ lb}{62.4\ \frac{lb}{ft^3}} = 6.73 \times 10^{-4}\ ft^3$$

so that from Eq. (3)

$$\forall_2 = \frac{6.73 \times 10^{-4}\ ft^3}{1.10} = 6.12 \times 10^{-4}\ ft^3$$

Thus, $\forall_1 - \forall_2 = (6.73 - 6.12) \times 10^{-4}\ ft^3 = 0.61 \times 10^{-4}\ ft^3$

To obtain this difference the change in length, $\Delta \ell$, is

$$\left(\frac{\pi}{4}\right)(0.30\ in.)^2 \Delta \ell = (0.61 \times 10^{-4}\ ft^3)\left(1728\ \frac{in.^3}{ft^3}\right)$$

$$\Delta \ell = 1.49\ in.$$

With the new liquid the stem would protrude

3.15 in. + 1.49 in. = $\underline{\underline{4.64\ in.}}$ above the surface.

2.90

2.90 The thin-walled, 1-m-diameter tank of Fig. P2.90 is closed at one end and has a mass of 90 kg. The open end of the tank is lowered into the water and held in the position shown by a steel block having a density of 7840 kg/m³. Assume that the air that is trapped in the tank is compressed at a constant temperature. Determine: **(a)** the reading on the pressure gage at the top of the tank, and **(b)** the volume of the steel block.

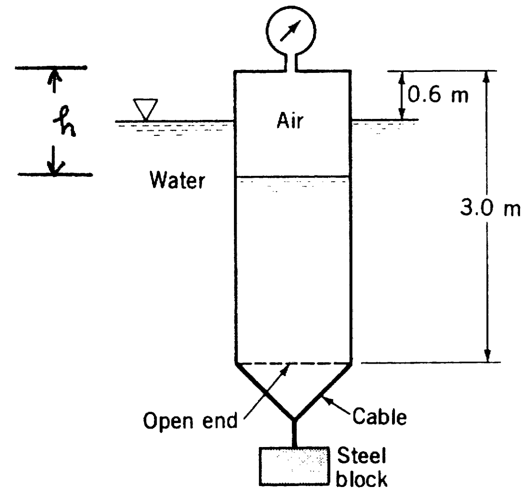


FIGURE P2.90

(a) For constant temperature compression,

$$p_i \mathcal{V}_i = p_f \mathcal{V}_f \quad \text{where } i \sim \text{initial state and } f \sim \text{final state.}$$

Let $\mathcal{V}_f = A_t h$ (see figure) where A_t is the cross sectional area of tank, and

$$p_f = \gamma(h - 0.6) + p_{atm} \quad \text{(where all lengths are in m).} \qquad (1)$$

Thus,

$$\mathcal{V}_f = A_t h = \frac{p_i \mathcal{V}_i}{p_f}$$

Since $p_i = p_{atm}$ and $\mathcal{V}_i = A_t(3)$

$$h = \frac{p_{atm}}{p_f}\frac{A_t(3)}{A_t} = \frac{3\,p_{atm}}{\gamma(h-0.6) + p_{atm}}$$

so that

$$h^2 + \left(\frac{p_{atm}}{\gamma} - 0.6\right)h - \frac{3\,p_{atm}}{\gamma} = 0$$

For $\gamma = 9.80 \frac{kN}{m^3}$ and $p_{atm} = 101\,kPa$,

$$h^2 + \left(\frac{101\,kPa}{9.80\,\frac{kN}{m^3}} - 0.6\,m\right)h - \frac{3\,(101\,kPa)}{9.80\,\frac{kN}{m^3}} = 0$$

or

$$h^2 + 9.71\,h - 30.9 = 0$$

So that

$$h = \frac{-9.71 \pm \sqrt{(9.71)^2 + 4(30.9)}}{2} = 2.53\,m$$

Thus, from Eq. (1)

$$p_f\,(\text{gage}) = \left(9.80\,\frac{kN}{m^3}\right)(2.53\,m - 0.6\,m) = \underline{\underline{18.9\,kPa}}$$

(con't)

2.90 (con't)

(b) For equilibrium of tank (see free-body-diagram),

$$T = p_f A_t - \mathcal{W}_t$$

where $W_t \sim$ tank weight, and for steel block

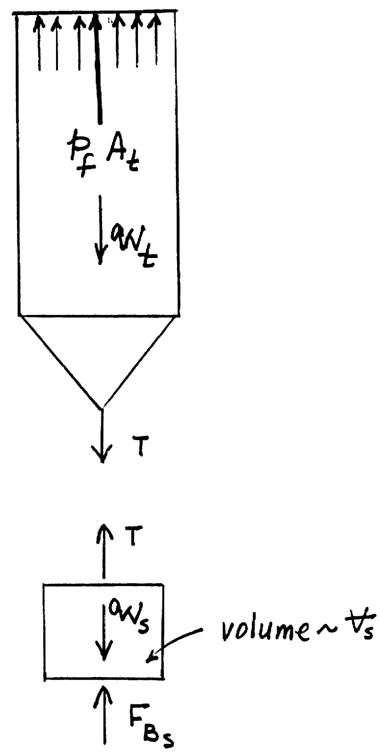
$$T = \mathcal{W}_s - F_{B_s} = \forall_s (\gamma_s - \gamma)$$

Thus,

$$\forall_s = \frac{T}{\gamma_s - \gamma} = \frac{p_f A_t - W_t}{\gamma_s - \gamma}$$

$$= \frac{\left(18.9 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(1\,m)^2 - (90\,kg)\left(9.81\,\frac{m}{s^2}\right)}{\left(7.840 \times 10^3 \frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right) - 9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.208\ m^3}}$$



2.91 *

2.91 * An inverted hollow cone is pushed into the water as is shown in Fig. P2.91. Determine the distance, ℓ, that the water rises in the cone as a function of the depth, d, of the lower edge of the cone. Plot the results for $0 \le d \le H$, when H is equal to 1 m. Assume the temperature of the air within the cone remains constant.

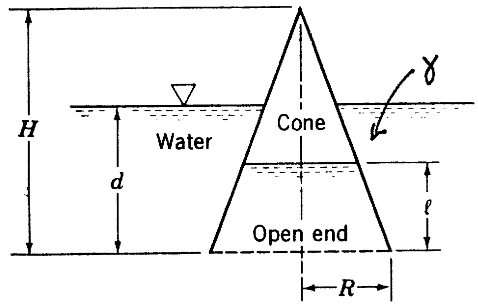


FIGURE P2.91

For constant temperature compression of the air within the cone,

$$p_i \forall_i = p_f \forall_f \qquad (1)$$

where $\forall$ is the air volume, and i and f refer to initial and final states, respectively. It follows that (see figure):

$$p_i = p_{atm} \qquad p_f = \gamma(d-\ell) + p_{atm}$$

$$\forall_i = \frac{\pi}{3}R^2 H \qquad \forall_f = \frac{\pi}{3}\left(\frac{H-\ell}{H}\right)^2 R^2 (H-\ell) = \frac{\pi}{3}\left(\frac{R}{H}\right)^2 (H-\ell)^3$$

Thus, from Eq. (1)

$$p_{atm}\left(\frac{\pi}{3}R^2 H\right) = \left[\gamma(d-\ell) + p_{atm}\right]\frac{\pi}{3}\left(\frac{R}{H}\right)^2 (H-\ell)^3$$

which simplifies to

$$\ell = d - \frac{p_{atm}}{\gamma}\left[\left(\frac{H}{H-\ell}\right)^3 - 1\right] \qquad (2)$$

For $p_{atm} = 101\ kPa$, $\gamma = 9.80\ kN/m^3$, and $H = 1\ m$,

$$\ell = d - \frac{101\ \frac{kN}{m^2}}{9.80\ \frac{kN}{m^3}}\left[\left(\frac{1}{1-\ell}\right)^3 - 1\right]$$

or

$$\ell = d - 10.3\left[(1-\ell)^{-3} - 1\right] \qquad (3)$$

where ℓ and d are in meters.
A computer program follows for calculating ℓ as a function of d.

(con't)

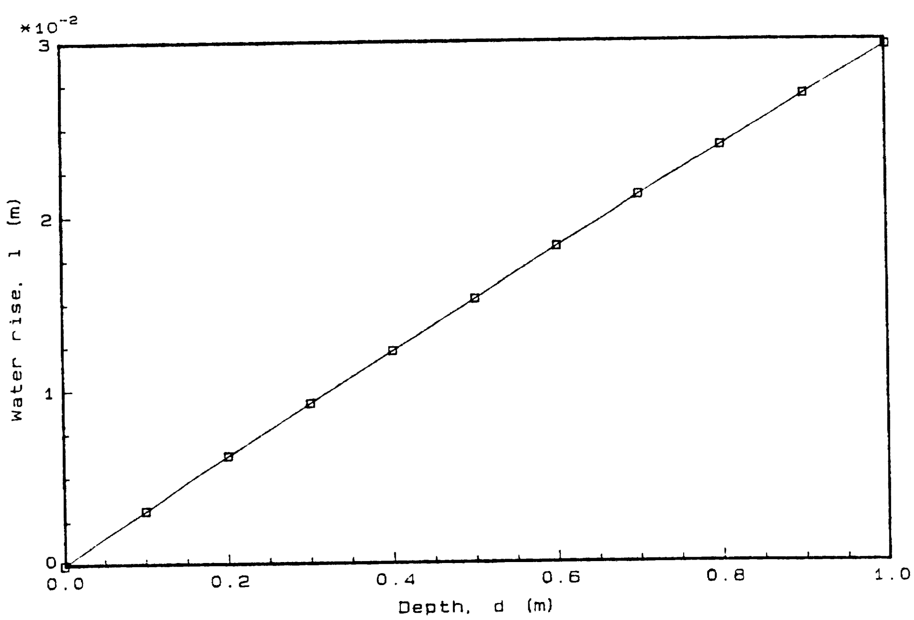
2.91 * (con't)

```
100 cls
110 print "**********************************************"
120 print "** This program solves iteratively a fourth **"
130 print "** order equation to give the water rise,   **"
135 print "** l, as a function of the depth, d         **"
140 print "**********************************************"
150 print
160 print "  Depth, d (m)    Water rise, l (m)"
170 for d=0.0 to 1.01 step 0.1
180 l=0.0
185 if d=0 then goto 220
190 lp=l
200 l=1-((d-lp)/10.3+1)^(-1/3)
210 if abs(1-lp/l)>0.001 goto 190
220 print using "     #.###                 #.####";d,l
230 next d
```

Tabulated data and a plot of the data are shown below.

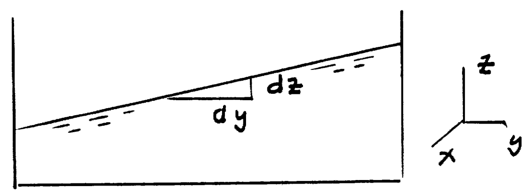
```
**********************************************
** This program solves iteratively a fourth **
** order equation to give the water rise,   **
** l, as a function of the depth, d         **
**********************************************

  Depth, d (m)    Water rise, l (m)
     0.000             0.0000
     0.100             0.0031
     0.200             0.0062
     0.300             0.0092
     0.400             0.0122
     0.500             0.0152
     0.600             0.0182
     0.700             0.0211
     0.800             0.0239
     0.900             0.0268
     1.000             0.0296
```



2.92

2.92 An open container of oil rests on the flatbed of a truck that is traveling along a horizontal road at 55 mi/hr. As the truck slows uniformly to a complete stop in 5 s, what will be the slope of the oil surface during the period of constant deceleration?



$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad (\text{Eq. } 2.28)$$

$$a_y = \frac{\text{final velocity} - \text{initial velocity}}{\text{time interval}}$$

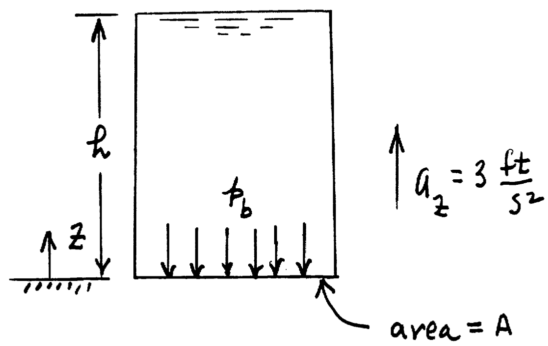
$$= \frac{0 - (55\,mph)\left(0.4470\,\frac{\frac{m}{s}}{mph}\right)}{5\,s} = -4.92\,\frac{m}{s^2}$$

Thus,

$$\frac{dz}{dy} = -\frac{\left(-4.92\,\frac{m}{s^2}\right)}{9.81\,\frac{m}{s^2} + 0} = \underline{\underline{0.502}}$$

2.93

2.93 A 5-gal, cylindrical open container with a bottom area of 120 in.^{2} is filled with glycerin and rests on the floor of an elevator. **(a)** Determine the fluid pressure at the bottom of the container when the elevator has an upward acceleration of 3 ft/s^2. **(b)** What resultant force does the container exert on the floor of the elevator during this acceleration? The weight of the container is negligible. (Note: 1 gal = 231 in.^{3})



(a) $\frac{dp}{dz} = -\rho(g + a_z)$ (Eq. 2.26)

Thus,

$$\int_0^{p_b} dp = -\rho(g+a_z)\int_h^0 dz$$

and

$$p_b = \rho(g+a_z)h$$

$$= \left(2.44\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2} + 3\ \frac{ft}{s^2}\right)\left(\frac{9.63}{12}\ ft\right)$$

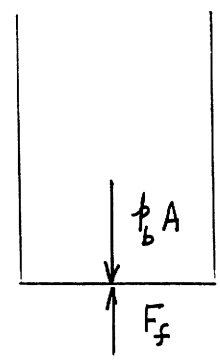
$$= \underline{\underline{68.9\ \frac{lb}{ft^2}}}$$

hA = volume

$$h\,(120\ in.^2) = (5\ gal)\left(\frac{231\ in.^3}{gal}\right)$$

$$h = 9.63\ in.$$

(b) From free-body-diagram of container,



$$F_f = p_b A$$

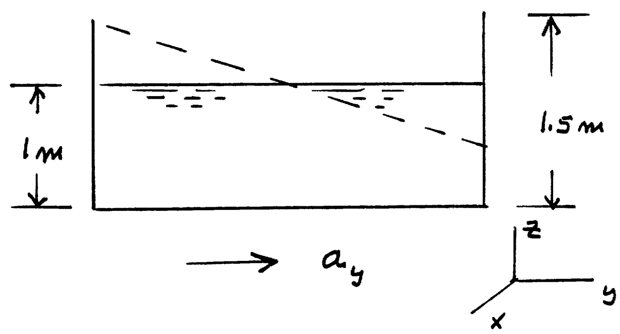
$$= \left(68.9\ \frac{lb}{ft^2}\right)(120\ in.^2)\left(\frac{1\ ft^2}{144\ in.^2}\right)$$

$$= 57.4\ lb$$

Thus, force of container on floor is $\underline{\underline{57.4\ lb\ downward.}}$

2.94

2.94 An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1 m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?



To prevent spilling,

$$\frac{dz}{dy} \geq -\frac{1.5\,m - 1.0\,m}{1\,m} = -0.50$$

(see figure).

Since,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad (Eq.\ 2.28)$$

or, with $a_z = 0$,

$$a_y = -\left(\frac{dz}{dy}\right) g$$

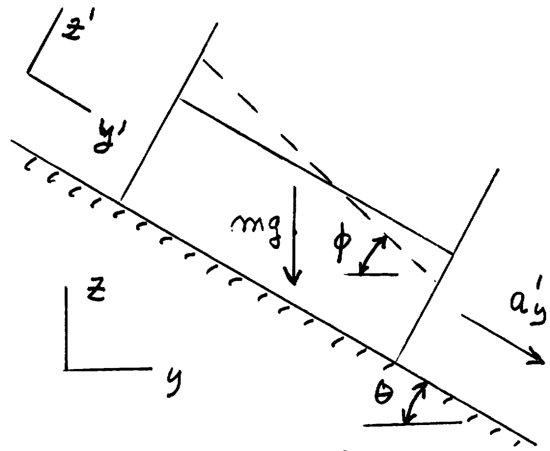
so that

$$(a_y)_{max} = -(-0.50)\left(9.81\,\frac{m}{s^2}\right) = \underline{\underline{4.91\,\frac{m}{s^2}}}$$

(Note: Acceleration could be either to the right or the left.)

2.95

2.95 If the tank of Problem 2.94 slides down a frictionless plane that is inclined at 30° with the horizontal, determine the angle the free surface makes with the horizontal.



$m \sim$ mass of tank and gasoline

From Newton's 2nd law,

$$\sum F_{y'} = m\, a_y'$$

Since the only force in the y'-direction is the component of weight $(mg)\sin\theta$,

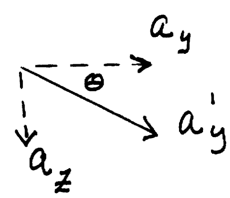
$$(mg)\sin\theta = m\, a_y'$$

so that

$$a_y' = g \sin\theta$$

and therefore

$$a_y = a_y' \cos\theta \qquad a_z = -\, a_y' \sin\theta$$



Also,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

$$= -\frac{a_y' \cos\theta}{g - a_y' \sin\theta} = -\frac{g \sin\theta \cos\theta}{g - g \sin\theta \cos\theta}$$

$$= -\frac{\frac{1}{2}\sin 2\theta}{1 - \frac{1}{2}\sin 2\theta}$$

and for $\theta = 30°$

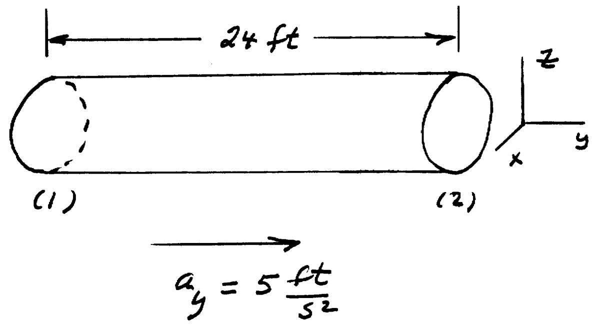
$$\frac{dz}{dy} = -\frac{\frac{1}{2}\sin 60°}{1 - \frac{1}{2}\sin 60°} = -0.764$$

Thus, $\tan\phi = 0.764$ (see figure)

and $\underline{\underline{\phi = 37.4°}}$

2.96

2.96 A closed cylindrical tank that is 8 ft in diameter and 24 ft long is completely filled with gasoline. The tank, with its long axis horizontal, is pulled by a truck along a horizontal surface. Determine the pressure difference between the ends (along the long axis of the tank) when the truck undergoes an acceleration of 5 ft/s^2.



$$\frac{\partial p}{\partial y} = -\rho a_y \qquad (Eq. 2.25)$$

Thus,

$$\int_{p_1}^{p_2} dp = -\rho a_y \int_0^{24} dy$$

where $p = p_1$ at $y = 0$ and $p = p_2$ at $y = 24\ ft$,

and

$$p_2 - p_1 = -\rho a_y (24\ ft)$$

$$= -\left(1.32 \frac{slugs}{ft^3}\right)\left(5 \frac{ft}{s^2}\right)(24\ ft)$$

$$= -158 \frac{lb}{ft^2}$$

or

$$p_1 - p_2 = \underline{\underline{158 \frac{lb}{ft^2}}}$$

2.97

2.97 The open U-tube of Fig. P2.97 is partially filled with a liquid. When this device is accelerated with a horizontal acceleration, a, a differential reading, h, develops between the manometer legs which are spaced a distance ℓ apart. Determine the relationship between a, ℓ, and h.

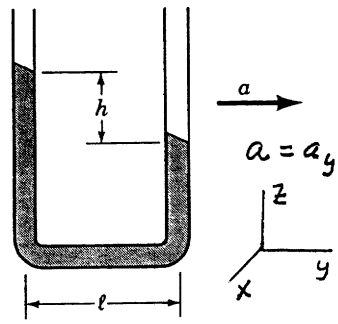


FIGURE P2.97

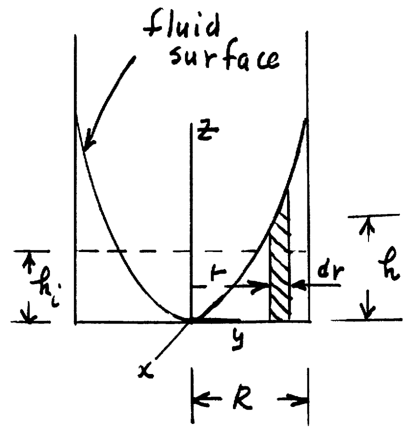
$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

Since, $\frac{dz}{dy} = -\frac{h}{\ell}$ and $a_z = 0$

then $-\frac{h}{\ell} = -\frac{a}{g + 0}$

or $$\underline{\underline{h = \frac{a\ell}{g}}}$$

2.98 An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.



$h_i \sim$ initial depth

Equation for surfaces of constant pressure (Eq. 2.32):

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h=0$ at $r=0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = \frac{2\pi\,\omega^2}{2g}\int_0^R r^3 dr = \frac{\pi \omega^2 R^4}{4g}$$

Since the initial volume, $\forall_i = \pi R^2 h_i$, must equal the final volume,

$$\forall_f = \forall_i$$

so that

$$\frac{\pi \omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4 g h_i}{R^2}} = \sqrt{\frac{4\left(9.81 \frac{m}{s^2}\right)(0.7 m)}{(0.5 m)^2}} = \underline{\underline{10.5 \frac{\text{rad}}{s}}}$$

2.99

2.99 The U-tube of Fig. P2.99 is partially filled with water and rotates around the axis a–a. Determine the angular velocity that will cause the water to start to vaporize at the bottom of the tube (point A).

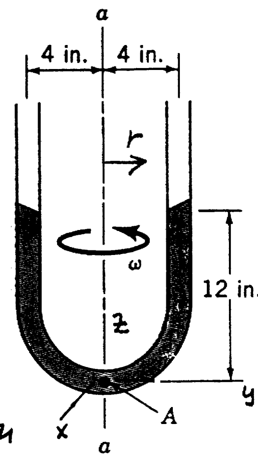


FIGURE P2.99

Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho \omega^2 r^2}{2} - \gamma z + \text{Constant} \qquad \text{(Eq. 2.33)}$$

With the coordinate system shown,

$p = 0$ at $r = 4$ in. and $z = 12$ in., so that

$$\text{constant} = -\frac{\rho \omega^2 \left(\frac{4}{12}\,ft\right)^2}{2} + \gamma\left(\tfrac{12}{12}\,ft\right) = -\frac{\rho\omega^2}{18} + \gamma$$

Thus,

$$p = \frac{\rho\omega^2}{2}\left(r^2 - \frac{1}{9}\right) - \gamma(z-1)$$

At point A, $r = 0$ and $z = 0$, and

$$p_A = -\frac{\rho\omega^2}{18} + \gamma \qquad (1)$$

If p_A = vapor pressure = 0.256 psia, or

$$p_A = (0.256\ psi - 14.7\ psi)\left(144\,\frac{in.^2}{ft^2}\right) = -2080\,\frac{lb}{ft^2}\ (gage)$$

then from Eq. (1)

$$\omega = \sqrt{\frac{18(\gamma - p_A)}{\rho}}$$

$$= \sqrt{\frac{18\left[62.4\,\frac{lb}{ft^3} - \left(-2080\,\frac{lb}{ft^2}\right)\right]}{1.94\,\frac{slugs}{ft^3}}} = \underline{\underline{141\ \frac{rad}{s}}}$$

2.100

2.100 The U-tube of Fig. P2.100 contains mercury and rotates about the off-center axis *a–a*. At rest, the depth of mercury in each leg is 150 mm as illustrated. Determine the angular velocity for which the difference in heights between the two legs is 75 mm.

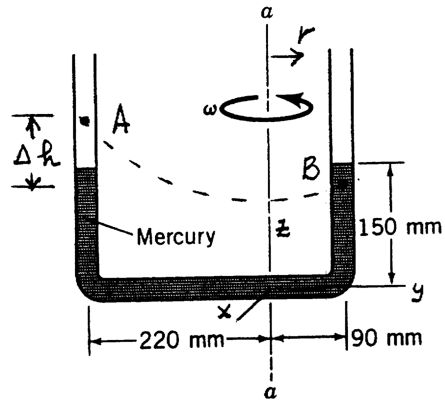


FIGURE P2.100

The equation of the free surface passing through A and B is

$$z = \frac{\omega^2 r^2}{2g} + \text{constant} \qquad (\text{Eq. } 2.32)$$

Thus,

$$z_A - z_B = \Delta h = \frac{\omega^2}{2g}\left(r_A^2 - r_B^2\right)$$

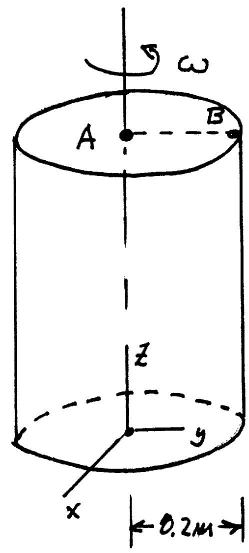
so that

$$\omega = \sqrt{\frac{2g(\Delta h)}{r_A^2 - r_B^2}}$$

$$= \sqrt{\frac{2\left(9.81\,\frac{m}{s^2}\right)(0.075\,m)}{(0.220\,m)^2 - (0.090\,m)^2}} = \underline{\underline{6.04\,\frac{rad}{s}}}$$

2.101

2.101 A closed, 0.4-m-diameter cylindrical tank is completely filled with oil ($SG = 0.9$) and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.



Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{constant} \qquad (\text{Eq. } 2.33)$$

Since $z_A = z_B$,

$$p_B - p_A = \frac{\rho\omega^2}{2}\left(r_B^2 - r_A^2\right)$$

$$= \frac{(0.9)\left(10^3\,\frac{kg}{m^3}\right)\left(40\,\frac{rad}{s}\right)^2}{2}\left[(0.2\,m)^2 - 0\right]$$

$$= \underline{\underline{28.8\ kPa}}$$

2.102

2.102 Force Needed to Open a Submerged Gate

Objective: A gate, hinged at the top, covers a hole in the side of a water filled tank as shown in Fig. P2.102 and is held against the tank by the water pressure. The purpose of this experiment is to compare the theoretical force needed to open the gate to the experimentally measured force.

Equipment: Rectangular tank with a rectangular hole in its side; gate that covers the hole and is hinged at the top; force transducer to measure the force needed to open the gate; ruler to measure the water depth.

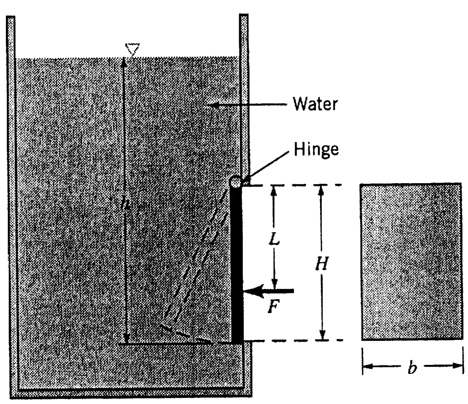
Experimental Procedure: Measure the height, H, and width, b, of the hole in the tank and the distance, L, from the hinge to the point of application of the force, F, that opens the gate. Fill the tank with water to a depth h above the bottom of the gate. Use the force transducer to determine the force, F, needed to slowly open the gate. Repeat the force measurements for various water depths.

Calculations: For arbitrary water depths, h, determine the theoretical force, F, needed to open the gate by equating the moment about the hinge from the water force on the gate to the moment produced by the applied force, F.

Graph: Plot the experimentally determined force, F, needed to open the gate as ordinates and the water depth, h, as abscissas.

Results: On the same graph, plot the theoretical force as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P2.102

(con't)

2.102 (Con't)

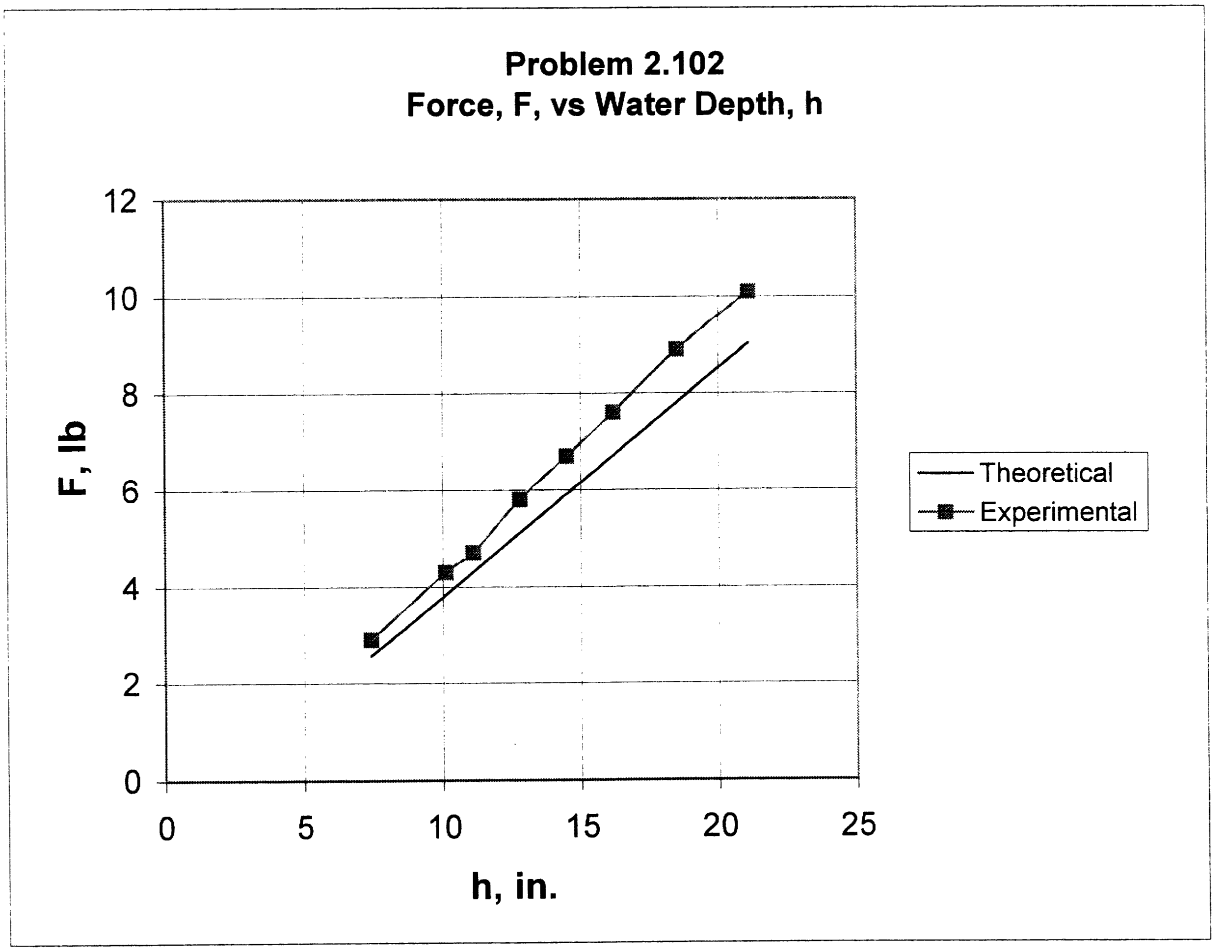
Solution for Problem 2.102: Force Needed to Open a Submerged Gate

L, in.	H, in.	b, in.	γ, lb/ft^3	I_{xc}, ft^4
5.5	6.0	4.0	62.4	0.003472

h, in.	F, lb	F_1, lb	$y_r - y_c$, ft	d, ft	F, lb
21.1	10.1	15.69	0.0138	0.264	9.03
18.5	8.9	13.43	0.0161	0.266	7.80
16.2	7.6	11.44	0.0189	0.269	6.71
14.5	6.7	9.97	0.0217	0.272	5.91
12.8	5.8	8.49	0.0255	0.276	5.11
11.1	4.7	7.02	0.0309	0.281	4.30
10.1	4.3	6.15	0.0352	0.285	3.83
7.4	2.9	3.81	0.0568	0.307	2.55

Since h > H, A = H*b = constant and I_{xc} = b*H^3/12 = constant.

F = F_1*d/L, where F_1 = γ*(h - H/2)*A, d = H/2 + (y_r - y_c), and y_r - y_c = I_{xc}/(h - H/2)*A



2.103

2.103 Hydrostatic Force on a Submerged Rectangle

Objective: A quarter-circle block with a vertical rectangular end is attached to a balance beam as shown in Fig. P2.103. Water in the tank puts a hydrostatic pressure force on the block which causes a clockwise moment about the pivot point. This moment is balanced by the counterclockwise moment produced by the weight placed at the end of the balance beam. The purpose of this experiment is to determine the weight, W, needed to balance the beam as a function of the water depth, h.

Equipment: Balance beam with an attached quarter-circle, rectangular cross-section block; pivot point directly above the vertical end of the beam to support the beam; tank; weights; ruler.

Experimental Procedure: Measure the inner radius, R_1, outer radius, R_2, and width, b, of the block. Measure the length, L, of the moment arm between the pivot point and the weight. Adjust the counter weight on the beam so that the beam is level when there is no weight on the beam and no water in the tank. Hang a known mass, m, on the beam and adjust the water level, h, in the tank so that the beam again becomes level. Repeat with different masses and water depths.

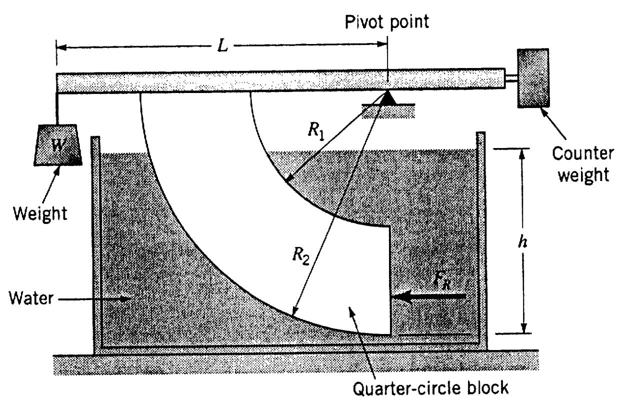
Calculations: For a given water depth, h, determine the hydrostatic pressure force, $F_R = \gamma h_c A$, on the vertical end of the block. Also determine the point of action of this force, a distance $y_R - y_c$ below the centroid of the area. Note that the equations for F_R and $y_R - y_c$ are different when the water level is below the end of the block ($h < R_2 - R_1$) than when it is above the end of the block ($h > R_2 - R_1$).

For a given water depth, determine the theoretical weight needed to balance the beam by summing moments about the pivot point. Note that both F_R and W produce a moment. However, because the curved sides of the block are circular arcs centered about the pivot point, the pressure forces on the curved sides of the block (which act normal to the sides) do not produce any moment about the pivot point. Thus the forces on the curved sides do not enter into the moment equation.

Graph: Plot the experimentally determined weight, W, as ordinates and the water depth, h, as abscissas.

Result: On the same graph plot the theoretical weight as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P2.103

(con't)

2.103 (Con't)

Solution for Problem 2.103: Hydrostatic Force on a Submerged Rectangle

R_1, in.	R_2, in.	L, in.	b, in.	g, ft/s^2	γ, lb/ft^3
5.0	9.0	12.0	3.0	32.2	62.4

		Experimental				Theoretical
m, kg	h, in.	W, lb	F_R, lb	y_r -y_c, ft	d, ft	W, lb
0.00	0.00	0.00	0.00		0.750	0.000
0.02	1.11	0.04	0.07		0.719	0.048
0.04	1.58	0.09	0.14		0.706	0.095
0.06	1.92	0.13	0.20		0.697	0.139
0.10	2.51	0.22	0.34		0.680	0.232
0.12	2.76	0.26	0.41		0.673	0.278
0.14	2.99	0.31	0.48		0.667	0.323
0.16	3.20	0.35	0.55		0.661	0.367
0.18	3.41	0.40	0.63		0.655	0.413
0.20	3.60	0.44	0.70		0.650	0.456
0.22	3.80	0.48	0.78		0.644	0.504
0.24	3.99	0.53	0.86		0.639	0.551
0.26	4.17	0.57	0.94	0.0512	0.634	0.597
0.28	4.33	0.62	1.01	0.0476	0.631	0.637
0.30	4.50	0.66	1.08	0.0444	0.628	0.680
0.35	4.95	0.77	1.28	0.0376	0.621	0.794
0.40	5.39	0.88	1.47	0.0328	0.616	0.905
0.45	5.83	0.99	1.66	0.0290	0.612	1.016
0.50	6.27	1.10	1.85	0.0260	0.609	1.127
0.55	6.70	1.21	2.04	0.0236	0.607	1.236

W = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Sum moments about pivot to give $W*L = F_R*d$

For $h < R_2 - R_1$:

$F_R = \gamma*(h/2)*h*b$

$d = R_2 - (h/3)$

For $h > R_2 - R_1$:

$F_R = \gamma*(h - (R_2 - R_1)/2)*(R_2 - R_1)*b$

$d = R_2 - (R_2 - R_1)/2 + (y_r - y_c)$

$y_r - y_c = I_{xc}/h_c*A$

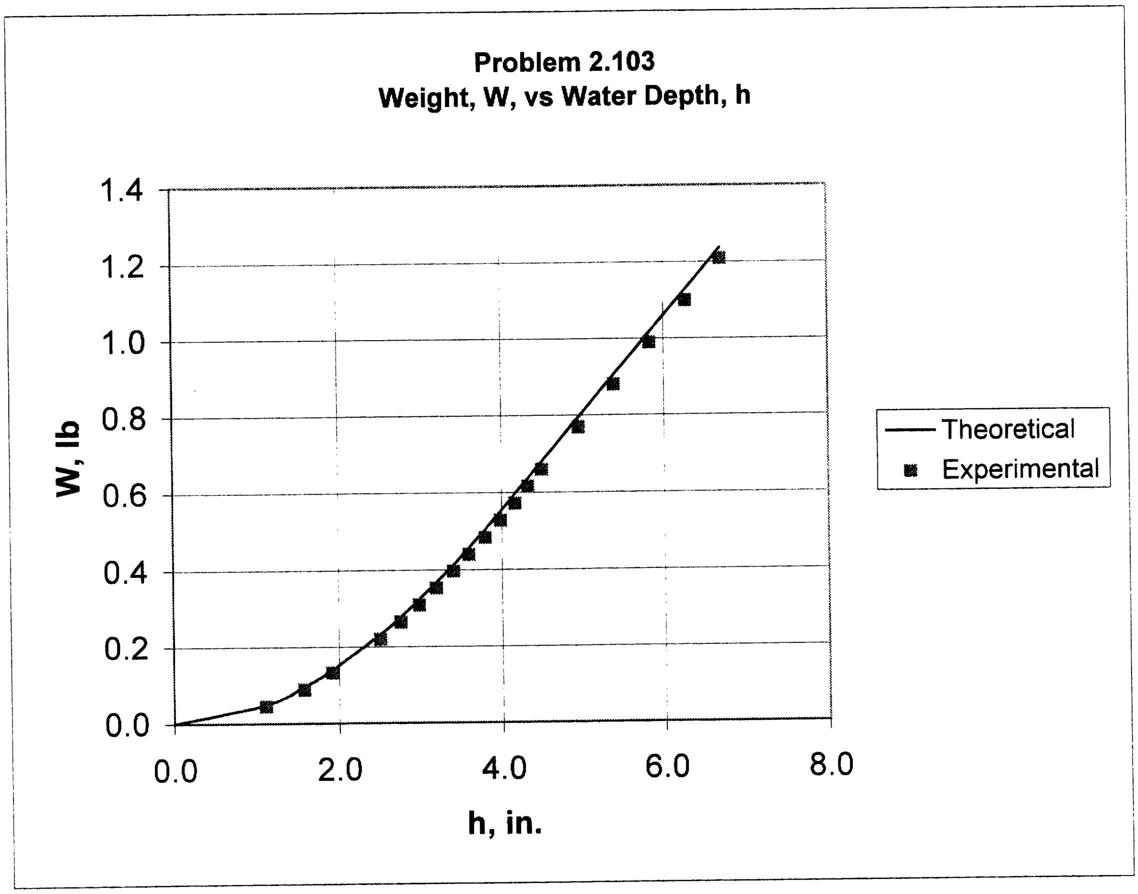
$I_{xc} = b*(R_2 - R_1)^3/12 = 0.000771$ ft^4

$h_c = h - (R_2 - R_1)/2$

$A = b*(R_2 - R_1)$

(con't)

2.103 (con't)



Problem 2.103
Weight, W, vs Water Depth, h
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
W, lb
0.0
2.0
4.0
6.0
8.0
h, in.
Theoretical
Experimental

2.104

2.104 Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

Objective: When a box or form as shown in Fig. P2.104 is filled with a liquid, the vertical force of the liquid on the box tends to lift it off the surface upon which it sits, thus allowing the liquid to drain from the box. The purpose of this experiment is to determine the minimum weight, W, needed to keep the box from lifting off the surface.

Equipment: An open-bottom box that has vertical side walls and slanted end walls; weights; ruler; scale.

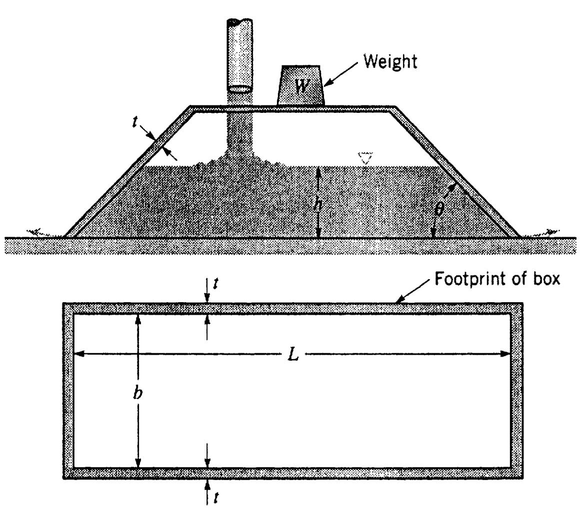
Experimental Procedure: Determine the weight, W_{box}, of the empty box and measure its length, L, width, b, wall thickness, t, and the angle of the ends, θ. Set the box on a smooth surface and place a known mass, m, on it. Slowly fill the box with water and note the depth, h, at which the net upward water force is equal to the total weight, $W + W_{box}$, where $W = mg$. This condition will be obvious because the friction force between the box and the surface on which it sits will be zero and the box will "float" effortlessly along the surface. Repeat for various masses and water levels.

Calculations: For an arbitrary water depth, h, determine the theoretical weight, W, needed to maintain equilibrium with no contact force between the box and the surface below it. This can be done by equating the total weight, $W + W_{box}$, to the net vertical hydrostatic pressure force on the box. Calculate this vertical pressure force for two different situations. (1) Assume the vertical pressure force is the vertical component of the pressure forces acting on the slanted ends of the box. (2) Assume the vertical upward force is that from part (1) plus the pressure force acting under the sides and ends of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma h/2$ acting on the "foot print" area of the box walls.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the water depth, h, as abscissas.

Results: On the same graph plot two theoretical total weight verses water depth curves—one involving only the slanted-end pressure force, and the other including the slanted end and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P2.104

(con't)

2.104 (con't)

Solution for Problem 2.104: Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

θ, deg	L, in.	b, in.	t, in.	W_{box}, lb	γ, lb/ft^3
45	10.3	4.0	0.25	0.942	62.4

m, kg	h, in.	Experimental $W + W_{box}$, lb	h, in.	Theory 1 $W + W_{box}$, lb	p_{avg}, lb/ft^2	Theory 2 $W + W_{box}$, lb
0.00	2.06	0.942	0.00	0.000	0.00	0.000
0.05	2.23	1.052	0.25	0.009	0.65	0.047
0.10	2.42	1.162	0.50	0.036	1.30	0.111
0.15	2.53	1.272	0.75	0.081	1.95	0.194
0.20	2.67	1.382	1.00	0.144	2.60	0.295
0.25	2.81	1.491	1.25	0.226	3.25	0.414
0.30	2.94	1.601	1.50	0.325	3.90	0.551
0.35	3.06	1.711	1.75	0.442	4.55	0.706
0.40	3.16	1.821	2.00	0.578	5.20	0.879
			2.25	0.731	5.85	1.070
			2.50	0.903	6.50	1.279
			2.75	1.092	7.15	1.506
			3.00	1.300	7.80	1.752
			3.25	1.526	8.45	2.015

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Including only the slanted-end pressure force:

$W + W_{box} = \gamma$*Vol

Vol = b*h*h

Theory 2. Including the slanted-end pressure force and the finite-thickness wall pressure force:

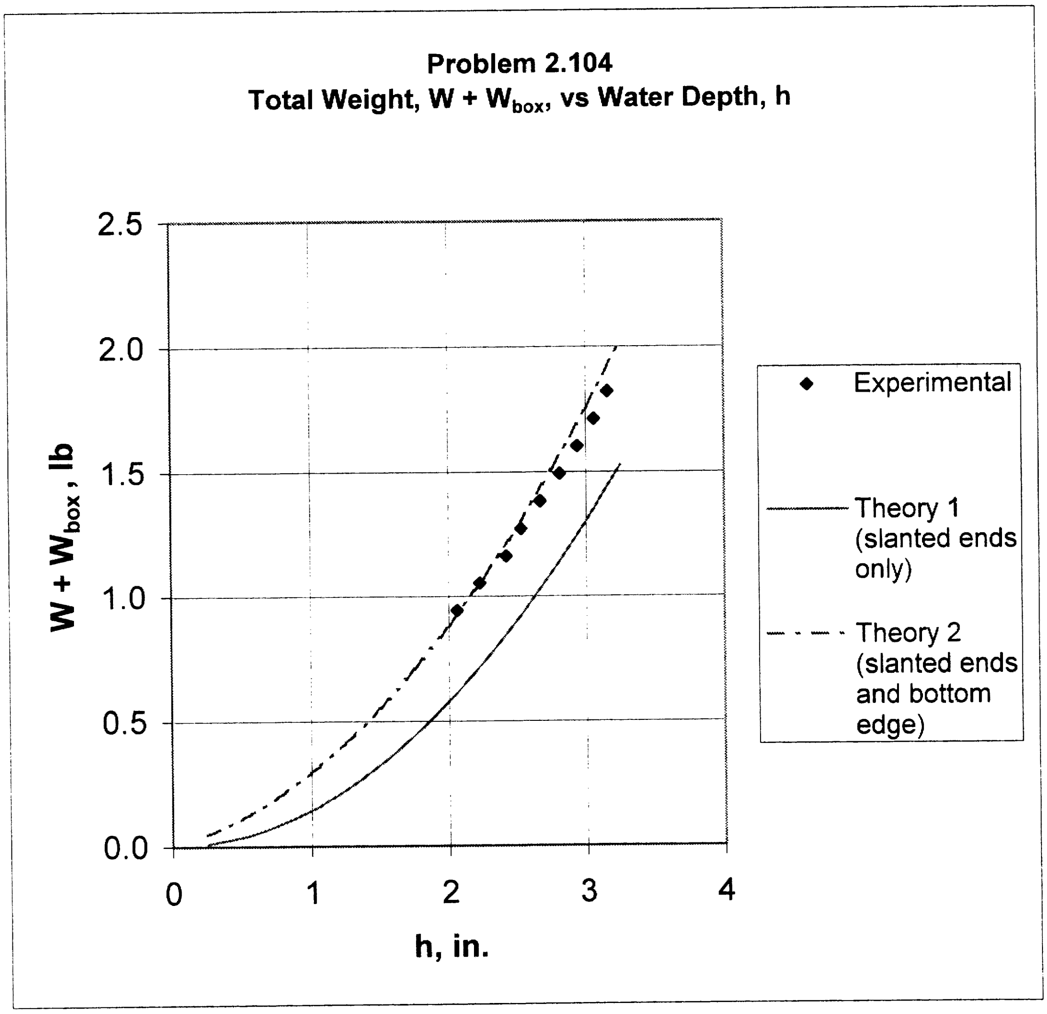
$W + W_{box} = \gamma$*Vol + p_{avg}*A

$p_{avg} = 0.5*\gamma*h$

A = (b + 2*t)*(L + 2*t/sinθ) - b*L = 8.33 in.^2 = 0.0579 ft^2

(con't)

2.104



Problem 2.104
Total Weight, W + W$_{box}$, vs Water Depth, h
W + W$_{box}$, lb
h, in.
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
Experimental
Theory 1 (slanted ends only)
Theory 2 (slanted ends and bottom edge)

2.105

2.105 Air Pad Lift Force

Objective: As shown in Fig. P2.105, it is possible to lift objects by use of an air pad consisting of an inverted box that is pressurized by an air supply. If the pressure within the box is large enough, the box will lift slightly off the surface, air will flow under its edges, and there will be very little frictional force between the box and the surface. The purpose of this experiment is to determine the lifting force, W, as a function of pressure, p, within the box.

Equipment: Inverted rectangular box; air supply; weights; manometer.

Experimental Procedure: Connect the air source and the manometer to the inverted square box. Determine the weight, W_{box}, of the square box and measure its length and width, L, and the wall thickness, t. Set the inverted box on a smooth surface and place a known mass, m, on it. Increase the air flowrate until the box lifts off the surface slightly and "floats" with negligible frictional force. Record the manometer reading, h, under these conditions. Repeat the measurements with various masses.

Calculations: Determine the theoretical weight that can be lifted by the air pad by equating the total weight, $W + W_{box}$, to the net vertical pressure force on the box. Here $W = mg$. Calculate this pressure force for two different situations. (1) Assume the pressure force is equal to the area of the box, $A = L^2$, times the pressure, $p = \gamma_m h$, within the box, where γ_m is the specific weight of the manometer fluid. (2) Assume that the net pressure force is that from part (1) plus the pressure force acting under the edges of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma_m h/2$ acting on the "foot print" area of the box walls, $4t(L + t)$.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the pressure within the box, p, as abscissas.

Results: On the same graph, plot two theoretical total weight verses pressure curves—one involving only the pressure times box area pressure force, and the other including the pressure times box area and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

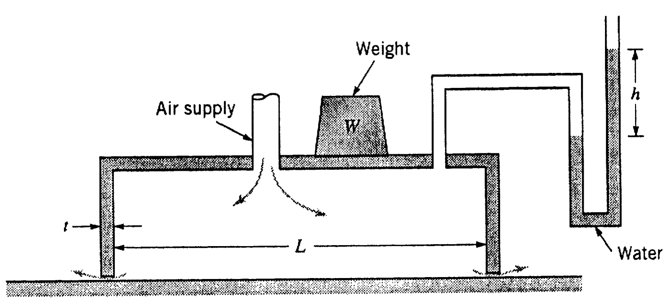


FIGURE P2.105

(Con't)

Solution for Problem 2.105: Air Pad Lift Force

L, in.	t, in.	W_{box}, lb	γ_{H2O}, lb/ft^3
7.5	0.25	1.25	62.4

m, kg	h, in.	Experiment $W + W_{box}$, lb	p, lb/ft^2	Theory 1 $W + W_{box}$, lb	Theory 2 $W + W_{box}$, lb
0.0	0.54	1.25	2.81	1.10	1.17
0.1	0.64	1.47	3.33	1.30	1.39
0.2	0.74	1.69	3.85	1.50	1.61
0.3	0.82	1.91	4.26	1.67	1.78
0.4	0.94	2.13	4.89	1.91	2.04
0.5	1.04	2.35	5.41	2.11	2.26
0.6	1.12	2.57	5.82	2.28	2.43
0.7	1.23	2.79	6.40	2.50	2.67
0.8	1.32	3.01	6.86	2.68	2.87
0.9	1.42	3.23	7.38	2.88	3.08
1.0	1.52	3.45	7.90	3.09	3.30
1.1	1.63	3.67	8.48	3.31	3.54
1.2	1.72	3.89	8.94	3.49	3.73
1.3	1.83	4.11	9.52	3.72	3.97
1.4	1.96	4.33	10.19	3.98	4.26
1.5	2.06	4.55	10.71	4.18	4.47
1.6	2.12	4.77	11.02	4.31	4.60
1.7	2.23	4.99	11.60	4.53	4.84
1.8	2.32	5.21	12.06	4.71	5.04

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Involving only the pressure times the box area:

$W + W_{box}$ = p*L^2

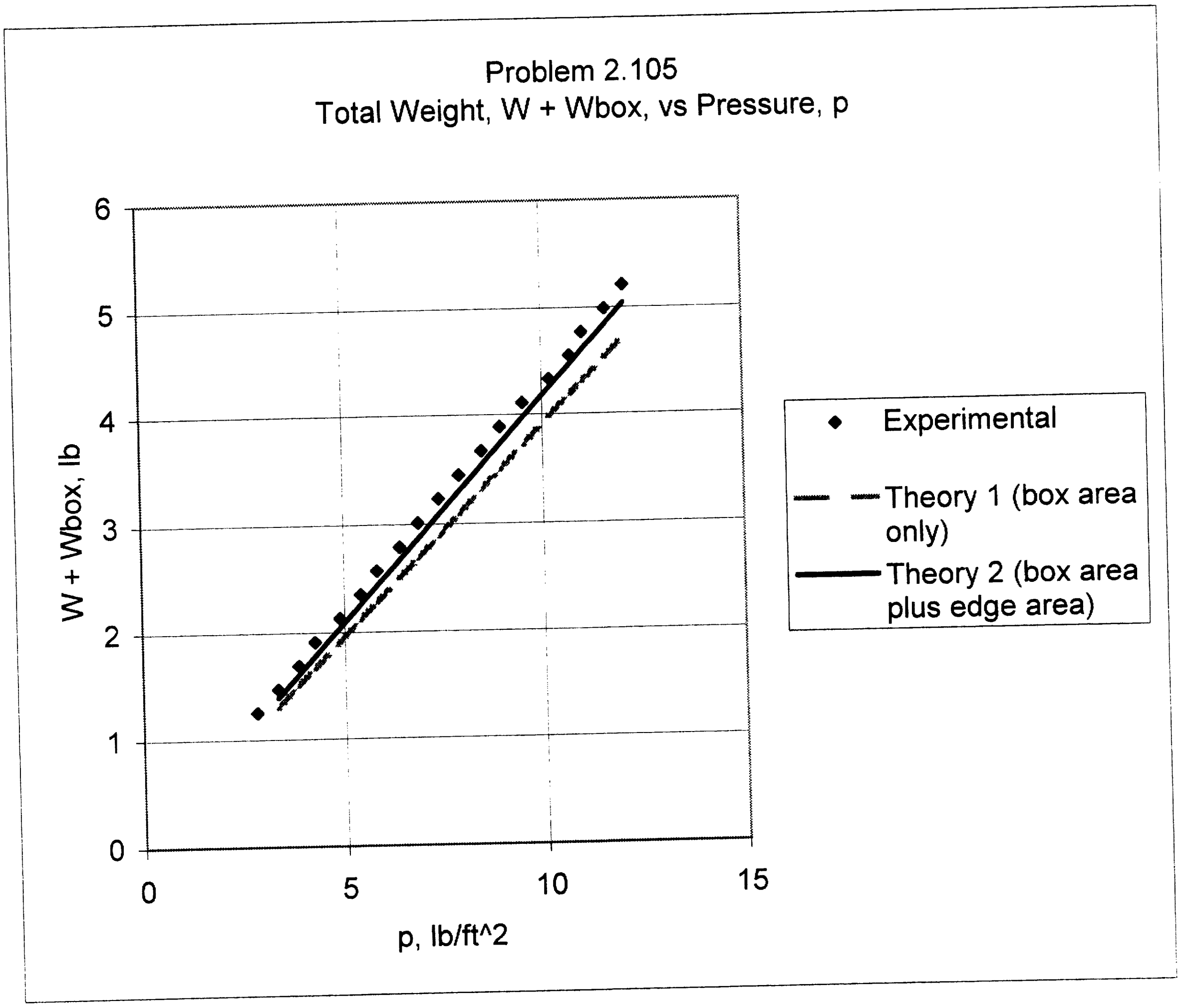
p = γ_{H2O}*h

Theory 2. Involving the pressure times the box area plus the average pressure times the edge area:

$W + W_{box}$ = p*L^2 + (p/2)*((L + 2t)^2 - L^2)

(con't)

2.105 (con't)



Problem 2.105
Total Weight, W + Wbox, vs Pressure, p
W + Wbox, lb
p, lb/ft^2
0
1
2
3
4
5
6
0
5
10
15
Experimental
Theory 1 (box area only)
Theory 2 (box area plus edge area)

3.1

3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

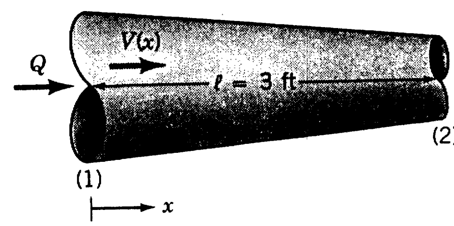


FIGURE P3.1

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} \quad \text{or} \quad \frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{slugs}{ft^3}\left(10 \frac{ft}{s}\right)^2 (1+x)$, with x in feet

$$= \underline{\underline{-194(1+x) \frac{lb}{ft^3}}}$$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50\,psi}^{p_2} dp = -194 \int_{x_1=0}^{x_2=3} (1+x)\,dx$

or $p_2 = 50\,psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10 \frac{ft}{s}$

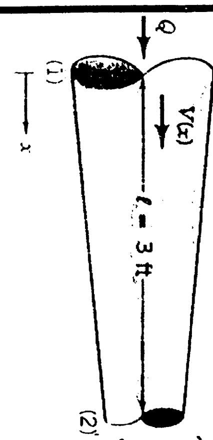
$V_2 = 10(1+3) = 40 \frac{ft}{s}$

Thus,

$$p_2 = 50\,psi + \frac{1}{2}\left(1.94 \frac{slugs}{ft^3}\right)(10^2 - 40^2)\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in^2}\right) = \underline{\underline{39.9\,psi}}$$

3.2

3.2 Repeat Problem 3.1 if the pipe is vertical with the flow *down*.



(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ with $\theta = -90^\circ$ and $V = 10(1+x)\,\frac{ft}{s}$

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} + \gamma$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} + \gamma = -\rho(10(1+x))(10) + \gamma$

Thus, $\frac{\partial p}{\partial x} = -1.94\,\frac{slugs}{ft^3}\left(10\,\frac{ft}{s}\right)^2 (1+x) + 62.4\,\frac{lb}{ft^3}$, with x in feet

$$= \underline{\underline{-194(1+x) + 62.4\,\frac{lb}{ft^3}}}$$

(b)(i) $\frac{dp}{dx} = -194(1+x) + 62.4$ so that $\int_{p_1 = 50psi}^{p_2} dp = \int_{x_1=0}^{x_2=3} [-194(1+x) + 62.4]\,dx$

or $p_2 = 50psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) + 62.4(3)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right)$

$= 50 - 10.1 + 1.3 = \underline{\underline{41.2\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = 0$, $z_2 = -3\,ft$

and $V_1 = 10(1+0) = 10\,\frac{ft}{s}$, $V_2 = 10(1+3) = 40\,\frac{ft}{s}$

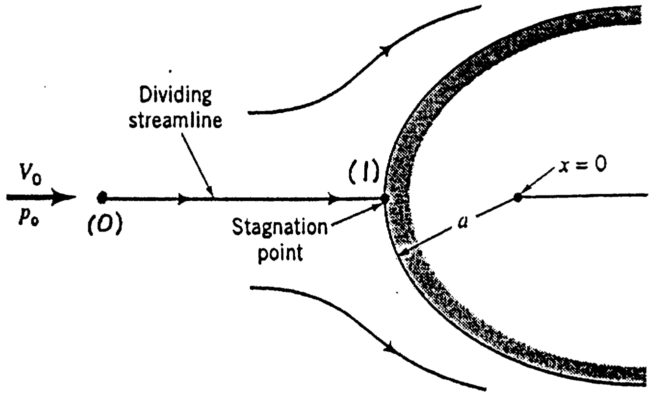
$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) - \gamma z_2$

$= 50psi + \frac{1}{2}\left(1.94\,\frac{slug}{ft^3}\right)(10^2 - 40^2) - 62.4\,\frac{lb}{ft^3}(-3\,ft)$

$= \underline{\underline{41.2\,psi}}$

3.3

3.3 An incompressible fluid with density ρ flows steadily past the object shown in Video V3.3 and Fig. P3.3. The fluid velocity along the horizontal dividing streamline ($-\infty \le x \le -a$) is found to be $V = V_0(1 + a/x)$, where a is the radius of curvature of the front of the object and V_0 is the upstream velocity. **(a)** Determine the pressure gradient along this streamline. **(b)** If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \le x \le -a$. **(c)** Show from the result of part **(b)** that the pressure at the stagnation point ($x = -a$) is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.



■ FIGURE P3.3

(a) $\frac{dp}{ds} = -\rho V \frac{dV}{ds}$ where $V = V_0\left(1 + \frac{a}{x}\right)$

Thus, $\frac{dV}{ds} = \frac{dV}{dx} = -\frac{V_0 a}{x^2}$

or

$$\frac{dp}{ds} = \frac{dp}{dx} = -\rho V_0\left(1 + \frac{a}{x}\right)\left(-\frac{V_0 a}{x^2}\right) = \underline{\underline{\rho a V_0^2\left(\frac{1}{x^2} + \frac{a}{x^3}\right)}}$$

(b) $$\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}\,dx = \rho a V_0^2 \int_{-\infty}^{x}\left(\frac{1}{x^2} + \frac{a}{x^3}\right)dx$$ Note: $p = p_0$ at $x = -\infty$

or

$$p - p_0 = \rho a V_0^2\left[-\frac{1}{x} - \frac{1}{2}\frac{a}{x^2}\right]_{-\infty}^{x}$$

Thus,

$$\underline{\underline{p = p_0 - \rho a V_0^2\left[\frac{1}{x} + \frac{a}{2x^2}\right]}}$$

(c) From part (b), when $x = -a$

$$p\Big|_{x=-a} = p_0 - \rho a V_0^2\left[-\frac{1}{a} + \frac{a}{2a^2}\right] = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$

From the Bernoulli equation $p_0 + \frac{1}{2}\rho V_0^2 = p_1 + \frac{1}{2}\rho V_1^2$

where

$$V_1 = V\Big|_{x=-a} = V_0\left(1 + \frac{a}{(-a)}\right) = 0$$

Thus, $p_1 = p_0 + \frac{1}{2}\rho V_0^2$ as expected.

3.4

3.4 What pressure gradient along the streamline, dp/ds, is required to accelerate water in a horizontal pipe at a rate of 30 m/s²?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s} \quad \text{where} \quad \theta = 0 \quad \text{and} \quad V\frac{\partial V}{\partial s} = a_s = 0\,\frac{m}{s^2}$$

Thus,

$$\frac{\partial p}{\partial s} = -\rho a_s = -999\,\frac{kg}{m^3}\left(30\,\frac{m}{s^2}\right) = -30{,}000\left(\frac{N}{m^2}\right)/m$$

or

$$\frac{\partial p}{\partial s} = \underline{\underline{-30.0\ kPa/m}}$$

3.5

3.5 At a given location the air speed is 20 m/s and the pressure gradient along the streamline is 100 N/m³. Estimate the air speed at a point 0.5 m further along the streamline.

If neglect gravity, $\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial V}{\partial s} = -\frac{\partial p}{\partial s}/\rho V$

or $\frac{\partial V}{\partial s} = -100\,\frac{N}{m^3}\Big/\left(1.23\,\frac{kg}{m^3}\right)\left(20\,\frac{m}{s}\right) = -4.07\,\frac{1}{s}$

Thus,

$\delta V \approx \frac{\partial V}{\partial s}\delta s = \left(-4.07\,\frac{1}{s}\right)(0.5\,m) = -2.03\,\frac{m}{s}$, so that $V + \delta V = 20\,\frac{m}{s} - 2.03\,\frac{m}{s}$

or $V \approx \underline{\underline{18.0\,\frac{m}{s}}}$

3.6

3.6 What pressure gradient along the streamline, dp/ds, is required to accelerate water upward in a vertical pipe at a rate of 30 ft/s²? What is the answer if the flow is downward?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s}$$

where $\theta = 90°$ for up flow,
$\theta = -90°$ for down flow,
and $V\frac{\partial V}{\partial s} = a_s = 30\,\frac{ft}{s^2}$

Thus, for up flow

$$\frac{\partial p}{\partial s} = -62.4(1)\,\frac{lb}{ft^3} - 1.94\,\frac{slugs}{ft^3}\left(30\,\frac{ft}{s^2}\right) = -120.6\left(\frac{lb}{ft^2}\right)/ft = \underline{\underline{-0.838\,\frac{psi}{ft}}}$$

and for down flow

$$\frac{\partial p}{\partial s} = -62.4(-1)\,\frac{lb}{ft^3} - 1.94\,\frac{slugs}{ft^3}\left(30\,\frac{ft}{s}\right) = 4.20\left(\frac{lb}{ft^3}\right)/ft = \underline{\underline{0.0292\,\frac{psi}{ft}}}$$

3.7

3.7 Consider a compressible fluid for which the pressure and density are related by $p/\rho^n = C_0$, where n and C_0 are constants. Integrate the equation of motion along the streamline, Eq. 3.6, to obtain the "Bernoulli equation" for this compressible flow as $[n/(n-1)]p/\rho + V^2/2 + gz =$ constant.

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant along a streamline}$$

and

$$\rho^n = \frac{p}{C_0} \quad \text{or} \quad \rho = \frac{p^{1/n}}{C_0^{1/n}} \quad \text{so that}$$

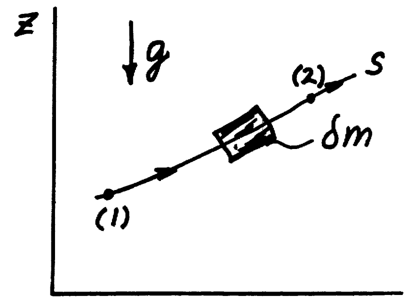
$$\int \frac{dp}{\rho} = C_0^{\frac{1}{n}} \int \frac{dp}{p^{\frac{1}{n}}} = C_0^{\frac{1}{n}} \int p^{-\frac{1}{n}} dp = C_0^{\frac{1}{n}} \frac{1}{(1-\frac{1}{n})} p^{1-\frac{1}{n}} + \text{const.}$$

Thus,

$$\int \frac{dp}{\rho} = \frac{n}{n-1} p \left(\frac{C_0}{p}\right)^{\frac{1}{n}} = \frac{n}{n-1} \frac{p}{\rho}$$

Hence: $\underline{\underline{\frac{n}{n-1} \frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{constant along a streamline}}}$

3.8 The Bernoulli equation is valid for steady, inviscid, incompressible flows with constant acceleration of gravity. Consider flow on a planet where the acceleration of gravity varies with height so that $g = g_0 - cz$, where g_0 and z are constants. Integrate "$\mathbf{F} = m\mathbf{a}$" along a streamline to obtain the equivalent of the Bernoulli equation for this flow.



From $\sum \delta F_s = \delta m\, a_s$ one obtains

$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz$ where $\gamma = \rho g$

(see Eq. 3.5)

Thus,

$dp + d(\frac{1}{2}\rho V^2) + \rho(g_0 - cz)\,dz = 0$, or by integrating from (1) to (2):

$$\int_{(1)}^{(2)} dp + \int_{(1)}^{(2)} d(\tfrac{1}{2}\rho V^2) + \rho\int_{(1)}^{(2)} (g_0 - cz)\,dz = 0$$

or

$$p_2 - p_1 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g_0(z_2 - z_1) - \frac{1}{2}\rho c(z_2^2 - z_1^2) = 0$$

Thus,

$p + \frac{1}{2}\rho V^2 + \rho g_0 z - \frac{1}{2}\rho c z^2 =$ constant along a streamline.

3.9

3.9 Consider a compressible liquid that has a constant bulk modulus. Integrate "**F** = m**a**" along a streamline to obtain the equivalent of the Bernoulli equation for this flow. Assume steady, inviscid flow.

From Eq. 3.6

$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$ where $\gamma = \rho g$

and $dp = E_v \frac{d\rho}{\rho}$ where

E_v = bulk modulus = constant

(see Eq. 1.13)

Thus, along a streamline:

$E_v \frac{d\rho}{\rho} + \frac{1}{2}\rho\, d(V^2) + \rho g\, dz = 0$ or

$E_v \frac{d\rho}{\rho^2} + d(\frac{1}{2}V^2) + g\, dz = 0$ which can be integrated between between points (1) and (2) to give

$$E_v \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho^2} + \int_{V_1}^{V_2} d(\tfrac{1}{2}V^2) + \int_{z_1}^{z_2} g\, dz = 0$$

or

$$-E_v\left[\frac{1}{\rho_2} - \frac{1}{\rho_1}\right] + \frac{1}{2}\left[V_2^2 - V_1^2\right] + g\left[z_2 - z_1\right] = 0$$

Hence:

$gz - \frac{E_v}{\rho} + \frac{V^2}{2}$ = constant along a streamline

3.10

3.10 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.10. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

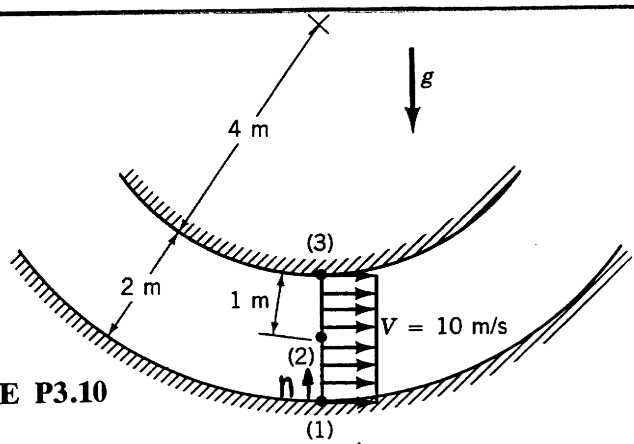


FIGURE P3.10

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{dz}{dn} = 1 \text{ and } V = 10\,m/s$$

Thus, with $\mathcal{R} = 6 - n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6-n} \quad \text{or}$$

$$\int_{n=0}^{n} \frac{dp}{dn}\, dn = -\int_{n=0}^{n} \gamma\, dn - \int_{n=0}^{n} \frac{\rho V^2 dn}{6-n}$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6-n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6-n}\right)$$

With $p_1 = 40\,kPa$ and $n_2 = 1\,m$: $p_2 = 40\,kPa - 9.8\times10^3 \frac{N}{m^3}(1\,m) - 999 \frac{kg}{m^3}\left(10\frac{m}{s}\right)^2 \ln\left(\frac{6}{5}\right)$

or

$$p_2 = \underline{\underline{12.0\ kPa}}$$

and

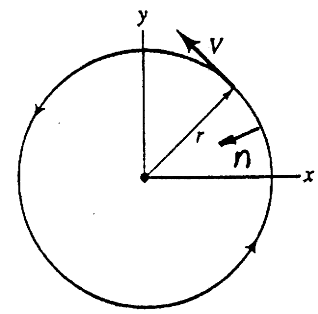
with $p_1 = 40\,kPa$ and $n_3 = 2m$: $p_3 = 40\,kPa - 9.80\times10^3 \frac{N}{m^3}(2m) - 999 \frac{kg}{m^3}\left(10\frac{m}{s}\right)^2 \ln\left(\frac{6}{4}\right)$

or

$$p_3 = \underline{\underline{-20.1\ kPa}}$$

3.12

3.12 Water in a container and air in a tornado flow in horizontal circular streamlines of radius r and speed V as shown in Video V3.2 and Fig. P3.12. Determine the radial pressure gradient, $\partial p/\partial r$, needed for the following situations: (a) The fluid is water with $r = 3$ in. and $V = 0.8$ ft/s. (b) The fluid is air with $r = 300$ ft and $V = 200$ mph.



■ FIGURE P3.12

For curved streamlines,

$$-\frac{dp}{dn} = \frac{\rho V^2}{\mathcal{R}} + \gamma \frac{dz}{dn}\text{, or with } \frac{dz}{dn}=0 \text{ (horizontal streamlines), } \mathcal{R}=r,$$

and $\frac{d}{dn} = -\frac{d}{dr}$ this becomes

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

a) With $r = \frac{3}{12}$ ft and $V = 0.8\,\frac{ft}{s}$ and water $\left(\rho = 1.94\,\frac{slugs}{ft^3}\right)$,

$$\frac{dp}{dr} = \frac{1.94\,\frac{slugs}{ft^3}\left(0.8\,\frac{ft}{s}\right)^2}{\left(\frac{3}{12}\,ft\right)} = 4.97\,\frac{slugs}{ft^2\cdot s^2} = \underline{\underline{4.97\,\frac{lb}{ft^3}}}$$

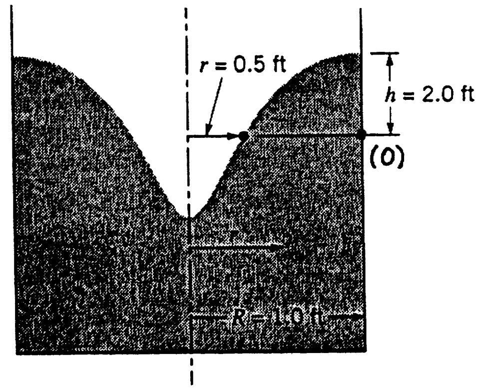
(b) With $r = 300\,ft$ and $V = 200\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 293\,\frac{ft}{s}$

and air $\left(\rho = 0.00238\,\frac{slugs}{ft^3}\right)$,

$$\frac{dp}{dr} = \frac{0.00238\,\frac{slugs}{ft^3}\left(293\,\frac{ft}{s}\right)^2}{300\,ft} = 0.681\,\frac{slugs}{ft^2\cdot s^2} = \underline{\underline{0.681\,\frac{lb}{ft^3}}}$$

3.13

3.13 As shown in Fig. P3.13 and Video V3.2, the swirling motion of a liquid can cause a depression in the free surface. Assume that an inviscid liquid in a tank with an $R = 1.0$ ft radius is rotated sufficiently to produce a free surface that is $h = 2.0$ ft below the liquid at the edge of the tank at a position $r = 0.5$ ft from the center of the tank. Also assume that the liquid velocity is given by $V = K/r$, where K is a constant. **(a)** Show that $h = K^2[(1/r^2) - (1/R^2)]/(2g)$. **(b)** Determine the value of K for this problem.



■ FIGURE P3.13

(a) $-\frac{dp}{dn} = \frac{\rho V^2}{R}$ or $\frac{dp}{dr} = \frac{\rho V^2}{r} = \frac{\rho K^2}{r^3}$

Thus, $\int_p^{p_0} dp = \rho K^2 \int_r^R \frac{dr}{r^3}$ or $p_0 - p = -\frac{\rho K^2}{2}\left[\frac{1}{R^2} - \frac{1}{r^2}\right]$

But $p_0 = \gamma h$ and $p = 0$ at r on the free surface.

Thus,

$$\gamma h = -\frac{\rho K^2}{2}\left[\frac{1}{R^2} - \frac{1}{r^2}\right] \quad \text{or since } \gamma = \rho g,$$

$$h = \frac{K^2}{2g}\left[\frac{1}{r^2} - \frac{1}{R^2}\right] \qquad (1)$$

(b) With $h = 2$ ft, $R = 1$ ft, and $r = 0.5$ ft Eqn. (1) gives

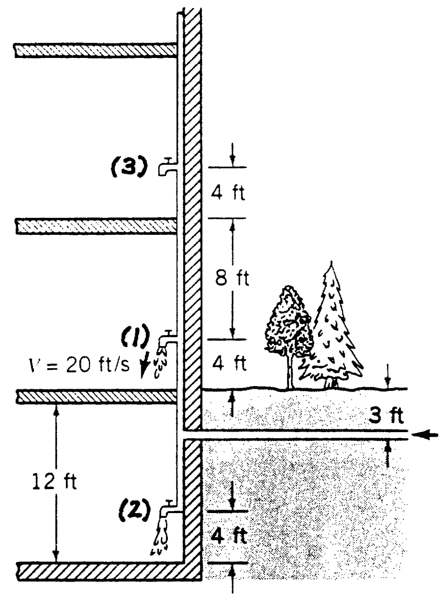
$$2\text{ ft} = \frac{K^2}{2(32.2\text{ ft/s}^2)}\left[\frac{1}{(0.5\text{ ft})^2} - \frac{1}{(1\text{ ft})^2}\right]$$

or

$$K = 6.55\ \frac{\text{ft}^2}{\text{s}}$$

3.14

3.14 Water flows from the faucet on the first floor of the building shown in Fig. P3.14 with a maximum velocity of 20 ft/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall).



■ FIGURE P3.14

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

Thus, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $p_2 = p_1 = 0$ (free jet) and $V_1 = 20\ ft/s$, $z_1 = 4\ ft$, $z_2 = -8\ ft$

or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\ ft = \frac{V_2^2}{2(32.2\frac{ft}{s^2})} + (-8\ ft)$$

or $V_2 = \underline{\underline{34.2\ \frac{ft}{s}}}$

and $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $p_3 = p_1 = 0$ (free jet) and $V_1 = 20\ \frac{ft}{s}$, $z_1 = 4\ ft$, $z_3 = 16\ ft$

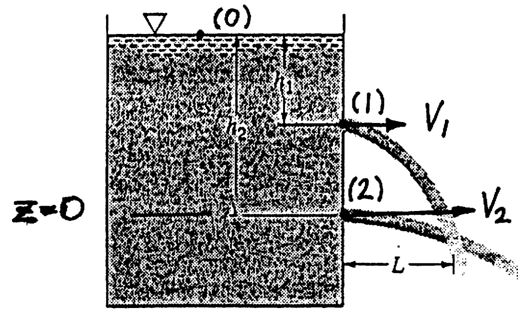
or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\ ft = \frac{V_3^2}{2(32.2\frac{ft}{s^2})} + 16\ ft$$

or $V_3 = \sqrt{20^2 - 2(32.2)(12)} = \sqrt{-373}$ Impossible! No flow from second floor faucet.

3.15

3.15 Water flows from a pop bottle that has holes in it as shown in Video V3.5 and Fig. P3.15. Two streams coming from holes located distances h_1 and h_2 below the free surface intersect at a distance L from the side of the bottle. If viscous effects are negligible and the flow is quasi-steady, show that $L = 2(h_1h_2)^{1/2}$. Compare this result with experimental data measured from the paused video for which the holes are 2 inches apart.



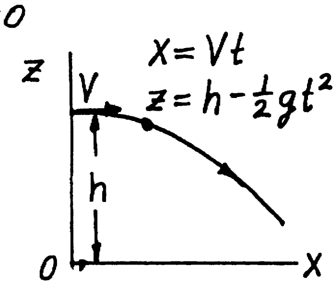
■ FIGURE P3.15

For steady inviscid flow, the velocities of the horizontal jets of water at points (1) and (2) are obtained from the Bernoulli equation as:

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2 \quad \text{where}$$

$p_0 = p_1 = p_2 = 0$; $V_0 = 0$; $z_0 = h_2$; $z_1 = h_2 - h_1$; and $z_2 = 0$

Thus, $V_1 = \sqrt{2gh_1}$ and $V_2 = \sqrt{2gh_2}$ (1)



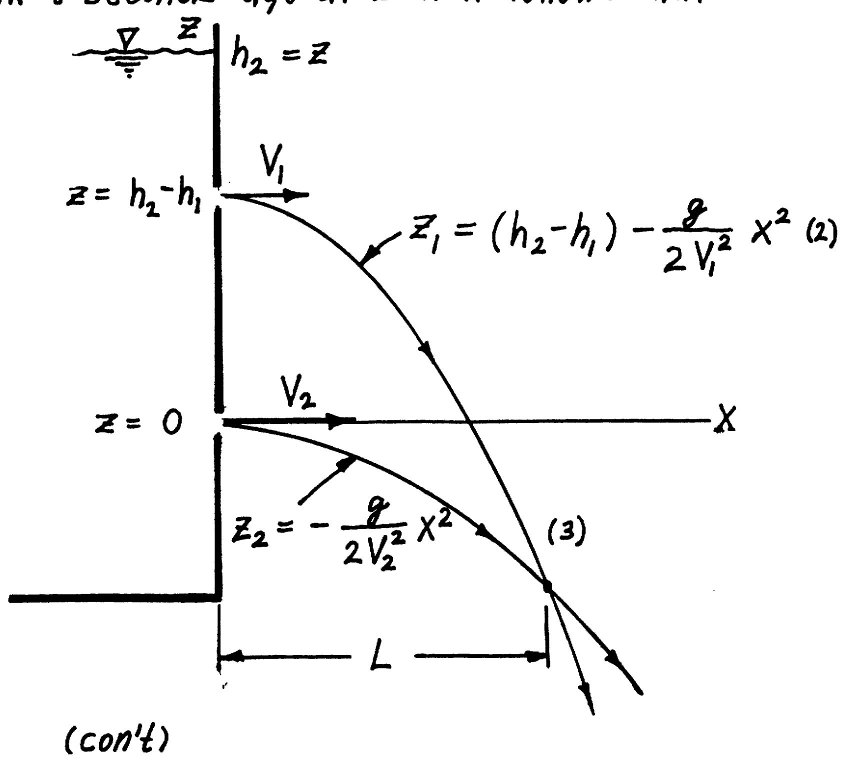
Once outside the tank, gravity is the only force on the fluid particles. Hence the horizontal component of velocity remains constant, but the particles accelerate downward with the acceleration of gravity. Thus, for a fluid particle that exited the tank t seconds ago at $z=h$ it follows that

$x = Vt$ and $z = h - \frac{1}{2}gt^2$.

By eliminating t, this gives the particle path (i.e. the shape of the water jet as

$$z = h - \frac{g}{2V^2}x^2$$

Thus, the shapes of the two water jets are as shown in the figure.



(con't)

3.15 (con't)

The streams intersect when $x = L$ and $z_1 = z_2$. Thus, from Eqns. (2) and (3),

$(h_2 - h_1) - \frac{g}{2V_1^2} L^2 = -\frac{g}{2V_2^2} L^2$ which can be rearranged to give

$$L = \sqrt{\frac{2(h_2 - h_1)}{g}} \Big/ \sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}} \tag{4}$$

From Eqn. (1),

$$\frac{1}{V_1^2} - \frac{1}{V_2^2} = \frac{1}{2gh_1} - \frac{1}{2gh_2} = \frac{1}{2g}\left[\frac{1}{h_1} - \frac{1}{h_2}\right] = \frac{(h_2 - h_1)}{2g\, h_1 h_2} \tag{5}$$

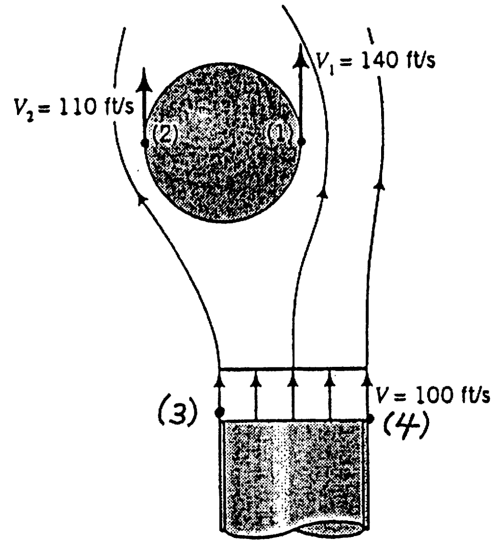
Thus, by combining Eqns. (4) and (5) we obtain

$$L = \sqrt{\frac{2(h_2 - h_1)}{g}} \Big/ \sqrt{\frac{(h_2 - h_1)}{2g\, h_1 h_2}} = \underline{\underline{2\sqrt{h_1 h_2}}}$$

Note that although V_1 and V_2 are a function of g, the distance L is not. Two tanks, one on Earth, the other on Mars would drain at different rates, but the intersection distance, L, of two streams would be the same.

3.16

3.16 A 100 ft/s jet of air flows past a ball as shown in Video V3.1 and Fig. P3.16. When the ball is not centered in the jet, the air velocity is greater on the side of the ball near the jet center [point (1)] than it is on the other side of the ball [point (2)]. Determine the pressure difference, $p_2 - p_1$, across the ball if $V_1 = 140$ ft/s and $V_2 = 110$ ft/s. Neglect gravity and viscous effects.



■ FIGURE P3.16

The Bernoulli equation from point (3) to (2) and (4) to (1) with gravity neglected gives

$$p_3 + \frac{1}{2}\rho V_3^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{and} \quad p_4 + \frac{1}{2}\rho V_4^2 = p_1 + \frac{1}{2}\rho V_1^2$$

But $p_3 = p_4 = 0$ and $V_3 = V_4$

Thus, even though points (1) and (2) are not on the same streamline,

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

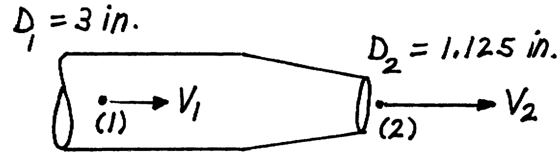
or

$$p_1 - p_2 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}\left(0.00238\,\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(140\,\frac{\text{ft}}{\text{s}}\right)^2 - \left(110\,\frac{\text{ft}}{\text{s}}\right)^2\right]$$

$$= 8.93\,\frac{\text{slugs}}{\text{ft}\cdot\text{s}^2} = \underline{\underline{8.93\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.18

3.18 A fire hose nozzle has a diameter of $1\frac{1}{8}$ in. According to some fire codes, the nozzle must be capable of delivering at least 250 gal/min. If the nozzle is attached to a 3-in.-diameter hose, what pressure must be maintained just upstream of the nozzle to deliver this flowrate?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $z_1 = z_2$, $p_2 = 0$

and $Q = \left(250\ \frac{gal}{min}\right)\left(231\ \frac{in^3}{gal}\right)\left(\frac{1\ ft^3}{1728\ in^3}\right)\left(\frac{1\ min}{60\ s}\right) = 0.557\ \frac{ft^3}{s}$

Thus,

$$p_1 = \frac{\gamma}{2g}\left[V_2^2 - V_1^2\right] \quad \text{where} \quad V_2 = \frac{Q}{A_2} = \frac{0.557\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{1.125}{12}\right)^2 ft^2} = 80.7\ \frac{ft}{s}$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.557\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2 ft^2} = 11.34\ \frac{ft}{s}$$

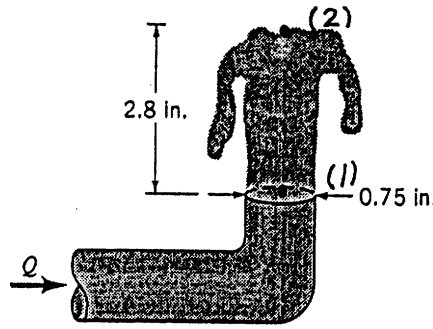
so that with $\frac{\gamma}{g} = \rho$

$$p_1 = \frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)\left[80.7^2 - 11.34^2\right]\frac{ft^2}{s^2}$$

$$= 6190\ \frac{lb}{ft^2} = \underline{\underline{43.0\ psi}}$$

3.19

3.19 Water flowing from the 0.75-in.-diameter outlet shown in Video V8.6 and Fig. P3.19 rises 2.8 inches above the outlet. Determine the flowrate.



■ FIGURE P3.19

The flowrate is $Q = A_1 V_1$, where from the Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Thus, with $p_1 = p_2 = z_1 = V_2 = 0$ we obtain

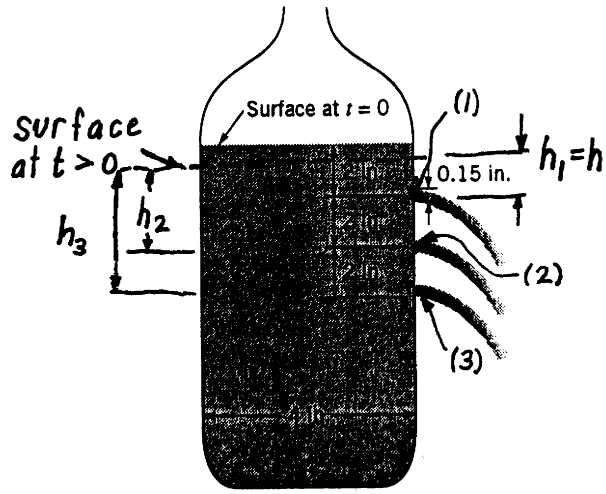
$$V_1 = \sqrt{2 g z_2} = \sqrt{2\,(32.2\ \text{ft/s}^2)\,(2.8/12)\text{ft}} = 3.88\ \text{ft/s}$$

so that

$$Q = A_1 V_1 = \frac{\pi}{4}\left(\frac{0.75}{12}\text{ft}\right)^2 \left(3.88\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.0119\ \frac{\text{ft}^3}{\text{s}}}}$$

3.20

3.20 Pop (with the same properties as water) flows from a 4-in. diameter pop container that contains three holes as shown in Fig. P3.20 (see Video 3.5). The diameter of each fluid stream is 0.15 in., and the distance between holes is 2 in. If viscous effects are negligible and quasi-steady conditions are assumed, determine the time at which the pop stops draining from the top hole. Assume the pop surface is 2 in. above the top hole when $t = 0$. Compare your results with the time you measure from the video.



■ FIGURE P3.20

$$Q = Q_1 + Q_2 + Q_3 = -A_T \frac{dh}{dt}$$

where $Q_i = V_i A_i = \sqrt{2gh_i}\, A_i \quad (i = 1,2,3)$ and $A_1 = A_2 = A_3 = \frac{\pi}{4}\left(\frac{0.15}{12}\,ft\right)^2 = 1.227 \times 10^{-4}\,ft^2$

$$A_T = \frac{\pi}{4}\left(\frac{4}{12}\,ft\right)^2 = 0.0873\,ft^2$$

Thus,

$$\sqrt{2g}\,A_1\left[\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3}\,\right] = -A_T \frac{dh}{dt},$$ where $h_1 = h$, $h_2 = h + L$, $h_3 = h + 2L$ and $L = 2\,in.$

Hence,

$$-(\sqrt{2g}\,A_1/A_T)\int_0^t dt = \int_L^0 \frac{dh}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

where t is the time it take for the free surface to reach the upper hole ($h=0$),

or

$$t = \frac{A_T}{A_1\sqrt{2g}} \int_0^L \frac{dh}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

$$= \frac{0.0873\,ft^2}{(1.227 \times 10^{-4}\,ft^2)\left[(2)(32.2\,ft/s^2)\right]^{1/2}} \int_0^L \frac{dh}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

Thus,

$$t = 88.7 \int_0^L \frac{dh}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$ where $L = \frac{2}{12}\,ft = 0.1667\,ft$

Note: With L in feet, this equation gives t in seconds.

Since there is no closed form

(con't)

3.20	(con't)

The numerical value of the integral is obtained by using the trapezoidal rule since the closed form analytical solution is not given in integral tables. The EXCEL spread sheet used for this is given below.

$$t = 88.7\int_0^L f(h)\,dh \quad \text{where } f(h) = \frac{1}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

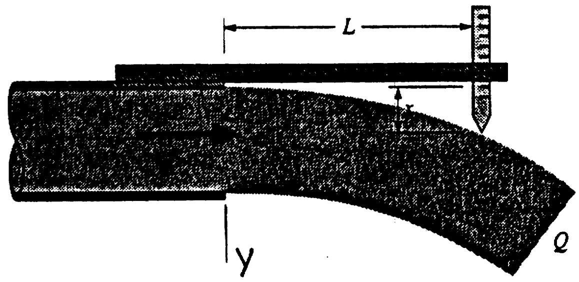
$$\approx 88.7\left[\frac{1}{2}\sum_{i=1}^{20}(f_i + f_{i+1})(h_{i+1} - h_i)\right] = \left(88.7\ \frac{s}{\sqrt{ft}}\right)\left[0.120\sqrt{ft}\right] = \underline{\underline{10.7\,s}}$$

h, in.	h, ft	f(h), 1/ft$^{1/2}$	(1/2)*(f_i + f_{i+1})*(h_{i+1} - h_i), ft$^{1/2}$	*i*
0.0	0.0000	1.015	0.00804	1
0.1	0.0083	0.914	0.00743	2
0.2	0.0167	0.870	0.00711	3
0.3	0.0250	0.837	0.00686	4
0.4	0.0333	0.810	0.00665	5
0.5	0.0417	0.786	0.00646	6
0.6	0.0500	0.764	0.00629	7
0.7	0.0583	0.745	0.00614	8
0.8	0.0667	0.728	0.00600	9
0.9	0.0750	0.712	0.00587	10
1.0	0.0833	0.697	0.00575	11
1.1	0.0917	0.684	0.00564	12
1.2	0.1000	0.671	0.00554	13
1.3	0.1083	0.659	0.00544	14
1.4	0.1167	0.647	0.00535	15
1.5	0.1250	0.637	0.00526	16
1.6	0.1333	0.627	0.00518	17
1.7	0.1417	0.617	0.00510	18
1.8	0.1500	0.608	0.00503	19
1.9	0.1583	0.599	0.00496	20
2.0	0.1667	0.591		21
	Sum of column = integral =		0.12011	

Thus, t = 88.7*0.12011 = 10.7 s

3.21

3.21 Water flowing from a pipe or a tank is acted upon by gravity and follows a curved trajectory as shown in Fig. P3.21 and Videos V3.5 and V4.3. A simple flow meter can be constructed as shown in Fig. P3.21. A point gage mounted a distance L from the end of the horizontal pipe is adjusted to indicate that the top of the water stream is a distance x below the outlet of the pipe. Show that the flowrate from this pipe of diameter D is given by $Q = \pi D^2 L\, g^{1/2}/(2^{5/2}\, x^{1/2})$.



■ FIGURE P3.21

The only force acting of any water particle in the free jet is that due to gravity — the particle's weight.
Thus, for the x-y axes shown

$\frac{d^2x}{dt^2} = g$ and $\frac{d^2y}{dt^2} = 0$ which for a particle starting at $x = y = 0$ at $t = 0$ give

$x = \frac{1}{2} g t^2$ and $y = Vt$

Eliminate t to give the water trajectory as

$x = \frac{1}{2} g \left(\frac{y}{V}\right)^2$ or $x = \frac{1}{2} g \frac{y^2}{V^2}$

Thus, with $y = L$:

$V = \sqrt{\frac{g}{2x}}\, L$ and

$Q = AV = \frac{\pi}{4} D^2 V = \underline{\underline{\pi D^2 L \sqrt{g} / (2^{5/2} \sqrt{x})}}$

3.22

3.22 A person holds her hand out of an open car window while the car drives through still air at 65 mph. Under standard atmospheric conditions, what is the maximum pressure on her hand? What would be the maximum pressure if the "car" were an Indy 500 racer traveling 220 mph?

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

with $z_1=z_2$

$V_1 = 65\ mph\left(\frac{88\ \frac{ft}{s}}{60\ mph}\right)=95.3\ \frac{ft}{s}$

$p_1=0,\ V_2=0$

Thus,

$$p_2=\frac{\gamma}{2g}V_1^2=\frac{1}{2}\rho V_1^2 \quad \text{or} \quad p_2=\frac{1}{2}\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(95.3\ \frac{ft}{s}\right)^2=\underline{\underline{10.8\ \frac{lb}{ft^2}}}$$

If $V_1 = 220\ mph\left(\frac{88\ \frac{ft}{s}}{60\ mph}\right)=323\ \frac{ft}{s}$, then

$$p_2=\frac{1}{2}\left(2.38\times10^{-3}\ \frac{slugs}{ft^3}\right)\left(323\ \frac{ft}{s}\right)^2=\underline{\underline{124\ \frac{lb}{ft^2}}}$$

3.23

3.23 A differential pressure gage attached to a Pitot-static tube (see Video V3.4) is calibrated to give speed rather than the difference between the stagnation and static pressures. The calibration is done so that the speed indicated on the gage is the actual fluid speed if the fluid flowing past the Pitot-static tube is air at standard sea level conditions. Assume the same device is used in water and the gage indicates a speed of 200 knots. Determine the water speed.

$$\Delta p=\frac{1}{2}\rho V^2$$

In air, $\Delta p_{air}=\frac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)(200\ knots)^2$

In water, $\Delta p_{water}=\frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)(V)^2$

so that with $\Delta p_{air}=\Delta p_{water}$,

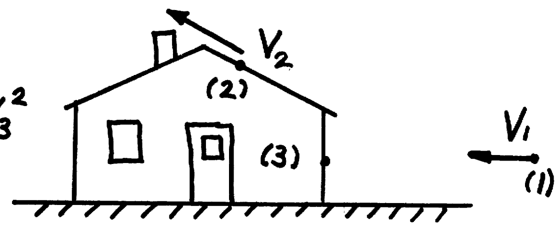
$$\frac{1}{2}(0.00238)(200)^2=\frac{1}{2}(1.94)V^2$$

or

$V=\underline{\underline{7.01\ knots}}$

3.24 A 40-mph wind blowing past your house speeds up as it flows up and over the roof. If elevation effects are negligible, determine **(a)** the pressure at the point on the roof where the speed is 60 mph if the pressure in the free stream blowing toward your house is 14.7 psia. Would this effect tend to push the roof down against the house, or would it tend to lift the roof? **(b)** Determine the pressure on a window facing the wind if the window is assumed to be a stagnation point.

The Bernoulli equation gives



$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = p_3 + \frac{1}{2}\rho V_3^2$$

a) Thus, from (1) to (2):

$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ or with $V_1 = 40\frac{mi}{hr}\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 58.7\,ft/s$

and $V_2 = 60\frac{mi}{hr} = 88.0\,ft/s$

$$p_2 = p_1 + \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left[(58.7\,ft/s)^2 - (88.0\,ft/s)^2\right]$$

or

$p_2 - p_1 = \underline{\underline{-5.12\,\frac{lb}{ft^2}}}$ This negative pressure tends <u>to lift the roof.</u>

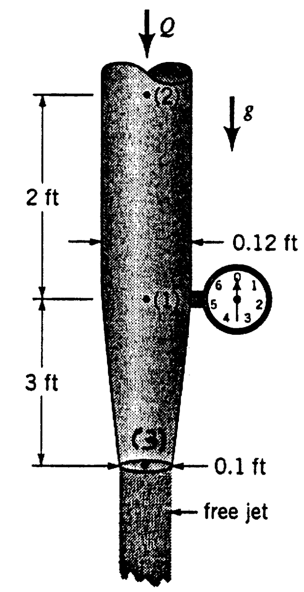
b) From (1) to (3): Since $V_3 = 0$,

$p_3 = p_1 + \frac{1}{2}\rho V_1^2$ or

$$p_3 - p_1 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)(58.7\,ft/s)^2 = \underline{\underline{4.10\,\frac{lb}{ft^2}}}$$

3.25

3.25 Water flows steadily downward through the pipe shown in Fig. P3.25. Viscous effects are negligible, and the pressure gage indicates the pressure is zero at point (1). Determine the flowrate and the pressure at point (2).



■ FIGURE P3.25

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where $z_1 = 3\text{ ft}$, $z_3 = 0$, $p_1 = p_3 = 0$

and

$$V_1 = \frac{A_3}{A_1} V_3 = \left(\frac{\frac{\pi}{4}(0.1\text{ ft})^2}{\frac{\pi}{4}(0.12\text{ ft})^2} \right) V_3 = 0.694\, V_3$$

Thus,

$$\frac{(0.694)^2 V_3^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft} = \frac{V_3^2}{2(32.2\text{ ft/s}^2)} \quad \text{or} \quad V_3 = 19.3\,\frac{\text{ft}}{\text{s}}$$

so that

$$Q_3 = A_3 V_3 = \frac{\pi}{4}(0.1\text{ ft})^2 (19.3\,\frac{\text{ft}}{\text{s}}) = \underline{\underline{0.152\,\frac{\text{ft}^3}{\text{s}}}}$$

Also,

$$\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}$$

where $p_1 = 0$ and since $A_1 = A_2$ it follows that $V_2 = V_1$

Thus,

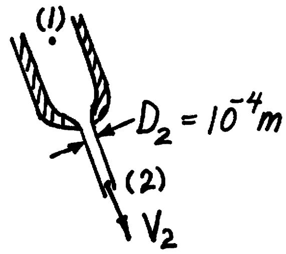
$$z_2 - z_1 = -\frac{p_2}{\gamma} \quad \text{or} \quad \frac{p_2}{\gamma} = -2\text{ ft}$$

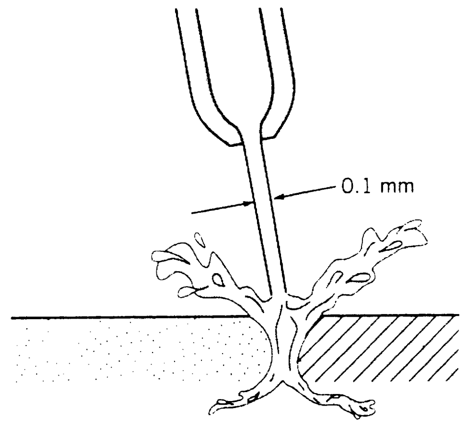
or

$$p_2 = -2\text{ ft}\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right) = \underline{\underline{-125\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.26

3.26 Small-diameter, high-pressure liquid jets can be used to cut various materials as shown in Fig. P3.26. If viscous effects are negligible, estimate the pressure needed to produce a 0.10-mm-diameter water jet with a speed of 700 m/s. Determine the flowrate.





■ FIGURE P3.26

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } V_1 \approx 0,\ z_1 \approx z_2, \text{ and } p_2 = 0$$

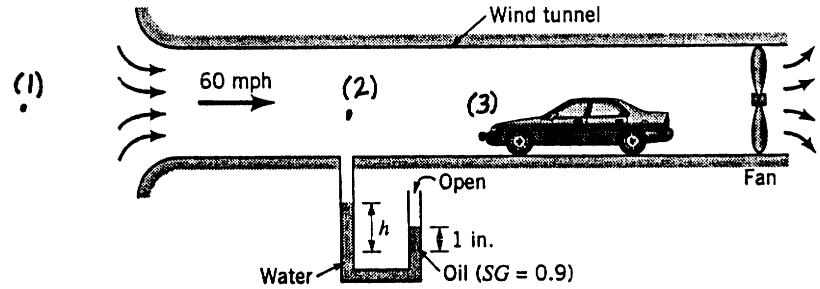
Thus $p_1 = \frac{1}{2}\frac{\gamma}{g}V_2^2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(999\,\frac{kg}{m^3})(700\,\frac{m}{s})^2 = \underline{\underline{2.45 \times 10^5\,\frac{kN}{m^2}}}$

Also,

$$Q = V_2 A_2 = 700\,\frac{m}{s}\left[\frac{\pi}{4}(10^{-4}m)^2\right] = \underline{\underline{5.50 \times 10^{-6}\,\frac{m^3}{s}}}$$

3.27

3.27 Air is drawn into a wind tunnel used for testing automobiles as shown in Fig. P3.27. (a) Determine the manometer reading, h, when the velocity in the test section is 60 mph. Note that there is a 1-in. column of oil on the water in the manometer. (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.



■ FIGURE P3.27

(a) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where

$z_1 = z_2$, $p_1 = 0$, and $V_1 \approx 0$

Thus, with $V_2 = 60\,mph = 88\,\frac{ft}{s}$,

$\frac{p_2}{\gamma} = -\frac{V_2^2}{2g}$ or

$p_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(88\,\frac{ft}{s})^2 = -9.22\,\frac{lb}{ft^2}$

But $p_2 + \gamma_{H_2O}h - \gamma_{oil}(\frac{1}{12}ft) = 0$ where $\gamma_{oil} = 0.9\,\gamma_{H_2O} = 0.9(62.4\,\frac{lb}{ft^3})$
$= 56.2\,\frac{lb}{ft^3}$

Thus,

$-9.22\,\frac{lb}{ft^2} + 62.4\,\frac{lb}{ft^3}(h\,ft) - 56.2\,\frac{lb}{ft^3}(\frac{1}{12}ft) = 0$, or $h = \underline{\underline{0.223\,ft}}$

(b) $\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$

where

$z_2 = z_3$ and $V_3 = 0$

Thus,

$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_3}{\gamma}$ or

$p_3 - p_2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(88\,\frac{ft}{s})^2 = \underline{\underline{9.22\,\frac{lb}{ft^2}}}$

3.28

3.28 A loon is a diving bird equally at home "flying" in the air or water. What swimming velocity under water will produce a dynamic pressure equal to that when it flies in the air at 40 mph?

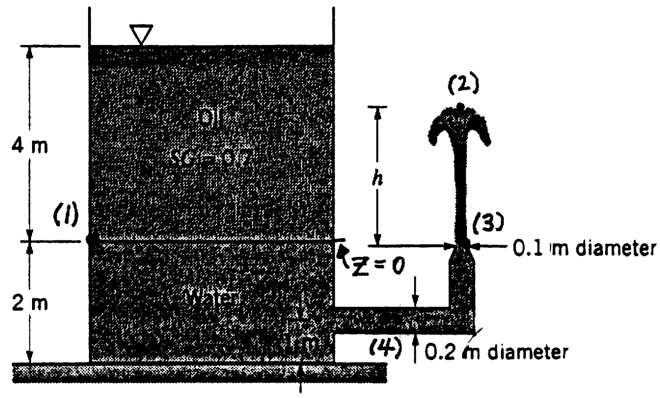
$$\frac{1}{2}\rho_{air}V_{air}^{2} = \frac{1}{2}\rho_{H_2O}V_{H_2O}^{2} \quad \text{or} \quad V_{H_2O} = \left[\frac{\rho_{air}}{\rho_{H_2O}}\right]^{\frac{1}{2}} V_{air}$$

Thus,

$$V_{H_2O} = \left[\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right](40\ mph) = \underline{\underline{1.40\ mph}}$$

3.29

3.29 A large open tank contains a layer of oil floating on water as shown in Fig. P3.29. The flow is steady and inviscid. (a) Determine the height, h, to which the water will rise. (b) Determine the water velocity in the pipe. (c) Determine the pressure in the horizontal pipe.



■ FIGURE P3.29

(a) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$ (1)

where

$z_1 = 0$, $p_2 = 0$, $V_1 = V_2 = 0$, $z_2 = h$, and $p_1 = 4m\,(\gamma_{oil})$

Thus, with $\gamma_{oil} = SG\,\gamma_{H_2O} = 0.7\left(9.80\,\frac{kN}{m^3}\right) = 6.86\,\frac{kN}{m^3}$

and from Eq. (1)

$\frac{p_1}{\gamma} = z_2$ or $p_1 = \gamma h$ so that

$$h = \frac{4m\,\gamma_{oil}}{\gamma} = 4m\,\frac{6.86\,\frac{kN}{m^3}}{9.80\,\frac{kN}{m^3}} = \underline{\underline{2.80\,m}}$$

(b) $V_4 A_4 = V_3 A_3$ or $V_4 = \frac{A_3}{A_4} V_3 = \frac{\frac{\pi}{4}(0.1m)^2}{\frac{\pi}{4}(0.2m)^2} V_3 = \frac{1}{4} V_3$

But from the Bernoulli equation,

$$V_3 = \sqrt{2gh} = \sqrt{2(9.81\,m/s^2)(2.80m)} = 7.41\,\frac{m}{s}$$

Thus,

$$V_4 = \frac{1}{4}\left(7.41\,\frac{m}{s}\right) = \underline{\underline{1.85\,\frac{m}{s}}}$$

(c) $\frac{p_4}{\gamma} + z_4 + \frac{V_4^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$

where

$z_4 = -1m$, $V_4 = 1.85\,\frac{m}{s}$, $p_2 = 0$, $z_2 = 2.8m$, $V_2 = 0$

Thus,

$$\frac{p_4}{\gamma} - 1m + \frac{\left(1.85\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} = 2.8m \quad \text{or} \quad \frac{p_4}{\gamma} = 3.63\,m$$

Thus,

$$p_4 = 3.63\,m\left(9.80\,\frac{kN}{m^3}\right) = \underline{\underline{35.5\,kPa}}$$

3.30

3.30 Water flows through the pipe contraction shown in Fig. P3.30. For the given 0.2-m difference in manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

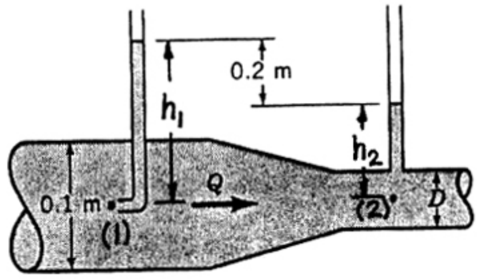


FIGURE P3.30

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{or with } z_1 = z_2 \text{ and } V_1 = 0$$

$$V_2 = \sqrt{2g\,\frac{(p_1 - p_2)}{\gamma}}$$

but $p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$V_2 = \sqrt{2g\,\frac{0.2\gamma}{\gamma}} = \sqrt{2g(0.2)}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4} D^2 V_2 = \frac{\pi}{4} D^2 \sqrt{2(9.81)(0.2)} = \underline{\underline{1.56\, D^2 \frac{m^3}{s}}} \text{ when } D \sim m$$

3.31 Water flows through the pipe contraction shown in Fig. P3.31. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

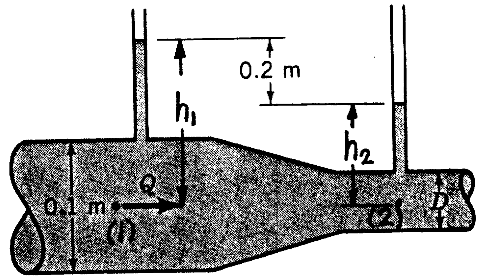


FIGURE P3.31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{with } A_1V_1 = A_2V_2$$

$$\text{or} \quad V_2 = \frac{\left(\frac{\pi}{4}D_1^2\right)}{\left(\frac{\pi}{4}D_2^2\right)} V_1 = \left(\frac{0.1}{D}\right)^2 V_1$$

Thus, with $z_1 = z_2$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

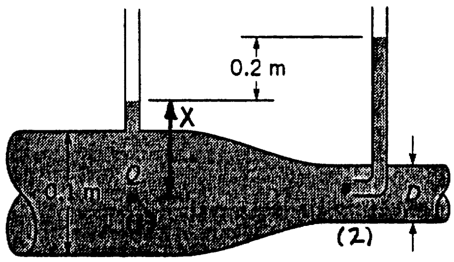
$$Q = A_1V_1 = \frac{\pi}{4}(0.1)^2\sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

or

$$Q = \frac{0.0156\,D^2}{\sqrt{(0.1)^4 - D^4}}\ \frac{m^3}{s} \quad \text{when } D \sim m$$

3.32

3.32 Water flows through the pipe contraction shown in Fig. P3.32. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.



■ FIGURE P3.32

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and $V_2 = 0$.

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$

But

$\frac{p_1}{\gamma} = x$ and $\frac{p_2}{\gamma} = 0.2\,m + x$ so that

$$x + \frac{V_1^2}{2g} = 0.2\,m + x \quad \text{or}$$

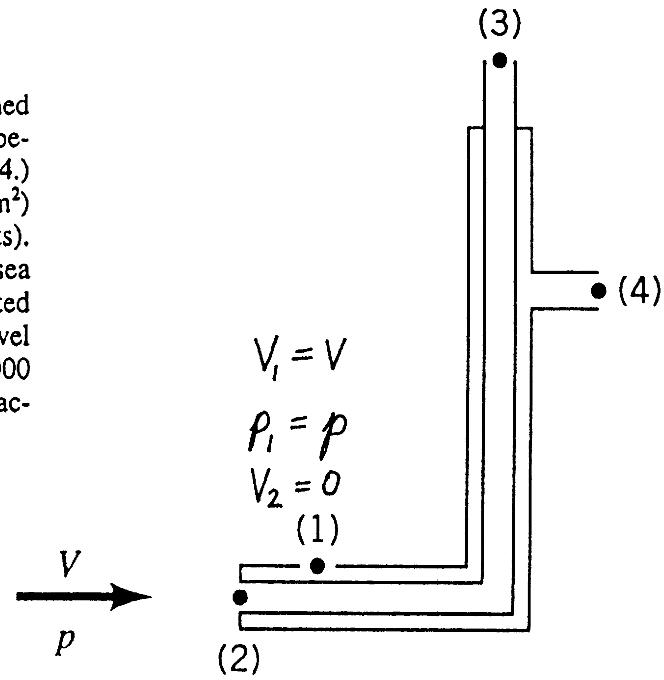
$$V_1 = \sqrt{2g(0.2\,m)} = \left(2\left(9.81\,\frac{m}{s^2}\right)(0.2\,m)\right)^{1/2} = 1.98\,\frac{m}{s}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.1\,m)^2\left(1.98\,\frac{m}{s}\right) = \underline{\underline{0.0156\,\frac{m^3}{s} \text{ for any } D}}$$

3.33

3.33 The speed of an airplane through the air is obtained by use of a Pitot-static tube that measures the difference between the stagnation and static pressures. (See Video V3.4.) Rather than indicating this pressure difference (psi or N/m^2) directly, the indicator is calibrated in speed (mph or knots). This calibration is done using the density of standard sea level air. Thus, the air speed displayed (termed the indicated air speed) is the actual air speed only at standard sea level conditions. If the aircraft is flying at an altitude of 20,000 ft and the indicated air speed is 220 knots, what is the actual air speed?



For the Pitot-static tube shown

$V = \sqrt{2(p_3 - p_4)/\rho}$. Thus, $p_3 - p_4 = \frac{1}{2}\rho V^2$ so that with the same indicated airspeed $(p_3 - p_4)_{standard} = (p_3 - p_4)_{20,000}$ or

$\frac{1}{2}\rho_{standard} V_{standard}^2 = \frac{1}{2}\rho_{20,000} V_{20,000}^2$. Hence,

$$V_{20,000} = V_{standard}\left[\frac{\rho_{standard}}{\rho_{20,000}}\right]^{\frac{1}{2}} = 220\,\text{knots}\left[\frac{0.00238\,\frac{slugs}{ft^3}}{0.001267\,\frac{slugs}{ft^3}}\right]^{\frac{1}{2}}$$

or

$$V_{20,000} = \underline{\underline{302\text{ knots}}}$$

3.34

3.34 Streams of water from two tanks impinge upon each other as shown in Fig. P3.34. If viscous effects are negligible and point A is a stagnation point, determine the height h.

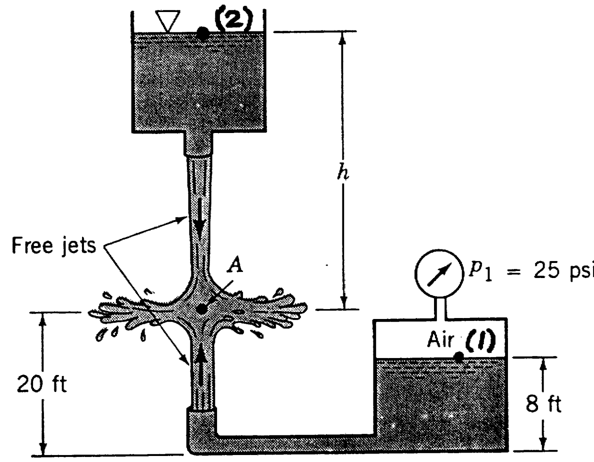


FIGURE P3.34

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A$$ where $p_2=0$, $V_2=0$, $z_2=h+20\,ft$, $V_A=0$, and $z_A=20\,ft$

Thus,

$$h+20\,ft=\frac{p_A}{\gamma}+20\,ft$$

or

$$h=\frac{p_A}{\gamma} \qquad (1)$$

Also,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A$$ where $p_1=25\,psi$, $V_1=0$ and $z_1=8\,ft$

Thus,

$\frac{p_A}{\gamma}=\frac{p_1}{\gamma}+z_1-z_A$ which when combined with Eq. (1) gives

$$h=\frac{p_1}{\gamma}+z_1-z_A=\frac{25\,\frac{lb}{in^2}\left(144\,\frac{in^2}{ft^2}\right)}{62.4\,\frac{lb}{ft^3}}+8\,ft-20\,ft=\underline{\underline{45.7\,ft}}$$

3.35

3.35 A 0.15-m-diameter pipe discharges into a 0.10-m-diameter pipe. Determine the velocity head in each pipe if they are carrying 0.12 m^3/s of kerosene.

$$V_1 = \frac{Q}{A_1} = \frac{0.12 \frac{m^3}{s}}{\frac{\pi}{4}(0.15m)^2} = 6.79 \frac{m}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{0.12 \frac{m^3}{s}}{\frac{\pi}{4}(0.10m)^2} = 15.27 \frac{m}{s}$$

Thus,

$$\frac{V_1^2}{2g} = \frac{(6.79 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{2.35\ m}}$$

and

$$\frac{V_2^2}{2g} = \frac{(15.27 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{11.9\ m}}$$

3.36

3.36 Water flows upward through a variable area pipe with a constant flowrate, Q, as shown in Fig. P3.36. If viscous effects are negligible, determine the diameter, $D(z)$, in terms of D_1 if the pressure is to remain constant throughout the pipe. That is, $p(z) = p_1$.

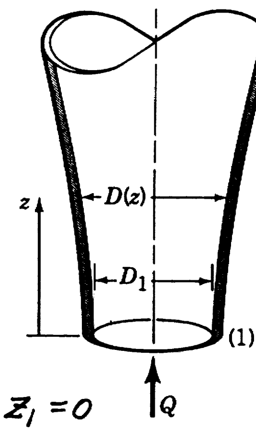


FIGURE P3.36

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or with } p = p_1 \text{ and } z_1 = 0$$

$$\frac{V^2}{2g} + z = \frac{V_1^2}{2g}, \text{ or } V_1^2 - V^2 = 2gz$$

but $V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2}$ and $V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

Thus,

$$\left(\frac{4Q}{\pi D_1^2}\right)^2 - \left(\frac{4Q}{\pi D^2}\right)^2 = 2gz$$

or

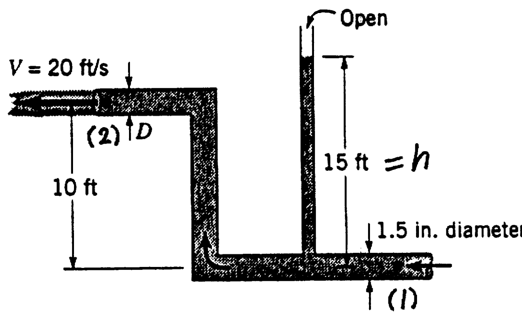
$$\frac{1}{D_1^4} - \frac{1}{D^4} = \frac{\pi^2 g z}{8\,Q^2}$$

Thus,

$$D = \frac{D_1}{\left[1 - \frac{\pi^2 g D_1^4 z}{8\,Q^2}\right]^{1/4}}$$

3.37

3.37 Water flows steadily with negligible viscous effects through the pipe shown in Fig. P3.37. Determine the diameter, D, of the pipe at the outlet (a free jet) if the velocity there is 20 ft/s.



■ FIGURE P3.37

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$z_1 = 0$, $z_2 = 10\,ft$, $V_2 = 20\,\frac{ft}{s}$, and $p_1 = \gamma h$ so that $\frac{p_1}{\gamma} = 15\,ft$

Thus,

$$15\,ft + 0 + \frac{V_1^2}{2(32.2\,\frac{ft}{s^2})} = 0 + 10\,ft + \frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$V_1 = 8.83\,\frac{ft}{s}$$

But $A_1V_1 = A_2V_2$ so that

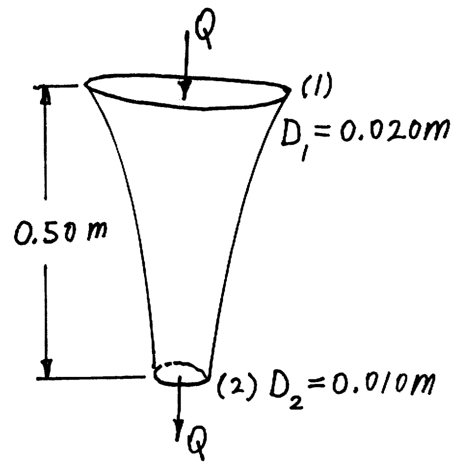
$$\frac{\pi}{4}D_1^2\left(8.83\,\frac{ft}{s}\right) = \frac{\pi}{4}D_2^2\left(20\,\frac{ft}{s}\right)$$

or

$$D_2 = \left(\frac{8.83}{20}\right)^{1/2}\left(\frac{1.5}{12}\,ft\right) = 0.0831\,ft = \underline{\underline{0.997\,in.}}$$

3.38

3.38 The circular stream of water from a faucet is observed to taper from a diameter of 20 mm to 10 mm in a distance of 50 cm. Determine the flowrate.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$, $z_2 = 0$, $z_1 = 0.50m$

and

$$V_1 = \frac{Q}{A_1} \quad , \quad V_2 = \frac{Q}{A_2}$$

Thus,

$$\left(\frac{Q}{A_1}\right)^2 + 2gz_1 = \left(\frac{Q}{A_2}\right)^2 \text{ or } Q = \left[\frac{2gz_1}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}\right]^{\frac{1}{2}} = \frac{A_2\sqrt{2gz_1}}{\sqrt{1-(A_2/A_1)^2}}$$

or since

$$\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 \text{ we obtain}$$

$$Q = A_2 \frac{\sqrt{2gz_1}}{\sqrt{1-(D_2/D_1)^4}} = \frac{\pi}{4}(0.010\,m)^2\left[\frac{2(9.81\frac{m}{s^2})(0.50m)}{1-\left(\frac{0.010}{0.020}\right)^4}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{2.54 \times 10^{-4} \frac{m^3}{s}}}$$

3.39

3.39 Water is siphoned from the tank shown in Fig. P3.39. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.

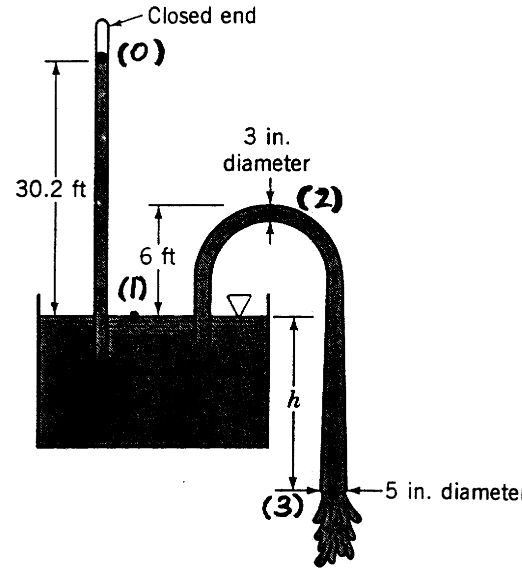


FIGURE P3.39

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ V_1 = 0,\ p_2 = p_{vapor}$$

Thus, $\quad z_1 = 0,\ z_2 = 6\ ft$

$$0 = \frac{p_{vapor}}{\gamma} + \frac{V_2^2}{2g} + 6\ ft$$

but $p_0 + 30.2\ ft\ \gamma = p_1$ or since $p_0 = p_{vapor}$, $\frac{p_{vapor}}{\gamma} = -30.2\ ft$

Hence,

$$0 = -30.2\ ft + \frac{V_2^2}{2g} + 6\ ft \quad \text{or} \quad \frac{V_2^2}{2g} = 24.2\ ft \quad \text{or} \quad V_2^2 = \left[2\left(32.2\frac{ft}{s^2}\right)(24.2\ ft)\right]$$

Thus,

$$V_2 = 39.5\ \frac{ft}{s}$$

Since $V_3 A_3 = V_2 A_2$, $\quad V_3 = \frac{A_2}{A_3} V_2 = \frac{D_2^2}{D_3^2} V_2 = \left(\frac{3\ in.}{5\ in.}\right)^2 \left(39.5\ \frac{ft}{s}\right)$

or

$$V_3 = 14.2\ \frac{ft}{s}$$

However,

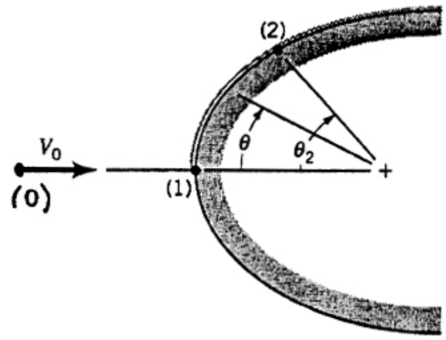
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{or} \quad V_3 = \sqrt{2gh}$$

Thus,

$$14.2\ \frac{ft}{s} = \sqrt{2\left(32.2\frac{ft}{s^2}\right) h\ ft} \quad \text{or} \quad \underline{\underline{h = 3.13\ ft}}$$

3.40

3.40 An inviscid fluid flows steadily along the stagnation streamline shown in Fig. P3.40 and Video V3.3, starting with speed V_0 far upstream of the object. Upon leaving the stagnation point, point (1), the fluid speed along the surface of the object is assumed to be given by $V = 2\,V_0 \sin\theta$, where θ is the angle indicated. At what angular position, θ_2, should a hole be drilled to give a pressure difference of $p_1 - p_2 = \rho V_0^2/2$? Gravity is negligible.



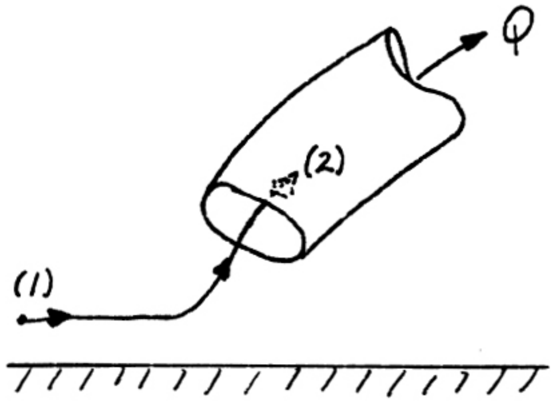
■ FIGURE P3.40

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$
where $V_1 = 0$

Thus,
$$p_1 - p_2 = \tfrac{1}{2}\rho\,(V_2^2 - V_1^2) = \tfrac{1}{2}\rho V_2^2$$
so that if
$$p_1 - p_2 = \tfrac{1}{2}\rho V_0^2 \quad \text{then } V_2 = V_0$$
That is:
$$V_2 = 2\,V_0 \sin\theta_2 = V_0 \quad \text{or} \quad \sin\theta_2 = \tfrac{1}{2}$$
Hence, $\theta_2 = \underline{\underline{30^\circ}}$

3.41

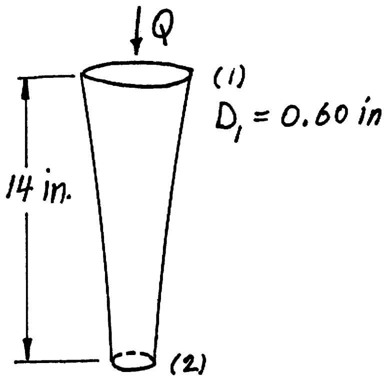
3.41 A cetain vacuum cleaner can create a vacuum of 2 kPa just inside the hose. What is the velocity of the air inside the hose?



$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$
where $p_1 = 0$, $V_1 \approx 0$ so that
$$p_2 = -\tfrac{1}{2}\rho V_2^2$$
Hence,
$$-2\times 10^3\,\frac{N}{m^2} = -\tfrac{1}{2}\left(1.23\,\frac{kg}{m^3}\right) V_2^2 \quad \text{or} \quad V_2 = \underline{\underline{57.0\,\frac{m}{s}}}$$

3.42

3.42 Water from a faucet fills a 16-oz glass (volume = 28.9 in.^{3}) in 10 s. If the diameter of the jet leaving the faucet is 0.60 in., what is the diameter of the jet when it strikes the water surface in the glass which is positioned 14 in. below the faucet?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $p_1 = p_2 = 0$, $z_1 = 14\text{ in.}$, $z_2 = 0$

Thus,

$$V_2 = \sqrt{2g\left(z_1 + \frac{V_1^2}{2g}\right)} \quad \text{where } V_1 = \frac{Q}{A_1} = \frac{\mathcal{V}}{A_1 t}$$

or

$$V_1 = \frac{(28.9\text{ in.}^3)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3}{\frac{\pi}{4}\left(\frac{0.60}{12}\right)^2\text{ft}^2(10\text{s})} = 0.852\ \frac{\text{ft}}{\text{s}}$$

Hence,

$$V_2 = \sqrt{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{14}{12}\text{ ft} + \frac{(0.852\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}\right)} = 8.71\ \frac{\text{ft}}{\text{s}}$$

But,

$$A_1 V_1 = A_2 V_2 \quad \text{so that} \quad D_1^2 V_1 = D_2^2 V_2$$

or

$$D_2 = \left(\frac{V_1}{V_2}\right)^{\frac{1}{2}} D_1 = \left(\frac{0.852\frac{\text{ft}}{\text{s}}}{8.71\frac{\text{ft}}{\text{s}}}\right)^{\frac{1}{2}}(0.60\text{ in.}) = \underline{\underline{0.188\text{ in.}}}$$

3.43

3.43 A smooth plastic, 10-m-long garden hose with an inside diameter of 20 mm is used to drain a wading pool as is shown in Fig. P3.43. If viscous effects are neglected, what is the flowrate from the pool?

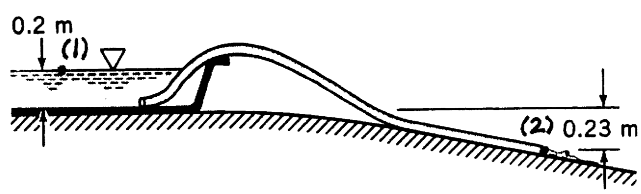


FIGURE P3.43

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$, $z_1 = 0.2\text{m}$, $z_2 = -0.23\text{m}$, and $V_1 = 0$

Thus,

$$V_2 = \sqrt{2g(z_1 - z_2)} = \left(2\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.2\text{ m} - (-0.23\text{m}))\right)^{\frac{1}{2}}$$

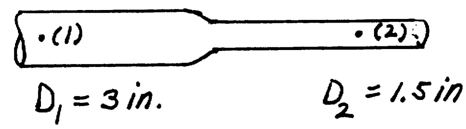
$$= 2.90\frac{\text{m}}{\text{s}}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4}(0.020\text{m})^2\left(2.90\frac{\text{m}}{\text{s}}\right) = \underline{\underline{9.11 \times 10^{-4}\frac{\text{m}^3}{\text{s}}}}$$

3.44

3.44 Carbon dioxide flows at a rate of 1.5 ft³/s from a 3-in. pipe in which the pressure and temperature are 20 psi (gage) and 120 °F into a 1.5-in. pipe. If viscous effects are neglected and incompressible conditions are assumed, determine the pressure in the smaller pipe.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and

$$V_1 = \frac{Q}{A_1} = \frac{1.5\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2 ft^2} = 30.6\ \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{1.5\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{1.5}{12}\right)^2 ft^2} = 122\ \frac{ft}{s}$$

Thus,

$$p_2 = p_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) \quad \text{where } \rho = \frac{p}{RT} = \frac{(20+14.7)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{\left(1130\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(460+120)\,{}^\circ R}$$

$$= 7.62 \times 10^{-3}\ \frac{slug}{ft^3}$$

or

$$p_2 = 20\ \frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right) + \frac{1}{2}\left(7.62 \times 10^{-3}\ \frac{slugs}{ft^3}\right)\left[\left(30.6\ \frac{ft}{s}\right)^2 - \left(122\ \frac{ft}{s}\right)^2\right]$$

$$= 2880\ \frac{lb}{ft^2} - 53.1\ \frac{lb}{ft^2} = 2{,}827\ \frac{lb}{ft^2}$$

or

$$p_2 = \underline{\underline{19.63\ psi\ gage}}$$

3.45

3.45 Oil of specific gravity 0.83 flows in the pipe shown in Fig. P3.45 If viscous effects are neglected, what is the flowrate?

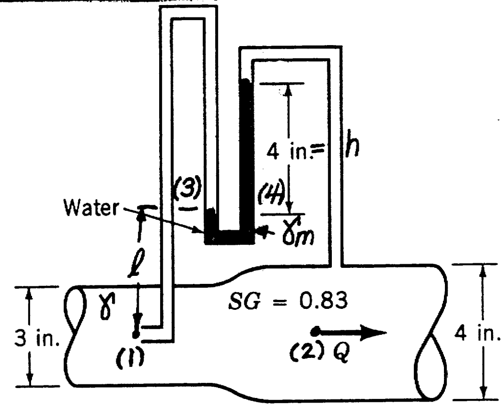


FIGURE P3.45

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_1 = 0$$

Thus,

$$\frac{V_2^2}{2g} = \frac{p_1 - p_2}{\gamma} \qquad (1)$$

but,

$$p_1 = p_3 + \gamma \ell = p_4 + \gamma \ell$$

and

$$p_2 = \gamma(\ell + h) - \gamma_m h + p_4$$

Thus,

$$p_1 - p_2 = (\gamma_m - \gamma)h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$V_2 = \sqrt{2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{2g\left(\frac{\gamma_m}{\gamma} - 1\right)h} = \sqrt{2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{62.4\,\frac{lb}{ft^3}}{0.83\left(62.4\,\frac{lb}{ft^3}\right)} - 1\right)\left(\frac{4}{12}\,ft\right)}$$

or

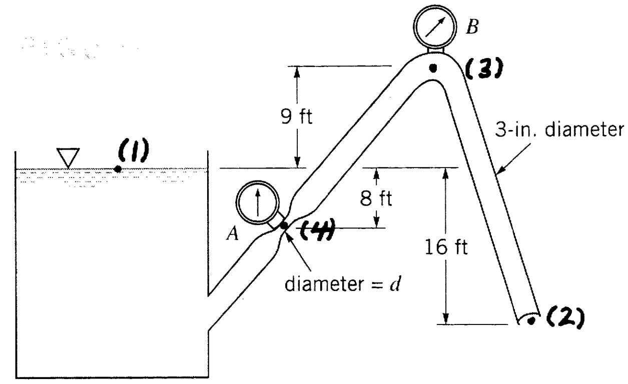
$$V_2 = 2.10\,\frac{ft}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{4}{12}\,ft\right)^2\left(2.10\,\frac{ft}{s}\right) = \underline{\underline{0.183\,\frac{ft^3}{s}}}$$

3.46

3.46 Water flows steadily from a large open tank and discharges into the atmosphere through a 3-in.-diameter pipe as shown in Fig. P3.46. Determine the diameter, d, in the narrowed section of the pipe at A if the pressure gages at A and B indicate the same pressure.



$$p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{ , where } z_2 = 0 \text{ and } p_2 = 0$$

Thus, since $p_3 = p_4$

$$p_3 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = \frac{1}{2}\rho V_2^2 \qquad (1)$$

However, $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$, where $p_1 = p_2 = V_1 = z_2 = 0$

so that

$$\frac{1}{2}\rho V_2^2 = \gamma z_1 \text{ or } V_2 = \sqrt{2\frac{\gamma}{\rho}z_1} = \sqrt{2 g z_1} = \left[2(32.2\tfrac{ft}{s^2})(16\,ft)\right]^{\frac{1}{2}} = 32.1\,ft/s$$

But

$$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{ where } V_2 = V_3 \text{ since } A_2 = A_3$$

Thus,

$$p_3 = -\gamma z_3 = -(16+9)ft(62.4\,lb/ft^3) = -1560\,lb/ft^2 \qquad (2)$$

From Eqs. (1) and (2):

$$-1560\,\frac{lb}{ft^2} + \frac{1}{2}(1.94\,\frac{slugs}{ft^3})V_4^2 = \frac{1}{2}(1.94\,\frac{slugs}{ft^3})(32.1\,\frac{ft}{s})^2$$

or

$$V_4 = 46.1\,ft/s$$

Since $A_4 V_4 = A_2 V_2$ it follows that

$$\frac{\pi}{4}d^2 V_4 = \frac{\pi}{4}D_2^2 V_2$$

or

$$d = D_2\sqrt{\frac{V_2}{V_4}} = (3\,in.)\sqrt{\frac{32.1\,ft/s}{46.1\,ft/s}} = \underline{\underline{2.50\,in.}}$$

3.47

3.47 Determine the flowrate through the pipe in Fig. P3.47.

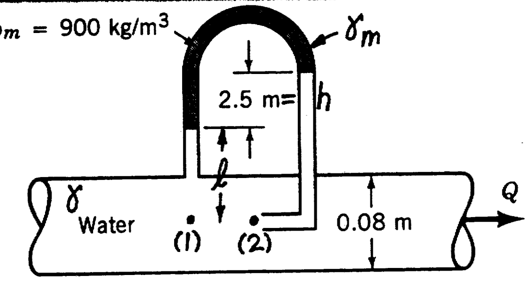


FIGURE P3.47

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2 \text{ and } V_2=0$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma} \quad \text{or} \quad V_1=\sqrt{2g\frac{(p_2-p_1)}{\gamma}}$$

but,

$$p_1-\gamma\ell-\gamma_m h+\gamma(\ell+h)=p_2 \quad \text{or} \quad p_2-p_1=(\gamma-\gamma_m)h$$

so that

$$V_1=\sqrt{2g\left(1-\frac{\gamma_m}{\gamma}\right)h}=\left[2\left(9.81\frac{m}{s^2}\right)\left(1-\frac{900\ \frac{kg}{m^3}}{999\ \frac{kg}{m^3}}\right)(2.5\ m)\right]^{1/2}$$

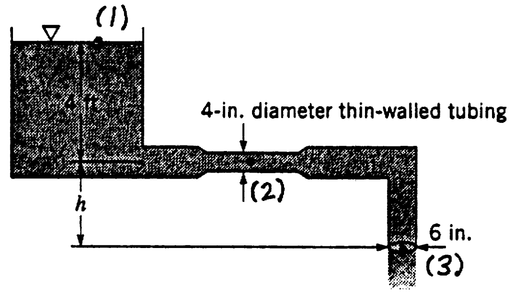
$$=2.20\ \frac{m}{s}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}(0.08\ m)^2\left(2.20\ \frac{m}{s}\right)=\underline{\underline{0.0111\ \frac{m^3}{s}}}$$

3.48

3.48 Water flows steadily with negligible viscous effects through the pipe shown in Fig. P3.48. It is known that the 4-in. diameter section of thin-walled tubing will collapse if the pressure within it becomes less than 10 psi below atmospheric pressure. Determine the maximum value that h can have without causing collapse of the tubing.



■ FIGURE P3.48

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

where

$p_1=0$, $V_1=0$, $z_2=0$, and $p_2=-10\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)=-1440\frac{lb}{ft^2}$

Thus, with $z_1=4\,ft$,

$$4\,ft=\frac{-1440\,lb/ft^2}{62.4\,lb/ft^3}+\frac{V_2^2}{2(32.2\,ft/s^2)}$$

or $V_2=41.7\frac{ft}{s}$

Also,

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_3}{\gamma}+z_3+\frac{V_3^2}{2g}$$

where

$p_3=0$, $z_3=-h$, and $V_3=\frac{A_2V_2}{A_3}=\left(\frac{D_2}{D_3}\right)^2V_2=\left(\frac{4\,in.}{6\,in.}\right)^2\left(41.7\frac{ft}{s}\right)$

$=18.5\frac{ft}{s}$

Thus,

$$4\,ft=-h+\frac{(18.5\,ft/s)^2}{2(32.2\,ft/s^2)}$$

or

$h=\underline{\underline{1.31\,ft}}$

3.49

3.49 For the pipe enlargement shown in Fig. P3.49, the pressures at sections (1) and (2) are 56.3 and 58.2 psi, respectively. Determine the weight flow rate (lb/s) of the gasoline in the pipe.

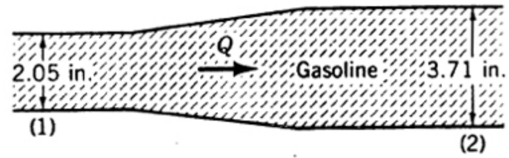


FIGURE P3.49

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $z_1=z_2$ and $A_1V_1=A_2V_2$

or $V_2=\left(\frac{D_1}{D_2}\right)^2 V_1$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{\left(\frac{D_1}{D_2}\right)^4 V_1^2}{2g}$$

or

$$V_1=\sqrt{\frac{2g(p_2-p_1)}{\gamma\left(1-\left(\frac{D_1}{D_2}\right)^4\right)}}=\left[\frac{2\left(32.2\frac{ft}{s^2}\right)\left(58.2\frac{lb}{in.^2}-56.3\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{42.5\frac{lb}{ft^3}\left(1-\left(\frac{2.05\,in}{3.71\,in}\right)^4\right)}\right]^{1/2}$$

or

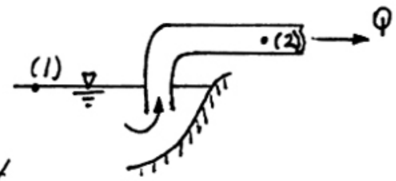
$V_1=21.4\frac{ft}{s}$ and $Q=A_1V_1=\frac{\pi}{4}\left(\frac{2.05}{12}ft\right)^2\left(21.4\frac{ft}{s}\right)=0.490\ \frac{ft^3}{s}$

Thus,

$$\gamma Q=42.5\frac{lb}{ft^3}\left(0.490\frac{ft^3}{s}\right)=\underline{\underline{20.8\ \frac{lb}{s}}}$$

3.50

3.50 Water is pumped from a lake through an 8-in. pipe at a rate of 10 ft³/s. If viscous effects are negligible, what is the pressure in the suction pipe (the pipe between the lake and the pump) at an elevation 6 ft above the lake?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $V_1 = 0$, $z_1 = 0$, $z_2 = 6.0\ ft$

and

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = \frac{4\left(10\ \frac{ft^3}{s}\right)}{\pi\left(\frac{8}{12}\ ft\right)^2} = 28.6\ \frac{ft}{s}$$

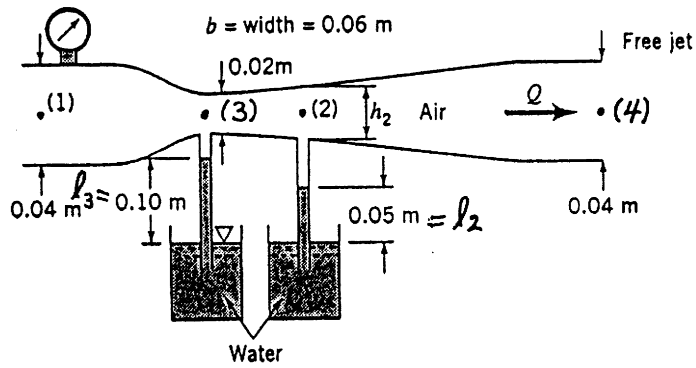
Thus,

$$p_2 = -\gamma z_2 - \frac{1}{2}\rho V_2^2 = -62.4\ \frac{lb}{ft^3}(6.0\ ft) - \frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)\left(28.6\ \frac{ft}{s}\right)^2$$

$$= -1168\ \frac{lb}{ft^2} = \underline{\underline{-8.11\ psi}}$$

3.51

3.51 Air flows through a Venturi channel of rectangular cross section as shown in Video V3.6 and Fig. P3.51 The constant width of the channel is 0.06 m and the height at the exit is 0.04 m. Compressibility and viscous effects are negligible. (a) Determine the flowrate when water is drawn up 0.10 m in a small tube attached to the static pressure tap at the throat where the channel height is 0.02 m. (b) Determine the channel height, h_2, at section (2) where, for the same flowrate as in part (a), the water is drawn up 0.05 m. (c) Determine the pressure needed at section (1) to produce this flow.



■ FIGURE P3.51

(a) For steady, inviscid, incompressible flow: $(\gamma = 12.0 \frac{N}{m^3})$

(1) $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_3 = -\gamma_{H_2O}\ell_3 = 9.80\times10^3\frac{N}{m^3}(0.1m)$
$= -980\frac{N}{m^2}$

Also,
$A_3V_3 = A_4V_4$ so that $V_3 = \frac{(0.04m \times 0.06m)}{(0.02m\times0.06m)}V_4 = 2V_4$

Thus, Eqn. (1) becomes

$$\frac{-980\frac{N}{m^2}}{12.0\frac{N}{m^3}} + \frac{4V_4^2}{2(9.81\frac{m}{s^2})} = \frac{V_4^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_4 = 23.1\frac{m}{s}$$

Hence,

$Q = A_4V_4 = (0.04m\times0.06m)(23.1\frac{m}{s}) = \underline{\underline{0.0554\frac{m^3}{s}}}$

(2) (b) $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_2 = -\gamma_{H_2O}\ell_2 = 9.80\times10^3\frac{N}{m^3}(0.05m)$
$= -490\frac{N}{m^2}$

From part (a), $V_4 = 23.1\frac{m}{s}$

Thus, Eqn. (2) becomes

$$\frac{-490\frac{N}{m^2}}{12.0\frac{N}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})} = \frac{(23.1\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_2 = 36.5\frac{m}{s}$$

But $V_2A_2 = V_4A_4$ so that

$(36.5\frac{m}{s})(0.06m)h_2 = (23.1\frac{m}{s})(0.06m)(0.04m)$ or $h_2 = \underline{\underline{0.0253\,m}}$

(3) (c) Also, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$ and $A_1V_1 = A_4V_4$

But since $A_1 = (0.04m\times0.06m) = A_4$ then $V_1 = V_4$ and Eqn. (3) gives

$p_1 = p_4 = \underline{\underline{0}}$

3.52

3.52 An inviscid, incompressible liquid flows steadily from the large pressurized tank shown in Fig. P.3.52. The velocity at the exit is 40 ft/s. Determine the specific gravity of the liquid in the tank.

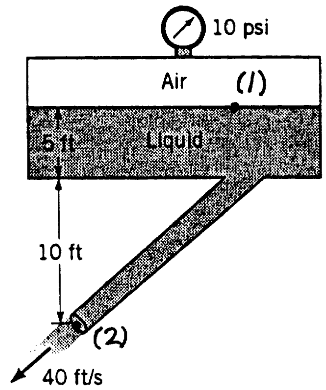


FIGURE P3.52

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = 10 \frac{lb}{in.^2}\left(144 \frac{in.^2}{ft^2}\right) = 1440 \frac{lb}{ft^2}$, $p_2 = 0$,

$z_1 = 15\,ft$, $z_2 = 0$, $V_1 = 0$, and $V_2 = 40 \frac{ft}{s}$

Thus,

$$\frac{1440\ lb/ft^2}{\gamma} + 15\,ft = \frac{(40\ ft/s)^2}{2(32.2\ ft/s^2)}$$

or

$$\gamma = 146.3 \frac{lb}{ft^3}$$

Hence,

$$SG = \frac{\gamma}{\gamma_{H_2O}} = \frac{146\ lb/ft^3}{62.4\ lb/ft^3} = \underline{\underline{2.34}}$$

3.53

3.53 Air (assumed frictionless and incompressible) flows steadily through the device shown in Fig. P3.53. The exit velocity is 100 ft/s, and the differential pressure across the nozzle is 6 lb/ft^2. (a) Determine the reading, H, for the water-filled manometer attached to the Pitot tube. (b) Determine the diameter, d, of the nozzle.

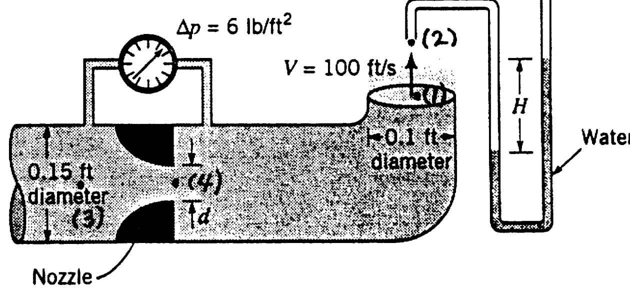


FIGURE P3.53

(a) $p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_2 + z_2 + \frac{1}{2}\rho V_2^2$

where

$z_1 \approx z_2$, $p_1 = 0$, and $V_2 = 0$

Thus,

$p_2 = \frac{1}{2}\rho V_1^2$, but $p_2 = \gamma_{H_2O} H$ so that

$(62.4 \frac{lb}{ft^3}) H = \frac{1}{2}(0.00238 \frac{slug}{ft^3})(100 \frac{ft}{s})^2$ or $H = \underline{\underline{0.191 \ ft}}$

(b) $p_3 + \frac{1}{2}\rho V_3^2 = p_4 + \frac{1}{2} V_4^2$

where $p_3 - p_4 = 6 \frac{lb}{ft^2}$

Also,

$Q_1 = Q_3 = Q_4$ where $Q_1 = A_1 V_1 = \frac{\pi}{4}(0.1 ft)^2 (100 \frac{ft}{s}) = 0.785 \frac{ft^3}{s}$

so that

$V_3 = \frac{Q_3}{A_3} = \frac{0.785 \frac{ft^3}{s}}{\frac{\pi}{4}(0.15 ft)^2} = 44.4 \frac{ft}{s}$

Hence,

$6 \frac{lb}{ft^2} + \frac{1}{2}(0.00238 \frac{slugs}{ft^3})(44.4 \frac{ft}{s})^2 = \frac{1}{2}(0.00238 \frac{slugs}{ft^3}) V_4^2$

or

$V_4 = 83.7 \frac{ft}{s}$

so that with $A_4 V_4 = Q_4 = Q_1$,

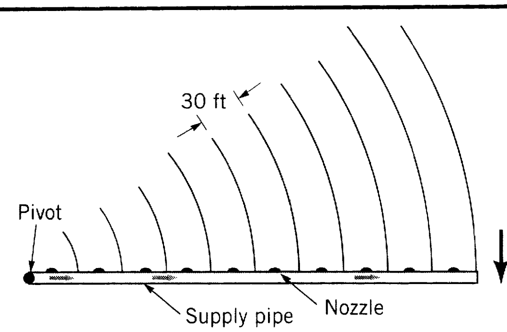
$\frac{\pi}{4} d^2 (83.7 \frac{ft}{s}) = 0.785$

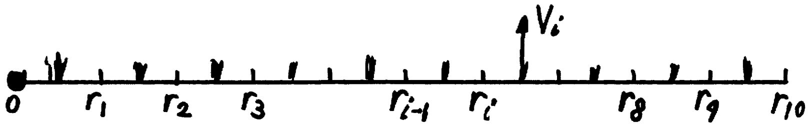
or

$d = \underline{\underline{0.109 \ ft}}$

3.54

3.54 The center pivot irrigation system shown in Fig. P3.54 is to provide uniform watering of the entire circular field. Water flows through the common supply pipe and out through 10 evenly spaced nozzles. Water from each nozzle is to cover a strip 30 feet wide as indicated. If viscous effects are negligible, determine the diameter of each nozzle, d_i, $i = 1$ to 10, in terms of the diameter, d_{10}, of the nozzle at the outer end of the arm.





In time t = time for one revolution of supply pipe about center pivot a uniform depth, h, of water is to be applied throughout. Thus,

Q_i = volume flowrate from nozzle i = $A_i h/t$ where (1)

A_i = area covered by i^{th} nozzle = $\pi(r_i^2 - r_{i-1}^2)$

But $V_i = \sqrt{\dfrac{2p_{sp}}{\rho}}$ where p_{sp} = pressure in the supply pipe = constant.

so that

$Q_i = A_{n_i} V_i = \frac{\pi}{4} d_i^2 V_i = \frac{\pi}{4} d_i^2 \sqrt{\dfrac{2p_{sp}}{\rho}}$ where d_i = diameter of i^{th} nozzle.

or

$$\frac{Q_i}{Q_{10}} = \frac{\frac{\pi}{4} d_i^2 \sqrt{\frac{2p_{sp}}{\rho}}}{\frac{\pi}{4} d_{10}^2 \sqrt{\frac{2p_{sp}}{\rho}}} = \left(\frac{d_i}{d_{10}}\right)^2 \qquad (2)$$

But, from Eq. (1):

$$\frac{Q_i}{Q_{10}} = \frac{A_i h/t}{A_{10} h/t} = \frac{A_i}{A_{10}} = \frac{(r_i^2 - r_{i-1}^2)}{(r_{10}^2 - r_9^2)} = \frac{(r_i^2 - r_{i-1}^2)}{(300^2 - 270^2)} \qquad (3)$$

Thus, from Eqs. (2) and (3):

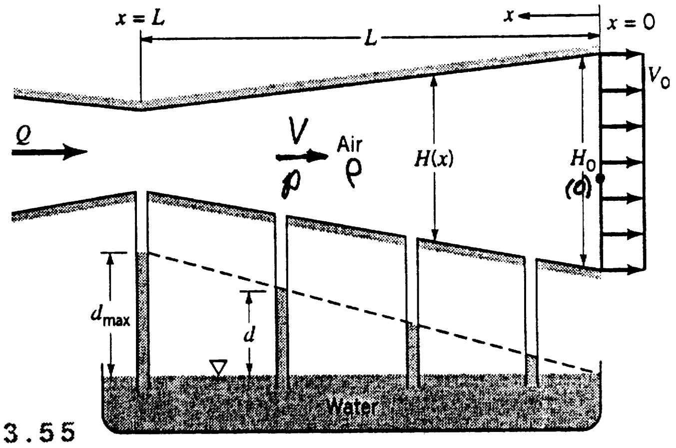
$$\underline{\underline{\frac{d_i}{d_{10}} = \left[\frac{r_i^2 - r_{i-1}^2}{17100}\right]^{1/2}}}$$

These results are given in the table.

i	r_i, ft	d_i/d_{10}
1	30	0.229
2	60	0.397
3	90	0.513
4	120	0.607
5	150	0.688
6	180	0.761
7	210	0.827
8	240	0.889
9	270	0.946
10	300	1.00

3.55

3.55 Air flows steadily through a converging–diverging rectangular channel of constant width as shown in Fig. P3.55 and Video V3.6. The height of the channel at the exit and the exit velocity are H_0 and V_0, respectively. The channel is to be shaped so that the distance, d, that water is drawn up into tubes attached to static pressure taps along the channel wall is linear with distance along the channel. That is, $d = (d_{max}/L)\,x$, where L is the channel length and d_{max} is the maximum water depth (at the minimum channel height; $x = L$). Determine the height, $H(x)$, as a function of x and the other important parameters.



■ FIGURE P3.55

$$p + z\gamma + \tfrac{1}{2}\rho V^2 = p_0 + z_0\gamma + \tfrac{1}{2}\rho V_0^2 \quad \text{where } \rho = \text{air density}$$

where

$$z = z_0, \quad p_0 = 0, \quad p = -\gamma_{H_2O}\, d = -\gamma_{H_2O}\frac{d_{max}}{L}x$$

Thus,

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho V_0^2$$

But

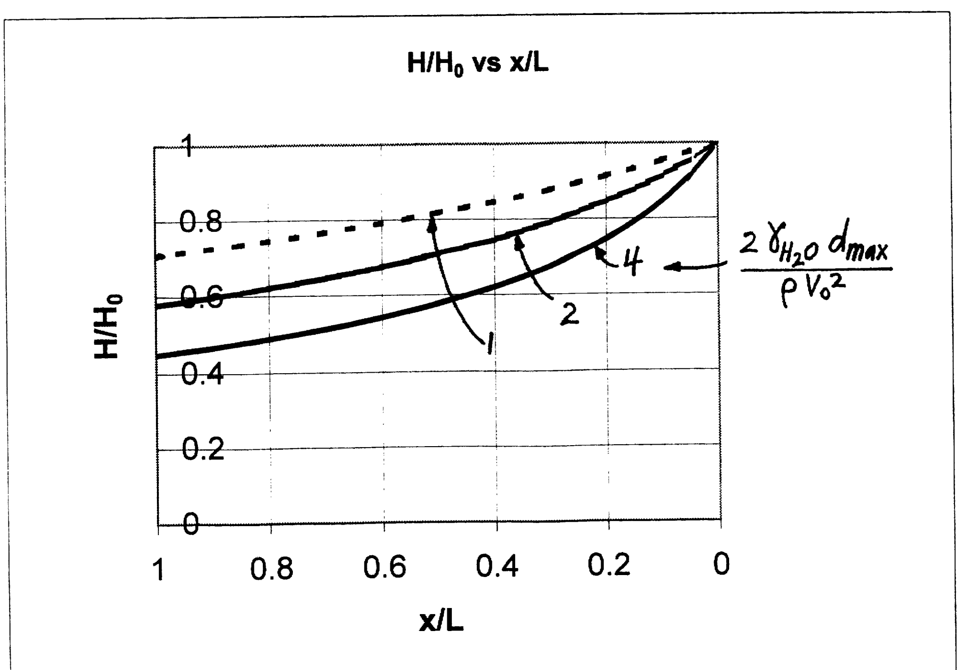
$$AV = A_0V_0, \quad \text{or} \quad V = \frac{A_0}{A}V_0 = \frac{H_0}{H}V_0 \quad \text{so that}$$

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho\left(\frac{H_0}{H}V_0\right)^2 = \tfrac{1}{2}\rho V_0^2$$

or

$$\frac{H}{H_0} = \frac{1}{\sqrt{1 + \left(\frac{2\gamma_{H_2O}d_{max}}{\rho V_0^2}\right)\frac{x}{L}}}$$

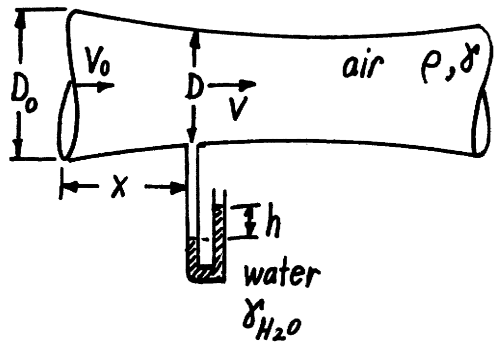
Typical shapes are shown below.



*3.56

*3.56 Air flows through a horizontal pipe of variable diameter, $D = D(x)$, at a rate of 1.5 ft^3/s. The static pressure distribution obtained from a set of 12 static pressure taps along the pipe wall is as shown below. Plot the pipe shape, $D(x)$, if the diameter at $x = 0$ is 1, 2, or 3 in. Neglect viscous and compressibility effects.

x (in.)	p (in. H_2O)	x (in.)	p (in. H_2O)
0	1.00		
1	0.72	7	0.44
2	0.16	8	0.51
3	−0.96	9	0.65
4	−0.31	10	0.78
5	0.27	11	0.90
6	0.39	12	1.00



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p}{\gamma} + \frac{V^2}{2g} + z \text{ , where } z_0 = z$$

Thus,

$$V = \sqrt{V_0^2 + \frac{2(p_0 - p)}{\rho}} \quad \text{with } V_0 = \frac{Q}{A_0} = \frac{1.5 \frac{ft^3}{s}}{\frac{\pi}{4} D_0^2} = \frac{1.91}{D_0^2} \frac{ft}{s} \text{, where } D_0 \sim ft$$

and

$$p_0 - p = \gamma_{H_2O}(h_0 - h) = \frac{62.4 \frac{lb}{ft^3}}{12 \frac{in.}{ft}}(1 in. - h) = 5.20(1-h) \frac{lb}{ft^2} \text{, with } h \sim in.$$

Hence, with $\rho = 2.38 \times 10^{-3} \frac{slug}{ft^3}$ we obtain

$$V = \left[\left(\frac{1.91}{D_0^2}\right)^2 + \frac{10.4(1-h)}{2.38 \times 10^{-3}}\right]^{\frac{1}{2}} = \left[\frac{3.65}{D_0^4} + 4370(1-h)\right]^{\frac{1}{2}} \qquad (1)$$

Also, $AV = Q$ or $\frac{\pi}{4}D^2 V = Q$ so that

$$D = \left[\frac{4Q}{\pi V}\right]^{\frac{1}{2}} = \left[\frac{4(1.5 \frac{ft^3}{s})}{\pi V \frac{ft}{s}}\right]^{\frac{1}{2}} = \frac{1.382}{\sqrt{V}} \text{, or when combined with Eq.(1)}$$

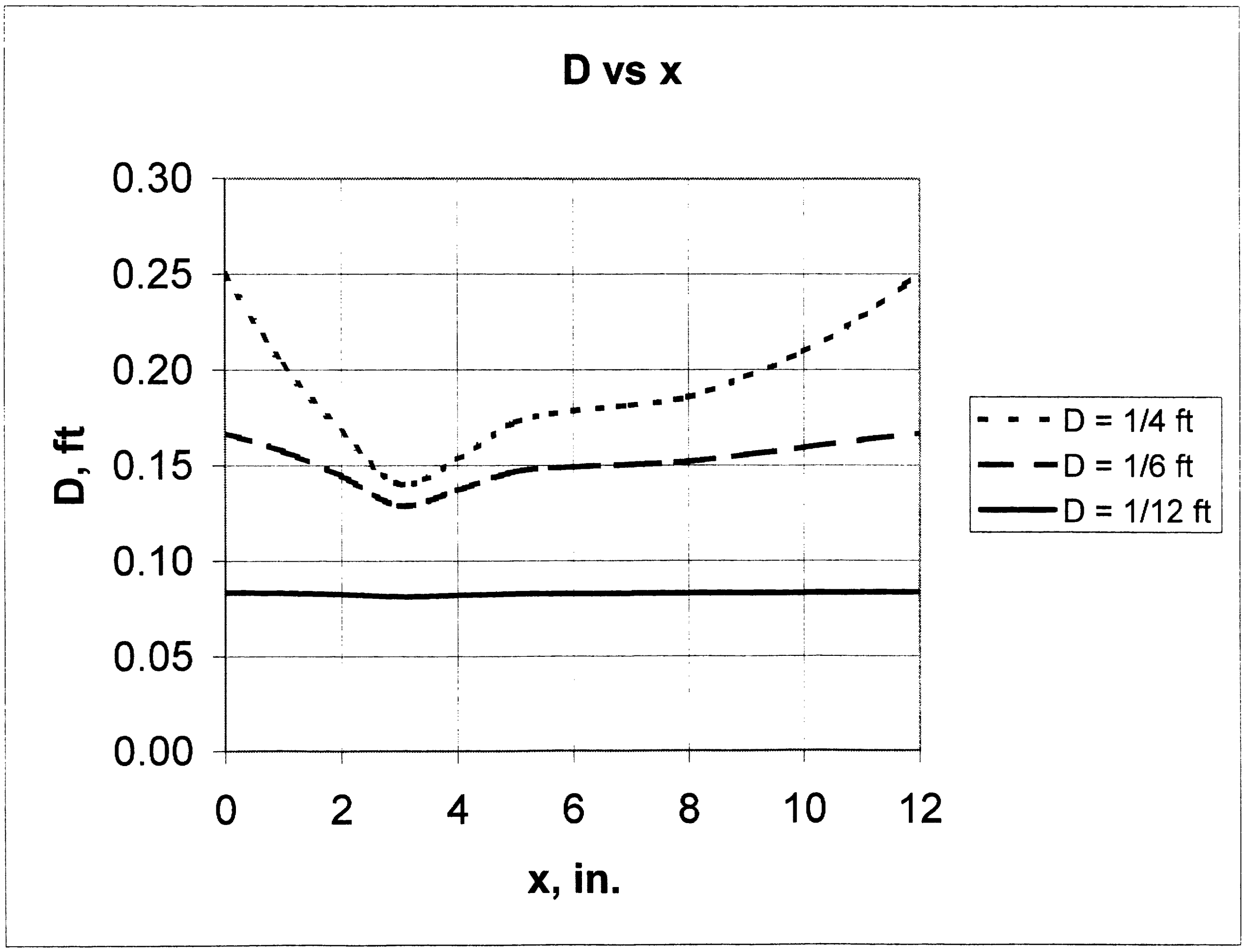
$$D = \frac{1.382}{\left[\frac{3.65}{D_0^4} + 4370(1-h)\right]^{\frac{1}{4}}} \text{ ft, where } D_0 \sim ft, h \sim in. \qquad (2)$$

Plot $D = D(x)$ with $D_0 = \frac{1}{12}, \frac{1}{6}$, and $\frac{1}{4}$ ft, using the values of $h = h(x)$ from the table. Note: h is the same a "p (in. H_2O)" in the table. An EXCEL program was used to obtain the following results.

(con't)

*3.56 (con't)

x, in.	D, ft (D_0 = 1/4 ft)	D, ft (D_0 = 1/6 ft)	D, ft (D_0 = 1/12 ft)	p, in. H_2O
0	0.24996	0.16664	0.08332	1.00
1	0.20277	0.15733	0.08299	0.72
2	0.16776	0.14435	0.08234	0.16
3	0.13999	0.12870	0.08112	-0.96
4	0.15299	0.13667	0.08182	-0.31
5	0.17245	0.14649	0.08247	0.27
6	0.17841	0.14902	0.08260	0.39
7	0.18123	0.15015	0.08266	0.44
8	0.18558	0.15179	0.08274	0.51
9	0.19616	0.15537	0.08291	0.65
10	0.20944	0.15911	0.08306	0.78
11	0.22710	0.16300	0.08320	0.90
12	0.24996	0.16664	0.08332	1.00



3.57

3.57 The vent on the tank shown in Fig. P3.57 is closed and the tank pressurized to increase the flowrate. What pressure, p_1, is needed to produce twice the flowrate of that when the vent is open?

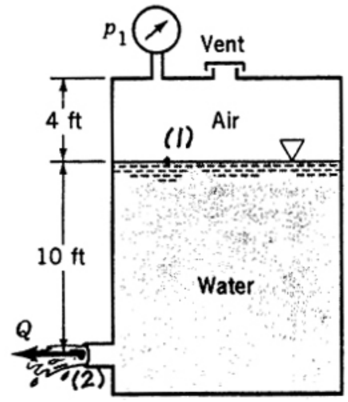


FIGURE P3.57

With the vent open:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_1=p_2=0,\ V_1=0,\ z_2=0$$

Thus,

$$z_1=\frac{V_2^2}{2g} \quad \text{or} \quad V_2=\sqrt{2gz_1}=\sqrt{2(32.2\tfrac{ft}{s^2})(10ft)}=25.4\tfrac{ft}{s}$$

To have double the flowrate with the vent closed ($p_1 \neq 0$):

$$\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g} \quad \text{where for this case } V_2=2(25.4\tfrac{ft}{s})=50.8\tfrac{ft}{s}$$

Thus,

$$\frac{p_1}{(62.4\frac{lb}{ft^3})}+10ft=\frac{(50.8\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$p_1=1876\tfrac{lb}{ft^2}=\underline{\underline{13.0\,psi}}$$

3.58

3.58 Water flows steadily through the large tanks shown in Fig. P3.58. Determine the water depth, h_A.

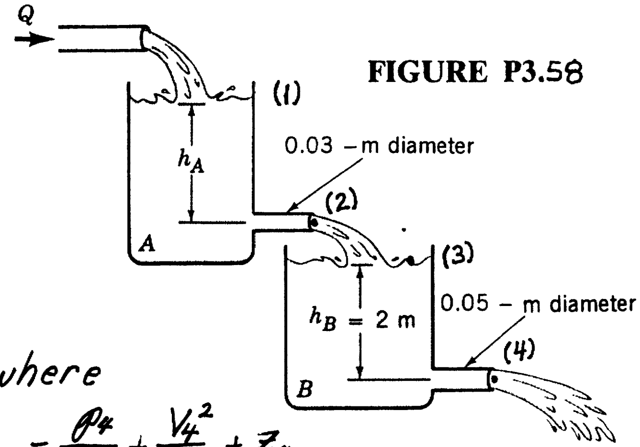


FIGURE P3.58

For steady flow, $Q_2 = Q_4$ where

$$Q_4 = A_4 V_4 \quad \text{with} \quad \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 = \frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4$$

where $p_3 = p_4 = 0$ and $V_3 = 0$

Thus,

$$V_4 = \sqrt{2g(z_3 - z_4)} = \sqrt{2(9.81\ \tfrac{m}{s^2})(2m)} = 6.26\ \tfrac{m}{s}$$

or

$$Q_4 = \frac{\pi}{4}(0.05m)^2(6.26\ \tfrac{m}{s}) = 0.0123\ \tfrac{m^3}{s}$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = p_2 = 0 \text{ and } V_1 = 0$$

so that

$$V_2 = \sqrt{2g h_A}$$

Thus,

$$A_2 V_2 = Q_4 \quad \text{or} \quad \frac{\pi}{4}(0.03m)^2\sqrt{2(9.81\tfrac{m}{s^2})h_A} = 0.0123\ \tfrac{m^3}{s}$$

or

$$h_A = \underline{\underline{15.4\,m}}$$

3.59

3.59 Air at 80 °F and 14.7 psia flows into the tank shown in Fig. P3.59. Determine the flowrate in ft^3/s, lb/s, and slugs/s. Assume incompressible flow.

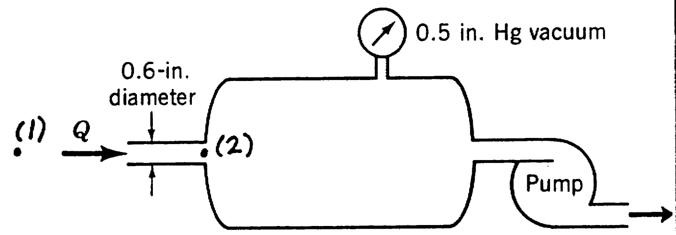


FIGURE P3.59

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2, \; p_1=0, \; V_1=0$$

Thus,

$$V_2=\sqrt{-2g\frac{p_2}{\gamma}}=\sqrt{-2\frac{p_2}{\rho}}$$

where $\rho=\frac{p}{RT}=\frac{\left(14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1716\,\frac{ft\cdot lb}{slugs\cdot{}^\circ R}\right)(460+80)^\circ R}=2.28\times10^{-3}\,\frac{slugs}{ft^3}$

Hence, with $p_2=-\gamma_{Hg}h=-\left(847\,\frac{lb}{ft^3}\right)\left(\frac{0.5}{12}\,ft\right)=-35.3\,\frac{lb}{ft^2}$

$$V_2=\left[-2\,\frac{\left(-35.3\,\frac{lb}{ft^2}\right)}{2.28\times10^{-3}\,\frac{slugs}{ft^3}}\right]^{1/2}=176\,\frac{ft}{s}$$

Thus,

$$Q=A_2V_2=\frac{\pi}{4}\left(\frac{0.6}{12}\,ft\right)^2\left(176\,\frac{ft}{s}\right)=\underline{\underline{0.346\,\frac{ft^3}{s}}}$$

$$\dot{m}=\rho Q=\left(2.28\times10^{-3}\,\frac{slugs}{ft^3}\right)\left(0.346\,\frac{ft^3}{s}\right)=\underline{\underline{7.89\times10^{-4}\,\frac{slugs}{s}}}$$

and

$$g\dot{m}=\left(32.2\,\frac{ft}{s^2}\right)\left(7.89\times10^{-4}\,\frac{slugs}{s}\right)=\underline{\underline{0.0254\,\frac{lb}{s}}}$$

3.60

3.60 Water flows from a large tank as shown in Fig. P3.60. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, h, will cavitation begin? To avoid cavitation should the value of D_1 be increased or decreased? To avoid cavitation should the value of D_2 be increased or decreased? Explain.

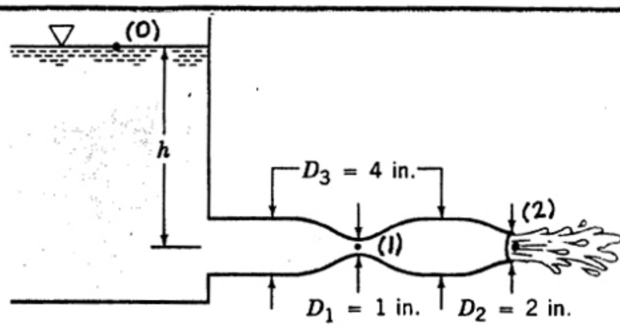


FIGURE P3.60

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1$$ where $p_0=14.5\text{ psia}$, $p_1=1.60\text{ psia}$, $z_0=h$, $z_1=0$, and $V_0=0$

Thus,

$$h=\frac{p_1-p_0}{\gamma}+\frac{V_1^2}{2g} \qquad (1)$$

However,

$$A_1V_1=A_2V_2 \quad \text{or} \quad V_1=\left(\frac{D_2}{D_1}\right)^2 V_2$$

where

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ with $p_0=p_2$ and $z_2=0$

Thus,

$$\frac{V_2^2}{2g}=h$$

so that

$$\frac{V_1^2}{2g}=\frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g}=\left(\frac{D_2}{D_1}\right)^4 h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$h=\frac{p_1-p_0}{\gamma}+\left(\frac{D_2}{D_1}\right)^4 h$$

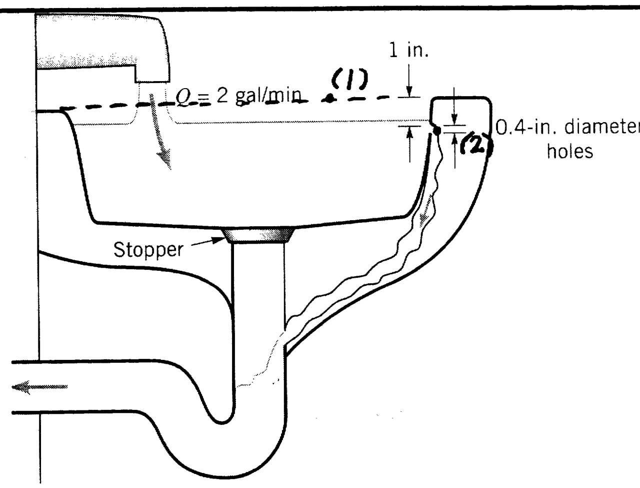
or

$$h=\frac{p_0-p_1}{\gamma\left[\left(\frac{D_2}{D_1}\right)^4-1\right]}=\frac{(14.5-1.60)\frac{\text{lb}}{\text{in.}^2}\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)}{62.4\frac{\text{lb}}{\text{ft}^3}\left[\left(\frac{2\text{ in.}}{1\text{ in.}}\right)^4-1\right]}=\underline{\underline{1.98\text{ ft}}} \qquad (3)$$

From Eq. (3) it is seen that h increases in increasing D_1 and decreasing D_2. Thus, to avoid cavitation (i.e. to have h small enough) <u>D_1 should be increased and D_2 decreased.</u>

3.61

3.61 Water flows into the sink shown in Fig. P3.61 at a rate of 2 gal/min. If the drain is closed, the water will eventually flow through the overflow drain holes rather than over the edge of the sink. How many 0.4-in.-diameter drain holes are needed to ensure that the water does not overflow the sink? Neglect viscous effects.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = 0,\ V_1 = 0, \text{ and } z_2 = 0,\ p_2 = 0$$

Thus,

$$z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2gz_1} = \left[2\left(32.2\,\frac{ft}{s}\right)\left(\frac{1+0.2}{12}\,ft\right)\right]^{1/2} = 2.54\,\frac{ft}{s}$$

Also,

$$Q = n A_2 V_2 = n\, C_c \frac{\pi}{4} d_2^2 V_2 \text{ , where } n = \text{number of holes required,}$$

$d_2 = 0.4$ in., and C_c = contraction coef. $= 0.61$ (see Fig. 3.14)

Thus, with

$$Q = 2\,\frac{gal}{min}\left(\frac{1\ min}{60\,s}\right)\left(\frac{231\ in.^3}{1\ gal}\right)\left(\frac{1\ ft^3}{1728\ in.^3}\right) = 4.46\times10^{-3}\,\frac{ft^3}{s},$$

$$n = \frac{4Q}{\pi C_c d_2^2 V_2} = \frac{4\,(4.46\times10^{-3}\,ft^3/s)}{\pi(0.61)\left(\frac{0.4}{12}\right)^2 ft^2\,(2.54\,ft/s)} = 3.30$$

Thus, $\underline{\underline{4}}$ holes are needed.

3.62

3.62 What pressure, p_1, is needed to produce a flowrate of 0.09 ft^3/s from the tank shown in Fig. P3.62?

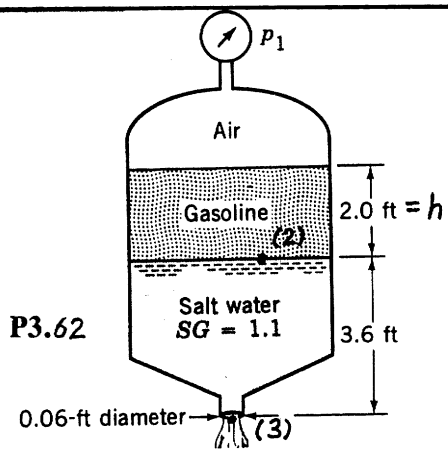


FIGURE P3.62

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$ where $p_2 = p_1 + \gamma_0 h$, $p_3 = 0$, $z_2 = 3.6\ ft$, $z_3 = 0$ and $V_2 = 0$

Thus,

$$\frac{p_1 + \gamma_0 h}{\gamma} + z_2 = \frac{V_3^2}{2g}$$

where $Q = A_3 V_3 = \frac{\pi}{4} D_3^2 V_3$

or

$$V_3 = \frac{4Q}{\pi D_3^2} = \frac{4\left(0.09\ \frac{ft^3}{s}\right)}{\pi (0.06\ ft)^2} = 31.8\ \frac{ft}{s}$$

Thus,

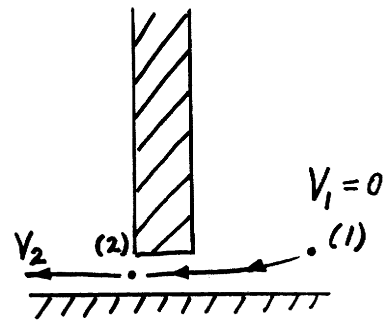
$$p_1 = \gamma\left(\frac{V_3^2}{2g} - z_2\right) - \gamma_0 h = \left(1.1\left(62.4\ \frac{lb}{ft^3}\right)\right)\left[\frac{\left(31.8\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} - 3.6\ ft\right]$$
$$- 42.5\ \frac{lb}{ft^3}\ (2.0\ ft)$$

or

$$p_1 = 746\ \frac{lb}{ft^2} = \underline{\underline{5.18\ psi}}$$

3.63

3.63 Laboratories containing dangerous materials are often kept at a pressure slightly less than ambient pressure so that contaminants can be filtered through an exhaust system rather than leaked through cracks around doors, etc. If the pressure in such a room is 0.1 in. of water below that of the surrounding rooms, with what velocity will air enter the room through an opening? Assume viscous effects are negligible.



If viscous effects are negligible,

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2, \text{ where } V_1 = 0 \text{ and } p_1 - p_2 = \gamma_{H_2O} h$$

or

$$p_1 - p_2 = \left(\frac{0.1}{12}\,ft\right)\left(62.4\,\frac{lb}{ft^3}\right) = 0.520\,\frac{lb}{ft^2}$$

Thus,

$$V_2 = \left[\frac{2(p_1 - p_2)}{\rho}\right]^{1/2} = \left[\frac{2\,(0.520\,lb/ft^2)}{0.00238\,slugs/ft^3}\right]^{1/2} = \underline{\underline{20.9\,\frac{ft}{s}}}$$

3.64

3.64 Water is siphoned from the tank shown in Fig. P3.64. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.

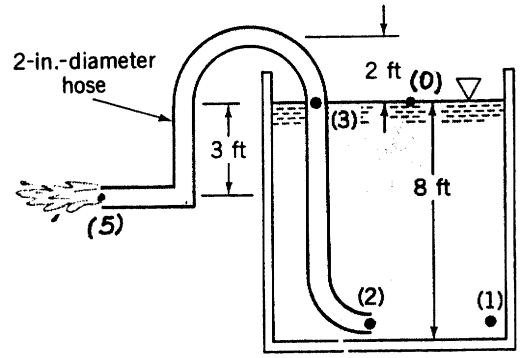


FIGURE P3.64

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i \quad \text{for } i = 1, \ldots, 5 \qquad (1)$$

For $i = 5$ and $p_0 = 0$, $V_0 = 0$, $p_5 = 0$ this becomes

$$z_0 = \frac{V_5^2}{2g} + z_5 \quad \text{or} \quad V_5 = \sqrt{2g(z_0 - z_5)} = \sqrt{2(32.2\,\frac{ft}{s^2})(3\,ft)}$$
$$= 13.9\,\frac{ft}{s}$$

Thus,

$$Q = A_5 V_5 = \frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2 (13.9\,\tfrac{ft}{s}) = \underline{\underline{0.303\,\frac{ft^3}{s}}}$$

From Eq.(1) with $i = 1$ and $V_1 = 0$, $p_1 = \gamma(z_0 - z_1)$

$$= (62.4\,\tfrac{lb}{ft^3})(8\,ft) = \underline{\underline{499\,\frac{lb}{ft^2}}}$$

From Eq.(1) with $i = 2$, $z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where $A_2 V_2 = A_5 V_5$

Since $A_2 = A_5$ it follows that $V_2 = V_5$ or $\frac{V_2^2}{2g} = \frac{V_5^2}{2g} = z_0 - z_5$

Thus,

$$\frac{p_2}{\gamma} = z_0 - z_2 - \frac{V_2^2}{2g} = z_0 - z_2 - \frac{V_5^2}{2g} = z_0 - z_2 - (z_0 - z_5)$$
$$= z_5 - z_2$$

or

$$p_2 = \gamma(z_5 - z_2) = (62.4\,\tfrac{lb}{ft^3})(5\,ft) = \underline{\underline{312\,\frac{lb}{ft^2}}}$$

From Eq. (1) with $i = 3$, $z_0 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$

where $A_3 V_3 = A_5 V_5$

Since $A_3 = A_5$ it follows that $V_3 = V_5$ or $\frac{V_3^2}{2g} = \frac{V_5^2}{2g} = z_0 - z_5$

Thus,

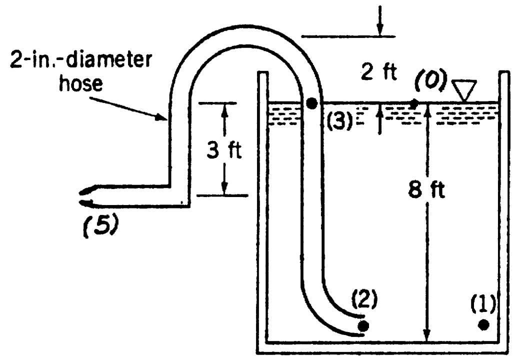
$$\frac{p_3}{\gamma} = z_0 - z_3 - \frac{V_3^2}{2g} = z_0 - z_3 - \frac{V_5^2}{2g} = z_0 - z_3 - (z_0 - z_5)$$
$$= z_5 - z_3$$

or

$$p_3 = \gamma(z_5 - z_3) = (62.4\,\tfrac{lb}{ft^3})(-3\,ft) = \underline{\underline{-187\,\frac{lb}{ft^2}}}$$

3.65

3.65 Redo Problem 3.64 if a 1-in.-diameter nozzle is placed at the end of the tube.



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i \quad \text{for } i = 1, \dots, 5 \qquad (1)$$

For $i=5$ and $p_0=0$, $V_0=0$, $p_5=0$ this becomes

$z_0 = \frac{V_5^2}{2g}$ or $V_5 = \sqrt{2g(z_0 - z_5)} = \sqrt{2(32.2\,\frac{ft}{s^2})(3\,ft)}$

$= 13.9\,\frac{ft}{s}$

Thus,

$Q = A_5 V_5 = \frac{\pi}{4}(\frac{1}{12}\,ft)^2(13.9\,\frac{ft}{s}) = \underline{\underline{0.0758\,\frac{ft^3}{s}}}$

From Eq. (1) with $i=1$ and $V_1=0$, $p_1 = \gamma(z_0 - z_1)$

$= (62.4\,\frac{lb}{ft^3})(8\,ft) = \underline{\underline{499\,\frac{lb}{ft^2}}}$

From Eq. (1) with $i=2$, $z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where $A_2 V_2 = A_5 V_5$

Since $A_2 = (\frac{D_2}{D_5})^2 A_5 = (\frac{2}{1})^2 A_5 = 4A_5$ it follows that

$V_2 = \frac{1}{4} V_5$ or $\frac{V_2^2}{2g} = \frac{1}{2g}(\frac{1}{4}V_5)^2 = \frac{1}{16}\frac{V_5^2}{2g} = \frac{1}{16}(z_0 - z_5)$

Thus,

$\frac{p_2}{\gamma} = z_0 - z_2 - \frac{V_2^2}{2g} = z_0 - z_2 - \frac{1}{16}(z_0 - z_5) = 8\,ft - \frac{1}{16}(3\,ft)$

$= 7.81\,ft$

or

$p_2 = (62.4\,\frac{lb}{ft^3})(7.81\,ft) = \underline{\underline{488\,\frac{lb}{ft^2}}}$

From Eq. (1) with $i=3$, $z_0 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$

where $A_3 V_3 = A_5 V_5$

or since $A_3 = A_2$ then $V_3 = V_2$ and $\frac{V_3^2}{2g} = \frac{V_2^2}{2g} = \frac{1}{16}(z_0 - z_5)$

Thus,

$\frac{p_3}{\gamma} = z_0 - z_3 - \frac{V_3^2}{2g} = z_0 - z_3 - \frac{1}{16}(z_0 - z_5) = -\frac{1}{16}(3\,ft) = -\frac{3}{16}\,ft$

or

$p_3 = (62.4\,\frac{lb}{ft^3})(-\frac{3}{16}\,ft) = \underline{\underline{-11.7\,\frac{lb}{ft^2}}}$

3.66

3.66 Determine the manometer reading, h, for the flow shown in Fig. P3.66.

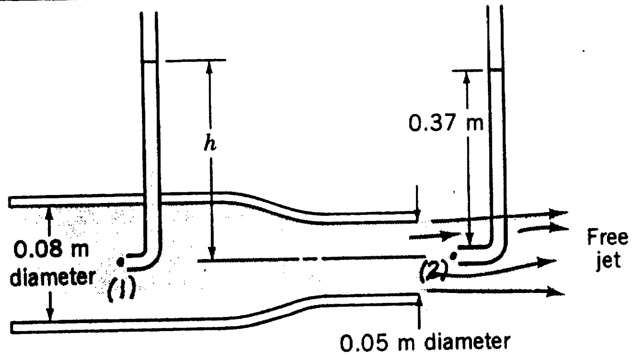


FIGURE P3.66

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = z_2$, $V_1 = 0$, and $V_2 = 0$

Thus,

$$p_1 = p_2$$

However, $p_1 = \gamma h$ and $p_2 = \gamma (0.37 m)$
so that

$$h = \underline{\underline{0.37\ m}}$$

3.67

3.67 The specific gravity of the manometer fluid shown in Fig. P3.67 is 1.07. Determine the volume flowrate, Q, if the flow is inviscid and incompressible and the flowing fluid is (a) water, (b) gasoline, or (c) air at standard conditions.

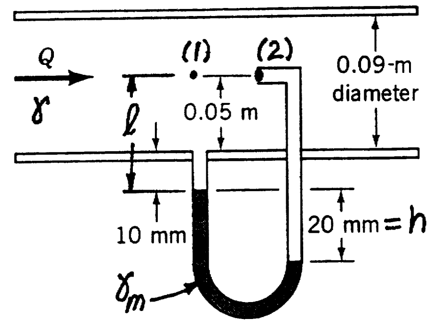


FIGURE P3.67

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2 \text{ and } V_2=0$$

Thus,

$$V_1=\sqrt{2g\frac{(p_2-p_1)}{\gamma}} \tag{1}$$

But

$$p_1+\gamma\ell+\gamma_m h=p_2+\gamma(\ell+h)$$

or

$p_2-p_1=(\gamma_m-\gamma)h$ so that Eq. (1) becomes

$$V_1=\sqrt{2g\frac{(\gamma_m-\gamma)}{\gamma}h}=\sqrt{2\left(9.81\frac{m}{s^2}\right)\left(\frac{1.07\left(9.8\times10^3\frac{N}{m^3}\right)}{\gamma}-1\right)(0.02\,m)}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(0.09\,m)^2\sqrt{2(9.81)\left(\frac{10.49\times10^3}{\gamma}-1\right)(0.02)}$$

or

$$Q=3.99\times10^{-3}\sqrt{\frac{10.49}{\gamma}-1}\ \frac{m^3}{s} \quad \text{where } \gamma\sim\frac{kN}{m^3}$$

For the given fluids this gives:

	fluid	$\gamma,\ \frac{kN}{m^3}$	$Q,\ \frac{m^3}{s}$
(a)	water	9.80	1.06×10^{-3}
(b)	gasoline	6.67	3.02×10^{-3}
(c)	air	12×10^{-3}	0.118

3.68

3.68 JP-4 fuel ($SG = 0.77$) flows through the Venturi meter shown in Fig. P3.68 with a velocity of 15 ft/s in the 6-in. pipe. If viscous effects are negligible determine the elevation, h, of the fuel in the open tube connected to the throat of the Venturi meter.

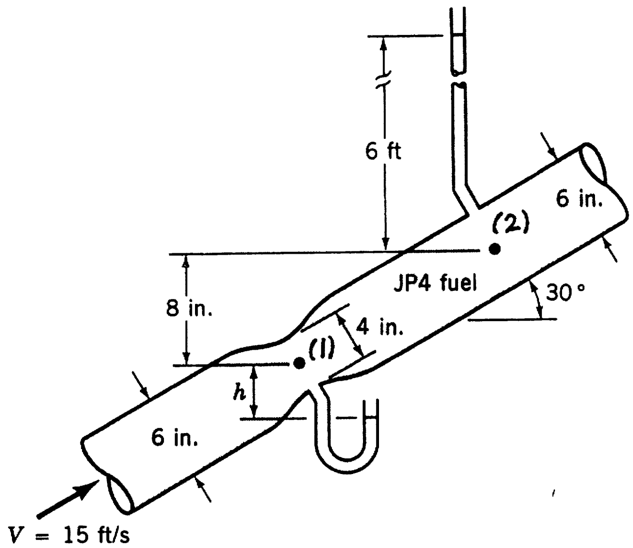


FIGURE P3.68

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = 0,\ z_2 = \frac{8}{12}\,ft, \text{ and } V_2 = 15\ ft/s \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{6\,in.}{4\,in.}\right)^2 \left(15\,\frac{ft}{s}\right) = 33.75\,\frac{ft}{s}$$

Thus, with $\frac{p_2}{\gamma} = 6\,ft$ Eq. (1) becomes

$$\frac{p_1}{\gamma} + \frac{(33.75\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = 6\,ft + \frac{(15\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + \frac{8}{12}\,ft$$

or

$$\frac{p_1}{\gamma} = -7.53\,ft$$

But $\frac{p_1}{\gamma} = -h$ so that $h = \underline{\underline{7.53\,ft}}$

3.69

3.69 Repeat Problem 3.68 if the flowing fluid is water rather than JP-4 fuel.

Note from the solution to Problem 3.68 that the value of γ is not needed. Thus, h = 7.53 ft for either water or JP-4 fuel.

3.70

3.70 Air at standard conditions flows through the cylindrical drying stack shown in Fig. P3.70. If viscous effects are negligible and the inclined water-filled manometer reading is 20 mm as indicated, determine the flowrate.

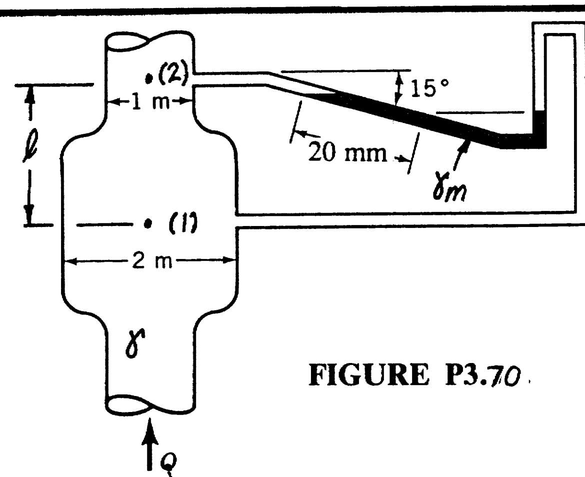


FIGURE P3.70

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $z_1=0$, $z_2=\ell$ and

$$V_2=\frac{A_1}{A_2}V_1=\left(\frac{D_1}{D_2}\right)^2V_1=\left(\frac{2\,m}{1\,m}\right)^2V_1=4V_1$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{(4V_1)^2}{2g}+\ell$$

or

$$\frac{15V_1^2}{2g}=\frac{p_1-p_2}{\gamma}-\ell \qquad (1)$$

However, $p_2+\gamma\ell_2+\gamma_m h=p_1-\gamma(\ell-h-\ell_2)$ where $h=(20\,mm)\sin 15$

or

$$\frac{p_1-p_2}{\gamma}=\left(\frac{\gamma_m}{\gamma}-1\right)h+\ell \qquad (2)$$

By combining Eqs.(1) and (2)

$$\frac{15V_1^2}{2g}=\left(\frac{\gamma_m}{\gamma}-1\right)h$$

or

$$V_1=\sqrt{\frac{2g\left(\frac{\gamma_m}{\gamma}-1\right)h}{15}}=\sqrt{\frac{2\left(9.81\,\frac{m}{s^2}\right)\left(\frac{9.80\times10^3\,\frac{N}{m^3}}{12.0\,\frac{N}{m^3}}-1\right)(0.02\sin 15)}{15}}$$

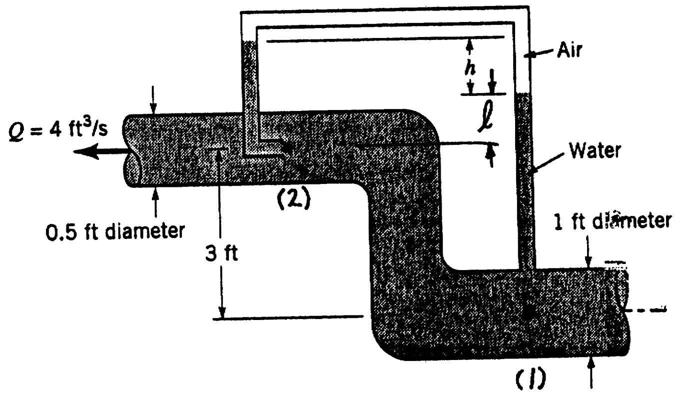
$$=2.35\,\frac{m}{s}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(2m)^2\left(2.35\,\frac{m}{s}\right)=\underline{\underline{7.38\,\frac{m^3}{s}}}$$

3.71

3.71 Water, considered an inviscid, incompressible fluid, flows steadily as shown in Fig. P3.71. Determine h.



■ FIGURE P3.71

$$p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho V_2^2$$

where $z_1 = 0$, $z_2 = 3\,ft$, $V_2 = 0$, and $V_1 = \frac{Q}{A_1} = \frac{4\,\frac{ft^3}{s}}{\frac{\pi}{4}(1\,ft)^2} = 5.09\,\frac{ft}{s}$

Thus,

$$p_1 + \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left(5.09\,\frac{ft}{s}\right)^2 = p_2 + 62.4\,\frac{lb}{ft^3}\,(3\,ft)$$

or

$$p_1 - p_2 = 162\,\frac{lb}{ft^2} \qquad (1)$$

But from the manometer,

$$p_1 - \gamma(\ell + 3\,ft) + \gamma(h + \ell) = p_2$$

or

$$p_1 - 62.4\,\frac{lb}{ft^3}\,(3\,ft) + 62.4\,\frac{lb}{ft^3}\,h = p_2$$

Hence,

$p_1 = p_2 + 187 - 62.4h$ which when combined with Eq. (1) gives

$$p_2 + 187 - 62.4h - p_2 = 162$$

or

$$h = \underline{\underline{0.400\,ft}}$$

3.72

3.72 Determine the flowrate through the submerged orifice shown in Fig. P3.72 if the contraction coefficient is $C_c = 0.63$.

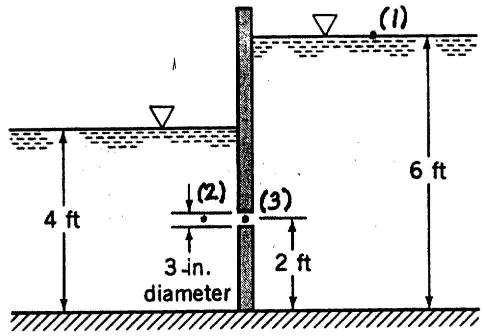


FIGURE P3.72

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_1 = 0$, $z_1 = 4\text{ ft}$, $z_2 = 0$, and $\frac{p_2}{\gamma} = 2\text{ ft}$

Thus,

$$4\text{ ft} = 2\text{ ft} + \frac{V_2^2}{2\left(32.2\frac{\text{ft}}{\text{s}}\right)}$$

or

$$V_2 = 11.34\ \frac{\text{ft}}{\text{s}}$$

so that

$$Q = A_2 V_2 = C_c A_3 V_2 = (0.63)\frac{\pi}{4}\left(\frac{3}{12}\text{ ft}\right)^2\left(11.34\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.351\ \frac{\text{ft}^3}{\text{s}}}}$$

3.73

3.73 Determine the flowrate through the Venturi meter shown in Fig. P3.73 if ideal conditions exist.

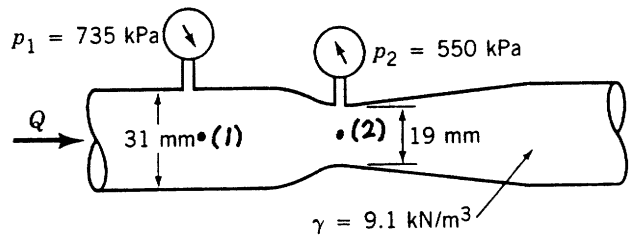


FIGURE P3.73

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $z_1=z_2$ and $A_1V_1=A_2V_2$

or

$$V_1=\frac{A_2}{A_1}V_2=\left(\frac{D_2}{D_1}\right)^2V_2$$

Thus,

$$\frac{p_1}{\gamma}+\frac{\left(\frac{D_2}{D_1}\right)^4V_2^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$$

or

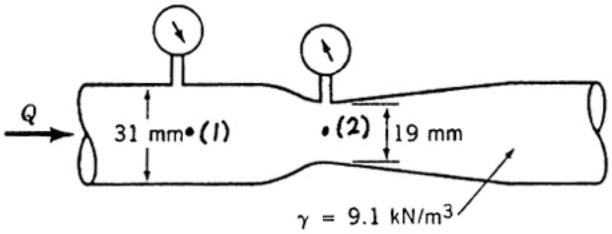
$$V_2=\sqrt{\frac{2g\frac{(p_1-p_2)}{\gamma}}{1-\left(\frac{D_2}{D_1}\right)^4}}=\sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)\frac{(735-550)kPa}{\left(9.1\frac{kN}{m^3}\right)}}{1-\left(\frac{19\ mm}{31\ mm}\right)^4}}=21.5\frac{m}{s}$$

so that

$$Q=A_2V_2=\frac{\pi}{4}D_2^2V_2=\frac{\pi}{4}(0.019m)^2\left(21.5\frac{m}{s}\right)=\underline{\underline{6.10\times10^{-3}\frac{m^3}{s}}}$$

3.74

3.74 For what flowrate through the Venturi meter of Prob. 3.73 will cavitation begin if $p_1 = 275$ kPa gage, atmospheric pressure is 101 kPa (abs), and the vapor pressure is 3.6 kPa (abs)?



$$(1) \quad \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$, $p_2 = 3.6\ kPa(abs)$ and $p_1 = (275 + 101)\ kPa(abs) = 376\ kPa(abs)$

Thus, with $A_1 V_1 = A_2 V_2$

or

$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$ Eq. (1) becomes

$$V_2 = \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\gamma}\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \left[\frac{2(9.81\frac{m}{s})\frac{(376-3.6)kPa}{9.1\ kN/m^3}}{1 - \left(\frac{19\ mm}{31\ mm}\right)^4}\right]^{1/2}$$

or

$V_2 = 30.6\ \frac{m}{s}$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}(0.019\ m)^2 (30.6\ \frac{m}{s}) = \underline{\underline{8.68 \times 10^{-3}\ \frac{m^3}{s}}}$$

3.75

3.75 What diameter orifice hole, d, is needed if under ideal conditions the flowrate through the orifice meter of Fig. P3.75 is to be 30 gal/min of seawater with $p_1 - p_2 = 2.37$ lb/in.^{2}? The contraction coefficient is assumed to be 0.63.

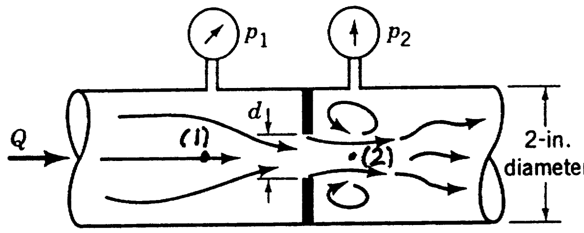


FIGURE P3.75

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $C_c = 0.63$, and $p_1 - p_2 = 2.37\,psi$

With

$$Q = \left(30\,\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,ft^3}{1728\,in^3}\right) = 0.0668\,\frac{ft^3}{s} \quad \text{and } \gamma = 64.0\,\frac{lb}{ft^3}$$

it follows that

$$V_1 = \frac{Q}{A_1} = \frac{0.0668\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2} = 3.06\,\frac{ft}{s}$$

Thus, Eq(1) gives

$$V_2 = \sqrt{V_1^2 + 2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{\left(3.06\,\frac{ft}{s}\right)^2 + 2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{2.37 \times 144\,\frac{lb}{ft^2}}{64.0\,\frac{lb}{ft^3}}\right)}$$

or

$$V_2 = 18.8\,\frac{ft}{s}$$

Thus, since

$$Q = A_2 V_2 = C_c \frac{\pi}{4} d^2 V_2 \quad \text{it follows that}$$

$$d = \left[\frac{4Q}{\pi C_c V_2}\right]^{1/2} = \left[\frac{4 \times 0.0668\,\frac{ft^3}{s}}{\pi\,(0.63)(18.8\,\frac{ft}{s})}\right]^{1/2} = 0.0847\,ft = \underline{\underline{1.016\,in.}}$$

3.76

3.76 An ancient device for measuring time is shown in Fig. P3.76. The axisymmetric vessel is shaped so that the water level falls at a constant rate. Determine the shape of the vessel, $R = R(z)$, if the water level is to decrease at a rate of 0.10 m/hr and the drain hole is 5.0 mm in diameter. The device is to operate for 12 hr without needing refilling. Make a scale drawing of the shape of the vessel.

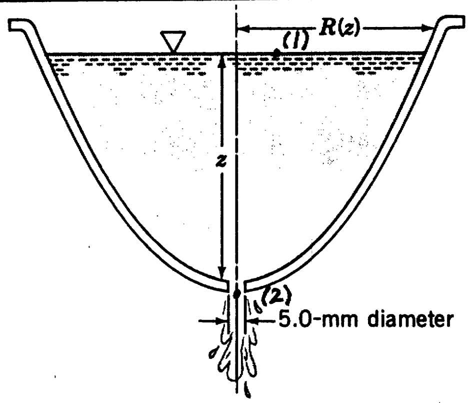


FIGURE P3.76

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ if the flow is assumed to be quasi-steady.

Also, $p_1 = 0$, $p_2 = 0$, $z_1 = z$, and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + z$$ which, if $V_1 \ll V_2$ (i.e. $R \gg 5.0\,mm$), becomes

$$V_2 = \sqrt{2gz}$$

Since $A_1 V_1 = A_2 V_2$ and $V_1 = \left|\frac{dz}{dt}\right| = 0.1\,\frac{m}{hr}\left(\frac{1\,hr}{3{,}600\,s}\right) = 2.78 \times 10^{-5}\,\frac{m}{s}$

we obtain

$$\pi R^2 \left(2.78 \times 10^{-5}\,\frac{m}{s}\right) = \frac{\pi}{4}(0.005\,m)^2 \sqrt{2\left(9.81\,\frac{m}{s^2}\right) z}\ ,$$

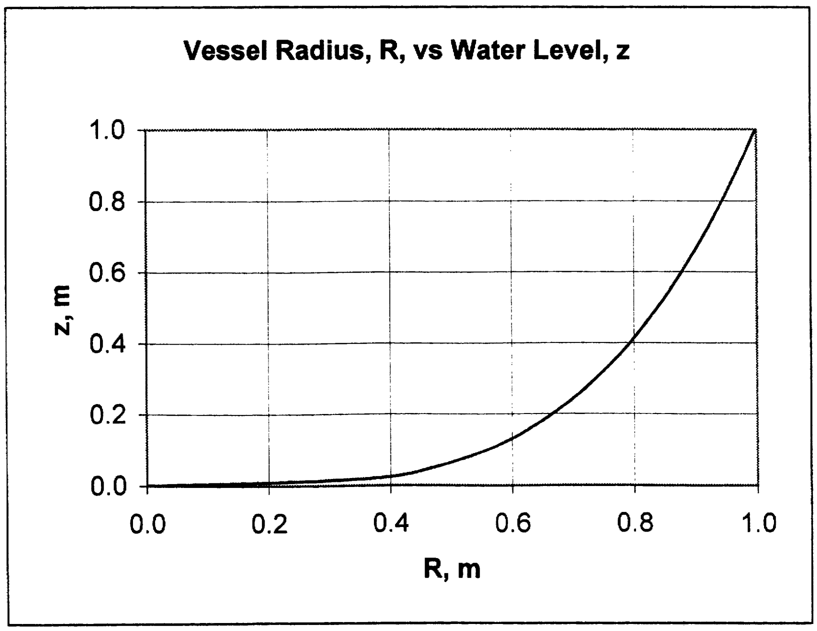
where R and z are ~m

Thus,

$$\underline{\underline{R = 0.998\, z^{1/4}}}$$

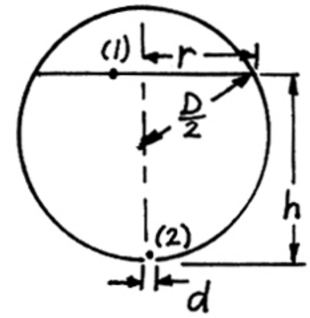
or

z, m	R, m
0	0
0.02	0.375
0.05	0.472
0.12	0.587
0.22	0.683
0.32	0.751
0.42	0.803
0.52	0.847
0.62	0.886
0.72	0.919
0.82	0.950
0.92	0.977
1.02	1.003
1.12	1.027
1.22	1.049



3.78*

3.78* A spherical tank of diameter D has a drain hole of diameter d at its bottom. A vent at the top of the tank maintains atmospheric pressure within the tank. The flow is quasisteady and inviscid and the tank is full of water initially. Determine the water depth as a function of time, $h = h(t)$, and plot graphs of $h(t)$ for tank diameters of 1, 5, 10, and 20 ft if $d = 1$ in.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = h$, $z_2 = 0$ and $V_1 = -\frac{dh}{dt} << V_2$ if $r >> d$

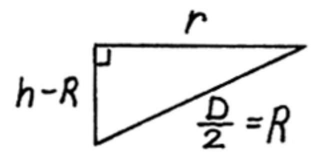
Thus,

$V_2 = \sqrt{2gh}$ which when combined with $A_1 V_1 = A_2 V_2$ gives

$$-A_1 \frac{dh}{dt} = A_2 \sqrt{2gh} \quad \text{or} \quad -\pi r^2 \frac{dh}{dt} = \frac{\pi}{4} d^2 \sqrt{2gh} \qquad (1)$$

where $R^2 = r^2 + (h-R)^2$

with $R = \frac{D}{2}$ = radius of tank



Thus, $r = \sqrt{R^2 - (h-R)^2}$ so that Eq. (1) becomes

$$-[R^2 - (h-R)^2] \frac{dh}{dt} = \frac{d^2}{4} \sqrt{2gh}$$

or

$$(h^{3/2} - 2Rh^{1/2})\, dh = \frac{d^2 \sqrt{2g}}{4} dt$$

which can be integrated from the initial time and depth ($t = 0$, $h = 2R$) to an arbitrary time and depth (t, h) as

$$\int_{2R}^{h} (h^{3/2} - 2Rh^{1/2})\, dh = \frac{d^2\sqrt{2g}}{4} \int_0^t dt$$

or

$$\frac{2}{5}\left(h^{5/2} - (2R)^{5/2}\right) - \frac{4}{3} R \left(h^{3/2} - (2R)^{3/2}\right) = \frac{d^2 \sqrt{2g}}{4} t \qquad (2)$$

Use $d = \frac{1}{12}$ ft and $g = 32.2 \frac{ft}{s^2}$ and plot $h = h(t)$ for values of $R = 0.5$, 2.5, 5, and 10 ft

Note: It is easier to solve Eq. (2) as $t = t(h)$ rather than $h = h(t)$

Note: The time taken to empty the tank, t_e, is obtained from Eq. (2) with $h = 0$ as

$$t_e = \frac{64 R^{5/2}}{15 d^2 \sqrt{g}}$$

(con't)

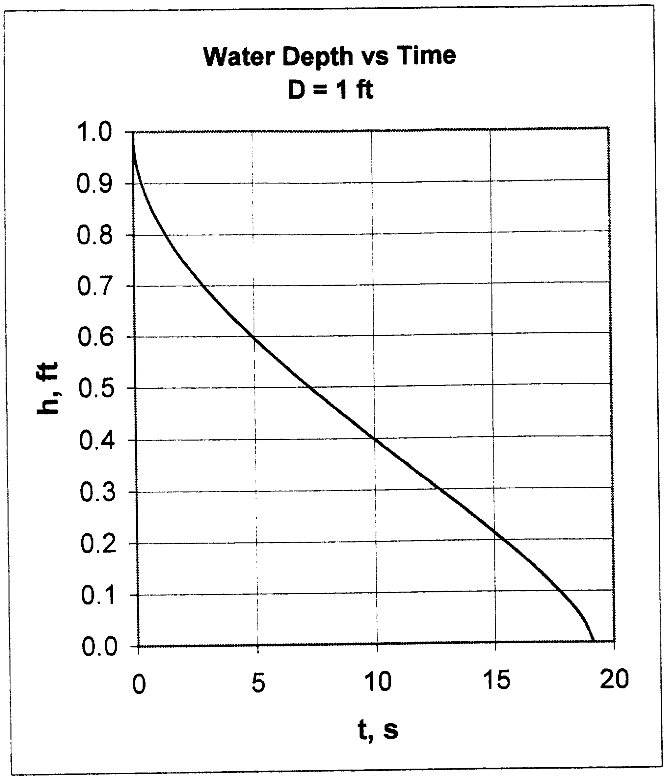
3.78* (con't.)

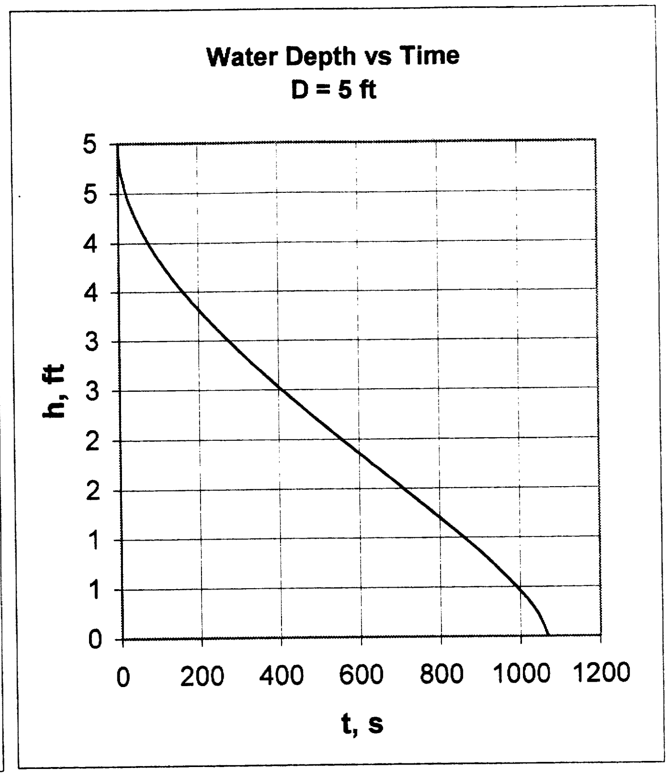
Results of an EXCEL Program to calculate h(t) from Eqn. (2):

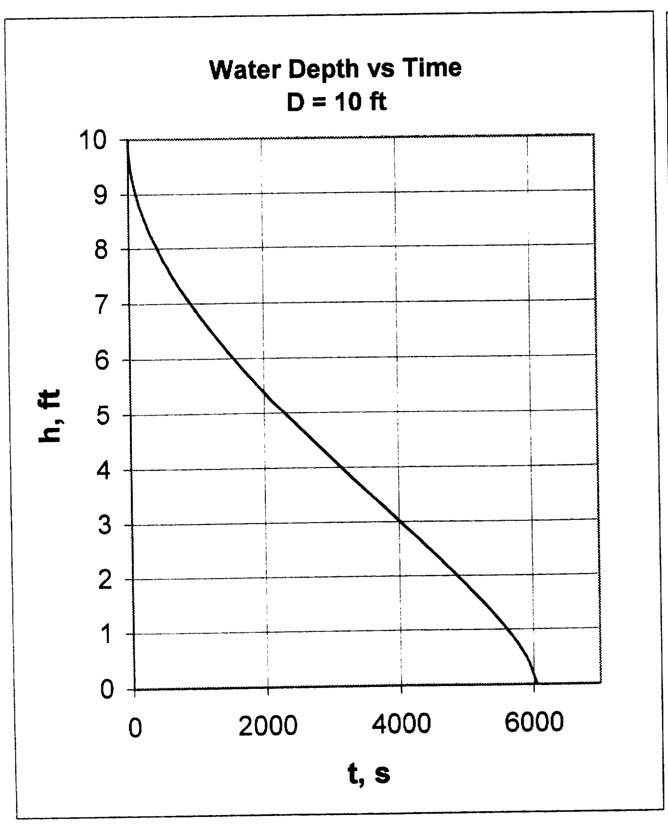
D = 1 ft		D = 5 ft		D = 10 ft		D = 20 ft	
t, s	h, ft	t, s	h, ft	t, s	h, ft	t, s	h, ft
0.00	1.000	0	5.000	0	10.00	0	20
0.09	0.950	5	4.750	28	9.50	158	19
0.35	0.900	19	4.500	110	9.00	620	18
0.77	0.850	43	4.250	242	8.50	1370	17
1.34	0.800	75	4.000	422	8.00	2390	16
2.05	0.750	114	3.750	647	7.50	3661	15
2.89	0.700	161	3.500	913	7.00	5163	14
3.84	0.650	215	3.250	1216	6.50	6876	13
4.91	0.600	274	3.000	1552	6.00	8778	12
6.06	0.550	339	2.750	1917	5.50	10846	11
7.30	0.500	408	2.500	2308	5.00	13055	10
8.60	0.450	481	2.250	2718	4.50	15376	9
9.94	0.400	556	2.000	3143	4.00	17782	8
11.31	0.350	632	1.750	3577	3.50	20237	7
12.69	0.300	710	1.500	4014	3.00	22706	6
14.06	0.250	786	1.250	4445	2.50	25144	5
15.37	0.200	859	1.000	4862	2.00	27502	4
16.61	0.150	929	0.750	5253	1.50	29714	3
17.72	0.100	990	0.500	5603	1.00	31695	2
18.62	0.050	1041	0.250	5889	0.50	33311	1
19.14	0.000	1070	0.000	6053	0.00	34239	0

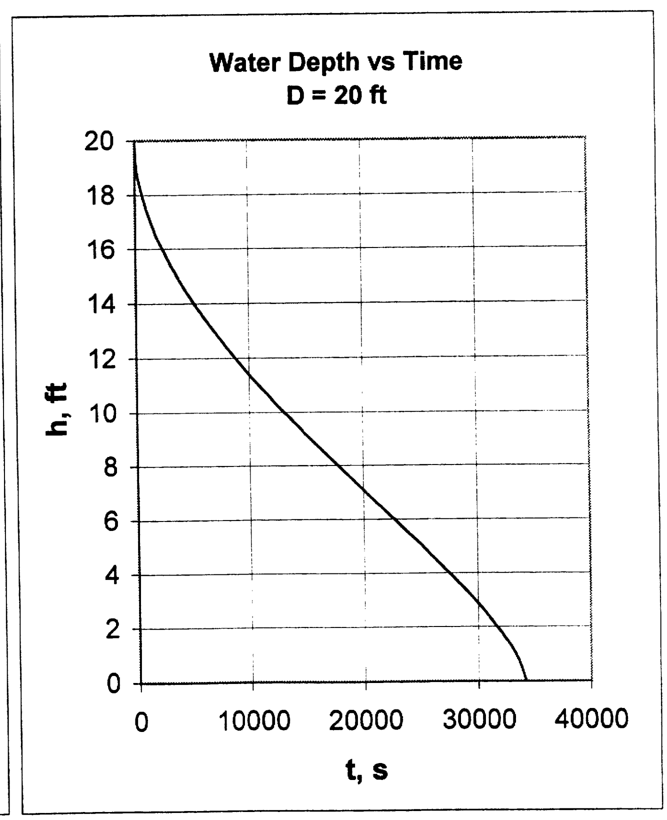
See next page for graphs of above results.

3.78* (con't)


Water Depth vs Time
D = 1 ft
h, ft
t, s
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20


Water Depth vs Time
D = 5 ft
h, ft
t, s
5
5
4
4
3
3
2
2
1
1
0
0
200
400
600
800
1000
1200


Water Depth vs Time
D = 10 ft
h, ft
t, s
10
9
8
7
6
5
4
3
2
1
0
0
2000
4000
6000


Water Depth vs Time
D = 20 ft
h, ft
t, s
20
18
16
14
12
10
8
6
4
2
0
0
10000
20000
30000
40000

3.79*

3.79* An inexpensive timer is to be made from a funnel as indicated in Fig. P3.79. The funnel is filled to the top with water and the plug is removed at time $t = 0$ to allow the water to run out. Marks are to be placed on the wall of the funnel indicating the time in 15-s intervals, from 0 to 3 min (at which time the funnel becomes empty). If the funnel outlet has a diameter of $d = 0.1$ in., draw to scale the funnel with the timing marks for funnels with angles of $\theta = 30$, 45, and 60°. Repeat the problem if the diameter is changed to 0.05 in.

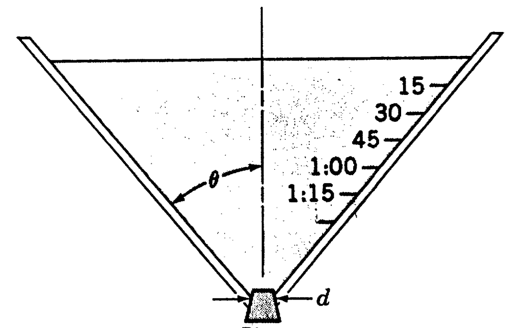
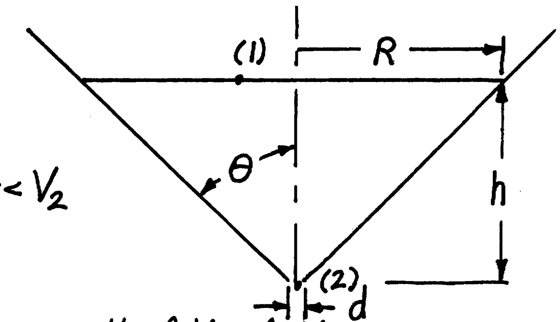


FIGURE P3.79



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 0$, $z_2 = 0$, and $V_1 = -\frac{dh}{dt} << V_2$ if $R >> \frac{d}{2}$

Thus,

$V_2 = \sqrt{2gh}$ which when combined with $A_1 V_1 = A_2 V_2$ gives

$$-A_1 \frac{dh}{dt} = A_2 \sqrt{2gh} \quad \text{or} \quad -\pi R^2 \frac{dh}{dt} = \frac{\pi}{4} d^2 \sqrt{2gh} \qquad (1)$$

where $R = h \tan\theta$

Thus, Eq. (1) becomes $\quad -h^2 \tan^2\theta \frac{dh}{dt} = \frac{d^2}{4}\sqrt{2gh}$

or

$h^{3/2} dh = \frac{-d^2\sqrt{2g}}{4\tan^2\theta} dt$ which can be integrated from $h = h_0$ at $t = 0$ as

$$\int_{h_0}^{h} h^{3/2} dh = -\frac{d^2\sqrt{2g}}{4\tan^2\theta}\int_0^t dt \quad \text{or} \quad \frac{2}{5}\left[h^{5/2} - h_0^{5/2}\right] = -\frac{d^2\sqrt{2g}}{4\tan^2\theta} t$$

Thus,

$$h = \left[h_0^{5/2} - \frac{5 d^2 \sqrt{2g}\, t}{8\tan^2\theta}\right]^{2/5} \qquad (2)$$

Since $h = 0$ when $t = 3\,\text{min} = 180\,\text{s}$ it follows that,

$$h_0^{5/2} = \frac{5 d^2 \sqrt{2(32.2 \frac{ft}{s^2})}\,(180\,s)}{8 \tan^2\theta}$$

which when combined with Eq. (2) gives

$$h = \left[\frac{5 d^2 \sqrt{2(32.2\frac{ft}{s^2})}\,(180\,s)}{8\tan^2\theta}\right]^{2/5}\left(1 - \frac{t}{180}\right)^{2/5}$$

or

$$h = 15.2\left(\frac{d}{\tan\theta}\right)^{4/5}\left(1 - \frac{t}{180}\right)^{2/5} \quad \text{where } h \sim ft,\ d \sim ft, \text{ and } t \sim s \qquad (3)$$

(con't)

3.79* (con't)

The results of an EXCEL Program using Eqn. (3) to calculate h as a function of t are shown below. The time interval markings for the six funnels are shown in the figures on the following page.

d = 0.1 in., $\theta = 30$ deg

t, s	h, ft
0	0.512
15	0.495
30	0.476
45	0.456
60	0.435
75	0.413
90	0.388
105	0.361
120	0.330
135	0.294
150	0.250
165	0.190
180	0.000

d = 0.1 in., $\theta = 45$ deg

t, s	h, ft
0	0.330
15	0.319
30	0.307
45	0.294
60	0.281
75	0.266
90	0.250
105	0.232
120	0.213
135	0.190
150	0.161
165	0.122
180	0.000

d = 0.1 in., $\theta = 45$ deg

t, s	h, ft
0	0.213
15	0.205
30	0.198
45	0.190
60	0.181
75	0.171
90	0.161
105	0.150
120	0.137
135	0.122
150	0.104
165	0.079
180	0.000

d = 0.05 in., $\theta = 30$ deg

t, s	h, ft
0	0.294
15	0.284
30	0.273
45	0.262
60	0.250
75	0.237
90	0.223
105	0.207
120	0.190
135	0.169
150	0.144
165	0.109
180	0.000

d = 0.05 in., $\theta = 45$ deg

t, s	h, ft
0	0.190
15	0.183
30	0.176
45	0.169
60	0.161
75	0.153
90	0.144
105	0.134
120	0.122
135	0.109
150	0.093
165	0.070
180	0.000

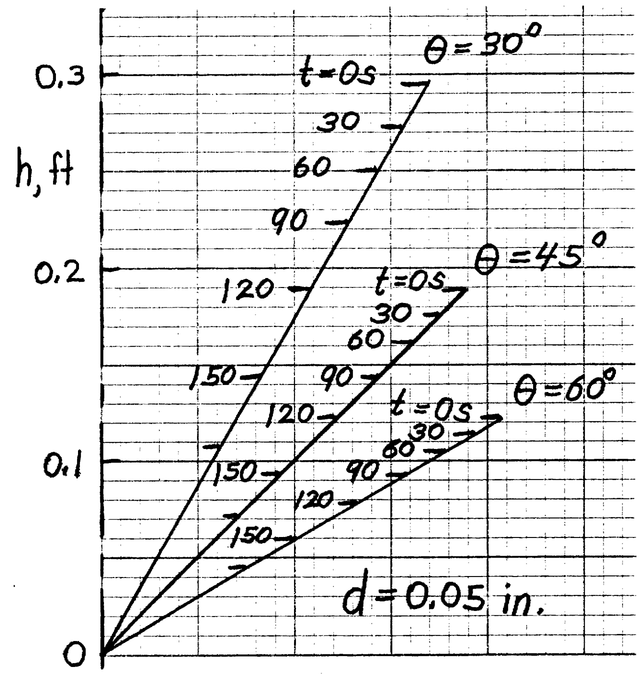
d = 0.05 in., $\theta = 45$ deg

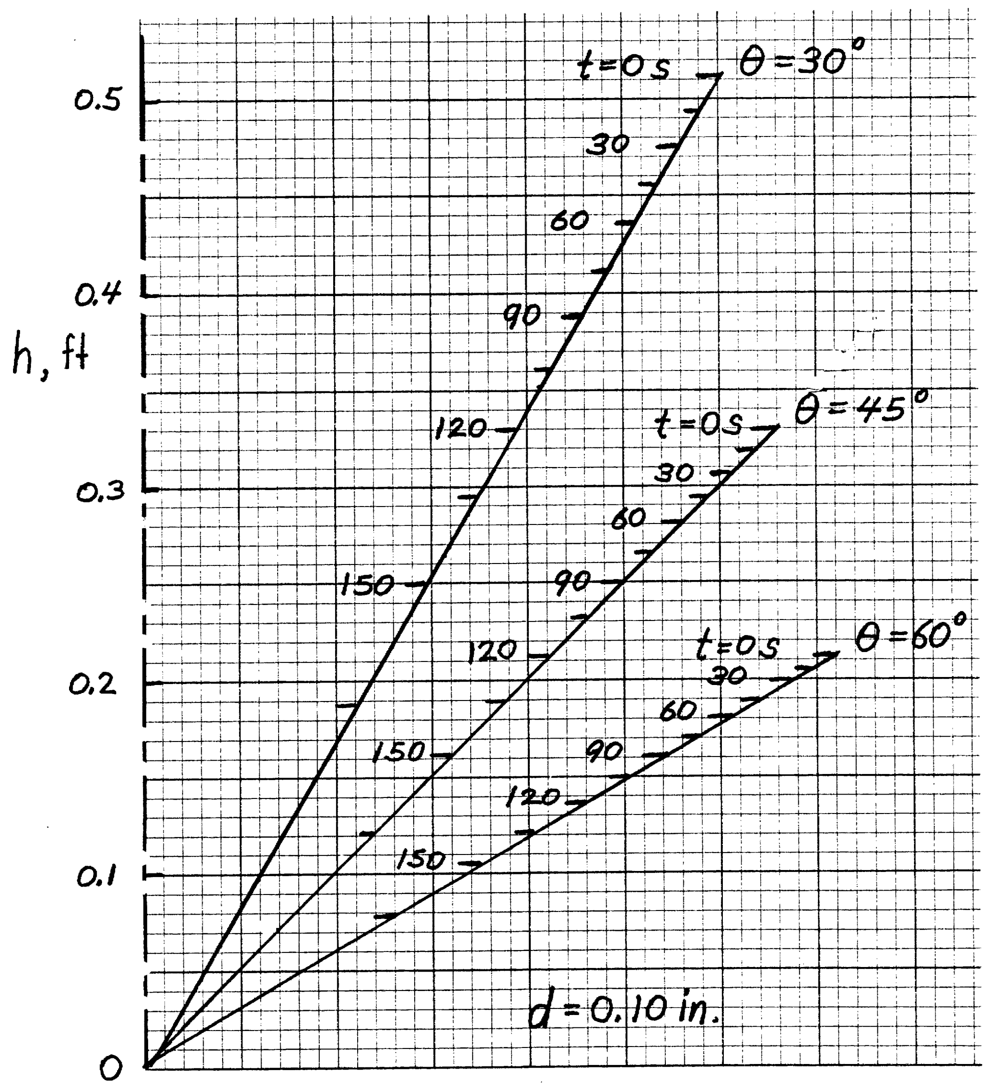
t, s	h, ft
0	0.122
15	0.118
30	0.114
45	0.109
60	0.104
75	0.098
90	0.093
105	0.086
120	0.079
135	0.070
150	0.060
165	0.045
180	0.000

(con't)

3.79* (con't)

The time interval markings for the six funnels ($d = 0.05$ in. or $d = 0.10$ in. and $\theta = 30, 45,$ or 60 degrees) are shown to scale below.





3.80

3.80 The surface area, A, of the pond shown in Fig. P3.80 varies with the water depth, h, as shown in the table. At time $t = 0$ a valve is opened and the pond is allowed to drain through a pipe of diameter D. If viscous effects are negligible and quasisteady conditions are assumed, plot the water depth as a function of time from when the valve is opened ($t = 0$) until the pond is drained for pipe diameters of $D = 0.5, 1.0, 1.5, 2.0, 2.5$, and 3.0 ft. Assume $h = 18$ ft at $t = 0$.

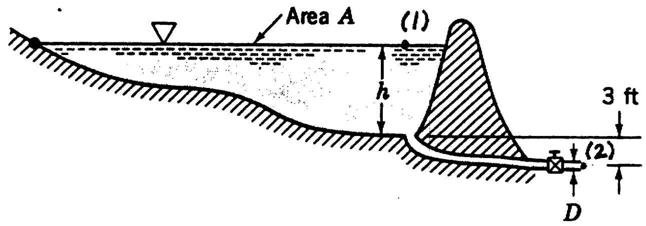


FIGURE P3.80

h (ft)	A [acres (1 acre = 43,560 ft^2)]
0	0
2	0.3
4	0.5
6	0.8
8	0.9
10	1.1
12	1.5
14	1.8
16	2.4
18	2.8

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = h,\ z_2 = -3\text{ft}$$

and $V_1 = -\frac{dh}{dt} \ll V_2$

Thus, $V_2 = \sqrt{2g(h+3)}$ which when combined with $A_1V_1 = A_2V_2$ gives

$$-A_1\frac{dh}{dt} = \frac{\pi}{4}D^2\sqrt{2g(h+3)}$$ where $A_1 = A_1(h)$ as given.

This can be rearranged and integrated to give

$$\int_{18\text{ ft}}^{h} A_1\frac{dh}{\sqrt{h+3}} = -\frac{\pi}{4}\sqrt{2g}\,D^2\int_0^t dt = -\frac{\pi}{4}D^2\sqrt{2g}\,t = -\frac{\pi}{4}D^2\sqrt{2 \times 32.2}\,t$$

or

$$t = \frac{0.159}{D^2}\int_h^{18} A_1\frac{dh}{\sqrt{h+3}} \quad , \text{ where } t \sim \text{s},\ A_1 \sim \text{ft}^2, \text{ and } h \sim \text{ft} \qquad (1)$$

Note: It is easier to determine t as a function of h rather than h as a function of t

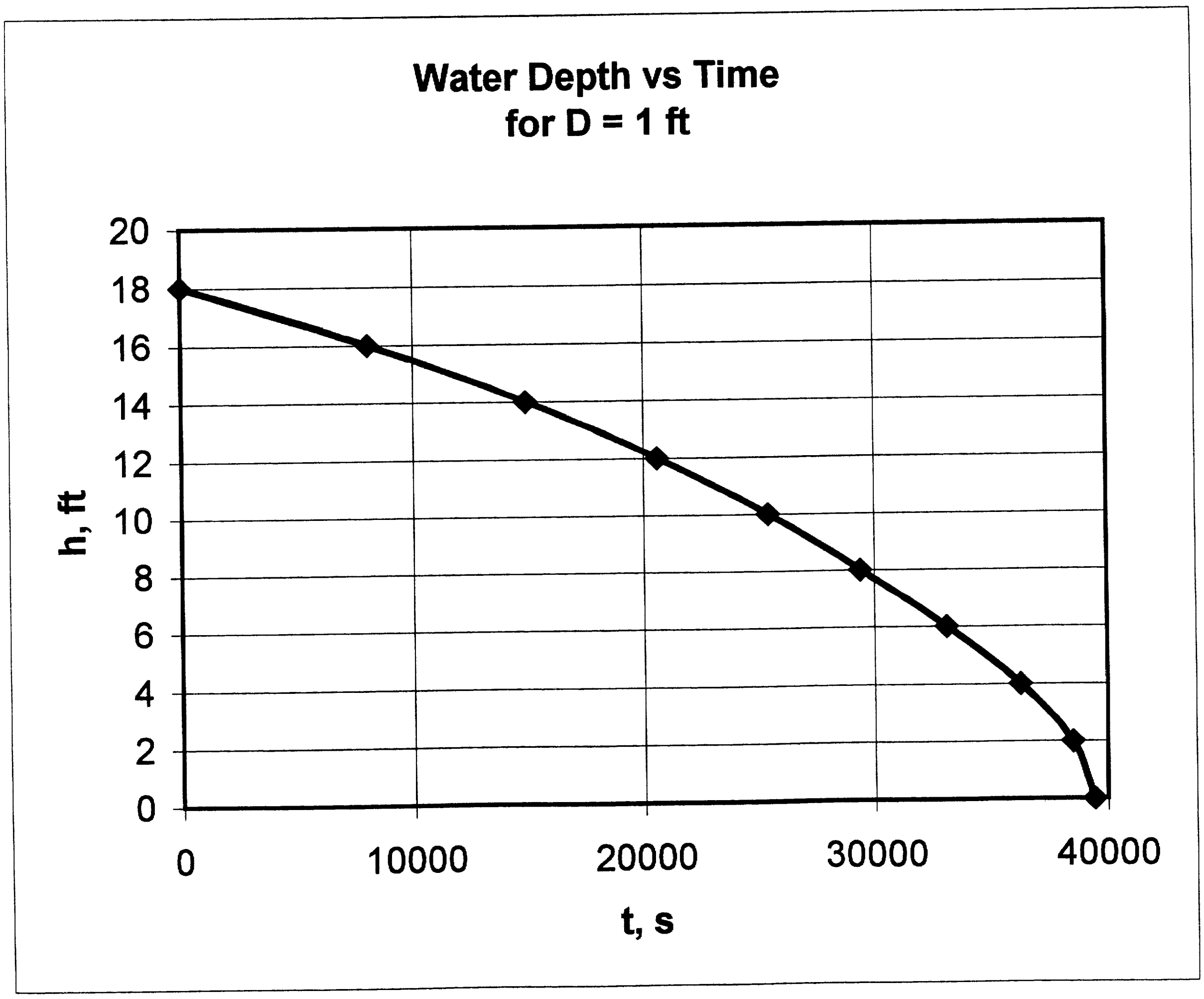
Note: $t \sim D^{-2}$

(con't)

An EXCEL Program using a trapezoidal integration approzimation was used to calculate the results shown below.

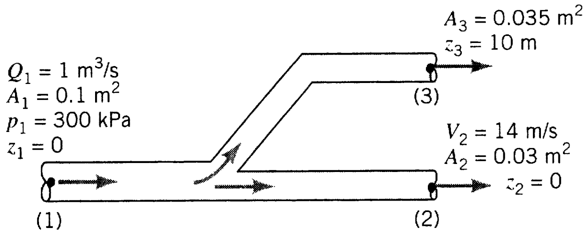
			D = 0.5 ft	D = 1.0 ft	D = 1.5 ft	D = 2.0 ft	D = 2.5 ft	D = 3.0 ft
h, ft	A, acres	A, ft^2	t, s	t, s	t, s	t, s	t, s	t, s
18	2.8	121968	0	0	0	0	0	0
16	2.4	104544	32181	8045	3576	2011	1287	894
14	1.8	78408	59530	14882	6614	3721	2381	1654
12	1.5	65340	82354	20589	9150	5147	3294	2288
10	1.1	47916	101536	25384	11282	6346	4061	2820
8	0.9	39204	117506	29377	13056	7344	4700	3264
6	0.8	34848	132412	33103	14712	8276	5296	3678
4	0.5	21780	145035	36259	16115	9065	5801	4029
2	0.3	13068	153988	38497	17110	9624	6160	4277
0	0	0	157704	39426	17523	9857	6308	4381

The graph for D = 1 ft is shown below. The shape of the curve is the same for any D.



3.81

3.81 Water flows through the branching pipe shown in Fig. P3.81. If viscous effects are negligible, determine the pressure at section (2) and the pressure at section (3).



Along the streamline from (1) to (2):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2=0 \text{ and}$$

$$V_1=\frac{Q_1}{A_1}=\frac{1\,m^3/s}{0.1\,m^2}=10\,\frac{m}{s}$$

Thus,

$$\frac{300\times10^3\,N/m^2}{9.80\times10^3\,N/m^3}+\frac{(10\,m/s)^2}{2(9.81\,m/s^2)}=\frac{p_2}{9.80\times10^3\,N/m^3}+\frac{(14\,m/s)^2}{2(9.81\,m/s^2)}$$

or

$$p_2=2.52\times10^5\,\frac{N}{m^2}=\underline{\underline{252\,kPa}}$$

Along the streamline from (1) to (3):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \quad \text{where since } Q_1=Q_2+Q_3 \text{ then} \qquad (1)$$

$$Q_3=A_3V_3=Q_1-Q_2=Q_1-A_2V_2 \quad \text{so that}$$

$$V_3=\frac{Q_1-A_2V_2}{A_3}=\frac{1\,m^3/s-0.03\,m^2(14\,m/s)}{0.035\,m^2}=16.6\,\frac{m}{s}$$

Thus, Eq. (1) becomes (with $z_1=0$, $z_3=10\,m$)

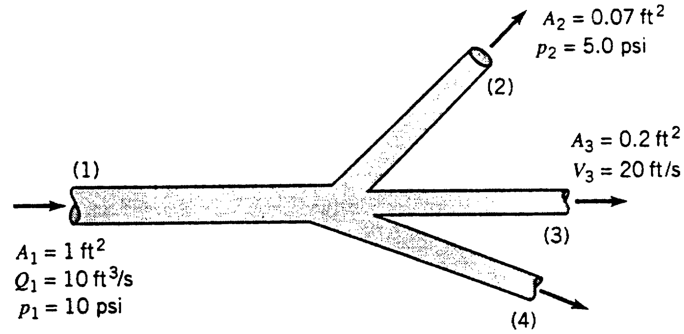
$$\frac{300\times10^3\,N/m^2}{9.80\times10^3\,N/m^3}+\frac{(10\,m/s)^2}{2(9.81\,m/s^2)}=\frac{p_3}{9.80\times10^3\,N/m^3}+\frac{(16.6\,m/s)^2}{2(9.81\,m/s^2)}+10\,m$$

or

$$p_3=1.14\times10^5\,\frac{N}{m^2}=\underline{\underline{114\,kPa}}$$

3.82

3.82 Water flows through the horizontal branching pipe shown in Fig. P3.82 at a rate of 10 ft³/s. If viscous effects are negligible, determine the water speed at section (2), the pressure at section (3), and the flowrate at section (4).



■ FIGURE P3.82

From (1) to (2): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_1 = 10\,psi$, $p_2 = 5\,psi$, and $V_1 = \frac{Q_1}{A_1}$ or $V_1 = (10\,\frac{ft^3}{s})/(1\,ft^2) = 10\,\frac{ft}{s}$

Thus, with $\gamma = \rho g$

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{(10\,\frac{ft}{s})^2}{2} = \frac{(5\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{V_2^2}{2} \text{ or } V_2 = \underline{\underline{29.0\,\frac{ft}{s}}}$$

From (1) to (3): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ where $z_1 = z_3$, $p_1 = 10\,psi$, $V_1 = 10\,\frac{ft}{s}$ and $V_3 = 20\,\frac{ft}{s}$

Thus,

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + \frac{(10\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = \frac{p_3}{62.4\,\frac{lb}{ft^3}} + \frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$p_3 = 1150\,\frac{lb}{ft^2} = \underline{\underline{7.98\,psi}}$$

Also,

$$Q_4 = Q_1 - Q_2 - Q_3 = Q_1 - A_2V_2 - A_3V_3$$

or

$$Q_4 = 10\,\frac{ft^3}{s} - 0.07\,ft^2(29.0\,\frac{ft}{s}) - 0.2\,ft^2(20\,\frac{ft}{s}) = \underline{\underline{3.97\,\frac{ft^3}{s}}}$$

3.83

3.83 Water flows from a large tank through a large pipe that splits into two smaller pipes as shown in Fig. P3.83. If viscous effects are negligible, determine the flowrate from the tank and the pressure at point (1).

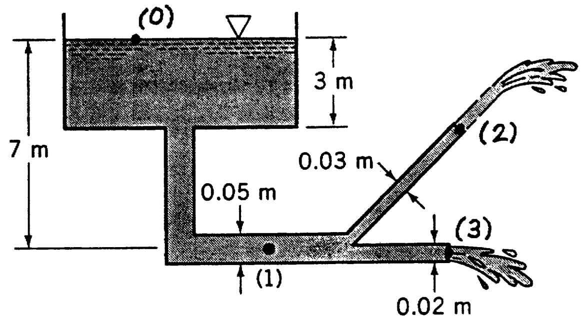


FIGURE P3.83

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_0=0,\ p_2=0,\ V_0=0,\ z_0=7m$$

and $z_2=4m$

Thus,

$$V_2=\sqrt{2g(z_0-z_2)}=\sqrt{2(9.81\tfrac{m}{s^2})(7-4)m}=7.67\tfrac{m}{s}$$

Similarly

$$V_3=\sqrt{2g(z_0-z_3)}=\sqrt{2(9.81\tfrac{m}{s^2})(7m)}=11.7\tfrac{m}{s}$$

Thus, $Q=Q_2+Q_3=\frac{\pi}{4}D_2^2V_2+\frac{\pi}{4}D_3^2V_3^2$

or

$$Q=\frac{\pi}{4}\left[(0.03m)^2(7.67\tfrac{m}{s})+(0.02m)^2(11.7\tfrac{m}{s})\right]=\underline{\underline{9.10\times10^{-3}\tfrac{m^3}{s}}}$$

Also,

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \quad \text{where } z_1=0 \text{ and}$$

or

$$V_1=\frac{Q}{A_1}=\frac{9.10\times10^{-3}\tfrac{m^3}{s}}{\frac{\pi}{4}(0.05m)^2}=4.63\tfrac{m}{s}$$

$$p_1=\gamma\left[z_0-\frac{V_1^2}{2g}\right]=9.80\times10^3\tfrac{N}{m^3}\left[7m-\frac{(4.63\tfrac{m}{s})^2}{2(9.81\tfrac{m}{s^2})}\right]=5.79\times10^4\tfrac{N}{m^2}$$

or

$$p_1=\underline{\underline{57.9\ kPa}}$$

3.84

3.84 Water flows through the horizontal Y-fitting shown in Fig. P3.84. If the flowrate and pressure in pipe (1) are $Q_1 = 2.3\ \text{ft}^3/\text{s}$ and $p_1 = 50\ \text{lb/in.}^2$, determine the pressures, p_2 and p_3, in pipes (2) and (3) under the assumption that the flowrate divides evenly between pipes (2) and (3).

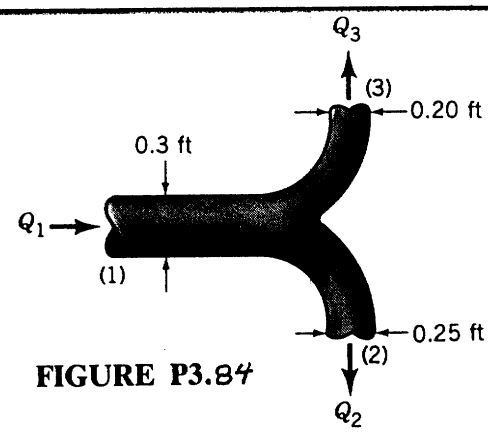


FIGURE P3.84

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_1 = \frac{Q_1}{A_1} \qquad (1)$$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.5\,Q_1}{A_2}$$

Thus, $V_1 = \dfrac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.3\,ft)^2} = 32.5\,\frac{ft}{s}$

and

$$V_2 = \frac{(0.5)(2.3\,\frac{ft^3}{s})}{\frac{\pi}{4}(0.25\,ft)^2} = 23.4\,\frac{ft}{s} \quad \text{so that Eq. (1) becomes}$$

$$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) = 50\text{ psi} + \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left[(32.5)^2 - (23.4)^2\right]\frac{ft^2}{s^2}$$

$$= 50\text{psi} + \left(493\,\frac{lb}{ft^2}\right)\left(\frac{1\,ft^2}{144\,in.^2}\right) = \underline{\underline{53.4\text{ psi}}}$$

Similarly

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{where } z_1 = z_3 \text{ and } V_3 = \frac{Q_3}{A_3} = \frac{0.5\,Q_1}{A_3}$$

Thus, $V_3 = \dfrac{(0.5)(2.3\,\frac{ft^3}{s})}{\frac{\pi}{4}(0.20\,ft)^2} = 36.6\,\frac{ft}{s}$

so that

$$p_3 = p_1 + \frac{1}{2}\rho(V_1^2 - V_3^2) = 50\text{psi} + \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left[(32.5)^2 - (36.6)^2\right]\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in.^2}\right)$$

$$= \underline{\underline{48.1\text{ psi}}}$$

3.85

3.85 Water flows from the pipe shown in Fig. P3.85 as a free jet and strikes a circular flat plate. The flow geometry shown is axisymmetrical. Determine the flowrate and the manometer reading, H.

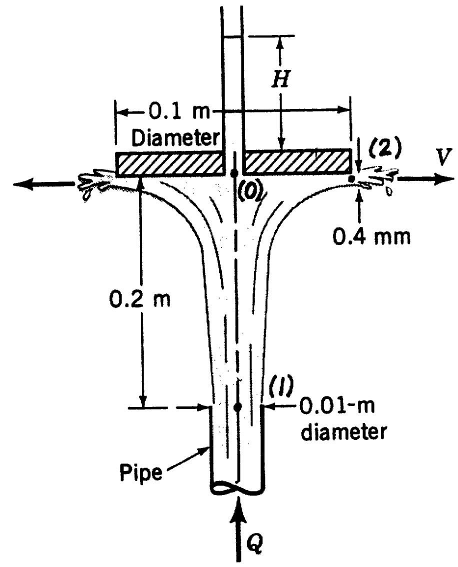


FIGURE P3.85

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\ , \text{ where } p_1=0,\ p_2=0,\ z_1=0, \text{ and } z_2=0.2\,m$$

Thus,

$$\frac{V_1^2}{2g}=\frac{V_2^2}{2g}+z_2 \quad \text{where} \quad A_1V_1=A_2V_2=Q \tag{1}$$

or $V_1=\frac{A_2}{A_1}V_2=\frac{\pi D_2 h}{\frac{\pi}{4}D_1^2}V_2=\frac{4D_2h}{D_1^2}V=\frac{4(0.1m)(4\times10^{-4}m)}{(0.01m)^2}V_2=1.6V_2$

Hence, Eq. (1) gives

$(1.60V_2)^2=V_2^2+2(9.81\frac{m}{s^2})(0.2m)$ or $V_2=1.59\frac{m}{s}$

so that

$Q=A_2V_2=\pi(0.1m)(4\times10^{-4}m)(1.59\frac{m}{s})=\underline{\underline{2.00\times10^{-4}\frac{m^3}{s}}}$

Also,

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0$, where $V_0=0$, $z_0=0.2m$, $V_1=1.60\,V_2$

or $V_1=1.60(1.59\frac{m}{s})=2.54\frac{m}{s}$, and $p_1=0$

Thus,

$H=\frac{p_0}{\gamma}=\frac{V_1^2}{2g}-z_0=\frac{(2.54\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}-0.2m=\underline{\underline{0.129\ m}}$

3.86

3.86 Air, assumed incompressible and inviscid, flows into the outdoor cooking grill through nine holes of 0.40-in. diameter as shown in Fig. P3.86. If a flowrate of 40 in.^{3}/s into the grill is required to maintain the correct cooking conditions, determine the pressure within the grill near the holes.

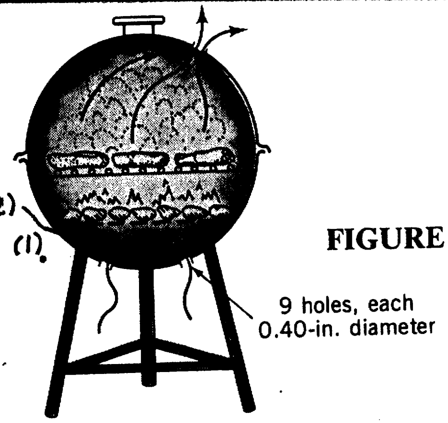


FIGURE P3.86

$$Q = 9A_2 V_2 \quad \text{where} \quad Q = \frac{40 \frac{in.^3}{s}}{1728 \frac{in.^3}{ft^3}} = 0.0231 \frac{ft^3}{s} \text{ and } A_2 = \frac{\pi}{4} D_2^2$$

Thus,

$$V_2 = \frac{Q}{9A_2} = \frac{4Q}{9\pi D_2^2} = \frac{4\left(0.0231 \frac{ft^3}{s}\right)}{9\pi \left(\frac{0.4}{12} ft\right)^2} = 2.94 \frac{ft}{s}$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad p_1 = 0,\ z_1 = z_2,\ \text{and } V_1 = 0$$

Thus,

$$p_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\left(2.38 \times 10^{-3} \frac{slugs}{ft^3}\right)\left(2.94 \frac{ft}{s}\right)^2 = -1.03 \times 10^{-2} \frac{lb}{ft^2}$$

or

$$p_2 = \underline{\underline{-7.14 \times 10^{-5}\ psi}}$$

3.87

3.87 A conical plug is used to regulate the air flow from the pipe shown in Fig. P3.87. The air leaves the edge of the cone with a uniform thickness of 0.02 m. If viscous effects are negligible and the flowrate is 0.50 m³/s, determine the pressure within the pipe.

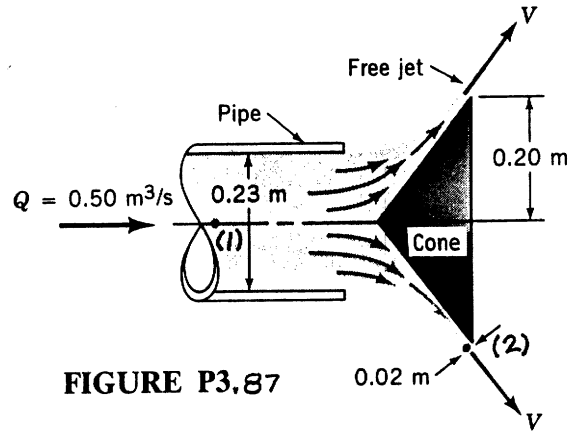


FIGURE P3.87

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad \text{where } z_1=z_2 \text{ and } p_2=0$$

Also,

$$V_1=\frac{Q}{A_1}=\frac{0.5\,\frac{m^3}{s}}{\frac{\pi}{4}(0.23m)^2}=12.0\,\frac{m}{s}$$

and

$$V_2=\frac{Q}{A_2}=\frac{Q}{2\pi Rh}=\frac{0.5\,\frac{m^3}{s}}{2\pi(0.2m)(0.02m)}=19.9\,\frac{m}{s}$$

Thus,

$$p_1=\frac{1}{2}\rho\left(V_2^2-V_1^2\right)=\frac{1}{2}\left(1.23\,\frac{kg}{m^3}\right)\left(19.9^2-12.0^2\right)\frac{m^2}{s^2}=\underline{\underline{155\,\frac{N}{m^2}}}$$

3.88

3.88 An air cushion vehicle is supported by forcing air into the chamber created by a skirt around the periphery of the vehicle as shown in Fig. P3.88. The air escapes through the 3-in. clearance between the lower end of the skirt and the ground (or water). Assume the vehicle weighs 10,000 lb and is essentially rectangular in shape, 30 by 50 ft. The volume of the chamber is large enough so that the kinetic energy of the air within the chamber is negligible. Determine the flowrate, Q, needed to support the vehicle. If the ground clearance were reduced to 2 in., what flowrate would be needed? If the vehicle weight were reduced to 5000 lb and the ground clearance maintained at 3 in., what flowrate would be needed?

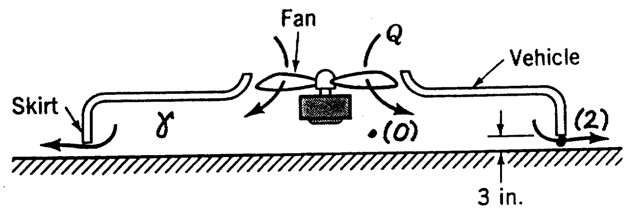


FIGURE P3.88

To support the load $p_0 = \frac{W}{A_0}$ where W = vehicle weight and $A_0 = (30ft)(50ft) = 1500\ ft^2$

Also,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_2 = 0,\ V_0 = 0, \text{ and } z_0 = z_2$$

so that

$$V_2 = \sqrt{\frac{2p_0}{\rho}} \quad \text{or} \quad V_2 = \sqrt{\frac{2W}{A_0\rho}}$$

With h = ground clearance it follows that

$$Q = A_2 V_2 = 2h(L+b)V_2 \quad \text{where } L = 50\ ft \text{ and } b = 30\ ft$$

Thus,

$$Q = 2h(50ft + 30ft)\sqrt{\frac{2W}{(1500\ ft^2)(2.38\times10^{-3}\ \frac{slug}{ft^3})}}$$

or

$$Q = 119.8\, h\sqrt{W}\ \frac{ft^3}{s} \quad \text{where } h \sim ft \text{ and } W \sim lb$$

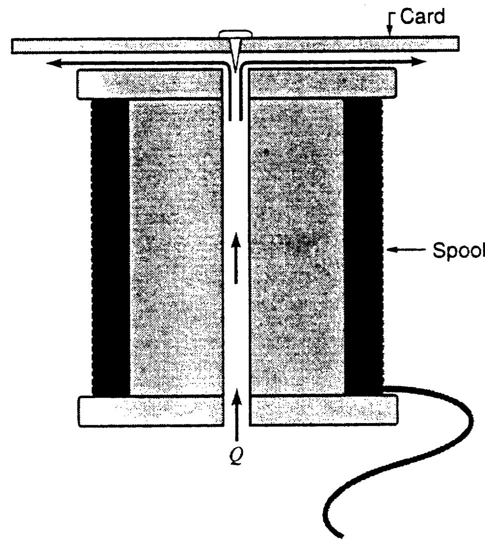
Thus, if $h = \frac{3}{12}$ ft and $W = 10{,}000$ lb, then $Q = \underline{\underline{3000\ \frac{ft^3}{s}}}$

if $h = \frac{2}{12}$ ft and $W = 10{,}000$ lb, then $Q = \underline{\underline{2000\ \frac{ft^3}{s}}}$

and if $h = \frac{3}{12}$ ft and $W = 5{,}000$ lb, then $Q = \underline{\underline{2120\ \frac{ft^3}{s}}}$

3.89

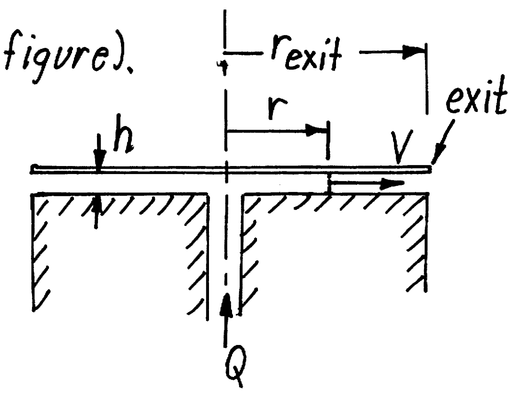
3.89 A small card is placed on top of a spool as shown in Fig. P3.89. It is not possible to blow the card off the spool by blowing air through the hole in the center of the spool. The harder one blows, the harder the card "sticks" to the spool. In fact, by blowing hard enough it is possible to keep the card against the spool with the spool turned upside down. (*Note:* It may be necessary to use a thumb tack to prevent the card from sliding from the spool.) Explain this phenomenon.



■ FIGURE P3.89

As the air flows radially outward in the gap between the card and the spool it slows down since the flow area increases with r, the radial distance from the center. That is,

$Q = 2\pi r h V$, or $V = \dfrac{Q}{2\pi h r}$ (see the figure). (1)



If viscous effects are not important, then

$$\frac{p}{\gamma} + \frac{V^2}{2g} = \text{constant} = \frac{p_{exit}}{\gamma} + \frac{V_{exit}^2}{2g}$$

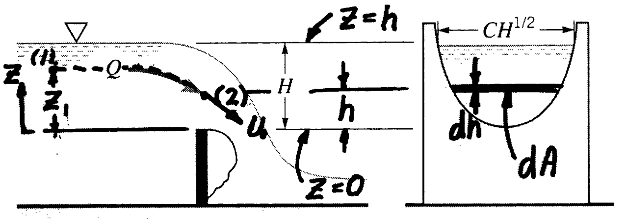
or since $p_{exit} = 0$ (a free jet) it follows that

$$p = \frac{1}{2}\rho\left(V_{exit}^2 - V^2\right), \text{ where from Eq. (1) } V_{exit}^2 - V^2 = \left(\frac{Q}{2\pi h}\right)^2\left[\frac{1}{r_{exit}^2} - \frac{1}{r^2}\right]$$

But $r_{exit} > r$ so that $p < 0$. There is a vacuum within the gap. The card is sucked against the spool. The harder one blows through the spool (larger Q), the larger the vacuum, and the harder the card is held against the spool.

3.90

3.90 Water flows over a weir plate (see Video V10.7) which has a parabolic opening as shown in Fig. P3.90. That is, the opening in the weir plate has a width $CH^{1/2}$, where C is a constant. Determine the functional dependence of the flowrate on the head, $Q = Q(H)$.



$Q = \int u \, dA$ where u is a function of h.

That is, from $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $\frac{p_1}{\gamma} = H - z_1$, $V_2 \equiv u$

$\frac{p_2}{\gamma} = 0$ ("free jet")

and $z_2 = H - h$

or

$(H - z_1) + \frac{V_1^2}{2g} + z_1 = 0 + \frac{u^2}{2g} + (H - h)$

Thus,

$u = \sqrt{2gh + V_1^2} \approx \sqrt{2gh}$ if V_1 is "small"

Also,

$dA = C\sqrt{z}\, dz$ (i.e. $dA = 0\, dz$ for $z = 0$; $dA = C\sqrt{H}$ for $z = H$) so that

$$Q = \int_{z=0}^{z=H} \sqrt{2g}\sqrt{h}\, C\sqrt{z}\, dz \quad \text{where } h = H - z.$$

Thus, $Q = C\sqrt{2g}\int_0^H \sqrt{zH - z^2}\, dz$, where

$$\int_0^H \sqrt{zH - H^2}\, dz = \frac{1}{2}\left[\left(z - \frac{H}{2}\right)\sqrt{Hz - z^2} + \left(\frac{H}{2}\right)^2 \sin^{-1}\left[\left(z - \frac{H}{2}\right)/(H/2)\right]\right]_{z=0}^{z=H}$$

which reduces to:

$Q = \frac{\pi C}{8}\sqrt{2g}\, H^2$ That is $\underline{\underline{Q \sim H^2}}$

Alternatively, $Q = VA$ where the average velocity is proportional to $\sqrt{H}$ (i.e. $V \sim \sqrt{2gH}$) and the total flow area is proportional to $H^{3/2}$ (i.e. $A \sim H \times (CH^{1/2}) = CH^{3/2}$). Thus,

$Q \sim \sqrt{2gH}\,(CH^{3/2}) = C\sqrt{2g}\, H^2$

That is, $Q \sim H^2$ as obtained above.

3.91

3.91 A weir (see Video V10.7) of trapezoidal cross section is used to measure the flowrate in a channel as shown in Fig. P3.91. If the flowrate is Q_0 when $H = \ell/2$, what flowrate is expected when $H = \ell$?

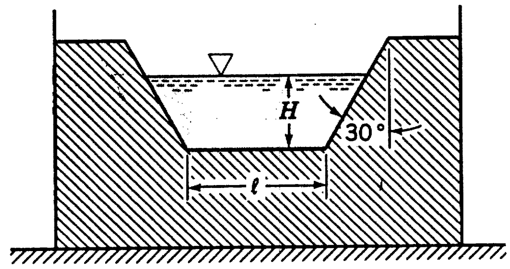


FIGURE P3.91

$Q = AV$ where it is expected that V is a function of the head, H. That is, $V \sim \sqrt{2gH}$

Also, from the geometry $A = \frac{1}{2}H(\ell + \ell_T)$ where $\ell_T = \ell + 2H\tan 30°$

Thus, $A = H(\ell + H\tan 30°)$ so that

$$Q = C_1\sqrt{2g}\,(\ell + H\tan 30°)\,H^{3/2}$$ where C_1 is a constant

Let Q_0 = flowrate when $H = \frac{\ell}{2}$

and Q_ℓ = flowrate when $H = \ell$

Thus,

$$\frac{Q_0}{Q_\ell} = \frac{C_1\sqrt{2g}\,(\ell + \frac{\ell}{2}\tan 30°)(\frac{\ell}{2})^{3/2}}{C_1\sqrt{2g}\,(\ell + \ell\tan 30°)(\ell)^{3/2}} = \frac{(1 + \frac{1}{2}\tan 30°)}{(1 + \tan 30°)(2^{3/2})} = 0.289$$

or

$$\underline{\underline{Q_\ell = 3.46\,Q_0}}$$

3.92

3.92 Water flows down the sloping ramp shown in Fig. P3.92 with negligible viscous effects. The flow is uniform at sections (1) and (2). For the conditions given show that three solutions for the downstream depth, h_2, are obtained by use of the Bernoulli and continuity equations. However, show that only two of these solutions are realistic. Determine these values.

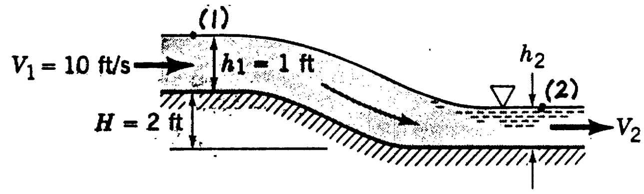


FIGURE P3.92

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 3\text{ ft}$, and $z_2 = h_2$

Also, $A_1V_1 = A_2V_2$

or

$$V_2 = \frac{h_1}{h_2}V_1 = \frac{(1\text{ ft})(10\frac{\text{ft}}{\text{s}})}{h_2} = \frac{10}{h_2}$$

Thus, Eq. (1) becomes

$$\frac{(10\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})} + 3\text{ ft} = \frac{\left(\frac{10}{h_2}\right)^2}{2(32.2\frac{\text{ft}}{\text{s}^2})} + h_2$$

or

$$64.4h_2^3 - 293h_2^2 + 100 = 0$$

By using a root finding program the three roots to this cubic equation are found to be:

$h_2 = 0.630\text{ ft}$

$h_2 = 4.48\text{ ft}$

or

$h_2 =$ a negative root

Clearly it is not possible (physically) to have $h_2 < 0$. Thus, $\underline{\underline{h_2 = 0.630\text{ ft}}}$ or $\underline{\underline{h_2 = 4.48\text{ ft}}}$

3.93

3.93 The flowrate in a water channel is sometimes determined by use of a device called a Venturi flume. As shown in Fig. P3.93, this device consists simply of a hump on the bottom of the channel. If the water surface dips a distance of 0.07 m for the conditions shown, what is the flowrate per width of the channel? Assume the velocity is uniform and viscous effects are negligible.

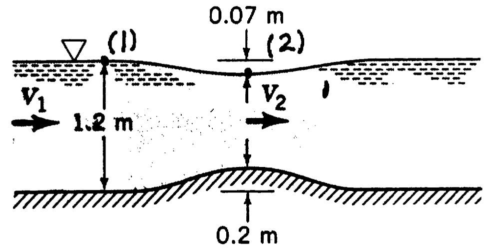


FIGURE P3.93

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

with $p_1 = 0$, $p_2 = 0$, $z_1 = 1.2\,m$, and $z_2 = 1.2\,m - 0.07\,m = 1.13\,m$

Also, $A_1V_1 = A_2V_2$

or

$$V_2 = \frac{h_1}{h_2}V_1 = \frac{1.2\,m \quad V_1}{(1.2-0.07-0.2)\,m} = 1.29\,V_1$$

Thus, from Eq. (1):

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 \quad \text{or} \quad \left[(1.29)^2 - 1\right]V_1^2 = 2(9.81\,\tfrac{m}{s^2})(1.2-1.13)\,m$$

or $V_1 = 1.438\,\frac{m}{s}$

Hence,

$$q = h_1V_1 = (1.438\,\tfrac{m}{s})(1.2\,m) = \underline{\underline{1.73\,\tfrac{m^2}{s}}}$$

3.94

3.94 Water flows in a rectangular channel that is 2.0 m wide as shown in Fig. P3.94. The upstream depth is 70 mm. The water surface rises 40 mm as it passes over a portion where the channel bottom rises 10 mm. If viscous effects are negligible, what is the flowrate?

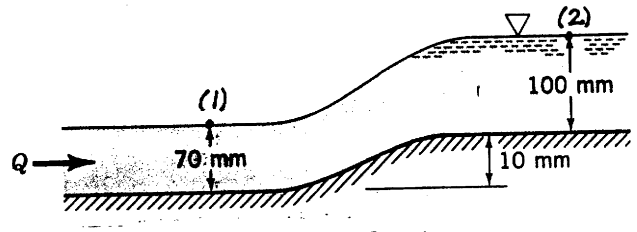


FIGURE P3.94

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 0.07 m$, and $z_2 = (0.01 + 0.10) m = 0.11 m$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{0.07 m}{0.10 m} V_1 = 0.7 V_1$$

Thus, Eq. (1) becomes

$$[1 - 0.7^2] V_1^2 = 2(9.81 \tfrac{m}{s^2})(0.11 - 0.07) m \quad \text{or} \quad V_1 = 1.24 \tfrac{m}{s}$$

Hence,

$$Q = A_1 V_1 = (0.07 m)(2.0 m)(1.24 \tfrac{m}{s}) = \underline{\underline{0.174 \tfrac{m^3}{s}}}$$

3.95

3.95 Water flows under the inclined sluice gate shown in Fig. P3.95. Determine the flowrate if the gate is 8 ft wide.

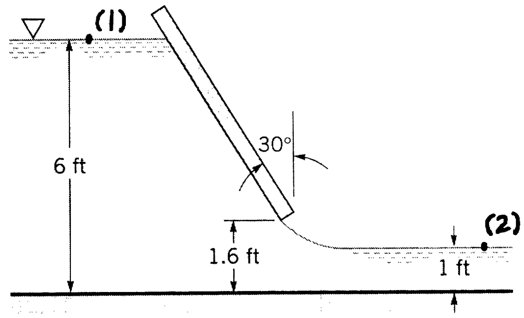


FIGURE P3.95

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $p_2 = 0$, $z_1 = 6\,ft$, and $z_2 = 1\,ft$

Thus,

$$\frac{V_1^2}{2g} + 6\,ft = \frac{V_2^2}{2g} + 1\,ft \qquad (1)$$

But $A_1 V_1 = A_2 V_2$, or

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{6\,ft}{1\,ft} V_1 = 6\,V_1$$

Hence, Eq. (1) becomes

$$\frac{V_1^2}{2g} + 6\,ft = \frac{(6)^2 V_1^2}{2g} + 1\,ft$$

or

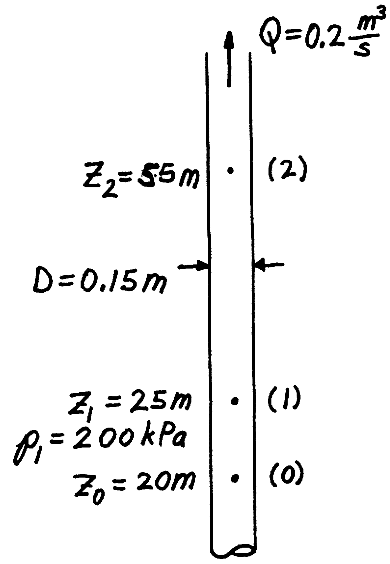
$$[6^2 - 1]V_1^2 = 2(32.2\,\tfrac{ft}{s^2})(6-1)\,ft \quad \text{or} \quad V_1 = 3.03\,\tfrac{ft}{s}$$

Hence,

$$Q = A_1 V_1 = 6\,ft\,(8\,ft)(3.03\,\tfrac{ft}{s}) = \underline{\underline{145\,\tfrac{ft^3}{s}}}$$

3.96

3.96 Water flows in a vertical pipe of 0.15-m diameter at a rate of 0.2 m³/s and a pressure of 200 kPa at an elevation of 25 m. Determine the velocity head and pressure head at elevations of 20 and 55 m.



$$V = \frac{Q}{A} = \frac{0.2 \frac{m^3}{s}}{\frac{\pi}{4}(0.15m)^2} = 11.3 \frac{m}{s} = V_0 = V_2$$

At point (0):

$$\frac{V_0^2}{2g} = \frac{(11.3 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{6.51 m}}$$

and

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_0}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_0$$

or

$$\frac{p_0}{\gamma} = \frac{200 \frac{kN}{m^2}}{9.80 \frac{kN}{m^3}} + (25-20)m = \underline{\underline{25.4 m}}$$

Similarly at point (2):

$$\frac{V_0^2}{2g} = \frac{V_2^2}{2g} = \underline{\underline{6.51 m}}$$

and

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2$$

or

$$\frac{p_2}{\gamma} = \frac{200 \frac{kN}{m^2}}{9.80 \frac{kN}{m^3}} + (25-55)m = \underline{\underline{-9.59 m}}$$

3.97

3.97 Draw the energy line and hydraulic grade line for the flow shown in Problem 3.64.

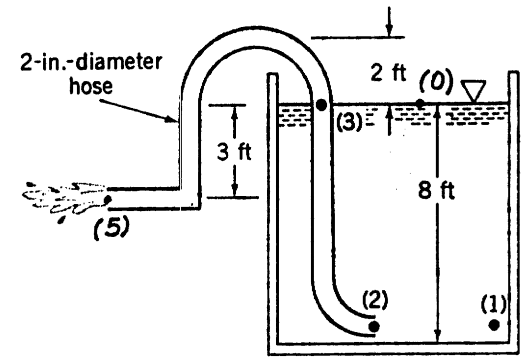
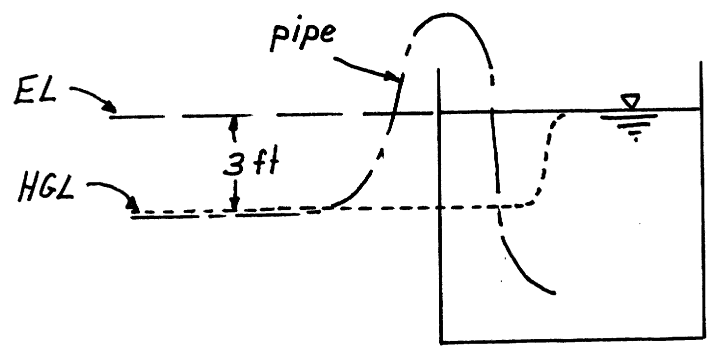


FIGURE P3.64

For inviscid flow with no pumps or turbines, the energy line (EL) is horizontal, at an elevation of the free surface. The hydraulic grade line (HGL) is one velocity head lower, even with the pipe outlet. Since the fluid velocity is constant throughout the pipe with $\frac{V^2}{2g} = 3\ \text{ft}$, the following is obtained:



3.98

3.98 Draw the energy line and the hydraulic grade line for the flow of Problem 3.60

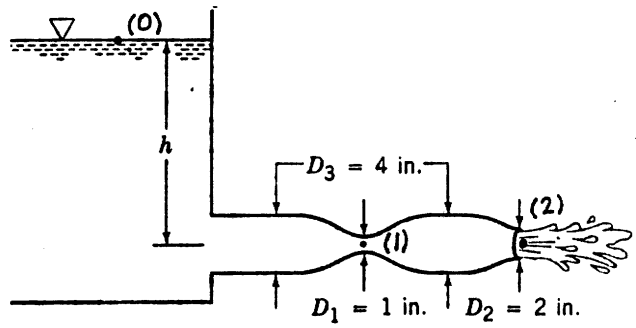


FIGURE P3.60

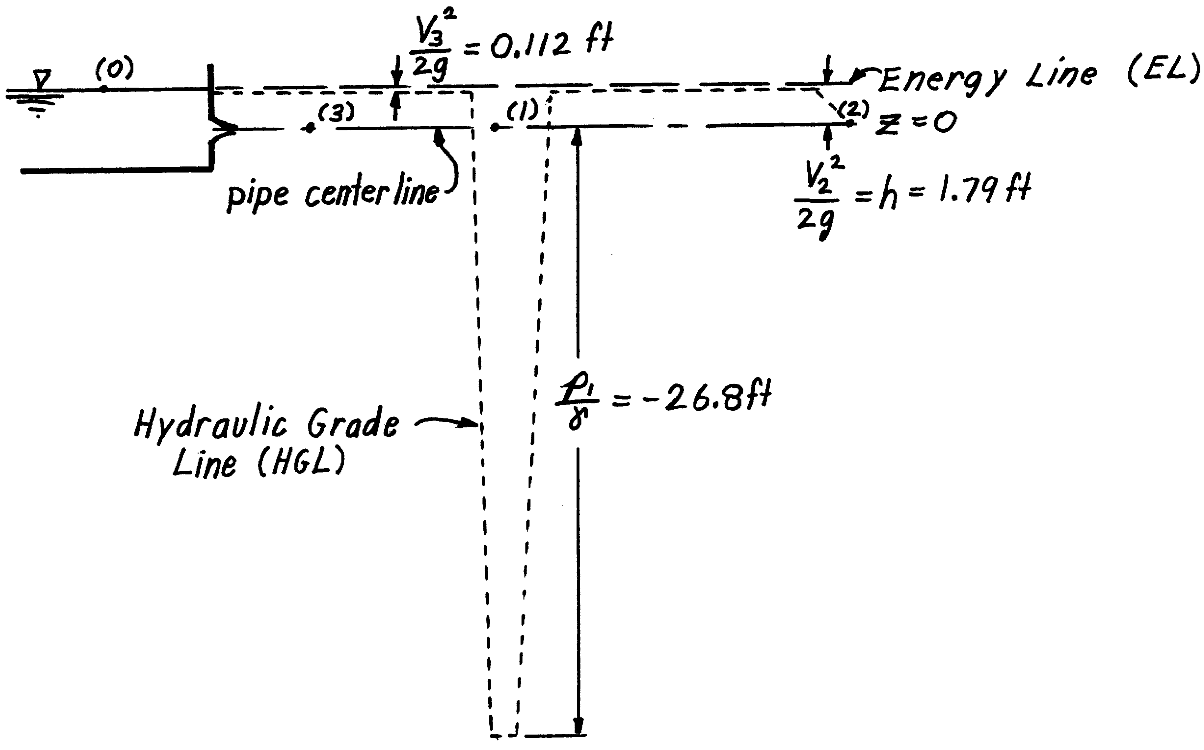
For inviscid flow with no pumps or turbines, the energy line is horizontal, a distance h above the outlet. From Problem 3.60 we obtain $h = 1.79$ ft.

The hydraulic grade line is $\frac{V^2}{2g}$ below the energy line, starting at the free surface where $V_0 = 0$ and ending at the pipe exit where $p_2 = 0$ and $\frac{V_2^2}{2g} = h$. At point (1) the pressure head is $p_1/\gamma = (2.88 - 14.5)\frac{\text{lb}}{\text{in.}^2}\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right)/62.4\,\frac{\text{lb}}{\text{ft}^3} = -26.8$ ft, and $z_1 = 0$.

In the 4 in. pipe $V_3 = A_2 V_2/A_3 = \left(\frac{D_2}{D_3}\right)^2 V_2$ so that

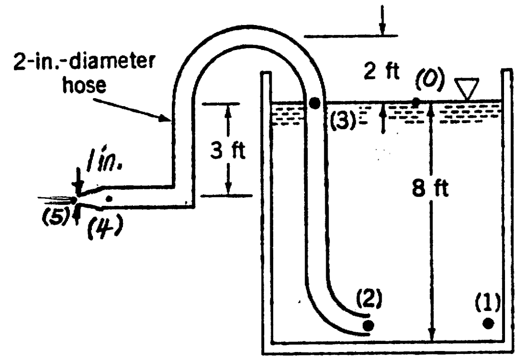
$$\frac{V_3^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 \frac{V_2^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 h = \left(\frac{2}{4}\right)^4 (1.79\,\text{ft}) = 0.112\text{ ft}$$

The corresponding EL and HGL are drawn to scale below.



3.99

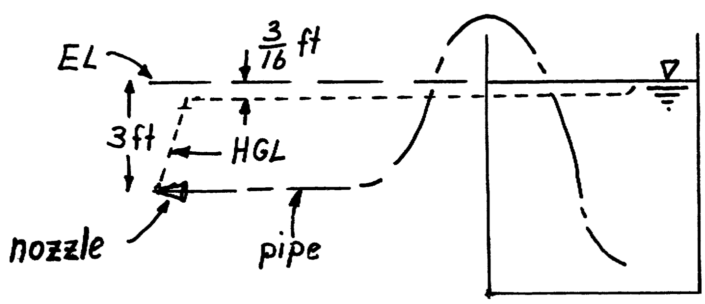
3.99 Draw the energy line and hydraulic grade line for the flow shown in Problem 3.65.



For inviscid flow with no pumps or turbines, the enery line (EL) is horizontal, at an elevation of the free surface. The hydraulic grade line (HGL) is one velocity head lower. Since $\frac{V_5^2}{2g} = 3\text{ ft}$ it follows that the HGL passes through the tip of the nozzle.

Also, since $V_4 = \frac{V_5 A_5}{A_4} = \left(\frac{D_5}{D_4}\right)^2 V_5$ it follows that

$\frac{V_4^2}{2g} = \left(\frac{D_5}{D_4}\right)^4 \frac{V_5^2}{2g} = \left(\frac{1}{2}\right)^4 (3\text{ft}) = \frac{3}{16}\text{ ft}$. Throughout the pipe the velocity head is constant so that the following is obtained:



3.100*

3.100* Water flows up the ramp shown in Fig. P3.100 with negligible viscous losses. The upstream depth and velocity are maintained at $h_1 = 0.3$ m and $V_1 = 6$ m/s. Plot a graph of the downstream depth, h_2, as a function of the ramp height, H, for $0 \le H \le 2$ m. Note that for each value of H there are three solutions, not all of which are realistic.

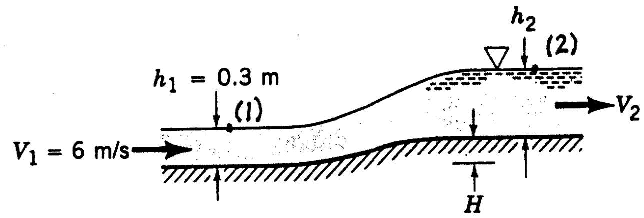


FIGURE P3.100

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 0.3m$, and $z_2 = H + h_2$

Also, $A_1V_1 = A_2V_2$ so that

$$V_2 = \frac{A_1}{A_2}V_1 = \frac{(0.3m)(6\frac{m}{s})}{h_2} = \frac{1.8}{h_2} \quad \text{where } h_2 \sim m$$

Thus, Eq. (1) becomes

$$\frac{V_1^2}{2g} + 0.3\,m = \frac{\left(\frac{1.8}{h_2}\right)^2}{2g} + (H + h_2) \quad \text{or with } V_1 = 6\frac{m}{s},$$

$$(6\tfrac{m}{s})^2 + 2(9.81\tfrac{m}{s^2})(0.3 - H - h_2)m = \left(\frac{1.8}{h_2}\right)^2 \frac{m^2}{s^2}$$

which can be written as:

$$h_2^3 - (2.135 - H)h_2^2 + 0.1651 = 0 \qquad (2)$$

For $0 \le H \le 2m$ solve Eq. (2) for h_2

Rather than solving a cubic equation for h_2 (give H), one can directly solve for H (given h_2). From Eq. (2):

$$H = 2.135 - h_2 - \frac{0.1651}{h_2^2} \qquad (3)$$

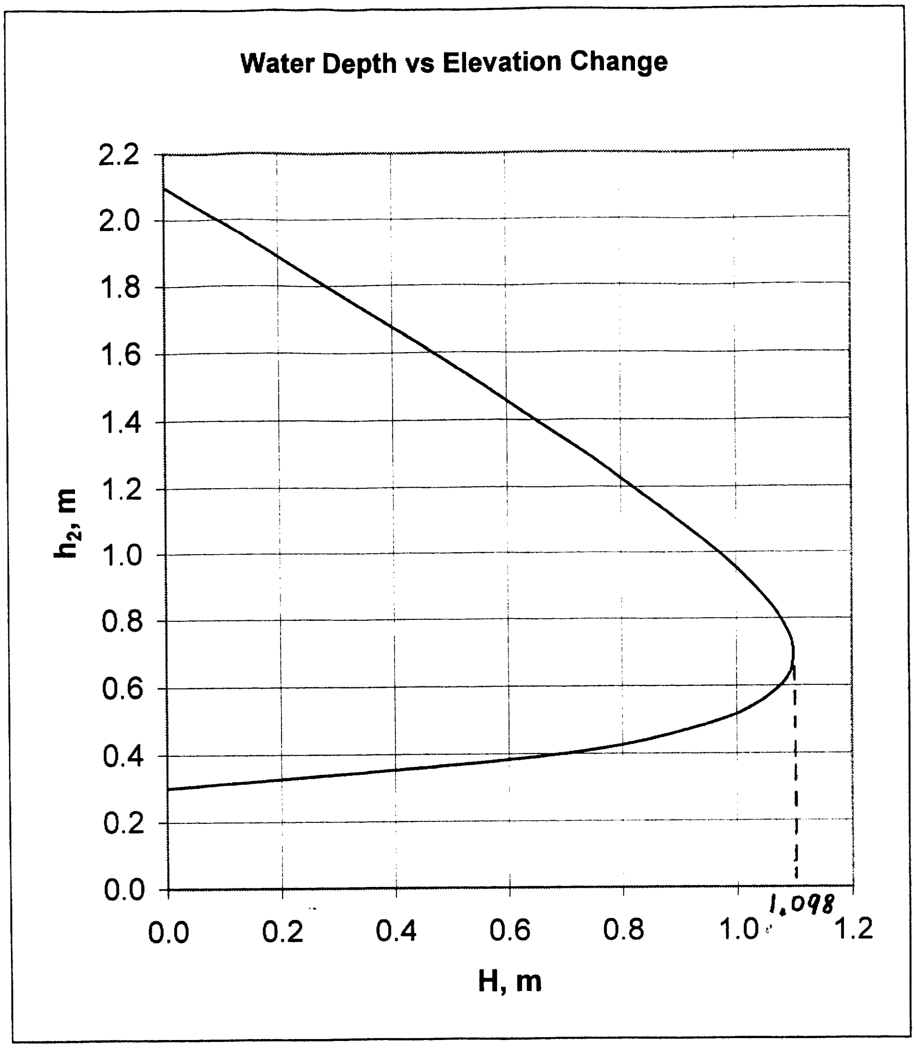
A graph of Eq. (2) or (3) is given on the following page.

(con't)

3.100* (con't)

The results of an EXCEL Program to calculate H for given values of h_2 are shown below.

h_2, m	H, m
0.3	0.001
0.4	0.703
0.5	0.975
0.6	1.076
0.7	1.098
0.8	1.077
0.9	1.031
1.0	0.970
1.1	0.899
1.2	0.820
1.3	0.737
1.4	0.651
1.5	0.562
1.6	0.471
1.7	0.378
1.8	0.284
1.9	0.189
2.0	0.094
2.1	-0.002



For $H \geq 1.098$ m there are no real, positive roots of Eq. (2). That is, for the given upstream conditions ($V_1 = 6 \frac{m}{s}$ and $h_1 = 0.3m$) we must have $H < 1.098$ m. It would not be possible to have the flow go up a ramp of greater height than this without increasing either V_1 and/or h_1. The two possible water depths for a given H are plotted below.

3.101 Pressure Distribution between Two Circular Plates

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing radially outward in the gap between two closely spaced flat plates as shown in Fig. P3.101.

Equipment: Air supply with a flow meter; two circular flat plates with static pressure taps at various radial locations from the center of the plates; spacers to maintain a gap of height b between the plates; manometer; barometer; thermometer.

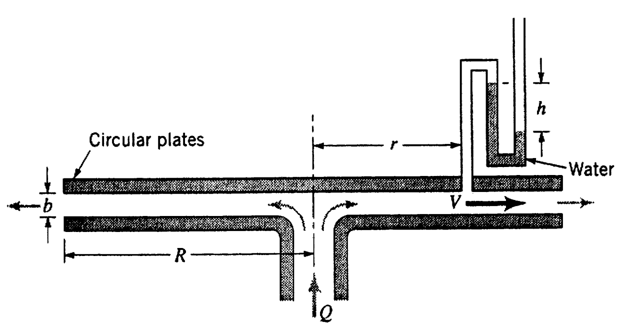
Experimental Procedure: Measure the radius, R, of the plates and the gap width, b, between them. Adjust the air supply to provide the desired, constant flowrate, Q, through the inlet pipe and the gap between the flat plates. Attach the manometer to the static pressure tap located a radial distance r from the center of the plates and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at different radial locations. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings to obtain the experimentally determined pressure distribution, $p = p(r)$, within the gap. That is, $p = -\gamma_m h$, where γ_m is the specific weight of the manometer fluid. Also use the Bernoulli equation ($p/\gamma + V^2/2g$ = constant) and the continuity equation (AV = constant, where $A = 2\pi rb$) to determine the theoretical pressure distribution within the gap between the plates. Note that the flow at the edge of the plates ($r = R$) is a free jet ($p = 0$). Also note that an increase in r causes an increase in A, a decrease in V, and an increase in p.

Graph: Plot the experimentally measured pressure head, p/γ, in feet of air as ordinates and radial location, r, as abscissas.

Results: On the same graph, plot the theoretical pressure head distribution as a function of radial location.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.101

(con't)

3.101 (con't)

Solution for Problem 3.101: Pressure Distribuition between Two Circular Plates

Q, ft^3/s	R, in.	b, in.	H_{atm}, in. Hg	T, deg F	γ_{H20}, lb/ft^3
0.879	5.0	0.125	29.09	83	62.4

		Experiment	Theory	
r, in.	h, in.	p/γ, ft	V, ft/s	p/γ, ft
0.7	-9.05	-663.75	220.8	-740.7
1.0	-6.02	-441.52	161.2	-387.2
1.5	-2.02	-148.15	107.4	-163.1
2.0	-0.96	-70.41	80.6	-84.7
2.5	-0.48	-35.20	64.5	-48.4
3.0	-0.24	-17.60	53.7	-28.7
3.5	-0.13	-9.53	46.0	-16.8
4.0	-0.03	-2.20	40.3	-9.1
4.5	-0.01	-0.73	35.8	-3.8
5.0	0.00	0.00	32.2	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.09/12ft) = 2053 lb/ft^2

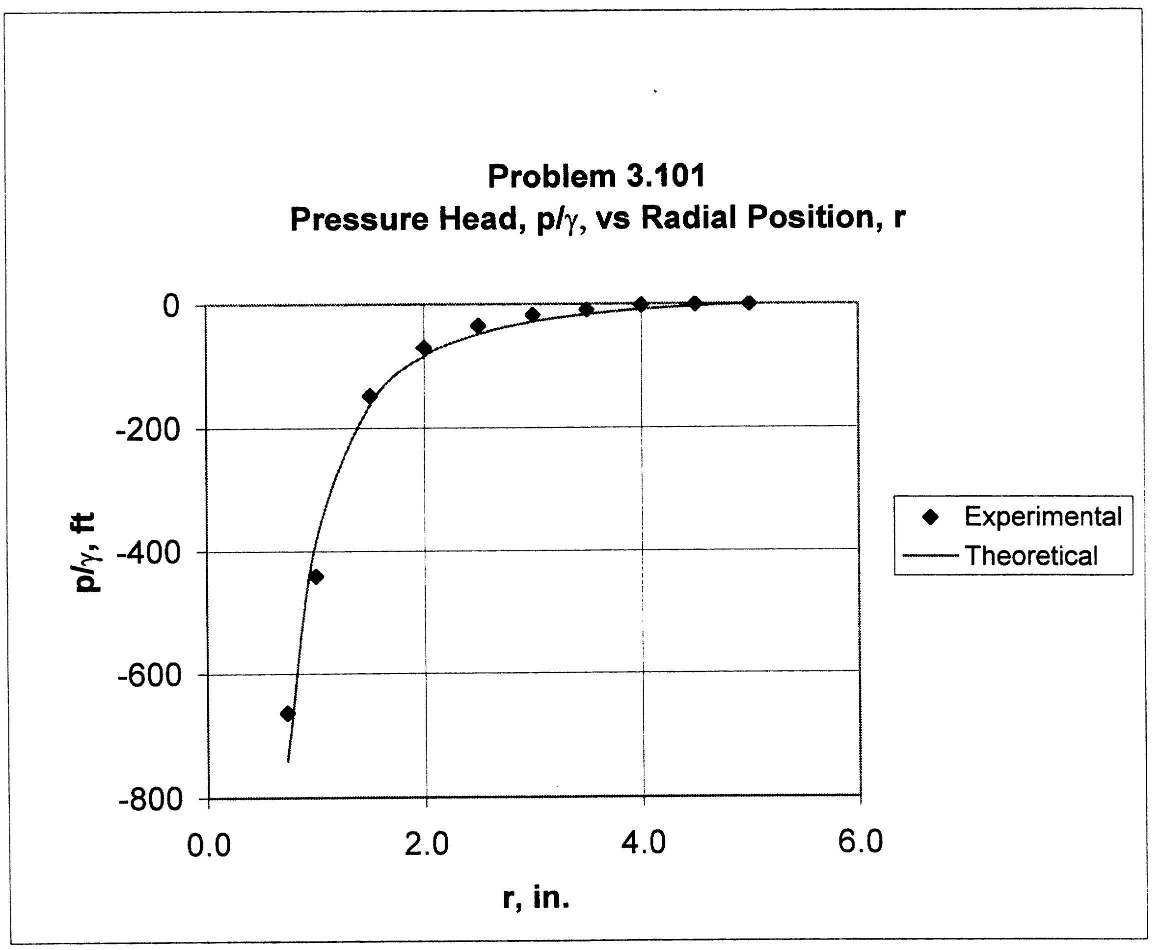
R = 1716 ft lb/slug deg R

T = 83 + 460 = 543 deg R

Thus, ρ = 0.00220 slug/ft^3 and γ = ρ*g = 0.00220*32.2 = 0.0709 lb/ft^3

$p/\gamma = \gamma_{H20} * h/\gamma$

V = Q/(2πrb) = 0.879 ft^s/(2*3.1415*(0.125/12)ft*r)



3.102 Calibration of a Nozzle Flow Meter

Objective: As shown in Section 3.6.3 of the text, the volumetric flowrate, Q, of a given fluid through a nozzle flow meter is proportional to the square root of the pressure drop across the meter. Thus, $Q = Kh^{1/2}$, where K is the meter calibration constant and h is the manometer reading that measures the pressure drop across the meter (see Fig. P3.102). The purpose of this experiment is to determine the value of K for a given nozzle flow meter.

Equipment: Pipe with a nozzle flow meter; variable speed fan; exit nozzle to produce a uniform jet of air; Pitot static tube; manometers; barometer; thermometer.

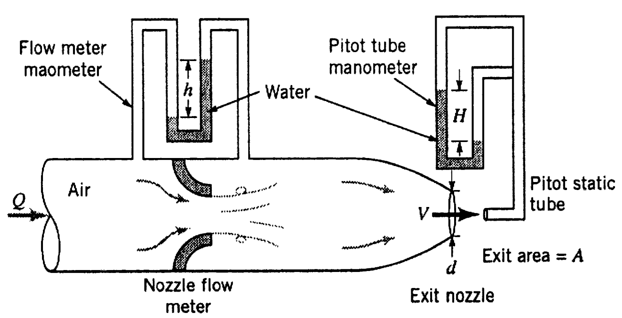
Experimental Procedure: Adjust the fan speed control to give the desired flowrate, Q. Record the flow meter manometer reading, h, and the Pitot tube manometer reading, H. Repeat the measurements for various fan settings (i.e., flowrates). Record the nozzle exit diameter, d. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated from the perfect gal law.

Calculations: For each fan setting determine the flowrate, $Q = VA$, where V and A are the air velocity at the exit and the nozzle exit area, respectively. The velocity, V, can be determined by using the Bernoulli equation and the Pitot tube manometer data, H (see Equation 3.16).

Graph: Plot flowrate, Q, as ordinates and flow meter manometer reading, h, as abscissas on a log-log graph. Draw the best-fit straight line with a slope of ½ through the data.

Results: Use your data to determine the calibration constant, K, in the flow meter equation $Q = Kh^{1/2}$.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.102

(con't)

Solution for Problem 3.102: Calibration of a Nozzle Flow Meter

d, in.	H_{atm}, in. Hg	T, deg F
1.169	29.01	75

h, in.	H, in.	Δp, lb/ft^2	V, ft/s	Q, ft^3/s
11.6	5.6	29.1	162	1.20
11.1	5.4	28.1	159	1.18
10.7	5.2	27.0	156	1.16
10.1	4.9	25.5	151	1.13
9.6	4.7	24.4	148	1.10
8.8	4.3	22.4	142	1.06
7.9	3.9	20.3	135	1.00
7.2	3.6	18.7	130	0.97
6.1	3.1	16.1	120	0.90
5.4	2.7	14.0	112	0.84
4.5	2.3	12.0	104	0.77
3.8	2.0	10.4	97	0.72
2.9	1.5	7.8	84	0.62
2.1	1.1	5.7	72	0.53
1.0	0.6	3.1	53	0.39

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.01/12 ft) = 2048 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

Thus, ρ = 0.00223 slug/ft^3

$V = (2*\Delta p/\rho)^{1/2}$

Q = AV where

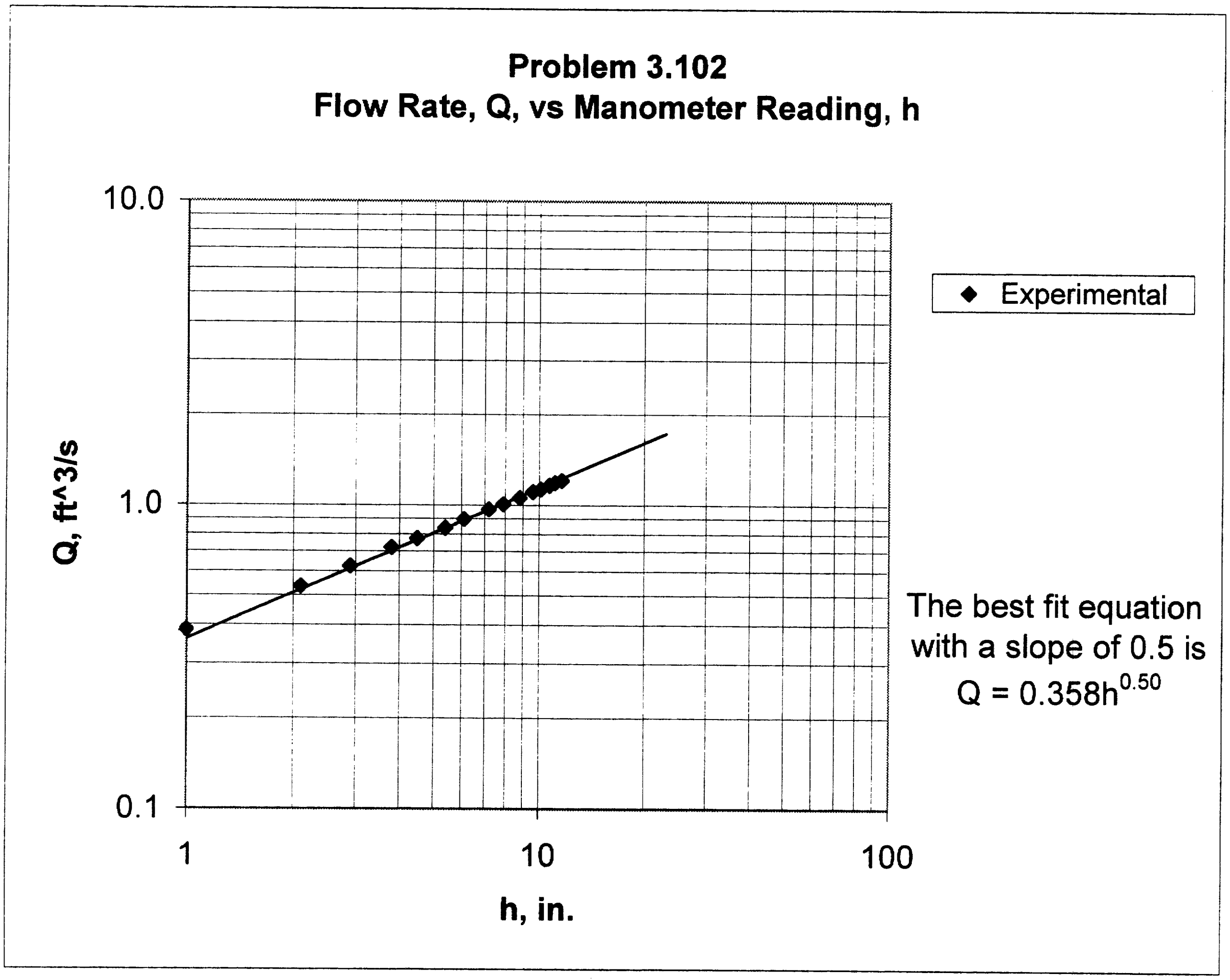
$A = \pi d^2/4 = \pi$*(1.169/12 ft)^2/4 = 7.45E-3 ft^2

From the graph, $Q = K\, h^{1/2} = 0.358\, h^{1/2}$ where Q is in ft^3/s and h is in in.

Thus, K = $\underline{0.358\ ft^3/(s*in.^{1/2})}$

(con't)

3.102 (con't)



Problem 3.102
Flow Rate, Q, vs Manometer Reading, h
10.0
1.0
0.1
Q, ft^3/s
1
10
100
h, in.
Experimental
The best fit equation with a slope of 0.5 is
Q = 0.358h^0.50

3.103 Pressure Distribution in a Two-Dimensional Channel

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing within a two-dimensional, variable area channel as shown in Fig. P3.103.

Equipment: Air supply with a flow meter; two-dimensional channel with one curved side and one flat side; static pressure taps at various locations along both walls of the channel; ruler; manometer; barometer; thermometer.

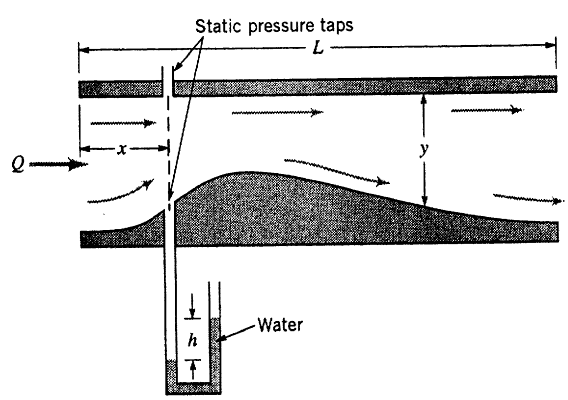
Experimental Procedure: Measure the constant width, b, of the channel and the channel height, y, as a function of distance, x, along the channel. Adjust the air supply to provide the desired, constant flowrate, Q, through the channel. Attach the manometer to the static pressure tap located a distance, x, from the origin and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at various locations on both the flat and the curved sides of the channel. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to calculate the pressure within the channel, $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Convert this pressure into the pressure head, p/γ, where $\gamma = g\rho$ is the specific weight of air. Also use the Bernoulli equation ($p/\gamma + V^2/2g$ = constant) and the continuity equation ($AV = Q$, where $A = yb$) to determine the theoretical pressure distribution within the channel. Note that the air leaves the end of the channel ($x = L$) as a free jet ($p = 0$).

Graph: Plot the experimentally determined pressure head, p/γ, as ordinates and the distance along the channel, x, as abscissas. There will be two curves—one for the curved side of the channel and another for the flat side.

Results: On the same graph, plot the theoretical pressure distribution within the channel.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.103

(con't)

3.103 (con't)

Solution for Problem 3.103: Pressure Distribution in a Two-Dimensional Channel

b, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F	L, in.
2.0	1.32	28.96	71	21.75

x, in.	y, in.	h, in. flat side	h, in. curved side	Experimental p/γ, ft flat side	Experimental p/γ, ft curved side	Theory p/γ, ft
0.75	2.00	0.28	0.31	20.2	22.3	0.0
2.50	2.00	0.21	0.37	15.1	26.6	0.0
4.00	1.28	-0.42	0.03	-30.2	2.3	-50.5
4.63	1.05	-0.77	-1.63	-55.5	-117.4	-92.2
5.38	1.05	-1.01	-1.05	-72.7	-75.6	-92.2
8.14	1.29	-0.63	-0.62	-45.4	-44.7	-49.2
10.75	1.54	-0.32	-0.31	-23.0	-22.3	-24.1
13.25	1.77	-0.15	-0.15	-10.8	-10.8	-9.7
15.78	2.00	-0.05	0.00	-3.6	0.0	0.0
21.75	2.00	0.00	0.00	0.0	0.0	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} \cdot H_{atm}$ = 847 lb/ft^3*(28.96/12 ft) = 2044 lb/ft^2

R = 1716 ft lb/slug deg R

T = 71 + 460 = 531 deg R

Thus, ρ = 0.00224 slug/ft^3 and $\gamma = \rho$*g = 0.00224 slug/ft^3*(32.2 ft/s^2) = 0.0722 lb/ft^3

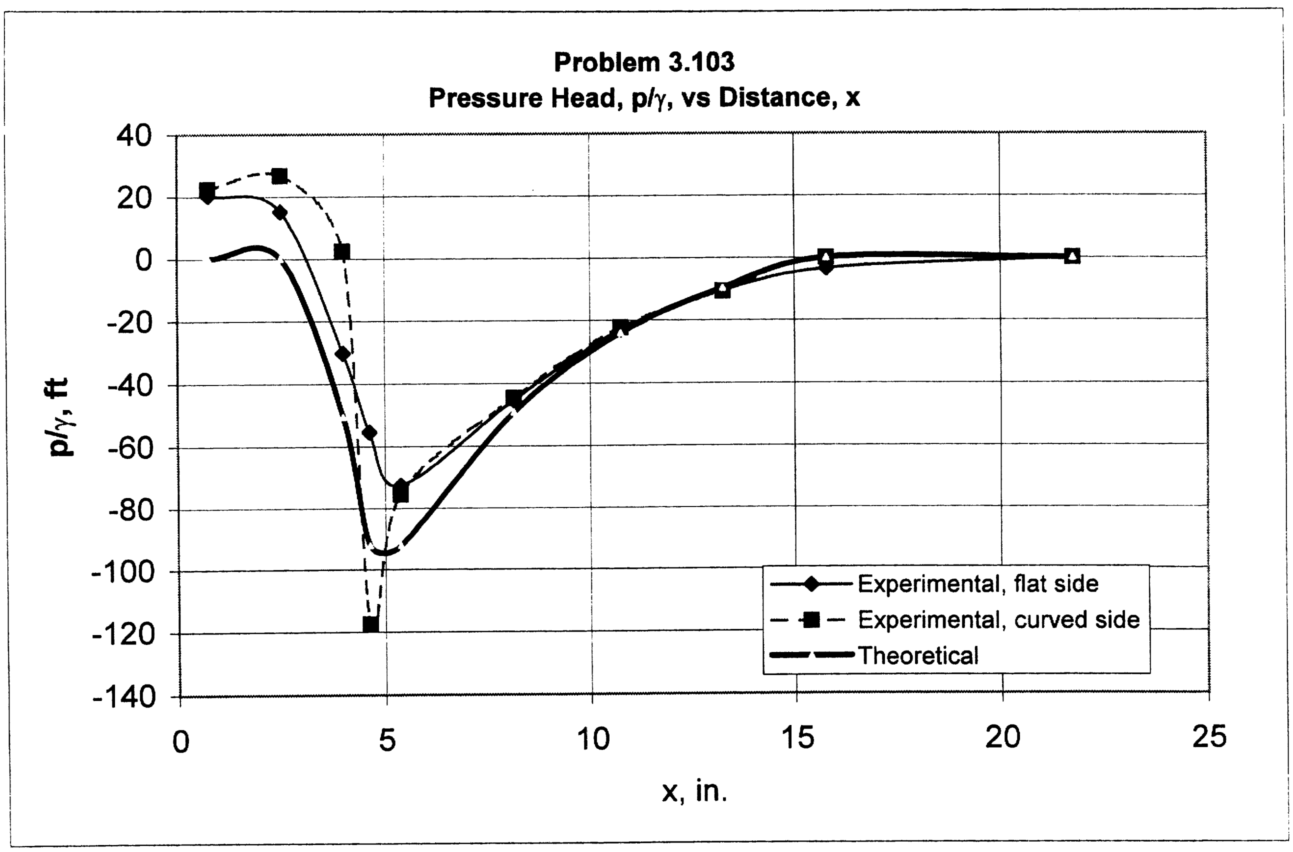
$p/\gamma = \gamma_{H2O}*h/\gamma$

Theoretical:

$p/\gamma = V_{exit}^2/2g - V^2/2g$ where

V = Q/A = Q/(b*y) and

$V_{exit} = Q/A_{exit}$ = (1.32 ft^3/s/)*(2 *2 /144 ft^2) = 47.5 ft/s



3.104

3.104 Sluice Gate Flowrate

Objective: The flowrate of water under a sluice gate as shown in Fig. P3.104 is a function of the water depths upstream and downstream of the gate. The purpose of this experiment is to compare the theoretical flowrate with the experimentally determined flowrate.

Equipment: Flow channel with pump and control valve to provide the desired flowrate in the channel; sluice gate; point gage to measure water depth; float; stop watch.

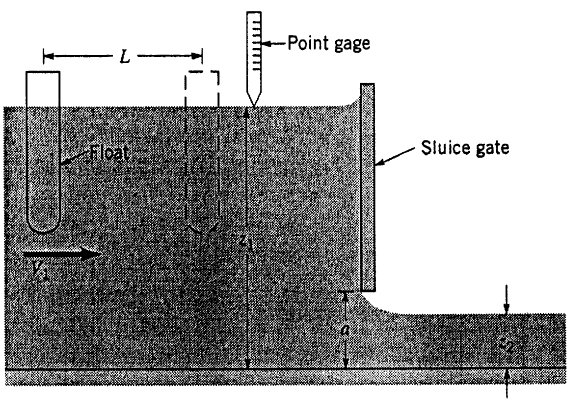
Experimental Procedure: Adjust the vertical position of the sluice gate so that the bottom of the gate is the desired distance, a, above the channel bottom. Measure the width, b, of the channel (which is equal to the width of the gate). Turn on the pump and adjust the control valve to produce the desired water depth upstream of the sluice gate. Insert a float into the water upstream of the gate and measure the water velocity, V_1, by recording the time, t, it takes the float to travel a distance L. That is, $V_1 = L/t$. Use a point gage to measure the water depth, z_1, upstream of the gate. Adjust the control valve to produce various water depths upstream of the gate and repeat the measurements.

Calculations: For each water depth used, determine the flowrate, Q, under the sluice gate by using the continuity equation $Q = A_1V_1 = b\,z_1V_1$. Use the Bernoulli and continuity equations to determine the theoretical flowrate under the sluice gate (see Equation 3.21). For these calculations assume that the water depth downstream of the gate, z_2, remains at 61% of the distance between the channel bottom and the bottom of the gate. That is, $z_2 = 0.61a$.

Graph: Plot the experimentally determined flowrate, Q, as ordinates and the water depth, z_1, upstream of the gate as abscissas.

Results: On the same graph, plot the theoretical flowrate as a function of water depth upstream of the gate.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.104

(con't)

3.104 (con't)

Solution for problem 3.104: Sluice Gate Flowrate

a, in.	b, in.	L, ft	z_2, ft
1.2	6.0	4.0	0.061

		Experimental		Theoretical
z_1, ft	t, s	V_1, ft/s	Q, ft^3/s	Q, ft^3/s
0.183	4.2	0.952	0.087	0.091
0.267	5.0	0.800	0.107	0.114
0.343	5.2	0.769	0.132	0.132
0.453	6.2	0.645	0.146	0.155
0.569	6.4	0.625	0.178	0.175
0.725	7.0	0.571	0.207	0.200
0.877	8.6	0.465	0.204	0.222

Experimental:

$V_1 = L/t$

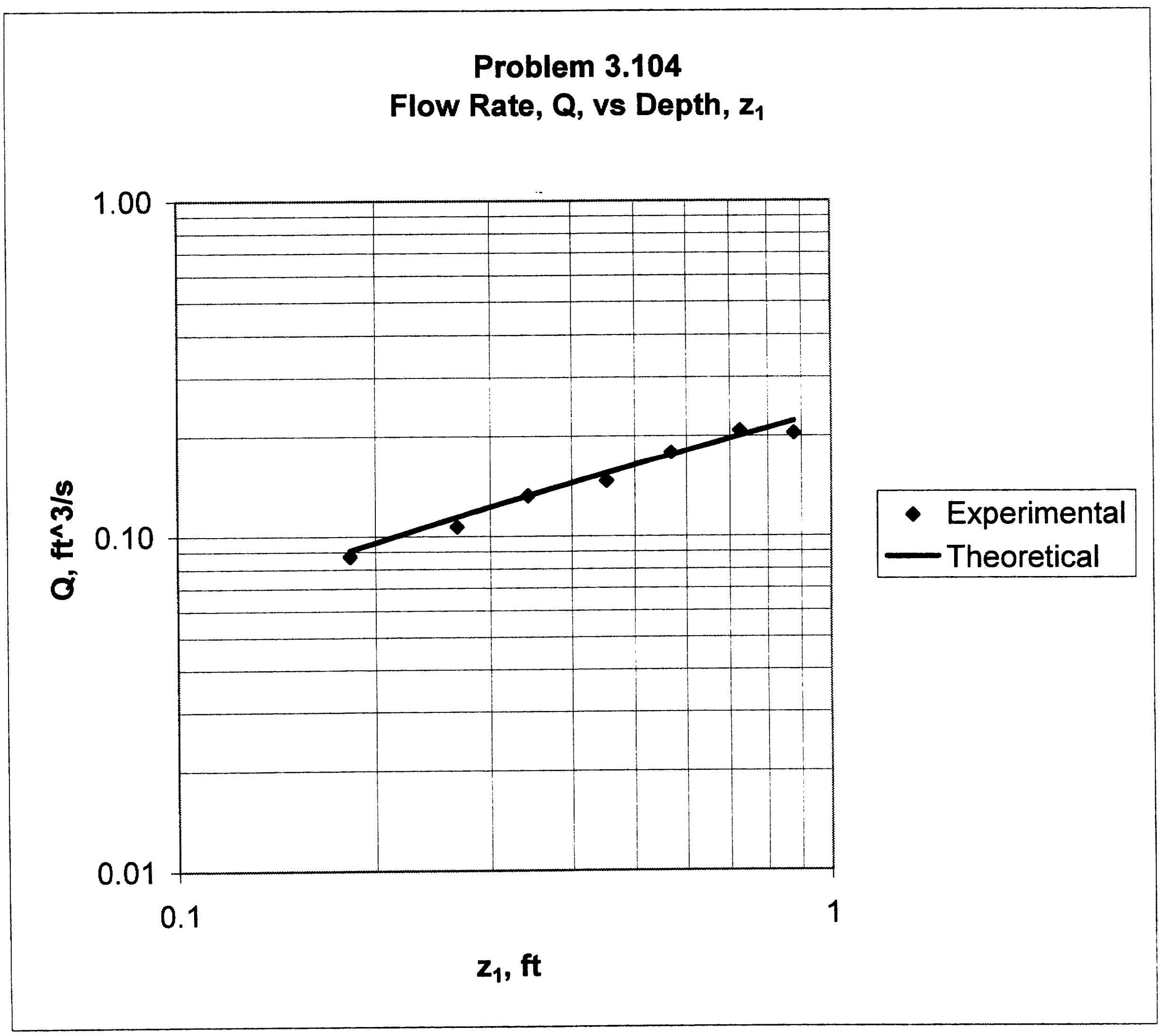
$Q = V_1 b z_1$

Theoretical:

$Q = b*z_2^{3/2}*(2*g)^{1/2}*[((z_1/z_2) - 1)/(1 - (z_2/z_1)^2)]^{1/2}$

where

$z_2 = 0.61*a$



4.1

4.1 The velocity field of a flow is given by $\mathbf{V} = (3y + 2)\hat{\mathbf{i}} + (x - 8)\hat{\mathbf{j}} + 5z\hat{\mathbf{k}}$ ft/s, where x, y, and z are in feet. Determine the fluid speed at the origin ($x = y = z = 0$) and on the y axis ($x = z = 0$).

$u = 3y+2$, $v = x-8$, $w = 5z$

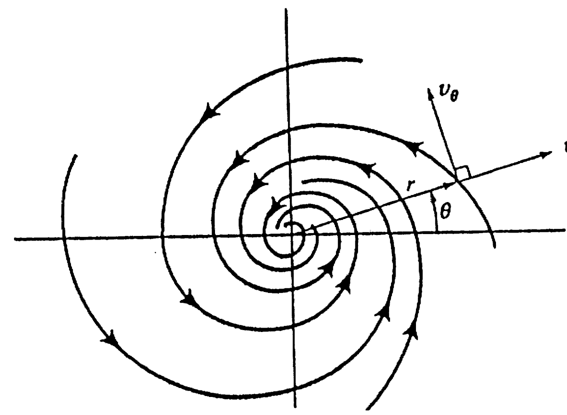
Thus, at $x=y=z=0$ $\quad V = \sqrt{u^2+v^2+w^2} = \sqrt{2^2+(-8)^2} = \underline{\underline{8.25\,\frac{ft}{s}}}$

and on the line $x=z=0$,

$V = \sqrt{(3y+2)^2+(-8)^2} = \underline{\underline{\sqrt{9y^2+12y+68}\,\frac{ft}{s}}}$ where $y \sim ft$

4.2

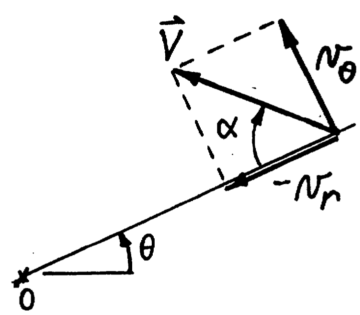
4.2 A flow can be visualized by plotting the velocity field as velocity vectors at representative locations in the flow as shown in Video V4.1 and Fig. E4.1. Consider the velocity field given in polar coordinates by $v_r = -10/r$ and $v_\theta = 10/r$. This flow approximates a fluid swirling into a sink as shown in Fig. P4.2. Plot the velocity field at locations given by $r = 1$, 2, and 3 with $\theta = 0$, 30, 60, and 90 deg.



■ FIGURE P4.2

With $v_r = -10/r$ and $v_\theta = 10/r$ then

$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-10/r)^2 + (10/r)^2} = \frac{14.14}{r}$



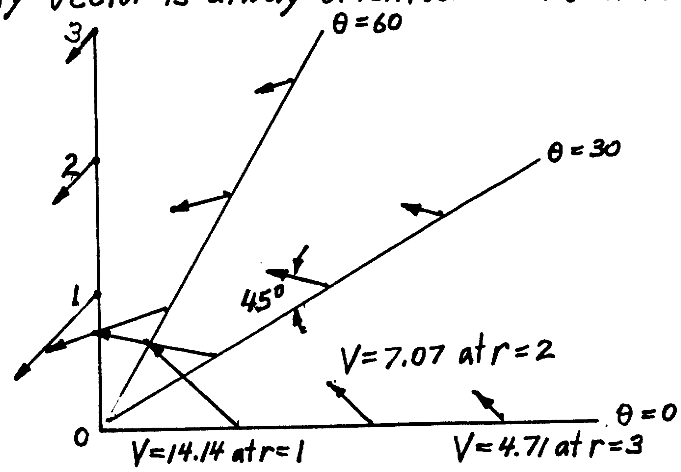
The angle α between the radial direction and the velocity vector is given by

$\tan\alpha = \frac{v_\theta}{-v_r} = \frac{10/r}{-(-10/r)} = 1$

Thus, $\alpha = 45°$ for any r, θ

(i.e. the velocity vector is alway oriented 45° relative to radial lines)

Note: V is independent of θ.



4.3

4.3 The velocity field of a flow is given by $\mathbf{V} = 20y/(x^2 + y^2)^{1/2}\hat{\mathbf{i}} - 20x/(x^2 + y^2)^{1/2}\hat{\mathbf{j}}$ ft/s, where x and y are in feet. Determine the fluid speed at points along the x axis; along the y axis. What is the angle between the velocity vector and the x axis at points $(x, y) = (5, 0)$, $(5, 5)$, and $(0, 5)$?

$$u = \frac{20y}{(x^2+y^2)^{1/2}} \quad , \quad v = -\frac{20x}{(x^2+y^2)^{1/2}}$$

Thus, $V = \sqrt{u^2+v^2}$ or

$$V = \left[\frac{400x^2+400y^2}{(x^2+y^2)}\right]^{1/2} = \underline{\underline{20\,\frac{ft}{s} \text{ for any } x, y}}$$

Also,

$$\tan\theta = \frac{v}{u} = \frac{\dfrac{-20x}{(x^2+y^2)^{1/2}}}{\dfrac{20y}{(x^2+y^2)^{1/2}}}$$

or

$$\tan\theta = -\frac{x}{y}$$

Thus, for $(x,y) = (5,0)$

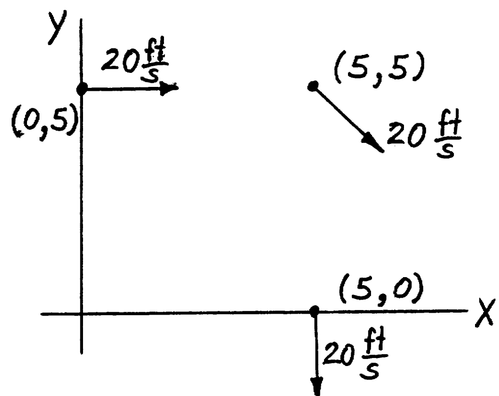
$\tan\theta = -\infty$ or $\theta = \underline{\underline{-90^\circ}}$

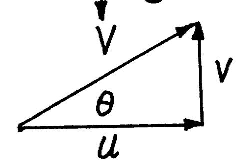
for $(x,y) = (5,5)$

$\tan\theta = -1$ or $\theta = \underline{\underline{-45^\circ}}$

for $(x,y) = (0,5)$

$\tan\theta = 0$ or $\theta = \underline{\underline{0^\circ}}$





4.4

4.4 The x and y components of a velocity field are given by $u = x - y$ and $v = x^2y - 8$. Determine the location of any stagnation points in the flow field. That is, at what point(s) is the velocity zero?

$V = 0$ provided that both $u = 0$ and $v = 0$.

Thus, $u = x - y = 0$ or $x = y$ and $v = x^2y - 8 = 0$ or $x^2y = 8$

By combining obtain $x^3 = 8$ or $x = 2$. Since $x = y$ it follows that $y = 2$ also. Thus $(x,y) = \underline{\underline{(2,2)}}$

4.5

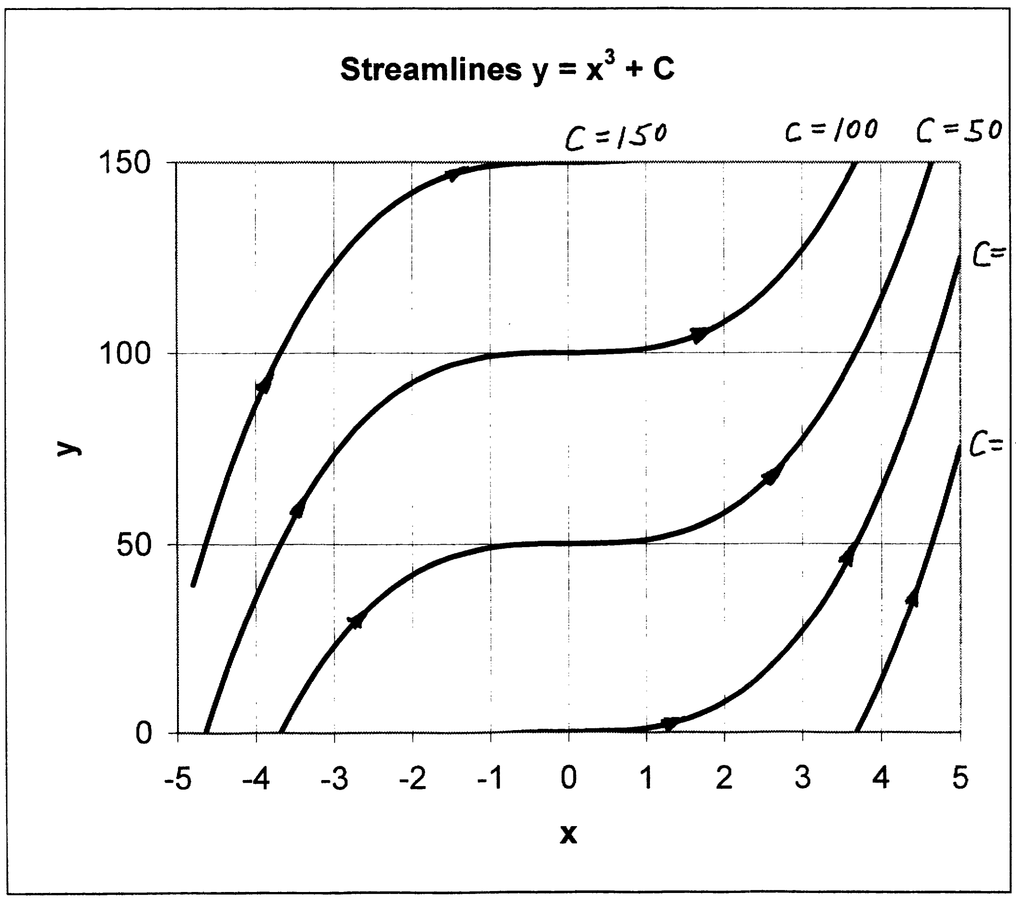
4.5 The x and y components of velocity for a two-dimensional flow are $u = 3$ ft/s and $v = 9x^2$ ft/s, where x is in feet. Determine the equation for the streamlines and graph representative streamlines in the upper half plane.

$u = 3$ and $v = 9x^2$ so that streamlines are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{9x^2}{3} = 3x^2 \quad \text{or} \quad \int dy = \int 3x^2 dx$$

Thus, $\underline{\underline{y = x^3 + C}}$, where C is a constant.

Representative streamlines corresponding to different values of C are shown below.



4.6

4.6 Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and $v = -2cxy$, where c is a constant, are given by the equation $x^2y - y^3/3 = \text{constant}$. At which point (points) is the flow parallel to the y axis? At which point (points) is the fluid stationary?

$u = c(x^2 - y^2)$, $v = -2cxy$

Streamlines given by $y = f(x)$ are such that $\frac{dy}{dx} = \frac{v}{u}$

Consider the function $x^2y - \frac{y^3}{3} = \text{const.}$ (1)

Note: It is not easy to write this explicitly as $y = f(x)$

However, we can differentiate Eq. (1) to give

$$2xy\,dx + x^2dy - y^2dy = 0 \text{ , or}$$

$$(x^2 - y^2)dy + 2xy\,dx = 0$$

Thus, the lines in the x-y plane given by Eq. (1) have a slope

$\frac{dy}{dx} = \frac{-2xy}{(x^2-y^2)}$ or for any constant c, $\underline{\underline{\frac{dy}{dx} = \frac{-2cxy}{c(x^2-y^2)} \equiv \frac{v}{u}}}$

i.e. the function $x^2y - \frac{y^3}{3} = \text{const.}$ represents the streamlines of the given flow.

The flow is parallel to the x-axis when $\frac{dy}{dx} = 0$, or $v = 0$.

This occurs when either $x = 0$ or $y = 0$, i.e., the x-axis or the y-axis

The flow is parallel to the y-axis when $\frac{dy}{dx} = \infty$, or $u = 0$.

This occurs when $\underline{\underline{x = \pm y}}$

The fluid has zero velocity at $\underline{\underline{x = y = 0}}$

4.7

4.7 The velocity field of a flow is given by $u = -V_0 y/(x^2 + y^2)^{1/2}$ and $v = V_0 x/(x^2 + y^2)^{1/2}$, where V_0 is a constant. Where in the flow field is the speed equal to V_0? Determine equation of the streamlines and discuss the various characteristics of this flow.

$$u = -V_0 \frac{y}{(x^2+y^2)^{1/2}} \quad , \quad v = V_0 \frac{x}{(x^2+y^2)^{1/2}} \quad \text{so that}$$

$$V = \sqrt{u^2+v^2} = \left[\frac{V_0^2 (y^2+x^2)}{(x^2+y^2)}\right]^{1/2} = V_0$$

Thus, $\underline{\underline{V = V_0 \text{ throughout the entire flow field}}}$

Streamlines are given by

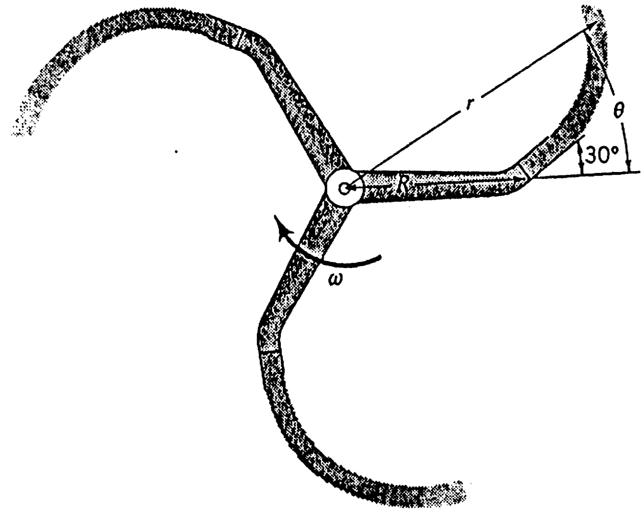
$$\frac{dy}{dx} = \frac{v}{u} = \frac{x}{-y} \quad \text{or} \quad -y\,dy = x\,dx \quad \text{which can be integrated}$$

to give $\underline{\underline{x^2+y^2 = \text{const.}}}$

Thus, the fluid flow with circular streamlines and the speed is constant throughout.

4.8

4.8 Water flows from a rotating lawn sprinkler as shown in Video V4.6 and Figure P4.8. The end of the sprinkler arm moves with a speed of ωR, where $\omega = 10$ rad/s is the angular velocity of the sprinkler arm and $R = 0.5$ ft is its radius. The water exits the nozzle with a speed of $V = 10$ ft/s

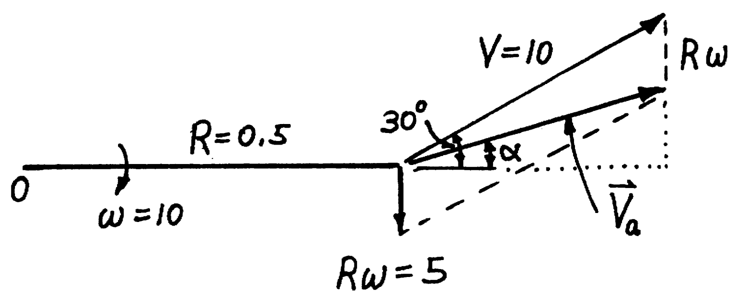


■ FIGURE P4.8

(a) Water leaves the nozzle with a velocity of $V = 10$ ft/s at an angle of $30°$ relalive to the radial direction — for an observer riding on the sprinkler arm. This is the relative velocity. As shown in the sketch, the sprinkler arm has a circumferencial velocity of $R\omega = 0.5\,\text{ft}\,(10\,\text{rad/s}) = 5$ ft/s. The absolute velocity, $\vec{V}_a$, as observed by a person standing on the lawn is the vector sum of relative velocity and the nozzle velocity.

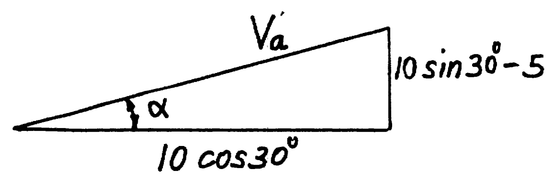
From the geometry of the figure:

$$\tan\alpha = \frac{10\sin 30° - 5}{10\cos 30°} = 0$$



That is $\alpha = 0$

i.e., the absolute water velocity is in the radial direction. Since there is no force acting on the water after it leaves, the water particles continue to move in the radial direction. Thus, the pathlines are straight radial lines.

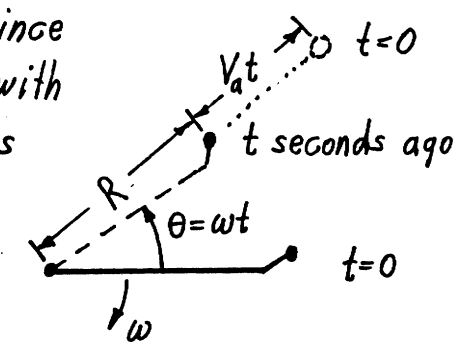


(b) The shape of the water stream at a given instant (i.e. a "snap shot" of the water) can be obtained as follows. Consider the water stream emanating from the end of the nozzle at $r = R$ and $\theta = 0$ at time $t = 0$

(con't)

4.8 (con't)

A particle in this stream that left from the nozzle t seconds ago did so when the nozzle was at $\theta = \omega t$. Since the particles in straight, radial paths with speed V_a (see part (a)), this particle is at a distance of $r = R + V_a t$ from the origin.



Thus, the stream shape is

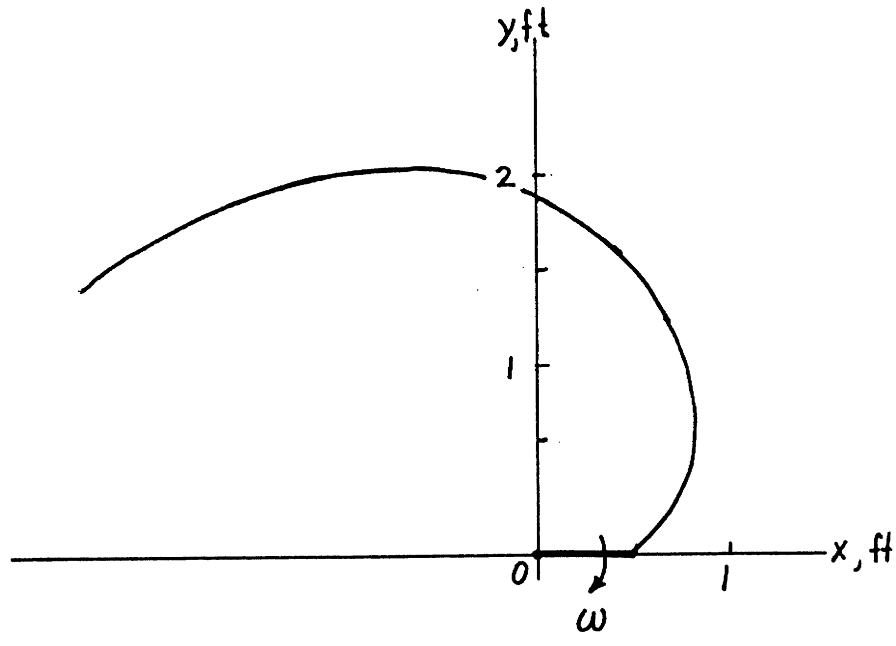
$r = R + V_a t$ and $\theta = \omega t$, or by eliminating t

$$\underline{\underline{r = R + \left(\frac{V_a}{\omega}\right)\theta}}$$

For the given data with $V_a = V \cos 30° = (10 \frac{ft}{s}) \cos 30° = 8.66 \frac{ft}{s}$ (see part(a)) and $\omega = 10\ rad/s$ this becomes

$r = 0.5 + 0.866\,\theta$, where $r \sim ft$ and $\theta \sim rad$.

This stream shape is plotted below.



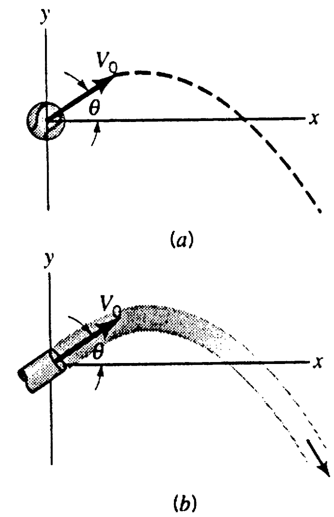
4.9

*4.9 Consider a ball thrown with initial speed V_0 at an angle of θ as shown in Fig. P4.9*a*. As discussed in beginning physics, if friction is negligible the path that the ball takes is given by

$$y = (\tan\theta)x - [g/(2\,V_0^2\cos^2\theta)]x^2$$

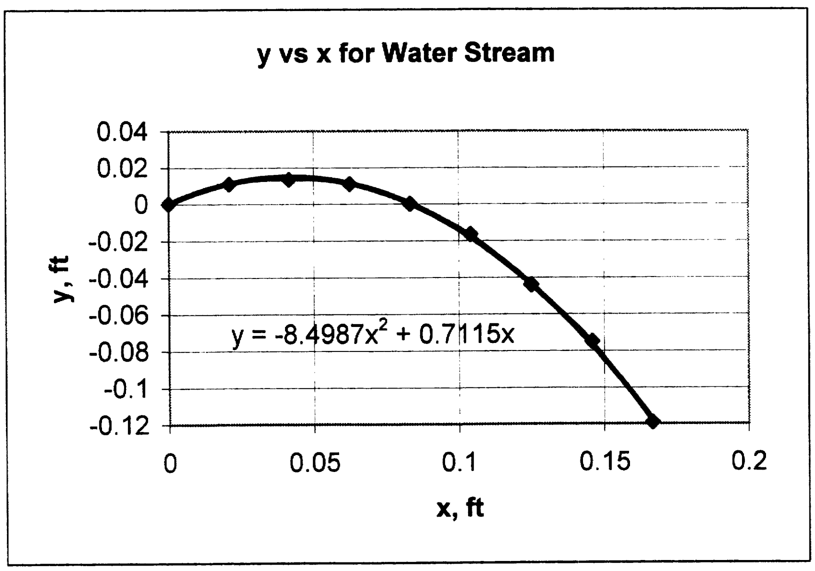
That is, $y = c_1x + c_2x^2$, where c_1 and c_2 are constants. The path is a parabola. The pathline for a stream of water leaving a small nozzle is shown in Fig. P4.9*b* and Video V4.3. The coordinates for this water stream are given in the following table. **(a)** Use the given data to determine appropriate values for c_1 and c_2 in the above equation and, thus, show that these water particles also follow a parabolic pathline. **(b)** Use your values of c_1 and c_2 to determine the speed of the water, V_0, leaving the nozzle.

x, in.	y, in.
0	0
0.25	0.13
0.50	0.16
0.75	0.13
1.0	0.00
1.25	−0.20
1.50	−0.53
1.75	−0.90
2.00	−1.43



■ FIGURE P4.9

An EXCEL Program was used to plot the x-y data and to fit a second order curve to the data. The results are shown below.



Thus, with $y = c_1x + c_2x^2$ it follows that

$c_1 = 0.7115 = \tan\theta$ or $\theta = 35.4°$

and

$$c_2 = -8.4987 = -\frac{g}{2V_0^2\cos^2\theta}$$

or

$$V_0^2 = \frac{32.2}{2(8.4987)\cos^2(35.4°)} = 2.85\,\frac{ft^2}{s^2}$$

Thus, $\underline{\underline{V_0 = 1.69\,\frac{ft}{s}}}$

4.10

4.10 The x and y components of a velocity field are given by $u = x^2y$ and $v = -xy^2$. Determine the equation for the streamlines of this flow and compare with those in Example 4.2. Is the flow in this problem the same as that in Example 4.2? Explain.

Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$

or $\frac{dy}{y} = -\frac{dx}{x}$ which can be integrated as:

$\int \frac{dy}{y} = -\int \frac{dx}{x}$ Thus, $\ln y = -\ln y + \tilde{C}$, where $\tilde{C}$ is a constant.

Thus, $\underline{\underline{xy = C}}$

Note: These streamlines are the same shape (same "flow pattern") as in Example 4.2 — but the velocity fields are different. However, the ratios $\frac{v}{u}$ are the same:

$$\frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$$

and

$$\frac{v}{u} = \frac{(V_0/\ell)(-y)}{(V_0/\ell)(x)} = -\frac{y}{x}$$

4.12

4.12 In addition to the customary horizontal velocity components of the air in the atmosphere (the "wind"), there often are vertical air currents (thermals) caused by buoyant effects due to uneven heating of the air as indicated in Fig. P4.12. Assume that the velocity field in a certain region is approximated by $u = u_0$, $v = v_0(1 - y/h)$ for $0 < y < h$, and $u = u_0$, $v = 0$ for $y > h$. Plot the shape of the streamline that passes through the origin for values of $u_0/v_0 = 0.5$, 1, and 2.

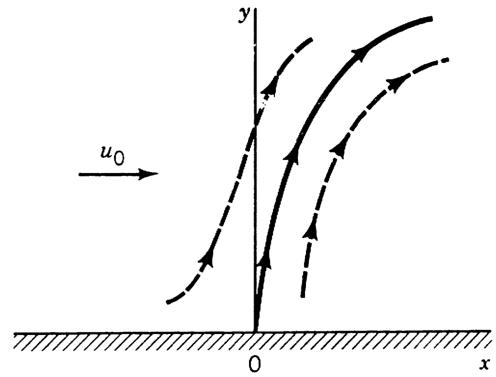


FIGURE P4.12

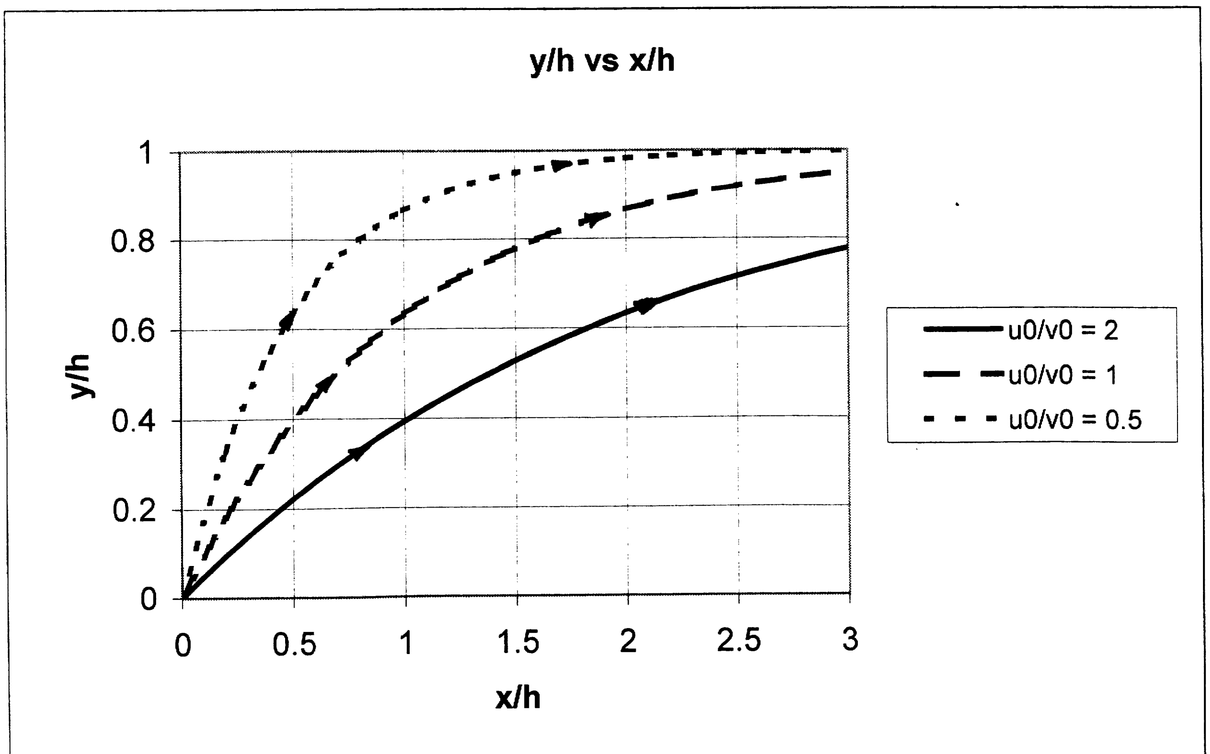
$u = u_0$, $v = v_0(1-\frac{y}{h})$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0(1-\frac{y}{h})}{u_0} \quad \text{or} \quad \int_0^y \frac{dy}{(1-\frac{y}{h})} = \frac{v_0}{u_0}\int_0^x dx$$

Thus, $-h\ln(1-\frac{y}{h}) = \frac{v_0}{u_0}x$ Note: The lower limits of integration ($x=0$, $y=0$) insure that this equation is for the streamline through the origin.

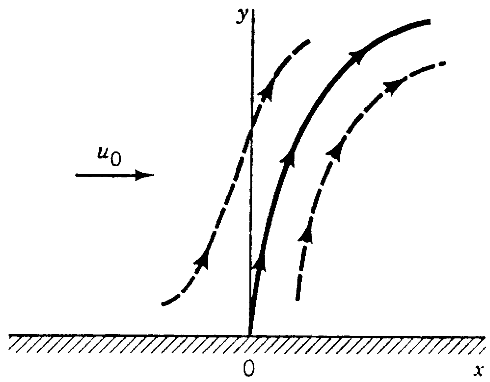
This streamline

$\underline{\underline{x = -h\left(\frac{u_0}{v_0}\right)\ln(1-\frac{y}{h})}}$ is plotted below.



4.13*

4.13* Repeat Problem 4.12 using the same information except that $u = u_0 y/h$ for $0 \le y \le h$ rather than $u = u_0$. Use values of $u_0/v_0 = 0$, 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0.



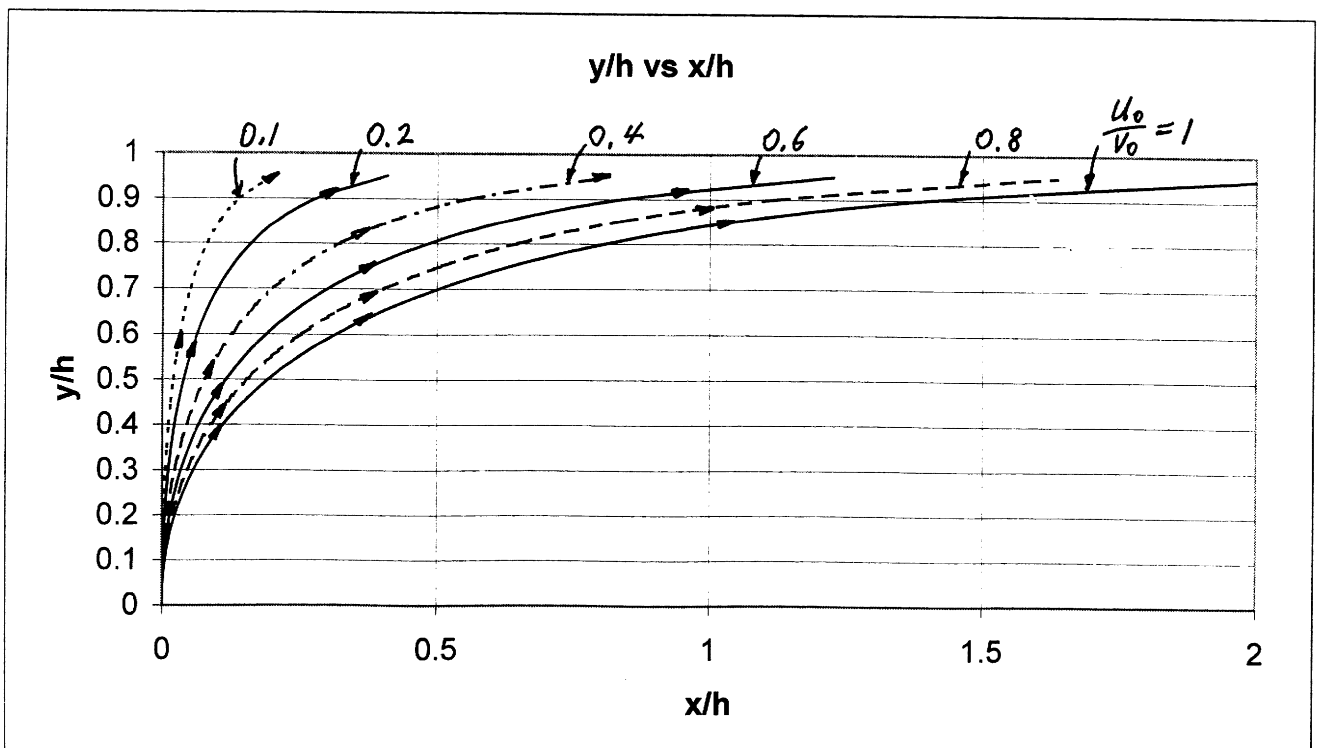
$u = \frac{u_0 y}{h}$, $v = v_0\left(1-\frac{y}{h}\right)$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0\left(1-\frac{y}{h}\right)}{u_0 \frac{y}{h}} = \frac{v_0}{u_0}\,\frac{(h-y)}{y}$$ or with $x=0$ when $y=0$

$$\int_0^y \frac{y}{(h-y)}\,dy = \int_0^x \frac{v_0}{u_0}\,dx$$ This integrates to give

$$-y - h\ln(h-y) + h\ln(h) = \frac{v_0}{u_0}x$$ or $$\frac{x}{h} = \left(\frac{u_0}{v_0}\right)\left[\ln\left(\frac{h}{h-y}\right) - \frac{y}{h}\right]$$

This streamline is plotted below for $0 \le \frac{y}{h} \le 1$, with $\frac{u_0}{v_0} = 0, 0.1, 0.2, 0.4, 0.6, 0.8,$ and 1.0 The values were calculated and plotted using an EXCEL Program.



4.14

4.14 A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow field is the acceleration zero?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (cx^2)(2cx) = \underline{\underline{2c^2x^3}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (cy^2)(2cy) = \underline{\underline{2c^2y^3}}$$

Thus, $\vec{a} = a_x\hat{i} + a_y\hat{j} = 0$ at $\underline{\underline{(x,y) = (0,0)}}$

4.15

4.15 A three-dimensional velocity field is given by $u = x^2$, $v = -2xy$, and $w = x + y$. Determine the acceleration vector.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= x^2(2x) = 2x^3$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= x^2(-2y) + (-2xy)(-2x) = 2x^2y$$

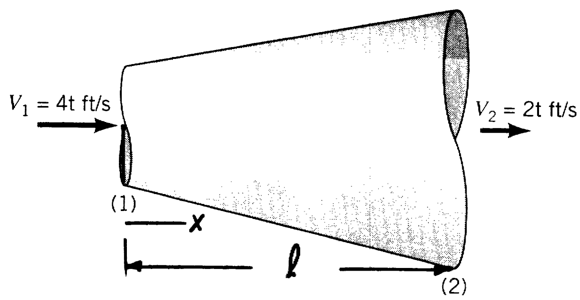
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= x^2(1) + (-2xy)(1) = x^2 - 2xy$$

Thus,

$$\vec{a} = \underline{\underline{2x^3\hat{i} + 2x^2y\,\hat{j} + (x^2 - 2xy)\,\hat{k}}}$$

4.17

4.17 The velocity of air in the diverging pipe shown in Fig. P4.17 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.



a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = \underline{\underline{4\,\frac{ft}{s^2}}}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = \underline{\underline{2\,\frac{ft}{s^2}}}$

b) convective acceleration along the pipe $= u\frac{\partial u}{\partial x}$
where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)
$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$
Hence, $u\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is $\underline{\underline{\text{negative}}}$.

4.18

4.18 Water flows through a constant diameter pipe with a uniform velocity given by $\mathbf{V} = (8/t + 5)\hat{\mathbf{j}}$ m/s, where t is in seconds. Determine the acceleration at time $t = 1$, 2, and 10 s.

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ With $u = 0$, $v = (\frac{8}{t} + 5)\frac{m}{s}$, $w = 0$

this becomes

$\vec{a} = (\frac{\partial V}{\partial t} + v\frac{\partial V}{\partial y})\hat{j} = \frac{\partial V}{\partial t}\hat{j} = -\frac{8}{t^2}\hat{j}\,\frac{m}{s^2}$

Thus, $\vec{a} = \underline{\underline{-8\hat{j}\,\frac{m}{s^2}}}$ at $t = 1$ s

$\vec{a} = \underline{\underline{-2.0\hat{j}\,\frac{m}{s^2}}}$ at $t = 2$ s

and

$\vec{a} = \underline{\underline{-0.08\hat{j}\,\frac{m}{s^2}}}$ at $t = 10$ s

4.19

4.19 When a valve is opened, the velocity of water in a certain pipe is given by $u = 10(1 - e^{-t})$, $v = 0$, and $w = 0$, where u is in ft/s and t is in seconds. Determine the maximum velocity and maximum acceleration of the water.

$V = \sqrt{u^2 + v^2 + w^2} = 10(1 - e^{-t})$ so that $\frac{dV}{dt} = 10e^{-t} > 0$ for all t

Thus, $V_{max} = V\Big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s}}}$

Also, $\vec{a} = a_x\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$ with $\frac{\partial u}{\partial x} = 0$

Thus, $a_x = \frac{\partial u}{\partial t} = 10e^{-t}$, so that $a_{x_{max}} = a_x\Big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s^2}}}$

4.20*

4.20* Water flows through a pipe with $\mathbf{V} = u(t)\hat{\mathbf{i}}$ where the approximate measured values of $u(t)$ are shown in the table. Plot the acceleration as a function of time for $0 \le t \le 20$ s. Plot the acceleration as a function of time if all of the values of $u(t)$ are increased by a factor of 2; by a factor of 5.

t (s)	u (ft/s)	t (s)	u (ft/s)
0	0	11.2	8.1
1.8	1.7	12.3	8.4
3.1	3.2	13.9	8.3
4.0	3.8	15.0	8.1
5.5	4.6	16.4	7.9
6.9	5.8	17.5	7.0
8.1	6.3	18.4	6.6
10.0	7.1	20.0	5.7

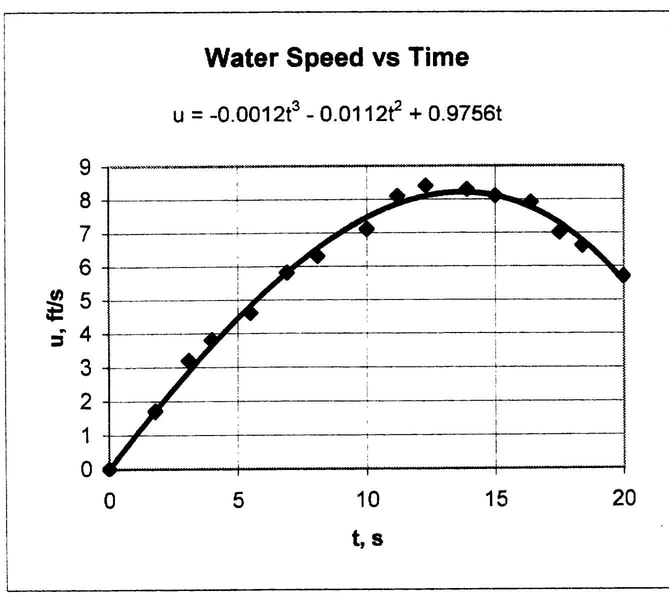
$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(t)$, $v = 0$, $w = 0$
this becomes

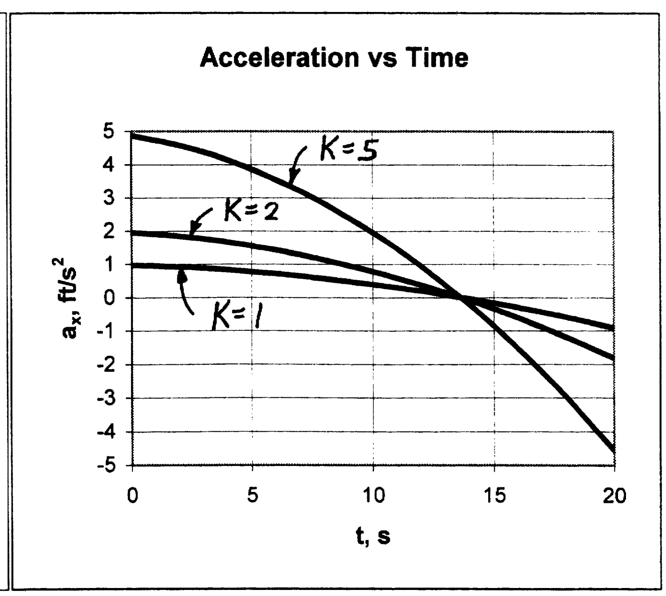
$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = \frac{\partial u}{\partial t}\hat{\imath} \quad \text{or} \quad a_x = \frac{\partial u}{\partial t} \qquad (1)$$

The $u = u(t)$ graph and the best fit cubic equation shown below was plotted using an EXCEL Program.
Thus, with $u = -0.0012t^3 - 0.0112t^2 + 0.9756t$ it follows that

$$a_x = \frac{\partial u}{\partial t} = -0.0036t^2 - 0.0224t + 0.9756 \ \frac{ft}{s^2}, \text{ where } t \sim s$$

This acceleration is also plotted below.
Note that if u increases by a factor of K (i.e, K=2 or K=5), the acceleration, $a_x = \frac{\partial u}{\partial t}$, does also.





4.2 The fluid velocity along the x axis shown in Fig. P4.21 changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.

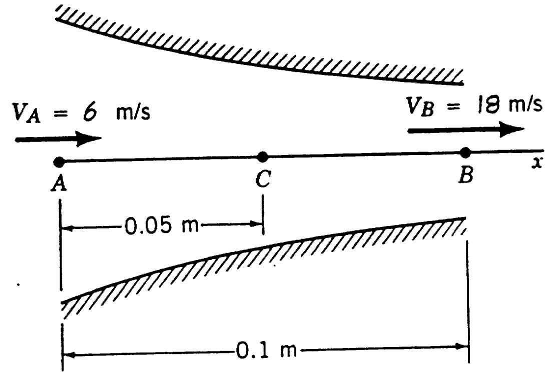


FIGURE P4.21

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ With $u = u(x)$, $v = 0$, and $w = 0$

this becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = u\frac{\partial u}{\partial x}\,\hat{i} \qquad (1)$$

Since u is a linear function of x, $u = c_1 x + c_2$ where the constants c_1, c_2 are given as: $u_A = 6 = c_2$

and $u_B = 18 = 0.1c_1 + c_2$

or $c_1 = 120$, $c_2 = 6.$

Thus, $u = (120x + 6)\frac{m}{s}$ with $x \sim m$

From Eq. (1)

$$\vec{a} = u\frac{\partial u}{\partial x}\hat{i} = (120x + 6)\frac{m}{s}\left(120\frac{m}{m \cdot s}\right)\hat{i}$$

or

for $x_A = 0$, $\vec{a}_A = 720\,\hat{i}\,\frac{m}{s^2}$

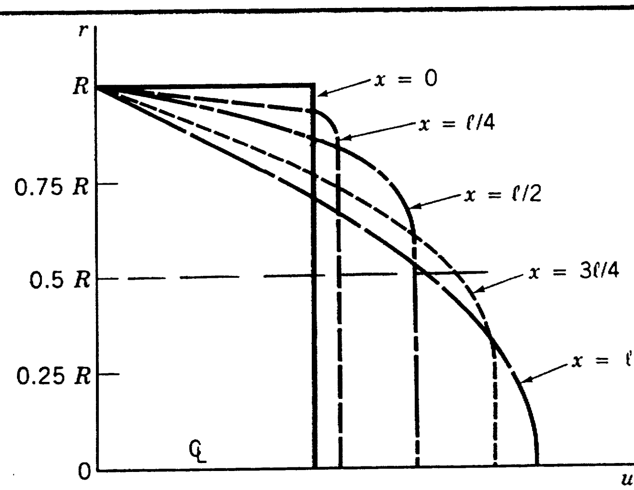
for $x_B = 0.05\,m$, $\vec{a}_B = 1440\,\hat{i}\,\frac{m}{s^2}$

and

for $x_C = 0.1\,m$, $\vec{a}_C = 2160\,\hat{i}\,\frac{m}{s^2}$

4.22

4.22 When a fluid flows into a round pipe as shown in Fig. P4.22, viscous effects may cause the velocity profile to change from a uniform profile ($\mathbf{V} = V_0\hat{\mathbf{i}}$) at the entrance of the pipe to a parabolic profile $\{\mathbf{V} = 2V_0[1 - (r/R)^2]\hat{\mathbf{i}}\}$ at $x = \ell$. Velocity profiles for various values of x are as indicated in the figure. Use this graph to show that a fluid particle moving along the centerline ($r = 0$) experiences an acceleration, but a particle close to the edge of the pipe ($r \approx R$) experiences a deceleration. Does a particle traveling along the line $r = 0.5$ R experience an acceleration or deceleration, or both? Explain.



(b)

FIGURE P4.22

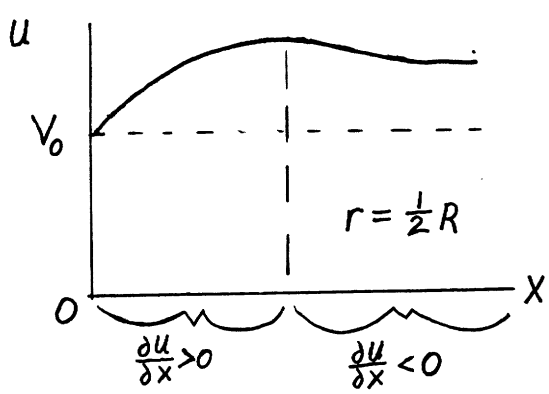
$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(r,x)$, $v=0$, and $w=0$ we obtain

$\vec{a} = a_x\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial x}$

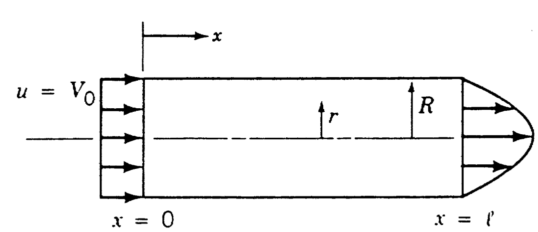
a) Along the centerline ($r=0$) we find $u>0$ and $\frac{\partial u}{\partial x}>0$
Thus, $a_x > 0$ on $r=0$.

b) Near the pipe wall ($r \approx R$) we find $u>0$ but $\frac{\partial u}{\partial x}<0$ (i.e., the velocity changes from $u=V_0$ to $u<V_0$ as x increases)
Thus, $a_x<0$ for $r\approx R$.

c) For $r=\frac{1}{2}R$ we find $u>0$ and $\frac{\partial u}{\partial x}>0$ near the pipe entrance, but $\frac{\partial u}{\partial x}<0$ elsewhere. This is indicated in the figure below.



Thus, for $r=\frac{1}{2}R$,
$a_x>0$ near the entrance and
$a_x<0$ elsewhere



(a)

4.23

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

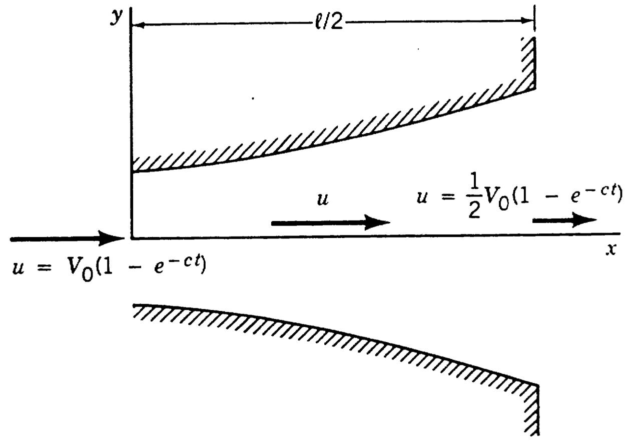


FIGURE P4.23

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$

this becomes

$\vec{a} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{i} = a_x\hat{i}$, where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$a_x = V_0(1 - \frac{x}{\ell})c\,e^{-ct} + V_0^2(1 - e^{-ct})^2(1 - \frac{x}{\ell})(-\frac{1}{\ell})$

or

$$a_x = V_0(1 - \frac{x}{\ell})\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right]$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1 - e^{-c})^2 = 0$ The solution (root) of this equation is $c = 0.490\ \frac{1}{s}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.

4.24

4.24 A fluid flows along the x axis with a velocity given by $\mathbf{V} = (x/t)\hat{\mathbf{i}}$, where x is in feet and t in seconds. **(a)** Plot the speed for $0 \le x \le 10$ ft and $t = 3$ s. **(b)** Plot the speed for $x = 7$ ft and $2 \le t \le 4$ s. **(c)** Determine the local and convective acceleration. **(d)** Show that the acceleration of any fluid particle in the flow is zero. **(e)** Explain physically how the velocity of a particle in this unsteady flow remains constant throughout its motion.

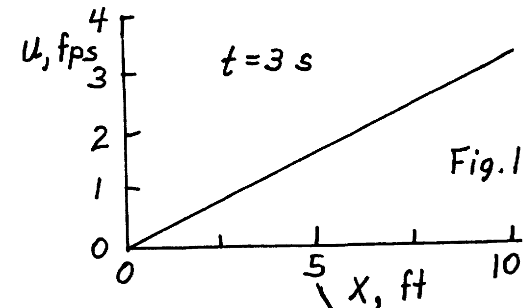


Fig. 1

(a) $u = \frac{x}{t}\ \frac{ft}{s}$ so at $t = 3\,s$, $u = \frac{x}{3}\ \frac{ft}{s}$

(b) For $x = 7$ ft, $u = \frac{7}{t}\ \frac{ft}{s}$

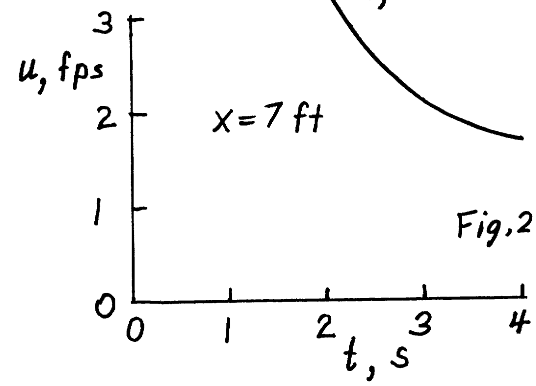


Fig. 2

(c) $\frac{\partial u}{\partial t} = \underline{\underline{-\frac{x}{t^2}}}$ and $u\frac{\partial u}{\partial x} = \frac{x}{t}\left(\frac{1}{t}\right) = \underline{\underline{\frac{x}{t^2}}}$

(d) For any fluid particle $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$
which with $v = 0$, $w = 0$ becomes
$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = \left(-\frac{x}{t^2} + \frac{x}{t^2}\right)\hat{i} \equiv 0$

(e) The particles flow into areas of higher velocity (see Fig. 1), but at any given location the velocity is decreasing in time (see Fig. 2). For the given velocity field the local and convective accelerations are equal and opposite, giving zero acceleration throughout.

4.25

4.25 A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Fig. P4.25 and Video V10.6. In a relatively short distance (thickness = ℓ) the liquid depth changes from z_1 to z_2, with a corresponding change in velocity from V_1 to V_2. If V_1 = 1.20 ft/s, V_2 = 0.30 ft/s, and ℓ = 0.02 ft, estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many g's deceleration does this represent?

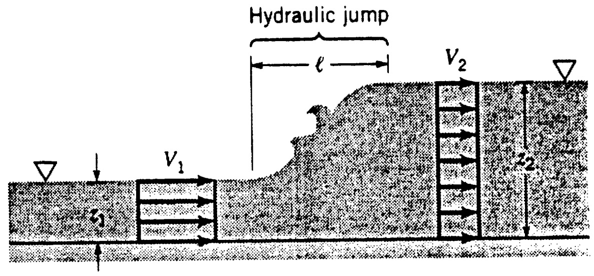


FIGURE P4.25

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ so with $\vec{V} = u(x)\hat{i}$, $\vec{a} = a_x\hat{i} = u\frac{\partial u}{\partial x}\hat{i}$

Without knowing the actual velocity distribution, $u = u(x)$, the acceleration can be approximated as

$$a_x = u\frac{\partial u}{\partial x} \approx \frac{1}{2}(V_1+V_2)\frac{(V_2-V_1)}{\ell} = \frac{1}{2}(1.20+0.30)\frac{ft}{s}\ \frac{(0.30-1.20)\frac{ft}{s}}{0.02\,ft}$$

$$= \underline{\underline{-33.8\frac{ft}{s^2}}}$$

Thus, $\frac{|a_x|}{g} = \frac{33.8\frac{ft}{s^2}}{32.2\frac{ft}{s^2}} = \underline{\underline{1.05}}$

4.26

4.26 A fluid particle flowing along a stagnation streamline, as shown in Video V4.5 and Fig. P4.26, slows down as it approaches the stagnation point. Measurements of the dye flow in the video indicate that the location of a particle starting on the stagnation streamline a distance $s = 0.6$ ft upstream of the stagnation point at $t = 0$ is given approximately by $s = 0.6e^{-0.5t}$, where t is in seconds and s is in ft. (a) Determine the speed of a fluid particle as a function of time, $V_{particle}(t)$, as it flows along the steamline. (b) Determine the speed of the fluid as a function of position along the streamline, $V = V(s)$. (c) Determine the fluid acceleration along the streamline as a function of position, $a_s = a_s(s)$.

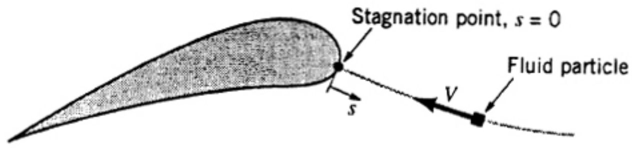


FIGURE P4.26

(a) With $s = 0.6\,e^{-0.5t}$ it follows that

$$V_{particle} = \frac{ds}{dt} = 0.6\,(-0.5)\,e^{-0.5t} = \underline{\underline{-0.3\,e^{-0.5t}}} \ \text{ft/s}$$

(b) From part (a),

$$V = (-0.5)\left[0.6\,e^{-0.5t}\right] \quad \text{where } s = 0.6\,e^{-0.5t}$$

Thus,

$$V = (-0.5)[s], \quad \text{or} \quad V = \underline{\underline{-0.5s}} \ \text{ft/s where } s \sim \text{ft}$$

(c) For steady flow, $a_s = V\frac{dV}{ds}$

Thus, with $V = -0.5s$ and $\frac{dV}{ds} = -0.5$,

$$a_s = (-0.5s)(-0.5) = \underline{\underline{0.25\,s}} \ \text{ft/s}^2 \ \text{where } s \sim \text{ft}$$

Note: For $s > 0$, a_s is positive — the particle's acceleration is to the right. Since the particle is moving to the left, a positive a_s for this case implies that the particle is decelerating (as it must be for this stagnation point flow).

4.27

4.27 A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. If the flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$ m, determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2).

With $u = ax + b$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ can be written as

$$\vec{a} = a_x \hat{i} \quad \text{where} \quad a_x = u \frac{\partial u}{\partial x}. \tag{1}$$

Since $u = V_1 = 10 \frac{m}{s}$ at $x = 0$ and $u = V_2 = 25 \frac{m}{s}$ at $x = 1$ we obtain

$$10 = 0 + b$$

$$25 = a + b \quad \text{so that } a = 15 \text{ and } b = 10$$

That is, $u = (15x + 10) \frac{m}{s}$, where $x \sim m$, so that from Eq.(1)

$$a_x = (15x + 10) \frac{m}{s} \left(15 \frac{1}{s}\right) = \underline{\underline{(225x + 150) \frac{m}{s^2}}}$$

Note: The local acceleration is zero, $\underline{\underline{\frac{\partial \vec{V}}{\partial t} = 0}}$, and the convective acceleration is $u \frac{\partial u}{\partial x} \hat{i} = \underline{\underline{(225x + 150) \hat{i} \frac{m}{s^2}}}$

At $x = 0$, $\vec{a} = \underline{\underline{150 \hat{i} \frac{m}{s^2}}}$; at $x = 1\,m$, $\vec{a} = \underline{\underline{375 \hat{i} \frac{m}{s^2}}}$

4.29

4.29 Repeat Problem 4.27 with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1/\partial t = 20$ m/s^2 and $\partial V_2/\partial t = 60$ m/s^2.

With $u = u(x,t)$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$\vec{a} = a_x \hat{\imath}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$, with $u = a(t)x + b(t)$. (1)

At the given time $(t=t_0)$ $u = V_1 = 10\frac{m}{s}$ at $x = 0$ and $u = V_2 = 25\frac{m}{s}$ at $x = 1m$

Thus, $10 = 0 + b(t_0)$

$25 = a(t_0) + b(t_0)$ so that $a(t_0) = 15$ and $b(t_0) = 10$

Also at $t = t_0$, $\frac{\partial u}{\partial t} = \frac{\partial V_1}{\partial t} = \underline{\underline{20\frac{m}{s^2}}}$ at $x = 0$

and $\frac{\partial u}{\partial t} = \frac{\partial V_2}{\partial t} = \underline{\underline{60\frac{m}{s^2}}}$ at $x = 1m$ Note: These are local accelerations at time $t = t_0$

The convective acceleration at $x = 0$ (Eq. (1) is

$$u\frac{\partial u}{\partial x} = (ax+b)(a) = (15(0)+10)\frac{m}{s}(15\frac{1}{s}) = \underline{\underline{150\frac{m}{s^2}}}$$

while at $x = 1$ it is

$$u\frac{\partial u}{\partial x} = (15(1)+10)\frac{m}{s}(15\frac{1}{s}) = \underline{\underline{375\frac{m}{s^2}}}$$

The fluid acceleration at $t = t_0$ is

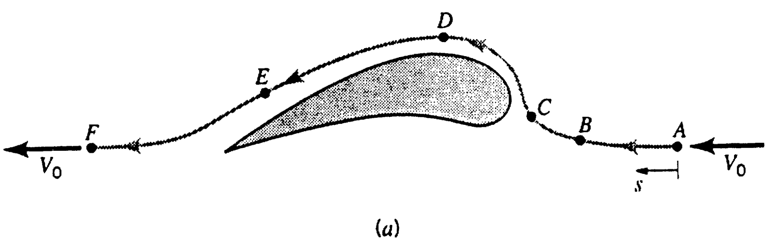
$$\vec{a} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{\imath} = (20+150)\hat{\imath}\frac{m}{s^2} = \underline{\underline{170\hat{\imath}\frac{m}{s^2} \text{ at } x=0}}$$

and

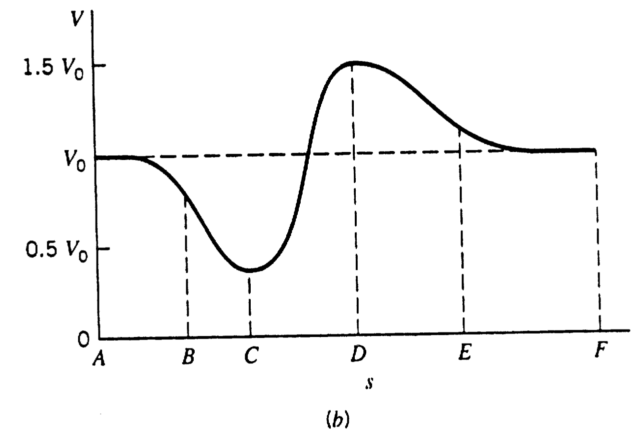
$$\vec{a} = (60+375)\hat{\imath}\frac{m}{s^2} = \underline{\underline{435\hat{\imath}\frac{m}{s^2} \text{ at } x=1m}}$$

4.30

4.30 An incompressible fluid flows past a turbine blade as shown in Fig. P4.30*a* and Video V4.5. Far upstream and downstream of the blade the velocity is V_0. Measurements show that the velocity of the fluid along streamline *A–F* near the blade is as indicated in Fig. P4.30*b*. Sketch the streamwise component of acceleration, a_s, as a function of distance, *s*, along the streamline. Discuss the important characteristics of your result.



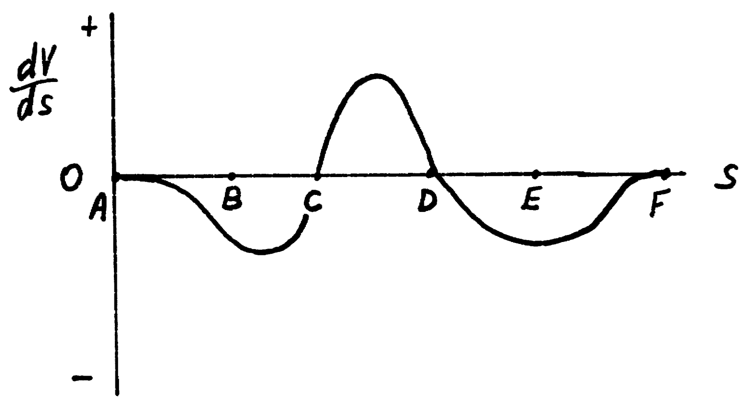
(*a*)



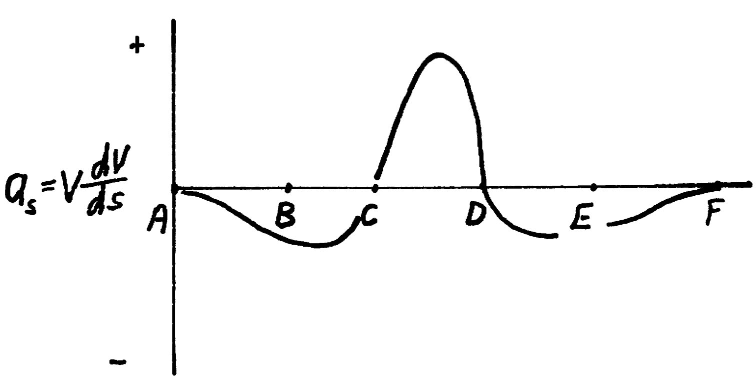
(*b*)

■ FIGURE P4.30

$a_s = V\frac{dV}{ds}$ where from the figure of $V = V(s)$ the function $\frac{dV}{ds}$ has the following shape.



Hence, the product $V\frac{dV}{ds}$ has the shape shown below.



The fluid decelerates from A to C, accelerates from C to D, and the decelerates again from D to F. The net acceleration from A to F is zero (i.e., $V_A = V_0 = V_F$).

4.31 ★

4.31* Air flows steadily through a variable area pipe with a velocity of $\mathbf{V} = u(x)\hat{\mathbf{i}}$ ft/s, where the approximate measured values of $u(x)$ are given in the table. Plot the acceleration as a function of x for $0 \le x \le 12$ in. Plot the acceleration if the flowrate is increased by a factor of N (i.e., the values of u are increased by a factor of N), for $N = 2, 4, 10$.

x (in.)	u (ft/s)	x (in.)	u (ft/s)
0	10.0	7	20.1
1	10.2	8	17.4
2	13.0	9	13.5
3	20.1	10	11.9
4	28.3	11	10.3
5	28.4	12	10.0
6	25.8	13	10.0

Since $u = u(x)$, $v=0$, and $w=0$ it follows that $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ simplifies to $\vec{a} = a_x \hat{i}$ where $a_x = u\frac{\partial u}{\partial x}$ (1)

The values u are given in the table; the corresponding values of $\frac{\partial u}{\partial x}$ can be obtained by an approximate numerical differentiation as given in Program P4#31 shown below.

Note that since $a_x = u\frac{\partial u}{\partial x}$ it follows that and increase in velocity from u to Nu increases the acceleration from a_x to $N^2 a_x$

```
100 cls
110 open "prn" for output as #1
120 dim u(14), n(4)
125 u(1)=10.0 : u(2)=10.2 : u(3)=13.0 : u(4)=20.1 : u(5)=28.3
130 u(6)=28.4 : u(7)=25.8 : u(8)=20.1 : u(9)=17.4 : u(10)=13.5
135 u(11)=11.9 : u(12)=10.3 : u(13)=10.0 : u(14)=10.0
140 n(1)=1 : n(2)=2 : n(3)=4 : n(4)=10
150 print#1, "**********************************************"
160 print#1, "**  This program calculates the acceleration  **"
170 print#1, "**  as a function of position.                **"
180 print#1, "**********************************************"
200 print#1, " "
210 for i = 1 to 4
220 print#1, " "
230 print#1, using "For N = ##";n(i)
240 print#1, " x, in.   u, ft/s    a, ft/s2"
300 for j = 1 to 13
310 a = n(i)^2*((u(j+1) + u(j))/2)*((u(j+1) - u(j))/(1/12))
320 uavg = (u(j+1) + u(j))/2
330 x = j - 0.5
340 print#1, using "###.#       ###.#     +#.###^^^^";x,uavg,a
350 next j
360 next i
```

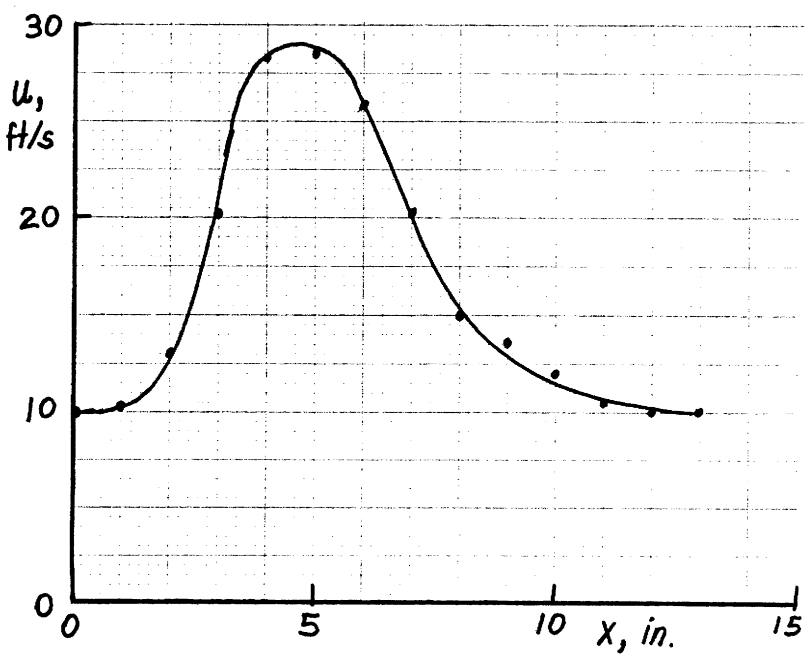
Note that although the velocity data, $u = u(x)$, appears to be quite "smooth", the acceleration result, $a_x = u\frac{\Delta u}{\Delta x}$, is somewhat irregular (especially for $x > 7$ in.).

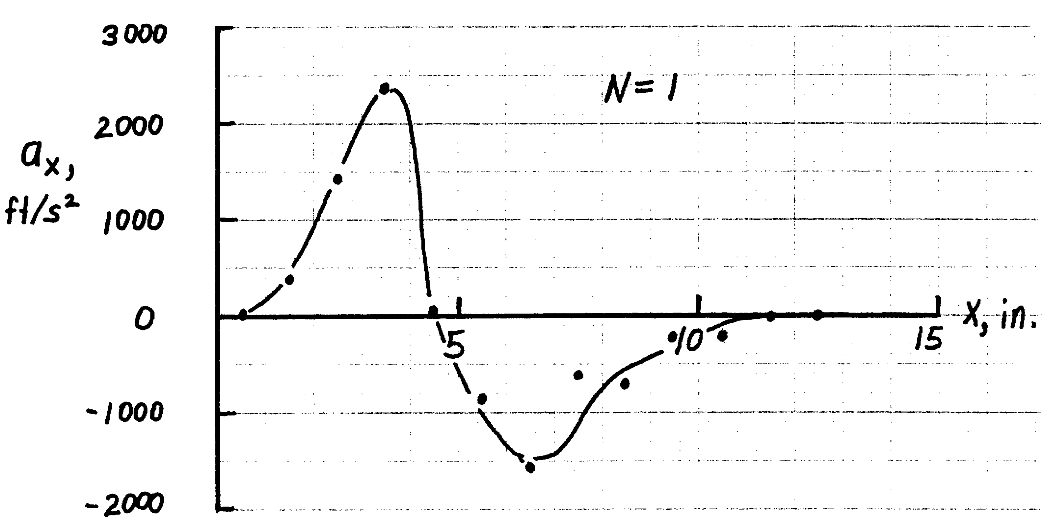
(con't)

4.31 * (con't)

```
**********************************************
**  This program calculates the acceleration **
**  as a function of position.               **
**********************************************
```

For N = 1			For N = 10		
x, in.	u, ft/s,	a, ft/s2	x, in.	u, ft/s,	a, ft/s2
0.5	10.1	+2.424E+01	0.5	10.1	+2.424E+03
1.5	11.6	+3.898E+02	1.5	11.6	+3.898E+04
2.5	16.5	+1.410E+03	2.5	16.5	+1.410E+05
3.5	24.2	+2.381E+03	3.5	24.2	+2.381E+05
4.5	28.3	+3.402E+01	4.5	28.3	+3.402E+03
5.5	27.1	-8.455E+02	5.5	27.1	-8.455E+04
6.5	23.0	-1.570E+03	6.5	23.0	-1.570E+05
7.5	18.8	-6.075E+02	7.5	18.8	-6.075E+04
8.5	15.4	-7.231E+02	8.5	15.4	-7.231E+04
9.5	12.7	-2.438E+02	9.5	12.7	-2.438E+04
10.5	11.1	-2.131E+02	10.5	11.1	-2.131E+04
11.5	10.1	-3.654E+01	11.5	10.1	-3.654E+03
12.5	10.0	+0.000E+00	12.5	10.0	+0.000E+00





4.32

4.32 Assume the temperature of the exhaust in an exhaust pipe can be approximated by $T = T_0(1 + ae^{-bx})[1 + c\cos(\omega t)]$, where $T_0 = 100\ °C$, $a = 3$, $b = 0.03\ m^{-1}$, $c = 0.05$, and $\omega = 100$ rad/s. If the exhaust speed is a constant 2 m/s, determine the time rate of change of temperature of the fluid particles at $x = 0$ and $x = 4$ m when $t = 0$.

Since $u = 2\ \frac{m}{s}$, $v = 0$, and $w = 0$ it follows that

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$$

Thus,

$$\frac{DT}{Dt} = T_0(1 + a e^{-bx})(-c\,\omega \sin(\omega t)) + u T_0 (1 + c\cos(\omega t))(-ab\,e^{-bx})$$

When $t = 0$:

$$\frac{DT}{Dt} = -abuT_0(1+c)\,e^{-bx}$$, or with the given data,

$$\frac{DT}{Dt} = -(3)(0.03\tfrac{1}{m})(2\tfrac{m}{s})(100\,°C)(1+0.05)\,e^{-0.03x}$$

$$= -18.9\ e^{-0.03x}\ \frac{°C}{s}$$, where $x \sim m$

Thus, $\underline{\underline{\frac{DT}{Dt} = -18.9\,\frac{°C}{s}}}$ at $x = 0$, $t = 0$

and

$\underline{\underline{\frac{DT}{Dt} = -16.8\,\frac{°C}{s}}}$ at $x = 4\,m$, $t = 0$

4.33*

4.33* As is indicated in Fig. P4.33, the speed of exhaust in a car's exhaust pipe varies in time and distance because of the periodic nature of the engine's operation and the damping effect with distance from the engine. Assume that the speed is given by $V = V_0[1 + ae^{-bx}\sin(\omega t)]$, where $V_0 = 8$ fps, $a = 0.05$, $b = 0.2\ \text{ft}^{-1}$, and $\omega = 50$ rad/s. Calculate and plot the fluid acceleration at $x = 0, 1, 2, 3, 4,$ and 5 ft for $0 \le t \le \pi/25$ s.

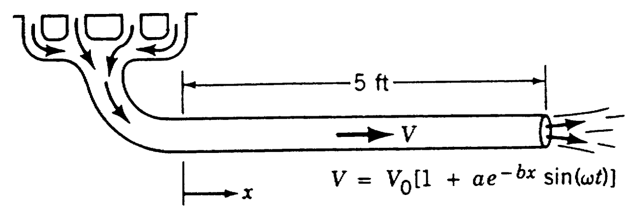


FIGURE P4.33

Since $u = u(x,t)$, $v = 0$, and $w = 0$ it follows that

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = a_x\hat{i}\ , \quad \text{where } a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \qquad (1)$$

Thus, with $u = V_0[1 + a e^{-bx}\sin(\omega t)]$ Eq. (1) gives

$$a_x = V_0 a \omega e^{-bx}\cos(\omega t) + V_0[1 + a e^{-bx}\sin(\omega t)]V_0 a(-b)e^{-bx}\sin(\omega t)$$

$$= V_0 a e^{-bx}[\omega\cos(\omega t) - V_0 b\sin(\omega t)(1 + a e^{-bx}\sin(\omega t))]$$

With $V_0 = 8\frac{ft}{s}$, $a = 0.05$, $b = 0.2\frac{1}{ft}$, and $\omega = 50\frac{rad}{s}$ this becomes

$$a_x = 0.4e^{-0.2x}[50\cos(50t) - 1.6\sin(50t)(1 + 0.05e^{-0.2x}\sin(50t))]\ \frac{ft}{s^2} \qquad (2)$$

where $t \sim s$ and $x \sim ft$

Plot a_x from Eq. (2) for $0 \le t \le \frac{\pi}{25}$ s with $x = 0, 1, 2, 3, 4,$ and 5 ft.

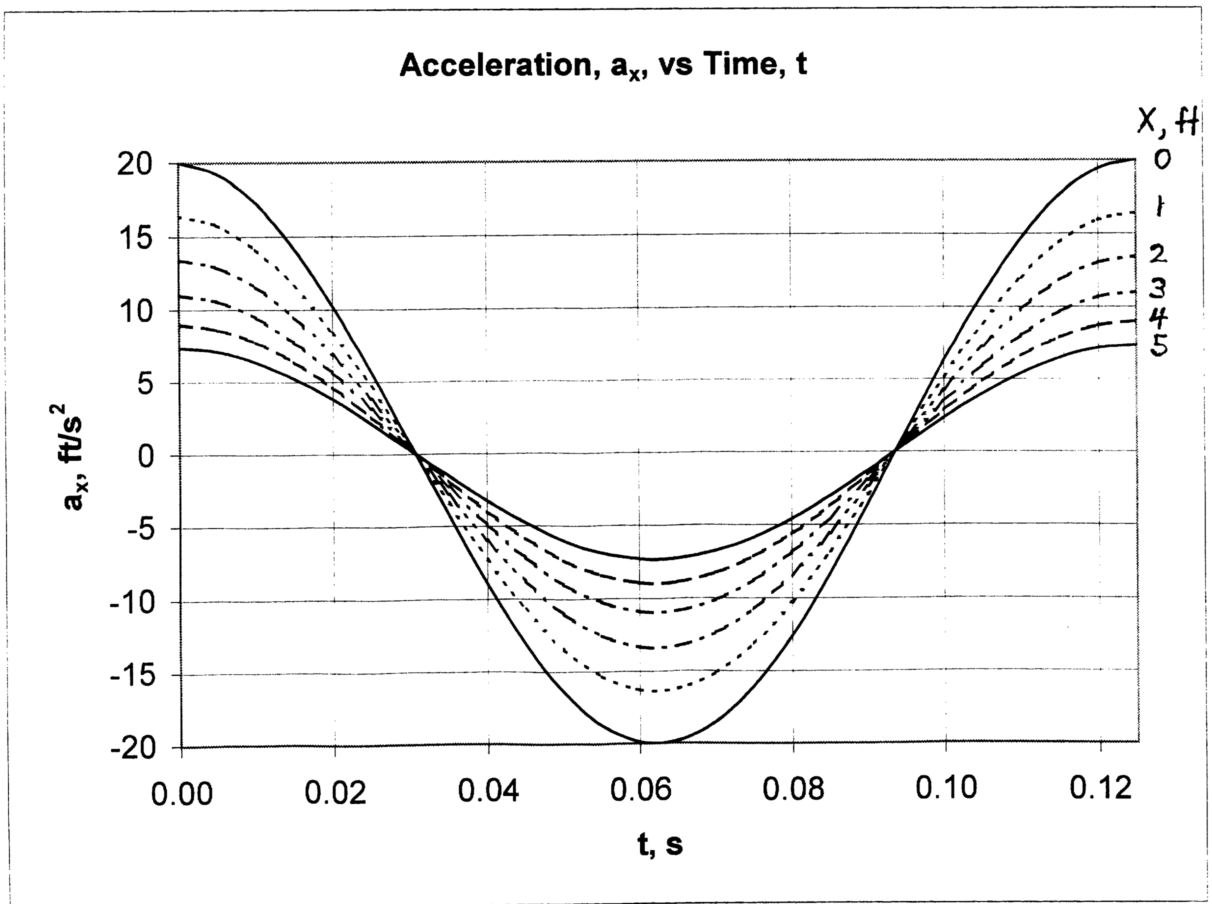
An Excel Program was used to calculate a_x from Eq. (2). The results are shown on the next page.

(con't)

4.33* (con't)

Acceleration at various x locations, ft/s^2

t, s	x = 0 ft	x = 1 ft	x = 2 ft	x= 3 ft	x = 4 ft	x = 5 ft
0.000	20.00	16.37	13.41	10.98	8.99	7.36
0.005	19.22	15.73	12.88	10.55	8.64	7.07
0.010	17.24	14.11	11.56	9.46	7.75	6.34
0.015	14.18	11.61	9.51	7.79	6.38	5.22
0.020	10.24	8.39	6.87	5.63	4.61	3.77
0.025	5.67	4.65	3.81	3.12	2.55	2.09
0.030	0.74	0.61	0.51	0.42	0.34	0.28
0.035	-4.23	-3.46	-2.83	-2.31	-1.89	-1.55
0.040	-8.93	-7.31	-5.98	-4.90	-4.01	-3.28
0.045	-13.08	-10.71	-8.76	-7.17	-5.87	-4.81
0.050	-16.42	-13.44	-11.00	-9.01	-7.37	-6.04
0.055	-18.73	-15.34	-12.56	-10.28	-8.42	-6.89
0.060	-19.89	-16.29	-13.33	-10.92	-8.94	-7.32
0.065	-19.81	-16.22	-13.28	-10.87	-8.90	-7.29
0.070	-18.51	-15.15	-12.41	-10.16	-8.32	-6.81
0.075	-16.06	-13.14	-10.76	-8.81	-7.21	-5.90
0.080	-12.61	-10.32	-8.45	-6.91	-5.66	-4.63
0.085	-8.37	-6.85	-5.61	-4.59	-3.76	-3.07
0.090	-3.62	-2.96	-2.42	-1.98	-1.62	-1.32
0.095	1.36	1.12	0.92	0.75	0.62	0.51
0.100	6.26	5.13	4.20	3.44	2.82	2.31
0.105	10.77	8.82	7.22	5.92	4.84	3.97
0.110	14.61	11.96	9.80	8.02	6.57	5.38
0.115	17.54	14.36	11.76	9.63	7.88	6.45
0.120	19.38	15.87	12.99	10.64	8.71	7.13
0.125	20.01	16.38	13.41	10.98	8.99	7.36



4.34

4.34 A gas flows along the x-axis with a speed of $V = 5x$ m/s and a pressure of $p = 10x^2$ N/m^2, where x is in meters. **(a)** Determine the time rate of change of pressure at the fixed location $x = 1$. **(b)** Determine the time rate of change of pressure for a fluid particle flowing past $x = 1$. **(c)** Explain without using any equations why the answers to parts (a) and (b) are different.

a) Since $p = 10x^2$ it follows that $\frac{\partial p}{\partial t} = \underline{\underline{0}}$ for all x.

b) With $u = 5x$, $v = 0$, $w = 0$, and $p = 10x^2$ it follows that

$$\frac{Dp}{Dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = u\frac{\partial p}{\partial x} = \left(5x\frac{m}{s}\right)\left(20x\frac{N}{m^3}\right) = 100x^2\ \frac{N}{m^2}/s$$

Thus $$\left.\frac{Dp}{Dt}\right|_{x=1m} = \underline{\underline{100\ \frac{N}{m^2\cdot s}}}$$

c) For this steady flow the pressure at a point is constant (part (a)), but the pressure for a given particle changes with time (part (b)) because the particle flows into a higher pressure region.

4.35

4.35 The temperature distribution in a fluid is given by $T = 10x + 5y$, where x and y are the horizontal and vertical coordinates in meters and T is in degrees centigrade. Determine the time rate of change of temperature of a fluid particle traveling (a) horizontally with $u = 20$ m/s, $v = 0$ or (b) vertically with $u = 0$, $v = 20$ m/s.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}, \text{ where } \frac{\partial T}{\partial t} = 0$$

Thus, if $u = 20\frac{m}{s}$ and $v = 0$, then $\frac{DT}{Dt} = u\frac{\partial T}{\partial x} = \left(20\frac{m}{s}\right)\left(10\frac{°C}{m}\right) = \underline{\underline{200\ \frac{°C}{s}}}$

and if $u = 0$ and $v = 20\frac{m}{s}$, then $\frac{DT}{Dt} = v\frac{\partial T}{\partial y} = \left(20\frac{m}{s}\right)\left(5\frac{°C}{m}\right) = \underline{\underline{100\ \frac{°C}{s}}}$

4.36

4.36 At the top of its trajectory, the stream of water shown in Fig. P4.36 and Video V4.3 flows with a horizontal velocity of 1.80 ft/s. The radius of curvature of its streamline at that point is approximately 0.10 ft. Determine the normal component of acceleration at that location.

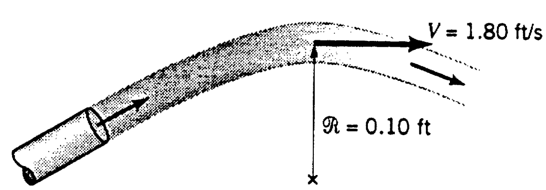
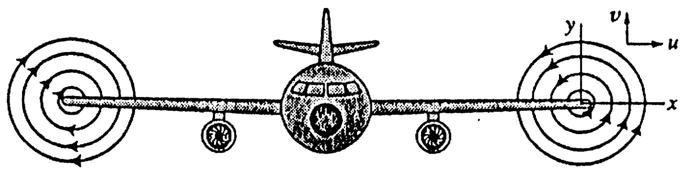


FIGURE P4.36

$$a_n = \frac{V^2}{\mathcal{R}} = \frac{\left(1.8\frac{ft}{s}\right)^2}{0.10 ft} = \underline{\underline{32.2\frac{ft}{s^2}}}$$

4.37

4.37 As shown in Video V4.2 and Fig. P4.37, a flying airplane produces swirling flow near the end of its wings. In certain circumstances this flow can be approximated by the velocity field $u = -Ky/(x^2 + y^2)$ and $v = Kx/(x^2 + y^2)$, where K is a constant depending on various parameters associated with the airplane (i.e., its weight, speed, etc.) and x and y are measured from the center of the swirl. (a) Show that for this flow the velocity is inversely proportional to the distance from the origin. That is, $V = K/(x^2 + y^2)^{1/2}$. (b) Show that the streamlines are circles.



■ FIGURE P4.37

(a) $V = \sqrt{u^2 + v^2} = \left[\frac{(-Ky)^2}{(x^2+y^2)^2} + \frac{(Kx)^2}{(x^2+y^2)^2}\right]^{1/2} = \frac{K}{\sqrt{x^2+y^2}}$

or

$\underline{\underline{V = \frac{K}{r}}}$, where $r = \sqrt{x^2+y^2}$

(b) Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = \frac{\frac{Kx}{(x^2+y^2)}}{\frac{-Ky}{(x^2+y^2)}} = -\frac{x}{y}$

Thus,

$y\,dy = -x\,dx$ which when integrated gives

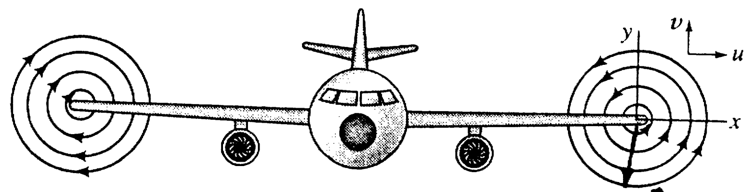
$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$, where C_1 is a constant.

or

$\underline{\underline{x^2 + y^2 = \text{Constant}}}$

4.38

4.38 Assume that the streamlines for the wingtip vortices from an airplane (see Fig. P4.37 and Video V4.2) can be approximated by circles of radius r and that the speed is $V = K/r$, where K is a constant. Determine the streamline acceleration, a_s, and the normal acceleration, a_n, for this flow.



■ FIGURE P4.13

$a_s = V\frac{dV}{ds}$ where since $V = \frac{K}{r}$, $\frac{dV}{ds} = 0$

Thus,

$$a_s = \underline{\underline{0}}$$

Also,

$$a_n = \frac{V^2}{\mathcal{R}} = \frac{(K/r)^2}{r} = \underline{\underline{\frac{K}{r^3}}}$$

4.39

4.39 A fluid flows past a sphere with an upstream velocity of $V_0 = 40$ m/s as shown in Fig. P4.39. From a more advanced theory it is found that the speed of the fluid along the front part of the sphere is $V = \frac{3}{2}V_0 \sin\theta$. Determine the streamwise and normal components of acceleration at point A if the radius of the sphere is $a = 0.20$ m.

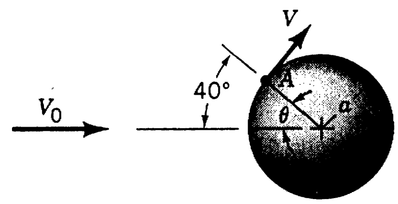


FIGURE P4.39

$$V = \frac{3}{2} V_0 \sin\theta = \frac{3}{2}\left(40\,\frac{m}{s}\right)\sin\theta = 60 \sin\theta\ \frac{m}{s} \qquad (1)$$

$$a_n = \frac{V^2}{R} = \frac{(60 \sin 40°)^2\ \frac{m^2}{s^2}}{0.2m} = \underline{\underline{7440\ \frac{m}{s^2}}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = (60 \sin\theta)\frac{\partial V}{\partial s}\ , \text{ where } \frac{\partial V}{\partial s} = \frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$$

From Eq. (1), $\frac{\partial V}{\partial \theta} = 60 \cos\theta$

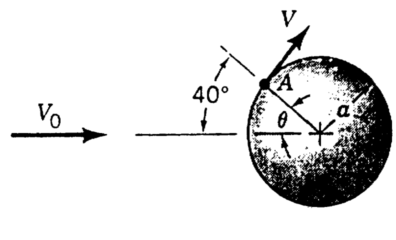
Also $s = a\theta = 0.2\,\theta$ m, where $\theta \sim$ rad, so that $\frac{\partial \theta}{\partial s} = \frac{1}{0.2m}$

Thus, for $\theta = 40°$

$$a_s = \left(60 \sin 40°\,\frac{m}{s}\right)\left(60 \cos 40°\,\frac{m}{s}\right)\left(\frac{1}{0.2m}\right) = \underline{\underline{8860\ \frac{m}{s^2}}}$$

4.40*

4.40* For flow past a sphere as discussed in Problem 4.39, plot a graph of the streamwise acceleration, a_s, the normal acceleration, a_n, and the magnitude of the acceleration as a function of θ for $0 \le \theta \le 90°$ with $V_0 = 50$ ft/s and $a = 0.1$, 1.0, and 10 ft. Repeat for $V_0 = 5$ ft/s. At what point is the acceleration a maximum; a minimum?



$$a_n = \frac{V^2}{\mathcal{R}} = \frac{\left(\frac{3}{2}V_0 \sin\theta\right)^2}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin^2\theta}} \qquad (1)$$

and $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$, where $\frac{\partial V}{\partial \theta} = \frac{3}{2}V_0 \cos\theta$ and $s = a\theta$

or $\frac{\partial \theta}{\partial s} = \frac{1}{a}$

Thus,

$$a_s = \left(\tfrac{3}{2}V_0 \sin\theta\right)\left(\tfrac{3}{2}V_0 \cos\theta\right)\frac{1}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin\theta\cos\theta}} \qquad (2)$$

Hence the magnitude of the acceleration is

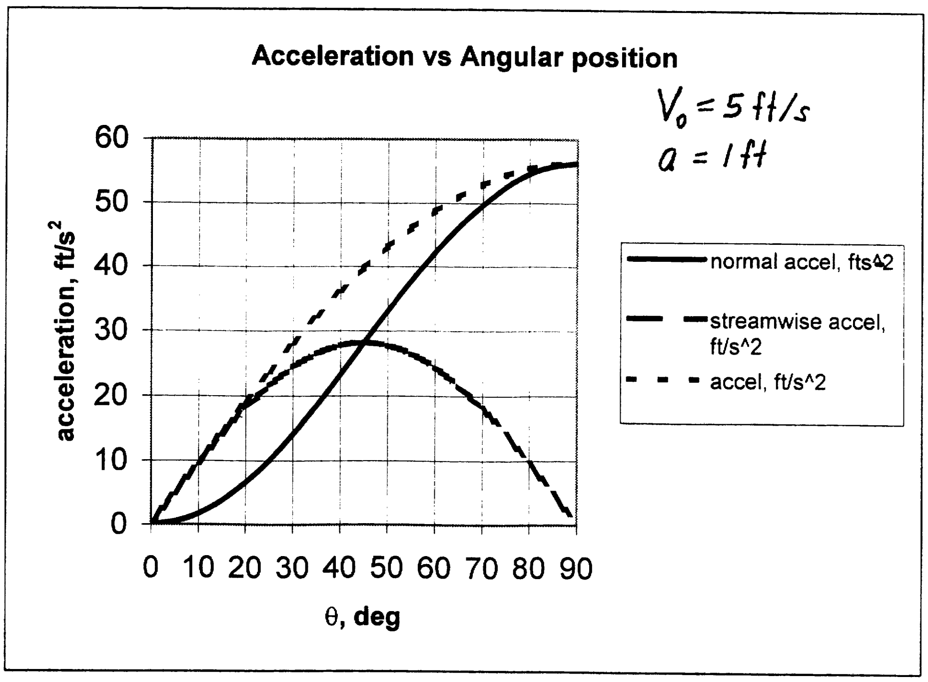
$$|\vec{a}| = \sqrt{a_n^2 + a_s^2} = \frac{9V_0^2}{4a}\sqrt{\sin^4\theta + \sin^2\theta\cos^2\theta} = \frac{9V_0^2}{4a}\sin\theta\sqrt{\sin^2\theta + \cos^2\theta}$$

or

$$(3) \qquad \underline{\underline{|\vec{a}| = \frac{9V_0^2}{4a}\sin\theta}} \qquad \text{Thus, } \underline{\underline{|\vec{a}|_{min} = 0 \text{ at } \theta = 0,\ |\vec{a}|_{max} = \frac{9V_0^2}{4a} \text{ at } \theta = 90°}}$$

An Excel Program was used to calculate a_s, a_n, and a from Eqns. (1), (2), and (3). The results are shown below. The results for other values are similar if the factor V_0^2/a is accounted for.
The following data is for $V_0 = 5$ ft/s, $a = 1$ ft

θ, deg	a_n, ft/s^2	a_s, ft/s^2	a, ft/s^2
0	0.0	0.0	0.0
5	0.4	4.9	4.9
10	1.7	9.6	9.8
15	3.8	14.1	14.6
20	6.6	18.1	19.2
25	10.0	21.5	23.8
30	14.1	24.4	28.1
35	18.5	26.4	32.3
40	23.2	27.7	36.2
45	28.1	28.1	39.8
50	33.0	27.7	43.1
55	37.7	26.4	46.1
60	42.2	24.4	48.7
65	46.2	21.5	51.0
70	49.7	18.1	52.9
75	52.5	14.1	54.3
80	54.6	9.6	55.4
85	55.8	4.9	56.0
90	56.3	0.0	56.3



4.41

4.41 A fluid flows past a circular cylinder of radius a with an upstream speed of V_0 as shown in Fig. P4.41. A more advanced theory indicates that if viscous effects are negligible, the velocity of the fluid along the surface of the cylinder is given by $V = 2V_0 \sin\theta$. Determine the streamline and normal components of acceleration on the surface of the cylinder as a function of V_0, a, and θ.

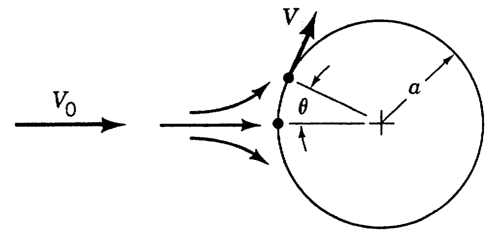


FIGURE P4.41

$$a_n = \frac{V^2}{R} = \frac{(2V_0 \sin\theta)^2}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin^2\theta}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}, \text{ where } \frac{\partial V}{\partial \theta} = 2V_0\cos\theta \text{ and } s = a\theta$$

$$\text{or } \frac{\partial \theta}{\partial s} = \frac{1}{a}$$

Thus,

$$a_s = (2V_0 \sin\theta)(2V_0\cos\theta)\frac{1}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin\theta\cos\theta}}$$

4.42*

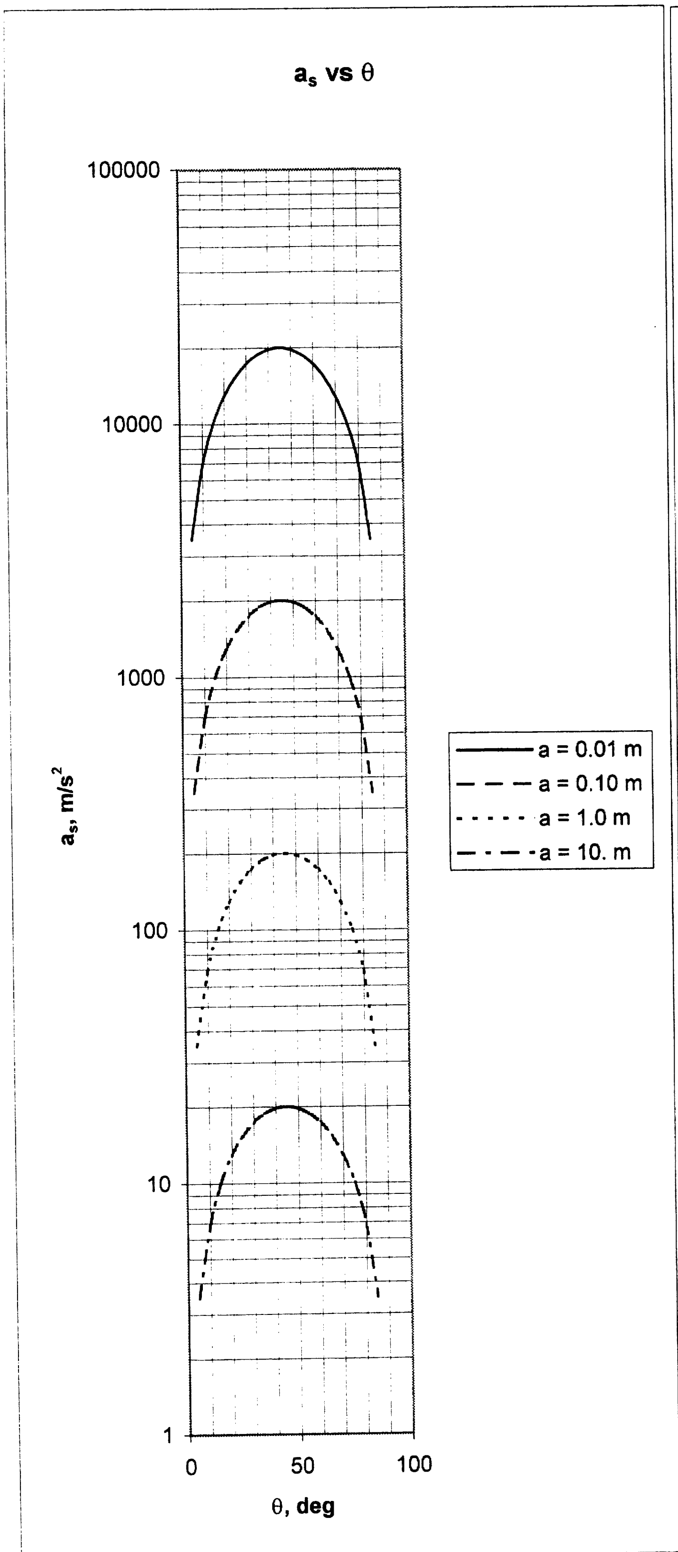
4.42* Use the results of Problem 4.41 to plot graphs of a_s and a_n for $0 \le \theta \le 90°$ with $V_0 =$ 10 m/s and a = 0.01, 0.10, 1.0, and 10.0 m.

From Problem 4.41, $a_n = \frac{4V_0^2}{a}\sin^2\theta$ and $a_s = \frac{4V_0^2}{a}\sin\theta\cos\theta$. These results with $V_0 = 10\frac{m}{s}$ and a = 0.01, 0.10, 1.0, and 10.0 m are plotted below.

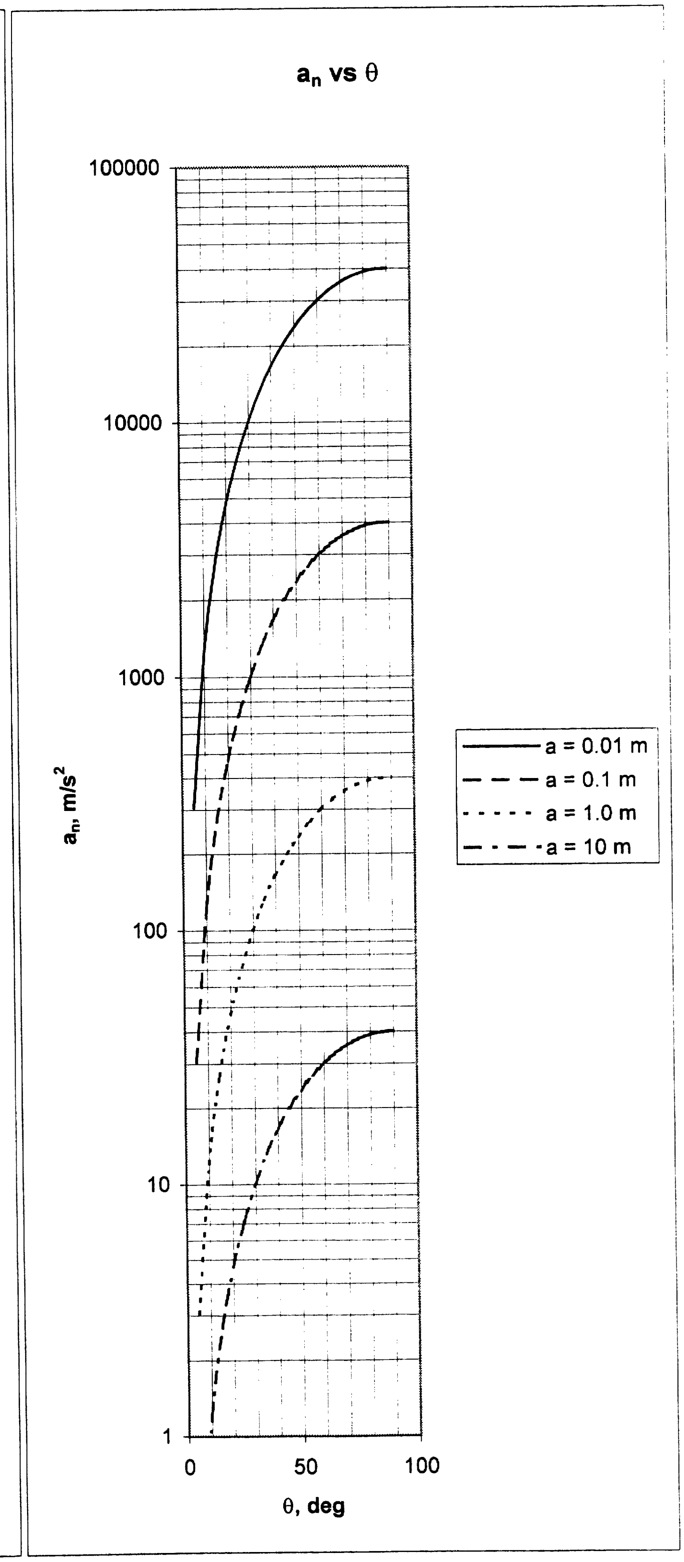
	a = 0.01 m	a = 0.10 m	a = 1.0 m	a = 10 m	a = 0.10 m	a = 0.10 m	a = 1.0 m	a = 10 m
θ, deg	a_s, ft/s²	a_s, ft/s²	a_s, ft/s²	a_s, ft/s²	a_n, ft/s²	a_n, ft/s²	a_n, ft/s²	a_n, ft/s²
0	0	0	0	0.00	0	0	0	0.00
5	3473	347	35	3.47	304	30	3	0.30
10	6840	684	68	6.84	1206	121	12	1.21
15	10000	1000	100	10.00	2679	268	27	2.68
20	12856	1286	129	12.86	4679	468	47	4.68
25	15321	1532	153	15.32	7144	714	71	7.14
30	17321	1732	173	17.32	10000	1000	100	10.00
35	18794	1879	188	18.79	13160	1316	132	13.16
40	19696	1970	197	19.70	16527	1653	165	16.53
45	20000	2000	200	20.00	20000	2000	200	20.00
50	19696	1970	197	19.70	23473	2347	235	23.47
55	18794	1879	188	18.79	26840	2684	268	26.84
60	17321	1732	173	17.32	30000	3000	300	30.00
65	15321	1532	153	15.32	32856	3286	329	32.86
70	12856	1286	129	12.86	35321	3532	353	35.32
75	10000	1000	100	10.00	37321	3732	373	37.32
80	6840	684	68	6.84	38794	3879	388	38.79
85	3473	347	35	3.47	39696	3970	397	39.70
90	0	0	0	0.00	40000	4000	400	40.00

(con't)

4.42* (con't)



a_s vs θ
100000
10000
1000
100
10
1
0
50
100
θ, deg
a_s, m/s^2
a = 0.01 m
a = 0.10 m
a = 1.0 m
a = 10. m



a_n vs θ
100000
10000
1000
100
10
1
0
50
100
θ, deg
a_n, m/s^2
a = 0.01 m
a = 0.1 m
a = 1.0 m
a = 10 m

4.43

4.43 Determine the x and y components of acceleration for the flow given in Problem 4.6. If $c > 0$, is the particle at point $x = x_0 > 0$ and $y = 0$ accelerating or decelerating? Explain. Repeat if $x_0 < 0$.

Since $u = c(x^2 - y^2)$ and $v = -2cxy$ it follows that

$\vec{a} = a_x \hat{i} + a_y \hat{j}$, where

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = c(x^2-y^2)(2cx) + (-2cxy)(-2cy)$$

or

$$\underline{\underline{a_x = 2c^2x(x^2+y^2)}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = c(x^2-y^2)(-2cy) + (-2cxy)(-2cx)$$

or

$$\underline{\underline{a_y = 2c^2y(x^2+y^2)}}$$

For $x = x_0$ and $y = 0$ we obtain:

$u = cx_0^2$, $v = 0$

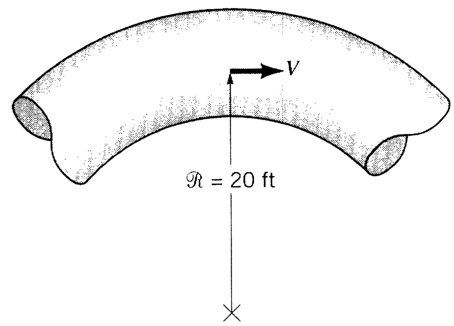
and

$a_x = 2c^2x_0^3$, $a_y = 0$

Thus, with $c > 0$ and $x_0 > 0$ it follows that $u > 0$, $a_x > 0$; i.e., the fluid is <u>accelerating.</u>

With $c > 0$ and $x_0 < 0$ it follows that $u > 0$, $a_x < 0$; i.e., the fluid is <u>decelerating.</u>

4.44 Water flows through the curved hose shown in Fig. P4.44 with an increasing speed of $V = 10t$ ft/s, where t is in seconds. For $t = 2$ s determine **(a)** the component of acceleration along the streamline, **(b)** the component of acceleration normal to the streamline, and **(c)** the net acceleration (magnitude and direction).



a) $a_s = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$, but $\frac{\partial V}{\partial s} = 0$

Thus,

$a_s = \frac{\partial V}{\partial t} = \underline{\underline{10 \frac{ft}{s^2}}}$ for all t.

b) $a_n = \frac{V^2}{\mathcal{R}} = \frac{(10t)^2 ft^2/s^2}{20\ ft} = 5t^2 \frac{ft}{s^2}$

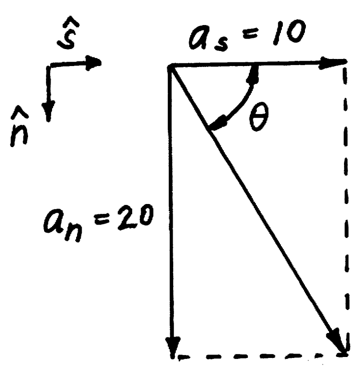
or

$a_n\Big|_{t=2s} = 5(2)^2 = \underline{\underline{20 \frac{ft}{s^2}}}$

c) At $t = 2s$ $\vec{a} = a_s\hat{s} + a_n\hat{n} = \underline{\underline{10\hat{s} + 20\hat{n} \frac{ft}{s^2}}}$

or

$|\vec{a}| = [a_s^2 + a_n^2]^{1/2} = [10^2 + 20^2]^{1/2} = \underline{\underline{22.4 \frac{ft}{s^2}}}$



and

$\theta = \arctan\left(\frac{20}{10}\right) = 63.4°$

4.45

4.45 Water flows steadily through the funnel shown in Fig. P4.45. Throughout most of the funnel the flow is approximately radial (along rays from O) with a velocity of $V = c/r^2$, where r is the radial coordinate and c is a constant. If the velocity is 0.4 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

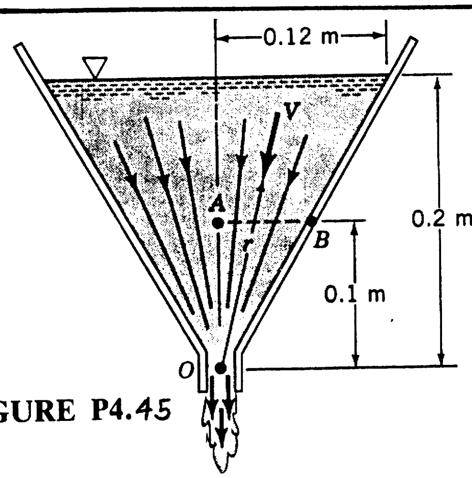


FIGURE P4.45

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r^2}$

Since $V = 0.4\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = V r^2 = (0.4\frac{m}{s})(0.1\,m)^2 = 4\times10^{-3}\,\frac{m^3}{s}$, or $V = \frac{4\times10^{-3}}{r^2}\,\frac{m}{s}$, where $r \sim m$

Thus,

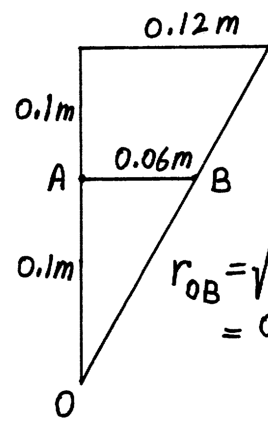
$$a_s = -\left(\frac{c}{r^2}\right)\left(-\frac{2c}{r^3}\right) = \frac{2c^2}{r^5}$$

At point A:

$$a_s = \frac{2(4\times10^{-3}\,\frac{m^3}{s})^2}{(0.1m)^5} = \underline{\underline{3.20\,\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{2(4\times10^{-3}\,\frac{m^3}{s})^2}{(0.1167m)^5} = \underline{\underline{1.48\,\frac{m}{s^2}}}$$



$r_{OB} = \sqrt{(0.1)^2 + (0.06)^2} = 0.1167m$

4.46

4.46 Water flows through the slit at the bottom of a two-dimensional water trough as shown in Fig. P4.46. Throughout most of the trough the flow is approximately radial (along rays from O) with a velocity of $V = c/r$, where r is the radial coordinate and c is a constant. If the velocity is 0.04 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

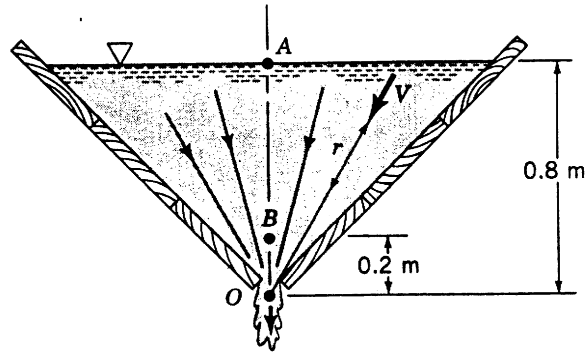


FIGURE P4.46

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r}$

Since $V = 0.04\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = Vr = (0.04\frac{m}{s})(0.1m) = 4\times10^{-3}\frac{m^2}{s}$, or $V = \frac{4\times10^{-3}}{r}\frac{m}{s}$, where $r \sim m$

Thus,

$$a_s = -\left(\frac{c}{r}\right)\left(-\frac{c}{r^2}\right) = \frac{c^2}{r^3}$$

At point A:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.8m)^3} = \underline{\underline{3.13\times10^{-5}\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.2m)^3} = \underline{\underline{2.00\times10^{-3}\frac{m}{s^2}}}$$

4.47

4.47 Air flows from a pipe into the region between two parallel circular disks as shown in Fig. P4.47. The fluid velocity in the gap between the disks is closely approximated by $V = V_0 R/r$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for r = 1, 2, or 3 ft if V_0 = 5 ft/s and R = 3 ft.

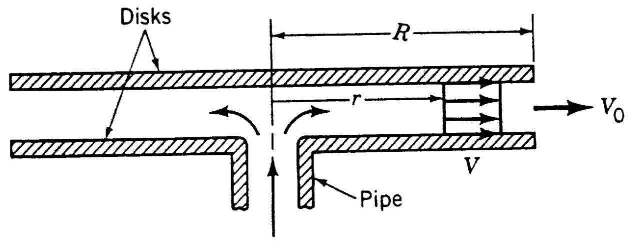


FIGURE P4.47

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial r}$, where $V = \frac{V_0 R}{r}$

Since $V_0 = 5\frac{ft}{s}$ and $R = 3ft$, $V = \frac{15}{r}\frac{ft}{s}$, where $r \sim ft$

Thus,

$$a_s = \left(\frac{V_0 R}{r}\right)\left(-\frac{V_0 R}{r^2}\right) = -\frac{V_0^2 R^2}{r^3} = -\frac{(5\frac{ft}{s})^2(3\,ft)^2}{r^3\,ft^3} = -\frac{225}{r^3}\frac{ft}{s^2}, \quad r \sim ft$$

At $r = 1\,ft$, $a_s = \underline{\underline{-225\,\frac{ft}{s^2}}}$

At $r = 2\,ft$, $a_s = \underline{\underline{-28.1\,\frac{ft}{s^2}}}$

At $r = 3\,ft$, $a_s = \underline{\underline{-8.33\,\frac{ft}{s^2}}}$

4.48

4.48 Air flows from a pipe into the region between a circular disk and a cone as shown in Fig. P4.48. The fluid velocity in the gap between the disk and the cone is closely approximated by $V = V_0 R^2/r^2$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for $r = 0.5$ and 2 ft if $V_0 = 5$ ft/s and $R = 2$ ft.

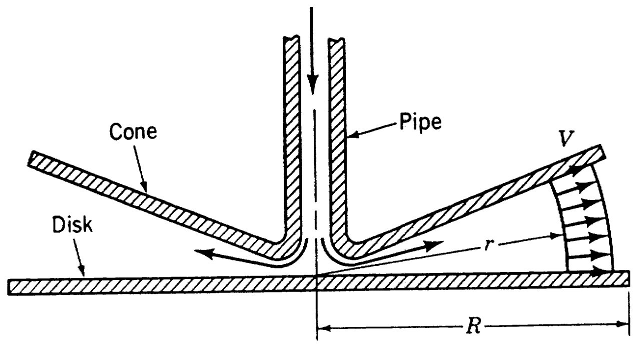


FIGURE P4.48

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial r}$, where $V = \frac{V_0 R^2}{r^2}$

Thus,

$$a_s = \left(\frac{V_0 R^2}{r^2}\right)\left(-\frac{2V_0 R^2}{r^3}\right) = -\frac{2V_0^2 R^4}{r^5} = -\frac{2(5\frac{ft}{s})^2(2\,ft)^4}{r^5\,ft^5} = -\frac{800}{r^5}\frac{ft}{s}$$

where $r \sim ft$

At $r = 0.5$ ft, $a_s = \underline{\underline{-25{,}600\ \frac{ft}{s^2}}}$

At $r = 2$ ft, $a_s = \underline{\underline{-25\ \frac{ft}{s^2}}}$

4.49

4.49 Water flows through a duct of square cross section as shown in Fig. P4.49 with a constant, uniform velocity of $V = 20$ m/s. Consider fluid particles that lie along line A–B at time $t = 0$. Determine the position of these particles, denoted by line A'–B', when $t = 0.20$ s. Use the volume of fluid in the region between lines A–B and A'–B' to determine the flowrate in the duct. Repeat the problem for fluid particles originally along line C–D; along line E–F. Compare your three answers.

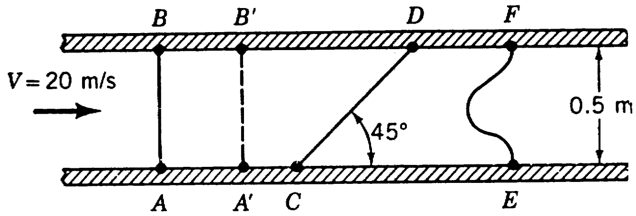


FIGURE P4.49

Since V is constant in time and space, all particles on line AB move a distance $\ell = V\Delta t \doteq (20\frac{m}{s})(0.2s) = 4m$ from $t=0$ to $t=0.2s$. Thus, the volume of $ABA'B'$ is $\forall_{ABA'B'} = (0.5m)^2(4m) = 1.00\ m^3$ so that

$$Q = \frac{\forall_{ABA'B'}}{\Delta t} = \frac{1.00\ m^3}{0.2s} = \underline{\underline{5.0\ \frac{m^3}{s}}}$$

Similarly from $t=0$ to $t=0.2s$ the fluid along lines CD and EF move to $C'D'$ and $E'F'$, respectively. Also, $\forall_{CDC'D'} = \forall_{EFE'F'} = \forall_{ABA'B'}$ so that we obtain $Q = \frac{\forall}{\Delta t} = 5.0\frac{m^3}{s}$ regardless which line we consider.

4.50

4.50 Repeat Problem 4.49 if the velocity profile is linear from 10 to 20 m/s across the duct as shown in Fig. P4.50.

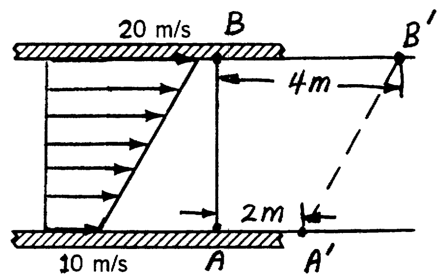


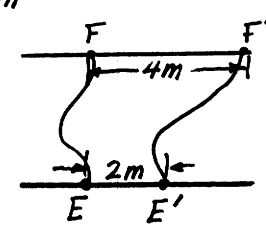
FIGURE P4.50

From $t=0$ to $t=0.2s$ the particle initially at B travels a distance $\ell_B = V_B\Delta t = (20\frac{m}{s})(0.2s) = 4m$ as shown while one at A travels a distance $\ell_A = V_A\Delta t = (10\frac{m}{s})(0.2s) = 2m$. Since the velocity profile is linear line AB remains straight, but "tilts" into line $A'B'$. Thus, the volume of fluid crossing the initial line AB is

$$\forall_{ABB'A'} = \tfrac{1}{2}(\ell_A + \ell_B)A = \tfrac{1}{2}(2m+4m)(0.5m)^2 = 0.75\ m^3$$

so that

$$Q = \frac{\forall_{ABB'A'}}{\Delta t} = \frac{0.75\ m^3}{0.2s} = \underline{\underline{3.75\ \frac{m^3}{s}}}$$



For any curved line EF (which moves to $E'F'$) $\forall_{EFF'E'} = \forall_{ABB'A'}$ so that the same volume flowrate, Q, is obtained for any line considered.

4.51

4.51 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.51 and Video V10.5. The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

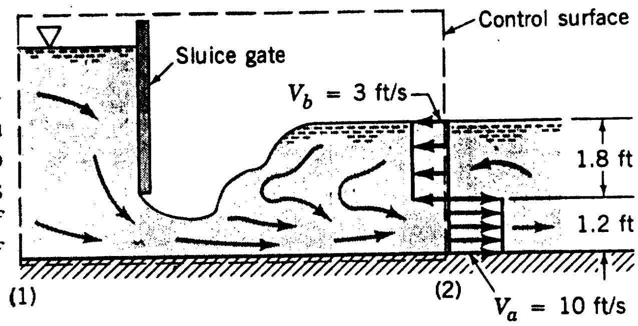


FIGURE P4.51

$$Q = V_a A_a - V_b A_b = (10\,\tfrac{ft}{s})(1.2ft)(20ft) - (3\,\tfrac{ft}{s})(1.8ft)(20ft)$$
$$= \underline{\underline{132\ \tfrac{ft^3}{s}}}$$

4.52

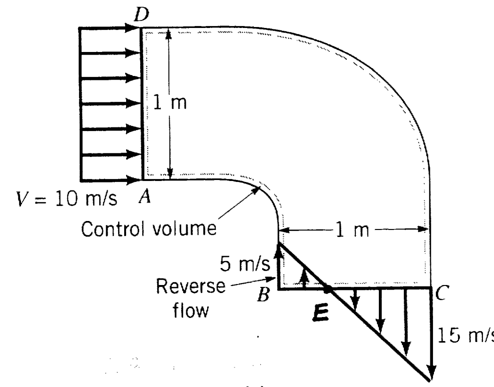
4.52 At time $t = 0$ the valve on an initially empty (perfect vacuum, $\rho = 0$) tank is opened and air rushes in. If the tank has a volume of $\forall_0$ and the density of air within the tank increases as $\rho = \rho_\infty(1 - e^{-bt})$, where b is a constant, determine the time rate of change of mass within the tank.

For $t \geq 0$, $\rho = \rho_0[1 - e^{-bt}]$ so that M = mass of air in tank $= \rho\forall_0 = \rho_0\forall_0[1 - e^{-bt}]$

Thus, $\dfrac{dM}{dt} = \underline{\underline{\rho_0 \forall_0 b\, e^{-bt}}}$

4.54

4.54 Air enters an elbow with a uniform speed of 10 m/s as shown in Fig. P4.54. At the exit of the elbow the velocity profile is not uniform. In fact, there is a region of separation or reverse flow. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.01$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



From $t=0$ to $t=0.01s$ particles A, B, C, D, and E move the following distances:

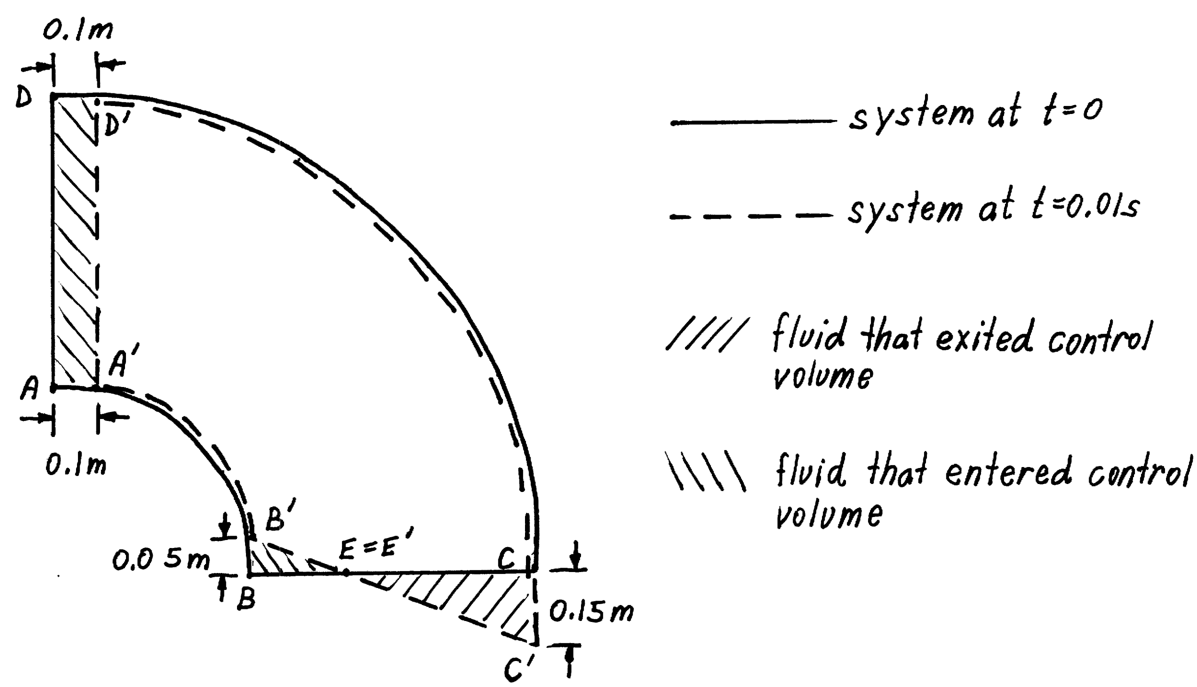
$\delta_A = V_A \delta t = (10\frac{m}{s})(0.01s) = 0.1m = \delta_D$

$\delta_B = V_B \delta t = (5\frac{m}{s})(0.01s) = 0.05m$

$\delta_C = V_C \delta t = (15\frac{m}{s})(0.01s) = 0.15m$, and

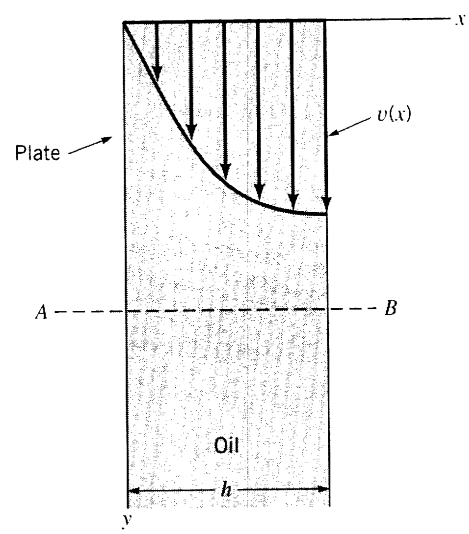
$\delta_E = 0$

Thus, fluid along lines AD and BEC originally moves to lines $A'D'$ and $B'E'C'$ shown below.



4.55

4.55 A layer of oil flows down a vertical plate as shown in Fig. P4.55 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\,\hat{\mathbf{j}}$ where V_0 and h are constants. **(a)** Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. **(b)** Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.6.)



a) $v = \dfrac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\Big|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\Big|_{x=h} = \mu \frac{dv}{dx}\Big|_{x=h} = \mu \frac{V_0}{h^2}\Big[2h - 2x\Big]_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

b) $$Q_{AB} = \int v\, dA = \int_{x=0}^{x=h} v\, b\, dx = \int_0^h \frac{V_0}{h^2}(2hx - x^2)\, b\, dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\Big[hx^2 - \tfrac{1}{3}x^3\Big]_0^h = \underline{\underline{\tfrac{2}{3}V_0 h b}}$$

4.56

4.56 Water flows in the branching pipe shown in Fig. P4.56 with uniform velocity at each inlet and outlet. The fixed control volume indicated coincides with the system at time $t = 20$ s. Make a sketch to indicate **(a)** the boundary of the system at time $t = 20.2$ s, **(b)** the fluid that left the control volume during that 0.2-s interval, and **(c)** the fluid that entered the control volume during that time interval.

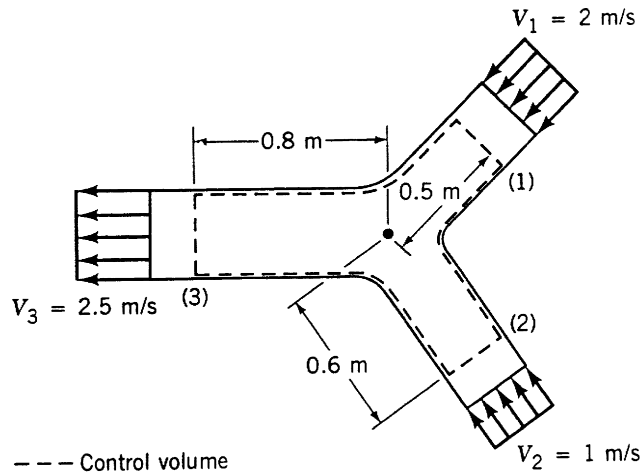


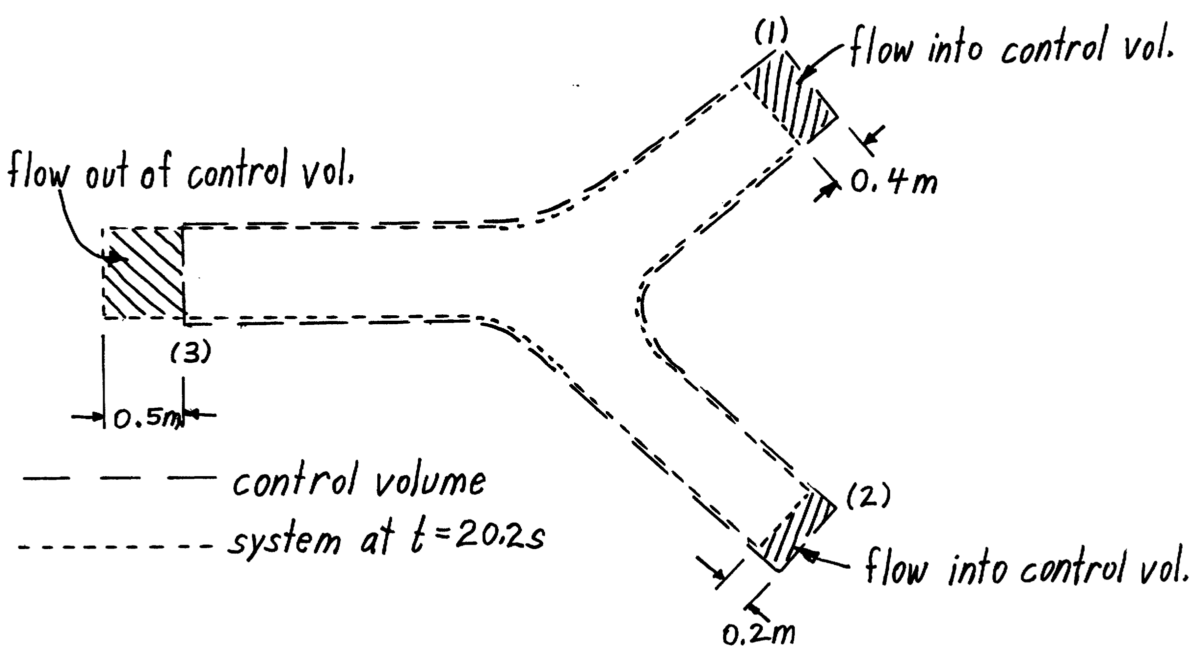
FIGURE P4.56

Since V is constant, the fluid travels a distance $\ell = V\delta t$ in time Δt. Thus, $\ell_1 = V_1\,\delta t = (2\,\frac{m}{s})(20.\ -20)s = 0.4\,m$

$$\ell_2 = V_2\,\delta t = (1\,\tfrac{m}{s})(20.\ -20)s = 0.2m$$

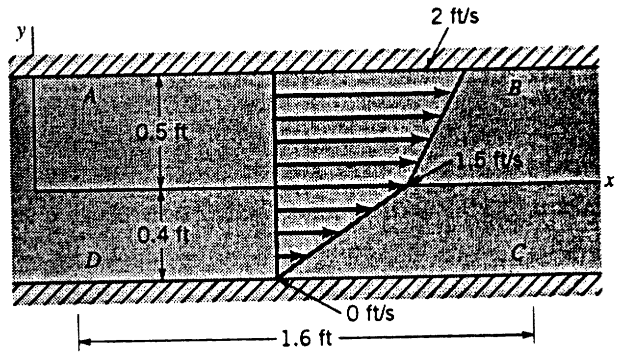
and $\ell_3 = V_3\,\delta t = (2.5\,\frac{m}{s})(20.\ -20)s = 0.50\,m$

The system at $t = 20.2s$ and the fluid that has entered or exited the control volume are indicated in the figure below.



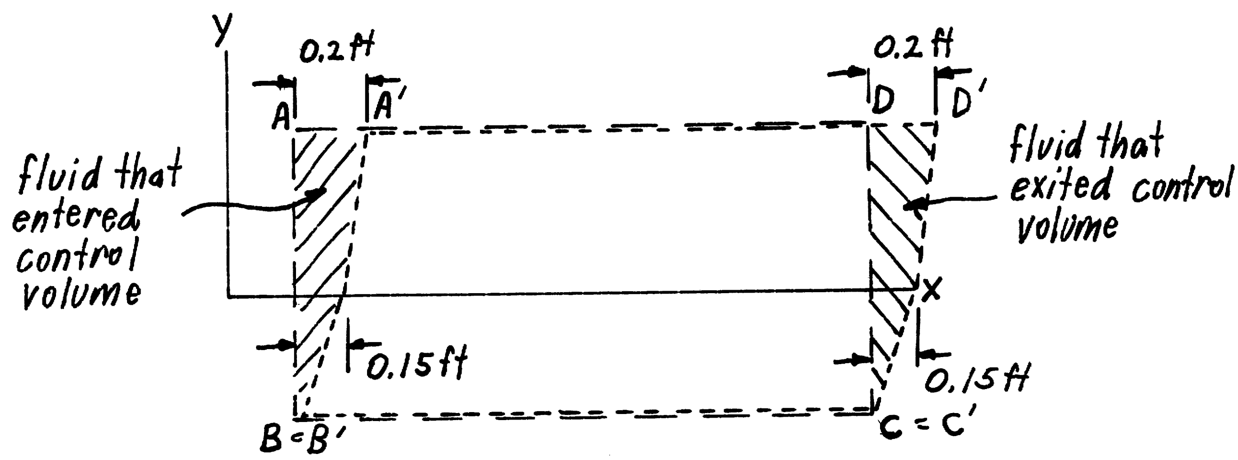
4.57

4.57 Two liquids with different densities and viscosities fill the gap between parallel plates as shown in Fig. P4.57. The bottom plate is fixed; the top plate moves with a speed of 2 ft/s. The velocity profile consists of two linear segements as indicated. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 0.1$ s and (b) the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.57

The fluid at $y = -0.4$ ft (the bottom plate) remains stationary. At $y = 0$ the fluid speed is 1.5 ft/s so that at time $t = 0.1$s it has moved to the right a distance $x = Vt = 1.5\frac{ft}{s}(0.1s) = 0.15$ ft. In the same time period the top plate and the fluid stuck to it has moved a distance $x = 2\frac{ft}{s}(0.1s) = 0.2$ ft. Since the velocity profile is piecewise linear, the ends of the system will move so that lines AD and BC remain straight. This is indicated in the sketch below.



— — — control volume

- - - - - - - - - system at $t = 0.1$s

4.58

4.58 Water is squirted from a syringe with a speed of $V = 5$ m/s by pushing in the plunger with a speed of $V_p = 0.03$ m/s as shown in Fig. P4.58. The surface of the deforming control volume consists of the sides and end of the cylinder and the end of the plunger. The system consists of the water in the syringe at $t = 0$ when the plunger is at section (1) as shown. Make a sketch to indicate the control surface and the system when $t = 0.5$ s.

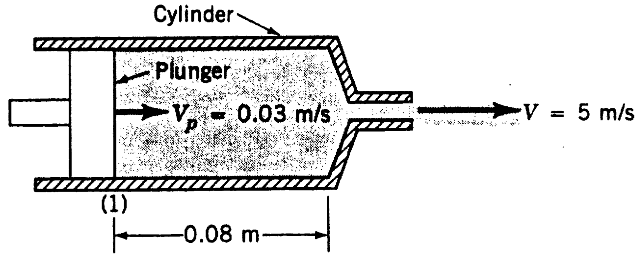
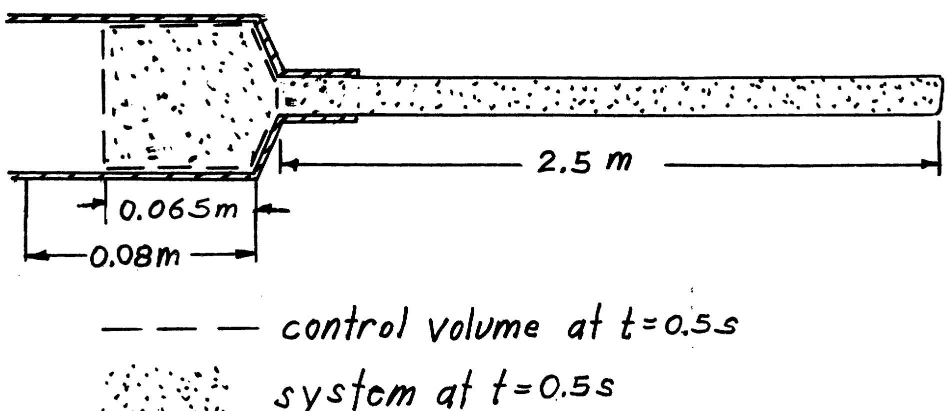


FIGURE P4.58

During the $t = 0.5$s time interval the plunger moves $\ell_1 = V_p\,\delta t = 0.015$m and the water initially at the exit moves $\ell_2 = V\,\delta t = 2.5$m. The corresponding control surfaces and systems at $t=0$ and $t=0.5$s shown in the figure below.



4.59

4.59 Water enters a 5-ft-wide, 1-ft-deep channel as shown in Fig. P4.59. Across the inlet the water velocity is 6 ft/s in the center portion of the channel and 1 ft/s in the remainder of it. Farther downstream the water flows at a uniform 2 ft/s velocity across the entire channel. The fixed control volume $ABCD$ coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 0.5$ s and (b) the fluid that has entered and exited the control volume in that time period.

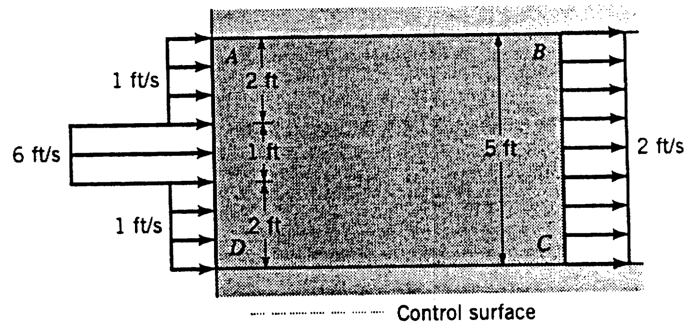
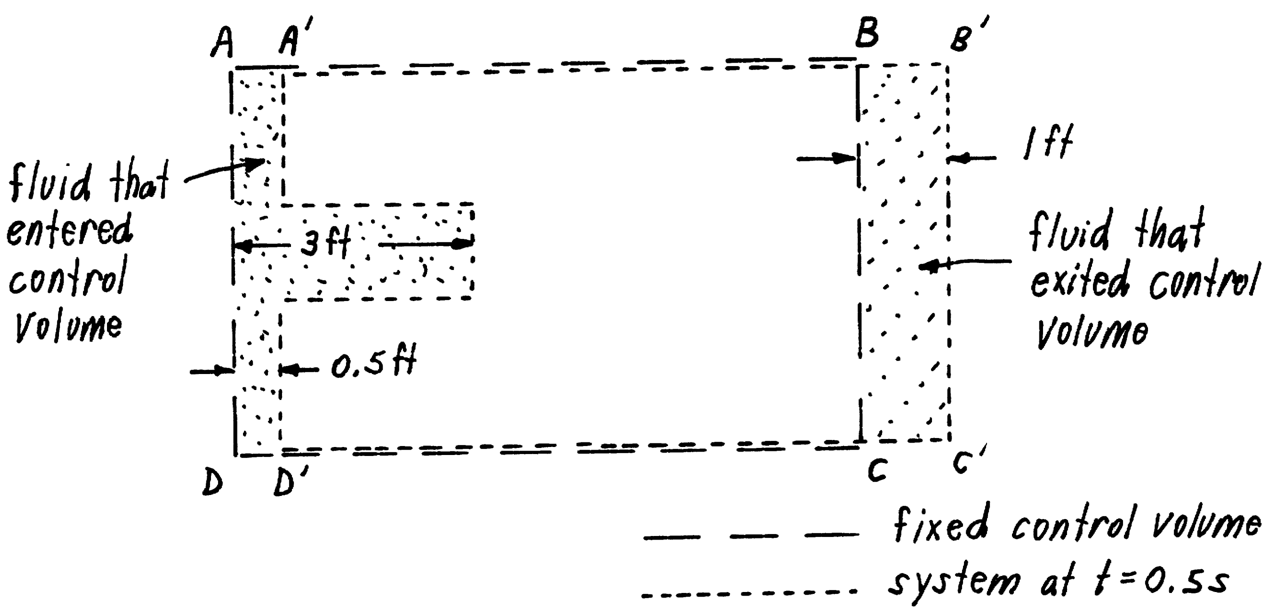
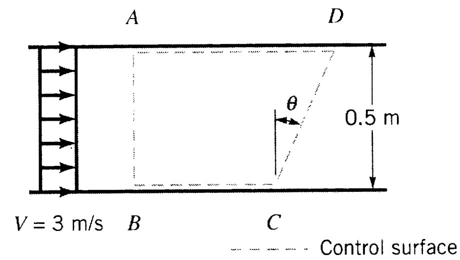


FIGURE P4.59

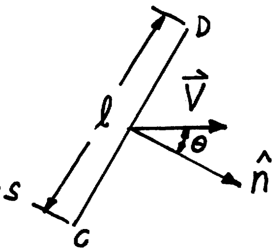
During the $t = 0.5$ s time interval the fluid that was along line BC at time $t=0$ has moved to the right a distance $\ell = V\,t = 2\frac{ft}{s}(0.5s) = 1 ft$. Similarly, portions of the fluid along line AD have moved $\ell = 1\frac{ft}{s}(0.5s) = 0.5 ft$ and $\ell = 6\frac{ft}{s}(0.5s) = 3 ft$. This assumes the $1\frac{ft}{s}$ and $6\frac{ft}{s}$ fluid streams do not mix or intermingle during the 0.5 s time interval. See figure below.



4.60 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.60 with a uniform velocity of 3 m/s. **(a)** Directly integrate Eq. 4.16 with $b = 1$ to determine the mass flowrate (kg/s) across section CD of the control volume. **(b)** Repeat part (a) with $b = 1/\rho$, where ρ is the density. Explain the physical interpretation of the answer to part (b).



a) $$\dot{B}_{out} = \int_{cs_{out}} \rho b \, \vec{V}\cdot\hat{n} \, dA \qquad (1)$$



With $b = 1$ and $\vec{V}\cdot\hat{n} = V\cos\theta$ this becomes

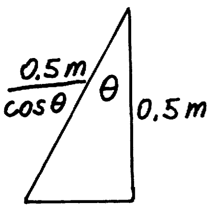
$$\dot{B}_{out} = \int_{CD} \rho V\cos\theta \, dA = \rho V\cos\theta \int_{CD} dA$$

$$= \rho V\cos\theta \, A_{CD}\,, \quad \text{where } A_{CD} = \ell\,(2m)$$

$$= \left(\frac{0.5\,m}{\cos\theta}\right)(2m)$$

$$= \left(\frac{1}{\cos\theta}\right) m^2$$

$\ell = \dfrac{0.5m}{\cos\theta}$



Thus, with $V = 3\,m/s$,

$$\dot{B}_{out} = \left(3\,\tfrac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2\left(999\,\tfrac{kg}{m^3}\right) = \underline{\underline{3000\,\tfrac{kg}{s}}}$$

b) With $b = 1/\rho$ Eq. (1) becomes

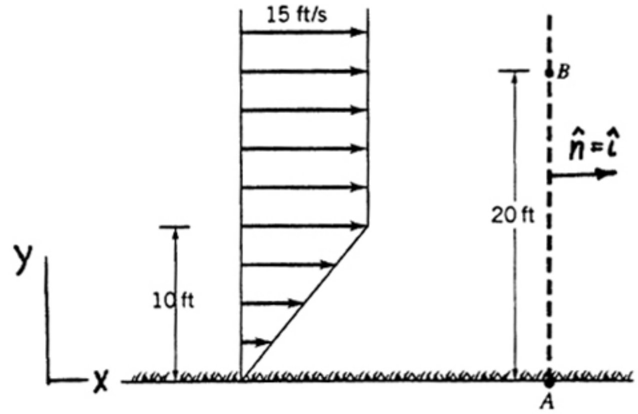
$$\dot{B}_{out} = \int_{CD} \vec{V}\cdot\hat{n}\, dA = \int_{CD} V\cos\theta\, dA = V\cos\theta\, A_{CD}$$

$$= \left(3\,\tfrac{m}{s}\right)\cos\theta\left(\frac{1}{\cos\theta}\right)m^2 = \underline{\underline{3.00\,\tfrac{m^3}{s}}}$$

With $b = 1/\rho = \dfrac{1}{\left(\frac{mass}{vol}\right)} = \dfrac{vol}{mass}$ it follows that "B = volume" (i.e., $b = \frac{B}{mass}$) so that $\int \vec{V}\cdot\hat{n}\, dA = \dot{B}_{out}$ represents the volume flowrate (m^3/s) from the control volume.

4.61

4.61 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.61. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface A–B, which is of unit depth into the paper.



■ FIGURE P4.61

$$\dot{\vec{B}}_{AB} = \int_{AB} \rho \vec{b}\, \vec{V}\cdot\hat{n}\, dA = \int_{AB} \rho \vec{V}\, \vec{V}\cdot\hat{n}\, dA = \rho \int_{y=0}^{y=20\,ft} (V\hat{i})[(V\hat{i})\cdot\hat{i}](1\,ft)\,dy$$

$$= \rho\hat{i}\int_0^{20} V^2\, dy$$

But, $V = \frac{15}{10}y\ \frac{ft}{s}$ for $0 \le y \le 10\,ft$ (i.e., $V=0$ at $y=0$; $V=15\,\frac{ft}{s}$ at $y=10$)
and $V = 15\,\frac{ft}{s}$ for $y \ge 10\,ft$

Thus,

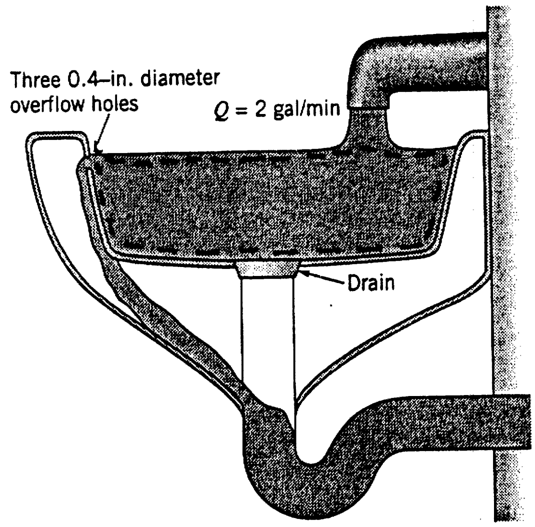
$$\dot{\vec{B}}_{AB} = \rho\hat{i}\left[\int_0^{10}\left(\frac{15}{10}y\right)^2 dy + \int_{10}^{20}(15)^2 dy\right] = \rho\hat{i}\left[2.25\,\frac{y^3}{3}\Big|_0^{10} + 225\,y\Big|_0^{10}\right]$$

$$= 0.00238\,\frac{slugs}{ft^3}\left[750\,\frac{ft^4}{s^2} + 2250\,\frac{ft^4}{s^2}\right]\hat{i}$$

$$= \underline{\underline{7.14\hat{i}\,\frac{slug\ ft}{s^2}}}$$

5.1

5.1 Water flows into a sink as shown in Video V5.1 and Fig. P5.1 at a rate of 2 gallons per minute. Determine the average velocity through each of the three 0.4 in. diameter overflow holes if the drain is closed and the water level in the sink remains constant.



■ FIGURE P5.1

$Q_1 = Q_2$ for the control volume indicated, where

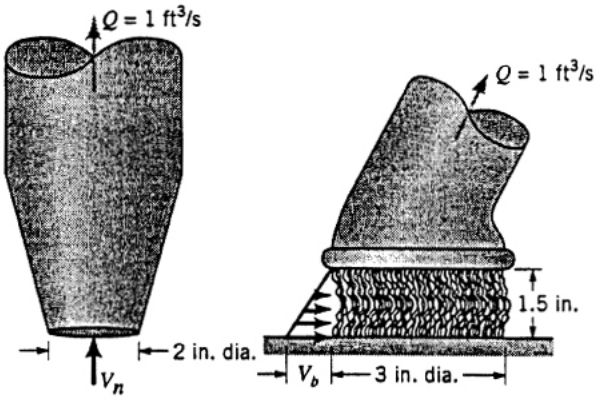
$$Q_1 = 2\,\frac{gal}{min}\left(\frac{1\,min}{60\,s}\right)\frac{1}{7.48\,\frac{gal}{ft^3}} = 0.00446\,\frac{ft^3}{s}$$

Thus,

$$Q_1 = A_2 V_2 \quad \text{or} \quad V_2 = \frac{Q_1}{A_2} = \frac{0.00446\,\frac{ft^3}{s}}{3\left[\frac{\pi}{4}\left(\frac{0.4}{12}ft\right)^2\right]} = \underline{\underline{1.70\,\frac{ft}{s}}}$$

5.2

5.2 Various types of attachments can be used with the shop vac shown in Video V5.2. Two such attachments are shown in Fig. P5.2—a nozzle and a brush. The flowrate is 1 ft³/s. (a) Determine the average velocity through the nozzle entrance, V_n. (b) Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to V_b along the length of the bristles as shown in the figure. Determine the value of V_b.



■ FIGURE P5.2

(a) $Q_1 = Q_2$ where $Q_2 = 1\,\frac{ft^3}{s}$

Thus,

$$A_1 V_1 = Q_2 \quad \text{or} \quad V_1 \equiv V_n = \frac{1\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2}$$

so

$$V_n = \underline{\underline{45.8\,\frac{ft}{s}}}$$

(b) $Q_3 = Q_4$ where $Q_4 = 1\,\frac{ft^3}{s}$ and $Q_3 = \overline{V}_3 A_3$ where

$\overline{V}_3$ = average velocity at (3) $= \frac{1}{2} V_b$ and

$A_3 = \pi D_3 h_3$

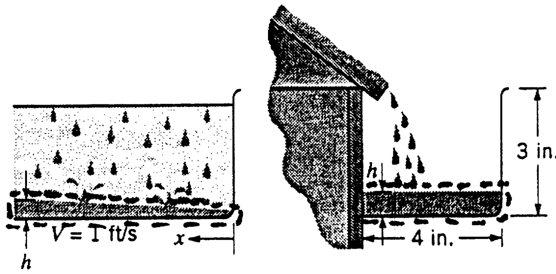
Thus,

$$\frac{1}{2} V_b \left[\pi \left(\frac{3}{12}\,ft\right)\left(\frac{1.5}{12}\,ft\right)\right] = 1\,\frac{ft^3}{s}, \text{ or}$$

$$V_b = \underline{\underline{20.4\,\frac{ft}{s}}}$$

5.3

5.3 Water flows into a rain gutter on a house as shown in Fig. P5.3 and in Video V10.3 at a rate of 0.0040 ft^3/s per foot of length of the gutter. At the beginning of the gutter ($x = 0$) the water depth is zero. (**a**) If the water flows with a velocity of 1.0 ft/s throughout the entire gutter, determine an equation for the water depth, h, as a function of location, x. (**b**) At what location will the gutter overflow?



■ FIGURE P5.3

(a) For the control volume shown in the sketch above

$$Q_{in} = Q_{out}$$

or $$\left(0.0040 \frac{ft^3}{s}\right)(x\ ft) = V_{out} A_{out} = \left(1 \frac{ft}{s}\right)\left(\frac{4}{12} ft\right)(h\ ft)$$

so $$h = \underline{\underline{0.012\ x}}$$

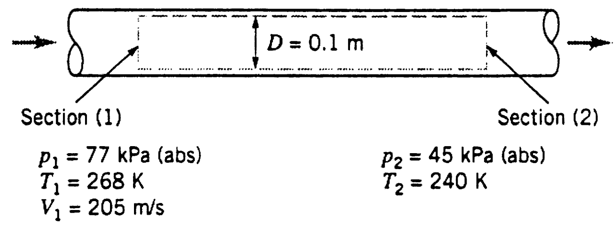
(b) The gutter will overflow when $h = \frac{3}{12}$ ft

So $$\frac{3}{12} ft = 0.012\ x$$

and $$x = \underline{\underline{20.8\ ft}}$$

5.4

5.4 Air flows steadily between two cross sections in a long, straight section of 0.1-m inside-diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.4. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).



■ FIGURE P5.4

This analysis is similar to the one of Example 5.2. For steady flow between sections (1) and (2)

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \bar{V}_2 = \rho_1 A_1 \bar{V}_1$$

Thus

$$\bar{V}_2 = \frac{\rho_1}{\rho_2}\frac{A_1}{A_2}\bar{V}_1 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\frac{T_2}{T_1} \qquad (2)$$

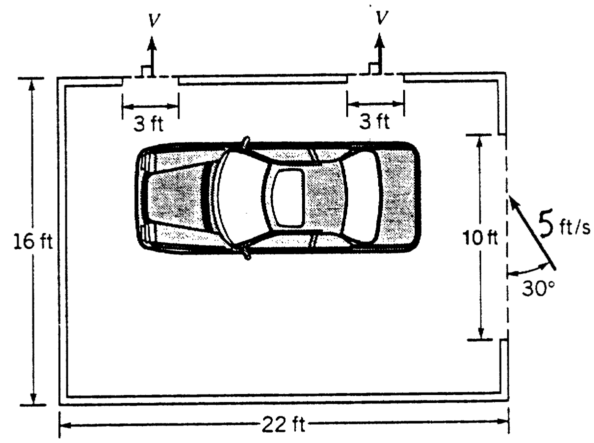
Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\bar{V}_2 = \frac{p_1}{p_2}\frac{T_2}{T_1}\bar{V}_1 = \frac{[77\ kPa(abs)](240 K)}{[45\ kPa(abs)](268 K)}\left(205\ \frac{m}{s}\right)$$

$$\bar{V}_2 = \underline{\underline{314\ \frac{m}{s}}}$$

5.5

5.5 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.5. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.



■ FIGURE P5.5

For steady incompressible flow

$$Q_{\substack{garage\\door}} = Q_{window} + Q_{window}$$

or

$$A_{\substack{garage\\door}} V_{\substack{normal\ to\\garage\ door}} = A_{window} V + A_{window} V$$

so the average speed, V, of the air through the two windows is

$$V = \frac{A_{\substack{garage\\door}} V_{normal\ to\ garage\ door}}{2A_{window}} = \frac{(7\,ft)(10\,ft)(5\,\frac{ft}{s})\sin 30^\circ}{2(3\,ft)(4\,ft)} = \underline{\underline{7.3\ \frac{ft}{s}}}$$

5.6

5.6 A hydroelectric turbine passes 4 million gal/min through its blades. If the average velocity of the flow in the circular cross-section conduit leading to the turbine is not to exceed 30 ft/s, determine the minimum allowable diameter of the conduit.

For incompressible flow through the conduit and turbine

$$Q_{conduit} = Q_{turbine}$$

Thus

$$A_{conduit} \overline{V}_{conduit} = Q_{turbine}$$

and

$$d_{conduit} = \sqrt{\frac{4}{\pi}\frac{Q_{turbine}}{\overline{V}_{conduit}}} = \sqrt{\frac{(4)(4\times10^6\,\frac{gal}{min})}{\pi\,(30\,\frac{ft}{s})(60\,\frac{s}{min})(7.48\,\frac{gal}{ft^3})}}$$

$$d_{conduit} = \underline{\underline{19.5\ ft}}$$

5.7

5.7 The cross-sectional area of the test section of a large water tunnel is 100 ft^2. For a test velocity of 50 ft/s, what volume flowrate capacity in gal/min is needed?

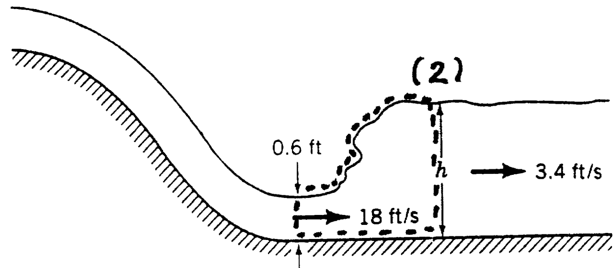
$$Q = A\bar{V}$$

$$Q = (100\,ft^2)\left(50\,\frac{ft}{s}\right)\left(7.48\,\frac{gal}{ft^3}\right)\left(\frac{1}{60\,\frac{s}{min}}\right)$$

$$Q = \underline{\underline{623}}\ gpm$$

5.8

5.8 A hydraulic jump (see Video V10.5) is in place downstream from a spill-way as indicated in Fig. P5.8. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



■ FIGURE P5.8

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{(18 \frac{ft}{s})(0.6\, ft)}{(3.4 \frac{ft}{s})} = \underline{\underline{3.18\, ft}}$$

5.9

5.9 A water jet pump (see Fig. P5.9) involves a jet cross section area of 0.01 m², and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross section area associated with the jet and entrained streams is 0.075 m². These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross section area of 0.075 m². Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.

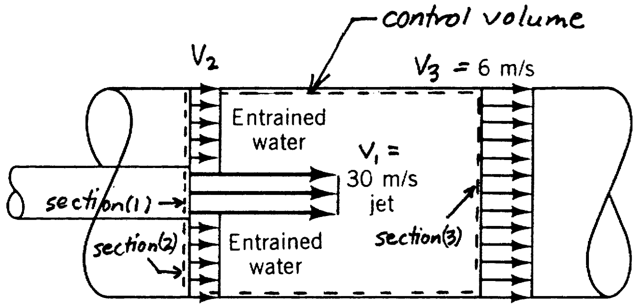


FIGURE P5.9

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\ \frac{m}{s}\right)(0.075\ m^2) - \left(30\ \frac{m}{s}\right)(0.01\ m^2)\right]\left(1000\ \frac{liters}{m^3}\right)$$

$$Q_2 = \underline{\underline{150\ \frac{liters}{s}}}$$

5.10

5.10 Water enters a cylindrical tank through two pipes at rates of 250 and 100 gal/min (see Fig. P5.10). If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through an 8-in.-inside diameter pipe.

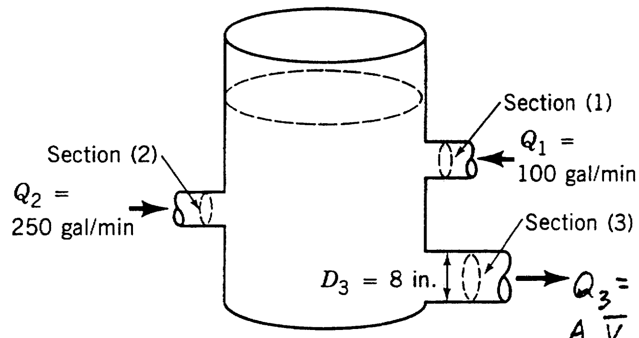


FIGURE P5.10

For steady and incompressible flow

$$Q_3 = Q_1 + Q_2$$

or

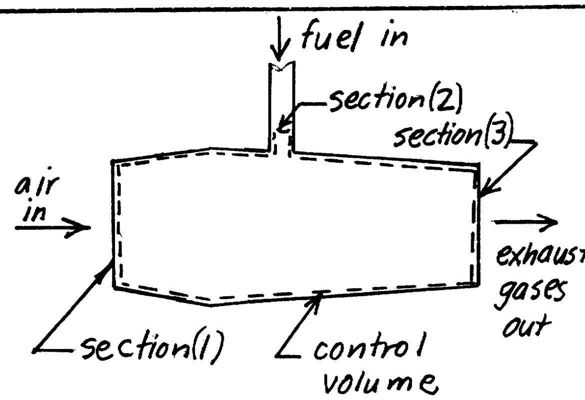
$$\bar{V}_3 = \frac{1}{A_3}\left(Q_1 + Q_2\right) = \frac{1}{\frac{\pi d_3^2}{4}}\left(Q_1 + Q_2\right)$$

$$\bar{V}_3 = \frac{1}{\frac{\pi (8\text{ in.})^2}{4}}\left(100\text{ gpm} + 250\text{ gpm}\right)\left(231\ \frac{\text{in.}^3}{\text{gal}}\right)\left(\frac{1}{60\ \frac{\text{s}}{\text{min}}}\right)\left(\frac{1}{12\ \frac{\text{in.}}{\text{ft}}}\right)$$

$$\bar{V}_3 = \underline{\underline{2.23}}\ \frac{\text{ft}}{\text{s}}$$

5.11

5.11 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross section area is 3.5 ft², estimate the density of the exhaust gases in lbm/ft³.



For steady flow

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

$$\rho_3 A_3 \bar{V}_3 = \dot{m}_1 + \dot{m}_2$$

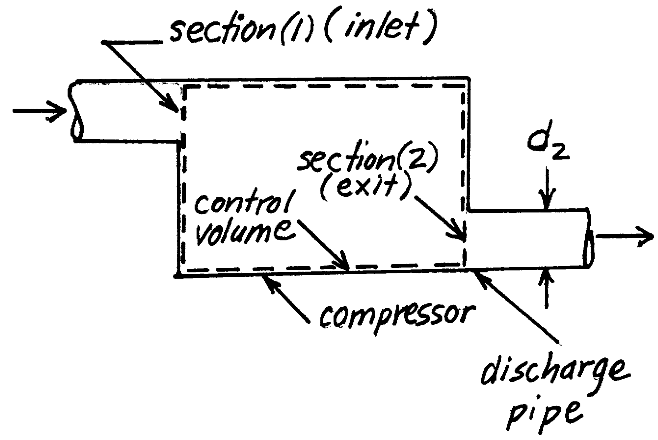
Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \bar{V}_3} = \frac{65\ \frac{\text{lbm}}{\text{s}} + 0.60\ \frac{\text{lbm}}{\text{s}}}{(3.5\text{ ft}^2)\left(1500\ \frac{\text{ft}}{\text{s}}\right)}$$

$$\rho_3 = \underline{\underline{0.0125}}\ \frac{\text{lbm}}{\text{ft}^3}$$

5.12

5.12 Air at standard atmospheric conditions is drawn into a compressor at the steady rate of 30 m^3/min. The compressor pressure ratio, p_{exit}/p_{inlet}, is 10 to 1. Through the compressor p/ρ^n remains constant with $n = 1.4$. If the average velocity in the compressor discharge pipe is not to exceed 30 m/s, calculate the minimum discharge pipe diameter required.



For steady flow

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \overline{V}_2 = \rho_1 Q_1$$

Thus

$$\rho_2 \frac{\pi d_2^2}{4} \overline{V}_2 = \rho_1 Q_1$$

and

$$d_2 = \sqrt{\frac{\rho_1 Q_1}{\rho_2 \frac{\pi}{4} \overline{V}_2}}$$

However

$$\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

so

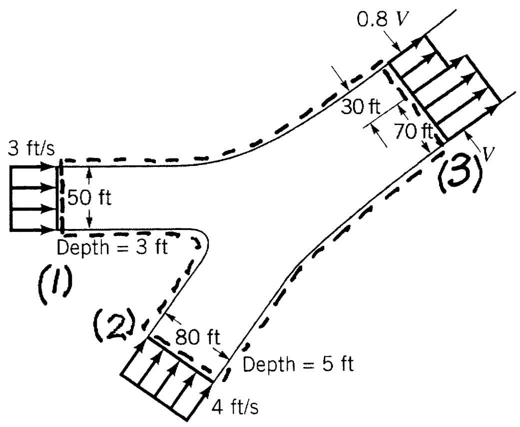
$$d_2 = \sqrt{\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} \frac{Q_1}{\frac{\pi}{4} \overline{V}_2}} = \sqrt{\left(\frac{1}{10}\right)^{\frac{1}{1.4}} \frac{30 \frac{m^3}{min}}{\frac{\pi}{4}\left(30 \frac{m}{s}\right) 60 \frac{s}{min}}}$$

Finally

$$\underline{\underline{d_2 = 0.064 \text{ m}}}$$

5.13

Two rivers merge to form a larger river as shown in Fig. P5.13. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown. Determine the value of V.



Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V} 0.8 V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50 ft)(3 ft)(3 \frac{ft}{s}) + (80 ft)(5 ft)(4 \frac{ft}{s})}{(30 ft)(6 ft)(0.8) + (70 ft)(6 ft)}$$

$$V = \underline{\underline{3.63}} \frac{ft}{s}$$

5.14

5.14 Oil having a specific gravity of 0.9 is pumped as illustrated in Fig. P5.14 with a water jet pump (see Video V3.6). The water volume flowrate is 2 m^3/s. The water and oil mixture has an average specific gravity of 0.95. Calculate the rate, in m^3/s, at which the pump moves oil.

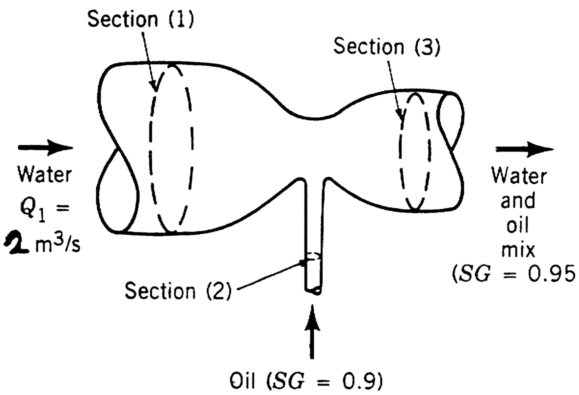


FIGURE P5.14

For steady flow

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

or

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 Q_3 \qquad (1)$$

Also, since the water and oil may be considered incompressible

$$Q_1 + Q_2 = Q_3 \qquad (2)$$

Combining Eqs. 1 and 2 we get

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 (Q_1 + Q_2)$$

or

$$Q_1 + SG_2 Q_2 = SG_3 (Q_1 + Q_2)$$

and

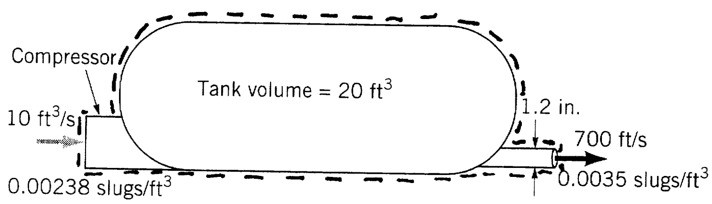
$$Q_2 = \frac{Q_1 (1 - SG_3)}{SG_3 - SG_2}$$

Thus

$$Q_2 = \frac{\left(2 \frac{m^3}{s}\right)(1 - 0.95)}{0.95 - 0.90} = \underline{\underline{2.00 \frac{m^3}{s}}}$$

5.15

Air at standard conditions enters the compressor shown in Fig. P5.15 at a rate of 10 ft³/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 slugs/ft³ and a uniform speed of 700 ft/s. **(a)** Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. **(b)** Determine the average time rate of change of air density within the tank.



Use the control volume within the broken lines.

(a) From the conservation of mass principle we get

$$\frac{DM_{sys}}{Dt} = \dot{m}_{in} - \dot{m}_{out} = \rho_{in} Q_{in} - \rho_{out} A_{out} V_{out}$$

$$\frac{DM_{sys}}{Dt} = \left(0.00238 \frac{slug}{ft^3}\right)\left(10 \frac{ft^3}{s}\right) - \left(0.0035 \frac{slug}{ft^3}\right)\frac{\pi (1.2\text{ in.})^2}{\left(144 \frac{in^2}{ft^2}\right)}\left(700 \frac{ft}{s}\right)$$

$$\frac{DM_{sys}}{Dt} = \underline{\underline{0.00456}} \frac{slug}{s} \quad \text{increasing}$$

(b) $$\frac{DM_{sys}}{Dt} = \frac{D(\rho V_{sys})}{Dt} = V_{sys}\frac{D\rho}{Dt} = 0.00456 \frac{slug}{s}$$

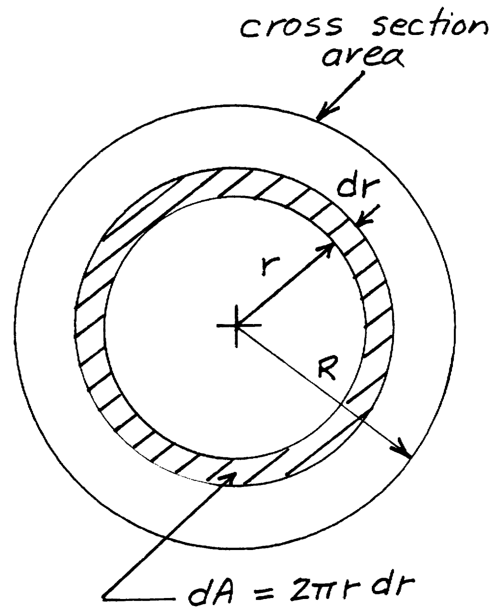
so $$\frac{D\rho}{Dt} = \frac{0.00456 \frac{slug}{s}}{V_{sys}} = \frac{0.00456 \frac{slug}{s}}{20 ft^3} = \underline{\underline{2.28 \times 10^{-4}}} \frac{slug}{ft^3 \, s}$$

5.16

5.16 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity u_c for **(a)** $n = 4$; **(b)** $n = 6$; **(c)** $n = 8$; **(d)** $n = 10$



For any cross section area

$$\dot{m} = \rho A \bar{u} = \int_A \rho\, \vec{V}\cdot\hat{n}\, dA$$

Also

$$\vec{V}\cdot\hat{n} = \vec{V}\cdot\hat{i} = u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}}$$

Thus for a uniformly distributed density, ρ, over area A

$$\bar{u} = \frac{\int_0^R u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}} 2\pi r\, dr}{\pi R^2}$$

and

$$\frac{\bar{u}}{u_c} = 2\int_0^R\left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = \frac{2n^2}{2n^2+3n+1}$$

n	$\frac{\bar{u}}{u_c}$
4	0.711
6	0.791
8	0.837
10	0.866

5.17

5.17 The velocity and temperature profiles for one circular cross section in laminar pipe flow of air with heat transfer are

$$\mathbf{V} = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]\hat{\mathbf{i}}$$

where the unit vector $\hat{\mathbf{i}}$ is along the pipe axis, and

$$T = T_c\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]$$

The subscript c refers to centerline value, r = local radius, R = pipe radius and T = local temperature. Show how you would evaluate the mass flowrate through this cross section area.

The mass flowrate is

$$\dot{m} = \int_A \rho \, \vec{V} \cdot \hat{n} \, dA$$

For air acting as an ideal gas

$$\rho = \frac{p}{R_{air} T}$$

For a circular cross-section area

$$dA = 2\pi r \, dr$$

Thus

$$\dot{m} = \int_0^R \frac{p}{R_{air} T} u_c \left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r \, dr$$

and for a uniformly distributed static pressure

$$\dot{m} = \frac{2\pi p u_c R^2}{R_{air} T_c} \int_0^R \frac{\left[1 - \left(\frac{r}{R}\right)^2\right] \frac{r}{R} d\left(\frac{r}{R}\right)}{\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]}$$

If we set

$$\beta = 1 + \frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4$$

then

$$\dot{m} = \frac{2\pi p u_c R^2}{R_{air} T_c} \int_1^{\frac{5}{4}} \frac{d\beta}{\beta} = \underline{\underline{0.701 \frac{p u_c R^2}{R_{air} T_c}}}$$

5.18

5.18* To measure the mass flowrate of air through a 6-in.-inside diameter pipe, local velocity data are collected at different radii from the pipe axis (see Table). Determine the mass flowrate corresponding to the data listed below.

r (in.)	Axial Velocity (ft/s)
0	30
0.2	29.71
0.4	29.39
0.6	29.06
0.8	28.70
1.0	28.31
1.2	27.89
1.4	27.42
1.6	26.90
1.8	26.32
2.0	25.64
2.2	24.84
2.4	23.84
2.6	22.50
2.8	20.38
2.9	18.45
2.95	16.71
2.98	14.66
3.00	0

The mass flowrate is calculated with

$$\dot{m} = \int_0^R \rho u \, 2\pi r dr = 2\pi\rho \int_0^R u r dr$$

where

$R = 3 \text{ in.}$

$\rho = 0.00238 \ \frac{\text{slug}}{\text{ft}^3}$

u = local axial velocity in $\frac{\text{ft}}{\text{s}}$

r = local radius in in.

and $\int_0^R u r dr$ is evaluated numerically with the trapezoidal rule with unequal intervals. The computer program used to solve this problem is listed on the next page.

(con't)

5.18 (con't)

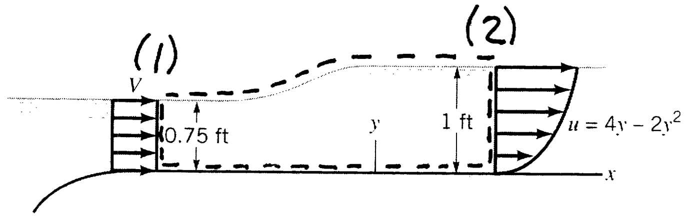
```
100 CLS
110 PRINT "**********************************************"
120 PRINT "** This program computes the mass flow rate    **"
130 PRINT "** for problem 5.18 using the trapezoidal rule **"
140 PRINT "** applied to unequal intervals.               **"
150 PRINT "**********************************************"
160 PRINT
170 DIM U(19), R(19)
180 '
190 'Initialize the variables
200 N = 19
210 RHO = .00238
220 PI = 4! * ATN(1!)
230 FOR I = 1 TO N
240 READ R(I), U(I)
250 R(I) = R(I) / 12!
260 NEXT I
270 DATA 0.0, 30.00, 0.2, 29.71, 0.4, 29.39, 0.6, 29.06
280 DATA 0.8, 28.70, 1.0, 28.31, 1.2, 27.89, 1.4, 27.42
290 DATA 1.6, 26.90, 1.8, 26.32, 2.0, 25.64, 2.2, 24.84
300 DATA 2.4, 23.84, 2.6, 22.50, 2.8, 20.38, 2.9, 16.71
310 DATA 2.95,16.71, 2.98,14.66, 3.0, 00.00
320 '
330 'Compute integral using trapezoidal rule
340 FOR I = 2 TO N
350 SUM = SUM+(U(I-1)*R(I-1)+U(I)*R(I))*(R(I)-R(I-1))/2!
360 NEXT I
370 MDOT = RHO * 2! * PI * SUM
380 '
390 'Print the results
400 PRINT
410 PRINT USING "The mass flow rate is ##.#### slugs/s"; MDOT
```

```
**********************************************
** This program computes the mass flow rate    **
** for problem 5.18 using the trapezoidal rule **
** applied to unequal intervals.               **
**********************************************

The mass flow rate is  0.0114 slugs/s
```

5.19

As shown in Fig. P5.19, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.



Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\, dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\,b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\ \frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78}}\ \frac{ft}{s}$$

5.20

5.20 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.20. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.

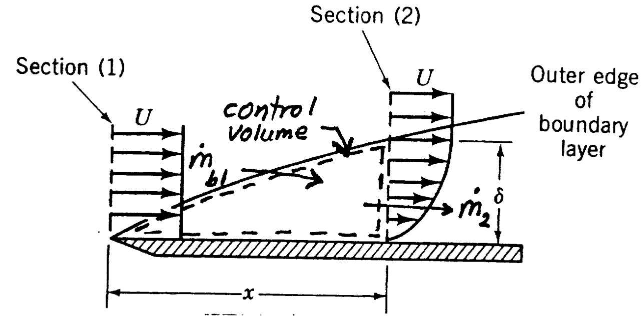


FIGURE P5.20

From the conservation of mass principle applied to the flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \vec{V} \cdot \hat{n} \, dA$$

For incompressible flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

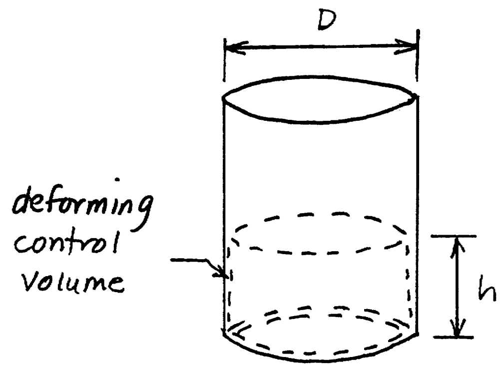
ℓ = width of the plate

and thus

$$Q_{bl} = \frac{7}{8} U \ell \delta$$

5.22

5.22 How long would it take to fill a cylindrical shaped swimming pool having a diameter of 10 m to a depth of 1.5 m with water from a garden hose if the flowrate is 1.0 liter/s?



From application of the conservation of mass principle to the control volume containing water only as shown in the figure we have

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \, \vec{V} \cdot \hat{n} \, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q \int_0^t dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{4 Q} = \frac{\pi (10\,m)^2 (1.5\,m)\left(1000 \frac{liters}{m^3}\right)}{4 \left(1.0 \frac{liter}{s}\right)\left(3600 \frac{s}{hr}\right)}$$

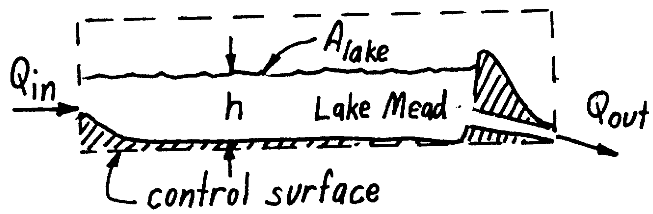
or

$$t = \underline{\underline{32.7 \text{ hrs}}}$$

5.23

5.23 The Hoover Dam backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. (See Video V2.3.) If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8,000 cfs, how many feet per 24-hour day will the lake level rise?

For the control volume shown:



$$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}\int_{cv\,water} \rho \, d\forall$$

or since $\dot{m} = \rho Q$,

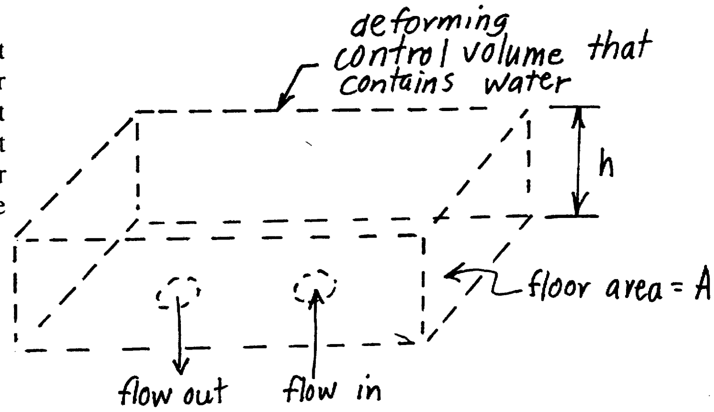
$$Q_{in} - Q_{out} = \frac{\partial}{\partial t}(A_{lake} h) = A_{lake}\frac{dh}{d}$$

Thus,
$$\frac{dh}{dt} = \frac{Q_{out} - Q_{in}}{A_{lake}} = \frac{(45{,}000 - 8{,}000)\frac{ft^3}{s}}{225\ mi^2\left(5280\frac{ft}{mi}\right)^2} = 5.90\times10^{-6}\ \frac{in.}{s}$$

$$= 5.90\times10^{-6}\ \frac{in.}{s}\left(3{,}600\frac{s}{hr}\right)\left(24\frac{hr}{day}\right) = \underline{\underline{0.510\ \frac{ft}{day}}}$$

5.24

5.24 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to (**a**) keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, (**b**) reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?



For a deforming control volume that contains the water over the basement floor (see sketch above), the conservation of mass principle (Eq. 5.17) leads to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}_r \cdot \hat{n} \, dA = 0$$

or for constant fluid density and area (A)

$$A \frac{dh}{dt} - Q_{in} + Q_{out} = 0 \qquad (1)$$

For part a, Eq. 1 leads to

$$Q_{out} = Q_{in}$$

To evaluate Q_{in}, we use Eq. 1 with $Q_{out} = 0$. Thus,

$$Q_{in} = A\frac{dh}{dt} = (1500 \text{ ft}^2)\left(1 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right) = 125 \frac{\text{ft}^3}{\text{hr}}$$

and

$$Q_{out} = \left(125 \frac{\text{ft}^3}{\text{hr}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right) = \underline{\underline{15.6 \frac{\text{gal}}{\text{min}}}}$$

For part b, Eq. 1 yields

$$Q_{out} = Q_{in} - A\frac{dh}{dt}$$

$$Q_{out} = 15.6 \frac{\text{gal}}{\text{min}} - (1500 \text{ ft}^2)\left(-3 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right)$$

$$Q_{out} = \underline{\underline{62.4}} \frac{\text{gal}}{\text{min}}$$

5.25

5.25 A hypodermic syringe (see Fig. P5.25) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks pass the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.

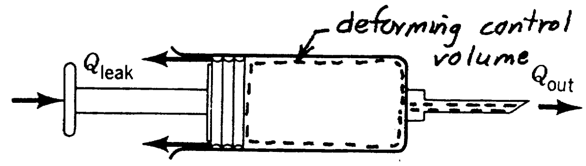


FIGURE P5.25

Using a deforming control volume and the conservation of mass principle (Eq. 5.17) as outlined in Example 5.8, we obtain (see Eq. 8 of Example 5.8)

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \qquad (1)$$

Since $\rho =$ constant, $Q_{leak} = 0.1\, Q_2$ and $Q_2 = A_2 V_2$ we obtain from Eq. 1

$$1.1\, A_2 V_2 = A_1 V_p$$

or

$$V_2 = \left(\frac{A_1}{A_2}\right)\frac{V_p}{1.1} = \left(\frac{d_1^2}{d_2^2}\right)\frac{V_p}{1.1} = \frac{(20\,mm)^2(20\,mm/s)}{(0.7\,mm)^2(1.1)\left(1000\,\frac{mm}{m}\right)}$$

and

$$V_2 = \underline{\underline{14.8}}\ \frac{m}{s}$$

5.27

5.27 It takes you 1 min to fill your car's fuel tank with 13.5 gallons of gasoline. What is the approximate average velcoity of the gasoline leaving the nozzle at this pump?

$$\bar{V}_{nozzle} A_{nozzle} = Q = \frac{(13.5 \text{ gal})}{(1 \text{ min})\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(60 \frac{\text{s}}{\text{min}}\right)}$$

and $A_{nozzle} = \dfrac{\pi d_{nozzle}^2}{4} = \dfrac{\pi\left(\frac{9}{16} \text{ in.}\right)^2}{(4)\left(12 \frac{\text{in.}}{\text{ft}}\right)^2}$ assuming a nozzle diameter of $\frac{9}{16}$ in.

so $\bar{V}_{nozzle} = \dfrac{(13.5)(4)(12)^2}{(\pi)\left(\frac{9}{16}\right)^2(7.48)(60)}$

$$\bar{V}_{nozzle} = \underline{\underline{17.4 \frac{\text{ft}}{\text{s}}}}$$

5.28

5.28 A gas flows steadily through a duct of varying cross section area. If the gas density is assumed to be uniformly distributed at any cross section, show that the conservation of mass principle leads to

$$\frac{d\rho}{\rho} + \frac{d\overline{V}}{\overline{V}} + \frac{dA}{A} = 0$$

where ρ = gas density, $\overline{V}$ = average speed of gas, and A = cross section area.

For a steady, one-dimensional flow, the conservation of mass principal leads to Eq. 5.12 or

$$\rho A \overline{V} = \text{constant}$$

Thus

$$d(\rho A \overline{V}) = 0$$

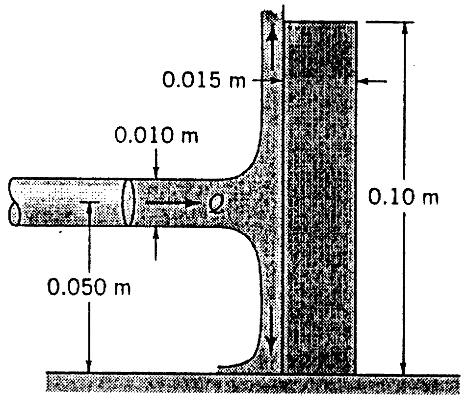
or $d\rho \, A \overline{V} + \rho A \, d\overline{V} + \rho \, dA \, \overline{V} = 0$ (1)

Dividing Eq. 1 by $\rho A \overline{V}$ we obtain

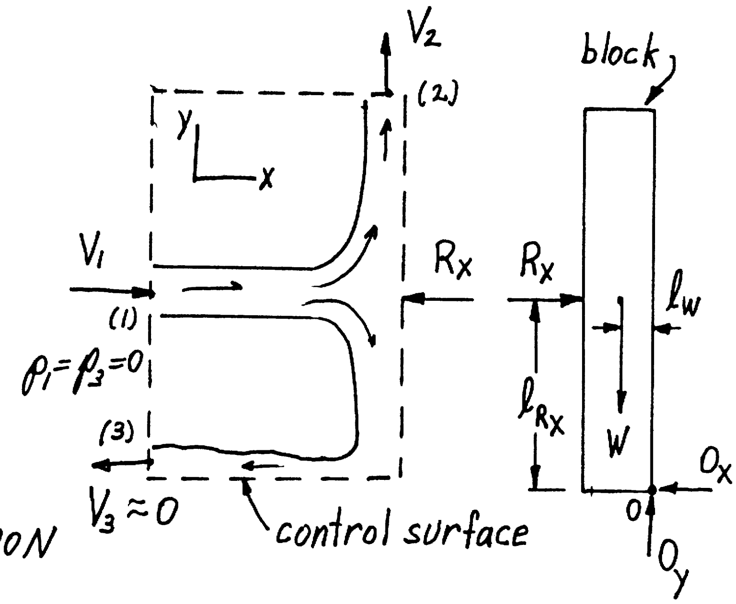
$$\frac{d\rho}{\rho} + \frac{d\overline{V}}{\overline{V}} + \frac{dA}{A} = 0$$

5.29

5.29 A 10-mm diameter jet of water is deflected by a homogeneous rectangular-block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.4 and Fig. P5.29 Determine the minimum volume flowrate needed to tip the block.



■ FIGURE P5.29



From the free body diagram of the block when it is ready to tip $\Sigma M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where R_x is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6\,N\left(\frac{0.015}{2}\,m\right)}{0.050\,m} = 0.90\,N$$

For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

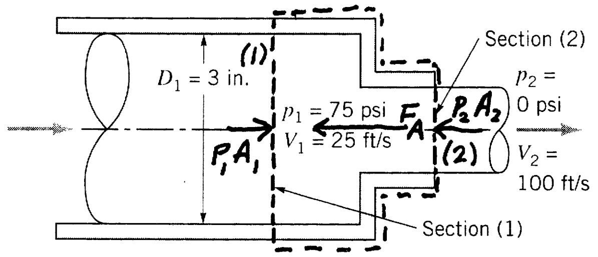
$$V_1 = \sqrt{\frac{0.9\,N}{\left(999\,\frac{kg}{m^3}\right)\frac{\pi}{4}(0.01\,m)^2}} = 3.39\,\frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01\,m)^2\left(3.39\,\frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4}\,\frac{m^3}{s}}}$$

5.30

Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.30 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.



For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = p_1 A_1 - F_A - p_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expressed as

$$\dot{m}(u_2 - u_1) = p_1 A_1 - F_A - p_2 A_2$$

or

$$F_A = p_1 A_1 - p_2 A_2 + \dot{m}(u_2 - u_1) = p_1 \frac{\pi D_1^2}{4} - p_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi D_1^2}{4}(u_2 - u_1)$$

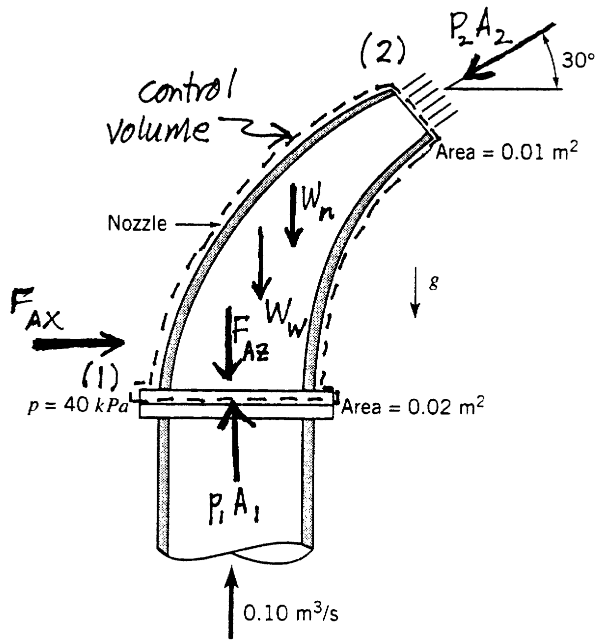
and

$$F_A = \left(75 \frac{lb}{in.^2}\right)\frac{\pi (3\ in.)^2}{4} - 0\ lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right)\frac{\pi (3\ in.)^2}{4\left(144 \frac{in.^2}{ft^2}\right)}\left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352\ lb}}$$

5.31

5.31 A nozzle is attached to a vertical pipe and discharges water into the atmosphere as shown in Fig. P5.31. When the discharge is 0.1 m^3/s, the gage pressure at the flange is 40 kPa. Determine the vertical component of the anchoring force required to hold the nozzle in place. The nozzle has a weight of 200 N, and the volume of water in the nozzle is 0.012 m^3. Is the anchoring force directed upward or downward?



■ FIGURE P5.31

The analysis leading to the solution of this problem is similar to the one outlined in Example 5.10. Included in the control volume are the nozzle and the water in the nozzle at an instant. Application of the vertical or z-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume leads to

$$\dot{m}(V_2 \sin 30° - V_1) = p_1 A_1 - F_{Az} - W_n - W_w - \cancelto{0 \text{ gage}}{p_2} A_2 \sin 30° \qquad (1)$$

Solving Eq. 1 for F_{Az} yields

$$F_{Az} = p_1 A_1 - W_n - W_w - \dot{m}(V_2 \sin 30° - V_1) \qquad (2)$$

For $\dot{m}$ we use $\dot{m} = \rho Q$

For W_w we use $W_w = \forall_w \gamma_w$

From conservation of mass we obtain

$$Q_2 = Q_1$$

or

$$V_2 = \frac{Q_1}{A_2}$$

(con't)

5.31 (con't)

Also, we not that $V_1 = \dfrac{Q_1}{A_1}$

Thus, Eq. 2 becomes

$$F_{Az} = P_1 A_1 - W_n - \forall_w \gamma_w - \rho Q\left(\frac{Q}{A_2}\sin 30^\circ - \frac{Q}{A_1}\right)$$

or

$$F_{Az} = (40\,kPa)\left(1\frac{N}{m^2\cdot Pa}\right)\left(1000\frac{Pa}{kPa}\right)\left(0.02\,m^2\right) - 200\,N$$

$$-\left(0.012\,m^3\right)\left(9.8\frac{kN}{m^3}\right)\left(1000\frac{N}{kN}\right)$$

$$-\left(999\frac{kg}{m^3}\right)\left(0.01\frac{m^3}{s}\right)\left(1\frac{N\cdot s^2}{kg\cdot m}\right)\left[\left(\frac{0.01\frac{m^3}{s}}{0.01\,m^2}\right)\sin 30^\circ - \left(\frac{0.01\frac{m^3}{s}}{0.02\,m^2}\right)\right]$$

and

$$F_{Az} = 800N - 200N - 117.6N - 0N = \underline{\underline{482}}\,N \text{ downward}$$

5.32

5.32 Determine the magnitude and direction of the x and y components of the anchoring force required to hold in place the horizontal 180° elbow and nozzle combination shown in Fig. P5.32. Also determine the magnitude and direction of the x and y components of the reaction force exerted by the 180° elbow and nozzle on the flowing water.

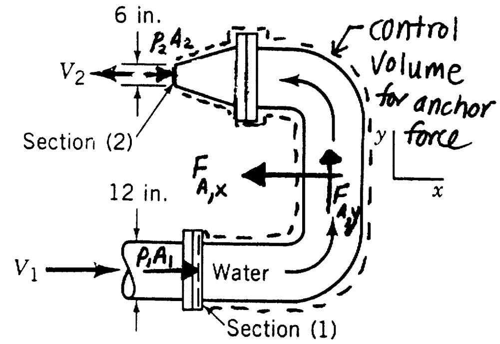


FIGURE P5.32 $p_1 = 15$ psi, $V_1 = 5$ ft/s

For determining the x and y direction components of the anchoring force a control volume that contains the elbow, nozzle and water between sections (1) and (2) is used. Th control volume and the forces involved are shown in the sketch above. Application of the y direction component of the linear momentum equation (Eq. 5.22) leads to

$$F_{A,y} = \underline{\underline{0}}$$

Application of the x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - F_{A,x} + p_2 A_2 \qquad (1)$$

From the conservation of mass equation

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2 \qquad (2)$$

Thus Eq. 1 may be expressed as

$$-\rho u_1 A_1 (u_1 + u_2) = p_1 A_1 - F_{A,x} + p_2 A_2$$

and

$$F_{A,x} = \rho u_1 A_1 (u_1 + u_2) + p_1 A_1 + p_2 A_2 = \rho u_1 \frac{\pi D_1^2}{4}(u_1 + u_2) + p_1 \frac{\pi D_1^2}{4} + (0) A_2$$

Also from Eq. 2

$$u_2 = \frac{A_1}{A_2} u_1 = \frac{D_1^2}{D_2^2} u_1$$

Thus

$$F_{A,x} = \rho u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1 \frac{\pi D_1^2}{4}$$

(con't)

5.32 (con't)

$$F_{A,x} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(5\ \frac{ft}{s}\right)\frac{\pi(12\ in.)^2}{4\left(144\ \frac{in.^2}{ft^2}\right)}\left[5\ \frac{ft}{s} + \frac{(12\ in.)^2}{(6\ in.)^2}\ 5\ \frac{ft}{s}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$+ \left(15\ \frac{lb}{in.^2}\right)\pi\ \frac{(12\ in.)^2}{4}$$

$$F_{A,x} = \underline{\underline{1890\ lb}}$$

For determining the x and y components of the reaction force a control volume that contains only the water between sections (1) and (2) is used. Application of the y direction component of the linear momentum equation yields

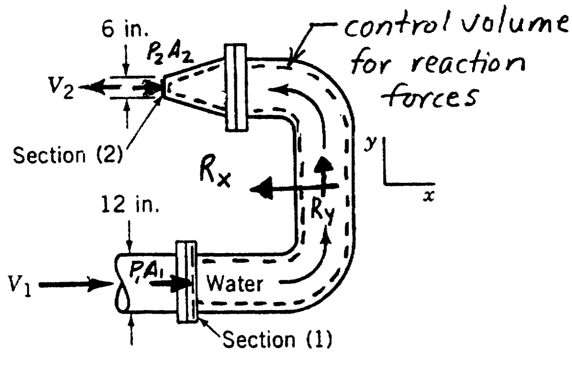
$$R_y = \underline{\underline{0}}$$

Application of the x direction component of the linear momentum equation leads to

$$R_x = \rho_1 u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2}u_1\right) + p_1 \frac{\pi D_1^2}{4}$$

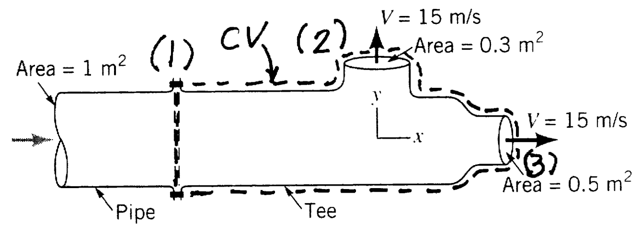
or

$$R_x = \underline{\underline{1890\ lb}}$$



5.33

Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.



Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3\rho V_3 A_3 = p_1 A_1 - p_3 A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= (p_{1\,gage} + p_{atm})A_1 - (p_{3\,gage} + p_{atm})A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= p_{1\,gage} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $V_1 = \dfrac{A_2 V_2 + A_3 V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,\frac{m}{s}) + (0.5\,m^2)(15\,\frac{m}{s})}{1\,m^2} = 12\,\frac{m}{s}$

To estimate $p_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{p_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{p_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$p_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right)$$

$$p_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $F_x = \underline{\underline{72{,}000\,N}} \leftarrow$

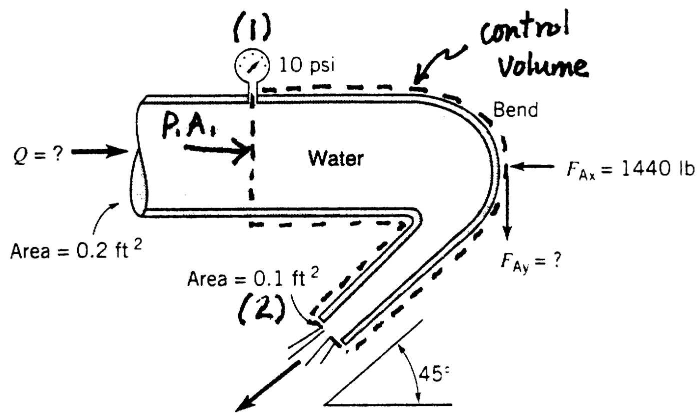
For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2\rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$

5.34

5.34 Water flows through a horizontal bend and discharges into the atmosphere as shown in Fig. P5.34. When the pressure gage reads 10 psi, the resultant x-direction anchoring force, F_{Ax}, in the horizontal plane required to hold the bend in place is shown on the figure. Determine the flowrate through the bend and the y direction anchoring force, F_{Ay}, required to hold the bend in place. The flow is not frictionless.



■ FIGURE P5.34

A control volume that contains the bend and the water within the bend between sections (1) and (2) as shown in the sketch above is used. Application of the x-direction component of the linear momentum equation yields

$$-u_1\rho Q - V_2\cos 45^\circ \rho Q = p_1A_1 - F_{Ax} + \overset{0\text{ gage}}{p_2}A_2\cos 45^\circ \qquad (1)$$

With

$$u_1 = \frac{Q}{A_1} \quad \text{and} \quad V_2 = \frac{Q}{A_2}$$

Eq. 1 becomes

$$-\frac{Q^2\rho}{A_1} - \frac{Q^2\rho\cos 45^\circ}{A_2} = p_1A_1 - F_{Ax}$$

or for part (a)

$$Q = \sqrt{\frac{-p_1A_1 + F_{Ax}}{\rho\left(\frac{\cos 45^\circ}{A_2} + \frac{1}{A_1}\right)}}$$

$$Q = \sqrt{\frac{-\left(10\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)\left(0.2\text{ ft}^2\right) + 1440\text{ lb}}{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)\left(\frac{\cos 45^\circ}{0.1\text{ ft}^2} + \frac{1}{0.2\text{ ft}^2}\right)}}$$

$$Q = \underline{\underline{7.01}}\ \frac{\text{ft}^3}{\text{s}}$$

(con't)

For part (b) we use the y-direction component of the linear momentum equation to get

$$F_{AY} = V_2 \sin 45^\circ \rho Q = \frac{Q}{A_2} \sin 45^\circ \rho Q$$

or

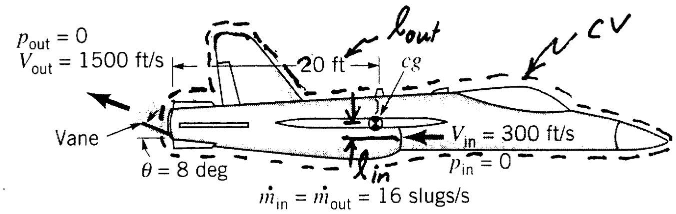
$$F_{AY} = \frac{Q^2}{A_2} \sin 45^\circ \rho$$

and

$$F_{AY} = \frac{\left(7.01 \frac{ft^3}{s}\right)^2}{(0.01\ ft^2)} \sin 45^\circ \left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right) = \underline{\underline{674\ lb}}$$

5.35

Thrust vector control is a new technique that can be used to greatly improve the maneuverability of military fighter aircraft. It consists of using a set of vanes in the exit of a jet engine to deflect the exhaust gases as shown in Fig. P5.35. **(a)** Determine the pitching moment (the moment tending to rotate the nose of the aircraft up) about the aircraft's mass center (cg) for the conditions indicated in the figure. **(b)** By how much is the thrust (force along the centerline of the aircraft) reduced for the case indicated compared to normal flight when the exhaust is parallel to the centerline?



For part (a) we apply the component of the moment-of-momentum equation that is perpendicular to the plane of the sketch of the aircraft to the contents of the control volume shown to get

$$\ell_{out} V_{out} \sin\theta \, \dot{m}_{out} - \ell_{in} V_{in} \dot{m}_{in} = \text{pitching moment}$$

$$\frac{(20 \text{ ft})(1500 \frac{\text{ft}}{\text{s}}) \sin 8^\circ (16 \frac{\text{slugs}}{\text{s}})}{\left(1 \frac{\text{slug}\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)} - \frac{\ell_{in}(300 \frac{\text{ft}}{\text{s}})(16 \frac{\text{slugs}}{\text{s}})}{\left(1 \frac{\text{slug}\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)} = \text{pitching moment}$$

$$66{,}800 - 4800\,\ell_{in} \quad \text{ft}\cdot\text{lb} = \text{pitching moment}$$

For part (b) we apply the horizontal component of the linear momentum equation to the contents of the control volume to get

$$V_{out} \cos\theta \, \dot{m}_{out} - V_{in} \dot{m}_{in} = \text{thrust}$$

So

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = V_{out}(\cos 0^\circ - \cos 8^\circ)\dot{m}_{out}$$

or

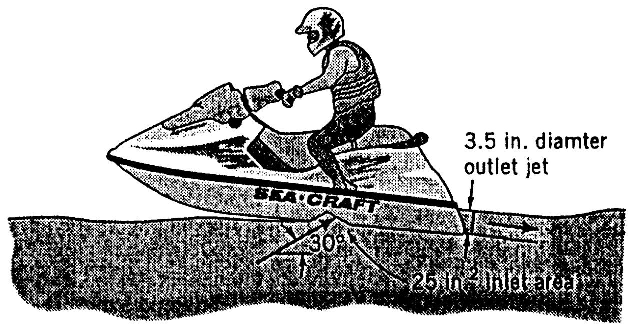
$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \frac{(1500 \frac{\text{ft}}{\text{s}})(\cos 0^\circ - \cos 8^\circ)(16 \frac{\text{slugs}}{\text{s}})}{\left(1 \frac{\text{slug}\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)}$$

and

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \underline{\underline{234}} \text{ lb}$$

5.36

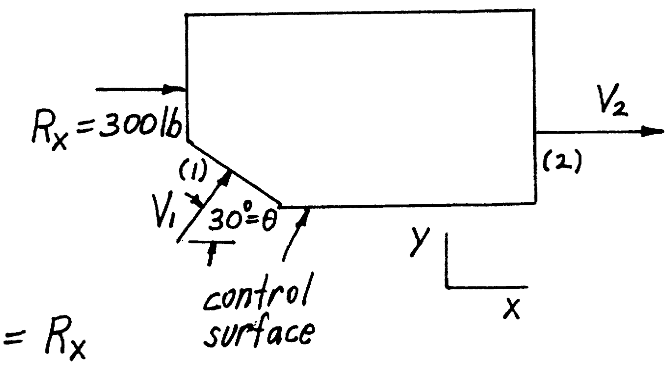
5.36 The thrust developed to propel the jet ski shown in Video V9.7 and Fig. P5.36 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300 lb thrust? Assume the inlet and outlet jets of water are free jets.



■ FIGURE P5.36

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \sum F_x \text{ becomes}$$



(1) $(V_1 \cos 30°)\rho(-V_1)A_1 + V_2\,\rho(+V_2)A_2 = R_x$

where we have assumed that $p=0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1\cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \quad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25\,in.^2}{\frac{\pi}{4}(3.5\,in.)^2} V_1 = 2.60\,V_1 \quad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300\,lb}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(\frac{25}{144}\,ft^2\right)(2.60-\cos 30°)}\right]^{1/2} = 22.7\,\frac{ft}{s}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144}\,ft^2\right)\left(22.7\,\frac{ft}{s}\right) = \underline{\underline{3.94\,\frac{ft^3}{s}}}$$

5.37

5.37 Water is sprayed radially outward over 180° as indicated in Fig. P5.3 The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.

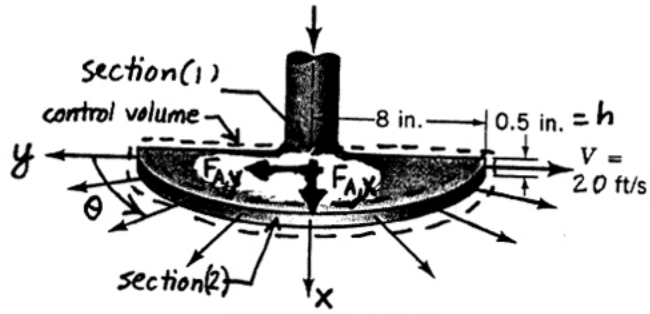


FIGURE P5.37

The control volume includes the nozzle and water between sections (1) and (2) as indicated in the sketch above. Application of the y direction component of the linear momentum equation yields

$$\int_{cs} v \rho \vec{V} \cdot \hat{n} \, dA = -F_{A,y}$$

or

$$F_{A,y} = -\rho \int_0^{\pi} (-V_2 \cos\theta)(V_2)\, h R \, d\theta = \rho h R V_2^2 (\sin\pi - \sin 0)$$

and

$$F_{A,y} = \underline{\underline{0}}$$

Application of the x direction component of the linear momentum equation leads to

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = F_{A,x}$$

or

$$F_{A,x} = \rho \int_0^{\pi} (V_2 \sin\theta)(V_2)\, h R \, d\theta = \rho h R V_2^2 (\cos 0 - \cos\pi)$$

and

$$F_{A,x} = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{(0.5 \text{ in.})(8 \text{ in.})\left(20 \frac{\text{ft}}{\text{s}}\right)^2 (2)}{\left(12 \frac{\text{in.}}{\text{ft}}\right)\left(12 \frac{\text{in.}}{\text{ft}}\right)} \left(1 \frac{\text{lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

$$F_{A,x} = \underline{\underline{43 \text{ lb}}}$$

5.38

5.38 A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.38. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.

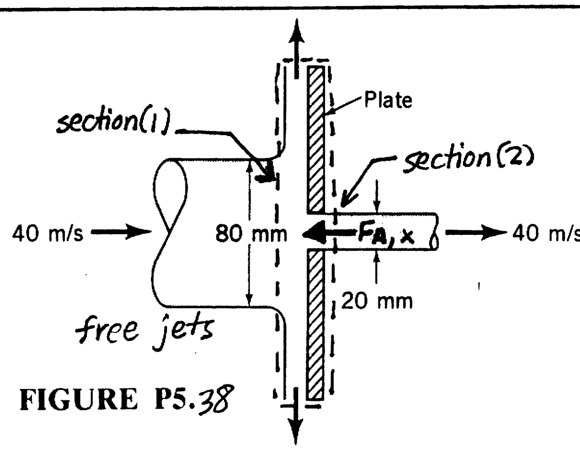


FIGURE P5.38

The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

Thus

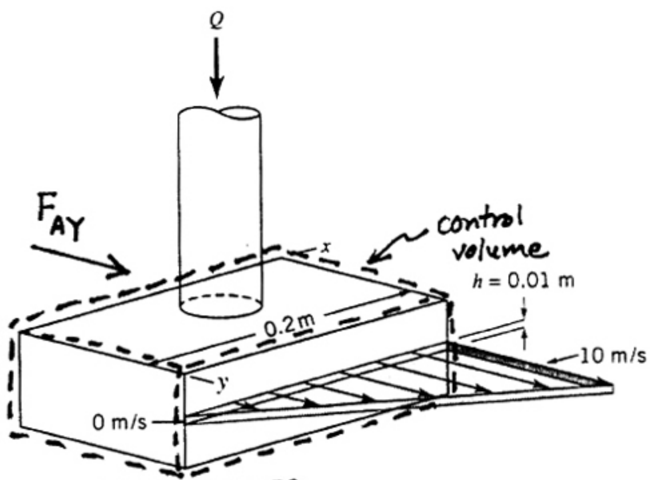
$$F_{A,x} = \left(40 \frac{m}{s}\right)^2 \left(1.23 \frac{kg}{m^3}\right) \frac{\pi}{4}\left[\frac{(80 \text{ mm})^2 - (20 \text{ mm})^2}{\left(1000 \frac{mm}{m}\right)^2}\right]\left(\frac{1 \text{ N}}{\frac{kg \cdot m}{s^2}}\right)$$

and

$$F_{A,x} = \underline{\underline{9.27 \text{ N}}}$$

5.39

5.39 A sheet of water of uniform thickness ($h = 0.01$ m) flows from the device shown in Fig. P5.39. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2 m length of the slit. Determine the y component of anchoring force necessary to hold this device stationary.



■ FIGURE P5.39

A control volume that contains the box portion of the device and the water in the box as shown in the sketch above is used. Application of the y-direction component of the linear momentum equation yields

$$F_{AY} = \int_{A_{slit}} v \rho \vec{V} \cdot \hat{n} \, dA = \rho \int_0^{0.2} v^2 h \, dx$$

The variation of v with x is linear or

$$v = 50x \ \frac{m}{s}$$

Thus

$$F_{AY} = \rho \int_0^{0.2} (50x)^2 h \, dx = \rho (50)^2 h \left. \frac{x^3}{3} \right|_0^{0.2}$$

or

$$F_{AY} = \left(999 \frac{kg}{m^3}\right)\left(50 \frac{1}{s}\right)^2 (0.01 m) \frac{(0.2 m)^3}{3} \left(1 \frac{N \cdot s^2}{kg \cdot m}\right)$$

and

$$F_{AY} = \underline{\underline{66.6 N}}$$

5.40

5.40 The results of a wind tunnel test to determine the drag on a body (see Fig. P5.40) are summarized below. The upstream [section (1)] velocity is uniform at 100 ft/s. The static pressures are given by $p_1 = p_2 = 14.7$ psia. The downstream velocity distribution which is symmetrical about the centerline is given by

$$u = 100 - 30\left(1 - \frac{|y|}{3}\right) \qquad |y| \le 3 \text{ ft}$$

$$u = 100 \qquad |y| > 3 \text{ ft}$$

where u is the velocity in ft/s and y is the distance on either side of the centerline in feet (see Fig. P5.42). Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.

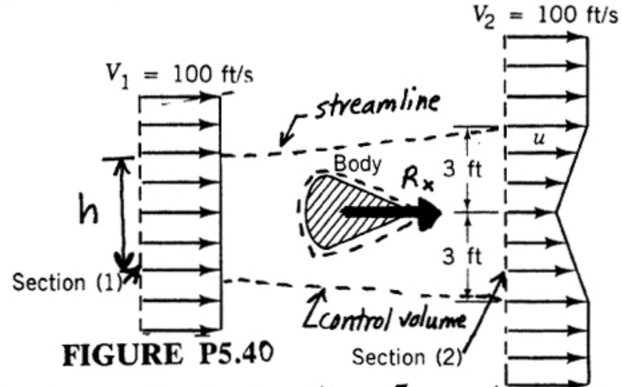


FIGURE P5.40

The control volume containing air only as shown in the figure is used. Application of the x direction component of the linear momentum equation leads to

$$-V_1 \rho V_1 A_1 + 2\int_0^{3\,ft} u\rho u\, dy = -R_x$$

or

$$R_x = \rho V_1^2 h - 2\rho \int_0^{3\,ft} \left[100 - 30\left(1 - \frac{y}{3}\right)\right]^2 dy \qquad (1)$$

To determine the distance h the conservation of mass equation is applied between sections (1) and (2) as follows

$$\rho h V_1 = 2\int_0^{3\,ft} \rho u\, dy$$

Thus

$$h = \frac{2}{V_1}\int_0^{3\,ft} \left[100 - 30\left(1 - \frac{y}{3}\right)\right] dy$$

or

$$h = \frac{(2)\left(255\ \frac{ft^3}{s}\right)}{\left(100\ \frac{ft}{s}\right)(1\ ft)} = 5.1\ ft$$

Then from Eq. 1

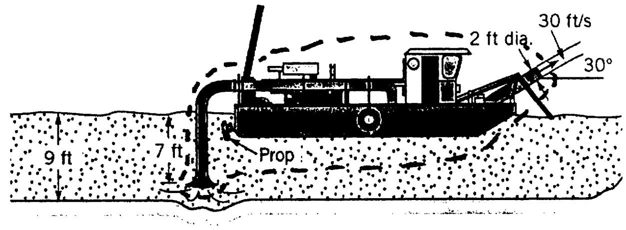
$$R_x = \left(0.00238 \frac{slugs}{ft^3}\right)\left(100\ \frac{ft}{s}\right)^2 (5.1\ ft)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)(1\ ft)$$

$$- 2\left(0.00238 \frac{slugs}{ft^3}\right)\left(21{,}900\ \frac{ft^4}{s^2}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$R_x = \underline{\underline{17.1\ lb}} \text{ per ft of length normal to the plane of the sketch}$$

5.41

5.41 The hydraulic dredge shown in Fig. P5.41 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.



■ FIGURE P5.41

Using the control volume shown by the broken line in the sketch above we use the horizontal or x component of the linear momentum equation to get

$$F_x = \rho_2 A_2 V_2 V_{2x} = \rho_{H_2O} (sg)_2 \frac{\pi d_2^2}{4} V_2 V_2 \cos 30^\circ$$

where section 1 is where flow enters the control volume vertically and section 2 is where flow leaves the control volume at an angle of 30° from the horizontal direction. Note that there is no horizontal direction linear momentum flow at section 1.

$$F_x = \left(1.94 \frac{slugs}{ft^3}\right)(1.4)\frac{\pi (2\,ft)^2}{4}\left(30 \frac{ft}{s}\right)\left(30\frac{ft}{s}\right)\cos 30^\circ \left(1 \frac{lb}{\frac{ft\, slug}{s^2}}\right)$$

$$F_x = \underline{\underline{6650}}\ lb$$

5.42

5.42 Water flows vertically upward in a circular cross section pipe as shown in Fig. P5.42. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R-r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe radius, and r = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

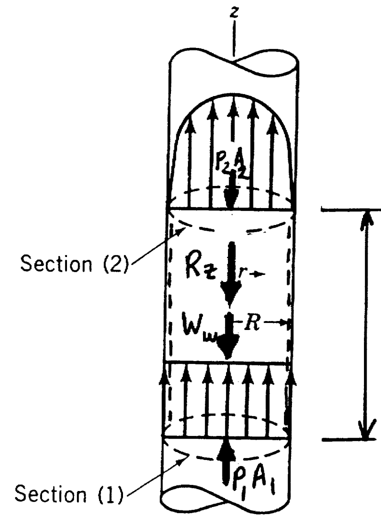


FIGURE P5.42

The analysis for this problem is similar to the one of Example 5.13. The control volume contains the fluid only between sections (1) and (2) as indicated in the sketch. Application of the vertical or z component of the linear momentum equation leads to

$$-w_1 \rho w_1 A_1 + \int_0^R w_2 \rho w_2 \, 2\pi r \, dr = p_1 A_1 - R_z + p_2 A_2 - W_w$$

Thus

$$p_1 - p_2 = \frac{R_z}{A} - \rho w_1^2 + \frac{\rho 2\pi}{A} \int_0^R \left[w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}} \right]^2 r \, dr + \frac{W_w}{A} \qquad (1)$$

The weight of the water in the control volume may be expressed as

$$W_w = g \rho A h$$

The value of $w_℄$ may be obtained from the conservation of mass equation as follows

$$\rho w_1 A_1 = \int_0^R \rho w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}} 2\pi r \, dr$$

or

$$w_c = \frac{w_1 A_1}{2\pi \int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r \, dr} \qquad (2)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r \, dr$ we substitute

$$\alpha = \frac{R-r}{R} \qquad (3)$$

then

$$d\alpha = -\frac{dr}{R} \qquad (4)$$

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5.42 (con't)

and

$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr = -\int_1^0 \alpha^{\frac{1}{7}}(1-\alpha) R^2 d\alpha = \frac{49}{120} R^2 \qquad (5)$$

Combining Eqs. 2 and 5 we obtain

$$w_c = \frac{60}{49} w_1$$

Thus from Eq. 1

$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \frac{\rho(2)(60)^2 w_1^2}{R^2(49)^2} \int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr + g\rho h \qquad (6)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr$ we use Eqs. 3 and 4.

Thus

$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr = -\int_1^0 \alpha^{2/7}(1-\alpha) R^2 d\alpha = \frac{49}{144} R^2$$

and Eq. 6 becomes

$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \rho(1.02) w_1^2 + g\rho h$$

or

$$\underline{\underline{p_1 - p_2 = \frac{R_z}{\pi R^2} + 0.02\rho w_1^2 + g\rho h}}$$

Note that in contrast to the result of Example 5.13, only a very small portion of the pressure drop is due to a change in the momentum flow between sections 1 and 2 in this case.

5.43 In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic, that is,

$$u = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]$$

as is illustrated in Fig. P5.43. Compare the axial direction momentum flowrate calculated with the average velocity, $\bar{u}$, with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.

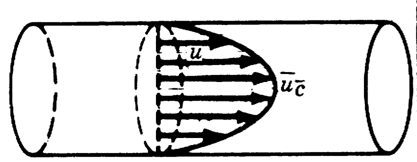

FIGURE P5.43

The axial direction momentum flowrate based on a uniform velocity profile with $u = \bar{u}$ is

$$MF_{x,\,uniform} = \bar{u}\rho\bar{u}A = \rho\bar{u}^2\pi R^2$$

The axial direction momentum flowrate based on the non-uniform parabolic velocity profile is

$$MF_{x,\,non\text{-}uniform} = \int_0^R u\rho u 2\pi r\,dr = \rho u_c^2 2\pi R^2\int_0^1\left[1-\left(\frac{r}{R}\right)^2\right]^2\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

$$MF_{x,\,non\text{-}uniform} = \frac{\rho u_c^2\pi R^2}{3}$$

To obtain a relationship between $\bar{u}$ and $u_{\mathfrak{c}}$ we use the conservation of mass equation as follows

$$\rho\bar{u}\pi R^2 = \rho 2\pi R^2 u_c\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

Thus

$$\bar{u} = \frac{u_c}{2}$$

and

$$MF_{x,\,non\text{-}uniform} = \frac{4}{3}\rho\bar{u}^2\pi R^2 = \frac{4}{3}MF_{x,\,uniform}$$

5.44

***5.44** For the pipe (6-in.-inside diameter) air flow data of Problem 5.18, calculate the rate of flow of axial direction momentum. How large would the error be if the average axial velocity were used to calculate axial direction momentum flow?

The rate of flow of axial direction momentum, MF_x, is calculated with

$$MF_x = \int_0^R \rho u^2 2\pi r dr = 2\pi\rho \int_0^R u^2 r dr \qquad (1)$$

where

$\rho = 0.00238 \ \frac{slug}{ft^3}$

u = local velocity, $\frac{ft}{s}$ (from Table of Problem 5.18)

r = local radius, in. (from Table of Problem 5.18)

R = pipe radius, 3 in.

The integral $\int_0^R u^2 r dr$ is evaluated by numerical integration using the trapezoidal rule with unequal intervals. The computer program used for this purpose is listed on the next page. The result of numerical integration and solution of Eq. 1 is

$$MF_x = \underline{\underline{0.284}} \ \frac{slug \cdot ft}{s^2}$$

The average axial velocity, $\bar{V}$, can also be used to approximate the momentum flow, $\overline{MF_x}$, with

$$\overline{MF_x} = \rho \bar{V}^2 A$$

where

From solution of Problem 5.

$$\bar{V} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \pi R^2} = \frac{\left(0.0114 \ \frac{slug}{s}\right)}{\left(0.00238 \ \frac{slug}{ft^3}\right)\frac{\pi (3 \text{ in.})^2}{\left(12 \ \frac{in.}{ft}\right)^2}} = 24.4 \ \frac{ft}{s}$$

Thus,

$$\overline{MF_x} = \frac{\left(0.00238 \ \frac{slug}{ft^3}\right)\left(24.4 \ \frac{ft}{s}\right)^2 \pi (3 \text{ in.})^2}{\left(12 \ \frac{in.}{ft}\right)^2} = 0.278 \ \frac{slug \cdot ft}{s^2}$$

The percent error in approximating the momentum flow with average velocity is

$$\left(\frac{\overline{MF_x} - MF_x}{MF_x}\right)100 = \left[\frac{\left(0.278 \ \frac{slug \cdot ft}{s^2} - 0.284 \ \frac{slug \cdot ft}{s^2}\right)}{\left(0.284 \ \frac{slug \cdot ft}{s^2}\right)}\right](100) = \underline{\underline{-2.11\%}}$$

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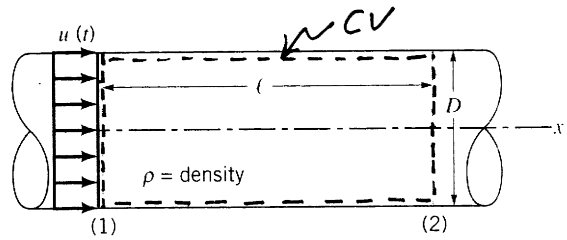
5.44 (Con't)

```
100 CLS
110 PRINT "*******************************************"
120 PRINT "** This program computes the axial-direction **"
130 PRINT "** momentum flow rate for problem 5.44 using **"
140 PRINT "** the trapezoidal rule applied to unequal   **"
150 PRINT "** intervals.                                **"
160 PRINT "*******************************************"
170 PRINT
180 DIM U(19), R(19)
190 '
200 'Initialize the variables
210 N = 19
220 RHO = .00238
230 PI = 4! * ATN(1!)
240 FOR I = 1 TO N
250 READ R(I), U(I)
260 R(I) = R(I) / 12!
270 NEXT I
280 DATA 0.0, 30.00, 0.2, 29.71, 0.4, 29.39, 0.6, 29.06
290 DATA 0.8, 28.70, 1.0, 28.31, 1.2, 27.89, 1.4, 27.42
300 DATA 1.6, 26.90, 1.8, 26.32, 2.0, 25.64, 2.2, 24.84
310 DATA 2.4, 23.84, 2.6, 22.50, 2.8, 20.38, 2.9, 16.71
320 DATA 2.95,16.71, 2.98,14.66, 3.0, 00.00
330 '
340 'Compute integral using trapezoidal rule
350 FOR I = 2 TO N
360 SUM = SUM+(U(I-1)^2*R(I-1)+U(I)^2*R(I))*(R(I)-R(I-1))/2!
370 NEXT I
380 MFX = RHO * 2! * PI * SUM
390 '
400 'Print the results
410 PRINT
420 PRINT "The axial-direction momentum flow "
430 PRINT USING "rate is ##.### slug·ft/s²"; MFX
```

```
*******************************************
** This program computes the axial-direction **
** momentum flow rate for problem 5.44 using **
** the trapezoidal rule applied to unequal   **
** intervals.                                **
*******************************************


The axial-direction momentum flow
rate is  0.284 slug·ft/s²
```

5.45

Consider unsteady flow in the constant diameter, horizontal pipe shown in Fig. P5.45. The velocity is uniform throughout the entire pipe, but it is a function of time: $\mathbf{V} = u(t)\,\hat{\mathbf{i}}$. Use the x component of the unsteady momentum equation to determine the pressure difference $p_1 - p_2$. Discuss how this result is related to $F_x = ma_x$.



Using the control volume shown in the sketch and applying the x- component of the unsteady linear momentum equation to the contents of this CV we get

$$\frac{\partial}{\partial t}\int_{cv} \rho u \, d\forall + \int_{cs} \rho u \, \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$$

or

$$\frac{\partial}{\partial t}\left(\rho u \frac{\pi}{4} D^2 \ell\right) - \rho u_1^2 A_1 + \rho u_2^2 A_2 = p_1 A_1 - p_2 A_2 + F_x$$

$$\rho u_1^2 A_1 = \rho u_2^2 A_2 \quad \text{assuming } u_1 = u_2 \text{ at every instant}$$

$$F_x = 0 \quad \text{assuming frictionless flow}$$

Thus,

$$\underbrace{\rho \frac{\pi D^2}{4}\ell}_{\text{mass in CV}} \underbrace{\frac{\partial u}{\partial t}}_{\text{local acceleration}} = \underbrace{(p_1 - p_2)\frac{\pi D^2}{4}}_{\text{force}}$$

and

$$p_1 - p_2 = \rho \ell \frac{\partial u}{\partial t}$$

5.46

5.46 The propeller on a swamp boat produces a jet of air having a diameter of 3 ft as illustrated in Fig. P5.46. The ambient air temperature is 80 °F, and the axial velocity of the flow is 85 ft/s relative to the boat. What propulsive forces are produced by the propeller when the boat is stationary and when the boat moves forward with a constant velocity of 20 ft/s?

FIGURE P5.46

For the stationary boat the horizontal component of the linear momentum equation applied to the contents of the control volume shown in the sketch above yields

$$F_{thrust} = \dot{m}(V_2 - V_1) = \rho A_2 V_2 V_2 \quad \text{since } V_1 << V_2$$

$$F_{thrust} = \left(\frac{p}{RT}\right)\left(\frac{\pi d_2^2}{4}\right)V_2^2 = \frac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in^2}{ft^2}\right)}{\left(53.3\,\frac{ft\cdot lb}{lbm\cdot {}^\circ R}\right)(540^\circ R)} \frac{\pi(3\,ft)^2\left(85\,\frac{ft}{s}\right)^2}{4\left(32.2\,\frac{lbm\cdot ft}{lb\cdot s^2}\right)}$$

$$F_{thrust} = \underline{\underline{117\ lb}}$$

For the boat moving forward with a speed of 20 ft/s, the same control volume shown in the sketch above is used, however, the relative velocity W is now important. From the horizontal component of the linear momentum equation we get

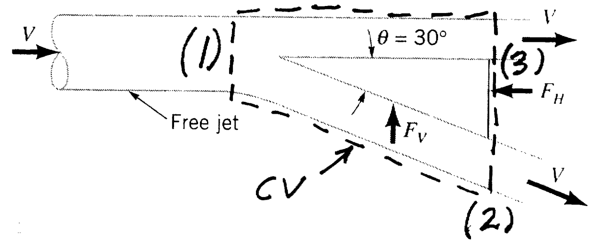
$$F_{thrust} = \dot{m}(W_2 - W_1) = \rho A_2 W_2 (W_2 - W_1) = \frac{p}{RT}\frac{\pi d_2^2}{4} W_2 (W_2 - W_1)$$

For the moving boat $W_2 = 85\,\frac{ft}{s}$ and $W_1 = 20\,\frac{ft}{s}$ and so

$$F_{thrust} = \underline{\underline{89.2\ lb}}$$

5.47

A free jet of fluid strikes a wedge as shown in Fig. P5.47. Of the total flow, a portion is deflected 30°; the remainder is not deflected. The horizontal and vertical components of force needed to hold the wedge stationary are F_H and F_V, respectively. Gravity is negligible, and the fluid speed remains constant. Determine the force ratio, F_H/F_V.



The horizontal and vertical components of the linear momentum equation are applied to the contents of the control volume shown to get

$$-V_1\rho V_1 A_1 + V_2 \rho V_2 A_2 + V_3 \cos 30^\circ \rho V_3 A_3 = -F_H \qquad (1)$$

$$-V_3 \sin 30^\circ \rho V_3 A_3 = F_V \qquad (2)$$

However $V_1 = V_2 = V_3 = V$ so eqs. (1) and (2) become

$$V^2\rho\left(A_2 + A_3 \cos 30^\circ - A_1\right) = -F_H$$

$$V^2\rho A_3 \sin 30^\circ = -F_V$$

and

$$\frac{F_H}{F_V} = \frac{A_2 + A_3 \cos 30^\circ - A_1}{A_3 \sin 30^\circ} \qquad (3)$$

From conservation of mass we get

$$Q_1 = Q_2 + Q_3$$

or

$$A_1 V = A_2 V + A_3 V$$

and

$$A_1 = A_2 + A_3 \qquad (4)$$

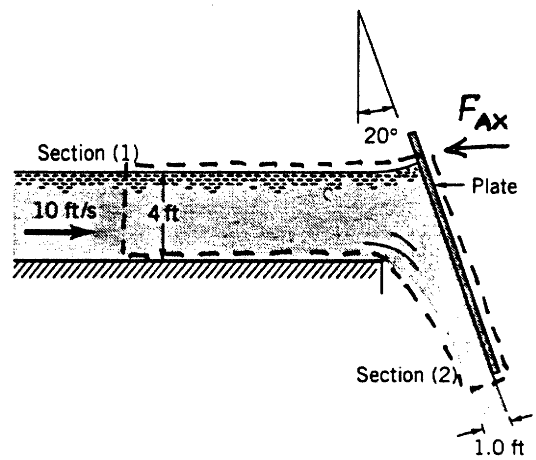
Combining Eqs. (3) and (4) we get

$$\frac{F_H}{F_V} = \frac{A_2 + A_3 \cos 30^\circ - A_2 - A_3}{A_3 \sin 30^\circ} = \frac{A_3(\cos 30^\circ - 1)}{A_3 \sin 30^\circ} = \underline{\underline{-0.27}}$$

The negative sign indicates that F_V is down rather than up as shown in the sketch.

5.48

5.48 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.48

A control volume that contains most of the plate and the water being turned by the plate as shown in the sketch above is used. Application of the horizontal x-direction component of the linear momentum equation yields

$$-V_1 \rho V_1 A_1 + V_2 \sin 20^\circ \rho V_2 A_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1 A_1 \qquad (1)$$

From conservation of mass we obtain

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{h_1}{h_2} V_1$$

Thus, Eq. 1 becomes for unit width

$$-V_1^2 \rho h_1 + \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1^2$$

or

$$F_{Ax} = \frac{1}{2}\gamma_w h_1^2 + V_1^2 \rho h_1 - \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2$$

Then

$$F_{Ax} = \frac{1}{2}\left(62.4 \frac{lb}{ft^3}\right)(4\ ft)^2 + \left(10 \frac{ft}{s}\right)^2 \left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)(4\ ft)$$
$$- \left[\left(\frac{4\ ft}{1\ ft}\right)\left(10 \frac{ft}{s}\right)\right]^2 \sin 20^\circ \left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)(1\ ft)$$

and

$$F_{Ax} = \underline{\underline{213}}\ lb$$

5.50

5.50 A vertical, circular cross section jet of air strikes a conical deflector as indicated in Fig. P5.50. A vertical anchoring force of 0.1 N is required to hold the deflector in the place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.

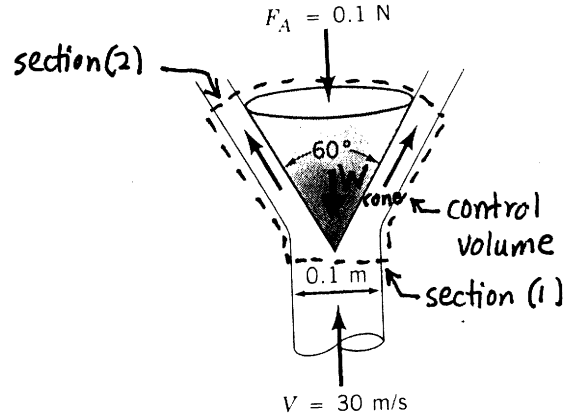


FIGURE P5.50

To determine the mass of the conical deflector we use the stationary, non-deforming control volume shown in the sketch above. Application of the vertical direction component of the linear momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\dot{m}(-V_1 + V_2 \cos 30°) = -F_A - W_{cone}$$

or

$$W_{cone} = m_{cone}\, g = \dot{m}(V_1 - V_2 \cos 30°) - F_A = \rho A_1 V_1 (V_1 - V_2 \cos 30°) - F_A \quad (1)$$

However

$$V_1 = V_2$$

and

$$A_1 = \frac{\pi D_1^2}{4}$$

Thus Eq. 1 can be expressed as

$$m_{cone} = \rho \frac{\pi D_1^2}{4g} V_1 (V_1 - V_1 \cos 30°) - \frac{F_A}{g}$$

or

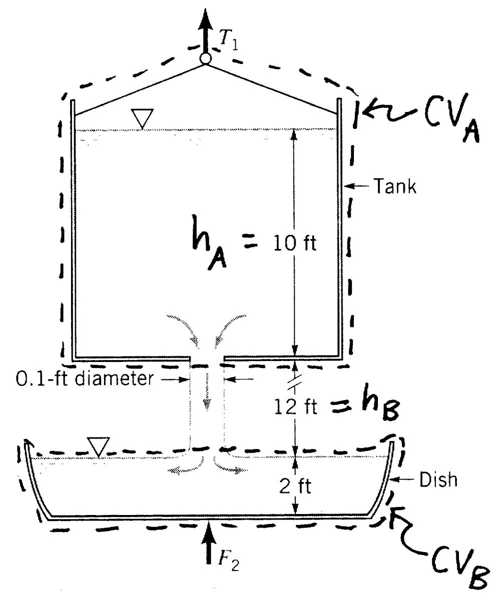
$$m_{cone} = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.1\,m)^2 \left(30 \frac{m}{s}\right)\left[30 \frac{m}{s} - \left(30 \frac{m}{s}\right)\cos 30°\right]}{(4)\left(9.81 \frac{m}{s^2}\right)} - \frac{0.1\,N}{\left(9.81 \frac{m}{s^2}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}$$

and

$$m_{cone} = \underline{\underline{0.108\ kg}}$$

5.51

Water flows from a large tank into a dish as shown in Fig. P5.51. **(a)** If at the instant shown the tank and the water in it weigh W_1 lb, what is the tension, T_1, in the cable supporting the tank? **(b)** If at the instant shown the dish and the water in it weigh W_2 lb, what is the force, F_2, needed to support the dish?



For part (a) we apply the vertical component of the linear momentum equation to the contents of control volume A, CV_A, to get

$$-V_{out}\,\rho V_{out} A_{out} = T_1 - W_1 \qquad (1)$$

To get value of V_{out} we apply Bernoulli's equation to the flow from the free surface of the water in the tank to the tank outlet to get

$$V_{out} = \sqrt{2gh_A} = \sqrt{(2)(32.2\ \tfrac{ft}{s^2})(10\ ft)} = 25.4\ \tfrac{ft}{s}$$

Then from Eq.(1) we get

$$\frac{-(25.4\ \frac{ft}{s})(1.94\ \frac{slugs}{ft^3})(25.4\ \frac{ft}{s})\ \pi\frac{(0.1\ ft)^2}{4}}{1\ \frac{slug\cdot ft}{lb\cdot s^2}} = T_1 - W_1$$

and

$$\underline{\underline{T_1 = W_1 - 9.8\ lb}}$$

For part (b) we apply the vertical component of the linear momentum equation to the contents of CV_B to get

$$V_{\substack{into\\CV_B}}\ \dot m_{\substack{into\\CV_B}} = F_2 - W_2 \qquad (2)$$

To get $V_{\substack{into\\CV_B}}$ we use Bernoulli's equation between free surface of water in tank to free surface of water in dish to get

$$V_{\substack{into\\CV_B}} = \sqrt{2g(h_A + h_B)} = \sqrt{2(32.2\ \tfrac{ft}{s^2})(10\ ft + 12\ ft)} = 37.6\ \tfrac{ft}{s}$$

For $\dot m_{\substack{into\\CV_B}}$ we use from conservation of mass, $\dot m_{\substack{into\\CV_B}} = \dot m_{\substack{out\ of\\tank}} = \rho V_{out} A_{out}$

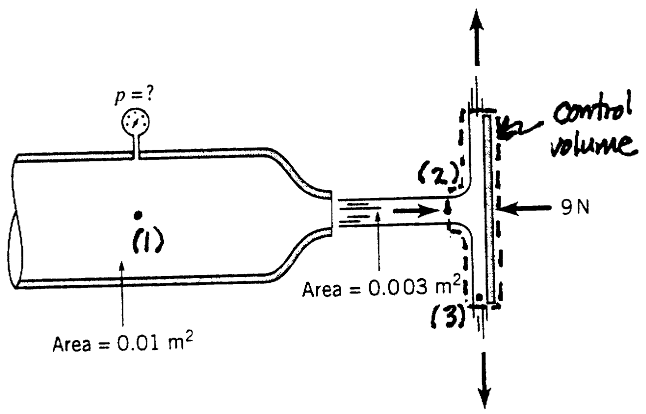
So from Eq. (2) we get

$$(37.6\ \tfrac{ft}{s})(1.94\ \tfrac{slug}{ft^3})(25.4\ \tfrac{ft}{s})\ \pi\frac{(0.1\ ft)^2}{4}\left(1\ \frac{lb\cdot s^2}{slug\cdot ft}\right) = F_2 - W_2$$

and $\underline{\underline{F_2 = W_2 + 14.7\ lb}}$

5.52

5.52 Air flows into the atmosphere from a nozzle and strikes a vertical plate as shown in Fig. P5.52. A horizontal force of 12 N is required to hold the plate in place. Determine the reading on the pressure gage. Assume the flow to be incompressible and frictionless.



■ FIGURE P5.52

To determine the static gage pressure at station (1) we first consider the frictionless and incompressible flow of air from (1) to (2). The Bernoulli equation for this flow is

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{\overset{0\text{ gage}}{\cancel{p_2}}}{\rho} + \frac{V_2^2}{2} \tag{1}$$

We note that V_1 and V_2 are linked by the continuity (conservation of mass) equation

$$Q_1 = Q_2 \quad \text{or} \quad A_1V_1 = A_2V_2 \tag{2}$$

Combining Eqs. 1 and 2 we obtain

$$\frac{p_1}{\rho} + \frac{\left(\frac{A_2}{A_1}V_2\right)^2}{2} = \frac{V_2^2}{2} \tag{3}$$

To determine V_2 we use the linear momentum equation for the flow from (2) to (3). For the control volume sketched above the linear momentum principle yields

$$-V_2\,\rho\,V_2\,A_2 = -9\text{ N}$$

or

$$V_2 = \sqrt{\frac{(12\text{ N})}{\rho A_2}} = \sqrt{\frac{12\text{ N}}{\left(1.23\,\frac{\text{kg}}{\text{m}^3}\right)\left(1\,\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)(0.003\text{ m}^2)}}$$

and

$$V_2 = 57\ \frac{\text{m}}{\text{s}}$$

(con't)

5.52 (con't)

Now, with Eq. 3

$$p_1 = \rho \left[\frac{V_2^2}{2} - \frac{\left(\frac{A_2}{A_1} V_2\right)^2}{2} \right]$$

or

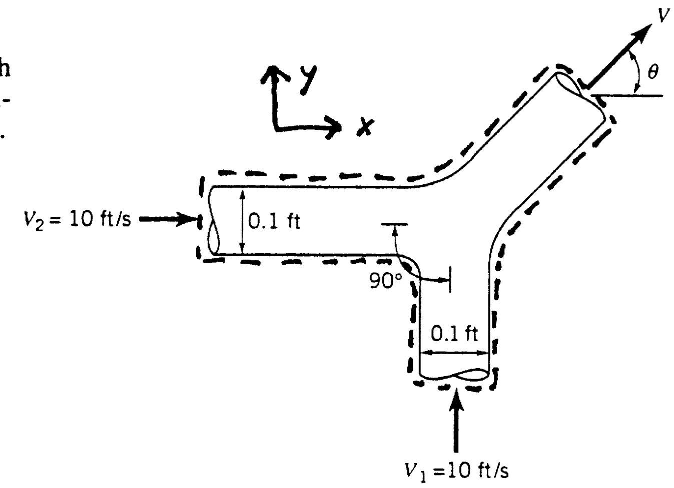
$$p_1 = \left(1.23 \frac{kg}{m^3}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right) \left\{ \frac{\left(57 \frac{m}{s}\right)^2}{2} - \frac{\left[\left(\frac{0.003 m^2}{0.01 m^2}\right)\left(57 \frac{m}{s}\right)\right]^2}{2} \right\}$$

and

$$p_1 = 1820 \frac{N}{m^2} = 1820 \ Pa = \underline{\underline{1.82}} \ kPa$$

5.54

5.54 Two water jets of equal size and speed strike each other as shown in Fig. P5.54. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



■ FIGURE P5.54

For the control volume shown in the sketch above the linear momentum equation for the x and y directions are, for the x direction

$$-V_2 \rho V_2 A_2 + (V\cos\theta)\rho V A = 0 \quad (1)$$

and for the y direction

$$-V_1 \rho V_1 A_1 + (V\sin\theta)\rho V A = 0 \quad (2)$$

Also for conservation of mass we have

$$\rho_1 V_1 A_1 + \rho V_2 A_2 - \rho V A = 0 \quad (3)$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

so

$$\theta = \cot^{-1}\frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1}\left[\frac{\left(10\,\frac{ft}{s}\right)^2 \pi\,\frac{(0.1ft)^2}{4}}{\left(10\,\frac{ft}{s}\right)^2 \pi\,\frac{(0.1ft)^2}{4}}\right] = 45^\circ$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V\sin\theta\,(V_1 A_1 + V_2 A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin\theta\,(V_1 A_1 + V_2 A_2)}$$

$$V = \frac{\left(10\,\frac{ft}{s}\right)^2 \pi\,\frac{(0.1ft)^2}{4}}{(\sin 45^\circ)\left[\left(10\,\frac{ft}{s}\right)\frac{\pi(0.1ft)^2}{4} + \left(10\,\frac{ft}{s}\right)\pi\,\frac{(0.1ft)^2}{4}\right]}$$

and

$$V = \underline{\underline{7.07}}\ \frac{ft}{s}$$

5.55

5.55 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.55, estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.

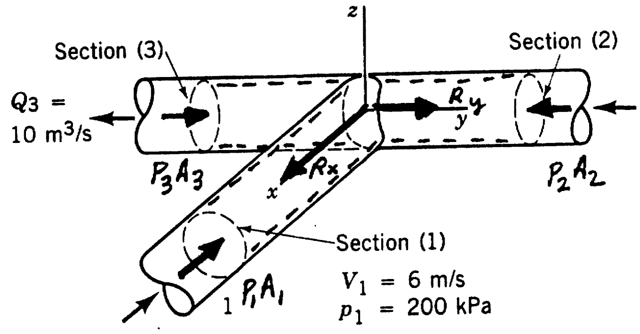


FIGURE P5.55

We can use the x and y components of the linear momentum equation (Eq. 5.22) to determine the x and y components of the reaction force exerted by the water on the tee. For the control volume containing water in the tee, Eq. 5.22 leads to

$$R_x = p_1 A_1 + V_1 \rho Q_1 = p_1 \frac{\pi D_1^2}{4} + V_1 \rho Q_1 \qquad (1)$$

and

$$R_y = p_2 \frac{\pi D_2^2}{4} - p_3 \frac{\pi D_3^2}{4} + V_2 \rho Q_2 - V_3 \rho Q_3 \qquad (2)$$

The reaction forces in Eqs. 1 and 2 are actually exerted by the tee on the water in the control volume. The reaction of the water on the tee is equal in magnitude but opposite in direction.

Conservation of mass (Eq. 5.4) leads to

$$Q_2 = Q_3 - Q_1 = Q_3 - V_1 \frac{\pi D_1^2}{4} = 10\,\frac{m^3}{s} - \left(6\,\frac{m}{s}\right)\frac{\pi}{4}(1\,m)^2 = 5.288\,\frac{m^3}{s}$$

Also

$$Q_1 = V_1 \frac{\pi D_1^2}{4} = \left(6\,\frac{m}{s}\right)\frac{\pi}{4}(1\,m)^2 = 4.712\,\frac{m^3}{s}$$

Further

$$V_2 = \frac{Q_2}{\frac{\pi D_2^2}{4}} = \frac{\left(5.288\,\frac{m^3}{s}\right)}{\frac{\pi (1\,m)^2}{4}} = 6.733\,\frac{m}{s}$$

and

$$V_3 = \frac{Q_3}{\frac{\pi D_3^2}{4}} = \frac{\left(10\,\frac{m^3}{s}\right)}{\frac{\pi (1\,m)^2}{4}} = 12.73\,\frac{m}{s}$$

(con't)

5.55 (con't)

Because the flow is incompressible and frictionless we assume that Bernoulli's equation (Eq. 5.74) is valid throughout the control volume. Thus

$$p_3 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_3^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(12.73\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_3 = 137\ kPa$$

Also

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(6.733\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_2 = 195.3\ kPa$$

With Eq. 1

$$R_x = \left(200{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(4.712\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = 185{,}000\ N = 185\ kN$$

and the x-direction component of force exerted by the water on the tee is $\underline{\underline{-185\ kN}}$.

With Eq. 2

$$R_y = \left(195{,}300\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 - \left(137{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)5.2$$
$$+ \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(5.288\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

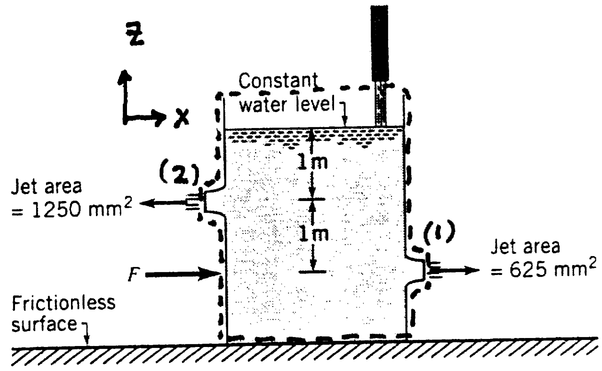
or

$$R_y = -45{,}800\ N = -45.8\ kN$$

and the y-direction component of force exerted by the water on the tee is $\underline{\underline{+45.8}}\ kN$.

5.56

5. Water is added to the tank shown in Fig. P5.56 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force, F, required to hold the tank stationary. Neglect all losses.



■ FIGURE P5.56

Applying the x-direction component of the linear momentum equation to the contents of the control volume sketched above we get

$$V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F \qquad (1)$$

Using Bernoulli's equation to describe the frictionless flow from the constant water surface level to the flow leaving at stations (1) and (2) we obtain

$$V_2 = \sqrt{2gh_2} \qquad (2)$$

and

$$V_1 = \sqrt{2gh_1} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$F = 2gh_1\rho A_1 - 2gh_2\rho A_2$$

or

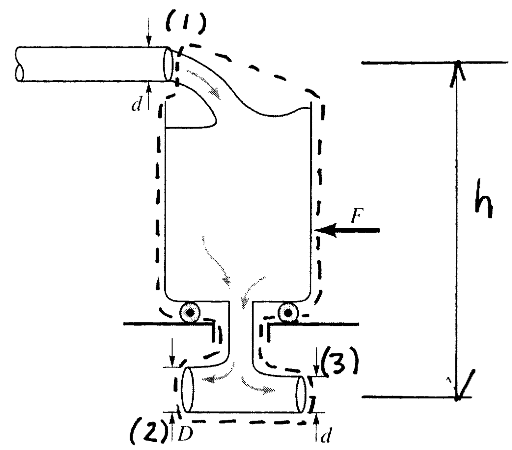
$$F = 2\left(9.81\,\frac{m}{s^2}\right)\left(999\,\frac{kg}{m^3}\right)\left[\frac{(2\,m)(625\,mm^2)}{\left(1000\,\frac{mm}{m}\right)^2} - \frac{(1\,m)(1250\,mm^2)}{\left(1000\,\frac{mm}{m}\right)^2}\right]$$

and

$$F = \underline{\underline{0\,N}}$$

5.57

Water flows steadily into and out of a tank that sits on frictionless wheels as shown in Fig. P5.57. Determine the diameter D so that the tank remains motionless if $F = 0$.



Applying the horizontal component of the linear momentum equation to the contents of the control volume shown in the sketch we get:

$$\int_{cs} \vec{V} \rho \vec{V} \cdot \hat{n}\, dA = \Sigma \vec{F}$$

or

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 + V_3 \rho V_3 A_3 = 0$$

and

$$-V_1^2 \rho \frac{\pi d^2}{4} - V_2^2 \rho \frac{\pi D^2}{4} + V_3^2 \rho \frac{\pi d^2}{4} = 0$$

Since $V_2 = V_3 = \sqrt{2gh}$ we obtain

$$V_1^2 d^2 = V_3^2 d^2 - V_3 D^2 \qquad (1)$$

From the conservation of mass equation we get

$$Q_1 = Q_3 + Q_3$$

or

$$V_1 d^2 = V_2 D^2 + V_3 d^2$$

Again, since $V_2 = V_3 = \sqrt{2gh}$ we get

$$V_1 d^2 = V_3 D^2 + V_3 d^2 \qquad (2)$$

Looking at Eqs. (1) and (2) together we conclude

If $V_3 < V_1$ eq. (1) cannot be satisfied
eq. (2) can be satisfied

If $V_3 > V_1$ eq. (1) can be satisfied
eq. (2) cannot be satisfied

If $V_3 = V_1$ eq. (1) can be satisfied with $D = 0$
eq. (2) can be satisfied with $D = 0$

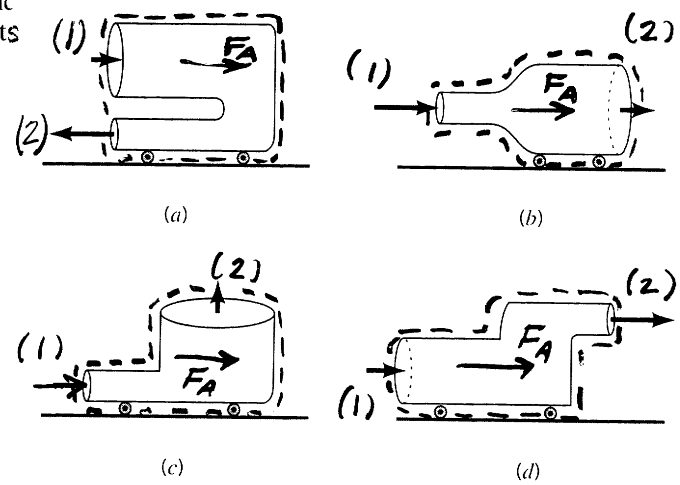
So $V_3 = V_1$ and $D = 0$

For $V_3 = V_1$, h must be set so that

$$V_3 = \sqrt{2gh} = V_1$$

5.58

The four devices shown in Fig. P5.58 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.



(a) (b) (c) (d)

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into CV at (1))

$$-V_1 \rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A + V_2 \rho V_2 A_2 = F_A$$

and from conservation of mass

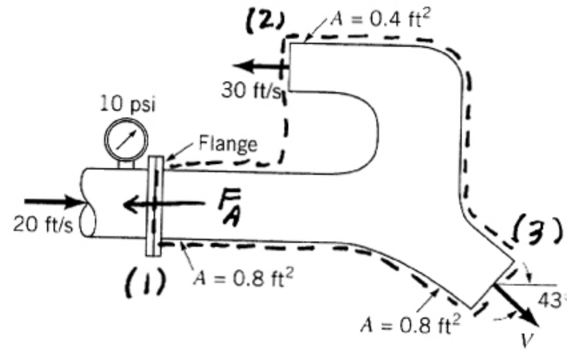
$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so F_A is to the right and motion is to the left.

5.59

Water discharges into the atmosphere through the device shown in Fig. P5.59. Determine the x component of force at the flange required to hold the device in place. Neglect the effect of gravity and friction.



To calculate the x-direction anchoring force required to hold the device in place, the x-direction component of the linear momentum equation is used on the contents of the control volume shown in the sketch to obtain:

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 + V_3 \cos 43^\circ \rho V_3 A_3 = -F_A + p_1 A_1 \qquad (1)$$

To determine V_3, the conservation of mass equation is used to obtain:

$$Q_1 = Q_2 + Q_3$$

or

$$V_1 A_1 = V_2 A_2 + V_3 A_3$$

and

$$\left(20\,\frac{ft}{s}\right)(0.8\,ft^2) = \left(30\,\frac{ft}{s}\right)(0.4\,ft^2) + V_3(0.8\,ft^2)$$

so

$$V_3 = 5\,\frac{ft}{s}$$

Then from Eq. 1 we get

$$-\frac{\left(20\,\frac{ft}{s}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(20\,\frac{ft}{s}\right)(0.8\,ft^2)}{\left(1\,\frac{slug\cdot ft}{lb\cdot s^2}\right)} - \frac{\left(30\,\frac{ft}{s}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(30\,\frac{ft}{s}\right)(0.4\,ft^2)}{\left(1\,\frac{slug\cdot ft}{lb\cdot s^2}\right)}$$

$$+\frac{\left(5\,\frac{ft}{s}\right)(\cos 43^\circ)\left(1.94\,\frac{slugs}{ft^3}\right)\left(5\,\frac{ft}{s}\right)(0.8\,ft^2)}{\left(1\,\frac{slug\cdot ft}{lb\cdot s^2}\right)} = -F_A + \left(10\,\frac{lb}{in^2}\right)\left(144\,\frac{in^2}{ft^2}\right)(0.8\,ft^2)$$

or

$$F_A = \underline{\underline{2440\,lb}}$$ to the left as shown in the sketch

5.60

5.60 A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated in Fig. P5.60. What is the vertical distance h?

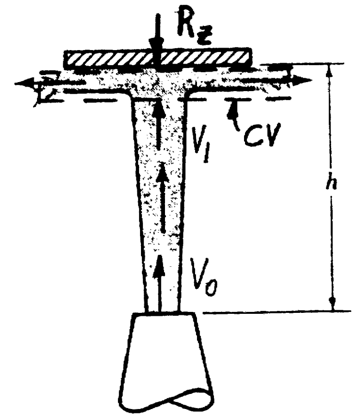


FIGURE P5.60

To determine the vertical distance h we apply the vertical direction component of the linear momentum equation (Eq. 5.22) to the water in the control volume shown in the sketch above. Thus,

$$-R_z - \rho g \forall_{water} = -V_1 \rho A_1 V_1 = -\rho V_1^2 \frac{\pi D_1^2}{4} \qquad (1)$$

The vertical reaction force of the plate on the water is equal in magnitude to the weight of the plate, or

$$R_z = g\, m_{plate} = (9.81 \tfrac{m}{s})(1.5\,kg) = 14.7\ N$$

Also, the weight of the water within the control volume, $\rho g \forall_{water}$, is negligible, and the mass flowrate is

$$\dot{m} = \rho A_1 V_1 = \rho A_0 V_0 = (999 \tfrac{kg}{m^3}) \tfrac{\pi}{4} (0.02 m)^2 (10 \tfrac{m}{s}) = 3.13 \tfrac{kg}{s}$$

Thus, Eq 1 becomes

$$-14.7\ N = -V_1 \dot{m} \quad \text{or} \quad V_1 = \frac{14.7\ N}{3.13\ kg/s} = 4.70 \tfrac{m}{s}$$

From the Bernouli Equation (Eq. 3.7) we have

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1, \text{ where } p_0 = p_1 = 0$$
$$z_0 = 0, \ z_1 = h$$

Thus,

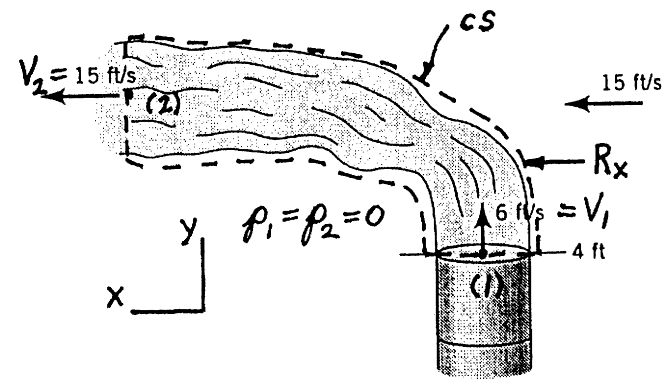
$$\tfrac{1}{2}\rho V_0^2 = \tfrac{1}{2}\rho V_1^2\ h$$

or since $\gamma = \rho g$

$$h = \frac{1}{2g}(V_0^2 - V_1^2) = \frac{1}{2(9.81 \frac{m}{s^2})}(10^2 - 4.70^2)\frac{m^2}{s^2} = \underline{\underline{3.97\ m}}$$

5.61

5.61 Exhaust (assumed to have the properties of standard air) leaves the 4-ft diameter chimney shown in Video V5.3 and Fig. P5.61 with a speed of 6 ft/s. Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind, 15 ft/s. Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



■ FIGURE P5.61

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_x \text{ becomes}$$

$V_2 \rho V_2 A_2 = R_x$, where R_x is the net horizontal force that the wind puts on the exhaust gases.

Thus,

$$R_x = \dot{m}_2 V_2 \quad \text{where } \dot{m}_2 = \rho A_2 V_2 = \rho A_1 V_1 \quad (\text{i.e. } \dot{m}_1 = \dot{m}_2)$$

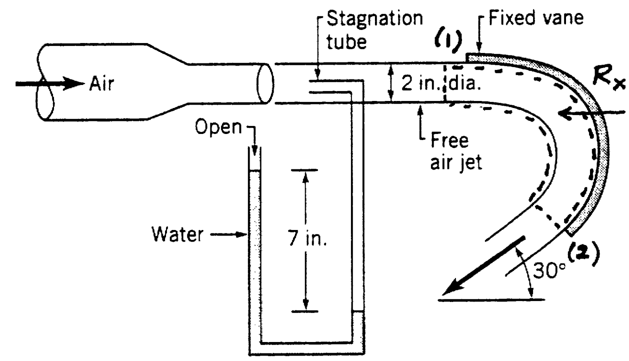
or $\dot{m}_2 = (0.00238 \frac{slugs}{s})\left[\frac{\pi}{4}(4ft)^2\right](6\frac{ft}{s}) = 0.179 \frac{slugs}{s}$

Hence,

$$R_x = 0.179 \frac{slugs}{s}(15\frac{ft}{s}) = 2.69 \frac{slug\cdot ft}{s^2} = \underline{\underline{2.69\ lb}}$$

5.62

5.62 Air discharges from a 2-in.-diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in Fig. P5.62. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.



■ FIGURE P5.62

Note that we ignore the effect of atmospheric pressure on the value of R_x in our solution below and use gage pressures. As indicated in Example 5.10, the atmospheric pressure force may need consideration when identifying reaction forces. For the air flowing through the control volume sketched above, the x-direction component of the linear momentum equation is

$$-V_1 \rho_{air} V_1 A_1 - V_2 \cos 30^\circ \rho_{air} V_2 A_2 = -R_x \qquad (1)$$

Application of Bernoulli's equation for the flow from (1) to (2) yields

$$V_2 = V_1 \qquad (2)$$

Then, from the conservation of mass principle

$$A_1 V_1 = A_2 V_2 \qquad (3)$$

We use the Bernoulli equation again to obtain the following equation for the stagnation tube deceleration

$$\frac{p_1}{\rho_{air}} + \frac{V_1^2}{2} = \frac{p_{stag}}{\rho_{air}} \qquad (4)$$

For the manometer, we obtain with the equation of hydrostatics

$$p_{atm} + h_{mano}\,\gamma_{water} - h_{mano}\,\gamma_{air} = p_{stag} \qquad (5)$$

With $p_1 = p_{atm}$, we get by combining Eqs. 4 and 5

$$V_1 = \sqrt{2 h_{mano}\left(\frac{\gamma_{water}}{\rho_{air}}\right)} \qquad (6)$$

(con't)

Combining Eqs. 1,2,3 and 6 we obtain

$$R_x = 2h_{mano}\left(\frac{\gamma_{water}}{\rho_{air}}\right)\rho_{air}\frac{\pi d_1^2}{4}(1+\cos 30^\circ)$$

or

$$R_x = \frac{2(7\text{ in.})(62.4\ \frac{\text{lb}}{\text{ft}^3})\ \pi\ (2\text{ in.})^2(1+\cos 30^\circ)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)\qquad\left(12\ \frac{\text{in.}}{\text{ft}}\right)^2\ 4}$$

and

$$R_x = \underline{\underline{2.96\text{ lb}}}$$

This is the force exerted by the vane on the flowing air. The force exerted by the flowing air exerts on the vane is equal in magnitude but opposite in direction (to the right)

5.65

5.65 A 3-in.-diameter horizontal jet of water strikes a flat plate as indicated in Fig. P5.65. Determine the jet velocity if a 10-lb horizontal force is required to: **(a)** hold the plate stationary; **(b)** allow the plate to move at a constant speed of 10 ft/s to the right.

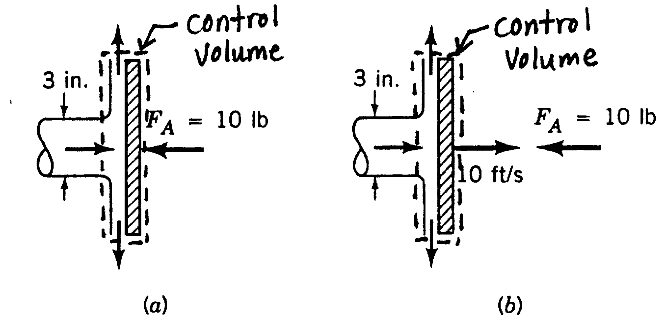


FIGURE P5.65

The control volume shown in the sketch is used. The stationary plate case is considered first. Application of the horizontal or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 = -F_{A,x}$$

or

$$u_1 = \left(\frac{F_{A,x}}{\rho A_1}\right)^{\frac{1}{2}} = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}}$$

Thus

$$u_1 = \left[\frac{(10\ lb)}{\left(1.94\ \frac{slugs}{ft^3}\right)\frac{\pi(3\ in.)^2}{4\left(12\ \frac{in.}{ft}\right)^2}\left(\frac{1\ lb}{slug\ \frac{ft}{s^2}}\right)}\right]^{\frac{1}{2}}$$

and $u_1 = \underline{\underline{10.2\ \frac{ft}{s}}}$ stationary plate

When the plate moves to the right with a speed, $U = 10\ \frac{ft}{s}$, the x-direction component of the linear momentum equation yields

$$-(u_1 - U)\rho(u_1 - U)A_1 = -F_{A,x}$$

or

$$u_1 - U = \left(\frac{F_{A,x}}{\rho A_1}\right)^{\frac{1}{2}} = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}}$$

and

$$u_1 = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}} + U = 10.2\ \frac{ft}{s} + 10\ \frac{ft}{s} = \underline{\underline{20.2\ \frac{ft}{s}}}$$

moving plate

5.66

5.66 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.66 and Video V5.4. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

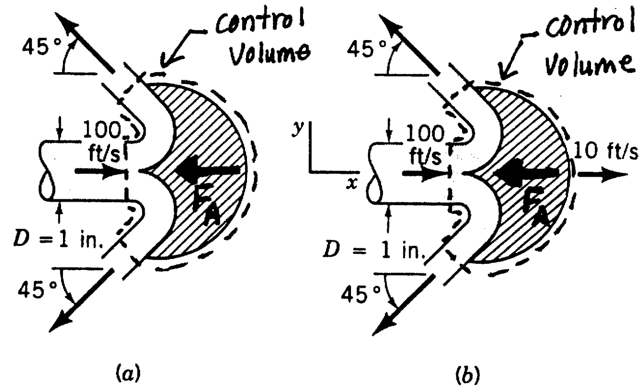


FIGURE P5.66

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

$$F_A = \dot{m}(V_1 + V_2 \cos 45°) = \rho A_1 V_1 (V_1 + V_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} V_1 (V_1 + V_2 \cos 45°)$$

or

$$F_A = \frac{\left(1.94 \frac{slugs}{ft^3}\right) \pi (1 \text{ in.})^2 \left(100 \frac{ft}{s}\right)}{(4)\left(12 \frac{in.}{ft}\right)^2} \left[\left(100 \frac{ft}{s}\right) + \left(100 \frac{ft}{s}\right) \cos 45°\right] \left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$F_A = \underline{\underline{181 \ lb}}$$

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of 10 $\frac{ft}{s}$ to the right we use a control volume moving to the right with a speed of 10 $\frac{ft}{s}$ and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

$$F_A = \rho A_1 W_1 (W_1 + W_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} W_1 (W_1 + W_2 \cos 45°) \qquad (1)$$

We note that

$$W_1 = V_1 - 10 \frac{ft}{s} = 100 \frac{ft}{s} - 10 \frac{ft}{s} = 90 \frac{ft}{s}$$

Thus, Eq. 1 leads to

$$F_A = \left(1.94 \frac{slugs}{ft^3}\right) \frac{\pi (1 \text{ in.})^2}{4 \left(12 \frac{in.}{ft}\right)^2} \left(90 \frac{ft}{s}\right) \left[90 \frac{ft}{s} + \left(90 \frac{ft}{s}\right) \cos 45°\right] \left(\frac{1 \ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

or

$$F_A = \underline{\underline{146 \ lb}}$$

5.67

5.67 How much power is transferred to the moving vane of Problem 5.66?

Power $= F_A V$, where from Problem 5.66 $F_A = 146\text{ lb}$

Thus,

$$\text{Power} = \frac{(146\text{ lb})(10\ \frac{\text{ft}}{\text{s}})}{\left(550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)} = \underline{\underline{2.65}}\text{ hp}$$

5.68

5.68 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.68. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

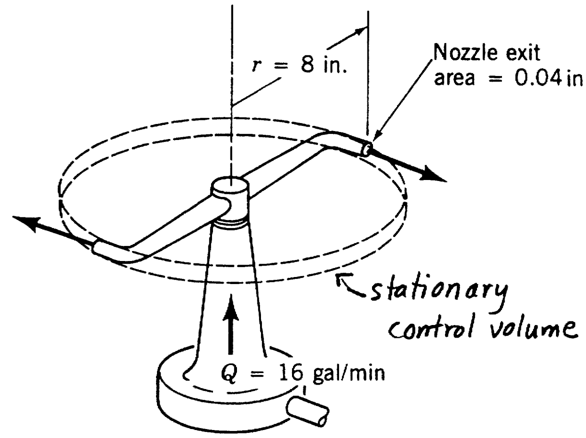


FIGURE P5.68

This is similar to Example 5.17.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.50). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta,2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta,2} = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{2\left(0.04\,in.^2\right)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\,\frac{slugs}{ft^3}\right)\left(16\,\frac{gal}{min}\right)\left(8\,in.\right)\left(64.17\,\frac{ft}{s}\right)\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\left(12\,\frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\,ft\cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of $500\,\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For W_2 we use

$$W_2 = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{(2)\left(0.04\,in.^2\right)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)} = 64.17\,\frac{ft}{s}$$

(con't)

For U_2 we use

$$U_2 = r_2\omega = \frac{(8\text{ in.})\left(500\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)}{\left(12\frac{\text{in.}}{\text{ft}}\right)\left(60\frac{\text{s}}{\text{min}}\right)} = 34.91\frac{\text{ft}}{\text{s}}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17\frac{\text{ft}}{\text{s}} - 34.91\frac{\text{ft}}{\text{s}} = 29.26\frac{\text{ft}}{\text{s}}$$

and with Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left(16\frac{\text{gal}}{\text{min}}\right)(8\text{ in.})\left(29.26\frac{\text{ft}}{\text{s}}\right)}{\left(7.48\frac{\text{gal}}{\text{ft}^3}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}\left(1\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$T_{shaft} = \underline{\underline{1.35\text{ ft}\cdot\text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

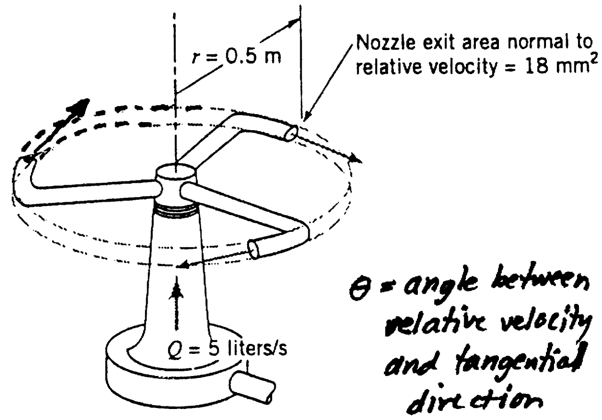
$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17\frac{\text{ft}}{\text{s}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}{(8\text{ in.})} = 96.3\frac{\text{rad}}{\text{s}}$$

The rotor speed, N, is thus

$$N = \left(96.3\frac{\text{rad}}{\text{s}}\right)\frac{\left(60\frac{\text{s}}{\text{min}}\right)}{\left(2\pi\frac{\text{rad}}{\text{rev}}\right)} = \underline{\underline{920\frac{\text{rev}}{\text{min}}}}$$

5.69

5.69 Five liters/s of water enters the rotor shown in Video V5.5 and Fig. P5.69 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm². How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 60°$?



■ FIGURE P5.69

θ = angle between relative velocity and tangential direction

To determine the torque required to hold the rotor stationary we use the moment-of-momentum torque equation (Eq. 5.50) to obtain

$$T_{shaft} = \dot{m}\, r_{out} V_{out} \cos\theta \qquad (1)$$

We note that

$$\dot{m} = \rho Q \qquad (2)$$

and

$$V_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$T_{shaft} = \frac{\rho Q^2 r_{out} \cos\theta}{3\, A_{\substack{nozzle\\exit}}} \qquad (4)$$

To determine the rotor angular velocity associated with zero shaft torque we again use the moment-of-momentum torque equation (Eq. 5.50) to obtain, this time with rotation,

$$T_{shaft} = \dot{m}\, r_{out} \left(W_{out} \cos\theta - U_{out} \right) \qquad (5)$$

We note that

$$U_{out} = r_{out}\, \omega \qquad (6)$$

and

$$W_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (7)$$

(con't)

5.69	(con't)

Combining Eqs. 2, 5, 6 and 7 we get

$$T_{shaft} = \rho Q r_{out} \left(\frac{Q \cos\theta}{3 A_{\substack{nozzle \\ exit}}} - r_{out}\,\omega \right) \qquad (8)$$

(a) For $\theta = 0°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5\,m)(\cos 0°)\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18\,mm^2)}$$

or

$$T_{shaft} = \underline{\underline{231}}\ N \cdot m$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{Q \cos\theta}{3 A_{\substack{nozzle \\ exit}}\, r_{out}} = \frac{\left(5 \frac{liters}{s}\right)(\cos 0°)\left(1000 \frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{185 \frac{rad}{s}}}$$

(b) For $\theta = 30°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5\,m)(\cos 30°)\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18\,mm^2)}$$

or

$$T_{shaft} = 200\ N \cdot m$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5 \frac{liters}{s}\right)(\cos 30°)\left(1000 \frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{160}}\ \frac{rad}{s}$$

(c) For $\theta = 60°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999 \frac{kg}{m^3}\right)\left(5 \frac{liters}{s}\right)^2 (0.5\,m)(\cos 60°)\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1\,N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1000 \frac{liters}{m^3}\right)^2 (3)(18\,mm^2)}$$

or

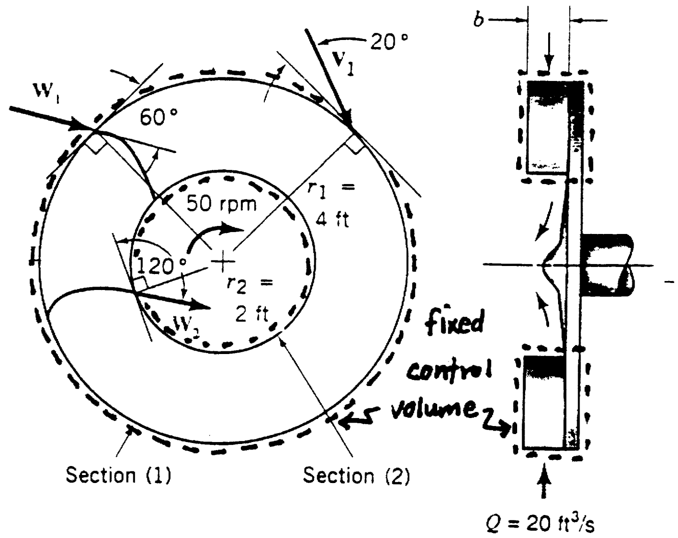
$$T_{shaft} = \underline{\underline{116}}\ N \cdot m$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5 \frac{liters}{s}\right)(\cos 60°)\left(1000 \frac{mm}{m}\right)^2}{(3)(18\,mm^2)\left(1000 \frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{92.5}}\ \frac{rad}{s}$$

5.71

5.71 A water turbine wheel rotates at the rate of 50 rpm in the direction shown in Fig. P5.71. The inner radius, r_2, of the blade row is 2 ft, and the outer radius, r_1, is 4 ft. The absolute velocity vector at the turbine rotor entrance makes an angle of 20° with the tangential direction. The inlet blade angle is 60° relative to the tangential direction. The blade outlet angle is 120°. The flowrate is 20 ft³/s. For the flow tangent to the rotor blade surface at inlet and outlet, determine an appropriate constant blade height, b, and the corresponding power available at the rotor shaft.



■ FIGURE P5.71

since

$$Q = 2\pi r_1 b V_{R,1}$$

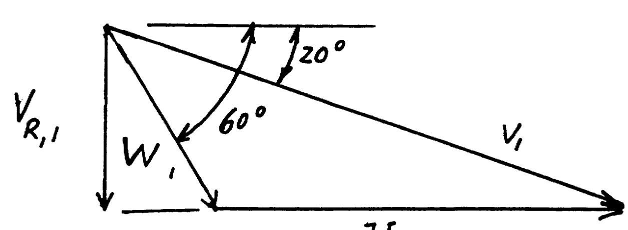
then the blade height, b, is determined with

$$b = \frac{Q}{2\pi r_1 V_{R,1}} \qquad (1)$$

The shaft power, $\dot{W}_{\substack{shaft \\ net\ out}}$, is obtained with the moment-of-momentum power equation (Eq. 5.53). Thus,

$$\dot{W}_{\substack{shaft \\ net\ out}} = \dot{m}\left(U_1 V_{\theta,1} \pm U_2 V_{\theta,2}\right) = \rho Q\left(U_1 V_{\theta,1} \pm U_2 V_{\theta,2}\right) \qquad (2)$$

and the use of "+" or "−" with $U_2 V_{\theta,2}$ depends on whether $V_{\theta,2}$ is opposite to or in the same direction as U_2 respectively. To determine the value of $V_{R,1}$ we use the velocity triangle at section (1). Thus, we have



With the velocity triangle we have

$$\frac{V_{R,1}}{\tan 20°} = \frac{V_{R,1}}{\tan 60°} + U_1 \qquad (3)$$

However

$$U_1 = r_1 \omega$$

(con't)

thus Eq. 3 leads to

$$V_{R,1} = \frac{r_1 \omega}{\left(\frac{1}{\tan 20^\circ} - \frac{1}{\tan 60^\circ}\right)} = \frac{(4\,ft)(50\,rpm)\left(2\pi \frac{rad}{rev}\right)}{\left(\frac{1}{\tan 20^\circ} - \frac{1}{\tan 60^\circ}\right)\left(60 \frac{s}{min}\right)} = 9.651 \frac{ft}{s}$$

With Eq. 1 we obtain

$$b = \frac{\left(20 \frac{ft^3}{s}\right)}{2\pi(4\,ft)\left(9.651 \frac{ft}{s}\right)} = 0.0825\ ft$$

For the blade velocities in Eq. 2 we get

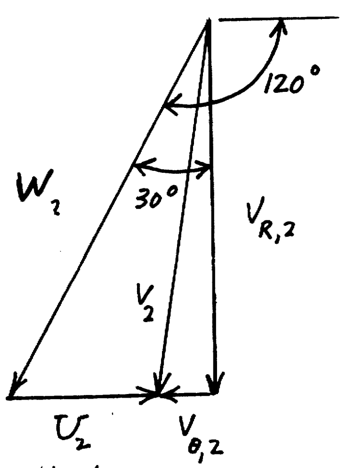
$$U_1 = r_1\omega = \frac{(4\,ft)(50\,rpm)\left(2\pi \frac{rad}{rev}\right)}{\left(60 \frac{s}{min}\right)} = 20.94 \frac{ft}{s}$$

$$U_2 = r_2\omega = \frac{(2\,ft)(50\,rpm)\left(2\pi \frac{rad}{rev}\right)}{60 \frac{s}{min}} = 10.47 \frac{ft}{s}$$

For $V_{\theta,1}$ we use the velocity triangle at section (1) to obtain

$$V_{\theta,1} = \frac{V_{R,1}}{\tan 20^\circ} = \frac{9.651 \frac{m}{s}}{\tan 20^\circ} = 26.52 \frac{ft}{s}$$

For $V_{\theta,2}$ we construct the section (2) velocity triangle sketched below ($V_{\theta,2}$ not to scale)



and we realize that

$$V_{\theta,2} = V_{R,2} \tan 30^\circ - U_2 \qquad (4)$$

From conservation of mass

$$V_{R,2} = V_{R,1}\frac{A_1}{A_2} = V_{R,1}\left(\frac{r_1}{r_2}\right) = \left(9.651 \frac{ft}{s}\right)\left(\frac{4\,ft}{2\,ft}\right) = 19.3 \frac{ft}{s}$$

(con't)

5.71 (Con't)

so with Eq. 4 we obtain

$$V_{\theta,2} = \left(19.3 \frac{ft}{s}\right)\tan 30° - 10.47 \frac{ft}{s} = 0.673 \frac{ft}{s}$$

Finally with Eq. 2 we obtain

$$\dot{W}_{\substack{shaft \\ net\ out}} = \left(1.94 \frac{slugs}{ft^3}\right)\left(20 \frac{ft^3}{s}\right)\left[\left(20.94 \frac{ft}{s}\right)\left(26.52 \frac{ft}{s}\right) + \left(10.47 \frac{ft}{s}\right)\left(0.673 \frac{ft}{s}\right)\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

or

$$\dot{W}_{\substack{shaft \\ net\ out}} = 2.18 \times 10^4 \frac{ft \cdot lb}{s}$$

and

$$\dot{W}_{\substack{shaft \\ net\ out}} = \frac{2.18 \times 10^4 \frac{ft \cdot lb}{s}}{550 \frac{ft \cdot lb}{s \cdot hp}} = \underline{\underline{39.6\ hp}}$$

5.72

5.72 An incompressible fluid flows outward through a blower as indicated in Fig. P5.72. The shaft torque involved, T_{shaft}, is estimated with the following relationship:

$$T_{shaft} = \dot{m} r_2 V_{\theta,2}$$

where $\dot{m}$ = mass flowrate through the blower, r_2 = outer radius of blower, and $V_{\theta,2}$ = tangential component of absolute fluid velocity leaving the blower. State the flow conditions that make this formula valid.

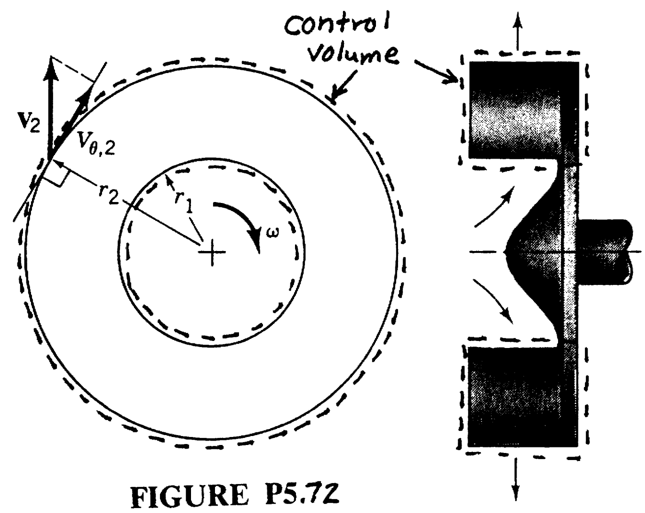


FIGURE P5.72

The flow conditions that make

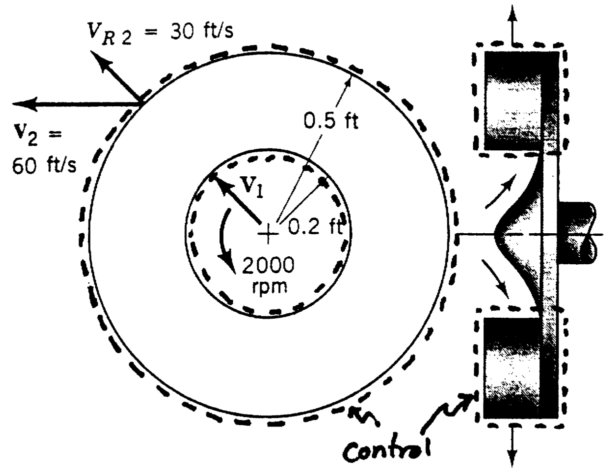
$$T_{shaft} = \dot{m} r_2 V_{\theta,2} \qquad (1)$$

valid may be identified by comparing Eq. 1 with the axial component of Eq. 5.42. These conditions are

a. stationary and non-deforming control volume (see sketch above)
b. steady-in-the-mean flow
c. negligible shear stress torque with respect to axis of rotation
d. $V_{\theta,1} = 0$
e. no torque with respect to axis of rotation due to normal stresses
f. uniform distribution of $V_{\theta,2}$

5.73

5.73 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P5.73 is 30 ft/s. The magnitude of the absolute velocity at the pump exit is 60 ft/s. The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



■ FIGURE P5.73

The stationary and non-deforming control volume shown in the sketch above is used. To determine the shaft work per unit mass, w_{shaft}, we can use Eq. 5.54. Thus

$$w_{shaft} = U_2 V_{\theta,2} \quad (1)$$

The blade speed, U_2, can be obtained as follows,

$$U_2 = r_2 \omega = (0.5\ ft)\left(2000\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)\left(\frac{1}{60\ \frac{s}{min}}\right) = 105\ \frac{ft}{s}$$

The tangential velocity, $V_{\theta,2}$, can be obtained as follows,

$$V_{\theta,2} = \left(V_2^2 - V_{R,2}^2\right)^{\frac{1}{2}} = \left[\left(60\ \frac{ft}{s}\right)^2 - \left(30\ \frac{ft}{s}\right)^2\right]^{\frac{1}{2}} = 52\ \frac{ft}{s}$$

Thus, from Eq. 1

$$w_{shaft} = \left(105\ \frac{ft}{s}\right)\left(52\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{5460}}\ \frac{ft \cdot lb}{slug}$$

5.74

5.74 A fan (see Fig. P5.74) has a bladed rotor of 12-in.-outside diameter and 5-in.-inside diameter and runs at 1725 rpm. The width of each rotor blade is 1 in. from blade inlet to outlet. The volume flowrate is steady at 230 ft³/min and the absolute velocity of the air at blade inlet, V_1, is purely radial. The blade discharge angle is 30° measured with respect to the tangential direction at the outside diameter of the rotor. **(a)** What would be a reasonable blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor)? **(b)** Find the power required to run the fan.

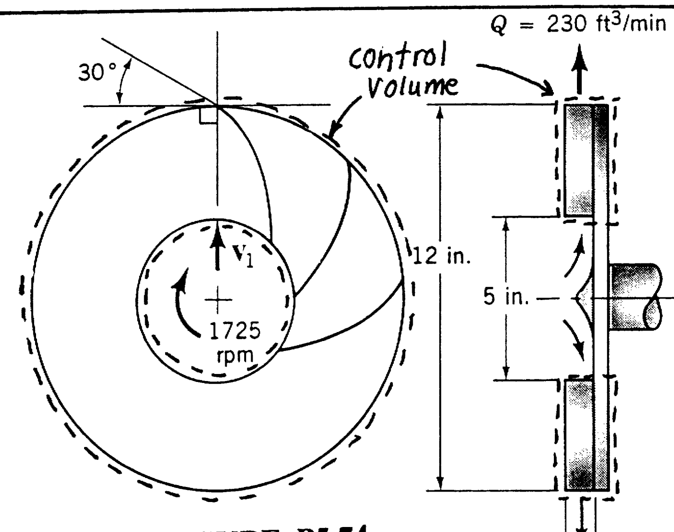
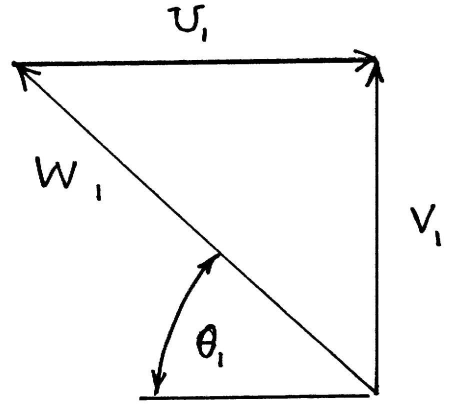


FIGURE P5.74

The stationary and non-deforming control volume shown in the sketch above is used. To determine a reasonable blade inlet angle we assume that the blade should be tangent to the relative velocity at the inlet. The inlet velocity triangle is sketched below.



With the velocity triangle, we conclude that

$$\theta_1 = \tan^{-1}\left(\frac{V_1}{U_1}\right) \tag{1}$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 h_1} = \frac{\left(230\ \frac{ft^3}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{2\pi\,(2.5\ in.)(1\ in.)\left(60\ \frac{s}{min}\right)} = 35.1\ \frac{ft}{s}$$

and

$$U_1 = r_1\omega = \frac{(2.5\ in.)\left(1725\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{\left(12\ \frac{in.}{ft}\right)\left(60\ \frac{s}{min}\right)} = 37.6\ \frac{ft}{s}$$

Thus with Eq. 1

$$\theta_1 = \tan^{-1}\left[\frac{\left(35.1\ \frac{ft}{s}\right)}{\left(37.6\ \frac{ft}{s}\right)}\right] = \underline{\underline{43^\circ}}$$

(con't)

5.74 (con't)

The power required, $\dot{W}_{shaft}$, may be obtained with Eq. 5.53. Thus

$$\dot{W}_{shaft} = \dot{m}_2 U_2 V_{\theta,2} \qquad (2)$$

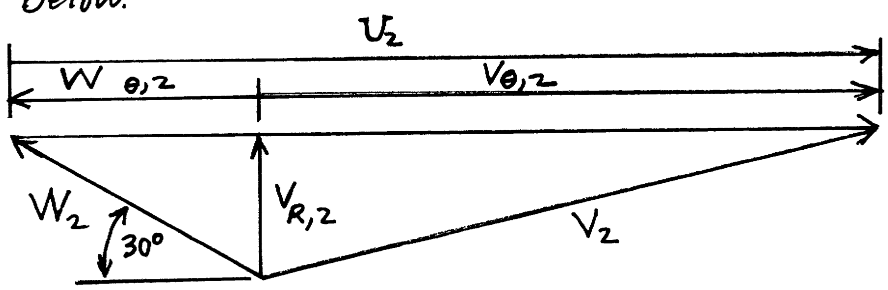
The mass flowrate, $\dot{m}_2$, may be obtained as follows.

$$\dot{m}_2 = \rho Q = \left(2.38 \times 10^{-3} \frac{slugs}{ft^3}\right)\left(230 \frac{ft^3}{min}\right)\left(\frac{1}{60 \frac{s}{min}}\right) = 9.12 \times 10^{-3} \frac{slug}{s}$$

Also

$$U_2 = r_2 \omega = \frac{(6 \text{ in.})\left(1725 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(12 \frac{in.}{ft}\right)\left(60 \frac{s}{min}\right)} = 90.3 \frac{ft}{s}$$

The value of $V_{\theta,2}$ may be obtained by considering the velocity triangle for the flow leaving the rotor at section (2). The relative velocity at the rotor exit is considered to be tangent to the blade there. The rotor exit flow velocity triangle is sketched below.



Now

$$V_{\theta,2} = U_2 - W_{\theta,2}$$

and

$$W_{\theta,2} = \frac{V_{R,2}}{\tan 30°} = \frac{\frac{Q}{2\pi r_2 h_2}}{\tan 30°} = \frac{\frac{\left(230 \frac{ft^3}{min}\right)\left(144 \frac{in.^2}{ft^2}\right)}{2\pi (6 \text{ in.})(1 \text{ in.})\left(60 \frac{s}{min}\right)}}{\tan 30°} = 25.4 \frac{ft}{s}$$

Thus

$$V_{\theta,2} = 90.3 \frac{ft}{s} - 25.4 \frac{ft}{s} = 64.9 \frac{ft}{s}$$

and from Eq. 2

$$\dot{W}_{shaft} = \left(9.12 \times 10^{-3} \frac{slug}{s}\right)\left(90.3 \frac{ft}{s}\right)\left(64.9 \frac{ft}{s}\right)\left(\frac{1 \text{ lb}}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{53.4}} \frac{ft \cdot lb}{s}$$

5.75

5.75 An axial flow gasoline pump (see Fig. P5.75) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are 60° and 45° from the axial direction. The pump annulus passage cross section area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline?

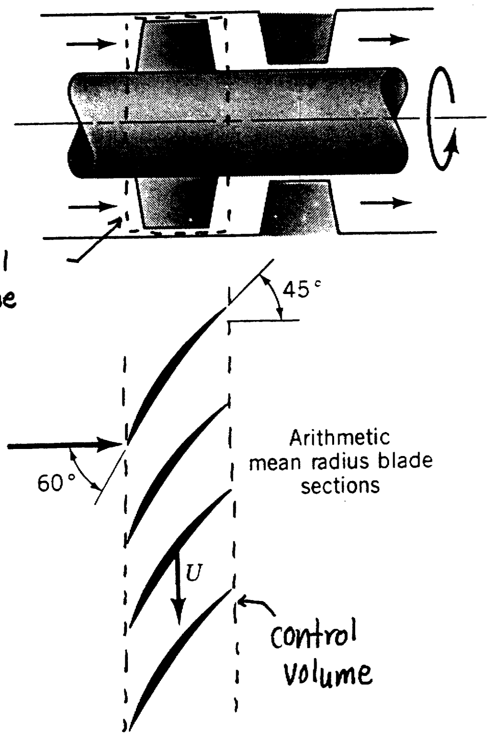
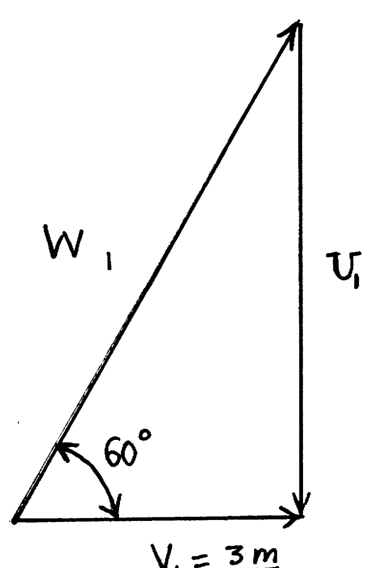


FIGURE P5.75

The velocity triangle for flow just upstream of the rotor is sketched below for the arithmetic mean radius.



With the triangle we conclude that

$$W_1 = \frac{V_1}{\cos 60^\circ} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 60^\circ} = 6\ \frac{m}{s}$$

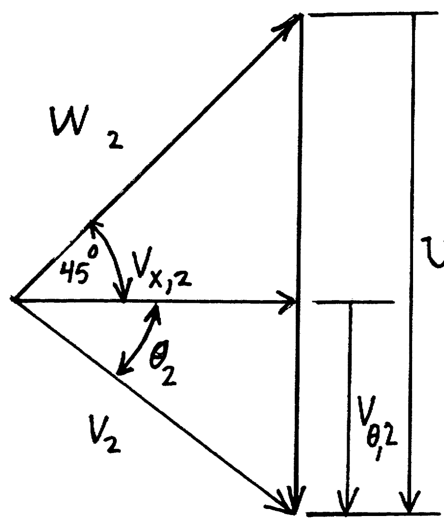
and

$$U_1 = W_1 \sin 60^\circ = \left(6\ \frac{m}{s}\right) \sin 60^\circ = 5.2\ \frac{m}{s}$$

(con't)

5.75 (con't)

The velocity triangle for flow just downstream of the rotor is sketched below for the arithmetic mean radius. For incompressible flow $V_{x,2} = V_1$. For mean radius flow $U_2 = U_1$. Thus for relative flow tangent to the blade we obtain the velocity triangle sketched below.



With the triangle we conclude that

$$V_{\theta,2} = U_2 - W_{\theta,2} = U_2 - V_{x,2}\tan 45^\circ = 5.2\,\frac{m}{s} - \left(3\,\frac{m}{s}\right)\tan 45^\circ = 2.2\,\frac{m}{s}$$

Also

$$\theta_2 = \tan^{-1}\left(\frac{V_{\theta,2}}{V_{x,2}}\right) = \tan^{-1}\left[\frac{\left(2.2\,\frac{m}{s}\right)}{\left(3\,\frac{m}{s}\right)}\right] = 36.2^\circ$$

$$W_2 = \frac{V_{x,2}}{\cos 45^\circ} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 45^\circ} = 4.24\,\frac{m}{s}$$

$$V_2 = \frac{V_{x,2}}{\cos\theta_2} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 36.2^\circ} = 3.72\,\frac{m}{s}$$

Using the stationary and non-deforming control volume shown above in the first sketch of this solution and Eq. 5.54 we can calculate the energy added to each kg of gasoline

$$w_{shaft} = U_2 V_{\theta,2} = \left(5.2\,\frac{m}{s}\right)\left(2.2\,\frac{m}{s}\right)\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{11.4\,\frac{N\cdot m}{kg}}}$$

The velocity triangle for the stator exit flow is sketched below.

$V_3 = 3\,\frac{m}{s}$

5.76

5.76 A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P5.76. The rotor speed is 1000 rpm. (a) Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. Assume flow enters and leaves each blade row at the blade angles shown. (b) Calculate the work per unit mass delivered at the shaft.

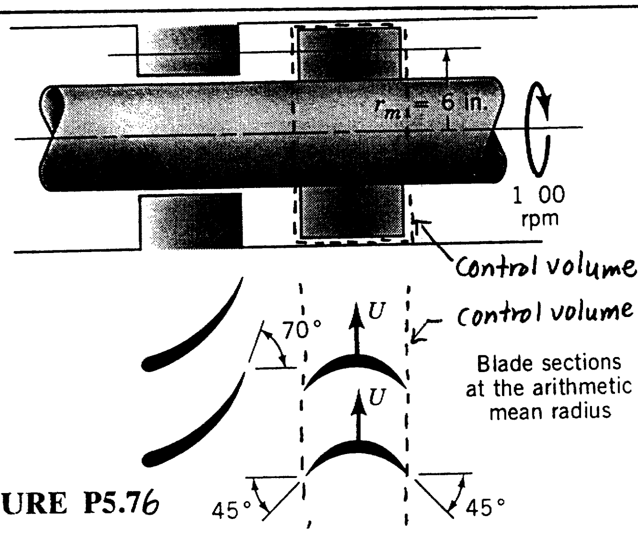
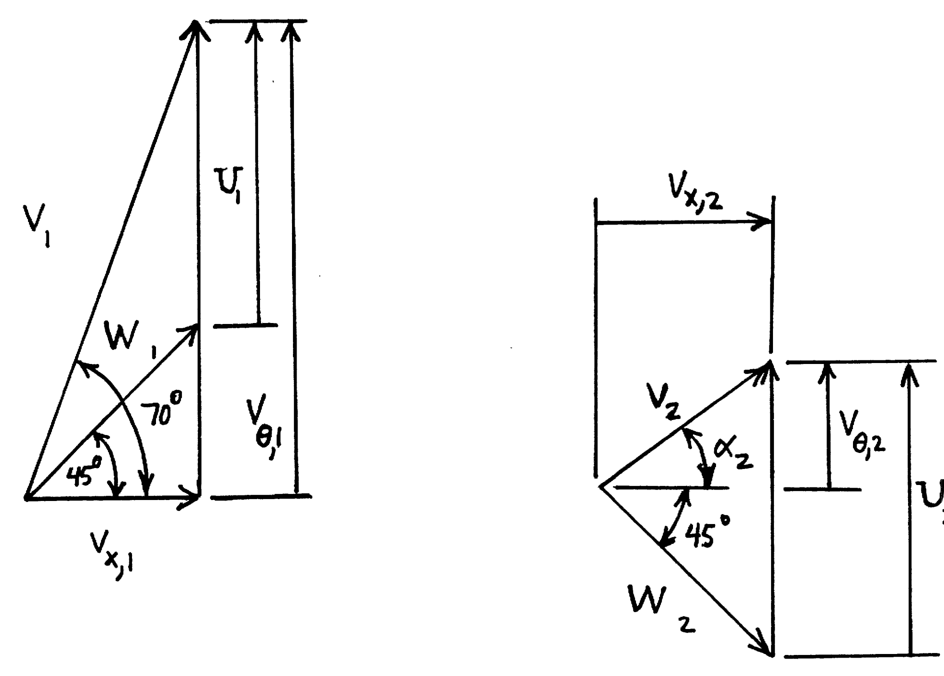


FIGURE P5.76

The velocity triangles for the flow entering and the flow leaving the rotor row at the arithmetic mean radius are sketched below.



At the arithmetic mean radius, the blade velocity, U, is

$$U_1 = U_2 = r_m \omega = \frac{(6\ in.)\ 1000 \frac{rev}{min} \left(2\pi \frac{rad}{rev}\right)}{\left(12 \frac{in.}{ft}\right)\left(60 \frac{s}{min}\right)} = 52.3 \frac{ft}{s}$$

With the velocity triangle for the flow entering the rotor we conclude that

$$V_1 \sin 70^\circ = V_{\theta,1} \qquad (1)$$

$$V_1 \cos 70^\circ = V_{x,1} \qquad (2)$$

$$W_1 \sin 45^\circ = V_{\theta,1} - U \qquad (3)$$

$$W_1 \cos 45^\circ = V_{x,1} \qquad (4)$$

(con't)

From the ratio of Eqs. 3 and 4 we obtain

$$\tan 45^\circ = \frac{V_{\theta,1} - U}{V_{x,1}}$$

which when combined with Eqs. 1 and 2 yields

$$\tan 45^\circ = \frac{V_1 \sin 70^\circ - U}{V_1 \cos 70^\circ}$$

or

$$V_1 = \frac{U}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]} = \frac{52.3\ \frac{ft}{s}}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]}$$

$$V_1 = 87.6\ \frac{ft}{s}$$

Then

$$V_{\theta_1} = V_1 \sin 70^\circ = \left(87.6\ \frac{ft}{s}\right) \sin 70^\circ = 82.3\ \frac{ft}{s}$$

$$V_{x,1} = V_1 \cos 70^\circ = \left(87.6\ \frac{ft}{s}\right) \cos 70^\circ = 29.9\ \frac{ft}{s}$$

and

$$W_1 = \frac{V_{x,1}}{\cos 45^\circ} = \frac{\left(29.9\ \frac{ft}{s}\right)}{\cos 45^\circ} = 42.4\ \frac{ft}{s}$$

With the velocity triangle for the flow leaving the rotor we conclude that

$$W_2 \cos 45^\circ = V_{x,2} \tag{5}$$

$$V_{\theta,2} = U_2 - W_2 \sin 45^\circ \tag{6}$$

$$V_2 \sin \alpha_2 = V_{\theta,2} \tag{7}$$

$$V_2 \cos \alpha_2 = V_{x,2} \tag{8}$$

From the conservation of mass equation

$$V_{x,1} = V_{x,2} = 29.9\ \frac{ft}{s}$$

(con't)

Thus from Eq. 5

$$W_2 = \frac{V_{x,2}}{\cos 45°} = \frac{(29.9 \frac{ft}{s})}{\cos 45°} = 42.4 \frac{ft}{s}$$

and from Eq. 6

$$V_{\theta,2} = U_2 - W_2 \sin 45° = 52.3 \frac{ft}{s} - (42.4 \frac{ft}{s}) \sin 45° = 22.4 \frac{ft}{s}$$

The ratio of Eqs. 7 and 8 yields

$$\alpha_2 = \tan^{-1}\left(\frac{V_{\theta,2}}{V_{x,2}}\right) = \tan^{-1}\left[\frac{(22.4 \frac{ft}{s})}{(29.9 \frac{ft}{s})}\right] = 37°$$

and from Eq. 7

$$V_2 = \frac{V_{\theta,2}}{\sin \alpha_2} = \frac{(22.4 \frac{ft}{s})}{\sin(37°)} = 37.2 \frac{ft}{s}$$

We can use Eq. 5.54 to calculate the work per unit mass delivered at the shaft. Thus

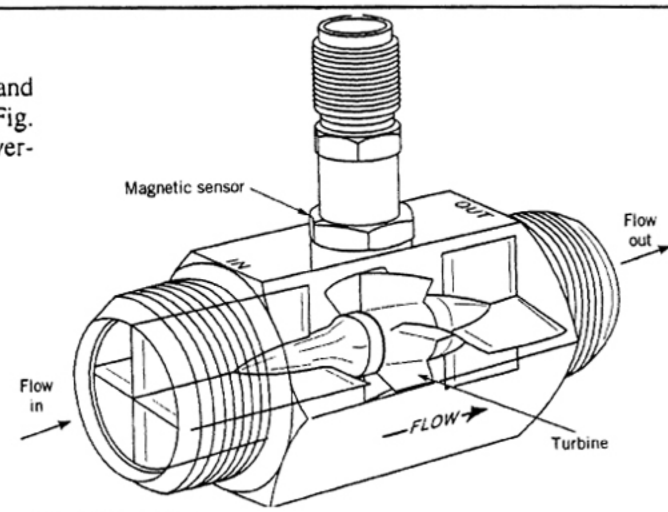
$$w_{shaft} = -U_1 V_{\theta,1} + U_2 V_{\theta,2}$$

$$w_{shaft} = \left[-(52.3 \tfrac{ft}{s})(82.3 \tfrac{ft}{s}) + (52.3 \tfrac{ft}{s})(22.4 \tfrac{ft}{s})\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$w_{shaft} = -\underline{\underline{3130}}\ \frac{ft \cdot lb}{slug}$$

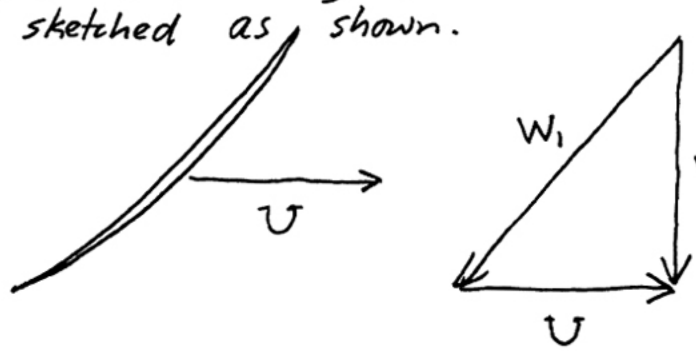
5.77

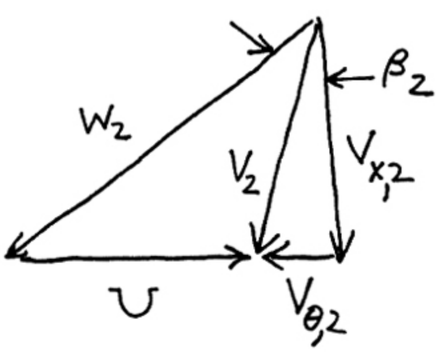
5.77 Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.77. Show how rotor angular velocity is proportional to average fluid velocity.



■ FIGURE P5.77 (Courtesy of EG&G Flow Technology, Inc.)

For a section of the turbine blade at radius r, the blade moves tangentially with a velocity $U = r\omega$. The velocity triangles may be sketched as shown.





Using Eq. 5.50 we get

$$T_{shaft} = r_2 V_{\theta,2} \dot{m}_2 = r_2 \left(V_{x,2} \tan\beta_2 - U\right) \dot{m}_2$$

For nearly zero T_{shaft}

$$0 = V_{x,2} \tan\beta_2 - U = V_{x,2} \tan\beta_2 - r\omega$$

so

$$\omega = \frac{V_{x,2} \tan\beta_2}{r}$$

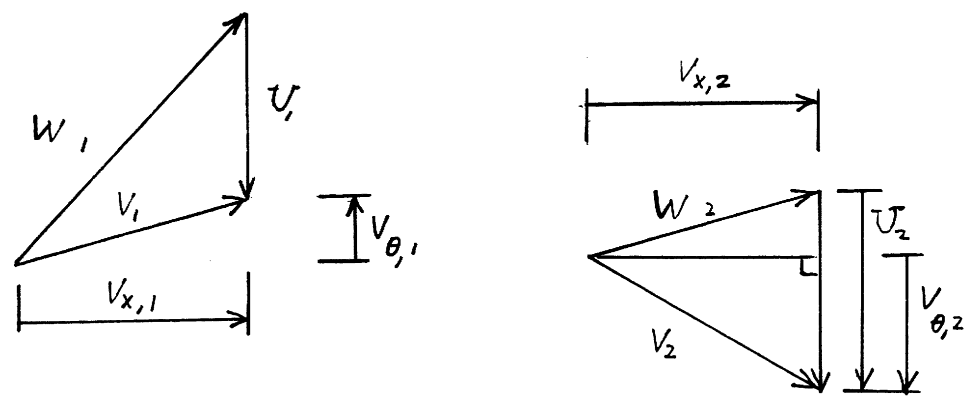
5.78

5.78 By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed.

Any set of velocity triangle for flow through a turbomachine rotor row would give the same result. We use the triangles of Fig. P5.77.



From the inlet flow velocity triangle we get

$$V_{x,1}^2 = V_1^2 - V_{\theta,1}^2 \qquad (1)$$

and

$$V_{x,1}^2 = W_1^2 - (V_{\theta,1} + U_1)^2 = W_1^2 - V_{\theta,1}^2 - 2U_1V_{\theta,1} - U_1^2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$U_1 V_{\theta,1} = \frac{{}_1^2 - V_1^2 - U_1^2}{2} \qquad (3)$$

From the outlet flow velocity triangle we get

$$V_{x,2}^2 = V_2^2 - V_{\theta,2}^2 \qquad (4)$$

and

$$V_{x,2}^2 = W_2^2 - (U_2 - V_{\theta,2})^2 = W_2^2 - U_2^2 + 2U_2V_{\theta,2} - V_{\theta,2}^2 \qquad (5)$$

(con't)

Combining Eqs. 4 and 5 we obtain

$$U_2 V_{\theta,2} = \frac{V_2^2 - W_2^2 + U_2^2}{2} \qquad (6)$$

For the set of velocity triangles

$$w_{\substack{\text{shaft}\\\text{net in}}} = U_1 V_{\theta,1} + U_2 V_{\theta,2} \qquad (7)$$

Combining Eqs. 3, 6 and 7 we obtain

$$w_{\substack{\text{shaft}\\\text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

5.79

5.79* Summarized below are air flow data for flow across a low-speed axial flow fan. Calculate the change in rate of flow of axial direction angular momentum across this rotor and evaluate the shaft power input involved. The inner and outer radii of the fan annulus are 142 and 203 mm. The rotor speed is 2400 rpm.

	Upstream of Rotor		Downstream of Rotor	
Radius (mm)	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)
142	0	0	0	0
148	32.03	0	32.28	12.64
169	32.03	0	32.37	12.24
173	32.04	0	31.78	11.91
185	32.03	0	31.50	11.35
197	31.09	0	29.64	11.66
203	0	0	0	0

the change in rate of flow of axial direction angular momentum across the rotor, ΔFAM_x, is evaluated with

$$\Delta FAM_x = \int_{r_i}^{r_o} r_2 V_{\theta,2} \rho V_{x,2} 2\pi r_2 dr_2 - \int_{r_i}^{r_o} r_1 V_{\theta,1} \rho V_{x,1} 2\pi r_1 dr_1$$

or

$$\Delta FAM_x = 2\pi\rho \left(\int_{r_i}^{r_o} V_{\theta,2} V_{x,2} r_2^2 dr_2 - \int_{r_i}^{r_o} V_{\theta,1} V_{x,1} r_1^2 dr_1 \right) \qquad (1)$$

where

r_i and r_o are fan annulus inner and outer radii

r_2 and r_1 are local radii at section (2) downstream of fan rotor and section (1) upstream of fan rotor

$V_{\theta,2}$ and $V_{\theta,1}$ are local absolute tangential velocity at sections (2) and (1)

$V_{x,2}$ and $V_{x,1}$ are local axial velocities at sections (2) and (1)

As suggested by Eq. 5.45

$$T_{shaft} = \Delta FAM_x \qquad (2)$$

and Eq. 2 is evaluated numerically with a computer program that utilizes the trapezoidal rule with uneven intervals. The program list and results are on the next page.

The shaft power input, $\dot{W}_{shaft}$, is evaluated with Eq. 5.47. Thus,

$$\dot{W}_{shaft} = T_{shaft}\,\omega \qquad (3)$$

Eq. 3 is evaluated by the computer program listed on the next page.

(con't)

5.79 (Con't)

```
100 CLS
110 PRINT "**************************************************"
120 PRINT "** This program computes the change in rate of  **"
130 PRINT "** axial-direction angular momentum and power   **"
140 PRINT "** input for problem 5.79 using the trapezoidal **"
150 PRINT "** rule applied to unequal intervals.           **"
160 PRINT "**************************************************"
170 PRINT
180 DIM UXU(19), UTU(19), UXD(19), UTD(19), R(19)
190 '
200 'Initialize the variables
210 N = 7
220 RHO = 1.23
230 PI = 4! * ATN(1!)
240 RPM = 2400!
250 FOR I = 1 TO N
260 READ R(I), UXU(I), UTU(I), UXD(I), UTD(I)
270 R(I) = R(I) / 1000!
280 NEXT I
290 DATA 142.0, 00.00, 00.00, 00.00, 00.00
300 DATA 148.0, 32.03, 00.00, 32.28, 12.64
310 DATA 169.0, 32.03, 00.00, 32.37, 12.24
320 DATA 173.0, 32.04, 00.00, 31.78, 11.91
330 DATA 185.0, 32.03, 00.00, 31.50, 11.35
340 DATA 197.0, 31.09, 00.00, 29.64, 11.66
350 DATA 203.0, 00.00, 00.00, 00.00, 00.00
360 '
370 'Compute integral using trapezoidal rule
380 SUMU = 0!
390 SUMD = 0!
400 FOR I = 2 TO N
410 TEMPU=UTU(I)*UXU(I)*R(I)^2+UTU(I-1)*UXU(I-1)*R(I-1)^2
420 TEMPD=UTD(I)*UXD(I)*R(I)^2+UTD(I-1)*UXD(I-1)*R(I-1)^2
430 SUMU = SUMU + TEMPU * (R(I) - R(I - 1)) / 2!
440 SUMD = SUMD + TEMPD * (R(I) - R(I - 1)) / 2!
450 NEXT I
460 MFXU = RHO * 2! * PI * SUMU
470 MFXD = RHO * 2! * PI * SUMD
480 POWER = (MFXD - MFXU) * 2! * PI * RPM / (60! * 1000!)
490 '
500 'Print the results
510 PRINT
520 PRINT USING "The shaft torque is ##.## N·m"; MFXD - MFXU
530 PRINT USING "The power input is ##.## KW"; POWER
```

```
**************************************************
** This program computes the change in rate of  **
** axial-direction angular momentum and power   **
** input for problem 5.79 using the trapezoidal **
** rule applied to unequal intervals.           **
**************************************************

The shaft torque is  4.79 N·m
The power input is  1.20 KW
```

5.80

5.80 Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity, V_θ, of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\underbrace{z_{in} - z_{out}}_{0,\text{ neglect}}) + w_{\substack{shaft\\ net\ in}}$$

or

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} + w_{\substack{shaft\\ net\ in}} \qquad (1)$$

The shaft work in, $w_{\substack{shaft\\ net\ in}}$ can be obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft\\ net\ in}} = U_{out} V_{\theta,out} \qquad (2)$$

Combining Eqs. 1 and 2 leads to

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} + U_{out} V_{\theta,out}$$

or

$$\text{loss} = -\frac{(0.4\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)} + \left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

and

$$\text{loss} = \underline{\underline{9800}}\ \frac{ft\cdot lb}{slug}$$

As was done in Example 5.24, we calculate rotor efficiency from

$$\text{rotor efficiency} = \frac{w_{\substack{shaft\\ net\ in}} - \text{loss}}{w_{\substack{shaft\\ net\ in}}} = \frac{U_{out}V_{\theta,out} - \text{loss}}{U_{out}V_{\theta,0}}$$

$$\text{rotor efficiency} = \frac{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) - 9800\ \frac{ft\cdot lb}{slug}}{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)} = \underline{\underline{0.71}}$$

5.81

5.81 Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

The analysis of Example 5.27 is applicable to solving this problem. Using Eq. 6 of Example 5.27 we obtain

$$loss = U_2 V_{\theta,2} - \frac{\text{actual total pressure rise across impeller}}{\rho}$$

However,

$$U_2 = r_2 \omega = \frac{(60\ mm)}{(2)(1000\ \frac{mm}{m})} \frac{(1800\ \frac{rev}{min})(2\pi\ \frac{rad}{rev})}{(60\ \frac{s}{min})} = 5.66\ \frac{m}{s}$$

Thus

$$loss = (5.66\ \frac{m}{s})(10\ \frac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) - \left(45 \times 10^3\ \frac{N}{m^2}\right)\frac{1}{\left(999\ \frac{kg}{m^3}\right)}$$

$$loss = \underline{\underline{11.6}}\ \frac{N \cdot m}{kg}$$

From Eq. 5 of Example 5.27 we obtain

$$\eta = \frac{\frac{\text{actual total pressure rise across impeller}}{\rho}}{U_2 V_{\theta,2}}$$

or

$$\eta = \frac{\left[\frac{(45 \times 10^3\ \frac{N}{m^2})}{(999\ \frac{kg}{m^3})}\right]}{(5.66\ \frac{m}{s})(10\ \frac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)} = \underline{\underline{0.796}}$$

5.82

5.82 Water enters an axial-flow turbine rotor with an absolute velocity tangential component, V_θ, of 15 ft/s. The corresponding blade velocity, U, is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

To determine the efficiency of the turbine we use

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} \qquad (1)$$

The actual work out, $w_{\substack{shaft\\ net\ out}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft\\ net\ out}} = -w_{\substack{shaft\\ net\ in}} = U_{in} V_{\theta, in} \qquad (2)$$

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$loss = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + \underbrace{\cancel{g(z_{in} - z_{out})}}_{\text{neglect}} + w_{\substack{shaft\\ net\ in}} \qquad (3)$$

Combining Eqs. 2 and 3 we obtain

$$loss = \frac{p_{0,in} - p_{0,out}}{\rho} - U_{in} V_{\theta, in} \qquad (4)$$

Combining Eqs. 1, 2 and 4 we obtain

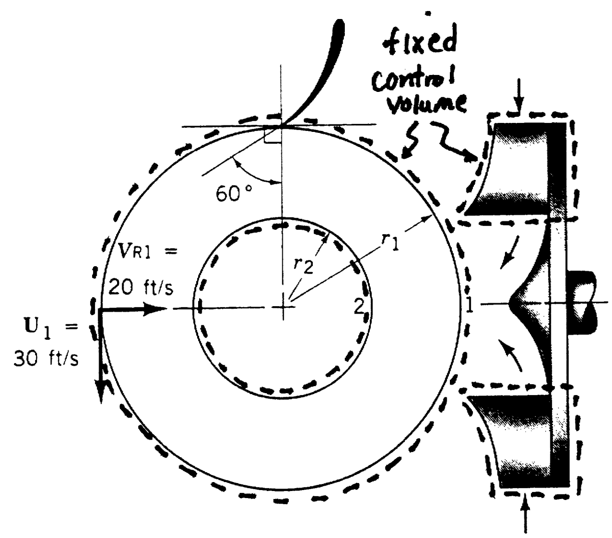
$$\eta = \frac{U_{in} V_{\theta, in}}{U_{in} V_{\theta, in} + loss} = \frac{U_{in} V_{\theta, in}}{\dfrac{p_{0,in} - p_{0,out}}{\rho}} = \frac{\left(50 \frac{ft}{s}\right)\left(15 \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\dfrac{(12\ psi)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)}}$$

and

$$\eta = \underline{\underline{0.842}}$$

5.83

5.83 An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.



■ FIGURE P5.83

An analysis like the one of Example 5.28 would be appropriate for solving this problem. Since a turbine is involved in this problem, $w_{\substack{shaft \\ net\ in}} = -w_{\substack{shaft \\ net\ out}}$ and from Eq. 1 of Example 5.28 we can conclude that

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - w_{\substack{shaft \\ net\ out}}$$

However from Eq. 5.54 we see that

$$w_{shaft} = w_{\substack{shaft \\ net\ in}} = -U_1 V_{\theta,1} = -w_{\substack{shaft \\ net\ out}}$$

and thus

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - U_1 V_{\theta,1} \qquad (1)$$

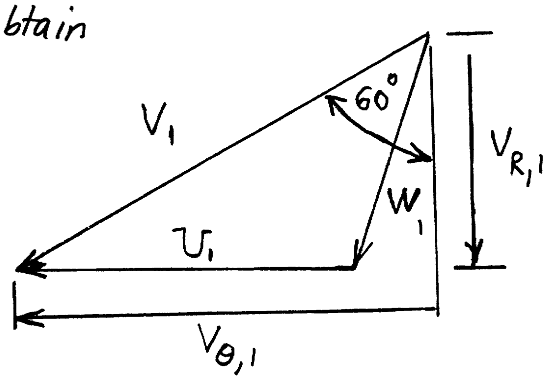
To determine the value of $V_{\theta,1}$ we examine the velocity triangle for the flow entering the rotor that is sketched below.

From the velocity triangle we obtain

$$V_{\theta,1} = V_{R,1} \tan 60^\circ$$

or

$$V_{\theta,1} = \left(20 \frac{ft}{s}\right) \tan 60^\circ = 34.64 \frac{ft}{s}$$



(Con't)

From Eq. 1 we obtain

$$loss = \frac{\left(16\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)} - \left(30\,\frac{ft}{s}\right)\left(34.64\,\frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{148}}\ \frac{ft\cdot lb}{slug}$$

From Eq. 5.82, we can conclude that

$$w_{\substack{shaft\\net\ out}} + loss = \frac{\text{stagnation pressure drop across the rotor}}{\rho}$$

or in other words, the stagnation pressure drop across the rotor results in shaft work and loss of available energy.

Thus a meaningful efficiency is

$$\eta = \frac{w_{\substack{shaft\\net\ out}}}{\left(\frac{\text{stagnation pressure drop across the rotor}}{\rho}\right)}$$

or

$$\eta = \frac{\left(30\,\frac{ft}{s}\right)\left(34.64\,\frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{\frac{\left(16\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)}} = \underline{\underline{0.875}}$$

5.84 An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82). Thus,

$$loss = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + \underbrace{g(\cancel{z_1 - z_2})}_{\text{neglect}} + w_{\substack{shaft \\ net\ in}} \qquad (1)$$

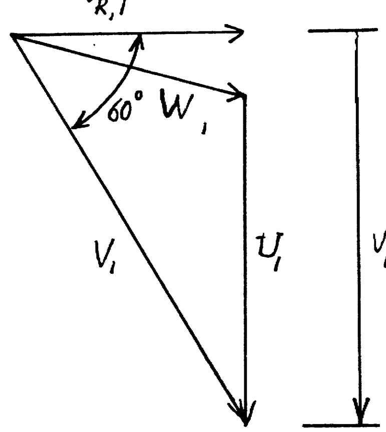
The shaft work, $w_{\substack{shaft \\ net\ in}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft \\ net\ in}} = -U_1 V_{\theta,1} = -w_{\substack{shaft \\ net\ out}} \qquad (2)$$

and combining Eqs. 1 and 2 yields

$$loss = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} - U_1 V_{\theta,1} \qquad (3)$$

To determine V_1 and $V_{\theta,1}$ we construct the velocity triangle sketched below.



(con't)

With the velocity triangle we conclude that

$$V_1 = \frac{\left(20\ \frac{ft}{s}\right)}{\cos 60^\circ} = 40\ \frac{ft}{s}$$

and

$$V_{\theta,1} = V_1 \sin 60^\circ = \left(40\ \frac{ft}{s}\right)\sin 60^\circ = 34.64\ \frac{ft}{s}$$

Since the flow leaving the rotor is radial, then

$$V_2 = V_{R,2} = 20\ \frac{ft}{s}$$

From Eq. 3 we obtain

$$\text{loss} = \frac{\left(0.01\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)} + \frac{\left[\left(40\ \frac{ft}{s}\right)^2 - \left(20\ \frac{ft}{s}\right)^2\right]\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{2} - \left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$\text{loss} = \underline{\underline{166}}\ \frac{ft\cdot lb}{slug}$$

The efficiency may be obtained with

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} = \frac{U_1 V_{\theta,1}}{U_1 V_{\theta,1} + \text{loss}}$$

or

$$\eta = \frac{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right) + 166\ \frac{ft\cdot lb}{slug}} = \underline{\underline{0.86}}$$

5.85

5.85 How much available energy is lost during the process shown in Video V5.7?

All of the potential energy lost in moving from the top of the toy to the bottom.

5.86

What is the size of the head loss that is needed to raise the temperature of water by 1°F?

This is similar to Example 5.22. From Eq. 5.78 we have

$$\check{u}_{out} - \check{u}_{in} - \overset{\text{0 assumed}}{\cancel{q_{\substack{net\\in}}}} = loss$$

However

$$\check{u}_{out} - \check{u}_{in} = c\,(T_{out} - T_{in})$$

Thus

$$loss = c\,(T_{out} - T_{in})$$

or

$$g h_L = c\,(T_{out} - T_{in})$$

and

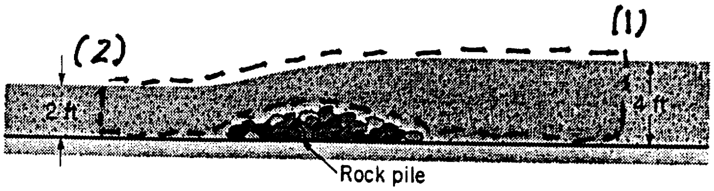
$$h_L = \frac{c}{g}\,(T_{out} - T_{in})$$

$$h_L = \left(1\ \frac{Btu}{lb_m\,{}^\circ F}\right)(1\,{}^\circ F)\left(32.2\ \frac{lb_m \cdot ft}{lb \cdot s^2}\right)\left(778\ \frac{ft \cdot lb}{Btu}\right)$$

$$h_L = \underline{\underline{778}}\ ft$$

5.87

5.87 A 100-ft-wide river with a flowrate of 2400 ft³/s flows over a rock pile as shown in Fig. P5.87. Determine the direction of flow and the head loss associated with the flow across the rock pile.



■ FIGURE P5.87

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\overbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}^{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \overbrace{\cancel{h_s}}^{0,\ \text{no shaft work}} - h_L$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(4\ ft)(100\ ft)} = 6\ \frac{ft}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(2\ ft)(100\ ft)} = 12\ \frac{ft}{s}$$

So

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} - \frac{\left(12\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} + 4\ ft - 2\ ft$$

$h_L = 0.32$ ft and since h_L is positive, our assumed right to left flow is correct

5.88

5.88 If a $\frac{3}{4}$-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.88, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.

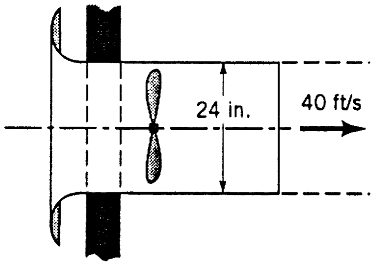


FIGURE P5.88

(a) The solution to this part of the problem is like Example 5.24. We use

$$\eta = \frac{w_{shaft} - loss}{w_{shaft}}$$

to calculate the fan efficiency.

We use the energy equation (Eq. 5.82) for flow through the control volume sketched above to calculate the loss as follows

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft\\ net\ in}} - loss$$

But $p_2 = p_1$ and $z_2 = z_1$; $V_1 \simeq 0$; $w_{\substack{shaft\\ net\ in}} = \frac{hp}{\dot{m}}$

Also $\dot{m} = \rho A_2 V_2 = \frac{p}{RT}\frac{\pi d_2^2}{4} V_2$

So

$$loss = w_{\substack{shaft\\ net\ in}} - \frac{V_2^2}{2} = \frac{hp}{\frac{p}{RT}\left(\frac{\pi d_2^2}{4}\right)V_2} - \frac{V_2^2}{2}$$

$$loss = \frac{\left(\frac{3}{4}\,hp\right)\left(550\,\frac{ft.lb}{s.hp}\right)}{\left\{\frac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\pi\left[\frac{24\,in.}{\left(12\,\frac{in.}{ft}\right)}\right]^2\left(40\,\frac{ft}{s}\right)}{\left(53.3\,\frac{ft.lb}{lbm.^{\circ}R}\right)(530^{\circ}R)\;4}\right\}} - \frac{\left(40\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{lbm\cdot ft}{lb.\,s^2}\right)}$$

(con't)

$$\text{loss} = 44\ \frac{ft\cdot lb}{lbm} - 24.8\ \frac{ft\cdot lb}{lbm} = 19.2\ \frac{ft\cdot lb}{lbm}$$

So

$$\eta = \frac{44\ \frac{ft\cdot lb}{lbm} - 19.2\ \frac{ft\cdot lb}{lbm}}{44\ \frac{ft\cdot lb}{lbm}} = \underline{\underline{0.56}}$$

For

(b) We use the horizontal component of the linear momentum equation to evaluate the anchoring force required to hold the fan in place

$$F_{AX} = V_2\, \dot{m}$$

From part (a)

$$\dot{m} = \frac{P}{RT}\frac{\pi d_2^2}{4} V_2 = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\pi\left(\frac{24\ in.}{12\ \frac{in.}{ft}}\right)^2\left(40\ \frac{ft}{s}\right)}{\left(53.3\ \frac{ft\cdot lb}{lbm\,{}^\circ R}\right)(530\,{}^\circ R)\ 4}$$

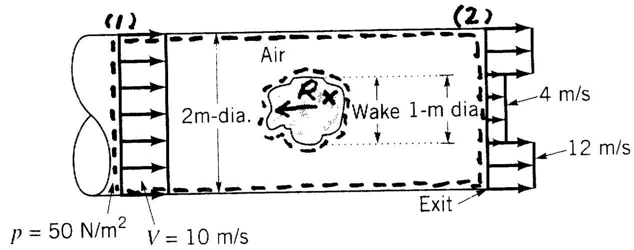
$$\dot{m} = 9.41\ \frac{lbm}{s}$$

So

$$F_{AX} = \frac{\left(40\ \frac{ft}{s}\right)\left(9.41\ \frac{lbm}{s}\right)}{\left(32.2\ \frac{lbm\cdot ft}{lb\cdot s^2}\right)} = \underline{\underline{11.7\ lb}}$$

5.89

Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.89. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m^2, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.



(a) To determine the loss suffered by a fluid particle as it flows from (1) to a location in the wake at (2) we apply the energy equation (Eq. 5.84) to that particle flow to get:

$$\cancelto{0}{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancelto{0}{\frac{w_{shaft\ net\ in}}{g}} - h_L \qquad (1)$$

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

and

$$h_L = \frac{\left(50\ \frac{N}{m^2}\right)}{\left(12\ \frac{N}{m^3}\right)} + \frac{\left(10\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} - \frac{\left(4\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} = \underline{\underline{8.45\ m}}$$

To determine the head loss associated with the entire flow across the object we use the non-uniform flow energy equation (Eq. 5.89) for flow from (1) to (2) through the control volume shown in the sketch to get:

$$\cancelto{0}{\frac{p_2}{\gamma}} + \frac{\alpha_2 \bar{V}_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{\alpha_1 \bar{V}_1^2}{2g} + \cancel{z_1} + \cancelto{0}{\frac{w_{shaft\ net\ in}}{g}} - h_L \qquad (2)$$

From Eq. 5.86 we get:

$$\frac{\alpha \bar{V}^2}{2g} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\dot{m}} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\rho \bar{V} A}$$

Eq. (2) becomes

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{\int_{A_2} \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{(\rho V A)_{4\frac{m}{s}} + (\rho V A)_{12\frac{m}{s}}}$$

(con't)

5.89 (Con't)

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{1}{2g}\left[\frac{V^3_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^3_{3\frac{m}{s}} A_{3\frac{m}{s}}}{V_{4\frac{m}{s}} A_{4\frac{m}{s}} + V_{12\frac{m}{s}} A_{12\frac{m}{s}}}\right]$$

and

$$h_L = \frac{\left(50\,\frac{N}{m^2}\right)}{\left(12\,\frac{N}{m^3}\right)} + \frac{\left(10\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} - \frac{1}{2\left(9.81\,\frac{m}{s^2}\right)}\left\{\frac{\left(12\,\frac{m}{s}\right)^3 \pi\left[\frac{(2m)^2-(1m)^2}{4}\right] + \left(4\,\frac{m}{s}\right)^3 \frac{\pi(1m)^2}{4}}{\left(4\,\frac{m}{s}\right)\frac{\pi(1m)^2}{4} + \left(12\,\frac{m}{s}\right)\pi\left[\frac{(2m)^2-(1m)^2}{4}\right]}\right\}$$

$$h_L = \underline{\underline{2.58\,m}}$$

(b) To determine the force that the air puts on the object, R_x, we use the horizontal component of the linear momentum equation to get:

$$-\rho V_1^2 A_1 + \rho V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + \rho V^2_{4\frac{m}{s}} A_{4\frac{m}{s}} = p_1 A_1 - R_x$$

and thus

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho\left(V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^2_{4\frac{m}{s}} A_{4\frac{m}{s}}\right)$$

So

$$R_x = \left(50\,\frac{N}{m^2}\right)\frac{\pi(2m)^2}{4} + \left(1.23\,\frac{kg}{m^3}\right)\left(10\,\frac{m}{s}\right)^2 \frac{\pi(2m)^2}{4}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$
$$-1.23\,\frac{kg}{m^3}\left\{\left(12\,\frac{m}{s}\right)^2 \frac{\pi\left[(2m)^2-(1m)^2\right]}{4} + \left(4\,\frac{m}{s}\right)^2 \frac{\pi(1m)^2}{4}\right\}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$

and

$$R_x = \underline{\underline{110\,N}}$$

5.90

5.90 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.90. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

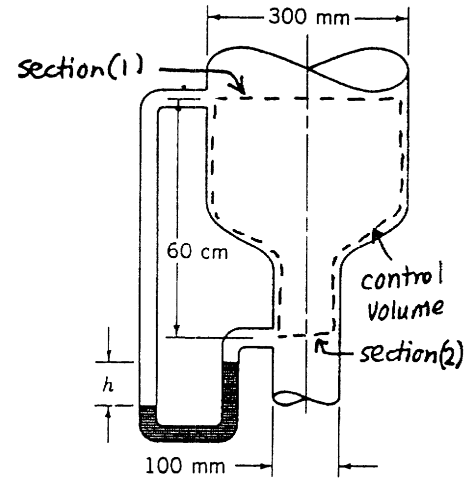


FIGURE P5.90

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either V_1 or V_2 we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \cancel{w_{\text{shaft net in}}}^{\,0} - \cancel{\text{loss}}^{\,\text{neglect}} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{p_1 - p_2}{\rho}$ we use the manometer equation from section 2.6 to obtain

$$\frac{p_1 - p_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

$$V_2 = \sqrt{\frac{(2)(9.81\ \frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29\ \frac{m}{s}$$

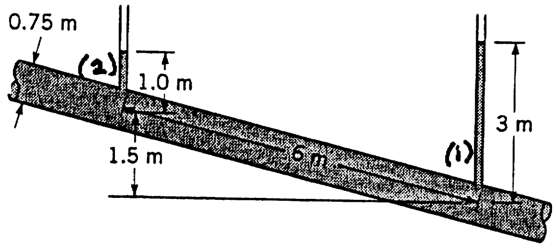
and from Eq. 1 we have

$$\therefore \quad Q = \frac{\pi (0.1 m)^2}{4}\left(5.29\ \frac{m}{s}\right) = \underline{\underline{0.042\ \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.

5.91

5.91 An incompressible liquid flows steadily along the pipe shown in Fig. P5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.



■ FIGURE P5.91

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_s}^{\,0} - h_\ell$$

Thus

$$h_\ell = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = 0.5\,m$$

and since $h_\ell > 0$, the assumed direction of flow is correct.

5.92

5.92 A siphon is used to draw water at 70°F from a large container as indicated in Fig. P5.92. The inside diameter of the siphon line is 1 in. and the pipe centerline rises 3 ft above the essentially constant water level in the tank. Show that by varying the length of the siphon below the water level, h, the rate of flow through the siphon can be changed. Assuming frictionless flow, determine the maximum flowrate possible through the siphon. The limiting condition is the occurrence of cavitation in the siphon. Will the actual maximum flow be more or less than the frictionless value? Explain.

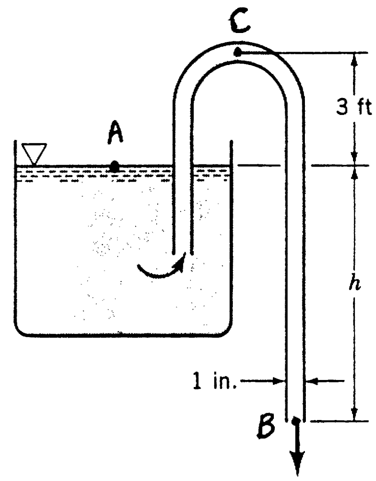


FIGURE P5.92

The flowrate, Q, can be determined with

$$Q = A_B V_B = \frac{\pi D_B^2}{4} V_B \qquad (1)$$

To obtain V_B we apply the energy equation (Eq. 5.82) between points A and B in the sketch above to obtain

$$\cancelto{0}{\frac{p_B}{\rho}} + \frac{V_B^2}{2} + gz_B = \cancelto{0}{\frac{p_A}{\rho}} + \cancelto{0}{\frac{V_A^2}{2}} + gz_A + \cancelto{0}{w_{\substack{\text{shaft}\\ \text{net in}}}} - \text{loss} \qquad (2)$$

or

$$\frac{V_B^2}{2} = g(z_A - z_B) - \text{loss}$$

and

$$V_B = \sqrt{2\left[g(h) - \text{loss}\right]} \qquad (3)$$

With Eq. 3 we conclude that as h varies, so does V_B and thus Q. For no loss, the maximum flow will occur when the pressure at point C is just equal to the vapor pressure of water at 0°C. We apply the energy equation (Eq. 5.82) between points A and C to get

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_A}{\rho} + \cancelto{0}{\frac{V_A^2}{2}} + gz_A + \cancelto{0}{w_{\substack{\text{shaft}\\ \text{net in}}}} - \cancelto{0}{\text{loss}} \qquad (4)$$

Using absolute instead of gage pressures we obtain with Eq. 4

$$V_C = \sqrt{2g(z_A - z_C) + \frac{p_A - p_C}{\rho}}$$

or

$$V_C = \sqrt{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(-3\,\text{ft})\left(0.3048\,\frac{\text{m}}{\text{ft}}\right) + \frac{\left(101{,}000\,\frac{\text{N}}{\text{m}^2} - 1228\,\frac{\text{N}}{\text{m}^2}\right)}{\left(999.7\,\frac{\text{kg}}{\text{m}^3}\right)\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = 9.048\,\frac{\text{m}}{\text{s}}$$

(Con't)

5.92 (con't)

Since

$$Q = A_c V_c = \frac{\pi D_c^2}{4} V_c$$

we have for the maximum flowrate through the siphon,

$$Q = \frac{\pi (1 \text{ in.})^2}{4\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)} \left(0.3048 \frac{\text{m}}{\text{ft}}\right)^2 \left(9.048 \frac{\text{m}}{\text{s}}\right) = \underline{\underline{4.58 \times 10^{-3} \frac{\text{m}^3}{\text{s}}}}$$

With Eqs. 3 and 4 we conclude that any loss would act to lower the value of V in the siphon and thus make the actual maximum flowrate with friction less than the maximum flowrate without friction.

5.93

5.93 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.93. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

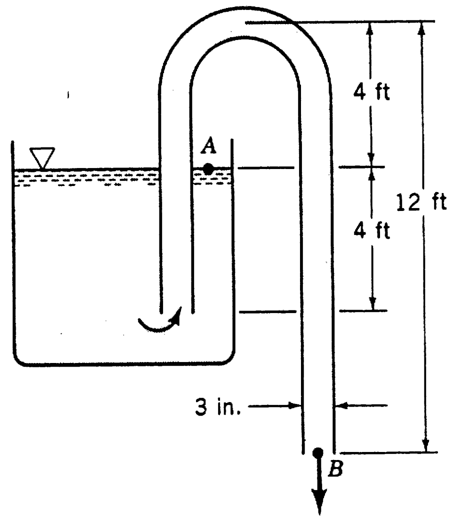


FIGURE P5.93

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancelto{0}{\frac{p_B}{\rho}} + \frac{V_B^2}{2} + g z_B = \cancelto{0}{\frac{p_A}{\rho}} + \cancelto{0}{\frac{V_A^2}{2}} + g z_A + w_{\substack{shaft \\ net\ in}} - loss$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8 \frac{V^2}{2}$$

Thus

$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{\left(32.2 \frac{ft}{s^2}\right)(8\ ft)}{0.9}} = 16.9 \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi (3 in.)^2}{4\left(144 \frac{in.^2}{ft^2}\right)} \left(16.9 \frac{ft}{s}\right) = \underline{\underline{0.83}} \frac{ft^3}{s}$$

5.95

5.95 Water flows through a vertical pipe as is indicated in Fig. P5.95. Is the flow up or down in the pipe? Explain.

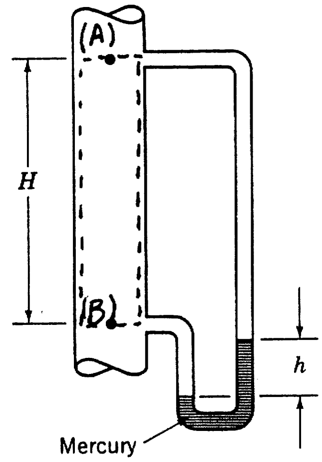


FIGURE P5.95

The control volume shown in the sketch above is used. For steady, incompressible flow downward from (A) to (B) we obtain from Eq. 5.79

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A - {}_A\text{loss}_B$$

From conservation of mass we conclude that

$$V_A = V_B$$

Thus from Eq. 1

$$_A\text{loss}_B = gH + \frac{p_A - p_B}{\rho}$$

However the manometer equation (see Section 2.6) yields

$$\frac{p_A - p_B}{\rho} = g\left[h\left(1 - SG_{Hg}\right) - H\right]$$

and

$$_A\text{loss}_B = gh\left(1 - SG_{Hg}\right)$$

which is a negative quantity since $SG_{Hg} = 13.6$. A negative loss is not physically possible so the flow must be upward from B to A. For <u>upward flow</u> the above analysis leads to

$$_B\text{loss}_A = gh\left(SG_{Hg} - 1\right)$$

which is positive and therefore physically reasonable.

5.96

5.96 A fire hose nozzle is designed to deliver water that will rise 40 m vertically. Calculate the stagnation pressure required at the nozzle inlet if: **(a)** no loss is assumed; **(b)** a loss of 30 N·m/kg is assumed.

To determine the stagnation pressure at the nozzle inlet we assume that the stagnation pressure at the nozzle exit is the same as the stagnation pressure at the nozzle inlet and we apply the energy equation (Eq. 5.84) to the flow from the nozzle exit to the maximum elevation of the water flow to get

$$p_0 = \gamma \Delta z + \rho(\text{loss}) \qquad (1)$$

(a) For no loss, Eq. 1 leads to

$$p_0 = \left(9.80\ \frac{kN}{m^3}\right)(40\ m) = 392\ \frac{kN}{m^2} = \underline{\underline{392\ kPa}}$$

(b) For loss $= 30\ \frac{N \cdot m}{kg}$, Eq. 1 yields

$$p_0 = \left(9.80\ \frac{kN}{m^3}\right)(40\ m) + \left(999\ \frac{kg}{m^3}\right)\left(30\ \frac{N \cdot m}{kg}\right)\frac{1}{\left(1000\ \frac{N}{kN}\right)} = \underline{\underline{422\ kPa}}$$

5.97

5.97 For the 180° elbow and nozzle flow shown in Fig. P5.97, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

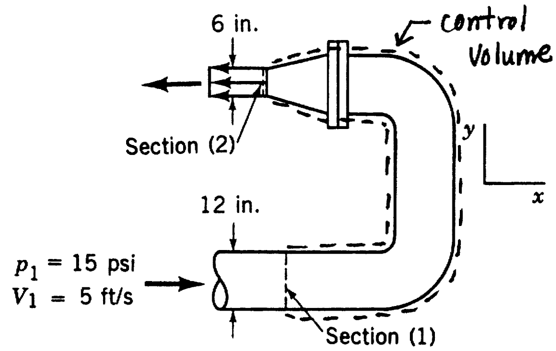


FIGURE P5.97

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.

From the conservation of mass principle we conclude that

$$V_2 = \frac{V_1 A_1}{A_2} = V_1\left(\frac{D_1^2}{D_2^2}\right)$$

Thus

$$_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

or

$$_1loss_2 = \frac{\left(15\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)} + \frac{\left(5\ \frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\ in.}{6\ in.}\right)^4\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$_1loss_2 = \underline{\underline{926}}\ \frac{ft.lb}{slug}$$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$_2loss_a = \frac{V_2^2}{2}$$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.

Thus

$$_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\ \frac{ft}{s}\right)^2\left(\frac{12\ in.}{6\ in.}\right)^4\left(\frac{1\ lb}{slug.\ \frac{ft}{s^2}}\right)}{2}$$

$$_2loss_A = \underline{\underline{200}}\ \frac{ft.lb}{slug}$$

5.98

5.98 An automobile engine will work best when the back pressure at the exhaust manifold, engine block interface is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

We apply the energy equation (Eq. 5.83) to the flow from the engine block, exhaust manifold interface to the exhaust system exit to get

$$p_{in} = p_{out} + \frac{\rho V_{out}^2}{2} - \frac{\rho V_{in}^2}{2} + \rho(\text{loss}) \qquad (1)$$

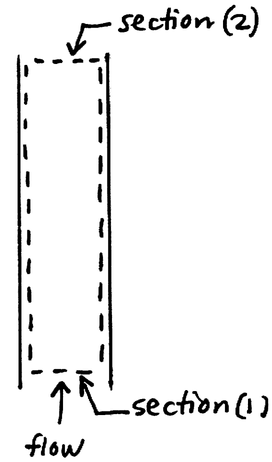
With Eq. 1 we see that reduction of loss in the exhaust system results in a lower value of p_{in} and thus the engine back pressure. Losses in the exhaust system could be reduced by eliminati g major loss components such as the catalytic converter and the muffler as is often done in race cars. However, noise and emissions legislation limits the extent to which this kind of loss reduction can occur in conventional vehicles. Some loss reduction can also occur by configuring the exhaust system piping with few bends and appropriate area distributions. However, requirements often leads to bends and turns in the piping and costs limit the extend of optimizing area distributions.

5.99

5.99 Water flows vertically upward in a circular cross section pipe. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R - r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).



For determining loss we use the energy equation for non-uniform flows, Eq. 5.87. Thus,

$$loss = \frac{p_1 - p_2}{\rho} + \frac{\alpha_1 \bar{V}_1^2 - \alpha_2 \bar{V}_2^2}{2} + g(z_1 - z_2) \qquad (1)$$

From conservation of mass (Eq. 5.13) we have

$$\bar{V}_1 = \bar{V}_2$$

Also, with Eq. 5.86 for the kinetic energy coefficient, α, we have

$$\alpha_1 = 1.0$$

since the velocity profile at section (1) is uniform. At section (2) we solve Eq. 5.86 (see solution for problem 5.125(c)) and obtain

$$\alpha_2 = 1.06$$

Thus, Eq. 1 yields

$$loss = \frac{p_1 - p_2}{\rho} - 0.06 \frac{\bar{V}_1^2}{2} + g(z_1 - z_2)$$

5.100

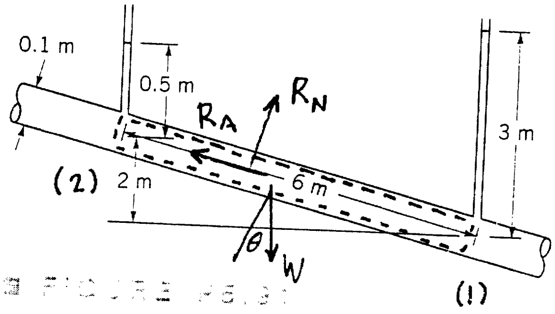
5.100 Discuss the causes of loss of available energy in a fluid flow.

Some causes of loss of available energy in a fluid flow

1. friction
2. heat transfer across a temperature difference
3. flow across a shock

5.101

Consider the flow shown in Fig. P5.91. If the flowing fluid is water, determine the axial (along the pipe) and normal (perpendicular to the pipe) components of force that the pipe puts on the fluid in the 6-m section shown.



Using the control volume shown by broken lines we apply the axial and normal components of the linear momentum equation to get:

$\Sigma F_N = 0$ since there is no momentum flow in the normal direction

and $\Sigma F_A = 0$ since the flow is assumed fully developed and the net amount of axial direction momentum flow out of the CV is zero

So

$$R_N - W\cos\theta = 0 \quad \text{or} \quad R_N = W\cos\theta$$

Now $W = \gamma \forall = \gamma A \ell = \gamma \frac{\pi d^2}{4}\ell = \left(9.8\times10^3 \frac{N}{m^3}\right)\frac{\pi (0.1m)^2(6m)}{4} = 462\,N$

and $\theta = \sin^{-1}\frac{2}{6} = 19.5°$

Then $R_N = (462N)(\cos 19.5°) = \underline{\underline{436}}\,N$

For the axial direction

$$p_2 A_2 + R_A + W\sin\theta - p_1 A_1 = 0 \quad \text{or}$$

$$R_A = p_1A_1 - p_2A_2 + W\sin\theta = (p_1 - p_2)A + W\sin\theta$$

From the manometer readings

$$p_2 = \gamma h_2 \quad \text{and} \quad p_1 = \gamma h_1$$

thus

$$p_1 - p_2 = \gamma(h_1 - h_2)$$

and

$$R_A = \gamma(h_1 - h_2)A - (W\sin 19.5°)$$

$$R_A = \left(9.8\times10^3\frac{N}{m^3}\right)(3.0m - 0.5m)\frac{\pi(0.1m)^2}{4} - (462N)(\sin 19.5°)$$

$$R_A = \underline{\underline{38}}\,N$$

5.102

5.102 Water flows steadily down the inclined pipe as indicated in Fig. P5.102. Determine the following: **(a)** The difference in pressure $p_1 - p_2$. **(b)** The loss per unit mass between sections (1) and (2). **(c)** The net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

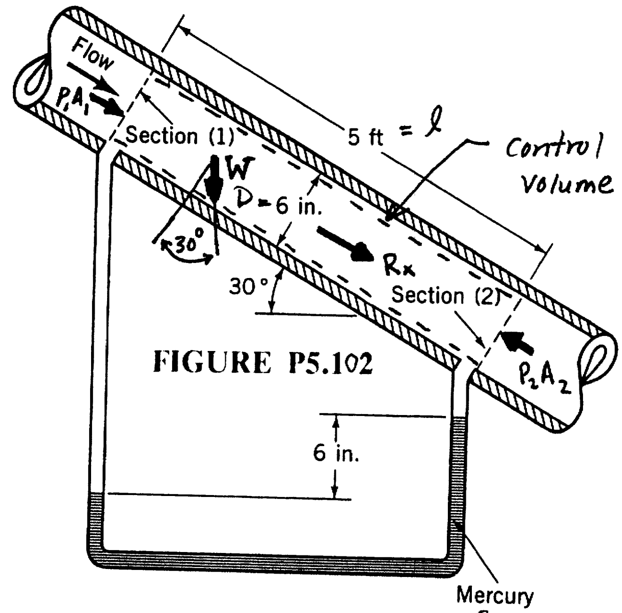


FIGURE P5.102

(a) The difference in pressure, $P_1 - P_2$, may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$P_1 - P_2 = -\gamma_{H_2O}\left[(5\,ft)\sin 30^\circ + \frac{(6\,in.)}{\left(12\,\frac{in.}{ft}\right)}\right] + \gamma_{Hg}\frac{(6\,in.)}{\left(12\,\frac{in.}{ft}\right)}$$

or

$$P_1 - P_2 = -\left(62.4\,\frac{lb}{ft^3}\right)\left[(5\,ft)\sin 30^\circ + (0.5\,ft)\right] + (13.6)\left(62.4\,\frac{lb}{ft^3}\right)(0.5\,ft) = 237\,\frac{lb}{ft^3}$$

and

$$P_1 - P_2 = 237\,\frac{lb}{ft^3}\,\frac{1}{\left(144\,\frac{in.^2}{ft^2}\right)} = \underline{\underline{1.65\,psi}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.79. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left(237\,\frac{lb}{ft^2}\right)\frac{1}{\left(1.94\,\frac{slugs}{ft^3}\right)} + \left(32.2\,\frac{ft}{s^2}\right)(5\,ft)(\sin 30^\circ)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{203\,\frac{ft\cdot lb}{slug}}}$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.22). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4}(P_1 - P_2) - \gamma\frac{\pi D^2}{4}(\ell)\sin 30^\circ = -\frac{\pi D^2}{4}\left[(P_1 - P_2) + \gamma\ell\sin 30^\circ\right]$$

or

$$R_x = -\frac{\pi}{4}\left(\frac{6\,in.}{12\,\frac{in.}{ft}}\right)^2\left[237\,\frac{lb}{ft^2} + \left(62.4\,\frac{lb}{ft^3}\right)(5\,ft)\sin 30^\circ\right]$$

and

$$R_x = -77.2\,lb = \underline{\underline{77.2\,lb \text{ opposite to flow direction.}}}$$

5.103

5.103 Water flows through a 2-ft-diameter pipe arranged horizontally in a circular arc as shown in Fig. P5.103. If the pipe discharges to the atmosphere (p = 14.7 psia), determine the x and y components of the resultant force exerted by the water on the piping between sections (1) and (2). The steady flowrate is 3000 ft^3/min. The loss in pressure due to fluid friction between sections (1) and (2) is 25 psi.

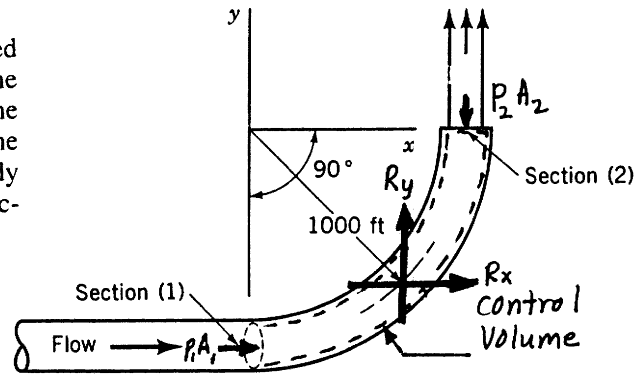


FIGURE P5.103

To determine the x and y components of the resultant force exerted by the water on the piping between section (1) and (2) we use the x and y components of the linear momentum equation (Eq 5.22). For the control volume containing the water in the pipe between section (1) and (2), Eq. 22 leads to

$$R_x = -p_1 A_1 - V_1 \rho Q = -p_1 \frac{\pi D_1^2}{4} - V_1 \rho Q \qquad (1)$$

and

$$R_y = p_2 A_2 + V_2 \rho Q \qquad (2)$$

The resultant force components in Eqs. 1 and 2 are exerted by the pipe on the water. The resultant force of water on pipe is equal in magnitude but opposite in direction.

To determine p_1 we use the energy equation, Eq. 5.83. Thus,

$$p_1 = \rho(\text{loss}) = 25 \text{ psi}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{\left(3000 \frac{ft^3}{min}\right)}{\frac{\pi (2 ft)^2}{4}\left(60 \frac{s}{min}\right)} = 15.92 \frac{ft}{s}$$

and

$$V_2 = V_1 = 15.92 \frac{ft}{s}.$$

With Eq. 1 we obtain

$$R_x = -(25 \text{ psi}) \frac{\pi (2 ft)^2}{4}\left(144 \frac{in.^2}{ft^2}\right) - \frac{\left(15.92 \frac{ft}{s}\right)\left(1.94 \frac{slugs}{ft^3}\right)\left(3000 \frac{ft^3}{min}\right)\left(\frac{1 \text{ lb}}{slug \cdot \frac{ft}{s^2}}\right)}{\left(60 \frac{s}{min}\right)}$$

or

$$R_x = -12{,}850 \text{ lb}$$

and the x direction component of the force exerted by the water on the pipe between sections (1) and (2) is +12,850 lb.

(con't)

5.103 (con't)

With Eq. 2 we obtain

$$R_y = \left(15.92\ \frac{ft}{s}\right)\left(1.94\ \frac{slugs}{ft^3}\right)\left(3000\ \frac{ft^3}{min}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(60\ \frac{s}{min}\right)} = 1540\ lb$$

and the y-direction component of the force exerted by the water on the pipe between sections (1) and (2) is $-$ 1540 lb.

5.104 When fluid flows through an abrupt expansion as indicated in Fig. P5.104, the loss in available energy across the expansion, loss $_{ex}$, is often expressed as

$$\text{loss}_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross section area upstream of expansion, A_2 = cross section area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

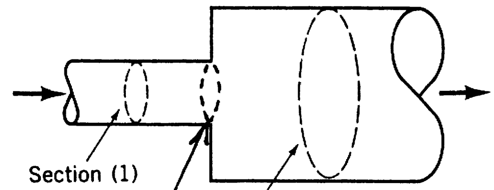


FIGURE P5.104

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + P_1 A_1 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + P_1 A_2 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, P_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.104 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\,\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

5.105

5.105 Near the downstream end of a river spillway, a hydraulic jump often forms, as illustrated in Fig. P5.105 and Video V10.5. The velocity of the channel flow is reduced abruptly across the jump. Using the conservation of mass and linear momentum principles, derive the following expression for h_2,

$$h_2 = -\frac{h_1}{2} + \sqrt{\left(\frac{h_1}{2}\right)^2 + \frac{2V_1^2 h_1}{g}}$$

The loss of available energy across the jump can also be determined if energy conservation is considered. Derive the loss expression

$$\text{jump loss} = \frac{g(h_2 - h_1)^3}{4h_1h_2}$$

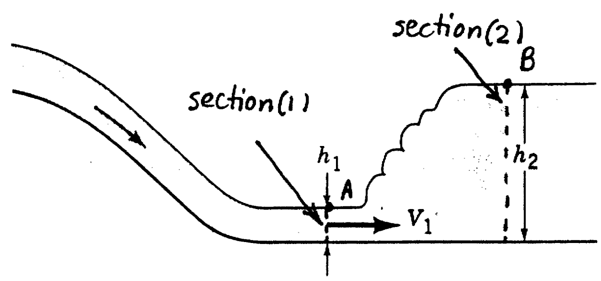


FIGURE P5.105

Application of the horizontal component of the linear momentum equation (Eq. 5.22) to the water in the control volume from section (1) to section (2) leads to, for unit width of flow,

$$-R_x + \frac{\gamma h_1^2}{2} - \frac{\gamma h_2^2}{2} = -V_1 \rho h_1 V_1 + V_2 \rho h_2 V_2 \qquad (1)$$

Since the jump occurs over a short distance we drop R_x from Eq. 1. Also from conservation of mass (Eq. 5.13) we obtain

$$V_2 = V_1 \frac{h_1}{h_2} \qquad (2)$$

(con't)

5.105 (Con't)

Combining Eqs. 1 and 2 we obtain

$$1-\left(\frac{h_2}{h_1}\right)^2 = \frac{2V_1^2}{gh_1}\left[\frac{1}{\left(\frac{h_2}{h_1}\right)} - 1\right]$$

or

$$\left(1-\frac{h_2}{h_1}\right)\left(1+\frac{h_2}{h_1}\right) = \frac{2V_1^2}{gh_1}\frac{\left(1-\frac{h_2}{h_1}\right)}{\left(\frac{h_2}{h_1}\right)}$$

and

$$\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - \frac{2V_1^2}{gh_1} = 0 \qquad (3)$$

From Eq. 3 we obtain

$$\frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1+\frac{8V_1^2}{gh_1}}}{2}$$

or

$$h_2 = -\frac{h_1}{2} + \sqrt{\left(\frac{h_1}{2}\right)^2 + \frac{2V_1^2}{g}h_1}$$

The other quadratic root is not meaningful.

Application of the energy equation (Eq. 5.82) to the flow from point A to point B shown on the sketch above leads to

$$\text{jump loss} = \frac{V_A^2 - V_B^2}{2} + g(z_A - z_B) = \frac{V_1^2 - V_2^2}{2} + g(h_1 - h_2) \qquad (4)$$

Combining Eqs. 2, 3 and 4 we obtain

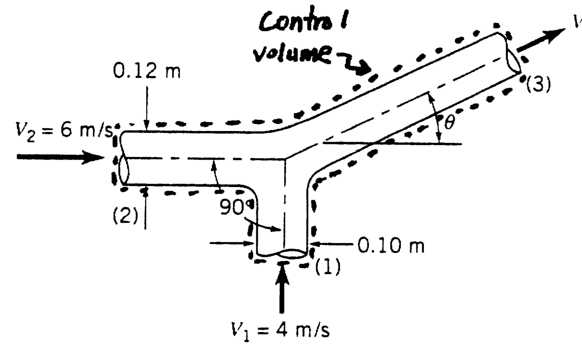
$$\text{jump loss} = \frac{gh_1}{4}\left[\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1}\right]\left[1-\left(\frac{h_1}{h_2}\right)^2\right] + g(h_1 - h_2)$$

or

$$\text{jump loss} = \frac{g}{4h_2h_1}(h_2 - h_1)^3$$

5.106

5.106 Two water jets collide and form one homogeneous jet as shown in Fig. P5.106. **(a)** Determine the speed, V, and direction, θ, of the combined jet. **(b)** Determine the head loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.106

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4 \frac{m}{s}\right)^2 \frac{\pi (0.1 m)^2}{4}}{\left(6 \frac{m}{s}\right)^2 \frac{\pi (0.12 m)^2}{4}} = 0.3086$$

so

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \left(V_1 A_1 + V_2 A_2\right)} = \frac{V_2^2 d_2^2}{\cos\theta \left(V_1 d_1^2 + V_2 d_2^2\right)}$$

Thus

$$V_3 = \frac{\left(6 \frac{m}{s}\right)^2 (0.12 m)^2}{(\cos 17.2^\circ)\left[\left(4 \frac{m}{s}\right)(0.1 m)^2 + \left(6 \frac{m}{s}\right)(0.12 m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29}} \ \frac{m}{s}$$

(con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1 + \frac{V_1^2}{2}\right)\dot{m}_1 - \left(\check{u}_2 + \frac{V_2^2}{2}\right)\dot{m}_2 + \left(\check{u}_3 + \frac{V_3^2}{2}\right)\dot{m}_3 = 0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1 - \dot{m}_2 + \dot{m}_3 = 0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3 - \check{u}_1) + \dot{m}_2(\check{u}_3 - \check{u}_2) = \dot{m}_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \dot{m}_2\left(\frac{V_2^2 - V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss} = \rho V_1 A_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \rho V_2 A_2\left(\frac{V_2^2 - V_3^2}{2}\right)$$

or

$$\text{rate of loss} = \frac{\rho\pi}{4}\left[d_1^2 V_1\left(\frac{V_1^2 - V_3^2}{2}\right) + d_2^2 V_2\left(\frac{V_2^2 - V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss} = \frac{\left(999\,\frac{\text{kg}}{\text{m}^3}\right)(3.14)\left(1\,\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)}{4}\left\{(0.10\,\text{m})^2\left(4\,\frac{\text{m}}{\text{s}}\right)\left[\frac{\left(4\,\frac{\text{m}}{\text{s}}\right)^2 - \left(4.29\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right] + (0.12\,\text{m})^2\left(6\,\frac{\text{m}}{\text{s}}\right)\left[\frac{\left(6\,\frac{\text{m}}{\text{s}}\right)^2 - \left(4.29\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right]\right\}$$

and

$$\text{rate of loss} = \underline{\underline{558}}\ \frac{\text{N}\cdot\text{m}}{\text{s}}$$

5.107

5.107 The pumper truck shown in Fig. P5.107 is to deliver 1.5 ft^3/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in. diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.

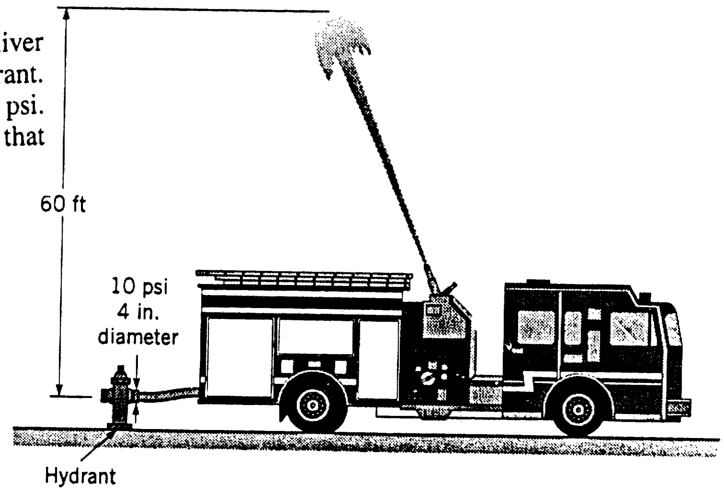


FIGURE P5.107

To solve this problem we first use the energy equation (Eq. 5.84) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get h_s or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

$$\cancelto{0}{\frac{p_2}{\rho}} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_s - \cancelto{0}{h_L}$$

$$h_s = z_2 - z_1 - \frac{p_1}{\rho} - \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{\left(1.5\ \frac{ft^3}{s}\right)(4)}{\pi\left(\frac{4\ in.}{12\ \frac{in.}{ft}}\right)^2} = 17.2\ \frac{ft}{s}$$

$$h_s = 60\ ft - \frac{\left(10\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} - \frac{\left(17.2\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)}$$

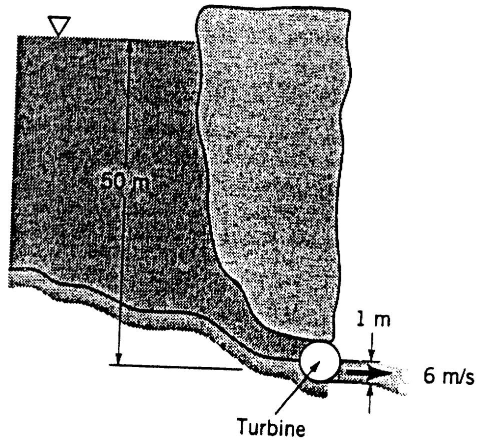
$$h_s = 32.3\ ft$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma Q h_s = \left(62.4\ \frac{lb}{ft^3}\right)\left(1.5\ \frac{ft^3}{s}\right)\frac{(32.2\ ft)}{\left(550\ \frac{ft.lb}{s.hp}\right)}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{5.48}}\ hp$$

5.108

5.108 What is the maximum possible power output of the hydroelectric turbine shown in Fig. P5.108?



■ FIGURE P5.108

For flow from section (1) to section (2), Eq. 5.82 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft\\net\ in}} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$, $\quad w_{\substack{shaft\\net\ in}} = -w_{\substack{shaft\\net\ out}}$ Eq. 1 can be expressed as

$$w_{\substack{shaft\\net\ out}} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when loss = 0.

Thus

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\, w_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\left[g(z_1 - z_2) - \frac{V_2^2}{2}\right]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = \left(999\,\frac{kg}{m^3}\right)\left(6\,\frac{m}{s}\right)\frac{\pi (1\,m)^2}{4} = 4710\,\frac{kg}{s}$$

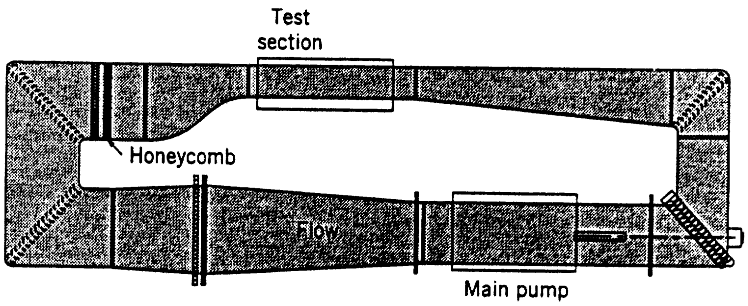
and

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \left(4710\,\frac{kg}{s}\right)\left[\left(9.81\,\frac{m}{s^2}\right)(50\,m) - \frac{\left(6\,\frac{m}{s}\right)^2}{2}\right]\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \underline{\underline{2.23\times10^6\,\frac{N\cdot m}{s}}} = \underline{\underline{2.23\times10^6\,W}} = \underline{\underline{2.23\,MW}}$$

5.109

5.109 Estimate the power in hp needed to drive the main pump of the large-scale water tunnel shown in Fig. P5.109. The design condition head loss is specified as 14 ft of water for a flowrate of 4900 ft^3/s.



■ FIGURE P5.109

The solution of this problem is similar to the one of Example 8.6. Looping around the water tunnel from any cross section of the tunnel back to the same cross section we conclude using the energy equation, Eq. 5.84

$$h_s = h_L$$

Then from Eq. 5.85

$$\dot{W}_{\substack{shaft\\net\ in}} = h_s \gamma Q$$

So

$$\dot{W}_{\substack{shaft\\net\ in}} = (14\ ft)\left(62.4\ \frac{lb}{ft^3}\right)\left(4900\ \frac{ft^3}{s}\right)\frac{1}{550\ \frac{ft \cdot lb}{s \cdot hp}}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{7780}}\ hp$$

5.110

5.110 Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3-ft-inside diameter inlet pipe as indicated in Fig. P5.110. The turbine discharge pipe has a 4-ft-inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10 in. Hg vacuum. If the turbine develops 2500 hp, determine the rate of loss of available energy between sections (1) and (2).

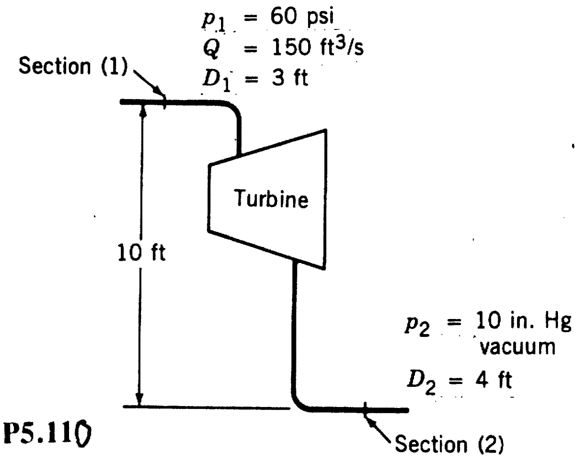


FIGURE P5.110

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{p_1-p_2}{\rho}\right)+g(z_1-z_2)+\frac{(V_1^2-V_2^2)}{2}\right]-\dot{W}_{\substack{\text{shaft}\\ \text{net out}}} \qquad (1)$$

From given data

$$p_2 = \frac{(-10\text{ in. Hg})(13.6)\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)}{\left(12\,\frac{\text{in.}}{\text{ft}}\right)}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right) = -708\,\frac{\text{lb}}{\text{ft}^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\pi(3\text{ft})^2} = 21.22\,\frac{\text{ft}}{\text{s}}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\,\frac{\text{ft}}{\text{s}}\right)\frac{(3\text{ft})^2}{(4\text{ft})^2} = 11.94\,\frac{\text{ft}}{\text{s}}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\left(550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)}\left\{\frac{\left(60\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)+\left(708\,\frac{\text{lb}}{\text{ft}^2}\right)}{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)}\right.$$

$$\left.+\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(10\text{ ft})\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)+\frac{\left[\left(21.22\,\frac{\text{ft}}{\text{s}}\right)^2-\left(11.94\,\frac{\text{ft}}{\text{s}}\right)^2\right]}{2}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right\}$$

$$-2500\text{hp}$$

or

$$\text{power loss} = \underline{\underline{301}}\text{ hp}$$

5.111

5.111 A steam turbine receives steam having a static pressure, p_1, of 400 psia, an enthalpy, $\check{h}_1$, of 1407 Btu/lbm, and a velocity, V_1, of 100 ft/s. The steam leaves the turbine as a mixture of vapor and liquid having an enthalpy, $\check{h}_2$, of 1098 Btu/lbm, a pressure, p_2, of 2 psia, and a velocity, V_2, of 200 ft/s. If the flow through the turbine is essentially adiabatic and the change in elevation of the steam is negligible, calculate: **(a)** the actual work output per unit mass of steam; **(b)** the efficiency of the turbine if the ideal work output is 467 Btu/lbm.

(a) This problem is similar to Example 5.21. From Eq. 5.69 we obtain

$$w_{\substack{\text{shaft}\\ \text{net out}}} = \check{h}_1 - \check{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

or

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 1407\,\frac{\text{Btu}}{\text{lbm}} - 1098\,\frac{\text{Btu}}{\text{lbm}} + \frac{\left(100\,\frac{\text{ft}}{\text{s}}\right)^2 - \left(200\,\frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)\left(778\,\frac{\text{ft}\cdot\text{lb}}{\text{Btu}}\right)}$$

and

$$w_{\substack{\text{shaft}\\ \text{net out}}} = \underline{\underline{308}}\,\frac{\text{Btu}}{\text{lbm}}$$

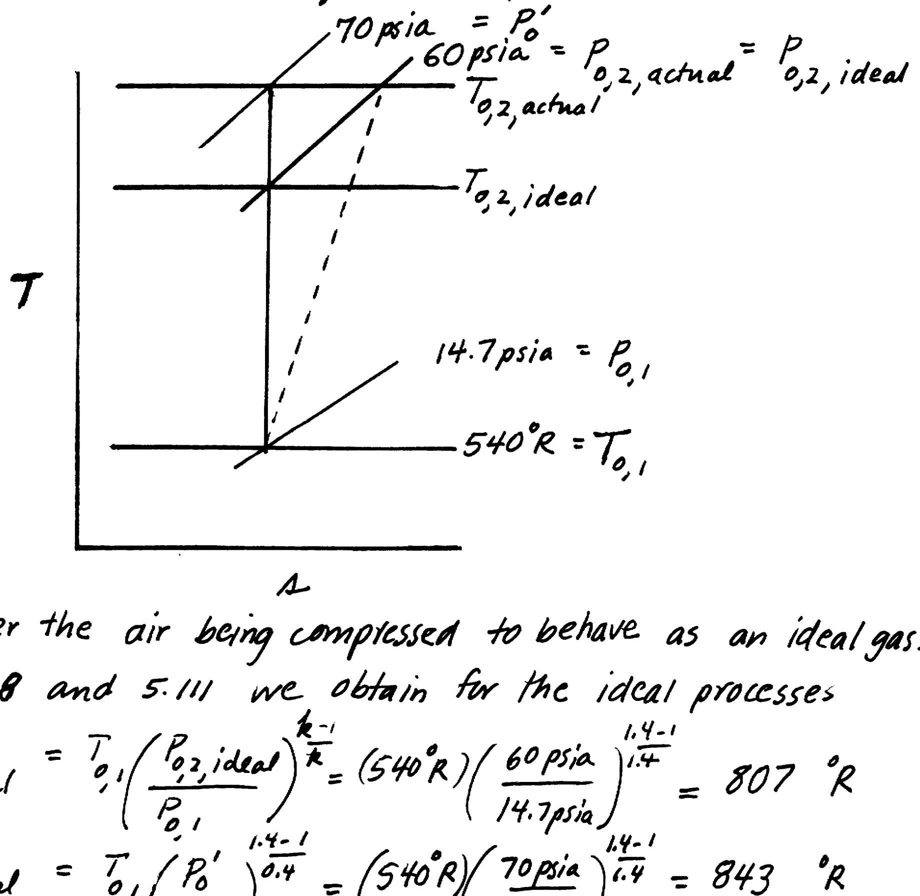
(b) A reasonable efficiency is the ratio of actual work output to ideal work output or

$$\eta = \frac{308\,\frac{\text{Btu}}{\text{lbm}}}{467\,\frac{\text{Btu}}{\text{lbm}}} \times 100 = \underline{\underline{66\%}}$$

5.112

5.112 A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, calculate the actual and ideal stagnation temperature rise through the compressor. Calculate the ratio of ideal to actual temperature rise to obtain efficiency.

We assume that the air compressor operates adiabatically. An ideal compression process is frictionless and adiabatic and thus according to Eq. 5.101, it is a constant entropy or isentropic process. With Eq. 5.101 we also conclude that an actual adiabatic compression process with friction must involve an entropy increase. On temperature - entropy coordinates, the ideal and actual compression processes appear as indicated in the sketch below. Also shown is the 10 psi loss in stagnation pressure due to friction.



We consider the air being compressed to behave as an ideal gas. Then from Eqs. 1.8 and 5.111 we obtain for the ideal processes

$$T_{0,2,ideal} = T_{0,1}\left(\frac{p_{0,2,ideal}}{p_{0,1}}\right)^{\frac{k-1}{k}} = (540°R)\left(\frac{60\ psia}{14.7\ psia}\right)^{\frac{1.4-1}{1.4}} = 807\ °R$$

and

$$T_{0,2,actual} = T_{0,1}\left(\frac{p_0'}{p_{0,1}}\right)^{\frac{1.4-1}{0.4}} = (540°R)\left(\frac{70\ psia}{14.7\ psia}\right)^{\frac{1.4-1}{1.4}} = 843\ °R$$

(con't)

5.112 (Con't)

Then

actual stagnation temperature rise $= T_{0,2,actual} - T_{0,1} = 843°R - 540°R = \underline{\underline{303°R}}$

and

ideal stagnation temperature rise $= T_{0,2,ideal} - T_{0,1} = 807°R - 540°R = \underline{\underline{267°R}}$

Also

$$\text{efficiency} = \frac{T_{0,2,ideal} - T_{0,1}}{T_{0,2,actual} - T_{0,1}} = \frac{267°R}{303°R} = \underline{\underline{0.88}}$$

5.114

5.114* Total head-rise values measured for air flowing across a fan are listed below as a function of volume flowrate.

Q (m^3/s)	Total Head Rise (mm H_2O)
0	79
0.14	79
0.28	76
0.42	67
0.57	65
0.71	70
0.85	76
0.99	79
1.13	75
1.27	64

Determine the flowrate that will result when this fan is connected to a piping system whose loss in total head is described by loss $= K_L Q^2$ when: **(a)** $K_L = 49$ mm $H_2O/(m^3/s)^2$; **(b)** $K_L = 91$ mm $H_2O/(m^3/s)^2$; **(c)** $K_L = 140$ mm $H_2O/(m^3/s)^2$.

The flowrate of the combination of a fan or pump and a connected piping system is determined by the intersection of the fan or pump head rise vs. volume flowrate curve and the system loss vs volume flowrate curve. To determine the flowrate resulting when the fan of this problem is connected to the three [(a), (b), and (c)] piping systems, the intersections of the piping system loss vs. Q curves and the fan total head rise vs. Q curve fit were determined with the computer program listed on the following pages. A polynomial least squares curve fit of the tabulated data is used. The intersection points were determined with the Newton – Raphson technique.

(con't)

```
100 CLS
110 PRINT "*******************************************************"
120 PRINT "** This program determines the intersection of the   **"
130 PRINT "** head loss and head rise curves for problem 5.114 **"
140 PRINT "** A least square fit polynomial of the form:       **"
150 PRINT "**     y = d0 + d1*x + d2*x^2 + d3*x^3 + ...        **"
160 PRINT "** is used to describe the head rise data.          **"
170 PRINT "*******************************************************"
180 PRINT :
190 DIM B(21), D(21), S(21), X(101), W(101), Y(101), F(101)
200 DIM ERRF(101), PJ(101), PJM1(101), YBAR(101)
210 '
220 'intialize the variables
230 NTERMS = 8: NTERMSAVE = NTERMS
240 NPOINT = 10
250 INPUT "Enter the head loss coefficient"; KL
260 PRINT
270 FOR I = 1 TO NPOINT
280 READ X(I), Y(I)
290 W(I) = 1
300 F(I) = Y(I)
310 NEXT I
320 DATA 0.00, 79.0, 0.14, 79.0, 0.28, 76.0, 0.42, 67.0
330 DATA 0.57, 65.0, 0.71, 70.0, 0.85, 76.0, 0.99, 79.0
340 DATA 1.13, 75.0, 1.27, 64.0
350 PRINT "The polynomial fit to the head rise data is of order";
360 PRINT USING "##"; NTERMS - 1
370 '
380 'determine the polynomial coefficients
390 PRINT "The coefficients of the polynomial are:"
400 FOR I = 1 TO NPOINT
410 F(I) = F(I) - D(NTERMS + 1) * X(I) ^ (NTERMS)
420 NEXT I
430 FOR J = 1 TO NTERMS
440 B(J) = 0
450 D(J) = 0
460 S(J) = 0
470 NEXT J
480 C(1) = 0
490 FOR I = 1 TO NPOINT
500 D(1) = D(1) + F(I) * W(I)
510 B(1) = B(1) + X(I) * W(I)
520 S(1) = S(1) + W(I)
530 NEXT I
540 D(1) = D(1) / S(1)
550 FOR I = 1 TO NPOINT
560 ERRF(I) = F(I) - D(1)
570 NEXT I
580 IF NTERMS = 1 THEN GOTO 850
590 B(1) = B(1) / S(1)
600 FOR I = 1 TO NPOINT
610 PJM1(I) = 1
620 PJ(I) = X(I) - B(1)
630 NEXT I
```

(Con't)

```
640 FOR J = 2 TO NTERMS
650 FOR I = 1 TO NPOINT
660 P = PJ(I) * W(I)
670 D(J) = D(J) + ERRF(I) * P
680 P = P * PJ(I)
690 B(J) = B(J) + X(I) * P
700 S(J) = S(J) + P
710 NEXT I
720 D(J) = D(J) / S(J)
730 FOR I = 1 TO NPOINT
740 ERRF(I) = ERRF(I) - D(J) * P(I)
750 NEXT I
760 IF J = NTERMS THEN GOTO 850
770 B(J) = B(J) / S(J)
780 C(J) = S(J) / S(J - 1)
790 FOR I = 1 TO NPOINT
800 P = PJ(I)
810 PJ(I) = (X(I) - B(J)) * PJ(I) - C(J) * PJM1(I)
820 PJM1(I) = P
830 NEXT I
840 NEXT J
850 PRINT USING "  d# = +#.####^^^^"; NTERMS - 1; D(NTERMS)
860 NTERMS = NTERMS - 1
870 IF NTERMS > 0 THEN GOTO 400
880 '
890 'determine the intersection using the
900 'Newton-Raphson method
910 QNP = 1!
920 QN = QNP
930 F = 0!
940 FP = 0!
950 FOR I = 1 TO NTERMSAVE STEP 1
960 F = F + D(I) * QN ^ (I - 1)
970 NEXT I
980 FOR I = 2 TO NTERMSAVE STEP 1
990 FP = FP + I * D(I) * QN ^ (I - 1)
1000 NEXT I
1010 F = KL * QN ^ 2 - F
1020 FP = 2! * KL * QN - FP
1030 QNP = QN - F / FP
1040 IF (ABS(QNP - QN) > .0001) THEN GOTO 920
1050 F = 0!
1060 FOR I = 1 TO NTERMSAVE STEP 1
1070 F = F + D(I) * QN ^ (I - 1)
1080 NEXT I
1090 PRINT
1100 PRINT USING "Head loss coefficient:  ###.##"; KL
1110 PRINT USING "Volume flow rate ----: ##.### m^3/s"; QN
1120 PRINT USING "Operating head ------: ### mm of H2O"; F
```

(con't)

```
*****************************************************
** This program determines the intersection of the  **
** head loss and head rise curves for problem 5.114 **
** A least square fit polynomial of the form:       **
**    y = d0 + d1*x + d2*x^2 + d3*x^3 + ...         **
** is used to describe the head rise data.          **
*****************************************************
```

(c)

```
Enter the head loss coefficient? 140.

The polynomial fit to the head rise data is of order 7
The coefficients of the polynomial are:
  d7 = -1.7369E+03
  d6 = +8.2623E+03
  d5 = -1.5353E+04
  d4 = +1.3788E+04
  d3 = -5.9543E+03
  d2 = +1.0551E+03
  d1 = -6.2329E+01
  d0 = +7.8983E+01

Head loss coefficient:  140.00
Volume flow rate ----:  0.705 m^3/s
Operating head ------:  70 mm of H2O
```

```
*****************************************************
** This program determines the intersection of the  **
** head loss and head rise curves for problem 5.114 **
** A least square fit polynomial of the form:       **
**    y = d0 + d1*x + d2*x^2 + d3*x^3 + ...         **
** is used to describe the head rise data.          **
*****************************************************
```

(b)

```
Enter the head loss coefficient? 91.

The polynomial fit to the head rise data is of order 7
The coefficients of the polynomial are:
  d7 = -1.7369E+03
  d6 = +8.2623E+03
  d5 = -1.5353E+04
  d4 = +1.3788E+04
  d3 = -5.9543E+03
  d2 = +1.0551E+03
  d1 = -6.2329E+01
  d0 = +7.8983E+01

Head loss coefficient:   91.00
Volume flow rate ----:  0.928 m^3/s
Operating head ------:  78 mm of H2O
```

(con't)

5.114 (con't)

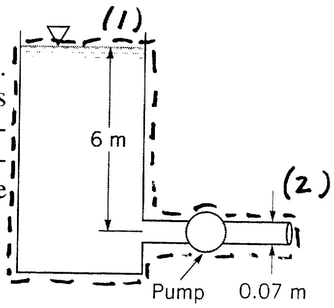
```
******************************************************
** This program determines the intersection of the  **
** head loss and head rise curves for problem 5.114 **
** A least square fit polynomial of the form:       **
**    y = d0 + d1*x + d2*x^2 + d3*x^3 + ...         **
** is used to describe the head rise data.          **
******************************************************
```

(a)

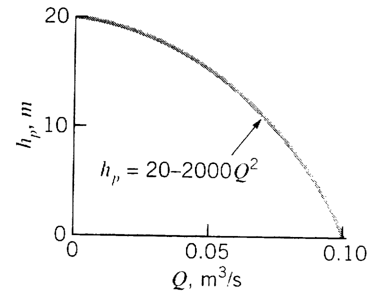
```
Enter the head loss coefficient? 49.

The polynomial fit to the head rise data is of order 7
The coefficients of the polynomial are:
  d7 = -1.7369E+03
  d6 = +8.2623E+03
  d5 = -1.5353E+04
  d4 = +1.3788E+04
  d3 = -5.9543E+03
  d2 = +1.0551E+03
  d1 = -6.2329E+01
  d0 = +7.8983E+01

Head loss coefficient:   49.00
Volume flow rate ----:  1.203 m^3/s
Operating head ------:  71 mm of H2O
```

5.115

Water is pumped from the tank shown in Fig. P5.115*a*. The head loss is known to be 1.2 $V^2/2g$, where V is the average velocity in the pipe. According to the pump manufacturer, the relationship between the pump head and the flowrate is as shown in Fig. P5.115*b*: $h_p = 20 - 2000\,Q^2$, where h_p is in meters and Q is in m^3/s. Determine the flowrate, Q.



(*a*)



(*b*)

We want to know the flowrate Q.

For the control volume shown, application of the energy equation (Eq. 5.84) yields:

$$\cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_s - h_\ell \qquad (1)$$

However

$$h_\ell = 1.2\,\frac{V_2^2}{2g} \qquad (2)$$

and

$$h_s = h_p = 20 - 2000\,Q^2 \qquad (3)$$

Since $Q = V_2 A_2$ we have from eq. 2

$$h_\ell = \frac{1.2}{2g}\left(\frac{Q}{A}\right)^2 \qquad (4)$$

and combining Eqs. (1), (3) and (4) we get:

$$\frac{1}{2g}\left(\frac{Q}{A_2}\right)^2 + z_2 = z_1 + 20 - 2000\,Q^2 - \frac{1.2}{2g}\left(\frac{Q}{A_2}\right)^2 \qquad (5)$$

or

$$Q^2\left(\frac{1}{2gA_2^2} + \frac{1.2}{2gA_2^2} + 2000\right) = z_1 - z_2 + 20$$

So

$$Q = \left[\frac{z_1 - z_2 + 20}{\frac{1}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + \frac{1.2}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + 2000}\right]^{\frac{1}{2}} = \left[\frac{6\,m + 20\,m}{\frac{1}{(2)\left(9.81\frac{m}{s^2}\right)\left[\frac{\pi(0.07\,m)^2}{4}\right]^2} + \frac{1.2}{2\left(9.81\frac{m}{s}\right)\left[\frac{\pi(0.07\,m)^2}{4}\right]^2} + 2000}\right]^{\frac{1}{2}}$$

$$Q = \underline{\underline{0.052\,\frac{m^3}{s}}}$$

5.116

5.116 Water flows by gravity from one lake to another as sketched in Fig. P5.116 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.

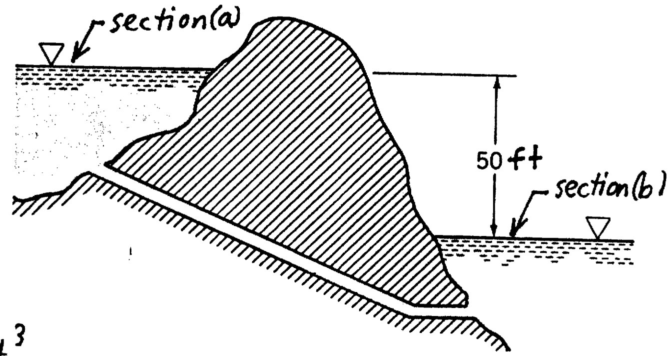


FIGURE P5.116

$$Q = \frac{80\ \frac{gal}{min}}{\left(60\ \frac{s}{min}\right)\left(7.48\ \frac{gal}{ft^3}\right)} = 0.178\ \frac{ft^3}{s}$$

For the flow from section (a) to section (b) Eq. 5.82 leads to

$$loss = g(z_a - z_b) = \left(32.2\ \frac{ft}{s^2}\right)(50 ft)\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{1610}}\ \frac{ft\,lb}{slug}$$

For pumped flow from section b to section a Eq. 5.82 yields

$$\dot{W}_{\substack{shaft\\ net\ in}} = \rho Q\left[g(z_a - z_b) + loss\right] = \left(1.94\ \frac{slugs}{ft^3}\right)\left(0.178\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50 ft)\left(\frac{1\ lb}{slug\ \frac{ft}{s^2}}\right) + 1610\ \frac{ft.lb}{slug}\right]$$

or

$$\dot{W}_{\substack{shaft\\ net\ in}} = \underline{\underline{1110}}\ \frac{ft.lb}{s} = \underline{\underline{2.02\ hp}}$$

5.117

5.117 A $\frac{3}{4}$-hp motor is required by an air ventilating fan to produce a 24-in.-diameter stream of air having a uniform speed of 40 ft/s. Determine the aerodynamic efficiency of the fan.

The aerodynamic efficiency of the fan, η, is

$$\eta = \frac{ideal\ power\ required}{actual\ power\ required}$$

The actual shaft power required, $\dot{W}_{actual}$, is 0.75 hp.

The ideal shaft power required, $\dot{W}_{ideal}$, is obtained from Eq. 5.82 for flow without loss across the fan. Thus

$$\dot{W}_{ideal} = \dot{m}\frac{V_{out}^2}{2} = \rho A_{out} V_{out}\frac{V_{out}^2}{2} = \rho\frac{\pi D_{out}^2}{4}\frac{V_{out}^3}{2} = \left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\frac{\pi(2 ft)^2}{4} \times \frac{\left(40\ \frac{ft}{s}\right)^3}{2}\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft \cdot lb}{s \cdot hp}}\right)$$

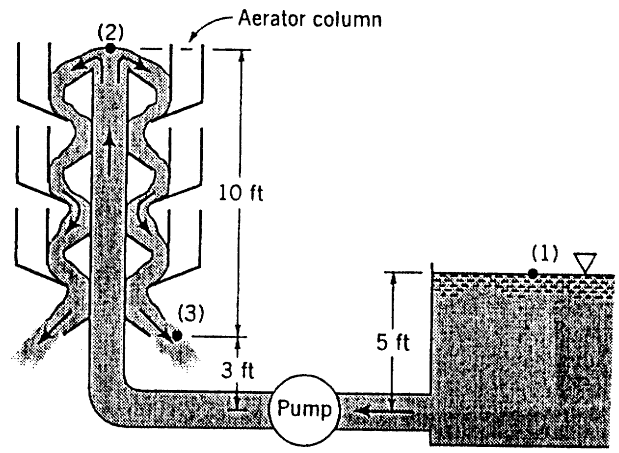
or $\dot{W}_{ideal} = 0.435\ hp$

Then

$$\eta = \frac{0.435\ hp}{0.75\ hp} = \underline{\underline{0.58}}$$

5.118

5.118 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in Video V5.8 and Fig. P5.118 at a rate of 3.0 ft^3/s. (a) Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. (b) Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



■ FIGURE P5.118

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_p = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_p - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_p = 0$ gives

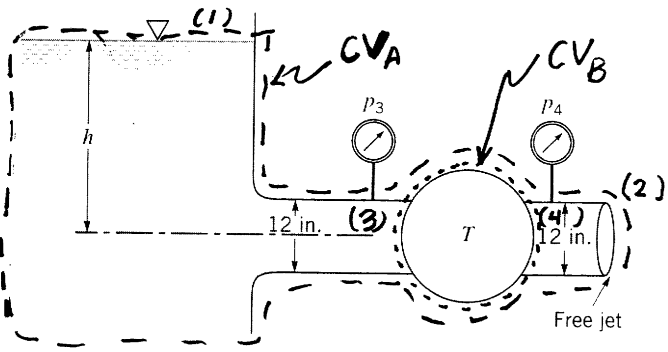
$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

or

$$h_L = \underline{\underline{9.94\,ft}}$$

5.119

The turbine shown in Fig. P5.119 develops 100 hp when the flowrate of water is 20 ft^3/s. If all losses are negligible, determine **(a)** the elevation h, **(b)** the pressure difference across the turbine, and **(c)** the flowrate expected if the turbine were removed.



(a) Using control volume A and the energy equation (Eq. 5.84) we get:

$$\cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 + h_s - \cancel{h_\ell}^{0} \qquad (1)$$

For a turbine, $h_T = -h_s$ and from Eq. 5.85 we get:

$$h_T = \frac{\dot{W}_{\substack{shaft\\ net\,out}}}{\gamma Q} = \frac{(100\text{ hp})\left(550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)}{\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(20\,\frac{\text{ft}^3}{\text{s}}\right)} = 44.1\text{ ft}$$

Since $Q = AV$ we have

$$V_2 = \frac{Q}{A_2} = \frac{20\,\frac{\text{ft}^3}{\text{s}}}{\frac{\pi d_2^2}{4}} = \frac{20\,\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}\left(\frac{12\text{ in.}}{12\,\frac{\text{in.}}{\text{ft}}}\right)^2} = 25.5\,\frac{\text{ft}}{\text{s}}$$

Then from eq. 1

$$z_1 - z_2 = h = \frac{V_2^2}{2g} - h_s = \frac{\left(25.5\,\frac{\text{ft}}{\text{s}}\right)^2}{2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)} + 44.1\text{ ft} = \underline{\underline{54.1}}\text{ ft}$$

(b) For control volume B the energy equation yields

$$p_3 - p_4 = \gamma h_T = \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(44.1\text{ ft}) = \underline{\underline{2.75}}\,\frac{\text{lb}}{\text{ft}^2}$$

(c) Since $Q = VA = V_2A_2$, if we knew value of V_2 with the turbine removed, we could calculate Q with the turbine removed. Without the turbine, Eq. (1) reduces to

$$\frac{V_2^2}{2g} = z_1 - z_2 = h$$

and

$$V_2 = \sqrt{2gh} = \sqrt{2\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(54.1\text{ ft})} = 59\,\frac{\text{ft}}{\text{s}}$$

Thus

$$Q_{\substack{w/o\\ turbine}} = \frac{\pi d_2^2}{4} V_2 = \frac{\pi}{4}\left(\frac{12\text{ in.}}{12\,\frac{\text{in.}}{\text{ft}}}\right)^2\left(59\,\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{46.3}}\,\frac{\text{ft}^3}{\text{s}}$$

5.120

5.120 A liquid enters a fluid machine at sections (1) and (2) and leaves at section (3) as shown in Fig. P5.120. The density of the fluid is constant at 2 slugs/ft³. All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. 5.120, determine the amount of shaft power involved.

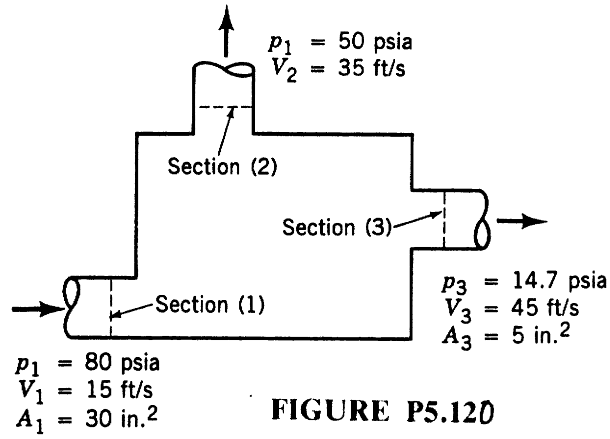


FIGURE P5.120

For the frictionless and adiabatic flow through this fluid machine Eqs. 5.64, 5.65 and 5.76 lead to

$$\dot{W}_{\substack{shaft\\net\ in}} = \dot{m}_3\left(\frac{p_3}{\rho}+\frac{V_3^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho}+\frac{V_1^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}\right) \quad (1)$$

since

$$\dot{m}_1\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = (\dot{m}_2+\dot{m}_3)\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = \dot{m}_2(\check{u}_1-\check{u}_2) + \dot{m}_3(\check{u}_1-\check{u}_2) = 0$$

At section (3)

$$\dot{m}_3 = \rho A_3 V_3 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{5\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(45\ \frac{ft}{s}\right) = 3.125\ \frac{slugs}{s}$$

At section (1)

$$\dot{m}_1 = \rho A_1 V_1 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{30\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(15\ \frac{ft}{s}\right) = 6.25\ \frac{slugs}{s}$$

From conservation of mass

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 6.25\ \frac{slugs}{s} - 3.125\ \frac{slugs}{s} = 3.125\ \frac{slugs}{s}$$

With Eq. 1 we obtain

$$\dot{W}_{\substack{shaft\\net\ in}} = \left\{\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(45\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]\right.$$
$$-\left(6.25\ \frac{slugs}{s}\right)\left[\frac{\left(80\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(15\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]$$
$$\left.+\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(50\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(35\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]\right\}\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{-31.1\ hp}}$$ the net shaft power is out

5.121

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss in available energy associated with 2.5 ft³/s being pumped from sections (1) to (2) is $61\overline{V}^2/2$ where $\overline{V}$ is the average velocity of water in the 8-in.-inside diameter piping involved. Determine the amount of shaft power required.

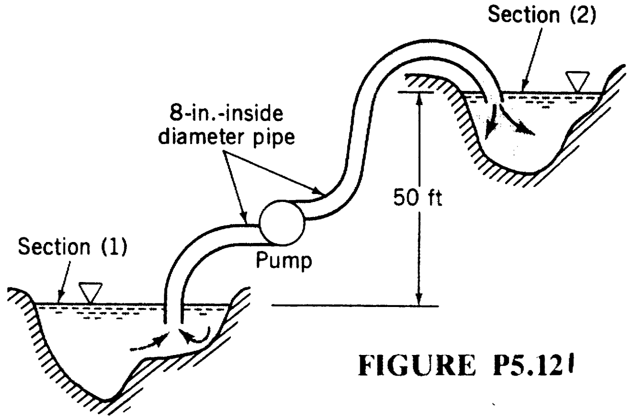


FIGURE P5.121

For the flow from section (1) to section (2) Eq. 5.82 leads to

$$\dot{W}_{\substack{shaft\\net\ in}} = \rho Q\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2 - z_1) + 61\frac{\overline{V}^2}{2}\right] \quad (1)$$

From the volume flowrate we obtain

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{\left(2.5\ \frac{ft^3}{s}\right)}{\frac{\pi}{4}\left(\frac{8\ in.}{12\ \frac{in.}{ft}}\right)^2} = 7.162\ \frac{ft}{s}$$

Thus, from Eq. 1

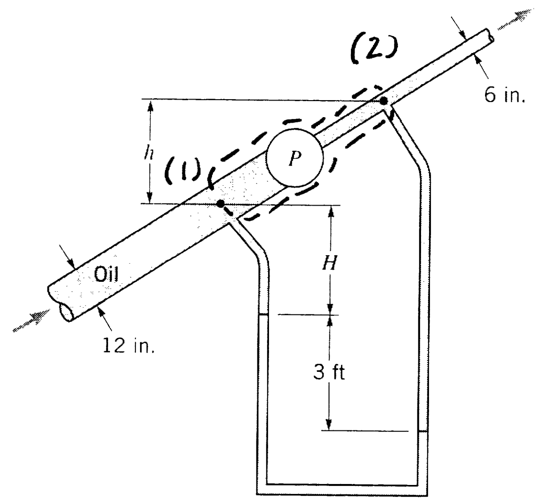
$$\dot{W}_{\substack{shaft\\net\ in}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\ ft) + \frac{(61)\left(7.162\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{28\ hp}}$$

5.122

Oil ($SG = 0.88$) flows in an inclined pipe at a rate of 5 ft^3/s as shown in Fig. P5.122. If the differential reading in the mercury manometer is 3 ft, calculate the power that the pump supplies to the oil if head losses are negligible.



Using the control volume shown and the energy equation (Eq. 5.84) we get:

$$\frac{p_2}{\gamma_{oil}} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - h_\ell \quad (1)$$

The power supplied by the pump to the oil is, from Eq. 5.85:

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma_{oil} Q h_s = SG_{oil}\, \gamma_{H_2O}\, Q h_s \quad (2)$$

Since $V = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}}$ we get

$$V_1 = \frac{5\,\frac{ft^3}{s}}{\frac{\pi}{4}(1\,ft)^2} = 6.37\,\frac{ft}{s} \quad \text{and} \quad V_2 = \frac{5\,\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{1}{2}ft)^2} = 25.5\,\frac{ft}{s}$$

From the manometer equation (see section 2.6) we get:

$$p_1 + \gamma_{oil} H + 3\gamma_{Hg} - (3 + H + h)\gamma_{oil} = p_2$$

Thus

$$\frac{p_1}{\gamma_{oil}} + H + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - (3 + H + h) = \frac{p_2}{\gamma_{oil}}$$

or $$\frac{p_1}{\gamma_{oil}} + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - 3 - h = \frac{p_2}{\gamma_{oil}} \quad (3)$$

combining Eqs. (1) and (3) we get:

$$\frac{p_1}{\gamma_{oil}} + (3\,ft)\frac{SG_{Hg}}{SG_{oil}} - (3\,ft) - h + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - \cancel{h_\ell}^{\,0}$$

or $$h_s = z_2 - z_1 + 3ft\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - h + \frac{V_2^2 - V_1^2}{2g} = (3ft)\left(\frac{13.6}{0.88} - 1\right) + \frac{(25.5\frac{ft}{s})^2 - (6.37\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

$$h_s = 52.9\ ft$$

Finally from Eq. (2)

$$\dot{W}_{\substack{shaft\\net\ in}} = (0.88)\left(62.4\,\frac{lb}{ft^3}\right)\left(5\,\frac{ft^3}{s}\right)(52.9\,ft) = \underline{\underline{14,500}}\ \frac{ft\cdot lb}{s}$$

5.124

5.124 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for: **(a)** $n = 5$; **(b)** $n = 6$; **(c)** $n = 7$; **(d)** $n = 8$; **(e)** $n = 9$; **(f)** $n = 10$.

For the kinetic energy coefficient, α, we may use Eq. 5.86. Thus,

$$\alpha = \frac{\int_0^R \frac{u^2}{2}\rho u\, 2\pi r\, dr}{\rho \bar{u}\, \pi R^2 \frac{\bar{u}^2}{2}} = \frac{2\int_0^1 u^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} = \frac{2u_c^3\int_0^1\left(1-\frac{r}{R}\right)^{\frac{3}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} \qquad (1)$$

For the average velocity, $\bar{u}$, we may use Eq. 5.7. Thus,

$$\bar{u} = \frac{\int_0^R \rho u\, 2\pi r\, dr}{\rho \pi R^2} = 2\int_0^1 u\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2u_c\int_0^1\left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \qquad (2)$$

To facilitate the integrations we make the substitution

$$\beta = 1 - \frac{r}{R} \qquad (3)$$

Thus,

$$d\beta = -d\left(\frac{r}{R}\right) \qquad (4)$$

and Eq. 2 becomes

$$\bar{u} = -2u_c\int_1^0 \beta^{\frac{1}{n}}(1-\beta)\, d\beta = \frac{2n^2}{(n+1)(2n+1)}\, u_c \qquad (5)$$

Combining Eqs. 1, 3, 4 and 5 we obtain

$$\alpha = \frac{-2\int_1^0 \beta^{\frac{3}{n}}(1-\beta)\, d\beta}{\left[\frac{2n^2}{(n+1)(2n+1)}\right]^3} = \left[\frac{2n^2}{(3+n)(3+2n)}\right]\left[\frac{(n+1)(2n+1)}{2n^2}\right]^3 \qquad (6)$$

(a) For $n = 5$, Eq. 6 yields

$$\alpha = \left\{\frac{2(5)^2}{(3+5)[3+2(5)]}\right\}\left\{\frac{(5+1)[(2)(5)+1]}{2(5)^2}\right\}^3 = \underline{\underline{1.11}}$$

(b) For $n = 6$

$\alpha = \underline{\underline{1.08}}$

(c) For $n = 7$

$\alpha = \underline{\underline{1.06}}$

(d) For $n = 8$

$\alpha = \underline{\underline{1.05}}$

(e) For $n = 9$

$\alpha = \underline{\underline{1.04}}$

(f) For $n = 10$

$\alpha = \underline{\underline{1.03}}$

5.125

5.125 A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient, α_2, is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: **(a)** assuming uniform velocity distributions; **(b)** considering actual velocity distributions.

(a) For uniform velocity distributions upstream and downstream of the fan, Eq. 5.82 is applicable. Thus,

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + \overbrace{g(z_{in} - z_{out})}^{0 \text{ for air}} + w_{\substack{shaft \\ net\ in}} \qquad (1)$$

We obtain the shaft work, $w_{\substack{shaft \\ net\ in}}$ from the given shaft power, $\dot{W}_{\substack{shaft \\ net\ in}}$, with

$$w_{\substack{shaft \\ net\ in}} = \frac{\dot{W}_{\substack{shaft \\ net\ in}}}{\dot{m}} = \frac{(0.00024\ hp)\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)}{0.004 \frac{lbm}{s}} = 33 \frac{ft \cdot lb}{lbm}$$

For V_{in} and V_{out} we use Eq. 5.11. Thus,

$$V_{in} = \frac{\dot{m}}{\rho A_{in}} = \frac{\dot{m}}{\rho \frac{\pi D_{in}^2}{4}} = \frac{\left(0.004 \frac{lbm}{s}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left(32.2 \frac{lbm}{slug}\right)\frac{\pi (2.5\ in.)^2}{4}} = 1.53 \frac{ft}{s}$$

and

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{\dot{m}}{\rho \frac{\pi D_{out}^2}{4}} = \frac{\left(0.004 \frac{lbm}{s}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left(32.2 \frac{lbm}{slug}\right)\frac{\pi (1\ in.)^2}{4}} = 9.57 \frac{ft}{s}$$

Now from Eq. 1 we obtain

$$\text{loss} = \frac{(-0.015\ psi)\left(144 \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left(32.2 \frac{lbm}{slug}\right)} + \left[\frac{\left(1.53 \frac{ft}{s}\right)^2 - \left(9.57 \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2 \frac{lbm}{slug}\right)} + 33 \frac{ft \cdot lb}{lbm}$$

or

$$\text{loss} = \underline{\underline{3.43 \frac{ft \cdot lb}{lbm}}}$$

(b) For non-uniform velocity distributions upstream and downstream of the fan Eq. 5.87 is applicable. Thus

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{\alpha_{in} \bar{V}_{in}^2 - \alpha_{out} \bar{V}_{out}^2}{2} + \overbrace{g(z_{in} - z_{out})}^{0 \text{ for air}} + w_{\substack{shaft \\ net\ in}}$$

or

$$\text{loss} = -28.18 \frac{ft \cdot lb}{lbm} + \left[\frac{(2)(1.53\ \ )^2}{2} - \frac{(1.08)\left(9.57 \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2 \frac{lbm}{slug}\right)} + 33 \frac{ft \cdot lb}{lbm}$$

and

$$\text{loss} = \underline{\underline{3.36 \frac{ft \cdot lb}{lbm}}}$$

5.126

5.126 Force from a Jet of Air Deflected by a Flat Plate

Objective: A jet of a fluid striking a flat plate as shown in Fig. P5.126 exerts a force on the plate. It is the equal and opposite force of the plate on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the plate with the experimentally measured force.

Equipment: Air source with an adjustable flowrate and a flow meter; nozzle to produce a uniform air jet; balance beam with an attached flat plate; weights; barometer; thermometer.

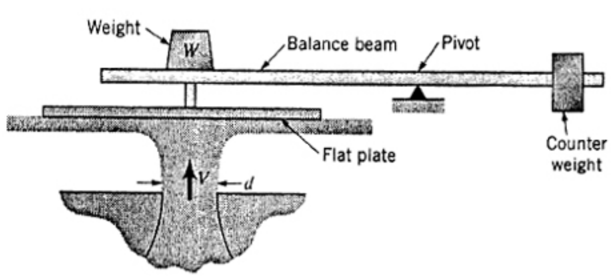
Experimental Procedure: Adjust the counter weight so that the beam is level when there is no mass, m, on the beam and no flow through the nozzle. Measure the diameter, d, of the nozzle outlet. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Place a known mass, m, on the flat plate and adjust the fan speed control to produce the necessary flowrate, Q, to make the balance beam level again. The flowrate is related to the flow meter manometer reading, h, by the equation $Q = 0.358\, h^{1/2}$, where Q is in ft^3/s and h is in inches of water. Repeat the measurements for various masses on the plate.

Calculations: For each flowrate, Q, calculate the weight, $W = mg$, needed to balance the beam and use the continuity equation, $Q = VA$, to determine the velocity, V, at the nozzle exit. Use the momentum equation for this problem, $W = \rho V^2 A$, to determine the theoretical relationship between velocity and weight.

Graph: Plot the experimentally measured force on the plate, W, as ordinates and air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.126

(con't)

Solution for Problem 5.126: Force from a Jet of Air Deflected by a Flat Plate

d, in.	H_{atm}, in. Hg	T, deg F	Q = 0.358 h^0.5, with Q in cfs and h in inches of water
1.174	29.25	70	

		Experimental				Theoretical
m, kg	h, in.	Q, ft^3/s	V, ft/s	m, slug	W, lb	W, lb
0.010	0.54	0.263	35.0	0.00069	0.022	0.021
0.020	1.08	0.372	49.5	0.00137	0.044	0.042
0.030	1.52	0.441	58.7	0.00206	0.066	0.059
0.040	2.18	0.529	70.3	0.00274	0.088	0.084
0.050	2.72	0.590	78.5	0.00343	0.110	0.105
0.060	3.25	0.645	85.8	0.00411	0.132	0.126
0.070	3.81	0.699	92.9	0.00480	0.154	0.147
0.080	4.32	0.744	98.9	0.00548	0.177	0.167
0.090	4.92	0.794	105.6	0.00617	0.199	0.190
0.100	5.46	0.837	111.2	0.00685	0.221	0.211
0.150	8.13	1.021	135.7	0.01028	0.331	0.315
0.200	10.85	1.179	156.8	0.01370	0.441	0.420
0.250	13.72	1.326	176.3	0.01713	0.552	0.531

Experimental:

V = Q/A where

$A = \pi d^2/4 = \pi$*(1.174/12 ft)^2/4 = 7.52E-3 ft^2

W = mg

Theoretical:

$W = \rho V^2 A$ where

$\rho = p_{atm}/RT$ with

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2

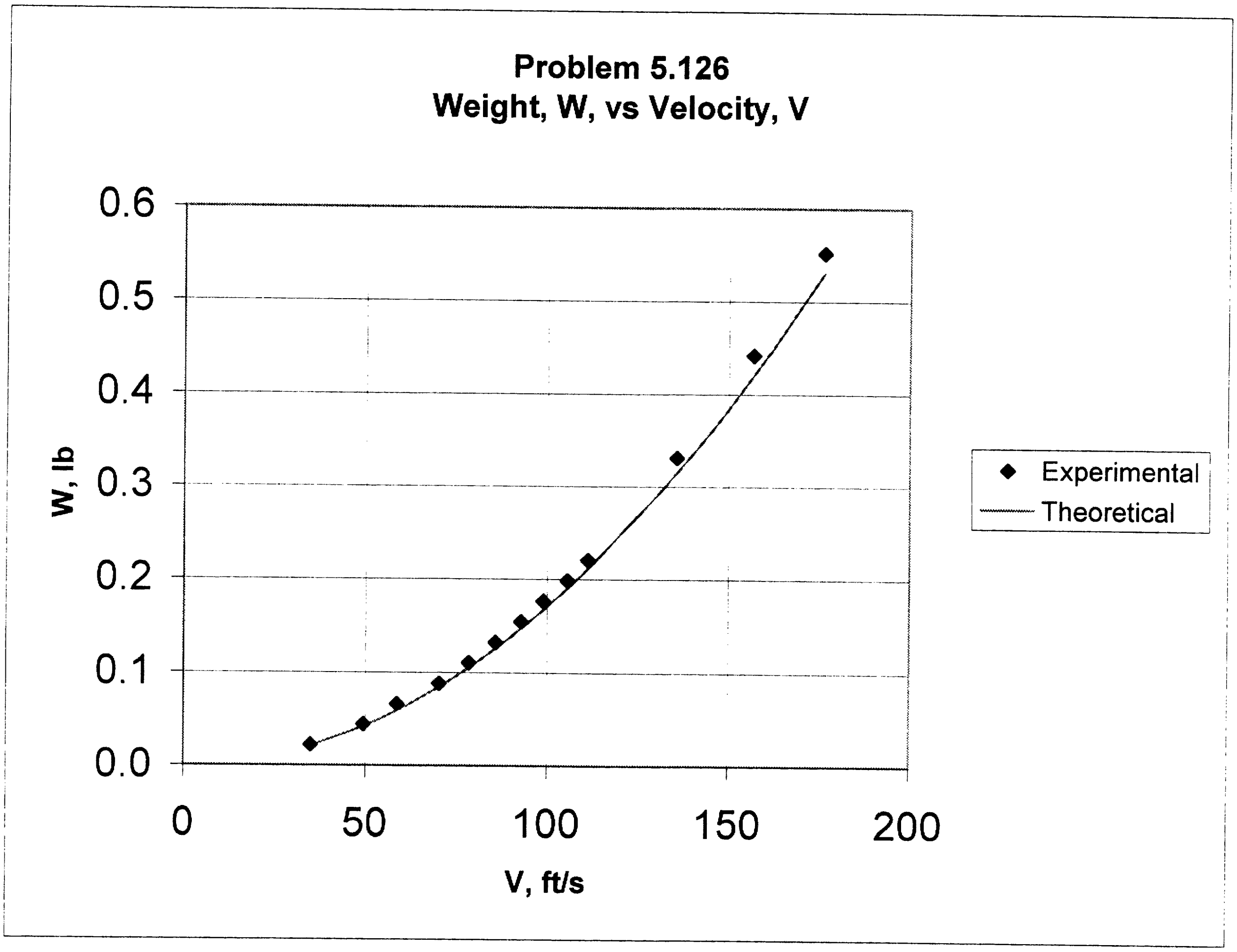
R = 1716 ft lb/slug deg R

T = 70 + 460 = 530 deg R

Thus, ρ = 0.00227 slug/ft^3

(con't)

5.126 (con't)



Problem 5.126
Weight, W, vs Velocity, V
0.6
0.5
0.4
0.3
0.2
0.1
0.0
W, lb
0
50
100
150
200
V, ft/s
Experimental
Theoretical

5.127

5.127 Pressure Distribution on a Flat Plate Due to the Deflection of an Air Jet

Objective: In order to deflect a jet of air as shown in Fig. P5.127, the flat plate must push against the air with a sufficient force to change the momentum of the air. This causes an increase in pressure on the plate. The purpose of this experiment is to measure the pressure distribution on the plate and to compare the resultant pressure force to that needed, according to the momentum equation, to deflect the air.

Equipment: Air supply with a flow meter; nozzle to produce a uniform jet of air; circular flat plate with static pressure taps at various radial locations; manometer; barometer; thermometer.

Experimental Procedure: Measure the diameters of the plate, D, and the nozzle exit, d, and the radial locations, r, of the various static pressure taps on the plate. Carefully center the plate over the nozzle exit and adjust the air flowrate, Q, to the desired constant value. Record the static pressure tap manometer readings, h, at various radial locations, r, from the center of the plate. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to determine the pressure on the plate as a function of location, r. That is, calculate $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid.

Graph: Plot pressure, p, as ordinates and radial location, r, as abscissas.

Results: Use the experimentally determined pressure distribution to determine the net pressure force, F, that the air jet puts on the plate. That is, numerically or graphically integrate the pressure data to obtain a value for $F = \int p\, dA = \int p\,(2\pi r\, dr)$, where the limits of the integration are over the entire plate, from $r = 0$ to $r = D/2$. Compare this force obtained from the pressure measurements to that obtained from the momentum equation for this flow, $F = \rho V^2 A$, where V and A are the velocity and area of the jet, respectively.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

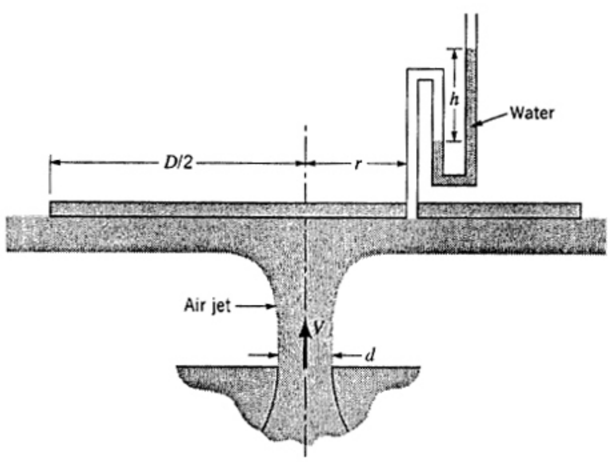


FIGURE P5.127

(con't)

5.127 (Con't)

Solution for Problem 5.127: Pressure Distribution on a Flat Plate due to the Deflection of an Air Jet

D, in.	d, in.	H_{atm}, in. Hg	T, deg F	Q. ft^3/s
8.0	1.174	29.25	77	1.41

r, in.	h, in.	p, lb/ft^2	p, lb/in.^2	p*r, lb/in.	i	pr_i+pr_{i+1}	$r_{i+1} - r_i$
0.00	6.62	34.42	0.2391	0.0000	1	0.0834	0.39
0.39	5.92	30.78	0.2138	0.0834	2	0.1701	0.40
0.79	3.04	15.81	0.1098	0.0867	3	0.1114	0.45
1.24	0.55	2.86	0.0199	0.0246	4	0.0355	0.35
1.59	0.19	0.99	0.0069	0.0109	5	0.0205	0.45
2.04	0.13	0.68	0.0047	0.0096	6	0.0174	0.37
2.41	0.09	0.47	0.0033	0.0078	7	0.0130	0.44
2.85	0.05	0.26	0.0018	0.0051	8	0.0086	0.38
3.23	0.03	0.16	0.0011	0.0035	9	0.0035	0.44
3.67	0.00	0.00	0.0000	0.0000			

$p = \gamma_{H2O}*h$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2
R = 1716 ft lb/slug deg R
T = 77 + 460 = 537 deg R

Thus, ρ = 0.00224 slug/ft^3

Using the trapezoidal rule for integration

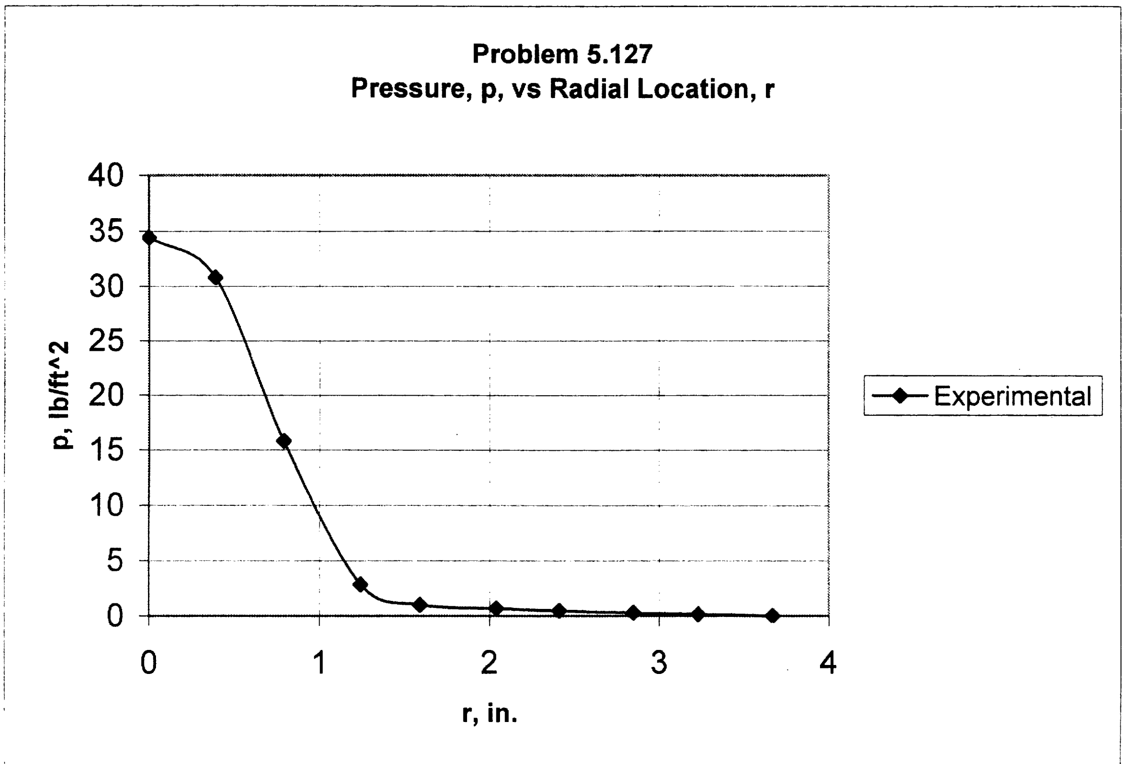
$F_{exp} = 2\pi*0.5*\sum_{1\text{ to }9}[(pr_i + pr_{i+1})*(r_{i+1} - r_i)] = 2\pi*0.5*0.189 = \underline{0.594\text{ lb}}$

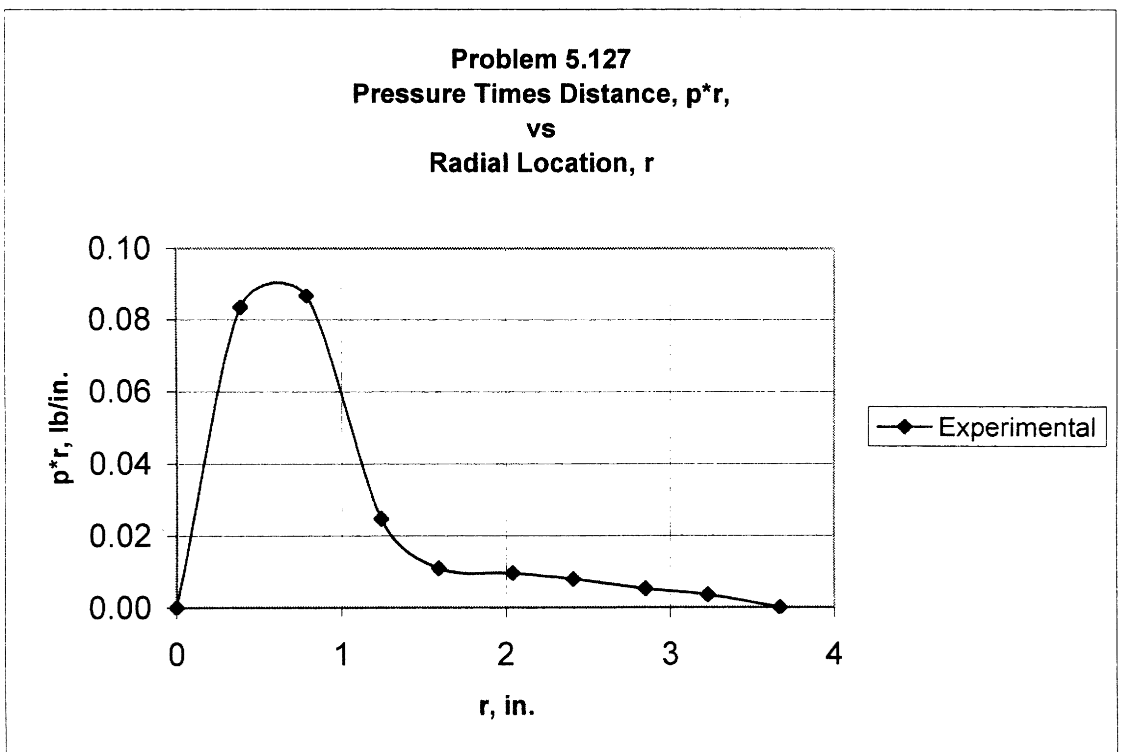
Theory:

$F = \rho V^2 A$ where
$A = \pi d^2/4 = \pi*(1.174/12\text{ ft})^2/4 = 0.00752\text{ ft}^2$
$V = Q/A = (1.41\text{ ft}^3/\text{s})/(0.00752\text{ ft}^2) = 188\text{ ft/s}$
Thus,
$F_{th} = 0.00224\text{ slug/ft}^3*(188\text{ ft/s})^2*(0.00752\text{ ft}^2) = \underline{0.595\text{ lb}}$

(con't)

5.127 (con't)


Problem 5.127
Pressure, p, vs Radial Location, r
p, lb/ft^2
r, in.
Experimental
0
5
10
15
20
25
30
35
40
0
1
2
3
4


Problem 5.127
Pressure Times Distance, p*r,
vs
Radial Location, r
p*r, lb/in.
r, in.
Experimental
0.00
0.02
0.04
0.06
0.08
0.10
0
1
2
3
4

5.128 Force from a Jet of Water Deflected by a Vane

Objective: A jet of a fluid striking a vane as shown in Fig. P5.128 exerts a force on the vane. It is the equal and opposite force of the vane on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the vane with the experimentally measured force.

Equipment: Water source; nozzle to produce a uniform jet of water; vanes to deflect the water jet; weigh tank to collect a known amount of water in a measured time period; stop watch; force balance system.

Experimental Procedure: Measure the outlet diameter, d, of the nozzle. Fasten the $\theta = 90$ degree vane to its support and adjust the balance spring to give a zero reading when there is no weight, W, on the platform and no flow through the nozzle. Place a known mass, m, on the platform and adjust the control valve on the pump to provide the necessary flowrate from the nozzle to return the platform to a zero reading. Determine the flowrate by collecting a known weight of water, W_{water}, in the weigh tank during a measured amount of time, t. Repeat the measurements for various masses, m. Repeat the experiment using a $\theta = 180$ degree vane.

Calculations: For each data set, determine the weight, $W = mg$, on the platform and the volume flowrate, $Q = W_{water}/(\gamma t)$, through the nozzle. Determine the exit velocity from the nozzle, V, by using $Q = VA$. Use the momentum equation to determine the theoretical weight that can be supported by the water jet as a function of V and θ.

Graph: For each vane, plot the experimentally determined weight, W, as ordinates and the water velocity, V, as abscissas.

Results: On the same graph plot the theoretical weight as a function of velocity for each vane.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

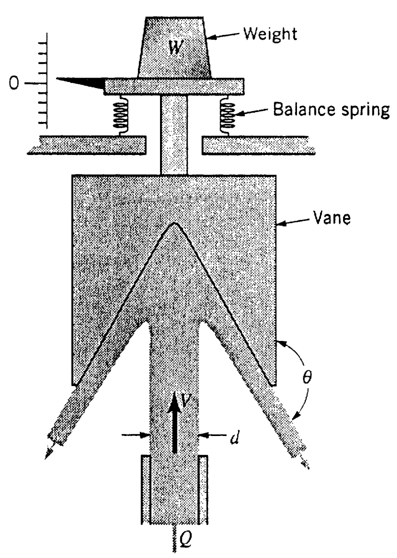


FIGURE P5.128

(con't)

5.128 (con't)

Solution for Problem 5.128: Force from a Jet of Water Deflected by a Vane

d, in.
0.40

m, kg	W_{water}, lb	t, s	m, slug	Experimental W, lb	Q, ft^3/s	V, ft/s	Theoretical W, lb
Data for θ = 90 deg:							
0.02	7.71	29.8	0.0014	0.044	0.0041	4.7	0.038
0.07	8.66	18.2	0.0048	0.154	0.0076	8.7	0.129
0.17	8.87	10.1	0.0116	0.375	0.0141	16.1	0.440
0.12	8.92	12.6	0.0082	0.265	0.0113	13.0	0.286
0.22	9.66	10.6	0.0151	0.485	0.0146	16.7	0.474
Data for θ = 180 deg:							
0.05	6.81	24.5	0.0034	0.110	0.0045	5.1	0.088
0.10	9.02	20.8	0.0069	0.221	0.0069	8.0	0.215
0.20	8.84	13.2	0.0137	0.441	0.0107	12.3	0.512
0.25	7.88	10.9	0.0171	0.552	0.0116	13.3	0.597
0.30	8.86	11.1	0.0206	0.662	0.0128	14.7	0.727
0.35	7.97	9.5	0.0240	0.772	0.0134	15.4	0.803
0.40	6.37	7.6	0.0274	0.883	0.0134	15.4	0.802

W = mg

$Q = W_{water}/(\gamma * t)$

V = Q/A where

$A = \pi d^2/4 = \pi*(0.40/12 \text{ ft})^2/4 = 0.000873 \text{ ft}^2$

Theoretical:

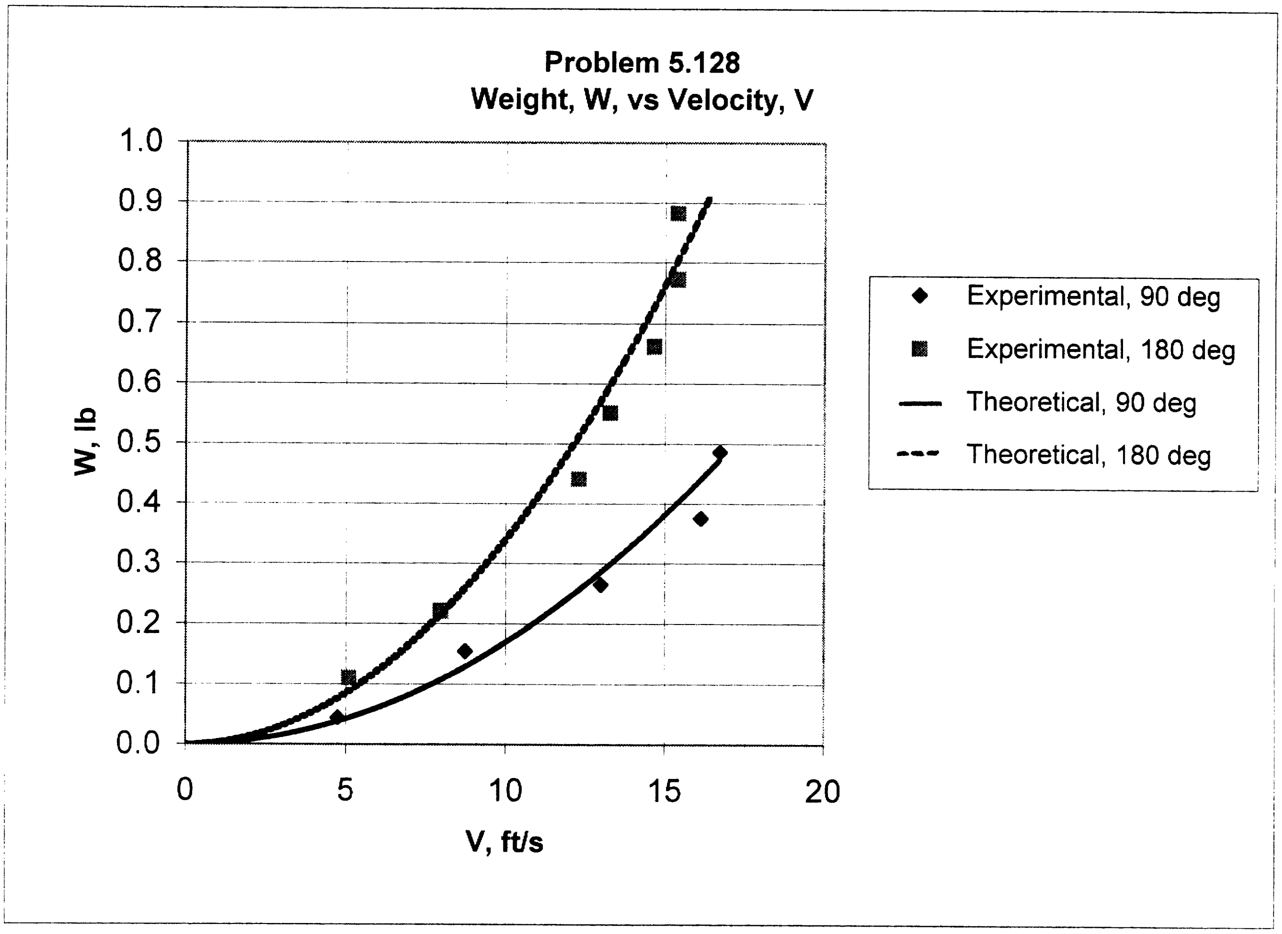
$W = \rho V^2 A$ for θ = 90 deg

and

$W = 2\rho V^2 A$ for θ = 180 deg

(con't)

5.128 (Con't)



Problem 5.128
Weight, W, vs Velocity, V
W, lb
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
V, ft/s
Experimental, 90 deg
Experimental, 180 deg
Theoretical, 90 deg
Theoretical, 180 deg

5.129

5.129 Force of a Flowing Fluid on a Pipe Elbow

Objective: When a fluid flows through an elbow in a pipe system as shown in Fig. P5.129, the fluid's momentum is changed as the fluid changes direction. Thus, the elbow must put a force on the fluid. Similarly, there must be an external force on the elbow to keep it in place. The purpose of this experiment is to compare the theoretical vertical component of force needed to hold an elbow in place with the experimentally measured force.

Equipment: Variable speed fan; Pitot static tube; air speed indicator; air duct and 90-degree elbow; scale; barometer; thermometer.

Experimental Procedure: Measure the diameter, d, of the air duct and adjust the scale to read zero when the elbow rests on it and there is no flow through it. Note that the duct is connected to the fan outlet by a pivot mechanism that is essentially friction free. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Adjust the variable speed fan to give the desired flowrate. Record the velocity, V, in the pipe as given by the Pitot static tube which is connected to an air speed indicator that reads directly in feet per minute. Record the force, F, indicated on the scale at this air speed. Repeat the measurements for various air speeds. Obtain data for two types of elbows: (1) a long radius elbow and (2) a mitered elbow (see Figs. 8.30 and 8.31).

Calculations: For a given air speed, V, use the momentum equation to calculate the theoretical vertical force, $F = \rho V^2 A$, needed to hold the elbow stationary.

Graph: Plot the experimentally measured force, F, as ordinates and the air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

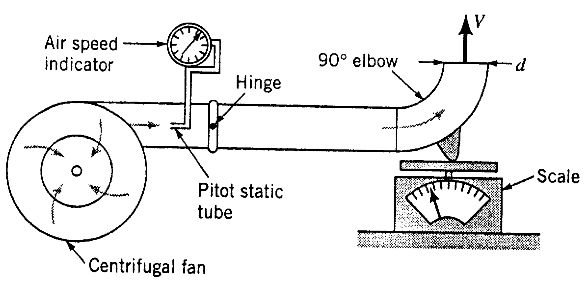


FIGURE P5.129

(Con't)

Solution for Problem 5.129: Force of a Flowing Fluid on a Pipe Elbow

d, in.	H_{atm}, in. Hg	T, deg F
8.0	29.07	73

Experiment			Theory	
V, ft/min	F, lb	V, ft/s	V, ft/s	F_{th}, lb
Long Radius Elbow Data			0	0
0	0	0.0	5.0	0.02
1200	0.38	20.0	10.0	0.08
1420	0.51	23.7	15.0	0.18
1800	0.79	30.0	20.0	0.31
2160	1.05	36.0	25.0	0.49
2440	1.38	40.7	30.0	0.70
2700	1.65	45.0	35.0	0.96
2900	1.91	48.3	40.0	1.25
3100	2.19	51.7	45.0	1.58
3520	2.83	58.7	50.0	1.95
3750	3.12	62.5	55.0	2.36
3950	3.38	65.8	60.0	2.81
			65.0	3.30
Mitered Elbow Data				
1400	0.30	23.3		
1780	0.55	29.7		
2000	0.74	33.3		
2300	1.12	38.3		
2630	1.44	43.8		
2900	1.72	48.3		
3150	2.06	52.5		
3360	2.38	56.0		
3550	2.62	59.2		
3620	2.74	60.3		

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.07/12ft) = 2052 lb/ft^2

R = 1716 ft lb/slug deg R

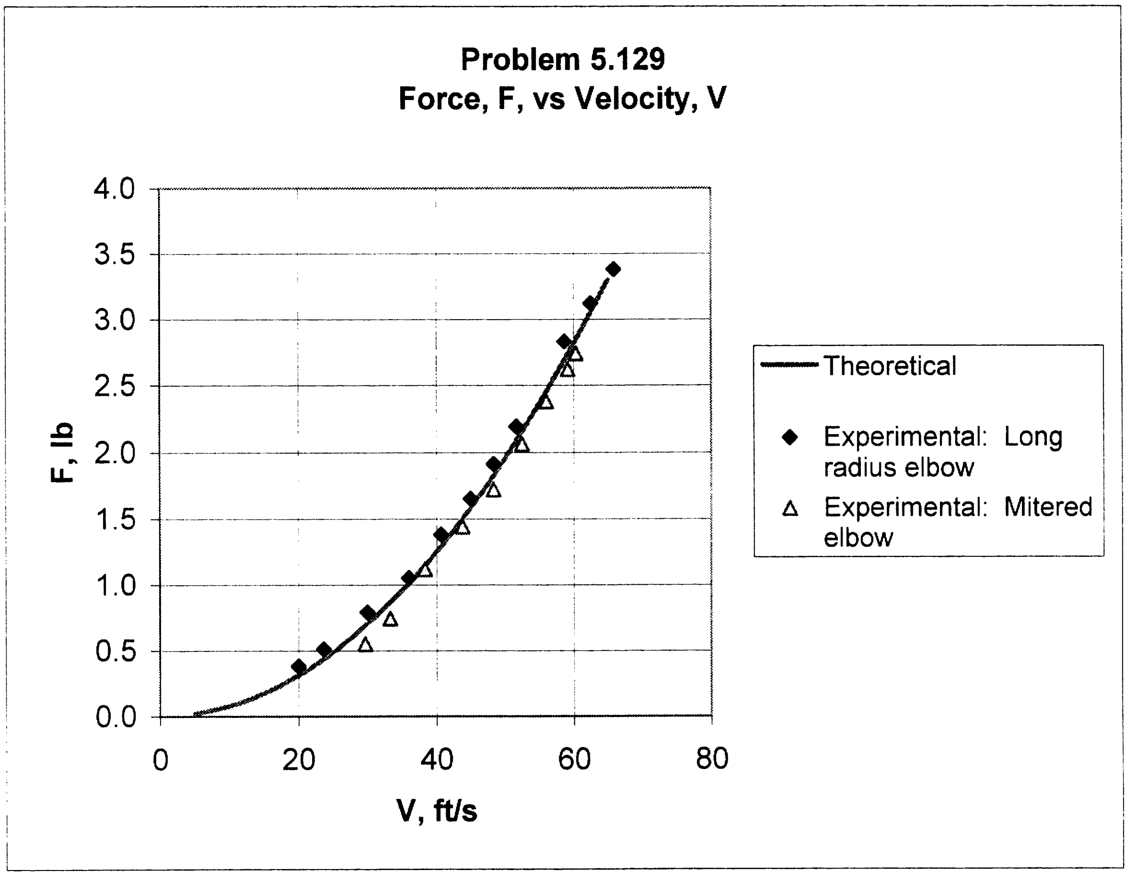
T = 73 + 460 = 533 deg R

Thus, ρ = 0.00224 slug/ft^3

A = πd^2/4 = π*(8/12)^2/4 = 0.349 ft^2

(con't)

5.129 (con't)



Problem 5.129
Force, F, vs Velocity, V
F, lb
V, ft/s
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
Theoretical
Experimental: Long radius elbow
Experimental: Mitered elbow

6.1

6.1 The velocity in a certain two-dimensional flow field is given by the equation

$$\mathbf{V} = 2xt\hat{\mathbf{i}} - 2yt\hat{\mathbf{j}}$$

where the velocity is in ft/s when x, y, and t are in feet and seconds, respectively. Determine expressions for the local and convective components of acceleration in the x and y directions. What is the magnitude and direction of the velocity and the acceleration at the point $x = y = 2$ ft at the time $t = 0$?

From expression for velocity, $u = 2xt$ and $v = -2yt$.

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

then

$$a_x\,(\text{local}) = \frac{\partial u}{\partial t} = \underline{\underline{2x}}$$

and

$$a_x\,(\text{conv}) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (2xt)(2t) + (-2yt)(0)$$

$$= \underline{\underline{4xt^2}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

and

$$a_y\,(\text{local}) = \frac{\partial v}{\partial t} = \underline{\underline{-2y}}$$

$$a_y\,(\text{conv.}) = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (2xt)(0) + (-2yt)(-2t)$$

$$= \underline{\underline{4yt^2}}$$

At $x = y = 2\text{ ft}$ and $t = 0$

$$u = 2(2)(0) = 0 \qquad v = -2(2)(0) = 0$$

So that $\underline{\underline{\vec{V} = 0}}$

and

$$a_x = 2x + 4xt^2 = 2(2) + 4(2)(0) = 4\ \text{ft/s}^2$$

$$a_y = -2y + 4yt^2 = -2(2) + 4(2)(0) = -4\ \text{ft/s}^2$$

Thus,

$$\vec{a} = \underline{\underline{4\hat{i} - 4\hat{j}\ \text{ft/s}^2}} \quad \text{with } |\vec{a}| = \sqrt{(4)^2 + (-4)^2} = \underline{\underline{5.66\ \text{ft/s}^2}}$$

6.2

6.2 Repeat Problem 6.1 if the flow field is described by the equation

$$\mathbf{V} = 3(x^2 - y^2)\hat{\mathbf{i}} - 6xy\hat{\mathbf{j}}$$

where the velocity is in ft/s when x and y are in feet.

From expression for velocity, $u = 3(x^2-y^2)$ and $v = -6xy$.

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

then

$$a_x\,(local) = \frac{\partial u}{\partial t} = \underline{\underline{0}}$$

and

$$a_x\,(conv) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 3(x^2-y^2)(6x) + (-6xy)(-6y)$$

$$= \underline{\underline{18(x^3 + xy^2)}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

and

$$a_y\,(local) = \frac{\partial v}{\partial t} = \underline{\underline{0}}$$

$$a_y\,(conv) = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 3(x^2-y^2)(-6y) + (-6xy)(-6x)$$

$$= \underline{\underline{18(x^2y + y^3)}}$$

At $x = y = 1\ ft$ and $t = 0$

$$u = 3\left[(1)^2 - (1)^2\right] = 0 \qquad v = -6\,(1)(1) = -6$$

so that

$$\vec{V} = \underline{\underline{-6\,\hat{j}\ \ ft/s}} \qquad \text{with } |\vec{V}| = \underline{\underline{6\ ft/s}}$$

and

$$a_x = 18\,(x^3 + xy^2) = 18\left[(1)^3 + (1)(1)^2\right] = 36\ ft/s^2$$

$$a_y = 18\,(x^2y + y^3) = 18\left[(1)^2(1) + (1)^3\right] = 36\ ft/s^2$$

Thus,

$$\vec{a} = \underline{\underline{36\,\hat{i} + 36\,\hat{j}\ \ ft/s^2}}$$

with

$$|\vec{a}| = \sqrt{(36)^2 + (36)^2} = \underline{\underline{50.9\ ft/s^2}}$$

6.3

6.3 The velocity in a certain flow field is given by the equation

$$\mathbf{V} = x\hat{\mathbf{i}} + x^2 z\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$$

Determine the expressions for the three rectangular components of acceleration.

From expression for velocity, $u = x$ $\quad v = x^2 z$ $\quad w = yz$

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

then

$$a_x = 0 + (x)(1) + (x^2 z)(0) + (yz)(0)$$

$$= \underline{\underline{x}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

and

$$a_y = 0 + (x)(2xz) + (x^2 z)(0) + (yz)(x^2)$$

$$= \underline{\underline{2x^2 z + x^2 yz}}$$

Also,

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

so that

$$a_z = 0 + (x)(0) + (x^2 z)(z) + (yz)(y)$$

$$= \underline{\underline{x^2 z^2 + y^2 z}}$$

6.4

6.4 The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate, and interpret the results. **(b)** Determine an expression for the rotation vector. Is this an irrotational flow field?

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ (Eq. 6.9)

Thus, for velocity components given

volumetric dilatation rate $= 2x + (x+z) + (-3x-z) = \underline{\underline{0}}$

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(y - 2y) = -\frac{y}{2}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[0 - (y+2z)\right] = -\left(\frac{y}{2}+z\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left[2z - (-3z)\right] = \frac{5z}{2}$$

Thus,

$$\vec{\omega} = \underline{\underline{-\left(\frac{y}{2}+z\right)\hat{i} + \frac{5z}{2}\hat{j} - \frac{y}{2}\hat{k}}}$$

Since $\vec{\omega}$ is not zero everywhere the flow field is not irrotational. $\underline{\underline{\text{No.}}}$

6.5

6.5 Determine an expression for the vorticity of the flow field described by

$$\mathbf{V} = -\,xy^3\,\hat{\mathbf{i}} + y^4\hat{\mathbf{j}}$$

Is the flow irrotational?

$$\vec{\zeta} = 2\vec{\omega} \qquad \text{(Eq. 6.17)}$$

From expression for velocity, $u = -xy^3$, $v = y^4$, and $w = 0$, and with

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \text{(Eq. 6.13)}$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \text{(Eq. 6.14)}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

it follows that

$$\omega_x = 0, \qquad \omega_y = 0, \quad \text{and} \quad \omega_z = \frac{1}{2}\left[0 - (-3xy^2)\right] = \frac{3}{2}xy^2$$

Thus,

$$\vec{\zeta} = 2\left(\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}\right)$$

$$= 2\left[(0)\hat{i} + (0)\hat{j} + \left(\tfrac{3}{2}xy^2\right)\hat{k}\right]$$

$$= \underline{\underline{3xy^2\,\hat{k}}}$$

Since $\vec{\zeta}$ is not zero everywhere the flow is not irrotational. <u>No.</u>

6.6

6.6 A one-dimensional flow is described by the velocity field

$$u = ay + by^2$$
$$v = w = 0$$

where a and b are constants. Is the flow irrotational? For what combination of constants (if any) will the rate of angular deformation as given by Eq. 6.18 be zero?

For irrotational flow $\vec{\omega} = 0$, and for the velocity distribution given:

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = 0$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\left(\frac{a}{2} + by\right)$$

Thus, $\vec{\omega}$ is not zero everywhere and the flow is not irrotational. No.

Since (from Eq. 6.18)

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

it follows for the velocity distribution given that

$$\dot{\gamma} = a + 2by$$

Thus, there are no values of a and b (except both equal to zero) that will give $\dot{\gamma} = 0$ for all values of y. None.

6.7

6.7 For incompressible fluids the volumetric dilatation rate must be zero; that is, $\nabla \cdot \mathbf{V} = 0$. For what combination of constants a, b, c, and e can the velocity components

$$u = ax + by$$
$$v = cx + ey$$
$$w = 0$$

be used to describe an incompressible flow field?

For an incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

With the velocity distribution given

$$\frac{\partial u}{\partial x} = a \qquad \frac{\partial v}{\partial y} = e \qquad \frac{\partial w}{\partial z} = 0$$

Thus, for an incompressible flow field

$$\underline{\underline{a + e = 0}}$$

6.8

6.8 An incompressible viscous fluid is placed between two large parallel plates as shown in Fig. P6.8. The bottom plate is fixed and the upper plate moves with a constant velocity, U. For these conditions the velocity distribution between the plates is linear, and can be expressed as

$$u = U\frac{y}{b}$$

Determine: **(a)** the volumetric dilatation rate, **(b)** the rotation vector, **(c)** the vorticity, and **(d)** the rate of angular deformation.

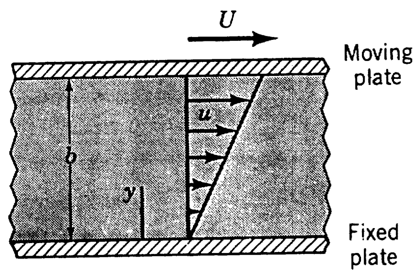


FIGURE P6.8

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \underline{\underline{0}}$

(b) For velocity distribution given,

$$\vec{\omega} = \omega_z \hat{k}$$

and

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\frac{U}{2b}$$

Thus,

$$\underline{\underline{\vec{\omega} = -\frac{U}{2b}\hat{k}}}$$

(c) $\vec{\zeta} = 2\vec{\omega} = \underline{\underline{-\frac{U}{b}\hat{k}}}$

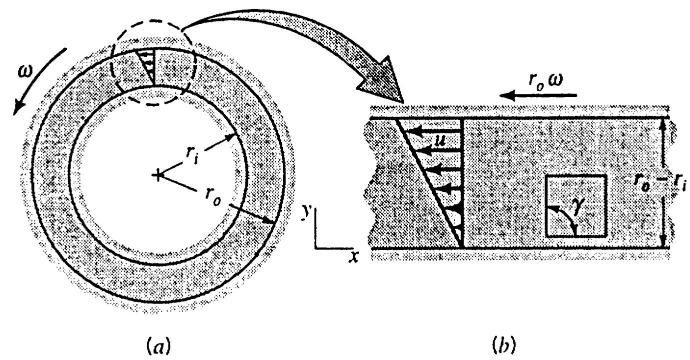
(d) $\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ (Eq. 6.18)

Thus,

$$\dot{\gamma} = \underline{\underline{\frac{U}{b}}}$$

6.9

6.9 A viscous fluid is contained in the space between concentric cylinders. The inner wall is fixed, and the outer wall rotates with an angular velocity ω. (See Fig. P6.9*a* and Video V6.1.) Assume that the velocity distribution in the gap is linear as illustrated in Fig. P6.9*b*. For the small rectangular element shown in Fig. P6.9*b*, determine the rate of change of the right angle γ due to the fluid motion. Express your answer in terms of r_0, r_i, and ω.



■ FIGURE P6.9

From Eq. 6.18

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

For the linear distribution

$$u = -\frac{r_0\omega}{r_0 - r_i}\, y$$

so that

$$\frac{\partial u}{\partial y} = -\frac{r_0\omega}{r_0 - r_i}$$

and since $v = 0$

$$\dot{\gamma} = -\frac{r_0\omega}{r_0 - r_i}$$

The negative sign indicates that the original right angle is increasing.

6.10

6.10 Some velocity measurements in a three-dimensional incompressible flow field indicate that $u = 6xy^2$ and $v = -4y^2z$. There is some conflicting data for the velocity component in the z direction. One set of data indicates that $w = 4yz^2$ and the other set indicates that $w = 4yz^2 - 6y^2z$. Which set do you think is correct? Explain.

To satisfy the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

Since $\frac{\partial u}{\partial x} = 6y^2$ and $\frac{\partial v}{\partial y} = -8yz$

then from Eq. (1)

$$6y^2 - 8yz + \frac{\partial w}{\partial z} = 0$$

Thus,

$$\frac{\partial w}{\partial z} = 8yz - 6y^2 \qquad (2)$$

Equation (2) can be integrated with respect to z to obtain

$$\int dw = \int 8yz\,dz - \int 6y^2 dz + f(x,y)$$

or

$$w = 4yz^2 - 6y^2z + f(x,y)$$

The set of data (with $f(x,y) = 0$)

$$\underline{\underline{w = 4yz^2 - 6y^2z}}$$

would appear to be the correct set.

6.11

6.11 The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$u = 2xy$$
$$v = x^2 - y^2$$

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

For the velocity distribution given,

$$\frac{\partial v}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = 2x$$

Thus,

$$\omega_z = \frac{1}{2}(2x - 2x) = 0$$

and the <u>flow is irrotational.</u>

To satisfy conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since,

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = -2y$$

then

$$2y - 2y = 0$$

and <u>conservation of mass is satisfied.</u>

6.12

6.12 For each of the following stream functions, with units of m^2/s, determine the magnitude and the angle the velocity vector makes with the x-axis at $x = 1$ m, $y = 2$ m. Locate any stagnation points in the flow field.
(a) $\psi = xy$
(b) $\psi = -2x^2 + y$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

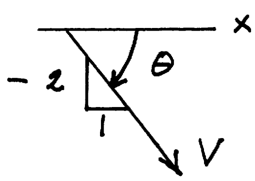
(a) For $\psi = xy$,

$$u = \frac{\partial \psi}{\partial y} = x \qquad v = -\frac{\partial \psi}{\partial x} = -y$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = -2\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2+v^2} = \sqrt{(1m)^2+(-2m)^2} = \underline{\underline{2.24\frac{m}{s}}}$$



$$\tan\theta = \frac{-2}{1} \qquad \theta = \underline{\underline{-63.4°}}$$

Since $u = 0$ at $x = 0$ and $v = 0$ at $y = 0$, a stagnation point occurs at $x = y = 0$.

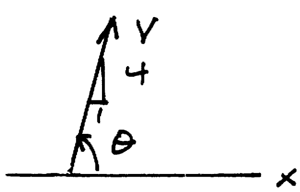
(b) For $\psi = -2x^2 + y$,

$$u = \frac{\partial \psi}{\partial y} = 1\frac{m}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 4x$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = 4\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2+v^2} = \sqrt{(1\tfrac{m}{s})^2+(4\tfrac{m}{s})^2} = \underline{\underline{4.12\frac{m}{s}}}$$



$$\tan\theta = \frac{4}{1} \qquad \underline{\underline{\theta = 76.0°}}$$

Since $u \neq 0$, there are no stagnation points.

6.13 The stream function for a certain incompressible flow field is

$$\psi = 10y + e^{-y} \sin x$$

Is this an irrotational flow field? Justify your answer with the necessary calculations.

For the flow to be irrotational (see Eq. 6.12),

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

and for the stream function given

$$u = \frac{\partial \psi}{\partial y} = 10 - e^{-y} \sin x$$

$$v = -\frac{\partial \psi}{\partial x} = -e^{-y} \cos x$$

Thus,

$$\frac{\partial u}{\partial y} = e^{-y} \sin x \qquad \frac{\partial v}{\partial x} = e^{-y} \sin x$$

so that

$$\omega_z = \frac{1}{2}\left(e^{-y} \sin x - e^{-y} \sin x\right) = 0$$

Since $\omega_z = 0$, this is an irrotational flow field. <u>Yes.</u>

6.14

6.14 The stream function for an incompressible, two-dimensional flow field is

$$\psi = ay^2 - bx$$

where a and b are constants. Is this an irrotational flow? Explain.

For the flow to be irrotational (see Eq. 6.12),

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

and for the stream function given,

$$u = \frac{\partial \psi}{\partial y} = 2ay$$

$$v = -\frac{\partial \psi}{\partial x} = b$$

Thus,

$$\frac{\partial u}{\partial y} = 2a \qquad \frac{\partial v}{\partial x} = 0$$

so that

$$\omega_z = \frac{1}{2}\left[0 - (2a)\right] = -a$$

Since $\omega_z \neq 0$ flow is not irrotational (unless $a = 0$). $\underline{\underline{\text{No}}}$.

6.15

6.15 The velocity components for an incompressible, plane flow are

$$v_r = Ar^{-1} + Br^{-2}\cos\theta$$

$$v_\theta = Br^{-2}\sin\theta$$

where A and B are constants. Determine the corresponding stream function.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = -\frac{\partial\psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that for the velocity distribution given,

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = Ar^{-1} + Br^{-2}\cos\theta \qquad (1)$$

$$\frac{\partial\psi}{\partial r} = -Br^{-2}\sin\theta \qquad (2)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int (A + Br^{-1}\cos\theta)\,d\theta + f_1(r)$$

or

$$\psi = A\theta + Br^{-1}\sin\theta + f_1(r) \qquad (3)$$

Similarly, integrate Eq. (2) with respect to r to obtain

$$\int d\psi = -\int Br^{-2}\sin\theta\,dr + f_2(\theta)$$

or

$$\psi = Br^{-1}\sin\theta + f_2(\theta) \qquad (4)$$

Thus, to satisfy both Eqs. (3) and (4)

$$\psi = \underline{\underline{A\theta + Br^{-1}\sin\theta + C}}$$

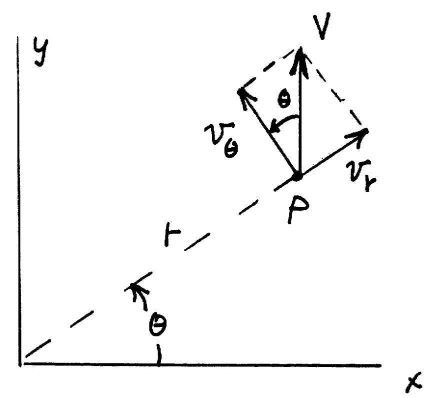
where C is an arbitrary constant.

6.16

6.16 For a certain two-dimensional flow field

$$u = 0$$
$$v = V$$

(a) What are the corresponding radial and tangential velocity components? **(b)** Determine the corresponding stream function expressed in Cartesian coordinates and in cylindrical polar coordinates.



(a) At an arbitrary point P (see figure)

$$\underline{\underline{v_r = V \sin\theta}}$$

$$\underline{\underline{v_\theta = V \cos\theta}}$$

(b) Since

$$u = \frac{\partial \psi}{\partial y} = 0 \qquad v = -\frac{\partial \psi}{\partial x} = V$$

it follows that ψ is not a function of y and

$$\underline{\underline{\psi = -Vx + C}}$$

where C is an arbitrary constant.

Also, with $x = r\cos\theta$

$$\underline{\underline{\psi = -Vr\cos\theta + C}}$$

6.17

6.17 Make use of the control volume shown in Fig. P6.17 to derive the continuity equation in cylindrical coordinates (Eq. 6.33 in text).

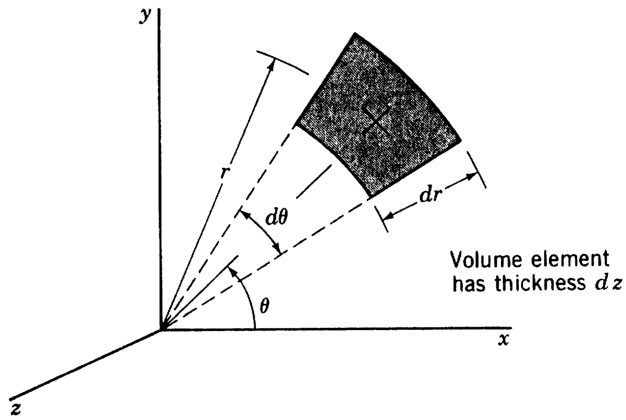


FIGURE P6.17

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = 0 \qquad \text{(Eq. 6.19)}$$

For the differential control volume shown

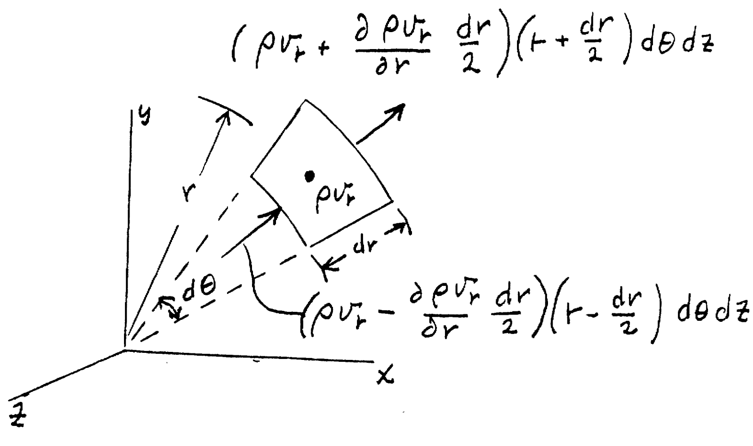
$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall \approx \frac{\partial \rho}{\partial t}\, r\, d\theta\, dr\, dz \qquad (1)$$

and

$$\int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = \text{net rate of mass outflow through surfaces of control volume}$$

From figure at right:

Net rate of mass outflow in r-direction =



$$\left(\rho v_r + \frac{\partial \rho v_r}{\partial r}\frac{dr}{2}\right)\left(r + \frac{dr}{2}\right) d\theta\, dz - \left(\rho v_r - \frac{\partial \rho v_r}{\partial r}\frac{dr}{2}\right)\left(r - \frac{dr}{2}\right) d\theta\, dz$$

$$= \frac{\partial \rho v_r}{\partial r}\, r\, dr\, d\theta\, dz + \rho v_r\, dr\, d\theta\, dz \qquad (2)$$

(con't)

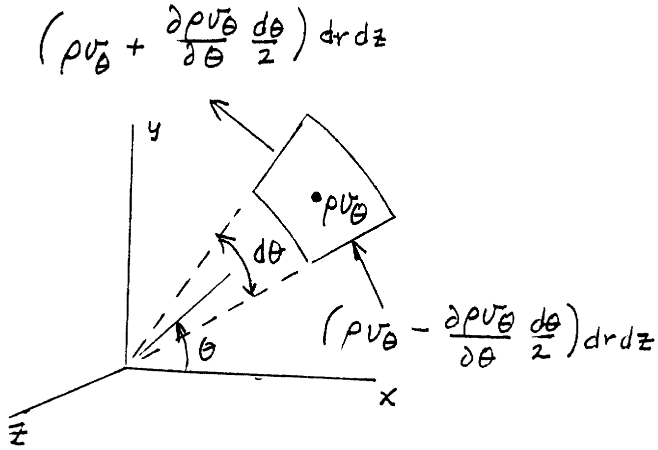
6.17 (Con't)

From figure at right:

Net rate of mass outflow in θ-direction =

$$\left(\rho v_\theta + \frac{\partial \rho v_\theta}{\partial \theta}\frac{d\theta}{2}\right) dr\,dz - \left(\rho v_\theta - \frac{\partial \rho v_\theta}{\partial \theta}\frac{d\theta}{2}\right) dr\,dz$$

$$= \frac{\partial \rho v_\theta}{\partial \theta}\, dr\, d\theta\, dz \qquad (3)$$

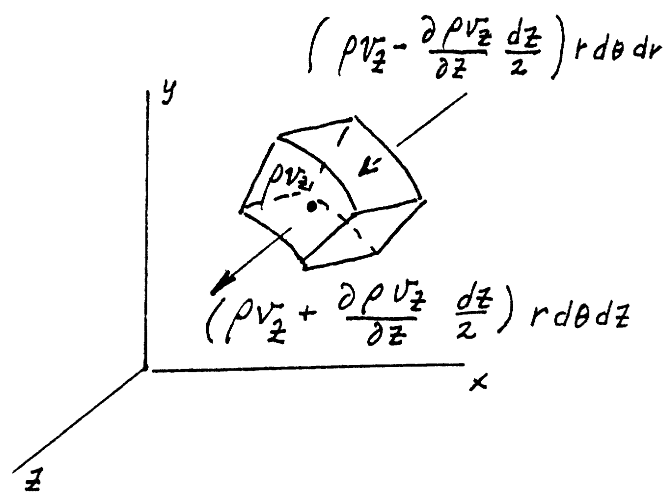


From figure at right:

Net rate of mass outflow in z-direction =

$$\left(\rho v_z + \frac{\partial \rho v_z}{\partial z}\frac{dz}{2}\right) r\,d\theta\,dr - \left(\rho v_z - \frac{\partial \rho v_z}{\partial z}\frac{dz}{2}\right) r\,d\theta\,dr$$

$$= \frac{\partial \rho v_z}{\partial z}\, r\,dr\,d\theta\,dz \qquad (4)$$



Substitution of Eqs. (1) thru (4) into Eq. 6.19 yields

$$\frac{\partial \rho}{\partial t} r\,dr\,d\theta\,dz + \frac{\partial \rho v_r}{\partial r} r\,dr\,d\theta\,dz + \rho v_r\, dr\,d\theta\,dz + \frac{\partial \rho v_\theta}{\partial \theta} dr\,d\theta\,dz + \frac{\partial \rho v_z}{\partial z} r\,dr\,d\theta\,dz = 0$$

or

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_r}{\partial r} + \frac{\rho v_r}{r} + \frac{1}{r}\frac{\partial \rho v_\theta}{\partial \theta} + \frac{\partial \rho v_z}{\partial z} = 0 \qquad (5)$$

Since

$$\frac{\partial \rho v_r}{\partial r} + \frac{\rho v_r}{r} = \frac{1}{r}\frac{\partial}{\partial r}(r \rho v_r)$$

Eq. (5) can be written as

$$\underline{\underline{\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0}}$$

which is Eq. 6.33.

6.18

6.18 It is proposed that a two-dimensional, incompressible flow field be described by the velocity components

$$u = Ay$$
$$v = Bx$$

where A and B are both positive constants. **(a)** Will the continuity equation be satisfied? **(b)** Is the flow irrotational? **(c)** Determine the equation for the streamlines and show a sketch of the streamline that passes through the origin. Indicate the direction of flow along this streamline.

(a) To satisfy the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since, for the velocity distribution given

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

the continuity equation is satisfied. Yes.

(b) In order for the flow to be irrotational $\omega_z = 0$, where

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

Since

$$\frac{\partial v}{\partial x} = B \qquad \frac{\partial u}{\partial y} = A$$

$$\omega_z = \frac{1}{2}(B - A)$$

Thus, flow will only be irrotational if $A = B$.

(c) Along a streamline

$$\frac{dy}{dx} = \frac{v}{u}$$

so that for the velocity distribution given

$$\frac{dy}{dx} = \frac{Bx}{Ay}$$

and therefore

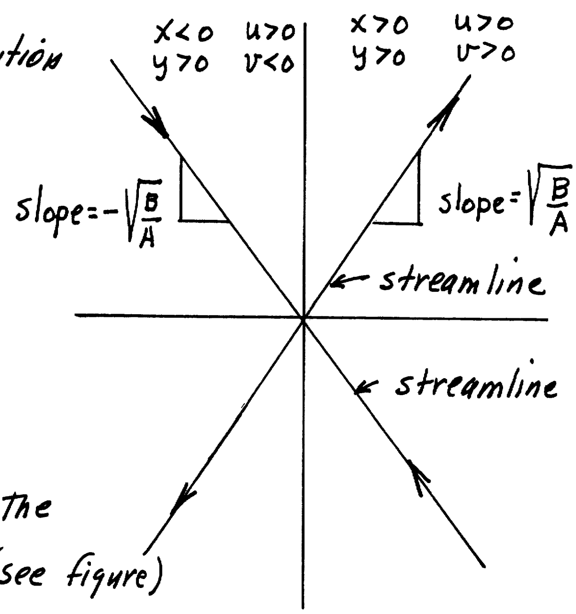
$$y\,dy = \frac{B}{A}x\,dx$$

Integration yields

$$y^2 = \frac{B}{A}x^2 + C$$

where C is a constant.

For the streamline passing through the origin ($C=0$) and $y = \pm\sqrt{\frac{B}{A}}\,x$ (see figure)



6.19

6.19 In a certain steady, two-dimensional flow field the fluid density varies linearly with respect to the coordinate x; that is, $\rho = Ax$ where A is a constant. If the x component of velocity u is given by the equation $u = y$, determine an expression for v.

For a variable density flow,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} \qquad \text{(Eq. 6.29)}$$

With

$$\rho u = (Ax)(y) = Axy$$

it follows that

$$\frac{\partial(\rho u)}{\partial x} = Ay$$

Thus,

$$\frac{\partial(\rho v)}{\partial y} = -Ay \qquad (1)$$

Integrate Eq.(1) with respect to y to obtain

$$\int d(\rho v) = -\int Ay\,dy + f_1(x)$$

or

$$\rho v = -\frac{Ay^2}{2} + f_1(x)$$

With $\rho = Ax$

$$v = -\left(\frac{1}{Ax}\right)\left(\frac{Ay^2}{2}\right) + \frac{f_1(x)}{Ax}$$

or

$$\underline{\underline{v = -\frac{y^2}{2x} + f(x)}}$$

where $f(x)$ is an arbitrary function of x.

6.20

6.20 In a two-dimensional, incompressible flow field, the x component of velocity is given by the equation $u = 2x$. **(a)** Determine the corresponding equation for the y component of velocity if $v = 0$ along the x axis. **(b)** For this flow field what is the magnitude of the average velocity of the fluid crossing the surface OA of Fig. P6.20? Assume that the velocities are in ft/s when x and y are in feet.

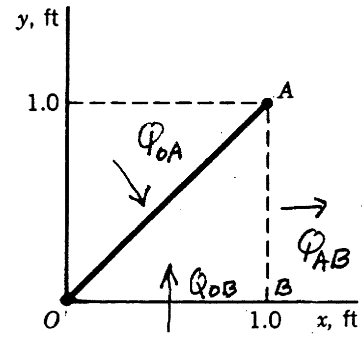


FIGURE P6.20

(consider a unit thickness = 1 ft)

(a) To satisfy the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since $\frac{\partial u}{\partial x} = 2$

it follows that

$$\frac{\partial v}{\partial y} = -2 \qquad (1)$$

Integration of Eq. (1) with respect to y yields

$$v = -2y + f(x)$$

If $v = 0$ along x-axis ($y=0$) then $f(x) = 0$ so that

$$\underline{\underline{v = -2y}}$$

(b) To satisfy conservation of mass

$$q_{OA} = q_{AB} - q_{OB} \qquad \text{(see figure)}$$

Along AB $\quad u = 2(1) = 2\ \frac{ft}{s}$ so that

$$q_{AB} = u A_{AB} = (2\ ft/s)(1\ ft)(1\ ft) = 2\ \frac{ft^3}{s}$$

Along OB $\quad v = 0$ so that $q_{OB} = 0$.

Thus,

$$q_{OA} = q_{AB} = 2\ \frac{ft^3}{s}$$

and

$$V_{AV} = \frac{q_{OA}}{\text{area}_{OA}} = \frac{2\ \frac{ft^3}{s}}{\sqrt{2}\ ft^2} = \underline{\underline{1.41\ \frac{ft}{s}}}$$

6.21

6.21 The radial velocity component in an incompressible, two-dimensional flow field ($v_z = 0$) is

$$v_r = 2r + 3r^2 \sin\theta$$

Determine the corresponding tangential velocity component, v_θ, required to satisfy conservation of mass.

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{(Eq. 6.35)}$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = -\frac{\partial(r v_r)}{\partial r} \qquad (1)$$

and with

$$r v_r = 2r^2 + 3r^3 \sin\theta$$

it follows that

$$\frac{\partial(r v_r)}{\partial r} = 4r + 9r^2 \sin\theta$$

Thus, Eq. (1) becomes

$$\frac{\partial v_\theta}{\partial \theta} = -(4r + 9r^2 \sin\theta) \qquad (2)$$

Equation (2) can be integrated with respect to θ to obtain

$$\int d v_\theta = -\int (4r + 9r^2 \sin\theta)\, d\theta + f(r)$$

or

$$v_\theta = \underline{\underline{-4r\theta - 9r^2 \cos\theta + f(r)}}$$

where $f(r)$ is an undetermined function of r.

6.22

6.22 The stream function for an incompressible flow field is given by the equation

$$\psi = 3x^2y - y^3$$

where the stream function has the units of m^2/s with x and y in meters. **(a)** Sketch the streamline(s) passing through the origin. **(b)** Determine the rate of flow across the straight path AB shown in Fig. P6.22.

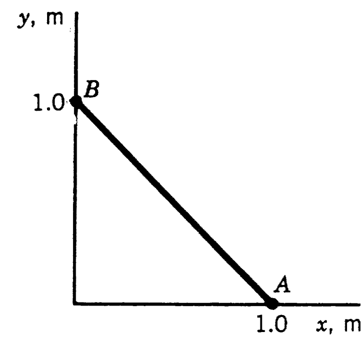
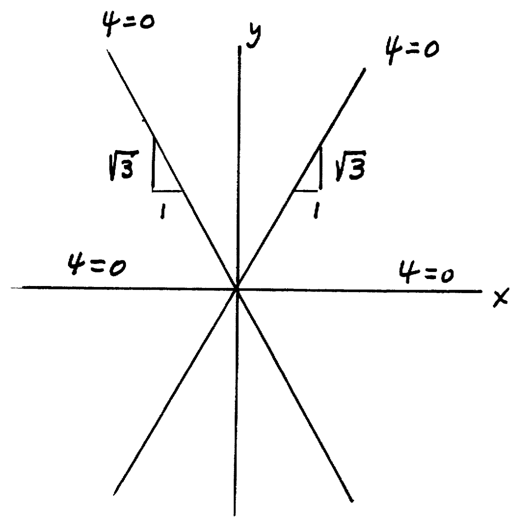


FIGURE P6.22

(a) Lines of constant ψ are streamlines. For $\psi = 3x^2y - y^3$ the streamline passing through the origin $(x=0, y=0)$ has a value $\psi = 0$. Thus, the equation for the streamlines through the origin is



$$0 = 3x^2y - y^3$$

or

$$y = \pm\sqrt{3}\, x$$

A sketch of these streamlines is shown in the figure.

(b) $$q = \psi_B - \psi_A$$

At B $\quad x = 0, \; y = 1\,m$ so that

$$\psi_B = 3(0)^2(1) - (1)^3 = -1\ m^3/s \text{ (per unit width)}$$

At A $\quad x = 1\,m, \; y = 0$ so that

$$\psi_A = 3(1)^2(0) - (0)^3 = 0$$

Thus,

$$q = \psi_B = \underline{\underline{-1\ m^3/s \text{ (per unit width)}}}$$

The negative sign indicates that the flow is from <u>right</u> to <u>left</u> as we look from A to B.

6.23

6.23 The streamlines in a certain incompressible, two-dimensional flow field are all concentric circles so that $v_r = 0$. Determine the stream function for **(a)** $v_\theta = Ar$ and for **(b)** $v_\theta = Ar^{-1}$, where A is a constant.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that with $v_r = 0$ it follows that $\frac{\partial \psi}{\partial \theta} = 0$

and therefore

$$\psi = f(r)$$

(a) For $v_\theta = Ar$

$$\frac{\partial \psi}{\partial r} = -Ar \qquad (1)$$

Integrate Eq.(1) with respect to r to obtain

$$\int d\psi = -\int Ar\,dr$$

or

$$\psi = -\frac{Ar^2}{2} + f_1(\theta)$$

However, since ψ is not a function of θ, it follows that

$$\underline{\underline{\psi = -\frac{Ar^2}{2} + C}}$$

where C is an arbitrary constant.

(b) Similarly, for $v_\theta = Ar^{-1}$

$$\int d\psi = -\int Ar^{-1}\,dr$$

or

$$\underline{\underline{\psi = -A\ln r + C}}$$

6.24*

6.24* The stream function for an incompressible, two-dimensional flow field is

$$\psi = 3x^2y + y$$

For this flow field plot several streamlines.

The equation for a streamline is found by setting ψ = constant in the equation for the stream function. Thus, for the given stream function

$$\psi = 3x^2y + y$$

it follows that the equation of a streamline is

$$y = \frac{\psi}{1 + 3x^2}$$

where various constant values can be assigned to ψ to obtain a family of streamlines. A program for calculating the x, y coordinates of various streamlines follows.

```
100 cls
110 print "***************************************************"
120 print "** This program calculates the x,y points for     **"
130 print "** various streamlines                            **"
150 print "***************************************************"
160 print
162 dim y(4)
165 print      "    x      y(Psi=1)     y(Psi=2)     y(Psi=3)     y(Psi=4)"
170 for x=-10 to 10
180 for psi=1 to 4
190 y(psi)=psi/(1+3*x^2)
200 next psi
210 print using "###.#    ###.####     ###.####     ###.####     ###.####";x,y(1),y(
2),y(3),y(4)
220 next x
```

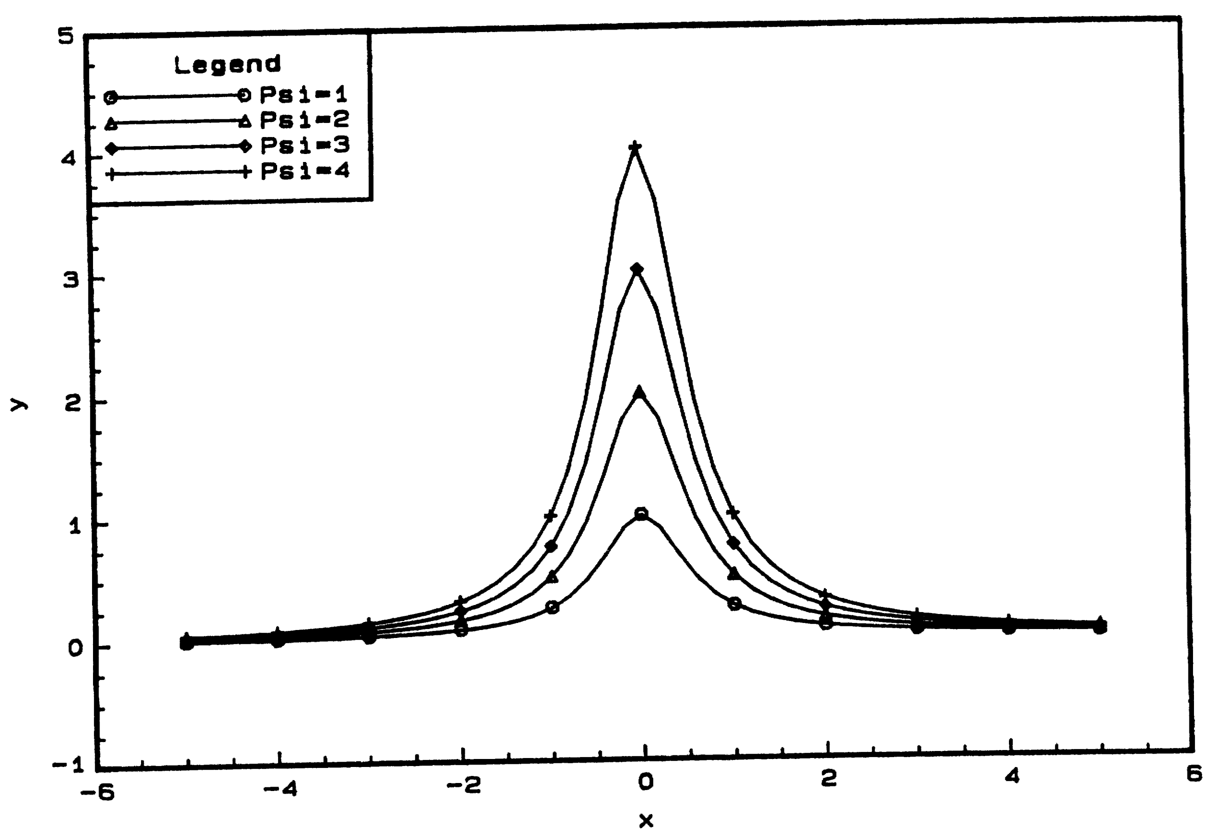
(con't)

6.24* (con't)

Tabulated results for $\psi = 1, 2, 3, 4$ and a plot showing the streamlines are given below.

****

x	y(Psi=1)	y(Psi=2)	y(Psi=3)	y(Psi=4)
-5.0	0.0132	0.0263	0.0395	0.0526
-4.5	0.0162	0.0324	0.0486	0.0648
-4.0	0.0204	0.0408	0.0612	0.0816
-3.5	0.0265	0.0530	0.0795	0.1060
-3.0	0.0357	0.0714	0.1071	0.1429
-2.5	0.0506	0.1013	0.1519	0.2025
-2.0	0.0769	0.1538	0.2308	0.3077
-1.5	0.1290	0.2581	0.3871	0.5161
-1.0	0.2500	0.5000	0.7500	1.0000
-0.5	0.5714	1.1429	1.7143	2.2857
0.0	1.0000	2.0000	3.0000	4.0000
0.5	0.5714	1.1429	1.7143	2.2857
1.0	0.2500	0.5000	0.7500	1.0000
1.5	0.1290	0.2581	0.3871	0.5161
2.0	0.0769	0.1538	0.2308	0.3077
2.5	0.0506	0.1013	0.1519	0.2025
3.0	0.0357	0.0714	0.1071	0.1429
3.5	0.0265	0.0530	0.0795	0.1060
4.0	0.0204	0.0408	0.0612	0.0816
4.5	0.0162	0.0324	0.0486	0.0648
5.0	0.0132	0.0263	0.0395	0.0526



6.25*

6.25* The stream function for an incompressible, two-dimensional flow field is

$$\psi = 2r^3 \sin 3\theta$$

For this flow field plot several streamlines for $0 \leq \theta \leq \pi/3$.

The equation for a streamline is found by setting ψ = constant in the equation for the stream function. Thus, for the given stream function

$$\psi = 2r^3 \sin 3\theta$$

it follows that the equation of a streamline is

$$r = \left(\frac{\psi}{2 \sin 3\theta}\right)^{1/3}$$

where various constant values can be assigned to ψ to obtain a family of streamlines. A program for calculating the x, y coordinates (where $x = r\cos\theta$ and $y = r\sin\theta$) of various streamlines follows.

```
100 cls
110 print "*************************************************"
120 print "** This program calculates the x,y points for     **"
130 print "** various streamlines                            **"
150 print "*************************************************"
160 print
162 dim psi(4),x(4),y(4)
164 print "    Psi=1              Psi=5              Psi=10             Psi=20"
165 print "   x      y          x      y          x      y          x      y"
166 pi=4*atn(1.0)
167 data 1,5,10,20
168 for i=1 to 4
169 read psi(i)
170 next i
175 for theta=pi/180 to 59*pi/180 step pi/45
180 for i=1 to 4
182 r=(psi(i)/(2*sin(3*theta)))^(1/3)
185 x(i)=r*cos(theta)
190 y(i)=r*sin(theta)
200 next i
210 print using "#.###   #.###   #.###   #.###   #.###   #.###   #.###   #.###";
x(1),y(1),x(2),y(2),x(3),y(3),x(4),y(4)
220 next theta
```

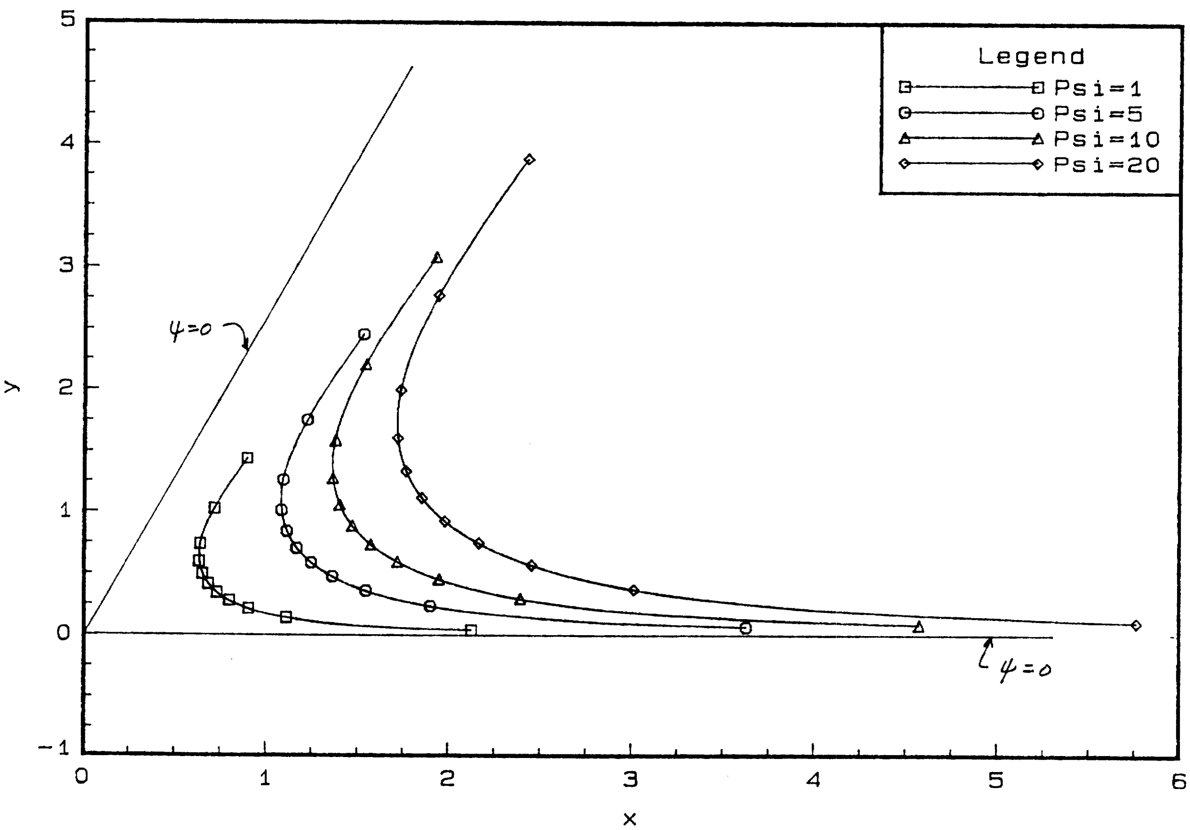
(con't)

6.25* (con't)

Tabulated results for $\psi = 1, 5, 10, 20$ and a plot showing the streamlines are given below.

```
***************************************************
** This program calculates the x,y points for   **
** various streamlines                           **
***************************************************
```

Psi=1 x	Psi=1 y	Psi=5 x	Psi=5 y	Psi=10 x	Psi=10 y	Psi=20 x	Psi=20 y
2.122	0.037	3.628	0.063	4.571	0.080	5.759	0.101
1.241	0.109	2.122	0.186	2.673	0.234	3.368	0.295
1.020	0.162	1.744	0.276	2.197	0.348	2.769	0.439
0.902	0.208	1.543	0.356	1.944	0.449	2.450	0.566
0.826	0.252	1.412	0.432	1.779	0.544	2.241	0.685
0.770	0.296	1.317	0.505	1.659	0.637	2.090	0.802
0.728	0.339	1.244	0.580	1.568	0.731	1.975	0.921
0.695	0.385	1.188	0.658	1.496	0.829	1.885	1.045
0.668	0.434	1.143	0.742	1.440	0.935	1.814	1.178
0.649	0.489	1.109	0.836	1.397	1.053	1.760	1.327
0.635	0.552	1.086	0.944	1.368	1.190	1.724	1.499
0.630	0.630	1.077	1.077	1.357	1.357	1.710	1.710
0.638	0.734	1.090	1.254	1.374	1.580	1.731	1.991
0.672	0.892	1.150	1.526	1.449	1.923	1.825	2.422
0.802	1.235	1.372	2.112	1.728	2.662	2.178	3.353



6.26

6.26 A two-dimensional flow field for a nonviscous, incompressible fluid is described by the velocity components

$$u = U_0 + 2y$$
$$v = 0$$

where U_0 is a constant. If the pressure at the origin (Fig. P6.26) is p_0, determine an expression for the pressure at **(a)** point A, and **(b)** point B. Explain clearly how you obtained your answer. Assume the units are consistent and body forces may be neglected.

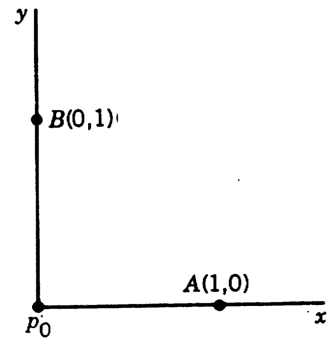


FIGURE P6.26

Check to see if flow is irrotational. Since

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and for the given velocity distribution, $\frac{\partial v}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 2$, it follows that $\omega_z \neq 0$. Since flow is not irrotational cannot apply the Bernoulli equation between any two points in the flow field.

(a) Since $v = 0$, the origin and point A are on the same streamline. Thus,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} \qquad (1)$$

At the origin $V_0 = U_0$ and at A $V_A = U_0$ so that from Eq. (1)

$$\underline{\underline{p_A = p_0}}$$

(b) Point B is not on same streamline as origin so cannot apply Bernoulli equation between B and 0. To find p_B use the y-component of Euler's equations:

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] \qquad \text{(Eq. 6.51b)}$$

Since $v = 0$ and $g_y = 0$,

$$\frac{\partial p}{\partial y} = 0$$

so that

$$\underline{\underline{p_B = p_0}}$$

6.27

6.27 In a certain two-dimensional flow field the velocity is constant with components $u = -4$ ft/s and $v = -2$ ft/s. Determine the corresponding stream function and velocity potential for this flow field. Sketch the equipotential line $\phi = 0$ which passes through the origin of the coordinate system.

From the definition of the stream function

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

so that for the velocity components given

$$\frac{\partial \psi}{\partial y} = -4 \qquad (1)$$

$$\frac{\partial \psi}{\partial x} = 2 \qquad (2)$$

Integrate Eq.(1) with respect to y to obtain

$$\int d\psi = \int -4\, dy + f_1(x)$$

or

$$\psi = -4y + f_1(x) \qquad (3)$$

Similarly, integrate Eq.(2) with respect to x to obtain

$$\int d\psi = \int 2\, dx + f_2(y)$$

or

$$\psi = 2x + f_2(y) \qquad (4)$$

Thus, to satisfy both Eqs.(3) and (4)

$$\psi = \underline{\underline{2x - 4y + C}}$$

where C is an arbitrary constant.

From the definition of the velocity potential

$$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y} \qquad \text{(Eqs. 6.64)}$$

so that for the velocity components given

$$\frac{\partial \phi}{\partial x} = -4 \qquad (5)$$

$$\frac{\partial \phi}{\partial y} = -2 \qquad (6)$$

(con't)

Integrate Eq. (5) with respect to x to obtain

$$\int d\phi = \int -4\,dx + f_3(y)$$

or

$$\phi = -4x + f_3(y) \tag{7}$$

Integrate Eq. (6) with respect to y to obtain

$$\int d\phi = \int -2\,dy + f_4(x)$$

or

$$\phi = -2y + f_4(x) \tag{8}$$

Thus, to satisfy both Eqs. (7) and (8)

$$\phi = \underline{\underline{-4x - 2y + C}} \tag{9}$$

where C is an arbitrary constant.

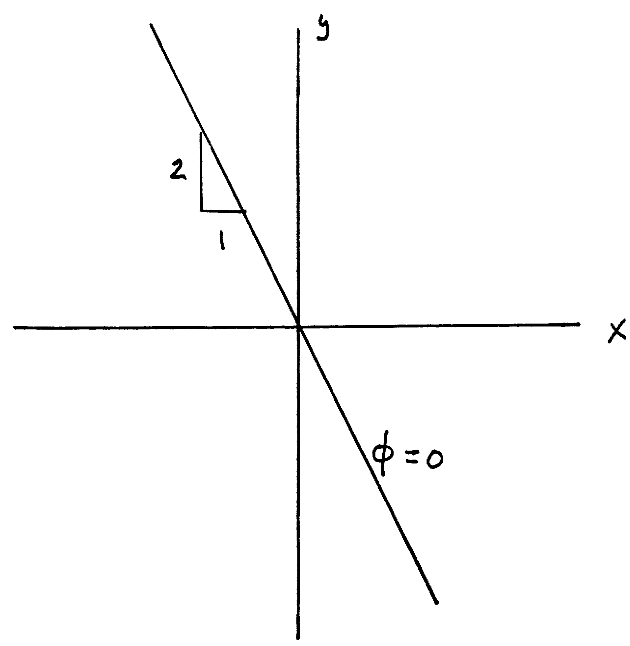
Since the equipotential line, $\phi = 0$, passes through the origin ($x = y = 0$), then $C = 0$ in Eq. (9) so that the equation of the $\phi = 0$ equipotential line is

$$2y = -4x$$

or

$$y = -2x$$

A sketch of this line is shown in the figure.



6.28

6.28 The velocity potential for a given two-dimensional flow field is

$$\phi = (\tfrac{5}{3})x^3 - 5xy^2$$

Show that the continuity equation is satisfied and determine the corresponding stream function.

To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For the given velocity potential,

$$u = \frac{\partial \phi}{\partial x} = (3)(\tfrac{5}{3})x^2 - 5y^2 = 5x^2 - 5y^2$$

and

$$\frac{\partial u}{\partial x} = 10x$$

Similarly,

$$v = \frac{\partial \phi}{\partial y} = -10xy$$

and

$$\frac{\partial v}{\partial y} = -10x$$

Thus,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10x - 10x = 0$$

and the continuity equation is satisfied.

Since

$$u = \frac{\partial \psi}{\partial y} = 5x^2 - 5y^2$$

and integrating with respect to y gives

$$\int d\psi = \int (5x^2 - 5y^2)\, dy$$

or

$$\psi = 5(x^2 y - \frac{y^3}{3}) + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = -10xy$$

and integration with respect to x gives

$$\psi = 5x^2 y + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2),

$$\underline{\underline{\psi = 5x^2 y - \frac{5}{3}y^3 + C}}$$ with C a constant.

6.29

6.29 Determine the stream function corresponding to the velocity potential

$$\phi = x^3 - 3xy^2$$

Sketch the streamline $\psi = 0$, which passes through the origin.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 3x^2 - 3y^2$$

Integrate with respect to y to obtain

$$\int d\psi = \int (3x^2 - 3y^2)\, dy$$

or

$$\psi = 3\left(x^2 y - \frac{y^3}{3}\right) + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -6xy$$

and integrating with respect to x yields

$$\int d\psi = \int 6xy\, dx$$

or

$$\psi = 3x^2 y + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 3x^2 y - y^3 + C$$

where C is an arbitrary constant. Since the streamline $\psi = 0$ passes through the origin $(x=0, y=0)$ it follows that $C = 0$ and

$$\underline{\underline{\psi = 3x^2 y - y^3}} \qquad (3)$$

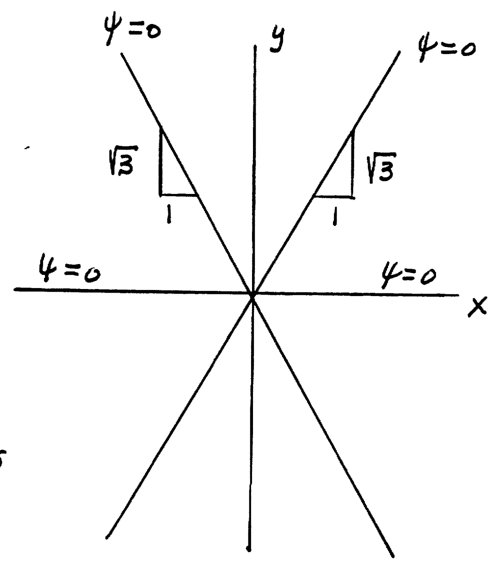
The equation of the streamline passing through the origin is found by setting $\psi = 0$ in Eq. (3) to yield

$$y(3x^2 - y^2) = 0$$

which is satisfied for $y = 0$

and

$$y = \pm\sqrt{3}\, x$$



A sketch of the $\psi = 0$ streamlines are shown in the figure.

6.30

6.30 A certain flow field is described by the stream function

$$\psi = A\theta + B r \sin\theta$$

where A and B are positive constants. Determine the corresponding velocity potential and locate any stagnation points in this flow field.

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{A}{r} + B\cos\theta \qquad (1)$$

Integrate with respect to r to obtain

$$\int d\phi = \int \left(\frac{A}{r} + B\cos\theta\right) dr$$

or

$$\phi = A \ln r + B r \cos\theta + f_1(\theta) \qquad (2)$$

Similarly,

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -B\sin\theta \qquad (3)$$

and

$$\int d\phi = -\int B r \sin\theta \, d\theta$$

or

$$\phi = B r \cos\theta + f_2(r) \qquad (4)$$

To satisfy both Eqs. (2) and (4)

$$\underline{\underline{\phi = A \ln r + B r\cos\theta + C}}$$

where C is an arbitrary constant.

Stagnation points occur where $v_r = 0$ and $v_\theta = 0$. From Eq. (3) $v_\theta = 0$ at $\theta = 0$ and $\theta = \pi$. From Eq. (1) with $\theta = 0$

$$v_r = \frac{A}{r} + B$$

so that $v_r = 0$ for $r = -\frac{A}{B}$. However, since A and B are both positive constants this result indicates a negative value for r which is not defined.

At $\theta = \pi$

$$v_r = \frac{A}{r} + B\cos\pi = \frac{A}{r} - B$$

so that $v_r = 0$ for $r = \frac{A}{B}$. Thus, a stagnation point occurs at

$$\underline{\underline{\theta = \pi \quad \text{and} \quad r = \frac{A}{B}}}$$

6.31

6.31 It is known that the velocity distribution for two-dimensional flow of a viscous fluid between wide parallel plates (Fig. P6.31) is parabolic; that is

$$u = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

with $v = 0$. Determine, if possible, the corresponding stream function and velocity potential.

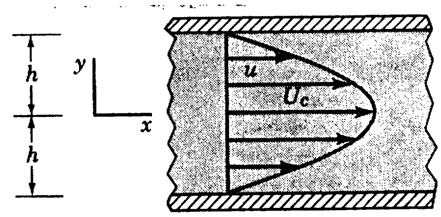


FIGURE P6.31

To determine the stream function let

$$u = \frac{\partial \psi}{\partial y} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

and integrate with respect to y to obtain

$$\int d\psi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dy$$

or

$$\psi = U_c\left[y - \frac{y^3}{3h^2}\right] + f_1(x)$$

Since $v = -\frac{\partial \psi}{\partial x} = 0$, ψ is not a function of x so that

$$\psi = U_c\, y\left[1 - \frac{1}{3}\left(\frac{y}{h}\right)^2\right] + C$$

where C is an arbitrary constant.

To determine the velocity potential let

$$u = \frac{\partial \phi}{\partial x} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$$

and integrate with respect to x to obtain

$$\int d\phi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dx$$

or

$$\phi = U_c\left[x - \left(\frac{y}{h}\right)^2 x\right] + f_2(y)$$

However,

$$v = \frac{\partial \phi}{\partial y} = 0 = -\frac{2U_c\, x\, y}{h^2} + \frac{\partial f_2(y)}{\partial y}$$

and this relationship cannot be satisfied for all values of x and y. Thus, there is not a velocity potential that describes this flow (the flow is not irrotational).

6.32

6.32 The velocity potential for a certain inviscid flow field is

$$\phi = -(3x^2y - y^3)$$

where ϕ has the units of ft^2/s when x and y are in feet. Determine the pressure difference (in psi) between the points (1, 2) and (4, 4), where the coordinates are in feet, if the fluid is water and elevation changes are negligible.

Since the flow field is described by a velocity potential the flow is irrotational and the Bernoulli equation can be applied between any two points. Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \quad (1)$$

Also,

$$u = \frac{\partial \phi}{\partial x} = -6xy \qquad v = \frac{\partial \phi}{\partial y} = -3x^2 + 3y^2$$

At $x = 1\ ft$, $y = 2\ ft$

$$u_1 = -6(1)(2) = -12\ \frac{ft}{s}$$

$$v_1 = -3(1)^2 + 3(2)^2 = 9\ \frac{ft}{s}$$

So that $V_1^2 = u_1^2 + v_1^2 = \left(-12\ \frac{ft}{s}\right)^2 + \left(9\ \frac{ft}{s}\right)^2 = 225\left(\frac{ft}{s}\right)^2$

At $x = 4\ ft$, $y = 4\ ft$

$$u_2 = -6(4)(4) = -96\ \frac{ft}{s}$$

$$v_2 = -3(4)^2 + 3(4)^2 = 0$$

So that $V_2^2 = \left(-96\ \frac{ft}{s}\right)^2$

Thus, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\frac{\gamma}{g}\left[V_2^2 - V_1^2\right]$$

$$= \frac{1}{2}\frac{\left(62.4\ \frac{lb}{ft^3}\right)}{\left(32.2\ \frac{ft}{s^2}\right)}\left[\left(-96\ \frac{ft}{s}\right)^2 - 225\left(\frac{ft}{s}\right)^2\right]$$

$$= 8710\ \frac{lb}{ft^2} = \left(8710\ \frac{lb}{ft^2}\right)\left(\frac{ft^2}{144\ in.^2}\right) = \underline{\underline{60.5\ psi}}$$

6.33

6.33 Consider the incompressible, two-dimensional flow of a nonviscous fluid between the boundaries shown in Fig. P6.33. The velocity potential for this flow field is

$$\phi = x^2 - y^2$$

(a) Determine the corresponding stream function. **(b)** What is the relationship between the discharge, q, (per unit width normal to plane of paper) passing between the walls and the coordinates x_i, y_i of any point on the curved wall? Neglect body forces.

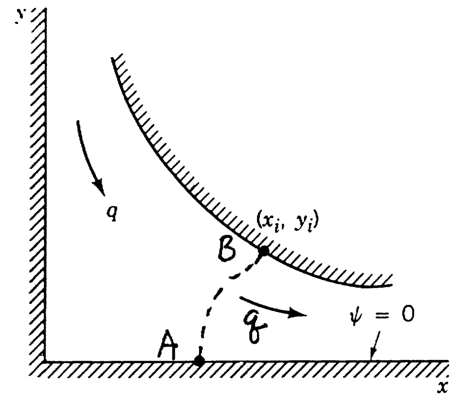


FIGURE P6.33

(a) $$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 2x$$

To determine ψ integrate with respect to y to obtain

$$\int d\psi = \int 2x\, dy$$

or

$$\psi = 2xy + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -2y$$

so that

$$\int d\psi = \int 2y\, dx$$

or

$$\psi = 2xy + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 2xy + C$$

where C is an arbitrary constant. Since $\psi = 0$ along $y = 0$, $C = 0$ and

$$\underline{\underline{\psi = 2xy}} \qquad (3)$$

(b) The discharge, q, passing through any surface connecting the two walls, such as AB (see figure), is

$$q = \psi_B - \psi_A$$

From Eq. (3), $\psi_A = 0$ and $\psi_B = 2x_i y_i$. It follows that

$$\underline{\underline{q = 2x_i y_i}}$$

6.34

6.34 The stream function for a two-dimensional, nonviscous, incompressible flow field is given by the expression

$$\psi = -2(x - y)$$

where the stream function has the units of ft^2/s with x and y in feet. **(a)** Is the continuity equation satisfied? **(b)** Is the flow field irrotational? If so, determine the corresponding velocity potential. **(c)** Determine the pressure gradient in the horizontal x direction at the point $x = 2$ ft, $y = 2$ ft.

(a) To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For the stream function given,

$$u = \frac{\partial \psi}{\partial y} = 2\ \frac{ft}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 2\ \frac{ft}{s}$$

so that

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

and the continuity equation is satisfied. Yes.
(Note: When a flow field is defined by a stream function the continuity equation is always identically satisfied.)

(b) Since

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and

$$\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial v}{\partial x} = 0$$

it follows that $\omega_z = 0$ and the flow field is irrotational. Yes.

Thus,

$$u = \frac{\partial \phi}{\partial x} = 2 \qquad v = \frac{\partial \phi}{\partial y} = 2$$

and integration yields

$$\phi = 2(x + y) + C$$

Where C is an arbitrary constant.

(c) With the x-axis horizontal, $g_x = 0$, and

$$-\frac{\partial p}{\partial x} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.51a)}$$

and at $x = 2ft$, $y = 2ft$

$$\frac{\partial p}{\partial x} = -\rho\left[2\ \frac{ft}{s}(0) + 2\ \frac{ft}{s}(0)\right] = \underline{0}$$

6.35

6.35 In a certain steady, two-dimensional flow field the fluid may be assumed to be ideal and the weight of the fluid (specific weight = 50 lb/ft³) is the only body force. The x component of velocity is known to be $u = 6x$ which gives the velocity in ft/s when x is measured in feet, and the y component of velocity is known to be a function of only y. The y axis is vertical, and at the origin the velocity is zero. **(a)** Determine the y component of velocity so that the continuity equation is satisfied. **(b)** Can the difference in pressures between the points x = 1 ft, y = 1 ft and x = 1 ft, y = 4 ft be determined from the Bernoulli equation? If so, determine the value in lb/ft². If not, explain why not.

(a) To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and with $\frac{\partial u}{\partial x} = 6$ it follows that

$$\frac{\partial v}{\partial y} = -6 \qquad (1)$$

Equation (1) can be integrated with respect to y to yield

$$v = -6y + f_1(x)$$

Since v is not a function of x and is zero at the origin,

$$\underline{\underline{v = -6y}}$$

(b) The Bernoulli equation can be applied between any two points if the flow is irrotational. Since

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

and $\frac{\partial u}{\partial y} = 0$, $\frac{\partial v}{\partial x} = 0$ it follows that $\omega_z = 0$ and the flow is irrotational. Thus,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

or

$$p_1 - p_2 = \frac{1}{2}\frac{\gamma}{g}\left(V_2^2 - V_1^2\right) + \gamma\left(z_2 - z_1\right) \qquad (2)$$

With

$$V_1^2 = u_1^2 + v_1^2 = \left[6(1)\frac{ft}{s}\right]^2 + \left[-6(1)\frac{ft}{s}\right]^2 = 72\frac{ft^2}{s^2}$$

and

$$V_2^2 = u_2^2 + v_2^2 = \left[6(1)\frac{ft}{s}\right]^2 + \left[-6(4)\frac{ft}{s}\right]^2 = 612\frac{ft^2}{s^2}$$

Thus, from Eq. (2)

$$p_1 - p_2 = \frac{1}{2}\left(\frac{50\frac{lb}{ft^3}}{32.2\frac{ft}{s^2}}\right)\left(612\frac{ft^2}{s^2} - 72\frac{ft^2}{s^2}\right) + \left(50\frac{lb}{ft^3}\right)\left(4\,ft - 1\,ft\right)$$

$$= \underline{\underline{569\frac{lb}{ft^2}}}$$

6.36

6.36 The velocity potential for a certain inviscid, incompressible flow field is given by the equation

$$\phi = 2x^2y - (\tfrac{2}{3})y^3$$

where ϕ has the units of m^2/s when x and y are in meters. Determine the pressure at the point $x = 2$ m, $y = 2$ m if the pressure at $x = 1$ m, $y = 1$ m is 200 kPa. Elevation changes can be neglected and the fluid is water.

Since the flow is irrotational,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

with $V^2 = u^2 + v^2$. For the velocity potential given,

$$u = \frac{\partial \phi}{\partial x} = 4xy \qquad v = \frac{\partial \phi}{\partial y} = 2x^2 - 2y^2$$

At point 1 let $x = 1\,m$ and $y = 1\,m$ so that

$$u_1 = 4(1)(1) = 4\,\frac{m}{s} \qquad v_1 = 2(1)^2 - 2(1)^2 = 0$$

and

$$V_1^2 = \left(4\,\frac{m}{s}\right)^2 = 16\,\frac{m^2}{s^2}$$

At point 2 $x = 2m$ and $y = 2m$ so that

$$u_2 = 4(2)(2) = 16\,\frac{m}{s} \qquad v_2 = 2(2)^2 - 2(2)^2 = 0$$

and

$$V_2^2 = \left(16\,\frac{m}{s}\right)^2 = 256\,\frac{m^2}{s^2}$$

Thus, from Eq. (1)

$$p_2 = p_1 + \frac{\gamma}{2g}\left(V_1^2 - V_2^2\right)$$

$$= 200 \times 10^3\,\frac{N}{m^2} + \frac{\left(9.80 \times 10^3\,\frac{N}{m^3}\right)}{2\left(9.81\,\frac{m}{s^2}\right)}\left(16\,\frac{m^2}{s^2} - 256\,\frac{m^2}{s^2}\right)$$

$$= \underline{\underline{80.1\ kPa}}$$

6.37

6.37 (a) Determine the velocity potential and the stream function for a steady, uniform, incompressible, inviscid, two-dimensional flow that makes an angle of 30° with the horizontal x-axis. (b) Determine an expression for the pressure gradient in the vertical y direction. What is the physical interpretation of this result?

(a) From Eqs. 6.80 and 6.81

$$\phi = U(x\cos\alpha + y\sin\alpha) \qquad \text{(Eq. 6.80)}$$

and for $\alpha = 30°$

$$\phi = U(x\cos 30° + y\sin 30°) = \underline{\underline{U(0.866x + 0.500y)}}$$

Similarly,

$$\psi = U(y\cos\alpha - x\sin\alpha) \qquad \text{(Eq. 6.81)}$$

and for $\alpha = 30°$

$$\psi = U(y\cos 30° - x\sin 30°) = \underline{\underline{U(0.866y - 0.500x)}}$$

(b) Since

$$u = \frac{\partial\phi}{\partial x} \quad \text{and} \quad v = \frac{\partial\phi}{\partial y}$$

it follows that

$$u = 0.866U \quad \text{and} \quad v = 0.500U$$

From the Euler equation in the vertical y-direction

$$\rho g_y - \frac{\partial p}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \qquad \text{(Eq. 6.51b)}$$

and with v = constant and $g_y = -g$

$$\frac{\partial p}{\partial y} = -\rho g$$

or

$$\underline{\underline{\frac{\partial p}{\partial y} = -\gamma}}$$

This result indicates that the pressure distribution is hydrostatic. This is not a surprising result since the Bernoulli equation indicates that if there is no change in velocity the change in pressure is simply due to the weight of the fluid, i.e., a hydrostatic variation.

6.38

6.38 The streamlines for an incompressible, inviscid, two-dimensional flow field are all concentric circles and the velocity varies directly with the distance from the common center of the streamlines; that is

$$v_\theta = Kr$$

where K is a constant. **(a)** For this *rotational* flow determine, if possible, the stream function. **(b)** Can the pressure difference between the origin and any other point be determined from the Bernoulli equation? Explain.

(a) $$v_\theta = -\frac{\partial \psi}{\partial r} = Kr \qquad (1)$$

Integrate Eq. (1) with respect to r to obtain

$$\int d\psi = -\int Kr\,dr$$

or

$$\psi = -\frac{Kr^2}{2} + f_1(\theta)$$

Since

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0$$

it follows that ψ is not a function of θ and therefore

$$\underline{\underline{\psi = -\frac{Kr^2}{2} + C}}$$

where C is an arbitrary constant.

(b) The flow is rotational and therefore the Bernoulli equation cannot be applied between the origin and any point, since these points are not on the same streamline. <u>No</u>.
(Refer to discussion associated with derivation of Eq. 6.57.)

6.39

6.39 The velocity potential

$$\phi = -k(x^2 - y^2) \qquad (k = \text{constant})$$

may be used to represent the flow against an infinite plane boundary as illustrated in Fig. P6.39. For flow in the vicinity of a stagnation point it is frequently assumed that the pressure gradient along the surface is of the form

$$\frac{\partial p}{\partial x} = Ax$$

where A is a constant. Use the given velocity potential to show that this is true.

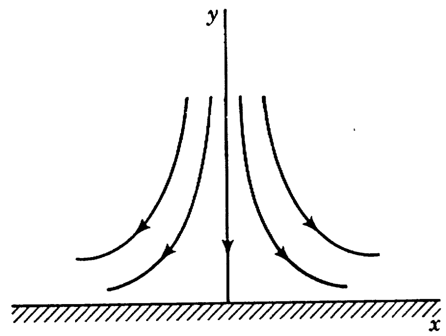


FIGURE P6.39

For the velocity potential given

$$u = \frac{\partial \phi}{\partial x} = -2kx \qquad (1)$$

$$v = \frac{\partial \phi}{\partial y} = -2ky \qquad (2)$$

and the stagnation point occurs at the origin.
For this steady, two-dimensional flow

$$-\frac{\partial p}{\partial x} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.51a)}$$

and along the surface $(y=0)$ $v=0$ so that

$$\frac{\partial p}{\partial x} = \rho u \frac{\partial u}{\partial x} \qquad (3)$$

From Eq. (1) $u = -2kx$ and therefore

$$\frac{\partial u}{\partial x} = -2k$$

and Eq. (3) becomes

$$\frac{\partial p}{\partial x} = \rho(-2kx)(-2k) = 4k^2 x$$

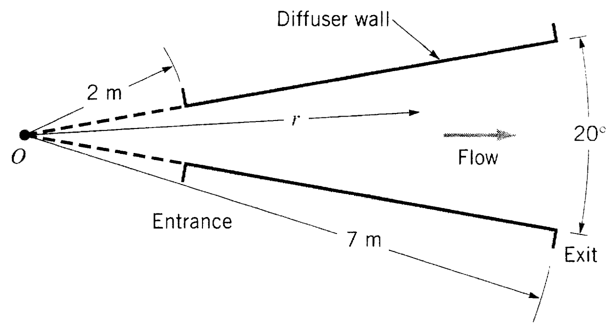
or

$$\underline{\underline{\frac{\partial p}{\partial x} = A\,x}}$$

where $A = 4k^2$.

6.40

6.40 Water flows through a two-dimensional diffuser having a 20° expansion angle as shown in Fig. P6.40. Assume that the flow in the diffuser can be treated as a radial flow emanating from a source at the origin O. **(a)** If the velocity at the entrance is 20 m/s, determine an expression for the pressure gradient along the diffuser walls. **(b)** What is the pressure rise between the entrance and exit?



(a) For radial flow

$$v_r = \frac{m}{2\pi r} \qquad \text{(see Table 6.1)}$$

For $r = 2\,m$ $\quad v_r = 20\ m/s$ so that

$$m = 2\pi r v_r = 2\pi\,(2m)\left(20\,\frac{m}{s}\right) = 80\pi\ \frac{m^2}{s}$$

From the Bernoulli equation

$$p + \frac{1}{2}\rho v_r^2 = \text{constant}$$

So that

$$\frac{\partial p}{\partial r} = -\rho v_r \frac{\partial v_r}{\partial r} \qquad (1)$$

Since $v_r = \dfrac{m}{2\pi r}$ then $\dfrac{\partial v_r}{\partial r} = -\dfrac{m}{2\pi r^2}$

and Eq. (1) can be written as

$$\frac{\partial p}{\partial r} = -\rho\left(\frac{m}{2\pi r}\right)\left(-\frac{m}{2\pi r^2}\right) = \frac{\rho m^2}{4\pi^2 r^3}$$

Thus, for $\rho = 999\ \frac{kg}{m^3}$, $m = 80\pi\ \frac{m^2}{s}$

$$\frac{\partial p}{\partial r} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(80\pi\,\frac{m^2}{s}\right)^2}{4\pi^2 r^3} = \frac{1.60\times10^6}{r^3}\,\frac{N}{m^3} = \underline{\underline{\frac{1.60\times10^3}{r^3}\,\frac{kPa}{m}}}$$

(b) Since $(v_r)_{entrance} = 20\ m/s$ and

$$(v_r)_{exit} = \frac{m}{2\pi r} = \frac{80\pi\ \frac{m^2}{s}}{2\pi\ (7m)} = 5.71\ \frac{m}{s}$$

then from the Bernoulli equation

$$p_{exit} - p_{entrance} = \frac{1}{2}\rho\left[(v_r)^2_{entrance} - (v_r)^2_{exit}\right]$$

$$= \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left[\left(20\,\frac{m}{s}\right)^2 - \left(5.71\,\frac{m}{s}\right)^2\right] = \underline{184\ kPa}$$

6.41

6.41 An ideal fluid flows between the inclined walls of a two-dimensional channel into a sink located at the origin (Fig. P6.41). The velocity potential for this flow field is

$$\phi = \frac{m}{2\pi} \ln r$$

where m is a constant. **(a)** Determine the corresponding stream function. Note that the value of the stream function along the wall OA is zero. **(b)** Determine the equation of the streamline passing through the point B, located at $x = 1$, $y = 4$.

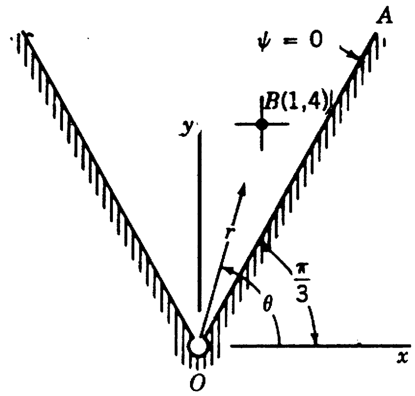


FIGURE P6.41

(a) $$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{m}{2\pi r} \qquad (1)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int \frac{m}{2\pi} d\theta$$

or

$$\psi = \frac{m\theta}{2\pi} + f_1(r)$$

Since

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0 \qquad (2)$$

ψ is not a function of r so Eq. (2) becomes

$$\psi = \frac{m\theta}{2\pi} + C$$

where C is a constant. Also, $\psi = 0$ for $\theta = \frac{\pi}{3}$ so that

$$C = -\frac{m}{6}$$

and

$$\underline{\underline{\psi = m\left(\frac{\theta}{2\pi} - \frac{1}{6}\right)}} \qquad (3)$$

(b) At B $\tan\theta = \frac{4}{1}$ so that $\theta = 1.33 \text{ rad}$. From Eq. (3) the value of ψ passing through this point is

$$\psi = m\left(\frac{1.33}{2\pi} - \frac{1}{6}\right) = 0.0450\,m$$

and therefore the equation of the streamline passing through B is

$$0.0450\,m = m\left(\frac{\theta}{2\pi} - \frac{1}{6}\right)$$

or

$$\underline{\underline{\theta = 1.33 \text{ rad}}}$$

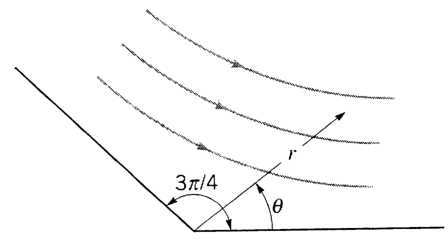
(Note: It can be seen from Eq. (3) that the streamlines are all straight lines passing through the origin.)

6.42

6.42 It is suggested that the velocity potential for the flow of an incompressible, nonviscous, two-dimensional flow along the wall shown in Fig. P6.42 is

$$\phi = r^{4/3} \cos \tfrac{4}{3}\theta$$

Is this a suitable velocity potential for flow along the wall? Explain.



If this is a suitable ϕ the corresponding ψ must have a constant value along the wall (since the wall must correspond to a streamline).

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} = \frac{4}{3} r^{1/3} \cos \frac{4}{3}\theta \qquad (1)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int \frac{4}{3} r^{4/3} \cos \frac{4}{3}\theta$$

or

$$\psi = r^{4/3} \sin \frac{4}{3}\theta + f_1(r) \qquad (2)$$

Similarly,

$$v_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{4}{3} r^{1/3} \sin \frac{4}{3}\theta$$

and

$$\int d\psi = \int \frac{4}{3} r^{1/3} \sin \frac{4}{3}\theta \, dr$$

or

$$\psi = r^{\frac{4}{3}} \sin \frac{4}{3}\theta + f_2(\theta) \qquad (3)$$

To satisfy both Eqs. (2) and (3)

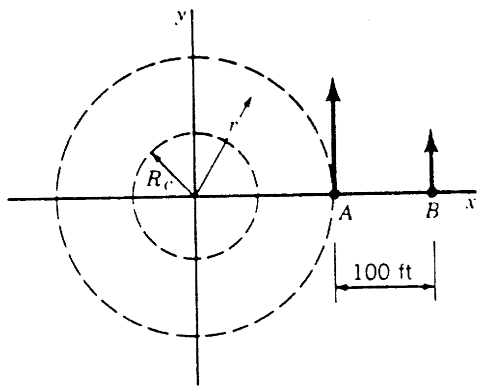
$$\psi = r^{4/3} \sin \frac{4}{3}\theta + C$$

where C is an arbitrary constant.

Along one section of the wall, $\theta = 0$, and $\psi = C$. Along the other section $\theta = \frac{3\pi}{4}$ and $\psi = C$. Thus, ψ has a constant value along the wall and the given velocity potential can be used to represent flow along the wall. Yes.

6.43

6.43 As illustrated in Fig. P6.43 a tornado can be approximated by a free vortex of strength Γ for $r > R_c$, where R_c is the radius of the core. Velocity measurements at points A and B indicate that $V_A = 125$ ft/s and $V_B = 60$ ft/s. Determine the distance from point A to the center of the tornado. Why can the free vortex model not be used to approximate the tornado throughout the flow field ($r \geq 0$)?



■ FIGURE P6.43

For a free vortex

$$v_\theta = \frac{K}{r} \qquad \text{(Eq. 6.86)}$$

Thus, at r_A, $v_\theta = 125 \frac{ft}{s}$, so that $K = 125\, r_A$

and at r_B, $v_\theta = 60 \frac{ft}{s}$, so that $K = 60\, r_B$.

Therefore,

$$125\, r_A = 60\, r_B$$

and since

$$r_B - r_A = 100 \text{ ft}$$

it follows that

$$125\, r_A = 60(100 + r_A)$$

or

$$\underline{\underline{r_A = 92.3 \text{ ft}}}$$

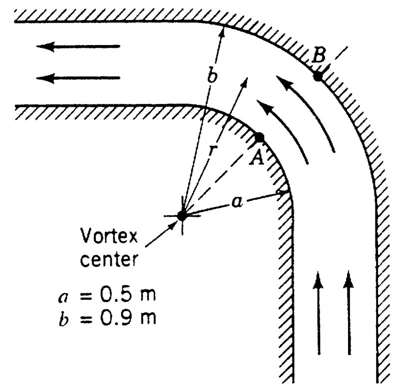
The free vortex cannot be used to approximate a tornado throughout the flow field since at $r=0$ the velocity becomes infinite.

6.44

6.44 The velocity distribution in a horizontal, two-dimensional bend through which an ideal fluid flows can be approximated with a free vortex as shown in Fig. P6.44. Show how the discharge (per unit width normal to plane of paper) through the channel can be expressed as

$$q = C\sqrt{\frac{\Delta p}{\rho}}$$

where $\Delta p = p_B - p_A$. Determine the value of the constant C for the bend dimensions given.



■ FIGURE P6.44

For free vortex $v_\theta = \frac{K}{r}$, so that

$$v_{\theta A} = \frac{K}{a} \qquad v_{\theta B} = \frac{K}{b}$$

From the Bernoulli equation

$$\frac{p_A}{\gamma} + \frac{v_{\theta A}^2}{2g} = \frac{p_B}{\gamma} + \frac{v_{\theta B}^2}{2g}$$

or

$$\Delta p = p_B - p_A = \frac{1}{2}\frac{\gamma}{g}\left(v_{\theta A}^2 - v_{\theta B}^2\right) = \frac{1}{2}\rho K^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

Since

$$q = \int_a^b v_\theta\, dr = K\int_a^b \frac{dr}{r} = K \ln\frac{b}{a}$$

or

$$K = \frac{q}{\ln\frac{b}{a}}$$

Thus,

$$\Delta p = \frac{1}{2}\rho \frac{q^2}{\left(\ln\frac{b}{a}\right)^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

or

$$q = \sqrt{2}\ln\frac{b}{a}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)^{-1/2}\sqrt{\frac{\Delta p}{\rho}}$$

and therefore

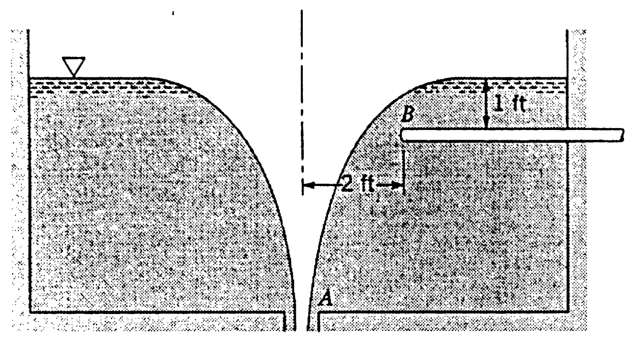
$$q = C\sqrt{\frac{\Delta p}{\rho}}$$

with

$$C = \frac{\sqrt{2}\ln\frac{b}{a}}{\sqrt{\frac{1}{a^2} - \frac{1}{b^2}}} = \frac{\sqrt{2}\ln\frac{0.9\text{ m}}{0.5\text{ m}}}{\sqrt{\frac{1}{(0.5\text{ m})^2} - \frac{1}{(0.9\text{ m})^2}}} = \underline{\underline{0.500\text{ m}}}$$

6.45

6.45 When water discharges from a tank through an opening in its bottom, a vortex may form with a curved surface profile as shown in Fig. P6.45 and Video V6.2. Assume that the velocity distribution in the vortex is the same as that for a free vortex. At the same time the water is being discharged from the tank at point A it is desired to discharge a small quantity of water through the pipe B. As the discharge through A is increased, the strength of the vortex, as indicated by its circulation, is increased. Determine the maximum strength that the vortex can have in order that no air is sucked in at B. Express your answer in terms of the circulation. Assume that the fluid level in the tank at a large distance from the opening at A remains constant and viscous effects are negligible.



■ FIGURE P6.45

From Example 6.6,

$$z_s = -\frac{\Gamma^2}{8\pi^2 r^2 g}$$

Air will be sucked into pipe when $z_s = -1\ ft$ for $r = 2\ ft$.

Thus,

$$\Gamma^2 = -8\pi^2 r^2 g\, z_s = -8\pi^2 (2\ ft)^2 \left(32.2\ \frac{ft}{s^2}\right)(-1\ ft)$$

or

$$|\Gamma| = \underline{\underline{101\ \frac{ft^2}{s}}}$$

6.46

6.46 The streamlines in a particular two-dimensional flow field are all concentric circles, as shown in Fig. P6.46. The velocity is given by the equation $v_\theta = \omega r$ where ω is the angular velocity of the rotating mass of fluid. Determine the circulation around the path *ABCD*.

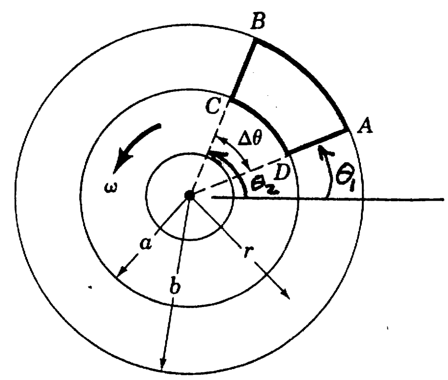


FIGURE P6.46

$$\Gamma = \oint_{ABCD} \vec{V} \cdot d\vec{s}$$

$$= \int_{AB} v_\theta \, b \, d\theta + \int_{BC} v_r \, dr + \int_{CD} v_\theta \, a \, d\theta + \int_{DA} v_r \, dr \qquad (1)$$

Since $v_r = 0$ and $v_\theta = \omega r$, Eq. (1) becomes

$$\Gamma = \int_{\theta_1}^{\theta_2} \omega b^2 d\theta + 0 + \int_{\theta_2}^{\theta_1} \omega a^2 d\theta + 0$$

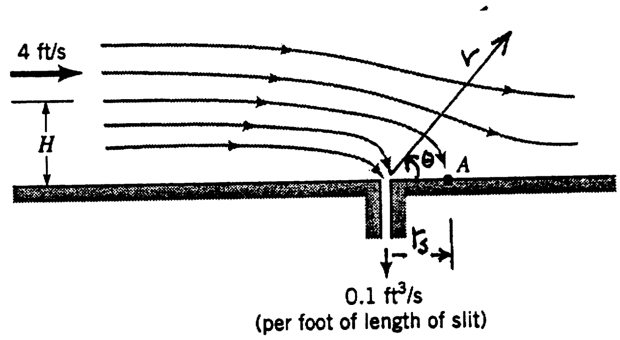
$$= \omega b^2 (\theta_2 - \theta_1) + \omega a^2 (\theta_1 - \theta_2)$$

or

$$\Gamma = \omega (\theta_2 - \theta_1)(b^2 - a^2) = \underline{\underline{\omega \Delta\theta (b^2 - a^2)}}$$

6.47

6.47 Water flows over a flat surface at 4 ft/s as shown in Fig. P6.47. A pump draws off water through a narrow slit at a volume rate of 0.1 ft^3/s per foot length of the slit. Assume that the fluid is incompressible and inviscid and can be represented by the combination of a uniform flow and a sink. Locate the stagnation point on the wall (point A) and determine the equation for the stagnation streamline. How far above the surface, H, must the fluid be so that it does not get sucked into the slit?



■ FIGURE P6.47

$$\psi = \psi_{\substack{uniform \\ flow}} + \psi_{sink} = Ur\sin\theta - \frac{m}{2\pi}\theta \qquad (1)$$

Thus,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta - \frac{m}{2\pi r} \qquad (2)$$

and

$$v_\theta = -\frac{\partial\psi}{\partial r} = -U\sin\theta$$

Along the wall $v_\theta = 0$, and the stagnation point occurs where $v_r = 0$, so that from Eq. (2)

$$0 = U\cos(0^\circ) - \frac{m}{2\pi r_s}$$

and therefore

$$r_s = \frac{m}{2\pi U}$$

For $U = 4\,\frac{ft}{s}$ and $m = 0.2\,\frac{ft^2}{s}$ (note that a source strength of $0.2\,\frac{ft^2}{s}$ must be used to obtain $0.1\,\frac{ft^2}{s}$ through slit which is only one half of a "full" sink). Thus,

$$r_s = \frac{0.2\,\frac{ft^2}{s}}{2\pi\left(4\,\frac{ft}{s}\right)} = 0.00796\ ft$$

and the stagnation point is on the wall $\underline{\underline{0.00796\ ft}}$ $\underline{\underline{\text{to the right of slit}}}$.

(con't)

The value of ψ at the stagnation point ($r = 0.00796\ ft$, $\theta = 0°$) is zero (Eq. 1) so that the equation of the stagnation streamline is

$$0 = U r \sin\theta - \frac{m}{2\pi}\theta$$

or

$$r\sin\theta = \frac{m}{2\pi U}\theta$$

Since $y = r\sin\theta$ the equation of the stagnation streamline can be written as

$$\underline{\underline{y = \frac{m}{2\pi U}\theta}}$$

Fluid above the stagnation streamline will not be sucked into slit. The maximum distance, H, for the stagnation streamline occurs as $\theta \rightarrow \pi$ so that

$$H = \frac{m\pi}{2\pi U} = \frac{0.2\ \frac{ft^2}{s}}{2\left(4\ \frac{ft}{s}\right)} = \underline{\underline{0.0250\ ft}}$$

(Note: All the fluid below the stagnation streamline must pass through the slit. Thus, from conservation of mass

$$HU = \text{flow into slit}$$

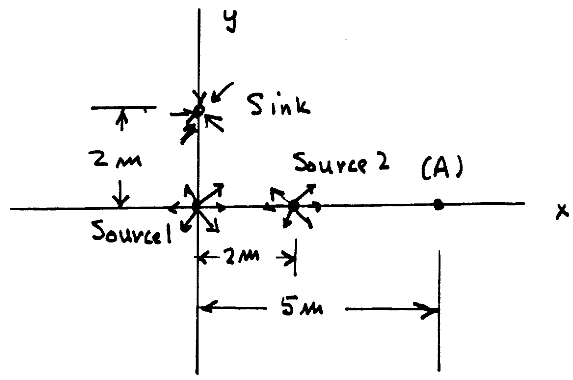
or

$$H = \frac{0.1\ \frac{ft^2}{s}}{4\ \frac{ft}{s}} = 0.0250\ ft$$

which checks with the answer above.)

6.48

6.48 Consider two sources having equal strengths located along the x axis at $x = 0$ and $x = 2$ m, and a sink located on the y axis at $y = 2$ m. Determine the magnitude and direction of the fluid velocity at $x = 5$ m and $y = 0$ due to this combination if the flowrate from each of the sources is $0.5\ m^3/s$ per m and the flowrate into the sink is $1.0\ m^3/s$ per m.



At point A along the x-axis at $x = 5\,m$ the velocities due to the two sources and the sink are as follows:

For source 1

$$(v_{r1})_A = \frac{m}{2\pi r} = \frac{0.5\ \frac{m^2}{s}}{2\pi\,(5\,m)} = 0.0159\ \frac{m}{s} \rightarrow$$

For source 2

$$(v_{r_1})_A = \frac{m}{2\pi\,(r - 2\,m)} = \frac{0.5\ \frac{m^2}{s}}{2\pi\,(5\,m - 2\,m)} = 0.0265\ \frac{m}{s} \rightarrow$$

For the sink

$$(v_r)_A = -\frac{m}{2\pi r} \quad \text{where} \quad r = \sqrt{(2\,m)^2 + (5\,m)^2} = \sqrt{29\,m^2}$$

so that

$$(v_r)_A = -\frac{1.0\ \frac{m^2}{s}}{2\pi\sqrt{29}\ m} = 0.0296\ \frac{m}{s}$$

Thus, at A the horizontal velocity component, u, is

$$u = 0.0159\ \frac{m}{s} + 0.0265\ \frac{m}{s} - \frac{5}{\sqrt{29}}\,(0.0296)\ \frac{m}{s}$$

$$= 0.0149\ \frac{m}{s} \rightarrow$$

and the vertical velocity component, v, is

$$v = \frac{2}{\sqrt{29}}\,(0.0296)\ \frac{m}{s} = 0.0110\ \frac{m}{s} \uparrow$$

The velocity at A is therefore

$$V_A = \sqrt{u^2 + v^2} = \sqrt{\left(0.0149\ \frac{m}{s}\right)^2 + \left(0.0110\ \frac{m}{s}\right)^2}$$

$$= 0.0185\ \frac{m}{s} \quad 0.0110,\ 0.0149 \quad = \underline{\underline{0.0185\ \frac{m}{s}\quad 36.4^\circ}}$$

6.49

6.49 The velocity potential for a spiral vortex flow is given by $\phi = (\Gamma/2\pi)\,\theta - (m/2\pi)\ln r$, where Γ and m are constants. Show that the angle, α, between the velocity vector and the radial direction is constant throughout the flow field (see Fig. P6.49).

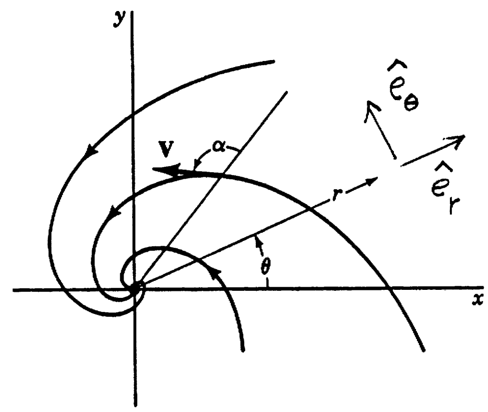


FIGURE P6.49

For the velocity potential given,

$$v_r = \frac{\partial\phi}{\partial r} = -\frac{m}{2\pi r} \qquad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{\Gamma}{2\pi r}$$

Since $\quad \vec{V}\cdot\hat{e}_r = |\vec{V}|\cos\alpha$

and $\quad \vec{V} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$

then

$$\cos\alpha = \frac{\vec{V}\cdot\hat{e}_r}{|\vec{V}|} = \frac{v_r}{\sqrt{v_r^2+v_\theta^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{v_\theta}{v_r}\right)^2}} = \frac{1}{\sqrt{1+\frac{\left(\frac{\Gamma}{2\pi r}\right)^2}{\left(-\frac{m}{2\pi r}\right)^2}}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{\Gamma}{m}\right)^2}}$$

Thus, for a given Γ and m the angle α is a constant.

6.50

6.50 For a free vortex (see Video V6.2) determine an expression for the pressure gradient **(a)** along a streamline, and **(b)** normal to a streamline. Assume the streamline is in a horizontal plane, and express your answer in terms of the circulation.

For a free vortex

$$\psi = -\frac{\Gamma}{2\pi} \ln r \qquad \text{(Eq. 6.91)}$$

so that

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0 \qquad v_\theta = -\frac{\partial \psi}{\partial r} = \frac{\Gamma}{2\pi r}$$

Since the free vortex represents an irrotational flow field, the Bernoulli equation

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant} \qquad (1)$$

is valid between any two points.

(a) Along a streamline ($r = \text{constant}$), v_θ is constant and $v_r = 0$ so that from Eq. (1) with z constant the pressure is constant, i.e.,

$$\underline{\underline{\frac{\partial p}{\partial \theta} = 0}}$$

(b) Normal to the streamline with $v_r = 0$ and $z = \text{constant}$

$$\frac{p}{\gamma} + \frac{v_\theta^2}{2g} + z = \text{constant}$$

so that

$$\frac{\partial p}{\partial r} = -\frac{\gamma}{2g}\frac{\partial (v_\theta^2)}{\partial r} = -\rho v_\theta \frac{\partial v_\theta}{\partial r}$$

$$= -\rho\left(\frac{\Gamma}{2\pi r}\right)\left(-\frac{\Gamma}{2\pi r^2}\right)$$

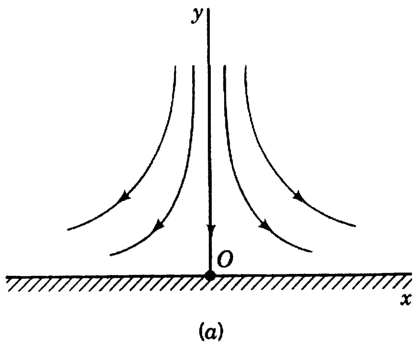
$$= \underline{\underline{\frac{\rho \Gamma^2}{4\pi^2 r^3}}}$$

6.51

6.51 Potential flow against a flat plate (Fig. P6.51*a*) can be described with the stream function

$$\psi = Axy$$

where A is a constant. This type of flow is commonly called a "stagnation point" flow since it can be used to describe the flow in the vicinity of the stagnation point at O. By adding a source of strength, m, at O, stagnation point flow against a flat plate with a "bump" is obtained as illustrated in Fig. P6.51*b*. Determine the relationship between the bump height, h, the constant, A; and the source strength, m.



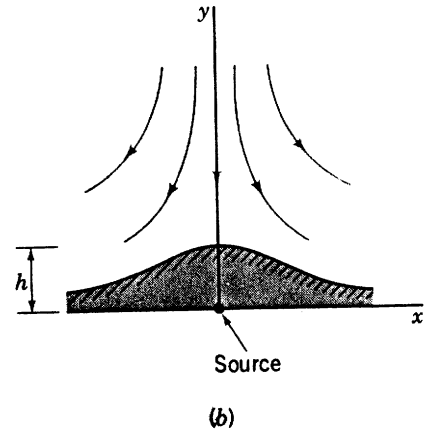


FIGURE P6.51

$$\psi = Axy + \frac{m}{2\pi}\theta = \frac{A}{2} r^2 \sin 2\theta + \frac{m}{2\pi}\theta$$

For the bump the stagnation point will occur at $x = 0$, $y = h$ $\left(\theta = \frac{\pi}{2}, r = h\right)$. For the given stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = Ar \cos 2\theta + \frac{m}{2\pi r} \qquad (1)$$

and

$$v_\theta = -\frac{\partial \psi}{\partial r} = Ar \sin 2\theta$$

The point, $\theta = \frac{\pi}{2}$, $r = h$, will be a stagnation point if $v_r = 0$ since $v_\theta = 0$ at this point. Thus, from Eq. (1)

$$0 = Ah \cos \pi + \frac{m}{2\pi h}$$

or

$$Ah = \frac{m}{2\pi h}$$

and therefore

$$\underline{\underline{h^2 = \frac{m}{2\pi A}}}$$

6.52

6.52 The combination of a uniform flow and a source can be used to describe flow around a streamlined body called a half-body. (See **Video V6.3.**) Assume that a certain body has the shape of a half-body with a thickness of 0.5 m. If this body is placed in an air stream moving at 15 m/s, what source strength is required to simulate flow around the body?

The width of half-body $= 2\pi b$ (See Fig. 6.24)
so that

$$b = \frac{(0.5\,m)}{2\pi}$$

From Eq. 6.99

$$b = \frac{m}{2\pi U}$$

where m is the source strength, and therefore

$$m = 2\pi U b = 2\pi \left(15\,\frac{m}{s}\right)\left(\frac{0.5\,m}{2\pi}\right)$$

$$= \underline{\underline{7.50\,\frac{m^2}{s}}}$$

6.53

6.53 A body having the general shape of a half-body is placed in a stream of fluid. At a great distance upstream the velocity is U as shown in Fig. P6.53. Show how a measurement of the differential pressure between the stagnation point and point A can be used to predict the free-stream velocity, U. Express the pressure differential in terms of U and fluid density. Neglect body forces and assume that the fluid is nonviscous and incompressible.

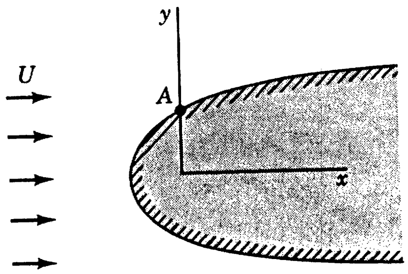


FIGURE P6.53

Write Bernoulli equation between stagnation point and point A to obtain

$$p_{stag} = p_A + \frac{1}{2}\rho V_A^2 \qquad (1)$$

Also,

$$V_A^2 = U^2\left(1 + 2\frac{b}{r_A}\cos\theta + \frac{b^2}{r_A^2}\right) \qquad \text{(Eq. 6.101)}$$

and

$$r = \frac{b(\pi-\theta)}{\sin\theta} \qquad \text{(Eq. 6.100)}$$

At point A $\theta = \frac{\pi}{2}$ so that

$$r_A = \frac{b\left(\pi - \frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = \frac{\pi b}{2}$$

or

$$\frac{b}{r_A} = \frac{2}{\pi} \qquad (2)$$

Substitution of Eq.(2) into Eq. 6.101 yields

$$V_A^2 = U^2\left(1 + 0 + \frac{4}{\pi^2}\right)$$

and therefore from Eq. (1)

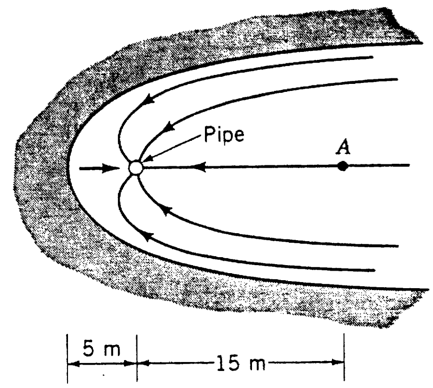
$$p_{stag} = p_A + \frac{1}{2}\rho U^2\left(1 + \frac{4}{\pi^2}\right) = p_A + 0.703\rho U^2$$

Thus,

$$\underline{\underline{p_{stag} - p_A = 0.703\rho U^2}}$$

6.54

6.54 One end of a pond has a shoreline that resembles a half-body as shown in Fig. P6.54. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.08 m³/s through a 3-m-long pipe, what will be the velocity at point A? *Hint*: Consider the flow *inside* a half-body. (See Video V6.3.)



■ FIGURE P6.54

For a half-body,

$$\psi = U r \sin\theta + \frac{m}{2\pi}\theta \qquad \text{(Eq. 6.97)}$$

so that

$$v_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U\cos\theta + \frac{m}{2\pi r}$$

Thus, at point A, $\theta = 0$, $r = 15\,m$ and

$$v_\theta = 0$$

$$v_r = V_A = U + \frac{m}{2\pi(15)} \qquad (1)$$

For a flowrate of $0.06\,\frac{m^3}{s}$ in a 3-m long pipe, the source strength is $\frac{0.06}{3}\,\frac{m^2}{s}$. Since

$$b = \frac{m}{2\pi U} \qquad \text{(Eq. 6.99)}$$

then with $b = 5m$

$$U = \frac{m}{2\pi b} = \frac{\left(\frac{0.06}{3}\,\frac{m^2}{s}\right)}{2\pi(5m)} = 6.37\times 10^{-4}\,\frac{m}{s}$$

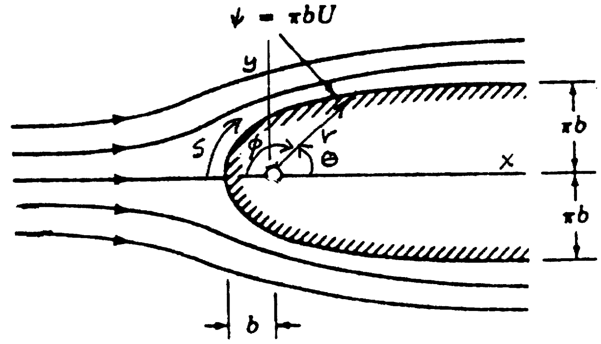
From Eq. (1)

$$V_A = 6.37\times10^{-4}\,\frac{m}{s} + \frac{\left(\frac{0.06}{3}\,\frac{m^2}{s}\right)}{2\pi\,(15m)}$$

$$= \underline{\underline{8.49\times10^{-4}\,\frac{m}{s}}}$$

6.55 *

6.55* For the half-body described in Section 6.6.1 show on a plot how the magnitude of the velocity on the surface, V_s, varies as a function of the distance, s (measured along the surface), from the stagnation point. Use the dimensionless variables V_s/U and s/b where U and b are defined in Fig. 6.24.



On the surface of the half-body

$$r = \frac{b(\pi - \theta)}{\sin\theta} \qquad \text{(Eq. 6.100)}$$

and

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

with $x = r\cos\theta$ and $y = r\sin\theta$. It follows that

$$dx = r(-\sin\theta)\,d\theta + \cos\theta\,dr$$
$$dy = r(\cos\theta)\,d\theta + \sin\theta\,dr$$

and therefore

$$ds = \sqrt{r^2(d\theta)^2 + (dr)^2}$$

or

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Let $s^* = s/b$ and $r^* = r/b$ so that

$$ds^* = \sqrt{(r^*)^2 + \left(\frac{dr^*}{d\theta}\right)^2}\,d\theta \qquad (1)$$

From Eq. 6.100

$$\frac{dr^*}{d\theta} = -\frac{\sin\theta + (\pi - \theta)\cos\theta}{\sin^2\theta} \qquad (2)$$

Thus, the arc length s^* is given by

$$s^* = \int_{\pi}^{\pi-\phi} \sqrt{(r^*)^2 + \left(\frac{dr^*}{d\theta}\right)^2}\,d\theta \qquad (3)$$

for $0 \le \phi \le \pi$.

(con't)

6.55* (con't)

The velocity, V_s, on the surface of the half-body can be obtained from Eq. 6.101 written in the form

$$V^* = \frac{V_s}{U} = \left[1 + \frac{2\cos\theta}{r^*} + \frac{1}{(r^*)^2}\right]^{1/2} \qquad (4)$$

Thus, for a given θ, r^* can be obtained from Eq. 6.100, s^* from Eq. (3), and V^* from Eq. (4). A program for calculating V^* as a function of s^* follows. (Note: In the program V^* is designated as v and s^* as s.)

```
100 cls
110 print "*******************************************************"
120 print "** This program calculates the velocity distribution **"
130 print "** over the arc-length of a half body                **"
150 print "*******************************************************"
160 print
170 dim th(18),r(18),s(18),intgd(18),v(18)
171 pi=4.0*atn(1.0)
180 n=18
190 dth=pi/18
200 s(1)=0.
210 for i=1 to n
220 th(i)=pi-(i-1)*dth
222 if i>1 then goto 230
224 r(i)=1.
226 drdth=0.
228 goto 236
230 r(i)=(pi-th(i))/sin(th(i))
232 drdth=-(sin(th(i))+(pi-th(i))*cos(th(i)))/sin(th(i))^2
236 intgd(i)=(r(i)^2+drdth^2)^0.5
238 v(i)=(1+2*cos(th(i))/r(i)+1/r(i)^2)^.5
240 next i
250 for i=2 to n
260 sum=(intgd(1)+intgd(i))/2
270 im1=i-1
280 for j=2 to im1
290 sum=sum+intgd(j)
300 next j
310 s(i)=dth*sum
320 next i
330 print "   Theta      Arc-length     Velocity"
340 for i=1 to n
350 print using "     ###.#       ###.####       ###.####";180/pi*th(i),s(i),v(i)
360 next i
```

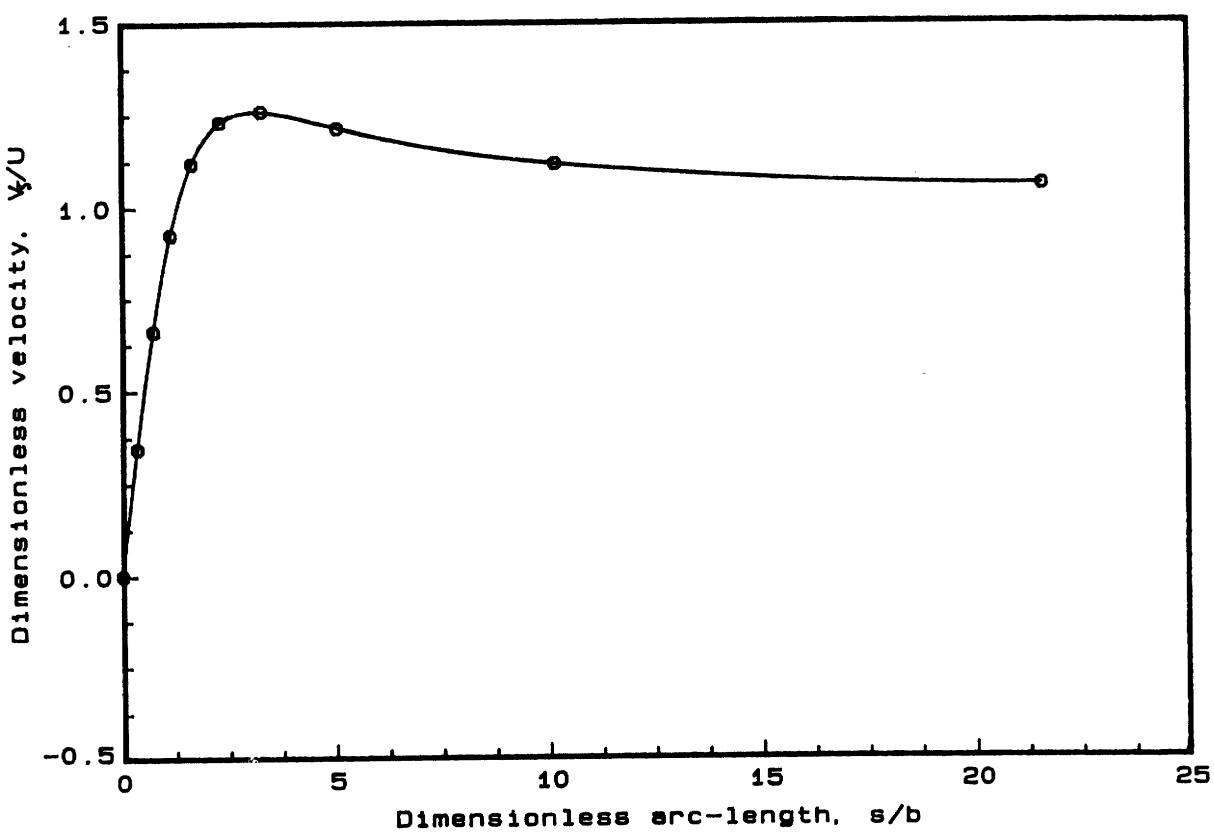
(con't)

6.55 * (con't)

Tabulated data and a plot of the data are given below.

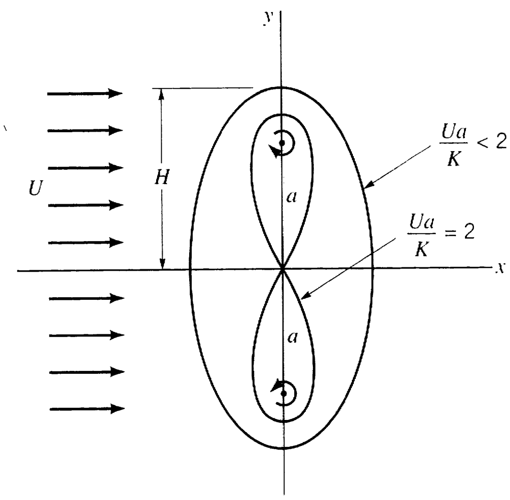
```
**************************************************************
** This program calculates the velocity distribution **
** over the arc-length of a half body                 **
**************************************************************
```

Theta	Arc-length, $\frac{s}{b}$	Velocity, $\frac{V_s}{U}$
180.0	0.0000	0.0000
170.0	0.1751	0.1739
160.0	0.3527	0.3444
150.0	0.5352	0.5078
140.0	0.7255	0.6611
130.0	0.9269	0.8013
120.0	1.1437	0.9257
110.0	1.3811	1.0322
100.0	1.6464	1.1192
90.0	1.9495	1.1854
80.0	2.3052	1.2306
70.0	2.7366	1.2547
60.0	3.2814	1.2588
50.0	4.0079	1.2442
40.0	5.0539	1.2134
30.0	6.7487	1.1693
20.0	10.1419	1.1159
10.0	21.5487	1.0577



6.56*

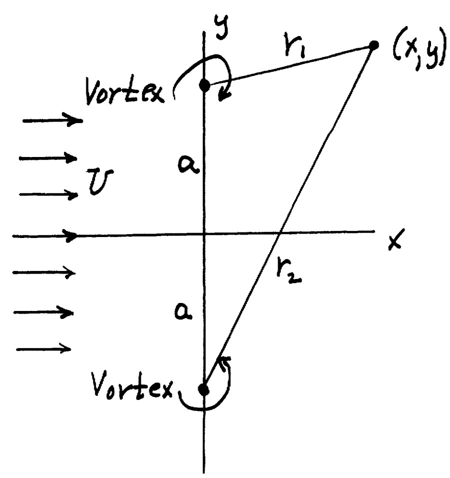
*6.56 Consider a uniform flow with velocity U in the positive x-direction combined with two free vortices of equal strength located along the y-axis. Let one vortex located at $y = a$ be a clockwise vortex ($\psi = K \ln r$) and the other at $y = -a$ be a counterclockwise vortex, where K is a positive constant. It can be shown by plotting streamlines that for $Ua/K < 2$ the streamline $\psi = 0$ forms a closed contour, as shown in Fig. P6.56. Thus, this combination can be used to represent flow around a family of bodies (called *Kelvin ovals*). Show, with the aid of a graph, how the dimensionless height, H/a, varies with the parameter Ua/K in the range $0.3 < Ua/K < 1.75$.



The stream function for the uniform flow and the two vortices is

$$\psi = Uy + K\ln r_1 - K\ln r_2 = Uy - K\ln\frac{r_2}{r_1}$$

Since $r_1^2 = x^2 + (y-a)^2$ and $r_2^2 = x^2+(y+a)^2$ (see figure) then



$$\psi = Uy - K\ln\left[\frac{x^2+(y+a)^2}{x^2+(y-a)^2}\right]^{1/2}$$

or

$$\psi = Uy - \frac{K}{2}\ln\left[\frac{x^2+(y+a)^2}{x^2+(y-a)^2}\right]$$

Thus, the equation for the $\psi = 0$ streamline is

$$U = \frac{K}{2y}\ln\left[\frac{x^2+(y+a)^2}{x^2+(y-a)^2}\right]$$

and in non-dimensional form

$$\frac{Ua}{K} = \frac{1}{2}\left(\frac{y}{a}\right)^{-1}\ln\left[\frac{\left(\frac{x}{a}\right)^2+\left(\frac{y}{a}+1\right)^2}{\left(\frac{x}{a}\right)^2+\left(\frac{y}{a}-1\right)^2}\right] \qquad (1)$$

(con't)

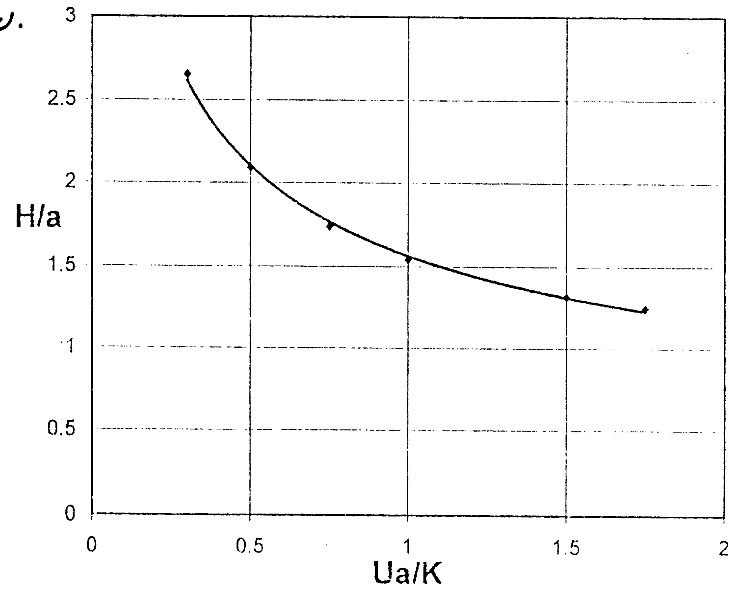
6.56* (Con't)

To find H/a, set $x/a = 0$ in Eq. (1) and $y/a = H/a$. Equation (1) reduces to

$$\frac{Ua}{K} = \frac{1}{2}\left(\frac{H}{a}\right)^{-1} \ln \frac{\left(\frac{H}{a}+1\right)^2}{\left(\frac{H}{a}-1\right)^2} \qquad (2)$$

For a specified value of Ua/K, Eq. (2) can be solved by a trial and error solution to obtain H/a. Some tabulated values and the corresponding graph are shown below.

Ua/K	H/a
0.30	2.65
0.50	2.09
0.75	1.74
1.00	1.54
1.50	1.32
1.75	1.25



6.57

6.57 A Rankine oval is formed by combining a source–sink pair, each having a strength of 36 ft²/s, and separated by a distance of 12 ft along the x axis, with a uniform velocity of 10 ft/s (in the positive x direction). Determine the length and thickness of the oval.

$$\frac{\ell}{a} = \left[\frac{m}{\pi U a} + 1\right]^{1/2} \qquad \text{(Eq. 6.107)}$$

$$\frac{h}{a} = \frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right] \tan\left[2\left(\frac{\pi U a}{m}\right)\frac{h}{a}\right] \qquad \text{(Eq. 6.109)}$$

For $m = 36 \frac{ft^2}{s}$, $a = 6\, ft$, and $U = 10 \frac{ft}{s}$,

$$\frac{\pi U a}{m} = \frac{\pi \left(10 \frac{ft}{s}\right)(6\, ft)}{36 \frac{ft^2}{s}} = 5.24$$

Thus, length $= 2\ell$ and from Eq. 6.107

$$\underline{\underline{\text{length}}} = 2\,(6\,ft)\left[\frac{1}{5.24} + 1\right]^{1/2} = \underline{\underline{13.1\, ft}}$$

The thickness, $2h$, can be determined from Eq. 6.109 by trial and error. Assume value for h/a and compare with right hand side of Eq. 6.109. (See table below.)

$\frac{h}{a}$	$\frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right] \tan\left[2\,(5.24)\frac{h}{a}\right]$
0.250	0.269
0.251	0.262
0.252	0.256
0.253	0.250 ← use

Thus, $\frac{h}{a} \approx 0.253$

and $\underline{\underline{\text{thickness}}} = 2h = 2\,(6\,ft)(0.253) = \underline{\underline{3.04\, ft}}$

6.58*

6.58* Make use of Eqs. 6.107 and 6.109 to construct a table showing how ℓ/a, h/a, and ℓ/h for Rankine ovals depend on the parameter $\pi Ua/m$. Plot ℓ/h versus $\pi Ua/m$ and describe how this plot could be used to obtain the required values of m and a for a Rankine oval having a specific value of ℓ and h when placed in a uniform fluid stream of velocity, U.

For a Rankine oval

$$\frac{\ell}{a} = \left[\frac{m}{\pi U a} + 1\right]^{1/2} \qquad \text{(Eq. 6.107)}$$

and

$$\frac{h}{a} = \frac{1}{2}\left[\left(\frac{h}{a}\right)^2 - 1\right] \tan\left[2\left(\frac{\pi U a}{m}\right)\frac{h}{a}\right] \qquad \text{(Eq. 6.109)}$$

where the length of the body is 2ℓ and the width is $2h$. For a given value of $\pi Ua/m$, Eq. 6.107 can be solved for ℓ/a, and Eq. 6.109 can be solved (using an iteration procedure) for h/a. The ratio ℓ/h can then be determined. A program for calculating ℓ/a, h/a, and ℓ/h as a function of $\pi Ua/m$ follows.

```
100 cls
110 print "**********************************************************"
120 print "** This program calculates l/a, h/a, and l/h as a       **"
130 print "** function of pi*U*a/m for Rankine ovals               **"
150 print "**********************************************************"
160 print
162 print "pi*U*a/m       l/a          h/a          l/h"
168 data 10.0,5.0,1.0,0.5,0.1,0.05,0.01
170 for i=1 to 7
172 start=0.001
175 read a
180 la=(1/a+1)^.5
190 for has=start to 10.0 step 0.0001
210 ha=0.5*(has^2-1)*tan(2*a*has)
220 if abs(1-has/ha)<0.002 and ha>0 then goto 230
222 next has
230 lh=la/ha
250 print using "##.####      ##.####      ##.####      ##.####";a,la,ha,lh
255 start=ha
260 next i
```

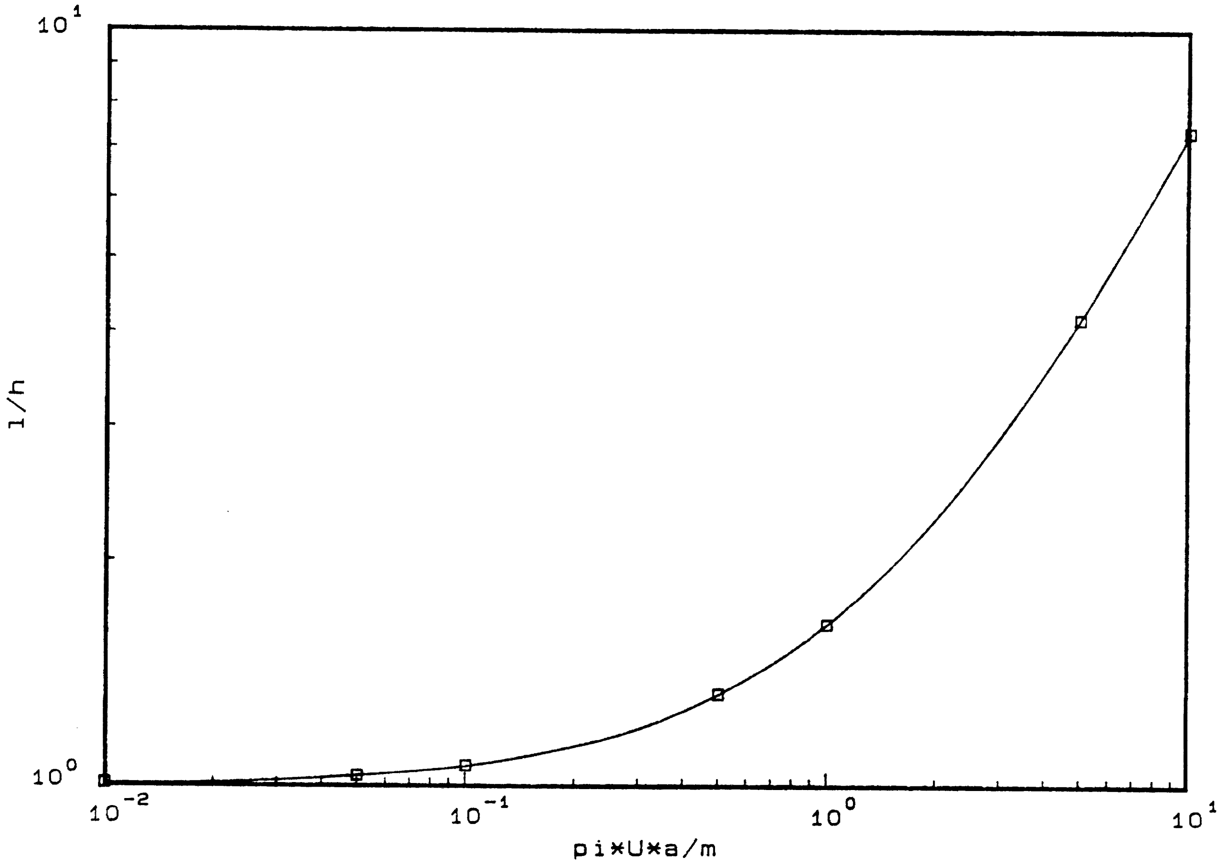
(con't)

6.58* (con't)

Tabulated data and a plot of ℓ/h as a function of $\pi U a/m$ are given below.

```
*******************************************************
** This program calculates l/a, h/a, and l/h as a    **
** function of pi*U*a/m for Rankine ovals            **
*******************************************************
```

pi*U*a/m	l/a	h/a	l/h
10.0000	1.0488	0.1427	7.3483
5.0000	1.0954	0.2632	4.1623
1.0000	1.4142	0.8604	1.6437
0.5000	1.7321	1.3042	1.3281
0.1000	3.3166	3.1022	1.0691
0.0500	4.5826	4.4227	1.0362
0.0100	10.0499	9.9538	1.0096



For a Rankine oval with ℓ and h specified the following steps could be followed to determine m and a:

(1) For a given ℓ/h determine the required value of $\pi U a/m$ from the graph.

(2) Using this value of $\pi U a/m$ calculate ℓ/a from Eq. 6.107.

(3) With the value of ℓ/a determined, and ℓ specified, determine the value of a.

(4) With $\pi U a/m$ and a determined, the value of U/m is known, and for a given U the value of m is fixed.

6.59

6.59 Assume that the flow around the long circular cylinder of Fig. P6.59 is nonviscous and incompressible. Two pressures, p_1 and p_2, are measured on the surface of the cylinder, as illustrated. It is proposed that the free-stream velocity, U, can be related to the pressure difference $\Delta p = p_1 - p_2$ by the equation

$$U = C\sqrt{\frac{\Delta p}{\rho}}$$

where ρ is the fluid density. Determine the value of the constant C. Neglect body forces.

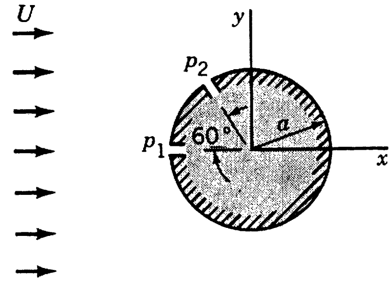


FIGURE P6.59

Since p_1 is the stagnation pressure,

$$\frac{p_1}{\gamma} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

where

$$V_2 = v_{\theta s} = -2U\sin\theta$$

For $\theta = 60°$,

$$V_2^2 = \left(-2U\sin 60°\right)^2 = 3U^2$$

Thus, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\frac{\gamma}{g}V_2^2 = \frac{1}{2}\rho\left(3U^2\right) = \frac{3}{2}\rho U^2$$

so that

$$U = \sqrt{\frac{2}{3}}\sqrt{\frac{p_1 - p_2}{\rho}} = \sqrt{\frac{2}{3}}\sqrt{\frac{\Delta p}{\rho}}$$

and therefore

$$C = \underline{\underline{\sqrt{\frac{2}{3}}}}$$

6.60

6.60 An ideal fluid flows past an infinitely long semicircular "hump" located along a plane boundary as shown in Fig. P6.60. Far from the hump the velocity field is uniform, and the pressure is p_0. **(a)** Determine expressions for the maximum and minimum values of the pressure along the hump, and indicate where these points are located. Express your answer in terms of ρ, U, and p_0. **(b)** If the solid surface is the $\psi = 0$ streamline, determine the equation of the streamline passing through the point $\theta = \pi/2$, $r = 2a$.

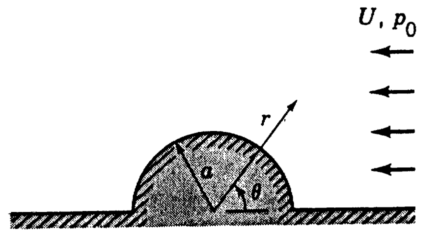


FIGURE P6.60

(a) On the surface of the hump,

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad \text{(Eq. 6.116)}$$

The maximum pressure occurs where $\sin\theta = 0$, or at $\theta = 0, \pi$, and at these points

$$\underline{\underline{p_s(\max) = p_0 + \frac{1}{2}\rho U^2}} \qquad \underline{\underline{(\text{at } \theta = 0 \text{ and } \pi)}}$$

The minimum pressure occurs where $\sin\theta = 1$, or at $\theta = \frac{\pi}{2}$, and at this point

$$\underline{\underline{p_s(\min) = p_0 - \frac{3}{2}\rho U^2}} \qquad \underline{\underline{(\text{at } \theta = \frac{\pi}{2})}}$$

(b) For uniform flow in the negative x-direction,

$$\psi = -Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

(refer to discussion associated with the derivation of Eq. 6.112).

At $\theta = \frac{\pi}{2}$, $r = 2a$

$$\psi = -2aU\left(1 - \frac{a^2}{(2a)^2}\right)\sin\frac{\pi}{2} = -\frac{3}{2}aU$$

and thus the equation of the streamline passing through this point is

$$-\frac{3}{2}aU = -Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

or

$$\underline{\underline{\frac{2}{3}\frac{r}{a}\left(1 - \frac{a^2}{r^2}\right)\sin\theta = 1}}$$

6.61

6.61 Water flows around a 6-ft diameter bridge pier with a velocity of 12 ft/s. Estimate the force (per unit length) that the water exerts on the pier. Assume that the flow can be approximated as an ideal fluid flow around the front half of the cylinder, but due to flow separation (see Video V6.4), the average pressure on the rear half is constant and approximately equal to $\frac{1}{2}$ the pressure at point A (see Fig. P6.61).

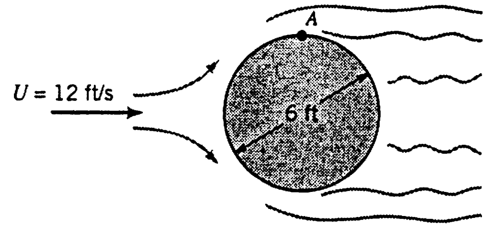
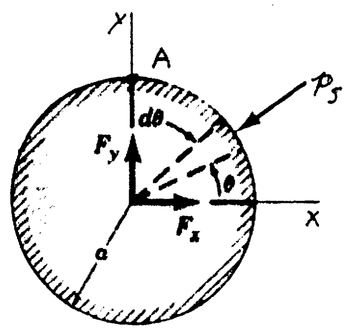


FIGURE P6.61



From Fig. 6.28 it follows that the drag on a section (between $\theta=0$ and $\theta=\alpha$) of a circular cylinder is given by the equation

$$Drag = F_x = -\int_0^{\alpha} p_s \cos\theta \, a \, d\theta$$

For the force on the front half of the cylinder (per unit length)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} p_s \cos\theta \, a \, d\theta \qquad (1)$$

and due to symmetry $F_y = 0$. From Eq. 6.116

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad (Eq. 6.116)$$

and since we are only interested in the force due to the flowing fluid we will let $p_0 = 0$. Thus, from Eq. (1)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta)\cos\theta \, a \, d\theta \qquad (2)$$

Since $\displaystyle\int_{\pi/2}^{\pi} \cos\theta \, d\theta = \sin\theta \Big]_{\pi/2}^{\pi} = -1$

and $\displaystyle\int_{\pi/2}^{\pi} \sin^2\theta\cos\theta \, d\theta = \frac{\sin^3\theta}{3}\Big]_{\pi/2}^{\pi} = -\frac{1}{3}$

(con't)

6.61 (con't)

It follows from Eq.(2) that

$$F_{x_1} = -\frac{\rho U^2 a}{3}$$

Note that the negative sign indicates that the water is actually "pulling" on the cylinder (front half) in the upstream direction. However, when the effect of the rear half of the cylinder is taken into account (in a real fluid) there will be a net drag in the direction of flow.

The pressure at the top of the cylinder (point A) is given by

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad \text{(Eq. 6. 116)}$$

and with $\theta = \pi/2$

$$p_A = p_0 - \frac{3}{2}\rho U^2$$

Since $p_0 = 0$

$$p_A = -\frac{3}{2}\rho U^2$$

Note that the negative pressure will give a positive F_x and

$$F_{x_2} = -\frac{p_A}{2} \times \text{projected area} = -\frac{p_A}{2} \times 2a(1)$$

So that

$$F_{x_2} = \frac{3}{4}\rho U^2 (2a)(1) = \frac{3}{2}\rho U^2 a$$

Thus,

$$F_x = F_{x_1} + F_{x_2}$$

$$= -\frac{\rho U^2 a}{3} + \frac{3\rho U^2 a}{2}$$

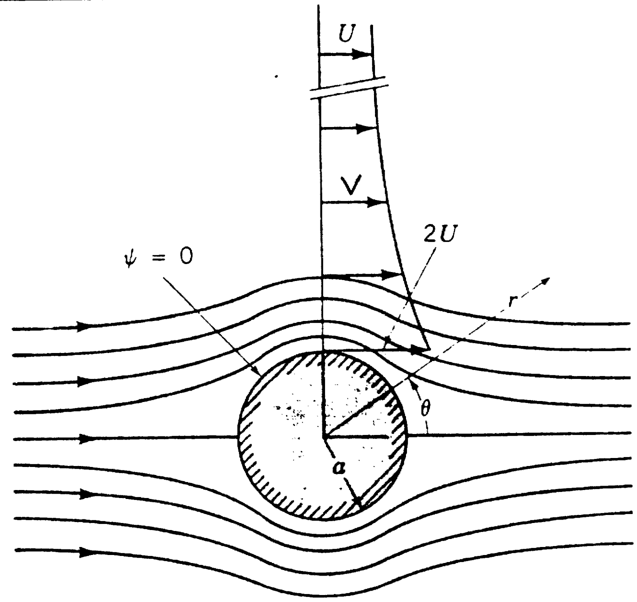
$$= \frac{7}{6}\rho U^2 a$$

With the data given,

$$F_x = \frac{7}{6}\left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(12\ \frac{\text{ft}}{\text{s}}\right)^2 (3\ \text{ft}) = \underline{\underline{978\ \frac{\text{lb}}{\text{ft}}}}$$

6.62*

6.62* Consider the steady potential flow around the circular cylinder shown in Fig. 6.26. Show on a plot the variation of the magnitude of the dimensionless fluid velocity, V/U, along the positive y axis. At what distance, y/a (along the y axis), is the velocity within 1% of the free-stream velocity?



Along the y-axis $v_r = 0$ so that the magnitude of the velocity, V, is equal to $|v_\theta|$. Since

$$v_\theta = -U\left(1+\frac{a^2}{r^2}\right)\sin\theta \qquad \text{(Eq. 6.115)}$$

it follows that along the positive y-axis ($\theta = \frac{\pi}{2}$, $r = y$)

$$V = |v_\theta| = U\left(1+\frac{a^2}{y^2}\right)$$

or

$$\frac{V}{U} = 1 + \frac{a^2}{y^2} = 1 + \frac{1}{\left(\frac{y}{a}\right)^2}$$

A program for calculating V/U as a function of y/a follows.

```
100 cls
110 print "****************************************************"
120 print "** This program calculates the velocity profile **"
130 print "** on the +y-axis for flow around a cylinder    **"
140 print "****************************************************"
150 print
155 print       " y/a      V/U"
160 for ya=1.0 to 10.0
170 u=1+1/ya^2
180 print using "##.##      #.####";ya,u
190 next ya
```

(con't)

6.62* (con't)

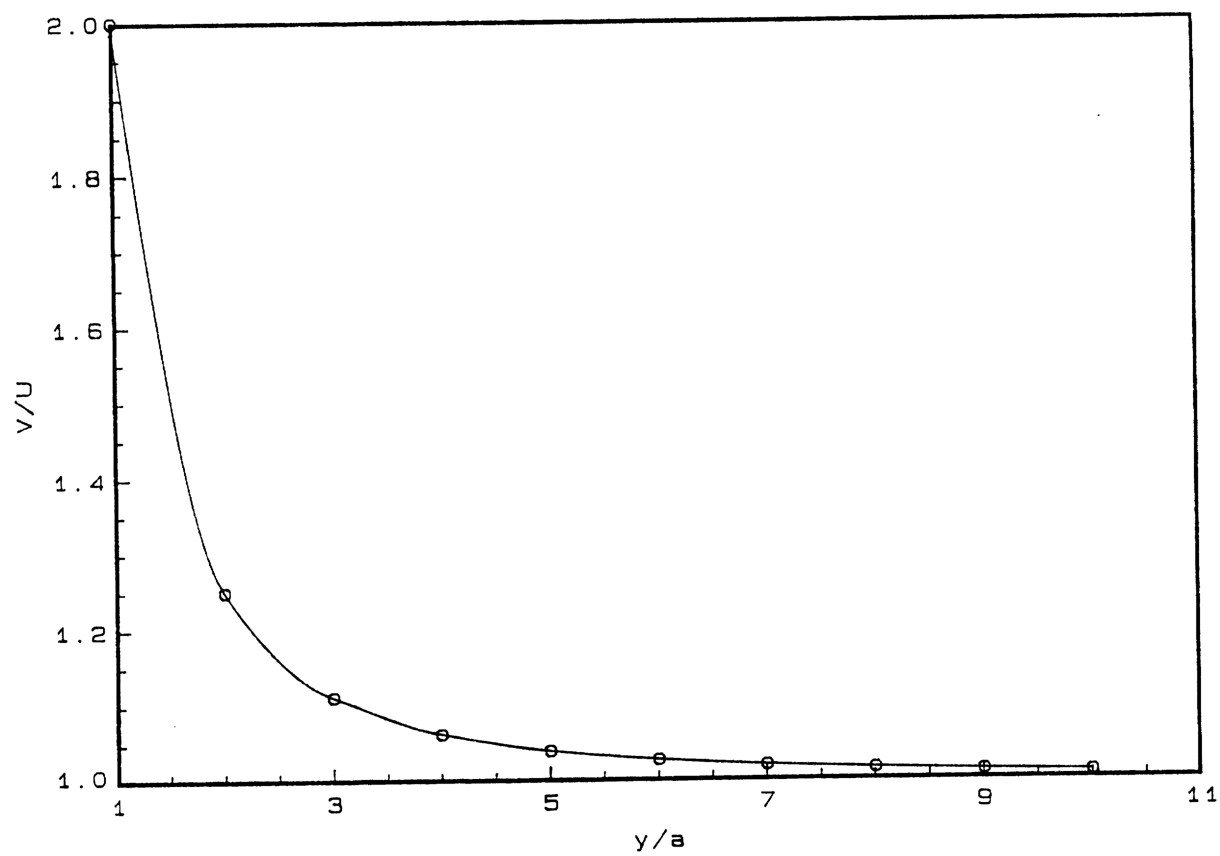
Tabulated data and a plot of the data are given below. It can be seen from these results that for

$$\underline{\underline{\frac{y}{a} \geq 10}}$$

the velocity V is within 1% of the free-stream velocity U.

```
**************************************************
** This program calculates the velocity profile **
** on the +y-axis for flow around a cylinder    **
**************************************************
```

y/a	V/U
1.00	2.0000
2.00	1.2500
3.00	1.1111
4.00	1.0625
5.00	1.0400
6.00	1.0278
7.00	1.0204
8.00	1.0156
9.00	1.0123
10.00	1.0100



6.63

6.63 The velocity potential for a cylinder (Fig. P6.63) rotating in a uniform stream of fluid is

$$\phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta + \frac{\Gamma}{2\pi}\theta$$

where Γ is the circulation. For what value of the circulation will the stagnation point be located at: **(a)** point A, **(b)** point B?

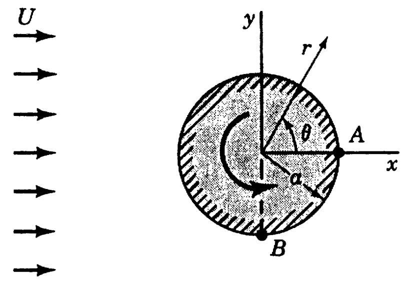


FIGURE P6.63

(a) $$\sin\theta_{stag} = \frac{\Gamma}{4\pi U a} \qquad \text{(Eq. 6.122)}$$

At point A, $\theta_{stag} = 0$ and it follows that $\underline{\underline{\Gamma = 0}}$.

(b) At point B, $\theta_{stag} = \frac{3\pi}{2}$, and from Eq. 6.122

$$\Gamma = 4\pi U a \sin\frac{3\pi}{2} = \underline{\underline{-4\pi U a}}$$

6.64

6.64 A fixed circular cylinder of infinite length is placed in a steady, uniform stream of an incompressible, nonviscous fluid. Assume that the flow is irrotational. Prove that the drag on the cylinder is zero. Neglect body forces.

$$\text{Drag} = F_x = -\int_0^{2\pi} p_s \cos\theta \, a \, d\theta \qquad \text{(Eq. 6.117)}$$

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta) \qquad \text{(Eq. 6.116)}$$

Thus,

$$\text{Drag} = -\left\{ a p_0 \int_0^{2\pi} \cos\theta \, d\theta + \frac{a}{2}\rho U^2 \int_0^{2\pi} \cos\theta \, d\theta - 2a\rho U^2 \int_0^{2\pi} \sin^2\theta \cos\theta \, d\theta \right\}$$

Since,

$$\int_0^{2\pi} \cos\theta \, d\theta = \sin\theta \Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^2\theta \cos\theta \, d\theta = \frac{\sin^3\theta}{3}\Big]_0^{2\pi} = 0$$

it follows that

$$\underline{\underline{\text{Drag} = 0}}$$

6.65

6.65 Repeat Problem 6.64 for a rotating cylinder for which the stream function and velocity potential are given by Eqs. 6.119 and 6.120, respectively. Verify that the lift is not zero and can be expressed by Eq. 6.124.

$$\text{Drag} = F_x = -\int_0^{2\pi} p_s \cos\theta \, a\,d\theta \qquad (\text{Eq. } 6.117)$$

$$p_s = p_0 + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi a U} - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\right) \qquad (\text{Eq. } 6.123)$$

Thus,

$$\text{Drag} = -\left\{ a p_0 \int_0^{2\pi} \cos\theta\,d\theta + \frac{a}{2}\rho U^2\left[\int_0^{2\pi}\cos\theta\,d\theta - 4\int_0^{2\pi}\sin^2\theta\cos\theta\,d\theta + \frac{2\Gamma}{\pi a U}\int_0^{2\pi}\cos\theta\sin\theta\,d\theta - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\int_0^{2\pi}\cos\theta\,d\theta\right]\right\}$$

Since,

$$\int_0^{2\pi}\cos\theta\,d\theta = \sin\theta\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi}\sin^2\theta\cos\theta\,d\theta = \frac{\sin^3\theta}{3}\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi}\cos\theta\sin\theta\,d\theta = \frac{\sin^2\theta}{2}\Big]_0^{2\pi} = 0$$

it follows that

$$\underline{\underline{\text{Drag} = 0}}$$

(con't)

6.65 (con't)

$$\text{Lift} = F_y = -\int_0^{2\pi} p_s \sin\theta \; a\,d\theta \qquad \text{(Eq. 6.118)}$$

With p_s given by Eq. 6.123 it follows that

$$\text{Lift} = -\left\{ a\,p_0 \int_0^{2\pi} \sin\theta\,d\theta + \frac{a}{2}\rho U^2 \left[\int_0^{2\pi} \sin\theta\,d\theta - 4\int_0^{2\pi} \sin^3\theta\,d\theta + \frac{2\Gamma}{\pi a U}\int_0^{2\pi} \sin^2\theta\,d\theta - \frac{\Gamma^2}{4\pi^2 a^2 U^2}\int_0^{2\pi} \sin\theta\,d\theta \right] \right\}$$

Since,

$$\int_0^{2\pi} \sin\theta\,d\theta = -\cos\theta\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^3\theta\,d\theta = -\frac{\cos\theta}{3}\left(\sin^2\theta + 2\right)\Big]_0^{2\pi} = 0$$

and

$$\int_0^{2\pi} \sin^2\theta\,d\theta = \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Big]_0^{2\pi} = \pi$$

it follows that

$$\text{Lift} = -\frac{a}{2}\rho U^2 \left(\frac{2\Gamma}{\pi a U}\right)(\pi)$$

Thus,

$$\underline{\underline{\text{Lift} = -\rho U \Gamma}}$$

(which is Eq. 6.124).

6.66

6.66 A source of strength m is located a distance ℓ from a vertical solid wall as shown in Fig. P6.66. The velocity potential for this incompressible, irrotational flow is given by

$$\phi = \frac{m}{4\pi}\{\ln[(x-\ell)^2 + y^2] + \ln[(x+\ell)^2 + y^2]\}$$

(a) Show that there is no flow through the wall. **(b)** Determine the velocity distribution along the wall. **(c)** Determine the pressure distribution along the wall, assuming $p = p_0$ far from the source. Neglect the effect of the fluid weight on the pressure.

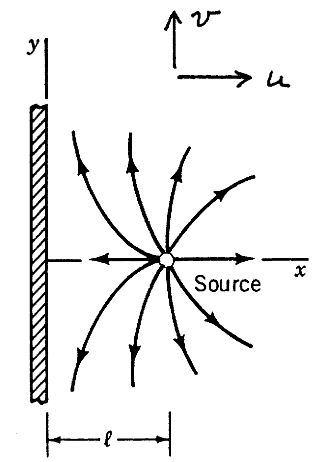


FIGURE P6.66

(a) $u = \frac{\partial \phi}{\partial x}$

Since, $\frac{\partial}{\partial x} \ln\left[(x-\ell)^2 + y^2\right] = \frac{2(x-\ell)}{(x-\ell)^2+y^2}$

and $\frac{\partial}{\partial x} \ln\left[(x+\ell)^2 + y^2\right] = \frac{2(x+\ell)}{(x+\ell)^2+y^2}$

it follows that

$$u = \frac{m}{4\pi}\left[\frac{2(x-\ell)}{(x-\ell)^2+y^2} + \frac{2(x+\ell)}{(x+\ell)^2+y^2}\right]$$

Along the wall, $x=0$, so that

$$u = \frac{m}{4\pi}\left[\frac{-2\ell}{\ell^2+y^2} + \frac{2\ell}{\ell^2+y^2}\right] = 0$$

Thus, there is no flow through the wall.

(b) The velocity along wall, $V_w = v$ since $u=0$. Also

$$v = \frac{\partial \phi}{\partial y}$$

and with the given velocity potential

$$v = \frac{m}{4\pi}\left[\frac{2y}{(x-\ell)^2+y^2} + \frac{2y}{(x+\ell)^2+y^2}\right] \qquad (1)$$

(con't)

 (con't)

Along the wall, $x=0$, and from Eq.(1)

$$V_w = v = \frac{m}{4\pi}\left[\frac{2y}{\ell^2+y^2} + \frac{2y}{\ell^2+y^2}\right]$$

or

$$\underline{\underline{V_w = \frac{m}{\pi}\left(\frac{y}{\ell^2+y^2}\right)}} \qquad (2)$$

(c) Far from the source, $p=p_0$ and $V \approx 0$. Thus,

$$\frac{p_0}{\gamma} = \frac{p_w}{\gamma} + \frac{V_w^2}{2g}$$

where p_w is the pressure at the wall, so that

$$p_w = p_0 - \frac{1}{2}\rho V_w^2$$

With V_w given by Eq.(2),

$$\underline{\underline{p_w = p_0 - \frac{\rho m^2}{2\pi^2}\left(\frac{y}{\ell^2+y^2}\right)^2}}$$

6.67 A long porous pipe runs parallel to a horizontal plane surface as shown in Fig. P6.67. The longitudinal axis of the pipe is perpendicular to the plane of the paper. Water flows radially from the pipe at a rate of $0.5\,\pi$ ft³/s per foot of pipe. Determine the difference in pressure (in lb/ft²) between point B and point A. The flow from the pipe may be approximated by a two-dimensional source. *Hint:* To develop the stream function or velocity potential for this type of flow, place (symmetrically) another equal source on the other side of the wall. With this combination there is no flow across the x-axis, and this axis can be replaced with a solid boundary. This technique is called the *method of images.*

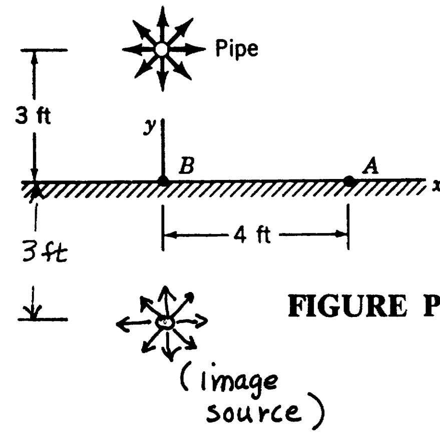


FIGURE P6.67

For a source,

$$\phi = \frac{m}{2\pi}\ln r = \frac{m}{4\pi}\ln r^2$$

where r is measured from the source. With the coordinate system shown in figure

$$r^2 = x^2 + (y-3)^2 \qquad (\text{for upper source})$$

and

$$r^2 = x^2 + (y+3)^2 \qquad (\text{for lower source})$$

so that for the combined sources

$$\phi = \frac{m}{4\pi}\left\{\ln\left[x^2+(y-3)^2\right] + \ln\left[x^2+(y+3)^2\right]\right\}$$

Since,

$$u = \frac{\partial \phi}{\partial x}$$

and

$$\frac{\partial}{\partial x}\ln\left[x^2+(y-3)^2\right] = \frac{2x}{x^2+(y-3)^2}$$

$$\frac{\partial}{\partial x}\ln\left[x^2+(y+3)^2\right] = \frac{2x}{x^2+(y+3)^2}$$

it follows that

$$u = \frac{m}{4\pi}\left[\frac{2x}{x^2+(y-3)^2} + \frac{2x}{x^2+(y+3)^2}\right]$$

Along the wall, $y=0$, $v=0$ and therefore

$$V_w = u = \frac{m}{4\pi}\left(\frac{4x}{x^2+9}\right)$$

(con't)

6.67 (Con't)

At point A, $x = 4\,ft$, and with $m = 0.5\pi \frac{ft^2}{s}$,

$$V_{wA} = \frac{0.5\pi \frac{ft^2}{s}}{4\pi}\left[\frac{4\,(4\,ft)}{(4\,ft)^2 + 9\,ft^2}\right] = \frac{2}{25}\,\frac{ft}{s}$$

At point B, $x = 0$, and

$$V_{wB} = 0$$

Thus, from the Bernoulli equation

$$\frac{p_B}{\gamma} + \frac{V_{wB}^2}{2g} = \frac{p_A}{\gamma} + \frac{V_{wA}^2}{2g}$$

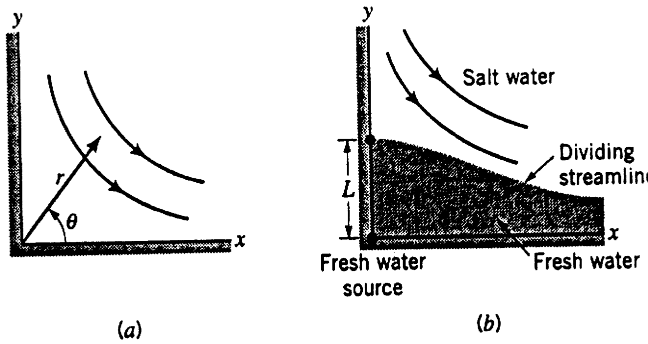
or

$$p_B - p_A = \frac{1}{2}\frac{\gamma}{g} V_{wA}^2$$

$$= \frac{\left(62.4 \frac{lb}{ft^3}\right)}{2\left(32.2 \frac{ft}{s^2}\right)}\left(\frac{2}{25}\,\frac{ft}{s}\right)^2 = \underline{\underline{0.00620\ psf}}$$

6.68

6.68 At a certain point at the beach, the coast line makes a right angle bend as shown in Fig. 6.68*a*. The flow of salt water in this bend can be approximated by the potential flow of an incompressible fluid in a right angle corner. **(a)** Show that the stream function for this flow is $\psi = A r^2 \sin 2\theta$, where A is a positive constant. **(b)** A fresh water reservoir is located in the corner. The salt water is to be kept away from the reservoir to avoid any possible seepage of salt water into the fresh water (Fig. 6.68*b*). The fresh water source can be approximated as a line source having a strength m, where m is the volume rate of flow (per unit length) emanating from the source. Determine m if the salt water is not to get closer than a distance L to the corner. *Hint:* Find the value of m (in terms of A and L) so that a stagnation point occurs at $y = L$. **(c)** The streamline passing through the stagnation point would represent the line dividing the fresh water from the salt water. Plot this streamline.



■ FIGURE P6.68

(a) For the given stream function,

$$\psi = A r^2 \sin 2\theta$$

along $\theta = 0$ $\psi = 0$ and $\theta = \pi/2$ $\psi = 0$.

Thus, the rays $\theta = 0$ and $\theta = \pi/2$ can be replaced with a solid boundary along which the stream function must be constant. This boundary forms a right angle and therefore this stream function can be used to represent flow in a right angle corner.

(b) Since

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 2Ar\cos 2\theta$$

at $\theta = \pi/2$

$$v_r = 2Ar\cos\pi = -2Ar$$

For a source located at the origin

$$\psi = \frac{m}{2\pi}\theta$$

and $v_{r_s} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{m}{2\pi r}$

To create a stagnation point at $r = L$ and $\theta = \frac{\pi}{2}$ let $v_r = v_{r_s}$.

(con't)

6.68 (Con't)

Thus,

$$2AL = \frac{m}{2\pi L}$$

and

$$\underline{\underline{m = 4\pi A L^2}}$$

gives a stagnation point at $r = L$, $\theta = \pi/2$.

(c) The combined stream function is

$$\psi = A r^2 \sin 2\theta + \frac{m}{2\pi}\theta$$

and with $m = 4\pi A L^2$

$$\psi = A r^2 \sin 2\theta + 2AL^2\theta$$

The value of ψ at the stagnation point $(r = L,\ \theta = \pi/2)$ is

$$\psi_{stag} = A L^2 \sin\pi + 2AL^2\left(\frac{\pi}{2}\right)$$

$$= AL^2\pi$$

Thus, the equation for the streamline passing through the stagnation point is

$$AL^2\pi = A r^2 \sin 2\theta + 2AL^2\theta$$

or

$$r = \sqrt{\frac{\pi L^2 - 2L^2\theta}{\sin 2\theta}}$$

and

$$r' = \frac{r}{L} = \sqrt{\frac{\pi - 2\theta}{\sin 2\theta}} \qquad (1)$$

For plotting let

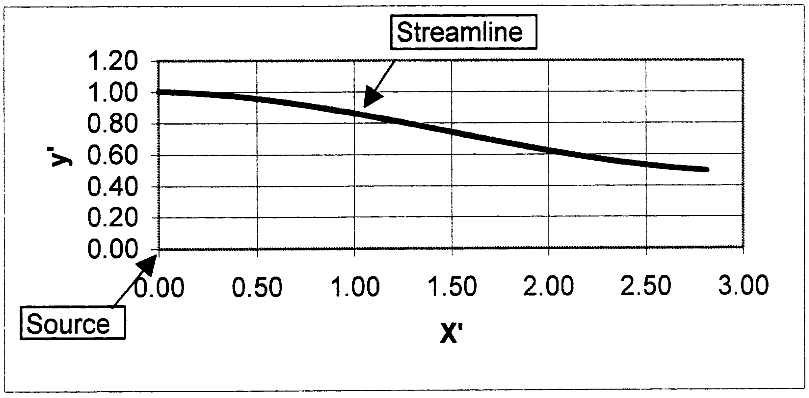
$$x' = r'\cos\theta \quad \text{and} \quad y' = r'\sin\theta$$

and a plot of the dividing streamline from Eq. (1) is shown on the following page.

(con't)

6.68 (con't)

Theta(deg)	Theta(rad)	r/L	x'	y'
10	0.175	2.857	2.814	0.496
20	0.349	1.950	1.832	0.667
30	0.524	1.555	1.347	0.778
40	0.698	1.331	1.020	0.856
50	0.873	1.191	0.765	0.912
60	1.047	1.100	0.550	0.952
70	1.222	1.042	0.356	0.979
80	1.396	1.010	0.175	0.995
90	1.571	1.000	0.000	1.000



6.69

6.69 The two-dimensional velocity field for an incompressible, Newtonian fluid is described by the relationship

$$\mathbf{V} = (12xy^2 - 6x^3)\hat{\mathbf{i}} + (18x^2y - 4y^3)\hat{\mathbf{j}}$$

where the velocity has units of m/s when x and y are in meters. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} at the point $x = 0.5$ m, $y = 1.0$ m if pressure at this point is 6 kPa and the fluid is glycerin at 20 °C. Show these stresses on a sketch.

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x} \qquad \text{(Eq. 6.125a)}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y} \qquad \text{(Eq. 6.125b)}$$

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \qquad \text{(Eq. 6.125d)}$$

For the given velocity distribution, with $x = 0.5\,m$ and $y = 1.0\,m$:

$$\frac{\partial u}{\partial x} = 12y^2 - 18x^2 = 12(1.0)^2 - 18(0.5)^2 = 7.50\,\frac{1}{s}$$

$$\frac{\partial u}{\partial y} = 24xy = 24(0.5)(1.0) = 12.0\,\frac{1}{s}$$

$$\frac{\partial v}{\partial x} = 36xy = 36(0.5)(1.0) = 18.0\,\frac{1}{s}$$

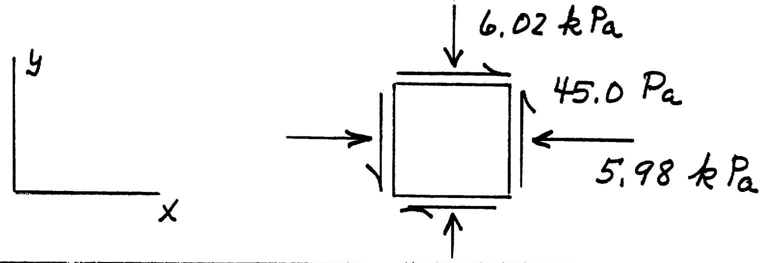
$$\frac{\partial v}{\partial y} = 18x^2 - 12y^2 = 18(0.5)^2 - 12(1.0)^2 = -7.50\,\frac{1}{s}$$

Thus, for $p = 6 \times 10^3 \frac{N}{m^2}$ and $\mu = 1.50 \frac{N \cdot s}{m^2}$,

$$\sigma_{xx} = -6 \times 10^3 \frac{N}{m^2} + 2\left(1.50 \frac{N \cdot s}{m^2}\right)\left(7.50\,\frac{1}{s}\right) = \underline{\underline{-5.98\,kPa}}$$

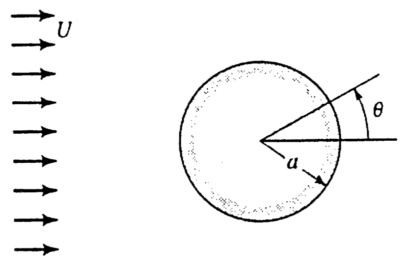
$$\sigma_{yy} = -6 \times 10^3 \frac{N}{m^2} + 2\left(1.50 \frac{N \cdot s}{m^2}\right)\left(-7.50\,\frac{1}{s}\right) = \underline{\underline{-6.02\,kPa}}$$

$$\tau_{xy} = \left(1.50 \frac{N \cdot s}{m^2}\right)\left(12.0\,\frac{1}{s} + 18.0\,\frac{1}{s}\right) = \underline{\underline{45.0\,Pa}}$$



6.70

6.70 Typical inviscid flow solutions for flow around bodies indicate that the fluid flows smoothly around the body, even for blunt bodies as shown in Video V6.4. However, experience reveals that due to the presence of viscosity, the main flow may actually separate from the body creating a wake behind the body. As discussed in a later section (Section 9.2.6), whether or not separation takes place depends on the pressure gradient along the surface of the body, as calculated by inviscid flow theory. If the pressure decreases in the direction of flow (a *favorable* pressure gradient), no separation will occur. However, if the pressure increases in the direction of flow (an *adverse* pressure gradient), separation may occur. For the circular cylinder of Fig. P6.70 placed in a uniform stream with velocity, U, determine an expression for the pressure gradient in the direction flow on the surface of the cylinder. For what range of values for the angle θ will an adverse pressure gradient occur?



■ FIGURE P6.70

From Eq. 6.116

$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta)$$

Thus,

$$\frac{\partial p_s}{\partial \theta} = \underline{\underline{4\rho U^2 \sin\theta\cos\theta}} \qquad (1)$$

Since an adverse pressure gradient occurs for a positive $\partial p_s/\partial\theta$, it follows from Eq. (1) that θ falls in the range of $\pm 90°$ for an adverse pressure gradient. This range corresponds to the rear half of the cylinder.

6.71

6.71 For a two-dimensional incompressible flow in the x–y plane show that the z component of the vorticity, ζ_z, varies in accordance with the equation

$$\frac{D\zeta_z}{Dt} = \nu \nabla^2 \zeta_z$$

What is the physical interpretation of this equation for a nonviscous fluid? *Hint:* This *vorticity transport equation* can be derived from the Navier–Stokes equations by differentiating and eliminating the pressure between Eqs. 6.127a and 6.127b.

For two-dimensional flow with $w=0$, Eq. 6.127a reduces to

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (1)$$

and Eq. 6.127b reduces to

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (2)$$

Differentiate Eq. (1) with respect to y and Eq. (2) with respect to x, and subtract Eq. (1) from Eq. (2) to obtain

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) =$$

$$\frac{\mu}{\rho}\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right] \qquad (3)$$

By definition (see Eq. 6.17)

$$\zeta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Re-write Eq. (3) to obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) =$$

$$\frac{\mu}{\rho}\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right] \qquad (4)$$

(con't)

 (con't)

Since each term in parenthesis in Eq. (4) is ζ_z it follows that

$$\frac{\partial \zeta_z}{\partial t} + u\frac{\partial \zeta_z}{\partial x} + v\frac{\partial \zeta_z}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 \zeta_z}{\partial x^2} + \frac{\partial^2 \zeta_z}{\partial y^2}\right) \qquad (5)$$

The left side of Eq. (5) can be expressed as (see Eq. 4.5) $\frac{D\zeta_z}{Dt}$ where the operator $\frac{D(\;)}{Dt}$ is the material derivative. The right hand side of Eq. (5) can be expressed as

$$\nu \nabla^2 \zeta_z$$

where $\nu = \mu/\rho$ so that Eq. (5) can be written as

$$\underline{\underline{\frac{D\zeta_z}{Dt} = \nu \nabla^2 \zeta_z}}$$

For a nonviscous fluid, $\nu = 0$, and in this case

$$\frac{D\zeta_z}{Dt} = 0$$

Thus, for a two-dimensional flow of an incompressible, nonviscous fluid, the change in the vorticity of a fluid particle as it moves through the flow field is zero.

6.72

6.72 The velocity of a fluid particle moving along a horizontal streamline that coincides with the x axis in a plane, two-dimensional incompressible flow field was experimentally found to be described by the equation $u = x^2$. Along this streamline determine an expression for: **(a)** the rate of change of the v-component of velocity with respect to y; **(b)** the acceleration of the particle; and **(c)** the pressure gradient in the x direction. The fluid is Newtonian.

(a) From the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

so that with $u = x^2$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \underline{\underline{-2x}} \qquad (1)$$

Also, Eq. (1) can be integrated with respect to y to obtain

$$\int dv = \int -2x\, dy$$

or

$$v = -2xy + f(x)$$

Since the x-axis is a streamline, $v=0$ along this axis and therefore $f(x)=0$ so that

$$v = -2xy$$

(b)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (x^2)(2x) + (-2xy)(0) = 2x^3$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (x^2)(-2y) + (-2xy)(-2x) = 2x^2y$$

Along x-axis, $y=0$, and therefore $a_y = 0$. Thus,

$$\vec{a} = \underline{\underline{2x^3\,\hat{i}}}$$

(c) From Eq. 6.127a (with $g_x = 0$),

$$a_x = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

so that

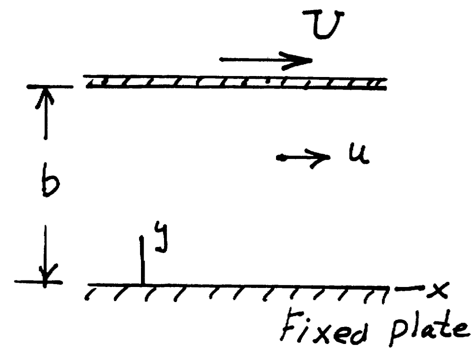
$$2x^3 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}(2+0)$$

and

$$\frac{\partial p}{\partial x} = \underline{\underline{2\mu - 2\rho x^3}}$$

6.73

6.73 Two horizontal, infinite, parallel plates are spaced a distance b apart. A viscous liquid is contained between the plates. The bottom plate is fixed and the upper plate moves parallel to the bottom plate with a velocity U. Because of the no-slip boundary condition (see **Video V6.5**), the liquid motion is caused by the liquid being dragged along by the moving boundary. There is no pressure gradient in the direction of flow. Note that this is a so-called simple *Couette flow* discussed in Section 6.9.2. **(a)** Start with the Navier–Stokes equations and determine the velocity distribution between the plates. **(b)** Determine an expression for the flowrate passing between the plates (for a unit width). Express your answer in terms of b and U.



(a) For steady flow with $v = w = 0$ it follows that the Navier-Stokes equations reduce to (in direction of flow)

$$0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(Eq. 6.129)}$$

Thus, for zero pressure gradient

$$\frac{\partial^2 u}{\partial y^2} = 0$$

so that

$$u = C_1 y + C_2$$

At $y = 0$ $u = 0$ and it follows that $C_2 = 0$. Similarly, at $y = b$ $u = U$ and $C_1 = \frac{U}{b}$

Therefore,

$$\underline{\underline{u = \frac{U}{b}\,y}}$$

(b) $$q = \int_0^b u(1)\,dy = \frac{U}{b}\int_0^b y\,dy = \frac{U}{b}\,\frac{y^2}{2}\Big|_0^b = \underline{\underline{\frac{Ub}{2}}}$$

where q is the flowrate per unit width.

6.74

6.74 Oil (SAE 30) at 15.6 °C flows steadily between fixed, horizontal, parallel plates. The pressure drop per unit length along the channel is 20 kPa/m, and the distance between the plates is 4mm. The flow is laminar. Determine: **(a)** the volume rate of flow (per meter of width), **(b)** the magnitude and direction of the shearing stress acting on the bottom plate, and **(c)** the velocity along the centerline of the channel.

(a) $$q = \frac{2h^3}{3\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.136)}$$

For $h = \frac{4\,mm}{2} = 2 \times 10^{-3}\,m$, $\mu = 0.38\,\frac{N\cdot s}{m^2}$, and $\frac{\Delta p}{\ell} = 20 \times 10^3\,\frac{N}{m^3}$,

$$q = \frac{2\,(2\times 10^{-3}\,m)^3 (20 \times 10^3\,\frac{N}{m^3})}{3\,(0.38\,\frac{N\cdot s}{m^2})} = \underline{\underline{2.81 \times 10^{-4}\,\frac{m^2}{s}}}$$

(b) $$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \text{(Eq. 6.125d)}$$

Since
$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - h^2) \qquad \text{(Eq. 6.134)}$$

and $$v = 0$$

it follows that
$$\frac{\partial u}{\partial y} = \frac{1}{2\mu}\frac{\partial p}{\partial x}(2y) \qquad \frac{\partial v}{\partial x} = 0$$

and therefore
$$\tau_{yx} = \frac{\partial p}{\partial x}(y)$$

At the bottom plate, $y = -h$, and since $\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$,

$$\tau_{yx} = \frac{\Delta p}{\ell}(h) = (20 \times 10^3\,\frac{N}{m^3})(2 \times 10^{-3}\,m)$$

$$= \underline{\underline{40\,\frac{N}{m^2}\ \text{acting in the direction of flow}}}$$

(c)
$$u_{max} = \frac{3}{2}V \qquad \text{(Eq. 6.138)}$$

$$= \frac{3}{2}\left(\frac{q}{2h}\right) = \frac{3}{2}\,\frac{(2.81 \times 10^{-4}\,\frac{m^2}{s})}{(2)(2 \times 10^{-3}\,m)} = \underline{\underline{0.105\,\frac{m}{s}}}$$

6.75

6.75 Two fixed, horizontal, parallel plates are spaced 0.2 in. apart. A viscous liquid ($\mu = 8 \times 10^{-3}$ lb · s/ft^2, $SG = 0.9$) flows between the plates with a mean velocity of 0.7 ft/s. Determine the pressure drop per unit length in the direction of flow. What is the maximum velocity in the channel?

$$V = \frac{h^2}{3\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.137)}$$

Thus,

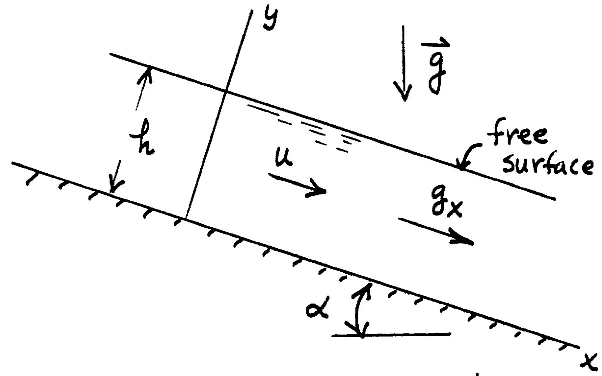
$$\frac{\Delta p}{\ell} = \frac{3\mu V}{h^2} = \frac{3\left(8\times 10^{-3}\,\frac{lb\cdot s}{ft^2}\right)\left(0.7\,\frac{ft}{s}\right)}{\left(\frac{0.1\ in.}{12\ \frac{in.}{ft}}\right)^2} = \underline{\underline{242\ \frac{lb}{ft^2}\ \text{per ft}}}$$

$$u_{max} = \frac{3}{2}V \qquad \text{(Eq. 6.138)}$$

$$= \frac{3}{2}\left(0.7\,\frac{ft}{s}\right) = \underline{\underline{1.05\ \frac{ft}{s}}}$$

6.76

6.76 A layer of viscous liquid of constant thickness (no velocity perpendicular to plate) flows steadily down an infinite, inclined plane. Determine, by means of the Navier–Stokes equations, the relationship between the thickness of the layer and the discharge per unit width. The flow is laminar, and assume air resistance is negligible so that the shearing stress at the free surface is zero.



$g_x = g \sin\alpha$

With the coordinate system shown in the figure $v = 0$, $w = 0$, and from the continuity equation $\frac{\partial u}{\partial x} = 0$. Thus, from the x-component of the Navier-Stokes equations (Eq. 6.127a),

$$0 = -\frac{\partial p}{\partial x} + \rho g \sin\alpha + \mu \frac{d^2 u}{dy^2} \qquad (1)$$

Also, since there is a free surface, there cannot be a pressure gradient in the x-direction so that $\frac{\partial p}{\partial x} = 0$ and Eq. (1) can be written as

$$\frac{d^2 u}{dy^2} = -\frac{\rho g}{\mu} \sin\alpha$$

Integration yields

$$\frac{du}{dy} = -\left(\frac{\rho g}{\mu} \sin\alpha\right) y + C_1 \qquad (2)$$

Since the shearing stress

$$\tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

equals zero at the free surface ($y = h$) it follows that

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = h$$

so that the constant in Eq. (2) is

$$C_1 = \frac{\rho g}{\mu} \sin\alpha$$

Integration of Eq. (2) yields

$$u = -\left(\frac{\rho g}{\mu} \sin\alpha\right) \frac{y^2}{2} + \left(\frac{\rho g}{\mu} \sin\alpha\right) y + C_2$$

Since $u = 0$ at $y = 0$, it follows that $C_2 = 0$, and therefore

$$u = \frac{\rho g}{\mu} \sin\alpha \left(h y - \frac{y^2}{2}\right)$$

The flowrate per unit width can be expressed as $q = \int_0^h u\, dy$ so that

$$q = \int_0^h \frac{\rho g}{\mu} \sin\alpha \left(h y - \frac{y^2}{2}\right) dy = \underline{\underline{\frac{\rho g h^3 \sin\alpha}{3\mu}}}$$

6.77

6.77 A viscous, incompressible fluid flows between the two infinite, vertical, parallel plates of Fig. P6.77. Determine, by use of the Navier–Stokes equations, an expression for the pressure gradient in the direction of flow. Express your answer in terms of the mean velocity. Assume that the flow is laminar, steady, and uniform.

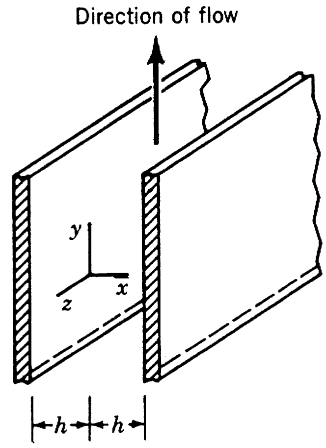


FIGURE P6.77

With the coordinate system shown $u=0$, $w=0$ and from the continuity equation $\frac{\partial v}{\partial y}=0$. Thus, from the y-component of the Navier-Stokes equations (Eq. 6.127b), with $g_y=-g$,

$$0 = -\frac{\partial p}{\partial y} - \rho g + \mu\frac{d^2 v}{dx^2} \qquad (1)$$

Since the pressure is not a function of x, Eq. (1) can be written as

$$\frac{d^2 v}{dx^2} = \frac{P}{\mu}$$

(where $P = \frac{\partial p}{\partial y} + \rho g$) and integrated to obtain

$$\frac{dv}{dx} = \frac{P}{\mu}x + C_1 \qquad (2)$$

From symmetry $\frac{dv}{dx}=0$ at $x=0$ so that $C_1=0$. Integration of Eq. (2) yields

$$v = \frac{P}{\mu}\frac{x^2}{2} + C_2$$

Since at $x=\pm h$, $v=0$ it follows that $C_2 = -\frac{P}{2\mu}(h^2)$

and therefore

$$v = \frac{P}{2\mu}(x^2-h^2)$$

The flowrate per unit width in the z-direction can be expressed as

$$q = \int_{-h}^{h} v\,dx = \int_{-h}^{h}\frac{P}{2\mu}(x^2-h^2)\,dx = -\frac{2}{3}\frac{Ph^3}{\mu}$$

Thus, with V (mean velocity) given by the equation

$$V = \frac{q}{2h} = -\frac{1}{3}\frac{Ph^2}{\mu}$$

it follows that

$$\underline{\underline{\frac{\partial p}{\partial y} = -\frac{3\mu V}{h^2} - \rho g}}$$

6.78

6.78 A fluid of density ρ flows steadily *downward* between the two vertical infinite, parallel plates shown in the figure for Problem 6.77. The flow is fully developed and laminar. Make use of the Navier–Stokes equation to determine the relationship between the discharge and the other parameters involved, for the case in which the change in pressure along the channel is zero.

See solution for Problem 6.83 to obtain

$$q = -\frac{2}{3}\frac{\mathcal{P} h^3}{\mu}$$

where q is the discharge per unit width and $\mathcal{P} = \frac{\partial p}{\partial y} + \rho g$. Thus,

$$\frac{\partial p}{\partial y} + \rho g = -\frac{3}{2}\frac{\mu q}{h^3}$$

or

$$\frac{\partial p}{\partial y} = -\frac{3}{2}\frac{\mu q}{h^3} - \rho g$$

For $\frac{\partial p}{\partial y} = 0$

$$\underline{\underline{q = -\frac{2}{3}\frac{\rho g h^3}{\mu}}}$$

(Note: The negative sign indicates that the direction of flow must be <u>downward</u> to create a zero pressure gradient.)

6.79

6.79 Due to the no-slip condition, as a solid is pulled out of a viscous liquid some of the liquid is also pulled along as described in Example 6.9 and shown in Video V6.5. Based on the results given in Example 6.9, show on a dimensionless plot the velocity distribution in the fluid film (v/V_0 vs. x/h) when the average film velocity, V, is 10% of the belt velocity, V_0.

From Example 6.9, the average velocity is given by the equation

$$V = V_0 - \frac{\gamma h^2}{3\mu} \qquad (1)$$

with the velocity distribution

$$v = \frac{\gamma}{2\mu} x^2 - \frac{\gamma h}{\mu} x + V_0 \qquad (2)$$

If $V = 0.1 V_0$, then from Eq. (1)

$$0.1 V_0 = V_0 - \frac{\gamma h^2}{3\mu}$$

or

$$V_0 = \frac{\gamma h^2}{2.7\mu} \qquad (3)$$

In dimensionless form Eq. (2) becomes

$$\frac{v}{V_0} = \frac{\gamma h^2}{2\mu V_0}\left(\frac{x}{h}\right)^2 - \frac{\gamma h^2}{\mu V_0}\left(\frac{x}{h}\right) + 1 \qquad (4)$$

From Eq. (3)

$$\frac{\gamma h^2}{\mu V_0} = 2.7$$

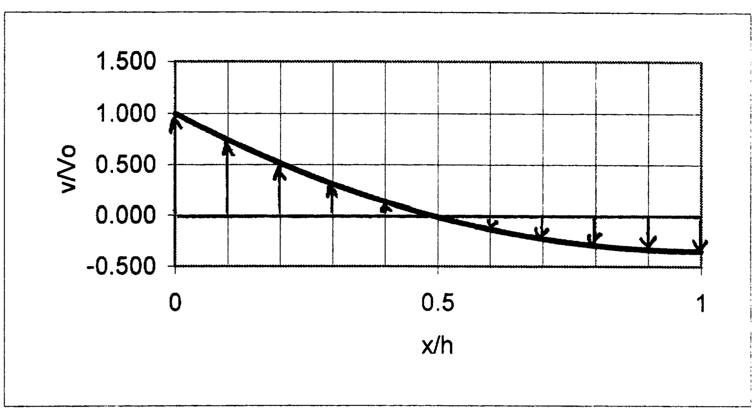
and Eq. (4) can be written as

$$\frac{v}{V_0} = 1.35\left(\frac{x}{h}\right)^2 - 2.7\left(\frac{x}{h}\right) + 1 \qquad (5)$$

A plot of the velocity distribution is shown below.

x/h	v/Vo
0	1.000
0.1	0.744
0.2	0.514
0.3	0.312
0.4	0.136
0.5	-0.013
0.6	-0.134
0.7	-0.229
0.8	-0.296
0.9	-0.337
1	-0.350

Calculated from Eq. (5)



6.80 An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig. P6.80. The two plates move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier–Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

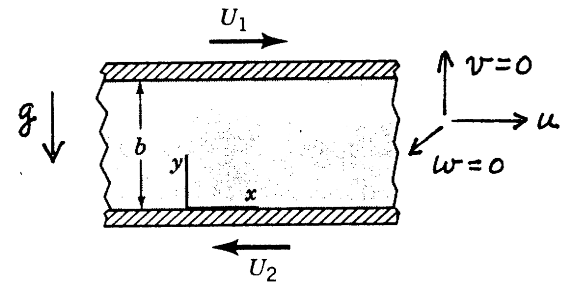


FIGURE P6.80

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) reduces to

$$\frac{d^2u}{dy^2}=0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = C_1 y + C_2 \qquad (2)$$

For $y=0$, $u=-U_2$ and therefore from Eq. (2)

$$C_2 = -U_2$$

For $y=b$, $u=U_1$ so that

$$U_1 = C_1 b - U_2$$

or

$$C_1 = \frac{U_1+U_2}{b}$$

Thus,

$$\underline{\underline{u = \left(\frac{U_1+U_2}{b}\right) y - U_2}}$$

6.81

6.81 Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates (Fig. P6.81. The bottom plate is fixed and the upper plate moves with a constant velocity U. Determine the velocity at the interface. Express your answer in terms of U, μ_1, and μ_2. The motion of the fluid is caused entirely by the movement of the upper plate; that is, there is no pressure gradient in the x direction. The fluid velocity and shearing stress is continuous across the interface between the two fluids. Assume laminar flow.

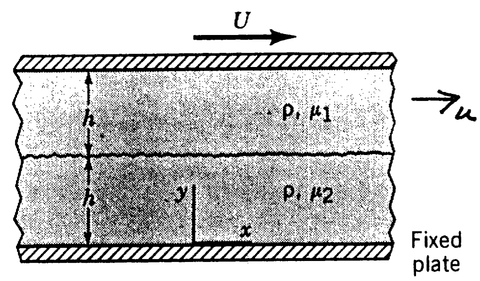


FIGURE P6.81

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) for either the upper or lower layer reduces to

$$\frac{d^2u}{dy^2}=0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = Ay + B$$

which gives the velocity distribution in either layer.
In the upper layer at $y=2h$, $u=U$ so that

$$B_1 = U - A_1(2h)$$

where the subscript 1 refers to the upper layer.
For the lower layer at $y=0$, $u=0$ so that

$$B_2 = 0$$

where the subscript 2 refers to the lower layer. Thus,

$$u_1 = A_1(y-2h)+U$$

and

$$u_2 = A_2 y$$

At $y=h$, $u_1=u_2$ so that

$$A_1(h-2h)+U = A_2 h$$

or

$$A_2 = -A_1 + \frac{U}{h} \qquad (2)$$

(con't)

Since the velocity distribution is linear in each layer the shearing stress

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu \frac{du}{dy}$$

is constant throughout each layer. For the upper layer

$$\tau_1 = \mu_1 A_1$$

and for the lower layer

$$\tau_2 = \mu_2 A_2$$

At the interface $\tau_1 = \tau_2$ so that

$$\mu_1 A_1 = \mu_2 A_2$$

or

$$\frac{A_1}{A_2} = \frac{\mu_2}{\mu_1} \qquad (3)$$

Substitution of Eq. (3) into Eq. (2) yields

$$A_2 = -\frac{\mu_2}{\mu_1} A_2 + \frac{U}{h}$$

or

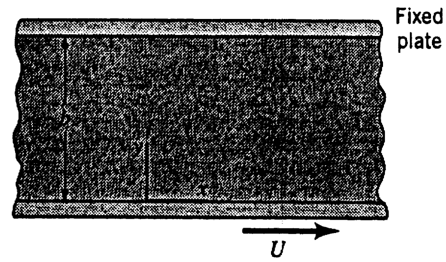
$$A_2 = \frac{U/h}{1 + \mu_2/\mu_1}$$

Thus, velocity at the interface is

$$u_2(y=h) = A_2 h = \underline{\underline{\frac{U}{1+\frac{\mu_2}{\mu_1}}}}$$

6.82

6.82 The viscous, incompressible flow between the parallel plates shown in Fig. P6.82 is caused by both the motion of the bottom plate and a pressure gradient, $\partial p/\partial x$. As noted in Section 6.9.2, an important dimensionless parameter for this type of problem is $P = -(b^2/2\,\mu U)\,(\partial p/\partial x)$ where μ is the fluid viscosity. Make a plot of the dimensionless velocity distribution (similar to that shown in Fig. 6.31b) for $P = 3$. For this case where does the maximum velocity occur?



■ FIGURE P6.82

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 + C_1 y + C_2 \qquad \text{(Eq. 6.133)}$$

At $y=0$, $u=U$ so that $C_2 = U$. At $y=b$, $u=0$ and therefore

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b^2 + C_1 b + U$$

or

$$C_1 = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b - \frac{U}{b}$$

Thus,

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) + U\left(1 - \frac{y}{b}\right)$$

or in dimensionless form

$$\frac{u}{U} = \frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (1)$$

Since,

$$P = -\frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)$$

Eq. (1) can be written as

$$\frac{u}{U} = -P\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (2)$$

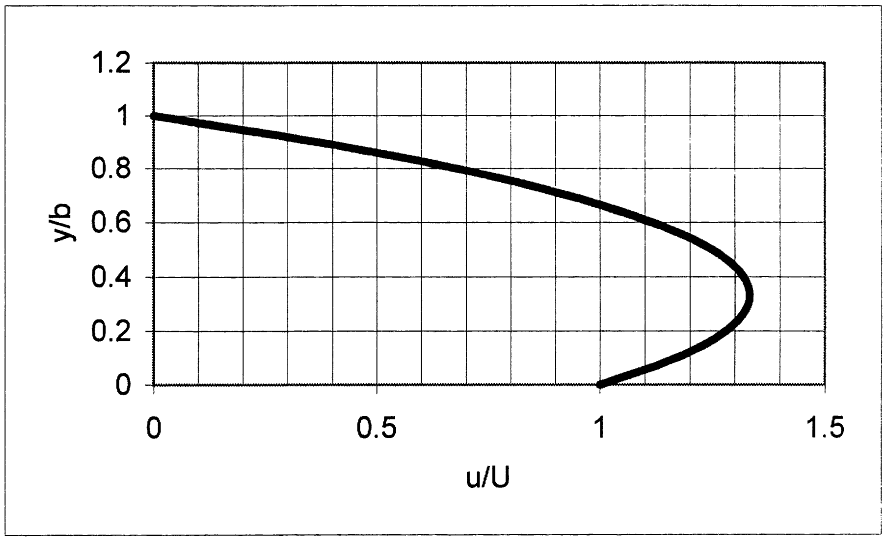
A plot of this velocity distribution for $P = 3$ is shown on the following page.

(con't)

6.82 (con't)

u/U	y/b
1	0
1.17	0.1
1.28	0.2
1.33	0.3
1.32	0.4
1.25	0.5
1.12	0.6
0.93	0.7
0.68	0.8
0.37	0.9
0	1

Calculated from Eq. (2) with P =3.



To determine where the maximum velocity occurs differentiate Eq. (2) and set equal to zero. Thus,

$$\frac{d(u/U)}{dy} = -P\left[2\left(\frac{y}{b^2}\right) - \frac{1}{b}\right] - \frac{1}{b} = 0$$

and with $P = 3$

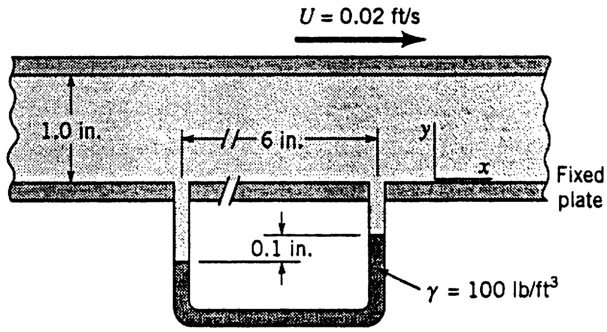
$$\frac{d(u/U)}{dy} = -3\left[\frac{1}{b}\left(2\frac{y}{b} - 1\right)\right] - \frac{1}{b} = 0$$

so that

$$\frac{y}{b} = \frac{1}{3}$$

6.83

6.83 A viscous fluid (specific weight = 80 lb/ft^3; viscosity = 0.03 lb · s/ft^2) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.83. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



■ FIGURE P6.83

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) \qquad \text{(Eq. 6.140)}$$

Maximum velocity will occur at distance y_m where $\frac{du}{dy} = 0$.

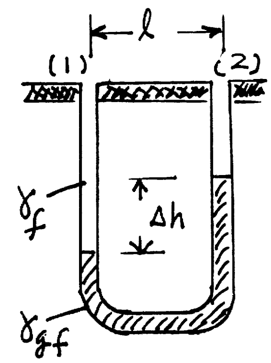
Thus,

$$\frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$$

and for $\frac{du}{dy} = 0$

$$y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2} \qquad (1)$$

For manometer (see figure to right),



$$p_1 + \gamma_f\,\Delta h - \gamma_{gf}\,\Delta h = p_2$$

or

$$p_1 - p_2 = (\gamma_{gf} - \gamma_f)\,\Delta h$$

$$= \left(100\,\frac{lb}{ft^3} - 80\,\frac{lb}{ft^3}\right)\left(\frac{0.1\text{ in.}}{12\,\frac{in.}{ft}}\right) = 0.167\,\frac{lb}{ft^2}$$

Also,

$$-\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\ell} = \frac{0.167\,\frac{lb}{ft^2}}{\left(\frac{6\text{ in.}}{12\,\frac{in.}{ft}}\right)} = 0.334\,\frac{lb}{ft^3}$$

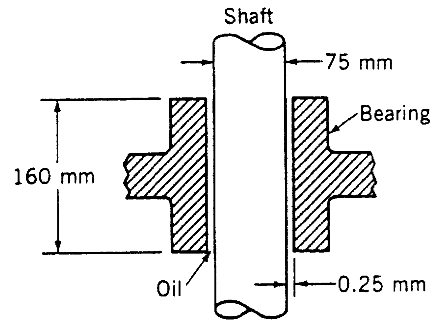
Thus, from Eq. (1)

$$y_m = -\frac{\left(0.03\,\frac{lb\cdot s}{ft^2}\right)\left(0.02\,\frac{ft}{s}\right)}{\left(\frac{1.0\text{ in.}}{12\,\frac{in.}{ft}}\right)\left(-0.334\,\frac{lb}{ft^3}\right)} + \frac{\frac{1.0\text{ in.}}{12\,\frac{in.}{ft}}}{2}$$

$$= 0.0632\text{ ft}\left(\frac{12\text{ in.}}{ft}\right) = \underline{\underline{0.759\text{ in.}}}$$

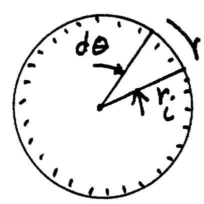
6.84

6.84 A vertical shaft passes through a bearing and is lubricated with an oil having a viscosity of $0.2\ \mathrm{N \cdot s/m^2}$ as shown in Fig. P6.84. Assume that the flow characteristics in the gap between the shaft and bearing are the same as those for laminar flow between infinite parallel plates with zero pressure gradient in the direction of flow. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rev/min.



■ **FIGURE P6.84**

The torque due to force dF acting on a differential area, $dA = r_i \ell \, d\theta$, is (see figure at right)



$$dF = \tau \, dA = \tau r_i \ell \, d\theta$$

$\ell \sim$ shaft length

$$d\mathcal{T} = r_i \, dF = r_i^2 \tau \ell \, d\theta$$

where τ is the shearing stress. Thus,

$$\mathcal{T} = r_i^2 \tau \ell \int_0^{2\pi} d\theta = 2\pi r_i^2 \tau \ell \qquad (1)$$

In the gap,

$$u = U \frac{y}{b} \qquad \text{(Eq. 6.142)}$$

where $U = r_i \omega$ and b is the gap width. Also,

$$\tau = \mu \frac{du}{dy} = \frac{\mu U}{b}$$

Thus, from Eq. (1)

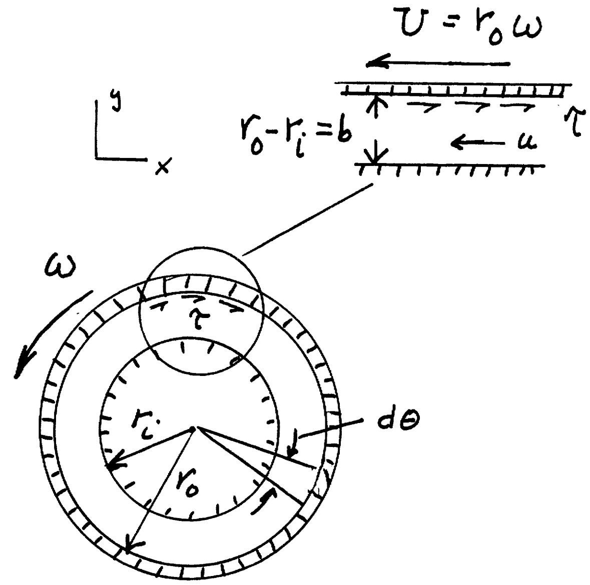
$$\mathcal{T} = 2\pi r_i^2 \left(\mu \frac{U}{b}\right) \ell = 2\pi r_i^3 \mu \omega \frac{\ell}{b}$$

$$= 2\pi \left(\frac{0.075}{2}\ \mathrm{m}\right)^3 \left(0.2\ \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}\right) \left[\left(80\ \frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{2\pi\ \mathrm{rad}}{\mathrm{rev}}\right)\left(\frac{\mathrm{min}}{60\ \mathrm{s}}\right)\right] \frac{(0.160\ \mathrm{m})}{(0.25 \times 10^{-3}\ \mathrm{m})}$$

$$= \underline{\underline{0.355\ \mathrm{N \cdot m}}}$$

6.85

6.85 A viscous fluid is contained between two long concentric cylinders. The geometry of the system is such that the flow between the cylinders is approximately the same as the laminar flow between two infinite parallel plates. (a) Determine an expression for the torque required to rotate the outer cylinder with an angular velocity ω. The inner cylinder is fixed. Express your answer in terms of the geometry of the system, the viscosity of the fluid, and the angular velocity. (b) For a small rectangular element located at the fixed wall determine an expression for the rate of angular deformation of this element. (See Video V6.1 and Fig. P6.9.)



$\ell \sim$ cylinder length
$\tau \sim$ shearing stress

(a) The torque which must be applied to outer cylinder to overcome the force due to the shearing stress is (see figure)

$$d\mathcal{T} = r_o\, dF = r_o\,(\tau\, r_o\, \ell\, d\theta) = r_o^2\, \tau\, \ell\, d\theta$$

so that

$$\mathcal{T} = r_o^2\, \tau\, \ell \int_0^{2\pi} d\theta = 2\pi r_o^2\, \tau\, \ell \qquad (1)$$

In the gap

$$u = U\frac{y}{b} \qquad \text{(Eq. 6.142)}$$

Since,

$$\tau = \mu\frac{du}{dy} = \frac{\mu U}{b}$$

and $b = r_o - r_i$, $U = r_o\omega$ (see figure), it follows from Eq. (1) that

$$\mathcal{T} = 2\pi r_o^2\left(\frac{\mu\, r_o\omega}{r_o - r_i}\right)\ell = \underline{\underline{\frac{2\pi r_o^3\, \mu\omega\ell}{r_o - r_i}}}$$

(con't)

6.85 (con't)

(b) From Eq. 6.18

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

For the linear distribution

$$u = -\frac{r_o \omega}{r_o - r_i}\, y = -\frac{U y}{b}$$

so that

$$\frac{\partial u}{\partial y} = -\frac{U}{b}$$

and $\quad v = 0$

$$\underline{\underline{\dot{\gamma} = -\frac{U}{b}}}$$

The negative sign indicates that the original right angle shown in Fig. P6.9b is increasing.

6.86*

6.86* Oil (SAE 30) flows between parallel plates spaced 5 mm apart. The bottom plate is fixed but the upper plate moves with a velocity of 0.2 m/s in the positive x direction. The pressure gradient is 60 kPa/m, and is negative. Compute the velocity at various points across the channel and show the results on a plot. Assume laminar flow.

The velocity distribution is given by the equation

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left(y^2 - by\right) \qquad \text{(Eq. 6.140)}$$

and for the given data,

$$u = \frac{\left(0.2\,\frac{m}{s}\right)}{(0.005\,m)}\,y + \frac{1}{2\left(0.38\,\frac{N\cdot s}{m^2}\right)}\left(-60\times10^3\,\frac{N}{m^3}\right)\left[y^2 - (0.005\,m)\,y\right]$$

so that

$$u = 40\,y + 7.89\times10^4\left(0.005\,y - y^2\right)$$

with u in m/s when y is in m. A program for calculating u as a function of y follows.

```
100 cls
110 print "*****************************************************"
120 print "** This program calculates the velocity profile **"
130 print "** for Couette flow                              **"
140 print "*****************************************************"
150 print
155 print "    y        u(y)"
160 for y=0.0 to 0.0051 step 0.0005
170 u=40*y+78900*(0.005*y-y^2)
180 print using "#.####    #.####";y,u
190 next y
```

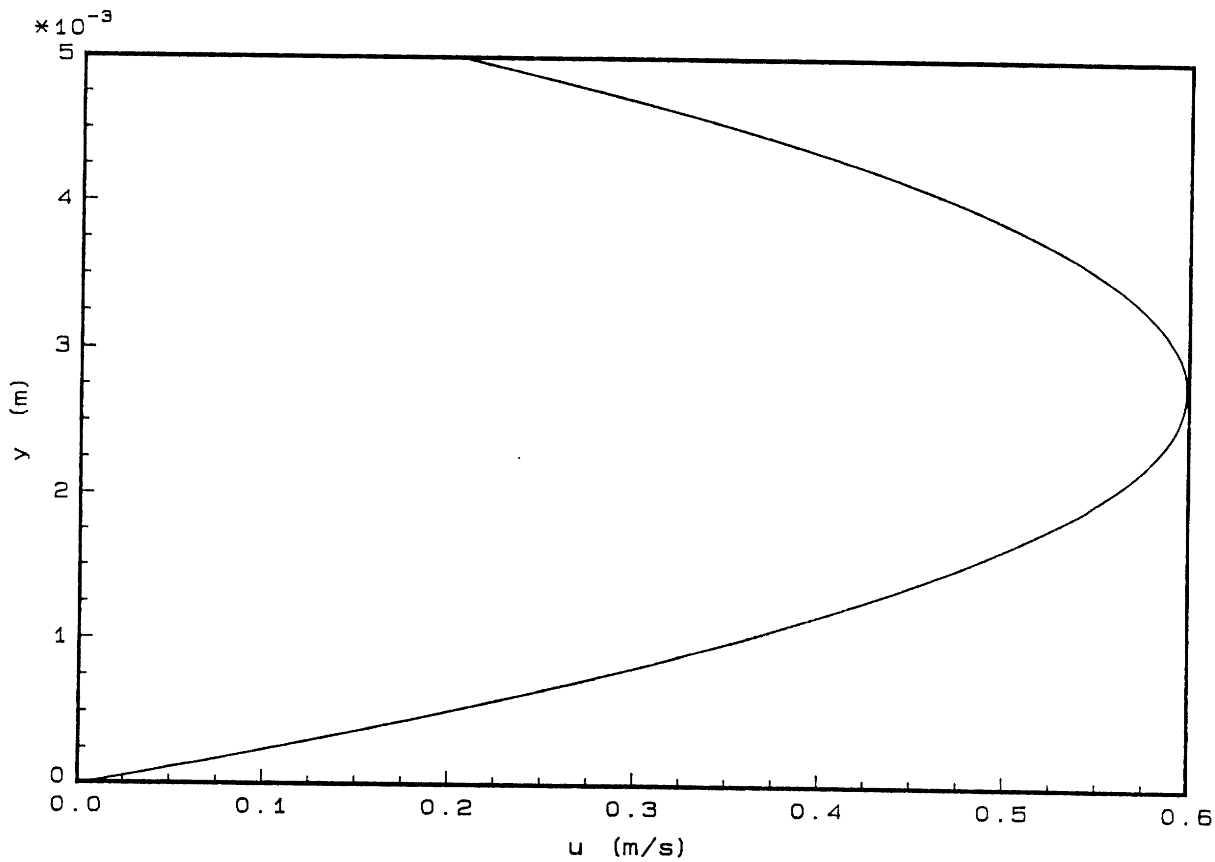
(con't)

6.86* (con't)

Tabulated data and a plot of the data are given below.

```
***************************************************
** This program calculates the velocity profile **
** for Couette flow                             **
***************************************************
```

y	u(y)
0.0000	0.0000
0.0005	0.1975
0.0010	0.3556
0.0015	0.4742
0.0020	0.5534
0.0025	0.5931
0.0030	0.5934
0.0035	0.5542
0.0040	0.4756
0.0045	0.3575
0.0050	0.2000



6.87

6.87 Consider a steady, laminar flow through a straight horizontal tube having the constant elliptical cross section given by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The streamlines are all straight and parallel. Investigate the possibility of using an equation for the z component of velocity of the form

$$w = A\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)$$

as an exact solution to this problem. With this velocity distribution what is the relationship between the pressure gradient along the tube and the volume flowrate through the tube?

From the description of the problem, $u=0$, $v=0$, $g_z=0$, $w \neq f(t)$, and the continuity equation indicates that $\frac{\partial w}{\partial z}=0$. With these conditions the z-component of the Navier-Stokes equations (Eq. 6.127c) reduces to

$$\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \qquad (1)$$

Due to the no-slip boundary condition, $w=0$ on the elliptical boundary

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Thus, the proposed velocity distribution satisfies this condition since on the boundary

$$w = A\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = A\left[1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)\right] = A\left[1-(1)\right] = 0$$

This result indicates that the proposed velocity distribution can be used as a solution. Substitution of the velocity distribution into Eq. (1) gives the relationship between the pressure gradient, $\frac{\partial p}{\partial z}$, and the velocity. Since,

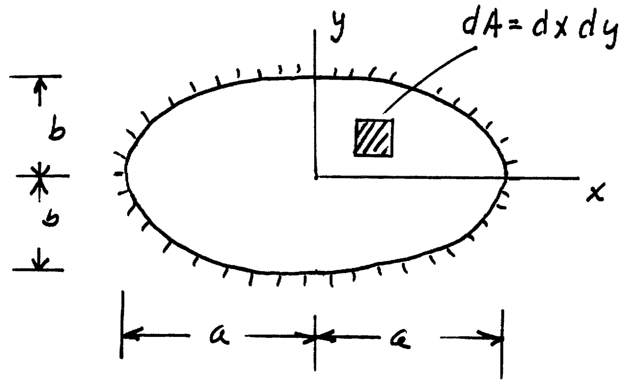
$$\frac{\partial^2 w}{\partial x^2} = -\frac{2A}{a^2} \qquad \frac{\partial^2 w}{\partial y^2} = -\frac{2A}{b^2}$$

it follows that

$$\frac{\partial p}{\partial z} = -2A\mu\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \qquad (2)$$

(con't)

The volume flowrate, Q, through the tube is given by the equation



$$Q = \int_{\text{area}} w \, dA$$

$$= 4 \int_0^b \int_0^{a\sqrt{1-\frac{y^2}{b^2}}} w \, dx \, dy$$

Thus,

$$Q = 4A \int_0^b \int_0^{a\sqrt{1-\frac{y^2}{b^2}}} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx \, dy$$

$$= 4A \int_0^b \left[x - \frac{x^3}{3a^2} - \frac{y^2}{b^2} x \right]_0^{a\sqrt{1-\frac{y^2}{b^2}}} dy$$

$$= 4A \int_0^b \left[a\sqrt{1-\frac{y^2}{b^2}} \left(1 - \frac{y^2}{b^2}\right) - \frac{1}{3} a \sqrt{1 - \frac{y^2}{b^2}} \left(1 - \frac{y^2}{b^2}\right) \right] dy$$

$$= \frac{8Aa}{3} \int_0^b \left(1 - \frac{y^2}{b^2}\right)^{3/2} dy = \frac{8Aa}{3} \left(\frac{3b\pi}{16}\right) = \frac{A\pi ab}{2}$$

and therefore

$$A = \frac{2Q}{\pi ab}$$

From Eq. (2)

$$\frac{\partial p}{\partial z} = -\frac{4\mu Q}{\pi ab} \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

6.88

6.88 A fluid is initially at rest between two horizontal, infinite, parallel plates. A constant pressure gradient in a direction parallel to the plates is suddenly applied and the fluid starts to move. Determine the appropriate differential equation(s), initial condition, and boundary conditions that govern this type of flow. You need not solve the equation(s).

Differential equations are the same as Eqs. 6.129, 6.130, and 6.131 except that $\frac{\partial u}{\partial t} \neq 0$ (since the flow is unsteady). Thus, Eq. 6.129 must include the local acceleration term, $\frac{\partial u}{\partial t}$, and the governing differential equations are:

(x- direction) $$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$ (with $\frac{\partial p}{\partial x}$ = constant)

(y- direction) $$0 = -\frac{\partial p}{\partial y} - \rho g$$

(z- direction) $$0 = -\frac{\partial p}{\partial z}$$

Initial condition: $u = 0$ for $t = 0$ for all y.

Boundary conditions: $u = 0$ for $y = \pm h$ for $t \geq 0$.

6.89

6.89 It is known that the velocity distribution for steady, laminar flow in circular tubes (either horizontal or vertical) is parabolic. (See Video V6.6.) Consider a 10-mm diameter horizontal tube through which ethyl alcohol is flowing with a steady mean velocity 0.15 m/s. **(a)** Would you expect the velocity distribution to be parabolic in this case? Explain. **(b)** What is the pressure drop per unit length along the tube?

(a) Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V(2R)}{\mu} = \frac{\left(789\,\frac{kg}{m^3}\right)\left(0.15\,\frac{m}{s}\right)(0.010\,m)}{1.19\times10^{-3}\,\frac{N\cdot s}{m^2}} = 995 < 2100$$

Thus, the flow is laminar and velocity distribution would be parabolic. $\underline{\underline{\text{Yes.}}}$

(b) Since the flow is laminar

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.152)}$$

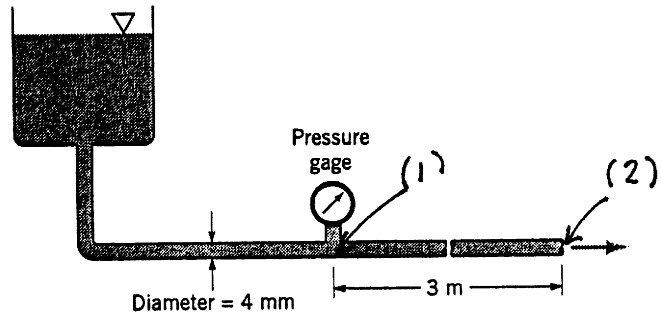
so that

$$\frac{\Delta p}{\ell} = \frac{8\mu V}{R^2} = \frac{8\left(1.19\times10^{-3}\,\frac{N\cdot s}{m^2}\right)\left(0.15\,\frac{m}{s}\right)}{\left(\frac{0.010\,m}{2}\right)^2}$$

$$= \underline{\underline{57.1\,\frac{N}{m^2}\text{ per m}}}$$

6.90

6.90 A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4-mm-diameter tubing as shown in Fig. P6.90. The liquid has a viscosity of 0.015 N · s/m², a density of 1200 kg/m³, and discharges into the atmosphere with a mean velocity of 2 m/s. **(a)** Verify that the flow will be laminar. **(b)** The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? **(c)** What is the magnitude of the wall shearing stress, τ_{rz}, in the fully developed region?



■ FIGURE P6.90

(a) Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V (2R)}{\mu} = \frac{(1200 \frac{kg}{m^3})(2 \frac{m}{s})(0.004 m)}{0.015 \frac{N \cdot s}{m^2}} = 640$$

Since the Reynolds number is well below 2100 the flow is laminar.

(b) For laminar flow,

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.152)}$$

Since $\Delta p = p_1 - p_2 = p_1 - 0$ (see figure)

$$p_1 = \frac{8\mu V \ell}{R^2} = \frac{8(0.015 \frac{N \cdot s}{m^2})(2 \frac{m}{s})(3 m)}{(\frac{0.004}{2} m)^2} = \underline{\underline{180 \text{ kPa}}}$$

(c)

$$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

For fully developed pipe flow, $v_r = 0$, so that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Also,

$$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{(Eq. 6.154)}$$

and with $v_{max} = 2V$, where V is the mean velocity

$$\tau_{rz} = 2V\mu\left(-\frac{2r}{R^2}\right)$$

Thus, at the wall, $r = R$,

$$\left|(\tau_{rz})_{wall}\right| = \left|-\frac{4V\mu}{R}\right| = \left|-\frac{4(2 \frac{m}{s})(0.015 \frac{N \cdot s}{m^2})}{(\frac{0.004}{2} m)}\right| = \underline{\underline{60.0 \frac{N}{m^2}}}$$

6.91

6.91 A highly viscous Newtonian liquid (ρ = 1,300 kg/m^3; μ = 6.0 N · s/m^2) is contained in a long, vertical, 150-mm diameter tube. Initially the liquid is at rest but when a valve at the bottom of the tube is opened flow commences. Although the flow is slowly changing with time, at any instant the velocity distribution is parabolic, that is, the flow is quasi-steady. (See Video V6.6.) Some measurements show that the average velocity, V, is changing in accordance with the equation V = 0.1 t, with V in m/s when t is in seconds. **(a)** Show on a plot the velocity distribution (v_z vs. r) at t = 2 s, where v_z is the velocity and r is the radius from the center of the tube. **(b)** Verify that the flow is laminar at this instant.

(a) For parabolic velocity distribution

$$\frac{v_z}{V_{max}} = 1 - \left(\frac{r}{R}\right)^2 \qquad \text{(Eq. 6.154)}$$

Since $V_{max} = 2V$

$$v_z = 2V\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (1)$$

With $V = 0.1t$, at $t = 2s$ $V = 0.2\,\frac{m}{s}$ and $R = \frac{150\,mm}{2} = 75\,mm$. Thus, Eq. (1) becomes

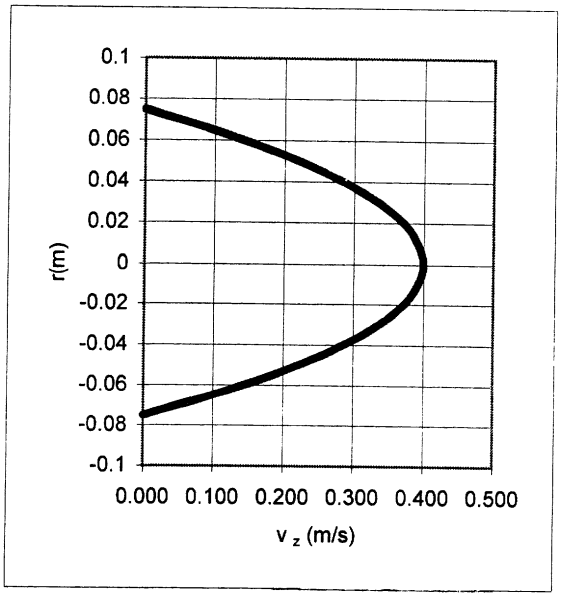
$$v_z = 2\left(0.2\,\frac{m}{s}\right)\left[1 - \frac{r^2}{(0.075m)^2}\right]$$

and

$$v_z = 0.4\,(1 - 178\,r^2)$$

A plot of this velocity distribution is shown below.

v_z (m/s)	r(m)
0.000	0.075
0.100	0.065
0.185	0.055
0.256	0.045
0.313	0.035
0.356	0.025
0.384	0.015
0.400	0
0.384	-0.015
0.356	-0.025
0.313	-0.035
0.256	-0.045
0.256	-0.045
0.185	-0.055
0.100	-0.065
0.000	-0.075



(b)

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1300\,\frac{kg}{m^3}\right)\left(0.2\,\frac{m}{s}\right)(0.150m)}{6.0\,\frac{N\,s}{m^2}}$$

$$= 6.5 \ll 2100 \quad \text{(Flow is laminar)}$$

6.92 **(a)** Show that for Poiseuille flow in a tube of radius R the magnitude of the wall shearing stress, τ_{rz}, can be obtained from the relationship

$$|(\tau_{rz})_{\text{wall}}| = \frac{4\mu Q}{\pi R^3}$$

for a Newtonian fluid of viscosity μ. The volume rate of flow is Q. **(b)** Determine the magnitude of the wall shearing stress for a fluid having a viscosity of 0.003 N·s/m² flowing with an average velocity of 100 mm/s in a 2-mm-diameter tube.

(a) $$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

For Poiseuille flow in a tube, $v_r = 0$, and therefore

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Since, $$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{(Eq. 6.154)}$$

and $v_{max} = 2V$, where V is the mean velocity, it follows that

$$\frac{\partial v_z}{\partial r} = -\frac{4Vr}{R^2}$$

Thus, at the wall $(r=R)$,

$$(\tau_{rz})_{wall} = -\frac{4\mu V}{R}$$

and with $Q = \pi R^2 V$

$$\left|(\tau_{rz})_{wall}\right| = \underline{\underline{\frac{4\mu Q}{\pi R^3}}}$$

(b) $$\left|(\tau_{rz})_{wall}\right| = \frac{4\mu V}{R} = \frac{4\left(0.003\ \frac{N\cdot s}{m^2}\right)\left(0.100\ \frac{m}{s}\right)}{\left(\frac{0.002}{2}\ m\right)}$$

$$= \underline{\underline{1.20\ Pa}}$$

6.93

6.93 An incompressible, Newtonian fluid flows steadily between two infinitely long, concentric cylinders as shown in Fig. P6.93 . The outer cylinder is fixed, but the inner cylinder moves with a longitudinal velocity V_0 as shown. For what value of V_0 will the drag on the inner cylinder be zero? Assume that the flow is laminar, axisymmetric, and fully developed.

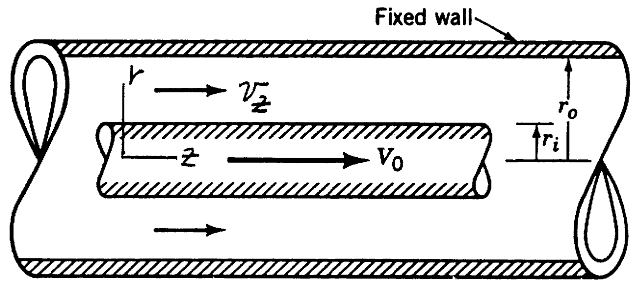


FIGURE P6.93

Equation 6.147, which was developed for flow in circular tubes, applies in the annular region. Thus,

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r^2 + c_1 \ln r + c_2 \qquad (1)$$

With boundary conditions, $r = r_o$, $v_z = 0$, and $r = r_i$, $v_z = V_0$, it follows that:

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_o^2 + c_1 \ln r_o + c_2 \qquad (2)$$

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_i^2 + c_1 \ln r_i + c_2 \qquad (3)$$

Subtract Eq. (2) from Eq. (3) to obtain

$$V_0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right) + c_1 \ln \frac{r_i}{r_o}$$

so that

$$c_1 = \frac{V_0 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{\ln \frac{r_i}{r_o}}$$

The drag on the inner cylinder will be zero if

$$\left(\tau_{rz}\right)_{r=r_i} = 0$$

Since,

$$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

and with $v_r = 0$, it follows that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

(con't)

6.93 (Con't)

Differentiate Eq. (1) with respect to r to obtain

$$\frac{\partial v_z}{\partial r} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r + \frac{c_1}{r}$$

so that at $r = r_i$

$$\left(\tau_{rz}\right)_{r=r_i} = \mu\left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{r_i \ln\frac{r_i}{r_o}}\right]$$

Thus, in order for the drag to be zero,

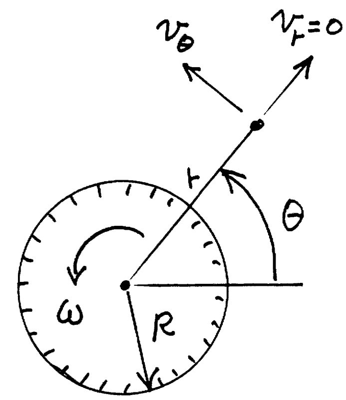
$$\frac{1}{2\mu}\left(\frac{\partial p}{\partial z}\right)r_i + \frac{V_o - \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_i^2 - r_o^2\right)}{r_i \ln\frac{r_i}{r_o}} = 0$$

or

$$V_o = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[2\, r_i^2 \ln\frac{r_i}{r_o} - \left(r_i^2 - r_o^2\right)\right]$$

6.94

6.94 An infinitely long, solid, vertical cylinder of radius R is located in an infinite mass of an incompressible fluid. Start with the Navier–Stokes equation in the θ direction and derive an expression for the velocity distribution for the steady flow case in which the cylinder is rotating about a fixed axis with a constant angular velocity ω. You need not consider body forces. Assume that the flow is axisymmetric and the fluid is at rest at infinity.



For this flow field, $v_r=0$, $v_z=0$, and from the continuity equation,

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0 \qquad \text{(Eq. 6.35)}$$

it follows that

$$\frac{\partial v_\theta}{\partial \theta}=0 \qquad \text{(See figure for notation.)}$$

Thus, the Navier-Stokes equation in the θ-direction (Eq. 6.128b) for steady flow reduces to

$$0=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)-\frac{v_\theta}{r^2}\right]$$

Due to the symmetry of the flow,

$$\frac{\partial p}{\partial \theta}=0$$

so that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)-\frac{v_\theta}{r^2}=0$$

or

$$\frac{\partial^2 v_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r^2}=0 \qquad (1)$$

Since v_θ is a function of only r, Eq. (1) can be expressed as an ordinary differential equation, and re-written as

$$\frac{d^2 v_\theta}{dr^2}+\frac{d}{dr}\left(\frac{v_\theta}{r}\right)=0 \qquad (2)$$

Equation (2) can be integrated to yield

$$\frac{d v_\theta}{dr}+\frac{v_\theta}{r}=C_1$$

or

$$r\frac{d v_\theta}{dr}+v_\theta=C_1 r \qquad (3)$$

(con't)

Equation (3) can be expressed as

$$\frac{d(rv_\theta)}{dr} = c_1 r$$

and a second integration yields

$$r v_\theta = \frac{c_1 r^2}{2} + c_2$$

or

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r}$$

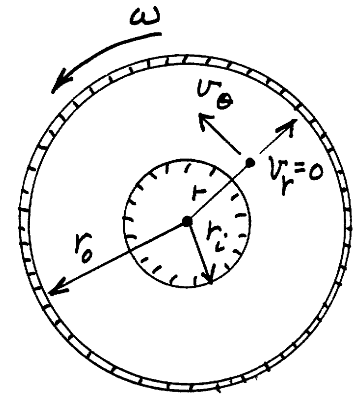
As $r \to \infty$, $v_\theta \to 0$, (since fluid is at rest at infinity) so that $c_1 = 0$. Thus,

$$v_\theta = \frac{c_2}{r}$$

and since at $r = R$, $v_\theta = R\omega$, it follows that $c_2 = R^2\omega$ and

$$\underline{\underline{v_\theta = \frac{R^2\omega}{r}}}$$

6.95 A viscous fluid is contained between two infinitely long vertical concentric cylinders. The outer cylinder has a radius r_o and rotates with an angular velocity ω. The inner cylinder is fixed and has a radius r_i. Make use of the Navier–Stokes equations to obtain an exact solution for the velocity distribution in the gap. Assume that the flow in the gap is axisymmetric (neither velocity nor pressure are functions of angular position θ within gap) and that there are no velocity components other than the tangential component. The only body force is the weight.



The velocity distribution in the annular space is given by the equation

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r} \qquad (1)$$

(See solution to Problem 6.94 for derivation.)

With the boundary conditions $r = r_i$, $v_\theta = 0$, and $r = r_o$, $v_\theta = r_o\omega$ (see figure for notation), it follows from Eq. (1) that:

$$0 = \frac{c_1 r_i}{2} + \frac{c_2}{r_i}$$

$$r_o\omega = \frac{c_1 r_o}{2} + \frac{c_2}{r_o}$$

Therefore,

$$c_1 = \frac{2\omega}{1 - \frac{r_i^2}{r_o^2}}$$

and

$$c_2 = \frac{-r_i^2\omega}{1 - \frac{r_i^2}{r_o^2}}$$

so that

$$v_\theta = \frac{r\omega}{1 - \frac{r_i^2}{r_o^2}} - \frac{r_i^2\omega}{r\left(1 - \frac{r_i^2}{r_o^2}\right)}$$

or

$$\underline{\underline{v_\theta = \frac{r\omega}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r^2}\right]}}$$

6.96

6.96 For flow between concentric cylinders, with the outer cylinder rotating at an angular velocity ω and the inner cylinder fixed, it is commonly assumed that the tangential velocity (v_θ) distribution in the gap between the cylinders is linear. Based on the exact solution to this problem (see Problem 6.95) the velocity distribution in the gap is not linear. For an outer cylinder with radius $r_o = 2.00$ in. and an inner cylinder with radius $r_i = 1.80$ in., show, with the aid of a plot, how the dimensionless velocity distribution, $v_\theta/r_o\omega$, varies with the dimensionless radial position, r/r_o, for the exact and approximate solutions.

For a linear velocity distribution (approximate solution)

$$v_\theta = (r_o\omega)\left(\frac{r-r_i}{r_o-r_i}\right)$$

and in nondimensional form

$$\frac{v_\theta}{r_o\omega} = \frac{\frac{r}{r_o} - \frac{r_i}{r_o}}{1 - \frac{r_i}{r_o}} \tag{1}$$

For the exact solution (see Problem 6.95)

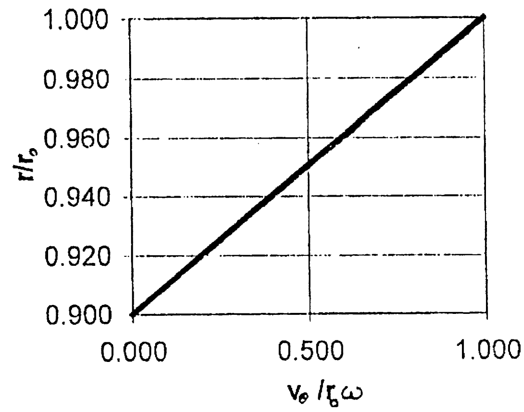
$$v_\theta = \frac{r\omega}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r^2}\right]$$

and in nondimensional form

$$\frac{v_\theta}{r_o\omega} = \frac{\frac{r}{r_o}}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r_o^2}\left(\frac{r}{r_o}\right)^{-2}\right] \tag{2}$$

For $r_i = 1.80$ in. and $r_o = 2.00$ in., some tabulated values and a graph are shown below. Note that there is little difference between the exact and approximate solutions for this small gap width. For all practical purposes both solutions fall on the single curve shown.

Linear	Exact	
$v_\theta/r_o\omega$	$v_\theta/r_o\omega$	r/r_o
0.000	0.000	0.900
0.125	0.131	0.913
0.250	0.260	0.925
0.375	0.387	0.938
0.500	0.512	0.950
0.625	0.637	0.963
0.750	0.759	0.975
0.875	0.880	0.988
1.000	1.000	1.000



6.97

6.97 A viscous liquid ($\mu = 0.012 \text{ lb} \cdot \text{s/ft}^2$, $\rho = 1.79 \text{ slugs/ft}^3$) flows through the annular space between two horizontal, fixed, concentric cylinders. If the radius of the inner cylinder is 1.5 in. and the radius of the outer cylinder is 2.5 in., what is the pressure drop along the axis of the annulus per foot when the volume flowrate is 0.14 $\text{ft}^3\text{/s}$?

Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V D_h}{\mu}$$

where $D_h = 2(r_o - r_i)$ and $V = \dfrac{Q}{\pi(r_o^2 - r_i^2)}$.

Thus,

$$Re = \frac{2\rho Q}{\pi \mu (r_o + r_i)} = \frac{2\left(1.79 \frac{\text{slugs}}{\text{ft}^3}\right)\left(0.14 \frac{\text{ft}^3}{\text{s}}\right)}{\pi\left(0.012 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(\frac{2.5\text{in.} + 1.5\text{in.}}{12 \frac{\text{in.}}{\text{ft}}}\right)}$$

$$= 39.9 < 2100$$

Since the Reynolds number is well below 2100 the flow is laminar and

$$Q = \frac{\pi}{8\mu}\frac{\Delta p}{\ell}\left[r_o^4 - r_i^4 - \frac{(r_o^2 - r_i^2)^2}{\ln \frac{r_o}{r_i}}\right] \qquad \text{(Eq. 6.156)}$$

so that

$$\frac{\Delta p}{\ell} = \frac{\dfrac{8\mu Q}{\pi}}{r_o^4 - r_i^4 - \dfrac{(r_o^2 - r_i^2)^2}{\ln \frac{r_o}{r_i}}}$$

$$= \frac{8\left(0.012 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(0.14 \frac{\text{ft}^3}{\text{s}}\right)/\pi}{\left(\frac{2.5\text{ in.}}{12 \frac{\text{in.}}{\text{ft}}}\right)^4 - \left(\frac{1.5\text{ in.}}{12 \frac{\text{in.}}{\text{ft}}}\right)^4 - \dfrac{\left[\left(\frac{2.5\text{in}}{12 \frac{\text{in.}}{\text{ft}}}\right)^2 - \left(\frac{1.5\text{in}}{12 \frac{\text{in.}}{\text{ft}}}\right)^2\right]^2}{\ln \frac{2.5\text{ in.}}{1.5\text{ in.}}}}$$

$$= \underline{\underline{33.1 \frac{\text{lb}}{\text{ft}^2} \text{ per ft}}}$$

6.98 *

6.98 * Plot the velocity profile for the fluid flowing in the annular space described in Problem P6.97. Determine from the plot the radius at which the maximum velocity occurs and compare with the value predicted from Eq. 6.157.

The velocity distribution in the annulus is given by the equation

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[r^2 - r_o^2 + \frac{r_i^2 - r_o^2}{\ln \frac{r_o}{r_i}} \ln \frac{r}{r_o}\right] \quad \text{(Eq. 6.155)}$$

From Problem 6.97

$$\frac{\partial p}{\partial z} = -\frac{\Delta p}{\ell} = -28.4\ \frac{\text{lb}}{\text{ft}^3}$$

Thus, with $\mu = 0.016\ \text{lb}\cdot\text{s/ft}^2$, $r_i = 1.5\ \text{in.}$, and $r_o = 2.5\ \text{in.}$ it follows that

$$v_z = -\frac{\left(28.4\ \frac{\text{lb}}{\text{ft}^3}\right)}{4\left(0.016\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)}\left[r^2 - \left(\frac{2.5}{12}\text{ft}\right)^2 + \frac{\left(\frac{1.5}{12}\text{ft}\right)^2 - \left(\frac{2.5}{12}\text{ft}\right)^2}{\ln \frac{2.5}{1.5}} \ln \frac{r}{\frac{2.5}{12}\text{ft}}\right]$$

or

$$v_z = -444\left(r^2 - 0.0434 - 0.0544 \ln \frac{r}{0.208}\right)$$

where v_z in ft/s with r in ft. A program for calculating v_z as a function of r in the range $\frac{1.5}{12}\text{ft} \le r \le \frac{2.5}{12}\text{ft}$ follows.

```
100 cls
110 print "**********************************************************"
120 print "** This program calculates the velocity profile **"
130 print "** for flow in an annulus                        **"
140 print "**********************************************************"
150 print
155 print " r (ft)         v (ft/s)"
160 for r=1.5/12 to 2.501/12 step 0.1/18
170 v=-444*(r^2-0.043403-0.05438*log(r/0.208333))
180 print using "##.###          ##.###";r,v
190 next r
```

(con't)

6.98* (con't)

Tabulated data and a plot of the data are given below. From these data it is seen that the maximum velocity occurs at

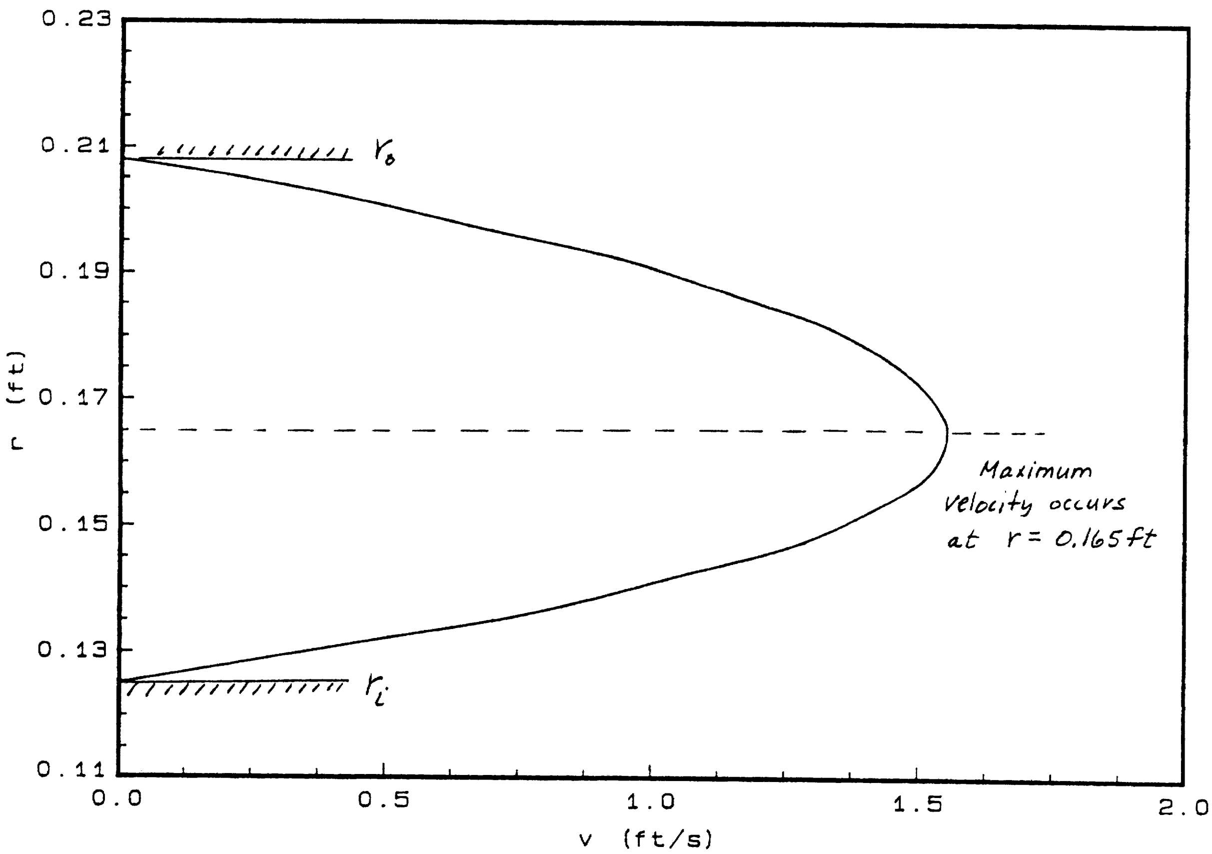
$$r_m \approx \underline{\underline{0.165\ ft}}$$

This value corresponds to the value calculated from Eq. 6.157:

$$r_m = \left[\frac{r_o^2 - r_i^2}{2 \ln \frac{r_o}{r_i}}\right]^{1/2} = \left[\frac{\left(\frac{2.5}{12} ft\right)^2 - \left(\frac{1.5}{12} ft\right)^2}{2 \ln \frac{2.5}{1.5}}\right]^{1/2} = 0.165\ ft$$

```
***********************************************
** This program calculates the velocity profile **
** for flow in an annulus                       **
***********************************************
```

r (ft)	v (ft/s)	(con't)	
0.125	0.000	0.169	1.534
0.131	0.419	0.175	1.464
0.136	0.768	0.181	1.341
0.142	1.048	0.186	1.169
0.147	1.265	0.192	0.947
0.153	1.419	0.197	0.678
0.158	1.514	0.203	0.362
0.164	1.552	0.208	0.000



6.99 *

6.99* As is shown by Eq. 6.150 the pressure gradient for laminar flow through a tube of constant radius is given by the expression:

$$\frac{\partial p}{\partial z} = -\frac{8\mu Q}{\pi R^4}$$

For a tube whose radius is changing very gradually, such as the one illustrated in Fig. P6.99, it is expected that this equation can be used to approximate the pressure change along the tube if the actual radius, $R(z)$, is used at each cross section. The following measurements were obtained along a particular tube.

z/ℓ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$R(z)/R_o$	1.00	0.73	0.67	0.65	0.67	0.80	0.80	0.71	0.73	0.77	1.00

Compare the pressure drop over the length ℓ for this nonuniform tube with one having the constant radius R_o. *Hint:* To solve this problem you will need to numerically integrate the equation for the pressure gradient given above.

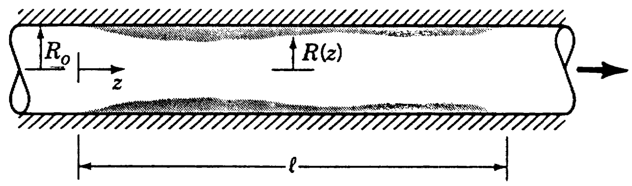


FIGURE P6.99

From the equation given for the pressure gradient,

$$\int_{p_1}^{p_2} dp = -\int_0^{\ell} \frac{8\mu Q}{\pi [R(z)]^4}\, dz$$

Since $p_1 - p_2 = \Delta p$ (the pressure drop) it follows that

$$\Delta p = \frac{8\mu Q}{\pi}\int_0^{\ell} [R(z)]^{-4} dz$$

or, with $z^* = z/\ell$ and $R^* = R/R_o$,

$$\Delta p = \frac{8\mu Q \ell}{\pi R_o^4}\int_0^1 (R^*)^{-4} dz^*$$

For a constant radius tube (see Eq. 6.151),

$$\Delta p_{R=R_o} = \frac{8\mu Q \ell}{\pi R_o^4}$$

so that

$$\frac{\Delta p \text{ (nonuniform tube)}}{\Delta p \text{ (uniform tube)}} = \int_0^1 (R^*)^{-4} dz^*$$

This integral can be evaluated numerically using SIMPSON and the data given.

(con't)

6.99 * (con't)

```
************************************************
** This program performs numerical integration  **
** over a set a set of an odd number of equally **
** spaced points using Simpson's Rule           **
************************************************

Enter number of data points: 11
Enter data points (X , Y)
? 0.0,1.00
? 0.1,3.52
? 0.2,4.96
? 0.3,5.60
? 0.4,4.96
? 0.5,2.44
? 0.6,2.44
? 0.7,3.94
? 0.8,3.52
? 0.9,2.84
? 1.0,1.00

The approximate value of the integral is: +3.5707E+00
```

Note: $X \sim z^*$ and $Y \sim (R^*)^{-4}$

Thus,

$$\frac{\Delta p \text{ (nonuniform tube)}}{\Delta p \text{ (uniform tube)}} = \underline{\underline{3.57}}$$

6.100

6.100 Show how Eq. 6.155 is obtained.

From Eq. 6.147

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r^2 + c_1 \ln r + c_2 \qquad \text{(Eq. 6.147)}$$

For flow in an annulus, $v_z = 0$ at $r = r_o$ and $v_z = 0$ at $r = r_i$. Thus, from Eq. 6.147

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_o^2 + c_1 \ln r_o + c_2$$

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right) r_i^2 + c_1 \ln r_i + c_2$$

and solving for c_1 and c_2 we have

$$c_1 = \frac{-\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_o^2 - r_i^2\right)}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (1)$$

$$c_2 = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r_o^2 + \frac{r_o^2 - r_i^2}{\ln\left(\frac{r_o}{r_i}\right)} \ln r_o\right) \qquad (2)$$

Substitution of Eqs. (1) and (2) into Eq. 6.147 gives

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left[r^2 - r_o^2 + \frac{r_i^2 - r_o^2}{\ln\left(\frac{r_o}{r_i}\right)} \ln \frac{r}{r_o}\right]$$

which is the desired equation (Eq. 6.155).

6.101

6.101 A wire of diameter d is stretched along the centerline of a pipe of diameter D. For a given pressure drop per unit length of pipe, by how much does the presence of the wire reduce the flowrate if (a) $d/D = 0.1$; (b) $d/D = 0.01$?

The volume flowrate is given by Eq. 6.156

$$Q = \frac{\pi \Delta p}{8\mu \ell}\left[r_o^4 - r_i^4 - \frac{(r_o^2 - r_i^2)^2}{\ln\left(\frac{r_o}{r_i}\right)}\right] \qquad \text{(Eq. 6.156)}$$

which can be written as

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - \left(\frac{r_i}{r_o}\right)^4 + \frac{\left[1 - \left(\frac{r_i}{r_o}\right)^2\right]^2}{\ln\left(\frac{r_i}{r_o}\right)}\right\} \qquad (1)$$

Since $\frac{r_i}{r_o} = \frac{d}{D}$, Eq. (1) can also be written as

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - \left(\frac{d}{D}\right)^4 + \frac{\left[1 - \left(\frac{d}{D}\right)^2\right]^2}{\ln\left(\frac{d}{D}\right)}\right\} \qquad (2)$$

Note that for $\frac{d}{D} = 0$ (no wire)

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}$$

which corresponds to Poiseuille's Law (Eq. 6.151).

(a) For $\frac{d}{D} = 0.1$, Eq. (2) gives

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - (0.1)^4 + \frac{\left[1 - (0.1)^2\right]^2}{\ln(0.1)}\right\} = 0.574$$

Thus, for the same Δp the flowrate is reduced by

$$\%\text{ reduction in } Q = (1 - 0.574) \times 100 = \underline{\underline{42.6\,\%}}$$

(b) Similarly, for $\frac{d}{D} = 0.01$ Eq. (2) gives

$$Q = \frac{\pi r_o^4 \Delta p}{8\mu \ell}\left\{1 - (0.01)^4 + \frac{\left[1 - (0.01)^2\right]^2}{\ln(0.01)}\right\} = 0.783$$

and

$$\%\text{ reduction in } Q = (1 - 0.783) \times 100 = \underline{\underline{21.7\,\%}}$$

Note that the presence of even a very small wire along the tube centerline has a significant effect on the flowrate.

7.1

7.1 The Reynolds number, $\rho VD/\mu$, is a very important parameter in fluid mechanics. Verify that the Reynolds number is dimensionless, using both the *FLT* system and the *MLT* system for basic dimensions, and determine its value for water (at 70 °C) flowing at a velocity of 2 m/s through a 2-in.-diameter pipe.

$$\text{Reynolds number} = \frac{\rho V D}{\mu} \doteq \frac{(FL^{-4}T^{2})(LT^{-1})(L)}{FL^{-2}T} \doteq \underline{\underline{F^0L^0T^0}}$$

$$\doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq \underline{\underline{M^0L^0T^0}}$$

For water at 70 °C, $\mu = 4.042 \times 10^{-4} \ \frac{N \cdot s}{m^2}$ and $\rho = 977.8 \ \frac{kg}{m^3}$ (Table B.2 in Appendix B).

Thus,

$$\frac{\rho V D}{\mu} = \frac{\left(977.8 \ \frac{kg}{m^3}\right)\left(2 \ \frac{m}{s}\right)\left(\frac{2}{12} \ ft\right)\left(0.3048 \ \frac{m}{ft}\right)}{4.042 \times 10^{-4} \ \frac{N \cdot s}{m^2}}$$

$$= \underline{\underline{2.46 \times 10^{5}}}$$

7.2 What are the dimensions of density, pressure, specific weight, surface tension, and dynamic viscosity in **(a)** the *FLT* system, and **(b)** the *MLT* system? Compare your results with those given in Table 1.1 in Chapter 1.

$$\rho = \text{density} = \frac{\text{mass}}{\text{unit volume}} \doteq \frac{M}{L^3} \doteq \frac{FT^2}{L^4} \quad (\text{since } F \doteq MLT^{-2})$$

$$p = \text{pressure} = \frac{\text{force}}{\text{area}} \doteq \frac{F}{L^2} \doteq \frac{MLT^{-2}}{L^2} \doteq \frac{M}{LT^2}$$

$$\gamma = \text{specific weight} = \frac{\text{weight}}{\text{unit volume}} \doteq \frac{F}{L^3} \doteq \frac{MLT^{-2}}{L^3} \doteq \frac{M}{T^2L^2}$$

$$\sigma = \text{surface tension} = \frac{\text{force}}{\text{unit length}} \doteq \frac{F}{L} \doteq \frac{MLT^{-2}}{L} \doteq \frac{M}{T^2}$$

$$\mu = \text{dynamic viscosity} = \frac{\text{stress}}{\text{velocity gradient}} \doteq \frac{FL^{-2}}{T^{-1}} \doteq \frac{(MLT^{-2})L^{-2}}{T^{-1}} \doteq \frac{M}{LT}$$

Thus,

(a) in the FLT system,

$\rho \doteq \underline{\underline{FL^{-4}T^2}}$

$p \doteq \underline{\underline{FL^{-2}}}$

$\gamma \doteq \underline{\underline{FL^{-3}}}$

$\sigma \doteq \underline{\underline{FL^{-1}}}$

$\mu \doteq \underline{\underline{FL^{-2}T}}$

(b) in the MLT system,

$\rho \doteq \underline{\underline{ML^{-3}}}$

$p \doteq \underline{\underline{ML^{-1}T^{-2}}}$

$\gamma \doteq \underline{\underline{ML^{-2}T^{-2}}}$

$\sigma \doteq \underline{\underline{MT^{-2}}}$

$\mu \doteq \underline{\underline{ML^{-1}T^{-1}}}$

7.3

7.3 For the flow of a thin film of a liquid with a depth h and a free surface, two important dimensionless parameters are the Froude number, $V/\sqrt{gh}$, and the Weber number, $\rho V^2 h/\sigma$. Determine the value of these two parameters for glycerin (at 20 °C) flowing with a velocity of 0.7 m/s at a depth of 3 mm.

$$\frac{V}{\sqrt{gh}} = \frac{0.7\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.003\ m)}} = \underline{\underline{4.08}}$$

$$\frac{\rho V^2 h}{\sigma} = \frac{\left(1260\ \frac{kg}{m^3}\right)\left(0.7\ \frac{m}{s}\right)^2 (0.003\ m)}{6.33 \times 10^{-2}\ \frac{N}{m}} = \underline{\underline{29.3}}$$

7.4

7.4 The Mach number for a body moving through a fluid with velocity V is defined as V/c, where c is the speed of sound in the fluid. This dimensionless parameter is usually considered to be important in fluid dynamics problems when its value exceeds 0.3. What would be the velocity of a body at a Mach number of 0.3 if the fluid is: **(a)** air at standard atmospheric pressure and 20 °C, and **(b)** water at the same temperature and pressure?

(a) $$\frac{V}{c} = 0.3$$

For air at 20°C, $c = 343.3\ \frac{m}{s}$ (Table B.4 in Appendix B)
so that

$$V = 0.3\left(343.3\ \frac{m}{s}\right) = \underline{\underline{103\ \frac{m}{s}}}$$

(b) For water at 20°C, $c = 1481\ \frac{m}{s}$ (Table B.2 in Appendix B)
so that

$$V = 0.3\left(1481\ \frac{m}{s}\right) = \underline{\underline{444\ \frac{m}{s}}}$$

7.5

7.5 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, 6-3 = 3 dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F)$$
$$-2 + a + b - 2c = 0 \qquad \text{(for } L)$$
$$-b + c = 0 \qquad \text{(for } T)$$

It follows that $a = 1$, $b = -1$, $c = -1$, and therefore

$$\Pi_1 = \frac{\Delta p\, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p\, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2:

$$\Pi_2 = D_2\, D_1^a V^b \mu^c$$

$$L(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L)$$
$$-b + c = 0 \qquad \text{(for } T)$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.5 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^a V^b \mu^c$$

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$1+c=0 \qquad \text{(for } F\text{)}$$

$$-4+a+b-2c=0 \qquad \text{(for } L\text{)}$$

$$2-b+c=0 \qquad \text{(for } T\text{)}$$

It follows that $a=1$, $b=1$, $c=-1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \quad \therefore OK$$

Thus,

$$\underline{\underline{\frac{\Delta p\, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1}, \frac{\rho D_1 V}{\mu}\right)}}$$

From the continuity equation,

$$V\frac{\pi}{4}D_1^2 = V_s\frac{\pi}{4}D_2^2$$

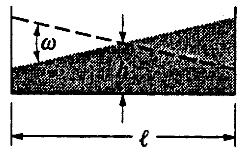
where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should <u>not</u> be included as an independent variable.

7.6

7.6 Water sloshes back and forth in a tank as shown in Fig. P7.6. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, ℓ. Develop a suitable set of dimensionless parameters for this problem using g and ℓ as repeating variables.



■ FIGURE P7.6

$$\omega = f(g, h, \ell)$$

$$\omega \doteq T^{-1} \quad g \doteq LT^{-2} \quad h \doteq L \quad \ell \doteq L$$

From the pi theorem, $4-2=2$ dimensionless parameters required. Use g and ℓ as repeating variables, Thus,

$$\Pi_1 = \omega g^a \ell^b$$

and

$$(T^{-1})(LT^{-2})^a (L)^b \doteq L^0 T^0$$

so that

$$a + b = 0 \qquad \text{(for } L)$$

$$-1 - 2a = 0 \qquad \text{(for } T)$$

It follows that $a = -1/2$, $b = 1/2$, and therefore

$$\Pi_1 = \omega \sqrt{\frac{\ell}{g}}$$

Check dimensions:

$$\omega \sqrt{\frac{\ell}{g}} \doteq \frac{1}{T}\sqrt{\frac{L}{LT^{-2}}} \doteq L^0 T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h\, g^a \ell^b$$

$$L(LT^{-2})^a (L)^b \doteq L^0 T^0$$

$$1 + a + b = 0 \qquad \text{(for } L)$$

$$-2a = 0 \qquad \text{(for } T)$$

It follows that $a = 0$, $b = -1$, and therefore

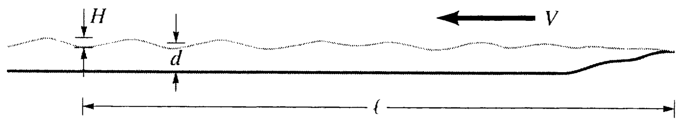
$$\Pi_2 = \frac{h}{\ell}$$

and Π_2 is obviously dimensionless. Thus,

$$\omega \sqrt{\frac{\ell}{g}} = \phi\left(\frac{h}{\ell}\right)$$

7.7

7.7 It is desired to determine the wave height when wind blows across a lake. The wave height, H, is assumed to be a function of the wind speed, V, the water density, ρ, the air density, ρ_a, the water depth, d, the distance from the shore, ℓ, and the acceleration of gravity, g, as shown in Fig. P7.7. Use d, V, and ρ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.



$$H = f(V, \rho, \rho_a, d, \ell, g)$$

$H \doteq L$ $V \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^2$ $\rho_a \doteq FL^{-4}T^2$ $d \doteq L$ $\ell \doteq L$ $g \doteq LT^{-2}$

From the pi theorem, $7-3=4$ pi terms required. Use d, V, and ρ as repeating variables. Thus,

$$\Pi_1 = H d^a V^b \rho^c$$

and

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

so that

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_1 = \frac{H}{d}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \rho_a d^a V^b \rho^c$$

and

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-4 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$2 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=0$, $b=0$, $c=-1$ so that

$$\Pi_2 = \frac{\rho_a}{\rho}$$

which is obviously dimensionless.

(con't)

7.7 (Con't)

For Π_3:

$$\Pi_3 = \ell\, d^a V^b \rho^c$$

and as for Π_1, $a=-1$, $b=0$, $c=0$ so that

$$\Pi_3 = \frac{\ell}{d}$$

For Π_4:

$$\Pi_4 = g\, d^a V^b \rho^c$$

$$(LT^{-2})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=$, $b=-2$, $c=0$, and therefore

$$\Pi_4 = \frac{gd}{V^2}$$

Check dimensions:

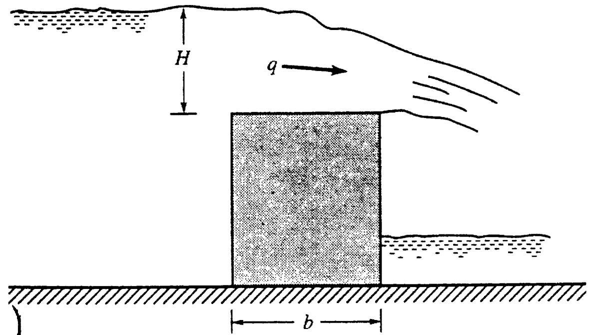
$$\frac{gd}{V^2} = \frac{(LT^{-2})(L)}{(LT^{-1})^2} \doteq L^0T^0 \qquad \therefore \text{ ok}$$

Thus,

$$\underline{\underline{\frac{H}{d} = \phi\left(\frac{\rho_a}{\rho}, \frac{\ell}{d}, \frac{gd}{V^2}\right)}}$$

7.8

7.8 Water flows over a dam as illustrated in Fig. P7.8. Assume the flowrate, q, per unit length along the dam depends on the head, H, width, b, acceleration of gravity, g, fluid density, ρ, and fluid viscosity, μ. Develop a suitable set of dimensionless parameters for this problem using b, g, and ρ as repeating variables.



■ FIGURE P7.8

$$q = f(H, b, g, \rho, \mu)$$

$q \doteq L^2T^{-1}$ $\quad H \doteq L$ $\quad b \doteq L$ $\quad g \doteq LT^{-2}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem $6-3=3$ pi terms required. Use b, g, and ρ as repeating variables. Thus,

$$\Pi_1 = q\, b^a g^b \rho^c$$

and

$$(L^2T^{-1})(L)^a(LT^{-2})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$c = 0 \quad \text{(for } F)$$
$$2 + a + b - 4c = 0 \quad \text{(for } L)$$
$$-1 - 2b + 2c = 0 \quad \text{(for } T)$$

It follows that $a = -3/2$, $b = -1/2$, $c = 0$, and therefore

$$\Pi_1 = \frac{q}{b^{3/2} g^{1/2}}$$

Check dimensions using MLT system:

$$\frac{q}{b^{3/2} g^{1/2}} \doteq \frac{L^2T^{-1}}{L^{3/2}(LT^{-2})^{1/2}} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

For Π_2:

$$\Pi_2 = H\, b^a g^b \rho^c$$

$$(L)(L)^a(LT^{-2})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \quad \text{(for } F)$$
$$1 + a + b - 4c = 0 \quad \text{(for } L)$$
$$-2b + 2c = 0 \quad \text{(for } T)$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{H}{b}$$

which is obviously dimensionless.

(con't)

7.8 (con't)

For Π_3:

$$\Pi_3 = \mu\, b^a g^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-2})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F)$$

$$-2 + a + b - 4c = 0 \qquad \text{(for } L)$$

$$1 - 2b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -\frac{3}{2}$, $b = -\frac{1}{2}$, $c = -1$, and therefore

$$\Pi_3 = \frac{\mu}{b^{3/2} g^{1/2} \rho}$$

Check dimensions using MLT system:

$$\frac{\mu}{b^{3/2} g^{1/2} \rho} \doteq \frac{(ML^{-1}T^{-1})}{(L)^{3/2}(LT^{-2})^{1/2}(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{q}{b^{3/2}\sqrt{g}} = \phi\left(\frac{H}{b}, \frac{\mu}{b^{3/2}\sqrt{g}\,\rho}\right)$$

7.9

7.9 The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^{a} \rho^{b} \omega^{c}$$

and

so that $\quad (FL^{-2})(L)^{a}(FL^{-4}T^{2})^{b}(T^{-1})^{c} \doteq F^{0}L^{0}T^{0}$

$$1 + b = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = -1$, $c = -2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^{2}\rho\,\omega^{2}}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^{2}\rho\,\omega^{2}} \doteq \frac{ML^{-1}T^{-2}}{(L)^{2}(ML^{-3})(T^{-1})^{2}} \doteq M^{0}L^{0}T^{0} \qquad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = Q\, D^{a} \rho^{b} \omega^{c}$$

$$(L^{3}T^{-1})(L)^{a}(FL^{-4}T^{2})^{b}(T^{-1})^{c} \doteq F^{0}L^{0}T^{0}$$

$$b = 0 \qquad \text{(for } F\text{)}$$
$$3 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$-1 + 2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{Q}{D^{3}\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^{3}\omega} \doteq \frac{L^{3}T^{-1}}{(L)^{3}(T^{-1})} \doteq M^{0}L^{0}T^{0} \qquad \therefore \text{ok}$$

Thus,

$$\underline{\underline{\frac{\Delta p}{D^{2}\rho\,\omega^{2}} = \phi\left(\frac{Q}{D^{3}\omega}\right)}}$$

7.10

7.10 The drag, $\mathfrak{D}$, on a washer shaped plate placed normal to a stream of fluid can be expressed as

$$\mathfrak{D} = f(d_1, d_2, V, \mu, \rho)$$

where d_1 is the outer diameter, d_2 the inner diameter, V the fluid velocity, μ the fluid viscosity, and ρ the fluid density. Some experiments are to be performed in a wind tunnel to determine the drag. What dimensionless parameters would you use to organize these data?

$\mathscr{D} \doteq F \quad d_1 \doteq L \quad d_2 \doteq L \quad V \doteq LT^{-1} \quad \mu \doteq FL^{-2}T \quad \rho \doteq FL^{-4}T^2$

From the pi theorem, $6-3=3$ pi terms required. Use d_1, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathscr{D}\, d_1^a V^b \rho^c$$

and

$$(F)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$1+c=0 \qquad \text{(for } F\text{)}$$
$$a+b-4c=0 \qquad \text{(for } L\text{)}$$
$$-b+2c=0 \qquad \text{(for } T\text{)}$$

It follows that $a=-2$, $b=-2$, $c=-1$, and therefore

$$\Pi_1 = \frac{\mathscr{D}}{d_1^2 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{\mathscr{D}}{d_1^2V^2\rho} \doteq \frac{MLT^{-2}}{(L)^2(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = d_2\, d_1^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c=0 \qquad \text{(for } F\text{)}$$
$$1+a+b-4c=0 \qquad \text{(for } L\text{)}$$
$$-b+2c=0 \qquad \text{(for } T\text{)}$$

(con't)

7.10 (con't)

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{d_2}{d_1}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = \mu \, d_1^{\,a} V^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1+c=0 \qquad \text{(for } F\text{)}$$

$$-2+a+b-4c=0 \qquad \text{(for } L\text{)}$$

$$1-b+2c=0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=-1$, $c=-1$, and therefore

$$\Pi_3 = \frac{\mu}{d_1 V \rho}$$

Check dimensions using MLT system:

$$\frac{\mu}{d_1 V\rho} \doteq \frac{ML^{-1}T^{-1}}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\frac{\mathcal{D}}{d_1^{\,2}V^2\rho} = \phi\left(\frac{d_2}{d_1}, \frac{\mu}{d_1 V\rho}\right) \qquad (1)$$

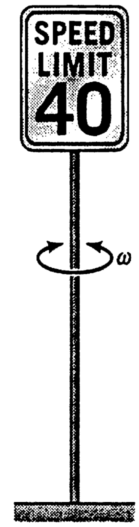
Since $\dfrac{\rho V d_1}{\mu}$ is a standard dimensionless parameter (Reynolds number), Eq. (1) would more commonly be expressed as

$$\underline{\underline{\frac{\mathcal{D}}{d_1^{\,2}V^2\rho} = \phi\left(\frac{d_2}{d_1}, \frac{\rho V d_1}{\mu}\right)}} \qquad (2)$$

As far as dimensional analysis is concerned, Eqs. (1) and (2) are equivalent.

7.11

7.11 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω. (See Fig. P7.11 and Video V9.6.) Assume that ω is a function of the sign width, b, sign height, h, wind velocity, V, air density, ρ, and an elastic constant, k, for the supporting pole. The constant, k, has dimensions of FL. Develop a suitable set of pi terms for this problem.



■ FIGURE P7.11

$$\omega = f(b, h, V, \rho, k)$$

$\omega \doteq T^{-1}$ $b \doteq L$ $h \doteq L$ $V \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^{2}$ $k \doteq FL$

From the pi theorem $6-3=3$ pi terms required. Use b, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \omega b^a V^b \rho^c$$

and
$$(T^{-1})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$c = 0 \quad \text{(for } F\text{)}$$
$$a + b - 4c = 0 \quad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = -1$, $c = 0$, and therefore

$$\Pi_1 = \frac{\omega b}{V}$$

Check dimensions:

$$\frac{\omega b}{V} \doteq \frac{(T^{-1})(L)}{(LT^{-1})} \doteq L^0T^0 \quad \therefore \text{OK}$$

For Π_2:
$$\Pi_2 = h b^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \quad \text{(for } F\text{)}$$
$$1 + a + b - 4c = 0 \quad \text{(for } L\text{)}$$
$$-b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{h}{b}$$

which is obviously dimensionless. (con't)

7.11 (Con't)

For Π_3:

$$\Pi_3 = k\, b^a V^b \rho^c$$

$$(FL)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = -2$, $c = -1$, and therefore

$$\Pi_3 = \frac{k}{b^3 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{k}{b^3V^2\rho} \doteq \frac{ML^2T^{-2}}{(L^3)(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \;\therefore \text{OK}$$

Thus,

$$\underline{\underline{\frac{\omega b}{V} = \phi\left(\frac{h}{b}, \frac{k}{b^3V^2\rho}\right)}}$$

7.12

7.12 The flowrate, Q, of water in an open channel is assumed to be a function of the cross-sectional area of the channel, A, the height of the roughness of the channel surface, ε, the acceleration of gravity, g, and the slope, S_o, of the hill on which the channel sits. Put this relationship into dimensionless form.

$$Q = f(A, e, g, S_o)$$

$Q \doteq L^3T^{-1}$ $\quad A \doteq L^2$ $\quad e \doteq L$ $\quad g \doteq LT^{-2}$ $\quad S_o = F^0L^0T^0$

From the pi Theorem, $5-2 = 3$ pi terms required. Use A and g as repeating variables. Thus,

$$\Pi_1 = Q A^a g^b$$

and

$$(L^3T^{-1})(L^2)^a(LT^{-2})^b = L^0T^0$$

so that

$$3 + 2a + b = 0 \quad \text{(for } L\text{)}$$

$$-1 - 2b = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -5/4$, $b = -1/2$, and therefore

$$\Pi_1 = \frac{Q}{A^{5/4}\sqrt{g}}$$

Check dimensions:

$$\frac{Q}{A^{5/4}\sqrt{g}} \doteq \frac{(L^3T^{-1})}{(L^2)^{5/4}\sqrt{LT^{-2}}} \doteq L^0T^0 \quad \therefore OK$$

For Π_2:

$$\Pi_2 = e A^a g^b$$

$$(L)(L^2)^a(LT^{-2})^b = L^0T^0$$

$$1 + 2a + b = 0 \quad \text{(for } L\text{)}$$

$$-2b = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1/2$, $b = 0$, and therefore

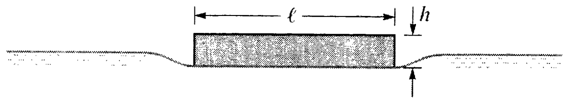
$$\Pi_2 = \frac{e}{\sqrt{A}}$$

which is obviously dimensionless. The third pi term is the slope since S_o is dimensionless. Thus,

$$\frac{Q}{A^{5/4}\sqrt{g}} = \phi\left(\frac{e}{\sqrt{A}}, S_o\right)$$

7.13

7.13 Because of surface tension, it is possible, with care, to support an object heavier than water on the water surface as shown in Fig. P7.13. (See Video V1.5.) The maximum thickness, h, of a square of material that can be supported is assumed to be a function of the length of the side of the square, ℓ, the density of the material, ρ, the acceleration of gravity, g, and the surface tension of the liquid, σ. Develop a suitable set of dimensionless parameters for this problem.



$$h = f(\ell, \rho, g, \sigma)$$

$h \doteq L \quad \ell \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad g \doteq LT^{-2} \quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3 = 2$ pi terms required. Use ℓ, g, and ρ as repeating variables. Thus,

$$\Pi_1 = h\,\ell^a g^b \rho^c$$

and

so that

$$(L)(L)^a(LT^{-2})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_1 = \frac{h}{\ell}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \sigma\,\ell^a g^b \rho^c$$

$$(FL^{-1})(L)^a(LT^{-2})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = -1$, $c = -1$, and therefore

$$\Pi_2 = \frac{\sigma}{\ell^2 g \rho}$$

Check dimensions using MLT system:

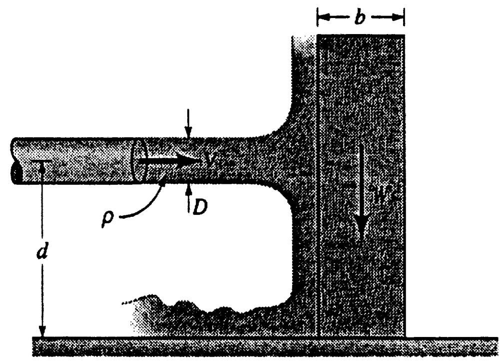
$$\frac{\sigma}{\ell^2 g\rho} \doteq \frac{(MT^{-2})}{(L^2)(LT^{-2})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Thus,

$$\underline{\underline{\frac{h}{\ell} = \phi\left(\frac{\sigma}{\ell^2 g\rho}\right)}}$$

7.14

7.14 As shown in Fig. P7.14 and Video V5.4, a jet of liquid directed against a block can tip over the block. Assume that the velocity, V, needed to tip over the block is a function of the fluid density, ρ, the diameter of the jet, D, the weight of the block, $\mathcal{W}$, the width of the block, b, and the distance, d, between the jet and the bottom of the block. **(a)** Determine a set of dimensionless parameters for this problem. Form the dimensionless parameters by inspection. **(b)** Use the momentum equation to determine an equation for V in terms of the other variables. **(c)** Compare the results of parts (a) and (b).



■ FIGURE P7.14

(a) $$V = f(\rho, D, \mathcal{W}, b, d)$$

$$V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L \quad \mathcal{W} \doteq F \quad b \doteq L \quad d \doteq L$$

From the pi theorem, $6-3 = 3$ pi terms required.
By inspection for Π_1 (containing V)

$$\Pi_1 = VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{FL^{-4}T^{2}}{F}}\right) \doteq F^0L^0T^0$$

Check using MLT:

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = (LT^{-1})(L)\left(\sqrt{\frac{ML^{-3}}{MLT^{-2}}}\right) \doteq M^0L^0T^0 \therefore ok$$

For Π_2 let

$$\Pi_2 = \frac{b}{d}$$

and for Π_3

$$\Pi_3 = \frac{d}{D}$$

and both Π_2 and Π_3 are obviously dimensionless.

Thus,

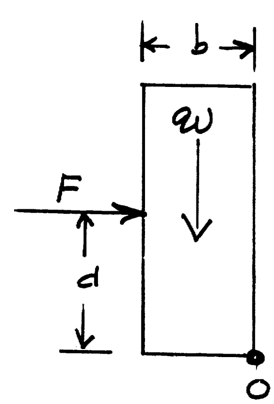
$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

(b) For impending tipping around O

$$\sum M_O = 0$$

so that

$$Fd = \mathcal{W}\left(\frac{b}{2}\right) \qquad (1)$$



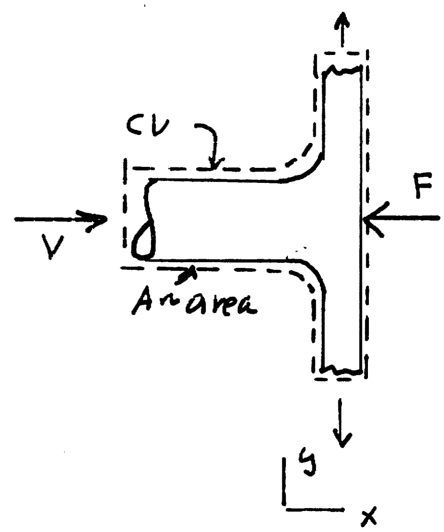
(con't)

7.14 (con't)

From momentum considerations using the CV shown

$$\xrightarrow{+} \quad \int \rho u \vec{V} \cdot \hat{n} \, dA = \sum F_x$$

$$\rho V^2 A = F$$



Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

so that

$$V = \sqrt{\frac{\mathcal{W}(b)}{2\rho A d}}$$

and with $A = \pi/4 \, D^2$

$$\underline{\underline{V = \sqrt{\frac{2\mathcal{W} b}{\pi \rho d D^2}}}} \qquad (2)$$

(c) From part (a)

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \, \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

Eq. (2) can be written as

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \left(\sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}\right) \qquad (3)$$

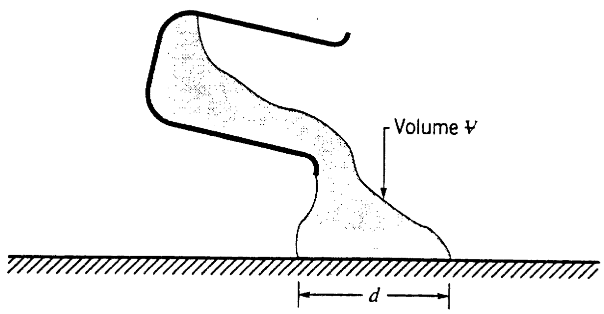
It follows by comparing Eqs. (2) and (3) that

$$\phi\left(\frac{b}{d}, \frac{d}{D}\right) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}$$

so that $\phi\left(\frac{b}{d}, \frac{d}{D}\right)$ is actually independent of $\frac{d}{D}$.

7.15

7.15 A viscous fluid is poured onto a horizontal plate as shown in Fig. P7.15. Assume that the time, t, required for the fluid to flow a certain distance, d, along the plate is a function of the volume of fluid poured, $\forall$, acceleration of gravity, g, fluid density, ρ, and fluid viscosity, μ. Determine an appropriate set of pi terms to describe this process. Form the pi terms by inspection.



■ FIGURE P7.15

$$t = f(d, \forall, g, \rho, \mu)$$

$t \doteq T \quad d \doteq L \quad \forall \doteq L^3 \quad g \doteq LT^{-2} \quad \rho \doteq FL^{-4}T^2 \quad \mu \doteq FL^{-2}T$

From the pi theorem $6-3=3$ pi terms required.

By inspection, for Π_1 (containing t):

$$\Pi_1 = t\sqrt{\frac{g}{d}} \doteq \frac{(T)(LT^{-2})^{1/2}}{(L)^{1/2}} \doteq F^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 (containing $\forall$):

$$\Pi_2 = \frac{\forall}{d^3} \doteq \frac{(L^3)}{(L)^3} \doteq F^0L^0T^0 \quad \therefore \text{ok}$$

For Π_3 (containing ρ and μ):

$$\Pi_3 = \frac{\rho\sqrt{g}\, d^{3/2}}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-2})^{1/2}(L)^{3/2}}{FL^{-2}T} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{\rho\sqrt{g}\, d^{3/2}}{\mu} \doteq \frac{(ML^{-3})(LT^{-2})^{1/2}(L)^{3/2}}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

Thus,

$$\underline{\underline{t\sqrt{\frac{g}{d}} = \phi\left(\frac{\forall}{d^3}, \frac{\rho\sqrt{g}\, d^{3/2}}{\mu}\right)}}$$

7.16

7.16 Assume that the drag, $\mathfrak{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathcal{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathcal{D} \doteq F \quad V = LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1}$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3,$ etc.).

By inspection, for Π_1 (containing $\mathcal{D}$):

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

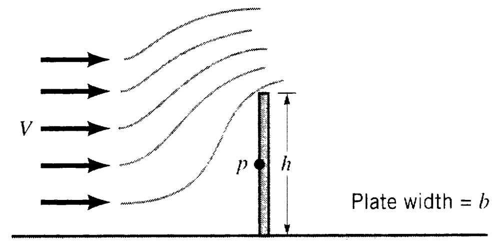
Thus,

$$\underline{\underline{\frac{\mathcal{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)}}$$

Where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1},$ etc.

7.17

7.17 When a fluid flows slowly past a vertical plate of height h and width b (see Fig. P7.17), pressure develops on the face of the plate. Assume that the pressure, p, at the midpoint of the plate is a function of plate height and width, the approach velocity, V, and the fluid viscosity, μ. Make use of dimensional analysis to determine how the pressure, p, will change when the fluid velocity, V, is doubled.



$$p = f(h, b, V, \mu)$$

$$p \doteq FL^{-2} \quad h \doteq L \quad b \doteq L \quad V \doteq LT^{-1} \quad \mu \doteq FL^{-2}T$$

From the pi Theorem $5-3=2$ pi terms required.
By inspection, for Π_1 (containing p):

$$\Pi_1 = \frac{ph}{V\mu} \doteq \frac{(FL^{-2})(L)}{(LT^{-1})(FL^{-2}T)} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{ph}{V\mu} = \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore OK$$

For Π_2 (containing b):

$$\Pi_2 = \frac{b}{h}$$

Which is obviously dimensionless. Thus,

$$\frac{ph}{V\mu} = \phi\left(\frac{b}{h}\right)$$

so that

$$p = \frac{V\mu}{h}\,\phi\left(\frac{b}{h}\right) \qquad (1)$$

From Eq. (1) it follows that for a given geometry and viscosity, if the velocity, V, is doubled the pressure, p, will be doubled.

7.18

7.18 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \tag{1}$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K \frac{\ell}{D}$$

where K is some constant. It thus follows that

$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

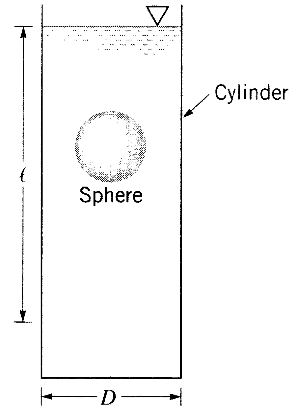
for a given velocity.

7.19

7.19 The viscosity, μ, of a liquid can be measured by determining the time, t, it takes for a sphere of diameter, d, to settle slowly through a distance, ℓ, in a vertical cylinder of diameter, D, containing the liquid (see Fig. P7.19). Assume that

$$t = f(\ell, d, D, \mu, \Delta\gamma)$$

where $\Delta\gamma$ is the difference in specific weights between the sphere and the liquid. Use dimensional analysis to show how t is related to μ, and describe how such an apparatus might be used to measure viscosity.



$$t \doteq T \quad \ell \doteq L \quad d \doteq L \quad D \doteq L \quad \Delta\gamma \doteq FL^{-3} \quad \mu \doteq FL^{-2}T$$

From the pi theorem, $6-3 = 3$ pi terms required. By inspection for Π_1 (containing t):

$$\Pi_1 = \frac{t\,\Delta\gamma\, d}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{(FL^{-2}T)} = F^0L^0T^0$$

Check using MLT:

$$\frac{t\,\Delta\gamma\, d}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2 (containing D):

$$\Pi_2 = \frac{D}{d}$$

which is obviously dimensionless.

For Π_3 (containing ℓ):

$$\Pi_3 = \frac{\ell}{d}$$

which is obviously dimensionless. Thus,

$$\frac{t\,\Delta\gamma\, d}{\mu} = \phi\left(\frac{D}{d}, \frac{\ell}{d}\right)$$

and for a fixed geometry

$$\frac{t\,\Delta\gamma\, d}{\mu} = C$$

where C is a constant, or

$$\mu = \frac{d}{C}\,\Delta\gamma\, t = C_1\,\Delta\gamma\, t$$

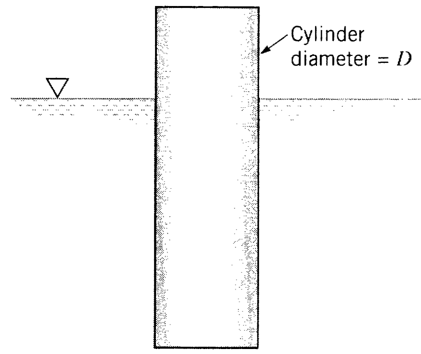
where $C_1 = \frac{d}{C}$ a constant for a fixed geometry. Thus, for this device

$$\underline{\underline{\mu = C_1\,\Delta\gamma\, t}} \qquad (1)$$

The constant C_1 can be determined by calibration with a fluid of known viscosity. With C_1 known the viscosity of other fluids can be determined through a measurement of the time t in conjunction with Eq. (1).

7.20

7.20 A cylinder with a diameter, D, floats upright in a liquid as shown in Fig. P7.20. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, and the specific weight, γ, of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?



$$\omega = f(D, m, \gamma)$$

$$\omega \doteq T^{-1} \qquad D \doteq L \qquad m \doteq FL^{-1}T^2 \qquad \gamma = FL^{-3}$$

From the pi theorem, $4-3 = 1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{FL^{-1}T^2}{FL^{-3}}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{M}{ML^{-2}T^{-2}}} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Since there is only 1 pi term, it follows that

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = C$$

where C is a constant. Thus,

$$\underline{\underline{\omega = CD\sqrt{\frac{\gamma}{m}}}}$$

From this result it follows that if m is increased ω will <u>decrease</u>.

7.21*

*7.21 The pressure drop, Δp, over a certain length of horizontal pipe is assumed to be a function of the velocity, V, of the fluid in the pipe, the pipe diameter, D, and the fluid density and viscosity, ρ and μ. **(a)** Show that this flow can be described in dimensionless form as a "pressure coefficient," $C_p = \Delta p/(0.5\,\rho V^2)$ that depends on the Reynolds number, $\mathrm{Re} = \rho VD/\mu$. **(b)** The following data were obtained in an experiment involving a fluid with $\rho = 2$ slugs/ft^3, $\mu = 2 \times 10^{-3}$ lb · s/ft^2, and $D = 0.1$ ft. Plot a dimensionless graph and use a power law equation to determine the functional relationship between the pressure coefficient and the Reynolds number.

V, ft/s	Δp, lb/ft^2
3	192
11	704
17	1088
20	1280

(c) What are the limitations on the applicability of your equation obtained in part **(b)**?

(a) $\Delta p = f(V, D, \rho, \mu)$

$\Delta p \doteq FL^{-2} \quad V \doteq LT^{-1} \quad D \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad \mu \doteq FL^{-2}T$

From the pi theorem, 5-3 = 2 pi terms required.

By inspection for Π_1,

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0 \quad \therefore \text{ OK}$$

Check using MLT system:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

For Π_2:

$$\Pi_2 = \frac{\rho V D}{\mu} \doteq \frac{(FL^{-4}T^{2})(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0 \quad \therefore \text{ ok}$$

Check using MLT system:

$$\frac{\rho V D}{\mu} = \frac{(ML^{-3})(LT^{-1})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Thus,
$$\frac{\Delta p}{\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}\right)$$

Since $\tilde{\phi}$ is an unknown function, a factor of 0.5 can be included in Π_1 (if desired) so that

$$\frac{\Delta p}{0.5\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Thus,
$$C_p = \phi(Re)$$

where C_p is the pressure coefficient and Re the Reynolds number.

(con't)

7.21 (con't)

(b) Using the data given,

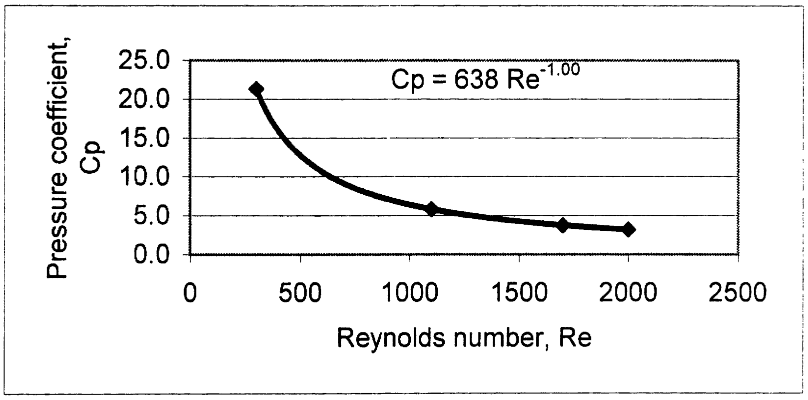
$$C_p = \frac{\Delta p}{0.5\rho V^2} = \frac{\Delta p}{(0.5)\left(2\ \frac{slugs}{ft^3}\right)V^2} = \frac{\Delta p}{V^2}$$

and

$$R_e = \frac{\rho V D}{\mu} = \frac{2\left(\frac{slugs}{ft^3}\right)(V)(0.1ft)}{2\times10^{-3}\ \frac{lb\cdot s}{ft^2}} = 100V$$

Tabulated values for C_p and R_e and a plot of the data are shown below.

V, ft/s	Δ p, psf	Re	Cp
3	192	300	21.3
11	704	1100	5.82
17	1090	1700	3.77
20	1280	2000	3.20



The power law relationship is

$$\underline{\underline{C_p = \frac{638}{R_e}}} \qquad (1)$$

(c) Based on the variables used and the given data, the empirical relationship, Eq. (1), would only be applicable in the Reynolds number range

$$300 \leq R_e \leq 2000$$

Note: Although the equation might be valid outside this range, results should not be extrapolated beyond the range of data used.

7.22

7.22 The height, h, that a liquid will rise in a capillary tube is a function of the tube diameter, D, the specific weight of the liquid, γ, and the surface tension, σ. Perform a dimensional analysis using both the FLT and MLT systems for basic dimensions. Note: The results should obviously be the same regardless of the system of dimensions used. If your analysis indicates otherwise, go back and check your work giving particular attention to the required number of reference dimensions.

$$h = f(D, \gamma, \sigma)$$

Using FLT system:

$$h \doteq L \qquad D \doteq L \qquad \gamma \doteq FL^{-3} \qquad \sigma \doteq FL^{-1}$$

From the pi theorem, $4-2 = 2$ pi terms required.

By inspection, for Π_1 (containing h):

$$\Pi_1 = \frac{h}{D}$$

which is obviously dimensionless.

For Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} \doteq \frac{FL^{-1}}{(FL^{-3})(L)^2} = F^0L^0$$

Thus,

$$\underline{\underline{\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)}}$$

Using MLT system:

$$h \doteq L \qquad D \doteq L \qquad \gamma \doteq ML^{-2}T^{-2} \qquad \sigma = MT^{-2}$$

Although there appears to be 3 reference dimensions, only 2 reference dimensions are actually required (L and MT^{-2}) to describe the variables. By inspection, for Π_1 (see above)

$$\Pi_1 = \frac{h}{D}$$

and for Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{MT^{-2}}{(ML^{-2}T^{-2})(L)^2} = M^0L^0T^0$$

Thus, (as above)

$$\underline{\underline{\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)}}$$

7.23

7.23 The speed of sound in a gas, c, is a function of the gas pressure, p, and density, ρ. Determine, with the aid of dimensional analysis, how the velocity is related to the pressure and density. Be careful when you decide on how many reference dimensions are required.

$$c = f(p, \rho)$$

$$c \doteq LT^{-1} \qquad p \doteq FL^{-2} \qquad \rho \doteq FL^{-4}T^{2}$$

Although there appears to be 3 reference dimensions (which would indicate that there are no possible pi terms), only 2 reference dimensions (LT^{-1} and FL^{-2}) are actually required since

$$\rho \doteq (FL^{-2})(LT^{-1})^{-2}$$

Thus, from the pi theorem, $3-2=1$ pi term required.
By inspection:

$$\Pi_1 = \frac{c^2\rho}{p} \doteq \frac{(LT^{-1})^2\left[(FL^{-2})(LT^{-1})^{-2}\right]}{FL^{-2}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{c^2\rho}{p} \doteq \frac{(LT^{-1})^2(ML^{-3})}{ML^{-1}T^{-2}} = M^0L^0T^0 \quad \therefore \text{ ok}$$

Since there is only 1 pi term, it follows that

$$\frac{c^2\rho}{p} = C$$

where C is a constant. Thus,

$$c = \sqrt{C\frac{p}{\rho}}$$

or

$$\underline{\underline{c = C_1\sqrt{\frac{p}{\rho}}}}$$

where C_1 is a constant ($C_1 = \sqrt{C}$).

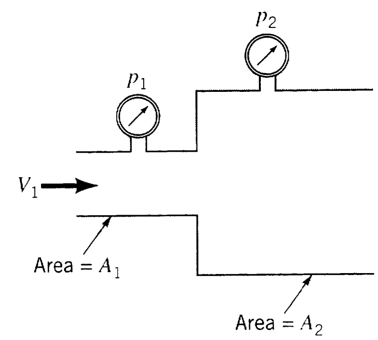
7.24*

*7.24 The pressure rise, $\Delta p = p_2 - p_1$, across the abrupt expansion of Fig. P7.24 through which a liquid is flowing can be expressed as

$$\Delta p = f(A_1, A_2, \rho, V_1)$$

where A_1 and A_2 are the upstream and downstream cross-sectional areas, respectively, ρ is the fluid density, and V_1 is the upstream velocity. Some experimental data obtained with $A_2 = 1.25\ \text{ft}^2$, $V_1 = 5.00$ ft/s, and using water with $\rho = 1.94\ \text{slugs/ft}^3$ are given in the following table:

A_1 (ft^2)	0.10	0.25	0.37	0.52	0.61
Δp (lb/ft^2)	3.25	7.85	10.3	11.6	12.3



Plot the results of these tests using suitable dimensionless parameters. With the aid of a standard curve fitting program determine a general equation for Δp and use this equation to predict Δp for water flowing through an abrupt expansion with an area ratio $A_1/A_2 = 0.35$ at a velocity $V_1 = 3.75$ ft/s.

$\Delta p \doteq FL^{-2}$ $\quad A_1 \doteq L^2$ $\quad A_2 \doteq L^2$ $\quad \rho \doteq FL^{-4}T^2$ $\quad V_1 \doteq LT^{-1}$

From the pi theorem, 5 - 3 = 2 pi terms required.

By inspection for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V_1^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p}{\rho V_1^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing A_1 and A_2):

$$\Pi_2 = \frac{A_1}{A_2}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p}{\rho V_1^2} = \phi\left(\frac{A_1}{A_2}\right)$$

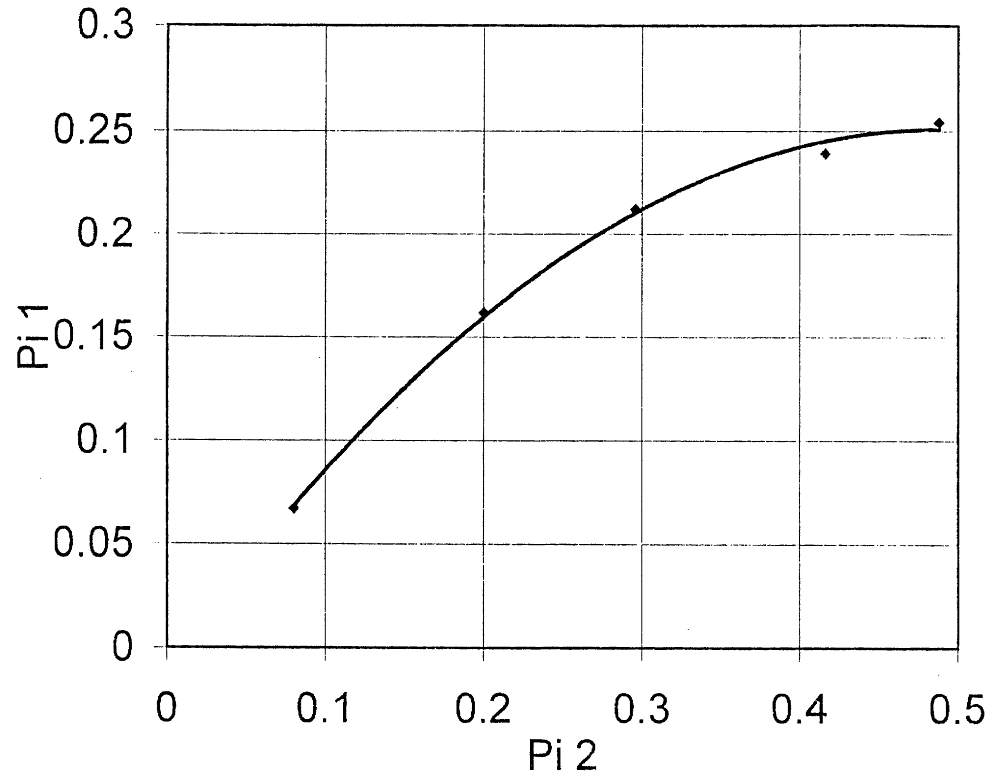
Using the data given, it follows that:

$\Delta p/\rho V_1^2$	0.067	0.162	0.212	0.239	0.254
A_1/A_2	0.080	0.200	0.296	0.416	0.488

A plot of these data is shown on the next page.

(con't)

7.24* (Con't)



The curve drawn on the graph is a 2nd order polynomial giving the equation

$$\frac{\Delta p}{\rho V_1^2} = -1.10\left(\frac{A_1}{A_2}\right)^2 + 1.07\left(\frac{A_1}{A_2}\right) - 0.0103$$

Thus, for $A_1/A_2 = 0.35$ and $V_1 = 3.75 \text{ ft/s}$ with water ($\rho = 1.94 \text{ slugs/ft}^3$)

$$\Delta p = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(3.75 \frac{\text{ft}}{\text{s}}\right)^2\left[-1.10(0.35)^2 + 1.07(0.35) - 0.0103\right]$$

$$= 6.26 \frac{\text{lb}}{\text{ft}^2}$$

7.25

7.25 A liquid flows with a velocity V through a hole in the side of a large tank. Assume that

$$V = f(h, g, \rho, \sigma)$$

where h is the depth of fluid above the hole, g is the acceleration of gravity, ρ the fluid density, and σ the surface tension. The following data were obtained by changing h and measuring V, with a fluid having a density $= 10^3$ kg/m³ and surface tension $= 0.074$ N/m.

V (m/s)	3.13	4.43	5.42	6.25	7.00
h (m)	0.50	1.00	1.50	2.00	2.50

Plot these data by using appropriate dimensionless variables. Could any of the original variables have been omitted?

$V \doteq LT^{-1}$ $h \doteq L$ $g \doteq LT^{-2}$ $\rho \doteq FL^{-4}T^2$ $\sigma \doteq FL^{-1}$

From the pi theorem, 5-3 = 2 pi terms required.

By inspection for Π_1 (containing V):

$$\Pi_1 = \frac{V}{\sqrt{gh}} \doteq \frac{LT^{-1}}{(LT^{-2})^{1/2}(L)^{1/2}} \doteq L^0T^0$$

For Π_2 (containing ρ and σ):

$$\Pi_2 = \frac{\rho g h^2}{\sigma} \doteq \frac{(FL^{-4}T^2)(LT^{-2})(L)^2}{FL^{-1}} \doteq F^0L^0T^0$$

Check using MLT:

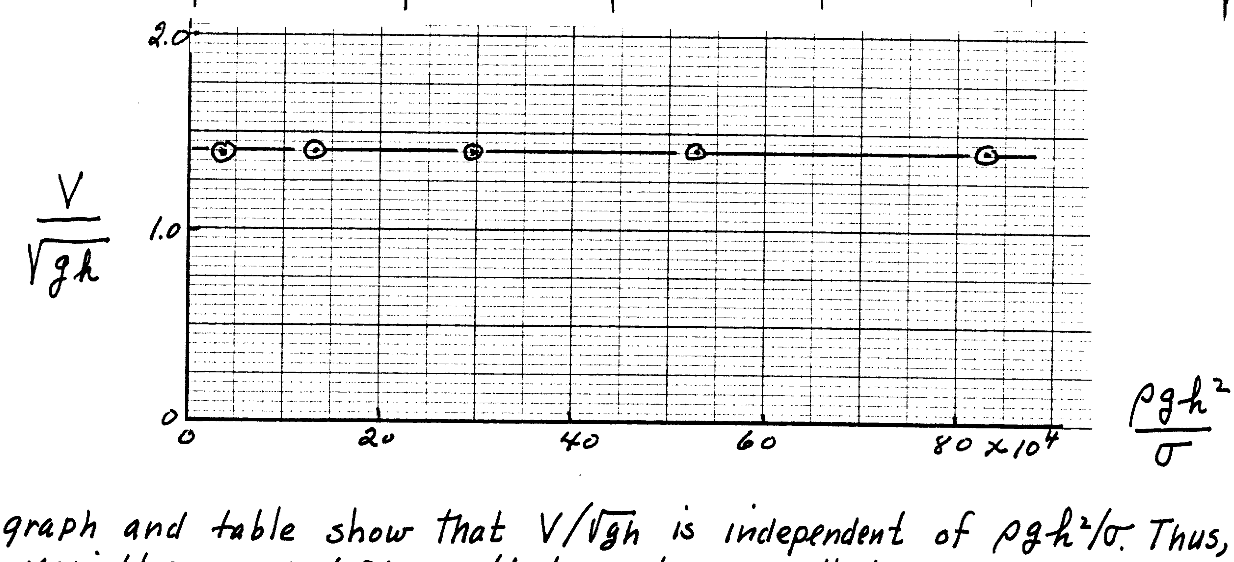
$$\frac{\rho g h^2}{\sigma} \doteq \frac{(ML^{-3})(LT^{-2})(L)^2}{MT^{-2}} \doteq M^0L^0T^0 \therefore \text{OK}$$

Thus,

$$\frac{V}{\sqrt{gh}} = \phi\left(\frac{\rho g h^2}{\sigma}\right)$$

For the data given:

$\rho g h^2/\sigma$	3.31×10^4	13.3×10^4	29.8×10^4	53.0×10^4	82.9×10^4
$V/\sqrt{gh}$	1.41	1.41	1.41	1.41	1.41



The graph and table show that $V/\sqrt{gh}$ is independent of $\rho g h^2/\sigma$. Thus, <u>the variables ρ and σ could have been omitted</u>.

7.26

7.26 The time, t, it takes to pour a certain volume of liquid from a cylindrical container depends on several factors, including the viscosity of the liquid. (See **Video V1.1**.) Assume that for very viscous liquids the time it takes to pour out 2/3 of the initial volume depends on the initial liquid depth, ℓ, the cylinder diameter, D, the liquid viscosity, μ, and the liquid specific weight, γ. The data shown in the following table were obtained in the laboratory. For these tests $\ell = 45$ mm, $D = 67$ mm, and $\gamma = 9.60$ kN/m^3. **(a)** Perform a dimensional analysis and based on the data given, determine if variables used for this problem appear to be correct. Explain how you arrived at your answer. **(b)** If possible, determine an equation relating the pouring time and viscosity for the cylinder and liquids used in these tests. If it is not possible, indicate what additional information is needed.

μ (N·s/m^2)	11	17	39	61	107
t(s)	15	23	53	83	145

$$t = f(\ell, D, \mu, \gamma)$$

(a) $t \doteq T \quad \ell \doteq L \quad D \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3}$

From the pi theorem $5-3=2$ pi terms required.

By inspection, for Π_1 (containing t)

$$\Pi_1 = \frac{t\gamma D}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{t\gamma D}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing ℓ)

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{t\gamma D}{\mu} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

For the data given $\frac{\ell}{D} = \frac{45\,mm}{67\,mm} = 0.672$ (a constant).

Thus, from Eq. (1) with ℓ/D a constant it follows that $\frac{t\gamma D}{\mu}$ = constant. For the data given:

(con't)

7.26 (con't)

$\frac{t\gamma D}{\mu}$	877	870	874	875	872

Since Π_1 is essentially constant over the range of the experimental data the variables used for the problem appear to be correct.

(b) The average value for Π_1 is 874 so that

$$\frac{t\gamma D}{\mu} = 874$$

and therefore

$$t = \frac{874}{\gamma D}\,\mu = \frac{874\,\mu}{\left(9.6\times10^{3}\,\frac{N}{m^3}\right)\left(67\times10^{-3}\,m\right)}$$

$$t = 1.36\,\mu$$

with t in seconds when μ is in units of $N\cdot s/m^2$. Note that this restricted equation is only valid for $\ell/D = 0.672$, $D = 67\,mm$, and $\gamma = 9.60\,kN/m^3$ with 2/3 of the initial volume being poured.

7.27

7.27 The pressure drop per unit length, Δp_ℓ, for the flow of blood through a horizontal small diameter tube is a function of the volume rate of flow, Q, the diameter, D, and the blood viscosity, μ. For a series of tests in which $d = 2$ mm, and $\mu = 0.004\ \text{N·s/m}^2$, the following data were obtained, where the Δp listed was measured over the length, $\ell = 300$ mm.

Q (m³/s)	Δp (N/m²)
3.6×10^{-6}	1.1×10^4
4.9×10^{-6}	1.5×10^4
6.3×10^{-6}	1.9×10^4
7.9×10^{-6}	2.4×10^4
9.8×10^{-6}	3.0×10^4

Perform a dimensional analysis for this problem, and make use of the data given to determine a general relationship between Δp_ℓ and Q (one that is valid for other values of D, ℓ, and μ).

$$\Delta p_\ell = f(Q, D, \mu)$$

$$\Delta p_\ell \doteq FL^{-3} \qquad Q \doteq L^3T^{-1} \qquad D \doteq L \qquad \mu \doteq FL^{-2}T$$

From the pi theorem, $4-3=1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(FL^{-3})(L)^4}{(FL^{-2}T)(L^3T^{-1})} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(ML^{-2}T^{-2})(L)^4}{(ML^{-1}T^{-1})(L^3T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

Since there is only 1 pi term, it follows that

$$\frac{\Delta p_\ell D^4}{\mu Q} = C$$

where C is a constant. For the data given

$$\frac{\Delta p_\ell D^4}{\mu Q} = \left(\frac{\Delta p}{0.3\,m}\right)\frac{(0.002\,m)^4}{(0.004\ N\cdot s/m^2)\,Q} = 1.33 \times 10^{-8}\,\frac{\Delta p}{Q}$$

and therefore using the data in the table

$\dfrac{\Delta p_\ell D^4}{\mu Q}$	40.6	40.7	40.1	40.4	40.7

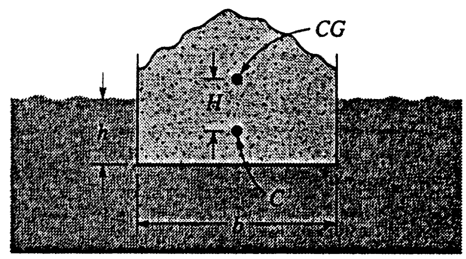
Thus, the average value for $C = 40.5$ and

$$\underline{\underline{\Delta p_\ell = 40.5\,\frac{\mu Q}{D^4}}}$$

7.28 *

*7.28 As shown in Fig. 2.26, Fig. P7.28, and Video V2.7, a rectangular barge floats in a stable configuration provided the distance between the center of gravity, *CG*, of the object (boat and load) and the center of buoyancy, *C*, is less than a certain amount, *H*. If this distance is greater than *H* the boat will tip over. Assume *H* is a function of the boat's width, *b*, length, ℓ, and draft, *h*. **(a)** Put this relationship into dimensionless form. **(b)** The results of a set of experiments with a model barge with a width of 1.0 m is shown in the table. Plot this data in dimensionless form and determine a power-law equation relating the dimensionless parameters.

ℓ, m	h, m	H, m
2.0	0.10	0.833
4.0	0.10	0.833
2.0	0.20	0.417
4.0	0.20	0.417
2.0	0.35	0.238
4.0	0.35	0.238



■ FIGURE P7.28

(a) $H = f(b, \ell, h)$

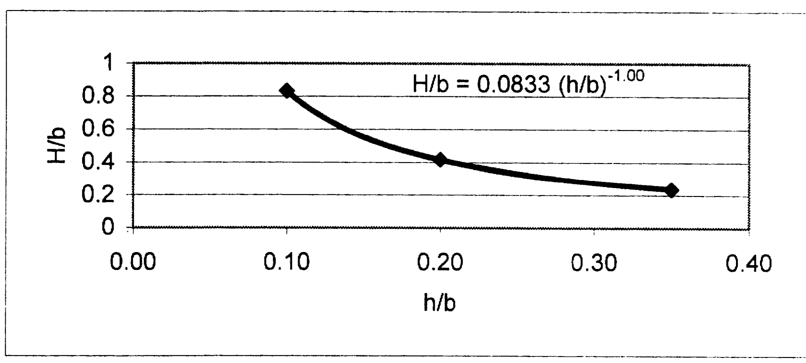
From the pi theorem, 4−1 = 3 pi terms required. By inspection:

$$\frac{H}{b} = \phi\left(\frac{h}{b}, \frac{\ell}{b}\right)$$

All of the pi terms are obviously dimensionless.

(b) For the data given, tabulated values for H/b, h/b, and ℓ/b are shown below.

h/b	H/b	ℓ/b
0.10	0.833	2.0
0.10	0.833	4.0
0.20	0.417	2.0
0.20	0.417	4.0
0.35	0.238	2.0
0.35	0.238	4.0



An inspection of these data reveals that H/b does not depend on ℓ/b, i.e., the same value of H/b is obtained for different values of ℓ/b. Thus,

$$\frac{H}{b} = \phi\left(\frac{h}{b}\right)$$

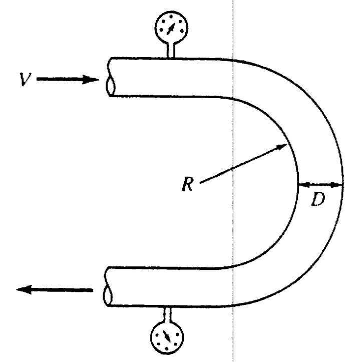
and from the plot of the data, using a power-law equation

$$\frac{H}{b} = 0.0833\left(\frac{h}{b}\right)^{-1.00}$$

7.29

7.29 A fluid flows through the horizontal curved pipe of Fig. P7.29 with a velocity V. The pressure drop, Δp, between the entrance and the exit to the bend is thought to be a function of the velocity, bend radius, R, pipe diameter, D, and fluid density, ρ. The data shown in the following table were obtained in the laboratory. For these tests $\rho = 2.0$ slugs/ft^3, $R = 0.5$ ft, and $D = 0.1$ ft. Perform a dimensional analysis and based on the data given, determine if the variables used for this problem appear to be correct. Explain how you arrived at your answer.

V (ft/s)	2.1	3.0	3.9	5.1
Δp (lb/ft^2)	1.2	1.8	6.0	6.5



■ FIGURE P7.29

$$\Delta p = f(V, R, D, \rho)$$

$\Delta p \doteq FL^{-2}$ $\quad V \doteq LT^{-1}$ $\quad R \doteq L$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$

From the pi theorem, $5-3 = 2$ pi terms required.
By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{(FL^{-2})}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{(ML^{-1}T^{-2})}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 (containing R and D):

$$\Pi_2 = \frac{D}{R}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{D}{R}\right) \qquad (1)$$

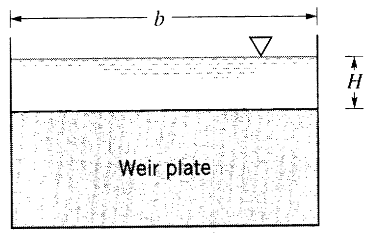
For the data given $\frac{D}{R} = \frac{0.1\text{ ft}}{0.5\text{ ft}} = \frac{1}{5}$ (a constant). Thus, from Eq. (1) with $\frac{D}{R}$ a constant it follows that $\frac{\Delta p}{\rho V^2} = $ Constant. However, for the data given:

$\frac{\Delta p}{\rho V^2}$	0.136	0.100	0.197	0.125

Since $\frac{\Delta p}{\rho V^2}$ is not constant, it follows that the variables used for the problem are not correct.

7.30

7.30 The water flowrate, Q, in an open rectangular channel can be measured by placing a plate across the channel as shown in Fig. P7.30. This type of a device is called a *weir*. The height of the water, H, above the weir crest is referred to as the head and can be used to determine the flowrate through the channel. Assume that Q is a function of the head, H, the channel width, b, and the acceleration of gravity, g. Determine a suitable set of dimensionless variables for this problem.



$$Q = f(H, b, g)$$

$$Q \doteq L^3T^{-1} \quad b \doteq L \quad g \doteq LT^{-2}$$

From the pi theorem, $4 - 2 = 2$ pi terms required.

By inspection for Π_1 (containing Q):

$$\Pi_1 = \frac{Q}{H^{5/2}g^{1/2}} \doteq \frac{L^3T^{-1}}{(L)^{5/2}(LT^{-2})^{1/2}} \doteq L^0T^0$$

For Π_2 (containing b):

$$\Pi_2 = \frac{b}{H}$$

which is obviously dimensionless. Thus,

$$\frac{Q}{H^{5/2}g^{1/2}} = \phi\left(\frac{b}{H}\right)$$

7.31

7.31 From theoretical considerations it is known that for the weir described in Problem 7.30 the flowrate, Q, must be directly proportional to the channel width, b. In some laboratory tests it was determined that if $b = 3$ ft and $H = 4$ in., then $Q = 1.96\ \text{ft}^3/\text{s}$. Based on these limited data, determine a general equation for the flowrate over this type of weir.

From Problem 7.30,

$$\frac{Q}{H^{5/2} g^{1/2}} = \phi\left(\frac{b}{H}\right) \qquad (1)$$

Since $Q \propto b$ it follows from Eq. (1) that

$$\frac{Q}{H^{5/2} g^{1/2}} = C\left(\frac{b}{H}\right)$$

where C is a constant. Thus, for the data given

$$C = \frac{Q}{H^{3/2} g^{1/2}\, b} = \frac{1.96\ \frac{\text{ft}^3}{\text{s}}}{\left(\frac{4}{12}\ \text{ft}\right)^{3/2}\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)^{1/2}(3\ \text{ft})}$$

$$= 0.598$$

so that the general equation is

$$\underline{\underline{Q = 0.598\, b\sqrt{g H^3}}}$$

7.32

7.32 SAE 30 oil at 60 °F is pumped through a 3-ft-diameter pipeline at a rate of 6400 gal/min. A model of this pipeline is to be designed using a 3-in.-diameter pipe and water at 60 °F as the working fluid. To maintain Reynolds number similarity between these two systems, what fluid velocity will be required in the model?

For Reynolds number similarity,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

or

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V \qquad (1)$$

Since,

$$V = \frac{Q}{\text{area}}$$

and

$$Q = \frac{\left(6400 \frac{gal}{min}\right)\left(231 \frac{in.^3}{gal}\right)\left(\frac{1 \, ft^3}{1728 \, in.^3}\right)}{60 \frac{s}{min}} = 14.3 \frac{ft^3}{s}$$

then

$$V = \frac{14.3 \frac{ft^3}{s}}{\frac{\pi}{4}(3 \, ft)^2} = 2.02 \frac{ft}{s}$$

Thus, from Eq. (1)

$$V_m = \frac{\left(1.21 \times 10^{-5} \frac{ft^2}{s}\right)(3 \, ft)}{\left(4.5 \times 10^{-3} \frac{ft^2}{s}\right)\left(\frac{3}{12} \, ft\right)} \left(2.02 \frac{ft}{s}\right) = \underline{\underline{6.52 \times 10^{-2} \frac{ft}{s}}}$$

7.33

7.33 Glycerin at 20 °C flows with a velocity of 4 m/s through a 30-mm-diameter tube. A model of this system is to be developed using standard air as the model fluid. The air velocity is to be 2 m/s. What tube diameter is required for the model if dynamic similarity is to be maintained between model and prototype?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

so that

$$D_m = \frac{\nu_m}{\nu}\frac{V}{V_m} D = \frac{\left(1.46\times10^{-5}\,\frac{m^2}{s}\right)\left(4\,\frac{m}{s}\right)}{\left(1.19\times10^{-3}\,\frac{m^2}{s}\right)\left(2\,\frac{m}{s}\right)}(0.030\,m)$$

$$= 0.736\times10^{-3}\,m = \underline{\underline{0.736\,mm}}$$

7.34 The drag characteristics of a torpedo are to be studied in a water tunnel using a 1:5 scale model. The tunnel operates with freshwater at 20 °C, whereas the prototype torpedo is to be used in seawater at 15.6 °C. To correctly simulate the behavior of the prototype moving with a velocity of 30 m/s, what velocity is required in the water tunnel?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

so that

$$V_m = \frac{\nu_m}{\nu}\frac{D}{D_m} V$$

Since, ν_m (water @ 20°C) $= 1.004 \times 10^{-6}\ m^2/s$ (Table B.2), ν (seawater @ 15.6°C) $= 1.17 \times 10^{-6}\ m^2/s$ (Table 1.6), and $D/D_m = 5$, it follows that

$$V_m = \frac{\left(1.004 \times 10^{-6}\ \frac{m^2}{s}\right)}{\left(1.17 \times 10^{-6}\ \frac{m^2}{s}\right)}(5)\left(30\ \frac{m}{s}\right) = \underline{\underline{129\ \frac{m}{s}}}$$

7.35 The design of a river model is to be based on Froude number similarity, and a river depth of 3 m is to correspond to a model depth of 50 mm. Under these conditions what is the prototype velocity corresponding to a model velocity of 1.2 m/s?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m d_m}} = \frac{V}{\sqrt{g d}}$$

where d is the fluid depth. Thus,

$$V = \sqrt{\frac{g\, d}{g_m\, d_m}}\, V_m$$

and with $g = g_m$

$$V = \sqrt{\frac{d}{d_m}}\, V_m = \sqrt{\frac{3\,m}{0.050\,m}}\left(1.2\,\frac{m}{s}\right) = \underline{\underline{9.30\,\frac{m}{s}}}$$

7.36

7.36 For a certain fluid flow problem it is known that both the Froude number and the Weber number are important dimensionless parameters. If the problem is to be studied by using a 1:15 scale model, determine the required surface tension scale if the density scale is equal to 1. The model and prototype operate in the same gravitational field.

For dynamic similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}} \qquad \text{(Froude number similarity)}$$

and

$$\frac{\rho_m V_m^2 \ell_m}{\sigma_m} = \frac{\rho V^2 \ell}{\sigma} \qquad \text{(Weber number similarity)}$$

To satisfy Froude number similarity (with $g = g_m$),

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

and therefore for Weber number similarity

$$\frac{\sigma_m}{\sigma} = \frac{\rho_m}{\rho}\left(\frac{V_m}{V}\right)^2 \frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)\frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)^2$$

Thus, with $\ell_m / \ell = 1/15$ and $\rho_m / \rho = 1$,

$$\frac{\sigma_m}{\sigma} = (1)\left(\frac{1}{15}\right)^2 = \underline{\underline{4.44 \times 10^{-3}}}$$

7.37

7.37 The fluid dynamic characteristics of an airplane flying at 240 mph at 10,000 ft are to be investigated with the aid of a 1:20 scale model. If the model tests are to be performed in a wind tunnel using standard air, what is the required air velocity in the wind tunnel? Is this a realistic velocity?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{\ell}{\ell_m} V \qquad (1)$$

Since,

$$\mu = 3.534 \times 10^{-7} \frac{lb \cdot s}{ft^2} \; ; \; \rho = 1.756 \times 10^{-3} \frac{slugs}{ft^3} \quad (\text{Table C.1})$$

$$\mu_m = 3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2} \; ; \; \rho_m = 2.38 \times 10^{-3} \frac{slugs}{ft^3} \quad (\text{Table 1.7})$$

and $\ell / \ell_m = 20$, it follows from Eq. (1) that

$$V_m = \frac{\left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)}{\left(3.534 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} \frac{\left(1.756 \times 10^{-3} \frac{slugs}{ft^3}\right)}{\left(2.38 \times 10^{-3} \frac{slugs}{ft^3}\right)} (20)(240 \text{ mph})$$

$$= \underline{\underline{3750 \text{ mph}}}$$

No, it is not a realistic velocity — much too high.

7.38

7.38 If an airplane travels at a speed of 1120 km/hr at an altitude of 15 km, what is the required speed at an altitude of 8 km to satisfy Mach number similarity? Assume the air properties correspond to those for the U.S. standard atmosphere.

For Mach number similarity,

$$\left(\frac{V}{c}\right)_{15\,km} = \left(\frac{V}{c}\right)_{8\,km} \qquad (1)$$

The speed of sound can be calculated from the equation

$$c = \sqrt{kRT} \qquad (Eq.\ 1.20)$$

and for air, $k = 1.40$, $R = 286.9\ J/kg\cdot K$.

At 15 km altitude,

$$T = -56.50\,°C + 273.15 = 216.7\ K \qquad (Table\ C.2)$$

and at 8 km

$$T = -36.94\,°C + 273.15 = 236.2\ K \qquad (Table\ C.2)$$

Thus, at 15 km altitude

$$c_{15\,km} = \sqrt{(1.40)\left(286.9\,\frac{J}{kg\cdot K}\right)(216.7\,K)} = 295\ \frac{m}{s}$$

and at 8 km

$$c_{8\,km} = \sqrt{(1.40)\left(286.9\,\frac{J}{kg\cdot K}\right)(236.2\,K)} = 308\ \frac{m}{s}$$

From Eq. (1)

$$V_{8\,km} = \frac{c_{8\,km}}{c_{15\,km}}\,V_{15\,km} = \left(\frac{308\,\frac{m}{s}}{295\,\frac{m}{s}}\right)\left(1120\,\frac{km}{hr}\right)$$

$$= \underline{\underline{1170\ \frac{km}{hr}}}$$

7.40

7.40 The lift and drag developed on a hydrofoil are to be determined through wind tunnel tests using standard air. If full scale tests are to be run, what is the required wind tunnel velocity corresponding to a hydrofoil velocity in seawater of 15 mph? Assume Reynolds number similarity is required.

For Reynolds number similarity,

$$\frac{V_m \ell_m}{\nu_m} = \frac{V \ell}{\nu}$$

where ℓ is some characteristic length of the hydrofoil. Thus,

$$V_m = \frac{\nu_m}{\nu} \frac{\ell}{\ell_m} V$$

and with $\ell/\ell_m = 1$ (full scale test)

$$V_m = \frac{\nu_m}{\nu} V = \frac{\left(1.57 \times 10^{-4} \frac{ft^2}{s}\right)}{\left(1.26 \times 10^{-5} \frac{ft^2}{s}\right)} (15 \text{ mph})$$

$$= \underline{\underline{187 \text{ mph}}}$$

7.41

7.41 A 1/50 scale model is to be used in a towing tank to study the water motion near the bottom of a shallow channel as a large barge passes over. (See Video V7.7.) Assume that the model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype barge moves at a typical speed of 15 knots. **(a)** At what speed (in ft/s) should the model be towed? **(b)** Near the bottom of the model channel a small particle is found to move 0.15 ft in one second so that the fluid velocity at that point is approximately 0.15 ft/s. Determine the velocity at the corresponding point in the prototype channel.

(a) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

Thus,

$$V_m = \sqrt{\frac{1}{50}}\,(15\text{ knots}) = 2.12\text{ knots}$$

From Table A.1 $\quad 1\text{ knot} = \left(0.514\,\frac{m}{s}\right)\left(3.281\,\frac{\frac{ft}{s}}{\frac{m}{s}}\right) = 1.69\,\frac{ft}{s}$

so that

$$V_m = (2.12\text{ knots})\left(1.69\,\frac{ft/s}{\text{knot}}\right) = \underline{\underline{3.58\,\frac{ft}{s}}}$$

(b) Since from Eq. (1)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\frac{1}{50}}$$

so that

$$V = \sqrt{50}\left(0.15\,\frac{ft}{s}\right) = \underline{\underline{1.06\,\frac{ft}{s}}}$$

7.42

7.42 A 1:40 scale model of a ship is to be tested in a towing tank. Determine the required kinematic viscosity of the model fluid so that both the Reynolds number and the Froude number are the same for model and prototype. Assume the prototype fluid to be seawater at 60 °F. Could any of the liquids with viscosities given in Fig. B.2 in Appendix B be used as the model fluid?

As discussed in Section 7.8.3, to maintain both Reynolds number and Froude number similarity

$$\frac{\nu_m}{\nu} = (\lambda_\ell)^{\frac{3}{2}}$$

For the data given

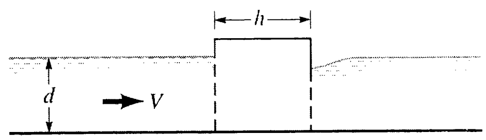
$$\nu_m = \left(\frac{1}{40}\right)^{\frac{3}{2}} \left(1.17 \times 10^{-6} \frac{m^2}{s}\right)$$

$$= \underline{\underline{4.62 \times 10^{-9} \frac{m^2}{s}}}$$

<u>No</u>. The values of ν for the liquids given in Fig. B.2 are all much larger than the required value.

7.43

7.43 A solid block in the shape of a cube rests partially submerged on the bottom of a river as shown in Fig. P7.43. The drag, $\mathcal{D}$, on the block depends on the river depth, d, the block dimension, h, the stream velocity, V, the fluid density, ρ, and the acceleration of gravity, g. **(a)** Perform a dimensional analysis for this problem. **(b)** The drag is to be determined from a model study using a length scale of 1/5. What model velocity should be used to predict the drag on the prototype located in a river with a velocity of 9 ft/s? Water is to be used for the model fluid. Determine the expected prototype drag in terms of the model drag.



(a) $$\mathcal{D} = f(d, h, V, \rho, g)$$

$\mathcal{D} \doteq F \quad d \doteq L \quad h \doteq L \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad g \doteq LT^{-2}$

From the pi theorem, $6-3=3$ pi terms required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 h^2} = \phi\left(\frac{d}{h}, \frac{V}{\sqrt{gd}}\right)$$

(b) For similarity between model and prototype

$$\frac{d_m}{h_m} = \frac{d}{h} \quad \text{and} \quad \frac{V_m}{\sqrt{g_m d_m}} = \frac{V}{\sqrt{gd}}$$

Thus, $$\frac{d_m}{d} = \frac{h_m}{h} = \frac{1}{5}$$

and $$V_m = \sqrt{\frac{g_m}{g}}\sqrt{\frac{d_m}{d}}\,V = \sqrt{\frac{1}{5}}\left(9\,\frac{ft}{s}\right) = 4.02\,\frac{ft}{s}$$

The prediction equation is

$$\frac{\mathcal{D}}{\rho V^2 h^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 h_m^2}$$

so that $$\mathcal{D} = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{h}{h_m}\right)^2 \mathcal{D}_m$$

and with $\rho = \rho_m$

$$\mathcal{D} = \left(\frac{9\,\frac{ft}{s}}{4.02\,\frac{ft}{s}}\right)^2 (5)^2\,\mathcal{D}_m = 125\,\mathcal{D}_m$$

7.44

7.44 The drag on a 2-m-diameter satellite dish due to an 80 km/hr wind is to be determined through a wind tunnel test using a geometrically similar 0.4-m-diameter model dish. Assume standard air for both model and prototype. **(a)** At what air speed should the model test be run? **(b)** With all similarity conditions satisfied, the measured drag on the model was determined to be 170 N. What is the predicted drag on the prototype dish?

(a) From Eq. 7.19, Reynolds number similarity is required. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

where D is the dish diameter. It follows that

$$V_m = \frac{\nu_m}{\nu}\frac{D}{D_m} V$$

and with $\nu_m/\nu = 1$

$$V_m = \left(\frac{2\ m}{0.4\,m}\right)\left(80\ \frac{km}{hr}\right) = \underline{\underline{400\ \frac{km}{hr}}}$$

(b) From Eq. 7.19,

$$\frac{\mathcal{D}_m}{\frac{1}{2}\rho_m V_m^2 D_m^2} = \frac{\mathcal{D}}{\frac{1}{2}\rho V^2 D^2}$$

so that (with $\rho_m = \rho$)

$$\mathcal{D} = \frac{V^2}{V_m^2}\frac{D^2}{D_m^2}\mathcal{D}_m$$

$$= \frac{\left(80\ \frac{km}{hr}\right)^2}{\left(400\ \frac{km}{hr}\right)^2}\frac{(2m)^2}{(0.4m)^2}(170\ N) = \underline{\underline{170\ N}}$$

(Note that $\mathcal{D} = \mathcal{D}_m$ in this problem, since from the condition of Reynolds number similarity, $V^2/V_m^2 = D_m^2/D^2$. This is not true in general.)

7.45

7.45 The pressure drop between the entrance and exit of a 150-mm-diameter 90° elbow, through which ethyl alcohol at 20 °C is flowing, is to be determined with a geometrically similar model. The velocity of the alcohol is 5 m/s. The model fluid is to be water at 20 °C, and the model velocity is limited to 10 m/s. **(a)** What is the required diameter of the model elbow to maintain dynamic similarity? **(b)** A measured pressure drop of 20 kPa in the model will correspond to what prototype value?

For flow in a closed conduit,

$$\text{Dependent pi term} = \phi\left(\frac{\ell_i}{\ell}, \frac{\epsilon}{\ell}, \frac{\rho V \ell}{\mu}\right) \qquad \text{(Eq. 7.16)}$$

For this particular problem the dependent variable is the pressure drop, Δp, so that

$$\text{Dependent pi term} = \frac{\Delta p}{\rho V^2}$$

Also, the characteristic length for flow through a 90° elbow is the diameter, D, so that

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{\epsilon}{D}, \frac{\rho V D}{\mu}\right)$$

(a) To maintain dynamic similarity,

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu}$$

or

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

and therefore

$$D_m = \frac{\nu_m}{\nu}\frac{V}{V_m} D$$

For water at 20°C, $\nu_m = 1.004 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table B.2), and for ethyl alcohol at 20°C, $\nu = 1.51 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table 1.6), so that

$$D_m = \frac{\left(1.004 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}\right)}{\left(1.51 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}\right)} \frac{\left(5\ \frac{\text{m}}{\text{s}}\right)}{\left(10\ \frac{\text{m}}{\text{s}}\right)} (150\ \text{mm})$$

$$= \underline{\underline{49.9\ \text{mm}}}$$

(con't)

(b) With the same Reynolds number for model and prototype, and with geometric similarity (which implies that $\epsilon_m / D_m = \epsilon / D$) then

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{\Delta p}{\rho V^2}$$

so that

$$\Delta p = \frac{\rho}{\rho_m} \frac{V^2}{V_m^2} \Delta p_m$$

For water at 20°C, $\rho_m = 998.2\ kg/m^3$ (Table B.2), and for ethyl alcohol at 20°C, $\rho = 789\ kg/m^3$ (Table 1.6). Thus,

$$\Delta p = \frac{\left(789 \frac{kg}{m^3}\right)}{\left(998.2 \frac{kg}{m^3}\right)} \frac{\left(5 \frac{m}{s}\right)^2}{\left(10 \frac{m}{s}\right)^2} \left(20\ kPa\right) = \underline{\underline{3.95\ kPa}}$$

7.46

7.46 For a certain model study involving a 1:5 scale model it is known that Froude number similarity must be maintained. The possibility of cavitation is also to be investigated, and it is assumed that the cavitation number must be the same for model and prototype. The prototype fluid is water at 30 °C, and the model fluid is water at 70 °C. If the prototype operates at an ambient pressure of 101 kPa (abs), what is the required ambient pressure for the model system?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that (with $g = g_m$)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

For cavitation number similarity,

$$\frac{(p_r - p_v)_m}{\frac{1}{2}\rho_m V_m^2} = \frac{(p_r - p_v)}{\frac{1}{2}\rho V^2}$$

It follows that

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{V_m^2}{V^2}(p_r - p_v)$$

and making use of Eq. (1)

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{\ell_m}{\ell}(p_r - p_v) \qquad (2)$$

For water (from Table B.2):

@ 70°C $\rho_m = 977.8\ kg/m^3$; $p_{v_m} = 3.116 \times 10^4\ N/m^2$ (abs)

@ 30°C $\rho = 995.7\ kg/m^3$; $p_v = 4.243 \times 10^3\ N/m^2$ (abs)

Thus, from Eq. (2)

$$p_{rm} = \frac{\left(977.8\ \frac{kg}{m^3}\right)}{\left(995.7\ \frac{kg}{m^3}\right)}\left(\frac{1}{5}\right)\left(101 \times 10^3\ \frac{N}{m^2} - 4.243 \times 10^3\ \frac{N}{m^2}\right) + 3.116 \times 10^4\ \frac{N}{m^2}$$

$$= \underline{\underline{50.2\ kPa\ (abs)}}$$

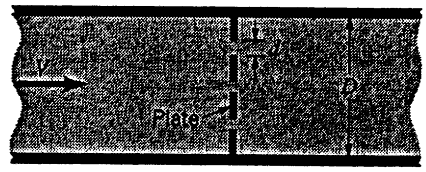
7.47

7.47 As shown in Fig. P7.47, a thin, flat plate containing a series of holes is to be placed in a pipe to filter out any particles in the liquid flowing through the pipe. There is some concern about the large pressure drop that may develop across the plate, and it is proposed to study this problem with a geometrically similar model. The following data apply.

(a) Assuming that the pressure drop, Δp, depends on the variables listed above, use dimensional analysis to develop a suitable set of dimensionless parameters for this problem.

(b) Determine values for the model indicated in the list above with a question mark. What will be the pressure drop scale, $\Delta p_m/\Delta p$?

Prototype	Model
d—hole diameter = 1.0 mm	d = ?
D—pipe diameter = 50 mm	D = 10 mm
μ—viscosity = 0.002 $N \cdot s/m^2$	μ = 0.002 $N \cdot s/m^2$
ρ—density = 1000 kg/m^3	ρ = 1000 kg/m^3
V—velocity = 0.1 m/s to 2 m/s	V = ?



■ FIGURE P7.47

(a) $$\Delta p = f(d, D, \mu, \rho, V)$$

$\Delta p \doteq FL^{-2}$ $d \doteq L$ $D \doteq L$ $\mu \doteq FL^{-2}T$ $\rho \doteq FL^{-4}T^2$ $V \doteq LT^{-1}$

From the pi theorem, 6-3= 3 pi terms required, and a dimensional analysis yields

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{d}{D}, \frac{\rho V D}{\mu}\right)$$

(b) For similarity,

$$\frac{d_m}{D_m} = \frac{d}{D}$$

and with the data given

$$d_m = \frac{D_m}{D} d = \left(\frac{10\,mm}{50\,mm}\right)(1.0\,mm) = \underline{\underline{0.200\,mm}}$$

Also,

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu}$$

and with $\rho_m = \rho$, $\mu_m = \mu$ it follows that

$$V_m = \frac{D}{D_m} V = \left(\frac{50\,mm}{10\,mm}\right) V = 5\,V$$

$$= 5\left(0.1\,\frac{m}{s} \text{ to } 2\,\frac{m}{s}\right) = \underline{\underline{0.500\,\frac{m}{s} \text{ to } 10.0\,\frac{m}{s}}}$$

With the similarity requirements satisfied, the prediction equation is

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{\Delta p}{\rho V^2}$$

so that

$$\frac{\Delta p_m}{\Delta p} = \left(\frac{\rho_m}{\rho}\right)\left(\frac{V_m}{V}\right)^2 = (1)(5)^2 = \underline{\underline{25.0}}$$

7.48

7.48 At a large fish hatchery the fish are reared in open, water-filled tanks. Each tank is approximately square in shape with curved corners, and the walls are smooth. To create motion in the tanks, water is supplied through a pipe at the edge of the tank. The water is drained from the tank through an opening at the center. (See Video V7.3.) A model with a length scale of 1:13 is to be used to determine the velocity, V, at various locations within the tank. Assume that $V = f(\ell, \ell_i, \rho, \mu, g, Q)$ where ℓ is some characteristic length such as the tank width, ℓ_i represents a series of other pertinent lengths, such as inlet pipe diameter, fluid depth, etc., ρ is the fluid density, μ is the fluid viscosity, g is the acceleration of gravity, and Q is the discharge through the tank. **(a)** Determine a suitable set of dimensionless parameters for this problem and the prediction equation for the velocity. If water is to be used for the model, can all of the similarity requirements be satisfied? Explain and support your answer with the necessary calculations. **(b)** If the flowrate into the full-sized tank is 250 gpm, determine the required value for the model discharge assuming Froude number similarity. What model depth will correspond to a depth of 32 in. in the full-sized tank?

(a) $$V = f(\ell, \ell_i, \rho, \mu, g, Q)$$

From the pi theorem, 7−3 = 4 pi terms required and a dimensional analysis yields

$$\frac{V\ell^2}{Q} = \phi\left(\frac{\ell_i}{\ell}, \frac{Q^2}{\ell^5 g}, \frac{\rho Q}{\ell \mu}\right)$$

Thus, the similarity requirements are

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g} \qquad \frac{\rho_m Q_m}{\ell_m \mu_m} = \frac{\rho Q}{\ell \mu}$$

and the prediction equation is

$$\frac{V\ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

From the last similarity requirement with $\rho_m = \rho$ and $\mu_m = \mu$

$$\frac{Q_m}{Q} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{\ell_m}{\ell} = \frac{\ell_m}{\ell}$$

However, from the second similarity requirement with $g_m = g$

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

Since these two requirements are in conflict it follows that the similarity requirements cannot be satisfied. No.

(con't)

7.48 (Con't)

(b) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Thus, from the prediction equation

$$\frac{V \ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

it follows that

$$\frac{Q_m}{Q} = \frac{V_m}{V}\left(\frac{\ell_m}{\ell}\right)^2 = \sqrt{\frac{\ell_m}{\ell}}\left(\frac{\ell_m}{\ell}\right)^2 = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

so that with $\ell_m/\ell = 1/13$

$$Q_m = \left(\frac{1}{13}\right)^{5/2}(250 \text{ gpm}) = \underline{\underline{0.410 \text{ gpm}}}$$

Note that this same result can be obtained from the second similarity requirement (which corresponds to Froude number similarity) since

$$\frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g}$$

and therefore

$$Q_m = \left(\frac{\ell_m}{\ell}\right)^{5/2} Q$$

Geometric similarity requires that

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell}$$

or

$$\frac{\ell_{im}}{\ell_i} = \frac{\ell_m}{\ell} = \frac{1}{13}$$

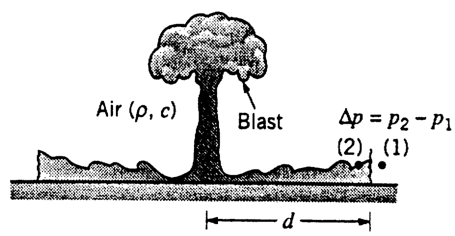
so that all lengths scale as the length scale. Thus,

$$(\text{depth})_{\text{model}} = \left(\frac{1}{13}\right)(\text{depth})_{\text{prototype}}$$

$$= \left(\frac{1}{13}\right)(32 \text{ in.}) = \underline{\underline{2.46 \text{ in.}}}$$

7.49

7.49 The pressure rise, Δp, across a blast wave, as shown in Fig. P7.49 and Video V11.5, is assumed to be a function of the amount of energy released in the explosion, E, the air density, ρ, the speed of sound, c, and the distance from the blast, d. (a) Put this relationship in dimensionless form. (b) Consider two blasts: the prototype blast with energy release E and a model blast with 1/1000th the energy release ($E_m = 0.001\ E$). At what distance from the model blast will the pressure rise be the same as that at a distance of 1 mile from the prototype blast?



■ FIGURE P7.49

(a) $$\Delta p = f(E, \rho, c, d)$$

$\Delta p \doteq FL^{-2}$ $\quad E \doteq FL$ $\quad \rho \doteq FL^{-4}T^2$ $\quad c \doteq LT^{-1}$ $\quad d \doteq L$

From the pi theorem, $5-3=2$ pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{\Delta p}{\rho c^2} = \phi\left(\frac{E}{\rho c^2 d^3}\right)}}$$

(b) For similarity,

$$\frac{E_m}{\rho_m c_m^2 d_m^3} = \frac{E}{\rho c^2 d^3}$$

and with $\rho_m = \rho$, $c_m = c$, it follows that

$$d_m^3 = \frac{E_m}{E} d^3$$

For $E_m/E = 0.001$ and $d = 1\ mi$

$$d_m^3 = (0.001)(1\ mi)^3$$

$$d_m = 0.100\ mi$$

With this similarity requirement satisfied, the prediction equation is

$$\frac{\Delta p_m}{\rho_m c_m^2} = \frac{\Delta p}{\rho c^2}$$

and therefore

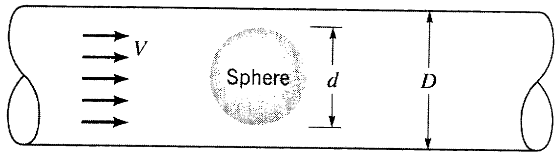
$$\Delta p_m = \Delta p$$

at

$$\underline{\underline{d_m = 0.100\ mi}}$$

7.50

7.50 The drag, $\mathscr{D}$, on a sphere located in a pipe through which a fluid is flowing is to be determined experimentally (see Fig. P7.50). Assume that the drag is a function of the sphere diameter, d, the pipe diameter, D, the fluid velocity, V, and the fluid density, ρ. **(a)** What dimensionless parameters would you use for this problem? **(b)** Some experiments using water indicate that for $d = 0.2$ in., $D = 0.5$ in., and $V = 2$ ft/s, the drag is 1.5×10^{-3} lb. If possible, estimate the drag on a sphere located in a 2-ft-diameter pipe through which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained. If it is not possible, explain why not.



(a)

$$\mathscr{D} = f(d, D, V, \rho)$$

$$\mathscr{D} \doteq F \quad d \doteq L \quad D \doteq L \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2}$$

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\mathscr{D}}{\rho V^2 D^2} = \phi\left(\frac{d}{D}\right)$$

(b) The similarity requirement is

$$\frac{d_m}{D_m} = \frac{d}{D}$$

so that

$$\frac{0.2\text{ in.}}{0.5\text{ in.}} = \frac{d\ (\text{ft})}{2\text{ ft}}$$

and $d = 0.8$ ft (required diameter).

Thus, the prediction equation is

$$\frac{\mathscr{D}}{\rho V^2 D^2} = \frac{\mathscr{D}_m}{\rho_m V_m^2 D_m^2}$$

so that

$$\mathscr{D} = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathscr{D}_m \quad (\text{and with } \rho = \rho_m)$$

$$\mathscr{D} = \left(\frac{6\ \frac{ft}{s}}{2\ \frac{ft}{s}}\right)^2\left(\frac{2\text{ ft}}{0.5/12\text{ ft}}\right)^2(1.5\times10^{-3}\text{ lb}) = \underline{\underline{31.1\text{ lb}}}$$

7.51

7.51 Flow patterns that develop as winds blow past a vehicle, such as a train, are often studied in low-speed environmental (meteorological) wind tunnels. (See **Video V7.5**.) Typically, the air velocities in these tunnels are in the range of 0.1 m/s to 30 m/s. Consider a cross wind blowing past a train locomotive. Assume that the local wind velocity, V, is a function of the approaching wind velocity (at some distance from the locomotive), U, the locomotive length, ℓ, height, h, and width, b, the air density, ρ, and the air viscosity, μ. **(a)** Establish the similarity requirements and prediction equation for a model to be used in the wind tunnel to study the air velocity, V, around the locomotive. **(b)** If the model is to be used for cross winds gusting to U = 25 m/s, explain why it is not practical to maintain Reynolds number similarity for a typical length scale 1:50.

(a) $$V = f(U, \ell, h, b, \rho, \mu)$$

$V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\ell \doteq L$ $h \doteq L$ $b \doteq L$ $\rho \doteq FL^{-4}T^{2}$ $\mu \doteq FL^{-2}T$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{V}{U} = \phi\left(\frac{\ell}{h}, \frac{b}{h}, \frac{\rho h U}{\mu}\right)$$

Thus, the similarity requirements are

$$\underline{\underline{\frac{\ell_m}{h_m} = \frac{\ell}{h}}} \qquad \underline{\underline{\frac{b_m}{h_m} = \frac{b}{h}}} \qquad \underline{\underline{\frac{\rho_m h_m U_m}{\mu_m} = \frac{\rho h U}{\mu}}}$$

The prediction equation is

$$\underline{\underline{\frac{V}{U} = \frac{V_m}{U_m}}}$$

(b) Since the density and viscosity of the air flowing around the train and the air in the wind tunnel would be practically the same ($\rho_m \simeq \rho$, $\mu_m \simeq \mu$), it follows from the last similarity requirement (which is the Reynolds number) that

$$U_m = \left(\frac{h}{h_m}\right) U$$

Thus, with a length scale of 1:50 and with $U = 25\,m/s$

$$U_m = (50)(25\,m/s) = 1{,}250\,m/s$$

This required model velocity is much higher than can be acheived in the wind tunnel and therefore it is not practical to maintain Reynolds number similarity. <u>The required model velocity is too high.</u>

7.52

7.52 An orifice flowmeter uses a pressure drop measurement to determine the flowrate through a pipe. A particular orifice flowmeter, when tested in the laboratory, yielded a pressure drop of 8 psi for a flow of 2.9 ft^3/s through a 6-in. pipe. For a geometrically similar system using the same fluid with a 24-in. pipe, what is the required flow if similarity between the two systems is to be maintained? What is the corresponding pressure drop?

Assume $\quad Q = f(\Delta p, d, D, \rho, \mu)$

where: $Q \sim$ flowrate $\doteq L^3T^{-1}$, $\Delta p \sim$ pressure drop $\doteq FL^{-2}$, $d \sim$ orifice diameter $\doteq L$, $D \sim$ pipe diameter $\doteq L$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, and $\mu \sim$ fluid viscosity $\doteq FL^{-2}T$.

From the pi theorem, $6-3=3$ pi terms required, and a dimensional analysis yields

$$\frac{\rho Q}{\mu D} = \phi\left(\frac{d}{D}, \frac{\Delta p D^2 \rho}{\mu^2}\right)$$

For similarity $\quad \frac{d_m}{D_m} = \frac{d}{D} \quad$ and $\quad \frac{d_m}{d} = \frac{D_m}{D} = \frac{6\text{ in.}}{24\text{ in.}} = \frac{1}{4}$

and

$$\frac{\Delta p_m D_m^2 \rho_m}{\mu_m^2} = \frac{\Delta p D^2 \rho}{\mu^2}$$

so that

$$\Delta p = \left(\frac{\mu}{\mu_m}\right)^2\left(\frac{D_m}{D}\right)^2\left(\frac{\rho_m}{\rho}\right)\Delta p_m$$

and with $\mu = \mu_m$, $\rho = \rho_m$

$$\Delta p = \left(\frac{D_m}{D}\right)^2 \Delta p_m = \left(\frac{1}{4}\right)^2(8\text{ psi}) = \underline{\underline{0.500\text{ psi}}}$$

Also,

$$\frac{\rho_m Q_m}{\mu_m D_m} = \frac{\rho Q}{\mu D}$$

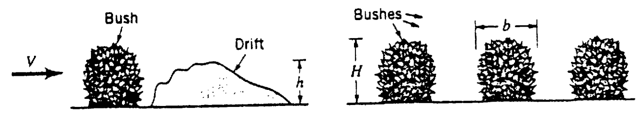
or

$$Q = \left(\frac{\mu}{\mu_m}\right)\left(\frac{\rho_m}{\rho}\right)\left(\frac{D}{D_m}\right)Q_m$$

$$= \left(\frac{D}{D_m}\right)Q_m = (4)\left(2.9\,\frac{ft^3}{s}\right) = \underline{\underline{11.6\,\frac{ft^3}{s}}}$$

7.53

7.53 During a storm, a snow drift is formed behind some bushes as shown in Fig. P7.53 and Video V9.4. Assume that the height of the drift, h, is a function of the number of inches of snow deposited by the storm, d, the height of the bush, H, the width of the bush, b, the wind speed, V, the acceleration of gravity, g, the air density, ρ, the specific weight of the snow, γ_s, and the porosity of the bush, η. Note that porosity is defined as percent open area of the bush. **(a)** Determine a suitable set of dimensionless variables for this problem. **(b)** A storm with 30 mph winds deposits 16 in. of snow having a specific weight of 5.0 lb/ft³. A half-sized scale model bush is to be used to investigate the drifting behind the bush. If the air density is the same for the model and the storm, determine the required specific weight of the model snow, the required wind speed for the model, and the number of inches of model snow to be deposited.



■ FIGURE P7.53

(a) $$h = f(d, H, b, V, g, \rho, \gamma_s, \eta)$$

$$h \doteq L \quad d \doteq L \quad H \doteq L \quad b \doteq L \quad V \doteq LT^{-1} \quad g \doteq LT^{-2}$$

$$\rho \doteq FL^{-4}T^{2} \quad \gamma_s \doteq FL^{-3} \quad \eta = F^0L^0T^0$$

From the pi theorem, $9-3=6$ pi terms required, and a dimensional analysis yields

$$\frac{h}{H} = \phi\left(\frac{d}{H}, \frac{b}{H}, \frac{\rho g}{\gamma_s}, \frac{V}{\sqrt{gH}}, \eta\right)$$

(b) Thus, for similarity between the model and prototype

$$\frac{\rho_m g_m}{\gamma_{sm}} = \frac{\rho g}{\gamma_s}$$

and for $\rho_m = \rho$ and $g_m = g$

$$\gamma_{sm} = \gamma_s = \underline{\underline{5.00 \frac{lb}{ft^3}}}$$

Also,

$$\frac{V_m}{\sqrt{g_m H_m}} = \frac{V}{\sqrt{gH}}$$

so that with $g_m = g$ and $H_m/H = \frac{1}{2}$

$$V_m = \sqrt{\frac{H_m}{H}}\, V = \sqrt{\left(\frac{1}{2}\right)}\,(30\text{ mph}) = \underline{\underline{21.2\text{ mph}}}$$

and

$$\frac{d_m}{H_m} = \frac{d}{H}$$

$$d_m = \left(\frac{H_m}{H}\right) d = \left(\frac{1}{2}\right)(16\text{ in.}) = \underline{\underline{8.00\text{ in.}}}$$

7.54

7.54 As illustrated in **Video V7.2**, models are commonly used to study the dispersion of a gaseous pollutant from an exhaust stack located near a building complex. Similarity requirements for the pollutant source involve the following independent variables: the stack gas speed, V, the wind speed, U, the density of the atmospheric air, ρ, the difference in densities between the air and the stack gas, $\rho - \rho_s$, the acceleration of gravity, g, the kinematic viscosity of the stack gas, ν_s, and the stack diameter, D. **(a)** Based on these variables, determine a suitable set of similarity requirements for modeling the pollutant source. **(b)** For this type of model a typical length scale might be 1:200. If the same fluids were used in model and prototype, would the similarity requirements be satisfied? Explain and support your answer with the necessary calculations.

(a) Since $V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^{2}$ $\rho - \rho_s \doteq FL^{-4}T^{2}$ $g \doteq LT^{-2}$ $\nu_s \doteq L^2T^{-1}$ $D \doteq L$, it follows from the pi theorem that $7-3=4$ pi terms are required. A dimensional analysis yields $\frac{V}{U}$, $\frac{VD}{\nu_s}$, $\frac{V^2}{gD}$, and $\frac{\rho-\rho_s}{\rho}$ as a possible set of pi terms. Thus, the similarity requirements would be:

$$\underline{\underline{\frac{V_m}{U_m} = \frac{V}{U}}} \qquad \underline{\underline{\frac{V_m D_m}{\nu_{sm}} = \frac{VD}{\nu_s}}} \qquad \underline{\underline{\frac{V_m^2}{g_m D_m} = \frac{V^2}{gD}}} \qquad \underline{\underline{\frac{(\rho-\rho_s)_m}{\rho_m} = \frac{(\rho-\rho_s)}{\rho}}}$$

(b) For $\frac{D_m}{D} = \frac{1}{200}$ and $\nu_{sm} = \nu_s$ the second similarity requirement is

$$\frac{V_m}{V} = \frac{\nu_{sm}}{\nu_s}\frac{D}{D_m} = 200 \quad \text{(see above)}$$

However, from the third similarity requirement with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{D_m}{D}} = \sqrt{\frac{1}{200}}$$

This result conflicts with that from the second similarity requirement, and therefore the similarity requirements cannot be satisfied under the stated conditions. <u>No</u>.

7.55

7.55 The drag on a small, completely submerged solid body having a characteristic length of 2.5 mm and moving with a velocity of 10 m/s through water is to be determined with the aid of a model. The length scale is to be 50, which indicates that the model is to be *larger* than the prototype. Investigate the possibility of using either an unpressurized wind tunnel or a water tunnel for this study. Determine the required velocity in both the wind and water tunnels, and the relationship between the model drag and the prototype drag for both systems. Would either type of test facility be suitable for this study?

As demonstrated in Eq. 7.19, for flow around immersed bodies, Reynolds number similarity is required so that

$$\frac{V_m \ell_m}{\nu_m} = \frac{V\ell}{\nu}$$

or

$$V_m = \frac{\nu_m}{\nu}\frac{\ell}{\ell_m}V$$

If model tests are run in unpressurized wind tunnel, then ν_m (standard air) $= 1.46 \times 10^{-5}\ m^2/s$, and ν (water) $= 1.12 \times 10^{-6}\ m^2/s$, so that

$$V_m = \frac{\left(1.46 \times 10^{-5}\ \frac{m^2}{s}\right)}{\left(1.12 \times 10^{-6}\ \frac{m^2}{s}\right)}\left(\frac{1}{50}\right)\left(10\ \frac{m}{s}\right) = \underline{\underline{2.61\ \frac{m}{s}}} \quad \text{(for wind tunnel)}$$

If model tests are run in water tunnel with $\nu_m = \nu$, then

$$V_m = (1)\left(\frac{1}{50}\right)\left(10\ \frac{m}{s}\right) = \underline{\underline{0.200\ \frac{m}{s}}} \quad \text{(for water tunnel)}$$

Since V_m is reasonable in both cases, either the wind tunnel or the water tunnel could be used.

With geometric and dynamic similarity, it follows that

$$\frac{\mathcal{D}}{\rho V^2 \ell^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 \ell_m^2}$$

or

$$\frac{\mathcal{D}}{\mathcal{D}_m} = \frac{\rho}{\rho_m}\frac{V^2}{V_m^2}\frac{\ell^2}{\ell_m^2}$$

Thus, for wind tunnel tests

$$\frac{\mathcal{D}}{\mathcal{D}_m} = \frac{\left(999\ \frac{kg}{m^3}\right)\left(10\ \frac{m}{s}\right)^2}{\left(1.23\ \frac{kg}{m^3}\right)\left(3.91\ \frac{m}{s}\right)^2}\left(\frac{1}{50}\right)^2 = \underline{\underline{2.13}} \quad \text{(for wind tunnel)}$$

and for water tunnel tests

$$\frac{\mathcal{D}}{\mathcal{D}_m} = (1.0)\frac{\left(10\ \frac{m}{s}\right)^2}{\left(0.300\ \frac{m}{s}\right)^2}\left(\frac{1}{50}\right)^2 = \underline{\underline{0.444}} \quad \text{(for water tunnel)}$$

7.56

7.56 The drag characteristics for a newly designed automobile having a maximum characteristic length of 20 ft are to be determined through a model study. The characteristics at both low speed (approximately 20 mph) and high speed (90 mph) are of interest. For a series of projected model tests an unpressurized wind tunnel that will accommodate a model with a maximum characteristic length of 4 ft is to be used. Determine the range of air velocities that would be required for the wind tunnel if Reynolds number similarity is desired. Are the velocities suitable? Explain.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{\ell}{\ell_m} V \qquad (1)$$

Since the wind tunnel is unpressurized, the air properties will be approximately the same for model and prototype. Thus, Eq. (1) reduces to

$$V_m = \frac{\ell}{\ell_m} V$$

and for the data given

$$V_m = \frac{(20\,ft)}{(4\,ft)} V = 5V$$

Therefore, at low speed

$$V_m = 5\,(20\,mph) = 100\,mph$$

and at high speed

$$V_m = 5\,(90\,mph) = 450\,mph$$

so that the model velocity range is $\underline{\underline{100\,mph \text{ to } 450\,mph}}$.

At the high velocity in the wind tunnel, compressibility of the air would start to become an important factor, whereas compressibility is not important for the prototype. Thus, the higher velocity required for the model would not be suitable. No.

7.57

7.57 If the unpressurized wind tunnel of Problem 7.56 were replaced with a tunnel in which the air can be pressurized isothermally to 8 atm (abs), what range of air velocities would be required to maintain Reynolds number similarity for the same prototype velocities given in Problem 7.56? For the pressurized tunnel the maximum characteristic model length that can be accommodated is 2 ft, whereas the maximum characteristic prototype length remains at 20 ft.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{\ell}{\ell_m}V \qquad (1)$$

For an ideal gas, $p = \rho R T$, and for isothermal compression

$$\frac{p}{\rho} = \text{constant}$$

Thus,

$$\frac{p_m}{\rho_m} = \frac{p}{\rho}$$

or

$$\frac{\rho}{\rho_m} = \frac{p}{p_m}$$

From Eq. (1) (assuming $\mu_m = \mu$)

$$V_m = \frac{p}{p_m}\frac{\ell}{\ell_m}V$$

where p is atmospheric pressure (pressure at which prototype operates), and p_m is pressure of compressed air in the wind tunnel.

For $p_m = 8p$

$$V_m = \left(\frac{1}{8}\right)\frac{(20\text{ ft})}{(2\text{ ft})}V = 1.25\,V$$

Thus, at low speed

$$V_m = 1.25\,(20\text{ mph}) = 25\text{ mph}$$

and at high speed

$$V_m = 1.25\,(90\text{ mph}) = 112.5\text{ mph}$$

Therefore, the required model velocity range is

25 mph to 112.5 mph.

7.58

7.58 The drag characteristics of an airplane are to be determined by model tests in a wind tunnel operated at an absolute pressure of 1300 kPa. If the prototype is to cruise in standard air at 385 km/hr, and the corresponding speed of the model is not to differ by more than 20% from this (so that compressibility effects may be ignored), what range of length scales may be used if Reynolds number similarity is to be maintained? Assume the viscosity of air is unaffected by pressure, and the temperature of the air in the tunnel is equal to the temperature of the air in which the airplane will fly.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$\frac{\ell_m}{\ell} = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{V}{V_m} \qquad (1)$$

For an ideal gas, $p = \rho R T$, and with constant temperature,

$$\frac{p}{\rho} = \text{constant}$$

or

$$\frac{p}{p_m} = \frac{\rho}{\rho_m}$$

and Eq. (1) can be written as (with $\mu_m = \mu$)

$$\frac{\ell_m}{\ell} = \frac{p}{p_m}\frac{V}{V_m}$$

For the data given

$$\frac{\ell_m}{\ell} = \frac{(101\ kPa)}{(1300\ kPa)}\frac{V}{V_m}$$

and with $V_m = (1 \pm 0.2)V$, it follows that

$$\frac{\ell_m}{\ell} = \frac{(101\ kPa)}{(1300\ kPa)}\frac{1}{(1 \pm 0.2)}$$

Thus, the range of length scales is $\underline{\underline{0.0647 \text{ to } 0.0971}}$.

7.59

7.59 Wind blowing past a flag causes it to "flutter in the breeze." The frequency of this fluttering, ω, is assumed to be a function of the wind speed, V, the air density, ρ, the acceleration of gravity, g, the length of the flag, ℓ, and the "area density," ρ_A, (with dimensions of ML^{-2}) of the flag material. It is desired to predict the flutter frequency of a large $\ell = 40$ ft flag in a $V = 30$ ft/s wind. To do this a model flag with $\ell = 4$ ft is to be tested in a wind tunnel. **(a)** Determine the required area density of the model flag material if the large flag has $\rho_A = 0.006$ slugs/ft^2. **(b)** What wind tunnel velocity is required for testing the model? **(c)** If the model flag flutters at 6 Hz, predict the frequency for the large flag.

$$\omega = f(V, \rho, g, \ell, \rho_A)$$

$$\omega \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq ML^{-3} \quad g \doteq LT^{-2} \quad \ell \doteq L \quad \rho_A \doteq ML^{-2}$$

From the pi Theorem, 6-3 = 3 pi terms required, and a dimensional analysis yields

$$\omega\sqrt{\frac{\ell}{g}} = \phi\left(\frac{V}{\sqrt{g\ell}}, \frac{\rho_A}{\rho\ell}\right)$$

(a) For similarity

$$\frac{\rho_{Am}}{\rho_m \ell_m} = \frac{\rho_A}{\rho\ell}$$

and since $\rho_m = \rho$

$$\rho_{Am} = \frac{\ell_m}{\ell}\rho_A = \left(\frac{4\text{ ft}}{40\text{ ft}}\right)\left(0.006\ \frac{\text{slugs}}{\text{ft}^2}\right) = \underline{\underline{0.0006\ \frac{\text{slugs}}{\text{ft}^2}}}$$

(b) For similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

and with $g_m = g$

$$V_m = \sqrt{\frac{\ell_m}{\ell}}\,V = \sqrt{\frac{4\text{ ft}}{40\text{ ft}}}\left(30\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{9.49\ \frac{\text{ft}}{\text{s}}}}$$

(c) With the similarity requirements satisfied the prediction equation is

$$\omega\sqrt{\frac{\ell}{g}} = \omega_m\sqrt{\frac{\ell_m}{g_m}}$$

so that

$$\omega = \sqrt{\frac{g}{g_m}}\sqrt{\frac{\ell_m}{\ell}}\,\omega_m = \sqrt{\frac{4\text{ ft}}{40\text{ ft}}}\,(6\text{ Hz}) = \underline{\underline{1.90\text{ Hz}}}$$

7.60 River models are used to study many different types of flow situations. (See, for example, **Video V7.6.**) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flowrate of 700 ft^3/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flowrate would the model operate?

For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{VA} = \sqrt{\frac{\ell_m}{\ell}} \frac{A_m}{A}$$

Also,

$$\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2$$

so that

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \qquad (1)$$

Thus,

$$\frac{\ell_m}{\ell} = 0.110$$

and for a prototype depth of 4 ft the corresponding model depth is

$$\ell_m = (0.110)(4\,ft) = \underline{\underline{0.440\,ft}}$$

The model flowrate is obtained from Eq. (1):

$$Q_m = \left(\frac{1}{250}\right)\left(700 \frac{ft^3}{s}\right) = \underline{\underline{2.80 \frac{ft^3}{s}}}$$

7.61

7.61 As winds blow past buildings, complex flow patterns can develop due to various factors such as flow separation and interactions between adjacent buildings. (See Video V7.4.) Assume that the local gage pressure, p, at a particular location on a building is a function of the air density, ρ, the wind speed, V, some characteristic length, ℓ, and all other pertinent lengths, ℓ_i, needed to characterize the geometry of the building or building complex. **(a)** Determine a suitable set of dimensionless parameters that can be used to study the pressure distribution. **(b)** An eight-story building that is 100 ft tall is to be modeled in a wind tunnel. If a length scale of 1:300 is to be used, how tall should the model building be? **(c)** How will a measured pressure in the model be related to the corresponding prototype pressure? Assume the same air density in model and prototype. Based on the assumed variables, does the model wind speed have to be equal to the prototype wind speed? Explain.

(a)
$$p = f(\rho, V, \ell, \ell_i)$$

$$p \doteq FL^{-2} \qquad \rho \doteq FL^{-4}T^2 \qquad V \doteq LT^{-1} \qquad \ell \doteq L \qquad \ell_i \doteq L$$

From the pi theorem, 5-3=2 pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{p}{\rho V^2} = \phi\left(\frac{\ell}{\ell_i}\right)}}$$

(b) For geometric similarity

$$\frac{\ell_m}{\ell_{im}} = \frac{\ell}{\ell_i}$$

So that

$$\frac{\ell_m}{\ell} = \frac{\ell_{im}}{\ell_i}$$

and it follows that all pertinent lengths are scaled with the length scale ℓ_m/ℓ. Thus, with $\ell_m/\ell = 1/300$

$$\text{model height} = \frac{100\ \text{ft}}{300} = \underline{\underline{0.333\ \text{ft}}}$$

(c) With geometric similarity satisfied it follows that

$$\frac{p}{\rho V^2} = \frac{p_m}{\rho_m V_m^2}$$

Thus, with $\rho_m = \rho$

$$\underline{\underline{p = \left(\frac{V}{V_m}\right)^2 p_m}}$$

With the set of given variables there is no requirement for the velocity scale, V_m/V, so the model wind speed does not have to be equal tho the prototype wind speed. <u>No.</u>

7.62

7.62 A $\frac{1}{50}$ scale model is to be used in a towing tank to determine the drag on the hull of a ship. The model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype ship is designed to cruise at 18 knots. At what velocity (in m/s) should the model be towed? Under these conditions what will be the ratio of the prototype drag to the model drag? Assume the water in the towing tank to have the same properties as those for the prototype and that shear drag is negligible.

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

and with $g = g_m$

$$V_m = \sqrt{\frac{\ell_m}{\ell}}\, V = \sqrt{\frac{1}{50}}\,(18 \text{ knots})\left(0.5144 \,\frac{m/s}{\text{knot}}\right) = \underline{\underline{1.31 \,\frac{m}{s}}}$$

With Froude number similarity and geometric similarity, then

$$\frac{\mathcal{D}}{\rho V^2 \ell^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 \ell_m^2}$$

where $\mathcal{D}$ is the drag. Thus,

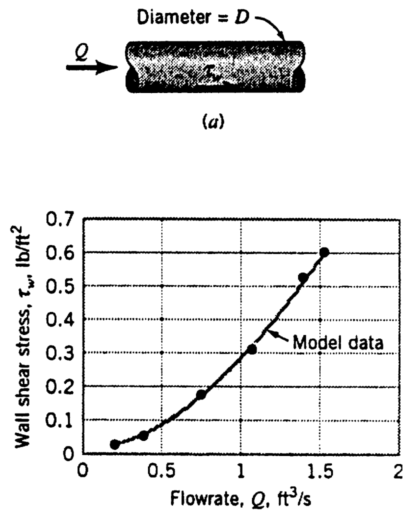
$$\frac{\mathcal{D}}{\mathcal{D}_m} = \frac{\rho}{\rho_m}\,\frac{V^2}{V_m^2}\,\frac{\ell^2}{\ell_m^2}$$

Since $\rho = \rho_m$ and $V/V_m = \sqrt{\ell/\ell_m}$, it follows that

$$\frac{\mathcal{D}}{\mathcal{D}_m} = (1)\left(\frac{\ell}{\ell_m}\right)^3 = (1)(50)^3 = \underline{\underline{1.25 \times 10^5}}$$

7.64

7.64 Assume that the wall shear stress, τ_w, created when a fluid flows through a pipe (see Fig. P7.64*a*) depends on the pipe diameter, D, the flowrate, Q, the fluid density, ρ, and the kinematic viscosity, ν. Some model tests run in a laboratory using water in a 0.2-ft-diameter pipe yield the τ_w vs. Q data shown in Fig. 7.64*b*. Perform a dimensional analysis and use the model data to predict the wall shear stress in a 0.3-ft-diameter pipe through which water flows at the rate of 1.5 ft³/s.



(*a*)

(*b*)

■ FIGURE P7.64

$$\tau_w = f(D, Q, \rho, \nu)$$

$\tau_w \doteq FL^{-2}$ $D \doteq L$ $Q \doteq L^3T^{-1}$ $\rho \doteq FL^{-4}T^2$ $\nu \doteq L^2T^{-1}$

From the pi theorem, $5-3=2$ pi terms required, and a dimensional analysis yields

$$\frac{\tau_w D^4}{\rho Q^2} = \phi\left(\frac{Q}{D\nu}\right)$$

Thus, the similarity requirement is

$$\frac{Q_m}{D_m \nu_m} = \frac{Q}{D\nu}$$

so that with $\nu_m = \nu$ and $Q = 1.5\ \frac{ft^3}{s}$

$$Q_m = \left(\frac{D_m}{D}\right)\left(\frac{\nu_m}{\nu}\right) Q = \left(\frac{0.2\,ft}{0.3\,ft}\right)(1)\left(1.5\ \frac{ft^3}{s}\right)$$

$$= 1.00\ \frac{ft^3}{s}$$

From the graph (Fig. P7.64b), for $Q_m = 1.00\ \frac{ft^3}{s}$, $\tau_m \doteq 0.29\ \frac{lb}{ft^2}$.

Thus,
$$\frac{\tau_{wm} D_m^4}{\rho_m Q_m^2} = \frac{\tau_w D^4}{\rho Q^2}$$

and
$$\tau_w = \left(\frac{\rho}{\rho_m}\right)\left(\frac{Q}{Q_m}\right)^2\left(\frac{D_m}{D}\right)^4 \tau_{wm}$$

$$= (1)\left(\frac{1.5\ \frac{ft^3}{s}}{1.0\ \frac{ft^3}{s}}\right)^2\left(\frac{0.2\,ft}{0.3\,ft}\right)^4\left(0.29\ \frac{lb}{ft^2}\right)$$

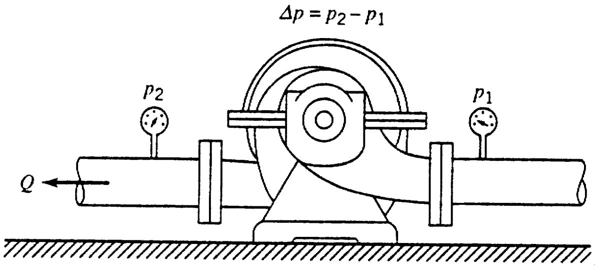
$$= \underline{\underline{0.129\ \frac{lb}{ft^2}}}$$

7.65

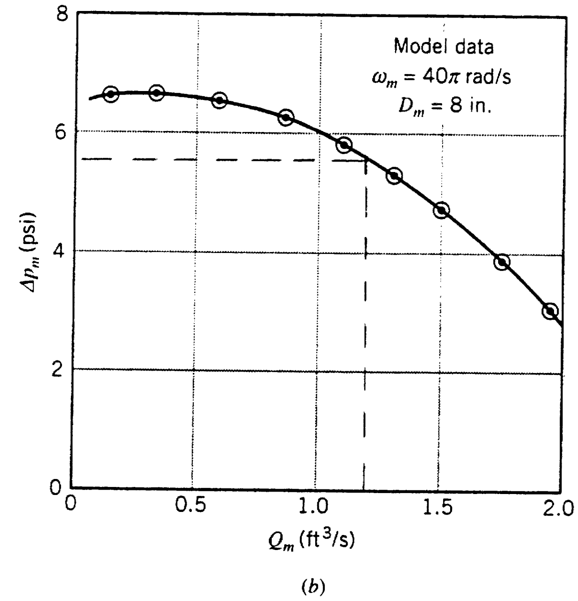
7.65 The pressure rise, Δp, across a centrifugal pump of a given shape (see Fig. P7.65a) can be expressed as

$$\Delta p = f(D, \omega, \rho, Q)$$

where D is the impeller diameter, ω the angular velocity of the impeller, ρ the fluid density, and Q the volume rate of flow through the pump. A model pump having a diameter of 8 in. is tested in the laboratory using water. When operated at an angular velocity of 40π rad/s the model pressure rise as a function of Q is shown in Fig. P7.65b. Use this curve to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flowrate of 6 ft³/s. The prototype has a diameter of 12 in. and operates at an angular velocity of 60π rad/s. The prototype fluid is also water.



(a)



(b)

■ FIGURE P7.65

$$\Delta p = f(D, \omega, \rho, Q)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \omega \doteq T^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\Delta p}{\rho \omega^2 D^2} = \phi\left(\frac{Q}{\omega D^3}\right)$$

Thus, the similarity requirement is

$$\frac{Q_m}{\omega_m D_m^3} = \frac{Q}{\omega D^3}$$

so that

$$Q_m = \left(\frac{\omega_m}{\omega}\right)\left(\frac{D_m}{D}\right)^3 Q$$

and for the data given

$$Q_m = \frac{(40\pi \frac{rad}{s})}{(60\pi \frac{rad}{s})}\left(\frac{8 in.}{12 in.}\right)^3\left(6\frac{ft^3}{s}\right) = 1.19\frac{ft^3}{s}$$

From the graph (Fig. P7.65b) $\Delta p_m = 5.50$ psi for $Q_m = 1.19\frac{ft^3}{s}$. Thus,

$$\frac{\Delta p}{\rho \omega^2 D^2} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2}$$

and with $\rho = \rho_m$

$$\Delta p = \left(\frac{\omega}{\omega_m}\right)^2\left(\frac{D}{D_m}\right)^2 \Delta p_m = \frac{(60\pi \frac{rad}{s})^2}{(40\pi \frac{rad}{s})^2}\left(\frac{12 in.}{8 in.}\right)^2 (5.50 \text{ psi})$$

$$= \underline{\underline{27.8 \text{ psi}}}$$

7.66 Start with the two-dimensional continuity equation and the Navier–Stokes equations (Eqs. 7.35, 7.36, and 7.37) and verify the nondimensional forms of these equations (Eqs. 7.38, 7.41, and 7.42).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Eq. 7.35)}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(Eq. 7.36)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \rho g + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad \text{(Eq. 7.37)}$$

As indicated in Section 7.10 let

$$u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad p^* = \frac{p}{p_0}$$

$$x^* = \frac{x}{\ell} \qquad y^* = \frac{y}{\ell} \qquad t^* = \frac{t}{\tau}$$

The various transformations can be made as follows:

$$\frac{\partial u}{\partial x} = \frac{\partial (Vu^*)}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{V}{\ell}\frac{\partial u^*}{\partial x^*}$$

and similarly,

$$\frac{\partial v}{\partial x} = \frac{V}{\ell}\frac{\partial v^*}{\partial x^*} \qquad \frac{\partial u}{\partial y} = \frac{V}{\ell}\frac{\partial u^*}{\partial y^*} \qquad \frac{\partial v}{\partial y} = \frac{V}{\ell}\frac{\partial v^*}{\partial y^*}$$

Also,

$$\frac{\partial^2 u}{\partial x^2} = \frac{V}{\ell}\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

and similarly,

$$\frac{\partial^2 v}{\partial x^2} = \frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial x^{*2}} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad \frac{\partial^2 v}{\partial y^2} = \frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial y^{*2}}$$

For the local acceleration,

$$\frac{\partial u}{\partial t} = \frac{\partial (Vu^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{V}{\tau}\frac{\partial u^*}{\partial t^*}$$

and similarly,

$$\frac{\partial v}{\partial t} = \frac{V}{\tau}\frac{\partial v^*}{\partial t^*}$$

(con't)

7.66 (con't)

For the pressure terms,

$$\frac{\partial p}{\partial x} = \frac{\partial\, p_0 p^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial x^*}$$

and similarly,

$$\frac{\partial p}{\partial y} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial y^*}$$

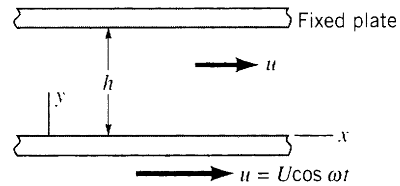
Substitution of the various terms, expressed in terms of the dimensionless variables, can be made into the original differential equations (Eqs. 7.35, 7.36, and 7.37) to yield Eqs. 7.38, 7.39, and 7.40. To obtain the final form for Eqs. 7.41 and 7.42 divide each term by $\rho V^2/\ell$.

7.67

7.67 A viscous fluid is contained between wide, parallel plates spaced a distance h apart as shown in Fig. P7.67. The upper plate is fixed, and the bottom plate oscillates harmonically with a velocity amplitude U and frequency ω. The differential equation for the velocity distribution between the plates is

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

where u is the velocity, t is time, and ρ and μ are fluid density and viscosity, respectively. Rewrite this equation in a suitable nondimensional form using h, U, and ω as reference parameters.



Let $y^* = \frac{y}{h}$, $u^* = \frac{u}{U}$, and $t^* = \omega t$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (U u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = U \frac{\partial u^*}{\partial t^*}(\omega) = U\omega \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (U u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = U \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \frac{U}{h}\frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{h}\frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \frac{U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Thus, the original differential equation becomes

$$\rho U\omega \frac{\partial u^*}{\partial t^*} = \frac{\mu U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\left[\frac{\rho\omega h^2}{\mu}\right]\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}}$$

7.68

7.68 The deflection of the cantilever beam of Fig. P7.68 is governed by the differential equation

$$EI\frac{d^2y}{dx^2} = P(x - \ell)$$

where E is the modulus of elasticity and I is the moment of inertia of the beam cross section. The boundary conditions are $y = 0$ at $x = 0$ and $dy/dx = 0$ at $x = 0$. **(a)** Rewrite the equation and boundary conditions in dimensionless form using the beam length, ℓ, as the reference length. **(b) Based on the results of part (a) what are the similarity requirements and the prediction equation for a model to predict deflections?**

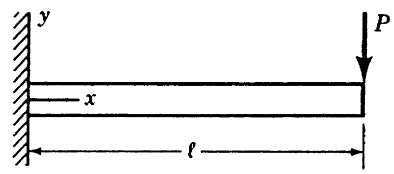


FIGURE P7.68

(a) Let $y^* = \frac{y}{\ell}$ and $x^* = \frac{x}{\ell}$ so that

$$\frac{dy}{dx} = \frac{d(\ell y^*)}{dx^*}\frac{dx^*}{dx} = \ell\frac{dy^*}{dx^*}\left(\frac{1}{\ell}\right) = \frac{dy^*}{dx^*}$$

and

$$\frac{d^2y}{dx^2} = \frac{d}{dx^*}\left(\frac{dy^*}{dx^*}\right)\frac{dx^*}{dx} = \frac{1}{\ell}\frac{d^2y^*}{dx^{*2}}$$

Thus, the original differential equation becomes

$$\left[\frac{EI}{\ell}\right]\frac{d^2y^*}{dx^{*2}} = P(\ell x^* - \ell)$$

or

$$\frac{d^2y^*}{dx^{*2}} = \left[\frac{P\ell^2}{EI}\right](x^* - 1)$$

and the boundary conditions are

$$y^* = 0 \text{ at } x^* = 0 \quad \text{and} \quad \frac{dy^*}{dx^*} = 0 \text{ at } x^* = 0.$$

(b) The similarity requirements are

$$x_m^* = x^* \quad \text{or} \quad \frac{x_m}{\ell_m} = \frac{x}{\ell} \quad \text{and} \quad \frac{P_m \ell_m^2}{E_m I_m} = \frac{P\ell^2}{EI}$$

The prediction equation is

$$y^* = y_m^*$$

or

$$\frac{y}{\ell} = \frac{y_m}{\ell_m}$$

7.69

7.69 A liquid is contained in a pipe that is closed at one end as shown in Fig. P7.69. Initially the liquid is at rest, but if the end is suddenly opened the liquid starts to move. Assume the pressure p_1 remains constant. The differential equation that describes the resulting motion of the liquid is

$$\rho \frac{\partial v_z}{\partial t} = \frac{p_1}{\ell} + \mu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right)$$

where v_z is the velocity at any radial location, r, and t is time. Rewrite this equation in dimensionless form using the liquid density, ρ, the viscosity, μ, and the pipe radius, R, as reference parameters.

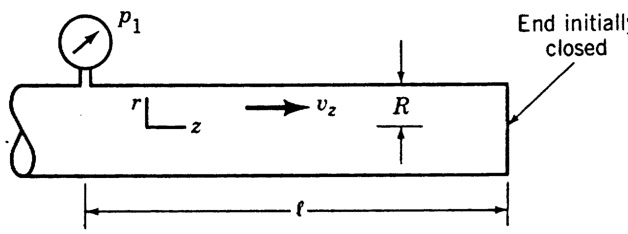


FIGURE P7.69

Let $r^* = \frac{r}{R}$, $t^* = \frac{t}{\tau}$, and $v_z^* = \frac{v_z}{V}$ where τ is some combinations of the parameters ρ, μ, and R having the dimensions of time, and V is some combination of the same parameters having the dimensions of a velocity. Let

$$\tau = \frac{\rho R^2}{\mu} \doteq \frac{(FL^{-4}T^{2})(L)^2}{FL^{-2}T} \doteq T$$

and

$$V = \frac{\mu}{\rho R} \doteq \frac{FL^{-2}T}{(FL^{-4}T^{2})(L)} \doteq LT^{-1}$$

With these dimensionless variables:

$$\frac{\partial v_z}{\partial t} = \frac{\partial (V v_z^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = V\frac{\partial v_z^*}{\partial t^*}\left(\frac{1}{\tau}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{\mu}{\rho R^2}\right)\frac{\partial v_z^*}{\partial t^*} = \left(\frac{\mu}{\rho}\right)^2\frac{1}{R^3}\frac{\partial v_z^*}{\partial t^*}$$

$$\frac{\partial v_z}{\partial r} = \frac{\partial (V v_z^*)}{\partial r^*}\frac{\partial r^*}{\partial r} = V\frac{\partial v_z^*}{\partial r^*}\left(\frac{1}{R}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{1}{R}\right)\frac{\partial v_z^*}{\partial r^*} = \frac{\mu}{\rho R^2}\frac{\partial v_z^*}{\partial r^*}$$

$$\frac{\partial^2 v_z}{\partial r^2} = \frac{\mu}{\rho R^2}\frac{\partial}{\partial r^*}\left(\frac{\partial v_z^*}{\partial r^*}\right)\frac{\partial r^*}{\partial r} = \frac{\mu}{\rho R^2}\frac{\partial^2 v_z^*}{\partial r^{*2}}\left(\frac{1}{R}\right) = \frac{\mu}{\rho R^3}\frac{\partial^2 v_z^*}{\partial r^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho\left(\frac{\mu}{\rho}\right)^2\frac{1}{R^3}\right]\frac{\partial v_z^*}{\partial t^*} = \frac{p_1}{\ell} + \left[\mu\left(\frac{\mu}{\rho R^3}\right)\right]\left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}\right)$$

or

$$\frac{\partial v_z^*}{\partial t^*} = \frac{p_1 \rho R^3}{\ell \mu^2} + \frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}$$

7.70 An incompressible fluid is contained between two infinite parallel plates as illustrated in Fig. P7.70. Under the influence of a harmonically varying pressure gradient in the x direction, the fluid oscillates harmonically with a frequency ω. The differential equation describing the fluid motion is

$$\rho \frac{\partial u}{\partial t} = X \cos \omega t + \mu \frac{\partial^2 u}{\partial y^2}$$

where X is the amplitude of the pressure gradient. Express this equation in nondimensional form using h and ω as reference parameters.

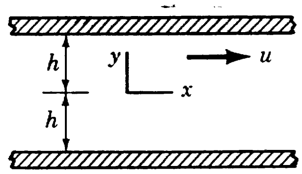


FIGURE P7.70

Let $y^* = \frac{y}{h}$, $t^* = \omega t$, and $u^* = \frac{u}{h\omega}$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (h\omega u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = h\omega \frac{\partial u^*}{\partial t^*}(\omega) = h\omega^2 \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (h\omega u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = h\omega \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \omega \frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \omega \frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \omega \frac{\partial^2 u^*}{\partial y^{*2}}\left(\frac{1}{h}\right) = \frac{\omega}{h}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho h \omega^2\right]\frac{\partial u^*}{\partial t^*} = X \cos t^* + \left[\frac{\mu\omega}{h}\right]\frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\frac{\partial u^*}{\partial t^*} = \left[\frac{X}{\rho h \omega^2}\right]\cos t^* + \left[\frac{\mu}{\rho h^2 \omega}\right]\frac{\partial^2 u^*}{\partial y^{*2}}$$

7.71

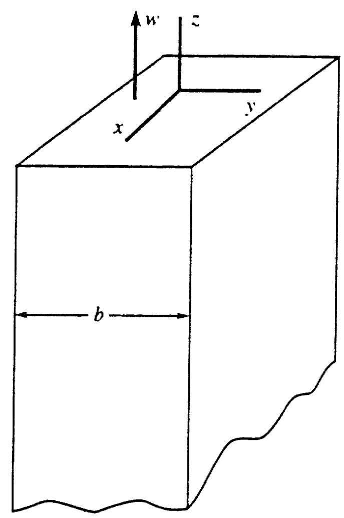
7.71 A viscous fluid flows through a vertical, square channel as shown in Fig. P7.71. The velocity w can be expressed as

$$w = f(x, y, b, \mu, \gamma, V, \partial p/\partial z)$$

where μ is the fluid viscosity, γ the fluid specific weight, V the mean velocity, and $\partial p/\partial z$ the pressure gradient in the z direction. **(a)** Use dimensional analysis to find a suitable set of dimensionless variables and parameters for this problem. **(b)** The differential equation governing the fluid motion for this problem is

$$\frac{\partial p}{\partial z} = -\gamma + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Write this equation in a suitable dimensionless form, and show that the similarity requirements obtained from this analysis are the same as those resulting from the dimensional analysis of part (a).



■ FIGURE P7.71

(a) $$w = f\left(x, y, b, \mu, \gamma, V, \frac{\partial p}{\partial z}\right)$$

$w \doteq LT^{-1}$ $\quad x \doteq L$ $\quad y \doteq L$ $\quad b \doteq L$ $\quad \mu \doteq FL^{-2}T$ $\quad \gamma \doteq FL^{-3}$ $\quad V \doteq LT^{-1}$ $\quad \frac{\partial p}{\partial z} \doteq FL^{-3}$

From the pi theorem, $8-3=5$ pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{w}{V} = \phi\left(\frac{x}{b}, \frac{y}{b}, \frac{1}{\gamma}\frac{\partial p}{\partial z}, \frac{\gamma b^2}{\mu V}\right)}} \qquad (1)$$

(b) Let $w^* = \frac{w}{V}$, $x^* = \frac{x}{b}$, $y^* = \frac{y}{b}$ so that:

$$\frac{\partial w}{\partial x} = \frac{\partial (V w^*)}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{V}{b}\frac{\partial w^*}{\partial x^*}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{V}{b}\frac{\partial}{\partial x^*}\left(\frac{\partial w^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{V}{b^2}\frac{\partial^2 w^*}{\partial x^{*2}}$$

Similarly, $$\frac{\partial^2 w}{\partial y^2} = \frac{V}{b^2}\frac{\partial^2 w^*}{\partial y^{*2}}$$

The original differential equation can now be expressed as

$$\frac{\partial p}{\partial z} = -\gamma + \left[\frac{\mu V}{b^2}\right]\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right)$$

or

$$\frac{b^2}{\mu V}\frac{\partial p}{\partial z} = -\frac{\gamma b^2}{\mu V} + \left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) \qquad (2)$$

(con't)

7.71 (Con't)

Eq. (2) indicates that

$$w^* = \frac{w}{V} = \phi\left(x^*, y^*, \frac{\gamma b^2}{\mu V}, \frac{b^2}{\mu V}\frac{\partial p}{\partial z}\right) \qquad (3)$$

Although this result does not appear to match the equation obtained by dimensional analysis (Eq. 1), the last two pi terms in Eq. (1) can be combined to yield

$$\left(\frac{1}{\gamma}\frac{\partial p}{\partial z}\right)\left(\frac{\gamma b^2}{\mu V}\right) = \frac{b^2}{\mu V}\frac{\partial p}{\partial z}$$

so that Eq. (1) can also be written as

$$\frac{w}{V} = \phi\left(\frac{x}{b}, \frac{y}{b}, \frac{\gamma b^2}{\mu V}, \frac{b^2}{\mu V}\frac{\partial p}{\partial z}\right) \qquad (4)$$

and this result is the same as that in Eq. (3). Thus, the similarity requirements indicated by Eqs. (3) and (4) are the same.

7.72

7.72 Flow from a Tank

Objective: When the drain hole in the bottom of the tank shown in Fig. P7.72 is opened, the liquid will drain out at a rate which is a function of many parameters. The purpose of this experiment is to measure the liquid depth, h, as a function of time, t, for two geometrically similar tanks and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Two geometrically similar cylindrical tanks; stop watch; thermometer; ruler.

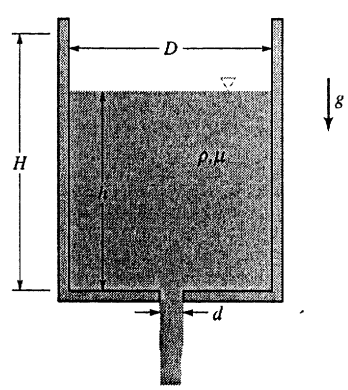
Experimental Procedure: Make appropriate measurements to show that the two tanks are geometrically similar. That is, show that the large tank is twice the size of the small tank (twice the height; twice the diameter; twice the hole diameter in the bottom). Fill the large tank with cold water of a known temperature, T, and determine the water depth, h, in the tank as a function of time, t, after the drain hole is opened. Thus, obtain $h = h(t)$. Note that t ranges from $t = 0$ when $h = H$ (where H is the initial depth of the water), to $t = t_{\text{final}}$ then the tank is completely drained ($h = 0$). Repeat the measurements using the small tank with the same temperature water. To ensure geometric similarity, the initial water level in the small tank must be one-half of what it was in the large tank. Repeat the experiment for each tank with hot water. Thus you will have a total of four sets of $h(t)$ data.

Calculations: Assume that the depth, h, of water in the tank is a function of its initial depth, H, the diameter of the tank, D, the diameter of the drain hole in the bottom of the tank, d, the time, t, after the drain is opened, the acceleration of gravity, g, and the fluid density, ρ, and viscosity, μ. Develop a suitable set of dimensionless parameters for this problem using H, g, and ρ as repeating variables. Use t as the dependent parameter. For each of the four conditions tested, calculate the dimensionless time, $tg^{1/2}/H^{1/2}$, as a function of the dimensionless depth, h/H.

Graph: On a single graph, plot the depth, h, as ordinates and time, t, as abscissas for each of the four sets of data.

Results: On another graph, plot the dimensionless water depth, h/H, as a function of dimensionless time, $tg^{1/2}/H^{1/2}$, for each of the four sets of data. Based on your results, comment on the importance of density and viscosity for your experiment and on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P7.72

(con't)

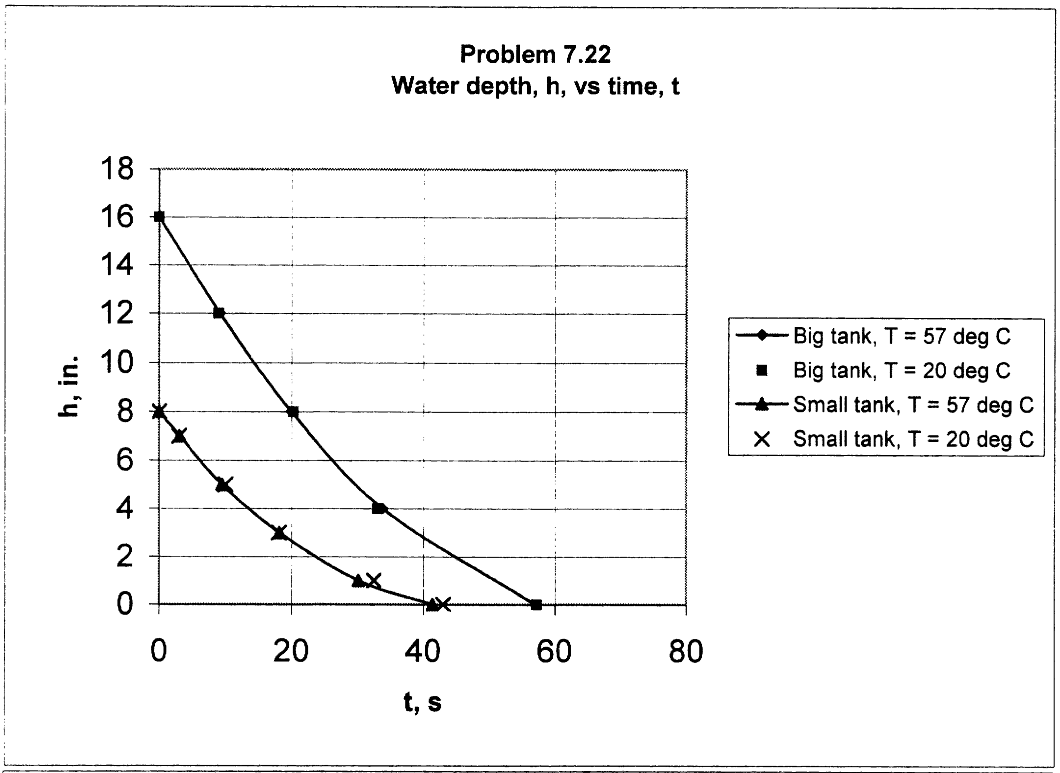
Solution for Problem 7.72: Flow from a Tank

H for big tank, in.	H for small tank, in.
16.0	8.0

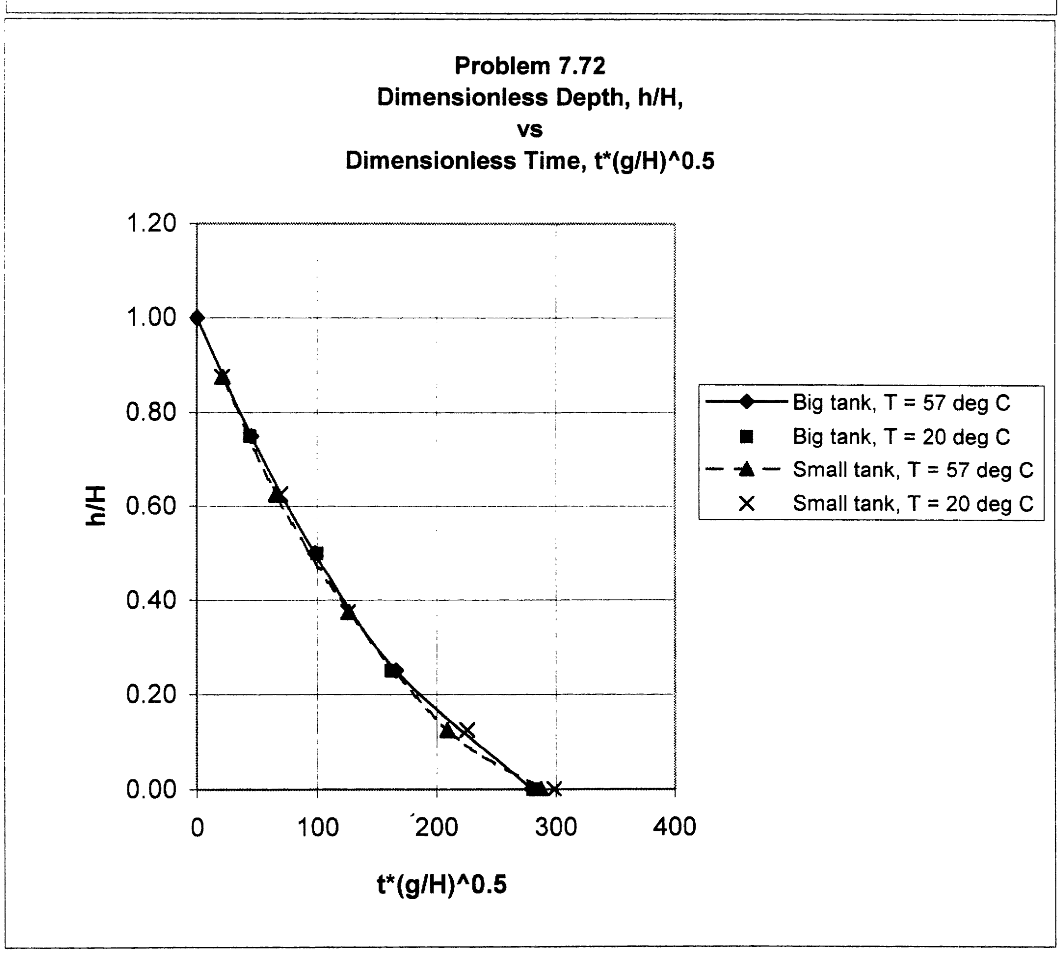
	h, in.	t, s	$tg^{1/2}/H^{1/2}$	h/H
Big Tank with T = 57 deg C				
	16.0	0.0	0.0	1.000
	12.0	9.2	45.2	0.750
	8.0	20.0	98.3	0.500
	4.0	33.8	166.1	0.250
	0.0	57.0	280.1	0.000
Big Tank with T = 20 deg C				
	16.0	0.0	0.0	1.000
	12.0	9.0	44.2	0.750
	8.0	20.3	99.8	0.500
	4.0	33.0	162.2	0.250
	0.0	57.2	281.1	0.000
Small Tank with T = 57 deg C				
	8.0	0.0	0.0	1.000
	7.0	3.1	21.5	0.875
	5.0	9.5	66.0	0.625
	3.0	18.2	126.5	0.375
	1.0	30.1	209.2	0.125
	0.0	41.4	287.7	0.000
Small Tank with T = 20 deg C				
	8.0	0.0	0.0	1.000
	7.0	3.0	20.8	0.875
	5.0	10.0	69.5	0.625
	3.0	18.1	125.8	0.375
	1.0	32.5	225.9	0.125
	0.0	43.0	298.8	0.000

(Con't)

7.72 (con't)



Problem 7.22
Water depth, h, vs time, t
18
16
14
12
10
8
6
4
2
0
h, in.
0
20
40
60
80
t, s
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C



Problem 7.72
Dimensionless Depth, h/H,
vs
Dimensionless Time, t*(g/H)^0.5
1.20
1.00
0.80
0.60
0.40
0.20
0.00
h/H
0
100
200
300
400
t*(g/H)^0.5
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C

7.73

7.73 Vortex Shedding from a Circular Cylinder

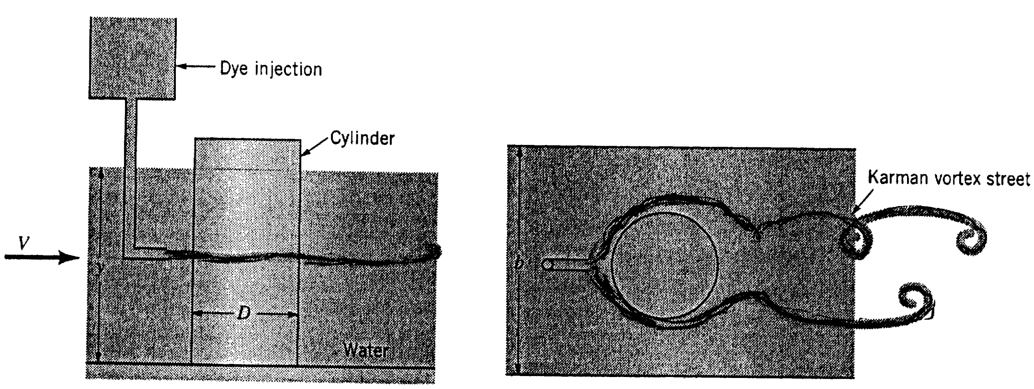
Objective: Under certain conditions, the flow of fluid past a circular cylinder will produce a Karman vortex street behind the cylinder. As shown in Fig. P7.73, this vortex street consists of a set of vortices (swirls) that are shed alternately from opposite sides of the cylinder and then swept downstream with the fluid. The purpose of this experiment is to determine the shedding frequency, ω cycles (vortices) per second, of these vortices as a function of the Reynolds number, Re, and to compare the measured results with published data.

Equipment: Water channel with an adjustable flowrate; flow meter; set of four different diameter cylinders; dye injection system; stopwatch.

Experimental Procedure: Insert a cylinder of diameter D into the holder on the bottom of the water channel. Adjust the control valve and the downstream gate on the channel to produce the desired flowrate, Q, and velocity, V. Make sure that the flow-straightening screens (not shown in the figure) are in place to reduce unwanted turbulence in the flowing water. Measure the width, b, of the channel and the depth, y, of the water in the channel so that the water velocity in the channel, $V = Q/(by)$, can be determined. Carefully adjust the control valve on the dye injection system to inject a thin stream of dye slightly upstream of the cylinder. By viewing down onto the top of the water channel, observe the vortex shedding and measure the time, t, that it takes for N vortices to be shed from the cylinder. For a given velocity, repeat the experiment for different diameter cylinders. Repeat the experiment using different velocities. Measure the water temperature so that the viscosity can be looked up in Table B.1.

Calculations: For each of your data sets calculate the vortex shedding frequency, $\omega = N/t$, which is expressed as vortices (or cycles) per second. Also calculate the dimensionless frequency called the Strouhl number, $\mathrm{St} = \omega D/V$, and the Reynolds number, $\mathrm{Re} = \rho VD/\mu$.

Graph: On a single graph, plot the vortex shedding frequency, ω, as ordinates and the water velocity, V, as abscissas for each of the four cylinders you tested. On another graph, plot the Strouhl number as ordinates and the Reynolds number as abscissas for each of the four sets of data.



■ FIGURE P7.73

Side view   Top view

(con't)

7.73 (con't)

Results: On your Strouhl number verses Reynolds number graph, plot the results taken from the literature and shown in the following table.

St	Re
0	<50
0.16	100
0.18	150
0.19	200
0.20	300
0.21	400
0.21	600
0.21	800

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

Solution for Problem 7.73: Vortex Shedding from a Circular Cylinder

T, deg F	b, ft
70	0.50

Q, ft^3/s	y, ft	D, ft	N	t, s	ω, cycles/s	V, ft/s	Re	St
0.036	0.82	0.0202	10.0	13.2	0.758	0.0878	169	0.174
0.036	0.82	0.0314	10.0	19.9	0.503	0.0878	263	0.180
0.036	0.82	0.0421	10.0	24.5	0.408	0.0878	352	0.196
0.036	0.82	0.0518	10.0	30.1	0.332	0.0878	433	0.196
0.062	0.79	0.0202	10.0	6.3	1.587	0.1570	302	0.204
0.062	0.79	0.0314	10.0	9.6	1.042	0.1570	469	0.208
0.062	0.79	0.0421	10.0	12.5	0.800	0.1570	629	0.215
0.062	0.79	0.0518	10.0	15.1	0.662	0.1570	774	0.219
0.029	0.86	0.0202	10.0	19.2	0.521	0.0674	130	0.156
0.029	0.86	0.0314	10.0	28.2	0.355	0.0674	202	0.165
0.029	0.86	0.0421	10.0	33.1	0.302	0.0674	270	0.189
0.029	0.86	0.0518	10.0	36.7	0.272	0.0674	333	0.209
0.018	0.92	0.0202	10.0	31.2	0.321	0.0391	75	0.165
0.018	0.92	0.0314	10.0	41.3	0.242	0.0391	117	0.194
0.018	0.92	0.0421	10.0	52.2	0.192	0.0391	157	0.206
0.018	0.92	0.0518	10.0	65.3	0.153	0.0391	193	0.203

Data from Literature

Re	St
50	0.00
100	0.16
150	0.18
200	0.19
300	0.20
400	0.21
600	0.21
800	0.21

ω = N/t

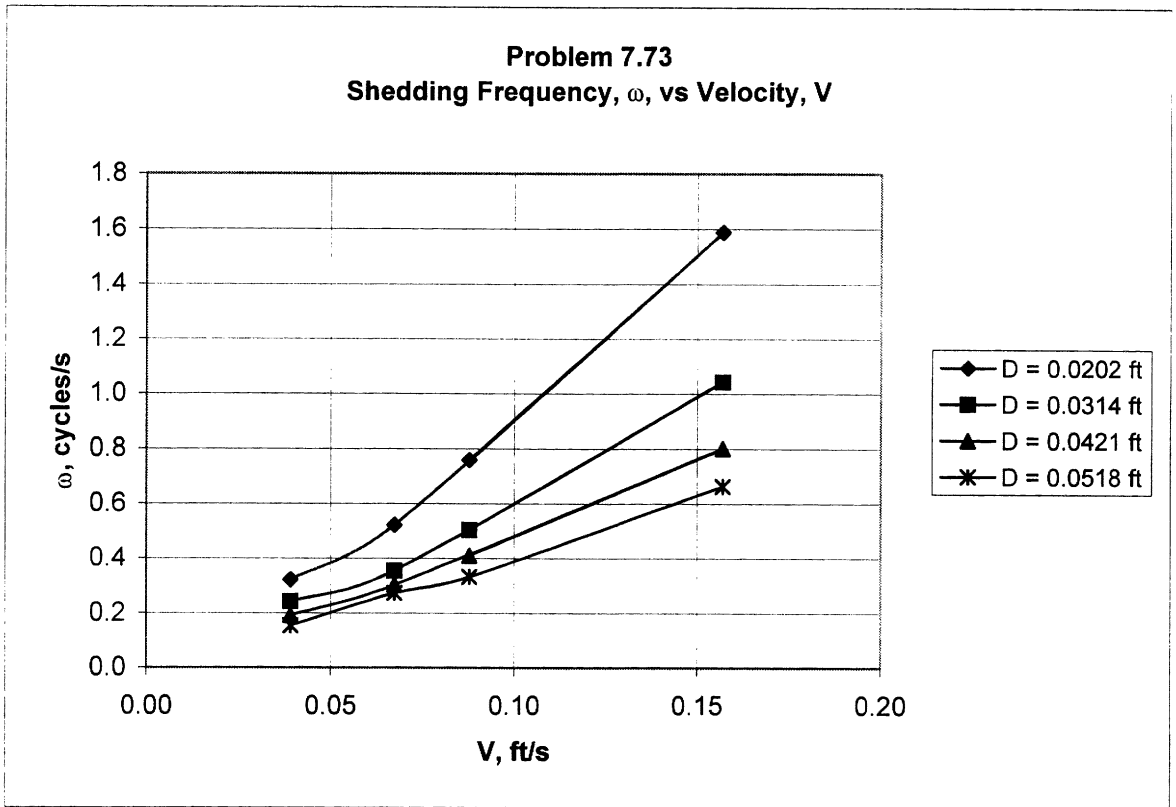
V = Q/(by)

St = ωD/V and Re = DV/ν, where

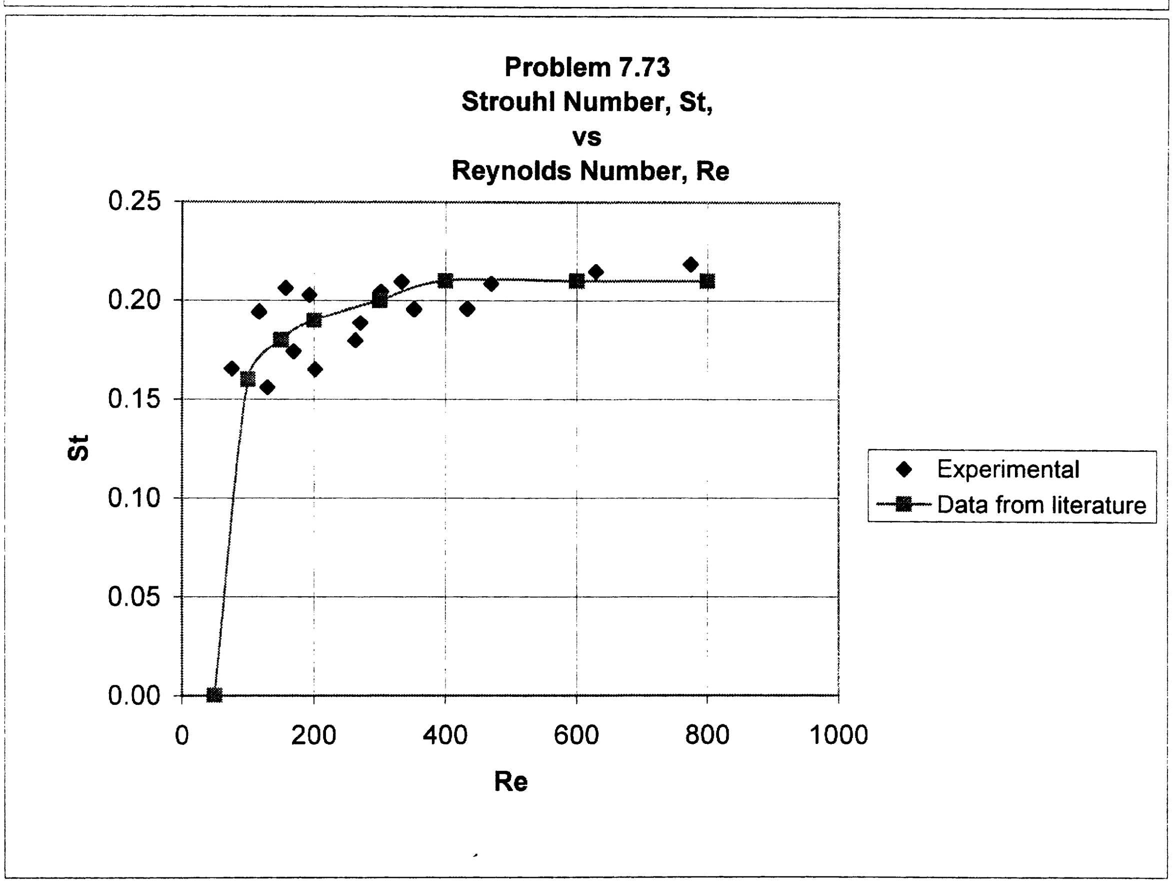
ν = 1.052E-5 ft^2/s

(con't)

7.73 (con't)



Problem 7.73
Shedding Frequency, ω, vs Velocity, V
ω, cycles/s
V, ft/s
D = 0.0202 ft
D = 0.0314 ft
D = 0.0421 ft
D = 0.0518 ft
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20



Problem 7.73
Strouhl Number, St,
vs
Reynolds Number, Re
St
Re
Experimental
Data from literature
0.25
0.20
0.15
0.10
0.05
0.00
0
200
400
600
800
1000

7.74

7.74 Head Loss across a Valve

Objective: A valve in a pipeline like that shown in Fig. P7.74 acts like a variable resistor in an electrical circuit. The amount of resistance or head loss across a valve depends on the amount that the valve is open. The purpose of this experiment is to determine the head loss characteristics of a valve by measuring the pressure drop, Δp, across the valve as a function of flowrate, Q, and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Air supply with flow meter; valve connected to a pipe; manometer connected to a static pressure tap upstream of the valve; barometer; thermometer.

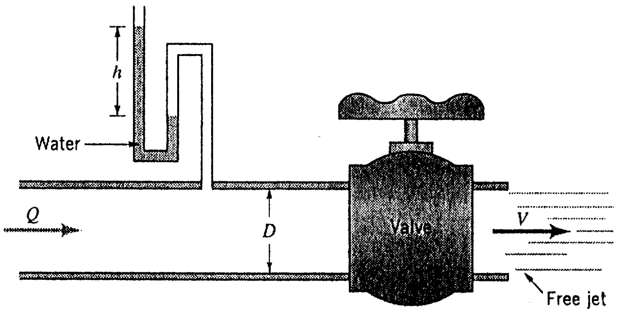
Experimental Procedure: Measure the pipe diameter, D. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Completely close the valve and then open it N turns from its closed position. Adjust the air supply to provide the desired flowrate, Q, of air through the valve. Record the manometer reading, h, so that the pressure drop, Δp, across the valve can be determined. Repeat the measurements for various flowrates. Repeat the experiment for various valve settings, N, ranging from barely open to wide open.

Calculations: For each data set calculate the average velocity in the pipe, $V = Q/A$, where $A = \pi D^2/4$ is the pipe area. Also calculate the pressure drop across the valve, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. For each data set also calculate the loss coefficient, K_L, where the head loss is given by $h_L = \Delta p/\gamma = K_L V^2/2g$ and γ is the specific weight of the flowing air.

Graph: On a single graph, plot the pressure drop, Δp, as ordinates and the flowrate, Q, as abscissas for each of the valve settings, N, tested.

Results: On another graph, plot the loss coefficient, K_L, as a function of valve setting, N, for all of the data sets.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P7.74

(con't)

7.74 (con't)

Solution for Problem 7.74: Head Loss across a Valve

D, in.	H_{atm}, in. Hg	T, deg F
0.81	28.7	70

h, in.	Q, ft^3/s	Δp, lb/ft^2	V, ft/s	N	K_L
N = 2 Turns Open Data					
9.20	0.235	47.8	65.7	2	9.95
6.50	0.195	33.8	54.5	2	10.21
5.04	0.169	26.2	47.2	2	10.54
N = 3 Turns Open Data					
9.40	0.479	48.9	133.9	3	2.45
6.33	0.386	32.9	107.9	3	2.54
5.01	0.341	26.1	95.3	3	2.57
3.62	0.289	18.8	80.8	3	2.59
1.92	0.214	10.0	59.8	3	2.50
N = 4 Turns Open Data					
9.35	0.827	48.6	231.1	4	0.816
7.65	0.767	39.8	214.3	4	0.777
6.01	0.691	31.3	193.1	4	0.752
4.32	0.578	22.5	161.5	4	0.772
3.24	0.504	16.8	140.8	4	0.762
2.62	0.456	13.6	127.4	4	0.752
1.85	0.391	9.6	109.3	4	0.723
0.98	0.283	5.1	79.1	4	0.731
N = 5 Turns Open Data					
3.03	0.897	15.8	250.7	5	0.225
2.37	0.799	12.3	223.3	5	0.222
1.79	0.701	9.3	195.9	5	0.218
1.39	0.618	7.2	172.7	5	0.217
0.97	0.517	5.0	144.5	5	0.217
0.64	0.426	3.3	119.0	5	0.211

$\Delta p = \gamma_{H2O}*h$

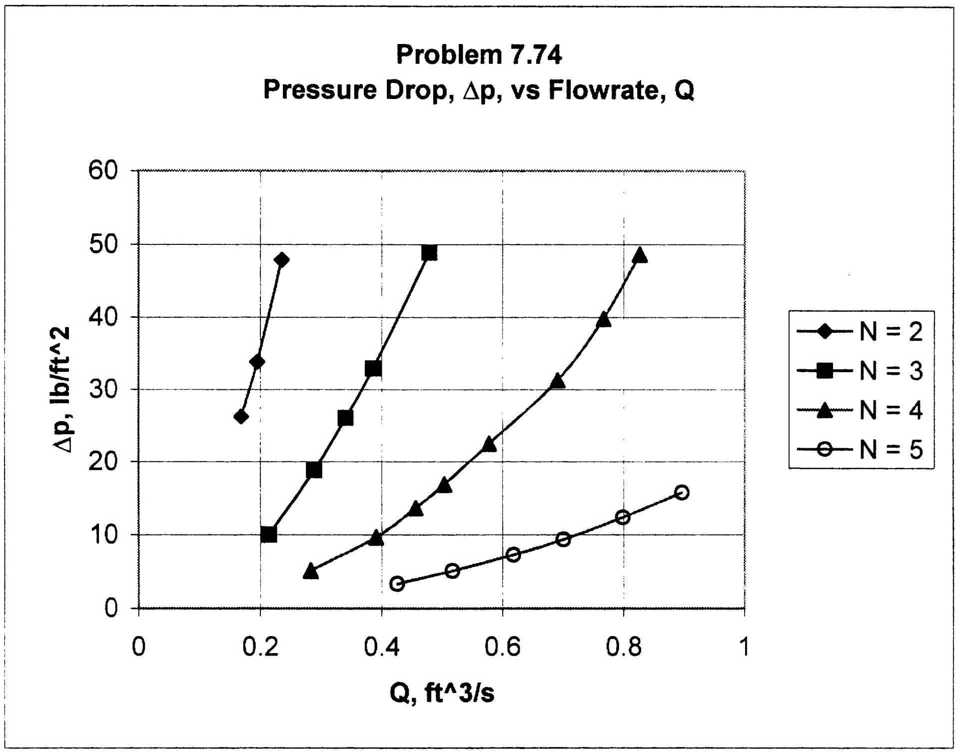
$K_L = \Delta p/(\rho V^2/2)$ where
$V = Q/A = Q/(\pi*D^2/4)$
and
$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(28.7/12 ft) = 2026 lb/ft^2
R = 1716 ft lb/slug deg R
T = 70 + 460 = 530 deg R

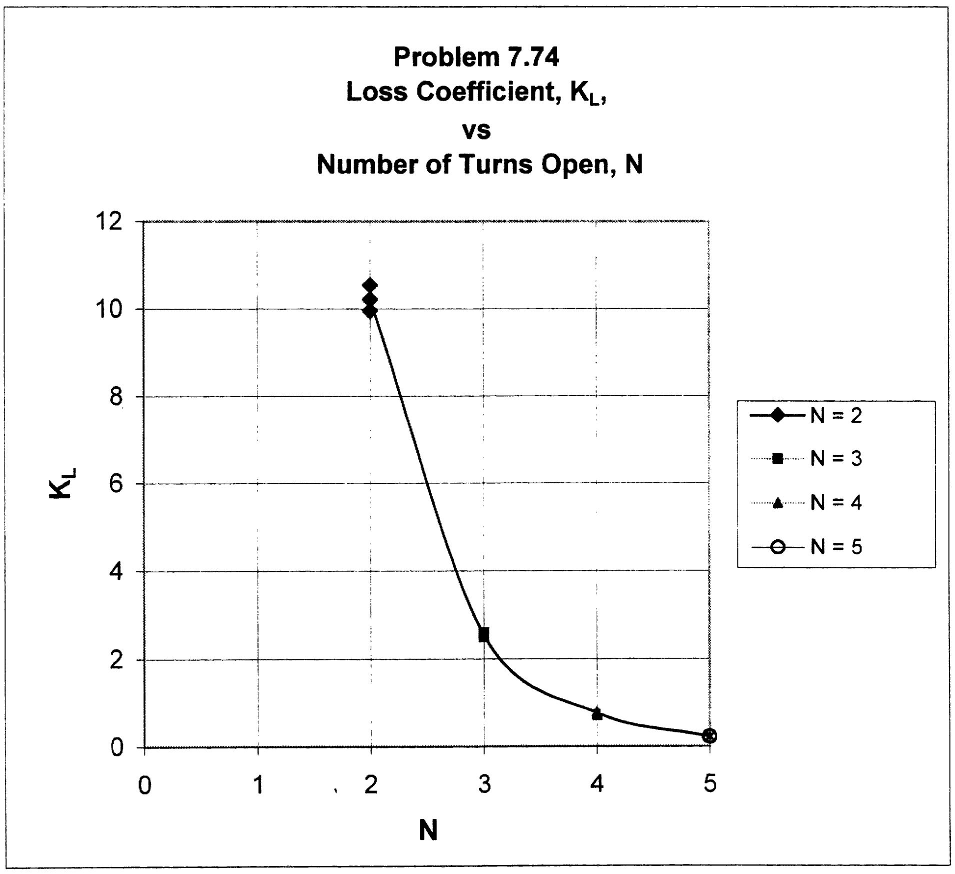
Thus, ρ = 0.00223 slug/ft^3

(con't)

7.74 (con't)



Problem 7.74
Pressure Drop, Δp, vs Flowrate, Q
Δp, lb/ft^2
0
10
20
30
40
50
60
0
0.2
0.4
0.6
0.8
1
Q, ft^3/s
N = 2
N = 3
N = 4
N = 5



Problem 7.74
Loss Coefficient, K_L,
vs
Number of Turns Open, N
K_L
0
2
4
6
8
10
12
0
1
2
3
4
5
N
N = 2
N = 3
N = 4
N = 5

7.75 Calibration of a Rotameter

Objective: The flowrate, Q, through a rotameter can be determined from the scale reading, SR, which indicates the vertical position of the float within the tapered tube of the rotameter as shown in Fig. P7.75. Clearly, for a given scale reading, the flowrate depends on the density of the flowing fluid. The purpose of this experiment is to calibrate a rotameter so that it can be used for both water and air.

Equipment: Rotameter, air supply with a calibrated flow meter, water supply, weighing scale, stop watch, thermometer, barometer.

Experimental Procedure: Connect the rotameter to the water supply and adjust the flowrate, Q, to the desired value. Record the scale reading, SR, on the rotameter and measure the flowrate by collecting a given weight, W, of water that passes through the rotameter in a given time, t. Repeat for several flow rates.

Connect the rotameter to the air supply and adjust the flowrate to the desired value as indicated by the flow meter. Record the scale reading on the rotameter. Repeat for several flowrates. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For the water portion of the experiment, use the weight, W, and time, t, data to determine the volumetric flowrate, $Q = W/\gamma t$. The equilibrium position of the float is a result of a balance between the fluid drag force on the float, the weight of the float, and the buoyant force on the float. Thus, a typical dimensionless flowrate can be written as $Q/[d(\rho/Vg(\rho_f - \rho))^{1/2}]$, where d is the diameter of the float, V is the volume of the float, g is the acceleration of gravity, ρ is the fluid density, and ρ_f is the float density. Determine this dimensionless flowrate for each condition tested.

Graph: On a single graph, plot the flowrate, Q, as ordinates and scale reading, SR, as abscissas for both the water and air data.

Results: On another graph, plot the dimensionless flowrate as a function of scale reading for both the water and air data. Note that the scale reading is a percent of full scale and, hence, is a dimensionless quantity. Based on your results, comment on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

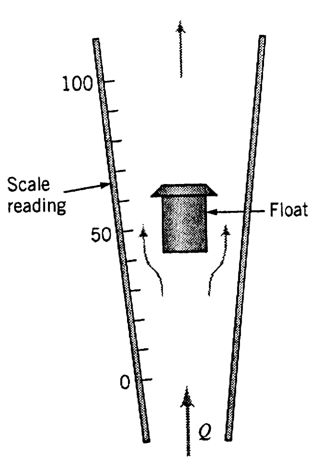


FIGURE P7.75

(con't)

7.75 (con't)

Solution for Problem 7.75: Calibration of a Rotameter

d, in.	V, in.^3	ρ_f, slug/ft^3	H_{atm}, in.	T, deg F
1.40	1.50	15.1	29.05	78

Air Flow Data

SR	Q, ft^3/s	$(Q/d)[\rho/(\mathbf{V}g(\rho_f-\rho))]1/2$
14.6	0.229	0.142
21.5	0.321	0.200
28.1	0.413	0.257
33.6	0.491	0.305
39.2	0.564	0.351
44.8	0.644	0.400
50.2	0.714	0.444
55.9	0.798	0.496
63.1	0.888	0.552
68.6	0.973	0.605
73.5	1.05	0.653
76.2	1.08	0.671

Water Flow Data

SR	W, lb	t, s	Q, ft^3/s	$(Q/d)[\rho/(\mathbf{V}g(\rho_f-\rho))]1/2$
13.1	6.52	19.9	0.0053	0.103
18.5	8.01	17.7	0.0073	0.143
24.2	7.02	10.4	0.0108	0.213
28.2	7.81	10.1	0.0124	0.244
37.1	8.20	8.4	0.0156	0.308
45.7	9.21	7.5	0.0197	0.387
52.6	8.19	5.7	0.0230	0.453

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.05/12 ft) = 2050 lb/ft^2

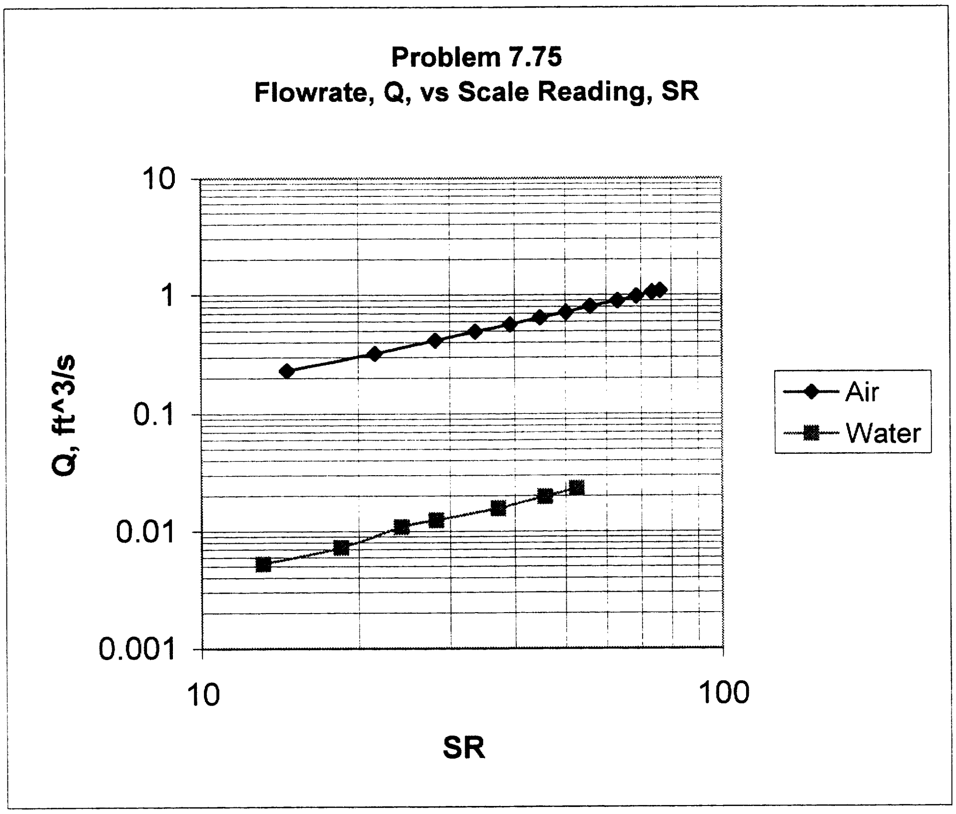
R = 1716 ft lb/slug deg R

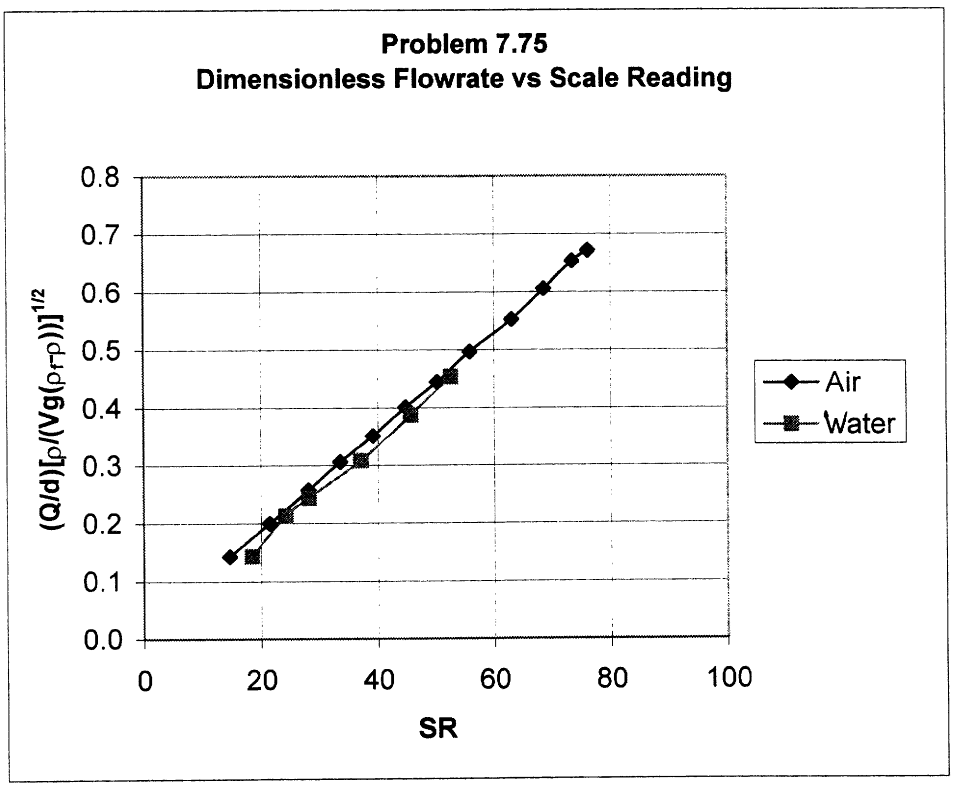
T = 78 + 460 = 538 deg R

Thus, ρ = 0.00222 slug/ft^3

(con't)

7.75 (con't)


Problem 7.75
Flowrate, Q, vs Scale Reading, SR
10
1
0.1
0.01
0.001
Q, ft^3/s
10
100
SR
Air
Water


Problem 7.75
Dimensionless Flowrate vs Scale Reading
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
(Q/d)[ρ/(Vg(ρf-ρ))]^1/2
0
20
40
60
80
100
SR
Air
Water

8.1

8.1 Rainwater runoff from a parking lot flows through a 3-ft-diameter pipe, completely filling it. Whether flow in a pipe is laminar or turbulent depends on the value of the Reynolds number. (See Video V8.1.) Would you expect the flow to be laminar or turbulent? Support your answer with appropriate calculations.

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$ If $Re > 4000$ the flow is turbulent. The corresponding velocity is

$$V = \frac{Re\,\nu}{D} = \frac{(4000)(1.21\times10^{-5}\ \frac{ft^2}{s})}{3\ ft} = 0.0161\ \frac{ft}{s}$$

Most likely the velocity will be greater than this, i.e., <u>turbulent flow.</u>

8.3

8.3 The flow of water in a 3-mm-diameter pipe is to remain laminar. Plot a graph of the maximum flowrate allowed as a function of temperature for $0 < T < 100$ °C.

For laminar flow $Re = \frac{VD}{\nu} \le 2100$, where $V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

Thus, the maximum Q is given by

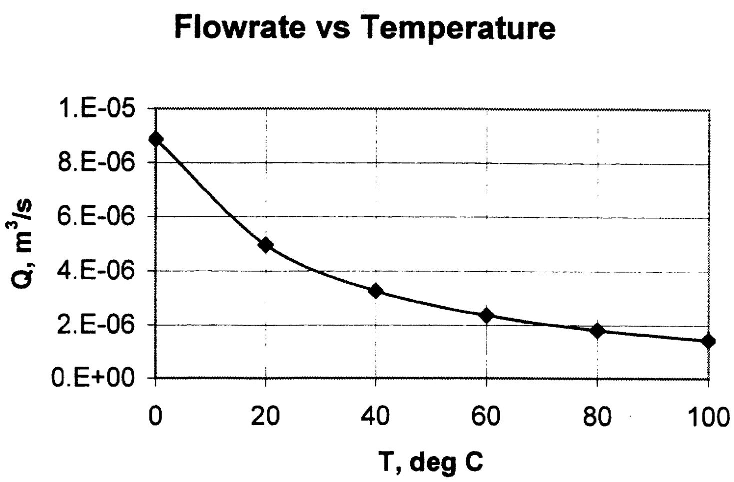
$$Re = \frac{\left(\frac{4Q}{\pi D^2}\right)D}{\nu} = \frac{4Q}{\pi \nu D} = 2100, \text{ or } Q = \frac{2100\pi \nu D}{4}$$

or

$$Q = \frac{2100\pi(0.003\,m)}{4}\nu = 4.95\,\nu, \text{ where } \nu \sim \frac{m^2}{s} \text{ and } Q \sim \frac{m^3}{s}$$

With values of ν from Table B.2 we obtain

T, deg C	ν, m^2/s	Q, m^3/s
0	1.79E-06	8.86E-06
20	1.00E-06	4.95E-06
40	6.58E-07	3.26E-06
60	4.75E-07	2.35E-06
80	3.65E-07	1.81E-06
100	2.90E-07	1.44E-06



8.4

8.4 Air at 100 °F flows at standard atmospheric pressure in a pipe at a rate of 0.08 lb/s. Determine the maximum diameter allowed if the flow is to be turbulent.

Minimum $Re = \frac{\rho V D}{\mu}$ for turbulent flow is $Re = 4000$.

or with

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}, \quad Re = \frac{\rho\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{4\rho Q}{\pi \mu D} = 4000$$

Hence,

$$Q = \frac{4000\pi\mu D}{4\rho} \tag{1}$$

Given $\gamma Q = 0.08 \frac{lb}{s}$, where $\gamma = g\rho$ and $\rho = \frac{p}{RT}$

Thus,

$$\rho = \frac{(14.7 \times 144 \frac{lb}{ft^2})}{(1716 \frac{ft \cdot lb}{slug \cdot °R})(460 + 100)°R} = 0.00220 \frac{slugs}{ft^3}$$

so that

$$Q = \frac{0.08 \frac{lb}{s}}{(32.2 \frac{ft}{s^2})(0.00220 \frac{slugs}{ft^3})} = 1.13 \frac{ft^3}{s}$$

Hence, with $\mu = 3.94 \times 10^{-7} \frac{lb \cdot s}{ft^2}$ (see Table B.3), Eq. (1) gives

$$D = \frac{4\rho Q}{4000\pi\mu} = \frac{4(0.00220 \frac{slugs}{ft^3})(1.13 \frac{ft^3}{s})}{4000\pi(3.49 \times 10^{-7} \frac{lb \cdot s}{ft^2})} = \underline{\underline{2.27 ft}}$$

8.5

8.5 Carbon dioxide at 20 °C and a pressure of 550 kPa (abs) flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter allowed if the flow is to be turbulent.

For turbulent flow, $Re = \frac{\rho V D}{\mu} > 4000$, where $Q = VA = \frac{\pi}{4} D^2 V$

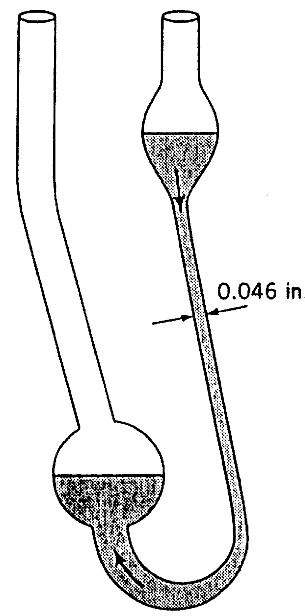
or $Re = \frac{4\rho Q D}{\pi \mu D^2} = \frac{4 \rho Q}{\pi \mu D} = 4000$

Thus, $D = \frac{4 \rho Q}{4000 \pi \mu}$, where $g\rho Q = 0.04 \frac{N}{s}$ and $\mu = 1.4 \times 10^{-5} \frac{N \cdot s}{m^2}$ (Table 1.8)

Hence,
$$D = \frac{4 \left(0.04 \frac{N}{s}\right)\left(\frac{1}{9.81 \frac{m}{s^2}}\right)}{4000 \pi \left(1.47 \times 10^{-5} \frac{N \cdot s}{m^2}\right)} = \underline{\underline{0.0883 \ m}}$$

8.6

8.6 It takes 20 seconds for 0.5 cubic inch of water to flow through the 0.046-in. diameter tube of the capillary tube viscometer shown in Video V1.3 and Fig. P8.6. Is the flow in the tube laminar or turbulent? Explain.



■ FIGURE P8.6

If $Re = \frac{VD}{\nu} < 2100$ the flow is laminar, where

$$V = \frac{Q}{A} = \frac{\mathcal{V}/t}{\frac{\pi}{4}D^2} = \frac{\frac{0.5\ in.^3}{(12\ in./ft)^3}/20s}{\frac{\pi}{4}\left(\frac{0.046}{12}ft\right)^2}$$

or

$$V = 1.25\ \frac{ft}{s}$$

Thus, with $\nu = 1.21 \times 10^{-5}\ \frac{ft^2}{s}$ (see Table 1.5)

$$Re = \frac{1.25\ \frac{ft}{s}\left(\frac{0.046}{12}ft\right)}{1.21 \times 10^{-5}\ \frac{ft^2}{s}} = 396 < 2100$$

The flow is laminar.

8.7

8.7 To cool a given room it is necessary to supply 5 ft³/s of air through an 8-in.-diameter pipe. Approximately how long is the entrance length in this pipe?

$$V = \frac{Q}{A} = \frac{5\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{8}{12}\,ft\right)^2} = 14.3\,\frac{ft}{s}$$ Thus, with $\nu = 1.57\times10^{-4}\,\frac{ft^2}{s}$ (see Table 1.7)

$$Re = \frac{VD}{\nu} = \frac{14.3\,\frac{ft}{s}\left(\frac{8}{12}\,ft\right)}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 60{,}700 > 4000$$ so the flow is turbulent.

Hence,

$$\frac{\ell_e}{D} = 4.4\,Re^{1/6},\quad \text{or}\quad \ell_e = 4.4\,(60{,}700)^{1/6}\left(\frac{8}{12}\right) = \underline{\underline{18.4\,ft}}$$

8.8

8.8 The wall shear stress in a fully developed flow portion of a 12-in.-diameter pipe carrying water is 1.85 lb/ft^2. Determine the pressure gradient, $\partial p/\partial x$, where x is in the flow direction, if the pipe is **(a)** horizontal, **(b)** vertical with flow up, or **(c)** vertical with flow down.

In general, $\dfrac{\Delta p - \gamma \ell \sin\theta}{\ell} = \dfrac{2\tau}{r}$

Thus, with $\tau = \tau_w$ at $r = \frac{D}{2}$ and $\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$ this becomes

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma \sin\theta$$

a) For a horizontal pipe $\theta = 0$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} = \underline{\underline{-7.40\,\frac{lb}{ft^3}}}$$

b) For vertical flow up $\theta = 90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} - 62.4\,\frac{lb}{ft^3} = \underline{\underline{-69.8\,\frac{lb}{ft^3}}}$$

and

c) For vertical flow down $\theta = -90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} + \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} + 62.4\,\frac{lb}{ft^3} = \underline{\underline{55.0\,\frac{lb}{ft^3}}}$$

8.9

8.9 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

For a horizontal pipe $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ or $\tau = \frac{r}{2}\frac{\Delta p}{\ell}$

Thus,

$$\tau = r\,\frac{(0.6 \times 144\,\frac{lb}{ft^2})}{2(12\ ft)} = 3.6\,r\,\frac{lb}{ft^2}\text{, where } r \sim ft$$

Hence,

$$\tau_w = 3.6\left(\frac{0.5}{12}\right) = \underline{\underline{0.15\,\frac{lb}{ft^2}}}$$

and with $r = (0.5 - 0.3)\,in. = 0.2\,in.$,

$$\tau = 3.6\left(\frac{0.2}{12}\right) = \underline{\underline{0.06\,\frac{lb}{ft^2}}}$$

Finally, with $r = (0.5 - 0.5)\,in. = 0\,in.$ $\quad \underline{\underline{\tau = 0}}$

8.10

8.10 Repeat Problem 8.9 if the pipe is on a 20° hill. Is the flow up or down the hill? Explain.

For a pipe on a hill $\frac{\Delta p}{\ell} = \frac{2\tau}{r} + \gamma \sin\theta$, where $\theta = \pm 20°$

Assume the flow is uphill: $\theta = +20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma \sin\theta\right]$ or $\tau_w = \frac{1}{2}\left(\frac{0.5}{12}\,ft\right)\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\,ft} - 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

or

$\tau_w = -0.295\,\frac{lb}{ft^2}$ Since we must have $\tau_w > 0$, the flow must not be uphill.

Assume the flow is downhill: $\theta = -20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma \sin\theta\right]$ or $\tau = \frac{r}{2}\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\,ft} + 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

$= 14.3\,r\,\frac{lb}{ft^2}$, where $r \sim ft$. The $\underline{\underline{\text{flow is downhill}}}$

Hence, with $r = \frac{D}{2}$

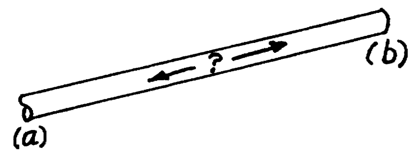
$$\tau_w = 14.3\left(\frac{0.5}{12}\right) = \underline{\underline{0.596\,\frac{lb}{ft^2}}}$$

With $r = (0.5 - 0.3)\,in. = 0.2\,in.$,

$$\tau = 14.3\left(\frac{0.2}{12}\right) = \underline{\underline{0.238\,\frac{lb}{ft^2}}}$$

With $r = (0.5 - 0.5)\,in. = 0$, $\quad \underline{\underline{\tau = 0}}$

8.11 Water flows in a constant diameter pipe with the following conditions measured: At section (a) $p_a = 32.4$ psi and $z_a = 56.8$ ft; at section (b) $p_b = 29.7$ psi and $z_b = 68.2$ ft. Is the flow from (a) to (b) or from (b) to (a)? Explain.



Assume the flow is uphill. Thus, $\frac{p_a}{\gamma} + \frac{V_a^2}{2g} + z_a = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} + z_b + h_L$

or with $V_a = V_b$,

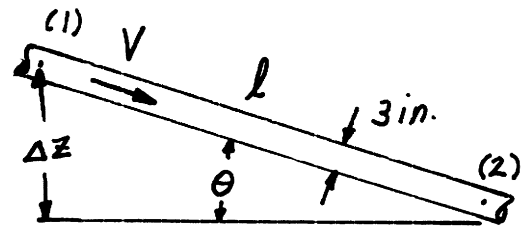
$$h_L = \frac{p_a}{\gamma} + z_a - \frac{p_b}{\gamma} - z_b = \frac{(32.4\,psi - 29.7\,psi)(144\,\frac{in^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + 56.8\,ft - 68.2\,ft$$

or

$h_L = -5.17\,ft < 0$, which is impossible. Thus, the flow is downhill, from (b) to (a).

8.12

8.12 Water flows downhill through a 3-in.-diameter steel pipe. The slope of the hill is such that for each mile (5280 ft) of horizontal distance, the change in elevation is Δz ft. Determine the maximum value of Δz if the flow is to remain laminar and the pressure all along the pipe is constant.



For laminar flow $V = \dfrac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell}$ (1)

where for this case

$\Delta p = 0$, $\ell = 5280\ ft$, $D = 0.25\ ft$ and for maximum Δz, $Re = 2100$

Thus,

$\dfrac{\rho V D}{\mu} = 2100$ or $V = \dfrac{2100 \mu}{\rho D} = \dfrac{2100 (2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2})}{1.94 \frac{slug}{ft^3} (0.25 ft)} = 0.101 \dfrac{ft}{s}$

Hence, from Eq. (1):

$\sin\theta = -\dfrac{32 \mu V}{\gamma D^2} = -\dfrac{32 (2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2})(0.101 \frac{ft}{s})}{62.4 \frac{lb}{ft^3} (0.25 ft)^2} = -1.94 \times 10^{-5}$

or

$\Delta z = -\ell \sin\theta = -(5280 ft) \times (-1.94 \times 10^{-5}) = \underline{\underline{0.102\ ft}}$

Note: Could use the energy equation

$z_1 - f \dfrac{\ell}{D} \dfrac{V^2}{2g} = z_2$ with $Re = 2100$ so that $V = 0.101 \frac{ft}{s}$ and

$f = \dfrac{64}{Re} = \dfrac{64}{2100} = 0.0305$

and obtain the same result: $z_1 - z_2 = 0.102\ ft$

8.13

8.13 Some fluids behave as a non-Newtonian power-law fluid characterized by $\tau = -C(du/dr)^n$, where $n = 1, 3, 5$, and so on, and C is a constant. (If $n = 1$, the fluid is the customary Newtonian fluid.) For flow in a round pipe of a diameter D, integrate the force balance equation (Eq. 8.3) to obtain the velocity profile

$$u(r) = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{1/n}\left[r^{(n+1)/n} - \left(\frac{D}{2}\right)^{(n+1)/n}\right]$$

For any fluid $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ so that with $\tau = -C\left(\frac{du}{dr}\right)^n$ we obtain

$$\frac{\Delta p}{\ell} = -\frac{2C}{r}\left(\frac{du}{dr}\right)^n \quad \text{or} \quad \frac{du}{dr} = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} r^{\frac{1}{n}} \quad *$$

or

$-\int du = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\int r^{\frac{1}{n}}dr$ which integrates to give

$u = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}\, r^{\left(\frac{n+1}{n}\right)} + C_1$, where C_1 is a constant. (1)

The fluid sticks to the pipe so that $u = 0$ at $r = \frac{D}{2}$.

Hence, from Eq. (1)

$$C_1 = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$$

so that

$$u = \frac{n}{(n+1)}\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\left[-r^{\frac{(n+1)}{n}} + \left(\frac{D}{2}\right)^{\frac{(n+1)}{n}}\right]$$

*Note: Since we are considering only odd integer values for n we can use the fact that if

$\left(\frac{du}{dr}\right)^n = -K$, where $K > 0$, then $\frac{du}{dr} = -K^{\frac{1}{n}}$

so that $\frac{du}{dr} < 0$.

8.14*

8.14* For the flow discussed in Problem 8.13, plot the dimensionless velocity profile u/V_c, where V_c is the centerline velocity (at $r = 0$), as a function of the dimensionless radial coordinate $r/(D/2)$, where D is the pipe diameter. Consider values of $n = 1, 3, 5,$ and 7.

From Problem 8.13,

$$u(r) = \frac{n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left[-r^{\left(\frac{n+1}{n}\right)} + \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}\right] \quad (1)$$

Let $V_c = u(r=0)$, or $V_c = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$ (2)

Note: For $\tau = C\left(\frac{du}{dr}\right)^n$ with $\frac{du}{dr} < 0$ and n an odd integer, to have $\tau > 0$, we must have $C < 0$. Thus, from Eq. (2), $V_c > 0$ as it must.

By dividing Eq. (1) by Eq. (2) we obtain

$$\frac{u}{V_c} = 1 - \left[\frac{r}{\left(\frac{D}{2}\right)}\right]^{\left(\frac{n+1}{n}\right)}$$

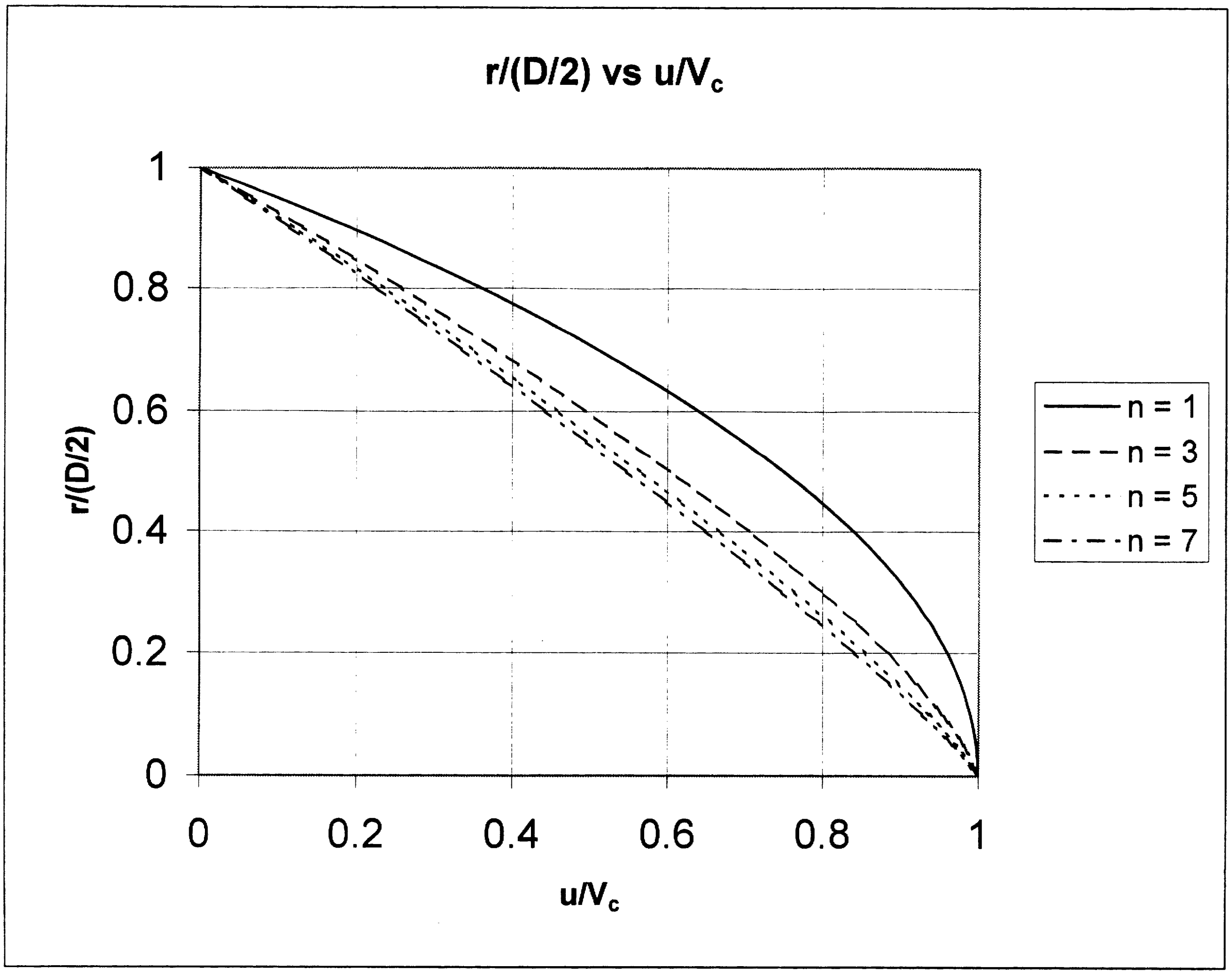
This result is plotted below for $n = 1, 3, 5,$ and 7, with $0 \le \frac{r}{\left(\frac{D}{2}\right)} \le 1$.

An EXCEL program was used to do the calculations and plotting.

	n = 1	n = 3	n = 5	n = 7
r/(D/2)	u/V$_c$	u/V$_c$	u/V$_c$	u/V$_c$
0	1	1	1	1
0.05	0.998	0.982	0.973	0.967
0.1	0.990	0.954	0.937	0.928
0.15	0.978	0.920	0.897	0.886
0.2	0.960	0.883	0.855	0.841
0.25	0.938	0.843	0.811	0.795
0.3	0.910	0.799	0.764	0.747
0.35	0.878	0.753	0.716	0.699
0.4	0.840	0.705	0.667	0.649
0.45	0.798	0.655	0.616	0.599
0.5	0.750	0.603	0.565	0.547
0.55	0.698	0.549	0.512	0.495
0.6	0.640	0.494	0.458	0.442
0.65	0.578	0.437	0.404	0.389
0.7	0.510	0.378	0.348	0.335
0.75	0.438	0.319	0.292	0.280
0.8	0.360	0.257	0.235	0.225
0.85	0.278	0.195	0.177	0.170
0.9	0.190	0.131	0.119	0.113
0.95	0.097	0.066	0.060	0.057
1	0.000	0.000	0.000	0.000

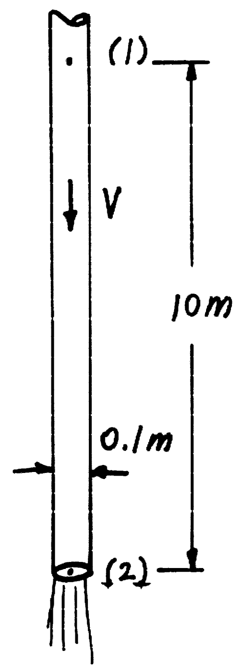
(con't)

8.14* (con't)



8.15

8.15 A fluid of density $\rho = 1000\ \text{kg/m}^3$ and viscosity $\mu = 0.30\ \text{N}\cdot\text{s/m}^2$ flows steadily down a vertical 0.10-m-diameter pipe and exits as a free jet from the lower end. Determine the maximum pressure allowed in the pipe at a location 10 m above the pipe exit if the flow is to be laminar.



Re = 2100 for maximum pressure.

Thus,

$$2100 = \frac{\rho V D}{\mu} = \frac{1000 \frac{kg}{m^3} V (0.1m)}{0.30 \frac{N \cdot s}{m^2}}$$

or

$$V = 6.30 \frac{m}{s}$$

But for laminar flow,

$$V = \frac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell} \quad \text{where } D = 0.1m,\ \ell = 10m, \text{ and } \theta = -90°$$

$$\gamma = \rho g = 9{,}810 \frac{N}{m^3}$$

Thus,

$$6.30 \frac{m}{s} = \frac{(\Delta p - 9{,}810 \frac{N}{m^3}(10m)\sin(-90°))(0.1m)^2}{32 \left(0.30 \frac{N \cdot s}{m^2}\right)(10m)}$$

so that

$$\Delta p = -3.76 \times 10^4 \frac{N}{m^2} = \underline{\underline{-37.6\ kPa}}$$

8.16

8.16 Water is pumped steadily from one large, open tank to another at the same elevation as shown in Fig. P8.16. Determine the maximum power the pump can add to the water if the flow is to remain laminar.

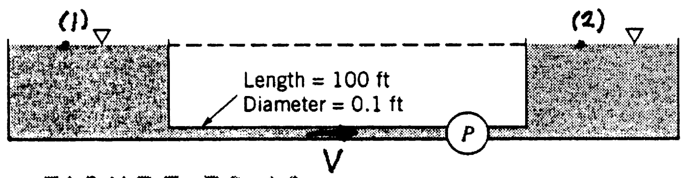


FIGURE P8.16

Maximum pump power with laminar flow means that $Re = 2100$.
Thus,

$$\frac{\rho V D}{\mu} = 2100 \quad \text{or}$$

$$V = \frac{2100\mu}{\rho D} = \frac{2100\left(2.34\times10^{-5}\ \frac{lb\cdot s}{ft^2}\right)}{1.94\ \frac{slug}{ft^3}(0.1\ ft)} = 0.253\ \frac{ft}{s}$$

Also,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_s = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$$p_1 = p_2 = 0,\quad V_1 = V_2 = 0,\quad z_1 = z_2$$

Thus,

$$h_s = h_L = f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{where } f = \frac{64}{Re} = \frac{64}{2100} = 0.0305$$

so that

$$h_s = 0.0305\ \frac{100\ ft}{0.1\ ft}\ \frac{\left(0.253\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} = 0.0303\ ft$$

Hence,

$$\dot{W}_s = \gamma Q h_s = \left(62.4\ \frac{lb}{ft^3}\right)\frac{\pi}{4}(0.1\ ft)^2\left(0.253\ \frac{ft}{s}\right)(0.0303\ ft)$$

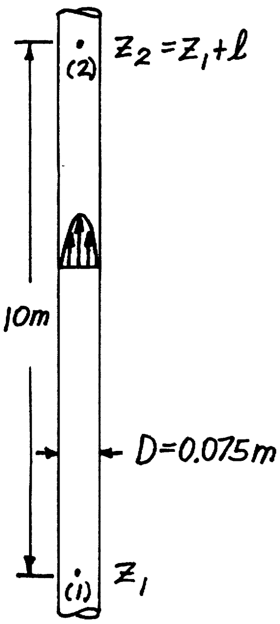
$$= \underline{\underline{0.00376\ \frac{ft\cdot lb}{s}}}$$

8.17

8.17 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

$\rho = 1260 \frac{kg}{m^3}$

$\mu = 1.50 \frac{N \cdot s}{m^2}$



For laminar flow in a pipe,

V = average velocity $= \frac{1}{2} V_{max} = \frac{1}{2}(1 \frac{m}{s}) = 0.5 \frac{m}{s}$

Thus,

$$Re = \frac{\rho V D}{\mu} = \frac{(1260 \frac{kg}{m^3})(0.5 \frac{m}{s})(0.075\,m)}{1.50 \frac{N \cdot s}{m^2}} = 31.5 < 2100$$

The flow is laminar so that

$$V = \frac{(\Delta p - \gamma \ell \sin\theta) D^2}{32 \mu \ell}, \text{ where } \theta = 90°$$

Thus,

$$\Delta p = \frac{32 \mu \ell V}{D^2} + \gamma \ell = \frac{32(1.50 \frac{N \cdot s}{m^2})(10m)(0.5 \frac{m}{s})}{(0.075\,m)^2} + (9.81 \frac{m}{s^2})(1260 \frac{kg}{m^3})(10m)$$

$$= 1.66 \times 10^5 \frac{N}{m^2}, \text{ or } \Delta p = \underline{\underline{166\ kPa}}$$

Also,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L, \text{ or with } V_1 = V_2,\ z_2 - z_1 = \ell, \text{ and}$$

$p_1 = p_2 + \Delta p$ this gives

$$h_L = \frac{\Delta p}{\gamma} - \ell = \frac{1.66 \times 10^5 \frac{N}{m^2}}{(9.81 \frac{m}{s^2})(1260 \frac{kg}{m^3})} - 10m = \underline{\underline{3.43\,m}}$$

8.18

8.18 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

$h_L = f \frac{\ell}{D} \frac{V^2}{2g}$, where since $Re = 1500 < 2100$ the flow is laminar.

Thus, $f = 64/Re = 64/1500 = 0.0427$ so that

$$6.4\,ft = 0.0427 \frac{20\,ft}{(0.1/12\,ft)} \frac{V^2}{2(32.2\,ft/s^2)}$$

or

$V = \underline{\underline{2.01 \frac{ft}{s}}}$

8.19

8.19 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

For laminar flow in a pipe

$u(r) = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$, where $D = 0.1\,m$ and $u = 0.8\,\frac{m}{s}$ at

$r = \frac{0.1m}{2} - 0.012m = 0.038m$

Thus,

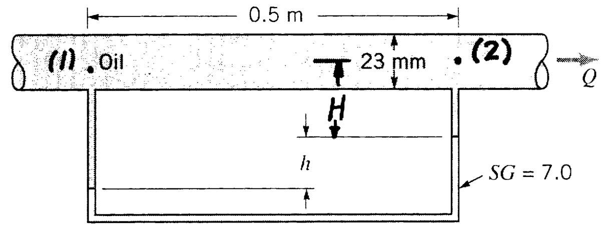
$0.8\,\frac{m}{s} = V_c\left[1 - \left(\frac{2(0.038m)}{0.10m}\right)^2\right]$ or $V_c = \underline{\underline{1.89\,\frac{m}{s}}}$

so that

$Q = \frac{\pi}{4}D^2 V = \frac{\pi}{4}D^2(0.5V_c) = \frac{\pi}{4}(0.1m)^2(0.5)(1.89\,\frac{m}{s}) = \underline{\underline{7.42 \times 10^{-3}\,\frac{m^3}{s}}}$

8.20

8.20 Oil (specific weight = 8900 N/m^3, viscosity = 0.10 N·s/m^2) flows through a horizontal 23-mm-diameter tube as shown in Fig. P8.20. A differential U-tube manometer is used to measure the pressure drop along the tube. Detrmine the range of values for h for laminar flow.



For laminar flow $Re \leq 2100$, or $\frac{\rho V D}{\mu} \leq 2100$

where $\rho = \gamma / g$. Thus, the minimum h is $h = 0$ (no flow) and the maximum h is for $Re = 2100$.

Hence,

$$2100 = \frac{\left(\frac{8900\ N/m^3}{9.81\ m/s^2}\right) V (0.023\ m)}{0.1\ N\cdot s/m^2}$$

or

$V = 10.06\ m/s$

For the flowing fluid, $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f \frac{\ell}{D} \frac{V^2}{2g}$

where $z_1 = z_2$ and $V_1 = V_2 = V$

Thus,

$p_1 - p_2 = f \frac{\ell}{D} \frac{V^2}{2g} \gamma$ where for laminar flow

$f = \frac{64}{Re}$, or $f = \frac{64}{2100} = 0.0305$

Hence,

$$\Delta p = p_1 - p_2 = 0.0305 \frac{0.5\ m}{0.023\ m} \frac{(10.06\ m/s)^2}{2(9.81\ m/s^2)} (8900\ N/m^3)$$

or

$\Delta p = 30{,}400\ N/m^2$

From manometer equations:

$p_1 + \gamma (H + h) - SG\, \gamma_{H_2O}\, h - \gamma H = p_2$, or

$\Delta p = p_1 - p_2 = (SG\, \gamma_{H_2O} - \gamma) h$

Thus,

$$h = \frac{30{,}400\ N/m^2}{(7(9800\ N/m^3) - 8900\ N/m^3)} = 0.509\ m$$

Hence $\underline{\underline{0 \leq h \leq 0.509\ m}}$

8.21

8.21 A fluid flows in a smooth pipe with a Reynolds number of 6000. By what percent would the head loss be reduced if the flow could be maintained as laminar flow rather than the expected turbulent flow?

For either laminar or turbulent flow

$h_L = f \frac{\ell}{D} \frac{V^2}{2g}$. Thus, with the same $V, \ell, D,$ and g

$$\frac{h_{L\,lam}}{h_{L\,turb}} = \frac{f_{lam}}{f_{turb}}$$

If the flow is laminar $f_{lam} = \frac{64}{Re} = \frac{64}{6000} = 0.0107$

If the flow is turbulent with $Re = 6000$ and $\frac{\varepsilon}{D} = 0$, then from the Moody chart (Fig. 8.20) $f_{turb} = 0.035$

Thus,

$$\frac{h_{L\,lam}}{h_{L\,turb}} = \frac{0.0107}{0.035} = 0.486$$

The headloss would be reduced by

$(h_{L\,turb} - h_{L\,lam})/h_{L\,turb} = 1 - 0.486 = 0.514$, or $\underline{\underline{51.4\,\%}}$

8.22

8.22 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}\ m^2/s$ flows through the vertical pipe shown in Fig. P8.22 at a rate of $4 \times 10^{-4}\ m^3/s$. Determine the manometer reading, h.

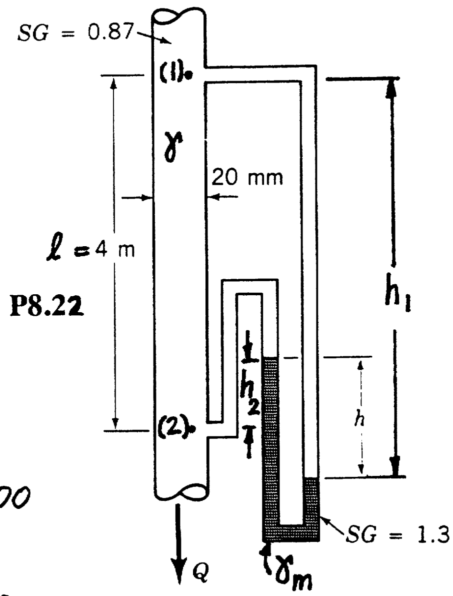


FIGURE P8.22

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27\frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2\times10^{-4}\frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p + \gamma \ell)D^4}{128\mu \ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128\mu \ell Q}{\pi D^4} - \gamma \ell \qquad (1)$$

Hence, with $\gamma = SG\,\gamma_{H_2O} = 0.87(9.81\frac{kN}{m^3}) = 8.53\frac{kN}{m^3}$ and

$$\mu = \nu\rho = \nu\, SG\,\rho_{H_2O} = (2.2\times10^{-4}\frac{m^2}{s})(0.87)(1000\frac{kg}{m^3}) = 0.191\ \frac{N\cdot s}{m^2}$$

Eq. (1) gives

$$\Delta p = \frac{128(0.191\frac{N\cdot s}{m^2})(4m)(4\times10^{-4}\frac{m^3}{s})}{\pi(0.020m)^4} - (8.53\frac{kN}{m^3})(4m)(10^3\frac{N}{kN})$$

or $\Delta p = 4.37\times10^4\frac{N}{m^2} = 43.7\frac{kN}{m^2}$ (2)

From manometer considerations

$p_1 + \gamma h_1 - \gamma_m h + \gamma h_2 = p_2$, where $\gamma_m = SG_m \gamma_{H_2O} = 1.3(9.81\frac{kN}{m^3}) = 12.74\frac{kN}{m^3}$

and $h_1 = h - h_2 + \ell$, or $h_2 + h_1 = h + \ell$

Thus,

$$p_1 - p_2 = \Delta p = -\gamma(h_2 + h_1) + \gamma_m h = (\gamma_m - \gamma)h - \gamma\ell \qquad (3)$$

Combine Eqs. (2) and (3) to give

$$43.7\frac{kN}{m^2} = (12.74 - 8.53)\frac{kN}{m^3}h - (8.53\frac{kN}{m^3})(4m)$$

or

$$h = \underline{\underline{18.5\ m}}$$

8.23

8.23 Determine the manometer reading, h, for Problem 8.22 if the flow is up rather than down the pipe. Note: The manometer reading will be reversed.

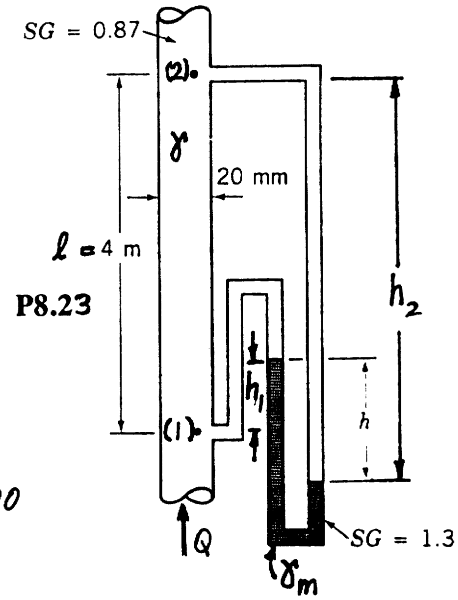


FIGURE P8.23

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27\frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2\times10^{-4}\frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p - \gamma \ell)D^4}{128\mu\ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} + \gamma\ell \qquad (1)$$

Hence, with $\gamma = SG\,\gamma_{H_2O} = 0.87(9.81\frac{kN}{m^3}) = 8.53\frac{kN}{m^3}$ and

$$\mu = \nu\rho = \nu\, SG\, \rho_{H_2O} = (2.2\times10^{-4}\frac{m^2}{s})(0.87)(1000\frac{kg}{m^3}) = 0.191\frac{N\cdot s}{m^2}$$

Eq. (1) gives

$$\Delta p = \frac{128(0.191\frac{N\cdot s}{m^2})(4m)(4\times10^{-4}\frac{m^3}{s})}{\pi(0.020m)^4} + (8.53\frac{kN}{m^3})(4m)(10^3\frac{N}{kN})$$

or

$$\Delta p = 1.119\times10^5\frac{N}{m^2} = 111.9\frac{kN}{m^2} \qquad (2)$$

From manometer considerations

$$p_1 - \gamma h_1 + \gamma_m h - \gamma h_2 = p_2, \text{ where } \gamma_m = SG_m\gamma_{H_2O} = 1.3(9.81\frac{kN}{m^3}) = 12.74\frac{kN}{m^3}$$

$$\text{and } h_2 = \ell + h - h_1 \text{ or } h_2 + h_1 = \ell + h$$

Thus,

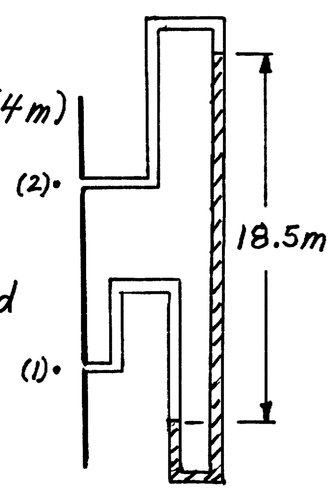
$$p_1 - p_2 = \Delta p = \gamma(h_2 + h_1) - \gamma_m h = -(\gamma_m - \gamma)h + \gamma\ell \qquad (3)$$

Combine Eqs. (2) and (3) to give

$$111.9\frac{kN}{m^2} = -(12.74 - 8.53)\frac{kN}{m^3}h + 8.53\frac{kN}{m^3}(4m)$$

or

$$\underline{\underline{h = -18.5m}}$$



Note: Since $h < 0$ the manometer is displaced in the direction opposite that shown in the original figure.

8.24

8.24 For Problem 8.22, what flowrate (magnitude and direction) will cause $h = 0$?

From Prob. 8.22, $\Delta p = (\gamma_m - \gamma)h - \gamma \ell$ and $Q = \dfrac{\pi(\Delta p + \gamma \ell)D^4}{128 \mu \ell}$

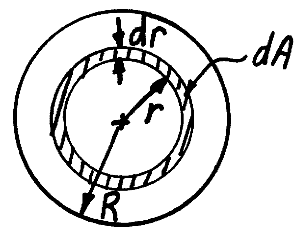
Thus, with $h=0$, $\Delta p = -\gamma \ell$ and

$$Q = \frac{\pi(-\gamma\ell + \gamma\ell)D^4}{128\mu\ell} = \underline{\underline{0}}$$

Note that $\Delta p \neq 0$, but $Q = 0$ since $\Delta p + \gamma \ell = 0$

8.25

8.25 The kinetic energy coefficient, α, is defined in Eq. 5.86. Show that its value for a power-law turbulent velocity profile (Eq. 8.31) is given by $\alpha = (n+1)^3(2n+1)^3/[4n^4(n+3)(2n+3)]$.



From Eq. 5.86, $\alpha = \dfrac{\rho \int \bar{u}^3 dA}{\rho A V V^2}$ where V = average velocity, $A = \pi R^2$,

and $\bar{u} = V_c\left[1 - \frac{r}{R}\right]^{\frac{1}{n}}$. From Example 8.4, $V = \dfrac{2n^2 V_c}{(n+1)(2n+1)}$ (0)

Thus, with $dA = 2\pi r\, dr$

$$\alpha = \frac{\int \bar{u}^3 dA}{A V^3}, \text{ where } \int \bar{u}^3 dA = 2\pi \int_{r=0}^{R} V_c^3 \left[1 - \frac{r}{R}\right]^{\frac{3}{n}} r\, dr = 2\pi R^2 V_c^3 \int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy \quad (1)$$

where $y = \frac{r}{R}$.

Let $x = 1-y$ so that $y = 1-x$ and $dy = -dx$.

Hence,
$$\int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy = -\int_{x=1}^{0} x^{3/n}(1-x)\, dx = \int_{0}^{1} (x^{\frac{3}{n}} - x^{\frac{3}{n}+1}) dx$$

$$= \frac{n}{n+3} x^{\frac{n+3}{n}} - \frac{n}{2n+3} x^{\frac{2n+3}{n}} \Bigg|_{x=0}^{x=1}$$

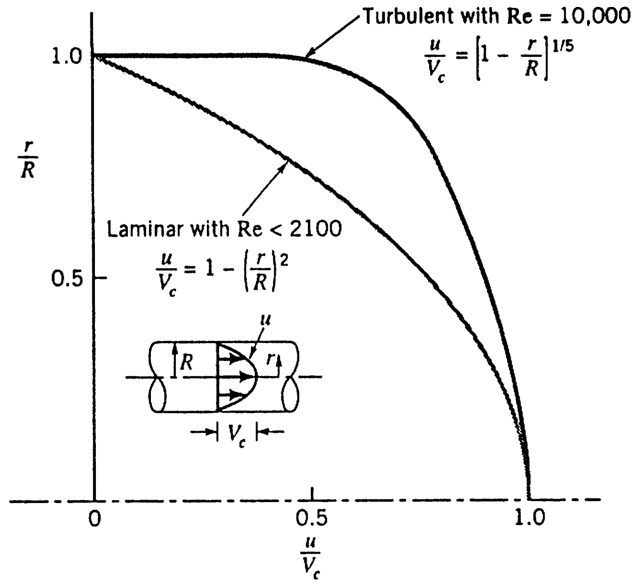
Thus,
$$\int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy = \frac{n^2}{(n+3)(2n+3)} \quad (2)$$

From Eqs. (0), (1), and (2):

$$\alpha = \frac{2\pi R^2 V_c^3 \dfrac{n^2}{(n+3)(2n+3)}}{\pi R^2 \left[\dfrac{2n^2 V_c}{(n+1)(2n+1)}\right]^3} = \underline{\underline{\frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)}}}$$

8.26

8.26 As shown in Video V8.3 and Fig. P8.26, the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. **(a)** For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. **(b)** Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.26

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\, dA = \int u(2\pi r\, dr) = 2\pi \int_{r=0}^{R} u r\, dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \left(\frac{r}{R}\right)^2\right] dr = 2\pi V_c\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2} V_c$. For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2, \text{ or } \left(\frac{r}{R}\right)^2 = \frac{1}{2} \quad \text{Thus, } r = \frac{1}{\sqrt{2}} R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \frac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\frac{r}{R}\right)\left[1 - \left(\frac{r}{R}\right)\right]^{1/5} d\left(\frac{r}{R}\right)$$

Let $y \equiv 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1 - y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y) y^{1/5} (-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right) dy$$

$$= 2\pi R^2 V_c \left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c \left(\frac{25}{66}\right)$$

or $V = \frac{50}{66} V_c$. For $u = V = \frac{50}{60}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \text{ or } \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750R}}$$

8.27

8.27 Water at 80 °F flows in a 6-in.-diameter pipe with a flowrate of 2.0 cfs. What is the approximate velocity at a distance 2.0 in. away from the wall? Determine the centerline velocity.

$$V = \frac{Q}{A} = \frac{2.0\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12}\,ft\right)^2} = 10.2\,\frac{ft}{s} \text{ so that } Re = \frac{VD}{\nu} = \frac{\left(10.2\,\frac{ft}{s}\right)\left(\frac{6}{12}\,ft\right)}{9.26\times10^{-6}\,\frac{ft^2}{s}} = 5.51\times10^5$$

The flow is turbulent with $\frac{\bar{u}}{V_c} = \left(1-\frac{r}{R}\right)^{\frac{1}{n}}$, where $n \approx 8.3$ (see Fig. 8.1)

Thus, (see Example 8.4)

$$\frac{V}{V_c} = \frac{2n^2}{(n+1)(2n+1)} = \frac{2(8.3)^2}{(8.3+1)(2\times8.3+1)} = 0.842$$

or

$$V_c = \frac{10.2\,\frac{ft}{s}}{0.842} = \underline{\underline{12.1\,\frac{ft}{s}}}$$

Also, at $r = 3\,in. - 2.0\,in. = 1.0\,in.$, $\bar{u} = V_c\left(1-\frac{r}{R}\right)^{\frac{1}{n}} = 12.1\,\frac{ft}{s}\left(1-\frac{1.0\,in.}{3\,in.}\right)^{\frac{1}{8.3}} = \underline{\underline{11.5\,\frac{ft}{s}}}$

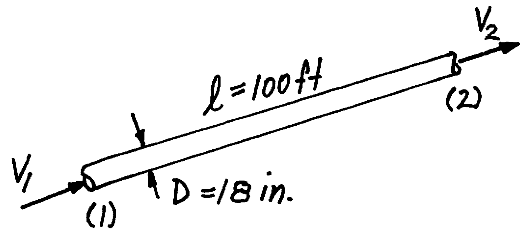
8.28

8.28 During a heavy rainstorm, water from a parking lot completely fills an 18-in.-diameter, smooth, concrete storm sewer. If the flowrate is 10 ft^3/s, determine the pressure drop in a 100-ft horizontal section of the pipe. Repeat the problem if there is a 2-ft change in elevation of the pipe per 100 ft of its length.

In general, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$ (1)

where $p_1 - p_2 = \Delta p$, $\gamma = \rho g$, and

$V = V_1 = V_2 = \frac{Q}{A} = \frac{10\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{18}{12}\,ft\right)^2} = 5.66\,\frac{ft}{s}$



Thus, Eq. (1) gives:

$\Delta p = \gamma(z_2 - z_1) + f\frac{\ell}{D}\frac{1}{2}\rho V^2$ (2)

For a smooth concrete pipe $\frac{\varepsilon}{D} = \frac{0.001\,ft}{\left(\frac{18}{12}\,ft\right)} = 6.67 \times 10^{-4}$ (see Table 8.1)

and $Re = \frac{VD}{\nu} = \frac{(5.66\,\frac{ft}{s})(\frac{18}{12}\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 7.02 \times 10^5$

Thus, from Fig. 8.20 $f = 0.0185$

(a) With $z_1 = z_2$ Eq. (2) gives

$\Delta p = 0.0185\,\frac{(100\,ft)(1.94\,\frac{slugs}{ft^3})}{(\frac{18}{12}\,ft)\;(2)}\left(5.66\,\frac{ft}{s}\right)^2 = 38.3\,\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = \underline{\underline{0.266\,psi}}$

(b) With flow uphill $z_2 - z_1 = 2\,ft$ so that

$\Delta p = \left(62.4\,\frac{lb}{ft^3}\right)(2\,ft)\left(\frac{1\,ft^2}{144\,in.^2}\right) + 0.266\,\frac{lb}{in.^2} = \underline{\underline{1.13\,psi}}$

(c) With flow downhill $z_2 - z_1 = -2\,ft$ so that

$\Delta p = \left(62.4\,\frac{lb}{ft^3}\right)(-2\,ft)\left(\frac{1\,ft^2}{144\,in.^2}\right) + 0.266\,\frac{lb}{in.^2} = \underline{\underline{-0.601\,psi}}$

8.29

8.29 Carbon dioxide at a temperature of 0 °C and a pressure of 600 kPa (abs) flows through a horizontal 40-mm-diameter pipe with an average velocity of 2 m/s. Determine the friction factor if the pressure drop is 235 N/m^2 per 10-m length of pipe.

For a horizontal pipe, $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$, or $f = \frac{2D\Delta p}{\rho \ell V^2}$

where

$$\rho = \frac{p}{RT} = \frac{600\times10^3\,\frac{N}{m^2}}{\left(188.9\,\frac{N\cdot m}{kg\cdot K}\right)(273\,K)} = 11.63\,\frac{kg}{m^3}$$

Thus,

$$f = \frac{2(0.04m)\left(235\frac{N}{m^2}\right)}{\left(11.63\frac{kg}{m^3}\right)(10m)\left(2\frac{m}{s}\right)^2} = \underline{\underline{0.0404}}$$

8.30

8.30 Water flows through a 6-in.-diameter horizontal pipe at a rate of 2.0 cfs and a pressure drop of 4.2 psi per 100 ft of pipe. Determine the friction factor.

For a horizontal pipe, $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$,

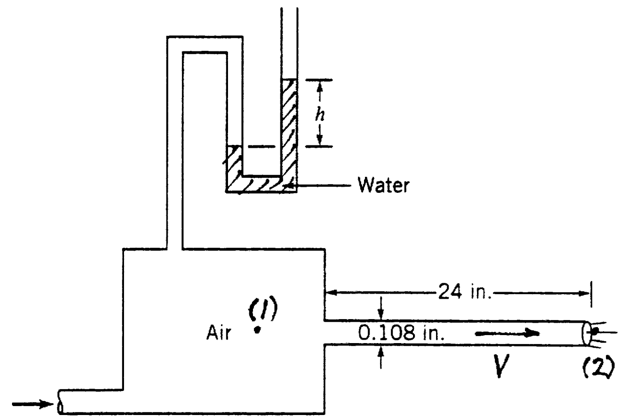
where $V = \frac{Q}{A} = \frac{2.0\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12}ft\right)^2} = 10.2\,\frac{ft}{s}$

Thus,

$$f = \frac{2D\Delta p}{\rho \ell V^2} = \frac{2\left(\frac{6}{12}ft\right)\left(4.2\times144\frac{lb}{ft^2}\right)}{\left(1.94\frac{slugs}{ft^3}\right)(100ft)\left(10.2\frac{ft}{s}\right)^2} = \underline{\underline{0.0300}}$$

8.31

8.31 Air flows through the 0.108-in.-diameter, 24-in.-long tube shown in Fig. P8.31. Determine the friction factor if the flowrate is $Q = 0.00191$ cfs when $h = 1.70$ in. Compare your results with the expression $f = 64/\text{Re}$. Is the flow laminar or turbulent?



■ FIGURE P8.31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

where $z_1 = z_2$, $p_2 = 0$, $V_1 = 0$ and

$$p_1 = \gamma_{H_2O}\, h = (62.4\,\tfrac{lb}{ft^3})(\tfrac{1.7}{12}\,ft) = 8.84\,\tfrac{lb}{ft^2}$$

Also,

$$V = V_2 = \frac{Q}{A} = \frac{0.00191\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.108}{12}\,ft\right)^2} = 30.0\,\frac{ft}{s}$$

Hence, Eq. (1) becomes

$$p_1 = \tfrac{1}{2}\rho V^2\left(1 + f\frac{\ell}{D}\right)$$

or

$$8.84\,\tfrac{lb}{ft^2} = \tfrac{1}{2}(0.00238\,\tfrac{slugs}{ft^3})\left[1 + f\left(\tfrac{24\,in.}{0.108\,in.}\right)\right](30.0\,\tfrac{ft}{s})^2 \quad \text{or} \quad f = \underline{\underline{0.0326}}$$

Also $\text{Re} = \frac{DV}{\nu}$

or

$$\text{Re} = \frac{\left(\frac{0.108}{12}\,ft\right)(30.0\,\frac{ft}{s})}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 1720 < 2100\text{, the flow is } \underline{\underline{\text{laminar}}}$$

Note:

$$\frac{64}{\text{Re}} = \frac{64}{1720} = 0.0372$$

8.33

8.33 Determine the thickness of the viscous sublayer in a smooth 8-in.-diameter pipe if the Reynolds number is 25,000.

$\delta_s = \frac{5\nu}{u^*}$, where $u^* = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$
we obtain $\tau_w = \frac{\rho f V^2}{8}$ and $u^* = \sqrt{\frac{f}{8}}\,V$

Thus,

$$\delta_s = \frac{5\nu}{\sqrt{\frac{f}{8}}\,V} = \frac{5\nu D}{\sqrt{\frac{f}{8}}\,VD}, \text{ or } \delta_s = \frac{5D}{Re\sqrt{\frac{f}{8}}} \qquad (1)$$

From Fig. 8.20, for a smooth pipe with $Re = 2.5\times10^4$, $f = 0.024$

Thus, from Eq. (1)

$$\delta_s = \frac{5\sqrt{8}\,(\frac{8}{12}\,ft)}{2.5\times10^4\sqrt{0.024}} = \underline{\underline{0.00243\ ft}}$$

8.34

8.34 Water at 60 °F flows through a 6-in.-diameter pipe with an average velocity of 15 ft/s. Approximately what is the height of the largest roughness element allowed if this pipe is to be classified as smooth?

Let h = roughness height. Thus, $h = \delta_s$, where $\delta_s = \frac{5\nu}{u^*}$
with $u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ we obtain
$\tau_w = \frac{\rho f V^2}{8}$ or $u^* = \sqrt{\frac{f}{8}}\,V$

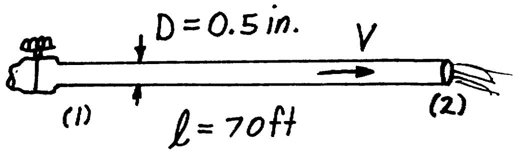
For a smooth pipe with $Re = \frac{VD}{\nu} = \frac{(15\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}} = 6.19\times10^5$ we obtain
from Fig. 8.20 $f = 0.0125$

Thus, $u^* = \left(\frac{0.0125}{8}\right)^{1/2}(15\frac{ft}{s}) = 0.593\frac{ft}{s}$

or $\delta_s = \frac{5\nu}{u^*} = \frac{5(1.21\times10^{-5}\frac{ft^2}{s})}{0.593\frac{ft}{s}} = \underline{\underline{1.02\times10^{-4}\,ft}}$

8.35

8.35 A 70-ft-long, 0.5-in.-diameter hose with a roughness of $\varepsilon = 0.0009$ ft is fastened to a water faucet where the pressure is p_1. Determine p_1 if there is no nozzle attached and the average velocity in the hose is 6 ft/s. Neglect minor losses and elevation changes.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$$, where $z_1 = z_2$, $V_1 = V_2 = V = 6\,\frac{ft}{s}$, and $p_2 = 0$

Thus,

$$p_1 = f\frac{\ell}{D}\frac{1}{2}\rho V^2 \qquad (1)$$

From Fig. 8.20 with $\frac{\varepsilon}{D} = \frac{0.0009\,ft}{\left(\frac{0.5}{12}\,ft\right)} = 2.16 \times 10^{-2}$
and

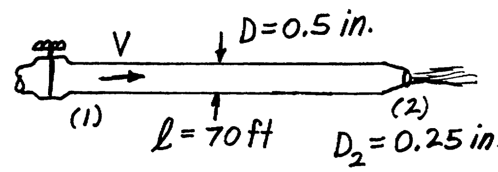
$Re = \frac{VD}{\nu} = \frac{(6\,\frac{ft}{s})(\frac{0.5}{12}\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 2.07 \times 10^4$ we obtain $f = 0.052$

Hence, from Eq. (1)

$$p_1 = (0.052)\frac{70\,ft}{\left(\frac{0.5}{12}\,ft\right)}\frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left(6\,\frac{ft}{s}\right)^2 = 3050\,\frac{lb}{ft^2} = \underline{\underline{21.2\,psi}}$$

8.36

8.36 Repeat Problem 8.35 if there is a nozzle of diameter 0.25 in. attached to the end of the hose.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1 = z_2,\ V_1 = V = 6\,\frac{ft}{s},\ p_2 = 0$$

$$\text{and } V_2 = \frac{V_1 A_1}{A_2} = V_1\left(\frac{D}{D_2}\right)^2 = \left(6\,\frac{ft}{s}\right)(2)^2 = 24\,\frac{ft}{s}$$

Thus,

$$p_1 = \left(V_2^2 + f\frac{\ell}{D}V^2\right)\frac{\gamma}{2g} = \frac{1}{2}\rho\left(V_2^2 + f\frac{\ell}{D}V^2\right) \qquad (1)$$

From Fig. 8.20 with $\frac{\varepsilon}{D} = \frac{0.0009\,ft}{\left(\frac{0.5}{12}\,ft\right)} = 2.16 \times 10^{-2}$

and

$Re = \frac{VD}{\nu} = \frac{\left(6\,\frac{ft}{s}\right)\left(\frac{0.5}{12}\,ft\right)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 2.07 \times 10^4$ we obtain $f = 0.052$

Hence, from Eq. (1)

$$p_1 = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left(\left(24\,\frac{ft}{s}\right)^2 + (0.052)\frac{70\,ft}{\left(\frac{0.5}{12}\,ft\right)}\left(6\,\frac{ft}{s}\right)^2\right) = 3609\,\frac{lb}{ft^2} = \underline{\underline{25.1\,\frac{lb}{in^2}}}$$

Note: To maintain the same flowrate with the nozzle attached as compared to that without the nozzle (see Prob. 8.35) the pressure must be increased from 21.2 psi to 25.1 psi.

8.37*

8.37* The following equation is sometimes used in place of the Colebrook equation (Eq. 8.35):

$$f = \frac{1.325}{[\ln[(\varepsilon/3.7D) + (5.74/\mathrm{Re}^{0.9})]]^2}$$

for $10^{-6} < \varepsilon/D < 10^{-2}$ and $5000 < \mathrm{Re} < 10^{+8}$ (Ref. 22, pg. 220). An advantage of this equation is that given Re and ε/D, it does not require an iteration procedure to obtain f. Plot a graph of the percent difference in f as given by this equation and the original Colebrook equation for Reynolds numbers in the range of validity of the above equation, with $\varepsilon/D = 10^{-4}$.

Let $\Delta f = \dfrac{f_{app} - f}{f_{app}}$, where f_{app} = approximate result obtained from

$$f_{app} = \frac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$ and f = result given by Eq. 8.35

$$\frac{1}{\sqrt{f}} = -2.0 \log\left[\frac{\varepsilon}{3.7D} + \frac{2.5}{Re\sqrt{f}}\right]$$

Thus, with $\frac{\varepsilon}{D} = 10^{-4}$ these become

$$f_{app} = \frac{1.325}{\left[\ln\left(2.70\times10^{-5} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad (1)$$

and

$$\frac{1}{\sqrt{f}} = -2.0 \log\left[2.70\times10^{-5} + \frac{2.5}{Re\sqrt{f}}\right] \qquad (2)$$

For $5\times10^3 \le Re \le 10^8$, calculate and plot $100\Delta f = \dfrac{100(f_{app} - f)}{f}$, where f_{app} and f are obtained from Eqs. (1) and (2).

Program P8#37 shown below was used for the calculations.

```
100 cls
120 print "**********************************************************"
130 print "**  This program calculates the difference            **"
140 print "**  between the friction factor given by the          **"
150 print "**  Colebrook equation and that given by the          **"
160 print "**  approximate formula provided.  The Cole-          **"
170 print "**  brook result is determined by an iterative        **"
180 print "**  routine.                                          **"
190 print "**********************************************************"
200 rr = 1E-4
210 Re = 2500
220 print "     Re          f       faprox      f - faprox, %"
230 for i = 1 to 16
240 Re = Re*2
250 faprox = 1.325/(log(rr/3.7 + (5.74/Re^0.9)))^2
```

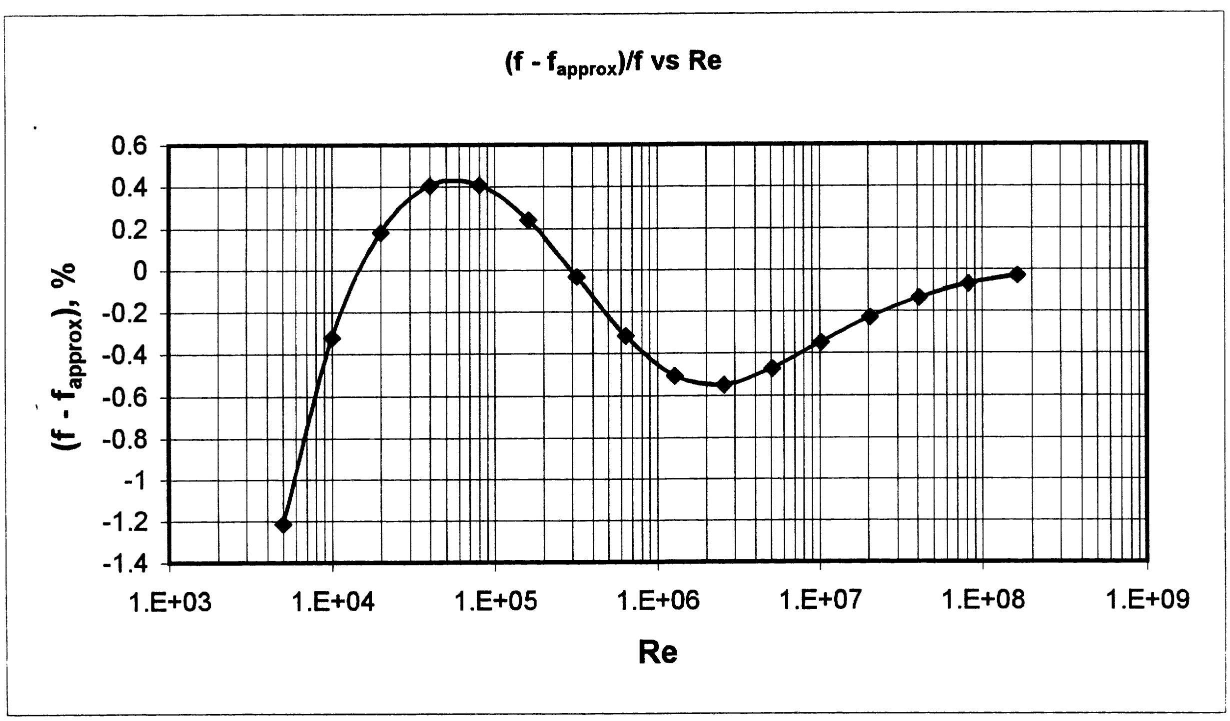
(con't)

8.37* (con't)

```
260 fp = faprox
270 goto 290
280 fp = f
290 f = 1/(-2.0*log(rr/3.7 + 2.51/(Re*fp^0.5))/log(10))^2
300 if abs(1 - f/fp) > 0.000001 then goto 280
310 diff = ((f - faprox)/f)*100
320 print using " #.###^^^^   #.######  #.######   +#.###^^^^";Re,f,faprox,
diff
330 next i
```

```
************************************************
**  This program calculates the difference    **
**  between the friction factor given by the  **
**  Colebrook equation and that given by the  **
**  approximate formula provided.  The Cole-  **
**  brook result is determined by an iterative **
**  routine.                                  **
************************************************
```

Re	f	faprox	f - faprox, %
5.000E+03	0.037505	0.037961	-1.216E+00
1.000E+04	0.031037	0.031138	-3.233E-01
2.000E+04	0.026101	0.026054	+1.809E-01
4.000E+04	0.022286	0.022196	+4.017E-01
8.000E+04	0.019319	0.019241	+4.047E-01
1.600E+05	0.017026	0.016985	+2.397E-01
3.200E+05	0.015290	0.015295	-3.227E-02
6.400E+05	0.014032	0.014077	-3.176E-01
1.280E+06	0.013179	0.013246	-5.091E-01
2.560E+06	0.012643	0.012713	-5.513E-01
5.120E+06	0.012332	0.012391	-4.748E-01
1.024E+07	0.012162	0.012204	-3.499E-01
2.048E+07	0.012072	0.012100	-2.298E-01
4.096E+07	0.012027	0.012043	-1.362E-01
8.192E+07	0.012003	0.012012	-7.108E-02
1.638E+08	0.011992	0.011995	-2.875E-02



8.38

8.38 Water flows at a rate of 10 gallons per minute in a new horizontal 0.75-in.-diameter galvanized iron pipe. Determine the pressure gradient, $\Delta p/\ell$, along the pipe.

$$Q = 10\,\frac{gal}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,gal}{1728\,in.^3}\right) = 0.0223\,\frac{ft^3}{s}$$

Thus,

$$V = \frac{Q}{A} = \frac{0.0223\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.75}{12}\,ft\right)^2} = 7.27\,\frac{ft}{s}$$

Now, for a horizontal pipe

$$\Delta p = f\,\frac{\ell}{D}\,\frac{1}{2}\rho V^2 \quad \text{where since}$$

$$Re = \frac{VD}{\nu} = \frac{7.27\,\frac{ft}{s}\left(\frac{0.75}{12}\,ft\right)}{1.21\times 10^{-5}\,\frac{ft^2}{s}} = 3.76\times 10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\,ft\right)} = 0.008$$

it follows from Fig. 8.20 that $f = 0.037$

Thus,

$$\frac{\Delta p}{\ell} = \frac{0.037\,(1.94\,slugs/ft^3)\,(7.27\,ft/s)^2}{\left(\frac{0.75}{12}\,ft\right)(2)} = 30.4\,\frac{lb}{ft^3}\left(\frac{1\,ft^2}{144\,in.^2}\right)$$

$$= \underline{\underline{0.211\,psi/ft}}$$

8.41

8.41 Air at standard temperature and pressure flows through a 1-in.-diameter galvanized iron pipe with an average velocity of 8 ft/s. What length of pipe produces a head loss equivalent to (a) a flanged 90° elbow, (b) a wide-open angle valve, or (c) a sharp-edged entrance?

$\ell_{eq} = \frac{K_L D}{f}$, where with $Re = \frac{VD}{\nu} = \frac{(8\frac{ft}{s})(\frac{1}{12}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.25\times10^3$ Thus, with $\frac{\varepsilon}{D} = \frac{0.0005\ ft}{(1/12\ ft)} = 0.006$ (see Table 8.1) we obtain $f = 0.045$ (Fig. 8.20)

Thus, $\ell_{eq} = \frac{K_L(\frac{1}{12}ft)}{0.045} = 1.852 K_L$ or

a) 90° elbow: $K_L = 0.3$ or $\ell_{eq} = \underline{0.556\ ft}$

b) globe valve: $K_L = 2$ or $\ell_{eq} = \underline{3.70\ ft}$

c) sharp entrance: $K_L = 0.5$ or $\ell_{eq} = \underline{0.926\ ft}$

8.42*

8.42* Water at 40 °C flows through drawn tubings with diameters of 0.025, 0.050, or 0.075 m. Plot the head loss in each meter length of pipe for flowrates between 5×10^{-4} m³/s and 50×10^{-4} m³/s. In your solution obtain the friction factor from the Colebrook formula.

$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$, where $V = \frac{Q}{\frac{\pi}{4}D^2}$ Thus, $h_L = f\frac{\ell}{D}\frac{1}{2g}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{8f\ell Q^2}{\pi^2 g D^5}$

or with $\ell = 1m$,

$h_L = \frac{8f(1m)Q^2}{\pi^2(9.81\frac{m}{s^2})D^5}$ or $h_L = 0.0826\frac{fQ^2}{D^5}$, where $h_L \sim m, D \sim m, Q \sim \frac{m^3}{s}$ (1)

For drawn tubing $\varepsilon = 0.0015\ mm = 1.5\times10^{-6}\ m$

or $\frac{\varepsilon}{D} = \frac{1.5\times10^{-6}}{D}$, where $D \sim m$ (2)

Also, $Re = \frac{VD}{\nu} = \frac{4Q}{\pi D\nu} = \frac{4Q}{\pi(6.58\times10^{-7}\frac{m^2}{s})D}$, or $Re = 1.94\times10^6\frac{Q}{D}$. (3)

Note: The minimum Re occurs for Q_{min} and D_{max}. Thus,

$Re_{min} = 1.94\times10^6\frac{5\times10^{-4}}{0.075} = 12{,}900$ The flow is turbulent over the range of parameters considered.

Hence, from Eq. (8.35)

$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right]$ or with $\varepsilon = 1.5\times10^{-6}m$

$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{4.05\times10^{-7}}{D} + \frac{2.51}{Re\sqrt{f}}\right]$ (4)

Thus, for $5\times10^{-4}\frac{m^3}{s} \le Q \le 50\times10^{-4}\frac{m^3}{s}$ and with $D = 0.025\ m$, $0.050\ m$, or $0.075\ m$, determine Re from Eq. (3), f from Eq. (4),

(con't)

8.42* (con't)

and h_L from Eq.(1). These results are calculated and plotted below. ($h_L = h_L(Q, D)$), See Program P8#42 shown below.

```
100 cls
110 open "prn" for output as #1
120 print#1, "*********************************************"
130 print#1, "**  This program calculates the head loss   **"
140 print#1, "**  as a function of flowrate and pipe dia- **"
150 print#1, "**  meter, using an iterative scheme to     **"
160 print#1, "**  determine the friction factor from the  **"
170 print#1, "**  Colebrook formula.                      **"
180 print#1, "*********************************************"
200 D = 0
210 for i = 1 to 3
220 D = D + 0.025
230 rr = 1.5E-6/D
240 print#1, " "
250 print#1, using "For D = #.#### m with e/D = #.###^^^^";D,rr
260 print#1, "  Q, m3/s        Re         f       hL, m"
270 Q = 0
280 for j = 1 to 10
285 f = 0.02
290 Q = Q + 5.0E-4
300 Re = 1.94E+6*Q/D
310 fp = f
320 f = 1/(-2.0*log(rr/3.7+2.51/(Re*fp^0.5))/log(10))^2
330 if abs(1 - f/fp) > 0.0001 then goto 310
340 h = 0.0826*f*Q^2/D^5
350 print#1, using " #.###^^^^  #.###^^^^  #.####  #.###^^^^";Q,Re,f,h
360 next j
370 next i
```

```
*********************************************
**  This program calculates the head loss   **
**  as a function of flowrate and pipe dia- **
**  meter, using an iterative scheme to     **
**  determine the friction factor from the  **
**  Colebrook formula.                      **
*********************************************
```

For D = 0.0250 m with e/D = 6.000E-05

Q, m3/s	Re	f	hL, m
5.000E-04	3.880E+04	0.0223	4.718E-02
1.000E-03	7.760E+04	0.0193	1.629E-01
1.500E-03	1.164E+05	0.0178	3.384E-01
2.000E-03	1.552E+05	0.0169	5.702E-01
2.500E-03	1.940E+05	0.0162	8.563E-01
3.000E-03	2.328E+05	0.0157	1.195E+00
3.500E-03	2.716E+05	0.0153	1.586E+00
4.000E-03	3.104E+05	0.0150	2.028E+00
4.500E-03	3.492E+05	0.0147	2.520E+00
5.000E-03	3.880E+05	0.0145	3.062E+00

(con't)

8.42* (con't)

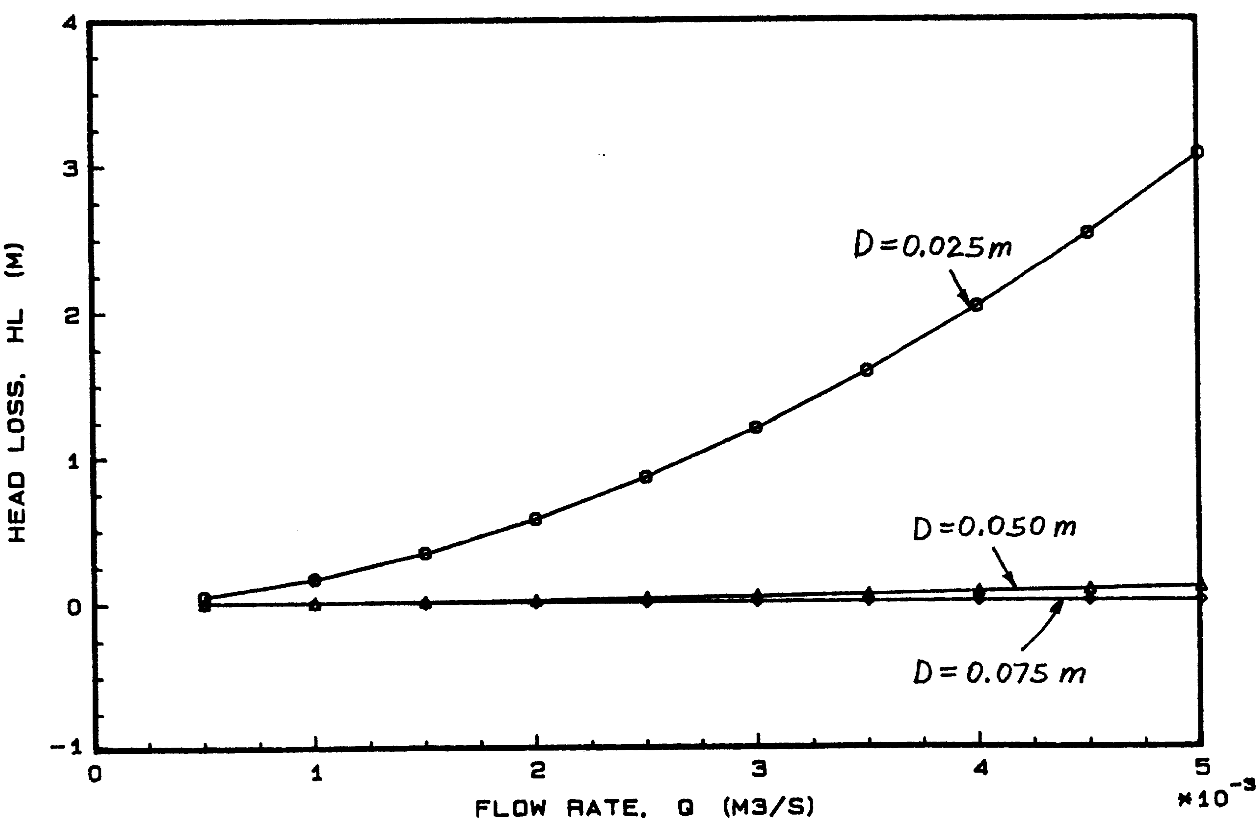
For D = 0.0500 m with e/D = 3.000E-05

Q, m3/s	Re	f	hL, m
5.000E-04	1.940E+04	0.0261	1.727E-03
1.000E-03	3.880E+04	0.0222	5.873E-03
1.500E-03	5.820E+04	0.0203	1.208E-02
2.000E-03	7.760E+04	0.0191	2.021E-02
2.500E-03	9.700E+04	0.0183	3.017E-02
3.000E-03	1.164E+05	0.0176	4.189E-02
3.500E-03	1.358E+05	0.0171	5.532E-02
4.000E-03	1.552E+05	0.0167	7.042E-02
4.500E-03	1.746E+05	0.0163	8.717E-02
5.000E-03	1.940E+05	0.0160	1.055E-01

For D = 0.0750 m with e/D = 2.000E-05

Q, m3/s	Re	f	hL, m
5.000E-04	1.293E+04	0.0289	2.516E-04
1.000E-03	2.587E+04	0.0244	8.483E-04
1.500E-03	3.880E+04	0.0222	1.738E-03
2.000E-03	5.173E+04	0.0208	2.897E-03
2.500E-03	6.467E+04	0.0198	4.313E-03
3.000E-03	7.760E+04	0.0191	5.975E-03
3.500E-03	9.053E+04	0.0185	7.876E-03
4.000E-03	1.035E+05	0.0180	1.001E-02
4.500E-03	1.164E+05	0.0176	1.237E-02
5.000E-03	1.293E+05	0.0172	1.495E-02

PROBLEM P8#42



Note the strong dependence of h_L on D.

8.43

8.43 Air at standard temperature and pressure flows at a rate of 7.0 cfs through a horizontal, galvanized iron duct that has a rectangular cross-sectional shape of 12 in. by 6 in. Estimate the pressure drop per 200 ft of duct.

For a horizontal duct $\Delta p = \gamma h_L = f \frac{\ell}{D_h} \frac{1}{2} \rho V^2$, where $V = \frac{Q}{A}$ (1)

or
$$V = \frac{7 \frac{ft^3}{s}}{(12\,in.)(6\,in.)\left(\frac{1\,ft^2}{144\,in.^2}\right)} = 14.0 \frac{ft}{s} \quad \text{and} \quad Re_h = \frac{V D_h}{\nu}$$

with
$$D_h = \frac{4A}{P} = \frac{4\,(0.5\,ft^2)}{(2+1)\,ft} = 0.667\,ft$$

Thus,
$$Re_h = \frac{(14.0 \frac{ft}{s})(0.667\,ft)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 5.95 \times 10^4$$

Also, for galvanized iron $\varepsilon = 0.0005\,ft$, or $\frac{\varepsilon}{D_h} = \frac{0.0005\,ft}{0.667\,ft} = 0.000750$

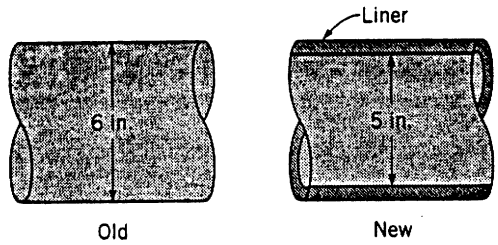
From Fig. 8.20 we obtain $f = 0.0227$

Thus, from Eq. (1) with $\ell = 200\,ft$,

$$\Delta p = (0.0227) \frac{200\,ft}{0.667\,ft} \frac{1}{2} (2.38 \times 10^{-3} \frac{slug}{ft^3})(14.0 \frac{ft}{s})^2 = 1.59 \frac{lb}{ft^2} = \underline{\underline{0.0110\ psi}}$$

8.44

8.44 Water flows at a rate of 2.0 ft³/s in an old, rusty 6-in.-diameter pipe that has a relative roughness of 0.010. It is proposed that by inserting a smooth plastic liner with an inside diameter of 5 in. into the old pipe as shown in Fig. P8.44, the pressure drop per mile can be reduced. Is it true that the lined pipe can carry the required 2.0 ft³/s at a lower pressure drop than in the old pipe? Support your answer with appropriate calculations.



■ FIGURE P8.44

Old pipe: $\frac{\Delta p_o}{\gamma} = f_o \frac{\ell}{D_o}\frac{V_o^2}{2g}$

New pipe: $\frac{\Delta p_n}{\gamma} = f_n \frac{\ell}{D_n}\frac{V_n^2}{2g}$

Thus, $\eta \equiv (\Delta p_n / \Delta p_o) = \left(\frac{f_n V_n^2}{D_n}\right) / \left(\frac{f_o V_o^2}{D_o}\right) = \frac{f_n}{f_o}\frac{D_o}{D_n}\left(\frac{V_n}{V_o}\right)^2$

But,

$V_o = \frac{Q}{A_o} = \frac{2.0\,\frac{ft^3}{s}}{\frac{\pi}{4}(6/12\,ft)^2} = 10.19\,\frac{ft}{s}$ so that

$Re_o = \frac{D_o V_o}{\nu} = \frac{(6/12\,ft)(10.19\,\frac{ft}{s})}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 4.21\times10^5$

From the Moody chart:

$Re = 4.21\times10^5$ and $\frac{\varepsilon}{D} = 0.01$ gives $f_o = 0.038$

Similarly,

$V_n = \frac{Q}{A_n} = \frac{2.0\,\frac{ft^3}{s}}{\frac{\pi}{4}(5/12\,ft)^2} = 14.7\,\frac{ft}{s}$ so that

$Re_n = \frac{D_n V_n}{\nu} = \frac{(5/12\,ft)(14.7\,\frac{ft}{s})}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 5.06\times10^5$

From the Moody chart:

$Re = 5.06\times10^5$ and $\frac{\varepsilon}{D} = 0$ gives $f_n = 0.013$

Hence,

$$\eta = \left(\frac{0.013}{0.038}\right)\left(\frac{6\text{ in.}}{5\text{ in.}}\right)\left(\frac{14.7\,\frac{ft}{s}}{10.19\,\frac{ft}{s}}\right)^2 = 0.854$$

or $\Delta p_n = 0.854\,\Delta p_o$ Yes, the new pipe has a lower Δp.

8.46

8.46 To conserve water and energy, a "flow reducer" is installed in the shower head as shown in Fig. P8.46. If the pressure at point (1) remains constant and all losses except for that in the "flow reducer" are neglected, determine the value of the loss coefficient (based on the velocity in the pipe) of the "flow reducer" if its presence is to reduce the flowrate by a factor of 2. Neglect gravity.

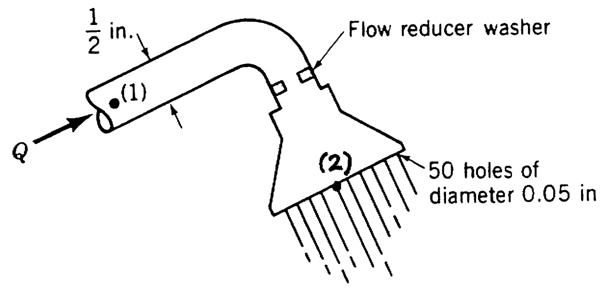


FIGURE P8.46

Without the reducer $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$, where $p_2=0$, $z_1=z_2$ and

$$V_1=\frac{Q}{A_1}=\frac{4Q}{\pi D_1^2}=\frac{4Q}{\pi\left(\frac{0.5}{12}ft\right)^2}=733Q$$

$$V_2=\frac{Q}{A_2}=\frac{4Q}{50\pi\left(\frac{0.05}{12}ft\right)^2}=1467Q \quad \left(V_1 \text{ and } V_2\sim\frac{ft}{s} \text{ and } Q\sim\frac{ft^3}{s}\right)$$

Thus, $p_1=\frac{1}{2}\rho(V_2^2-V_1^2)=\frac{1}{2}\rho(1467^2Q^2-733Q^2)=8.07\times10^5\rho Q^2\ \frac{lb}{ft^2}$, where $\rho\sim\frac{slug}{ft^3}$, $Q\sim\frac{ft^3}{s}$ (1)

With the flow reducer the flowrate is reduced by a factor of two.

Thus, $V_1=\frac{1}{2}(733Q)$ and $V_2=\frac{1}{2}(1467Q)$ with (2)

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{V_2^2}{2g}+K_L\frac{V_1^2}{2g} \text{ or } p_1=\frac{1}{2}\rho\left(V_2^2+(K_L-1)V_1^2\right) \quad (3)$$

Hence, by combing Eqs. (1), (2), and (3) we obtain

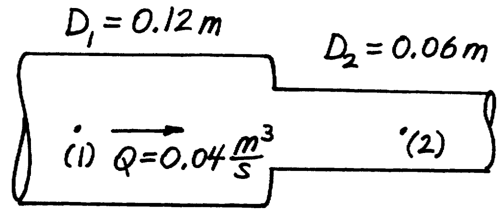
$$8.07\times10^5\rho Q^2=\frac{1}{2}\rho\left[\left(\frac{1467}{2}Q\right)^2+(K_L-1)\left(\frac{733}{2}Q\right)^2\right]$$

or

$$\underline{\underline{K_L=9.00}}$$

8.47

8.47 Water flows at a rate of 0.040 m^3/s in a 0.12-m-diameter pipe that contains a sudden contraction to a 0.06-m-diameter pipe. Determine the pressure drop across the contraction section. How much of this pressure difference is due to losses and how much is due to kinetic energy changes?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + K_L \frac{V_2^2}{2g}, \quad \text{where } z_1 = z_2 \qquad (1)$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.12\,m)^2} = 3.54\,\frac{m}{s}, \quad V_2 = \frac{Q}{A_2} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2} = 14.1\,\frac{m}{s}$$

Thus, with $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{0.06\,m}{0.12\,m}\right)^2 = 0.25$ we obtain from Fig. 8.30

$K_L = 0.40$

Hence, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\rho\left[K_L V_2^2 + V_2^2 - V_1^2\right] = \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left[0.40\left(14.1\,\frac{m}{s}\right)^2 + \left(14.1\,\frac{m}{s}\right)^2 - \left(3.54\,\frac{m}{s}\right)^2\right]$$

or

$$p_1 - p_2 = 39.7\times 10^3\,\frac{N}{m^2} + 93.0\times 10^3\,\frac{N}{m^2} = \underline{\underline{133\,kPa}}$$

This represents a <u>39.7 kPa</u> drop from losses and a <u>93.0 kPa</u> drop due to an increase in kinetic energy.

8.49

8.49 At time $t = 0$ the level of water in tank A shown in Fig. P8.49 is 2 ft above that in tank B. Plot the elevation of the water in tank A as a function of time until the free surfaces in both tanks are at the same elevation. Assume quasisteady conditions—that is, the steady pipe flow equations are assumed valid at any time, even though the flowrate does change (slowly) in time. Neglect minor losses. *Note*: Verify and use the fact that the flow is laminar.

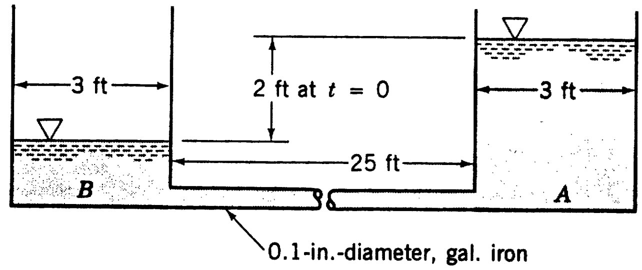
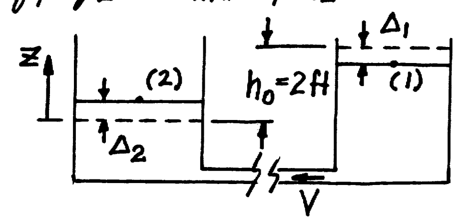


FIGURE P8.49

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}\text{, where } p_1=p_2=0 \text{ and } V_1=V_2\approx 0 \qquad (1)$$

At $t=0$, $z_2=0$ and $z_1=h_0=2$ ft



Because the tanks are the same diameter

$\Delta_1=\Delta_2$ and with $z_2=\Delta_2$, $z_1=h_0-\Delta_2$

we obtain $z_1=h_0-z_2$ Thus, Eq.(1) becomes

$$z_1=z_2+f\frac{\ell}{D}\frac{V^2}{2g} \text{ or } 2z_1-h_0=f\frac{\ell}{D}\frac{V^2}{2g} \qquad (2)$$

Also, $A_1\left(-\frac{dz_1}{dt}\right)=Q=\frac{\pi}{4}D^2V$, where $A_1=\frac{\pi}{4}D_T^2$ with $D_T=3ft$ = tank diameter

Thus, $V=-\left(\frac{D_T}{D}\right)^2\frac{dz_1}{dt}$ (3)

The maximum $Re=\frac{\rho VD}{\mu}$ occurs when the head, z_1-z_2, is greatest.

From Eq.(2) (with $z_1=h_0$), $h_0=f\frac{\ell}{D}\frac{V_{max}^2}{2g}$

Assume laminar flow so that $f=\frac{64}{Re}$ or $f=\frac{64\mu}{\rho VD}$ (4)

Thus, from Eq.(4)

$$h_0=\frac{64\mu}{\rho V_{max}D}\frac{\ell}{D}\frac{V_{max}^2}{2g}=\frac{32\mu\ell V_{max}}{\gamma D^2}\text{, or } V_{max}=\frac{\gamma D^2 h_0}{32\mu\ell}=\frac{(62.4\frac{lb}{ft^3})(\frac{0.1}{12}ft)^2(2ft)}{32(2.34\times10^{-5}\frac{lb\cdot s}{ft^2})(25ft)}$$

$$=0.462\frac{ft}{s}$$

or $Re_{max}=\frac{(1.94\frac{slugs}{ft^3})(0.462\frac{ft}{s})(\frac{0.1}{12}ft)}{2.34\times10^{-5}\frac{lb\cdot s}{ft^2}}=319<2100$ The flow remains laminar.

Thus, Eqs.(2) and (4) give

$$2z_1-h_0=\frac{64\mu}{\rho VD}\frac{\ell}{D}\frac{V^2}{2g}=\frac{32\mu\ell V}{\gamma D^2}\text{, or by using Eq.(3)}$$

$$2z_1-h_0=-\left(\frac{D_T}{D}\right)^2\frac{32\mu\ell}{\gamma D^2}\frac{dz_1}{dt} \qquad (5)$$

Let $F\equiv z_1-\frac{h_0}{2}$ so that $\frac{dF}{dt}=\frac{dz_1}{dt}$ and Eq.(5) becomes

$$2F=-\left(\frac{D_T}{D}\right)^2\frac{32\mu\ell}{\gamma D^2}\frac{dF}{dt}$$

(con't)

or $\alpha \frac{dF}{dt} + F = 0$, where $\alpha = \frac{16\mu \ell}{\gamma D^2}\left(\frac{D_T}{D}\right)^2$

Thus,

$$\alpha \int \frac{dF}{F} = -\int dt \quad \text{or} \quad \alpha \ln F = -t + \tilde{C}, \text{ where } \tilde{C} = \text{constant}$$

Hence,

$F = C e^{-(t/\alpha)}$ That is, $z_1 - \frac{h_0}{2} = C e^{-(t/\alpha)}$ with the initial condition $z_1 = h_0$ when $t = 0$, or $C = \frac{h_0}{2}$

Thus, $z_1 - \frac{h_0}{2} = \frac{h_0}{2} e^{-(t/\alpha)}$

or

$z_1 = \frac{h_0}{2}\left[1 + e^{-(t/\alpha)}\right]$ Note: As $t \to \infty$, $z_1 \to \frac{h_0}{2}$

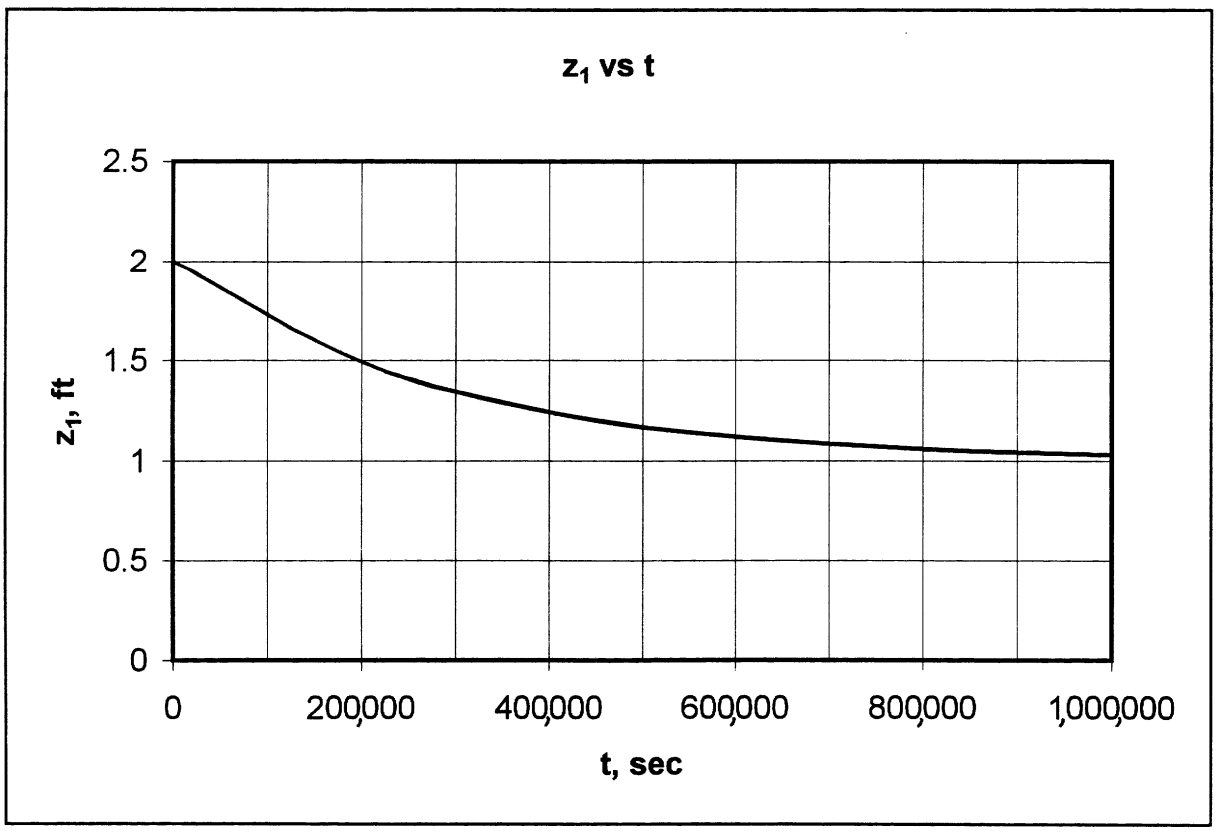
For the conditions given, $h_0 = 2$ ft and

$$\alpha = \frac{16\left(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right)(25 ft)}{\left(62.4 \frac{lb}{ft^3}\right)\left(\frac{0.1}{12} ft\right)^2}\left(\frac{3 ft}{\left(\frac{0.1}{12} ft\right)}\right)^2 = 2.80 \times 10^5 s$$

Hence,

$\underline{\underline{z_1 = 1 + e^{-\left(\frac{t}{2.8 \times 10^5 s}\right)}}}$, where $z_1 \sim$ ft and $t \sim s$

This result is plotted below. (Note: $\lim_{t \to \infty} z_1 = 1$ ft)



8.50*

8.50* Repeat Problem 8.49 if the pipe diameter is changed to 0.1 ft rather than 0.1 in. *Note*: The flow may not be laminar for this case.

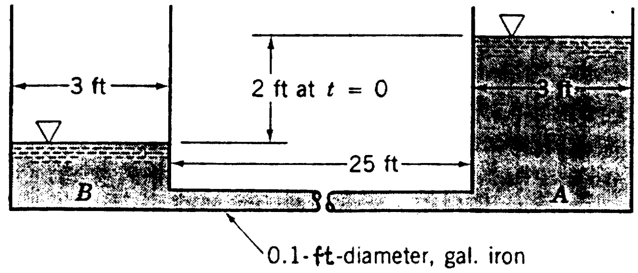
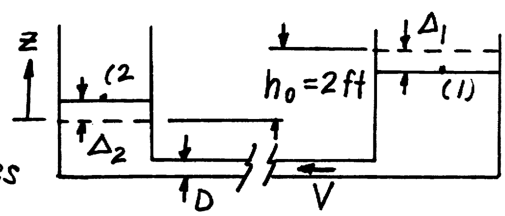


FIGURE P8.49

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L, \text{ where } p_1=p_2=0 \text{ and } V_1=V_2\approx 0 \qquad (1)$$

At $t=0$, $z_2=0$ and $z_1=h_0=2\text{ ft}$

Because the tanks are the same diameter $\Delta_1=\Delta_2$ and with $z_2=\Delta_2$, $z_1=h_0-\Delta_2$ we obtain $z_1=h_0-z_2$ Thus, Eq. (1) gives



$$z_1=z_2+h_L \text{ or } 2z_1-h_0=f\frac{\ell}{D}\frac{V^2}{2g} \qquad (2)$$

Also, $A_1\left(-\frac{dz_1}{dt}\right)=Q=\frac{\pi}{4}D^2V$, where $A_1=\frac{\pi}{4}D_T^2$ with $D_T=$ tank diameter $=3\text{ ft}$

Thus, $V=-\left(\frac{D_T}{D}\right)^2\frac{dz_1}{dt}=-\left(\frac{3\text{ ft}}{0.1\text{ ft}}\right)^2\frac{dz_1}{dt}=-900\frac{dz_1}{dt}$ (3)

By combining Eqs. (2) and (3) we obtain

$$2z_1-h_0=f\frac{\ell}{D}\frac{1}{2g}\left(\frac{D_T}{D}\right)^4\left(\frac{dz_1}{dt}\right)^2 \text{ or } 2z_1-2\text{ ft}=f\left(\frac{25\text{ ft}}{0.1\text{ ft}}\right)\frac{\left(\frac{3\text{ ft}}{0.1\text{ ft}}\right)^4}{2(32.2\frac{\text{ft}}{\text{s}^2})}\left(\frac{dz_1}{dt}\right)^2$$

or

$$\frac{dz_1}{dt}=\frac{-7.98\times10^{-4}}{\sqrt{f}}\sqrt{z_1-1}, \text{ where } z_1\sim\text{ft}, \frac{dz_1}{dt}\sim\frac{\text{ft}}{\text{s}} \text{ (Note: } \frac{dz_1}{dt}<0) \qquad (4)$$

To integrate this and obtain z_1 as a function of t we must know f.

For $Re<2100$ the flow is laminar and $f=\frac{64}{Re}$

Note that $Re=\frac{\rho VD}{\mu}=\frac{VD}{\nu}=2100$ gives $V=\frac{2100\nu}{D}=\frac{2100(1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}})}{0.1\text{ ft}}$

or $V=0.254\frac{\text{ft}}{\text{s}}$ and $f=\frac{64}{2100}$

Thus, from Eq.(2) $2z_1-2\text{ ft}=\frac{64}{2100}\left(\frac{25\text{ ft}}{0.1\text{ ft}}\right)\frac{(0.254\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$ or $z_1=(1+0.00382)\text{ft}$

At $t=0$, $z_1=2\text{ ft}$; at $t=\infty$, $z_1=z_2=1\text{ ft}$.

For $z_1<1.00382$ ft the flow is laminar. Thus, for all practical purposes the flow is turbulent. Obtain f from the Colebrook formula (Eq. 8.35) as

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\left(\frac{\varepsilon}{D}\right)}{3.7}+\frac{2.51}{Re\sqrt{f}}\right], \text{ where } \frac{\varepsilon}{D}=\frac{0.005\text{ ft}}{0.1\text{ ft}}=0.05$$

(see Table 8.1)

and

$$Re=\frac{VD}{\nu}=\frac{(0.1\text{ ft})V}{1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}}}=8.26\times10^3\,V$$

(con't)

8.50* (con't)

Hence,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left[1.35\times10^{-3} + \frac{3.04\times10^{-4}}{V\sqrt{f}}\right], \text{ where } V \sim \frac{ft}{s} \qquad (5)$$

Solve (i.e. integrate) Eq. (4) for $z_1 = z(t)$, starting with initial condition $z_1 = 2$ ft at $t=0$. Obtain f (which is a function of t because $V = -900\frac{dz_1}{dt}$ (Eq. (3)) is a function of t) from Eq. (5). Program P8#50 shown below was used to obtain the results.

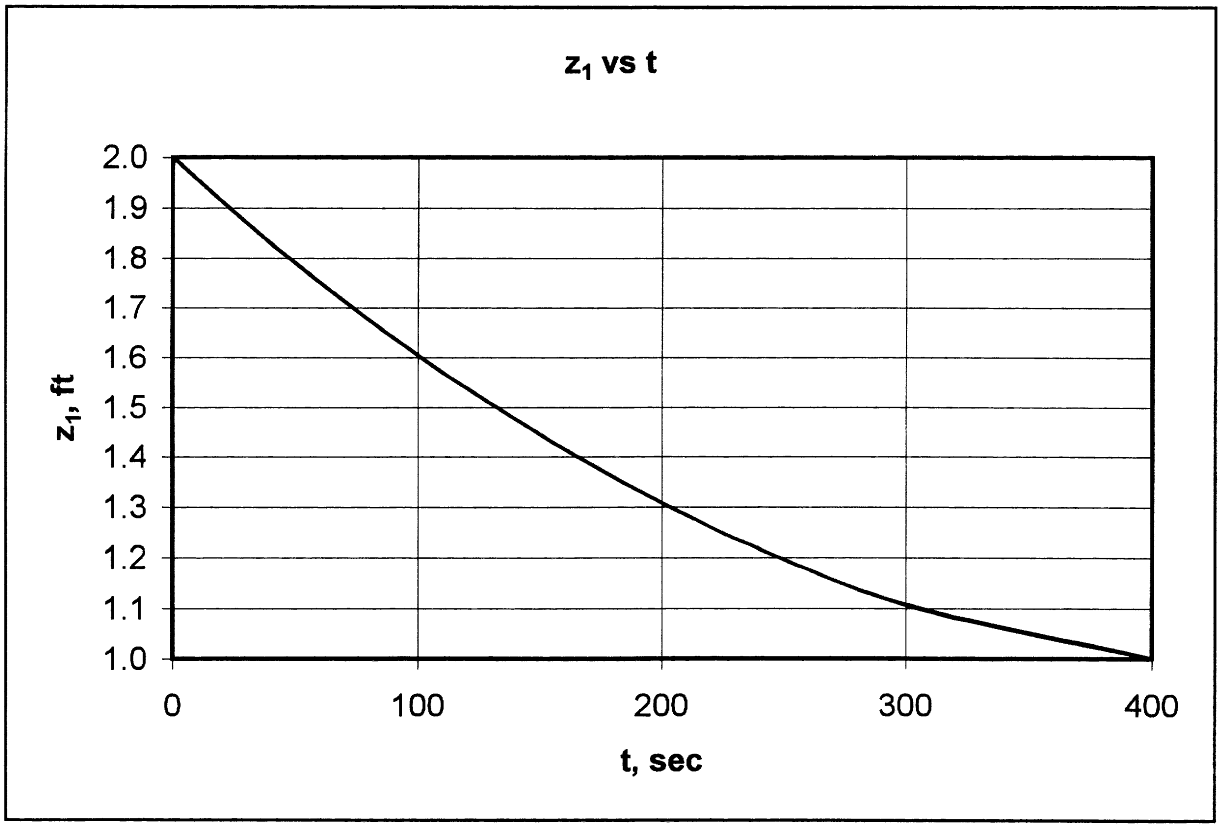
```
100 cls
110 open "prn" for output as #1
120 print#1, "**********************************************"
130 print#1, "**  This program calculates the water depth  **"
140 print#1, "**  as a function of time.  The friction     **"
150 print#1, "**  factor is obtained by itteration from    **"
160 print#1, "**  the Colebrook formula.                   **"
170 print#1, "**********************************************"
200 print#1, " "
210 t = 0
220 VP = 0
230 rr = 0.005
240 dz = 0.05
250 print#1, "  z, ft  dz/dt, ft/s   f       t, s"
260 for i = 1 to 20
270 z = 2 - (i-1)*dz
280 f = 0.02
300 dzdt = -7.98E-4*(z - 1)^0.5/f^0.5
310 V = -900*dzdt
330 Re = 8.26E+3*V
340 fp = f
350 f = 1/(-2.0*log(rr/3.7 + 2.51/(Re*fp^0.5))/log(10))^2
360 if abs(1 - f/fp) > 0.001 then goto 340
380 if abs(1 - VP/V) < 0.01 then goto 400
385 VP = V
390 goto 300
400 t = t - dz/dzdt
405 zn = z - dz
410 print#1, using " #.####  +##.#####  #.####  +#.##^^^^";zn,dzdt,f,t
420 next i
```

(con't)

8.50* (con't)

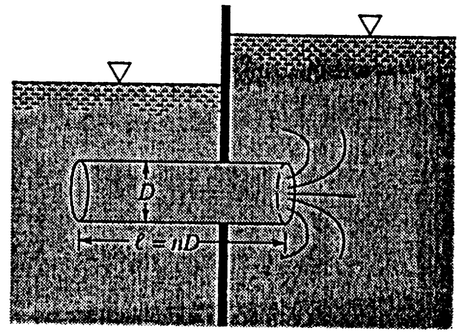
```
*********************************************
**  This program calculates the water depth  **
**  as a function of time.  The friction     **
**  factor is obtained by itteration from    **
**  the Colebrook formula.                   **
*********************************************
```

z, ft	dz/dt, ft/s	f	t, s
1.9500	-0.00440	0.0329	+1.14E+01
1.9000	-0.00429	0.0329	+2.30E+01
1.8500	-0.00417	0.0330	+3.50E+01
1.8000	-0.00405	0.0331	+4.74E+01
1.7500	-0.00392	0.0332	+6.01E+01
1.7000	-0.00379	0.0333	+7.33E+01
1.6500	-0.00366	0.0334	+8.70E+01
1.6000	-0.00352	0.0335	+1.01E+02
1.5500	-0.00337	0.0336	+1.16E+02
1.5000	-0.00322	0.0337	+1.32E+02
1.4500	-0.00307	0.0339	+1.48E+02
1.4000	-0.00290	0.0341	+1.65E+02
1.3500	-0.00273	0.0343	+1.83E+02
1.3000	-0.00254	0.0345	+2.03E+02
1.2500	-0.00234	0.0348	+2.24E+02
1.2000	-0.00213	0.0352	+2.48E+02
1.1500	-0.00189	0.0358	+2.74E+02
1.1000	-0.00162	0.0365	+3.05E+02
1.0500	-0.00130	0.0377	+3.44E+02
1.0000	-0.00089	0.0402	+4.00E+02



8.51

8.51 As shown in Fig. P8.51, water flows from one tank to another through a short pipe whose length is n times the pipe diameter. Head losses occur in the pipe and at the entrance and exit. (See Video V8.4.) Determine the maximum value of n if the major loss is to be no more than 10% of the minor loss and the friction factor is 0.02.



■ FIGURE P8.51

If $h_{L_{major}} = 10\% h_{L_{minor}}$; then

$$10 f \frac{\ell}{D} \frac{V^2}{2g} = \Sigma K_L \frac{V^2}{2g} \quad \text{or} \quad \frac{\ell}{D} = \frac{\Sigma K_L}{10 f} \qquad (1)$$

where $\Sigma K_L = K_{L\,entrance} + K_{L\,exit} = 0.8 + 1 = 1.8$

Thus, with $f = 0.02$ and $\ell = nD$ Eq. (1) becomes

$$\frac{nD}{D} = \frac{1.8}{10(0.02)}$$

or

$$\underline{\underline{n = 9}}$$

8.52

8.52 Gasoline flows in a smooth pipe of 40-mm diameter at a rate of 0.001 m^3/s. If it were possible to prevent turbulence from occurring, what would be the ratio of the head loss for the actual turbulent flow compared to that if it were laminar flow?

Let $(\;)_t$ denote the turbulent flow and $(\;)_\ell$ the laminar flow.

Thus, $h_{L_t} = f_t \frac{\ell}{D}\frac{V^2}{2g}$ and $h_{L_\ell} = f_\ell \frac{\ell}{D}\frac{V^2}{2g}$ (1)

where

$$V = V_t = V_\ell = \frac{Q}{A} = \frac{0.001\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04m)^2} = 0.796\,\frac{m}{s}$$

From Table 1.6 $\rho = 680\,\frac{kg}{m^3}$ and $\mu = 3.1\times10^{-4}\,\frac{N\cdot s}{m^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(680\,\frac{kg}{m^3})(0.796\,\frac{m}{s})(0.04m)}{3.1\times10^{-4}\,\frac{N\cdot s}{m^2}} = 6.98\times10^4$$

Hence, from Fig. 8.20, for a smooth pipe $f_t = 0.0192$

while for laminar flow $f_\ell = \frac{64}{Re} = \frac{64}{6.98\times10^4} = 9.16\times10^{-4}$

Thus, from Eq. (1)

$$\frac{h_{L_t}}{h_{L_\ell}} = \frac{f_t}{f_\ell} = \frac{0.0192}{9.16\times10^{-4}} = \underline{\underline{21.0}}$$

8.53

8.53 A 3-ft-diameter duct is used to carry ventilating air into a vehicular tunnel at a rate of 9000 ft³/min. Tests show that the pressure drop is 1.5 in. of water per 1500 ft of duct. What is the approximate size of the equivalent roughness of the surface of the duct?

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g},\ \text{where } z_1=z_2,\ V_1=V_2,\ \text{and} \quad (1)$$

$$p_1-p_2=\gamma_{H_2O}h=(62.4\tfrac{lb}{ft^3})(\tfrac{1.5}{12}ft)=7.80\tfrac{lb}{ft^2}$$

Also, $V=\frac{Q}{A}=\frac{(9000\frac{ft^3}{min})(\frac{1\,min}{60s})}{\frac{\pi}{4}(3ft)^2}=21.2\frac{ft}{s}$

Thus, from Eq.(1) $\quad p_1-p_2=f\frac{\ell}{D}\frac{1}{2}\rho V^2$ or

$$f=\frac{2D(p_1-p_2)}{\rho\ell V^2}=\frac{2(3ft)(7.80\frac{lb}{ft^2})}{(2.38\times10^{-3}\frac{slugs}{ft^3})(1500ft)(21.2\frac{ft}{s})^2}=\underline{\underline{0.0292}}$$

From Fig. 8.20 with $f=0.0292$ and $Re=\frac{VD}{\nu}=\frac{(21.2\frac{ft}{s})(3ft)}{1.57\times10^{-4}\frac{ft^2}{s}}=4.05\times10^5$

we obtain $\frac{\varepsilon}{D}=0.0044$ Thus, $\varepsilon=0.0044\,(3ft)=\underline{\underline{0.0132\ ft}}$

8.54

8.54 Natural gas (ρ = 0.0044 slugs/ft³ and ν = 5.2 × 10⁻⁵ ft²/s) is pumped through a horizontal 6-in.-diameter cast-iron pipe at a rate of 800 lb/hr. If the pressure at section (1) is 50 psi (abs), determine the pressure at section (2) 8 mi downstream if the flow is assumed incompressible. Is the incompressible assumption reasonable? Explain.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g},\ \text{where } z_1=z_2 \text{ and } V_1=V_2 \quad (1)$$

Also, $\gamma Q=800\frac{lb}{hr}\left(\frac{1\,hr}{3600s}\right)=0.222\frac{lb}{s}$, or $Q=\frac{0.222\frac{lb}{s}}{(32.2\frac{ft}{s^2})(4.4\times10^{-3}\frac{slug}{ft^3})}=1.57\frac{ft^3}{s}$

Thus, $V=\frac{Q}{A}=\frac{1.57\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=8.00\frac{ft}{s}$

With $Re=\frac{\rho VD}{\mu}=\frac{VD}{\nu}=\frac{(8.00\frac{ft}{s})(\frac{6}{12}ft)}{5.2\times10^{-5}\frac{ft^2}{s}}=7.69\times10^4$ and (from Table 8.1)

$\frac{\varepsilon}{D}=\frac{0.00085ft}{(6/12\,ft)}=0.0017$ we obtain $f=0.0245$ Thus, from Eq.(1)

$$p_2=p_1-f\frac{\ell}{D}\frac{1}{2}\rho V^2=50\frac{lb}{in^2}-(0.0245)\frac{(8mi)(5280\frac{ft}{mi})}{0.5ft}\frac{1}{2}(4.4\times10^{-3}\frac{slug}{ft^3})(8.00\frac{ft}{s})^2$$

$$=50psi-291\frac{lb}{ft^2}=(50-2.02)psi \quad \text{or } p_2=\underline{\underline{48.0\ psi}}$$

Note: $\frac{p_1-p_2}{p_1}=\frac{2.02psi}{50psi}=0.0404$, a 4.0 % change in pressure.

Since $p=\rho RT$, with T essentially constant, a small change in p gives a small change in ρ. Thus, the incompressible assumption is valid.

8.55*

8.55* Water flows in a 20-mm-diameter galvanized iron pipe with average velocities between 0.01 and 10.0 m/s. Plot the head loss per meter of pipe length over this velocity range.

$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$, or with $\ell = 1m$ and $D = 0.020\,m$

$$h_L = f\left(\frac{1m}{0.02m}\right)\frac{V^2}{2(9.81\frac{m}{s^2})} \text{ , or } h_L = 2.55\,f\,V^2 \qquad (1)$$

Also, $Re = \frac{VD}{\nu} = \frac{V\,(0.02m)}{1.12\times10^{-6}\frac{m^2}{s}}$, or $Re = 1.79\times10^4\,V$ (2)

For this pipe, $\frac{\varepsilon}{D} = \frac{0.15\,mm}{20\,mm} = 7.5\times10^{-3}$ (see Table 8.1) so that the Colebrook formula becomes (Eq. 8.35)

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.5}{Re\sqrt{f}}\right] \text{ or } \frac{1}{\sqrt{f}} = -2.0\log\left[2.03\times10^{-3} + \frac{2.5}{Re\sqrt{f}}\right] \qquad (3)$$

Note: If $Re \le 2100$ the flow is laminar and $f = \frac{64}{Re}$ (4)
If $Re \ge 4000$ the flow is turbulent and f is obtained from Eq. (3).
For $2100 < Re < 4000$ it is not clear which value of f to use. For simplicity assume laminar flow is maintained up to $Re = 4000$.
From Eq. (2), $V = \frac{4000}{1.79\times10^4} = 0.223\,\frac{m}{s}$ when $Re = 4000$.

Thus, for $0.01 \le V \le 0.223\,\frac{m}{s}$ obtain Re and f from Eqs. (2) and (4) and h_L from Eq. (1). For $0.223 < V \le 10\,\frac{m}{s}$ obtain Re and f from Eqs. (2) and (4). The values of h_L are calculated and plotted below (see program P8#55).

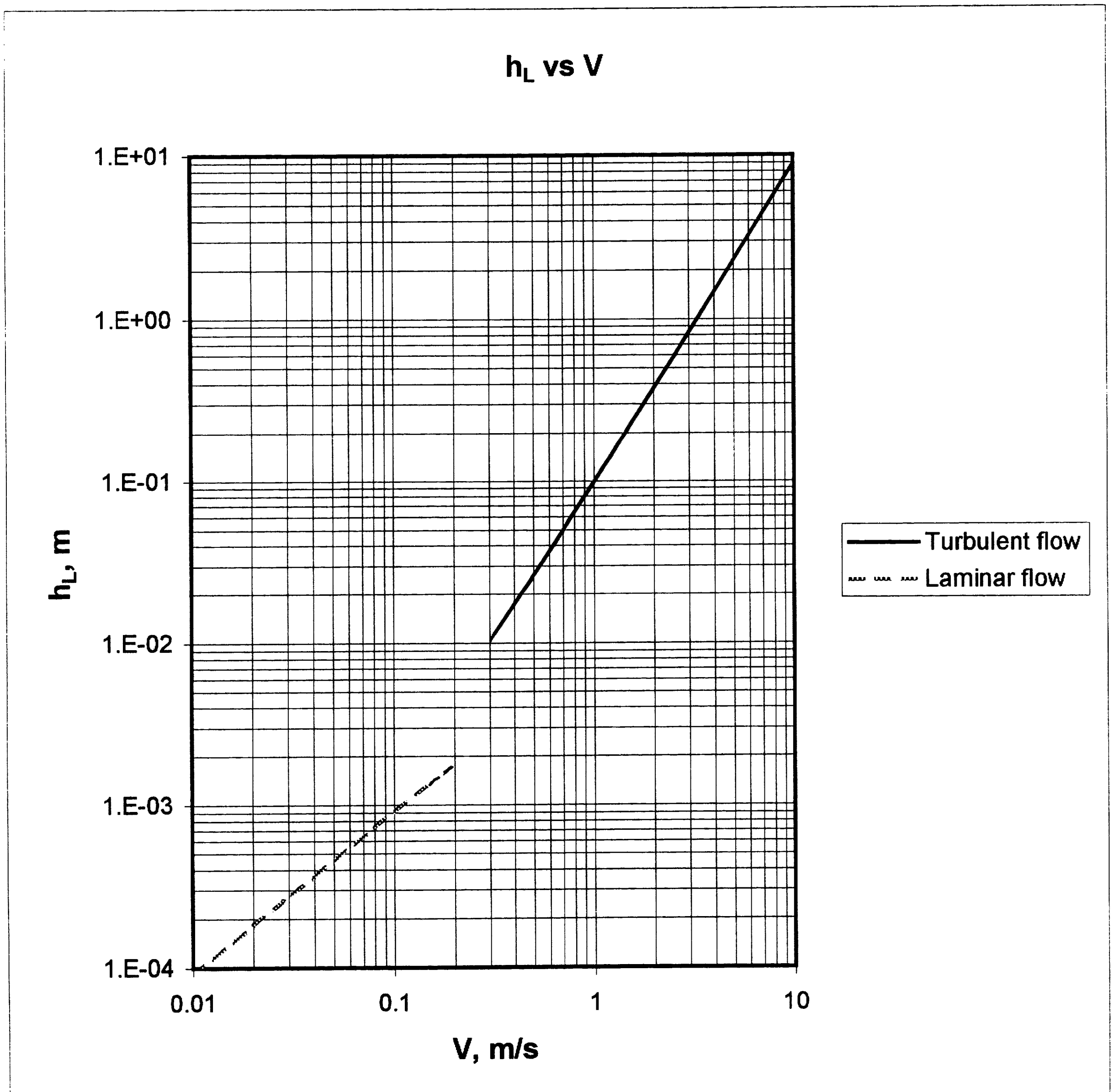
```
100 cls
110 open "prn" for output as #1
120 print#1, "***********************************************"
130 print#1, "**  This program calculates the head loss in  **"
140 print#1, "**  pipe as a function of velocity.  The      **"
150 print#1, "**  friction factor is obtained by the Cole-  **"
160 print#1, "**  formula.                                  **"
170 print#1, "***********************************************"
200 rr = 0.0075
210 V = 0.005
220 print#1, " "
230 print#1, "  V/ m/s     Re         f        hL, m"
300 V = 2*V
310 Re = 1.79E+4*V
320 f = 64/Re
330 if Re < 4000 then goto 400
340 fp = f
350 f = 1/(-2.0*log(rr/3.7+2.51/(Re*fp^0.5))/log(10))^2
360 if abs(1 - f/fp) > 0.0001 then goto 340
400 h = 2.55*f*V^2
410 print#1, using " ###.###  #.###^^^^  #.###   #.###^^^^";V,Re,f,h
420 if V < 10 then goto 300
```

(con't)

8.55* (con't)

```
*************************************************
**  This program calculates the head loss in   **
**  pipe as a function of velocity.  The       **
**  friction factor is obtained by the Cole-   **
**  formula.                                   **
*************************************************
```

V m/s	Re	f	hL, m
0.010	1.790E+02	0.3575	9.117E-05
0.020	3.580E+02	0.1788	1.823E-04
0.040	7.160E+02	0.0894	3.647E-04
0.080	1.432E+03	0.0447	7.294E-04
0.160	2.864E+03	0.0223	1.459E-03
0.320	5.728E+03	0.0439	1.147E-02
0.640	1.146E+04	0.0398	4.161E-02
1.280	2.291E+04	0.0374	1.562E-01
2.560	4.582E+04	0.0360	6.016E-01
5.120	9.165E+04	0.0353	2.357E+00
10.240	1.833E+05	0.0349	9.323E+00



8.56

8.56 A fluid flows through a smooth horizontal 2-m-long tube of diameter 2 mm with an average velocity of 2.1 m/s. Determine the head loss and the pressure drop if the fluid is **(a)** air, **(b)** water, or **(c)** mercury.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L \text{ , where } h_L=f\frac{\ell}{D}\frac{V^2}{2g}\text{ , } z_1=z_2\text{ , and } V_1=V_2$$

Thus, $h_L=f\left(\frac{2\,m}{0.002\,m}\right)\frac{(2.1\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}$ or $h_L=225f\ m$ (1)

and

$$\Delta p=p_1-p_2=\gamma h_L$$

Also, $Re=\frac{VD}{\nu}=\frac{(2.1\frac{m}{s})(0.002\,m)}{\nu}=\frac{4.2\times10^{-3}}{\nu}$, where $\nu\sim\frac{m^2}{s}$

Thus:

fluid	ν, $\frac{m^2}{s}$	Re	flow	f	h_L, m	γ, $\frac{N}{m^3}$	Δp, $\frac{N}{m^2}$
a) air	1.46×10^{-5}	287	laminar	$\frac{64}{Re}=0.223$	50.2	12.0	602
b) water	1.12×10^{-6}	3750	~turbulent	0.0404	9.09	9800	8.91×10^4
c) mercury	1.15×10^{-7}	36,500	turbulent	0.0220	4.95	133,000	6.58×10^5

8.57

8.57 Air at standard temperature and pressure flows through a horizontal 2 ft by 1.3 ft rectangular galvanized iron duct with a flowrate of 8.2 cfs. Determine the pressure drop in inches of water per 200-ft length of duct.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D_h}\frac{V^2}{2g}\text{ , where } z_1=z_2 \text{ and } V_1=V_2$$

Also, $D_h=\frac{4A}{P}=\frac{4(2ft)(1.3ft)}{2[2ft+1.3ft]}=1.576\ ft$

and

$$V=\frac{Q}{A}=\frac{8.2\frac{ft^3}{s}}{(2ft)(1.3ft)}=3.15\frac{ft}{s}$$

Thus, $p_1-p_2=f\frac{\ell}{D_h}\frac{1}{2}\rho V^2$, where for galvanized iron $\varepsilon=0.0005\ ft$ (Table 8.1)

Hence, $\frac{\varepsilon}{D_h}=\frac{0.0005\,ft}{1.576\,ft}=0.000317$ and $Re_h=\frac{VD_h}{\nu}=\frac{(1.576\,ft)(3.15\frac{ft}{s})}{1.57\times10^{-4}\frac{ft^2}{s}}=31,600$

so from Fig. 8.20, $f=0.025$

Thus, $p_1-p_2=(0.025)\left(\frac{200\,ft}{1.576\,ft}\right)\frac{1}{2}(2.38\times10^{-3}\frac{slug}{ft^3})(3.15\frac{ft}{s})^2=0.0374\frac{lb}{ft^2}$

or with $p_1-p_2=\gamma_{H_2O}h$,

$$h=\frac{p_1-p_2}{\gamma_{H_2O}}=\frac{0.0374\frac{lb}{ft^2}}{62.4\frac{lb}{ft^3}}=6.00\times10^{-4}\,ft=0.00720\text{ in. of water}$$

8.58

8.58 Air flows through a rectangular galvanized iron duct of size 0.30 m by 0.15 m at a rate of 0.068 m³/s. Determine the head loss in 12 m of this duct.

$h_L = f \frac{\ell}{D_h}\frac{V^2}{2g}$, where $D_h = \frac{4A}{P} = \frac{4(0.3m)(0.15m)}{2[0.3m+0.15m]} = 0.2\ m$

and

$V = \frac{Q}{A} = \frac{0.068\frac{m^3}{s}}{(0.3m)(0.15m)} = 1.51\frac{m}{s}$ Also, $Re_h = \frac{VD_h}{\nu} = \frac{(1.51\frac{m}{s})(0.2m)}{1.46\times10^{-5}\frac{m^2}{s}} = 20{,}700$

and from Table 8.1,

$\frac{\varepsilon}{D_h} = \frac{0.15\times10^{-3}m}{0.2m} = 7.5\times10^{-4}$ Hence, from Fig. 8.20 $f = 0.027$

so that

$h_L = (0.027)\left(\frac{12m}{0.2m}\right)\frac{(1.51\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} = \underline{\underline{0.188m}}$

8.59

8.59 Air at standard conditions flows through a horizontal 1 ft by 1.5 ft rectangular wooden duct at a rate of 5000 ft³/min. Determine the head loss, pressure drop, and power supplied by the fan to overcome the flow resistance in 500 ft of the duct.

$h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{(5000\frac{ft^3}{min})(\frac{1\,min}{60s})}{(1ft)(1.5ft)} = 55.6\frac{ft}{s}$

and $D_h = \frac{4A}{P} = \frac{4(1ft)(1.5ft)}{2[1ft+1.5ft]} = 1.2\ ft$

Also, $Re_h = \frac{VD_h}{\nu} = \frac{(55.6\frac{ft}{s})(1.2ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.25\times10^5$ and from Table 8.1

$\varepsilon \approx 0.0006\ ft$ to $0.003\ ft$. Use an "average" $\varepsilon = 0.0018$ ft so that

$\frac{\varepsilon}{D_h} = \frac{0.0018\ ft}{1.2ft} = 0.0015$ Thus, from Fig. 8.20 $f = 0.022$, or

$h_L = (0.022)\left(\frac{500\ ft}{1.2ft}\right)\frac{(55.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = \underline{\underline{440\,ft}}$

For this horizontal pipe $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$, where $z_1 = z_2$ and $V_1 = V_2$.

Thus, $p_1 - p_2 = \gamma h_L = (7.65\times10^{-2}\frac{lb}{ft^3})(440ft) = 33.7\frac{lb}{ft^2} = \underline{\underline{0.234 psi}}$

$\mathcal{P} = \gamma Q h_L = Q(p_1 - p_2) = (5000\frac{ft^3}{min})\left(\frac{1\,min}{60s}\right)(33.7\frac{lb}{ft^2}) = \left(2810\frac{ft\cdot lb}{s}\right)\left[\frac{1\,hp}{(550\frac{ft\cdot lb}{s})}\right]$

or

$\mathcal{P} = \underline{\underline{5.11\ hp}}$

8.60

8.60 When the valve is closed the pressure throughout the horizontal pipe shown in Fig. P8.60 is 400 kPa, and the water level in the closed surge chamber is $h = 0.4$ m. If the valve is fully opened and the pressure at point (1) remains 400 kPa, determine the new level of the water in the surge chamber. Assume the friction factor is $f = 0.02$ and the fittings are threaded fittings.

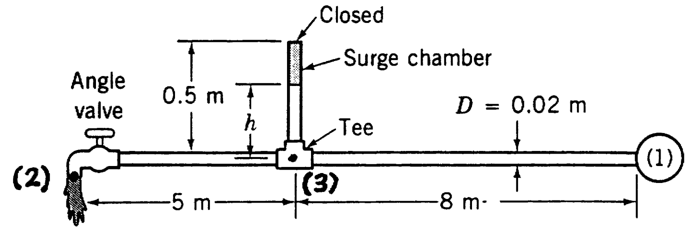


FIGURE P8.60

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1=z_2,\ V_1=0,\ p_2=0$$

Thus, $\frac{p_1}{\gamma}=\frac{V_2^2}{2g}+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}$ with $V=V_2$.

With $K_L=2$ for an angle valve and $K_L=0.9$ for the tee (see Table 8.2) we obtain

$$\frac{400\frac{kN}{m^2}}{9.80\frac{kN}{m^3}}=\frac{1}{2(9.81\frac{m}{s^2})}\left[1+(0.02)\left(\frac{(8+5)m}{0.02\,m}\right)+2+0.9\right]V^2$$

or
$V=6.88\frac{m}{s}$

Thus, p_3 is determined from

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1=z_3 \text{ and } V_1=0$$

Also, $V_3=V$ Hence,

$$\frac{p_1}{\gamma}=\frac{p_3}{\gamma}+\left(1+f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } \ell=8m \text{ and } K_L=0$$

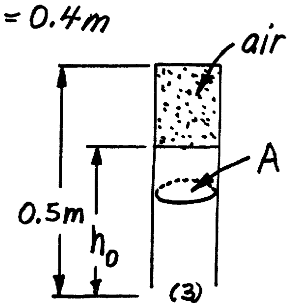
Thus,

$$p_3=p_1-\left(1+f\frac{\ell}{D}\right)\frac{V^2}{2g}\gamma=p_1-\left(1+f\frac{\ell}{D}\right)\frac{1}{2}\rho V^2$$

$$=400\,kP-\left(1+(0.02)\left(\frac{8\,m}{0.02\,m}\right)\right)\frac{1}{2}\left(999\frac{kg}{m^3}\right)\left(6.88\frac{m}{s}\right)^2=400\,kP-2.13\times10^5\frac{N}{m^2}$$

or $p_3=187\,kP$

Thus, $p_3=400$ kPa with the valve closed when $h=0.4m$
and $p_3=187$ kPa with the valve open and $h=h_0$



M = mass of air in surge chamber $=\rho\forall=$ constant,
where $\forall=A(0.5m-h)$ and $p=\rho RT$, or $\rho=\frac{p}{RT}$
Thus, with $(\ )_c$ denoting the closed valve condition,

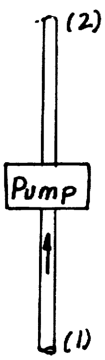
$$\rho_c\forall_c=\rho_0\forall_0, \text{ or } \frac{p_c}{RT_c}A(0.5-0.4)=\frac{p_0}{RT_0}A(0.5-h_0) \quad \text{Assume } T_c=T_0$$

or
$0.1\,p_c=(0.5-h_0)\,p_0$, where $p_c=400kPa-\gamma h_c=400kPa-9.80\frac{kN}{m^3}(0.4m)=396\,kPa$
and $p_0=187kPa-\gamma h_0=187kPa-9.80\frac{kN}{m^2}h_0=(187-9.8h_0)kPa$

Thus, $0.1(396)=(0.5-h_0)(187-9.8h_0)$, or $h_0=\underline{\underline{0.285\ m}}$

8.61

8.61 What horsepower is added to water to pump it vertically through a 200-ft-long, 1.0-in.-diameter drawn tubing at a rate of 0.060 ft³/s if the pressures at the inlet and outlet are the same?



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1=p_2$$

and $V_1=V_2$. Thus,

$$h_p=z_2-z_1+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_2-z_1=\ell=200\text{ ft} \text{ and} \qquad (1)$$

$$V=\frac{Q}{A}=\frac{0.06\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{1}{12}\text{ft})^2}=11.0\frac{\text{ft}}{\text{s}}$$

Also, $\frac{\varepsilon}{D}=\frac{5\times10^{-6}\text{ ft}}{(1/12\text{ ft})}=6\times10^{-5}$ (Table 8.1)

and $Re=\frac{VD}{\nu}=\frac{(11.0\frac{\text{ft}}{\text{s}})(\frac{1}{12}\text{ft})}{1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}}}=7.58\times10^4$ we obtain from Fig. 8.20

$f=0.019$

From Eq. (1)

$$h_p=200\text{ ft}+(0.019)\left(\frac{200\text{ ft}}{\frac{1}{12}\text{ft}}\right)\frac{(11.0\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}=286\text{ ft}$$

Thus,

$$\mathcal{P}=\gamma Q h_p=(62.4\frac{\text{lb}}{\text{ft}^3})(0.06\frac{\text{ft}^3}{\text{s}})(286\text{ft})=1071\frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\text{ hp}}{550\frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right)=\underline{\underline{1.95\text{ hp}}}$$

8.62

8.62 Water flows from a lake as is shown in Fig. P8.62 at a rate of 4.0 cfs. Is the device inside the building a pump or a turbine? Explain and determine the horsepower of the device. Neglect all minor losses and assume the friction factor is 0.025.

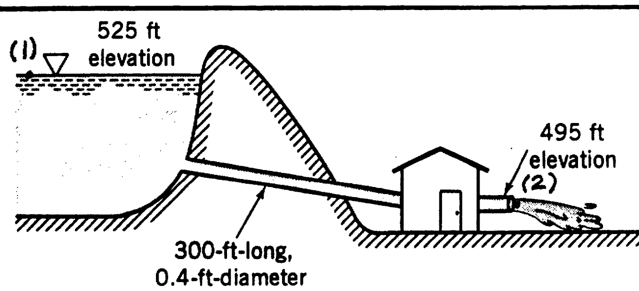


FIGURE P8.62

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_t+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ V_1=0$$

and

$$V_2=V=\frac{Q}{\frac{\pi}{4}D^2}=\frac{4\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(0.4\text{ft})^2}=31.8\frac{\text{ft}}{\text{s}}$$

Assume the device is a pump ($h_t=0$).

Thus, $z_1+h_p=\frac{V^2}{2g}(1+f\frac{\ell}{D})+z_2$, or

$$h_p=495\text{ ft}-525\text{ft}+\frac{(31.8\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}\left(1+0.025\left(\frac{300\text{ ft}}{0.4\text{ft}}\right)\right)=280\text{ ft}$$

Note: Since $h_p>0$ <u>the device is a pump</u>.

Also, $\mathcal{P}=g\dot{m}h_p=\gamma Q h_p=(62.4\frac{\text{lb}}{\text{ft}^3})(4\frac{\text{ft}^3}{\text{s}})(280\text{ft})=69{,}900\frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\text{ hp}}{550\frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right)$

or

$\mathcal{P}=\underline{\underline{127\text{ hp}}}$

8.63

8.63 Repeat Problem 8.62 if the flowrate is 1.0 cfs.

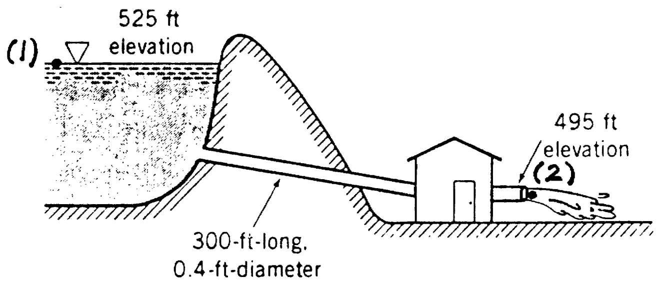


FIGURE P8.62

$$\cancel{\frac{p_1}{\gamma}} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1 = p_2 = 0,\ V_1 = 0$$

and

$$V_2 = V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{1\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.4\,ft)^2} = 7.96\,\frac{ft}{s}$$

Assume the device is a turbine ($h_p = 0$)

Thus,

$$h_t = z_1 - z_2 - \frac{V^2}{2g}\left(1 + f\frac{\ell}{D}\right)$$

or

$$h_t = 525\,ft - 495\,ft - \frac{(7.96\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}\left(1 + 0.025\left(\frac{300\,ft}{0.4\,ft}\right)\right) = 10.57\,ft$$

Note: Since $h_t > 0$ the device is a turbine.

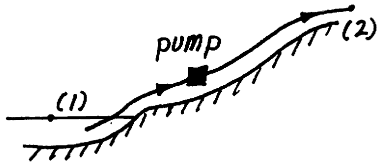
Also,

$$\mathcal{P} = g\dot{m}h_t = \gamma Q h_t = (62.4\,\frac{lb}{ft^3})(1\,\frac{ft^3}{s})(10.57\,ft) = (660\,\frac{ft\cdot lb}{s})\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

or $\mathcal{P} = 1.20\,hp$

8.64

8.64 At a ski resort water at 40 °F is pumped through a 3-in.-diameter, 2000-ft-long steel pipe from a pond at an elevation of 4286 ft to a snow-making machine at an elevation of 4623 ft at a rate of 0.26 ft³/s. If it is necessary to maintain a pressure of 180 psi at the snow-making machine, determine the horsepower added to the water by the pump. Neglect minor losses.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_2=180\,\frac{lb}{in.^2},\ p_1=0,\ V_1=0 \quad (1)$$

and $V=V_2=\frac{Q}{\frac{\pi}{4}D^2}=\frac{0.26\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{3}{12}ft)^2}=5.30\frac{ft}{s}$. From Table 8.1, $\frac{\varepsilon}{D}=\frac{0.00015\,ft}{(3/12\,ft)}=6\times10^{-4}$

Also, $Re=\frac{VD}{\nu}=\frac{(5.30\frac{ft}{s})(\frac{3}{12}ft)}{1.664\times10^{-5}\frac{ft^2}{s}}=7.96\times10^4$ so that $f=0.0212$ (see Fig. 8.20)

Thus, from Eq. (1)

$$h_p=\frac{p_2}{\gamma}+z_2-z_1+\left(1+f\frac{\ell}{D}\right)\frac{V^2}{2g}$$

or

$$h_p=\frac{(180\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+4623\,ft-4286\,ft+\left(1+(0.0212)\left(\frac{2000\,ft}{\frac{3}{12}ft}\right)\right)\frac{(5.30\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

Hence, $h_p=827\,ft$ so that

$$\mathcal{P}=\gamma Q h_p=(62.4\tfrac{lb}{ft^3})(0.26\tfrac{ft^3}{s})(827\,ft)=(13{,}420\tfrac{ft\cdot lb}{s})\left(\frac{1\,hp}{550\frac{ft\cdot lb}{s}}\right)=\underline{\underline{24.4\,hp}}$$

8.65

8.65 Water flows through the screen in the pipe shown in Fig. P8.65 as indicated. Determine the loss coefficient for the screen.

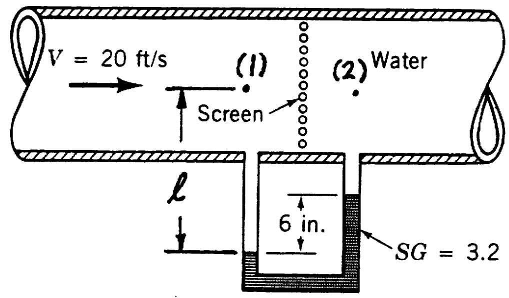


FIGURE P8.65

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+K_L\frac{V^2}{2g}\text{, where } z_1=z_2\text{, } V_1=V_2=V=20\,\frac{ft}{s}$$

Thus, $K_L=\dfrac{2(p_1-p_2)}{\rho V^2}$ where $p_1+\gamma\ell=p_2+\gamma(\ell-6\text{in.})+SG\gamma(6\text{in.})$

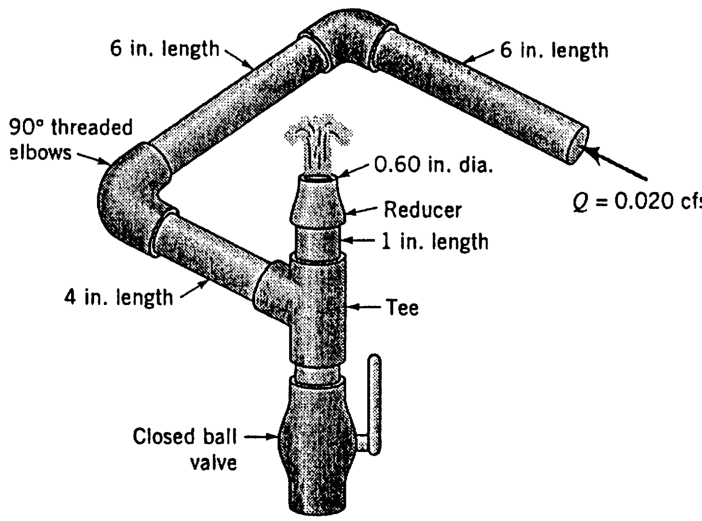
or $p_1-p_2=\gamma(SG-1)(6\text{in.})$

Hence,

$$K_L=\frac{2(62.4\,\frac{lb}{ft^3})(3.2-1)(\frac{6}{12}\,ft)}{(1.94\,\frac{slugs}{ft^3})(20\,\frac{ft}{s})^2}=\underline{\underline{0.177}}$$

8.66

8.66 Water flows steadily through the 0.75-in. diameter galvanized iron pipe system shown in Video V8.6 and Fig. P8.66 at a rate of 0.020 cfs. Your boss suggests that friction losses in the straight pipe sections are negligible compared to losses in the threaded elbows and fittings of the system. Do you agree or disagree with your boss? Support your answer with appropriate calculations.



■ FIGURE P8.66

Major loss $= f\frac{\ell}{D}\frac{V^2}{2g}$ where

$\ell = (6+6+4+1)\text{ in.} = 17\text{ in.}$, $D = 0.75\text{ in.}$

and

$$V = \frac{Q}{A} = \frac{0.02\ \frac{ft^3}{s}}{\frac{\pi}{4}(0.75/12)^2 ft^2} = 6.52\ \frac{ft}{s}$$

Thus, with

$$Re = \frac{VD}{\nu} = \frac{6.52\frac{ft}{s}\left(\frac{0.75}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 3.37\times10^4 \text{ and}$$

$$\frac{\varepsilon}{D} = \frac{0.0005\ ft}{\left(\frac{0.75}{12}ft\right)} = 8\times10^{-3} \text{ (see Table 8.1) we obtain (see Fig. 8.20)}$$

$$f = 0.038 \text{ so that } f\frac{\ell}{D}\frac{V^2}{2g} = 0.038\,\frac{17\text{ in.}}{0.75\text{ in.}}\frac{V^2}{2g} = 0.861\frac{V^2}{2g} \qquad (1)$$

Also,

$$\text{Minor loss} = \sum K_L\frac{V^2}{2g} = \left[\underbrace{2(1.5)}_{90^\circ \text{ elbow}} + \underbrace{2}_{\text{tee}} + \underbrace{0.15}_{\text{reducer}}\right]\frac{V^2}{2g} = 5.15\frac{V^2}{2g} \qquad (2)$$

reducer with $\frac{A_2}{A_1} = \left(\frac{0.6\text{ in.}}{0.75\text{ in.}}\right)^2 = 0.64$
(see Fig. 8.26)

Thus, from Eqs. (1) and (2):

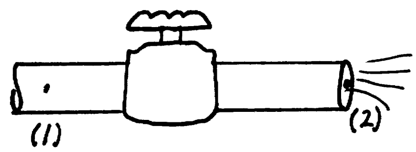
$$\frac{\text{major loss}}{\text{minor loss}} = \frac{0.861\frac{V^2}{2g}}{5.15\frac{V^2}{2g}} = 0.167 = 16.7\%$$

Probably <u>disagree</u> with boss because pipe friction is about 17% of other losses.

8.67

8.67 Because of a worn-out washer in a kitchen sink faucet, water drips at a steady rate even through the faucet is "turned off." Readings from a water meter of the type shown in Video V8.7 indicate that during a one-week time period when the homeowners were away, 200 gallons of water dripped from the faucet. (a) If the pressure within the 0.50-in-diameter pipe is 50 psi, determine the loss coefficient for the leaky faucet. (b) What length of the pipe would be needed to produce a head loss equivalent to the leaky faucet?

(a) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$



where

$p_1 = 50\,psi$, $p_2 = 0$, $V_1 = V_2$, and $z_1 = z_2$

Thus,

$\frac{p_1}{\gamma} = h_L = K_L \frac{V^2}{2g}$ with $V = \frac{Q}{A} = \frac{200\frac{gal}{week}\left(\frac{1\,week}{7\times24\times3600\,s}\right)\left(\frac{ft^3}{7.48\,gal}\right)}{\frac{\pi}{4}\left(\frac{0.5}{12}\,ft\right)^2}$

or $V = 0.0324\,\frac{ft}{s}$

Hence,

$$\frac{\left(50\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}} = K_L \frac{\left(0.0324\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)}$$

or

$K_L = \underline{\underline{7.08\times10^6}}$

(b) $K_L \frac{V^2}{2g} = f\frac{\ell_{eq}}{D}\frac{V^2}{2g}$ or $\ell_{eq} = \frac{K_L D}{f}$

But

$Re = \frac{VD}{\nu} = \frac{\left(0.0324\frac{ft}{s}\right)\left(\frac{0.5}{12}\,ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 112 < 2100$ so the flow is laminar

Hence,

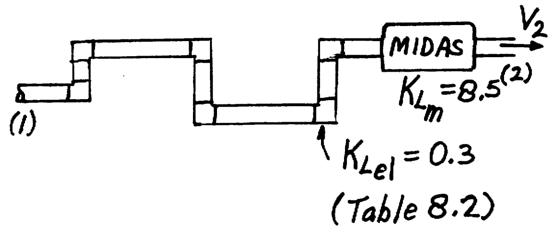
$f = \frac{64}{Re} = \frac{64}{112} = 0.571$

so that

$\ell_{eq} = \frac{(7.08\times10^6)\left(\frac{0.5}{12}\,ft\right)}{0.571} = \underline{\underline{5.17\times10^5\,ft}} = 97.9\,miles!$

8.68

8.68 Assume a car's exhaust system can be approximated as 14 ft of 0.125-ft-diameter cast-iron pipe with the equivalent of six 90° flanged elbows and a muffler. (See Video V8.5.) The muffler acts as a resistor with a loss coefficient of $K_L = 8.5$. Determine the pressure at the beginning of the exhaust system if the flowrate is 0.10 cfs, the temperature is 250 °F, and the exhaust has the same properties as air.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1 = z_2,\ p_2 = 0,$$

and

$$V = V_1 = V_2 = \frac{Q}{A} = \frac{0.1\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.125\,ft)^2} = 8.15\,\frac{ft}{s}$$

Thus,

$$p_1 = \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{1}{2}\rho V^2, \text{ where } \rho = \frac{p}{RT} = \frac{(14.7\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1716\,\frac{ft\cdot lb}{slug\cdot{}^\circ R})(460+250)^\circ R} = 1.74\times10^{-3}\,\frac{slug}{ft^3}$$

Also, $\frac{\varepsilon}{D} = \frac{0.00085\ ft}{0.125\,ft} = 0.0068$ (Table 8.1)

so that with $Re = \frac{\rho V D}{\mu} = \frac{(1.74\times10^{-3}\,\frac{slug}{ft^3})(8.15\,\frac{ft}{s})(0.125\,ft)}{4.7\times10^{-7}\,\frac{lb\cdot s}{ft^2}} = 3770$ we

obtain from Fig. 8.20, $f = 0.047$

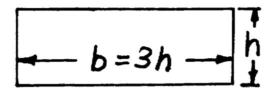
Hence,

$$p_1 = \left(0.047\left(\frac{14\,ft}{0.125\,ft}\right) + 6(0.3) + 8.5\right)\left(\frac{1}{2}\right)\left(1.74\times10^{-3}\,\frac{slug}{ft^3}\right)\left(8.15\,\frac{ft}{s}\right)^2$$

$$= \underline{\underline{0.899\,\frac{lb}{ft^2}}}$$

8.69

8.69 Air is to flow through a smooth horizontal rectangular duct at a rate of 100 m³/s with a pressure drop of not more than 40 mm of water per 50 m of duct. If the aspect ratio (width to height) is 3 to 1, determine the size of the duct.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D_h}\frac{V^2}{2g}, \text{ where } z_1=z_2 \text{ and } V_1=V_2=V \quad (1)$$

with

$$V=\frac{Q}{A}=\frac{100\frac{m^3}{s}}{(3h)h}=\frac{100}{3h^2}, \text{ where } h\sim m,\ V\sim\frac{m}{s}$$

Also, $D_h=\frac{4A}{P}=\frac{4(3h^2)}{2(3h+h)}=1.5h$

Hence, from Eq. (1)

$$p_1-p_2=f\frac{\ell}{D_h}\frac{1}{2}\rho V^2, \text{ or with } p_1-p_2=\gamma_{H_2O}H=9800\frac{N}{m^3}(0.04m)=392\frac{N}{m^2},$$

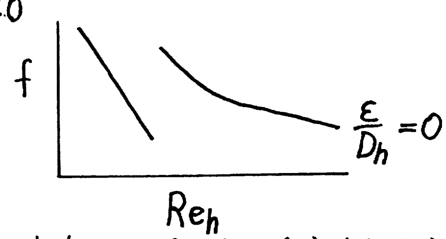
$$392\frac{N}{m^2}=f\left(\frac{50m}{1.5h}\right)\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(\frac{100}{3h^2}\frac{m}{s}\right)^2$$

Thus,

$$f=0.0172\,h^5 \quad (2)$$

Also, $Re=\frac{VD_h}{\nu}=\frac{(\frac{100}{3}h^{-2})(1.5h)}{1.46\times10^{-5}\frac{m^2}{s}}$ or $Re_h=\frac{3.42\times10^6}{h}$ (3)

and from Fig. 8.20



(4)

Trial and error solution of Eqs. (2), (3), (4) for f, Re, and V:

Assume $f=0.02$ so that $0.02=0.0172h^5$ or $h=1.03\,m$. From Eq. (3),

$Re_h=\frac{3.42\times10^6}{1.03}=3.32\times10^6$ which from Fig. 8.20 gives $f=0.0096\neq0.02$

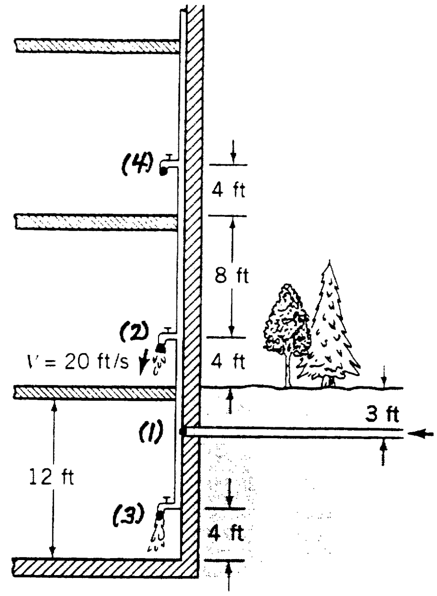
Assume $f=0.0096$ which gives $h=0.890\,m$. Thus, $Re_h=3.84\times10^6$ or $f=0.0093\neq0.0096$

Assume $f=0.0093$, or $h=0.884\,m$. Thus, $Re_h=3.87\times10^6$, of $f=0.0093$ which agrees with the assumed value.

Thus, the duct is $h=0.884\,m$ by $3h=2.65\,m$ in size.

8.70

8.70 Repeat Problem 3.14 if all head losses are included. The pipes are 1-in. copper pipes with regular flanged fittings. The faucets are globe valves.



■ FIGURE P3.14

With 1st floor faucet open, $V = 20\frac{ft}{s}$. Assume $V_1 = V_2 = V = 20\frac{ft}{s}$ also. Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g} \quad \text{with } z_1 = 0,\ z_2 = 7\,ft,\ \ell = 7\,ft,$$

$p_2 = 0$ and $\Sigma K_L = K_{L_{tee}} + K_{L_{globe\ valve}} = 1 + 10 = 11$ becomes

$$\frac{p_1}{\gamma} = 7\,ft + \left(f\frac{7\,ft}{\left(\frac{1}{12}ft\right)} + 11\right)\frac{\left(20\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 7\,ft + 6.21(84f + 11)\,ft \qquad (1)$$

But from Table 8.1, $\frac{\varepsilon}{D} = \frac{5\times10^{-6}\,ft}{\left(\frac{1}{12}ft\right)} = 6.0\times10^{-5}$

and since

$$Re = \frac{DV}{\nu} = \frac{\left(\frac{1}{12}ft\right)\left(20\frac{ft}{s}\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 1.38\times10^5$$ it follows from Fig. 8.20 that

$f = 0.0165$

Hence, from Eq. (1), $\frac{p_1}{\gamma} = 7\,ft + 6.21(84(0.0165) + 11)\,ft = 83.9\,ft$

or $p_1 = (83.9\,ft)\left(62.4\frac{lb}{ft^3}\right)\left(\frac{1\,ft^2}{144\,in.^2}\right) = 36.4\,psi$

Now, assume p_1 remains the same regardless which faucet is open. This is essentially true if the supply line leading to the pipes is relatively large compared to the pipes in the house.

(con't)

8.70 (con't)

(a) Open basement faucet:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\,,\quad \text{where } z_1=0,\ z_3=-5\,ft,$$

$p_3=0$, $V_1=V_3=V$, $\frac{p_1}{\gamma}=83.9\,ft$, $\ell=5\,ft$, and

$$\Sigma K_L = K_{L_{90^\circ \text{ elbow}}} + K_{L_{\text{globe valve}}} = 0.3+10=10.3$$

Thus,

$$83.9\,ft=-5\,ft+\left(f\frac{5\,ft}{(\frac{1}{12}ft)}+10.3\right)\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

or

$$5730=(60f+10.3)V^2,\ \text{where } V\sim\frac{ft}{s} \qquad (1)$$

Also $\frac{\varepsilon}{D}=6.0\times10^{-5}$ and

$$Re=\frac{DV}{\nu}=\frac{(\frac{1}{12}ft)\,V}{1.21\times10^{-5}\frac{ft^2}{s}}=6.89\times10^3\,V \text{ where } V\sim\frac{ft}{s} \qquad (2)$$

Trial and error solution: Assume $f=0.0165$ as before.
Thus, from Eq.(1)

$$V=\sqrt{\frac{5730}{(60)(0.0165)+10.3}}=22.5\,\frac{ft}{s} \text{ and from Eq.(2)}$$

$Re=6.89\times10^3(22.5)=1.55\times10^5$ so that from Fig. 8.20, $f=0.0165$. This agrees with the assumed value.

Hence, $V=\underline{\underline{22.5\,\frac{ft}{s}}}$

(b) Open 2^{nd} floor faucet: As above (except between points (1) and (4)),

$$z_4+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}=\frac{p_1}{\gamma} \text{ where } z_4=19\,ft,\ \ell=19\,ft,\ \frac{p_1}{\gamma}=83.9\,ft,$$

and

$$\Sigma K_L=K_{L_{\text{tee}}}+K_{L_{90^\circ \text{ elbow}}}+K_{L_{\text{globe valve}}}=0.2+0.3+10=10.5$$

Hence, $19+\left(f\frac{19}{(\frac{1}{12})}+10.5\right)\frac{V^2}{2(32.2)}=83.9$, or $4180=(228f+10.5)V^2$ (3)
where $V\sim\frac{ft}{s}$

Assume $f=0.0175$. From Eq. (3), $V=17.0\,\frac{ft}{s}$ so from Eq.(2) $Re=1.17\times10^5$. Thus, from Fig. 8.20 with $\frac{\varepsilon}{D}=6.0\times10^{-5}$, $f=0.0175$ which agrees with the assumed value.

Hence, $V=\underline{\underline{17.0\,\frac{ft}{s}}}$

8.71

8.71 Water at 40 °F flows through the coils of the heat exchanger as shown in Fig. P8.71 at a rate of 0.9 gal/min. Determine the pressure drop between the inlet and outlet of the horizontal device.

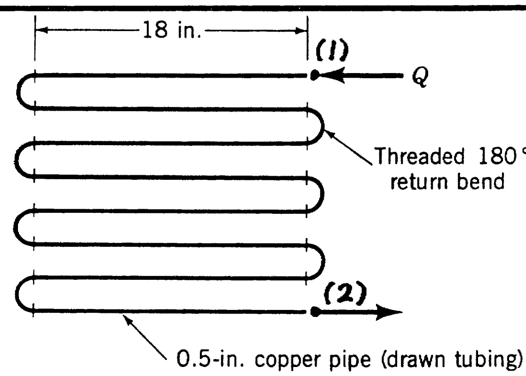


FIGURE P8.71

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1 = z_2,$$

$$V = V_1 = V_2 = \frac{Q}{A} = \frac{(0.9\,\frac{gal}{min})(231\,\frac{in.^3}{gal})(\frac{1\,ft^3}{1728\,in.^3})(\frac{1\,min}{60\,s})}{\frac{\pi}{4}(\frac{0.5}{12}\,ft)^2} = 1.47\,\frac{ft}{s}$$

Thus,

$$p_1 - p_2 = \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{1}{2}\rho V^2, \text{ with } \ell = 8\left(\frac{18}{12}\,ft\right) = 12\,ft \quad (1)$$

and $\Sigma K_L = 7(1.5) = 10.5$ (see Table 8.2)

Also, from Table 8.1 $\frac{\varepsilon}{D} = (0.000005\,ft/(0.5/12\,ft)) = 1.2 \times 10^{-4}$

and $Re = \frac{VD}{\nu} = \frac{(1.47\,\frac{ft}{s})(\frac{0.5}{12}\,ft)}{1.66 \times 10^{-5}\,\frac{ft^2}{s}} = 3690$ (see Table B.1 for ν)

Hence, from Fig. 8.20

$f = 0.041$

and from Eq. (1)

$$p_1 - p_2 = \left(0.041\left(\frac{12\,ft}{\frac{0.5}{12}\,ft}\right) + 10.5\right)\left(\frac{1}{2}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(1.47\,\frac{ft}{s}\right)^2$$

or

$$p_1 - p_2 = 46.8\,\frac{lb}{ft^2} = \underline{\underline{0.325\,psi}}$$

8.72

8.72 Water at 40 °F is pumped from a lake as shown in Fig. P8.72. What is the maximum flowrate possible without cavitation occurring?

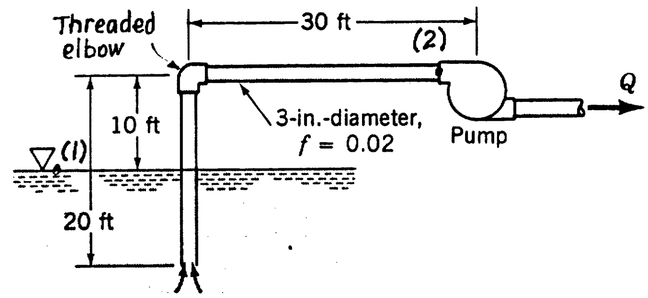


FIGURE P8.72

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1 = 0,\ z_2 = 10\text{ft}, \quad (1)$$

$p_1 = 14.7\frac{lb}{in.^2}(abs)$, $V_1 = 0$, $V_2 = V$, and from Table B.1 $p_2 = 0.1217\ \frac{lb}{in^2}(abs) = 17.52\ \frac{lb}{ft^2}$

Thus, with the given $f = 0.02$ we obtain from Eq. (1)

$$\frac{(14.7\frac{lb}{in^2})(144\frac{in.^2}{ft^2}) - 17.52\ \frac{lb}{ft^2}}{62.4\frac{lb}{ft^3}} = 10ft + \left(0.02\left(\frac{50\ ft}{\frac{3}{12}ft}\right) + 1 + 1.5 + 0.8\right)\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

where we have used $K_L = 0.8$ for the entrance, $K_L = 1.5$ for the $90°$ elbow (see Fig. 8.22 and Table 8.2)

Thus, $V = 14.46\frac{ft}{s}$ so that $Q = AV = \frac{\pi}{4}\left(\frac{3}{12}ft\right)^2(14.46\frac{ft}{s}) = \underline{\underline{0.710\ \frac{ft^3}{s}}}$

8.73

8.73 The $\frac{1}{2}$-in.-diameter hose shown in Fig. P8.73 can withstand a maximum pressure of 200 psi without rupturing. Determine the maximum length, ℓ, allowed if the friction factor is 0.022 and the flowrate is 0.010 cfs. Neglect minor losses.

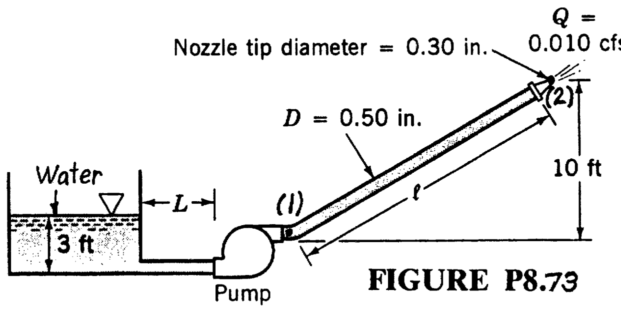


FIGURE P8.73

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1=0,\ z_2=10\,ft,\ p_1=200\,psi, \quad (1)$$

$$p_2=0,\ V_1=\frac{Q}{A_1}=\frac{0.01\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{0.5}{12}ft^2)}=7.33\frac{ft}{s}, \text{ and } V_2=\frac{Q}{A_2}=\frac{0.01\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{0.3}{12}ft)^2}=20.4\frac{ft}{s}$$

Thus, with $f=0.022$ Eq. (1) becomes (using $V=V_1$)

$$\frac{(200\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\frac{(7.33\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}=\frac{(20.4\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}+10ft+0.022\left(\frac{\ell}{\frac{0.5}{12}ft}\right)\frac{(7.33\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$\underline{\underline{\ell=1012\ ft}}$$

8.74

8.74 The hose shown in Fig. P8.73 will collapse if the pressure within it is lower than 10 psi below atmospheric pressure. Determine the maximum length, L, allowed if the friction factor is 0.015 and the flowrate is 0.010 cfs. Neglect minor losses.

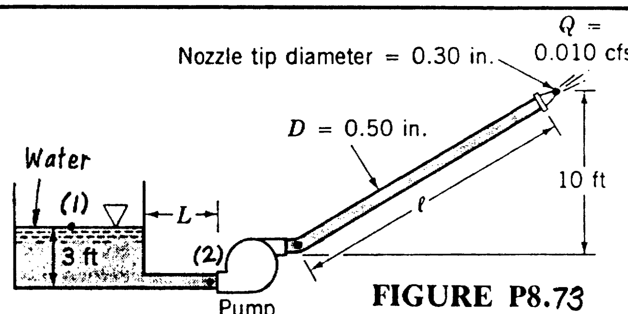


FIGURE P8.73

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{L}{D}\frac{V^2}{2g}, \text{ where } p_1=0,\ V_1=0,\ z_1=3\,ft, \quad (1)$$

$$z_2=0,\ p_2=-10\frac{lb}{in.^2}, \text{ and } V_2=\frac{Q}{A_2}=\frac{0.01\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{0.5}{12}ft)^2}=7.33\frac{ft}{s}=V$$

Thus, with $f=0.015$ Eq. (1) becomes

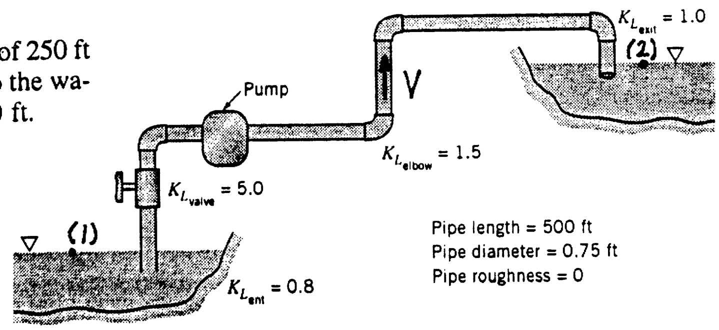
$$3ft=\frac{(-10\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+(0.015)\left(\frac{L}{\frac{0.5}{12}ft}\right)\right)\frac{(7.33\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$\underline{\underline{L=84.0\ ft}}$$

8.75

8.75 The pump shown in Fig. P8.75 delivers a head of 250 ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200 ft.



■ FIGURE P8.75

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = 0$, $z_2 = 200\text{ ft}$, $h_p = 250\text{ ft}$

Thus,

$$-f\frac{\ell}{D}\frac{V^2}{2g} - \sum_i K_{L_i}\frac{V^2}{2g} + h_p = z_2 \quad \text{so that with } \sum_i K_{L_i}\frac{V^2}{2g} = (0.8 + 4(1.5) + 5.0 + 1)\frac{V^2}{2g} = 12.8\frac{V^2}{2g}$$

$$\left[-f\left(\frac{500}{0.75}\right) - 12.8\right]\frac{V^2}{2(32.2)} + 250 = 200$$

or

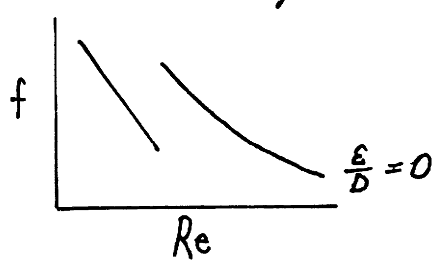
(1) $$(667f + 12.8)V^2 = 3220$$

Also, $$Re = \frac{\rho V D}{\mu} = \frac{\left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)V(0.75\text{ ft})}{2.34\times10^{-5}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}$$

or

(2) $$Re = 6.22\times10^4\, V$$

and from Fig. 8.20:

(3) 

Trial and error solution. Assume $f = 0.02 \xrightarrow{(1)} V = 11.1\ \frac{\text{ft}}{\text{s}} \xrightarrow{(2)} Re = 6.9\times10^5 \xrightarrow{(3)} f = 0.012 \neq 0.02$

Assume $f = 0.012 \xrightarrow{(1)} V = 12.4\ \frac{\text{ft}}{\text{s}} \xrightarrow{(2)} Re = 7.7\times10^5 \xrightarrow{(3)} f = 0.0121 \approx 0.012$

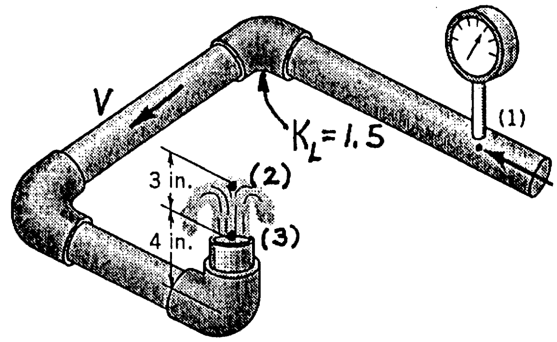
Thus, $V = 12.4\ \frac{\text{ft}}{\text{s}}$ and

$$\dot{W}_s = \gamma Q h_p = \left(62.4\ \frac{\text{lb}}{\text{ft}^3}\right)\frac{\pi}{4}(0.75\text{ ft})^2\left(12.4\ \frac{\text{ft}}{\text{s}}\right)(250\text{ ft}) = 8.55\times10^4\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$= 8.55\times10^4\ \frac{\text{ft}\cdot\text{lb}}{\text{s}} \times 1\ \frac{\text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}} = \underline{\underline{155\text{ hp}}}$$

8.76

8.76 As shown in Video V8.6 and Fig. P8.76, water "bubbles up" 3 in. above the exit of the vertical pipe attached to three horizontal pipe segments. The total length of the 0.75-in.-diameter galvanized iron pipe between point (1) and the exit is 21 inches. Determine the pressure needed at point (1) to produce this flow.



■ FIGURE P8.76

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = 0$, $p_2 = 0$, $V_2 = 0$ Thus,

(1) $$\frac{p_1}{\gamma} = z_2 + h_L - \frac{V_1^2}{2g} \quad \text{where } V_1 = V_3 = V$$

With no head loss from (3) to (2) and $p_2 = p_3 = V_2 = 0$ we obtain

$$\frac{V_3^2}{2g} + z_3 = z_2, \text{ or } V_3 = \sqrt{2g(z_2 - z_3)} = \sqrt{2(32.2\,\tfrac{ft}{s^2})(\tfrac{3}{12}\,ft)} = 4.01\,\tfrac{ft}{s}$$

Thus,

$$Re = \frac{VD}{\nu} = \frac{V_3 D}{\nu} = \frac{4.01\,\frac{ft}{s}\left(\frac{0.75}{12}\,ft\right)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 2.07\times10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\right)ft} = 0.008$$ (see Table 10.1), so that (see Fig. 8.20)

$f = 0.039$

Also, $h_L = f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$ where $\sum K_L = 3(1.5) = 4.5$

Hence, Eq. (1) becomes

$$\frac{p_1}{\gamma} = z_2 + \left[f\frac{\ell}{D} + \sum K_L\right]\frac{V^2}{2g} - \frac{V_1^2}{2g} \quad \text{where } V_1 = V$$

or

$$\frac{p_1}{\gamma} = \frac{7}{12}\,ft + \left[0.039\,\frac{21\,in.}{0.75\,in.} + 4.5 - 1\right]\frac{\left(4.01\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = (0.583 + 1.147)\,ft$$

$$= 1.73\,ft$$

Thus,

$$p_1 = (62.4\,\tfrac{lb}{ft^3})(1.73\,ft) = 108\,\tfrac{lb}{ft^2} = \underline{\underline{0.750\,psi}}$$

8.77

8.77 The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flowrate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, h, of the water tank under the assumption that (a) minor losses are negligible, (b) minor losses are not negligible.

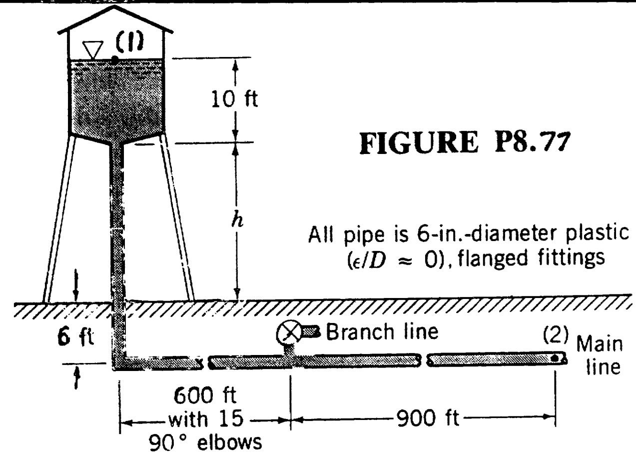


FIGURE P8.77

All pipe is 6-in.-diameter plastic ($\epsilon/D \approx 0$), flanged fittings

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_1=0,\ V_1=0,\ z_1=16\,ft+h,$$

and $z_2=0$ Thus, with $V=V_2$

$$16+h=\frac{p_2}{\gamma}+\frac{V^2}{2g}+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}$$. Note: h must be no less than that with $p_{2min}=60\,psi$ and $Q_{max}=1\,cfs$, or

$$V_2=V=\frac{Q}{A_2}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12}ft\right)^2}=5.09\frac{ft}{s}$$

Hence,

$$h=-16\,ft+\frac{\left(60\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}}+\left(1+f\left(\frac{h+6+600+900}{\frac{6}{12}}\right)+\Sigma K_L\right)\frac{\left(5.09\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)}$$

or

$$h=122.5+\left(1+f\left(\frac{1506+h}{0.5}\right)+\Sigma K_L\right)(0.402)\ ft,\ \text{where } h\sim ft \qquad (1)$$

With $\frac{\varepsilon}{D}=0$ and $Re=\frac{VD}{\nu}=\frac{\left(5.09\frac{ft}{s}\right)\left(\frac{6}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$ we obtain

$f=0.0155$ (see Fig. 8.20)

a) Neglect minor losses ($\Sigma K_L=0$):

From Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)\right)(0.402)$$

or $h=\underline{\underline{143\ ft}}$

b) Include minor losses:

$$\Sigma K_L=K_{L_{entrance}}+15K_{L_{elbow}}+K_{L_{tee}}=0.5+15(0.3)+0.2=5.2$$

(see Table 8.2, assume flanged fittings)

Thus, from Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)+5.2\right)(0.402)$$

or

$h=\underline{\underline{146\,ft}}$

Note: For this case minor losses are not very important.

8.78

8.78 Repeat Problem 8.77 with the assumption that the branch line is open so that half of the flow from the tank goes into the branch, and half continues in the main line.

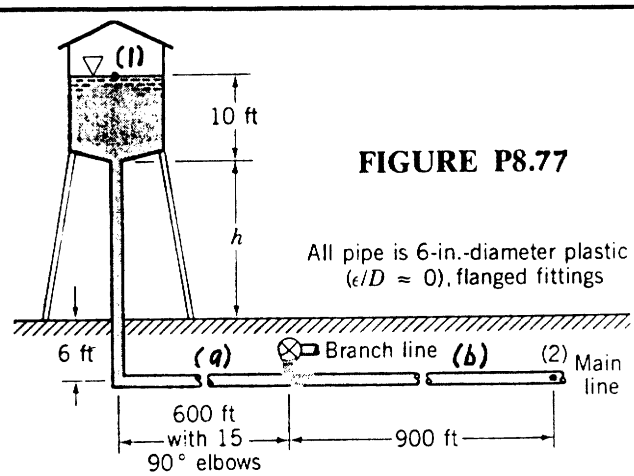


FIGURE P8.77

For the flow from (1) to (2):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f_a\frac{\ell_a}{D_a}+\Sigma K_{La}\right)\frac{V_a^2}{2g}+\left(f_b\frac{\ell_b}{D_b}+\Sigma K_{Lb}\right)\frac{V_b^2}{2g} \quad (1)$$

where $(\ )_a$ and $(\ )_b$ denote pipes "a" and "b" as indicated in the figure. Thus, with $p_1=0$, $V_1=0$, $z_1=16\text{ ft}+h$, $z_2=0$, and $p_2=60\text{ psi}$. Also,

$$V_a=\frac{Q_a}{A_a}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=5.09\frac{ft}{s},\ V_b=\frac{Q_b}{A_b}=\frac{0.5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=2.55\frac{ft}{s},$$ Eq. (1) becomes

$$16+h=\frac{(60\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+f_a\left(\frac{h+6+600}{\frac{6}{12}}\right)+\Sigma K_{La}\right)\frac{(5.09\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$
$$+\left(f_b\left(\frac{900}{\frac{6}{12}}\right)+\Sigma K_{Lb}\right)\frac{(2.55\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h=122.5+\left(1+f_a\left(\frac{606+h}{0.5}\right)+\Sigma K_{La}\right)(0.402)+(1800f_b+\Sigma K_{Lb})(0.101),\ \text{where } h\sim ft \quad (2)$$

With $\frac{\epsilon}{D}=0$, $Re_a=\frac{V_aD_a}{\nu}=\frac{(5.09\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$, and

$Re_b=\frac{V_bD_b}{\nu}=\frac{1}{2}Re_a=1.05\times10^5$ we obtain $f_a=0.0155$ and $f_b=0.0175$ (Fig. 8.20)

a) Neglect minor losses ($\Sigma K_{La}=\Sigma K_{Lb}=0$):

From Eq. (2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)\right)(0.402)+(1800(0.0175)(0.101))$$

or

$$\underline{\underline{h=135\text{ ft}}}$$

b) Include minor losses:

$\Sigma K_{La}=K_{L\,entrance}+15K_{L\,elbow}=0.5+15(0.3)=5.0$ (see Table 8.2; assume flanged fittings)

and

$\Sigma K_{Lb}=K_{L\,tee}=0.2$

From Eq. (2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)+5.0\right)(0.402)+(1800(0.0175)+0.2)(0.101)$$

or

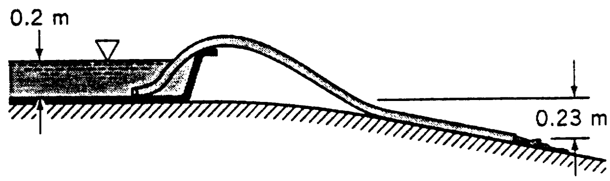
$$\underline{\underline{h=137\text{ ft}}}$$

Note: For this case minor losses are not very important.

8.79

8.79 Repeat Problem 3.43 if head losses are included.

3.43 A smooth plastic, 10-m-long garden hose with an inside diameter of 20 mm is used to drain a wading pool as is shown in Fig. P3.43. If viscous effects are neglected, what is the flowrate from the pool?



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g} \qquad (1)$$

where $p_1=p_2=0$, $V_1=0$, $z_2=0$, $z_1=0.43\,m$ and $V_2=V$

Thus, with $\Sigma K_L = K_{L\,ent} = 0.8$ Eq. (1) becomes

$$0.43\,m=\frac{V^2}{2(9.81\frac{m}{s^2})}\left(1+f\frac{10\,m}{0.02\,m}+0.8\right)$$

or

$$8.44=(1.8+500f)V^2, \quad \text{where } V\sim\frac{m}{s} \qquad (2)$$

Also, $$Re=\frac{DV}{\nu}=\frac{(0.02\,m)\,V}{1.12\times10^{-6}\frac{m^2}{s}}=1.79\times10^4\,V \qquad (3)$$

Trial and error solution:

Assume $f=0.02$ or from Eq. (2)

$$V=\left[\frac{8.44}{(1.8+500(0.02))}\right]^{\frac{1}{2}}=0.846\,\frac{m}{s}$$

so from Eq. (3), $Re=1.79\times10^4(0.846)=1.51\times10^4$

With this Re value and $\frac{\varepsilon}{D}=0$ we obtain from the Moody chart (Fig. 8.20), $f=0.027$ which is not the assumed value. Thus try again.

Assume $f=0.027$ or from Eqs. (2) and (3),
$V=0.742\,\frac{m}{s}$ and $Re=1.33\times10^4$. Thus, from the Moody chart $f=0.028\neq0.027$

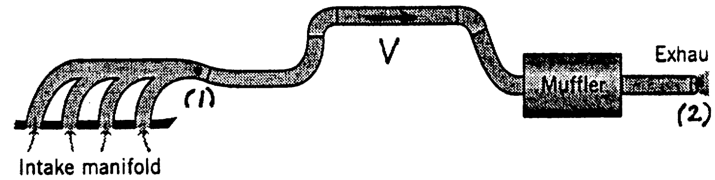
Assume $f=0.028$ which gives $V=0.731\,\frac{m}{s}$, $Re=1.31\times10^4$, and from the Moody chart, $f=0.028$, the assumed value.

Hence, $V=0.731\,\frac{m}{s}$ and

$$Q=AV=\frac{\pi}{4}(0.020\,m)^2\left(0.731\,\frac{m}{s}\right)=\underline{\underline{2.30\times10^{-4}\,\frac{m^3}{s}}}$$

8.80

8.80 The exhaust from your car's engine flows through a complex pipe system as shown in Fig. P8.80 and Video V8.5. Assume that the pressure drop through this system is Δp_1 when the engine is idling at 1000 rpm at a stop sign. Estimate the pressure drop (in terms of Δp_1) with the engine at 3000 rpm when you are driving on the highway. List all assumptions you made to arrive at your answer.



■ FIGURE P8.80

For steady flow,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

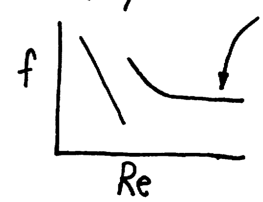
Assume $z_1 = z_2$ and $V_1 = V_2$ so that with $h_L = \left[f\frac{\ell}{D} + K_L\right]\frac{V^2}{2g}$ and $\Delta p \equiv p_1 - p_2$ we obtain

$$\Delta p = \gamma h_L = \gamma\left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g} = \frac{1}{2}\rho V^2\left(f\frac{\ell}{D} + K_L\right)$$

Hence,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \frac{\frac{1}{2}\rho_{3000} V_{3000}^2 \left(f_{3000}\frac{\ell}{D} + K_L\right)}{\frac{1}{2}\rho_{1000} V_{1000}^2 \left(f_{1000}\frac{\ell}{D} + K_L\right)}$$

Assume $\rho_{1000} = \rho_{3000}$ and $f_{1000} = f_{3000}$ (i.e. f independent of Re)



Thus,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \left(\frac{V_{3000}}{V_{1000}}\right)^2$$

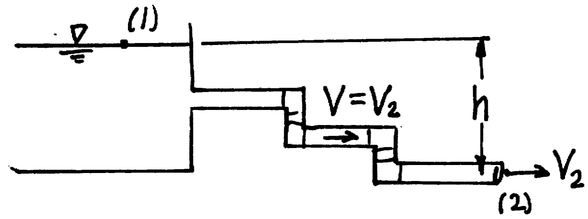
But $V = \frac{Q}{A}$ where Q is assumed proportional to engine rpm.

That is $V_{3000} = 3\,V_{1000}$ so that

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = (3)^2 = \underline{\underline{9}}$$

8.81

8.81 Water flows from a large open tank, through a 50-ft-long, 0.10-ft-diameter pipe and exits with a velocity of 5 ft/s when the water level in the tank is 10 ft above the pipe exit. The sum of the minor loss coefficients for the pipe system is 12. Determine the new water level needed in the tank if the velocity is to remain 5 ft/s when 20 ft of the pipe is removed (i.e., when the length is reduced to 30 ft). The minor loss coefficients remain the same.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = p_2 = 0$, $z_2 = 0$, $z_1 = h$, $V_1 = 0$, $V = V_2$

and
$$h_L = f\frac{\ell}{D} + \sum_i K_{L_i} \frac{V^2}{2g} \quad \text{with } \sum_i K_{L_i} = 12$$

Thus, originally $h = 10\,ft$ and $\ell = 50\,ft$,

$$h = h_L + \frac{V^2}{2g} \quad \text{or}$$

$$10\,ft = \left[\frac{50\,ft}{0.1\,ft} f + 12 + 1\right] \frac{(5\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} \quad \text{or} \quad f = 0.0255$$

With ℓ reduced to 30 ft but V remaining at 5 ft/s (and thus the same f)

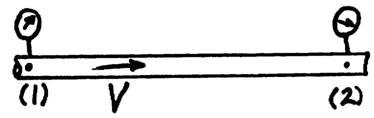
$$h = \left[\frac{30\,ft}{0.1\,ft}(0.0255) + 12 + 1\right] \frac{(5\,ft/s)^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h = \underline{\underline{8.02\,ft}}$$

8.82

8.82 Water is to flow at a rate of 3.5 ft^3/s in a horizontal aluminum pipe ($\varepsilon = 5 \times 10^{-6}$ ft). The inlet and outlet pressures are 65 psi and 30 psi, respectively, and the pipe length is 500 ft. Determine the diameter of this water pipe.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} \;; \quad z_1 = z_2 \,,\; V_1 = V_2 = V$$

where

$$V = \frac{Q}{A} = \frac{3.5}{\frac{\pi}{4}D^2} = \frac{4.46}{D^2} \quad \text{where } D \sim ft,\; V \sim ft/s$$

Thus,

$$\frac{p_1 - p_2}{\gamma} = f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{or} \quad \frac{(65-30)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}} = f\,\frac{500}{D}\,\frac{\left(\frac{4.46}{D^2}\right)^2}{2(32.2)}$$

which simplifies to

$$D = 1.138\, f^{1/5} \qquad (1)$$

Also, from Table 8.1, $\quad \frac{\varepsilon}{D} = \frac{5 \times 10^{-6}}{D} \qquad (2)$

and

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\left(\frac{4.46}{D^2}\right)D}{2.34 \times 10^{-5}} \quad \text{or} \quad Re = \frac{3.70 \times 10^5}{D} \qquad (3)$$

Trial and error solution: 4 unknowns (D, $\frac{\varepsilon}{D}$, Re, f); 4 equations ((1), (2), (3), and Moody chart (Fig. 8.20))

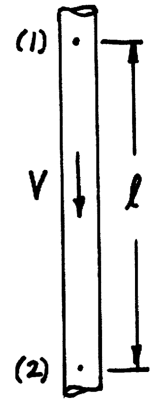
Assume $f = 0.02$ so from Eq. (1) $D = 0.520$ ft. Thus, from Eqs. (2) and (3) $Re = 7.11 \times 10^5$ and $\frac{\varepsilon}{D} = 9.6 \times 10^{-6}$, so from Fig. 8.20, $f = 0.0128 \neq 0.02$

Assume $f = 0.0128$ which gives $D = 0.476$ ft, $Re = 7.77 \times 10^5$, and $\frac{\varepsilon}{D} = 1.1 \times 10^{-5}$. Thus, from Fig. 8.20, $f = 0.0128$ which agrees with the assumed value.

Thus, $D = \underline{\underline{0.476 \text{ ft}}}$

8.83

8.83 Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft^3/s there is no change in pressure along the pipe. Determine the diameter of the pipe.



$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}$$

where $p_1=p_2$, $V_1=V_2=V$, and $z_1-z_2=\ell$

Thus,

$$\ell=f\frac{\ell}{D}\frac{V^2}{2g}, \text{ or } 1=\frac{f}{D}\frac{V^2}{2g} \qquad (1)$$

Also,

$V=\frac{Q}{A}=\frac{Q}{\frac{\pi}{4}D^2}$ so that Eq. (1) becomes $1=\frac{f}{D}\frac{\left(\frac{4Q}{\pi D^2}\right)^2}{2g}$

or

$$D^5=\frac{8}{\pi^2}f\frac{Q^2}{g}=\frac{8}{\pi^2}f\frac{(0.5)^2}{32.2} \text{ or } D=0.363\,f^{1/5} \qquad (2)$$

Also,

$$Re=\frac{\rho VD}{\mu}=\frac{1.94\left(\frac{4Q}{\pi D^2}\right)D}{\mu}=\frac{1.94\left(\frac{4(0.5)}{\pi}\right)}{2.34\times10^{-5}\,D} \text{ or } Re=\frac{5.28\times10^4}{D} \qquad (3)$$

From Fig. 8.20 with $\frac{\varepsilon}{D}=0$ we have $f=f(Re, \frac{\varepsilon}{D}=0)$

Trial and error solution: 3 unknowns (D, Re, f) and 3 equations ((2), (3), and Fig. 8.20)

Assume $f=0.02$ so from Eq. (2), $D=0.166$ ft and from Eq. (3), $Re=3.18\times10^5$. Thus, from Fig. 8.20, $f=0.014\neq0.02$

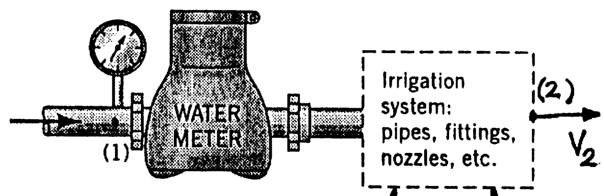
Assume $f=0.014$ so that $D=0.155$ ft and $Re=3.42\times10^4$

Thus, from Fig. 8.20, $f\approx0.14$ which checks with the assumed value.

Thus, $D=\underline{\underline{0.155\text{ ft}}}$

8.84

8.84 As shown in Fig. P8.84, a standard household water meter is incorporated into a lawn irrigation system to measure the volume of water applied to the lawn. Note that these meters measure volume, not volume flowrate. (See Video V8.7.) With an upstream pressure of $p_1 = 50$ psi the meter registered that 120 ft^3 of water was delivered to the lawn during an "on" cycle. Estimate the upstream pressure, p_1, needed if it is desired to have 150 ft^3 delivered during an "on" cycle. List any assumptions needed to arrive at your answer.



■ FIGURE P8.84

$f\frac{\ell}{D}$ $\qquad \Sigma K_L$

The energy equation for this flow is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - \left[f\frac{\ell}{D} + \Sigma K_L\right]\frac{V^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $p_2 = 0$, $V_1 = V$, and $V_2 = \frac{A_1}{A_2}V_1$

Thus, from Eq. (1)

$$p_1 = \frac{1}{2}\rho V_1^2\left[f\frac{\ell}{D} + \Sigma K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right] \qquad (2)$$

But $Q = A_1V_1 = \frac{\forall}{t}$, where $\forall$ is the volume of water supplied during an "on" cycle and t is the length of the cycle.

For a given system ΣK_L is independent of Q. Similarly, for large Re pipe flow, f is independent of Re (or Q). Thus,

$\left[f\frac{\ell}{D} + \Sigma K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right]$ is constant, independent of Q.

Hence, from Eq. (2), if the length of the cycle is constant,

$$\frac{p_1)_{150\,ft^3}}{p_1)_{120\,ft^3}} = \frac{\frac{1}{2}\rho V_1^2)_{150}}{\frac{1}{2}\rho V_1^2)_{120}} = \left[\frac{V_1)_{150}}{V_1)_{120}}\right]^2 = \left(\frac{\forall_{150}}{\forall_{120}}\right)^2 = \left(\frac{150}{120}\right)^2 = 1.563$$

or

$$p_1)_{150} = 1.563\,p_1)_{120} = 1.563(50\,psi) = \underline{\underline{78.1\,psi}}$$

8.85

8.85 When water flows from the tank shown in Fig. P8.85, the water depth in the tank as a function of time is as indicated. Determine the cross-sectional area of the tank. The total length of the 0.60-in.-diameter pipe is 20 ft, and the friction factor is 0.03. The loss coefficients are: 0.50 for the entrance, 1.5 for each elbow, and 10 for the valve.

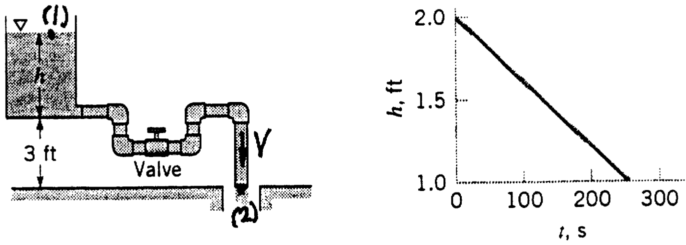


FIGURE P8.85

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0,\ z_2 = 0,\ z_1 = 3\,ft + h,\ V_1 = 0,\ V_2 = V$ and

$h_L = \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g}$ with $\sum_i K_{L_i} = 0.5 + 5(1.5) + 10 = 18$

Thus,

$$z_1 = h_L + \frac{V_2^2}{2g} = \left(f\frac{\ell}{D} + \sum_i K_{L_i} + 1\right)\frac{V^2}{2g}$$

Consider the flow when $h = 1.5\,ft$ so that $z_1 = 4.5\,ft$

Hence,

$$4.5\,ft = \left(0.03\,\frac{20\,ft}{\left(\frac{0.6}{12}ft\right)} + 18 + 1\right)\frac{V^2}{2\left(32.2\,\frac{ft}{s^2}\right)}$$

or

$V = 3.06\,\frac{ft}{s}$ so that $Q = AV = \frac{\pi}{4}\left(\frac{0.6}{12}ft\right)^2\left(3.06\,\frac{ft}{s}\right) = 0.00601\,\frac{ft^3}{s}$

But $Q = A_{tank}\left(-\frac{dh}{dt}\right)$

where from the graph

$$\frac{dh}{dt} \approx \frac{(-1\,ft)}{250\,s} = -0.004\,\frac{ft}{s}$$

Hence,

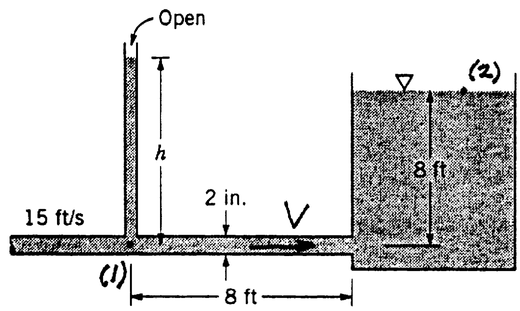
$$0.00601\,\frac{ft^3}{s} = A_{tank}\left(0.004\,\frac{ft}{s}\right)$$

or

$$A_{tank} = \underline{\underline{1.50\,ft^2}}$$

8.86

8.86 Water flows through a 2-in.-diameter pipe with a velocity of 15 ft/s as shown in Fig. P8.86. The relative roughness of the pipe is 0.004, and the loss coefficient for the exit is 1.0. Determine the height, h, to which the water rises in the piezometer tube.



■ FIGURE P8.86

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$\frac{p_1}{\gamma} = h$, $z_1 = 0$, $p_2 = 0$, $z_2 = 8\,ft$, $V_2 = 0$ and

$h_L = \left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g}$ with $V = V_1$ and $K_L = 1$

Thus,

(1) $$h + \frac{V^2}{2g} - \left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g} = z_2$$

But $Re = \frac{\rho V D}{\mu} = \frac{1.94\frac{slug}{ft^3}(15\frac{ft}{s})(\frac{2}{12}ft)}{2.34\times10^{-5}\frac{lb\cdot s}{ft^2}} = 2.07\times10^5$

Hence from Fig. 8.20 with $\varepsilon/D = 0.004$ we obtain $f = 0.029$

so that Eq. (1) becomes

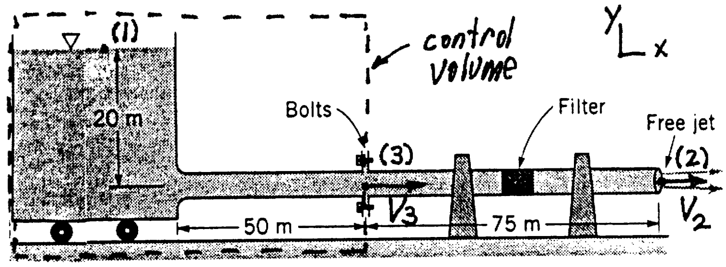
$$h + \left[1 - 0.029\frac{14\,ft}{(\frac{2}{12}ft)} - 1\right]\frac{(15\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = 8\,ft$$

or

$h = \underline{\underline{16.5\,ft}}$

8.87

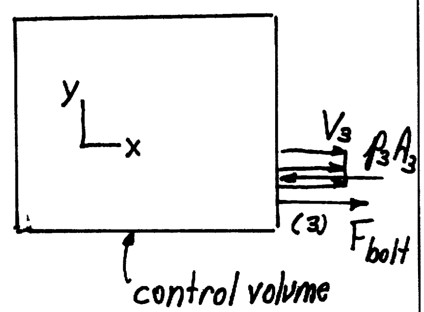
8.87 Water flows from a large tank that sits on frictionless wheels as shown in Fig. P8.87. The pipe has a diameter of 0.50 m and a roughness of 9.2×10^{-5} m. The loss coefficient for the filter is 8; other minor losses are negligible. The tank and the first 50-m section of the pipe are bolted to the last 75-m section of the pipe which is clamped firmly to the floor. Determine the tension in the bolts.



■ FIGURE P8.87

The x-component of the momentum equation for the control volume shown is

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x \quad \text{or} \quad \rho A_3 V_3^2 = F_{bolt} - p_3 A_3$$



(0) Thus, $F_{bolt} = p_3 A_3 + \rho A_3 V_3^2$

Determine V_3 from the energy equation:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - \left(f \frac{\ell}{D} + K_L\right) \frac{V^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = p_2 = 0$, $V_1 = 0$ and $V = V_2 = V_3$ Thus, with $z_2 = 0$

$$z_1 = \left(f \frac{\ell}{D} + K_L + 1\right) \frac{V^2}{2g} \quad \text{or}$$

$$20\,m = \left[f \frac{(50+75)\,m}{0.5\,m} + 8 + 1 \right] \frac{V^2}{2(9.81 \frac{m}{s^2})}$$

or

(1) $$V = \frac{19.8}{\sqrt{9 + 250 f}}$$

Also $$Re = \frac{\rho V D}{\mu} = \frac{(999 \frac{kg}{m^3}) V (0.5\,m)}{1.12 \times 10^{-3} \frac{N \cdot s}{m^2}}$$

or

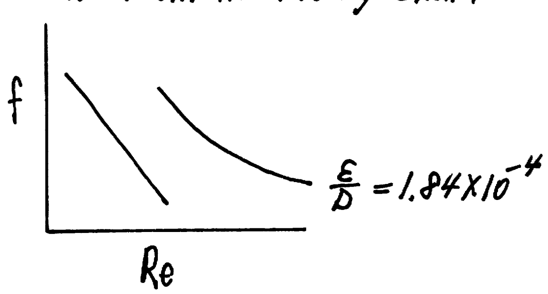
(2) $$Re = 4.46 \times 10^5 \, V$$

and

$$\frac{\varepsilon}{D} = \frac{9.2 \times 10^{-5}\,m}{0.5\,m} = 1.84 \times 10^{-4}$$

Thus from the Moody chart:

(3)



(con't)

8.87 (con't)

Trial and error solution:

Assume $f = 0.02 \xrightarrow{(1)} V = 5.29\,\frac{m}{s} \xrightarrow{(2)} Re = 2.4 \times 10^6 \xrightarrow{(3)} f = 0.014 \neq 0.02$

Assume $f = 0.014 \xrightarrow{(1)} V = 5.60\,\frac{m}{s} \xrightarrow{(2)} Re = 2.5 \times 10^6 \xrightarrow{(3)} f = 0.014$

Thus, $V = 5.60\,\frac{m}{s}$

Also:

$$\frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g} - h_{L_{3-2}} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$z_2 = z_3$, $V_2 = V_3$ and $p_2 = 0$ so that

$$\frac{p_3}{\gamma} = h_{L_{3-2}} = \left(f\,\frac{\ell_{3-2}}{D} + K_L\right)\frac{V^2}{2g} = \left(0.014\left(\frac{75m}{0.5m}\right) + 8\right)\frac{(5.60\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} = 16.1m$$

or

$$p_3 = 9.80 \times 10^3\,\frac{N}{m^3}(16.1m) = 1.58 \times 10^5\,\frac{N}{m^2}$$

Thus, from Eq. (0):

$$F_{bolt} = p_3 A_3 + \rho A_3 V_3^2$$

$$= (1.58 \times 10^5\,\frac{N}{m^2})\,\frac{\pi}{4}(0.5m)^2 + (999\,\frac{kg}{m^3})\frac{\pi}{4}(0.5m)^2(5.60\,\frac{m}{s})^2$$

$$= 3.10 \times 10^4 N + 6.15 \times 10^3 N$$

or

$$F_{bolt} = \underline{\underline{37.2\ kN}}$$

8.88

8.88 Water flows through two sections of the vertical pipe shown in Fig. P8.88. The bellows connection cannot support any force in the vertical direction. The 0.4-ft-diameter pipe weighs 0.2 lb/ft and the friction factor is assumed to be 0.02. At what velocity will the force, F, required to hold the pipe be zero?

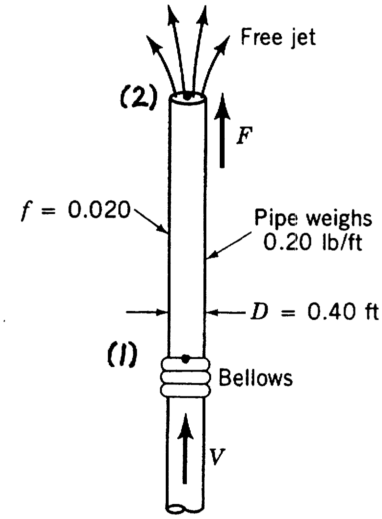
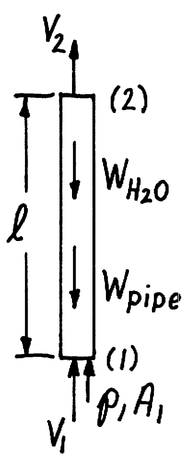


FIGURE P8.88

From the momentum equation applied to the control volume indicated



$$p_1 A_1 - W_{H_2O} - W_{pipe} = \dot{m}(V_2 - V_1) = 0 \text{ since } V_1 = V_2$$

Thus, $p_1 = \dfrac{W_{H_2O} + W_{pipe}}{A_1} = \dfrac{\gamma \ell A_1 + \ell\left(\frac{W_{pipe}}{\ell}\right)}{A_1}$

or

$$p_1 = \gamma \ell + \frac{(0.20\frac{lb}{ft})\ell}{\frac{\pi}{4}(0.4ft)^2} = \gamma \ell + 1.59\ell, \text{ where } p_1 \sim \tfrac{lb}{ft^2},\ \ell \sim ft$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_2 = 0,$$

$V_1 = V_2 = V$, $z_1 = 0$, and $z_2 = \ell$

Thus, $p_1 = \gamma z_2 + f\frac{\ell}{D}\frac{1}{2}\rho V^2$,

or when combined with the above force balance result

$$p_1 = \gamma \ell + f\frac{\ell}{D}\frac{1}{2}\rho V^2 = \gamma \ell + 1.59\ell$$

That is, $\dfrac{f\rho V^2}{2D} = 1.59$ or $V = \sqrt{\dfrac{2D(1.59)}{\rho f}} = \sqrt{\dfrac{2(0.4)(1.59)}{(1.94)(0.02)}} = \underline{\underline{5.73\frac{ft}{s}}}$

Note: This answer is independent of the pipe length, ℓ.

8.89

8.89 The pump shown in Fig. P8.89 adds 25 kW to the water and causes a flowrate of 0.04 m^3/s. Determine the flowrate expected if the pump is removed from the system. Assume $f = 0.016$ for either case and neglect minor losses.

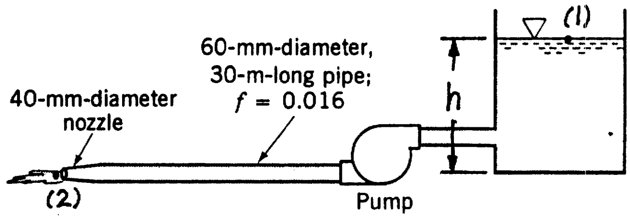


FIGURE P8.89

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ z_1=h,\ z_2=0,$$

$$V_1=0,\quad V_2=\frac{Q}{A_2}=\frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04\,m)^2}=31.8\,\frac{m}{s},\quad V=\frac{Q}{A}=\frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2}=14.15\,\frac{m}{s}$$

Thus,

$$h+h_p=\frac{(31.8\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})}+0.016\left(\frac{30\,m}{0.06\,m}\right)\frac{(14.15\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})}=133.2\,m$$

but,

$$h_p=\frac{P}{\gamma Q}=\frac{25\times10^3\,\frac{N\cdot m}{s}}{(9.80\times10^3\,\frac{N}{m^3})(0.04\,\frac{m^3}{s})}=63.8\,m$$

Hence,

$$h=133.2\,m-63.8\,m=69.5\,m$$

Without the pump $h_p=0$ and $z_1=\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}$ where $h=69.5\,m=z_1$

and

$$V_2=\frac{AV}{A_2}=\left(\frac{D}{D_2}\right)^2V \quad\text{or}\quad V_2=\left(\frac{60\,mm}{40\,mm}\right)^2V=2.25\,V$$

Thus,

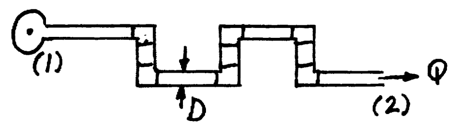
$$69.5\,m=\frac{(2.25\,V)^2+0.016\left(\frac{30\,m}{0.06\,m}\right)V^2}{2(9.81\,\frac{m}{s^2})} \quad\text{or}\quad V=10.22\,\frac{m}{s}$$

so that

$$Q=AV=\frac{\pi}{4}(0.06\,m)^2(10.22\,\frac{m}{s})=\underline{\underline{0.0289\,\frac{m^3}{s}}}$$

8.90

8.90 A certain process requires 2.3 cfs of water to be delivered at a pressure of 30 psi. This water comes from a large diameter supply main in which the pressure remains at 60 psi. If the galvanized iron pipe connecting the two locations is 200 ft long and contains six threaded 90° elbows, determine the pipe diameter. Elevation differences are negligible.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_2=30\text{ psi},\ p_1=60\text{ psi},$$

$z_1=z_2$, $V_1=0$, $V_2=V=\frac{Q}{A}=\frac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{2.93}{D^2}\,\frac{ft}{s}$, with $D\sim ft$

Thus,

$$p_1-p_2=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2$$

or

$$(60-30)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)=\left(1+f\left(\frac{200ft}{D}\right)+6(1.5)+0.5\right)\left(\frac{2.93}{D^2}\frac{ft}{s}\right)^2\left(\frac{1}{2}\right)\left(1.94\frac{slugs}{ft^3}\right)$$

where we have used

$$\Sigma K_L=6K_{L_{elbow}}+K_{L_{entrance}}=6(1.5)+0.5$$

Thus,

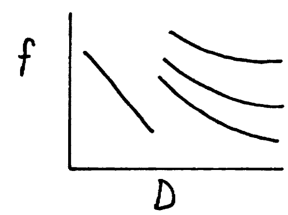
$$49.4=\left(1+\frac{19.0f}{D}\right)\frac{1}{D^4} \qquad (1)$$

Also, $Re=\frac{VD}{\nu}=\frac{\left(\frac{2.93}{D^2}\right)D}{\nu}=\frac{2.93\,\frac{ft^2}{s}}{1.21\times10^{-5}\frac{ft^2}{s}D}$ or $Re=2.42\times10^5\,\frac{1}{D}$ (2)

and from Table 8.1

$$\frac{\varepsilon}{D}=\frac{0.0005\,ft}{D} \qquad (3)$$

Finally, from Fig. 8.20:



(4)

Trial and error solution of Eqs. (1), (2), (3), and (4) for f, D, $\frac{\varepsilon}{D}$, and Re.

Normally it is easiest to guess a value of f, calculate D, etc. In this case (because of minor losse), Eq.(1) is not easy to use in this fashion. Thus, assume D, calculate f (Eq.(1)), Re (Eq.(2)), and $\frac{\varepsilon}{D}$ (Eq.(3)). Look up f in Fig. 8.20 (Eq.(4)) and compare with that from Eq.(1).

Assume $D=0.4ft$. Thus, $f=0.00557$, $Re=6.05\times10^5$, $\frac{\varepsilon}{D}=0.00125$
or from Fig. 8.20 $f=0.021\neq0.00557$

Assume $D=0.5ft$; $f=0.0551$, $Re=4.84\times10^5$, $\frac{\varepsilon}{D}=0.001$ or $f=0.0203\neq0.0551$

Assume $D=0.45ft$; $f=0.0243$, $Re=5.38\times10^5$, $\frac{\varepsilon}{D}=0.00111$ or $f=0.0205\neq0.0243$

Assume $D=0.44ft$; $f=0.0197$, $Re=5.50\times10^5$, $\frac{\varepsilon}{D}=0.00114$ or $f=0.0205\neq0.0197$

After enough trials obtain $D=\underline{\underline{0.442\ ft}}$

Note: If Fig. 8.20 (Eq.(4)) is replaced by the Colebrook equation this problem could be solved easily with a computer.

8.91

8.91 The turbine shown in Fig. P8.91 develops 400 kW. Determine the flowrate if **(a)** head losses are negligible or **(b)** head loss due to friction in the pipe is considered. Assume $f = 0.02$. *Note*: There may be more than one solution or there may be no solution to this problem.

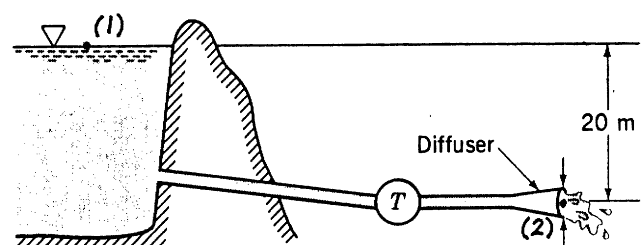


FIGURE P8.91

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}+h_T \text{, where } p_1=p_2=0,\ z_1=20\,m,$$

$z_2=0$ Thus, $z_1=\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}+h_T$ (1)

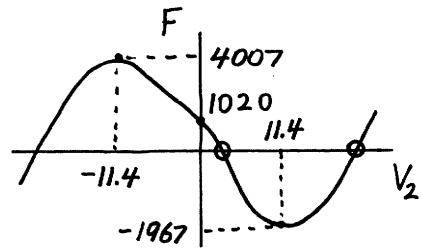
a) Neglect head losses ($f=0$):

$$z_1=\frac{V_2^2}{2g}+h_T \text{, where } h_T=\frac{P}{\gamma Q}=\frac{400\times10^3\,\frac{N\cdot m}{s}}{(9.80\times10^3\frac{N}{m^3})\frac{\pi}{4}(1m)^2 V_2}=\frac{52.0}{V_2}\ m$$

Thus,

$$20m=\frac{V_2^2}{2(9.81\frac{m}{s^2})}+\frac{52.0}{V_2} \quad\text{or}\quad V_2^3-392V_2+1020=0 \qquad (2)$$

Determine the roots of this cubic equation. Let $V_2^3-392V_2+1020\equiv F$
Want $F=0$. Note that $\frac{dF}{dV_2}=3V_2^2-392$ so that $\frac{dF}{dV_2}=0$ at $V_2=\pm11.4\frac{m}{s}$
Also, $F=1020$ when $V_2=0$, $F=4007$ when $V_2=-11.4$, and
$F=-1967$ when $V_2=11.4$. As $V_2\to\infty$, $F\to\infty$.
As $V_2\to-\infty$, $F\to-\infty$. This information indicates there are two positive real roots (see the figure). The negative root has no physical meaning. Solution of Eq. (2) gives

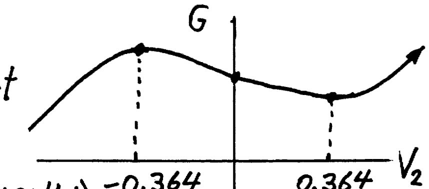


$V_2=2.65\frac{m}{s}$ or $V_2=18.3\frac{m}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}(1m)^2V_2$
or
$\underline{Q=2.08\frac{m^3}{s} \text{ or } Q=14.4\frac{m^3}{s}}$

b) Include head loss ($f=0.02$): From Eq. (1) $V=\frac{V_2A_2}{A}=V_2\left(\frac{D_2}{D}\right)^2=V_2\left(\frac{1m}{0.3m}\right)^2$
$=11.1V_2$
or

$$20m=\left(1+0.02\left(\frac{120m}{0.3m}\right)(11.1)^2\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})}+\frac{52.0}{V_2}\ m$$

Thus,
$V_2^3-0.398V_2+1.034=0$ Let $G\equiv V_2^3-0.398V_2+1.034$; determine V_2 that gives $G=0$. As above, $G\to\pm\infty$ as $V_2\to\pm\infty$; $\frac{dG}{dV_2}=3V_2^2-0.398=0$ for $V_2=\pm0.364$; $G=1.034$ for $V_2=0$, and $G=1.13$ for $V_2=-0.364$, $G=0.937$ at $V_2=+0.364$. Thus, the graph of G looks as shown. A cubic equation has at most two min. or max. As shown, there is no positive real root. The flow cannot occur (must have $P<400\,kW$)



8.92

8.92 A fan is to produce a constant air speed of 40 m/s throughout the pipe loop shown in Fig. P8.92. The 3-m-diameter pipes are smooth, and each of the four 90-degree elbows has a loss coefficient of 0.30. Determine the power that the fan adds to the air.

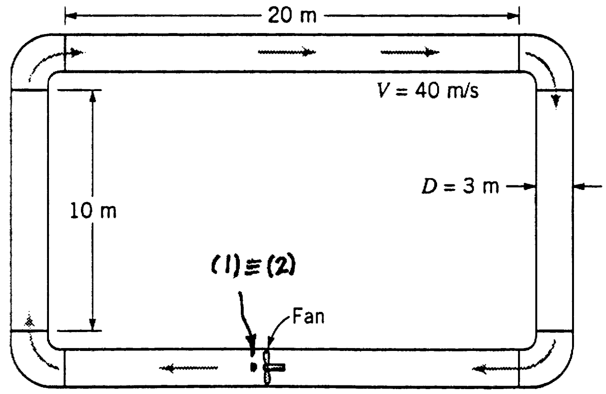


FIGURE P8.92

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}-h_L+h_s=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

If we locate (1) and (2) at the same place it follows that

$p_1=p_2$, $V_1=V_2$, and $z_1=z_2$.

Thus,

$$h_s=h_L=\left(f\frac{\ell}{D}+\sum_i K_{L_i}\right)\frac{V^2}{2g} \quad \text{where } \sum_i K_{L_i}=4(0.30)=1.2$$

Also, $$Re=\frac{\rho VD}{\mu}=\frac{1.23\frac{kg}{m^3}\left(40\frac{m}{s}\right)(3m)}{1.79\times10^{-5}\frac{N\cdot s}{m^2}}=8.25\times10^6$$

and $\frac{\varepsilon}{D}=0$ so that from Fig. 8.20, $f=0.0083$

Hence,

$$h_s=\left(0.0083\,\frac{(20+20+10+10)m}{3\,m}+1.2\right)\frac{\left(40\frac{m}{s}\right)^2}{2\left(9.81\frac{m}{s^2}\right)}=111\,m$$

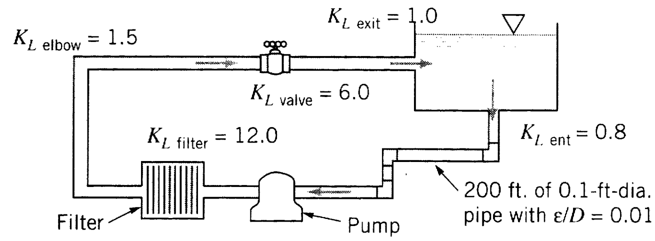
so that

$$\dot{W}_s=\gamma Q h_s=\rho g Q h_s=\left(1.23\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)\left[\frac{\pi}{4}(3m)^2\left(40\frac{m}{s}\right)\right]111\,m$$

$$=3.79\times10^5\,\frac{N\cdot m}{s}=\underline{\underline{379\,kW}}$$

8.93

8.93 Water is circulated from a large tank, through a filter, and back to the tank as shown in Fig. P8.93. The power added to the water by the pump is 200 ft·lb/s. Determine the flowrate through the filter.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \quad (1)$$

where

$p_1 = p_2$, $V_1 = V_2 = 0$, and $z_1 = z_2$

Also, $\dot{W}_p = \gamma Q h_p$ or

$$h_p = \frac{200\ \frac{ft\cdot lb}{s}}{62.4\frac{lb}{ft^3}\left(\frac{\pi}{4}(0.1ft)^2\right)V} = \frac{408}{V}$$

Thus, Eq. (1) becomes

$$\frac{408}{V} = \left(\frac{200ft}{0.1ft}f + (0.8 + 5(1.5) + 12 + 6 + 1)\right)\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

or

$$V^3 = \frac{13.13}{(f + 0.01365)} \quad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\frac{slugs}{ft^3}(V\frac{ft}{s})(0.1ft)}{2.34\times10^{-5}\ \frac{lb\cdot s}{ft^2}} \text{ or } Re = 8290V \quad (3)$$

Trial and error solution:

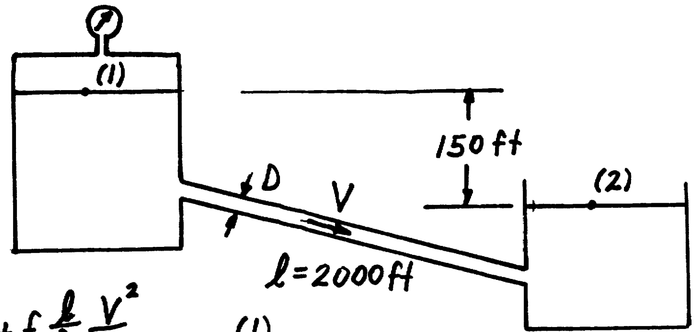
Assume $f = 0.04$. From Eq. (2), $V = 6.26\frac{ft}{s}$; from Eq. (3), $Re = 5.20\times10^4$. Thus, from Fig. 8.20, $f = 0.039 \neq 0.04$

Assume $f = 0.039$, or $V = 6.29\frac{ft}{s}$ and $Re = 5.21\times10^4$ and $f = 0.039$ (Checks)

Thus, $Q = AV = \frac{\pi}{4}(0.1ft)^2(6.29\frac{ft}{s}) = \underline{\underline{0.0494\frac{ft^3}{s}}}$

8.94

8.94 Water is to be moved from a large, closed tank in which the air pressure is 20 psi into a large, open tank through 2000 ft of smooth pipe at the rate of 3 ft^3/s. The fluid level in the open tank is 150 ft below that in the closed tank. Determine the required diameter of the pipe. Neglect minor losses.



$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

where

$V_1=V_2=0$, $z_1-z_2=150\text{ ft}$, and $p_1=20\text{ psi}$, $p_2=0$

Also,

$$V=\frac{Q}{A}=\frac{3\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{3.82}{D^2}, \text{ where } V\sim\frac{ft}{s},\ D\sim ft$$

Thus, Eq. (1) becomes

$$\frac{(20\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+150ft=f\,\frac{2000ft}{D}\,\frac{(\frac{3.82}{D^2})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$D=1.18\,f^{1/5} \qquad (2)$$

Also,

$$Re=\frac{\rho VD}{\mu}=\frac{\rho(\frac{3.82}{D^2})D}{\mu}=\frac{1.94\,(3.82)}{2.34\times10^{-5}\,D}, \text{ or } Re=\frac{3.17\times10^5}{D} \qquad (3)$$

Trial and error solution:

Assume $f=0.02$ so from Eq. (2), $D=0.540$ ft and from Eq. (3) $Re=5.87\times10^5$. Thus, from Fig. 8.20 (with $\frac{\varepsilon}{D}=0$) $f=0.013\neq0.02$

Assume $f=0.013$ which gives $D=0.495$ ft, $Re=6.40\ 10^5$, and $f=0.0125$

Assume $f=0.0125$, so $D=0.491$ ft, $Re=6.46\times10^5$, $f=0.0125$ (Checks)

Thus, $D=\underline{\underline{0.491\text{ ft}}}$

8.95

8.95 Rainwater flows through the galvanized iron downspout shown in Fig. P8.95 at a rate of 0.006 m³/s. Determine the size of the downspout cross section if it is a rectangle with an aspect ratio of 1.7 to 1 and it is completely filled with water. Neglect the velocity of the water in the gutter at the free surface and the head loss associated with the elbow.

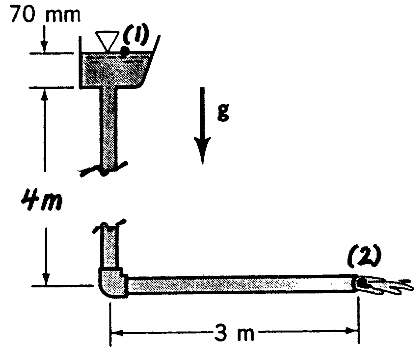


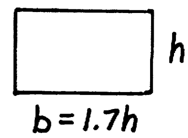
FIGURE P8.95

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D_h}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ V_1=0,\ V_2=V, \quad (1)$$

$z_1=4.07m$, and $z_2=0$

Also, $D_h=\frac{4A}{P}=\frac{4(1.7h^2)}{2(1.7h+h)}=1.26h$

and $V=\frac{Q}{A}=\frac{0.006\frac{m^3}{s}}{1.7h^2}=0.00353h^{-2}\ \frac{m}{s}$, where $h\sim m$



Thus, from Eq.(1)

$$4.07m=\left(1+f\left(\frac{7m}{1.26h\,m}\right)\right)\left(\frac{3.53\times10^{-3}}{h^2}\right)^2\frac{m^2}{s^2}\left(\frac{1}{2(9.81\frac{m}{s^2})}\right)$$

or

$$6.41\times10^6\,h^4=1+5.55\frac{f}{h} \quad (2)$$

From Table 8.1 $\frac{\varepsilon}{D_h}=\frac{0.15\times10^{-3}m}{1.26h\ m}=\frac{1.19\times10^{-4}}{h}$, where $h\sim m$ (3)

and $Re_h=\frac{VD_h}{\nu}=\frac{(0.00353h^{-2}\frac{m}{s})(1.26h\ m)}{1.12\times10^{-6}\frac{m^2}{s}}$ or $Re_h=\frac{3970}{h}$ (4)

Finally, from Fig. 8.20:

(5)

Trial and error solution of Eqs. (2),(3),(4), and (5) for $f, h, Re_h, \frac{\varepsilon}{D_h}$.

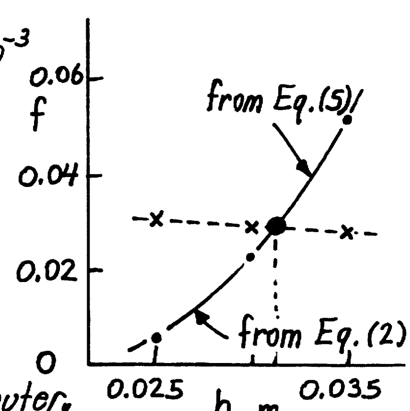
Assume $h=0.04m$; from (2) $f=0.111$, from (3) $\frac{\varepsilon}{D_h}=1.07\times10^{-3}$, and from (4) $Re_h=9.93\times10^4$. Hence, from (4) $f=0.0223\neq0.111$

Assume $h=0.03m$; from (2) $f=0.0227$, $\frac{\varepsilon}{D_h}=4.0\times10^{-3}$ and $Re_h=1.32\times10^5$
Hence, from (5) $f=0.0290\neq0.0227$

Assume $h=0.025m$; or $f=0.00677$, $\frac{\varepsilon}{D_h}=4.76\times10^{-3}$ and $Re_h=1.59\times10^5$
Hence, from (5) $f=0.0303\neq0.00677$

Assume $h=0.035m$; or $f=0.0544$; $\frac{\varepsilon}{D_h}=3.40\times10^{-3}$
$Re_h=1.13\times10^5$. Hence from (5) $f=0.0280$

Plot f from Eq.(2) and f from Eq.(5) as a function of h. Solution is where the two curves intersect.

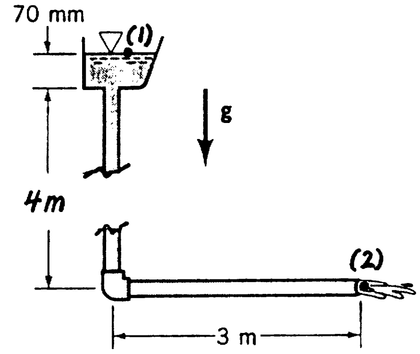


Thus $h=0.031m$ and $b=1.7(0.031m)$

or $\underline{0.031\ m \text{ by } 0.053\ m}$ Note: Replace Fig. 8.20 by the Colebrook eqn. (Eq. 8.35); Use computer.

8.96 *

8.96 * Repeat Problem 8.95 if the downspout is circular.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ V_1=0,\ V_2=V,$$

$z_1 = 4.07\ m$, and $z_2=0$ Thus, $z_1=\left(1+f\frac{\ell}{D}\right)\frac{V^2}{2g}$ or

$$(4.07\,m)(2)(9.81\tfrac{m}{s^2})=\left(1+f\left(\frac{7\,m}{D}\right)\right)V^2 \qquad (1)$$

Hence, with $V=\dfrac{Q}{\frac{\pi}{4}D^2}$ or $V=\dfrac{4(0.006\frac{m^3}{s})}{\pi D^2}=\dfrac{0.00764}{D^2}$, Eq. (1) becomes

$$79.9=\left(1+\frac{7f}{D}\right)\left(\frac{0.00764}{D^2}\right)^2$$

or $f=1.956\times10^5 D^5-0.1429\,D$, where $D\sim m$ (2)

Also, $Re=\dfrac{VD}{\nu}=\dfrac{\left(\frac{0.00764}{D^2}\right)D}{\nu}=\dfrac{0.00764\,\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})D}$

or $Re=\dfrac{6.82\times10^3}{D}$ (3)

From Table 8.1 $\dfrac{\varepsilon}{D}=\dfrac{0.15\times10^{-3}}{D}$ so that Eq. 8.35 becomes (4)

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7}+\frac{2.51}{Re\sqrt{f}}\right] \text{ or when combined with Eqs. (3) and (4)}$$

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{\sqrt{f}}\right] \qquad (5)$$

Solve Eqs. (2) and (5) for f and D as follows:

Rewrite Eqs. (2) and (5) as

$$D=\left[\frac{0.1429\,D+f}{1.956\times10^5}\right]^{1/5} \qquad (6)$$

$$f=\left\{\frac{1}{-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{\sqrt{f}}\right]}\right\}^2 \qquad (7)$$

Solve Eqs. (6) and (7) iteratively. Start with assumed values $D=0.1$, $f=0.02$. From Eq. (6) obtain a new D value. With this new D calculate a new f value from Eq. (7). Repeat such calculations until the n^{th} and $n-1^{st}$ values satisfy the convergence criterion

$$\left|1-\frac{f_n}{f_{n-1}}\right|<0.001 \quad \text{and} \quad \left|1-\frac{D_n}{D_{n-1}}\right|<0.001$$

(con't)

8.96 * (con't)

Program P8#96 .BAS shown below was used to solve Eqs. (6) and (7) as indicated above to give $D = \underline{\underline{0.0445\,m}}$ and $f = 0.0278$

```
100 cls
110 print "*****************************************************"
120 print "** This program determines the friction factor, f, **"
130 print "** and the diameter, D, solving iteratively       **"
140 print "** Colebrook's equation                            **"
170 print "*****************************************************"
180 print
190 f=0.02
200 d=0.1
210 dp=d
220 fp=f
230 d=((f+0.1429*dp)/195600)^0.2
240 f=1/(-2.0*log(0.0000405/d+0.000368*d/fp^.5)/log(10))^2
250 if abs(1-f/fp)>0.001 or abs(1-d/dp)>0.001 then goto 210
260 print
270 print using "The friction factor is f = +#.####^^^^";f
280 print using "       The diameter is D = +#.####^^^^";d
```

```
*****************************************************
** This program determines the friction factor, f, **
** and the diameter, D, solving iteratively       **
** Colebrook's equation                            **
*****************************************************


The friction factor is f = +2.7842E-02
       The diameter is D = +4.4518E-02
```

8.97

8.97 Air, assumed incompressible, flows through the two pipes shown in Fig. P8.97. Determine the flowrate if minor losses are neglected and the friction factor in each pipe is 0.015. Determine the flowrate if the 0.5-in.-diameter pipe were replaced by a 1-in.-diameter pipe. Comment on the assumption of incompressibility.

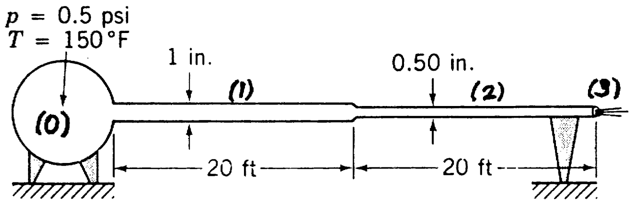


FIGURE P8.97

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3, \text{ where } V_0=0,\ z_0=z_2,\ p_3=0, \quad (1)$$

$V_2=V_3$, $h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}$, $h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$, and $V_1=V_2\frac{A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2$
$=0.25V_2$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \quad (2)$$

With $p_0=\rho_0RT_0$ or $\rho_0=\frac{p_0}{RT_0}=\frac{(0.5\frac{\text{lb}}{\text{in.}^2}+14.7\frac{\text{lb}}{\text{in.}^2})(144\frac{\text{in.}^2}{\text{ft}^2})}{(1716\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}})(150+460)^\circ\text{R}}=0.00209\frac{\text{slug}}{\text{ft}^3}$

and $f_1=f_2=0.015$ Eq. (2) gives

$$(0.5\frac{\text{lb}}{\text{in.}^2})(144\frac{\text{in.}^2}{\text{ft}^2})=\frac{1}{2}(0.00209\frac{\text{slug}}{\text{ft}^3})V_2^2\left[(0.015)\left(\left(\frac{20\text{ ft}}{\frac{1}{12}\text{ft}}\right)(0.25)^2+\left(\frac{20\text{ft}}{\frac{1}{24}\text{ft}}\right)\right)+1\right]$$

or $V_2=90.4\frac{\text{ft}}{\text{s}}$ Thus, $Q=A_2V_2=\frac{\pi}{4}\left(\frac{1}{24}\text{ft}\right)^2(90.4\frac{\text{ft}}{\text{s}})=\underline{\underline{0.123\frac{\text{ft}^3}{\text{s}}}}$

If both pipes were 1 in. diameter, then $V_1=V_2$ and Eq. (1) becomes

$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}+f\frac{\ell_2}{D_2}+1\right]$ or with $f_1=f_2$, $\ell_1=\ell_2$, and $D_1=D_2$

$$p_0=\frac{1}{2}\rho V_2^2\left[f_2\frac{(2\ell_2)}{D_2}+1\right]$$

Hence,

$$(0.5\frac{\text{lb}}{\text{in.}^2})(144\frac{\text{in.}^2}{\text{ft}^2})=\frac{1}{2}(0.00209\frac{\text{slug}}{\text{ft}^3})V_2^2\left[0.015\left(\frac{40\text{ ft}}{\frac{1}{12}\text{ft}}\right)+1\right]$$

or

$V_2=91.7\frac{\text{ft}}{\text{s}}$ Thus, $Q=A_2V_2=\frac{\pi}{4}\left(\frac{1}{12}\text{ft}\right)^2(91.7\frac{\text{ft}}{\text{s}})=\underline{\underline{0.500\frac{\text{ft}^3}{\text{s}}}}$

Since $p=\rho RT$ it follows that

$$\frac{\rho_3}{\rho_0}=\frac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)}=\frac{p_3}{p_0}\frac{T_0}{T_3}$$ If we assume $T_3=T_0$ (it probably will not be, but it should be a reasonable approximation) then

$$\frac{\rho_3}{\rho_0}\approx\frac{p_3}{p_0}=\frac{14.7\text{ psi}}{(0.5+14.7)\text{psi}}=0.967$$ The flow is nearly incompressible.

8.98 ★

***8.98** Repeat Problem 8.97 if the pipes are galvanized iron and the friction factors are not known a priori.

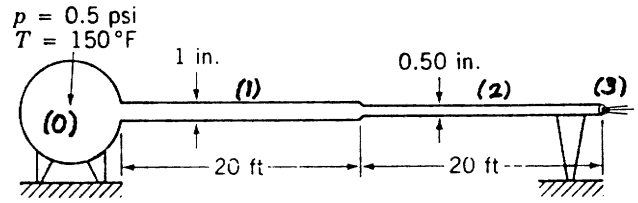


FIGURE P8.98

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3\,,\ \text{where } V_0=0,\ z_0=z_2,\ p_3=0,\ V_2=V_3, \quad (1)$$

$$h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g},\ h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g},\ \text{and } V_1=\frac{V_2A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2=0.25\,V_2$$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \quad (2)$$

With $p_0=\rho_0RT_0$ or $\rho_0=\dfrac{p_0}{RT_0}=\dfrac{(0.5\frac{\text{lb}}{\text{in.}^2}+14.7\frac{\text{lb}}{\text{in.}^2})(144\frac{\text{in.}^2}{\text{ft}^2})}{(1716\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}})(150+460)^\circ\text{R}}=0.00209\ \dfrac{\text{slug}}{\text{ft}^3}$

Eq. (2) becomes

$$(0.5\tfrac{\text{lb}}{\text{in.}^2})(144\tfrac{\text{in.}^2}{\text{ft}^2})=\tfrac{1}{2}(0.00209\tfrac{\text{slug}}{\text{ft}^3})V_2^2\left[(0.25)^2f_1\left(\frac{20\text{ ft}}{\frac{1}{12}\text{ ft}}\right)+f_2\left(\frac{20\text{ ft}}{\frac{1}{24}\text{ ft}}\right)+1\right]$$

or

$$6.89\times10^4=V_2^2(15f_1+480f_2+1) \quad (3)$$

Also from Table 8.1, $\dfrac{\varepsilon}{D_1}=\dfrac{0.0005\text{ ft}}{D_1}=\dfrac{0.0005\text{ ft}}{\frac{1}{12}\text{ ft}}=0.006$ (4)

and $\dfrac{\varepsilon}{D_2}=\dfrac{0.0005\text{ ft}}{\frac{1}{24}\text{ ft}}=0.012$ (5)

and

$Re_1=\dfrac{V_1D_1}{\nu}$, $Re_2=\dfrac{V_2D_2}{\nu}$, where from Table B.3

$$\nu=\frac{\mu}{\rho_0}=\frac{4.18\times10^{-7}\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{0.00209\frac{\text{slug}}{\text{ft}^3}}=2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}$$

Hence,

$$Re_1=\frac{(0.25V_2)(\frac{1}{12}\text{ ft})}{2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}}=104\,V_2 \quad (6)$$

and

$$Re_2=\frac{V_2(\frac{1}{24}\text{ ft})}{2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}}=208\,V_2 \quad (7)$$

For turbulent flow Eq. 8.35 gives $\dfrac{1}{\sqrt{f}}=-2.0\log\left[\dfrac{\frac{\varepsilon}{D}}{3.7}+\dfrac{2.51}{Re\sqrt{f}}\right]$ (8)

By combining Eqs. (4) through (8) we obtain

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left[1.62\times10^{-3}+\frac{2.41\times10^{-2}}{V_2\sqrt{f_1}}\right] \quad (9)$$

and

$$\frac{1}{\sqrt{f_2}}=-2.0\log\left[3.24\times10^{-3}+\frac{1.21\times10^{-2}}{V_2\sqrt{f_2}}\right] \quad (10)$$

(con't)

8.98 (con't)

Solve Eqs.(3), (9), and (10) for the unknowns f_1, f_2, and V_2 (see below).

If $D_1 = D_2$, then $V_1 = V_2$, $f_1 = f_2$ since $\frac{\varepsilon}{D_1} = \frac{\varepsilon}{D_2} = 0.006$, and

$$Re_1 = Re_2 = \frac{V_2 D_2}{\nu} = \frac{V_2\left(\frac{1}{12}\text{ft}\right)}{2.00\times 10^{-4}\frac{\text{ft}^2}{\text{s}}} = 416\,V_2$$

Thus, Eq.(1) becomes

$$p_0 = \frac{1}{2}\rho V_2^2\left[f_2\left(\frac{\ell_1+\ell_2}{D_2}\right)+1\right]$$

or

$$\left(0.5\frac{\text{lb}}{\text{in}^2}\right)\left(144\frac{\text{in}^2}{\text{ft}^2}\right) = \frac{1}{2}\left(0.00209\frac{\text{slug}}{\text{ft}^3}\right)V_2^2\left[f_2\left(\frac{40\text{ft}}{\frac{1}{12}\text{ft}}\right)+1\right]$$

Hence,

$$6.89\times 10^4 = V_2^2\left[480 f_2 + 1\right] \qquad (11)$$

Also, from Eq. (8)

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left[1.62\times 10^{-3} + \frac{6.03\times 10^{-3}}{V_2\sqrt{f_2}}\right] \qquad (12)$$

Solve Eqs. (11) and (12) for f_2 and V_2 (see below)

Note: Since $p = \rho R T$ it follows that

$$\frac{\rho_3}{\rho_0} = \frac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)} = \frac{p_3}{p_0}\frac{T_0}{T_3}$$ If we assume $T_3 = T_0$ (it probably will not be, but it should be a reasonable approximation) then

$$\frac{\rho_3}{\rho_0} = \frac{p_3}{p_0} = \frac{14.7\,\text{psi}}{(0.5+14.7)\text{psi}} = 0.967$$ The flow is nearly incompressible.

Program P8#98 shown below was used to obtain the following results:

a) With the two different diameter pipes: $Q = 0.0746\,\frac{\text{ft}^3}{\text{s}}$

b) With the single pipe: $Q = 0.339\,\frac{\text{ft}^3}{\text{s}}$

```
100 cls
110 print "*******************************************************"
120 print "** This program determines the friction factors,    **"
130 print "** f1 and f2, and the velocity V, solving           **"
140 print "** iteratively Colebrook's equation                 **"
170 print "*******************************************************"
180 print
190 f1=0.002
200 f2=0.002
210 f1p=f1
220 f2p=f2
```

(con't)

8.98 * (con't)

```
230 v=(68900/(15*f1+480*f2+1))^0.5
240 f1=1/(-2.0*log(0.00162+0.0241/(v*f1p^.5))/log(10))^2
245 f2=1/(-2.0*log(0.00324+0.0121/(v*f2p^.5))/log(10))^2
250 if abs(1-f1/f1p)>0.001 or abs(1-f2/f2p)>0.001 then goto 210
260 print
265 print "For the case of unequal diameter pipes:"
270 print using "The friction factors are f1 = +#.####^^^^";f1
275 print using "                       and f2 = +#.####^^^^";f2
280 print using "          The velocity is V = +#.####^^^^ ft/s";v
290 Q = 3.14159*(0.5/12)^2*v/4
300 print using "          The flowrate is Q = +#.####^^^^ ft3/s";Q
380 print
385 print
390 print "For the case of equal diameter pipes:"
400 f2=0.002
420 f2p=f2
430 v=(68900/(480*f2+1))^0.5
445 f2=1/(-2.0*log(0.00162+0.00603/(v*f2p^0.5))/log(10))^2
450 if abs(1-f2/f2p)>0.001 then goto 420
470 print using "  The friction factor is f2 = +#.####^^^^";f2
480 print using "          The velocity is V = +#.####^^^^ ft/s";v
490 Q=3.14159*(1/12)^2*v/4
500 print using "          The flowrate is Q = +#.###^^^^ ft3/s";Q
```

```
*****************************************************
** This program determines the friction factors.   **
** f1 and f2, and the velocity V, solving          **
** iteratively Colebrook's equation                **
*****************************************************


For the case of unequal diameter pipes:
The friction factors are f1 = +4.2508E-02
                     and f2 = +4.4593E-02
          The velocity is V = +5.4682E+01 ft/s
          The flowrate is Q = +7.4561E-02 ft3/s


For the case of equal diameter pipes:
  The friction factor is f2 = +3.5069E-02
          The velocity is V = +6.2160E+01 ft/s
          The flowrate is Q = +3.390E-01 ft3/s
```

8.100

8.100 With the valve closed, water flows from tank A to tank B as shown in Fig. P8.100. What is the flowrate into tank B when the valve is opened to allow water to flow into tank C also? Neglect all minor losses and assume that the friction factor is 0.02 for all pipes.

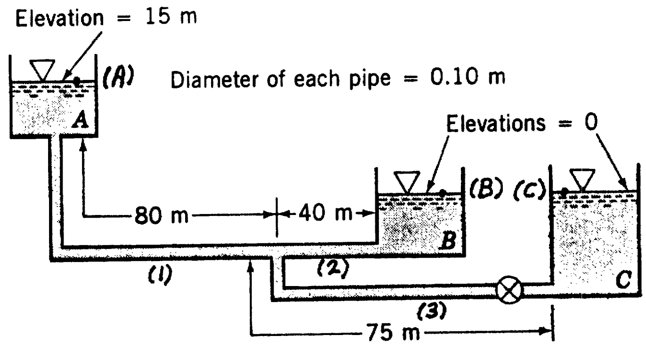


FIGURE P8.100

$Q_1 = Q_2 + Q_3$ where $Q_i = A_i V_i = \frac{\pi}{4} D_i^2 V_i$, $i = 1,2,3$

Thus, since $D_1 = D_2 = D_3$ it follows that

$$V_1 = V_2 + V_3 \tag{1}$$

Also, for fluid flowing from A to B,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}, \text{ where } p_A = p_B = 0,$$

$V_A = V_B = 0$, $z_A = 15$ m, and $z_B = 0$.

Thus,
$$z_A = f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}, \text{ or} \tag{2}$$

$$15\text{ m} = \frac{(0.02)}{(0.1\text{m})(2)(9.81\frac{\text{m}}{\text{s}^2})}\left[(80\text{m})V_1^2 + (40\text{m})V_2^2\right]$$

Hence,

$$18.4 = V_1^2 + 0.5V_2^2, \quad V_i \sim \frac{\text{m}}{\text{s}} \tag{3}$$

Similarly, for fluid flowing from A to C,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}, \text{ where } p_A = p_C = 0,$$

$V_A = V_C = 0$, $z_A = 15$ m, and $z_C = 0$

Thus,
$$z_A = f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g} \tag{4}$$

By comparing Eqs. (2) and (4) we find $f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g} = f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$

or since $f_2 = f_3$ and $D_2 = D_3$,

$$\ell_2 V_2^2 = \ell_3 V_3^2 \quad \text{Thus, } 40V_2^2 = 75V_3^2 \text{ or } V_2 = 1.369V_3 \tag{5}$$

Solve Eqs. (1), (3), and (5) for V_1, V_2, and V_3.

From Eq. (1) and (5): $V_1 = 1.369V_3 + V_3 = 2.369V_3$ and from Eq. (3)

$18.4 = (2.369V_3)^2 + 0.5(1.369V_3)^2$ or $V_3 = 1.676\frac{\text{m}}{\text{s}}$, $V_2 = 1.369(1.676\frac{\text{m}}{\text{s}}) = 2.29\frac{\text{m}}{\text{s}}$

and $V_1 = 2.29\frac{\text{m}}{\text{s}} + 1.676\frac{\text{m}}{\text{s}} = 4.00\frac{\text{m}}{\text{s}}$

Thus

$$Q_2 = A_2V_2 = \frac{\pi}{4}(0.1\text{m})^2(2.29\frac{\text{m}}{\text{s}}) = \underline{\underline{0.0180\frac{\text{m}^3}{\text{s}}}}$$

8.101 *

*8.101 Repeat Problem 8.100 if the friction factors are not known, but the pipes are steel pipes.

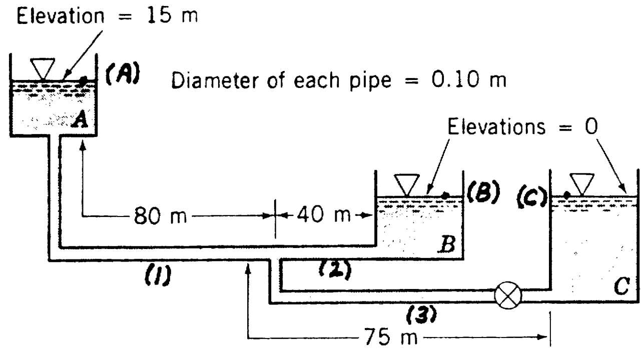


FIGURE P8.100

$Q_1 = Q_2 + Q_3$ where $Q_i = A_i V_i = \frac{\pi}{4} D_i^2 V_i$, $i = 1,2,3$

Thus, since $D_1 = D_2 = D_3$ it follows that

$V_1 = V_2 + V_3$ (1)

Also, for fluid flowing from A to B,

$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g}$, where $p_A = p_B = 0$,

$V_A = V_B = 0$, $z_A = 15\ m$, and $z_B = 0$

Thus, $z_A = f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g}$, or (2)

$15\,m = \left[f_1\left(\frac{80\,m}{0.1\,m}\right)V_1^2 + f_2\left(\frac{40\,m}{0.1\,m}\right)V_2^2\right]\frac{1}{2(9.81\frac{m}{s^2})}$

or

$0.368 = f_1 V_1^2 + 0.5 f_2 V_2^2$ (3)

Similarly, for fluid flowing from A to C,

$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g}$, where $p_A = p_C = 0$,

$V_A = V_C = 0$, $z_A = 15\ m$, and $z_C = 0$

Thus, $z_A = f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g}$ (4)

By comparing Eqs. (2) and (4) we find $f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g} = f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g}$

or since $D_2 = D_3$,

$f_2 l_2 V_2^2 = f_3 l_3 V_3^2$ Thus, $40 f_2 V_2^2 = 75 f_3 V_3^2$ or $V_2 = 1.369\left(\frac{f_3}{f_2}\right)^{1/2} V_3$ (5)

From Eq. 8.135, $\frac{1}{\sqrt{f}} = -2.0 \log\left[\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right]$ where from Table 8.1

$\varepsilon = 0.045\ mm$ so that for each pipe, $\frac{\varepsilon}{D} = \frac{0.045\ mm}{100\ mm} = 4.5 \times 10^{-4}$

Also, $Re = \frac{VD}{\nu}$ or for $i = 1,2,3$

$Re_i = \frac{V_i D_i}{\nu} = \frac{V_i\,(0.1m)}{1.12 \times 10^{-6}\frac{m^2}{s}} = 8.93 \times 10^4 V_i$

(con't)

 (con't)

Thus, $\frac{1}{\sqrt{f_i}} = -2.0 \log\left[1.22\times10^{-4} + \frac{2.81\times10^{-5}}{V_i\sqrt{f_i}}\right]$ for $i = 1,2,3$ (6),(7),(8)

Solve 6 equations for 6 unknowns: Eqs. (1),(3),(5),(6),(7), and (8) for f_1, f_2, f_3, V_1, V_2, V_3. Trial and error solution as follows:

From Eq. (5), $V_3 = 0.730\left(\frac{f_2}{f_3}\right)^{\frac{1}{2}} V_2$, which when combined with Eq. (1) gives

$$V_1 = \left[1 + 0.730\left(\frac{f_2}{f_3}\right)^{\frac{1}{2}}\right] V_2 \qquad (9)$$

Thus, by combining Eqs. (3) and (9) we obtain

$$0.368 = \left\{f_1\left[1+0.730\left(\frac{f_2}{f_3}\right)^{\frac{1}{2}}\right]^2 + 0.5\, f_2\right\} V_2^2$$

or

$$V_2 = \left\{\frac{0.368}{f_1\left[1+0.730\left(\frac{f_2}{f_3}\right)^{\frac{1}{2}}\right]^2 + 0.5\, f_2}\right\}^{\frac{1}{2}} \qquad (10)$$

Also, from Eq. (1),

$$V_3 = V_1 - V_2 \qquad (11)$$

Solution method: a) Guess values of f_1, f_2, and f_3 (A good starting value is the large Re value for $\frac{\varepsilon}{D} = 4.5\times10^{-4}$, or $f_1 = f_2 = f_3 = 0.017$); b) Calculate V_1, V_2, and V_3 from Eqs. (9), (10), and (11); c) Calculate f_1, f_2, and f_3 from Eqs. (6), (7), (8); d) Compare the new f_i with the previous ones; e) If not good enough agreement, repeat with the new f_i as the "guess".

Program P8#101 shown below was used to calculate the following results:

$$\underline{\underline{Q_1 = 0.0331\ \frac{m^3}{s}, \quad Q_2 = 0.0193\ \frac{m^3}{s}, \quad Q_3 = 0.0138\ \frac{m^3}{s}}}$$

(con't)

8.101* (con't)

```
100 cls
110 open "prn" for output as #1
120 print "*********************************************"
130 print "**  This program calculates the flowrates in  **"
140 print "**  the three pipes using the Colebrook form- **"
150 print "**  ula to determine the friction factors.    **"
160 print "**  An i teration scheme is used.             **"
170 print "*********************************************"
200 dim f(3), fp(3), V(3), VP(3), Re(3)
210 for i = 1 to 3
220 f(i) = 0.017
230 VP(i) = 0
240 next i
250 rr = 4.5E-4
260 print " "
270 print "pipe no.   Re        f     V, m/s   Q, m3/s"
300 del=0
305 V(2)=(0.368/(f(1)*(1+0.730*(f(2)/f(3))^0.5)^2+0.5*f(2)))^0.5
310 V(1)=(1+0.730*(f(2)/f(3))^0.5)*V(2)
320 V(3)=V(1)-V(2)
330 for i = 1 to 3
340 fp(i)=f(i)
350 Re(i)=8.93E+4*V(i)
360 if Re(i)<2100 then goto 400
370 f(i)=1/(-2.0*log(rr/3.7+2.51/(Re(i)*fp(i)^0.5))/log(10))^2
380 if abs(1-fp(i)/f(i))>0.001 then goto 340
390 goto 410
400 f(i)=64/Re(i)
410 del=del+abs(1-VP(i)/V(i))
420 next i
500 if del<0.001 then goto 600
510 for i = 1 to 3
520 VP(i)=V(i)
530 next i
540 goto 300
600 for i = 1 to 3
610 Q=(3.14159*0.1^2/4)*V(i)
620 print using " ##     #.##^^^^  #.####  ##.###  #.##^^^^";i,Re(i),f(i),
                                                            V(i),Q
630 next i
```

```
*********************************************
**  This program calculates the flowrates in  **
**  the three pipes using the Colebrook form- **
**  ula to determine the friction factors.    **
**  An i teration scheme is used.             **
*********************************************
```

pipe no.	Re	f	V, m/s	Q, m3/s
1	3.76E+05	0.0176	4.211	3.31E-02
2	2.19E+05	0.0184	2.451	1.93E-02
3	1.57E+05	0.0190	1.760	1.38E-02

8.102

8.102 The three water-filled tanks shown in Fig. P8.102 are connected by pipes as indicated. If minor losses are neglected, determine the flowrate in each pipe.

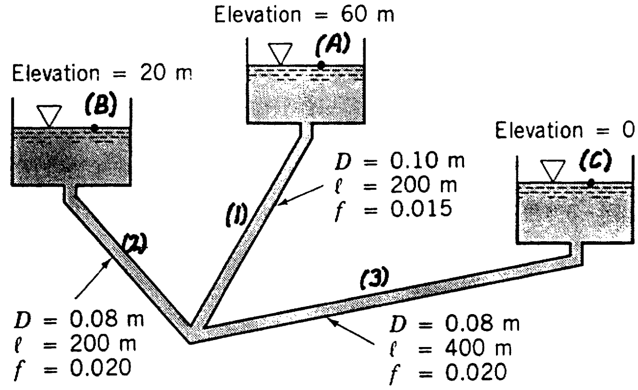


FIGURE P8.102

Assume the fluid flows from A to B and A to C. Thus, $Q_1 = Q_2 + Q_3$

or $\frac{\pi}{4}(0.1m)^2 V_1 = \frac{\pi}{4}(0.08m)^2 V_2 + \frac{\pi}{4}(0.08m)^2 V_3$

Thus, $V_1 = 0.64\, V_2 + 0.64\, V_3$ (1)

For fluid flowing from A to B with $p_A = p_B = 0$ and $V_A = V_B = 0$,

$$z_A = z_B + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}$$

or

$$60m - 20m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{200m}{0.08m}\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})}$$

Hence,

$40 = 1.529\, V_1^2 + 2.55\, V_2^2$ (2)

Similarly, for fluid flowing from A to C with $p_A = p_C = 0$ and $V_A = V_C = 0$,

$$z_A = z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

or

$$60m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{400m}{0.08m}\right)\frac{V_3^2}{2(9.81\frac{m}{s^2})}$$

Hence,

$60 = 1.529\, V_1^2 + 5.10\, V_3^2$ (3)

Solve Eqs. (1), (2), and (3) for V_1, V_2, and V_3. From Eqs. (1) and (3):

$60 = 1.529(0.64)^2(V_2 + V_3)^2 + 5.10\, V_3^2$, or $95.8 = (V_2 + V_3)^2 + 8.14\, V_3^2$ (4)

Subtract Eq. (2) from Eq. (3):

$60 - 40 = 5.10\, V_3^2 + 2.55\, V_2^2$ or $V_2 = \sqrt{2V_3^2 - 7.84}$ (5)

Thus, from Eqs. (4) and (5): $8.14 V_3^2 + \left(\sqrt{2V_3^2 - 7.84} + V_3\right)^2 - 95.8 = 0$

This can be simplified to

$2V_3\sqrt{2V_3^2 - 7.84} = 103.6 - 11.14 V_3^2$ Square both sides and (6)

rearrange to give $V_3^4 - 19.63\, V_3^2 + 92.5 = 0$ which can be solved by the quadratic formula to give

$$V_3^2 = \frac{19.63 \pm \sqrt{19.63^2 - 4(92.5)}}{2} = 11.77 \text{ or } 7.86$$ Thus $V_3 = 3.43 \frac{m}{s}$

or $V_3 = 2.80 \frac{m}{s}$

(con't)

8.102 (con't)

Note: The value $V_3 = 3.43\,\frac{m}{s}$ is not a solution of the original equations, Eqs. (1), (2), and (3). With this value the right hand side of Eq. (6) is negative (i.e. $103.6 - 11.14 V_3^2 = 103.6 - 11.14(3.43)^2 = -24.5$). As seen from the left hand side of Eq. (6), this cannot be. This extra root was introduced by squaring Eq. (6).

Thus, $Q_3 = A_3 V_3 = \frac{\pi}{4}(0.08\,m)^2 (2.80\,\frac{m}{s}) = \underline{\underline{0.0141\,\frac{m^3}{s}}}$

Also, from Eq. (3):

$60 = 1.529\,V_1^2 + 5.10\,(2.80)^2$ or $V_1 = 3.62\,\frac{m}{s}$

or $Q_1 = A_1 V_1 = \frac{\pi}{4}(0.10\,m)^2 (3.62\,\frac{m}{s}) = \underline{\underline{0.0284\,\frac{m^3}{s}}}$

and from Eq. (1):

$3.62 = 0.64\,V_2 + 0.64(2.80)$ or $V_2 = 2.86\,\frac{m}{s}$

or $Q_2 = A_2 V_2 = \frac{\pi}{4}(0.08\,m)^2 (2.86\,\frac{m}{s}) = \underline{\underline{0.0143\,\frac{m^3}{s}}}$

8.103

8.103 Water is pumped from a lake, into a large pressurized tank, and out through two pipes as shown in Fig. P8.103. The pump head is $h_p = 45 + 27.5Q - 54Q^2$, where h_p is in feet and Q (the total flowrate through the pump) is in ft^3/s. Minor losses and gravity are negligible, and the friction factor in each pipe is 0.02. Determine the flowrates through each of the pipes, Q_1, and Q_2.

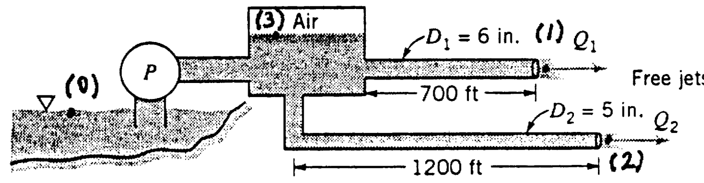


FIGURE P8.103

Since $V_3 = 0$ and gravity is negligible,

(1) $$\frac{p_3}{\gamma} = \frac{V_1^2}{2g} + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g}$$

and

(2) $$\frac{p_3}{\gamma} = \frac{V_2^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}$$ Equate Eqs. (1) and (2) and solve for V_2 to obtain

$$V_2 = V_1\sqrt{\frac{1+f_1\frac{\ell_1}{D_1}}{1+f_2\frac{\ell_2}{D_2}}} = V_1\sqrt{\frac{1+0.02(700\,ft/(6/12\,ft))}{1+0.02(1200\,ft/(5/12\,ft))}} = 0.703V_1$$

Thus,

$$\frac{Q_2}{Q_1} = \frac{A_2V_2}{A_1V_1} = \frac{\frac{\pi}{4}(\frac{5}{12}ft)^2(0.703V_1)}{\frac{\pi}{4}(\frac{6}{12}ft)^2\ V_1}$$ or $Q_2 = 0.488Q_1$ so that

$$Q = Q_1 + Q_2 = Q_1 + 0.488Q_1 = 1.488Q_1$$

Also, with no losses from the lake to the tank,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + h_p = \frac{p_3}{\gamma} + \frac{V_3^2}{2g}$$ so that with $p_0 = 0$ and $V_0 = V_3 = 0$,

(3) $$h_p = \frac{p_3}{\gamma}$$ which can be combined with Eq. (1) to give

(4) $$h_p = \frac{V_1^2}{2g}\left[1+f_1\frac{\ell_1}{D_1}\right]$$ where $V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4}(\frac{6}{12}ft)^2} = 5.09Q_1$

and $h_p = 45 + 27.5 - 54Q^2$ so that Eq. (4) becomes

$$45 + 27.5(1.488Q_1) - 54(1.488Q_1)^2 = \frac{(5.09Q_1)^2}{2(32.2)}\left[1 + 0.02\frac{700}{(\frac{6}{12})}\right]$$

or

$131Q_1^2 - 40.9Q_1 - 45 = 0$ which has the solution

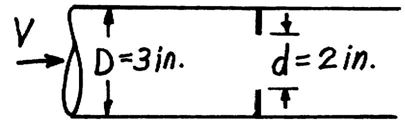
$$Q_1 = \frac{40.9 \pm \sqrt{(40.9)^2 + 4(45)(131)}}{2(131)} = \underline{\underline{0.763\ \frac{ft^3}{s}}}$$ (or -0.450 which has no physical meaning)

Also,

$$Q_2 = 0.488\,Q_1 = 0.488(0.763\frac{ft^3}{s}) = \underline{\underline{0.372\ \frac{ft^3}{s}}}$$

8.104

8.104 A 2-in.-diameter orifice plate is inserted in a 3-in.-diameter pipe. If the water flowrate through the pipe is 0.90 cfs, determine the pressure difference indicated by a manometer attached to the flow meter.



$$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}\ , \quad \text{where } \beta = \frac{d}{D} = \frac{2\ in.}{3\ in.} = \frac{2}{3}\ , \quad Q = 0.70\ \frac{ft^3}{s}\ , \text{ and}$$

$$A_o = \frac{\pi}{4} d$$

Also,

$$Re = \frac{VD}{\nu}\ , \quad \text{where } V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.7\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}\ ft\right)^2} = 14.26\ \frac{ft}{s}$$

Thus,

$$Re = \frac{\left(14.26\ \frac{ft}{s}\right)\left(\frac{3}{12}\ ft\right)}{1.21\times10^{-5}\ \frac{ft^2}{s}} = 2.95\times10^5$$

Hence, from Fig. 8.41 : $C_o = 0.608$

so that,

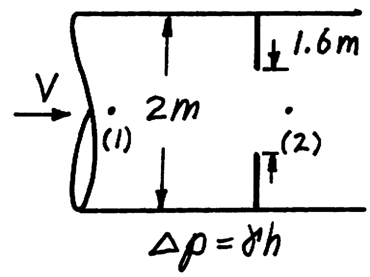
$$0.9\ \frac{ft^3}{s} = (0.608)\frac{\pi}{4}\left(\frac{2}{12}\ ft\right)^2\sqrt{\frac{2(p_1-p_2)}{\left(1.94\ \frac{slugs}{ft^3}\right)\left(1-\left(\frac{2}{3}\right)^4\right)}}$$

or

$$p_1 - p_2 = 3590\ \frac{lb}{ft^2} = \underline{\underline{24.9\ \frac{lb}{in.^2}}}$$

8.105

8.105 Air to ventilate an underground mine flows through a large 2-m-diameter pipe. A crude flowrate meter is constructed by placing a sheet metal "washer" between two sections of the pipe. Estimate the flowrate if the hole in the sheet metal has a diameter of 1.6 m and the pressure difference across the sheet metal is 8.0 mm of water.



$$Q = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}} = C_o \frac{\pi}{4}(1.6\,m)^2 \sqrt{\frac{2(0.008\,m)(9.80\times10^3 \frac{N}{m^3})}{(1.23\frac{kg}{m^3})\left[1-\left(\frac{1.6\,m}{2.0\,m}\right)^4\right]}}$$

or

$$Q = 29.5\,C_o \ \frac{m^3}{s} \qquad (1)$$

Also, $Re = \frac{DV}{\nu} = \frac{(2\,m)\ V}{1.46\times10^{-5}\frac{m^2}{s}}$ or $Re = 1.37\times10^5\, V$ where $V \sim \frac{m}{s}$ (2)

and

$$\beta = \frac{d}{D} = \frac{1.6\,m}{2.0\,m} = 0.8$$

Trial and error solution:

Assume $C_o = 0.61$ so that from Eq. (1), $Q = 29.5\,(0.61) = 18.0\,\frac{m^3}{s}$

Hence, $V = \frac{Q}{A} = \frac{18.0\frac{m^3}{s}}{\frac{\pi}{4}(2.0m)^2} = 5.73\,\frac{m}{s}$

From Eq. (2), $Re = 1.37\times10^5\,(5.73) = 7.85\times10^5$

This Re and β give $C_o = 0.61$ (see Fig. 8.41) which agrees with the assumed value.

Thus, $Q = \underline{\underline{18.0\,\frac{m^3}{s}}}$

8.106

8.106 Gasoline flows through a 35-mm-diameter pipe at a rate of 0.0032 m³/s. Determine the pressure drop across a flow nozzle placed in the line if the nozzle diameter is 20 mm.

$$Q = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}} \text{ , where } \beta = \frac{d}{D} = \frac{20\,mm}{35\,mm} = 0.571 \text{ , } A_n = \frac{\pi}{4}d^2 \qquad (1)$$

From Table 1.6 $\rho = 680 \frac{kg}{m^3}$ and $\mu = 3.1 \times 10^{-4} \frac{N \cdot s}{m^2}$

Thus, $Re = \frac{\rho V D}{\mu}$, where $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.0032 \frac{m^3}{s}}{\frac{\pi}{4}(0.035\,m)^2} = 3.33 \frac{m}{s}$

so that
$$Re = \frac{(680 \frac{kg}{m^3})(3.33 \frac{m}{s})(0.035\,m)}{3.1 \times 10^{-4} \frac{N \cdot s}{m^2}} = 2.56 \times 10^5$$

Hence, from Fig. 8.43, $C_n = 0.986$

From Eq. (1)
$$0.0032 \frac{m^3}{s} = (0.986)\frac{\pi}{4}(0.020\,m)^2 \sqrt{\frac{2(p_1 - p_2)}{(680 \frac{kg}{m^3})(1 - 0.571^4)}}$$
or
$$p_1 - p_2 = 3.24 \times 10^4 \frac{N}{m^2} = \underline{\underline{32.4\,kPa}}$$

8.107

8.107 Air at 200 °F and 60 psia flows in a 4-in.-diameter pipe at a rate of 0.52 lb/s. Determine the pressure at the 2-in-diameter throat of a Venturi meter placed in the pipe.

$$Q = C_v A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}} \text{ , where } \beta = \frac{d}{D} = \frac{2\,in.}{4\,in.} = 0.5 \text{ and } \gamma Q = 0.52 \frac{lb}{s} \qquad (1)$$

Also, $\rho = \frac{p}{RT} = \frac{(60 \frac{lb}{in.^2})(144 \frac{in.^2}{ft^2})}{(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(200+460)^\circ R} = 7.63 \times 10^{-3} \frac{slug}{ft^3}$

so that
$$\gamma = \rho g = (7.63 \times 10^{-3} \frac{slug}{ft^3})(32.2 \frac{ft}{s}) = 0.246 \frac{lb}{ft^3}$$

Thus, $Q = \frac{0.52 \frac{lb}{s}}{0.246 \frac{lb}{ft^3}} = 2.11 \frac{ft^3}{s}$ and $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{2.11 \frac{ft^3}{s}}{\frac{\pi}{4}(\frac{4}{12}ft)^2} = 24.2 \frac{ft}{s}$

Also, from Table B.3, $\mu = 4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2}$ so that
$$Re = \frac{\rho V D}{\mu} = \frac{(7.63 \times 10^{-3} \frac{slug}{ft^3})(24.2 \frac{ft}{s})(\frac{4}{12}ft)}{4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2}} = 1.37 \times 10^5$$

Hence, from Fig. 8.45,
$$C_v \approx 0.98$$

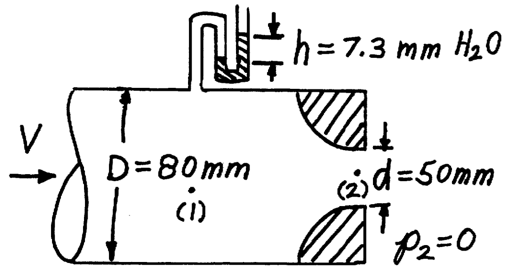
From Eq. (1): $2.11 \frac{ft^3}{s} = (0.98)\frac{\pi}{4}(\frac{2}{12}ft)^2 \sqrt{\frac{2(p_1 - p_2)}{(7.63 \times 10^{-3} \frac{slug}{ft^3})(1 - 0.5^4)}}$

or
$$p_1 - p_2 = 34.8 \frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 0.242 \frac{lb}{in.^2}$$

Thus, $p_2 = (60 - 0.242)\,psia = \underline{\underline{59.76\,psia}}$

8.108

8.108 A 50-mm-diameter nozzle is installed at the end of a 80-mm-diameter pipe through which air flows. A manometer attached to the static pressure tap just upstream from the nozzle indicates a pressure of 7.3 mm of water. Determine the flowrate.



$Q = C_n A_n \sqrt{\dfrac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$, where $\beta = \dfrac{50mm}{80mm} = 0.625$ and

$p_1 - p_2 = \gamma_{H_2O} h = (9800 \frac{N}{m^3})(7.3 \times 10^{-3} m) = 71.5 \frac{N}{m^2}$

Thus, with $A_n = \frac{\pi}{4} d^2$,

$$Q = C_n \frac{\pi}{4}(0.050m)^2 \sqrt{\frac{2(71.5 \frac{N}{m^2})}{(1.23 \frac{kg}{m^3})(1-0.625^4)}}$$

or

$Q = 0.0230\, C_n$ Assume $C_n = 0.97$ so that $Q = 0.0223 \frac{m^3}{s}$

and $V = \dfrac{Q}{\frac{\pi}{4}D^2} = \dfrac{0.0223 \frac{m^3}{s}}{\frac{\pi}{4}(0.08m)^2} = 4.44 \frac{m}{s}$

or

$Re = \dfrac{VD}{\nu} = \dfrac{(4.44 \frac{m}{s})(0.08m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 2.43 \times 10^4$. With the Re and β we obtain

$C_n = 0.963 \neq 0.97$ (the assumed value) (See Fig. 8.43)

Thus, assume $C_n = 0.963$ so that $Q = 0.0230(0.963) = 0.0221 \frac{m^3}{s}$

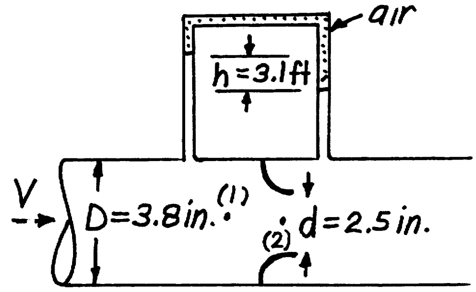
and $V = \dfrac{0.0221 \frac{m^3}{s}}{\frac{\pi}{4}(0.08m)^2} = 4.40 \frac{m}{s}$

Check C_n: With $Re = \dfrac{(4.40 \frac{m}{s})(0.08m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 2.41 \times 10^4$ we obtain $C_n = 0.963$ from Fig. 8.43 (checks)

Thus, $Q = \underline{\underline{0.0221 \frac{m^3}{s}}}$

8.109

8.109 A 2.5-in.-diameter nozzle meter is installed in a 3.8-in.-diameter pipe that carries water at 160 °F. If the inverted air-water U-tube manometer used to measure the pressure difference across the meter indicates a reading of 3.1 ft, determine the flowrate.



$$Q = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{2.5 \text{ in.}}{3.8 \text{ in.}} = 0.658 \qquad (1)$$

From Table B.1: $\rho = 1.896 \frac{\text{slugs}}{\text{ft}^3}$, $\mu = 8.32 \times 10^{-6} \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(1.896 \frac{\text{slugs}}{\text{ft}^3}) V (\frac{3.8}{12} \text{ ft})}{8.32 \times 10^{-6} \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}$$

or

$$Re = 7.22 \times 10^4 V, \text{ where } V \sim \frac{\text{ft}}{\text{s}} \qquad (2)$$

Also, with $Q = \frac{\pi}{4} D^2 V$ Eq.(1) becomes (using $p_1 - p_2 = \gamma h$):

$$\frac{\pi}{4}\left(\frac{3.8}{12} \text{ ft}\right)^2 V = C_n \frac{\pi}{4}\left(\frac{2.5}{12} \text{ ft}\right)^2 \left[\frac{2(32.2 \frac{\text{ft}}{\text{s}^2})(1.896 \frac{\text{slugs}}{\text{ft}^3})(3.1 \text{ ft})}{(1.896 \frac{\text{slugs}}{\text{ft}^3})(1 - 0.658^4)}\right]^{1/2}$$

or

$$V = 6.78\, C_n \qquad (3)$$

Trial and error solution using Fig. 8.43 for $C_n = C_n(Re, \beta = 0.658)$:

Assume $C_n = 0.99$ From Eq.(3) $V = 6.78(0.99) = 6.71 \frac{\text{ft}}{\text{s}}$

From Eq. (2) $Re = 7.22 \times 10^4 (6.71 \frac{\text{ft}}{\text{s}}) = 4.84 \times 10^5$ which from Fig. 8.47 gives $C_n = 0.99$ (checks with assumed value)

Thus, $V = 6.71 \frac{\text{ft}}{\text{s}}$ and $Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4}\left(\frac{3.8}{12} \text{ ft}\right)^2 \left(6.71 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.528 \frac{\text{ft}^3}{\text{s}}}}$

8.110

8.110 Water flows through the Venturi meter shown in Fig. P8.110. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.

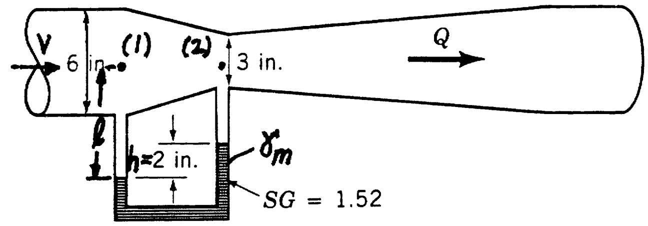


FIGURE P8.110

$$Q = C_V A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{3\text{ in.}}{6\text{ in.}} = 0.5$$

Also,

$$p_1 + \gamma \ell = p_2 + \gamma(\ell - h) + \gamma(SG)h \quad \text{or} \quad p_1 - p_2 = \gamma(SG-1)h = \rho g(SG-1)h$$

Hence,

$$Q = C_V A_T \sqrt{\frac{2\rho g(SG-1)h}{\rho(1-\beta^4)}} \quad \text{or}$$

$$Q = C_V \frac{\pi}{4}\left(\frac{3}{12}\text{ft}\right)^2 \left[\frac{2(32.2\frac{\text{ft}}{\text{s}^2})(1.52-1)(\frac{2}{12}\text{ft})}{(1-0.5^4)}\right]^{1/2}$$

Thus,

$Q = 0.1198\, C_V$ Assume $C_V \approx 0.98$ so that $Q = 0.1198(0.98) = 0.117 \frac{\text{ft}^3}{\text{s}}$

Hence,

$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.117\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2} = 0.596\frac{\text{ft}}{\text{s}} \quad \text{so that}$$

$$Re = \frac{VD}{\nu} = \frac{(0.596\frac{\text{ft}}{\text{s}})(\frac{6}{12}\text{ft})}{1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}}} = 2.46\times10^4$$

From Fig. 8.45 at this Re, $C_V \approx 0.96 \neq 0.98$, the assumed value.

Hence, assume $C_V = 0.96$, or

$$Q \approx 0.1198(0.96) = 0.115\frac{\text{ft}^3}{\text{s}} \quad \text{and} \quad V = \frac{0.115}{\frac{\pi}{4}(\frac{6}{12})^2} = 0.586\frac{\text{ft}}{\text{s}}$$

Therefore, $Re = \frac{0.586(\frac{6}{12})}{1.21\times10^{-5}} = 2.42\times10^4$ so that from Fig. 8.45,

$C_V \approx 0.96$ Checks with assumed value.

Hence, $Q = \underline{\underline{0.115\frac{\text{ft}^3}{\text{s}}}}$

8.111

8.111 If the fluid flowing in Problem 8.110 were air, what would the flowrate be? Would compressibility effects be important? Explain.

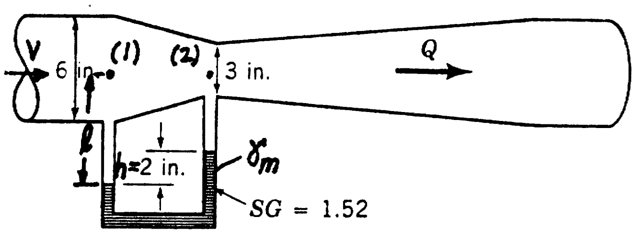


FIGURE P8.110

$$Q = C_V A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{3\,in.}{6\,in.} = 0.5 \qquad (1)$$

Also, since $\rho << \rho_{H_2O}$, $p_1 - p_2 = SG\,\gamma_{H_2O}\,h = 1.52\,(62.4\frac{lb}{ft^3})(\frac{2}{12}ft) = 15.8\frac{lb}{ft^2}$

With $p_1 = 14.7\,psia$ and $p_1 - p_2 = \frac{15.8}{144}psia = 0.110\,psia$, compressibility effects are not important.

Thus, from Eq. (1)

$$Q = C_V \frac{\pi}{4}(\frac{3}{12}ft)^2 \left[\frac{2(15.8\frac{lb}{ft^2})}{(2.38\times10^{-3}\frac{slug}{ft^3})(1-0.5^4)}\right]^{1/2}$$

or

$$Q = 5.84\,C_V \qquad (2)$$

Assume $C_V = 0.96$, or $Q = 5.61\frac{ft^3}{s}$ so that $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{5.61\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2} = 28.6\frac{ft}{s}$

Hence,

$Re = \frac{VD}{\nu} = \frac{(28.6\frac{ft}{s})(\frac{6}{12}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 9.11\times10^4$ From Fig. 8.45 this gives

$C_V \approx 0.975$ or $Q = 5.84\,(0.975) = \underline{\underline{5.69\frac{ft^3}{s}}}$

Note: With $V = \frac{Q}{A} = \frac{5.69}{\frac{\pi}{4}(\frac{6}{12})^2} = 29.0\frac{ft}{s}$ we obtain

$Re = \frac{29.0(\frac{6}{12})}{1.57\times10^{-4}} = 9.24\times10^4$. Thus, from Fig. 8.45 $C_V \approx 0.975$, which agrees with the assumed value.

8.112

8.112 Water flows through the orifice meter shown in Fig. P8.112 at a rate of 0.10 cfs. If $d = 0.1$ ft, determine the value of h.

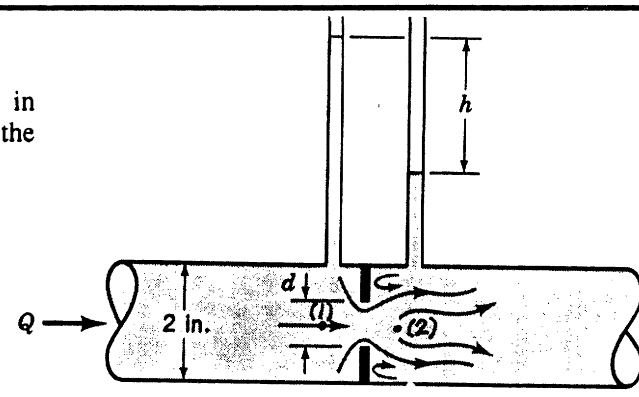


FIGURE P8.112

$$Q = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{0.1\,ft}{\frac{2}{12}\,ft} = 0.6,\ p_1 - p_2 = \gamma h = \rho g h \qquad (1)$$

Also, $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.10\,\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{2}{12}\,ft)^2} = 4.58\,\frac{ft}{s}$ so that

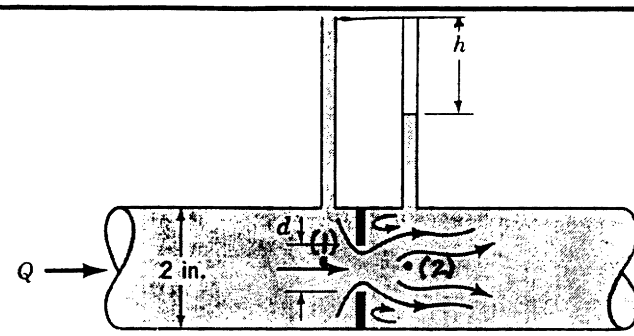
$$Re = \frac{VD}{\nu} = \frac{(4.58\,\frac{ft}{s})(\frac{2}{12}\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 6.31 \times 10^4$$ Hence, from Fig. 8.41, $C_o = 0.616$

Therefore, from Eq. (1):

$$0.10\,\frac{ft^3}{s} = (0.616)\frac{\pi}{4}(0.1\,ft)^2 \sqrt{\frac{2\rho(32.2\,\frac{ft}{s^2})h}{\rho(1-0.6^4)}} \quad \text{or} \quad h = \underline{\underline{5.77\,ft}}$$

8.113

8.113 Water flows through the orifice meter shown in Fig. P8.112 at a rate of 0.10 cfs. If $h = 3.8$ ft, determine the value of d.



$$Q = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{d}{\frac{2}{12}\,ft}, \quad p_1 - p_2 = \gamma h = \rho g h \qquad (1)$$

and $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.10\,\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{2}{12}\,ft)^2} = 4.58\,\frac{ft}{s}$, $Re = \frac{VD}{\nu} = \frac{(4.58\,\frac{ft}{s})(\frac{2}{12}\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 6.31 \times 10^4$

Trial and error solution: From Eq. (1)

$$0.1\,\frac{ft^3}{s} = C_o \frac{\pi}{4}(d\,ft)^2 \left[\frac{2\rho(32.2\,\frac{ft}{s^2})(3.8\,ft)}{\rho(1-\beta^4)}\right]^{\frac{1}{2}} \text{ or } 0.00814 = \frac{C_o d^2}{\sqrt{1-\beta^4}}, \text{ where } d \sim ft \qquad (2)$$

Assume $\beta = 0.6$, or $d = \frac{2}{12}\beta = \frac{2}{12}(0.6) = 0.10\,ft$. Thus, from Eq. (2)
$C_o = 0.759$. However, from Fig. 8.41 for this Re and β, $C_o = 0.615 \neq 0.759$

Assume $\beta = 0.65$, or $d = \frac{2}{12}(0.65) = 0.108\,ft$. From Eq. (2) $C_o = 0.633$.
From Fig. 8.41, $C_o = 0.618 \neq 0.633$.

Assume $\beta = 0.67$, or $d = \frac{2}{12}(0.67) = 0.112\,ft$. From Eq. (2) $C_o = 0.580$
From Fig. 8.41, $C_o = 0.619 \neq 0.580$ Thus, $d \approx \underline{\underline{0.109\,ft}}$

8.114

8.114 Water flows through the orifice meter shown in Fig. P8.112 such that $h = 1.6$ ft with $d = 1.5$ in. Determine the flowrate.

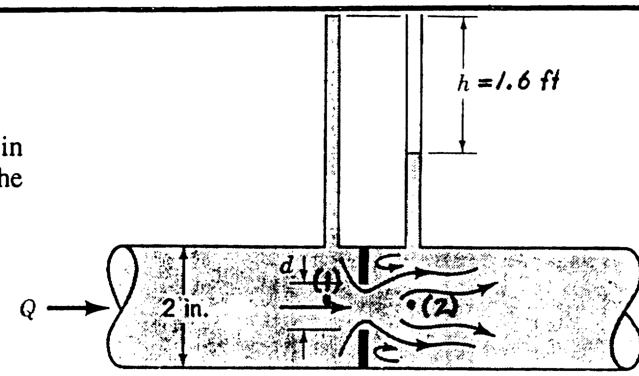


FIGURE P8.112

$Q = C_o A_o \sqrt{\dfrac{2(p_1-p_2)}{\rho(1-\beta^4)}}$, where $\beta = \dfrac{d}{D} = \dfrac{1.5\,in.}{2\,in.} = 0.75$ and $p_1 - p_2 = \gamma h = \rho g h$

Thus,

$$Q = C_o \frac{\pi}{4}\left(\frac{1.5}{12}ft\right)^2\left[\frac{2\,\rho(32.2\frac{ft}{s})(1.6ft)}{\rho(1-0.75^4)}\right]^{\frac{1}{2}}$$

or

$Q = 0.151\,C_o$ (1)

Also, $Re = \dfrac{VD}{\nu} = \dfrac{V(\frac{2}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}} = 1.38\times10^4\,V$, where $V = \dfrac{Q}{\frac{\pi}{4}D^2} = 45.8Q$ (2)

Trial and error solution:

Assume $C_o = 0.6$; or from Eq.(1), $Q = 0.151(0.6) = 0.0906\frac{ft^3}{s}$

Hence, from Eq.(2), $V = 45.8(0.0906) = 4.15$ and $Re = 5.73\times10^4$

From Fig. 8.41 with this Re and β, $C_o = 0.62 \neq 0.6$ (the assumed value)

Assume $C_o = 0.62$ or $Q = 0.151(0.62) = 0.0936\frac{ft^3}{s}$, Thus $V = 45.8(0.0936)$ or $V = 4.29\frac{ft}{s}$ and $Re = 5.92\times10^4$, From Fig. 8.41, $C_o = 0.62$, the assumed value.

Hence, $Q = \underline{\underline{0.0936\frac{ft^3}{s}}}$

8.115

8.115 The scale reading on the rotameter shown in Fig. P8.115 and Video V8.6 (also see Fig. 8.46) is directly proportional to the volumetric flowrate. With a scale reading of 2.6 the water bubbles up approximately 3 in. How far will it bubble up if the scale reading is 5.0?

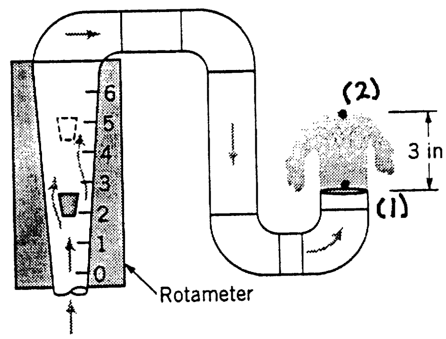


FIGURE P8.115

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0$, $z_1 = 0$, $V_2 = 0$, so that with no losses ($h_L = 0$),

(1) $$\frac{V_1^2}{2g} = z_2$$

For the rotameter $Q = K \times SR$ where SR = scale reading and K is a constant.

Thus,

$$V_1 = \frac{Q_1}{A_1} = \frac{K \times SR}{A_1}$$ so that when combined with Eq. (1),

$$\frac{K^2 (SR)^2}{A_1^2 (2g)} = z_2 \quad \text{or} \quad \frac{K^2 (2.6)^2}{A_1 (2g)} = \left(\tfrac{3}{12}\,\text{ft}\right) \quad \text{and} \quad \frac{K^2 (5.0)^2}{A_1 (2g)} = h$$

By dividing these two equations,

$$\frac{(5.0)^2}{(2.6)^2} = \frac{h}{\left(\tfrac{3}{12}\,\text{ft}\right)} \quad \text{or} \quad h = 0.925\ \text{ft} = \underline{\underline{11.1\ \text{in.}}}$$

8.116

8.116 Friction Factor for Laminar and Transitional Pipe Flow

Objective: Theoretically, the friction factor, f, for laminar pipe flow is given by $f = 64/\mathrm{Re}$, where the Reynolds number, $\mathrm{Re} = \rho VD/\mu$, is based on the average velocity, V, within the pipe and the pipe diameter, D. Also, the flow is normally laminar for $\mathrm{Re} < 2100$. The purpose of this experiment is to use the device shown in Fig. P8.116 to investigate these two properties.

Equipment: Small diameter metal tubes (pipes), air supply with flow regulator, rotameter flow meter, manometer.

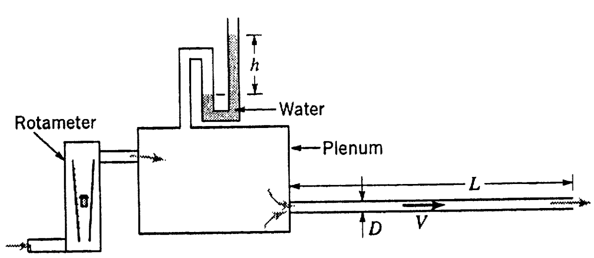
Experimental Procedure: Attach a tube of length L and diameter D to the plenum. Adjust the flow regulator to obtain the desired flowrate as measured by the rotameter. Record the manometer reading, h, so that the pressure difference between the plenum (tank) and the free jet at the end of the tube can be determined. Repeat for several different flowrates and tube diameters. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each of the data sets determine the pressure difference, $\Delta p = \gamma_{\mathrm{m}} h$, between the plenum pressure and the free jet pressure. Here γ_{m} is the specific weight of the manometer fluid. Use the energy equation, Eq. 5.84, to determine the friction factor, f. Assume the loss coefficient for the pipe entrance is $K_L = 0.8$. Also calculate the Reynolds number, Re, for each data set.

Graph: On a log-log graph, plot the experimentally determined friction factor, f, as ordinates and the Reynolds number, Re, as abscissas.

Results: On the same graph, plot the theoretical friction factor for laminar flow, $f = 64/\mathrm{Re}$, as a function of the Reynolds number. Based on the experimental data, determine the maximum value of the Reynolds number for which the flow in these pipes is laminar.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.116

(con't)

8.116 (con't)

Solution for Problem 8.116: Friction Factor for Laminar and Transitional Pipe Flow

L, in.	H_{atm}, in. Hg	T, deg F
24	28.9	73

h, in.	Q, ml/min	Q, cfs	V, fps	Re	f	Theoretical Re	Theoretical f
D = 0.108 in. Data						100	0.6400
7.5	6600	0.003887	61.11	3202	0.0341	2100	0.0305
6.75	6200	0.003652	57.40	3008	0.0349		
6.26	6000	0.003534	55.55	2911	0.0345		
5.54	5650	0.003328	52.31	2741	0.0344		
4.66	5150	0.003033	47.68	2499	0.0349		
4.29	5000	0.002945	46.29	2426	0.0339		
3.92	4860	0.002863	45.00	2358	0.0325		
3.48	4600	0.002709	42.59	2232	0.0322		
3.21	4500	0.002651	41.66	2183	0.0307		
2.34	3700	0.002179	34.26	1795	0.0338		
1.86	2900	0.001708	26.85	1407	0.0461		
1.11	1800	0.001060	16.67	873	0.0758		
0.63	1100	0.000648	10.18	534	0.1194		
D = 0.046 in. Data							
9.52	560	0.000330	28.58	638	0.1007		
7.68	475	0.000280	24.24	541	0.1134		
7.08	425	0.000250	21.69	484	0.1311		
5.26	315	0.000186	16.08	359	0.1785		
3.39	221	0.000130	11.28	252	0.2348		
2.61	165	0.000097	8.42	188	0.3256		
D = 0.063 in. Data							
4.58	925	0.000545	25.17	770	0.0838		
3.32	680	0.000401	18.50	566	0.1140		
2.51	530	0.000312	14.42	441	0.1431		
1.48	325	0.000191	8.84	270	0.2270		
0.86	190	0.000112	5.17	158	0.3893		

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O}{*}H_{atm}$ = 847 lb/ft^3*(28.9/12 ft) = 2040 lb/ft^2

R = 1716 ft lb/slug deg R

T = 73 + 460 = 533 deg R

Thus, ρ = 0.00223 slug/ft^3 and $\gamma = \rho{*}g$ = 0.0718 lb/ft^3

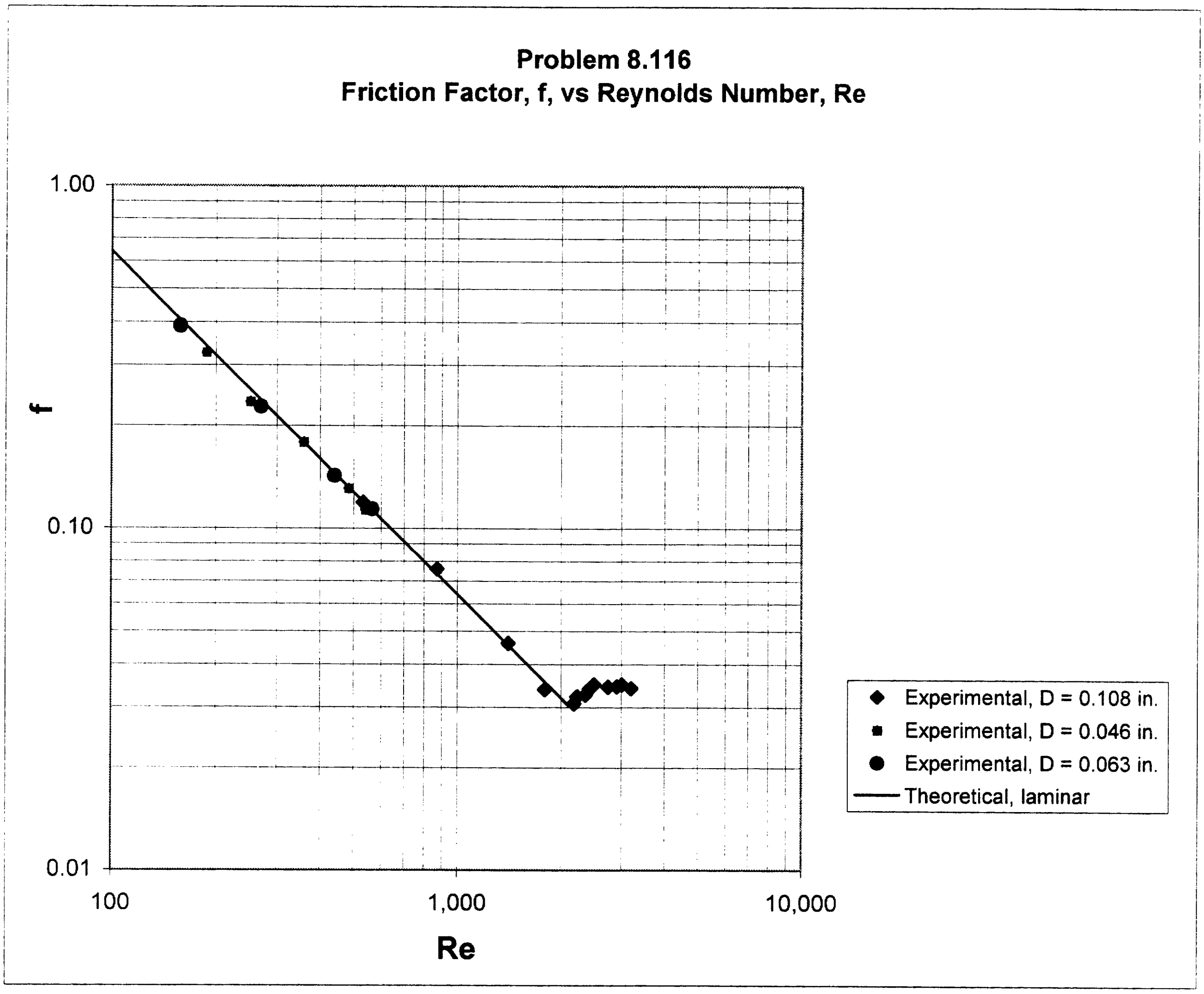
Also, μ = 3.83E-7 lb s/ft^2

Theoretical for laminar flow: f = 64/Re = 64/(ρDV/μ)

$\Delta p/\gamma$ = (fL/D + K_L + 1)(V^2/2g) where K_L = entrance loss coefficient = 0.8 and V = Q/(πD^2/4)

(con't)

8.116 (con't)



Problem 8.116
Friction Factor, f, vs Reynolds Number, Re
1.00
0.10
0.01
f
100
1,000
10,000
Re
Experimental, D = 0.108 in.
Experimental, D = 0.046 in.
Experimental, D = 0.063 in.
Theoretical, laminar

8.117

8.117 Calibration of an Orifice Meter and a Venturi Meter

Objective: Because of various real-world, nonideal conditions, neither orifice meters nor Venturi meters operate exactly as predicted by a simple theoretical analysis. The purpose of this experiment is to use the device shown in Fig. P8.117 to calibrate an orifice meter and a Venturi meter.

Equipment: Water tank with sight gage, pump, Venturi meter, orifice meter, manometers.

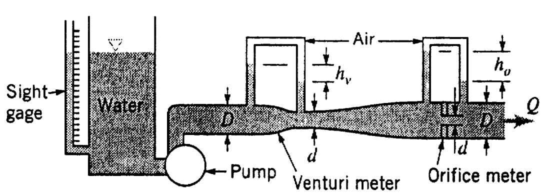
Experimental Procedure: Determine the pipe diameter, D, and the throat diameter, d, for the flow meters. Note that each meter has the same values of D and d. Make sure that the tubes connecting the manometers to the flow meters do not contain any unwanted air bubbles. This can be verified by noting that the manometer readings, h_v, and h_o, are zero when the system is full of water and the flowrate, Q, is zero. Turn on the pump and adjust the valve to give the desired flowrate. Record the time, t, it takes for a given volume, V, of water to be pumped from the tank. The volume can be determined from using the sight gage on the tank. At this flowrate record the manometer readings. Repeat for several different flowrates.

Calculations: For each data set determine the volumetric flowrate, $Q = V/t$, and the pressure differences across each meter, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Use the flow meter equations (see Section 8.6.1) to determine the orifice discharge coefficient, C_o, and the Venturi discharge coefficient, C_v, for these meters.

Graph: On a log-log graph, plot flowrate, Q, as ordinates and pressure difference, Δp, as abscissas.

Result: On the same graph, plot the ideal flowrate, Q_{ideal} (see Eq. 8.37), as a function of pressure difference.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.117

(con't)

Solution for Problem 8.117: Calibration of an Orifice Meter and a Venturi Meter

d, in.	D, in.	*V*, gallons
0.625	1.025	2.00

t, s	h_o, in.	h_v, in.	Δp_o, lb/ft^2	Δp_v, lb/ft^2	Q, ft^3/s	C_o	C_v	Ideal C = 1 Δp, lb/ft^2
27.0	9.3	3.8	48.4	19.8	0.0099	0.611	0.956	18.0
13.2	37.1	14.5	192.9	75.4	0.0203	0.626	1.001	75.5
34.2	5.5	1.9	28.6	9.9	0.0078	0.627	1.067	11.2
16.6	23.9	10.1	124.3	52.5	0.0161	0.620	0.953	47.7
12.0	43.2	18.1	224.6	94.1	0.0223	0.638	0.985	91.4
11.7	51.3	21.7	266.8	112.8	0.0229	0.600	0.923	96.1
15.4	27.9	11.2	145.1	58.2	0.0174	0.618	0.976	55.5
25.1	10.1	4.2	52.5	21.8	0.0107	0.631	0.978	20.9
20.4	14.7	6.2	76.4	32.2	0.0131	0.643	0.990	31.6
17.3	21.4	8.7	111.3	45.2	0.0155	0.629	0.986	44.0
15.7	26.7	11.2	138.8	58.2	0.0170	0.620	0.957	53.4
			Average discharge coefficient:			0.624 orifice	0.979 venturi	

Q = **V** gal/t s x (231 in.^3/gal)x(1 ft^3/1728 in.^3)

$\Delta p = \gamma_{H2O}$*h = 62.4 lb/ft^3 *h ft

$Q_v = A_2/[1 - (A_2/A_1)^2]^{0.5} * C_v * (2*g*\Delta p_v/\gamma_{H2O})^{0.5}$

and

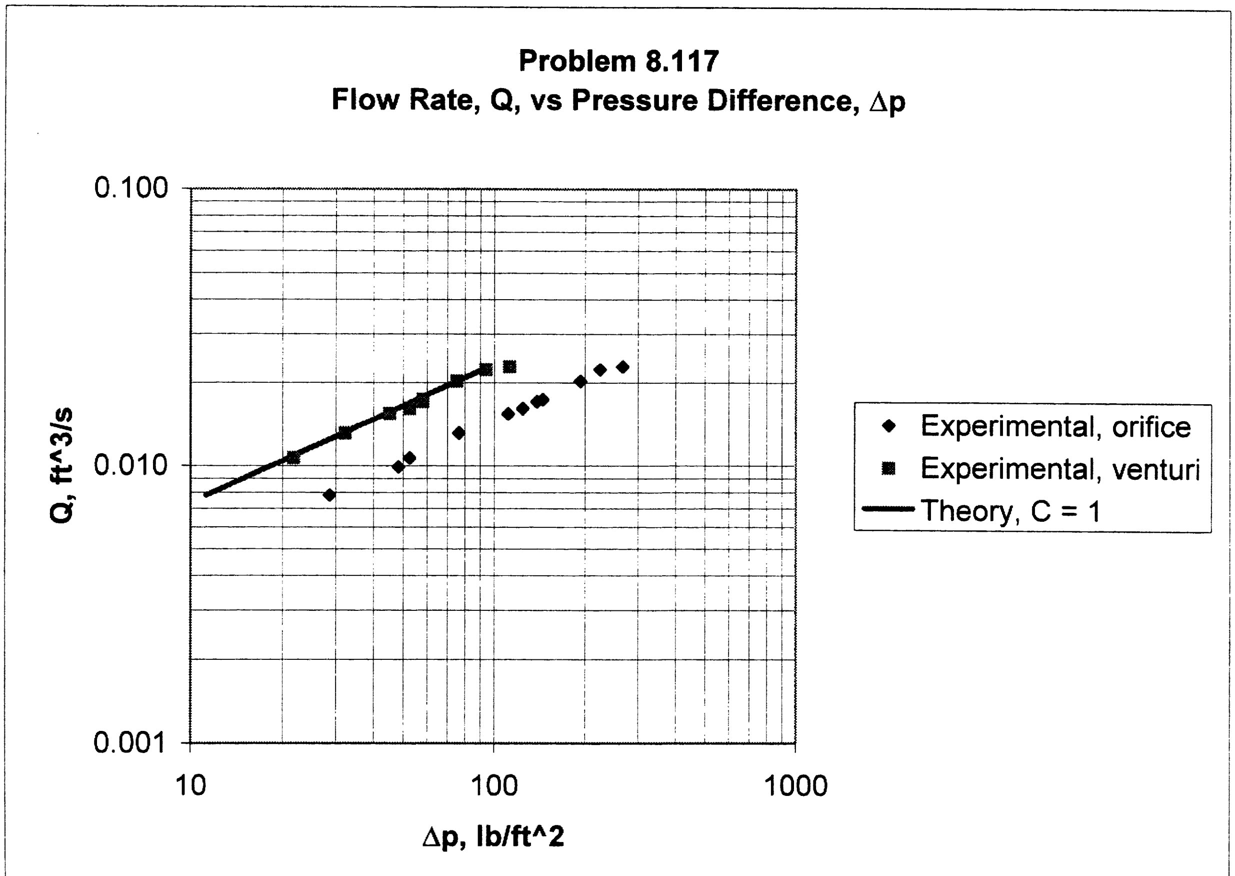
$Q_o = A_2/[1 - (A_2/A_1)^2]^{0.5} * C_o * (2*g*\Delta p_o/\gamma_{H2O})^{0.5}$

where

$A_1 = \pi$ D^2/4 = π (1.025/12 ft)^2/4 = 0.00573 ft^2

and

$A_2 = \pi$ d^2/4 = π (0.625/12 ft)^2/4 = 0.00213 ft^2



8.118 Flow from a Tank through a Pipe System

Objective: The rate of flow of water from a tank is a function of the pipe system used to drain the tank. The purpose of this experiment is to use a pipe system as shown in Fig. P8.118 to investigate the importance of major and minor head losses in a typical pipe flow situation.

Equipment: Water tank; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, etc.); pipe wrenches; stop watch; thermometer.

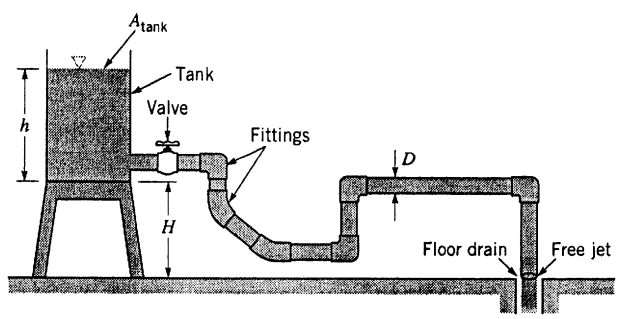
Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may flow into a floor drain. Measure the pipe diameter, D, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, H, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, T. With the pipeline valve wide open, measure the water depth, h, in the tank as a function of time, t, as the tank drains.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{ex} = -A_{tank}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the h versus t graph. Select a typical water depth, h_1, for this calculation.

Graph: Plot the water depth, h, in the tank as ordinates and time, t, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the theoretical flowrate, Q_{th}, based on three different assumptions. Use the same typical water depth, h_1, for the theoretical calculations as was used in determining Q_{ex}. First, calculate Q_{th} under the assumption that all losses are negligible. Second, calculate Q_{th} if only major losses (pipe friction) are important. Third, calculate Q_{th} if both major and minor losses are important.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.118

(con't)

Solution for Problem 8.118: Flow from a Tank Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp edged entrance
- one fully open globe valve
- two 45-deg elbows
- four 90-deg elbows

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.595	0.654	1.00	135	71

h, ft	t, s
1.00	0
0.90	13
0.80	26
0.70	40
0.60	54
0.50	67
0.40	81

Experimental: Q_{ex} = -(dh/dt)*A_{tank} = -(0.0074 ft/s)*(0.654 ft^2) = <u>0.00484 ft^3/s</u>

Theoretical with no losses: Q_{th} = V_2*A_2, where when h = 0.90 ft

V_2 = (2g*(h + H))^0.5 = (2*32.2*(0.9 +1.0))^0.5 = 11.06 ft/s

and with A_2 = πD^2/4 = π*(0.595/12 ft)^2/4 = 0.00193 ft^2

Q_{th} = 0.00193 ft^2*(11.06 ft/s) = <u>0.0213 ft^3/s</u>

Theoretical with major losses: Q_{th} = V_2*A_2, where the energy equation gives

h + H = V_2^2/2g(1 + fL/D), where again use h = 0.90 ft and f is a function of Re and ε/D

Thus, with h = 0.90 ft,

1.9 = (V_2^2/64.4)*(1 + f*135/0.595), or

122.4 = V_2^2*(1 + 227f)

Re = V_2D/ν = V_2*(0.595/12 ft)/(1.04E-5 ft^2/s) = 4768*V_2

and

ε/D =0.0005 ft/(0.595/12 ft) = 0.0101

Trial and error solution: Guess f, solve for V_2, calculate Re, obtain new f from Moody chart

The solution is: f = 0.041, V_2 = 3.44 ft/s, Re = 16,430

Q_{th} = 0.00193 ft^2*(3.44 ft/s) = <u>0.00664 ft^3/s</u>

Theoretical with major and minor losses: The energy equation gives

h + H = (1 + fL/D +ΣK_L)V_2^2/2g

where ΣK_L = 0.5 + 10 + 2*0.4 + 4*1.5 = 17.3

Thus, with h = 0.9 ft

1.9 = (V_2^2/64.4)*(17.3 + f*135/0.595), or

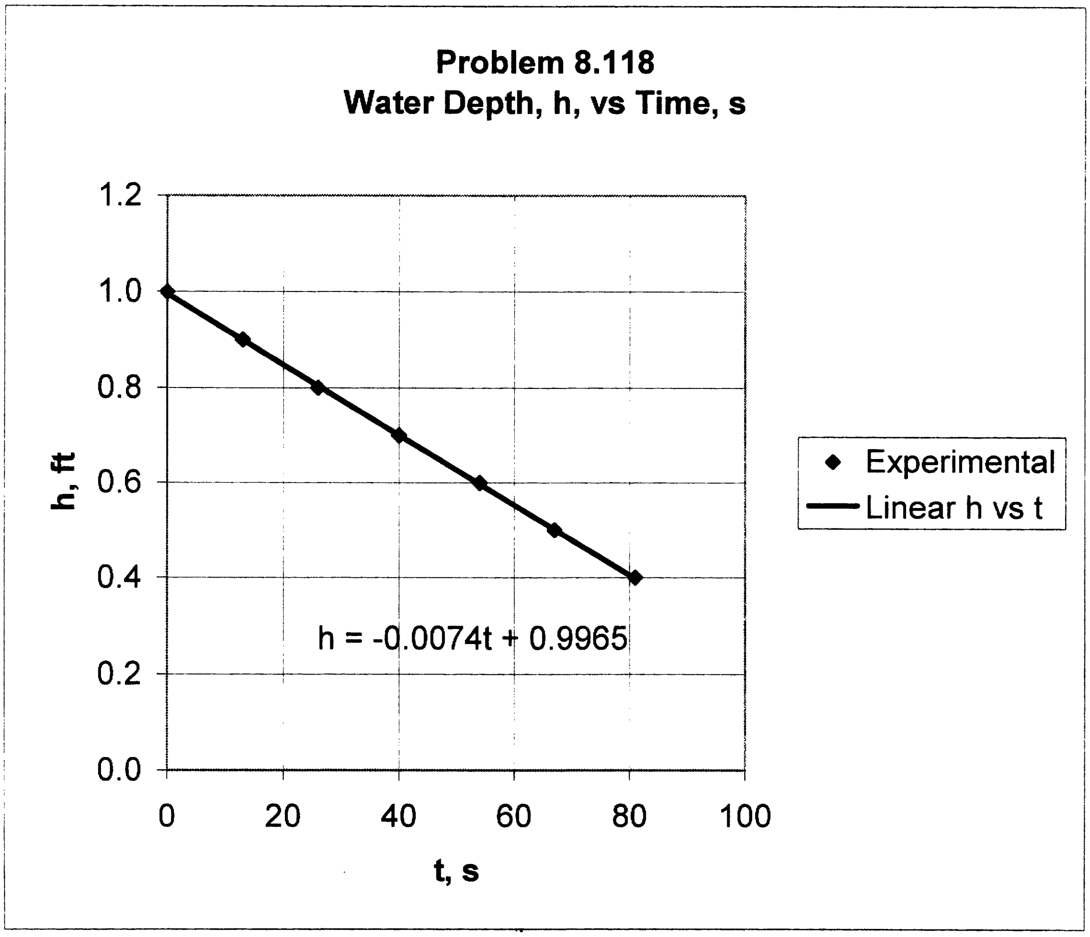
122.4 = V_2^2*(17.3 + 227f)

Trial and error solution gives: f = 0.42, V_2 = 2.14 ft/s, Re = 10,200

Q_{th} = 0.00193 ft^2*(2.14 ft/s) = <u>0.00413 ft^3/s</u>

(con't)

8.118 (con't)



Problem 8.118
Water Depth, h, vs Time, s
h, ft
t, s
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
h = -0.0074t + 0.9965
Experimental
Linear h vs t

8.119

8.119 Flow of Water Pumped from a Tank and through a Pipe System

Objective: The rate of flow of water pumped from a tank is a function of the pump properties and of the pipe system used. The purpose of this experiment is to use a pump and pipe system as shown schematically in Fig. P8.119 to investigate the rate at which the water is pumped from the tank.

Equipment: Water tank; centrifugal pump; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, unions, etc.); pipe wrenches; stop watch; thermometer.

Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may be pumped into a sink. Measure the pipe diameter, D, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, H, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, T. With the pipeline valves wide open, measure the water depth, h, in the tank as a function of time, t, as water is pumped from the tank.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{ex} = -A_{tank}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the h versus t graph.

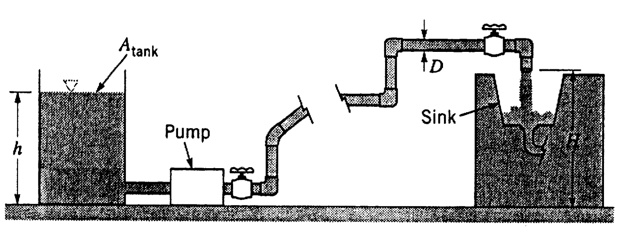
Graph: Plot the water depth, h, in the tank as ordinates and time, t, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the pump head, h_p, needed to in order to produce a given flowrate, Q. For these calculations include all major and minor losses in the pipe system. Plot the system curve (i.e., pump head as ordinates and flowrate as abscissas) based on the results of these calculations. On the same graph, plot the pump curve (i.e., h_p as a function of Q) as supplied by the pump manufacturer. For the pump used this curve is given by

$$h_p = -2.44 \times 10^5 Q^2 + 51.0\,Q - 12.5$$

where Q is in ft^3/s and h_p is in ft. From the intersection of the system curve and the pump curve, determine the theoretical flowrate that the pump should provide for the pipe system used.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.119

(con't)

Solution for Problem 8.119: Flowrate of Water Pumped from a Tank and Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp entrance
- eight 90-deg elbows
- two 45-deg elbows
- two globe valves
- one union

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.625	0.647	3.50	242	62

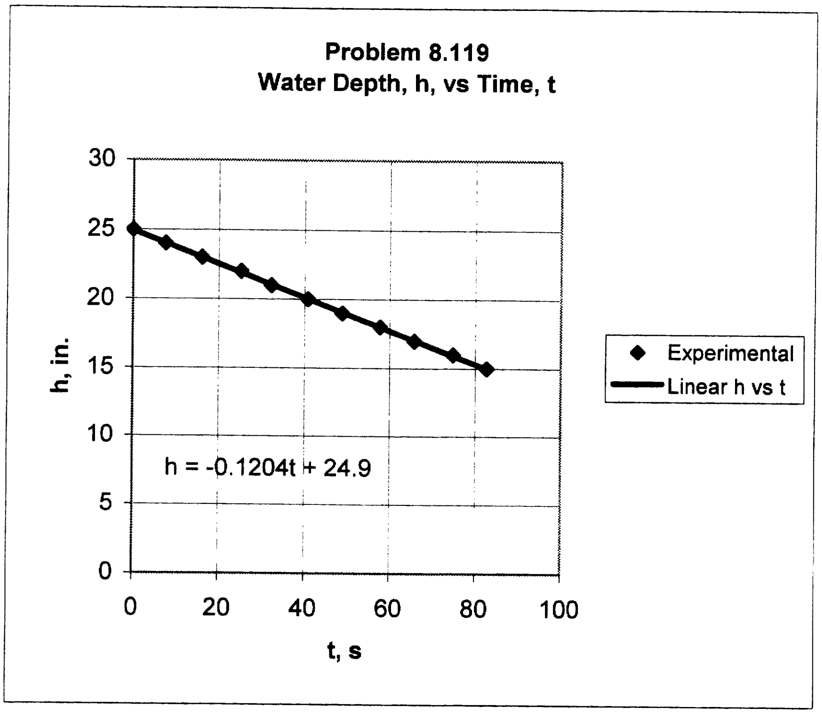
		Pump equation			System equation		
h, in.	t, s	h_p, ft	Q, ft^3/s	V, ft/s	Re	f	h_p, ft
25	0	12.50	0.000	0.00	0		2
24	7.6	12.31	0.001	0.47	2070	0.0309	2.16
23	16.1	11.63	0.002	0.94	4140	0.0490	2.73
22	25.2	10.46	0.003	1.41	6210	0.0470	3.62
21	32.3	8.80	0.004	1.88	8281	0.0450	4.84
20	40.8	6.66	0.005	2.35	10351	0.0430	6.37
19	48.9	4.02	0.006	2.81	12421	0.0425	8.27
18	57.7	0.90	0.007	3.28	14491	0.0420	10.50
17	65.7						
16	74.9						
15	82.7						

Experimental:
$Q_{ex} = -A_{tank}$*(dh/dt) where from the graph, dh/dt = -0.1204 in./s
Thus,
Q_{ex} = -(0.647 ft^2)*(-0.1204/12 ft/s) = <u>0.00669 ft^3/s</u>

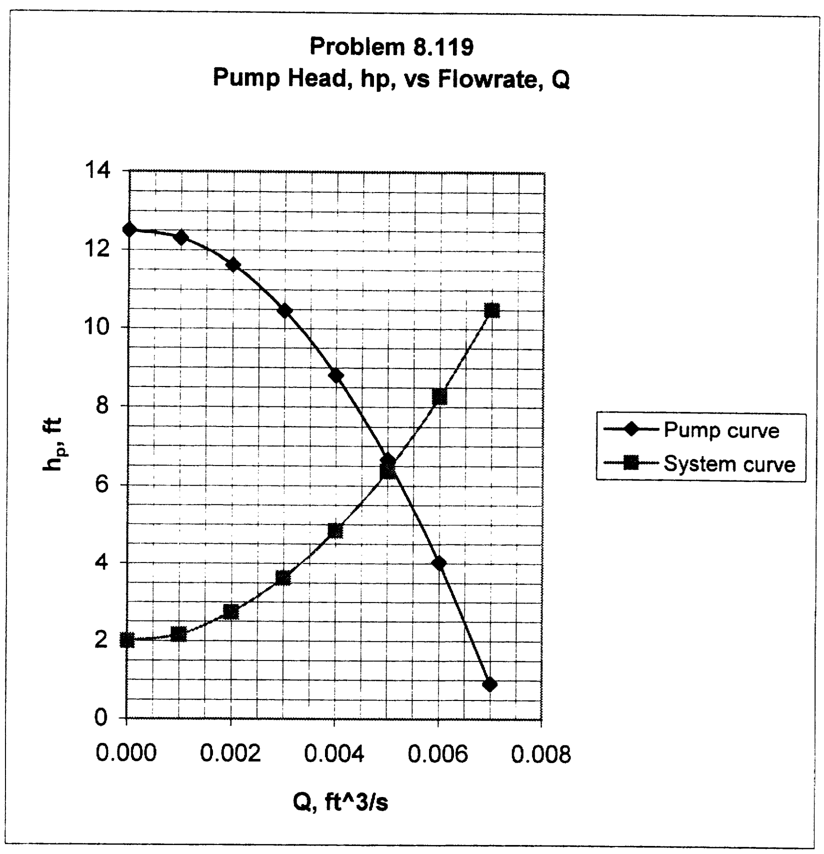
Theoretical:
The energy equation gives
$h + h_p - h_L = H + V^2/2g$, where
$h_L = (fL/D + \Sigma K_L)*V^2/2g = (f*(242\text{ in.}/0.625\text{ in.}) + 0.5 + 8*1.5 + 2*0.4 + 2*10 + 0.08)*V^2/2g$
$= (387*f + 33.4)*V^2/(2*32.2) = (6.01*f + 0.519)*V^2$
Thus, with h = 18 in. = 1.5 ft,
$h_p = H - h + h_L + V^2/2g = 3.5 - 1.5 + (6.01*f + 0.519)*V^2 + V^2/(64.4)$
or
$h_p = 2.0 + (6.01*f + 0.535)*V^2$
But $V = Q/A = Q/(\pi D^2/4) = Q/(\pi*(0.625/12\text{ ft})^2/4) = 469*Q$
Thus, the system equation is
$h_p = 2.0 + (6.01*f + 0.535)*(469*Q)^2 = 2.0 + (1.32E{+}6*f + 1.18E{+}5)*Q^2$
Also, obtain f from the Moody chart with
$Re = VD/\nu = V*(0.625/12\text{ ft})/(1.18E{-}5\text{ ft}^2/s) = 4414*V$
$\varepsilon/D = 0.0005\text{ ft}/(0.625/12\text{ ft}) = 0.0096$
From the graph, the pump and system equations intersect at Q_{th} = <u>0.0051 ft^3/s</u>

(con't)

8.119 (con't)



Problem 8.119
Water Depth, h, vs Time, t
h, in.
t, s
h = -0.1204t + 24.9
Experimental
Linear h vs t



Problem 8.119
Pump Head, hp, vs Flowrate, Q
h_p, ft
Q, ft^3/s
Pump curve
System curve

8.120

8.120 Pressure Distribution in the Entrance Region of a Pipe

Objective: The pressure distribution in the entrance region of a pipe is different than that in the fully developed portion of the pipe. The purpose of this experiment is to use an apparatus, as shown in Fig. P8.120, to determine the pressure distribution and the head loss in the pipe entrance region.

Equipment: Air supply with flow meter, pipe with static pressure taps, manometer, ruler, barometer, thermometer.

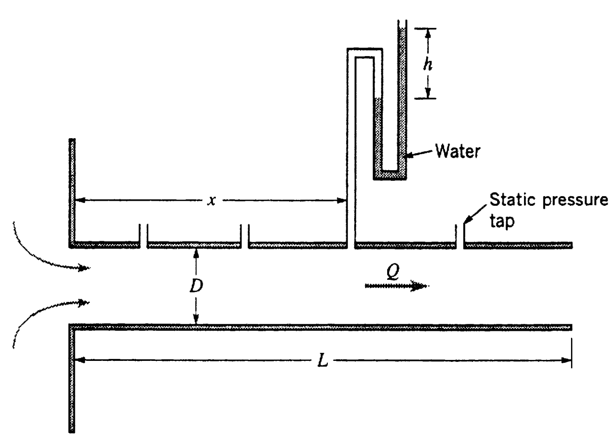
Experimental Procedure: Measure the diameter, D, and length, L, of the pipe and the distance, x, from the pipe inlet to the various static pressure taps. Adjust the flowrate, Q, to the desired value. Record the manometer readings, h, at the various distances from the pipe entrance. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Determine the average velocity, $V = Q/A$, in the pipe and the pressure $p = \gamma_m h$ at the various locations, x, along the pipe. Here γ_m is the specific weight of the manometer fluid.

Graph: Plot the pressure, p, within the pipe as ordinates and the axial location, x, as abscissas.

RESULT: Use the graph to determine the entrance length, L_e, for the pipe. This can be done by noting the approximate location at which the pressure distribution becomes linear with distance along the pipe (i.e., where dp/dx becomes constant). Use the experimental data to determine the friction factor for fully developed flow in this pipe. Also determine the entrance loss coefficient, K_{Lent}.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.120

(con't)

8.120 (con't)

Solution for Problem 8.120: Pressure Distribution in the Entrance Region of a Pipe

D, in.	L, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F
0.74	50	0.481	29.7	75

x, in.	h, in.	p, lb/ft^2
0	9.98	51.9
1	7.21	37.5
2	6.61	34.4
4	6.19	32.2
6	5.82	30.3
10	5.15	26.8
15	4.23	22.0
20	3.64	18.9
30	2.28	11.9
40	1.09	5.7
50	0	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.7/12 ft) = 2096 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

Thus, ρ = 0.00228 slug/ft^3

V = Q/A =(0.481 ft^3/s)/(π*(0.74/12 ft)^2/4) = 161 ft/s

$p = \gamma_{H2O}*h$

From the graph, the p vs x results are linear after (approximately) x = 15 in. Thus, L_e = 15 in.

For the fully developed flow portion, dp/dx = -fρV^2/2D and from the graph dp/dx = -0.635 (lb/ft^2)/in.
Thus,

f = 0.635 (lb/ft^2)/in.*2*0.74 in./(0.00228 slugs/ft^3*(161 ft/s)^2) = 0.0159

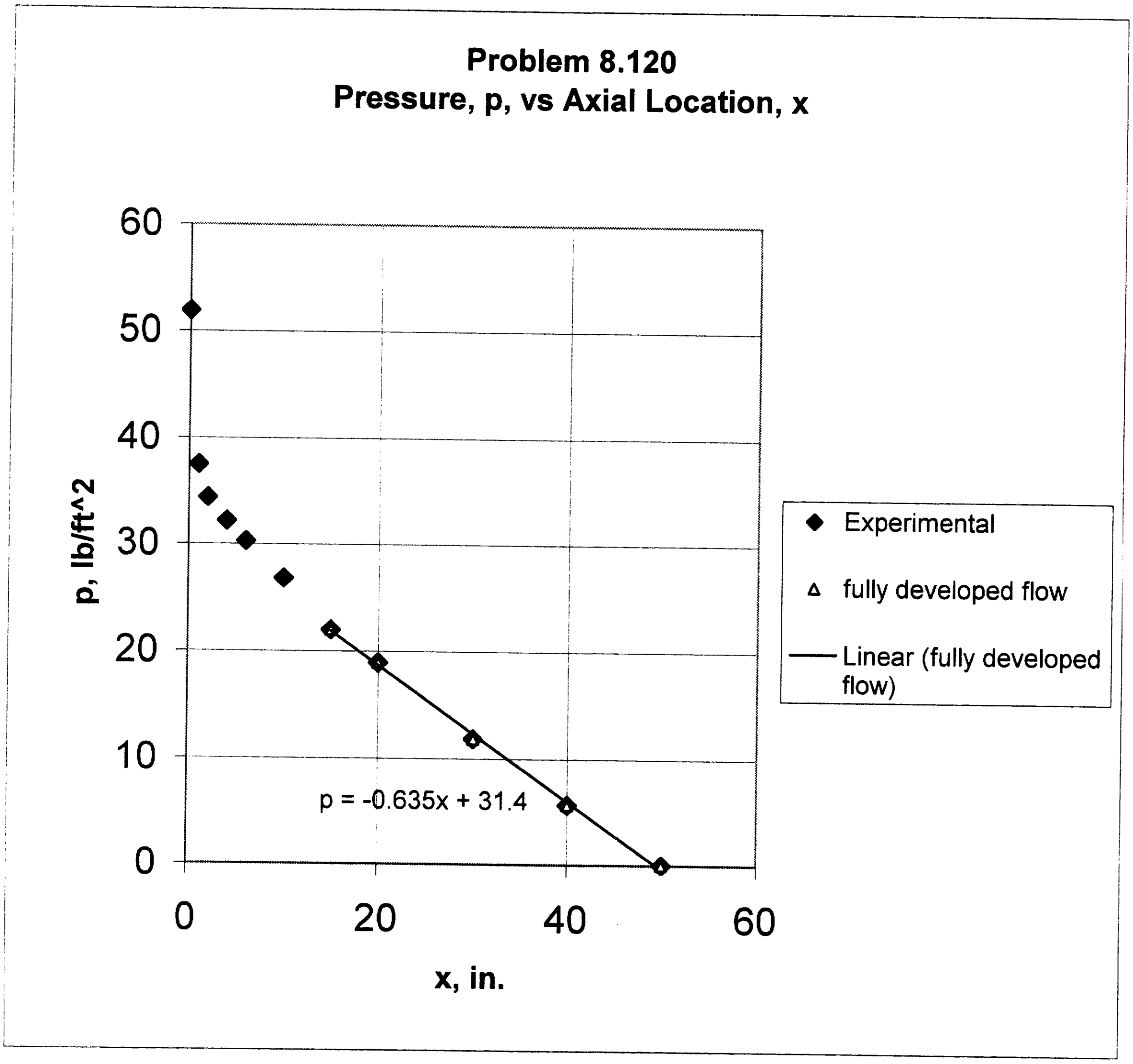
From the entrance to the exit of the pipe p_{ent} = (K_L + fL/D)ρV^2/2
Thus,

K_L = 2p_{ent}/(ρV^2) -fL/D= 2*51.9 lb/ft^2/(0.00228 slugs/ft^3*(161 ft/s)^2) - 0.0159*50in./0.74 in.
=0.682

Results: L_e = 15 in.; f = 0.0159, and K_L = 0.682.

(con't)

8.120 (con't)



Problem 8.120
Pressure, p, vs Axial Location, x
60
50
40
30
20
10
0
p, lb/ft^2
0
20
40
60
x, in.
p = -0.635x + 31.4
Experimental
fully developed flow
Linear (fully developed flow)

8.121 Power Loss in a Coiled Pipe

Objective: The amount of power, P, dissipated in a pipe depends on the head loss, h_L, and the flowrate, Q. The purpose of this experiment is to use an apparatus as shown in Fig. P8.121 to determine the power loss in a coiled pipe and to determine how the coiling of the pipe affects the power loss.

Equipment: Air supply with a flow meter; flexible pipe that can be used either as a straight pipe or formed into a coil; manometer; barometer; thermometer.

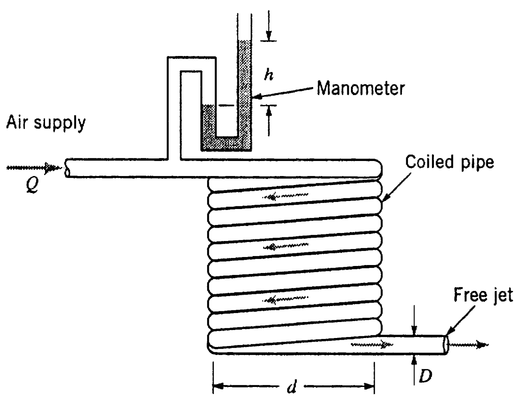
Experimental Procedure: Straighten the pipe and fasten it to the air supply exit. Measure the diameter, D, and length, L, of the pipe. Adjust the flowrate, Q, to the desired value and determine the manometer reading, h. Repeat the measurements for various flowrates. Form the pipe into a coil of diameter d and repeat the flowrate-pressure measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer data to determine the pressure drop, $\Delta p = \gamma_m h$, and head loss, $h_L = \Delta p/\gamma$, as a function of flowrate, Q, for both the straight and coiled pipes. Here γ_m is the specific weight of the manometer fluid and γ is the specific weight of the flowing air. Also calculate the power loss, $P = \gamma Q h_L$, for both the straight and coiled pipes.

Graph: Plot head loss, h_L, as ordinates and flowrate, Q, as abscissas.

Results: On a log-log graph, plot the power loss, P, as a function of flowrate for both the straight and coiled pipes. Determine the best-fit straight lines through the data.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.121

(con't)

Solution for Problem 8.121: Power Loss in a Coiled Pipe

D, in.	L, ft	H_{atm}, in. Hg	T, deg F
1.44	18	29.9	80

h, in.	Q, ft^3/s	Δp, lb/ft^2	h_L, ft	P, hp
Straight Pipe Data (d = infinity)				
10	1.19	52.0	709	0.1125
8	1.06	41.6	568	0.0802
6	0.913	31.2	426	0.0518
4	0.731	20.8	284	0.0276
2	0.505	10.4	142	0.0095
Coiled Pipe Data (d = 8 in.)				
10	0.835	52.0	709	0.0789
8	0.745	41.6	568	0.0563
6	0.641	31.2	426	0.0364
4	0.517	20.8	284	0.0196
2	0.357	10.4	142	0.0068

$\Delta p = \gamma_{H2O} h$ where γ_{H2O} = 62.4 lb/ft^3

$h_L = \Delta p/\gamma$ where $\gamma = g\rho$

$\rho = p_{atm}/RT$ where

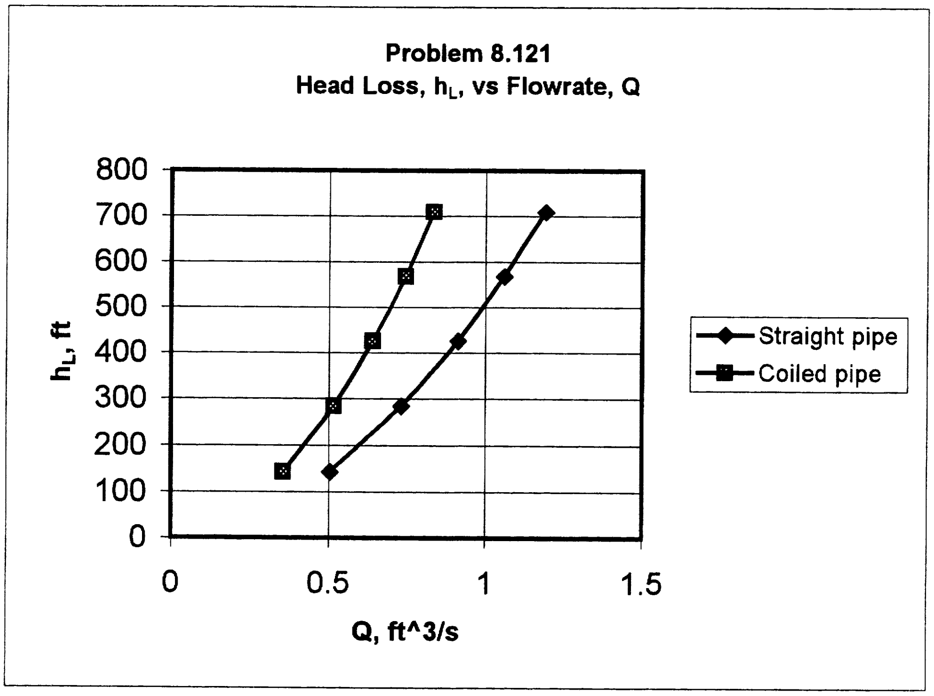
$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.9/12 ft) = 2110 lb/ft^2
R = 1716 ft lb/slug deg R
T = 80 +460 =540 deg R

Thus, ρ = 0.00228 slug/ft^3 and γ = 0.0733 lb/ft^3

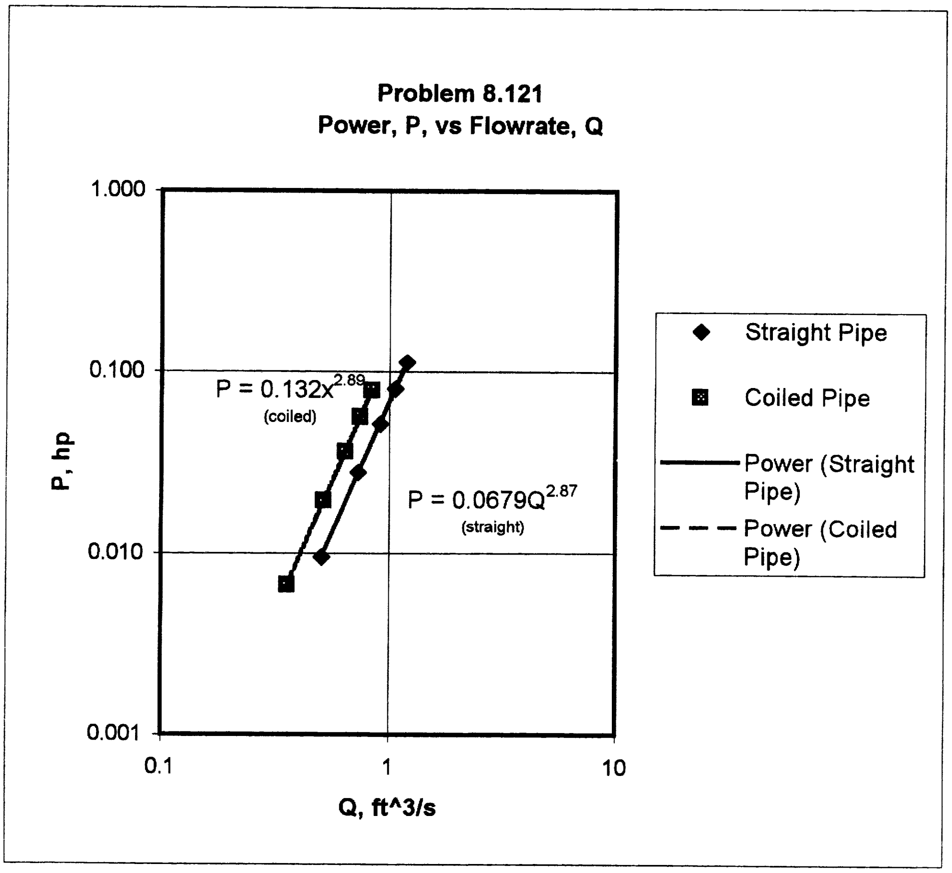
P = ($\gamma Q h_L$)ft lb/s*(1hp/550 ft lb/s)

(con't)

8.121 (con't)



Problem 8.121
Head Loss, h_L, vs Flowrate, Q
800
700
600
500
400
300
200
100
0
h_L, ft
0
0.5
1
1.5
Q, ft^3/s
Straight pipe
Coiled pipe



Problem 8.121
Power, P, vs Flowrate, Q
1.000
0.100
0.010
0.001
P, hp
$P = 0.132x^{2.89}$
(coiled)
$P = 0.0679Q^{2.87}$
(straight)
0.1
1
10
Q, ft^3/s
Straight Pipe
Coiled Pipe
Power (Straight Pipe)
Power (Coiled Pipe)

9.1

9.1 Assume that water flowing past the equilateral triangular bar shown in Fig. P9.1 produces the pressure distributions indicated. Determine the lift and drag on the bar and the corresponding lift and drag coefficients (based on frontal area). Neglect shear forces.

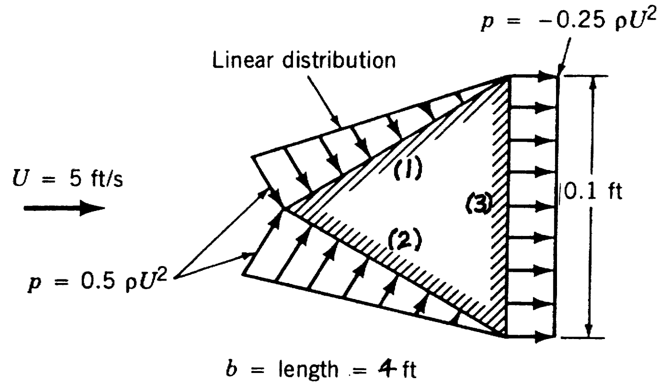


FIGURE P9.1

$\mathcal{D} = \int p \cos\theta \, dA + \int \tau_w \sin\theta \, dA$, where $\tau_w = 0$

Thus,

$$\mathcal{D} = \int_1 p \cos\theta \, dA + \int_2 p \cos\theta \, dA + \int_3 p \cos\theta \, dA$$

$$= 2\int_1 p \cos 60^\circ dA - \int_3 p \, dA = 2 \frac{(0.5\rho U^2)}{2} \cos 60^\circ \, \ell b - (-0.25\,\rho U^2)\,\ell b$$

or

$$\mathcal{D} = 0.5\,\rho U^2 \ell b \qquad (1)$$

so that $\mathcal{D} = 0.5\left(1.94 \frac{slugs}{ft^3}\right)\left(5\frac{ft}{s}\right)^2 (0.1\,ft)(4\,ft) = \underline{\underline{9.70\ lb}}$

Because of symmetry of the object, $\mathcal{L} = \underline{\underline{0}}$

Also, from Eq. (1)

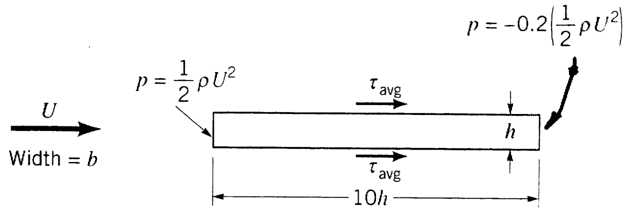
$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} = \frac{0.5\rho U^2 \ell b}{\frac{1}{2}\rho U^2 \ell b} = \underline{\underline{1.00}}$$

and since $\mathcal{L} = 0$

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \underline{\underline{0}}$$

9.2

9.2 Fluid flows past the two-dimensional bar shown in Fig. P9.2. The pressures on the ends of the bar are as shown, and the average shear stress on the top and bottom of the bar is τ_{avg}. Assume that the drag due to pressure is equal to the drag due to viscous effects. **(a)** Determine τ_{avg} in terms of the dynamic pressure, $\rho U^2/2$. **(b)** Determine the drag coefficient for this object.



a) $\mathscr{D}_f$ = friction drag $= 2\,\tau_{avg}\,(10h\,b) = 20\,hb\,\tau_{avg}$

and

$\mathscr{D}_p$ = pressure drag $= \frac{1}{2}\rho U^2(bh) - (-\frac{1}{2}\rho U^2(0.2))(bh) = 1.2\,(\frac{1}{2}\rho U^2)(bh)$

Thus, if $\mathscr{D}_f = \mathscr{D}_p$ then

$20\,hb\,\tau_{avg} = 1.2\,(bh)\frac{1}{2}\rho U^2$

or

$\tau_{avg} = \underline{\underline{0.06\,(\frac{1}{2}\rho U^2)}}$

b) $\mathscr{D} = \mathscr{D}_f + \mathscr{D}_p = C_D\,\frac{1}{2}\rho U^2 A = C_D\,\frac{1}{2}\rho U^2 bh$

Thus,

$20hb\,\tau_{avg} + 1.2\,(\frac{1}{2}\rho U^2)(bh) = C_D\,\frac{1}{2}\rho U^2 bh$

or

$20\,(0.06)(\frac{1}{2}\rho U^2) + 1.2\,(\frac{1}{2}\rho U^2) = C_D\,(\frac{1}{2}\rho U^2)$

Thus,

$C_D = \underline{\underline{2.40}}$

9.3*

9.3* The pressure distribution on the 1-m-diameter circular disk Fig. P9.3 is given in the table below. Determine the drag on the disk.

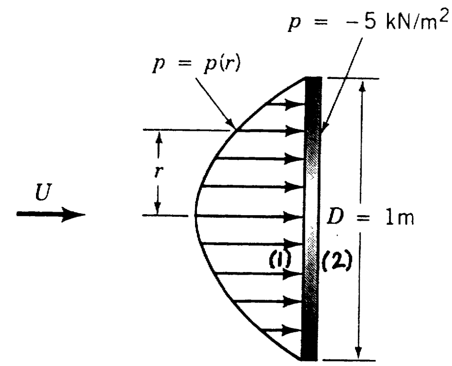


FIGURE P9.3

$$\mathcal{D} = \int_1 p\,dA - \int_2 p\,dA = \int_{r=0}^{r=\frac{D}{2}} p(2\pi r\,dr) - p_2 \frac{\pi}{4}D^2, \text{ since } dA = 2\pi r\,dr$$

Thus,

$$\mathcal{D} = 2\pi\int_0^{0.5\,m} p\,r\,dr - (-5\tfrac{kN}{m^2})\tfrac{\pi}{4}(1m^2) = 2\pi\int_0^{0.5} p\,r\,dr + 3.93\text{ kN}$$

where $p \sim \frac{kN}{m^2}$, $r \sim m$

Evaluate the integral numerically using the following integrand:

r, m	pr, kN/m
0	0
0.05	0.214
0.10	0.406
0.15	0.558
0.20	0.620
0.25	0.695
0.30	0.711
0.35	0.662
0.40	0.564
0.45	0.333
0.50	0.000

r (m)	p (kN/m²)
0	4.34
0.05	4.28
0.10	4.06
0.15	3.72
0.20	3.10
0.25	2.78
0.30	2.37
0.35	1.89
0.40	1.41
0.45	0.74
0.50	0.0

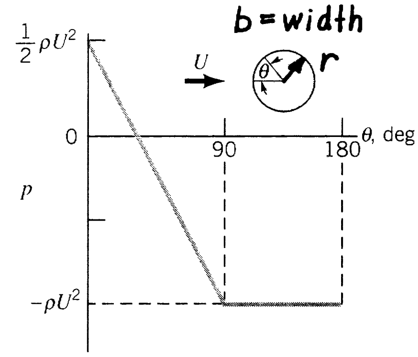
By using the program SIMPSON.BAS we obtain $\int_0^{0.5} p\,r\,dr = 0.241$

```
*************************************************
** This program performs numerical integration  **
** over a set a set of an odd number of equally **
** spaced points using Simpson's Rule            **
*************************************************

Enter number of data points: 11
Enter data points (X , Y)
? 0.00,0.000     ? 0.45,0.333
? 0.05,0.214     ? 0.50,0.000
? 0.10,0.406
? 0.15,0.558
? 0.20,0.620
? 0.25,0.695
? 0.30,0.711
? 0.35,0.662
? 0.40,0.564

The approximate value of the integral is: +2.4083E-01
```

Thus, $\mathcal{D} = 2\pi(0.241) + 3.93 = \underline{\underline{5.44\text{ kN}}}$

9.4

9.4 The pressure distribution on a cylinder is approximated by the two straight line segments shown in Fig. P9.4. Determine the drag coefficient for the cylinder. Neglect shear forces.



$$\mathcal{D} = \int p \cos\theta \, dA = \int_{\theta=0}^{\theta=2\pi} p \cos\theta \, (b r \, d\theta) = 2\int_0^{\pi} p \cos\theta \, (br) d\theta \qquad (1)$$

where

$p = -\rho U^2$ for $\frac{\pi}{2} \le \theta \le \pi$ and

$p = \frac{1}{2}\rho U^2 [1 - \frac{6}{\pi}\theta]$ for $0 \le \theta \le \frac{\pi}{2}$ (i.e. $p = \frac{1}{2}\rho U^2$ if $\theta = 0$

$p = -\rho U^2$ if $\theta = \frac{\pi}{2}$)

Thus,

$$\int_{\pi/2}^{\pi} p \cos\theta \, d\theta = -\rho U^2 \int_{\pi/2}^{\pi} \cos\theta \, d\theta = -\rho U^2 \sin\theta \Big|_{\pi/2}^{\pi} = \rho U^2 \qquad (2)$$

and

$$\int_0^{\pi/2} p \cos\theta \, d\theta = \frac{1}{2}\rho U^2 \int_0^{\pi/2} [1 - \frac{6}{\pi}\theta] \cos\theta \, d\theta = \frac{1}{2}\rho U^2 [\sin\theta - \frac{6}{\pi}(\cos\theta + \theta \sin\theta)] \Big|_0^{\pi/2}$$

$$= \frac{1}{2}\rho U^2 [1 - \frac{6}{\pi}(\frac{\pi}{2}) - (-\frac{6}{\pi})] = \frac{1}{2}\rho U^2 [\frac{6}{\pi} - 2] \qquad (3)$$

Thus, from Eqs. (1), (2), and (3)

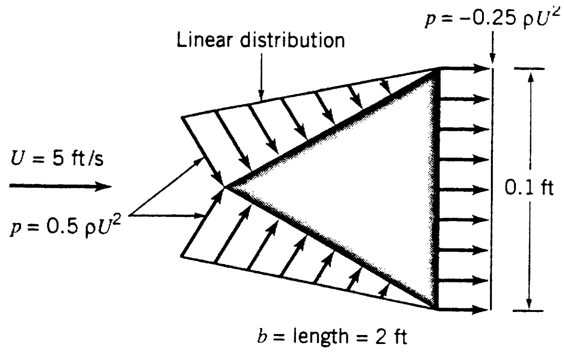
$$\mathcal{D} = 2br \int_0^{\pi} p \cos\theta \, d\theta = 2br[\frac{1}{2}\rho U^2(\frac{6}{\pi} - 2) + \rho U^2] = \frac{1}{2}\rho U^2 (\frac{12br}{\pi})$$

so that

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 (2rb)} = \frac{\frac{1}{2}\rho U^2 (\frac{12br}{\pi})}{\frac{1}{2}\rho U^2 (2br)} = \frac{6}{\pi} = \underline{\underline{1.91}}$$

9.5

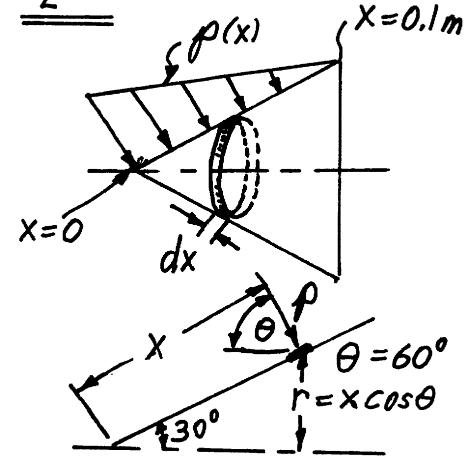
9.5 Repeat Problem 9.1 if the object is a cone (made by rotating the equilateral triangle about the horizontal axis through its tip) rather than a triangular bar.



■ FIGURE P9.1

By symmetry the lift is zero. Thus, $\underline{\underline{\mathcal{L}=0}}$ and $\underline{\underline{C_L=0}}$

Also $\mathcal{D}=\mathcal{D}_{front}+\mathcal{D}_{rear}$ where



$\mathcal{D}_{front}=\int p\cos\theta\, dA$ and $dA=2\pi r\,dx$

Thus, with $p=0.5\rho U^2(1-10x)$

i.e. $p\big|_{x=0}=0.5\rho U^2$

and $p\big|_{x=0.1}=0$, we have

$$\mathcal{D}_{front}=\int_{x=0}^{0.1}0.5\rho U^2(1-10x)2\pi(x\cos^2\theta)\,dx=2\pi\cos^2 60^\circ(0.5\rho U^2)\int_0^{0.1}(x-10x^2)\,dx$$

or

$$\mathcal{D}_{front}=2\pi(0.5)^2(0.5\rho U^2)\left[\frac{x^2}{2}-\frac{10}{3}x^3\right]_0^{0.1}\text{, or } \mathcal{D}_{front}=0.001309\,\rho U^2$$

Also,

$$\mathcal{D}_{rear}=-(0.25\rho U^2)\frac{\pi}{4}(0.1)^2=0.00196\,\rho U^2$$

so that

$$\mathcal{D}=\mathcal{D}_{front}+\mathcal{D}_{rear}=(0.001309+0.00196)\rho U^2=0.00327\,\rho U^2$$

so with $\rho=1.94\ \frac{slugs}{ft^3}$ and $U=5\ \frac{ft}{s}$,

$$\mathcal{D}=0.00327(1.94)(5)^2=\underline{\underline{0.159\ lb}}$$

Also,

$$C_D=\frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}=\frac{0.00327\,\rho U^2}{\frac{1}{2}(1.94)(5)^2\frac{\pi}{4}(0.1)^2}=\underline{\underline{0.833}}$$

9.6 A 17-ft-long kayak moves with a speed of 5 ft/s (see Video V9.2). Would a boundary layer type flow be developed along the sides of the boat? Explain.

$Re = \frac{U\ell}{\nu}$, so with $\ell = 17\,ft$ and $U = 5\,\frac{ft}{s}$ and $60°F$ water with $\nu = 1.21 \times 10^{-5}\,\frac{ft^2}{s}$ we have

$$Re = \frac{(17\,ft)(5\,\frac{ft}{s})}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 7.02 \times 10^6$$

Since $Re \approx 1000$ is often assumed to be the lower limit for boundary layer type flow, it is clear that a boundary layer would develop along the sides of the kayak. Yes.

9.7

9.7 Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar; turbulent? Explain.

Animal	Speed	Re
(a) large whale	10 m/s	300,000,000
(b) flying duck	20 m/s	300,000
(c) large dragonfly	7 m/s	30,000
(d) invertebrate larva	1 mm/s	0.3
(e) bacterium	0.01 mm/s	0.00003

Inertia important if $Re \gtrsim 1$ (i.e whale, duck, dragonfly)

Viscous effects dominate if $Re \lesssim 1$ (i.e larva, bacterium)

Boundary layer flow becomes turbulent for Re on the order of 10^5 to 10^6. (i.e. whale and perhaps the duck)
The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.

9.9 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

$Re = \frac{UD}{\nu} < 1$ or $U < \frac{\nu}{D}$ if viscous effects are to be important throughout the flow.

For standard air $\nu = 1.57 \times 10^{-4} \frac{ft^2}{s}$

Thus,

$U < \frac{1.57 \times 10^{-4}}{D}$, where D is the diameter in feet.

object	D, ft	U, $\frac{ft}{s}$
twig	2.08×10^{-2}	7.54×10^{-3}
hair	3.33×10^{-4}	0.471
smokestack	6	2.62×10^{-5}

9.10

9.10 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.
Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3} m}{\sqrt{1.3 m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m, \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

9.11

9.11 If the upstream velocity of the flow in Problem 9.10 is U = 1.5 m/s, determine the kinematic viscosity of the fluid.

For laminar flow $\delta = 5\sqrt{\frac{\nu x}{U}}$, or $\nu = \frac{U\delta^2}{25\, x}$

Thus,

$$\nu = \frac{(1.5 \frac{m}{s})(12 \times 10^{-3} m)^2}{25\,(1.3 m)} = \underline{\underline{6.65 \times 10^{-6} \frac{m^2}{s}}}$$

9.12

9.12 Water flows past a flat plate with an upstream velocity of $U = 0.02$ m/s. Determine the water velocity a distance of 10 mm from the plate at distances of $x = 1.5$ m and $x = 15$ m from the leading edge.

From the Blasius solution for boundary layer flow on a flat plate, $u = U f'(\eta)$, where η, the similarity variable, is $\eta = y\sqrt{\frac{U}{\nu x}}$. Values of $f'(\eta)$ are given in Table 9.1.

Since $Re_x = \frac{Ux}{\nu} = \frac{(0.02\frac{m}{s})(15m)}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5$ is less than the critical $Re_{x_{cr}} = 5\times10^5$, it follows that the boundary layer flow is laminar.

At $x_1 = 1.5\,m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_1 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(1.5m)}} = 1.091$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.2647 + \frac{(0.3938-0.2647)}{(1.2-0.8)}(1.091-0.8) = 0.359$$

Hence,

$$u_1 = Uf'(\eta_1) = (0.02\tfrac{m}{s})(0.359) = \underline{\underline{0.00718\tfrac{m}{s}}}$$

Similarly, at $x_2 = 15m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_2 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(15m)}} = 0.345$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.0 + \frac{(0.1328-0.0)}{(0.8-0.4)}(0.345-0.0) = 0.1145$$

Hence,

$$u_2 = Uf'(\eta_2) = (0.02\tfrac{m}{s})(0.1145) = \underline{\underline{0.00229\tfrac{m}{s}}}$$

9.13*

*9.13 A Pitot tube connected to a water-filled U-tube manometer is used to measure the total pressure within a boundary layer. Based on the data given in the table below, determine the boundary layer thickness, δ, the displacement thickness, δ^*, and the momentum thickness, Θ.

y (mm)	h (mm)
0	0
2.1	10.6
4.3	21.1
6.4	25.6
10.7	32.5
15.0	36.9
19.3	39.4
23.6	40.5
26.8	41.0
29.3	41.0
32.7	41.0

From the Bernoulli equation, with $\rho_{air} \ll \rho_{H_2O}$ it follows that

$$p_1 + \frac{1}{2}\rho_{air} V_1^2 = p_2 + \frac{1}{2}\rho_{air} V_2^2, \text{ where}$$

$V_1 = u$, $V_2 = 0$, $p_1 = 0$, and $p_2 = \gamma_{H_2O} h$

Thus,

$$u = \sqrt{\frac{2\gamma_{H_2O} h}{\rho_{air}}} = \sqrt{\frac{2(9800 \frac{N}{m^3}) h\, m}{1.23 \frac{kg}{m^3}}}$$

or

$$u = 126.2\sqrt{h}, \text{ where } h \sim m, u \sim \frac{m}{s}$$

For $y > 26.8\,mm$ we see that $h = 41.0\,mm$

Thus, $U = 126.2\sqrt{(0.041)} = 25.55 \frac{m}{s}$

For $y = 23.6\,mm$, $u = 126.2\sqrt{(0.0405)}$
$= 25.40 \frac{m}{s}$

or $\frac{u}{U} = \frac{25.40}{25.55} = 0.994$

Thus, $\delta \approx \underline{\underline{23.6\ mm}}$

The displacement thickness, δ^*, is

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \text{ or since}$$

$$\frac{u}{U} = \frac{126.2\sqrt{h}}{25.55} = 4.94\sqrt{h} \text{ this becomes}$$

$$\delta^* = \int_{y=0}^{0.0268\,m} (1 - 4.94\sqrt{h})\, dy$$

Numerical integration of the tabulated data gives $\delta^* = \underline{\underline{4.18 \times 10^{-3}\ m}}$
(See next page.)

(con't)

9.13* (con't)

Also, the momentum thickness, θ, is

$$\theta = \int_0^{\infty} \frac{u}{U}\left(1-\frac{u}{U}\right) dy = 4.94 \int_{y=0}^{0.0268\,m} \sqrt{h}\left(1-4.94\sqrt{h}\right) dy \,.$$

Numerical integration of the tabulated data gives $\theta = \underline{\underline{2.23\times10^{-3}\,m}}$

Use program TRAPEZOI to integrate the integrands tabulated below:

y, m	$(1-4.94\sqrt{h})$	$\sqrt{h}\,(1-4.94\sqrt{h})$
0	1	0
0.0021	0.491	0.0506
0.0043	0.282	0.0410
0.0064	0.210	0.0335
0.0107	0.109	0.0197
0.0150	0.0511	0.00981
0.0193	0.0194	0.00386
0.0236	0.00584	0.00118
0.0268	0	0
0.0293	0	0
0.0327	0	0

```
*****************************************************
** This program performs numerical integration    **
** over a set of points using the Trapezoidal Rule **
*****************************************************

Enter number of data points: 9
Enter data points (X , Y)
? 0,1
? 0.0021,0.491
? 0.0043,0.282
? 0.0064,0.210
? 0.0107,0.109
? 0.0150,0.0511,
? 0.0193,0.0194
? 0.0236,0.00584
? 0.0268,0

The approximate value of the integral is: +4.1777E-03
```

Thus, $\int_0^{0.0268} (1-4.94\sqrt{h})\,dy = 0.00418$

(con't)

9.13* (con't)

```
****************************************************
** This program performs numerical integration    **
** over a set of points using the Trapezoidal Rule **
****************************************************

Enter number of data points: 9
Enter data points (X , Y)
? 0,0
? 0.0021,0.0506
? 0.0043,0.0410
? 0.0064,0.0335
? 0.0107,0.0197
? 0.0150,0.00981
? 0.0193,0.00386
? 0.0236,0.00118
? 0.0268,0

The approximate value of the integral is: +4.5206E-04
```

Thus, $\int_0^{0.026} \sqrt{h}\,(1-4.94\sqrt{h})\,dy = 4.52\times10^{-4}$

or $\Theta = 4.94\times 4.52\times10^{-4} = 2.23\times10^{-3}\,m$

9.14

9.14 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, δ^*. For air blowing past the flat plate shown in Fig. P9.14, plot the streamline $A-B$ that passes through the edge of the boundary layer ($y = \delta_B$ at $x = \ell$) at point B. That is, plot $y = y(x)$ for streamline $A-B$. Assume laminar boundary layer flow.

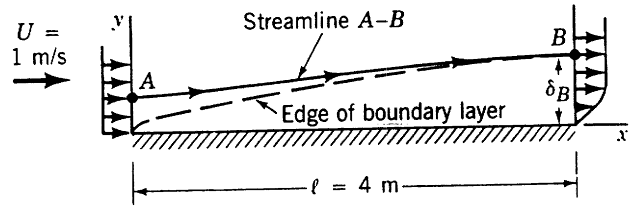
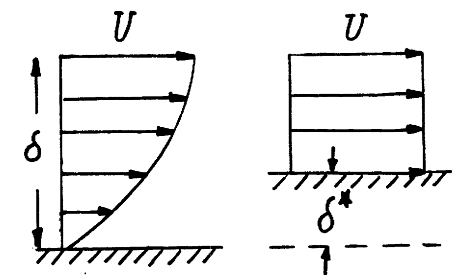


FIGURE P9.14

Since $Re_\ell = \dfrac{U\ell}{\nu} = \dfrac{(1\frac{m}{s})(4m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.74\times10^5 < 5\times10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta_B = 5\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.0382\,m$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with the plate displaced by an amount δ^*. Since there is no flow through the plate or streamline A-B,



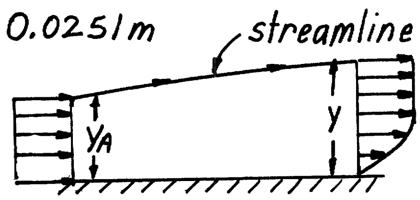
$$Q_A = Q_B, \text{ or } U y_A = (\delta_B - \delta_B^*)U \qquad (1)$$

where $\delta^* = 1.721\sqrt{\dfrac{\nu x}{U}}$

or $\delta_B^* = 1.721\left[\dfrac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.01315\,m$

Thus,

$$y_A = \delta_B - \delta_B^* = 0.0382\,m - 0.01315\,m = 0.0251\,m$$



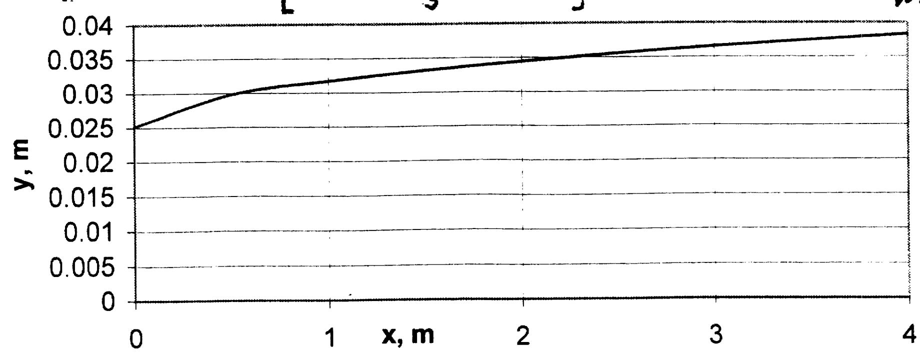
Hence, for any x-location

$Q_A = Q$ or $U y_A = U(y - \delta^*)$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\dfrac{\nu x}{U}}$

$$= 0.0251\,m + 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})\,x\,m}{1\frac{m}{s}}\right]^{\frac{1}{2}} = \underline{\underline{0.0251 + 6.58\times10^{-3}\sqrt{x}\ m,}}$$

where $x \sim m$



9.15

9.15 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.15. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \le x \le 10$ ft if U is to remain constant. Assume laminar flow.

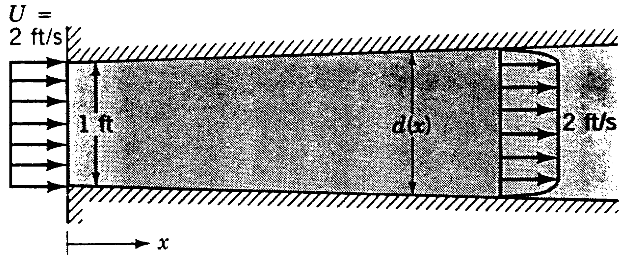


FIGURE P9.15

For incompressible flow $Q_0 = Q(x)$ where Q_0 = flowrate into the duct $= UA_0 = (2\,\frac{ft}{s})(1\,ft^2) = 2\,\frac{ft^3}{s}$

and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1\,ft + 2\delta^*, \qquad (1)$$

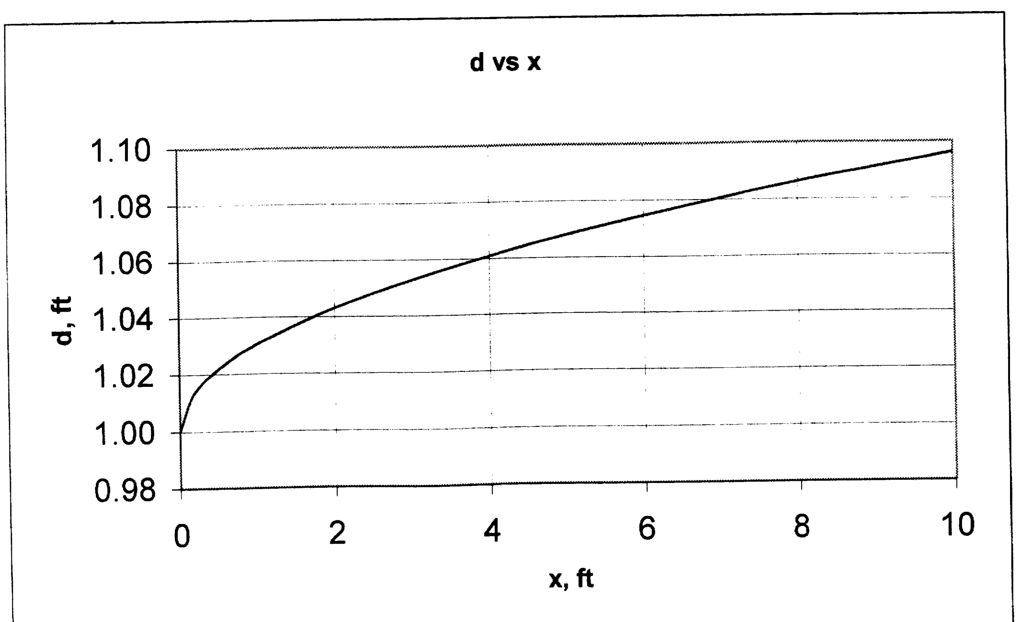
where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57\times 10^{-4}\,\frac{ft^2}{s})\,x}{2\,\frac{ft}{s}}\right]^{\frac{1}{2}} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq. (1)

$$\underline{\underline{d = 1 + 0.0304\sqrt{x}\ ft}}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.



9.16

9.16 A smooth flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{\left(1.12\times10^{-6}\,\frac{m^2}{s}\right)x}{0.5\,\frac{m}{s}}} = 7.48\times10^{-3}\sqrt{x}\ m,\ \text{where } x\sim m$$

and

$$\tau_w = 0.332\,U^{3/2}\sqrt{\frac{\rho\mu}{x}} = 0.332\left(0.5\,\frac{m}{s}\right)^{3/2}\sqrt{\frac{\left(999\,\frac{kg}{m^3}\right)\left(1.12\times10^{-3}\,\frac{N\cdot s}{m^2}\right)}{x}}$$

$$= \frac{0.124}{\sqrt{x}}\ \frac{N}{m^2},\ \text{where } x\sim m$$

Thus, at $x = 3$ m $\quad \delta = 7.48\times10^{-3}\sqrt{3} = \underline{\underline{0.0130\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{3}} = \underline{\underline{0.0716\ \frac{N}{m^2}}}$$

while at $x = 6$ m $\quad \delta = 7.48\times10^{-3}\sqrt{6} = \underline{\underline{0.0183\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{6}} = \underline{\underline{0.0506\ \frac{N}{m^2}}}$$

9.17

9.17 An atmospheric boundary layer is formed when the wind blows over the earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. P9.17, typical values are $n = 0.40$ for urban areas, $n = 0.28$ for woodland or suburban areas, and $n = 0.16$ for flat open country (Ref. 23). **(a)** If the velocity is 20 ft/s at the bottom of the sail on your boat ($y = 4$ ft), what is the velocity at the top of the mast ($y = 30$ ft)? **(b)** If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

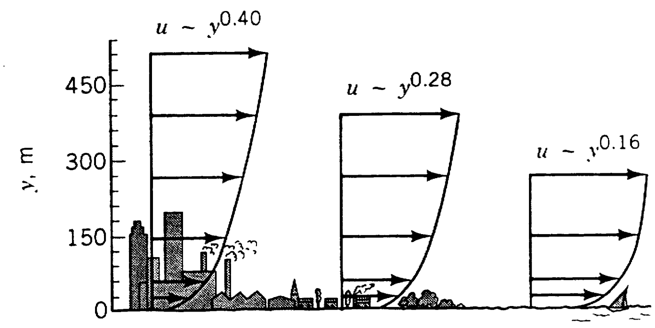


FIGURE P9.17

(a) $u = C\,y^{0.16}$, where C is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.16}$ or $u_2 = 20\,\frac{ft}{s}\left(\dfrac{30\,ft}{4\,ft}\right)^{0.16} = \underline{\underline{27.6\,\frac{ft}{s}}}$

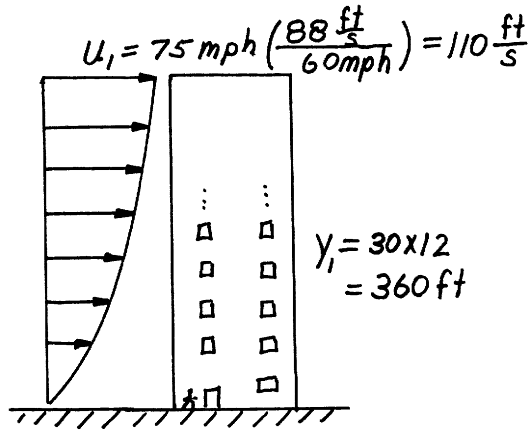
(b) $u = \tilde{C}\,y^{0.4}$, where $\tilde{C}$ is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.40}$ or $u_2 = 10\,mph\left(\dfrac{60}{10}\right)^{0.4} = \underline{\underline{20.5\,mph}}$

9.18

9.18 A 30-story office building (each story is 12 ft tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at hurricane strength (75 mph) at the top of the building. Use the atmospheric boundary layer information of Problem 9.17.

From Fig. P9.17 the boundary layer velocity profile is given by $u \sim y^{0.28}$, or $u = C\,y^{0.28}$, where C is a constant.



Thus, $\dfrac{u}{u_1} = \left(\dfrac{y}{y_1}\right)^{0.28}$

or

$u = 110\left(\dfrac{y}{360}\right)^{0.28} \dfrac{ft}{s}$ where $y \sim ft$

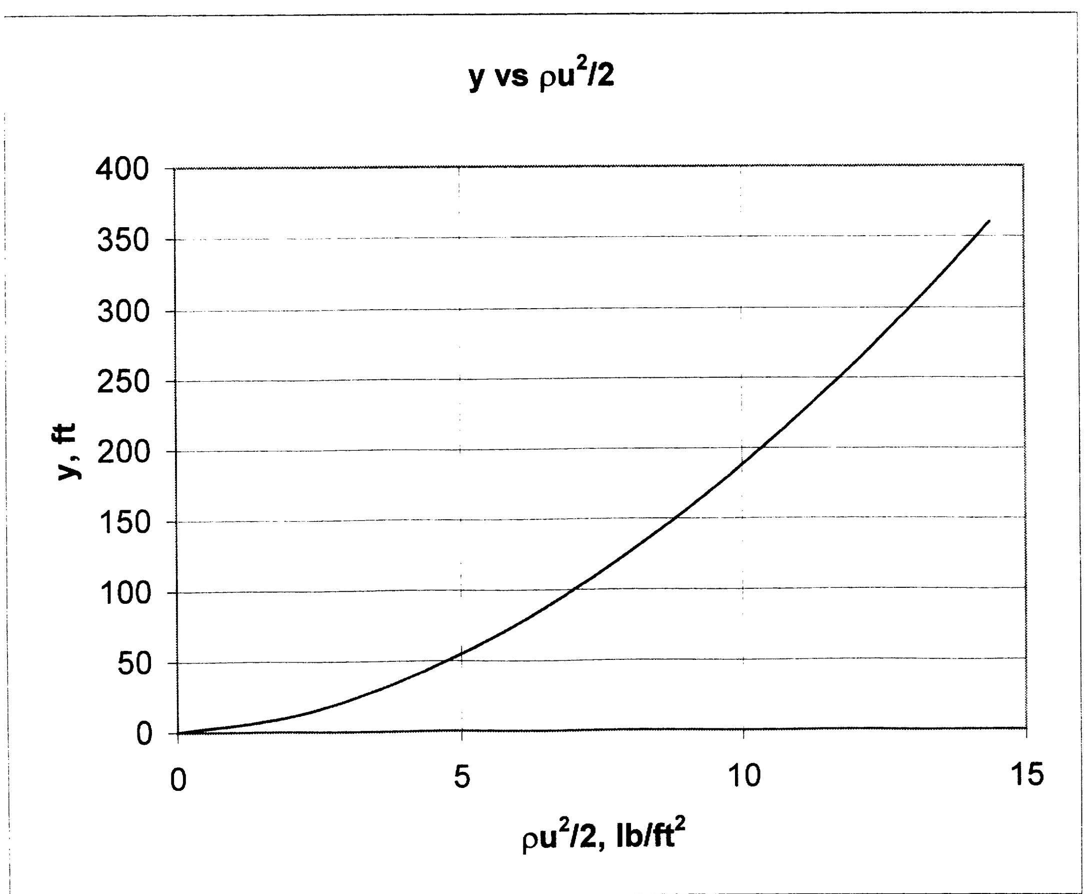
Hence,

$$\frac{1}{2}\rho u^2 = \frac{1}{2}\left(2.38\times10^{-3}\,\frac{slug}{ft^3}\right)\left[110\left(\frac{y}{360}\right)^{0.28}\frac{ft}{s}\right]^2$$

or

$$\frac{1}{2}\rho u^2 = 14.4\left(\frac{y}{360}\right)^{0.56}\frac{lb}{ft^2}, \text{ where } y \sim ft$$

This is plotted in the figure below.



9.19

9.19 The typical shape of small cumulous clouds is as indicated in Fig. P9.19. Based on boundary layer ideas, explain why it is clear that the wind is blowing from right to left as indicated.

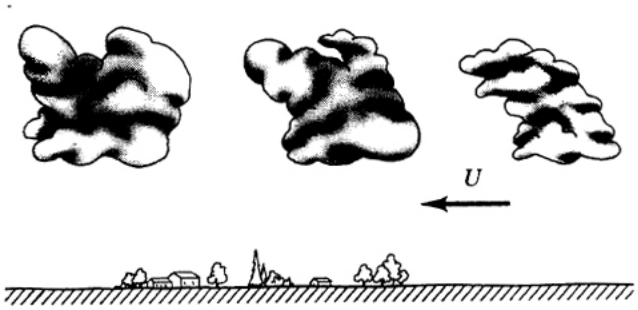


FIGURE P9.19

As indicated in Fig. P9.17, because of the atmospheric boundary layer the velocity of the wind generally increases with altitude. Thus, the top portions of a cloud travels faster than its base — the clouds tend to "tip" toward the direction of the wind. That is, the wind is from right to left.

9.20 Show that by writing the velocity in terms of the similarity variable η and the function $f(\eta)$ the momentum equation for boundary layer flow on a flat plate (Eq. 9.9) can be written as the ordinary differential equation given by Eq. 9.14.

The governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{and} \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{2}$$

Consider $u = U f'(\eta)$ and $v = \left(\frac{\nu U}{4x}\right)^{1/2}(\eta f' - f)$ where $(\ )' \equiv \frac{d}{d\eta}$ (2.1)
and $\eta = \left(\frac{U}{\nu x}\right)^{1/2} y$

Thus, $\frac{\partial \eta}{\partial x} = -\frac{1}{2}\sqrt{\frac{U}{\nu}}\, y x^{-3/2} = -\frac{1}{2}\frac{\eta}{x}$ and $\frac{\partial \eta}{\partial y} = \sqrt{\frac{U}{\nu}}\, x^{-1/2}$ (3)

so that

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(Uf') = U\frac{\partial f'}{\partial x} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial x} = -\frac{1}{2}\frac{U}{x}\eta f'' \tag{4}$$

and

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{1/2}\frac{\partial}{\partial y}(\eta f' - f) = \left(\frac{\nu U}{4x}\right)^{1/2}(\eta f'' + f' - f')\frac{\partial \eta}{\partial y} = \left(\frac{\nu U}{4x}\right)^{1/2}\eta f''\sqrt{\frac{U}{\nu}}\, x^{-1/2}$$

or

$$\frac{\partial v}{\partial y} = \frac{1}{2}\frac{U}{x}\eta f'' \tag{5}$$

Thus, by using Eqs. (4) and (5) we see that Eq (1) is satisfied for any function $f(\eta)$.

Also, $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial}{\partial \eta}(Uf')\left[\sqrt{\frac{U}{\nu}}\, x^{-1/2}\right] = \left(\frac{U^3}{\nu x}\right)^{1/2} f''$ (6)

and $\frac{\partial^2 u}{\partial y^2} = \left(\frac{U^3}{\nu x}\right)^{1/2}\frac{\partial f''}{\partial y} = \left(\frac{U^3}{\nu x}\right)^{1/2} f'''\frac{\partial \eta}{\partial y} = \frac{U^2}{\nu x} f'''$ (7)

Thus, by using Eqs. (2.1), (6), and (7) with Eq. (2) we obtain

$$(Uf')\left(\frac{-1}{2}\frac{U}{x}\eta f''\right) + \left(\frac{\nu U}{4x}\right)^{1/2}(\eta f' - f)\left(\frac{U^3}{\nu x}\right)^{1/2} f'' = \nu\frac{U^2}{\nu x} f'''$$

which simplifies to:

$$\underline{\underline{2f''' - ff'' = 0}}$$

From Eq. (2.1) the boundary conditions at $y=0$ (i.e. $\eta = 0$) become
$u = 0 = Uf'(0)$ and $v = 0 = \left(\frac{\nu U}{4x}\right)^{1/2}(0\, f'(0) - f(0))$

That is, $\underline{\underline{f(0) = 0 \text{ and } f'(0) = 0}}$

Similarly, as $y \to \infty$ (i.e., $\eta \to \infty$) we require $u \to U$. Thus, from Eq. (2.1) $\underline{\underline{f' \to 1 \text{ as } \eta \to \infty.}}$

9.21*

9.21* Integrate the Blasius equation (Eq. 9.14) numerically to determine the boundary layer profile for laminar flow past a flat plate. Compare your results with those of Table 9.1.

Solve the following third order differential equation by a numerical integration technique:

$$2f''' + ff'' = 0 \quad \text{with boundary conditions}$$
$$f = f' = 0 \text{ at } \eta = 0 \text{ and } f' \to 1 \text{ as } \eta \to \infty \qquad \left((\,)' \equiv \frac{d}{d\eta} \right)$$

Write this third order equation as 3 first order equations and use a Runga-Kutta numerical technique to integrate them. Thus, let
$y_1 \equiv f$, $y_1' = f' \equiv y_2$, $y_2' = f'' \equiv y_3$, and $y_3' = f''' = -\frac{1}{2} ff'' = -\frac{1}{2} y_1 y_3$
That is:

$y_1' = y_2$
$y_2' = y_3$ and
$y_3' = -y_1 y_3 / 2$

These can be approximated as
$\Delta y_1 = y_2 \Delta\eta$, $\Delta y_2 = y_3 \Delta\eta$, and $\Delta y_3 = (-y_1 y_3/2)\Delta\eta$
Start with $y_1 = y_2 = 0$ at $\eta = 0$. Assume $y_3 = C$ at $\eta = 0$ (where C is some given constant) and "integrate to $\eta = \infty$" by $y_i = y_i(0) + \sum_j \Delta y_{ij} \Delta\eta$
If $y_2(\infty) \neq 1$ (i.e., $f'(\infty) \neq 1$) adjust the value of C (i.e., $f''(0)$) and try again. The two-point boundary value problem (i.e., $f(0) = f'(0) = 0$ and $f'(\infty) = 1$) is solved by iteration as an initial value problem (i.e., $f(0) = f'(0) = 0$, $f''(0) = C$).
Program P8#21 shown below was used for the calculations. The final value of $C = 0.332$ and the velocity profile, $u = U f'(\eta)$, agree very well with the standard values given in Table 9.1

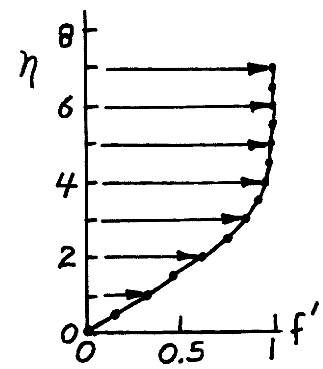
```
100 cls
110 open "prn" for output as #1
120 print "*********************************************"
130 print "**  This program integrates the boundary layer   **"
140 print "**  equation (Blasius equation) for a flat plate **"
150 print "**  using a Runga-Kutta type routine.  The user  **"
160 print "**  must specify an initial value of f''(0) so   **"
170 print "**  the boundary condition 'at infinity' (f'''=  **"
```

(con't)

9.21* (con't)

```
180 print "**  1) is satisfied.                                    **"
190 print "******************************************************"
200 print " "
210 print " "
300 print "Input a value for f''(0)"
310 input c
320 print "Input stepsize and number of steps"
330 input dx, n
335 print "Input how often to print output (number of steps)"
336 input nn
340 print "    eta        f         f'         f''"
350 y1 = 0
360 y2 = 0
370 y3 = c
380 x = 0
385 m = 0
390 for i = 1 to n
395 m = m +1
400 x = x + dx
410 y1 = y1 + y2*dx
420 y2 = y2 + y3*dx
430 y3 = y3 - (y1*y3/2)*dx
435 if m < nn goto 450
440 print using " ##.####  +#.##^^^^ +#.##^^^^ +#.##^^^^";x,y1,y2,y3
445 m = 0
450 next i
460 goto 210
```

```
****************************************************
**  This program integrates the boundary layer    **
**  equation (Blasius equation) for a flat plate  **
**  using a Runga-Kutta type routine.  The user   **
**  must specify an initial value of f''(0) so    **
**  the boundary condition 'at infinity' (f'''=   **
**  1) is satisfied.                              **
****************************************************

Input a value for f''(0)
? 0.332
Input stepsize and number of steps
? 0.01 700
Input how often to print output (number of steps)
? 50
   eta        f          f'         f''
  0.5000  +4.07E-02  +1.66E-01  +3.31E-01
  1.0000  +1.64E-01  +3.30E-01  +3.23E-01
  1.5000  +3.68E-01  +4.87E-01  +3.03E-01
  2.0000  +6.47E-01  +6.30E-01  +2.67E-01
  2.5000  +9.93E-01  +7.52E-01  +2.17E-01
  3.0000  +1.39E+00  +8.47E-01  +1.61E-01
  3.5000  +1.83E+00  +9.14E-01  +1.07E-01
  4.0000  +2.30E+00  +9.56E-01  +6.38E-02
  4.5000  +2.79E+00  +9.80E-01  +3.36E-02
  5.0000  +3.28E+00  +9.92E-01  +1.56E-02
  5.5000  +3.78E+00  +9.97E-01  +6.41E-03
  6.0000  +4.28E+00  +9.99E-01  +2.32E-03
  6.5001  +4.78E+00  +1.00E+00  +7.36E-04
  7.0001  +5.28E+00  +1.00E+00  +2.06E-04
```



9.22

9.22 An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of $Re_{xcr} = 5 \times 10^5$. If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

At 10,000 ft:

(a) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and from Table C.1, $\nu = \frac{\mu}{\rho} = \frac{3.534 \times 10^{-7}\,\frac{lb\cdot s}{ft^2}}{1.756 \times 10^{-3}\,\frac{slug}{ft^3}}$

$= 2.01 \times 10^{-4}\,\frac{ft^2}{s}$

Hence, with $Re_{x_{cr}} = 5 \times 10^5$,

$$x_{cr} = \frac{\nu\, Re_{x_{cr}}}{U} = \frac{(2.01 \times 10^{-4}\,\frac{ft^2}{s})(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.171\,ft}}$$

At sea-level:

(b) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and $\nu = 1.57 \times 10^{-4}\,\frac{ft^2}{s}$

Hence,

$$x_{cr} = \frac{\nu\, Re_{cr}}{U} = \frac{(1.57 \times 10^{-4}\,\frac{ft^2}{s})(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.134\,ft}}$$

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.

9.24

9.24 A laminar boundary layer velocity profile is approximated by $u/U = [2 - (y/\delta)](y/\delta)$ for $y \le \delta$, and $u = U$ for $y > \delta$. **(a)** Show that this profile satisfies the appropriate boundary conditions. **(b)** Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$.

(a) $\frac{u}{U} = g(Y) = 2Y - Y^2$ where $Y = y/\delta$

Thus, $\left.\frac{u}{U}\right|_{y=0} = 0$ as it must, $\left.\frac{u}{U}\right|_{y=\delta} = 2 - 1 = 1$ or $u = U$ at $y = \delta$ as it must.

Also, $\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$ so that $\left.\frac{du}{dy}\right|_{y=\delta} = U\left[\frac{2}{\delta} - \frac{2}{\delta}\right] = 0$

(b) From the momentum integral equation,

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

Thus,

$$C_1 = \int_0^1 (2Y - Y^2)(1 - 2Y + Y^2)\,dY = \int_0^1 (2Y - 5Y^2 + 4Y^3 - Y^4)\,dY$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15}$$

and

$$C_2 = (2 - 2Y|_{Y=0} = 2$$

so that

$$\delta = \sqrt{\frac{2(2)\nu x}{\frac{2}{15} U}} = \sqrt{\frac{30 \nu x}{U}}$$

Hence, with $Re_x = \frac{Ux}{\nu}$,

$$\frac{\delta}{x} = \frac{\sqrt{30}}{\sqrt{Re_x}} = \underline{\underline{\frac{5.48}{\sqrt{Re_x}}}}$$

9.25

9.25 A laminar boundary layer velocity profile is approximated by the two straight-line segments indicated in Fig. P9.25. Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$, and wall shear stress, $\tau_w = \tau_w(x)$. Compare these results with those in Table 9.2.

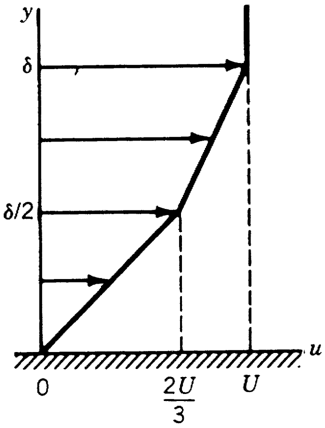


FIGURE P9.25

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0} \qquad (1)$$

and $\frac{u}{U} = g(Y)$ with $Y = \frac{y}{\delta}$.

For $0 \le Y < \frac{1}{2}$, $g = a_1 + b_1 Y$ with the constants a_1 and b_1 obtained from $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 0$ at $Y = 0$. Thus, $a_1 = 0$, $b_1 = \frac{4}{3}$

or $g = \frac{4}{3}Y$ for $0 \le Y < \frac{1}{2}$

Hence, $C_2 = \frac{4}{3}$ (2)

Similarly, for $\frac{1}{2} \le Y \le 1$, $g = a_2 + b_2 Y$ with $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 1$ at $Y = 1$

Thus, $\frac{2}{3} = a_2 + \frac{1}{2}b_2$ and $1 = a_2 + b_2$ which give $a_2 = \frac{1}{3}$, $b_2 = \frac{2}{3}$

or

$g = \frac{1}{3} + \frac{2}{3}Y$, for $\frac{1}{2} \le Y < 1$

Hence, $C_1 = \int_0^1 g(1-g)\,dY = \int_0^{1/2} \frac{4}{3}Y\left(1 - \frac{4}{3}Y\right)dY + \int_{1/2}^1 \left(\frac{1}{3} + \frac{2}{3}Y\right)\left(1 - \frac{1}{3} - \frac{2}{3}Y\right)dY$

$= \frac{4}{9}\int_0^{1/2}(3Y - 4Y^2)\,dY + \frac{2}{9}\int_{1/2}^1 (1+2Y)(1-Y)\,dY$ which upon integration gives

$C_1 = 0.1574$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$$\delta = \left[\frac{2\left(\frac{4}{3}\right)\nu x}{0.1574\,U}\right]^{1/2} = \underline{\underline{4.12\sqrt{\frac{\nu x}{U}}}} \quad \text{or} \quad \frac{\delta\, Re_x^{1/2}}{x} = 4.12$$

Also,

$$\tau_w = \frac{\mu U}{\delta} C_2 = \underline{\underline{\frac{4\mu U}{3\delta}}} \quad \text{or} \quad c_f = \frac{\sqrt{2C_1C_2}}{\sqrt{Re_x}} = \frac{\sqrt{2(0.1574)\left(\frac{4}{3}\right)}}{\sqrt{Re_x}} = \frac{0.648}{\sqrt{Re_x}}$$

Compare these results to those in Table 9.2.

9.26★

9.26* An assumed dimensionless laminar boundary layer profile for flow past a flat plate is given in the table below. Use the momentum integral equation to determine $\delta = \delta(x)$. Compare your result with the exact Blasius solution result (see Table 9.2).

y/δ	u/U
0	0
0.080	0.133
0.16	0.265
0.24	0.394
0.32	0.517
0.40	0.630
0.48	0.729
0.56	0.811
0.64	0.876
0.72	0.923
0.80	0.956
0.88	0.976
0.96	0.988
1.00	1.000

From the momentum integral equation

$$\delta = \sqrt{\frac{2 C_2 \nu x}{U C_1}}, \quad \text{where } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

and

$$C_1 = \int_0^1 g(1-g)\,dY \quad \text{with } \frac{u}{U} = g(Y) \text{ and } Y = \frac{y}{\delta}$$

The value of C_2 can be approximated as $C_2 \approx \left.\dfrac{\Delta(\frac{u}{U})}{\Delta(\frac{y}{\delta})}\right|_{\frac{y}{\delta}=0} = \dfrac{0.133}{0.080} = 1.66$

and the value of C_1 can be obtained from numerical integration (program TRAPEZOI)

Y	$g(1-g)$
0	0
0.8	0.115
0.16	0.195
0.24	0.239
0.32	0.250
0.40	0.233
0.48	0.198
0.56	0.153
0.64	0.109
0.72	0.071
0.80	0.042
0.88	0.023
0.96	0.012
1.00	0

As indicated below:

$$C_1 = \int_0^1 g(1-g)\,dY \approx 0.131 \quad \text{so that}$$

$$\delta = \left[\frac{2\nu x\,(1.66)}{U\,(0.131)}\right]^{\frac{1}{2}} = 5.03\left(\frac{\nu x}{U}\right)^{\frac{1}{2}}$$

or

$$\frac{\delta}{x} = \frac{5.03}{\sqrt{Re_x}}, \quad \text{where } Re_x = \frac{\rho V x}{\mu}$$

Note: The Blasius solution has 5, not 5.03

```
*************************************************
** This program performs numerical integration     **
** over a set of points using the Trapezoidal Rule **
*************************************************

Enter number of data points: 14
Enter data points (X , Y)
? 0.00,0.000
? 0.08,0.115
? 0.16,0.195
? 0.24,0.239
? 0.32,0.250
? 0.40,0.233
? 0.48,0.198
? 0.56,0.153
? 0.64,0.109
? 0.72,0.071
? 0.80,0.042
? 0.88,0.023
? 0.96,0.012
? 1.00,0.000

The approximate value of the integral is: +1.3096E-01
```

9.27*

9.27* For a fluid of specific gravity $SG = 0.86$ flowing past a flat plate with an upstream velocity of $U = 5$ m/s, the wall shear stress on a flat plate was determined to be as indicated in the table below. Use the momentum integral equation to determine the boundary layer momentum thickness, $\Theta = \Theta(x)$. Assume $\Theta = 0$ at the leading edge, $x = 0$.

x (m)	τ_w (N/m^2)
0	—
0.2	13.4
0.4	9.25
0.6	7.68
0.8	6.51
1.0	5.89
1.2	6.57
1.4	6.75
1.6	6.23
1.8	5.92
2.0	5.26

Since $\tau_w = \rho U^2 \frac{d\Theta}{dx}$ it follows that $d\Theta = \frac{\tau_w}{\rho U^2} dx$

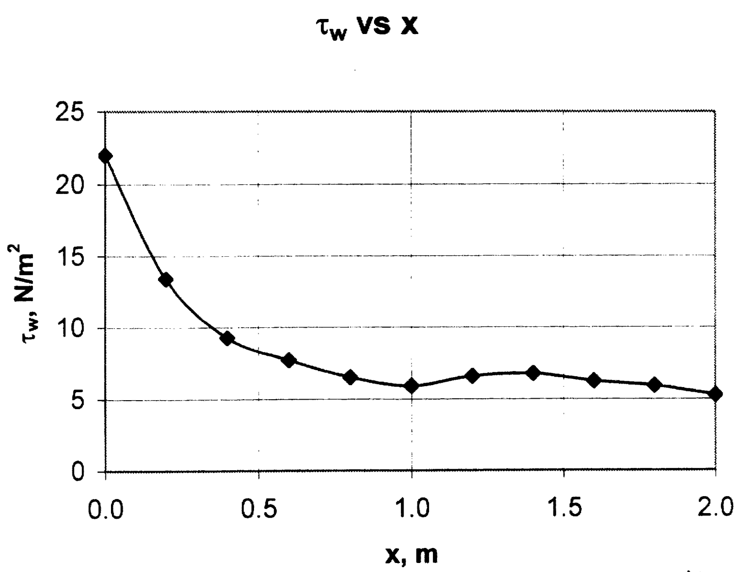
which can be integrated to give (using $\Theta = 0$ at $x = 0$)

$$\Theta = \frac{1}{\rho U^2}\int_0^x \tau_w \, dx = \frac{1}{(0.86)(1000 \frac{kg}{m^3})(5 \frac{m}{s})^2}\int_0^x \tau_w \, dx$$

or

$$\Theta = 4.65 \times 10^{-5} \int_0^x \tau_w \, dx \text{ , where } \Theta \sim m, \; x \sim m, \text{ and } \tau_w \sim \frac{N}{m^2} \qquad (1)$$

For $0 \le x \le 2.0$ m, integrate Eq. (1) to determine Θ as a function of x. To do so, we need the value of τ_w at $x = 0$, which is not given in the table. Theoretically, $\tau_w = \infty$ at the leading. For our purposes, based on the extrapolated curve below, assume $\tau_w = 22 \frac{N}{m^2}$ at $x = 0$



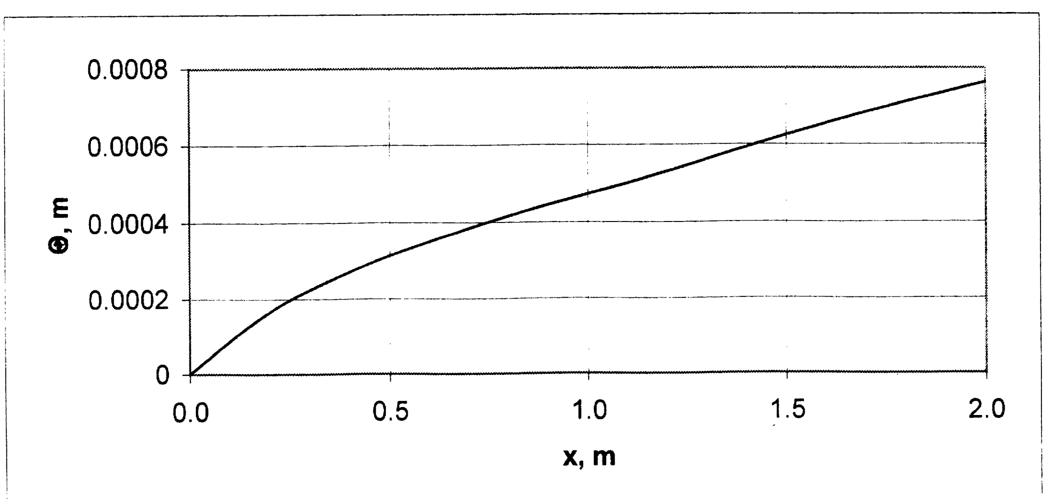
Program P9#27 shown below was used for the calculations.

(con't)

9.27* (con't)

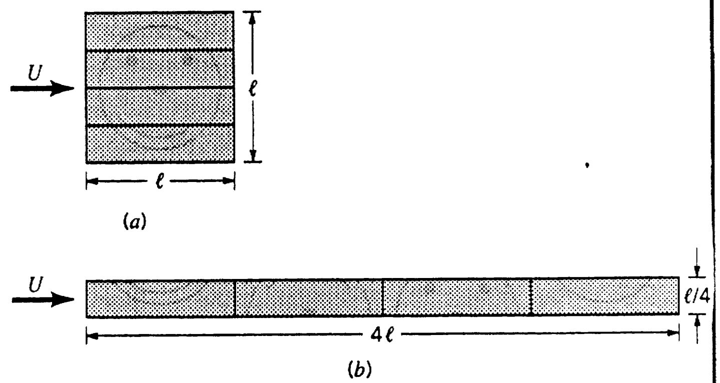
```
100 cls
110 open "prn" for output as #1
120 print#1, "**************************************************"
130 print#1, "**  This program calculates the momentum        **"
140 print#1, "**  boundary layer thickness as a function of    **"
150 print#1, "**  x from the given wall shear stress distri-   **"
160 print#1, "**  bution.                                      **"
170 print#1, "**************************************************"
200 dim tau(11)
210 tau(1)=22.0 : tau(2)=13.4 : tau(3)=9.25 : tau(4)=7.68
220 tau(5)=6.51 : tau(6)=5.89 : tau(7)=6.57 : tau(8)=6.75
230 tau(9)=6.23 : tau(10)=5.92 : tau(11)=5.26
240 print#1, " "
250 print#1, "    x, m   momentum thickness, m"
260 for i = 1 to 11
270 x = 0.2*(i-1)
280 if i = 1 then goto 400
290 intgrl = 0
300 for j = 1 to i-1
310 intgrl = intgrl + 0.5*0.2*(tau(j+1) + tau(j))
320 next j
330 theta = 4.65E-5*intgrl
400 print#1, using "  ##.###         #.###^^^^";x,theta
410 next i
```

```
**************************************************
**  This program calculates the momentum        **
**  boundary layer thickness as a function of    **
**  x from the given wall shear stress distri-   **
**  bution.                                      **
**************************************************

  x, m   momentum thickness, m
 0.000        0.000E+00
 0.200        1.646E-04
 0.400        2.699E-04
 0.600        3.487E-04
 0.800        4.146E-04
 1.000        4.723E-04
 1.200        5.302E-04
 1.400        5.922E-04
 1.600        6.525E-04
 1.800        7.090E-04
 2.000        7.610E-04
```



9.28

9.28 The square flat plate shown in Fig. P9.28a is cut into four equal-sized prices and arranged as shown in Fig. P9.28b. Determine the ratio of the drag on the original plate [case **(a)**] to the drag on the plates in the configuration shown in **(b)**. Assume laminar boundary flow. Explain your answer physically.



■ FIGURE P9.28

For case (a):

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \text{ and } A = \ell^2$$

Thus,

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\,\ell^2 = 0.664\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b):

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \quad \text{and } A = (4\ell)\left(\frac{\ell}{4}\right) = \ell^2$$

Thus,

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}\,\ell^2 = \frac{1}{2}\left(0.664\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2}\right) \qquad (2)$$

By comparing Eqs. (1) and (2) we see that

$$\mathscr{D}_{f_a} = \underline{\underline{2.0\,\mathscr{D}_{f_b}}}$$

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.

9.29

9.29 A plate is oriented parallel to the free stream as is indicated in Fig. 9.29. If the boundary layer flow is laminar, determine the ratio of the drag for case (*a*) to that for case (*b*). Explain your answer physically.

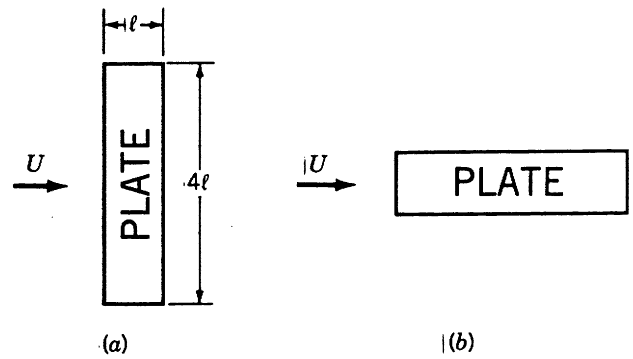


FIGURE P9.29

For case (a):

$$\mathscr{D}_{fa} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where} \quad C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \quad \text{and} \quad A = 4\ell^2$$

Thus,

$$\mathscr{D}_{fa} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}(4\ell^2) = 2.56\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b)

$$\mathscr{D}_{fb} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where} \quad C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \quad \text{and} \quad A = 4\ell^2$$

Thus,

$$\mathscr{D}_{fb} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}(4\ell^2) = \frac{1}{2}\left(2.56\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2}\right) \qquad (2)$$

From Eqs. (1) and (2) we see that

$$\frac{\mathscr{D}_{fa}}{\mathscr{D}_{fb}} = \underline{\underline{2}}$$

The shear stress decreases with distance from the leading edge of the plate (i.e., the thickening of the boundary layer). Thus, even though the plate area is the same for case (a) or (b), the average shear stress (and the drag) is greater for case (a).

9.30

9.30 If the drag on one side of a flat plate parallel to the upstream flow is $\mathcal{D}$ when the upstream velocity is U, what will the drag be when the upstream velocity is $2U$; or $U/2$? Assume laminar flow.

For laminar flow $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$, where $C_{Df} = \dfrac{1.328}{\sqrt{\frac{U\ell}{\nu}}}$

Thus,

$$\mathcal{D} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}} A = 0.664\,\rho A \frac{\sqrt{\nu}}{\sqrt{\ell}} U^{3/2} \sim U^{3/2}$$

Hence,

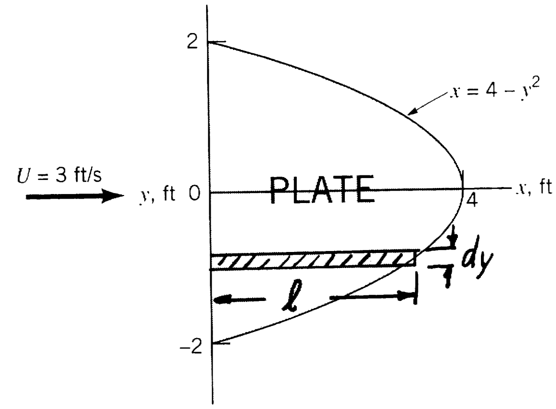
$$\frac{\mathcal{D}_U}{\mathcal{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{2U} = 2.83\,\mathcal{D}_U}}$$

and

$$\frac{\mathcal{D}_U}{\mathcal{D}_{U/2}} \quad \frac{U^{3/2}}{\left(\frac{U}{2}\right)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{U/2} = 0.354\,\mathcal{D}_U}}$$

9.31

9.31 Air flows past a parabolic-shaped flat plate oriented parallel to the free stream shown in Fig. P9.31. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar flow.



Treat each strip of thickness dy and length $\ell = \ell(y)$ as a small flat plate with drag $d\mathcal{D}$ where for laminar flow

$$d\mathcal{D} = C_{Df}\,\tfrac{1}{2}\rho U^2\,dA \quad \text{with } dA = \ell\,dy \text{ and } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{\rho U \ell}{\mu}}}$$

Thus,

$$d\mathcal{D} = \frac{1.328}{\sqrt{\frac{\rho U \ell}{\mu}}}\,\tfrac{1}{2}\rho U^2 \ell\,dy = 0.664\sqrt{\mu\rho}\,U^{3/2}\sqrt{\ell}\,dy$$

But $\ell = 4 - y^2$ so that

$$\mathcal{D} = \int d\mathcal{D} = \int_{y=-2}^{+2} 0.664\sqrt{\mu\rho}\,U^{3/2}\sqrt{4-y^2}\,dy$$

$$= 2(0.664)\sqrt{\mu\rho}\,U^{3/2}\int_0^2 \sqrt{4-y^2}\,dy$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}\left[y\sqrt{4-y^2} + 4\sin^{-1}\left(\frac{y}{2}\right)\right]_0^2 \left(\frac{1}{2}\right)$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}\left[4\sin^{-1}(1)\right]\left(\frac{1}{2}\right)$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}(2\pi)\left(\frac{1}{2}\right)$$

Note: The units on the integral are $ft^{3/2}$ (i.e. $2\pi \doteq L^{3/2}$)

Thus,

$$\mathcal{D} = 1.328\left[3.74\times10^{-7}\,\frac{lb\cdot s}{ft^2}\left(2.38\times10^{-3}\,\frac{slugs}{ft^3}\right)\right]^{1/2}\left(3\,\frac{ft}{s}\right)^{3/2}(\pi)\,ft^{3/2}$$

$$= \underline{\underline{6.47\times10^{-4}\,lb}}$$

9.32

9.32 It is often assumed that "sharp objects can cut through the air better than blunt ones." Based on this assumption, the drag on the object shown in Fig. P9.32 should be less when the wind blows from right to left than when it blows from left to right. Experiments show that the opposite is true. Explain.

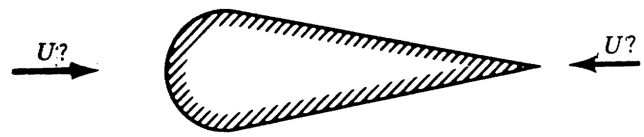
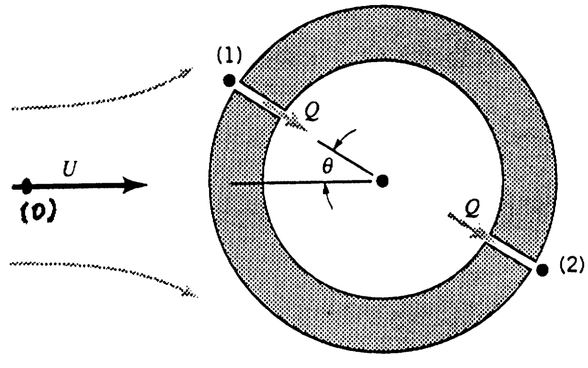


FIGURE P9.32

A significant portion of the drag on an object can be from the relatively low pressure developed in the wake region behind the object. By making the object streamlined (i.e., flow from left to right, not right to left in the above figure) boundary layer separation is avoided and a relatively thin wake with low drag is obtained. Whether the front of the object is "sharp" or "blunt" does not affect the contribution to the drag from the front part of the body—at least not as much a the width of the wake affects the drag.

9.33

9.33 Two small holes are drilled opposite each other in a circular cylinder as shown in Fig. P9.33. Thus, when air flows past the cylinder, air will circulate through the interior of the cylinder at a rate of $Q = K(p_1 - p_2)$, where the constant K depends on the geometry of the passage connecting the two holes. It is assumed that the flow around the cylinder is not affected by either the presence of the two holes or the small flowrate through the passage. Let Q_0 denote the flowrate when $\theta = 0$. Plot a graph of Q/Q_0 as a function of θ for $0 \le \theta \le \pi/2$ if (a) the flow is inviscid, and (b) if the boundary layer on the cylinder is turbulent (see Fig. 9.17c for pressure data).



■ FIGURE P9.33

(a) For inviscid flow:

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2} = 1 - 4\sin^2\theta$$

Thus, $Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)]$

$$= K\left[\tfrac{1}{2}\rho U^2(1 - 4\sin^2\theta) - \tfrac{1}{2}\rho U^2(1 - 4\sin^2(\theta + \pi))\right]$$

but $\sin^2\theta = \sin^2(\theta + \pi)$

Hence, $\underline{\underline{Q = 0 \text{ for inviscid flow}}}$. Note: This is to be expected because of the symmetrical pressure distribution.

(b) For a turbulent boundary layer:

$$Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)] = \tfrac{1}{2}\rho U^2 K[C_{p_1} - C_{p_2}]$$

where C_{p_1} is for θ and C_{p_2} is for $180 - \theta$ deg.

Obtain C_p data from Fig. 9.17.

Note: $C_p = 1$ for $\theta = 0$ and $C_p \approx -0.4$ for $\theta = 180$ deg

Thus, $Q_0 = Q\big|_{\theta=0} = \tfrac{1}{2}\rho U^2 K[1 - (-0.4)] = 1.4(\tfrac{1}{2}\rho U^2 K)$

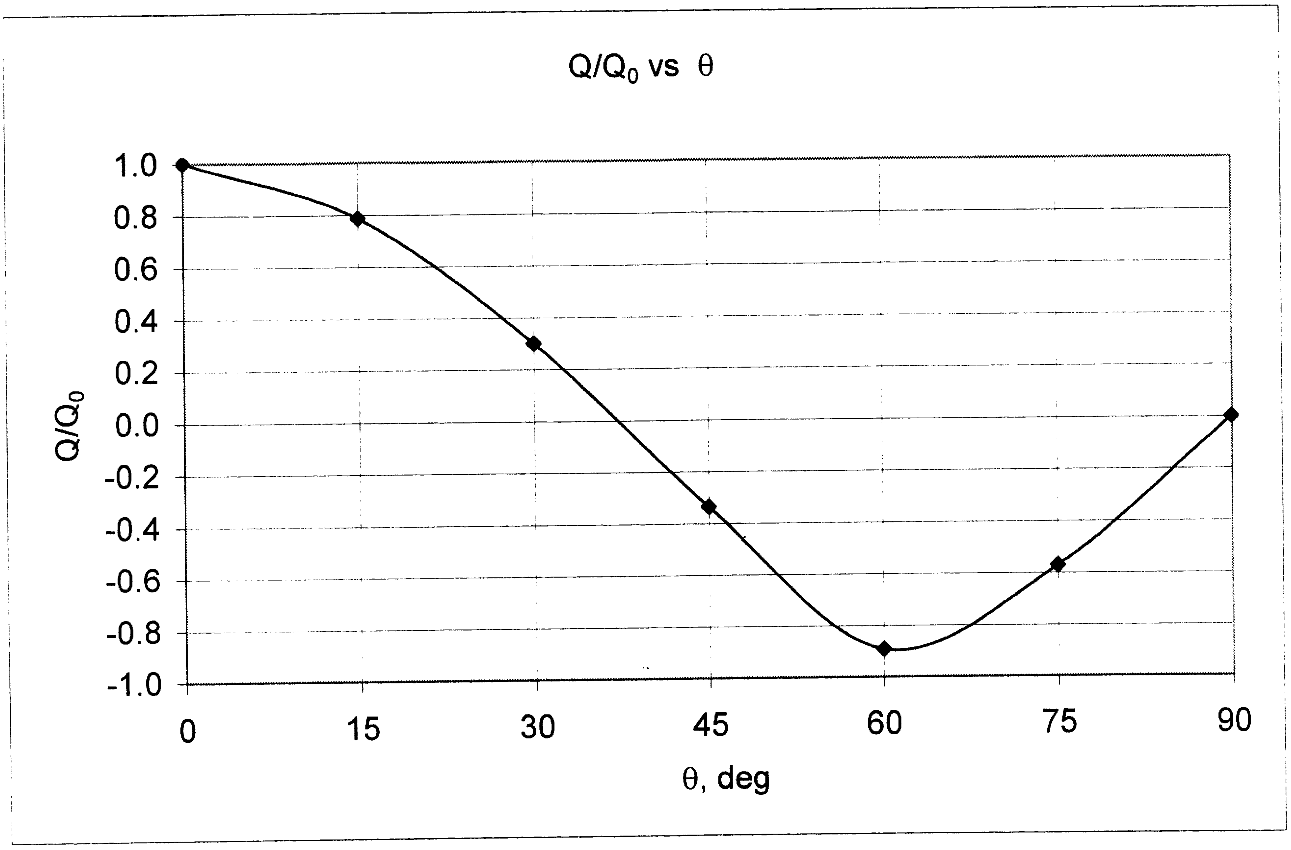
so that

$$\frac{Q}{Q_0} = \frac{\frac{1}{2}\rho U^2 K[C_{p_1} - C_{p_2}]}{1.4(\frac{1}{2}\rho U^2 K)} = \frac{C_{p_1} - C_{p_2}}{1.4}$$

The results are tabulated and plotted below.

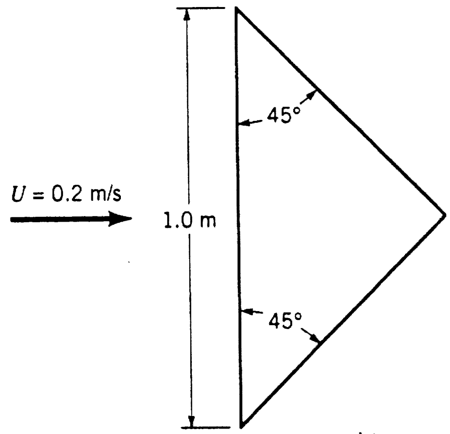
(con't)

θ, deg	180 - θ, deg	C_{p1}	C_{p2}	Q/Q_0
0	180	1.00	-0.40	1.00
15	165	0.70	-0.40	0.79
30	150	0.00	-0.42	0.30
45	135	-0.90	-0.43	-0.34
60	120	-1.70	-0.45	-0.89
75	105	-2.10	-1.30	-0.57
90	90	-1.90	-1.90	0.00

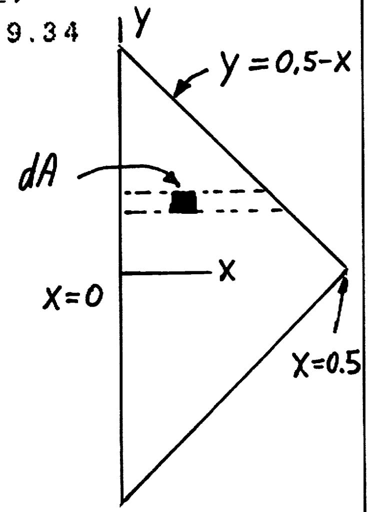


9.34

9.34 Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.34. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.



■ FIGURE P9.34



$\mathscr{D} = \int \tau_w \, dA$ where $\tau_w = 0.332\, U^{3/2} \sqrt{\dfrac{\rho\mu}{x}}$

Thus,

$$\mathscr{D} = 0.332\, U^{3/2}\sqrt{\rho\mu}\int \frac{1}{\sqrt{x}}\, dA$$

$$= 0.332\, U^{3/2}\sqrt{\rho\mu}\,(2)\int_{x=0}^{x=0.5}\int_{y=0}^{y=0.5-x} \frac{dy\,dx}{\sqrt{x}}$$

$$= 0.332\, U^{3/2}\sqrt{\rho\mu}\,(2)\int_{x=0}^{0.5} \frac{0.5-x}{\sqrt{x}}\, dx$$

$$= 0.332\, U^{3/2}\sqrt{\rho\mu}\,(2)\left[0.5(2)x^{\frac{1}{2}} - \frac{2}{3}x^{3/2}\right]_0^{0.5}$$

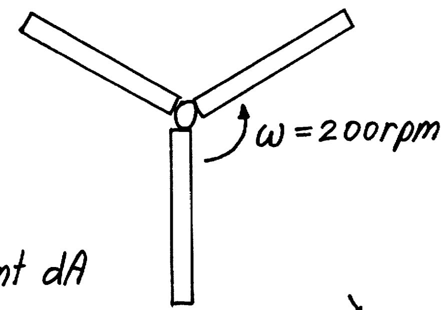
$$= 0.664\left(0.2\,\frac{m}{s}\right)^{3/2}\sqrt{999\,\frac{kg}{m^3}\left(1.12\times10^{-3}\,\frac{N\cdot s}{m^2}\right)}\left[\sqrt{0.5} - \frac{2}{3}(0.5)^{3/2}\right]$$

or

$$\mathscr{D} = \underline{\underline{0.0296\ N}}$$

9.35

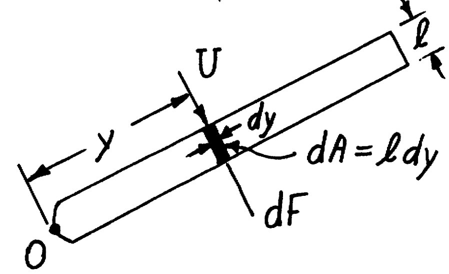
9.35 A three-bladed helicopter blade rotates at 200 rpm. If each blade is 12 ft long and 1.5 ft wide, estimate the torque needed to overcome the friction on the blades if they act as flat plates.



Let dM = torque from the drag on area element dA

or

$$dM = (\mathscr{D}_{top} + \mathscr{D}_{bottom})\,y = 2\left(\tfrac{1}{2}\rho U^2 C_{Df}\, dA\right) y$$



where

$U = \omega y$ and for laminar flow*

$$C_{Df} = \frac{1.328}{Re_\ell^{1/2}} \quad \text{with } Re_\ell = \frac{U\ell}{\nu}$$

Thus,

$$dM = \rho U^2 \frac{1.328}{\left(\frac{U\ell}{\nu}\right)^{\frac{1}{2}}}\, y\,\ell\, dy = 1.328\, \rho\, U^{3/2} \ell^{\frac{1}{2}} \nu^{\frac{1}{2}}\, y\, dy$$

or with

$U = \omega y$

$$dM = 1.328\ \rho\, \omega^{3/2} \ell^{\frac{1}{2}} \nu^{1/2}\, y^{5/2}\, dy$$

$$= (1.328)(2.38\times10^{-3}\ \tfrac{slug}{ft^3})\left[(200\ \tfrac{rev}{min})(\tfrac{1\ min}{60s})(\tfrac{2\pi\ rad}{1\ rev})\right]^{\frac{3}{2}}(1.5ft)^{\frac{1}{2}}(1.57\times10^{-4}\ \tfrac{ft^2}{s})^{\frac{1}{2}}\, y^{\frac{5}{2}} dy$$

or

$dM = 0.0125\ y^{5/2}\ dy$ ft·lb , where $y \sim ft$

Thus, the net torque for the three blades

$$M = 3\int dM = 3(0.0125)\int_{y=0}^{12} y^{\frac{5}{2}} dy = 3(0.0125)\left(\tfrac{2}{7}\right)(12)^{7/2}$$

or

$M =$ 64.1 ft·lb

Note: The torque could be greater if the boundary layer is turbulent.

---

* At the tip $y = 12\ ft$ so that $U = \omega y = \left(\frac{200\ \frac{rev}{min}}{60\ \frac{s}{min}}\right)(2\pi\ \tfrac{rad}{rev})(12\ ft) = 251\ \tfrac{ft}{s}$

and $Re_{\ell\, tip} = \dfrac{(251\ \frac{ft}{s})(1.5\ ft)}{1.57\times10^{-4}\ \frac{ft^2}{s}} = 2.40\times10^6$, which is greater than the critical value. Thus, the boundary is turbulent at the tip and laminar at the hub. For simplicity assume a laminar boundary layer throughout.

9.36

9.36 A ceiling fan consists of five blades of 0.80-m length and 0.10-m width which rotate at 100 rpm. Estimate the torque needed to overcome the friction on the blades if they act as flat plates.

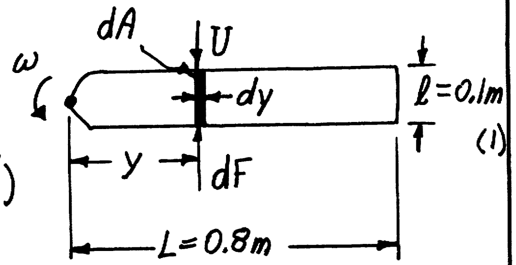
Let $d\mathcal{M}$ = torque from the drag on element dA of the blade

or

$d\mathcal{M} = (\mathcal{D}_{top} + \mathcal{D}_{bottom})y = 2(\frac{1}{2}\rho U^2 C_{Df}\, dA)y$ (1)

where $U = \omega y$ and $\omega = 100\frac{rev}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$

or $\omega = 10.47\,\frac{rad}{s}$



The maximum Re_ℓ will occur at point (1) where $y = L$ or

$$Re_{\ell_1} = \frac{U\ell}{\nu} = \frac{\omega L \ell}{\nu} = \frac{(10.47\,\frac{rad}{s})(0.8\,m)(0.1\,m)}{1.46\times10^{-5}\,\frac{m^2}{s}} = 5.74\times10^4$$

Thus, at all points on the blade $Re_x < Re_{x\,cr} = 5\times10^5$ and the flow is laminar.

$$C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}$$

so that from Eq. (1)

$$d\mathcal{M} = \rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}(\ell\, dy)y = 1.328\,\rho U^{3/2}\sqrt{\nu\ell}\; y\,dy \text{ , but with } U = \omega y$$

$$d\mathcal{M} = 1.328\,\rho\,\omega^{3/2}\sqrt{\nu\ell}\; y^{5/2}\,dy$$
$$= 1.328\left(1.23\,\frac{kg}{m^3}\right)\left(10.47\,\frac{rad}{s}\right)^{3/2}\left[\left(1.46\times10^{-5}\,\frac{m^2}{s}\right)(0.1\,m)\right]^{1/2} y^{5/2}\,dy$$

or

$$d\mathcal{M} = 0.0669\, y^{5/2}\,dy \quad N\cdot m \text{ , where } y \sim m$$

Thus, the net torque on the four blades is

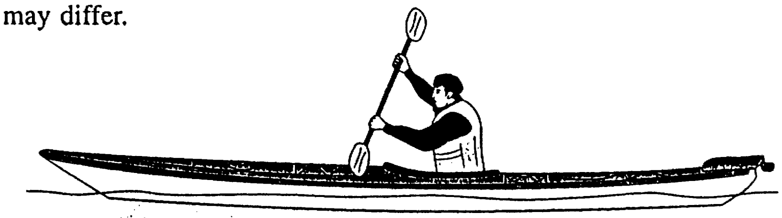
$$\mathcal{M} = 5\int d\mathcal{M} = 5\int_{y=0}^{0.8m} 0.0669\, y^{5/2}\,dy = 5(0.0669)\left(\tfrac{2}{7}\right) y^{7/2}\Big|_0^{0.8}$$

or

$\mathcal{M} = \underline{\underline{0.0438\ N\cdot m}}$

9.37

9.37 As shown in Video V9.2 and Fig. P9.37*a*, a kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in Fig. P9.37*b*. Comment on reasons why the two sets of values may differ.



For a flat plate $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$ where (1)
$A = 17\,ft(2\,ft) = 34\,ft^2$ and C_{Df} is a function of $Re_\ell = \frac{U\ell}{\nu}$
Thus,

$$Re_\ell = \frac{17\,ft\ U}{1.21\times10^{-5}\,ft^2/s} = 1.40\times10^6\,U \quad (2)$$

Consider $1 \le U \le 8\,\frac{ft}{s}$, or $1.40\times10^6 \le Re_\ell \le 1.12\times10^7$
From Fig. 9.15 we see that in this Re_ℓ range the boundary layer flow is in the transitional range. Thus, from Table 9.3

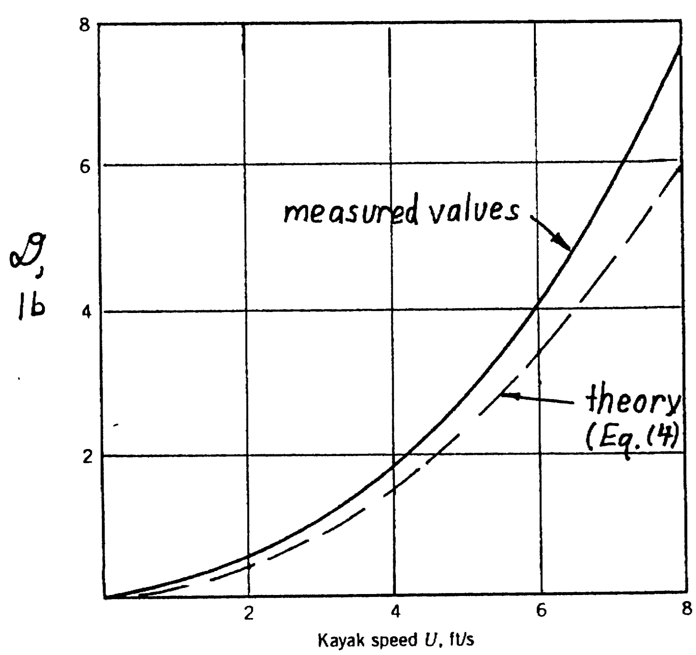
$$C_{Df} = 0.455/(\log Re_\ell)^{2.58} - 1700/Re_\ell \quad (3)$$

By combining Eqs. (1), (2), and (3):

$$\mathcal{D} = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)U^2 C_{Df}\,(34\,ft^2) \quad \text{or}$$

$$\mathcal{D} = 33.0\,U^2\left[0.455/(\log(1.40\times10^6 U))^{2.58} - 1700/(1.40\times10^6\,U)\right] \quad (4)$$

The results from this equation are plotted below.

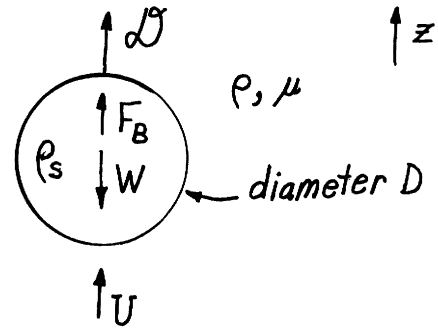


U, ft/s	$\mathcal{D}$, lb
1	0.0986
2	0.410
3	0.909
4	1.58
5	2.42
6	3.43
7	4.59
8	5.90

■ FIGURE P9.37

9.38

9.38 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, $\mathrm{Re} = \rho D U/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathcal{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathcal{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since $\mathrm{Re} < 1$

$\mathcal{D} = 3\pi D U \mu$

Thus,

$$3\pi D U \mu + \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$$

which can be rearranged to give

$$\underline{\underline{\mu = \frac{g D^2(\rho_s - \rho)}{18\,U}}}$$

9.39

9.39 Determine the drag on a small circular disk of 0.01 ft diameter moving 0.01 ft/s through oil with a specific gravity of 0.87 and a viscosity 10,000 times that of water. The disk is oriented normal to the upstream velocity. By what percent is the drag reduced if the disk is oriented parallel to the flow?

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$, where $\rho = (0.87)(1.94 \frac{slug}{ft^3}) = 1.688 \frac{slugs}{ft^3}$ (1)

and $\mu = 10^4 \mu_{H_2O} = 10^4(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}) = 0.234 \frac{lb \cdot s}{ft^2}$

Thus, $Re = \frac{UD}{\nu} = \frac{\rho U D}{\mu} = \frac{(1.688 \frac{slugs}{ft^3})(0.01 \frac{ft}{s})(0.01 ft)}{0.234 \frac{lb \cdot s}{ft^2}} = 7.21 \times 10^{-4} << 1$

so that the low Re data of Table 9.4 is valid.

For the disk normal to the flow, $C_D = \frac{20.4}{Re} = \frac{20.4}{7.21 \times 10^{-4}} = 28,300$

so that from Eq.(1)

$\mathscr{D} = 28,300(\frac{1}{2})(1.688 \frac{slugs}{ft^3})(0.01 \frac{ft}{s})^2 \frac{\pi}{4}(0.01 ft)^2 = \underline{\underline{1.88 \times 10^{-4} lb}}$

If the disk is parallel to the flow, $C_D = \frac{13.6}{Re}$ so that

$$\frac{\mathscr{D}_{parallel}}{\mathscr{D}_{normal}} = \frac{C_{D\,parallel}}{C_{D\,normal}} = \frac{\left(\frac{13.6}{Re}\right)}{\left(\frac{20.4}{Re}\right)} = 0.667 \text{, a } \underline{\underline{33.3\% \text{ reduction}}}$$

9.40

9.40 For small Reynolds number flows the drag coefficient of an object is given by a constant divided by the Reynolds number (see Table 9.4). Thus, as the Reynolds number tends to zero, the drag coefficient becomes infinitely large. Does this mean that for small velocities (hence, small Reynolds numbers) the drag is very large? Explain.

For a given object $C_D = \frac{C}{Re}$ (where the value of C depends on the shape of the object), provided $Re \lesssim 1$. Thus, as $Re \rightarrow 0$, $C_D \rightarrow \infty$.

However,

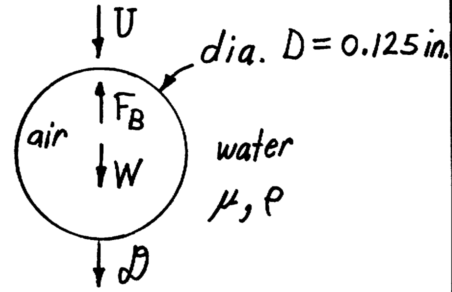
$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A = \frac{C}{\left(\frac{\rho U \ell}{\mu}\right)} \frac{1}{2}\rho U^2 A \sim U$

That is, as $U \rightarrow 0$ (i.e. $Re \rightarrow 0$), then $\mathscr{D} \sim U$

Thus, does $C_D \rightarrow \infty$ mean that $\mathscr{D} \rightarrow 0$? $\underline{\underline{\text{No.}}}$

9.41

9.41 Compare the rise velocity of an $\frac{1}{8}$-in.-diameter air bubble in water to the fall velocity of an $\frac{1}{8}$-in.-diameter water drop in air. Assume each to behave as a solid sphere.



(a) air bubble in water: For steady rise $\sum F_z = 0$
or

$$F_B = W + \mathcal{D}, \text{ where } \mathcal{D} = \text{drag} = C_D \tfrac{1}{2}\rho U^2 \tfrac{\pi}{4} D^2$$

$$W = \text{weight} = \gamma_{air} \forall = \gamma_{air} \tfrac{4\pi}{3}\left(\tfrac{D}{2}\right)^3$$

and

$$F_B = \text{buoyant force} = \gamma_{H_2O} \forall = \gamma_{H_2O} \tfrac{4\pi}{3}\left(\tfrac{D}{2}\right)^3$$

However, since $\gamma_{air} \ll \gamma_{H_2O}$ it follows that $W \ll F_B$
or

$$F_B = \mathcal{D}$$

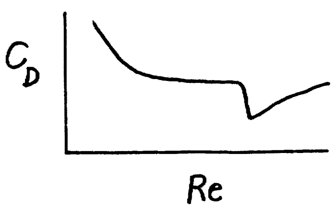
Hence, $g\rho \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$ or $U = \sqrt{\frac{4Dg}{3C_D}} = \sqrt{\frac{4\left(\frac{0.125}{12}\right)\text{ft}\left(32.2\frac{\text{ft}}{\text{s}^2}\right)}{3C_D}}$

or $U = \dfrac{0.669}{\sqrt{C_D}} \dfrac{\text{ft}}{\text{s}}$, where $C_D = C_D(Re)$ and $Re = \dfrac{UD}{\nu}$ (1)

or

$$Re = \frac{\left(\frac{0.125}{12}\text{ft}\right)U}{1.21 \times 10^{-5}\frac{\text{ft}^2}{\text{s}}} = 861\,U \quad (2)$$

From Fig. 9.21:



Trial and error solution for U: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph. (3)

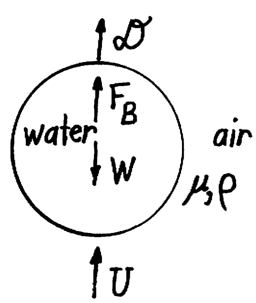
Assume $C_D = 1 \rightarrow U = 0.669\frac{\text{ft}}{\text{s}} \rightarrow Re = 576 \rightarrow C_D = 0.5 \neq 1$

Assume $C_D = 0.5 \rightarrow U = 0.946\frac{\text{ft}}{\text{s}} \rightarrow Re = 815 \rightarrow C_D = 0.5$ (checks)

Thus, $\underline{\underline{U = 0.946\,\frac{\text{ft}}{\text{s}}}}$

(b) water drop in air: Since $\gamma_{air} \ll \gamma_{H_2O}$, $F_B \ll W$

Thus, $W = \mathcal{D}$, or $\gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$



or $U = \sqrt{\dfrac{4D\gamma_{H_2O}}{3\rho C_D}} = \left[\dfrac{4\left(\frac{0.125}{12}\text{ft}\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)}{3\left(2.38 \times 10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)C_D}\right]^{\frac{1}{2}} = \dfrac{19.1}{\sqrt{C_D}}\dfrac{\text{ft}}{\text{s}}$ (4)

Also,

$$Re = \frac{UD}{\nu} = \frac{\left(\frac{0.125}{12}\text{ft}\right)U}{1.57 \times 10^{-4}\frac{\text{ft}^2}{\text{s}}} = 66.3\,U \quad (5)$$

Trial and error solution of Eqs. (4), (5), and graph (3):

Assume $C_D = 0.5 \rightarrow U = 27.0\frac{\text{ft}}{\text{s}} \rightarrow Re = 1790 \rightarrow C_D = 0.4 \neq 0.5$

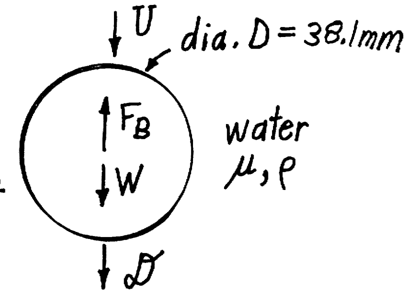
Assume $C_D = 0.4 \rightarrow U = 30.2\frac{\text{ft}}{\text{s}} \rightarrow Re = 2000 \rightarrow C_D = 0.4$ (checks)

Thus, $U = \underline{\underline{30.2\,\frac{\text{ft}}{\text{s}}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.

9.42

9.42 A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.



For steady rise $\Sigma F_z = 0$

or

$F_B = W + \mathcal{D}$, where $\mathcal{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$W = \text{weight} = 0.0245\ N$

$F_B = \text{buoyant force} = \gamma \forall = \gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

Thus,

$$\gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = W + C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$$

or

$$\left(9.80\times 10^3 \frac{N}{m^3}\right)\frac{4\pi}{3}\left(\frac{0.0381}{2}\right)^3 m = 0.0245\ N + \frac{1}{2} C_D \left(999 \frac{kg}{m^3}\right) U^2 \frac{\pi}{4}(0.0381\ m)^3$$

or

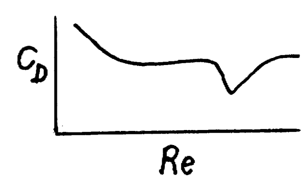
$$C_D U^2 = 0.455, \text{ where } U \sim \frac{m}{s} \qquad (1)$$

Also, $Re = \frac{UD}{\nu}$

or

$$Re = \frac{U\,(0.0381\,m)}{1.12\times 10^{-6}\,\frac{m^2}{s}} = 3.40\times 10^4\, U, \text{ where } U \sim \frac{m}{s} \qquad (2)$$

Finally, from Fig. 9.21: (3)



Trial and error solution: Assume C_D; obtain U from Eq. (1), Re from Eq. (2); check C_D from Eq. (3), the graph.

Assume $C_D = 0.5 \longrightarrow U = 0.954 \frac{m}{s} \longrightarrow Re = 3.24\times 10^4 \longrightarrow C_D = 0.4 \neq 0.5$

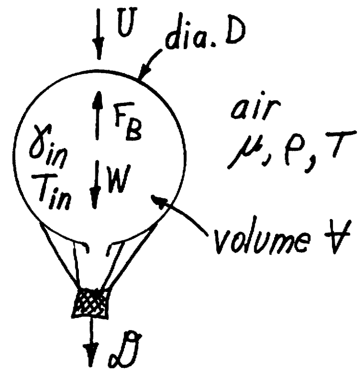
Assume $C_D = 0.4 \longrightarrow U = 1.06 \frac{m}{s} \longrightarrow Re = 3.62\times 10^4 \longrightarrow C_D = 0.4$ (checks)

Thus, $U = \underline{\underline{1.06 \frac{m}{s}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.

9.44

9.44 A hot air balloon roughly spherical in shape has a volume of 70,000 ft³ and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). If the outside air temperature is 80 °F and the temperature within the balloon is 165 °F, estimate the rate at which it will rise under steady state conditions if the atmospheric pressure is 14.7 psi.



For steady rise $\sum F_z = 0$, or $F_B = W + \mathscr{D}$
where

$\mathscr{D} = \text{drag} = C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2$

F_B = buoyant force $= \gamma \forall$

and

W = total weight $= 500 \text{ lb} + \gamma_{in} \forall$

Now $\rho = \dfrac{p}{RT} = \dfrac{(14.7 \frac{lb}{in.^2})(12 \frac{in.}{ft})^2}{(1715 \frac{ft \cdot lb}{slug \cdot °R})(460+80)°R} = 0.00229 \frac{slugs}{ft^3}$

$\gamma = \rho g = (0.00229 \frac{slugs}{ft^3})(32.2 \frac{ft}{s^2}) = 0.0736 \frac{lb}{ft^3}$

and

$\gamma_{in} = \dfrac{p g}{R\, T_{in}} = \dfrac{(14.7 \frac{lb}{in.^2})(12 \frac{in.}{ft})^2 (32.2 \frac{ft}{s^2})}{(1715 \frac{ft \cdot lb}{slug \cdot °R})(460+165)°R} = 0.0636 \frac{lb}{ft^3}$

Note: Since the balloon is open at the bottom, the pressure within the balloon is nearly the same as it is outside.

Thus, with $\forall = 7 \times 10^4 \text{ ft}^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$, or $D = 51.1$ ft we obtain

$\mathscr{D} = C_D \frac{1}{2}(0.00229) U^2 \frac{\pi}{4}(51.1)^2$

$= 2.36\, C_D U^2 \text{ lb}$, where $U \sim \frac{ft}{s}$

Also,

$W = 500 \text{ lb} + (0.0636 \frac{lb}{ft^3})(70{,}000 \text{ ft}^3) = 4952 \text{ lb}$

and

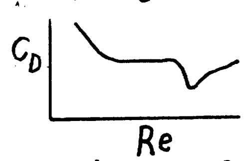
$F_B = (0.0736 \frac{lb}{ft^3})(70{,}000 \text{ ft}^3) = 5152 \text{ lb}$ Thus, $F_B = W + \mathscr{D}$ gives

$5152 \text{ lb} = 4952 \text{ lb} + 2.36\, C_D U^2$ or $C_D U^2 = 84.7$ (1)

Also, $Re = \frac{UD}{\nu}$

or $Re = \dfrac{51.1 \text{ ft}\, U}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 3.25 \times 10^5\, U$ (2)

and from Fig. 9.21 (3)



Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

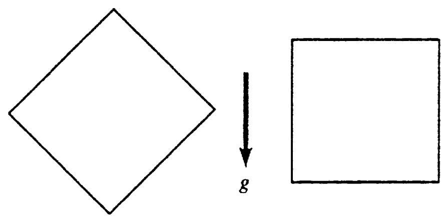
Assume $C_D = 0.5 \rightarrow U = 13.0 \frac{ft}{s} \rightarrow Re = 4.23 \times 10^6 \rightarrow C_D = 0.24 \neq 0.5$

Assume $C_D = 0.24 \rightarrow U = 18.8 \frac{ft}{s} \rightarrow Re = 6.11 \times 10^6 \rightarrow C_D = 0.30 \neq 0.24$

Assume $C_D = 0.30 \rightarrow \underline{\underline{U = 16.8 \frac{ft}{s}}} \rightarrow Re = 5.46 \times 10^6 \rightarrow C_D = 0.30$ (checks)

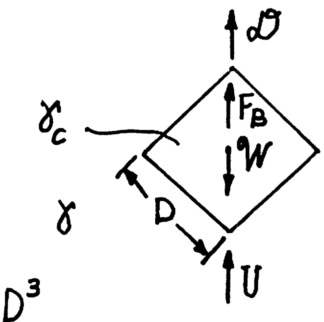
9.45

9.45 A 500-N cube of specific gravity $SG = 1.8$ falls through water at a constant speed U. Determine U if the cube falls (a) as oriented in Fig. P9.45a, (b) as oriented in Fig. P9.45b.



(a) (b)

■ FIGURE P9.45



For steady fall, $\Sigma F = ma = 0$

or

$$\mathcal{W} = \mathcal{D} + F_B \; , \quad \text{where } \mathcal{W} = \text{weight} = 500\,N \qquad (1)$$

$$F_B = \text{buoyant force} = \gamma D^3$$

$$\text{and } \mathcal{D} = \tfrac{1}{2}\rho U^2 C_D A = \text{drag}$$

But,

$$\mathcal{W} = \gamma_c D^3 = SG\,\gamma D^3, \text{ or } \quad 500\,N = 1.8\left(9.80\times10^3 \tfrac{N}{m^3}\right) D^3$$

Thus, $D = 0.305\,m$ so that from Eq. (1)

$$500\,N = \tfrac{1}{2}\left(999 \tfrac{kg}{m^3}\right) U^2 C_D \,(0.305\,m)^2 + \left(9.80\times10^3 \tfrac{N}{m^3}\right)(0.305\,m)^3$$

or

$$U^2 C_D = 4.78 \quad \text{where } U \sim \tfrac{m}{s}$$

(a) For case (a) $C_D = 0.80$ (see Fig. 9.29)

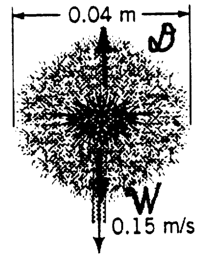
Hence, $U = \left(\dfrac{4.78}{0.80}\right)^{1/2} = \underline{\underline{2.44 \tfrac{m}{s}}}$

(b) For case (b) $C_D = 1.05$

Hence, $U = \left(\dfrac{4.78}{1.05}\right)^{1/2} = \underline{\underline{2.13 \tfrac{m}{s}}}$

9.46

9.46 The 5×10^{-6} kg dandelion seed shown in Fig. P9.46 settles through the air with a constant speed of 0.15 m/s. Determine the drag coefficient for this object.



■ FIGURE P9.46

For steady falling at a constant speed,

$$\mathcal{D} = \mathcal{W} \quad \text{or} \quad mg = C_D \tfrac{1}{2} \rho U^2 A$$

Thus,

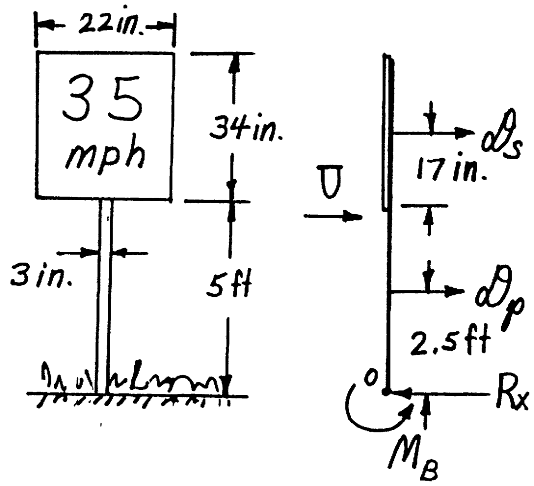
$$5 \times 10^{-6}\,\text{kg}\,(9.81\,\tfrac{\text{m}}{\text{s}^2}) = C_D\,(\tfrac{1}{2})(1.23\,\tfrac{\text{kg}}{\text{m}^3})(0.15\,\tfrac{\text{m}}{\text{s}})^2\,\tfrac{\pi}{4}(0.04\,\text{m})^2$$

or

$$C_D = \underline{\underline{2.82}}$$

9.47

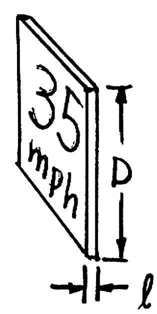
9.47 A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.6.) List any assumptions used in your calculations.



For equalibrium, $\Sigma M_0 = 0$ or

$$M_B = 2.5\,ft\,\mathscr{D}_p + (5 + \tfrac{17}{12})ft\,\mathscr{D}_s \text{, where} \tag{1}$$

$\mathscr{D}_p$ = drag on the pole and $\mathscr{D}_s$ = drag on the sign



From Fig. 9.28 with $\ell/D < 0.1$ for the sign,

$C_{Ds} = 1.9$

From Fig. 9.19 if the post acts as a square rod with sharp corners $C_{Dp} = 2.2$. Thus, with $U = 30\,mph = 44\,\frac{ft}{s}$,

$$\mathscr{D}_s = \tfrac{1}{2}\rho U^2 C_{Ds} A_s = \tfrac{1}{2}(0.00238\,\tfrac{slugs}{ft^3})(44\,\tfrac{ft}{s})^2(1.9)\left(\tfrac{22\,(34)}{144}ft^2\right) = 22.7\,lb$$

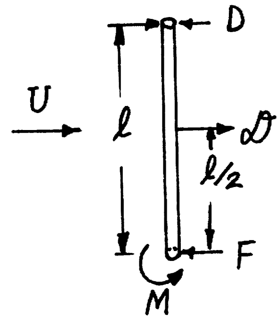
and

$$\mathscr{D}_p = \tfrac{1}{2}\rho U^2 C_{Dp} A_p = \tfrac{1}{2}(0.00238\,\tfrac{slugs}{ft^3})(44\,\tfrac{ft}{s})^2(2.2)\left(\tfrac{3}{12}(5)\,ft^2\right) = 6.34\,lb$$

Thus, from Eq. (1):

$$M_B = 2.5\,ft\,(6.34\,lb) + (5 + \tfrac{17}{12})ft\,(22.7\,lb) = \underline{\underline{162\,ft\cdot lb}}$$

9.48 Determine the moment needed at the base of 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.



For equilibrium, $M = \frac{\ell}{2}\mathcal{D}$ where (1)

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 \ell D$

Since $Re = \frac{UD}{\nu} = \frac{(20\frac{m}{s})(0.12\,m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.64\times10^5$, it follows from Fig. 9.21

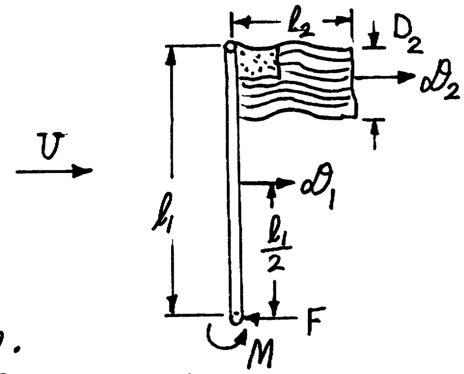
that $C_D = 1.2$

Thus, $\mathcal{D} = 1.2\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(20\frac{m}{s})^2(20m)(0.12m) = 708N$

Hence, from Eq. (1)

$M = \frac{20m}{2}(708N) = \underline{\underline{7{,}080\,N\cdot m}}$

9.49 Repeat Problem 9.48 if a 2-m by 2.5-m flag is attached to the top of the pole. See Fig. 9.30 for drag coefficient data for flags.



For equilibrium, $M = \frac{\ell_1}{2}\mathcal{D}_1 + \left(\ell_1 - \frac{D_2}{2}\right)\mathcal{D}_2$ (1)

where $\ell_1 = 20\,m$, $\ell_2 = 2.5\,m$, and $D_2 = 2\,m$.

From the solution to Problem 9.48, $\frac{\ell_1}{2}\mathcal{D}_1 = 7{,}080\ N\cdot m$ (2)

Also,

$\mathcal{D}_2 = C_D \frac{1}{2}\rho U^2 \ell_2 D_2$, where from Fig. 9.30 with $\frac{\ell_2}{D_2} = \frac{2.5}{2} = 1.25$

we obtain $C_D = 0.08$.

Thus,

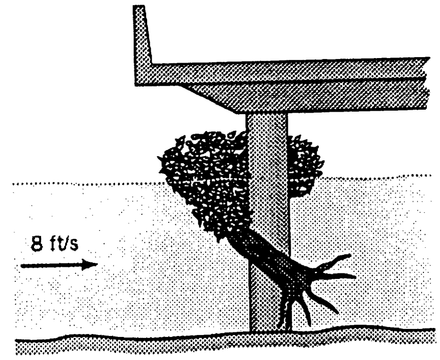
$\mathcal{D}_2 = 0.08\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(20\frac{m}{s}\right)^2(2.5\,m)(2\,m) = 98.4\,N$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$M = 7{,}080\ N\cdot m + (20\,m - 1\,m)(98.4\,N) = \underline{\underline{8{,}950\ N\cdot m}}$

9.50

† 9.50 During a flood, a 30-ft-tall, 15-ft-wide tree is uprooted, swept downstream, and lodged against a bridge pillar as shown in Fig. P9.50 and Video V7.6. Estimate the force that the tree puts on the bridge pillar. Assume the tree is half submerged and the river is flowing at 8 ft/s. See Fig. 9.30 for drag coefficient data.



■ FIGURE P9.50

Force of tree on bridge = drag on tree = $\mathcal{D} = C_D \frac{1}{2} \rho V^2 A$

where $V = 8\ \text{ft/s}$

Assume the tree is shaped as an ellipse and $\frac{1}{2}$ is in the water.

Thus,

$$A = (\tfrac{1}{2}) \tfrac{\pi}{4} (30\ \text{ft})(15\ \text{ft}) = 177\ \text{ft}^2$$

From Fig. 9.30 as the wind past a tree increases, the drag coefficient decreases (the leaves "fold back"). Assume the same thing happens in water and use $C_D = 0.20$.

Thus,

$$\mathcal{D} = 0.20\,(\tfrac{1}{2})(1.94\tfrac{\text{slugs}}{\text{ft}^3})(8\tfrac{\text{ft}}{\text{s}})^2(177\ \text{ft}^2) = \underline{\underline{2200\ \text{lb}}}$$

9.51

9.51 If for a given vehicle it takes 20 hp to overcome aerodynamic drag while being driven at 65 mph, estimate the horsepower required at 75 mph.

$\mathcal{P} = \text{power} = U\mathcal{D} = U C_D \frac{1}{2}\rho U^2 A = C_D \frac{1}{2}\rho U^3 A$

Thus,

$$\frac{\mathcal{P}_{75}}{\mathcal{P}_{65}} = \frac{C_D \frac{1}{2}\rho(65)^3 A}{C_D \frac{1}{2}\rho(55)^3 A} = \left(\frac{75}{65}\right)^3 = 1.54,$$ provided the values of C_D are independent of U (i.e., Re).

Hence,

$\mathcal{P}_{75} = 1.54\,\mathcal{P}_5 = 1.54(20\,hp) = \underline{\underline{30.8\,hp}}$

9.52

9.52 Two bicycle racers ride 30 km/hr through still air. By what percentage is the power required to overcome aerodynamic drag for the second cyclist reduced if she drafts closely behind the first cyclist rather than riding alongside her? Neglect any forces other than aerodynamic drag. (See Fig. 9.30).

$\mathcal{P}_{ND}$ = power when not drafting $= U\mathcal{D}_{ND} = U C_{D_{ND}} \frac{1}{2}\rho U^2 A = C_{D_{ND}} \frac{1}{2}\rho U^3 A$

and

$\mathcal{P}_D$ = power when drafting $= C_{D_D} \frac{1}{2}\rho U^3 A$

From Fig. 9.30 $C_{D_{ND}} = 0.88$ and $C_{D_D} = 0.50$

Thus,

$$\frac{\mathcal{P}_{ND} - \mathcal{P}_D}{\mathcal{P}_{ND}} = \frac{C_{D_{ND}} - C_{D_D}}{C_{D_{ND}}} = \frac{0.88 - 0.50}{0.88} = 0.432, \text{ i.e., a } \underline{\underline{43.2\% \text{ decrease}}}$$

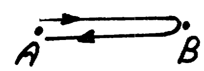
9.54

9.54 On a day without any wind, your car consumes x gallons of gasoline when you drive at a constant speed, U, from point A to point B and back to point A. Assume that you repeat the journey, driving at the same speed, on another day when there is a steady wind blowing from B to A. Would you expect your fuel consumption to be less than, equal to, or greater than x gallons for this windy round-trip? Support your answer with appropriate analysis.

Trip with the larger power lost due to aerodynamic drag will use the most gas. Let $(\ )_1$ mean "no wind" and $(\ )_2$ mean "wind".

(1) No wind:

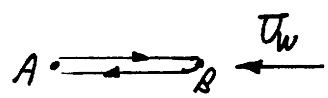
$\mathcal{D}_1 = C_D \frac{1}{2}\rho U^2 A$ for both $A \rightarrow B$ and $B \rightarrow A$



Thus,

$\mathcal{P}_1 = \text{power} = U\mathcal{D}_1 = \frac{1}{2}\rho U^3 C_D A$

(2) Wind (U_w = wind speed; assume $U_w < U$):



$\mathcal{D}_2 = C_D \frac{1}{2}\rho (U+U_w)^2 A$ for $A \rightarrow B$

$\mathcal{D}_2 = C_D \frac{1}{2}\rho (U-U_w)^2 A$ for $B \rightarrow A$

Thus,

$\mathcal{P}_2 = \frac{1}{2}\rho (U+U_w)^2 U C_D A$ for $A \rightarrow B$

$\mathcal{P}_2 = \frac{1}{2}\rho (U-U_w)^2 U C_D A$ for $B \rightarrow A$

Energy used $= \mathcal{P}t$, where t = time to go from $A \rightarrow B$ or $B \rightarrow A$

Thus,

$E_1 = 2\left(\frac{1}{2}\rho U^3 C_D A\right)t$ (Note: Factor of 2 for $A \rightarrow B + B \rightarrow A$)

and

$E_2 = \frac{1}{2}\rho (U+U_w)^2 U C_D A t + \frac{1}{2}\rho (U-U_w)^2 U C_D A t$

Thus,

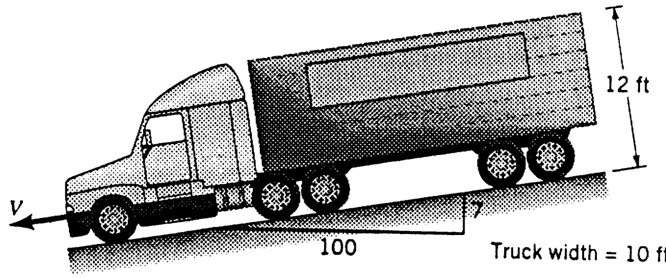
$$\frac{E_1}{E_2} = \frac{2U^3}{(U+U_w)^2 U + (U-U_w)^2 U} = \frac{2U^3}{2U^3 + 2U_w^2 U} = \frac{1}{1+(U_w/U)^2} < 1$$

Hence,

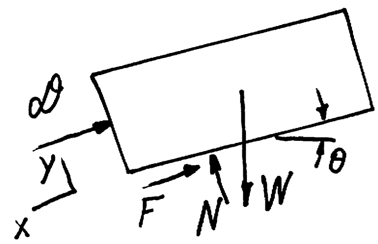
$\frac{E_1}{E_2} < 1$, i.e. more fuel needed when windy

9.55

9.55 A 25-ton (50,000-lb) truck coasts down a steep 7% mountain grade without brakes, as shown in Fig. P9.55. The truck's ultimate steady-state speed, V, is determined by a balance between weight, rolling resistance, and aerodynamic drag. Assume that the rolling resistance for a truck on concrete is 1.2% of the weight and the drag coefficient is 0.96 for a truck without an air deflector, but 0.70 if it has an air deflector (see Fig. P9.56 and Video V9.8). Determine V for these two situations.



■ FIGURE P9.55



For constant speed, $\Sigma F_x = m a_x = 0$

or

$$W \sin\theta = \mathcal{D} + F$$

where $\theta = \arctan\left(\frac{7}{100}\right) = 4.00 \text{ deg}$, $\mathcal{D} = \frac{1}{2}\rho V^2 C_D A$

Thus,

$$50{,}000 \text{ lb} (\sin 4.00\text{deg}) = \frac{1}{2}\left(0.00238 \frac{\text{slugs}}{\text{ft}^3}\right) V^2 C_D (12\text{ft} \times 10\text{ft}) + 0.012(50{,}000 \text{ lb})$$

or

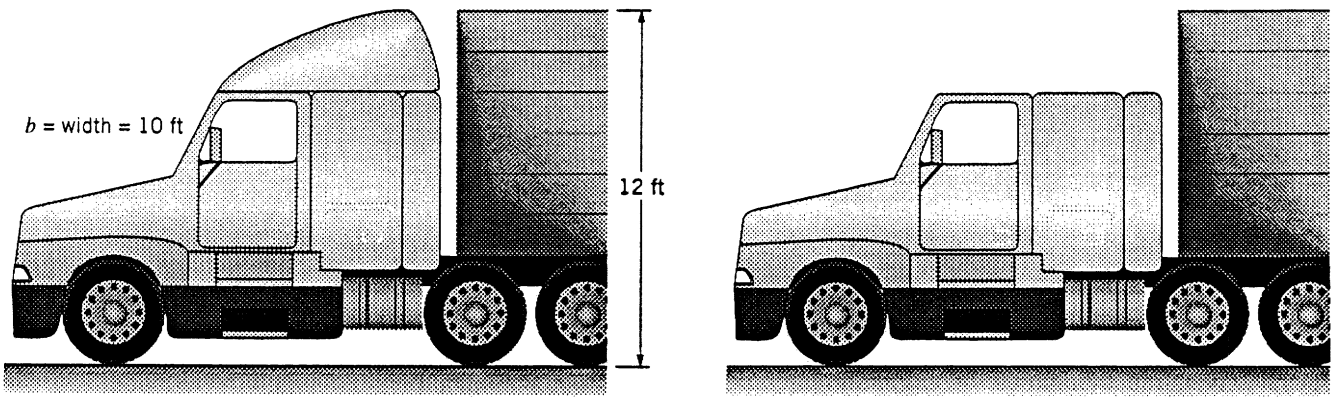
$$3488 \text{ lb} = 0.143 V^2 C_D + 600 \text{ lb}$$

(a) If $C_D = 0.96$, then $V = 145 \frac{\text{ft}}{\text{s}} = \underline{\underline{98.9 \text{mph}}}$

(b) If $C_D = 0.70$, then $V = 170 \frac{\text{ft}}{\text{s}} = \underline{\underline{116 \text{mph}}}$

9.56

9.56 As shown in Video V9.8 and Fig. P9.56, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction of how many horsepoer needed at a highway speed of 65 mph?



(a) $C_D = 0.70$ (b) $C_D = 0.96$

■ FIGURE P9.56

$\mathcal{P}$ = power = $\mathcal{D}V$ where

$\mathcal{D} = \frac{1}{2}\rho V^2 C_D A$

Thus, $\Delta\mathcal{P}$ = reduction in power

$= \mathcal{P}_b - \mathcal{P}_a$

$= \frac{1}{2}\rho V^3 A\left[C_{D_b} - C_{D_a}\right]$

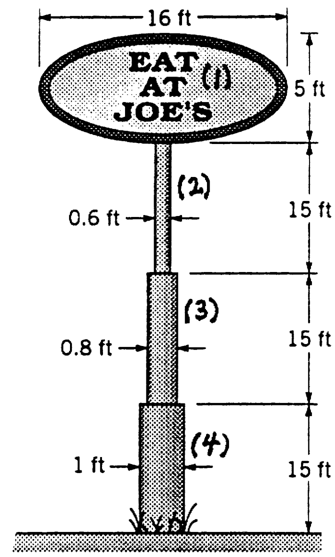
With $V = 65$ mph $= 95.3$ fps,

$$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left(95.3\,\frac{ft}{s}\right)^3(10\,ft)(12\,ft)\left[0.96-0.70\right]$$

$$= 32{,}100\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{58.4\,hp}}$$

9.57

9.57 The structure shown in Fig. P9.57 consists of three cylindrical support posts to which an elliptical flat-plate sign is attached. Estimate the drag on the structure when a 50 mph wind blows against it.



■ FIGURE P9.57

For the composite body:

(1) $\mathscr{D}=\sum_{i=1}^{4}\mathscr{D}_i = \frac{1}{2}\rho U^2\left[C_{D_1}A_1 + C_{D_2}A_2 + C_{D_3}A_3 + C_{D_4}A_4\right]$

where if we assume the sign is an ellipse,

$A_1=\frac{\pi}{4}(10ft)(5ft) = 39.3\ ft^2$

$A_2 = 0.6\,ft(15ft) = 9.00\ ft^2$

$A_3 = 0.8\,ft(15ft) = 12.0\ ft^2$ and

$A_4 = 1\,ft(15ft) = 15.0\ ft^2$

From Fig. 9.29, for a thin disc $C_{D_1}=1.1$

For the cylindrical post, obtain C_D from Fig. 9.21 as: $(U=50mph=73.3\frac{ft}{s})$

$$Re_2 = \frac{UD_2}{\nu} = \frac{73.3\frac{ft}{s}(0.6ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 2.8\times10^5 \longrightarrow C_{D_2}=0.6$$

Similarly,

$Re_3 = 3.7\times10^5 \longrightarrow C_{D_3} = 0.5$

$Re_4 = 4.7\times10^5 \longrightarrow C_{D_4} = 0.25$

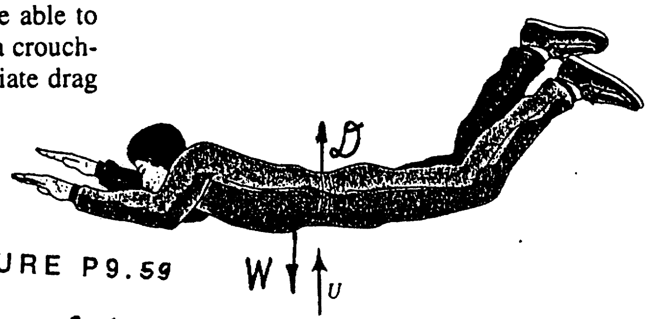
Thus, from Eq.(1):

$$\mathscr{D}=\frac{1}{2}(0.00233\frac{slugs}{ft^3})(73.3\frac{ft}{s})^2\left[1.1(39.3ft^2)+0.6(9.0ft^2)+0.5(12ft^2)+0.25(15ft^2)\right]$$

$$= \underline{\underline{378\ lb}}$$

9.59

9.59 As shown in Video V9.5 and Fig. P9.59, a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) curls up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.



■ FIGURE P9.59

For equilibrium conditions

$$W = \mathcal{D} = C_D \tfrac{1}{2} \rho U^2 A$$

Assume $W = 160\text{ lb}$ and $C_D A = 9\text{ ft}^2$ (see Fig. 9.30)

Thus,

$$160\text{ lb} = \left(\tfrac{1}{2}\right)\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right) U^2 (9\text{ ft}^2) \quad \text{where } U \sim \frac{\text{ft}}{\text{s}}$$

or

$$U = \left(122\frac{\text{ft}}{\text{s}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = \underline{\underline{83.2\text{ mph}}}$$

Note: If the skydiver "curled up into a ball", then $C_D A \approx 2.5\text{ ft}$ (see Fig. 9.30) and $U = 158\text{ mph}$

9.60*

9.60* The helium-filled balloon shown in Fig. P9.60 is to be used as a wind speed indicator. The specific weight of the helium is $\gamma = 0.011\ \text{lb/ft}^3$, the weight of the balloon material is 0.20 lb, and the weight of the anchoring cable is negligible. Plot a graph of θ as a function of U for $1 \le U \le 50$ mph. Would this be an effective device over the range of U indicated? Explain.

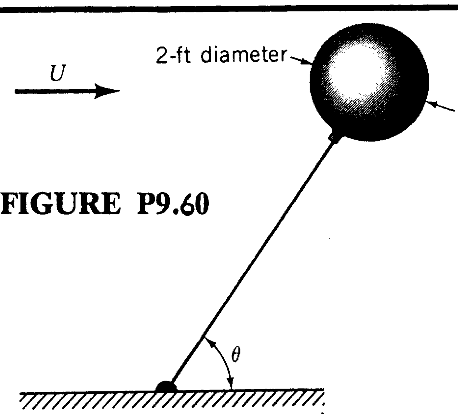


FIGURE P9.60

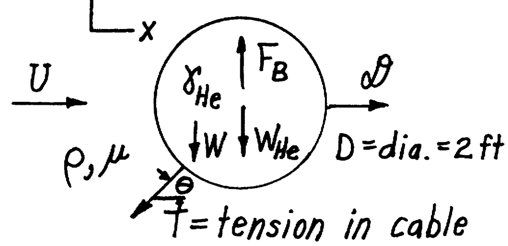
For the balloon to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathcal{D} = T\cos\theta$ or $T = \dfrac{\mathcal{D}}{\cos\theta}$

and $F_B = W + T\sin\theta + W_{He}$

which combine to give



$$F_B = W + \mathcal{D}\tan\theta + W_{He} \tag{1}$$

But $W = 0.2\ lb$, $F_B = \rho g \forall = \left(7.65\times10^{-2}\ \frac{lb}{ft^3}\right)\frac{4\pi}{3}\left(\frac{2}{2}ft\right)^3 = 0.3204\ lb$

and $W_{He} = \gamma_{He}\forall = \left(0.011\ \frac{lb}{ft^3}\right)\frac{4\pi}{3}\left(\frac{2}{2}ft\right)^3 = 0.0461\ lb$

Thus, Eq. (1) becomes

$0.3204\ lb = 0.2\ lb + \mathcal{D}\tan\theta + 0.0461\ lb$

or

$\mathcal{D}\tan\theta = 0.0743\ lb$ Also, $\mathcal{D} = C_D\frac{1}{2}\rho U^2\frac{\pi}{4}D^2$

$= C_D U^2\left(0.00238\ \frac{slugs}{ft^3}\right)\frac{\pi}{8}(2\,ft)^2$

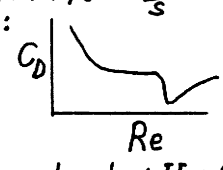
$= 0.00374\, C_D U^2\ lb$, where $U \sim \frac{ft}{s}$

Hence,

$$0.00374\, C_D U^2\tan\theta = 0.0743 \quad \text{or} \quad \tan\theta = \frac{19.9}{C_D U^2} \tag{2}$$

$$\text{Also,}\ Re = \frac{UD}{\nu} = \frac{(2\,ft)\,U}{1.57\times10^{-4}\ \frac{ft^2}{s}} \quad \text{or} \quad Re = 1.27\times10^4\, U \tag{3}$$

and from Fig. 9.21 : (4)



Thus, select various $1\ mph \le U \le 50\ mph$ (i.e., $1.47\ \frac{ft}{s} \le U \le 73.3\ \frac{ft}{s}$) and use Eqs. (2), (3), (4) to obtain θ. Plotted results are shown below, along with the Program P9#60.

```
100 cls
110 open "prn" for output as #1
120 print#1, "**************************************************"
130 print#1, "**  This program calculates the angle of the     **"
140 print#1, "**  cable as a function of the velocity of the   **"
150 print#1, "**  air.  Values of the drag coefficient are     **"
160 print#1, "**  obtained from Fig. 9.23 as a function of     **"
170 print#1, "**  Reynolds number.                             **"
```

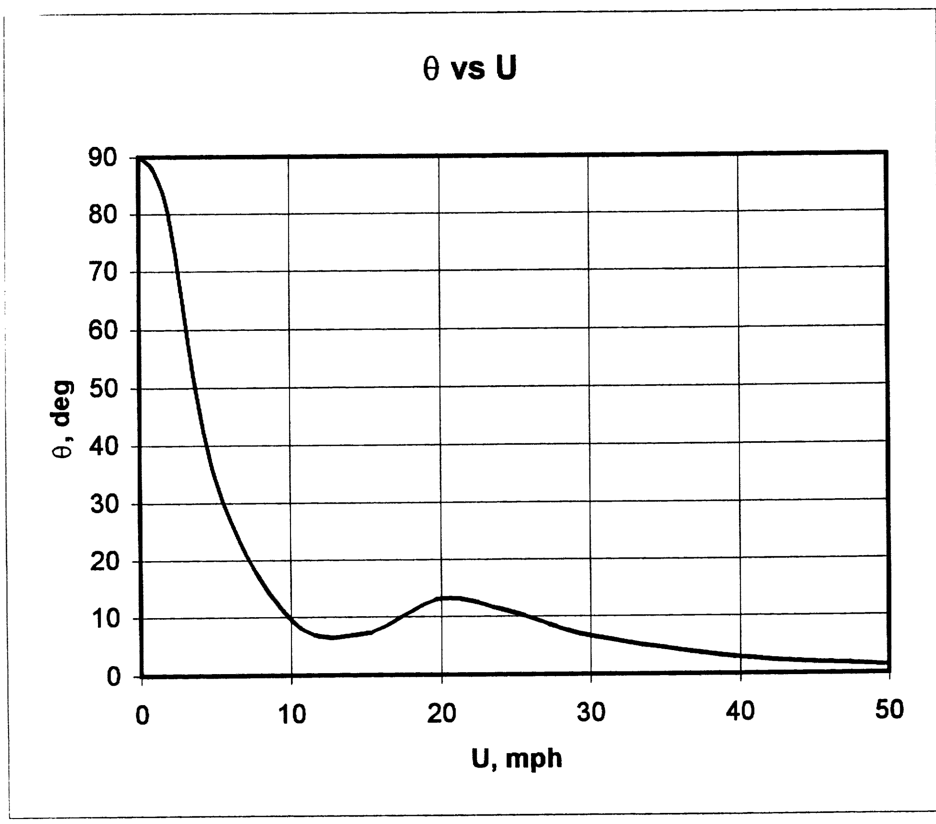
(con't)

9.60* (con't)

```
180 print#1, "*****************************************************"
190 print#1, " "
200 pi = 4*atn(1)
210 print#1, "    U, mph    CD     theta, deg"
220 for i = 1 to 10
230 print " "
240 input "For a velocity of (in mph) U=",U
250 Re = 1.27E4*(88*U/60)
260 print using "the Reynolds number is Re=#.###^^^^";Re
270 input "Enter the drag coefficient:  CD=",CD
280 theta = (atn(19.9/(CD*(88*U/60)^2)))*180/pi
290 print using "For U = ###.## mph theta = ##.#### deg";U,theta
300 print#1, using " ####.#   ##.###      ###.###";U,CD,theta
310 next i
```

```
**************************************************
**  This program calculates the angle of the     **
**  cable as a function of the velocity of the   **
**  air.  Values of the drag coefficient are     **
**  obtained from Fig. 9.23 as a function of     **
**  Reynolds number.                             **
**************************************************
```

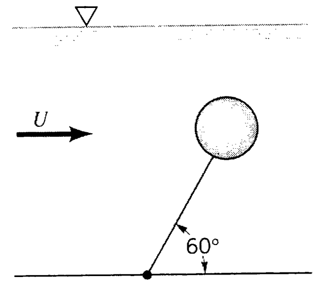
U, mph	CD	theta, deg
1.0	0.400	87.524
2.0	0.420	79.707
5.0	0.540	34.421
10.0	0.550	9.548
15.0	0.330	7.102
20.0	0.100	13.022
25.0	0.080	10.482
30.0	0.090	6.516
40.0	0.120	2.759
50.0	0.160	1.325



Note: Because of the sudden change in C_D when the boundary layer becomes turbulent (at about 15 mph), the θ vs U curve is highly non-linear. In fact, for some values of θ there is more than one possible value of U. It would not work well as a wind speed indicator in this range.

9.61

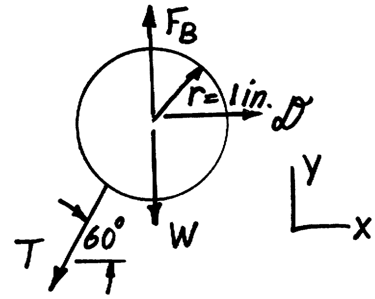
9.61 A 2-in.-diameter cork sphere (specific weight = 13 lb/ft³) is attached to the bottom of a river with a thin cable, as is illustrated in Fig. P9.61. If the sphere has a drag coefficient of 0.5, determine the river velocity. Both the drag on the cable and its weight are negligible.



$\Sigma F_x = 0$ or $\mathscr{D} = T\cos 60°$ (1)

and

$\Sigma F_y = 0$ or $F_B - W = T\sin 60°$ (2)



Since

$\forall$ = volume of cork = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{1}{12}\,ft\right)^3 = 2.42\times10^{-3}\,ft^3$ it follows from Eq.(2) that

$\gamma_{H_2O}\forall - \gamma_{cork}\forall = T\sin 60°$, or

$(62.4 - 13)\frac{lb}{ft^3}(2.42\times10^{-3})ft^3 = T\sin 60°$

Thus, $T = 0.138\,lb$

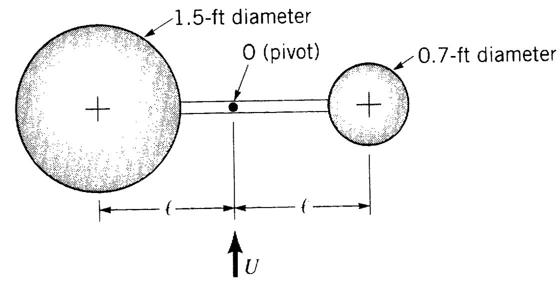
From Eq.(1), $\mathscr{D} = C_D\frac{1}{2}\rho U^2 A = T\cos 60°$ where $A = \pi r^2$

Thus,

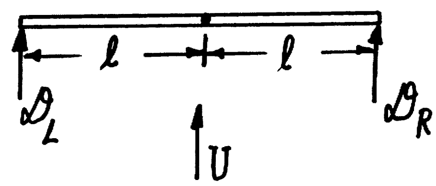
$$U = \left[\frac{2T\cos 60°}{C_D\rho\pi r^2}\right]^{1/2} = \left[\frac{2(0.138\,lb)\cos 60°}{0.5\left(1.94\frac{slugs}{ft^3}\right)\pi\left(\frac{1}{12}ft\right)^2}\right]^{1/2} = \underline{\underline{2.55\frac{ft}{s}}}$$

9.62

9.62 Two smooth spheres are attached to a thin rod that is free to rotate in the horizontal plane about point O as shown in Fig. P9.62. The rod is held stationary until the air speed reaches 50 ft/s. Which direction will the rod rotate (clockwise or counterclockwise) when the holding force is released? Explain your answer.



Let $\mathscr{D}_L$ and $\mathscr{D}_R$ denote the drag forces on the left and right spheres, respectively. If $\mathscr{D}_R > \mathscr{D}_L$ the rod will rotate counter clockwise.



$\mathscr{D}_L = C_{D_L} \frac{1}{2} \rho U^2 A_L$ and $\mathscr{D}_R = C_{D_R} \frac{1}{2} \rho U^2 A_R$ so that

$$\frac{\mathscr{D}_R}{\mathscr{D}_L} = \frac{C_{DR} A_R}{C_{DL} A_L} \quad \text{where } C_D = C_D(Re). \qquad (1)$$

Now,

$$Re_L = \frac{\rho U D_L}{\mu} = \frac{0.00238 \frac{slugs}{ft^3} (50 \frac{ft}{s})(1.5 ft)}{3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}} = 4.77 \times 10^5$$

and

$$Re_R = \frac{\rho U D_R}{\mu} = \frac{0.00238 \frac{slugs}{ft^3} (50 \frac{ft}{s})(0.7 ft)}{3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}} = 2.23 \times 10^5$$

Thus, from Fig. 8.21 or 8.25, for smooth spheres

$C_{D_L} = 0.06$ and $C_{D_R} = 0.5$ so that Eq.(1) gives

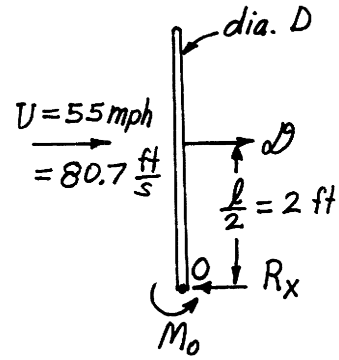
$$\frac{\mathscr{D}_R}{\mathscr{D}_L} = \frac{C_{DR} \frac{\pi}{4} D_R^2}{C_{DL} \frac{\pi}{4} D_L^2} = \frac{0.5 (0.7 ft)^2}{0.06 (1.5 ft)^2} = 1.81$$

Since $\mathscr{D}_R > \mathscr{D}_L$ the rod will rotate <u>counter clockwise</u>.

Note: Although the right sphere is smaller than the left, it has more drag because it has a large drag coefficient (laminar boundary layer, wide wake). The large sphere has a smaller drag coefficient (turbulent boundary layer, narrow wake).

9.63

9.63 A radio antenna on a car consists of a circular cylinder $\frac{1}{4}$ in. in diameter and 4 ft long. Determine the bending moment at the base of the antenna if the car is driven 55 mph through still air.



For equilibrium, $\Sigma M_0 = 0$, or $M_0 = \frac{\ell}{2}\mathcal{D}$

where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

Since $Re = \frac{UD}{\nu} = \frac{(80.7\frac{ft}{s})(\frac{1}{48}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 1.07\times10^4$,

it follows from Fig. 9.21 that $C_D = 1.3$

Hence $\mathcal{D} = 1.3(\frac{1}{2})(0.00238\frac{slugs}{ft^3})(80.7\frac{ft}{s})^2(4ft)(\frac{1}{48}ft) = 0.840\,lb$

Thus, $M_0 = (2ft)(0.840lb) = \underline{\underline{1.68\ ft\cdot lb}}$

9.65

9.65 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$, where $U = (55\,mph)\left(\frac{88\frac{ft}{s}}{60\,mph}\right) = 80.7\frac{ft}{s}$

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with $C_D \approx 1.1$ (see Fig. 9.29).

Thus,

$\mathcal{D} = (1.1)(\frac{1}{2})(0.00238)(80.7\frac{ft}{s})^2(\frac{4}{12}ft)(\frac{6}{12}ft) = \underline{\underline{1.42\ lb}}$

If your hand is normal to the the lift force is zero.

For $U = 550\,mph = 807\frac{ft}{s}$ (i.e., a 10 fold increase in U) the drag will increase by a factor of 100 (i.e., $\mathcal{D} \sim U^2$), or $\mathcal{D} = \underline{\underline{142\,lb}}$

Note: We have assumed that C_D is not a function of U. That is, it is not a function of either $Re = \frac{UD}{\nu}$ or $Ma = \frac{U}{c}$.

9.66★

9.66* Let $\mathcal{P}_0$ be the power required to fly a particular airplane at 500 mph at sea-level conditions. Plot a graph of the ratio $\mathcal{P}/\mathcal{P}_0$, where $\mathcal{P}$ is the power required at a speed of U, for 500 mph $\le U \le$ 3000 mph at altitudes of sea level, 10,000 ft, 20,000 ft, and 30,000 ft. Assume that the drag coefficient for the aircraft behaves similarly to that of the sharp-pointed ogive indicated in Fig. 9.24.

$$\mathcal{P} = U\mathscr{D} = C_D \tfrac{1}{2}\rho U^3 A \quad \text{so that} \quad \frac{\mathcal{P}}{\mathcal{P}_0} = \frac{\frac{1}{2}\rho U^3 C_D A}{\frac{1}{2}\rho_0 U_0^3 C_{D_0} A} = \frac{\rho C_D}{\rho_0 C_{D_0}}\left(\frac{U}{U_0}\right)^3 \qquad (1)$$

Now, $U_0 = (500\,\text{mph})\dfrac{(88\,\frac{ft}{s})}{(60\,\text{mph})} = 733\,\frac{ft}{s}$

and

$Ma = \dfrac{U}{c}$ where $c = \sqrt{kRT}$ so that $c_0 = \sqrt{1.4(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R})(460+59)^\circ R} = 1117\,\frac{ft}{s}$

or $Ma_0 = \dfrac{U_0}{c_0} = \dfrac{733\,\frac{ft}{s}}{1117\,\frac{ft}{s}} = 0.656$

Hence, from Fig. 9.24, $C_{D_0} = 0.2$ so that Eq. (1) becomes

$$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{\rho C_D}{(0.00238\,\frac{slugs}{ft^3})(0.2)}\left(\frac{U}{733\,\frac{ft}{s}}\right)^3 \quad \text{or} \quad \frac{\mathcal{P}}{\mathcal{P}_0} = 5.33\times10^{-6}\rho C_D U^3, \text{ where } U \sim \tfrac{ft}{s} \text{ and } \rho \sim \tfrac{slugs}{ft^3} \qquad (2)$$

Also, $C_D = C_D(Ma)$ from Fig. 9.24 (3)

where

$$Ma = \frac{U}{\sqrt{1.4(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R})(460+T)^\circ R}}$$

or

$$Ma = \frac{U}{\sqrt{2400(460+T)}}, \text{ where } U \sim \tfrac{ft}{s} \text{ and } T \sim {}^\circ F \qquad (4)$$

Thus, for the given altitude obtain T and ρ in Table C.1. Select 500 mph $\le U \le$ 3,000 mph (i.e., $733\,\frac{ft}{s} \le U \le 4400\,\frac{ft}{s}$), determine Ma from Eq. (4), C_D from Eq. (3) (the graph), and $\frac{\mathcal{P}}{\mathcal{P}_0}$ from Eq. (2). The results are plotted below.

a) At 10,000 ft, $\rho = 1.76\times10^{-3}\,\frac{slug}{ft^3}$, $T = 23.4\,^\circ F$

Thus, $\dfrac{\mathcal{P}}{\mathcal{P}_0} = 9.38\times10^{-9}\,C_D U^3$

$Ma = \dfrac{U}{1080}$

b) At 20,000 ft, $\rho = 1.27\times10^{-3}\,\frac{slug}{ft^3}$, $T = -12.3\,^\circ F$

Thus $\dfrac{\mathcal{P}}{\mathcal{P}_0} = 6.77\times10^{-9}\,C_D U^3$

$Ma = \dfrac{U}{1040}$

(con't)

9.66* (con't)

and

c) At 30,000 ft, $\rho = 8.91 \times 10^{-4} \frac{slugs}{ft^3}$, $T = -47.8\,°F$

Thus, $\frac{\mathcal{P}}{\mathcal{P}_0} = 4.75 \times 10^{-9} C_D U^3$

$Ma = \frac{U}{995}$

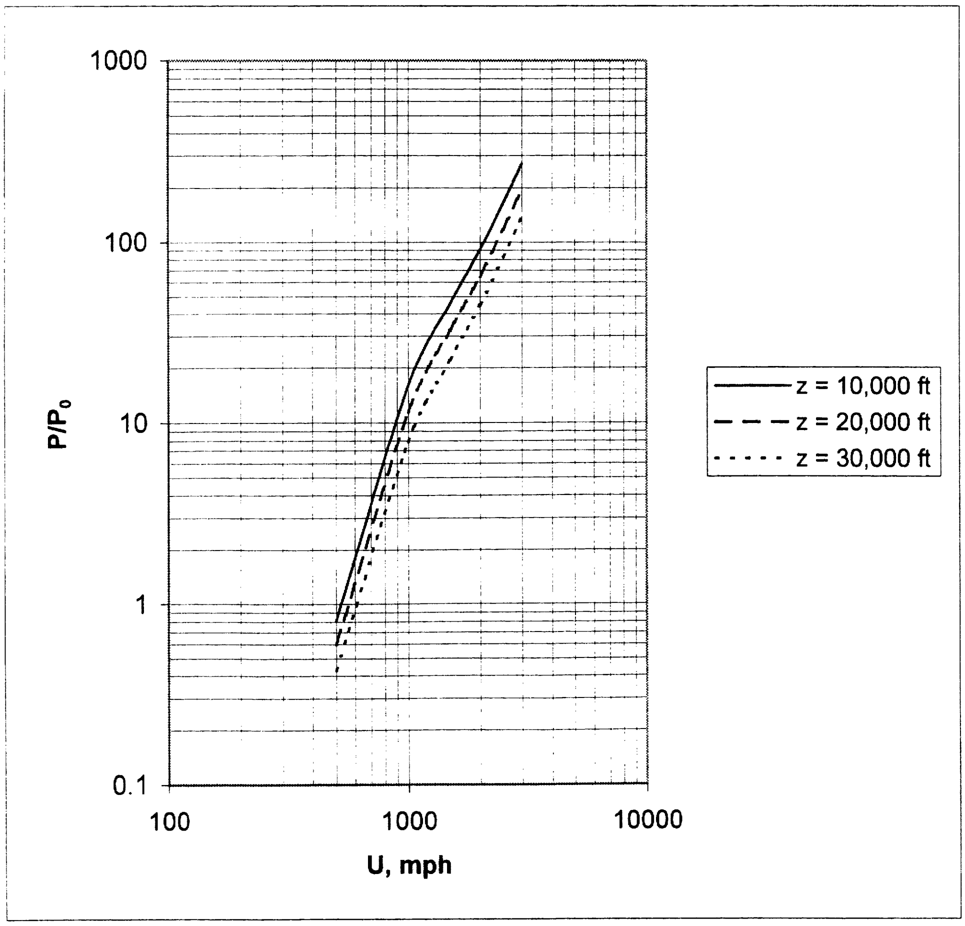
Program P9#66 shown below was used for the calculations. C_D vs Ma data was taken from Fig. 9.24.

```
100 cls
110 open "prn" for output as #1
120 dim M(17), C(17), A(3), AA(3)
130 print#1, "**************************************************"
140 print#1, "**  This program calculates the power ratio     **"
150 print#1, "**  at different altitudes of flight for        **"
160 print#1, "**  flight speeds from 500 to 3000 mph.         **"
170 print#1, "**************************************************"
200 C(1)=0.19 : C(2)=0.23 : C(3)=0.42 : C(4)=0.55 : C(5)=0.52
210 C(6)=0.49 : C(7)=0.46 : C(8)=0.43 : C(9)=0.40 : C(10)=0.38
220 C(11)=0.37 : C(12)=0.36 : C(13)=0.35 : C(14)=0.35
230 C(15)=0.34 : C(16)=0.34 : C(17)=0.34
240 A(1)=1080 : A(2)=1040 : A(3)=995
250 AA(1)=9.38E-9 : AA(2)=6.77E-9 : AA(3)=4.75E-9
260 for i = 1 to 17
270 M(i) = 0.25*(i+1)
280 next i
300 for i = 1 to 3
310 z = 10000*i
320 print#1, " "
330 print#1, using "For an altitude of z = ######. ft";z
340 print#1, "  U, mph    Ma       CD       P/Po"
350 U = 0
360 for j = 1 to 6
370 B = 0
380 U = U + 500/60*88
390 Ma = U/A(i)
400 for k = 1 to 17
410 if B = 1 then goto 450
420 if Ma > M(k) then goto 450
430 CD = C(k-1) + (C(k) -C(k-1))*(Ma - M(k-1))/(M(k) - M(k-1))
440 B = 1
450 next k
460 ratio = AA(i)*CD*U^3
465 UM = U*60/88
470 print#1, using " #####.#  #.####  #.####  #.###^^^^";UM,Ma,CD,ratio
480 next j
490 next i
```

(con't)

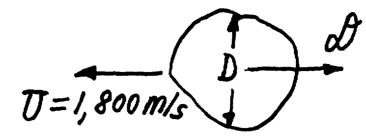
9.66* (con't)

```
***************************************************
**  This program calculates the power ratio      **
**  at different altitudes of flight for         **
**  flight speeds from 500 to 3000 mph.          **
***************************************************

For an altitude of z =   10000 ft
  U, mph     Ma        CD        P/Po
   500.0   0.6790   0.2186   8.088E-01
  1000.0   1.3580   0.5370   1.589E+01
  1500.0   2.0370   0.4556   4.550E+01
  2000.0   2.7160   0.3827   9.061E+01
  2500.0   3.3951   0.3542   1.638E+02
  3000.0   4.0741   0.3400   2.717E+02

For an altitude of z =   20000 ft
  U, mph     Ma        CD        P/Po
   500.0   0.7051   0.2228   5.949E-01
  1000.0   1.4103   0.5308   1.134E+01
  1500.0   2.1154   0.4462   3.216E+01
  2000.0   2.8205   0.3772   6.445E+01
  2500.0   3.5256   0.3500   1.168E+02
  3000.0   4.2308   0.3400   1.961E+02

For an altitude of z =   30000 ft
  U, mph     Ma        CD        P/Po
   500.0   0.7370   0.2279   4.270E-01
  1000.0   1.4740   0.5231   7.839E+00
  1500.0   2.2111   0.4347   2.198E+01
  2000.0   2.9481   0.3721   4.461E+01
  2500.0   3.6851   0.3500   8.196E+01
  3000.0   4.4221   0.3400   1.376E+02
```



9.67

9.67 A 0.50-m-diameter meteor streaks through the earth's atmosphere with a speed of 1800 m/s at an altitude of 20,000 m where the air density is 9×10^{-2} kg/m^3 and the speed of sound is 300 m/s. The specific gravity of the meteor is 7.65. Use the data in Fig. 9.24 to determine the rate at which the meteor is decelerating.



$\Sigma \vec{F} = m\vec{a}$ or $\mathcal{D} = ma$

Thus,

$$\frac{1}{2}\rho U^2 C_D \frac{\pi}{4} D^2 = \rho_m \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 a \qquad \rho_m = SG\,\rho_{H_2O}$$

where

$$m = \rho_m \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \left[7.65\left(1000\frac{kg}{m^3}\right)\right]\frac{4}{3}\pi\left(\frac{0.5m}{2}\right)^3 = 501\, kg$$

Also,

$$Ma = \frac{U}{c} = \frac{1800\frac{m}{s}}{300\frac{m}{s}} = 6.0 \text{ so that from Fig. 9.24, } C_D = 0.95$$

Thus,

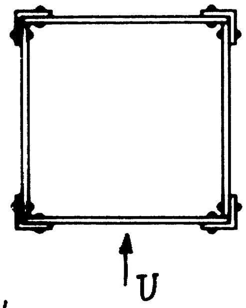
$$\mathcal{D} = \frac{1}{2}\left(9\times10^{-2}\frac{kg}{m^3}\right)\left(1800\frac{m}{s}\right)^2\frac{\pi}{4}(0.5m)^2(0.95) = 2.72\times10^4 N$$

so that,

$$a = \frac{\mathcal{D}}{m} = \frac{2.72\times10^4 N}{501\, kg} = 54.3\frac{N}{kg} = 54.3\frac{\frac{kg\cdot m}{s^2}}{kg} = \underline{\underline{54.3\frac{m}{s^2}}}$$

9.68

9.68 A 30-ft-tall tower is constructed of equal 1-ft segments as is indicated in Fig. P9.68. Each of the four sides is similar. Estimate the drag on the tower when a 75-mph wind blows against it.



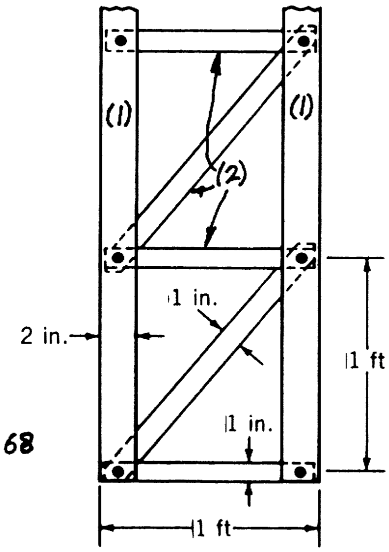


FIGURE P9.68

Assume no interference between the front and back portions of the tower. Also, neglect the drag on the sides of the tower. Hence, for thirty one-foot segments

$$\mathcal{D} = 30\left(\tfrac{1}{2}\rho U^2\right)\left[\left(C_{D_1}A_1 + C_{D_2}A_2\right)_{front} + \left(C_{D_1}A_1 + C_{D_2}A_2\right)_{back}\right] \quad (1)$$

where

$$U = 75\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 110\,\frac{ft}{s}$$

From Fig. 9.28, $C_{D_{1_{front}}} = 1.98$, $C_{D_{1_{back}}} = 1.82$

$C_{D_2} \approx 1.9$

Thus, from Eq. (1)

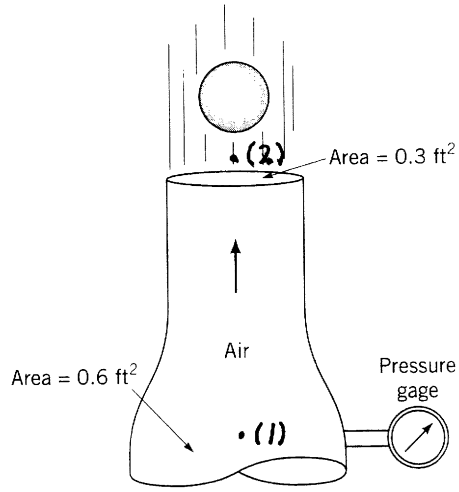
$$\mathcal{D} = 30\left(\tfrac{1}{2}(0.00238\,\tfrac{slugs}{ft^3})(110\,\tfrac{ft}{s})^2\right)\Big[(1.98)(2)(1\,ft)\left(\tfrac{2}{12}\,ft\right)+(1.9)\left(\tfrac{1}{12}\,ft\right)\left(\tfrac{8+8+8\sqrt{2}}{12}\,ft\right)$$
$$+ (1.82)(2)(1\,ft)\left(\tfrac{2}{12}\,ft\right)+(1.9)\left(\tfrac{1}{12}\,ft\right)\left(\tfrac{8+8+8\sqrt{2}}{12}\,ft\right)\Big]$$

or

$$\mathcal{D} = \underline{\underline{859\ lb}}$$

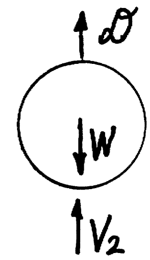
9.69

9.69 A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.69 and **Video V3.1**. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.



For equilibrium, $\mathcal{D} = W$ or

$C_D \frac{1}{2}\rho V_2^2 A = W$, where $A = \frac{\pi}{4}D^2$



Thus,

$$V_2 = \left[\frac{2W}{C_D \rho \pi D^2/4}\right]^{1/2}$$

$$= \left[\frac{8\,(0.14\,lb)}{0.5(0.00238\,\frac{slugs}{ft^3})\pi(\frac{2}{12}\,ft)^2}\right]^{1/2} = 104\,\frac{ft}{s}$$

Also,

$$V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_1 = V_2 \frac{A_2}{A_1} = \left(104\,\frac{ft}{s}\right)\frac{0.3\,ft^2}{0.6\,ft^2} = 52.0\,\frac{ft}{s}$$

and

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{where } p_2 = 0$$

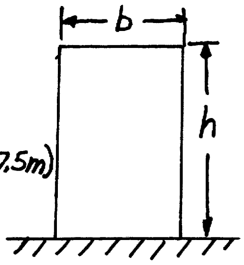
Thus,

$$p_1 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(104\,\frac{ft}{s}\right)^2 - \left(52.0\,\frac{ft}{s}\right)^2\right]$$

$$= \underline{\underline{9.65\,\frac{lb}{ft^2}}}$$

9.70

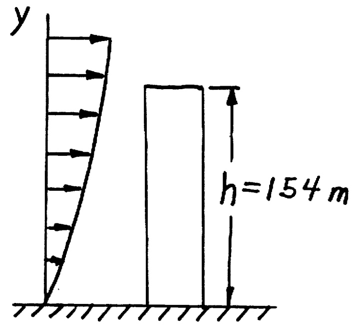
9.70 The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. **(a)** Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. **(b)** Repeat your calculations if the velocity profile against the building is a typical profile for an urban area (see Problem 9.17) and the wind speed halfway up the building is 20 m/s.



(a) $\mathcal{D} = C_D \frac{1}{2} \rho V^2 A = 1.3\left(\frac{1}{2}\right)\left(1.23\ \frac{kg}{m^3}\right)\left(20\frac{m}{s}\right)^2(154m)(87.5m)$

or

$\mathcal{D} = 4.31 \times 10^6\ N = \underline{\underline{4.31\ MN}}$



(b) For an urban area, $u = C\,y^{0.4}$

Thus, with $u = 20\frac{m}{s}$ at $y = \frac{h}{2} = 77m$

we obtain

$C = \frac{20}{77^{0.4}} = 3.52$, or $u = 3.52\ y^{0.4}$ with $u \sim \frac{m}{s}$, $y \sim m$

The total drag is

$$\mathcal{D} = \int d\mathcal{D} = \int C_D \frac{1}{2}\rho u^2\, dA = \frac{1}{2}\rho C_D \int_{y=0}^{y=154} (3.52\ y^{0.4})^2 (87.5)\, dy$$

or

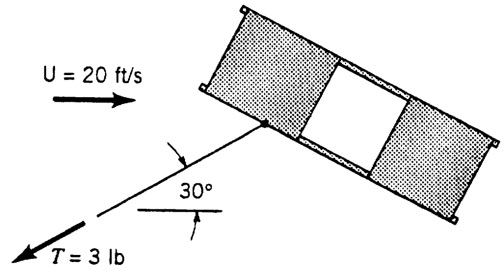
$$\mathcal{D} = \frac{1}{2}(1.23)(1.3)(3.52)^2(87.5)\int_0^{154} y^{0.8}dy = 867\left(\frac{1}{1.8}\right)(154)^{1.8} = 4.17 \times 10^6\ N$$

Thus,

$\mathcal{D} = \underline{\underline{4.17\ MN}}$

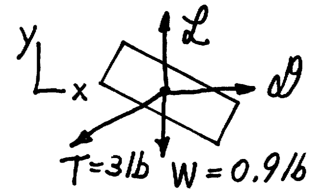
9.72

9.72 When the 0.9-lb box kite shown in Fig. P9.72 is flown in a 20 ft/s wind, the tension in the string, which is at a 30° angle relative to the ground, is 3.0 lb. (a) Determine the lift and drag coefficients for the kite based on the frontal area of 6.0 ft^2. (b) If the wind speed increased to 30 ft/s, would the kite rise or fall? That is, would the 30° angle shown in the figure increase or decrease? Assume the lift and drag coefficients remain the same. Support your answer with appropriate calculations.



■ FIGURE P9.72

(a) $\Sigma F_x = ma_x = 0$ or $\mathcal{D} = T\cos 30^\circ$
$= (3\text{ lb})\cos 30^\circ = 2.60\text{ lb}$



Thus,

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A} = \frac{2.60\text{ lb}}{\frac{1}{2}(0.00238\frac{\text{slugs}}{\text{ft}^3})(20\frac{\text{ft}}{\text{s}})^2(6\text{ ft}^2)}$$

or

$C_D = \underline{\underline{0.910}}$

Also,

$\Sigma F_y = ma_y = 0$ or $\mathcal{L} = W + T\sin 30^\circ = 0.9\text{ lb} + (3\text{ lb})\sin 30^\circ = 2.40\text{ lb}$

Thus,

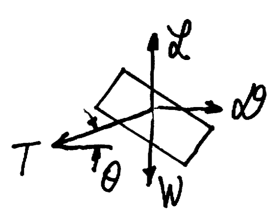
$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{2.40\text{ lb}}{\frac{1}{2}(0.00238\frac{\text{slugs}}{\text{ft}^3})(20\frac{\text{ft}}{\text{s}})^2(6\text{ ft}^2)}$$

or

$C_L = \underline{\underline{0.840}}$

(b) For a wind speed U and string angle θ, for equilibrium

$\mathcal{D} = T\cos\theta$ and
$\mathcal{L} = W + T\sin\theta$
or



(1) $T\cos\theta = \mathcal{D}$ and
(2) $T\sin\theta = \mathcal{L} - W$ so that if we divide Eq.(2) by Eq.(1),

$$\tan\theta = \frac{\mathcal{L} - W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} - \frac{W}{\mathcal{D}}$$

But,

$$\frac{\mathcal{L}}{\mathcal{D}} = \frac{\frac{1}{2}\rho U^2 A C_L}{\frac{1}{2}\rho U^2 A C_D} = \frac{C_L}{C_D} \text{ so that}$$

$\tan\theta = \frac{C_L}{C_D} - \frac{W}{\mathcal{D}} = \frac{C_L}{C_D} - \frac{W}{\frac{1}{2}\rho U^2 C_D A}$. Thus, for constant C_L and C_D, $\tan\theta$ increases as U increases. The kite will $\underline{\underline{\text{rise}}}$ as U increases.

9.73

9.73 A regulation football is 6.78 in. in diameter and weighs 0.91 lb. If its drag coefficient is $C_D = 0.2$, determine its deceleration if it has a speed of 20 ft/s at the top of its trajectory.

$\mathcal{D} = ma$, where $m = \frac{W}{g} = \frac{0.91\ lb}{32.2\ \frac{ft}{s^2}} = 0.0283\ slugs$

and

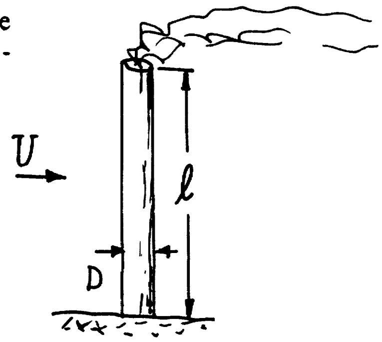
$\mathcal{D} = C_D \frac{1}{2} \rho U^2 A = 0.2\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left(20\ \frac{ft}{s}\right)^2\left(\frac{\pi}{4}\left(\frac{6.78}{12}\ ft^2\right)\right) = 0.0239\ lb$

Thus,

$a = \frac{\mathcal{D}}{m} = \frac{0.0238\ lb}{0.0283\ slugs} = \underline{\underline{0.841\ \frac{ft}{s^2}}}$

9.74

9.74 Explain how the drag on a given smokestack could be the same in a 2 mph wind as in a 4 mph wind. Assume the values of ρ and μ are the same for each case.



$$\mathcal{D} = C_D \tfrac{1}{2} \rho U^2 A = C_D \tfrac{1}{2} \rho U^2 \pi D \ell$$

Let $(\ )_1$ denote conditions with $U = 1\,\frac{m}{s}$ and $(\ )_2$ with $U = 2\,\frac{m}{s}$

Thus, with $\rho_1 = \rho_2$, to have $\mathcal{D}_1 = \mathcal{D}_2$ we have

$$C_{D_1} \tfrac{1}{2} \rho_1 U_1^2 \pi D \ell = C_{D_2} \tfrac{1}{2} \rho_2 U_2^2 \pi D \ell \quad \text{or} \quad C_{D_1} U_1^2 = C_{D_2} U_2^2$$

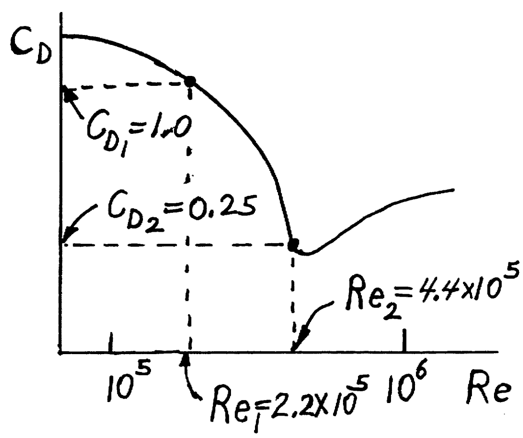
That is,

$C_{D_1} = 4 C_{D_2}$ where C_{D_1} and C_{D_2} are functions of $Re = \frac{UD}{\nu}$ as shown in Figure 9.21 (a).

Since $\nu_1 = \nu_2$ and $U_1 = \frac{1}{2} U_2$ it follows that

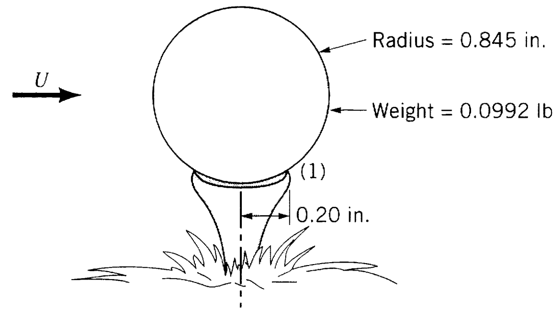
$$Re_1 = 0.5\, Re_2$$

From Fig. 9.21 (a) we can determine a value of Re_2 such that $Re_1 = 0.5\, Re_2$ and $C_{D_1} = 4 C_{D_2}$; hence the drags are equal eventhough the velocities are unequal. This occurs because of the sudden drop in C_D as the boundary layer becomes turbulent.

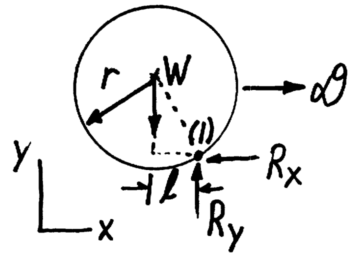


9.77

9.77 A strong wind can blow a golf ball off the tee by pivoting it about point 1 as shown in Fig. P9.77. Determine the wind speed necessary to do this.



When the ball is about to be blown from the tee the free body diagram is as shown. Hence, by summing moments about (1):

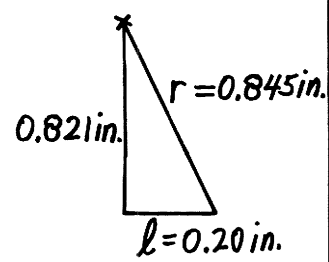


$\Sigma M_1 = 0$, or $W\ell = \mathcal{D} r$

Thus,

$(0.0992\ lb)(0.20 in.) = \mathcal{D}(0.821\ in.)$

or



$\mathcal{D} = 0.0242\ lb$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \pi r^2$

Thus,

$0.0242\ lb = C_D \frac{1}{2}(0.00238 \frac{slugs}{ft^3}) U^2 \pi \left(\frac{0.845\ in.}{12 \frac{in.}{ft}}\right)^2$

or

$C_D U^2 = 1305$, where $U \sim \frac{ft}{s}$ (1)

For a sphere* $C_D = C_D(Re)$ (see Fig. 9.25) where (2)

$$Re = \frac{\rho U D}{\mu} = \frac{(0.00238\ slugs/ft^3)\, U\, (2(0.845)/12\ ft)}{3.47 \times 10^{-7} (lb \cdot s/ft^2)}$$

or

$Re = 966\ U$, where $U \sim \frac{ft}{s}$ (3)

Trial and error solution:

Assume $C_D = 0.4$ so that from Eq. (1), $U = 57.1 \frac{ft}{s}$ and from Eq.(3), $Re = 966(57.1) = 5.52 \times 10^4$. Thus, from Fig. 9.25, $C_D \approx 0.25 \neq 0.40$ Try again.

Assume $C_D = 0.22$ so that $U = 77.0 \frac{ft}{s}$ and $Re = 7.44 \times 10^4$

Thus, from Fig. 9.25, $C_D \approx 0.22$ Checks.

Hence, $U \approx \underline{\underline{77.0 \frac{ft}{s}}}$

* golf ball (i.e. with dimples)

9.78

9.78 An airplane tows a banner that is b = 0.8 m tall and ℓ = 25 m long at a speed of 150 km/hr. If the drag coefficient based on the area $b\ell$ is C_D = 0.06, estimate the power required to tow the banner. Compare the drag force on the banner with that on a rigid flat plate of the same size. Which has the larger drag force and why?

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $A = b\ell$.

Thus, with $C_D = 0.06$ and $U = (150\ \frac{km}{hr})(\frac{1\ hr}{3600\ s})(\frac{1000m}{1\ km}) = 41.7\ \frac{m}{s}$

this gives

$\mathcal{P} = (0.06)(\frac{1}{2})(1.23\ \frac{kg}{m^3})(41.7\ \frac{m}{s})^3(0.8m)(25m) = 53.5 \times 10^3\ W = \underline{\underline{53.5\ kW}}$

For a rigid flat plate

$\mathcal{P} = \mathcal{D}U = 2C_D \frac{1}{2}\rho U^3 b\ell$ (the factor of two is needed because the drag coefficient is based on the drag on one side of the plate)

With $Re_\ell = \frac{U\ell}{\nu} = \frac{(41.7\frac{m}{s})(25m)}{1.46 \times 10^{-5}\ \frac{m^2}{s}} = 7.14 \times 10^7$ we obtain from Fig. 9.15 a value of $C_D \approx 0.0025$ for a smooth plate.

Thus,

$\mathcal{P} = 2(0.0025)(\frac{1}{2})(1.23\frac{kg}{m^3})(41.7\frac{m}{s})^3(0.8m)(25m) = 4.46 \times 10^3 W = \underline{\underline{4.46\ kW}}$

For the flat plate case the drag is relatively small because it is due entirely to shear (viscous) forces. Due to the "fluttering" of the banner, a good portion of its drag (and hence power) is a result of pressure forces. It is not as streamlined as a rigid flat plate.

9.79

9.79 By appropriate streamlining the drag coefficient for an airplane is reduced by 12% while the frontal area remains the same. For the same power output, by what percentage is the flight speed increased?

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

Let $(\;)_0$ denote the original configuration and $(\;)_s$ the streamlied one. Thus, with $\mathcal{P}_0 = \mathcal{P}_s$ we obtain

$C_{D_0} \frac{1}{2}\rho_0 U_0^2 A_0 U_0 = C_{D_s} \frac{1}{2}\rho_s U_s^2 A_s U_s$ or with $A_0 = A_s$, $\rho_0 = \rho_s$

$U_0^3 C_{D_0} = U_s^3 C_{D_s}$ Thus, $\frac{U_s}{U_0} = \left[\frac{C_{D_0}}{C_{D_s}}\right]^{\frac{1}{3}} = \left[\frac{C_{D_0}}{C_{D_0} - 0.12 C_{D_0}}\right]^{\frac{1}{3}} = 1.0435$

i.e., a 4.35 % speed increase

Note: $\mathcal{P} \sim U^3 C_D$ so that $\delta\mathcal{P} = 3U^2 C_D \delta U + U^3 \delta C_D$. Thus, with $\delta\mathcal{P} = 0$, this gives $\frac{\delta U}{U} = -\frac{1}{3}\frac{\delta C_D}{C_D} = -\frac{-0.12}{3} = +0.04 = 4\%$

9.80

9.80 The dirigible Akron had a length of 239 m and a maximum diameter of 40.2 m. Estimate the power required at its maximum speed of 135 km/hr if the drag coefficient based on frontal area is 0.060.

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

Thus, with $U = (135 \frac{km}{hr})(\frac{1\,hr}{3600\,s})(\frac{1000\,m}{1\,km}) = 37.5 \frac{m}{s}$

$\mathcal{P} = C_D \frac{1}{2}\rho U^3 A$

$= (0.060)(\frac{1}{2})(1.23 \frac{kg}{m^3})(37.5 \frac{m}{s})^3 (\frac{\pi}{4})(40.2\,m)^2 = 2.47 \times 10^6 \frac{kg \cdot m^2}{s^3}$

or

$\mathcal{P} = 2.47 \times 10^3\,kW \; (1.34 \frac{hp}{kW}) = 3310\,hp$

9.81 Estimate the power needed to overcome the aerodynamic drag of a person who runs at a rate of 100 yds in 10 s in still air. Repeat the calculations if the race is run into a 20-mph headwind; a 20-mph tailwind. Explain.

In still air $\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $U = \frac{300\,ft}{10\,s} = 30\,\frac{ft}{s}$.
From Fig. 9.30, $C_D A \approx 9\,ft^2$

Thus, $\mathcal{D} = \left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left(30\,\frac{ft}{s}\right)^2\left(9\,ft^2\right) = 9.64\,lb$

and

$$\mathcal{P} = (9.64\,lb)\left(30\,\frac{ft}{s}\right)\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{0.526\,hp}}$$

Into a $20\,mph = 29.3\,\frac{ft}{s}$ headwind $\mathcal{P} = \mathcal{D}U_r$, where $U_r = 30\,\frac{ft}{s}$ = runner's speed

and $\mathcal{D} = C_D \frac{1}{2}\rho\left(U_r + 29.3\,\frac{ft}{s}\right)^2 A$ or

$$\mathcal{D} = \left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)(30+29.3)^2\,\frac{ft^2}{s^2}\left(9\,ft^2\right) = 37.7\,lb$$

Thus,

$$\mathcal{P} = (37.7\,lb)\left(30\,\frac{ft}{s}\right)\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{2.06\,hp}}$$

With a $20\,mph = 29.3\,\frac{ft}{s}$ tailwind the relative headwind that the runner feels is $U_r - 29.3\,\frac{ft}{s} = (30 - 29.3)\,\frac{ft}{s} = 0.7\,\frac{ft}{s}$

Thus,

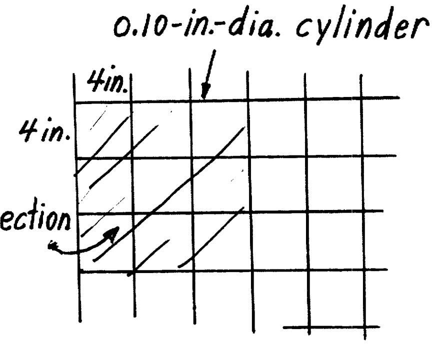
$$\mathcal{D} = \left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left(0.7\,\frac{ft}{s}\right)^2\left(9\,ft^2\right) = 0.00525\,lb$$

Thus,

$$\mathcal{P} = (0.00525\,lb)\left(30\,\frac{ft}{s}\right)\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{0.000286\,hp}}$$

Note: The tailwind essentially cancels the relative wind speed produced by the runner's forward motion.

9.83 A fishnet consists of 0.10-in.-diameter strings tied into squares 4 in. per side. Estimate the force needed to tow a 15 ft by 30 ft section of this net through seawater at 5 ft/s.



The net can be treated as one long 0.10-in.-diameter circular cylinder with

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$, where $U = 5\,\frac{ft}{s}$. Each 1 ft² section of the net contains 6 feet of string (donot count the edges twice). Thus, the total string length is approximately $\ell = (6\,\frac{ft}{ft^2})(15\,ft)(30\,ft) = 2700\,ft$

Also, since $\rho = 1.99\,\frac{slugs}{ft^3}$ and $\nu = 1.26\times10^{-5}\,\frac{ft^2}{s}$ (see Table 1.5)

$$Re = \frac{UD}{\nu} = \frac{(5\,\frac{ft}{s})(\frac{0.10}{12}\,ft)}{1.26\times10^{-5}\,\frac{ft^2}{s}} = 3310$$

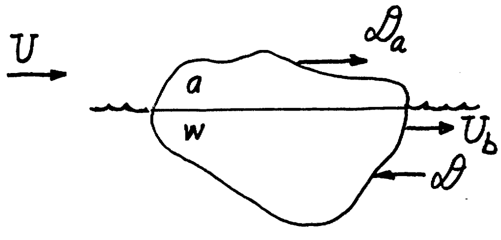
Hence, from Fig. 9.21 that $C_D = 1.1$

Thus,

$$\mathcal{D} = (1.1)(\tfrac{1}{2})(1.99\,\tfrac{slugs}{ft^3})(5\,\tfrac{ft}{s})^2(\tfrac{0.1}{12}\,ft)(2700\,ft) = \underline{\underline{616\,lb}}$$

9.84

9.84 An iceberg floats with approximately $\frac{1}{7}$ of its volume in the air as is shown in Fig. P9.84. If the wind velocity is U and the water is stationary, estimate the speed at which the wind forces the iceberg through the water.



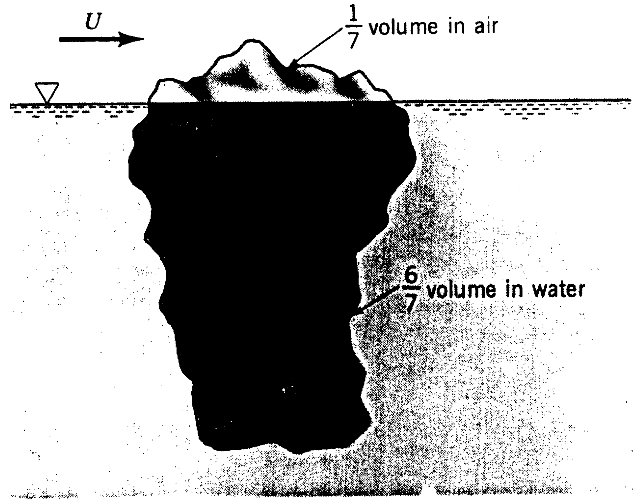


FIGURE P9.84

Let $(\ )_a$ denote the portion of the iceberg in the air and $(\ )_w$ that portion in the water.

Thus, $\forall_a = \frac{1}{7}\forall$ and $\forall_w = \frac{6}{7}\forall$, where $\forall$ = volume of the iceberg

For steady motion, $\mathcal{D}_a = \mathcal{D}_w$, where $\mathcal{D}_a = C_{D_a}\frac{1}{2}\rho_a(U-U_b)^2 A_a$

and $\mathcal{D}_w = C_{D_w}\frac{1}{2}\rho_w U_b^2 A_w$

with U_b = speed of the iceberg

Thus,

$$C_{D_a}\frac{1}{2}\rho_a(U-U_b)^2 A_a = C_{D_w}\frac{1}{2}\rho_w U_b^2 A_w \quad \text{or}$$

$$\frac{(U-U_b)^2}{U_b^2} = \frac{C_{Dw}\rho_w A_w}{C_{Da}\rho_a A_a} = \frac{\rho_w A_w}{\rho_a A_a} \quad \text{if we assume } C_{Da} = C_{Dw} \qquad (1)$$

If D is a characteristic length, then $\forall \sim D^3$ and $A \sim D^2$

Hence, $\frac{\forall_a}{\forall_w} = \frac{\frac{1}{7}\forall}{\frac{6}{7}\forall} = \frac{D_a^3}{D_w^3}$, or $\frac{D_a}{D_w} = \left(\frac{1}{6}\right)^3$

so that

$$\frac{A_a}{A_w} = \left(\frac{D_a}{D_w}\right)^2 = \left(\frac{1}{6}\right)^{\frac{2}{3}}$$

Thus, from Eq.(1)

$$\frac{(U-U_b)^2}{U_b^2} = \frac{(1.99\ \frac{slugs}{ft^3})}{(0.00238\ \frac{slugs}{ft^3})}(6)^{2/3} = 2760.$$

or

$$\frac{U}{U_b} = 53.5 \qquad \text{Thus,} \quad \underline{\underline{U_b = 0.0187\,U}}$$

9.85

9.85 A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft². Determine the lift coefficient of this airplane for these conditions.

For equilibrium $\mathcal{L} = W = 1750\ lb$, where $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, with $U = (115\ mph)\dfrac{(88\ \frac{ft}{s})}{(60\ mph)} = 169\ \frac{ft}{s}$

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{1750\ lb}{\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(169\ \frac{ft}{s})^2(179\ ft^2)} = \underline{\underline{0.288}}$$

9.86

9.86 A light aircraft with a wing area of 200 ft² and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium $\mathcal{L} = W = 2000\ lb = C_L \frac{1}{2}\rho U^2 A$

or

$$2000\ lb = (0.40)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})\,U^2(200\ ft^2)$$

Hence,

$$U = 145\ \frac{ft}{s}$$

Also, $\mathcal{P} = \text{power} = \mathcal{D}\,U$, where

$$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = (0.05)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(145\ \frac{ft}{s})^2(200\ ft^2) = 250\ lb$$

Note: This value of $\mathcal{D}$ could be obtained from

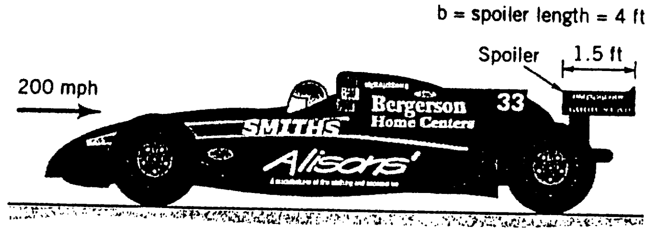
$$\frac{W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8, \text{ or } \mathcal{D} = \frac{W}{8} = \frac{2000\ lb}{8} = 250\ lb$$

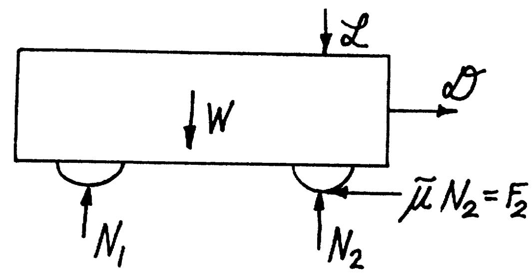
Thus,

$$\mathcal{P} = 250\ lb\,(145\ \frac{ft}{s}) = 3.63 \times 10^4\ \frac{ft \cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right) = \underline{\underline{65.9\ hp}}$$

9.87

9.87 As shown in Video V9.9 and Fig. P9.87, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is $C_L = 1.1$ and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.





Tractive force $= F_2 = \tilde{\mu} N_2$

where $\tilde{\mu}$ = coefficient of friction $= 0.6$

Thus,

$\Delta F_2 = \tilde{\mu}\, \Delta N_2 = \tilde{\mu}\, \mathcal{L}$, where ΔF_2 is the increase in tractive force due to the (downward) lift.

Hence, with $U = 200\,mph = 293\,ft/s$,

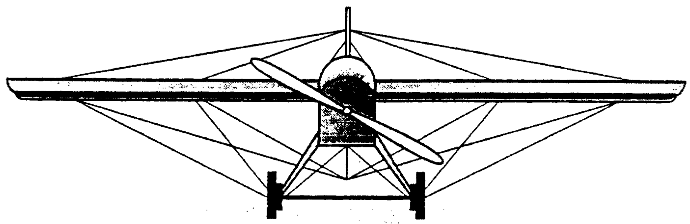
$$\mathcal{L} = \frac{1}{2}\rho U^2 C_L A = \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(293\,\frac{ft}{s})^2(1.1)(1.5\,ft)(4\,ft) = 674\,lb,$$

and

$\Delta F_2 = 0.6\,(674\,lb) = \underline{\underline{405\,lb}}$

9.88

9.88 The wings of old airplanes are often strengthened by the use of wires that provided cross-bracing as shown in Fig. P9.88. If the drag coefficient for the wings was 0.020 (based on the planform area), determine the ratio of the drag from the wire bracing to that from the wings.



Speed: 70 mph
Wing area: 148 ft^2
Wire: length = 160 ft
diameter = 0.05 in.

■ FIGURE P9.88

$\mathcal{D}_{wing} = \frac{1}{2}\rho U^2 C_{D_{wing}} A_{wing}$
and
$\mathcal{D}_{wire} = \frac{1}{2}\rho U^2 C_{D_{wire}} A_{wire}$ so that

$\frac{\mathcal{D}_{wire}}{\mathcal{D}_{wing}} = \frac{C_{D_{wire}} A_{wire}}{C_{D_{wing}} A_{wing}}$, where $A_{wing} = 148\, ft^2$, $C_{D_{wing}} = 0.02$

Also, $A_{wire} = \ell D = (160\, ft)\left(\frac{0.05}{12}\, ft\right) = 0.667\, ft^2$

and since
$Re = \frac{UD}{\nu} = \frac{(70\,mph)\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right)\left(\frac{0.05}{12}\,ft\right)}{1.57\times 10^{-4}\,\frac{ft^2}{s}} = 2720.$

From Fig. 9.21, with $Re = 2720$ we obtain $C_D = 1.0$

Hence,

$\frac{\mathcal{D}_{wire}}{\mathcal{D}_{wing}} = \frac{(1.0)(0.667\,ft^2)}{(0.02)(148\,ft^2)} = 0.225$, or $\underline{\underline{22.5\,\%}}$

9.89 The jet engines on a Boeing 757 must develop a certain amount of power to propel the airplane through the air with a speed of 570 mph at a cruising altitude of 35,000 ft. By what percent must the power be increased if the same airplane were to maintain its 570 mph flight speed at sea level?

$\mathcal{P}$ = power $= \mathcal{D}U = \frac{1}{2}\rho U^3 C_D A$

Let $(\ )_0$ and $(\ )_{35}$ denote conditions at sea level and 35,000 ft, respectively. Thus, $U_0 = U_{35}$ so that

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{\frac{1}{2}\rho_0 U_0^3 C_{D_0} A_0}{\frac{1}{2}\rho_{35} U_{35}^3 C_{D_{35}} A_{35}}$$ so if $A_0 = A_{35}$ and $C_{D_0} = C_{D_{35}}$, then

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{\rho_0}{\rho_{35}}$$, or with ρ values from Table C.1,

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{0.00238\ \frac{slugs}{ft^3}}{0.000738\ \frac{slugs}{ft^3}} = 3.22 = \underline{\underline{322\%\ \text{increase}}}$$

9.90

9.90 A wing generates a lift $\mathscr{L}$ when moving through sea-level air with a velocity U. How fast must the wing move through the air at an altitude of 35,000 ft with the same lift coefficient if it is to generate the same lift?

$\mathscr{L} = C_L \frac{1}{2} \rho U^2 A$ so with $\mathscr{L}$, C_L, and A constant

$$(\rho U^2)_{sea\ level} = (\rho U^2)_{35{,}000\,ft}$$

Hence,

$$U_{35{,}000\,ft} = \left(\frac{\rho_{sea\ level}}{\rho_{35{,}000\,ft}} \right)^{1/2} U_{sea\ level} = \left(\frac{2.38 \times 10^{-3}\ \frac{slug}{ft^3}}{7.38 \times 10^{-4}\ \frac{slug}{ft^3}} \right)^{1/2} U_{sea\ level}$$

or

$$U_{35{,}000\,ft} = \underline{\underline{1.80\ U_{sea\ level}}}$$

9.91 *

9.91 * When air flows past the airfoil shown in Fig. P9.91 the velocity just outside the boundary layer, u, is as indicated. Estimate the lift coefficient for these conditions.

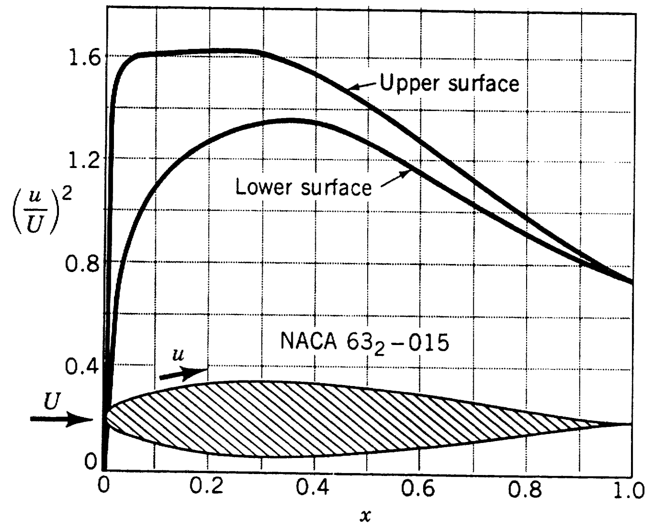


FIGURE P9.91

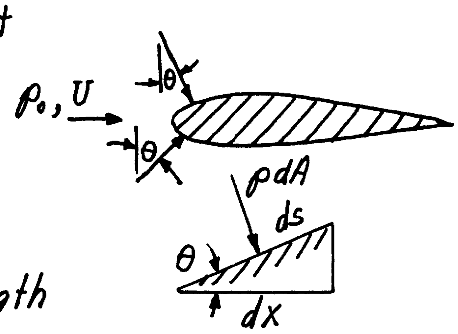
If shear forces are negligible

$\mathcal{L} = \int \pm p \cos\theta \, dA$, where the + sign is used on the lower surface; - sign on upper surface. Also,

$p = p_0 + \frac{1}{2}\rho U^2 - \frac{1}{2}\rho u^2$. Since the constant atmospheric pressure does not contribute to the lift, we set $p_0 = 0$

Thus, $p = \frac{1}{2}\rho U^2\left[1 - \left(\frac{u}{U}\right)^2\right]$ so that

$\mathcal{L} = \int \frac{\pm 1}{2}\rho U^2\left[1 - \left(\frac{u}{U}\right)^2\right]\cos\theta \, dA$



However, $dA = \ell \, ds$ where ℓ = wing length

or $\cos\theta \, dA = \cos\theta \, \ell ds = \ell \, dx$

Hence,

$$\mathcal{L} = -\frac{1}{2}\rho U^2 \int_{upper}\left[1 - \left(\frac{u}{U}\right)^2\right]\ell \, dx + \frac{1}{2}\rho U^2 \int_{lower}\left[1 - \left(\frac{u}{U}\right)^2\right]\ell \, dx$$

or

$$\mathcal{L} = \frac{1}{2}\rho U^2 \ell \int\left[\left(\frac{u}{U}\right)^2_{upper} - \left(\frac{u}{U}\right)^2_{lower}\right]dx$$

Also, since $C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 \ell c}$, where c = chord length

it follows that

$$C_L = \int_{\frac{x}{c}=0}^{1}\left[\left(\frac{u}{U}\right)^2_{upper} - \left(\frac{u}{U}\right)^2_{lower}\right]d\left(\frac{x}{c}\right)$$

This integral is obtained by numerical integration of the data given in the figure.

The following table of data is obtained:

(con't)

9.91 * (con't)

$\frac{x}{c}$	$\left[\left(\frac{u}{V}\right)^2_{upper} - \left(\frac{u}{V}\right)^2_{lower}\right]$
0	0
0.05	0.65
0.10	0.52
0.15	0.41
0.20	0.36
0.25	0.32
0.30	0.29
0.35	0.24
0.40	0.20
0.45	0.18
0.50	0.17
0.55	0.15
0.60	0.14
0.65	0.12
0.70	0.11
0.75	0.09
0.80	0.07
0.85	0.05
0.90	0.03
0.95	0.01
1.00	0

By using the program TRAPEZOI with the above values of the integrand we obtain

```
*****************************************************
** This program performs numerical integration     **
** over a set of points using the Trapezoidal Rule **
*****************************************************

Enter number of data points: 21

Enter data points (X , Y)
? 0,0
? 0.05,0.65
? 0.10,0.52
? 0.15,0.41
? 0.20,0.36
? 0.25,0.32
? 0.30,0.29
? 0.35,0.24
? 0.40,0.20
? 0.45,0.18
? 0.50,0.17
? 0.55,0.15
? 0.60,0.14
? 0.65,0.12
? 0.70,0.11
? 0.75,0.09
? 0.80,0.07
? 0.85,0.05
? 0.90,0.03
? 0.95,0.01
? 1.00,0

The approximate value of the integral is: +2.0550E-01
```

Thus, $C_L = \underline{\underline{0.206}}$

9.93

9.93 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A = W$ (1)

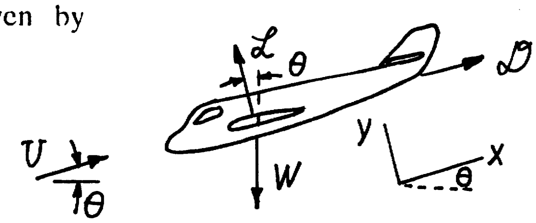
Let $(\ )_{100}$ denote conditions with 100 passengers and $(\ )_{372}$ with 372 passengers. Thus, with $C_{L_{100}} = C_{L_{372}}$, $A_{100} = A_{372}$, and $\rho_{100} = \rho_{372}$ Eq. (1) gives

$$\frac{\mathcal{L}_{100}}{\mathcal{L}_{372}} = \frac{U_{100}^2}{U_{372}^2} \quad \text{or} \quad U_{372} = U_{100}\left\{\frac{[580{,}000 + (372-100)(200)]\ lb}{580{,}000\ lb}\right\}^{1/2}, \text{ with } U_{100} = 140\ mph$$

Thus, $U_{372} = \underline{\underline{146\ mph}}$

9.94

9.94 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, θ, is given by $\tan\theta = C_D/C_L$.



For steady unpowered flight

$\Sigma F_x = 0$ gives $\mathcal{D} = W\sin\theta$

and

$\Sigma F_y = 0$ gives $\mathcal{L} = W\cos\theta$

Thus,

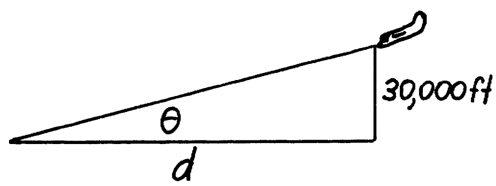
$$\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta, \text{ where } \frac{\mathcal{D}}{\mathcal{L}} = \frac{\frac{1}{2}\rho U^2 A C_D}{\frac{1}{2}\rho U^2 A C_L} = \frac{C_D}{C_L}$$

Hence, $\underline{\underline{\tan\theta = \frac{C_D}{C_L}}}$

9.95

9.95 If the lift coefficient for a Boeing 777 aircraft is 15 times greater than its drag coefficient, can it glide from an altitude of 30,000 ft to an airport 80 mi away if it loses power from its engines? Explain. (See Problem 9.94.)

From Problem 9.94, $\tan\theta = \frac{C_D}{C_L} = \frac{1}{15}$



Hence,

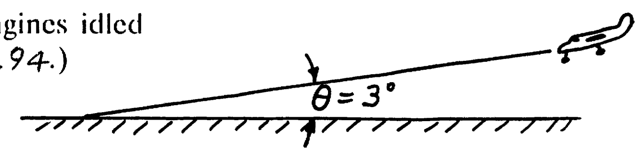
$\frac{30{,}000}{d} = \frac{1}{15}$, or $d = 4.5\times10^5\,ft$

$= 85.2\,mi$

Hence, the plane can glide 80 mi.

9.96

9.96 On its final approach to the airport an airplane flies on a flight path that is 3.0° relative to the horizontal. What lift-to-drag ratio is needed if the airplane is to land with its engines idled back to zero power? (See Problem 9.94.)



From Problem 9.94,

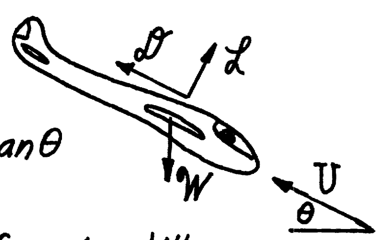
$\tan\theta = \frac{C_D}{C_L}$

or

$\frac{C_D}{C_L} = \tan 3° = 0.0524$ $\qquad \frac{C_L}{C_D} = 19.1$

9.97

9.97 A sail plane with a lift-to-drag ratio of 25 flies with a speed of 50 mph. It maintains or increases its altitude by flying in thermals, columns of vertically rising air produced by buoyancy effects of nonuniformly heated air. What vertical air speed is needed if the sail plane is to maintain a constant altitude?



With no vertical air motion the sailplane would glide with a slope angle θ, where since $\Sigma \vec{F} = 0$
$\mathcal{D} = \mathcal{W} \sin\theta$ and $\mathcal{L} = \mathcal{W}\cos\theta$. Hence, $\frac{\mathcal{D}}{\mathcal{L}} = \frac{\mathcal{W}\sin\theta}{\mathcal{W}\cos\theta} = \tan\theta$
or since $\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$ and
$\mathcal{L} = \frac{1}{2}\rho U^2 C_L A$ it follows that $\tan\theta = \frac{C_D}{C_L}$. Therefore in still air the sailplane would loose altitude at a rate of $U\sin\theta$, where
$\theta = \tan^{-1}\left(\frac{C_D}{C_L}\right) = \tan^{-1}\left(\frac{1}{25}\right) = 2.29°$. Hence, an upward wind of $(50\,mph)\sin 2.29°$
$= \underline{\underline{2.00\ mph}}$ will allow horizontal flight.

9.98

9.98 Over the years there has been a dramatic increase in the flight speed (U) and altitude (h), weight ($\mathcal{W}$), and wing loading ($\mathcal{W}/A$ = weight divided by wing area) of aircraft. Use the data given in the table below to determine the lift coefficient for each of the aircraft listed.

Aircraft	Year	$\mathcal{W}$, lb	U, mph	$\mathcal{W}/A$, lb/ft^2	h, ft
Wright Flyer	1903	750	35	1.5	0
Douglas DC-3	1935	25,000	180	25.0	10,000
Douglas DC-6	1947	105,000	315	72.0	15,000
Boeing 747	1970	800,000	570	150.0	30,000

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{2}{\rho U^2}\left(\frac{W}{A}\right)$$

Thus,

	ρ, $slugs/ft^3$	U, ft/s	W/A, lb/ft^2	C_L
Wright Flyer	2.38×10^{-3}	51.3	1.5	0.480
DC-3	1.76×10^{-3}	264	25.0	0.409
DC-6	1.50×10^{-3}	462	72.0	0.451
747	8.91×10^{-4}	836	150	0.482

9.99

9.99 The landing speed of an airplane such as the Space Shuttle is dependent on the air density. (See Video V9.1.) By what percent must the landing speed be increased on a day when the temperature is 110 deg F compared to a day when it is 50 deg F? Assume the atmospheric pressure remains constant.

For equalibrium, lift = weight, or

$$\frac{1}{2}\rho U^2 C_L A = W$$

Thus, with constant W, C_L, and A,

$$(\rho U^2)_{T=110°} = (\rho U^2)_{T=50°} \quad \text{or}$$

$$U_{110°} = \left(\frac{\rho_{50}}{\rho_{110}}\right)^{\frac{1}{2}} U_{50°}$$

But $p = \rho R T$ so that $\frac{\rho_{50}}{\rho_{110}} = \frac{(p_{50}/RT_{50})}{(p_{110}/RT_{110})} = \frac{(460+110)}{(460+50)} = 1.1176$

Thus,

$$U_{110°} = \sqrt{1.1176}\, U_{50°} = 1.0572\, U_{50°} \quad \text{or a } \underline{\underline{5.72\% \text{ increase}}}$$

9.100

9.100 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight W = aircraft weight $= \mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, for given W, C_L, and A the dynamic pressure is constant, independent of altitude. That is

$$\left.\tfrac{1}{2}\rho U^2\right)_{10,000\,ft} = \left.\tfrac{1}{2}\rho U^2\right)_{30,000\,ft}, \text{ or } U_{30,000} = \left(\frac{\rho_{10,000}}{\rho_{30,000}}\right)^{\frac{1}{2}} U_{10,000}$$

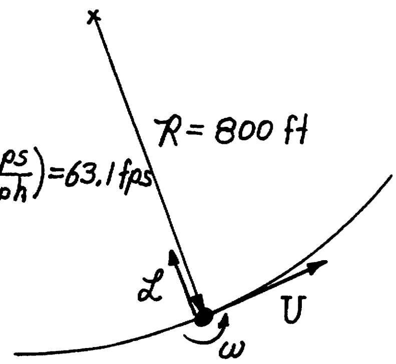
Hence, $U_{30,000} > U_{10,000}$

Also, since the drag is $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ it follows that

$$\mathcal{D}_{30,000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right)_{30,000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right)_{10,000} \text{ since } \tfrac{1}{2}\rho U^2_{30,000} = \tfrac{1}{2}\rho U^2_{10,000}$$

Hence, the aircrart can fly faster at highe altituded with the same amount of drag ($\mathcal{D}_{30,000} = \mathcal{D}_{10,000}$)

9.102 Repeated controversy regarding the ability of a baseball to curve appeared in the literature for years. According to a test (*Life*, July 27, 1953) a baseball (assume the diameter is 2.9 in. and weight 5.25 oz) spinning 1400 rpm while traveling 43 mph was observed to follow a path with an 800-ft horizontal radius of curvature. Based on the data of Fig. 9.39 do you agree with this test result? Explain.



For steady motion along the curved path $\Sigma F_r = m\,a_r$

or

$$\mathcal{L} = m\frac{U^2}{\mathcal{R}} = \frac{W}{g}\frac{U^2}{\mathcal{R}}, \text{ where } U = (43\,\text{mph})\left(\frac{88\,\text{fps}}{60\,\text{mph}}\right) = 63.1\,\text{fps}$$

Thus,

$$\mathcal{L} = \frac{\left(\frac{5.25}{16}\,\text{lb}\right)\left(63.1\,\frac{\text{ft}}{\text{s}}\right)^2}{\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(800\,\text{ft})} = 0.0507\ \text{lb}$$

But,

$$\mathcal{L} = C_L\,\tfrac{1}{2}\rho U^2 A = C_L\,\tfrac{1}{2}\rho U^2 \tfrac{\pi}{4} D^2$$

or

$$C_L = \frac{8\mathcal{L}}{\pi\rho U^2 D^2} = \frac{8\,(0.0507\,\text{lb})}{\pi\left(0.00238\,\frac{\text{slug}}{\text{ft}^3}\right)\left(63.1\,\frac{\text{ft}}{\text{s}}\right)^2\left(\frac{2.9}{12}\,\text{ft}\right)^2} = 0.233 \qquad (1)$$

From Fig. 9.39 with

$$\frac{\omega D}{2U} = \frac{\left(1400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{2.9}{12}\,\text{ft}\right)}{2\left(63.1\,\frac{\text{ft}}{\text{s}}\right)} = 0.281 \text{ we obtain}$$

$C_L \approx 0.08$ which is less than the $C_L = 0.233$ in Eq. (1).

Hence a smooth sphere would not curve as much as indicated, but perhaps a rough ball (i.e., one with seams) would.

9.103 Boundary Layer on a Flat Plate

Objective: A boundary layer is formed on a flat plate when air blows past the plate. The thickness, δ, of the boundary layer increases with distance, x, from the leading edge of the plate. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.103, to measure the boundary layer thickness.

Equipment: Wind tunnel; flat plate; boundary layer mouse consisting of ten Pitot tubes positioned at various heights, y, above the flat plate; inclined multiple manometer; measuring calipers; barometer, thermometer.

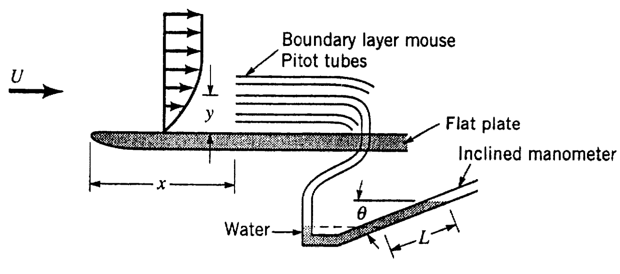
Experimental Procedure: Position the tips of the Pitot tubes of the boundary layer mouse a known distance, x, downstream from the leading edge of the plate. Use calipers to determine the distance, y, between each Pitot tube and the plate. Fasten the tubing from each Pitot tube to the inclined multiple manometer and determine the angle of inclination, θ, of the manometer board. Adjust the wind tunnel speed, U, to the desired value and record the manometer readings, L. Move the boundary layer mouse to a new distance, x, downstream from the leading edge of the plate and repeat the measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each distance, x, from the leading edge, use the manometer data to determine the air speed, u, as a function of distance, y, above the plate (see Eq. 3.13). That is, obtain $u = u(y)$ at various x locations. Note that both the wind tunnel test section and the open end of the manometer tubes are at atmospheric pressure.

Graph: Plot speed, u, as ordinates and distance from the plate, y, as abscissas for each location, x, tested.

Results: Use the $u = u(y)$ results to determine the approximate boundary layer thickness as a function of distance, $\delta = \delta(x)$. Plot a graph of boundary layer thickness as a function of distance from the leading edge. Note that the air flow within the wind tunnel is quite turbulent so that the measured boundary layer thickness is not expected to match the theoretical laminar boundary layer thickness given by the Blassius solution (see Eq. 9.15).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.103

(con't)

9.103 (con't)

Solution for Problem 9.103: Boundary Layer on a Flat Plate

θ, deg	H_{atm}, in. Hg	T, deg F	γ_{H2O}, lb/ft^3
25	29.09	80	62.4

y, in.	L, in.	u, ft/s	y, in.	L, in.	u, ft/s
Data for x = 7.75 in.			Data for x = 3.75 in.		
0.020	0.20	19.9	0.020	0.15	17.2
0.035	0.35	26.3	0.035	0.35	26.3
0.044	0.48	30.8	0.044	0.45	29.8
0.060	0.70	37.2	0.060	0.71	37.5
0.096	0.95	43.4	0.096	1.20	48.7
0.110	1.06	45.8	0.110	1.30	50.7
0.138	1.21	48.9	0.138	1.56	55.6
0.178	1.44	53.4	0.178	1.77	59.2
0.230	1.70	58.0	0.230	1.95	62.1
0.270	1.85	60.5	0.270	2.00	62.9
Data for x = 5.75 in.			Data for x = 1.75 in.		
0.020	0.20	19.9	0.020	0.20	19.9
0.035	0.42	28.8	0.035	0.50	31.5
0.044	0.50	31.5	0.044	0.68	36.7
0.060	0.71	37.5	0.060	0.90	42.2
0.096	0.98	44.0	0.096	1.51	54.7
0.110	1.06	45.8	0.110	1.70	58.0
0.138	1.30	50.7	0.138	1.90	61.3
0.178	1.54	55.2	0.178	1.95	62.1
0.230	1.76	59.0	0.230	2.00	62.9
0.270	1.88	61.0	0.270	2.00	62.9

$\rho u^2/2 = \gamma_{H2O}$*L $\sin\theta$
where
$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O}$*H_{atm} = 847 lb/ft^3*(29.09/12 ft) = 2053 lb/ft^2
R = 1716 ft lb/slug deg R
T = 80 + 460 = 540 deg R

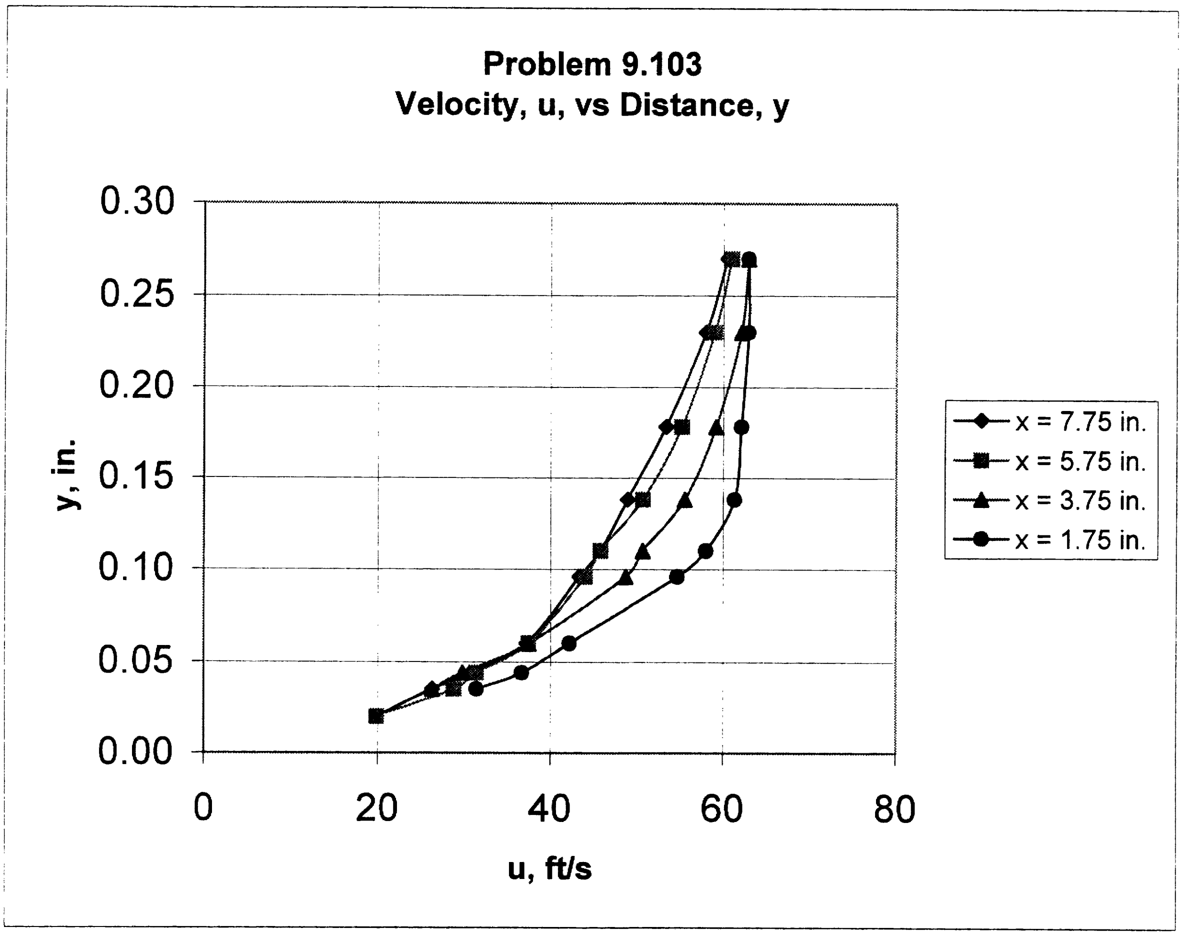
Thus, ρ = 0.00222 slug/ft^3

Approximate boundary layer thickness as obtained from the graph:

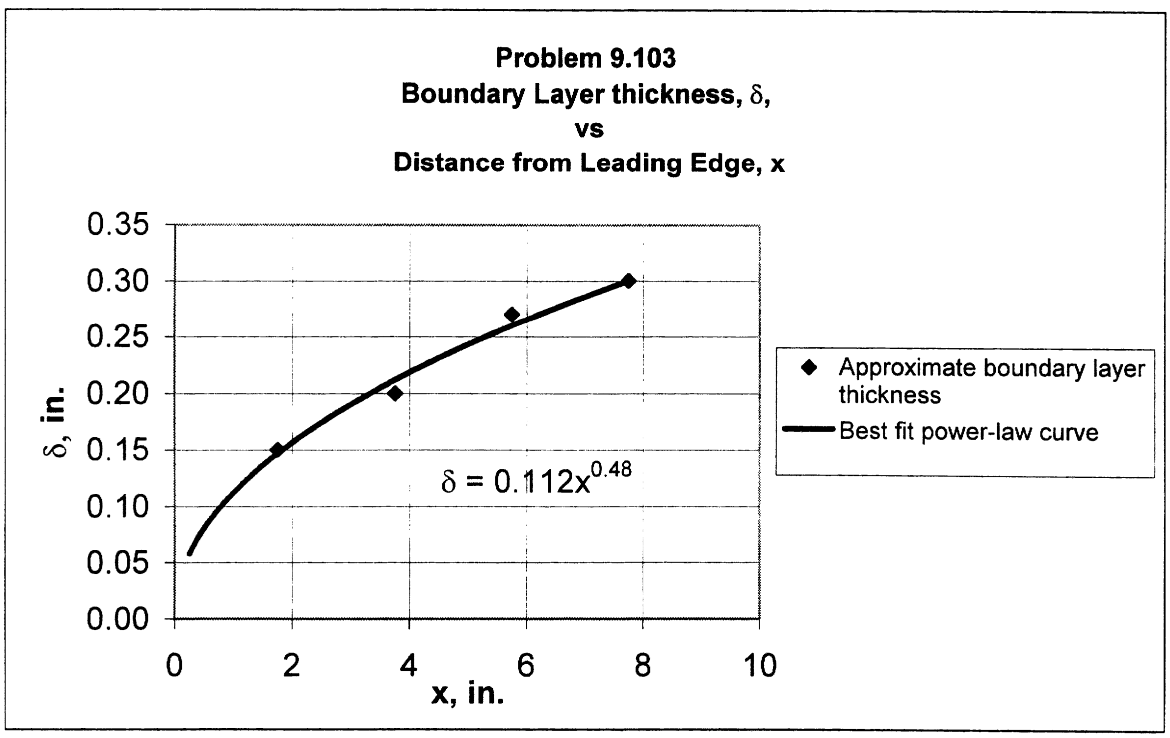
x, in.	δ, in.
1.75	0.15
3.75	0.20
5.75	0.27
7.75	0.30

(con't)

9.103 (con't)



Problem 9.103
Velocity, u, vs Distance, y
0.30
0.25
0.20
0.15
0.10
0.05
0.00
y, in.
0
20
40
60
80
u, ft/s
x = 7.75 in.
x = 5.75 in.
x = 3.75 in.
x = 1.75 in.



Problem 9.103
Boundary Layer thickness, δ,
vs
Distance from Leading Edge, x
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
δ, in.
0
2
4
6
8
10
x, in.
δ = 0.112x^0.48
Approximate boundary layer thickness
Best fit power-law curve

9.104

9.104 Pressure Distribution on a Circular Cylinder

Objective: Viscous effect within the boundary layer on a circular cylinder cause boundary layer separation, thereby causing the pressure distribution on the rear half of the cylinder to be different than that on the front half. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.104, to determine the pressure distribution on a circular cylinder.

Equipment: Wind tunnel; circular cylinder with 18 static pressure taps arranged equally from the front to the back of the cylinder; inclined multiple manometer; barometer; thermometer.

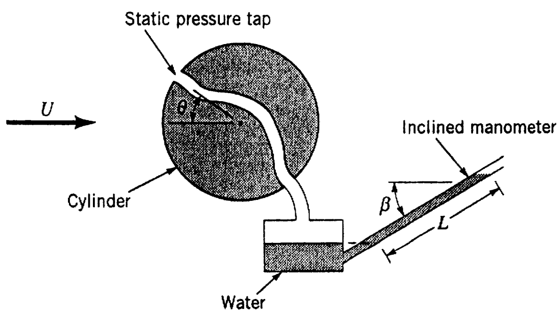
Experimental Procedure: Mount the circular cylinder in the wind tunnel so that a static pressure tap points directly upstream. Measure the angle, β, of the inclined manometer. Adjust the wind tunnel fan speed to give the desired free stream speed, U, in the test section. Attach the tubes from the static pressure taps to the multiple manometer and record the manometer readings, L, as a function of angular position, θ. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the data to determine the pressure coefficient, $C_p = (p - p_0)/(\rho U^2/2)$, as a function of position, θ. Here $p_0 = 0$ is the static pressure upstream of the cylinder in the free stream of the wind tunnel, and $p = \gamma_m L \sin\beta$ is the pressure on the surface of the cylinder.

Graph: Plot the pressure coefficient, C_p, as ordinates and the angular location, θ, as abscissas.

Results: On the same graph, plot the theoretical pressure coefficient, $C_p = 1 - 4\sin^2\theta$, obtained from ideal (inviscid) theory (see Section 6.6.3).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.104

(con't)

9.104 (con't)

Solution for Problem 9.104: Pressure Distribution on a Circular Cylinder

β, deg	H_{atm}, in. Hg	T, deg F	U, ft/s
25	29.97	75	47.9

θ, deg	L, in.	Experiment p, lb/ft^2	Experiment C_p	Theory C_p
0	1.2	2.64	1.00	1.00
10	1.1	2.42	0.92	0.88
20	0.7	1.54	0.58	0.53
30	0.1	0.22	0.08	0.00
40	-0.6	-1.32	-0.50	-0.65
50	-1.6	-3.52	-1.33	-1.35
60	-2.4	-5.27	-2.00	-2.00
70	-3.1	-6.81	-2.58	-2.53
80	-3.0	-6.59	-2.50	-2.88
90	-2.7	-5.93	-2.25	-3.00
100	-2.7	-5.93	-2.25	-2.88
110	-2.6	-5.71	-2.17	-2.53
120	-2.6	-5.71	-2.17	-2.00
130	-2.6	-5.71	-2.17	-1.35
140	-2.6	-5.71	-2.17	-0.65
150	-2.6	-5.71	-2.17	0.00
160	-2.7	-5.93	-2.25	0.53
170	-2.7	-5.93	-2.25	0.88
180	-2.8	-6.15	-2.33	1.00

$p = \gamma_{H20} * L \sin\beta$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.97/12 ft) = 2115 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

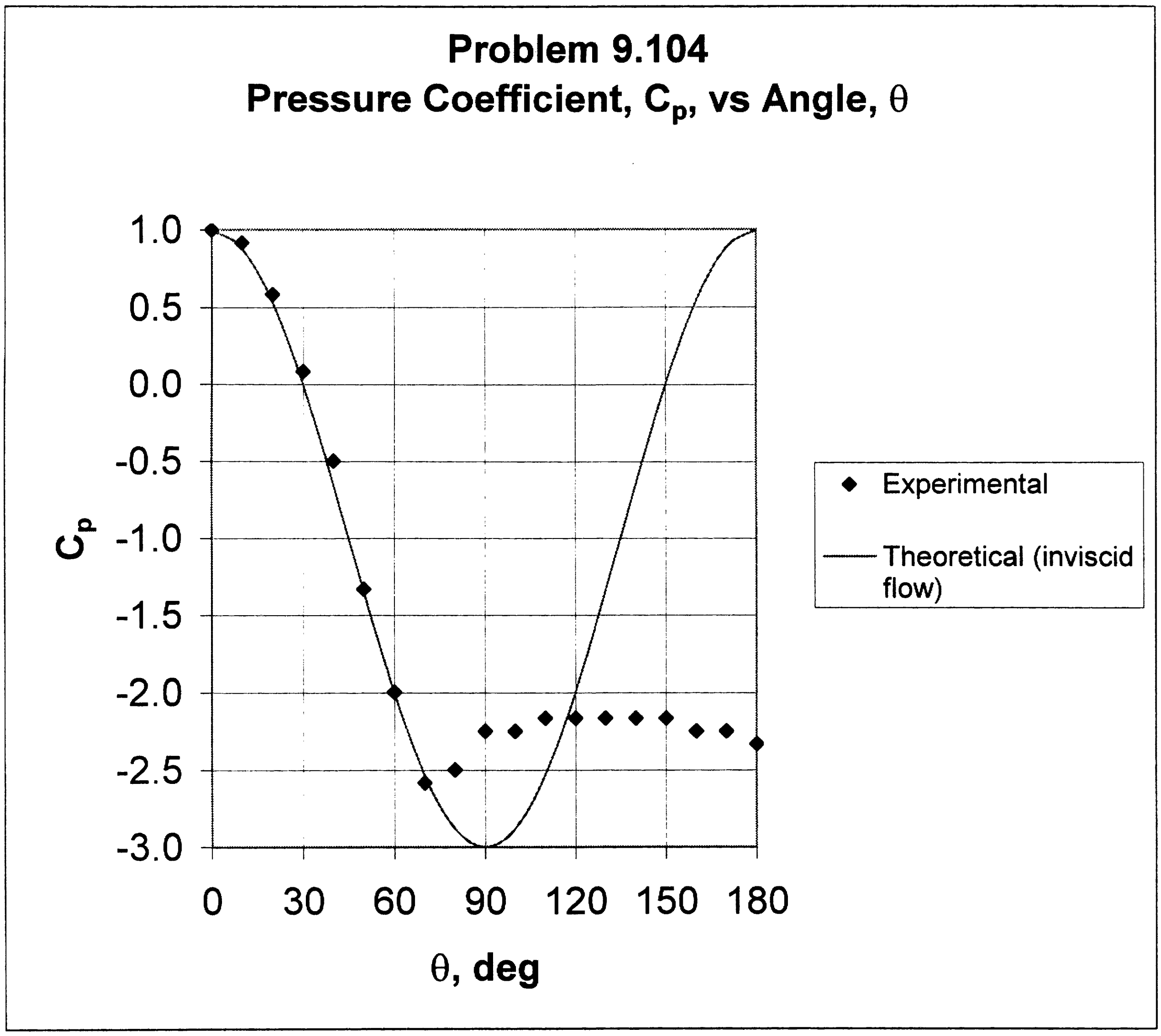
Thus, ρ = 0.00230 slug/ft^3

$C_p = p/(\rho U^2/2)$

Theory: $C_p = 1 - 4\sin^2\theta$

(con't)

9.104 (con't)



Problem 9.104
Pressure Coefficient, C_p, vs Angle, θ
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
C_p
0
30
60
90
120
150
180
θ, deg
Experimental
Theoretical (inviscid flow)

10.1

10.1 Water flows at a depth of 2 ft in a 10-ft-wide channel. Determine the flowrate if the flow is critical.

$Fr = \frac{V}{\sqrt{gy}}$ where $y = 2\,ft$, $Q = V(by)$ with $b = 10\,ft$.

Thus, if $Fr = 1$, $V = Fr\sqrt{gy} = \sqrt{32.2\frac{ft}{s^2}(2ft)} = 8.02\,ft/s$

so that

$Q = 8.02\frac{ft}{s}(10ft)(2ft) = \underline{\underline{160\frac{ft^3}{s}}}$

10.2

10.2 The flowrate per unit width in a wide channel is $q = 2.3\ m^2/s$. Is the flow subcritical or supercritical if the depth is **(a)** 0.2 m, **(b)** 0.8m, or **(c)** 2.5 m?

$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y}$ so that $Fr = \frac{V}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}} = \frac{q}{\sqrt{g}\,y^{3/2}}$

or

$Fr = \frac{2.3\frac{m^2}{s}}{\sqrt{9.81\frac{m}{s^2}}\,y^{3/2}} = \frac{0.734}{y^{3/2}}$, where $y \sim m$

	y, m	Fr	flow type
a)	0.2	8.21	supercritical
b)	0.8	1.03	supercritical
c)	2.5	0.186	subcritical

10.3

10.3 Water flows in a canal at a depth of 2.8 ft and a velocity of 5.3 ft/s. Will waves produced by throwing a stick into the canal travel both upstream and downstream, or will they all be washed downstream? Explain.

$c = \sqrt{gy} = \sqrt{(32.2\frac{ft}{s^2})(2.8ft)} = 9.50\frac{ft}{s}$

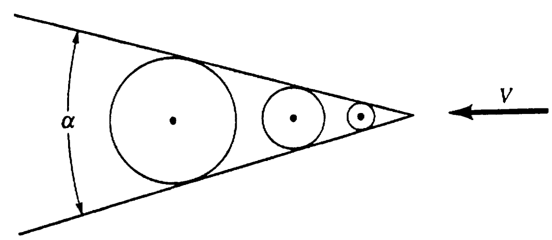
Thus, with $V = 5.3\frac{ft}{s} < c$ the wave can travel upstream against the current. Relative to the stream bank the wave travels upstream with velocity $c - V = 9.50\frac{ft}{s} - 5.3\frac{ft}{s} = 4.2\frac{ft}{s}$. It also travels downstream.

Note: $Fr = \frac{V}{c} = \frac{5.3\frac{ft}{s}}{9.50\frac{ft}{s}} = 0.558$; the flow is subcritical.

Waves travel <u>both upstream and downstream.</u>

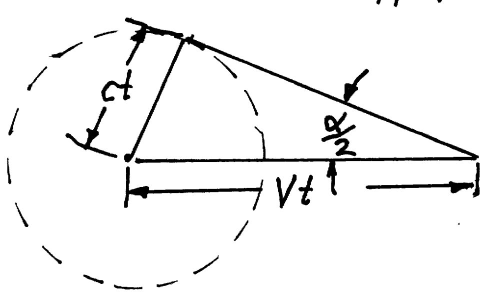
10.4

10.4 Consider waves made by dropping objects (one after another from a fixed location) into a stream of depth y that is moving with speed V as shown in Fig. P10.4 (see Video V9.1). The circular wave crests that are produced travel with speed $c = (gy)^{1/2}$ relative to the moving water. Thus, as the circular waves are washed downstream, their diameters increase and the center of each circle is fixed relative to the moving water. **(a)** Show that if the flow is supercritical, lines tangent to the waves generate a wedge of half-angle $\alpha/2 = \arcsin(1/Fr)$, where $Fr = V/(gy)^{1/2}$ is the Froude number. **(b)** Discuss what happens to the wave pattern when the flow is subcritical, $Fr < 1$.



■ FIGURE P10.4

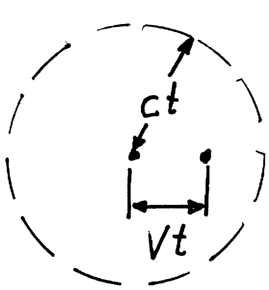
(a) In a time interval of t since the object hit the water (and iniatiated the wave), the center of the wave has been swept downstream a distance Vt and the wave has expanded to be a distance ct from its center. This is shown in the figure below. Note that $Vt > ct$ if $V > c$ (i.e. $Fr > 1$).



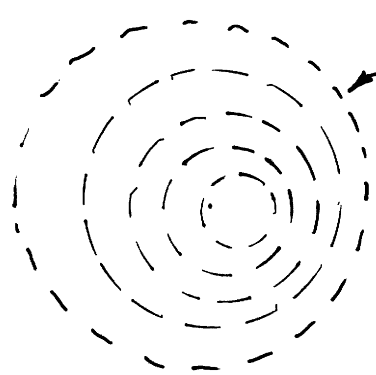
Thus, from the figure, $\sin\frac{\alpha}{2} = \dfrac{ct}{Vt} = \dfrac{c}{V} = \dfrac{\sqrt{gy}}{V} = \dfrac{1}{Fr}$

or $\frac{\alpha}{2} = \arcsin(1/Fr)$

(b) If $Fr < 1$ the above result gives $\sin\frac{\alpha}{2} > 1$, which is impossible. For $Fr < 1$ the following wave pattern would result. There is no "wedge" produced.



$Vt < ct$ if $Fr < 1$



non-concentric circular waves

10.5

10.5 Waves on the surface of a tank are observed to travel at a speed of 2 m/s. How fast would these waves travel if (**a**) the tank were in an elevator accelerating upward at a rate of 4 m/s^2, (**b**) the tank accelerates horizontally at a rate of 9.81 m/s^2, (**c**) the tank were aboard the orbiting Space Shuttle. Explain.

Since $c=\sqrt{gy}$ it follows that the tank depth is

$$y=\frac{c^2}{g}=\frac{(2\frac{m}{s})^2}{9.81\frac{m}{s^2}}=0.408\ m$$

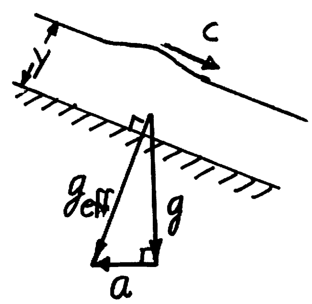
(a) If the tank accelerates upward with acceleration a, the effective acceleration of gravity is $g_{eff}=g+a=(9.81+4)\frac{m}{s^2}=13.81\frac{m}{s^2}$

Thus,

$$c=\sqrt{g_{eff}\,y}=\sqrt{(13.81\frac{m}{s^2})(0.408m)}=\underline{\underline{2.37\frac{m}{s}}}$$

(b) If the tank accelerates horizontally with acceleration a, the effective acceleration is

$$g_{eff}=\sqrt{g^2+a^2}=\sqrt{9.81^2+9.81^2}=13.87\frac{m}{s^2}$$



Thus,

$$c=\sqrt{(13.87\frac{m}{s^2})(0.408m)}=\underline{\underline{2.38\frac{m}{s}}}$$

(c) In orbit $g_{eff}=0$ (weightless) so $c=\underline{\underline{0}}$

10.6

10.6 In flowing from section (1) to section (2) along an open channel, the water depth decreases by a factor of two and the Froude number changes from a subcritical value of 0.5 to a supercritical value of 3.0. Determine the channel width at (2) if it is 12 ft wide at (1).

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = 0.5, \text{ or } \sqrt{g y_1} = 2.0\, V_1 \qquad (1)$$

and

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = 3.0 \quad \text{where } y_2 = 0.5\, y_1$$

$$\text{Thus, } \frac{V_2}{\sqrt{0.5\, g y_1}} = 3.0, \text{ or } \sqrt{g y_1} = V_2 / (3\sqrt{0.5}) \qquad (2)$$

By equating Eq. (1) and (2): $2.0\, V_1 = V_2 / (3\sqrt{0.5})$

or

$V_2 = 4.24\, V_1$

However, $Q_1 = Q_2$ or $b_1 y_1 V_1 = b_2 y_2 V_2$ where b = channel width.

Thus, with $b_1 = 12\ ft$:

$$(12\,ft)\, y_1 (V_1) = b_2 (0.5\, y_1)(4.24\, V_1) \text{ or } b_2 = \frac{12\,ft}{0.5(4.24)} = \underline{\underline{5.66\,ft}}$$

10.7

10.7 Observations at a shallow sandy beach show that even though the waves several hundred yards out from the shore are not parallel to the beach, the waves often "break" on the beach nearly parallel to the shore as is indicated in Fig. P10.7. Explain this behavior based on the wave speed $c = (gy)^{1/2}$.

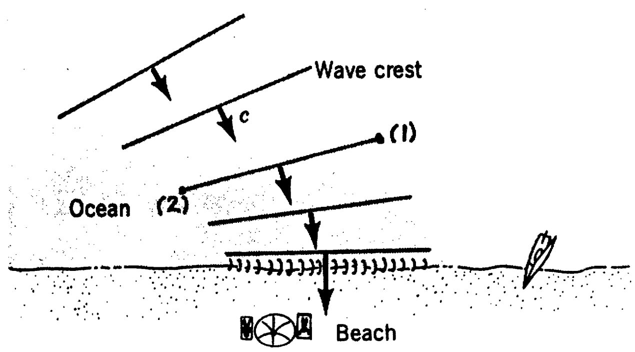


FIGURE P10.7

Since $c = \sqrt{gy}$ it follows that $c_1 > c_2$ because of the fact that $y_1 > y_2$. Therefore, as the waves move, that portion in the deeper water tends to "catch up" with that portion closer to shore in the shallower water. The wave crest tends to become more nearly parallel to the shore line. The waves "break" on the shore as if the wind were blowing normal to the shore.

10.8

10.8 Waves on the surface of a tank containing water are observed to move with a velocity of 1.8 m/s. If the water is replaced by mercury, with all other conditions the same, determine the wave speed expected. Determine the wave speed if the tank were in a laboratory on the surface of a planet where the acceleration of gravity is 4 times that on earth.

Since $c = \sqrt{gy}$ it follows that the wave speed is independent of the fluid density. Thus, $c_{H_2O} = c_{Hg} = \underline{\underline{1.8\ \frac{m}{s}}}$ on earth.

However, on the planet

$$c_{planet} = \sqrt{g_{planet}\,y} = \left(\frac{g_{earth}}{g_{earth}}\right)^{\frac{1}{2}}(g_{planet}\,y)^{\frac{1}{2}} = \left(\frac{g_{planet}}{g_{earth}}\right)^{\frac{1}{2}}(g_{earth}\,y)^{\frac{1}{2}}$$

or

$$c_{planet} = (4)^{\frac{1}{2}} c_{earth} = (4)^{\frac{1}{2}}\left(1.8\ \frac{m}{s}\right) = \underline{\underline{3.60\ \frac{m}{s}}} \text{ for water or mercury}$$

10.9

10.9 Often when an earthquake shifts a segment of the ocean floor, a relatively small amplitude wave of very long wavelength is produced. Such waves go unnoticed as they move across the open ocean; only when they approach the shore do they become dangerous (a tsunami or "tidal wave"). Determine the wave speed if the wavelength, λ, is 6000 ft and the ocean depth is 15,000 ft.

From Eq. 10.4: $c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{\frac{1}{2}}$

or

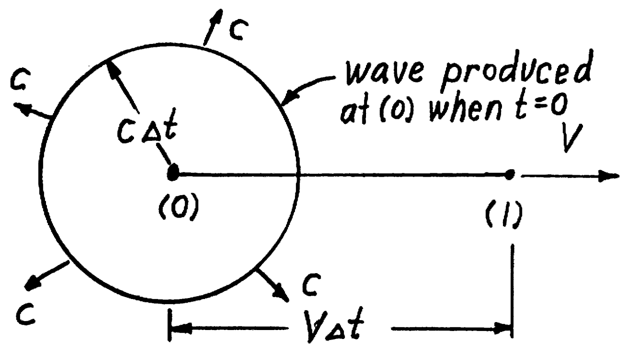
$$c = \left[\frac{\left(32.2\ \frac{ft}{s^2}\right)(6000\ ft)}{2\pi}\tanh\left(\frac{2\pi(15{,}000\ ft)}{6000\ ft}\right)\right]^{\frac{1}{2}} = \underline{\underline{175\ \frac{ft}{s}}}$$

10.10

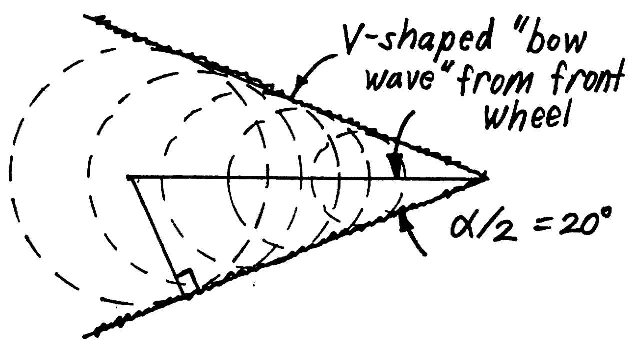
10.10 A bicyclist rides through a 3-in. deep puddle of water as shown in Video V10.1 and Fig. P10.10 If the angle made by the V-shaped wave pattern produced by the front wheel is observed to be 40 deg, estimate the speed of the bike through the puddle. *Hint:* Make a sketch of the current location of the bike wheel relative to where it was Δt seconds ago. Also indicate on this sketch the current location of the wave that the wheel made Δt seconds ago. Recall that the wave moves radially outward in all directions with speed c relative to the stationary water.

■ FIGURE P10.10

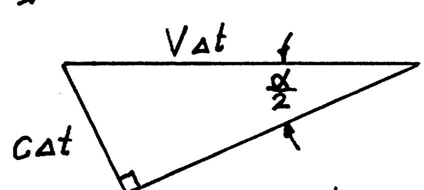
At time $t=0$ the front wheel was at point (0). At the current time, $t=\Delta t$, the wheel has traveled a distance $d = V\Delta t$ and is at point (1). At time $t=\Delta t$, a wave produced by the wheel when it was at (0) will be a distance $c\Delta t$ from (0) as indicated in the figure.



Waves produced at various times (from $t=0$ to $t=\Delta t$) by the front wheel will form a V-shaped wave as shown in the second figure (provided $V > c$; supercritical bike speed).



From the geometry of the figure



$$\sin\frac{\alpha}{2} = \frac{c\Delta t}{V\Delta t}$$

or

$$V = \frac{c}{\sin\frac{\alpha}{2}} \quad \text{where } c = \sqrt{gy} = \left[32.2\frac{ft}{s^2}\left(\frac{3}{12}ft\right)\right]^{\frac{1}{2}} = 2.84\frac{ft}{s}$$

Thus,

$$V = \frac{2.84\frac{ft}{s}}{\sin 20^\circ} = \underline{\underline{8.30\frac{ft}{s}}}$$

10.11 Water flows in a rectangular channel with a flowrate per unit width of $q = 2.5\ \text{m}^2/\text{s}$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 2.5$ m.

$$E = y + \frac{q^2}{2g\,y^2} = y + \frac{\left(2.5\,\frac{m^2}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)y^2} = y + \frac{0.319}{y^2}$$

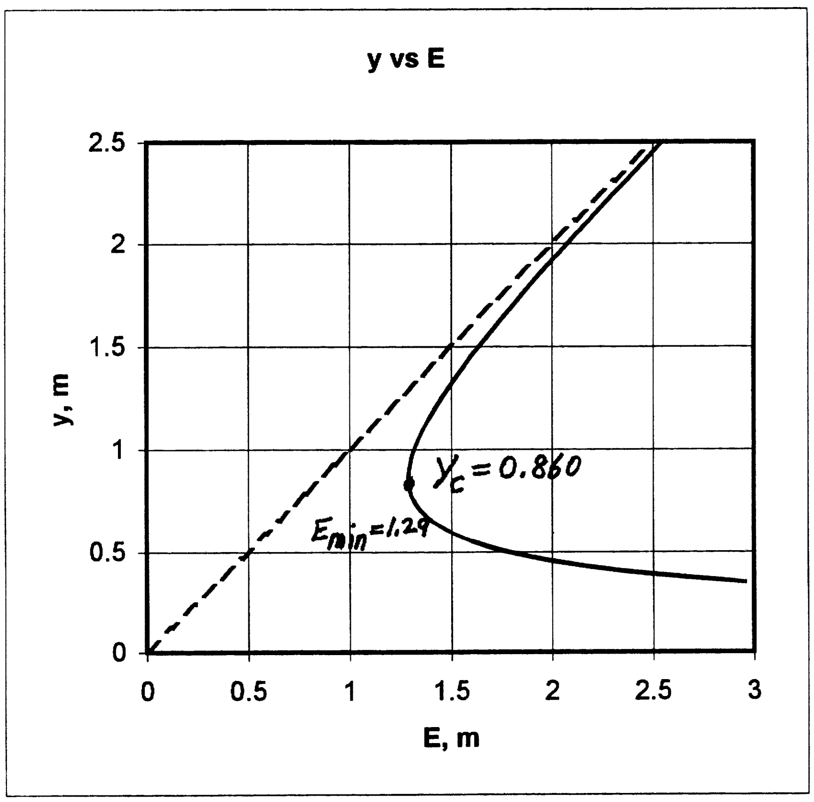
Thus, plot

$$E = y + \frac{0.319}{y^2}\,, \quad \text{where } E \sim m,\ y \sim m$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left(2.5\,\frac{m^2}{s}\right)^2}{9.81\,\frac{m}{s^2}}\right)^{1/3} = 0.860\,m$

and

$$E_{min} = \frac{3}{2}\,y_c = \frac{3}{2}(0.860m) = 1.29\,m$$



For $E = 2.5m$, Eq.(1) is $2.5 = y + \frac{0.319}{y^2}$

or $y^3 - 2.5y^2 + 0.319 = 0$

The roots to this equation are $y = 2.45$, 0.338, and -0.335

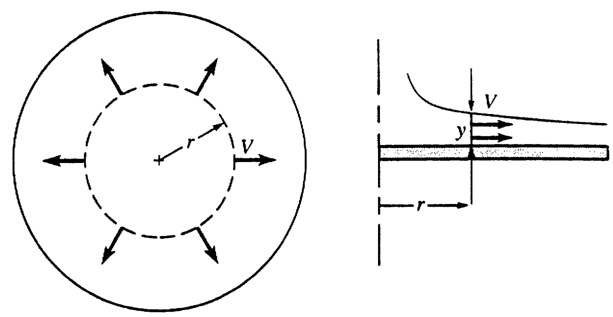
Thus, $\underline{\underline{y = 2.45\ m \text{ or } y = 0.388\ m}}$

10.12

10.12 Water flows radially outward on a horizontal round disk as is shown in Video V10.6 and Fig. P10.12. (a) Show that the specific energy can be written in terms of the flowrate, Q, the radial distance from the axis of symmetry, r, and the fluid depth, y, as

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) For a constant flowrate, sketch the specific energy diagram. Recall Fig. 10.7, but note that for the present case r is a variable. Explain the important characteristics of your sketch. (c) Based on the results of Part (b), show that the water depth increases in the flow direction if the flow is subcritical, but that it decreases in the flow direction if the flow is supercritical.



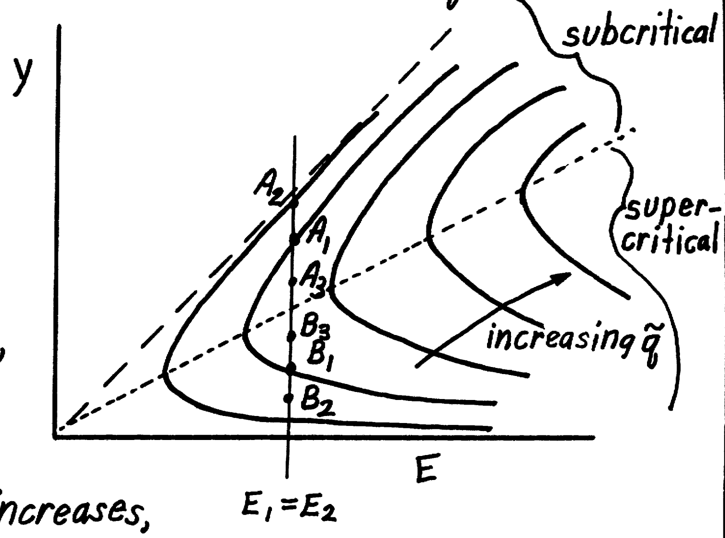
■ FIGURE P10.12

(a) The specific energy is $E = y + \frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{Q}{2\pi r y}$

Thus,

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) Let $\tilde{q} \equiv \frac{Q}{2\pi r}$ so that $E = y + \frac{\tilde{q}^2}{2gy^2}$ which is the same as for two dimensional flow with $q = \frac{Q}{b}$ being replaced by $\tilde{q}$. However, for two dimensional flow q is constant; for radial flow $\tilde{q}$ is a variable since r varies. But E vs y curves for constant $\tilde{q}$ would look as shown below (Fig. 10.7).



(c) From the Bernoulli equation $E_1 = E_2$ or $E = constant$ for this flow. Consider subcritical flow — point A. For outflow r increases so that $\tilde{q}$ decreases. Thus since $E = const.$, the flow goes from state A_1 to A_2; the depth increases. For subcritical inflow r decreases, $\tilde{q}$ increases, the flow goes from A_1 to A_3, and the depth decreases. For supercritical flow + is true. Thus, outfow increases r, decreases $\tilde{q}$; or from B_1 to B_2 — decreasing depth. Supercritical inflow from B_1 to B_3 — increasing depth.

	subcritical	supercritical
inflow	depth decreases	depth increases
outflow	depth increases	depth decreases

10.13*

10.13* Water flows in a rectangular channel with a specific energy of $E = 5$ ft. If the flowrate per unit width is $q = 30$ ft²/s, determine the two possible flow depths and the corresponding Froude numbers. Plot the specific energy diagram for this flow. Repeat the problem for $E = 1, 2, 3$, and 4 ft.

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(30\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{14.0}{y^2} \qquad (0)$$

Thus,
$$y^3 - Ey^2 + 14.0 = 0, \text{ where } E \sim ft,\ y \sim ft \qquad (1)$$

Also, $Fr = \frac{V}{\sqrt{gy}}$ where $V = \frac{q}{y}$ Thus, $Fr = \frac{q}{\sqrt{g}\,y^{3/2}} = \frac{30\,\frac{ft^2}{s}}{\sqrt{32.2\,\frac{ft}{s^2}}\,y^{3/2}}$

or
$$Fr = \frac{5.29}{y^{3/2}} \qquad (2)$$

Determine y and Fr from Eqs. (1) and (2) for $E = 1, 2, 3, 4, 5$

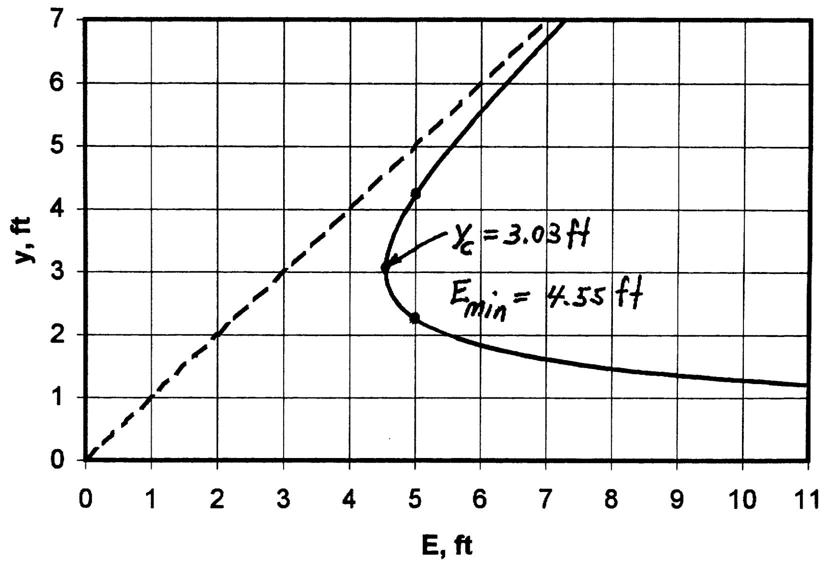
The program CUBIC was used to determine the values of y listed below.

E, ft	y, ft (subcritical)	Fr	y, ft (supercritical)	Fr
1	no solution possible			
2				
3				
4				
5	4.21	0.612	2.26	1.56

Note that $E_{min} = \frac{3}{2} y_c$, where $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(30\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right]^{1/3} = 3.03$ ft

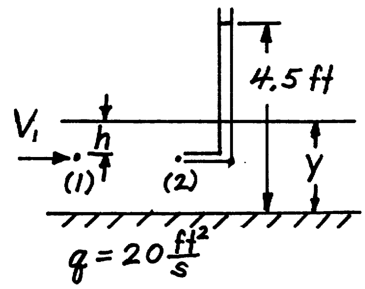
Thus, $E_{min} = \frac{3}{2}(3.03\,ft) = 4.55$ ft

As shown on the graph below, there are no positive real roots of Eq. (1) if $E < E_{min} = 4.55$ ft.



10.14

10.14 Water flows in a rectangular channel at a rate of $q = 20$ cfs/ft. When a Pitot tube is placed in the stream, water in the tube rises to a level of 4.5 ft above the channel bottom. Determine the two possible flow depths in the channel. Illustrate this flow on a specific energy diagram.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\text{, where } z_1=z_2,$$

$V_2=0$, $\frac{p_1}{\gamma}=h$, and $\frac{p_2}{\gamma}=4.5-(y-h)$

Thus,

$$h+\frac{V_1^2}{2g}=4.5\,\text{ft}-y+h\text{ , or } \frac{V_1^2}{2g}=4.5-y$$

but,

$$V_1=\frac{q}{y}=\frac{20\frac{ft^2}{s}}{y}$$

Hence,

$$\frac{\left(\frac{20}{y}\right)^2}{2(32.2\frac{ft}{s^2})}=4.5-y \quad\text{or}\quad y^3-4.5y^2+6.21=0\text{ , where } y\sim ft$$

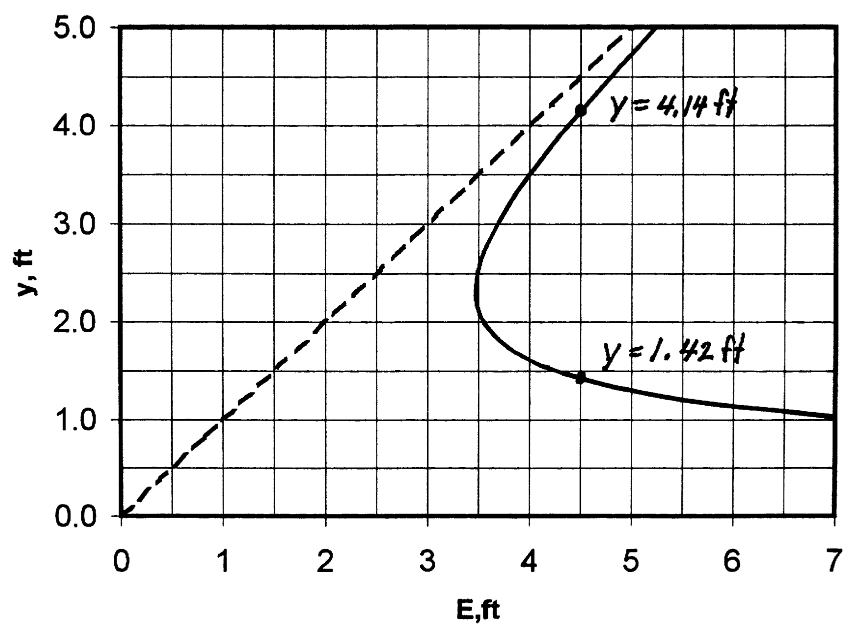
The roots of this equation are $y = 4.14, 1.42,$ and -1.06

Thus,

$$\underline{\underline{y = 4.14\,\text{ft or } y = 1.42\,\text{ft}}}$$

$$E=y+\frac{q^2}{2gy^2}=y+\frac{(20\frac{ft^2}{s})^2}{2(32.2\frac{ft}{s^2})y^2} \quad\text{or } E=y+\frac{6.21}{y^2} \qquad (1)$$

The specific energy diagram (plot of Eq. (1)) is shown below.



10.15 Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q = 30\ \text{ft}^3/\text{s}$ and an upstream depth of $y_1 = 2.5$ ft as is shown in Fig. P10.15. Determine the flow depth and the surface elevation at section (2).

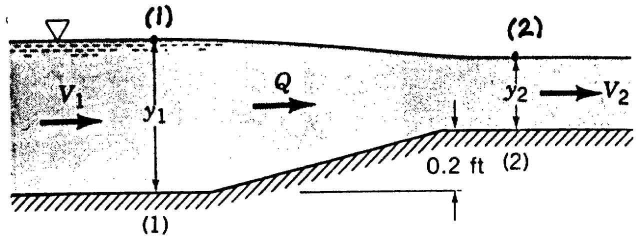


FIGURE P10.15

$$\cancel{\frac{p_1}{\gamma}} + \frac{V_1^2}{2g} + z_1 = \cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 2\,ft,\ z_2 = 0.2\,ft + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{(30\frac{ft^3}{s})}{(2ft)(5ft)} = 3\,\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30\frac{ft^3}{s}}{(5ft)y_2} = \frac{6}{y_2}$$

Thus,

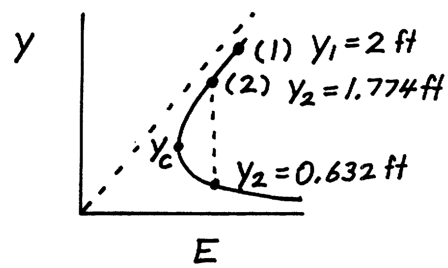
$$\frac{(3\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 2ft = \frac{(\frac{6}{y_2}\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 0.2ft + y_2$$

or

$$y_2^3 - 1.94y_2^2 + 0.559 = 0 \quad \text{which has roots } y_2 = 1.774,\ 0.632, \text{ and } -0.632$$

Note: $Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3\frac{ft}{s}}{[(32.2\frac{ft}{s^2})(2ft)]^{1/2}} = 0.374 < 1$

If $y_2 = 0.632$, then $Fr_2 > 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2 = 1.774\ ft \text{ and } z_2 = 1.974\ ft}}$

10.16 Repeat Problem 10.15 if the upstream depth is $y_1 = 0.5$ ft.

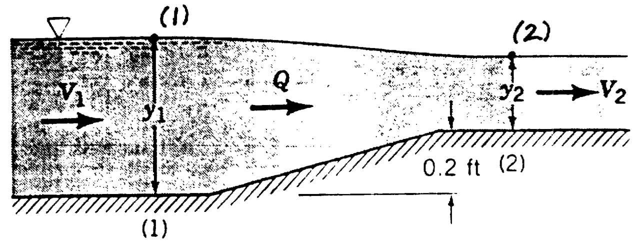


FIGURE P10.16

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\,ft,\ z_2 = 0.2\,ft + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{30\,\frac{ft^3}{s}}{(0.5\,ft)(5\,ft)} = 12\,\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30\,\frac{ft^3}{s}}{(5\,ft)y_2} = \frac{6}{y_2}$$

Thus,

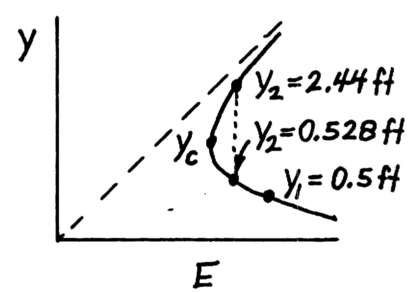
$$\frac{(12\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} + 0.5\,ft = \frac{(\frac{6}{y_2})^2}{2(32.2\,\frac{ft}{s^2})} + 0.2\,ft + y_2$$

or

$y_2^3 - 2.53\,y_2^2 + 0.559 = 0$ which has roots $y_2 = 2.44,\ 0.528,$ and -0.434

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12\,\frac{ft}{s}}{[(32.2\,\frac{ft}{s^2})(0.5\,ft)]^{1/2}} = 2.99 > 1$

If $y_2 = 2.44$ ft, then $Fr_2 < 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2 = 0.528\ \text{ft and } z_2 = 0.728\ \text{ft}}}$

10.17*

10.17* Water flows over the bump in the bottom of the rectangular channel shown in Fig. P10.17 with a flowrate per unit width of $q = 4\ m^2/s$. The channel bottom contour is given by $z_B = 0.2e^{-x^2}$, where z_B and x are in meters. The water depth far upstream of the bump is $y_1 = 2$ m. Plot a graph of the water depth, $y = y(x)$, and the surface elevation, $z = z(x)$, for $-4\ m \le x \le 4$ m. Assume one-dimensional flow.

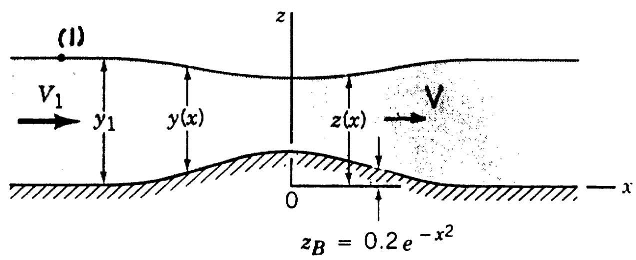


FIGURE P10.17

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p}{\gamma}+\frac{V^2}{2g}+z, \text{ where } p_1=p=0,\ z_1=y_1=2m,\ z_2=y+z_B$$

or $z = y + 0.2\,e^{-x^2}$, $V_1=\frac{q}{y_1}=\frac{4\frac{m^2}{s}}{2m}=2\frac{m}{s}$, and $V=\frac{q}{y}=\frac{4}{y}$

Thus, $$\frac{(2\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+2m=\frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+y+0.2\,e^{-x^2}$$

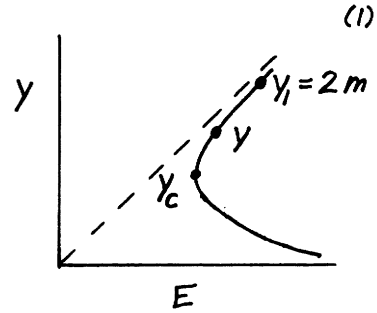
or

$$y^3-(2.20-0.2e^{-x^2})y^2+0.815=0 \text{ where } y\sim m \qquad (1)$$

Solve for y with $-4 \le x \le 4m$

Note: $Fr_1=\frac{V_1}{\sqrt{gy_1}}=\frac{2\frac{m}{s}}{[(9.81\frac{m}{s^2})(2m)]^{\frac{1}{2}}}=0.452<1$

Thus, the flow will remain subcritical throughout — the largest root of Eq. (1) will be the correct one.



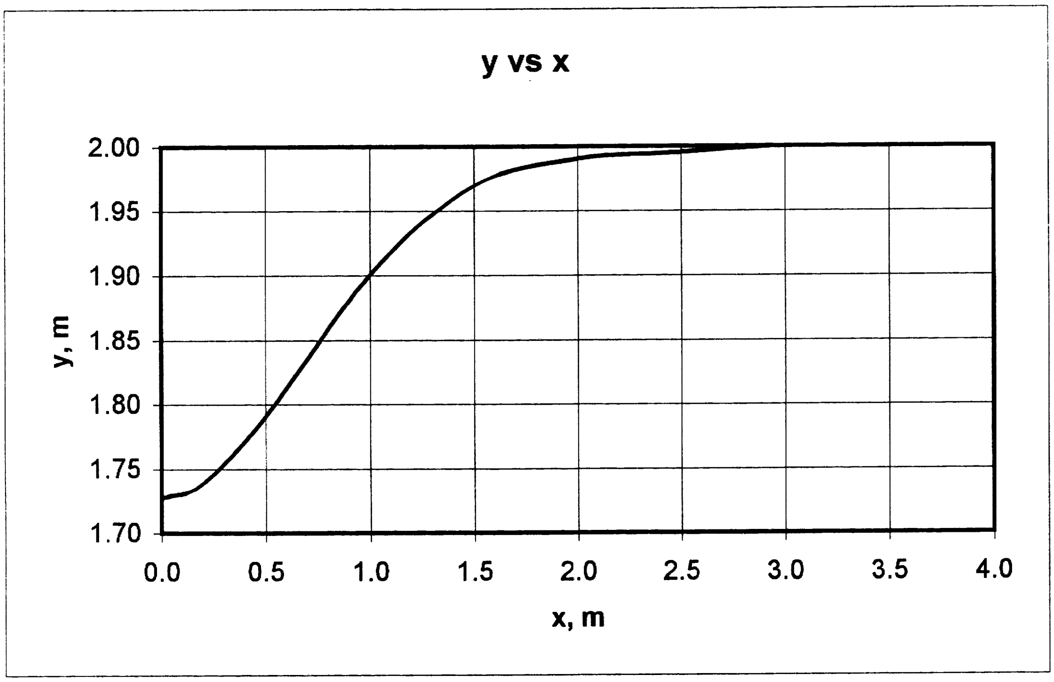
Use program CUBIC to solve for y(x) from Eq. (1) (largest root) and then $z=y+0.2\,e^{-x^2}$ for $-4m \le x \le 4m$.

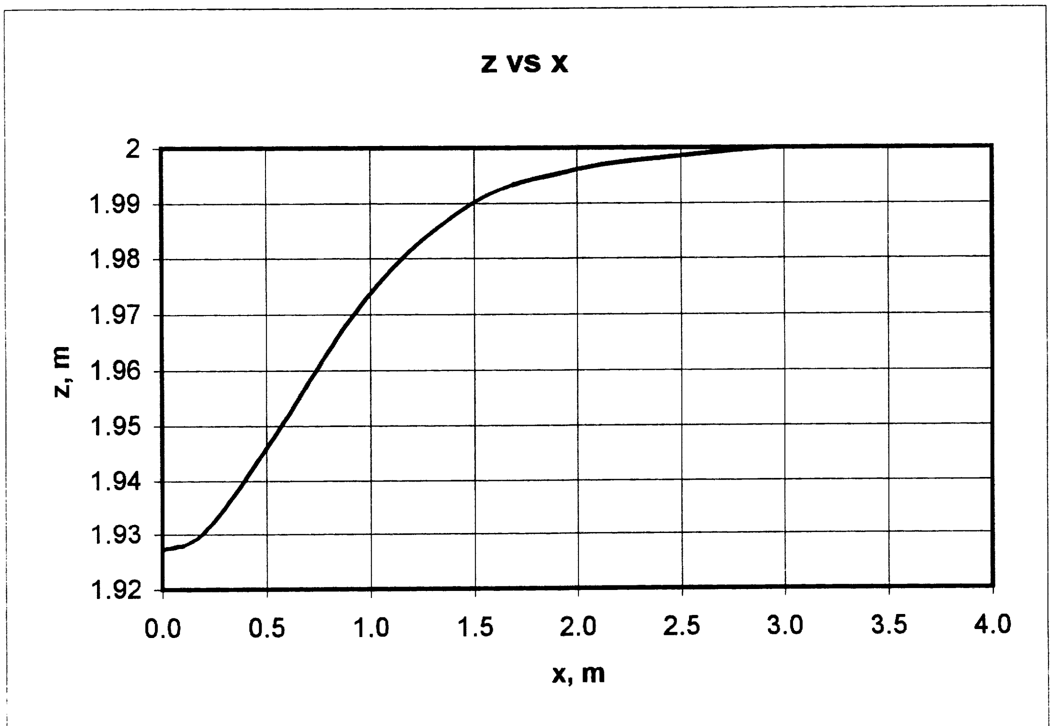
x, m	$-(2.20-0.2\,e^{-x^2})$	y	$z=y+0.2e^{-x^2}$, m
± 4.0	-2.200	2.000	2.000
± 3.5	-2.200	2.000	2.000
± 3.0	-2.200	2.000	2.000
± 2.5	-2.199	1.995	1.995
± 2.0	-2.196	1.990	1.994
± 1.5	-2.179	1.969	1.990
± 1.0	-2.126	1.900	1.974
± 0.5	-2.044	1.790	1.946
0	-2.000	1.727	1.927

(con't)

10.17* (con't)

The above results are plotted in the graph below.





10.18*

*10.18 Repeat Problem 10.17 if the upstream depth is 0. 4 m.

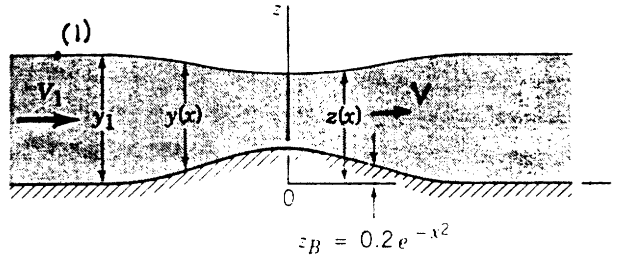


FIGURE P10.18

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\gamma} + \frac{V^2}{2g} + z, \text{ where } p_1 = p = 0, \; z_1 = y_1 = 0.4\,m, \; z_2 = y + z_B$$

or $z_2 = y + 0.2e^{-x^2}$, $V_1 = \frac{q}{y_1} = \frac{4\frac{m^2}{s}}{0.4m} = 10\frac{m}{s}$, and $V = \frac{q}{y} = \frac{4}{y}$

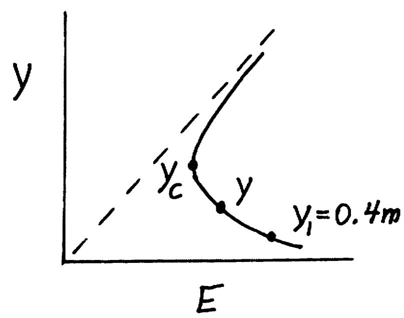
Thus, $$\frac{(10\frac{m}{s})^2}{2(9.81\frac{m}{s})^2} + 0.4m = \frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s})^2} + y + 0.2e^{-x^2}$$

or
$$y^3 - (5.50 - 0.2e^{-x^2})y^2 + 0.815 = 0 \quad \text{where } y \sim m \tag{1}$$

Solve for y with $-4 \le x \le 4\,m$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{10\frac{m}{s}}{[(9.81\frac{m}{s^2})(0.4m)]^{1/2}} = 5.05 > 1$

Thus, the flow will remain supercritical throughout — the smallest positive root of Eq.(1) will be the correct one.



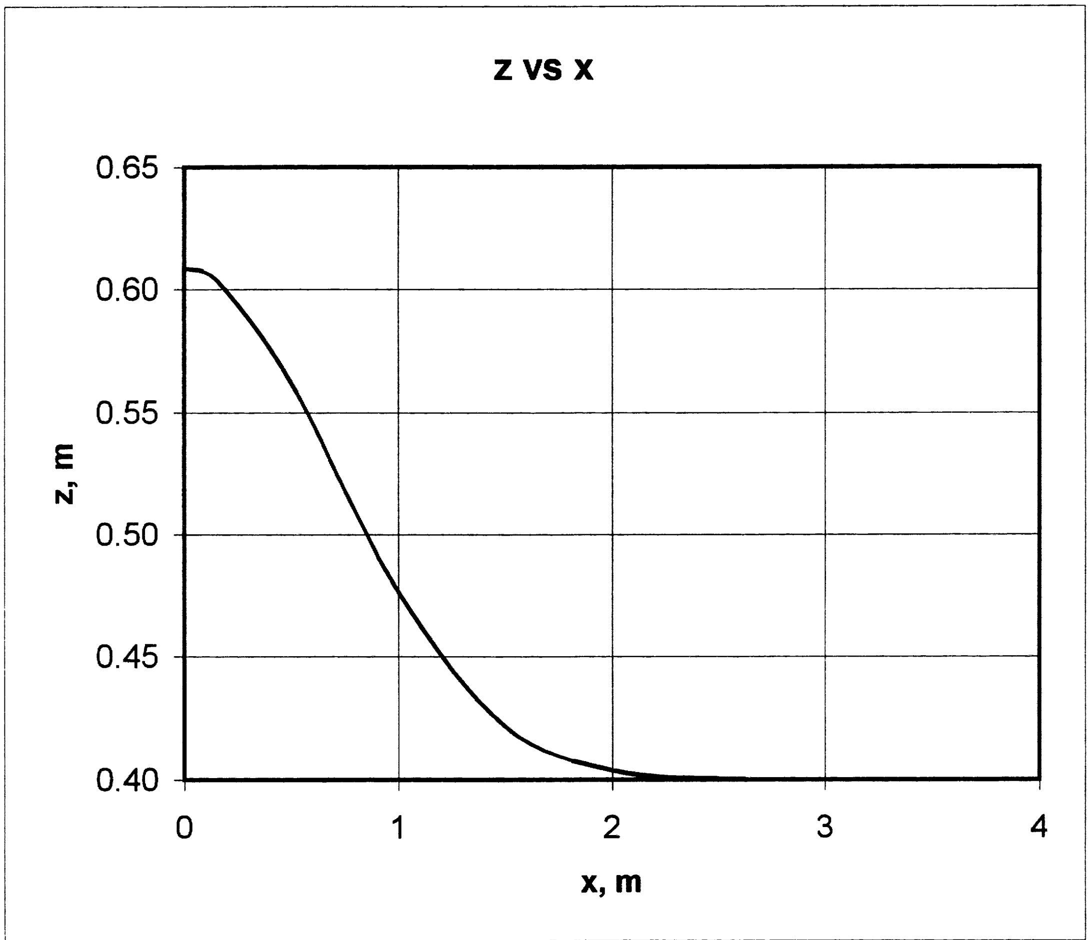
Use program CUBIC to solve for y(x) from Eq.(1) (smallest positive root) and then $z = y + 0.2e^{-x^2}$ for $-4m \le x \le 4m$.

x, m	$-(5.50-0.2e^{-x^2})$	y	$z = y + 0.2e^{-x^2}$, m
± 4.0	-5.500	0.4000	0.4000
± 3.5	-5.500	0.4000	0.4000
± 3.0	-5.500	0.4000	0.4000
± 2.5	-5.499	0.3998	0.4004
± 2.0	-5.496	0.3999	0.4036
± 1.5	-5.479	0.4006	0.4217
± 1.0	-5.426	0.4028	0.4764
± 0.5	-5.344	0.4063	0.5621
0	-5.300	0.4082	0.6082

(con't)

10.18* (con't)

The above results are plotted on the graph below.



10.19

10.19 Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.19. If the flowrate is $Q = 25\ \text{ft}^3/\text{s}$ and the upstream depth is $y_1 = 2$ ft, determine the downstream depth, y_2.

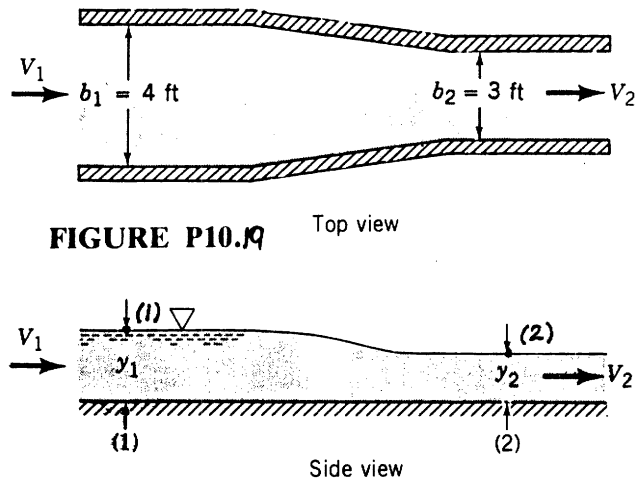


FIGURE P10.19

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 2\,\text{ft},\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\,\frac{\text{ft}^3}{\text{s}}}{(4\,\text{ft})(2\,\text{ft})} = 3.13\,\frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25\,\frac{\text{ft}^3}{\text{s}}}{(3\,\text{ft})\,y_2} = \frac{8.33}{y_2}$$

Thus,

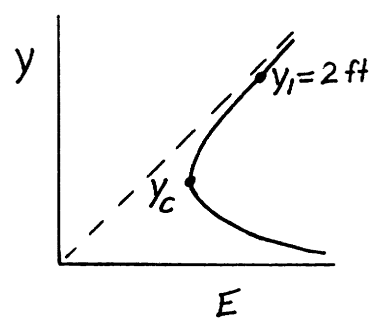
$$\frac{(3.13\,\frac{\text{ft}}{\text{s}})^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} + 2\,\text{ft} = \frac{\left(\frac{8.33}{y_2}\right)^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} + y_2$$

or

$$y_2^3 - 2.15\,y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 1.828,\ 0.946, \text{ and } -0.623 \qquad (1)$$

Note: $Fr_1 = \dfrac{V_1}{\sqrt{g y_1}} = \dfrac{3.13\,\frac{\text{ft}}{\text{s}}}{\left[(32.2\,\frac{\text{ft}}{\text{s}^2})(2\,\text{ft})\right]^{\frac{1}{2}}} = 0.390 < 1$

Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 < 1$ also. Thus, it is not possible to have $y_2 = 0.946$



Thus, $\underline{\underline{y_2 = 1.828\,\text{ft}}}$

10.20

10.20 Sketch the specific energy diagram for the flow of Problem 10.19 and indicate its important characteristics. Note that $q_1 \neq q_2$.

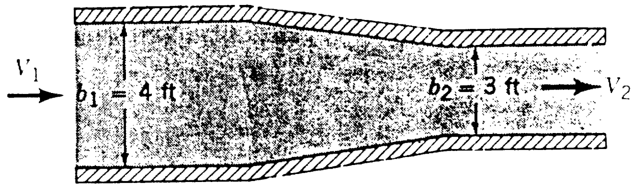
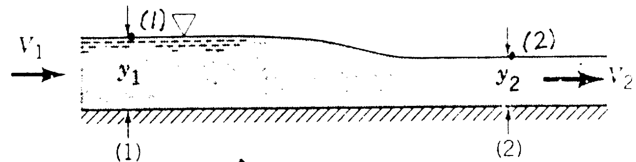


FIGURE P10.20



$$E = y + \frac{q^2}{2gy^2}$$

Thus, for the $b_1 = 4$ ft channel, $q_1 = \frac{Q}{b_1} = \frac{25\,\frac{ft^3}{s}}{4\,ft} = 6.25\,\frac{ft^2}{s}$

or

$$E = y + \frac{(6.25\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{0.607}{y^2} \qquad (1)$$

For the $b_2 = 3$ ft channel, $q_2 = \frac{Q}{b_2} = \frac{25\,\frac{ft^3}{s}}{3\,ft} = 8.33\,\frac{ft^2}{s}$

or

$$E = y + \frac{(8.33\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{1.077}{y^2} \qquad (1)$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ so that $y_{c_1} = \left(\frac{q_1^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(6.25\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.067\,ft$

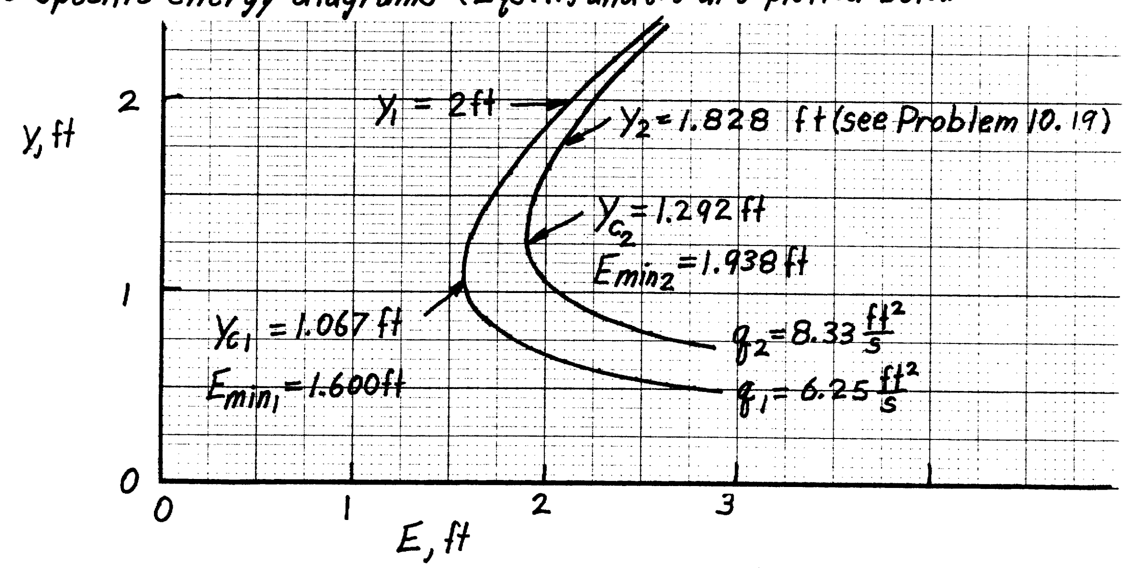
and

$$y_{c_2} = \left(\frac{q_2^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(8.33\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.292\,ft$$

Also, $E_{min} = \frac{3}{2}\,y_c$, or $E_{min_1} = \frac{3}{2}(1.067\,ft) = 1.600\,ft$

$$E_{min_2} = \frac{3}{2}(1.292\,ft) = 1.938\,ft$$

The specific energy diagrams (Eqs. (1) and (2) are plotted below:



Note: $E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g} = 2\,ft + \frac{(3.13\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = 2.15\,ft$

10.21

10.21 Repeat Problem 10.19 if the upstream depth is $y_1 = 0.5$ ft. Assume that there are no losses between sections (1) and (2).

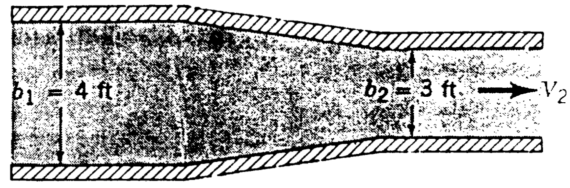
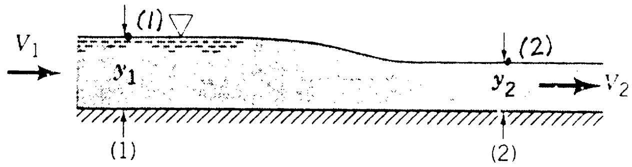


FIGURE P10.21



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{, where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\,ft,\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\,\frac{ft^3}{s}}{(4ft)(0.5ft)} = 12.5\,\frac{ft}{s} \text{, and } V_2 = \frac{Q}{A_2} = \frac{25\,\frac{ft^3}{s}}{(3ft)\,y_2} = \frac{8.33}{y_2}$$

Thus,

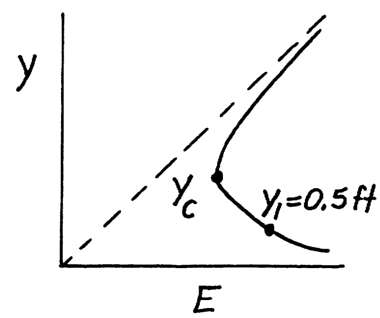
$$\frac{(12.5\,\frac{ft}{s})^2}{2\,(32.2\,\frac{ft}{s^2})} + 0.5\,ft = \frac{\left(\frac{8.33}{y_2}\right)^2}{2\,(32.2\,\frac{ft}{s^2})} + y_2$$

or

$$y_2^3 - 2.93\,y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 2.79,\ 0.694, \text{ and } -0.555$$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12.5\,\frac{ft}{s}}{[(32.2\,\frac{ft}{s^2})(0.5ft)]^{1/2}} = 3.12 > 1$

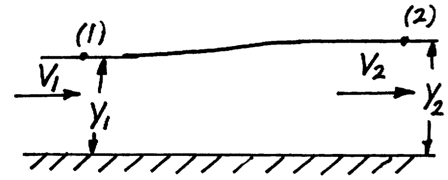
Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 > 1$ also. Thus, it is not possible to have $y_2 = 2.79$ (the subcritical root).



Thus, $y_2 = \underline{\underline{0.694\,ft}}$

10.22

10.22 Water flows in a rectangular channel with a flowrate per unit width of $q = 1.5\ \text{m}^2/\text{s}$ and a depth of 0.5 m at section (1). The head loss between sections (1) and (2) is 0.03 m. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram. Is it possible to have a head loss of 0.06 m? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0,\ z_1 = 0.5\,m,\ z_2 = y_2, \quad (1)$$

$$V_1 = \frac{q}{y_1} = \frac{1.5\frac{m^2}{s}}{0.5m} = 3\frac{m}{s}, \text{ and } V_2 = \frac{q}{y_2} = \frac{1.5\frac{m^2}{s}}{y_2}$$

Thus, with $E = y + \frac{V^2}{2g}$ and $h_L = 0.03m$ Eq.(1) is

$$E_1 = E_2 + 0.03$$

Also, $E = y + \frac{q^2}{2g\,y^2} = y + \frac{(1.5\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y^2}$

or

$$E = y + \frac{0.1146}{y^2} \quad (2)$$

Eq.(2) is plotted below.

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(1.5\frac{m^2}{s})^2}{9.81\frac{m}{s^2}}\right)^{1/3} = 0.612\ m$

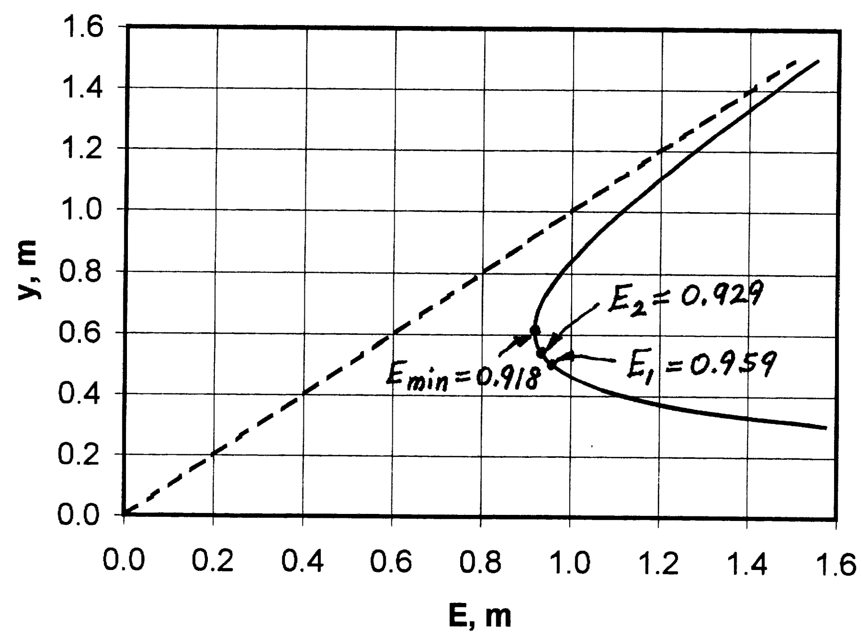
and

$$E_{min} = \frac{3}{2}y_c = \frac{3}{2}(0.612m) = 0.918m$$

Also,

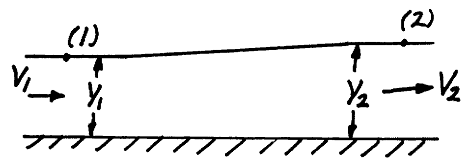
$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.5 + \frac{(1.5\frac{m^2}{s})}{2(9.81\frac{m}{s^2})(0.5m)^2} = 0.959m$$

and $E_2 = E_1 - 0.03 = 0.929\,m$



Note: If $h_L = 0.06m$ with $E_1 = 0.959m$ so that $E_2 = E_1 - 0.06$, then $E_2 = 0.899\ m < E_{min}$. Thus, it is not possible to have $h_L = 0.06$ with the given q and y_1.

10.23 Water flows in a horizontal rectangular channel with a flowrate per unit width of $q = 10$ ft^2/s and a depth of 1.0 ft at the downstream section (2). The head loss between section (1) upstream and section (2) is 0.2 ft. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L\text{, where } p_1=p_2=0,\ V_2=\frac{q}{y_2}=\frac{10\frac{ft^2}{s}}{1\,ft}=10\frac{ft}{s}, \quad (1)$$

and $y_2 = 1\,ft$

Thus, with $E = y+\frac{V^2}{2g}$

$$E = y+\frac{q^2}{2gy^2} = y+\frac{(10\frac{ft^2}{s})^2}{2(32.2\frac{ft}{s^2})y^2}$$

or

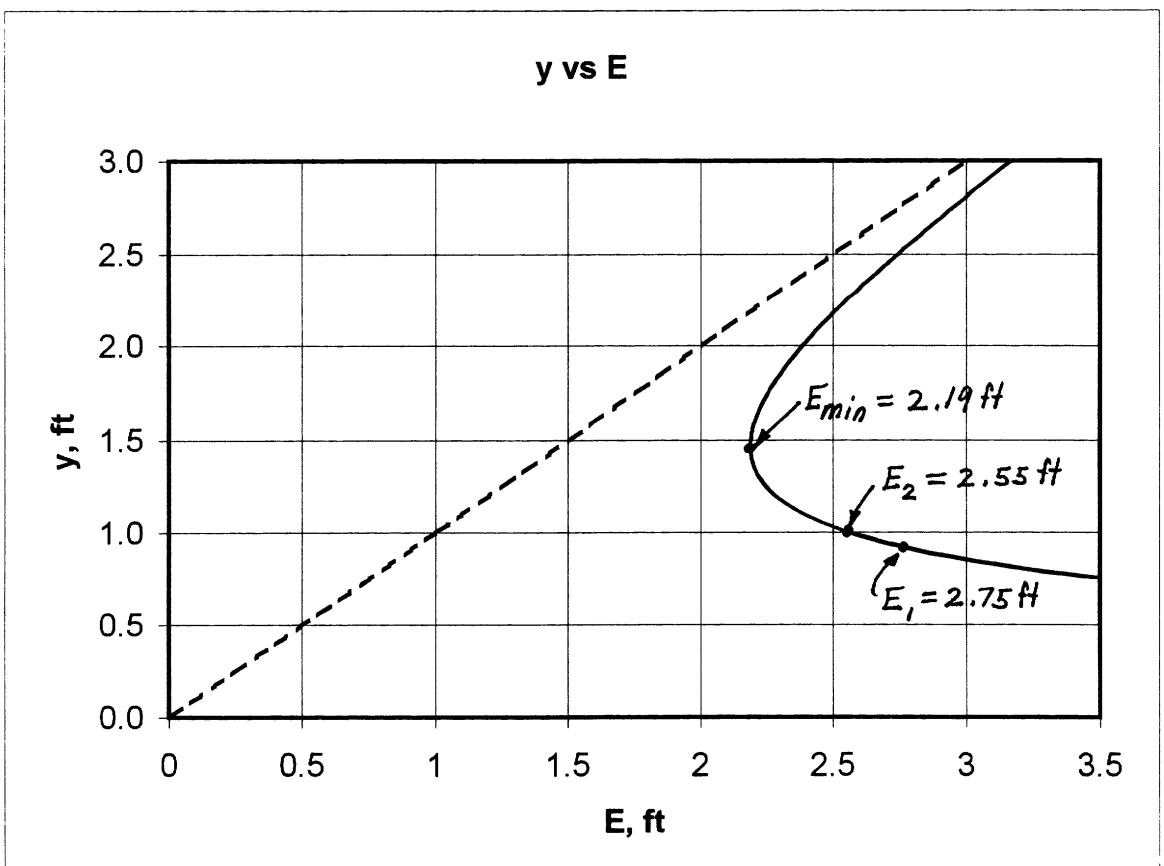
$$E = y+\frac{1.553}{y^2} \text{ where } E\sim ft,\ y\sim ft. \quad (2)$$

and Eq.(1) gives $E_1 = E_2+h_L = E_2+0.2\,ft$

Eq.(2) is plotted below.

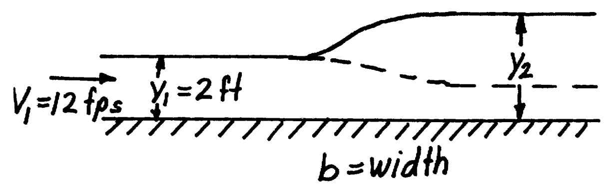
Note: $y_c=\left(\frac{q^2}{g}\right)^{1/3}=\left(\frac{(10\frac{ft^2}{s})^2}{32.2\frac{ft}{s^2}}\right)^{1/3}=1.459\,ft,\ E_{min}=\frac{3}{2}y_c=\frac{3}{2}(1.459\,ft)=2.19\,ft,$

$$E_2=y_2+\frac{1.553}{y_2^2}=1\,ft\ \frac{1.553}{1^2}=2.55\,ft, \text{ and } E_1=E_2+h_L=2.75\,ft$$



10.24

10.24 Water flows in a horizontal, rectangular channel with an initial depth of 2 ft and initial velocity of 12 ft/s. Determine the depth downstream if losses are negligible. Note that there may be more than one solution. Repeat the problem if the initial depth remains the same, but the initial velocity is 6 ft/s.



$$E_1 = E_2\ ,\ or\ \ y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \qquad (1)$$

but $Q_1 = Q_2$ or $V_1 b y_1 = V_2 b y_2$

so that

$$V_2 = \frac{V_1 y_1}{y_2} = \frac{(12\ \frac{ft}{s})(2\ ft)}{y_2} = \frac{24}{y_2}\ \frac{ft}{s}\ ,\ where\ y_2 \sim ft$$

Thus, Eq. (1) becomes

$$2\ ft + \frac{(12\ \frac{ft}{s})^2}{2\,(32.2\ \frac{ft}{s^2})} = y_2 + \frac{(\frac{24}{y_2}\ \frac{ft}{s})^2}{2\,(32.2\ \frac{ft}{s^2})}$$

or

$y_2^3 - 4.24\,y_2^2 + 8.94 = 0$ which has 3 roots; one negative (no physical meaning), one is $y_2 = \underline{\underline{2\ ft}}$ (no change in depth), and $y_2 = \underline{\underline{3.51\ ft}}$ (an increase in depth).

If $V_1 = 6\ \frac{ft}{s}$, then $V_2 = \frac{(6\ \frac{ft}{s})(2\ ft)}{y_2} = \frac{12}{y_2}$

and Eq. (1) becomes

$$2 + \frac{6^2}{2(32.2)} = y_2 + \frac{(\frac{12}{y_2})^2}{2(32.2)}$$

or

$y_2^3 - 2.56\,y_2^2 + 2.24 = 0$ The positive real roots are $y_2 = \underline{\underline{2\ ft}}$, or $y_2 = \underline{\underline{1.38\ ft}}$ (a decrease in depth)

10.25

10.25 A smooth transition section connects two rectangular channels as shown in Fig. P10.25. The channel width increases from 6.0 to 7.0 ft and the water surface elevation is the same in each channel. If the upstream depth of flow is 3.0 ft, determine h, the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.

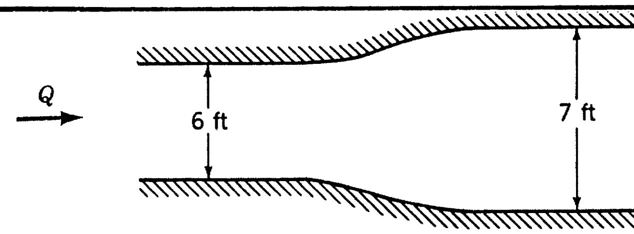
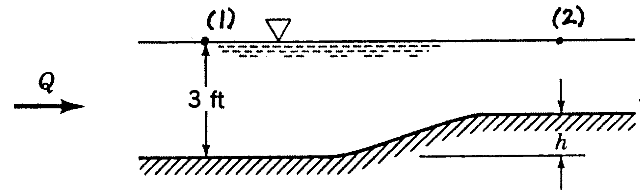


FIGURE P10.25 Top view



Side view

$\frac{\cancel{p_1}}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{\cancel{p_2}}{\gamma}+\frac{V_2^2}{2g}+z_2$, where $p_1=p_2=0$ and $z_1=z_2$

Thus, $V_1=V_2$ or

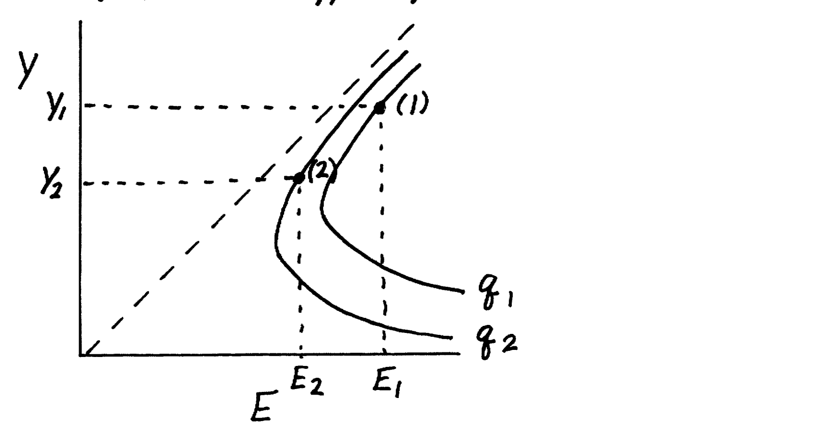
$\frac{Q}{A_1}=\frac{Q}{A_2}$ Hence, $A_1=A_2$ or $(6ft)(3ft)=(7ft)(3ft-h)$

or $h=\underline{\underline{0.429\ ft}}$

Note: $q_1=\frac{Q}{b_1}=\frac{Q}{6}$ and $q_2=\frac{Q}{b_2}=\frac{Q}{7}<q_1$

and $E_1=y_1+\frac{V_1^2}{2g}$ and $E_2=y_2+\frac{V_2^2}{2g}$ Thus, since $V_1=V_2$ it follows that $E_1-E_2=y_1-y_2$

The corresponding specific energy diagram is as indicated below:



10.26

10.26 Water flows over a bump of height $h = h(x)$ on the bottom of a wide rectangular channel as is indicated in Fig. P10.26. If energy losses are negligible, show that the slope of the water surface is given by $dy/dx = -(dh/dx)/[1 - (V^2/gy)]$, where $V = V(x)$ and $y = y(x)$ are the local velocity and depth of flow. Comment on the sign (i.e., <0, $=0$, or >0) of dy/dx relative to the sign of dh/dx.

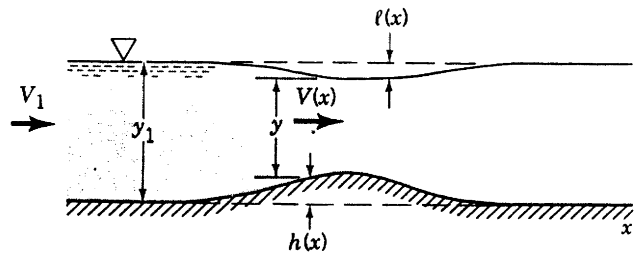


FIGURE P10.26

For any two points on the free surface:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = p_2 = 0 \text{ , } z_1 = y_1 \text{ , and } z_2 = h + y_2$$

Thus, $\frac{V^2}{2g} + h + y = \text{constant}$ so that by differentiating

$$\frac{2V\frac{dV}{dx}}{2g} + \frac{dh}{dx} + \frac{dy}{dx} = 0 \tag{1}$$

Also, for conservation of mass

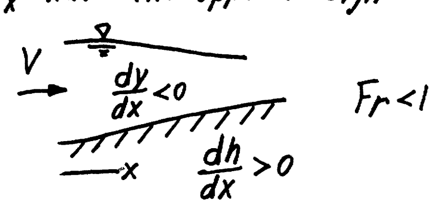
$$V_1 y_1 = Vy \text{ or } V\frac{dy}{dx} + y\frac{dV}{dx} = 0 \text{ or } \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx} \tag{2}$$

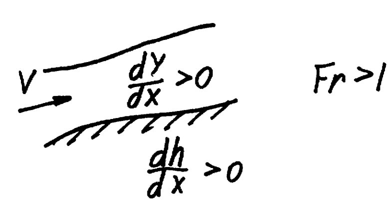
Combine Eqs. (1) and (2):

$$\frac{V}{g}\left(-\frac{V}{y}\frac{dy}{dx}\right) + \frac{dh}{dx} + \frac{dy}{dx} = 0 \text{ , or } \frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left(1 - \left(\frac{V^2}{gy}\right)\right)}$$

Note: If $Fr = \frac{V}{\sqrt{gy}} < 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the opposite sign

If $Fr > 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the same sign.





10.27

10.27 Integrate the differential equation obtained in Problem 10.26 to determine the "draw-down" distance, $\ell = \ell(x)$, indicated in Fig. P10.26. Comment on your results.

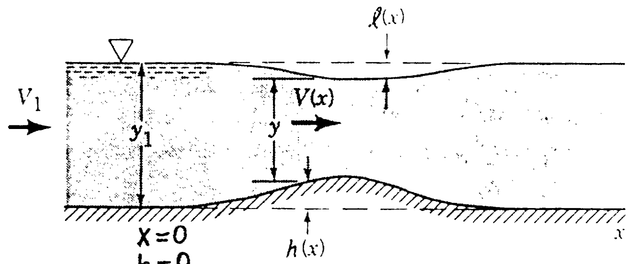


FIGURE P10.26

From Problem 10.26:

$$\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left(1-\left(\frac{V^2}{gy}\right)\right)}, \text{ where } V_1 y_1 = Vy, \text{ or } V = \frac{V_1 y_1}{y}$$

Thus, $\frac{V^2}{gy} = \frac{\left(\frac{V_1 y_1}{y}\right)^2}{gy} = \frac{V_1^2 y_1^2}{gy^3}$ so that

$$\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left[1-\frac{V_1^2 y_1^2}{gy^3}\right]}, \text{ or } \left[1-\left(\frac{V_1^2 y_1^2}{g}\right)y^{-3}\right]dy = -\left(\frac{dh}{dx}\right)dx$$

Integrate from $y = y_1$ and $x = 0$, with $\frac{dh}{dx}$ a given function of x.

$$\int_{y=y_1}^{y}\left[1-\left(\frac{V_1^2 y_1^2}{g}\right)y^{-3}\right]dy = -\int_{x=0}^{x}\left(\frac{dh}{dx}\right)dx = -\int_{h=0}^{h} dh = -h$$

or

$$\left[y - \left(\frac{V_1^2 y_1^2}{g}\right)\left(-\frac{1}{2}\right)y^{-2}\right]_{y_1}^{y} = -h \quad \text{Thus,} \quad y + \left(\frac{V_1^2 y_1^2}{2g}\right)y^{-2} - y_1 - \frac{V_1^2}{2g} = -h$$

or

$$y^3 - \left(y_1 + \frac{V_1^2}{2g} - h\right)y^2 + \left(\frac{V_1^2 y_1^2}{2g}\right) = 0 \qquad (1)$$

Obtain $y = y(x)$ from Eq. (1) and then $\ell = \ell(x)$ from $y_1 = h + y + \ell$ or $\ell = y_1 - h - y$

Note: Eq. (1) is nothing more than the Bernoulli equation:

$$\frac{V_1^2}{2g} + y_1 = \frac{V^2}{2g} + y + h \quad \text{with } V = \frac{V_1 y_1}{y} \text{ so that}$$

$$\frac{V_1^2}{2g} + y_1 = \frac{\left(\frac{V_1 y_1}{y}\right)^2}{2g} + y + h \quad \text{which simplifies to Eq. (1).}$$

10.28 Determine the maximum depth in a 3-m-wide rectangular channel if the flow is to be supercritical with a flowrate of $Q = 60\ m^3/s$.

$$V = \frac{Q}{A} = \frac{60\frac{m^3}{s}}{(3\,m)\,y} = \frac{20}{y}\ ,\ \text{where } y = \text{depth}$$

Also,

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\left(\frac{20}{y}\right)}{\left[\left(9.81\frac{m}{s}\right)y\right]^{1/2}} = \frac{6.39}{y^{3/2}}$$

Note: Fr decreases as y increases.

Thus, with $Fr = 1$, $y = (6.39)^{2/3} = \underline{\underline{3.44\ m}}$

10.30

10.30 The following data are taken from measurements on a river: $A = 200\ \text{ft}^2$, $P = 80$ ft, and $S_0 = 0.015$ ft/50 ft. Determine the average shear stress on the wetted perimeter of this channel.

$$\tau_w = \gamma R_h S_0\,, \quad \text{where} \quad R_h = \frac{A}{P} = \frac{200\,ft^2}{80\,ft} = 2.50\,ft$$

and

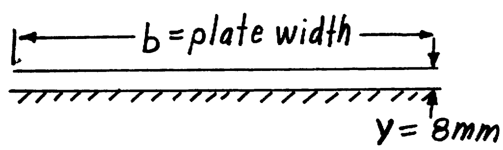
$$S_0 = \frac{0.015\,ft}{50\,ft} = 0.00030$$

Thus,

$$\tau_w = 62.4\,\frac{lb}{ft^3}(2.50\,ft)(0.00030) = \underline{\underline{0.0468\,\frac{lb}{ft^2}}}$$

10.31

10.31 A viscous oil flows down a wide plate with a uniform depth of 8 mm and an average velocity of 50 mm/s. The plate is on a 3° hill and the specific gravity of the oil is 0.85. Determine the average shear stress between the oil and the plate.



$$\tau_w = \gamma R_h S_0\,, \quad \text{where} \quad \gamma = 0.85\,\gamma_{H_2O} = 0.85\left(9800\,\frac{N}{m^3}\right) = 8330\,\frac{N}{m^3}$$

For a wide flat plate, $A = by$ and $P = b$ so that $R_h = \frac{A}{P} = y = 8\times10^{-3}\,m$

Also, $S_0 = \sin 3°$ so that

$$\tau_w = \left(8330\,\frac{N}{m^3}\right)(8\times10^{-3}\,m)\sin 3° = \underline{\underline{3.49\,\frac{N}{m^2}}}$$

10.32

10.32 The following data are obtained for a particular reach of the Provo River in Utah: $A = 183\ \text{ft}^2$, free-surface width = 55 ft, average depth = 3.3 ft, $R_h = 3.22$ ft, $V = 6.56$ ft/s, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine the **(a)** average shear stress on the wetted perimeter, **(b)** the Manning coefficient, n, and **(c)** the Froude number of the flow.

a) $\tau_w = \gamma R_h S_o$, where $S_o = \dfrac{1.04\text{ ft}}{116\text{ ft}} = 0.00897$

Thus, $\tau_w = (62.4\frac{\text{lb}}{\text{ft}^3})(3.22\text{ ft})(0.00897) = \underline{\underline{1.80\frac{\text{lb}}{\text{ft}^2}}}$

b) $Q = \dfrac{\kappa}{n} A R_h^{2/3} S_o^{1/2} = AV$, where $\kappa = 1.49$

Thus,
$$n = \frac{1.49 R_h^{2/3} S_o^{1/2}}{V} = \frac{(1.49)(3.22)^{2/3}(0.00897)^{1/2}}{6.56} = \underline{\underline{0.0469}}$$

c) $Fr = \dfrac{V}{\sqrt{gy}} = \dfrac{6.56\frac{\text{ft}}{\text{s}}}{[(32.2\frac{\text{ft}}{\text{s}^2})(3.3\text{ ft})]^{1/2}} = \underline{\underline{0.636}}$ < 1 (subcritical)

10.33

10.33 By what percent is the flowrate reduced in the rectangular channel shown in Fig. P10.33 because of the addition of the thin center board? All surfaces are of the same material.

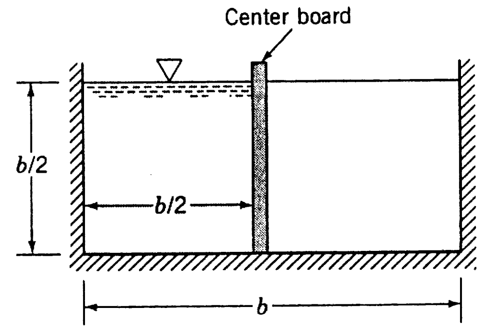


FIGURE P10.33

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$$

Without the centerboard $A = b\left(\frac{b}{2}\right) = \frac{b^2}{2}$, $R_h = \frac{A}{P} = \frac{\frac{b^2}{2}}{2b} = \frac{b}{4}$

or

$$Q_{without} = \frac{K}{n}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3} S_o^{1/2} \qquad (1)$$

With the centerboard $Q_{with} = 2Q_2$, where $A_2 = \left(\frac{b}{2}\right)^2$,

$$R_{h_2} = \frac{A_2}{P_2} = \frac{\left(\frac{b}{2}\right)^2}{3\left(\frac{b}{2}\right)} = \frac{b}{6}$$

or

$$Q_{with} = 2\frac{K}{n}\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3} S_o^{1/2} \qquad (2)$$

Divide Eq. (2) by Eq. (1) to obtain $\frac{Q_{with}}{Q_{without}} = \frac{2\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3}}{\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}} = 0.763$

a $100 - 76.3 = \underline{\underline{23.7\ \%\ \text{reduction}}}$

10.34

10.34 Water flows in an unfinished concrete channel at a rate of 30 m^3/s. What flowrate can be expected if the concrete were finished and the depth remains constant?

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$ Let $(\ )_f$ denote finished; $(\ )_u$ denote unfinished.

Thus, since $A_u = A_f$, $R_{h_u} = R_{h_f}$ and $S_{ou} = S_{of}$, it follows that

$$\frac{Q_u}{Q_f} = \frac{\frac{K}{n_u} A_u R_{h_u}^{2/3} S_{ou}^{1/2}}{\frac{K}{n_f} A_f R_{h_f}^{2/3} S_{of}^{1/2}} = \frac{n_f}{n_u}$$

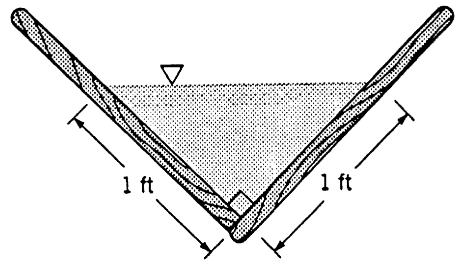
From Table 10.1 $n_u = 0.014$, $n_f = 0.012$

or

$$Q_f = \frac{n_u}{n_f} Q_u = \frac{0.014}{0.012}\left(30\frac{m^3}{s}\right) = \underline{\underline{35.0\frac{m^3}{s}}}$$

10.35

10.35 The great Kings River flume in Fresno County, California, was used from 1890 to 1923 to carry logs from an elevation of 4500 ft where trees were cut to an elevation of 300 ft at the railhead. The flume was 54 miles long, constructed of wood, and had a V-cross section as indicated in Fig. P10.35. It is claimed that logs would travel the length of the flume in 15 hours. Do you agree with this claim? Provide appropriate calculations to support your answer.



■ FIGURE P10.35

ℓ = distance traveled $= V_{log}\, t$. Thus,

$$V_{log} = \frac{\ell}{t} = \frac{(54\,mi)(5280\,ft/mi)}{(15\,hr)(3600\,s/hr)} = 5.28\,\frac{ft}{s}$$

Determine the average water velocity, V, and compare it with this V_{log}.

$V = \frac{K}{n} R_h^{2/3} \sqrt{S_0}$, where $K = 1.49$, $A = \frac{1}{2}(1\,ft^2) = 0.5\,ft^2$, $P = 2\,ft$

so that $R_h = \frac{A}{P} = \frac{0.5\,ft^2}{2\,ft} = 0.25\,ft$

Also,

$$S_0 = \frac{\Delta z}{\ell} = \frac{(4500 - 300)\,ft}{(54\,mi)(5280\,ft/mi)} = 0.0147$$

Thus, with $n = 0.012$ (see Table 10.1, planed wood),

$$V = \frac{1.49}{0.012}(0.25)^{2/3}\sqrt{0.0147} = 5.97\,\frac{ft}{s}$$

Note: V is slightly larger than V_{log}. Thus, the claim appears to be correct. Yes.

10.36

10.36 Water lows in a river with a speed of 3 ft/s. The river is a clean, straight natural channel, 400 ft wide with a nearly uniform 3-ft depth. Is the slope of this river greater than or less than the average slope of the Mississippi River which drops a distance of 1475 ft in its 2552-mi length? Support your answer with appropriate calculations.

(1) $V = \frac{\kappa}{n} R_h^{2/3} \sqrt{S_0}$, where $\kappa = 1.49$

$V = 3\ \text{ft/s}$, $y = 3\ \text{ft}$, $b = 400\ \text{ft}$, $A = by = 1200\ \text{ft}^2$, $P = b + 2y = 406\ \text{ft}$

Thus,

$$R_h = \frac{A}{P} = \frac{1200\ \text{ft}^2}{406\ \text{ft}} = 2.96\ \text{ft}$$

Also, from Table 10.1, $n = 0.03$ so that from Eq. (1):

$$3 = \frac{1.49}{0.03} (2.96)^{2/3} \sqrt{S_0}$$

or

$$S_0 = 0.000858$$

The average Mississippi slope is

$$S_{0\,Miss} = \frac{1475\ \text{ft}}{2552\ \text{m}\left(5280\ \frac{\text{ft}}{\text{mi}}\right)} = 0.000110 < 0.000858$$

The unknown river has a <u>greater</u> slope.

10.37

10.37 At a particular location the cross section of the Columbia River is as indicated in Fig. P10.37. If on a day without wind it takes 5 min to float 0.5 mi along the river, which drops 0.46 ft in that distance, determine the value of the Manning coefficient, n.

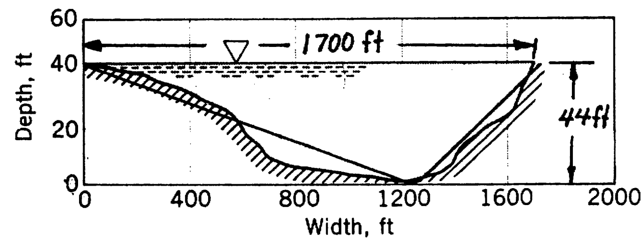


FIGURE P10.37

From the given data, $V = \dfrac{(0.5\,mi)(5280\,\frac{ft}{mi})}{(5\,min)(60\,\frac{s}{min})} = 8.8\,\frac{ft}{s}$.

From the Manning equation,

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49,\ S_o = \frac{0.46\,ft}{(0.5\,mi)(5280\,\frac{ft}{mi})} = 0.000174, \quad (1)$$

and $R_h = \dfrac{A}{P}$.

Approximate A and P from the figure as

$$A \approx \tfrac{1}{2} b y = \tfrac{1}{2}(1700\,ft)(44\,ft) = 37{,}400\,ft^2$$

and

$$P \approx 1800\,ft \quad \text{Thus, } R_h \approx \frac{37{,}400\,ft^2}{1800\,ft} = 20.8\,ft$$

Hence, from Eq. (1):

$$8.8 = \frac{1.49}{n}(20.8)^{2/3}(0.000174)^{1/2}$$

or

$$n = \underline{\underline{0.0169}}$$

10.38

10.38 If the free surface of the Columbia River shown in Fig. P10.37 were 20 ft above the bottom rather than 44 ft, as is indicated in the figure, how long would it take to float the 0.5-mi stretch considered in Problem 10.37? Assume the elevation change remains 0.46 ft.

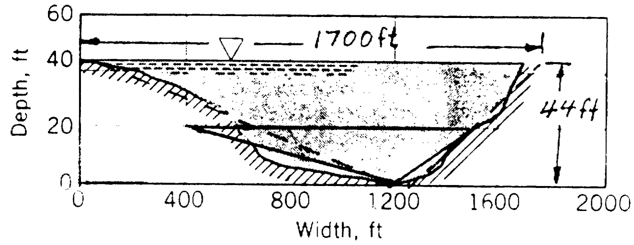


FIGURE P10.37

Let $(\ )_{20}$ denote the 20 ft depth and $(\ )_{44}$ the 44 ft depth.
Thus,

$$V_{20} = \frac{K}{n_{20}} R_{h_{20}}^{2/3} S_{o_{20}}^{1/2} \quad \text{and} \quad V_{44} = \frac{K}{n_{44}} R_{h_{44}}^{2/3} S_{o_{44}}^{1/2}, \text{ where } n_{20} = n_{44}$$

and $S_{o_{20}} = S_{o_{44}}$

Hence,

$$\frac{V_{20}}{V_{44}} = \left(\frac{R_{h_{20}}}{R_{h_{44}}}\right)^{2/3} \qquad (1)$$

From the figure $A_{44} \approx \frac{1}{2} b y = \frac{1}{2}(1700\,ft)(44\,ft)$
$= 37{,}400\,ft^2$

$P_{44} \approx 1800\,ft$

and

$$A_{20} \approx \frac{1}{2} b y = \frac{1}{2}(1550-400)(20)\,ft^2 = 11{,}500\,ft^2$$

$$P_{20} \approx 1550 - 600 \approx 1000\,ft$$

Thus,

$$R_{h_{20}} \approx \frac{A_{20}}{P_{20}} = \frac{11{,}500\,ft^2}{1000\,ft} = 11.5\,ft$$

and

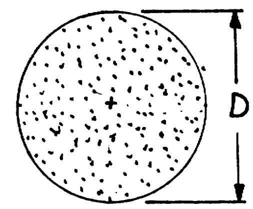
$$R_{h_{44}} \approx \frac{A_{44}}{P_{44}} = \frac{37{,}400\,ft^2}{1800\,ft} = 20.8\,ft$$

Therefore, from Eq. (1):

$$\frac{V_{20}}{V_{40}} = \left(\frac{11.5\,ft}{20.8\,ft}\right)^{2/3} = 0.674$$ so that with $\ell = Vt$ and $\ell_{20} = \ell_{44} = 0.5\,mi$,

$$t_{20} = \left(\frac{V_{44}}{V_{20}}\right) t_{44} = \frac{5\,min}{0.674} = \underline{\underline{7.42\,min}}$$

10.39 Rainwater runoff from a 200-ft by 500-ft parking lot is to drain through a circular concrete pipe that is laid on a slope of 3 ft/mi. Determine the pipe diameter if it is to be full with a steady rainfall of 1.5 in./hr.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49, \; S_o = \frac{3 \text{ ft}}{5280 \text{ ft}} = 0.000568, \tag{1}$$

$$A = \frac{\pi}{4} D^2 \quad \text{and} \quad R_h = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

From Table 10.1, $n = 0.012$

Also, $Q = A_{lot}\, r$, where r = rainfall rate $= 1.5 \frac{\text{in.}}{\text{hr}}$

Thus,

$$Q = (200\text{ft})(500\text{ft})\left(1.5 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12} \frac{\text{ft}}{\text{in.}}\right)\left(\frac{1 \text{ hr}}{3600 \text{s}}\right) = 3.47 \frac{\text{ft}^3}{\text{s}}$$

Hence, from Eq. (1):

$$3.47 = \frac{1.49}{0.012}\left(\frac{\pi}{4} D^2\right)\left(\frac{D}{4}\right)^{2/3} (0.000568)^{1/2}$$

or

$$D = \underline{\underline{1.64 \text{ft}}}$$

10.40

10.40 To prevent weeds from growing in a clean earthen-lined canal, it is recommended that the velocity be no less than 2.5 ft/s. For the symmetrical canal shown in Fig. P10.40, determine the minimum slope needed.

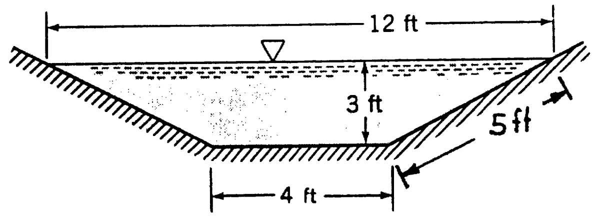


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \quad (1)$$

$$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2 \text{ and } P = 4\,ft + 2(5\,ft) = 14\,ft$$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.022$ so that Eq.(1) gives (with $V = 2.5\,\frac{ft}{s}$)

$$2.5 = \frac{1.49}{0.022}(1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000664}}$$

10.41

10.41 The smooth concrete-lined symmetrical channel shown in Video V10.3 and Fig. P10.40 carries water from the silt-laden Colorado River. If the velocity must be 4.0 ft/s to prevent the silt from settling out (and eventually clogging the channel), determine the minimum slope needed.

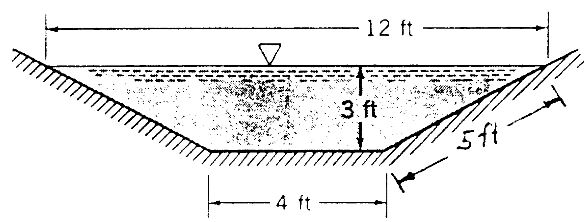


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \quad (1)$$

$$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2 \text{ and } P = 4\,ft + 2(5\,ft) = 14\,ft$$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.012$ so that Eq.(1) gives (with $V = 4\,\frac{ft}{s}$)

$$4.0 = \frac{1.49}{0.012}(1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000505}}$$

10.42

10.42 The symmetrical channel shown in Fig. P10.40 is dug in sandy loam soil with $n = 0.020$. For such surface material it is recommended that to prevent scouring of the surface the average velocity be no more than 1.75 ft/s. Determine the maximum slope allowed.

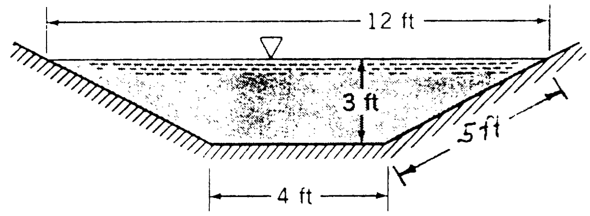


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \qquad (1)$$

$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \dfrac{24\,ft^2}{14\,ft} = 1.714\,ft$

With $n = 0.020$ and $V = 1.75\,\frac{ft}{s}$ Eq.(1) gives

$1.75 = \dfrac{1.49}{0.020}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000269}}$

10.43

10.43 The flowrate in the clay-lined channel ($n = 0.025$) shown in Fig. P10.43 is to be 300 ft³/s. To prevent erosion of the sides, the velocity must not exceed 5 ft/s. For this maximum velocity, determine the width of the bottom, b, and the slope, S_0.

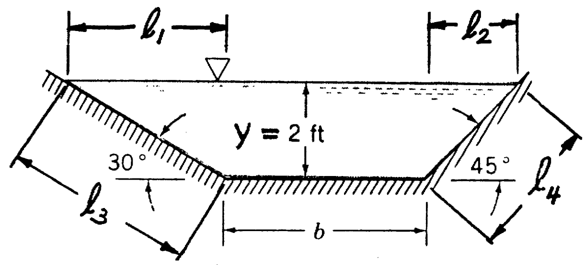


FIGURE P10.43

$$V = \frac{Q}{A}, \text{ where } A = \tfrac{1}{2}[b + (b + \ell_1 + \ell_2)]y \text{ with } \ell_1 = \frac{2\,ft}{\tan 30^\circ} = 3.46\,ft \qquad (1)$$

and $\ell_2 = \dfrac{2\,ft}{\tan 45^\circ} = 2\,ft$

Thus, $5\,\frac{ft}{s} = \dfrac{300\,\frac{ft^3}{s}}{\frac{1}{2}[b + (b + 3.46\,ft + 2\,ft)](2\,ft)}$, or $b = \underline{\underline{27.3\text{ ft}}}$

Also, $V = \dfrac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1, $n = 0.025$ (2)

From Eq.(1), $A = \frac{1}{2}[2(27.3\,ft) + 3.46\,ft + 2\,ft](2\,ft) = 60.0\,ft^2$

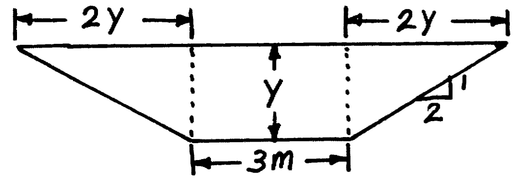
Also, $P = b + \ell_3 + \ell_4 = 27.3\,ft + \dfrac{2\,ft}{\sin 30^\circ} + \dfrac{2\,ft}{\sin 45^\circ} = 34.1\,ft$

Thus, $R_h = \dfrac{A}{P} = \dfrac{60.0\,ft^2}{34.1\,ft} = 1.76\,ft$ so that Eq.(2) becomes

$5 = \dfrac{1.49}{0.025}(1.76)^{2/3} S_o^{1/2}$, or $S_o = \underline{\underline{0.00331}}$

10.44

10.44 A trapezoidal channel with a bottom width of 3.0 m and sides with a slope of 2:1 (horizontal:vertical) is lined with fine gravel (n = 0.020) and is to carry 10 m³/s. Can this channel be built with a slope of S_0 = 0.00010 if it is necessary to keep the velocity below 0.75 m/s to prevent scouring of the bottom? Explain.



Determine V with $Q = 10\frac{m^3}{s}$ and $S_o = 0.00010$.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}\ , \text{ where } A = \tfrac{1}{2}y[3+(3+4y)] = 2y^2+3y \qquad (0)$$

and $R_h = \frac{A}{P}$, with $P = 3+2(\sqrt{5}\,y)$

Thus,

$$10 = \frac{1}{0.02}(2y^2+3y)\left[\frac{2y^2+3y}{3+2\sqrt{5}\,y}\right]^{2/3}(0.0001)^{1/2}$$

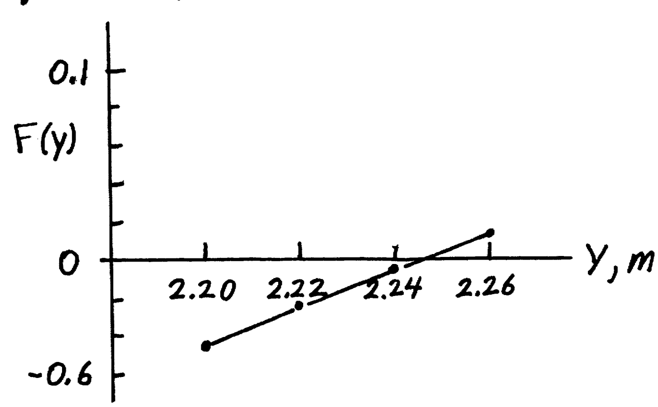
or

$$20 = \frac{(2y^2+3y)^{5/3}}{(3+2\sqrt{5}\,y)^{2/3}}$$ which can be written as

$$2y^2+3y-6.03\,(3+2\sqrt{5}\,y)^{0.4} = 0 = F(y) \qquad (1)$$

Solve (by trial and error) Eq. (1) for y:

y, m	F (y)
2.20	-0.459
2.22	-0.269
2.24	-0.077
2.26	0.117



From the graph we see that $y \approx 2.25$ m

Hence, from Eq. (0) $A = 2(2.25)^2 + 3(2.25) = 16.9\ m^2$

so that

$$V = \frac{Q}{A} = \frac{10\frac{m^3}{s}}{16.9\ m^2} = 0.592\ \frac{m}{s}$$

Thus, $V < 0.75\ \frac{m}{s}$ so that scouring will not occur.

10.45

10.45 Water flows in a 2-m-diameter finished concrete pipe so that it is completely full and the pressure is constant all along the pipe. If the slope is $S_0 = 0.005$, determine the flowrate by using open-channel flow methods. Compare this result with that obtained by using pipe flow methods of Chapter 8.

For open channel flow $Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}$, where $\kappa = 1$

Also, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(2m)^2 = 3.14\,m^2$ and $P = \pi D = 6.28\,m$ so that

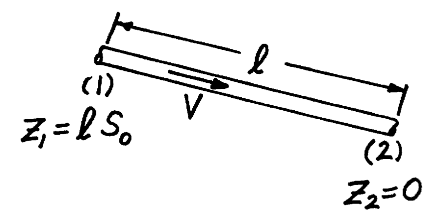
$$R_h = \frac{A}{P} = \frac{3.14\,m^2}{6.28\,m} = 0.5\,m$$

Hence, with $n = 0.012$ for finished concrete (see Table 10.1)

$$Q = \frac{1}{0.012}(3.14)(0.5)^{2/3}(0.005)^{1/2} = \underline{\underline{11.7\,\frac{m^3}{s}}} \text{ (open channel)}$$

For pipe flow with constant pressure:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$$



where $p_1 = p_2$ and $V_1 = V_2$

Thus, with $z_1 - z_2 = \ell S_0$,

$$\ell S_0 = f\frac{\ell}{D}\frac{V^2}{2g}$$

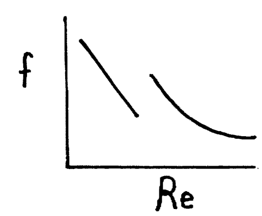
or

$$fV^2 = 2gDS_0 = 2(9.81\tfrac{m}{s^2})(2m)(0.005) \quad \text{Thus, } fV^2 = 0.196 \qquad (1)$$

From Fig. 8.22, for smooth concrete $\frac{\varepsilon}{D} = 1.5 \times 10^{-4}$

$$\text{Also, } Re = \frac{VD}{\nu} = \frac{V(2m)}{1.12 \times 10^{-6}\,\frac{m^2}{s}} = 1.79 \times 10^6 V \qquad (2)$$

and from the Moody chart (Fig. 8.23): (3)



Solve Eqs. (1), (2), and (3) for f, V, Re:

Assume $f = 0.015$ so that from Eq. (1)

$$V = \left[\frac{0.196}{0.015}\right]^{1/2} = 3.61\,\frac{m}{s}$$

or

$Re = 1.79 \times 10^6 (3.61) = 6.46 \times 10^6$ Thus, from Eq. (3) (Moody chart)

$f = 0.013 \neq 0.015$. Assume $f = 0.013$, or $V = \left[\frac{0.196}{0.013}\right]^{1/2} = 3.88\,\frac{m}{s}$

so that $Re = 1.79 \times 10^6 (3.88) = 6.95 \times 10^6$ Thus, from Eq. (3) $f = 0.013$ (checks with the assumed value) Hence, $V = 3.88\,\frac{m}{s}$ or

$$Q = AV = \frac{\pi}{4}(2m)^2(3.88\tfrac{m}{s}) = \underline{\underline{12.2\,\frac{m^3}{s}}} \text{ (pipe flow)} \approx 11.7\,\frac{m^3}{s} \text{ (open channel flow)}$$

10.47 Because of neglect, an irrigation canal has become weedy and the maximum flowrate possible is only 90% of the desired flowrate. Would removing the weeds, thus making the surface gravel, allow the canal to carry the desired flowrate? Support your answer with appropriate calculations.

Let $(\ )_w$ and $(\ )_g$ denote weedy and gravel conditions, respectively.

Thus,

(1) $Q_w = \frac{K}{n_w} A_w R_{hw}^{2/3} \sqrt{S_{ow}}$ and

(2) $Q_g = \frac{K}{n_g} A_g R_{hg}^{2/3} \sqrt{S_{og}}$

where with the channel full, $A_w = A_g$, $R_{hw} = R_{hg}$, and $S_{ow} = S_{og}$

Hence, by dividing Eq (2) by Eq (1):

$$\frac{Q_g}{Q_w} = \frac{\frac{1}{n_g}}{\frac{1}{n_w}} = \frac{n_w}{n_g}$$, where from Table 10.1: $n_w = 0.030$, $n_g = 0.025$

Thus,

$$Q_g = \frac{0.030}{0.025} Q_w = 1.2\, Q_w$$

But $Q_w = 0.9\, Q_{desired}$ so that

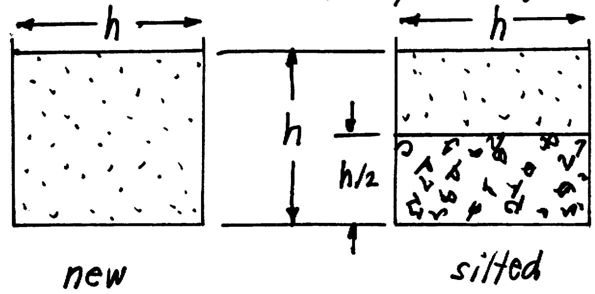
$$Q_g = 1.2\,(0.9\, Q_{desired}) = 1.08\, Q_{desired}$$

Yes, it would work.

10.48

10.48 An open channel of square cross section had a flowrate of 80 ft^3/s when first used. After extended use, the channel became half-filled with silt. Determine the flowrate for this silted condition. Assume the Manning coefficient is the same for all the surfaces.

Let $(\ )_n$ and $(\ )_s$ denote the new and silted conditions, respectively.



Thus,

(1) $$Q_n = \frac{K}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{on}}$$

(2) $$Q_s = \frac{K}{n_s} A_s R_{h_s}^{2/3} \sqrt{S_{os}}$$

where

$n_s = n_n$, $A_n = h^2$, $P_n = 3h$, $A_s = \frac{1}{2}h^2$, $P_s = h + 2(h/2) = 2h$, $S_{on} = S_{os}$

Thus,

$$R_{h_n} = \frac{A_n}{P_n} = \frac{h^2}{3h} = \frac{h}{3} \quad \text{and} \quad R_{h_s} = \frac{A_s}{P_s} = \frac{\frac{1}{2}h^2}{2h} = \frac{h}{4}$$

Hence, divide Eq.(2) by Eq.(1) to obtain:

$$\frac{Q_s}{Q_n} = \frac{n_n A_s R_{hs}^{2/3}}{n_s A_n R_{hs}^{2/3}} = \left(\frac{A_s}{A_n}\right)\left(\frac{R_{hs}}{R_{hn}}\right)^{2/3} = \left(\frac{\frac{1}{2}h^2}{h^2}\right)\left(\frac{\frac{h}{4}}{\frac{h}{3}}\right)^{2/3} = 0.413$$

so that

$$Q_s = 0.413\, Q_n = 0.413\left(80\ \frac{ft^3}{s}\right) = \underline{\underline{33.0\ \frac{ft^3}{s}}}$$

10.49

10.49 A circular, finished concrete culvert is to carry a discharge of 50 ft³/s on a slope of 0.0010. It is to flow not more than half-full. The culvert pipes are available from the manufacturer with diameters that are multiples of 1 ft. Determine the smallest suitable culvert diameter.

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = 0.001$, and (from Table 10.1) $n = 0.012$

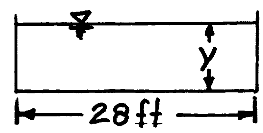
For a circular pipe half full $A = \frac{\pi}{8}D^2$, $P = \frac{\pi}{2}D$ so that $R_h = \frac{A}{P} = \frac{D}{4}$

Thus, $50 = \frac{1.49}{0.012}\left(\frac{\pi}{8}D^2\right)\left(\frac{D}{4}\right)^{2/3}(0.001)^{1/2}$, or $D = 5.21\ ft$

To make sure it is not more than half full use the <u>6 ft diameter</u> pipe.

10.50

10.50 A rectangular unfinished concrete channel of 28-ft-width is laid on a slope of 8 ft/mi. Determine the flow depth and Froude number of the flow if the flowrate is 400 ft³/s.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{8\ ft}{5280 ft} = 0.001515$, and from Table 10.1 $n = 0.014$

Also, $A = 28y$ and $P = 2y + 28$ so that $R_h = \frac{A}{P} = \frac{28y}{2y+28}$

Thus, $400 = \frac{1.49}{0.014}\left(\frac{28y}{2y+28}\right)^{2/3}(28y)(0.001515)^{1/2}$

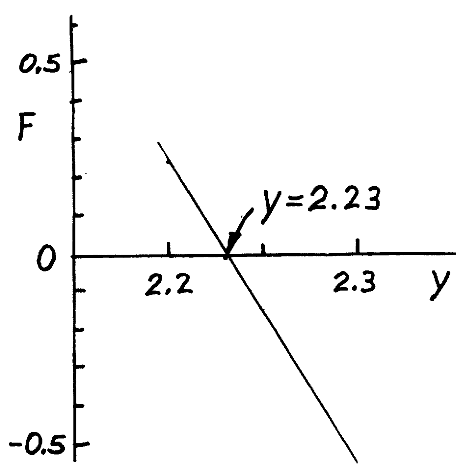
or

$0.594 = \frac{y^{5/3}}{(y+14)^{2/3}}$

Hence, $0.458(y+14) - y^{5/2} = 0 \equiv F(y)$

Trial and error solution foy $F(y) = 0$

From the graph $F = 0$ when $y \simeq 2.23$



Thus, $y = \underline{2.23\ ft}$

$V = \frac{Q}{A} = \frac{400 \frac{ft^3}{s}}{(28 ft)(2.23 ft)} = 6.41 \frac{ft}{s}$

so that

$Fr = \frac{V}{\sqrt{gy}} = \frac{6.41 \frac{ft}{s}}{\left[(32.2 \frac{ft}{s^2})(2.23 ft)\right]^{1/2}} = \underline{0.756}$

10.51

10.51 A 10-ft-wide rectangular channel is built to bypass a dam so that fish can swim upstream during their migration. During normal conditions when the water depth is 4 ft, the water velocity is 5 ft/s. Determine the velocity during a flood when the water depth is 8 ft.

Let $(\ )_n$ and $(\ )_f$ denote normal and flood conditions, respectively. Thus,

(1) $V_n = \frac{\kappa}{n_n} R_{h_n}^{2/3} \sqrt{S_{on}}$ and

(2) $V_f = \frac{\kappa}{n_f} R_{h_f}^{2/3} \sqrt{S_{of}}$

where $n_n = n_f$, $S_{on} = S_{of}$ and

$A_n = 10\,ft(4\,ft) = 40\,ft^2$, $A_f = 10\,ft(8\,ft) = 80\,ft^2$

$P_n = 10\,ft + 2(4\,ft) = 18\,ft$, $P_f = 10\,ft + 2(8\,ft) = 26\,ft$

Thus, $R_{h_n} = \frac{A_n}{P_n} = \frac{40\,ft^2}{18\,ft} = 2.22\,ft$

and

$R_{h_f} = \frac{A_f}{P_f} = \frac{80\,ft^2}{26\,ft} = 3.08\,ft$

Hence, divide Eq(2) by Eq(1) to obtain:

$$\frac{V_f}{V_n} = \left(\frac{R_{hf}}{R_{hn}}\right)^{2/3} = \left(\frac{3.08\,ft}{2.22\,ft}\right)^{2/3} = 1.24$$

so that

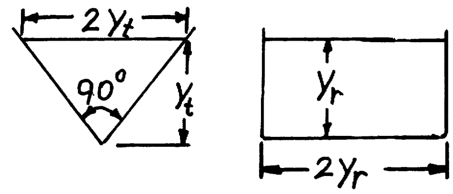
$$V_f = 1.24\,V_n = 1.24\left(5\,\frac{ft}{s}\right) = \underline{\underline{6.22\,\frac{ft}{s}}}$$

10.52

10.52 An engineer is to design a channel lined with planed wood to carry water at a flowrate of 2 m^3/s on a slope of 10 m/800 m. The channel cross section can be either a 90° triangle or a rectangle with a cross section twice as wide as its depth. Which would require less wood and by what percent?

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2} \qquad (1)$$

Let $(\ )_t$ denote the triangular cross-section and $(\ )_r$ denote the rectangular cross-section



Thus, $Q_r = Q_t = 2\,\frac{m^3}{s}$, $S_{or} = S_{ot} = \frac{10}{800}$ and $n_r = n_t$ so that Eq. (1) gives

$$A_r R_{hr}^{2/3} = A_t R_{ht}^{2/3}, \text{ where } R_h = \frac{A}{P} \qquad (2)$$

Hence,

$A_r = 2y_r^2$, $P_r = 4y_r$ so that $R_{hr} = \frac{2y_r^2}{4y_r} = \frac{1}{2}y_r$

Also,

$A_t = \frac{1}{2}(2y_t)y_t = y_t^2$, $P_t = 2(\sqrt{2}\,y_t)$ so that $R_{ht} = \frac{y_t}{2\sqrt{2}}$

Thus, from Eq. (2):

$2y_r^2\left(\frac{1}{2}y_r\right)^{2/3} = y_t^2\left(\frac{1}{2\sqrt{2}}y_t\right)^{2/3}$, or $y_r = 0.707\,y_t$

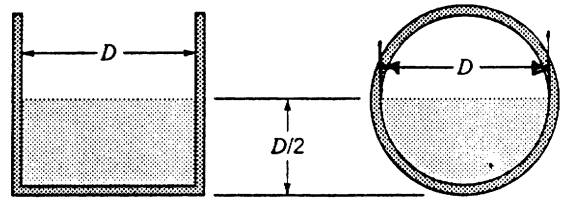
The amount of wood is proportional to the wetted perimeter, P. Since

$$\frac{P_t}{P_r} = \frac{2\sqrt{2}\,y_t}{4y_r} = \frac{2\sqrt{2}\,y_t}{4(0.707)y_t} = 1.00$$

<u>the triangle requires the same amount of wood as the rectangle</u>

10.53

10.53 The two channels shown in Fig. P10.53 are laid on the same slope and lined with the same material. When these channels are flowing half full, in which one will the flowrate be the greatest? Show any calculations needed to obtain your answer.



■ FIGURE P10.53

Let $(\ )_s$ and $(\ )_r$ denote the square and round channels, respectively. Thus,

(1) $Q_s = \frac{\kappa}{n_s} A_s R_{h_s}^{2/3} \sqrt{S_{os}}$ and

(2) $Q_r = \frac{\kappa}{n_r} A_r R_{h_r}^{2/3} \sqrt{S_{or}}$

where $n_s = n_r$ and $S_{os} = S_{or}$

Also,

$$A_s = \frac{D^2}{2} \text{ and } R_{h_s} = \frac{A_s}{P_s} = \frac{\frac{D^2}{2}}{2D} = \frac{D}{4}$$

$$A_r = \frac{\pi D^2}{8} \text{ and } R_{h_r} = \frac{A_r}{P_r} = \frac{\frac{\pi D^2}{8}}{\frac{\pi}{2} D} = \frac{D}{4}$$

Thus, divide Eq (1) by Eq (2) to obtain:

$$\frac{Q_s}{Q_r} = \frac{A_s}{A_r}\left(\frac{R_{hs}}{R_{hr}}\right)^{2/3} = \left(\frac{\frac{D^2}{2}}{\frac{\pi D^2}{8}}\right)\left(\frac{D/4}{D/4}\right)^{2/3} = \frac{4}{\pi}$$

or

$$Q_s = \frac{4}{\pi} Q_r = 1.27\, Q_r$$

The rectangular channel has the larger flowrate.

10.54

10.54 Water flows in a channel with an equilateral triangle cross section as shown in Fig. P10.54. For a given Manning coefficient, n, and channel slope, determine the depth that gives the maximum flowrate.

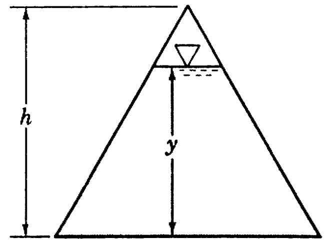
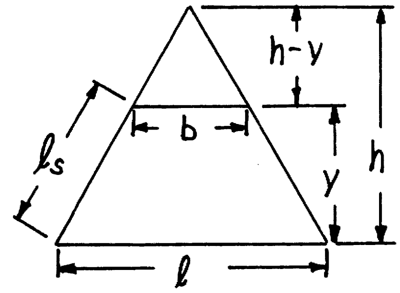


FIGURE P10.54



$Q = \frac{K}{n} A R_h^{2/3} S_0^{1/2}$, where

$A = \frac{1}{2}\ell h - \frac{1}{2}b(h-y) = \frac{1}{\tan 60^\circ}\left[h^2 - (h-y)^2\right]$

or

$A = \frac{1}{\tan 60^\circ}\left[2hy - y^2\right]$

and

$P = \ell + 2\ell_s = 2\left(\frac{h}{\tan 60^\circ} + \frac{y}{\sin 60^\circ}\right)$

$\ell = \frac{2h}{\tan 60^\circ}$

$b = \frac{2(h-y)}{\tan 60^\circ}$

$\ell_s = \frac{y}{\sin 60^\circ}$

Thus, $R_h = \frac{A}{P}$ or

$R_h = \frac{2hy - y^2}{2\left(h + \frac{y}{\cos 60^\circ}\right)}$

Therefore,

$Q = \frac{K}{n}\frac{1}{\tan 60^\circ}(2hy - y^2)\left[\frac{2hy - y^2}{2\left(h + \frac{y}{\cos 60^\circ}\right)}\right]^{2/3} S_0^{1/2}$

For the maximum flowrate, $\frac{dQ}{dy} = 0$, which is equivalent to $\frac{dF}{dy} = 0$, where $F(y) \equiv \frac{(2hy - y^2)^{5/3}}{(y + h\cos 60^\circ)^{2/3}}$ Upon differentiation and simplification this gives

$5(y + h\cos 60^\circ)(h-y) - (2hy - y^2) = 0$

or

$4y^2 + (5h\cos 60^\circ - 3h)y - 5h^2\cos 60^\circ = 0$

which can be written as

$8\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right) - 5 = 0$

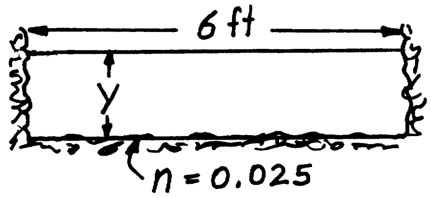
Hence, $\frac{y}{h} = \frac{1 \pm \sqrt{1 + 4(8)(5)}}{16} = -0.731$ or $+0.856$

The negative root has no physical meaning.

Thus, $y = \underline{\underline{0.856\ h}}$

10.55

10.55 At what depth will 50 ft^3/s of water flow in a 6-ft-wide rectangular channel lined with rubble masonry set on a slope of 1 ft in 500 ft? Is a hydraulic jump possible under these conditions? Explain.



$Q = \frac{1.49}{n} A R_h^{2/3} \sqrt{S_0}$ where

$A = 6y$, $R_h = \frac{A}{P} = \frac{6y}{2y+6}$, $S_0 = \frac{1\ ft}{500\ ft}$

and

$n = 0.025$ (see Table 10.1)

Thus,

$$50 = \frac{1.49}{0.025}(6y)\left[\frac{6y}{2y+6}\right]^{2/3}(0.002)^{1/2}$$

which becomes

$$y^{5/3} = (2y+6)^{2/3}(0.948)$$

The trial and error solution to this equation is

$y = \underline{\underline{2.53\ ft}}$

Thus, $V = \frac{Q}{A} = \frac{50\ ft^3/s}{6(2.53)\ ft^2} = 3.29\ ft/s$

so that

$$Fr = \frac{V}{\sqrt{gy}} = \frac{3.29\ ft/s}{[(32.2\ ft/s^2)(2.53\ ft)]^{1/2}} = 0.365$$

Since $Fr < 1$ it is $\underline{\underline{\text{not possible}}}$ to have a hydraulic jump.

10.56 Water flows in the symmetrical, unfinished concrete trapezoidal channel shown in Fig. P10.56 at a rate of 120 ft^3/s. The slope is 4.2 ft/2000 ft. Determine the number of cubic yards of concrete needed to line each 1000 ft of the channel.

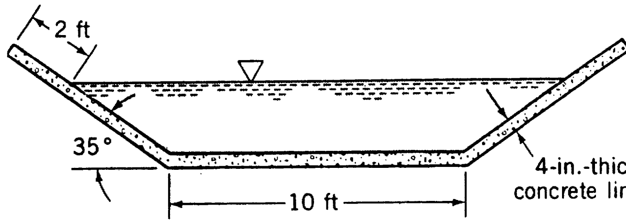
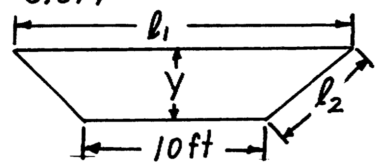


FIGURE P10.56

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where from Table 10.1 } n = 0.014 \qquad (1)$$

Also,



$$A = \tfrac{1}{2}(\ell_1 + 10\,ft)y = \tfrac{1}{2}(20\,ft + 2.86y)y$$

$$\ell_1 = 10\,ft + 2\frac{y}{\tan 35°}$$

or

$$A = (10 + 1.43y)y$$

$$\ell_2 = \frac{y}{\sin 35°} = 1.74y$$

$$R_h = \frac{A}{P} = \frac{A}{(10\,ft + 2\ell_2)}$$

or $R_h = \dfrac{(10+1.43y)y}{(10+3.48y)}$

Hence, with $K = 1.49$ Eq. (1) becomes

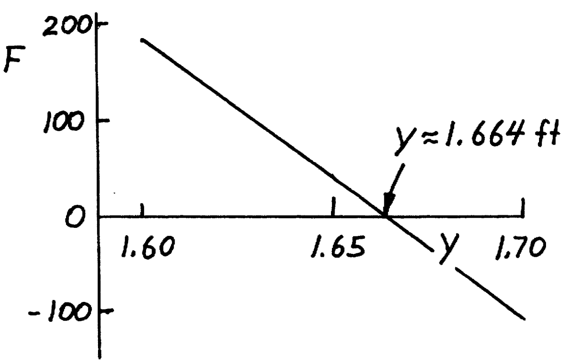
$$120 = \frac{1.49}{0.014}(10+1.43y)y\left[\frac{(10+1.43y)y}{(10+3.48y)}\right]^{2/3}\left(\frac{4.2}{2000}\right)^{1/2}$$

or

$$14{,}890 = \frac{(10y+1.43y^2)^5}{(10+3.48y)^2}, \text{ or } \; 122\,(10+3.48y) - (10y+1.43y^2)^{5/2} = 0 \equiv F(y)$$

Solve (trial and error) for $F(y) = 0$

y	F
1.60	185
1.65	42.5
1.70	-111



Thus, $y = 1.664$ ft

$\forall$ = volume of concrete per 1,000 ft

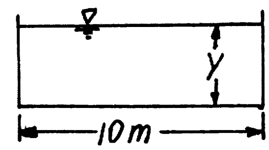
$= (P + 4\,ft)(1{,}000\,ft)(\tfrac{4}{12}\,ft)$

where $P = 10\,ft + 2\ell_2 = 10\,ft + 2(1.74)(1.664\,ft) = 15.8\,ft$

Hence,

$$\forall = (15.8\,ft + 4\,ft)(1{,}000\,ft)(\tfrac{4}{12}\,ft) = 6{,}600\;ft^3\left(\frac{1\,yd^3}{27\,ft^3}\right) = \underline{\underline{244\;yd^3}}$$

10.57 Determine the critical depth for a flow of 200 m^3/s through a rectangular channel of 10-m width. If the water flows 3.8 m deep, is the flow supercritical? Explain.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$ where for critical flow $Fr = 1$ or $V = \sqrt{gy}$

Thus, with $V = \frac{Q}{A} = \frac{200 \frac{m^3}{s}}{10y\ m^2} = \frac{20}{y} \frac{m}{s}$ we have

$\frac{20}{y} = \sqrt{9.81\,y}$ or $y = \underline{\underline{3.44\,m}}$

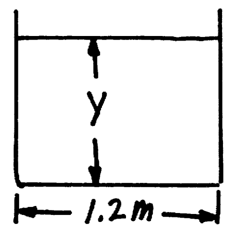
If $y = 3.8\,m$, then $V = \frac{20}{3.8} = 5.26 \frac{m}{s}$ and $Fr = \frac{V}{\sqrt{gy}} = \frac{5.26 \frac{m}{s}}{\left[(9.81 \frac{m}{s^2})(3.8m)\right]^{1/2}}$

$= 0.862$

The flow is $\underline{\underline{\text{subcritical.}}}$

10.58

10.58 Water flows in a rectangular, brick-lined aqueduct of width 1.2 m at a rate of 73,000 m^3/day. Determine the water depth if the change in elevation over the 16-km length of this channel is 9.6 m.



$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o} \quad \text{where} \qquad (1)$$

$K = 1$, $n = 0.015$ (see Table 10.1), and

$$Q = 73{,}000 \frac{m^3}{day}\left(\frac{1\,day}{24hr}\right)\left(\frac{1\,hr}{3600\,s}\right) = 0.845 \frac{m^3}{s}$$

Also, $S_o = (9.6m)/(16 \times 10^3 m) = 0.0006$

$$A = 1.2y \quad \text{and} \quad R_h = \frac{A}{P} = \frac{1.2y}{2y+1.2}$$

Thus, Eq. (1) becomes:

$$0.845 = \frac{1}{0.015}(1.2y)\left[\frac{1.2y}{2y+1.2}\right]^{2/3} \sqrt{0.0006}$$

or

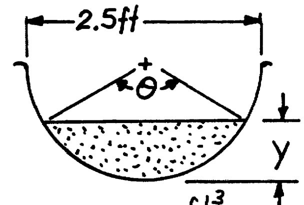
$$0.431 = y\left[\frac{1.2y}{2y+1.2}\right]^{2/3}$$

A trial and error solution of this equation gives

$y = \underline{\underline{0.861\ m}}$

10.59

10.59 A smooth steel water slide at an amusement park is of semicircular cross section with a diameter of 2.5 ft. The slide descends a vertical distance of 35 ft in its 420 ft length. If pumps supply water to the slide at a rate of 6 cfs, determine the depth of flow. Neglect the effects of the curves and bends of the slide.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{35\text{ ft}}{420\text{ ft}} = 0.0833$, $Q = 6.0\ \frac{ft^3}{s}$

and from Table 10.1 $n = 0.012$

Also (see Example 10.5), $A = \frac{D^2}{8}(\theta - \sin\theta)$ and

$R_h = \frac{D(\theta - \sin\theta)}{4\theta}$, where $D = 2.5$ ft

Thus,

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \right], \text{ where } \theta \sim \text{rad},$$

or

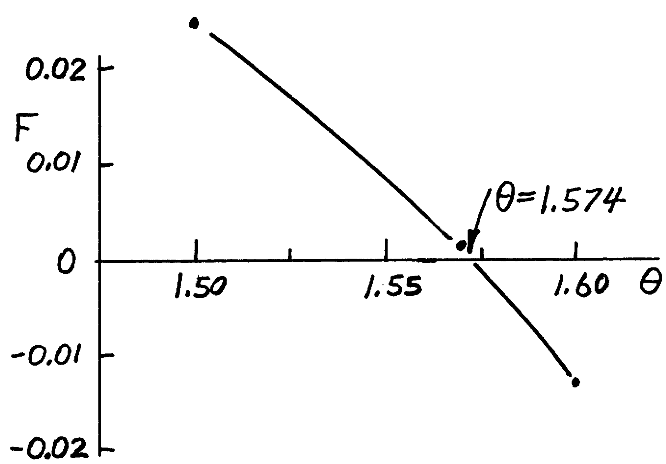
$$6.0 = \frac{1.49}{0.012}(0.0833)^{1/2} \frac{(2.5)^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \right]$$

Hence,

$$0.293\, \theta^{2/3} = (\theta - \sin\theta)^{5/3} \qquad 0.0252\, \theta^2 - (\theta - \sin\theta)^5 = 0 \equiv F(\theta)$$

Trial and error solution for $F(\theta) = 0$

θ, rad	F
1.50	0.0247
1.57	0.00195
1.60	-0.0135

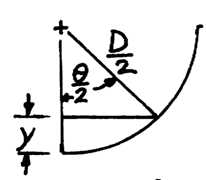


Thus, $\theta = (1.574\text{ rad})\left(\frac{180\text{ deg}}{\pi\text{ rad}}\right) = 90.2°$

or since

$y = \frac{D}{2}\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ it follows that

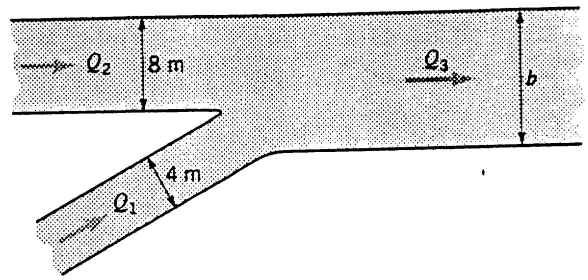
$y = \left(\frac{2.5}{2}\text{ ft}\right)\left(1 - \cos\left(\frac{90.2}{2}\right)\right) = \underline{\underline{0.368\text{ ft}}}$



$y + \frac{D}{2}\cos\frac{\theta}{2} = \frac{D}{2}$

10.60

10.60 Two canals join to form a larger canal as shown in Video V10.2 and Fig. P10.60. Each of the three rectangular canals is lined with the same material and has the same bottom slope. The water depth in each is to be 2 m. Determine the width of the merged canal, b. Explain physically (i.e., without using any equations) why it is expected that the width of the merged canal is less than the combined widths of the two original canals (i.e., $b < 4\text{ m} + 8\text{ m} = 12\text{ m}$).



■ FIGURE P10.60

$Q_3 = Q_1 + Q_2$ where for $i = 1, 2, 3$

$$Q_i = \frac{\kappa}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{o_i}}$$

Thus,

$$\frac{\kappa}{n_3} A_3 R_{h_3}^{2/3} \sqrt{S_{o_3}} = \frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{o_2}} + \frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{o_1}} \qquad (1)$$

But $n_1 = n_2 = n_3$ and $S_{o_1} = S_{o_2} = S_{o_3}$ so that Eq. (1) becomes

$$A_3 R_{h_3}^{2/3} = A_2 R_{h_2}^{2/3} + A_1 R_{h_1}^{2/3} \qquad (2)$$

where

$A_1 = 2m(4m) = 8m^2$, $P_1 = (2+2+4) = 8m$ so that $R_{h_1} = \frac{A_1}{P_1} = \frac{8m^2}{8m} = 1\,m$

$A_2 = 2m(8m) = 16m^2$, $P_2 = (2+2+8) = 12m$ so that $R_{h_2} = \frac{A_2}{P_2} = \frac{16m^2}{12m} = 1.333m$

and

$A_3 = 2b\,m^2$, $P_3 = (2+2+b) = (b+4)m$ so that $R_{h_3} = \frac{A_3}{P_3} = \frac{2b}{(b+4)}$

Thus, Eq. (2) becomes

$$(2b)\left[\frac{2b}{(b+4)}\right]^{2/3} = 16\,(1.333)^{2/3} + 8\,(1)^{2/3} = 27.4$$

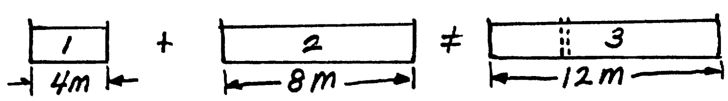
or

$$b^{5/3} = 8.63\,(b+4)^{2/3} \qquad (3)$$

A trial and error or equation solver solution to Eq. (3) gives

$b = \underline{\underline{10.66\ m}}$

If the two original canals merged to form a 12m wide canal, the water depth would be less than 2m because without the two walls there would be less friction force hold the water back. Thus, to maintain the 2m depth we must have $b < 12m$.



10.61*

10.61* Water flows in the painted steel rectangular channel with rounded corners shown in Fig. P10.61. The bottom slope is 1 ft/200 ft. Plot a graph of flowrate as a function of water depth for $0 \le y \le 1$ ft with corner radii of $r = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0 ft.

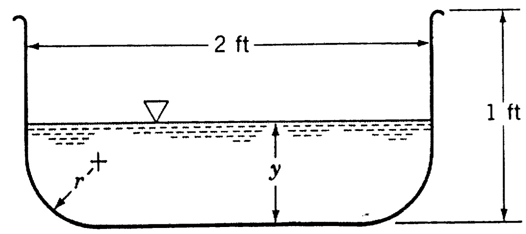
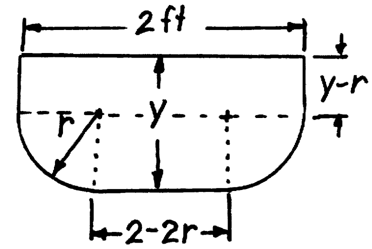


FIGURE P10.61

$Q = \frac{K}{n} A R_h S_o^{1/2}$, where $K = 1.49$, from Table 10.1 $n = 0.014$, and (1)

$S_o = \frac{1\,ft}{200\,ft} = 0.005$



(a) Assume $y \ge r$:

Thus, $A = 2(y-r) + r(2-2r) + \frac{1}{2}\pi r^2$

or $A = 2y - (2 - \frac{\pi}{2})r^2$ (2)

and $P = 2(y-r) + (2-2r) + \pi r$

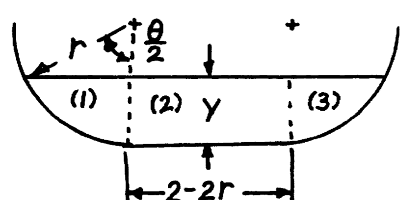
or $P = 2y - (4-\pi)r + 2$ (3)

Hence, with $R_h = \frac{A}{P}$ Eqs. (1), (2), and (3) give

$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$

or

$Q = 7.53 \frac{[2y - (2-\frac{\pi}{2})r^2]^{5/3}}{[2y - (4-\pi)r + 2]^{2/3}}$ for $r \le y \le 1$, where $r \sim ft$, $y \sim ft$, $Q \sim \frac{ft^3}{s}$ (4)



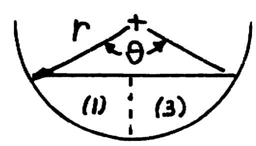
(b) Assume $y \le r$:

Thus, $A = A_1 + A_2 + A_3$

From Example 10.5, with $D = 2r$

$A_1 + A_3 = \frac{(2r)^2}{8}(\theta - \sin\theta)$ where $\theta \sim rad$ and

$\cos\frac{\theta}{2} = \frac{r-y}{r}$



Hence, $A = \frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y$ (5)

Also, $P = 2 - 2r + P_1 + P_3$, where

from Example 10.5, $P_1 + P_3 = \frac{(2r)\theta}{2} = r\theta$

Thus, $P = 2 - 2r + r\theta = 2 + (\theta - 2)r$ (6)

By combing Eqs. (1), (5), and (6) we obtain:

$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$

or

$Q = 7.53 \frac{[\frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y]^{5/3}}{[2 + (\theta-2)r]^{2/3}}$ for $0 \le y \le r$, where $r \sim ft$, $y \sim ft$, (7)

$Q \sim \frac{ft^3}{s}$, and $\theta = 2\cos^{-1}(\frac{r-y}{r}) \sim rad$

(con't)

10.61* (con't)

For $r=0, 0.2, 0.4, 0.6, 0.8$, and 1 ft plot $Q=Q(y)$ from either Eq. (4) or Eq. (7) for $0 \le y \le 1$ ft. Program P10#61 shown below was used to calculate the sample results shown.

```
100 cls
110 open "prn" for output as #1
120 print#1, "**********************************************"
130 print#1, "**  This program calculates the flowrate as  **"
140 print#1, "**  a function of depth for various values   **"
150 print#1, "**  of the radius of curvature of the        **"
160 print#1, "**  corners.                                 **"
170 print#1, "**********************************************"
180 r = -0.2
190 pi = 4*atn(1)
200 for i = 1 to 6
220 r = r + 0.2
230 print#1, " "
240 print#1, using "With r = ##.## ft";r
260 print#1, "  y, ft    Q, ft3/s"
280 y = -0.1 + 0.00001
290 for j = 1 to 11
295 y = y + 0.1
300 if y < = r then goto 500
320 Q = 7.53*(2*y-(2-pi/2)*r*r)^(5/3)/(2*y-(4-pi)*r+2)^(2/3)
340 goto 600
500 th = 2*atn((r*r-(r-y)^2)^0.5/(r-y))
520 Q = 7.53*(r*r*(th-sin(th))/2+(2-2*r)*y)^(5/3)/(2+(th-2)*r)^(2/3)
600 print#1, using " ##.##     #.###^^^^";y,Q
620 next j
640 next i
```

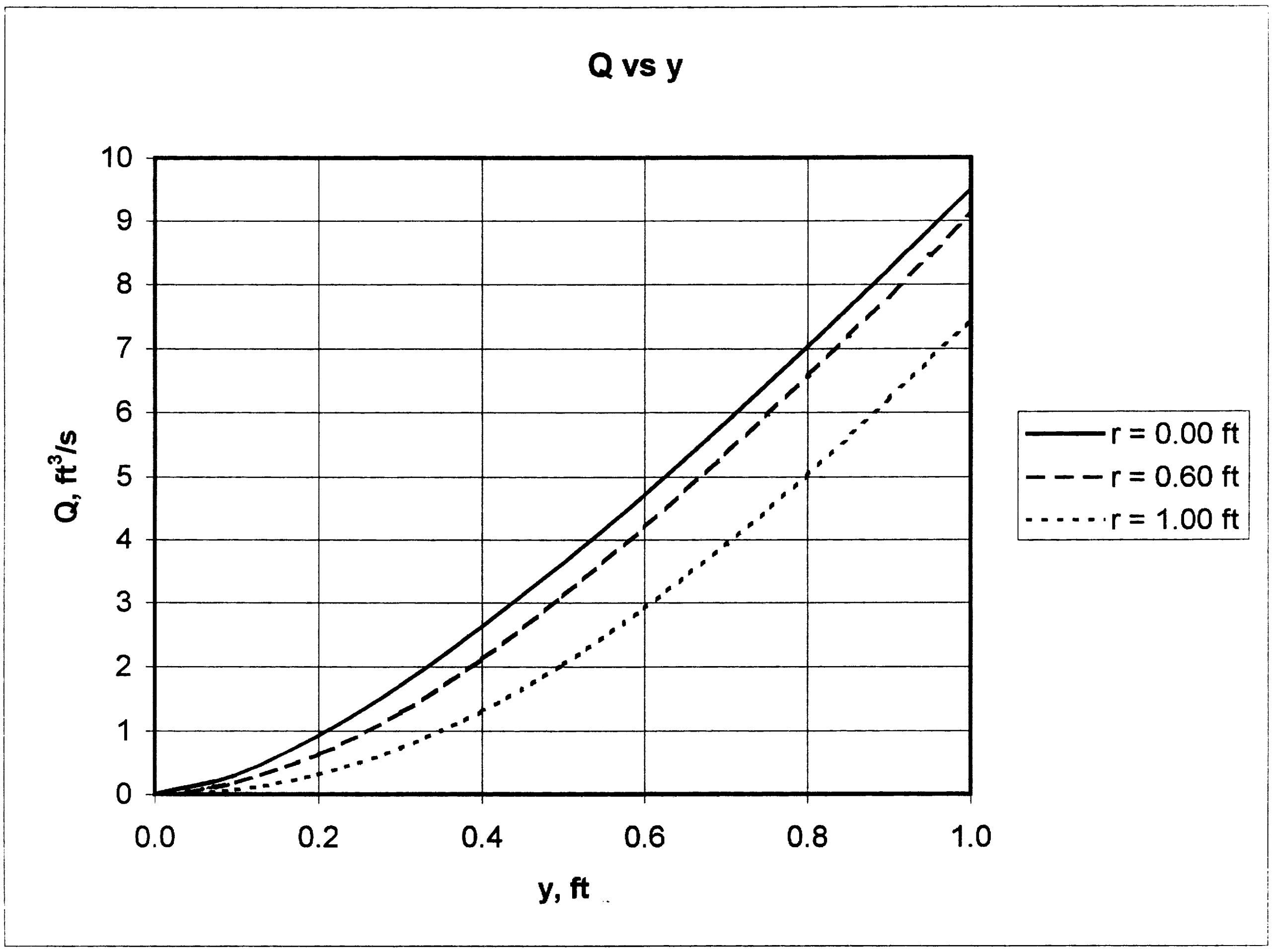
```
**********************************************
**  This program calculates the flowrate as  **
**  a function of depth for various values   **
**  of the radius of curvature of the        **
**  corners.                                 **
**********************************************
```

With r = -0.00 ft		With r = 0.60 ft		With r = 1.00 ft	
y, ft	Q, ft3/s	y, ft	Q, ft3/s	y, ft	Q, ft3/s
0.00	6.987E-08	0.00	2.806E-08	0.00	1.589E-10
0.10	3.045E-01	0.10	1.794E-01	0.10	7.158E-02
0.20	9.123E-01	0.20	6.220E-01	0.20	3.112E-01
0.30	1.700E+00	0.30	1.281E+00	0.30	7.244E-01
0.40	2.613E+00	0.40	2.122E+00	0.40	1.305E+00
0.50	3.620E+00	0.50	3.107E+00	0.50	2.041E+00
0.60	4.699E+00	0.60	4.198E+00	0.60	2.918E+00
0.70	5.835E+00	0.70	5.357E+00	0.70	3.919E+00
0.80	7.017E+00	0.80	6.566E+00	0.80	5.022E+00
0.90	8.236E+00	0.90	7.815E+00	0.90	6.207E+00
1.00	9.487E+00	1.00	9.096E+00	1.00	7.451E+00

(con't)

10.61* (con't)

The results, $Q = Q(y)$, are plotted below for $r = 0$, 0.6, and 1 ft.



10.62*

10.62* Water flows in the fiberglass ($n = 0.014$) triangular channel with a round bottom shown in Fig. P10.62. The channel slope is 0.1 m/90 m. Plot a graph of flowrate as a function of water depth for $0 \le y \le 0.50$ m with bottom radii of $r = 0, 0.25, 0.50, 0.75$, and 1.0 m.

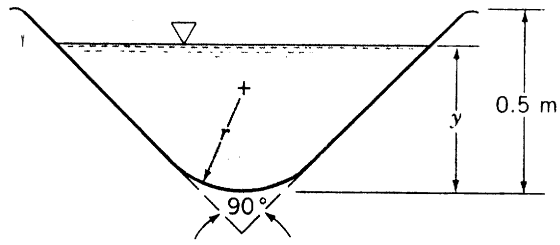
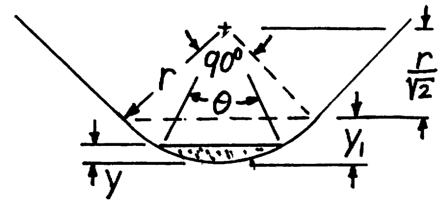


FIGURE P10.62

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \quad \text{where } K = 1,\ n = 0.014, \text{ and } S_o = \frac{0.1\,m}{90\,m} = 0.00111 \qquad (1)$$

(a) Assume $y \le y_1$ where the depth y_1 is shown in the figure. That is



$y_1 = r - \frac{r}{\sqrt{2}} = (1 - \frac{1}{\sqrt{2}})r$

In this case the flow is the same as that in a circular pipe. From Example 10.5

$$Q = \frac{K}{n} S_o^{1/2} \frac{(2r)^{8/3}}{8\,(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} = \frac{1}{0.014}(0.00111)^{1/2} \frac{(2r)^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$$

or

$$Q = 0.750\, r^{8/3} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}, \quad \text{where } \theta \sim rad,\ Q \sim \frac{m^3}{s},\ r \sim m, \qquad (2)$$

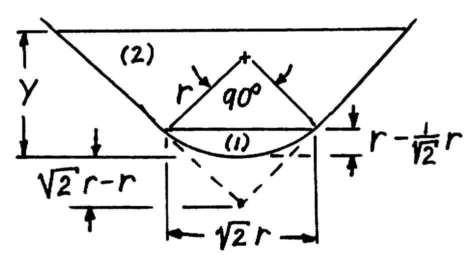
and

$$\cos\frac{\theta}{2} = \frac{r - y}{r} \qquad (\text{valid for } y \le (1 - \tfrac{1}{\sqrt{2}})r)$$

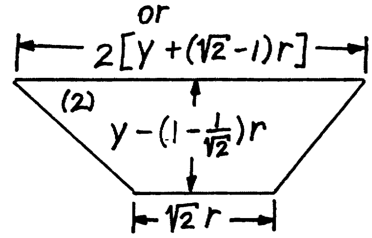
(b) Assume $y \ge y_1$

$A = A_1 + A_2$, where from Example 10.5 with $\theta = \frac{\pi}{2}$

$A_1 = \frac{(2r)^2}{8}(\frac{\pi}{2} - \sin\frac{\pi}{2}) = \frac{(\pi - 2)}{4} r^2$



or



Also, for the trapezoidal area A_2 (see figures):

$$A_2 = \frac{1}{2}\left\{\sqrt{2}r + 2[y + (\sqrt{2} - 1)r]\right\}[y - (1 - \tfrac{1}{\sqrt{2}})r]$$
$$= [y + (\tfrac{3}{\sqrt{2}} - 1)r][y - (1 - \tfrac{1}{\sqrt{2}})r]$$

Thus,

$$A = [y + (\tfrac{3}{\sqrt{2}} - 1)r][y - (1 - \tfrac{1}{\sqrt{2}})r] + \frac{(\pi - 2)}{4} r^2 \qquad (3)$$

Also,

$P = P_1 + P_2$, where $P_1 = \frac{\pi}{2} r$ and $P_2 = 2(\sqrt{2})[y - (1 - \frac{1}{\sqrt{2}})r]$

Thus,

$$P = \frac{\pi}{2} r + 2\sqrt{2}[y - (1 - \tfrac{1}{\sqrt{2}})r] \qquad (4)$$

(con't)

Thus, with $R_h = \frac{A}{P}$ Eq. (1) becomes

$$Q = \frac{1}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.00111)^{1/2}$$

or

$$Q = 2.38 \frac{A^{5/3}}{P^{2/3}} \qquad (5)$$

Hence, for $y \geq (1-\frac{1}{\sqrt{2}})r$ calculate Q from Eq. (5), with A and P from Eqs. (3) and (4).

Thus, plot $Q = Q(y)$ for $0 \leq y \leq 0.5\,m$ with $r = 0,\ 0.25,\ 0.50,\ 0.75,\ 1.0\,m$, where

a) $Q = 0.750\, r^{8/3} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$ with $\cos\frac{\theta}{2} = \frac{r-y}{r}$ if $y \leq (1-\frac{1}{\sqrt{2}})r$

and

b) $Q = 2.38 \frac{A^{5/3}}{P^{2/3}}$ with $A = [y + (\frac{3}{\sqrt{2}} - 1)r][y - (1-\frac{1}{\sqrt{2}})r] + \frac{(\pi - 2)}{4} r$ and

$P = \frac{\pi}{2} r + 2\sqrt{2}[y - (1-\frac{1}{\sqrt{2}})r]$ if $y > (1-\frac{1}{\sqrt{2}})r$

These results are calculated and plotted below using Program P10#62.

```
100 cls
110 open "prn" for output as #1
120 print#1, "*********************************************"
130 print#1, "**  This program calculates the flowrate in  **"
140 print#1, "**  a vee shaped open channel with a rounded **"
150 print#1, "**  bottom.                                  **"
160 print#1, "*********************************************"
190 pi = 4*atn(1)
195 r2 = 2^0.5
200 r = -0.25
210 for i = 1 to 5
220 r = r + 0.25
225 print#1, " "
230 print#1, using "With r = ##.## m";r
240 print#1, "  y, m       Q, m3/s"
250 y = -0.05 + 0.000001
260 for j = 1 to 11
270 y = y + 0.05
280 if y < = (1 - 1/r2)*r then goto 400
300 A = (y+(3/r2-1)*r)*(y-(1-1/r2)*r) + (pi-2)*r*r/4
320 P = pi*r/2 + 2*r2*(y - (1 - 1/r2)*r)
340 Q = 2.38*A^(5/3)/P^(2/3)
360 goto 500
400 th = 2*atn((r*r - (r - y)^2)^0.5/(r-y))
420 Q = 0.750*r^(8/3)*(th - sin(th))^(5/3)/th^(2/3)
500 print#1, using " ##.###    #.###^^^^";y,Q
510 next j
520 next i
```

(con't)

10.62* (con't)

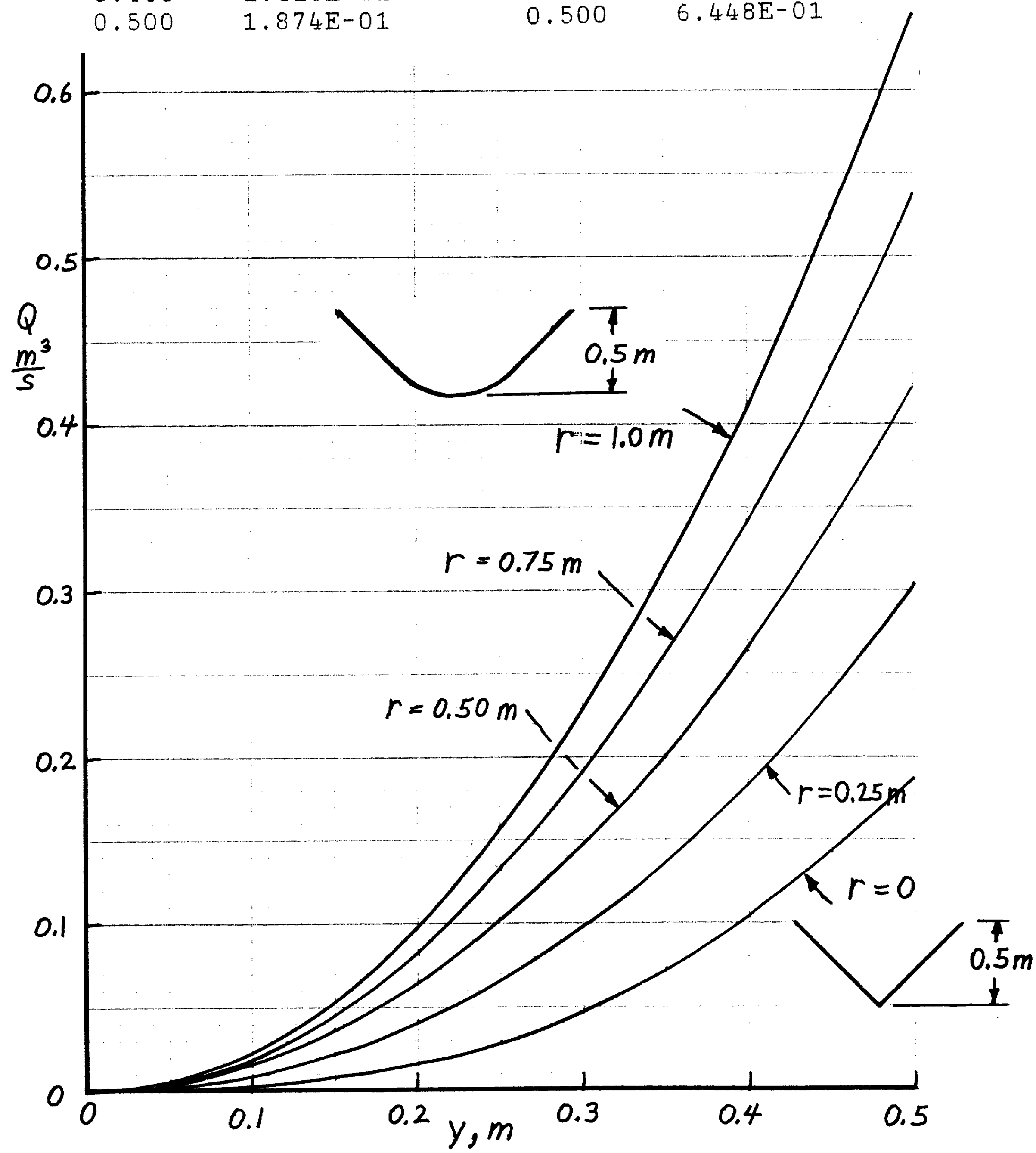
```
*********************************************
**  This program calculates the flowrate in  **
**  a vee shaped open channel with a rounded **
**  bottom.                                  **
*********************************************
```

With r = 0.00 m

y, m	Q, m3/s
0.000	1.194E-16
0.050	4.038E-04
0.100	2.564E-03
0.150	7.559E-03
0.200	1.628E-02
0.250	2.952E-02
0.300	4.800E-02
0.350	7.240E-02
0.400	1.034E-01
0.450	1.415E-01
0.500	1.874E-01

With r = 1.00 m

y, m	Q, m3/s
0.000	3.436E-13
0.050	5.120E-03
0.100	2.263E-02
0.150	5.361E-02
0.200	9.839E-02
0.250	1.569E-01
0.300	2.290E-01
0.350	3.138E-01
0.400	4.113E-01
0.450	5.215E-01
0.500	6.448E-01



10.63

10.63 The cross section of an ancient Roman aqueduct is drawn to scale in Fig. P10.63. When it was new the channel was essentially rectangular and for a flowrate of 100,000 m^3/day, the water depth was as indicated. Archeological evidence indicates that after many years of use, calcium carbonate deposits on the sides and bottom modified the shape to that shown in the figure. Estimate the flowrate for the modified shape if the slope and surface roughness did not change.

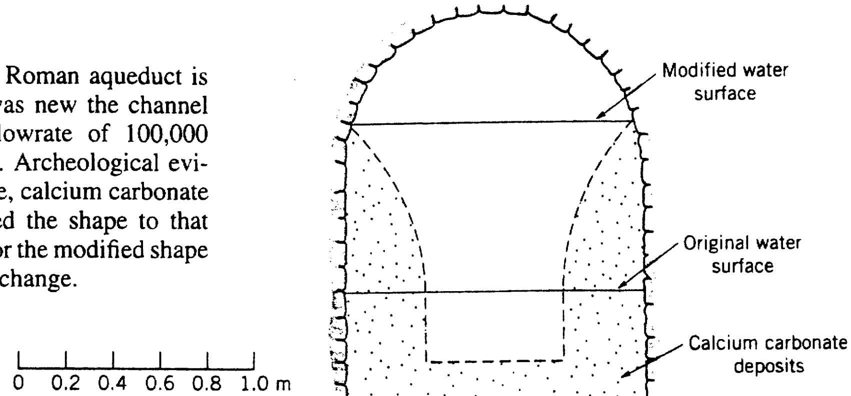


FIGURE P10.63

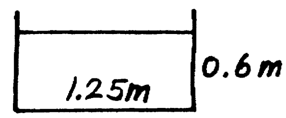
Originally:

$$Q_o = \frac{K}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{oo}} = 100{,}000 \frac{m^3}{day}\left(\frac{1\,day}{24hr}\right)\left(\frac{1\,hr}{3600\,s}\right) = 1.16 \frac{m^3}{s} \quad (1)$$

where from measurements on the figure

$A_o \approx 0.6m(1.25m) = 0.75m^2$ and

$$R_{h_o} = \frac{A_o}{P_o} \approx \frac{0.75m^2}{1.25m + 2(0.6m)} = 0.306m$$



original

Thus, from Eq. (1):

$$1.16 = \frac{K}{n_o}(0.75)(0.306)^{2/3}\sqrt{S_{oo}}$$

or

$$\frac{K}{n_o}\sqrt{S_{oo}} = 3.41$$

Modified:

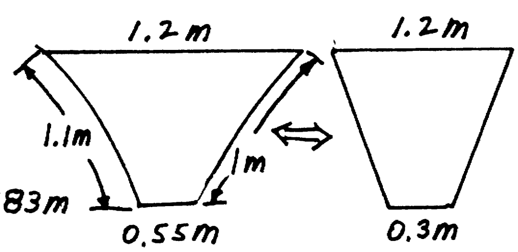
$$Q_m = \frac{K}{n_m} A_m R_{hm}^{2/3}\sqrt{S_{om}}$$, where $n_o = n_m$, $S_{oo} = S_{om}$, and

from the figure,

$$A_m \approx 1m\left(\frac{1.2m + 0.3m}{2}\right) = 0.75m^2$$

and

$$R_{hm} = \frac{A_m}{P_m} \approx \frac{0.75m^2}{(0.55m + 1.1m + 1m)} = 0.283m$$



modified

Thus, with $\frac{K}{n_m}\sqrt{S_{om}} = \frac{K}{n_o}\sqrt{S_{oo}} = 3.41$,

$$Q_m = 3.41(0.75)(0.283)^{2/3} = 1.10 \frac{m^3}{s}\left(\frac{3600s}{hr}\right)\left(\frac{24hr}{day}\right)$$

or

$$Q_m = 95{,}200 \frac{m^3}{day}$$

10.64

10.64 The smooth concrete-lined channel shown in Fig. P10.64 is built on a slope of 2 m/km. Determine the flowrate if the depth is $y = 1.5$ m.

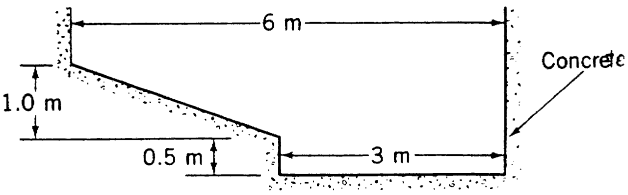


FIGURE P10.64

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = \frac{2\,m}{1000\,m} = 0.002, \text{ and from Table 10.1} \quad (1)$$

$n = 0.012$

With $y = 1.5\,m$, $A = (3m)(0.5m) + \frac{1}{2}(3m+6m)(1.0m) = 6\,m^2$

and

$$P = 1.5\,m + 3\,m + 0.5\,m + (1^2+3^2)^{\frac{1}{2}}\,m = 8.16\,m$$

Thus, $R_h = \frac{A}{P} = \frac{6\,m^2}{8.16\,m} = 0.735\,m$, and Eq. (1) gives

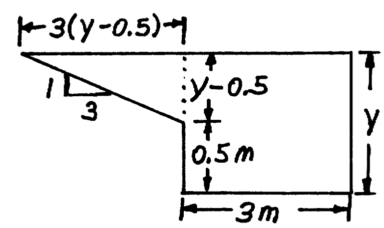
$$Q = \frac{1}{0.012}(6)(0.735)^{2/3}(0.002)^{1/2} = \underline{\underline{18.2\,\frac{m^3}{s}}}$$

10.65

10.65 Determine the flow depth for the channel shown in Fig. P10.64 if the flowrate is 15 m^3/s.

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = \frac{3\,m}{3000\,m} = 0.003, \text{ and from Table 10.1 } n = 0.012$$

Also, $A = 3y + \frac{1}{2}[3(y-0.5)](y-0.5) = \frac{3}{2}y^2 + \frac{3}{2}y + \frac{3}{8}$



and

$$P = y + 3 + 0.5 + \left[(y-\tfrac{1}{2})^2 + 9(y-\tfrac{1}{2})^2\right]^{\frac{1}{2}}$$

$$= y + 3.5 + \sqrt{10}\,(y-0.5) = 4.16y + 1.92$$

Hence, with $R_h = \frac{A}{P}$ and $Q = 15\,\frac{m^3}{s}$ we obtain

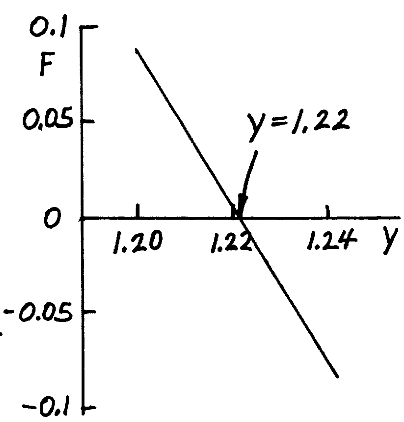
$$15 = \frac{1}{0.012}(1.5y^2 + 1.5y + 0.375)^{5/3}\frac{1}{(4.16y+1.92)^{2/3}}(0.003)^{\frac{1}{2}}$$

or

$$2.04(4.16y+1.92)^{0.4} - 1.5y^2 - 1.5y - 0.375 = 0 \equiv F(y)$$

Trial and error solution for $F(y) = 0$

y	F
1.20	0.0855
1.22	0.0041
1.24	-0.0786



Thus, $y \approx \underline{\underline{1.22\,m}}$

Note: Since $y < 1.5\,m$ the water does not contact the left vertical wall

10.66*

10.66* The cross section of a creek valley is shown in Fig. P10.66. Plot a graph of flowrate as a function of depth, y, for $0 \le y \le 10$ ft. The slope is 5 ft/mi.

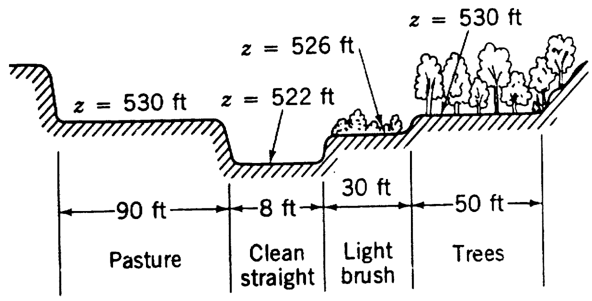
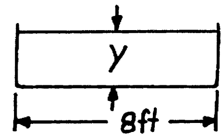


FIGURE P10.66

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } S_o = \frac{5\text{ ft}}{5280\text{ ft}} = 0.000947 \qquad (1)$$

Assume the cross-section consists of rectangular segments.

(a) For $y \le 526\text{ ft} - 522\text{ ft} = 4\text{ ft}$:



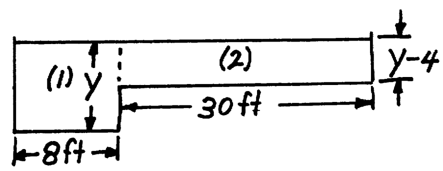
From Table 10.1, $n = 0.03$, $A = 8y$, $P = 8 + 2y$ so that Eq. (1) gives

$$Q = \frac{1.49}{0.03}(8y)\left[\frac{8y}{2y+8}\right]^{2/3}(0.000947)^{1/2}$$

or

$$Q = 30.8\,\frac{y^{5/3}}{(y+4)^{2/3}} \quad \text{for } 0 \le y \le 4\text{ft, where } y \sim \text{ft}, Q \sim \frac{\text{ft}^3}{\text{s}} \qquad (2)$$

(b) For $4 \le y \le 530\text{ft} - 522\text{ft} = 8\text{ ft}$:



$Q = Q_1 + Q_2$, where $Q_i = \frac{1.49}{n_i} A_i R_{h_i}^{2/3}(0.000947)^{1/2}$

From Table 10.1, $n_1 = 0.03$ and $n_2 = 0.050$

Also, $A_1 = 8y$, $P_1 = y + 8 + 4 = y + 12$ or $R_{h_1} = \frac{A_1}{P_1} = \frac{8y}{y+12}$

and $A_2 = 30(y-4) = 30y - 120$, $P_2 = 30 + (y-4) = y + 26$, or $R_{h_2} = \frac{30y-120}{y+26}$

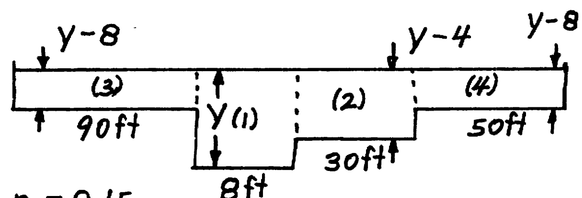
Hence,

$$Q = \frac{1.49}{0.03}(8y)\left[\frac{8y}{y+12}\right]^{2/3}(0.000947)^{1/2} + \frac{1.49}{0.05}(30y-120)\left[\frac{30y-120}{y+26}\right]^{2/3}(0.000947)^{1/2}$$

or

$$Q = 48.9\,\frac{y^{5/3}}{(y+12)^{2/3}} + 265\,\frac{(y-4)^{5/3}}{(y+26)^{2/3}} \quad \text{for } 4 \le y \le 8\text{ft, where } Q \sim \frac{\text{ft}^3}{\text{s}} \qquad (3)$$

(c) For $y \ge 8$ ft:



$Q = Q_1 + Q_2 + Q_3 + Q_4$

with $n_1 = 0.03$, $n_2 = 0.05$, $n_3 = 0.035$, $n_4 = 0.15$

Also, $A_1 = 8y$, $A_2 = 30(y-4)$, $A_3 = 90(y-8)$, and $A_4 = 50(y-8)$

and

$P_1 = 8 + 8 + 4 = 20$, $P_2 = 30 + 4 = 34$, $P_3 = 90 + (y-8) = y + 82$, and

$P_4 = 50 + (y-8) = y + 42$ so that with $R_{h_i} = \frac{A_i}{P_i}$ and

(con't)

10.66* (con't)

$$Q_i = \frac{1.49}{n_i} A_i R_{h_i}^{2/3} S_o^{1/2} = \frac{1.49}{n_i} A_i R_{h_i}^{2/3} (0.000947)^{1/2} = \frac{0.0459}{n_i} A_i R_{h_i}^{2/3}$$

Thus,

$$Q = \frac{0.0459}{0.03}\,\frac{(8y)^{5/3}}{20^{2/3}} + \frac{0.0459}{0.05}\,\frac{[30(y-4)]^{5/3}}{34^{2/3}} + \frac{0.0459}{0.035}\,\frac{[90(y-8)]^{5/3}}{(y+82)^{2/3}}$$

$$+ \frac{0.0459}{0.15}\,\frac{[50(y-8)]^{5/3}}{(y+42)^{2/3}}$$

or

$$Q = 6.64y^{5/3} + 25.3(y-4)^{5/3} + 2370\,\frac{(y-8)^{5/3}}{(y+82)^{2/3}} + 208\,\frac{(y-8)^{5/3}}{(y+42)^{2/3}} \quad \text{for } y \geq 8,\; Q \sim \frac{ft^3}{s} \qquad (4)$$

For $0 \leq y \leq 10$ plot $Q = Q(y)$ from Eqs. (2), (3), or (4).

Program P10#66 shown below was used to calculate the results.

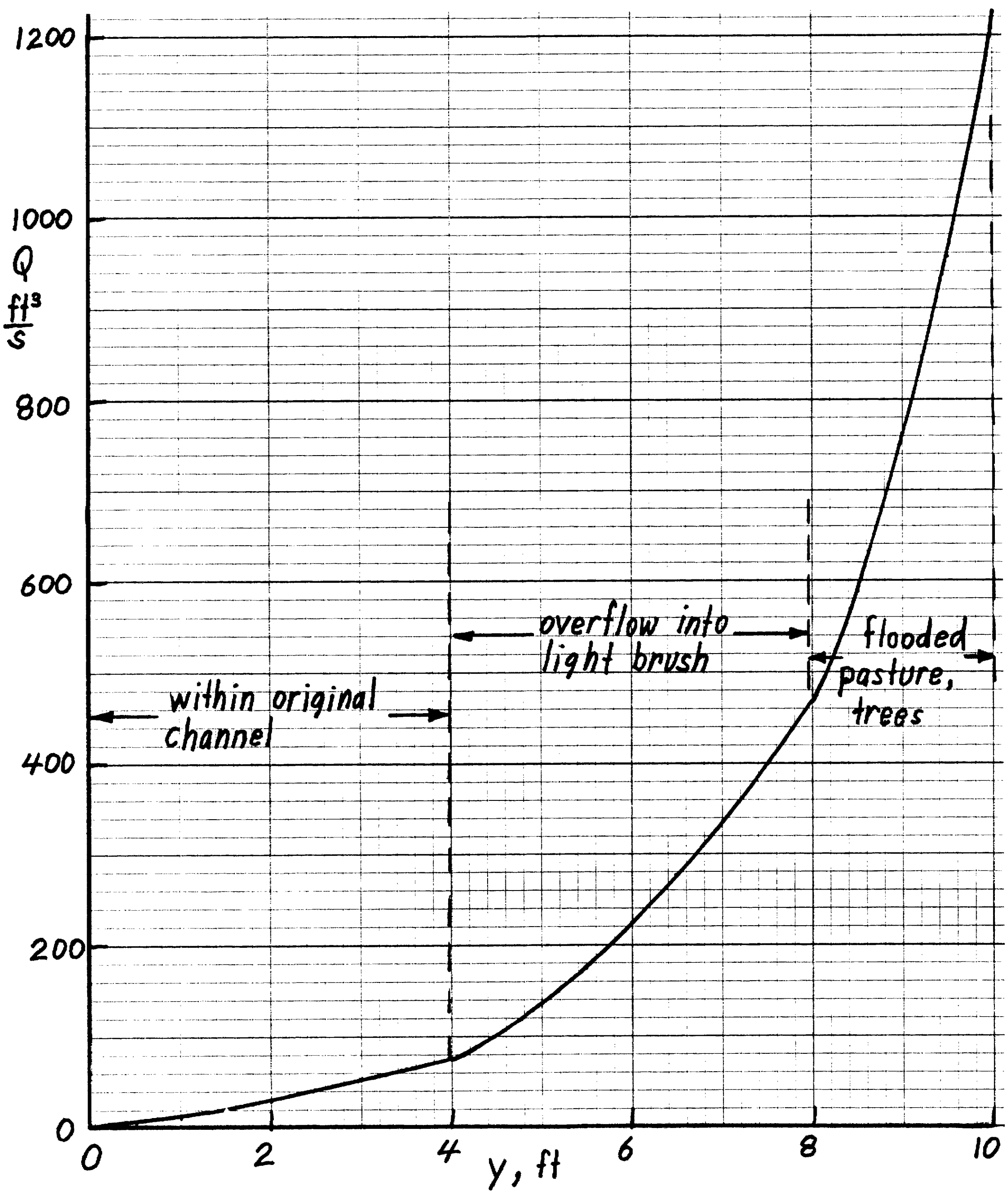
```
100 cls
110 open "prn" for output as #1
120 print#1, "**********************************************"
130 print#1, "**  This program calculates the flowrate in    **"
140 print#1, "**  the creek at various depths of flow.       **"
150 print#1, "**********************************************"
160 y = -0.5
170 print#1, " "
180 print#1, "  y, ft    Q, cfs"
190 for i = 1 to 21
200 y = y +0.5
210 if y < = 4 then goto 300
230 if y < = 8 then goto 400
240 goto 500
300 Q = 30.8*y^(5/3)/(y + 4)^(2/3)
310 goto 600
400 Q = 48.9*y^(5/3)/(y + 12)^(2/3)+265*(y - 4)^(5/3)/(y + 26)^(2/3)
410 goto 600
500 a = 5/3
510 b = (y-8)^a
520 Q=6.64*y^a+25.3*(y-4)^a+2370*b/(y+82)^(2/3)+208*b/(y+42)^(2/3)
600 print#1, using " ###.#    #.###^^^^";y,Q
700 next i
```

```
**********************************************
**  This program calculates the flowrate in    **
**  the creek at various depths of flow.       **
**********************************************

  y, ft     Q, cfs
   0.0    0.000E+00
   0.5    3.559E+00
   1.0    1.053E+01
   1.5    1.943E+01
   2.0    2.961E+01
   2.5    4.072E+01
   3.0    5.252E+01
   3.5    6.486E+01
   4.0    7.761E+01
   4.5    1.011E+02
   5.0    1.350E+02
   5.5    1.765E+02
   6.0    2.245E+02
   6.5    2.781E+02
   7.0    3.366E+02
   7.5    3.997E+02
   8.0    4.669E+02
   8.5    5.872E+02
   9.0    7.607E+02
   9.5    9.755E+02
  10.0    1.226E+03
```

(con't)

10.66* (con't)

The flowrate as a function of depth is plotted below.



10.67

10.67 Repeat Problem 10.64 if the surfaces are smooth concrete as is indicated except for the diagonal surface, which is gravelly with $n = 0.025$.

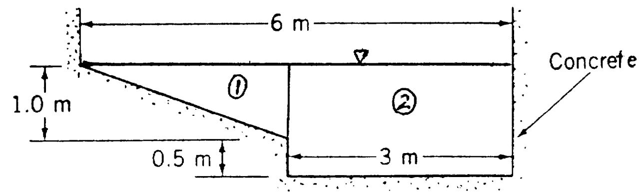


FIGURE P10.64

$$Q = Q_1 + Q_2 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2} + \frac{K}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = 0.002, \tag{1}$$

$n_1 = 0.025$, and from Table 10.1 $n_2 = 0.012$

Also, $A_1 = \frac{1}{2}(1.0\,m)(3\,m) = 1.50\,m^2$, $P_1 = (1.0^2 + 3.0^2)^{\frac{1}{2}}\,m = 3.16\,m$

or $R_{h_1} = \frac{A_1}{P_1} = \frac{1.50\,m^2}{3.16\,m} = 0.475\,m$

and

$A_2 = (3\,m)(1.5\,m) = 4.5\,m^2$, $P_2 = 0.5\,m + 3\,m + 1.5\,m = 5\,m$

or $R_{h_2} = \frac{A_2}{P_2} = \frac{4.5\,m^2}{5\,m} = 0.90\,m$

Hence, from Eq. (1):

$$Q = \frac{1}{0.025}(1.50)(0.475)^{2/3}(0.002)^{1/2} + \frac{1}{0.012}(4.5)(0.90)^{2/3}(0.002)^{1/2}$$

or

$$Q = \underline{\underline{17.3\ \frac{m^3}{s}}}$$

Note: With all surfaces concrete, $Q = 18.2\ \frac{m^3}{s}$ (see Problem 10.64).

10.68*

10.68* Water flows through the storm sewer shown in Fig. P10.68. The slope of the bottom is 2 m/400 m. Plot a graph of the flowrate as a function of depth for $0 \le y \le 1.7$ m. On the same graph plot the flowrate expected if the entire surface were lined with material similar to that of a clay tile.

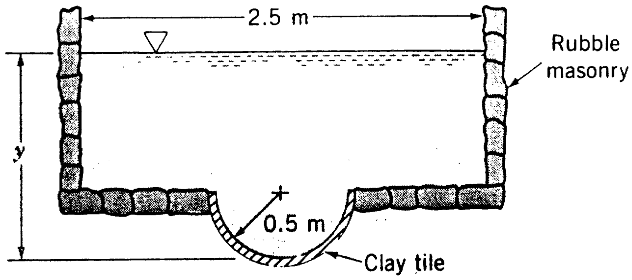


FIGURE P10.68

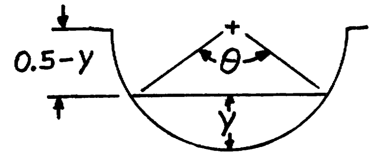
(a) For $0 \le y = 0.5\,m$: The flow is the same as that in a circular pipe.
Thus, from Example 10.5 with $D = 1m$, $K = 1$, and $n = 0.014$ (Table 10.1):

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} = \frac{1}{0.014}\left(\frac{2m}{400m}\right)^{1/2} \frac{(1)^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$$

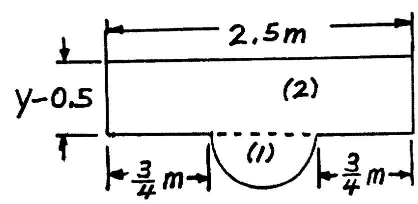
or

$$Q = 0.251 \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \frac{m^3}{s}, \text{ where } \theta \sim rad \qquad (1)$$

and $\theta = 2\cos^{-1}\left(\frac{0.5 - y}{0.5}\right)$



(b) For $y \ge 0.5m$:



$Q = Q_1 + Q_2$, where

$Q_1 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2}$, with $n_1 = 0.014$,

$A_1 = \frac{\pi}{2}(0.5m)^2 = 0.393\,m^2$, $P_1 = \pi(0.5) = 1.57\,m$ so that

$R_{h_1} = \frac{A_1}{P_1} = \frac{0.393\,m^2}{1.57\,m} = 0.250\ m$

Thus,

$$Q_1 = \frac{1}{0.014}(0.393)(0.250)^{2/3}\left(\frac{2}{400}\right)^{1/2} = 0.787 \frac{m^3}{s}$$

Also,

$$Q_2 = \frac{K}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}, \text{ with } n_2 = 0.025 \text{ (see Table 10.1)} \qquad (2)$$

$A_2 = (2.5m)(y - 0.5) = 2.5y - 1.25$ and $P_2 = 2(y - 0.5) + 2(\tfrac{3}{4}) = 2y + 0.5$

Hence, with $R_{h_2} = \frac{A_2}{P_2}$, Eq. (2) becomes

$$Q_2 = \frac{1}{0.025}(2.5y - 1.25)^{5/3} \frac{1}{(2y+0.5)^{2/3}} \left(\frac{2}{400}\right)^{1/2} = 13.0 \frac{(y - 0.5)^{5/3}}{(2y + 0.5)^{2/3}}$$

Therefore,

$$Q = 0.787 + 13.0 \frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}} \frac{m^3}{s} \text{ for } y \ge 0.5m \qquad (3)$$

Plot $Q = Q(y)$ for $0 \le y \le 1.7m$ using Eqs. (1) and (3).

(con't)

10.68* (con't)

If the entire surface were lined with material with $n_1 = n_2 = 0.014$, Eqn. (1) would remain valid. The coefficient "13.0" in Eq. (3) would become $13.0\left(\frac{0.025}{0.014}\right) = 23.2$. For this case,

$$Q = 0.787 + 23.2\,\frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}}\ \ \frac{m^3}{s}\ \text{ for } y \geq 0.5\,m \qquad (4)$$

This result is also plotted (i.e. Q from Eq. (1) for $0 \leq y \leq 0.5$, and Q from Eq. (4) for $0.5 < y \leq 1.7\,m$). See Program P10#68 below.

```
100 cls
110 open "prn" for output as #1
120 print#1, "**************************************************"
130 print#1, "**  This program calculates the flowrate in    **"
140 print#1, "**  the channel as a function of depth.        **"
150 print#1, "**************************************************"
160 dim a(2)
170 a(1) = 13.0
180 a(2) = 23.2
190 for i = 1 to 2
200 print#1, " "
210 if i = 1 then goto 260
220 print#1, "With n = 0.014 for the entire channel"
230 goto 280
260 print#1, "With n = 0.025 for part of the channel"
280 y = -.1 + 0.00001
290 print#1, "  y, m     Q, m3/s"
300 for j = 1 to 18
320 y = y + 0.1
340 if y < = 0.5 goto 500
360 Q = 0.787 + a(i)*(y - 0.5)^(5/3)/(2*y + 0.5)^(2/3)
380 goto 600
500 th = 2*atn(((0.5^2 - (0.5 - y)^2)^0.5)/(0.5 - y))
520 Q = 0.251*(th - sin(th))^(5/3)/th^(2/3)
600 print#1, using " ##.#    #.###^^^^";y,Q
610 next j
620 next i
```

```
************************************************
**  This program calculates the flowrate in    **
**  the channel as a function of depth.        **
************************************************

With n = 0.025 for part of the channel
  y, m      Q, m3/s
  0.0     7.552E-11
  0.1     3.293E-02
  0.2     1.381E-01
  0.3     3.089E-01
  0.4     5.315E-01
  0.5     7.870E-01
  0.6     9.837E-01
  0.7     1.367E+00
  0.8     1.853E+00
```

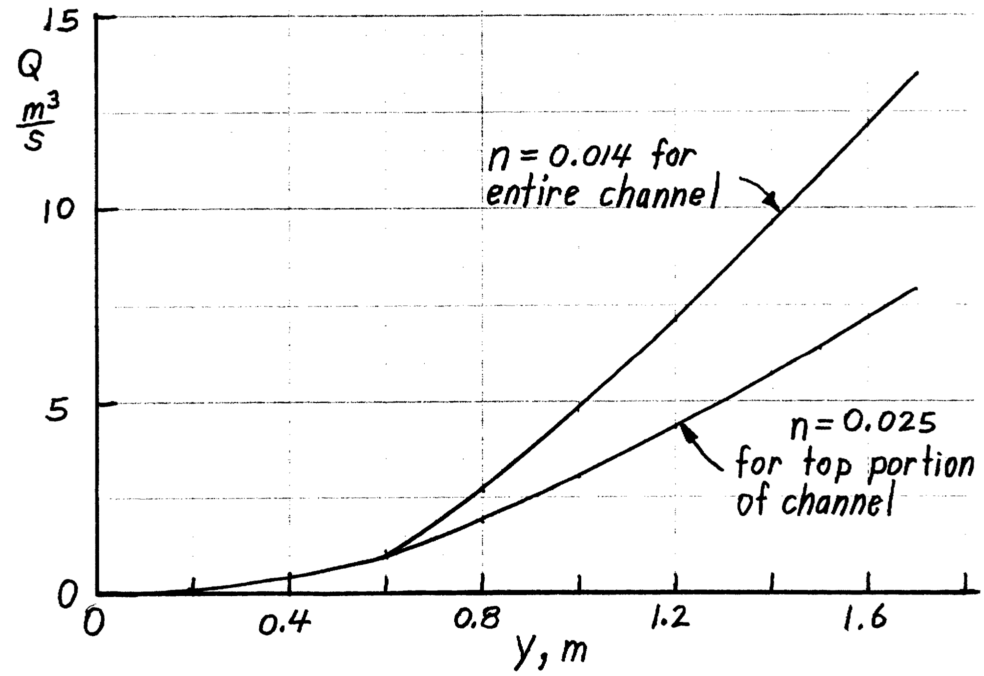
(con't)

10.68* (con't)

0.9	2.407E+00
1.0	3.010E+00
1.1	3.649E+00
1.2	4.315E+00
1.3	5.003E+00
1.4	5.708E+00
1.5	6.426E+00
1.6	7.157E+00
1.7	7.897E+00

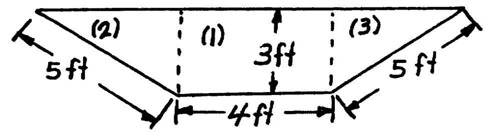
With n = 0.014 for the entire channel

y, m	Q, m3/s
0.0	7.552E-11
0.1	3.293E-02
0.2	1.381E-01
0.3	3.089E-01
0.4	5.315E-01
0.5	7.870E-01
0.6	1.138E+00
0.7	1.822E+00
0.8	2.689E+00
0.9	3.678E+00
1.0	4.754E+00
1.1	5.894E+00
1.2	7.083E+00
1.3	8.310E+00
1.4	9.568E+00
1.5	1.085E+01
1.6	1.215E+01
1.7	1.348E+01



10.69

10.69 Determine the flowrate for the symmetrical channel shown in Fig. P10.40 if the bottom is smooth concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$.



$Q = Q_1 + Q_2 + Q_3 = Q_1 + 2Q_2$, where $Q_i = \frac{K}{n_i} A_i R_{h_i}^{2/3} S_0^{1/2}$ with $K = 1.49$

Also, $A_1 = (3\,ft)(4\,ft) = 12\,ft^2$, $A_2 = \frac{1}{2}(3\,ft)(4\,ft) = 6\,ft^2$, $P_1 = 4\,ft$, and $P_2 = 5\,ft$,

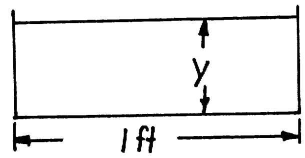
so that $R_{h_1} = \frac{A_1}{P_1} = \frac{12\,ft^2}{4\,ft} = 3\,ft$ and $R_{h_2} = \frac{A_2}{P_2} = \frac{6\,ft^2}{5\,ft} = 1.2\,ft$

Hence, with $n_1 = 0.012$ and $n_2 = 0.030$ (see Table 10.1) we obtain:

$$Q = \frac{1.49}{0.012}(12)(3)^{2/3}(0.001)^{1/2} + (2)\frac{1.49}{0.030}(6)(1.2)^{2/3}(0.001)^{1/2} = \underline{\underline{119\ \frac{ft^3}{s}}}$$

10.70

10.70 Water in a rectangular painted steel channel of width $b = 1$ ft and depth y is to flow at critical conditions, Fr = 1. Plot a graph of the critical slope, S_{0c}, as a function of y for 0.05 ft $\leq y \leq 5$ ft. What is the maximum slope allowed if critical flow is not to occur regardless of the depth?



$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1 $n = 0.014$

Also, $R_h = \frac{A}{P} = \frac{y}{2y+1}$ and with $Fr = \frac{V}{\sqrt{gy}} = 1$, $V = \sqrt{gy}$

Thus,

$$\sqrt{32.2\,y} = \frac{1.49}{0.014}\left(\frac{y}{2y+1}\right)^{2/3} S_{oc}^{1/2} \quad \text{or} \quad S_{oc} = 0.00284\left[\frac{(2y+1)^4}{y}\right]^{1/3} \qquad (1)$$

Equation (1) is plotted below. To determine the minimum critical slope set $\frac{dS_{oc}}{dy} = 0$. That is:

$$\frac{dS_{oc}}{dy} = \left(\frac{1}{3}\right)(0.00284)\left[\frac{(2y+1)^4}{y}\right]^{-2/3}\left[\frac{4(2y+1)^3(2)y - (2y+1)^4}{y^2}\right] = 0$$

Thus, $y = \frac{1}{6}$ so that from Eq. (1)

$$S_{oc_{min}} = 0.00284\left[\frac{(\frac{2}{6}+1)^4}{\frac{1}{6}}\right]^{1/3} = \underline{\underline{0.00757}}$$

If $S_o < 0.00757$ critical flow cannot occur at any depth.

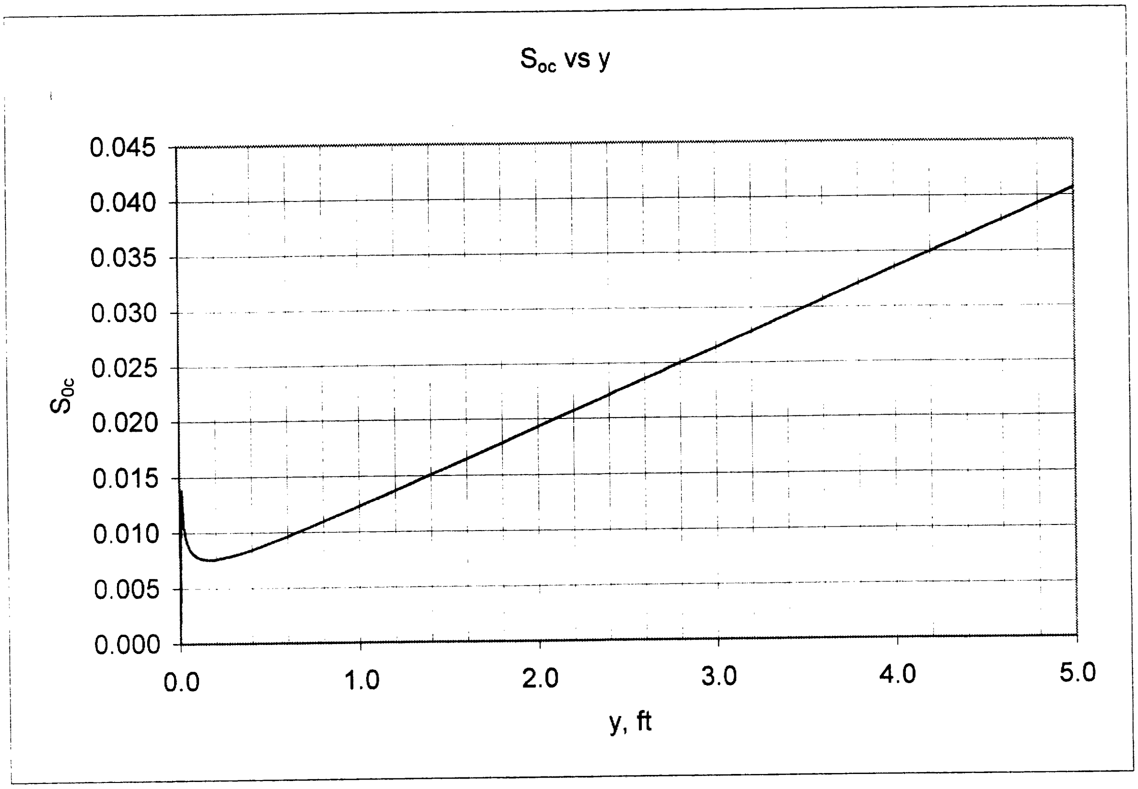
The following values are obtained from Eq. (1). Note that

$$\lim_{y \to 0} S_{oc} = 0.00284 \lim_{y \to 0}\left[\frac{(2y+1)^4}{y}\right]^{1/3} = \infty \quad \text{and} \quad \lim_{y \to \infty} S_{oc} = \infty$$

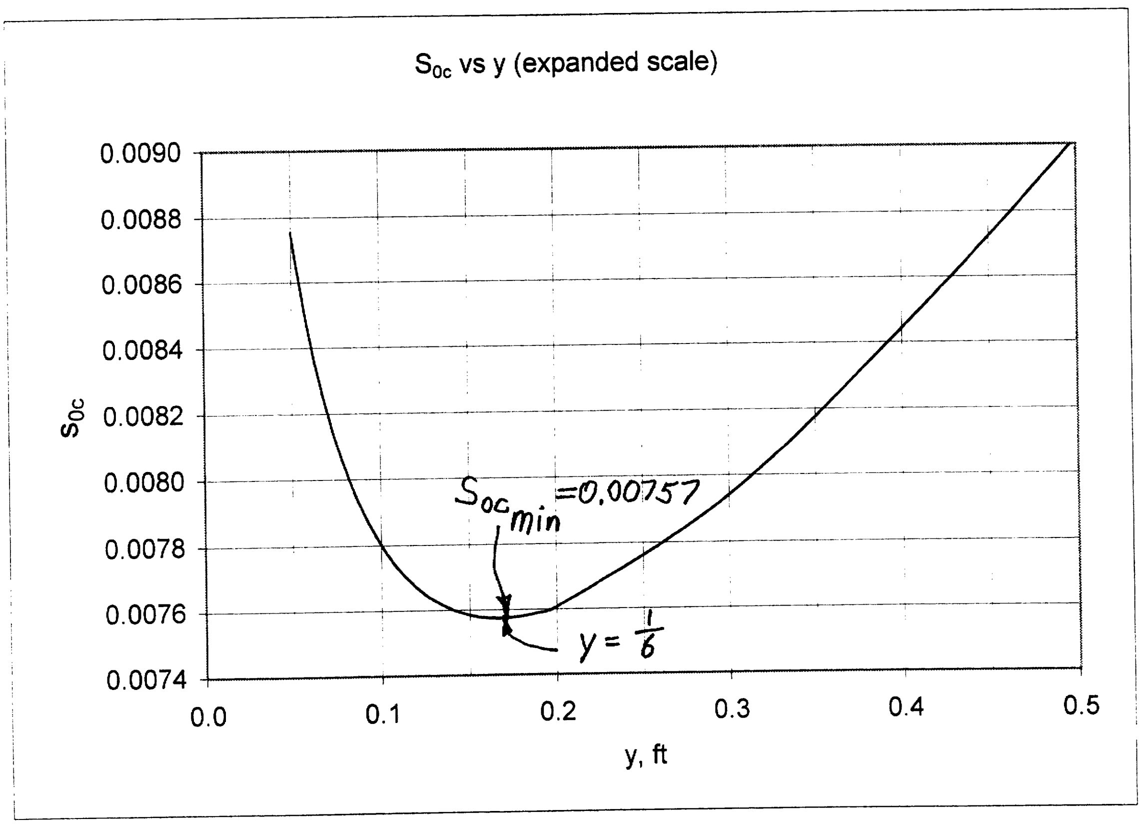
See next page for graphs.

(con't)

10.70 (con't)



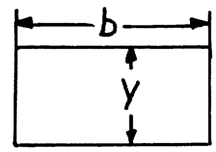
S_{oc} vs y
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
S_{0c}
0.0
1.0
2.0
3.0
4.0
5.0
y, ft



S_{0c} vs y (expanded scale)
0.0090
0.0088
0.0086
0.0084
0.0082
0.0080
0.0078
0.0076
0.0074
S_{0c}
0.0
0.1
0.2
0.3
0.4
0.5
y, ft
$S_{oc_{min}} = 0.00757$
$y = \frac{1}{6}$

10.71 Water flows in a rectangular channel of width b and depth y with a Froude number of unity. The slope, S_{0c}, of the channel needed to produce this critical flow is a function of y. Show that as $y \to \infty$ the slope becomes proportional to y (i.e., $S_{0c} = C_1 y$, where C_1 is a constant) and that as $y \to 0$ the slope becomes proportional to $y^{-1/3}$ (i.e., $S_{0c} = C_2/y^{1/3}$, where C_2 is a constant). Show that the channel with an aspect ratio of $b/y = 6$ gives the minimum value of S_{0c}.

$V = \frac{\kappa}{n} R_h^{2/3} S_o^{1/2}$ with $Fr = \frac{V}{(gy)^{1/2}} = 1$, or $V = (gy)^{1/2}$



Also, $R_h = \frac{A}{P} = \frac{by}{2y+b}$ Thus,

$$(gy)^{1/2} = \frac{\kappa}{n}\left[\frac{by}{2y+b}\right]^{2/3} S_{oc}^{1/2}\ , \text{ where } S_{oc} = \text{critical slope}$$

Thus, $$\frac{n^2 g}{\kappa^2} y = \left[\frac{by}{2y+b}\right]^{4/3} S_{oc}\ , \text{ or } S_{oc} = \left(\frac{n^2 g}{\kappa^2 b^{4/3}}\right)\left[\frac{(2y+b)^4}{y}\right]^{1/3} \qquad (1)$$

Hence, as $y \to \infty$, $\frac{(2y+b)^4}{y} \to \frac{16y^4}{y} = 16y^3$ and

$$S_{oc} \to \left(\frac{ng}{\kappa b^{4/3}}\right)(16y^3)^{1/3} = \left(\frac{16^{1/3} ng}{\kappa b^{4/3}}\right) y = C_1 y$$

As $y \to 0$, $\frac{(2y+b)^4}{y} \to \frac{b^4}{y}$ so that

$$S_{oc} \to \left(\frac{ng}{\kappa b^{4/3}}\right)\left(\frac{b^4}{y}\right)^{1/3} = \frac{C_2}{y^{1/3}}$$

To determine the minimum S_{oc}, calculate $\frac{dS_{oc}}{dy} = 0$ from Eq.(1):

$$\frac{dS_{oc}}{dy} = \left(\frac{ng}{\kappa b^{4/3}}\right)\left(\frac{1}{3}\right)\left[\frac{(2y+b)^4}{y}\right]^{-2/3}\left[\frac{4(2y+b)^3(2)y - (2y+b)^4}{y^2}\right] = 0$$

or $$(2y+b)^3[8y - (2y+b)] = 0$$

Thus,

$$\underline{\underline{y = \frac{b}{6}}}$$

10.72

10.72 Water flows in a rectangular channel with a bottom slope of 4.2 ft/mi and a head loss of 2.3 ft/mi. At a section where the depth is 5.8 ft and the average velocity 5.9 ft/s, does the flow depth increase or decrease in the direction of flow? Explain.

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \quad \text{where } S_f = \frac{h_L}{\ell} = \frac{2.3\,ft}{5280\,ft}, \quad S_o = \frac{4.2\,ft}{5280\,ft},$$

$$\text{and } Fr = \frac{V}{(gy)^{1/2}} = \frac{5.9\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(5.8\,ft)\right]^{1/2}} = 0.432$$

Thus,

$$\frac{dy}{dx} = \frac{\left(\frac{2.3 - 4.2}{5280}\right)}{1 - (0.432)^2} = -0.000442 < 0$$

The flow depth decreases in flow direction.

There is less head loss than change in elevation for this subcritical flow. The fluid speeds up and gets shallower.

10.73

10.73 Water flows in the river shown in Fig. P10.73 with a uniform bottom slope. The total head at each section is measured by using Pitot tubes as indicated. Determine the value of dy/dx at a location where the Froude number is 0.357.

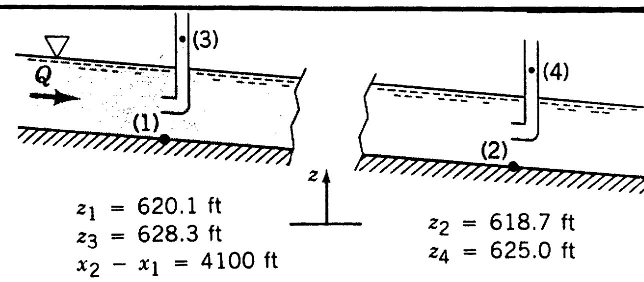


FIGURE P10.73

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \quad \text{where from the figure } S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\,ft}{4100\,ft}$$

$$\text{or } S_f = 8.05 \times 10^{-4} \text{ and } S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)\,ft}{4100\,ft} = 3.41 \times 10^{-4}$$

Thus,

$$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (0.357)^2} = 0.000532 \quad \left(i.e.,\ 2.81\,\frac{ft}{mi}\right)$$

10.74

10.74 Repeat Problem 10.73 if the Froude number is 2.75.

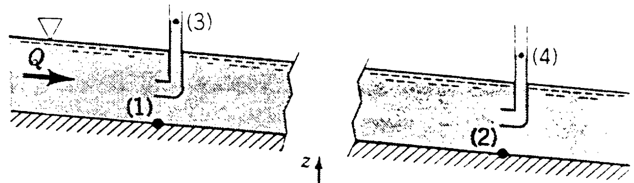


FIGURE P10.73

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \text{ where from the figure } S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\text{ ft}}{4100\text{ ft}}$$

or $S_f = 8.05 \times 10^{-4}$ and $S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)\text{ ft}}{4100\text{ ft}} = 3.41 \times 10^{-4}$

Thus,

$$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (2.75)^2} = \underline{\underline{-7.07 \times 10^{-5}}} \quad \left(\text{i.e., } -0.373 \frac{ft}{mi}\right)$$

10.75

10.75 Assume that the conditions given in Fig. P10.73 are as indicated except that the value of z_4 is not known. Determine the value of z_4 if the flow is uniform depth.

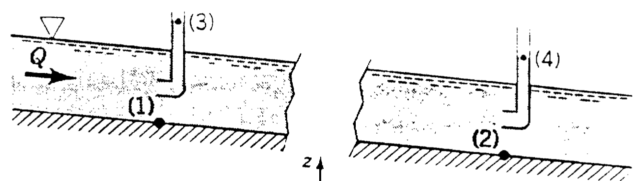


FIGURE P10.73

For uniform flow $\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2} = 0$, or $S_f = S_o$

Thus,

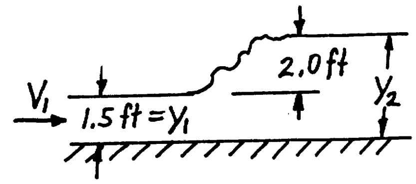
$$\frac{z_3 - z_4}{\ell} = \frac{z_1 - z_2}{\ell}, \text{ or } z_4 = z_3 + z_2 - z_1 = (628.3 + 618.7 - 620.1)\text{ ft}$$

or

$z_4 = \underline{\underline{626.9\text{ ft}}}$

10.76

10.76 A 2.0-ft standing wave is produced at the bottom of the rectangular channel in an amusement park water ride. If the water depth upstream of the wave is estimated to be 1.5 ft, determine how fast the boat is traveling when it passes through this standing wave (hydraulic jump) for its final "splash."



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

or $\left(\frac{2.0\,ft + 1.5\,ft}{1.5\,ft}\right) = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$

Thus, $Fr_1 = 1.97$, or since $Fr_1 = \frac{V_1}{\sqrt{g y_1}}$

$$V_1 = Fr_1\sqrt{g y_1} = 1.97\sqrt{(32.2\,\tfrac{ft}{s^2})(1.5\,ft)} = \underline{\underline{13.7\,\tfrac{ft}{s}}}$$

10.77

10.77 The water depths upstream and downstream of a hydraulic jump are 0.3 and 1.2 m, respectively. Determine the upstream velocity and the power dissipated if the channel is 50 m wide.

$\frac{y_2}{y_1} = \frac{1.2\,m}{0.3\,m} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$ or $Fr_1 = 3.16$ Thus, since $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$

it follows that $V_1 = (3.16)\left[(9.81\,\tfrac{m}{s^2})(0.3\,m)\right]^{1/2} = \underline{\underline{5.42\,\tfrac{m}{s}}}$

The power dissipated is given by

$\mathcal{P} = \gamma Q h_L$, where $\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)$

or $h_L = (0.3\,m)\left[1 - \frac{1.2\,m}{0.3\,m} + \frac{(3.16)^2}{2}\left(1 - \left(\frac{0.3\,m}{1.2\,m}\right)^2\right)\right] = 0.504\,m$

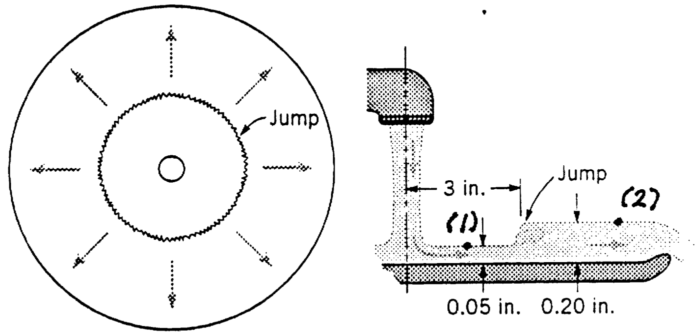
Also, $Q = A_1 V_1 = y_1 b V_1 = (0.3\,m)(50\,m)(5.42\,\tfrac{m}{s}) = 81.3\,\tfrac{m^3}{s}$

Thus,

$\mathcal{P} = (9.8\,\tfrac{kN}{m^3})(81.3\,\tfrac{m^3}{s})(0.504\,m) = 401\,\tfrac{kN\cdot m}{s} = \underline{\underline{401\,kW}}$

10.78

10.78 Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. P10.78 and **Video V10.6**. Consider a situation where a jump forms 3.0 in. from the center of the plate with depths upstream and downstream of the jump of 0.05 in. and 0.20 in., respectively. Determine the flowrate from the faucet.



■ FIGURE P10.78

For a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{or}$$

$$\frac{0.20\text{ in.}}{0.05\text{ in.}} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{so that} \quad Fr_1 = 3.16 = \frac{V_1}{\sqrt{g y_1}}$$

Thus,

$$V_1 = 3.16\sqrt{32.2\frac{ft}{s^2}(0.05/12)ft} = 1.16\frac{ft}{s}$$

and

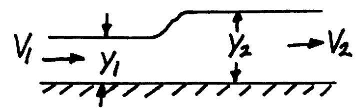
$$Q = A_1 V_1 = 2\pi R_1 y_1 V_1 = 2\pi\left(\frac{3}{12}ft\right)\left(\frac{0.05}{12}ft\right)\left(1.16\frac{ft}{s}\right) = \underline{\underline{0.00759\frac{ft^3}{s}}}$$

10.79

10.79 In order to have a hydraulic jump, the flow upstream of the jump must be supercritical. This implies that a wave made by a disturbance upstream of the jump cannot travel upstream; it gets washed downstream (see Video V10.6). Show that for a hydraulic jump in a rectangular channel, the Froude number upstream, Fr_1, and the Froude number downstream, Fr_2, are related by

$$Fr_2^2 = \frac{8\,Fr_1^2}{[(1 + 8\,Fr_1^2)^{1/2} - 1]^3}$$

Plot Fr_2 as a function of Fr_1 and show that the flow downstream of a jump is subcritical.



Since $A_1 V_1 = A_2 V_2$, or $V_2 = \frac{A_1}{A_2} V_1 = \frac{y_1}{y_2} V_1$ it follows that

$$Fr_2 = \frac{V_2}{(g y_2)^{1/2}} = \left(\frac{y_1}{y_2}\right)\frac{V_1}{(g y_2)^{1/2}} = \left(\frac{y_1}{y_2}\right)^{3/2}\frac{V_1}{(g y_1)^{1/2}} = \left(\frac{y_1}{y_2}\right)^{3/2} Fr_1 \qquad (1)$$

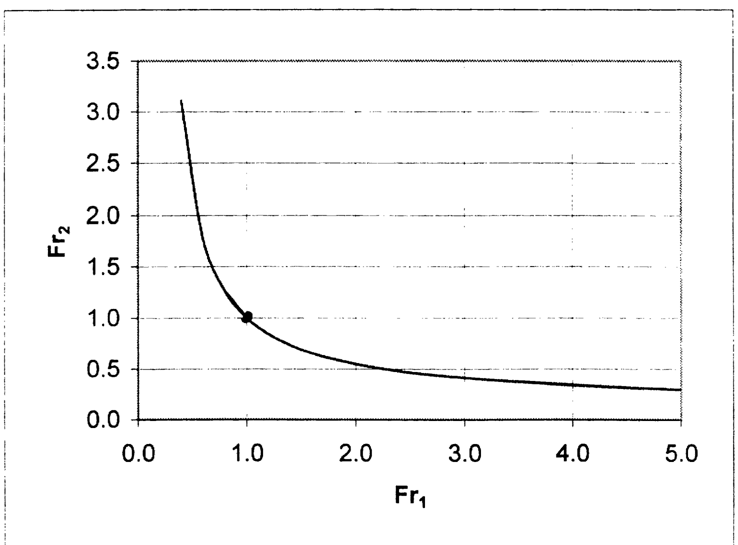
Also, $\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$ or $\left(\frac{y_1}{y_2}\right)^3 = \frac{8}{\left[-1 + \sqrt{1 + 8Fr_1^2}\right]^3}$ (2)

Combine Eqs. (1) and (2) to obtain:

$$Fr_2^2 = \frac{8\,Fr_1^2}{\left[\sqrt{1 + 8Fr_1^2} - 1\right]^3} \qquad (3)$$

This result is plotted below.

Fr_1	Fr_2
0.5	2.26
1	1
2	0.547
3	0.410
4	0.339
5	0.296

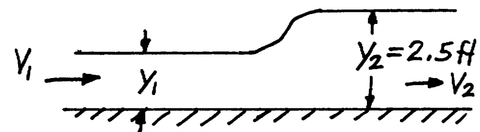


Note: To have a jump we must have $Fr_1 > 1$. From the graph $Fr_2 < 1$ if $Fr_1 > 1$.

Note: $Fr_1 = 1$ gives $Fr_2 = 1$. Also can show from Eq. (3) that $dFr_2/dFr_1 < 0$. Hence, $Fr_2 < 1$ for a jump.

10.80

10.80 Water flows in a 2-ft-wide rectangular channel at a rate of 10 ft³/s. If the water depth downstream of a hydraulic jump is 2.5 ft, determine **(a)** the water depth upstream of the jump, **(b)** the upstream and downstream Froude numbers, and **(c)** the head loss across the jump.



$Q = 10\,\frac{ft^3}{s}$, $b = \text{width} = 2\,ft$

(a) $V_2 = \frac{Q}{A_2} = \frac{10\,\frac{ft^3}{s}}{(2.5ft)(2ft)} = 2.0\,\frac{ft}{s}$ so that $Fr_2 = \frac{V_2}{(g y_2)^{1/2}} = \frac{2\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(2.5ft)\right]^{1/2}}$

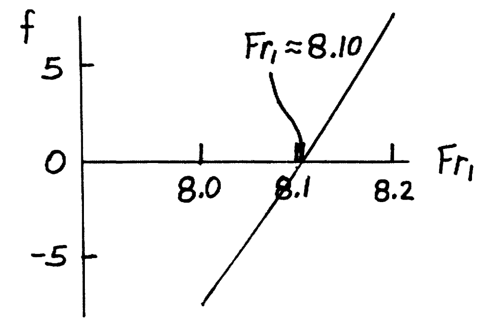
or

$Fr_2 = 0.223$ Obtain Fr_1 from the result of Problem 10.79: *

$$Fr_2^2 = \frac{8\,Fr_1^2}{\left[(1+8Fr_1^2)^{1/2}-1\right]^3} \text{ or } (0.223)^2\left[(1+8Fr_1^2)^{1/2}-1\right]^3 - 8Fr_1^2 = 0 \equiv f(Fr_1) \qquad (1)$$

Trial and error solution for $f=0$:

Fr_1	f
8.0	-7.39
8.1	-0.26
8.2	7.25



Thus, $Fr_1 \approx 8.10 = \frac{V_1}{(g\,y_1)^{1/2}}$, where

$V_1 = \frac{Q}{A_1} = \frac{10\,\frac{ft^3}{s}}{(2ft)y_1} = \frac{5}{y_1}$ so that $8.10 = \frac{\left(\frac{5}{y_1}\right)}{(32.2\,y_1)^{1/2}}$

Hence,

$y_1 = \underline{\underline{0.228\,ft}}$ and $V_1 = \frac{5}{0.228} = 21.9\,\frac{ft}{s}$

(b) From part (a) $Fr_1 = \underline{\underline{8.10}}$, $Fr_2 = \underline{\underline{0.223}}$

(c) Also,

$$h_L = y_1\left[1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right] = 0.228ft\left[1 - \frac{2.5}{0.228} + \frac{(8.10)^2}{2}\left(1-\left(\frac{0.228}{2.5}\right)^2\right)\right]$$

or

$h_L = \underline{\underline{5.15\,ft}}$

* Or could use $\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8\,Fr_1^2}\right]$ with $y_2 = 2.5ft$ so that

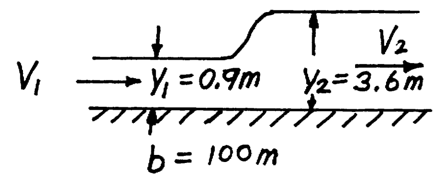
$5 + y_1 = y_1\sqrt{1+8\,Fr_1^2}$ Now, with $Fr_1^2 = \frac{V_1^2}{g y_1} = \frac{(Q/(b y_1))^2}{g\,y_1} = \frac{(10/(2y_1))^2}{32.2\,y_1}$,

or $Fr_1^2 = \frac{0.776}{y_1^3}$, we obtain

$5 + y_1 = y_1\left[1+8\left(\frac{0.776}{y_1^3}\right)\right]^{\frac{1}{2}}$ By squaring both sides and simplifying we obtain $y_1^2 + 2.5y_1 - 0.621 = 0$ which gives $y_1 = 0.228$ ft as above.

10.81

10.81 A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively (see Video V10.5). If the spillway is 10 m wide, what is the flowrate over the spillway?



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right], \text{ or } \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$, but $Fr_1 = \frac{V_1}{(g\,y_1)^{1/2}}$ so that

$$V_1 = 3.16\left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2} = 9.39\ \tfrac{m}{s}$$

Thus,

$$Q = A_1V_1 = by_1V_1 = (10.0m)(0.9m)(9.39\tfrac{m}{s}) = \underline{\underline{84.5\ \tfrac{m^3}{s}}}$$

10.82

10.82 Determine the head loss and power dissipated by the hydraulic jump of Problem 10.81.

$$h_L = y_1\left[1-\frac{y_2}{y_1}+\frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right], \text{ where from } \frac{y_2}{y_1} = \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$ so that

$$h_L = (0.9m)\left[1-\frac{3.6m}{0.9m}+\frac{(3.16)^2}{2}\left(1-\left(\frac{0.9m}{3.6m}\right)^2\right)\right] = \underline{\underline{1.51\ m}}$$

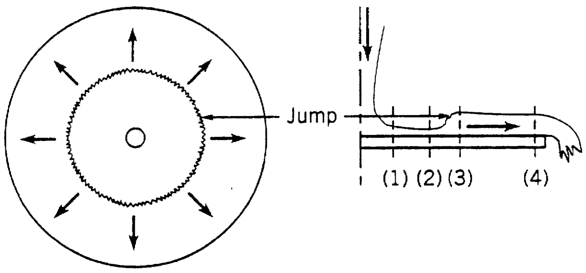
Also, $\mathcal{P} = \gamma Q h_L$, where $V_1 = (gy_1)^{\frac{1}{2}}Fr_1 = \left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2}(3.16) = 9.39\tfrac{m}{s}$

Hence,

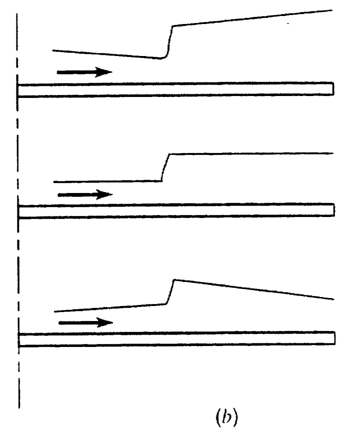
$$\mathcal{P} = (9.80\tfrac{kN}{m^3})\left[(0.9m)(100m)(9.39\tfrac{m}{s})\right](1.51\ m) = 12,500\frac{kN\cdot m}{s} = \underline{\underline{12,500kW}}$$

10.83

10.83 Water flowing radially outward along a circular plate forms a circular hydraulic jump as is shown in Fig. P10.83*a*. This is shown easily by holding a dinner plate under the faucet of the kitchen sink (see **Video V10.6**). **(a)** Sketch a typical specific energy diagram for this flow (see Problem 10.12) and locate points 1, 2, 3, and 4 on the diagram. **(b)** Which of the water depth profiles shown in Fig. P10.83*b* represents the actual situation? Explain.



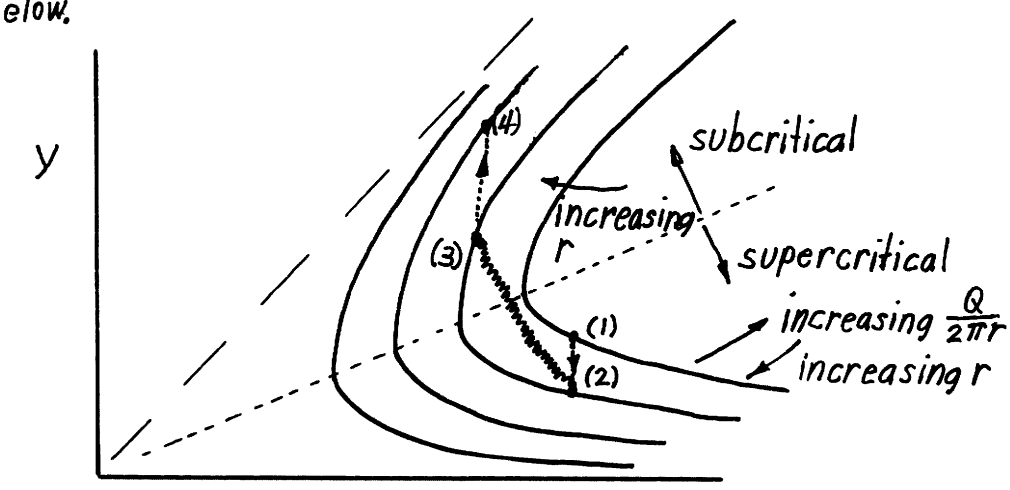
(*a*)



(*b*)

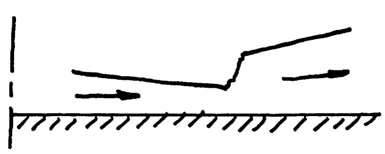
FIGURE P10.83

From Problem 10.12 the specific energy diagram for this radial flow is shown below.



Upstream of the jump the flow must be supercritical so (1) and (2) are located. Energy is conserved — $E_1 = E_2$. The depth decreases from (1) to (2). In the jump energy decreases — $E_3 = E_4 < E_2$. The flow is subcritical downstream of the jump and the depth increases. (See the above graph.)

Thus, the flow is like the following:



10.84

10.84 Water flows in a wide finished concrete channel as is shown in Fig. P10.84 such that a hydraulic jump occurs at the transition of the change in slope of the channel bottom. If the upstream Froude number and depth are 4.0 and 0.2 ft, respectively, determine the slopes upstream, S_{01}, and downstream, S_{02}, of the jump to maintain uniform flows in those regions. The jump can be treated as a jump on a horizontal surface.

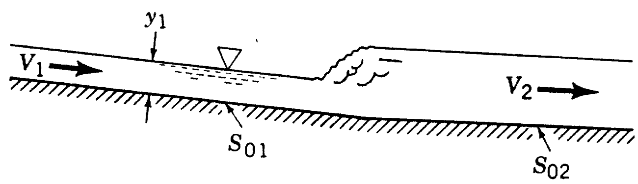
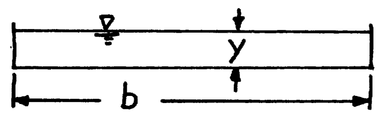


FIGURE P10.84

For uniform flow $V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and for a wide channel (1)

$R_h = \frac{A}{P} = \frac{yb}{2y+b} \approx y$ since $b \gg y$



From Table 10.1 $n = 0.012$

Upstream of the jump $Fr_1 = \frac{V_1}{(g y_1)^{1/2}} = 4$ so that

$V_1 = (g y_1)^{1/2} Fr_1 = \left[(32.2 \frac{ft}{s^2})(0.2 ft)\right]^{1/2}(4) = 10.2 \frac{ft}{s}$

Hence, from Eq. (1):

$10.2 = \frac{1.49}{0.012}(0.2)^{2/3} S_{o_1}^{1/2}$, or $S_{o_1} = \underline{\underline{0.0577}}$

Downstream of the jump $\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right]$

or

$y_2 = (\frac{1}{2})(0.2 ft)\left[-1 + \sqrt{1 + 8(4)^2}\right] = 1.036 ft$

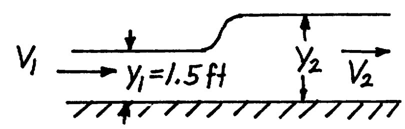
Thus, $A_1 V_1 = A_2 V_2$ or $V_2 = \frac{y_1}{y_2} V_1 = \left(\frac{0.2 ft}{1.036 ft}\right)(10.2 \frac{ft}{s}) = 1.97 \frac{ft}{s}$

so that Eq. (1) gives

$1.97 = \frac{1.49}{0.012}(1.036)^{2/3} S_{o_2}^{1/2}$, or $S_{o_2} = \underline{\underline{0.000240}}$

10.85*

10.85* A rectangular channel of width b is to carry water at flowrates from $30 \le Q \le 600$ cfs. The water depth upstream of the hydraulic jump that occurs (if one does occur) is to remain 1.5 ft for all cases. Plot the power dissipated in the jump as a function of flowrate for channels of width b = 10, 20, 30, and 40 ft.



$$\mathcal{P} = \gamma Q h_L \text{, where } h_L = y_1\left[1 - \left(\frac{y_2}{y_1}\right) + \frac{Fr_1^2}{2}\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)\right] \tag{1}$$

and
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right] \text{, provided } Fr_1 \ge 0 \tag{2}$$

Also,
$$Fr_1 = \frac{V_1}{(g y_1)^{1/2}} \text{, where } V_1 = \frac{Q}{A_1} = \frac{Q}{1.5b} \text{ so that}$$

$$Fr_1 = \frac{\left(\frac{Q}{1.5b}\right)}{\left[(32.2\frac{ft}{s^2})(1.5ft)\right]^{1/2}} = 0.0959\frac{Q}{b} \quad \text{Hence, from Eq.(1)}$$

$$h_L = (1.5)\left[1 - \left(\frac{y_2}{y_1}\right) + (0.00460)\left(\frac{Q}{b}\right)^2\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)\right] \text{ ft, where } b \sim ft,\ Q \sim \frac{ft^3}{s} \tag{3}$$

and from Eq.(2)
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \left(1 + 0.0736\left(\frac{Q}{b}\right)^2\right)^{1/2}\right] \tag{4}$$

For the given values of plot $\mathcal{P}$ from
$$\mathcal{P} = 62.4 Q h_L \ \frac{ft \cdot lb}{s} \quad \text{for } 30 \le Q \le 600 \ \frac{ft \cdot lb}{s} \tag{5}$$

Note: If $Fr_1 < 1$ there is no jump and $\mathcal{P} = 0$. From above, $Fr_1 = 1$
$$\text{when } Q = \frac{b}{0.0959} = 10.4b \tag{6}$$

Let Q_1 = flowrate when $Fr_1 = 1$. From Eq.(6) we obtain

b, ft	Q_1, $\frac{ft^3}{s}$
10	104
20	208
30	312
40	416

With b = 10, 20, 30, or 40 ft calculate and plot $\mathcal{P}$ from:

a) $\mathcal{P} = 0$ if $Q < Q_1$

b) $\mathcal{P} = 62.4\, Q h_L \ \frac{ft \cdot lb}{s}$, where obtain h_L from Eq. (3) with $\frac{y_2}{y_1}$ from Eq. (4) if $Q_1 \le Q \le 600 \ \frac{ft^3}{s}$

The program and results are given below. (See program P10#85)

(con't)

10.85* (con't)

```
100 cls
105 open "prn" for output as #1
110 print#1, "*********************************************"
120 print#1, "**  This program calculates the power       **"
130 print#1, "**  dissipated by a hydraulic jump for      **"
140 print#1, "**  various width channels and various      **"
150 print#1, "**  flowrates.                              **"
160 print#1, "*********************************************"
180 b = 0
190 for i = 1 to 4
200 b = b + 10
220 print#1, " "
230 print#1, using "With b = ###.##";b
240 Q1 = 10.4*b
250 print#1, using "If Q < ###.# then P = 0 (no jump possible)";Q1
260 print#1, "  Q, cfs   P, ft.lb/s"
270 Q = 50
280 for j = 1 to 11
300 Q = Q + 50
310 if Q < Q1 then goto 360
320 y2y1 = 0.5*(-1 + (1 + 0.0736*(Q/b)^2)^0.5)
330 h = 1.5*(1- y2y1 + 0.00460*(Q/b)^2*(1 - y2y1^(-2)))
340 P = 62.4*Q*h
350 print#1, using " ####.##   +#.###^^^^";Q,P
360 next j
370 next i
```

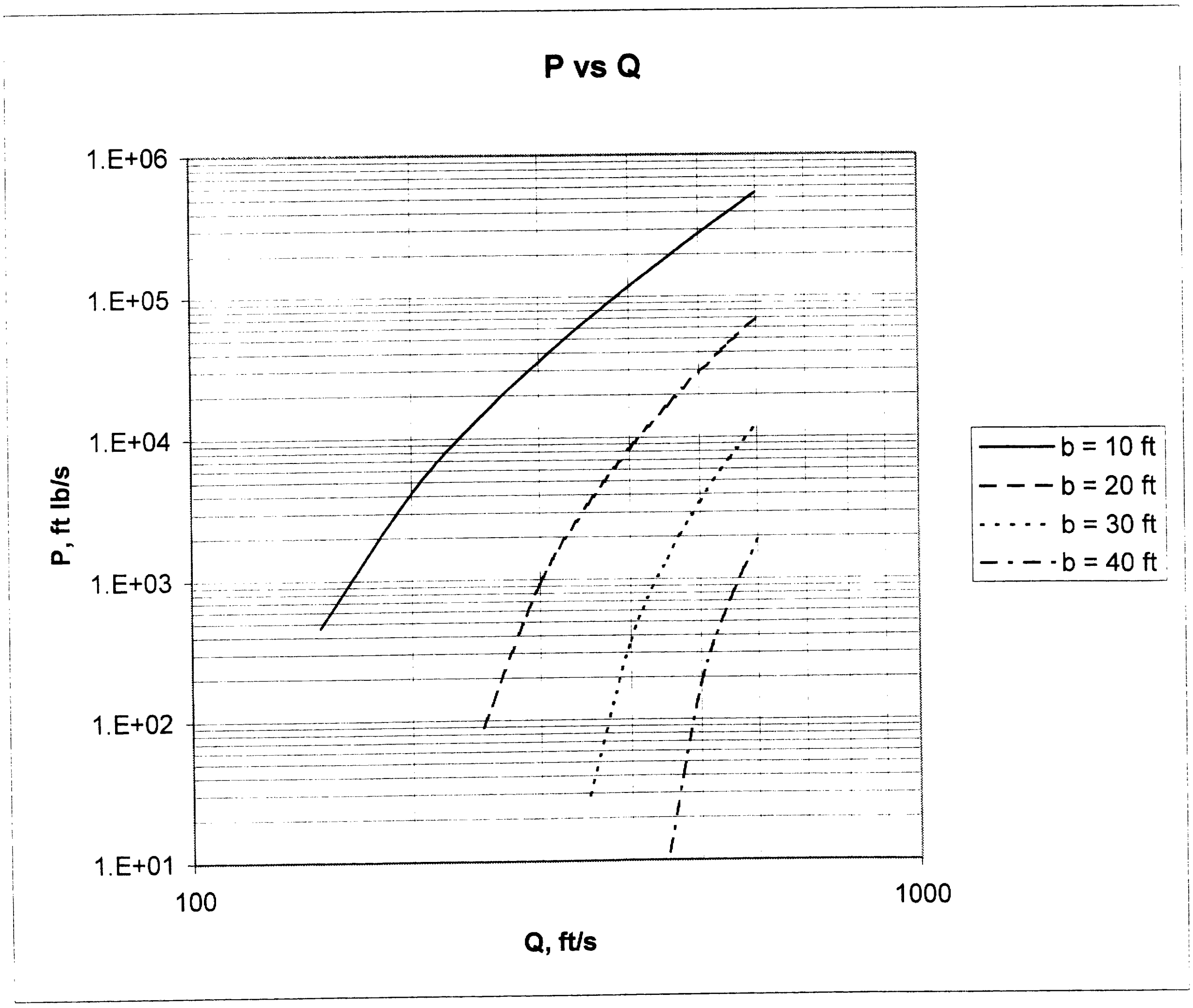
Sample output:

```
*********************************************
**  This program calculates the power      **
**  dissipated by a hydraulic jump for     **
**  various width channels and various     **
**  flowrates.                             **
*********************************************

With b =  10.00
If Q < 104.0 then P = 0 (no jump possible)
  Q, cfs   P, ft.lb/s
  150.00   +4.640E+02
  200.00   +4.131E+03
  250.00   +1.432E+04
  300.00   +3.427E+04
  350.00   +6.724E+04
  400.00   +1.165E+05
  450.00   +1.852E+05
  500.00   +2.766E+05
  550.00   +3.939E+05
  600.00   +5.404E+05

With b =  20.00
If Q < 208.0 then P = 0 (no jump possible)
  Q, cfs   P, ft.lb/s
  250.00   +8.859E+01
  300.00   +9.281E+02
  350.00   +3.376E+03
  400.00   +8.263E+03
  450.00   +1.641E+04
  500.00   +2.863E+04
  550.00   +4.574E+04
  600.00   +6.855E+04
```

(con't)

10.85* (con't)

```
With b =  30.00
If Q < 312.0 then P = 0 (no jump possible)
  Q, cfs    P, ft.lb/s
  350.00    +2.874E+01
  400.00    +3.628E+02
  450.00    +1.392E+03
  500.00    +3.494E+03
  550.00    +7.039E+03
  600.00    +1.239E+04

With b =  40.00
If Q < 416.0 then P = 0 (no jump possible)
  Q, cfs    P, ft.lb/s
  450.00    +1.129E+01
  500.00    +1.772E+02
  550.00    +7.201E+02
  600.00    +1.856E+03
```



10.86

10.86 Water flows in a rectangular channel at a depth of $y = 1$ ft and a velocity of $V = 20$ ft/s. When a gate is suddenly placed across the end of the channel, a wave (a moving hydraulic jump) travels upstream with velocity V_w as is indicated in Fig. P10.86. Determine V_w. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

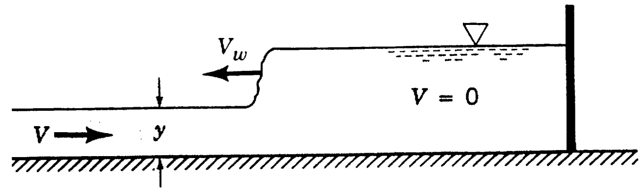
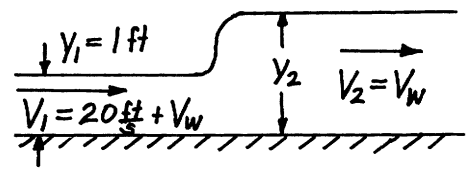


FIGURE P10.86

For an observer moving to the left with speed V_w the flow appears as shown below.

Thus, treat the flow as a jump with



$$Fr_1 = \frac{V_1}{(g y_1)^{1/2}} = \frac{(20+V_w)}{\left[(32.2\frac{ft}{s})(1\,ft)\right]^{1/2}}$$

or

$$Fr_1 = 0.176\,(20+V_w) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$, or $\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{20+V_w}{V_w}$ (2)

and

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$ which when combined with Eqs. (1) and (2) becomes

$$\frac{20+V_w}{V_w} = \frac{1}{2}\left[-1+\sqrt{1+8(0.176)^2(20+V_w)^2}\right]$$

or

$$2(20+V_w)+V_w = V_w\left(1+(0.248)(20+V_w)^2\right)^{\frac{1}{2}}$$

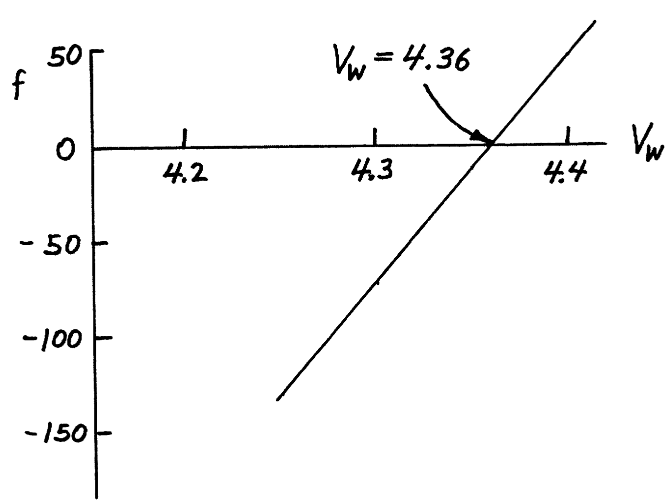
or

$$(40+3V_w)^2 = V_w^2\left[1+(0.248)(20+V_w)^2\right]$$, which can be written as

$$0.248\,V_w^4 + 9.92\,V_w^3 + 91.2\,V_w^2 - 240\,V_w - 1600 = 0 \equiv f(V_w) \qquad (3)$$

Trial and error solution of Eq. (3) for $f(V_w) = 0$:

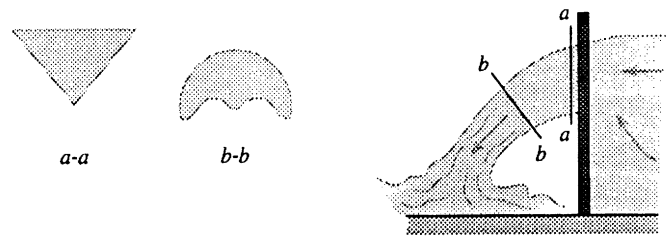
V_w, $\frac{ft}{s}$	$f(V_w)$
4.20	-187
4.25	-130
4.30	-72.2
4.35	-12.9
4.40	47.6



Thus, $\underline{\underline{V_w = 4.36\,\frac{ft}{s}}}$

10.87

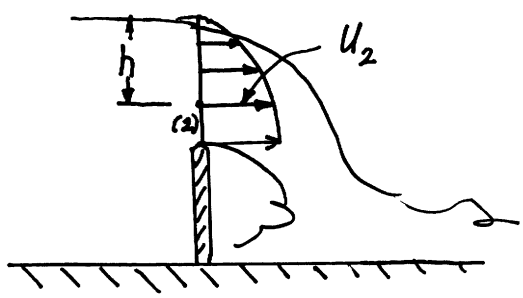
10.87 When water flows over a triangular weir as shown in Fig. P10.87 and Video V10.7, the cross-sectional shape of the water stream is clearly triangular in the plane of the weir (Section *a-a*). Farther downstream (Section *b-b*) the shape of the water stream is definitely not triangular. Explain why this is so. *Hint:* Consider the water velocity profile at Section *a-a*.



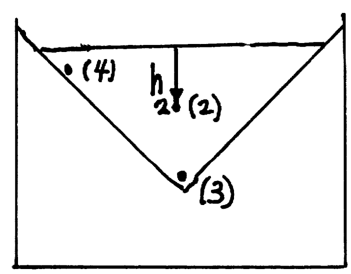
■ FIGURE P10.87

As discussed in Section 10.6.2, the speed of the water flowing over a weir is a function of h (see the figure):

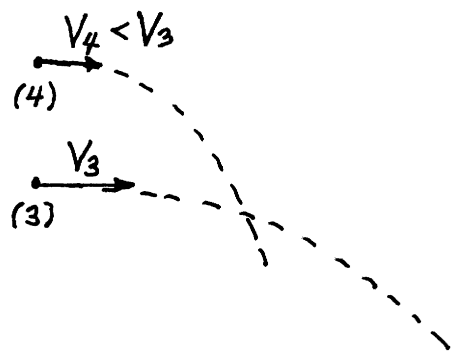
$$u_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)}$$



Thus, for a triangular weir the water speed at (3) is greater than that at (4).



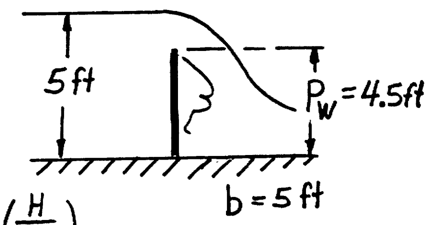
The higher the speed, the farther the water "shoots out" as it falls under the action of gravity. The trajectories of the water, therefore, are as shown below.



The result is a distortion of the original triangular cross-section of the water stream as shown in the video.

10.88

10.88 Water flows over a 5-ft-wide, rectangular sharp-crested weir that is $P_w = 4.5$ ft tall. If the depth upstream is 5 ft, determine the flowrate.



$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\,b\,H^{3/2}, \quad \text{where } C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$$

with

$$H = 5\text{ ft} - 4.5\text{ ft} = 0.5\text{ ft}$$

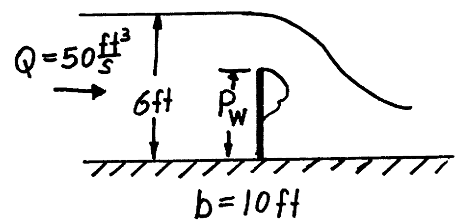
Hence, $C_{wr} = 0.611 + 0.075\left(\frac{0.5\text{ ft}}{4.5\text{ ft}}\right) = 0.619$

and

$$Q = (0.619)\left(\frac{2}{3}\right)\left(2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\right)^{1/2}(5\text{ ft})(0.5)^{3/2} = \underline{\underline{5.85\,\frac{\text{ft}^3}{\text{s}}}}$$

10.89

10.89 A rectangular sharp crested weir is used to measure the flowrate in a channel of width 10 ft. It is desired to have the channel flow depth be 6 ft when the flowrate is 50 cfs. Determine the height, P_w, of the weir plate.



$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\,b\,H^{3/2}, \quad \text{where } H = 6\text{ ft} - P_w \text{ and}$$

$$C_{wr} = 0.611 + 0.075\,\frac{H}{P_w}$$

Thus,

$$Q = \left(0.611 + 0.075\left(\frac{6-P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)(2g)^{1/2}b\,(6-P_w)$$

or

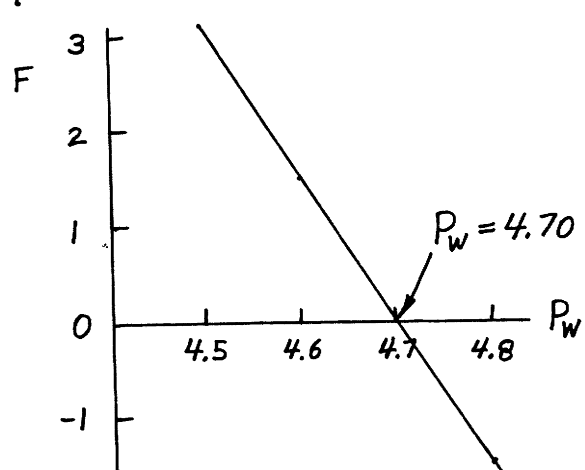
$$50\frac{\text{ft}^3}{\text{s}} = \left(0.611 + 0.075\left(\frac{6-P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)\left(64.4\frac{\text{ft}}{\text{s}^2}\right)^{1/2}(10\text{ ft})(6-P_w), \quad \text{where } P_w \sim \text{ft}$$

Hence,

$$\left[8.15 + \frac{(6-P_w)}{P_w}\right](6-P_w)^{3/2} - 12.5 = 0 \equiv F(P_w) \qquad (1)$$

Trial and error solution of Eq. (1) for $F(P_w) = 0$:

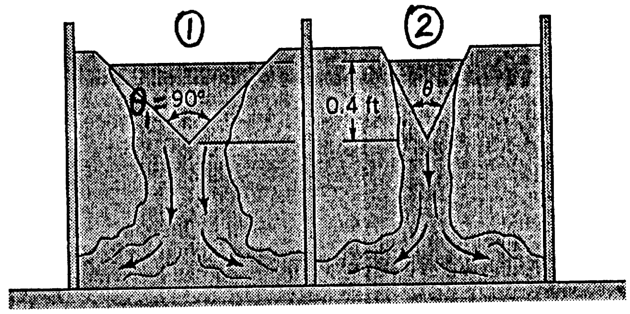
P_w	$F(P_w)$
4.5	3.08
4.6	1.50
4.7	0.0099
4.8	-1.46



Thus, $P_w = \underline{\underline{4.70\text{ ft}}}$

10.90

10.90 Water flows from a storage tank, over two triangular weirs, and into two irrigation channels as shown in Video V10.7 and Fig. P10.90 The head for each weir is 0.4 ft and the flowrate in the channel fed by the 90-degree V-notch weir is to be twice the flowrate in the other channel. Determine the angle θ for the second weir.



■ FIGURE P10.90

$$Q = C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} \qquad (1)$$

where

$$\theta_1 = 90°,\ H_1 = H_2 = 0.4\,ft,\ \text{and}\ Q_1 = 2Q_2 \qquad (2)$$

Thus, from Fig. 10.25,

$$C_{wt_1} = 0.590$$

From Eqs. (1) and (2),

$$C_{wt_1}\frac{8}{15}\tan\left(\frac{\theta_1}{2}\right)\sqrt{2g}\,H_1^{5/2} = C_{wt_2}\frac{8}{15}\tan\left(\frac{\theta_2}{2}\right)\sqrt{2g}\,H_2^{5/2}\times 2$$

or

$$0.590\tan 45° = C_{wt_2}\tan\left(\frac{\theta_2}{2}\right)\times 2$$

or

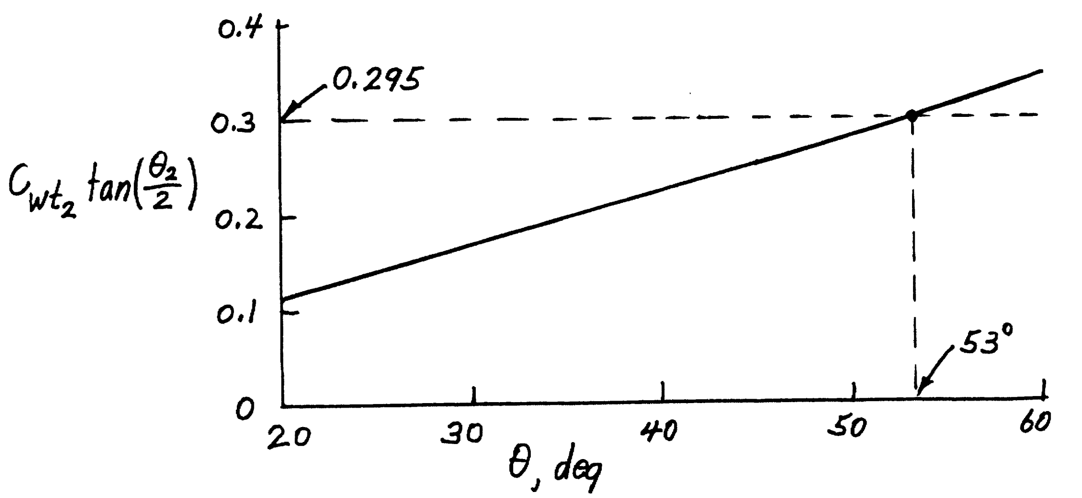
$$C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.295 \qquad (3)$$

Trial and error solution: Assume $\theta_2 = 20°$. From Fig. 10.16, $C_{wt_2} = 0.626$

Thus, $C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.626\tan(10°) = 0.110 = 0.295$. Thus, $\theta_2 \neq 20°$

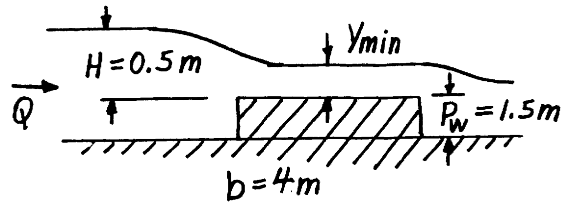
Repeated tries result in the graph below from which we conclude that

$\theta_2 = \underline{\underline{53°}}$



10.91

10.91 Water flows over a broad-crested weir that has a width of 4 m and a height of P_w = 1.5 m. The free-surface well upstream of the weir is at a height of 0.5 m above the surface of the weir. Determine the flowrate in the channel and the minimum depth of the water above the weir block.



$$Q = C_{wb}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}, \text{ where}$$

$$C_{wb} = \frac{0.65}{\left(1+\frac{H}{P_w}\right)^{1/2}} = \frac{0.65}{\left(1+\frac{0.5\,m}{1.5\,m}\right)^{1/2}} = 0.563$$

Thus,

$$Q = (0.563)(4m)\left(9.81\frac{m}{s^2}\right)^{1/2}\left(\frac{2}{3}\right)^{3/2}(0.5m)^{3/2} = \underline{\underline{1.36\ \frac{m^3}{s}}}$$

Also,

$$y_{min} = y_c = \frac{2}{3}H = \left(\frac{2}{3}\right)(0.5m) = \underline{\underline{0.333\ m}}$$

10.92

10.92 Determine the flowrate per unit width, q, over a broad-crested weir that is 3.0 m tall if the head, H, is 0.60 m.

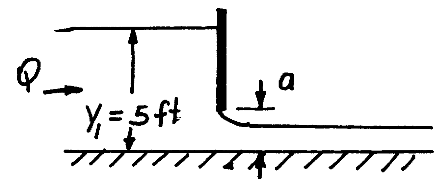
$$q = \frac{Q}{b} = \frac{0.65}{\left(1+\frac{H}{P_w}\right)^{1/2}} \sqrt{g} \left(\frac{2}{3}\right)^{3/2} H^{3/2}, \text{ where } H = 0.6\,m \text{ and } P_w = 3.0\,m$$

Thus,

$$q = \frac{0.65}{\left(1+\frac{0.6}{3.0}\right)^{1/2}} \left(9.81\,\frac{m}{s^2}\right)^{1/2} \left(\frac{2}{3}\right)^{3/2} (0.6\,m)^{3/2} = \underline{\underline{0.470\ \frac{m^2}{s}}}$$

10.93

10.93 Water flows under a sluice gate in a channel of 10-ft width. If the upstream depth remains constant at 5 ft, plot a graph of flowrate as a function of the distance between the gate and channel bottom as the gate is slowly opened. Assume free outflow.

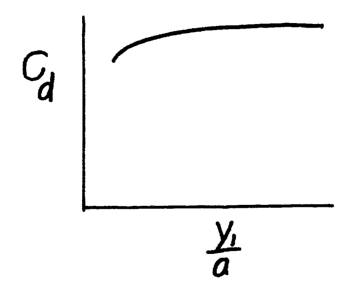


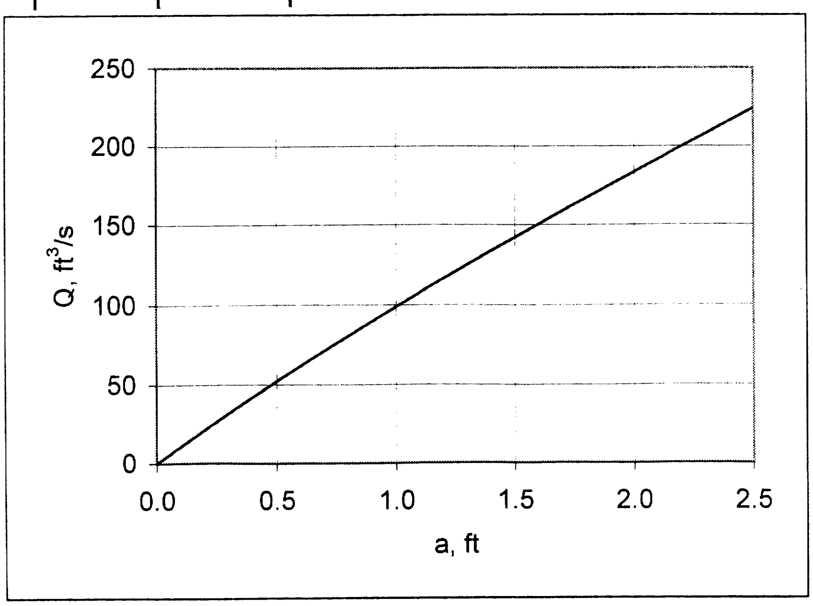
$$Q = qb = b\,C_d\,a\sqrt{2gy_1}, \text{ where } y_1 = 5\,ft,\ b = 10\,ft, \text{ and } C_d \text{ is from Fig. 10.29.}$$

Thus,

$$Q = C_d\,(10\,ft)\,a\left[2\left(32.2\,\frac{ft}{s^2}\right)(5\,ft)\right]^{1/2} = 179\,C_d\,a\ \frac{ft^3}{s}, \text{ where } a \sim ft$$

a, ft	$\frac{y_1}{a}$	C_d	Q, $\frac{ft^3}{s}$
0	∞	0.6	0
0.5	10	0.58	51.9
1.0	5	0.55	98.5
1.5	3.33	0.53	142
2.0	2.5	0.51	183
2.5	2	0.50	224





10.94

10.94 Water flows over the rectangular sharp crested weir in a wide channel as shown in Fig. P10.94. If the channel is lined with unfinished concrete with a bottom slope of 2 m/300 m, will it be possible to produce a hydraulic jump in the channel downstream of the weir? Explain.

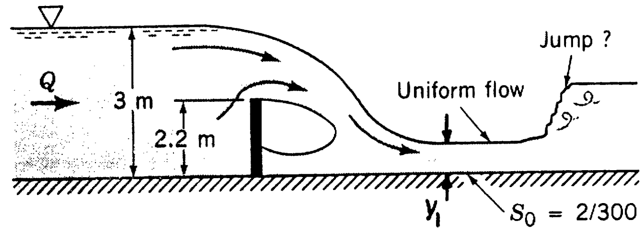


FIGURE P10.94

$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;b\,H^{3/2}$, where $C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$ with

$H = 3\,m - 2.2\,m = 0.8\,m$ and $P_w = 2.2\,m$

Thus,

$$Q = \left[0.611 + 0.075\left(\frac{0.8\,m}{2.2\,m}\right)\right]\left(\frac{2}{3}\right)\left[2\left(9.81\,\frac{m}{s^2}\right)\right]^{1/2} b\,(0.8\,m)^{3/2}$$

or

$Q = 1.349\,b\;\frac{m^3}{s}$, where $b \sim m$

Hence,

$$V_1 = \frac{Q}{A_1} = \frac{Q}{b y_1} = \frac{1.349\,b}{b\,y_1} \quad \text{or} \quad V_1 = \frac{1.349}{y_1} \qquad (1)$$

For uniform flow

$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2}$, or $V_1 = \frac{\kappa}{n} R_{h_1}^{2/3} S_o^{1/2}$, where $\kappa = 1$, $S_o = \frac{2\,m}{300\,m} = 0.00667$

Also, for a wide channel $A_1 = b y_1$ and $P_1 = 2y_1 + b$ so that

$$R_{h_1} = \frac{A_1}{P_1} = \frac{b y_1}{(2y_1 + b)} \approx y_1 \quad \text{if } b \gg y_1$$

Thus, with $n = 0.014$ (see Table 10.1)

$$Q = 1.349\,b = \frac{1}{0.014}(b y_1)(y_1)^{2/3}(0.00667)^{1/2}$$

or

$y_1 = 0.415\,m$

Hence, from Eq. (1)

$$V_1 = \frac{1.349}{0.415} = 3.25\,\frac{m}{s} \text{ so that } Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.25\,\frac{m}{s}}{\left[\left(9.81\,\frac{m}{s^2}\right)(0.415\,m)\right]^{1/2}}$$

or

$Fr_1 = 1.61$ Since $Fr_1 > 1$ it is possible to produce a jump.

10.95

10.95 Water flows in a rectangular channel of width $b = 20$ ft at a rate of 100 ft³/s. The flowrate is to be measured by using either a rectangular weir of height $P_w = 4$ ft or a triangular ($\theta = 90°$) sharp crested weir. Determine the head, H, necessary. If measurement of the head is accurate to only ±0.04 ft, determine the accuracy of the measured flowrate expected for each of the weirs. Which weir would be the most accurate? Explain.

(a) Rectangular weir:

$$Q = \left(0.611 + 0.075\left(\frac{H}{P_w}\right)\right)\left(\frac{2}{3}\right)\sqrt{2g}\, b\, H^{3/2}, \text{ where } P_w = 4 \text{ ft}$$

Thus,

$$Q = \left[0.611 + 0.075\left(\frac{H}{4}\right)\right]\left(\frac{2}{3}\right)\left[2\left(32.2\frac{ft}{s^2}\right)\right]^{1/2}(20\,ft)\, H^{3/2}$$

or

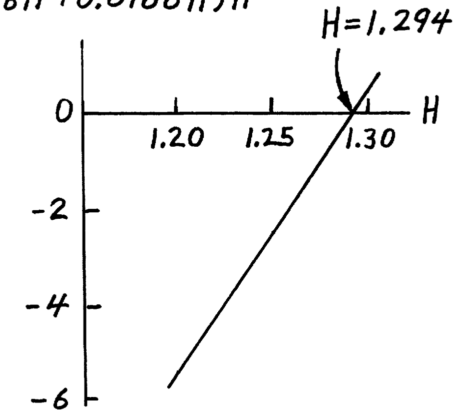
$$Q = 107\,(0.611 + 0.0188\,H)\,H^{3/2}, \text{ where } Q \sim \frac{ft^3}{s} \text{ and } H \sim ft \qquad (1)$$

With $Q = 100\,\frac{ft^3}{s}$ this gives $0.935 = (0.611 + 0.0188H)H^{3/2}$

or $(32.5 + H)H^{3/2} - 49.7 = 0 \equiv F(H)$

Trial and error solution for $F(H) = 0$:

H	F(H)
1.20	-5.40
1.25	-2.53
1.30	0.40



Thus, $H = \underline{\underline{1.294\,ft}}$

(b) Triangular weir:

$$Q = C_{wt}\,\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} = C_{wt}\left(\frac{8}{15}\right)(\tan 45°)\left[2\left(32.2\frac{ft}{s^2}\right)\right]^{1/2} H^{5/2}$$

or

$$Q = 4.28\,C_{wt}\,H^{5/2}\ \frac{ft^3}{s}, \text{ where } H \sim ft \text{ and } C_{wt} \text{ is from Fig. 10.25} \qquad (2)$$

For $Q = 100\,\frac{ft^3}{s}$, assume $C_{wt} = 0.58$ so that

$4.28\,(0.58)\,H^{5/2}$, or $H = \underline{\underline{4.39\,ft}}$ Note: The assumed $C_{wt} = 0.58$ checks (see Fig. 10.25)

Calculate Q for $H = H_{100}$, $H_{100} + 0.04$, and $H_{100} - 0.04$ from Eqs. (1) and (2):

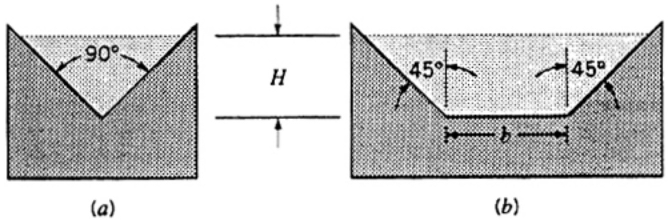
(Rectangular) H, ft	Q, cfs
1.254	95.3
$H_{100} =$ 1.294	100
1.334	104.9

(Triangular) H, ft	Q, cfs
4.35	98.0
$H_{100} =$ 4.39	100
4.43	102.5

With $H \pm 0.04$ ft it is seen that <u>triangular weir is more accurate</u> (i.e. smaller variation in Q).

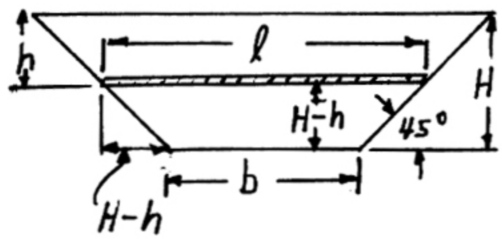
10.96

10.96 Water flows over a triangular weir as shown in Fig. P10.96*a* and Video V10.7. It is proposed that in order to increase the flowrate, Q, for a given head, H, the triangular weir should be changed to a trapezoidal weir as shown in Fig. P10.96*b*. (**a**) Derive an equation for the flowrate as a function of the head for the trapezoidal weir. Neglect the upstream velocity head and assume the weir coefficient is 0.60, independent of H. (**b**) Use the equation obtained in part (**a**) to show that when $b \ll H$ the trapezoidal weir functions as if it were a triangular weir. Similarly, show that when $b \gg H$ the trapezoidal weir functions as if it were a rectangular weir.



■ FIGURE P10.96

(a) $Q = C_w \int_{h=0}^{h=H} u_2 \, \ell \, dh$, where $u_2 = \sqrt{2gh}$



and

$\ell = b + 2(H-h) = (2H+b) - 2h$

Thus,

$$Q = C_w \int_0^H \sqrt{2gh}\,\left[(2H+b) - 2h\right] dh$$

$$= C_w\sqrt{2g}\,(2H+b)\int_0^H \sqrt{h}\,dh - C_w\sqrt{2g}\,(2)\int_0^H h^{3/2}\,dh$$

$$= C_w\sqrt{2g}\,(2H+b)\,\tfrac{2}{3}H^{3/2} - C_w\sqrt{2g}\,\tfrac{2}{5}H^{5/2}$$

or

(1) $$Q = C_w\left[\tfrac{2}{3}\sqrt{2g}\,b\,H^{3/2} + \tfrac{8}{15}\sqrt{2g}\,H^{5/2}\right], \text{ where } C_w = 0.6$$

Note: This equation is simply the sum of Q for a rectangular weir and Q for a triangular weir. That is $Q_{\text{trapezoid}} = Q_{\square} + Q_{\triangledown}$.

(b) From Eq. (1)

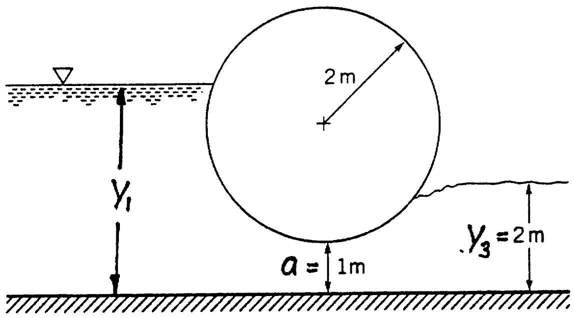
$$Q = C_w\sqrt{2g}\,H^{3/2}\left[\tfrac{2}{3}b + \tfrac{8}{15}H\right]$$

Thus, if $b \ll H$, $Q \approx C_w\sqrt{2g}\,H^{3/2}\left[\tfrac{8}{15}H\right] = C_w\sqrt{2g}\,\tfrac{8}{15}H^{5/2}$ which is the equation for a triangular wier.

Similarily, if $b \gg H$, $Q \approx C_w\sqrt{2g}\,b\,\tfrac{2}{3}H^{3/2}$ which is the equation for a rectangular weir.

10.97

10.97 A water-level regulator (not shown) maintains a depth of 2.0 m downstream from a 50-ft-wide drum gate as shown in Fig. P10.97 . Plot a graph of flowrate, Q, as a function of water depth upstream of the gate, y_1, for $2.0 \leq y_1 \leq 5.0$ m.



■ FIGURE P10.97

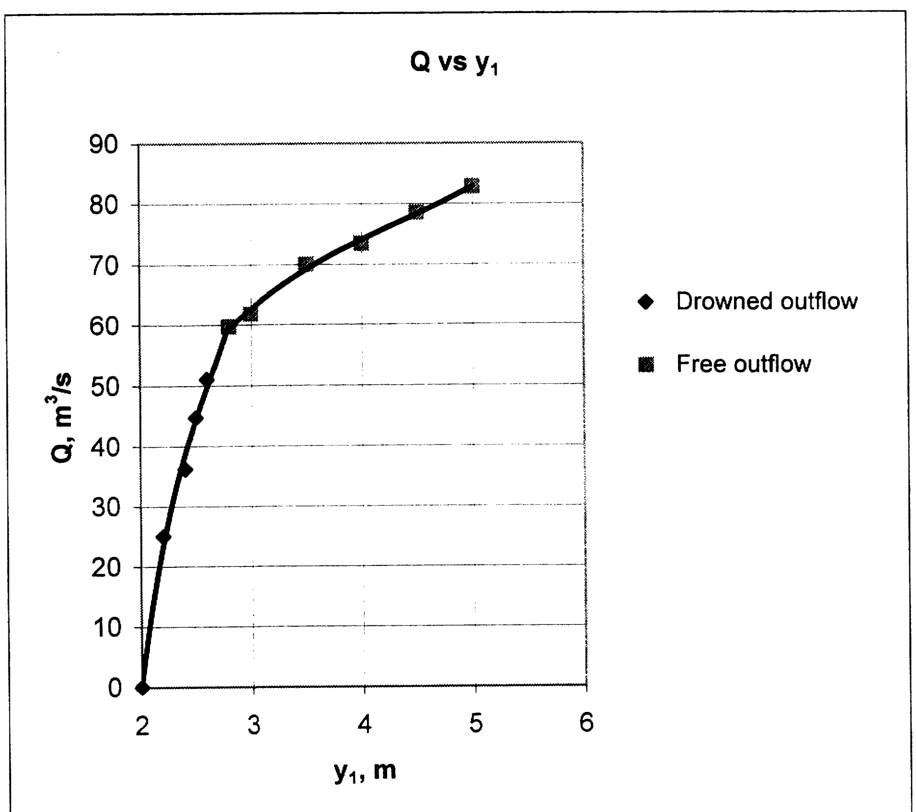
$Q = bq = b\,C_d\,a\sqrt{2gy_1}$ where $a = 1m$ and $b = 50ft\,(0.3048\,\frac{m}{ft}) = 15.2m$

Thus,

$Q = (15.2m)\,C_d\,(1m)\sqrt{2(9.81\frac{m}{s^2})(y_1\,m)} = 67.3\,C_d\sqrt{y_1}\;\frac{m^3}{s}$ where $y_1 \sim m$

Obtain C_d from Fig. 10.29 with $\frac{y_3}{a} = 2$.

y_1, m	$\frac{y_1}{a}$	C_d	Q, $\frac{m^3}{s}$
2.0	2.0	0	0
2.5	2.5	0.42	44.7
3.0	3.0	0.53	61.8
3.5	3.5	0.54	70.0
4.0	4.0	0.545	73.4
4.5	4.5	0.55	78.5
5.0	5.0	0.55	82.8
2.2	2.2	0.25	25.0
2.4	2.4	0.35	36.5
2.6	2.4	0.47	51.0
2.8	2.8	0.53	59.7



10.98 Calibration of a Triangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.98 to calibrate a triangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; triangular weir; float; point gage; stop watch.

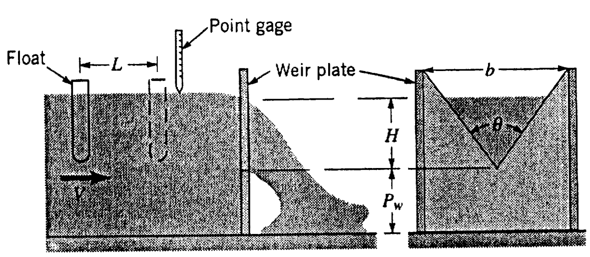
Experimental Procedure: Measure the width, b, of the channel, the distance, P_w, between the channel bottom and the bottom of the V-notch in the weir plate, and the angle, θ, of the V-notch. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 5/2 through the data.

Results: Use the flowrate-weir head data to determine the triangular weir coefficient, C_{wt}, for this weir (see Eq. 10.32). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.98

(con't.)

10.98 (con't)

Solution for Problem 10.98: Calibration of a Triangluar Weir

θ, deg	b, in.	P_w, in.	L, ft
90	6.00	6.55	1.50

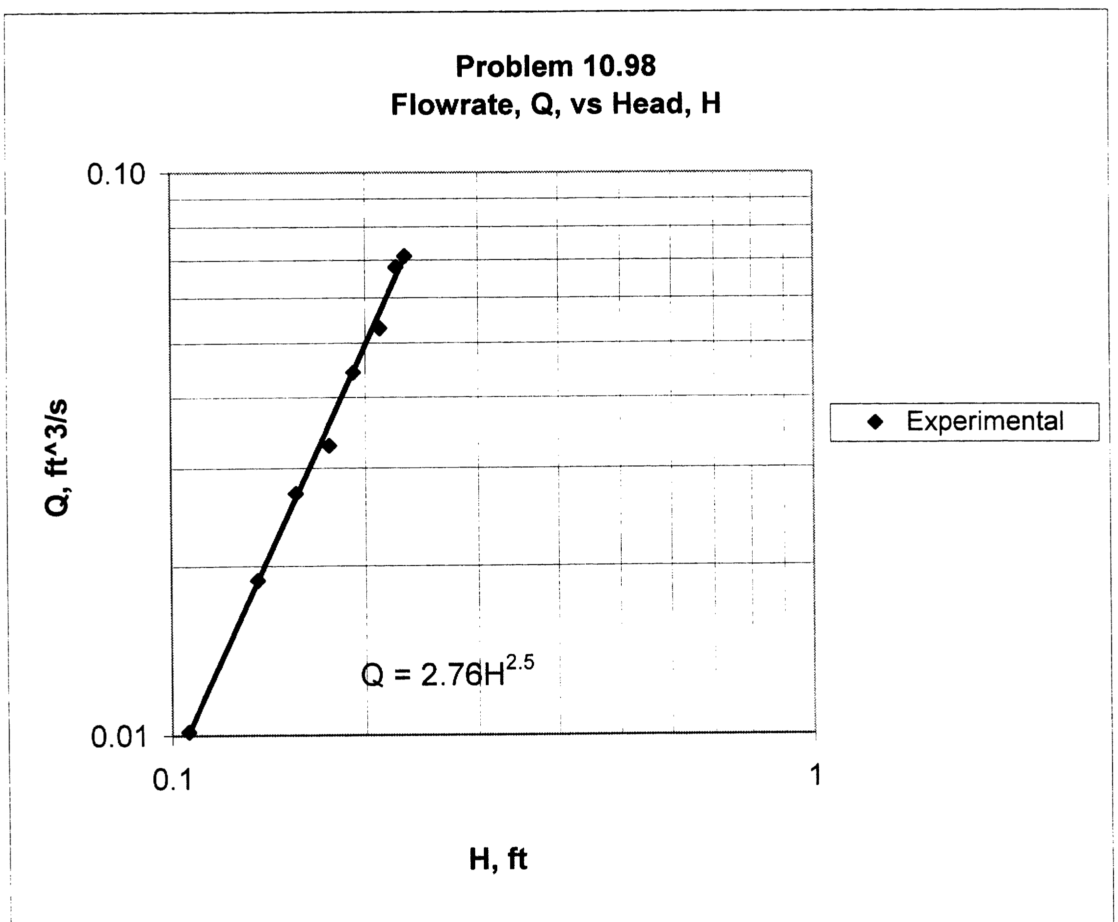
H, ft	t, s	V, ft/s	Q, ft^3/s
0.231	8.2	0.183	0.0711
0.224	8.5	0.176	0.0679
0.211	10.7	0.140	0.0530
0.192	12.5	0.120	0.0443
0.176	16.5	0.091	0.0328
0.156	19.5	0.077	0.0270
0.136	27.1	0.055	0.0189
0.106	48.2	0.031	0.0101
0.091	62.9	0.024	0.0076
0.088	68.1	0.022	0.0070

$Q = VA = V{*}b(P_w + H)$ where $V = L/t$

$Q = C_{wt}\,(8/15)\tan(\theta/2)\,(2g)^{1/2}\,H^{5/2}$ where from the graph

$Q = 2.76\,H^{2.5}$

Thus, $C_{wt} = (15/8){*}2.76/(2{*}32.2)^{1/2} = \underline{0.645}$



10.99

10.99 Calibration of a Rectangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.99 to calibrate a rectangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; rectangular weir; float; point gage; stop watch.

Experimental Procedure: Measure the width, b, of the channel and the distance, P_w, between the channel bottom and the top of the weir plate. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 3/2 through the data.

Results: Use the flowrate-weir head data to determine the rectangular weir coefficient, C_{wr}, for this weir (see Eq. 10.30). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

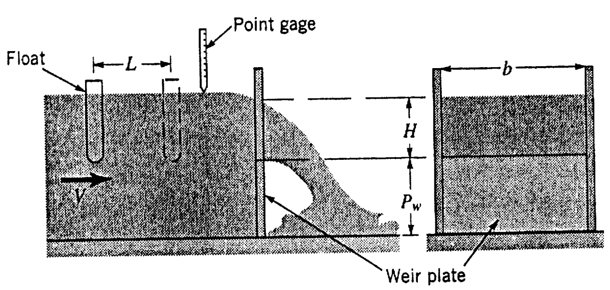


FIGURE P10.99

(con't)

10.99 (con't)

Solution for Problem 10.99: Calibration of a Rectangular Weir

b, in.	P_w, in.	L, ft
6.00	6.00	1.40

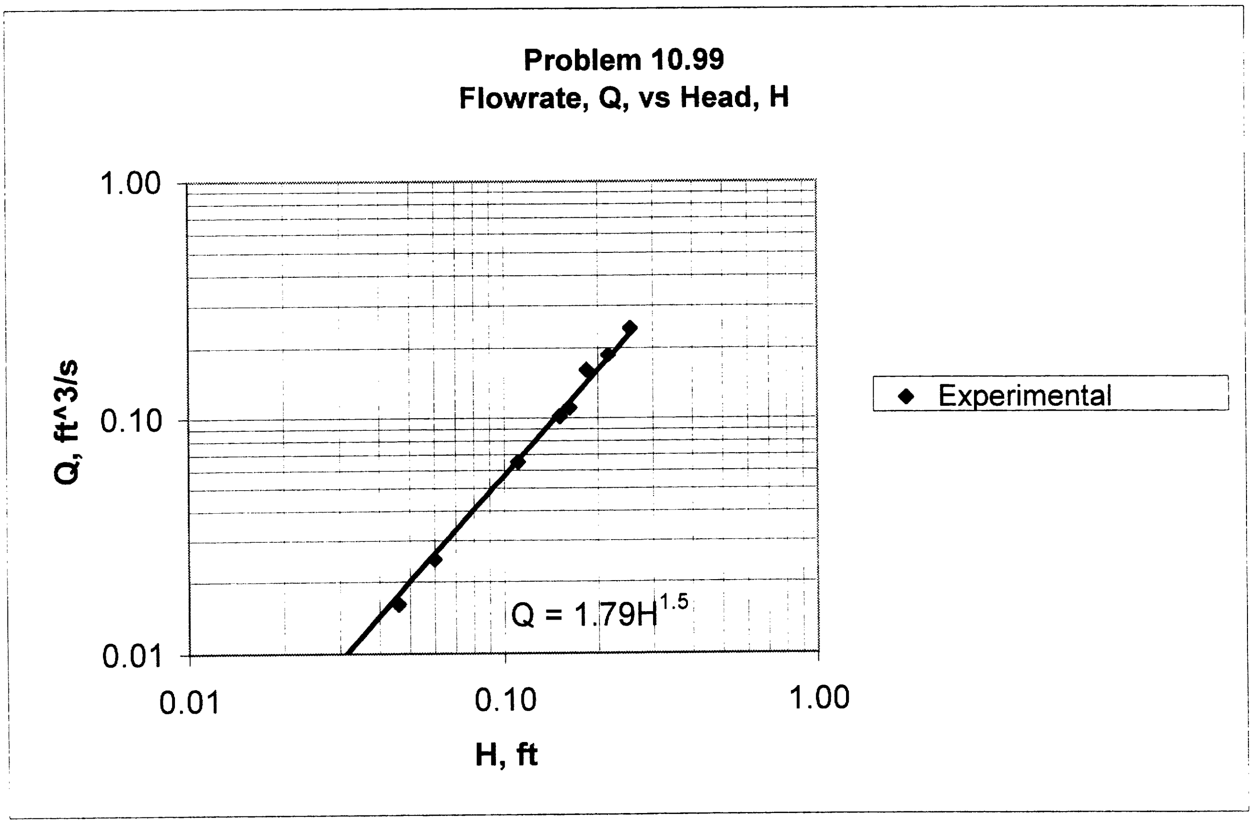
H, ft	t, s	V, ft/s	Q, ft^3/s
0.254	2.2	0.636	0.240
0.216	2.7	0.519	0.186
0.184	3.0	0.467	0.160
0.162	4.2	0.333	0.110
0.151	4.5	0.311	0.101
0.111	6.6	0.212	0.065
0.060	15.8	0.089	0.025
0.046	23.8	0.059	0.016
0.031	38.4	0.036	0.010

$Q = VA = V*b(P_w + H)$ where $V = L/t$

$Q = C_{wr}\ (2/3)\ (2g)^{1/2}\ H^{3/2}\ b$ where from the graph

$Q = 1.79\ H^{1.5}$

Thus, $C_{wr} = (3/2)*1.79/(0.5*(2*32.2)^{1/2}) = \underline{0.669}$



10.100

10.100 Hydraulic Jump Depth Ratio

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.100 to determine the depth ratio, y_2/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; adjustable tail gate.

Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, the depth just upstream from the jump, y_1, and the depth downstream from the jump, y_2. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump (see Eq. 3.21). Also use the measured depths to determine the depth ratio, y_2/y_1, across the jump.

Graph: Plot the depth ratio, y_2/y_1, as ordinates and Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical depth ratio as a function of Froude number (see Eq. 10.24).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

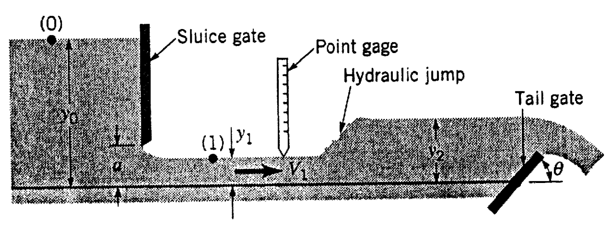


FIGURE P10.100

(con't)

10.100 (con't)

Solution for Problem 10.100: Hydraulic Jump Depth Ratio

y_0, ft	y_1, ft	y_2, ft.	Experimental V_1, ft/s	Experimental Fr_1	Experimental y_2/y_1	Theoretical Fr_1	Theoretical y_2/y_1
0.855	0.055	0.404	7.19	5.40	7.35	1	1.00
0.759	0.055	0.386	6.75	5.07	7.02	2	2.37
0.691	0.055	0.367	6.42	4.82	6.67	3	3.77
0.578	0.055	0.337	5.83	4.38	6.13	4	5.18
0.492	0.055	0.308	5.34	4.01	5.60	5	6.59
0.414	0.055	0.280	4.85	3.65	5.09	6	8.00
0.289	0.055	0.233	3.95	2.97	4.24		
0.248	0.055	0.211	3.62	2.72	3.84		

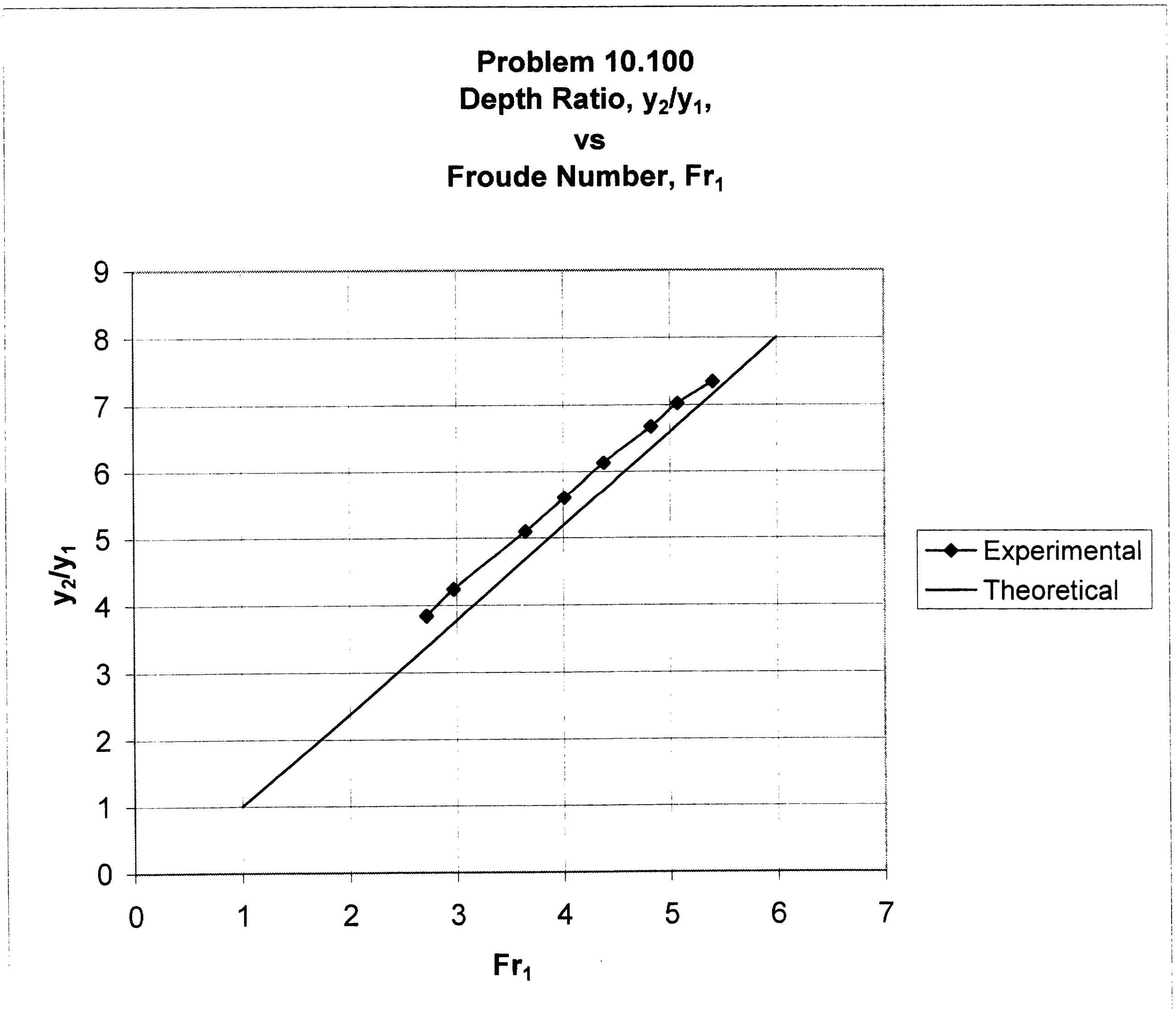
For flow under a sluice gate:

$$V_1 = [2g*(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$

$$Fr_1 = V_1/(gy_1)^{1/2}$$



10.101 Hydraulic Jump Head Loss

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.101 to determine the head loss ratio, h_L/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; Pitot tubes; adjustable tail gate.

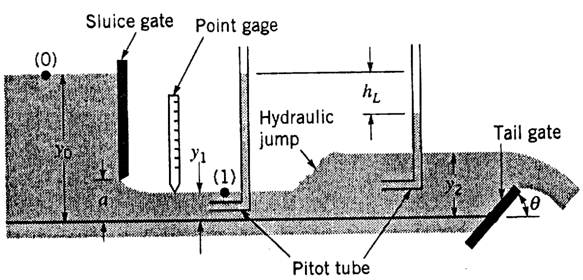
Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, and the depth just upstream from the jump, y_1. Also measure the head loss, h_L, as the difference in the water elevations in the piezometer tubes connected to the two Pitot tubes located upstream and downstream of the jump. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and the Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump. Also calculate the dimensionless head loss, h_L/y_1, for each data set.

Graph: Plot the dimensionless head loss across the jump, h_L/y_1, as ordinates and the Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical dimensionless head loss as a function of Froude number (see Eqs. 10.24 and 10.25).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.101

(con't)

10.101 (con't)

Solution for Problem 10.101: Hydraulic Jump Head Loss

				Experimental			Theoretical		
y_0, ft	y_1, ft	y_2, ft.	h_L, ft	V_1, ft/s	Fr_1	h_L/y_1	Fr_1	y_2/y_1	h_L/y_1
0.855	0.055	0.404	0.364	7.19	5.40	6.62	1	1.00	0.00
0.759	0.055	0.386	0.313	6.75	5.07	5.69	2	2.37	0.27
0.691	0.055	0.367	0.271	6.42	4.82	4.93	3	3.77	1.41
0.578	0.055	0.337	0.201	5.83	4.38	3.65	4	5.18	3.52
0.492	0.055	0.308	0.152	5.34	4.01	2.76	5	6.59	6.62
0.414	0.055	0.280	0.117	4.85	3.65	2.13	6	8.00	10.72
0.289	0.055	0.233	0.058	3.95	2.97	1.05			
0.248	0.055	0.211	0.042	3.62	2.72	0.76			

For flow under a sluice gate:

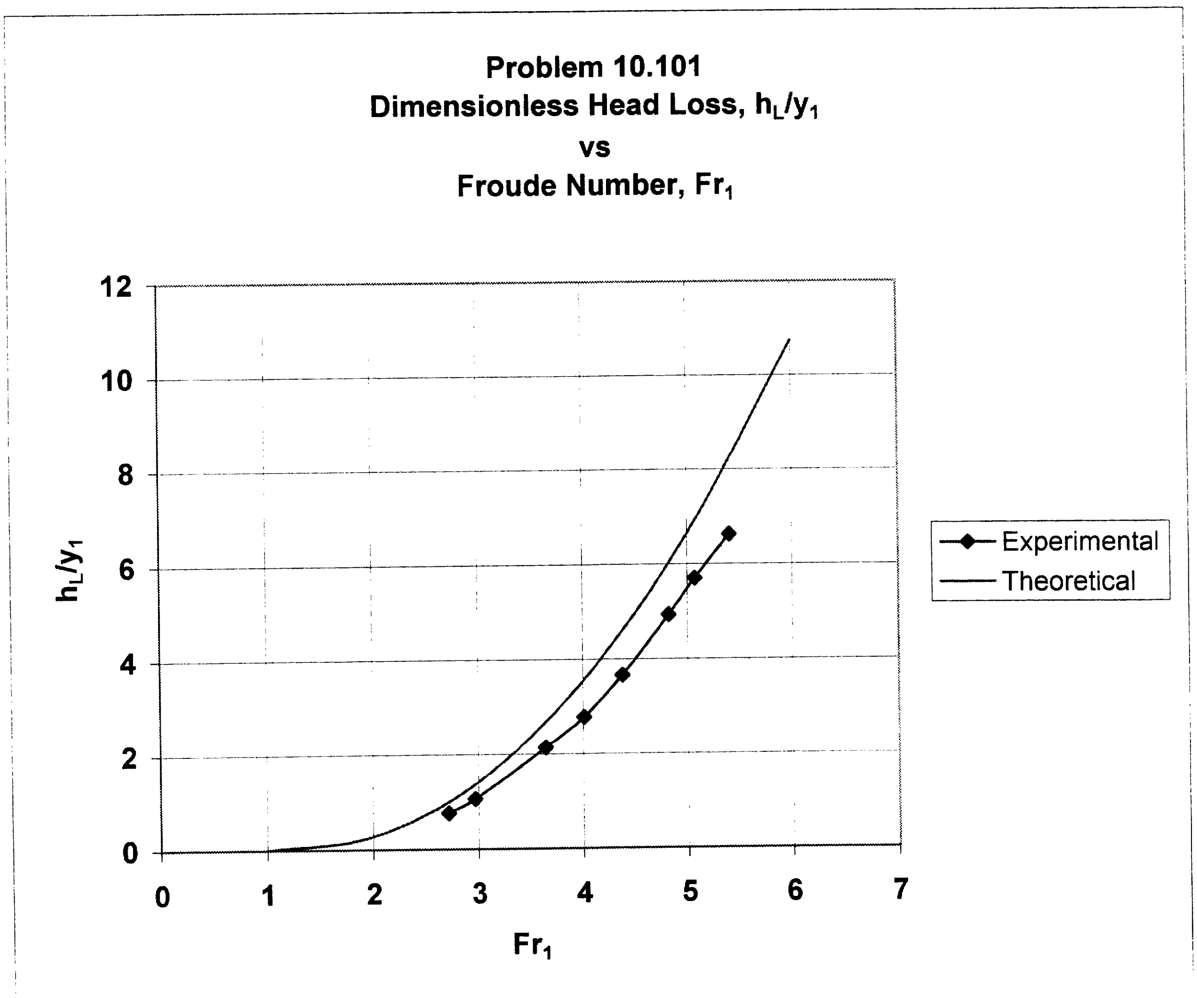
$$V_1 = [2g^*(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$h_L/y_1 = 1 - (y_2/y_1) + Fr_1^2[1 - (y_1/y_2)^2]/2$$

where

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$



11.1 As demonstrated in Video V11.1, fluid density differences in a flow may be seen with the help of a Schlieren optical system. Discuss what variables affect fluid density and how.

For an ideal gas:

$$\rho = \frac{p}{RT}$$

so changes in density, ρ, will accompany changes in pressure, p, gas composition, R, and/or temperature, T. Variations in fluid velocity and/or heating and cooling may result in pressure and temperature changes. Changes in gas composition that affect the value of the gas constant, R, will result in changes of density, ρ.

11.2

11.2 Describe briefly how a Schlieren optical visualization system (Videos V11.1 and V11.2, also Fig. 11.4) works.

A Schlieren vizualization system detects even small variations in the index of refraction of a transparent material. In fluid mechanics, this system is often used to vizualize shocks and other variations in fluid density and thus variations in index of refraction. Variations in the density and index of refraction in a fluid flow result in some light rays being "bent" around an intentional light blocker called a "knife edge" resulting in vizualization of density variations. For more information including a schematic diagram see Engineering Fluid Mechanics, a Wiley book by John Roberson and Clayton Crowe.

11.3

11.3 Are the flows shown in Videos V11.1 and V11.2 compressible? Do they involve high-speed flow velocities? Discuss.

Variations in fluid density are evident so compressible flows are involved. All of these flows are low speed except for the nozzle exit flow which is high speed.

11.4

11.4 In cities where it can get very hot, airplanes are not allowed to take off when the ambient temperature exceeds a cap level. Does this make sense?

On a hot day, the density of the air is less than on a colder day since $\rho = \frac{p}{RT}$.

The lift forces exerted on the lifting surfaces of an airplane are related to density by

$$\mathcal{L} = \frac{1}{2}\rho U^2 A C_L$$

So, for the same airplane weight, a higher speed U is required on a hot day than on a cold day. To achieve this higher speed, more runway length is needed.

11.5

11.5 Air flows steadily between two sections in a duct. At section (1), the temperature and pressure are $T_1 = 80$ °C, $p_1 = 301$ kPa(abs), and at section (2), the temperature and pressure are $T_2 = 180$ °C, $p_2 = 181$ kPa(abs). Calculate the **(a)** change in internal energy between sections (1) and (2), **(b)** change in enthalpy between sections (1) and (2), **(c)** change in density between sections (1) and (2), **(d)** change in entropy between sections (1) and (2). How would you estimate the loss of available energy between the two sections of this flow?

(a) Eq. 11.5 may be used to evaluate the change in internal energy, $\check{u}_2 - \check{u}_1$. Thus,

From problem 11.1 (a)

$$\check{u}_2 - \check{u}_1 = c_v (T_2 - T_1) = \left(717.2 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{71{,}720 \frac{J}{kg}}}$$

(b) Eq. 11.9 may be used to evaluate the change in enthalpy, $\check{h}_2 - \check{h}_1$. Thus,

From problem 11.1 (a)

$$\check{h}_2 - \check{h}_1 = c_p (T_2 - T_1) = \left(1004 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{100{,}400 \frac{J}{kg}}}$$

(c) The ideal gas equation (Eq. 11.1) may be used to evaluate the density at each section. Thus,

$$\rho_2 - \rho_1 = \frac{p_2}{RT_2} - \frac{p_1}{RT_1} = \frac{1}{R}\left(\frac{p_2}{T_2} - \frac{p_1}{T_1}\right)$$

or

$$\rho_2 - \rho_1 = \frac{1}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)} \left[\frac{\left(181{,}000 \frac{N}{m^2}\right)}{(453\,K)} - \frac{\left(301{,}000 \frac{N}{m^2}\right)}{(353\,K)} \right] = \underline{\underline{-1.58 \frac{kg}{m^3}}}$$

From Table 1.8

(d) Eq. 11.22 may be used to evaluate the change in entropy, $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = \left(1004 \frac{J}{kg \cdot K}\right) \ln \left[\frac{(453\,K)}{(353\,K)}\right] - \left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \ln \left[\frac{(181\,kPa)}{(301\,kPa)}\right]$$

or

$$s_2 - s_1 = \underline{\underline{396}} \ \frac{J}{kg \cdot K}$$

(con't)

11.5 (con't)

Since the flow involves a significant change in density, see solution to part (c) above, it is compressible and Eq. 5.108 must be used to evaluation the loss in available energy between sections (1) and (2). So from Eq. 5.108 we get

$$\text{loss} = \check{u}_2 - \check{u}_1 + \int_1^2 p\,d\left(\frac{1}{\rho}\right) - q_{\substack{net\\in}}$$

and to complete this solution we need more information so we can evaluate the integral and $q_{\substack{net\\in}}$.

11.6

11.6 Helium is compressed isothermally from 121 kPa (abs) to 301 kPa (abs). Determine the entropy change associated with this process.

Eq. 11.22 may be used to evaluate the entropy change for this isothermal process. Thus,

$$s_2 - s_1 = -R \ln \frac{p_2}{p_1} = -\left(2077 \frac{J}{kg \cdot K}\right) \ln\left(\frac{301 \text{ kPa (abs)}}{121 \text{ kPa (abs)}}\right)$$

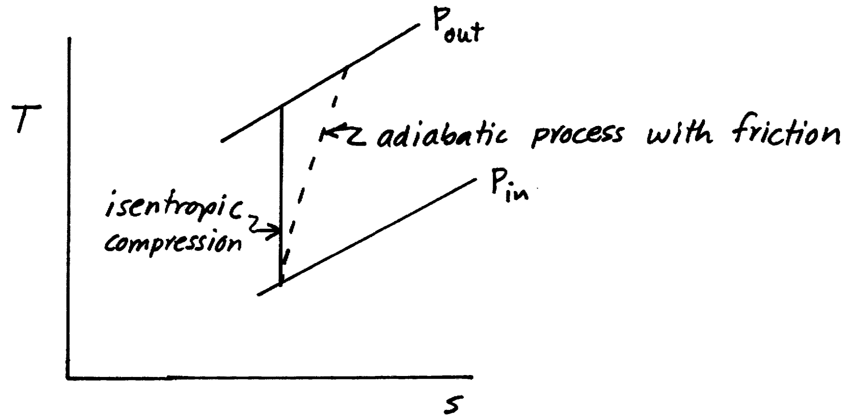
and

$$s_2 - s_1 = \underline{\underline{-1890}} \frac{J}{kg \cdot K}$$

11.7

11.7 Air at 14.7 psia and 70 °F is compressed adiabatically by a centrifugal compressor to a pressure of 100 psia. What is the minimum temperature rise possible? Explain.

The minimum temperature rise would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a minimum temperature rise.



For the isentropic process, Eq. 11.24 is valid. Thus,

$$T_{\substack{out\\minimum}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (530\,°R)\left(\frac{100\,psia}{14.7\,psia}\right)^{\frac{1.4-1}{1.4}} = 917\,°R$$

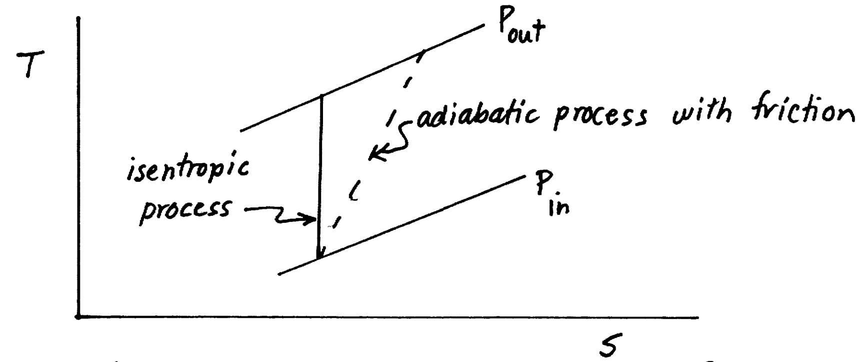
and

$$T_{\substack{out\\minimum}} - T_{in} = 917\,°R - 530\,°R = \underline{\underline{387\,°R}}$$

11.8

11.8 Methane is compressed adiabatically from 100 kPa (abs), 25 °C to 200 kPa (abs). What is the minimum compressor exit temperature possible? Explain.

The minimum compressor exit temperature would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a lower exit temperature than any actual adiabatic process between the same pressures.



For the isentropic compression, we conclude from Eq. 11.24 that

$$T_{\substack{out\\ minimum}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}}$$

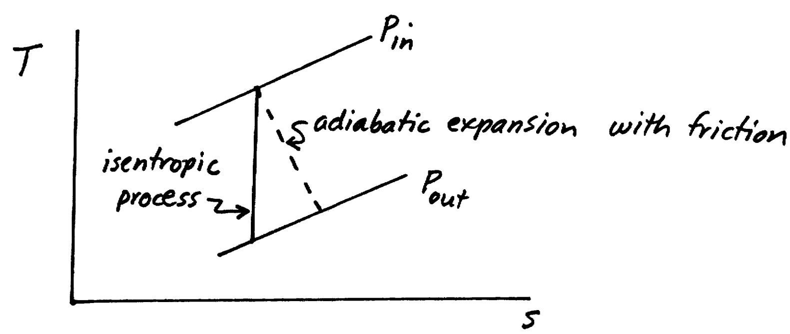
or

$$T_{\substack{out\\ minimum}} = (298\,K)\left(\frac{200\,kPa}{100\,kPa}\right)^{\frac{1.31-1}{1.31}} = \underline{\underline{351\,K}}$$

11.9

11.9 Air expands adiabatically through a turbine from a pressure and temperature of 180 psia, 1600 °R to a pressure of 14.7 psia. If the actual temperature change is 85% of the ideal temperature change, determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine.

To determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine we need first to determine the ideal temperature change across the turbine. The ideal temperature change across the turbine is associated with an adiabatic and frictionless and thus isentropic turbine expansion. The actual process involves a smaller temperature change as illustrated with the T-s diagram sketch below.



Eq. 11.24 is valid for the isentropic expansion. Thus,

$$T_{\substack{out\\ideal}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (1600\,°R)\left(\frac{14.7\,psia}{180\,psia}\right)^{\frac{1.4-1}{1.4}} = 782\,°R$$

Since

$$\left(T_{\substack{out\\actual}} - T_{in}\right) = 0.85\left(T_{\substack{out\\ideal}} - T_{in}\right)$$

then

$$T_{\substack{out\\actual}} = 0.85(782\,°R - 1600\,°R) + 1600\,°R = \underline{\underline{905\,°R}}$$

The actual enthalpy difference, $\check{h}_{\substack{out\\actual}} - \check{h}_{in}$, may be obtained with Eq. 11.9. Thus,

$$\check{h}_{\substack{out\\actual}} - \check{h}_{in} = c_p\left(T_{\substack{out\\actual}} - T_{in}\right) = \left(6006\,\frac{ft.lb}{slug.°R}\right)(905\,°R - 1600\,°R) = \underline{\underline{-4.17\times10^{6}\,\frac{ft.lb}{slug}}}$$

The actual entropy difference, $s_{\substack{out\\actual}} - s_{in}$, may be calculated with Eq. 11.22. Thus,

$$s_{\substack{out\\actual}} - s_{in} = c_p \ln\left(\frac{T_{\substack{out\\actual}}}{T_{in}}\right) - R\ln\left(\frac{P_{out}}{P_{in}}\right) = \left(6006\,\frac{ft.lb}{slug.°R}\right)\ln\left(\frac{905\,R}{1600\,°R}\right) - \left(1716\,\frac{ft.lb}{slug.°R}\right)\ln\left(\frac{14.7\,psia}{180\,psia}\right)$$

From problem 11.2 (a) — From Table 1.7

or

$$s_{\substack{out\\actual}} - s_{in} = \underline{\underline{877}}\,\frac{ft.lb.}{slug.°R}$$

11.10

11.10 An expression for the value of c_p for carbon dioxide as a function of temperature is

$$c_p = 9210 - \frac{3.71 \times 10^6}{T} + \frac{8.02 \times 10^8}{T^2}$$

where c_p is in (ft · lb)/(slug · °R) and where T is in °R. Compare the change in enthalpy of carbon dioxide using the constant value of c_p from Problem 11.2 with the change in enthalpy of carbon dioxide using the expression above, for $T_2 - T_1$ equal to **(a)** 10 °R; **(b)** 1000 °R and **(c)** 3000 °R. Set $T_1 = 540$ °R.

For constant c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.9. Thus,

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{constant} \\ c_p}} = c_p (T_2 - T_1)$$

For varying c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.8. Thus,

$$\check{h}_2 - \check{h}_1 = \int_{T_1}^{T_2} c_p \, dT = \int_{T_1}^{T_2} \left(9210 - \frac{3.71 \times 10^6}{T} + \frac{8.02 \times 10^8}{T^2}\right) dT$$

or

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying} \\ c_p}} = 9210 (T_2 - T_1) - 3.71 \times 10^6 \ln\left(\frac{T_2}{T_1}\right) - 8.02 \times 10^8 \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

(a) For $T_1 = 540$ °R and $T_2 = 550$ °R

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{constant} \\ c_p}} = \left(4897 \frac{\text{ft} \cdot \text{lb}}{\text{slug } °\text{R}}\right)(550 °\text{R} - 540 °\text{R}) = \underline{\underline{49{,}000 \frac{\text{ft} \cdot \text{lb}}{\text{slug}}}}$$

and

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying} \\ c_p}} = \left(9210 \frac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °\text{R}}\right)(550 °\text{R} - 540 °\text{R}) - \left(3.71 \times 10^6 \frac{\text{ft} \cdot \text{lb}}{\text{slug}}\right) \ln\left(\frac{550 °\text{R}}{540 °\text{R}}\right) - \left(8.02 \times 10^8 \frac{\text{ft} \cdot \text{lb} \cdot °\text{R}}{\text{slug}}\right)\left(\frac{1}{550 °\text{R}} - \frac{1}{540 °\text{R}}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying} \\ c_p}} = \underline{\underline{51{,}000}} \frac{\text{ft} \cdot \text{lb}}{\text{slug}}$$

(con't)

11.10 (con't)

(b) For $T_1 = 540\,°R$ and $T_2 = 1540\,°R$

$$(\check{h}_2 - \check{h}_1)_{\text{constant } c_p} = \left(4897 \frac{ft \cdot lb}{slug \cdot °R}\right)(1540\,°R - 540\,°R) = \underline{\underline{4.90 \times 10^6}} \frac{ft \cdot lb}{slug}$$

and

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \left(9210 \frac{ft \cdot lb}{slug \cdot °R}\right)(1540\,°R - 540\,°R)$$

$$-\left(3.71 \times 10^6 \frac{ft \cdot lb}{slug \cdot}\right) \ln\left(\frac{1540\,°R}{540\,°R}\right)$$

$$-\left(8.02 \times 10^8 \frac{ft \cdot lb \cdot °R}{slug}\right)\left(\frac{1}{1540\,°R} - \frac{1}{540\,°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \underline{\underline{6.29 \times 10^6}} \frac{ft \cdot lb}{slug}$$

(c) For $T_1 = 540\,°R$ and $T_2 = 3540\,°R$

$$(\check{h}_2 - \check{h}_1)_{\text{constant } c_p} = \left(4897 \frac{ft \cdot lb}{slug \cdot °R}\right)(3540\,°R - 540\,°R) = 1.47 \times 10^7 \frac{ft \cdot lb}{slug}$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \left(9210 \frac{ft \cdot lb}{slug \cdot °R}\right)(3540\,°R - 540\,°R) - \left(3.71 \times 10^6 \frac{ft \cdot lb}{slug}\right) \ln\left(\frac{3540\,°R}{540\,°R}\right)$$

$$-\left(8.02 \times 10^8 \frac{ft \cdot lb \cdot °R}{slug}\right)\left(\frac{1}{3540\,°R} - \frac{1}{540\,°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \underline{\underline{2.19 \times 10^7}} \frac{ft \cdot lb}{slug}$$

11.11

11.11 Does sound travel faster in the winter or summer? Why?

For air, the speed of sound from Eq. 11.36 is

$$c = \sqrt{RTk}$$

So when the ambient temperature, T, is lower as during the winter, the speed of sound, c, is lower than during the summer, when T and thus c is higher.

11.12

11.12 Estimate how fast sound travels at an altitude of 250,000 ft above the surface of the earth.

From Eq. 11.36

$$c = \sqrt{RTk}$$

At an altitude of 250,000 ft, from Table C.1 we estimate temperature to be $-88.77\,°F$ so

$$c = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(460\,°R - 88.77)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{944 \frac{ft}{s}}}$$

11.13

11.13 Determine the Mach number of a car moving in standard air at a speed of **(a)** 25 mph, **(b)** 55 mph, and **(c)** 100 mph.

The Mach number is the ratio of local velocity to speed of sound. Thus

$$Ma = \frac{V}{c}$$

For standard air

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\ {}^\circ R)(1.4)} = 1117\ \frac{ft}{s}$$

or

$$c = \left(1117\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 761.6\ mph$$

(a) For $V = 25\ mph$

$$Ma = \frac{25\ mph}{761.6\ mph} = \underline{\underline{0.0328}}$$

(b) For $V = 55\ mph$

$$Ma = \frac{55\ mph}{761.6\ mph} = \underline{\underline{0.0722}}$$

(c) For $V = 100\ mph$

$$Ma = \frac{100\ mph}{761.6\ mph} = \underline{\underline{0.131}}$$

11.15

11.15 How would you estimate the distance between you and an approaching storm front involving lightning and thunder?

One way to estimate the distance between you and approaching storm clouds, x, is to count the number of seconds, t, between seeing the lightning and hearing thunder. Using an approximate value of the speed of sound, $1145\ \frac{ft}{s}$ (see Table B.3) we can approximate distance, x from

$$x = \left(1145\ \frac{ft}{s}\right)(t)$$

11.16

11.16 If a high-performance aircraft is able to cruise at a Mach number of 3.0 at an altitude of 80,000 ft, how fast is this in: **(a)** mph; **(b)** ft/s; **(c)** m/s?

(b) With Eq. 11.46

$$V = (Ma)\,c$$

and at 80,000 ft in U.S. standard atmosphere, we have from the solution of problem 11.16

$$c = 978\ \frac{ft}{s}$$

Thus

$$V = (3.0)\left(978\ \frac{ft}{s}\right) = \underline{\underline{2930}}\ \frac{ft}{s}$$

(a) Then

$$V = \left(2930\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = \underline{\underline{2000}}\ mph$$

(c) Also

$$V = \left(2930\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = \underline{\underline{893}}\ \frac{m}{s}$$

11.17

11.17 Compare values of the speed of sound in m/s at 20°C in the following gases: **(a)** air, **(b)** carbon dioxide, **(c)** helium, **(d)** hydrogen, **(e)** methane. Give one example of why knowing this may be important.

To calculate the speed of sound in an ideal gas we can use Eq. 11.36. Thus

$$c = \sqrt{RTk}$$

With values of R and k from Table 1.7 we obtain

(a) ~~for~~ air

$$c = \sqrt{\frac{\left(286.9 \frac{J}{kg \cdot K}\right)(293\ K)(1.4)}{\left(1 \frac{J}{N \cdot m}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)}} = \underline{\underline{343 \frac{m}{s}}}$$

(b) for carbon dioxide

$$c = \sqrt{\frac{\left(188.9 \frac{J}{kg \cdot K}\right)(293\ K)(1.3)}{\left(1 \frac{J}{N \cdot m}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)}} = \underline{\underline{268}} \frac{m}{s}$$

(c) for helium

$$c = \sqrt{\frac{\left(2077 \frac{J}{kg \cdot K}\right)(293\ K)(1.66)}{\left(1 \frac{J}{N \cdot m}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)}} = \underline{\underline{1005}} \frac{m}{s}$$

(d) for hydrogen

$$c = \sqrt{\frac{\left(4124 \frac{J}{kg \cdot K}\right)(293\ K)(1.41)}{\left(1 \frac{J}{N \cdot m}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)}} = \underline{\underline{1306}} \frac{m}{s}$$

(e) for methane

$$c = \sqrt{\frac{\left(518.3 \frac{J}{kg \cdot K}\right)(293\ K)(1.31)}{\left(1 \frac{J}{N \cdot m}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)}} = \underline{\underline{446}} \frac{m}{s}$$

(con't)

11.17 (Con't)

In tests of turbomachines a gas that has a lower speed of sound is sometimes used so high-Mach-number operation can be achieved with less rotational speed and hence less stressing of the mechanical parts of the test rig.

11.18

11.38 If a person inhales helium and then talks, his or her voice sounds like "Donald Duck." Explain why this happens.

The speed of sound in helium is nearly three times the speed of sound in air so a persons voice sounds like it does when speaking through helium.

11.19

11.19 Explain how you could vary the Mach number but not the Reynolds number in air flow past a sphere. For a constant Reynolds number of 300,000, estimate how much the drag coefficient will increase as the Mach number is increased from 0.3 to 1.0.

Considering air as an ideal gas, we can express the Mach number Ma, as

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (1)$$

The Reynolds number, Re, is

$$Re = \frac{\rho V d}{\mu} = \frac{pVd}{RT\mu} \qquad (2)$$

Looking at equations 1 and 2 we reason that we can vary Ma while holding Re constant by varying V and p only with pV held constant.

From the graph below we conclude that at $Re = 3\times10^5$, the drag coefficient increases from 0.47 to 0.75 as Ma increases from 0.3 to 1.0.

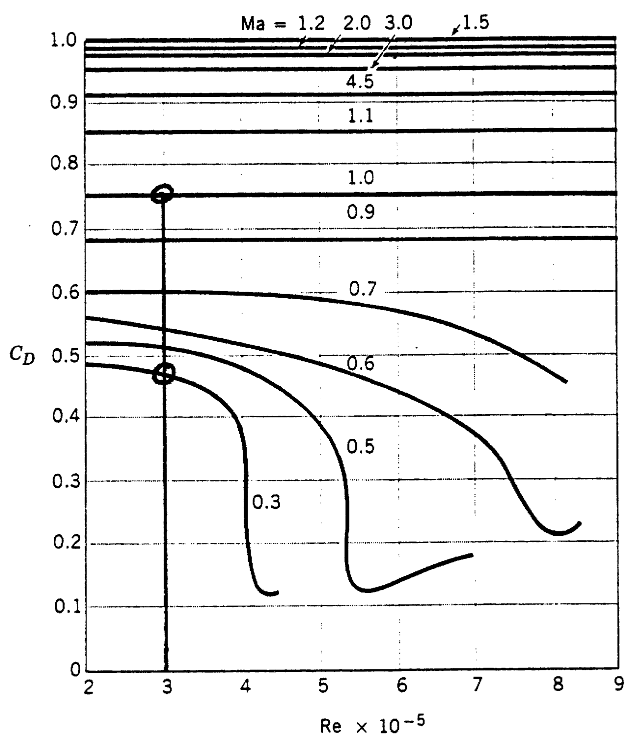


FIGURE 11.2 The variation of the drag coefficient of a sphere with Reynolds number and Mach number. (Adapted from Fig. 1.8 in Ref. 1 of Chapter 9)

11.20 The flow of an ideal gas may be considered incompressible if the Mach number is less than 0.3. Determine the velocity level in ft/s and in m/s for Ma = 0.3 in the following gases: **(a)** standard air; **(b)** hydrogen at 68 °F.

From Eq. 11.46 we have

$$V = (Ma)\,c$$

which when combined with Eq. 11.36, leads to

$$V = Ma\sqrt{RTk} \qquad (1)$$

(a) For standard air, $R = 1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}$, $k = 1.40$ and $T = 519\,^\circ R$ (from Table 1.7). Thus with Eq. 1

$$V = 0.3\sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(519\,^\circ R)(1.40)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{335\ \frac{ft}{s}}}$$

Also

$$V = \left(335\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = \underline{\underline{102\ \frac{m}{s}}}$$

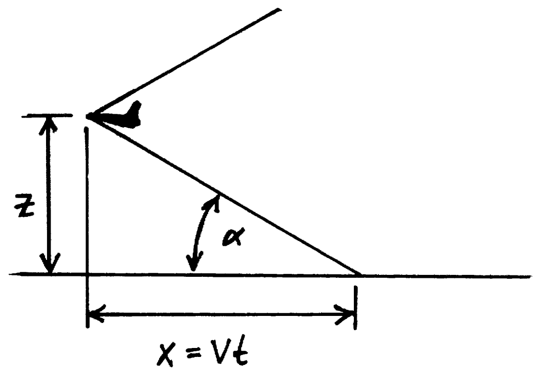
(b) For hydrogen at 68 °F, $R = 2.466\times10^4\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}$ and $k = 1.41$ from Table 1.7. Also $T = 528\,^\circ R$. Thus with Eq. 1

$$V = 0.3\sqrt{\frac{\left(2.466\times10^4\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(528\,^\circ R)(1.41)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1280\ \frac{ft}{s}}}$$

Also

$$V = \left(1280\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = \underline{\underline{390\ \frac{m}{s}}}$$

11.21 At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 10,000 ft. You hear the plane 8 s after it passes directly overhead. Using a nominal air temperature of 40 °F, estimate the Mach number and speed of the aircraft.



The Mach number is related to the angle α by Eq. 11.39. Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{V}{c} \qquad (1)$$

Also

$$\tan\alpha = \frac{z}{Vt} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{\sin\alpha}{\cos\alpha} = \frac{z\ \sin\alpha}{c\ t}$$

or

$$\alpha = \cos^{-1}\left(\frac{c\,t}{z}\right)$$

Now

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(500\ {}^\circ R)(1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1096\ \frac{ft}{s}$$

Then

$$\alpha = \cos^{-1}\left[\frac{\left(1096\ \frac{ft}{s}\right)(8\ s)}{(10000\ ft)}\right] = 28.7^\circ$$

and

$$Ma = \frac{1}{\sin 28.7^\circ} = \underline{\underline{2.08}}$$

Further

$$V = (Ma)\ c = (2.08)\left(1096\ \frac{ft}{s}\right) = \underline{\underline{2280}}\ \frac{ft}{s}$$

11.22 A schlieren photo of a bullet moving through air at 14.7 psia and 68 °F indicates a Mach cone angle of 28°. How fast was the bullet moving in: (a) m/s, (b) ft/s, (c) mph?

With Eqs. 11.39 and 11.36 we obtain

$$V = \frac{c}{\sin\alpha} = \frac{\sqrt{RTk}}{\sin\alpha}$$

For air at 14.7 psia and 68°F, $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}$ and $k = 1.40$ from Table 1.7. Also, $T = 528^\circ R$.

(b) Thus,

$$V = \frac{\sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \dfrac{(528^\circ R)(1.40)}{\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)}}}{\sin 28^\circ} = \underline{\underline{2400 \frac{ft}{s}}}$$

(a) or

$$V = \left(2400 \frac{ft}{s}\right)\left(0.3048 \frac{m}{ft}\right) = \underline{\underline{732 \frac{m}{s}}}$$

(c) and

$$V = \left(2400 \frac{ft}{s}\right) \frac{\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = \underline{\underline{1636 \text{ mph}}}$$

11.23

11.23 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.23 Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

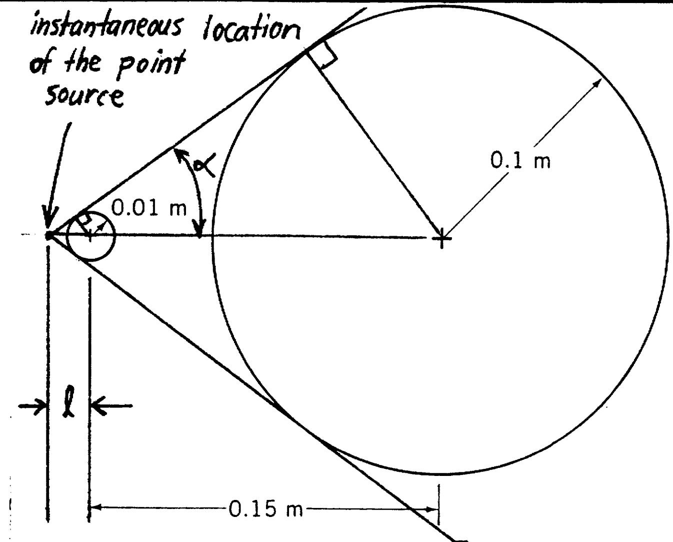


FIGURE P11.23

The Mach number associated with the motion of the point source involved in the sketch above is easily obtained with Eq. 11.39 as shown below.

$$Ma = \frac{1}{\sin\alpha}$$

From the sketch above we note that

$$\sin\alpha = \frac{0.01\ m}{l} = \frac{0.1\ m}{0.15\ m + l}$$

Thus

$$(0.01\ m)(0.15\ m + l) = (0.1\ m)\, l$$

or

$$l = \frac{(0.01\ m)(0.15\ m)}{(0.09\ m)} = \underline{\underline{0.0167\ m}}$$

and

$$\sin\alpha = \frac{0.01\ m}{0.0167\ m} = 0.599$$

Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{1}{0.599} = \underline{\underline{1.67}}$$

11.24

11.24 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.24. Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

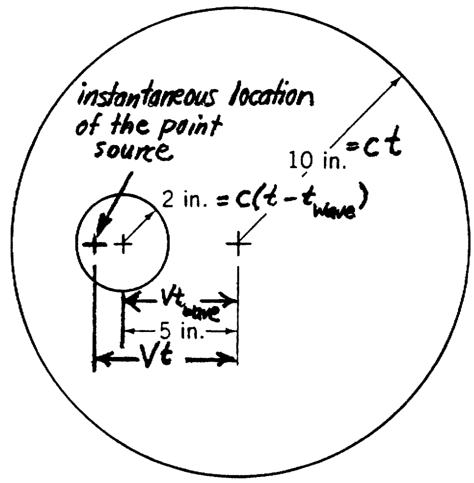


FIGURE P11.24

To determine the Mach number, Ma, we use

$$Ma = \frac{Vt_{wave}}{c\,t_{wave}} \qquad (1)$$

However, from the sketch above we have

$$c(t - t_{wave}) = 2 \text{ in.} = ct - ct_{wave} = 10 \text{ in.} - ct_{wave}$$

Thus,

$$ct_{wave} = 10 \text{ in.} - 2 \text{ in.} = 8 \text{ in.}$$

and with Eq. 1

$$Ma = \frac{5 \text{ in.}}{8 \text{ in.}} = \underline{\underline{0.625}}$$

Also

$$Ma = \frac{Vt}{ct} = \frac{Vt}{10 \text{ in.}} = 0.625$$

Thus,

$$Vt = (0.625)(10 \text{ in.}) = \underline{\underline{6.25 \text{ in.}}}$$

11.25

11.25 How much time in seconds will it take for the "bang" of a firecracker exploding to be heard after the blast from 200 yards away on a standard day (see Video V11.5)?

We can use the speed of sound in standard air, $1117 \frac{ft}{s}$, to get the time estimate sought, t, with

$$t = \frac{x}{c} = \frac{(200\,yds)\left(3\frac{ft}{yd}\right)}{\left(1117\frac{ft}{s}\right)} = 0.5\,s$$

11.26

11.26 Sound waves are very small amplitude pressure pulses that travel at the "speed of sound." Do very large amplitude waves such as a blast wave caused by an explosion (see Video V11.5) travel less than, equal to, or greater than the speed of sound? Explain.

The speed of sound is the speed at which an infinitesimal pressure disturbance travels through a fluid and it represents the minimum speed of this disturbance. Finite pressure disturbances travel faster than sound waves because the larger pressure difference acts as a driver of faster movement.

11.27

11.27 Starting with the enthalpy form of the energy equation (Eq. 5.69) show that for isentropic flows, the stagnation temperature remains constant.

starting with Eq. 5.69 we have

$$\dot{m}\left[\check{h}_{out} - \check{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{shaft\\net\ in}}$$

For isentropic flow the entropy remains constant and $\dot{Q}_{net} = 0$. Stagnation enthalpy is defined as

$$\check{h}_0 = \check{h} + \frac{V^2}{2}$$

So, for negligible change in elevation (okay for gases) and no shaft work, $\dot{W}_{shaft}$, then

$\check{h}_0$ remains constant.

11.28

11.28 Explain how fluid pressure varies with cross section area change for the isentropic flow of an ideal gas when the flow is **(a)** subsonic; **(b)** supersonic.

With the help of Eq. 11.47 we can comment on how pressure varies with area change in an isentropic flow. From Eq. 11.47 we obtain

$$dp = \frac{\rho V^2}{(1-Ma^2)} \frac{dA}{A} \qquad (1)$$

(a) For subsonic flow, Eq. 1 suggests that changes of p follow changes of A. If A increases, p increases and vice versa.

(b) For supersonic flow, Eq. 1 suggests that changes of p are opposite to changes of A. If A increases, p decreases and vice versa.

11.29

11.29 For any ideal gas, prove that the slope of constant pressure lines on a temperature–entropy diagram is positive and that higher pressure lines are above lower pressure lines.

From the second Tds equation (Eq. 11.18) we note that for a constant pressure line

$$\frac{d\check{h}}{ds} = T$$

and since for an ideal gas Eq. 11.7 is valid, we have

$$d\check{h} = c_p \, dT$$

and thus

$$\frac{dT}{ds} = \frac{T}{c_p} \qquad (1)$$

With Eq. 1 we conclude that the slope of a constant pressure line on a temperature - entropy diagram is positive.

Further, from Eq. 11.24 we conclude that

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{k}{k-1}\right)}$$

for any isentropic process and thus higher pressure lines are above lower pressure lines in temperature - entropy diagrams.

11.30

11.30 Determine the critical pressure and temperature ratios for: **(a)** air; **(b)** carbon dioxide; **(c)** helium; **(d)** hydrogen; **(e)** methane; **(f)** nitrogen; **(g)** oxygen.

The critical pressure ratio and the critical temperature ratio for an ideal gas are, from Eqs. 11.61 and 11.63

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (1)$$

and

$$\frac{T^*}{T_0} = \frac{2}{k+1} \qquad (2)$$

(a) For air, $k = 1.40$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.40+1}\right)^{\frac{1.40}{1.40-1}} = \underline{\underline{0.5283}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.40+1} = \underline{\underline{0.8333}}$$

(b) For carbon dioxide, $k = 1.30$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.30+1}\right)^{\frac{1.30}{1.30-1}} = \underline{\underline{0.5457}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.30+1} = \underline{\underline{0.8696}}$$

(c) For helium, $k = 1.66$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.66+1}\right)^{\frac{1.66}{1.66-1}} = \underline{\underline{0.4881}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.66+1} = \underline{\underline{0.7519}}$$

(con't)

11.30 (con't)

(d) For hydrogen, $k = 1.41$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.41+1}\right)^{\frac{1.41}{1.41-1}} = \underline{\underline{0.5266}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.41+1} = \underline{\underline{0.8299}}$$

(e) For methane, $k = 1.31$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.31+1}\right)^{\frac{1.31}{1.31-1}} = \underline{\underline{0.5439}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.31+1} = \underline{\underline{0.8658}}$$

(f) For nitrogen, $k = 1.40$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.40+1}\right)^{\frac{1.40}{1.40-1}} = \underline{\underline{0.5283}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.40+1} = \underline{\underline{0.8333}}$$

(g) For oxygen, $k = 1.40$ from Table 1.7. Thus,

$$\frac{p^*}{p_0} = \left(\frac{2}{1.40+1}\right)^{\frac{1.40}{1.40-1}} = \underline{\underline{0.5283}}$$

and

$$\frac{T^*}{T_0} = \frac{2}{1.40+1} = \underline{\underline{0.8333}}$$

11.31

11.31 Air flows steadily and isentropically from standard atmospheric conditions to a receiver pipe through a converging duct. The cross section area of the throat of the converging duct is 0.05 ft^2. Determine the mass flowrate through the duct if the receiver pressure is **(a)** 10 psia; **(b)** 5 psia. Sketch temperature–entropy diagrams for situations (a) and (b). Verify results obtained with values from the appropriate graph in Appendix D with calculations involving ideal gas equations.

This problem is similar to Example 11.5

The mass flowrate is obtained at the throat with Eq. 11.40. Thus,

$$\dot{m} = \rho_{th} A_{th} V_{th} \qquad (1)$$

The throat density can be obtained with Eq. 11.60. Thus,

$$\rho_{th} = \rho_0 \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \right]^{\frac{1}{k-1}} \qquad (2)$$

To determine the throat Mach number we use Eq. 11.59. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{k-1}\right) \left[\left(\frac{p_0}{p_{th}}\right)^{\frac{k-1}{k}} - 1 \right]} \qquad (3)$$

The critical throat pressure is obtained with Eq. 11.61. Thus,

$$p_{th}^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = (14.7\text{ psia}) \left(\frac{2}{1.40+1}\right)^{\frac{1.4}{1.4-1}} = 7.76\text{ psia}$$

If the receiver pressure, p_{re}, is greater than or equal to p_{th}^*, then $p_{th} = p_{re}$ and the flow is not choked. If $p_{re} < p_{th}^*$, then $p_{th} = p_{th}^*$ and the flow is choked.

The velocity at the throat is obtained with Eqs. 11.36 and 11.46 combined to yield

$$V_{th} = Ma_{th} \sqrt{R T_{th} k} \qquad (4)$$

where T_{th} is obtained with Eq. 11.56. Thus,

$$T_{th} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \qquad (5)$$

(con't)

11.31 (con't)

(a) For $p_{re} = 10$ psia $> p_{th}^* = 7.76$ psia, $p_{th} = 10$ psia and we use Eq. 3 to calculate the throat Mach number. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{1.40-1}\right)\left[\left(\frac{14.7\text{ psia}}{10\text{ psia}}\right)^{\frac{1.40-1}{1.40}} - 1\right]} = 0.7628$$

From Eq. 2 we obtain

$$\rho_{th} = \left(2.38\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2}\right]^{\frac{1}{1.40-1}} = 1.807\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}$$

From Eq. 5 we get

$$T_{th} = \frac{519\,^\circ R}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2} = 464.9\,^\circ R$$

and with Eq. 4

$$V_{th} = (0.7628)\sqrt{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ R}\right)\frac{(1.40)(464.9\,^\circ R)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 806.2\,\frac{\text{ft}}{\text{s}}$$

With Eq. 1 we obtain

$$\dot{m} = \left(1.807\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}\right)\left(0.05\text{ ft}^2\right)\left(806.2\,\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.0728}}\,\frac{\text{slug}}{\text{s}}$$

Alternatively, using Fig. D.1 with

$$\frac{p_{th}}{p_0} = \frac{10\text{ psia}}{14.7\text{ psia}} = 0.68$$

The value of Ma_{th} is

$$Ma_{th} = 0.76$$

For $Ma_{th} = 0.76$, we get from Fig. D.1

$$T_{th} = (0.9\quad)T_0 = (0.9\quad)(519\,^\circ R) = 467\,^\circ R$$

Then with Eq. 4

$$V_{th} = 0.76\sqrt{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ R}\right)\frac{(1.40)(467\,^\circ R)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 805\,\frac{\text{ft}}{\text{s}}$$

(con't)

For $Ma_{th} = 0.76$ we get from Fig. D.1

$$\rho_{th} = 0.76086\rho_o = (0.76)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right) = 1.8\times10^{-3}\frac{slug}{ft^3}$$

Now, with Eq. 1 we obtain

$$\dot{m} = \left(1.8\times10^{-3}\frac{slug}{ft^3}\right)(0.05\ ft^2)\left(805\frac{ft}{s}\right) = \underline{\underline{0.076}}\ \frac{slug}{ft^3}$$

(b) For $p_{re} = 5\,psia < p^* = 7.76\,psia$, $p_{th} = 7.76\,psia$ and $Ma_{th} = 1.0$. From Eq. 2,

$$\rho_{th} = \left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)}\right]^{\frac{1}{1.40-1}} = 1.509\times10^{-3}\frac{slug}{ft^3}$$

From Eq. 5 we obtain

$$T_{th} = \frac{519\,°R}{1+\left(\frac{1.40-1}{2}\right)} = 432.5\,°R$$

and with Eq. 4

$$V_{th} = \sqrt{\frac{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(432.5\,°R)}{\left(1\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1019\frac{ft}{s}$$

With Eq. 1 we obtain

$$\dot{m} = \left(1.509\times10^{-3}\frac{slug}{ft^3}\right)(0.05\ ft^2)\left(1019\frac{ft}{s}\right) = 0.0769\frac{slug}{s}$$

Alternatively, from Fig. D.1 for $Ma = 1.0$

$$T_{th} = (0.83)(519\,°R) = 431\,°R$$

and

$$\rho_{th} = (0.64)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right) = 1.52\times10^{-3}\frac{slus}{ft^3}$$

Then with Eq. 4

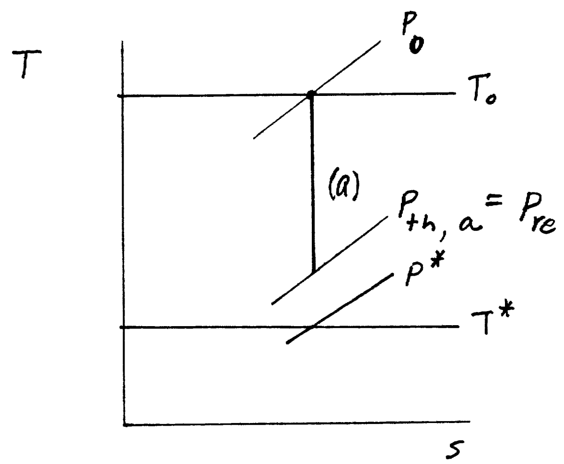
$$V_{th} = \sqrt{\frac{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(431\,°R)}{\left(1\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1020\,ft/s$$

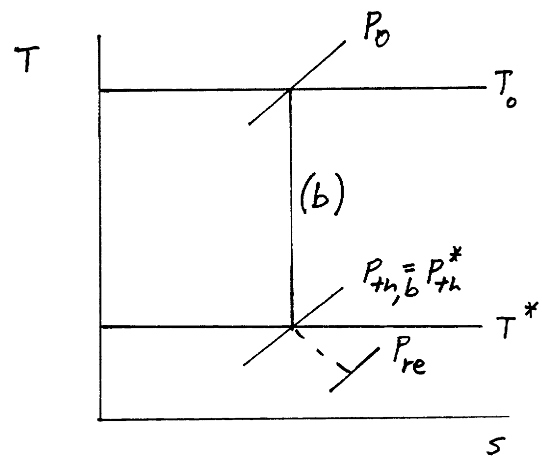
(con't)

11.31 (con't)

and with Eq. 1 we obtain

$$\dot{m} = \left(1.52 \times 10^{-3} \frac{slug}{ft^3}\right)\left(0.05\ ft^2\right)\left(1020 \frac{ft}{s}\right) = 0.078 \frac{slug}{s}$$





11.32

11.32 Helium at 68 °F and 14.7 psia in a large tank flows steadily and isentropically through a converging nozzle to a receiver pipe. The cross section area of the throat of the converging passage is 0.05 ft². Determine the mass flowrate through the duct if the receiver pressure is **(a)** 10 psia; **(b)** 5 psia. Sketch temperature–entropy diagrams for situations (a) and (b).

This problem is similar to Example 11.5, except helium is involved. The mass flowrate is obtained at the throat with Eq. 11.40. Thus,

$$\dot{m} = \rho_{th} A_{th} V_{th} \qquad (1)$$

The throat density can be obtained with Eq. 11.60. Thus,

$$\rho_{th} = \rho_0 \left[\frac{1}{1+\left(\frac{k-1}{2}\right) Ma_{th}^2} \right]^{\frac{1}{k-1}} \qquad (2)$$

To determine the throat Mach number we use Eq. 11.59. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{k-1}\right)\left[\left(\frac{p_0}{p_{th}}\right)^{\frac{k-1}{k}} - 1\right]} \qquad (3)$$

The critical throat pressure is obtained with Eq. 11.61. Thus,

$$p_{th}^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad \left(\frac{2}{1.66+1}\right)^{\frac{1.66}{1.66-1}} = \qquad 7.175 \text{ psia}$$

If the receiver pressure, p_{re}, is greater than or equal to p_{th}^*, then $p_{th} = p_{re}$ and the flow is not choked. If $p_{re} < p_{th}^*$, then $p_{th} = p_{th}^*$ and the flow is choked.

The velocity at the throat is obtained with Eqs. 11.36 and 11.46 combined to yield

$$V_{th} = Ma_{th} \sqrt{R T_{th} k} \qquad (4)$$

where T_{th} is obtained with Eq. 11.56. Thus,

$$T_{th} = \frac{T_0}{1+\left(\frac{k-1}{2}\right) Ma_{th}^2} \qquad (5)$$

(Con't)

(a) For $p_{re} = 10\ psia > p_{th}^* = 7.175\ psia$, $p_{th} = 10\,psia$ and we use Eq. 3 to calculate the throat Mach number. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{1.66-1}\right)\left[\left(\frac{14.7\ psia}{10\ psia}\right)^{\frac{1.66-1}{1.66}} - 1\right]} = 0.7082$$

We use the ideal gas equation of state (Eq. 11.1) to obtain ρ_0. Thus,

$$\rho_0 = \frac{p_0}{RT_0} = \frac{(14.7\ psia)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.242 \times 10^4\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(528\,{}^\circ R)} = 3.228 \times 10^{-4}\ \frac{slug}{ft^3}$$

From Eq. 2 we obtain

$$\rho_{th} = \left(3.228 \times 10^{-4}\ \frac{slug}{ft^3}\right)\left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)(0.7082)^2}\right]^{\frac{1}{1.66-1}} = 2.56 \times 10^{-4}\ \frac{slug}{ft^3}$$

From Eq. 5 we get

$$T_{th} = \frac{528\,{}^\circ R}{1 + \left(\frac{1.66-1}{2}\right)(0.7082)^2} = 453\,{}^\circ R$$

and with Eq. 4

$$V_{th} = (0.7082)\sqrt{\left(1.242 \times 10^4\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)\frac{(1.66)(453\,{}^\circ R)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 2164\ \frac{ft}{s}$$

With Eq. 1 we obtain

$$\dot{m} = \left(2.56 \times 10^{-4}\ \frac{slug}{ft^3}\right)\left(0.05\ ft^2\right)\left(2164\ \frac{ft}{s}\right) = \underline{\underline{0.0277\ \frac{slug}{s}}}$$

(b) For $p_{re} = 5\ psia < p_{th}^* = 7.175\ psia$, $p_{th} = 7.175\ psia$ and $Ma_{th} = 1.0$.

From Eq. 2 we obtain

$$\rho_{th} = \left(3.228 \times 10^{-4}\ \frac{slug}{ft^3}\right)\left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)}\right]^{\frac{1}{1.66-1}} = 2.096 \times 10^{-4}\ \frac{slug}{ft^3}$$

From Eq. 5 we get,

$$T_{th} = \frac{528\,{}^\circ R}{1 + \left(\frac{1.66-1}{2}\right)} = 397\,{}^\circ R$$

(con't)

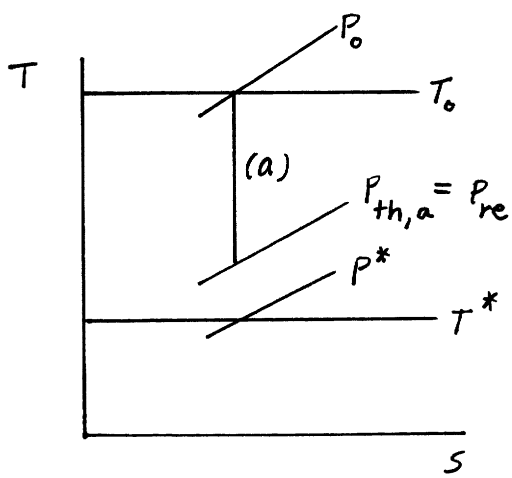
11.32 (con't)

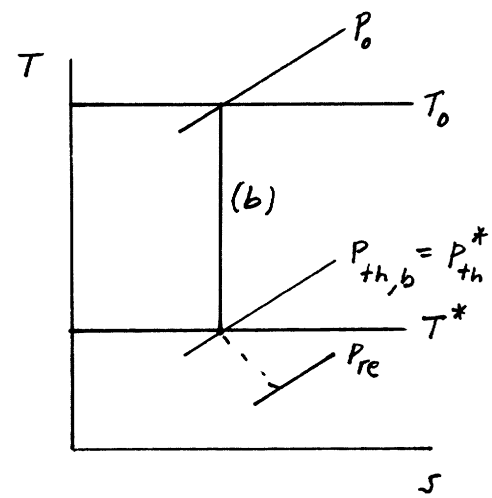
and with Eq. 4

$$V_{th} = \sqrt{\left(1.242 \times 10^4 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \frac{(1.66)(397^\circ R)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 2861 \frac{ft}{s}$$

With Eq. 1 we obtain

$$\dot{m} = \left(2.096 \times 10^{-4} \frac{slug}{ft^3}\right)\left(0.05\ ft^2\right)\left(2861 \frac{ft}{s}\right) = \underline{\underline{0.03}}\ \frac{slug}{s}$$





11.33 What is the static pressure to stagnation pressure ratio associated with the following motion in standard air: **(a)** a runner moving at the rate of 20 mph; **(b)** a cyclist moving at the rate of 40 mph; **(c)** a car moving at the rate of 65 mph; **(d)** an airplane moving at the rate of 500 mph.

With a value of Mach number calculated with

$$Ma = \frac{V}{c} \qquad (1)$$

we can calculate $\frac{p}{p_0}$ with

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]^{\frac{k}{k-1}} \qquad (11.59)$$

For c we use

$$c = \sqrt{RTk} = \frac{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\,{}^\circ R)(1.40)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)} = 1117 \frac{ft}{s}$$

or

$$c = \frac{\left(1117 \frac{ft}{s}\right)\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = 761.6 \text{ mph}$$

(a) For $V = 20$ mph

$$Ma = \frac{20 \text{ mph}}{761.6 \text{ mph}} = 0.0262$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.0262)^2}\right]^{\frac{1.4}{1.4-1}} = \left[\frac{1}{1+(0.2)(0.0262)^2}\right]^{3.5} = 0.9995$$

(b) For $V = 40$ mph

$$Ma = \frac{40 \text{ mph}}{761.6 \text{ mph}} = 0.0525$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0525)^2}\right]^{3.5} = 0.998$$

(c) For $V = 65$ mph

$$Ma = \frac{65 \text{ mph}}{761.6 \text{ mph}} = 0.0854$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0854)^2}\right]^{3.5} = 0.9949$$

(d) For $V = 500$ mph

$$Ma = \frac{500 \text{ mph}}{761.6 \text{ mph}} = 0.656$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.656)^2}\right]^{3.5} = 0.749$$

11.34

11.34 The static pressure to stagnation pressure ratio at a point in an ideal gas flow field is measured with a Pitot-static probe as being equal to 0.6. The stagnation temperature of the gas is 20 °C. Determine the flow speed in m/s and Mach number if the gas is **(a)** air; **(b)** carbon dioxide; **(c)** hydrogen.

(a) To determine the flow speed and Mach number having been given the static pressure to stagnation pressure ratio, $\frac{p}{p_0}$, and stagnation temperature, T_0, for air we enter Fig. D.1 with the given value of $\frac{p}{p_0}$ and read the corresponding value of Ma. Thus with $\frac{p}{p_0} = 0.6$, the corresponding value in Fig. D.1 is

$$Ma = \underline{\underline{0.89}}$$

For $Ma = 0.89$, Fig. D.1 gives

$$\frac{T}{T_0} = 0.86$$

and thus

$$T = \left(\frac{T}{T_0}\right) T_0 = (0.86)(293\,K) = 252\ K$$

Then

$$V = (Ma)\,c = Ma\sqrt{RTk} = 0.89\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(252\ K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}}$$

or

$$V = \underline{\underline{283}}\ \frac{m}{s}$$

(b) For carbon dioxide, we use Eq. 11.59 to determine Ma knowing $\frac{p}{p_0}$. Thus we have

$$Ma = \sqrt{\left[\frac{1}{\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}} - 1\right]\left(\frac{2}{k-1}\right)} = \sqrt{\left[\frac{1}{(0.6)^{\frac{1.3-1}{1.3}}} - 1\right]\left(\frac{2}{1.3-1}\right)}$$

or

$$Ma = \underline{\underline{0.913}}$$

(con't)

11.34 (con't)

To determine V we use

$$V = Ma\sqrt{RTk}$$

For obtaining T we use Eq. 11.56. Thus, we have

$$T = T_0\left(\frac{1}{1+\frac{k-1}{2}Ma^2}\right) = (293\,K)\left[\frac{1}{1+\left(\frac{1.3-1}{2}\right)(0.913)^2}\right] = 260.4\,K$$

and

$$V = (0.913)\sqrt{\frac{\left(188.9\ \frac{N\cdot m}{kg\cdot K}\right)(260.4\,K)(1.3)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{231}}\ \frac{m}{s}$$

(c) For hydrogen, we proceed as we did for carbon dioxide above. Thus,

$$Ma = \sqrt{\left[\frac{1}{(0.6)^{\frac{1.41-1}{1.41}}}-1\right]\left(\frac{2}{1.41-1}\right)} = \underline{\underline{0.884}}$$

Also,

$$T = (293\,K)\left[\frac{1}{1+\left(\frac{1.41-1}{2}\right)(0.884)^2}\right] = 252.5\,K$$

Then

$$V = (0.884)\sqrt{\frac{\left(4124\ \frac{N\cdot m}{kg\cdot K}\right)(252.5\,K)(1.41)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{1070}}\ \frac{m}{s}$$

11.35

11.35 The stagnation pressure and temperature of air flowing past a probe are 120 kPa (abs) and 100 °C, respectively. The air pressure is 80 kPa (abs). Determine the air speed and Mach number considering the flow to be **(a)** incompressible; **(b)** compressible.

(a) Assuming incompressible flow we use Bernoulli's equation (Eq. 3.7) to connect the static and stagnation states and get

$$V = \sqrt{\frac{2(p_0 - p)}{\rho_0}} \qquad (1)$$

With the ideal gas equation of state (Eq. 1) we obtain

$$\rho_0 = \frac{p_0}{RT_0} \qquad (2)$$

and combining Eqs. 1 and 2 we obtain

$$V = \sqrt{\frac{2(p_0 - p)RT_0}{p_0}}$$

or

$$V = \sqrt{\frac{2\,[120\text{ kPa(abs)} - 80\text{ kPa(abs)}]\left(286.9\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(373\text{ K})}{[120\text{ kPa(abs)}]\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = \underline{\underline{267\,\frac{\text{m}}{\text{s}}}}$$

For Mach number we need

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (3)$$

To determine T we use the equation of motion (Eq. 11.54) to obtain

$$T = T_0 - \frac{V^2(k-1)}{2kR} = 373\text{K} - \frac{\left(267\,\frac{\text{m}}{\text{s}}\right)^2(1.4-1)}{2(1.4)\left(286.9\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left(1\,\frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)$$

or $T = 337.5\text{ K}$

(con't)

11.35 (con't)

With Eq. 3 we obtain

$$Ma = \frac{267\ \frac{m}{s}}{\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(337.5\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}}} = \underline{\underline{0.725}}$$

(b) For compressible flow

$$\frac{p}{p_0} = \frac{80\ kPa(abs)}{120\ kPa\ (abs)} = 0.67$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.78}}$$

Also from Fig. D.1 we read

$$\frac{T}{T_0} = 0.89$$

and thus

$$T = (0.89)(373\ K) = 332\ K$$

Thus,

$$V = Ma\sqrt{RTk} = (0.78)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(332\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{285}}\ \frac{m}{s}$$

11.36

11.36 The stagnation pressure indicated by a Pitot tube mounted on an airplane in flight is 45 kPa (abs). If the aircraft is cruising in standard atmosphere at an altitude of 10,000 m, determine the speed and Mach number involved.

For 10,000 m standard atmosphere we get from Table C.2

$$p = 26.50 \ \text{kPa (abs)}$$

and

$$T = 223.1 \text{K}$$

Thus

$$\frac{p}{p_0} = \frac{26.50 \ \text{kPa (abs)}}{45 \ \text{kPa (abs)}} = 0.59$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.9}}$$

Thus

$$V = (Ma)c = Ma\sqrt{RTk} = (0.9)\sqrt{\frac{\left(286.9 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(223.1\text{K})(1.4)}{\left(1 \frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}}$$

or

$$V = \underline{\underline{269}} \ \frac{\text{m}}{\text{s}}$$

11.38

*11.38 An ideal gas enters subsonically and flows isentropically through a choked converging-diverging duct having a circular cross-section area A that varies with axial distance from the throat, x, according to the formula

$$A = 0.1 + x^2$$

where A is in square feet and x is in feet. For this flow situation, sketch the side view of the duct and graph the variation of Mach number, static temperature to stagnation temperature ratio, T/T_0, and static pressure to stagnation pressure ratio, p/p_0, through the duct from $x = -0.6$ ft to $x = +0.6$ ft. Also show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Consider the gas as being helium (use $0.051 \leq \text{Ma} \leq 5.193$). Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see Video V11.4) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is like Example 11.8.

Since

$$A = \pi r^2$$

and

$$A = 0.1 + x^2$$

then

$$r = \frac{0.1 + x^2}{\pi} \qquad (1)$$

With Eq. 1 we can determine r values corresponding to values of x. The are summarized in the graph and tables

duct is choked,

$$A^* = 0.1 \text{ ft}^2$$

and

$$\frac{A}{A^*} = 1 + \frac{x^2}{0.1} \qquad (2)$$

With Eq. 2 we can determine $\frac{A}{A^*}$ values corresponding to values of x. These $\frac{A}{A^*}$ values are tabulated

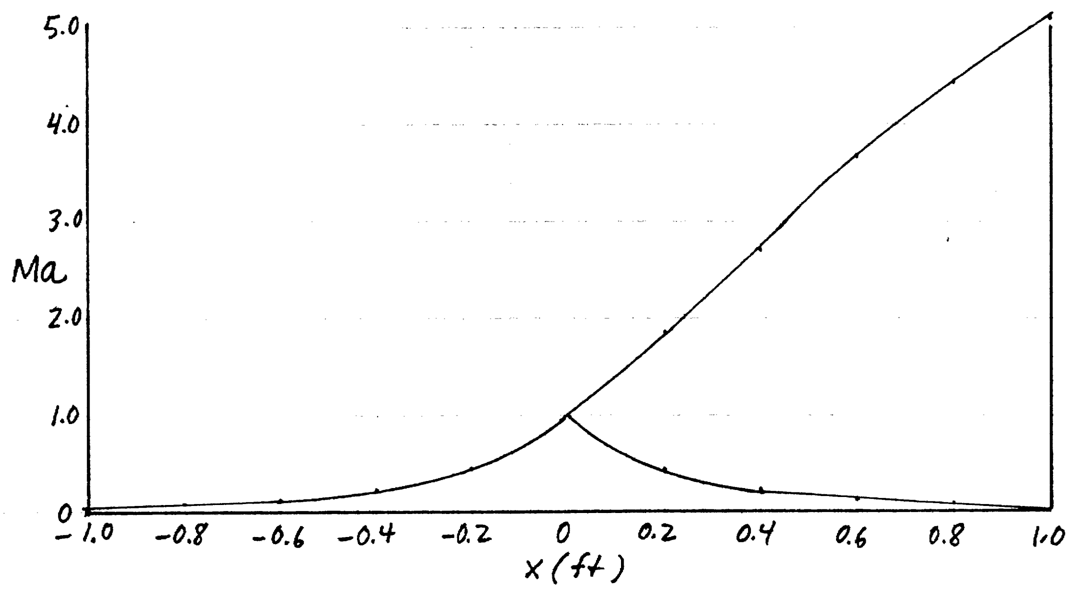
For helium we enter program ISENTROP with $k=1.66$ and with Ma values within the range specified in the problem statement and obtain values of $\frac{A}{A^*}$ (Eq. 11.71), x (Eq. 2), $\frac{T}{T_0}$ (Eq. 11.56) and $\frac{p}{p_0}$ (Eq. 11.59). These values are tabulated and graphed on pages that follow.

(con't)

11.38 (con't)

From program ISENTROP with $k = 1.66$; x from Eq. 2

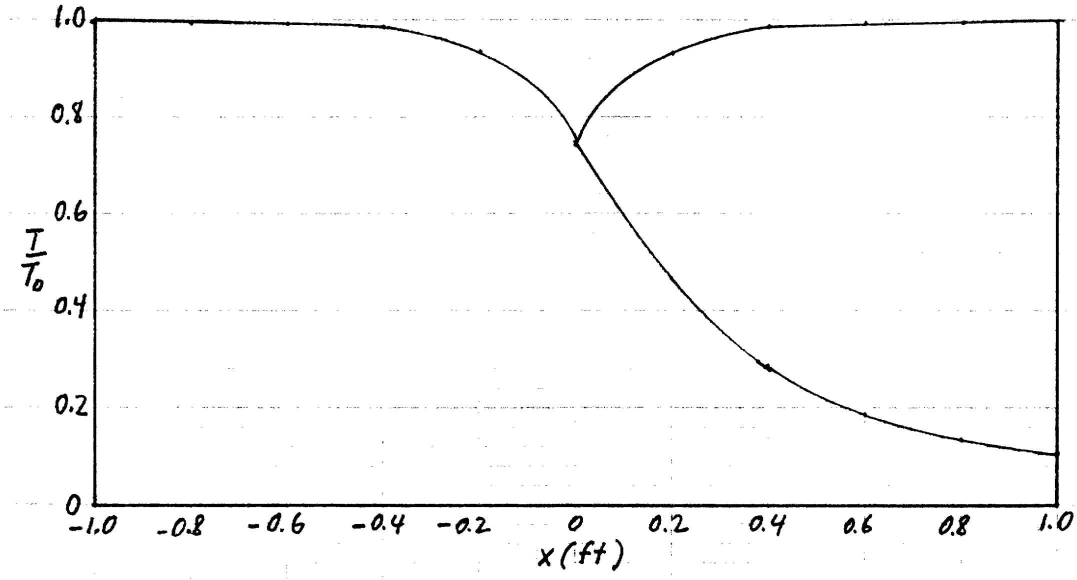
Ma	$\frac{A}{A^*}$	x (ft)	$\frac{T}{T_0}$	$\frac{P}{P_0}$	state
		subsonic solution			
0.051	11.06	±1.00	0.99914	0.99784	a, c
0.076	7.43	±0.80	0.99809	0.99522	
0.123	4.62	±0.60	0.99503	0.98755	
0.223	2.61	±0.40	0.98385	0.95989	
0.460	1.40	±0.20	0.93473	0.84386	
1.00	1.00	0	0.75188	0.48808	b
		supersonic solution			
1.855	1.40	0.20	0.46827	0.14833	
2.778	2.60	0.40	0.28195	0.04141	
3.647	4.60	0.60	0.18556	0.01446	
4.448	7.40	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	d



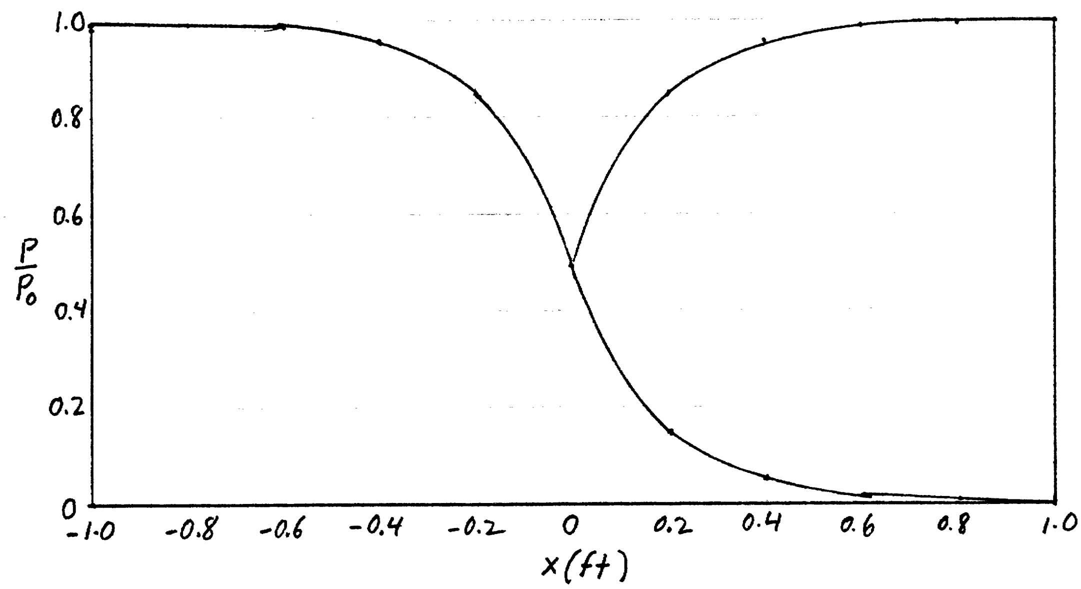
Variation of Mach number for helium

(con't)

11.38 (con't)



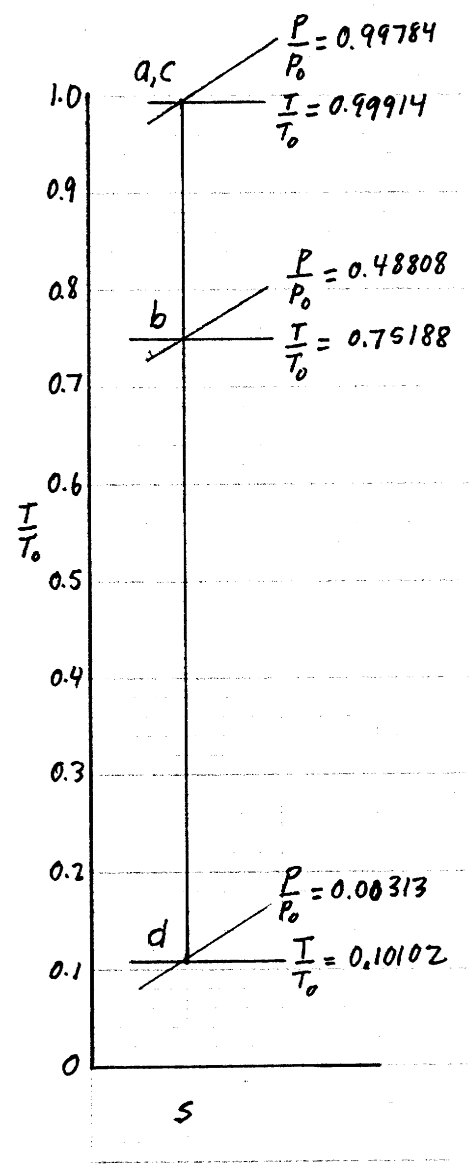
Variation of static temperature to stagnation temperature ratio for helium



Variation of static pressure to stagnation pressure ratio for helium

(con't)

11.38 (con't)

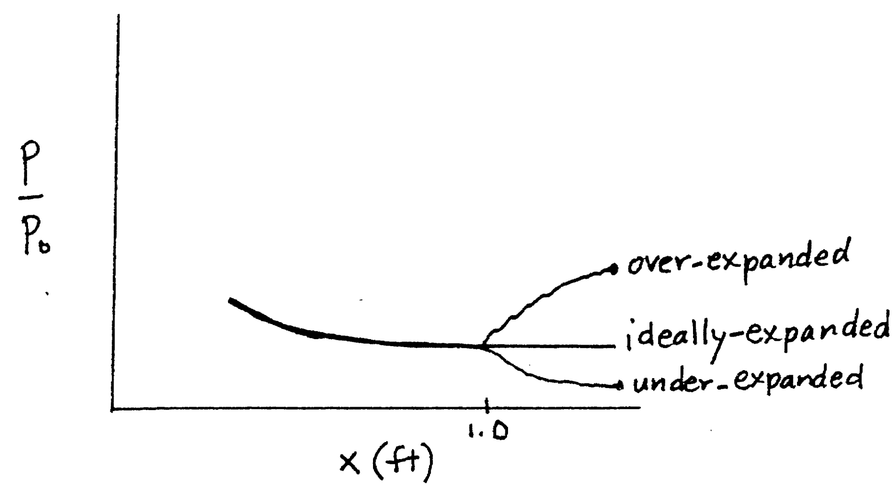


Temperature entropy diagram for helium

(con't)

11.38 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit. This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.39

*11.39 An ideal gas enters supersonically and flows isentropically through the choked converging-diverging duct described in Problem 11.38. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for helium (use $0.051 \leq \text{Ma} \leq 5.193$). Show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see Video V11.4) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is similar to Example 11.9.
This problem involves the duct of Problem 11.39. However the flow enters supersonically. We can use values from the tables of problem 11.39 with a little rearrangement to account for the supersonic entering flow.

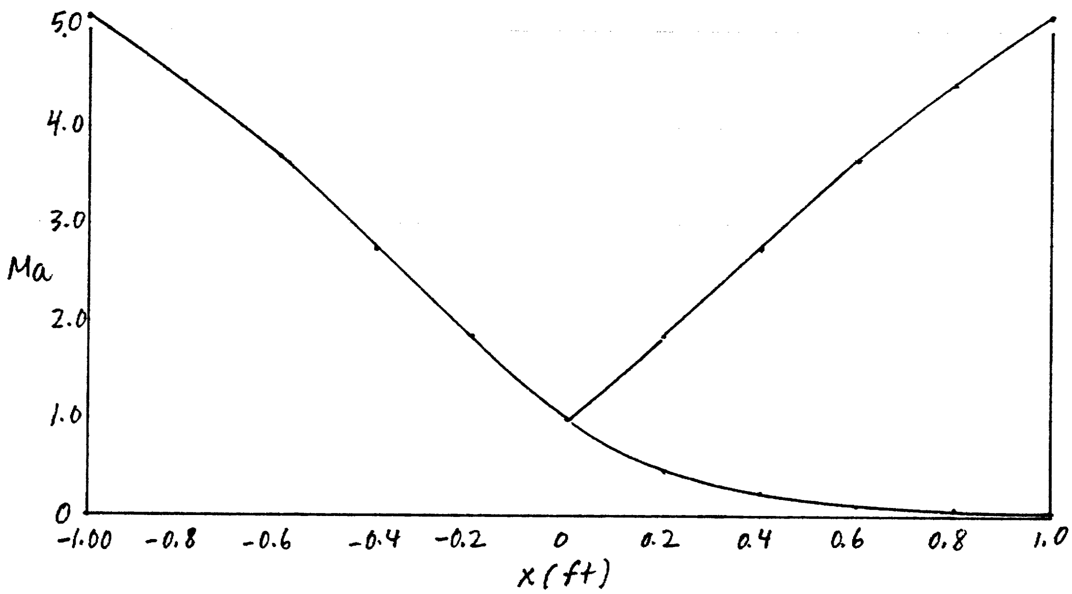
For helium we have

From program ISENTROP with $k = 1.66$

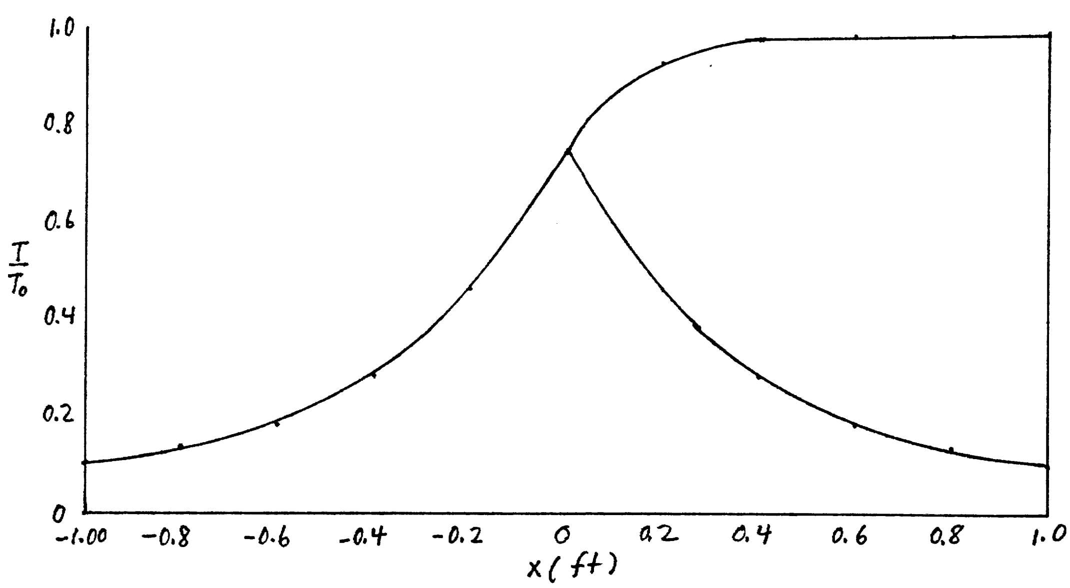
Ma	$\frac{A}{A^*}$	Eq. 2 of 11.39 x (ft)	$\frac{T}{T_0}$	$\frac{p}{p_0}$	state
		supersonic solution			
5.193	11.0	−1.00	0.10102	0.00313	a
4.448	7.4	−0.80	0.13282	0.00624	
3.647	4.6	−0.60	0.18556	0.01446	
2.778	2.6	−0.40	0.28195	0.04141	
1.855	1.4	−0.20	0.46827	0.14833	
1.0	1.0	0	0.75188	0.48808	b
1.855	1.4	0.20	0.46827	0.14833	
2.778	2.6	0.40	0.28195	0.04141	
3.647	4.6	0.60	0.18556	0.01446	
4.448	7.4	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	c
		subsonic solution			
0.460	1.40	0.20	0.93473	0.84386	
0.223	2.61	0.40	0.98385	0.95989	
0.123	4.62	0.60	0.99503	0.98755	
0.076	7.43	0.80	0.99809	0.99522	
0.051	11.06	1.00	0.99914	0.99784	d

(Con't)

11.39 (con't)



Variation of Mach number for helium

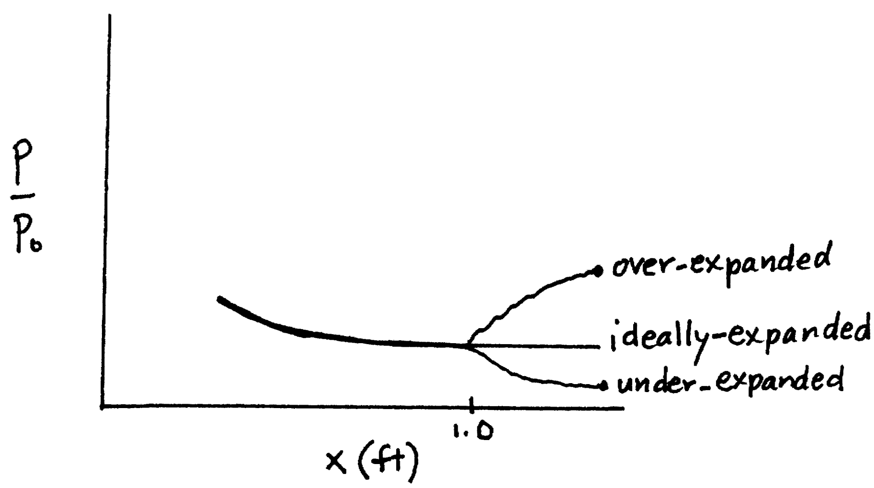


Variation of static temperature to stagnation temperature ratio for helium

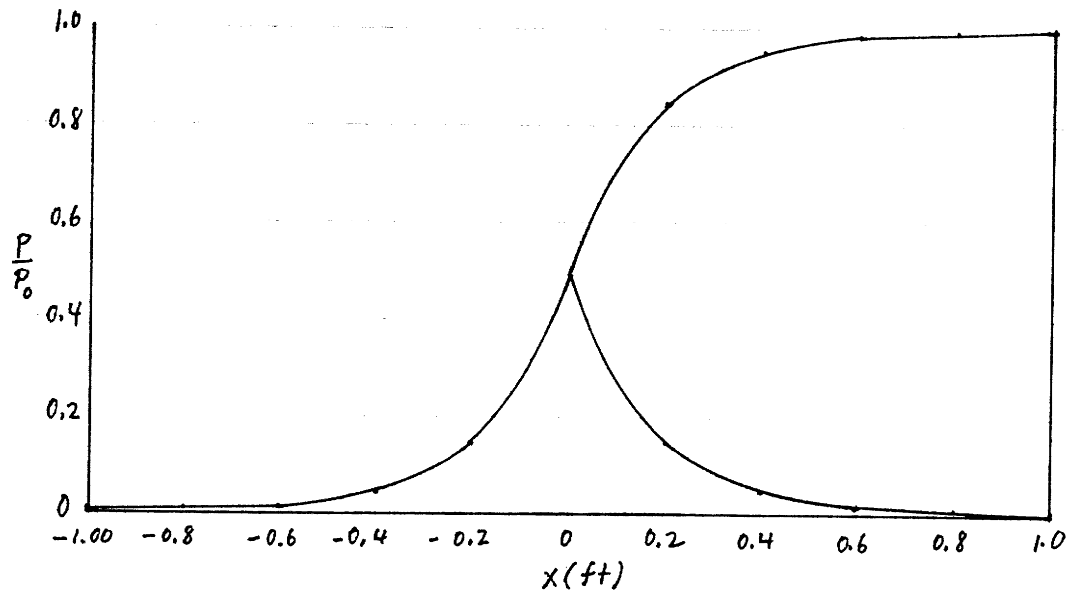
(con't)

11.39 (con't)

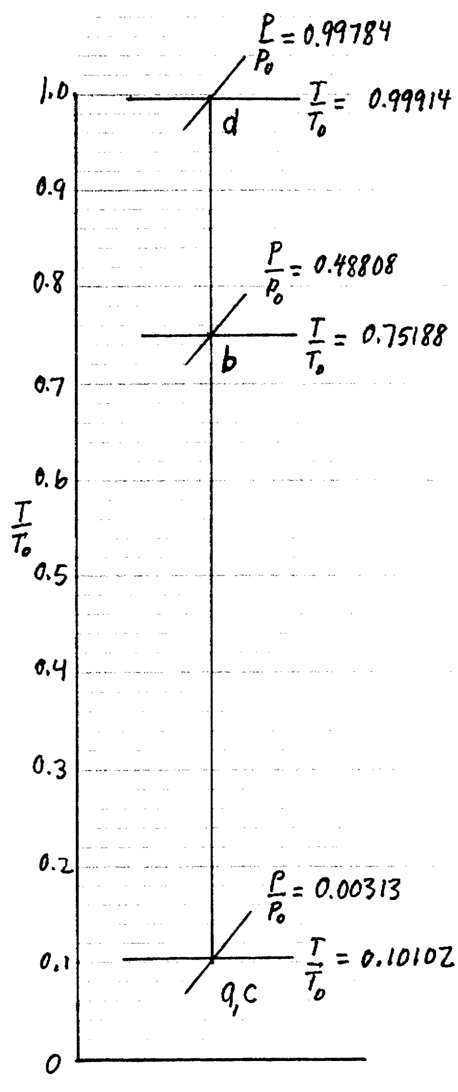
Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit.This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.39 (con't)



Variation of static pressure to stagnation pressure ratio for helium



(con't) T-s diagram for helium

11.40

*11.40 Helium enters supersonically and flows isentropically through the choked converging-diverging duct described in Example 11.8. Compare the variation of Ma, T/T_0, and p/p_0 for helium with the variation of these parameters for air throughout the duct. Use $0.163 \leq \text{Ma} \leq 3.221$. Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see Video V11.4) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

The variation of $\frac{A}{A^*}$, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ with Ma for helium was obtained with program ISENTROP with $k = 1.66$. Values of Mach numbers in the range $0.163 \leq \text{Ma} \leq 3.221$ were used as input. Values of x were obtained with

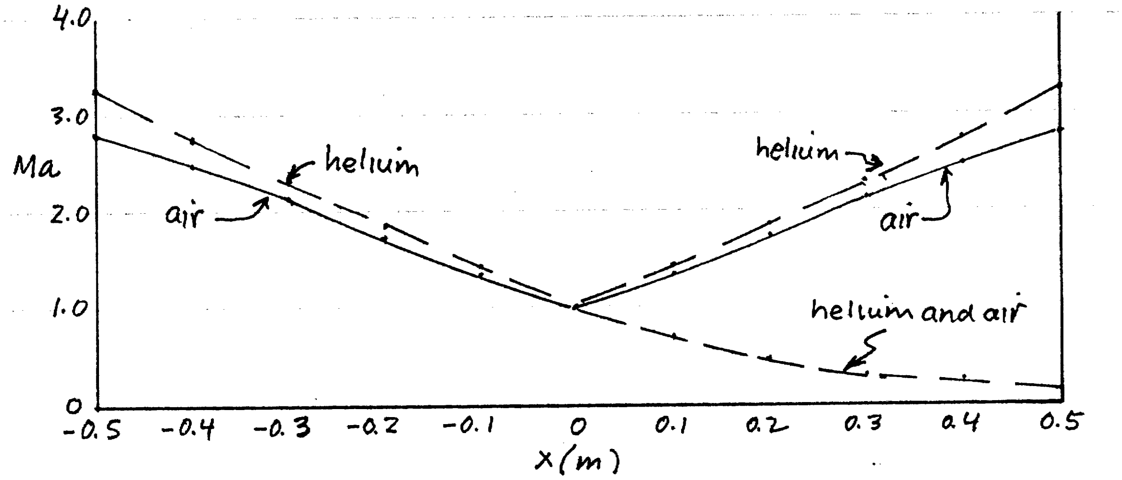
$$x = \sqrt{\left(\frac{A}{A^*}\right) 0.1 - 0.1}$$

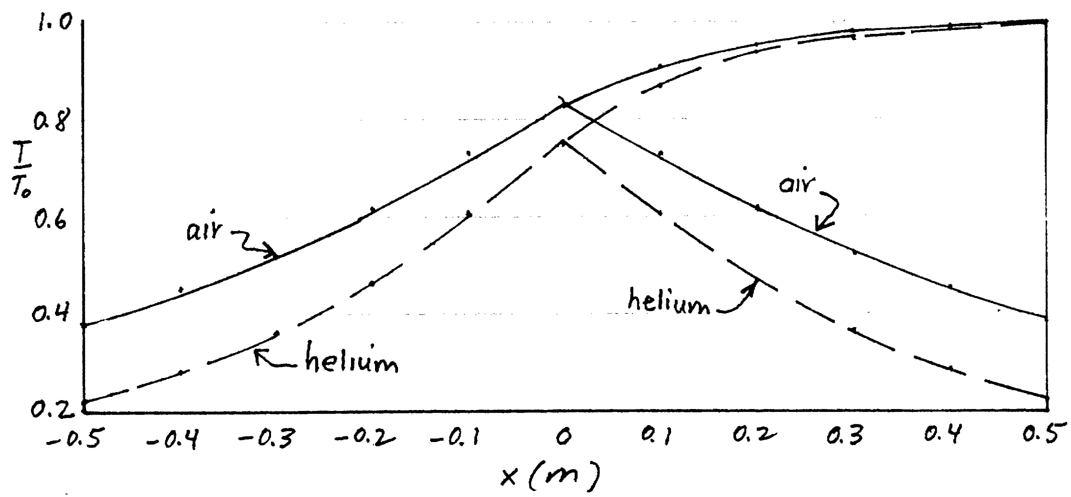
which is Eq. 5 of Example 11.8 rearranged to yield x. The variation of Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ with x for air obtained from Example 11.8.

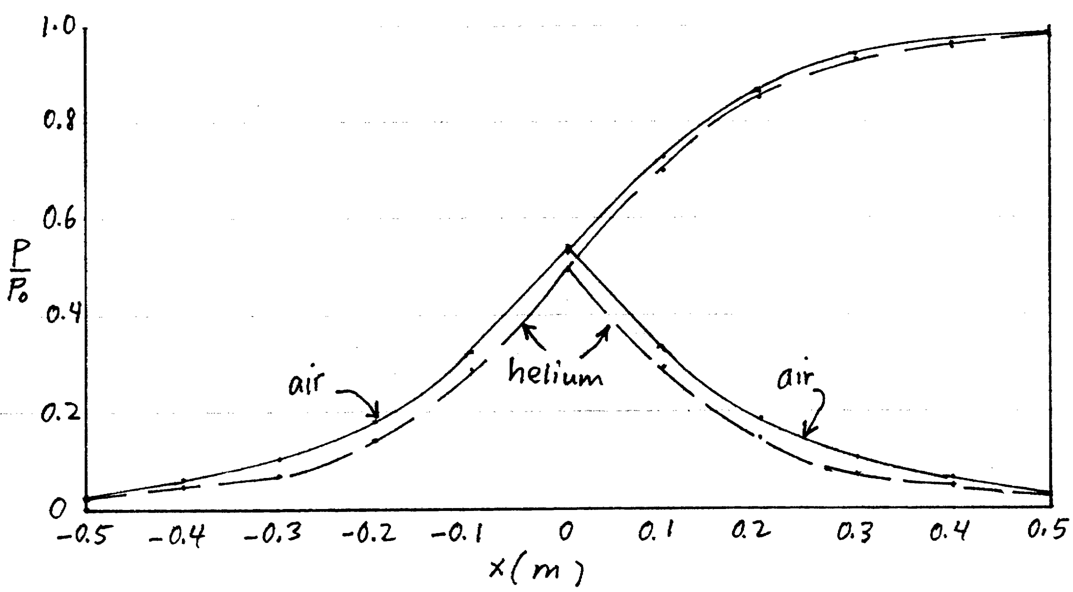
	helium			air		
x (m)	Ma	$\frac{T}{T_0}$	$\frac{p}{p_0}$	Ma	$\frac{T}{T_0}$	$\frac{p}{p_0}$
-0.5	3.221	0.2261	0.0238	2.80	0.3894	0.0369
-0.4	2.778	0.2820	0.0414	2.48	0.4484	0.0604
-0.3	2.320	0.3602	0.0767	2.14	0.5219	0.1027
-0.2	1.858	0.4683	0.1483	1.76	0.6175	0.1850
-0.1	1.402	0.6066	0.2844	1.37	0.7271	0.3278
0	1.0	0.7519	0.4881	1.0	0.8333	0.5283
			supersonic solution			
0.1	1.402	0.6066	0.2844	1.37	0.7271	0.3278
0.2	1.858	0.4683	0.1483	1.76	0.6175	0.1850
0.3	2.320	0.3602	0.0767	2.14	0.5219	0.1027
0.4	2.778	0.2820	0.0414	2.48	0.4484	0.0604
0.5	3.221	0.2261	0.0238	2.80	0.3894	0.0369
			subsonic solution			
0.1	0.682	0.8669	0.6983	0.69	0.9131	0.7274
0.2	0.460	0.9347	0.8439	0.47	0.9577	0.8596
0.3	0.316	0.9681	0.9217	0.32	0.9799	0.9315
0.4	0.223	0.9839	0.9599	0.23	0.9895	0.9638
0.5	0.163	0.9913	0.9783	0.17	0.9943	0.9800

(con't)

11.40 (con't)

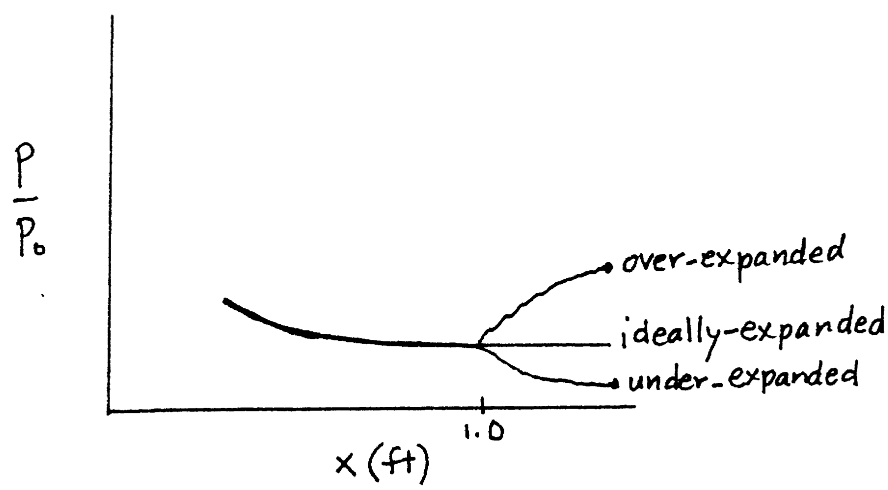






(con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit. This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.41

*11.41 Helium enters subsonically and flows isentropically through the converging-diverging duct of Example 11.8. Compare the values of Ma, T/T_0, and p/p_0 for helium with those for air at several locations in the duct. Use $0.163 \le \text{Ma} \le 3.221$. Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see Video V11.4) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

The variation of $\frac{A}{A^*}$, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ with Ma for helium was obtained with program ISENTROP with $k = 1.66$. Values of Mach numbers in the range $0.163 \le Ma \le 3.221$ were used as input. Values of x were obtained with

$$x = \sqrt{\left(\frac{A}{A^*}\right) 0.1 - 0.1}$$

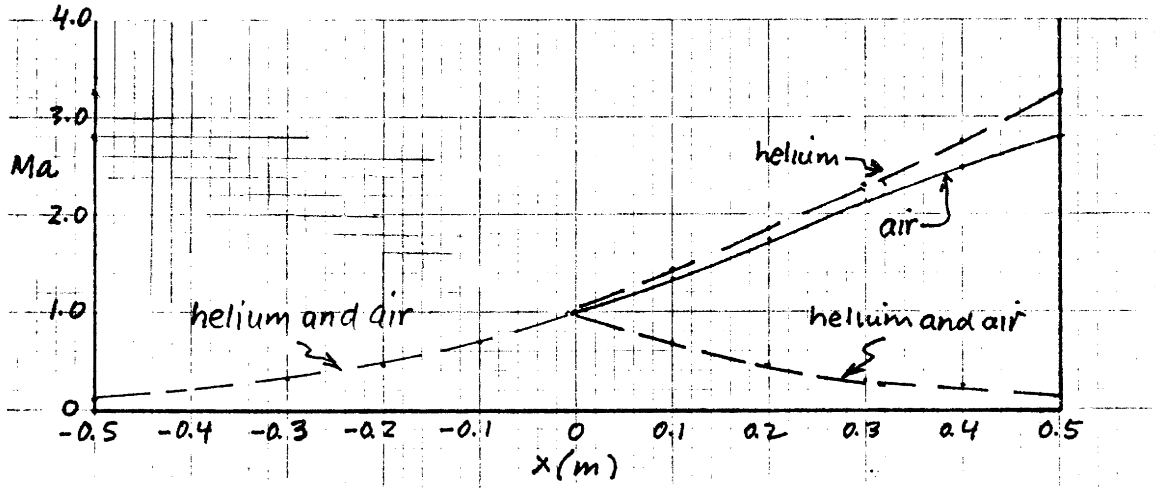
which is Eq. 5 of Example 11.8 rearranged to yield x.

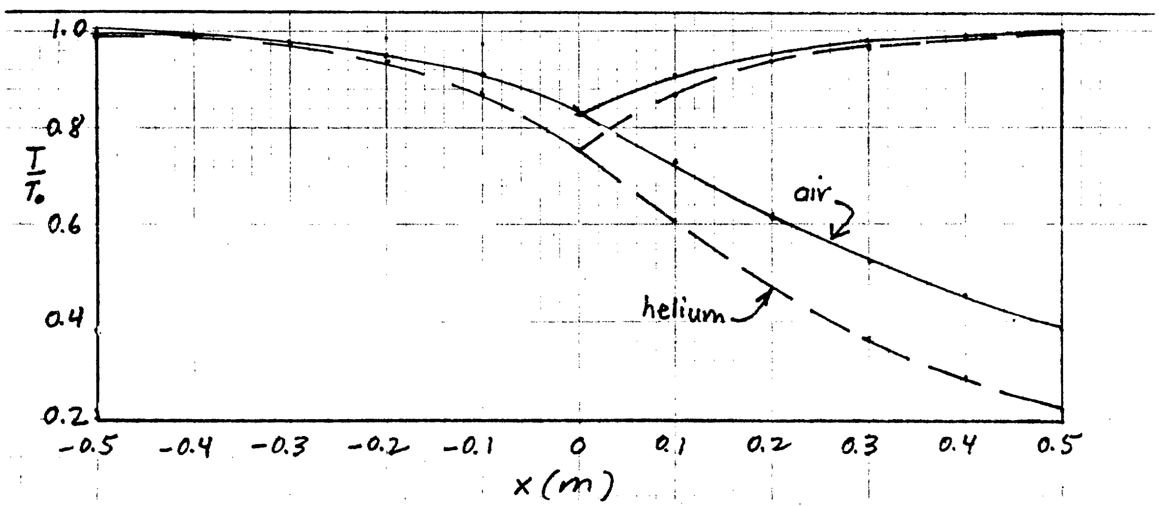
The variation of Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ with x for air was obtained from Example 11.8.

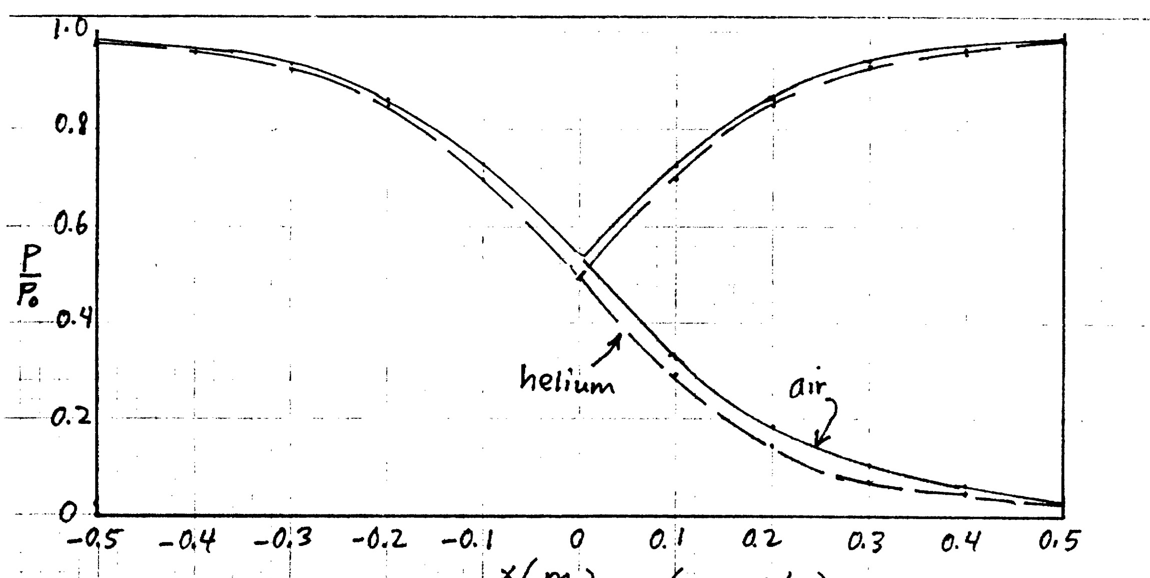
	helium			air		
x (m)	Ma	$\frac{T}{T_0}$	$\frac{p}{p_0}$	Ma	$\frac{T}{T_0}$	$\frac{p}{p_0}$
−0.5	0.163	0.9913	0.9783	0.17	0.9943	0.9800
−0.4	0.223	0.9839	0.9599	0.23	0.9895	0.9638
−0.3	0.316	0.9681	0.9217	0.32	0.9799	0.9315
−0.2	0.460	0.9347	0.8439	0.47	0.9577	0.8596
−0.1	0.682	0.8669	0.6983	0.69	0.9131	0.7274
0	1.0	0.7519	0.4881	1.0	0.8333	0.5283
			supersonic solution			
0.1	1.402	0.6066	0.2844	1.37	0.7271	0.3278
0.2	1.858	0.4683	0.1483	1.76	0.6175	0.1850
0.3	2.320	0.3602	0.0767	2.14	0.5219	0.1027
0.4	2.778	0.2820	0.0414	2.48	0.4484	0.0604
0.5	3.221	0.2261	0.0238	2.80	0.3894	0.0369
			subsonic solution			
0.1	0.682	0.8669	0.6983	0.69	0.9131	0.7274
0.2	0.460	0.9347	0.8439	0.47	0.9577	0.8596
0.3	0.316	0.9681	0.9217	0.32	0.9799	0.9315
0.4	0.223	0.9839	0.9599	0.23	0.9895	0.9638
0.5	0.163	0.9913	0.9783	0.17	0.9943	0.9800

(Con't)

11.41 (con't)



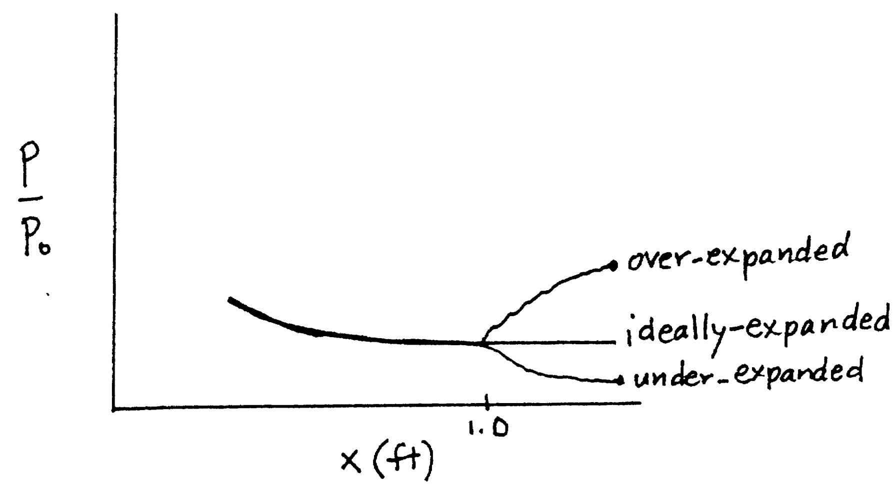




(con't)

11.41 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit. This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.42

11.42* Helium flows subsonically and isentropically through the converging–diverging duct of Example 11.8. Graph the variation of Ma, T/T_0, and p/p_0 through the duct from $x = -0.5$ m to $x = +0.5$ m for $p/p_0 = 0.99$ at $x = -0.5$ m. Sketch the corresponding $T - s$ diagram. Use $0.110 \leq Ma \leq 0.430$.

This is like Example 11.10.

For helium we use program ISENTROP with $k=1.66$ to determine values of $\frac{A}{A^*}$, $\frac{T}{T_0}$ and $\frac{P}{P_0}$ corresponding to values of Ma within the range $0.110 \leq Ma \leq 0.430$. We calculate x with

$$x = \sqrt{A^*\left(\frac{A}{A^*}\right) - 0.1} \qquad (1)$$

which is based on Eq. 2 of Example 11.8. Since the flow is not choked, $A^* \neq A_{throat}$. Thus, we determine A^* with

$$A^* = \frac{A_{throat}}{\left(\frac{A}{A^*}\right)_{throat}}$$

where $\left(\frac{A}{A^*}\right)_{throat}$ is obtained with program ISENTROP for $Ma = 0.430$.

Thus,

$$A^* = \frac{0.1\,m^2}{1.47} = 0.068\ m^2$$

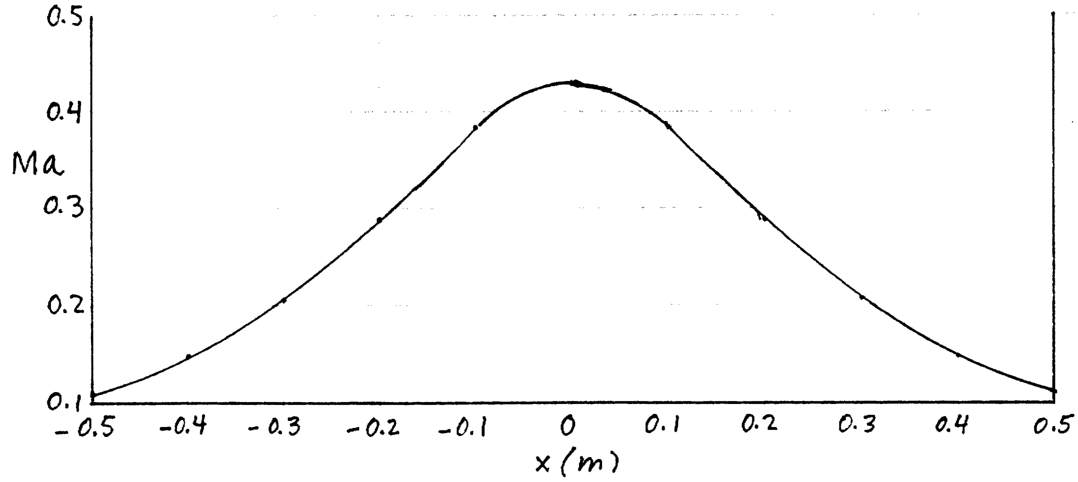
and Eq. 1 becomes

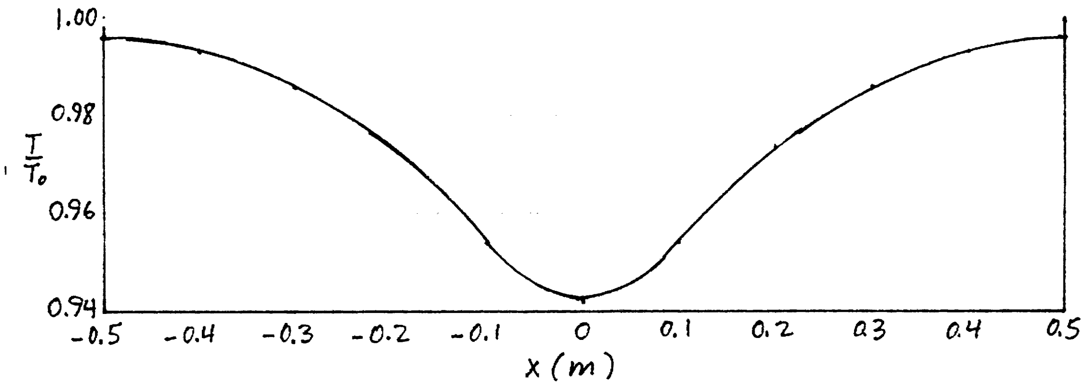
$$x = \sqrt{0.068\left(\frac{A}{A^*}\right) - 0.1} \qquad (2)$$

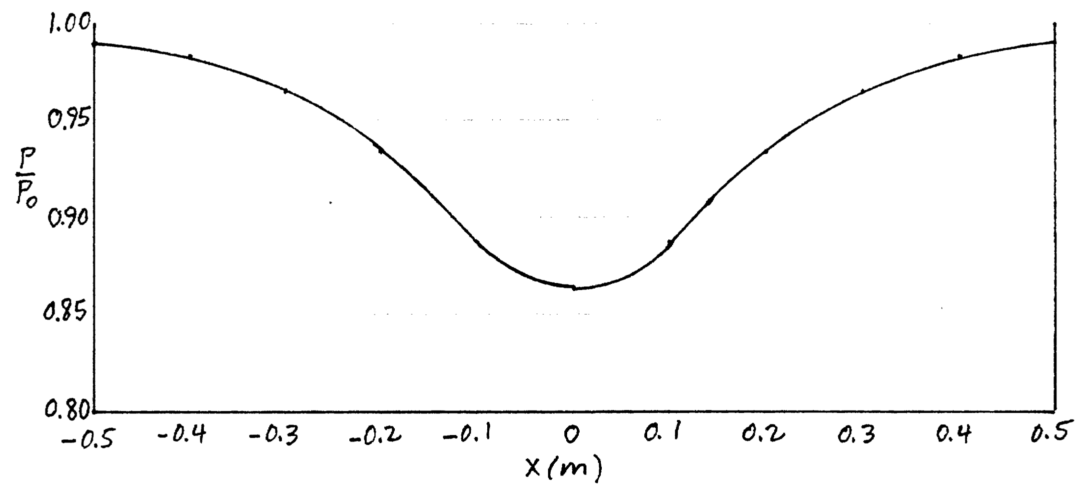
With Eq. 2 x(m)	With program ISENTROP with k=1.66 Ma	$\frac{A}{A^*}$	$\frac{T}{T_0}$	$\frac{P}{P_0}$	state
-0.50	0.110	5.16	0.9960	0.9900	a
-0.40	0.149	3.83	0.9927	0.9818	
-0.30	0.206	2.81	0.9862	0.9656	
-0.20	0.288	2.06	0.9734	0.9343	
-0.10	0.381	1.62	0.9543	0.8890	
0	0.430	1.47	0.9425	0.8616	b
0.10	0.381	1.62	0.9543	0.8890	
0.20	0.288	2.06	0.9734	0.9343	
0.30	0.206	2.81	0.9862	0.9656	
0.40	0.149	3.83	0.9927	0.9818	
0.50	0.110	5.16	0.9960	0.9900	c

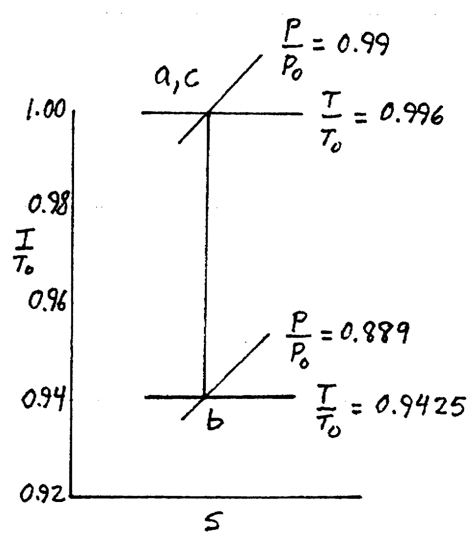
(con't)

11.42 (con't)


Ma
0.5
0.4
0.3
0.2
0.1
−0.5 −0.4 −0.3 −0.2 −0.1 0 0.1 0.2 0.3 0.4 0.5
x (m)


$\frac{T}{T_0}$
1.00
0.98
0.96
0.94
−0.5 −0.4 −0.3 −0.2 −0.1 0 0.1 0.2 0.3 0.4 0.5
x (m)


$\frac{p}{p_0}$
1.00
0.95
0.90
0.85
0.80
−0.5 −0.4 −0.3 −0.2 −0.1 0 0.1 0.2 0.3 0.4 0.5
x (m)


$\frac{p}{p_0} = 0.99$
a, c
$\frac{T}{T_0} = 0.996$
1.00
0.98
$\frac{T}{T_0}$
0.96
$\frac{p}{p_0} = 0.889$
0.94
b
$\frac{T}{T_0} = 0.9425$
0.92
s

11.43

11.43 An ideal gas flows subsonically and isentropically through the converging–diverging duct described in Problem 11.39. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for **(a)** air; **(b*)** helium (use $0.047 \leq \text{Ma} \leq 0.722$). The value of p/p_0 is 0.6708 at $x = 0$ ft. Sketch important states on a $T - s$ diagram.

This is like Example 11.10.

Since $\frac{p}{p_0} = 0.6708$ at $x = 0$ is greater than $\frac{p^*}{p_0} = 0.5283$ for air (see Problem 11.30(a) solution) and 0.4881 for helium (see Problem 11.30(c) solution) the air and helium flows through the converging-diverging duct are not choked. For values of $\frac{A}{A^*}$ at different values of x we obtain corresponding values of Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$.

(a) For air we enter Fig. D.1 with values of $\frac{A}{A^*}$ to get Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$. For A^* we use

$$A^* = \frac{A}{\left(\frac{A}{A^*}\right)}$$

evaluated at $x = 0$ where $A = 0.1\ ft^2$. We determine $\frac{A}{A^*}$ at $x = 0$ from Fig. D.1 for the subsonic flow value of $\frac{p}{p_0} = 0.6708$, we get

$$\frac{A}{A^*} = 1.05 \quad \text{and thus}$$

$$A^* = \frac{0.1\ ft^2}{1.05} = 0.095\ ft^2$$

We then determine the $\frac{A}{A^*}$ variation through the duct with

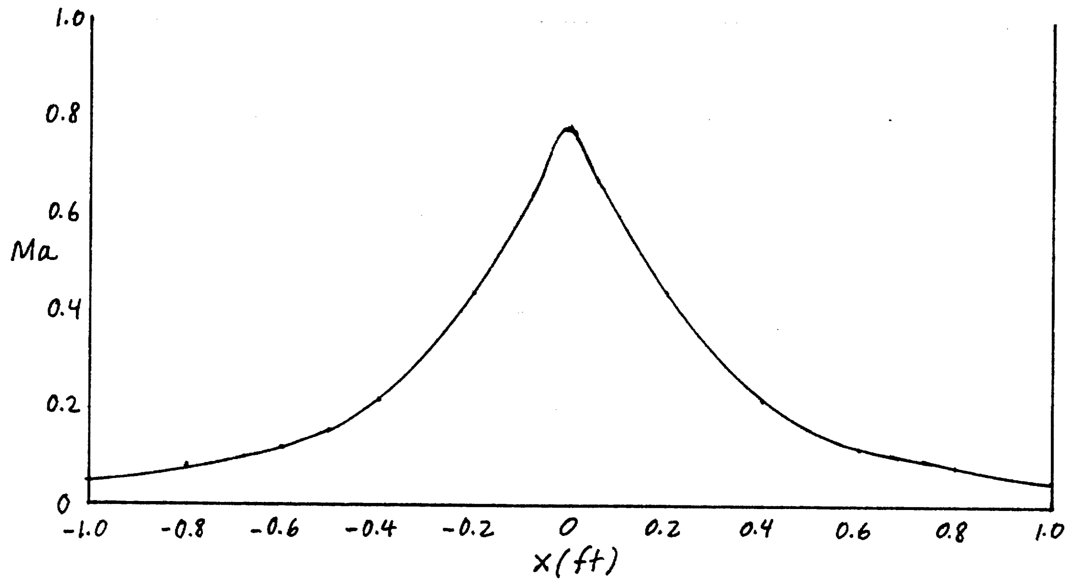
$$\frac{A}{A^*} = \frac{x^2 + 0.1}{A^*} = \frac{x^2 + 0.1}{0.095} \qquad (1)$$

The corresponding values of $\frac{A}{A^*}$, Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ from Fig. D.1 are also tabulated on the next page.

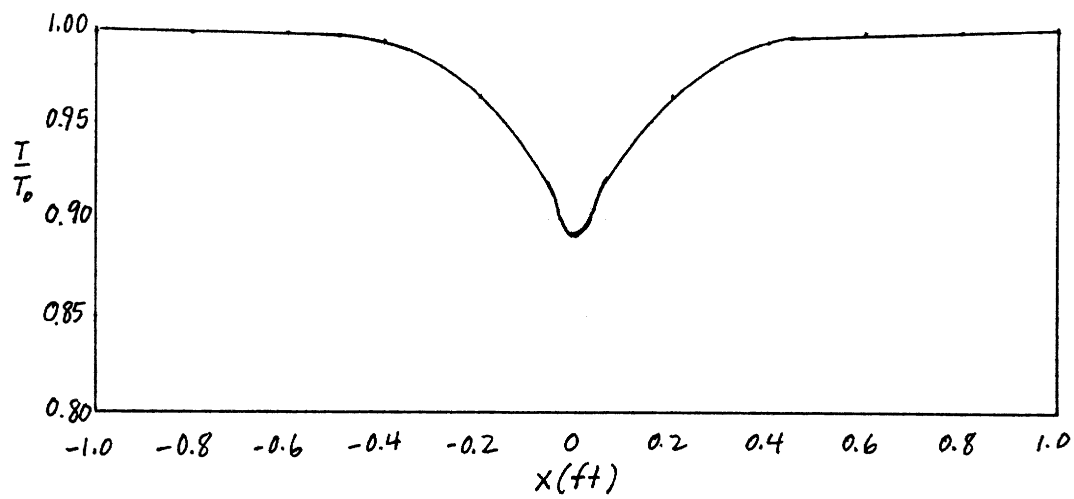
(con't)

11.43 (con't)

x (ft)	$\frac{A}{A^*}$ With Eq. 1	Ma (From Fig. D.1)	$\frac{T}{T_0}$ (From Fig. D.1)	$\frac{p}{p_0}$ (From Fig. D.1)	state
-1.0	11.6	0.05	0.99	0.99	a
-0.8	7.8	0.08	0.99	0.99	
-0.6	4.8	0.12	0.99	0.98	
-0.4	2.7	0.22	0.99	0.966	
-0.2	1.5	0.44	0.96	0.87	
0	1.0	0.78	0.89	0.66	b
0.2	1.5	0.44	0.96	0.87	
0.4	2.7	0.22	0.99	0.96	
0.6	4.8	0.12	0.99	0.98	
0.8	7.8	0.08	0.99	0.99	
1.0	11.6	0.05	0.99	0.99	c



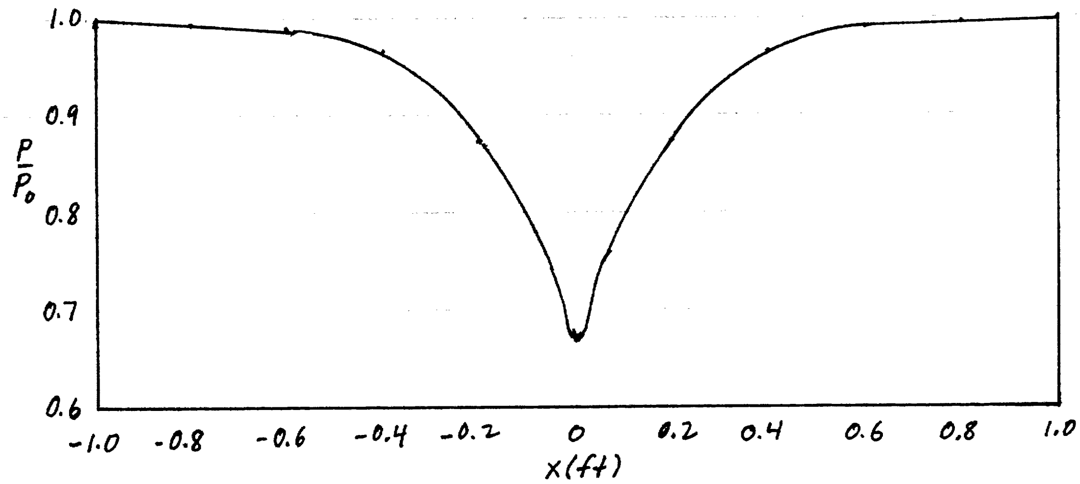
Variation of Mach number for air



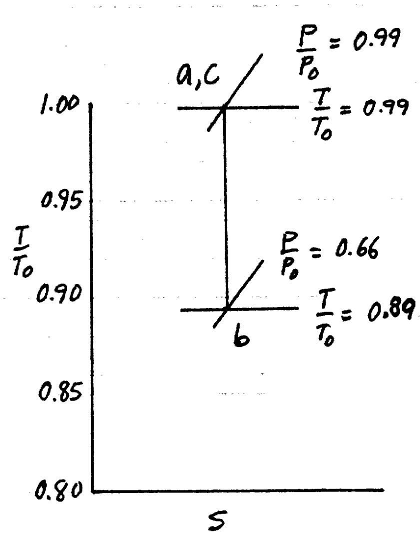
Variation of static temperature to stagnation temperature ratio for air

(con't)

11.43 (Con't)



Variation of static pressure to stagnation pressure ratio for air



T-s diagram for air

(b) For helium, we use program ISENTROP with $k = 1.66$ to determine values of $\frac{A}{A^*}$, $\frac{T}{T_0}$ and $\frac{P}{P_0}$ corresponding to values of Ma within the range $0.47 \leq Ma \leq 0.722$. We calculate x with

$$x = \sqrt{A^*\left(\frac{A}{A^*}\right) - 0.1} \qquad (1)$$

Which is based on Eq. 2 of Problem 11.39. Since this flow is not choked, $A^* \neq A_{throat}$. Thus we determine A^* with

$$A^* = \frac{A_{throat}}{\left(\frac{A}{A^*}\right)_{throat}}$$

(con't)

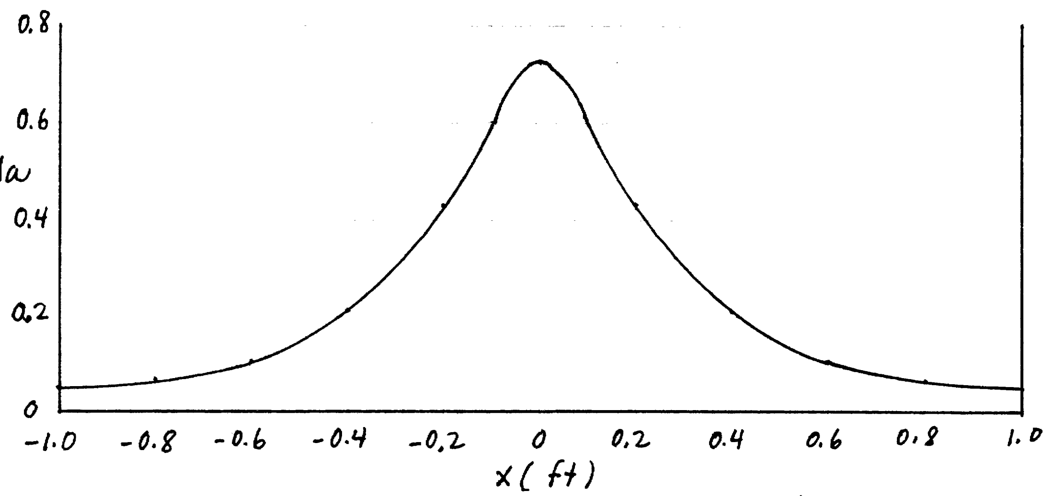
where $\left(\frac{A}{A^*}\right)_{throat}$ is obtained with program ISENTROP for $Ma = 0.722$. Thus,

$$A^* = \frac{0.1\ ft^2}{1.0735} = 0.09315\ ft^2$$

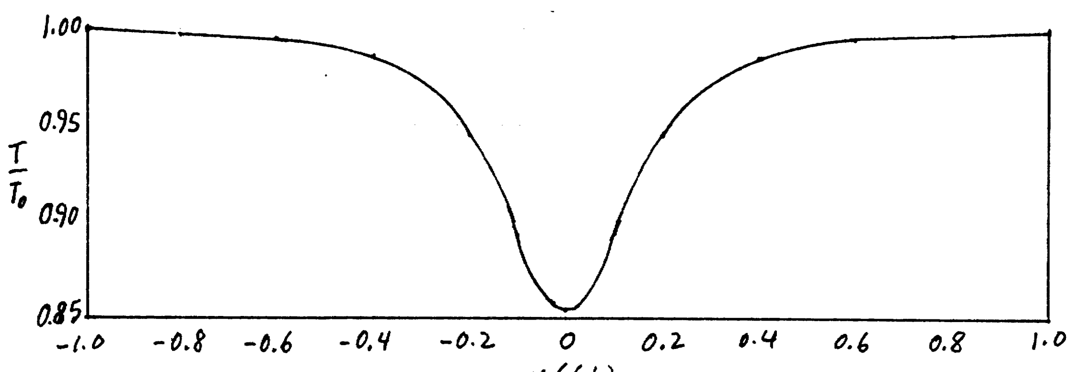
and Eq. 1 becomes

$$x = \sqrt{(0.09315)\left(\frac{A}{A^*}\right) - 0.1} \qquad (2)$$

With Eq. 2 x (ft)	Ma	With ISENTROP $\frac{A}{A^*}$	$\frac{T}{T_0}$	$\frac{p}{p_0}$	state
-1.0	0.047	11.9939	0.9993	0.9982	a
-0.8	0.071	7.9546	0.9983	0.9958	
-0.6	0.115	4.9378	0.9957	0.9891	
-0.4	0.207	2.7973	0.9861	0.9653	
-0.2	0.419	1.5049	0.9452	0.8679	
0	0.722	1.0735	0.8532	0.6708	b
0.2	0.419	1.5049	0.9452	0.8679	
0.4	0.207	2.7973	0.9861	0.9653	
0.6	0.115	4.9378	0.9957	0.9891	
0.8	0.071	7.9546	0.9983	0.9958	
1.0	0.047	11.9939	0.9993	0.9982	c



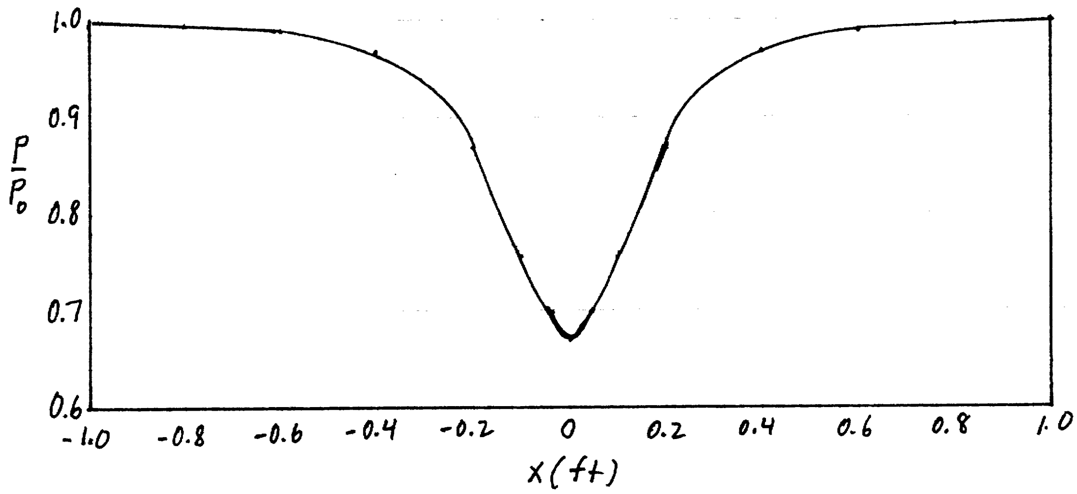
Variation of Mach number for helium



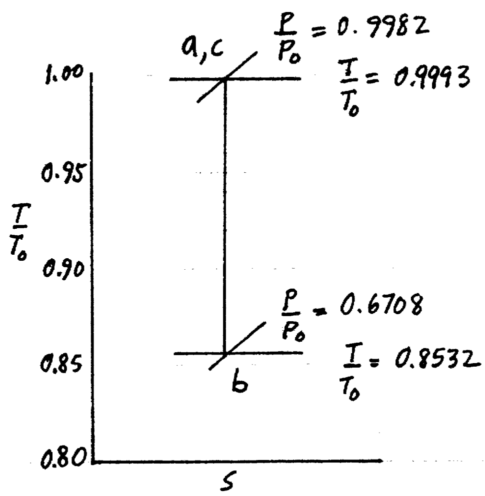
Variation of static temperature to stagnation temperature ratio for helium

(con't)

11.43 (con't)



Variation of static pressure to stagnation pressure ratio for helium



T- s diagram for helium

11.44

11.44 An ideal gas contained in a large storage container at a constant temperature and pressure of 59 °F and 25 psia is to be expanded isentropically through a duct to standard atmospheric discharge conditions. Describe in general terms the kind of duct required and determine the duct exit cross section area if the discharge mass flowrate required is 1.0 lbm/s and the gas is **(a)** air; **(b)** carbon dioxide; **(c)** helium.

To determine the duct exit flow area we use

$$A_{exit} = \frac{\dot{m}}{\rho_{exit} V_{exit}} \qquad (1)$$

For the exit flow density, ρ_{exit}, we use Eq. 11.60, or for air, Fig. D.1. Thus,

$$\rho_{exit} = \rho_0 \left[\frac{1}{1+\left(\frac{k-1}{2}\right) Ma_{exit}^2}\right]^{\left(\frac{1}{k-1}\right)}$$

where $\rho_0 = \frac{p_0}{RT_0}$ so

$$\rho_{exit} = \frac{p_0}{RT_0}\left[\frac{1}{1+\left(\frac{k-1}{2}\right) Ma_{exit}^2}\right]^{\left(\frac{1}{k-1}\right)} \qquad (2)$$

or for air

$$\rho_{exit} = \frac{p_0}{RT_0}\left(\text{Fig. D.1 value of } \frac{\rho_{exit}}{\rho_0}\right) \qquad (3)$$

We obtain Ma_{exit} from $\frac{p_{exit}}{p_0}$ using Eq. 11.59, or for air, Fig. D.1. Thus

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{p_{exit}}{p_0}\right)^{\left(\frac{k-1}{k}\right)}} - 1\right]\left(\frac{2}{k-1}\right)} \qquad (4)$$

or for air

$$Ma_{exit} = \text{Fig. D.1 value as a function of } \frac{p_{exit}}{p_0} \qquad (5)$$

To obtain V_{exit} we use

$$V_{exit} = (Ma_{exit})\,c = (Ma_{exit})\sqrt{R\,T_{exit}\,k}$$

(con't)

11.44 (con't)

where from Eq. 11.56, or for air, from Fig. D.1

$$T_{exit} = \frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma_{exit}^2}$$

or for air

$$T_{exit} = T_0\left(\frac{T_{exit}}{T_0}\text{ value from Fig. D.1 for } Ma_{exit}\right)$$

and thus

$$V_{exit} = (Ma_{exit})\sqrt{\frac{R\,T_0\,k}{\left[1+\left(\frac{k-1}{2}\right)Ma_{exit}^2\right]}} \qquad (5)$$

or for air

$$V_{exit} = (Ma_{exit})\sqrt{R\,T_0\left(\frac{T_{exit}}{T_0}\text{ value from Fig. D.1}\right)k} \qquad (6)$$

(a) For air

$$\frac{p_{exit}}{p_0} = \frac{14.7\text{ psia}}{25\text{ psia}} = 0.59$$

The corresponding values from Fig. D.1 are

$$Ma_{exit} = 0.9$$

$$\frac{T_{exit}}{T_0} = 0.86$$

and

$$\frac{\rho_{exit}}{\rho_0} = 0.68$$

Now with Eq. 3

$$\rho_{exit} = \frac{(25\text{ psia})\left(144\ \frac{in.^2}{ft^2}\right)(0.68)}{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(519\ {}^\circ R)} = 2.75\times 10^{-3}\ \frac{slug}{ft^3}$$

(con't)

11.44 (con't)

With Eq. 6 we obtain

$$V_{exit} = 0.9\sqrt{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)\frac{(519\,^\circ R)(0.86\quad)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 932\ \frac{ft}{s}$$

With Eq. 1 we get

$$A_{exit} = \frac{\left(1\ \frac{lbm}{s}\right)}{\left(2.75 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(932\ \frac{ft}{s}\right)\left(32.2\ \frac{lbm}{slug}\right)} = \underline{\underline{0.012\ ft^2}}$$

A converging duct would suffice since the exit flow is subsonic.

(b) For carbon dioxide, Eq. 4 yields

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{14.7\ psia}{25\ psia}\right)^{\frac{1.3-1}{1.3}}} - 1\right]\left(\frac{2}{1.3-1}\right)} = 0.9323$$

With Eq. 2 we obtain

$$\rho_{exit} = \frac{(25\ psia)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1130\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\,^\circ R)}\left[\frac{1}{1+\left(\frac{1.3-1}{2}\right)(0.9323)^2}\right]^{\frac{1}{1.3-1}}$$

or

$$\rho_{exit} = 4.08 \times 10^{-3}\ \frac{slug}{ft^3}$$

With Eq. 5 we get

$$V_{exit} = (0.9323)\sqrt{\frac{\left(1130\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\,^\circ R)(1.3)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)\left[1+\left(\frac{1.3-1}{2}\right)(0.9323)^2\right]}} = 765.7\ \frac{ft}{s}$$

With Eq. 1 we obtain

$$A_{exit} = \frac{\left(1\ \frac{lbm}{s}\right)}{\left(4.08 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(765.7\ \frac{ft}{s}\right)\left(32.2\ \frac{lbm}{slug}\right)} = \underline{\underline{0.00994\ ft^2}}$$

A converging nozzle would do since the exit flow is subsonic.

(con't)

(c) For helium, Eq. 4 yields

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{14.7\,psia}{25\,psia}\right)^{\frac{1.66-1}{1.66}}} - 1\right]\left(\frac{2}{1.66-1}\right)} = 0.844$$

With Eq. 2 we obtain

$$\rho_{exit} = \frac{(25\,psia)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.242\times10^4\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)(519\,{}^\circ R)}\left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(0.844)^2}\right]^{\frac{1}{1.66-1}}$$

or

$$\rho_{exit} = 4.056\times10^{-4}\,\frac{slug}{ft^3}$$

With Eq. 5 we get

$$V_{exit} = (0.844)\sqrt{\frac{\left(1.242\times10^4\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)(519\,{}^\circ R)(1.66)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)\left[1+\left(\frac{1.66-1}{2}\right)(0.844)^2\right]}} = 2484\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$A_{exit} = \frac{\left(1\,\frac{lbm}{s}\right)}{\left(4.056\times10^{-4}\,\frac{slug}{ft^3}\right)\left(2484\,\frac{ft}{s}\right)\left(32.2\,\frac{lbm}{slug}\right)} = \underline{\underline{0.0308\,ft^2}}$$

A converging nozzle will do since the exit flow is subsonic.

11.45

11.45 An ideal gas is to flow isentropically from a large tank where the air is maintained at a temperature and pressure of 59 °F and 80 psia to standard atmospheric discharge conditions. Describe in general terms the kind of duct involved and determine the duct exit Mach number and velocity in ft/s if the gas is **(a)** air; **(b)** methane; **(c)** helium.

To determine the duct exit Mach number, Ma_{exit}, we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{p_{exit}}{p_0}\right)^{\frac{k-1}{k}}} - 1\right]\left(\frac{2}{k-1}\right)} \qquad (1)$$

or for air

$$Ma_{exit} = \text{Fig. D.1 value as a function of } \frac{p_{exit}}{p_0} \qquad (2)$$

To determine exit velocity, V_{exit}, we use

$$V_{exit} = (Ma_{exit})\, c_{exit} = Ma_{exit}\sqrt{R\,T_{exit}\,k} \qquad (3)$$

where

$$T_{exit} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right)Ma_{exit}^2} \qquad (4)$$

or for air

$$T_{exit} = T_0\left(\frac{T_{exit}}{T_0} \text{ value from Fig. D.1 for } Ma_{exit}\right) \qquad (5)$$

(a) For air

$$\frac{p_{exit}}{p_0} = \frac{14.7 \text{ psia}}{80 \text{ psia}} = 0.1838$$

and thus from Fig. D.1, the corresponding values are

$$Ma_{exit} = \underline{\underline{1.8}}$$

and

$$\frac{T_{exit}}{T_0} = 0.62$$

(con't)

Then with Eq. 5 we obtain

$$T_{exit} = (519°R)(0.62\quad) = 322°R.$$

and with Eq. 3 we conclude that

$$V_{exit} = (1.8\,)\sqrt{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)\frac{(322\ °R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{m}{s^2}}\right)}} = \underline{\underline{1580\ \frac{ft}{s}}}$$

A converging-diverging nozzle is required because the exit flow is supersonic.

(b) For methane we obtain from Eq. 1

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{14.7\,psia}{80\,psia}\right)^{\frac{1.31-1}{1.31}}} - 1\right]\left(\frac{2}{1.31-1}\right)} = \underline{\underline{1.78}}$$

Then with Eq. 4 we get

$$T_{exit} = \frac{519°R}{1+\left(\frac{1.31-1}{2}\right)(1.78)^2} = 348.1\,°R$$

and with Eq. 3 we obtain

$$V_{exit} = (1.78)\sqrt{\left(3099\ \frac{ft \cdot lb}{slug \cdot °R}\right)\frac{(348.7°R)(1.31)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{2120\ \frac{ft}{s}}}$$

A converging-diverging nozzle is required because the exit flow is supersonic.

(c) For helium we obtain from Eq. 1

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{14.7\,psia}{80\,psia}\right)^{\frac{1.66-1}{1.66}}} - 1\right]\left(\frac{2}{1.66-1}\right)} = \underline{\underline{1.71}}$$

Then with Eq. 4 we get

$$T_{exit} = \frac{519°R}{1+\left(\frac{1.66-1}{2}\right)(1.71)^2} = 264.1\,°R$$

(con't)

11.45 (Con't)

and with Eq. 3 we obtain

$$V_{exit} = (1.71)\sqrt{\frac{\left(1.242\times10^{4}\ \frac{ft\cdot lb}{slug\cdot {}^{\circ}R}\right)(264.1^{\circ}R)(1.66)}{\left(1\ \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}}$$

or

$$V_{exit} = \underline{\underline{3990}}\ \frac{ft}{s}$$

A converging-diverging nozzle is required since the exit flow is supersonic.

11.46

11.46 An ideal gas flows isentropically through a converging–diverging nozzle. At a section in the converging portion of the nozzle, $A_1 = 0.1\ \text{m}^2$, $p_1 = 600$ kPa (abs), $T_1 = 20\ °\text{C}$, and $\text{Ma}_1 = 0.6$. For section (2) in the diverging part of the nozzle determine A_2, p_2, and T_2 if $\text{Ma}_2 = 3.0$ and the gas is **(a)** air; **(b)** helium.

To determine A_2 we use Eq. 11.71 or for air, Fig. D.1 Thus,

$$A_2 = A_1 \frac{\left(\frac{A_2}{A^*}\right)}{\left(\frac{A_1}{A^*}\right)} = A_1 \left\{ \frac{\frac{1}{Ma_2}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_2^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}}{\frac{1}{Ma_1}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}} \right\} \quad (1)$$

or for air

$$A_2 = A_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{A_2}{A^*} \text{ for } Ma_2\right)}{\left(\text{Fig. D.1 value of } \frac{A_1}{A^*} \text{ for } Ma_1\right)}\right] \quad (2)$$

To determine P_2 we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$P_2 = P_1 \frac{\left(\frac{P_2}{P_0}\right)}{\left(\frac{P_1}{P_0}\right)} = P_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]^{\frac{k}{k-1}}}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k}{k-1}}} \right\} \quad (3)$$

or for air,

$$P_2 = P_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{P_2}{P_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{P_1}{P_0} \text{ for } Ma_1\right)}\right] \quad (4)$$

To determine T_2 we use Eq. 11.56 or for air, Fig. D.1. Thus,

$$T_2 = T_1 \frac{\left(\frac{T_2}{T_0}\right)}{\left(\frac{T_1}{T_0}\right)} = T_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]} \right\} \quad (5)$$

or for air

$$T_2 = T_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{T_2}{T_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{T_1}{T_0} \text{ for } Ma_1\right)}\right] \quad (6)$$

(con't)

11.46 (con't)

(a) For air, Eq. 2 leads to

$$A_2 = (0.1\,m^2)\frac{(4.3\quad)}{(1.2\quad)} = \underline{\underline{0.36\ m^2}}$$

Eq. 4 yields

$$P_2 = [600\ kPa(abs)]\frac{(0.03\quad)}{(0.78\quad)} = \underline{\underline{23\ kPa(abs)}}$$

and Eq. 6 gives

$$T_2 = (293\,K)\frac{(0.36\quad)}{(0.93\quad)} = \underline{\underline{113\ K}}$$

(b) For helium, Eq. 1 leads to

$$A_2 = (0.1\,m^2)\left\{\frac{\left(\frac{1}{3.0}\right)\left[\frac{1+\left(\frac{1.66-1}{2}\right)(3.0)^2}{1+\left(\frac{1.66-1}{2}\right)}\right]^{\frac{1.66+1}{2(1.66-1)}}}{\frac{1}{0.6}\left[\frac{1+\left(\frac{1.66-1}{2}\right)(0.6)^2}{1+\left(\frac{1.66-1}{2}\right)}\right]^{\frac{1.66+1}{2(1.66-1)}}}\right\} = \underline{\underline{0.257\ m^2}}$$

Eq. 3 yields

$$P_2 = [600\ kPa(abs)]\left\{\frac{\left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(3.0)^2}\right]^{\frac{1.66}{1.66-1}}}{\left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(0.6)^2}\right]^{\frac{1.66}{1.66-1}}}\right\} = \underline{\underline{24.8\ kPa(abs)}}$$

Eq. 5 gives

$$T_2 = (293\,K)\left\{\frac{\left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(3.0)^2}\right]}{\left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(0.6)^2}\right]}\right\} = \underline{\underline{82.6\ K}}$$

11.47

11.47 Upstream of the throat of an isentropic converging–diverging nozzle at section (1), $V_1 = 150$ m/s, $p_1 = 100$ kPa (abs), and $T_1 = 20$ °C. If the discharge flow is supersonic and the throat area is 0.1 m^2, determine the mass flowrate in kg/s for the flow of **(a)** air; **(b)** methane; **(c)** helium.

We determine the Mach number at section (1) with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{R T_1 k}} \qquad (1)$$

For all of the gases involved it is likely that Ma_1 is less than 1.0 because V_1 is low. Thus, the flow at the throat is choked since the entering flow is subsonic and the leaving flow is supersonic. For mass flowrate we use Eq. 11.40 to obtain

$$\dot{m} = \rho^* A^* V^* \qquad (2)$$

For throat velocity, V^*, we use

$$V^* = \sqrt{R T^* k} \qquad (3)$$

To obtain T^* we use Eq. 11.63. Thus,

$$T^* = T_0 \left(\frac{2}{k+1}\right) \qquad (4)$$

or for air,

$$T^* = T_0 \left(\text{value of } \frac{T}{T_0} \text{ from Fig. D.1 for } Ma = 1.0\right) \qquad (5)$$

To determine T_0 we use Eq. 11.56. Thus,

$$T_0 = T_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right] \qquad (6)$$

or for air,

$$T_0 = \frac{T_1}{\left(\text{Value of } \frac{T_1}{T_0} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (7)$$

(con't)

11.47 (con't)

To determine ρ^* we use the ideal gas equation of state (Eq. 11.1). Thus,

$$\rho^* = \frac{p^*}{RT^*} \qquad (8)$$

For p^* we use Eq. 11.61. Thus

$$p^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (9)$$

or for air,

$$p^* = p_0 \left(\text{value of } \frac{p^*}{p_0} \text{ from Fig. D.1 for Ma} = 1.0\right) \qquad (10)$$

For p_0 we use Eq. 11.59. Thus,

$$p_0 = p_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right]^{\frac{k}{k-1}} \qquad (11)$$

or for air

$$p_0 = \frac{p_1}{\left(\text{value of } \frac{p_1}{p_0} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (12)$$

(a) For air we use Eq. 1 to obtain

$$Ma_1 = \frac{\left(150 \frac{m}{s}\right)}{\sqrt{\frac{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(293 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.4372$$

Thus, the flow is choked at the throat. From Eq. 7 we obtain for corresponding value in Fig. D.1 for $Ma_1 = 0.44$

$$T_0 = \frac{293 K}{(0.96)} = 305 K$$

With Eq. 5 we obtain

$$T^* = (305 K)(0.83333) = 254 K$$

Thus

$$V^* = \sqrt{\frac{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(254 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 319 \frac{m}{s}$$

(con't)

11.47 (con't)

From Eq. 12 we obtain with the help of Fig. D.1

$$p_0 = \frac{100 \text{ kPa(abs)}}{0.87} = 115 \text{ kPa(abs)}$$

and with Eq. 10

$$p^* = [115 \text{ kPa(abs)}](0.52828) = 60.8 \text{ kPa(abs)}$$

Then with Eq. 8

$$\rho^* = \frac{\left(60.8 \times 10^3 \frac{N}{m^2}\right)}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(254 \text{ K})} = 0.83 \frac{kg}{m^3}$$

Finally, with Eq. 2 we obtain

$$\dot{m} = \left(0.83 \frac{kg}{m^3}\right)(0.1 m^2)\left(319 \frac{m}{s}\right) = \underline{\underline{26.5 \frac{kg}{s}}}$$

(b) For methane we use Eq. 1 to obtain

$$Ma_1 = \frac{\left(150 \frac{m}{s}\right)}{\sqrt{\left(518.3 \frac{N \cdot m}{kg \cdot K}\right)\frac{(293 K)(1.31)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.3363$$

Thus, the flow is choked at the throat. From Eq. 7 we obtain for $Ma_1 = 0.3363$

$$T_0 = (293 K)\left[1 + \left(\frac{1.31-1}{2}\right)(0.3363)^2\right] = 298.1 K$$

and with Eq. 4 we obtain

$$T^* = (298.1 K)\left(\frac{2}{1.31+1}\right) = 258.1 K$$

Thus

$$V^* = \sqrt{\left(518.3 \frac{N \cdot m}{kg \cdot K}\right)\frac{(258.1 K)(1.31)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 418.6 \frac{m}{s}$$

From Eq. 11 we obtain

$$p_0 = [100 \text{ kPa(abs)}]\left[1 + \left(\frac{1.31-1}{2}\right)(0.3363)^2\right]^{\frac{1.31}{1.31-1}} = 107.6 \text{ kPa(abs)}$$

and with Eq. 9 we get

$$p^* = [107.6 \text{ kPa(abs)}]\left(\frac{2}{1.31+1}\right)^{\frac{1.31}{1.31-1}} = 58.53 k$$

Then with Eq. 8 we have

$$\rho^* = \frac{\left(58.3 \times 10^3 \frac{N}{m^2}\right)}{\left(518.3 \frac{N \cdot m}{kg \cdot K}\right)(258.1 K)} = 0.438 \frac{kg}{m^3}$$

(con't)

11.47 (con't)

Finally, with Eq. 2 we obtain

$$\dot{m} = \left(0.438\ \frac{kg}{m^3}\right)\left(0.1\ m^2\right)\left(418.6\ \frac{m}{s}\right) = \underline{\underline{18.3}}\ \frac{kg}{s}$$

(c) For helium we use Eq. 1 to obtain

$$Ma_1 = \frac{\left(150\ \frac{m}{s}\right)}{\sqrt{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(293K)(1.66)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}}} = 0.1492$$

Thus the flow is choked at the throat. From Eq. 7 we obtain

$$T_0 = (293\ K)\left[1 + \left(\frac{1.66-1}{2}\right)(0.1492)^2\right] = 295.2\ K$$

and with Eq. 4 we obtain

$$T^* = (295.2\ K)\left(\frac{2}{1.66+1}\right) = 222\ K$$

Thus

$$V^* = \sqrt{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(222K)(1.66)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 874.9\ \frac{m}{s}$$

From Eq. 11 we have

$$p_0 = [100\ kPa(abs)]\left[1 + \left(\frac{1.66-1}{2}\right)(0.1492)^2\right]^{\frac{1.66}{1.66-1}} = 101.9\ kPa(abs)$$

and with Eq. 9

$$p^* = [101.9\ kPa(abs)]\left(\frac{2}{1.66+1}\right)^{\frac{1.66}{1.66-1}} = 49.74\ kPa(abs)$$

Then with Eq. 8

$$\rho^* = \frac{\left(49.74\times10^3\ \frac{N}{m^2}\right)}{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(222K)} = 0.1079\ \frac{kg}{m^3}$$

Finally with Eq. 2 we obtain

$$\dot{m} = \left(0.1079\ \frac{kg}{m^3}\right)\left(0.1\ m^2\right)\left(874.9\ \frac{m}{s}\right) = \underline{\underline{9.44}}\ \frac{kg}{s}$$

11.48

11.48 The flow blockage associated with the use of an intrusive probe can be important. Determine the percentage increase in section velocity corresponding to a 0.5% reduction in flow area due to probe blockage for air flow if the section area is 1.0 m², T_0 = 20 °C, and the unblocked flow Mach numbers are **(a)** Ma = 0.2; **(b)** Ma = 0.8; **(c)** Ma = 1.5; **(d)** Ma = 3.0.

We want to ascertain

$$\frac{V_{blocked} - V_{unblocked}}{V_{unblocked}} \times 100$$

To determine the unblocked area velocity, $V_{unblocked}$, we use

$$V_{unblocked} = Ma_{unblocked}\sqrt{R T_{unblocked} k} \quad (1)$$

For $T_{unblocked}$ we use

$$T_{unblocked} = T_0 \left(\frac{T}{T_0} \text{ for } Ma_{unblocked} \text{ from Eq. 11.56 for } Ma_{unblocked} \right) \quad (2)$$

To determine the blocked area velocity, $V_{blocked}$, we use

$$V_{blocked} = Ma_{blocked}\sqrt{R T_{blocked} k} \quad (3)$$

For $Ma_{blocked}$ we use $\frac{A_{blocked}}{A^*}$ and determine $Ma_{blocked}$ from Eq. 11.71.

Solution of Eq. 11.71 for $Ma_{blocked}$ from $\frac{A_{blocked}}{A^*}$ requires trial and error.

To determine $\frac{A_{blocked}}{A^*}$ we set

$$\frac{A_{blocked}}{A^*} = 0.995 \frac{A_{unblocked}}{A^*} \quad (4)$$

We obtain $\frac{A_{unblocked}}{A^*}$ from Eq. 11.71 with the given value of $Ma_{unblocked}$.

To determine $T_{blocked}$ we use Eq. 11.56 to obtain

$$T_{blocked} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{blocked}^2} \quad (5)$$

(con't)

(a) For $Ma_{unblocked} = 0.2$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.99206) = 290.7\,K$$

Then with Eq. 1 we have

$$V_{unblocked} = (0.2)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.7\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.34\,\frac{ft}{s}$$

we use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(2.9635) = 2.949$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.201$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.201)^2} = 290.6\,K$$

With Eq. 3 we have

$$V_{blocked} = (0.201)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.6\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.67\,\frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})\times 100}{V_{unblocked}} = \frac{\left(68.67\,\frac{m}{s} - 68.34\,\frac{m}{s}\right)(100)}{68.34\,\frac{m}{s}} = \underline{\underline{0.483\,\%}}$$

(b) For $Ma = 0.8$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.88652) = 259.8\,K$$

Then with Eq. 1 we get

$$V_{unblocked} = 0.8\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(259.8\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 258.4\,\frac{m}{s}$$

we use Eqs. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(1.03823) = 1.033$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.813$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.813)^2} = 258.8\,K$$

(con't)

With Eq. 3 we have

$$V_{blocked} = (0.813)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(258.8\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 262.1\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(262.1\ \frac{m}{s} - 258.4\ \frac{m}{s}\right)}{\left(258.4\ \frac{m}{s}\right)}(100) = \underline{\underline{1.43\ \%}}$$

(c) For Ma = 1.5, we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.68965) = 202.1\,K$$

Then with Eq. 1 we get

$$V_{unblocked} = (1.5)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(202.1\ )(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 427.4\ \frac{m}{s}$$

We use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(1.1762) = 1.17$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 1.491$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(1.491)^2} = 202.8\,K$$

With Eq. 3 we have

$$V_{blocked} = (1.491)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(202.8\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 425.5\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(425.5\ \frac{m}{s} - 427.4\ \frac{m}{s}\right)(100)}{427.4\ \frac{m}{s}} = \underline{\underline{-0.445\%}}$$

(d) For Ma = 3.0 we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.35714) = 104.6\,K$$

Then with Eq. 1 we get

$$V_{unblocked} = (3.0)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(104.6\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 614.9\ \frac{m}{s}$$

(con't)

11.48 (Con't)

We use Eqs. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(4.2346) = 4.213$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 2.995$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\ K}{1+\left(\frac{1.4-1}{2}\right)(2.995)^2} = 104.9K$$

With Eq. 3 we have

$$V_{blocked} = (2.995)\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(104.9K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 614.8\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(614.8\ \frac{m}{s} - 614.9\ \frac{m}{s}\right)(100)}{\left(614.9\ \frac{m}{s}\right)} = -0.0163\%$$

11.50

11.50 For Fanno flow, prove that

$$\frac{dV}{V} = \frac{fk(\mathrm{Ma}^2/2)(dx/D)}{1 - \mathrm{Ma}^2}$$

and in so doing show that when the flow is subsonic, friction accelerates the fluid, and when the flow is supersonic, friction decelerates the fluid.

Starting with Eq. 11.95 we have

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \frac{d(Ma^2)}{Ma^2} + \frac{fk}{2}\,Ma^2\,\frac{dx}{D} = 0 \qquad (1)$$

From Eq. 11.93 we have

$$\frac{d(Ma^2)}{Ma^2} = \frac{d(V^2)}{V^2}\left[1 + \left(\frac{k-1}{2}\right)Ma^2\right] \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \left[1 + \left(\frac{k-1}{2}\right)Ma^2\right]\frac{d(V^2)}{V^2} + \frac{fk}{2}\,Ma^2\frac{dx}{D} = 0 \qquad (3)$$

or

$$\frac{1}{2}\left(Ma^2 - 1\right)\frac{d(V^2)}{V^2} = -\frac{fk}{2}\,Ma^2\,\frac{dx}{D}$$

and

$$\frac{d(V^2)}{V^2} = \frac{Ma^2}{(Ma^2-1)}\,fk\,\frac{dx}{D} \qquad (4)$$

However

$$d(V^2) = 2V\,dV \qquad (5)$$

Thus combining Eqs. 4 and 5 we get

$$\frac{dV}{V} = \frac{fk\left(\frac{Ma^2}{2}\right)\left(\frac{dx}{D}\right)}{1 - Ma^2} \qquad (6)$$

When the flow is subsonic ($Ma < 1.0$), Eq. 6 leads to $\frac{dV}{V} = +$ and thus friction accelerates the fluid. On the other hand when the flow is supersonic ($Ma > 1.0$), Eq. 6 leads to $\frac{dV}{V} = -$ and in this case friction decelerates the fluid.

11.51 Standard atmospheric air ($T_0 = 59$ °F, $p_0 = 14.7$ psia) is drawn steadily through a frictionless and adiabatic converging nozzle into an adiabatic, constant cross section area duct. The duct is 10 ft long and has an inside diameter of 0.5 ft. The average friction factor for the duct may be estimated as being equal to 0.03. What is the maximum mass flowrate in slugs/s through the duct? For this maximum flowrate determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

This is similar to Example 11.12. As explained in Example 11.12, the maximum flowrate through the duct will occur when the constant area duct chokes and the Mach number at the duct exit [section (2)] is 1.0. The maximum flowrate can be obtained with

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (1)$$

We note that T_0 is constant throughout the entire flow since the flow is adiabatic. Thus, $T_{0,1} = T_{0,2} = 519\,°R$. Also, p_0 is constant in the converging nozzle but decreases through the constant area duct because of friction. Thus, $p_{0,1} = 14.7$ psia.

For choked flow

$$\frac{f(\ell_2 - \ell_1)}{D} = \frac{(0.03)(10\text{ ft})}{0.5\text{ ft}} = 0.6 = \frac{f(\ell^* - \ell_1)}{D}$$

and from Fig. D.2

we can read values of Ma_1, $\frac{T_1}{T^*}$, $\frac{V_1}{V^*}$, $\frac{p_1}{p^*}$ and $\frac{p_{0,1}}{p^*}$. Then $T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right)T_0 = \left(\frac{2}{1.4+1}\right)(519\,°R) = \underline{\underline{432\,°R}} = T_2$$

and $V^* = V_2$ can be determined with

$$V^* = \sqrt{RT^*k} = \sqrt{\frac{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(432\,°R)(1.4)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = \underline{\underline{1020\,\frac{\text{ft}}{\text{s}}}} = V_2$$

(con't)

For $\frac{f(\ell^*-\ell)}{D} = 0.6$, from Fig. D.2 we read

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.6 \qquad (3)$$

$$\frac{p_1}{p^*} = 1.86 \qquad (4)$$

$$\frac{p_{0,1}}{p_0^*} = 1.22 \qquad (5)$$

From Eq. 2 we get

$$T_1 = (1.13)(432^\circ R) = \underline{\underline{488^\circ R}}$$

With Eq. 3 we obtain

$$V_1 = (0.6)\left(1020\ \frac{ft}{s}\right) = \underline{\underline{612}}\ \frac{ft}{s}$$

With Eq. 5 we have

$$p_0^* = p_{0,2} = \frac{p_{0,1}}{1.22} = \frac{14.7\text{ psia}}{1.22} = \underline{\underline{12}}\text{ psia}$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.57$ and read

$$\frac{p_1}{p_{0,1}} = 0.8$$

and

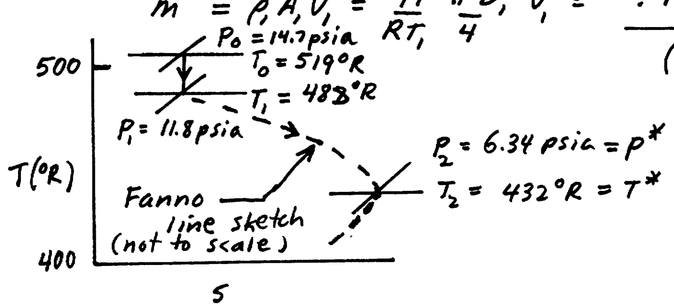
$$p_1 = (0.8)(14.7\text{ psia}) = 11.8\text{ psia}$$

With Eq. 4 we obtain

$$p^* = p_2 = \frac{p_1}{1.86} = \frac{11.8\text{ psia}}{1.86} = 6.34\text{ psia}$$

With Eq. 1 we have

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(11.8\text{ psia})\left(144\ \frac{in.^2}{ft^2}\right)\pi(0.5\text{ ft})^2\left(612\ \frac{ft}{s}\right)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(488^\circ R)(4)} = \underline{\underline{0.244\ \frac{slug}{s}}}$$



11.52

11.52 The upstream pressure of a Fanno flow venting to the atmosphere is increased until the flow chokes. What will happen to the flowrate when the upstream pressure is further increased?

For a Fanno flow

$$\rho V = \frac{p}{RT} Ma\sqrt{RTk} = \text{constant}$$

Also at any one axial location in the flow, from Eq. 11.56

$$\frac{T}{T_0} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}$$

Combining we get

$$\rho V = \frac{p}{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]} Ma \sqrt{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]k} = \text{constant}$$

So for any one axial location of the flow where the Ma level is the same, T_0 is also the same but p is higher. Thus ρV is also higher and we conclude that increasing the inlet pressure of a choked Fanno flow into the atmosphere results in an increase of flowrate also.

Following the procedure of Example 11.11 one could plot a series of Fanno lines for different values of increased inlet pressure.

11.53

11.53 The duct in Problem 11.51 is shortened by 50%. The duct discharge pressure is maintained at the choked flow value determined in Problem 11.51. Determine the change in mass flowrate through the duct associated with the 50% reduction in length. The average friction factor remains constant at a value of 0.03.

This is like Example 11.13. We guess that the shortened duct will still choke and check our assumption by comparing p_d with p^*. If $p_d < p^*$, the flow is choked. If not, another assumption must be made. For choked flow we calculate the mass flowrate as we did in Example 11.12 or in the solution of problem 11.51. For unchoked flow, we must devise another strategy.

For choked flow

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{(0.03)(5\,ft)}{(0.5\,ft)} = 0.3 = \frac{f(\ell^*-\ell)}{D}$$

From Fig. D.2 we read

$$Ma_1 = 0.66$$

$$\frac{T_1}{T^*} = 1.1 \qquad (1)$$

$$\frac{V_1}{V^*} = 0.7 \qquad (2)$$

$$\frac{p_1}{p^*} = 1.6 \qquad (3)$$

With $Ma_1 = 0.66$, we enter Fig. D.1 and read

$$\frac{p_1}{p_{0,1}} = 0.75$$

Thus

$$p_1 = (0.75)(14.7\,psia) = 11 \quad psia$$

(con't)

11.53 (con't)

and with Eq. 3 we obtain

$$p^* = p_2 = \frac{p_1}{1.6} = \frac{11 \text{ psia}}{1.6} = 6.88 \text{ psia}$$

Since

$$p_2 = 6.88 \text{ psia} > p_d = 6.34 \text{ psia}$$

the flow is choked as assumed.

$T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right) T_0 = \left(\frac{2}{1.4+1}\right)(519°R) = 432°R = T_2$$

and $V_2 = V^*$ can be determined with

$$V_2 = V^* = \sqrt{RT^*k} = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432°R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1020 \frac{ft}{s}$$

With Eq. 1 we have

$$\dot{m} = \rho_2 A_2 V_2 = \frac{p_2}{RT_2} \frac{\pi D_2^2}{4} V_2 = \frac{(6.88 \text{ psia})\left(144 \frac{in.^2}{ft^2}\right)}{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432°R)} \frac{\pi (0.5 ft)^2}{(4)} \left(1020 \frac{ft}{s}\right)$$

or

$$\dot{m} = 0.268 \frac{slug}{s}$$

The change in mass flowrate is

$$\left(\frac{\dot{m}_{5\,ft} - \dot{m}_{10\,ft}}{\dot{m}_{10\,ft}}\right) \times 100 = \left(\frac{0.268 \frac{slug}{s} - 0.244 \frac{slug}{s}}{0.244 \frac{slug}{s}}\right)(100) = \underline{\underline{9.8\,\%}}$$

The mass flowrate increased by 9.8% when the tube was shortened by 50%.

11.54 If the same mass flowrate of air obtained in Problem 11.51 is desired through the shortened duct of Problem 11.53, determine the back pressure, p_2, required. Assume f remains constant at a value of 0.03.

This is similar to Example 11.14. Since the same mass flowrate achieved in Problem 11.51 is desired with the shortened duct of Problem 11.53, we need to achieve the value of Ma_1 obtained in Problem 11.51. Thus, for the same value of Ma_1 as in Problem 11.51 we have

$$f\frac{(\ell^* - \ell_1)}{D} = 0.6$$

However,

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_2 - \ell_1)}{D}$$

or

$$\frac{f(\ell^* - \ell_2)}{D} = 0.6 - \frac{(0.03)(5\text{ ft})}{0.5\text{ ft}} = 0.3$$

with $\frac{f(\ell^* - \ell_2)}{D} = 0.3$ we enter Fig. D.2 and read

$$\frac{p_2}{p^*} = 1.6 \qquad (1)$$

The value of p^* obtained in Problem 11.53 is still valid, so

$$p^* = 6.88 \text{ psia}$$

and with Eq. 1 we get

$$p_2 = (1.6)(6.88 \text{ psia}) = \underline{\underline{11}} \text{ psia}$$

11.55

11.55 If the average friction factor of the duct of Example 11.12 is changed to **(a)** 0.01 or **(b)** 0.03, determine the maximum mass flowrate of air through the duct associated with each new friction factor and compare with the maximum mass flowrate value of Example 11.12.

(a) For $f = 0.01$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.01)(2\,m)}{(0.1\,m)} = 0.2$$

and on Fig. D.2 we read

$$Ma_1 = 0.7 \qquad (1)$$

$$\frac{T_1}{T^*} = 1.1 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.73$$

From Example 11.12

$$T^* = 240\,K$$

and

$$V^* = 310\,\frac{m}{s}$$

Thus, with Eq. 1 we get

$$T_1 = (1.1)(240\,K) = 264\,K$$

and with Eq. 2 we obtain

$$V_1 = (0.73)(310\,\frac{m}{s}) = 226\,\frac{m}{s}$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.7$ and read

$$\frac{p_1}{p_{0,1}} = 0.72$$

Thus,

$$p_1 = (0.72)[101\,kPa\,(abs)] = 72.7\,kPa\,(abs)$$

To determine the mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(72.7\,10^3\,\frac{N}{m^2})\,\pi\,(0.1\,m)^2(226\,\frac{m}{s})}{(286.9\,\frac{N\cdot m}{kg\cdot K})(264\,K)(4)} = \underline{\underline{1.7}}\,\frac{kg}{s}$$

(con't)

11.55 (con't)

For $f = 0.03$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.03)(2m)}{(0.1m)} = 0.6$$

and on Fig. D.2 we read

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13$$

$$\frac{V_1}{V^*} = 0.6$$

Thus,

$$T_1 = (1.13)(240K) = 271K$$

$$V_1 = (0.6)(310\ \tfrac{m}{s}) = 186\ \tfrac{m}{s}$$

From Fig. D.1 we read for $Ma_1 = 0.57$

$$\frac{p_1}{p_{0,1}} = 0.8$$

Thus,

$$p_1 = (0.8)[101\ kPa(abs)] = 81\ kPa(abs)$$

To determine $\dot{m}$ we use

$$\dot{m} = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(81\times10^3\ \frac{N}{m^2})\pi(0.1m)^2(186\ \frac{m}{s})}{(286.9\ \frac{N\cdot m}{kg\cdot K})(271K)(4)} = \underline{\underline{1.52\ \frac{kg}{s}}}$$

The maximum (choked duct) flowrates for different values of f are

$$\dot{m}_{f=0.01} = 1.70\ \frac{kg}{s}$$

$$\dot{m}_{f=0.02} = 1.65\ \frac{kg}{s}$$

$$\dot{m}_{f=0.03} = 1.52\ \frac{kg}{s}$$

11.56

11.56 If the length of the constant area duct of Example 11.12 is changed to **(a)** 1 m or **(b)** 3 m, and all other specifications in the problem statement remain the same, determine the maximum mass flowrate of air through the duct associated with each new length and compare with the maximum mass flowrate of Example 11.12.

For maximum flowrate the duct is choked.

(a) For $\ell_2 - \ell_1 = 1\,m$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell_2 - \ell_1)}{D} = \frac{(0.02)(1\,m)}{0.1\,m} = 0.2$$

From Fig. D.2

$$Ma_1 = 0.7$$

$$\frac{T_1}{T^*} = 1.1 \quad (1)$$

$$\frac{V_1}{V^*} = 0.73 \quad (2)$$

From Example 11.12

$$T^* = 240\,K$$

and

$$V^* = 310\,\frac{m}{s}$$

Thus, with Eq. 1 we obtain

$$T_1 = (1.1)(240\,K) = 264\,K$$

and with Eq. 2 we get

$$V_1 = (0.73)(310\,\frac{m}{s}) = 226\,\frac{m}{s}$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.70$ and read

$$\frac{p_1}{p_{0,1}} = 0.72$$

Thus

$$p_1 = (0.72)[101\,kPa(abs)] = 72.7\,kPa(abs)$$

(con't)

11.56 (con't)

To determine mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(72.7\times10^3 \frac{N}{m^2})\pi(0.1m)^2(226\frac{m}{s})}{(286.9\ \frac{N\cdot m}{kg\cdot K})(264K)(4)} = \underline{\underline{1.70\ \frac{kg}{s}}}$$

(b) For $\ell_2 - \ell_1 = 3\,m$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell_2-\ell_1)}{D} = \frac{(0.02)(3m)}{(0.1m)} = 0.6$$

From Fig. D.2

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13$$

$$\frac{V_1}{V^*} = 0.6$$

Thus

$$T_1 = (1.13)(240K) = 271K$$

$$V_1 = (0.6)(310\frac{m}{s}) = 186\frac{m}{s}$$

From Fig. D.1 we read for $Ma_1 = 0.57$

$$\frac{p_1}{p_{0,1}} = 0.8$$

Thus

$$p_1 = (0.8)[101\ kPa(abs)] = 81\ kPa(abs)$$

To determine $\dot{m}$ we use

$$\dot{m} = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(81\times10^3\frac{N}{m^2})\pi(0.1m)^2(186\frac{m}{s})}{(286.9\ \frac{N\cdot m}{kg\cdot K})(271K)(4)} = \underline{\underline{1.52\ \frac{kg}{s}}}$$

Thus the maximum (choked flow) flowrates for different values of $\ell_2-\ell_1$ are

$$\dot{m}_{\ell_2-\ell_1=1m} = 1.70\ \frac{kg}{s}$$

$$\dot{m}_{\ell_2-\ell_1=2m} = 1.65\ \frac{kg}{s}$$

$$\dot{m}_{\ell_2-\ell_1=3m} = 1.52\ \frac{kg}{s}$$

11.57 The duct of Example 11.12 is lengthened by 50%. If the duct discharge pressure is set at a value of $p_d = 46.2$ kPa (abs), determine the mass flowrate of air through the lengthened duct. The average friction factor for the duct remains constant at a value of 0.02.

This is a trial and error solution. We begin with

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D}$$

or

$$\frac{(0.02)(3m)}{0.1m} = 0.6 = \frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D} \qquad (1)$$

We guess a value of Ma_2 and get a corresponding value of $\frac{f(\ell^*-\ell_2)}{D}$ from Fig. D.2. With Eq. 1 we then calculate a value of $\frac{f(\ell^*-\ell_1)}{D}$ and with this value of $\frac{f(\ell^*-\ell_1)}{D}$ we get from Fig. D.2 a value of Ma_1. With Ma_1 and Ma_2 we obtain from Figs. D.1 and D.2, $\frac{p_1}{p_{0,1}}$, $\frac{p_1}{p^*}$, and $\frac{p_2}{p^*}$. Then with

$$p_2 = p_{0,1}\left(\frac{p_1}{p_{0,1}}\right)\left(\frac{p^*}{p_1}\right)\left(\frac{p_2}{p^*}\right) \qquad (2)$$

we get a value of p_2 that we can compare with $p_2 = 46.2$ kPa(abs).
We guess a value of $Ma_2 = 0.95$. Corresponding to $Ma_2 = 0.95$ we read on Fig. D.2

$$\frac{f(\ell^*-\ell_2)}{D} = 0$$

Then with Eq. 1 we obtain

$$\frac{f(\ell^*-\ell_1)}{D} = 0.6 + 0.00328 = 0.6$$

for which from Fig. D.2

$$Ma_1 = 0.57$$

(con't)

Now with Eq. 2 we get

$$P_2 = [101\ kPa(abs)](0.81)\left(\frac{1}{1.86}\right)(1.06) = 46.6\ kPa(abs)$$

which is close enough to the given discharge pressure for us to accept the assumption of $Ma_2 = 0.95$.

Thus, $P_1 = \left(\frac{P_1}{P_{0,1}}\right) P_{0,1} = (0.81)\ [101\ kPa(abs)] = 82\ kPa(abs)$

and

$$T_1 = \left(\frac{T_1}{T_{0,1}}\right) T_{0,1} = (0.94)(288K) = 271\ K$$

Also from Fig. D.2 for $Ma_1 = 0.57$

$$\frac{V_1}{V^*} = 0.61$$

and thus

$$V_1 = (0.61)(310\ \tfrac{m}{s}) = 189\ \tfrac{m}{s}$$

Finally,

$$\dot{m} = \rho_1 A_1 V_1 = \frac{P_1}{RT_1}\frac{\pi D_1^2}{4} V_1 = \frac{(81{,}000\ \frac{N}{m^2})\pi(0.1m)^2(189\ \frac{m}{s})}{(286.9\ \frac{N \cdot m}{kg \cdot K})(271K)(4)}$$

and

$$\dot{m} = \underline{\underline{1.55}}\ \frac{kg}{s}$$

11.58 An ideal gas flows adiabatically with friction through a long, constant cross section area pipe. At upstream section (1), $p_1 = 60$ kPa (abs), $T_1 = 60$ °C and $V_1 = 200$ m/s. At downstream section (2), $T_2 = 30$ °C. Determine p_2, V_2, and the stagnation pressure ratio $p_{0,2}/p_{0,1}$ if the gas is **(a)** air; **(b)** helium.

(a) For air we first determine the Mach number at section (1) with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} = \frac{\left(200\ \frac{m}{s}\right)}{\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(333\,K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.55$$

Entering Fig. D.2 with $Ma_1 = 0.55$ we find

$$\frac{T_1}{T^*} = 1.14$$

$$\frac{p_1}{p^*} = 1.9 \qquad (1)$$

and

$$\frac{p_{0,1}}{p_0^*} = 1.25 \qquad (2)$$

From the temperature ratio above we obtain

$$T^* = \frac{T_1}{1.14} = \frac{(333\,K)}{(1.14)} = 292\,K$$

Thus

$$\frac{T_2}{T^*} = \frac{(303\,K)}{(292\,K)} = 1.04$$

We enter Fig. D.2 with $\frac{T_2}{T^*} = 1.04$ and read

$$Ma_2 = 0.9$$

$$\frac{p_2}{p^*} = 1.1 \qquad (3)$$

$$\frac{p_{0,2}}{p_0^*} = 1 \qquad (4)$$

(con't)

11.58 (con't)

With

$$p_2 = p_1\left(\frac{p^*}{p_1}\right)\left(\frac{p_2}{p^*}\right)$$

and Eqs. 1 and 3 we obtain

$$p_2 = [60\text{ kPa(abs)}]\left(\frac{1}{1.9}\right)(1.1\ \ ) = \underline{\underline{34.7\text{ kPa (abs)}}}$$

Also, with

$$\frac{p_{0,2}}{p_{0,1}} = \left(\frac{p_{0,2}}{p_0^*}\right)\left(\frac{p_0^*}{p_{0,1}}\right)$$

and Eqs. 2 and 4 we obtain

$$\frac{p_{0,2}}{p_{0,1}} = (1\qquad)\left(\frac{1}{1.25}\right) = \underline{\underline{0.8}}$$

Finally

$$V_2 = Ma_2\sqrt{RT_2k} = (0.9\ )\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(303K)(1.4)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{314}}\ \frac{m}{s}$$

(b) For helium, $k=1.66$ and $R = 2077\ \frac{N\cdot m}{kg\cdot K}$ from Table 1.8.
The Mach number at section (1) is

$$Ma_1 = \frac{V_1}{\sqrt{RT_1k}} = \frac{\left(200\ \frac{m}{s}\right)}{\sqrt{\frac{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(333K)(1.66)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}}} = 0.187$$

With $Ma_1 = 0.187$ and Eqs. 11.101, 11.107 and 11.109 we obtain

$$\frac{T_1}{T^*} = \frac{\left(\frac{k+1}{2}\right)}{1+\left(\frac{k-1}{2}\right)Ma_1^2} = \frac{\left(\frac{1.66+1}{2}\right)}{1+\left(\frac{1.66-1}{2}\right)(0.187)^2} = 1.315 \qquad (5)$$

$$\frac{p_1}{p^*} = \frac{1}{Ma_1}\left[\frac{\left(\frac{k+1}{2}\right)}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{1}{2}} = \frac{1}{(0.187)}\left[\frac{\left(\frac{1.66+1}{2}\right)}{1+\left(\frac{1.66-1}{2}\right)(0.187)^2}\right]^{\frac{1}{2}} = 6.13 \qquad (6)$$

$$\frac{p_{0,1}}{p_0^*} = \frac{1}{Ma_1}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}Ma_1^2\right)\right]^{\frac{(k+1)}{2(k-1)}} = \frac{1}{(0.187)}\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(0.187)^2\right]\right\}^{\frac{1.66+1}{2(1.66-1)}}$$

or

$$\frac{p_{0,1}}{p_0^*} = 3.08 \qquad (7)$$

(con't)

11.58 | (con't)

With Eq. 5 we obtain

$$T^* = \frac{(333K)}{(1.315)} = 253.2K$$

Thus

$$\frac{T_2}{T^*} = \frac{(303K)}{(253.2K)} = 1.197$$

With $\frac{T_2}{T^*} = 1.197$ we use Eqs. 11.101, 11.107 and 11.109 to obtain

$$Ma_2 = \sqrt{\left[\frac{\left(\frac{k+1}{2}\right)}{\left(\frac{T_2}{T^*}\right)} - 1\right]\left(\frac{2}{k-1}\right)} = \sqrt{\left[\frac{\left(\frac{1.66+1}{2}\right)}{(1.197)} - 1\right]\left(\frac{2}{1.66-1}\right)} = 0.5803$$

$$\frac{p_2}{p^*} = \frac{1}{Ma_2}\left[\frac{\left(\frac{k+1}{2}\right)}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]^{\frac{1}{2}} = \left(\frac{1}{0.5803}\right)\left[\frac{\left(\frac{1.66+1}{2}\right)}{1+\left(\frac{1.66-1}{2}\right)(0.5803)^2}\right]^{\frac{1}{2}} = 1.885 \qquad (8)$$

$$\frac{p_{0,2}}{p_0^*} = \frac{1}{Ma_2}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}Ma_2^2\right)\right]^{\frac{k+1}{2(k-1)}} = \left(\frac{1}{0.5803}\right)\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(0.5803)^2\right]\right\}^{\frac{1.66+1}{2(1.66-1)}} = 1.2 \qquad (9)$$

With

$$p_2 = p_1\left(\frac{p^*}{p_1}\right)\left(\frac{p_2}{p^*}\right)$$

and Eqs. 6 and 8 we have

$$p_2 = [60\,kPa(abs)]\left(\frac{1}{6.13}\right)(1.885) = \underline{\underline{18.4}}\,kPa(abs)$$

With

$$\frac{p_{0,2}}{p_{0,1}} = \left(\frac{p_{0,2}}{p_0^*}\right)\left(\frac{p_0^*}{p_{0,1}}\right)$$

and Eqs. 7 and 9 we have

$$\frac{p_{0,2}}{p_{0,1}} = (1.2)\left(\frac{1}{3.08}\right) = \underline{\underline{0.39}}$$

Finally,

$$V_2 = Ma_2\sqrt{RT_2k} = (0.5803)\sqrt{\frac{\left(2077\,\frac{N\cdot m}{kg\cdot K}\right)(303K)(1.66)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}}$$

or

$$V_2 = \underline{\underline{593}}\,\frac{m}{s}$$

11.59 For the air flow of Problem 11.58, determine T, p, and V for the section halfway between sections (1) and (2).

If section (A) is placed halfway between sections (1) and (2) we have

$$\frac{f(\ell^*-\ell_A)}{D} = \frac{\frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D}}{2} + \frac{f(\ell^*-\ell_2)}{D} \qquad (1)$$

and with $\frac{f(\ell^*-\ell_A)}{D}$ we enter Fig. D.2 and read corresponding values of Ma_A, $\frac{T_A}{T^*}$ and $\frac{p_A}{p^*}$. With $T^* = 294K$ from the solution of Problem 11.58 we obtain

$$T_A = \left(\frac{T_A}{T^*}\right)(294K) \qquad (2)$$

We get p_A from

$$p_A = \left(\frac{p_A}{p^*}\right)\left(\frac{p^*}{p_1}\right)p_1 \qquad (3)$$

where $\frac{p_1}{p^*}$ was obtained in the solution of Problem 11.58

Finally, V_A is obtained with

$$V_A = Ma_A\sqrt{RT_A k} \qquad (4)$$

To determine $\frac{f(\ell^*-\ell_1)}{D}$ and $\frac{f(\ell^*-\ell_2)}{D}$ we enter Fig. D.2 with $Ma_1 = 0.55$ and $Ma_2 = 0.91$ from the solution of Problem 11.58 and read

$$\frac{f(\ell^*-\ell_1)}{D} = 0.7$$

and

$$\frac{f(\ell^*-\ell_2)}{D} = 0.01$$

Then, with Eq. 1

$$\frac{f(\ell^*-\ell_A)}{D} = \frac{(0.7 - 0.01)}{2} + 0.01 = 0.36$$

(con't)

11.59 (con't)

With $\frac{f(\ell^*-\ell_A)}{D} = 0.36$ we enter Fig. D.2 and read

$$Ma_A = 0.63$$

$$\frac{T_A}{T^*} = 1.1$$

$$\frac{p_A}{p^*} = 1.7$$

Now with Eq. 2 we obtain

$$T_A = (1.1)(294\,K) = \underline{\underline{327\,K}}$$

and with Eq. 3 we get

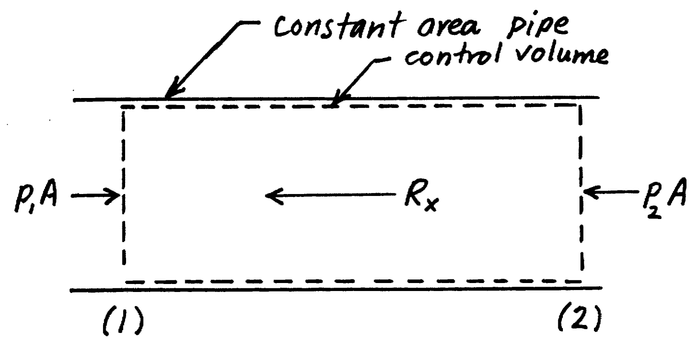
$$p_A = (1.7)\left(\frac{1}{1.9}\right)\left[60\,kPa(abs)\right] = \underline{\underline{54}}\ kPa\,(abs)$$

Finally with Eq. 4 we have

$$V_A = (0.63)\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(323K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = \underline{\underline{227}}\ \frac{m}{s}$$

11.60

11.60 An ideal gas flows adiabatically between two sections in a constant cross section area pipe. At upstream section (1), $p_{0,1}$ = 100 psia, $T_{0,1}$ = 600 °R, and Ma_1 = 0.5. At downstream section (2), the flow is choked. Determine the magnitude of the force per unit cross section area exerted by the inside wall of the pipe on the fluid between sections (1) and (2) if the gas is **(a)** air; **(b)** helium.



flow from (1) to (2)

The control volume sketched above is used. Applying the axial component of the linear momentum equation (Eq. 5.22) to the contents of this control volume we get for the force exerted by the pipe wall on the fluid, R_x,

$$R_x = p_1 A - p_2 A + \dot{m}(V_1 - V_2)$$

or

$$\frac{R_x}{A} = p_1 - p_2 + \rho_1 V_1 (V_1 - V_2) \qquad (1)$$

Thus we need p_1, p_2, ρ_1, V_1 and V_2.

(a) For air we enter Fig. D.1 with $\mathrm{Ma}_1 = 0.5$ and get

$$\frac{T_1}{T_{0,1}} = 0.95$$

and

$$\frac{p_1}{p_{0,1}} = 0.84$$

Thus

$$T_1 = (0.95)(600\,°R) = 570\,°R$$

and

$$p_1 = (0.84)(100\text{ psia}) = 84\text{ psia}$$

Then

$$V_1 = \mathrm{Ma}_1\sqrt{RT_1 k} = (0.5)\sqrt{\frac{\left(1716\ \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(570\,°R)(1.4)}{\left(1\ \frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 585\ \frac{\text{ft}}{\text{s}}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(84\text{ psia})\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(1716\ \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot°R}\right)(570\,°R)} = 0.0124\ \frac{\text{slug}}{\text{ft}^3}$$

(con't)

11.60 | (con't)

At section (2) the flow is choked. Thus we use the * state of the Fanno flow, Fig. D.2 for section (2). Entering Fig. D.2 with $Ma_1 = 0.5$ we read

$$\frac{p_1}{p^*} = 2.14 = \frac{p_1}{p_2}$$

and

$$\frac{V_1}{V^*} = 0.54 = \frac{V_1}{V_2}$$

Thus

$$p_2 = \frac{p_1}{2.14} = \frac{(84.3\ psia)}{(2.14\ )} = 39.4\ psia$$

and

$$V_2 = \frac{V_1}{0.54} = \frac{(586\ \frac{ft}{s})}{(0.54\ )} = 1080\ \frac{ft}{s}$$

Now with Eq. 1 we have

$$\frac{R_x}{A} = (84\ psia)\left(144\ \frac{in.^2}{ft^2}\right) - (39.4\ psia)\left(144\ \frac{in.^2}{ft^2}\right) + \left(0.0124\ \frac{slug}{ft^3}\right)\left(585\ \frac{ft}{s}\right)\left(585\ \frac{ft}{s} - 1080\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$\frac{R_x}{A} = \underline{\underline{2830}}\ \frac{lb}{ft^2}$$

(b) For helium ($R = 12{,}420\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}$ and $k = 1.66$ from Table 1.7) we use Eqs. 11.56 and 11.59 with $Ma_1 = 0.5$ and obtain

$$T_1 = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_1^2} = \frac{600\ {}^\circ R}{1 + \left(\frac{1.66-1}{2}\right)(0.5)^2} = 554\ {}^\circ R$$

and

$$p_1 = p_0 \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_1^2}\right]^{\frac{k}{k-1}} = (100\ psia)\left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)(0.5)^2}\right]^{\frac{1.66}{1.66-1}} = 81.9\ psia$$

Then

$$V_1 = Ma_1 \sqrt{R T_1 k} = (0.5)\sqrt{\frac{\left(12{,}420\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(554\ {}^\circ R)(1.66)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1690\ \frac{ft}{s}$$

and

$$\rho_1 = \frac{p_1}{R T_1} = \frac{(81.9\ psia)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(12{,}420\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(554\ {}^\circ R)} = 0.00171\ \frac{slug}{ft^3}$$

(con't)

11.60 (con't)

At section (2) the flow is choked. Thus, we use the * state of Fanno flow for section (2). With Eqs. 11.107 and 11.103 and $Ma_1 = 0.5$ we get

$$p^* = \frac{P_1}{\frac{1}{Ma_1}\left[\frac{\left(\frac{k+1}{2}\right)}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{1}{2}}} = \frac{81.9\ psia}{\frac{1}{(0.5)}\left[\frac{\left(\frac{1.66+1}{2}\right)}{1+\left(\frac{1.66-1}{2}\right)(0.5)^2}\right]^{\frac{1}{2}}}$$

or

$$p^* = 36.9\ psia = P_2$$

and

$$V^* = \frac{V_1}{\left[\frac{\left(\frac{k+1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{1}{2}}} = \frac{\left(1690\ \frac{ft}{s}\right)}{\left[\frac{\left(\frac{1.66+1}{2}\right)(0.5)^2}{1+\left(\frac{1.66-1}{2}\right)(0.5)^2}\right]^{\frac{1}{2}}} = 3050\ \frac{ft}{s} = V_2$$

Now with Eq. 1 we have

$$\frac{R_x}{A} = (81.9\,psia - 36.9\,psia)\left(144\ \frac{in.^2}{ft^2}\right) + \left(0.00171\ \frac{slug}{ft^3}\right)\left(1690\ \frac{ft}{s}\right)\left(1690\ \frac{ft}{s} - 3050\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

and

$$\frac{R_x}{A} = \underline{\underline{2550}}\ \frac{lb}{ft^2}$$

11.61 An ideal gas enters [section (1)] a frictionless, constant flow cross section area duct with the following properties:

$$T_0 = 293 \text{ K}$$
$$p_0 = 101 \text{ kPa (abs)}$$
$$\text{Ma}_1 = 0.2$$

For Rayleigh flow, determine corresponding values of fluid temperature and entropy change for various levels of pressure and plot the Rayleigh line if the gas is helium.

This is similar to Example 11.15.

To plot the Rayleigh line asked for we use Eq. 11.111

$$p + \frac{(\rho V)^2 RT}{p} = \text{constant} = p_1 + \frac{(\rho_1 V_1)^2 RT_1}{p_1} \qquad (1)$$

and Eq. 11.76

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1} \qquad (2)$$

to construct a table of values of temperature and entropy change corresponding to different levels of pressure downstream in a Rayleigh flow.

To determine p_1 we use Ma_1 and Eq. 11.59 to obtain $\frac{p_1}{p_{0,1}}$ and then p_1 from

$$p_1 = \left(\frac{p_1}{p_{0,1}}\right) p_{0,1} \qquad (3)$$

To determine T_1 we use Ma_1 and Eq. 11.56 to obtain $\frac{T_1}{T_{0,1}}$ and then T_1 from

$$T_1 = \left(\frac{T_1}{T_{0,1}}\right) T_{0,1} \qquad (4)$$

We obtain $\rho_1 V_1 = \rho V = \text{constant}$ from

$$\rho_1 V_1 = \frac{p_1}{RT_1} V_1 = \frac{p_1}{RT_1} Ma_1 \sqrt{RT_1 k} \qquad (5)$$

(con't)

11.61 (con't)

For helium ($k = 1.66$ and $R = 2077\ \frac{J}{kg\cdot K}$ from Table 1.8) we use $Ma_1 = 0.2$ and Eqs. 11.59 and 3 to get

$$p_1 = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k}{k-1}} p_{0,1} = \left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(0.2)^2}\right]^{\frac{1.66}{1.66-1}}\left[101\ kPa(abs)\right] = 97.72\ kPa(abs)$$

We use $Ma_1 = 0.2$ and Eqs. 11.56 and 4 to get

$$T_1 = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right] T_{0,1} = \left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(0.2)^2}\right](293K) = 289.2\ K$$

With Eq. 5 we have

$$\rho_1 V_1 = \frac{\left(97.72\times 10^3\ \frac{N}{m^2}\right)(0.2)}{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(289.2K)}\sqrt{\frac{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(289.2K)(1.66)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 32.49\ \frac{kg}{m^2\cdot s} = \rho V$$

Now Eq. 1 becomes

$$p + \frac{\left(32.49\ \frac{kg}{m^2\cdot s}\right)^2\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}\frac{T}{p} = 97.72\times 10^3\ \frac{N}{m^2} + \frac{\left(32.49\ \frac{kg}{m\cdot s^2}\right)^2\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(289.2K)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)}{\left(97.72\times 10^3\ \frac{N}{m^2}\right)}$$

or

$$p + \left(2.192\times 10^6\ \frac{N^2}{m^4\cdot K}\right)\frac{T}{p} = 1.042\times 10^5\ \frac{N}{m^2}$$

and

$$T = \frac{p}{\left(2.192\times 10^6\ \frac{N^2}{m^4\cdot K}\right)}\left(1.042\times 10^5\ \frac{N}{m^2} - p\right) \qquad (8)$$

With Eq. 2 we have

$$s - s_1 = \left(5224\ \frac{J}{kg\cdot K}\right)\ln\left(\frac{T}{289.2K}\right) - \left(2077\ \frac{J}{kg\cdot K}\right)\ln\left[\frac{p}{97.72\ kPa(abs)}\right] \qquad (9)$$

So for $p = 90\ kPa(abs)$ we use Eqs. 8 and 9 to obtain

$$T = \frac{\left(90\times 10^3\ \frac{N}{m^2}\right)}{\left(2.192\times 10^6\ \frac{N^2}{m^4\cdot K}\right)}\left(1.042\times 10^5\ \frac{N}{m^2} - 90\times 10^3\ \frac{N}{m^2}\right) = 583\ K$$

and

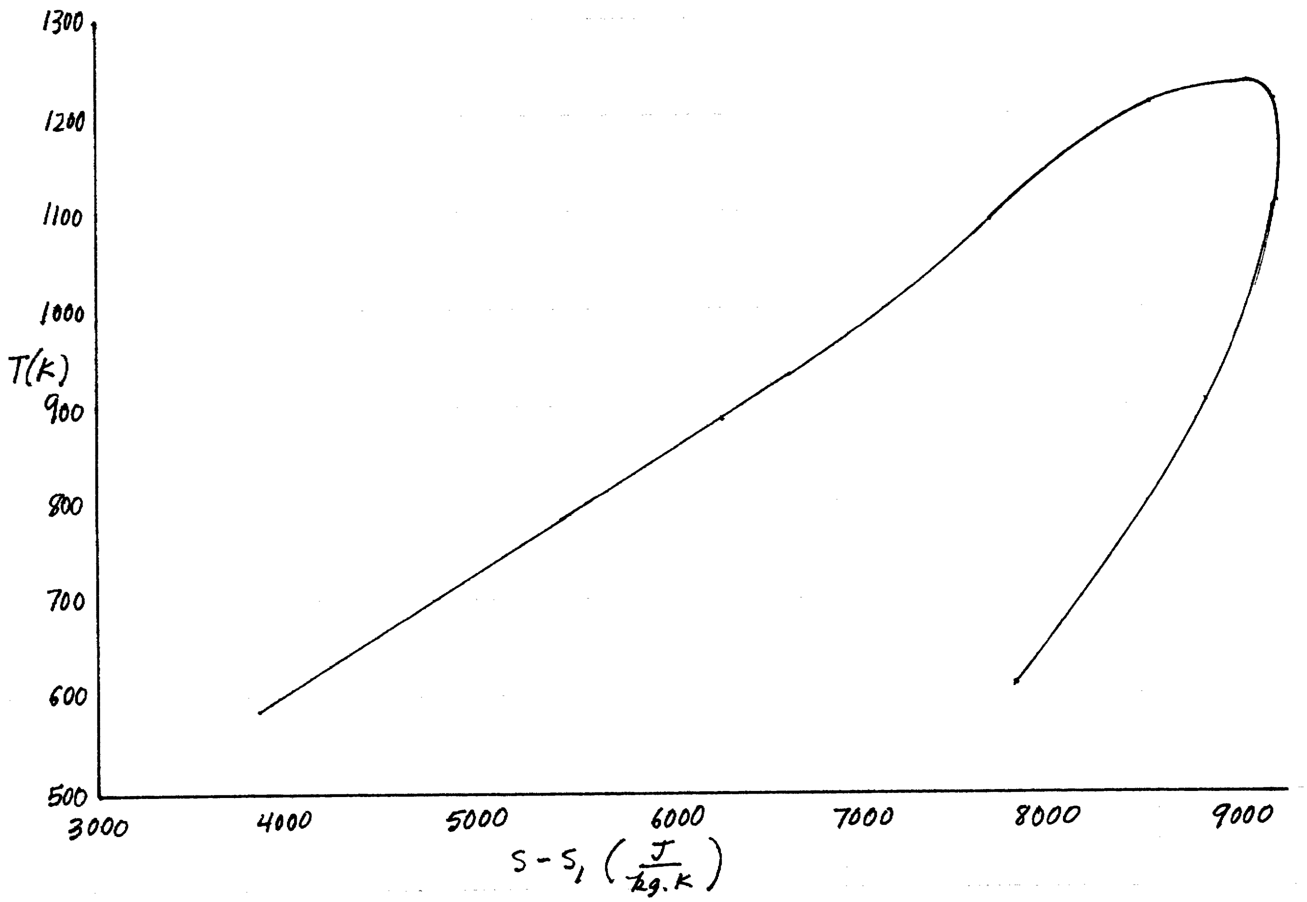
$$s - s_1 = \left(5224\ \frac{J}{kg\cdot K}\right)\ln\left(\frac{583K}{289.2K}\right) - \left(2077\ \frac{J}{kg\cdot K}\right)\ln\left[\frac{90\ kPa(abs)}{97.72\ kPa(abs)}\right] = 3833\ \frac{J}{kg\cdot K}$$

We proceed similarly for other values of p and construct the table and graph on the next page

(con't)

11.61 (con't)

p [kPa(abs)]	T (K)	$s - s_1$ $\left(\frac{J}{kg \cdot K}\right)$
90	583	3833
80	883.2	6248
70	1092	7634
60	1212	8490
50	1236	8980
45	1215	9109
40	1172	9165
35	1105	9135
25	903.3	8781
15	610.4	7795



Rayleigh line for helium.

11.62

11.62 Standard atmospheric air [T_0 = 288 K, p_0 = 101 kPa (abs)] is drawn steadily through an isentropic converging nozzle into a frictionless and diabatic (q = 500 kJ/kg) constant cross section area duct. For maximum flow determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and flow velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

For maximum flow, the Rayleigh flow is choked. For the isentropic nozzle

$$T_{0,1} = T_0 = \underline{\underline{288\ K}}$$

$$p_{0,1} = p_0 = \underline{\underline{101\ kPa(abs)}}$$

To determine the static state at the nozzle exit, Rayleigh flow inlet, we need the value of Ma_1. To determine Ma_1 we use

$$h_{0,2} - h_{0,1} = q = c_p\,(T_{0,2} - T_{0,1})$$

or

$$T_{0,2} = \frac{q}{c_p} + T_{0,1} = \frac{\left(500{,}000\ \frac{N.m}{kg}\right)}{\left(1004\ \frac{N.m}{kg.K}\right)} + 288K = \underline{\underline{786\ K}}$$

and noting that for choked flow, $T_{0,2} = T_{0,a}$ we get

$$\frac{T_{0,1}}{T_{0,2}} = \frac{T_{0,1}}{T_{0,a}} = \frac{288\ K}{786\ K} = 0.37$$

With $\frac{T_{0,1}}{T_{0,a}} = 0.37$ we enter Fig. D.3 and read

$$Ma_1 = 0.31$$

$$\frac{p_1}{p_a} = 2.1 \qquad (1)$$

$$\frac{T_1}{T_a} = 0.42 \qquad (2)$$

(con't)

11.62 (con't)

$$\frac{V_1}{V_a} = 0.2 \qquad (3)$$

$$\frac{p_{0,1}}{p_{0,a}} = 1.19 \qquad (4)$$

With Eq. 4 we obtain

$$p_{0,a} = \frac{p_{0,1}}{1.19} = \frac{101\ kPa(abs)}{1.19} = \underline{\underline{84.9\ kPa(abs)}} = p_{0,2}$$

With $Ma_1 = 0.31$ we read from Fig. D.1

$$\frac{p_1}{p_{0,1}} = 0.94 \qquad (5)$$

and

$$\frac{T_1}{T_{0,1}} = 0.98 \qquad (6)$$

With Eqs. 5 and 6 we get

$$p_1 = (0.94)[101\ kPa(abs)] = \underline{\underline{95\ kPa(abs)}} \qquad (7)$$

and

$$T_1 = (0.98114)(288K) = \underline{\underline{282K}}$$

Thus

$$V_1 = Ma_1\sqrt{RT_1k} = (0.31)\sqrt{\frac{\left(286.9\ \frac{N.m}{kg.K}\right)(282K)(1.4)}{\left(1\ \frac{N}{kg.\frac{m}{s^2}}\right)}} = \underline{\underline{104\ \frac{m}{s}}} \qquad (9)$$

Combining Eqs. 1 and 7 we obtain

$$p_a = \frac{p_1}{2.1} = \frac{[95\ kPa(abs)]}{(2.1)} = \underline{\underline{45\ kPa(abs)}} = p_2$$

Combining Eqs. 2 and 8 we have

$$T_a = \frac{T_1}{0.42} = \frac{(283K)}{(0.42)} = \underline{\underline{674K}} = T_2$$

Combining Eqs. 3 and 9 we have

$$V_a = \frac{V_1}{0.2} = \frac{104\ \frac{m}{s}}{0.2} = \underline{\underline{520\ \frac{m}{s}}} = V_2$$

(con't)

11.62 (con't)

To sketch a T-s diagram we obtain $s_2 - s_1$ from

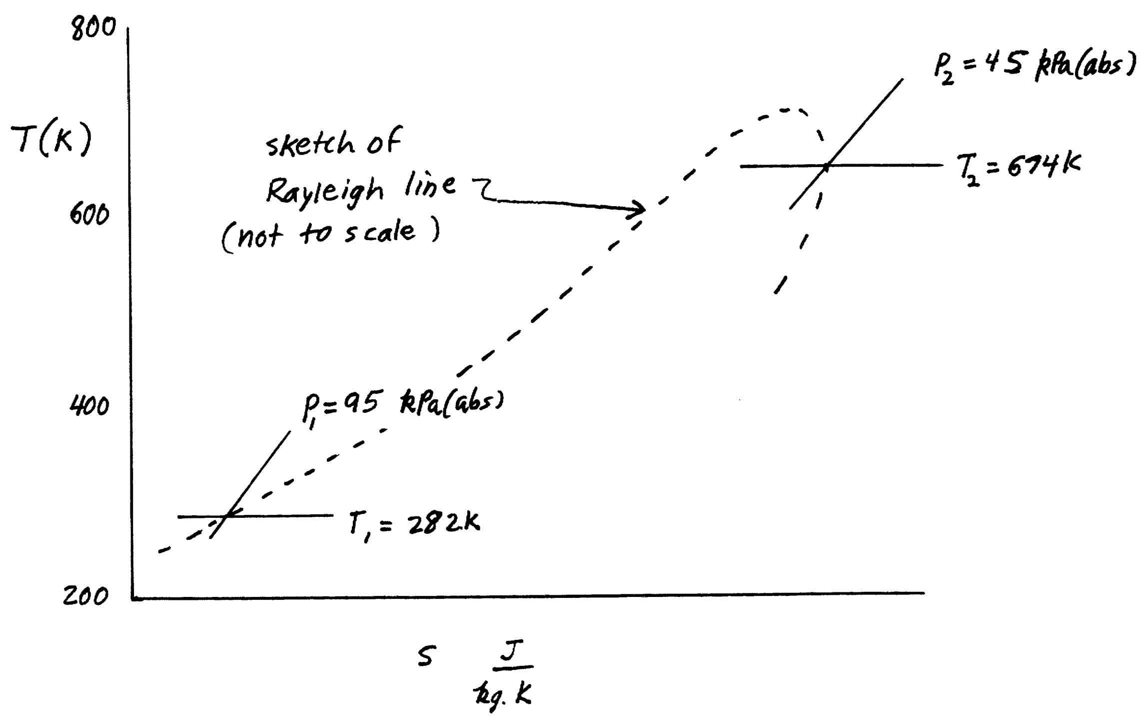
$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

or

$$s_2 - s_1 = \left(1004 \frac{N \cdot m}{kg \cdot K}\right) \ln\left(\frac{674}{282}\right) - \left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \ln\left[\frac{45\ kPa(abs)}{95\ kPa(abs)}\right]$$

and

$$s_2 - s_1 = 1090 \frac{N \cdot m}{kg \cdot K}$$



11.63

11.63 An ideal gas enters a 0.5-ft inside diameter duct with $p_1 = 20$ psia, $T_1 = 80$ °F, and $V_1 = 200$ ft/s. What frictionless heat addition rate in Btu/s is necessary for an exit gas temperature, $T_2 = 1500$ °F? Determine p_2, V_2, and Ma_2 also. The gas is **(a)** air; **(b)** helium.

To determine the heat transfer rate we use the energy equation (Eq. 5.69) to get

$$\dot{Q}_{\substack{net\\in}} = \dot{m}\left(h_{0,2} - h_{0,1}\right) = \dot{m}\, c_p \left(T_{0,2} - T_{0,1}\right) \qquad (1)$$

For mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\,\frac{\pi D_1^2}{4}\,V_1 \qquad (2)$$

To determine $T_{0,2}$ and $T_{0,1}$ we use Eq. 11.56. Thus,

$$\frac{T}{T_0} = \frac{1}{1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2} \qquad (3)$$

or for air

$$\frac{T}{T_0} = f(\mathrm{Ma}) \text{ in Fig. D.1} \qquad (4)$$

To determine p_2 we use

$$p_2 = p_1\left(\frac{p_a}{p_1}\right)\left(\frac{p_2}{p_a}\right) \qquad (5)$$

where with Eq. 11.123 for Rayleigh flow

$$\frac{p}{p_a} = \frac{1+k}{1+k\,\mathrm{Ma}^2} \qquad (6)$$

or for air

$$\frac{p}{p_a} = f(\mathrm{Ma}) \text{ in Fig. D.3} \qquad (7)$$

For exit velocity, V_2, we use

$$V_2 = \mathrm{Ma}_2\sqrt{RT_2 k} \qquad (8)$$

We determine Ma_1 with

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (9)$$

(con't)

11.63 (con't)

and we determine Ma_2 with

$$\frac{T_2}{T_a} = \left(\frac{T_2}{T_1}\right)\left(\frac{T_1}{T_a}\right) \tag{10}$$

and Eq. 11.128 for Rayleigh flow, namely

$$\frac{T}{T_a} = \left[\frac{(1+k)\,Ma}{1+k\,Ma^2}\right]^2 \tag{11}$$

or for air with

$$\frac{T}{T_a} = f(Ma) \text{ on Fig. D.3} \tag{12}$$

(a) For air we determine Ma_1 with Eq. 9. Thus,

$$Ma_1 = \frac{\left(200\,\frac{ft}{s}\right)}{\sqrt{\left(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\frac{(540\,{}^\circ R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}} = 0.18$$

For $Ma_1 = 0.18$ we read on Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.99$$

Thus

$$T_{0,1} = \frac{540\,{}^\circ R}{0.99} = 545\,{}^\circ R$$

With $Ma_1 = 0.18$ we read on Fig. D.3 the values

$$\frac{T_1}{T_a} = 0.17$$

and

$$\frac{p_1}{p_a} = 2.3$$

Thus with Eq. 10 we obtain

$$\frac{T_2}{T_a} = \left(\frac{1960\,{}^\circ R}{540\,{}^\circ R}\right)(0.17) = 0.62$$

(con't)

11.63 (con't)

For $\frac{T_2}{T_a} = 0.62$ we get from Fig. D.3

$$Ma_2 = \underline{\underline{0.40}}$$

and

$$\frac{P_2}{P_a} = 1.96$$

With $Ma_2 = 0.40$ we read on Fig. D.1

$$\frac{T_2}{T_{0,2}} = 0.97$$

Thus,

$$T_{0,2} = \frac{1960}{0.97} = 2020°R$$

Then with Eq. 5 we have

$$P_2 = (20\,psia)\left(\frac{1}{2.3}\right)(1.96) = \underline{\underline{17}}\ psia$$

With Eq. 8 we have

$$V_2 = (0.40)\sqrt{\frac{\left(1716\,\frac{ft.lb}{slug.°R}\right)(1960°R)(1.4)}{\left(1\,\frac{lb}{slug.\frac{ft}{s^2}}\right)}} = \underline{\underline{868}}\ \frac{ft}{s}$$

With Eq. 2 we get

$$\dot{m} = \frac{(20\,psia)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1716\,\frac{ft.lb}{slug.°R}\right)(540°R)} \frac{\pi\,(0.5\,ft)^2\left(200\,\frac{ft}{s}\right)}{(4)} = 0.122\,\frac{slug}{s}$$

and with Eq. 1 we obtain

$$\dot{Q}_{net\ in} = \left(0.122\,\frac{slug}{s}\right)\left(6006\,\frac{ft.lb}{slug.°R}\right)\frac{(2020°R - 545°R)}{\left(778\,\frac{ft.lb}{Btu}\right)} = \underline{\underline{1390}}\ \frac{Btu}{s}$$

(Con't)

(b) For helium, $k = 1.66$ and $R = 12{,}420 \ \frac{ft \cdot lb}{slug \cdot {}^\circ R}$ from Table 1.7.

With Eq. 9 we have

$$Ma_1 = \frac{\left(200 \ \frac{ft}{s}\right)}{\sqrt{\left(12{,}420 \ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)\frac{(540^\circ R)(1.66)}{\left(1 \ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}} = 0.0599$$

Thus with $Ma_1 = 0.0599$ and Eq. 3 we obtain

$$T_{0,1} = T_1\left[1 + \left(\frac{k-1}{2}\right)Ma_1^2\right] = (540^\circ R)\left[1 + \left(\frac{1.66-1}{2}\right)(0.0599)^2\right] = 540.6^\circ R$$

For $Ma_1 = 0.0599$, Eqs. 6 and 11 yield

$$\frac{P_1}{P_a} = \frac{1 + 1.66}{1 + (1.66)(0.0599)^2} = 2.644$$

and

$$\frac{T_1}{T_a} = \left[\frac{(1 + 1.66)(0.0599)}{1 + (1.66)(0.0599)^2}\right]^2 = 0.0251$$

Thus with Eq. 10 we obtain

$$\frac{T_2}{T_a} = \left(\frac{1960^\circ R}{540^\circ R}\right)(0.0251) = 0.0911$$

Now with Eq. 11 we have

$$\frac{T_2}{T_a} = 0.0911 = \left[\frac{(1 + 1.66)Ma_2}{1 + 1.66 \, Ma_2^2}\right]^2$$

or

$$0.501 \, Ma_2^2 - 2.66 \, Ma_2 + 0.3018 = 0$$

and

$$Ma_2 = \underline{\underline{0.116}} \quad \text{or} \quad 5.19$$

We use the subsonic solution, $Ma_2 = 0.116$ since heat is being added and with heat addition we cannot accelerate to supersonic Rayleigh flow from a subsonic condition upstream.

(con't)

11.63 (con't)

With $Ma_2 = 0.116$ and with Eq. 3 we obtain

$$T_{0,2} = (1960°R)\left[1+\left(\frac{1.66-1}{2}\right)(0.116)^2\right] = 1969°R$$

With Eq. 6 and $Ma_2 = 0.116$ we have

$$\frac{p_2}{p_a} = \frac{1+1.66}{1+(1.66)(0.116)^2} = 2.602$$

and thus with Eq. 5 we obtain

$$p_2 = (20\ psia)\left(\frac{1}{2.644}\right)(2.602) = \underline{\underline{19.7}}\ psia$$

With Eq. 8 we have

$$V_2 = (0.116)\sqrt{\frac{\left(12{,}420\ \frac{ft \cdot lb}{slug \cdot °R}\right)(1960°R)(1.66)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{737}}\ \frac{ft}{s}$$

With Eq. 2 we get

$$\dot{m} = \frac{(20\ psia)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(12{,}420\ \frac{ft \cdot lb}{slug \cdot °R}\right)(540°R)} \frac{\pi (0.5)^2}{(4)}\left(200\ \frac{ft}{s}\right) = 0.0169\ \frac{slug}{s}$$

and with Eq. 1 we have

$$\dot{Q}_{\substack{net \\ in}} = \left(0.0169\ \frac{slug}{s}\right)\left(31{,}240\ \frac{ft \cdot lb}{slug \cdot °R}\right)\frac{(1969°R - 540.6°R)}{\left(778\ \frac{ft \cdot lb}{Btu}\right)} = \underline{\underline{969}}\ \frac{Btu}{s}$$

11.64 Air enters a length of constant cross section area pipe with $p_1 = 200$ kPa (abs), $T_1 = 500$ K, and $V_1 = 400$ m/s. If 500 kJ/kg of energy is removed from the air by frictionless heat transfer between sections (1) and (2), determine p_2, T_2, and V_2. Sketch a temperature–entropy diagram for the flow between sections (1) and (2).

To determine the state of the air at section (2) we use the energy equation (Eq. 5.69) to calculate the value of $T_{0,2}$. Thus,

$$q_{\substack{net\\in}} = h_{0,2} - h_{0,1} = c_p(T_{0,2} - T_{0,1})$$

or

$$T_{0,2} = \frac{q_{\substack{net\\in}}}{c_p} + T_{0,1} = -\frac{q_{\substack{net\\out}}}{c_p} + T_{0,1} \qquad (1)$$

We obtain $T_{0,1}$ from $\frac{T_1}{T_{0,1}}$ which we read from Fig. D.1 with a value of Ma_1. We determine Ma_1 with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (2)$$

With Ma_1 we also enter Fig. D.3 and read values of $\frac{p_1}{p_a}$, $\frac{T_1}{T_a}$, $\frac{V_1}{V_a}$, and $\frac{T_{0,1}}{T_{0,a}}$. Then we determine $\frac{T_{0,2}}{T_{0,a}}$ with

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{T_{0,2}}{T_{0,1}}\right)\left(\frac{T_{0,1}}{T_{0,a}}\right) \qquad (3)$$

With this value of $\frac{T_{0,2}}{T_{0,a}}$ we enter Fig. D.3 and read corresponding values of $\frac{p_2}{p_a}$, $\frac{T_2}{T_a}$, and $\frac{V_2}{V_a}$. Then we determine p_2, T_2 and V_2 with

$$p_2 = \left(\frac{p_2}{p_a}\right)\left(\frac{p_a}{p_1}\right) p_1 \qquad (4)$$

$$T_2 = \left(\frac{T_2}{T_a}\right)\left(\frac{T_a}{T_1}\right) T_1 \qquad (5)$$

and

$$V_2 = \left(\frac{V_2}{V_a}\right)\left(\frac{V_a}{V_1}\right) V_1 \qquad (6)$$

(con't)

11.64 (con't)

We use Eq. 2 to get

$$Ma_1 = \frac{\left(400 \frac{m}{s}\right)}{\sqrt{\frac{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(500 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.89$$

For $Ma_1 = 0.89$ we get from Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.86$$

Thus,

$$T_{0,1} = \frac{(500 K)}{(0.86)} = 580 K$$

and with Eq. 1 we have

$$T_{0,2} = -\frac{\left(500{,}000 \frac{J}{kg}\right)}{\left(1004 \frac{J}{kg \cdot K}\right)} + 580 K = 82 K$$

With $Ma_1 = 0.893$ we enter Fig. D.3 and read

$$\frac{P_1}{P_a} = 1.14$$

$$\frac{T_1}{T_a} = 1.02$$

$$\frac{V_1}{V_a} = 0.9$$

and

$$\frac{T_{0,1}}{T_{0,a}} = 0.99$$

Now with $\frac{T_{0,1}}{T_{0,a}} = 0.99$ and Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{82 K}{579 K}\right)(0.99) = 0.14$$

(con't)

11.64 (con't)

which has as corresponding values in Fig. D.3 of

$$Ma_2 = 0.18$$

$$\frac{P_2}{P_a} = 2.3$$

$$\frac{T_2}{T_a} = 0.17$$

and

$$\frac{V_2}{V_a} = 0.07$$

With these ratios and those ratios corresponding to $Ma_1 = 0.89$ we use Eqs. 4, 5 and 6 to obtain

$$p_2 = (2.3)\left(\frac{1}{1.14}\right)\left[200\ \text{kPa(abs)}\right] = \underline{\underline{404}}\ \text{kPa(abs)}$$

$$T_2 = (0.17)\left(\frac{1}{1.02}\right)(500\text{K}) = \underline{\underline{83}}\ \text{K}$$

and

$$V_2 = (0.07)\left(\frac{1}{0.9}\right)\left(400\ \frac{\text{m}}{\text{s}}\right) = \underline{\underline{31}}\ \frac{\text{m}}{\text{s}}$$

Note that according to our calculations, $T_2 = 83.2\text{K}$ is slightly larger than $T_{0,2} = 82\text{K}$. This is not correct and is a result of the inaccuracy associated with using the graphs.

For more precision we ascertain the value of Ma_2 knowing $\frac{T_{0,2}}{T_{0,a}}$ using Eq. 11.131. First however, we determine $\frac{T_{0,1}}{T_{0,a}}$ knowing Ma_1 with Eq. 11.131. Thus,

$$\frac{T_{0,1}}{T_{0,a}} = \frac{2(k+1)\,Ma_1^2\left(1+\frac{k-1}{2}Ma_1^2\right)}{(1+kMa_1^2)^2} = \frac{2(1.4+1)(0.893)^2\left[1+\left(\frac{1.4-1}{2}\right)(0.893)^2\right]}{\left[1+(1.4)(0.893)^2\right]^2}$$

or

$$\frac{T_{0,1}}{T_{0,a}} = 0.9908$$

(con't)

11.64 (con't)

Now we use Eq. 11.56 to determine $\frac{T_1}{T_{0,1}}$. Thus,

$$\frac{T_1}{T_{0,1}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2} = \frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.893)^2} = 0.8624$$

and

$$T_{0,1} = \frac{T_1}{0.8624} = \frac{(500\,K)}{0.8624} = 579.8\,K$$

Now with Eq. 1 we have

$$T_{0,2} = \frac{-\left(500{,}000\ \frac{J}{kg}\right)}{\left(1004\ \frac{J}{kg \cdot K}\right)} + 579.8\,K = 81.79\,K$$

With Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{81.79\,K}{579.8\,K}\right)(0.9908) = 0.1398$$

With Eq. 11.131 and $\frac{T_{0,2}}{T_{0,a}} = 0.1398$ we get

$$Ma_2 = 0.1776$$

Then with Eq. 11.128 and $Ma_1 = 0.893$ and $Ma_2 = 0.1776$ we get

$$\frac{T_1}{T_a} = \left[\frac{(1+k)\,Ma_1}{1+k\,Ma_1^2}\right]^2 = \left[\frac{(1+1.4)(0.893)}{1+(1.4)(0.893)^2}\right]^2 = 1.026$$

and

$$\frac{T_2}{T_a} = \left[\frac{(1+1.4)(0.1776)}{1+(1.4)(0.1776)^2}\right]^2 = 0.1666$$

(con't)

11.64 (con't)

Now with Eq. 5 we have

$$T_2 = (0.1666)\left(\frac{1}{1.026}\right)(500 K) = 81.19 K$$

and

$$T_2 = 81.19 K < T_{0,2} = 81.79 K$$

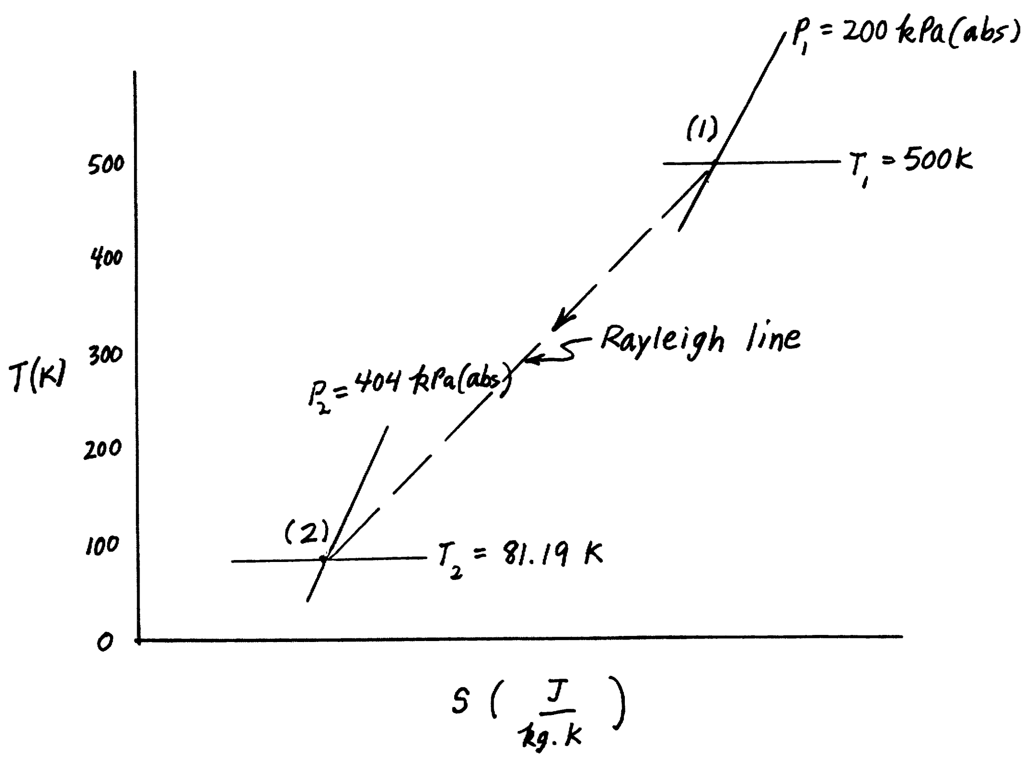
as it should be.

For our T-s sketch we use Eq. 11.76 to calculate $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1004 \frac{J}{kg\cdot K} \ln\left(\frac{81.19 K}{500 K}\right)$$

and

$$s_2 - s_1 = -2030 \frac{J}{kg\cdot K} \qquad - 286.9 \frac{J}{kg\cdot K} \ln\left[\frac{404 \, kPa(abs)}{200 \, kPa(abs)}\right]$$



11.65

11.65 Air flows through a constant cross section area pipe. At an upstream section (1), $p_1 = 15$ psia, $T_1 = 530\ °R$, and $V_1 = 200$ ft/s. Downstream at section (2), $p_2 = 10$ psia and $T_2 = 1760\ °R$. For this flow, determine the stagnation temperature and pressure ratios, $T_{0,2}/T_{0,1}$ and $p_{0,2}/p_{0,1}$, and the heat transfer per unit mass of air flowing between sections (1) and (2). Is the flow between sections (1) and (2) frictionless? Explain.

To determine the stagnation temperature and the stagnation pressure ratios we use

$$\frac{T_{0,2}}{T_{0,1}} = \left(\frac{T_{0,2}}{T_2}\right)\left(\frac{T_1}{T_{0,1}}\right)\left(\frac{T_2}{T_1}\right) \qquad (1)$$

and

$$\frac{p_{0,2}}{p_{0,1}} = \left(\frac{p_{0,2}}{p_2}\right)\left(\frac{p_1}{p_{0,1}}\right)\left(\frac{p_2}{p_1}\right) \qquad (2)$$

where

$$\frac{T}{T_0} = f(Ma) \quad \text{in Fig. D.1}$$

and

$$\frac{p}{p_0} = f(Ma) \quad \text{in Fig. D.1}$$

To determine the Mach number at each section we use

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (3)$$

For the velocity at section (2), V_2, we use the conservation of mass principal to obtain

$$\rho_1 V_1 = \rho_2 V_2$$

or

$$V_2 = \frac{\rho_1 V_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right) V_1 \qquad (4)$$

For the heat transfer per unit mass of air flowing between sections (1) and (2) we use the energy equation (Eq. 5.69) to obtain

$$q_{\substack{net\\in}} = h_{0,2} - h_{0,1} = c_p (T_{0,2} - T_{0,1}) = c_p T_{0,1} \left(\frac{T_{0,2}}{T_{0,1}} - 1\right) \qquad (5)$$

We obtain $T_{0,1}$ from

$$T_{0,1} = T_1 \left(\frac{T_{0,1}}{T_1}\right) \qquad (6)$$

(con't)

11.65 (con't)

To ascertain whether or not the air flow between sections (1) and (2) is frictionless we use the axial component of the linear momentum equation (Eq. 5.22) to get

$$R_x = A(p_1 - p_2) + \dot{m}(V_1 - V_2)$$

or

$$\frac{R_x}{A} = p_1 - p_2 + \rho_1 V_1 (V_1 - V_2) = p_1 - p_2 + \frac{p_1}{RT_1} V_1 (V_1 - V_2) \qquad (7)$$

First we use Eq. 4 to determine V_2. Thus,

$$V_2 = \left(\frac{15\text{ psia}}{10\text{ psia}}\right)\left(\frac{1760\,°R}{530\,°R}\right)\left(200\ \frac{ft}{s}\right) = 996\ \frac{ft}{s}$$

Now with Eq. 3 we calculate Ma_1 and Ma_2. We have

$$Ma_1 = \frac{\left(200\ \frac{ft}{s}\right)}{\sqrt{\left(1716\ \frac{ft\cdot lb}{slug\cdot °R}\right)\dfrac{(530\,°R)(1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}} = 0.18$$

and

$$Ma_2 = \frac{\left(996\ \frac{ft}{s}\right)}{\sqrt{\left(1716\ \frac{ft\cdot lb}{slug\cdot °R}\right)\dfrac{(1760\,°R)(1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}} = 0.48$$

With these values of Ma we enter Fig. D.1 and read for

$Ma_1 = 0.18$,

$$\frac{T_1}{T_{0,1}} = 0.99$$

$$\frac{p_1}{p_{0,1}} = 0.97$$

and for $Ma_2 = 0.48$,

$$\frac{T_2}{T_{0,2}} = 0.96$$

$$\frac{p_2}{p_{0,2}} = 0.85$$

(con't)

11.65 (con't)

These temperature and pressure ratios are used with Eqs. 1 and 2 to obtain

$$\frac{T_{0,2}}{T_{0,1}} = \left(\frac{1}{0.96}\right)(0.99)\left(\frac{1760\,°R}{530\,°R}\right) = \underline{\underline{3.42}}$$

and

$$\frac{p_{0,2}}{p_{0,1}} = \left(\frac{1}{0.85}\right)(0.97)\left(\frac{10\,psia}{15\,psia}\right) = \underline{\underline{0.76}}$$

With Eq. 6 we get

$$T_{0,1} = 530\,°R\left(\frac{1}{0.99}\right) = 535\,R$$

Then with Eq. 5 we obtain

$$q_{\substack{net\\in}} = \left(6006\,\frac{ft\cdot lb}{slug\cdot °R}\right)(535\,R)(3.42-1) = \underline{\underline{7.87\times10^6}}\,\frac{ft\cdot lb}{slug}$$

and with Eq. 7 we have

$$\frac{R_x}{A} = (15\,psia - 10\,psia)\left(144\,\frac{in.^2}{ft^2}\right) + \frac{(15\,psia)\left(144\,\frac{in.^2}{ft^2}\right)\left(200\,\frac{ft}{s}\right)\left(200\,\frac{ft}{s} - 996\,\frac{ft}{s}\right)}{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(530\,°R)}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$\frac{R_x}{A} = 342\,\frac{lb}{ft^2}$$

and we conclude that the flow is not frictionless.

11.66

11.66 The Mach number and stagnation pressure of an ideal gas are 2.0 and 200 kPa (abs) just upstream of a normal shock. Determine the stagnation pressure loss across the shock for the following gases: **(a)** air; **(b)** helium. Comment on the effect of specific heat ratio, k, on shock loss.

We want to determine the stagnation pressure loss across a normal shock, or

$$p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) \tag{1}$$

To determine the stagnation pressure ratio we use Eq. 11.156. Thus,

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left[\left(\frac{k+1}{2}\right)Ma_x^2\right]^{\frac{k}{k-1}}\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]^{\frac{k}{1-k}}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}} \tag{2}$$

or for air

$$\frac{p_{0,y}}{p_{0,x}} = f(Ma_x) \quad \text{in Fig. D.4.}$$

(a) For air ($k=1.4$) we have from Fig. D.4 for $Ma_x = 2.0$,

$$\frac{p_{0,y}}{p_{0,x}} = 0.72$$

Thus, with Eq. 1 we obtain

$$p_{0,x} - p_{0,y} = [200 \text{ kPa(abs)}](1 - 0.72) = \underline{\underline{56}} \text{ kPa}$$

(b) For helium ($k = 1.66$) we have with Eq. 2

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left[\left(\frac{1.66+1}{2}\right)(2.0)^2\right]^{\left(\frac{1.66}{1.66-1}\right)}\left[1+\left(\frac{1.66-1}{2}\right)(2.0)^2\right]^{\left(\frac{1.66}{1-1.66}\right)}}{\left\{\left[\frac{2(1.66)}{1.66+1}\right](2.0)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right\}^{\left(\frac{1}{1.66-1}\right)}} = 0.7621$$

and with Eq. 1 we get

$$p_{0,x} - p_{0,y} = [200 \text{ kPa(abs)}](1 - 0.7621) = \underline{\underline{47.6}} \text{ kPa}$$

From parts (a) and (b) we conclude that the loss of total pressure across a normal shock decreases with an increase in k.

11.67

11.67 The stagnation pressure ratio across a normal shock in an ideal gas flow is 0.8. Determine the Mach number of the flow entering the shock if the gas is air.

To determine the Mach number of the air flow entering a normal shock, Ma_x, given the stagnation pressure ratio, $\frac{p_{0,x}}{p_{0,y}}$, we enter Fig. D.4 with

$$\frac{p_{0,x}}{p_{0,y}} = 0.8$$

and read on Fig. D.4

$$Ma_x = \underline{\underline{1.83}}$$

11.68

11.68 Just upstream of a normal shock in an ideal gas flow, Ma = 3.0, T = 600 °R, and p = 30 psia. Determine values of Ma, T_0, T, p_0, p, and V downstream of the shock if the gas is **(a)** air; **(b)** helium.

To determine Ma_y knowing Ma_x we use Eq. 11.149. Thus,

$$Ma_y = \sqrt{\frac{Ma_x^2 + \left(\frac{2}{k-1}\right)}{\left(\frac{2k}{k-1}\right)Ma_x^2 - 1}} \qquad (1)$$

or for air we use Fig. D.4 for Ma_y as a function of Ma_x.
To determine $T_{0,y}$ we use Eq. 11.56. Thus,

$$T_{0,y} = T_y\left[1 + \left(\frac{k-1}{2}\right)Ma_y^2\right] \qquad (2)$$

or for air we use Fig. D.1 for $\frac{T_y}{T_{0,y}}$ as a function of Ma_y.
To obtain T_y we use Eq. 11.151. Thus,

$$T_y = T_x\left\{\frac{\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]\left[2\left(\frac{k}{k-1}\right)Ma_x^2 - 1\right]}{\left[\frac{(k+1)^2}{2(k-1)}\right]Ma_x^2}\right\} \qquad (3)$$

or for air we use Fig. D.4 for $\frac{T_y}{T_x}$ as a function of Ma_x.
For $p_{0,y}$ we use Eq. 2 of Example 11.19 to get

$$p_{0,y} = p_x\left\{\frac{\left[\left(\frac{k+1}{2}\right)Ma_x^2\right]^{\frac{k}{k-1}}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}}\right\} \qquad (4)$$

or for air we use Fig. D.4 for $\frac{p_{0,y}}{p_x}$ as a function of Ma_x.
For p_y we use Eq. 11.150 to obtain

$$p_y = p_x\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right] \qquad (5)$$

or for air we use Fig. D.4 for $\frac{p_y}{p_x}$ as a function of Ma_x.
For V_y we use

$$V_y = Ma_y\sqrt{RT_y k} \qquad (6)$$

(con't)

11.68 (con't)

(a) For air we read from Fig. D.4 for $Ma_x = 3.0$

$$Ma_y = \underline{\underline{0.475}}$$

$$\frac{p_y}{p_x} = 10.3 \qquad (7)$$

$$\frac{T_y}{T_x} = 2.7 \qquad (8)$$

$$\frac{p_{0,y}}{p_x} = 12 \qquad (9)$$

and we obtain from Fig. D.1 for $Ma_y = 0.475$ res

$$\frac{T_y}{T_{0,y}} = 0.96 \qquad (10)$$

From Eq. 8 we get

$$T_y = (2.7)(600°R) = \underline{\underline{1620°R}}$$

and thus with Eq. 10

$$T_{0,y} = \frac{T_y}{0.96} = \frac{1620°R}{0.96} = \underline{\underline{1690°R}}$$

With Eq. 7 we obtain

$$p_y = (10.3)(30\, psia) = \underline{\underline{309}}\ psia$$

and Eq. 9 yields

$$p_{0,y} = (12)(30\, psia) = \underline{\underline{360}}\ psia$$

Then with Eq. 6 we obtain

$$V_y = (0.475)\sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(1620°R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{937}}\ \frac{ft}{s}$$

(con't)

11.68 (con't)

(b) For helium we have with Eq. 1

$$Ma_y = \sqrt{\frac{(3.0)^2 + \left(\frac{2}{1.66-1}\right)}{\left[\frac{2(1.66)}{(1.66-1)}\right](3.0)^2 - 1}} = \underline{\underline{0.521}}$$

With Eq. 3 we obtain

$$T_y = (600°R)\left\{\frac{\left[1+\left(\frac{1.66-1}{2}\right)(3.0)^2\right]\left[2\left(\frac{1.66}{1.66-1}\right)(3.0)^2 - 1\right]}{\left[\frac{(1.66+1)^2}{2(1.66-1)}\right](3.0)^2}\right\} = \underline{\underline{2190}}°R$$

and with Eq. 2 we get

$$T_{0,y} = (2190°R)\left[1+\left(\frac{1.66-1}{2}\right)(0.521)^2\right] = \underline{\underline{2390}}°R$$

With Eq. 4 we have

$$p_{0,y} = (30\ psia)\frac{\left[\left(\frac{1.66+1}{2}\right)(3.0)^2\right]^{\left(\frac{1.66}{1.66-1}\right)}}{\left\{\left[\frac{2(1.66)}{1.66+1}\right](3.0)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right\}^{\left(\frac{1}{1.66-1}\right)}} = \underline{\underline{409}}\ psia$$

and with Eq. 5 we get

$$p_y = (30\ psia)\left[\frac{2(1.66)}{(1.66+1)}(3.0)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right] = \underline{\underline{330}}\ psia$$

With Eq. 6 we obtain

$$V_y = 0.521\sqrt{\frac{\left(1.242\times10^4\ \frac{ft\cdot lb}{slug\cdot °R}\right)(2190°R)(1.66)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{3500}}\ \frac{ft}{s}$$

11.69

like the one shown in video V3.4

11.69 A total pressure probe is inserted into a supersonic air flow. A shock wave forms just upstream of the impact hole. The probe measures a total pressure of 500 kPa(abs). The stagnation temperature at the probe head is 500 K. The static pressure upstream of the shock is measured with a wall tap to be 100 kPa(abs). From these data, determine the Mach number and velocity of the flow.

This is like Example 11.19.

We enter Fig. D.4 with

$$\frac{p_{0,y}}{p_x} = \frac{500 \text{ kPa(abs)}}{100 \text{ kPa(abs)}} = 5$$

and read

$$Ma_x = \underline{\underline{1.9}}$$

We determine the value of V_x with

$$V_x = Ma_x \sqrt{R T_x k} \qquad (1)$$

For T_x we read from Fig. D.1 for $Ma_x = 1.9$

$$\frac{T_x}{T_{0,x}} = 0.58$$

and since

$$T_{0,x} = T_{0,y} = 500 \text{ K}$$

we have

$$T_x = (0.58)\,500 \text{ K} = 290 \text{ K}$$

and with Eq. 1 we obtain

$$V_x = 1.9 \sqrt{\frac{\left(286.9 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290 \text{ K})(1.4)}{\left(1 \frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = \underline{\underline{648 \frac{\text{m}}{\text{s}}}}$$

11.70

(See Video V3.4)

11.70 The Pitot tube on a supersonic aircraft cruising at an altitude of 30,000 ft senses a stagnation pressure of 12 psia. If the atmosphere is considered standard, determine the air speed and Mach number of the aircraft. A shock wave is present just upstream of the probe impact hole.

At 30,000 ft, we read from Table C.1 for standard atmosphere

$$T = -47.83\,^{\circ}F = 412.2\,^{\circ}R$$

and

$$p = 4.373\ psia$$

Thus,

$$\frac{p_{0,y}}{p_x} = \frac{12\ psia}{4.373\ psia} = 2.74$$

and with this value of $\frac{p_{0,y}}{p_x}$ we read from Fig. D.4

$$Ma_y = \underline{\underline{1.25}}$$

Thus,

$$V_x = Ma_x\sqrt{RT_x k} = 1.25\sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^{\circ}R}\right)(412.2\,^{\circ}R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

and

$$V_x = \underline{\underline{1240}}\ \frac{ft}{s}$$

11.71

11.71 An aircraft cruises at a Mach number of 2.0 at an altitude of 15 km. Inlet air is decelerated to a Mach number of 0.4 at the engine compressor inlet. A normal shock occurs in the inlet diffuser upstream of the compressor inlet at a section where the Mach number is 1.2. For isentropic diffusion, except across the shock, and for standard atmosphere determine the stagnation temperature and pressure of the air entering the engine compressor.

The deceleration process in the inlet diffuser is assumed to be adiabatic since we are considering isentropic diffusion except across the shock. Thus,

$$T_0 = \text{constant}$$

and

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} \qquad (1)$$

To determine the diffuser inlet stagnation temperature we enter Fig. D.1 with Ma = 2.0 and read

$$\frac{T}{T_0} = 0.55 \qquad (2)$$

At 15 km elevation in standard atmosphere we read from Table C.2

$$T = -56.5\,^\circ C = 216.5\ K$$

Thus, with Eqs. 1 and 2 we obtain

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} = \frac{(216.5\ K)}{(0.55)} = \underline{\underline{394\ K}}$$

To determine the stagnation pressure at the compressor inlet we use

$$p_{0,\text{comp inlet}} = p_{0,\text{diffuser inlet}}\left(\frac{p_{0,x}}{p_{0,\text{diffuser inlet}}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right)\left(\frac{p_{0,\text{comp inlet}}}{p_{0,y}}\right) \qquad (3)$$

For $p_{0,\text{diffuser inlet}}$ we use

$$p_{0,\text{diffuser inlet}} = \left(\frac{p_{0,\text{diffuser inlet}}}{p_{\text{diffuser inlet}}}\right) p_{\text{diffuser inlet}} \qquad (4)$$

where $p_{\text{diffuser inlet}} = p_{atm}$ at 15 km or $p_{\text{diffuser inlet}} = 1.211 \times 10^4\ \frac{N}{m^2}\ (abs)$ from Table C.2.

(con't)

11.71 (con't)

We obtain $\dfrac{p_{\text{diffuser inlet}}}{p_{0,\text{diffuser inlet}}}$ from Fig. D.1 for $Ma_{\text{diffuser inlet}} = 2.0$.

Thus from Fig. D.1 we have

$$\frac{p_{\text{diffuser inlet}}}{p_{0,\text{diffuser inlet}}} = 0.13 \qquad (5)$$

Combining Eqs. 4 and 5 we obtain

$$p_{0,\text{diffuser inlet}} = \frac{1.211 \times 10^4 \frac{N}{m^2}(\text{abs})}{(0.13)} = 93{,}000 \ \frac{N}{m^2}(\text{abs})$$

For $Ma_x = 1.2$, we read from Fig. D.4

$$\frac{p_{0,y}}{p_{0,x}} = 0.99$$

Also, since the flow is isentropic except across the shock,

$$\frac{p_{0,x}}{p_{0,\text{diffuser inlet}}} = 1.0$$

and

$$\frac{p_{0,\text{comp inlet}}}{p_{0,y}} = 1.0$$

Thus, with Eq. 3 we obtain

$$p_{0,\text{comp inlet}} = \left[93{,}000 \ \frac{N}{m^2}(\text{abs})\right](1.0)(0.9928)(1.0) = \underline{\underline{92{,}000 \ \frac{N}{m^2}(\text{abs})}} = \underline{\underline{92}} \ \text{kPa(abs)}$$

To determine the static pressure at the compressor inlet we enter Fig. D.1 with $Ma_{\text{comp inlet}} = 0.4$ and read

$$\frac{p_{\text{comp inlet}}}{p_{0,\text{comp inlet}}} = 0.89$$

Thus,

$$p_{\text{comp inlet}} = (0.89)[92 \ \text{kPa(abs)}] = \underline{\underline{82}} \ \text{kPa(abs)}$$

11.72

11.72 Determine, for the air flow through the frictionless and adiabatic converging–diverging duct of Example 11.8, the ratio of duct exit pressure to duct inlet stagnation pressure that will result in a standing normal shock at: **(a)** $x = +0.1$ m; **(b)** $x = +0.2$ m; **(c)** $x = +0.4$ m. How large is the stagnation pressure loss in each case?

This is similar to Example 11.20.

(a) For a standing normal shock at $x = +0.1\text{ m}$ we note from the table of Example 11.8 that

$$Ma_x = 1.37$$

and

$$\frac{p_x}{p_{0,x}} = 0.33 \qquad (1)$$

From Fig. D.4, for $Ma_x = 1.37$ we obtain

$$Ma_y = 0.75$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.96 \qquad (2)$$

From Fig. D.1 we find for

$$Ma_y = 0.75$$

$$\frac{A_y}{A^*} = 1.1 \qquad (3)$$

For $x = +0.1\text{ m}$, the ratio of duct exit area to local area (A_2/A_y) is

$$\frac{A_2}{A_y} = \frac{0.1\text{ m}^2 + (0.5\text{ m})^2}{0.1\text{ m}^2 + (0.1\text{ m})^2} = 3.18 \qquad (4)$$

and using Eqs. 3 and 4 we get

$$\frac{A_2}{A^*} = \left(\frac{A_y}{A^*}\right)\left(\frac{A_2}{A_y}\right) = (1.1)(3.18) = 3.5$$

(con't)

11.72 (con't)

With $\frac{A_2}{A^*} = 3.5$ we get from Fig. D.1

$Ma_2 = 0.17$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.98$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.98)(0.96) = \underline{\underline{0.94}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.96) = \underline{\underline{4}}\ kPa$$

(b) For a standing normal shock at $x = +0.2\,m$ we note from the table of Example 11.8 that

$Ma_x = 1.76$

and

$$\frac{p_x}{p_{0,x}} = 0.18$$

From Fig. D.4, for $Ma_x = 1.76$ we obtain

$Ma_y = 0.62$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.83$$

From Fig. D.1 we find for

$Ma_y = 0.63$

$$\frac{A_y}{A^*} = 1.16$$

For $x = +0.2\,m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.2\,m)^2} = 2.5$$ (con't)

11.72 (con't)

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (2.5)(1.16\ ) = 2.9$$

With $\frac{A_2}{A^*} = 2.9$ we get from Fig. D.1

$$Ma_2 = 0.20$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.97$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.97\ )(0.83\ ) = \underline{\underline{0.8}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.83\ ) = \underline{\underline{17}}\ kPa$$

(c) For a standing normal shock at $x = +0.4\,m$ we note from the table of Example 11.8 that

$$Ma_x = 2.48$$

and

$$\frac{p_x}{p_{0,x}} = 0.06$$

From Fig. D.4, for $Ma_x = 2.48$ we obtain

$$Ma_y = 0.515$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.51$$

From Fig. D.1 we find

$$Ma_y = 0.51$$

$$\frac{A_y}{A^*} = 1.3$$

(con't)

11.72 (con't)

For $x = +0.4\,m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.4\,m)^2} = 1.35$$

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (1.35)(1.3\quad) = 1.8$$

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.92$$

Thus,

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.92\quad)(0.51) = \underline{\underline{0.47}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.51\quad) = \underline{\underline{50}}\ kPa$$

11.73

11.73 A normal shock is positioned in the diverging portion of a frictionless, adiabatic, converging–diverging air flow duct where the cross section area is 0.1 ft^2 and the local Mach number is 2.0. Upstream of the shock, p_0 = 200 psia and T_0 = 1200 °R. If the duct exit area is 0.15 ft^2, determine the exit area temperature and pressure and the duct mass flowrate.

To determine the duct exit temperature, T_2, and pressure, P_2, we need $\frac{T_2}{T_{0,2}}$ and $\frac{P_2}{P_{0,2}}$. We can obtain these ratios from Fig. D.1 knowing the value of Ma_2. The value of Ma_2 we obtain from Fig. D.1 with a known value of $\frac{A_2}{A^*}$ which we get from

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) \qquad (1)$$

The value of $\left(\frac{A_y}{A^*}\right)$ is obtained from Fig. D.1 with the value of Ma_y obtained from Fig. D.4 with a known value of Ma_x = 2.0. Thus from Fig. D.4 for $Ma_x = 2.0$

$$Ma_y = 0.58$$

and from Fig. D.1 we read for $Ma_y = 0.58$

$$\frac{A_y}{A^*} = 1.2$$

From the problem statement

$$\frac{A_2}{A_y} = \frac{0.15\ ft^2}{0.1\ ft^2} = 1.5$$

and thus with Eq. 1 we have

$$\frac{A_2}{A^*} = (1.5)(1.2\ \ ) = 1.8$$

(con't)

11.73 (con't)

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34 \quad (2)$$

$$\frac{T_2}{T_{0,2}} = 0.97 \quad (3)$$

and

$$\frac{P_2}{P_{0,2}} = 0.92 \quad (4)$$

The value of $T_{0,2}$ is obtained from

$$T_{0,2} = T_{0,x} = T_{0,y} = T_0 = 1200\,°R \quad (5)$$

The value of $P_{0,2}$ is obtained from

$$P_{0,2} = P_{0,y} = P_{0,x}\left(\frac{P_{0,y}}{P_{0,x}}\right)$$

where

$$\frac{P_{0,y}}{P_{0,x}} = 0.72$$

from Fig. D.4 for $Ma_x = 2.0$.

Thus

$$p_{0,2} = (200\text{ psia})(0.72) = 144\text{ psia} \quad (6)$$

With Eqs 3 and 5 we obtain

$$T_2 = T_{0,2}\left(\frac{T_2}{T_{0,2}}\right) = (1200°R)(0.97) = \underline{\underline{1160}}\,°R$$

With Eqs. 4 and 6 we have

$$P_2 = P_{0,2}\left(\frac{P_2}{P_{0,2}}\right) = (144\text{ psia})(0.92) = \underline{\underline{132}}\text{ psia}$$

For mass flowrate we use

$$\dot{m} = \rho_2 A_2 V_2 = \frac{P_2}{RT_2} A_2 Ma_2 c_2 = \frac{P_2}{RT_2} A_2 Ma_2 \sqrt{RT_2 k}$$

and

$$\dot{m} = \frac{(132\text{psia})\left(144\frac{in.^2}{ft^2}\right)(0.15\,ft^2)(0.34)}{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1160\,°R)} \sqrt{\frac{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1160°R)(1.4)}{\left(1\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}$$

$$\dot{m} = \underline{\underline{0.81}}\ \frac{slug}{s}$$

11.74

11.74 Supersonic air flow enters an adiabatic, constant cross section area (inside diameter = 1 ft) pipe 30 ft long with $Ma_1 = 3.0$. The pipe friction factor is estimated to be 0.02. What ratio of pipe exit pressure to pipe inlet stagnation pressure would result in a normal shock wave standing at **(a)** $x = 5$ ft, or **(b)** $x = 10$ ft, where x is the distance downstream from the pipe entrance? Determine also the duct exit Mach number and sketch the temperature–entropy diagram for each situation.

This is similar to Example 11.21.

With $Ma_1 = 3.0$ we enter Fig. D.2 and get

$$\frac{f(\ell^* - \ell_1)}{D} = 0.52$$

We note that

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell^* - \ell_x)}{D} + \frac{f(\ell_x - \ell_1)}{D} \qquad (1)$$

(a) With Eq. 1 we get for $\ell_x - \ell_1 = 5$ ft

$$\frac{f(\ell^* - \ell_x)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_x - \ell_1)}{D} = 0.52 - \frac{(0.02)(5\text{ ft})}{(1\text{ ft})}$$

or

$$\frac{f(\ell^* - \ell_x)}{D} = 0.42$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.42$ we enter Fig. D.2 and find

$$Ma_x = 2.5$$

With $Ma_x = 2.5$ we enter Fig. D.4 and read

$$Ma_y = 0.52$$

Now with $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{f(\ell^* - \ell_y)}{D} = 0.9$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$ (con't)

we get

$$f\frac{(\ell^* - \ell_2)}{D} = 0.9 \qquad \frac{(0.02)(25\,ft)}{(1\,ft)} = 0.4$$

and entering Fig. D.2 with $f\frac{(\ell^*-\ell_2)}{D} = 0.4$ we obtain

$$Ma_2 = \underline{\underline{0.62}} \text{ (subsonic flow)}$$

Now we note that

$$\frac{p_2}{p_{0,1}} = \left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p^*}\right)\left(\frac{p^*}{p_1}\right)\left(\frac{p_1}{p_{0,1}}\right) \tag{2}$$

With $Ma_2 = 0.62$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.7 \tag{3}$$

With $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 2.05 \tag{4}$$

With $Ma_x = 2.5$ we get from Fig. D.4

$$\frac{p_y}{p_x} = 7 \tag{5}$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.3 \tag{6}$$

For $Ma_1 = 3.0$ we get from Fig. D.2 .

$$\frac{p_1}{p^*} = 0.22 \tag{7}$$

and from Fig. D.1

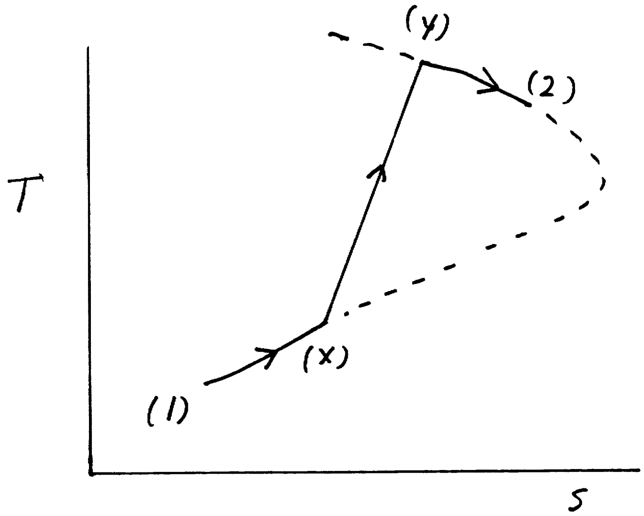
$$\frac{p_1}{p_{0,1}} = 0.03 \tag{8}$$

(con't)

11.74 (con't)

Combining Eqs. 2 through 8 we obtain

$$\frac{p_2}{p_{0,1}} = (1.7)\left(\frac{1}{2.05}\right)(7)(0.3)\left(\frac{1}{0.22}\right)(0.03) = \underline{\underline{0.213}}$$



Since we do not have values of temperature or pressure anywhere in the flow, we can only sketch qualitatively what happens on T-s coordinates. The T-s diagram will be similar to the one of Fig. E11.21(b) as indicated above.

(b) With Eq. 1 we get for $\ell_x - \ell_1 = 10\,ft$

$$\frac{f(\ell^* - \ell_x)}{D} = 0.52 - \frac{(0.02)(10\,ft)}{(1\,ft)} = 0.32$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.32$ we enter Fig. D.2 and find

$$Ma_x = 2$$

With $Ma_x = 2$ we enter Fig. D.4 and read

$$Ma_y = 0.58$$

Now with $Ma_y = 0.58$ we obtain from Fig. D.2

(con't)

11.74 (con't)

$$\frac{f(\ell^* - \ell_y)}{D} = 0.62$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$

we get

$$\frac{f(\ell^* - \ell_2)}{D} = 0.62 - \frac{(0.02)(20\,ft)}{(1\,ft)} = 0.22$$

and entering Fig. D.2 with $\frac{f(\ell^* - \ell_2)}{D} = 0.22$ we obtain

$$Ma_2 = \underline{\underline{0.89}}$$

With $Ma_2 = 0.89$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.14 \qquad (9)$$

With $Ma_y = 0.57$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 1.86 \qquad (10)$$

With $Ma_x = 2$ we get from Fig. D.4

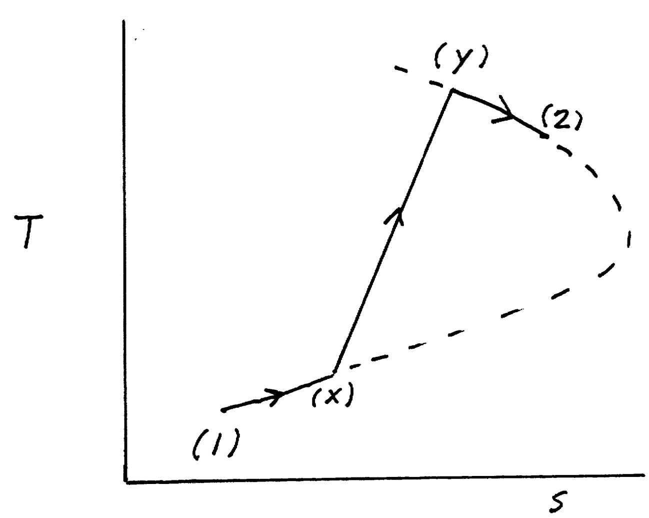
$$\frac{p_y}{p_x} = 4.8 \qquad (11)$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.4 \qquad (12)$$

Combining Eqs. 2, 7, 8, 9, 10, 11 and 12 we obtain

$$\frac{p_2}{p_{0,1}} = (1.14)\left(\frac{1}{1.86}\right)(4.8)(0.4)\left(\frac{1}{0.22}\right)(0.03) = \underline{\underline{0.16}}$$



11.75 Supersonic ideal gas flow enters an adiabatic, constant cross section area pipe (inside diameter = 0.1 m) with $Ma_1 = 2.0$. The pipe friction factor is 0.02. If a standing normal shock is located right at the pipe exit, and the Mach number just upstream of the shock is 1.2, determine the length of the pipe if the gas is **(a)** air; **(b)** helium.

We note that

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D} \qquad (1)$$

where according to Eq. 11.98

$$\frac{f(\ell^*-\ell)}{D} = \frac{1}{k}\frac{(1-Ma^2)}{(Ma^2)} + \left(\frac{k+1}{2k}\right)\ln\left[\frac{\left(\frac{k+1}{2}\right)Ma^2}{1+\left(\frac{k-1}{2}\right)Ma^2}\right] \qquad (2)$$

or for air, $\frac{f(\ell^*-\ell)}{D}$ is graphed as a function of Ma in Fig. D.2.

Thus, knowing Ma_1 and Ma_2 we can determine $\frac{f(\ell^*-\ell_1)}{D}$ and $\frac{f(\ell^*-\ell_2)}{D}$ and with Eq. 1 we obtain $\frac{f(\ell_2-\ell_1)}{D}$. With f and D also known we can determine $\ell_2-\ell_1$.

(a) For air, we find in Fig. D.2 corresponding to $Ma_1 = 2.0$ and $Ma_2 = 1.2$,

$$\frac{f(\ell^*-\ell_1)}{D} = 0.3$$

and

$$\frac{f(\ell^*-\ell_2)}{D} = 0.03$$

Thus, with Eq. 1 we have

$$\frac{f(\ell_2-\ell_1)}{D} = 0.3 - 0.03 = 0.27$$

and

$$\ell_2-\ell_1 = \frac{(0.27)(0.1\ m)}{0.02} = \underline{\underline{1.35\ m}}$$

(con't)

11.75 (con't)

(b) For helium ($k = 1.66$ from Table 1.8) we have with Eq. 2

$$\frac{f(\ell^* - \ell_1)}{D} = \left(\frac{1}{1.66}\right)\left[\frac{1-(2.0)^2}{(2.0)^2}\right] + \left[\frac{1.66+1}{2(1.66)}\right] \ln\left[\frac{\left(\frac{1.66+1}{2}\right)(2.0)^2}{1+\left(\frac{1.66-1}{2}\right)(2.0)^2}\right] = 0.2131$$

and

$$\frac{f(\ell^* - \ell_2)}{D} = \left(\frac{1}{1.66}\right)\left[\frac{1-(1.2)^2}{(1.2)^2}\right] + \left[\frac{1.66+1}{2(1.66)}\right] \ln\left[\frac{\left(\frac{1.66+1}{2}\right)(1.2)^2}{1+\left(\frac{1.66-1}{2}\right)(1.2)^2}\right] = 0.02507$$

With Eq. 1 we obtain

$$\frac{f(\ell_2 - \ell_1)}{D} = 0.2131 - 0.02507 = 0.188$$

and thus

$$\ell_2 - \ell_1 = \frac{(0.188)(0.1\,m)}{0.02} = \underline{\underline{0.94\,m}}$$

11.76

11.76 Air enters a frictionless, constant cross section area duct with $Ma_1 = 2.0$, $T_{0,1} = 59$ °F, and $p_{0,1} = 14.7$ psia. The air is decelerated by heating until a normal shock wave occurs where the local Mach number is 1.5. Downstream of the normal shock, the subsonic flow is accelerated with heating until it chokes at the duct exit. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock and at the duct exit. Sketch the temperature–entropy diagram for this flow.

At the duct entrance, section (1), we have

$$T_{0,1} = \underline{\underline{59\,°F}} = \underline{\underline{519\,°R}}$$

and

$$p_{0,1} = \underline{\underline{14.7 \text{ psia}}}$$

With $Ma_1 = 2.0$ we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.56 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.13 \qquad (2)$$

Thus with Eqs. 1 and 2 we obtain

$$T_1 = (0.56)(519\,°R) = \underline{\underline{291\,°R}}$$

and

$$p_1 = (0.13)(14.7 \text{ psia}) = \underline{\underline{1.91 \text{ psia}}}$$

Then

$$V_1 = Ma_1\sqrt{RT_1k} = (2.0)\sqrt{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)\frac{(288\,°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{1660\,\frac{ft}{s}}}$$

At section (x) just upstream of the shock

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

(con't)

11.76 (con't)

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \quad (4)$$

For $Ma_1 = 2.0$ and $Ma_x = 1.5$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.79 \quad (5)$$

$$\frac{p_{0,1}}{p_{0,a}} = 1.5 \quad (6)$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.91$$

$$\frac{p_{0,x}}{p_{0,a}} = 1.12$$

With these ratios and Eqs. 3 and 4 we obtain

$$T_{0,x} = (519\,°R)\left(\frac{1}{0.79}\right)(0.91) = \underline{\underline{598\,°R}}$$

$$p_{0,x} = (14.7\,psia)\left(\frac{1}{1.5}\right)(1.12) = \underline{\underline{11}}\ psia$$

With $Ma_x = 1.5$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.69$$

and

$$\frac{p_x}{p_{0,x}} = 0.27$$

Thus,

$$T_x = (0.69)(595\,°R) = \underline{\underline{411}}\,°R$$

and

$$p_x = (0.27)(11\,psia) = \underline{\underline{3}}\ psia$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.5)\sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(410\,°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = \underline{\underline{1490\,\frac{ft}{s}}}$$

(con't)

11.76 (con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.5$

$$Ma_y = 0.7$$

$$\frac{p_y}{p_x} = 2.5$$

$$\frac{T_y}{T_x} = 1.3$$

$$\frac{V_x}{V_y} = 1.9$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.93$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (2.5)(3.00\ psia) = \underline{\underline{7.5\ psia}}$$

$$T_y = (1.3)(410\,^\circ R) = \underline{\underline{533\,^\circ R}}$$

$$V_y = \frac{(1490\ \frac{ft}{s})}{1.9} = \underline{\underline{784\ \frac{ft}{s}}}$$

$$p_{0,y} = (0.93)(11.0\ psia) = \underline{\underline{10.2\ psia}}$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{598\,^\circ R}}$$

At the duct exit, section (2) we have the subscript "a" state in Fig. D.3 since the flow is choked there. Thus from Eqs. 5 and 6 we conclude that

$$T_{0,a} = \frac{T_{0,1}}{0.79} = \frac{(519\,^\circ R)}{(0.79)} = \underline{\underline{657\,^\circ R}} = T_{0,2}$$

and

$$p_{0,a} = \frac{p_{0,1}}{1.5} = \frac{(14.7\ psia)}{(1.5)} = \underline{\underline{9.8\ psia}} = p_{0,2}$$

(con't)

11.76 (con't)

With $Ma_1 = 2.0$ we read further from Fig. D.3

$$\frac{p_1}{p_a} = 0.36$$

$$\frac{T_1}{T_a} = 0.53$$

$$\frac{V_1}{V_a} = 1.45$$

Thus,

$$p_a = \frac{(1.91\,psia)}{(0.36)} = \underline{\underline{5.31\,psia}} = p_2$$

$$T_a = \frac{(291°R)}{(0.53)} = \underline{\underline{549°R}} = T_2$$

and

$$V_a = \frac{(1660\,\frac{ft}{s})}{(1.45)} = \underline{\underline{1140\,\frac{ft}{s}}} = V_2$$

To sketch a T-s diagram we need values of $s-s_1$ and we calculate these values with

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p_x}{p_1}$$

So, for example,

$$s_x - s_1 = \left(6006\,\frac{ft \cdot lb}{slug \cdot °R}\right) \ln\left(\frac{411°R}{519°R}\right) - \left(1716\,\frac{ft\,lb}{slug \cdot °R}\right) \ln\left(\frac{3\,psia}{14.7\,psia}\right) = 1310\,\frac{ft \cdot lb}{slug \cdot °R}$$

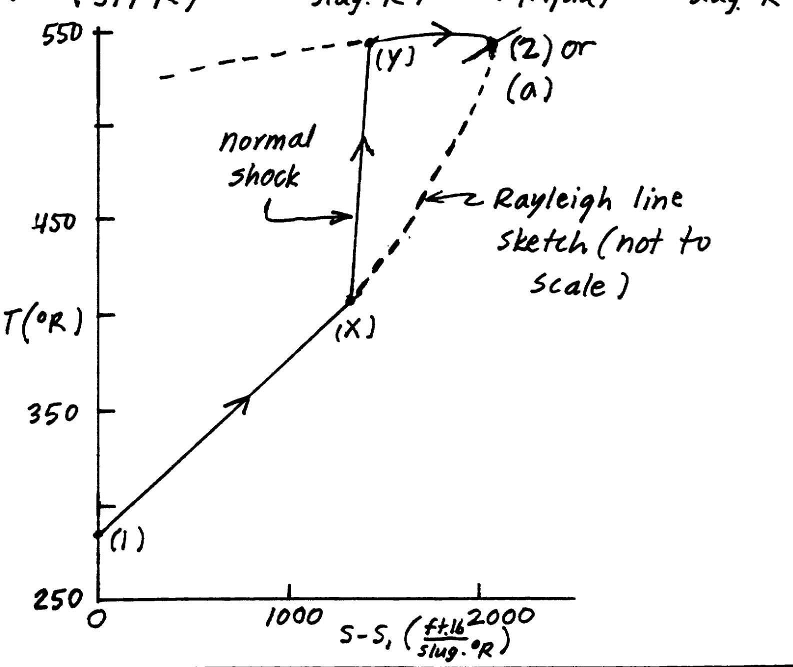
Similarly

$$s_y - s_1 = 6006 \ln \frac{533}{519} - 1716 \ln \frac{7.5}{14.7}$$

$$s_y - s_1 = 1320\,\frac{ft \cdot lb}{slug \cdot °R}$$

$$s_2 - s_1 = 6006 \ln \frac{549}{519} - 1716 \ln \frac{5.31}{14.7}$$

$$s_2 - s_1 = 2080\,\frac{ft \cdot lb}{slug \cdot °R}$$



11.77

11.77 An ideal gas enters a frictionless, constant cross section area duct with Ma = 2.5, T_0 = 20 °C, and p_0 = 101 kPa (abs). The gas is decelerated by heating until a normal shock occurs where the local Mach number is 1.3. Downstream of the shock, the subsonic flow is accelerated with heating until it exits with a Mach number of 0.9. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock and at the duct exit if the gas is **(a)** air or **(b)** helium. Sketch the temperature–entropy diagram for each flow.

(a) For air we have at the duct entrance, section (1)

$$Ma_1 = 2.5$$

$$T_{0,1} = 20°C = \underline{\underline{293\text{ K}}}$$

$$p_{0,1} = \underline{\underline{101\text{ kPa (abs)}}}$$

With $Ma_1 = 2.5$, we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.44 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.06 \qquad (2)$$

Thus we have with Eqs. 1 and 2

$$T_1 = (0.44)(293\text{ K}) = \underline{\underline{130\text{ K}}}$$

and

$$p_1 = (0.06)[101\text{ kPa(abs)}] = \underline{\underline{6.0}}\text{ kPa(abs)}$$

Then,

$$V_1 = Ma_1\sqrt{RT_1k} = (2.5)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(130\text{ K})(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{571}}\,\frac{m}{s}$$

At section (x) just upstream of the shock,

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \qquad (4)$$

(con't)

11.77 (con't)

For $Ma_1 = 2.5$ and $Ma_x = 1.3$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.71$$

$$\frac{p_{0,1}}{p_{0,a}} = 2.2$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.95$$

$$\frac{p_{0,x}}{p_{0,a}} = 1.04$$

With these values and Eqs. 3 and 4 we obtain

$$T_{0,x} = (293\,K)\left(\frac{1}{0.71}\right)(0.95) = \underline{\underline{396\,K}}$$

$$p_{0,x} = [101\,kPa(abs)]\left(\frac{1}{2.2}\right)(1.04) = \underline{\underline{47.7\,kPa(abs)}}$$

With $Ma_x = 1.3$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.75$$

and

$$\frac{p_x}{p_{0,x}} = 0.36$$

Thus,

$$T_x = (0.75)(395\,K) = \underline{\underline{296\,K}}$$

and

$$p_x = (0.36)[47.4\,kPa(abs)] = \underline{\underline{17\,kPa(abs)}}$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.3)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(296\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{448\,\frac{m}{s}}}$$

(con't)

11.77 (con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.3$

$$Ma_y = 0.79$$

$$\frac{p_y}{p_x} = 1.8$$

$$\frac{T_y}{T_x} = 1.2$$

$$\frac{V_x}{V_y} = 1.5$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.98$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (1.8)\,[17.1\ kPa(abs)] = \underline{\underline{30.8}}\ kPa(abs)$$

$$T_y = (1.2)(295\ K) = \underline{\underline{354}}\ K$$

$$V_y = \frac{(448\ \frac{m}{s})}{(1.5)} = \underline{\underline{299}}\ \frac{m}{s}$$

$$p_{0,y} = (0.98)\,[47.4\ kPa(abs)] = \underline{\underline{46.4}}\ kPa(abs)$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{396}}\ K$$

At the duct exit, section (2), we have

$$p_2 = p_y \left(\frac{p_a}{p_y}\right)\left(\frac{p_2}{p_a}\right) \qquad (5)$$

$$T_2 = T_y \left(\frac{T_a}{T_y}\right)\left(\frac{T_2}{T_a}\right) \qquad (6)$$

$$T_{0,2} = T_{0,y} \left(\frac{T_{0,a}}{T_{0,y}}\right)\left(\frac{T_{0,2}}{T_{0,a}}\right) \qquad (7)$$

$$p_{0,2} = p_{0,y} \left(\frac{p_{0,a}}{p_{0,y}}\right)\left(\frac{p_{0,2}}{p_{0,a}}\right) \qquad (8)$$

$$V_2 = V_y \left(\frac{V_a}{V_y}\right)\left(\frac{V_2}{V_y}\right) \qquad (9)$$

(con't)

11.77 (con't)

Appropriate ratios to use in Eqs. 5 through 9 are obtained from Fig. D.3 for $Ma_y = 0.79$ and $Ma_2 = 0.9$.

Thus,

$$\frac{p_y}{p_a} = 1.3$$

$$\frac{p_2}{p_a} = 1.12$$

$$\frac{T_y}{T_a} = 1.02$$

$$\frac{T_2}{T_a} = 1.02$$

$$\frac{T_{0,y}}{T_{0,a}} = 0.96$$

$$\frac{T_{0,2}}{T_{0,a}} = 0.99$$

$$\frac{p_{0,y}}{p_{0,a}} = 1.02$$

$$\frac{p_{0,2}}{p_{0,a}} = 1.01$$

$$\frac{V_y}{V_a} = 0.8$$

$$\frac{V_2}{V_a} = 0.91$$

With these ratios and Eqs. 5 through 9 we obtain

$$p_2 = [30.9\ kPa\,(abs)]\left(\frac{1}{1.3}\right)(1.12) = \underline{\underline{26.6\ kPa\,(abs)}}$$

$$T_2 = (351\,K)\left(\frac{1}{1.02}\right)(1.02) = \underline{\underline{351}}\ K$$

$$T_{0,2} = (395\,K)\left(\frac{1}{0.96}\right)(0.99) = \underline{\underline{407}}\ K$$

(con't)

11.77 (con't)

$$P_{0,2} = [46.4\ kPa(abs)]\left(\frac{1}{1.02}\right)(1.01) = \underline{\underline{45.9\ kPa(abs)}}$$

$$V_2 = \left(296\ \frac{m}{s}\right)\left(\frac{1}{0.8}\right)(0.91) = \underline{\underline{337\ \frac{m}{s}}}$$

For sketching a T-s diagram we need values of $s - s_1$.
We use,

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1}$$

Thus, for example,

$$s_x - s_1 = \left(1004\ \frac{J}{kg \cdot K}\right)\ln\left(\frac{296\ K}{130\ K}\right) - \left(286.9\ \frac{J}{kg \cdot K}\right)\ln\left[\frac{17\ kPa(abs)}{6.0\ kPa(abs)}\right]$$

or

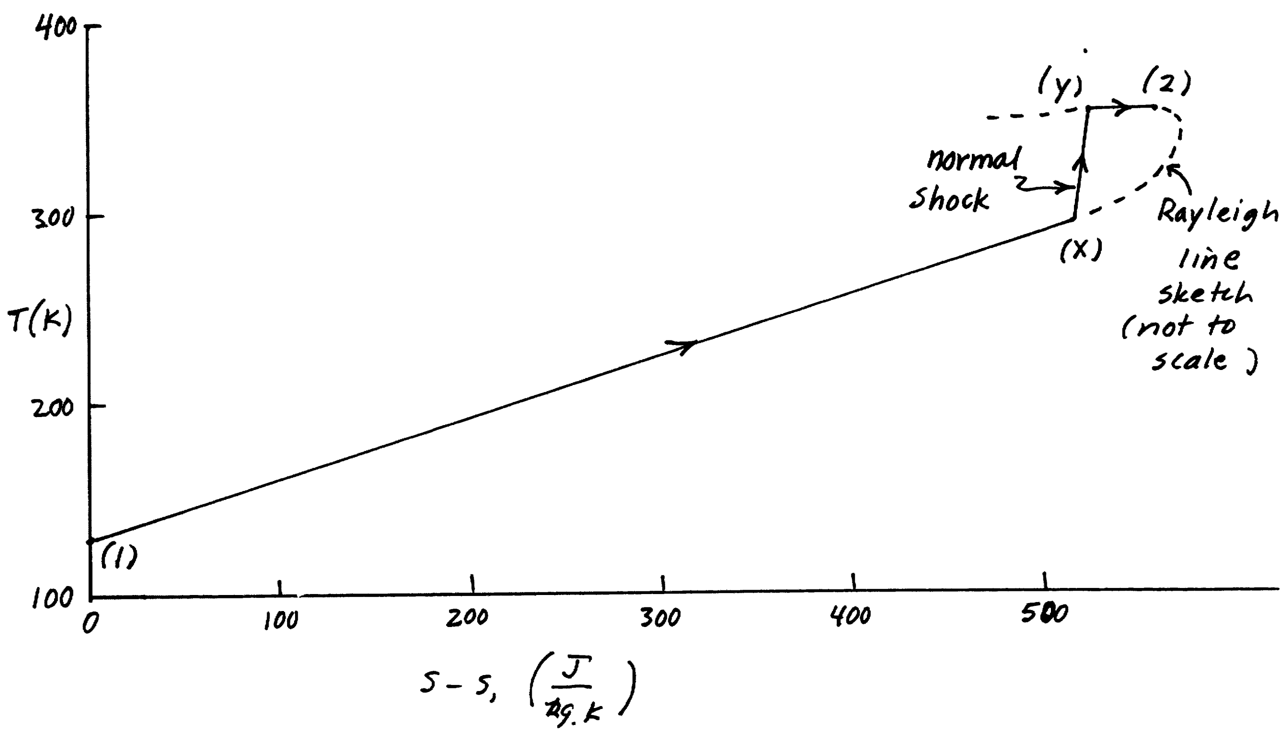
$$s_x - s_1 = 527\ \frac{J}{kg \cdot K}$$

Similarly

$$s_y - s_1 = 536\ \frac{J}{kg \cdot K}$$

and

$$s_2 - s_1 = 570\ \frac{J}{kg \cdot K}$$



11.77 (con't)

(b) For helium ($k = 1.66$ and $R = 2077 \frac{N \cdot m}{kg \cdot K}$ from Table 1.8) we have at the duct entrance, section (1)

$$Ma_1 = 2.5$$

$$T_{0,1} = 20^\circ C = \underline{\underline{293 K}}$$

$$p_{0,1} = \underline{\underline{101 \text{ kPa (abs)}}}$$

With $Ma_1 = 2.5$ we use Eqs. 11.56 and 11.59 to obtain

$$\frac{T_1}{T_{0,1}} = \frac{1}{1 + \frac{k-1}{2} Ma_1^2} = \frac{1}{1 + \left(\frac{1.66-1}{2}\right)(2.5)^2} = 0.3265 \qquad (10)$$

and

$$\frac{p_1}{p_{0,1}} = \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_1^2}\right]^{\frac{k}{k-1}} = \left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)(2.5)^2}\right]^{\frac{1.66}{1.66-1}} = 0.0599 \qquad (11)$$

Thus we have with Eqs. 10 and 11

$$T_1 = (0.3265)(293 K) = \underline{\underline{95.7 K}}$$

and

$$p_1 = (0.0599)\left[101 \text{ kPa (abs)}\right] = \underline{\underline{6.05 \text{ kPa (abs)}}}$$

Then

$$V_1 = Ma_1 \sqrt{R T_1 k} = (2.5)\sqrt{\frac{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(95.7 K)(1.66)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = \underline{\underline{1440 \frac{m}{s}}}$$

At section (x) just upstream of the shock,

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (12)$$

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \qquad (13)$$

For $Ma_1 = 2.5$ and $Ma_x = 1.3$ we use Eqs. 11.131 and 11.133 to get

$$\frac{T_{0,1}}{T_{0,a}} = 2(k+1) Ma_1^2 \frac{\left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right]}{(1 + k Ma_1^2)^2} = 2(1.66+1)(2.5)^2 \frac{\left[1 + \left(\frac{1.66-1}{2}\right)(2.5)^2\right]}{\left[1 + (1.66)(2.5)^2\right]^2}$$

or

$$\frac{T_{0,1}}{T_{0,a}} = 0.787$$

(con't)

11.77 (con't)

$$\frac{p_{0,1}}{p_{0,a}} = \frac{(1+k)}{(1+k\,Ma_1^2)}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}Ma_1^2\right)\right]^{\frac{k}{k-1}} = \frac{(1+1.66)}{1+(1.66)(2.5)^2}\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(2.5)^2\right]\right\}^{\left(\frac{1.66}{1.66-1}\right)}$$

or

$$\frac{p_{0,1}}{p_{0,a}} = 1.905$$

$$\frac{T_{0,x}}{T_{0,a}} = 2(1.66+1)(1.3)^2\,\frac{\left[1+\left(\frac{1.66-1}{2}\right)(1.3)^2\right]}{\left[1+(1.66)(1.3)^2\right]^2} = 0.9671$$

$$\frac{p_{0,x}}{p_{0,a}} = \frac{(1+1.66)}{1+(1.66)(1.3)^2}\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(1.3)^2\right]\right\}^{\left(\frac{1.66}{1.66-1}\right)} = 1.04$$

With these ratios and Eqs. 12 and 13 we get

$$T_{0,x} = (293\ K)\left(\frac{1}{0.787}\right)(0.9671) = \underline{\underline{360\ K}}$$

$$p_{0,x} = [101\ kPa(abs)]\left(\frac{1}{1.905}\right)(1.04) = \underline{\underline{55.1\ kPa(abs)}}$$

With $Ma_x = 1.3$ we use Eqs. 11.56 and 11.59 to get

$$\frac{T_x}{T_{0,x}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_x^2} = \frac{1}{1+\left(\frac{1.66-1}{2}\right)(1.3)^2} = 0.642 \qquad (14)$$

and

$$\frac{p_x}{p_{0,x}} = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_x^2}\right]^{\frac{k}{k-1}} = \left[\frac{1}{1+\left(\frac{1.66-1}{2}\right)(1.3)^2}\right]^{\left(\frac{1.66}{1.66-1}\right)} = 0.328 \qquad (15)$$

Thus with Eqs. 14 and 15

$$T_x = (0.642)(360\ K) = \underline{\underline{231\ K}}$$

and

$$p_x = (0.328)[55.1\ kPa(abs)] = \underline{\underline{18.1\ kPa(abs)}}$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.3)\sqrt{\frac{\left(2077\ \frac{N\cdot m}{kg\cdot K}\right)(231\ K)(1.66)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{1160\ \frac{m}{s}}}$$

(con't)

11.77 (con't)

At section (y) just downstream of the shock we obtain with Eqs. 11.149, 11.150, 11.151, 11.154 and 11.156 and $Ma_x = 1.3$

$$Ma_y = \sqrt{\frac{Ma_x^2 + \left(\frac{2}{k-1}\right)}{\left(\frac{2k}{k-1}\right)Ma_x^2 - 1}} = \sqrt{\frac{(1.3)^2 + \left(\frac{2}{1.66-1}\right)}{\frac{(2)(1.66)(1.3)^2}{(1.66-1)} - 1}} = 0.7933$$

$$\frac{p_y}{p_x} = \left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right) = \frac{(2)(1.66)(1.3)^2}{(1.66+1)} - \frac{(1.66-1)}{(1.66+1)} = 1.861$$

$$\frac{T_y}{T_x} = \frac{\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]\left[\left(\frac{2k}{k-1}\right)Ma_x^2 - 1\right]}{\left[\frac{(k+1)^2}{2(k-1)}\right]Ma_x^2} = \frac{\left[1+\left(\frac{1.66-1}{2}\right)(1.3)^2\right]\left[\frac{(2)(1.66)(1.3)^2}{(1.66-1)} - 1\right]}{\left[\frac{(1.66+1)^2}{2(1.66-1)}\right](1.3)^2} = 1.29$$

$$\frac{V_x}{V_y} = \frac{(k+1)Ma_x^2}{(k-1)Ma_x^2 + 2} = \frac{(1.66+1)(1.3)^2}{(1.66-1)(1.3)^2 + 2} = 1.443$$

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left[\frac{\left(\frac{k+1}{2}\right)Ma_x^2}{1+\left(\frac{k-1}{2}\right)Ma_x^2}\right]^{\left(\frac{k}{k-1}\right)}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\left(\frac{1}{k-1}\right)}} = \frac{\left[\frac{\left(\frac{1.66+1}{2}\right)(1.3)^2}{1+\left(\frac{1.66-1}{2}\right)(1.3)^2}\right]^{\left(\frac{1.66}{1.66-1}\right)}}{\left\{\left[\frac{2(1.66)}{1.66+1}\right](1.3)^2 - \left(\frac{1.66-1}{1.66+1}\right)\right\}^{\left(\frac{1}{1.66-1}\right)}} = 0.9812$$

With these ratios and values of properties at section (x) we have

$$p_y = (1.861)\left[18.1\ kPa(abs)\right] = \underline{\underline{33.7\ kPa(abs)}}$$

$$T_y = (1.29)(231\ K) = \underline{\underline{298\ K}}$$

$$V_y = \frac{\left(1160\ \frac{m}{s}\right)}{1.443} = \underline{\underline{804\ \frac{m}{s}}}$$

$$p_{0,y} = (0.9812)\left[55.1\ kPa(abs)\right] = \underline{\underline{54.1\ kPa(abs)}}$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{360K}}$$

At the duct exit, section (2) we have

$$p_2 = p_y\left(\frac{p_a}{p_y}\right)\left(\frac{p_2}{p_a}\right) \qquad (16)$$

$$T_2 = T_y\left(\frac{T_a}{T_y}\right)\left(\frac{T_2}{T_a}\right) \qquad (17)$$

(con't)

11.77 (con't)

$$T_{0,2} = T_{0,y}\left(\frac{T_{0,a}}{T_{0,y}}\right)\left(\frac{T_{0,2}}{T_{0,a}}\right) \tag{18}$$

$$p_{0,2} = p_{0,y}\left(\frac{p_{0,a}}{p_{0,y}}\right)\left(\frac{p_{0,2}}{p_{0,a}}\right) \tag{19}$$

and

$$V_2 = V_y\left(\frac{V_a}{V_y}\right)\left(\frac{V_2}{V_a}\right) \tag{20}$$

Appropriate ratios to use in Eqs. 16 through 20 are obtained with Eqs. 11.123, 11.128, 11.131, 11.133 and 11.129 for $Ma_y = 0.7933$ and $Ma_2 = 0.80$. Thus,

$$\frac{p_y}{p_a} = \frac{1+1.66}{1+(1.66)(0.7933)^2} = 1.301$$

$$\frac{p_2}{p_a} = \frac{1+1.66}{1+(1.66)(0.9)^2} = 1.134$$

$$\frac{T_y}{T_a} = \left[\frac{(1+1.66)(0.7933)}{1+(1.66)(0.7933)^2}\right]^2 = 1.065$$

$$\frac{T_2}{T_a} = \left[\frac{(1+1.66)(0.90)}{1+(1.66)(0.90)^2}\right]^2 = 1.043$$

$$\frac{T_{0,y}}{T_{0,a}} = \frac{2(1.66+1)(0.7933)^2\left[1+\left(\frac{1.66-1}{2}\right)(0.7933)^2\right]}{\left[1+(1.66)(0.7933)^2\right]^2} = 0.9671$$

$$\frac{T_{0,2}}{T_{0,a}} = \frac{2(1.66+1)(0.90)^2\left[1+\left(\frac{1.66-1}{2}\right)(0.90)^2\right]}{\left[1+(1.66)(0.90)^2\right]^2} = 0.9934$$

$$\frac{p_{0,y}}{p_{0,a}} = \frac{(1+1.66)}{1+(1.66)(0.7933)^2}\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(0.7933)^2\right]\right\}^{\left(\frac{1.66}{1.66-1}\right)} = 1.021$$

$$\frac{p_{0,2}}{p_{0,a}} = \frac{(1+1.66)}{1+(1.66)(0.90)^2}\left\{\left(\frac{2}{1.66+1}\right)\left[1+\left(\frac{1.66-1}{2}\right)(0.90)^2\right]\right\}^{\left(\frac{1.66}{1.66-1}\right)} = 1.005$$

(con't)

$$\frac{V_y}{V_a} = (0.7933)\left[\frac{(1+1.66)(0.7933)}{1+(1.66)(0.7933)^2}\right] = 0.8187$$

$$\frac{V_2}{V_a} = (0.90)\left[\frac{(1+1.66)(0.90)}{1+(1.66)(0.90)^2}\right] = 0.919$$

With these ratios and Eqs. 16 through 20 we obtain

$$p_2 = [33.7\ kPa(abs)]\left(\frac{1}{1.301}\right)(1.134) = \underline{\underline{29.4}}\ kPa(abs)$$

$$T_2 = (298\,K)\left(\frac{1}{1.065}\right)(1.043) = \underline{\underline{292}}\,K$$

$$T_{0,2} = (360\,K)\left(\frac{1}{0.9671}\right)(0.9934) = \underline{\underline{370}}\,K$$

$$p_{0,2} = [54.1\ kPa(abs)]\left(\frac{1}{1.021}\right)(1.005) = \underline{\underline{53.2}}\ kPa(abs)$$

$$V_2 = \left(804\ \frac{m}{s}\right)\left(\frac{1}{0.8187}\right)(0.919) = \underline{\underline{902}}\ \frac{m}{s}$$

For sketching a T-s diagram we need values of $s - s_1$. We use

$$s - s_1 = c_p \ln\frac{T}{T_1} - R\ln\frac{p}{p_1}$$

Thus, for example,

$$s_x - s_1 = \left(5224\ \frac{J}{kg\cdot K}\right)\ln\left(\frac{231\ K}{95.7\ K}\right) - \left(2077\ \frac{J}{kg\cdot K}\right)\ln\left[\frac{18.1\ kPa(abs)}{6.05\ kPa(abs)}\right]$$

or

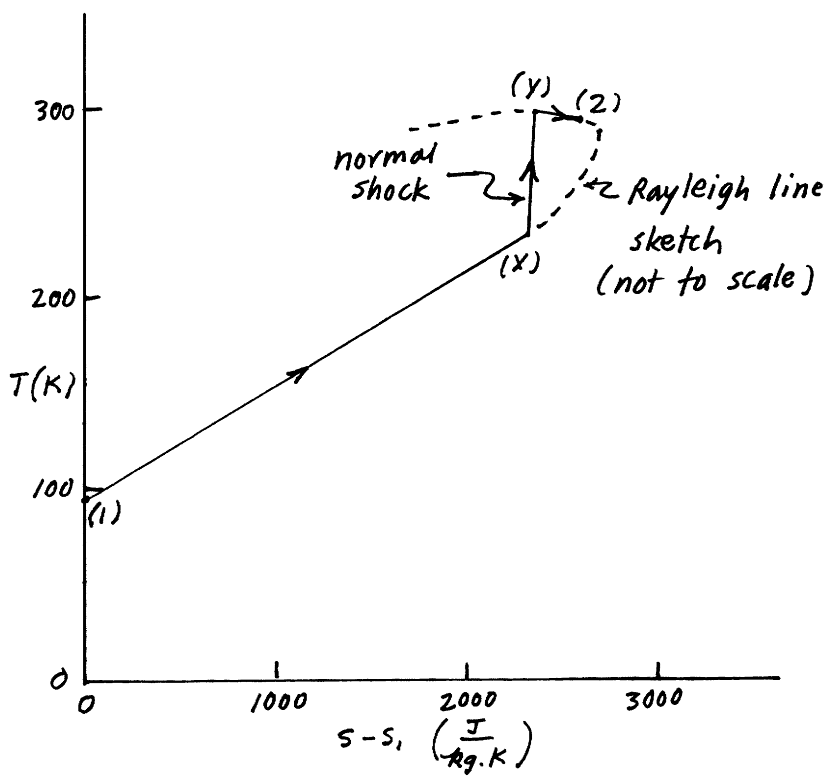
$$s_x - s_1 = 2327\ \frac{J}{kg\cdot K}$$

Similarly,

$$s_y - s_1 = 2367\ \frac{J}{kg\cdot K}$$

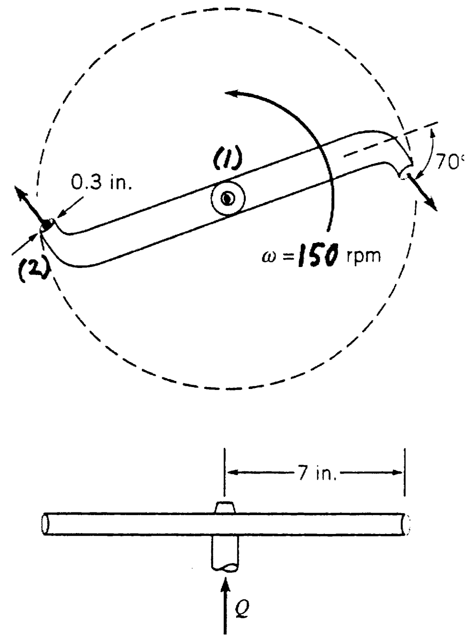
and

$$s_2 - s_1 = 2544\ \frac{J}{kg\cdot K}$$



12.1

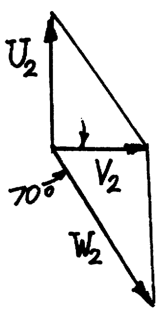
12.1 Water flows through a rotating sprinkler arm as shown in Fig. P12.1 and Video V12.2. Determine the flowrate if the angular velocity is 150 rpm. Friction is negligible. Is this a turbine or a pump? What is the maximum angular velocity for this flowrate?



■ FIGURE P12.1

$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = 0$ since there is no friction. Also, $V_{\theta 1} = 0$

Hence, $V_{\theta 2} = 0$



$$U_2 = \omega r_2 = 150\frac{rev}{min}\left(\frac{2\pi\ rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{7}{12}\ ft\right) = 9.16\ \frac{ft}{s}$$

Thus,

$$W_2 \sin\theta = U_2$$

or

$$W_2 = \frac{9.16\ \frac{ft}{s}}{\sin 70^\circ} = 9.75\ \frac{ft}{s}$$

Hence, $Q = 2\,W_2 A_2 = 2\left(\frac{\pi}{4}\left(\frac{0.3}{12}\ ft\right)^2\right)\left(9.75\frac{ft}{s}\right)$

$$= 0.00958\ \frac{ft^3}{s} = \underline{\underline{4.3\ gal/min}}$$

This is a turbine because the sprinkler moves in response to fluid flow forces.

Since friction is negligible the maximum angular velocity for this flowrate is the one corresponding to 150 rpm or

$$\omega = \left(150\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = \underline{\underline{15.7}}\ \frac{rad}{s}$$

12.2

12.2 Water flows axially up the shaft and out through the two sprinkler arms as sketched in Fig. P12.1 and as shown in Video V12.2. With the help of the moment-of-momentum equation explain why, at a threshold amount of water flow, the sprinkler arms begin to rotate. What happens when the flowrate increases above this threshold amount?

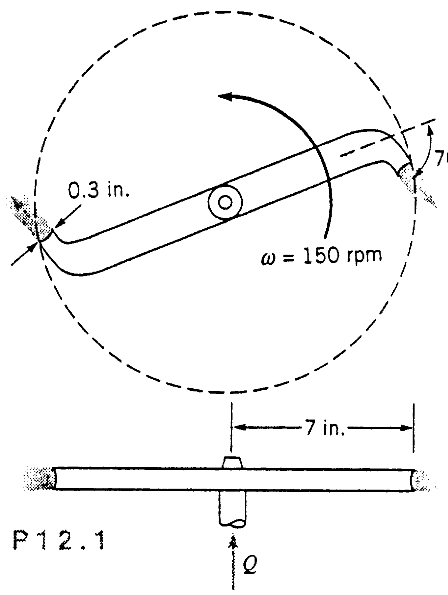


FIGURE P12.1

This sprinkler is similar to the one of Example 5.18. Thus,

$$T_{shaft} = -r_2 V_{\theta 2} \dot{m}$$

From the velocity triangle shown in the sketch above, we conclude that

$$V_{\theta 2} = (W_2 \sin 70° - U_2)$$

where

$$U_2 = r_2 \omega$$

Combining, we get

$$T_{shaft} = -r_2 (W_2 \sin 70° - r_2 \omega)$$

So, when W_2 is large enough with $\omega = 0$ to overcome T_{shaft}, the sprinkler rotor begins to rotate.

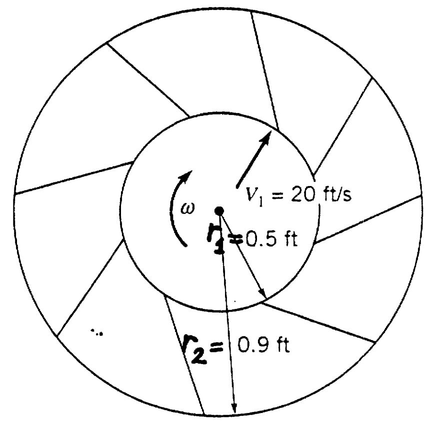
When flowrate increases further, ω is no longer zero and with T_{shaft} negligibly small,

$$W_2 \sin 70 \approx r\omega$$

and ω increases with increase of W_2 until at maximum value of W_2, we also have maximum value of ω.

12.3

12.3 The rotor shown in Fig. P12.3 rotates with an angular velocity of 2000 rpm. Assume that the fluid enters in the radial direction and the relative velocity is tangent to the blades across the entire rotor. Is the device a pump or a turbine? Explain.



■ FIGURE P12.3

$T = \dot{m}(r_2 V_{\theta 2} - r V_{\theta 1})$ where $V_{\theta 1} = 0$

If the rotor is of constant height, then $b_1 = b_2$ and the continuity equation is

$2\pi r_1 b_1 V_{r1} = 2\pi r_2 b_2 V_{r2}$

or

$$V_{r2} = \frac{r_1}{r_2} V_{r1} = \frac{0.5\,ft}{0.9\,ft}\left(20\,\frac{ft}{s}\right) = 11.1\,\frac{ft}{s}$$

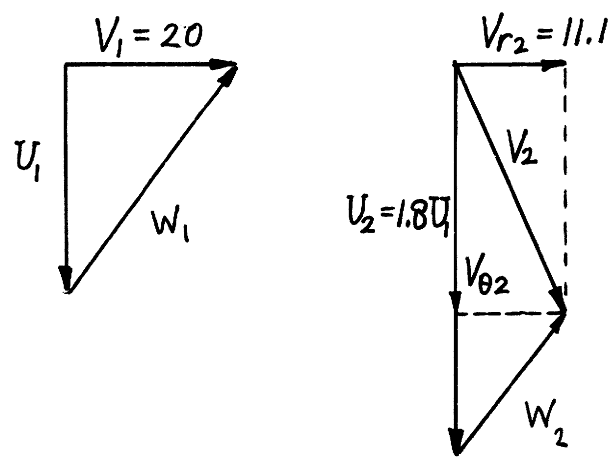
and

$$U_1 = r_1 \omega \,,\; U_2 = r_2 \omega \text{ so that } U_2 = \frac{r_2 U_1}{r_1} = \frac{0.9\,ft}{0.5\,ft} U_1$$

or

$U_2 = 1.8\,U_1$

Hence, the following velocity triangles can be drawn.



It is seen that the rotor turns the flow into the direction of the blade motion. <u>This is a pump.</u>

12.4

12.4 At a given radial location, a 15 ft/s wind against a windmill (see Video V12.1) results in the upstream (1) and downstream (2) velocity triangles shown in Fig. P12.4. Sketch an appropriate blade section at that radial location and determine the energy transferred per unit mass of fluid.

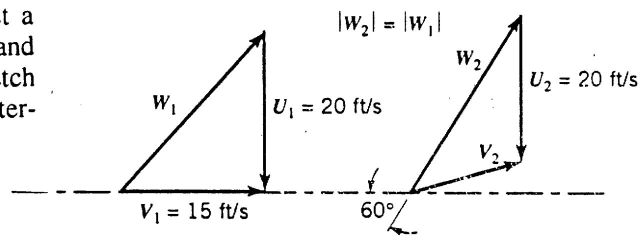


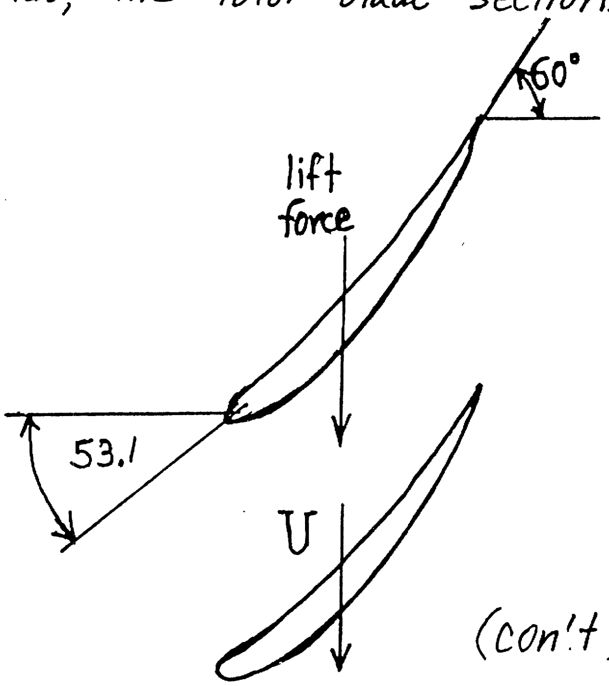
FIGURE P12.4

We can determine whether the axial flow turbomachine involved is a turbine or a fan by comparing the direction of the lift force on the rotor blade section with the direction of the blade velocity, U. If the lift force and the blade velocity are in the same direction a turbine is involved. If the lift force and blade velocity are in opposite directions, a fan is involved. The direction of the lift force can be inferred from the shape of the rotor blade section sketched to be tangent to the relative flows entering and leaving the rotor row.

The entering relative flow angle, β_1, is

$$\beta_1 = \tan^{-1}\frac{U_1}{V_1} = \tan^{-1}\frac{\left(20\frac{ft}{s}\right)}{\left(15\frac{ft}{s}\right)} = 53.1^\circ$$

Thus, the rotor blade sections sketched below are appropriate



(con't)

Since the lift force acting on each rotor blade section is in the same direction as the blade velocity we conclude that this turbomachine is a turbine. The energy transferred per unit mass is the shaft work per unit mass, w_{shaft}, which we can determine with Eq. 11.5. Thus

$$w_{shaft} = -U_2 V_{\theta,2} \tag{1}$$

From the velocity triangles we obtain

$$V_{\theta,2} = W_2 \sin 60° - U_2$$

and

$$W_2 = W_1 = \sqrt{V_1^2 + U_1^2}$$

Thus

$$w_{shaft} = -U_2\left(\sqrt{V_1^2 + U_1^2}\ \sin 60° - U_2\right)$$

$$w_{shaft} = -\left(20\,\frac{ft}{s}\right)\left[\sqrt{\left(15\,\frac{ft}{s}\right)^2 + \left(20\,\frac{ft}{s}\right)^2}\ \sin 60° - 20\,\frac{ft}{s}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

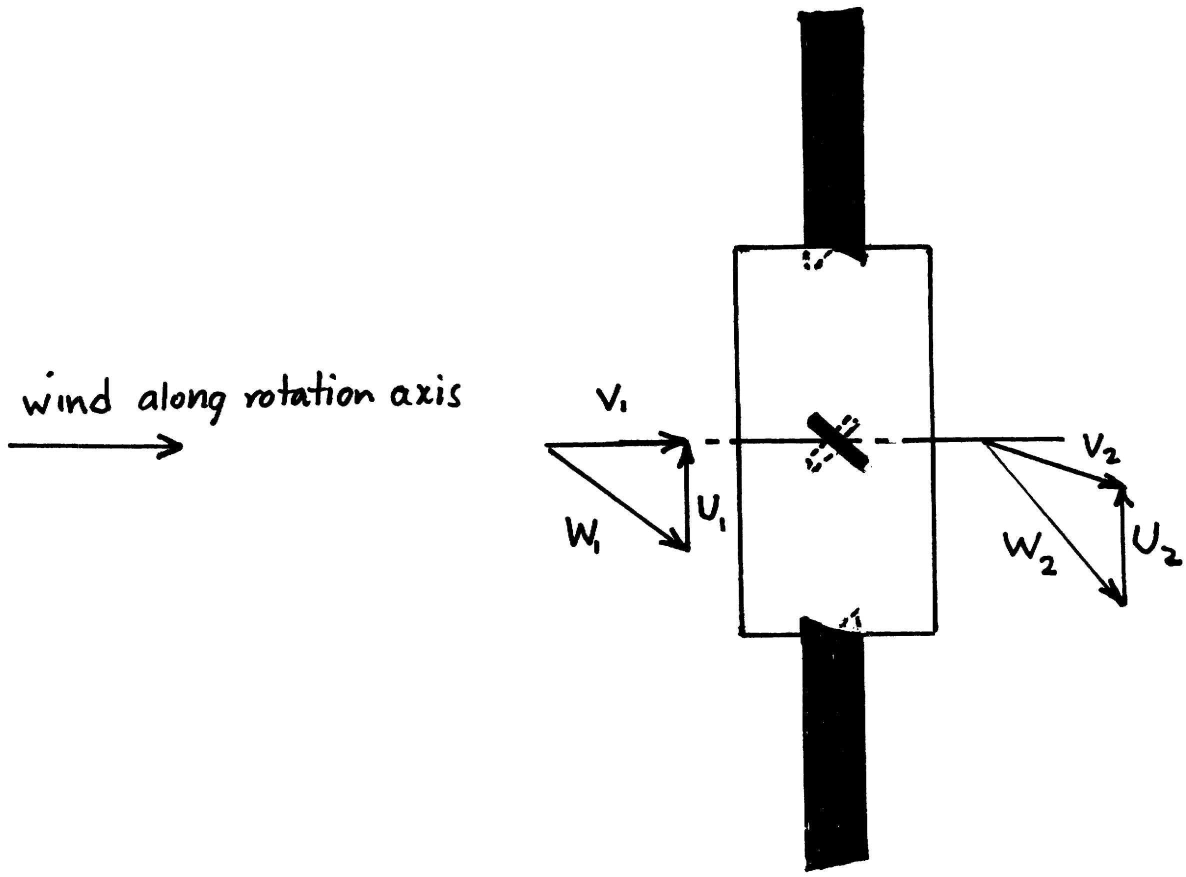
$$w_{shaft} = \underline{\underline{-33.0}}\ \frac{ft\cdot lb}{slug}$$

or

$$w_{shaft} = -33.0\ \frac{ft\cdot lb}{slug}\ \frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} = -1.02\ \frac{ft\cdot lb}{lbm}$$

12.5

12.5 Sketch how you would arrange four 3-in.-wide by 12-in.-long thin but rigid strips of sheet metal on a hub to create a windmill like the one shown in Video V12.1. Discuss, with the help of velocity triangles, how you would arrange each blade on the hub and how you would orient your windmill in the wind.



12.6 Sketched in Fig. P12.6 are the upstream [section (1)] and downstream [section (2)] velocity triangles at the arithmetic mean radius for flow through an axial-flow turbomachine rotor. The axial component of velocity is 50 ft/s at sections (1) and (2). (**a**) Label each velocity vector appropriately. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. (**b**) Are you dealing with a turbine or a fan? (**c**) Calculate the work per unit mass involved. (**d**) Sketch a reasonable blade section. Do you think the actual blade exit angle will need to be less or greater than 15°? Why?

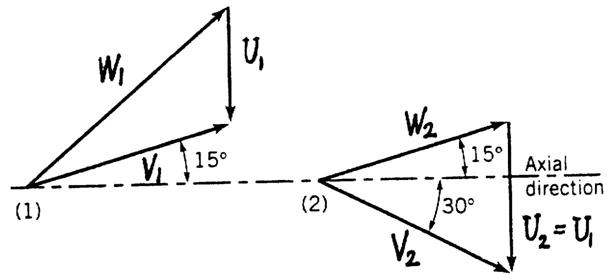


FIGURE P.12.6

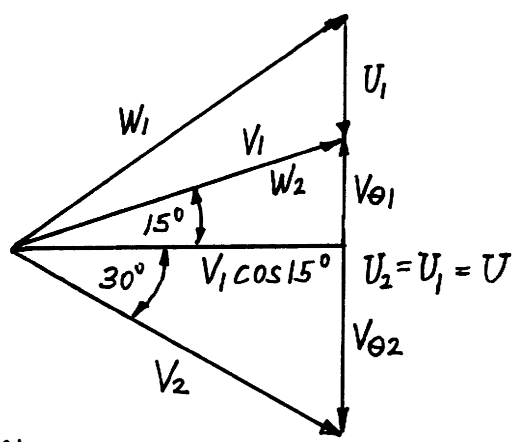
(a) See figure above.

(b) $T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \dot{m} r_{mean}(V_{\theta 2} - V_{\theta 1})$
where $V_{\theta 2} > 0$ and $V_{\theta 1} < 0$ (see figure above)

Thus, $T > 0$. The machine is a <u>fan.</u>

(c) $w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = U(V_{\theta 2} - V_{\theta 1})$ where $U = U_1 = U_2$

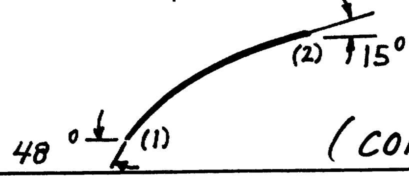
Since $V_{x1} = V_{x2} = 50 \frac{ft}{s}$, it follows from the figure that:

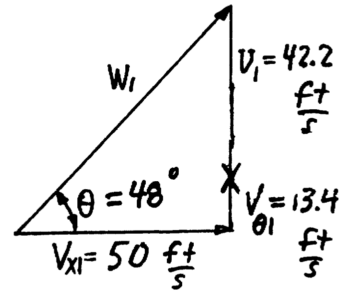


$V_1 \cos 15° = 50 \frac{ft}{s}$
or $V_1 = 51.8 \frac{ft}{s}$
and
$V_2 \cos 30° = 50 \frac{ft}{s}$ or $V_2 = 57.7 \frac{ft}{s}$
so that
$V_{\theta 1} = -V_1 \sin 15° = -51.8 \sin 15° = -13.4 \frac{ft}{s}$
and
$V_{\theta 2} = V_2 \sin 30° = 28.8 \frac{ft}{s}$
Also, $U = |V_{\theta 1}| + |V_{\theta 2}| = 42.2 \frac{ft}{s}$
Hence, $w_{shaft} = 42.2 \frac{ft}{s}\left(28.8 \frac{ft}{s} - \left(-13.4 \frac{ft}{s}\right)\right) = \underline{\underline{1780 \frac{ft^2}{s^2}}}$

(d) From the figure $\tan\theta = \dfrac{42.2 + 13.4}{50}$, or $\theta = 48°$

Thus, the blade shape is as shown:





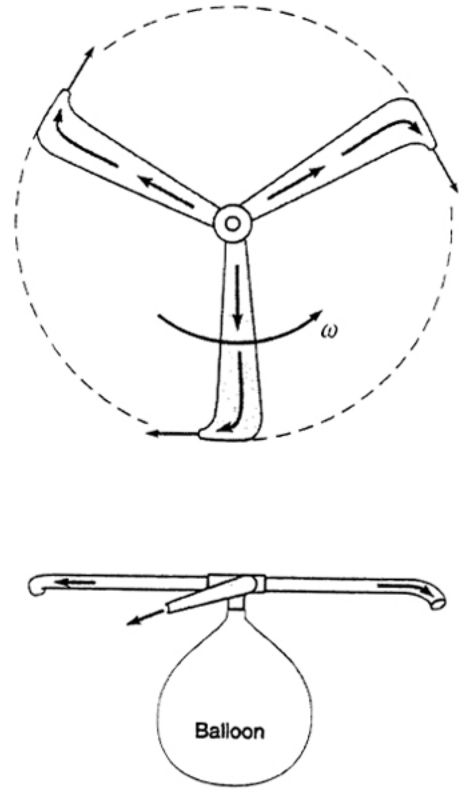
(con't)

12.6 (con't)

The actual blade angle will need to be less than 15°
to achieve a 15° flow angle at the blade exit.
Because of boundary layer development on both surfaces of the blade, the fluid angle will be different from the blade angle. Less turning than expected will be actually achieved.

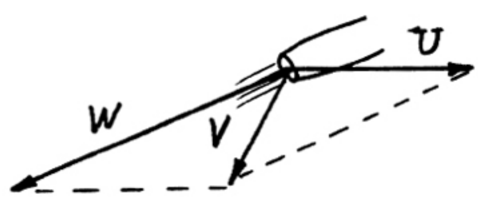
12.7

12.7 Shown in Fig. P12.7 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out small nozzles at the tips of the blades. The nozzles (along with the rotating propeller blades) are tilted at a small angle as indicated. Sketch the velocity triangle (i.e., blade, absolute, and relative velocities) for the flow from the nozzles. Explain why this toy tends to move upward. Is this a turbine? Pump?



■ FIGURE P12.7

If we assume the helicopter is stationary, then the blade speed is ωR in the horizontal plane as shown in the side view below. The relative velocity, $\vec{W}$, is directed along the nozzle, and the absolute velocity, $\vec{V} = \vec{W} + \vec{U}$, is as indicated.



The toy tends to move upward because the flow over the blades push up on them. The air from the balloon forces the blades to rotate like a turbine. However, the blades act on the ambient air as a pump.

12.8

A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, β_2, (see Fig. 12.8) is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 m^3/s. The uniform blade height is 50 mm.

$$\dot{W}_{shaft} = T_{shaft}\,\omega = T_{shaft}\,2\pi N$$

$$T_{shaft} = \rho Q (r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad \text{(Eq. 12.10)}$$

With $V_{\theta 1} = 0$

$$T_{shaft} = \rho Q r_2 V_{\theta 2} \qquad (1)$$

From Fig. 12.8c

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2 \qquad (2)$$

For $r_2 = \frac{0.5\,m}{2} = 0.25\,m$ with $\omega = \frac{(900\,\frac{rev}{min})(2\pi\,\frac{rad}{rev})}{60\,\frac{s}{min}} = 94.2\,\frac{rad}{s}$

then

$$U_2 = r_2\omega = (0.25\,m)(94.2\,\tfrac{rad}{s}) = 23.6\,\tfrac{m}{s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{(0.16\,\frac{m^3}{s})}{(2\pi)(0.25\,m)(0.05\,m)} = 2.04\,\frac{m}{s}$$

Thus, from Eq. (2)

$$V_{\theta 2} = (23.6 - 2.04\cot 25°)\,\tfrac{m}{s} = 19.2\,\tfrac{m}{s}$$

and from Eq. (1)

$$T_{shaft} = (999\,\tfrac{kg}{m^3})(0.16\,\tfrac{m^3}{s})(0.25\,m)(19.2\,\tfrac{m}{s}) = 768\,N\cdot m$$

so,

$$\dot{W}_{shaft} = (768\,N.m)(2\pi\,\tfrac{rad}{rev})(900\,\tfrac{rev}{min})\frac{1}{(60\,\frac{s}{min})}\left(\frac{1}{1000\,\frac{N.m}{s.\,kW}}\right) = \underline{\underline{0.08\,kW}}$$

12.9

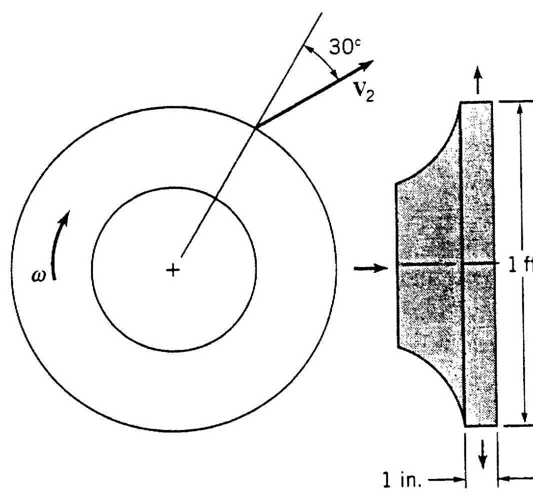
12.9 Discuss the differences between a centrifugal pump and a positive displacement pump (see Video V12.3 for an example of a positive displacement pump impeller that looks like a centrifugal pump impeller).

A centrifugal pump impeller pushes against the forces developed by a fluid flowing over its wetted surfaces and in so doing moves fluid through.

A positive displacement pump "traps" or "captures" a volume of fluid and moves the entire volume from one location to another.

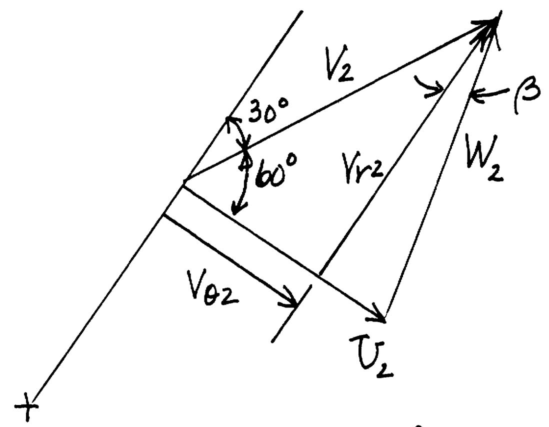
12.10

A centrifugal pump impeller is rotating at 1200 rpm in the direction shown in Fig. P12.10. The flow enters parallel to the axis of rotation and leaves at an angle of 30° to the radial direction. The absolute exit velocity, V_2, is 90 ft/s. **(a)** Draw the velocity triangle for the impeller exit flow. **(b)** Estimate the torque necessary to turn the impeller if the fluid density is 2.0 slugs/ft³. What will the impeller rotation speed become if the shaft breaks?



■ FIGURE P12.10

(a) The exit flow velocity triangle can be constructed graphically as indicated below,



with $U_2 = r_2 \omega = (0.5\,ft)\dfrac{(1200\,\frac{rev}{min})(2\pi\,\frac{rad}{rev})}{(60\,\frac{s}{min})} = 62.8\,\frac{ft}{s}$

From the velocity triangle

$$\tan\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

Since $V_{\theta 2} = V_2 \sin 30°$ and $V_{r2} = V_2 \cos 30°$ it follows that

$$\beta_2 = \tan^{-1}\left[\frac{U_2 - V_2 \sin 30°}{V_2 \cos 30°}\right]$$

$$= \tan^{-1}\left[\frac{62.8\,\frac{ft}{s} - (90\,\frac{ft}{s})\sin 30°}{(90\,\frac{ft}{s})\cos 30°}\right] = 12.9°$$

(con't)

12.10 (con't)

Thus, from the velocity triangle

$$W_2 = \frac{V_{r2}}{\cos 12.9^\circ} = \frac{V_2 \cos 30^\circ}{\cos 12.9^\circ} = \frac{(90 \frac{ft}{s}) \cos 30^\circ}{\cos 12.9^\circ}$$

$$= 80.0 \frac{ft}{s}$$

With β_2 and W known, the velocity triangle is completely specified.

(b) From Eq. 12.9 with $V_{\theta 1} = 0$

$$T_{shaft} = \dot{m} r_2 V_{\theta 2} \qquad (1)$$

Since

$$\dot{m} = \rho 2\pi r_2 b_2 V_{r2}$$

$$= (2.0 \frac{slugs}{ft^3})(2\pi)(0.5 ft)(\frac{1}{12} ft)(90 \frac{ft}{s}) \cos 30^\circ$$

$$= 40.8 \frac{slugs}{s}$$

so that from Eq. (1)

$$T_{shaft} = (40.8 \frac{slugs}{s})(0.5 ft)(90 \frac{ft}{s}) \sin 30^\circ$$

$$= \underline{\underline{918 \text{ ft} \cdot \text{lb}}}$$

A positive torque is in the same direction as the rotation.

When the shaft breaks, the torque becomes zero and the impeller eventually stops because there is no longer a driving torque to force it to rotate. In a pump, the shaft torque drives the impeller and the impeller moves fluid. On the other hand, in a turbine, the moving fluid drives the impeller.

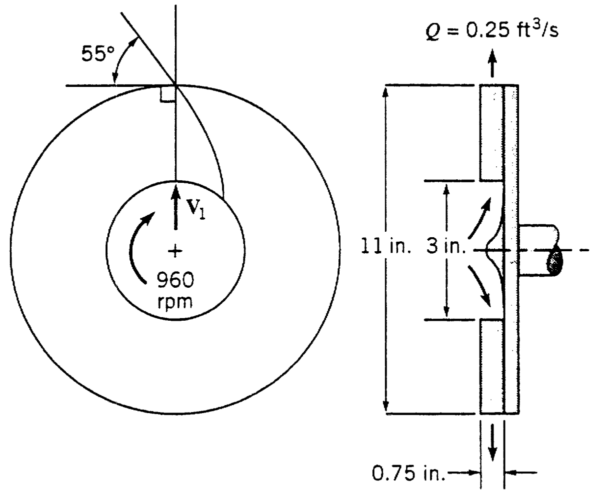
12.11

Discuss the main simplifying assumptions associated with Eq. 12.13 and explain why actual head rise is always less than ideal head rise. Discuss how ideal head rise is head "added" to the fluid and actual head rise is head "gained" by the fluid.

Eq. 12.13 is obtained assuming that no loss of available energy occurs in the flow through the pump impeller. The actual head rise across the pump is thus equal to the ideal head rise across the pump minus the loss of available energy suffered by the flowing fluid because of friction. The blades add the ideal head rise amount to the flowing fluid, however, the fluid flow loss results in the actual head rise realized by the flowing fluid being less than the ideal amount by the loss.

12.12

12.12 A centrifugal radial water pump has the dimensions shown in Fig. P12.12. The volume rate of flow is 0.25 ft³/s, and the absolute inlet velocity is directed radially outward. The angular velocity of the impeller is 960 rpm. The exit velocity as seen from a coordinate system attached to the impeller can be assumed to be tangent to the vane at its trailing edge. Calculate the power required to drive the pump.



■ FIGURE P12.12

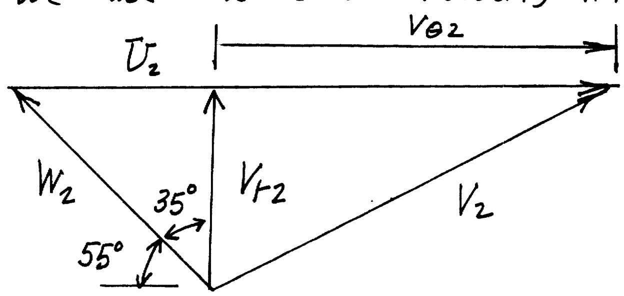
From Eq. 12.11, with $V_{\theta 1}=0$,

$$\dot{W}_{shaft} = \rho Q U_2 V_{\theta 2} \qquad (1)$$

To determine U_2 we use

$$U_2 = r_2\omega = \left(\frac{5.5\text{ in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)\left(960\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1}{60\ \frac{\text{s}}{\text{min}}}\right) = 46.1\ \frac{\text{ft}}{\text{s}}$$

To obtain $V_{\theta 2}$ we use the exit velocity triangle shown below.



Since $\quad V_{\theta 2} = U_2 - V_{r2}\tan 35°$

and

$$V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.25\ \frac{\text{ft}^3}{\text{s}}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{(2\pi)(5.5\text{ in.})(0.75\text{ in.})} = 1.39\ \frac{\text{ft}}{\text{s}}$$

it follows that

$$V_{\theta 2} = (46.1 - 1.39\tan 35°)\ \frac{\text{ft}}{\text{s}} = 45.1\ \frac{\text{ft}}{\text{s}}$$

Thus, from Eq. (1)

$$\dot{W}_{shaft} = \left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(0.25\ \frac{\text{ft}^3}{\text{s}}\right)\left(46.1\ \frac{\text{ft}}{\text{s}}\right)\left(45.1\ \frac{\text{ft}}{\text{s}}\right)$$

$$= 1010\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

or

$$\dot{W}_{shaft} = \frac{1010\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}} = \underline{\underline{1.84\text{ hp}}}$$

12.13

Water is pumped with a centrifugal pump, and measurements made on the pump indicate that for a flowrate of 240 gpm the required input power is 6 hp. For a pump efficiency of 62%, what is the actual head rise of the water being pumped?

From Eq. 12.23 the pump efficiency is given by the equation

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

so that

$$h_a = \frac{(\eta)(bhp)(550)}{\gamma Q}$$

$$= \frac{(0.62)(6\ hp)\left(550\ \frac{ft \cdot lb}{s \cdot hp}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)\left[\left(240\ \frac{gal}{min}\right) \Big/ \left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)\right]}$$

$$= \underline{\underline{61.3\ ft}}$$

12.14

The performance characteristics of a certain centrifugal pump are determined from an experimental setup similar to that shown in Fig. 12.10. When the flowrate of a liquid ($SG = 0.9$) through the pump is 120 gpm, the pressure gage at (1) indicates a vacuum of 95 mm of mercury and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. If $z_2 - z_1 = 0.5$ m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.

From Eq. 12.19

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} \qquad (1)$$

Since

$$V_1 = \frac{Q}{A_1} = \frac{(120\ gpm)\left(6.309 \times 10^{-5}\ \frac{m^3/s}{gpm}\right)}{\frac{\pi}{4}(0.110\ m)^2} = 0.797\ \frac{m}{s}$$

and

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \left(\frac{110\ mm}{55\ mm}\right)^2 = \left(0.797\ \frac{m}{s}\right)(2)^2 = 3.19\ \frac{m}{s}$$

Thus, from Eq. (1), with $p_1 = -(h_{Hg})(\gamma_{Hg}) = -(0.095\ m)\left(133 \times 10^3\ \frac{N}{m^3}\right)$
and $p_2 = 80 \times 10^3\ N/m^2$,

$$h_a = \frac{80 \times 10^3\ \frac{N}{m^2} + (0.095)(133 \times 10^3)\ \frac{N}{m^2}}{(0.9)(9.80 \times 10^3)\ \frac{N}{m^3}} + 0.5\ m + \frac{\left(3.19\ \frac{m}{s}\right)^2 - \left(0.797\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)}$$

$$h_a = \underline{\underline{11.5\ m}}$$

To estimate the pump motor power requirement use Eq. 12.23

$$\eta = \frac{\gamma Q h_a}{bhp\,(550)}$$

to get

$$bhp = \frac{\gamma Q h_a}{\eta (550)}$$

For differing values of η, a corresponding bhp can be calculated.

12.15

The performance characteristics of a certain centrifugal pump having a 9-in.-diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

Q (gpm)	20	40	60	80	100	120	140
$p_2 - p_1$ (psi)	40.2	40.1	38.1	36.2	33.5	30.1	25.8
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

Based on these data, show or plot how the actual head rise, h_a, and the pump efficiency, η, vary with the flowrate. What is the design flowrate for this pump?

From Eq. 12.19 with $z_1 = z_2$ and $V_1 = V_2$

$$h_a = \frac{p_2 - p_1}{\gamma}$$

Thus, for the first set of data in the table

$$h_a = \frac{\left(40.2 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.4 \frac{lb}{ft^3}} = 92.8\,ft$$

From Eq. 12.23

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

and for the first set of data in the table

$$\eta = \frac{\left(62.4 \frac{lb}{ft^3}\right)\left[(20\,gpm)/\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\right](92.8\,ft)}{(1.58\,hp)\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)}$$

$$= 0.297$$

or

$$\eta = 29.7\,\%$$

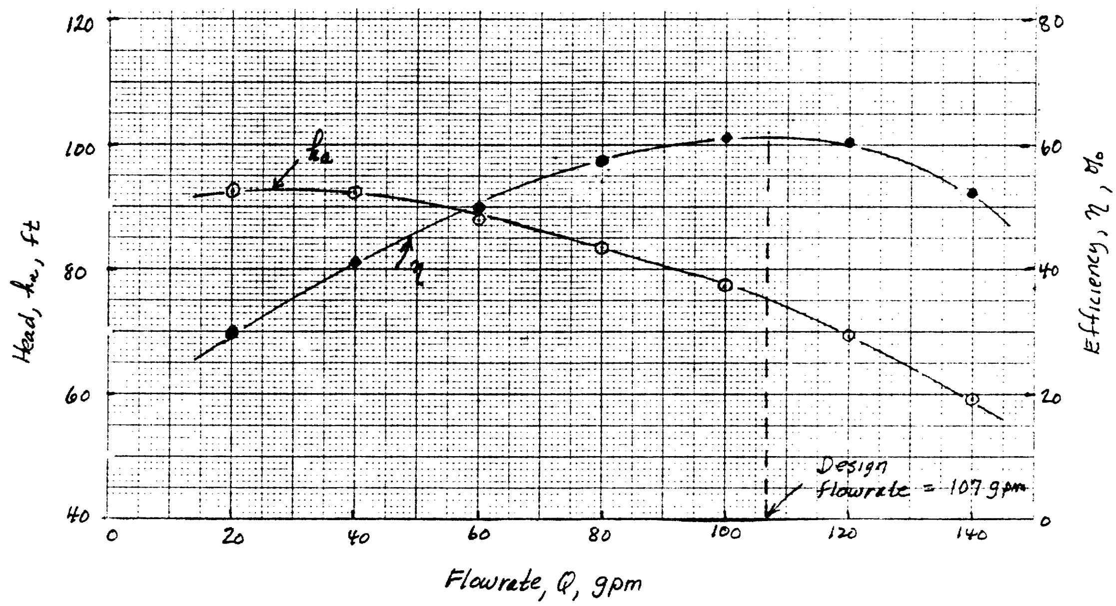
Remaining values for h_a and η can be calculated in a similar manner, and all values are tabulated in the table below.

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6

(con't)

12.15 (con't)

A plot of the data is shown below. The design flowrate occurs at peak efficiency and is $\underline{\underline{107 \text{ gpm}}}$.



12.16

12.16 It is sometimes useful to have $h_a - Q$ pump performance curves expressed in the form of an equation. Fit the $h_a - Q$ data given in Problem 12.15 to an equation of the form $h_a = h_o - kQ^2$ and compare the values of h_a determined from the equation with the experimentally determined values. (Hint: Plot h_a versus Q^2 and use the method of least squares to fit the data to the equation.)

Based on the data from Problem 12.15, the following table can be created and from a standard, linear regression curve fitting program the following results are obtained.

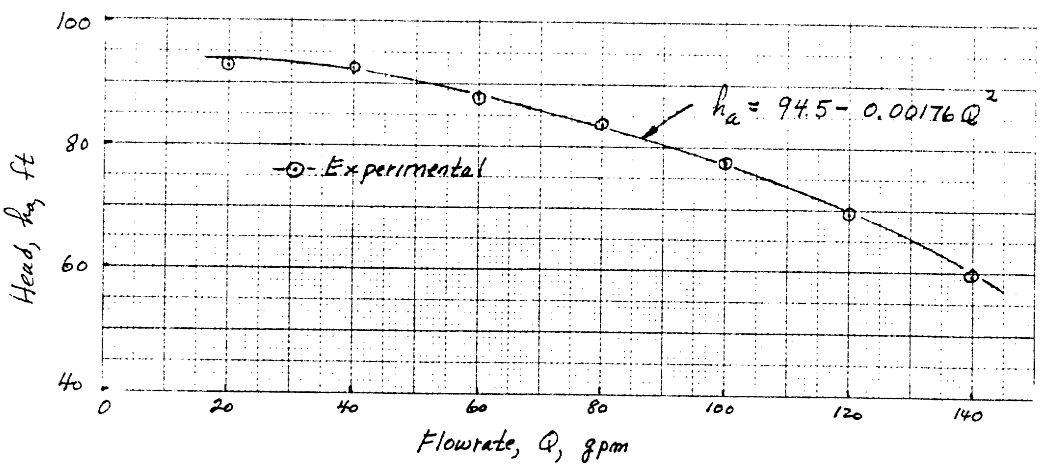
Q (gpm)	20	40	60	80	100	120	140
$[Q \text{ (gpm)}]^2$	4×10^2	16×10^2	36×10^2	64×10^2	100×10^2	144×10^2	196×10^2
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
Δh_a (ft)*	−1.00	0.81	−0.27	0.26	0.39	0.33	−0.52

* $\Delta h_a = h_a$ (experimental) $- h_a$ (predicted)

The equation obtained from the data using linear regression is

$$h_a = 94.5 - 0.00176\, Q^2 \qquad (1)$$

where h_a is in ft with Q in gpm. A plot showing the comparison between the experimental data and the predicted results (from Eq. 1) is shown below.



12.17

12.17 In Example 12.3, how will the maximum height, z_1, that the pump can be located above the water surface change if (**a**) the water temperature is increased to 120 °F, or (**b**) the fluid is changed from water to gasoline at 60 °F?

(a) From Table B.1 the water vapor pressure is 1.692 psia and $\gamma = 61.71\ lb/ft^3$. Thus, with this change in Eq.(2) in Example 12.3

$$(z_1)_{max} = \frac{(14.7\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{61.71\ \frac{lb}{ft^3}} - 10.2\ ft$$

$$- \frac{(1.692\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{61.71\ \frac{lb}{ft^3}} - 15\ ft$$

so that

$$(z_1)_{max} = \underline{\underline{5.15\ ft}}$$

Thus, there is a decrease in height from 7.65 ft to 5.15 ft.

(b) From Table 1.6 the gasoline vapor pressure at 60°F is 8.0 psia and $\gamma = 42.5\ lb/ft^3$. Thus, as above

$$(z_1)_{max} = \frac{(14.7\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{42.5\ \frac{lb}{ft^3}} - 10.2\ ft$$

$$- \frac{(8.0\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{42.5\ \frac{lb}{ft^3}} - 15\ ft$$

so that

$$(z_1)_{max} = \underline{\underline{-2.49\ ft}}$$

The negative sign for $(z_1)_{max}$ indicates that under the conditions specified the pump could not operate without cavitation unless it is located <u>below</u> the surface level of the gasoline.

12.18

12.18 A centrifugal pump with a 7-in.-diameter impeller has the performance characteristics shown in Fig. 12.12. The pump is used to pump water at 100 °F, and the pump inlet is located 12 ft above the open water surface. When the flowrate is 200 gpm the head loss between the water surface and the pump inlet is 6 ft of water. Would you expect cavitation in the pump to be a problem? Assume standard atmospheric pressure. Explain how you arrived at your answer.

From Eq. 12.25

$$NPSH_A = \frac{p_{atm}}{\gamma} - z_1 - \Sigma h_L - \frac{p_v}{\gamma} \qquad (1)$$

From Table B.1 the water vapor pressure at 100°F is 0.9493 psia and $\gamma = 62.00 \frac{lb}{ft^3}$. Thus, with $p_{atm} = 14.7$ psia, $z_1 = 12$ ft, and $\Sigma h_L = 6$ ft, Eq. (1) yields

$$NPSH_A = \frac{\left(14.7 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.00 \frac{lb}{ft^3}} - 12\text{ ft} - 6\text{ ft} - \frac{\left(0.9493 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.00 \frac{lb}{ft^3}}$$

$$= 13.9\text{ ft}$$

From Fig. 12.12 at 200 gpm

$$NPSH_R = \sim 12\text{ ft}$$

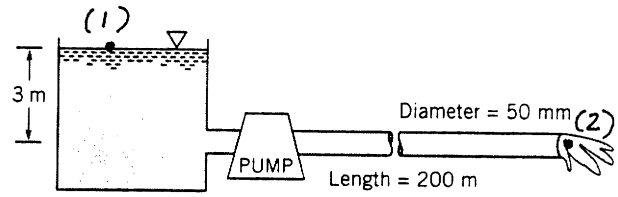
For proper pump operation

$$NPSH_A \geq NPSH_R$$

Since this is true in this case, we expect that cavitation in the pump would not be a problem. No.

12.19

12.19 Water at 40 °C is pumped from an open tank through 200 m of 50-mm-diameter smooth horizontal pipe as shown in Fig. P12.19 and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. (**a**) If the efficiency of the pump is 70%, how much power is being supplied to the pump? (**b**) What is the NPSH$_A$ at the pump inlet? Neglect losses in the short section of pipe connecting the pump to the tank. Assume standard atmospheric pressure.



■ **FIGURE P12.19**

(a) $$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V_2^2}{2g} \qquad (1)$$

where $p_1 = p_2 = 0$, $V_1 = 0$, $V_2 = 3\ m/s$, $z_1 = 3m$, and $z_2 = 0$. Thus, Eq. (1) becomes

$$z_1 + h_p = \frac{V_2^2}{2g}\left(1 + f\frac{\ell}{D}\right) \qquad (2)$$

Also,
$$Re = \frac{VD}{\nu} = \frac{(3\frac{m}{s})(0.05m)}{(6.580\times10^{-7}\frac{m^2}{s})} = 2.28\times10^5$$

and from Fig. 8.23 for smooth pipe $f = 0.0152$. Thus, from Eq. (2)

$$h_p = \frac{(3\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\left[1 + 0.0152\left(\frac{200\ m}{0.05m}\right)\right] - 3m = 25.3\ m$$

Hence,

$$\text{Power gained by fluid} = \gamma Q h_p$$
$$= (9.731\times10^3\frac{N}{m^3})(\frac{\pi}{4})(0.05m)^2(3\frac{m}{s})(25.3m)$$
$$= 1.45\times10^3\frac{N\cdot m}{s} = 1.45\ kW$$

and

$$\text{Power supplied to pump} = \frac{\text{Power gained by fluid}}{\text{Efficiency}}$$
$$= \frac{1.45\ kW}{0.7} = \underline{\underline{2.07\ kW}}$$

(b) From Eq. 12.24

$$NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \qquad (3)$$

where p_s and V_s refer to the pressure and velocity at the pump inlet, respectively. Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + h_L$$

so that with $p_1 = p_{atm}$, $V_1 = 0$, $z_s = 0$, and $h_L = 0$

(con't)

12.19 (Con't)

$$\frac{p_{atm}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

and therefore from Eq.(3) the available NPSH is

$$NPSH_A = \frac{p_{atm}}{\gamma} + z_1 - \frac{p_v}{\gamma} \qquad (4)$$

Note that this result corresponds to Eq. 12.25 with z_1 positive (since pump is below reservoir) and $\Sigma h_L = 0$.

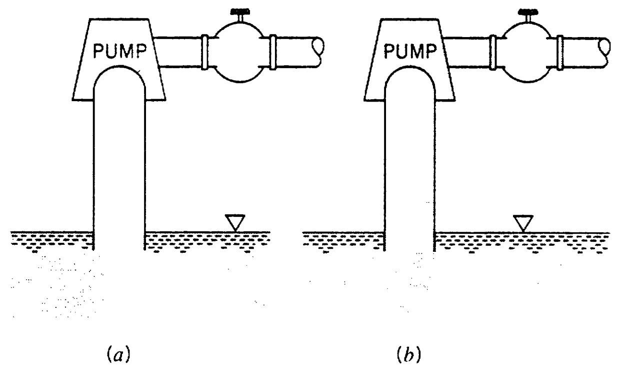
From Table B.2 the water vapor pressure at $40°C$ is $7.376 \times 10^3\ N/m^2$ (abs) and $\gamma = 9.731 \times 10^3\ N/m^3$. Thus, from Eq.(4) with $P_{atm} = 101\ kPa$

$$NPSH_A = \frac{(101 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})} + 3m - \frac{(7.376 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})}$$

$$= \underline{\underline{12.6\ m}}$$

12.20

12.20 The centrifugal pump shown in Fig. P12.20 is not self-priming. That is, if the water is drained from the pump and pipe as shown in Fig. P12.20(a), the pump will not draw the water into the pump and start pumping when the pump is turned on. However, if the pump is primed [i.e., filled with water as in Fig. P12.20(b)], the pump does start pumping water when turned on. Explain this behavior.



■ FIGURE P12.20

The head-flowrate characteristics for a typical centrifugal pump are shown in Fig. 12.11. The maximum head that the pump can add occurs when when $Q \approx 0$ (i.e., at start up for example). This head is in terms of the fluid in the pump. Neglecting losses and the velocity head (and cavitation effects) the pump can lift the fluid a height H equal to the head added by the pump. However, if the fluid in the pump is air (i.e., not primed) the head added is in terms of ft or m of air. For example, if $h_a = 30\,ft$ the pump could raise water that high if it is primed (filled with water). If the pump is not primed (filled with air) then the pump can only raise water up to a distance

$$H = 30\,ft\,\frac{\gamma_{air}}{\gamma_{water}} = 30\,ft\,\frac{\left(0.0765\,\frac{lb}{ft^3}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)} = 0.0368\,ft$$

Hence the water will not get into the pump.

12.21

Owing to fouling of the pipe wall, the friction factor for the pipe of Example 12.4 increases from 0.02 to 0.03. Determine the new flowrate, assuming all other conditions remain the same. What is the pump efficiency at this new flowrate? Explain how a line valve could be used to vary the flowrate through the pipe of Example 12.4. Would it be better to place the valve upstream or downstream of the pump? Why?

With $f=0.03$, Eq.(2) in Example 12.4 becomes

$$h_p = 10\text{ ft} + \left[0.03\frac{(200\text{ ft})}{(\frac{6}{12}\text{ ft})} + (0.5+1.5+1.0)\right]\frac{V^2}{2(32.2\frac{\text{ft}}{\text{s}^2})} \qquad (1)$$

Since,

$$V = \frac{Q}{A} = \frac{Q(\frac{\text{ft}^3}{\text{s}})}{(\frac{\pi}{4})(\frac{6}{12}\text{ft})^2}$$

Eq.(1) can be written as

$$h_p = 10 + 6.04\,Q^2$$

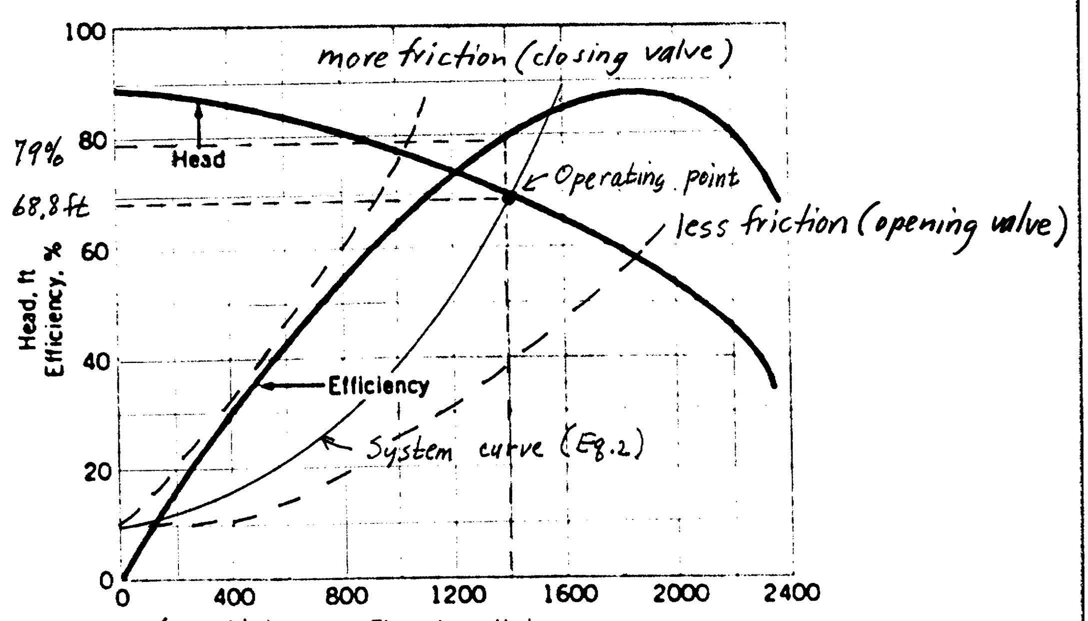
or with Q in gal/min

$$h_p = 10 + 3.00\times10^{-5}\left[Q(\text{gal/min})\right]^2 \qquad (2)$$

The intersection of Eq.(2) (the system equation) with the performance curve for the pump, as shown below, indicates that the new flowrate is

$$\underline{\underline{Q = 1400\ \frac{\text{gal}}{\text{min}}}}$$

and the efficiency at this flowrate is approximately $\underline{\underline{79.0\%}}$.



(con't)

12.21 (Con't)

A line valve acts as a variable frictional resistance to the flow. Closing the valve is equivalent to adding friction and moving the system curve to the left intersecting the head curve at an operational point involving less flowrate than with a more open valve setting. This system curve is sketched in the figure on the previous page and labeled "more friction (closing valve)." Opening the valve is similar to removing friction and moving the system curve to the right intersecting the head curve at an operating point involving more flowrate than with a less open valve setting. This system curve is sketched on the previous page and labeled "less friction (opening valve)."

It would be generally better to place the valve downstream of the pump to avoid the low suction pressure and cavitation possible with upstream placement of the valve.

12.22

12.22 A centrifugal pump having a head-capacity relationship given by the equation $h_a = 180 - 6.10 \times 10^{-4}Q^2$, with h_a in feet when Q is in gpm, is to be used with a system similar to that shown in Fig. 12.14. For $z_2 - z_1 = 50$ ft, what is the expected flowrate if the total length of constant-diameter pipe is 600 ft and the fluid is water? Assume the pipe diameter to be 4 in. and the friction factor to be equal to 0.02. Neglect all minor losses.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_2 - z_1 = 50\,ft$, $f = 0.02$, $D = 4/12\ ft$, and $\ell = 600\,ft$, Eq. (1) becomes

$$h_p = 50\,ft + 0.02\,\frac{(600\,ft)}{(\frac{4}{12}\,ft)}\,\frac{V^2}{(2)(32.2\,\frac{ft}{s^2})} \qquad (2)$$

Since

$$V = \frac{Q}{A} = \frac{Q(\frac{ft^3}{s})}{(\frac{\pi}{4})(\frac{4}{12}\,ft)^2}$$

Eq. (2) can be written as

$$h_p = 50 + 73.4\,Q^2$$

or with Q in gal/min

$$h_p = 50 + 3.64 \times 10^{-4}\,[Q\,(gal/min)]^2 \qquad (3)$$

The pump head-capacity relationship is

$$h_a = 180 - 6.10 \times 10^{-4}\,[Q\,(gal/min)]^2 \qquad (4)$$

Thus, the operating point will occur at the flowrate where $h_a = h_p$, or

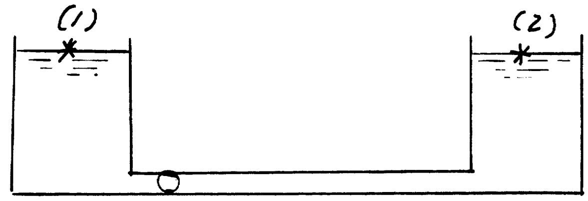
$$180 - 6.10 \times 10^{-4}\,Q^2 = 50 + 3.64 \times 10^{-4}\,Q^2$$

So that

$$Q = \underline{\underline{365\ gpm}}$$

12.23

12.23 A centrifugal pump having a 6-in.-diameter impeller and the characteristics shown in Fig. 12.12 is to be used to pump gasoline through 4000 ft of commercial steel 3-in.-diameter pipe. The pipe connects two reservoirs having open surfaces at the same elevation. Determine the flowrate. Do you think this pump is a good choice? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \tag{1}$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = z_2 = 0$, $\ell = 4000\,ft$, and $D = 3/12\,ft$ (neglecting minor losses), Eq. (1) becomes

$$h_p = f\,\frac{(4000\,ft)}{(3/12\,ft)}\,\frac{V^2}{(2)(32.2\,\frac{ft}{s^2})} \tag{2}$$

Since $V = \frac{Q}{A} = \frac{Q\,(ft^3/s)}{(\frac{\pi}{4})(\frac{3}{12}\,ft)^2}$

Eq. (2) can be written as

$$h_p = 1.03 \times 10^5 f\left[Q\,(ft^3/s)\right]^2 \tag{3}$$

The friction factor depends on $Re = VD/\nu = 4Q/\pi D\nu$ and with $\nu = 4.9 \times 10^{-6}\,ft^2/s$ for gasoline

$$Re = \frac{4\,Q\,(ft^3/s)}{(\pi)(3/12\,ft)(4.9 \times 10^{-6}\,\frac{ft^2}{s})} = 1.04 \times 10^6\,Q\,(ft^3/s)$$

For commercial steel 3-in. diameter pipe (from Fig. 8.22)

$$\frac{\varepsilon}{D} = 5.8 \times 10^{-4}$$

Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq. (3). Tabulated values are given in the following table.

(con't)

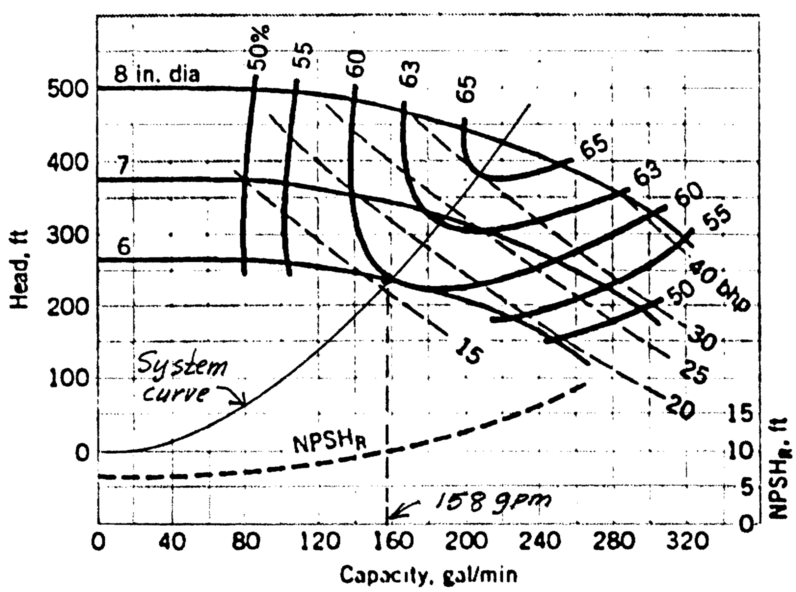
12.23 (Con't)

$Q\left(\frac{gal}{min}\right)$	$Q\left(\frac{ft^3}{s}\right)$	Re	f	h_p (ft)
40	0.0891	9.27×10^4	0.0208	17.0
80	0.178	1.85×10^5	0.0193	63.0
120	0.267	2.78×10^5	0.0187	137
160	0.357	3.71×10^5	0.0184	242
200	0.446	4.64×10^5	0.0182	373
240	0.535	5.56×10^5	0.0181	534

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced below), and the flowrate at the intersection of the system curve and the pump curve is

$$Q = 158\ \frac{gal}{min}$$

Since at this flowrate the pump operates near peak efficiency this type of pump would appear to be a good choice if the 158 gal/min flowrate is at or near the desired flowrate.



12.24

Determine the new flowrate for the system described in Problem 12.23 if the pipe diameter is increased from 3 in. to 4 in. Is this pump still a good choice? Explain.

Refer to solution to Problem 12.23. With $D = 4/12$ ft Eq. (2) becomes

$$h_p = f \frac{(4000\ ft)}{(\frac{4}{12}\ ft)} \frac{V^2}{(2)(32.2\ \frac{ft}{s^2})} \qquad (2)$$

and

$$V = \frac{Q}{A} = \frac{Q\ (ft^3/s)}{(\frac{\pi}{4})(\frac{4}{12}\ ft)^2}$$

so that

$$h_p = 2.45 \times 10^4 f \left[Q\ (ft^3/s) \right]^2 \qquad (3)$$

The Reynolds number becomes

$$Re = \frac{4Q}{\pi D \nu} = \frac{4\ Q\ (ft^3/s)}{(\pi)(4/12\ ft)(4.9 \times 10^{-6}\ \frac{ft^2}{s})} = 7.80 \times 10^5\ Q\ (ft^3/s)$$

For commercial steel 4-in. diameter pipe (from Fig. 8.22), $\frac{\varepsilon}{D} = 4.3 \times 10^{-4}$. Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq. (3). Tabulated values are given in the following table.

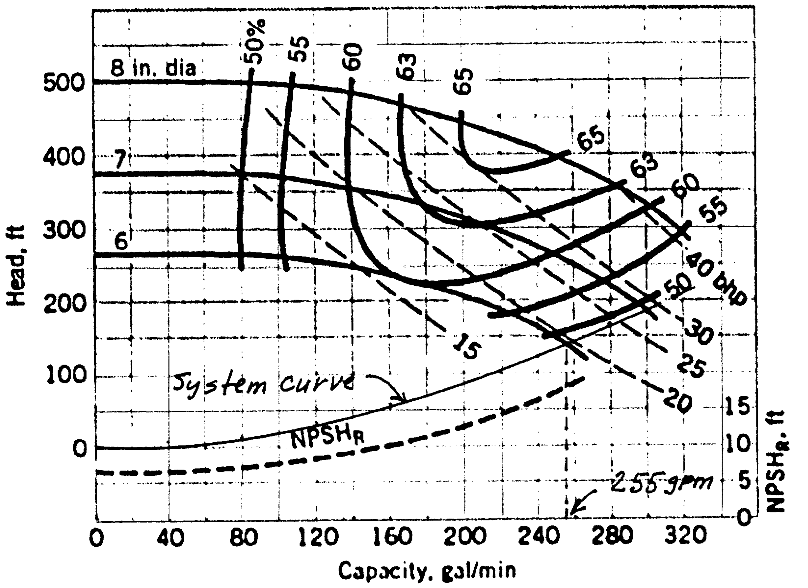
$Q\ (\frac{gal}{min})$	$Q\ (\frac{ft^3}{s})$	Re	f	h_p (ft)
40	0.0891	6.95×10^4	0.0211	4.1
80	0.178	1.39×10^5	0.0192	14.9
120	0.267	2.08×10^5	0.0183	32.0
160	0.357	2.78×10^5	0.0179	55.9
200	0.446	3.48×10^5	0.0176	85.8
240	0.535	4.17×10^5	0.0174	122
280	0.624	4.87×10^5	0.0172	164
320	0.713	5.56×10^5	0.0170	212

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced on the following page), and the flowrate at the intersection of the system curve and the pump curve is

$$\underline{\underline{Q = 255\ \frac{gal}{min}}}$$

(con't)

12.24 (Con't)



Since at this flowrate the pump efficiency is fairly low (~49%), this pump <u>is no longer a good choice</u>.

12.25

12.25 A centrifugal pump having the characteristics shown in Example 12.4 is used to pump water between two large open tanks through 100 ft of 8-in.-diameter pipe. The pipeline contains 4 regular flanged 90° elbows, a check value, and a fully open globe value. Assume the friction factor $f = 0.02$ for the 100-ft section of pipe. Other minor losses are negligible. If the static head (difference in height of fluid surfaces in the two tanks) is 30 ft, what is the expected flowrate? Do you think this pump is a good choice? Explain.

Application of the energy equation between the two free surfaces, points (1) and (2), gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 30\text{ ft}$, Eq. (1) becomes

$$h_p = 30\text{ ft} + \sum h_L \qquad (2)$$

The head loss term can be expressed as

$$\sum h_L = \left[4(0.3) + 10 + 2 + 0.02\frac{(100\text{ ft})}{(\frac{8}{12}\text{ ft})}\right]\frac{V^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

with the minor loss coefficients obtained from Table 8.3. Also,

$$V = \frac{Q}{A} = \frac{Q(\text{ft}^3/\text{s})}{(\frac{\pi}{4})(\frac{8}{12}\text{ ft})^2}$$

and Eq. (2) becomes

$$h_p = 30 + 2.06\left[Q(\text{ft}^3/\text{s})\right]^2$$

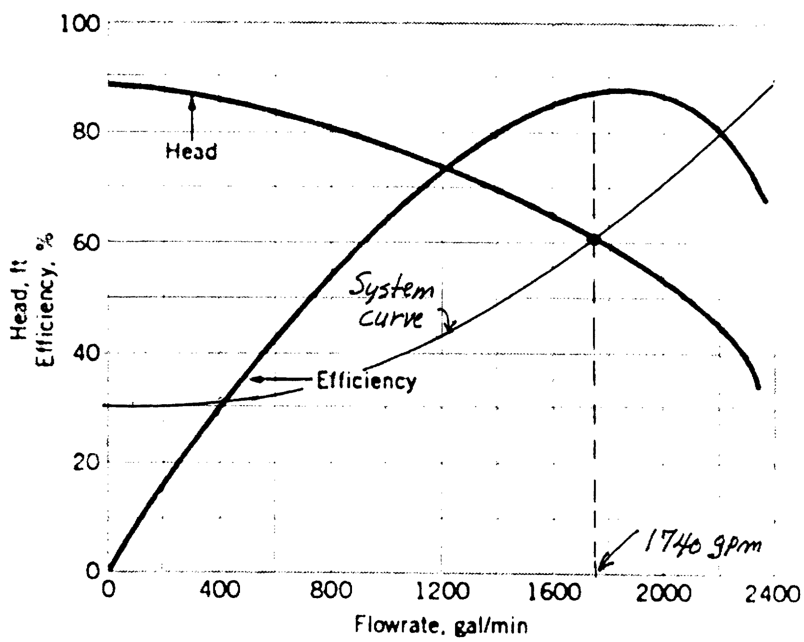
or the system equation can be written as

$$h_p = 30 + 1.02\times10^{-5}\left[Q(\tfrac{\text{gal}}{\text{min}})\right]^2 \qquad (3)$$

The intersection of the system curve (Eq. 3) with the pump curve, as shown on the figure, indicates that

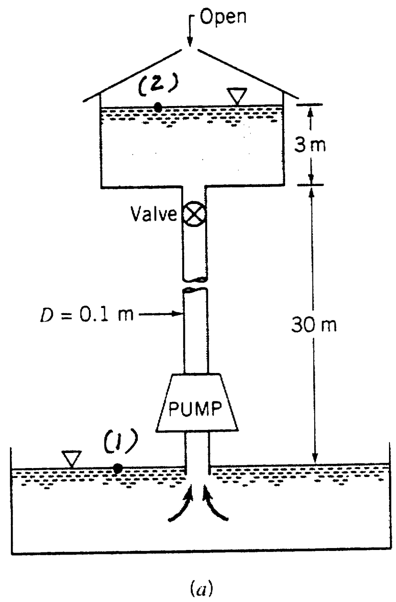
$$Q = 1740\ \frac{\text{gal}}{\text{min}}$$

Since the efficiency at this flowrate is near peak efficiency, as shown on the figure, this pump would be satisfactory.

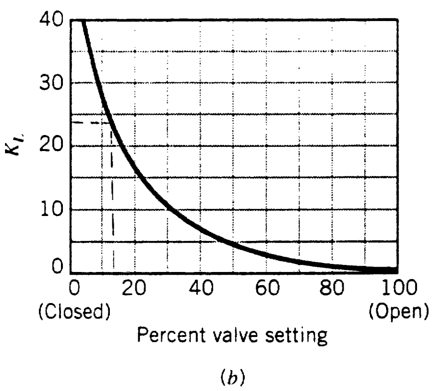


12.26

12.26 In a chemical processing plant a liquid is pumped from an open tank through a 0.1-m-diameter vertical pipe into another open tank as shown in Fig. P12.26(*a*). A valve is located in the pipe, and the minor loss coefficient for the valve as a function of the valve setting is shown in Fig. P12.26(*b*). The pump head-capacity relationship is given by the equation $h_a = 52.0 - 1.01 \times 10^3 Q^2$ with h_a in meters when Q is in m^3/s. Assume the friction factor $f = 0.02$ for the pipe, and all minor losses, except for the valve, are negligible. The fluid levels in the two tanks can be assumed to remain constant. **(a)** Determine the flowrate with the valve wide open. **(b)** Determine the required valve setting (percent open) to reduce the flowrate by 50%.



(*a*)



(*b*)

■ **FIGURE P12.26**

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \Sigma h_L \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 33\,m$, Eq. (1) becomes

$$h_p = 33\,m + \Sigma h_L \qquad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left(K_L + f\frac{\ell}{D}\right)\frac{V^2}{2g}$$

(a) With the valve open $K_L \approx 1.0$ (from Fig. P12.29 b) so that with $f = 0.02$, $\ell = 30m$, and $D = 0.1m$, Eq. (2) can be written as

$$h_p = 33m + \left[1.0 + 0.02\frac{(30m)}{(0.1m)}\right]\frac{V^2}{2\left(9.81\frac{m}{s^2}\right)} \qquad (3)$$

and with

$$V = \frac{Q}{A} = \frac{Q\left(\frac{m^3}{s}\right)}{\left(\frac{\pi}{4}\right)(0.1m)^2}$$

Eq. (3) becomes

$$h_p = 33m + [1.0 + 6.0](826)\left[Q\left(\frac{m^3}{s}\right)\right]^2 \qquad (4)$$

or

$$h_p = 33 + 5.78 \times 10^3\left[Q\left(\frac{m^3}{s}\right)\right]^2 \qquad (5)$$

(con't)

12.26 (con't)

Since the pump equation is

$$h_p = 52.0 - 1.01 \times 10^3 \left[Q\left(\frac{m^3}{s}\right) \right]^2 \qquad (6)$$

Eq. (5) and Eq. (6) can be equated to determine the flowrate. Thus,

$$33 + 5.78 \times 10^3 Q^2 = 52.0 - 1.01 \times 10^3 Q^2$$

and

$$Q = \underline{\underline{0.0529 \frac{m^3}{s}}}$$

(b) If the flowrate is to be cut in half so that $Q = 0.0529/2 = 0.0265\, m^3/s$, the head added by the pump is

$$h_p = 52.0 - 1.01 \times 10^3 \left(0.0265 \frac{m^3}{s}\right)^2$$
$$= 50.6\, m$$

From Eq. (4) with K_L unknown

$$50.6\, m = 33\, m + (K_L + 6.0)(826)\left(0.0265 \frac{m^3}{s}\right)^2$$

So that

$$K_L = 24.3$$

From Fig. 12.29 (b) the valve would be $\underline{\underline{13\%\ \text{open}}}$ to obtain this K_L

12.27

A centrifugal pump having an impeller diameter of 1 m is to be constructed so that it will supply a head rise of 200 m at a flowrate of 4.1 m^3/s of water when operating at a speed of 1200 rpm. To study the characteristics of this pump, a 1/5 scale, geometrically similar model operated at the same speed is to be tested in the laboratory. Determine the required model discharge and head rise. Assume both model and prototype operate with the same efficiency (and therefore the same flow coefficient).

For similarity the model pump must operate at the same flow coefficient, Eq. 12.32, so that

$$\left(\frac{Q}{\omega D^3}\right)_m = \left(\frac{Q}{\omega D^3}\right)_p$$

where the subscript (m) refers to the model and (p) to the prototype. Thus,

$$Q_m = \frac{\omega_m}{\omega_p}\left(\frac{D_m}{D_p}\right)^3 Q_p$$

and with $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $Q_p = 4.1\,m^3/s$, then

$$Q_m = (1)\left(\frac{1}{5}\right)^3\left(4.1\,\frac{m^3}{s}\right) = \underline{\underline{0.0328\,\frac{m^3}{s}}}$$

From Eq. 12.33

$$\left(\frac{g h_a}{\omega^2 D^2}\right)_m = \left(\frac{g h_a}{\omega^2 D^2}\right)_p$$

so that

$$h_{a_m} = \frac{g_p}{g_m}\left(\frac{\omega_m}{\omega_p}\right)^2\left(\frac{D_m}{D_p}\right)^2 h_{a_p}$$

and with $g_p = g_m$, $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $h_{a_p} = 200\,m$, then

$$h_{a_m} = (1)(1)^2\left(\frac{1}{5}\right)^2(200\,m) = \underline{\underline{8.00\,m}}$$

12.28

12.28 Explain how Fig. 12.18 was constructed from test data. Why is this use of specific speed important? Illustrate with a specific example.

A variety of pump configurations like the ones shown in Fig. 12.18 were tested over a range of flow rates. Performance data like those shown in Fig. 12.17 were acquired. For each pump configuration, the operation at maximum efficiency was noted and the specific speed, N_s, (Eq. 12. 43) was calculated for that condition of flow. These specific speed values calculated at maximum efficiency operation were then used to distribute the different pump configurations as shown in Fig. 12.18.

Specific speed is important because from desired design operational data (ω, Q, and h_a) a specific speed value can be determined. With that value of specific speed and Fig. 12.18 the designer can decide what kind of pump configuration to use for maximum efficiency operation. For example, at lower values of specific speed, a centrifugal pump is generally best. At higher values of specific speed, an axial-flow pump may be best. In between values of specific speed may suggest that a mixed-flow pump would serve most efficiently.

12.29

Use the data given in Problem 12.15 and plot the dimensionless coefficients C_H, $C_{\mathcal{P}}$, η versus C_Q for this pump. Calculate a meaningful value of specific speed, discuss its usefulness, and compare the result with data of Fig. 12.18.

From Problem 12.15 the following data were obtained:

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

For $\omega = \left(1750\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1}{60\,\frac{s}{min}}\right) = 183.3\,\frac{rad}{s}$ and $D = \frac{9}{12}\,ft$, it follows that

$$C_Q = \frac{Q}{\omega D^3} = \frac{Q(gpm)/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}{\left(183.3\,\frac{rad}{s}\right)\left(\frac{9}{12}\,ft\right)^3}$$

$$= 2.88 \times 10^{-5}\, Q\,(gpm)$$

$$C_H = \frac{g h_a}{\omega^2 D^2} = \frac{\left(32.2\,\frac{ft}{s^2}\right) h_a(ft)}{\left(183.3\,\frac{rad}{s}\right)^2\left(\frac{9}{12}\,ft\right)^2}$$

$$= 1.70 \times 10^{-3}\, h_a(ft)$$

$$C_{\mathcal{P}} = \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5} = \frac{\dot{W}_{shaft}(hp)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(183.3\,\frac{rad}{s}\right)^3\left(\frac{9}{12}\,ft\right)^5}$$

$$= 1.94 \times 10^{-4}\, \dot{W}_{shaft}(hp)$$

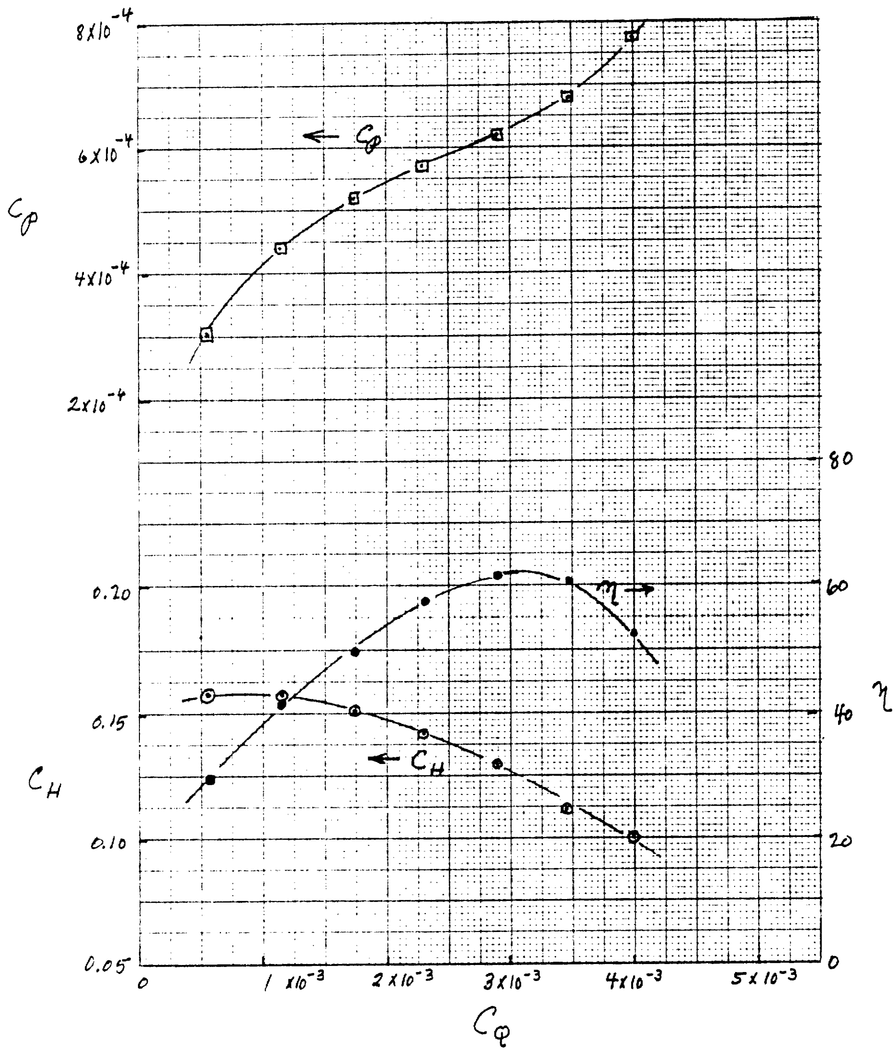
Based on the data above:

Q (gpm)	20	40	60	80	100	120	140
C_Q	5.76×10^{-4}	1.15×10^{-3}	1.73×10^{-3}	2.30×10^{-3}	2.88×10^{-3}	3.46×10^{-3}	4.03×10^{-3}
C_H	0.1581	0.1576	0.1498	0.1423	0.1317	0.1184	0.104
$C_{\mathcal{P}}$	3.07×10^{-4}	4.40×10^{-4}	5.18×10^{-4}	5.72×10^{-4}	6.19×10^{-4}	6.77×10^{-4}	7.76×10^{-4}
η	29.7	41.2	49.9	57.7	61.3	60.4	52.6

(con't)

12.29 (con't)

The plot of C_H, C_P, η versus C_Q is shown below.



$$N_{sd} = \frac{\omega(\text{rpm})\sqrt{Q(\text{gpm})}}{[h_a(\text{ft})]^{3/4}}$$

so for $Q = 100$ gpm at $\eta_{max} = 61.3\%$

$$N_{sd} = \frac{(1750\ \text{rpm})\sqrt{(100\ \text{gpm})}}{[(77.3\ \text{ft})]^{3/4}} = 671$$

which is within the range of N_{sd} values for radial flow pumps in Fig. 12.18

12.30

12.30 A centrifugal pump provides a flowrate of 500 gpm when operating at 1750 rpm against a 200-ft head. Determine the pump's flowrate and developed head if the pump speed is increased to 3500 rpm.

For a given pump the effect of a change in speed on Q and h_a is given by Eqs. 12.36 and 12.37. Thus,

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad \text{(Eq. 12.36)}$$

and with $Q_1 = 500\,gpm$, $\omega_1 = 1750\,rpm$, and $\omega_2 = 3500\,rpm$, then

$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \frac{(3500\,rpm)}{(1750\,rpm)}(500\,gpm)$$

$$= \underline{\underline{1000\,gpm}}$$

Similarly,

$$\frac{h_{a_1}}{h_{a_2}} = \frac{\omega_1^2}{\omega_2^2} \qquad \text{(Eq. 12.37)}$$

so that with $h_{a_1} = 200\,ft$

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{3500\,rpm}{1750\,rpm}\right)^2 (200\,ft)$$

$$= \underline{\underline{800\,ft}}$$

12.31

12.31 A centrifugal pump with a 12-in.-diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60-ft head. The impeller is changed to one with a 10-in. diameter. Determine the expected flowrate, head, and input power if the pump speed remains the same.

For geometrically similar pumps operating at the same speed the effect of a change in impeller diameter is given by Eqs. 12.39, 12.40, 12.41. Thus,

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad \text{(Eq. 12.39)}$$

and with $Q_1 = 3200\,gpm$, $D_1 = 12\,in.$, and $D_2 = 10\,in.$

$$Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{10\,in.}{12\,in.}\right)^3 (3200\,gpm) = \underline{\underline{1850\,gpm}}$$

From Eq. 12.40

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad \text{(Eq. 12.40)}$$

so that with $h_{a_1} = 60\,ft$

$$h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{10\,in.}{12\,in.}\right)^2 (60\,ft) = \underline{\underline{41.7\,ft}}$$

Similarly from Eq. 12.41

$$\frac{\dot{W}_{shaft_1}}{\dot{W}_{shaft_2}} = \frac{D_1^5}{D_2^5} \qquad \text{(Eq. 12.41)}$$

and with $\dot{W}_{shaft_1} = 60\,hp$

$$\dot{W}_{shaft_2} = \left(\frac{D_2}{D_1}\right)^5 \dot{W}_{shaft_1} = \left(\frac{10\,in.}{12\,in.}\right)^5 (60\,hp) = \underline{\underline{24.1\,hp}}$$

12.32 Do the head-flowrate data shown in Fig. 12.12 appear to follow the similarity laws as expressed by Eqs. 12.39 and 12.40? Explain.

The data in Fig. 12.12 show the effect of changing impeller diameter on head-flowrate characteristics. According to the similarity laws expressed by Eq. 12.39 and Eq. 12.40

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad \text{(Eq. 12.39)}$$

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad \text{(Eq. 12.40)}$$

Thus, as the diameter is increased from 6 in. to 7 in. to 8 in. the flowrate increases according to Eq. 12.39 as

(from 6 in. to 7 in.) $Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 1.59\, Q_1$

and

(from 6 in. to 8 in.) $Q_2 = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 2.37\, Q_1$

Similarly, from Eq. 12.40

(from 6 in. to 7 in.) $h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.36\, h_{a_1}$

and

(from 6 in. to 8 in.) $h_{a_2} = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.78\, h_{a_1}$

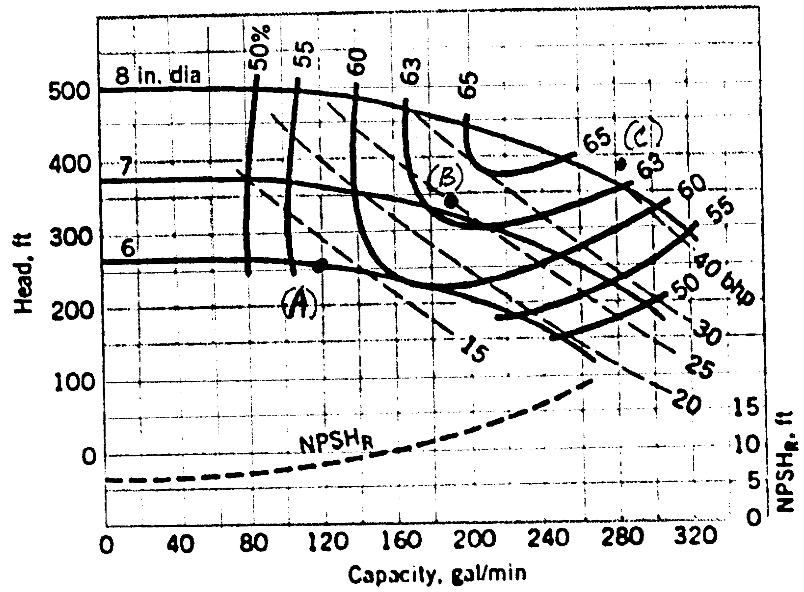
Thus, for any given point, such as (A) where $Q = 120$ gpm and $h_a = 250$ ft (see Fig. 12.12 on following page) for the 6-in. diameter impeller, the corresponding predicted point would be at (B) where

$$Q_2 = (1.59)(120\text{ gpm}) = 191\text{ gpm}$$

$$h_{a_2} = (1.36)(250\text{ ft}) = 340\text{ ft}$$

(con't)

12.32 (Con't)



Similarly, for the 8-in. diameter impeller the predicted point, point (C), would be at

$$Q_2 = (2.37)(120\ gpm) = 284\ gpm$$

and

$$h_{a_2} = (1.78)(250\ ft) = 445\ ft$$

Points (B) and (C) fall near the corresponding curves in Fig. 12.12 thereby demonstrating that they do appear to follow the similarity laws. Yes.

Note that according to the similarity laws the 6-in. diameter curve is simply translated to the right and upward to obtain the corresponding head-flowrate curves for the 7-in. and 8-in. diameter pumps. It is clear from Fig. 12.12 that this is generally how the three curves are related.

12.33

A centrifugal pump has the performance characteristics of the pump with the 6-in.-diameter impeller described in Fig. 12.12. What is the expected head gained if the speed of this pump is reduced to 2800 rpm while maintaining a flowrate equal to 200 gpm?

From Fig. 12.12 for the 6-in. diameter impeller operating at 3500 rpm, $Q = 170$ gpm and $h_a = 230$ ft when operating at peak efficiency (see figure below). Thus, if the pump is still operated at peak efficiency with the speed reduced to 2800 rpm then from Eq. 12.36

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad (\text{Eq. } 12.36)$$

so that

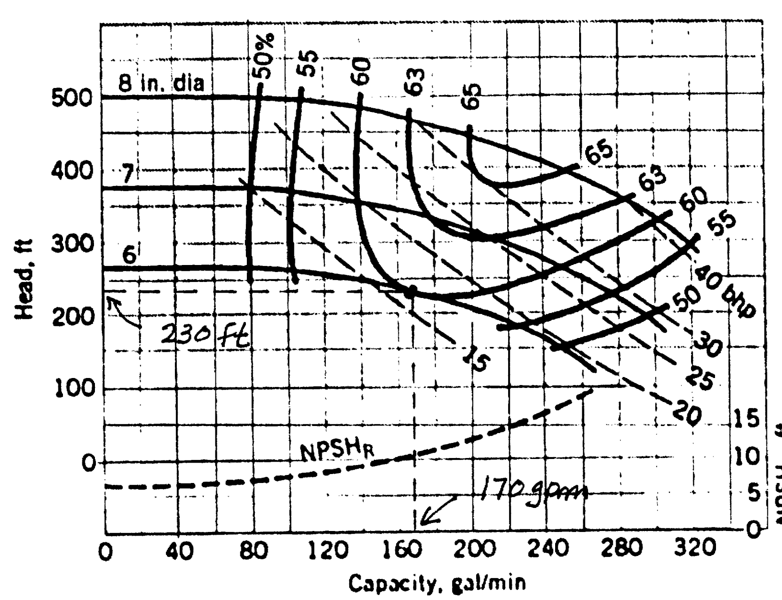
$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \left(\frac{2800 \text{ rpm}}{3500 \text{ rpm}}\right)(170 \text{ gpm}) = \underline{\underline{136 \text{ gpm}}}$$

From Eq. 12.37

$$\frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \qquad (\text{Eq. } 12.37)$$

so that

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{2800 \text{ rpm}}{3500 \text{ rpm}}\right)^2 (230 \text{ ft}) = \underline{\underline{147 \text{ ft}}}$$



12.34

12.34 In a certain application a pump is required to deliver 5000 gpm against a 300-ft head when operating at 1200 rpm. What type of pump would you recommend?

For $Q = 5000$ gpm, $h_a = 300$ ft, and $\omega = 1200$ rpm, the specific speed is

$$N_{sd} = \frac{\omega\,(\text{rpm})\sqrt{Q\,(\text{gpm})}}{\left[h_a\,(\text{ft})\right]^{3/4}}$$

$$= \frac{(1200\text{ rpm})\sqrt{5000\text{ gpm}}}{(300\text{ ft})^{3/4}}$$

$$= \underline{\underline{1180}}$$

From Fig. 12.18, at this specific speed a radial flow pump (<u>centrifugal pump</u>) would be recommended.

12.35

12.35 A certain axial-flow pump has a specific speed of $N_S = 5.0$. If the pump is expected to deliver 3000 gpm when operating against a 15-ft head, at what speed (rpm) should the pump be run?

Since

$$N_s = \frac{\omega\,(\mathrm{rad/s})\sqrt{Q\,(\mathrm{ft^3/s})}}{\left[g\,(\mathrm{ft/s^2})\;h_a\,(\mathrm{ft})\right]^{3/4}}$$

for $N_s = 5.0$, $g = 32.2\ \mathrm{ft/s^2}$; $h_a = 15\ \mathrm{ft}$, and with

$$Q = \frac{3000\ \frac{\mathrm{gal}}{\mathrm{min}}}{\left(7.48\ \frac{\mathrm{gal}}{\mathrm{ft^3}}\right)\left(60\ \frac{\mathrm{s}}{\mathrm{min}}\right)} = 6.68\ \frac{\mathrm{ft^3}}{\mathrm{s}}$$

it follows that

$$\omega\,(\mathrm{rad/s}) = \frac{(5.0)\left[\left(32.2\ \frac{\mathrm{ft}}{\mathrm{s^2}}\right)(15\ \mathrm{ft})\right]^{3/4}}{\sqrt{6.68\ \frac{\mathrm{ft^3}}{\mathrm{s}}}}$$

$$= 199\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Hence

$$\omega\,(\mathrm{rpm}) = \frac{\left(199\ \frac{\mathrm{rad}}{\mathrm{s}}\right)\left(60\ \frac{\mathrm{s}}{\mathrm{min}}\right)}{2\pi\ \frac{\mathrm{rad}}{\mathrm{rev}}}$$

$$= \underline{\underline{1900\ \mathrm{rpm}}}$$

12.36

A certain pump is known to have a capacity of 3 m^3/s when operating at a speed of 60 rad/s against a head of 20 m. Based on the information in Fig. 12.18, would you recommend a radial-flow, mixed-flow, or axial-flow pump?

Since

$$N_s = \frac{\omega\,(rad/s)\sqrt{Q\,(m^3/s)}}{\left[g\,(m/s^2)\;h_a\;(m)\right]^{3/4}}$$

for $\omega = 60\ rad/s$, $Q = 3\ m^3/s$, $g = 9.81\ m/s^2$, and $h_a = 20\,m$

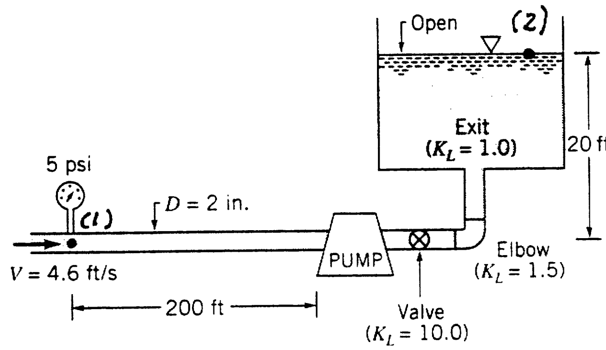
$$N_s = \frac{(60\ rad/s)\sqrt{3\ m^3/s}}{\left[(9.81\ m/s^2)(20\ m)\right]^{3/4}}$$

$$= \underline{\underline{1.98}}$$

From Fig. 12.18 with $N_s = 1.98$ the pump is a <u>mixed-flow pump</u>.

12.37

12.37 Fuel oil (sp. wt = 48.0 lb/ft³, viscosity = 2.0 × 10^{-5} lb·s/ft²) is pumped through the piping system of Fig. P12.37 with a velocity of 4.6 ft/s. The pressure 200 ft upstream from the pump is 5 psi. Pipe losses downstream from the pump are negligible, but minor losses are not (minor loss coefficients are given on the figure). **(a)** For a pipe diameter of 2 in. with a relative roughness $\varepsilon/D = 0.001$, determine the head that must be added by the pump. **(b)** For a pump operating speed of 1750 rpm, what type of pump (radial-flow, mixed-flow, or axial-flow) would you recommend for this application?



■ FIGURE P12.37

(a)
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

With $\gamma = 48.0\ lb/ft^3$, $p_1 = 5\,psi$, $p_2 = 0$, $V_1 = 4.6\ ft/s$, $V_2 = 0$, and $z_2 - z_1 = 20\ ft$, Eq. (1) becomes

$$\frac{(5\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{48.0\frac{lb}{ft^3}} + \frac{(4.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + h_p = 20\,ft + \sum h_L \qquad (2)$$

The head loss ~~term~~ can be expressed as

$$\sum h_L = \Big[\underbrace{10.0}_{valve} + \underbrace{1.5}_{elbow} + \underbrace{1.0}_{exit} + f\,\frac{200\,ft}{2/12\,ft}\Big]\frac{(4.6\frac{ft}{s})^2}{(2)(32.2\frac{ft}{s^2})}$$

The Reynolds number is

$$Re = \frac{\rho V D}{\mu} = \frac{\left(\frac{48.0\frac{lb}{ft^3}}{32.2\frac{ft}{s^2}}\right)(4.6\frac{ft}{s})(\frac{2}{12}ft)}{2.0\times10^{-5}\ \frac{lb\cdot s}{ft^2}} = 5.71\times10^4$$

and with $\epsilon/D = 0.001$ $\quad f = 0.024$ (from Fig. 8.23).

Thus, $h_L = 13.6\ ft$ and from Eq. (2)

$$\underline{\underline{h_p = 18.3\ ft}}$$

(b) Since

$$Q = VA = (4.6\frac{ft}{s})(\frac{\pi}{4})(\frac{2}{12}ft)^2 = 0.100\ \frac{ft^3}{s}$$

or

$$Q = (0.100\frac{ft^3}{s})(7.48\frac{gal}{ft^3})(60\frac{s}{min}) = 45.0\ gpm$$

the specific speed at 1750 rpm is

$$N_{sd} = \frac{\omega\,(rpm)\sqrt{Q\,(gpm)}}{[h_a(ft)]^{3/4}} = \frac{(1750\,rpm)\sqrt{45.0\,gpm}}{[18.3\,ft]^{3/4}} = 1330$$

For this specific speed a <u>radial-flow pump</u> would be recommended for this application (see Fig. 12.18).

12.39

12.39 The axial-flow pump shown in Fig. 12.19 is designed to move 5000 gal/min of water over a head rise of 5 ft of water. Estimate the motor power requirement and the $U_2 V_{\theta 2}$ needed to achieve this flowrate on a continuous basis. Comment on any cautions associated with where the pump is placed vertically in the pipe.

From Eq. 12.21 we get the power equivalent to the head rise and flowrate involved. This is the minimum power required to achieve the performance specified.

$$P = \gamma Q h_a$$

$$P = \left(62.4 \frac{lb}{ft^3}\right)\left(5000 \frac{gal}{min}\right) \frac{1}{\left(7.48 \frac{gal}{ft^3}\right)} \frac{1}{\left(60 \frac{s}{min}\right)} (4\ ft) \frac{1}{\left(550 \frac{ft.lb}{hp.s}\right)}$$

$$P = 5.1\ hp$$

To estimate the shaft or motor power requirement, we need to assume the efficiency of the conversion of shaft or motor power into the pump performance specified.

$$P_{shaft} = \frac{P}{\eta} \quad \text{or for } 80\% \text{ efficiency}$$

$$P_{shaft} = \frac{5.1\ hp}{0.8} = \underline{\underline{6.4\ hp}}$$

$U_2 V_{\theta 2}$ and P_{shaft} are related in Eq. 12.4 through

$$P_{Shaft} = \dot{m}\, U_2 V_{\theta 2} = \rho A V U_2 V_{\theta 2} = \rho Q U_2 V_{\theta 2}$$

so

$$U_2 V_{\theta 2} = \frac{P_{shaft}}{\rho Q} = \frac{(6.4\ hp)\left(32.2 \frac{lbm.s^2}{lbf.ft}\right)\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\left(550 \frac{ft.lb}{hp.s}\right)}{\left(62.4 \frac{lbm}{ft^3}\right)\left(5000 \frac{gal}{min}\right)}$$

$$U_2 V_{\theta 2} = \underline{\underline{163}} \frac{ft^2}{s^2}$$

(con't)

The main caution in placing the pump vertically in the intake pipe is to do so in a way to avoid cavitation in the pump. The collapse of cavitation bubbles in the pump can erode pump blade and other wetted surfaces. Applying the energy equation, Eq. 5.84, between the free surface (1) and the pump entrance (2) we get

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L$$

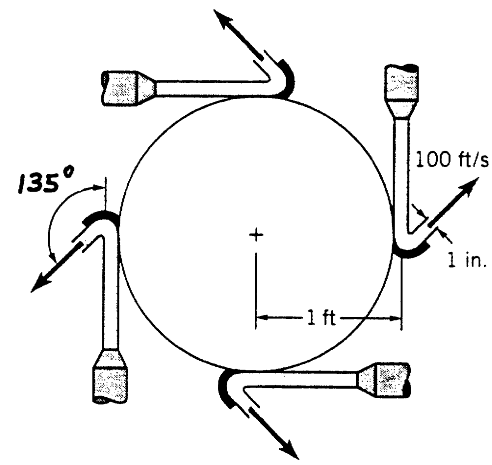
So

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2 - \frac{V_2^2}{2g} - h_L$$

and to maximize $\frac{p_2}{\gamma}$, we minimize $z_1 - z_2$. To achieve this we place the pump high vertically in the intake pipe. This will tend to keep p_2 high enough to avoid cavitation which occurs when p_2 and/or related pressures in the pump become less than the vapor pressure of the fluid.

12.41

A Pelton wheel turbine is illustrated in Fig. P12.41. The radius to the line of action of the tangential reaction force on each vane is 1 ft. Each vane deflects fluid by an angle of 135° as indicated. Assume all of the flow occurs in a horizontal plane. Each of the four jets shown strikes a vane with a velocity of 100 ft/s and a stream diameter of 1 in. The magnitude of velocity of the jet remains constant along the vane surface.
(a) How much torque is required to hold the wheel stationary?
(b) How fast will the wheel rotate if shaft torque is negligible and what practical situation is simulated by this condition?



■ FIGURE P12.41

$$T = n \dot{m} r_m (U - V_1)(1 - \cos\beta) \quad \text{where } n = 4 \tag{1}$$

(a) With the wheel stationary $U = 0$ so that

$T = -4 \dot{m} r_m V_1 (1 - \cos\beta)$ where

$\dot{m} = \rho A V = \left(1.94 \frac{slugs}{ft^3}\right) \frac{\pi}{4}\left(\frac{1}{12} ft\right)^2 \left(100 \frac{ft}{s}\right) = 1.057 \frac{slugs}{s}$

Thus, $T = -4\left(1.057 \frac{slugs}{s}\right)(1\, ft)\left(100 \frac{ft}{s}\right)(1 - \cos 135°) = \underline{\underline{-722\ ft \cdot lb}}$

(b) From Eq. (1), when $T = 0$, then $U = V_1$

Thus,

$U = \omega r_m = V_1$ or $\omega = \frac{V_1}{r_m} = \frac{100 \frac{ft}{s}}{1\, ft} = 100 \frac{rad}{s}\left(\frac{60\, s}{min}\right)\left(\frac{1\, rev}{2\pi\, rad}\right)$

$= \underline{\underline{955\ rpm}}$

The zero torque case represents a broken shaft situation.

12.42

12.42 Consider the Pelton wheel turbine illustrated in Figs. 12.24, 12.25, 12.26, and 12.27. This kind of turbine is used to drive the oscillating sprinkler shown in **Video V12.4**. Explain how this kind of sprinkler is started, and subsequently operated at constant oscillating speed. What is the physical significance of the zero torque condition with the Pelton wheel rotating?

As shown on page 795 below Eq. 12.50

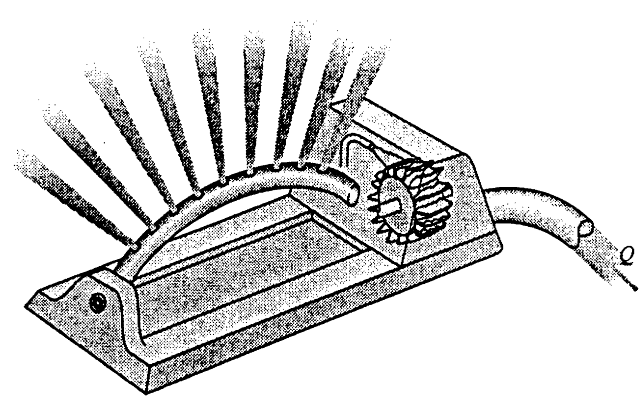
$$T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$$

So for no rotation of the wheel or $U = 0$, the variation of T_{shaft} with changing $\dot{m}$ is linear. When T_{shaft} is just larger than the resisting torque provided by the sprinkler, the Pelton wheel rotates and drives the oscillation of the sprinkler. After wheel rotation and sprinkler oscillation begins, any constant value of $\dot{m}$ and T_{shaft} results in a constant value of U and thus rotation speed and also oscillation period.

If the shaft connecting the oscillating sprinkler to the Pelton wheel breaks during operation, the sprinkler will cease oscillating and the Pelton wheel will run at constant rotation speed corresponding to $U = V_1$.

12.43

12.43 A small Pelton wheel is used to power an oscillating lawn sprinkler as shown in **Video** V12.4 and Fig. P12.43. The arithmetic mean radius of the turbine is 1 in. and the exit angle of the blade is 135 degrees relative to the blade motion. Water is supplied through a single 0.20-in. diameter nozzle at a speed of 50 ft/s. Determine the flowrate, the maximum torque developed, and the maximum power developed by this turbine.



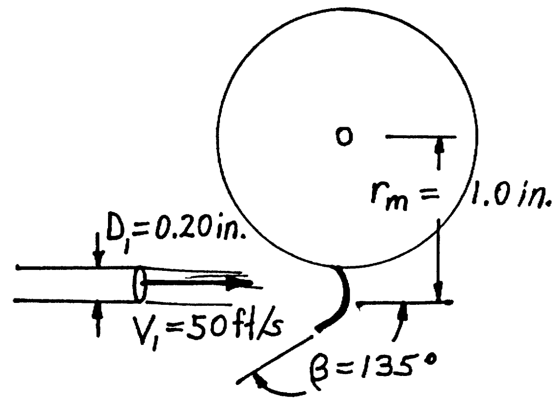
■ FIGURE P12.43

For the Pelton wheel shown

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4}\left(\frac{0.20}{12}\,ft\right)^2\left(50\,\frac{ft}{s}\right)$$

or

$$\underline{\underline{Q = 0.0109\ \frac{ft^3}{s}}}$$



From Fig. 11.22

$$T_{shaft_{max}} = \dot{m}\, r_m V_1 (1 - \cos\beta)$$

and

$$\dot{W}_{shaft_{max}} = 0.25\, \dot{m} V_1^2 (1 - \cos\beta)$$

where $\dot{m} = \rho Q = 1.94\ \frac{slugs}{ft^3}\left(0.0109\ \frac{ft^3}{s}\right) = 0.0211\ \frac{slugs}{s}$

Thus,

$$T_{shaft_{max}} = 0.0211\ \frac{slugs}{s}\left(\frac{1}{12}\,ft\right)\left(50\,\frac{ft}{s}\right)(1 - \cos 135^\circ) = 0.150\ \frac{slugs \cdot ft^2}{s^2}$$

$$= \underline{\underline{0.150\ ft \cdot lb}}$$

and

$$\dot{W}_{shaft_{max}} = 0.25\left(0.0211\ \frac{slugs}{s}\right)\left(50\,\frac{ft}{s}\right)^2 (1 - \cos 135^\circ) = 22.5\ \frac{slugs \cdot ft^2}{s^3}$$

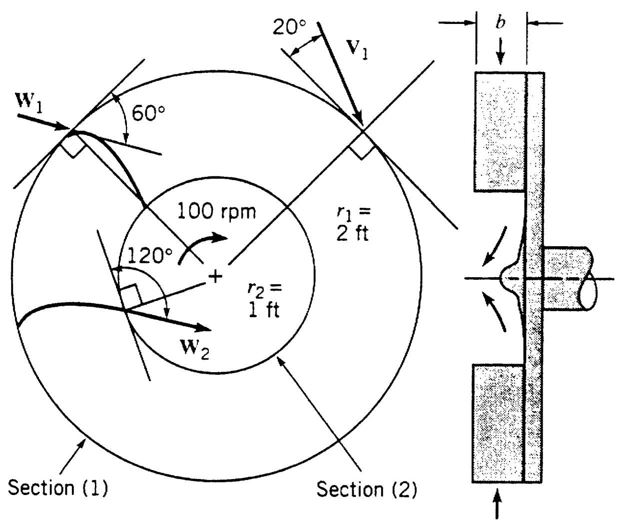
$$= 22.5\ \frac{ft \cdot lb}{s}$$

or

$$\dot{W}_{shaft_{max}} = 22.5\ \frac{ft \cdot lb}{s}\ \frac{1\ hp}{550\ \frac{ft \cdot lb}{s}} = \underline{\underline{0.0409\ hp}}$$

12.44

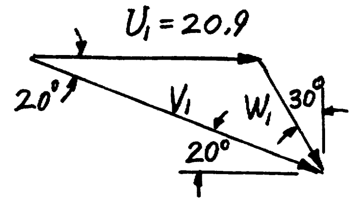
A water turbine wheel rotates at the rate of 100 rpm in the direction shown in Fig. P12.44. The inner radius, r_2, of the blade row is 1 ft, and the outer radius, r_1, is 2 ft. The absolute velocity vector at the turbine rotor entrance makes an angle of 20° with the tangential direction. The inlet blade angle is 60° relative to the tangential direction. The blade outlet angle is 120°. The flowrate is 10 ft^3/s. For the flow tangent to the rotor blade surface at inlet and outlet, determine an appropriate constant blade height, b, and the corresponding power available at the rotor shaft. Is the shaft power greater or less than the power lost by the fluid? Explain.



■ FIGURE P12.44

Note that the shaft power calculated below, $\dot{W}_{shaft}$, is less than the power lost by the fluid because some of the power lost by the fluid is due to fluid and shaft bearing friction while the rest is delivered at the shaft.

$Q = 2\pi r_1 b\, W_1 \cos 30°$ where $Q = 10 \frac{ft^3}{s}$ and $r_1 = 2\,ft$ (1)



Also, with $\omega = (100 \frac{rev}{min})(\frac{1 min}{60 s})(\frac{2\pi\, rad}{rev}) = 10.47 \frac{rad}{s}$

it follows that

$U_1 = r_1 \omega = (2 ft)(10.47 \frac{rad}{s}) = 20.9 \frac{ft}{s}$ and

$U_2 = r_2 \omega = (1 ft)(10.47 \frac{rad}{s}) = 10.47 \frac{ft}{s}$

From the Law of Sines (see figure): $\dfrac{W_1}{\sin 20°} = \dfrac{20.9 \frac{ft}{s}}{\sin(90° - 20° - 30°)}$

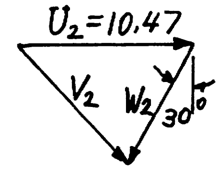
or $W_1 = 11.12 \frac{ft}{s}$ so that from Eq. (1)

$$b = \frac{Q}{2\pi r_1 W_1 \cos 50°} = \frac{10 \frac{ft^3}{s}}{2\pi (2 ft)(11.12 \frac{ft}{s}) \cos 30°} = \underline{\underline{0.0826\, ft}}$$

Also, $V_{\theta 1} = U_1 + W_1 \sin 30° = 20.9 + 11.12 \sin 30° = 26.5 \frac{ft}{s}$ and

$\dot{W}_{shaft} = \rho Q (U_2 V_{\theta 2} - U_1 V_{\theta 1})$ where $Q = 2\pi r_2 b W_2 \cos 30°$ (2)

or $W_2 = \dfrac{Q}{2\pi r_2 b \cos 30°} = \dfrac{10 \frac{ft^3}{s}}{2\pi (1 ft)(0.0826 ft) \cos 30°} = 22.2 \frac{ft}{s}$



From the Law of Cosines (see figure):

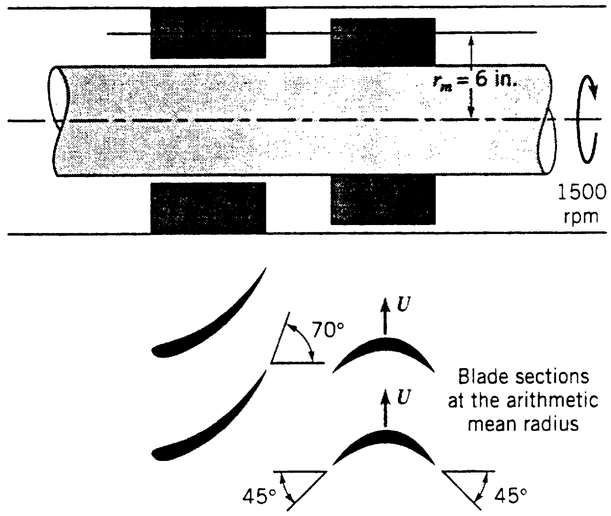
$V_2^2 = (10.47)^2 + (22.2)^2 - 2(10.47)(22.2)\cos 60°$ or $V_2 = 19.2 \frac{ft}{s}$

Thus, $V_{\theta 2} = 10.47 - 22.2 \sin 30° = -0.63 \frac{ft}{s}$ and Eq. (2) becomes

$$\dot{W}_{shaft} = (1.94 \frac{slugs}{ft^3})(10 \frac{ft^3}{s})\left[(10.47 \frac{ft}{s})(-0.63 \frac{ft}{s}) - (20.9 \frac{ft}{s})(26.5 \frac{ft}{s})\right] = -1.08 \times 10^4 \frac{ft \cdot lb}{s}$$

$$= \underline{\underline{-19.8\, hp}}$$

12.45 A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P12.45. The rotor speed is 1500 rpm. (**a**) Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. Assume flow enters and leaves each blade row at the blade angles shown. (**b**) Calculate the work per unit mass delivered at the shaft.



■ FIGURE P12.45

(a) $U_1 = r_1\omega$ and $U_2 = r_2\omega$ where $\omega = \left(1500\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right) = 157\,\frac{rad}{s}$

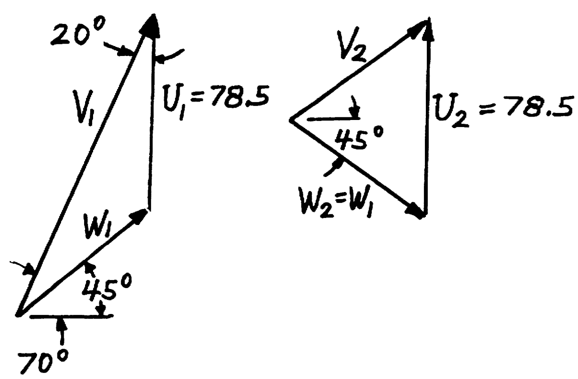
so that with $r_m = 0.5$ ft,

$U_1 = (0.5\,ft)(157\,\frac{rad}{s}) = 78.5\,\frac{ft}{s}$

The inlet and exit velocity triangles are as shown.

Note: $U_1 = U_2$ (same radius)

and $W_1 = W_2$ (from continuity eqn.)



(b) $w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = U(V_{\theta 2} - V_{\theta 1})$ (1)

From the figures: $V_1 \cos 70° = W_1 \cos 45°$

and from Law of Sines $\dfrac{W_1}{\sin 20°} = \dfrac{78.5}{\sin(70° - 45°)}$ or $W_1 = 63.5\,\frac{ft}{s}$

so that $V_1 = \dfrac{(63.5\,\frac{ft}{s})\cos 45°}{\cos 70°} = 131.3\,\frac{ft}{s}$

or

$V_{\theta 1} = V_1 \cos 20° = 123\,\frac{ft}{s}$

Also,

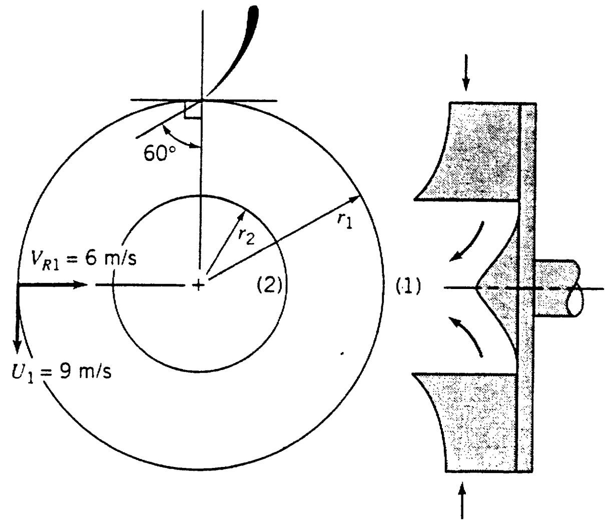
$V_{\theta 2} = U_2 - W_2 \sin 45° = 78.5\,\frac{ft}{s} - 63.5\,\frac{ft}{s}\sin 45° = 33.6\,\frac{ft}{s}$

Hence, from Eq. (1)

$w_{shaft} = 78.5\,\frac{ft}{s}\left[33.6\,\frac{ft}{s} - 123\,\frac{ft}{s}\right] = \underline{\underline{-7020\,\frac{ft^2}{s^2}}}$

12.46

12.46 An inward flow radial turbine (see Fig. P12.46) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 9 m/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 6 m/s through the rotor and the flow leaving the rotor at section (2) is without angular momentum. **(a)** If the flowing fluid is water and the stagnation pressure drop across the rotor is 110 kPa, determine the loss of available energy across the rotor and the efficiency involved. **(b)** If the flowing fluid is air and the static pressure drop across the rotor is 0.07 kPa, determine the loss of available energy across the rotor and the rotor efficiency.



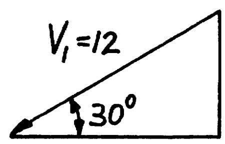
■ FIGURE P12.46

(a) $\text{loss} = \dfrac{p_{01}-p_{02}}{\rho} + w_{shaft}$, where $p_{01}-p_{02}$ = stagnation pressure drop across rotor $= \Delta p_s$

and

$$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = -U_1 V_{\theta 1} \text{ since } V_{\theta 2} = 0$$

Thus, $w_{shaft} = -\left(9\,\frac{m}{s}\right)\left(12\,\frac{m}{s}\cos 30°\right) = -93.5\,\frac{m^2}{s^2}$



so that

$$\text{loss} = \frac{110\times10^3\,\frac{N}{m^2}}{999\,\frac{kg}{m^3}} + \left(-93.5\,\frac{m^2}{s^2}\right) = \underline{\underline{16.6\,\frac{m^2}{s^2}}}$$

Also,

$$\eta = \frac{-w_{shaft}}{\frac{\Delta p_s}{\rho}} = \frac{93.5\,\frac{m^2}{s^2}}{\frac{\left(110\times10^3\,\frac{N}{m^2}\right)}{\left(999\,\frac{kg}{m^3}\right)}} = \underline{\underline{0.849}}$$

(con't)

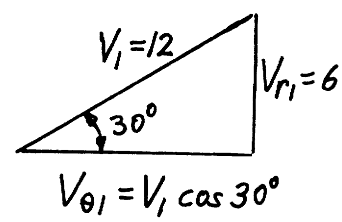
12.46 (con't)

(b) $loss = \dfrac{p_{01} - p_{02}}{\rho} + w_{shaft}$, where $p_{01} - p_{02}$ = stagnation pressure drop across rotor $= \Delta p_s$

and

$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = -U_1 V_{\theta 1}$ since $V_{\theta 2} = 0$

Thus, $w_{shaft} = -(9\,\frac{m}{s})(12\,\frac{m}{s}\cos 30^\circ) = -93.5\,\frac{m^2}{s^2}$



Also,

$$\Delta p_s = p_1 - p_2 + \tfrac{1}{2}\rho(V_1^2 - V_2^2)$$
$$= 0.07\,kPa + \tfrac{1}{2}(1.23\,\tfrac{kg}{m^3})((12\,\tfrac{m}{s})^2 - (6\,\tfrac{m}{s})^2)\left(\frac{1\,kPa}{10^3\,Pa}\right)$$
$$= (0.07 + 0.0664)\,kPa = 0.1364\,kPa$$

Thus,

$$loss = \frac{0.1364 \times 10^3\,\frac{N}{m^2}}{(1.23\,\frac{kg}{m^3})} - 93.5 = \underline{\underline{17.4\,\frac{m^2}{s^2}}}$$

and

$$\eta = \frac{-w_{shaft}}{\left(\frac{\Delta p_s}{\rho}\right)} = \frac{93.5\,\frac{m^2}{s^2}}{\left(\frac{1364\,\frac{N}{m^2}}{1.23\,\frac{kg}{m^3}}\right)} = \underline{\underline{0.843}}$$

12.47

12.47 For an air turbine of a dentist's drill like the one shown in Fig. E12.8 and Video V12.5, calculate the average blade speed asociated with a rotational speed of 350,000 rpm. Estimate the air pressure needed to run this turbine.

We calculate the average blade speed, U, from

$$U = r_m \omega = \left(\frac{r_i + r_o}{2}\right)\omega = \frac{(0.133 + 0.168)\text{ in}\left(350{,}000 \frac{\text{rev}}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{\text{s}}\right)}{(2)\left(12 \frac{\text{in}}{\text{ft}}\right)} \frac{1}{\left(60 \frac{\text{s}}{\text{min}}\right)}$$

$$U = 459 \frac{\text{ft}}{\text{s}}$$

To estimate the air pressure, p_0, needed to run this turbine, we estimate that the nozzle exit velocity is about twice as large as the average blade velocity or

$$V = 2U = 918 \text{ ft/s}$$

So, the corresponding Mach number, M, is approximately

$$M = \frac{V}{c} = \frac{918 \text{ ft/s}}{1100 \text{ ft/s}}$$ with c estimated to be about $1100 \frac{\text{ft}}{\text{s}}$

$$M = 0.83$$

Then from Fig. D.1 the value of $\frac{p}{p_0}$ corresponding to $M = 0.83$ is

$$\frac{p}{p_0} = 0.1 \quad \text{and} \quad p_0 = \frac{p}{0.1} = 10p$$

So $$p_0 \approx 10(14.7 \text{ psia}) = \underline{\underline{147 \text{ psia}}}$$

12.48

12.48 A high-speed turbine used to power a dentist's drill is shown in Video V12.5 and Fig. E12.8. With the conditions stated in Example 12.8, for every slug of air that passes through the turbine there is 310,000 ft · lb of energy available at the shaft to drive the drill. One of the assumptions made to obtain this numerical result is that the tangential component of the absolute velocity out of the rotor is zero. Suppose this assumption were not true (but all other parameter values remain the same). Discuss how and why the value of 310,000 ft · lb/slug would change for these new conditions.

From Example 12.8 we have

$$w_{shaft} = -U_1 V_{\theta 1} + U_2 V_{\theta 2}$$

So if $V_{\theta 2}$ is actually not zero, then depending on whether or not $V_{\theta 2}$ was in the direction of rotation, w_{shaft} would be smaller or larger. If blades do less turning than is the case in Example 12.8, $V_{\theta 2}$ will be in the direction of rotation, the lift force on each blade is less, and less work is extracted from the flowing fluid. Just the opposite is true when the blades do more turning of the fluid than is the case in Example 12.8.

12.50

12.50 A Pelton wheel has a diameter of 2 m and develops 500 kW when rotating 180 rpm. What is the average force of the water against the blades? If the turbine is operating at maximum efficiency, determine the speed of the water jet from the nozzle and the mass flowrate.

$$\dot{W}_{shaft} = T\omega = \frac{D}{2} F \omega \quad \text{or} \quad 500 \times 10^3 \ \frac{N \cdot m}{s} = \left(\frac{2}{2} m\right) F \left(180 \frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right)$$

Thus, $F = \underline{\underline{26{,}600\ N}}$

Also,

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$ so that at maximum efficiency with $\beta = 180°$ and $U = \frac{V_1}{2}$ this gives

$$\dot{W}_{shaft} = \rho Q \frac{V_1}{2}\left(-\frac{V_1}{2}\right)(2) = \frac{\rho Q V_1^2}{2} = \frac{\dot{m} V_1^2}{2} \qquad (1)$$

But

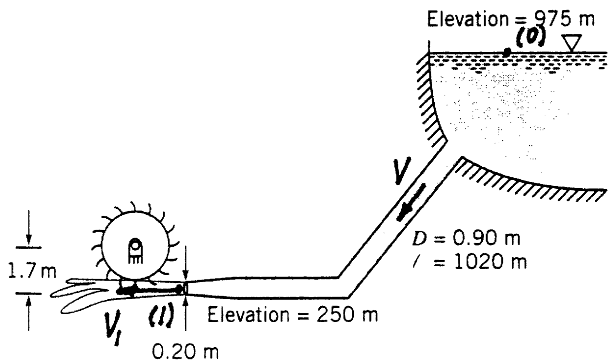
$$V_1 = 2U = 2\omega \frac{D}{2} = \omega D = \left(180 \frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right)(2m) = \underline{\underline{37.6 \ \frac{m}{s}}}$$

Thus, from Eq. (1):

$$\dot{m} = \frac{2 \dot{W}_{shaft}}{V_1^2} = \frac{2\left(500 \times 10^3 \ \frac{N \cdot m}{s}\right)}{\left(37.6 \ \frac{m}{s}\right)^2} = 707 \ \frac{N \cdot s}{m} = \underline{\underline{707 \ \frac{kg}{s}}}$$

12.51

12.51 Water for a Pelton wheel turbine flows from the headwater and through the penstock as shown in Fig. P12.51. The effective friction factor for the penstock, control valves, and the like is 0.032 and the diameter of the jet is 0.20 m. Determine the maximum power output.



■ FIGURE P12.51

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1-\cos\beta)$ or for maximum power $\beta = 180^\circ$, $U = \frac{V_1}{2}$

Thus,

$$\dot{W}_{shaft\,max} = -\rho Q \frac{V_1^2}{2} \qquad (1)$$

But $\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$ where $p_0 = p_1 = 0$, $z_0 = 975\,m$, $z_1 = 250\,m$, and $V_0 = 0$

Hence,

$$z_0 = z_1 + \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{where } A_1V_1 = AV \qquad (2)$$

or $\frac{\pi}{4}d_1^2 V_1 = \frac{\pi}{4}D^2 V$. That is $V = \left(\frac{d_1}{D}\right)^2 V_1 = \left(\frac{0.2\,m}{0.9\,m}\right)^2 V_1 = 0.0494\,V_1$

so that Eq. (2) becomes:

$$975\,m = 250\,m + \frac{V_1^2}{2(9.81\,\frac{m}{s^2})}\left[1 + 0.032\left(\frac{1020\,m}{0.9\,m}\right)(0.0494)^2\right] \quad \text{where } V_1 \sim \frac{m}{s}$$

or $V_1 = 114.3\,\frac{m}{s}$

Hence,

$$Q = A_1V_1 = \frac{\pi}{4}(0.2\,m)^2(114.3\,\frac{m}{s}) = 3.56\,\frac{m^3}{s}$$

Therefore, from Eq. (1):

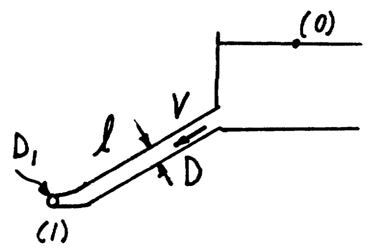
$$\dot{W}_{shaft\,max} = -(999\,\frac{kg}{m^3})(3.56\,\frac{m^3}{s})\frac{(114.3\,\frac{m}{s})^2}{2} = 23.2 \times 10^6\,\frac{N\cdot m}{s} = \underline{\underline{23{,}200\,kW}}$$

12.52

12.52 Water to run a Pelton wheel is supplied by a penstock of length ℓ and diameter D with a friction factor f. If the only losses associated with the flow in the penstock are due to pipe friction, shown that the maximum power output of the turbine occurs when the nozzle diameter, D_1, is given by $D_1 = D/(2f\ell/D)^{1/4}$.

$\dot{W}_{shaft} = \rho Q U(U-V_1)(1-\cos\beta)$ so the maximum power output occurs with $\beta = 180°$ and $U = \frac{V_1}{2}$. Thus,

$$\dot{W}_{shaft} = \rho Q \frac{V_1^2}{2} \quad \text{where} \qquad (1)$$



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$$

But $p_0 = p_1 = 0$, $V_0 = 0$, and $z_0 - z_1 = h$. Thus,

$$h = \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g}$$ where since $A_1V_1 = AV$ or $\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D^2V$ we have

$$V_1 = \left(\frac{D}{D_1}\right)^2 V$$

Therefore, $h = \frac{V_1^2}{2g}\left[1 + f\frac{\ell}{D}\frac{D_1^4}{D^4}\right]$ or $\frac{V_1^2}{2g} = \frac{h}{\left[1 + f\ell\frac{D_1^4}{D^5}\right]}$ and Eq. (1) gives

$$\dot{W}_{shaft} = \frac{\rho Q h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)} = \frac{\rho\frac{\pi}{4}D_1^2V_1 h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)}, \text{ but } V_1 = \frac{\sqrt{2gh}}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)^{\frac{1}{2}}} \qquad (2), (3)$$

For this problem f, ℓ, D, and h are constants; D_1 is variable. Thus, from Eqs. (2) and (3):

$$\dot{W}_{shaft} = \frac{KD_1^2}{(1 + cD_1^4)^{3/2}}$$ where K = const., and c = const. $= f\frac{\ell}{D^5}$

Note: $\dot{W}_{shaft} \to 0$ as $D_1 \to 0$ and as $D_1 \to \infty$. To find the D_1 that gives maximum power over all, set $\frac{d\dot{W}_{shaft}}{dD_1} = 0$

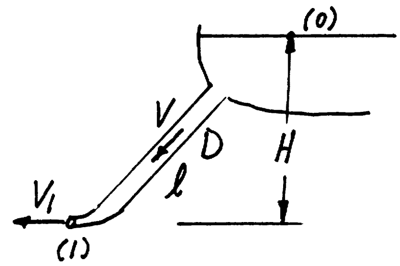
$$\frac{d\dot{W}_{shaft}}{dD} = \frac{2KD_1}{(1+cD_1^4)^{3/2}} + \frac{\left(-\frac{3}{2}\right)KD_1^2}{(1+cD_1^4)^{5/2}}(c)4D_1^3 = 0$$

or $\frac{2KD_1}{(1+cD_1^4)^{3/2}}\left[1 - \frac{3cD_1^4}{(1+cD_1^4)}\right] = 0$, or $1 + cD_1^4 = 3cD_1^4$, or $D_1^4 = \frac{1}{2c}$

Thus, $$D_1 = \frac{1}{\left(2f\frac{\ell}{D^5}\right)^{1/4}} = \underline{\underline{\frac{D}{\left(2f\frac{\ell}{D}\right)^{1/4}}}}$$

12.53

12.53 A Pelton wheel is supplied with water from a lake at an elevation H above the turbine. The penstock that supplies the water to the wheel is of length ℓ, diameter D, and friction factor f. Minor losses are negligible. Show that the power developed by the turbine is maximum when the velocity head at the nozzle exit is $2H/3$. Note: The result of Problem 12.52 may be of use.



For the flow through the penstock:

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$$

where $p_0 = p_1 = 0$, $V_0 = 0$, and $z_0 - z_1 = H$

Thus,

$$H = \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{but } A_1V_1 = AV \text{ or } \frac{\pi}{4}D_1^2 V_1 = \frac{\pi}{4}D^2 V \qquad (1)$$

so that

$$V^2 = \left(\frac{D_1}{D}\right)^4 V_1^2$$

From Problem 12.59, the maximum power occurs if $\frac{D_1}{D} = \frac{1}{(2f\frac{\ell}{D})^{1/4}}$

or

$$\left(\frac{D_1}{D}\right)^4 = \frac{1}{2f\frac{\ell}{D}} \quad \text{so that } V^2 = \frac{1}{(2f\frac{\ell}{D})} V_1^2$$

Thus, Eq. (1) becomes

$$H = \frac{V_1^2}{2g}\left[1 + \left(f\frac{\ell}{D}\right)\frac{1}{(2f\frac{\ell}{D})}\right] = \frac{3}{2}\frac{V_1^2}{2g}$$

or

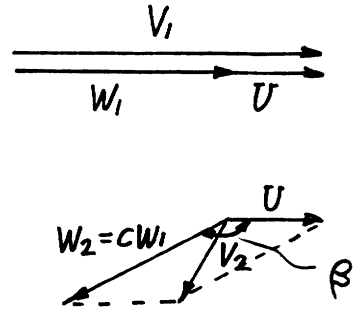
$$\frac{V_1^2}{2g} = \underline{\underline{\frac{2}{3}H}}$$

12.54

12.54 If there is negligible friction along the blades of a Pelton wheel, the relative speed remains constant as the fluid flows across the blades, and the maximum power output occurs when the blade speed is one-half the jet speed (see Eq. 12.52). Consider the case where friction is not negligible and the relative speed leaving the blade is some fraction, c, of the relative speed entering the blade. That is, $W_2 = cW_1$. Show that Eq. 12.52 is valid for this case also.

$$\dot{W}_{shaft} = \dot{m}\,(U_2 V_{\theta 2} - U_1 V_{\theta 1}) = \dot{m}\,U(V_{\theta 2} - V_{\theta 1})$$

The inlet and outlet velocity triangles are as shown.



Thus,

$V_{\theta 1} = V_1$ and

$V_{\theta 2} = U + W_2 \cos\beta$

but $W_2 = cW_1 = c(V_1 - U)$ so that $V_{\theta 2} = U + c(V_1 - U)\cos\beta$

Therefore,

$$\dot{W}_{shaft} = \dot{m}\,U\left[U + c(V_1 - U)\cos\beta - V_1\right] = \dot{m}\left[U(1 - c\cos\beta) - V_1(1 - c\cos\beta)\right]$$

$$= \dot{m}\,(1 - c\cos\beta)\left[U^2 - UV_1\right]$$

For maximum power, $\dfrac{d\dot{W}_{shaft}}{dU} = 0$ or

$\dot{m}(1 - c\cos\beta)\left[2U - V_1\right] = 0$ or $\underline{\underline{U = \dfrac{V_1}{2}}}$

12.55

12.55 A hydraulic turbine operating at 180 rpm with a head of 170 feet develops 20,000 horsepower. Estimate the power and speed if the turbine were to operate under a head of 300 ft.

$\omega_1 = 180\,rpm$, $h_{T1} = 170\,ft$, $\dot{W}_{shaft_1} = 20{,}000\,hp$

$\omega_2 = ?$, $h_{T2} = 190\,ft$, $\dot{W}_{shaft_2} = ?$

Assume the efficiency remains constant: $\left.\dfrac{g h_T}{\omega^2 D^2}\right)_1 = \left.\dfrac{g h_T}{\omega^2 D^2}\right)_2$

so with $D_1 = D_2$ and $g_1 = g_2$:

$$\frac{170}{(180)^2} = \frac{300}{\omega_2^2} \quad \text{or} \quad \omega_2 = \underline{\underline{239\,rpm}}$$

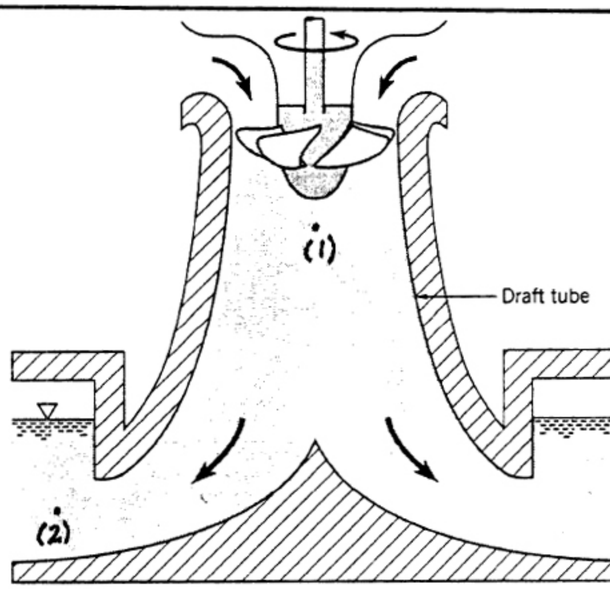
Assume the same power coefficient: $\left.\dfrac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_1 \quad \left.\dfrac{\dot{W}_{shaft}}{\rho \omega^3 D^5}\right)_2$

so with $D_1 = D_2$ and $\rho_1 = \rho_2$:

$$\frac{20{,}000}{(180)^3} = \frac{\dot{W}_{shaft_2}}{(239)^3} \quad \text{or} \quad \dot{W}_{shaft_2} = \underline{\underline{46{,}800\,hp}}$$

12.56

12.56 Draft tubes as shown in Fig. P12.56 are often installed at the exit of Kaplan and Francis turbines. Explain why such draft tubes are advantageous.



■ FIGURE P12.56

Without the draft tube there would be a relatively high speed exit jet (speed V_1, pressure $p_1 = 0$). With the draft tube (which acts as a diffuser) the exit speed is much smaller ($V_2 \approx 0$, $p_2 \approx 0$). From Bernoulli equation it follows that $p_1 < 0$ (with the draft tube). Hence there is a larger head available to the turbine. More energy can be removed from the fluid.

12.57

12.57 Turbines are to be designed to develop 30,000 horsepower while operating under a head of 70 ft and an angular velocity of 60 rpm. What type of turbines is best suited for this purpose? Estimate the flowrate needed.

$\dot{W}_{shaft} = 30{,}000\ hp$; $h_T = 70 ft$; and $\omega = 60\ rpm$ so that

$$N'_{sd} = \frac{\omega\sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = \frac{60\sqrt{3\times10^4}}{(70)^{5/4}} = 51.3$$

For this value a $\underline{\underline{\text{Francis}}}$ turbine would be appropiate.

Also, since $\dot{W}_{shaft} = \gamma Q h_T$ it follows that

$$Q = \frac{\dot{W}_{shaft}}{\gamma h_T} = \frac{(30{,}000\ hp)(550\ \frac{ft\ lb}{s}/hp)}{(62.4\ \frac{lb}{ft^3})(70 ft)} = \underline{\underline{378\ \frac{ft^3}{s}}}$$

12.58

12.58 Show how you would estimate the relationship between feature size and power production for a wind turbine like the one shown in **Video V12.1**.

To estimate the relationship between feature size and power production for a wind turbine we use the dimensionless pi terms of Eqs. 12.29 and 12.30 which are applicable for this incompressible flow. For similar turbines and operating conditions

$$\frac{\dot{W}_{shaft\,1}}{\rho_1 \omega_1^3 D_1^5} = \frac{\dot{W}_{shaft\,2}}{\rho_2 \omega_2^3 D_2^5}$$

and

$$\frac{g h_{a1}}{\omega_1^2 D_1^2} = \frac{g h_{a2}}{\omega_2^2 D_2^2}$$

Since $\rho_1 = \rho_2$ and $h_{a1} = h_{a2}$, we combine and get

$$\frac{\dot{W}_{shaft\,1}}{\dot{W}_{shaft\,2}} = \frac{D_1^2}{D_2^2}$$

Or power varies with feature size squared.

12.59

12.59 Water at 400 psi is available to operate a turbine at 1750 rpm. What type of turbine would you suggest to use if the turbine should have an output of approximately 200 hp?

With $p_0 = 400\,psi$, the maximum turbine head would be

$$h_T = \frac{p_0}{\gamma} = \frac{(400\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(62.4\,\frac{lb}{ft^3})} = 923\,ft$$

Hence,

$$N_{sd}' = \frac{\omega\sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = \frac{1750\sqrt{200}}{(923)^{5/4}} = 4.86$$ which is in the range appropriate for an impulse turbine.

12.60

12.60 What do you think are the major unresolved fluid dynamics problems for gas turbine engines?

Some major unresolved fluid mechanics problems for gas turbine engines include:

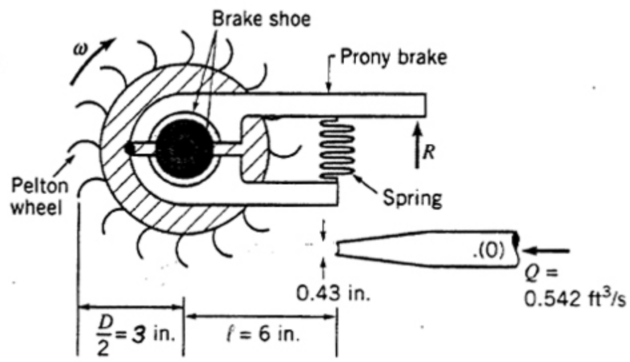
1. compressor stability prediction and control
2. fan and compressor blade and disk vibrations
3. noise -- front and back ends
4. seal leakage
5. high pressure turbine cooling (combination of fluid mechanics and heat transfer)
6. pollutant emissions (combination of fluid mechanics and combustion chemistry)
7. higher blade loads throughout

12.64

12.64 The device shown in Fig. P12.64 is used to investigate the power produced by a Pelton wheel turbine. Water supplied at a constant flowrate issues from a nozzle and strikes the turbine buckets as indicated. The angular velocity, ω, of the turbine wheel is varied by adjusting the tension on the Prony brake spring, thereby varying the torque, T_{shaft}, applied to the output shaft. This torque can be determined from the measured force, R, needed to keep the brake arm stationary as $T_{shaft} = F\ell$, where ℓ is the moment arm of the brake force.

Experimentally determined values of ω and R are shown in the following table. Use these results to plot a graph of torque as a function of the angular velocity. On another graph plot the power output, $\dot{W}_{shaft} = T_{shaft}\,\omega$, as a function of the angular velocity. On each of these graphs plot the theoretical curves for this turbine, assuming 100 percent efficiency.

Compare the experimental and theoretical results and discuss some possible reasons for any differences between them.



■ FIGURE P12.64

ω (rpm)	R (lb)
0	2.47
360	1.91
450	1.84
600	1.69
700	1.55
940	1.17
1120	0.89
1480	0.16

(a) Experimental: $T = R\ell = (0.5\ \text{ft})R$ or $T = 0.5R\ \text{ft}\cdot\text{lb}$, where $R \sim \text{lb}$ (1)

and $\dot{W}_{shaft} = T\omega = T\left(\omega \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)$

or

$\dot{W}_{shaft} = 0.1047\,T\omega\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$, where $T \sim \text{ft}\cdot\text{lb}$, $\omega \sim \text{rpm}$ (2)

Values of ω, T, and $\dot{W}_{shaft}$ are given in the table and graph below.

(b) Theoretical: $T = \dot{m}r(U - V_1)(1 - \cos\beta)$ where assume $\beta = 180°$,

$$V_1 = \frac{Q}{A_1} = \frac{0.542\ \frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}\left(\frac{0.43}{12}\text{ft}\right)^2} = 53.7\ \frac{\text{ft}}{\text{s}},\ \text{and}$$

$$\dot{m} = \rho Q = \left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(0.542\ \frac{\text{ft}^3}{\text{s}}\right) = 0.105\ \frac{\text{slugs}}{\text{s}}$$

Hence, with $U = \omega\frac{D}{2} = \left(\frac{3}{12}\text{ft}\right)\left(\frac{2\pi\omega}{60}\ \frac{\text{rad}}{\text{s}}\right) = 0.0262\,\omega\ \frac{\text{ft}}{\text{s}}$, $\omega \sim \text{rpm}$

$$T = \left(0.105\ \frac{\text{slugs}}{\text{s}}\right)\left(\frac{3}{12}\text{ft}\right)\left[0.0262\,\omega - 53.7\right]\frac{\text{ft}}{\text{s}}$$

or

$T = 1.41\left[4.88\times10^{-4}\omega - 1\right]\ \text{ft}\cdot\text{lb}$, where $\omega \sim \text{rpm}$ (3)

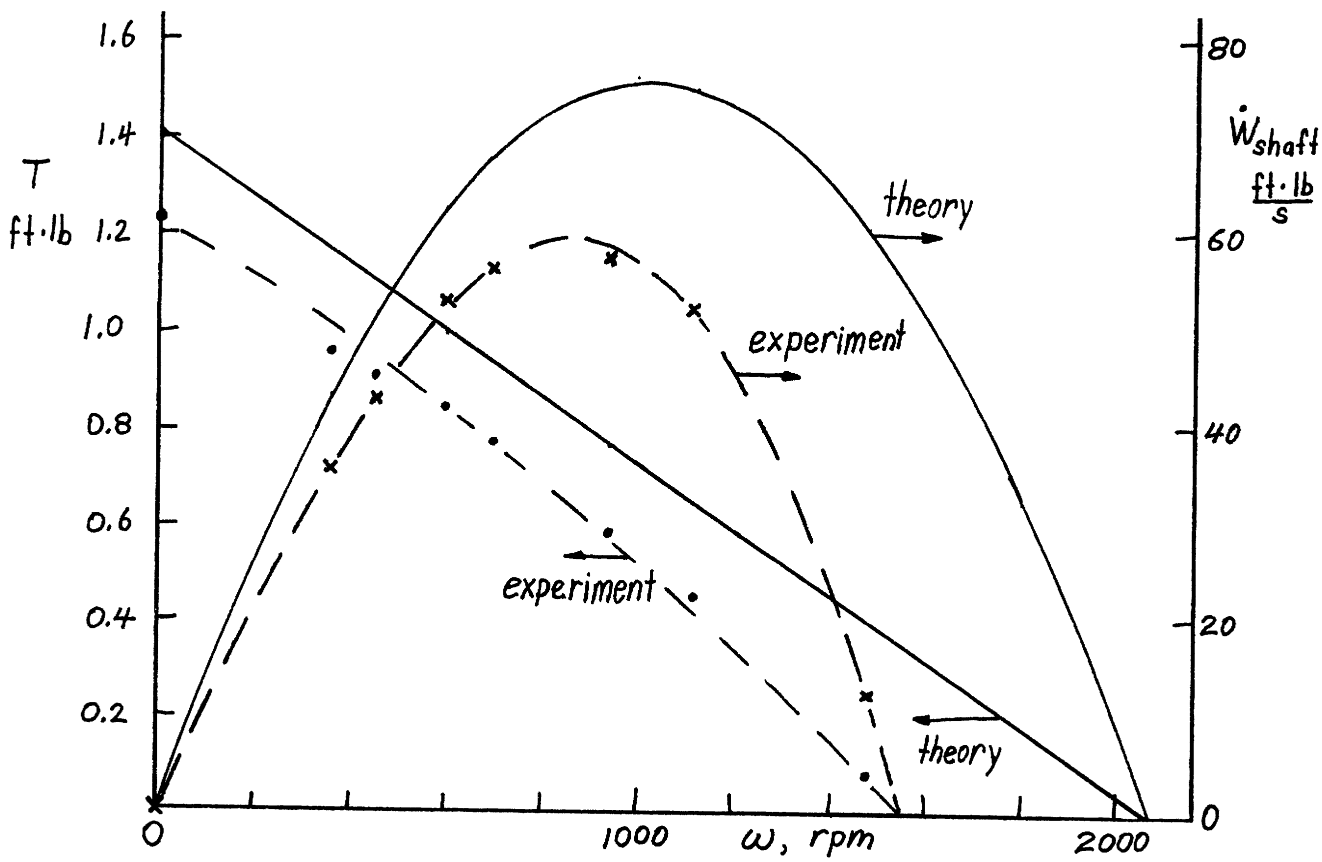
(con't)

12.64 (con't)

Also, $\dot{W}_{shaft} = T\omega = T\left(\frac{2\pi}{60}\omega\right) = 0.1047\, T\omega \ \frac{ft\cdot lb}{s}$, where $T \sim ft\cdot lb$, $\omega \sim rpm$ (4)

Values of T and $\dot{W}_{shaft}$ from Eqs. (3) and (4) are plotted in the graph below.

	experiment		theory	
ω, rpm	T, ft·lb	$\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$	$-T$, ft·lb	$-\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$
0	1.235	0	1.41	0
360	0.955	36.0	1.16	43.8
450	0.920	43.3	1.100	51.8
600	0.845	53.1	0.997	62.6
700	0.775	56.8	0.928	68.0
940	0.585	57.6	0.763	75.1
1120	0.445	52.2	0.639	75.0
1480	0.080	12.4	0.392	60.7



APPENDIX A

Listing of Standard Programs

EXPFIT.BAS

```
100 cls
110 print "*************************************************"
120 print "** This program determines the least squares fit **"
130 print "** for a function of the form  y = a * e ^ b*x   **"
140 print "*************************************************"
150 dim x(101),y(101),logy(101),ybar(101)
160 print
170 input "Number of points: ",n
180 print "Input X, Y"
190 for i=1 to n
200 input x(i),y(i)
210 logy(i)=log(y(i))
220 next i
230 sx=0
250 sy=0
260 sxy=0
270 sxsq=0
280 for i=1 to n
290 sx=sx+x(i)
300 sy=sy+logy(i)
310 sxy=sxy+x(i)*logy(i)
320 sxsq=sxsq+x(i)^2
330 next i
340 loga=(sxsq*sy-sxy*sx)/(n*sxsq-sx^2)
350 b=(n*sxy-sx*sy)/(n*sxsq-sx^2)
360 a=exp(loga)
370 print
380 print using "a = +#.###^^^^";a
390 print using "b = +#.###^^^^";b
400 print
410 print "     X              Y        Y(predicted)"
420 for i=1 to n
430 ybar(i)=a*exp(b*x(i))
440 print using "+#.####^^^^  +#.####^^^^  +#.####^^^^";x(i),y(i),ybar(i)
450 next i
```

LINREG1.BAS

```
5 cls
10 print "************************************************"
20 print "** This program determines the least squares fit **"
30 print "** for a function of the form  y = b * x         **"
40 print "************************************************"
45 print
50 dim x(101),y(101),ybar(101)
60 input "Number of points: ",n
70 print "Input X, Y"
80 for i=1 to n
90 input x(i),y(i)
100 next i
110 sxy=0
120 sxsq=0
130 for i=1 to n
140 sxy=sxy+x(i)*y(i)
150 sxsq=sxsq+x(i)^2
160 next i
170 b=sxy/sxsq
180 print
190 print using "b = +#.###^^^^";b
200 print
210 print "     X             Y        Y(predicted)"
220 for i=1 to n
230 ybar(i)=b*x(i)
240 print using "+#.####^^^^  +#.####^^^^  +#.####^^^^";x(i),y(i),ybar(i)
250 next i
```

LINREG2.BAS

```
5 cls
10 print "**************************************************"
20 print "** This program determines the least squares fit **"
30 print "** for a function of the form  y = a + b * x     **"
40 print "**************************************************"
50 dim x(101),y(101),ybar(101)
55 print
60 input "Number of points: ",n
70 print "Input X, Y"
80 for i=1 to n
90 input x(i),y(i)
100 next i
101 sx=0
102 sy=0
110 sxy=0
120 sxsq=0
130 for i=1 to n
131 sx=sx+x(i)
132 sy=sy+y(i)
140 sxy=sxy+x(i)*y(i)
150 sxsq=sxsq+x(i)^2
160 next i
161 a=(sxsq*sy-sxy*sx)/(n*sxsq-sx^2)
170 b=(n*sxy-sx*sy)/(n*sxsq-sx^2)
180 print
190 print using "a = +#.###^^^^";a
200 print using "b = +#.###^^^^";b
210 print
220 print "     X             Y        Y(predicted)"
230 for i=1 to n
240 ybar(i)=a+b*x(i)
250 print using "+#.####^^^^  +#.####^^^^  +#.####^^^^";x(i),y(i),ybar(i)
260 next i
```

POLREG.BAS[a]

```
100 cls
110 print "***************************************************"
120 print "** This program determines the least squares fit **"
130 print "** for any order polynomial of the form:         **"
140 print "**     y = d0 + d1*x + d2*x^2 + d3*x^3 + ...     **"
150 print "***************************************************"
160 print
170 dim b(21),d(21),s(21),x(101),y(101),f(101)
180 dim errf(101),pj(101),pjm1(101),ybar(101)
200 input "Enter number of terms in the polynomial: ",nterms
210 input "Enter number of data points: ",npoint
220 print:print "Enter data points (X , Y)"
230 for i=1 to npoint
240 input x(i),y(i)
250 d(i)=0
260 f(i)=y(i)
270 next i
280 print
290 print "The coefficients of the polynomial are:"
300 for i=1 to npoint
310 f(i)=f(i)-d(nterms+1)*x(i)^(nterms)
320 next i
330 for j=1 to nterms
340 b(j)=0
350 d(j)=0
360 s(j)=0
370 next j
380 c(1)=0
390 for i=1 to npoint
400 d(1)=d(1)+f(i)
410 b(1)=b(1)+x(i)
420 s(1)=s(1)+1
430 next i
440 d(1)=d(1)/s(1)
450 for i=1 to npoint
460 errf(i)=f(i)-d(1)
470 next i
480 if nterms=1 then goto 750
490 b(1)=b(1)/s(1)
```

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POLREG.BAS[a]

```
500 for i=1 to npoint
510 pjm1(i)=1
520 pj(i)=x(i)-b(1)
530 next i
540 for j=2 to nterms
550 for i=1 to npoint
560 p=pj(i)
570 d(j)=d(j)+errf(i)*p
580 p=p*pj(i)
590 b(j)=b(j)+x(i)*p
600 s(j)=s(j)+p
610 next i
620 d(j)=d(j)/s(j)
630 for i=1 to npoint
640 errf(i)=errf(i)-d(j)*p(i)
650 next i
660 if j=nterms then goto 750
670 b(j)=b(j)/s(j)
680 c(j)=s(j)/s(j-1)
690 for i=1 to npoint
700 p=pj(i)
710 pj(i)=(x(i)-b(j))*pj(i)-c(j)*pjm1(i)
720 pjm1(i)=p
730 next i
740 next j
750 print using "d# = +#.####^^^^";nterms-1,d(nterms)
760 nterms=nterms-1
770 if nterms>0 then goto 300
780 print
790 print "     X             Y        Y(predicted)"
800 for i=1 to npoint
810 print using "+#.####^^^^  +#.####^^^^  +#.####^^^^";x(i),y(i),y(i)-errf(i)
820 next i
```

[a]This program is based on an algorithm described in Conte, S.D. and de Boor, C., Elementary Numerical Analysis: An Algorithmic Approach, 3rd Ed., McGraw-Hill, New York, 1981, p. 259.

POWER1.BAS

```
5 cls
10 print "************************************************"
20 print "** This program determines the least squares fit **"
30 print "** for a function of the form  y = a * x ^ b     **"
40 print "************************************************"
50 dim x(101),y(101),logx(101),logy(101),ybar(101)
55 print
60 input "Number of points: ",n
70 print:print "Input X, Y"
80 for i=1 to n
90 input x(i),y(i)
98 logx(i)=log(x(i))
99 logy(i)=log(y(i))
100 next i
101 sx=0
102 sy=0
110 sxy=0
120 sxsq=0
130 for i=1 to n
131 sx=sx+logx(i)
132 sy=sy+logy(i)
140 sxy=sxy+logx(i)*logy(i)
150 sxsq=sxsq+logx(i)^2
160 next i
161 loga=(sxsq*sy-sxy*sx)/(n*sxsq-sx^2)
170 b=(n*sxy-sx*sy)/(n*sxsq-sx^2)
175 a=exp(loga)
180 print
190 print using "a = +#.###^^^^";a
200 print using "b = +#.###^^^^";b
210 print
220 print      "     X              Y        Y(predicted)"
230 for i=1 to n
240 ybar(i)=a*x(i)^b
250 print using "+#.####^^^^  +#.####^^^^  +#.####^^^^";x(i),y(i),ybar(i)
260 next i
```

SIMPSON.BAS

```
100 cls
110 print "***********************************************"
120 print "** This program performs numerical integration **"
130 print "** over a set of an odd number of equally      **"
140 print "** spaced points using Simpson's Rule          **"
150 print "***********************************************"
160 print
170 dim x(101),y(101)
180 input "Enter number of data points: ",n
190 print "Enter data points (X , Y)"
200 for i=1 to n
210 input x(i),y(i)
220 next i
230 h=(x(n)-x(1))/(n-1)
240 s=0
250 for i=2 to n-1 step 2
260 s=s+4*y(i)+2*y(i+1)
270 next i
280 intgrl=h/3*(s+y(1)-y(n))
290 print
300 print using "The approximate value of the integral is: +#.####^^^^";intgrl
```

TRAPEZOI.BAS

```
100 cls
110 print "*****************************************************"
120 print "** This program performs numerical integration     **"
130 print "** over a set of points using the Trapezoidal Rule **"
140 print "*****************************************************"
150 print
160 dim x(101),y(101)
170 input "Enter number of data points: ",n
180 print "Enter data points (X , Y)"
190 for i=1 to n
200 input x(i),y(i)
210 next i
230 intgrl=0
240 for i=1 to n-1
250 intgrl=intgrl+0.5*(x(i+1)-x(i))*(y(i)+y(i+1))
260 next i
280 print
290 print using "The approximate value of the integral is: +#.####^^^^";intgrl
```

COLEBROO.BAS

```
100 cls
110 print "*********************************************************"
120 print "** This program determines the friction factor, f, for **"
130 print "** pipe flow for the case of laminar or turbulent flow **"
140 print "** (solving iteratively Colebrook's equation), given   **"
150 print "** the Reynolds number and the relative roughness of   **"
160 print "** the pipe                                            **"
170 print "*********************************************************"
180 print
190 input "Enter Reynolds number, Re = ",re
200 f=64/re
210 if re < 2100 then goto 260
220 input "Enter relative roughness, rr = ",rr
230 fp=f
240 f=1/(-2.0*log(rr/3.7+2.51/(re*fp^.5))/log(10))^2
250 if abs(1-f/fp)>0.001 then goto 230
260 print
270 print using "The friction factor is f = +#.####^^^^";f
```

CUBIC.BAS

```
100 cls
110 print "**********************************************************"
120 print "** This program determines the real roots of a          **"
130 print "** cubic equation of the form x^3 + a*x^2 + b*x + c = 0 **"
140 print "**********************************************************"
150 print
160 input "                             a = ",a
170 input "                             b = ",b
180 input "                             c = ",c
190 '
200 'Check if the equation has complex roots
210 p=(3*b-a^2)/3
220 q=(2*a^3-9*a*b+27*c)/27
230 if q^2/4+p^3/27<=0 then goto 250
240 print:print "The equation has complex roots":stop
250 x0=-a/3+2/3*(a^2-3*b)^0.5
260 for i=1 to 20
270 x1=(2*x0^3+a*x0^2-c)/(3*x0^2+2*a*x0+b)
280 if abs(x1/x0-1)<0.0001 then goto 310
290 x0=x1
300 next i
310 m=a+x1
320 n=b+a*x1+x1^2
330 x2=(-m+(m^2-4*n)^.5)/2
340 x3=(-m-(m^2-4*n)^.5)/2
350 print
360 print "The roots of the cubic equation are:"
370 print using "x1=+#.####^^^^  x2=+#.####^^^^  x3=+#.####^^^^";x1,x2,x3
```

FAN_RAY.BAS

```
100 CLS
110 PRINT "*********************************************************"
120 PRINT "**  This program computes the one-dimensional Fanno or  **"
130 PRINT "**  Rayleigh flow functions for a gas with constant     **"
140 PRINT "**  specific heat and molecular weight. (NOTE: k > 1)   **"
150 PRINT "*********************************************************"
160 '
170 ' Fanno flow functions
180 DEF FNFTTSTAR (K, MA) = (K + 1#) / (2# + (K - 1#) * MA ^ 2)
190 DEF FNFVVSTAR (K, MA) = SQR(FNFTTSTAR(K, MA) * MA ^ 2)
200 DEF FNFPPSTAR (K, MA) = SQR(FNFTTSTAR(K, MA)) / MA
210 DEF FNFPOPOSTAR (K, MA) = (1# / FNFTTSTAR(K, MA)) ^ ((K + 1#) / (2# * (K - 1
#))) / MA
220 DEF FNFLD (K, MA) = (1# - MA ^ 2) / (K * MA ^ 2) + (K + 1#) * LOG(FNFVVSTAR(
K, MA) ^ 2) / (2# * K)
230 '
240 ' Rayleigh flow functions
250 DEF FNRTMP (K, MA) = 1# + (K - 1#) * MA ^ 2 / 2#
260 DEF FNRPPA (K, MA) = (1# + K) / (1# + K * MA ^ 2)
270 DEF FNRTTA (K, MA) = (FNRPPA(K, MA) * MA) ^ 2
280 DEF FNRVVA (K, MA) = FNRPPA(K, MA) * MA ^ 2
290 DEF FNRTOTOA (K, MA) = 2# * FNRPPA(K, MA) ^ 2 * MA ^ 2 * FNRTMP(K, MA) / (K
+ 1#)
300 DEF FNRPOPOA (K, MA) = FNRPPA(K, MA) * (2# * FNRTMP(K, MA) / (K + 1#)) ^ (K
/ (K - 1#))
310 '
320 ' Get functions desired
330 LOCATE 8:  PRINT "Program options"
340 LOCATE 9:  PRINT "  (1) Fanno flow calculations"
350 LOCATE 10: PRINT "  (2) Rayleigh flow calculations"
360 LOCATE 11: INPUT "Enter the number of the option desired: ", OPT
370 IF (OPT <> 1) AND (OPT <> 2) THEN LOCATE 11: PRINT SPACE$(79): GOTO 360
380 '
390 '--- Display banner specifing which flow calculation is being performed
400 CLS
410 IF OPT = 2 GOTO 480
420 PRINT "*********************************************************"
430 PRINT "**  Computing the one-dimensional Fanno flow functions  **"
440 PRINT "**  for a gas with constant specific heat and molecular **"
450 PRINT "**  weight. (NOTE: k > 1)                               **"
460 PRINT "*********************************************************"
470 GOTO 540
480 PRINT "************************************************************"
490 PRINT "**  Computing the one-dimensional Rayleigh flow functions  **"
500 PRINT "**  for a gas with constant specific heat and molecular    **"
510 PRINT "**  weight. (NOTE: k > 1)                                  **"
520 PRINT "************************************************************"
530 '
540 '--- Get the user specified specific heat ratio
550 LOCATE 7: INPUT "Enter the specific heat ratio, (k > 1): ", K
```

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FAN_RAY.BAS

```
560 IF K <= 1 THEN GOTO 550
570 LOCATE 7: PRINT SPACE$(79): LOCATE 7
580 PRINT USING "The specific heat ratio is k=##.###"; K
590 '
600 ' Get Mach number to solve for
610 FOR I = 8 TO 16: PRINT SPACE$(79): NEXT I
620 LOCATE 16: PRINT SPACE$(79): LOCATE 16
630 INPUT "Enter a Mach number to solve for (999 to quit): ", MA
640 IF MA = 999 THEN END
650 IF MA > 0 AND OPT = 2 THEN GOTO 710
660 IF MA > 0 AND OPT = 1 THEN GOTO 830
670 LOCATE 9: FOR I = 1 TO 6: PRINT SPACE$(79): NEXT I: LOCATE 14
680 PRINT "Valid Mach number range: Ma > 0"
690 GOTO 620
700 '
710 ' Solve Rayleigh flow functions for specified k and Ma
720 LOCATE 9: FOR I = 1 TO 6: PRINT SPACE$(79): NEXT I: LOCATE 9
730 PRINT USING "    Ma = #.####^^^^"; MA
740 PRINT USING "  P/Pa = #.####^^^^"; FNRPPA(K, MA)
750 PRINT USING "  T/Ta = #.####^^^^"; FNRTTA(K, MA)
760 PRINT USING "  V/Va = #.####^^^^"; FNRVVA(K, MA)
770 PRINT USING "To/Toa = #.####^^^^"; FNRTOTOA(K, MA)
780 PRINT USING "Po/Poa = #.####^^^^"; FNRPOPOA(K, MA)
790 '
800 ' Loop back for another Mach number
810 GOTO 620
820 '
830 ' Solve Fanno flow functions for specified k and Ma
840 LOCATE 9: FOR I = 1 TO 6: PRINT SPACE$(79): NEXT I: LOCATE 9
850 PRINT USING "         Ma = #.####^^^^"; MA
860 PRINT       "f(l*-l)/D = ";
865 PRINT USING "#.####^^^^"; FNFLD(K, MA)
870 PRINT       "     T/T* = ";
875 PRINT USING "#.####^^^^"; FNFTTSTAR(K, MA)
880 PRINT       "     V/V* = ";
885 PRINT USING "#.####^^^^"; FNFVVSTAR(K, MA)
890 PRINT       "     P/p* = ";
895 PRINT USING "#.####^^^^"; FNFPPSTAR(K, MA)
900 PRINT       "  Po/Po,* = ";
905 PRINT USING "#.####^^^^"; FNFPOPOSTAR(K, MA)
910 PRINT
920 '
930 ' Loop back for another Mach number
940 GOTO 620
```

ISENTROP.BAS

```
100 CLS
110 PRINT "***********************************************************"
120 PRINT "** This program computes the one-dimensional isentropic  **"
130 PRINT "** flow functions for a gas with constant specific heat  **"
140 PRINT "** and molecular weight.  (NOTE: k > 1)                  **"
150 PRINT "***********************************************************"
160 '
170 ' Isentropic flow functions
180 DEF FNITTO (K, MA) = 2# / (2# + (K - 1#) * MA ^ 2)
190 DEF FNIPPO (K, MA) = FNITTO(K, MA) ^ (K / (K - 1#))
200 DEF FNIRRO (K, MA) = FNITTO(K, MA) ^ (1# / (K - 1#))
210 DEF FNIAASTAR (K, MA) = (2# / ((K + 1#) * FNITTO(K, MA))) ^ ((K + 1#) / (2#
* (K - 1#))) / MA
220 '
230 ' Get user specific heat ratio and display
240 LOCATE 7: INPUT "Enter the specific heat ratio, (k > 1): ", K
250 IF K <= 1! THEN GOTO 240
260 LOCATE 7: PRINT SPACE$(79)
270 LOCATE 7: PRINT USING "The specific heat ratio is k=##.###"; K
280 '
290 ' Get Mach number to solve for
300 FOR I = 8 TO 15: PRINT SPACE$(79): NEXT I
310 LOCATE 15: PRINT SPACE$(79): LOCATE 15
320 INPUT "Enter a Mach number to solve for (999 to quit): ", MA
330 IF MA = 999 THEN END
340 IF MA > 0 THEN GOTO 390
350 LOCATE 9: FOR I = 1 TO 5: PRINT SPACE$(79): NEXT I: LOCATE 13
360 PRINT "Valid Mach number range: Ma > 0"
370 GOTO 310
380 !
390 ' Solve isentropic flow functions for specified k and Ma
400 LOCATE 9: FOR I = 1 TO 5: PRINT SPACE$(79): NEXT I: LOCATE 9
410 TTO = FNITTO(K, MA)
420 PPO = FNIPPO(K, MA)
430 AASTAR = FNIAASTAR(K, MA)
440 RRO = FNIRRO(K, MA)
450 '
460 ' Display computed results
470 PRINT USING "      Ma = #.####^^^^"; MA
480 PRINT USING "    T/To = #.####^^^^"; TTO
490 PRINT USING "    P/Po = #.####^^^^"; PPO
500 PRINT USING "RHO/RHOo = #.####^^^^"; RRO
510 PRINT "    A/A* = ";
515 PRINT USING "#.####^^^^";AASTAR
520 PRINT
530 '
540 ' Loop back for another Mach number
550 GOTO 310
```

SHOCK.BAS

```
100 CLS
110 PRINT "************************************************************"
120 PRINT "** This program computes the one-dimensional normal-shock **"
130 PRINT "** functions for a gas with constant specific heat and    **"
140 PRINT "** molecular weight.  (NOTE: k > 1)                       **"
150 PRINT "************************************************************"
160 '
170 ' Normal-shock functions
180 DEF FNSTMP (K, MAX) = (2# * K * MAX ^ 2 / (K - 1#)) - 1#
190 DEF FNSMAY (K, MAX) = SQR((MAX ^ 2 + (2# / (K - 1#))) / FNSTMP(K, MAX))
200 DEF FNSPYPX (K, MAX) = 2# * K * MAX ^ 2 / (K + 1#) - (K - 1#) / (K + 1#)
210 DEF FNSVYVX (K, MAX) = (K + 1#) * MAX ^ 2 / ((K - 1#) * MAX ^ 2 + 2#)
220 DEF FNSTYTX (K, MAX) = (1# + (K - 1#) * MAX ^ 2 / 2#) * FNSTMP(K, MAX) / (((
K + 1#) * MAX) ^ 2 / (2# * (K - 1#)))
230 DEF FNSPOYPX (K, MAX) = ((K + 1#) * MAX ^ 2 / 2#) ^ (K / (K - 1#)) * FNSPYPX
(K, MAX) ^ (1# / (1# - K))
240 DEF FNSPOYPOX (K, MAX) = (((K + 1#) * MAX ^ 2) / (2# + (K - 1#) * MAX ^ 2))
^ (K / (K - 1#)) / FNSPYPX(K, MAX) ^ (1# / (K - 1#))
250 '
260 ' Get user specific heat ratio and display
270 LOCATE 7: INPUT "Enter the specific heat ratio, (k > 1): ", K
280 IF K <= 1! THEN GOTO 270
290 LOCATE 7: PRINT SPACE$(79): LOCATE 7
300 PRINT USING "The specific heat ratio is k=##.###"; K
310 '
320 ' Get Mach number to solve for
330 FOR I = 8 TO 17: PRINT SPACE$(79): NEXT I
340 LOCATE 17: PRINT SPACE$(79): LOCATE 17
350 INPUT "Enter a Mach number to solve for (999 to quit): ", MAX
360 IF MAX = 999 THEN END
370 IF MAX >= 1! THEN GOTO 420
380 LOCATE 9: FOR I = 1 TO 7: PRINT SPACE$(79): NEXT I
390 LOCATE 15: PRINT "Valid Mach number range:  Ma,x >= 1"
400 GOTO 340
410 '
420 ' Solve normal-shock functions for specified k and Ma
430 LOCATE 9: FOR I = 1 TO 7: PRINT SPACE$(79): NEXT I: LOCATE 9
440 MAY = FNSMAY(K, MAX)
450 PYPX = FNSPYPX(K, MAX)
460 VYVX = FNSVYVX(K, MAX)
470 TYTX = FNSTYTX(K, MAX)
480 POYPOX = FNSPOYPOX(K, MAX)
490 POYPX = FNSPOYPX(K, MAX)
500 '
510 ' Display computed results
520 PRINT USING "      Max = #.####^^^^"; MAX
530 PRINT USING "      May = #.####^^^^"; MAY
540 PRINT USING "    Py/Px = #.####^^^^"; PYPX
550 PRINT USING "RHOy/RHOx = #.####^^^^"; VYVX
560 PRINT USING "    Ty/Tx = #.####^^^^"; TYTX
570 PRINT USING "  Poy/Pox = #.####^^^^"; POYPOX
580 PRINT USING "   Poy/Px = #.####^^^^"; POYPX
590 PRINT
600 '
610 ' Loop back for another Mach number
620 GOTO 340
```