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CHAPTER

24

Clutches

24.1 Introduction

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

24.2 Types of Clutches

Following are the two main types of clutches commonly used in engineering practice :

1. Positive clutches, and **2.** Friction clutches.

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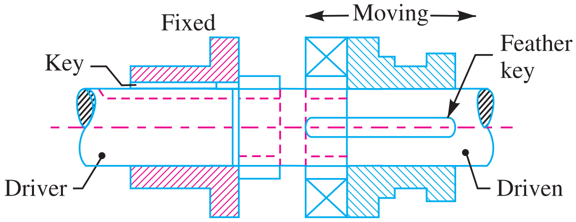
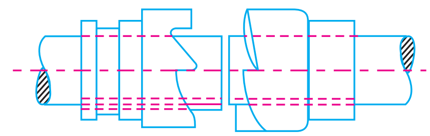
We shall now discuss these clutches in the following pages.

24.3 Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch.*** The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the



(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

Fig. 24.1. Jaw clutches.

driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. 24.1 (*a*) or of spiral type as shown in Fig. 24.1 (*b*).

A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

24.4 Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly *dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

24.5 Material for Friction Surfaces

The material used for lining of friction surfaces of a clutch should have the following characteristics :

* During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (*i.e.* for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch.

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1. It should have a high and uniform coefficient of friction.
2. It should not be affected by moisture and oil.
3. It should have the ability to withstand high temperatures caused by slippage.
4. It should have high heat conductivity.
5. It should have high resistance to wear and scoring.

The materials commonly used for lining of friction surfaces and their important properties are shown in the following table.

Table 24.1. Properties of materials commonly used for lining of friction surfaces.

Material of friction surfaces	*Operating condition*	*Coefficient of friction*	*Maximum operating temperature (°C)*	*Maximum pressure (N/mm^2)*
Cast iron on cast iron or steel	dry	0.15 – 0.20	250 – 300	0.25– 0.4
Cast iron on cast iron or steel	In oil	0.06	250 – 300	0.6 – 0.8
Hardened steel on Hardened steel	In oil	0.08	250	0.8 – 0.8
Bronze on cast iron or steel	In oil	0.05	150	0.4
Pressed asbestos on cast iron or steel	dry	0.3	150 – 250	0.2 – 0.3
Powder metal on cast iron or steel	dry	0.4	550	0.3
Powder metal on cast iron or steel	In oil	0.1	550	0.8

24.6 Considerations in Designing a Friction Clutch

The following considerations must be kept in mind while designing a friction clutch.

1. The suitable material forming the contact surfaces should be selected.
2. The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed service.
3. The clutch should not require any external force to maintain contact of the friction surfaces.
4. The provision for taking up wear of the contact surfaces must be provided.
5. The clutch should have provision for facilitating repairs.
6. The clutch should have provision for carrying away the heat generated at the contact surfaces.
7. The projecting parts of the clutch should be covered by guard.

24.7 Types of Friction Clutches

Though there are many types of friction clutches, yet the following are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

We shall now discuss these clutches, in detail, in the following pages.

Note : The disc and cone clutches are known as ***axial friction clutches,*** while the centrifugal clutch is called ***radial friction clutch.***

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24.8 Single Disc or Plate Clutch

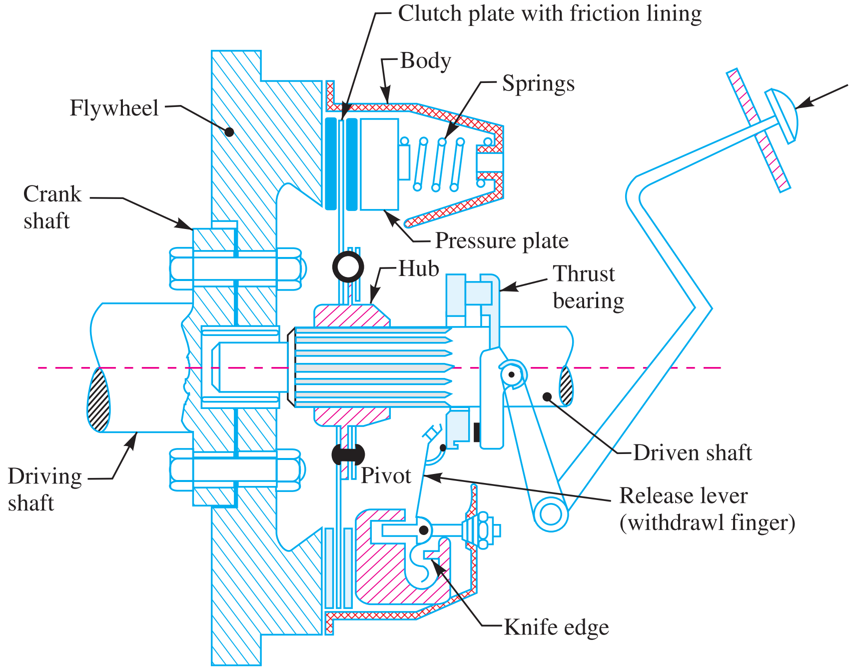

Fig. 24.2. Single disc or plate clutch.

A single disc or plate clutch, as shown in Fig 24.2, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

When a car hits an object and decelerates quickly the objects are thrown forward as they continue to move forwards due to inertia.

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The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

24.9 Design of a Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (W) as shown in Fig. 24.3 (*a*).

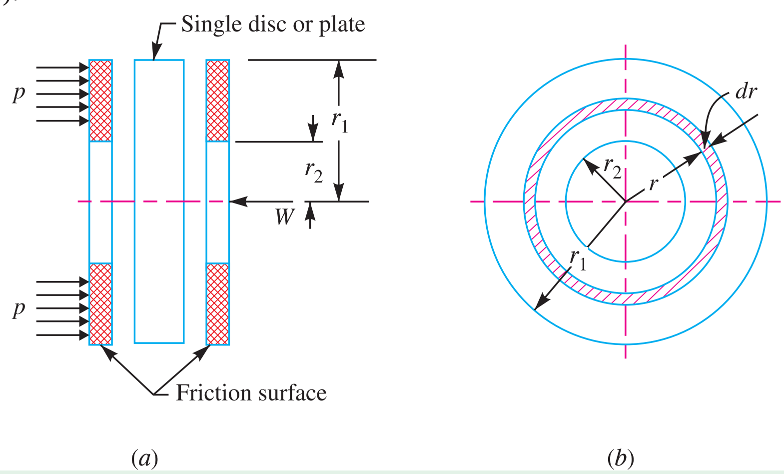

Fig. 24.3. Forces on a disc clutch.

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces,

r = Mean radius of the friction face, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 24.3 (*b*).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform axial wear.

1. *Considering uniform pressure*. When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (*a*), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

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where W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3-(r_2)^3}{3}\right] = 2\pi\,\mu\times\frac{W}{\pi\,[(r_1)^2-(r_2)^2]}\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\,\mu.\,W\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right] = \mu\,.W.R$$

where $R = \frac{2}{3}\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right]$ = Mean radius of the friction surface.

2. *Considering uniform axial wear.* The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear ∝ Work of friction ∝ $p.V$

or $p.V = K$ (a constant) or $p = K/V$ ...(*i*)

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product $p.V$ is constant over the entire surface. After this, the wear will be uniform as shown in Fig. 24.4.

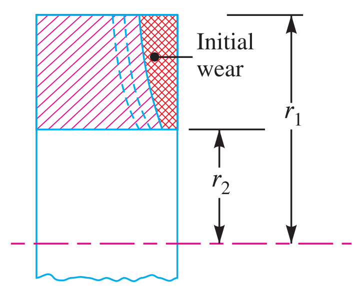

Fig. 24.4. Uniform axial wear.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r = C$ (a constant) or $p = C / r$ ...(*ii*)

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r}\times 2\pi r.dr = 2\pi C.dr$$

∴ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\,C\,dr = 2\pi\,C\,[r]_{r_2}^{r_1} = 2\pi\,C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

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We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because\ p = C/r)$$

$\therefore$ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\,C.r.dr = 2\pi\,\mu\,C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

Notes : 1. In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction (or contact) surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{... (For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation ***(ii)*** may be written as

$$p_{max} \times r_2 = C \qquad \text{or} \qquad p_{max} = C / r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation ***(ii)*** may be written as

$$p_{min} \times r_1 = C \qquad \text{or} \qquad p_{min} = C / r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

A twin disk clutch

24.10 Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 24.5, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

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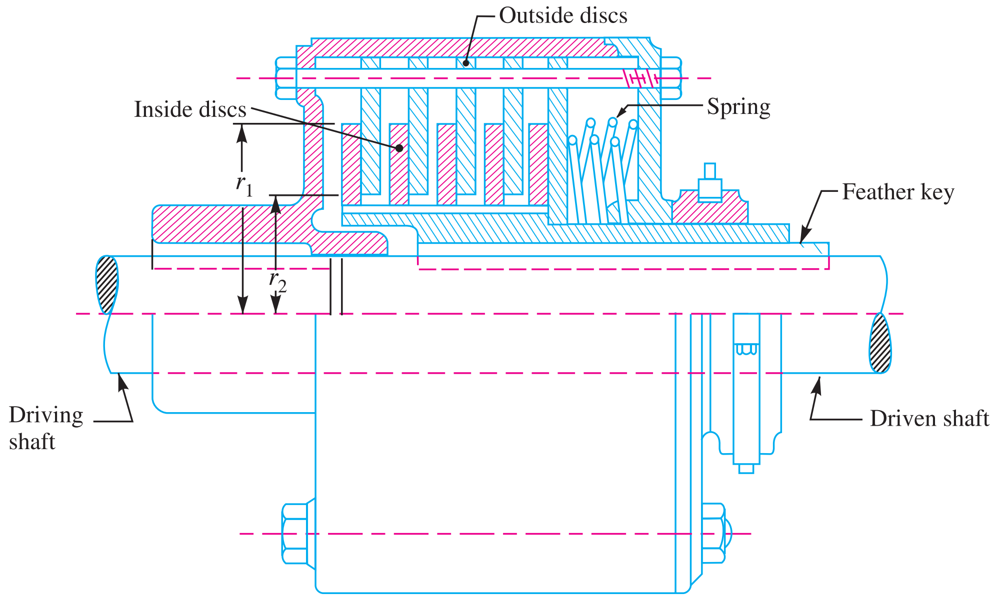

Fig. 24.5. Multiple disc clutch.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

Example 24.1. *Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : W = 4 kN = 4000 N ; r_2 = 50 mm ; r_1 = 100 mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We also know that total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4000 / 15\,710 = 0.2546 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore,

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4000 = 2\pi C (r_1 - r_2) = 2\pi \times 100\, p_{min}\, (100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4000 / 31\,420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$= \frac{4000}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.2. *A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.*

(a) Assuming a uniform pressure of 0.17 N/mm²; determine the inner diameter of the friction surfaces.

(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $N = 1250$ r.p.m. ; $d_1 = 300$ mm or $r_1 = 150$ mm ; $\mu = 0.4$; $p = 0.17$ N/mm²

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\, N} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi\, [(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi\, [(150)^2 - (r_2)^2] = 0.534\, [(150)^2 - (r_2)^2] \quad ...(i)$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

$\therefore$ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534\, [(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \quad ...(\because n = 2)$$

$$= 0.285\, [(150)^3 - (r_2)^3]$$

or $\quad (150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$

$\therefore \quad (r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6$ or $r_2 = 75$ mm

and $\quad d_2 = 2r_2 = 2 \times 75 = 150$ mm **Ans.**

(b) Maximum torque transmitted

We know that the axial thrust,

$$W = 0.534\,[(150)^2 - (r_2)^2] \qquad \text{... [From equation } (i)]$$

$$= 0.534\,[(150)^2 - (75)^2] = 9011 \text{ N}$$

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

∴ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$

$$= 811 \text{ N-m} \quad \textbf{Ans.}$$

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \qquad \text{or} \qquad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\,\pi\, C\,(r_1 - r_2) = 2\pi \times p_{max} \times 75\,(150 - 75) = 35\,347\; p_{max}$$

$$\therefore \quad p_{max} = 9011 / 35\,347 = 0.255 \text{ N/mm}^2 \qquad \textbf{Ans.}$$

Example 24.3. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.*

Solution. Given : $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. ; $\mu = 0.255$; $d_1 / d_2 = 1.25$ or $r_1 / r_2 = 1.25$; $p_{max} = 0.1$ N/mm²

Outer and inner diameters of frictional surface

Let d_1 and d_2 = Outer and inner diameters (in mm) of frictional surface, and

r_1 and r_2 = Corresponding radii (in mm) of frictional surface.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\, N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 3000} = 79.6 \text{ N-m} = 79\,600 \text{ N-mm}$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore.

$$p_{max} \times r_2 = C$$

or

$$C = 0.1\, r_2 \text{ N/mm}$$

and normal or axial load acting on the friction surface,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2)$$

$$= 0.157\,(r_2)^2 \qquad \text{... } (\because r_1 / r_2 = 1.25)$$

We know that mean radius of the frictional surface (for uniform wear),

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

and the torque transmitted (T),

$$79\,600 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2\; 1.125\, r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3 / 0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm}$$

and

$$r_1 = 1.25\, r_2 = 1.25 \times 96 = 120 \text{ mm}$$

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∴ Outer diameter of frictional surface,

$$d_1 = 2r_1 = 2 \times 120 = 240 \text{ mm} \quad \textbf{Ans.}$$

and inner diameter of frictional surface,

$$d_2 = 2r_2 = 2 \times 96 = 192 \text{ mm} \quad \textbf{Ans.}$$

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2)$$
$$= 0.157\,(r_2)^2 = 0.157\,(96)^2 = 1447 \text{ N} \quad \textbf{Ans.}$$

Example 24.4. *A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of the friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N/mm, determine the dimensions of the friction plate and initial compression in the springs.*

Solution. Given : P = 100 kW = 100 × 10^3 W ; *N = 2400 r.p.m. ; T = 500 N-m = 500 × 10^3 N-mm ; p = 0.07 N/mm² ; μ = 0.3 ; No. of springs = 8 ; Stiffness/spring = 40 N/mm

Dimensions of the friction plate

Let r_1 = Outer radius of the friction plate, and

r_2 = Inner radius of the friction plate.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and axial load acting on the friction plate,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.07\, r_2\,(1.25\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \qquad \ldots(i)$$

We know that mean radius of the friction plate, for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2} = 1.125\, r_2$$

∴ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2\, 1.125\, r_2 = 0.074\,(r_2)^3 \qquad \ldots(\because n = 2)$$

$$(r_2)^3 = 500 \times 10^3 / 0.074 = 6757 \times 10^3 \quad \text{or} \quad r_2 = 190 \text{ mm} \quad \textbf{Ans.}$$

and $$r_1 = 1.25\, r_2 = 1.25 \times 190 = 237.5 \text{ mm} \quad \textbf{Ans.}$$

Initial compression in the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \qquad \ldots \text{[From equation } (i)\text{]}$$

∴ Initial compression in the springs

$$= W / s = 3970 / 320 = 12.4 \text{ mm} \quad \textbf{Ans.}$$

* Superfluous data

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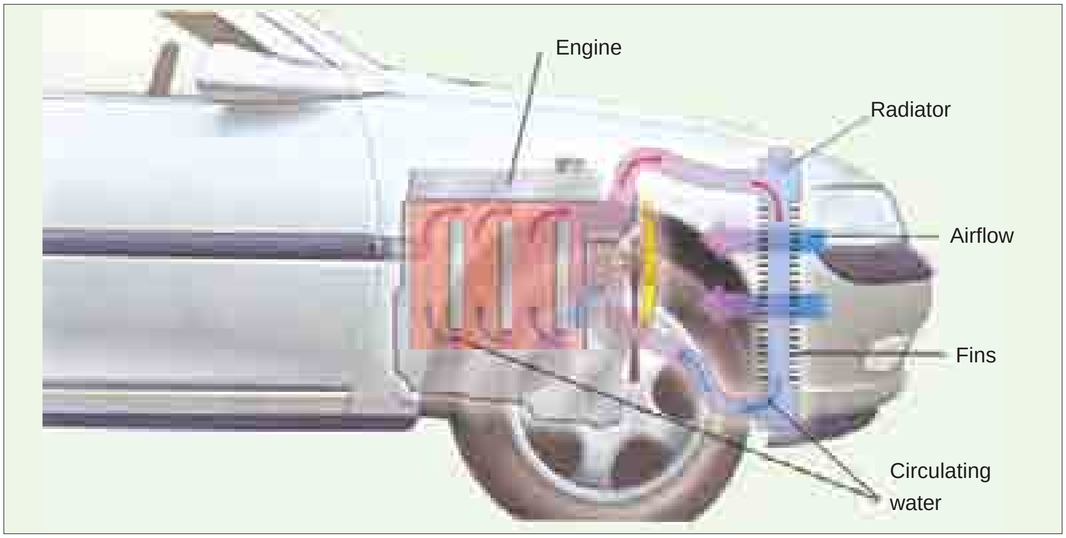

In car cooling system a pump circulates water through the engine and through the pipes of the radiator.

Example 24.5. *A single dry plate clutch is to be designed to transmit 7.5 kW at 900 r.p.m. Find :*

1. *Diameter of the shaft,*
2. *Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4,*
3. *Outer and inner radii of the clutch plate, and*
4. *Dimensions of the spring, assuming that the number of springs are 6 and spring index = 6. The allowable shear stress for the spring wire may be taken as 420 MPa.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 900 r.p.m. ; r/b = 4 ; No. of springs = 6 ; $C = D/d = 6$; τ = 420 MPa = 420 N/mm^2

1. *Diameter of the shaft*

Let d_s = Diameter of the shaft, and

τ_1 = Shear stress for the shaft material. It may be assumed as 40 N/mm^2.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{7500 \times 60}{2\,\pi \times 900} = 79.6 \text{ N-m} = 79\ 600 \text{ N-mm} \quad ...(i)$$

We also know that the torque transmitted (T),

$$79\ 600 = \frac{\pi}{16} \times \tau_1\,(d_s)^3 = \frac{\pi}{16} \times 40\,(d_s)^3 = 7.855\,(d_s)^3$$

$\therefore$ $(d_s)^3$ = 79 600 / 7.855 = 10 134 or d_s = 21.6 say 25 mm **Ans.**

2. *Mean radius and face width of the friction lining*

Let R = Mean radius of the friction lining, and

b = Face width of the friction lining = $R/4$ ... (Given)

We know that the area of the friction faces,

$$A = 2\pi\,R.b$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi\,R.b.p$$

and torque transmitted, $T = \mu\ W.R.n = \mu\ (2\pi\ Rb.p)\ R.n$

$$= \mu\left(2\,\pi R \times \frac{R}{4} \times p\right) R.n = \frac{\pi}{2} \times \mu.R^3.p.n \qquad \text{...}(ii)$$

Assuming the intensity of pressure (p) as 0.07 N/mm^2 and coefficient of friction (μ) as 0.25, we have from equations ***(i)*** and ***(ii)***,

$$79\ 600 = \frac{\pi}{2} \times 0.25 \times R^3 \times 0.07 \times 2 = 0.055\ R^3$$

... ($\because$ n = 2, for both sides of plate effective)

$\therefore$ $R^3 = 79\ 600 / 0.055 = 1.45 \times 10^6$ or $R = 113.2$ say 114 mm **Ans.**

and $b = R / 4 = 114 / 4 = 28.5$ mm **Ans.**

3. Outer and inner radii of the clutch plate

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the face width (or radial width) of the plate is equal to the difference of the outer and inner radii, therefore,

$$b = r_1 - r_2 \quad \text{or} \quad r_1 - r_2 = 28.5 \text{ mm} \qquad \text{...}(iii)$$

We know that for uniform wear, mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 114 = 228 \text{ mm} \qquad \text{...}(iv)$$

From equations ***(iii)***, and ***(iv)***, we find that

r_1 = 128.25 mm and r_2 = 99.75 mm **Ans.**

4. Dimensions of the spring

Let D = Mean diameter of the spring, and

d = Diameter of the spring wire.

We know that the axial force on the friction faces,

$$W = 2\pi\ R.b.p = 2\pi \times 114 \times 28.5 \times 0.07 = 1429.2 \text{ N}$$

In order to allow for adjustment and for maximum engine torque, the spring is designed for an overload of 25%.

$\therefore$ Total load on the springs

$$= 1.25\ W = 1.25 \times 1429.2 = 1786.5 \text{ N}$$

Since there are 6 springs, therefore maximum load on each spring,

$$W_s = 1786.5 / 6 = 297.75 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We also know that maximum shear stress induced in the wire (τ),

$$420 = K \times \frac{8\ W_s\ C}{\pi\ d^2} = 1.2525 \times \frac{8 \times 297.75 \times 6}{\pi\ d^2} = \frac{5697}{d^2}$$

$\therefore$ $d^2 = 5697 / 420 = 13.56$ or $d = 3.68$ mm

We shall take a standard wire of size *SWG* 8 having diameter (d) = 4.064 mm **Ans.**

and mean diameter of the spring,

$$D = C.d = 6 \times 4.064 = 24.384 \text{ say } 24.4 \text{ mm} \quad \textbf{Ans.}$$

Let us assume that the spring has 4 active turns (*i.e.* $n = 4$). Therefore compression of the spring,

$$\delta = \frac{8\,W_s.C^3.n}{G.d} = \frac{8 \times 297.75 \times 6^3 \times 4}{84 \times 10^3 \times 4.064} = 6.03 \text{ mm}$$

... (Taking $G = 84 \times 10^3$ N/mm^2)

Assuming squared and ground ends, total number of turns,

$$n' = n + 2 = 4 + 2 = 6$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$

$$= 6 \times 4.064 + 6.03 + 0.15 \times 6.03 = 31.32 \text{ mm} \quad \textbf{Ans.}$$

and pitch of the coils $= \dfrac{L_F}{n' - 1} = \dfrac{31.32}{6 - 1} = 6.264$ mm **Ans.**

Example 24.6. *Design a single plate automobile clutch to transmit a maximum torque of 250 N-m at 2000 r.p.m. The outside diameter of the clutch is 250 mm and the clutch is engaged at 55 km/h. Find : 1. the number of revolutions of the clutch slip during engagement; and 2. heat to be dissipated by the clutch for each engagement.*

The following additional data is available:

Engine torque during engagement = 100 N-m; Mass of the automobile = 1500 kg; Diameter of the automobile wheel = 0.7 m; Moment of inertia of combined engine rotating parts, flywheel and input side of the clutch = 1 kg-m^2; Gear reduction ratio at differential = 5; Torque at rear wheels available for accelerating automobile = 175 N-m; Coefficient of friction for the clutch material = 0.3; Permissible pressure = 0.13 N/mm^2.

Solution. Given : T = 250 N-m = 250 × 10^3 N-mm ; N = 2000 r.p.m. ; d_1 = 250 mm or r_1 = 125 mm ; V = 55 km/h = 15.3 m/s ; T_e = 100 N-m ; m = 1500 kg ; D_w = 0.7 m or R_w = 0.35 m ; I = 1 kg-m^2 ; T_a = 175 N-m ; Gear ratio = 5 ; μ = 0.3 ; p = 0.13 N/mm^2

1. Number of revolutions of the clutch slip during engagement

First of all, let us find the inside radius of the clutch (r_2). We know that, for uniform wear, mean radius of the clutch,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + r_2}{2} = 62.5 + 0.5\,r_2$$

and axial force on the clutch,

$$W = p.\pi\,[(r_1)^2 - (r_2)^2] = 0.13 \times \pi\,[(125)^2 - (r_2)^2]$$

We know that the torque transmitted (T),

$$250 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.13\,\pi\,[(125)^2 - (r_2)^2]\,[62.5 + 0.5\,r_2]$$

$$= 0.245\,[\,976.56 \times 10^3 + 7812.5\,r_2 - 62.5\,(r_2)^2 - 0.5\,(r_2)^3]$$

Solving by hit and trial, we find that

$$r_2 = 70 \text{ mm}$$

We know that angular velocity of the engine,

$$\omega_e = 2\pi N / 60 = 2\pi \times 2000 / 60 = 210 \text{ rad / s}$$

and angular velocity of the wheel,

$$\omega_W = \frac{\text{Velocity of wheel}}{\text{Radius of wheel}} = \frac{V}{R_w} = \frac{15.3}{0.35} = 43.7 \text{ rad/s}$$

Since the gear ratio is 5, therefore angular velocity of the clutch follower shaft,

$$\omega_0 = \omega_W \times 5 = 43.7 \times 5 = 218.5 \text{ rad / s}$$

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We know that angular acceleration of the engine during the clutch slip period of the clutch,

$$\alpha_e = \frac{T_e - T}{I} = \frac{100 - 250}{1} = -150 \text{ rad/s}^2$$

Let a = Linear acceleration of the automobile.

We know that accelerating force on the automobile,

$$F_a = \frac{T_a}{R} = \frac{175}{0.35} = 500 \text{ N}$$

We also know that accelerating force (F_a),

$$500 = m.a = 1500 \times a \quad \text{or} \quad a = 500 / 1500 = 0.33 \text{ m/s}^2$$

∴ Angular acceleration of the clutch output,

$$\alpha_0 = \frac{\text{Acceleration} \times \text{Gear ratio}}{\text{Radius of wheel}} = \frac{0.33 \times 5}{0.35} = 4.7 \text{ rad/s}^2$$

We know that clutch slip period,

$$\Delta t = \frac{\omega_0 - \omega_e}{\alpha_0 - \alpha_e} = \frac{218.5 - 210}{4.7 - (-150)} = 0.055 \text{ s}$$

Angle through which the input side of the clutch rotates during engagement time (Δt) is

$$\theta_e = \omega_e \times \Delta t + \frac{1}{2}\alpha_e (\Delta t)^2$$

$$= 210 \times 0.055 + \frac{1}{2}(-150)(0.055)^2 = 11.32 \text{ rad}$$

and angle through which the output side of the clutch rotates during engagement time (Δt) is

$$\theta_0 = \omega_0 \times \Delta t + \frac{1}{2}\alpha_0 (\Delta t)^2$$

$$= 218.5 \times 0.055 + \frac{1}{2} \times 4.7 (0.055)^2 = 12 \text{ rad}$$

∴ Angle of clutch slip,

$$\theta = \theta_0 - \theta_e = 12 - 11.32 = 0.68 \text{ rad}$$

We know that number of revolutions of the clutch slip during engagement

$$= \frac{\theta}{2\pi} = \frac{0.68}{2\pi} = 0.11 \text{ revolutions}$$ **Ans.**

Heat to be dissipated by the clutch for each engagement

We know that heat to be dissipated by the clutch for each engagement

$$= T.\theta = 250 \times 0.68 = 170 \text{ J}$$ **Ans.**

Example 24.7. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

A twin-disk clutch

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

We know that for uniform wear, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

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and axial force required to engage the clutch,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 9.525\,(125 - 75) = 2993 \text{ N}$$

Mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that the torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2993 \times 0.1 = 359 \text{ N-m}$$

$\therefore$ Power transmitted, $$P = \frac{T \times 2\,\pi\,N}{60} = \frac{359 \times 2\,\pi \times 500}{60} = 18\,800 \text{ W} = 18.8 \text{ kW } \textbf{Ans.}$$

Example 24.8. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The inside diameter of the contact surface is 120 mm. The maximum pressure between the surface is limited to 0.1 N/mm². Design the clutch for transmitting 25 kW at 1575 r.p.m. Assume uniform wear condition and coefficient of friction as 0.3.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_2 = 120$ mm or $r_2 = 60$ mm ; $p_{max} = 0.1$ N/mm² ; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. ; $\mu = 0.3$

Let r_1 = Outside radius of the contact surface.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 1575} = 151.6 \text{ N-m} = 151\,600 \text{ N-mm}$$

For uniform wear, we know that $p.r = C$. Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 0.1 \times 60 = 6 \text{ N/mm}$$

We know that the axial force on each friction surface,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 6(r_1 - 60) = 37.7\,(r_1 - 60) \qquad \text{...}(i)$$

For uniform wear, mean radius of the contact surface,

$$R = \frac{r_1 + r_2}{2} = \frac{r_1 + 60}{2} = 0.5\,r_1 + 30$$

We know that number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$\therefore$ Torque transmitted (T),

$$151\,600 = n.\mu.W.R = 4 \times 0.3 \times 37.7\,(r_1 - 60)\,(0.5\,r_1 + 30)$$

... [Substituting the value of W from equation (*i*)]

$$= 22.62\,(r_1)^2 - 81\,432$$

$$\therefore \quad (r_1)^2 = \frac{151\,600 + 81\,432}{22.62} = 10\,302$$

or $r_1 = 101.5$ mm **Ans.**

Example 24.9. *A multiple disc clutch, steel on bronze, is to transmit 4.5 kW at 750 r.p.m. The inner radius of the contact is 40 mm and outer radius of the contact is 70 mm. The clutch operates in oil with an expected coefficient of 0.1. The average allowable pressure is 0.35 N/mm². Find : 1. the total number of steel and bronze discs; 2. the actual axial force required; 3. the actual average pressure; and 4. the actual maximum pressure.*

Solution. Given : $P = 4.5$ kW $= 4500$ W ; $N = 750$ r.p.m. ; $r_2 = 40$ mm ; $r_1 = 70$ mm ; $\mu = 0.1$; $p_{av} = 0.35$ N/mm²

1. *Total number of steel and bronze discs*

Let n = Number of pairs of contact surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{4500 \times 60}{2\,\pi \times 750} = 57.3 \text{ N-m} = 57\,300 \text{ N-mm}$$

For uniform wear, mean radius of the contact surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{70 + 40}{2} = 55 \text{ mm}$$

and average axial force required,

$$W = p_{av} \times \pi\,[(r_1)^2 - (r_2)^2] = 0.35 \times \pi\,[(70)^2 - (40)^2] = 3630 \text{ N}$$

We also know that the torque transmitted (T),

$$57\,300 = n.\mu.W.R = n \times 0.1 \times 3630 \times 55 = 19\,965\, n$$

$$\therefore \quad n = 57\,300 / 19\,965 = 2.87$$

Since the number of pairs of contact surfaces must be even, therefore we shall use 4 pairs of contact surfaces with 3 steel discs and 2 bronze discs (because the number of pairs of contact surfaces is one less than the total number of discs). **Ans.**

2. *Actual axial force required*

Let W' = Actual axial force required.

Since the actual number of pairs of contact surfaces is 4, therefore actual torque developed by the clutch for one pair of contact surface,

$$T' = \frac{T}{n} = \frac{57\,300}{4} = 14\,325 \text{ N-mm}$$

We know that torque developed for one pair of contact surface (T'),

$$14\,325 = \mu.W'.R = 0.1 \times W' \times 55 = 5.5\,W'$$

$$\therefore \quad W' = 14\,325 / 5.5 = 2604.5 \text{ N}$$ **Ans.**

3. *Actual average pressure*

We know that the actual average pressure,

$$p'_{av} = \frac{W'}{\pi[(r_1)^2 - (r_2)^2]} = \frac{2604.5}{\pi\,[(70)^2 - (40)^2]} = 0.25 \text{ N/mm}^2$$ **Ans.**

4. *Actual maximum pressure*

Let p_{max} = Actual maximum pressure.

For uniform wear, $p.r = C$. Since the intensity of pressure is maximum at the inner radius, therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 40\,p_{max} \text{ N/mm}$$

We know that the actual axial force (W'),

$$2604.5 = 2\pi C\,(r_1 - r_2) = 2\pi \times 40\,p_{max}\,(70 - 40) = 7541\,p_{max}$$

$$\therefore \quad p_{max} = 2604.5 / 7541 = 0.345 \text{ N/mm}^2$$ **Ans.**

Example 24.10. *A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3, find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m.*

If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m.

Total spring load

Let W = Total spring load.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 1575} = 151.5 \text{ N-m}$$

$$= 151.5 \times 10^3 \text{ N-mm}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm}$$

and torque transmitted (T),

$$151.5 \times 10^3 = n.\mu.W.R = 4 \times 0.3 \times W \times 93.3 = 112\,W$$

$\therefore$ $W = 151.5 \times 10^3 / 112 = 1353$ N **Ans.**

Maximum power transmitted

Given : No. of springs = 6

$\therefore$ Contact surfaces of the spring = 8

Wear on each contact surface = 1.25 mm

$\therefore$ Total wear = 8 × 1.25 = 10 mm = 0.01 m

Stiffness of each spring = 13 kN/m = 13×10^3 N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

= $0.01 \times 13 \times 10^3 \times 6 = 780$ N

and new axial load, $W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

and torque transmitted, $T = n\,.\,\mu\,W\,.\,R = 4 \times 0.3 \times 573 \times 0.09 = 62$ N-m

$\therefore$ Power transmitted,

$$P = \frac{T \times 2\pi\,N}{60} = \frac{62 \times 2\pi \times 1575}{60} = 10\,227 \text{ W} = 10.227 \text{ kW}$$ **Ans.**

24.11 Cone Clutch

A cone clutch, as shown in Fig. 24.6, was extensively used in automobiles, but now-a-days it has been replaced completely by the disc clutch. It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring

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holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

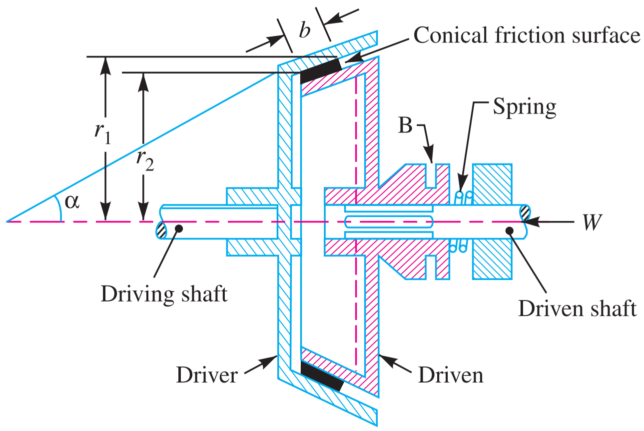

Fig. 24.6. Cone clutch.

24.12 Design of a Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Fig. 24.7. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.

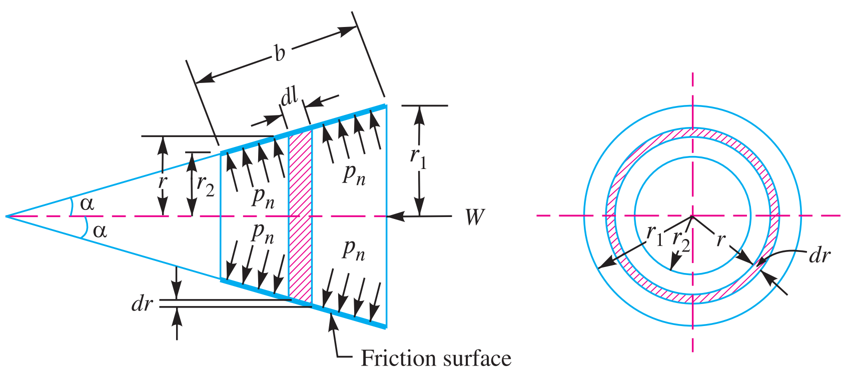

Fig. 24.7. Friction surfaces as a frustrum of a cone.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between the contact surfaces),

r_1 = Outer radius of friction surface,

r_2 = Inner radius of friction surface,

R = Mean radius of friction surface = $\dfrac{r_1 + r_2}{2}$,

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

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Consider a small ring of radius r and thickness dr as shown in Fig. 24.7. Let dl is the length of ring of the friction surface, such that,

$$dl = dr \operatorname{cosec} \alpha$$

$\therefore$ Area of ring $= 2\pi\, r.\, dl = 2\pi\, r.dr \operatorname{cosec} \alpha$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

1. *Considering uniform pressure*

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \ (i.e. \text{ in the direction of } W)$$
$$= \delta W_n \times \sin \alpha = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha \times \sin \alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\ p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$
$$= \pi\, p_n\, [(r_1)^2 - (r_2)^2]$$

and
$$p_n = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]} \qquad ...(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha \times r$$
$$= 2\pi\, \mu.p_n \operatorname{cosec} \alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n. \operatorname{cosec} \alpha.r^2\ dr = 2\pi\ \mu.p_n\ \operatorname{cosec} \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$
$$= 2\pi\ \mu.p_n \operatorname{cosec} \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

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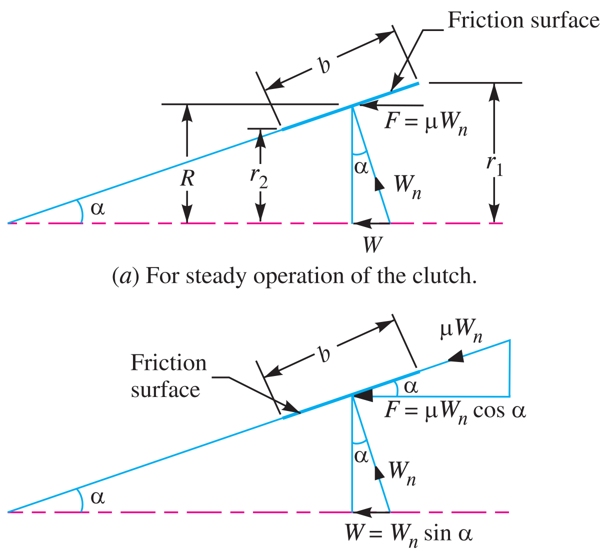
Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} \times \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\ \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \ldots(ii)$$



(*a*) For steady operation of the clutch.

(*b*) During engagement of the clutch.

Fig. 24.8. Forces on a friction surface.

2. *Considering uniform wear*

In Fig. 24.7, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \qquad p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C / r$$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\ \alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \times 2\pi\, r.dr\ \text{cosec}\ \alpha \times \sin\alpha$$

$$= 2\pi \times p_r.r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \qquad \ldots\left(\because p_r = \frac{C}{r}\right)$$

∴ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\ (r_1 - r_2)} \qquad \ldots(iii)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\ r.dr\ \text{cosec}\ \alpha$$

Top

A mammoth caterpillar dump truck for use in quarries and open-cast mines.

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r = 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr = 2\pi\mu.C \operatorname{cosec} \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu.C \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation **(*iii*)**, we have

$$T = 2\pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)} \times \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W \operatorname{cosec} \alpha \left[\frac{r_1 + r_2}{2}\right] = \mu\, WR \operatorname{cosec} \alpha \qquad \text{...(iv)}$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface.

Since the normal force acting on the friction surface, $W_n = W \operatorname{cosec} \alpha$, therefore the equation **(*iv*)** may be written as

$$T = \mu\, W_n\, R \qquad \text{...(v)}$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 24.8 (*a*) and (*b*) respectively.

From Fig. 24.8 (*a*), we find that

$$r_1 - r_2 = b \sin\alpha \qquad \text{and} \qquad R = \frac{r_1 + r_2}{2} \qquad \text{or} \qquad r_1 + r_2 = 2R$$

∴ From equation **(*i*)**, normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi R.b \sin\alpha}$$

or $\quad W = p_n \times 2\pi R.b \sin\alpha = W_n \sin\alpha$

where $\quad W_n$ = Normal load acting on the friction surface $= p_n \times 2\pi R.b$

Now the equation **(*iv*)** may be written as

$$T = \mu\,(p_n \times 2\pi R.\,b \sin\alpha)\,R \operatorname{cosec}\alpha = 2\pi\,\mu.p_n\,R^2.b$$

The following points may be noted for a cone clutch :

1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

2. If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. 24.8 (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu.W_n \cos\alpha$) acts on the clutch which resists the engagement, and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu.W_n \cos\alpha = W_n.\sin\alpha + \mu\,W_n \cos\alpha$$
$$= W_n(\sin\alpha + \mu\cos\alpha)$$

It has been found experimentally that the term ($\mu\,W_n.\cos\alpha$) is only 25 percent effective.

∴ $$W_e = W_n \sin\alpha + 0.25\,\mu\,W_n \cos\alpha = W_n(\sin\alpha + 0.25\,\mu\cos\alpha)$$

3. Under steady operation of the clutch, a decrease in the semi-cone angle (α) increases the torque produced by the clutch (T) and reduces the axial force (W). During engaging period, the axial force required for engaging the clutch (W_e) increases under the influence of friction as the angle α decreases. The value of α can not be decreased much because smaller semi-cone angle (α) requires larger axial force for its disengagement.

If the clutch is to be designed for free disengagement, the value of tan α must be greater than μ. In case the value of tan α is less than μ, the clutch will not disengage itself and axial force required to disengage the clutch is given by

$$W_d = W_n(\mu\cos\alpha - \sin\alpha)$$

Example 24.11. *The contact surfaces in a cone clutch have an effective diameter of 80 mm. The semi-angle of the cone is 15° and coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch, if the axial force applied is 200 N. The clutch is employed to connect an electric motor, running uniformly at 900 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 14 kg and its radius of gyration is 160 mm. Calculate the time required for the flywheel to attain full-speed and also the energy lost in slipping of the clutch.*

Solution. Given : D = 80 mm or R = 40 mm ; α = 15° ; μ = 0.3 ; W = 200 N ; N = 900 r.p.m. or ω = 2π × 900/60 = 94.26 rad/s ; m = 14 kg ; k = 160 mm = 0.16 m

Torque required to produce slipping of the clutch

We know that the torque required to produce slipping of the clutch,

$$T = \mu\,WR \operatorname{cosec}\alpha = 0.3 \times 200 \times 40 \operatorname{cosec} 15° = 9273 \text{ N-mm}$$
$$= 9.273 \text{ N-m}$$ **Ans.**

Time required for the flywheel to attain full-speed

Let t = Time required for the flywheel to attain full speed from the stationary position, and

α = Angular acceleration of the flywheel.

We know that mass moment of inertia of the flywheel,

$$I = m.k^2 = 14\,(0.16)^2 = 0.3584 \text{ kg-m}^2$$

We also know that the torque (T),

$$9.273 = I \times \alpha = 0.3584\,\alpha$$

$$\therefore \quad \alpha = 9.273 / 0.3584 = 25.87 \text{ rad / s}^2$$

and angular speed (ω),

$$94.26 = \omega_0 + \alpha.t = 0 + 25.87 \times t = 25.87\,t \qquad \dots (\because\ \omega_0 = 0)$$

$$\therefore \quad t = 94.26 / 25.87 = 3.64 \text{ s}$$ **Ans.**

Energy lost in slipping of the clutch

We know that angular displacement,

$$\theta = \text{Average angular speed} \times \text{time} = \frac{\omega_0 + \omega}{2} \times t$$

$$= \frac{0 + 94.26}{2} \times 3.64 = 171.6 \text{ rad}$$

$\therefore$ Energy lost in slipping of the clutch,

$$= T.\theta = 9.273 \times 171.6 = 1591 \text{ N-m}$$ **Ans.**

Example 24.12. *An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm^2. Determine : 1. the face width required, and 2. the axial spring force necessary to engage the clutch.*

Solution. Given : P = 45 kW = 45 × 10^3 W ; N = 1000 r.p.m. ; α = 12.5° ; D = 500 mm or R = 250 mm ; μ = 0.2 ; p_n = 0.1 N/mm^2

1. Face width

Let b = Face width of the clutch in mm.

We know that torque developed by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{45 \times 10^3 \times 60}{2\,\pi \times 1000} = 430 \text{ N-m} = 430 \times 10^3 \text{ N-mm}$$

We also know that torque developed by the clutch (T),

$$430 \times 10^3 = 2\pi.\,\mu.\,p_n.\,R^2.b = 2\pi \times 0.2 \times 0.1\,(250)^2 b = 7855\,b$$

$$\therefore \quad b = 430 \times 10^3 / 7855 = 54.7 \text{ say } 55 \text{ mm}$$ **Ans.**

2. Axial spring force necessary to engage the clutch

We know that the normal force acting on the contact surfaces,

$$W_n = p_n \times 2\pi R.b = 0.1 \times 2\pi \times 250 \times 55 = 8640 \text{ N}$$

$\therefore$ Axial spring force necessary to engage the clutch,

$$W_e = W_n\,(\sin\alpha + 0.25\,\mu\cos\alpha)$$

$$= 8640\,(\sin 12.5° + 0.25 \times 0.2 \cos 12.5°) = 2290 \text{ N}$$ **Ans.**

Example 24.13. *Determine the principal dimensions of a cone clutch faced with leather to transmit 30 kW at 750 r.p.m. from an electric motor to an air compressor. Sketch a sectional front view of the clutch and provide the main dimensions on the sketch.*

Assume : semi-angle of the cone = $12\frac{1}{2}°$; μ = 0.2 ; mean diameter of cone = 6 to 10 d where d is the diameter of shaft; allowable normal pressure for leather and cast iron = 0.075 to 0.1 N/mm^2; load factor = 1.75 and mean diameter to face width ratio = 6.

Solution. Given : P = 30 kW = 30 × 10^3 W ; N = 750 r.p.m. ; $\alpha = 12\frac{1}{2}°$; μ = 0.2 ; D = 6 to 10 d ; p_n = 0.075 to 0.1 N/mm^2 ; K_L = 1.75 ; D / b = 6

First of all, let us find the diameter of shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi\ N} \times K_L = \frac{30 \times 10^3 \times 60}{2\ \pi \times 750} \times 1.75 = 668.4 \text{ N-m}$$

$$= 668.4 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$668.4 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3 \quad \text{... (Taking } \tau = 42 \text{ N/mm}^2)$$

$$\therefore \quad d^3 = 668.4 \times 10^3 / 8.25 = 81 \times 10^3 \quad \text{or} \quad d = 43.3 \text{ say } 50 \text{ mm} \quad \textbf{Ans.}$$

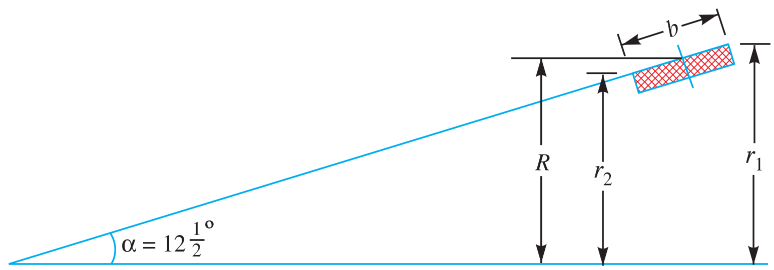


Fig. 24.9

Now let us find the principal dimensions of a cone clutch.

Let D = Mean diameter of the clutch,

R = Mean radius of the clutch, and

b = Face width of the clutch.

Since the allowable normal pressure (p_n) for leather and cast iron is 0.075 to 0.1 N/mm^2, therefore let us take p_n = 0.1 N/mm^2.

We know that the torque developed by the clutch (T),

$$668.4 \times 10^3 = 2\ \pi\ \mu .\ p_n .\ R^2 . b = 2\pi \times 0.2 \times 0.1 \times R^2 \times \frac{R}{3} = 0.042\ R^3$$

$$\text{... } (\because D / b = 6 \text{ or } 2R / b = 6 \text{ or } R / b = 3)$$

$$\therefore \quad R^3 = 668.4 \times 10^3 / 0.042 = 15.9 \times 10^6 \quad \text{or} \quad R = 250 \text{ mm}$$

and $D = 2R = 2 \times 250 = 500$ mm **Ans.**

Since this calculated value of the mean diameter of the clutch (D) is equal to 10 d and the given value of D is 6 to 10d, therefore the calculated value of D is safe.

We know that face width of the clutch,

$$b = D / 6 = 500 / 6 = 83.3 \text{ mm} \quad \textbf{Ans.}$$

From Fig. 24.9, we find that outer radius of the clutch,

$$r_1 = R + \frac{b}{2} \sin \alpha = 250 + \frac{83.3}{2} \sin 12\frac{1}{2}° = 259 \text{ mm} \quad \textbf{Ans.}$$

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and inner radius of the clutch,

$$r_2 = R - \frac{b}{2} \sin \alpha = 250 - \frac{83.3}{2} \sin 12\frac{1}{2}° = 241 \text{ mm} \quad \textbf{Ans.}$$

24.13 Centrifugal Clutch

Centrifugal clutch with three discs and four steel float plates.

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 24.10. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

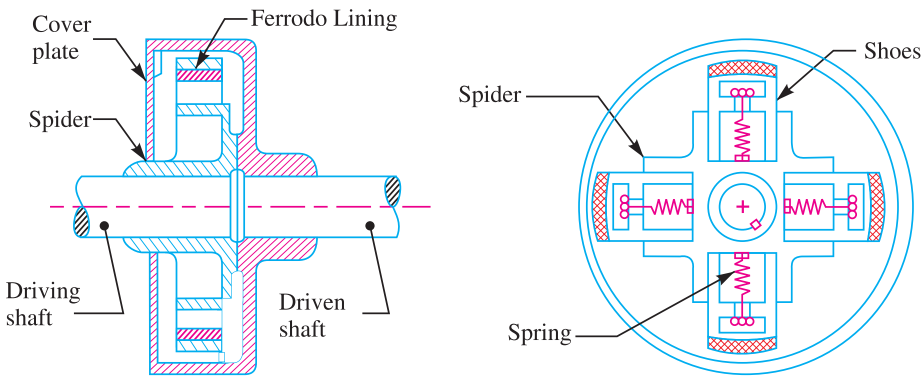

Fig. 24.10. Centrifugal clutch.

24.14 Design of a Centrifugal Clutch

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for the design of a centrifugal clutch.

1. Mass of the shoes

Consider one shoe of a centrifugal clutch as shown in Fig. 24.11.

Let m = Mass of each shoe,

n = Number of shoes,

Top

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad / s $= 2\pi N / 60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

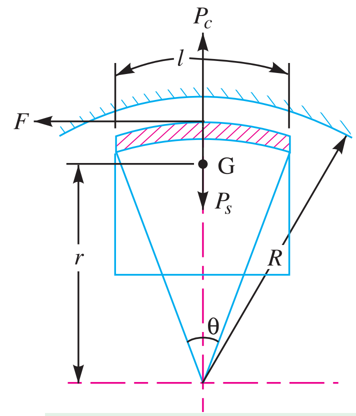

Fig. 24.11. Forces on a shoe of a centrifugal clucth.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\; r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\, m.\omega^2\cdot r$$

∴ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\, m.\omega^2.r = \frac{7}{16}\, m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\, R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\, R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ = Angle subtended by the shoes at the centre of the spider in radians, and

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm^2.

We know that $\theta = \dfrac{l}{R}$ or $l = \theta.R = \dfrac{\pi}{3} R$ ...(Assuming $\theta = 60° = \pi / 3$ rad)

* The radial clearance between the shoe and the rim is about 1.5 mm. Since this clearance is small as compared to r, therefore it is neglected for design purposes. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$$r_1 = r + c, \text{ where } c \text{ is the radial clearance,}$$

Then $P_c = m.\omega^2\, r_1$ and $P_s = m\,(\omega_1)^2\, r_1$

∴ Area of contact of the shoe

$$= l\,.\,b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

3. *Dimensions of the spring*

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.

Example 24.14. *A centrifugal clutch is to be designed to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. mass of the shoes, and 2. size of the shoes.*

Solution. Given : P = 15 kW = 15×10^3 W ; N = 900 r.p.m. ; n = 4 ; R = 150 mm = 0.15 m ; μ = 0.25

1. *Mass of the shoes*

Let m = Mass of the shoes.

We know that the angular running speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Since the speed at which the engagement begins is 3/4 th of the running speed, therefore angular speed at which engagement begins is

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Assuming that the centre of gravity of the shoe lies at a distance of 120 mm (30 mm less than R) from the centre of the spider, *i.e.*

$$r = 120 \text{ mm} = 0.12 \text{ m}$$

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2\, 0.12 = 1066\, m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed, ω_1,

$$P_s = m(\omega_1)^2\, r = m\,(70.7)^2\, 0.12 = 600\, m \text{ N}$$

We know that the torque transmitted at the running speed,

$$T = \frac{P \times 60}{2\pi\, N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159 \text{ N-m}$$

We also know that the torque transmitted (T),

$$159 = \mu\,(P_c - P_s)\, R \times n = 0.25\,(1066\, m - 600\, m)\, 0.15 \times 4 = 70\, m$$

$$\therefore\quad m = 159/70 = 2.27 \text{ kg}$$ **Ans.**

2. *Size of the shoes*

Let l = Contact length of shoes in mm, and

b = Width of the shoes in mm.

Assuming that the arc of contact of the shoes subtend an angle of $\theta = 60°$ or $\pi / 3$ radians, at the centre of the spider, therefore

$$l = \theta.R = \frac{\pi}{3} \times 150 = 157 \text{ mm}$$

Area of contact of the shoes

$$A = l.b = 157\, b \text{ mm}^2$$

Assuming that the intensity of pressure (p) exerted on the shoes is 0.1 N/mm^2, therefore force with which the shoe presses against the rim

$$= A.p = 157b \times 0.1 = 15.7\, b \text{ N} \quad ...(i)$$

We also know that the force with which the shoe presses against the rim

$$= P_c - P_s = 1066\, m - 600\, m = 466\, m$$

$$= 466 \times 2.27 = 1058 \text{ N} \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, we find that

b = 1058 / 15.7 = 67.4 mm **Ans.**

Special trailers are made to carry very long loads. The longest load ever moved was gas storage vessel, 83.8 m long.

EXERCISES

1. A single disc clutch with both sides of the disc effective is used to transmit 10 kW power at 900 r.p.m. The axial pressure is limited to 0.085 N/mm^2. If the external diameter of the friction lining is 1.25 times the internal diameter, find the required dimensions of the friction lining and the axial force exerted by the springs. Assume uniform wear conditions. The coefficient of friction may be taken as 0.3.

[Ans. 132.5 mm ; 106 mm ; 1500 N]

2. A single plate clutch with both sides of the plate effective is required to transmit 25 kW at 1600 r.p.m. The outer diameter of the plate is limited to 300 mm and the intensity of pressure between the plates not to exceed 0.07 N/mm^2. Assuming uniform wear and coefficient of friction 0.3, find the inner diameter of the plates and the axial force necessary to engage the clutch.

[Ans. 90 mm ; 2375 N]

3. Give a complete design analysis of a single plate clutch, with both sides effective, of a vehicle to transmit 22 kW at a speed of 2800 r.p.m. allowing for 25% overload. The pressure intensity is not to exceed 0.08 N/mm^2 and the surface speed at the mean radius is not to exceed 2000 m/min. Take coefficient of friction for the surfaces as 0.35 and the outside diameter of the surfaces is to be 1.5 times the inside diameter. The axial thrust is to be provided by 6 springs of about 24 mm coil diameter. For spring material, the safe shear stress is to be limited to 420 MPa and the modulus of rigidity may be taken as 80 kN/mm^2. **[Ans. 120 mm ; 80 mm ; 3.658 mm]**

4. A multiple disc clutch has three discs on the driving shaft and two on the driven shaft, providing four pairs of contact surfaces. The outer diameter of the contact surfaces is 250 mm and the inner diameter is 150 mm. Determine the maximum axial intensity of pressure between the discs for transmitting 18.75 kW at 500 r.p.m. Assume uniform wear and coefficient of friction as 0.3.

5. A multiple disc clutch employs 3 steel and 2 bronze discs having outer diameter 300 mm and inner diameter 200 mm. For a coefficient of friction of 0.22, find the axial pressure and the power transmitted at 750 r.p.m., if the normal unit pressure is 0.13 N/mm^2.

   Also find the axial pressure of the unit normal pressure, if this clutch transmits 22 kW at 1500 r.p.m. **[Ans. 5105 N ; 44.11 kW ; 0.0324 N/mm^2]**

6. A multiple disc clutch has radial width of the friction material as 1/5th of the maximum radius. The coefficient of friction is 0.25. Find the total number of discs required to transmit 60 kW at 3000 r.p.m. The maximum diameter of the clutch is 250 mm and the axial force is limited to 600 N. Also find the mean unit pressure on each contact surface. **[Ans. 13 ; 0.034 N/mm^2]**

7. An engine developing 22 kW at 1000 r.p.m. is fitted with a cone clutch having mean diameter of 300 mm. The cone has a face angle of 12°. If the normal pressure on the clutch face is not to exceed 0.07 N/mm^2 and the coefficient of friction is 0.2, determine :

   (*a*) the face width of the clutch, and

   (*b*) the axial spring force necessary to engage the clutch.

[Ans. 106 mm ; 1796 N]

8. A cone clutch is to be designed to transmit 7.5 kW at 900 r.p.m. The cone has a face angle of 12°. The width of the face is half of the mean radius and the normal pressure between the contact faces is not to exceed 0.09 N/mm^2. Assuming uniform wear and the coefficient of friction between the contact faces as 0.2, find the main dimensions of the clutch and the axial force required to engage the clutch.

[Ans. R = 112.4 mm ; b = 56.2 mm ; r_1 = 118.2 mm ; r_2 = 106.6 mm ; W_e = 917 N]

9. A soft cone clutch has a cone pitch angle of 10°, mean diameter of 300 mm and a face width of 100 mm. If the coefficient of friction is 0.2 and has an average pressure of 0.07 N/mm^2 for a speed of 500 r.p.m., find : (*a*) the force required to engage the clutch; and (*b*) the power that can be transmitted. Assume uniform wear. **[Ans. 1470 N ; 10.4 kW]**

10. A cone clutch is mounted on a shaft which transmits power at 225 r.p.m. The small diameter of the cone is 230 mm, the cone face is 50 mm and the cone face makes an angle of 15° with the horizontal. Determine the axial force necessary to engage the clutch to transmit 4.5 kW if the coefficient of friction of the contact surfaces is 0.25. What is the maximum pressure on the contact surfaces assuming uniform wear? **[Ans. 2414 N ; 0.216 N/mm^2]**

11. A soft surface cone clutch transmits a torque of 200 N-m at 1250 r.p.m. The larger diameter of the clutch is 350 mm. The cone pitch angle is 7.5° and the face width is 65 mm. If the coefficient of friction is 0.2, find :

1. the axial force required to transmit the torque;
2. the axial force required to engage the clutch;
3. the average normal pressure on the contact surfaces when the maximum torque is being transmitted; and
4. the maximum normal pressure assuming uniform wear.

[Ans. 764 N ; 1057 N ; 0.084 N/mm^2 ; 0.086 N/mm^2]

12. A centrifugal friction clutch has a driving member consisting of a spider carrying four shoes which are kept from contact with the clutch case by means of flat springs until increase of centrifugal force overcomes the resistance of the springs and the power is transmitted by the friction between the shoes and the case.

Determine the necessary mass and size of each shoe if 22.5 kW is to be transmitted at 750 r.p.m. with engagement beginning at 75% of the running speed. The inside diameter of the drum is 300 mm and the radial distance of the centre of gravity of each shoe from the shaft axis is 125 mm. Assume μ = 0.25. **[Ans. 5.66 kg ; l = 157.1 mm ; b = 120 mm]**

Clutches, brakes, steering and transmission need to be carefully designed to ensure the efficiency and safety of an automobile

QUESTIONS

1. What is a clutch? Discuss the various types of clutches giving at least one practical application for each.

2. Why a positive clutch is used? Describe, with the help of a neat sketch, the working of a jaw or claw clutch.

3. Name the different types of clutches. Describe with the help of neat sketches the working principles of two different types of friction clutches.

4. What are the materials used for lining of friction surfaces?

5. Why it is necessary to dissipate the heat generated when clutches operate?

6. Establish a formula for the frictional torque transmitted by a cone clutch.

7. Describe, with the help of a neat sketch, a centrifugal clutch and deduce an expression for the total frictional torque transmitted. How the shoes and springs are designed for such a clutch?

OBJECTIVE TYPE QUESTIONS

1. A jaw clutch is essentially a

(*a*) positive action clutch (*b*) cone clutch

(*c*) friction clutch (*d*) disc clutch

2. The material used for lining of friction surfaces of a clutch should have .... coefficient of friction.

(*a*) low (*b*) high

3. The torque developed by a disc clutch is given by

(*a*) $T = 0.25\ \mu.W.R$ (*b*) $T = 0.5\ \mu.W.R$

(*c*) $T = 0.75\ \mu.W.R$ (*d*) $T = \mu.W.R$

where W = Axial force with which the friction surfaces are held together ;

μ = Coefficient of friction ; and

R = Mean radius of friction surfaces.

4. In case of a multiple disc clutch, if n_1 are the number of discs on the driving shaft and n_2 are the number of the discs on the driven shaft, then the number of pairs of contact surfaces will be

(*a*) $n_1 + n_2$ (*b*) $n_1 + n_2 - 1$

(*c*) $n_1 + n_2 + 1$ (*d*) none of these

5. The cone clutches have become obsolete because of

(*a*) small cone angles (*b*) exposure to dirt and dust

(*c*) difficulty in disengaging (*d*) all of these

6. The axial force (W_e) required for engaging a cone clutch is given by

(*a*) $W_n \sin \alpha$ (*b*) $W_n (\sin \alpha + \mu \cos \alpha)$

(*c*) $W_n (\sin \alpha + 0.25\ \mu \cos \alpha)$ (*d*) none of these

where W_n = Normal force acting on the contact surfaces,

α = Face angle of the cone, and

μ = Coefficient of friction.

7. In a centrifugal clutch, the force with which the shoe presses against the driven member is the .... of the centrifugal force and the spring force.

(*a*) difference (*b*) sum

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*d*)
6. (*c*)	**7.** (*a*)			