# Chapter 6

**6-1** Eq. (2-21): $S_{ut} = 3.4H_B = 3.4(300) = 1020$ MPa

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(1020) = 510$ MPa

Table 6-2: $a = 1.58, b = -0.085$

Eq. (6-19): $k_a = aS_{ut}^b = 1.58(1020)^{-0.085} = 0.877$

Eq. (6-20): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$

Eq. (6-18): $S_e = k_a k_b S'_e = (0.877)(0.969)(510) = 433$ MPa *Ans.*

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**6-2** **(a)** Table A-20: $S_{ut}$ = 80 kpsi

Eq. (6-8): $S'_e = 0.5(80) = 40$ kpsi *Ans.*

**(b)** Table A-20: $S_{ut}$ = 90 kpsi

Eq. (6-8): $S'_e = 0.5(90) = 45$ kpsi *Ans.*

**(c)** Aluminum has no endurance limit. *Ans.*

**(d)** Eq. (6-8): $S_{ut}$ > 200 kpsi, $S'_e = 100$ kpsi *Ans.*

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**6-3** $S_{ut} = 120$ kpsi, $\sigma_{\text{rev}} = 70$ kpsi

Fig. 6-18: $f = 0.82$

Eq. (6-8): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.82(120)\right]^2}{60} = 161.4 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{60}\right) = -0.0716$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 116\,700 \text{ cycles} \quad \textit{Ans.}$$

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**6-4** $S_{ut} = 1600$ MPa, $\sigma_{\text{rev}} = 900$ MPa

Fig. 6-18: $S_{ut}$ = 1600 MPa = 232 kpsi. Off the graph, so estimate $f$ = 0.77.

Eq. (6-8): $S_{ut}$ > 1400 MPa, so $S_e$ = 700 MPa

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(1600)\right]^2}{700} = 2168.3 \text{ MPa}$$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$

Eq. (6-16): $N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400$ cycles *Ans.*

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**6-5** $S_{ut} = 230$ kpsi, $N = 150\,000$ cycles

Fig. 6-18, point is off the graph, so estimate: $f = 0.77$

Eq. (6-8): $S_{ut} > 200$ kpsi, so $S_e' = S_e = 100$ kpsi

Eq. (6-14): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.77(230)]^2}{100} = 313.6$ kpsi

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$

Eq. (6-13): $S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0$ kpsi *Ans.*

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**6-6** $S_{ut} = 1100$ MPa = 160 kpsi

Fig. 6-18: $f = 0.79$

Eq. (6-8): $S_e' = S_e = 0.5(1100) = 550$ MPa

Eq. (6-14): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.79(1100)]^2}{550} = 1373$ MPa

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.79(1100)}{550}\right) = -0.06622$

Eq. (6-13): $S_f = aN^b = 1373(150\,000)^{-0.06622} = 624$ MPa *Ans.*

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**6-7** $S_{ut} = 150$ kpsi, $S_{yt} = 135$ kpsi, $N = 500$ cycles

Fig. 6-18: $f = 0.798$

From Fig. 6-10, we note that below $10^3$ cycles on the *S-N* diagram constitutes the low-cycle region, in which Eq. (6-17) is applicable.

Eq. (6-17): $S_f = S_{ut}N^{(\log f)/3} = 150(500)^{[\log(0.798)]/3} = 122$ kpsi *Ans.*

The testing should be done at a completely reversed stress of 122 kpsi, which is below the yield strength, so it is possible. *Ans.*

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**6-8** The general equation for a line on a log $S_f$ - log N scale is $S_f = aN^b$, which is Eq. (6-13). By taking the log of both sides, we can get the equation of the line in slope-intercept form.

$$\log S_f = b\log N + \log a$$

Substitute the two known points to solve for unknowns *a* and *b*. Substituting point (1, $S_{ut}$),

$$\log S_{ut} = b\log(1) + \log a$$

From which $a = S_{ut}$. Substituting point $(10^3, f\,S_{ut})$ and $a = S_{ut}$

$$\log f\,S_{ut} = b\log 10^3 + \log S_{ut}$$

From which $b = (1/3)\log f$

$$\therefore S_f = S_{ut}N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

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**6-9** Read from graph: $(10^3, 90)$ and $(10^6, 50)$. From $S = aN^b$

$$\log S_1 = \log a + b\log N_1$$
$$\log S_2 = \log a + b\log N_2$$

From which

$$\log a = \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2 / N_1}$$
$$= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6 / 10^3}$$
$$= 2.2095$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0 \text{ kpsi}$$
$$b = \frac{\log 50/90}{3} = -0.0851$$
$$(S_f)_{ax} = 162N^{-0.0851} \qquad 10^3 \le N \le 10^6 \text{ in kpsi} \qquad \textit{Ans.}$$

Check:

$$\left[(S_f)_{ax}\right]_{10^3} = 162(10^3)^{-0.0851} = 90 \text{ kpsi}$$

$$\left[(S_f)_{ax}\right]_{10^6} = 162(10^6)^{-0.0851} = 50 \text{ kpsi}$$

The end points agree.

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**6-10** $d = 1.5$ in, $S_{ut} = 110$ kpsi

Eq. (6-8): $S'_e = 0.5(110) = 55$ kpsi

Table 6-2: $a = 2.70$, $b = -0.265$

Eq. (6-19): $k_a = aS_{ut}{}^b = 2.70(110)^{-0.265} = 0.777$

Since the loading situation is not specified, we'll assume rotating bending or torsion so Eq. (6-20) is applicable. This would be the worst case.

$$k_b = 0.879d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$$

Eq. (6-18): $S_e = k_a k_b S'_e = 0.777(0.842)(55) = 36.0$ kpsi *Ans.*

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**6-11** For AISI 4340 as-forged steel,

Eq. (6-8): $S_e = 100$ kpsi

Table 6-2: $a = 39.9$, $b = -0.995$

Eq. (6-19): $k_a = 39.9(260)^{-0.995} = 0.158$

Eq. (6-20): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi}$$

For AISI 1040:

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$

$$k_a = 39.9(113)^{-0.995} = 0.362$$

$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other modifying factors is unity

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength.

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**6-12** $D = 1$ in, $d = 0.8$ in, $T = 1800$ lbf·in, $f = 0.9$, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68$ kpsi, and $S_y = 57$ kpsi.

**(a)** Fig. A-15-15: $\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125,\ \dfrac{D}{d} = \dfrac{1}{0.8} = 1.25,\ K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-21, or from the curve-fit Eqs. (6-34) and (6-35b). We'll use the equations.

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(68) + 1.35\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.07335$$

$$q_s = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800$ lbf·in,

$$\tau_a = K_{fs}\frac{Tr}{J} = \frac{K_{fs}16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-8): $S'_e = 0.5(68) = 34$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Eq. (6-20): $k_b = 0.879(0.8)^{-0.107} = 0.900$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18) (labeling for shear): $S_{se} = 0.883(0.900)(0.59)(34) = 15.9$ kpsi

For purely reversing torsion, use Eq. (6-54) for the ultimate strength in shear.

Eq. (6-54): $S_{su} = 0.67\,S_{ut} = 0.67(68) = 45.6$ kpsi

Adjusting the fatigue strength equations for shear,

Eq. (6-14): $a = \dfrac{\left(f\,S_{su}\right)^2}{S_{se}} = \dfrac{[0.9(45.6)]^2}{15.9} = 105.9$ kpsi

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{su}}{S_{se}}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.9(45.6)}{15.9}\right) = -0.137\,27$

Eq. (6-16): $N = \left(\dfrac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\dfrac{23.3}{105.9}\right)^{\frac{1}{-0.137\,27}} = 61.7\left(10^3\right)$ cycles *Ans.*

**(b)** For an operating temperature of $750^\circ$F, the temperature modification factor, from Table 6-4 is $k_d = 0.90$.

$$S_{se} = 0.883(0.900)(0.59)(0.9)(34) = 14.3 \text{ kpsi}$$

$$a = \frac{(f\,S_{su})^2}{S_{se}} = \frac{[0.9(45.6)]^2}{14.3} = 117.8 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.3}\right) = -0.152\ 62$$

$$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.3}{117.8}\right)^{\frac{1}{-0.152\ 62}} = 40.9(10^3) \text{ cycles} \qquad \textit{Ans.}$$

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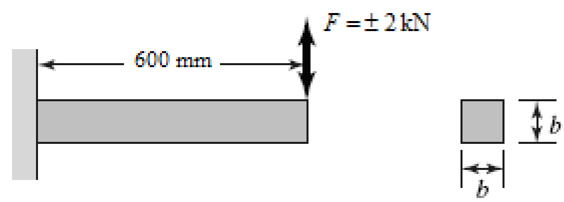
**6-13** $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)

First evaluate the fatigue strength.

$$S_e' = 0.5(770) = 385 \text{ MPa}$$
$$k_a = 57.7(770)^{-0.718} = 0.488$$




Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

Eq. (6-18): $S_e = 0.488(0.85)(385) = 160$ MPa

In kpsi, $S_{ut} = 770/6.89 = 112$ kpsi

Fig. 6-18: $f = 0.83$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{160} = 2553 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{160}\right) = -0.2005$$

Eq. (6-13): $$S_f = aN^b = 2553(10^4)^{-0.2005} = 403 \text{ MPa}$$

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary *b*.

$$n = \frac{S_f}{\sigma_a} = \frac{403(10^6)}{7200/b^3} = 1.5$$

$b = 0.0299$ m   Select $b = 30$ mm.

Since the size factor was guessed, go back and check it now.

Eq. (6-25): $d_e = 0.808(hb)^{1/2} = 0.808b = 0.808(30) = 24.24$ mm

Eq. (6-20): $k_b = \left(\frac{24.2}{7.62}\right)^{-0.107} = 0.88$

Our guess of 0.85 was slightly conservative, so we will accept the result of

$b = 30$ mm.   *Ans.*

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3}(10^{-6}) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

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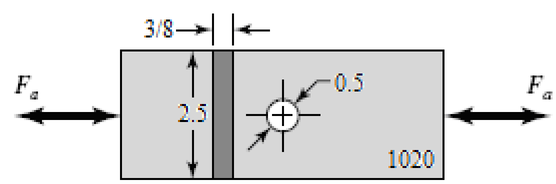
**6-14** Given: $w$ =2.5 in, $t$ = 3/8 in, $d$ = 0.5 in, $n_d$ = 2. From Table A-20, for AISI 1020 CD, $S_{ut}$ = 68 kpsi and $S_y$ = 57 kpsi.




Eq. (6-8): $S'_e = 0.5(68) = 34$ kpsi

Table 6-2: $k_a = 2.70(68)^{-0.265} = 0.88$

Eq. (6-21): $k_b$ = 1 (axial loading)

Eq. (6-26): $k_c$ = 0.85

Eq. (6-18): $S_e$ = 0.88(1)(0.85)(34) = 25.4 kpsi

Table A-15-1: $d / w = 0.5 / 2.5 = 0.2,\ K_t = 2.5$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). The relatively large radius is off the graph of Fig. 6-20, so we'll assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(68) + 1.51(10^{-5})(68)^2 - 2.67(10^{-8})(68^3) = 0.09799$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.09799}{\sqrt{0.25}}} = 0.836$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5-1) = 2.25$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5-0.5)} = 3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{25.4}{3F_a} = 2$$

$$F_a = 4.23 \text{ kips} \quad \textit{Ans.}$$

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**6-15** Given: $D = 2$ in, $d = 1.8$ in, $r = 0.1$ in, $M_{\max} = 25\,000$ lbf $\cdot$ in, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, $S_{ut} = 120$ kpsi and $S_y = 66$ kpsi.

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(120)^{-0.265} = 0.76$

Eq. (6-24): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.76)(0.92)(1)(60) = 42.0$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \therefore K_t = 2.1$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We'll use the equations.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1-1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eq. (6-36): $$\sigma_m = K_f \frac{\sigma_{\max} + \sigma_{\min}}{2} = (1.96)\frac{(43.7+0)}{2} = 42.8 \text{ kpsi}$$

$$\sigma_a = K_f \left|\frac{\sigma_{\max} - \sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7-0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-46): $$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{42.8}{42.0} + \frac{42.8}{120}$$

$$n_f = 0.73 \qquad \textit{Ans.}$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \qquad \textit{Ans.}$$

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**6-16** From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is $M$ = 2.1(200) = 420 kN·mm. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{420\,(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: $r/d$ = 3/35 = 0.086, $D/d$ = 50/35 = 1.43, $K_t$ =1.7

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We'll use the equations, with $S_{ut}$ = 470 MPa = 68.2 kpsi and $r$ = 3 mm = 0.118 in.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(68.2) + 1.51\left(10^{-5}\right)(68.2)^2 - 2.67\left(10^{-8}\right)(68.2)^3 = 0.09771$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.09771}{\sqrt{0.118}}} = 0.78$$

Eq. (6-32): $$K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$$

Eq. (6-8): $S_e' = 0.5S_{ut} = 0.5(470) = 235$ MPa

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(470)^{-0.265} = 0.88$

Eq. (6-24): $k_b = 1.24d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e' = (0.88)(0.85)(1)(235) = 176$ MPa

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{176}{1.55(99.8)} = 1.14 \quad \text{Infinite life is predicted.} \qquad \textit{Ans.}$$

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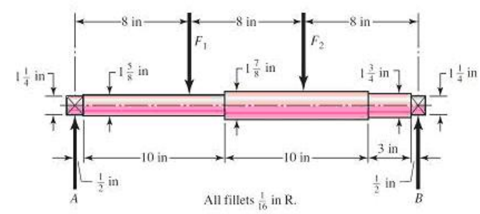
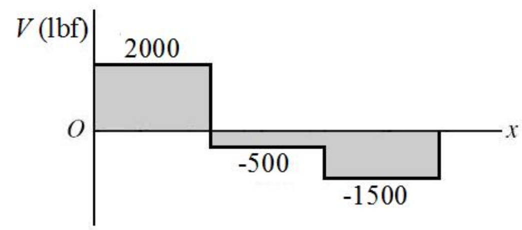
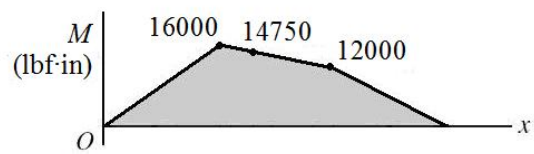
**6-17** From a free-body diagram analysis, the bearing reaction forces are found to be $R_A$ = 2000 lbf and $R_B$ = 1500 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.




$$M = 16\,000 - 500\,(2.5) = 14\,750 \text{ lbf} \cdot \text{in}$$




With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$
Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We will use the equations.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-8): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$

Eq. (6-20): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e' = (0.832)(0.835)(1)(42.5) = 29.5$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72(35.0)} = 0.49 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

Fig. 6-18: $f = 0.867$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$$

Eq. (6-16): $$N = \left(\frac{K_f \sigma_{\text{rev}}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(35.0)}{184.1}\right)^{\frac{1}{-0.1325}} = 4611 \text{ cycles}$$

$N = 4600$ cycles *Ans.*

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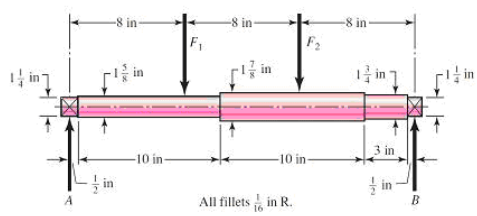
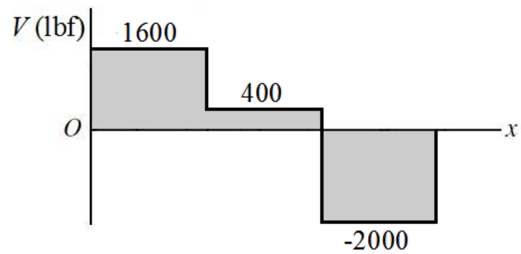
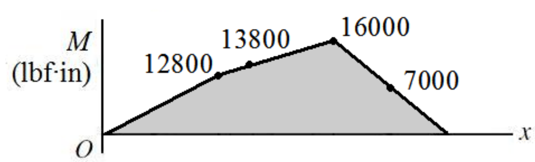
**6-18** From a free-body diagram analysis, the bearing reaction forces are found to be $R_A$ = 1600 lbf and $R_B$ = 2000 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.




$M = 12\,800 + 400\,(2.5) = 13\,800$ lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We will use the equations

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07690}{\sqrt{0.0625}}} = 0.76$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-8): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$

Eq. (6-20): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e' = (0.832)(0.835)(1)(42.5) = 29.5$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72(32.8)} = 0.52 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S*-*N* diagram to estimate the life.

Fig. 6-18: $f = 0.867$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$$

Eq. (6-16): $$N = \left(\frac{K_f \sigma_{\text{rev}}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{184.1}\right)^{\frac{1}{-0.1325}} = 7527 \text{ cycles}$$

$N = 7500$ cycles *Ans.*

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**6-19** Table A-20: $S_{ut} = 120$ kpsi, $S_y = 66$ kpsi

$N$ = (950 rev/min)(10 hr)(60 min/hr) = 570 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we'll evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are $R_1$ = 2 kips and $R_2$ = 6 kips. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2 \text{ kip}(10 \text{ in}) = 20 \text{ kip}\cdot\text{in}$$

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3} \text{ kpsi}$$

Now we'll get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, we'll estimate a value for $q$ = 0.85 from observation of Fig. 6-20, and check it later.

Fig. A-15-9: $D/d = 1.4d/d = 1.4, \quad r/d = 0.1d/d = 0.1, \quad K_t = 1.65$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now we will evaluate the fatigue strength.

$$S_e' = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.76$$

Since the diameter is not yet known, assume a typical value of $k_b$ = 0.85 and check later. All other modifiers are equal to one.

$$S_e = (0.76)(0.85)(60) = 38.8 \text{ kpsi}$$

Determine the desired fatigue strength from the *S-N* diagram.

Fig. 6-18: $f = 0.82$

Eq. (6-14): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.82(120)]^2}{38.8} = 249.6$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{38.8}\right) = -0.1347$

Eq. (6-13): $S_f = aN^b = 249.6(570\,000)^{-0.1347} = 41.9 \text{ kpsi}$

Compare strength to stress and solve for the necessary *d*.

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{41.9}{(1.55)(203.7/d^3)} = 1.6$$

$d = 2.29$ in

Since the size factor and notch sensitivity were guessed, go back and check them now.

Eq. (6-20): $k_b = 0.91d^{-0.157} = 0.91(2.29)^{-0.157} = 0.80$

Our guess of 0.85 was conservative. From Fig. 6-20 with $r = d/10 = 0.229$ in, we are off the graph, but it appears our guess for $q$ is low. Assuming the trend of the graph continues, we'll choose $q = 0.91$ and iterate the problem with the new values of $k_b$ and $q$. Intermediate results are $S_e = 36.5$ kpsi, $S_f = 39.6$ kpsi, and $K_f = 1.59$. This gives

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{39.6}{(1.59)(203.7/d^3)} = 1.6$$

$d = 2.36$ in *Ans.*

A quick check of $k_b$ and $q$ show that our estimates are still reasonable for this diameter.

---

**6-20** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[25^2 + 3(0)^2\right]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[25^2 + 3(15^2)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \quad \textit{Ans.}$$

**(b)** Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{25.00}{40}\right)\left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)}\right)^2}\,\right] = 1.31 \quad \textit{Ans.}$$

**(c)** ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.00/40)^2 + (25.98/60)^2}} = 1.32 \qquad \textit{Ans.}$$

---

**6-21** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[10^2 + 3\left(0\right)^2\right]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(20\right)^2\right]^{1/2} = 34.64 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[10^2 + 3\left(20^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \qquad \textit{Ans.}$$

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \qquad \textit{Ans.}$$

**(b)** Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{34.64}\right)^2 \left(\frac{10.00}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)}\right)^2}\right\} = 1.74 \qquad \textit{Ans.}$$

**(c)** ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(10.00/40)^2 + (34.64/60)^2}} = 1.59 \qquad \textit{Ans.}$$

---

**6-22** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[12^2 + 3\left(10\right)^2\right]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(15\right)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(12 + 0\right)^2 + 3\left(10 + 15\right)^2\right]^{1/2} = 44.93 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{44.93} = 1.34 \quad \textit{Ans.}$$

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(21.07/40) + (25.98/80)} = 1.17 \quad \textit{Ans.}$$

**(b)** Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{21.07}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(21.07)}\right)^2}\right\} = 1.47 \quad \textit{Ans.}$$

**(c)** ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(21.07/40)^2 + (25.98/60)^2}} = 1.47 \quad \textit{Ans.}$$

---

**6-23** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = 0 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{51.96} = 1.15 \quad \textit{Ans.}$$

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(51.96/40)} = 0.77 \quad \textit{Ans.}$$

**(b)** Gerber criterion of Table 6-7 is only valid for $\sigma_m > 0$; therefore use Eq. (6-47).

$$n_f \frac{\sigma'_a}{S_e} = 1 \quad \Rightarrow \quad n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \quad \textit{Ans.}$$

**(c)** ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(51.96/40)^2}} = 0.77 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the *S-N* diagram is applicable for $\sigma'_a$.
Fig. 6-18: $f = 0.875$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.875(80)\right]^2}{40} = 122.5$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.875(80)}{40}\right) = -0.08101$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{51.96}{122.5}\right)^{\frac{1}{-0.08101}} = 39\ 600 \text{ cycles} \quad \textit{Ans.}$$

---

**6-24** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3\left(15\right)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[15^2 + 3\left(0\right)^2\right]^{1/2} = 15.00 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(15\right)^2 + 3\left(15\right)^2\right]^{1/2} = 30.00 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \quad \textit{Ans.}$$

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \quad \textit{Ans.}$$

**(b)** Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{15.00}\right)^2\left(\frac{25.98}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)}\right)^2}\right\} = 1.43 \quad \textit{Ans.}$$

**(c)** ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.98/40)^2 + (15.00/60)^2}} = 1.44 \qquad \textit{Ans.}$$

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**6-25** Given: $F_{\max} = 28 \text{ kN}, F_{\min} = -28 \text{ kN}$. From Table A-20, for AISI 1040 CD, $S_{ut} = 590$ MPa, $S_y = 490$ MPa,

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-8): $S_e^{'} = 0.5(590) = 295$ MPa

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(590)^{-0.265} = 0.832$

Eq. (6-21): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.832)(1)(0.85)(295) = 208.6$ MPa

Fig. 6-20: $q = 0.83$

Fig. A-15-1: $d/w = 0.24, K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{324.2}{208.6} + \frac{0}{590}$$

$$n_f = 0.64 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. Since $\sigma_m = 0$, the stress state is completely reversed and the *S-N* diagram is applicable for $\sigma_a$.

$S_{ut} = 590/6.89 = 85.6$ kpsi

Fig. 6-18: $f = 0.87$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{\left[0.87(590)\right]^2}{208.6} = 1263$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(590)}{208.6}\right) = -0.1304$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{324.2}{1263}\right)^{\frac{1}{-0.1304}} = 33\,812 \text{ cycles}$

$N = 34\,000$ cycles *Ans.*

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**6-26** $S_{ut} = 590$ MPa, $S_y = 490$ MPa, $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life.
From Prob. 6-25: $S_e = 208.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f\left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000 - (12\,000)}{2(10)(25-6)}\right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f\,\frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000 + 12\,000}{2(10)(25-6)}\right] = 231.6 \text{ MPa}$$

Modified Goodman criteria:

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$n_f = 1.20$ *Ans.*

Gerber criteria:

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{590}{231.6}\right)^2\frac{92.63}{208.6}\left[-1 + \sqrt{1 + \left(\frac{2(231.6)(208.6)}{590(92.63)}\right)^2}\,\right]$$

$n_f = 1.49$ *Ans.*

ASME-Elliptic criteria:

$$n_f = \sqrt{\frac{1}{(\sigma_a / S_e)^2 + (\sigma_m / S_y)^2}} = \sqrt{\frac{1}{(92.63/208.6)^2 + (231.6/490)^2}}$$

$$= 1.54 \quad Ans.$$

The results are consistent with Fig. 6-27, where for a mean stress that is about half of the yield strength, the Modified Goodman line should predict failure significantly before the other two.

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**6-27** $S_{ut} = 590$ MPa, $S_y = 490$ MPa

**(a)** $F_{\max} = 28$ kN, $F_{\min} = 0$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad Ans.$$

From Prob. 6-25: $S_e = 208.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 0}{2(10)(25-6)} \right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[ \frac{28\,000 + 0}{2(10)(25-6)} \right] = 162.1 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{162.1}{208.6} + \frac{162.1}{590}$$

$$n_f = 0.95 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{162.1}{1 - (162.1/590)} = 223.5 \text{ MPa}$$

Fig. 6-18: $f = 0.87$

Eq. (6-14): $a = \dfrac{(f\, S_{ut})^2}{S_e} = \dfrac{[0.87(590)]^2}{208.6} = 1263$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{208.6}\right) = -0.1304$

Eq. (6-16): $N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{223.5}{1263}\right)^{\frac{1}{-0.1304}} = 586\,000 \text{ cycles}$ *Ans.*

**(b)** $F_{\max} = 28 \text{ kN}, F_{\min} = 12 \text{ kN}$

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \quad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000 - 12\,000}{2(10)(25-6)}\right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000 + 12\,000}{2(10)(25-6)}\right] = 231.6 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$$n_f = 1.20 \quad \textit{Ans.}$$

**(c)** $F_{\max} = 12 \text{ kN}, F_{\min} = -28 \text{ kN}$

The compressive load is the largest, so check it for yielding.

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{-28\,000}{10(25-6)} = -147.4 \text{ MPa}$$

$$n_y = \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \quad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{12\,000 - (-28\,000)}{2(10)(25-6)}\right| = 231.6 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{12\,000 + (-28\,000)}{2(10)(25-6)}\right] = -92.63 \text{ MPa}$$

For $\sigma_m < 0$, $n_f = \frac{S_e}{\sigma_a} = \frac{208.6}{231.6} = 0.90$ *Ans.*

Since infinite life is not predicted, estimate a life from the *S-N* diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress. Get *a* and *b* from part (a).

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{231.6}{1263}\right)^{\frac{1}{-0.1304}} = 446\,000 \text{ cycles} \qquad Ans.$$

---

**6-28** Eq. (2-21): $S_{ut} = 0.5(400) = 200$ kpsi

Eq. (6-8): $S_e^{'} = 0.5(200) = 100$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 14.4(200)^{-0.718} = 0.321$

Eq. (6-25): $d_e = 0.37d = 0.37(0.375) = 0.1388$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

Eq. (6-18): $S_e = (0.321)(1)(100) = 32.1$ kpsi

$$F_a = \frac{40-20}{2} = 10 \text{ lb} \qquad F_m = \frac{40+20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

**(a)** Modified Goodman criterion

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{32.1} + \frac{69.54}{200}$$

$$n_f = 0.94 \qquad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{23.18}{1-(69.54/200)} = 35.54 \text{ kpsi}$$

Fig. 6-18: $f = 0.775$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.775(200)]^2}{32.1} = 748.4$$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.775(200)}{32.1}\right) = -0.228$

Eq. (6-16): $N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{35.54}{748.4}\right)^{\frac{1}{-0.228}} = 637\ 000$ cycles *Ans.*

**(b)** Gerber criterion, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{200}{69.54}\right)^2 \frac{23.18}{32.1}\left[-1+\sqrt{1+\left(\frac{2(69.54)(32.1)}{200(23.18)}\right)^2}\right]$$

$= 1.16$ Infinite life is predicted *Ans.*

---

**6-29** $E = 207.0$ GPa

**(a)** $I = \frac{1}{12}(20)(4^3) = 106.7\ \text{mm}^4$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3\ \text{N} \qquad \textit{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9\ \text{N} \qquad \textit{Ans.}$$

**(b)** Get the fatigue strength information.

Eq. (2-21): $S_{ut} = =3.4H_B = 3.4(490) = 1666$ MPa

From problem statement: $S_y = 0.9S_{ut} = 0.9(1666) = 1499$ MPa

Eq. (6-8): $S'_e = 700$ MPa

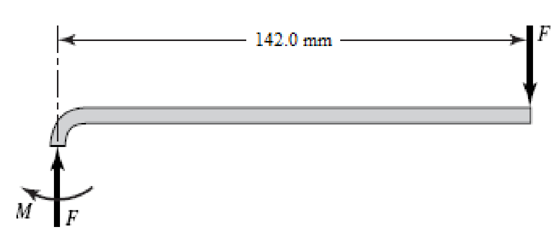
Eq. (6-19): $k_a = 1.58(1666)^{-0.085} = 0.84$

Eq. (6-25): $d_e = 0.808[20(4)]^{1/2} = 7.23$ mm

Eq. (6-20): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-18): $S_e = 0.84(1)(700) = 588$ MPa

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$M = 142F$ N·mm, $A = 4(20) = 80$ mm$^2$, $h = 4$ mm, $r_i = 4$ mm, $r_o = r_i + h = 8$ mm,
$r_c = r_i + h/2 = 6$ mm

Table 3-4: $$r_n = \frac{h}{\ln(r_o / r_i)} = \frac{4}{\ln(8/4)} = 5.7708 \text{ mm}$$

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$

$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$

$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-65) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$

$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$

$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$

$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$

$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$

$$(\sigma_i)_a = \left| \frac{-166.2 - (-498.6)}{2} \right| = 166.2 \text{ MPa}$$

$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$

$$(\sigma_o)_a = \left| \frac{310.8 - 103.6}{2} \right| = 103.6 \text{ MPa}$$

$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is –498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa, so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.
Inner radius:

Since $\sigma_m < 0$, $n_f = \dfrac{S_e}{\sigma_a} = \dfrac{588}{166.2} = 3.54$

Outer radius:

Since $\sigma_m > 0$, we will use the Modified Goodman line.

$$1/n_f = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{103.6}{588} + \frac{207.2}{1666}$$

$$n_f = 3.33$$

Infinite life is predicted at both inner and outer radii. *Ans.*

---

**6-30** From Table A-20, for AISI 1018 CD, $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-8): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = (0.897)(1)(0.85)(32) = 24.4$ kpsi

Fillet:

Fig. A-15-5: $D/d = 3.5/3 = 1.17, \quad r/d = 0.25/3 = 0.083, \quad K_t = 1.85$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for $q$. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(1.85 - 1) = 1.72$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{5}{3.0(0.5)} = 3.33 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{3.0(0.5)} = -10.67 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.72 \left| \frac{3.33 - (-10.67)}{2} \right| = 12.0 \text{ kpsi}$$

$$\sigma_m = K_f \left( \frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.72 \left( \frac{3.33 + (-10.67)}{2} \right) = -6.31 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.67} \right| = 5.06 \quad \therefore \text{Does not yield.}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{12.0} = 2.03$$

Hole:

Fig. A-15-1: $d / w_1 = 0.4 / 3.5 = 0.11 \quad \therefore K_t = 2.68$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for $q$. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.68 - 1) = 2.43$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{5}{0.5(3.5 - 0.4)} = 3.226 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-16}{0.5(3.5 - 0.4)} = -10.32 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.43 \left| \frac{3.226 - (-10.32)}{2} \right| = 16.5 \text{ kpsi}$$

$$\sigma_m = K_f \left( \frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.43 \left( \frac{3.226 + (-10.32)}{2} \right) = -8.62 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.32} \right| = 5.23 \quad \therefore \text{does not yield}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{16.5} = 1.48$$

Thus the design is controlled by the threat of fatigue at the hole with a minimum factor of safety of $n_f = 1.48$. *Ans.*

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**6-31** $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-8): $S_e^{'} = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = (0.897)(1)(0.85)(32) = 24.4$ kpsi

Fillet:

Fig. A-15-5: $D / d = 2.5 / 1.5 = 1.67, \quad r / d = 0.25 / 1.5 = 0.17, \quad K_t \doteq 2.1$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for $q$. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.1 - 1) = 1.94$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{16}{1.5(0.5)} = 21.3 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-4}{1.5(0.5)} = -5.33 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.94 \left| \frac{21.3 - (-5.33)}{2} \right| = 25.8 \text{ kpsi}$$

$$\sigma_m = K_f \left( \frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.94 \left( \frac{21.3 + (-5.33)}{2} \right) = 15.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{21.3} = 2.54 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{25.8}{24.4} + \frac{15.5}{64}$$

$$n_f = 0.77$$

Hole:

Fig. A-15-1: $d / w_1 = 0.4 / 2.5 = 0.16 \quad \therefore K_t = 2.55$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for $q$. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.55 - 1) = 2.32$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{16}{0.5(2.5 - 0.4)} = 15.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-4}{0.5(2.5 - 0.4)} = -3.81 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.32 \left( \frac{15.2 - (-3.81)}{2} \right) = 22.1 \text{ kpsi}$$

$$\sigma_m = K_f \left( \frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.32 \left( \frac{15.2 + (-3.81)}{2} \right) = 13.2 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{15.2} = 3.55 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{22.1}{24.4} + \frac{13.2}{64}$$

$$n_f = 0.90$$

Thus the design is controlled by the threat of fatigue at the fillet with a minimum factor of safety of $n_f = 0.77$ *Ans.*

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**6-32** $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 6-30, the fatigue factor of safety at the hole is $n_f = 1.48$. To match this at the fillet,

$$n_f = \frac{S_e}{\sigma_a} \quad \Rightarrow \quad \sigma_a = \frac{S_e}{n_f} = \frac{24.4}{1.48} = 16.5 \text{ kpsi}$$

where $S_e$ is unchanged from Prob. 6-30. The only aspect of $\sigma_a$ that is affected by the fillet radius is the fatigue stress concentration factor. Obtaining $\sigma_a$ in terms of $K_f$,

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = K_f \left| \frac{3.33 - (-10.67)}{2} \right| = 7.00 K_f$$

Equating to the desired stress, and solving for $K_f$,

$$\sigma_a = 7.00 K_f = 16.5 \quad \Rightarrow \quad K_f = 2.36$$

Assume since we are expecting to get a smaller fillet radius than the original, that $q$ will be back on the graph of Fig. 6-20, so we'll estimate $q = 0.8$.

$$K_f = 1 + 0.80(K_t - 1) = 2.36 \quad \Rightarrow \quad K_t = 2.7$$

From Fig. A-15-5, with $D / d = 3.5/3 = 1.17$ and $K_t = 2.6$, find $r / d$. Choosing $r / d = 0.03$, and with $d = w_2 = 3.0$,

$$r = 0.03 w_2 = 0.03(3.0) = 0.09 \text{ in}$$

At this small radius, our estimate for $q$ is too high. From Fig. 6-20, with $r = 0.09$, $q$ should be about 0.75. Iterating, we get $K_t = 2.8$. This is at a difficult range on Fig. A-15-5 to read the graph with any confidence, but we'll estimate $r / d = 0.02$, giving $r = 0.06$ in. This is a very rough estimate, but it clearly demonstrates that the fillet radius can be relatively sharp to match the fatigue factor of safety of the hole. *Ans.*

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**6-33** $S_y = 60$ kpsi, $S_{ut} = 110$ kpsi

Inner fiber where $r_c = 3/4$ in

$$r_o = \frac{3}{4} + \frac{3}{16(2)} = 0.84375$$

$$r_i = \frac{3}{4} - \frac{3}{32} = 0.65625$$

Table 3-4, p. 121,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{3/16}{\ln \frac{0.84375}{0.65625}} = 0.74608 \text{ in}$$

$$e = r_c - r_n = 0.75 - 0.74608 = 0.00392 \text{ in}$$

$$c_i = r_n - r_i = 0.74608 - 0.65625 = 0.08983$$

$$A = \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = 0.035156 \text{ in}^2$$

Eq. (3-65), p. 119,

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(0.08983)}{(0.035156)(0.00392)(0.65625)} = -993.3T$$

where $T$ is in lbf·in and $\sigma_i$ is in psi.

$$\sigma_m = \frac{1}{2}(-993.3)T = -496.7T$$

$$\sigma_a = 496.7T$$

Eq. (6-8): $S_e' = 0.5(110) = 55$ kpsi

Eq. (6-19): $k_a = 2.70(110)^{-0.265} = 0.777$

Eq. (6-25): $d_e = 0.808\left[(3/16)(3/16)\right]^{1/2} = 0.1515$ in

Eq. (6-20): $k_b = 0.879(0.1515)^{-0.107} = 1.08$ (round to 1)

Eq. (6-19): $S_e = (0.777)(1)(55) = 42.7$ kpsi

For a compressive midrange component, $\sigma_a = S_e / n_f$. Thus,

$$0.4967T = \frac{42.7}{3}$$

$$T = 28.7 \text{ lbf} \cdot \text{in}$$

Outer fiber where $r_c = 2.5$ in

$$r_o = 2.5 + \frac{3}{32} = 2.59375$$

$$r_i = 2.5 - \frac{3}{32} = 2.40625$$

$$r_n = \frac{3/16}{\ln \frac{2.59375}{2.40625}} = 2.49883$$

$$e = 2.5 - 2.49883 = 0.00117 \text{ in}$$

$$c_o = 2.59375 - 2.49883 = 0.09492 \text{ in}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{T(0.09492)}{(0.035156)(0.00117)(2.59375)} = 889.7T \text{ psi}$$

$$\sigma_m = \sigma_a = \frac{1}{2}(889.7T) = 444.9T \text{ psi}$$

**(a)** Using Eq. (6-46), for modified Goodman, we have

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{0.4449T}{42.7} + \frac{0.4449T}{110} = \frac{1}{3}$$

$T = 23.0$ lbf · in *Ans.*

**(b)** Gerber, Eq. (6-47), at the outer fiber,

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$\frac{3(0.4449T)}{42.7} + \left(\frac{3(0.4449T)}{110}\right)^2 = 1$$

$T = 28.2$ lbf · in *Ans.*

**(c)** To guard against yield, use $T$ of part (b) and the inner stress.

$$n_y = \frac{S_y}{\sigma_i} = \frac{60}{0.9933(28.2)} = 2.14 \quad \textit{Ans.}$$

---

**6-34** From Prob. 6-33, $S_e = 42.7$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 110$ kpsi

**(a)** Assuming the beam is straight,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} = \frac{6T}{(3/16)^3} = 910.2T$$

Goodman: $$\frac{0.4551T}{42.7} + \frac{0.4551T}{110} = \frac{1}{3}$$

$T = 22.5$ lbf · in *Ans.*

**(b)** Gerber: $$\frac{3(0.4551T)}{42.7} + \left(\frac{3(0.4551T)}{110}\right)^2 = 1$$

$T = 27.6$ lbf · in *Ans.*

**(c)** $$n_y = \frac{S_y}{\sigma_{\max}} = \frac{60}{0.9102(27.6)} = 2.39 \quad Ans.$$

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**6-35** $K_{f,\text{bend}} = 1.4,\ K_{f,\text{axial}} = 1.1,\ K_{f,\text{tors}} = 2.0,\ S_y = 300 \text{ MPa},\ S_{ut} = 400 \text{ MPa},\ S_e = 200 \text{ MPa}$

Bending: $\sigma_m = 0,\ \sigma_a = 60$ MPa

Axial: $\sigma_m = 20$ MPa, $\sigma_a = 0$

Torsion: $\tau_m = 25$ MPa, $\tau_a = 25$ MPa

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{[1.4(60)+0]^2 + 3[2.0(25)]^2} = 120.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{[0+1.1(20)]^2 + 3[2.0(25)]^2} = 89.35 \text{ MPa}$$

Using Modified Goodman, Eq. (6-46),

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{120.6}{200} + \frac{89.35}{400}$$

$$n_f = 1.21 \quad Ans.$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{120.6 + 89.35} = 1.43 \quad Ans.$$

---

**6-36** $K_{f,\text{bend}} = 1.4,\ K_{f,\text{tors}} = 2.0,\ S_y = 300 \text{ MPa},\ S_{ut} = 400 \text{ MPa},\ S_e = 200 \text{ MPa}$

Bending: $\sigma_{\max} = 150$ MPa, $\sigma_{\min} = -40$ MPa, $\sigma_m = 55$ MPa, $\sigma_a = 95$ MPa

Torsion: $\tau_m = 90$ MPa, $\tau_a = 9$ MPa

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{[1.4(95)]^2 + 3[2.0(9)]^2} = 136.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{[1.4(55)]^2 + 3[2.0(90)]^2} = 321.1 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{136.6}{200} + \frac{321.1}{400}$$

$$n_f = 0.67 \quad Ans.$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{136.6 + 321.1} = 0.66 \qquad Ans.$$

Since the conservative yield check indicates yielding, we will check more carefully with with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{(\sigma_{\max})^2 + 3(\tau_{\max})^2} = \sqrt{(150)^2 + 3(90+9)^2} = 227.8 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{300}{227.8} = 1.32 \qquad Ans.$$

Since yielding is not predicted, and infinite life is not predicted, we would like to estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{136.6}{1 - (321.1/400)} = 692.5 \text{ MPa}$$

This stress is much higher than the ultimate strength, rendering it impractical for the *S-N* diagram. We must conclude that the stresses from the combination loading, when increased by the stress concentration factors, produce such a high midrange stress that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

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**6-37** Table A-20: $S_{\text{ut}} = 64$ kpsi, $S_{\text{y}} = 54$ kpsi

From Prob. 3-68, the critical stress element experiences $\sigma = 15.3$ kpsi and $\tau = 4.43$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 15.3$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 4.43$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[15.3^2 + 3(0)^2\right]^{1/2} = 15.3 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(4.43)^2\right]^{1/2} = 7.67 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[15.3^2 + 3(4.43)^2\right]^{1/2} = 17.11 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{17.11} = 3.16$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $\quad S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(32) = 24.8$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{15.3}{24.8} + \frac{7.67}{64}$$

$$n_f = 1.36 \quad \textit{Ans.}$$

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**6-38** Table A-20: $S_{\mathrm{ut}} = 440$ MPa, $S_{\mathrm{y}} = 370$ MPa

From Prob. 3-69, the critical stress element experiences $\sigma = 263$ MPa and $\tau = 57.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 263$ MPa, $\sigma_m = 0$, $\tau_a = 0$ MPa, $\tau_m = 57.7$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[263^2 + 3\left(0\right)^2\right]^{1/2} = 263 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(57.7\right)^2\right]^{1/2} = 99.9 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[263^2 + 3\left(57.7\right)^2\right]^{1/2} = 281 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{281} = 1.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5\left(440\right) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(220) = 170$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{263}{170} + \frac{99.9}{440}$$

$n_f = 0.56$ Infinite life is not predicted. *Ans.*

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**6-39** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-70, the critical stress element experiences $\sigma = 21.5$ kpsi and $\tau = 5.09$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 21.5$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 5.09$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[21.5^2 + 3\left(0\right)^2\right]^{1/2} = 21.5 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(5.09\right)^2\right]^{1/2} = 8.82 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[21.5^2 + 3\left(5.09\right)^2\right]^{1/2} = 23.24 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{23.24} = 2.32$$

Obtain the modifying factors and endurance limit.

$$k_a = 2.70(64)^{-0.265} = 0.90$$

$$k_b = 0.879(1)^{-0.107} = 0.88$$

$$S_e = 0.90(0.88)(0.5)(64) = 25.3 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{21.5}{25.3} + \frac{8.82}{64}$$

$$n_f = 1.01 \quad \textit{Ans.}$$

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**6-40** Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-71, the critical stress element experiences $\sigma = 72.9$ MPa and $\tau = 20.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 72.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 20.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[72.9^2 + 3\left(0\right)^2\right]^{1/2} = 72.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(20.3\right)^2\right]^{1/2} = 35.2 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[72.9^2 + 3\left(20.3\right)^2\right]^{1/2} = 80.9 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{80.9} = 4.57$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(20)^{-0.107} = 0.90$

Eq. (6-18): $S_e = 0.90(0.90)(220) = 178.2 \text{ MPa}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{72.9}{178.2} + \frac{35.2}{440}$$

$$n_f = 2.04 \quad \textit{Ans.}$$

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**6-41** Table A-20: $S_{\text{ut}} = 64 \text{ kpsi}$, $S_{\text{y}} = 54 \text{ kpsi}$

From Prob. 3-72, the critical stress element experiences $\sigma = 35.2$ kpsi and $\tau = 7.35$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 35.2$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 7.35$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[35.2^2 + 3(0)^2\right]^{1/2} = 35.2 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(7.35)^2\right]^{1/2} = 12.7 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[35.2^2 + 3(7.35)^2\right]^{1/2} = 37.4 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{37.4} = 1.44$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(32) = 24.8 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{35.2}{24.8} + \frac{12.7}{64}$$

$n_f = 0.62$ Infinite life is not predicted. *Ans.*

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**6-42** Table A-20: $S_{\text{ut}} = 440$ MPa, $S_{\text{y}} = 370$ MPa

From Prob. 3-73, the critical stress element experiences $\sigma = 333.9$ MPa and $\tau = 126.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 333.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 126.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[333.9^2 + 3\left(0\right)^2\right]^{1/2} = 333.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(126.3\right)^2\right]^{1/2} = 218.8 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[333.9^2 + 3\left(126.3\right)^2\right]^{1/2} = 399.2 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{399.2} = 0.93$$

The sample fails by yielding, infinite life is not predicted. *Ans.*

The fatigue analysis will be continued only to obtain the requested fatigue factor of safety, though the yielding failure will dictate the life.

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(50)^{-0.107} = 0.82$

Eq. (6-18): $S_e = 0.90(0.82)(220) = 162.4$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{333.9}{162.4} + \frac{218.8}{440}$$

$n_f = 0.39$ Infinite life is not predicted. *Ans.*

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**6-43** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-74, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 9.495 \text{ kpsi}, \qquad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$
$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \qquad \sigma_{m,\text{axial}} = -0.362 \text{ kpsi}$$
$$\tau_a = 0 \text{ kpsi}, \qquad \tau_m = 11.07 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(9.495\right)^2 + 3\left(0\right)^2\right]^{1/2} = 9.495 \text{ kpsi}$$
$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(-0.362\right)^2 + 3\left(11.07\right)^2\right]^{1/2} = 19.18 \text{ kpsi}$$
$$\sigma_{\max}' = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(-9.495 - 0.362\right)^2 + 3\left(11.07\right)^2\right]^{1/2} = 21.56 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma_{\max}'} = \frac{54}{21.56} = 2.50$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.13)^{-0.107} = 0.87$

Eq. (6-18): $S_e = 0.90(0.87)(32) = 25.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{9.495}{25.1} + \frac{19.18}{64}$$

$$n_f = 1.47 \quad \textit{Ans.}$$

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**6-44** Table A-20: $S_{\text{ut}} = 64$ kpsi, $S_{\text{y}} = 54$ kpsi

From Prob. 3-76, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 33.99 \text{ kpsi}, \qquad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$
$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \qquad \sigma_{m,\text{axial}} = -0.153 \text{ kpsi}$$
$$\tau_a = 0 \text{ kpsi}, \qquad \tau_m = 7.847 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(33.99\right)^2 + 3\left(0\right)^2\right]^{1/2} = 33.99 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(-0.153\right)^2 + 3\left(7.847\right)^2\right]^{1/2} = 13.59 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(-33.99 - 0.153\right)^2 + 3\left(7.847\right)^2\right]^{1/2} = 36.75 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{36.75} = 1.47$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(0.88)^{-0.107} = 0.89$

Eq. (6-18): $S_e = 0.90(0.89)(32) = 25.6 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{33.99}{25.6} + \frac{13.59}{64}$$

$n_f = 0.65$ Infinite life is not predicted. *Ans.*

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**6-45** Table A-20: $S_{\text{ut}} = 440 \text{ MPa}$, $S_{\text{y}} = 370 \text{ MPa}$

From Prob. 3-77, the critical stress element experiences $\sigma$ = 68.6 MPa and $\tau$ = 37.7 MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a$ = 68.6 MPa, $\sigma_m$ = 0 MPa, $\tau_a$ = 0 MPa, $\tau_m$ = 37.7 MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[68.6^2 + 3\left(0\right)^2\right]^{1/2} = 68.6 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(37.7\right)^2\right]^{1/2} = 65.3 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[68.6^2 + 3\left(37.7\right)^2\right]^{1/2} = 94.7 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{94.7} = 3.91$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(220) = 170 \text{ MPa}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{68.6}{170} + \frac{65.3}{440}$$

$$n_f = 1.81 \quad \textit{Ans.}$$

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**6-46** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-79, the critical stress element experiences $\sigma = 3.46$ kpsi and $\tau = 0.882$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 3.46$ kpsi, $\sigma_m = 0$, $\tau_a = 0$ kpsi, $\tau_m = 0.882$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[3.46^2 + 3\left(0\right)^2\right]^{1/2} = 3.46 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(0.882\right)^2\right]^{1/2} = 1.53 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[3.46^2 + 3\left(0.882\right)^2\right]^{1/2} = 3.78 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{3.78} = 14.3$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.375)^{-0.107} = 0.85$

Eq. (6-18): $S_e = 0.90(0.85)(32) = 24.5 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{3.46}{24.5} + \frac{1.53}{64}$$

$$n_f = 6.06 \qquad \textit{Ans.}$$

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**6-47** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-80, the critical stress element experiences $\sigma = 16.3$ kpsi and $\tau = 5.09$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(64\right) + 1.51\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)\left(64\right) + 1.35\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[\left(1.38\right)\left(8.15\right)\right]^2 + 3\left[\left(1.88\right)\left(2.55\right)\right]^2\right\}^{1/2} = 13.98 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.98 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.98 + 13.98} = 1.93$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879 d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{13.98}{28.2} + \frac{13.98}{64}$$

$$n_f = 1.40 \quad \textit{Ans.}$$

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**6-48** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-81, the critical stress element experiences $\sigma = 16.4$ kpsi and $\tau = 4.46$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.4$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 4.46$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.20$ kpsi, $\tau_m = \tau_a = 2.23$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma_a' = \left\{\left[(1.38)(8.20)\right]^2 + 3\left[(1.88)(2.23)\right]^2\right\}^{1/2} = 13.45 \text{ kpsi}$$

$$\sigma_m' = \sigma_a' = 13.45 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma_{\max}' = \sigma_a' + \sigma_m'$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.45 + 13.45} = 2.01$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{13.45}{28.2} + \frac{13.45}{64}$$

$$n_f = 1.46 \quad \textit{Ans.}$$

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**6-49** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-82, the critical stress element experiences repeatedly applied bending, axial, and torsional stresses of $\sigma_{x,\text{bend}} = 20.2$ kpsi, $\sigma_{x,\text{axial}} = 0.1$ kpsi, and $\tau = 5.09$ kpsi.. Since the axial stress is practically negligible compared to the bending stress, we will simply combine the two and not treat the axial stress separately for stress concentration factor and load factor. This gives $\sigma_{\max} = 20.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 10.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(64\right) + 1.51\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)\left(64\right) + 1.35\left(10^{-5}\right)\left(64\right)^2 - 2.67\left(10^{-8}\right)\left(64\right)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.38)(10.15)\right]^2 + 3\left[(1.88)(2.55)\right]^2\right\}^{1/2} = 16.28 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 16.28 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{16.28 + 16.28} = 1.66$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{16.28}{28.2} + \frac{16.28}{64}$$

$n_f = 1.20$ *Ans.*

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**6-50** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-83, the critical stress element on the neutral axis in the middle of the longest side of the rectangular cross section experiences a repeatedly applied shear stress of $\tau_{\max} = 14.3$ kpsi, $\tau_{\min} = 0$ kpsi. Thus, $\tau_m = \tau_a = 7.15$ kpsi. Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y/2}{\tau_{\max}} = \frac{54/2}{14.3} = 1.89$$

Find the modifiers and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

The size factor for a torsionally loaded rectangular cross section is not readily available. Following the procedure on p. 289, we need an equivalent diameter based on the 95 percent stress area. However, the stress situation in this case is nonlinear, as described on p. 102. Noting that the maximum stress occurs at the middle of the longest side, or with a radius from the center of the cross section equal to half of the shortest side, we will simply choose an equivalent diameter equal to the length of the shortest side.

$$d_e = 0.25 \text{ in}$$

Eq. (6-20): $k_b = 0.879 d_e^{-0.107} = 0.879(0.25)^{-0.107} = 1.02$

We will round down to $k_b = 1$.

Eq. (6-26): $k_c = 0.59$

Eq. (6-18): $S_{se} = 0.9(1)(0.59)(32) = 17.0 \text{ kpsi}$

Since the stress is entirely shear, we choose to use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67 S_u = 0.67\ (64) = 42.9$ kpsi.

Using Modified Goodman,

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(7.15/17.0) + (7.15/42.9)} = 1.70 \qquad \textit{Ans.}$$

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**6-51** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-84, the critical stress element experiences $\sigma = 28.0$ kpsi and $\tau = 15.3$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{max} = 28.0$ kpsi, $\sigma_{min} = 0$ kpsi, $\tau_{max} = 15.3$ kpsi, $\tau_{min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 14.0$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78\left(1.60 - 1\right) = 1.47$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82\left(1.39 - 1\right) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.47)(14.0)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 27.0 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 27.0 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{27.0 + 27.0} = 1.00$$

Since stress concentrations are included in this quick yield check, the low factor of safety is acceptable.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{27.0}{28.1} + \frac{27.0}{64}$$

$$n_f = 0.72 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{27.0}{1 - (27.0/64)} = 46.7 \text{ kpsi}$$

Fig. 6-18: $f = 0.9$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.9(64)]^2}{28.1} = 118.07$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.9(64)}{28.1}\right) = -0.1039$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{46.7}{118.07}\right)^{\frac{1}{-0.1039}} = 7534 \text{ cycles} \doteq 7500 \text{ cycles} \qquad \textit{Ans.}$

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**6-52** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-85, the critical stress element experiences $\sigma_{x,\text{bend}}$ = 46.1 kpsi, $\sigma_{x,\text{axial}}$ = 0.382 kpsi and $\tau$ = 15.3 kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}}$ = 46.1 kpsi, $\sigma_{\text{min,bend}}$ = 0 kpsi, $\sigma_{\text{max,axial}}$ = 0.382 kpsi, $\sigma_{\text{min,axial}}$ = 0 kpsi, $\tau_{\text{max}}$ = 15.3 kpsi, $\tau_{\text{min}}$ = 0 kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}}$ = 23.05 kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}}$ = 0.191 kpsi, $\tau_m = \tau_a$ = 7.65 kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}}$ =0.78, $q_{\text{tors}}$ = 0.82
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60 - 1) = 1.47$$
$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75 - 1) = 1.59$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma_a' = \left\{\left[(1.47)(23.05) + (1.59)\frac{(0.191)}{0.85}\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.45 \text{ kpsi}$$

$$\sigma_m' = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.40 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\text{max}} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{38.45 + 38.40} = 0.70$$

Since the conservative yield check indicates yielding, we will check more carefully with with $\sigma'_{\text{max}}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\text{max}} = \sqrt{\left(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}}\right)^2 + 3\left(\tau_{\text{max}}\right)^2} = \sqrt{(46.1 + 0.382)^2 + 3(15.3)^2} = 53.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{54}{53.5} = 1.01 \qquad \textit{Ans.}$$

This shows that yielding is imminent, and further analysis of fatigue life should not be interpreted as a guarantee of more than one cycle of life.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{38.45}{28.1} + \frac{38.40}{64}$$

$$n_f = 0.51 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1-(\sigma'_m / S_{ut})} = \frac{38.45}{1-(38.40/64)} = 96.1 \text{ kpsi}$$

This stress is much higher than the ultimate strength, rendering it impractical for the *S-N* diagram. We must conclude that the fluctuating stresses from the combination loading, when increased by the stress concentration factors, are so far from the Goodman line that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

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**6-53** Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-86, the critical stress element experiences $\sigma_{x,\text{bend}} = 55.5$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\max,\text{bend}} = 55.5$ kpsi, $\sigma_{\min,\text{bend}} = 0$ kpsi, $\sigma_{\max,\text{axial}} = 0.382$ kpsi, $\sigma_{\min,\text{axial}} = 0$ kpsi, $\tau_{\max} = 15.3$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 27.75$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60-1) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75-1) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39-1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma_a' = \left\{ \left[ (1.47)(27.75) + (1.59)\frac{(0.191)}{0.85} \right]^2 + 3\left[(1.32)(7.65)\right]^2 \right\}^{1/2} = 44.71 \text{ kpsi}$$

$$\sigma_m' = \left\{ \left[ (1.47)(27.75) + (1.59)(0.191) \right]^2 + 3\left[(1.32)(7.65)\right]^2 \right\}^{1/2} = 44.66 \text{ kpsi}$$

Since these stresses are relatively high compared to the yield strength, we will go ahead and check for yielding using the distortion energy failure theory.

$$\sigma'_{\max} = \sqrt{\left(\sigma_{\max,\text{bend}} + \sigma_{\max,\text{axial}}\right)^2 + 3\left(\tau_{\max}\right)^2} = \sqrt{(55.5+0.382)^2 + 3(15.3)^2} = 61.8 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{61.8} = 0.87 \qquad \textit{Ans.}$$

This shows that yielding is predicted. Further analysis of fatigue life is just to be able to report the fatigue factor of safety, though the life will be dictated by the static yielding failure, i.e. $N$ = 1/2 cycle. *Ans.*

Eq. (6-8): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{44.71}{28.1} + \frac{44.66}{64}$$

$$n_f = 0.44 \qquad \textit{Ans.}$$

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**6-54** From Table A-20, for AISI 1040 CD, $S_{ut}$ = 85 kpsi and $S_y$ = 71 kpsi. From the solution to Prob. 6-17 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{\text{rev}}$ = 35.0 kpsi, producing $\sigma_a$ = 35.0 kpsi and $\sigma_m$ = 0 kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2500(1.625/2)}{\pi\left(1.625^4\right)/32} = 2967 \text{ psi} = 2.97 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d$ = 0.0625/1.625 = 0.04, $D/d$ = 1.875/1.625 = 1.15, $K_{t,\text{bend}}$ =1.95, $K_{t,\text{tors}}$ =1.60

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76(1.95 - 1) = 1.72$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.72)(35.0)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 60.2 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.97)\right]^2\right\}^{1/2} = 7.66 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{60.2 + 7.66} = 1.05$$

From the solution to Prob. 6-17, $S_e$ = 29.5 kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{60.2}{29.5} + \frac{7.66}{85}$$

$$n_f = 0.47 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{60.2}{1 - (7.66/85)} = 66.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.867$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.867(85)\right]^2}{29.5} = 184.1$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{66.2}{184.1}\right)^{\frac{1}{-0.1325}} = 2251 \text{ cycles}$$

$N$ = 2300 cycles $\qquad$ *Ans.*

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**6-55** From the solution to Prob. 6-18 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{\text{rev}}$ = 32.8 kpsi, producing $\sigma_a$ = 32.8 kpsi and $\sigma_m$ = 0 kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2200\left(1.625/2\right)}{\pi\left(1.625^4\right)/32} = 2611 \text{ psi} = 2.61 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d$ = 0.0625/1.625 = 0.04, $D/d$ = 1.875/1.625 = 1.15, $K_{t,\text{bend}}$ =1.95, $K_{t,\text{tors}}$ =1.60

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76\left(1.95 - 1\right) = 1.72$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81\left(1.60 - 1\right) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.72)(32.8)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 56.4 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.61)\right]^2\right\}^{1/2} = 6.74 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{56.4 + 6.74} = 1.12$$

From the solution to Prob. 6-18, $S_e$ = 29.5 kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{56.4}{29.5} + \frac{6.74}{85}$$

$$n_f = 0.50 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the $S$-$N$ diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{56.4}{1 - (6.74/85)} = 61.3 \text{ kpsi}$$

Fig. 6-18: $f = 0.867$

Eq. (6-14): $a = \dfrac{\left(f\,S_{ut}\right)^2}{S_e} = \dfrac{\left[0.867(85)\right]^2}{29.5} = 184.1$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.867(85)}{29.5}\right) = -0.1325$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{61.3}{184.1}\right)^{\frac{1}{-0.1325}} = 4022 \text{ cycles}$

$N = 4000$ cycles *Ans.*

---

**6-56** $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi, $K_{ts} = 1.6$, $L = 2$ ft, $F_{\min} = 150$ lbf, $F_{\max} = 500$ lbf

Eqs. (6-34) and (6-35*b*), or Fig. 6-21: $q_s = 0.80$

Eq. (6-32): $K_{fs} = 1 + q_s\left(K_{ts} - 1\right) = 1 + 0.80\left(1.6 - 1\right) = 1.48$

$$T_{\max} = 500(2) = 1000 \text{ lbf}\cdot\text{in}, \quad T_{\min} = 150(2) = 300 \text{ lbf}\cdot\text{in}$$

$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3} = \frac{16(1.48)(1000)}{\pi(0.875)^3} = 11\,251 \text{ psi} = 11.25 \text{ kpsi}$$

$$\tau_{\min} = \frac{16K_{fs}T_{\min}}{\pi d^3} = \frac{16(1.48)(300)}{\pi(0.875)^3} = 3375 \text{ psi} = 3.38 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{11.25 + 3.38}{2} = 7.32 \text{ kpsi}$$

$$\tau_a = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{11.25 - 3.38}{2} = 3.94 \text{ kpsi}$$

Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y/2}{\tau_{\max}} = \frac{30/2}{11.25} = 1.33$$

Find the modifiers and endurance limit.

Eq. (6-8): $S'_e = 0.5(55) = 27.5$ kpsi

Eq. (6-19): $k_a = 14.4(55)^{-0.718} = 0.81$

Eq. (6-24): $d_e = 0.370(0.875) = 0.324$ in

Eq. (6-20): $k_b = 0.879(0.324)^{-0.107} = 0.99$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18): $S_{se} = 0.81(0.99)(0.59)(27.5) = 13.0$ kpsi

Since the stress is entirely shear, we will use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67S_u = 0.67\ (55) = 36.9$ kpsi.

**(a)** Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(3.94/13.0) + (7.32/36.9)} = 1.99 \qquad Ans.$$

**(b)** Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{36.9}{7.32}\right)^2\left(\frac{3.94}{13.0}\right)\left[-1+\sqrt{1+\left(\frac{2(7.32)(13.0)}{36.9(3.94)}\right)^2}\right]$$

$$n_f = 2.49 \qquad Ans.$$

---

**6-57** $S_{ut} = 145$ kpsi, $S_y = 120$ kpsi

From Eqs. (6-34) and (6-35*a*), or Fig. 6-20, with a notch radius of 0.1 in, $q = 0.9$. Thus, with $K_t = 3$ from the problem statement,

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(3-1) = 2.80$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-2.476P) = -1.238P$$

$$T_{\max} = \frac{f\,P\left(D+d\right)}{4} = \frac{0.3P\left(6+1.2\right)}{4} = 0.54P$$

From Eqs. (6-34) and (6-35*b*), or Fig. 6-21, with a notch radius of 0.1 in, $q_s = 0.92$. Thus, with $K_{ts} = 1.8$ from the problem statement,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8-1) = 1.74$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(1.238P / 0.85)^2 + 3(1.385P)^2]^{1/2} = 2.81P$$

$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Eq. (6-8): $S'_e = 0.5(145) = 72.5$ kpsi

Eq. (6-19): $k_a = 2.70(145)^{-0.265} = 0.722$

Eq. (6-20): $k_b = 0.879(1.2)^{-0.107} = 0.862$

Eq. (6-18): $S_e = (0.722)(0.862)(72.5) = 45.12$ kpsi

Modified Goodman: $\dfrac{1}{n_f} = \dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}} = \dfrac{2.81P}{45.12} + \dfrac{2.70P}{145} = \dfrac{1}{3}$

$P = 4.12$ kips *Ans.*

Yield (conservative): $n_y = \dfrac{S_y}{\sigma'_a + \sigma'_m} = \dfrac{120}{(2.81)(4.12) + (2.70)(4.12)} = 5.29$ *Ans.*

---

**6-58** From Prob. 6-57, $K_f = 2.80$, $K_{fs} = 1.74$, $S_e = 45.12$ kpsi

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.80 \frac{4(18)}{\pi(1.2^2)} = -44.56 \text{ kpsi}$$

$$\sigma_{\min} = -K_f \frac{4P_{\min}}{\pi d^2} = -2.80 \frac{4(4.5)}{\pi(1.2)^2} = -11.14 \text{ kpsi}$$

$$T_{\max} = f\, P_{\max}\left(\frac{D+d}{4}\right) = 0.3(18)\left(\frac{6+1.2}{4}\right) = 9.72 \text{ kip}\cdot\text{in}$$

$$T_{\min} = f\, P_{\min}\left(\frac{D+d}{4}\right) = 0.3(4.5)\left(\frac{6+1.2}{4}\right) = 2.43 \text{ kip}\cdot\text{in}$$

$$\tau_{\max} = K_{fs} \frac{16T_{\max}}{\pi d^3} = 1.74 \frac{16(9.72)}{\pi(1.2)^3} = 49.85 \text{ kpsi}$$

$$\tau_{\min} = K_{fs} \frac{16T_{\min}}{\pi d^3} = 1.74 \frac{16(2.43)}{\pi(1.2)^3} = 12.46 \text{ kpsi}$$

$$\sigma_a = \left|\frac{-44.56 - (-11.14)}{2}\right| = 16.71 \text{ kpsi}$$

$$\sigma_m = \frac{-44.56 + (-11.14)}{2} = -27.85 \text{ kpsi}$$

$$\tau_a = \frac{49.85 - 12.46}{2} = 18.70 \text{ kpsi}$$

$$\tau_m = \frac{49.85 + 12.46}{2} = 31.16 \text{ kpsi}$$

Eqs. (6-55) and (6-56):

$$\sigma_a' = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(16.71/0.85)^2 + 3(18.70)^2]^{1/2} = 37.89 \text{ kpsi}$$
$$\sigma_m' = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-27.85)^2 + 3(31.16)^2]^{1/2} = 60.73 \text{ kpsi}$$

Modified Goodman: $\dfrac{1}{n_f} = \dfrac{\sigma_a'}{S_e} + \dfrac{\sigma_m'}{S_{ut}} = \dfrac{37.89}{45.12} + \dfrac{60.73}{145}$

$n_f = 0.79$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a'}{1-(\sigma_m'/S_{ut})} = \frac{37.89}{1-(60.73/145)} = 65.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.8$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.8(145)]^2}{45.12} = 298.2$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.8(145)}{45.12}\right) = -0.1367$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{65.2}{298.2}\right)^{\frac{1}{-0.1367}} = 67\,607$ cycles

$N = 67\,600$ cycles *Ans.*

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**6-59** For AISI 1020 CD, From Table A-20, $S_y = 390$ MPa, $S_{ut} = 470$ MPa. Given: $S_e = 175$ MPa.

First Loading: $(\sigma_m)_1 = \dfrac{360+160}{2} = 260$ MPa, $(\sigma_a)_1 = \dfrac{360-160}{2} = 100$ MPa

Goodman: $(\sigma_a)_{e1} = \dfrac{(\sigma_a)_1}{1-(\sigma_m)_1/S_{ut}} = \dfrac{100}{1-260/470} = 223.8 \text{ MPa} > S_e \therefore$ finite life

$$a = \frac{[0.9(470)]^2}{175} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(470)}{175} = -0.127\,767$$

$$N = \left(\frac{223.8}{1022.5}\right)^{-1/0.127\,767} = 145\ 920 \text{ cycles}$$

Second loading: $(\sigma_m)_2 = \dfrac{320 + (-200)}{2} = 60 \text{ MPa}, \qquad (\sigma_a)_2 = \dfrac{320 - (-200)}{2} = 260 \text{ MPa}$

$$(\sigma_a)_{e2} = \frac{260}{1 - 60/470} = 298.0 \text{ MPa}$$

(**a**) Miner's method: $N_2 = \left(\dfrac{298.0}{1022.5}\right)^{-1/0.127767} = 15\ 520 \text{ cycles}$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{80\ 000}{145\ 920} + \frac{n_2}{15\ 520} = 1 \quad \Rightarrow \quad n_2 = 7000 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method: The number of cycles remaining after the first loading

$N_{\text{remaining}}$ =145 920 − 80 000 = 65 920 cycles

Two data points: 0.9(470) MPa, $10^3$ cycles
223.8 MPa, 65 920 cycles

$$\frac{0.9(470)}{223.8} = \frac{a_2 (10^3)^{b_2}}{a_2 (65\ 920)^{b_2}}$$

$$1.8901 = (0.015170)^{b_2}$$

$$b_2 = \frac{\log 1.8901}{\log 0.015170} = -0.151\ 997$$

$$a_2 = \frac{223.8}{(65\ 920)^{-0.151\,997}} = 1208.7 \text{ MPa}$$

$$n_2 = \left(\frac{298.0}{1208.7}\right)^{1/-0.151\,997} = 10\ 000 \text{ cycles} \qquad \textit{Ans.}$$

---

**6-60** Given: $S_e$ = 50 kpsi, $S_{ut}$ = 140 kpsi, $f$ =0.8. Using Miner's method,

$$a = \frac{[0.8(140)]^2}{50} = 250.88 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.8(140)}{50} = -0.116\ 749$$

$$\sigma_1 = 95 \text{ kpsi}, \quad N_1 = \left(\frac{95}{250.88}\right)^{1/-0.116\ 749} = 4100 \text{ cycles}$$

$$\sigma_2 = 80 \text{ kpsi}, \quad N_2 = \left(\frac{80}{250.88}\right)^{1/-0.116\ 749} = 17\ 850 \text{ cycles}$$

$$\sigma_3 = 65 \text{ kpsi}, \quad N_3 = \left(\frac{65}{250.88}\right)^{1/-0.116\ 749} = 105\ 700 \text{ cycles}$$

$$\frac{0.2N}{4100} + \frac{0.5N}{17\ 850} + \frac{0.3N}{105\ 700} = 1 \quad \Rightarrow \quad N = 12\ 600 \text{ cycles} \quad \textit{Ans.}$$

---

**6-61** Given: $S_{ut}$ = 530 MPa, $S_e$ = 210 MPa, and $f$ = 0.9.

**(a)** Miner's method

$$a = \frac{[0.9(530)]^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(530)}{210} = -0.118\ 766$$

$$\sigma_1 = 350 \text{ MPa}, \quad N_1 = \left(\frac{350}{1083.47}\right)^{1/-0.118\ 766} = 13\ 550 \text{ cycles}$$

$$\sigma_2 = 260 \text{ MPa}, \quad N_2 = \left(\frac{260}{1083.47}\right)^{1/-0.118\ 766} = 165\ 600 \text{ cycles}$$

$$\sigma_3 = 225 \text{ MPa}, \quad N_3 = \left(\frac{225}{1083.47}\right)^{1/-0.118\ 766} = 559\ 400 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{13\ 550} + \frac{50\ 000}{165\ 600} + \frac{n_3}{559\ 400} = 184\ 100 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method:

The life remaining after the first series of cycling is $N_{R1}$ = 13 550 – 5000 = 8550 cycles. The two data points required to define $S'_{e,1}$ are [0.9(530), $10^3$] and (350, 8550).

$$\frac{0.9(530)}{350}=\frac{a_2\left(10^3\right)^{b_2}}{a_2\left(8550\right)^{b_2}} \quad \Rightarrow \quad 1.3629=\left(0.11696\right)^{b_2}$$

$$b_2=\frac{\log(1.362\ 9)}{\log(0.116\ 96)}=-0.144\,280$$

$$a_2=\frac{350}{(8550)^{-0.144\ 280}}=1292.3\ \text{MPa}$$

$$N_2=\left(\frac{260}{1292.3}\right)^{-1/0.144\ 280}=67\ 090\ \text{cycles}$$

$$N_{R2}=67\ 090-50\ 000=17\ 090\ \text{cycles}$$

$$\frac{0.9(530)}{260}=\frac{a_3\left(10^3\right)^{b_3}}{a_3\left(17\ 090\right)^{b_3}} \quad \Rightarrow \quad 1.834\ 6=\left(0.058\ 514\right)^{b_2}$$

$$b_3=\frac{\log(1.834\ 6)}{\log(0.058\ 514)}=-0.213\ 785, \quad a_3=\frac{260}{(17\ 090)^{-0.213\ 785}}=2088.7\ \text{MPa}$$

$$N_3=\left(\frac{225}{2088.7}\right)^{-1/0.213\ 785}=33\ 610\ \text{cycles} \qquad \textit{Ans.}$$

---

**6-62** Given: $S_e$ = 45 kpsi, $S_{ut}$ = 85 kpsi, $f$ = 0.86, and $\sigma_a$ = 35 kpsi and $\sigma_m$ = 30 kpsi for 12 ($10^3$) cycles.

Gerber equivalent reversing stress: $\sigma_{\text{rev}}=\dfrac{\sigma_a}{1-\left(\sigma_m/S_{ut}\right)^2}=\dfrac{35}{1-\left(30/85\right)^2}=39.98$ kpsi

(**a**) Miner's method: $\sigma_{\text{rev}} < S_e$. According to the method, this means that the endurance limit has not been reduced and the new endurance limit is $S'_e$ = 45 kpsi. Ans.

(**b**) Manson's method: Again, $\sigma_{\text{rev}} < S_e$. According to the method, this means that the material has not been damaged and the endurance limit has not been reduced. Thus, the new endurance limit is $S'_e$ = 45 kpsi. Ans.

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**6-63** Given: $S_e$ = 45 kpsi, $S_{ut}$ = 85 kpsi, $f$ = 0.86, and $\sigma_a$ = 35 kpsi and $\sigma_m$ = 30 kpsi for 12 ($10^3$) cycles.

Goodman equivalent reversing stress: $\sigma_{\text{rev}}=\dfrac{\sigma_a}{1-\left(\sigma_m/S_{ut}\right)}=\dfrac{35}{1-\left(30/85\right)}=54.09$ kpsi

Initial cycling

$$a = \frac{\left[0.86(85)\right]^2}{45} = 116.00 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.86(85)}{45} = -0.070\ 235$$

$$\sigma_1 = 54.09 \text{ kpsi}, \qquad N_1 = \left(\frac{54.09}{116.00}\right)^{1/-0.070\ 235} = 52\ 190 \text{ cycles}$$

(**a**) Miner's method (see discussion on p. 325): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}}$ = 52 190 − 12 000 = 40 190 cycles. The new coefficients are $b' = b$, and $a' = S_f / N^b$ = 54.09/(40 190)$^{-0.070\ 235}$ = 113.89 kpsi. The new endurance limit is

$$S'_{e,1} = a'N_e^{b'} = 113.89\left(10^6\right)^{-0.070\ 235} = 43.2 \text{ kpsi} \qquad \textit{Ans.}$$

(**b**) Manson's method (see discussion on p. 326): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}}$ = 52 190 − 12 000 = 40 190 cycles. At $10^3$ cycles, $S_f$ = 0.86(85) = 73.1 kpsi. The new coefficients are $b'$ = [log(73.1/54.09)]/log($10^3$/40 190) = − 0.081 540 and $a' = \sigma_1 / (N_{\text{remaining}})^{b'}$ = 54.09/(40 190)$^{-0.081\ 540}$ = 128.39 kpsi. The new endurance limit is

$$S'_{e,1} = a'N_e^{b'} = 128.39\left(10^6\right)^{-0.081\ 540} = 41.6 \text{ kpsi} \qquad \textit{Ans.}$$

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**6-64** Given $\mathbf{S}_{ut}$ =1030**LN**(1, 0.0508) MPa

From Table 6-10: $a$ = 1.58, $b$ = − 0.086, $C$ = 0.120

Eq. (6-72) and Table 6-10): $\mathbf{k}_a = 1.58\left(1030\right)^{-0.086}\mathbf{LN}\left(1,\ 0.120\right) = 0.870\mathbf{LN}\left(1,\ 0.120\right)$

From Prob. 6-1: $k_b$ = 0.97

Eqs. (6-70) and (6-71): $\mathbf{S}_e$ = [0.870**LN**(1, 0.120)] (0.97) [0.506(1030)**LN**(1, 0.138)]

$\bar{S}_e$ = 0.870 (0.97)(0.506)(1030) = 440 MPa

and, $C_{Se} \doteq (0.12^2 + 0.138^2)^{1/2} = 0.183$

$\mathbf{S}_e$ =440**LN**(1, 0.183) MPa *Ans.*

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**6-65** A Priori Decisions:

- Material and condition: 1020 CD, $\mathbf{S}_{ut}$ = 68 **LN**(1, 0.28), and $\mathbf{S}_y$ = 57 **LN**(1, 0.058) kpsi
- Reliability goal: $R = 0.99$ ($z = -2.326$, Table A-10)
- Function:
  Critical location—hole
- Variabilities:

$$C_{ka} = 0.058$$
$$C_{kc} = 0.125$$
$$C_{S'_e} = 0.138$$
$$C_{Se} = \left(C_{ka}^2 + C_{kc}^2 + C_{S'_e}^2\right)^{1/2} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$
$$C_{Kf} = 0.10$$
$$C_{Fa} = 0.20$$
$$C_{\sigma a} = (0.10^2 + 0.20^2)^{1/2} = 0.234$$
$$C_n = \sqrt{\frac{C_{Se}^2 + C_{\sigma a}^2}{1 + C_{\sigma a}^2}} = \sqrt{\frac{0.195^2 + 0.234^2}{1 + 0.234^2}} = 0.297$$

Resulting in a design factor $n_f$ of,

Eq. (6-59): $n_f = \exp[-(-2.326)\sqrt{\ln(1+0.297^2)} + \ln\sqrt{1+0.297^2}\,] = 2.05$

- Decision: Set $n_f = 2.05$

Now proceed deterministically using the mean values:

Table 6-10: $\bar{k}_a = 2.67\left(68\right)^{-0.265} = 0.873$

Eq. (6-21): $k_b = 1$

Table 6-11: $\bar{k}_c = 1.23\left(68\right)^{-0.0778} = 0.886$

Eq. (6-70): $\bar{S}'_e = 0.506\left(68\right) = 34.4$ kpsi

Eq. (6-71): $\bar{S}_e = 0.873\left(1\right)\left(0.886\right)34.4 = 26.6$ kpsi

From Prob. 6-14, $K_f = 2.26$. Thus,

$$\bar{\sigma}_a = \bar{K}_f \frac{\bar{F}_a}{A} = \bar{K}_f \frac{\bar{F}_a}{t(2.5-0.5)} = \bar{K}_f \frac{\bar{F}_a}{2t} = \frac{\bar{S}_e}{\bar{n}_f}$$

$$\therefore t = \frac{\bar{n}_f \bar{K}_f \bar{F}_a}{2\bar{S}_e} = \frac{2.05(2.26)3.8}{2(26.6)} = 0.331 \text{ in}$$

*Decision:* Use $t = \frac{3}{8}$ in *Ans.*

---

**6-66** Rotation is presumed. $M$ and $S_{ut}$ are given as deterministic, but notice that $\boldsymbol{\sigma}$ is not; therefore, a reliability estimation can be made.

From Eq. (6-70): $\mathbf{S}'_e = 0.506(780)\mathbf{LN}(1, 0.138) = 394.7\,\mathbf{LN}(1, 0.138)$

Table 6-13: $\mathbf{k}_a = 4.45(780)^{-0.265}\mathbf{LN}(1, 0.058) = 0.762\,\mathbf{LN}(1, 0.058)$
Based on $d = 32 - 6 = 26$ mm, Eq. (6-20) gives

$$k_b = \left(\frac{26}{7.62}\right)^{-0.107} = 0.877$$

Conservatism is not necessary

$$\mathbf{S}_e = \left[0.762\mathbf{LN}(1,\ 0.058)\right](0.877)(394.7)\left[\mathbf{LN}(1,\ 0.138)\right]$$
$$\bar{S}_e = 263.8 \text{ MPa}$$
$$C_{Se} = (0.058^2 + 0.138^2)^{1/2} = 0.150$$
$$\mathbf{S}_e = 263.8\mathbf{LN}(1,\ 0.150) \text{ MPa}$$

Fig. A-15-14: $D/d = 32/26 = 1.23$, $r/d = 3/26 = 0.115$. Thus, $K_t \doteq 1.75$, and Eq. (6-78) and Table 6-15 gives

$$\bar{K}_f = \frac{K_t}{1 + \dfrac{2(K_t - 1)}{K_t}\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1.75}{1 + \dfrac{2(1.75-1)}{1.75}\dfrac{104/780}{\sqrt{3}}} = 1.64$$

From Table 6-15, $C_{Kf} = 0.15$. Thus,

$$\mathbf{K}_f = 1.64\mathbf{LN}(1, 0.15)$$

The bending stress is

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{32M}{\pi d^3} = 1.64\mathbf{LN}(1,\ 0.15)\left[\frac{32(160)}{\pi(0.026)^3}\right]$$
$$= 152\left(10^6\right)\mathbf{LN}(1,\ 0.15) \text{ Pa} = 152\mathbf{LN}(1,\ 0.15) \text{ MPa}$$

From Eq. (5-43), p. 250,

$$z = -\frac{\ln\left(\dfrac{\mu_S}{\mu_\sigma}\sqrt{\dfrac{1+C_\sigma^2}{1+C_S^2}}\right)}{\sqrt{\ln\left[\left(1+C_S^2\right)\left(1+C_\sigma^2\right)\right]}}$$

$$= -\frac{\ln\left[\left(263.8/152\right)\sqrt{\left(1+0.15^2\right)/\left(1+0.15^2\right)}\right]}{\sqrt{\ln\left[\left(1+0.15^2\right)\left(1+0.15^2\right)\right]}} = -2.61$$

From Table A-10, $p_f$ = 0.004 53. Thus, $R$ = 1 − 0.004 53 = 0.995 *Ans.*

*Note:* The correlation method uses only the mean of $\mathbf{S}_{ut}$ ; its variability is already included in the 0.138. When a deterministic load, in this case *M*, is used in a reliability estimate, engineers state, “For a *Design* Load of *M*, the reliability is 0.995.” They are, in fact, referring to a Deterministic Design Load.

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**6-67** For completely reversed torsion, $\mathbf{k}_a$ and $k_b$ of Prob. 6-66 apply, but $\mathbf{k}_c$ must also be considered. $\bar{S}_{ut}$ = 780/6.89 = 113 kpsi

Eq. 6-74: $\mathbf{k}_c = 0.328(113)^{0.125}\mathbf{LN}(1, 0.125) = 0.592\mathbf{LN}(1, 0.125)$

Note 0.590 is close to 0.577.

$$\mathbf{S}_e = \mathbf{k}_a k_b \mathbf{k}_c \mathbf{S}'_e$$
$$= 0.762[\mathbf{LN}(1,\ 0.058)](0.877)[0.592\mathbf{LN}(1,\ 0.125)][394.7\mathbf{LN}(1,\ 0.138)]$$
$$\bar{S}_e = 0.762(0.877)(0.592)(394.7) = 156.2 \text{ MPa}$$
$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$
$$\mathbf{S}_e = 156.2\mathbf{LN}(1,\ 0.195) \text{ MPa}$$

Fig. A-15-15: $D/d$ = 1.23, $r/d$ = 0.115, then $K_{ts} \doteq 1.40$. From Eq. (6-78) and Table 7-8

$$\bar{K}_{fs} = \frac{K_{ts}}{1+\dfrac{2\left(K_{ts}-1\right)}{K_{ts}}\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1.40}{1+\dfrac{2\left(1.40-1\right)}{1.40}\dfrac{104/780}{\sqrt{3}}} = 1.34$$

From Table 6-15, $C_{Kf}$ = 0.15. Thus,

$$\mathbf{K}_{fs} = 1.34\mathbf{LN}(1, 0.15)$$

The torsional stress is

$$\boldsymbol{\tau} = \mathbf{K}_{fs}\frac{16T}{\pi d^3} = 1.34\mathbf{LN}(1,\ 0.15)\left[\frac{16\left(160\right)}{\pi\left(0.026\right)^3}\right]$$
$$= 62.1\left(10^6\right)\mathbf{LN}(1,\ 0.15) \text{ Pa} = 62.1\mathbf{LN}(1,\ 0.15) \text{ MPa}$$

From Eq. (5-43), p. 250,

$$z = -\frac{\ln\left[(156.2/62.1)\sqrt{(1+0.15^2)/(1+0.195^2)}\right]}{\sqrt{\ln[(1+0.195^2)(1+0.15^2)]}} = -3.75$$

From Table A-10, $p_f = 0.000\ 09$

$$R = 1 - p_f = 1 - 0.000\ 09 = 0.999\ 91 \quad \textit{Ans.}$$

For a design with completely-reversed torsion of 160 N · m, the reliability is 0.999 91. The improvement over bending comes from a smaller stress-concentration factor in torsion. See the note at the end of the solution of Prob. 6-66 for the reason for the phraseology.

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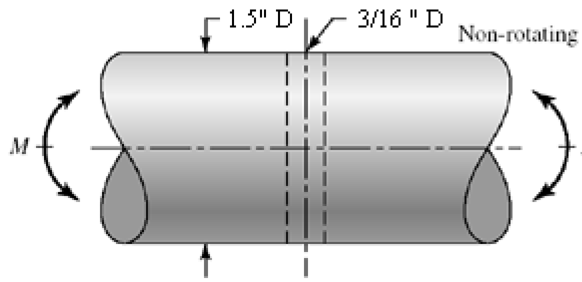
**6-68**




Given: $S_{ut} = 58$ kpsi.

Eq. (6-70): $\mathbf{S}'_e = 0.506(76)\ \mathbf{LN}(1, 0.138) = 38.5\ \mathbf{LN}(1, 0.138)$ kpsi

Table 6-13: $\mathbf{k}_a = 14.5(76)^{-0.719}\ \mathbf{LN}(1, 0.11) = 0.644\ \mathbf{LN}(1, 0.11)$

Eq. (6-24): $d_e = 0.370(1.5) = 0.555$ in

Eq. (6-20): $k_b = (0.555/0.3)^{-0.107} = 0.936$

Eq. (6-70): $\mathbf{S}_e = [0.644\ \mathbf{LN}(1, 0.11)](0.936)[38.5\ \mathbf{LN}(1, 0.138)]$

$$\bar{S}_e = 0.644(0.936)(38.5) = 23.2 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.138^2)^{1/2} = 0.176$$

$$\mathbf{S}_e = 23.2\ \mathbf{LN}(1, 0.176) \text{ kpsi}$$

Table A-16: $d/D = 0$, $a/D = (3/16)/1.5 = 0.125$, $A = 0.80$ $\therefore$ $K_t = 2.20$.

From Eqs. (6-78) and (6-79) and Table 6-15

$$K_f = \frac{2.20\mathbf{LN}(1,\ 0.10)}{1 + \frac{2(2.20-1)}{2.20}\frac{5/76}{\sqrt{0.125}}} = 1.83\mathbf{LN}(1,\ 0.10)$$

Table A-16:

$$Z_{\text{net}} = \frac{\pi AD^3}{32} = \frac{\pi(0.80)(1.5^3)}{32} = 0.265 \text{ in}^3$$

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{M}{Z_{\text{net}}} = 1.83\mathbf{LN}(1,\ 0.10)\left(\frac{1.5}{0.265}\right)$$

$$= 10.4\mathbf{LN}(1,\ 0.10) \text{ kpsi}$$

$$\bar{\sigma} = 10.4 \text{ kpsi}$$

$$C_\sigma = 0.10$$

Eq. (5-43), p. 250: $$z = -\frac{\ln\left[(23.2/10.4)\sqrt{(1+0.10^2)/(1+0.176^2)}\right]}{\sqrt{\ln[(1+0.176^2)(1+0.10^2)]}} = -3.94$$

Table A-10: $p_f = 0.000\ 041\ 5 \Rightarrow R = 1 - p_f = 1 - 0.000\ 041\ 5 = 0.999\ 96$ *Ans.*

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**6-69** From Prob. 6-68: $\mathbf{S}'_e = 23.2\ \mathbf{LN}(1, 0.138)$ kpsi

$\mathbf{k}_a = 0.644\mathbf{LN}(1, 0.11)$

$k_b = 0.936$

Eq. (6-74): $\mathbf{k}_c = 0.328(76)^{0.125}\mathbf{LN}(1, 0.125) = 0.564\ \mathbf{LN}(1, 0.125)$

Eq. (6-71): $\mathbf{S}_e = [0.644\mathbf{LN}(1, 0.11)](0.936)[0.564\ \mathbf{LN}(1, 0.125)][23.2\ \mathbf{LN}(1, 0.138)]$

$$\bar{S}_e = 0.644(0.936)(0.564)(23.2) = 7.89 \text{ kpsi}$$
$$C_{Se} = (0.11^2 + 0.125^2 + 0.138^3)^{1/2} = 0.216$$

Table A-16: $d/D = 0$, $a/D = (3/16)/1.5 = 0.125$, $A = 0.89$, $\boldsymbol{K}_{ts} = 1.64$
From Eqs. (6-78) and(7-79), and Table 6-15

$$\mathbf{K}_{fs} = \frac{1.64\mathbf{LN}(1,\ 0.10)}{1 + \frac{2(1.64-1)}{1.64}\frac{5/76}{\sqrt{3/32}}} = 1.40\mathbf{LN}(1,\ 0.10)$$

Table A-16:

$$J_{\text{net}} = \frac{\pi AD^4}{32} = \frac{\pi(0.89)(1.5^4)}{32} = 0.4423 \text{ in}^4$$

$$\tau_a = \mathbf{K}_{fs}\frac{T_a D}{2J_{\text{net}}} = 1.40[\mathbf{LN}(1,\ 0.10)]\frac{2(1.5)}{2(0.4423)} = 4.75\mathbf{LN}(1,\ 0.10) \text{ kpsi}$$

From Eq. (6-57):

$$z = -\frac{\ln(7.89/4.75)\sqrt{(1+0.10^2)/(1+0.216^2)}}{\sqrt{\ln[(1+0.10^2)(1+0.216^2)]}} = -2.08$$

Table A-10, $p_f = 0.0188$, $R = 1 - p_f = 1 - 0.0188 = 0.981$ *Ans.*

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**6-70** This is a very important task for the student to attempt before starting Part 3. It illustrates the drawback of the deterministic factor of safety method. It also identifies the a priori decisions and their consequences.

The range of force fluctuation in Prob. 6-30 is – 16 to + 5 kip, or 21 kip. Let the repeatedly-applied $F_a$ be 10.5 kip. The stochastic properties of this heat of AISI 1018 CD are given in the problem statement.

| Function | Consequences |
|---|---|
| Axial | $F_a = 10.5$ kip |
| Fatigue load | $C_{Fa} = 0$ |
| | $C_{kc} = 0.125$ |
| Overall reliability $R \geq 0.998$; | $z = -3.09$ |
| with twin fillets | $C_{Kf} = 0.11$ |
| $R \geq \sqrt{0.998} = 0.999$ | |
| Cold rolled or machined surfaces | $C_{ka} = 0.058$ |
| Ambient temperature | $C_{kd} = 0$ |
| Use correlation method | $C_\phi = 0.138$ |
| Stress amplitude | $C_{Kf} = 0.11$ |
| | $C_{\sigma a} = 0.11$ |
| Significant strength $S_e$ | $C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$ |

Choose the mean design factor which will meet the reliability goal. From Eq. (6-88)

$$C_n = \sqrt{\frac{0.195^2 + 0.11^2}{1+0.11^2}} = 0.223$$

$$\bar{n} = \exp\left[-(-3.09)\sqrt{\ln(1+0.223^2)} + \ln\sqrt{1+0.223^2}\right]$$

$$\bar{n} = 2.02$$

In Prob. 6-30, it was found that the hole was the significant location that controlled the analysis. Thus,

$$\boldsymbol{\sigma}_a = \frac{\mathbf{S}_e}{\mathbf{n}}$$

$$\bar{\sigma}_a = \frac{\bar{S}_e}{\bar{n}} \quad \Rightarrow \quad \bar{K}_f \frac{F_a}{h(w_1 - d)} = \frac{\bar{S}_e}{\bar{n}}$$

We need to determine $\bar{S}_e$

$$\bar{k}_a = 2.67\bar{S}_{ut}^{-0.265} = 2.67(64)^{-0.265} = 0.887$$
$$k_b = 1$$
$$\bar{k}_c = 1.23\bar{S}_{ut}^{-0.0778} = 1.23(64)^{-0.0778} = 0.890$$
$$\bar{k}_d = \bar{k}_e = 1$$
$$\bar{S}_e = 0.887(1)(0.890)(1)(1)(0.506)(64) = 25.6 \text{ kpsi}$$

From the solution to Prob. 6-30, the stress concentration factor at the hole is $K_t = 2.68$. From Eq. (6-78) and Table 6-15

$$\bar{K}_f = \frac{2.68}{1 + \dfrac{2(2.68-1)}{2.68}\dfrac{5/64}{\sqrt{0.2}}} = 2.20$$

$$h = \frac{\bar{K}_f \bar{n} F_a}{(w_1 - d)\bar{S}_e} = \frac{2.20(2.02)(10.5)}{(3.5-0.4)(25.6)} = 0.588 \qquad \textit{Ans.}$$

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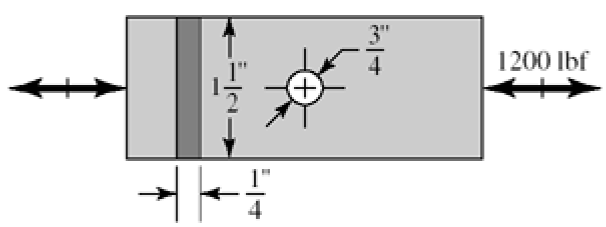
**6-71**

$$F_a = 1200 \text{ lbf}$$
$$S_{ut} = 80 \text{ kpsi}$$




**(a)** *Strength*

$\mathbf{k}_a = 2.67(80)^{-0265}\mathbf{LN}(1, 0.058) = 0.836\ \mathbf{LN}(1, 0.058)$

$k_b = 1$

$\mathbf{k}_c = 1.23(80)^{-0.0778}\mathbf{LN}(1, 0.125) = 0.875\ \mathbf{LN}(1, 0.125)$

$$\mathbf{S}'_e = 0.506(80)\mathbf{LN}(1,\ 0.138) = 40.5\mathbf{LN}(1,\ 0.138)\text{ kpsi}$$

$$\mathbf{S}_e = \left[0.836\mathbf{LN}(1,\ 0.058)\right](1)\left[0.875\mathbf{LN}(1,\ 0.125)\right]\left[40.5\mathbf{LN}(1,\ 0.138)\right]$$

$$\bar{S}_e = 0.836(1)(0.875)(40.5) = 29.6\text{ kpsi}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

*Stress:* Fig. A-15-1; $d/w = 0.75/1.5 = 0.5$, $K_t = 2.18$. From Eqs. (6-78), (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.18\mathbf{LN}(1,\ 0.10)}{1+\dfrac{2(2.18-1)}{2.18}\dfrac{5/80}{\sqrt{0.375}}} = 1.96\mathbf{LN}(1,\ 0.10)$$

$$\boldsymbol{\sigma}_a = \mathbf{K}_f \frac{F_a}{(w-d)t}, \quad C_\sigma = 0.10$$

$$\bar{\sigma}_a = \frac{\bar{K}_f F_a}{(w-d)t} = \frac{1.96(1.2)}{(1.5-0.75)(0.25)} = 12.54\text{ kpsi}$$

$$\bar{S}_a = \bar{S}_e = 29.6\text{ kpsi}$$

$$z = -\frac{\ln\left[(\bar{S}_a/\bar{\sigma}_a)\sqrt{\left(1+C_\sigma^2\right)\big/\left(1+C_S^2\right)}\right]}{\ln\left[\left(1+C_\sigma^2\right)\left(1+C_S^2\right)\right]}$$

$$= -\frac{\ln\left[\left(29.6/12.48\right)\sqrt{\left(1+0.10^2\right)/\left(1+0.195^2\right)}\right]}{\sqrt{\ln\left[\left(1+0.10^2\right)\left(1+0.195^2\right)\right]}} = -3.9$$

From Table A-20, $p_f = 4.81(10^{-5}) \Rightarrow R = 1 - 4.81(10^{-5}) = 0.999\ 955$ *Ans.*

**(b)** All computer programs will differ in detail.

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**6-72 to 6-78** Computer programs are very useful for automating specific tasks in the design process. All computer programs will differ in detail.