Chapter 5

Materials selection — the basics

5.1 Introduction and synopsis

This chapter sets out the basic procedure for selection, establishing the link between material and function (Figure 5.1). A material has *attributes*: its density, strength, cost, resistance to corrosion, and so forth. A design demands a certain profile of these: a low density, a high strength, a modest cost and resistance to sea water, perhaps. The problem is that of identifying the desired attribute profile and then comparing it with those of real engineering materials to find the best match. This we do by, first, *screening and ranking* the candidates to give a shortlist, and then seeking detailed *supporting information* for each shortlisted candidate, allowing a final choice. It is important to start with the full menu of materials in mind; failure to do so may mean a missed opportunity. If an innovative choice is to be made, it must be identified early in the design process. Later, too many decisions have been taken and commitments made to allow radical change: it is now or never.

The immensely wide choice is narrowed, first, by applying *property limits* which screen out the materials which cannot meet the design requirements. Further narrowing is achieved by ranking the candidates by their ability to maximize performance. Performance is generally limited not by a single property, but by a combination of them. The best materials for a light stiff tie-rod are those with the greatest value of the 'specific stiffness', E/ρ, where E is Young's modulus and ρ the density. The best materials for a spring, regardless of its shape or the way it is loaded, are those with the greatest value of σ_f^2/E, where σ_f is the failure stress. The materials which best resist thermal shock are those with the largest value of $\sigma_f/E\alpha$, where α is the thermal coefficient of expansion; and so forth. Combinations such as these are called *material indices*: they are groupings of material properties which, when maximized, maximize some aspect of performance. There are many such indices. They are derived from the design requirements for a component by an analysis of *function*, *objectives* and *constraints*. This chapter explains how to do this.

The materials property charts introduced in Chapter 4 are designed for use with these criteria. Property limits and material indices are plotted onto them, isolating the subset of materials which are the best choice for the design. The procedure is fast, and makes for lateral thinking. Examples of the method are given in Chapter 6.

5.2 The selection strategy

Material attributes

Figure 5.2 illustrates how the Kingdom of Materials can be subdivided into families, classes, subclasses and members. Each member is characterized by a set of attrributes: its properties. As an example, the Materials Kingdom contains the family 'Metals' which in turn contains the class 'Aluminium alloys', the subclass '5000 series' and finally the particular member 'Alloy 5083 in the

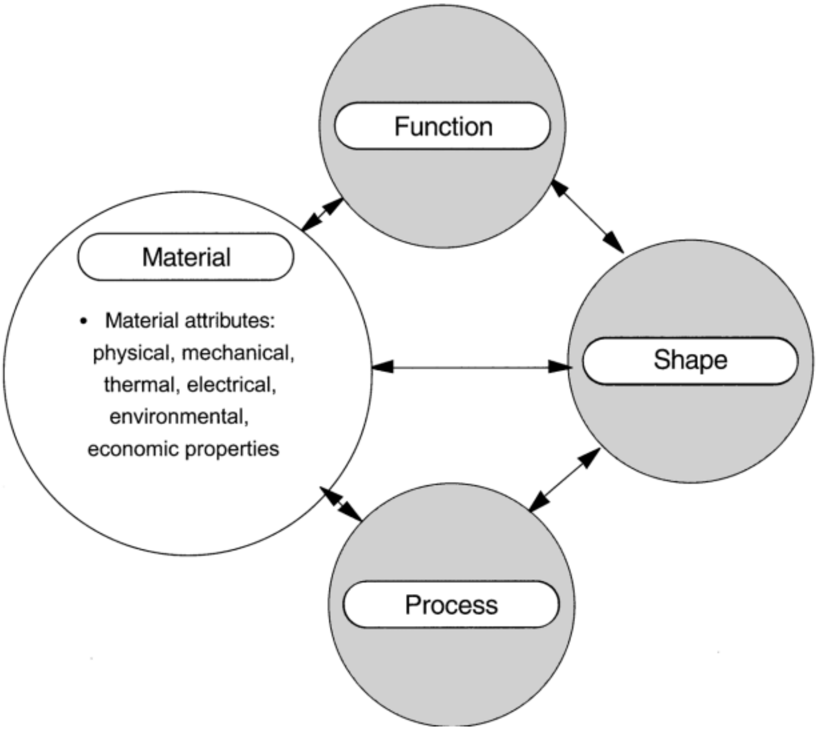

Fig. 5.1 Material selection is determined by function. Shape sometimes influences the selection. This chapter and the next deal with materials selection when this is independent of shape.

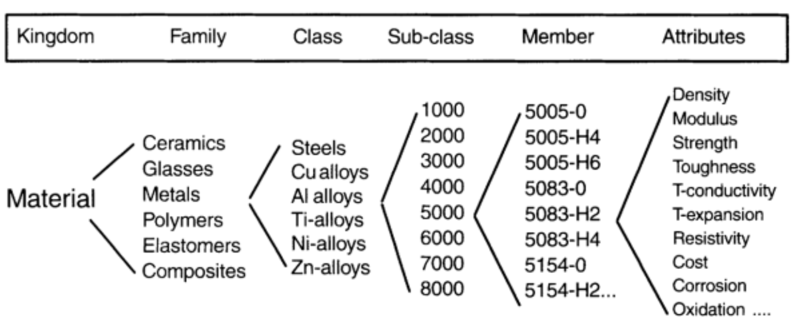

Fig. 5.2 The taxonomy of the kingdom of materials and their attributes.

H2 heat treatment condition'. It, and every other member of the materials kingdom, is characterized by a set of attributes which include its mechanical, thermal, electrical and chemical properties, its processing characteristics, its cost and availability, and the environmental consequences of its use. We call this its *property-profile*. Selection involves seeking the best match between the property-profile of materials in the kingdom and that required by the design.

There are two main steps which we here call *screening and ranking*, and *supporting information* (Figure 5.3). The two steps can be likened to those in selecting a candidate for a job. The job is first advertised, defining essential skills and experience ('essential attributes'), screening-out potential

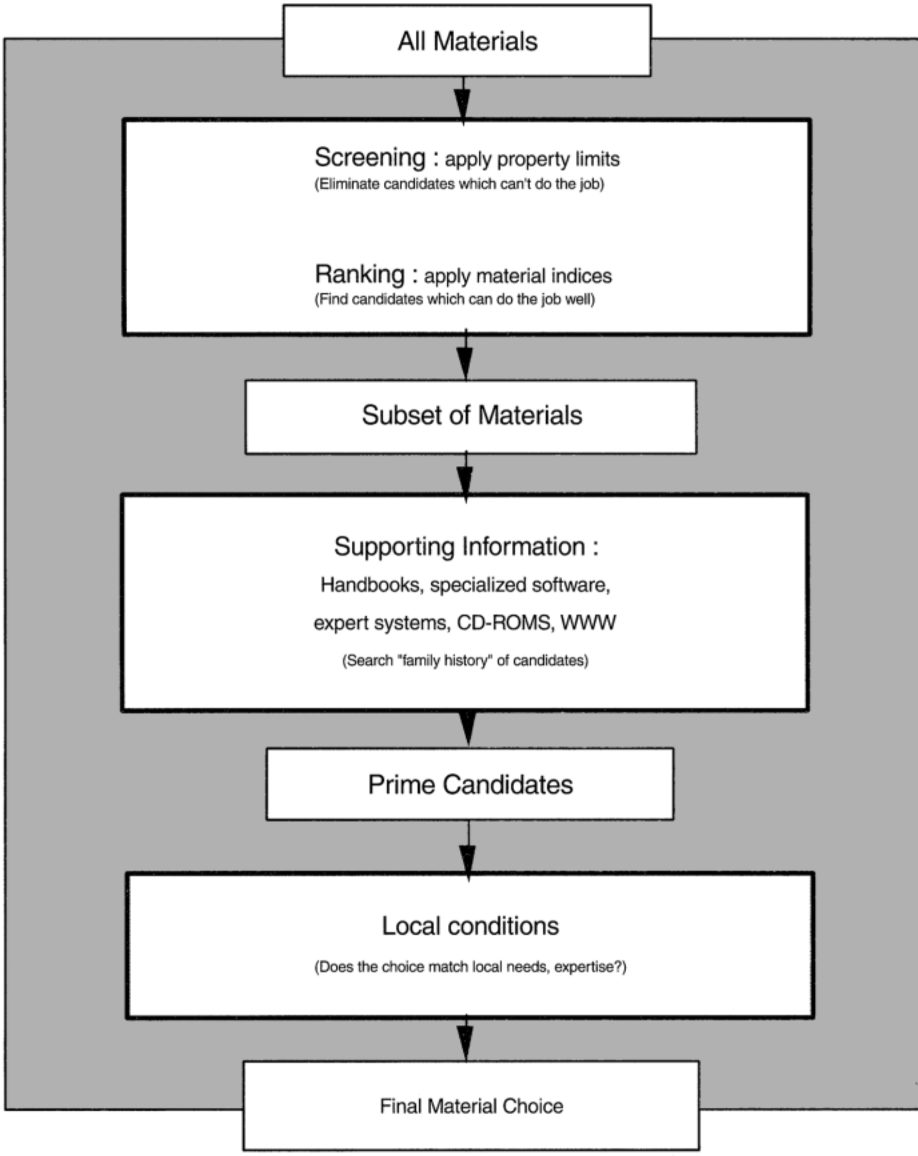

Fig. 5.3 The strategy for materials selection. The main steps are enclosed in bold boxes.

applicants whose attribute-profile does not match the job requirements and allowing a shortlist to be drawn up. References and interviews are then sought for the shortlisted candidates, building a file of supporting information.

Screening and ranking

Unbiased selection requires that all materials are considered to be candidates until shown to be otherwise, using the steps detailed in the boxes of Figure 5.3. The first of these, *screening,* eliminates candidates which cannot do the job at all because one or more of their attributes lies outside the limits imposed by the design. As examples, the requirement that 'the component must function at 250°C', or that 'the component must be transparent to light' imposes obvious limits on the attributes of *maximum service temperature* and *optical transparency* which successful candidates must meet. We refer to these as property limits. They are the analogue of the job advertisement which requires that the applicant 'must have a valid driving licence', or 'a degree in computer science', eliminating anyone who does not.

Property limits do not, however, help with ordering the candidates that remain. To do this we need optimization criteria. They are found in the material indices, developed below, which measure how well a candidate which has passed the limits can do the job. Familiar examples of indices are the specific stiffness E/ρ and the specific strength σ_f/ρ (E is the Young's modulus, σ_f is the failure strength and ρ is the density). The materials with the largest values of these indices are the best choice for a light, stiff tie-rod, or a light, strong tie-rod respectively. There are many others, each associated with maximizing some aspect of performance*. They allow ranking of materials by their ability to perform well in the given application. They are the analogue of the job advertisement which states that 'typing speed and accuracy are a priority', or that 'preference will be given to candidates with a substantial publication list', implying that applicants will be ranked by these criteria.

To summarize: property limits isolate candidates which are capable of doing the job; material indices identify those among them which can do the job well.

Supporting information

The outcome of the screening step is a shortlist of candidates which satisfy the quantifiable requirements of the design. To proceed further we seek a detailed profile of each: its *supporting information* (Figure 5.3, second heavy box).

Supporting information differs greatly from the property data used for screening. Typically, it is descriptive, graphical or pictorial: case studies of previous uses of the material, details of its corrosion behaviour in particular environments, information of availability and pricing, experience of its environmental impact. Such information is found in handbooks, suppliers data sheets, CD-based data sources and the World-Wide Web. Supporting information helps narrow the shortlist to a final choice, allowing a definitive match to be made between design requirements and material attributes. The parallel, in filling a job, is that of taking up references and conducting interviews — an opportunity to probe deeply into the character and potential of the candidate.

Without screening, the candidate-pool is enormous; there is an ocean of supporting information, and dipping into this gives no help with selection. But once viable candidates have been identified by screening, supporting information is sought for these few alone. The *Encyclopaedia Britannica* is an example of a source of supporting information; it is useful if you know what you are looking for, but overwhelming in its detail if you do not.

Local conditions

The final choice between competing candidates will often depend on local conditions: on the existing in-house expertise or equipment, on the availability of local suppliers, and so forth. A systematic procedure cannot help here — the decision must instead be based on local knowledge. This does

* Maximizing performance often means *minimizing* something: cost is the obvious example; mass, in transport systems, is another. A low-cost or light component, here, improves performance. Chapter 6 contains examples of both.

not mean that the result of the systematic procedure is irrelevant. It is always important to know which material is best, even if, for local reasons, you decide not to use it.

We will explore supporting information more fully in Chapter 13. Here we focus on the derivation of property limits and indices.

5.3 Deriving property limits and material indices

How are the design requirements for a component (which define what it must do) translated into a prescription for a material? To answer this we must look at the *function* of the component, the *constraints* it must meet, and the *objectives* the designer has selected to optimize its performance.

Function, objectives and constraints

Any engineering component has one or more *functions*: to support a load, to contain a pressure, to transmit heat, and so forth. In designing the component, the designer has an *objective*: to make it as cheap as possible, perhaps, or as light, or as safe, or perhaps some combination of these. This must be achieved subject to *constraints*: that certain dimensions are fixed, that the component must carry the given load or pressure without failure, that it can function in a certain range of temperature, and in a given environment, and many more. Function, objective and constraints (Table 5.1) define the boundary conditions for selecting a material and — in the case of load-bearing components — a shape for its cross-section.

Let us elaborate a little using the simplest of mechanical components as examples, helped by Figure 5.4. The loading on a component can generally be decomposed into some combination of axial tension or compression, bending, and torsion. Almost always, one mode dominates. So common is this that the functional name given to the component describes the way it is loaded: *ties* carry tensile loads; *beams* carry bending moments; *shafts* carry torques; and *columns* carry compressive axial loads. The words 'tie', 'beam', 'shaft' and 'column' each imply a function. Many simple engineering functions can be described by single words or short phrases, saving the need to explain the function in detail. In designing any one of these the designer has an objective: to make it as light as possible, perhaps (aerospace), or as safe (nuclear-reactor components), or as cheap — if there is no other objective, there is always that of minimizing cost. This must be achieved while meeting constraints: that the component carries the design loads without failing; that it survives in the chemical and thermal environment in which it must operate; and that certain limits on its dimensions must be met. The first step in relating design requirements to material properties is a clear statement of function, objectives and constraints.

Table 5.1 Function, objectives and constraints

Function	What does component do?
Objective	What is to be maximized or minimized?
Constraints*	What non-negotiable conditions must be met? What negotiable but desirable conditions ...?

* It is sometimes useful to distinguish between 'hard' and 'soft' constraints. Stiffness and strength might be absolute requirements (hard constraints); cost might be negotiable (a soft constraint).

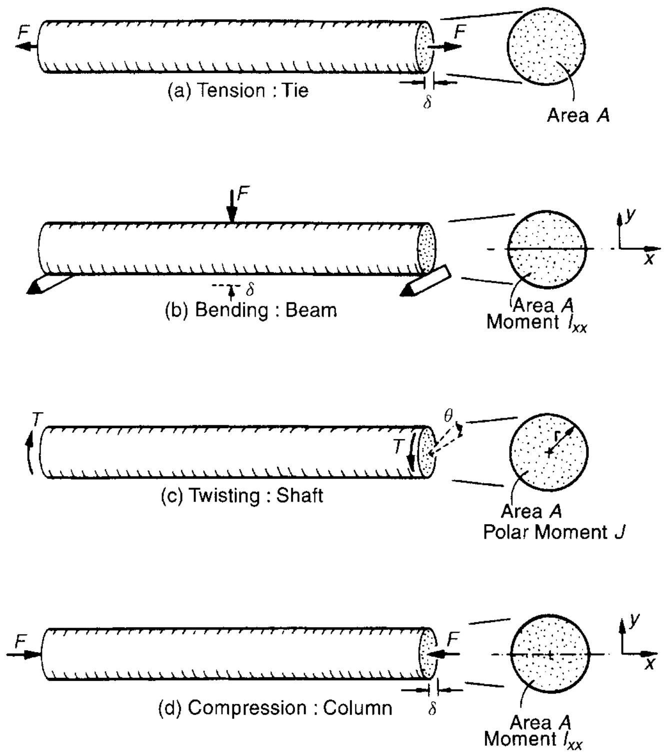

Fig. 5.4 A cylindrical tie-rod loaded (a) in tension, (b) in bending, (c) in torsion and (d) axially, as a column. The best choice of materials depends on the mode of loading and on the design goal; it is found by deriving the appropriate material index.

Property limits

Some constraints translate directly into simple *limits on material properties*. If the component must operate at 250°C, then all materials with a maximum service temperature less than this are eliminated. If it must be electrically insulating, then all material with a resistivity below 10^{20} μΩ cm are rejected. The screening step of the procedure of Figure 5.3 uses property limits derived in this way to reduce the kingdom of materials to an initial shortlist.

Constraints on stiffness, strength and many other component characteristics are used in a different way. This is because stiffness (to take an example) can be achieved in more than one way: by choosing a material with a high modulus, certainly; but also by simply increasing the cross-section; or, in the case of bending-stiffness or stiffness in torsion, by giving the section an efficient shape (a box or I-section, or tube). Achieving a specified stiffness (the constraint) involves a trade-off between these, and to resolve it we need to invoke an objective. The outcome of doing so is a material index. They are keys to optimized material selection. So how do you find them?

Material indices

A *material index* is a combination of material properties which characterizes the performance of a material in a given application.

First, a general statement of the scheme; then examples. *Structural elements* are components which perform a physical function: they carry loads, transmit heat, store energy and so on; in short, they satisfy *functional requirements*. The functional requirements are specified by the design: a tie must carry a specified tensile load; a spring must provide a given restoring force or store a given energy, a heat exchanger must transmit heat with a given heat flux, and so on.

The design of a structural element is specified by three things: the functional requirements, the geometry and the properties of the material of which it is made. The performance of the element is described by an equation of the form

$$p = f\left[\begin{pmatrix}\text{Functional} \\ \text{requirements,} \quad F\end{pmatrix}, \begin{pmatrix}\text{Geometric} \\ \text{parameters,} \quad G\end{pmatrix}, \begin{pmatrix}\text{Material} \\ \text{properties,} \quad M\end{pmatrix}\right] \tag{5.1}$$

or $$p = f(F, G, M)$$

where p describes some aspect of the performance of the component: its mass, or volume, or cost, or life for example; and 'f' means 'a function of'. *Optimum design* is the selection of the material and geometry which maximize or minimize p, according to its desirability or otherwise.

The three groups of parameters in equation (5.1) are said to be *separable* when the equation can be written

$$p = f_1(F)f_2(G)f_3(M) \tag{5.2}$$

where f_1, f_2 and f_3 are separate functions which are simply multiplied together. When the groups are separable, as they generally are, the optimum choice of material becomes independent of the details of the design; it is the same for all geometries, G, and for all the values of the functional requirement, F. Then the optimum subset of materials can be identified without solving the complete design problem, or even knowing all the details of F and G. This enables enormous simplification: the performance for all F and G is maximized by maximizing f_3 (M), which is called the material efficiency coefficient, or *material index* for short*. The remaining bit, $f_1(F)f_2(G)$, is related to the *structural efficiency coefficient*, or *structural index*. We don't need it now, but will examine it briefly in Section 5.5.

Each combination of function, objective and constraint leads to a material index (Figure 5.5); the index is characteristic of the combination. The following examples show how some of the indices are derived. The method is general, and, in later chapters, is applied to a wide range of problems. A catalogue of indices is given in Appendix C.

Example 1: The material index for a light, strong, tie

A design calls for a cylindrical tie-rod of specified length ℓ, to carry a tensile force F without failure; it is to be of minimum mass. Here, 'maximizing performance' means 'minimizing the mass while still carrying the load F safely'. Function, objective and constraints are listed in Table 5.2.

We first seek an equation describing the quantity to be maximized or minimized. Here it is the mass m of the tie, and it is a minimum that we seek. This equation, called the *objective function*, is

$$m = A\ell\rho \tag{5.3}$$

where A is the area of the cross-section and ρ is the density of the material of which it is made. The length ℓ and force F are specified and are therefore fixed; the cross-section A, is free. We can

* Also known as the 'merit index', 'performance index', or 'material factor'. In this book it is called the 'material index' throughout.

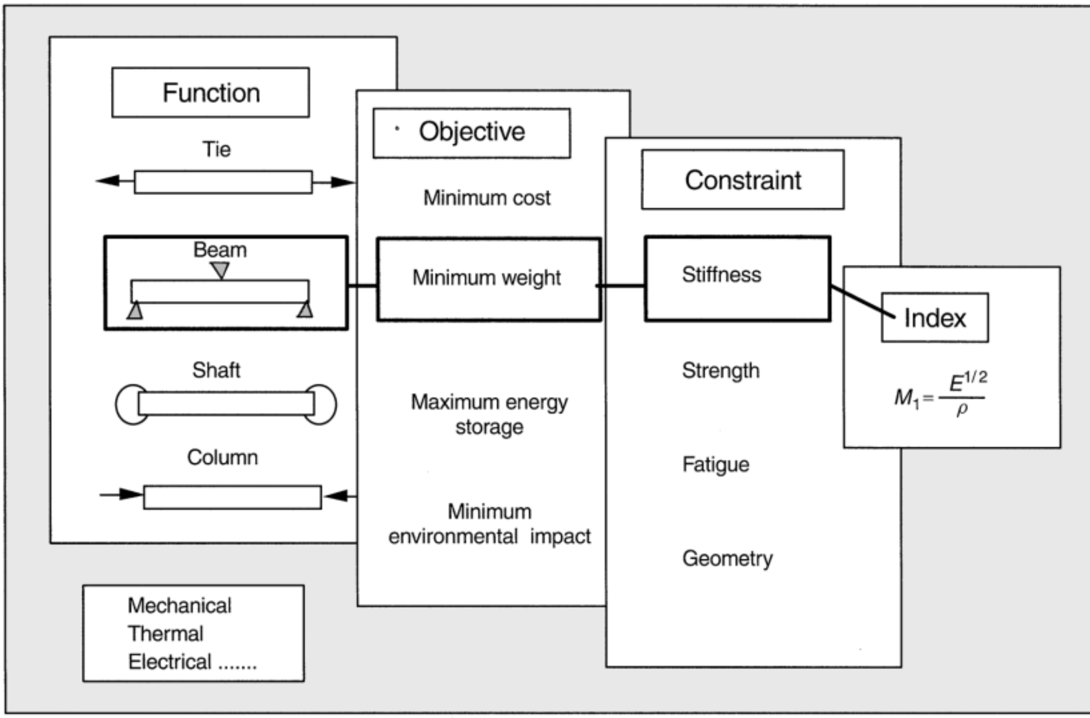

Fig. 5.5 The specification of function, objective and constraint leads to a materials index. The combination in the highlighted boxes leads to the index $E^{1/2}/\rho$.

Table 5.2 Design requirements for the light tie

Function	Tie-rod
Objective	Minimize the mass
Constraints	(a) Length ℓ specified (b) Support tensile load F without failing

reduce the mass by reducing the cross-section, but there is a constraint: the section-area A must be sufficient to carry the tensile load F, requiring that

$$\frac{F}{A} \le \sigma_f$$

where σ_f is the failure strength. Eliminating A between these two equations gives

$$m \ge (F)(\ell)\left(\frac{\rho}{\sigma_f}\right)$$

Note the form of this result. The first bracket contains the specified load F. The second bracket contains the specified geometry (the length ℓ of the tie). The last bracket contains the material properties. The lightest tie which will carry F safely* is that made of the material with the smallest value of ρ/σ_f. It is more natural to ask what must be *maximized* in order to maximize performance; we therefore invert the material properties in equation (5.5) and define the material index M as:

$$\boxed{M = \frac{\sigma_f}{\rho}}$$

The lightest tie-rod which will safely carry the load F without failing is that with the largest value of this index, the 'specific strength', mentioned earlier. A similar calculation for a light *stiff* tie leads to the index

$$\boxed{M = \frac{E}{\rho}}$$

where E is Young's modulus. This time the index is the 'specific stiffness'. But things are not always so simple. The next example shows how this comes about.

Example 2: The material index for a light, stiff beam

The mode of loading which most commonly dominates in engineering is not tension, but bending — think of floor joists, of wing spars, of golf-club shafts. Consider, then, a light beam of square section $b \times b$ and length ℓ loaded in bending which must meet a constraint on its stiffness S, meaning that it must not deflect more than δ under a load F (Figure 5.6). Table 5.3 itemizes the function, the objective and the constraints.

Appendix A of this book catalogues useful solutions to a range of standard problems. The stiffness of beams is one of these. Turning to Section A3 we find an equation for the stiffness of an elastic

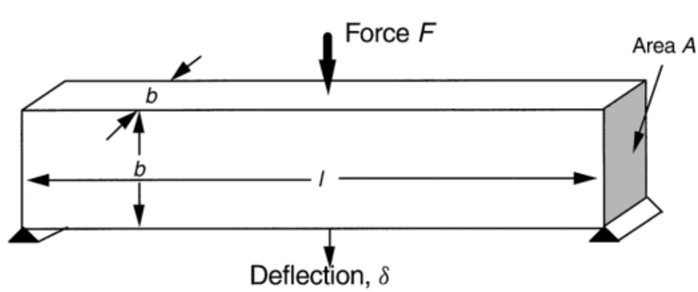

Fig. 5.6 A beam of square section, loaded in bending. Its stiffness is $S = F/\delta$, where F is the load and δ is the deflection. In Example 2, the active constraint is that of stiffness, S; it is this which determines the section area A. In Example 3, the active constraint is that of strength; it now determines the section area A.

* In reality a safety factor, S_f, is always included in such a calculation, such that equation (5.4) becomes $F/A \leq \sigma_f/S_f$. If the same safety factor is applied to each material, its value does not influence the choice. We omit it here for simplicity.

Table 5.3 Design requirements for the light stiff beam

Function	Beam
Objective	Minimize the mass
Constraints	(a) Length ℓ specified (b) Support bending load F without deflecting too much

beam. The constraint requires that $S = F/\delta$ be greater than this:

$$S = \frac{F}{\delta} \geq \frac{C_1 EI}{\ell^3} \tag{5.8}$$

where E is Young's modulus, C_1 is a constant which depends on the distribution of load and I is the second moment of the area of the section, which, for a beam of square section ('Useful Solutions', Appendix A, Section A2), is

$$I = \frac{b^4}{12} = \frac{A^2}{12} \tag{5.9}$$

The stiffness S and the length ℓ are specified; the section A is free. We can reduce the mass of the beam by reducing A, but only so far that the stiffness constraint is still met. Using these two equations to eliminate A in equation (5.3) gives

$$m \geq \left(\frac{12S}{C_1 \ell}\right)^{1/2} \ell^3 \left(\frac{\rho}{E^{1/2}}\right) \tag{5.10}$$

The brackets are ordered as before: functional requirement, geometry and material. The best materials for a light, stiff beam are those with large values of the material index

$$\boxed{M = \frac{E^{1/2}}{\rho}} \tag{5.11}$$

Here, as before, the properties have been inverted; to minimize the mass, we must maximize M. Note the procedure. The length of the rod or beam is specified but we are free to choose the section area A. The *objective* is to minimize its mass, m. We write an equation for m; it is called the *objective function*. But there is a *constraint*: the rod must carry the load F without yielding in tension (in the first example) or bending too much (in the second). Use this to eliminate the free variable A. Arrange the result in the format

$$p = f_1(F) f_2(G) f_3(M)$$

and read off the combination of properties, M, to be maximized. It sounds easy, and it is so long as you are clear from the start what you are trying to maximize or minimize, what the constraints are, which parameters are specified, and which are free. In deriving the index, we have assumed that the section of the beam remained square so that both edges changed in length when A changed. If one of the two dimensions is held fixed, the index changes. If only the height is free, it becomes

(via an identical derivation)

$$\boxed{M = \frac{E^{1/3}}{\rho}} \tag{5.12}$$

and if only the width is free, it becomes

$$\boxed{M = \frac{E}{\rho}} \tag{5.13}$$

Example 3: The material index for a light, strong beam

In stiffness-limited applications, it is elastic deflection which is the active constraint: it limits performance. In strength-limited applications, deflection is acceptable provided the component does not fail; strength is the active constraint. Consider the selection of a beam for a strength-limited application. The dimensions are the same as before. Table 5.4 itemizes the design requirements.

The objective function is still equation (5.3), but the constraint is now that of strength: the beam must support F without failing. The failure load of a beam (Appendix A, Section A4) is:

$$F_f = C_2 \frac{I\sigma_f}{y_m \ell} \tag{5.14}$$

where C_2 is a constant and y_m is the distance between the neutral axis of the beam and its outer filament ($C_2 = 4$ and $y_m = t/2$ for the configuration shown in the figure). Using this and equation (5.9) to eliminate A in equation (5.3) gives the mass of the beam which will just support the load F_f:

$$m_2 = \left(\frac{6}{C_2}\frac{F_f}{\ell^2}\right)^{2/3} \ell^3 \left[\frac{\rho}{\sigma_y^{2/3}}\right] \tag{5.15}$$

The mass is minimized by selecting materials with the largest values of the index

$$\boxed{M = \frac{\sigma_f^{3/2}}{\rho}} \tag{5.16}$$

This is the moment to distinguish more clearly between a *constraint* and an *objective*. A constraint is a feature of the design which must be met at a specified level (stiffness in the last example). An

Table 5.4 Design requirements for the light strong beam

Function	Beam
Objective	Minimize the mass
Constraints	(a) Length ℓ specified (b) Support bending load F without failing by yield or fracture

objective is a feature for which an extremum is sought (mass, just now). An important judgement is that of deciding which is to be which. It is not always obvious: for a racing bicycle, as an example, mass might be minimized with a constraint on cost; for a shopping bicycle, cost might be minimized with a constraint on the mass. It is the objective which gives the objective function; the constraints set the free variables it contains.

So far the objective has been that of minimizing weight. There are many others. In the selection of a material for a spring, the objective is that of maximizing the elastic energy it can store. In seeking materials for thermal-efficient insulation for a furnace, the best are those with the lowest thermal conductivity and heat capacity. And most common of all is the wish to minimize cost. So here is an example involving cost.

Example 4: The material index for a cheap, stiff column

Columns support compressive loads: the legs of a table; the pillars of the Parthenon. We seek materials for the cheapest cylindrical column of specified height, ℓ, which will safely support a load F (Figure 5.7). Table 5.5 lists the requirements.

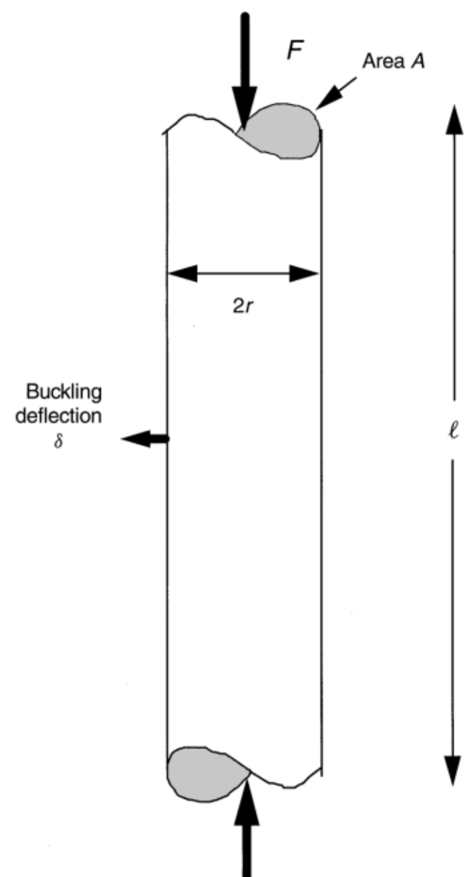

Fig. 5.7 A column carrying a compressive load F. The constraint that it must not buckle determines the section area A.

Table 5.5 Design requirements for the cheap column

Function	Column
Objective	Minimize the cost
Constraints	(a) Length ℓ specified (b) Support compressive load F without buckling

A slender column uses less material than a fat one, and thus is cheaper; but it must not be so slender that it will buckle under the design load, F. The objective function is the cost

$$C = A\ell C_m \rho \tag{5.17}$$

where C_m is the cost/kg of the material* of the column. It will buckle elastically if F exceeds the Euler load, F_{crit}, found in Appendix A, 'Useful Solutions', Section A5. The design is safe if

$$F \leq F_{\text{crit}} = \frac{n\pi^2 EI}{\ell^2} \tag{5.18}$$

where n is a constant that depends on the end constraints and $I = \pi r^2/4 = A^2/4\pi$ is the second moment of area of the column (see Appendix A for both). The load F and the length ℓ are specified; the free variable is the section-area A. Eliminating A between the last two equations, using the definition of I, gives:

$$C \geq \left(\frac{4}{n\pi}\right)^{1/2} \left(\frac{F}{\ell^2}\right)^{1/2} \ell^3 \left(\frac{C_m \rho}{E^{1/2}}\right) \tag{5.19}$$

The pattern is the usual one: functional requirement, geometry, material. The cost of the column is minimized by choosing materials with the largest value of the index

$$\boxed{M = \frac{E^{1/2}}{C_m \rho}} \tag{5.20}$$

From all this we distil the procedure for deriving a material index. It is shown in Table 5.6.

Table 5.7 summarizes a few of the indices obtained in this way. Appendix D contains a more complete catalogue. We now examine how to use them to select materials.

5.4 The selection procedure

Property limits: go/no-go conditions and geometric restrictions

Any design imposes certain non-negotiable demands on the material of which it is made. Temperature is one: a component which is to carry load at 500°C cannot be made of a polymer since all polymers lose their strength and decompose at lower temperatures than this. Electrical conductivity is another: components which must insulate cannot be made of metals because all metals conduct well. Corrosion resistance can be a third. Cost is a fourth: 'precious' metals are not used in structural applications simply because they cost too much.

* C_m is the cost/kg of the *processed* material, here, the material in the form of a circular rod or column.

Table 5.6 Procedure for deriving material indices

Step	*Action*
1	*Define the design requirements:* (a) Function: what does the component do? (b) Objective: what is to be maximized or minimized? (c) Constraints: essential requirements which must be met: stiffness, strength, corrosion resistance, forming characteristics...
2	Develop an *equation* for the objective in terms of the functional requirements, the geometry and the material properties (the *objective function*).
3	Identify the *free* (unspecified) *variables.*
4	Develop *equations* for the constraints (no yield; no fracture; no buckling, etc.).
5	*Substitute* for the free variables from the constraint equations into the objective function.
6	*Group the variables* into three groups: functional requirements, F, geometry, G, and material properties, M, thus Performance characteristic $\le f_1(F)f_2(G)f_3(M)$ or Performance characteristic $\ge f_1(F)f_2(G)f_3(M)$
7	*Read off* the material index, expressed as a quantity M, which optimizes the performance characteristic.

Table 5.7 Examples of material indices

Function, Objective and Constraint	*Index*
Tie, minimum weight, stiffness prescribed	$\dfrac{E}{\rho}$
Beam, minimum weight, stiffness prescribed	$\dfrac{E^{1/2}}{\rho}$
Beam, minimum weight, strength prescribed	$\dfrac{\sigma_y^{2/3}}{\rho}$
Beam, minimum cost, stiffness prescribed	$\dfrac{E^{1/2}}{C_m\rho}$
Beam, minimum cost, strength prescribed	$\dfrac{\sigma_y^{2/3}}{C_m\rho}$
Column, minimum cost, buckling load prescribed	$\dfrac{E^{1/2}}{C_m\rho}$
Spring, minimum weight for given energy storage	$\dfrac{\sigma_y^2}{E\rho}$
Thermal insulation, minimum cost, heat flux prescribed	$\dfrac{1}{\lambda C_m\rho}$
Electromagnet, maximum field, temperature rise prescribed	$\kappa C_p\rho$

(ρ = density; E = Young's modulus; σ_y = elastic limit; C_m = cost/kg; λ = thermal conductivity; κ = electrical conductivity; C_p = specific heat)

Geometric constraints also generate property limits. In the examples of the last section the length ℓ was constrained. There can be others. Here are two examples. The tie of Example 1, designed to carry a tensile force F without yielding (equation 5.4), requires a section

$$A \geq \frac{F}{\sigma_f}$$

If, to fit into a confined space, the section is limited to $A \leq A^*$, then the only possible candidate materials are those with strengths greater than

$$\sigma_f^* = \frac{F}{A^*} \tag{5.21}$$

Similarly, if the column of Example 4, designed to carry a load F without buckling, is constrained to have a diameter less than $2r^*$, it will require a material with modulus (found by inverting equation (5.18)) greater than

$$E^* = \frac{4F\ell^2}{n\pi^3 r^{*4}} \tag{5.22}$$

Property limits plot as horizontal or vertical lines on material selection charts. The restriction on r leads to a lower bound for E, given by equation (5.22). An upper limit on density (if one were desired) requires that

$$\rho < \rho^* \tag{5.23}$$

One way of applying the limits is illustrated in Figure 5.8. It shows a schematic $E-\rho$ chart, in the manner of Chapter 4, with a pair of limits for E and ρ plotted on it. The optimizing search is restricted to the window between the limits within which the next steps of the procedure operate. Less quantifiable properties such as corrosion resistance, wear resistance or formability can all appear as primary limits, which take the form

$$P > P^*$$

or

$$P < P^* \tag{5.24}$$

where P is a property (service temperature, for instance) and P^* is a critical value of that property, set by the design, which must be exceeded, or (in the case of cost or corrosion rate) must *not* be exceeded.

One should not be too hasty in applying property limits; it may be possible to engineer a route around them. A component which gets too hot can be cooled; one that corrodes can be coated with a protective film. Many designers apply property limits for fracture toughness, K_{Ic}, and ductility ε_f, insisting on materials with, as rules of thumb, $K_{Ic} > 15\,\mathrm{MPa\,m^{1/2}}$ and $\varepsilon_f > 2\%$ in order to guarantee adequate tolerance to stress concentrations. By doing this they eliminate materials which the more innovative designer is able to use to good purpose (the limits just cited for K_{Ic} and ε_f eliminate most polymers and all ceramics, a rash step too early in the design). At this stage, keep as many options open as possible.

Performance maximizing criteria

The next step is to seek, from the subset of materials which meet the property limits, those which maximize the performance of the component. We will use the design of light, stiff components as an example; the other material indices are used in a similar way.

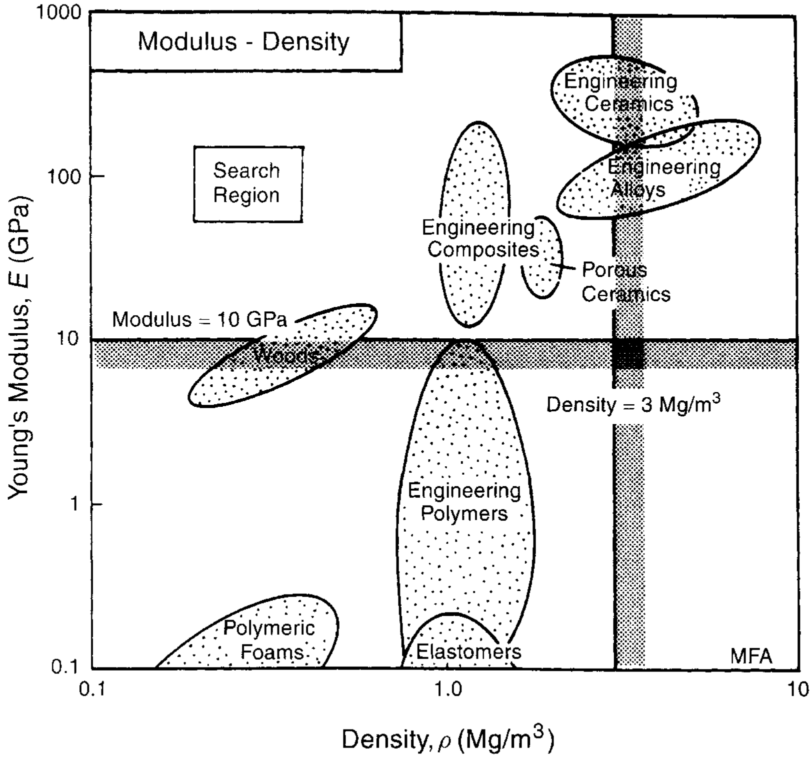

Fig. 5.8 A schematic $E-\rho$ chart showing a lower limit for E and an upper one for ρ.

Figure 5.9 shows, as before, the modulus E, plotted against density ρ, on log scales. The material indices E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ can be plotted onto the figure. The condition

$$E/\rho = C$$

or taking logs

$$\log E = \log \rho + \log C \tag{5.25}$$

is a family of straight parallel lines of slope 1 on a plot of $\log E$ against $\log \rho$; each line corresponds to a value of the constant C. The condition

$$E^{1/2}/\rho = C \tag{5.24}$$

gives another set, this time with a slope of 2; and

$$E^{1/3}/\rho = C \tag{5.25}$$

gives yet another set, with slope 3. We shall refer to these lines as selection *guide lines*. They give the slope of the family of parallel lines belonging to that index.

It is now easy to read off the subset materials which optimally maximize performance for each loading geometry. All the materials which lie on a line of constant $E^{1/2}/\rho$ perform equally well as a light, stiff beam (Example 2); those above the line are better, those below, worse. Figure 5.10 shows

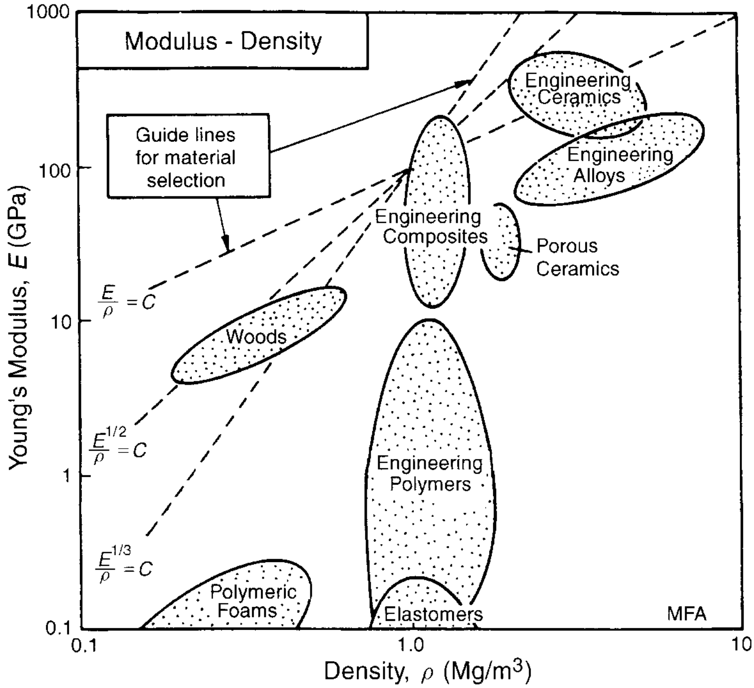

Fig. 5.9 A schematic $E-\rho$ chart showing guide lines for the three material indices for stiff, lightweight design.

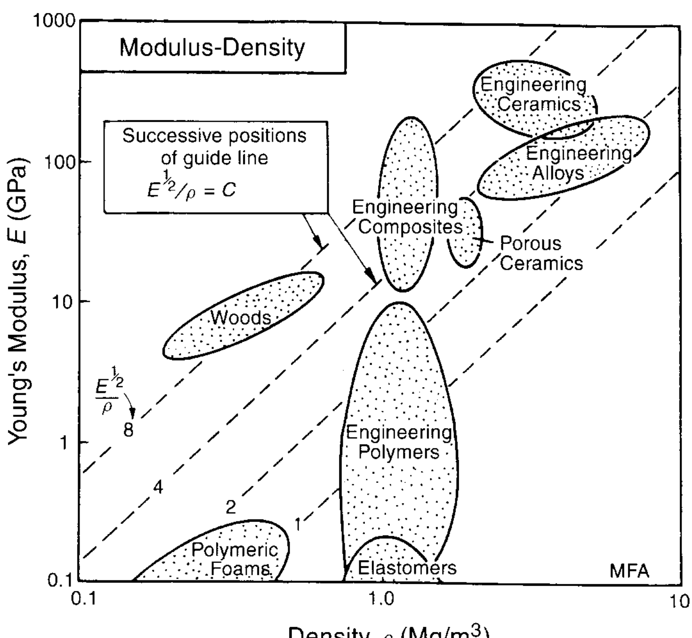

Fig. 5.10 A schematic $E-\rho$ chart showing a grid of lines for the material index $M = E^{1/2}/\rho$. The units are $(\mathrm{GPa})^{1/2}/(\mathrm{Mg/m^3})$.

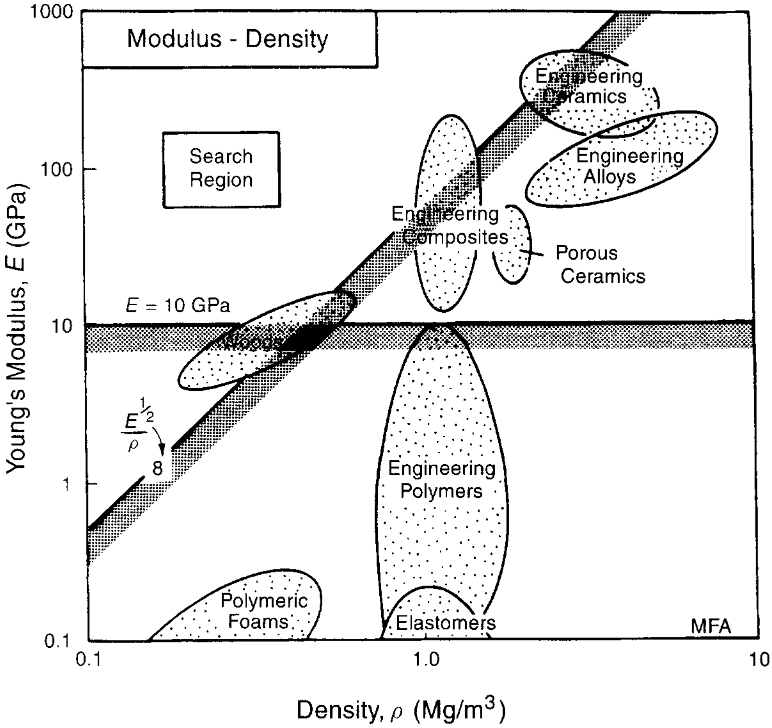

Fig. 5.11 A selection based on the index $M = E^{1/2}/\rho$, together with the property limit $E > 10\,\text{GPa}$. The shaded band with slope 2 has been positioned to isolate a subset of materials with high $E^{1/2}/\rho$; the horizontal ones lie at $E = 10\,\text{GPa}$. The materials contained in the Search Region become the candidates for the next stage of the selection process.

a grid of lines corresponding to values of $M = E^{1/2}/\rho$ from 1 to 8 in units of $\text{GPa}^{1/2}/(\text{Mg m}^{-3})$. A material with $M = 4$ in these units gives a beam which has half the weight of one with $M = 2$. One with $M = 8$ weighs one quarter as much. The subset of materials with particularly good values of the index is identified by picking a line which isolates a *search area* containing a reasonably small number of candidates, as shown schematically in Figure 5.11. Properly limits can be added, narrowing the search window: that corresponding to $E > 10\,\text{GPa}$ is shown. The shortlist of candidate materials is expanded or contracted by moving the index line.

The procedure is extended in Chapters 7 and 9 to include section shape and to deal with multiple constraints and objectives. Before moving on to these, it is a good idea to consolidate the ideas so far by applying them to a number of Case Studies. They follow in Chapter 6. But first a word about the structural index.

5.5 The structural index

Books on optimal design of structures (e.g. Shanley, 1960) make the point that the efficiency of material usage in mechanically loaded components depends on the product of three factors: the material index, as defined here; a factor describing section shape, the subject of our Chapter 7; and a *structural index**, which contains elements of the F and G of equation (5.1). The subjects of this book — material and process selection — focus attention on the material index and on shape; but we should examine the structural index briefly, partly to make the connection with the classical theory of optimal design, and partly because it becomes useful (even to us) when structures are scaled in size.

Consider, as an example, the development of the index for a cheap, stiff column, given as Example 4 in Section 5.2. The objective was that of minimizing cost. The *mechanical efficiency* is a measure of the load carried divided by the 'objective' — in this case, cost per unit length. Using equation (5.19) the efficiency of the column is given by

$$\frac{F}{(C/\ell)} = \left(\frac{n\pi}{4}\right)^{1/2} \left[\frac{F}{\ell^2}\right]^{1/2} \left[\frac{E^{1/2}}{C_m\rho}\right] \tag{5.26}$$

The first bracketed term on the right is merely a constant. The last is the material index. The structural index is the middle one: F/ℓ^2. It has the dimensions of stress; it is a measure of the intensity of loading. Design proportions which are optimal, minimizing material usage, are optimal for structures of any size provided they all have the same structural index. The performance equations (5.5), (5.10), (5.15) and (5.19) were all written in a way which isolated the structural index

The structural index for a column of minimum weight is the same as that for one which minimizes material cost; it is F/ℓ^2 again. For beams of minimum weight, or cost, or energy content, it is the same: F/ℓ^2. For ties it is simply 1 (try it: use equation (5.5) to calculate the load F divided by the mass per unit length, m/ℓ). For panels loaded in bending or such that they buckle it is $F/\ell b$ where ℓ and b are the (fixed) dimensions of the panel.

5.6 Summary and conclusions

The design requirements of a component which performs mechanical, thermal or electrical functions can be formulated in terms of one or more objective functions, limited by constraints. The objective function describes the quantity to be maximized or minimized in the design. One or more of the variables describing the geometry is 'free', that is, it (or they) can be varied to optimize the design. If the number of constraints is equal to the number of free variables, the problem is fully constrained; the constraints are substituted into the objective function identifying the group of material properties (the 'material index') to be maximized or minimized in selecting a material. The charts allow this using the method outlined in this chapter. Often, the index characterizes an entire class of designs, so that the details of shape or loading become unimportant in deriving it. The commonest of these indices are assembled in Appendix C of this book, but there are more. New problems throw up new indices, as the Case Studies of the next chapter will show.

5.7 Further reading

The books listed below discuss optimization methods and their application in materials engineering. None contains the approach developed here.

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* Also called the 'structural loading coefficient', the 'strain number' or the 'strain index'.

Dieter, G.E. (1991) *Engineering Design, A Materials and Processing Approach*, 2nd edition, Chapter 5, McGraw-Hill, New York.
Gordon, J.E. (1978) *Structures, or Why Things don't Fall through the Floor*, Penguin Books, Harmondsworth.
Johnson, R.C. (1980) *Optimum Design of Mechanical Elements*, 2nd edition, Wiley, New York.
Shanley, F.R. (1960) *Weight–Strength Analysis of Aircraft Structures*, 2nd edition, Dover Publications, New York.
Siddall, J.N. (1982) *Optimal Engineering Design*, Marcel Dekker, New York.