Appendix B

Material indices

Introduction

The performance, p, of a component is measured by a performance equation. The performance equation contains groups of material properties. These groups are the material indices. Sometimes the 'group' is a single property; thus if the performance of a beam is measured by its stiffness, the performance equation contains only one property, the elastic modulus E. It is the material index for this problem. More commonly the performance equation contains a group of two or more properties. Familiar examples are the specific stiffness, E/ρ, and the specific strength, σ_y/ρ, (where E is Young's modulus, σ_y is the yield strength or elastic limit, and ρ is the density), but there are many others. They are a key to the optimal selection of materials. Details of the method, with numerous examples are given in references [1] and [2]. PC-based software systems [3] which implement the method are available. This Appendix compiles indices for a range of common applications.

Uses of material indices

Material selection

Components have functions: to carry loads safely, to transmit heat, to store energy, to insulate, and so forth. Each function has an associated material index. Materials with high values of the appropriate index maximize that aspect of the performance of the component. For reasons given in reference [1], the material index is generally independent of the details of the design. Thus the indices for beams in the tables which follow are independent of the detailed shape of the beam; that for minimizing thermal distortion of precision instruments is independent of the configuration of the instrument. This gives them great generality.

Material deployment or substitution

A new material will have potential application in functions for which its indices have unusually high values. Fruitful applications for a new material can be identified by evaluating its indices and comparing these with those of incumbent materials. Similar reasoning points the way to identifying viable substitutes for an incumbent material in an established application.

Table B1 Stiffness-limited design at minimum mass (cost, energy, environmental impact*)

*Function and constraints**	*Maximize*†
Tie (tensile strut)	
stiffness, length specified; section area free	E/ρ
Shaft (loaded in torsion)	
stiffness, length, shape specified, section area free	$G^{1/2}/\rho$
stiffness, length, outer radius specified; wall thickness free	G/ρ
stiffness, length, wall-thickness specified; outer radius free	$G^{1/3}/\rho$
Beam (loaded in bending)	
stiffness, length, shape specified; section area free	$E^{1/2}/\rho$
stiffness, length, height specified; width free	E/ρ
stiffness, length, width specified; height free	$E^{1/3}/\rho$
Column (compression strut, failure by elastic buckling)	
buckling load, length, shape specified; section area free	$E^{1/2}/\rho$
Panel (flat plate, loaded in bending)	
stiffness, length, width specified, thickness free	$E^{1/3}/\rho$
Plate (flat plate, compressed in-plane, buckling failure)	
collapse load, length and width specified, thickness free	$E^{1/3}/\rho$
Cylinder with internal pressure	
elastic distortion, pressure and radius specified; wall thickness free	E/ρ
Spherical shell with internal pressure	
elastic distortion, pressure and radius specified, wall thickness free	$E/(1-\nu)\rho$

*To minimize cost, use the above criteria for minimum weight, replacing density ρ by $C_m\rho$, where C_m is the material cost per kg. To minimize energy content, use the above criteria for minimum weight replacing density ρ by $q\rho$ where q is the energy content per kg. To minimize environmental impact, replace density ρ by $I_e\rho$ instead, where I_e is the eco-indicator value for the material (references [1] and [4]).

†E = Young's modulus for tension, the flexural modulus for bending or buckling; G = shear modulus; ρ = density, q = energy content/kg; I_e = eco-indicator value/kg.

Table B2 Strength-limited design at minimum mass (cost, energy, environmental impact*)

*Function and constraints**‡	*Maximize*†
Tie (tensile strut)	
stiffness, length specified; section area free	σ_f/ρ
Shaft (loaded in torsion)	
load, length, shape specified, section area free	$\sigma_f^{2/3}/\rho$
load, length, outer radius specified; wall thickness free	σ_f/ρ
load, length, wall-thickness specified; outer radius free	$\sigma_f^{1/2}/\rho$
Beam (loaded in bending)	
load, length, shape specified; section area free	$\sigma_f^{2/3}/\rho$
load length, height specified; width free	σ_f/ρ
load, length, width specified; height free	$\sigma_f^{1/2}/\rho$
Column (compression strut)	
load, length, shape specified; section area free	σ_f/ρ
Panel (flat plate, loaded in bending)	
stiffness, length, width specified, thickness free	$\sigma_f^{1/2}/\rho$
Plate (flat plate, compressed in-plane, buckling failure)	
collapse load, length and width specified, thickness free	$\sigma_f^{1/2}/\rho$
Cylinder with internal pressure	
elastic distortion, pressure and radius specified; wall thickness free	σ_f/ρ
Spherical shell with internal pressure	
elastic distortion, pressure and radius specified, wall thickness free	σ_f/ρ
Flywheels, rotating discs	
maximum energy storage per unit volume; given velocity	ρ
maximum energy storage per unit mass; no failure	σ_f/ρ

*To minimize cost, use the above criteria for minimum weight, replacing density ρ by $C_m\rho$, where C_m is the material cost per kg. To minimize energy content, use the above criteria for minimum weight replacing density ρ by $q\rho$ where q is the energy content per kg. To minimize environmental impact, replace density ρ by $I_e\rho$ instead, where I_e is the eco-indicator value for the material (references [1] and [4]).

†σ_f = failure strength (the yield strength for metals and ductile polymers, the tensile strength for ceramics, glasses and brittle polymers loaded in tension; the flexural strength or modulus of rupture for materials loaded in bending); ρ = density.

‡For design for infinite fatigue life, replace σ_f by the endurance limit σ_e.

Table B3 Strength-limited design: springs, hinges etc. for maximum performance*

*Function and constraints**‡	*Maximize*†
Springs	
maximum stored elastic energy per unit volume; no failure	σ_f^2/E
maximum stored elastic energy per unit mass; no failure	$\sigma_f^2/E\rho$
Elastic hinges	
radius of bend to be minimized (max flexibility without failure)	σ_f/E
Knife edges, pivots	σ_f^3/E^2 and
minimum contact area, maximum bearing load	H
Compression seals and gaskets	$\sigma_f^{3/2}/E$ and
maximum conformability; limit on contact pressure	$1/E$
Diaphragms	
maximum deflection under specified pressure or force	$\sigma_f^{3/2}/E$
Rotating drums and centrifuges	
maximum angular velocity; radius fixed; wall thickness free	σ_f/ρ

*To minimize cost, use the above criteria for minimum weight, replacing density ρ by $C_m\rho$, where C_m is the material cost per kg. To minimize energy content, use the above criteria for minimum weight replacing density ρ by $q\rho$ where q is the energy content per kg. To minimize environmental impact, replace density ρ by $I_e\rho$ instead, where I_e is the eco-indicator value for the material (references [1] and [4]).

†σ_f = failure strength (the yield strength for metals and ductile polymers, the tensile strength for ceramics, glasses and brittle polymers loaded in tension; the flexural strength or modulus of rupture for materials loaded in bending); ρ = density; H = hardness.

‡For design for infinite fatigue life, replace σ_f by the endurance limit σ_e.

Table B4 Vibration-limited design

Function and constraints	*Maximize**
Ties, columns	
maximum longitudinal vibration frequencies	E/ρ
Beams, all dimensions prescribed	
maximum flexural vibration frequencies	E/ρ
Beams, length and stiffness prescribed	
maximum flexural vibration frequencies	$E^{1/2}/\rho$
Panels, all dimensions prescribed	
maximum flexural vibration frequencies	E/ρ
Panels, length, width and stiffness prescribed	
maximum flexural vibration frequencies	$E^{1/3}/\rho$
Ties, columns, beams, panels, stiffness prescribed	
minimum longitudinal excitation from external drivers, ties	$\eta E/\rho$
minimum flexural excitation from external drivers, beams	$\eta E^{1/2}/\rho$
minimum flexural excitation from external drivers, panels	$\eta E^{1/3}/\rho$

*E = Young's modulus for tension, the flexural modulus for bending; G = shear modulus; ρ = density; η = damping coefficient (loss coefficient).

Table B5 Damage-tolerant design

Function and constraints	*Maximize**
Ties (tensile member)	
Maximize flaw tolerance and strength, load-controlled design	K_{Ic} and σ_f
Maximize flaw tolerance and strength, displacement-control	K_{Ic}/E and σ_f
Maximize flaw tolerance and strength, energy-control	K_{Ic}^2/E and σ_f
Shafts (loaded in torsion)	
Maximize flaw tolerance and strength, load-controlled design	K_{Ic} and σ_f
Maximize flaw tolerance and strength, displacement-control	K_{Ic}/E and σ_f
Maximize flaw tolerance and strength, energy-control	K_{Ic}^2/E and σ_f
Beams (loaded in bending)	
Maximize flaw tolerance and strength, load-controlled design	K_{Ic} and σ_f
Maximize flaw tolerance and strength, displacement-control	K_{Ic}/E and σ_f
Maximize flaw tolerance and strength, energy-control	K_{Ic}^2/E and σ_f
Pressure vessel	
Yield-before-break	K_{Ic}/σ_f
Leak-before-break	K_{Ic}^2/σ_f

*K_{Ic} = fracture toughness; E = Young's modulus; σ_f = failure strength (the yield strength for metals and ductile polymers, the tensile strength for ceramics, glasses and brittle polymers loaded in tension; the flexural strength or modulus of rupture for materials loaded in bending).

Table B6 Thermal and thermo-mechanical design

Function and constraints	*Maximize**
Thermal insulation materials	
minimum heat flux at steady state; thickness specified	$1/\lambda$
minimum temp rise in specified time; thickness specified	$1/a = \rho C_p/\lambda$
minimize total energy consumed in thermal cycle (kilns, etc)	$\sqrt{a}/\lambda = \sqrt{1/\lambda\rho C_p}$
Thermal storage materials	
maximum energy stored/unit material cost (storage heaters)	C_p/C_m
maximize energy stored for given temperature rise and time	$\lambda/\sqrt{a} = \sqrt{\lambda\rho C_p}$
Precision devices	
minimize thermal distortion for given heat flux	λ/α
Thermal shock resistance	$\sigma_f/E\alpha$
maximum change in surface temperature; no failure	
Heat sinks	
maximum heat flux per unit volume; expansion limited	$\lambda/\Delta\alpha$
maximum heat flux per unit mass; expansion limited	$\lambda/\rho\Delta\alpha$
Heat exchangers (pressure-limited)	
maximum heat flux per unit area; no failure under Δp	$\lambda\sigma_f$
maximum heat flux per unit mass; no failure under Δp	$\lambda\sigma_f/\rho$

*λ = thermal conductivity; a = thermal diffusivity; C_p = specific heat capacity; C_m = material cost/kg; T_{max} = maximum service temperature; α = thermal expansion coeff.; E = Young's modulus; ρ = density; σ_f = failure strength (the yield strength for metals and ductile polymers, the tensile strength for ceramics, glasses and brittle polymers).

Table B7 Electro-mechanical design

Function and constraints	*Maximize**
Bus bars	
minimum life-cost; high current conductor	$1/\rho_e \rho C_m$
Electro-magnet windings	
maximum short-pulse field; no mechanical failure	σ_y
maximize field and pulse-length, limit on temperature rise	$C_p \rho/\rho_e$
Windings, high-speed electric motors	
maximum rotational speed; no fatigue failure	σ_e/ρ_e
minimum ohmic losses; no fatigue failure	$1/\rho_e$
Relay arms	
minimum response time; no fatigue failure	$\sigma_e/E\rho_e$
minimum ohmic losses; no fatigue failure	$\sigma_e^2/E\rho_e$

*C_m = material cost/kg; E = Young's modulus; ρ = density; ρ_e = electrical resistivity; σ_y = yield strength; σ_e = endurance limit.

References

Derivations for almost all the indices listed in this Appendix can be found in references [1] and [2].

[1] Ashby, M.F. (1999) *Materials Selection in Mechanical Design*, 2nd edn, Chapter 6, Butterworth-Heinemann, Oxford.
[2] Ashby, M.F. and Cebon, D. (1995) *Case Studies in Materials Selection*, Granta Design, Trumpington.
[3] CMS 2.1(1995) and CMS 3.0 (1999) Granta Design, Trumpington.
[4] Goedkoop, M.J., Demmers, M. and Collignon, M.X. 'Eco-Indicator '95, Manual', Pré Consultants, and the Netherlands Agency for Energy and the Environment, Amersfort, Holland (1995).