$$\frac{\sigma_3}{30.86 + 20.66} = \frac{P_c}{30.86 - 20.66}$$

$\underline{\sigma_3 = 5.05\ P_c}$

$$\frac{\sigma_2 + 50}{51 + 30.86} = \frac{P_c}{51 - 30.86}$$

$\sigma_2 + 50 = 4.06\ P_c$

$\underline{\sigma_2 = 4.065\ P_c - 50}$

$$w_E \quad \frac{18E-2}{2E5} \quad (5.05\ P_c + 0.3\ P_c)$$

$= 4.815E\text{-}6\ P_c$

$$w_I \quad = \quad \frac{18E-2}{1E5} \quad (4.065\ P_c - 50 + 0.4\ P_c)$$

$= 8.037E\text{-}6P_c - 9E - 5$

but due to pressure above,

$w_E = w_I$

$\therefore\ 4.815E\text{-}6\ P_c = 8.037E\text{-}6\ P_c - 9E\text{-}5$

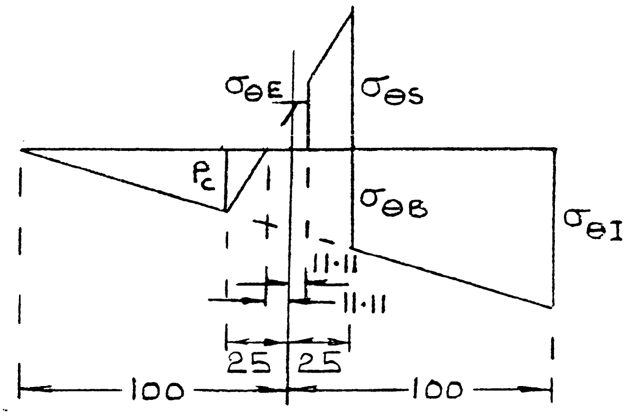
$\therefore\ \underline{P_c = 27.9\ MPa}$

$\therefore\ \underline{\sigma_3 = 141.1\ MPa}$

σ_{max} (on inner surface of outer cylinder)

$\underline{= 241.1\ MPa}$

8. $A_B = 0.0236 \text{ m}^2$



$A_s = 0.0393 \text{ m}^2$

At inner and outer surfaces $\sigma_r = 0$

& $\epsilon_{LS} = \epsilon_{LB}$

At outer surface

$$\epsilon_{LS} = \frac{1}{E_s}(\sigma_{LS} - \upsilon_s \sigma_{\theta E})$$

At inner surface

$$\epsilon_{LB} = \frac{1}{E_B}(\sigma_{LB} - \upsilon_B \sigma_{\theta I})$$

or $\frac{1}{E_s}(\sigma_{LS} - \upsilon_s \sigma\theta_E) = \frac{1}{E_B}(\sigma_{LB} - \upsilon_B \sigma_{\theta I})$

$\therefore \sigma_{LS} - 3\sigma_{\theta E} = 2\sigma_{LB} - 0.86\sigma_{\theta I}$

$\sigma_{LS} - 2\sigma_{LB} - 0.3\sigma_{\theta E} - 0.86\sigma_{\theta I} = 0$ 1

Due to axial load

$P = \sigma_{LS} A_s + \sigma_{LB} A_B$

$-5 = \sigma_{LS} * 0.0393 + \sigma_{LB} * 0.0236$

$\therefore \sigma_{LS} = -127.2 - 0.6\sigma_{LB}$ 2

From Lamé line

$$\frac{-P_c}{100-25} = \frac{\sigma_{\theta I}}{200}$$

$\underline{\sigma_{\theta I} = - 2.667\ P_c}$

$$\frac{\sigma_{\theta E}}{11.11 \times 2} = \frac{P_c}{25 - 11.11}$$

$\underline{\sigma_{\theta E} = 1.6\ P_c}$

$$\frac{\sigma_{\theta B}}{125} = \frac{-P_c}{75}$$

$\underline{\sigma_{\theta B} = -1.667\ P_c}$

$$\frac{\sigma_{\theta S}}{25 + 11.11} = \frac{P_c}{25 - 11.11}$$

$\underline{\sigma_{\theta S} = 2.6\ P_c}$

At the common surface

$$\frac{w}{R} = \frac{1}{E_B}(\sigma_{\theta B} - \upsilon_B\ \sigma_{LB} + \upsilon_B\ P_c) = \frac{1}{E_s}(\sigma_{\sigma s} - \upsilon_s\ \sigma_{LS} + \upsilon_s P_c)$$

or $2\ \sigma_{\theta B} - 0.8\ \sigma_{LB} + 0.8\ P_c = \sigma_{\theta s} - 0.3\ \sigma_{LS} + 0.3\ P_c$

$- 3.333\ P_c - 0.8\ \sigma_{LB} + 0.8\ P_c = 2.6\ P_c - 0.3\ \sigma_{LS} + 0.3\ P_c$

$- 2.533\ P_c - 0.8\sigma_{LB} = 2.9\ P_c - 0.3\ \sigma_{LS}$

$\sigma_{LS} = 18.11 P_c + 2.667\ \sigma_{LB}$ 3

From 1

$\sigma_{LS} - 2\sigma_{LB} - 0.3 * 1.6\ P_c + 0.8 * (-2.667\ P_c) = 0$

$\sigma_{LB} - 0.5\sigma_{LS} + 1.307\ P_c = 0$ 4

Rewriting 2, 3 and 4

$\sigma_{LS} + 0.6\ \sigma_{LB} = -\ 127.2$

$\sigma_{LS} - 2.667\ \sigma_{LB} - 18.11\ P_c = 0$

$\sigma_{LS} - 2\sigma_{LB} - 2.614\ P_c = 0$

Solving the above,

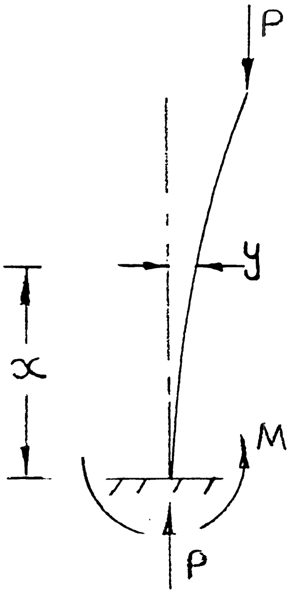
$\sigma_{LS} = -\ 96.52$ MPa

$\sigma_{LB} = -\ 51.14$ MPa

$P_c = 2.2$ MPa

CHAPTER 12

1. $EI \frac{d^2y}{dx^2} = - Py + M$



Let $\alpha^2 = P/EI$

$\therefore \frac{d^2y}{dx^2} + \alpha^2 y = M/EI$

The complete solution is

$y = A \cos \alpha x + B \sin \alpha x + M/(EI\alpha^2)$

$\frac{dy}{dx} = - \alpha A \sin \alpha x + \alpha B \cos \alpha x$

$\frac{d^2y}{d^2x} = - \alpha^2 A \cos \alpha x - \alpha^2 B \sin \alpha x$

@ $x = 0, y = 0$

$\therefore$ $\underline{A = - M/(EI\alpha^2)}$

@ $x = 0, \frac{dy}{dx} = 0 \quad \therefore B = 0$

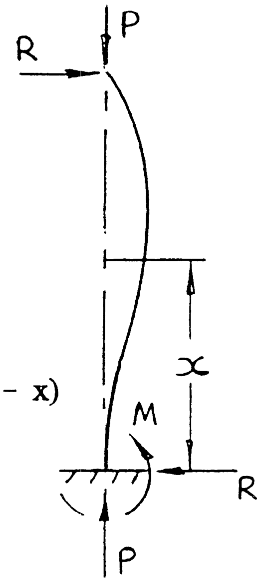
@ $x = \ell, \frac{d^2y}{dx^2} = 0$

$\therefore -\alpha^2 A \cos \alpha \ell = 0$ or $\cos \alpha \ell = 0$

ie $\alpha \ell = \pi/2$

$\therefore P_{cr} = \frac{\pi^2 EI}{4\ell^2}$

2. $EI \frac{d^2y}{dx^2} = -Py + R(\ell - x)$



$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{R}{EI}(\ell - x)$$

The complete solution is $y = A \cos \alpha x + B \sin \alpha x + \frac{R}{P}(\ell - x)$

@ $x = 0$, $y = 0$

$\therefore A = -R\,\ell/P$

@ $x = 0$, $\frac{dy}{dx} = 0$, $\therefore B = R/\alpha P$

$$\therefore y = \frac{R}{P}\left[-\ell \cos\alpha\ x + \frac{\sin\alpha\ell}{\alpha} + (\ell - x)\right]$$

@ $x = \ell$, $y = 0$

$$\therefore 0 = \frac{R}{P}\left[-\ell \cos\alpha\ell + \frac{\sin\alpha\ell}{\alpha}\right)$$

or $\tan \alpha \ell = \alpha \ell$

Hence $\alpha \ell = 4.5$ rads

so that $P_{cr} = 20.25\ EI/\ell^2$

3. $I = \frac{\pi(0.15^4 - 0.11^4)}{64} = 1.766E-5$

$A = 8.168E\text{-}3$

$k^2 = 2.162E\text{-}3$ $k = 0.046$ m

<u>$P_e = 363.1$ kN</u>

$$P_r = \frac{\sigma_c A}{\left[1 + a\left(\frac{\ell}{k}\right)^2\right]} = \frac{540E3 \text{ x } 8.16E-3}{\left[1 + \frac{1}{1600} * \left(\frac{36}{2.162E-3}\right)\right]}$$

$$= \frac{4410.7}{1 + 10.4} = \underline{386.7 \text{ kN}}$$

$$\frac{\pi^2 EI}{\ell^2} = \frac{\sigma_c A}{\left[1 + a\left(\frac{\ell}{k}\right)^2\right]}$$

$$\frac{1307230}{\ell^2} = \frac{540E6 \text{ x } 8.168E-3}{1 + \frac{1}{1600} * \frac{\ell^2}{2.162E-3}}$$

or $2.964\ (1 + 0.289\ \ell^2) = \ell^2$

$2.964 + 0.857\ \ell^2 = \ell^2$

$\underline{\ell = 4.55 \text{ m}}$

4.

$$30E3 = \frac{\pi^2 EI}{\ell^2}$$

$$I = \frac{30E3 \text{ x } 15^2}{\pi^2 \text{ x } 196.5E9} = \underline{3.48E-6m^4}$$

$$3.48E-6 = \frac{\pi(0.1^4 - d^4)}{64}$$

$7.09E-5 - 0.1^4 = -d^4$

$\underline{d = \sqrt{2.91E-5} = 0.0734 \text{ m}}$

$A = 3.617E\text{-}3$ $\qquad\qquad k^2 = 9.62E\text{-}4$

$$P_r = \frac{800E3}{\left[1 + a\left(\frac{\ell}{k}\right)^2\right]}$$

$$1 + a \quad \frac{15^2}{9.62E-4} = 26.67$$

$$a = 25.67 \text{ x } \frac{9.62E-4}{152} = 1.097E-4$$

$$a = \frac{1}{9113}$$

5. $$I = \frac{\pi}{64}\left[(36E-3)^4 - (24E-3)^4\right]$$

$\underline{I = 6.61E\text{-}8 \text{ m}^4}$

$\underline{A = 5.655E\text{-}4m^2}$ $\qquad\qquad k^2 = 1.17E\text{-}4$

$\sigma = \alpha\ell TE$

$P_R = \alpha\ell TEA = \underline{1255.4T}$ 1

$$P_R = \frac{325E6 \text{ x } 5.655E-4}{\left(1 + \frac{1}{7500} \text{ x } \frac{1^2}{1.17E-4}\right)} = \frac{183.79E-3}{2.1397}$$

$= \underline{85696 \text{ N}}$ 2

Equation 1 = Equation 2

$\therefore \underline{T = 68.4°C}$

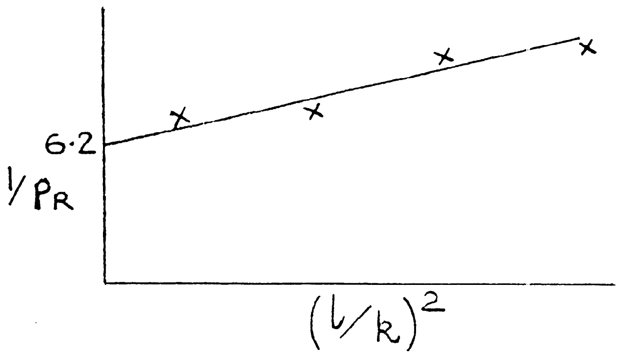
6. $I = \dfrac{\pi(35E\text{-}3^4 - 25E\text{-}3^4)}{64} = 5.449E\text{-}8m^4$

$A = 4.712E\text{-}4m^2$

$k^2 = 1.156E\text{-}4m^2 = 115.6\ mm^2$

ℓ (mm)	600	1000	1400	1800
$(\ell/k)^2$	3113	8650	16950	28020
$1/P_R$	6.667	8	9.09	11.36

Plot $1/P_R$ against $(\ell/k)^2$



From graph,

$$\text{Intercept} = \frac{1}{\sigma_c A} = 6.2 \ \& \ P_R = 0.162$$

$$\therefore \sigma_c = \frac{0.162}{A} = 344 \text{ MPa}$$

Also from graph

$$1 + a \left(\frac{\ell}{k}\right)^2 = \frac{\sigma_c A}{P_R}$$

From graph

$$\left(\frac{\ell}{R}\right)^2 = 30000$$

$1/P_R = 11.25$

$P_R = 0.0889$

$$a \times 30000 = \left(\frac{345 \times 4.712 \times 10^{-4}}{0.0889}\right) - 1$$

$$= 0.829$$

$$a = \frac{1}{36200}$$

7. $$P_R = \frac{\sigma_c A}{\left[1 + a \left(\frac{\ell}{k}\right)^2\right]}$$

or $$\frac{P_R}{A} = \frac{\sigma_c}{\left[1 + a \left(\frac{\ell}{k}\right)^2\right]}$$

Let $\underline{\sigma_c = 350 \text{ MN/m}^2}$

For $\left(\frac{\ell}{k}\right) = 50$ & $a = 1//8000$