Now,

$$\frac{d^2y}{dx^2} = \frac{+\,d^2y_o}{dx^2} - \alpha^2 y$$

The complete solution to this is

$$y = A\cos\alpha x + B\sin\alpha x - \frac{8\Delta}{\alpha^2 \ell}$$

$$\frac{dy}{dx} = -\alpha A\sin\alpha x + \alpha B\cos\alpha x$$

@ $x = 0$ $\quad y = 0 \therefore 0 = A - \dfrac{8\Delta}{\alpha^2 \ell}$

$\underline{A = 8\Delta/\alpha^2\,\ell^2}$

@ $x = \ell/2$, $\dfrac{dy}{dx} = 0 \;\therefore\; 0 = -\dfrac{\alpha 8\Delta}{\alpha^2\,\ell^2} \sin \dfrac{\alpha\ell}{2} + \alpha B\cos\dfrac{\alpha\ell}{2}$

$$\therefore B = \frac{8\Delta}{\alpha^2\ell^2}\tan\left(\frac{\alpha\ell}{2}\right)$$

$$y = \frac{8\Delta}{\alpha^2\ell 2}\cos\alpha x + \frac{8\Delta}{\alpha^2\,\ell^2}\tan\frac{\alpha\ell}{2}\sin\alpha x - \frac{8\Delta}{\alpha^2\ell^2}$$

The maximum deflection occurs at $x = \ell/2$

$$\delta = \frac{8\Delta}{\alpha^2\ell^2}\left[\cos\frac{\alpha\ell}{2} + \frac{\sin^2(\alpha\ell/2)}{\cos(\alpha\ell/2)} - 1\right]$$

$$= \frac{8\Delta}{\alpha^2\ell^2}\left[\cos\frac{\alpha\ell}{2} + \frac{1 - \cos^2\frac{\alpha\ell}{2}}{\cos\frac{\alpha\ell}{2}} - 1\right]$$

$$\delta = \frac{8\Delta}{\alpha^2\ell^2}\left(\sec\frac{\alpha\ell}{2} - 1\right)$$

$$M_{max} = P\delta$$

$$\therefore\ \sigma_b = \frac{P\delta\bar{y}}{Ak^2}$$

$$\sigma(\text{direct}) = \frac{P}{A}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{\delta\bar{y}}{k^2}\right]$$

$$= \frac{P}{A}\left[1 + \frac{8\Delta\bar{y}}{\alpha^2\ell^2k^2}\left(\sec\frac{\alpha\ell}{2} - 1\right)\right]$$

$$= \frac{P}{A}\left[1 + \frac{\Delta\bar{y}}{k^2}\cdot\frac{8EI}{P\ell^2}\left(\sec\frac{\alpha\ell}{2} - 1\right)\right]$$

Now, $P_e = \pi^2 EI/\ell^2$

$$\therefore \sigma_{max} = \frac{P}{A}\left[1 + \frac{\Delta\bar{y}}{k^2}\cdot\frac{8P_e}{\pi^2P}\left(\sec\frac{\alpha\ell}{2} - 1\right)\right]$$

and as $\alpha^2 = P/EI$

$$\frac{\alpha\ell}{2} = \tfrac{1}{2}\sqrt{\frac{P\ell^2}{EI}} = \tfrac{1}{2}\sqrt{\frac{P\pi^2}{P_e}}$$

$$\therefore\ \sigma_{max} = \frac{P}{A}\left[1 + \frac{\Delta\bar{y}}{k^2}\cdot\frac{8P_e}{\pi^2P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_e}} - 1\right)\right]$$

$$\text{or } \hat{\sigma} = \frac{P}{A}\left[1 + \frac{\Delta\bar{y}}{k^2}\,\frac{8EI}{P\ell^2}\left(\sec\frac{\alpha\ell}{2} - 1\right)\right]$$

where $P_e = \pi^2 EI/\ell^2$

$EI = 2 \times 10'' \times 6.594 \times 10^{-5} = 1.318E7$. $\ell = 10$ m

$\alpha\ell = 69.88°$ $\hat{\sigma} = -75$ MPa

P = 0.196 MN A = 9.327E-3 m^2 $\bar{y}$ = 12.5 cm

$$\hat{\sigma} = -21E6 - \frac{0.196E6 \times 12.5E{-2} \times 8 \times 2E11.\Delta}{0.196E6 \times 100}\left(\sec\frac{\alpha\ell}{2} - 1\right)$$

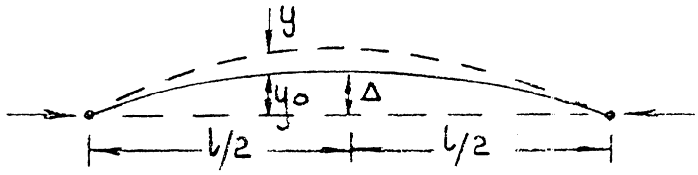
or

-75E6 = - 21E6 - 2000E6. Δ x 0.2199

- 54E6 = - 439.8E6 Δ

<u>Δ = 0.123 m - 12.3 m</u>

14.



$$\Delta(2R_o - \Delta) = \frac{\ell^2}{4}$$

$$2R_o\,\Delta - \Delta^2 = \frac{\ell^2}{4}$$

but $\Delta^2 \to 0$

$$\therefore R_o = \frac{\ell^2}{8\Delta}$$

but $\frac{1}{R_o} \doteq \frac{d^2y}{dx^2}$

$$\therefore EI\left(\frac{d^2y}{dx^2} - \frac{d^2y_o}{dx^2}\right) = -Py \text{ becomes}$$

$$EI\left(\frac{d^2y}{dx^2} - \frac{8\Delta}{\ell^2}\right) = -Py$$

$$\text{or } \frac{d^2y}{dx^2} + \alpha^2 y = \frac{8\Delta}{\ell^2}$$

$$y = A\cos\alpha x + B\sin\alpha x + \frac{8\Delta}{\alpha^2\ell^2}$$

$$@\, x = 0 \qquad y = 0 \;\therefore A = \frac{8\Delta}{\alpha^2\ell^2}$$

$$\frac{dy}{dx} = -\alpha A\sin\alpha x + \alpha B\cos\alpha x$$

$$@\, x = \ell/2, \quad \frac{dy}{dx} = 0$$

$$\therefore 0 = \alpha\frac{8\Delta}{\alpha^2\ell^2}\sin\frac{\alpha\ell}{2} + \alpha B\cos\frac{\alpha\ell}{2}$$

$$B = -\frac{8\Delta}{\alpha^2\ell^2}\tan\left(\frac{\alpha\ell}{2}\right)$$

$$\therefore y = -8\Delta/(\alpha^2\ell^2)\,[\cos\alpha x + \tan(\alpha\ell/2)\sin\alpha x - 1]$$

$$\frac{d^2y}{dx^2} = -\alpha^2 A\cos\alpha x - \alpha^2 B\sin\alpha x$$

$$\frac{d^2y}{dx^2} = +\alpha^2 \,.\, \frac{8\Delta}{\alpha^2\ell^2}\cos\alpha x + \alpha^2 \,.\, \frac{8\Delta}{\alpha^2\ell^2}\tan\left(\frac{\alpha\ell}{2}\right)\sin\alpha x$$

$$\frac{d^2y}{dx^2} = \frac{8\Delta}{\ell^2}\left[\cos\alpha x + \tan\left(\frac{\alpha\ell}{2}\right)\sin\alpha x\right]$$

M_{max} occurs @ $x = \ell/2$

$$M_{max} = EI \left(\frac{d^2y}{dx^2} - \frac{1}{R_o} \right)$$

$$EI \frac{d^2y}{dx^2} = \frac{2 \times 10^{11} \times 6.594 \times 10^{-5} \times 8\Delta}{100} [0.8197 + 0.6986 * 0.5727]$$

$$= \underline{1286925\ \Delta}$$

$$\frac{EI}{R_o} = 2 \times 10" \times 6.59 \times 10^{-5} \times \frac{8\Delta}{100} = \underline{1055040\ \Delta}$$

$\underline{M_{max} = 231885\ \Delta}$

$$\text{but } M_{max} = \frac{53.96 \times 10^6 \times 6.594 \times 10^{-5}}{12.5 \times 10^{-2}}$$

$\underline{M_{max} = 28465\ Nm}$

or $231885\Delta = 28465$

$\underline{\Delta = 0.123\ m = 12.3\ m}$

CHAPTER 13

1.

Section	a	y	ay	ay^2	i_H
1	1E-3	0.195	1.95E-4	3.803E-5	8.33E-9
2	2E-3	0.1	2E-4	2E-5	6.667E-6
Σ	3E-3	-	3.95E-4	5.803E-5	6.675E-6

$$\bar{y} = \frac{\sum ay}{\sum a} = 0.1317 \text{ m}$$

$$I_{XX} = \sum ay^2 + \sum i_H = 6.471E\text{-}5 m^4$$

$$I_{xx} = I_{XX} - \bar{y}^2 \sum a = 1.27E\text{-}5 \text{ m}^4$$

Section	a	x	ax	ax^2	i_V
1	1E-3	0.05	5E-5	2.5E-6	8.333E-7
2	2E-3	0.105	2.1E-4	2.205E-5	1.667E-8
Σ	3E-3	-	2.6E-4	2.455E-5	8.5E-8

$$\bar{x} = \frac{\sum ax}{\sum a} = 0.0867 \text{ m}$$

$$I_{YY} = \sum ax^2 + \sum i_V = 2.54E\text{-}5 \text{ m}^4$$

$$I_{yy} = I_{YY} - \bar{x}^2 \sum a = \underline{2.849E\text{-}6 \text{ m}^4}$$

$$I_{xy} = \sum A\overline{hk} = 1E\text{-}3 * (0.05 - 0.0867) * (0.195 - 0.1317)$$

$$+ 2E\text{-}3 * (0.105 - 0.0867) * (0.1 = 0.1317)$$

$$= -2.323E\text{-}6 - 1.16E\text{-}6$$

$$I_{xy} = -3.483E\text{-}6 \text{ m}^4$$

$$\tan 2\theta = \frac{2I_{xy}}{I_y - I_x} = \frac{-2 \times 3.483E-6}{2.849E-6 - 1.27E-5}$$

$$= 0.707$$

$$\therefore \theta = 17.64°$$

I_{uu} $= ½ (I_x = I_y) + ½ (I_x - I_y) \sec 2\theta$

$= 7.775E-6 + 4.9255E-6 * 1.225$

$\underline{I_{uu} = 1.381E\text{-}5m^4}$

$\underline{I_{vv} = 1.741E\text{-}6m^4}$

2.

Section	a	y	ay	ay^2	i_H
1	0.012	0.31	3.72E-3	1.153E-3	4E-7
2	4.5E-3	0.15	6.75E-4	1.0125E-4	3.375E-5
3	3E-3	7.5E-3	2.25E-5	1.6875E-7	5.625E-8
Σ	0.0195	-	4.418E-3	1.255E-3	3.42E-5

$$\bar{y} = \frac{\sum ay}{\sum a} = \underline{0.227m}$$

$I_{XX} = 1.2892E-3$

$\underline{I_{xx} = 2.844E\text{-}4m^4}$

Section	a	x	ax	ax^2	i_V
1	0.012	0.3	3.6E-3	1.08E-3	3.6E-4
2	4.5E-3	0.3	1.35E-3	4.05E-4	8.44E-8
3	3E-3	0.408	1.224E-3	4.99E-4	1E-5
Σ	0.0195	-	6.174E-3	1.984E-3	3.701E-4

$\underline{\bar{x} = 0.3166\ m}$

$\underline{I_{YY} = 2.354E\text{-}3}$

$\underline{I_{yy} = 3.994E\text{-}4m^4}$

$I_{xy} = 0.012 * (0.3 - 0.3166) * (0.31 - 0.227)$

$+ 4.5E\text{-}3 * (0.3 - 0.3166) * (0.15 - 0.227)$

$+ 3E\text{-}3 * (0.408 - 0.3166) * (7.5E\text{-}3 - 0.227)$

$= - 1.654E\text{-}5 + 5.752E\text{-}6 - 6.02E\text{-}8$

$I_{xy} = -7.097E\text{-}5m^4$

$$2\theta = \tan^{-1} \frac{2I_{xy}}{I_y - I_x}$$

$$= \tan^{-1} \left(\frac{-1.4194E-4}{1.15E-4}\right) = \tan^{-1} (-1.2340)$$

$$= - 50.99°$$

$$\theta = -25.49$$

$I_u = ½ (I_x + I_y) + ½(I_x - I_y) \sec (- 50.99)$

$= 3.419E\text{-}4 - 1.15 * E\text{-}4 * 1.589$

$= 3.419E\text{-}4 - 1.827E\text{-}4$

$I_u = 1.592E\text{-}4m^4$

$I_V = 5.246E\text{-}4m^4$

3. $$I_{yy} = \frac{BH^3}{36} = \frac{0.2 \text{ x } 0.,3^3}{36} = 1.5E-4m^4$$

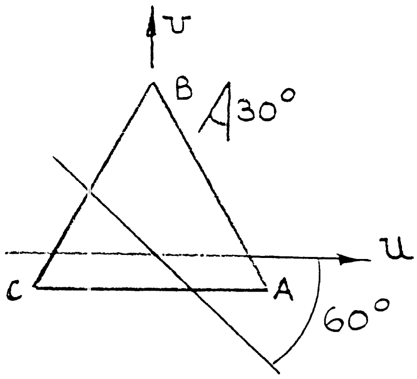
$$I_{vv} = \frac{2 \text{ x } 0.3 \text{ x } 0.1^3}{12} = 5E-5m^4$$

$\underline{M_{max} = 10 \times 3 = 30 \text{ kN.m}}$

$\underline{\theta = 0}$ $\underline{\text{as } I_{xy} = 0}$

$\beta = \tan^{-1}(-I_{uu} * \tan\alpha/I_{vv})$

$\underline{\beta = -60°}$



$\underline{\theta\ B}$

$\underline{u_B = 0}$ $\underline{v_B = 0.2 \text{ m}}$

$\underline{@\ C,\ u_C = -0.1,\ v_C = -0.1}$

$$\sigma_B = \frac{M\cos\alpha.v}{I_{uu}} + \frac{M\sin\alpha.u}{I_{vv}}$$

$= +30 \times 10^3 (1154.7 + 0) = \underline{+34.64 \text{ MN/m}^2}$

$\sigma_c = +30 \times 10^3 (-577.35 - 1000) = \underline{-47.32 \text{ MN/m}^2}$

$u_A = 0.1$ $v_A = -0.1$

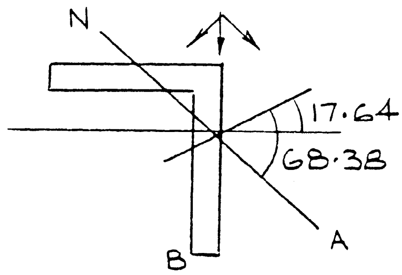
$\sigma_A = 30 \times 10^3 (-577.35 + 1000) = \underline{12.68 \text{ MN/m}^2}$

4. To find NA

$\beta = \tan^{-1}(-I_{uu} * \tan\theta/I_{vv})$

$= \tan^{-1}(-1.381E\text{-}5 * \tan(17.64)/1.741E\text{-}6)$

$\underline{\beta = -68.38°}$



$u_B = x_B\cos\theta + y_B\sin\theta$

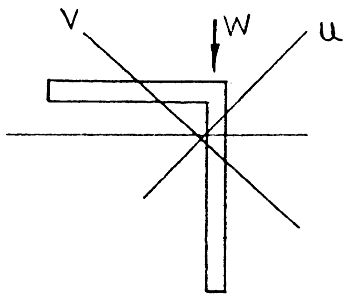
$= (0.1 - 0.0867)\cos 17.64 - 0.1317 \sin 17.64$

$= 0.0127 - 0.03991 = \underline{-0.0272 \text{ m}}$

$\upsilon_B \quad = y_B \cos\theta - x_B \sin\theta$

$= -0.1255 - 4.03E\text{-}3$

$\underline{\upsilon_B \quad = -0.1295 \text{ m}}$



$M = 4 \times 3 = 12 \text{ kN.m}$

$$\sigma = \frac{12\cos\theta\ \upsilon_B}{I_{uu}} + \frac{12\sin\theta\ u_B}{I_{vv}}$$

$$= \frac{12 \times 10^3 \times 0.953 \times (-0.1295)}{1.381E-5} + \frac{12 \times 10^3 \times 0.303 \times (-0.027)}{1.741E-6}$$

$= -107.24 - 56.39$

$\sigma_B = -\ 163.63 \text{ MN/m}^2$

5. $\beta \quad = \tan^{-1}(-I_{uu} \tan\theta / I_{vv})$

$= \tan^{-1}(-1.592E\text{-}4 \tan(-25.49)/5.246E\text{-}4)$

$\beta \quad = \tan^{-1}(0.1447) = \underline{8.23°}$

$u_B \quad = (0.6 - 0.3166)\cos(-25.49) + (0.32 - 0.227)\sin(-25.49)$

$= 0.2558 - 0.04 = \underline{0.2158 \text{ m}}$

$\upsilon_B \quad = (0.32 - 0.227)\cos(-25.49) - (0.6 - 0.3166) \times \sin(-25.49)$

$= \quad 0.084 + 0.122 = \underline{0.206 \text{ m}}$