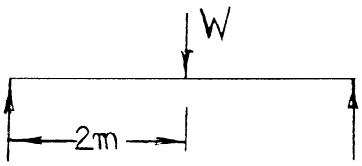
$w = 1.051E7/\ell^2 = 89561$

$\therefore \ell = 10.83$ m

4. $M_{max} = \frac{W}{2} \cdot x = 5x$



$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\therefore \sigma = \frac{My}{I} = 5x * \frac{d}{2} \frac{12}{0.03 * d^3}$$

$$\sigma = \frac{1000}{d^2} x$$

$$d^2 = \frac{1000}{\sigma} x = \frac{1000 * 1000}{100E6} x$$

$$d = 0.1x^{1/2}$$

5. $\epsilon_1 = \epsilon_d - \epsilon_b$

$\epsilon_2 = \epsilon_d + \epsilon_b$

Add 1 and 2

$\therefore \epsilon = 2\epsilon_d = 50 \times 10^{-6} - 200 \times 10^{-6}$

$\epsilon_d = 75 \times 10^{-6}$

$\underline{\sigma_d = -15 \text{ MN/m}^2}$

$$\therefore W = -15 * \pi \frac{(0.4^2 - 0.2^2)}{4} = \underline{-1.414 \text{ MN}}$$

$\epsilon_2 - \epsilon_1 = 2\epsilon_B = 250 \times 10^{-6}$

$\underline{\epsilon_b = 125 \times 10^{-6}}$

$\underline{\sigma_b = +/- 25 \text{ MN/m}^2}$

$$\frac{\sigma}{\bar{y}} = \frac{M}{I}$$

$$M = \frac{25}{0.2} \times \frac{\pi(0.4^4 - 0.2^4)}{64} = \underline{0.1473 \text{ MN.m}}$$

$$0.1473 = W\Delta$$

$$\therefore \Delta = 0.104 \text{ m}$$

6.

Section	a	y	ay	ay^2	i_o
1 2	1.5 1.0	2 0.25	3 0.25	6 0.0625	1.125 0.021
Σ	2.5	-	3.25	6.063	1.15

$$\bar{y} = \frac{3.25}{2.5} = 1.3 \text{ cm} = 1.3 \text{ E-2m}$$

$A = 2.5 \text{ cm}^2 = 2.5 \text{ E-4 m}^2$

$I_{XX} = 6.063 + 1.15 = 7.213 \text{ cm}^4$

$I_{NA} = 2.983 \text{ cm}^4 = 2.983\text{E-8 m}^4$

$M = 1 \text{ kN} \times 0.133 = \underline{0.133 \text{ kNm}}$

$$\sigma_d = \frac{1 \text{ kN}}{2.5 \times 10^{-4}} = \underline{4 \text{ MN/m}^2}$$

$$\sigma_{bottom} = \frac{0.133 \times 10^3 \times 1.3 \text{ E-2}}{2.983\text{E-8}} + 4$$

$\underline{\sigma_{bottom} = 61.96 \text{ MN/m}^2}$

$$\sigma_{top} = \frac{-0.133 \times 10^3 \times 0.022}{2.983\text{E-8}} + 4$$

$\underline{\sigma_{top} = - 94.1 \text{ MN/m}^2}$

7. $A_s = 1.885E\text{-}3\ m^2$

From (4.12)

$H = \sqrt{[8.883E\text{-}3 + 0.09425]} - 0.0943$

$\underline{H = 0.227\ m}$

From (4.13),

$\underline{M = 0.16\ MN.m}$

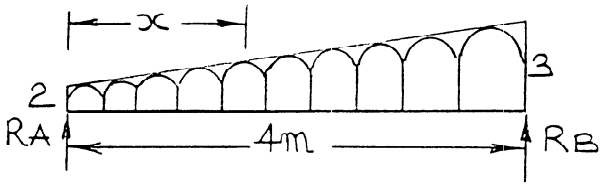
From (4.14),

$\underline{M = 0.144\ MN.m}$

$\underline{\therefore\ M\ (permissible) = 0.144\ MN.m}$

CHAPTER 5

1.



$w = 2 = x/4$

Moments about B

$R_A \times 4 = 2 \times 4 \times 2 + \frac{1}{2} \times 4 \times 1 \times \frac{1}{3} \times 4$

$= 16 + 2.667$

$R_A = 4.667$ kN

Resolving vertically

$$R_A + R_B = \frac{(2 + 3)}{2} \times 4 = 10$$

$R_B = 5.333$ kN

$$EI \frac{d^2y}{dx^2} = 4.667\,x - \frac{2 * x}{2} * \frac{2}{3}x - \frac{1}{2} * (2 + x/4) * x * \frac{1}{3} * x$$

$$= 4.667x - 0.667x^2 - \frac{1}{6}(2x^2 + x^3/4)$$

$$= 4.667x - 0.667x^2 - 0.333x^2 - 0.0417x^3$$

$$EI \frac{d^2y}{dx^2} = 4.667\,x - x^2 - 0.0417x^3$$

$$EI \frac{dy}{dx} = 2.333x^2 - 0.333x^3 - 0.01043x^4 + A$$

$$EIy = 0.778x^3 - 0.0833x^4 - 2.083E\text{-}3x^5 + Ax + B$$

@ $x = 0$, $y = 0$ $\therefore$ $B = 0$

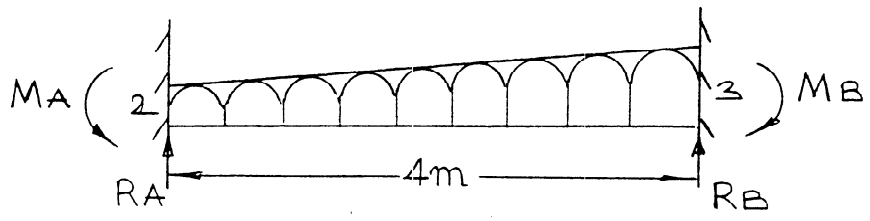
@ $x = 4$, $y = 0$

$\therefore\ 0 = 49.79 - 21.32 - 2.13 + 4A$

$\therefore\ A = -6.58$

ie $y = \frac{1}{EI}\ (0.778x^3 - 0.0833x^4 - 2.083E\text{-}3x^5 - 6.58x)$

2.



$EI\ \frac{d^2y}{dx^2} = R_A\,x - M_A - 0.667x^2 - 0.333x^2 - 0.0417x^3$

$EI\ \frac{d^2y}{dx^2} = R_Ax - M_A - x^2 - 0.0417x^3$

$EI\ \frac{dy}{dx} = \frac{R_Ax^2}{2} - M_Ax - \frac{x^3}{3} - \frac{0.0417x^4}{4} + A$

$EI\ \frac{dy}{dx} = 0.5\,R_Ax^2 - M_Ax - 0.333x^3 - 0.01043x^4 + A$

$EIy = 0.167\,R_Ax^3 - 0.5\,M_Ax^2 - 0.0833x^4 - 2.083E\text{-}3x^5 + Ax + B$

@ $x = 0,\ y = 0\ \therefore\ \underline{B = 0}$

@ $x = 0,\ \frac{dy}{dx} = 0,\ \therefore\ \underline{A = 0}$

$y = \frac{1}{EI}\ \{0.77\,x^3 - 1.62\,x^2 - 0.0833x^4 - 20.083E\text{-}3x^5\}$

@ x = 4, $\frac{dy}{dx} = 0$

$0 = 8\ R_A - 4\ M_A - 21.31 - 2.67$

$8\ R_A - 4M_A - 24 = 0$ ———— 1

@ x = 4, y = 0

$0 = 10.69\ R_A - 8M_A - 21.43 - 2.13$

$5.345\ R_A - 4M_A - 11.73 = 0$ ———— 2

Taking 2 from 1

$2.655\ R_A - 12.27 = 0$

$\underline{R_A = 4.62\ kN}$

Substituting 3 into 1

$36.97 - 4\ M_A - 24 = 0$

$\underline{M_A = 3.24\ kN.m}$

$\underline{R_B = 5.38\ kN}$

Moments about B

$M_B + 4.62 \times 4 = 3.24 + 2 \times 4 \times 2 + ½ \times 4 \times 1 \times ⅓ \times 4$

$M_B = -18.48 + 3.24 + 16 + 2.67$

$\underline{M_B = 3.43\ kN.m}$

$0 = 49.5 + 3A - 13.33 - 2.67 + 0.143 - 3.5$

$\underline{A = -10.05}$

$$\frac{R_A \ell}{6} = \frac{Wb^2}{2\ell} - \frac{Wb^3}{3\ell^2}$$

$$= \frac{Wb^2}{\ell}\left[\frac{1}{2} - \frac{b}{3\ell}\right]$$

$$= \frac{Wb^2}{\ell^2}\left[\frac{\ell}{2} - \frac{b}{3}\right]$$

$$= \frac{Wb^2}{6\ell^2}(3\ell - 2b)$$

$$= \frac{Wb^2}{6\ell^2}(\ell + 2a + 2b - 2b)$$

$$R_A = \frac{Wb^2}{\ell^3}(\ell + 2a)$$

$$R_B = W - \frac{Wb^2}{\ell^3}(\ell + 2a)$$

$$= W\,\frac{(\ell^3 - b^2\ell - 2ab^2)}{\ell^3}$$

3. Moments about B

$R_A \times 3.5 = 0.7 \times 1.5 \times 2.75$

$\underline{R_A = 0.825}$

$EI\ \frac{d^2y}{dx^2} = 0.825x - \frac{0.7x^2}{2}$	$+ \frac{0.7}{2}(x - 1.5)^2$
$EI\ \frac{dy}{dx} = 0.413x^2 - 0.117x^3 + A$	$+ 0.117\,(x - 1.5)^3$
$EIy = 0.138x^3 - 0.0292x^4 + Ax + B$	$+ 0.0292\,(x - 1.5)^4$

@ $x = 0, y = 0 \therefore \underline{B = 0}$

@ $x = 3.5, y = 0$

$\therefore\ 0 = 5.917 - 4.38 + 3.5A + 0.467$

$\underline{A = -0.573}$

For δ, $\dfrac{dy}{dx} = 0$ between C and B

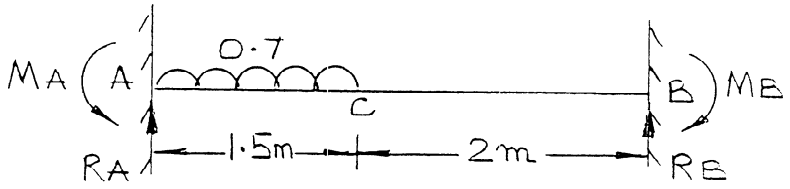
$0 = 0.413x^2 - 0.117x^3 - 0.573 + 0.117\,(x^3 + 2.25x - 1.5x^2 - 3.375)$

$0 = 0.238x^2 + 0.263x - 0.967$

$$x = \frac{-0.263 \pm \sqrt{0.069 + 0.921}}{0.476} = 1.540 \text{ m}$$

$\underline{\delta = 0.543/EI}$

4.



$EI\dfrac{d^2y}{dx^2} = R_A x - M_A - \dfrac{0.7x^2}{2}$	$+ \dfrac{0.7}{2}(x - 1.5)^2$
$EI\dfrac{dy}{dx} = \dfrac{R_A x^2}{2} - M_A x - 0.1167x^3 + A$	$+ 0.1167\,(x - 1.5)^3$
$EIy = \dfrac{R_A x^3}{6} - \dfrac{M_A x^2}{2} - 0.292x^4 + Ax + B$	$+ 0.0292\,(x - 1.5)^4$

@ $x = 0$, $y = 0$, $\therefore\ \underline{B = 0}$

@ $x = 0$, $\dfrac{dy}{dx} = 0$ $\therefore\ \underline{A = 0}$

@ x = 3.5, $\frac{dy}{dx} = 0$

$0 = 6.125\ R_A - 3.5\ M_A - 5 + 0.9336$

$M_A = 1.75\ R_A - 1.162$ ———— 1

@ x = 3.5, y = 0

$0 = 7.146\ R_A - 6.125\ M_A - 4.382 + 0.4672$

$M_A = 1.167\ R_A - 0.6392$ ———— 2

Equating 1 and 2

$(1.75 - 1.167)\ R_A = 1.162 - 0.6392$

$$R_A = \frac{0.5228}{0.583}$$

$\underline{R_A = 0.897\ kN}$

From 1

$\underline{M_A = 0.407\ kN.m}$

$\underline{R_B = 0.153\ kN}$

$\underline{M_B = 0.155\ kN.m}$

<u>To find "δ"</u>, try span CB

$0 = 0.4485x^2 - 0.407x - 0.1167x^3 + 0.1167\ (x^3 - 3.375 + 2.25x - 1.5x^2)$

$0 = 0.2735x^2 - 0.1444x - 0.394$

$$x = \frac{0.1444 + \sqrt{0.02085 + 0.431}}{0.547}$$

$= 1.49$ m

Not possible as $1.49 < 1.5$

Try span AC

$0 = 0.4485x^2 - 0.407x - 0.1167x^3$

$0 = -0.1167x^2 + 0.4485x^2 - 0.407$

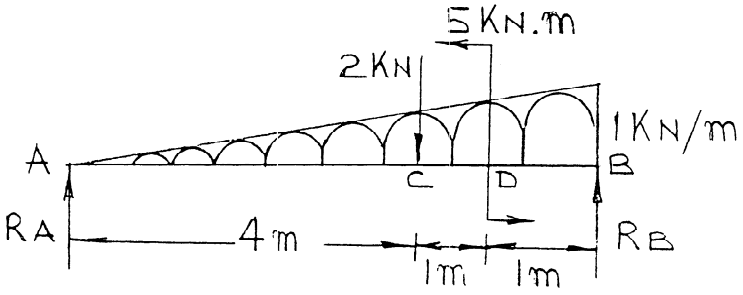
$$x = \frac{-0.4485 \pm \sqrt{0.2012 - 0.18999}}{-0.2334}$$

$x = 2.375$ or $x = \underline{1.47\ m}$ OK

$\delta = (0.4749 - 0.4397 - 0.1363)/EI$

$\underline{\delta = -0.101/EI}$

5.



$w = x/6$

Moments about B

$R_A \times 6 = 2 \times 2 + 5 + ½ \times 6 \times 1 \times ⅓ \times 6$

$\underline{R_A = 2.5}$

Moments about A

$R_B \times 6 + 5 = 2 \times 4 + ½ \times 6 \times 1 \times ⅔ \times 6$

$\underline{R_B = 2.5}$

RV $2.5 + 2.5 = ½ \times 6 \times 1 + 2 = 5$