$EI \frac{d^2y}{dx^2} = 2.5x - ½ * x * \frac{x}{6} * \frac{x}{3}$	$- 2 (x - 4)$	$- 5 (x - 5)^0$
$EI \frac{d^2y}{dx^2} = 2.5x - \frac{x^3}{36}$	$- 2 (x - 4)$	$- 5 (x - 5)^0$
$EI \frac{dy}{dx} = 1.25x^2 - \frac{x^4}{144} + A$	$- (x - 4)^2$	$- 5 (x - 5)$
$EIy = 0.417x^3 - \frac{x^5}{720} + Ax + B$	$- \frac{(x - 4)^3}{3}$	$- \frac{5}{2} (x - 5)^2$

@ $x = 0$, $y = 0$, $\therefore B = 0$

@ $x = 6$, $y = 0$

$0 = 90 - 10.8 + 6A - 2.667 - 2.5$

<u>$A = -12.34$</u>

@ $x = 4$

$$\delta_c = \frac{1}{4300} (26.69 - 1.422 - 49.36) = \underline{-5.6E\text{-}3m}$$

@ $x = 5$

$$\delta_D = \frac{1}{4300} (52.1 - 4.34 - 61.7 - 0.3) = \underline{- 3.32E\text{-}3m}$$

6.

$EI\frac{d^2y}{dx^2} = R_A x - M_A = x.\frac{x}{2}.\frac{x}{3}$	$+\frac{3(x-3)^2}{2} + \frac{(x-3)^3}{6}$	$+ 5(x-4)^0$
$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x - \frac{x^4}{24} + A$	$+\frac{3(x-3)^3}{6} + \frac{(x-3)^4}{24}$	$+ 5(x-4)$
@ $x = 0$, $\frac{dy}{dx} = 0 \therefore \underline{A = 0}$		
$EIy = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} - \frac{x^5}{120} + B$	$+\frac{3(x-3)^4}{24} + \frac{(x-3)^5}{120}$	$+\frac{5(x-4)^2}{2}$

@ $x = 0$ $y = 0$ $\therefore B = 0$

@ $x = 5$ $\frac{dy}{dx} = 0$

$0 \quad = 12.5\,R_A - 5\,M_A - 26.04 + 4 + 0.667 + 5$

$\quad = 12.5\,R_A - 5\,M_A - 16.373$

$R_A = 1.31 + 0.4\,M_A$ ———— 1

: $x = 5$, $y = 0$

$0 \quad = 20.833\,R_A - 12.5\,M_A - 26.04 + 2 + 0.27 + 2.5$

$\quad = 20.833\,R_A - 12.5\,M_A - 21.27$ ———— 2

Substituting 1 into 2

$0 = 27.29 - 4.167\,M_A - 21.27$

$M_A = \frac{6.02}{4.167} = 1.445$ kN.m ———— 3

Substituting 3 into 1

$\underline{R_A = 1.888 \text{ kN}}$

Resolving vertically

$$R_A + R_B = \frac{3 \times 3}{2} = 4.5$$

$\underline{R_B = 2.612 \text{ kN}}$

Moments about A

$$M_B + 5 + \frac{3 \times 3}{2} \times 2 = R_B \times 5 - M_A$$

$\underline{M_B = 13.06 + 1.445 - 5 - 9 = 0.505 \text{ kN.m}}$

7.

$EI \frac{d^2y}{d^2} = R_A x - M_A - \frac{2x^2}{2}$	$-5(x - 2) + \frac{(x - 2)^2}{2}.2$	$+ 10 (x - 4)^0$
$EI \frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x - \frac{x^3}{3} + A$	$- 5\frac{(x - 2)^2}{2} + \frac{(x - 2)^3}{3}$	$+ 10 (x - 4)$
@ $x = 0$ $\frac{dy}{dx} = 0 \therefore A = 0$		
$EIy = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} - \frac{x^4}{12} + B$	$-5 \frac{(x - 2)^3}{6} + \frac{(x - 2)^4}{12}$	$+ \frac{10 (x - 4)^2}{2}$

@ $x = 0$ $y = 0 \therefore B = 0$

@ $x = 6$, $\frac{dy}{dx} = 0$

$0 = 18 R_A - 6 M_A - 72 - 40 + 21.33 + 20$

$= 18 R_A - 6 M_A - 70.67$

$R_A = 0.333 M_A + 3.926$ ——— 1

@ $x = 6$, $y = 0$

$0 = 36 R_A - 18 M_A - 108 - 53.33 + 21.33 + 20$

$0 = 36\,R_A - 18M_A - 120$ ———————— 2

Substituting 1 into 2

$0 = 12\,M_A + 141.33 - 18\,M_A - 120$

$M_A = 3.556$ kN.m ———————— 3

Substitute 3 into 1

$R_A = 5.11$ kN

Resolving vertically

$R_A + R_B = 2 \times 2 + 5$

$R_b = 3.89$ kN

Moments about A

$M_B + 10 + 5 \times 2 + 2 \times 2 \times 1 = R_B \times 6 + M_A$

$M_B = -0.66 + 3.556$

$M_B = 2.89$ kN.m

8. Moments about B

$R_A \times 4 + 2 = 2 \times 2 + 2 \times 1 \times 3$

$R_A = 2$ kN

$EI\frac{d^2y}{dx^2} = 2x - \frac{x^2}{2}$	$-2(x-2) + \frac{(x-2)^2}{2}$	$+2(x-3)^0$
$EI\frac{dy}{dx} = x^2 - \frac{x^3}{6} + A$	$-(x-2)^2 + \frac{(x-2)^3}{6}$	$+2(x-3)$ 1
$EIy = \frac{x^3}{3} - \frac{x^4}{24} + Ax + B$	$-\frac{(x-2)^3}{3} + \frac{(x-2)^4}{24}$	$+(x-3)^2$ 2

@ $x = 0,\ y = 0 \therefore B = 0$

@ $x = 4,\ y = 0$

$\therefore = 21.33 - 10.667 + 4A - 2.667 + 0.667 + 1$

$A = - 2.416$

$$\therefore EI \frac{dy}{dx} = x^2 - \frac{x^3}{6} - 2.416 - (x - 2)^2 + \frac{(x - 2)^3}{6}$$

For maximum y, $\frac{dy}{dx} = 0$

Try second "span"

$$\therefore x^2 - \frac{x^3}{6} - 2.416 - x^2 + 4x - 4 + \frac{x^3}{6} - x^2 + \frac{12x}{6} - \frac{8}{6} = 0$$

or $2.416 + 4x - 4 - x^2 + 2x - 1.33 = 0$

$- x^2 + 6x - 7.746 = 0$

$$x = \frac{-6 \pm \sqrt{[36 - 4 \times 7.746]}}{-2} = \frac{-6 \pm 2.24}{-2}$$

$\underline{x = 1.88 \text{ m}}$

As $1.88 < 2$, the maximum deflection does not occur in the second "span".

Consider first "span"

$$0 = x^2 - \frac{x^3}{6} - 2.416$$

Try $x = 1.87$

$0 = 3.497 - 1.0899 - 2.416 = 0$

$x = 1.87$ m (approximately) ———————— 3

For maximum deflection, substitute 3 into 2

$$\delta = \frac{1}{EI} (2.18 - 0.51 - 4.518)$$

$\underline{\delta = - 0.0285 \text{ m}}$

9. $R_A \ell = \frac{3W\ell}{2} + \frac{w\ell^2}{2}$

$$R_A = \frac{3W}{2} - \frac{w\ell}{2}$$

$EI \frac{d^2y}{dx^2} = - Wx$	$+ R_A (x - \ell/2) - \frac{w}{2} (x - \ell/2)^2$
$EI \frac{dy}{dx} = - \frac{Wx^2}{2} + A$	$+ \frac{R_A}{2} (x - \ell/2)^2 - \frac{w}{6} (x - \ell/2)^3$
$EIy = - \frac{Wx^3}{6} + Ax + B$	$+ \frac{R_A}{6} (x - \ell/2)^3 - \frac{w}{24} (x - \ell/2)^4$

@ $x = 0, y = 0$ $\therefore$ $B = 0$

@ $x = \ell/2. y = 0$

$$\therefore 0 = - \frac{W\ell^3}{48} + \frac{A\ell}{2}$$

$$A = \frac{W\ell^2}{24}$$

@ $x = 3\ell/2, y = 0$

$$\therefore 0 = - \frac{27W\ell^3}{48} + \frac{3W\ell^3}{48} + \frac{R_A}{6} \ell^3 - \frac{w\ell^4}{24}$$

$$0 = -\frac{27W}{48} + \frac{3W}{48} + \frac{3W}{12} + \frac{w\ell}{12} - \frac{w\ell}{24}$$

$0 = - \frac{W}{4} + \frac{w\ell}{24}$ ie $w = 6W/\ell$

CHAPTER 6

1. $J_1 = 1.272E\text{-}6m^4$

$J_2 = 4.02E\text{-}6m^4$

$J_3 = 3.77E\text{-}6m^4$

$$\theta = \frac{3E3}{7.7E10} \times \left(\frac{0.6}{1.272E-6} + \frac{0.8}{4.02E-6} + \frac{0.7}{3.77E-6}\right)$$

$$= 3.896E-8 \times (471698 + 198944 + 185676)$$

$$\theta = 3.336E-2 \text{ rads} = 1.91 \text{ degrees}$$

$$\tau_1 = \frac{TR_1}{J_1} = 70.75 \text{ MN/m}^2$$

$$\tau_2 = \frac{TR_2}{J_2} = 29.85 \text{ MN/m}^2$$

$$\tau_3 = \frac{TR_3}{J_3} = 31.83 \text{ MN/m}^2$$

2. θ $= 3E3 \times (6.126E\text{-}6 + 7.96E\text{-}6 + 4.761E\text{-}6)$

$= 0.0565 \text{ rads} = \underline{3.24°}$

τ_1, τ_2 and τ_3 as in Example 1.

3. $\omega = 2\pi * 25 = 157.1$ Hz

$T\omega$ = Power

$$\therefore T = \frac{5E3}{152.1} = \underline{31.83 \text{ N.m}}$$

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\frac{35E6}{0.01} = \frac{31.83}{\frac{\pi}{2}(1E-8-r^4)}$$

$\therefore$ $(1E\text{-}8 - r^4) = 5.7896E\text{-}9$

$r^4 = 4.12E\text{-}9$

$r = 8.055$ mm

<u>d = 16.11 mm (say) 16 mm</u>

<u>Bolts</u>

$$\delta F = \frac{\pi d^2}{4} * \tau_F = \underline{35.34E6\ d^2}$$

$T = n * \delta F * R = 2.12E6 * d^2 = 31.83$

$\therefore$ <u>d = 3.88 mm (say) 4 mm</u>

4. $J_1 = 2.749E\text{-}6$

$J_2 = 8.545E\text{-}6$

$$\frac{\tau}{r} = \frac{G\theta}{\ell}$$

$$\theta = \frac{\tau\ell}{Gr}$$

$$\theta_{C1} = \frac{140E6 \text{ x } 0.6}{7.7E10 \text{ x } 0.04} = 0.02727 \text{ rads}$$

$$\theta_{C2} = \frac{90E6 \text{ x } 0.6}{2.6E10 \text{ x } 0.05} = 0.0415 \text{ rads}$$

$\therefore$ design criterion is $\tau_{(steel)} = 140E6$

$$\frac{T}{J} = \frac{G\theta}{\ell}$$

$$T_1 = \frac{G_1\ \theta_c\ J_1}{\ell_1} = \frac{7.7E10 \times 0.02727 \times 2.749E-6}{0.6}$$

$\underline{T_1 = 9620.5\ N.m}$

$$T_s = \frac{G_2 \theta_C J_2}{\ell_2} = \frac{2.6E10 \times 0.2727 \times 8.545E-6}{0.6}$$

$\underline{T_2 = 10\ 098\ Nm}$

$\underline{T = T_1 + T_2 = 19.72\ kN.m}$

5. $J_s = 8.545E\text{-}6m^4$

$J_a = 9.8175E\text{-}6m^4$

$$\theta_{ca} = \frac{T_a \ell_a}{G_a J_a} = \underline{3.918E-6\ T_a}$$

$\theta_{cs} = 1.5369E\text{-}6T_s$

$\underline{T_s = 2.549T_a}$

$T_s + T_a = 9$

$\underline{T_a = 2.536\ kN.m}$

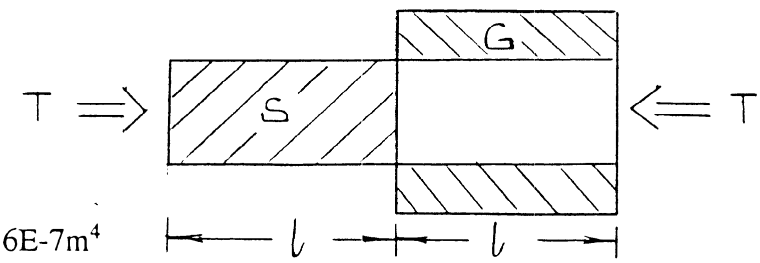
$\underline{T_s = 6.464\ kN.m}$

$\underline{\theta_c = 9.935\ rads = 0.569°}$

$$\tau_{al} = \frac{2.536E3}{9.8175E-6} \times 0.05 = \underline{12.92\ MN.m^2}$$

$\underline{\tau_{steel} = 37.82\ MN/m^2}$

6.



$J_s = 6.136E\text{-}7m^4$

$$J_G = \frac{\pi}{32}(D_2^4 - 0.05^4)$$

$$\frac{GJ}{\ell} = k$$

$$\frac{G_s J_s}{\ell_s} = \frac{G_G\ J_G}{\ell_G}$$

$$J_G = 1.534E\text{-}6m^4 = \frac{\pi}{32}(D_2^4 - 0.05^4)$$

$$\therefore D_2^4 = 1.5625E{-}5 + 6.25E{-}6$$

$$D_2 = 0.0684m = 68.4\ mm$$

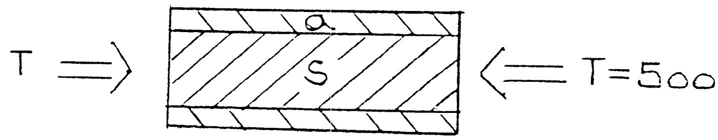
$$T_G = \frac{\tau_G}{r_G} \times J_G = 2018Nm$$

$$T_s = \frac{\tau_s \times J_s}{r_s} = 2209\ Nm$$

7.

$\underline{T_s = 333.3N.m}$

$\underline{T_{a\ell} = 166.7\ m}$



$$\frac{T}{J} = \frac{G\theta}{\ell}$$

$$\theta_s = \frac{T_s}{G_s\ J_s}\ \ell = \frac{T_a \ell}{G_a\ J_a}$$

$$\therefore \frac{T_s}{G_s\ J_s} = \frac{166.7}{3.8E10J_a}$$

$$\frac{333.3}{7.7E10J_s} = \frac{166.7}{3.8E10J_a}$$