$\underline{J_a = 1.0134 J_s}$ 1

$$J_a = \frac{\pi(D^4 - d^4)}{32}$$ 2

$$J_s = \frac{\pi d^4}{32}$$ 3

$D^4 - d^4 = 1.0134\ d^4$

$D^4 = 2.0134\ d^4$

$\underline{D = 1.191\ d}$

$$\frac{\tau}{r} = \frac{G\theta}{\ell}$$

Steel

$$\theta = \frac{\tau_s \ell}{G_s\ r_s} = \frac{36E6\ \ell \times 2}{7.7E10 \times d}$$

$$\frac{\theta}{\ell} = \frac{9.35E{-4}}{d}$$ 4

Aluminium bronze alloy

$$\theta = \frac{\tau_a \ell}{G_a\ r_a} = \frac{18E6\ \ell \times 2}{3.8E10 \times D}$$

$$\frac{\theta}{\ell} = \frac{7.954E{-4}}{d}$$ 5

Al bronze alloy is the design criterion

$$J_a = \frac{\pi}{32}(1.191^4 - 1^4)d^4 = \underline{0.0994 d^4}$$

$$\frac{\tau_a}{r_a} = \frac{T_a}{J_a}$$

$$\frac{18E6}{1.191d} \times 2 = \frac{166.7}{0.994d^4}$$

d = 0.0381m = <u>38.1 mm</u>

<u>D = 45.4 mm</u>

8.

$$\frac{\tau}{r} = \frac{G\theta}{\ell}$$

$$r = \frac{\tau\ell}{G\theta} = \frac{180 \times 10^6 \times 1}{7.7E10 \times 0.06109} = 0.03827 \text{ m}$$

$$\frac{T_e}{J} = \frac{\tau}{r}$$

$$T_e = \frac{180E6 \times 3.3687E{-6}}{0.03827} \cdot \frac{1}{1E6} = 0.0158 \text{ MN.m}$$

$$T_p = \frac{2\pi}{3} \tau_{yp} (R^3 - r^3)$$

$$= \frac{2\pi}{3} \cdot 180E6\ (0.1^3 - 0.03827^3) \cdot \frac{1}{1E6}$$

$$= 0.356 \text{ MN.m}$$

$$T = T_e + T_p = 0.37\text{MN.m}$$

9. $$\tau = \frac{T}{2At}$$

$$170E6 = \frac{T}{2 \times 0.1 \times 0.2 \times 1E{-2}}$$

<u>T = 68 000 N.m</u>

$$\theta = \frac{TL}{4A^2G} \oint \frac{ds}{t}$$

$$\theta = \frac{68000 \text{ x } 1}{4 \text{ x } 4E-4 \text{ x}7.7E10} \text{ x } \frac{180}{\pi} \left( \frac{0.5}{1E-2} + \frac{0.1}{2E-2} \right)$$

$$= 0.0316 \text{ x } 55$$

$\underline{\theta = 1.738°}$

10. $$J = \Sigma \frac{bt^3}{3}$$

$$= \frac{0.1 \text{ x } (1E - 2)^3 + 0.2 \text{ x } (1E - 2)^3 + 0.2 \text{ x } (2E - 2)^3}{3}$$

$$J = \frac{3E-7 + 1.6E-6}{3}$$

$\underline{J = 6.333E\text{-}7 \text{ m}}$

$$\tau_{max} = \frac{T \, t_{max}}{J}$$

$$T = \frac{170E6 \text{ x } 6.333E-7}{2E-2} = 5383 \text{ M.m}$$

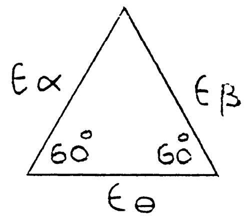
$$T = GJ \left( \frac{\theta}{\ell} \right)$$

$$\frac{\theta}{\ell} = \frac{T}{GJ} = \frac{5383}{7.7E10 \text{ x } 6.333E-7} \text{ x } \frac{180}{\pi}$$

$$\frac{\theta}{\ell} = 6.325°/\text{metre}$$

**CHAPTER 7**

1.




$\epsilon_\theta = ½ (\epsilon_1 + \epsilon_2) + ½ (\epsilon_1 - \epsilon_2) \cos 2\theta$ 1

$\epsilon_\alpha = ½ (\epsilon_1 + \epsilon_2) + ½ (\epsilon_1 - \epsilon_2) \cos (2\,\theta + 120°)$ 2

$\epsilon_\beta = ½ (\epsilon_1 + \epsilon_2) + ½ (\epsilon_1 - \epsilon_2) \cos (2\,\theta + 240°)$ 3

Equations 2 and 3 can be rewritten in the following form:

$\epsilon_\alpha = ½ (\epsilon_1 + \epsilon_2) + ½ (\epsilon_1 - \epsilon_2) (- \cos 2\theta.\cos 60 - \sin 2\theta) \sin 60)$ 4

$\epsilon_\beta = ½ (\epsilon_1 + \epsilon_2) + ½ (\epsilon_1 - \epsilon_2) (- \cos 2\theta.\cos 60 + \sin 2\theta.\sin 60)$ 5

Adding together equations 1, 4 and 5

$$\epsilon_\theta + \epsilon_\alpha + \epsilon_\beta = \frac{3}{2} (\epsilon_1 + \epsilon_2)$$

or $(\epsilon_1 + \epsilon_2) = ⅔ (\epsilon_\theta + \epsilon_\alpha + \epsilon_\beta)$ 6

Taking 4 from 5

$\epsilon_\beta - \epsilon_\alpha = (\epsilon_1 - \epsilon_2) \sin 2\theta.\ \sin 60$ 7

Taking 1 from 5

$$\epsilon_\beta - \epsilon_\theta = ½ (\epsilon_1 - \epsilon_2) (- \frac{3}{2} \cos 2\theta + \sin 2\theta.\ \frac{\sqrt{3}}{2})$$ 8

Dividing 8 by 7

$$\frac{\epsilon_\beta - \epsilon_\theta}{\epsilon_\beta - \epsilon_\alpha} = \frac{1}{2} \frac{(-\frac{3}{2} \cos 2\theta + \sin 2\theta \cdot \frac{\sqrt{3}}{2})}{(\sin 2\theta \cdot \frac{\sqrt{3}}{2})}$$

$$= \frac{1}{2} \left(- \sqrt{3} \cot 2\theta + 1\right)$$

$$\frac{2(\epsilon_\beta - \epsilon_\theta)}{\epsilon_\beta - \epsilon_\alpha} = - \sqrt{3} \cot 2\theta + 1$$

or

$$- \sqrt{3} \cot 2\theta = \frac{2(\epsilon_\beta - \epsilon_\theta)}{\epsilon_\beta - \epsilon_\alpha} - 1$$

$$= \frac{2\epsilon_\beta - 2\epsilon_\theta - \epsilon_\beta + \epsilon_\alpha}{\epsilon_\beta - \epsilon_\alpha}$$

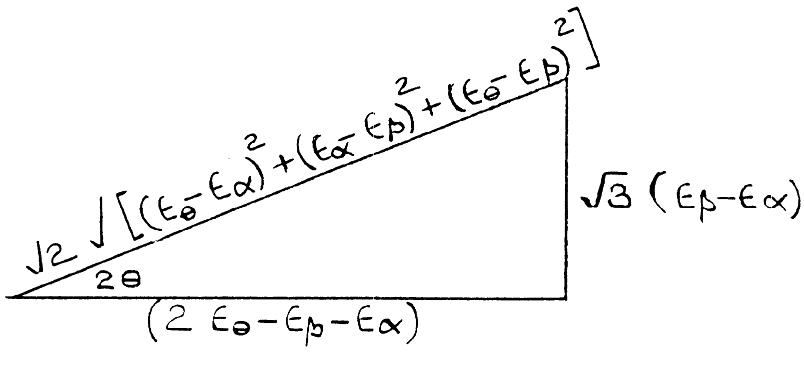
$$= \frac{-2\epsilon_\theta + \epsilon_\beta + E_\alpha}{\epsilon_\beta - \epsilon_\alpha}$$

or

$$\tan 2\theta = \frac{\sqrt{3} (\epsilon_\beta - \epsilon_\alpha)}{2\epsilon_\theta - \epsilon_\beta - \epsilon_\alpha} \qquad 9$$

<u>To determine $E_1$ & $E_2$</u>

From the mathematical triangle



$$\cos 2\theta = \frac{(2\epsilon_\theta - \epsilon_\beta - \epsilon_\alpha)}{\sqrt{2}\sqrt{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)]^2]}} \qquad 10$$

$$\sin 2\theta \; \frac{\sqrt{3}\,(\epsilon_\beta - \epsilon_\alpha)}{\sqrt{2}\sqrt{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)^2]}} \qquad 11$$

Substituting equation 11 into 7

$$\epsilon_\beta - \epsilon_\alpha = (\epsilon_1 - \epsilon_2).\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}\,(\epsilon_\beta - \epsilon_\alpha)}{\sqrt{2}\sqrt{[(\epsilon_\theta - \epsilon_\alpha^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)^2]}}$$

$$\text{or } \epsilon_1 - \epsilon_2 = \frac{2}{3}\sqrt{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)^2]} \qquad 12$$

Adding equations 6 and 12

$$\epsilon_1 = ⅓\sqrt{[(\epsilon_\theta + \epsilon_\alpha + \epsilon_\beta) + \frac{\sqrt{2}}{3}\sqrt{[(\epsilon_\theta - \epsilon_\alpha)^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)^2]}$$

$$\epsilon_2 = ⅓\,(\epsilon_\theta + \epsilon_\alpha + \epsilon_\beta) - \frac{\sqrt{2}}{3}\sqrt{[(\epsilon_\theta - \epsilon\alpha)^2 + (\epsilon_\alpha - \epsilon_\beta)^2 + (\epsilon_\theta - \epsilon_\beta)^2]}$$

2.

$\sigma_\theta = ½(\sigma_x + \sigma_y) + ½\,(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$

Let $\underline{\sigma_x = -10\ \text{MN/m}^2}$

<u>For $\theta = 30°$</u>

$\therefore \quad \sigma_{30} = 30 = - ½ (-10 + \sigma_y) + ½ (-10 - \sigma_y) \cos 60 + \tau_{xy} \sin 60$

$$30 = - 5 + \frac{\sigma_y}{2} - (5 + \frac{\sigma_y}{2}) 0.5 + 0.866 \tau_{xy}$$

$30 = - 5 + 0.5\sigma_y - 0.25 \sigma_y - 2.5 + 0.866 \tau_{xy}$

$37.5 = 0.25 \sigma_y + 0.866 \tau_{xy}$ 1

<u>For $\theta = 100°$</u>

$40 = ½ (-10 + \sigma_y) + ½ (- 10 - \sigma_y) \cos 200 + \tau_{xy} \sin 200$

$40 = - 5 + 0.5\sigma_y - (5 + \sigma_y/2) (-0.9397) - 0.342 \tau_{xy}$

$40 = - 0.3015 + 0.9698\sigma_y - 0.342 \tau_{xy}$

$40.302 = 0.9698 \sigma_y - 0.342 \tau_{xy}$ 2

Divide 1 by 0.25

$150 = \sigma_y + 3.464 \tau_{xy}$ 1a

Divide 2 by 0.9698

$41.55 = \sigma_y - 0.353 \tau_{xy}$ 2a

Take 5a from 1a

$108.45 = 3.817 \tau_{xy}$

<u>$\therefore \tau_{xy} = 28.41$ MN/m$^2$</u>

From 1a

<u>$\sigma_y = 51.58$ MN/m$^2$</u>

$$\theta = ½ \tan^{-1} \left( \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

<u>$\theta = - 21.35°$</u>

$\sigma_1 = ½ (\sigma_x + \sigma_y) + ½ \sqrt{[(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}^2]}$

$= 20.79 + 41.89$

$\underline{\sigma_1 = 62.68 \text{ MN/m}^2}$

$\underline{\sigma_2 = -21.1 \text{ MN/m}^2}$

3.

$\epsilon_\theta = ½ (\epsilon_x + \epsilon_y) + ½ (\epsilon_x - \epsilon_y) \cos 2\theta + ½ \gamma_{xy} \sin 2\theta$

Let $\epsilon_x = 300 \times 10^{-6}$

$\underline{\text{At } \theta = 30°}$

$E_{30} = -100 \times 10^{-6} = 150 \times 10^{-6} + 0.5\epsilon_y + 0.5 (300 \times 10^{-6} - \epsilon_y)$

$+ 0.5 \gamma_{xy}$ 1

$-100 \times 10^{-6} = 225E\text{-}6 + 0.25 \epsilon_y + 0.433 \gamma_{xy}$

$-325E\text{-}6 = 0.25 \epsilon_y + 0.433 \gamma_{xy}$

$\underline{\text{At } \theta = 100°}$

$- 200 E\text{-}6 + 150 E\text{-}6 + 0.5 \epsilon_y = 0.5 (300E\text{-}6-\epsilon_y) (-0.9397)$

$+ 0.5 \gamma_{xy} * (-0.342)$

$= 9.05E\text{-}6 + 0.9699 \epsilon_y - 0.171 \gamma_{xy}$

$- 209.1E\text{-}6 = 0.9699 \epsilon_y - 0.171 \gamma_{xy}$ 2

$- 1300 E\text{-}6 = \epsilon_y - 0.1732 \gamma_{xy}$ 1a

$- 215.6E\text{-}6 = \epsilon_y - 0.176 \gamma_{xy}$ 2a

Take 2a from 1a

$- 1084.4 - 1.908 \gamma_{xy}$

$\underline{\gamma_{xy} = - 568.3 \times 10^{-6}}$

<u>From 1a</u>

$\epsilon_y = - 315.6E\text{-}6$

$$\epsilon_1 = \frac{(\epsilon_x + \epsilon_y)}{2} + ½ \sqrt{[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}]^2}$$

$= - 7.8E-6 + \frac{1}{2} \times 837.8 \times 10^{-6}$

$\epsilon_1 = - 7.8E-6 + 418.9E-6$

$\underline{\epsilon_1 = 411.1 \times 10^{-6}}$

$\underline{\epsilon_2 = -426.7E-6}$

$$\sigma_1 = \frac{2 \times 10^{11}}{0.91} (411.1 - 0.3 \times 426.7) \times 10^{-6}$$

$\underline{\sigma_1 = 62.2 \text{ MN/m}^2}$

$\underline{\sigma_2 = -66.67 \text{ MN/m}^2}$

$\underline{\hat{\tau} = 64.44 \text{ MN/m}^2}$

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \right)$$

$\underline{\theta = - 21.36°}$

4.

$$G = \frac{2E11}{(2.6)} = 7.69E10$$

$\gamma = \epsilon_1 - \epsilon_3$

$= (-300 - 200) E-6$

$= - 500 E-6$

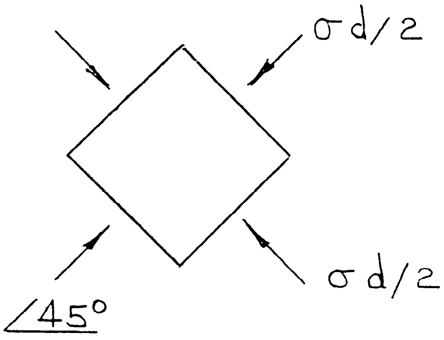
$\underline{\tau = 38.45 \text{ MN/m}^2}$

$$J = \frac{\pi(2E-2)^4}{32} = \underline{1.571E-8m^4}$$

$$T = \frac{\tau * J}{r} = \frac{38.45E6 \times 1.571E-8}{1E-2}$$

$\underline{T = 60.4 \text{ Nm}}$

$A = 3.142E\text{-}4m^2$




$\epsilon_1 = \epsilon_{IT} + \epsilon_{45D}$

$\epsilon_3 = -\epsilon_{IT} + \epsilon_{45D}$

Adding together

$(-300 + 200)\ E\text{-}6 = 2\epsilon_{45D}$

$\therefore\ \epsilon_{45D} = -\ 50E\text{-}6$ @ 45° to axis

$$= \frac{1}{2E}\ (\sigma_d - \upsilon\sigma_d)$$

$$\therefore\ \frac{(-100\ E-6)\ 2E11}{0.7} = \sigma_d$$

$\sigma_d = -28.57\ MN/m^2$

$\underline{\text{Thrust} = -\ 8.98\ \text{kN}}$

5.

$50 \sin\alpha\ ab = 150\ ac + 75\ bc = 0$

$$50 \sin\alpha + 150\ \frac{ac}{ab} + 75\ \frac{bc}{ab} = 0$$

$50 \sin\alpha + 150 \sin\alpha + 75 \cos\alpha = 0$

$200 \sin\alpha = -\ 75 \cos\alpha$

$\underline{\alpha = -\ 20.56°}$

$50 \cos\alpha\ ab + 75\ ac + \sigma_y\ bc = 0$

$46.82 - 26.34 + 0.936\sigma_y = 0$

$\therefore\ \underline{\sigma_y = -21.88}$

$\sigma_1 = ½\ (150 - 21.88) + ½\ \sqrt{[\ (150 + 21.88)^2 + 4 \times 75^2]}$