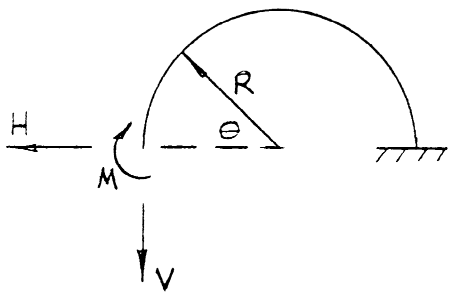
CHAPTER 9

1.



$M = M + HR \sin\theta - VR(1 - \cos\theta)$

$$\delta V = \frac{1}{EI} \int M \frac{\partial M}{\partial V} dx$$

$$= \frac{1}{EI} \int [M + HR \sin\theta - VR(1 - \cos\theta)][-R(1 - \cos\theta)] Rd\theta$$

but $H = V = 0$

$$\therefore \delta V = -\frac{MR^2}{EI} \int M(1 - \cos\theta)\, d\theta$$

$$= -\frac{MR^2}{EI} [\theta - \sin\theta]_0^{\pi}$$

$$\delta V = \frac{-\pi MR^2}{EI} \text{ ie upwards}$$

$$\delta H = \frac{1}{EI} \int M \frac{\partial M}{\partial H} Rd\theta$$

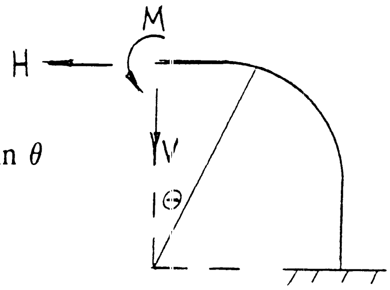
$$= \frac{1}{EI} \int [M] \; R\sin\theta \; Rd\theta$$

$$= \frac{-MR^2}{EI} [\cos \theta]_0^\pi$$

$$= \frac{-MR^2}{EI} [-1 \; -1]$$

$$\delta H = \frac{2MR^2}{EI} \text{ to the left}$$

1b.



$$M = M + HR\,(1 - \cos\theta) + V\,R\,\sin\theta$$

$$\delta V = \frac{1}{EI} \int [M] \; R\sin\theta \; Rd\theta$$

$$= \frac{MR^2}{EI} [-\cos\theta]_0^{\pi/2}$$

$$= \frac{MR^2}{EI} [0 + 1]$$

$$\delta V = \frac{MR^2}{EI} \text{ downwards}$$

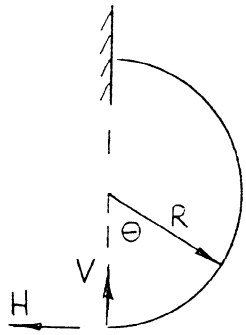
$$\delta_H = \frac{1}{EI} \int [M] \; R\,(1 - \cos\theta) \; Rd\theta$$

$$= \frac{MR^2}{EI} [\theta - \sin\theta]_0^{\pi/2}$$

$$= \frac{MR^2}{EI}\left\{\left[\frac{\pi}{2} - 1\right] - [0]\right.$$

$$\delta_H = \frac{MR^2}{EI}\left[\frac{\pi}{2} - 1\right] \text{ to the left}$$

1c



$$M = HR(1 - \cos\theta) + VR\sin\theta$$

$$\delta V = \frac{1}{EI}\int_0^{\pi}[HR\,(1 - \cos\theta) + VR\sin\theta]\,R\sin\theta.Rd\theta$$

$$= \frac{R^2}{EI}\int \{HR\,(\sin\theta - \sin\theta\cos\theta) + VR\sin^2\theta\}\,d\theta$$

$$= \frac{R^2}{EI}\left\{HR\,[-\cos\theta - \frac{\sin^2\theta}{2}] + VR\int\frac{(1 - \cos 2\theta)}{2}\,d\theta\right\}$$

$$= \frac{R^2}{EI}\left\{HR\,[-\cos\theta - \frac{\sin^2\theta}{2}] + \frac{VR}{2}\,[\theta - \frac{\sin 2\theta}{2}]\right\}_0^{\pi}$$

$$= \frac{R^3}{EI}\left\{H\left[(+1 - 0) - (-1 - 0) + \frac{V}{2}\,[(\pi - 0) - (0 - 0)\right]\right.$$

$$\delta V = \frac{R^3}{EI}\left[2H + \frac{\pi V}{2}\right]$$

$$\delta H = \frac{R^3}{EI} \int_0^{\pi} [H(1 - \cos\theta) + V\sin\theta] \, R(1 - \cos\theta) \, d\theta$$

$$= \frac{R^3}{EI} \int H(1 - 2\cos\theta + \cos^2\theta) + V(\sin\theta - \sin\theta\cos\theta) d\theta$$

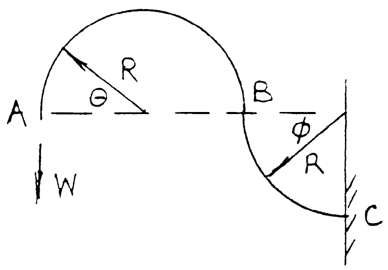
$$= \frac{R^3}{EI} \left\{ H[\theta - 2\sin\theta] + V\left[-\cos\theta - \frac{\sin^2\theta}{2}\right] + H \int \frac{(1 + \cos 2\theta)}{2} d\theta \right\}_0^{\pi}$$

$$= \frac{R^3}{EI} \left\{ H\left[\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right] + V\left[-\cos\theta - \frac{\sin^2\theta}{2}\right] \right\}$$

$$= \frac{R^3}{EI} \left\{ H\left[\left(\frac{3\pi}{2} - 0 + 0\right] - (0) + V[(+1 - 0) - (-1 + 0)]\right] \right\}$$

$$\delta H = \frac{R^3}{EI} \left[\frac{3\pi H}{2} + 2V\right]$$

2.



<u>AB</u>

$$M = WR(1 - \cos\theta)$$

<u>BC</u>

$$M = W[2R + R(1 - \cos\phi)]$$

$$\delta W = \frac{1}{EI}\int_0^{\pi} WR\,(1 - \cos\theta).\ R(1 - \cos\theta)\ Rd\theta$$

$$+ \frac{W}{EI}\int_0^{\pi/2} [2R + R\,(1 - \cos\phi)]^2\ Rd\theta$$

$$= \frac{WR^3}{EI}\int_0^{\pi} (1 - 2\cos\theta + \cos^2\theta)\ d\theta$$

$$+ \frac{WR^3}{EI}\int_0^{\pi/2} [4 + (1 - 2\cos\phi + \cos^2\phi) + 4\,(1 - \cos\phi)]d\phi$$

$$= \frac{WR^3}{EI}\left\{\left[1 - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi} + \int [4 + 1 - 2\cos\phi + \cos^2 + 4 - 4\cos\phi]_0^{\pi/2}\ d\phi\right\}$$

$$= \frac{WR^3}{EI}\left\{\left[\pi - 0 + \frac{\pi}{2} + 0\right] - [0] + \left[9\phi - 6\sin\phi + \frac{\phi}{2} + \frac{\sin 2\phi}{4}\right]_0^{\frac{\pi}{2}}\right\}$$

$$= \frac{WR^3}{EI}\left\{\frac{3\pi}{2} + \left[\frac{9\pi}{2} - 6 + \frac{\pi}{4} + 0\right] - 0\right\}$$

$$\delta w = \frac{WR^3}{EI}\left(\frac{25\pi}{4} - 6\right)$$

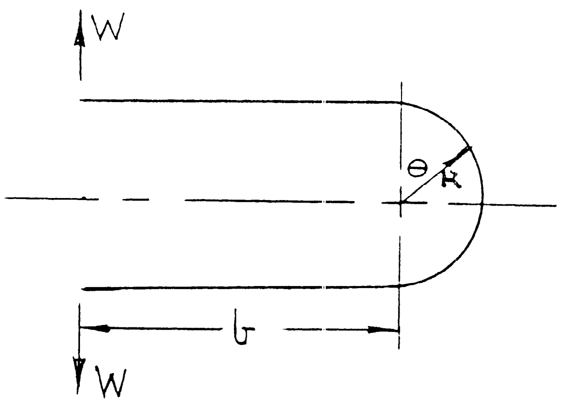
3.

Straight

$M = Wx$

Curved

$M = W\,(b + R\sin\theta)$



$$\delta W = \frac{1}{EI}\int_0^b Wx \,.\, x\,dx + \frac{W}{EI}\int (b + R\sin\theta)^2\, Rd\theta$$

$$= \frac{Wb^3}{3EI} + \frac{W}{EI}\int [b^2 + 2b\,R\sin\theta + R^2\sin^2\theta]\, Rd\theta$$

$$= \frac{Wb^3}{3EI} + \frac{WR}{EI}\left[b^2\,\theta - 2bR\cos\theta + \frac{R^2\theta}{2} + \frac{R^2\sin 2\theta}{4}\right]\int_0^{\pi/2}$$

$$= \frac{Wb^3}{3EI} + \frac{WR}{EI}\left\{\left[\frac{b^2\pi}{2} - 0 + \frac{R^2\pi}{4} + 0\right] + R\right\}$$

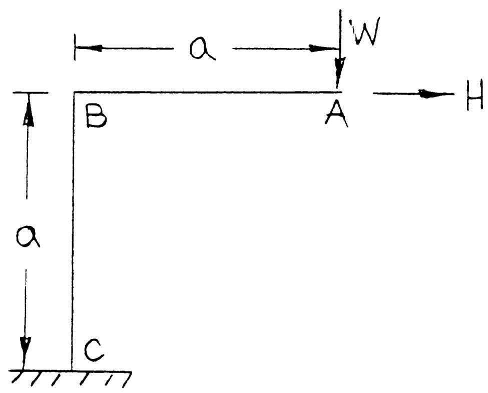
$$= \frac{Wb^3}{3EI} + \frac{\pi WR}{2EI}\,\frac{(b^2 + R^2)}{2} + \frac{2bRW}{EI}$$

$$\delta W = \frac{W}{EI}\left[\frac{b^3}{3} + R\left(\frac{b^2\pi}{2} + \frac{R^2\pi}{4}\right) 2bR\right]$$

Distance by which the two end separate

$$= \frac{2W}{EI}\left[\frac{b^3}{3} + R\left(\frac{b^2\pi}{2} + \frac{R^2\pi}{4}\right) + 2bR\right]$$

4.



<u>AB</u>

M = Wx

<u>BC</u>

M = Wa + Hy

$$\delta W = \frac{1}{EI}\int_0^a Wx.^2\,dx + \frac{1}{EI}\int_0^a (Wa + Hy)\,a\,dx$$

$$= \frac{Wa^3}{3EI} + \frac{Wa^2}{EI}.[y]_0^a$$

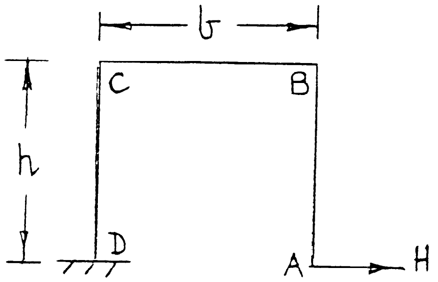
$$\delta W = \frac{4Wa^3}{3EI}$$

$$\delta H = \frac{1}{EI}\int W.x.o\,dx + \frac{1}{EI}\int (Wa + Hy)\,y.\,dy$$

$$=0 + \frac{1}{EI}\left\{\frac{Way^2}{2} + \frac{Hy^3}{3}\right\}_o^a$$

$$\delta H = \frac{Wa^3}{2EI} \text{ to the right}$$

5.



<u>AB</u>

$M = Hx$

<u>BC</u>

$M = Hh$

<u>CD</u>

$M = H\,(h - x)$

$$\delta H = \frac{1}{EI}\int_0^h Hx^2\,dx + \frac{1}{EI}\int_0^b Hh^2\,dx + \frac{1}{EI}\int_0^h H(h - x)^2dx$$

$$= \frac{H}{EI}\left\{\frac{h^3}{3} + h^2b + \int_0^h (h^2 - 2hx + x^2)\,dx\right\}$$

$$= \frac{H}{EI}\left\{\frac{h^3}{3} + h^2b + h^3 - h^3 + \frac{h^3}{3}\right\}$$

$$\delta H = \frac{Hh^2}{EI}\left(\frac{2h}{3} + b\right)$$

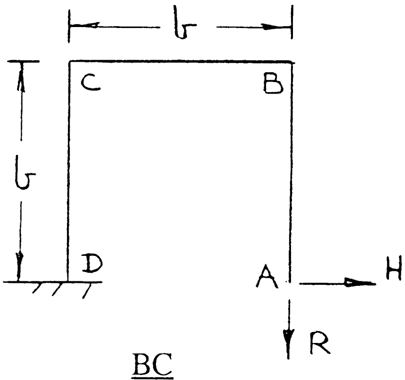
$$\delta H = \frac{1}{EI}\int_0^h Hx^2\,dx + \frac{1}{EI}\int_0^b Hh^2\,dx + \frac{1}{EI}\int_0^h H(h-x)^2 dx$$

$$= \frac{H}{EI}\left\{\frac{h^3}{3} + h^2 b + \int_0^h (h^2 - 2hx + x^2)\,dx\right\}$$

$$= \frac{H}{EI}\left\{\frac{h^3}{3} + h^2 b + h^3 - h^3 + \frac{h^3}{3}\right\}$$

$$\delta H = \frac{Hh^2}{EI}\left[\frac{2h}{3} + b\right]$$

6.



<u>AB</u>

$M = (Hx)$

<u>BC</u>

$M = Hb - Rx$

<u>CD</u>

$M = H(b - x) - Rb$

$$\frac{\partial u}{\partial R} = 0 = \frac{1}{EI}\int M\,\frac{\partial M}{\partial R}\,dx$$

$$0 = \frac{1}{EI}\left\{\int [Hx].0\,dx \int [Hb - Rx]\,.\,-x\,dx + \int [H(b-x) - Rb].\,-\,bdx\right\}$$

$$0 = \left\{-\frac{Hbx^2}{2} + \frac{Rx^3}{3} - Hb^2x + \frac{Hbx^2}{2} + Rb^2x\right\}_0^b$$

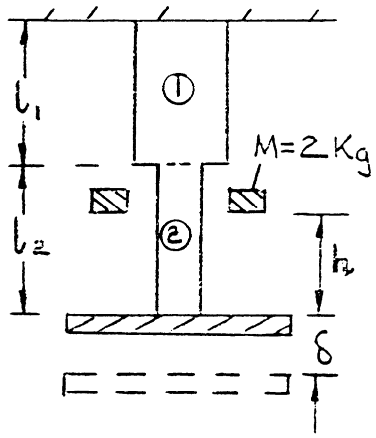
$$0 = -\frac{Hb^3}{2} + \frac{Rb^3}{3} - Hb^3 + \frac{Hb^3}{2} + Rb^3$$

$$\frac{4}{3}R = H$$

$$R = \frac{3}{4}H \text{ acting downwards}$$

7. $\sigma_1 A_1 = \sigma_2 A_2 \therefore \sigma_1 = \sigma_2 A_2/A_1$

$$Mg\ (h + \delta) = \frac{\sigma_1^2}{2E} * A_1\ \ell_1 + \frac{\sigma_2^2}{2E} * A_2\ \ell_2$$



or

$$Mg\ (h + \delta_1 + \delta_2) = \frac{\sigma_2^2 * A_2^2\ \ell_1}{2A_1E} + \frac{\sigma_2^2}{2E} * A_2\ell_2$$

$$Mg\ (h + \epsilon_1\ \ell_1 + \epsilon_2\ \ell_2) = \frac{\sigma_2^2}{2E}\left[\frac{A_2^2\ell_1}{A_1} + A_2\ell_2\right]$$

$$Mg\left[h + \frac{\sigma_1}{E}\ \ell_1 + \frac{\sigma_2\ell_2}{E}\right] = \frac{\sigma_2^2}{2E}\left[\frac{A_2^2\ell_1}{A_1} + A_2\ \ell_2\right]$$

$$Mg\ \left[h + \frac{\sigma_2\ A_2\ \ell_2}{A_1E} + \frac{\sigma_2\ \ell_2}{E}\right] = \frac{\sigma_2^2}{2E}\left[\frac{A_2^2\ell_1}{A_1} + A_2\ \ell_2\right]$$

$$2 * 9.81\ \left[0.4 + \frac{\sigma_2 * 0.72}{E} + \frac{\sigma_2 * 0.8}{E}\right] = \frac{\sigma_2^2}{2E}\left[\frac{216}{1E6} + 2.4E{-}4\right]$$

$$19.62\ (0.4 + 3.6E{-}12\ \sigma_2 + 4E{-}12\ \sigma_2) = 2.5E{-}12\sigma_2^2\ (4.56E{-}4)$$

$$\therefore\ 1.14E{-}15\sigma_2^2 - 1.491E{-}10\sigma_2 - 7.848 = 0$$

$$\sigma_2 = \frac{1.491E{-}10 \pm \sqrt{(2.223E{-}20 + 3.579E{-}14)}}{2.28E{-}15}$$

$$= \frac{1.491E{-}10 + 1.892E{-}7}{2.28E{-}15}$$

$\sigma_2 = 83.04\ MN/m^2$; $\quad \sigma_1 = \sigma_2\ A_2/A_1$

$\sigma_1 = 49.82\ MN/m^2$

8. Let W_e = equivalent static load

$$W_e = \sigma_c\ A_c + \sigma_s\ A_s \qquad 1$$

$$\delta = \epsilon_c = \epsilon_s$$

$$\delta = \frac{\sigma_1}{E}\ell_1 + \frac{\sigma_2}{E}\ell_2 = \frac{49.82E6 \text{ x } 1.2}{2E11} + \frac{83E6 \text{ x } 0.8}{2E11}$$

$$= 2.989E{-4} + 3.32E{-4}$$

$$\delta = 0.631 \text{ mm}$$

$$= \epsilon_c \ell = \epsilon_s \ell$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

$\sigma_c = E_c\ \sigma_s / E_s = \underline{0.07\ \sigma_s}$ 2

$\therefore\ W_e = 0.07\sigma_s \text{ x } 0.2 + \sigma_s \text{ x } 0.01$

$W_e = 0.014\ \sigma_s = 0.01\ \sigma_s$

$\underline{W_e = 0.024\ \sigma_s}$ 3

$$Mg\ (h + \delta) = \frac{\sigma_c^2}{2E_c} * A_c\ell + \frac{\sigma_s^2}{2E_s} * A_s\ell$$

$$20 \text{ x } 9.81\ \ (0.4 + \frac{\sigma_s}{E_s}\ell) = \frac{0.07^2\sigma_s^2}{2E_c} * A_c\ell + \frac{\sigma_s^2}{2E_s} * A_s\ell$$

$196.2\ (0.4 + 1.5E\text{-}11\sigma_s) = 1.05E\text{-}13\ \sigma_s^2 + 7.5E\text{-}14\sigma_s^2$

$78.48 + 2.943E\text{-}9\sigma_s = 1.8E\text{-}13\sigma_s^2$

$1.8E\text{-}13\sigma_s^2 - 2.943E\text{-}9\sigma_s - 78.48 = 0$

$$\sigma_s = \frac{2.943E{-9} + \sqrt{8.66E{-18} + 5.65E{-11}}}{3.6E{-13}}$$

$$= \frac{2.943E{-9} + 7.517E{-6}}{3.6E{-13}}$$

$\underline{\sigma_s = -\ 20.89\ MN/m^2}$

$\underline{\sigma_c = -\ 1.46\ MN/m^2}$