9. <u>EI for beam = 2E6</u>

$$\sigma_s = \frac{W_e}{2A_s} = \underline{2500\ W_e}$$

$$\sigma_a = \frac{W_e}{2A_a} = \underline{1250\ W_e}$$

or $W_e = \underline{8E\text{-}4\sigma_a}$

$\underline{\sigma_s = 2\sigma_a}$

δ_b = deflection of the beam due to the flexure alone

$$= \frac{W_e \ell^3}{48EI} = 8.333E\text{-}8\ W_e$$

$\underline{\delta_b = 6.6667E\text{-}11\sigma_a}$

$$\delta_a = \frac{\sigma_a}{E_a} \ell_a = \underline{1.4286E\text{-}11\sigma_a}$$

$$\delta_s = \frac{\delta_s \ell_s}{E_s} = \underline{2E\text{-}11\sigma_a}$$

$$Mg(h + \delta_b + \frac{\delta_a}{2} + \frac{\delta_s}{2}) = \frac{\sigma_s^2}{2E_s} * A_s\ell_s + \frac{\sigma_a^2}{2E_a} * A_a\ell_a + \frac{W_e^2 \ell^3}{96EI}$$

$2000\ (0.1 + 6.6667E\text{-}11\ \sigma_a + 7.143E\text{-}12\sigma_a + 1E\text{-}11\sigma_a)$

$= 4E\text{-}15\sigma_a^2 + 2.857E\text{-}15\sigma_a^2 + 2.667E\text{-}14\sigma_a^2$

$200 + 1.67E\text{-}8\sigma_a = 3.353E\text{-}14\sigma_a^2$

$3.353E\text{-}14\sigma_a^2 - 1.676E\text{-}8\sigma_a - 200 = 0$

$$\sigma_a = \frac{1.676E-8 + 5.179E-6}{6.706E-14}$$

$\underline{\sigma_a = 77.48\ MN/m^2}$

$\underline{\sigma_s = 154.96\ MN/m^2}$

$$\delta = \delta_b + \frac{\delta_a}{2} + \frac{\delta_s}{2}$$

$$= 5.16\ mm + \frac{1.107\ mm}{2} + \frac{1.55\ mm}{2}$$

$$= 5.16 + 0.55 + 0.775$$

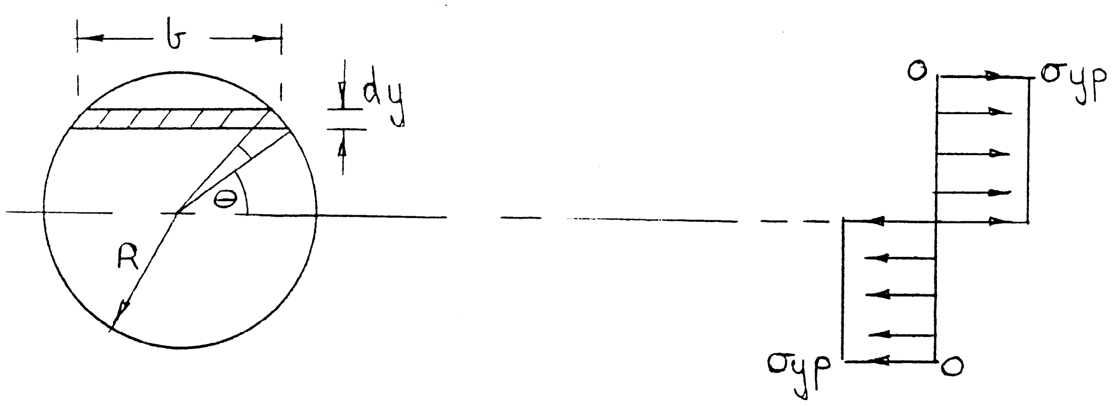
$\delta = 6.489$ mm

10a. The stress distribution of the solid circular section is shown below, where it can be seen that

$$M_p = \int_{-R}^{R} \sigma_{yp} * bdy * y$$

but

$b = 2R \cos \theta$



(a) Section (b) Stress distribution

and

$y = R \sin \theta$

and by differentiating y w.r.t. θ,

$dy = R \cos \theta . d\theta$

therefore

$$M_p = \sigma_{yp} \int_{-\pi/2}^{\pi/2} 2R^3 \cos^2 \theta \sin \theta \, d\theta$$

$$= -4R^3 \sigma_{yp} \int_{0}^{\pi/2} \cos^2 \theta \, d(\cos\theta)$$

$$= -4R^3 \sigma_{yp} \left[\frac{\cos^3\theta}{3} \right]_0^{\pi/2}$$

$\underline{M_p = 4\sigma_{yp}.R^3/3}$

From elementary elastic theory,

$$M_{yp} = \frac{\sigma_{yp}}{R} * I = \frac{\sigma_{yp}}{R} * \frac{\pi R^4}{4}$$

$\underline{M_{yp} = \pi\sigma_{yp}.R^3/4}$

Now,

$$S = \frac{M_p}{M_{yp}}$$

therefore

$\underline{S = 1.7}$

10b.

Section	a	y	ay	ay^2	i_o
1 2 3	1E-3 1E-3 2E-3	0.115 0.6 0.005	1.15E-4 6E-5 1E-5	1.323E-5 3.6E-6 5E-8	8.3E-9 8.33E-7 1.67E-8
Σ	4E-3	-	1.85E-4	1.688E-5	8.58E-7

$$\bar{y} = \frac{1.85E-4}{4E-3} = 0.0463$$

$$I_{xx} = 1.688E-5 + 8.58E-7 = 1.774E-5$$

$$I_{NA} = 1.7738E-5 - 0.0463^3 \times 4E-3$$

$$= 9.163E-6 \text{ m}^4$$

$$M_{yp} = \frac{\sigma_{yp}}{\bar{y}} \times I$$

$$= \frac{\sigma_{yp} \times 9.163E-6}{0.0737}$$

$\underline{M_{yp} = 1.2433E-4\sigma_{yp}}$

$$M_p = (1E-3 \times 0.105 + 1E-3 \times 0.05 + 2E-3 \times 0.005)\ \sigma_{yp}$$

$$= 1.65E-4\ \sigma_{yp}$$

$$S = \frac{M_p}{M_{yp}} = \frac{1.65E-4}{1.2433E-4} = 1.327$$

10c

Section	a	y	ay	ay^2	i_o
1 2	2E-3 2E-3	0.21 0.1	4.2E-4 2E-4	8.82E-5 2E-5	6.67E-8 6.667E-6
Σ	4E-3	-	6.2E-4	10.82E-5	6.734E-6

$\bar{y} = \frac{6.2E-4}{4E-3} = 0.155$

$I_{xx} = 10.82E\text{-}5 + 6.734E\text{-}6$

$= 1.1493E\text{-}4$

$I_{NA} = 1.1493E\text{-}4 - 0.155^2 \times 4E\text{-}3$

$\underline{I_{NA} = 1.883E\text{-}5 \ m^4}$

$M_{yp} = \frac{\sigma_{yp}}{0.155} \times 1.883E-5$

$\underline{M_{yp} = 1.2151E\text{-}4 \ \sigma_{yp}}$

$M_p = \sigma_{yp} \ (2E\text{-}3 \times 0.01 + 2E - 3 \times 0.1)$

$\underline{M_p = 2.2E\text{-}4 \ \sigma_{yp}}$

$S = \frac{M_p}{M_{yp}} = 1.811$

11a

$6 \times 5\theta = M_p\theta$

$M_p = 30$

Design $M_p = 30 \times 3 = 90$ kNm

$M_{yp} = \frac{90}{1.14} = 78.95$ kNm

$Z = \quad Z = \frac{M_{yp}}{\sigma_{yp}} = \frac{78.95E3}{300E6}$

$\underline{Z = 2.632E\text{-}4 m^3}$

11b

$5 \times 1 \times 2.5 \ \theta = M_p\theta$

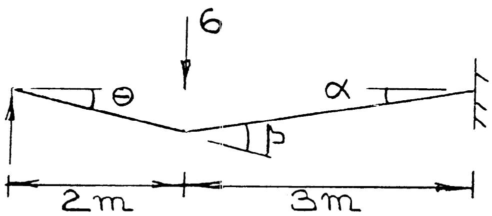
$M_p = 12.5$ kNm

Design $M_p = 12.5 \times 3 = 37.5$

$$M_{yp} = \frac{37.5}{1.14} = 32.89 \text{ kN.m}$$

$$Z = \frac{32.89}{300E3} = 1.096E{-4}m^3$$

11c.



$2\theta = 3\alpha$

$\alpha = 0.667\ \theta$

$\beta = \alpha + \theta = 1.667\theta$

$M_p\beta + M_p\alpha = 6 \times 2\theta$

$$\text{or } M_p = \frac{12}{2.333} = 5.143 \text{ kNm}$$

Design $M_p = 5.143 \times 3 = 15.43$ kN.m

$M_{yp} = 13.535$ kN.m

<u>$Z = 13.535/(300E3) = 4.511E\text{-}5\ m^3$</u>

11d.

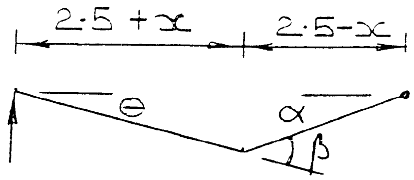
$M_p\ (\theta + 2\theta + \theta) = 5 \times 1 \times 1.25\ \theta$

$M_p = 1.563$ kN.m

Design $M_p = 4.688$ kN/m

$$Z = \frac{4.688}{300E3} = 1.563E{-5}m^3$$

11e $(2.5 + x)\theta$ - $(2.5 - x)\alpha$



$$\alpha = \left[\frac{2.5 + x}{2.5 - x}\right]\theta$$

$$\beta = \alpha + \theta = \left[\frac{2.5 + x}{2.5 - x}\right]\theta + \left[\frac{2.5 - x}{2.5 - x}\right]\theta$$

$$\beta = \frac{5\theta}{2.5 - x}$$

$$M_p\beta + 2M_p\alpha = 5 \text{ x } 1 \text{ x } \frac{(2 + 5 + x)}{2}\theta$$

$$\text{or } M_p \frac{5}{2.5 - x} + \frac{2\, M_p(2.5 + x)}{2.5 - x} = 2.5\ (2.5 + x)$$

$$\text{or } M_p \frac{5 + 5 + 2x}{2.5 - x} = 2.5\ (2.5 + x)$$

$$\text{or } M_p = \frac{2.5\ (2.5 + x)\ (2.5 - x)}{(10 + 2x)}$$

$$M_p = \frac{2.5\ (6.25 - x^2)}{(10 + 2x)}$$

For maximum M_p, $\frac{dM_p}{dx} = 0$

$\therefore (10 + 2x)\ 2.5\ (-2x) - 2.5\ (6.25 - x^2).2 = 0$

$- 20\ x - 4x^2 - 12.5 + 2x^2 = 0$

or $2x^2 + 20x + 12.5 = 0$

$$x = \frac{-20 + \pm\sqrt{(400 - 100)}}{4}$$

$\underline{= -0.67 \text{ m}}$

$\therefore M_p = 1.675$ kN.m

Design $M_p = 5.024$ kN.m

$M_{yp} = 5.024/1.14 = 4.407$ kN.m

$$Z = \frac{4.407}{300E3} = 1.469E{-5}\ m^3$$

11f

$$M_p\theta + M_p\,\beta + 2M_p\alpha = 5 * 1 * \frac{(2.5 + x)}{2}\,\theta$$

$$\text{or } M_p \frac{(2.5 - x)}{(2.5 - x)} + \frac{5}{(2.5 - x)} + \frac{2(2.5 + x)}{(2.5 - x)} = 2.5\ (2.5 + x)$$

$$\text{or } M_p \frac{2.5 - x + 5 + 5 + 2x}{2.5 - x} = 2.5(2.5 + x)$$

$$\text{or } \left(\frac{12.5 + x}{2.5 - x}\right) M_p = 2.5(2.5 + x)$$

$$\text{or } M_p = \frac{2.5\ (6.25 - x^2)}{(12.5 + x)}$$

$$\text{Now } \frac{dM_p}{dx} = 0$$

$$\text{or } 0 = (12.5 + x)\ 2.5\ (-2x) - 2.5\ (6.25 - x^2).1$$

$$\text{or } 0 = -25x - 2x^2 - 6.25 + x^2 = 0$$

$$\text{or } x^2 + 25\ x + 6.25 = 0$$

$$x = \frac{-25 \pm \sqrt{(625 - 25)}}{2}$$

$x = -0.253$ m

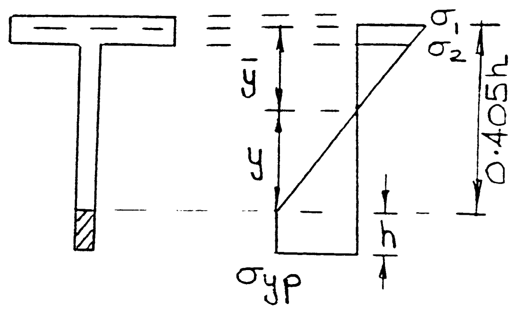
$\therefore\ M_p = 1.263$ kN.m

Design $M_p = 3.788$ kN.m

M_{yp} 3.323 kN.m

$Z = 1.108E{-5}m^3$

12. $M = 100 \times 2.1 = 210$ kN.m $= M_e + M_p$



Resolving forces

$$\sigma_1 \times 2E\text{-}3 + \frac{\sigma_2}{2} \times (\bar{y} - 0.005) \times 0.01 \quad \text{—— 1}$$

$$= \sigma_{yp} \times 0.01h + \frac{\sigma_{yp}}{2} (0.405 - h - \bar{y}) \times 0.01$$

By similar triangles,

$$\frac{\sigma_1}{\bar{y}} = \frac{\sigma_{yp}}{(0.405 - h - \bar{y})} \quad \text{—— 2}$$

$$\therefore \sigma_1 = \frac{\sigma_{yp}\bar{y}}{(0.405 - h - \bar{y})}$$

$$\& \frac{\sigma_2}{(\bar{y} - 0.005)} = \frac{\sigma_{yp}}{(0.405 - h - \bar{y})}$$

$$\text{or } \sigma_2 = \frac{\sigma_{yp}(\bar{y} - 0.005)}{(0.405 - h - \bar{y})} \quad \text{—— 3}$$

Substitute 2 and 3 into 1

$$\frac{2E\text{-}3 \times \bar{y}}{(0.405 - h - \bar{y})} + \frac{(\bar{y} - .005)^2 \times 0.01}{2\,(0.405 - h - \bar{y})} = 0.01h + \frac{(0.405 - h - \bar{y})}{2} \times 0.01$$

or

$$2E\text{-}3\bar{y} + 0.005\,(\bar{y}^2 - 0.01\,\bar{y} + 2.5E\text{-}5)$$

$= 0.01\ h\ (0.405 - h - \bar{y}) + 0.005\ (0.405 - h - \bar{y})^2$

or $1.95\ E\text{-}3\bar{y} + 0.005\ \bar{y} + 1.25E\text{-}7$

$= 4.05E\text{-}3h - 0.01h - 0.01hy$

$+ 0.005\ (0.164 + h^2 + \bar{y}^2 - 0.81\ h - 0.81\bar{y} + 2h\bar{y})$

or $1.95E\text{-}3\bar{y} + 0.005\bar{y} + 1.25E\text{-}7 =$

$4.05E\text{-}3h - 0.01h^2 - 0.01hy + 8.2E\text{-}4 + 0.005h^2$

$+ 0.005\bar{y}^2 - 4.05E\text{-}3h - 4.05E\text{-}3\bar{y} + 0.01\ hy$

or $1.95E\text{-}3\bar{y} + 1.25E\text{-}7 + 4.05E\text{-}3\bar{y} - 8.2E\text{-}4$

$= -\ 0.005h^2$

or $6E\text{-}3\bar{y} - 8.2E\text{-}4 = -\ 0.005h^2$

or $6E\text{-}3\bar{y} = 8.2E\text{-}4 - 0.005h^2$

$$\frac{\sigma_{yp}}{Y} = \frac{\sigma_1}{\bar{y}} \text{ or } Y = \sigma_{yp}.\bar{y}/\sigma_1$$

$\bar{y} = 0.1367 - 0.833h^2$

h	$\bar{y}$	σ_1	Y	$Y + \bar{y} + H$
0.05 0.1	0.1346 0.1284	$0.61\sigma_{yp}$ $0.727\sigma_{yp}$	0.220 0.1766	0.404 0.405

$\therefore$ <u>$\bar{y} = 0.1284$ m & h =0.1 m</u>

Now $M_p = \sigma_{yp} * 0.01 * h * (Y + h/2)$

$= 300 * 0.01 * 0.1 * (0.2266)$

$M_p = 68$ kN.m

$\therefore$ <u>$M_c = 142$ kN.m</u>