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# CHAPTER 1

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# INTRODUCTION TO THE HANDBOOK

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## CONCEPTS OF SHOCK AND VIBRATION

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*Vibration* is a term that describes oscillation in a mechanical system. It is defined by the frequency (or frequencies) and amplitude. Either the motion of a physical object or structure or, alternatively, an oscillating force applied to a mechanical system is vibration in a generic sense. Conceptually, the time-history of vibration may be considered to be sinusoidal or simple harmonic in form. The frequency is defined in terms of cycles per unit time, and the magnitude in terms of amplitude (the maximum value of a sinusoidal quantity). The vibration encountered in practice often does not have this regular pattern. It may be a combination of several sinusoidal quantities, each having a different frequency and amplitude. If each frequency component is an integral multiple of the lowest frequency, the vibration repeats itself after a determined interval of time and is called *periodic*. If there is no integral relation among the frequency components, there is no periodicity and the vibration is defined as *complex*.

Vibration may be described as *deterministic* or *random*. If it is deterministic, it follows an established pattern so that the value of the vibration at any designated future time is completely predictable from the past history. If the vibration is random, its future value is unpredictable except on the basis of probability. Random vibration is defined in statistical terms wherein the probability of occurrence of designated magnitudes and frequencies can be indicated. The analysis of random vibration involves certain physical concepts that are different from those applied to the analysis of deterministic vibration.

Vibration of a physical structure often is thought of in terms of a model consisting of a mass and a spring. The vibration of such a model, or system, may be “free” or “forced.” In *free vibration*, there is no energy added to the system but rather the vibration is the continuing result of an initial disturbance. An *ideal system* may be considered undamped for mathematical purposes; in such a system the free vibration is assumed to continue indefinitely. In any *real system*, damping (i.e., energy dissipation) causes the amplitude of free vibration to decay continuously to a negligible value. Such free vibration sometimes is referred to as *transient vibration*. *Forced vibration*, in contrast to free vibration, continues under “steady-state” conditions

because energy is supplied to the system continuously to compensate for that dissipated by damping in the system. In general, the frequency at which energy is supplied (i.e., the forcing frequency) appears in the vibration of the system. Forced vibration may be either deterministic or random. In either instance, the vibration of the system depends upon the relation of the excitation or forcing function to the properties of the system. This relationship is a prominent feature of the analytical aspects of vibration.

*Shock* is a somewhat loosely defined aspect of vibration wherein the excitation is nonperiodic, e.g., in the form of a pulse, a step, or transient vibration. The word *shock* implies a degree of suddenness and severity. These terms are relative rather than absolute measures of the characteristic; they are related to a popular notion of the characteristics of shock and are not necessary in a fundamental analysis of the applicable principles. From the analytical viewpoint, the important characteristic of shock is that the motion of the system upon which the shock acts includes both the frequency of the shock excitation and the natural frequency of the system. If the excitation is brief, the continuing motion of the system is free vibration at its own natural frequency.

The technology of shock and vibration embodies both theoretical and experimental facets prominently. Thus, methods of analysis and instruments for the measurement of shock and vibration are of primary significance. The results of analysis and measurement are used to evaluate shock and vibration environments, to devise testing procedures and testing machines, and to design and operate equipment and machinery. Shock and/or vibration may be either wanted or unwanted, depending upon circumstances. For example, vibration is involved in the primary mode of operation of such equipment as conveying and screening machines; the setting of rivets depends upon the application of impact or shock. More frequently, however, shock and vibration are unwanted. Then the objective is to eliminate or reduce their severity or, alternatively, to design equipment to withstand their influences. These procedures are embodied in the control of shock and vibration. Methods of control are emphasized throughout this Handbook.

## ***CONTROL OF SHOCK AND VIBRATION***

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Methods of shock and vibration control may be grouped into three broad categories:

### **1. Reduction at the Source**

- a. *Balancing of Moving Masses.*** Where the vibration originates in rotating or reciprocating members, the magnitude of a vibratory force frequently can be reduced or possibly eliminated by balancing or counterbalancing. For example, during the manufacture of fans and blowers, it is common practice to rotate each rotor and to add or subtract material as necessary to achieve balance.
- b. *Balancing of Magnetic Forces.*** Vibratory forces arising in magnetic effects of electrical machinery sometimes can be reduced by modification of the magnetic path. For example, the vibration originating in an electric motor can be reduced by skewing the slots in the armature laminations.
- c. *Control of Clearances.*** Vibration and shock frequently result from impacts involved in operation of machinery. In some instances, the impacts result from inferior design or manufacture, such as excessive clearances in bearings, and can be reduced by closer attention to dimensions. In other instances, such as the movable armature of a relay, the shock can be decreased by employing a rubber bumper to cushion motion of the plunger at the limit of travel.

## 2. Isolation

- a. *Isolation of Source.* Where a machine creates significant shock or vibration during its normal operation, it may be supported upon isolators to protect other machinery and personnel from shock and vibration. For example, a forging hammer tends to create shock of a magnitude great enough to interfere with the operation of delicate apparatus in the vicinity of the hammer. This condition may be alleviated by mounting the forging hammer upon isolators.
- b. *Isolation of Sensitive Equipment.* Equipment often is required to operate in an environment characterized by severe shock or vibration. The equipment may be protected from these environmental influences by mounting it upon isolators. For example, equipment mounted in ships of the navy is subjected to shock of great severity during naval warfare and may be protected from damage by mounting it upon isolators.

## 3. Reduction of the Response

- a. *Alteration of Natural Frequency.* If the natural frequency of the structure of an equipment coincides with the frequency of the applied vibration, the vibration condition may be made much worse as a result of resonance. Under such circumstances, if the frequency of the excitation is substantially constant, it often is possible to alleviate the vibration by changing the natural frequency of such structure. For example, the vibration of a fan blade was reduced substantially by modifying a stiffener on the blade, thereby changing its natural frequency and avoiding resonance with the frequency of rotation of the blade. Similar results are attainable by modifying the mass rather than the stiffness.
- b. *Energy Dissipation.* If the vibration frequency is not constant or if the vibration involves a large number of frequencies, the desired reduction of vibration may not be attainable by altering the natural frequency of the responding system. It may be possible to achieve equivalent results by the dissipation of energy to eliminate the severe effects of resonance. For example, the housing of a washing machine may be made less susceptible to vibration by applying a coating of damping material on the inner face of the housing.
- c. *Auxiliary Mass.* Another method of reducing the vibration of the responding system is to attach an auxiliary mass to the system by a spring; with proper tuning the mass vibrates and reduces the vibration of the system to which it is attached. For example, the vibration of a textile-mill building subjected to the influence of several hundred looms was reduced by attaching large masses to a wall of the building by means of springs; then the masses vibrated with a relatively large motion and the vibration of the wall was reduced. The incorporation of damping in this auxiliary mass system may further increase its effectiveness.

## CONTENT OF HANDBOOK

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The chapters of this Handbook each deal with a discrete phase of the subject of shock and vibration. Frequent references are made from one chapter to another, to refer to basic theory in other chapters, to call attention to supplementary information, and to give illustrations and examples. Therefore, each chapter when read with other referenced chapters presents one complete facet of the subject of shock and vibration.

Chapters dealing with similar subject matter are grouped together. The first 11 chapters following this introductory chapter deal with fundamental concepts of shock and vibration. Chapter 2 discusses the free and forced vibration of linear sys-

tems that can be defined by lumped parameters with similar types of coordinates. The properties of rigid bodies are discussed in Chap. 3, together with the vibration of resiliently supported rigid bodies wherein several modes of vibration are coupled. Nonlinear vibration is discussed in Chap. 4, and self-excited vibration in Chap. 5. Chapter 6 discusses two degree-of-freedom systems in detail—including both the basic theory and the application of such theory to dynamic absorbers and auxiliary mass dampers. The vibration of systems defined by distributed parameters, notably beams and plates, is discussed in Chap. 7. Chapters 8 and 9 relate to shock; Chap. 8 discusses the response of lumped parameter systems to step- and pulse-type excitation, and Chap. 9 discusses the effects of impact on structures. Chapter 10 discusses applications of the use of mechanical impedance and mechanical admittance methods. Then Chap. 11 presents statistical methods of analyzing vibrating systems.

The second group of chapters is concerned with instrumentation for the measurement of shock and vibration. Chapter 12 includes not only piezoelectric and piezoresistive transducers, but also other types such as force transducers (although strain gages are described in Chap. 17). The electrical instruments to which such transducers are connected (including various types of amplifiers, signal conditioners, and recorders) are considered in detail in Chap. 13. Chapter 14 is devoted to the important topics of spectrum analysis instrumentation and techniques. The use of all such equipment in making vibration measurements in the field is described in Chap. 15. There has been increasing use of vibration measurement equipment for monitoring the mechanical condition of machinery, as an aid in preventive maintenance; this is the subject of Chap. 16. The calibration of transducers, Chap. 18, is followed by Chap. 19 on national and international standards and test codes related to shock and vibration.

A discussion of test criteria and specifications is given in Chap. 20, followed by a comprehensive chapter on modal analysis and testing in Chap. 21. Chapters 22 and 23 discuss data analysis, in conjunction with Chap. 14; the first of these two chapters is primarily concerned with an analysis of vibration data and the second is concerned with shock data. Vibration that is induced in buildings, as a result of ground motion, is described in Chap. 24. Then Chap. 25 considers vibration testing machines, followed by Chap. 26 on conventional shock testing and pyrotechnic shock testing machines.

The next two chapters deal with computational methods. Chapter 27 is concerned with applications of computers, presenting information that is useful in both analytical and experimental work. This is followed by Chap. 28, which is in two parts: Part I describes modern matrix methods of analysis, dealing largely with the formulation of matrices for use with digital computers and other numerical calculation methods; the second part shows how finite element methods can be applied to the solution of shock and vibration problems by the use of computer techniques.

Part I of Chap. 29 describes vibration that is induced as a result of air flow, the second part discusses vibration that is induced by the flow of water, and the third part is concerned with the response of structures to acoustic environments.

The theory of vibration isolation is discussed in detail in Chap. 30, and an analogous presentation for the isolation of mechanical shock is given in Chap. 31. Various types of isolators for shock and vibration are described in Chap. 32, along with the selection and practical application of such isolators. The relatively new field of active vibration control is described in Chap. 33. A presentation is given in Chap. 34 on the engineering properties of rubber, followed by a presentation of the engineering properties of metals (including conventional fatigue) and the engineering properties of composite materials in Chap. 35.

An important method of controlling shock and vibration involves the addition of damping or energy-dissipating means to structures that are susceptible to vibration. Chapter 36 discusses the general concepts of damping together with the application

of such concepts to hysteresis and slip damping. The application of damping materials to devices and structures is described in Chap. 37.

The latter chapters of the Handbook deal with the specific application of the fundamentals of analysis, methods of measurement, and control techniques—where these are developed sufficiently to form a separate and discrete subject. Torsional vibration is discussed in Chap. 38, with particular application to internal-combustion engines. The balancing of rotating equipment is discussed in Chap. 39, and balancing machines are described. Chapter 40 describes the special vibration problems associated with the design and operation of machine tools. Chapter 41 describes procedures for the design of equipment to withstand shock and vibration—both the design and practical aspects. A comprehensive up-to-date discussion of the human aspects of shock and vibration is considered in Chap. 42, which describes the effects of shock and vibration on people.

## ***SYMBOLS AND ACRONYMS***

This section includes a list of symbols and acronyms generally used in the Handbook. Symbols of special or limited application are defined in the respective chapters as they are used.

Symbol	Meaning
$a$	radius
A/D	analog-to-digital
ANSI	American National Standards Institute
ASTM	American Society for Testing and Materials
$B$	bandwidth
$B$	magnetic flux density
BSI	British Standards Institution
$c$	damping coefficient
$c$	velocity of sound
$c_c$	critical damping coefficient
$C$	capacitance
CPU	central processing unit
CSIRO	Commonwealth Scientific and Industrial Research Organisation
$D$	diameter
D/A	digital-to-analog
DFT	discrete Fourier transform
DSP	discrete signal processor
$e$	electrical voltage
$e$	eccentricity
$E$	energy
$E$	modulus of elasticity in tension and compression (Young's modulus)
$f$	frequency
$f_n$	undamped natural frequency
$f_i$	undamped natural frequencies in a multiple degree-of-freedom system, where $i = 1, 2, \dots$
$f_d$	damped natural frequency
$f_r$	resonance frequency
$F$	force
$f_f$	Coulomb friction force
FEM	finite element method, finite element model
FFT	fast Fourier transform

$g$	acceleration of gravity
$G$	modulus of elasticity in shear
$h$	height, depth
$H$	magnetic field strength
Hz	hertz
$i$	electric current
$I_i$	area or mass moment of inertia (subscript indicates axis)
$I_p$	polar moment of inertia
$I_{ij}$	area or mass product of inertia (subscripts indicate axes)
IC	integrated circuit
ISO	International Standards Organization
$j$	imaginary part of
$j$	$\sqrt{-1}$
$J$	inertia constant (weight moment of inertia)
$J$	impulse
$k$	spring constant, stiffness, stiffness constant
$k_t$	rotational (torsional) stiffness
$l$	length
$L$	inductance
$m$	mass
$m_u$	unbalanced mass
$M$	torque
$M$	mutual inductance
$\mathfrak{M}$	mobility
MIMO	multiple input, multiple output
$n$	number of coils, supports, etc.
NEMA	National Electrical Manufacturers Association
NIST	National Institute of Standards and Technology
$p$	alternating pressure
$p$	probability density
$P$	probability distribution
$P$	static pressure
$q$	electric charge
$Q$	resonance factor (also ratio of reactance to resistance)
$r$	electrical resistance
$R$	radius
$\Re$	real part of
$s$	arc length
$S$	area of diaphragm, tube, etc.
SEA	statistical energy analysis
SIMO	single input, multiple output
SCC	Standards Council of Canada
$t$	thickness
$t$	time
$T$	transmissibility
$T$	kinetic energy
$v$	linear velocity
$V$	potential energy
$w$	width
$W$	weight
$W$	power
$W_e$	spectral density of the excitation
$W_r$	spectral density of the response
$x$	linear displacement in direction of $X$ axis
$y$	linear displacement in direction of $Y$ axis
$z$	linear displacement in direction of $Z$ axis

$Z$	impedance
$\alpha$	rotational displacement about $X$ axis
$\beta$	rotational displacement about $Y$ axis
$\gamma$	rotational displacement about $Z$ axis
$\gamma$	shear strain
$\gamma$	weight density
$\delta$	deflection
$\delta_{st}$	static deflection
$\Delta$	logarithmic decrement
$\epsilon$	tension or compression strain
$\zeta$	fraction of critical damping
$\eta$	stiffness ratio, loss factor
$\theta$	phase angle
$\lambda$	wavelength
$\mu$	coefficient of friction
$\mu$	mass density
$\mu$	mean value
$\nu$	Poisson's ratio
$\rho$	mass density
$\rho_i$	radius of gyration (subscript indicates axis)
$\sigma$	Poisson's ratio
$\sigma$	normal stress
$\sigma$	root-mean-square (rms) value
$\tau$	period
$\tau$	shear stress
$\phi$	magnetic flux
$\Phi$	phase angle
$\psi$	phase angle
$\Psi$	standard deviation
$\omega$	forcing frequency—angular
$\omega_n$	undamped natural frequency—angular
$\omega_i$	undamped natural frequencies—angular—in a multiple degree-of-freedom system, where $i = 1, 2, \dots$
$\omega_d$	damped natural frequency—angular
$\omega_r$	resonance frequency—angular
$\Omega$	rotational speed
$\approx$	approximately equal to

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## CHARACTERISTICS OF HARMONIC MOTION

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Harmonic functions are employed frequently in the analysis of shock and vibration. A body that experiences simple harmonic motion follows a displacement pattern defined by

$$x = x_0 \sin (2\pi ft) = x_0 \sin \omega t \quad (1.1)$$

where  $f$  is the *frequency* of the simple harmonic motion,  $\omega = 2\pi f$  is the corresponding *angular frequency*, and  $x_0$  is the *amplitude* of the displacement.

The velocity  $\dot{x}$  and acceleration  $\ddot{x}$  of the body are found by differentiating the displacement once and twice, respectively:

$$\dot{x} = x_0(2\pi f) \cos 2\pi ft = x_0\omega \cos \omega t \quad (1.2)$$

$$\ddot{x} = -x_0(2\pi f)^2 \sin 2\pi ft = -x_0\omega^2 \sin \omega t \quad (1.3)$$

The maximum absolute values of the displacement, velocity, and acceleration of a body undergoing harmonic motion occur when the trigonometric functions in Eqs. (1.1) to (1.3) are numerically equal to unity. These values are known, respectively, as displacement, velocity, and acceleration amplitudes; they are defined mathematically as follows:

$$x_0 = x_0 \quad \dot{x}_0 = (2\pi f)x_0 \quad \ddot{x}_0 = (2\pi f)^2 x_0 \tag{1.4}$$

It is common to express the displacement amplitude  $x_0$  in inches when the English system of units is used and in centimeters or millimeters when the metric system is used. Accordingly, the velocity amplitude  $\dot{x}_0$  is expressed in inches per second in the English system (centimeters per second or millimeters per second in the metric system). For example, consider a body that experiences simple harmonic

**TABLE 1.1** Conversion Factors for Translational Velocity and Acceleration

Multiply Value in → or → By ↘ To obtain value in ↓	<i>g</i> -sec, <i>g</i>	ft/sec ft/sec <sup>2</sup>	in./sec in./sec <sup>2</sup>	cm/sec cm/sec <sup>2</sup>	m/sec m/sec <sup>2</sup>
<i>g</i> -sec, <i>g</i>	1	0.0311	0.00259	0.00102	0.102
ft/sec ft/sec <sup>2</sup>	32.16	1	0.0833	0.0328	3.28
in./sec in./sec <sup>2</sup>	386	12.0	1	0.3937	39.37
cm/sec cm/sec <sup>2</sup>	980	30.48	2.540	1	100
m/sec m/sec <sup>2</sup>	9.80	0.3048	0.0254	0.010	1

**TABLE 1.2** Conversion Factors for Rotational Velocity and Acceleration

Multiply Value in → or → By ↘ To obtain value in ↓	rad/sec rad/sec <sup>2</sup>	degree/sec degree/sec <sup>2</sup>	rev/sec rev/sec <sup>2</sup>	rev/min rev/min/sec
rad/sec rad/sec <sup>2</sup>	1	0.01745	6.283	0.1047
degree/sec degree/sec <sup>2</sup>	57.30	1	360	6.00
rev/sec rev/sec <sup>2</sup>	0.1592	0.00278	1	0.0167
rev/min rev/min/sec	9.549	0.1667	60	1



motion having a frequency  $f$  of 50 Hz and a displacement amplitude  $x_0$  of 0.01 in. (0.000254 m). According to Eq. (1.4), the velocity amplitude  $\dot{x}_0 = (2\pi f) x_0 = 3.14$  in./sec (0.0797 m/s). The acceleration amplitude  $\ddot{x}_0 = (2\pi f)^2 x_0$  in./sec<sup>2</sup> = 986 in./sec<sup>2</sup> (25.0 m/s<sup>2</sup>). The acceleration amplitude  $\ddot{x}_0$  is often expressed as a dimensionless multiple of the gravitational acceleration  $g$  where  $g = 386$  in./sec<sup>2</sup> (9.8 m/s<sup>2</sup>). Therefore in this example, the acceleration amplitude may also be expressed as  $\ddot{x}_0 = 2.55g$ .

Factors for converting values of rectilinear velocity and acceleration to different units are given in Table 1.1; similar factors for angular velocity and acceleration are given in Table 1.2.

For certain purposes in analysis, it is convenient to express the amplitude in terms of the average value of the harmonic function, the root-mean-square (rms) value, or 2 times the amplitude (i.e., peak-to-peak value). These terms are defined mathematically in Chap. 22; numerical conversion factors are set forth in Table 1.3 for ready reference.

**TABLE 1.3** Conversion Factors for Simple Harmonic Motion

Multiply numerical value in terms of By To obtain value in terms of	→ ↘ ↓	Amplitude	Average value	Root-mean-square (rms) value	Peak-to-peak value
Amplitude		1	1.571	1.414	0.500
Average value		0.637	1	0.900	0.318
Root-mean-square (rms) value		0.707	1.111	1	0.354
Peak-to-peak value		2.000	3.142	2.828	1

## **APPENDIX 1.1 NATURAL FREQUENCIES OF COMMONLY USED SYSTEMS**

The most important aspect of vibration analysis often is the calculation or measurement of the natural frequencies of mechanical systems. Natural frequencies are discussed prominently in many chapters of the Handbook. Appendix 1.1 includes in tabular form, convenient for ready reference, a compilation of frequently used expressions for the natural frequencies of common mechanical systems:

1. Mass-spring systems in translation
2. Rotor-shaft systems
3. Massless beams with concentrated mass loads
4. Beams of uniform section and uniformly distributed load
5. Thin flat plates of uniform thickness
6. Miscellaneous systems

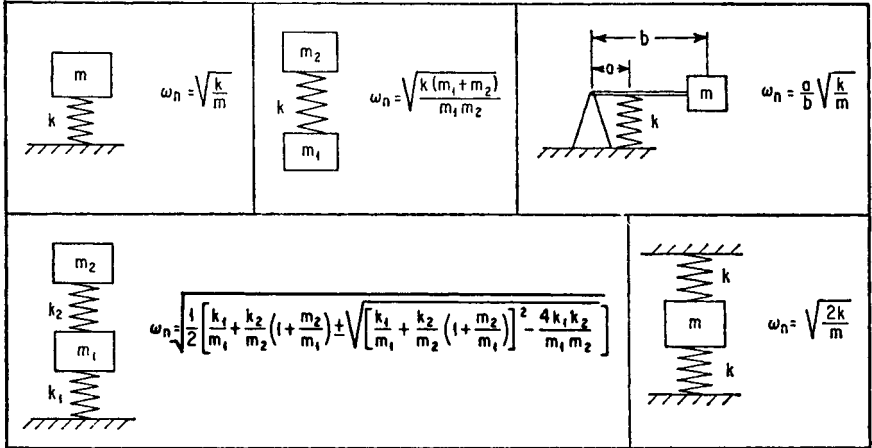
The data for beams and plates are abstracted from Chap. 7.

**MASS - SPRING SYSTEMS IN TRANSLATION**  
(RIGID MASS AND MASSLESS SPRING)

$k$  = SPRING STIFFNESS, LB/IN.

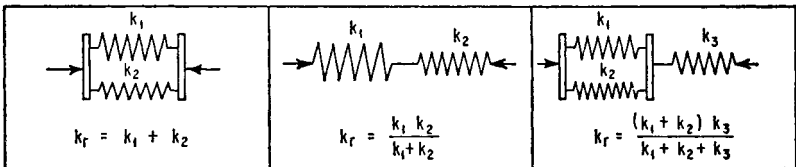
$m$  = MASS, LB-SEC<sup>2</sup>/IN.

$\omega_n$  = ANGULAR NATURAL FREQUENCY, RAD/SEC



SPRINGS IN COMBINATION

$k_r$  = RESULTANT STIFFNESS OF COMBINATION



HELICAL SPRINGS

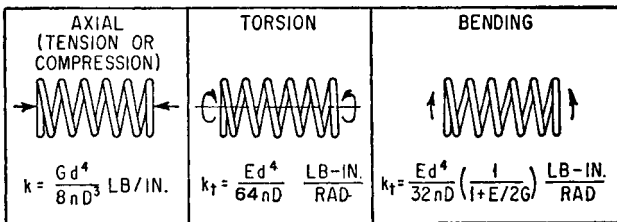
$d$  = WIRE DIAMETER, IN

$D$  = MEAN COIL DIAMETER, IN.

$n$  = NUMBER OF ACTIVE COILS

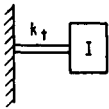
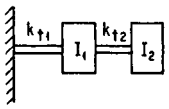
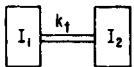
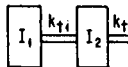
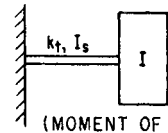
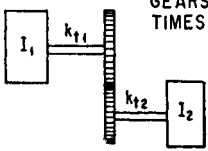
$E$  = YOUNG'S MODULUS, LB/IN<sup>2</sup>

$G$  = MODULUS OF ELASTICITY IN SHEAR, LB/IN<sup>2</sup>



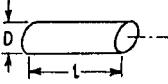
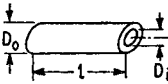
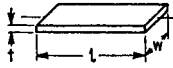
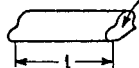
**ROTOR-SHAFT SYSTEMS**  
(RIGID ROTOR AND MASSLESS SHAFT)

$k_t$  = TORSIONAL STIFFNESS OF SHAFT, LB-IN./RAD  
 $I$  = MASS MOMENT OF INERTIA OF ROTOR, LB-IN.-SEC<sup>2</sup>  
 $\omega_n$  = ANGULAR NATURAL FREQUENCY, RAD/SEC

 $\omega_n = \sqrt{\frac{k_t}{I}}$	 $\omega_n = \sqrt{\frac{1}{2} \left[ \frac{k_{t1} + k_{t2}}{I_1 + I_2} \left( 1 + \frac{I_2}{I_1} \right) \pm \sqrt{\left[ \frac{k_{t1} + k_{t2}}{I_1 + I_2} \left( 1 + \frac{I_2}{I_1} \right) \right]^2 - \frac{4k_{t1}k_{t2}}{I_1I_2}} \right]}$
 $\omega_n = \sqrt{\frac{k_t(I_1 + I_2)}{I_1I_2}}$	 $\omega_n = \sqrt{\frac{1}{2} \left[ B \pm \sqrt{B^2 - \frac{4k_{t1}k_{t2}}{I_1I_2I_3} (I_1 + I_2 + I_3)} \right]}$ <p>WHERE <math>B = \frac{k_{t1}}{I_1} + \frac{k_{t2}}{I_3} + \frac{k_{t1} + k_{t2}}{I_2}</math></p>
 <p>(MOMENT OF INERTIA OF SHAFT = <math>I_s</math>)</p> $\omega_n = \sqrt{\frac{k_t}{I + \frac{1}{3}I_s}}$	 <p>GEARED SYSTEM WITH MASSLESS GEARS (SPEED OF ROTOR 2 IS <math>n</math> TIMES SPEED OF ROTOR 1)</p> $\omega_n = \sqrt{\frac{k_1k_2(I_1 + n^2I_2)}{I_1I_2(n^2k_2 + k_1)}}$

**STIFFNESS OF SHAFTS IN TORSION**

$G$  = MODULUS OF ELASTICITY IN SHEAR, LB/IN.<sup>2</sup>  
 $l$  = LENGTH OF SHAFT, IN.  
 $I_p$  = POLAR MOMENT OF INERTIA OF SHAFT CROSS-SECTION, IN.<sup>4</sup>

<p>SOLID CIRCULAR</p>  $k_t = \frac{\pi GD^4}{32l}$	<p>HOLLOW CIRCULAR</p>  $k_t = \frac{\pi G(D_o^4 - D_i^4)}{32l}$
<p>RECTANGULAR</p>  $k_t = \frac{Gwt^3}{3l}$	<p>ANY SOLID SECTION</p>  <p>S = AREA</p> $k_t = \frac{GS^4}{4\pi^2 l I_p}$

MASSLESS BEAMS WITH CONCENTRATED MASS LOADS

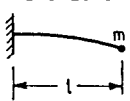
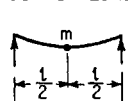
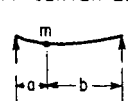
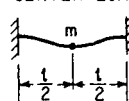
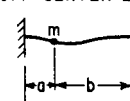
$m$  = MASS OF LOAD, LB-SEC<sup>2</sup>/IN.

$l$  = LENGTH OF BEAM, IN.

$I$  = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.<sup>4</sup>

$E$  = YOUNG'S MODULUS, LB/IN.<sup>2</sup>

$\omega_n$  = ANGULAR NATURAL FREQUENCY, RAD/SEC

<p><b>FIXED-FREE END LOAD</b></p>  <p><math>\omega_n = \sqrt{\frac{3EI}{ml^3}}</math></p>	<p><b>HINGED-HINGED CENTER LOAD</b></p>  <p><math>\omega_n = 4\sqrt{\frac{3EI}{ml^3}}</math></p>	<p><b>HINGED-HINGED OFF-CENTER LOAD</b></p>  <p><math>\omega_n = \frac{l}{ab}\sqrt{\frac{3EI}{m}}</math></p>	<p><b>FIXED-FIXED CENTER LOAD</b></p>  <p><math>\omega_n = 8\sqrt{\frac{3EI}{ml^3}}</math></p>	<p><b>FIXED-FIXED OFF-CENTER LOAD</b></p>  <p><math>\omega_n = \frac{1}{ab}\sqrt{\frac{3EI l^3}{mab}}</math></p>
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MASSIVE SPRINGS (BEAMS) WITH CONCENTRATED MASS LOADS

$m$  = MASS OF LOAD, LB-SEC<sup>2</sup>/IN.

$m_s(m_b)$  = MASS OF SPRING (BEAM), LB-SEC<sup>2</sup>/IN.

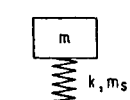
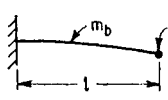
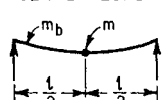
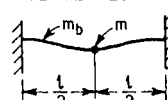
$k$  = STIFFNESS OF SPRING LB/IN.

$l$  = LENGTH OF BEAM, IN.

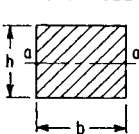
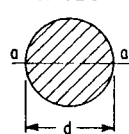
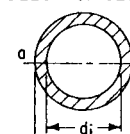
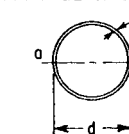
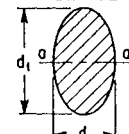
$I$  = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.<sup>4</sup>

$E$  = YOUNG'S MODULUS, LB/IN.<sup>2</sup>

$\omega_n$  = ANGULAR NATURAL FREQUENCY, RAD/SEC

<p><b>MASS - HELICAL SPRING</b></p>  <p><math>\omega_n = \sqrt{\frac{k}{(m + \frac{m_s}{3})}}</math></p>	<p><b>FIXED-FREE END LOAD</b></p>  <p><math>\omega_n = \sqrt{\frac{3EI}{l^3(m + 0.23m_b)}}</math></p>	<p><b>HINGED-HINGED CENTER LOAD</b></p>  <p><math>\omega_n = \sqrt{\frac{48EI}{l^3(m + 0.5m_b)}}</math></p>	<p><b>FIXED-FIXED CENTER LOAD</b></p>  <p><math>\omega_n = 14\sqrt{\frac{EI}{l^3(m + 0.375m_b)}}</math></p>
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AREA MOMENT OF INERTIA OF BEAM SECTIONS  
(WITH RESPECT TO AXIS a-a)

<p><b>RECTANGLE</b></p>  <p><math>I = \frac{bh^3}{12}</math></p>	<p><b>CIRCLE</b></p>  <p><math>I = \frac{\pi d^4}{64}</math></p>	<p><b>HOLLOW CIRCLE</b></p>  <p><math>I = \frac{\pi}{64}(d_o^4 - d_i^4)</math></p>	<p><b>THIN WALL CIRCLE</b></p>  <p><math>I = \frac{\pi d^3 t}{8}</math></p>	<p><b>ELLIPSE</b></p>  <p><math>I = \frac{\pi d_2 d_1^3}{64}</math></p>
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BEAMS OF UNIFORM SECTION AND UNIFORMLY DISTRIBUTED LOAD

ANGULAR NATURAL FREQUENCY  $\omega_n = A \sqrt{\frac{EI}{\mu l^4}}$  RAD/SEC

WHERE E = YOUNG'S MODULUS, LB/IN.<sup>2</sup>

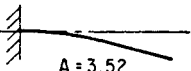
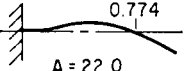
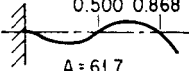
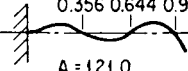
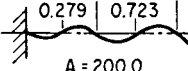
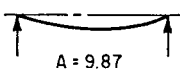
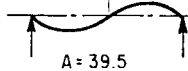
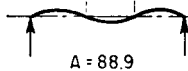
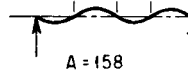
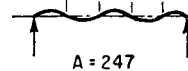
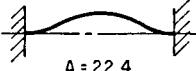
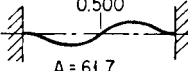
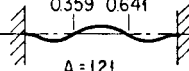
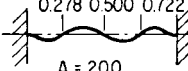
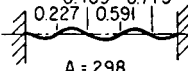
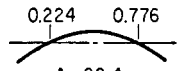
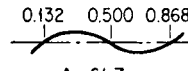
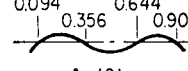
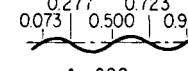
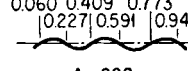
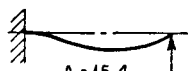
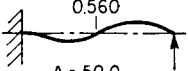
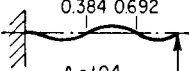
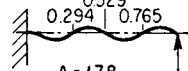
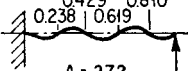
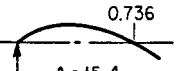
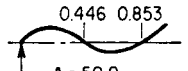
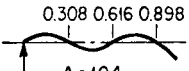
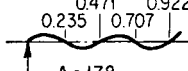
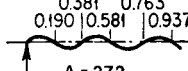
I = AREA MOMENT OF INERTIA OF BEAM CROSS SECTION, IN.

l = LENGTH OF BEAM, IN.

$\mu$  = MASS PER UNIT LENGTH OF BEAM, LB-SEC<sup>2</sup>/IN.<sup>2</sup>

A = COEFFICIENT FROM TABLE BELOW

NODES ARE INDICATED IN TABLE BELOW AS A PROPORTION OF LENGTH l MEASURED FROM LEFT END

FIXED-FREE (CANTILEVER)	 A = 3.52	 A = 22.0	 A = 61.7	 A = 121.0	 A = 200.0
HINGED-HINGED (SIMPLE)	 A = 9.87	 A = 39.5	 A = 88.9	 A = 158	 A = 247
FIXED-FIXED (BUILT-IN)	 A = 22.4	 A = 61.7	 A = 121	 A = 200	 A = 298
FREE-FREE	 A = 22.4	 A = 61.7	 A = 121	 A = 200	 A = 298
FIXED-HINGED	 A = 15.4	 A = 50.0	 A = 104	 A = 178	 A = 272
HINGED-FREE	 A = 15.4	 A = 50.0	 A = 104	 A = 178	 A = 272

NATURAL FREQUENCIES OF THIN FLAT PLATES OF UNIFORM THICKNESS

$$\omega_n = B \sqrt{\frac{E t^3}{\rho a^4 (1-\nu^2)}} \text{ RAD/SEC}$$

E = YOUNG'S MODULUS, LB / IN<sup>2</sup>

t = THICKNESS OF PLATE, IN.

 $\rho$  = MASS DENSITY, LB-SEC<sup>2</sup>/IN.<sup>3</sup>

a = DIAMETER OF CIRCULAR PLATE OR SIDE OF SQUARE PLATE, IN.

 $\nu$  = POISSON'S RATIO

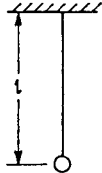
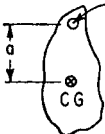
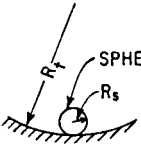
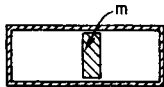
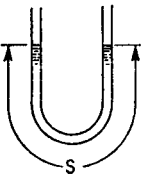
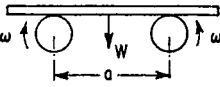
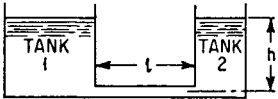
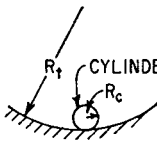
SHAPE OF PLATE	DIAGRAM	EDGE CONDITIONS	VALUE OF B FOR MODE:										
			1	2	3	4	5	6	7	8			
CIRCULAR		CLAMPED AT EDGE	11.84	24.61	40.41	46.14	103.12						
CIRCULAR		FREE	6.09	10.53	14.19	23.80	40.88	44.68	61.38	69.44			
CIRCULAR		CLAMPED AT CENTER	4.35	24.26	70.39	138.85							
CIRCULAR		SIMPLY SUPPORTED AT EDGE	5.90										
SQUARE		ONE EDGE CLAMPED-THREE EDGES FREE	1.01	2.47	6.20	7.94	9.01						
SQUARE		ALL EDGES CLAMPED	10.40	21.21	31.29	38.04	38.22	47.73					
SQUARE		TWO EDGES CLAMPED-TWO EDGES FREE	2.01	6.96	7.74	13.89	18.25						
SQUARE		ALL EDGES FREE	4.07	5.94	6.91	10.39	17.80	18.85					
SQUARE		ONE EDGE CLAMPED-THREE EDGES SIMPLY SUPPORTED	6.83	14.94	16.95	24.89	28.99	32.71					
SQUARE		TWO EDGES CLAMPED-TWO EDGES SIMPLY SUPPORTED	8.37	15.82	20.03	27.34	29.54	37.31					
SQUARE		ALL EDGES SIMPLY SUPPORTED	5.70	14.26	22.82	28.52	37.08	48.49					

## MASSLESS CIRCULAR PLATE WITH CONCENTRATED CENTER MASS

CLAMPED EDGES		$\omega_n = 4.09 \sqrt{\frac{E h^3}{m a^2 (1-\nu^2)}}$
SIMPLY SUPPORTED EDGES		$\omega_n = 4.09 \sqrt{\frac{E h^3}{m a^2 (1-\nu)(3+\nu)}}$

**NATURAL FREQUENCIES OF MISCELLANEOUS SYSTEMS**

$(\omega_n = \text{ANGULAR NATURAL FREQUENCY, RAD/SEC})$

<p align="center"><b>SIMPLE PENDULUM</b></p>  $\omega_n = \sqrt{\frac{g}{l}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math> (<math>g</math> AND <math>l</math> IN CONSISTENT UNITS)</p>	<p align="center"><b>COMPOUND PENDULUM</b></p>  $\omega_n = \sqrt{\frac{ag}{\rho_o^2}} = \sqrt{\frac{aW}{I_o}}$ <p><math>\rho_o = \text{RADIUS OF GYRATION ABOUT AXIS OF SUPPORT}</math>  <math>g = \text{ACCELERATION OF GRAVITY}</math>  <math>W = \text{WEIGHT OF PENDULUM}</math>  <math>I_o = \text{MOMENT OF INERTIA ABOUT AXIS OF SUPPORT}</math></p>	<p align="center"><b>SPHERE IN CYLINDRICAL TRACK</b></p>  $\omega_n = \sqrt{\frac{5g}{7(R_t - R_s)}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math>  <math>R_t = \text{RADIUS OF TRACK}</math>  <math>R_s = \text{RADIUS OF SPHERE}</math></p>	<p align="center"><b>PNEUMATIC SYSTEM</b></p>  $\omega_n = \sqrt{\frac{2p S^2}{m V_o}}$ <p><math>p = \text{PRESSURE AT EACH END OF CYLINDER, LB/IN}^2</math>  <math>S = \text{AREA OF PISTON, IN}^2</math>  <math>m = \text{MASS OF PISTON, LB-SEC}^2/\text{IN}</math>  <math>V_o = \text{VOLUME OF EACH END OF CYLINDER, IN}^3</math></p>
<p align="center"><b>U-TUBE WITH LIQUID</b></p>  $\omega_n = \sqrt{\frac{2g}{S}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math></p>	<p align="center"><b>PLANK ON ROTATING DRUMS</b></p>  $\omega_n = \sqrt{\frac{2\mu g}{a}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math>  <math>\mu = \text{COEFFICIENT OF FRICTION BETWEEN PLANK AND DRUM}</math></p>	<p align="center"><b>TANKS WITH CONNECTING CONDUIT</b></p>  $\omega_n = \sqrt{\frac{9(1 + S_1/S_2)}{h(1 + S_1/S_2) + l(S_1/S_0)}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math>  <math>S_1 = \text{AREA OF TANK 1}</math>  <math>S_2 = \text{AREA OF TANK 2}</math>  <math>S_0 = \text{AREA OF CONDUIT}</math></p>	<p align="center"><b>CYLINDER IN CYLINDRICAL TRACK</b></p>  $\omega_n = \sqrt{\frac{2g}{3(R_t - R_c)}}$ <p><math>g = \text{ACCELERATION OF GRAVITY}</math>  <math>R_t = \text{RADIUS OF TRACK}</math>  <math>R_c = \text{RADIUS OF CYLINDER}</math></p>

## APPENDIX 1.2 TERMINOLOGY

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For convenience, definitions of terms which are used frequently in the field of shock and vibration are assembled here. Many of these are identical with those developed by technical committees of the International Standards Organisation (ISO) and the International Electrotechnical Commission (IEC) in cooperation with the American National Standards Institute (ANSI). Copies of standards publications may be obtained from the Standards Secretariat, Acoustical Society of America, 120 Wall Street, 32d Floor, New York, NY 10005-3993; the e-mail address is [asastds@aip.org](mailto:asastds@aip.org). In addition to the following definitions, many more terms used in shock and vibration are defined throughout the Handbook—far too many to include in this appendix. The reader is referred to the Index.

**acceleration** Acceleration is a vector quantity that specifies the time rate of change of velocity.

**acceleration of gravity** (See *g*.)

**accelerometer** An accelerometer is a transducer whose output is proportional to the acceleration input.

**ambient vibration** Ambient vibration is the all-encompassing vibration associated with a given environment, being usually a composite of vibration from many sources, near and far.

**amplitude** Amplitude is the maximum value of a sinusoidal quantity.

**analog** If a first quantity or structural element is analogous to a second quantity or structural element belonging in another field of knowledge, the second quantity is called the analog of the first, and vice versa.

**analogy** An analogy is a recognized relationship of consistent mutual similarity between the equations and structures appearing within two or more fields of knowledge, and an identification and association of the quantities and structural elements that play mutually similar roles in these equations and structures, for the purpose of facilitating transfer of knowledge of mathematical procedures of analysis and behavior of the structures between these fields.

**angular frequency (circular frequency)** The angular frequency of a periodic quantity, in radians per unit time, is the frequency multiplied by  $2\pi$ .

**angular mechanical impedance (rotational mechanical impedance)** Angular mechanical impedance is the impedance involving the ratio of torque to angular velocity. (See *impedance*.)

**antinode (loop)** An antinode is a point, line, or surface in a standing wave where some characteristic of the wave field has maximum amplitude.

**antiresonance** For a system in forced oscillation, antiresonance exists at a point when any change, however small, in the frequency of excitation causes an increase in the response at this point.

**aperiodic motion** A vibration that is not periodic.

**apparent mass** (See *effective mass*.)

**audio frequency** An audio frequency is any frequency corresponding to a normally audible sound wave.

**autocorrelation coefficient** The autocorrelation coefficient of a signal is the ratio of the autocorrelation function to the mean-square value of the signal:

$$R(\tau) = \overline{x(t)x(t+\tau)} / \overline{[x(t)]^2}$$

**autocorrelation function** The autocorrelation function of a signal is the average of the product of the value of the signal at time  $t$  with the value at time  $t + \tau$ :

$$R(\tau) = \overline{x(t)x(t+\tau)}$$



For a stationary random signal of infinite duration, the power spectral density (except for a constant factor) is the cosine Fourier transform of the autocorrelation function.

**autospectral density** The limiting mean-square value (e.g., of acceleration, velocity, displacement, stress, or other random variable) per unit bandwidth, i.e., the limit of the mean-square value in a given rectangular bandwidth divided by the bandwidth, as the bandwidth approaches zero. Also called *power spectral density*.

**auxiliary mass damper (damped vibration absorber)** An auxiliary mass damper is a system consisting of a mass, spring, and damper which tends to reduce vibration by the dissipation of energy in the damper as a result of relative motion between the mass and the structure to which the damper is attached.

**background noise** Background noise is the total of all sources of interference in a system used for the production, detection, measurement, or recording of a signal, independent of the presence of the signal.

**balancing** Balancing is a procedure for adjusting the mass distribution of a rotor so that vibration of the journals, or the forces on the bearings at once-per-revolution, are reduced or controlled. (See Chap. 39 for a complete list of definitions related to *balancing*.)

**bandpass filter** A bandpass filter is a wave filter that has a single transmission band extending from a lower cutoff frequency greater than zero to a finite upper cutoff frequency.

**bandwidth, effective** (See *effective bandwidth*.)

**beat frequency** The absolute value of the difference in frequency of two oscillators of slightly different frequency.

**beats** Beats are periodic variations that result from the superposition of two simple harmonic quantities of different frequencies  $f_1$  and  $f_2$ . They involve the periodic increase and decrease of amplitude at the beat frequency  $(f_1 - f_2)$ .

**broadband random vibration** Broadband random vibration is random vibration having its frequency components distributed over a broad frequency band. (See *random vibration*.)

**calibration factor** The average sensitivity of a transducer over a specified frequency range.

**center-of-gravity** Center-of-gravity is the point through which passes the resultant of the weights of its component particles for all orientations of the body with respect to a gravitational field; if the gravitational field is uniform, the center-of-gravity corresponds with the *center-of-mass*.

**circular frequency** (See *angular frequency*.)

**complex angular frequency** As applied to a function  $\alpha = Ae^{\sigma t} \sin(\omega t - \phi)$ , where  $\sigma$ ,  $\omega$ , and  $\phi$  are constant, the quantity  $\omega_c = \sigma + j\omega$  is the complex angular frequency where  $j$  is an operator with rules of addition, multiplication, and division as suggested by the symbol  $\sqrt{-1}$ . If the signal decreases with time,  $\sigma$  must be negative.

**complex function** A complex function is a function having real and imaginary parts.

**complex vibration** Complex vibration is vibration whose components are sinusoids not harmonically related to one another. (See *harmonic*.)

**compliance** Compliance is the reciprocal of stiffness.

**compressional wave** A compressional wave is one of compressive or tensile stresses propagated in an elastic medium.

**continuous system (distributed system)** A continuous system is one that is considered to have an infinite number of possible independent displacements. Its configuration is specified by a function of a continuous spatial variable or variables in contrast to a discrete or lumped parameter system which requires only a finite number of coordinates to specify its configuration.

**correlation coefficient** The correlation coefficient of two variables is the ratio of the correlation function to the product of the averages of the variables:

$$\frac{\overline{x_1(t) \cdot x_2(t)}}{\overline{x_1(t)} \cdot \overline{x_2(t)}}$$

**correlation function** The correlation function of two variables is the average value of their product:

$$\overline{x_1(t) \cdot x_2(t)}$$

**Coulomb damping (dry friction damping)** Coulomb damping is the dissipation of energy that occurs when a particle in a vibrating system is resisted by a force whose magnitude is a constant independent of displacement and velocity and whose direction is opposite to the direction of the velocity of the particle.

**coupled modes** Coupled modes are modes of vibration that are not independent but which influence one another because of energy transfer from one mode to the other. (See *mode of vibration*.)

**coupling factor, electromechanical** The electromechanical coupling factor is a factor used to characterize the extent to which the electrical characteristics of a transducer are modified by a coupled mechanical system, and vice versa.

**crest factor** The crest factor is the ratio of the peak value to the root-mean-square value.

**critical damping** Critical damping is the minimum viscous damping that will allow a displaced system to return to its initial position without oscillation.

**critical speed** Critical speed is the speed of a rotating system that corresponds to a resonance frequency of the system.

**cross-talk** The signal observed in one channel due to a signal in another channel.

**cycle** A cycle is the complete sequence of values of a periodic quantity that occur during a period.

**damped natural frequency** The damped natural frequency is the frequency of free vibration of a damped linear system. The free vibration of a damped system may be considered periodic in the limited sense that the time interval between zero crossings in the same direction is constant, even though successive amplitudes decrease progressively. The frequency of the vibration is the reciprocal of this time interval.

**damper** A damper is a device used to reduce the magnitude of a shock or vibration by one or more energy dissipation methods.

**damping** Damping is the dissipation of energy with time or distance.

**damping ratio** (See *fraction of critical damping*.)

**decibel (dB)** The decibel is a unit which denotes the magnitude of a quantity with respect to an arbitrarily established reference value of the quantity, in terms of the logarithm (to the base 10) of the ratio of the quantities. For example, in electrical transmission circuits a value of power may be expressed in terms of a power level in decibels; the power level is given by 10 times the logarithm (to the base 10) of the ratio of the actual power to a reference power (which corresponds to 0 dB).

**degrees-of-freedom** The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time. In general, it is equal to the number of independent displacements that are possible.

**deterministic function** A deterministic function is one whose value at any time can be predicted from its value at any other time.

**displacement** Displacement is a vector quantity that specifies the change of position of a body or particle and is usually measured from the mean position or position of rest. In general, it can be represented as a rotation vector or a translation vector, or both.

**displacement pickup** Displacement pickup is a transducer that converts an input displacement to an output that is proportional to the input displacement.

**distortion** Distortion is an undesired change in waveform. Noise and certain desired changes

in waveform, such as those resulting from modulation or detection, are not usually classed as distortion.

**distributed system** (See *continuous system*.)

**driving point impedance** Driving point impedance is the impedance involving the ratio of force to velocity when both the force and velocity are measured at the same point and in the same direction. (See *impedance*.)

**dry friction damping** (See *Coulomb damping*.)

**duration of shock pulse** The duration of a shock pulse is the time required for the acceleration of the pulse to rise from some stated fraction of the maximum amplitude and to decay to this value. (See *shock pulse*.)

**dynamic stiffness** Dynamic stiffness is the ratio of the change of force to the change of displacement under dynamic conditions.

**dynamic vibration absorber (tuned damper)** A dynamic vibration absorber is an auxiliary mass-spring system which tends to neutralize vibration of a structure to which it is attached. The basic principle of operation is vibration out-of-phase with the vibration of such structure, thereby applying a counteracting force.

**effective bandwidth** The effective bandwidth of a specified transmission system is the bandwidth of an ideal system which (1) has uniform transmission in its pass band equal to the maximum transmission of the specified system and (2) transmits the same power as the specified system when the two systems are receiving equal input signals having a uniform distribution of energy at all frequencies.

**effective mass (apparent mass)** The complex ratio of force to acceleration during simple harmonic motion.

**electromechanical coupling factor** (See *coupling factor, electromechanical*.)

**electrostriction** Electrostriction is the phenomenon wherein some dielectric materials experience an elastic strain when subjected to an electric field, this strain being independent of the polarity of the field.

**ensemble** A collection of signals. (See also *process*.)

**environment** (See *natural environments* and *induced environment*.)

**equivalent system** An equivalent system is one that may be substituted for another system for the purpose of analysis. Many types of equivalence are common in vibration and shock technology: (1) equivalent stiffness, (2) equivalent damping, (3) torsional system equivalent to a translational system, (4) electrical or acoustical system equivalent to a mechanical system, etc.

**equivalent viscous damping** Equivalent viscous damping is a value of viscous damping assumed for the purpose of analysis of a vibratory motion, such that the dissipation of energy per cycle at resonance is the same for either the assumed or actual damping force.

**ergodic process** An ergodic process is a random process that is stationary and of such a nature that all possible time averages performed on one signal are independent of the signal chosen and hence are representative of the time averages of each of the other signals of the entire random process.

**excitation (stimulus)** Excitation is an external force (or other input) applied to a system that causes the system to respond in some way.

**filter** A filter is a device for separating waves on the basis of their frequency. It introduces relatively small insertion loss to waves in one or more frequency bands and relatively large insertion loss to waves of other frequencies. (See *insertion loss*.)

**force factor** The force factor of an electromechanical transducer is (1) the complex quotient of the force required to block the mechanical system divided by the corresponding current in the electric system and (2) the complex quotient of the resulting open-circuit voltage in the

electric system divided by the velocity in the mechanical system. Force factors (1) and (2) have the same magnitude when consistent units are used and the transducer satisfies the principle of reciprocity. It is sometimes convenient in an electrostatic or piezoelectric transducer to use the ratios between force and charge or electric displacement, or between voltage and mechanical displacement.

**forced vibration (forced oscillation)** The oscillation of a system is forced if the response is imposed by the excitation. If the excitation is periodic and continuing, the oscillation is steady-state.

**foundation (support)** A foundation is a structure that supports the gravity load of a mechanical system. It may be fixed in space, or it may undergo a motion that provides excitation for the supported system.

**fraction of critical damping** The fraction of critical damping (damping ratio) for a system with viscous damping is the ratio of actual damping coefficient  $c$  to the critical damping coefficient  $c_c$ .

**free vibration** Free vibration is that which occurs after the removal of an excitation or restraint.

**frequency** The frequency of a function periodic in time is the reciprocal of the period. The unit is the cycle per unit time and must be specified; the unit *cycle per second* is called *hertz* (Hz).

**frequency, angular** (See *angular frequency*.)

**fundamental frequency** (1) The fundamental frequency of a periodic quantity is the frequency of a sinusoidal quantity which has the same period as the periodic quantity. (2) The fundamental frequency of an oscillating system is the lowest natural frequency. The normal mode of vibration associated with this frequency is known as the fundamental mode.

**fundamental mode of vibration** The fundamental mode of vibration of a system is the mode having the lowest natural frequency.

**$g$**  The quantity  $g$  is the acceleration produced by the force of gravity, which varies with the latitude and elevation of the point of observation. By international agreement, the value  $980.665 \text{ cm/sec}^2 = 386.087 \text{ in./sec}^2 = 32.1739 \text{ ft/sec}^2$  has been chosen as the standard acceleration due to gravity.

**harmonic** A harmonic is a sinusoidal quantity having a frequency that is an integral multiple of the frequency of a periodic quantity to which it is related.

**harmonic motion** (See *simple harmonic motion*.)

**harmonic response** Harmonic response is the periodic response of a vibrating system exhibiting the characteristics of resonance at a frequency that is a multiple of the excitation frequency.

**high-pass filter** A high-pass filter is a wave filter having a single transmission band extending from some critical or cutoff frequency, not zero, up to infinite frequency.

**image impedances** The image impedances of a structure or device are the impedances that will simultaneously terminate all of its inputs and outputs in such a way that at each of its inputs and outputs the impedances in both directions are equal.

**impact** An impact is a single collision of one mass in motion with a second mass which may be either in motion or at rest.

**impedance** Mechanical impedance is the ratio of a force-like quantity to a velocity-like quantity when the arguments of the real (or imaginary) parts of the quantities increase linearly with time. Examples of force-like quantities are: force, sound pressure, voltage, temperature. Examples of velocity-like quantities are: velocity, volume velocity, current, heat flow. *Impedance* is the reciprocal of *mobility*. (See also *angular mechanical impedance*, *linear mechanical impedance*, *driving point impedance*, and *transfer impedance*.)

**impulse** Impulse is the product of a force and the time during which the force is applied; more specifically, the impulse is  $\int_{t_1}^{t_2} F dt$  where the force  $F$  is time dependent and equal to zero before time  $t_1$  and after time  $t_2$ .

**impulse response function** See Eq. (21.7).

**induced environments** Induced environments are those conditions generated as a result of the operation of a structure or equipment.

**insertion loss** The insertion loss, in decibels, resulting from insertion of an element in a transmission system is 10 times the logarithm to the base 10 of the ratio of the power delivered to that part of the system that will follow the element, before the insertion of the element, to the power delivered to that same part of the system after insertion of the element.

**isolation** Isolation is a reduction in the capacity of a system to respond to an excitation, attained by the use of a resilient support. In steady-state forced vibration, isolation is expressed quantitatively as the complement of transmissibility.

**isolator** (See *vibration isolator*.)

**jerk** Jerk is a vector that specifies the time rate of change of acceleration; jerk is the third derivative of displacement with respect to time.

**level** Level is the logarithm of the ratio of a given quantity to a reference quantity of the same kind; the base of the logarithm, the reference quantity, and the kind of level must be indicated. (The type of level is indicated by the use of a compound term such as *vibration velocity level*. The level of the reference quantity remains unchanged whether the chosen quantity is peak, rms, or otherwise.) Unit: decibel. Unit symbol: dB.

**line spectrum** A line spectrum is a spectrum whose components occur at a number of discrete frequencies.

**linear mechanical impedance** Linear mechanical impedance is the impedance involving the ratio of force to linear velocity. (See *impedance*.)

**linear system** A system is linear if for every element in the system the response is proportional to the excitation. This definition implies that the dynamic properties of each element in the system can be represented by a set of linear differential equations with constant coefficients, and that for the system as a whole superposition holds.

**logarithmic decrement** The logarithmic decrement is the natural logarithm of the ratio of any two successive amplitudes of like sign, in the decay of a single-frequency oscillation.

**longitudinal wave** A longitudinal wave in a medium is a wave in which the direction of displacement at each point of the medium is normal to the wave front.

**low-pass filter** A low-pass filter is a wave filter having a single transmission band extending from zero frequency up to some critical or cutoff frequency which is not infinite.

**magnetic recorder** A magnetic recorder is equipment incorporating an electromagnetic transducer and means for moving a ferromagnetic recording medium relative to the transducer for recording electric signals as magnetic variations in the medium.

**magnetostriction** Magnetostriction is the phenomenon wherein ferromagnetic materials experience an elastic strain when subjected to an external magnetic field. Also, magnetostriction is the converse phenomenon in which mechanical stresses cause a change in the magnetic induction of a ferromagnetic material.

**maximum value** The maximum value is the value of a function when any small change in the independent variable causes a decrease in the value of the function.

**mechanical admittance** (See *mobility*.)

**mechanical impedance** (See *impedance*.)

**mechanical shock** Mechanical shock is a nonperiodic excitation (e.g., a motion of the foundation or an applied force) of a mechanical system that is characterized by suddenness and severity and usually causes significant relative displacements in the system.

**mechanical system** A mechanical system is an aggregate of matter comprising a defined configuration of mass, stiffness, and damping.

**mobility (mechanical admittance)** Mobility is the ratio of a velocity-like quantity to a force-like quantity when the arguments of the real (or imaginary) parts of the quantities increase lin-

early with time. *Mobility* is the reciprocal of *impedance*. The terms *angular mobility*, *linear mobility*, *driving-point mobility*, and *transfer mobility* are used in the same sense as corresponding impedances.

**modal numbers** When the normal modes of a system are related by a set of ordered integers, these integers are called modal numbers.

**mode of vibration** In a system undergoing vibration, a mode of vibration is a characteristic pattern assumed by the system in which the motion of every particle is simple harmonic with the same frequency. Two or more modes may exist concurrently in a multiple degree-of-freedom system.

**modulation** Modulation is the variation in the value of some parameter which characterizes a periodic oscillation. Thus, amplitude modulation of a sinusoidal oscillation is a variation in the amplitude of the sinusoidal oscillation.

**multiple degree-of-freedom system** A multiple degree-of-freedom system is one for which two or more coordinates are required to define completely the position of the system at any instant.

**narrow-band random vibration (random sine wave)** Narrow-band random vibration is random vibration having frequency components only within a narrow band. It has the appearance of a sine wave whose amplitude varies in an unpredictable manner. (See *random vibration*.)

**natural environments** Natural environments are those conditions generated by the forces of nature and whose effects are experienced when the equipment or structure is at rest as well as when it is in operation.

**natural frequency** Natural frequency is the frequency of free vibration of a system. For a multiple degree-of-freedom system, the natural frequencies are the frequencies of the normal modes of vibration.

**natural mode of vibration** The natural mode of vibration is a mode of vibration assumed by a system when vibrating freely.

**neutral surface** That surface of a beam, in simple flexure, over which there is no longitudinal stress.

**node** A node is a point, line, or surface in a standing wave where some characteristic of the wave field has essentially zero amplitude.

**noise** Noise is any undesired signal. By extension, noise is any unwanted disturbance within a useful frequency band, such as undesired electric waves in a transmission channel or device.

**nominal bandwidth** The nominal bandwidth of a filter is the difference between the nominal upper and lower cutoff frequencies. The difference may be expressed (1) in cycles per second, (2) as a percentage of the passband center frequency, or (3) in octaves.

**nominal passband center frequency** The nominal passband center frequency is the geometric mean of the nominal cutoff frequencies.

**nominal upper and lower cutoff frequencies** The nominal upper and lower cutoff frequencies of a filter passband are those frequencies above and below the frequency of maximum response of a filter at which the response to a sinusoidal signal is 3 dB below the maximum response.

**nonlinear damping** Nonlinear damping is damping due to a damping force that is not proportional to velocity.

**normal mode of vibration** A normal mode of vibration is a mode of vibration that is uncoupled from (i.e., can exist independently of) other modes of vibration of a system. When vibration of the system is defined as an eigenvalue problem, the normal modes are the eigenvectors and the normal mode frequencies are the eigenvalues. The term *classical normal mode* is sometimes applied to the normal modes of a vibrating system characterized by vibration of each element of the system at the same frequency and phase. In general, classical normal modes exist only in systems having no damping or having particular types of damping.

**octave** The interval between two frequencies that have a frequency ratio of two.

**oscillation** Oscillation is the variation, usually with time, of the magnitude of a quantity with respect to a specified reference when the magnitude is alternately greater and smaller than the reference.

**partial node** A partial node is the point, line, or surface in a standing-wave system where some characteristic of the wave field has a minimum amplitude differing from zero. The appropriate modifier should be used with the words *partial node* to signify the type that is intended; e.g., displacement partial node, velocity partial node, pressure partial node.

**peak-to-peak value** The peak-to-peak value of a vibrating quantity is the algebraic difference between the extremes of the quantity.

**peak value** Peak value is the maximum value of a vibration during a given interval, usually considered to be the maximum deviation of that vibration from the mean value.

**period** The period of a periodic quantity is the smallest increment of the independent variable for which the function repeats itself.

**periodic quantity** A periodic quantity is an oscillating quantity whose values recur for certain increments of the independent variable.

**phase of a periodic quantity** The phase of a periodic quantity, for a particular value of the independent variable, is the fractional part of a period through which the independent variable has advanced, measured from an arbitrary reference.

**pickup** (See *transducer*.)

**piezoelectric (crystal) (ceramic) transducer** A piezoelectric transducer is a transducer that depends for its operation on the interaction between the electric charge and the deformation of certain asymmetric crystals having piezoelectric properties.

**piezoelectricity** Piezoelectricity is the property exhibited by some asymmetrical crystalline materials which when subjected to strain in suitable directions develop electric polarization proportional to the strain. Inverse piezoelectricity is the effect in which mechanical strain is produced in certain asymmetrical crystalline materials when subjected to an external electric field; the strain is proportional to the electric field.

**power spectral density** Power spectral density is the limiting mean-square value (e.g., of acceleration, velocity, displacement, stress, or other random variable) per unit bandwidth, i.e., the limit of the mean-square value in a given rectangular bandwidth divided by the bandwidth, as the bandwidth approaches zero. Also called *autospectral density*.

**power spectral density level** The spectrum level of a specified signal at a particular frequency is the level in decibels of that part of the signal contained within a band 1 cycle per second wide, centered at the particular frequency. Ordinarily this has significance only for a signal having a continuous distribution of components within the frequency range under consideration.

**power spectrum** A spectrum of mean-squared spectral density values.

**process** A process is a collection of signals. The word *process* rather than the word *ensemble* ordinarily is used when it is desired to emphasize the properties the signals have or do not have as a group. Thus, one speaks of a stationary process rather than a stationary ensemble.

**pulse rise time** The pulse rise time is the interval of time required for the leading edge of a pulse to rise from some specified small fraction to some specified larger fraction of the maximum value.

**$Q$  (quality factor)** The quantity  $Q$  is a measure of the sharpness of resonance or frequency selectivity of a resonant vibratory system having a single degree of freedom, either mechanical or electrical. In a mechanical system, this quantity is equal to one-half the reciprocal of the damping ratio. It is commonly used only with reference to a lightly damped system and is then approximately equal to the following: (1) Transmissibility at resonance, (2)  $\pi/\text{logarithmic decrement}$ , (3)  $2\pi W/\Delta W$  where  $W$  is the stored energy and  $\Delta W$  the energy dissipation per cycle, and (4)  $f_r/\Delta f$  where  $f_r$  is the resonance frequency and  $\Delta f$  is the bandwidth between the half-power points.

**quasi-ergodic process** A quasi-ergodic process is a random process which is not necessarily

stationary but of such a nature that some time averages performed on a signal are independent of the signal chosen.

**quasi-periodic signal** A quasi-periodic signal is one consisting only of quasi-sinusoids.

**quasi-sinusoid** A quasi-sinusoid is a function of the form  $\alpha = A \sin(2\pi ft - \phi)$  where either  $A$  or  $f$ , or both, is not a constant but may be expressed readily as a function of time. Ordinarily  $\phi$  is considered constant.

**random sine wave** (See *narrow-band random vibration*.)

**random vibration** Random vibration is vibration whose instantaneous magnitude is not specified for any given instant of time. The instantaneous magnitudes of a random vibration are specified only by probability distribution functions giving the probable fraction of the total time that the magnitude (or some sequence of magnitudes) lies within a specified range. Random vibration contains no periodic or quasi-periodic constituents. If random vibration has instantaneous magnitudes that occur according to the Gaussian distribution, it is called *Gaussian random vibration*.

**ratio of critical damping** (See *fraction of critical damping*.)

**Rayleigh wave** A Rayleigh wave is a surface wave associated with the free boundary of a solid, such that a surface particle describes an ellipse whose major axis is normal to the surface, and whose center is at the undisturbed surface. At maximum particle displacement away from the solid surface the motion of the particle is opposite to that of the wave.

**recording channel** The term *recording channel* refers to one of a number of independent recorders in a recording system or to independent recording tracks on a recording medium.

**recording system** A recording system is a combination of transducing devices and associated equipment suitable for storing signals in a form capable of subsequent reproduction.

**rectangular shock pulse** An ideal shock pulse for which motion rises instantaneously to a given value, remains constant for the duration of the pulse, then drops to zero instantaneously.

**relaxation time** Relaxation time is the time taken by an exponentially decaying quantity to decrease in amplitude by a factor of  $1/e = 0.3679$ .

**re-recording** Re-recording is the process of making a recording by reproducing a recorded signal source and recording this reproduction.

**resonance** Resonance of a system in forced vibration exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

**resonance frequency** Resonance frequency is a frequency at which resonance exists.

**response** The response of a device or system is the motion (or other output) resulting from an excitation (stimulus) under specified conditions.

**response spectrum** (See *shock response spectrum*.)

**rotational mechanical impedance** (See *angular mechanical impedance*.)

**seismic pickup; seismic transducer** A seismic pickup or transducer is a device consisting of a seismic system in which the differential movement between the mass and the base of the system produces a measurable indication of such movement.

**seismic system** A seismic system is one consisting of a mass attached to a reference base by one or more flexible elements. Damping is usually included.

**self-induced (self-excited) vibration** The vibration of a mechanical system is self-induced if it results from conversion, within the system, of nonoscillatory excitation to oscillatory excitation.

**sensing element** That part of a transducer which is activated by the input excitation and supplies the output signal.

**sensitivity** The sensitivity of a transducer is the ratio of a specified output quantity to a specified input quantity.

**shake table** (See *vibration machine*.)



**shear wave (rotational wave)** A shear wave is a wave in an elastic medium which causes an element of the medium to change its shape without a change of volume.

**shock** (See *mechanical shock*.)

**shock absorber** A shock absorber is a device which dissipates energy to modify the response of a mechanical system to applied shock.

**shock excitation** An excitation, applied to a mechanical system, that produces a mechanical shock.

**shock isolator (shock mount)** A shock isolator is a resilient support that tends to isolate a system from a shock motion.

**shock machine** A shock machine is a device for subjecting a system to controlled and reproducible mechanical shock.

**shock motion** Shock motion is an excitation involving motion of a foundation. (See *foundation* and *mechanical shock*.)

**shock mount** (See *shock isolator*.)

**shock pulse** A shock pulse is a substantial disturbance characterized by a rise of acceleration from a constant value and decay of acceleration to the constant value in a short period of time. Shock pulses are normally displayed graphically as curves of acceleration as functions of time.

**shock-pulse duration** (See *duration of shock pulse*.)

**shock response spectrum** A shock spectrum is a plot of the maximum response experienced by a single degree-of-freedom system, as a function of its own natural frequency, in response to an applied shock. The response may be expressed in terms of acceleration, velocity, or displacement.

**shock testing machine; shock machine** A shock testing machine is a device for subjecting a mechanical system to controlled and reproducible mechanical shock.

**signal** A signal is (1) a disturbance used to convey information; (2) the information to be conveyed over a communication system.

**simple harmonic motion** A simple harmonic motion is a motion such that the displacement is a sinusoidal function of time; sometimes it is designated merely by the term *harmonic motion*.

**single degree-of-freedom system** A single degree-of-freedom system is one for which only one coordinate is required to define completely the configuration of the system at any instant.

**sinusoidal motion** (See *simple harmonic motion*.)

**snubber** A snubber is a device used to increase the stiffness of an elastic system (usually by a large factor) whenever the displacement becomes larger than a specified value.

**spectrum** A spectrum is a definition of the magnitude of the frequency components that constitute a quantity.

**spectrum density** (See *power spectral density*.)

**standard deviation** Standard deviation is the square root of the variance; i.e., the square root of the mean of the squares of the deviations from the mean value of a vibrating quantity.

**standing wave** A standing wave is a periodic wave having a fixed distribution in space which is the result of interference of progressive waves of the same frequency and kind. Such waves are characterized by the existence of nodes or partial nodes and antinodes that are fixed in space.

**stationary process** A stationary process is an ensemble of signals such that an average of values over the ensemble at any given time is independent of time.

**stationary signal** A stationary signal is a random signal of such nature that averages over samples of finite time intervals are independent of the time at which the sample occurs.

**steady-state vibration** Steady-state vibration exists in a system if the velocity of each particle is a continuing periodic quantity.

**stiffness** Stiffness is the ratio of change of force (or torque) to the corresponding change on translational (or rotational) deflection of an elastic element.

**subharmonic** A subharmonic is a sinusoidal quantity having a frequency that is an integral submultiple of the fundamental frequency of a periodic quantity to which it is related.

**subharmonic response** Subharmonic response is the periodic response of a mechanical system exhibiting the characteristic of resonance at a frequency that is a submultiple of the frequency of the periodic excitation.

**superharmonic response** Superharmonic response is a term sometimes used to denote a particular type of harmonic response which dominates the total response of the system; it frequently occurs when the excitation frequency is a submultiple of the frequency of the fundamental resonance.

**time history** The magnitude of a quantity expressed as a function of time.

**transducer (pickup)** A transducer is a device which converts shock or vibratory motion into an optical, a mechanical, or most commonly to an electrical signal that is proportional to a parameter of the experienced motion.

**transfer impedance** Transfer impedance between two points is the impedance involving the ratio of force to velocity when force is measured at one point and velocity at the other point. The term *transfer impedance* also is used to denote the ratio of force to velocity measured at the same point but in different directions. (See *impedance*.)

**transient vibration** Transient vibration is temporarily sustained vibration of a mechanical system. It may consist of forced or free vibration or both.

**transmissibility** Transmissibility is the nondimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities, or accelerations.

**transmission loss** Transmission loss is the reduction in the magnitude of some characteristic of a signal, between two stated points in a transmission system.

**transverse wave** A transverse wave is a wave in which the direction of displacement at each point of the medium is parallel to the wave front.

**tuned damper** (See *dynamic vibration absorber*.)

**uncorrelated** Two signals or variables  $\alpha_1(t)$  and  $\alpha_2(t)$  are said to be uncorrelated if the average value of their product is zero:  $\alpha_1(t) \cdot \alpha_2(t) = 0$ . If the correlation coefficient is equal to unity, the variables are said to be completely correlated. If the coefficient is less than unity but larger than zero, they are said to be partially correlated. (See *correlation coefficient*.)

**uncoupled mode** An uncoupled mode of vibration is a mode that can exist in a system concurrently with and independently of other modes.

**undamped natural frequency** The undamped natural frequency of a mechanical system is the frequency of free vibration resulting from only elastic and inertial forces of the system.

**variance** Variance is the mean of the squares of the deviations from the mean value of a vibrating quantity.

**velocity** Velocity is a vector quantity that specifies the time rate of change of displacement with respect to a reference frame. If the reference frame is not inertial, the velocity is often designated "relative velocity."

**velocity pickup** A velocity pickup is a transducer that converts an input velocity to an output (usually electrical) that is proportional to the input velocity.

**velocity shock** Velocity shock is a particular type of shock motion characterized by a sudden velocity change of the foundation. (See *foundation* and *mechanical shock*.)

**vibration** Vibration is an oscillation wherein the quantity is a parameter that defines the motion of a mechanical system. (See *oscillation*.)

**vibration acceleration** Vibration acceleration is the rate of change of speed and direction of a vibration, in a specified direction. The frequency bandwidth must be identified. Unit meter per second squared. Unit symbol:  $m/s^2$ .

**vibration acceleration level** The vibration acceleration level is 10 times the logarithm (to the base 10) of the ratio of the square of a given vibration acceleration to the square of a reference acceleration, commonly  $1g$  or  $1 m/s^2$ . Unit: decibel. Unit symbol: dB.

**vibration isolator** A vibration isolator is a resilient support that tends to isolate a system from steady-state excitation.

**vibration machine** A vibration machine is a device for subjecting a mechanical system to controlled and reproducible mechanical vibration.

**vibration meter** A vibration meter is an apparatus for the measurement of displacement, velocity, or acceleration of a vibrating body.

**vibration mount** (See *vibration isolator*.)

**vibration pickup** (See *transducer*.)

**vibrograph** A vibrograph is an instrument, usually mechanical and self-contained, that provides an oscillographic recording of a vibration waveform.

**vibrometer** An instrument capable of indicating some measure of the magnitude (such as r.m.s. acceleration) on a scale.

**viscous damping** Viscous damping is the dissipation of energy that occurs when a particle in a vibrating system is resisted by a force that has a magnitude proportional to the magnitude of the velocity of the particle and direction opposite to the direction of the particle.

**viscous damping, equivalent** (See *equivalent viscous damping*.)

**wave** A wave is a disturbance which is propagated in a medium in such a manner that at any point in the medium the quantity serving as measure of disturbance is a function of the time, while at any instant the displacement at a point is a function of the position of the point. Any physical quantity that has the same relationship to some independent variable (usually time) that a propagated disturbance has, at a particular instant, with respect to space, may be called a wave.

**wave interference** Wave interference is the phenomenon which results when waves of the same or nearly the same frequency are superposed; it is characterized by a spatial or temporal distribution of amplitude of some specified characteristic differing from that of the individual superposed waves.

**wavelength** The wavelength of a periodic wave in an isotropic medium is the perpendicular distance between two wave fronts in which the displacements have a difference in phase of one complete period.

**white noise** White noise is a noise whose power spectral density is substantially independent of frequency over a specified range.