CHAPTER 5
SELF-EXCITED VIBRATION

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INTRODUCTION

Self-excited systems begin to vibrate of their own accord spontaneously, the amplitude increasing until some nonlinear effect limits any further increase. The energy supplying these vibrations is obtained from a uniform source of power associated with the system which, due to some mechanism inherent in the system, gives rise to oscillating forces. The nature of self-excited vibration compared to forced vibration is:[1]

In self-excited vibration the alternating force that sustains the motion is created or controlled by the motion itself; when the motion stops, the alternating force disappears.

In a forced vibration the sustaining alternating force exists independent of the motion and persists when the vibratory motion is stopped.

The occurrence of self-excited vibration in a physical system is intimately associated with the stability of equilibrium positions of the system. If the system is disturbed from a position of equilibrium, forces generally appear which cause the system to move either toward the equilibrium position or away from it. In the latter case the equilibrium position is said to be unstable; then the system may either oscillate with increasing amplitude or monotonically recede from the equilibrium position until nonlinear or limiting restraints appear. The equilibrium position is said to be stable if the disturbed system approaches the equilibrium position either in a damped oscillatory fashion or asymptotically.

The forces which appear as the system is displaced from its equilibrium position may depend on the displacement or the velocity, or both. If displacement-dependent forces appear and cause the system to move away from the equilibrium position, the system is said to be statically unstable. For example, an inverted pendulum is statically unstable. Velocity-dependent forces which cause the system to recede from a statically stable equilibrium position lead to dynamic instability.

Self-excited vibrations are characterized by the presence of a mechanism whereby a system will vibrate at its own natural or critical frequency, essentially *independent* of the *frequency* of any external stimulus. In mathematical terms, the motion is described by the unstable *homogeneous* solution to the homogeneous equations of motion. In contradistinction, in the case of "forced," or "resonant," vibrations, the *frequency* of the oscillation is *dependent* on (equal to, or a whole number ratio of) the frequency of a forcing function external to the vibrating system (e.g., shaft rotational speed in the case of rotating shafts). In mathematical terms, the forced vibration is the *particular* solution to the *nonhomogeneous* equations of motion.

Self-excited vibrations pervade all areas of design and operations of physical systems where motion or time-variant parameters are involved—aeromechanical systems (flutter, aircraft flight dynamics), aerodynamics (separation, stall, musical wind instruments, diffuser and inlet chugging), aerothermodynamics (flame instability, combustor screech), mechanical systems (machine-tool chatter), and feedback networks (pneumatic, hydraulic, and electromechanical servomechanisms).

ROTATING MACHINERY

One of the more important manifestations of self-excited vibrations, and the one that is the principal concern in this chapter, is that of rotating machinery, specifically, the self-excitation of lateral, or flexural, vibration of rotating shafts (as distinct from torsional, or longitudinal, vibration).

In addition to the description of a large number of such phenomena in standard vibrations textbooks (most typically and prominently, Ref. 1), the field has been subject to several generalized surveys.[2–4] The mechanisms of self-excitation which have been identified can be categorized as follows:

Whirling or Whipping

Hysteretic whirl

Fluid trapped in the rotor

Dry friction whip

Fluid bearing whip

Seal and blade-tip-clearance effect in turbomachinery

Propeller and turbomachinery whirl

Parametric Instability

Asymmetric shafting

Pulsating torque

Pulsating longitudinal loading

Stick-Slip Rubs and Chatter

Instabilities in Forced Vibrations

Bistable vibration

Unstable imbalance

In each instance, the physical mechanism is described and aspects of its prevention or its diagnosis and correction are given. Some exposition of its mathematical analytic modeling is also included.

WHIRLING OR WHIPPING

ANALYTIC MODELING

In the most important subcategory of instabilities (generally termed whirling or whipping), the unifying generality is the generation of a tangential force, normal to an arbitrary radial deflection of a rotating shaft, whose magnitude is proportional to (or varies monotonically with) that deflection. At some "onset" rotational speed, such a force system will overcome the stabilizing external damping forces which are generally present and induce a whirling motion of ever-increasing amplitude, limited only by nonlinearities which ultimately limit deflections.

A close mathematical analogy to this class of phenomena is the concept of "negative damping" in linear systems with constant coefficients, subject to *plane* vibration.

A simple mathematical representation of a self-excited vibration may be found in the concept of negative damping. Consider the differential equation for a damped, free vibration:

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{5.1}$$

This is generally solved by assuming a solution of the form

$$x = Ce^{st}$$

Substitution of this solution into Eq. (5.1) yields the characteristic (algebraic) equation

$$s^2 + \frac{c}{m}s + \frac{k}{m} = 0 \tag{5.2}$$

If $c < 2\sqrt{mk}$, the roots are complex:

$$s_{1,2} = -\frac{c}{2m} \pm iq$$

where
$$q = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$$

The solution takes the form

$$x = e^{-ct/2m}(A \cos qt + B \sin qt) \tag{5.3}$$

This represents a decaying oscillation because the exponential factor is negative, as illustrated in Fig. 5.1*A*. If $c < 0$, the exponential factor has a positive exponent and the vibration appears as shown in Fig. 5.1*B*. The system, initially at rest, begins to oscillate spontaneously with ever-increasing amplitude. Then, in any physical system, some nonlinear effect enters and Eq. (5.1) fails to represent the system realistically. Equation (5.4) defines a nonlinear system with negative damping at small amplitudes but with large positive damping at larger amplitudes, thereby limiting the amplitude to finite values:

$$m\ddot{x} + (-c + ax^2)\dot{x} + kx = 0 \tag{5.4}$$

Thus, the fundamental criterion of stability in linear systems is that the roots of the characteristic equation have negative real parts, thereby producing decaying amplitudes.

In the case of a whirling or whipping shaft, the equations of motion (for an idealized shaft with a single lumped mass m) are more appropriately written in polar coordinates for the radial force balance,

$$-m\omega^2 r + m\ddot{r} + c\dot{r} + kr = 0 \tag{5.5}$$

and for the tangential force balance,

$$2m\omega\dot{r} + c\omega r - F_n = 0 \tag{5.6}$$

where we presume a constant rate of whirl ω.

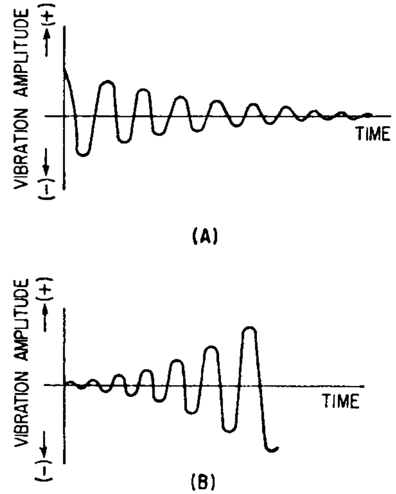

FIGURE 5.1 (*A*) Illustration showing a decaying vibration (stable) corresponding to negative real parts of the complex roots. (*B*) Increasing vibration corresponding to positive real parts of the complex roots (unstable).

In general, the whirling is predicated on the existence of some physical phenomenon which will induce a force F_n that is normal to the radial deflection r and is in the direction of the whirling motion—i.e., in opposition to the damping force, which tends to inhibit the whirling motion. Very often, this normal force can be characterized or approximated as being proportional to the radial deflection:

$$F_n = f_n r \tag{5.7}$$

The solution then takes the form

$$r = r_0 e^{at} \tag{5.8}$$

For the system to be stable, the coefficient of the exponent

$$a = \frac{f_n - c\omega}{2m\omega} \tag{5.9}$$

must be negative, giving the requirement for stable operation as

$$f_n < \omega c \tag{5.10}$$

As a rotating machine increases its rotational speed, the left-hand side of this inequality (which is generally also a function of shaft rotation speed) may exceed the right-hand side, indicative of the onset of instability. At this onset condition,

$$a = 0+ \tag{5.11}$$

so that whirl speed at onset is found to be

$$\omega = \left(\frac{k}{m}\right)^{1/2} \tag{5.12}$$

That is, the whirling speed at onset of instability is the shaft's natural or critical frequency, irrespective of the shaft's rotational speed (rpm). The direction of whirl may be in the same rotational direction as the shaft rotation (*forward* whirl) or opposite to the direction of shaft rotation (*backward* whirl), depending on the direction of the destabilizing force F_n.

When the system is unstable, the solution for the trajectory of the shaft's mass is, from Eq. (5.8), an exponential spiral as in Fig. 5.2. Any planar component of this two-dimensional trajectory takes the same form as the unstable planar vibration shown in Fig. 5.1*B*.

GENERAL DESCRIPTION

The most important examples of whirling and whipping instabilities are

Hysteretic whirl
Fluid trapped in the rotor

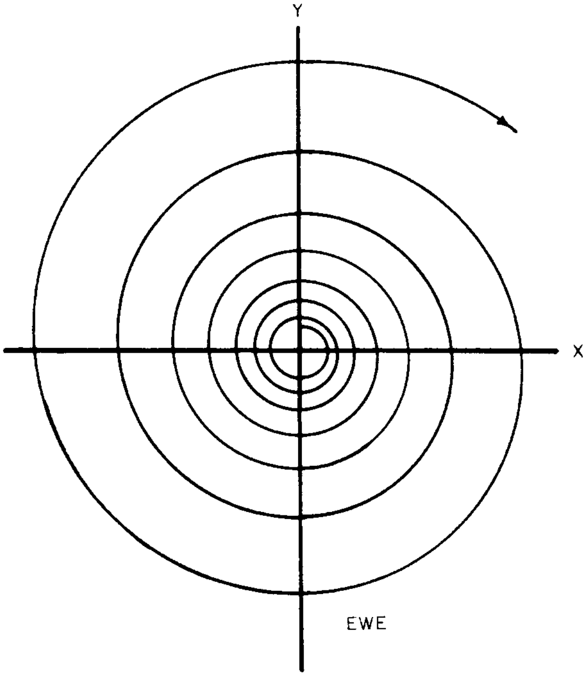

FIGURE 5.2 Trajectory of rotor center of gravity in unstable whirling or whipping.

Dry friction whip
Fluid bearing whip
Seal and blade-tip-clearance effect in turbomachinery
Propeller and turbomachinery whirl

All these self-excitation systems involve friction or fluid energy mechanisms to generate the destabilizing force.

These phenomena are rarer than forced vibration due to unbalance or shaft misalignment, and they are difficult to anticipate before the fact or diagnose after the fact because of their subtlety. Also, self-excited vibrations are potentially more destructive, since the asynchronous whirling of self-excited vibration induces alternating stresses in the rotor and can lead to fatigue failures of rotating components. Synchronous forced vibration typical of unbalance does not involve alternating stresses in the rotor and will rarely involve rotating element failure. The general attributes of these instabilities, insofar as they differ from forced excitations, are summarized in Table 5.1 and Figs. 5.3*A* and 5.3*B*.

HYSTERETIC WHIRL

The mechanism of hysteretic whirl, as observed experimentally,[5] defined analytically,[6] or described in standard texts,[7] may be understood from the schematic representation of Fig. 5.4. With some nominal radial deflection of the shaft, the flexure of the shaft would induce a neutral strain axis normal to the deflection direction. From

TABLE 5.1 Characterization of Two Categories of Vibration of Rotating Shafts

	Forced or resonant vibration	Whirling or whipping
Vibration frequency–rpm relationship	Frequency is equal to (i.e., synchronous with) rpm or a whole number or rational fraction of rpm, as in Fig. 5.3*A*.	Frequency is nearly constant and relatively independent of rotor rotational speed or any external stimulus and is at or near one of the shaft critical or natural frequencies, as in Fig. 5.3*B*.
Vibration amplitude–rpm relationship	Amplitude will peak in a narrow band of rpm wherein the rotor's critical frequency is equal to the rpm or to a whole-number multiple or a rational fraction of the rpm or an external stimulus, as in Fig. 5.3*A*.	Amplitude will suddenly increase at an onset rpm and continue at high or increasing levels as rpm is increased, as in Fig. 5.3*B*.
Influence of damping	Addition of damping may reduce peak amplitude but not materially affect rpm at which peak amplitude occurs, as in Fig. 5.3*A*.	Addition of damping may defer onset to a higher rpm but not materially affect amplitude after onset, as in Fig. 5.3*B*.
System geometry	Excitation level and hence amplitude are dependent on some lack of axial symmetry in the rotor mass distribution or geometry, or external forces applied to the rotor. Amplitudes may be reduced by refining the system to make it more perfectly axisymmetric.	Amplitudes are independent of system axial symmetry. Given an infinitesimal deflection to an otherwise symmetric system, the amplitude will self-propagate.
Rotor fiber stress	For synchronous vibration, the rotor vibrates in frozen, deflected state, without oscillatory fiber stress.	Rotor fibers are subject to oscillatory stress at a frequency equal to the difference between rotor rpm and whirling speed.
Avoidance or elimination	1. Tune the system's critical frequencies to be out of the rpm operating range. 2. Eliminate all deviations from axial symmetry in the system as built or as induced during operation (e.g., balancing). 3. Introduce damping to limit peak amplitudes at critical speeds which must be traversed.	1. Restrict operating rpm to below instability onset rpm. 2. Defeat or eliminate the instability mechanism. 3. Introduce damping to raise the instability onset speed to above the operating speed range. 4. Introduce stiffness anisotropy to the bearing support system.[8]

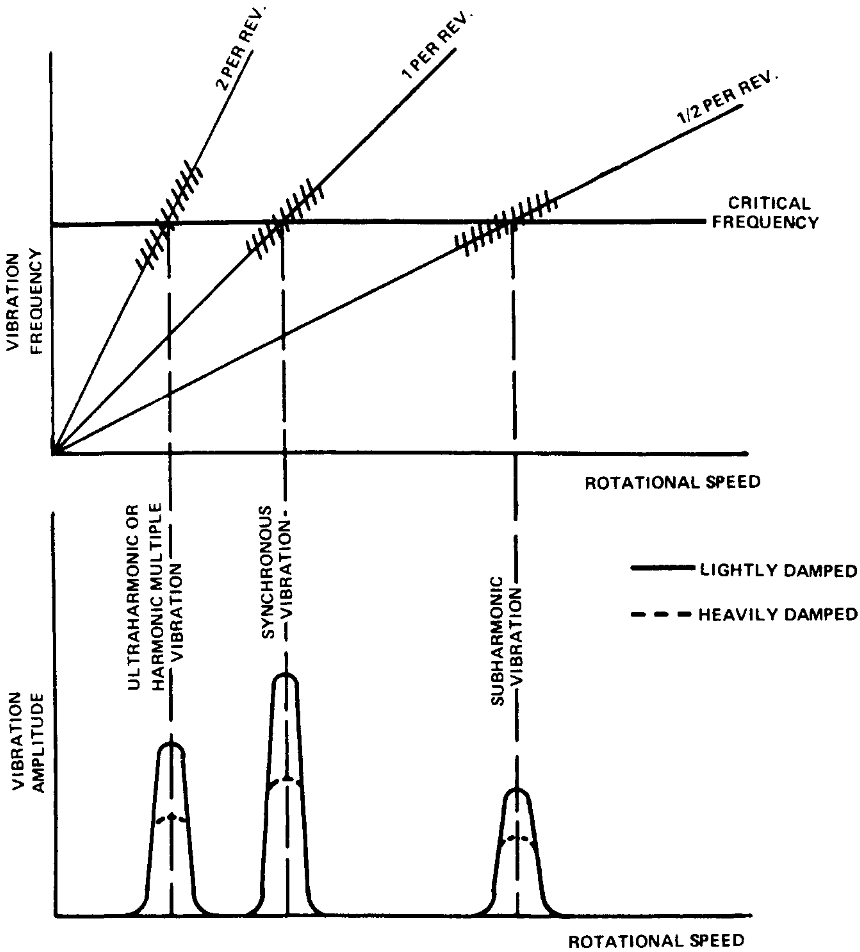

FIGURE 5.3*A* Attributes of forced vibration or resonance in rotating machinery.

first-order considerations of elastic-beam theory, the neutral axis of stress would be coincident with the neutral axis of strain. The net elastic restoring force would then be perpendicular to the neutral stress axis, i.e., parallel to and opposing the deflection. In actual fact, hysteresis, or internal friction, in the rotating shaft will cause a phase shift in the development of stress as the shaft fibers rotate around through peak strain to the neutral strain axis. The net effect is that the neutral stress axis is displaced in angle orientation from the neutral strain axis, and the resultant force is not parallel to the deflection. In particular, the resultant force has a tangential component *normal* to the deflection, which is the fundamental precondition for whirl. This tangential force component is in the direction of rotation and induces a *forward* whirling motion which increases centrifugal force on the deflected rotor, thereby increasing its deflection. As a consequence, induced stresses are increased, thereby increasing the whirl-inducing force component.

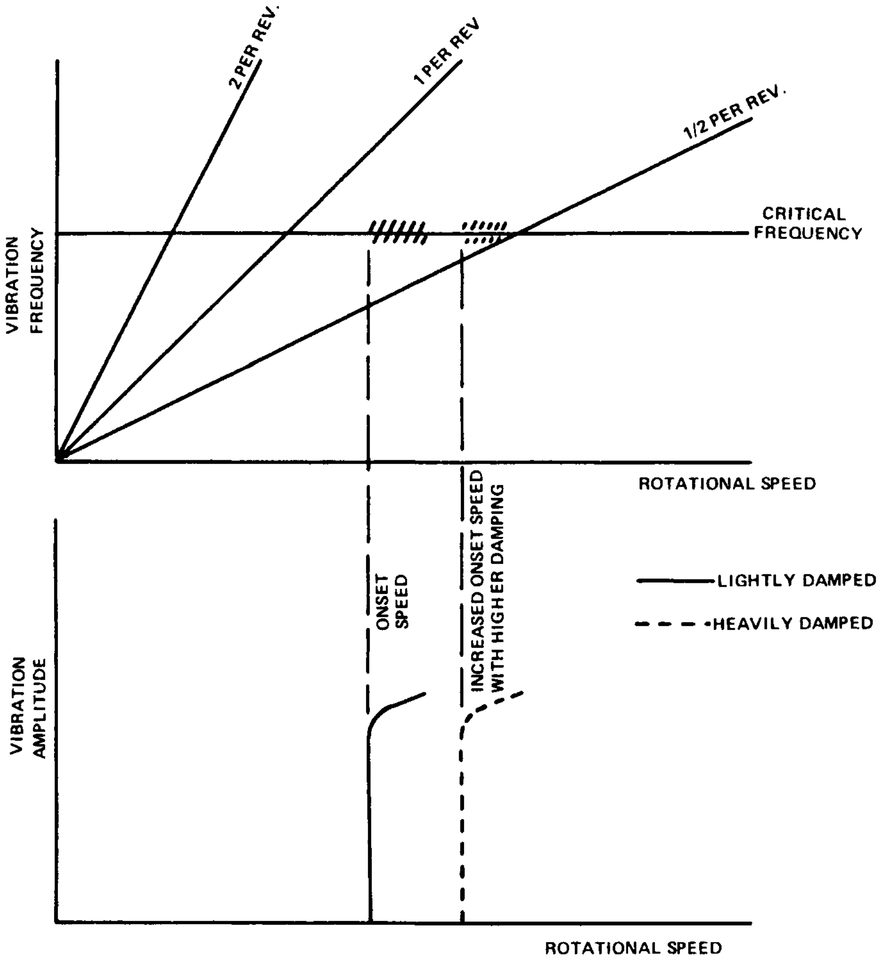

FIGURE 5.3*B* Attributes of whirling or whipping in rotating machinery.

Several surveys and contributions to the understanding of the phenomenon have been published in Refs. 9, 10, 11, and 12. It has generally been recognized that hysteretic whirl can occur only at rotational speeds above the first-shaft critical speed (the lower the hysteretic effect, the higher the attainable whirl-free operating rpm). It has been shown[13] that once whirl has started, the critical whirl speed that will be induced (from among the spectrum of criticals of any given shaft) will have a frequency approximately half the onset rpm.

A straightforward method for hysteretic whirl avoidance is that of limiting shafts to subcritical operation, but this is unnecessarily and undesirably restrictive. A more effective avoidance measure is to limit the hysteretic characteristic of the rotor. Most investigators (e.g., Ref. 5) have suggested that the essential hysteretic effect is

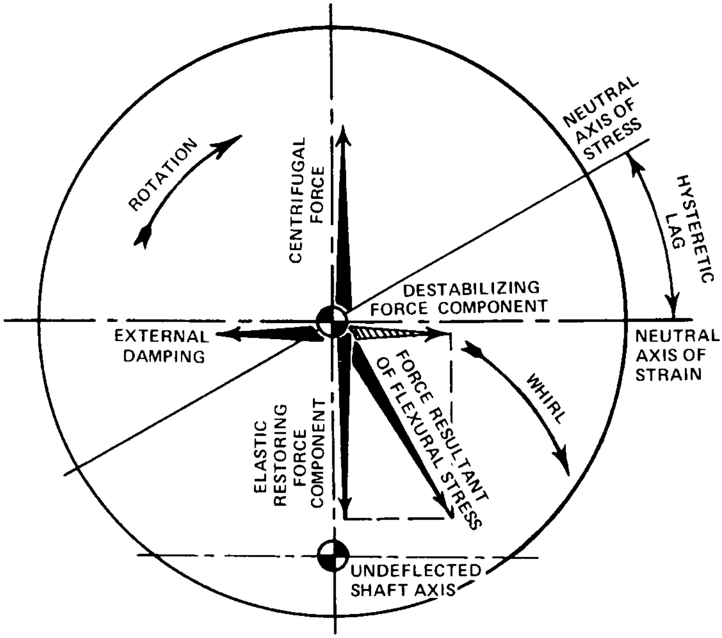

FIGURE 5.4 Hysteretic whirl.

caused by working at the interfaces of joints in a rotor rather than within the material of that rotor's components. Success in avoiding hysteretic whirl has been achieved by minimizing the number of separate elements, restricting the span of concentric rabbets and shrunk fitted parts, and providing secure lockup of assembled elements held together by tie bolts and other compression elements. Bearing-foundation characteristics also play a role in suppression of hysteretic whirl.[9]

WHIRL DUE TO FLUID TRAPPED IN ROTOR

There has always been a general awareness that high-speed centrifuges are subject to a special form of instability. It is now appreciated that the same self-excitation may be experienced more generally in high-speed rotating machinery where liquids (e.g., oil from bearing sumps, steam condensate, etc.) may be inadvertently trapped in the internal cavity of hollow rotors. The mechanism of instability is shown schematically in Fig. 5.5. For some nominal deflection of the rotor, the fluid is flung out radially in the direction of deflection. But the fluid does not remain in simple radial orientation. The spinning surface of the cavity drags the fluid (which has some finite viscosity) in the direction of rotation. This angle of advance results in the centrifugal force on the fluid having a component in the tangential direction in the direction of rotation. This force then is the basis of instability, since it induces forward whirl which increases the centrifugal force on the fluid and thereby increases the whirl-inducing force.

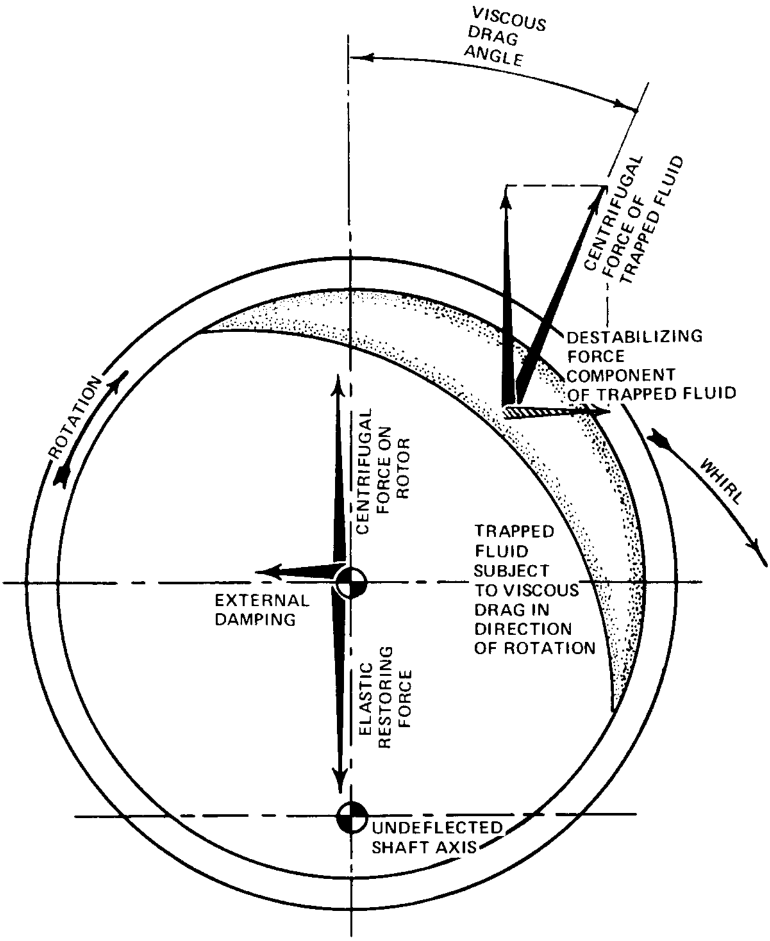

FIGURE 5.5 Whirl due to fluid trapped in rotor.

Contributions to the understanding of the phenomenon as well as a complete history of the phenomenon's study are available in Ref. 14. It has been shown[15] that onset speed for instability is always above critical rpm and below twice-critical rpm. Since the whirl is at the shaft's critical frequency, the ratio of whirl frequency to rpm will be in the range of 0.5 to 1.0.

Avoidance of this self-excitation can be accomplished by running shafting subcritically, although this is generally undesirable in centrifuge-type applications when further consideration is made of the role of trapped fluids as unbalance in forced vibration of rotating shafts (as described in Ref. 15). Where the trapped fluid is not fundamental to the machine's function, the appropriate avoidance measure, if the particular application permits, is to provide drain holes at the outermost radius of all hollow cavities where fluid might be trapped.

DRY FRICTION WHIP

As described in standard vibration texts (e.g., Ref. 16), dry friction whip is experienced when the surface of a rotating shaft comes in contact with an unlubricated stationary guide or shroud or stator system. This can occur in an unlubricated journal bearing or with loss of clearance in a hydrodynamic bearing or inadvertent closure and contact in the radial clearance of labyrinth seals or turbomachinery blading or power screws.[17]

The phenomenon may be understood with reference to Fig. 5.6. When radial contact is made between the surface of the rotating shaft and a static part, Coulomb friction will induce a tangential force on the rotor. Since the friction force is approximately proportional to the radial component of the contact force, we have the preconditions for instability. The tangential force induces a whirling motion which induces larger centrifugal force on the rotor, which in turn induces a large radial contact and hence larger whirl-inducing friction force.

It is interesting to note that this whirl system is *counter* to the shaft rotation direction (i.e., *backward* whirl). One may envision the whirling system as the rolling (accompanied by appreciable slipping) of the shaft in the stator system.

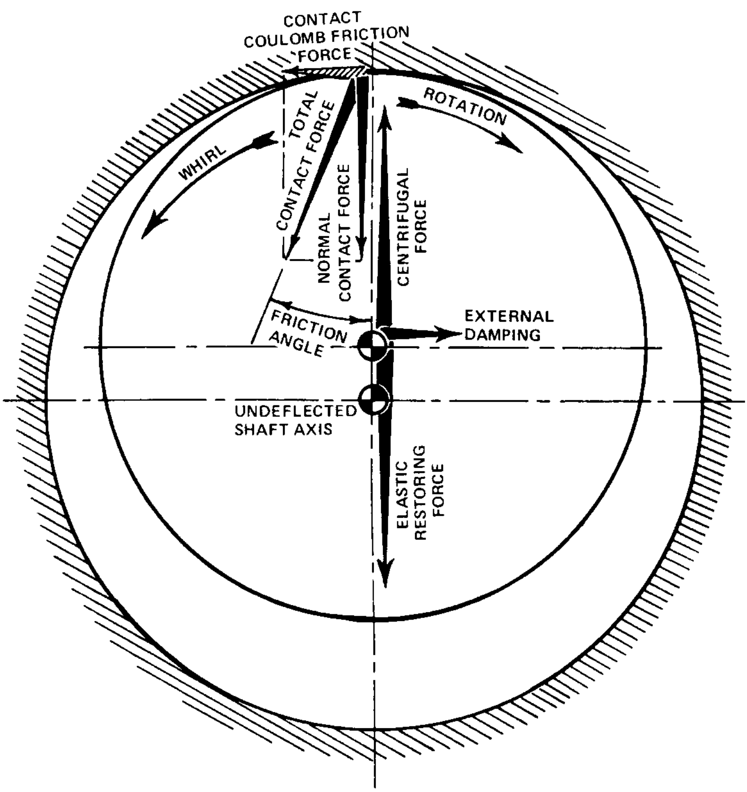

FIGURE 5.6 Dry friction whip.

The same situation can be produced by a thrust bearing where angular deflection is combined with lateral deflection.[18] If contact occurs on the same side of the disc as the virtual pivot point of the deflected disc, then backward whirl will result. Conversely, if contact occurs on the side of the disc opposite to the side where the virtual pivot point of the disc is located, then forward whirl will result.

It has been suggested (but not concluded)[19] that the whirling frequency is generally less than the critical speed.

The vibration is subject to various types of control. If contact between rotor and stator can be avoided or the contact area can be kept well lubricated, no whipping will occur. Where contact must be accommodated, and lubrication is not feasible, whipping may be avoided by providing abradability of the rotor or stator element to allow disengagement before whirl. When dry friction is considered in the context of the dynamics of the stator system in combination with that of the rotor system,[20] it is found that whirl can be inhibited if the independent natural frequencies of the rotor and stator are kept dissimilar, that is, a very stiff rotor should be designed with a very soft mounted stator element that may be subject to rubs. No first-order interdependence of whirl speed with rotational speed has been established.

FLUID BEARING WHIP

As described in experimental and analytic literature,[21] and in standard texts (e.g., Ref. 22), fluid bearing whip can be understood by referring to Fig. 5.7. Consider some nominal radial deflection of a shaft rotating in a fluid (gas- or liquid-) filled clearance. The entrained, viscous fluid will circulate with an average velocity of about half the shaft's surface speed. The bearing pressures developed in the fluid will not be symmetric about the radial deflection line. Because of viscous losses of the bearing fluid circulating through the close clearance, the pressure on the upstream side of the close clearance will be higher than that on the downstream side. Thus, the resultant bearing force will include a tangential force component in the direction of rotation which tends to induce *forward* whirl in the rotor. The tendency to instability is evident when this tangential force exceeds inherent stabilizing damping forces. When this happens, any induced whirl results in increased centrifugal forces; this, in turn, closes the clearance further and results in ever-increasing destabilizing tangential force. Detailed reviews of the phenomenon are available in Refs. 23 and 24.

These and other investigators have shown that to be unstable, shafting must rotate at an rpm equal to or greater than approximately twice the critical speed, so that one would expect the ratio of frequency to rpm to be equal to less than approximately 0.5.

The most obvious measure for avoiding fluid bearing whip is to restrict rotor maximum rpm to less than twice its lowest critical speed. Detailed geometric variations in the bearing runner design, such as grooving and tilt-pad configurations, have also been found effective in inhibiting instability. In extreme cases, use of rolling contact bearings instead of fluid film bearings may be advisable.

Various investigators (e.g., Ref. 25) have noted that fluid seals as well as fluid bearings are subject to this type of instability.

SEAL AND BLADE-TIP-CLEARANCE EFFECT IN TURBOMACHINERY

Axial-flow turbomachinery may be subject to an additional whirl-inducing effect by virtue of the influence of tip clearance on turbopump or compressor or turbine

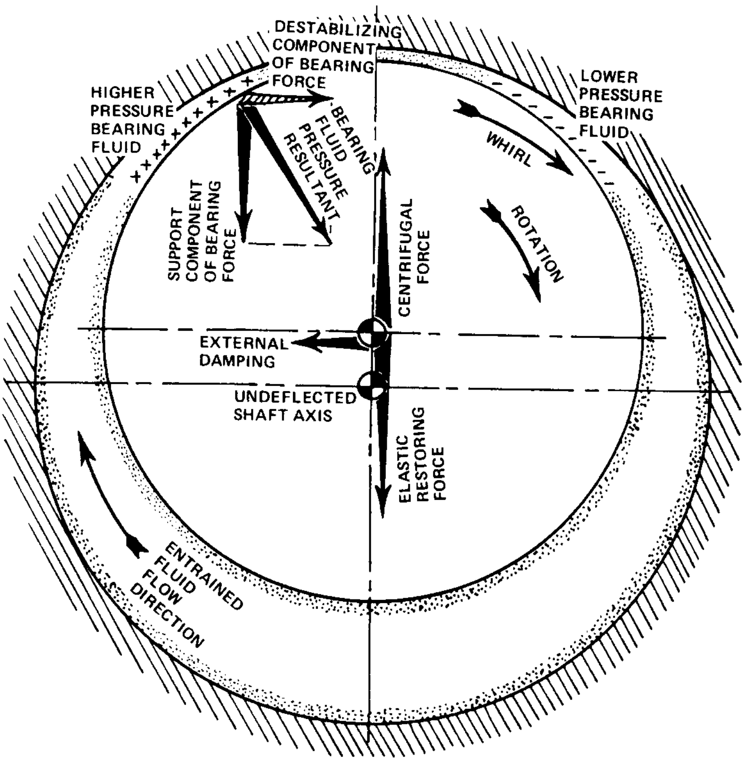

FIGURE 5.7 Fluid bearing whip.

efficiency.[26] As shown schematically in Fig. 5.8, some nominal radial deflection will close the radial clearance on one side of the turbomachinery component and open the clearance 180° away on the opposite side. We would expect the closer clearance zone to operate more efficiently than the open clearance zone. For a turbine, a greater work extraction and blade force level is achieved in the more efficient region for a given average pressure drop so that a resultant net tangential force is generated to induce whirl in the direction of rotor rotation (i.e., forward whirl). For an axial compressor, it has been found[27] that the magnitude and direction of the destabilizing forces are a very strong function of the operating point's proximity to the stall line. For operation close to the stall line, very large negative forces (i.e., inducing backward whirl) are generated. The magnitude of the destabilizing force declines sharply for lower operating lines, and stabilizes at a small positive value (i.e., making a small contribution to inducing forward whirl). In the case of radial-flow turbomachinery, it has been suggested[28] that destabilizing forces are exerted on an eccentric (i.e., dynamically deflected) impeller due to variations of loading of the diffuser vanes.

One text[29] describes several manifestations of this class of instability—in the thrust balance piston of a steam turbine; in the radial labyrinth seal of a radial-flow Ljungstrom counterrotating steam turbine; in the Kingsbury thrust bearing of a

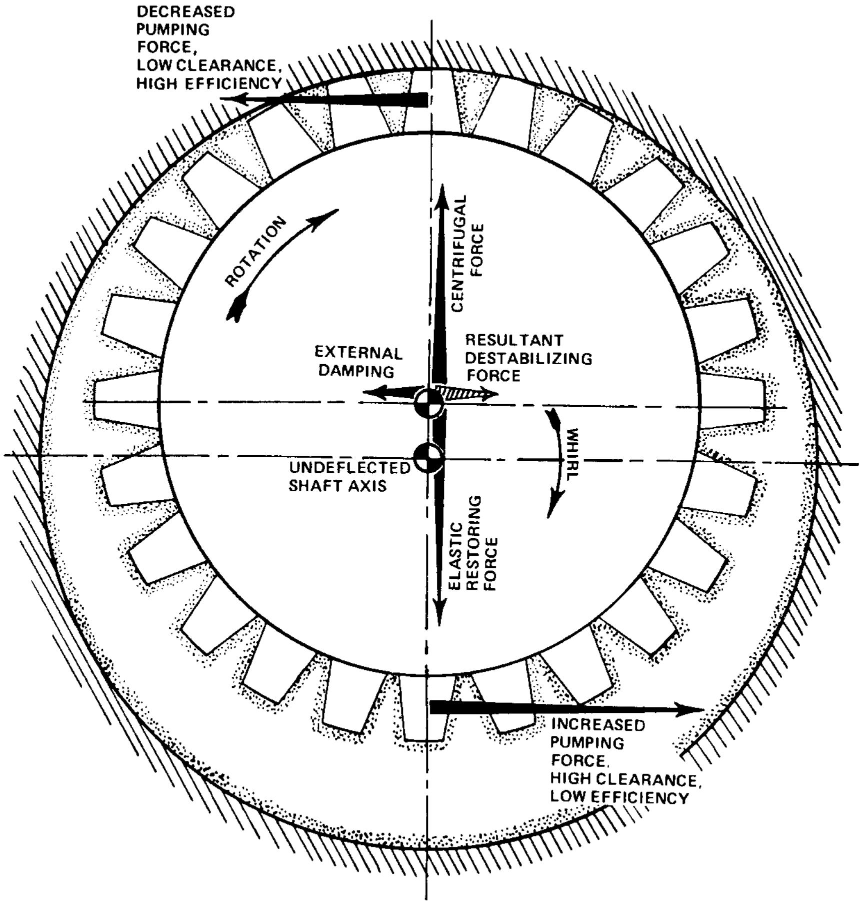

FIGURE 5.8 Turbomachinery tip clearance effect's contribution to whirl.

vertical-shaft hydraulic turbogenerator; and in the tip seals of a radial-inflow hydraulic Francis turbine.

A survey paper[3] includes a bibliography of several German papers on the subject from 1958 to 1969.

An analysis is available[30] dealing with the possibility of stimulating flexural vibrations in the seals themselves, although it is not clear if the solutions pertain to gross deflections of the entire rotor.

It is reasonable to expect that such destabilizing forces may at least contribute to instabilities experienced on high-powered turbomachines. If this mechanism were indeed a key contributor to instability, one would conjecture that very small or very large initial tip clearances would minimize the influence of tip clearance on the unit's performance and, hence, minimize the contribution to destabilizing forces.

PROPELLER AND TURBOMACHINERY WHIRL

Propeller whirl has been identified both analytically[31] and experimentally.[32] In this instance of shaft whirling, a small *angular* deflection of the shaft is hypothesized, as shown schematically in Fig. 5.9. The tilt in the propeller disc plane results at any instant at any blade in a small angle change between the propeller rotation velocity vector and the approach velocity vector associated with the aircraft's speed. The change in local relative velocity angle and magnitude seen by any blade results in an increment in its load magnitude and direction. The cumulative effect of these changes in load on all the blades results in a net moment whose vector has a significant component which is normal to and approximately proportional to the angular deflection vector. By analogy to the destabilizing cross-coupled *deflection* stiffness we noted in previously described instances of whirling and whipping, we have now identified the existence of a cross-

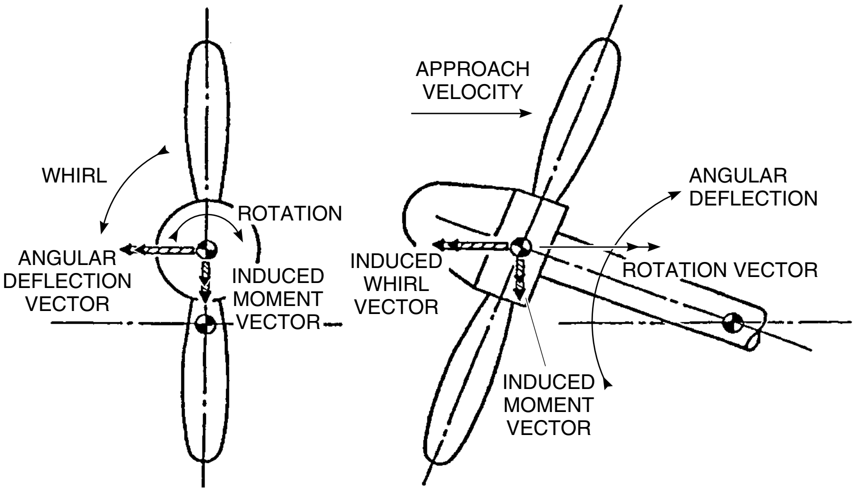

FIGURE 5.9 Propeller whirl.[2]

coupled destabilizing *moment* stiffness. At high airspeeds, the destabilizing moments can grow to the point where they may overcome viscous damping moments to cause destructive whirling of the entire system in a "conical" mode. This *propeller whirl* is generally found to be counter to the shaft rotation direction. It has been suggested[33] that equivalent stimulation is possible in turbomachinery. An attempt has been made[34] to generalize the analysis for axial-flow turbomachinery. Although it has been shown that this analysis is not accurate, the general deduction seems appropriate that forward whirl may also be possible if the virtual pivot point of the deflected rotor is forward of the rotor (i.e., on the side of the approaching fluid).

Instability is found to be load-sensitive in the sense of being a function of the velocity and density of the impinging flow. It is not thought to be sensitive to the torque level of the turbomachine since, for example, experimental work[32] was on an unloaded windmilling rotor. Corrective action is generally recognized to be stiffening the entire system and manipulating the effective pivot center of the whirling mode to inhibit angular motion of the propeller (or turbomachinery) disc as well as system damping.

PARAMETRIC INSTABILITY

ANALYTIC MODELING

There are systems in engineering and physics which are described by linear differential equations having periodic coefficients,

$$\frac{d^2y}{dz^2} + p(z)\frac{dy}{dz} + q(z)y = 0 \tag{5.13}$$

where $p(z)$ and $q(z)$ are periodic in z. These systems also may exhibit self-excited vibrations, but the stability of the system cannot be evaluated by finding the roots of a characteristic equation. A specialized form of this equation, which is representative of a variety of real physical problems in rotating machinery, is Mathieu's equation:

$$\frac{d^2f}{dz^2} + (a - 2q \cos 2z)f = 0 \tag{5.14}$$

Mathematical treatment and applications of Mathieu's equation are given in Refs. 35 and 36.

This general subcategory of self-excited vibrations is termed "parametric instability," since instability is induced by the effective periodic variation of the system's parameters (stiffness, inertia, natural frequency, etc.). Three particular instances of interest in the field of rotating machinery are

- Lateral instability due to asymmetric shafting and/or bearing characteristics
- Lateral instability due to pulsating torque
- Lateral instabilities due to pulsating longitudinal compression

LATERAL INSTABILITY DUE TO ASYMMETRIC SHAFTING

If a rotor or its stator contains sufficient levels of asymmetry in the flexibility associated with its two principal axes of flexure as illustrated in Fig. 5.10, self-excited vibration may take place. This phenomenon is completely independent of any unbalance, and independent of the forced vibrations associated with twice-per-revolution excitation of such shafting mounted horizontally in a gravitational field.

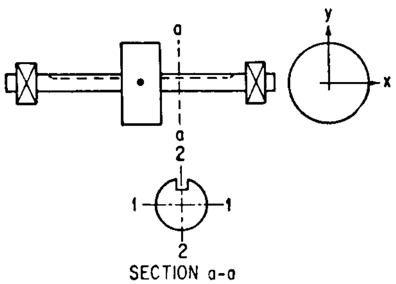

FIGURE 5.10 Shaft system possessing unequal rigidities, leading to a pair of coupled inhomogeneous Mathieu equations.

As described in standard vibration texts,[37] we find that presupposing a nominal whirl amplitude of the shaft at some whirl frequency, the rotation of the asymmetric shaft at an rpm different from the whirling speed will appear as periodic change in flexibility in the plane of the whirling shaft's radial deflection. This will result in an instability in certain specific ranges of rpm as a

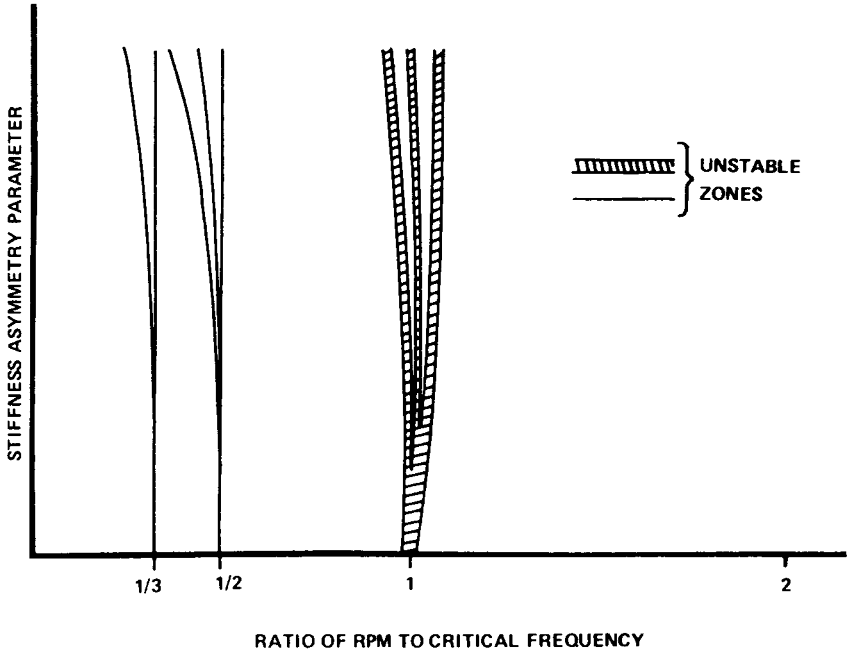

FIGURE 5.11 Instability regimes of rotor system induced by asymmetric stiffness (Ref. 39).

function of the degree of asymmetry. In general, instability is experienced when the rpm is approximately one-third and one-half the critical rpm and approximately equal to the critical rpm (where the critical rpm is defined with the average value of shaft stiffness), as in Fig. 5.11. The ratios of whirl frequency to rotational speed will then be approximately 3.0, 2.0, and 1.0. But with gross asymmetries, and with the additional complication of asymmetrical inertias with principal axes in arbitrary orientation to the shaft's principal axes' flexibility, no simple generalization is possible.

There is a considerable literature dealing with many aspects of the problem and substantial bibliographies.[38–40]

Stability is accomplished by minimizing shaft asymmetries and avoiding rpm ranges of instability.

LATERAL INSTABILITY DUE TO PULSATING TORQUE

Experimental confirmation[41] has been achieved that establishes the possibility of inducing first-order lateral instability in a rotor-disc system by the application of a proper combination of constant and pulsating torque. The application of torque to a shaft affects its natural frequency in lateral vibration so that the instability may also be characterized as "parametric." Analytic formulation and description of the phenomenon are available in Ref. 42 and in the bibliography of Ref. 3. The experimen-

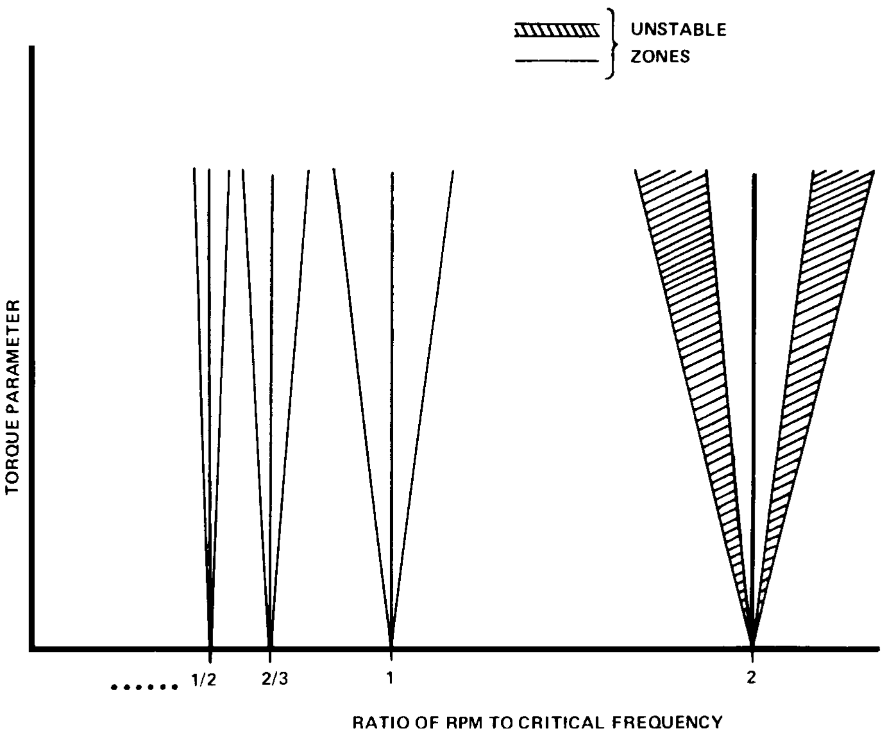

FIGURE 5.12 Instability regimes of rotor system induced by pulsating torque (Ref. 42).

tal work (Ref. 41) explored regions of shaft speed where the disc always whirled at the first critical speed of the rotor-disc system, regardless of the torsional forcing frequency or the rotor speed within the unstable region.

It therefore appears that combinations of ranges of steady and pulsating torque, which have been identified[40] as being sufficient to cause instability, should be avoided in the narrow-speed bands where instability is possible in the vicinity of twice the critical speed and lesser instabilities at 2/2, 2/3, 2/4, 2/5, . . . times the critical frequency, as in Fig. 5.12, implying frequency/speed ratios of approximately 0.5, 1.0, 1.5, 2.0, 2.5, ...

LATERAL INSTABILITY DUE TO PULSATING LONGITUDINAL LOADS

Longitudinal loads on a shaft which are of an order of magnitude of the buckling will tend to reduce the natural frequency of that lateral, flexural vibration of the shaft. Indeed, when the compressive buckling load is reached, the natural frequency goes to zero. Therefore pulsating longitudinal loads effectively cause a periodic variation in stiffness, and they are capable of inducing “parametric instability” in rotating as well as stationary shafts,[43] as noted in Fig. 5.13.

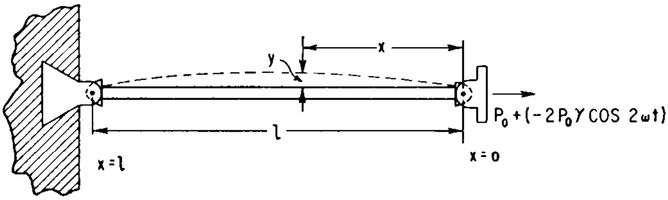

FIGURE 5.13 Long column with pinned ends. A periodic force is superimposed upon a constant axial pull. (*After McLachlan.*[43])

STICK-SLIP RUBS AND CHATTER

Mention is appropriate of another family of instability phenomena—stick-slip or chatter. Though the instability mechanism is associated with the dry friction contact force at the point of rubbing between a rotating shaft and a stationary element, it must not be confused with dry friction whip, previously discussed. In the case of stick-slip, as is described in standard texts (e.g., Ref. 44), the instability is caused by the irregular nature of the friction force developed at very low rubbing speeds.

At high velocities, the friction force is essentially independent of contact speed. But at very low contact speeds we encounter the phenomenon of "stiction," or breakaway friction, where higher levels of friction force are encountered, as in Fig. 5.14. Any periodic motion of the rotor's point of contact, superimposed on the basic relative contact velocity, will be self-excited. In effect, there is negative

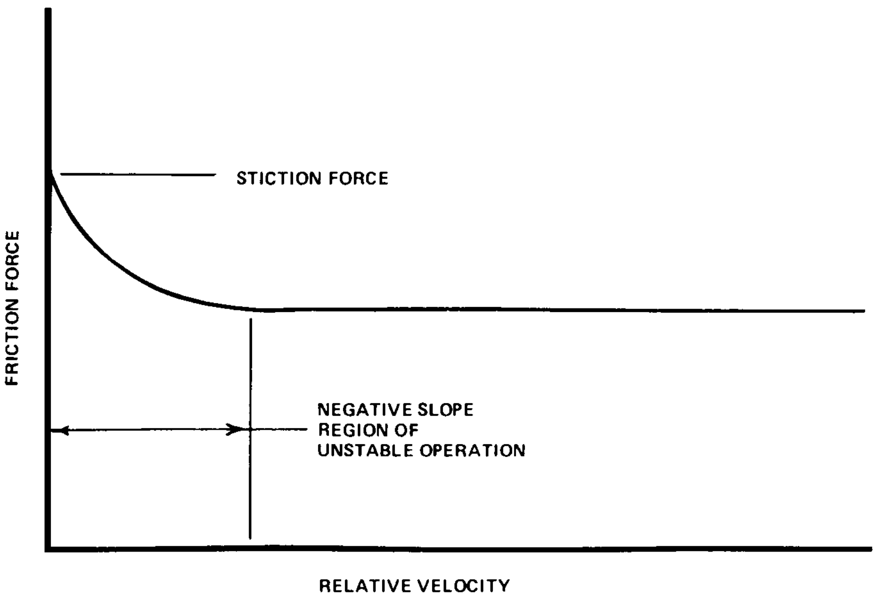

FIGURE 5.14 Dry friction characteristic giving rise to stick-slip rubs or chatter.

damping (as illustrated in Fig. 5.1*B*) since motion of the rotor's contact point in the direction of rotation will increase relative contact velocity and reduce stiction and the net force resisting motion. Rotor motion counter to the contact velocity will reduce relative velocity and increase friction force, again reinforcing the periodic motion. The ratio of vibration frequency to rotation speed will be much larger than unity.

While the vibration associated with stick-slip or chatter is often reported to be torsional, planar lateral vibrations can also occur. Surveys of the phenomenon are included in Refs. 45 and 46.

Measures for avoidance are similar to those prescribed for dry friction whip: avoid contact where feasible and lubricate the contact point where contact is essential to the function of the apparatus.

INSTABILITIES IN FORCED VIBRATIONS

In a middle ground between the generic categories of force vibrations and self-excited vibrations is the category of *instabilities in force vibrations.* These instabilities are characterized by forced vibration at a frequency equal to rotor rotation (generally induced by unbalance), but with the amplitude of that vibration being unsteady or unstable. Such unsteadiness or instability is induced by the interaction of the forced vibration on the mechanics of the system's response, or on the unbalance itself. Two manifestations of such instabilities and unsteadiness have been identified in the literature—bistable vibration and unstable imbalance.

BISTABLE VIBRATION

A classical model of one type of unstable motion is the "relaxation oscillator," or "*multivibrator.*" A system subject to relaxation oscillation has *two* fairly stable states, separated by a zone where stable operation is impossible.[47] Furthermore, in each of the stable states, a mechanism exists which will induce the system to drift toward the unstable state. The system will develop a periodic motion of the general form shown in Fig. 5.15.

An idealized formulation of this class of vibration with nonlinear damping is[48]

$$m\ddot{x} + c(x^2 - 1)\dot{x} + kx = 0 \tag{5.15}$$

When the deflection amplitude x is greater than +1 or less than −1, as in *A-B* and *C-D,* the damping coefficient is positive, and the system is stable, although presence of a spring system k will always tend to drag the mass to a smaller absolute deflection amplitude. When the deflection amplitude lies between −1 and +1, as in *B-C* or *D-A,* the damping coefficient is negative and the system will move violently until it stabilizes in one of the damped stable zones.

While such systems are common in electronic circuitry, they are rather rare in the field of rotating machinery. One instance has been observed[49] in a rotor system supported by rolling element bearings with finite internal clearance. In this situation, the effective stiffness of the rotor is small for small deflections (within the clearance) but large for large deflections (when full contact is made between the rollers and the rotor and stator). Such a nonlinearity in stiffness causes a "rightward leaning" peak in the response curve when the rotor is operating in the vicinity of its critical speed

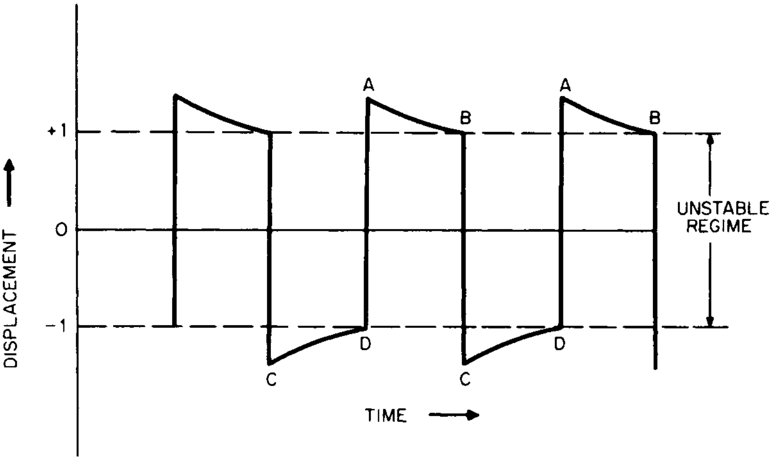

FIGURE 5.15 General form of relaxation oscillations.

and being stimulated by unbalance. In this region, two stable modes of operation are possible, as in Fig. 5.16. In region *A-B,* the rotor and stator are in solid contact through the rollers. In region *C-D,* the rotor is whirling within the clearance, out of contact. A jump in amplitude is experienced when operating from *B* to *C* or *D* to *A*. When operating at constant speed, either of the nominally stable states can drift toward instability by virtue of thermal effects on the rollers. When the rollers are unloaded, they will skid and heat up, thereby reducing the clearance. When the rollers are loaded, they will be cooled by lubrication and will tend to contract and increase clearance. In combination, these mechanisms are sufficient to cause a relaxation oscillation in the amplitude of the forced vibration.

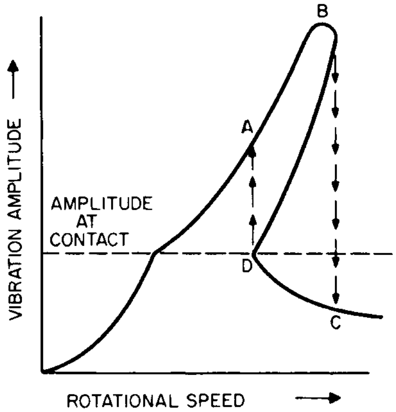

FIGURE 5.16 Response of a rotor, in bearings with (constant) internal clearance, to unbalance excitation in the vicinity of its critical speed.

The remedy for this type of self-excited vibration is to eliminate the precondition of skidding rollers by reducing bearing geometric clearance, by preloading the bearing, or by increasing the temperature of any recirculating lubricant.

UNSTABLE IMBALANCE

A standard text[50] describes the occurrence of unstable vibration of steam turbines where the rotor "would vibrate with the frequency of its rotation, obviously caused by unbalance, but the intensity of the vibration would vary periodically and extremely slowly." The instability in the vibration amplitude is attributable to thermal bowing of the shaft, which is caused by the heat input associated with rubbing at the rotor's deflected "high spot," or by the mass of accumulated steam condensate in the inside of a hollow rotor at the rotor's deflected high spot. In either case, there is basis for continuous variation of amplitude, since unbalance gives rise to deflection and the deflection is, in turn, a function of that imbalance.

The phenomenon is sometimes referred to as the Newkirk effect in reference to its early recorded experimental observation.[51] A manifestation of the phenomenon in a steam turbine has been diagnosed and reported in Ref. 52 and a bibliography is available in Ref. 53. An analytic study[54] shows the possibility of both spiraling, oscillating, and constant modes of amplitude variability.

TABLE 5.2 Diagnostic Table of Rotating Machinery Self-excited Vibrations

	R, characteristic ratio: whirl frequency/rpm	Whirl direction
Whirling or whipping:		
Hysteretic whirl	$R \approx 0.5$	Forward
Fluid trapped in rotor	$0.5 < R < 1.0$	Forward
Dry friction whip	No functional relationship; whirl frequency a function of coupled rotor-stator system; onset rpm is a function of rpm at contact	Backward—axial contact on disc side nearest virtual pivot; Forward—axial contact on disc side opposite to virtual pivot; Backward—radial contact
Fluid bearing whip	$R < 0.5$	Forward
Seal and blade-tip-clearance effect in turbomachinery	Load-dependent	Forward—blade tip clearance; Unspecified—for seal clearance
Propeller and turbomachinery whirl	Load-dependent	Backward—virtual pivot aft of rotor; Forward—virtual pivot front of rotor (where front is source of impinging flow)
Parametric instability:		
Asymmetric shafting	$R \approx 1.0, 2.0, 3.0, \ldots$	Unspecified
Pulsating torque	$R \approx 0.5, 1.0, 1.5, 2.0, \ldots$	Unspecified
Pulsating longitudinal load	A function of pulsating load frequency rather than rpm	Unspecified
Stick-slip rubs and chatter	$R << 1$	Essentially planar rather than whirl motion
Instabilities in forced vibrations:		
Bistable vibration	$R = 1$ with periodic square wave fluctuations in amplitude of frequency much lower than rotation rate	Forward
Unstable imbalance	$R = 1$ with slow variation in amplitude	Forward

IDENTIFICATION OF SELF-EXCITED VIBRATION

Even with the best of design practice and application of the most effective methods of avoidance, the conditions and mechanisms of self-excited vibrations in rotating machinery are so subtle and pervasive that incidents continue to occur, and the major task for the vibrations engineer is diagnosis and correction.

Figure 5.3*B* suggests the forms for display of experimental data to perceive the patterns characteristic of whirling or whipping, so as to distinguish it from forced vibration, Fig. 5.3*A*. Table 5.2 summarizes particular quantitative measurements that can be made to distinguish between the various types of whirling and whipping, and other types of self-excited vibrations. The table includes the characteristic ratio of whirl speed to rotation speed at onset of vibration, and the direction of whirl with respect to the rotor rotation. The latter parameter can generally be sensed by noting the phase relation between two stationary vibration pickups mounted at 90° to one another at similar radial locations in a plane normal to the rotor's axis of rotation. Table 5.1 and specific prescriptions in the foregoing text and references suggest corrective action based on these diagnoses. Reference 55 gives additional description of corrective actions.

REFERENCES

1. Den Hartog, J. P.: "Mechanical Vibrations," 4th ed., p. 346, McGraw-Hill Book Company, Inc., New York, 1956.
2. Ehrich, F. F.: "Handbook of Rotordynamics," 2d ed., pp. 1.72–1.106, Krieger Publishing Co., Malabar, Fla., 1999.
3. Kramer, E.: "Instabilities of Rotating Shafts," *Proc. Conf. Vib. Rotating Systems, Inst. Mech. Eng., London,* February 1972.
4. Vance, J. M.: "High Speed Rotor Dynamics—Assessment of Current Technology for Small Turboshaft Engines," USAAMRDL-TR-74-66, Ft. Eustis, Va., July 1974.
5. Newkirk, B. L.: *Gen. Elec. Rev.,* **27**:169 (1924).
6. Kimball, A. L.: *Gen. Elec. Rev.,* **17**:244 (1924).
7. Ref. 1, pp. 295–296.
8. Ehrich, F.: *ASME DE,* **18**:1, September 1989.
9. Gunter, E. J.: "Dynamic Stability of Rotor-Bearing Systems," NASA SP-113, chap. 4, 1966.
10. Bolatin, V. V.: "Non-Conservative Problems of the Theory of Elastic Stability," Pergamon Press, New York, 1964.
11. Bentley, D. E.: "The Re-Excitation of Balance Resonance Regions by Internal Friction," *ASME Paper* 72-PET-49, September 1972.
12. Vance, J. M., and J. Lee: "Stability of High Speed Rotors with Internal Friction," *ASME Paper* 73-DET-127, September 1973.
13. Ehrich, F. F.: *J. Appl. Mech.,* (E) **31**(2):279 (1964).
14. Wolf, J. A.: "Whirl Dynamics of a Rotor Partially Filled with Liquids," *ASME Paper* 68-WA/APM-25, December 1968.
15. Ehrich, F. F.: *J. Eng. Ind.,* (B) **89**(4):806 (1967).
16. Ref. 1, pp. 292–293.
17. Sapetta, L. P., and R. J. Harker: "Whirl of Power Screws Excited by Boundary Lubrication at the Interface," *ASME Paper* 67-Vibr-37, March 1967.

18. Ref. 1, pp. 293–295.
19. Begg, I. C.: "Friction Induced Rotor Whirl—A Study in Stability," *ASME Paper* 73-DET-10, September 1973.
20. Ehrich, F. F.: "The Dynamic Stability of Rotor/Stator Radial Rubs in Rotating Machinery," *ASME Paper* 69-Vibr-56, April 1969.
21. Newkirk, B. L., and H. D. Taylor: *Gen. Elec. Rev.,* **28**:559–568 (1925).
22. Ref. 1, pp. 297–298.
23. Ref. 9, chap. 5.
24. Pinkus, O., and B. Sternlight: "Theory of Hydrodynamic Lubrication," chap. 8, McGraw-Hill Book Company, Inc., New York, 1961.
25. Black, H. F., and D. N. Jenssen: "Effects of High Pressure Seal Rings on Pump Rotor Vibrations," *ASME Paper* 71-WA/FE-38, December 1971.
26. Alford, J. S.: *J. Eng. Power,* **87**(4):333, October 1965.
27. Ehrich, F F. et al.: "Unsteady Flow and Whirl-Inducing Forces in Axial Flow Compressors. Part II—Analysis," *ASME Paper* 2000-GT-0566, May 2000.
28. Black, H. F.: "Calculation of Forced Whirling and Stability of Centrifugal Pump Rotor Systems," *ASME Paper* 73-DET-131, September 1973.
29. Ref. 1, pp. 317–321.
30. Ehrich, F. F.: *Trans. ASME,* (A) **90**(4):369 (1968).
31. Taylor, E. S., and K. A. Browne: *J. Aeronaut. Sci.,* **6**(2):43–49 (1938).
32. Houbolt, J. C., and W. H. Reed: "Propeller Nacelle Whirl Flutter," *I.A.S. Paper* 61–34, January 1961.
33. Trent, R., and W. R. Lull: "Design for Control of Dynamic Behavior of Rotating Machinery," *ASME Paper* 72-DE-39, May 1972.
34. Ehrich, F. F.: "An Aeroelastic Whirl Phenomenon in Turbomachinery Rotors," *ASME Paper* 73-DET-97, September 1973.
35. Floquet, G.: *Ann. l'école normale supérieure,* **12**:47 (1883).
36. McLachlan, N. W.: "Theory and Applications of Mathieu Functions," p. 40, Oxford University Press, New York, 1947.
37. Ref. 1, pp. 336–339.
38. Brosens, P. J., and H. S. Crandall: *J. Appl. Mech.,* **83**(4):567 (1961).
39. Messal, E. E., and R. J. Bronthon: "Subharmonic Rotor Instability Due to Elastic Asymmetry," *ASME Paper* 71-Vibr-57, September 1971.
40. Arnold, R. C., and E. E. Haft: "Stability of an Unsymmetrical Rotating Cantilever Shaft Carrying an Unsymmetrical Rotor," *ASME Paper* 71-Vibr-57, September 1971.
41. Eshleman, R. L., and R. A. Eubanks: "Effects of Axial Torque on Rotor Response: An Experimental Investigation," *ASME Paper* 70-WA/DE-14, December 1970.
42. Wehrli, V. C.: "Uber Kritische Drehzahlen unter Pulsierender Torsion," *Ing. Arch.,* **33**:73–84 (1963).
43. Ref. 36, p. 292.
44. Ref. 1, p. 290.
45. Conn, H.: *Tool Eng.,* **45**:61–65 (1960).
46. Sadowy, M.: *Tool Eng.,* **43**:99–103 (1959).
47. Ref. 1, pp. 365–368.
48. Van der Pol, B.: *Phil. Mag.,* **2**:978 (1926).
49. Ehrich, F. F.: "Bi-Stable Vibration of Rotors in Bearing Clearance," *ASME Paper* 65-WA/MD-1, November 1965.

50. Ref. 1, pp. 245–246.
51. Newkirk, B. L.: "Shaft Rubbing," *Mech. Eng.,* **48**:830 (1926).
52. Kroon, R. P., and W. A. Williams: "Spiral Vibration of Rotating Machinery," *5th Int. Congr. Appl. Mech.*, p. 712, John Wiley & Sons, Inc., New York, 1939.
53. Dimarogonas, A. D., and G. N. Sander: *Wear,* **14**(3):153 (1969).
54. Dimarogonas, A. D.: "Newkirk Effect: Thermally Induced Dynamic Instability of High-Speed Rotors," *ASME Paper* 73-GT-26, April 1973.
55. Ehrich, F. and D. Childs: *Mech. Eng.,* **106**(5):66 (1984).