CHAPTER 18

CALIBRATION OF PICKUPS

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INTRODUCTION

This chapter describes various methods of calibrating shock and vibration transducers, commonly called *vibration pickups.* The objective of calibrating a transducer is to determine its sensitivity or calibration factor, as defined below. The chapter is divided into three major parts which discuss comparison methods of calibration, absolute methods of calibration, and calibration methods which employ high acceleration and shock. Field calibration techniques are described in Chap. 15.

PICKUP SENSITIVITY, CALIBRATION FACTOR, AND FREQUENCY RESPONSE

As defined in Chap. 12, the *sensitivity* of a vibration pickup is the ratio of electrical output to mechanical input applied along a specified axis.[1–3] The sensitivity of all pickups is a function of frequency, containing both amplitude and phase information, as illustrated in Fig. 18.1, and therefore is usually a complex quantity. If the sensitivity is practically independent of frequency over a range of frequencies, the value of its magnitude is referred to as the *calibration factor* for that range, but it is specified at a discrete frequency. The phase component of the sensitivity function likewise has a constant value in that range of frequencies, usually equal to zero or 180°, but it may also be proportional to frequency, as explained in Chap. 12.

The *frequency response* of a pickup is shown by plotting the magnitude and phase components of its sensitivity versus frequency. This information is usually presented relative to the value of sensitivity at a reference frequency within the flat range. A preferred frequency, internationally accepted, is 160 Hz.

Displacements are usually expressed as single-amplitude (peak) or double-amplitude (peak-to-peak) values, while velocities are usually expressed as peak, root-mean-square (rms), or average values. Acceleration and force generally are expressed as peak or rms values. The electrical output of the vibration pickup may be expressed as peak, rms, or average value. The sensitivity magnitude or calibration factor are commonly stated in similarly expressed values, i.e., the numerator and

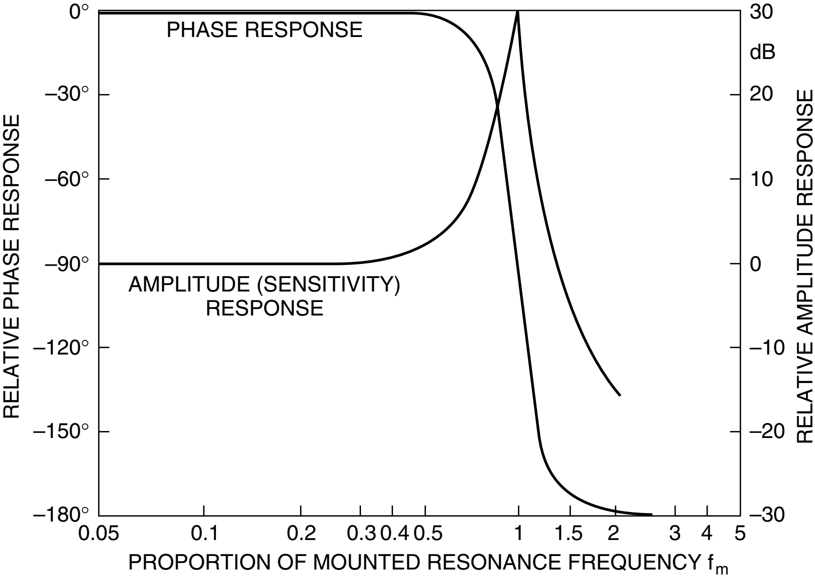


FIGURE 18.1 Pickup amplitude and phase response as functions of frequency. (*After M. Serridge and T.R. Licht.*[4])

denominator are both peak or both rms values. Examples of typical sensitivity specifications for an accelerometer: 2 pC per m/sec^2, 10 millivolts/m/sec^2, 5 milliV/g (where C is the symbol for coulomb, V is the symbol for volt, and g is the acceleration of gravity). For some special applications it may be desirable to express the sensitivity in mixed values, such as rms voltage per peak acceleration.

CALIBRATION TRACEABILITY

In calibrating an instrument, one measures the instrument's error relative to a reference which is traceable to the national standard of a country. A calibration is said to be *traceable*[5] to a national or international standard if it can be related to the standard through an unbroken chain of comparisons—all having stated uncertainties. In the U.S.A., for example, national vibration standards are maintained at the National Institute of Standards and Technology in Gaithersburg, Maryland. A number of other national metrology laboratories having known capabilities for maintaining national vibration standards are listed in Table 18.1. Countries whose national laboratories do not provide a national vibration standard may belong to a regional international association, such as NORAMET (North American Metrology Cooperation), EUROMET (European Metrology Cooperation), or OIML (Organization for Legal Metrology) that can assist transducer manufacturers in setting up steps necessary for establishing traceability to a national standard.

Vendors of transducers must be able to show that calibrations of their instruments are traceable to a national standard by means of calibration reports stating the value(s) of sensitivity, measurement uncertainty, environmental conditions, and

TABLE 18.1 National Standards Laboratories Responsible for the Calibration of Vibration Pickups

Institution	Laboratory	Location	Country
CSIRO	Natl. Measurement Laboratory	Lindfield	Australia
INMETRO	Laboratório de Vibrações	Rio de Janeiro	Brazil
NRC-CNRC	Inst. Natl. Meas. Stds.	Ottawa	Canada
NIM	Vibrations Laboratory	Beijing	China
CMU	Primary Stds. of Kinematics	Prague	Czech Repub.
B&K	Danish Prim. Lab. for Acoustics	Naerum	Denmark
BNM	CEA/CESTA	Belin-Beliet	France
PTB	Fachlabor. Beschleunigung	Braunschweig	Germany
IMGC	Sezione Meccanica	Torino	Italy
NRLM	Mechanical Metrology Dept.	Tsukuba	Japan
KSRI	Division of Appl. Metrology	Daedeog Danji	Rep. of Korea
NMC	SIRIM	Berhad	Malaysia
CENAM	Div. Acustica y Vibraciones	Queretaro	Mexico
DSIR	Measurement Stds. Laboratory	Lower Hutt	New Zealand
VNIIM	Mendeleyev Inst. for Metrology	St. Petersburg	Russia
ITRI	Center for Measurement Stds.	Hsinchu	Taiwan
NIST	Manufacturing Metrology Div.	Gaithersburg	U.S.A.

identification of the standard(s) used in the calibration procedure. Depending on the application, there may be one or more links to the national standard.

Primary and Secondary Standards. *Primary standards,*[6,7] maintained at national metrology institutes, are derived from *absolute measurements*[8] of the transducer's sensitivity, measured in terms of seven basic units. For example, the absolute measurement of "speed" must be made in terms of measurements of distance or distance and time, not by a speedometer. Thus the word *absolute* implies nothing about precision or accuracy. An example of a laboratory setup for the calibration of primary-standard accelerometers, derived from absolute measurements, is shown in Fig. 18.2.[9] A vibration exciter generates sinusoidal motion which is measured by a

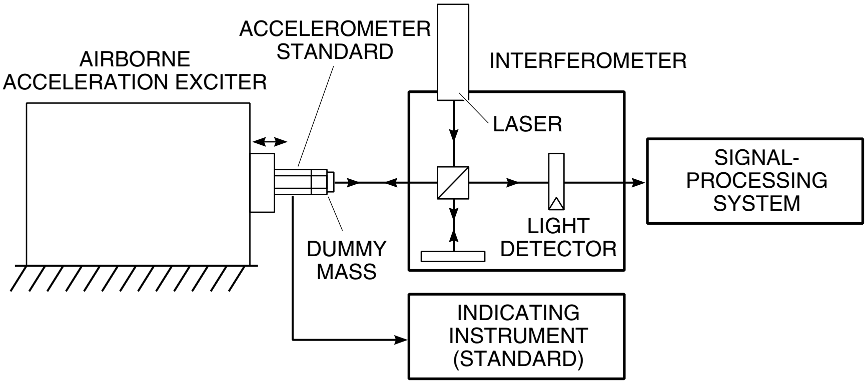


FIGURE 18.2 Primary (absolute) calibration of an accelerometer standard using laser interferometry. (*After von Martens.*[9])

Michelson interferometer (described later in this chapter). The vibration is applied to the base of the standard accelerometer whose output is measured. A dummy mass, mounted on its top surface, simulates the conditions when this standard accelerometer is used to calibrate a *secondary standard*[10] accelerometer by the comparison method described in the next section. *Secondary standards* (also referred to as *transfer standards* or *working standards*) are maintained at various government laboratories and industrial laboratories. A secondary standard accelerometer may be calibrated either from absolute measurements or from a comparison with a primary standard accelerometer. Such secondary standards are usually used for purposes of comparisons of calibrations between laboratories or for checking production and field units.

COMPARISON METHODS OF CALIBRATION

A rapid and convenient method of measuring the sensitivity of a vibration pickup to be tested is by direct comparison of the pickup's electrical output with that of a second pickup (used as a "reference" standard) that has been calibrated by one of the methods described in this chapter. A comparison method is used in most shock and vibration laboratories, which periodically send their standards to a primary standards laboratory for recalibration. This procedure should be followed on a yearly basis in order to establish a history of the accuracy and quality of its reference standard pickup.

In this method of calibration the two pickups usually are mounted back-to-back on a vibration exciter as shown in Fig. 18.3. It is essential to ensure that each pickup experiences the same motion. Any angular rotation of the table should be small to avoid any difference in excitation between the two pickup locations. The error due to rotation may be reduced by carefully locating the pickups firmly on opposite faces with the center-of-gravity of the pickups located at the center of the table. Relative differences in pickup excitation may be observed by reversing the pickup locations and observing if the voltage ratio is the same for both positions.

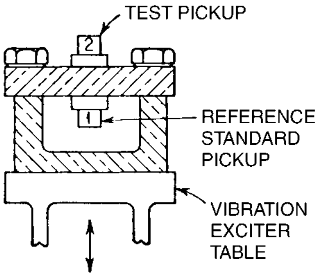


FIGURE 18.3 Comparison method of calibration: Pickup 2 is calibrated against Pickup 1 (the reference standard). The two pickups may be excited by any of the means described in this chapter. (*After ANSI Standard S2.2-1959, R 1997.*[1])

Calibration by the comparison method is limited to the range of frequencies and amplitudes for which the reference standard pickup has been previously calibrated. If both pickups are linear, the sensitivity of the test pickup can be calculated in both magnitude and phase from

$$S_t = \frac{e_t}{e_r} S_r \tag{18.1}$$

where S_t = sensitivity of test pickup
S_r = sensitivity of reference standard pickup
e_t = output voltage from test pickup
e_r = output voltage from reference standard pickup

Several calibration methods described below are variations on the implementation of Eq. (18.1); they differ mainly in the manner of vibration excitation.

USING THE COMPARISON METHOD

A simple and convenient way of performing a comparison calibration is to fix the test pickup and reference standard pickup so they experience identical motion, as in Fig. 18.3. Then, set the frequency of the vibration exciter at a desired value, adjust the amplitude of vibration of the vibration exciter to a desired value, and then compare the electrical outputs of the pickups. Often, instead of making a comparison at a fixed frequency, a graphical plot of the sensitivity versus frequency is obtained by incorporating a swept-frequency signal generator in the calibration system.

RANDOM-EXCITATION-TRANSFER-FUNCTION METHOD

The use of random-vibration-excitation and transfer-function analysis techniques can provide quick and accurate comparison calibrations.[11] The reference standard pickup and the test pickup are mounted back-to-back on a suitable vibration exciter. Their outputs are usually fed into a spectrum analyzer through a pair of low-pass (antialiasing) filters. The bandwidth of the random signal which drives the exciter is determined by settings of the analyzer.

This method provides a nearly continuous calibration over a desired frequency spectrum, with the resulting sensitivity function having both amplitude and phase information. Since purely sinusoidal motion is not a requirement as in the other calibration methods, this lessens the requirements for the power amplifier and exciter to maintain low values of harmonic distortion. A very useful measure of process quality is obtained by computing the input-output coherence function, which requires knowledge of the input and output power spectra, the cross-power spectrum, and the transfer function.

CALIBRATION BY ABSOLUTE METHODS

RECIPROCITY METHOD

The reciprocity calibration method is an absolute means for calibrating vibration exciters that have a velocity coil or reference accelerometer. This method relates the pickup sensitivity to measurements of voltage ratio, resistance, frequency, and mass. For this method to be applicable, it is necessary that the vibration exciter system be linear (e.g., that the displacement, velocity, acceleration, and current in the driver coil each increase linearly with force and driver-coil voltage). The reciprocity method is used chiefly with electrodynamic exciters[12] but also with piezoelectric vibration exciters.[13]

The reciprocity method generally is applied only under controlled laboratory conditions. Many precautions must be taken, and the process is usually time-consuming. Several variations of the basic approach have been developed at national standards laboratories.[14,15] The method described here has been used at the

National Institute of Standards and Technology.[16–19] The method consists of two laboratory experiments:

1. The measurement of the transfer admittance between the exciter's driver coil and the attached velocity coil or accelerometer.
2. The measurement of the voltage ratio of the open-circuit velocity coil or accelerometer and the driving coil while the exciter is driven by a second external exciter. The use of a piezoelectric accelerometer is assumed here. The electrical connections for the transfer admittance and voltage ratio measurements are shown in Fig. 18.4.

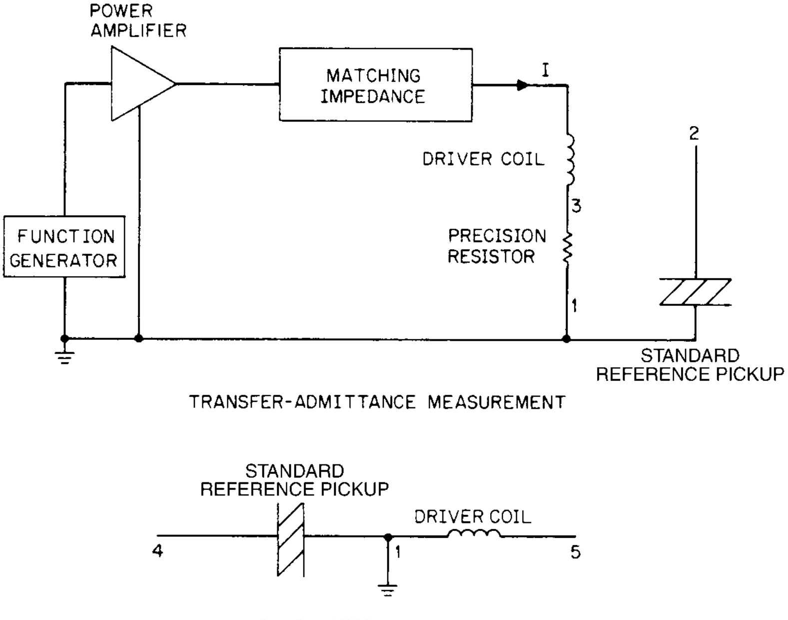


FIGURE 18.4 Transfer-admittance and voltage-ratio-measurement circuit connections for the reciprocity calibration method in the Levy-Bouche realization.[14]

The relationship defining the transfer admittance is

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{e}_{12}} \tag{18.2}$$

where $\mathbf{Y}$ = transfer admittance
$\mathbf{e}_{12}$ = voltage generated in standard accelerometer and amplifier
$\mathbf{I}$ = current in driver coil

and the bold letters denote phasor (complex) quantities. The current is determined by measuring the voltage drop across a standard resistor. The phase, ψ_Y, of $\mathbf{Y}$ is measured with a phase meter having an uncertainty of $\pm 0.1°$ or better. Transfer admittance measurements are made with a series of masses attached, one at a time, to the table of the exciter. Also, a zero-load transfer admittance measurement is made before and after attaching each mass. This zero-load measurement is denoted by $\mathbf{Y}_0$. Using the measured values of $\mathbf{Y}$ and $\mathbf{Y}_0$, graphs of the real and imaginary values of the ratio

$$\mathbf{T}_n = \frac{M_n}{\mathbf{Y} - \mathbf{Y}_0} \tag{18.3}$$

are plotted versus M_n for each frequency, where M_n is the value of the mass attached to the table. The zero intercepts, J_i and J_r, of the resulting nominally straight lines and their slopes, Q_i and Q_r, are computed by a weighted least-squares method.[12] The values of $\mathbf{Y}_0$ used in the calculations are obtained by averaging the values of the $\mathbf{Y}_0$ measurements before and after each measurement of $\mathbf{Y}$ using different masses. These computed values are used in determining the sensitivity of the standard.

The ratio of two voltages, measured while the exciter is driven with an external exciter, is given by

$$\mathbf{R} = \frac{\mathbf{e}_{14}}{\mathbf{e}_{15}} \tag{18.4}$$

where $\mathbf{e}_{14}$ = voltage generated in standard accelerometer and amplifier, and $\mathbf{e}_{15}$ = open-circuit voltage in driving coil.

After $\mathbf{R}$, J_r, J_i, Q_r, and Q_i have been determined for a number of frequencies, f, the sensitivity of the exciter is calculated from the following relationship:[12]

$$\mathbf{S} = \left[\frac{\mathbf{RJ}}{j2\pi f}\right]^{1/2}\left[1 + \frac{\mathbf{MQ}}{\mathbf{J}}\right] \tag{18.5}$$

where j = unit imaginary vector
$\mathbf{J} = J_r + jJ_i$
$\mathbf{Q} = Q_r + jQ_i$
$\mathbf{M} = \dfrac{\mathbf{J}(\mathbf{Y} - \mathbf{Y}_0)}{1 - \mathbf{Q}(\mathbf{Y} - \mathbf{Y}_0)}$

The sensitivity of the exciter is, therefore, determined from the measured quantities **Q, J, T,** and f and from the masses M_n which are attached to the exciter table. The sensitivity as computed from Eq. (18.5) has the units of volts per meter per second squared if the values of the measurements are in the SI system. If the masses M_n are not in kilograms, appropriate conversion factors must be applied to the quantities **J, Q,** and **M.** A commonly used engineering formula,[12] with the mass expressed in pounds and the sensitivity in millivolts per g, is

$$\mathbf{S} = 2635\left[\frac{\mathbf{RJ}}{jf}\right]^{1/2} \tag{18.6}$$

which also assumes that $MQ/J \ll 1$, a condition usually satisfied in practice but which should be verified experimentally. The use of a computer greatly facilitates the application of the reciprocity calibration process.

Assuming the errors to be uncorrelated, a typical estimate of uncertainty expected from a reciprocity calibration method is ±0.5 percent in the frequency range 100 to 1000 Hz. This is a twofold improvement over the earlier systems.[18,19] The critical component in a reciprocity-based calibration system is the vibration exciter. Electrodynamic exciters utilizing an air bearing are generally superior to other types, for this application.

CALIBRATION USING THE EARTH'S GRAVITATIONAL FIELD

The earth's gravitational field provides a convenient means of applying a small constant acceleration to a vibration pickup. It is particularly useful in calibrating accelerometers whose frequency range extends down to 0 Hz. A $2g$ change in acceleration may be obtained by first aligning the sensitivity axis of the transducer in one direction of the earth's gravitational field, as shown in Fig. 15.5*A*, and then inverting it so the sensitive axis is aligned in the opposite direction. This method of calibration is particularly useful in field work.

Accelerations in the 1–$10g$ range can be generated by several methods, which have been largely replaced by the *structural gravimetric calibrator* described in the next section. In the *tilting-support calibrator*[1] the pickup is fastened to one end of an arm attached to a platform. The arm may be set at any angle between 0° and 180° relative to the vertical, thus yielding different values of acceleration. The *pendulum calibrator*[1] generates transient accelerations as great as $10g$ for a duration of about one second. In the *rotating-table calibrator*[20,21] the disk on which the test pickup is mounted rotates at a uniform angular rate about a horizontal axis in such a way that the pickup's axis of sensitivity rotates in a vertical plane. This method makes it possible to obtain both the static and dynamic responses of the pickup in the same test setup.

Structural-Gravimetric Calibration. This technique provides a simple, robust, and low-cost method of calibrating pickups.[22,23] The structural-gravimetric-calibration (SGC) method is applicable over a broad frequency range because it relies on a quartz force transducer as the reference pickup and the behavior of the simplest of structures (i.e., a mass behaving as a rigid body). It references the acceleration of gravity and allows the measurement of sensitivity magnitude and phase. The results of calibration using this method agree within a fraction of 1 percent with those obtained by laser interferometry and reciprocity methods. The following steps are the procedure of SGC method:

Step 1. *Determine the acceleration sensitivity* S_r *of the reference force transducer.* Mount the reference force transducer, reference mass (can be built-in or external), and the test pickup to be calibrated on a drop-test fixture, as shown in Fig. 18.5. (For use at higher frequencies it is important to make the reference mass small in size in order to satisfy the rigid-body assumption.) Then subject the mass and the two pickups to a free fall of $1g$ by striking the junction of line, which causes the line to relax momentarily and impart a step-function gravitational acceleration to the assembly by allowing it to fall freely. Measure the output of the reference force transducer, e_g;

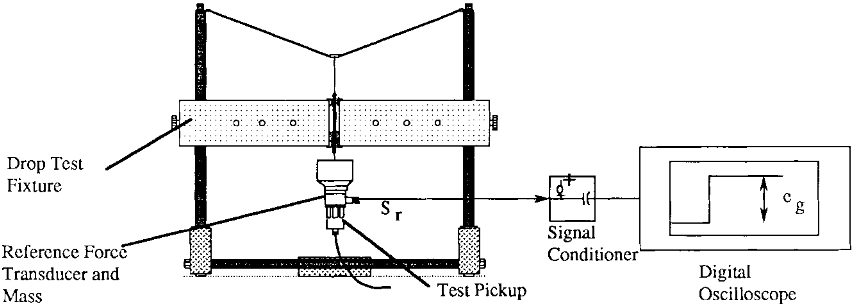


FIGURE 18.5 Gravimetric free-fall calibrator for scaling reference force gage. (*After D. Corelli and R. W. Lally.*[22])

in order to reduce the effect of measurement noise, curve fitting may be used to estimate the step value. Equation (18.7) shows how the sensitivity of the reference force transducer is related to the other parameters of the system.

$$S_r = S_{rf}M = \frac{e_g}{gM}M = \frac{e_g}{g} \qquad (18.7)$$

where S_r = acceleration sensitivity of the reference force transducer, in mV/ms^{-2}
S_{rf} = force sensitivity of the reference force transducer, in mV/N
M = total mass on the force transducer, in kg
e_g = output of the force transducer, in mV
g = acceleration of free fall due to gravity, in ms^{-2}

Note that e_g is numerically equal to S_r expressed in mV/g.

Step 2. *Measure the voltage ratio* e_t/e_r. Remove the reference force transducer, reference mass, and the pickup being calibrated from the drop-test fixture; then mount them on the vibration exciter, as shown in Fig. 18.6. By measuring the transfer function e_t/e_r (i.e., the ratio of the voltage output of the signal conditioner from the test pickup to the voltage output of the signal conditioner from the reference force transducer, shown in Fig. 18.6) the frequency response of the test pickup can be measured over 0.1 to 100,000 Hz, depending upon frequency range of the vibration exciter and signal-to-noise ratio of the system. For use at low frequencies, the discharge time constant of the reference force transducer should be ten times greater than that of the test pickup.

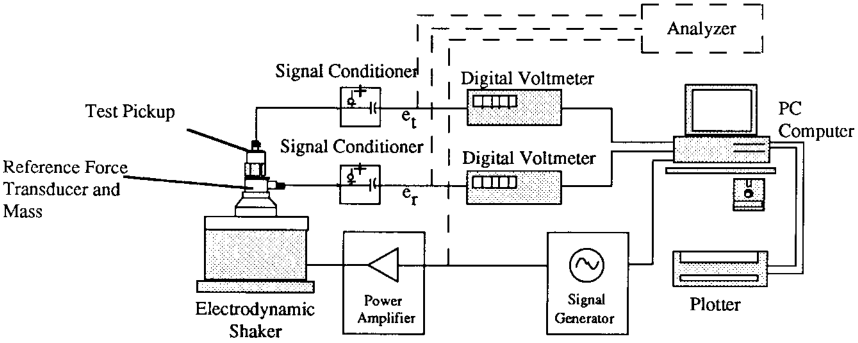


FIGURE 18.6 System configuration for frequency response calibration by measuring acceleration-to-force ratio.

Step 3. *Calculate the sensitivity* S_t *of the test pickup.* If the reference force transducer and the test pickup are linear, the acceleration sensitivity of the test pickup S_t, expressed in the same units as S_r, can be calculated from Eq. 18.1. If either velocity or displacement sensitivity of the test pickup is required, it can be obtained by dividing the acceleration sensitivity by $2\pi f$ or $(2\pi f)^2$, respectively.

CENTRIFUGE CALIBRATOR

A centrifuge provides a convenient means of applying constant acceleration to a pickup. Simple centrifuges can be obtained readily for acceleration levels up to 100g

and can be custom-made for use at much higher values because of the light load requirement by this application. They are particularly useful in calibrating rectilinear accelerometers whose frequency range extends down to 0 Hz and whose sensitivity to rotation is negligible. Centrifuges are mounted so as to rotate about a vertical axis. Cable leads from the pickup, as well as power leads, usually are brought to the table of the centrifuge through specially selected low-noise slip rings and brushes.

To perform a calibration, the accelerometer is mounted on the centrifuge with its axis of sensitivity carefully aligned along a radius of the circle of rotation. If the centrifuge rotates with an angular velocity of ω rad/sec, the acceleration a acting on the pickup is

$$a = \omega^2 r \tag{18.8}$$

where r is the distance from the center-of-gravity of the mass element of the pickup to the axis of rotation. If the exact location of the center-of-gravity of the mass in the pickup is not known, the pickup is mounted with its positive sensing axis first outward and then inward; then the average response is compared with the average acceleration acting on the pickup as computed from Eq. (18.8) where r is taken as the mean of the radii to a given point on the pickup case. The calibration factor is determined by plotting the output e of the pickup as a function of the acceleration a given by Eq. (18.8) for successive values of ω and then determining the slope of the straight line fitted through the data.

INTERFEROMETER CALIBRATORS

A primary (absolute) method of calibrating an accelerometer using standard laser interferometry is shown in Fig. 18.2. All systems in the following category of calibrators consist of three stages: modulation, interference, and demodulation. The differences are in the specific type of interferometer that is used (for example, a Michelson or Mach-Zehnder) and in the type of signal processing, which is usually dictated by the nature of the vibration. The vibratory displacement to be measured modulates one of the beams of the interferometer and is consequently encoded in the output signal of the photodetector in both magnitude and phase.

Figure 18.7 shows the principle of operation of the Michelson interferometer. One of the mirrors, *D* in Fig. 18.7*A,* is attached to the plate on which the device to be calibrated is mounted. Before exciting vibrations, it is necessary to obtain an interference pattern similar to that shown in Fig. 18.7*B.* The relationship underlying the illustrations to be presented is the classical interference formula for the time average intensity I of the light impinging on the photodetector surface.[24,25]

$$I = A + B \cos 4\pi\delta/\lambda \tag{18.9}$$

where A and B are system constants depending on the transfer function of the detector, the intensities of the interfering beams, and alignment of the interferometer. The vibration information is contained in the quantity δ, 2δ being the optical-path difference of the interfering beams. The absoluteness of the measurement comes from λ, the wavelength of the illumination, in terms of which the magnitude of vibratory displacement is expressed. Velocity and acceleration values are obtained from displacement measurements by differentiation with respect to time.

Fringe-Counting Interferometer. An optical interferometer is a natural instrument for measuring vibration displacement. The Michelson and Fizeau interferome-

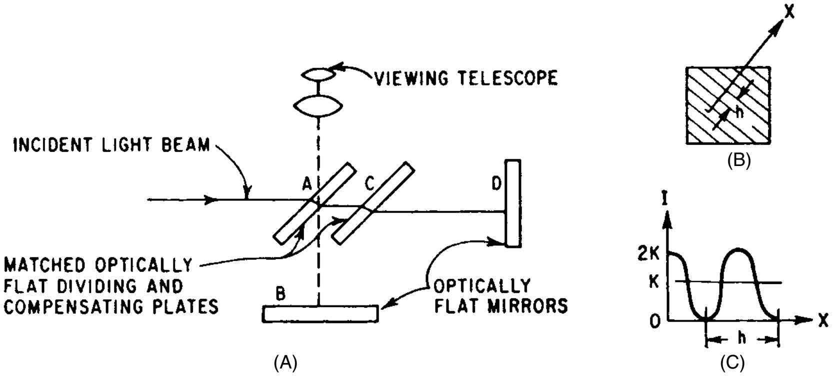


FIGURE 18.7 The principle of operation of a Michelson interferometer: (*A*) Optical system. (*B*) Observed interference pattern. (*C*) Variation of the light intensity along the *X* axis.

ters are the most popular configurations. A modified Michelson interferometer is shown in Fig. 18.8.[26] A corner cube reflector is mounted on the vibration-exciter table. A helium-neon laser is used as a source of illumination. The photodiode and its amplifier must have sufficient bandwidth (as high as 10 MHz) to accommodate the Doppler frequency shift associated with high velocities. An electrical pulse is generated by the photodiode for each optical fringe passing it. The vibratory displacement amplitude is directly proportional to the number of fringes per vibration cycle. The peak acceleration can be calculated from

$$a = \frac{\lambda \nu \pi^2 f^2}{2} \tag{18.10}$$

where λ = wavelength of light
ν = number of fringes per vibration cycle
f = vibration frequency

Interferometric fringe counting is useful for vibration-displacement measurement in the lower frequency ranges, perhaps to several hundred hertz depending on the characteristics of the vibration exciter.[27,28] At the low end of the frequency spectrum, conventional procedures and commercially available equipment are not able to meet all the present requirements. Low signal-to-noise ratios, cross-axis components of motion, and zero-drifts are some of the problems usually encountered. In response to those restrictions an electrodynamic exciter for the frequency range 0.01 to 20 Hz has been developed.[29] It features a maximum displacement amplitude of 0.5 meter, a transverse sensitivity less than 0.01 percent, and a maximum uncorrected distortion of 2 percent. These characteristics have been achieved by means of a specially designed air bearing, an electro-optic control, and a suitable foundation.

Figure 18.9 shows the main components of a computer-controlled low-frequency calibration system which employs this exciter. Its functions are (1) generation of sinusoidal vibrations, (2) measurement of rms and peak values of voltage and charge, (3) measurement of displacement magnitude and phase response, and (4) control of nonlinear distortion and zero correction for the moving element inside a tubelike magnet. Position of the moving element is measured by a fringe-counting interferometer. Uncertainties in accelerometer calibrations using this system have been reduced to about 0.25 to 0.5 percent, depending on frequency and vibration amplitude.

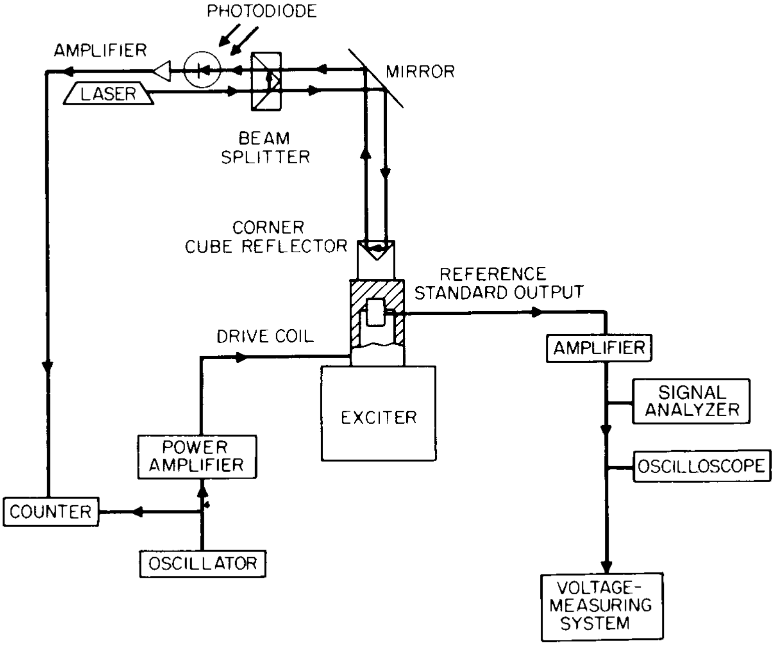


FIGURE 18.8 Typical laboratory setup for interferometric measurement of vibratory displacement by fringe counting. (*After R. S. Koyanagi.*[26])

Fringe-Disappearance Interferometer. The phenomenon of the interference band disappearance in an optical interferometer can be used to establish a precisely known amplitude of motion. Figure 18.7 shows the principle of operation of the Michelson interferometer employed in this technique. One of the mirrors *D,* in Fig. 18.7*A,* is attached to the mounting plate of the calibrator. Before exciting vibrations it is necessary to obtain an interference pattern similar to that shown in Fig. 18.7*B.*

When the mirror *D* vibrates sinusoidally[30] with a frequency *f* and a peak displacement amplitude *d,* the time average of the light intensity *I* at position *x,* measured from a point midway between two dark bands, is given by

$$I = A + BJ_0\left(\frac{4\pi d}{\lambda}\right)\cos\left(\frac{2\pi x}{h}\right) \tag{18.11}$$

where J_0 = zero-order Bessel function of the first kind
A and B = constants of measuring system
h = distance between fringes, as shown in Fig. 18.11*B* and *C*

For certain values of the argument, the Bessel function of zero order is zero; then the fringe pattern disappears and a constant illumination intensity *A* is present. Electronic methods for more precisely establishing the fringe disappearance value of the vibratory displacement have been successfully used at the National Institute of Standards and Technology[17,31] and elsewhere. The latter method has been fully automated using a desktop computer.

The use of piezoelectric exciters is common for high-frequency calibration of accelerometers.[32] They provide pistonlike motion of relatively high amplitude and

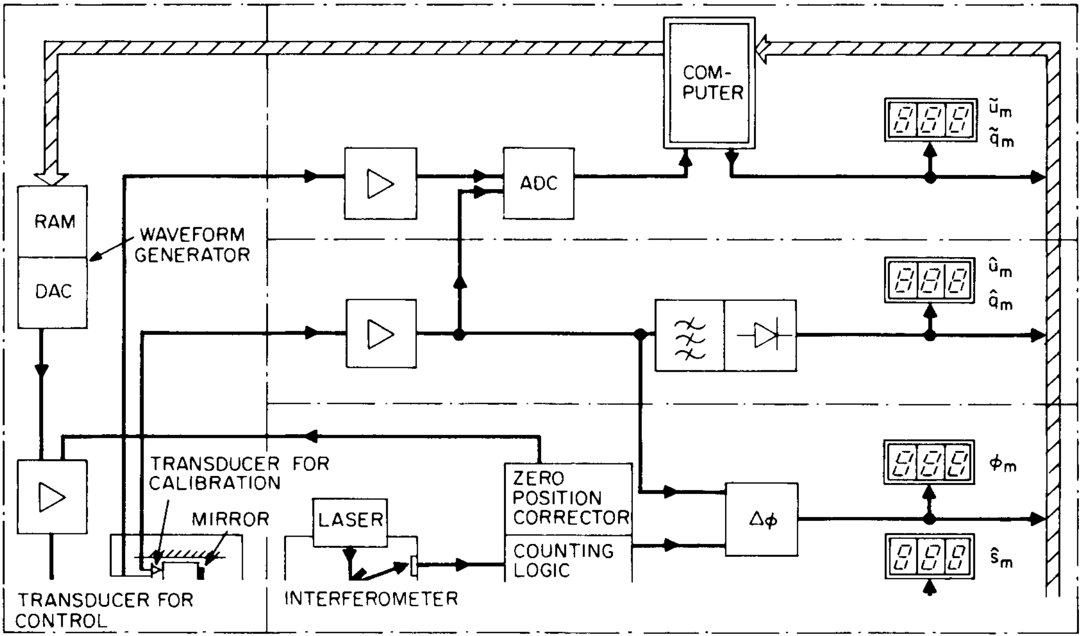


FIGURE 18.9 Simplified block diagram of a low-frequency vibration standard. (*After H. J. von Martens.*[29])

are structurally stiff at the lower frequencies where displacement noise is bothersome. When electrodynamic exciters are used with fringe disappearance methods, it is generally necessary to stiffen the armature suspensions to reduce the background displacement noise.

Signal-Nulling Interferometer. This method, although mathematically similar to fringe disappearance, relies on finding the nulls in the fundamental frequency component of the signal from a photodetector.[17,25,33] The instrumentation is, therefore, quite different, except for the interferometer. One successful arrangement is shown in Fig. 18.10. Laboratory environmental restrictions are much more severe for this method.

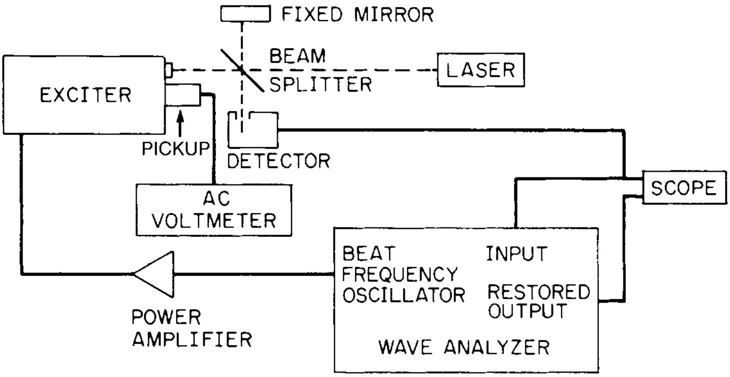


FIGURE 18.10 Interferometric measurement of displacement d as given by $J_1(4\pi d/\lambda) = 0$.

The interferometer apparatus should be well-isolated to ensure stability of the photodetector signals. Air currents in the room may contribute to noise problems by physically moving the interferometer components and by changing the refractive index of the air. An active method of stabilization has also been successfully employed.[34]

To make displacement amplitude measurements, a wave analyzer tuned to the frequency of vibration can be used to filter the photodetector signal. The filtered signal amplitude will pass through nulls as the vibration amplitude is increased, according to the following relationship:

$$I = 2BJ_1\left(\frac{4\pi d}{\lambda}\right) \tag{18.12}$$

where J_1 is the first-order Bessel function of the first kind, and the other terms are as previously defined. The signal nulls may be established using a wave analyzer. The null amplitude will generally be 60 dB below the maximum signal level of the photodetector output.

The accelerometer output may be measured by an accurate voltmeter at the same time that the nulls are obtained. The sensitivity is then calculated by dividing the output voltage by the displacement. Because the filtered output of the photodetector is a replica of the vibrational displacement, a phase calibration of the pickup can also be obtained with this arrangement.

Heterodyne Interferometer. A *homodyne interferometer* is an interferometer in which interfering light beams are created from the same beam by a process of beam splitting. All illumination is at the same optical frequency. In contrast, in the *heterodyne interferometer,*[35] light from a laser-beam source containing two components, each with a unique polarization, is separated into (1) a measurement beam and (2) a reference beam by a polarized beam splitter. When the mounting surface of the device under test is stationary, the interference pattern impinging on the photodetector produces a signal of varying intensity at the beat frequency of the two beams. When surface moves, the frequency of the measurement beam is shifted because of the Doppler effect, but that of the reference beam remains undisturbed. Thus, the photodetector output can be regarded as a carrier that is frequency modulated by the velocity waveform of the motion.

The main advantages of the heterodyne interferometer are greater measurement stability and lower noise susceptibility. Both advantages occur because displacement information is carried on ac waveforms; hence, a change in the average value of beam intensity cannot be interpreted as motion. Digitization and subsequent phase demodulation of the interferometer output reduce measurement uncertainties.[36] This can yield significant improvements in calibration results at high frequencies, where the magnitude of displacement typically is only a few nanometers. As in the case of homodyning, variations of the heterodyning technique have been developed to meet specific needs of calibration laboratories. Reference 37 describes an accelerometer calibration system, applicable in the frequency range from 1 mHz to 25 kHz and at vibration amplitudes from 1 nanometer to 10 meters. The method requires the acquisition of instantaneous position data as a function of the phase angle of the vibration signal and the use of Fourier analysis.

HIGH-ACCELERATION METHODS OF CALIBRATION

Some applications in shock or vibration measurement require that high amplitudes be determined accurately. To ensure that the pickups used in such applications meet certain performance criteria, calibrations must be made at these high amplitudes. The following methods are available for calibrating pickups subject to accelerations in excess of several hundred g.

SINUSOIDAL-EXCITATION METHODS

The use of a metal bar, excited at its fundamental resonance frequency, to apply sinusoidal accelerations for calibration purposes has several advantages: (1) an inherently constant frequency, (2) very large amplitudes of acceleration (as much as $4000g$, and (3) low waveform distortion. A disadvantage of this type of calibrator is that calibration is limited to the resonance frequencies of the metal bar.

The bar can be supported at its nodal points, and the pickup to be calibrated can be mounted at its mid-length location. The bar can be energized by a small electromagnet or can be self-excited. Acceleration amplitudes of several thousand g can thus be obtained at frequencies ranging from several hundred to several thousand hertz. The bar also may be calibrated by clamping it at its midpoint and mounting the pickup at one end.[38] The displacement at the point of attachment of the pickup can be measured optically since displacements encountered are adequately large.

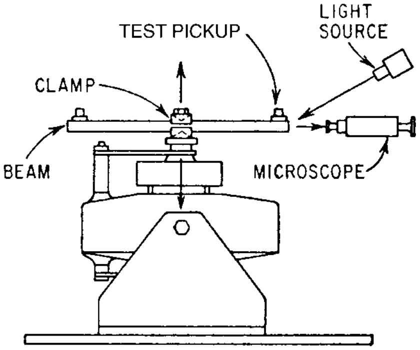


FIGURE 18.11 Resonant-bar calibrator with the pickup mounted at end and a counterbalancing weight at the other. (*After E. I. Feder and A. M. Gillen.*[38])

The resonant-bar calibrator shown in Fig. 18.11 is limited in amplitude primarily by the fatigue resistance of the bar.[38] Accelerations as much as 500g have been attained using aluminum bars without special designs. Peak accelerations as large as 4000g have been attained using tempered vanadium steel bar. The bar is mounted at its mid-length on a conventional electrodynamic exciter. The accelerometer being calibrated is mounted at one end of the bar, and an equivalent balance weight is mounted at the opposite end in the same relative position.

Axial resonances of long rods have been used to generate motion for accurate calibration of vibration pickups over a frequency range from about 1 to 20 kHz and at accelerations up to 12,000g.[39,40] The use of axially driven rods has an advantage over the beams discussed above in that no bending or lateral motion is present. This minimizes errors from the pickup response to such unwanted modes and also from the direct measurement of the displacement having nonrectilinear motion.

SHOCK-EXCITATION METHODS

There are several methods by which a sudden velocity change may be applied to pickups designed for high-frequency acceleration measurement, for example, the ballistic pendulum, drop-test, and drop-ball calibrators, described below. Any method which generates a reproducible velocity change as function of time can be used to obtain the calibration factor.[1] Impact techniques can be employed to obtain calibrations over an amplitude range from a few g to over 100,000g. An example of the latter is the Hopkinson bar, in which the test pickup is mounted at one end and stress pulses are generated by an air gun firing projectiles impacting at the other end, described below.

An accurate determination of shock performance of an accelerometer depends not only upon the mechanical and electrical characteristics of the test pickup but also upon the characteristics of the instrumentation and recording equipment. It is often best to perform system calibrations to determine the linearity of the test pickup as well as the linearity of the recording instrumentation in the range of intended use. Several of the following methods make use of the fact that the velocity change during a transient pulse is equal to the time integral of acceleration:

$$v = \int_{t_1}^{t_2} a\,dt \tag{18.13}$$

where the initial or final velocity is taken as reference zero, and the integration is performed to or from the time at which the velocity is constant. If the output closely resembles a half-sine pulse, the area is equal to approximately $2h(t_2 - t_1)/\pi$, where h is the height of the pulse, and $(t_2 - t_1)$ is its width.

In this section, several methods for applying known velocity changes v to a pickup are presented. The voltage output e and the acceleration a of the test pickup are related by the following linear relationship:

$$S = \frac{e}{a} \tag{18.14}$$

where S is the pickup calibration factor.

After Eq. (18.14) is substituted into Eq. (18.13), the calibration factor for the test pickup can be expressed as

$$S = \frac{A}{v} \tag{18.15}$$

where

$$A = \int_{t_1}^{t_2} e\, dt \tag{18.16}$$

the area under the acceleration-versus-time curve.

The calibration factor assumes that no significant spectral energy exists beyond the frequency region in which the test pickup has nominally constant complex sensitivity (uniform magnitude and phase response as functions of frequency). In general, this assumption becomes less valid with decreasing pulse duration resulting in increasing bandwidth in the excitation signal.

Sometimes it is convenient to express acceleration as a multiple of g. The corresponding calibration factor S_1 is in volts per g:

$$S_1 = \frac{e}{(a/g)} = \frac{Ag}{v} \tag{18.17}$$

In either case, the integrals representing A and v must first be evaluated. The linear range of a pickup is determined by noting the magnitude of the velocity change v at which the calibration factor S or S_1 begins to deviate from a constant value. The minimum pulse duration is similarly found by shortening the pulse duration and noting when S changes appreciably from previous values.

Hopkinson Bar Calibrator. An apparatus called a *Hopkinson bar*[41–43] provides very high levels of acceleration for use in the calibration and acceptance testing of shock accelerometers. As shown in Fig. 18.12, a controlled-velocity projectile strikes one end of the bar, at $x = 0$; a strain gage is placed at the middle of the bar, at $x = L/2$; and the accelerometer under test is mounted at the other end of the bar, at $x = L$. When the projectile strikes the bar, a strain wave is initiated at $x = 0$. This wave travels along the bar, producing a large acceleration at the accelerometer. The duration and shape of the strain wave can be controlled by varying the geometry and mate-

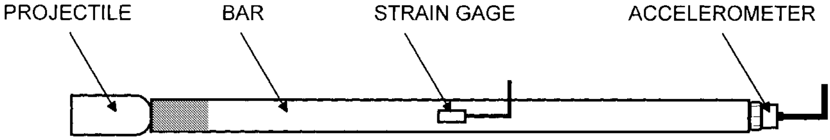


FIGURE 18.12 A Hopkinson bar, showing a projectile striking the bar at $x = 0$; a strain gage mounted on the bar at $x = L/2$; and the accelerometer under test is attached to the bar at $x = L$. Impact of the projectile on the bar generates a strain wave which travels down the bar.

rial of the projectile. And, to a limited extent, the duration of the pulse can be controlled by placing a piece of soft metal or rubber on the bar at the position where the projectile strikes the bar, $x = 0$. The acceleration at the accelerometer may be determined from equations given in Ref. 43, using measured values of strain.

Ballistic Pendulum Calibrator. A ballistic pendulum calibrator provides a means for applying a sudden velocity change to a test pickup. The calibrator consists of two masses which are suspended by wires or metal ribbons. These ribbons restrict the motion of the masses to a common vertical plane.[44] This arrangement, shown in Fig. 18.13, maintains horizontal alignment of the principal axes of the masses in the direction parallel to the direction of motion at impact. The velocity attained by the anvil mass as the result of the sudden impact is determined.

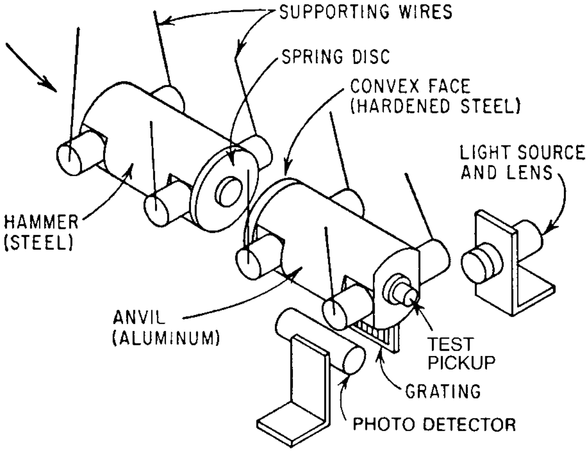


FIGURE 18.13 Components arrangement of the ballistic pendulum with photodetector and light grating to determine the anvil-velocity change during impact. (*After R. W. Conrad and I. Vigness.*[44])

The accelerometer to be calibrated is mounted to an adapter which attaches to the forward face of the anvil. The hammer is raised to a predetermined height and held in the release position by a solenoid-actuated clamp. Since the anvil is at rest prior to impact, it is necessary to record the measurement of the change in velocity of anvil and transient waveform on a calibrated time base. One method of measurement of velocity change is performed by focusing a light beam through a grating attached to the anvil, as shown in Fig. 18.13. The slots modulate the light beam intensity, thus varying the photodetector output, which is recorded with the pickup output. Since the distance between grating lines is known, the velocity of the anvil is calculated directly, assuming that the velocity is essentially constant over the distance between successive grating lines. The velocity of the anvil in each case is determined directly; the time relation between initiation of the velocity and the pulse at the output of the pickup is obtained by recording both signals on the same time base. The most frequently used method infers the anvil velocity from its vertical rise by measuring the maximum horizontal displacement and making use of the geometry of the pendulum system.

The duration of the pulse, which is the time during which the hammer and anvil are in contact, can be varied within close limits.[13] In Fig. 18.13 the hammer nosepiece is a disc with a raised spherical surface. It develops a contact time of 0.55 millisecond. For larger periods, ranging up to 1 millisecond, the stiffness of the nosepiece is decreased by bolting a hollow ring between it and the hammer. A pulse longer than 1 millisecond may be obtained by placing various compliant materials, such as lead, between the contacting surfaces.

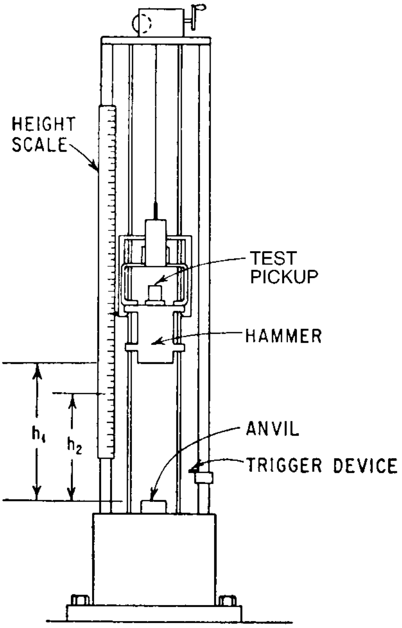


FIGURE 18.14 Component of a conventional drop tester used to apply a sudden velocity change to a vibration pickup. (*After R. W. Conrad and I. Vigness.*[44])

Drop-Test Calibrator. In the drop-test calibrator, shown in Fig. 18.14, the test pickup is attached to the hammer using a suitable adapter plate. An impact is produced as the guided hammer falls under the influence of gravity and strikes the fixed anvil. To determine the velocity change, measurement is made of the time required for a contactor to pass over a known region just prior to and after impact. The pickup output and the contactor indicator are recorded simultaneously in conjunction with a calibrated time base. The velocity change also may be determined by measuring the height h_1 of hammer drop before rebound and the height h_2 of hammer rise after rebound. The total velocity is calculated from the following relationship:

$$v = (2gh_1)^{1/2} + (2gh_2)^{1/2} \quad (18.18)$$

A total velocity change of 40 ft/sec (12.2 meters/sec) is typical.

Drop-Ball Shock Calibrator. Figure 18.15 shows a drop-ball shock calibrator.[12,45] The accelerometer is mounted on an anvil which is held in position by a magnet assembly. A large steel ball is dropped from the top of the calibrator, striking the anvil. The anvil (and mounted test pickup) are accelerated in a short free-flight path. A cushion catches the anvil and accelerometer. Shortly after impact, the anvil passes through an optical timing gate of a known distance. From this the velocity after impact can be calculated. Acceleration amplitudes and pulse durations can be varied by selecting the mass of the anvil, mass of the impacting ball, and resilient pads on top of the anvil where the ball strikes. Common accelerations and durations are $100g$ at 33 milliseconds, $500g$ at 1 millisecond, $1000g$ at 1 millisecond, $5000g$ at 2 milliseconds, and $10{,}000g$ at 0.1 millisecond.[45] With experience and care, shock calibrations can be performed with an uncertainty of about ±5 percent.

INTEGRATION OF ACCELEROMETER OUTPUT

Change-of-velocity methods for calibrating an accelerometer at higher accelerations than obtainable by the methods discussed above have been developed using specially modified ballistic pendulums, air guns, inclined troughs, and other devices.

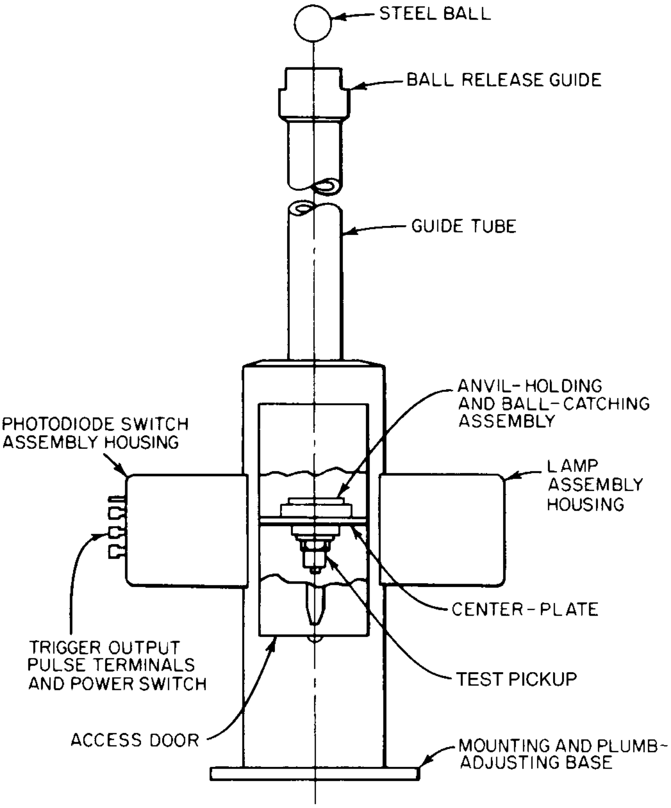


FIGURE 18.15 Diagram of a drop-ball shock calibrator. The accelerometer being calibrated is mounted on an anvil which is held in place by a small magnet. (*After R. R. Bouche.*[45])

Regardless of the device employed to generate the mechanical acceleration or the method used to determine the change of velocity, it is necessary to compare the measured velocity and the velocity derived from the integral of the acceleration waveform as described by Eq. (18.13). Electronic digitizers can be used to capture the waveform and produce a recording. Care must be exercised in selecting the time at which the acceleration waveform is considered complete, and its integral should be compared with the velocity. The calibration factor for the test pickup is computed from Eq. (18.15) or (18.17).

IMPACT-FORCE SHOCK CALIBRATOR

The impact-force shock calibrator has a free-fall carriage and a quartz load cell. The accelerometer to be calibrated is mounted onto the top of the carriage, as shown in Fig. 18.16. The carriage is suspended about ½ to 1 meter above the load cell and allowed to fall freely onto the cell.[46] The carriage's path is guided by a plastic tube. Cushion pads are attached at the top of the load cell to lengthen the impulse duration

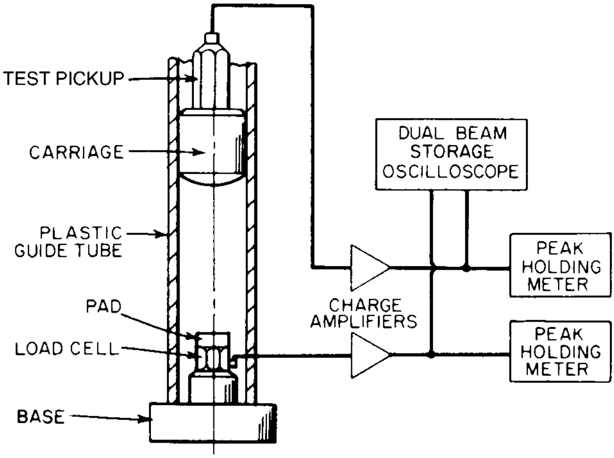


FIGURE 18.16 Impact-force calibrator with auxiliary instruments. (*After W. P. Kistler.*[46])

and to shape the pulse. Approximate haversines are generated by this calibrator. The outputs of the accelerometer and load cell are fed to two nominally identical charge amplifiers or power units. The outputs from load cell and test accelerometer are recorded or measured on a storage-type oscilloscope or peak-holding meters.

During impact, the voltage produced at the output of the accelerometer, $e_a(t)$, is

$$e_a(t) = a(t)S_aH_a \tag{18.19}$$

where $a(t)$ = acceleration
S_a = calibration factor for accelerometer
H_a = gain of charge amplifier or power unit

The output of load cell $e_f(t)$ is

$$e_f(t) = F(t)S_fH_f \tag{18.20}$$

where $F(t)$ = force
S_f = calibration factor for load cell
H_f = gain of charge amplifier or power unit

By using the relationship $F(t) = ma(t)$, where m is the falling mass, and combining Eqs. (18.19) and (18.20),

$$\frac{e_a(t)}{e_f(t)} = \frac{a(t)S_aH_a}{ma(t)S_fH_f} \tag{18.21}$$

and hence

$$S_a = \frac{e_a(t)\ H_fm}{e_f(t)\ H_ag} S_f \tag{18.22}$$

When calculating the mass, it is necessary to know the mass of the carriage, accelerometer, mounting stud, cable connector, and a short portion of the accelerometer cable. Experience has shown that for small coaxial cables, a length of about 2 to 4 cm is correct. Calibrations by this method can be accomplished with uncertainties generally between ±2 to ±5 percent.

FOURIER-TRANSFORM SHOCK CALIBRATION

The above calibration methods yield the approximate magnitude of the sensitivity function for the accelerometer being tested. For shock standards and other critical applications, more information may be required, for example, the accelerometer's sensitivity, both in magnitude and phase, as a function of frequency.[47–50] The equipment required for obtaining this information usually consists of a mechanical-shock-generating machine and a two-channel signal analyzer, in addition to the accelerometer being tested and a reference accelerometer. For a typical application, a signal analyzer with 12-bit resolution and 5 MHz sampling rate is adequate. The calibration results are obtained from the complex ratios of the output of the test accelerometer to that of the standard accelerometer (see Chap. 14, *FFT Analyzers*). The magnitude and phase of these ratios represent the sensitivity of the test accelerometer relative to the standard.

The range of usable frequencies is limited by the pulse shape and duration, sampling rate, and analyzer capability. Figure 18.17 shows a typical half-sine shock pulse whose spectral content is predominantly below about 2 kHz, but pulses of shorter duration contain sufficient energy up to 10 kHz, and even 30 kHz.[50] An important advantage of the spectral methods over the time-domain methods is that they do not require the waveform or pulse to be smooth and clean. Modern signal processing equipment has made it possible to calibrate shock accelerometers at amplitudes approaching 1 megameter per second^2 by using the FFT method with a Hopkinson bar,[50] shown in Fig. 18.12. The uncertainties in this type of calibration can be as low as 1 percent.[51,52]

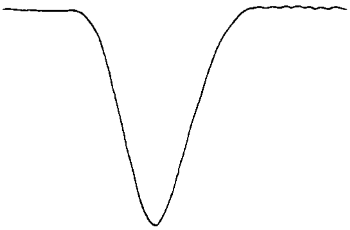


FIGURE 18.17 A typical half-sine shock pulse generated by a pneumatic shock machine. Deceleration amplitude is 900g and pulse duration is 1 millisecond. (*After J. D. Ramboz and C. Federman.*[47])

VIBRATION EXCITERS USED FOR CALIBRATION

A vibration exciter that is suitable for calibration of vibration pickups should provide:

- Distortion-free sinusoidal motion
- True rectilinear motion in a direction normal to the vibration-table surface without the presence of any other motion
- A table that is rigid for all design loads at all operating frequencies
- A table that remains at ambient temperature and does not provide either a source or sink for heat regardless of the ambient temperature
- A table whose mounting area is free from electromagnetic disturbances

- Stepless variation of frequency and amplitude of motion within specified limits, which is easily adjustable

ELECTRODYNAMIC EXCITERS

Electrodynamic exciters, described in Chap. 25, satisfactorily meet the requirements of the ideal calibrator, providing a constant-force (acceleration) output with little distortion over a rather wide frequency range from 1 to 10,000 Hz.[53] Ordinarily, to cover this frequency range, more than one exciter is required. Specially designed machines featuring long strokes for very low frequencies or ultralight moving elements for very high frequencies are commercially available. One national standards laboratory has a custom-built vibration exciter that has a low-frequency limit of 20 mHz.[29] This machine employs a special air bearing, real-time electro-optic control, and a suitable foundation.

A shaker system for the calibration of accelerometer sensitivity has been developed at the National Institute of Standards and Technology[54,55] with the goal of reducing the inherent uncertainties in the absolute measurements of accelerometer sensitivity. The shaker has dual retractable magnets equipped with optical ports to allow laser-beam access to the surface upon which the accelerometer is mounted and the one opposite to it. The purpose of the optical ports is to enable interferometric measurement of the surface displacement. The moving element of the shaker is physically compact for directional stability and good high-frequency response. At each end it is equipped with nominally identical coils and axially oriented mounting tables. The driving and sensing coils are located on the same moving element so that a separate shaker external to the calibration shaker is not needed when a reciprocity calibration is performed. The dual-coil feature eliminates complications resulting from mutual mechanical coupling between two separate shakers. Minimal distortion and cross-action motion were two of the most important design requirements of this vibration generator. These parameters are essential for the validity of the assumptions underlying the theory of electromechanical reciprocity.

PIEZOELECTRIC EXCITERS

The piezoelectric exciter (see Fig. 25.9 and Chap. 12) offers a number of advantages in the calibration of vibration pickups, particularly at high frequencies. Calibration is impracticable at low frequencies because of inherently small displacements in this frequency range. A design which has been used at the National Institute of Standards and Technology for many years is described in Ref. 32.

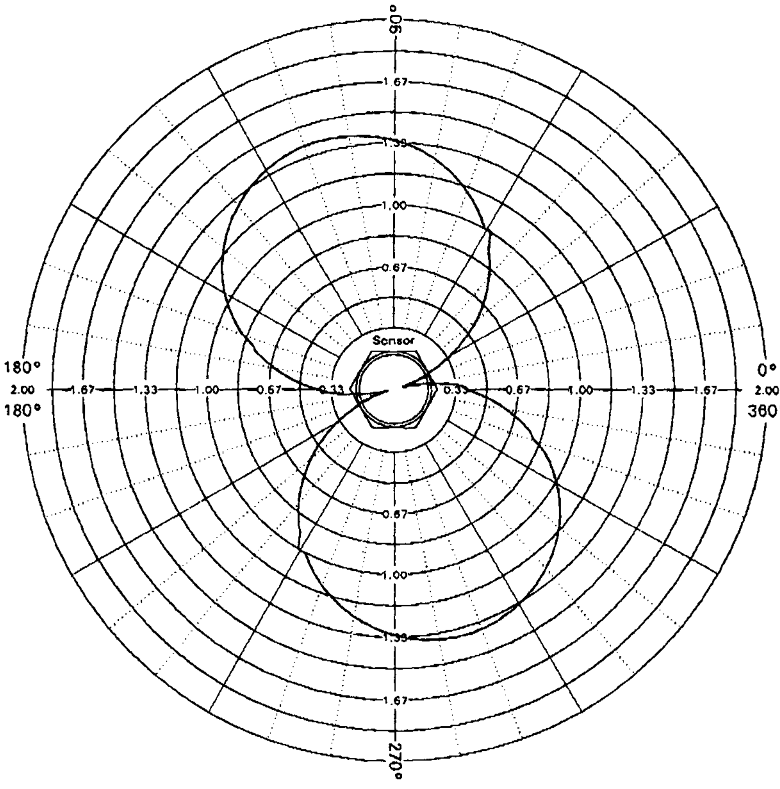
MECHANICAL EXCITERS

Rectilinear motion can be produced by mechanical exciter systems of the type described in Chap. 25 under *Direct-Drive Mechanical Vibration Machine.* Their usable frequency range is from few hertz to less than 100 Hz. Despite their relatively low cost, mechanical exciters are no longer used for high-quality calibrations of transducers because of their appreciable waveform distortion and background noise.

For generating vibratory motion at discrete frequencies (below 5 Hz), a linear oscillator can be employed. Reference 56 describes a calibrator consisting of a spring-supported table which is guided vertically by air bearings. Its advantages are a clean waveform, resulting from free vibration, and large rectilinear displacement with little damping, made possible by use of air bearings.

CALIBRATION OF TRANSVERSE SENSITIVITY

The characteristics of a vibration pickup may be such that an extraneous output voltage is generated as a result of vibration which is in a direction at right angles to the axis of designated sensitivity of the pickup. This effect, illustrated in Fig. 12.11, results



Model Number 353

Serial Number 1

Sensitivity 20.90 mV/g

Test Frequency 700 Hz

Test g level 10.93 g's

Maximum Transverse 1.40% @ 107°

Minimum Transverse 0.035% @ 195°

FIGURE 18.18 Transverse sensitivity of a piezoelectric accelerometer to vibration in the plane normal to the sensitive axis.[57]

in the axis of maximum sensitivity not being aligned with the axis of designated sensitivity. As indicated in Eq. (12.11), the cross-axis or *transverse sensitivity* of a pickup is expressed as the tangent of an angle, i.e., the ratio of the output resulting from the transverse motion divided by the output resulting from motion in the direction of designated sensitivity. This ratio varies with the azimuth angle in the transverse plane, as shown in Fig. 12.12, and also with frequency. In practice, tan θ has a value between 0.01 and 0.05 and is expressed as a percentage. Figure 18.18 presents a typical result of a transverse-sensitivity calibration.[57]

Knowledge of the transverse sensitivity is vitally important in making accurate vibration measurements, particularly at higher frequencies (i.e., at frequencies approaching the mounted resonance frequency of the pickup). Figure 18.19 shows the relative responses of an accelerometer to main-axis and transverse-axis vibration. It is noteworthy that the transverse resonance frequency is lower than the usually specified mounted resonance frequency.

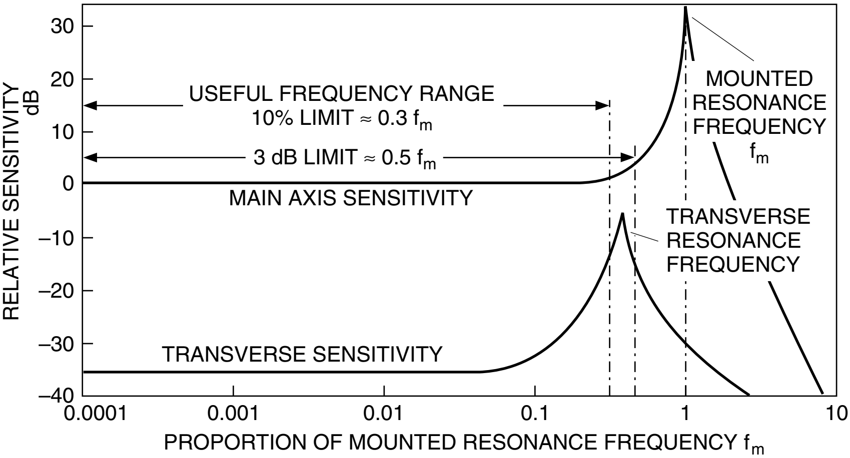


FIGURE 18.19 The relative response of an accelerometer to main-axis and transverse-axis vibrations.[4]

A direct measurement of the transverse sensitivity of a pickup requires a vibration exciter capable of pure unidirectional motion at the frequencies of interest. This usually means that any cross-axis motion of the mounting table should be less than 2 percent of the main-axis motion.[12] Resonance beam exciters[1] and air-bearing shakers[53] have been used for this purpose.

The *resonant-beam* method,[58] used by many testing laboratories to provide the sensitivity of a transducer automatically (in both magnitude and direction) yields a plot of its sensitivity versus angle (similar to the one shown in Fig. 18.18). The accelerometer under test is mounted at the free end of a circular-section steel beam which is cantilevered from a massive base. Motion of the accelerometer is generated by exciting the beam near resonance in its first bending mode, providing a large-amplitude vibration at the free end of the beam, typically at a frequency between 300 and 800 Hz. A pair of vibration exciters, and associated electronic equipment,

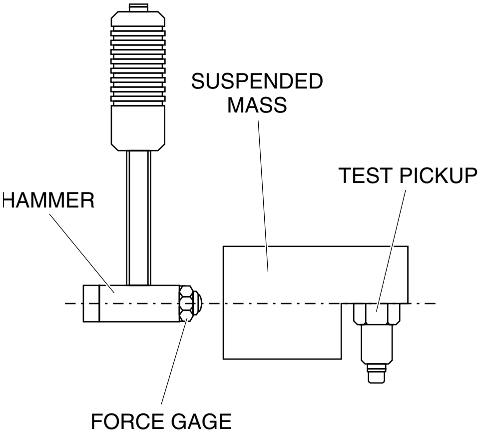


FIGURE 18.20 Schematic diagram for impact hammer method of measuring transverse sensitivity.[57]

permits the beam to be excited in any desired direction. Thus the transverse sensitivity may be obtained at any angle without reorientation of the accelerometer.

Another method for obtaining the transverse sensitivity of a pickup is by use of the impulse technique similar to that used in modal analysis (Chap. 21). An impulse is generated by the impact of a hammer against a suspended mass on which the test pickup is mounted. A force gage is mounted on the hammer, as illustrated in Fig. 18.20. From the characteristics of the force gage and its output when it strikes against the suspended mass, from the output signal of the test pickup, and from the magnitude of the suspended mass, the transverse sensitivity of the accelerometer under test S_{ta} may be calculated according to a procedure described in Ref. 57, using the following formula:

$$S_{ta} = mS_f\left(\frac{e_a}{e_f}\right) \tag{18.23}$$

where m = the mass of the suspended rigid block
S_f = the sensitivity of the force gage
e_a = the output of the accelerometer under test
e_f = the output of the force gage

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