CHAPTER 35
ENGINEERING PROPERTIES OF COMPOSITES

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INTRODUCTION

Composite materials are simply a combination of two or more different materials that may provide superior and unique mechanical and physical properties. The most attractive composite systems effectively combine the most desirable properties of their constituents and simultaneously suppress the least desirable properties. For example, a glass-fiber reinforced plastic combines the high strength of thin glass fibers with the ductility and environmental resistance of an epoxy resin; the inherent damage susceptibility of the fiber surface is thereby suppressed whereas the low stiffness and strength of the resin is enhanced.

The opportunity to develop superior products for aerospace, automotive, and recreational applications has sustained the interest in advanced composites. Currently composites are being considered on a broader basis, specifically, for applications that include civil engineering structures such as bridges and freeway pillar reinforcement, and for biomedical products such as prosthetic devices. The recent trend toward affordable composite structures with a somewhat decreased emphasis on performance will have a major impact on the wider exploitation of composites in engineering.

BASIC TYPES OF COMPOSITES

Composites typically comprise a high-strength synthetic fiber embedded within a protective matrix. The most mature and widely used composite systems are *polymer matrix composites* (PMCs), which will provide the major focus for this chapter. Contemporary PMCs typically use a ceramic type of reinforcing fiber such as carbon, Kevlar™, or glass in a resin matrix wherein the fibers make up approximately 60 percent of the PMC volume. Metal or ceramic matrices can be substituted for the resin matrix to provide a higher-temperature capability. These specialized systems are termed *metal matrix composites* (MMCs) and *ceramic matrix composites* (CMCs); a

TABLE 35.1 Composite Design Comparisons

	PMC	CMC	MMC
Specific strength and stiffness	Generally excellent if exclusively unidirectional reinforcement is avoided	Highest potential for high-temperature applications	Moderately high for dominantly axial loads and intermediate temperatures
Fatigue characteristics	Excellent for designs that avoid out-of-plane loads	Good for high-temperature applications loads	Potential concern for other than dominantly axial
Nonlinear effects	Usually not important for continuous fiber reinforcements	Significant effect after first matrix and interface cracks have developed	Can be significant, particularly for multidirectional and off-axis loads
Temperature capability	Less than 600°F	Potential for maximum values between 1000 and 2000°F	Potential for maximum values up to 1000°F
Degree of anisotropy	Extreme, particularly considering out-of-plane properties and consequent coupling effects in minimum-gage configurations	Can develop significantly during loading, due to matrix and interface breakdown	Not usually a major issue where interface effects are negligible

general qualitative comparison of the relative merits of all three categories is summarized in Table 35.1.

SHORT FIBER/PARTICULATE COMPOSITES

The fibrous reinforcing constituent of composites may consist of thin continuous fibers or relatively short fiber segments, or whiskers. However, reinforcing effectiveness is realized by using segments of relatively high *aspect ratio,* which is defined as the length-to-diameter ratio. Nevertheless, as a reinforcement for PMCs, these short fiber or whisker systems are structurally less efficient and very susceptible to damage from long-term and/or cyclic loading. On the other hand, the substantially lower cost and reduced anisotropy on the macroscopic scale render these composite systems appropriate in structurally less demanding industrial applications.

Randomly oriented short fiber or particulate-reinforced composites tend to exhibit a much higher dependence on polymer-based matrix properties, as compared to typical continuous fiber reinforced PMCs. Elastic modulus, strength, creep, and fatigue are most susceptible to the significant limitations of the polymer matrix constituent and fiber-matrix interface properties.[1]

CONTINUOUS FIBER COMPOSITES

Continuous fiber reinforcements are generally required for structural or high-performance applications. The *specific strength* (strength-to-density ratio) and *specific stiffness* (elastic modulus-to-density ratio) of continuous fiber reinforced PMCs, for example, can be vastly superior to conventional metal alloys, as illustrated in Fig. 35.1. These types of composite can also be designed to provide other attractive properties, such as high thermal or electrical conductivity and low coefficient of thermal expansion (CTE). In addition, depending on how the fibers are oriented or inter-

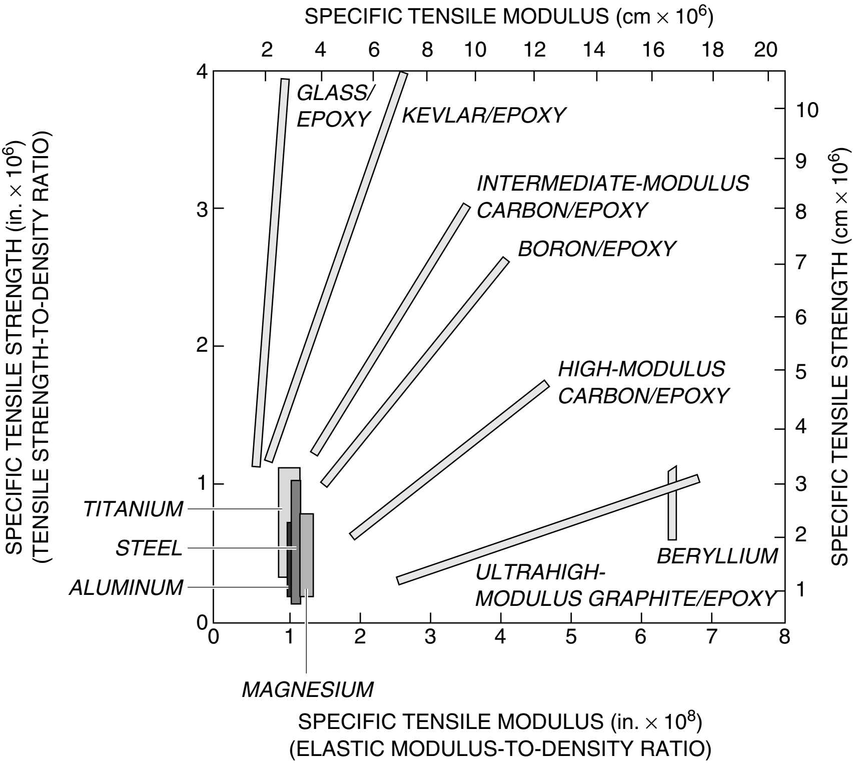


FIGURE 35.1 A weight-efficiency comparison.

woven within the matrix, these composites can be tailored to provide the desired structural properties for a specific structural component. *Anisotropy* is a term used to define such a material that can exhibit properties varying with direction. Thus designing for, and with, anisotropy is a unique aspect of contemporary composites in that the design engineer must simultaneously design the structure and the material of construction. Of course, anisotropy brings problems as well as unique opportunities, as is discussed in a later section. With reference to Fig. 35.1, it should be appreciated that the vertical bars representing the conventional metals signify the potential variation in specific strength that may be brought about by changes in alloy constituents and heat treatment. The angled bars for the continuous fiber composites represent the range of specific properties from the unidirectional, all 0° fiber orientation at the upper end to the pseudo-isotropic laminate with equal proportions of fibers in the 0°, +45°, –45°, and 90° orientations at the lower end. In the case of the composites, the variations between the upper or lower ends of the bars are achieved by tailoring in the form of laminate design.

SPECIAL DESIGN ISSUES AND OPPORTUNITIES

Product design that involves the utilization of composites is most likely to be effective when the aspects of materials, structures, and dynamics technologies are embraced in the process of the development of mechanical systems. One illustrative example was cited in the introductory chapter of this handbook (see Chap. 1), which introduces the technique of reducing the vibration response of a fan blade by alteration of the natural frequency. In the design of composite fan blades for aircraft, this approach has been achieved by tailoring the frequency and the associated mode shape.[2] Such a tailoring capability can assist the designer in adjusting flexural and torsional vibration and fatigue responses, as well as the damping characteristics explained later.

A more challenging issue that frequently arises in composite hardware design for a majority of the more geometrically complex products is the potential impact of the low secondary or matrix-influenced properties of these strongly nonisotropic material forms. The transverse (in-plane) tensile strength of the unidirectional composite laminate is merely a few percent of the longitudinal tensile strength (as observed from Tables 35.2 and 35.3). Consequently, it is of no surprise that the through-thickness or short-transverse tensile strength of a multidirectional laminate is of the same order, but even lower than the transverse tensile strength of the individual layers. Thus, the importance of the designer's awareness of such limitations cannot be overemphasized. In fact, the large majority of the failures in composite hardware development testing has arisen due to underestimated or unrecognized out-of-plane loading effects and interrelated regions of structural joints and attachments. Due to the many common adverse experiences with delaminations induced by out-of-plane

TABLE 35.2 Properties of Typical Continuous, Fiber-Reinforced Composites and Structural Metals

Property	Unidirectional composite (60% fiber/40% resin, by volume)				Metals	
	E-glass/ resin	Kevlar/ resin	HS carbon/ epoxy	UHM Gr./ epoxy	7075-T6 aluminum	4130 steel
			Elastic			
Density, lb/in.^{3} (10^3 kg/m^3)	0.070 (1.9)	0.047 (1.3)	0.058 (1.6)	0.060 (1.7)	0.100 (2.77)	0.284 (7.86)
E_L, 10^6 lb/in.^{2} (10^3 MPa)	6.5 (45)	11.0 (75.8)	19.5 (134)	40.0 (276)	10.3 (71.0)	30.0 (207)
E_T, 10^6 lb/in.^{2} (10^3 MPa)	1.8 (12)	1.0 (6.9)	1.5 (10)	1.2 (8.3)	10.3 (71.0)	30.0 (207)
G_{LT}, 10^6 lb/in.^{2} (10^3 MPa)	0.7 (4.8)	0.4 (2.8)	0.9 (6.2)	0.65 (4.5)	4.0 (27.6)	12.0 (82.7)
ν_{LT}	0.32	0.33	0.30	0.28	0.30	0.28
			Strength			
F_L^{tu}, 10^3 lb/in.^{2} (MPa)	180 (1240)	220 (1520)	200 (1380)	100 (689)	79 (545)	100 (689)
F_T^{tu}, 10^3 lb/in.^{2} (MPa)	6 (41)	4.5 (31)	7 (48)	5 (34)	77 (531)	100 (689)
F_L^{cu}, 10^3 lb/in.^{2} (MPa)	120 (827)	45 (310)	170 (1170)	90 (620)	70 (483)	130 (896)
F_T^{cu}, 10^3 lb/in.^{2} (MPa)	20 (138)	20 (138)	20 (138)	20 (138)	70 (483)	130 (896)
F_{LT}^{su}, 10^3 lb/in.^{2} (MPa)	8 (55)	4 (28)	10 (69)	9 (62)	47 (324)	60 (414)

TABLE 35.3 Typical Unidirectional Properties for a Carbon/Epoxy System

Stiffness properties		Strength properties		Thermal properties	
E_L, 10^6 lb/in.^{2}	20.0	F_L^{tu}, 10^3 lb/in.^{2}	240.0	α_L, με/°F	–0.3
(10^3 MPa)	(138)	(MPa)	(1650)	(με/K)	(–0.54)
E_T, 10^6 lb/in.^{2}	1.4	F_L^{cu}, 10^3 lb/in.^{2}	200.0	α_T, με/°F	17.0
(10^3 MPa)	(9.6)	(MPa)	(1380)	(με/K)	(30.6)
G_{LT}, 10^6 lb/in.^{2}	0.8	F_T^{tu}, 10^3 lb/in.^{2}	7.0	K_L, Btu in./h ft^2 °F	40.0
(10^3 MPa)	(5.5)	(MPa)	(48)	(W/m K)	(5.76)
ν_{LT}	0.28	F_T^{tu}, 10^3 lb/in.^{2}	20.0	K_T, Btu in./h ft^2 °F	4.5
		(MPa)	(138)	(W/m K)	(0.65)
		F_{LT}^{isu}, 10^3 lb/in.^{2}	10.0		
		(MPa)	(69)		
$\nu_{LT}/E_L = \nu_{TL}/E_T$		F^{isu}, 10^3 lb/in.^{2}	9.0		
		(MPa)	(62)		

load components, this section will be devoted to the identification of the numerous sources of out-of-plane load development and the candidate approaches to eliminate or minimize their influence.

First, a general overview of many of the common problems created for the engineering designer that are consequences of low-matrix-dominated, elastic, and strength properties are summarized in Table 35.4. Several of the most common sources will now be discussed in more detail. Figure 35.2 illustrates these major sources, which may be broadly categorized as follows:

Category A: Curved sections including curved segments, rings, hollow cylinders, and spherical vessels that are representative of angle bracket design details, curved frames, and internally or externally pressurized vessels.

TABLE 35.4 General Overview of Problems Created by the Low Secondary (Matrix-Dominated) Properties of Advanced Composites

Controlling property	Problem
F^{isu}	Failure induced by shear in beams under flexural loading.
	Premature torsional failures.
	Premature crippling failure in compression.*
	Failure of adherends in structural bonded joints.*
	Failure of laminae due to free-edge effects, e.g., cutouts, ply drops.*
F_T^{tu}	Failure induced by transverse tensile fracture of curved beams in flexure.
	Shock waves during normal impacts.
G_{LT}	Reduction in flexural and torsional stiffness.
	Reduction in resonant frequencies of plate and beam members.
	Reduction of elastic buckling capability.
	Interpretation of experimental stress analysis data.
α_T	Distortion at fillets due to high expansion coefficient (through-thickness).
$\alpha_T F_T^{tu}$	Failure due to thermal stresses in thick-walled composite cylinders.

*For these problems, the controlling properties are both F^{isu} and F_T^{tu}.

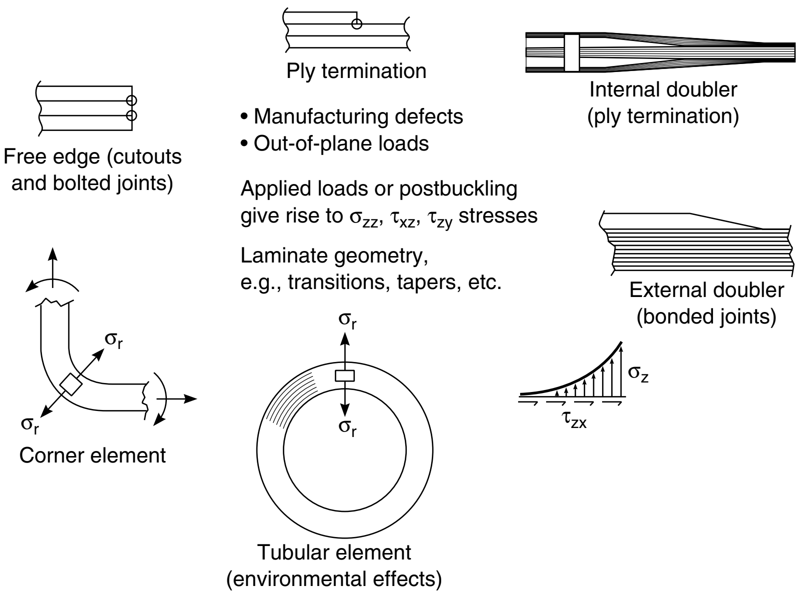


FIGURE 35.2 Generic sources of delamination.

Category B: Tapers and transitions including local changes of section that are representative of laminate layer terminations, doublers, and stiffener terminations, as well as the end details of bonded and bolted joints.

As mentioned earlier, commonplace structural details of both categories have contributed to numerous unanticipated failures in composite hardware components. In some cases, such failures can propagate catastrophically after initiation and may therefore be a serious safety threat. Other instances have arisen where initial failures may self-arrest resulting in benign failures, but with some degree of local stiffness degradation. Subsequent load distribution may, however, precipitate eventual catastrophic failure depending on the load spectrum characteristics.

COMPOSITE PROPERTIES

The class of composites which forms the focus of this chapter is polymer matrix composites (PMCs) with continuous fiber reinforcement. In this type of composite, the properties of an arbitrary laminated composite architecture are derived from the elastic and strength properties of a unidirectional layer. The unidirectional layer properties can be derived from the constituent properties of the fiber and matrix that typically range between 50 and 65 percent by volume of the fiber reinforcement phase. Here a nominal value of 60 percent by volume of fiber will be adopted.

Fiber reinforcements most commonly encountered in contemporary composites include carbon or graphite fibers, Kevlar fibers, and glass fibers, all of which can be obtained in similar diameters, i.e., 0.0003–0.0005 in. Both the carbon/graphite and Kevlar fibers are inherently anisotropic in themselves, although it is the axial (fiber direction) properties that dominate the in-plane behavior of unidirectional and, gen-

TABLE 35.5 Typical Fiber Properties

Fiber	Density, lb/in.^{3} (10^3 kg/m^3)	Axial elastic modulus, 10^6 lb/in.^{2} (10^3 MPa)	Transverse elastic modulus, 10^6 lb/in.^{2} (10^3 MPa)	Tensile strength, 10^3 lb/in.^{2} (10^3 MPa)
E-glass	0.091 (2.5)	10.5 (72.4)	10.5 (72.4)	500 (3.4)
S-glass	0.090 (2.5)	12.4 (85.5)	12.4 (85.5)	600 (4.1)
Kevlar 49	0.052 (1.4)	18.0 (124)	1.3 (8.96)	400 (2.8)
AS4 carbon	0.064 (1.8)	35.0 (241)	2.0 (13.8)	350 (2.4)

erally, multidirectional fiber arrays or laminates. Typical fiber properties are presented in Table 35.5, where the degree of individual fiber anisotropy is indicated.

GENERAL PROPERTIES

The properties of polymer matrices range over a much smaller spectrum in Table 35.6, and the relatively low stiffness and strength properties rarely dominate the composite behavior, with certain exceptions. The most notable exceptions are the interlaminar shear strength and the thickness-direction interlaminar tensile strength, to be discussed later, wherein the fiber-to-matrix interface may play an important role. For these reasons, the greatest attention is placed on the macroscopic composite properties that are of most direct interest to the mechanical or structural engineer. Typical values for such properties are provided in Table 35.2 for the three different, but all widely used composites. One well-established carbon fiber/epoxy composite system is chosen to illustrate typical properties and degrees of anisotropy in elastic, strength, and thermal properties in Table 35.3. Engineers responsible for design and structural evaluation should take particular note of the degree of anisotropy in both the strength and stiffness properties. Usually the matrix-dominated properties, such as the shear and transverse tensile strengths, are very low and the avoidance of matrix-dominated failure modes represents a major challenge for the structural designer. It is also worthy of note that compression strength in the fiber direction, F_L^{cu}, is significantly lower than the equivalent tensile strength, F_L^{tu}, due to a microfiber instability mechanism. In fact, the ratio of these two strengths, F_L^{cu}/F_L^{tu}, may be much lower for some other systems, e.g., Kevlar/epoxy and more recently developed high strain-to-failure carbon fibers. The lower compression strengths relative to the tensile strengths is also influenced by the fiber diameter and the matrix properties that are themselves affected by moisture, temperature, interface integrity, and porosity.

IN-SITU PROPERTIES

An important fundamental aspect of multidirectional composite laminates is the manner in which the individual unidirectional layer or lamina properties translate

TABLE 35.6 Typical Properties for Polymer Matrices

Polymer	Density, lb/in.^{3} (10^3 kg/m^3)	Elastic modulus, 10^6 lb/in.^{2} (10^3 MPa)	Tensile strength, 10^3 lb/in.^{2} (MPa)	Poisson's ratio
HERCULES 3501-6 epoxy	0.044 (1.2)	0.62 (4.3)	12.0 (82.7)	0.34
NARMCO 5208 epoxy	0.044 (1.2)	0.50 (3.4)	11.0 (75.8)	0.35
EPON 828 epoxy	0.044 (1.2)	0.47 (3.2)	13.0 (89.6)	0.35

into laminate properties. For all the thermoelastic properties, this translation is accomplished by the usual rules for transformation of stress and strain. However, the strength properties tend to be modified by the mutual constraint imposed by adjacent layers, and therefore is a function of the individual layer thickness. The result is a need to modify the basic unidirectional properties, one of the most significant being the ultimate transverse strain to failure in tension of individual layers. Unidirectional layer compressive strength and the associated ultimate strain to failure is also influenced to a significant degree by the mutual support offered by adjacent transverse or angled layers. As a consequence, correction factors are sometimes introduced to compensate for these effects, but more routine tests are conducted on the actual laminate configuration in an effort to establish reliable allowables for its use in design.

LAMINATED COMPOSITE DESIGN

For the simultaneous design of material and structure that is the basic philosophy for composite structures development, *laminated plate theory* (LPT) and the associated computer codes represent the fundamental tool for the composite designer. The anatomy of a composite laminate indicating the translation from the constituent fiber and matrix properties to those of a built-up laminate is illustrated in Fig. 35.3.

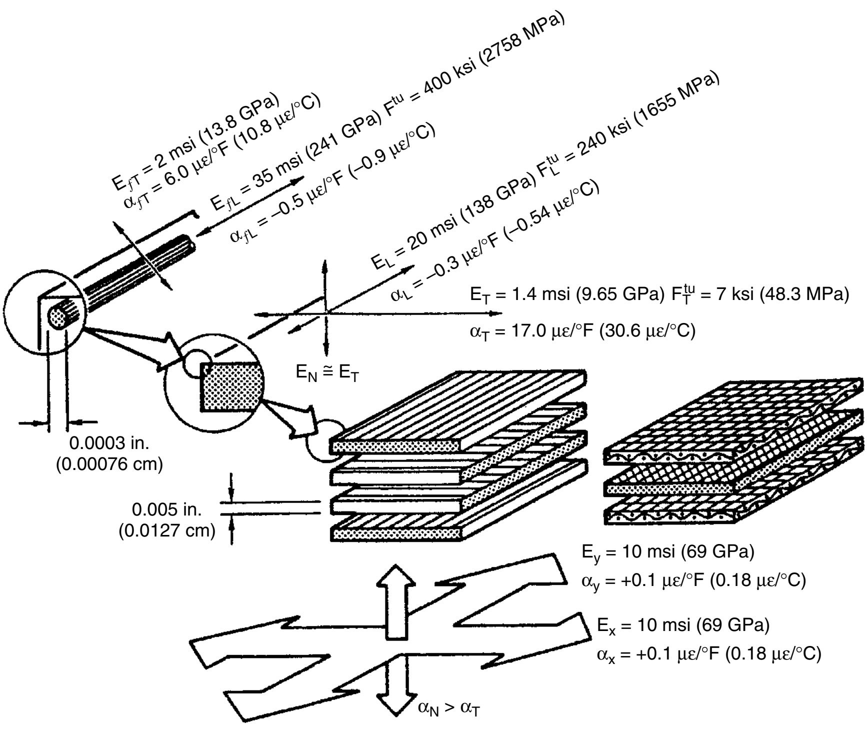


FIGURE 35.3 The anatomy of a composite laminate.

Values contained in this figure compare with those presented in Table 35.3. Figure 35.3 also illustrates the use of an alternative form of material, a fabric laminate that can provide similar, but slightly inferior, properties in a reduced thickness. The ability to produce a single layer comprised of equal proportions of fibers woven into 0° and 90° orientations is offered by this approach. Such a textile system therefore represents a valuable composite form. A state of plane stress and, for bending, plane sections remain plane, is assumed in most conventional theoretical treatments.

To remain within the scope and purpose of this chapter, the full treatment of conventional laminated plate theory will not be repeated here since it appears in numerous established texts on the subject (see Refs. 3 through 8). However, the essential information on conventional notations, whereby laminates are specified together with the physical behavioral insights concerning coupling phenomena, will be presented herein.

LAMINATE CONFIGURATION NOTATION

A method for specifying a given multidirectional laminate configuration has been established and is now routinely used on engineering drawings and documents. The following items essentially explain this laminate orientation notation:

1. Each layer or lamina is denoted by the angle representing the orientation (in degrees) between its fiber orientation and the reference structural axis in the x direction of the laminate.
2. Individual adjacent angles, if different, are separated by a slash (/).
3. Layers are listed in sequence starting with the first layer laid up, adjacent to the tool surface.
4. Adjacent layers of the same angle are denoted by a numerical subscript.
5. The total laminate is contained between square brackets with a subscript indicating that it is the total laminate (subscript T) or one-half of a symmetric laminate (subscript S).
6. Positive angles are assumed clockwise looking toward the lay-up tool surface, and adjacent layers of equal and opposite signs are specified with + or − signs as appropriate.
7. Symmetrical laminates with an odd number of layers are denoted as symmetric laminates with an even number of plies, but with the center layer overlined.

The notations for some commonly used laminate configurations are illustrated in Fig. 35.4.

In essence, lamination theory is involved in the transformation of the individual stiffnesses of each layer in the principal directions to the direction of orientation in the laminate, thereby providing the stiffness characterization for the specified laminate configuration. Subsequently, application of a given system of loads is broken down into individual layer contributions and referred back to the principal directions in each layer. A failure criterion is then used to assess the margin-of-safety arising in each layer. The complete process is illustrated in Fig. 35.5.

FAILURE CRITERIA

Although much debate and development has occurred with regard to the most appropriate failure criteria for composite laminates, the most widely adopted

Ply	Angle
8	0
7	−45
6	90
5	+45
4	+45
3	90
2	−45
1	0

Notation $[0/\pm45/90]_s$

or

Ply	Angle
6	9
5	60
4	120
3	120
2	60
1	0

$[0/60/120]_s$

Required
(i) When properties required to be equal in all directions in plane of laminate:
Tensile strength
Compressive strength
Modulus
Coefficient of thermal expansion (CTE)
(ii) In some cases of combined tension or compression and shear.

(a)

Ply	Angle
11,12	0
10	45
9	90
8	−45
6,7	0
5	−45
4	90
3	45
1,2	0

Notation $[0_3/\pm45/90]_s$

or

Ply	Angle
20	90
19	60
16,17,18	0
15	120
12,13,14	0
11	60
10	60
7,8,9	0
6	120
3,4,5	0
2	60
1	90

$[0_6/60_2/120/90]_s$

Used
(i) When load or stiffness requirement is highly directional.
(ii) To attain low CTE in one direction with low modulus fibers.

(b)

Ply	Angle
8	90
7	+θ
6	0
5	−θ
4	−θ
3	0
2	+θ
1	90

Laminate "balanced" about median plane

Notation $[90/\theta/0/-\theta]_s$

(i) Required to prevent warpage of flat laminate.
(ii) Can be departed from in contained circular parts: cylinders, cones, etc.
(iii) Small deviations near median plane possible; verify by test.

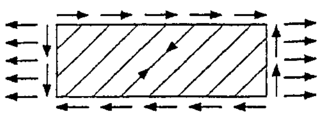
(c)

Ply	Angle
10	90
8 9	+θ
7	0
6	−θ
5	−θ
4	0
2 3	+θ
1	90

Balanced but more +θ than −θ

Notation $[90/+\theta_2/0/-\theta]_s$

(i) Will not warp, but



(ii) Applied axial load induces shear due to strain in excess +θ plies.

(d)

FIGURE 35.4 Examples of laminates and conventional notations.

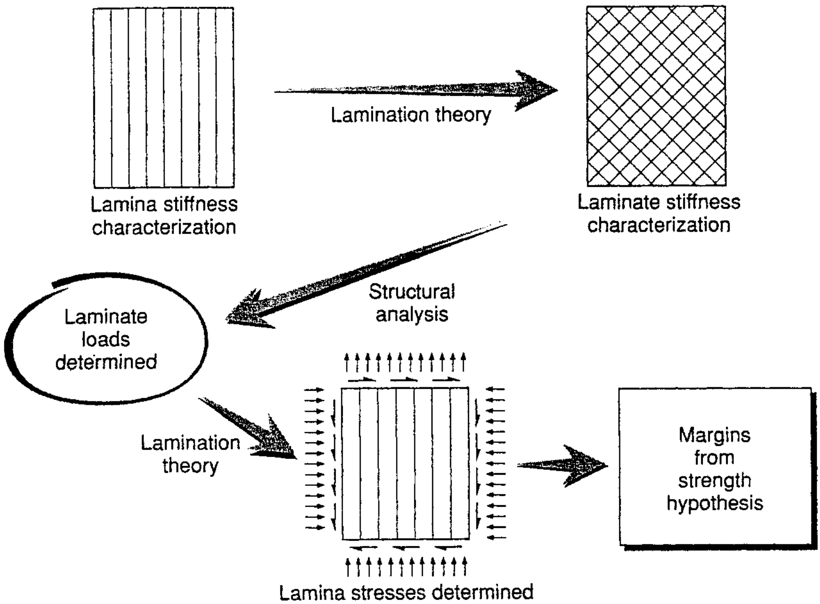


FIGURE 35.5 Procedure for strength determination.

approach in composite applications is the *maximum strain criterion.* The application of this relatively simple criterion requires an experimental database for the ultimate strains for each of the three fundamental loading directions for the individual orthotropic layer comprising the laminate. The three fundamental loading directions refer to axial loading in the fiber direction, axial loading transverse to the fiber direction, and in-plane shear associated with the former directions. However, it should be acknowledged that the ultimate strain values may be markedly different for tension and compression both in the fiber direction and transverse to it. Thus a total of the following five ultimate strains are required to facilitate application of the maximum strain criterion:

1. ε_L^{tu} is the ultimate tensile strain in the fiber direction.
2. ε_L^{cu} is the ultimate compressive strain in the fiber direction.
3. ε_T^{tu} is the ultimate tensile strain transverse to the fiber direction.
4. ε_T^{cu} is the ultimate compressive strain transverse to the fiber direction.
5. γ_{LT}^{su} is the ultimate shear strain associated with directions parallel and normal to the fiber direction.

In connection with the actual values used for (1) through (5), see the previous discussion on *In-Situ Properties,* which explains how the individual layer properties must be adjusted to represent the strength or ultimate strain values of a given layer that is contained within a multidirectional laminate. The prudent approach in engineering development work is to identify special laminate configurations that may be used to establish representative "in situ" properties for the range of potential candidate laminates for application to a specific design.

COUPLING, BALANCE, AND SYMMETRY

The mathematical relationships obtained in laminated plate theory define all the coupling relationships arising in the arbitrary laminate. However, a discussion of the physical aspects of such coupling phenomena and the laminate designs that may be invoked to suppress these responses is helpful to the structural engineer.

Extension-Shear Coupling. First, the in-plane coupling between extension and shear or vice versa arises in the case of any off-axis layer, for example,

$$\gamma_{xy} = S_{16}\sigma_x \qquad \text{or} \qquad \varepsilon_x = S_{16}\tau_{xy} \tag{35.1}$$

or, for the inverse situation,

$$\sigma_x = Q_{16}\gamma_{xy} \qquad \text{or} \qquad \tau_{xy} = Q_{16}\varepsilon_x \tag{35.2}$$

where S_{16} and Q_{16} are, respectively, the compliance and stiffness terms defining the coupling magnitudes.[3] From a physical point of view, the shear deformation induced by an axial tensile stress is caused by the tendency for the layer to contract along the diagonals by unequal amounts due to differences in the Poisson's ratio in these two directions. Alternatively, considering the special case of a +45° layer, the axial stress may be resolved into planes at +45° and −45° to the direction of applied stress. The resulting strains due to equal resolved stress components along these directions are obviously different.

Intuitively, it is easily rationalized that the use of a $[\pm\theta]_T$ laminate will result in the mutual suppression of the tension-induced shear deformation in each individual layer. In the general case, equal numbers of layers in the off-axis, $+\theta$ and $-\theta$, layers will suppress this coupling; the resulting laminate is termed a *balanced laminate.*

Extension-Torsion Coupling. For this the previous balanced laminate $[\pm\theta]_T$ is considered. The spatial separation in the thickness direction results in equal and opposite deformations in the shear deformation induced by an axial tensile stress. This deformation situation therefore results in twisting of the laminate, a condition that is illustrated in Fig. 35.6. From a simplistic viewpoint, the illustration presented in Fig. 35.7 provides a type of designers' guide to coupling evaluations, which facilitates rational judgments in laminate design. All the responses indicated in these two figures can be confirmed by use of conventional lamination theory. Suppression of the twisting deformation is achieved by use of a symmetric laminate in which the off-axis layers below the central plane are mirrored by an identical off-axis layer at the same distance above the central plane (see Fig. 35.7).

Extension-Bending Coupling (Related through B_{11} and B_{22} Matrix Components). The simplest form of laminate, exhibiting a coupling between in-plane extension (or compression) and bending deformation, is the $[0°, 90°]_T$ unsymmetrical laminate. This response can be rationalized, on a physical basis, by recognizing that the neutral plane for this two-layer laminate will be located within the stiffest 0° layer, giving rise to a bending moment produced by the in-plane forces applied at the midplane and the associated effect between the two planes. For this case, it is clearly seen that the coupling would be suppressed by use of a four-layer symmetric laminate, i.e., $[0°, 90°]_s$, or a three-layer symmetric laminate such as $[0°, \overline{90°}]_s$, where the bar over the 90° layers signifies that this layer orientation is not repeated.

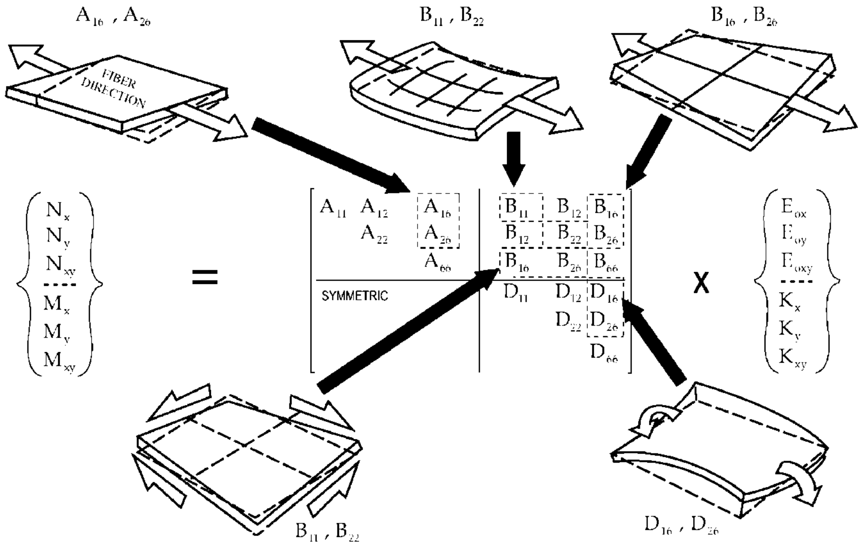


FIGURE 35.6 Illustration of coupling phenomena in laminated composite plates.

In-Plane Shear-Bending Coupling (Related through B_{16} and B_{26} Matrix Components). To visualize the mechanism associated with this mode of coupling, consider a $[\pm 45°]_T$ unsymmetrical, two-layer laminate subjected to in-plane shear loads. By recognizing that the in-plane shear is equivalent to a biaxial tension and compression loading with the tensile direction in the lower layer aligned with the fiber direction and, in the upper layer, transverse to the fiber direction, it will be realized that the plate will assume a torsional deformation (see Fig. 35.6).

Bending-Torsion Coupling (Related through D_{16} and D_{26} Matrix Components). For this mode of coupling, a four-layer balanced symmetric laminate, i.e., $[\pm\theta]_s$, is considered. The application of a bending moment, and an associated strain gradient, to this laminate will induce different degrees of shear coupling to the outer and inner layers. As a consequence, the response of the outer layers will dominate due to the higher strain levels in these layers, resulting in a net torsional deformation, as illustrated in Fig. 35.6. For qualitative assessment of this mode of coupling, the magnitude of the shear responses can be considered to exert an internal couple on the laminated plate as illustrated in Fig. 35.7. A similar rationale can be used to design a laminate that would not exhibit this coupling. For example, an eight-layer laminate of the configuration

$$[(\pm\theta)_s/(\mp\theta_s)]_T \qquad \text{or} \qquad [\pm\theta, \mp\theta, \mp\theta, \pm\theta]_T$$

will not exhibit bending-torsion coupling.

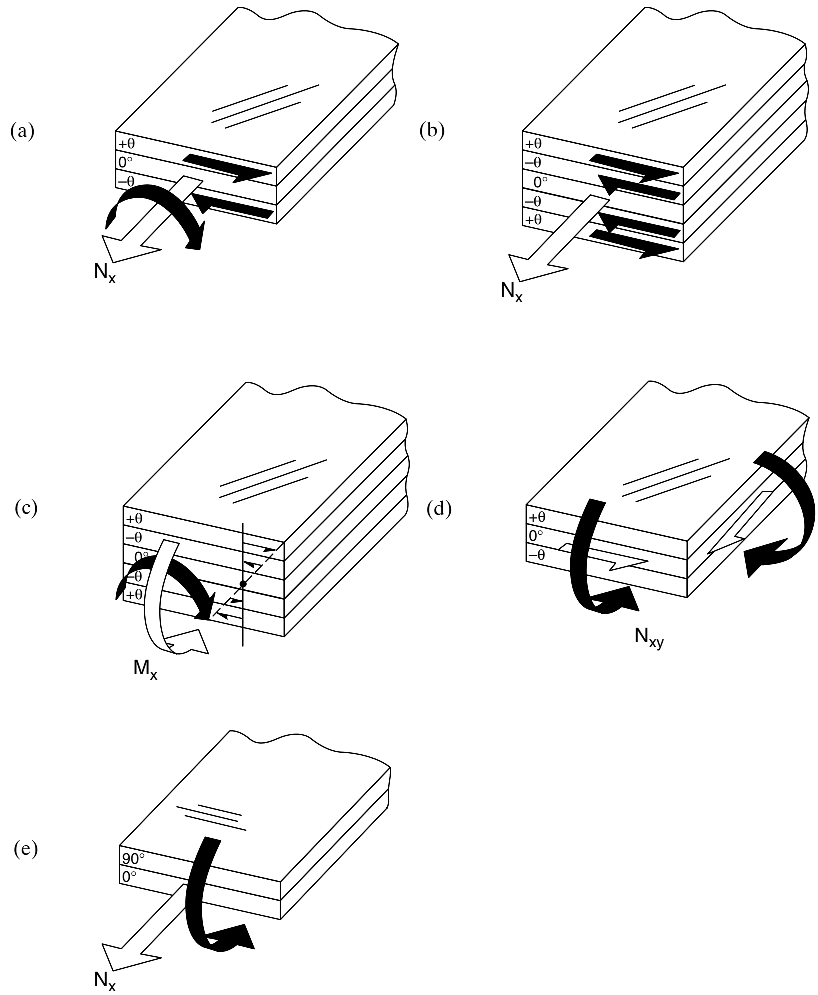


FIGURE 35.7 Designer's guide to coupling evaluation. (*a*) $B_{16} \neq 0$. (*b*) $B_{16} = 0$. (*c*) $D_{16} \neq 0$. (*d*) B_{16}, $B_{26} \neq 0$. (*e*) B_{11}, $B_{22} \neq 0$. Open arrows: applied force/moment. Shaded arrows: resulting displacement.

GENERAL LAMINATE DESIGN PHILOSOPHY

The recommended approach for laminates that are required to support biaxial loads is conveyed in the family of laminates represented by the shaded area in Fig. 35.8. This figure merely provides guidelines for selecting suitable laminates that have been shown to be durable and damage-tolerant. However, the form of presentation is also adopted for a system of *carpet plots* that can be very useful in the design and analysis of laminates for a specific composite system. These carpet plots facilitate reasonable predictions of the elastic and strength properties, and the coefficients of thermal expansion for a family of practicable laminates that comprise 0°, +45°, and

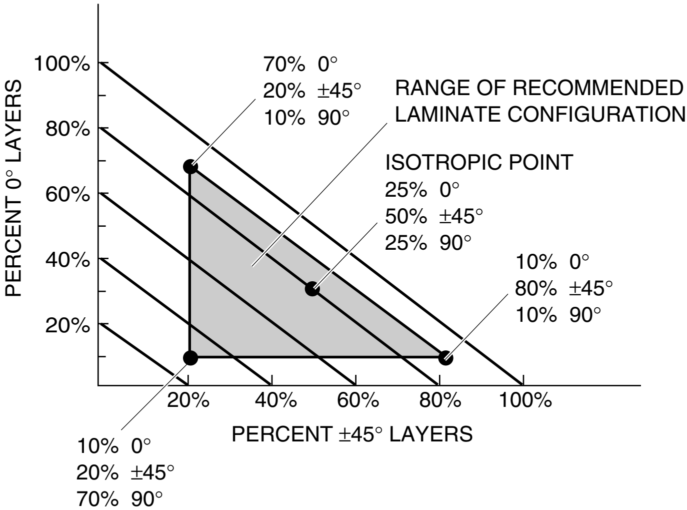


FIGURE 35.8 General guidelines for the selection of durable, damage-tolerant laminate design.

90° fiber orientations of any proportions in an assumed balanced, symmetric laminate arrangement. Examples of these carpet plots are presented in Ref. 3 and in most of the texts referenced previously. Even for highly directional loading, a nominal (approx. 10 percent) amount of layers, in each of the 0°, 90°, +45°, and –45° directions, should be included for the following reasons:

1. Providing restraints that inhibit development of microcracks that typically form in directions parallel to fibers.
2. Improved resistance to handling loads and enhanced damage tolerance (this is especially relevant for relatively thin laminates, i.e., less than 0.200 in. thick).
3. More manageable values of the major Poisson's ratio (ν_{xy}), particularly where interfaces exist with other materials or laminates with values in the 0.30 range.
4. Compatibility between the thermal expansion coefficients with respect to adjacent structure.

Other commonly adopted and recommended practices include laminate designs that minimize the subtended angle between adjacent layers and use of the minimum practicable number of layers of the same orientation in one group. To illustrate the former, a laminate configuration of $[0°, +45°, 0°, –45°, 90°]_s$ is preferred over a laminate such as $[0°, +45°, –45°, 0°, 90°]_s$ even though the in-plane thermoelastic properties would be identical for these two laminates. For the latter, the length of transverse microcracks tends to be limited by the existence of the layer boundaries; hence a $[0°, +45°, 0°, –45°, 0°, 90°]_s$ laminate is preferred over a $[0°_3, +45°, –45°, 90°]_s$ laminate.

FATIGUE PERFORMANCE

The treatment of fatigue and damage accumulation in composite design is greatly complicated by the heterogeneity and anisotropy of the material in the laminated form. As a consequence, there is a multiplicity of mechanisms for the initiation and propagation of damage and, understandably, the approaches, such as Miner's cumulative damage rule discussed in Chap. 34, are not recommended. For similar reasons the test results obtained from small laboratory test coupons can rarely be used directly in support of design for prediction of fatigue performance. Nevertheless, such test coupon data can serve the purpose of obtaining preliminary indications of the fatigue performance of specific laminate design configurations.

Basic failure mechanisms that occur in laminated composites, in general, include the following:

1. Transverse cracking of individual layers in multidirectional laminates which will typically arrest at the interlaminar boundaries.
2. Fiber-matrix debonding which often can contribute to premature transverse cracking.
3. Delamination between layers due to interlaminar shear and/or tensile stress components that can be initiated by the aforementioned transverse cracks. Out-of-plane or bending loads on the structure will tend to give rise to such delamination.
4. Fiber breakage which will usually occur in the later stages of damage growth under monotonic static loading or under cyclic loading. However, most reinforcing fibers are not, in themselves, fatigue sensitive.

The first two initiating mechanisms motivate the above general laminate design philosophy advocated in the previous section, as illustrated in Fig. 35.8. A common sequence of failure events is illustrated for a quasi-isotropic, $[\pm 45°, 0°, 90°]_s$, carbon/epoxy laminate, also summarized in Fig. 35.9 (adapted from Ref. 9).

It may be stated, with some confidence, that the composites industry is able to design polymer matrix composite (PMC) laminates of uniform thickness in a reliable manner. Extensive experience with PMCs has taught us to use fiber-dominated laminate designs, which are most often specified in the $[0°/\pm 45°/90°]_s$ or pseudo-isotropic form with respect to the in-plane directions. In-plane compression failure is somewhat of an exception since the matrix and the degradation thereof can develop delaminations and influence premature failure mechanisms. However, by far the largest number of development and in-service problems with composite hardware are associated with matrix-dominated phenomena, that is, interlaminar shear and out-of-plane tension forces. This is a major concern in that failure contributed by either one or a combination of these matrix-dominated phenomena are susceptible to the following:

1. High variability contributed by sensitivity to processing and environmental conditions.
2. Brittle behavior, particularly for early, i.e., 1970s era, epoxy matrix systems.
3. Inspectability of local details where flaws or defects may exist.
4. Low reliability associated with the lack of acceptable or representative test methods and complex, highly localized, stress states (the use of the transverse tensile strength of a unidirectional laminate for out-of-plane or thickness tensile strength is generally unconservative).
5. Potential degradation of residual static strength after fatigue/cyclic load exposure.

The development of stress components that induce interlaminar shear/out-of-plane tension failures was illustrated in Fig. 35.2, where commonplace generic features of composite hardware designs that frequently experience delaminations are

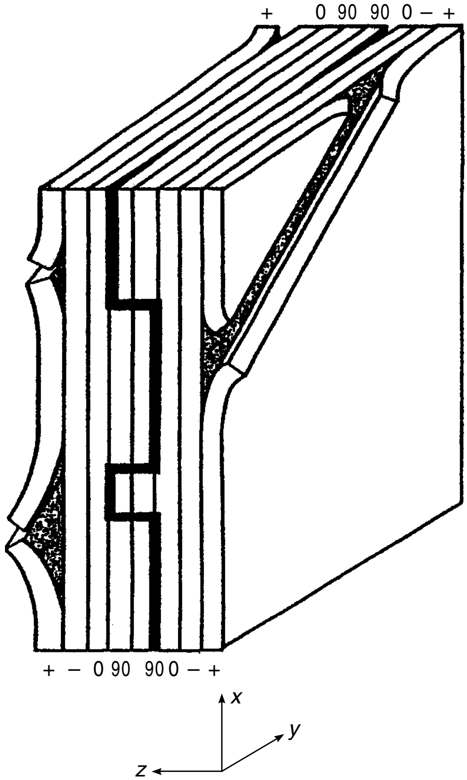


FIGURE 35.9 A common sequence of fatigue failure events for a $[\pm 45/0/90]_s$ pseudo-isotropic carbon/epoxy laminate: transverse cracking of 90° plies; edge delamination at 0° → 90° interfaces; transverse cracking of ±45° plies; delaminations at 45° → +45° then at 45° → 0° interfaces; fiber failures in 0° plies. (*Adapted from Ref. 9.*)

shown. It is at such details that PMC structures are particularly vulnerable both under static and fatigue loading. The propensity for delamination and localized matrix-dominated failures that represents a general characteristic of many PMCs is that notch sensitivity may be reduced after fatigue load cycling for local through-thickness penetrations. On the other hand, this demands that a fatigue life methodology should be available to deal with composite structures that are subjected to out-of-plane load components. Naturally, the capability of predicting the fatigue life is an essential element in the process of qualifying, or certifying, composite products and systems.

The design requirements generally specified for qualifying and/or certifying a composite product typically include (*a*) static strength, (*b*) fatigue/durability, and (*c*) damage tolerance. All of these requirements rely on a comprehensive appreciation of failure modes; the variability (or scatter); discontinuities caused by notches, holes, and fasteners; and environmental factors, particularly damage caused by the impact of foreign objects, machining, and assembly phenomena.

In the case of fatigue, three potential design approaches are considered. The particular selection may be based on the nature of usage, economics, safety implications, and the specific hardware configuration. Often some combination of approaches may be adopted particularly during the developmental phase. These three general categories of approach are the (*a*) Safe Life/Reliability Method, (*b*) Fail Safe/Damage Tolerance Method, and (*c*) Wearout Model.

SAFE LIFE/RELIABILITY METHOD

Statistically based qualification methodologies[9–11] provide a means for determining the strength, life, and reliability of composite structures. Such methods rely on the correct choice of population models and the generation of a sufficient behavioral database. Of the available models, the most commonly accepted for both static and fatigue testing is the two-parameter *Weibull distribution.* The Weibull distribution is attractive for a number of reasons, including the following:

1. Its simple functional form is easily manipulated.
2. Censoring and pooling techniques are available.
3. Statistical significance tests have been verified.

The cumulative probability of the survival function is given by

$$P_s(x) = \exp\,[(-x/\beta)^{\alpha_s}] \tag{35.3}$$

where α_s is the shape parameter and β is the scale parameter.

For composite materials, α_s and β are typically determined using the maximum-likelihood estimator.[15] In addition, the availability of pooling techniques is especially useful in composite structure test programs where tests conducted in different environments may be combined. Statistical significance tests are used in these cases to check data sets for similarity.

The following paragraphs present a review of the statistical method of Ref. 10. The development tests required to generate the behavioral database are outlined, followed by a discussion of the specific requirements for static strength and fatigue life testing. Special attention is given to the effect that matrix- and fiber-dominated failure modes have on test requirements.

A key to the successful application of any statistical methodology is the generation of a sufficiently complete database. The tests must range from the level of coupons and elements to full-scale test articles in a building-block approach. Additionally, the test program must examine the effects of the operating environment (temperature, moisture, etc.) on static and fatigue behavior. The coupon and subelement tests are used to establish the variability of the material properties. Although they typically focus on the in-plane behavior, it is also important to include the transverse properties. This is especially important in the case of research and development programs. The resulting data can be pooled as required and estimates of the Weibull parameters made. Thus, the level and scatter of the possible failure modes can be established. The transverse data are characterized by the highest degree of scatter. Element and subcomponent tests can be used to identify the structural failure modes. They may also be used to detect the presence of competing failure modes. Higher-level tests, such as tests of components, can be used to investigate the variability of the structural response resulting from fabrication techniques. The resulting database should describe, to the desired level of confidence, the failure mode, the data scatter, and the response variability of a composite structure. These data along with full-scale test articles can be used in the argument to justify qualification.

Out-of-plane failure modes can complicate the generation of the database. Well-proven and reliable transverse test methods are few. The typically high data scatter makes higher numbers of tests desirable. In addition, the increased environmental sensitivity in the thickness direction can cause failure mode changes, negating the ability to pool data and possibly resulting in competing failure modes. Thus, a design whose structural capability is limited by transverse strength can lead to increased testing requirements and qualification difficulties.

The static strength of a composite structure is typically demonstrated by a test to the *design ultimate load* (DUL), which is 1.5 times the maximum operating load, that is, the *design limit load* (DLL). Figure 35.10 shows the reliability achieved for a single static ultimate test to 150 percent of the DLL for values of the static strength shape parameter from 0 to 25. For fiber-dominated failure with α_s values near 20, such a test would demonstrate an *A-basis value,* which is defined as the value above which at least 99 percent of the population is expected to fall, with a confidence of 95 percent (a statistical tolerance limit as detailed in Chap. 20). However, for matrix-

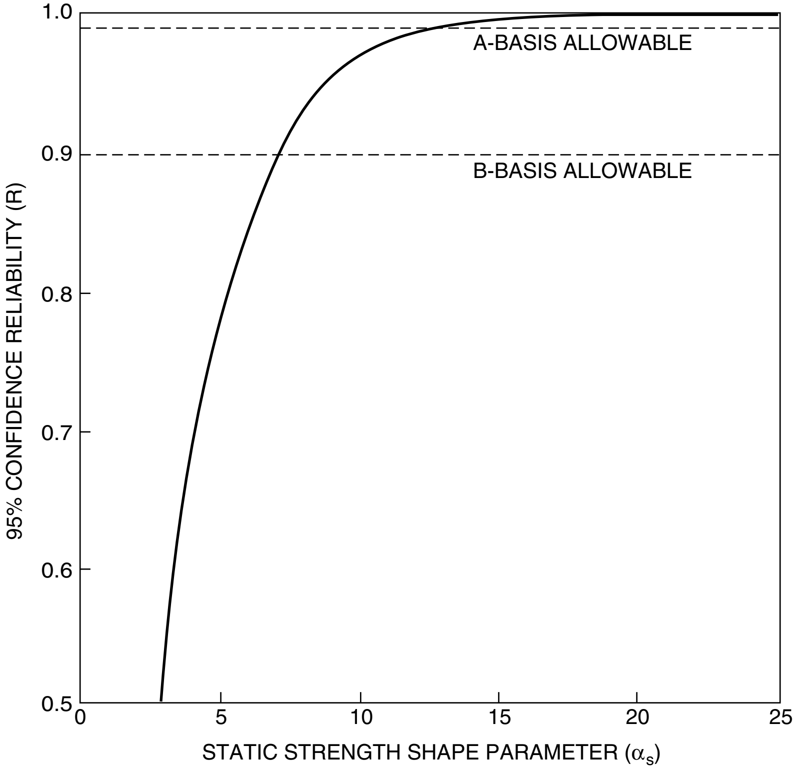


FIGURE 35.10 Plot of the 95 percent confidence reliability against the static strength shape parameter for a single full-scale static test to 150 percent of the design limit load.

dominated failure modes, with α_s ranging from 5 to 10, a test to 150 percent of the DLL would not demonstrate an A-basis value. Two options are available to increase the demonstrated reliability, namely, (*a*) increasing the number of test specimens, or (*b*) increasing the load level. The most effective choice is to increase the load level beyond 150 percent of the DLL, whereas increasing the number of test specimens yields little benefit and is expensive.

The two most applicable methods of statistical qualification approaches for fatigue are the *life factor* (also known as the *scatter factor*) and the *load enhancement factor.* The life factor approach relies on a knowledge of the fatigue life scatter factor from the development test program and full-scale test or tests. The factor gives the number of lives that must be demonstrated in tests to yield a given level of reliability at the end of one life. A plot of life factor N_F against the fatigue life shape parameter α_L is given in Fig. 35.11 for a typical scenario. A single full-scale test to demonstrate the reliability of the *B-basis* value, defined as that value above which at

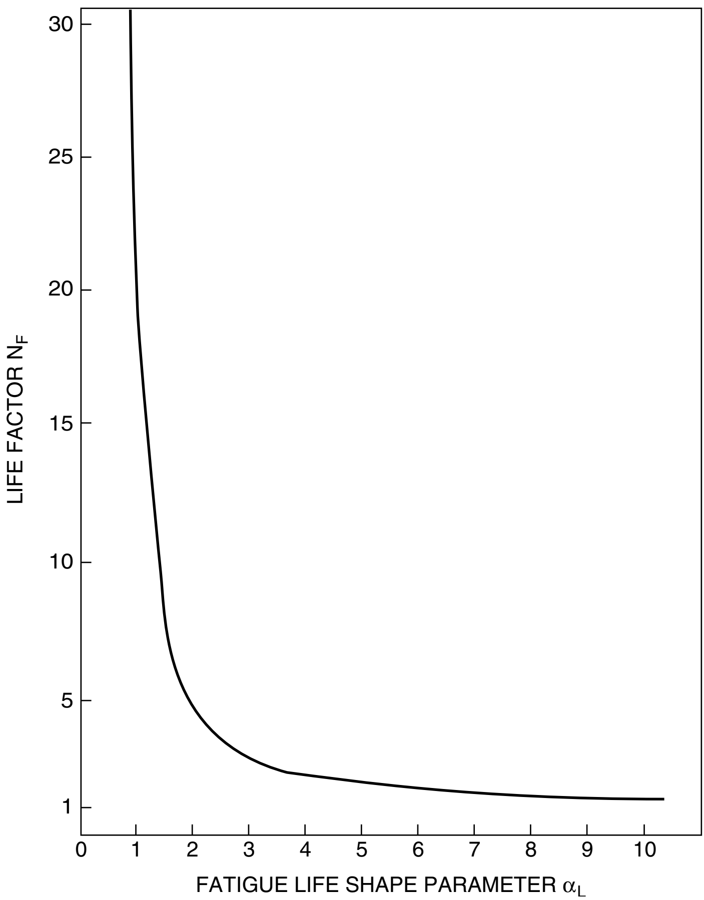


FIGURE 35.11 Plot of the life factor required to demonstrate the reliability of the B-basis results at the end of one life against the fatigue life shape parameter using a single full-scale test article.

least 90 percent of the population is expected to fall, with a confidence of 95 percent at the end of one life, is to be conducted. The curve shows that as the shape parameter approaches 1.0, the number of lives rapidly becomes excessive. Such is the case of an in-plane fatigue failure ($\alpha_L = 1.25$). Although few data for transverse fatigue are available, other than perhaps for bonded parts, it is reasonable to assume that the value of the shape parameter will be the same or less. Hence, it is apparent that the life factor approach is not acceptable for the certification of composites, especially where out-of-plane failure modes are dominant.

An alternative approach to life certification is the load enhancement factor, wherein the loads are increased during the fatigue test to demonstrate the desired level of reliability. Figure 35.12 illustrates the effect of the fatigue life shape parameter α_L and the residual-strength shape parameter α_R on the load enhancement factor F required to demonstrate B-basis reliability for one life using a single full-scale fatigue test to one lifetime. It is obvious that the required factor does not change significantly for fatigue life shape parameters in the range of 5 to 10. However, as the shape parameter approaches 1.0, as is the case for composites, the required load enhancement factor increases noticeably, especially for small values of the residual-strength shape parameter. This curve illustrates well the potential problems that may arise from dominant out-of-plane failure modes. Such failure modes tend to have low values of α_L (near 1.0) and also low values of α_R (in the range from 5.0 to 10.0). These values would make the required load enhancement factors prohibitively large. It is evident that for failure modes that exhibit a high degree of static and fatigue scatter, the life factor and load enhancement factor approaches can result in impossible test requirements. A combined approach can be achieved through the manipulation of the functional expressions. The resulting method allows some latitude in balancing the test duration and the load enhancement factor to demonstrate a desired level of reliability.

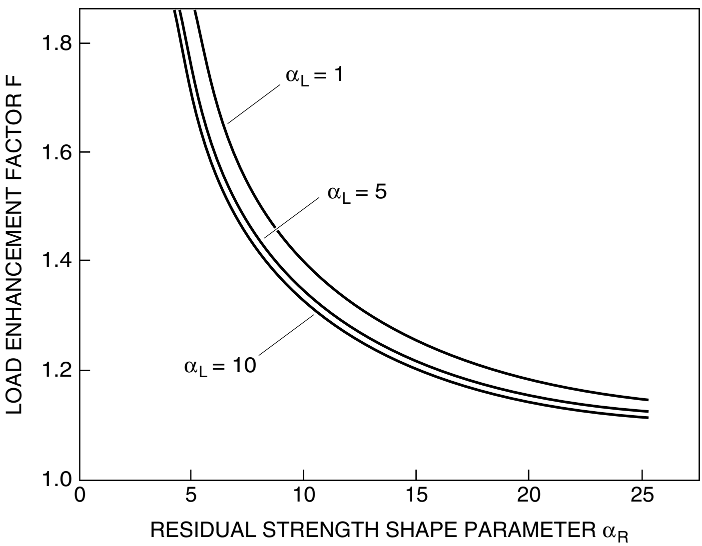


FIGURE 35.12 Plot of the load enhancement factor required to demonstrate the reliability of the B-basis results at the end of one life against the residual-strength shape parameter for three values of fatigue life shape parameter using a single fatigue test to one lifetime.

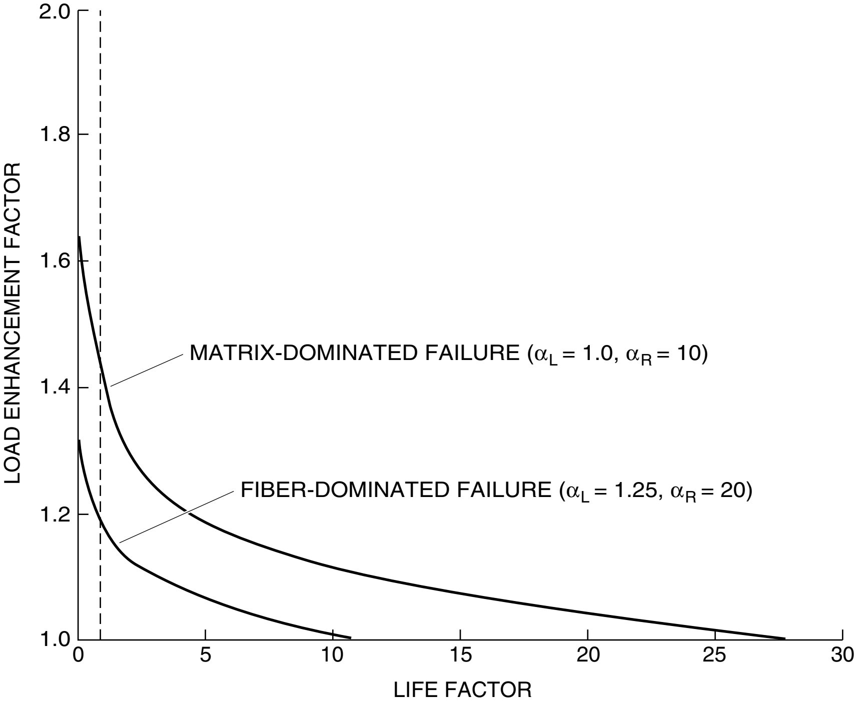


FIGURE 35.13 Plot of possible combinations of load enhancement factor and life factor necessary to demonstrate the reliability of the B-basis results at the end of one lifetime using a single full-scale test article for matrix- and fiber-dominated failure modes.

Figure 35.13 gives the curves of load enhancement factor against life factor for the cases of fiber- and matrix-dominated failures. Typical values for the fatigue life and residual-strength shape parameter were employed. The curves show the possible combinations of life factor (or test duration) and load enhancement factor to demonstrate the B-basis reliability at the end of one lifetime using a single full-scale fatigue test article. The curve for fiber-dominated failure modes exhibits quite reasonable values of life factor and load enhancement factor. For test durations ranging from 1 to 5 lifetimes, the load enhancement factor ranges from 1.18 down to 1.06. However, the test requirements for matrix-dominated failure are more severe. Over the range of life factor from 1 to 5, the load enhancement factor ranges from 1.4 down to 1.19. An environmental compensation factor would further complicate the test of a matrix-dominated failure. Such a factor must be combined with the load level. As is well known in composites, the adverse effects of environment on matrix properties are much more severe than on fiber-dominated properties, and the resulting factor may be significant.

Further illustration of the problems induced by a matrix-dominated failure is possible by assuming a limit exists on the load enhancement factor. Such limits may exist because of failure mode transitions at higher load levels. For instance, assuming a load enhancement factor of 1.2 is the maximum allowable value, it is obvious that a successful one-lifetime test for a fiber-dominated failure will demonstrate the reliability better than a B-basis test. For matrix-dominated failure, the same reliability would require a test duration of about 4.5 lives.

Two important aspects of the statistical qualification methodology are the generation of an adequate database and the proper execution of a full-scale demonstration test. The development test program must be conducted in a "building block" approach that produces confident knowledge of the material shape parameters, environmental effects, failure modes, and response variability. Perhaps the most important result should be the ability to predict the failure mode and know the scatter associated with it. Structures that exhibit transverse failures, which can result in competing modes and a high degree of scatter, may render the application of this fatigue methodology impractical. This result has been illustrated by the effect of shape parameters on both the static and fatigue test requirements. The requirements clearly show that a well-designed structure that exhibits fiber-dominated failure modes will be more easily qualified than one constrained by matrix-dominated effects.

FAIL SAFE/DAMAGE TOLERANCE METHOD

The damage tolerance philosophy assumes that the largest undetectable flaw exists at the most critical location in the structure, and the structural integrity is maintained throughout the flaw growth until detected by periodic inspection.[12] In this approach, the damage tolerance capability covering both the flaw growth potential and the residual strength is verified by both analysis and test. Analyses would assume the presence of flaw damage placed at the most unfavorable location and orientation with respect to applied loads and material properties. The assessment of each component should include areas of high strain, strain concentration, a minimum margin of safety, a major load path, damage-prone areas, and special inspection areas. The structure selected as critical by this review should be considered for inclusion in the experimental and test validation of the damage tolerance procedures. Those structural areas identified as critical after the analytical and experimental screening should form the basis for the subcomponent and full-scale component validation test program. Test data on the coupon, element, detail subcomponent, and full-scale component level, whichever is applicable, should be developed or be available to (*a*) verify the capability of the analysis procedure to predict damage growth/no growth and residual strength, (*b*) determine the effects of environmental factors, and (*c*) determine the effects of repeated loads. Flaws and damage will be assumed to exist initially in the structure as a result of the manufacturing process, or to occur at the most adverse time after entry into service.

A decision to employ proof testing must take the following factors into consideration:

1. The loading that is applied must accurately simulate the peak stresses and stress distributions in the area being evaluated.
2. The effect of the proof loading on other areas of the structure must be thoroughly evaluated.
3. Local effects must be taken into account in determining both the maximum possible initial flaw/damage size after testing and the subsequent flaw/damage growth.

The most probable life-limiting failure experienced in composite structure, particularly in nonplanar structures where interlaminar stresses are present, is delamination growth. Potential initiation sites are free edges, bolt-holes, and ply terminations (see Fig. 35.2), in addition to existing manufacturing defects and subsequent impact damage. Hence, an analysis technique for the evaluation of growth/no growth of delaminations is an essential tool for the evaluation of the damage tolerance of composite structures. A numerical method is available through the use of finite element analysis (see Chap. 28, Part II) and the crack closure integral technique from fracture mechanics.[13] Prerequisites for an evaluation are as follows:

1. A structural analysis made in sufficient detail to indicate the locations where the critical interlaminar stresses exist.
2. Experimentally based critical interlaminar strain energy release rates G_{Ic}, G_{IIc}, and a subcritical growth law, that is, *da/dN,* where *da/dN* is the rate of change of the crack length or damage zone size *a* with the number of cycles *N,* against ΔG for each mode (see Chap. 34).
3. A mixed mode I/mode II fracture criterion.

The test specimens used to generate the required mode I and mode II fracture toughness parameters are described in detail in Ref. 14. The application of this approach requires a significant analysis and test effort to evaluate hot spots within the structure and to generate the necessary fracture toughness data. One limitation is the absence of a reliable mixed-mode fracture criterion, and consequently this method is not considered sufficiently mature to warrant a recommendation for wide general application, particularly for developmental composite hardware evaluations.

THE WEAROUT MODEL

Wearout is defined as the deterioration of a composite structure to the point where it can no longer fulfill its intended purpose. The wearout methodology was developed in the early 1970s and is comprehensively summarized in Ref. 15. The essential features are portrayed in Fig. 35.14. This methodology was previously used by the military aircraft command for the certification of several composite aircraft components. In essence, the wearout approach recognizes the probability of progressive structural deterioration of a composite structure. The approach utilizes the development test data on the static strength and the residual strength, after a specified period of use, in conjunction with proof testing of all product hardware items to characterize this deterioration and protect the structure against premature failures. It has become evident that the residual stiffness is an indicator of the extent of the structural deterioration and can be an important performance parameter with regard to the natural frequencies of oscillation of the aerodynamic surfaces. Thus, in some instances, it may be prudent to incorporate a residual-stiffness requirement in an adopted methodology to evaluate the tolerance of the structure to component stiffness degradation.

The difficulties in the implementation of the methodology include the determination of the critical load conditions to be applied for static and residual strength and stiffness testing and for the proof load specification. Similar difficulties would arise in the case of all candidate methodologies considered here, and indeed emphasize the importance of a representative structural analysis. However, the advantage of the wearout approach for advanced composite hardware development projects resides in the ability to assign gates for safe flight testing as the flight envelope is progressively expanded.

Since the era of the initial development and interest in the wearout approach, there appears to have been minimal development or usage. Nevertheless, the poten-

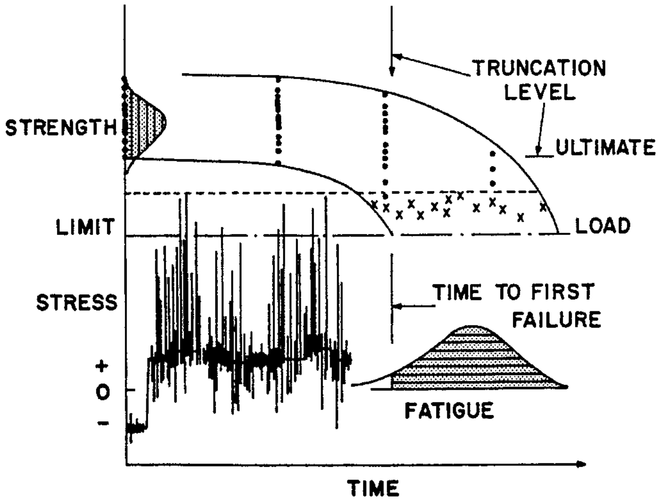


FIGURE 35.14 Essential features of the "wearout model" relating static failure, load history, and fatigue failure.

tial motivation for a methodology of this type calls for a brief review of the physical and theoretical basis for the important concepts. Further detail can be found in Refs. 15 and 16.

By combining several basic assumptions regarding the behavior of a composite structure under load with basic Weibull statistics, a *kinetic fracture model* can be derived. This model serves to assist in predicting the fatigue wearout behavior of composite structures. The first assumption concerns the growth rate of an inherent or real material flaw, da/dt, which is deemed to be proportional to the strain energy release rate G of the material system raised to some power r, where r is to be determined experimentally. Thus

$$da/dt \propto G^r \tag{35.4}$$

where a is the flaw length. As the cyclic load, $F(t)$, is applied to the flawed body, the internally stored strain energy will occasionally exceed the critical level required to overcome the local resistance of the material to flaw growth or damage accumulation, and flaw or damage growth will occur. Impediments to further development have been related to those cited in Chap. 34, as it pertained to the fracture mechanics method for metals, i.e., the need for further data to define the growth rate and/or threshold level below which the damage area does not grow. One important wearout parameter r is defined as the slope of the da/dN curve, or may be derived from the S-N curve for the failure/damage mode in question.

Various relationships have been proposed[15] relating the initial Weibull static shape parameter, α_0, and the fatigue life shape parameter, α_f, both of which tend to be a function of the damage size exponent alone. Specifically, available relationships are given by

$$\alpha_0 = 2r + 1 \qquad \text{and} \qquad \alpha_f = \frac{\alpha_0}{2(r-1)} \tag{35.5}$$

Postulating that the composite system will lose strength at a uniform rate with respect to a logarithmic scale of cycles or time, then from the specific fatigue curve expressed as

$$NF_b^\gamma = B_N \qquad \text{or} \qquad tF_b^\gamma = B_t \tag{35.6}$$

the slope of the fatigue curve is given by $\gamma = -1/2$. In Ref. 16, a compilation of data on damage growth rate exponents from a broad range of literature items, including various types of polymer composite systems and composite bonded structures, were found to range between 4.3 and 6.6.

DAMPING CHARACTERIZATION

The major sources of damping in polymer matrix composites (PMCs) are associated with the visco-elastic or microplastic phenomena of the polymer matrix constituent and, to some degree for some composite systems, with weak fiber-matrix interfaces to microslip mechanisms. Other sources of damping, such as matrix microcracking and delamination resulting from poor fabrication conditions or service damage, can also create increased damping in certain cases. Very little or no damping is contributed by the fiber-reinforcement constituent with the possible exception of aramid, i.e., Kevlar, fibers. Environmental factors, such as temperature, moisture, and frequency, on the other hand, can have a significant effect on damping.

Two-phase materials therefore tend to derive any damping from the polymer matrix phase in a large majority of composite systems. Consequently, matrix-influenced deformations, such as the interlaminar shear and tension components, are the significant contributors. For the basic unidirectional composite, some closed-form predictive methods are available, but generally the micromechanics theories have been found to be unreliable for damping determinations, although reasonable for modulus predictions. Structural imperfections at the constituent level are considered to be the main contributors to this situation.

As mentioned earlier, micromechanics-based theories are available to give some indication of the effects of fiber volume content on damping parameters for unidirectional materials. One example based on conventional visco-elasticity assumption was formulated in Ref. 11 for the case of longitudinal shear deformation. For this case the *specific damping capacity* (SDC), ψ_{12}, for longitudinal shear can be expressed[17] as

$$\psi_{12} = \frac{\psi_m(1 - V_f)[(G + 1)^2 + V_f(G - 1)^2]}{[G(1 - V_f) + (1 - V_f)][G(1 - V_f) + (1 + V_f)]} \tag{35.7}$$

where ψ_m = the SDC for the matrix
G = the ratio of fiber shear modulus to that of the matrix
V_f = the fiber volume fraction

For the condition of flexural vibration of composite beams, the damping due to transverse shear effects that are highly matrix-dominated exhibit up to two orders of magnitude greater damping than pure axial, fiber-direction effects. Specific data, adapted from Ref. 18, on the SDC for the flexural vibration of unidirectional beams,

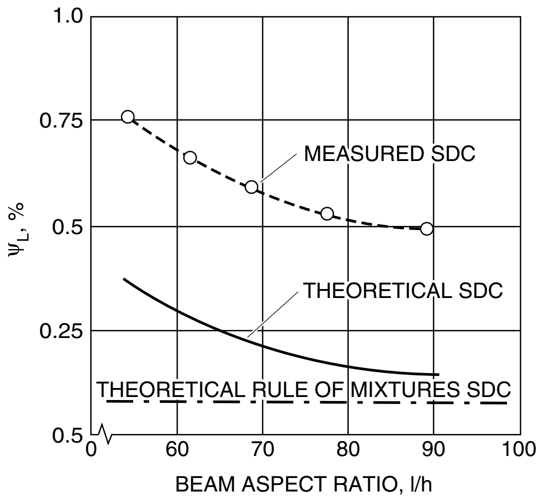


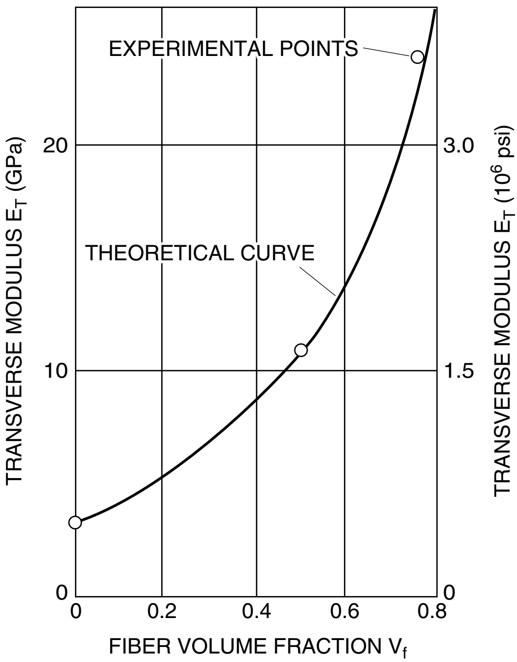
FIGURE 35.15 Variation of flexural damping with aspect *ratio* for high-modulus carbon fiber in DX209 epoxy resin $V_f =$ 0.5, SDC, shear damping contribution.

over a range of aspect ratios (length ℓ/thickness h), are compared to theoretical predictions in Fig. 35.15. Here the steady increase in damping for progressively lower beam aspect ratios is clearly due to the shear deformation which indicates a much stronger effect on damping than on the flexural modulus. The discrepancies in the theoretically predicted SDC in Fig. 35.15 is generally attributed again to imperfections in the composite at the constituent level.

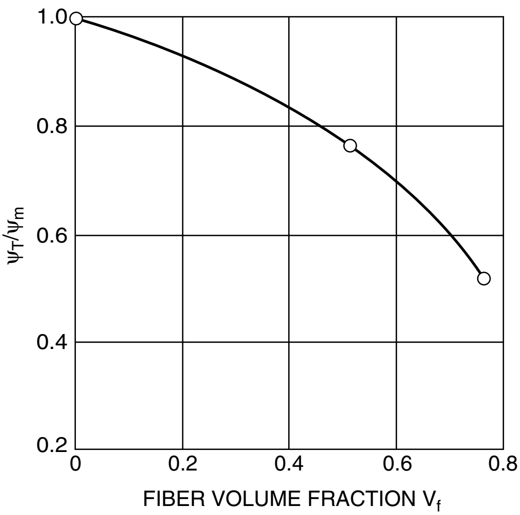
The damping trends for the other matrix-influenced deformational mode of transverse tension (at 90° to the fiber direction) in a unidirectional composite is illustrated in Fig. 35.16 for an E-glass fiber-reinforced epoxy over a wide range of fiber volume fractions V_f. Substantial damping can also occur in the deformation of an off-axis, unbalanced lamina or laminate, due to shear-induced deformation created by coupling under tension, compression, or flexural loading directed at an angle to the fiber direction. In Ref. 19, good correlation between the theoretical prediction and experimental measurements is demonstrated for a complete range of fiber orientations from 0° to 90° (see Fig. 35.17). Based on the flexural vibration of a high-modulus carbon-fiber/epoxy matrix system with $V_f = 0.5$, Fig. 35.17 compares both the flexural modulus and SDC. The latter damping parameter was predicted using the approximate relationship

$$\psi_\theta = E_x\left[\frac{\psi_2}{E_2}\sin^4\theta + \frac{\psi_{12}}{G_{12}}\sin^2\theta\cos^2\theta\right] \tag{35.8}$$

where
x = the axial direction of the beam
θ = the angle between the fiber direction and the axis of the beam
E_2, ψ_2 = the elastic modulus and SDC, respectively, in the transverse direction of the fiber
G_{12}, ψ_{12} = the shear modulus and shear-induced SDC, respectively, referred to directions parallel and perpendicular to the fibers



(a)



(b)

FIGURE 35.16 (*a*) Variation of transverse modulus with fiber volume fraction for unidirectional in glass/epoxy beam flexure. (*b*) Variation of transverse damping to matrix damping ratio with fiber volume fraction for unidirectional glass/epoxy beam flexure.

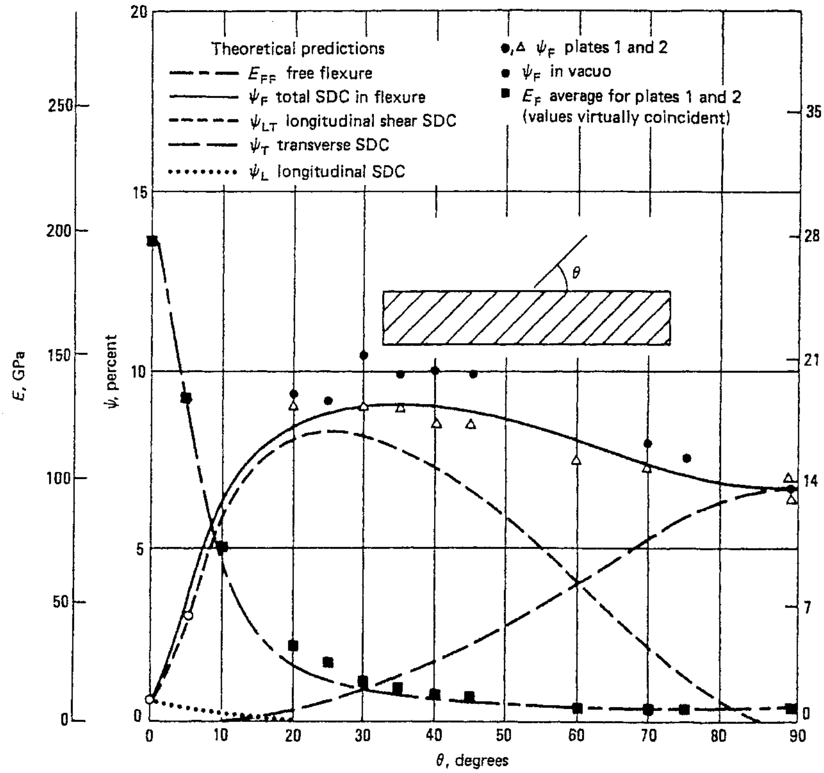


FIGURE 35.17 Variation of flexural modulus and specific damping capacity with fiber orientation for a carbon/epoxy, off-axis laminate in flexure.

In this relationship the modulus E_x is given by

$$\frac{1}{E_x} = \frac{\cos^4 \theta}{E_1} + \frac{\sin^4 \theta}{E_2} - \frac{2\nu_{12} \cos^2 \theta \sin^2 \theta}{E_1} + \frac{\cos^2 \theta \sin^2 \theta}{G_{12}} \tag{35.9}$$

With the above correlation as background, predictive methods for the damping of laminated beam specimens based on the classical laminate analysis method referenced above (see Ref. 3), the damping terms were incorporated and presented in Ref. 20 and summarized in Ref. 18. The approach involved formulation of the overall SDC, ψ_{ov}, to yield the total energy dissipated divided by the total energy stored as

$$\psi_{ov} = \frac{\Sigma \Delta Z}{\Sigma Z} = \frac{\psi_1 Z_1 + \psi_2 Z_2 + \psi_{21} Z_{12}}{Z_1 + Z_2 + Z_{12}} \tag{35.10}$$

where $\Delta Z_1 = \psi_1 \cdot Z_1$ is the energy dissipation in the 1-direction, the axial being parallel to the fiber direction in a given layer.

Predicted values obtained by this approach are compared with measured values for a balanced, angle-ply laminated beam of high-modulus carbon-fiber/epoxy in flexural vibration in Fig. 35.18. In this figure, the SDC approaches 10 percent maximum at a fiber orientation of ±45°, where the dynamic flexural modulus, however, is

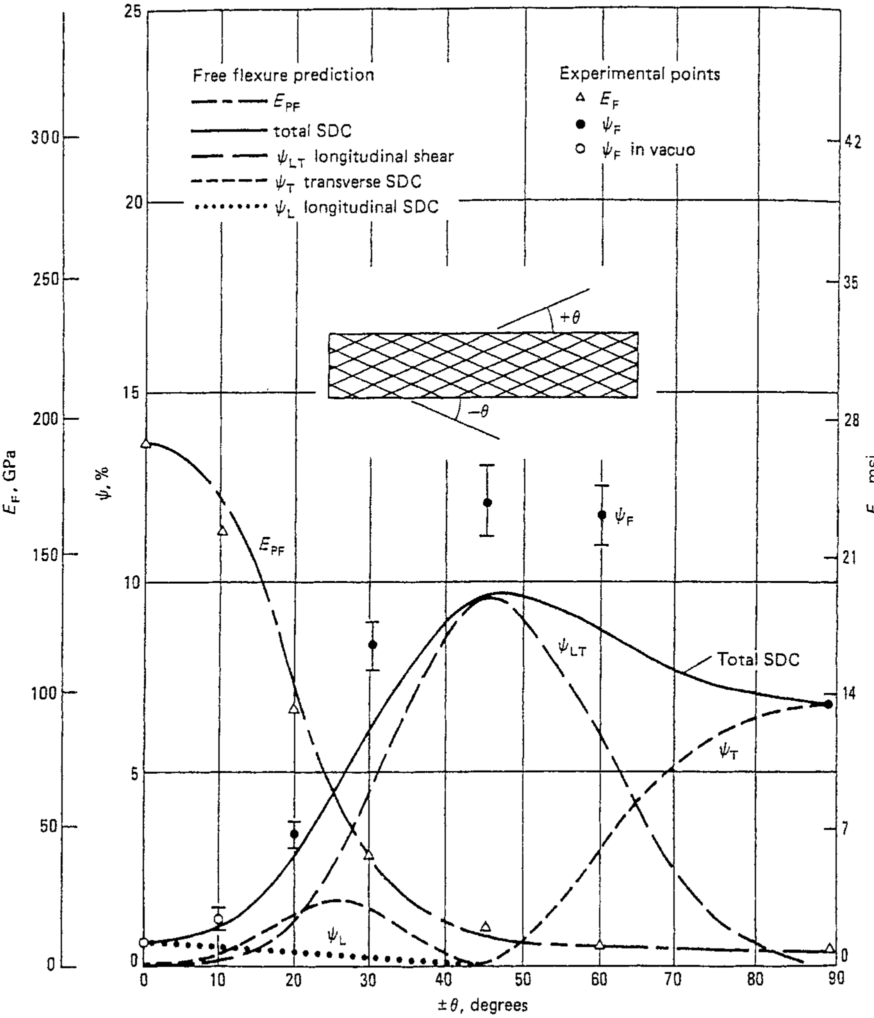


FIGURE 35.18 Variation of flexural modulus and specific damping capacity with fiber orientation for a carbon/epoxy, angle-ply laminate $[\pm\theta]_s$ in flexure.

very small. Damping predictions are again shown to be below measured values, but the discrepancy is much smaller in this case and the general trend with respect to fiber orientation is predicted extremely well.

The above theoretical treatment has subsequently been extended to laminated composite plates, again with reasonable correlation. SDC values ranged from just below 1 percent up to around 7 percent, with lower damping exhibited by the carbon/epoxy-laminated plates configured to provide essentially isotropic elastic modulus in the plane of the plate. Reference 18 contains extensive comparisons, including mode shapes, for both carbon/epoxy- and glass/epoxy-composite laminates.

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