CHAPTER 37

APPLIED DAMPING TREATMENTS

David I. G. Jones

INTRODUCTION TO THE ROLE OF DAMPING MATERIALS

The damping of an element of a structural system is a measure of the rate of energy dissipation which takes place during cyclic deformation. In general, the greater the energy dissipation, the less the likelihood of high vibration amplitudes or of high noise radiation, other things being equal. *Damping treatments* are configurations of mechanical or material elements designed to dissipate sufficient vibrational energy to control vibrations or noise.

Proper design of damping treatments requires the selection of appropriate damping materials, location(s) of the treatment, and choice of configurations which assure the transfer of deformations from the structure to the damping elements. These aspects of damping treatments are discussed in this chapter, along with relevant background information including:

- Internal mechanisms of damping
- External mechanisms of damping
- Polymeric and elastomeric materials
- Analytical modeling of complex modulus behavior
- Benefits of applied damping treatments
- Free-layer damping treatments
- Constrained-layer damping treatments
- Integral damping treatments
- Tuned dampers and damping links
- Measures or criteria of damping
- Methods for measuring complex modulus properties
- Commercial test systems

MECHANISMS AND SOURCES OF DAMPING

INTERNAL MECHANISMS OF DAMPING

There are many mechanisms that dissipate vibrational energy in the form of heat within the volume of a material element as it is deformed. Each such mechanism is associated with internal atomic or molecular reconstructions of the microstructure or with thermal effects. Only one or two mechanisms may be dominant for specific materials (metals, alloys, intermetallic compounds, etc.) under specific conditions, i.e., frequency and temperature ranges, and it is necessary to determine the precise mechanisms involved and the specific behavior on a phenomenological, experimental basis for each material specimen. Most structural metals and alloys have relatively little damping under most conditions, as demonstrated by the ringing of sheets of such materials after being struck. Some alloy systems, however, have crystal structures specifically selected for their relatively high damping capability; this is often demonstrated by their relative deadness under impact excitation. The damping behavior of metallic alloys is generally nonlinear and increases as cyclic stress amplitudes increase. Such behavior is difficult to predict because of the need to integrate effects of damping increments which vary with the cyclic stress amplitude distribution throughout the volume of the structure as it vibrates in a particular mode of deformation at a particular frequency. The prediction processes are complicated even further by the possible presence of external sources of damping at joints and interfaces within the structure and at connections and supports. For this reason, it is usually not possible, and certainly not simple, to predict or control the initial levels of damping in complex built-up structures and machines. Most of the current techniques of increasing damping involve the application of polymeric or elastomeric materials which are capable (under certain conditions) of dissipating far larger amounts of energy per cycle than the natural damping of the structure or machine without added damping.

EXTERNAL MECHANISMS OF DAMPING

Structures and machines can be damped by mechanisms which are essentially external to the system or structure itself. Such mechanisms, which can be very useful for vibration control in engineering practice (discussed in other chapters), include:

1. Acoustic radiation damping, whereby the vibrational response couples with the surrounding fluid medium, leading to sound radiation from the structure
2. Fluid pumping, in which the vibration of structure surfaces forces the fluid medium within which the structure is immersed to pass cyclically through narrow paths or leaks between different zones of the system or between the system and the exterior, thereby dissipating energy
3. Coulomb friction damping, in which adjacent touching parts of the machine or structure slide cyclically relative to one another, on a macroscopic or a microscopic scale, dissipating energy
4. Impacts between imperfectly elastic parts of the system

POLYMERIC AND ELASTOMERIC MATERIALS

A mechanism commonly known as *viscoelastic damping* is strongly displayed in many polymeric, elastomeric, and amorphous glassy materials. The damping arises from the relaxation and recovery of the molecular chains after deformation. A strong dependence exists between frequency and temperature effects in polymer behavior because of the direct relationship between temperature and molecular vibrations. A wide variety of commercial polymeric damping material compositions exist, most of which fit one of the main categories listed in Table 37.1.

TABLE 37.1 Typical Damping Material Types

Acrylic rubber	Natural rubber	Polysulfone
Butadiene rubber	Nitrile rubber (NBR)	Polyvinyl chloride (PVC)
Butyl rubber	Nylon	Silicone
Chloroprene (e.g., Neoprene)	Polyisoprene	Styrene-butadiene (SBR)
Fluorocarbon	Polymethyl methacrylate (Plexiglas)	Urethane
Fluorosilicone	Polysulfide	Vinyl

Polymeric damping materials are available commercially in the following categories:

1. Mastic treatment materials
2. Cured polymers
3. Pressure sensitive adhesives
4. Damping tapes
5. Laminates

Some manufacturers of damping material are given as a footnote.* Data related to the damping performance is provided in many formats. The current internationally recognized format, used in many databases, is the temperature-frequency nomogram, which provides modulus and loss factor as a function of both frequency and temperature in a single graph, such as that illustrated in Fig. 37.1.[1,2] The user requiring complex modulus data at, say, a frequency of 100 Hz and a temperature of 50°F (10°C) simply follows a horizontal line from the 100-Hz mark on the right vertical axis until it intersects the sloping 50°F (10°C) isotherm, and then projects vertically to read off the values of the Young's modulus E and loss factor η.

* Manufacturers of damping materials and systems, from whom information on specific materials and damping tapes may be obtained, include:

Antiphon Inc. (U.S.A.)
Arco Chemical Company (U.S.A.; www.arco.com)
Avery International (U.S.A.; www.avery.com)
CDF Chimie (France)
Dow Corning (U.S.A.; www.dowcorning.com)
EAR Corporation (U.S.A.)
El duPont deNemours (U.S.A.; www.DuPont.com)
Farbwercke-Hoechst (Germany)
Flexcon (U.S.A.; www.flexcon.com)
Goodyear (U.S.A.; www.goodyear.com)
Goodfellow (U.K.; www.goodfellow.com)
Imperial Chemical Industries (U.K.)
Leyland & Birmingham Rubber Company (U.K.)
MSC Laminates (U.S.A.)
Morgan Adhesives (U.S.A.; www.mactac.com)
Mystic Tapes (U.S.A.)
Shell Chemicals (U.S.A.; www.shellchemicals.com)
SNPE (France; www.snpe.com)
Sorbothane Inc. (U.S.A.; www.sorbothane.com)
Soundcoat Inc. (U.S.A.; www.soundcoat.com)
United McGill Corporation (U.S.A.; www.unitedmcgillcorp.com)
Uniroyal (U.S.A.; www.uniroyalchem.com)
Vibrachoc (France; www.vibrachoc.com)

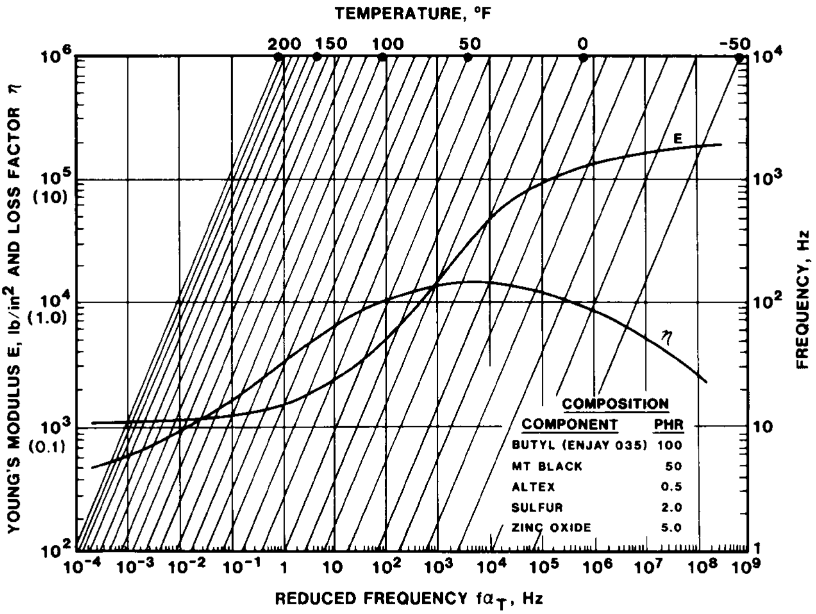

FIGURE 37.1 Temperature-frequency nomogram for butyl rubber composition.

ANALYTICAL MODELING OF COMPLEX MODULUS BEHAVIOR

It is very convenient to be able to mathematically describe the complex modulus properties of damping polymers, not only in the form of a nomogram as just described, but also by algebraic equations which can be folded into finite element and other computer codes for predicting dynamic response to external excitation (see Chap. 28). Such models include the standard model, comprising a distribution of springs and viscous dashpots in series and parallel configurations[3] for which the complex Young's modulus E^* (and equally the shear modulus G^*) can be described in the frequency domain by a series such as

$$E^* = \sum_{n=1}^{N} \frac{a_n + b_n(i f \alpha_T)}{1 + c_n(i f \alpha_T)} \tag{37.1}$$

or a fractional derivative model[4] for which the series becomes

$$E^* = \sum_{n=1}^{N} \frac{a_n + b_n(i f \alpha_T)^{\beta_n}}{1 + c_n(i f \alpha_T)^{\beta_n}} \tag{37.2}$$

where a_n, b_n, and c_n are numerical parameters, which may be real or complex, the β_n are fractions of the order of 0.5, and α_T is a shift factor which depends on temperature. Both models work, but Eq. (37.1) will usually require many terms, often 10 or more, to properly model actual material behavior, whereas Eq. (37.2) usually requires only one term for a good fit to the data. The shift factor α_T is determined as a function of temperature for each material from the test data, and is usually modeled by a Williams-Landel-Ferry (WLF) relationship[1,5] of the form

$$\log [\alpha_T] = \frac{-C_1(T - T_0)}{B_1 + T - T_0} \tag{37.3}$$

or by an Arrhenius relationship[1,5] of the form

$$\log [\alpha_T] = T_A \left(\frac{1}{T} - \frac{1}{T_0} \right) \tag{37.4}$$

where C_1 and B_1 are numerical parameters, the temperatures T and T_0 (the reference temperature) are in degrees absolute, and T_A is a numerical parameter related to the activation energy. The behavior of each specific polymer composition dictates which expression is most appropriate, and simple statistical methods may be applied for determining "best estimates" of each parameter in these equations.[6]

BENEFITS OF APPLIED DAMPING TREATMENTS

When the natural damping in a system is inadequate for its intended function, then an applied damping treatment may provide the following benefits:

Control of vibration amplitude at resonance. Damping can be used to control excessive resonance vibrations which may cause high stresses, leading to premature failure. It should be used in conjunction with other appropriate measures to achieve the most satisfactory approach. For random excitation it is not possible to detune a system and design to keep random stresses within acceptable limits without ensuring that the damping in each mode at least exceeds a minimum specified value. This is the case for sonic fatigue of aircraft fuselage, wing, and control surface panels when they are excited by jet noise or boundary layer turbulence-induced excitation. In these cases, structural designs have evolved toward semiempirical procedures, but damping levels are a controlling factor and must be increased if too low.

Noise control. Damping is very useful for the control of noise radiation from vibrating surfaces, or the control of noise transmission through a vibrating surface. The noise is not reduced by sound absorption, as in the case of an applied acoustical material, but by decreasing the amplitudes of the vibrating surface. For example, in a diesel engine, many parts of the surface contribute to the overall noise level, and the contribution of each part can be measured by the use of the acoustic intensity technique or by blanketing off, in turn, all parts except that of interest. If many parts of an engine contribute more or less equally to the noise, significant amplitude reductions of only one or two parts (whether by damping or other means) leads to only very small reductions of the overall noise, typically 1 or 2 dB.

Product acceptance. Damping can often contribute to product acceptance, not only by reducing the incidence of excessive noise, vibration, or resonance-induced failure but also by changing the "feel" of the product. The use of mastic damping treatments in car doors is a case in point. While the treatment may achieve some noise reduction, it may be the subjective evaluation by the customer of the solidity of the door which carries the greater weight.

Simplified maintenance. A useful by-product from reduction of resonance-induced fatigue by increased damping, or by other means, can be the reduction of maintenance costs.

TYPES OF DAMPING TREATMENTS

FREE-LAYER DAMPING TREATMENTS

The mechanism of energy dissipation in a free-, or unconstrained-, layer treatment is the cyclic extensional deformation of the imaginary fibers of the damping layer during each cycle of flexural vibration of the base structure, as illustrated in Fig. 37.2. The presence of the free layer changes the apparent flexural rigidity of the base structure in a manner which depends on the dimensions of the two layers involved and the elastic moduli of the two layers. The treatment depends for its effectiveness on the assumption, usually well-founded, that plane sections remain plane. The treatment fiber labeled *yy* is extended or compressed during each half of a cycle of flexural deformation of the base structure surface, in a manner which depends on the position of the fiber in the treatment and the radius of curvature of the element of length Δl, and can be calculated on the basis of purely geometric considerations. One fiber in particular does not change length during each cycle of deformation and is referred to as the *neutral axis.* For the uncoated plate or beam, the neutral axis is the center plane, but when the treatment is added, it moves in the direction of the treatment and its new position is calculated by the requirement that the net in-plane load across any section remain unchanged during deformation. The basic equations for predicting the modal loss factor η for the given damping layer loss factor η_2 and for predicting the direct flexural rigidity $(EI)_D$ as a function of the flexural rigidity E_1I_1 of the base beam are well known.[1,7]

The simplest expression relating the damping of a structure, in a particular mode, to the properties of the structure and the damping material layer is[8]

$$\frac{\eta}{\eta_2} = \frac{eh(3 + 6h + 4h^2 + 2eh^3 + e^2h^4)}{(1 + eh)(1 + 4eh + 6eh^2 + 4eh^3 + e^2h^4)} \tag{37.5}$$

where η is the damped structure modal loss factor, η_2 is the loss factor of the damping material, E_2 is the Young's modulus of the damping material and E_1 is that of the structure ($e = E_2/E_1$), and h_2 and h_1 are the thicknesses of damping layer and structure, respectively ($h = h_2/h_1$).

To calculate η, the user estimates η_2 and E_2 at the frequency and temperature of interest (from a nomogram), then calculates h and e, and then inserts these values into Eq. (37.5). Change thickness (h) or material (e) if the calculated value of η is not

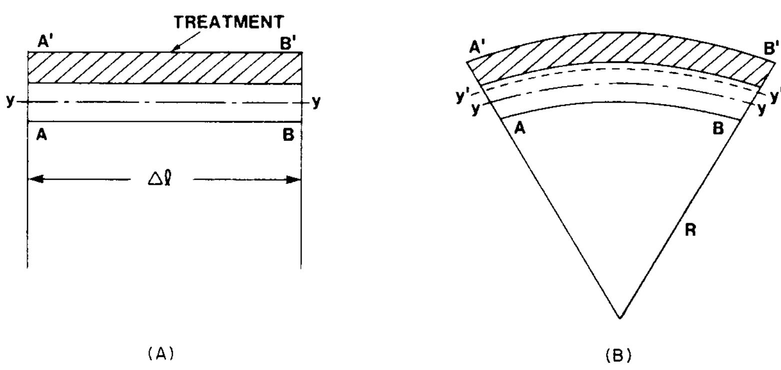

FIGURE 37.2 Free-layer treatment. (*A*) Undeformed. (*B*) Deformed.

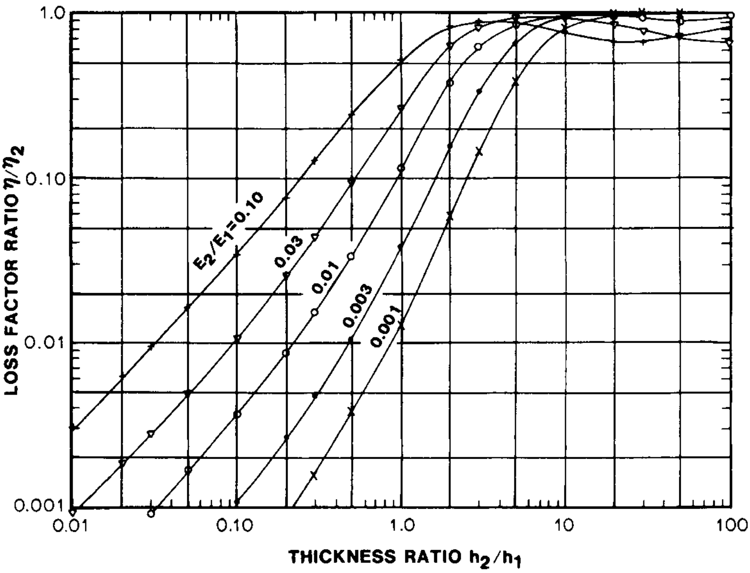

FIGURE 37.3 Graphs of η/η_2 vs. h_2/h_1, for a free-layer treatment.

adequate, and continue the process until satisfied. Figure 37.3 illustrates how η/η_2 varies with E_2/E_1 and with h_2/h_1, as calculated using the Oberst equations.

Limitations of Free-Layer Treatment Equations. The classical equations for free-layer treatment behavior are approximate. The main limitation is that the equations are applicable to beams or plates of uniform thickness and uniform stiff isotropic elastic characteristics with boundary conditions which do not dissipate or store energy during vibration. These boundary conditions include the classical pinned, free, and clamped conditions. Another limitation is that the deformation of the damping material layer is purely extensional with no in-plane shear, which would allow the "plane sections remain plane" criterion to be violated. This restriction is not very important unless the damping layer is very thick and very soft ($h_2/h_1 > 10$ and $E_2/E_1 < 0.001$). A third limitation is that the treatment must be uniformly applied to the full surface of the beam or plate, and especially that it be anchored well at the boundaries so that plane sections remain plane in the boundary areas where bending stresses can be very high and the effects of any cuts in the treatment can be very important. Other forms of the equations can be derived for partial coverage or for nonclassical boundary conditions.

Effect of Bonding Layer. Free-layer damping treatments are usually applied to the substrate surface through a thin adhesive or surface treatment coating. This adhesive layer should be very thin and stiff in comparison with the damping treatment layer in order to minimize shear strains in the adhesive layer which would alter the behavior of the damping treatment. The effect of a stiff thin adhesive layer is minimal, but a thick softer layer alters the treatment behavior significantly.

Amount of Material Required. Local panel weight increases up to 30 percent may often be needed to increase the damping of the structure in several modes of vibration to an acceptable level. Greater weight increases usually lead to diminishing returns. This weight increase can be offset to some degree if the damping is added early in the design, by judicious weight reductions achieved by proper sizing of the structure to take advantage of the damping.

CONSTRAINED-LAYER DAMPING TREATMENTS

The mechanism of energy dissipation in a constrained-layer damping treatment is quite different from the free-layer treatment, since the constraining layer helps induce relatively large shear deformations in the viscoelastic layer during each cycle of flexural deformation of the base structure, as illustrated in Fig. 37.4. The presence of the constraining viscoelastic layer-pair changes the apparent flexural rigidity of the base structure in a manner which depends on the dimensions of the three layers involved and the elastic moduli of the three layers, as for the free-layer treatment, but also in a manner which depends on the deformation pattern of the system, in contrast to the free-layer treatment. A useful set of equations which may be used to predict the flexural rigidity and modal damping of a beam or plate damped by a full-coverage constrained-layer treatment are given in Ref. 1. These equations give the direct (in-phase) component $(EI)_D$ of the flexural rigidity of the three-layer beam, and the quadrature (out-of-phase) component $(EI)_Q$ as a function of the various physical parameters of the system, including the thicknesses h_1, h_2, and h_3, the moduli $E_1\,(1 + j\eta_1)$, $E_2\,(1 + j\eta_2)$, $E_3\,(1 + j\eta_3)$, and the shear modulus of the damping layer $G_2\,(1 + j\eta_2)$.

Shear Parameter. The behavior of the damped system depends most strongly on the shear parameter

$$\mathbf{g} = \frac{G_2(\lambda/2)^2}{E_3 h_3 h_2 \pi^2} \tag{37.6}$$

which combines the effect of the damping layer modulus with the semiwavelength $(\lambda/2)$ of the mode of vibration, the modulus of the constraining layer, and the thicknesses of the damping and constraining layers. The other two parameters are the thickness ratios h_2/h_1 and h_3/h_1. Figure 37.5 illustrates the typical variation of η_n/η_2

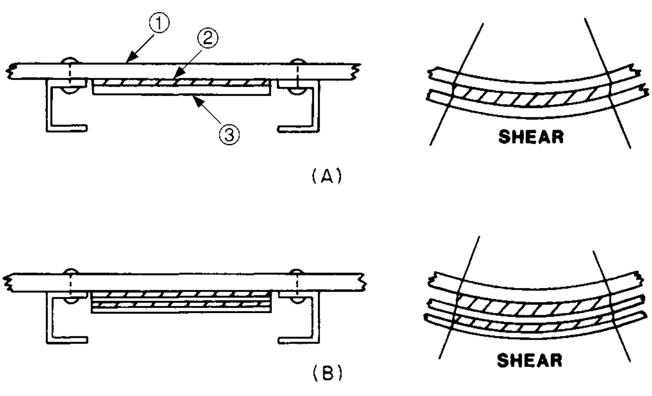

FIGURE 37.4 Additive layered damping treatments. (*A*) Constrained-layer treatment. (*B*) Multiple constrained-layer treatment.

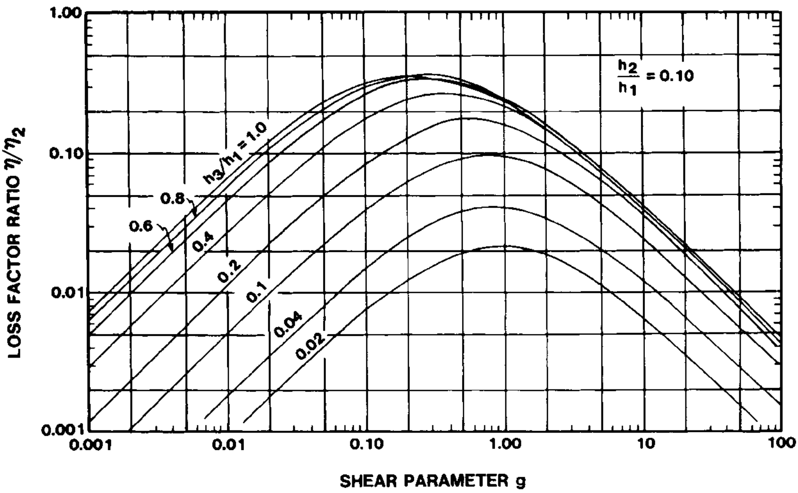

FIGURE 37.5 Typical plots of η/η_2 versus shear parameter $\mathbf{g}(h_2/h_1 = 0.10, \eta_2 = 0.1)$.

and $(EI)_D/E_1I_1$ with the shear parameter **g** for particular values of h_2/h_1 and h_3/h_1. These plots may be used for design of constrained layer treatments. Note that η_n will be small for both large and small values of **g**. For **g** approaching zero, G_2 or $\lambda/2$ may be very small or E_3, h_3, and h_2 may be very large. This could mean that while G_2 might appear at first sight to be sufficiently large, the dimensions h_2 and h_3 are nevertheless too large to achieve the needed value of **g**. This could happen for very large structures, especially for high-order modes. On the other hand, for **g** approaching infinity, G_2 or $\lambda/2$ may be large, or E_3, h_2, or h_3 may be very small.

Effects of Treatment Thickness. In general, increasing h_2 and h_3 will lead to increased damping of a beam or plate with a constrained-layer treatment, but the effect of the shear parameter will modify the specific values. The influence of h_3/h_1 is stronger than that of h_2/h_1, and as h_2/h_1 approaches zero, η_n/η_2 does not approach zero but a finite value. This behavior seems to occur in practice and accounts for the very thin damping layers, 0.002 in. (0.051 mm) or less, used in damping tapes. A practical limit of 0.001 in. (0.025 mm) is usually adopted to avoid handling problems.

Effect of Initial Damping. If the base beam is itself damped, with η_1 not equal to zero, then the damping from the constrained-layer treatment will be added to η_1 for small values of η_1. The general effect is readily visualized, but specific behavior depends on treatment dimensions and the value of the shear parameter.

Integral Damping Treatments. Some damping treatments are applied or added not after a structure has been partly or fully assembled but during the manufacturing process itself. Some examples are illustrated in Fig. 37.6. They include laminated sheets which are used for construction assembly, or for deep drawing of structural components in a manner similar to that for solid sheets, and also for faying surface damping which is introduced into the joints during assembly of built-up, bolted, riveted, or spot-welded structures. The conditions at the bolt, rivet, or weld areas critically influence the behavior of the damping configurations and make analysis

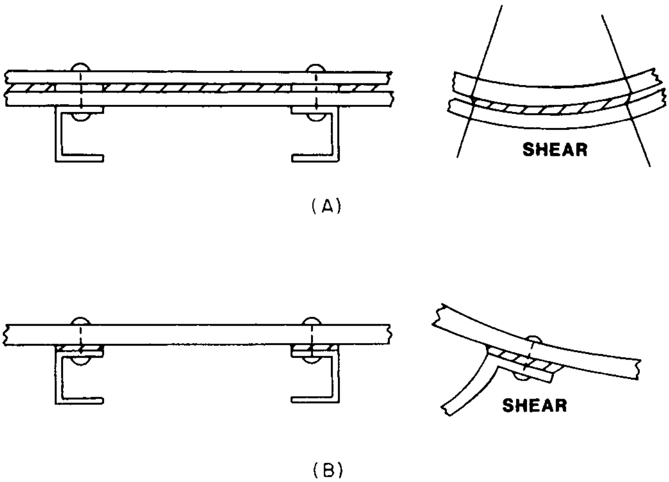

FIGURE 37.6 Some basic integral damping treatments. (*A*) Laminate. (*B*) Faying surface damping.

particularly difficult because of the limited control of conditions at these points. Finite element analysis may be one of the few techniques for such analysis.

Damping Tapes. Constrained layer treatments are sometimes available in the form of a premanufactured combination of an adhesive layer and a constraining layer, which may be applied to the surface of a vibrating panel in one step, as opposed to the several steps required when the adhesive and constraining layers are applied separately. Such damping tapes are available from several companies, including the 3M Company, Avery International, and Mystic Tapes, to name a few. An example of such a damping tape is the 3M™ 2552 damping foil product, which consists of a 0.005-in.-thick layer of a particular pressure-sensitive adhesive prebonded to a 0.010-in.-thick aluminum constraining layer, with an easy-release paper liner protecting the adhesive layer. One limitation of damping tapes is at once evident, namely, that the particular adhesive is effective over a specific temperature range and the adhesive and constraining layer thicknesses are fixed. The choice of adhesive is particularly important, since it must be selected in accordance with the required temperature range of operation, and the available thicknesses may not be ideal for all applications. Constrained layer treatments such as those illustrated in Fig. 37.4 could be built up conventionally, with adhesive and constraining layers applied separately, or by means of damping tapes. In each case, the adhesive material and thickness, and the constraining layer thickness, must be chosen to ensure optimal damping for the temperature range required by each application. The Ross-Kerwin-Ungar (RKU) equations[1] may be used to estimate, even if roughly, the best combination of dimensions and adhesive for each application, whether by means of damping tapes or conventional treatments, applying the complex modulus properties of the adhesive as described by a temperature-frequency nomogram or by a fractional derivative equation.

Tuned Dampers. The tuned damper is essentially a single degree-of-freedom mass-spring system having its resonance frequency close to the selected resonance frequency of the system to be damped, i.e., tuned. As the structure vibrates, the damper elastomeric element vibrates with much greater amplitude than the structure at the point of attachment and dissipates significant amounts of energy per cycle, thereby introducing large damping forces back to the structure which tend to reduce the amplitude. The system also adds another degree of freedom, so two peaks arise in place of the single original resonance. Proper tuning is required to ensure that the two new peaks are both lower in amplitude than the original single peak. The damper mass should be as large as practicable in order to maximize the damper effectiveness, up to perhaps 5 or 10 percent of the weight of the structure at most, and the damping capability of the resilient element should be as high as possible. The weight increase needed to add significant damping in a single mode is usually smaller than for a layered treatment, perhaps 5 percent or less.

Damping Links. The damping link is another type of discrete treatment, joining two appropriately chosen parts of a structure. Damping effectiveness depends on the existence of large relative motions between the ends of the link and on the existence of unequal stiffnesses or masses at each end. The deformation of the structure when it is bent leads to deformation of the viscoelastic elements. These deformations of the viscoelastic material lead to energy dissipation by the damper.

RATING OF DAMPING EFFECTIVENESS

MEASURES OR CRITERIA OF DAMPING

There are many measures of the damping of a system. Ideally, the various measures of damping should be consistent with each other, being small when the damping is low and large when the damping is high, and having a linear relationship with each other. This is not always the case, and care must be taken, when evaluating the effects of damping treatments, to ensure that the same measure is used for comparing behavior before and after the damping treatment is added. The measures discussed here include the loss factor η, the fraction of critical damping (damping ratio) ζ, the logarithmic decrement Δ, the resonance or quality factor Q, and the specific damping energy D. Table 37.2 summarizes the relationship between these parameters, in the ideal case of low damping in a single degree-of-freedom system. Some care must be taken in applying these measures for high damping and/or for multiple degree-of-freedom systems and especially to avoid using different measures to compare treated and untreated systems.

Loss Factor. The loss factor η is a measure of damping which describes the relationship between the sinusoidal excitation of a system and the corresponding sinusoidal response. If the system is linear, the response to a sinusoidal excitation is also sinusoidal and a loss factor is easily defined, but great care must be taken for nonlinear systems because the response is not sinusoidal and a unique loss factor cannot be defined. Consider first an inertialess specimen of linear viscoelastic material excited by a force $F(t) = F_0 \cos \omega t$, as illustrated in Fig. 37.7. The response $x(t) = x_0 \cos (\omega t - \delta)$ is also harmonic at the frequency ω as for the excitation but with a phase lag δ. The relationship between $F(t)$ and $x(t)$ can be expressed as

TABLE 37.2 Comparison of Damping Measures

Measure	Damping ratio	Loss factor	Log dec	Quality factor	Spec damping	Amp factor
Damping ratio	ζ	$\frac{\eta}{2}$	$\frac{\Delta}{\pi}$	$\frac{1}{2Q}$	$\frac{D}{4\pi U}$	$\frac{1^*}{2A}$
Loss factor	2ζ	η	$\frac{2\Delta}{\pi}$	$\frac{1}{Q}$	$\frac{D}{2\pi U}$	$\frac{1^*}{A}$
Log decrement	$\pi\zeta$	$2\pi\eta$	Δ	$\frac{\pi}{2Q}$	$\frac{D}{4U}$	$\frac{2\pi^*}{A}$
Quality factor	$\frac{1}{2\zeta}$	$\frac{1}{\eta}$	$\frac{\pi}{2\Delta}$	Q	$\frac{2\pi U}{D}$	A^*
Spec damping	$4\pi U\zeta$	$2\pi U\eta$	$4U\Delta$	$\frac{2\pi U}{Q}$	D	$\frac{2\pi U^*}{A}$
Amp factor	$\frac{1}{2\zeta}$	$\frac{1}{\eta}$	$\frac{\pi}{2\Delta}$	Q	$\frac{2\pi U}{D}$	A^*

* For single degree-of-freedom system only.

$$F = kx + \frac{k\eta}{|\omega|}\frac{\partial x}{\partial t} \tag{37.7}$$

where $k = F_0/x_0$ is a stiffness and $\eta = \tan\delta$ is referred to as the *loss factor.* The phase angle δ varies from 0° to 90° as the loss factor η varies from zero to infinity, so a one-to-one correspondence exists between η and δ. Equation (37.7) is a simple relationship between excitation and response which can be related to the stress-strain relationship because normal stress $\sigma = F/S$ and extensional strain $\varepsilon = x/l$. This is a generalized form of the classical Hooke's law which gives $F = kx$ for a perfectly elastic system. The loss factor, as a measure of damping, can be extended further to apply to a system possessing inertial as well as stiffness and damping characteristics. Consider, for example, the one degree-of-freedom linear viscoelastic system shown in Fig. 37.8*A*. The equation of motion is obtained by balancing the stiffness and damping forces from Eq. (37.7) to the inertia force $m(d^2x/dt^2)$:

$$m\frac{d^2x}{dt^2} + kx + \frac{k\eta}{\omega}\frac{dx}{dt} = F_0 \cos \omega t \tag{37.8}$$

The steady-state harmonic response, after any start-up transients have died away, is illustrated in Fig. 2.22. If k and η depend on frequency as is the case for real materials, then the maximum amplitude at the resonance frequency $\omega_r = \sqrt{k/m}$ is equal to $F_0/k(\omega_r)\eta(\omega_r)$, while the static response, at $\omega = 0$, is equal to $F_0/k(0)\sqrt{1+\eta^2(0)}$. The amplification factor A is approximately equal to $1/\eta(\omega_r)$, provided that $\eta^2(0) \ll 1$. Furthermore, the ratio $\Delta\omega/\omega_r$, where $\Delta\omega$ is the separation of the frequencies for which the response is $1/\sqrt{2}$ times the peak response, is known as the *half-power bandwidth* (see Fig. 2.22). It is also equal to η, provided that $\eta^2 \ll 1$. In summary, therefore,

$$\eta = \frac{1}{A} = \frac{\Delta\omega}{\omega_r} \tag{37.9}$$

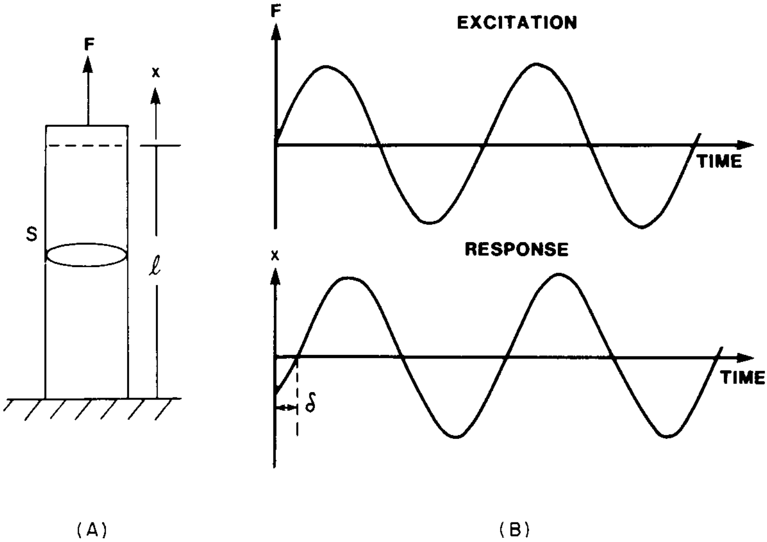

FIGURE 37.7 Linear viscoelastic behavior of a sample under sinusoidal loading, described in terms of response and excitation as functions of time. (*A*) Specimen. (*B*) Response and excitation.

This relationship between η and $1/A$ is applicable only for a single degree-of-freedom system and may not be directly applicable for more complex systems such as beams, plates, or more complex structures. The measure $\Delta\omega/\omega_r$ is applicable for more complex systems, as well as single degree-of-freedom systems. For large values of η, on the order of 0.2 or greater, none of these measures of damping agree exactly, even for an ideal linear single degree-of-freedom system, but each measure is at least self-consistent. The stiffness and loss-factor parameters defined here do not specify any particular model of material behavior. For example, k and η could be

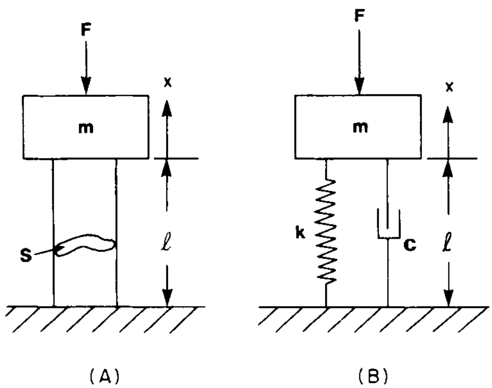

FIGURE 37.8 Single degree-of-freedom system with: (*A*) viscoelastic damping; (*B*) viscous damping.

constants as for hysteretic damping, or they could be functions of frequency, temperature, specimen composition and shape, or amplitude as for a nonlinear material. The model with constant k and η is not too useful over a wide frequency range, and such behavior is impossible over an infinite frequency range, but these parameters can vary quite slowly with frequency for some particular material compositions. If k and η vary strongly with frequency, or amplitude, then the various definitions of the loss factor must be used with great care, since each measure gives different results.

Fraction of Critical Damping. The fraction of critical damping (damping ratio) is a measure of one very specific mechanism of damping, i.e., viscous damping which is proportional to velocity. If the damping forces acting on a single degree-of-freedom mass-spring system, illustrated in Fig. 37.8*B*, satisfy this type of relationship, then the equation of motion for harmonic excitation is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F_0 \cos \omega t \tag{37.10}$$

The response depends on m, k, and a parameter $c/2\sqrt{km}$ which involves c, k, and m and is known as the *fraction of critical damping* (damping ratio). This parameter, labeled ζ, controls the peak amplitude, the half-power bandwidth, and the resonance frequency ω_r:

$$x_{\max} = \frac{F_0}{2k\zeta\sqrt{1-\zeta^2}} \qquad x(0) = \frac{F_0}{k}$$
$$\omega_r = \sqrt{(k/m)(1-\zeta^2)} \qquad \frac{\Delta\omega}{\omega_r} = 2\zeta \tag{37.11}$$

The plot of $x(\omega)$ versus frequency ω, for specific values of m and k is very similar to those for the viscoelastic damping, provided that $\eta \doteq 2\zeta$. The distinction between viscous and hysteretic damping (constant k, and η) is not at once apparent. Equations (37.8) and (37.10) convey the difference, since the damping coefficient in Eq. (37.8) decreases in proportion to $1/\omega$ as ω increases, while that in Eq. (37.10) is constant with frequency, at least for the hypothetical cases considered here. Figure 37.9 shows plots of response versus frequency based on the solutions of these equations of motion for each type of damping. Some differences arise at low frequency, but they are not very great except for very high values of damping. For high values of damping, neither η nor ζ are linearly related to the bandwidth ratio $\Delta\omega/\omega_r$. Figure 37.10 shows the variation of $\Delta\omega/\omega_r$ with η and 2ζ for values of η which are not small. Limits exist beyond which the ratio $\Delta\omega/\omega_r$ does not give a good estimate of η or ζ.

Logarithmic Decrement. When a damped system is struck by an impulsive load or is released from a displaced position relative to its equilibrium state, a decaying oscillation usually takes place as illustrated in Fig. 2.8. A measure of damping called *logarithmic decrement* Δ is defined as the natural logarithm of the ratio of amplitudes of successive peaks [see Eq. (2.19)]:

$$\Delta = \ln \frac{x_1}{x_2} = \ln \frac{x_n}{x_{n+1}} \tag{37.12}$$

This definition is useful only if these ratios are equal for the various cycles, i.e., for specific types and amounts of damping. The measure is useful for viscous and hys-

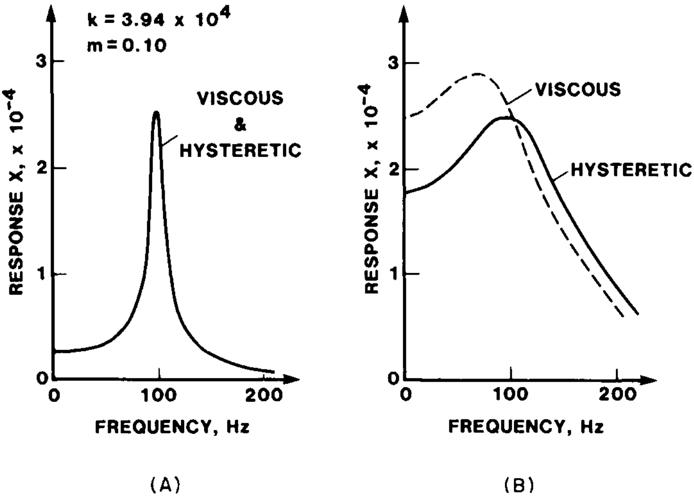

FIGURE 37.9 Comparison of viscous and hysteretic damping of a single degree-of-freedom system with (*A*) low damping ($\eta = 0.1$, $\zeta = 0.05$); (*B*) high damping ($\eta = 1.0$, $\zeta = 0.5$).

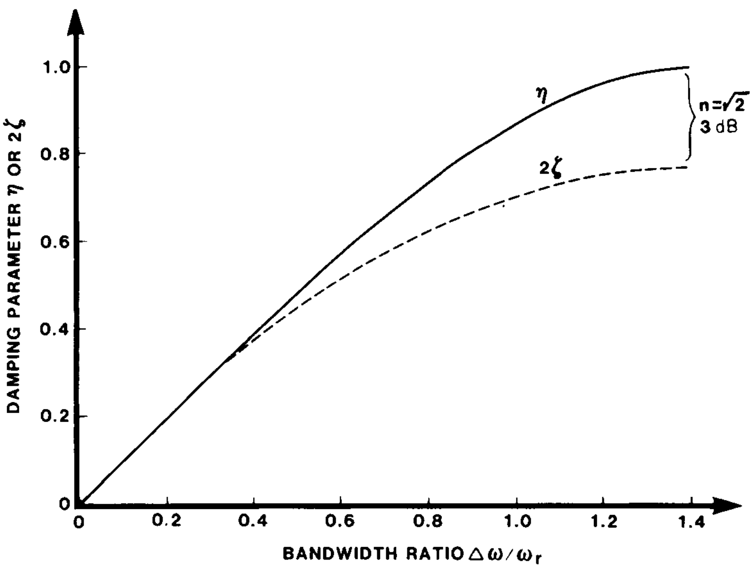

FIGURE 37.10 Variation of loss factor (η) and 2 times the fraction of critical damping (2ζ) of a single degree-of-freedom system with $\Delta\omega/\omega_r$.

teretic damping, within limits. For viscous damping, the solution of Eq. (37.10) for an impulsive excitation $F\,\delta(t)$ is obtained.

$$x = \frac{F}{\sqrt{km(1-\zeta^2)}}\; e^{-t\sqrt{k/m}}\; \sin t\sqrt{(k/m)(1-\zeta^2)} \tag{37.13}$$

so that

$$\Delta = \frac{\pi\zeta}{\sqrt{1-\zeta^2}} \tag{37.14}$$

for small ζ. If ζ approaches 1.0, the response becomes aperiodic and a logarithmic decrement cannot be defined or related to ζ. The loss factor in Eq. (37.7) also can be related to the transient response of a single degree-of-freedom mass-spring system, subject to an impulsive excitation. Consider the impulsive excitation $F(t)$ to be modeled as a spike of the form of a delta function at time $t = 0$. Then the equation of motion, in the form of Eq. (37.8), cannot be written directly, but if $F(t)$ and $x(t)$ are both described in terms of their corresponding Fourier transforms, then $\bar{F}(\omega) = \int_{-\infty}^{\infty} F(t)\exp(-j\omega t)\,dt = F$ and $\bar{x} = F/(k - m\omega^2 + jk\eta)$. The inverse Fourier transform gives

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{Fe^{j\omega t}\,d\omega}{k - m\omega^2 + jk\eta} \tag{37.15}$$

This equation contains real and imaginary parts, but using the fact that $\exp(j\omega t) = \cos \omega t + j \sin \omega t$ and if $k(\omega) = k(-\omega)$ and $\eta(-\omega) = -\eta(\omega)$, then it may be shown that $x(t)$ is given by

$$x(t) = \frac{F}{\pi}\int_{0}^{\infty} \frac{(k - m\omega^2)\cos \omega t + k\eta \sin \omega t}{(k - m\omega^2)^2 + (k\eta)^2}\, d\omega \tag{37.16}$$

For k and η constant over all frequencies from zero to infinity, problems arise regarding $x(t)$ being finite for values of t less than zero, i.e., before the impulse is applied, and this is physically impossible. The problem is that k and η cannot be constants for real systems over any extremely wide frequency range, no matter how close to constant they may be over a limited frequency range. For small values of η, however, a useful and accurate solution is given by

$$x(t) = \frac{F}{\sqrt{km}}\; e^{-1/2\eta t\sqrt{k/m}} \sin t\sqrt{k/m} \tag{37.17}$$

$$\Delta = \pi\eta/2 \tag{37.18}$$

Comparing Eqs. (37.14) and (37.18) gives

$$\eta = 2\zeta \tag{37.19}$$

Quality Factor. The quality factor Q is defined as $\omega_r/\Delta\omega$, so

$$Q = \frac{1}{\eta} \tag{37.20}$$

For a single degree-of-freedom $Q = A$ [where A is defined in Eq. (37.9)], but this is not the case for multiple degree-of-freedom systems.

Specific Damping Energy. Another useful measure of material damping is the amount of energy dissipated per unit volume per cycle, known as the *specific damping energy.* For a damping material specimen subject to an applied external force $F(t) = F_0 \cos \omega t$ the specific damping energy D is equal to

$$D = \oint F\, dx = \int_0^{2\pi/\omega} F \frac{dx}{dt}\, dt \qquad (37.21)$$

For a viscoelastic material obeying Eq. (37.7)

$$D = F_0 x_0 k\eta \sqrt{1 + \eta^2} \qquad (37.22)$$

But $F_0 = kx_0 \sqrt{1 + \eta^2}$, also from Eq. (37.7), so

$$D = \pi x_0^2\, k\eta \qquad (37.23)$$

The specific damping energy D increases as the square of the amplitude of vibration x_0 for linear viscoelastic materials, so it is clearly desirable to ensure that the damping material is strained as vigorously as possible in order to maximize D and hence the damping of the system. This has an important bearing on the choice of location within a vibrating system for application of a damping treatment. Furthermore, both k and η must be as large as possible to ensure maximum energy dissipation in the system, but this can be done only to the extent that further increases of k and η do not reduce x_0. While D is related to k and η for linear viscoelastic materials, this is not possible for nonlinear materials or for high cyclic strain levels where nonlinear behavior occurs; the value of D is then, of itself, often used as a measure of overall damping performance.

COMPARISON OF DAMPING MEASURES

The damping measures described in this section are related to each other as follows (Table 37.2):

$$\eta = 2\zeta = \frac{2\Delta}{\pi} = \frac{1}{Q} = \frac{D}{2\pi U} = \frac{1}{A} = \frac{\Delta\omega}{\omega_r} \qquad (37.24)$$

These various equations relate η, Δ, and ζ for viscous and viscoelastic damping of single degree-of-freedom systems. The relationships usually agree well for low values of η and ζ ($\eta < 0.2$ or so), but for higher values the comparisons are not so precise.

It is important, when analyzing tests to determine the effects of damping treatments on dynamic response, to be consistent in the use of these damping measures and to recognize that they are not completely equivalent, especially over wide frequency ranges or for multimodal response.

Effects of Mass and Stiffness. Changing the mass or stiffness of a single degree-of-freedom mass-spring system without changing any other parameters leads to a change of resonance frequency, and when the frequency changes over a wide range, the differences of viscous and hysteretic damping become more apparent. For viscous damping, the fraction of critical damping $\zeta = c/2\sqrt{km}$ changes as k or m change, whereas for hysteretic damping η does not change, at least within a limited frequency range. Although viscous and hysteretic damping measures are related by the simple relationship $\eta = 2\zeta$ for a single mode at a particular frequency, they do not remain equivalent as the frequency changes, and significant differences in response may be observed.

METHODS FOR MEASURING COMPLEX MODULUS PROPERTIES

Vibrating Beam Test Methods. The vibrating beam test methods are frequently used to measure the extensional or shear complex modulus properties of damping materials.[1,6] The dynamic response behavior of the beam, first in the undamped uncoated form and then with an added damping layer or added constrained configuration, is measured for several modes of vibration and over a range of temperatures. At each temperature, the measured damped resonance frequency f_n, the undamped resonance frequency f_{on}, and the loss factor η_n in the nth mode of vibration are measured and used in an appropriate set of equations to deduce the Young's modulus E, or the shear modulus G, and the loss factor η of the damping material at a number of discrete frequencies and temperatures.

Various configurations of cantilever beams are used to measure viscoelastic material damping properties in tension-compression or shear at low cyclic strain amplitudes. Figure 37.11 illustrates some of the configurations used. The damping layers are bonded to the base beams by means of a stiff adhesive such as an epoxy. This bonding is very important and must be done well using an adhesive which is stiff in comparison to the damping layer and is very thin. The thickness ratio h_2/h_1 generally lies in the range $0.1 \le h_2/h_1 \le 2.0$, and the length l is about 5 to 10 in. (12.7 to 25.4 cm). The base beam material is typically aluminum, steel, or a stiff epoxy or epoxy matrix composite material having low intrinsic damping. Great care must be taken to ensure that the temperature range of the tests is not excessive in relation to the behavior of the base beam, and in particular to allow for the effect of temperature on the base beam properties such as Young's modulus, the resonance frequencies, and the modal loss factor in the absence of the damping layer. The vibration test is conducted allowing the specimen to soak at each selected temperature for several minutes, often 30 minutes, to be sure of thermal equilibrium; then the beam is excited by means of a noncontacting transducer or by impact, and the resulting response in the frequency domain is measured, either through swept sine-wave excitation or FFT analysis of the transient response signal in the time domain. At each temperature, several resonance frequencies and modal loss factors are measured over a wide range of frequencies. The test is then repeated after thermal equilibrium has been reached at the next selected temperature. The data obtained for the first mode is usually not used because of the low frequency involved and the high amplitudes and high modal damping of the base beam, as well as because of errors in the analysis when sandwich beams are used. Such vibrating beam tests are widely used for measuring viscoelastic material damping properties for shear and extensional deformation.[9,10]

Geiger Thick-Plate Test Method. The Geiger thick-plate method is of importance because it is widely used to describe damping materials in the automotive industry. It makes use of a large flat plate, suspended freely from four points selected to be at or near the nodal lines of the first free-free mode, to which is bonded the damping layer being evaluated. The rate of decay of vibration amplitude (expressed in decibels per second) is measured and serves as a measure of the effectiveness of the damping layer. Figure 37.12 illustrates a typical test setup. The system can be

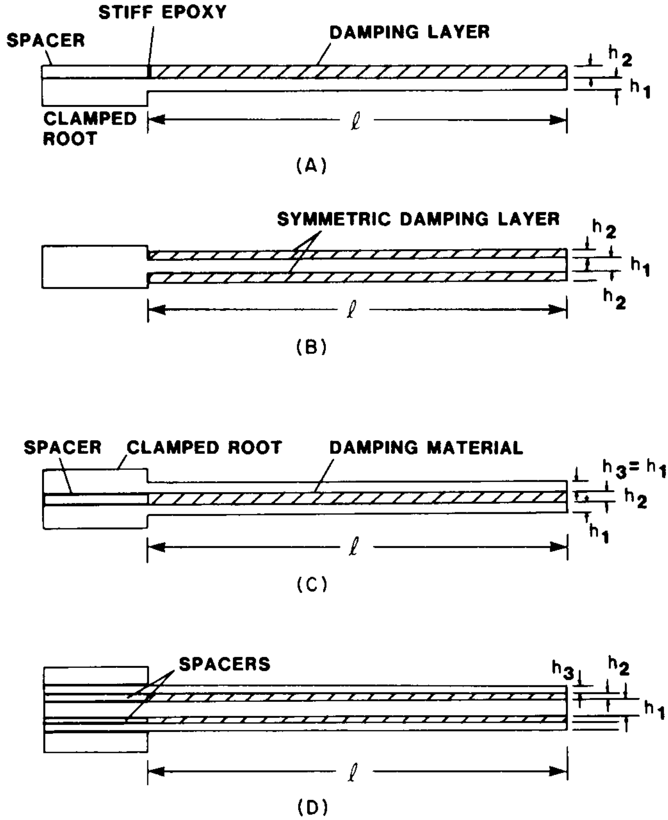

FIGURE 37.11 Cantilever beam damping material test configurations. (*A*) Nonsymmetric Oberst beam. (*B*) Symmetric modified Oberst beam. (*C*) Symmetric sandwich beam. (*D*) Symmetric constrained-layer beam.

excited by an impulsive force, measured through a force gage, and applied by a hammer, by an electromagnetic exciter, an electrically actuated impeller, or by sine-wave or random excitation. The response can be picked up by an electromagnetic transducer, in which case cross talk with the excitation transducer must be avoided by adequate separation or by the use of capacitative or electro-optical transducers or by a miniature accelerometer. The measured output can be displayed in many ways, including a decaying sinusoidal trace representing response to an impulsive excitation (a measure of the logarithmic decrement), or a frequency domain display in the region of the fundamental free-free mode (loss factor measure). The observed logarithmic decrement or loss factor value is a measure of the damping of the plate/damping material system and depends on the plate and treatment thicknesses. The free-layer treatment equations used for the vibrating beam tests may also be used with the Geiger plate test provided that the same conditions are satisfied. In particular, the treatment thickness must be sufficient to make the ratio of the stiffness of the coated plate to that of the uncoated plate greater than about 1.05. The size of the specimen and the use of only one mode makes this condition somewhat less restrictive than for the beam tests, for which the specimens are much smaller.

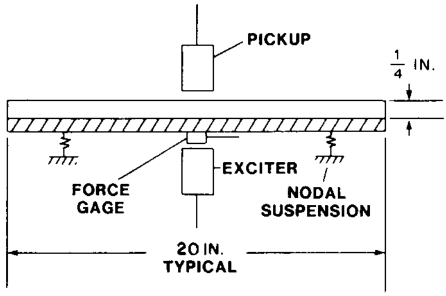

FIGURE 37.12 Geiger plate test configuration.

Single Degree-of-Freedom Resonance Tests. Digital test instrumentation and data analysis techniques make it relatively easy to conduct vibration tests directly on relatively small samples of damping materials and to readily determine the damping properties. Typical test configurations are illustrated in Fig. 37.13. For a resonance type of test, the specimen is driven inertially by a large vibration table (see Chap. 25), usually by swept sine-wave excitation. The input and output accelerations are usually measured by accelerometers, and the response parameter of interest is the amplification $A = x/x_0$ as a function of frequency, where x is the amplitude of displacement of the mass and x_0 is that of the shaker table. At resonance, the maximum value of $A = x/x_0$ is observed along with the resonance frequency ω_R for each temperature. The loss factor and modulus in both the tension-compression and shear loading of the specimen material are determined from

$$\eta = \frac{1}{\sqrt{A^2 - 1}} \tag{37.25}$$

$$E = \frac{m_e \omega_R^2 l}{S_1} \tag{37.26}$$

$$G = \frac{m_e \omega_R^2 h}{S_2} \tag{37.27}$$

For tension-compression loading, l is the length and $S_1 = wh$ is the cross-sectional area of the load-carrying member, where w is the cross-section width and h is the cross-section thickness, as illustrated in Fig. 37.13. For shear loading, h is the thickness of the shear layer and $S_2 = 2wl$ is the cross-sectional area of the shear member, where l is now the breadth of the load-carrying area, again as illustrated in Fig. 37.13. The effective mass m_e includes the added mass m and the effective mass of the specimen damping material, which is about one-third of its actual mass. For the extensional specimen, the ratio l/h or l/w, whichever is smaller, must be greater than 1.0 or shape effects will have to be taken into account. For the shear configuration, the ratio h/l must be less than 0.2 for the same reason. For highly damped materials, for which x/x_0 does not exceed 1.0 by a significant amount, considerable error in measuring A and η will be encountered, but the method is very effective for values of η less than about 0.5. In this method, data are obtained at only one frequency; the mass m must be changed to obtain data at other frequencies. Care must be taken to avoid sagging or creep of the specimen at high temperatures and to ensure that thermal equilibrium has been achieved. A thermocouple placed within the volume of the

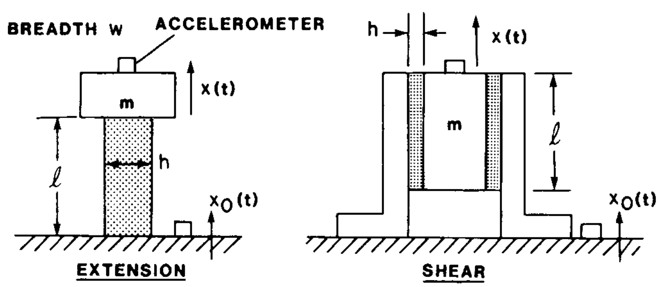

FIGURE 37.13 Resonance test concepts.

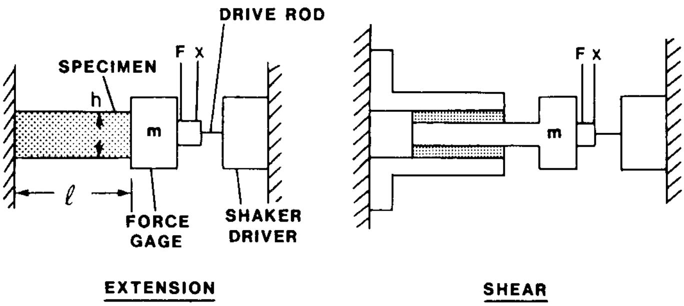

FIGURE 37.14 Impedance test concepts.

specimen material may be necessary, particularly for tests at high strain amplitudes where internal heating of the specimen by energy dissipation from damping may lead to wide differences between true specimen temperature and the temperature of the surroundings.

Impedance Tests. If the specimens are excited by a driver through a force gage, then the response measure used to characterize the system behavior is the compliance or receptance x/F, where F is the driving force measured by the force gage and x is the response at the same point, measured by an accelerometer, for example. If the mass m is large compared with the mass of the specimen, as illustrated in Fig. 37.14, then one may add one-third the mass of the specimen to m to give the effective mass m_e of the equivalent single degree-of-freedom system, so that

$$\frac{x}{F} = \frac{1}{k(1 + j\eta) - m_e\omega^2} \tag{37.28}$$

If this is expressed instead in terms of the ratio F/x, the dynamic stiffness at the driving point, which is directly related to the driving-point impedance, then

$$\kappa = k - m_e\omega^2 + jk\eta \tag{37.29}$$

which shows that the direct dynamic stiffness is a linear function of ω^2 and the quadrature dynamic stiffness $\kappa_Q = k\eta$. It is not difficult to obtain good measurements of k

and η by this type of test approach from about $0.2\omega_r$ to $3\omega_r$, so data can be obtained quite easily over about a decade of frequency instead of at only a single frequency as for the resonance method. Analytical mass corrections may also have to be made to account for inertial effects at the force gage.

COMMERCIAL TEST SYSTEMS

Many commercial systems are available for measuring the complex modulus properties of viscoelastic damping materials.[11–13] All are based on some kind of deformation mode of a sample of the material, measurement of the corresponding excitation forces and displacements, and analysis of the data to obtain the material properties. Each system has advantages and disadvantages, but when due care is exercised, good results usually can be obtained with each system. Particular care should be taken to read, understand, and follow the manufacturer's instructions. For example, in some tests such as monitoring cure cycles of epoxies, the temperature sweep rate can be quite high in order to keep up with the reaction. This is acceptable if one is monitoring the progress of the cure cycle, but it may not be acceptable if one seeks to measure the damping properties at a state approximating thermal equilibrium. For thermal equilibrium to be maintained, temperature sweep rates well below 1°F (0.5°C) per minute are usually recommended, and even lower rates may be required for large specimens. A dwell period at each temperature is recommended before performing the test.

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